Predicting Bank Movement of the Brahmaputra River in Bangladesh

by

MD. ABU OBAIDA ANSARI KHAN

A thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

Department of Civil & Geological Engineering
University of Manitoba
Winnipeg, Manitoba

ISBN 0-612-13244-7

Name

Dissertation Abstracts International is arranged by broad, general subject categories. Please select the one subject which most nearly describes the content of your dissertation. Enter the corresponding four-digit code in the spaces provided.

Civil Engineering — 0543 U·M·I

SUBJECT TERM — SUBJECT CODE

Subject Categories

THE HUMANITIES AND SOCIAL SCIENCES

COMMUNICATIONS AND THE ARTS
Architecture 0729
Art History 0377
Cinema 0900
Dance 0378
Fine Arts 0357
Information Science 0723
Journalism 0391
Library Science 0399
Mass Communications 0708
Music 0413
Speech Communication 0459
Theater 0465

EDUCATION
General 0515
Administration 0514
Adult and Continuing 0516
Agricultural 0517
Art 0273
Bilingual and Multicultural 0282
Business 0688
Community College 0275
Curriculum and Instruction 0727
Early Childhood 0518
Elementary 0524
Finance 0277
Guidance and Counseling 0519
Health 0680
Higher 0745
History of 0520
Home Economics 0278
Industrial 0521
Language and Literature 0279
Mathematics 0280
Music 0522
Philosophy of 0998
Physical 0523
Psychology 0525
Reading 0535
Religious 0527
Sciences 0714
Secondary 0533
Social Sciences 0534
Sociology of 0340
Special 0529
Teacher Training 0530
Technology 0710
Tests and Measurements 0288
Vocational 0747

LANGUAGE, LITERATURE AND LINGUISTICS
Language
- General 0679
- Ancient 0289
- Linguistics 0290
- Modern 0291

Literature
- General 0401
- Classical 0294
- Comparative 0295
- Medieval 0297
- Modern 0298
- African 0316
- American 0591
- Asian 0305
- Canadian (English) 0352
- Canadian (French) 0355
- English 0593
- Germanic 0311
- Latin American 0312
- Middle Eastern 0315
- Romance 0313
- Slavic and East European 0314

PHILOSOPHY, RELIGION AND THEOLOGY
Philosophy 0422
Religion
- General 0318
- Biblical Studies 0321
- Clergy 0319
- History of 0320
- Philosophy of 0322

Theology 0469

SOCIAL SCIENCES
American Studies 0323
Anthropology
- Archaeology 0324
- Cultural 0326
- Physical 0327

Business Administration
- General 0310
- Accounting 0272
- Banking 0770
- Management 0454
- Marketing 0338

Canadian Studies 0385
Economics
- General 0501
- Agricultural 0503
- Commerce-Business 0505
- Finance 0508
- History 0509
- Labor 0510
- Theory 0511

Folklore 0358
Geography 0366
Gerontology 0351
History
- General 0578
- Ancient 0579
- Medieval 0581
- Modern 0582
- Black 0328
- African 0331
- Asia, Australia and Oceania 0332
- Canadian 0334
- European 0335
- Latin American 0336
- Middle Eastern 0333
- United States 0337

History of Science 0585
Law 0398
Political Science
- General 0615
- International Law and Relations 0616
- Public Administration 0617

Recreation 0814
Social Work 0452
Sociology
- General 0626
- Criminology and Penology 0627
- Demography 0938
- Ethnic and Racial Studies 0631
- Individual and Family Studies 0628
- Industrial and Labor Relations 0629
- Public and Social Welfare 0630
- Social Structure and Development 0700
- Theory and Methods 0344

Transportation 0709
Urban and Regional Planning 0999
Women's Studies 0453

THE SCIENCES AND ENGINEERING

BIOLOGICAL SCIENCES
Agriculture
- General 0473
- Agronomy 0285
- Animal Culture and Nutrition 0475
- Animal Pathology 0476
- Food Science and Technology 0359
- Forestry and Wildlife 0478
- Plant Culture 0479
- Plant Pathology 0480
- Plant Physiology 0817
- Range Management 0777
- Wood Technology 0746

Biology
- General 0306
- Anatomy 0287
- Biostatistics 0308
- Botany 0309
- Cell 0379
- Ecology 0329
- Entomology 0353
- Genetics 0369
- Limnology 0793
- Microbiology 0410
- Molecular 0307
- Neuroscience 0317
- Oceanography 0416
- Physiology 0433
- Radiation 0821
- Veterinary Science 0778
- Zoology 0472

Biophysics
- General 0786
- Medical 0760

EARTH SCIENCES
Biogeochemistry 0425
Geochemistry 0996
Geodesy 0370
Geology 0372
Geophysics 0373
Hydrology 0388
Mineralogy 0411
Paleobotany 0345
Paleoecology 0426
Paleontology 0418
Paleozoology 0985
Palynology 0427
Physical Geography 0368
Physical Oceanography 0415

HEALTH AND ENVIRONMENTAL SCIENCES
Environmental Sciences 0768
Health Sciences
- General 0566
- Audiology 0300
- Chemotherapy 0992
- Dentistry 0567
- Education 0350
- Hospital Management 0769
- Human Development 0758
- Immunology 0982
- Medicine and Surgery 0564
- Mental Health 0347
- Nursing 0569
- Nutrition 0570
- Obstetrics and Gynecology 0380
- Occupational Health and Therapy 0354
- Ophthalmology 0381
- Pathology 0571
- Pharmacology 0419
- Pharmacy 0572
- Physical Therapy 0382
- Public Health 0573
- Radiology 0574
- Recreation 0575
- Speech Pathology 0460
- Toxicology 0383

Home Economics 0386

PHYSICAL SCIENCES
Pure Sciences

Chemistry
- General 0485
- Agricultural 0749
- Analytical 0486
- Biochemistry 0487
- Inorganic 0488
- Nuclear 0738
- Organic 0490
- Pharmaceutical 0491
- Physical 0494
- Polymer 0495
- Radiation 0754

Mathematics 0405
Physics
- General 0605
- Acoustics 0986
- Astronomy and Astrophysics 0606
- Atmospheric Science 0608
- Atomic 0748
- Electronics and Electricity 0607
- Elementary Particles and High Energy 0798
- Fluid and Plasma 0759
- Molecular 0609
- Nuclear 0610
- Optics 0752
- Radiation 0756
- Solid State 0611

Statistics 0463

Applied Sciences

Applied Mechanics 0346
Computer Science 0984
Engineering
- General 0537
- Aerospace 0538
- Agricultural 0539
- Automotive 0540
- Biomedical 0541
- Chemical 0542
- Civil 0543
- Electronics and Electrical 0544
- Heat and Thermodynamics 0348
- Hydraulic 0545
- Industrial 0546
- Marine 0547
- Materials Science 0794
- Mechanical 0548
- Metallurgy 0743
- Mining 0551
- Nuclear 0552
- Packaging 0549
- Petroleum 0765
- Sanitary and Municipal 0554
- System Science 0790

Geotechnology 0428
Operations Research 0796
Plastics Technology 0795
Textile Technology 0994

PSYCHOLOGY
General 0621
Behavioral 0384
Clinical 0622
Developmental 0620
Experimental 0623
Industrial 0624
Personality 0625
Physiological 0989
Psychobiology 0349
Psychometrics 0632
Social 0451

PREDICTING BANK MOVEMENT OF THE BRAHMAPUTRA RIVER IN BANGLADESH

BY

MD. ABU OBAIDA ANSARI KHAN

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

Abstract

Bank erosion along the Brahmaputra river in Bangladesh causes a large number of people to lose their land each year and thereby their only means of livelihood. The resulting migration to the already overcrowded towns and cities then creates severe social problems there. In addition, frequent bank erosion endangers any dyking along the river so that little can be done to contain the extensive and damaging floods that often cover large areas of Bangladesh.

At the present time, bank protection and containment works appear infeasible both technically and economically. The people will therefore have to live with the problem, at least for the time being. A better understanding of potential bank movement and an ability to assess the risk of large local movements, however, should assist in coping more effectively with what cannot be avoided. This thesis describes a probabilistic study that estimates the risk of a given local bank movement based on information that is available at the beginning of the year.

The Brahmaputra is a braided river. It forms a complex water and sediment transport system that can assume many possible states and the known laws of fluid mechanics and river behaviour are quite inadequate to predict the morphological changes that continually take place. Compared to the single channel alluvial river, which attains a dynamic equilibrium between erosion and deposition along its course by which it maintains a typical channel alignment and cross-sectional geometry, the braided river has an extra degree of freedom in that the flow of water and sediment can be divided over a variable number of different channels. The large changes that occur in this distribution of water and sediment due to channel joining and splitting are very difficult to predict. In this study some success has been achieved in predicting them *qualitatively* one year in advance. Quantitative and longer term prediction, however, does not appear possible at this time.

The required conceptual model for *quantitative* prediction of river bank movement was therefore conceived as a trend of *individual* channels in a given cross-section towards equilibrium conditions (as

with single channel rivers) on which a large random noise (the changes in water and sediment flow over the branch channels) is superimposed.

It was argued that average width/depth relations in the numerous branch channels over the period of record are probably not far from equilibrium conditions for the given water and sediment distribution. This allowed an assessment of the degree to which the channels in each river cross-section are out of equilibrium at any given time. The expected changes towards equilibrium could then be estimated. The expected river bank movement then follows from the corresponding changes in channel width.

Changes in width and depth towards the state of equilibrium can occur in many different ways without violating the basic energy and continuity conditions. This raises the question of which changes are most likely, given the known constraints. This is a typical problem for which the principle of maximum entropy provides an appropriate solution.

The following results were obtained. In a *qualitative* sense, good success was achieved in predicting the joining of two single channels in the following year. Predicting the splitting of a wider channel into two channels was less successful. In a *quantitative* sense, the amount of outer bank line movement could be predicted with a fair degree of accuracy one year in advance. The R^2 value based on a comparison of observed and predicted movement was on average 0.93. No systematic errors in prediction were observed.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisor, Professor Caspar Booy for his constant guidance and encouragement throughout the preparation of this thesis. His critical reviews and valuable comments helped shape this thesis at various stages. This study would not have been possible without the financial support provided by him. Above all, his continued moral support, during my share of personal crisis, made it possible to keep my faith in humanity. For this, I am ever grateful to him.

I am indebted to Professor D. H. Burn and Professor L. P. Stene whose comments and helpful suggestions contributed significantly towards the preparation of this thesis. Special thanks to Professor S. Simonovic for his helpful comments.

My gratitude to Irene Hamel for her help and friendship. I am grateful to Professor Leonard Lye for his encouragement and friendship.

My sincere thanks to my family and friends Shaila Khan, Protiti Khan, Nishad Jabeen Linsa, Anindo Choudhury and Nishat Haque for their help and constant moral support. Their encouragement and trust have been an immense motivation.

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE | PAGE

LIST OF TABLES

TABLE **PAGE**

Chapter One

INTRODUCTION

1.1 General

Alluvial rivers have their own characteristic channel dimensions, alignment and planform that reflect a dynamic equilibrium between erosion and deposition of the sediment the rivers transport and through which they flow. This equilibrium does not imply an absence of bank movement. In fact, continuous bank movement is a characteristic of most alluvial rivers as meanders progress and the process of delta formation proceeds. Bank movement is at times aggravated, however, and can take on catastrophic proportions as has happened in recent times along the Brahmaputra river in Bangladesh. This raises the question whether the bank movement is still within the normal spectrum of river variability or caused by more drastic changes in flow and sediment transport. Such changes could be human induced or natural.

To account for the huge bank movements that occur on the Brahmaputra, attention has been drawn to the deforestation that has taken place in the upper catchment areas. This is said to have greatly increased the sediment load in the river. Others have pointed to the continuing geological changes that are occurring in this tectonically active region and to the effect of the often underestimated variability of the natural climate. It is true that the result of such changes in water and sediment input into the system, would be changes in channel geometry as a behavioral response aimed at restoring a disturbed state of dynamic equilibrium. The question, however, is *to what degree* the equilibrium has in fact been affected by these factors.

For a braided river the response to changes in water and sediment flow is poorly understood which makes it difficult to distinguish between so-called normal bankline movement which reflects normal hydrologic variability and abnormal movement which reflects sporadic or progressive change of a climatologic, geographic or geologic nature. The purpose of this study is to contribute to the understanding of the behavioral response of the Brahmaputra river in Bangladesh and its effect on bankline movement under *current conditions*. The study thus aims at assessing *expected bank movement* not only in a qualitative but also in a quantitative sense. While the study focuses on the Brahmaputra river, the results are considered to be applicable to the analysis of bank movement of braided rivers in general.

1.2 Characteristics of a Braided river

The characteristic bank movement along a braided river is different from that of a single channel river which tends to meander. A meandering river invariably experiences erosion of the concave bank and deposition of sediment along the convex bank. The effect of the alternating zones of erosion and deposition is that the meandering river maintains a relatively stable, albeit travelling, sinuous planform.

Braided rivers, on the other hand, are divided into subchannels or anabranches separated by bars or islands, each of which carries part of the total flow and sediment load. Each of these channels experiences the kind of local erosion and deposition that is characteristic of single channel rivers. More important, however, the contribution of each channel to the total flow of water and sediment is also in a state of continual change. Thus some channels become wider and deeper, others get shallower. Some split into two channels, some unite to form a single channel, yet others tend to disappear. The erosion or deposition in the subchannels or anabranches within a cross section also causes erosion or deposition along the outer banks. Unlike meandering rivers the banks in a single cross-section of a braided river may both experience erosion or deposition. Thus, a stable bank line configuration does not exist for braided rivers. By the same token the islands between the anabranches are not stable either. Old islands may erode away and new ones may appear.

The bank movement in braided rivers is not gradual as in meandering rivers but sporadic. The Kosi river in India is known to have shifted 19 kilometres laterally in a single year (Leopold, Wolman & Miller, 1964). Lateral migration of as much as 5 kilometres in a single year has been reported by Khan (1988) for the Brahmaputra River in Bangladesh. Similar rapid movement has also been reported for the Yellow River in China (Chien, 1961, quoted by Leopold et al., 1964).

The common aspect of bank movement in meandering and braided rivers is that both tend to attain or restore a state of dynamic equilibrium that corresponds to the existing water and sediment flow. The difference is that the braided river has extra degrees of freedom in the distribution of the flow of water and sediment over the anabranches, which makes the phenomenon far more complex. Little success has therefore been achieved to date in explaining the morphology of a braided river.

Two different approaches have been adopted in the past, an empirical and an analytical approach. In the empirical approach, field observations and experimental data have been used to develop numerical relationships between the channel characteristics and the variables describing the transport of water and sediment (e.g., Leopold & Wolman, 1957). This approach has been reasonably successful for single river channels, although the mathematical form and the magnitude of the parameters of the relationships are obviously situation specific. The success for braided rivers has not been great because the relationships developed from observations tend to describe existing stable channel conditions while in a braided river stability of the river as a whole may involve radical morphological change in individual branch channels. Moreover, the empirical approach does not attempt to explain why braiding or meandering develops in the first place, nor why and how the channel configuration changes.

In the analytical approach, it is postulated that the braiding and the meandering processes are but different results of the same instability phenomena associated with the simple case of a single straight alluvial stream channel. The inherent instability of a straight alluvial channel leads inevitably to a situation which is non-steady in the sense that important changes occur over a period of time. In the longer term, however, many of these changes may be reversed and a state of dynamic equilibrium will *then* be established. The braided river is considered to be but the most pronounced example of such non-steady behaviour leading to dynamic stability.

In the analytical approach, a basic or undisturbed flow is assumed over which small travelling perturbations are superimposed. The effect on channel plan form is determined by computing the rate of growth of these perturbations. The phase difference between the shear stress gradient (or tractive force gradient) and the bed form gradient is considered to be the factor causing the instability of erodible beds in this approach.

Although such analytical stability models have succeeded in explaining a number of aspects of meandering and braiding, a major problem remains as far as the braided river is concerned. All models lead to the conclusion that the rate at which new braids form increases monotonically with the number of channels. This is contradicted by common sense and by observations. As pointed out by Jansen, Van Bendegom, Van den Berg, de Vries & Zanen (1979), the presently available analytical models can only assess whether stability or instability prevails. They are incapable of predicting what happens as the result of instability, for example, how many channels will form in the river. Thus, little or no information is provided on the extent and the geometry of the braiding. Neither is any of the analytical methods able to predict what channel widths will prevail or what lateral displacement of the channels one may expect. They, therefore, do not allow any *quantitative* prediction of bank erosion or channel migration.

The present study takes a radically different approach. It recognizes the random nature of the many changes that take place in a braided river as a result of the global and local changes in water and sediment flow and attempts to make probabilistic predictions regarding bank movement at specific locations. The underlying assumption is that there are two factors at work. There are, in the first place, changes in the flow of water and sediment that cause deviations from equilibrium conditions in any given river section. These are considered imposed by changes in water and sediment flow elsewhere in the river. The other factor is the trend in the river section to return to what would be equilibrium conditions for the changed flow regime. The return to equilibrium is the predictable part. It can be assessed by an analysis of the state of equilibrium for the given flow of water and sediment in a cross-section and the magnitude of any deviations from that state. The changes generated by changed conditions elsewhere in the river are considered to be a random noise imposed on the river section under consideration. The thesis will show that in this way it is possible to make probabilistic predictions of the bank movement that can be expected in a given year at any location along the river.

1.3 The Problem along the Brahmaputra River

The Brahmaputra river in Bangladesh (Fig. 1.1) has drawn worldwide attention because of the magnitude of the problems created by the flooding and river bank erosion of this vast river. The bank erosion is of particular concern because of its long term disastrous consequences for this small, densely populated country that is located for the greater part in the flood plain of the river. Almost every year a large number of people are dislocated from their homesteads, losing their only means of livelihood - the farm land. For example, in Kazipur Upazila (sub-district) alone, along a 25 kilometre stretch of the river, 18,032 out of 111,820 people (16.12%) were displaced during the 1974-1981 period as a result of erosion of 9,717 acres of land out of a total of 36,827 acres (26.39%) (Saleheen, 1991). These people gather in the already overpopulated cities and towns creating severe and unmanageable social problems and intense human suffering. Loss of human lives is also incurred by the bank erosion both directly and indirectly. Moreover, any developmental planning near the river is hampered by the uncertainty of the future position of the river banks. It is true that new shore land is also formed, including new islands (locally called char) as a consequence of the channel movement. This, however, adds a new dimension to the problem. The authorities (as in most developing countries) may favour others, rather than the displaced people, in distributing the newly formed land, causing severe dissatisfaction and tension.

The bank erosion problem of the Brahmaputra river has been studied extensively in the past (e.g., Burger, Klassen, & Prins, 1988; BWDB, 1977; BWDB, 1978a; BWDB, 1978b; Coleman, 1969; Galay, 1980; Goulter & Dubois, 1988; IECO, 1964; Khan, 1988; Khan & Booy, 1988; Khan & Booy, 1989; Khan & Booy, 1990; Suharyanto, 1992; Stene, 1988; Stene, 1993). Authors differ in their opinions on the general trend, magnitude and predictability of the erosion experienced by the river. Coleman (1969) concluded that the right bank of the river is consistently moving westward. This observation was supported by other studies done in the seventies. Indeed, a flood embankment that was constructed on the right bank of the river by the Bangladesh Water Development Board (BWDB) had to be rehabilitated and moved to the west (BWDB, 1978a; 1978b). Later studies (e.g., Burger et al., 1988;

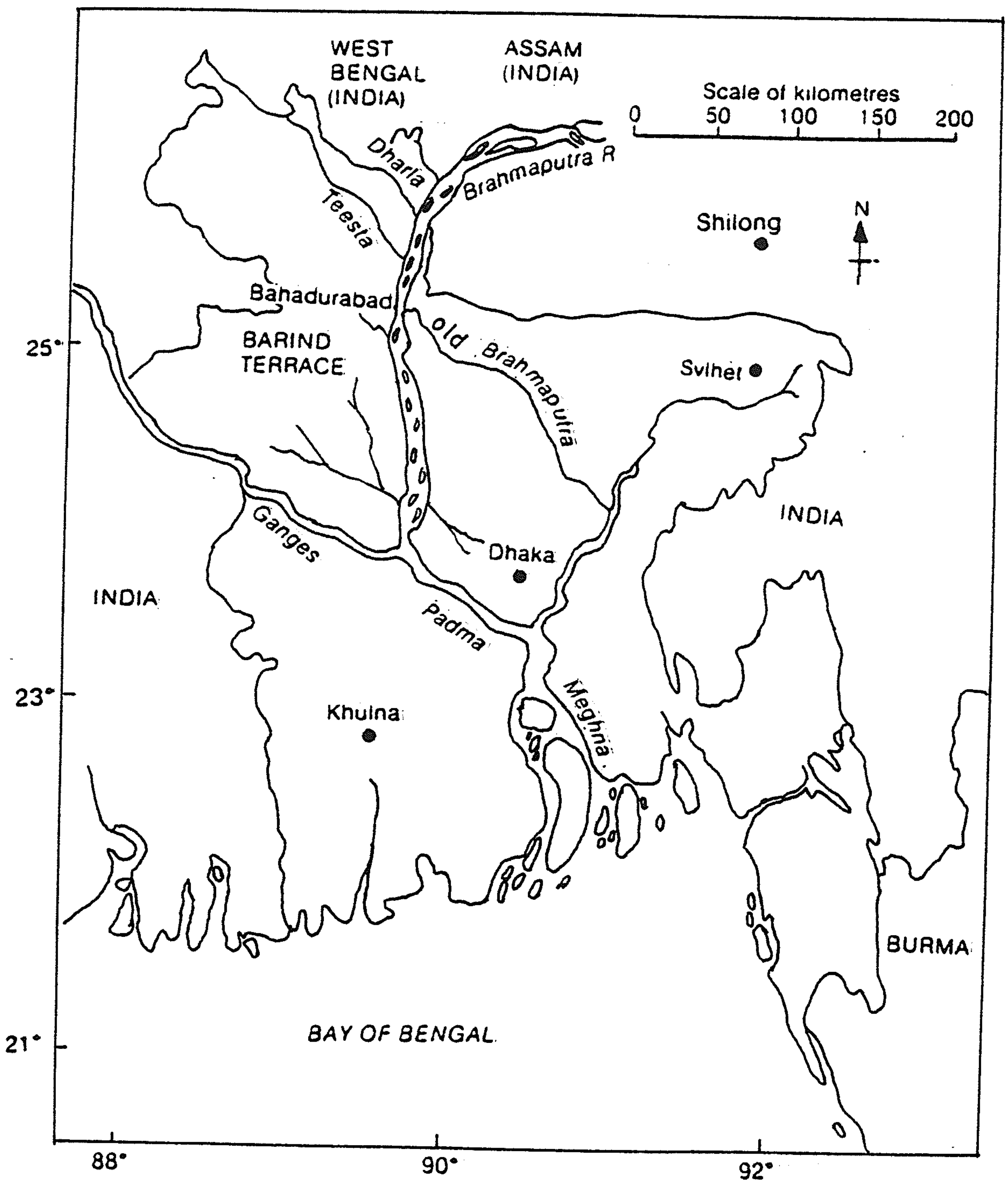

Fig. 1.1: The Brahmaputra River in Bangladesh

Khan & Booy, 1988; 1989), however, concluded that no single point on the right bank of the river moves continually in westward direction; they all move back and forth. It appears that the average right bank line has in recent times moved westward. It is questionable, however, whether this will be sustained over a longer period.

Regarding the magnitude of the erosion, while examples of very large annual movements abound, it is difficult to obtain reliable figures from earlier studies about the average annual movement because different researchers studied records of different lengths. Some studies, especially those that concentrated on the socio-economic aspect of the problem, suffered from generalization and ignored the fact that the bank lines did not move in a monotonous and unidirectional fashion. Reported yearly erosion rates are frequently unreliable because some of those were obtained simply by averaging the bank movement over several years at a very limited number of locations.

The disastrous consequences of flooding and bank erosion along the Brahmaputra river in Bangladesh has, of course, raised the question whether the situation can be rectified or improved by dyking and bank protection. At the present time containment works appear to be technically and economically infeasible and it is highly doubtful that attempts in this direction will be undertaken in the foreseeable future. It appears, therefore, that at least for the time being the people of Bangladesh will have to live with the flooding and erosion problems which the Brahmaputra river poses. Under these circumstances the best they can do is to protect lives and to mitigate the worst effects of flooding and bank erosion on the people who live close to the river banks.

Adapting to the bank movement problem is easier when the phenomenon is well understood and if the occurrence of large local erosion can be anticipated some time in advance. It is therefore highly desirable to analyze potential bank movements in an effort to quantify the risk that they exceed critical values at points along the river.

Such an analysis presents a difficult problem. The process of erosion and deposition of sediments as governed by stream flow dynamics is reasonably well understood. But the bank movement that results from this process in a braided stream is far too complex to trace. Consequently, bank movement has to a large degree the appearance of a random process. It means that statements about future bank movement must be formulated in probabilistic terms.

1.4 Basic Approach of the Study

The immediate purpose of the study is to develop a statistical model whereby the bank movements of the branches at specified cross-sections can be estimated a year in advance. The basic information used for this development consists of the hydrologic information regarding the river configuration and the annual changes therein for the successive years for which detailed observations are available. The basic constraints on the changes in cross-section are that the channels must together accommodate the flow of water and sediment in the river.

The first problem that arises in this regard is a problem that occurs also for single channels, namely that the same discharge and sediment flow can be handled by channels with different geometric cross sections. This is because the available analytical methods that relate water discharge and sediment flow to channels geometry do not provide sufficient relationships to uniquely determine the principal dimensions of width, average depth and channel slope. In other words, there are more independent variables involved in a change in channel dimensions than there are independent flow and frictional relationships. Researchers studying the equilibrium conditions for single channels were therefore obliged to resort to experience, in the form of observed equilibrium relations (regime flow relations) or to some extremum principle to supply the missing relationship in order to estimate cross-sectional change in response to changes in flow conditions. In the case of a braided river there are additional degrees of freedom since the total water and sediment flow can be distributed in many ways over branch channels. One can therefore conclude that the known constraints on the movement of water and sediment (provided by the hydrologic input and the available analytical relationships) allow the braided river to assume many possible states.

This raises two questions. The first is: What are the equilibrium conditions for the given regime of water discharge and sediment flow in a given section of river? The second is: What is the response of a given river section to a deviation from such equilibrium conditions?

It would seem plausible that in a braided river over a number of years deviations from equilibrium depth-width relationships occur with equal frequency on either side of the equilibrium. In other words, it is assumed that as many branches are in the process of widening as there are in the process of narrowing for given conditions of flow. Thus one might expect that the average depth-width relationships for given flow conditions, when taken over many branch channel cross-sections and several years, is close to the equilibrium relationship for such flow conditions. The first objective of the study was therefore to determine the so-called equilibrium depth-width relationships for branch channels as a function of overall flow conditions in the section. A comparison of individual branches with the equilibrium conditions will then provide an indication in what way and to what degree branch channels are out of equilibrium. This will indicate whether they are likely to widen or to become narrower. This leads to the second question, namely, the quantitative response of a particular river section to deviations from equilibrium conditions.

Here the problem arises that changes of the cross sections in the direction of equilibrium can be made in many different ways without violating the basic conditions of flow and sediment transport. In other words, at any given time a great many different changes to the next state of the system are compatible with our knowledge of the constraints that must be observed. This raises the question of what changes are the most likely, *given these constraints.*

Under these circumstances it appears attractive to resort to the principle of maximum entropy to determine what new state can be considered most probable given the preceding state and the constraints imposed by flow and frictional relationships. This principle recognizes the probabilistic nature of the morphological process and determines the most likely bank movement, based only on the available information about the state, the degrees of freedom and the constraints, without any other (arbitrary) assumption on the part of the investigator. The maximum entropy principle was therefore used.

It should be emphasized that the maximum entropy approach does not attempt to predict what in fact will happen – it only determines the most probable new state evolving from a process that is governed by the initial state and the constraints on the occurrence of new states. The method also assumes that the time needed to reach the most probable new state is not a factor because a time step of one year is assumed to be large enough.

Deviations from the assumptions that are made are expected to show up in a discrepancy between the determined most probable and observed bank movements of the branches in the available cross sections. If such deviations are serious and systematic then the maximum entropy method will, if nothing else, point objectively to the need for a revision of the assumptions.

The study thus provided a methodology to account for the erosion and deposition along all the branch channels of a braided river. Movements of the *outer bank lines* are, of course, more important to the people and the structures affected and are emphasized in this study.

The paucity of data both in temporal and spatial directions of the Brahmaputra river necessitated working with the average bankfull discharge. The method may be expanded to accommodate a more complete spectrum of water and sediment input when more data become available. For a similar reason, the time step was kept to one year. The predicted movement is, therefore, the most likely scenario as expected on a macro scale on a yearly basis.

The application of the result of this study is two-fold. First, this study is expected to bring further insight into the problem of bank erosion and deposition of a braided river. Secondly, the application of the principles to the Brahmaputra river will enable one to assess the general direction and amount of movement of bank lines at specific locations on a short term basis. This will help in planning developmental activities along the river.

1.5 Outline of Thesis

The study starts in Chapter Two with a brief literature review of the braiding process and its causes. The chapter then discusses the mathematical models of river channels behaviour that have been developed. A discussion of the variational principles and the principle of maximum entropy found in the literature is also provided.

Chapter Three gives a brief description of the physical setting of the Brahmaputra river, and the data available for the river in Bangladesh. Chapter Four describes the morphological and hydraulic characteristics of the Brahmaputra river. Chapter Five presents the equilibrium width-depth relationship of the river as developed in this study. In Chapter Six a bank movement model for the Brahmaputra river in Bangladesh is developed. Chapter Seven contains the computational procedure and the application of the developed model. The obtained results of quantitative bank movement prediction are discussed in Chapter Eight. Chapter Nine presents the conclusions drawn from this study and the recommendations made for further study and data collection. Additional relevant information and some derivations are contained in a number of appendices attached to the thesis.

Chapter Two

LITERATURE REVIEW

2.1 Scope

The literature consulted for the study can be divided into four main parts. The first part deals with the research that has been devoted to the morphology of different river types. This part starts with a review of the categorization of river types according to their morphological characteristics. Next, the research on whether a river will be meandering or braiding is discussed. Following this, the different opinions on the mechanism governing the process of braiding are discussed. Finally, different approaches to the determination of the so-called formative discharge are presented.

The second part of the review starts out with the empirical and analytical approaches to the determination of equilibrium conditions for alluvial rivers. Following this, the variational principles used by different researchers to obtain the extra relationship needed to solve the stable dimensions of a river are discussed.

The third part describes the maximum entropy principle and its use for the analysis of probabilistic phenomena of which the braided river may be considered an example.

The fourth part provides a discussion of the previous studies on the problem of bank movement of the Brahmaputra river in Bangladesh.

Finally, a summary is presented of the findings most relevant to the research described in this thesis.

2.2 River Type Categories

The typical geometric channel pattern of an alluvial river is the result of the adjustment of the channel form to the hydrodynamics of the channel flow and the accompanying processes of energy dissipation and sediment transport. Since the channel geometry also affects the resistance to flow per unit length of channel, the pattern can also be viewed as an adjustment of the effective slope of the river, at least in the short to medium time scale for which the valley slope can be regarded as constant. The ability to adjust the channel pattern reflects the ability of the river to modify its planform in response to changed conditions of water and sediment discharge, which may include changes caused directly or indirectly by human interference. The recognition and study of the response of an alluvial river to changed conditions is essential for proper river management.

The most commonly used classification divides rivers into three classes: straight, meandering or braided. The boundaries between the classes are not always clear. For example, the distinction between straight and meandering patterns is to some extent arbitrary, since it depends on the degree of regularity and sinuosity. At times, a braided pattern may be superimposed on a straight or meandering channel. A fundamental problem with this classification is that it is not based on a single characteristic. The distinction between a straight and a meandering pattern is based on the tortuosity of the course, whereas a braided channel is distinguished on the basis of the multiplicity of the flow channels.

A more logical classification divides the rivers into single channel and multiple channel types. These main classes can then be further sub-divided as needed.

2.2.1 Single Channel Pattern The single-channel pattern is further classified according to sinuosity, degree of regularity, and level of mobility. Sinuosity (P), defined as the ratio of channel length to the straight-line valley length, is used to distinguish straight channels from meandering channels. According to Leopold & Wolman (1957), channels with a value of $P \leq 1.5$ are considered to be straight, and those with $P > 1.5$ are meandering. Schumm (1963a) classified single-channel rivers to be straight ($P = 1.0$), regular ($P = 1.5$), and tortuous ($P = 2.1$). It is to be noted that alternate definitions of sinuosity exist. These are: ratio of thalweg length to the valley length (Leopold & Wolman, 1957); and ratio of channel length to meander belt axis length (Brice, 1964).

Kellerhals, Church & Bray (1976) identified three categories of meanders depending on the degree of regularity. These are: (1) regular meanders having a clear repeated pattern and a maximum deviation angle between the channel and the downvalley axis of less than *90°*; (2) tortuous meanders having a somewhat repeated pattern and a maximum deviation angle of more than *90°*; and (3) irregular meanders having a vague repeated pattern.

Comparing the level of mobility, Popov (1964, reported in Knighton, 1984) identified three patterns. These are: (1) incised non-meandering or embedded meanders, (2) confined or limited meandering, and (3) free meandering. Channels may also be distinguished as active and inactive on this criterion.

In a less generalized classification, Kondrat'yev & Popov (1967, reported in Richards, 1982) identified two categories distinguished by bedform characteristics.

2.2.2 Multiple Channel Pattern Braiding is not the only possible form of multiple channel type rivers. Braiding in a descriptive sense refers to the intertwining of multiple flow paths. Formation of multiple channels, however, occurs in three general situations: braiding, anastomosing or anabranching, and alluvial fan switching.

On the basis of stability of the branches, Schumm (1963a) distinguished between braiding and anastomosing patterns. The multiple flow pattern where the channels are stable and identifiable even with changing discharge and time is called the anastomosing pattern. This pattern would be created if, for example, a meandering river divides, forming an isolated distributary which may show many meanders before rejoining the main river. Some rivers with stable banks in cohesive material show this pattern of multiple flow channels.

The channels in the braiding pattern are constantly shifting. The separate branches rejoin after only a few simple curves. Rivers flowing on relatively steep slope through loose bank material may show this pattern of multiple flow channels.

Streams formed on alluvial fans often form multiple channels. Any diversion of channels on the alluvial fan results in a permanent lateral shift in flow path. Thus, alluvial fan switching differs from the other two multiple flow path patterns by the permanency of the splitting of channels although the underlying mechanism may be the same.

The multi-channel pattern may be further classified depending on the degree of island development. This ranges from occasional braided pattern with widely separated single islands to fully braided patterns with many channels divided by bars and islands. The form of multiple channel pattern is to a degree dependent on the river stage, which poses a special problem. For example, at higher discharges, bars which are visible at lower flows, may be inundated. A river with a multiple channel pattern may then look and begin to act more like one with a single channel. In this situation, an appropriate flow needs to be specified at which the multi-channel pattern becomes apparent. Further classification based on the degree of island development may only be relevant for the lower flows.

It is evident from this discussion that the classification in single and multiple channel types is not without its problems. The classifications based on sinuosity values are arbitrary. Perhaps, a more preferable approach is to use a common index such as 'total sinuosity' (Hong & Davies, 1979) to define channel pattern morphology. The total sinuosity is the ratio of total active channel length to the valley length. Note that the total active channel length is the summation of the lengths of all flow paths in a reach. Channel patterns may be identified by relating this index to stream power or any other measure of energy expenditure.

2.3 Factors that Determine River Channel Type

The conventional classification of straight, meandering and braided patterns continues to be the most widely used classification. This is perhaps due to its association with a concept introduced by Leopold & Wolman (1957) which regards all three as different manifestations of the same continuum of underlying causal factors. Leopold & Wolman (1957) state:

> If we assume that the pattern of a stream is controlled by the mutual interaction of a number of variables, and the range of these variables in nature is continuous, then we should expect to find a complete range of channel patterns (p. 59).

The concept assumes that streams are freely able to form their planimetric dimensions in response to flows and sedimentations. It does not directly deal with the possibility of a transition between channel patterns. However, the transitional patterns have been observed in nature, albeit over relatively short river reaches. Knighton & Nanson (1993) noted that anastomosis in certain cases represent a transitional channel pattern. This seems to support Leopold & Wolman's view.

Each channel pattern in this continuum of channel forms, is associated with a particular combination of controlling variables. Attempts have been made to relate the three basic patterns to numerical values of such variables. Slope-discharge relationships have been widely used for this purpose. Some of these relationships are given in Table 2.1.

Table 2.1 Slope, discharge, bed material size and channel pattern

Source	Equations	Comments
Leopold & Wolman (1957)	$S = 0.012\ Q^{-0.44}$	Braided channels plot above the line, meanders plot below. Natural streams
Lane (1957)	$S = 0.0007\ Q^{-0.25}$ $S = 0.004\ Q^{-0.25}$	Meanders Braided with sandy bed
Henderson (1963)	$S = 0.00012\ D^{1.15}\ Q^{-0.46}$	Slopes greater than this braids, less than this meanders
Ackers & Charlton (1970)	$S < 0.001\ Q^{-0.12}$ $S > 0.0014\ Q^{-0.12}$ $S = 0.00085 Q^{-0.21}$ $0.001\ Q^{-0.12} < S < 0.0014\ Q^{0.12}$	Straight channels Meanders Meanders plot above the line, straight plots below. Straight channel, alternating bars
Osterkamp (1978)	$S = 0.001\ Q^{-0.24}$ $S = 0.0019\ Q^{-0.31}$	Includes all patterns. Coefficient value increases as sinuosity decreases or material size increases. Braided channels

Note: In above equations, S = slope, Q = discharge, and D = bed material size (SI units)

A close look at the relationships listed on Table 2.1 shows considerable differences in their form and the values of the coefficients. These differences are, perhaps, due to the different environmental conditions under which the relations are developed. For example, some of the relations were developed from natural streams while the others result from laboratory experiments. All relations, however, suggest that the braided pattern occurs at a steeper slope than the meandering or the straight pattern. This suggests that a threshold value may exist for the slope beyond which the braided pattern prevails.

Laboratory experiments (e.g., Ackers & Charlton, 1970; Schumm & Khan, 1972) suggest that a similar threshold governs the occurrence of the straight and the meandering channels. If that is the case, then one could argue from the relationships of Table 2.1 that channels do assume their patterns ranging from straight via meandering to braiding as the stream power increases. The stream power is defined as γQS, where, Q is the discharge, S is the slope, and γ is the specific weight of water. It contains the variables discharge and slope as do the relationships of Table 2.1. Indeed, Chang (1979) showed analytically that the number of braids within a braided channel tends to increase with increasing stream power. Since the sediment transport also increases with increased stream power (Bagnold, 1977), the number of channels would also tend to increase with increasing sediment transport.

The dependency of channel patterns on slope and sediment load was shown in the laboratory experiments of Schumm & Khan (1972). In their experiments, the channels remained straight until a threshold slope (about 0.002) was reached. Then the pattern changed to meandering. This was associated with the development of secondary currents that eroded and redistributed the sediment so that alternating bars and a meandering thalweg were formed. These secondary currents were destroyed at an upper threshold slope (about 0.016) when the channel changed from a meandering to a braided pattern. At this point the stream power was high enough to induce braided channel development by widening the channel significantly. Between these two thresholds the bank resistance was sufficient to keep the channel in a meandering one. Schumm & Khan (1972) also observed that with an increase of slope in their experiment, an increasing amount of sediment supply was required for the stability of the channels. This observation suggests that another possible threshold may exist for the occurrence of different channel patterns, namely sediment availability.

Laboratory studies have shown that straight channels are unstable except at small sediment load. Field data supporting this observation are largely unavailable. Schumm (1963b) observed for the mid-west region of the United States that meandering is associated with cohesive sediments and wash-load transport whereas, braiding is associated with coarser non-cohesive sediments and bed-load transport. This finding emphasizes the relationship between channel pattern, composition of boundary material and sediment load. Gravel bed channels were not included in the observations.

Inherent resistance of the river bank to erosion is probably an important criterion for the development of different channel patterns since this influences the ability of channels to modify their patterns. Ackers & Charlton (1970) found from experiment that meandering channels tend to be wider than straight ones. Note that availability of sufficient energy for bank erosion and sediment transport controls to a degree the process of meandering. If the banks easily erode then the channels will assume a wide and shallow form with multiple cells of secondary circulation that possibly lead to braiding. Brotherton (1979) therefore related the occurrence of straight, meandering and braided patterns to the erodibility and transportability of bank particles. In an analytical approach in which the meandering and braiding is considered as an instability problem, Engleund & Skovgaard (1973) defined a threshold width above which the channel braids and below which the channel meanders. Following a similar approach and assuming that the rivers have an inherent tendency to form braids, Parker (1976) found that straight channels exist only at low width-depth ratio. According to him, braiding is prominent when the slope and the width-depth ratio are sufficiently high, and meandering is prominent when the slope and the width-depth ratio are sufficiently low. Empirical relationships between channel width (W) and shear stress (τ) developed by Antropovskiy (1972) distinguishes channel patterns as:

Free meandering: $W < 0.013\ \tau^{-1.40}$

Incomplete meandering: $0.013\ \tau^{-1.40} \leq W \leq 0.041\ \tau^{-1.40}$

Braiding: $W > 0.041\ \tau^{-1.40}$

From the above discussion, it may be concluded that the channel pattern sequences of straight, meandering and braided are related to:

(1) increasing stream power implied by an increase in slope at constant discharge or an increase in discharge at a constant slope,

(2) increasing sediment load, particularly, the bed load, and

(3) increasing width-depth ratio which is accompanied by an increase in bank erodibility and an increase in sediment load.

Slope here is taken as an imposed constraint which is adjustable only over a relatively long period of time.

Braided and straight patterns are in a sense two extremes. The braiding pattern with wide and shallow channels caused by bank erosion and localized circulation (Chitale, 1973) is the high energy-environment extreme. The straight channels are the low energy-environment extremes where the stream power is insufficient to cause bank erosion and secondary currents are insignificant. Meandering channels exist in between these two extremes. This and the fact that the variables considered are continuous in nature tend to confirm that the channel patterns can be expected to represent a continuum of channel forms as Leopold and Wolman suggested.

It has been observed, however, that the changes from one pattern to the other as the critical thresholds are crossed are more abrupt than the concept of a continuum would suggest (Schumm & Khan, 1972). There are also other problems with the continuum concept. First, it relies heavily on the results of laboratory experiments. Second, the classification of channel patterns underlying the concept oversimplifies the range found in nature. There are evidences of various transitional patterns with specific characteristics. For example, meandering streams in upland environments often have extensive unvegetated point bars across which chutes are formed that create pseudo-braided patterns at high stages and straight channel with mid-channel bars appear braided at low stages (Richards, 1982). Also, other distinctions such as active and inactive or confined and unconfined can be made between channels (Ferguson, 1981). Schumm & Khan (1972) reported that channel pattern may vary along a river even with the uniform condition of stream power and sediment load. This may indicate that the valley slope is close to a threshold value.

2.4 Characteristics of the Braided River Type

As discussed in the previous section, braided channel patterns are formed in a high energy fluvial environment with relatively steep valley slopes, large and variable discharges, dominant bedload transport, non-cohesive banks that are not stabilized by vegetation, and lack of river competence. They have a relatively high width-depth ratio. Individual channels are generally straight in alignment although some may be sinuous. The wide and shallow cross-sections develop secondary currents with multiple cells that results in bar formation. Braids may develop where erodible banks allow channel widening.

The geometry of braided channels has received little attention compared to that of meandering rivers. The literature on meandering rivers is vast and provides many relationships between channel geometry and variables of causative nature. Only a few such relationships have been developed for braided rivers.

Even the description of the degree of braiding is complicated by the fact that braiding tends to decrease with the higher river stages. For example, the Braiding Index, a measure of degree of braiding defined by Brice (1964), which is the ratio of twice the sum of the lengths of islands to the length of the reach, is dependent on the river stage since the lengths of the islands varies with the river stage. To avoid this problem of stage dependency, Howard, Keetch & Vincent (1970) used only the variables that can be determined from the topographic maps.

When a single channel divides into branch channels the result is a reduced hydraulic efficiency. Thus, braiding brings about modifications of flow dynamics and energy loss. The sum of the widths of the divided channels will be greater than the width of a single channel carrying the same water and sediment flow. Similarly, the average depth of the divided channels tend to be less than that of the single one. More stream power is needed therefore to maintain the sediment transport. This implies a steeper slope along the divided reach. Adjustment of several hydraulic variables is thus needed to maintain both water and sediment transport continuity. It can, therefore, be said that the braiding channel form is associated with the requirements of the physical transport processes. However, a unique one-to-one relationship between the channel form and the transport process may not exist. Krumbein & Orme (1972) are of the opinion that complex braided reaches may appear topologically random.

Shifting bar forms and abandoned distributary channels may occur in a braided reach that appears to be in equilibrium. However, on average, over a period of years, the total sinuosity and area of such a reach is maintained (Richards, 1982). Thus, a braided reach may continue to maintain this equilibrium locally after the bar formation or channel widening.

The instability of bedload transport in wide, shallow channels in high energy-environment streams may create braided patterns (Parker, 1976). The hydraulic inefficiency associated with the braided pattern then permits dissipation of the excess energy.

The conditions leading to the development of a braiding pattern can thus be summarized as; (1) highly variable discharge, (2) excessive bed load, (3) highly erodible banks, and (4) steep valley slopes. The effect of these variables on braiding are described briefly in the following.

Influence of Discharge Variability The regime of braided rivers is usually characterized by a highly variable discharge. It shows higher flood peakedness, higher total discharge range, and higher monthly variability (Morisawa, 1985). The variability of these flow characteristics tends to lead to bar formation, diversion of the flow and creation of new channels. Rapid fluctuation in discharge is often associated with high overall rates of sediment supply. This causes large fluctuation in bed load movement and bank erosion that lead to bar formation which in turn may initiate braiding. The rising as well as the falling river stages are both hydraulically important in braid formation. Secondary circulation cells with opposite direction of rotation meet and rise at the mid-section during the rising stage of the flood (Morisawa, 1985). This meeting causes the flow to decelerate at that section causing deposition of coarse particles. Secondary cells with opposite direction of rotation were found to exist in braided reaches of the Mistaya River in Alberta (Diemer, 1979, reported in Morisawa, 1985). Also, there is evidence, on a longer time scale, that braids have at times developed as a direct result of historical floods and continued to exist (Burkham, 1972; Schumm & Lichty, 1963). Goff & Ashmore (1994) reported that in the braided Sunwapta river in Alberta, major periods of morphological change occur during periods of high discharge.

It should be noted, however, that a rapid variation in discharge does not always lead to the development of braids but promotes it. Braiding reaches are sometimes interspersed with meandering reaches and in the laboratory experiments, braiding can be produced under steady state conditions (Hong & Davies, 1979; Leopold & Wolman, 1957).

Influence of Excessive Bed Load Braided rivers carry heavy bed loads as indicated by the existence of sand waves (see Coleman, 1969). While it is generally assumed that braiding is not symptomatic of overloading, availability of sediment in sufficient quantity is essential to obtain and maintain a braided form (Knighton, 1984). Hoey (1992) noted that in complex braided systems, fluctuations in bedload transport and changes in channel configuration may be autogenic (inherent) and may not require external forcing.

The process of braiding is enhanced by the occurrence of a coarse fraction in the sediment load. This forms the initial deposits due to local incompetency. The deposits, in the form of bars, cause the flow to attack the channel banks. The resulting bank erosion is needed for the development of a wide and shallow channel pattern.

According to Church (1972), braiding occurs when the resistance to flow of the single channel becomes too high to transport the water-sediment load. If the resistance to flow is not reducible by single channel adjustments, local aggradation occurs to increase the available specific energy. Thus, bars are formed due to deposition of excess load. This divides the flow into channels around the bar. These channels are steeper, relatively deeper and more efficient even though the combined resistance of these channels may be higher than that of the upstream single channel. This, in turn, increases velocity and energy of the river to be able to transport the incoming sediment.

Others (e.g., Knighton, 1972) have also suggested that braiding occurs when the river is locally incompetent to transport the bed load. In this situation, deposition of the bed load occurs which increases the gradient thereby increasing the competency. A bar formed due to deposition of the bedload divides the channel and braiding occurs. The effects of fluctuating water and sediment load are reflected by the oscillation of bars with single chute. During periods of low flow, sediment in transit is stored in these bars. These bars also provide the mechanism for removal of the stored sediment during periods of high flow. Old channels are reoccupied during high flow, and thus, secondary braiding occurs.

Leopold & Wolman (1957), on the other hand, are of the opinion that braiding is not necessarily indicative of excessive bed load. They point to observations of aggradation at constant slope without braiding. The process of braiding, in their opinion, is not a matter of lack of *overall* capacity but a lack of *local* competence. Schumm's (1968) statement that high suspended load is transported in meandering channels whereas bedload transport needs a straight and braided system, supports this. However, the importance of the sediment *availability* is evidenced by the changes in channel pattern of the Rangitata River in New Zealand (Schumm, 1980). Depending on the difference in the supply of sediment at different reaches, the pattern along the river changes from a braided to a meandering one. The laboratory experiments of Schumm & Khan (1972) demonstrate the need of an ample supply of sediment. The variability of bedload transport was also reported in a hydraulic model study of a braided river by Warburton & Davies (1994).

Influence of Slope The existence of a threshold value of the slope above which braiding occurs, is shown by theoretical and empirical studies (e.g., Leopold & Wolman, 1957; Parker, 1976; Schumm & Khan, 1972). When the slope is less than this threshold value the pattern is straight or meandering. Chang (1979) and Parker (1976) showed that the degree of braiding seems to increase with slope. There is, however, evidence of braiding of large rivers with very mild slopes (Leopold & Wolman, 1957). Also, braiding is observed to be a characteristic of channels in deltaic areas where the slope is very mild. It seems, therefore, that the critical factor in braiding is *high stream power* (γQS, where γ is the specific weight of the fluid, Q is the discharge, and S is the slope) rather than steep slope. A river must have sufficient power to achieve high bed mobility and to erode its banks.

Influence of Erodible Banks Fahnestock (1963) noted that highly erodible river banks are a prerequisite of braiding. Such banks are important sources of sediment and will permit the channel to widen allowing braided reaches to occur. The opportunity of bar formation is greater in a wider channel in which the non-uniformity of flow due to localized deposition tend to affect part of the channel rather than the whole cross-section. Without erodible banks, bars that are formed have a tendency to be wiped out rather than to grow.

It has been observed that rivers tend to meander, rather than assume a braided form, when banks are resistant to erosion even though the bed load transport may be dominant (Miall, 1977). Rivers may even assume a sequence of meandering and braided forms as they pass through deposits of varying resistance to erosion (Mackin, 1956). Also, a river was reported to have changed its pattern from braiding to meandering as a result of increased resistivity of bank material caused by planting vegetation at appropriate places (Nevins, 1969, reported in Knighton, 1984). On the other hand, both meandering and braided patterns have been observed with bank materials that are not noticeably different in terms of resistivity (Fahnestock, 1963). The erodibility of bank material is therefore not the only factor in the occurrence of braiding. It may be the deciding factor when other conditions are close to their threshold values.

None of the factors described above is capable of either creating or maintaining a braided pattern alone. It is the combination of the variables that is responsible for an environment that is conducive to a braided pattern.

Ackers & Charlton (1970) consider the braided pattern to be unstable. They argue that braiding is simply an adjustment made by rivers with erodible banks to carry the large sediment through multiple channels (which a single channel is incapable of transporting). The result of braiding is aggradation which occurs until the river achieves an equilibrium slope. Then braiding will stop. The channel will then assume either a straight or a meandering pattern. This view is not undisputed since increases in slope tend to accelerate braiding instead of stopping it. Many studies (e.g., Chang, 1979; Parker, 1976) consider the braided pattern to be a true equilibrium form even though the individual channels are transient. Leopold & Wolman (1957) observed that rivers maintain braided reaches with little changes over decades. In their opinion, braiding is the result of the adjustment made by the rivers with erodible banks to a sediment load too large to be carried by a single channel. Once established, braiding can be maintained with only slight modification. Some theoretical justification in support of this opinion has been provided by Parker(1976) and Chang (1979).

2.5 Mechanism of Braiding

The theories of the mechanism initiating and maintaining the formation of braids have long been dominated by the concept of bifurcation at the point of disjunction. The central proposition of this idea is the formation of a mid-channel bar which eventually splits the flow of the river into two or more active channels. However, the initiation of such a bar and its growth has not been adequately addressed. The following discussion summarizes the different opinions encountered in the literature on the mechanism of braiding.

There is no evidence of any particular sedimentary structure that would cause the special hydraulic conditions that initiate braiding. The analysis of parameters aimed at describing the flow behaviour leading to braiding is mostly a description of the conditions surrounding the phenomenon of braiding rather than an explanation of the phenomenon itself. There are a very few articles (e.g., Chien, 1961; Krigström, 1962, reported in Richards, 1982; and Smith, 1971) directly concerned with the mechanism of channel multiplication. Most investigators accept the idea that meandering and braiding are simply alternate forms of the same river behaviour so that braiding as well as meandering results from the general flow condition that exists in the river. In the same way the mechanism that leads to meandering is repeated at any bend, the potential for braiding is thought to be inherent and latent at every incipient point of disjunction when the overall flow conditions in the river reach a critical point.

The result is a growing bar in the middle of a river. Such a bifurcation is the simplest form of multiplication, and repeated splitting could produce any observed braided form.

According to this concept, bifurcation is imminent when the hydraulic conditions in a wide channel may lead to secondary currents that cause a deceleration of the current in the mid section of the channel sufficient to initiate the local deposition of coarser bedload material. This then serves as a locus for further deposition. The bars formed in this way are diamond shaped in outline and grow in the direction of flow. As further deposition occurs the bar grows vertically and in a downstream direction. When the bar is large enough, the main flow becomes concentrated in the channels on either side of the bar. This concentration of flow causes erosion of the banks and bed until the bar emerges as an island. With lower river stages vegetation starts to grow on the island which resist erosion and causes further deposition of finer material.

In addition to these longitudinal bars, Smith (1970) describes transverse bars. The transverse bars are tabular in shape, flat-topped and covered with bed forms (ripples and dunes). They have a steep slope face downstream and are dominantly composed of sand. Many such bars seem to be dune fields that were uncovered at low flows. Also, Brice (1964) suggested that bedforms created the bars of the Loup Rivers. According to him, large dunes were built across the river during flood stages. The crests of these dunes, as they grew bigger, were subsequently breached by the currents, and water pouring through the gaps produced a fanlike structure that became bars at low flow. Coleman (1969) noted that huge sand waves (7-16 m high with a wave length of 183-914 m) migrated at the rising stage on the Brahmaputra river. At the falling stage, one bedform climbed on the back of the other, and the troughs were filled up. These structures composed of both suspended and bedload material remained as bars.

Cadle & Cairncross (1992) from a study of Karoo river in South Africa, suggested that braid bars originate and develop through the initial deposition of sediment from unit bars (sand waves). Vertical accumulation of sediment occurs by the repeated deposition of sand waves to form an incipient braid bar. Subsequent growth occurs through sediment transport over the braid bars and deposition on the side and downstream end of the bar. Depending on the amount of vertical aggradation and channel scouring which takes place, sediment moves laterally or obliquely across the bar. These braid bars evolve into compound bars through oblique accretion. Also, periods of high flow cause sediments to be transported obliquely across the compound bar and deposited on the side and downstream. Flow divergence around the compound bar causes erosion of the first-order channel banks and channel widening takes place. And thus, channel bifurcation occurs.

Rundle (1985a; 1985b) strongly criticised the concept of mid-channel bar formation as the mechanism of braiding, and proposed that braiding occurs by the dissection at a lower flow of the tongue structure across the channel left behind by the flood. The central argument put forward by Rundle (1985a) is that the process of braiding is erosional rather than depositional contrary to the prevailing view in the literature. However, the mechanism he proposed and some of his criticisms of depositional concepts are questionable. He argues that a stream would accelerate around mid-channel deposition and hence, deposition around it would not be possible. This is of course true when an object is mechanically placed in a uniform equipotential flow pattern. However, the deposition of the mid-channel bar is supposedly initiated by a local reduction of competence that is caused by the occurrence of multiple circulation cells in the wide channel. Moreover, the blanket assertion that the changed flow pattern around an obstacle is not comparable with deposition is unrealistic. For example, when a bend in the river forces a change in direction deposition, nevertheless, occurs along the inside river bank. Moreover, Rundle's observation were mostly on river reaches with high Froude number (supercritical flow). The formation of a tongue shaped transverse bar in the reaches with lower velocity is not unexpected under this condition. This, however, is rather an unusual situation. Tongue shaped transverse bars also occur when a sediment laden concentrated stream enters into a lake or a sea. This does not lead to braiding but to delta formation which is a different process.

It seems that in spite of Rundle's criticism the most plausible explanation of the initiation of braiding is the formation of a mid-channel longitudinal bar which subsequently grows and causes bifurcation of the river, culminating in a braided pattern. This, however, does not rule out other mechanisms under special circumstances.

2.6 Formative Discharge

2.6.1 The Concept When the regime theory that was developed for irrigation canals was applied to rivers with variable flow the concept of the formative or dominant discharge was needed. This is by definition the steady discharge that would have the same effect on channel morphology as the spectrum of discharges that actually occurs. The regime theory of Blench (1952; 1969), reaffirmed by Ackers & Charlton (1970), posits that channel morphology does not adjust with every short-term variation of discharge. Rather, it depends on a discharge measure which typifies the range of competent discharges experienced by the river.

The formative discharge can be defined more precisely in a number of ways. For example, Ackers & Charlton (1970) has defined dominant discharge to be the steady flow that produces the same meander wavelength as the observed range of varying flow. Wolman & Miller (1960) defined the dominant discharge as the flow that performs most work in terms of sediment transport. They reasoned that the effect of a particular flow event in a longer period depends on its frequency of occurrence. Sediment transport is approximately a power function of discharge while the frequency distribution of the discharge is approximately log-normal. The total sediment transport that can be attributed to a particular flow is thus the product of transport and frequency of occurrence. This suggests that most of the work is done by events of intermediate magnitude. The single formative discharge then becomes the mean of the probability distribution of:

$$F[G(Q)] = G(Q)f(Q) \tag{2.1}$$

Where,

$f(Q)$ is the probability distribution of the discharge Q assumed to be known;

$G(Q)$ = sediment discharge = a function of Q, also assumed to be known for a river.

The NEDECO (1959) defines dominant discharge as the discharge corresponding to the water level h_D which follows (Jansen et al., 1979):

$$-\int_0^T (1-b)\frac{Q_s}{W}\frac{\partial W}{\partial x}dt + \int_0^T \frac{b}{b_D}\frac{Q_s}{h}\frac{(1-b_D)}{W_D} h_D \frac{\partial W_D}{\partial x}dt = 0 \qquad (2.2)$$

where,

Q_s = sediment discharge = $W\,a\,u^b$;

W = width; u = velocity;

a, b = constants of sediment property;

x = coordinate in flow direction;

T = time = 1 year, and subscript D marks the value corresponding to dominant discharge. Pickup & Rieger (1979), however, considered that every competent flow event has some influence on channel form. The channel form 'y' at a time 't' should be taken as a weighted sum of the effects of all input discharge events up to and including t, and can be expressed as:

$$y(t) = \int_0^t h(u)Q(t-u)du \qquad (2.3)$$

where,

$h(u)$ is the impulse response measuring the effect of discharge on channel form over various time lags u; and

Q is the discharge.

The flow which just fills the available cross-section without overtopping the banks has often been taken as the formative discharge. Several arguments can be advanced for this practice.

(a) There is evidence suggesting that the frequency with which bankfull discharge occurs is to a degree consistent among rivers (Wolman & Leopold, 1957). The average recurrence interval of bankfull discharge is found to be about 1.5 years (Leopold et al., 1964; Williams, 1978).

(b) There is an approximate correspondence between the bankfull discharge and the discharge which cumulatively transports most sediment (Wolman & Miller, 1960). Note that although flood flows individually transport greater loads, they recur too infrequently to have a greater cumulative effect.

(c) There is a marked process discontinuity associated with overbank flow. The water level corresponding to the bankfull discharge is considered by many to have significant influence of channel formation process (e.g., Harvey, 1969; Pickup & Warner, 1976). River stages above this level are considered to have less influence on the cross-sectional shape.

2.6.1 Estimating Bankfull Discharge

The bankfull discharge can be estimated from the bankfull stage by means of a stage-discharge relationship or a slope-area method (Brown, 1971). Williams (1978) provides a review of eleven definitions used for identifying bankfull stage, and compares sixteen methods of computing bankfull discharge. The definitions are: (1) the elevation of the valley flat; (2) the elevation of the active floodplain; (3) the elevation of the low bench; (4) the elevation of the 'middle bench' for rivers having three or four overflow surfaces; (5) the elevation of the most prominent bench; (6) the average elevation of the highest surfaces of the channel bars; (7) the height of the lower limit of perennial vegetation; (8) the elevation of the upper limit of sand-sized particles in the boundary sediment; (9) the elevation at which the width-depth ratio of the cross-section is a minimum; (10) the stage corresponding to a change in the relation of cross-sectional area to top width; and (11) the stage corresponding to the first maximum of the bench-index *(I)* defined as (Riley, 1972):

$$I = \frac{(W_i - W_{i+1})}{(d_i - d_{i+1})} \tag{2.4}$$

where,

W is the width;

d is the depth; and

subscript *i* represents positions of the wetted perimeter at equal distances.

Which definition to choose depends on the use to be made of the bankfull discharge. For fluvial geomorphologists, the active flood plain is the most meaningful bankfull level. The banks of the valley flats are the most important to engineers since a stage higher than this would cause damage to structures, crops etc. William (1978) computed bankfull discharge at 136 sites using active floodplain (*Definition 2*), and at 113 sites using valley flat (*Definition 1*) as the bankfull stage. Combining these two groups of data he found a general relationship of bankfull discharge (Q_b), average bankfull cross-sectional area (A_b) and slope of the water surface as surveyed at low flow over the entire reach (S) as:

$$Q_b = 4.0\, A_b^{1.21}\, S^{0.28} \tag{2.5}$$

Some other equations, that exist in the literature, are given in Table 2.2 along with their applicability.

Table 2.2 Equations of bankfull discharge

Reference	Equation for Q_b (m^3/s)	Applicability
Lacey (1930; 1934)	$10.8\, A_b\, d_b^{0.67}\, S^{0.33}$	All
Nixon (1959)	$0.57\, A_b^{1.21}$	All
Rundquist (1975)	$0.41\, A_b^{1.22}\, D_{50}^{0.96}$	All
Riggs (1976)	$3.39\, A_b^{1.295}\, S^{0.316}$	$D_{50} > 2$ mm
Riggs (1976)	$1.55\, A_b^{1.33} S^{0.05} 10^{-0.056S*S}$	All
Leopold et al. (1969)	*1.5 Year flood* (annual maximum)	Active floodplain
Kellerhals (1967)	$0.9\, D_{90}^{0.15}\, d_b^{0.25}\, A_b$	$D_{50} > 2$ mm
Emmett (1972)	$0.8\, A_b^{1.18}$	South Central Alaska
Emmett (1975)	$1.4\, A_b^{1.14}$	South central Idaho
Dury (1976)	$1.33\, S^{0.25}\, (0.14\, W_b^{1.81} + 0.83\, A_b^{1.09})^{1.19}$	Humid regions
Kellerhals (1967)	$(5.1 \times 10^{-6})\, W_b\, D_{90}^{1.15}\, S^{-1.25}$	$D_{50} > 2$ mm
Henderson (1961)	$(3.76 \times 10^{-9})\, D_{50}^{2.59}\, S^{-2.27}$	$5 < D_{50} < 270$ mm

Note that in the above table, Q_b is the bankfull discharge in m^3/sec,

A_b is the bankfull area in m^2,

W_b is the bankfull width in metres,

d_b is the bankfull depth in metres, and

D is the particle size in millimetres.

There are some objections to taking the bankfull discharge as the formative discharge. They are: (1) the bankfull channel is difficult to define if the valley bottom is narrow or if several benches exist, (2) the bankfull discharge which is defined in terms of elevation may not always be the most effective discharge in terms of sediment transport, (3) channel form parameters may not always relate well with bankfull discharge, and (4) bankfull discharge may not always have the same frequency of occurrence even within the same basin. Nevertheless, when these limitations are kept in mind, the bankfull discharge can serve as a good surrogate of the formative discharge.

2.7 Modelling River Channel Changes

A study of changes in river channel dimensions in response to deviations from equilibrium conditions must take into account the constraints imposed by the physics of fluid flow and sediment discharge. In this section, a brief description of the mathematical-physical analysis of water and sediment flow process is provided. Also, a brief review of modelling of the aggregate transportation process is provided. This includes a review of the available deterministic and stochastic modelling techniques.

2.7.1 Mathematical-physical Models The equations of fluid motion for a single channel with small curvature can be written as:

$$u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial n} + w\frac{\partial u}{\partial z} + \frac{uv}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + F_s \tag{2.6}$$

$$u\frac{\partial v}{\partial s} + v\frac{\partial v}{\partial n} + w\frac{\partial v}{\partial z} - \frac{u^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + F_n \tag{2.7}$$

$$u\frac{\partial w}{\partial s} + v\frac{\partial w}{\partial n} + w\frac{\partial w}{\partial z} + g = -\frac{1}{\rho}\frac{\partial p}{\partial z} + F_z \tag{2.8}$$

where,

s, n and z form an orthogonal curvilinear coordinate system;

s-axis is along the channel centreline, positive in downstream direction;

n-axis is perpendicular to the s-axis, positive towards the concave bank;

z-axis is vertically upward;

r is the local radius of curvature;

p is the pressure;

g is the acceleration due to gravity;

u, v and w are the time averaged velocities along s, n and z directions respectively;

ρ is the fluid density;

F_s, F_n and F_z are the friction terms in the s, n and z directions respectively.

The continuity equation of water and sediment can be written as:

$$\frac{\partial u}{\partial s} + \frac{\partial (vr)}{\partial n} + \frac{\partial w}{\partial z} = 0 \tag{2.9}$$

$$\frac{\partial q_s}{\partial s} + \frac{1}{r}\frac{\partial (q_n r)}{\partial n} = 0 \tag{2.10}$$

where,

q_n and q_s are volumetric bedload transport in n and s directions respectively.

Equations (2.6) through (2.10) can be used to obtain solutions for bank erosion rate. This requires some simplifying assumptions and the use of experimental results. One of the techniques, for instance, imposes a channel alignment perturbation of the form:

$$\eta(x,t) = A(t) sin[k(x - ct)] \tag{2.11}$$

where,

$\eta(x,t)$ is the channel centreline deviation at a straight line distance x at time t;

A is the amplitude;

k is the wave number $(2\pi/\lambda)$;

λ is the wave length; and

c is the celerity of the sinusoid.

This analytical approach is controversial and raises the question whether it makes sense to start with such complicated equations if the results are not any more accurate than what can be obtained with simpler formulations. Some of the problems are the following: Firstly, the reduction of the equations to those of a damped oscillating system through linearization neglects higher order terms and limits the application to channels with small curvatures only. The solution then essentially becomes that of a straight channel whereas the interest lies precisely in the deviation from a straight channel. Secondly, the critical stresses for incipient sediment motion is a questionable basis for sediment transport equations as has been addressed by Lavelle & Mofjeld (1987). Thirdly, an erosion constant is used in predicting rate and extent of erosion at any point. This constant can be very uncertain and will vary from place to place, even for the same river. Finally, there are many uncertainties regarding friction and velocity distribution, which raise questions about the adequacy of the analysis.

One may conclude that the morphology of a river system is far too complex to be described by analytical relations that describe the dynamics of water and sediment movement. Researchers have therefore sought for descriptions of river behaviour in simpler terms that provide approximate mathematical models of response to changed conditions. Such models of river behaviour can be deterministic or stochastic.

2.7.2 Deterministic Models The deterministic approach which one finds in the literature is based primarily on an evaluation of the stability of the channel alignment. Two distinct groups of studies are available here. The first group consists of the so-called regime theories which were originally developed for the design of stable canals in erodible soils (Lacey, 1930; Blench, 1952). Arguing that nature often provides relatively stable channel dimensions corresponding to given conditions of water and sediment transport, these theories determine empirical relationships between channel dimensions and flow parameters based on observed stable river channels. With the exception of a few (e.g., Hooke, 1980;

Hickin & Nanson, 1975; Nanson & Hickin, 1983), the investigators do not attempt to predict rate and direction of change if equilibrium conditions are violated. Although Stevens & Nordin (1987) have provided regime theories with some analytical basis employing the conservation law and Newton's law, they are basically empirical.

The second group of deterministic models attempts to evaluate channel alignment stability through a more rigorous analysis of perturbation. Three types of conceptual models have been employed to study the instability phenomenon. These are: (a) the potential flow model (e.g., Reynolds, 1965), (b) the shallow water flow model (e.g., Adachi, 1967; Callander, 1969; Parker, 1976), and (c) the shear flow model (e.g., Engelund & Skovgaard, 1973). The basic model is developed from the equations of conservation of mass and momentum, and from a stability criterion that relates to the movement of sediment particles on the stream bed. The models thus attempt to present the observed typical channel planforms as features that follow from the basic flow equations. Although these models have succeeded in explaining a number of aspects of meandering and braiding, a major problem remains as far as the braided river is concerned. All three models lead to the conclusion that the rate at which new braids form increases monotonically with the number of channels. Also, no information is provided on the extent and the geometry of braiding.

Two different categories of models have been developed to deal with planform geometry. The first one evaluates the conditions for the formation of alternating bars in straight channels and is known as the bar theory (e.g., Ikeda, Parker & Sawai, 1981; Kitanidis & Kennedy, 1984). The second one examines channel migration features in weakly meandering flows and is known as the bend theory (Odgaard, 1989a; 1989b). Blondeaux & Seminara (1985) presented a unified treatment of bar-bend theory. Authors on the bar theory differ in their treatment of bank erosion and of secondary flow and its effect on bed topography and primary flow. Some authors assume the rate of bank migration to be proportional to and in phase with the difference between near bank and section average velocity (e.g., Odgaard, 1987; Hasegawa, 1989), while others assume the rate to be proportional to and in phase with the secondary flow (e.g., Kitanidis & Kennedy, 1984). In the bend theory there also is a difference of opinion in the treatment of the secondary flow. Blondeaux & Seminara (1985) consider a secondary current to be controlled by an external stress relation, while Odgaard (1989a) considers it to be controlled by basic flow equations.

The major problem of deterministic modelling is that the relations between the sediment transport processes and morphological change of a river are not sufficiently developed. Except for a few specific bed forms, morphological change cannot yet be related adequately to the physical process of sedimentation. The process of morphological change as it occurs in natural channels is governed by three-dimensional turbulent flow characteristics. It is not yet possible to include these characteristics in a model for natural channels. Even if it were possible to adequately describe the flow mathematically, the numerical solutions would require discretization of the process (Alexander, 1979). The calculated erosion and deposition in each part of the cross section would be affected by the way the continuous boundary is discretized (Bridge, 1976).

The deterministic models described above were developed for single channel rivers. Application of such models to a braided river cannot be expected to be very successful for a number of reasons. First, the stability of a cross section of a braided river implies the stability of the anabranches. The stability of each of the anabranches, on the other hand, depends on the stability of all other branch channels as well as the cross section as a whole. Secondly, in the process of stabilization the number of branch channels may change over time. Two or more branches may reunite to form one channel or one branch may split into two or more channels. Thirdly, one branch may be stable and continue to be so while the others are changing. In brief, the behaviour of individual branch channels and the entire river are interrelated and, this relationship must be considered if the model is to be adequate.

In a recent study, Murray & Paola (1994) developed a deterministic numerical model of water flow over a cohesionless bed that captures the main features of real braided rivers. The model was similar in concept to those of cellular computer models to study naturally occurring self-organized patterns (e.g., Forrest & Haff, 1992; Werner & Fink, 1993; Werner & Hallet, 1993). In the cellular model of stream braiding of Murray & Paola (1994), the initial condition was a uniform slope with white-noise elevation perturbation. Iteration began when water was introduced into some upstream cells and started moving downstream, cell-by-cell, carrying discharge. Iteration ended when water reached the downstream end. Although their model reproduced many of the phenomena observed in natural braided rivers, it exhibited apparently unpredictable changes in configuration indefinitely even though no random influences were introduced after the run started. They compared this and the observed fluctuation in sediment to those of self-organized or "autogenic" behaviour of braided streams. Representation of this behaviour, however, was not achieved in their model.

2.7.3 Stochastic Models Stochastic models of bank erosion are not abundantly available in the literature. Khan & Booy (1988) presented a Transfer Function approach of modelling bankline erosion. In their model, the banklines at the end of a time step were assumed to be the output of a dynamic system where the input to the system was the bankline at the beginning of the time step. Variables other than bankline position were lumped in the system. In addition, the model parameters were related to the magnitude of the water discharge.

2.8 Variational Principles

2.8.1 The Need for a Variational Principle To transport a certain amount of water and sediment, a single channel alluvial river can adjust its width, depth and slope. Thus, it has three degrees-of-freedom and the problem is to establish relationships that determine these three quantities. If one imposes values of discharge and slope, then the equations produce a family of solutions containing different values of width and depth, and hence, different sediment transport rate and velocity. Two sets of equations defining the sediment transport and frictional characteristics are readily available. One extra equation is, therefore, needed to solve the system. What constitutes the appropriate choice of this extra equation is, however, unclear. Different approaches have been used to provide the extra equation needed to solve the system of equations. Some researchers related bank stability (e.g., Parker, 1978) with bank movement while others followed variational principle.

The variational principle approach is based on the argument that a channel achieves stability or equilibrium when a specified function of some of the variables defining the system has an extremum (either maximum or minimum) subject to given local constraints. The stable solution can be obtained by maximizing or minimizing the specified function related to the system.

The most commonly used variational principles are: minimum stream power, minimum unit stream power, minimum energy dissipation rate, maximum friction factor, and maximum sediment transport rate. The inter-relationship between these principles are discussed by Davies & Sutherland (1983), Yang, Song & Woldenberg (1981), and White, Bettess & Paris (1982). Some of these principles may be equivalent under certain conditions, one may be a special case of the other. The principles are briefly discussed in the following.

2.8.1 Principle of Minimum Unit Stream Power The principle of minimum unit stream power was advanced by Yang & Song (1979) who hypothesize that: an alluvial channel will adjust its velocity, slope, roughness and geometry to minimize amount of unit stream power used to transport a given sediment and water discharge. The unit stream power, U_p, is defined as:

$$U_p = VS \tag{2.12}$$

where,

V is the mean flow velocity, and

S is the slope.

This principle has been used to explain equilibrium flow conditions of alluvial streams both in field and laboratory conditions (Yang, 1976).

2.8.2 Principle of Minimum Stream Power Chang (1979) uses stream power instead of unit stream power as the quantity that must be minimized to obtain the equilibrium condition of an alluvial channel. The stream power is defined as:

$$\textit{Stream Power} = \gamma Q S \tag{2.13}$$

where,

Q is the discharge,

S is the slope, and

γ is the specific weight of water.

It may be noted that the unit stream power concept of Yang (1976) will produce the same result only if the cross-sectional area remains the same. Also, attempts have been made to relate the principle of minimum stream power and minimum unit stream power to that of minimum rate of energy dissipation (see Yang & Song, 1979; Song & Yang, 1980).

The principle has been used to explain patterns of natural rivers, width-depth ratios of alluvial rivers in regime and gravel bed streams (Chang, 1980a; 1980b; Chang & Hill, 1977). However, it is only applicable to a channel of fixed location, and is not intended to provide solutions for the planform changes.

2.8.3 Principle of Maximum Friction Factor The principle of maximum friction factor as stated by Davies & Sutherland (1980) is as follows: If the flow of a fluid past an originally plane boundary is able to deform the boundary to a non-planar shape, it will do so in such a way that the friction factor increases. The deformation will cease when the shape of the boundary is that which gives rise to a local maximum of friction factor. Thus, the equilibrium shape of a non-planar self-formed flow boundary or channel corresponds to a local maximum of the friction factor.

The friction factor f (using continuity $Q = WVd$) is defined as:

$$f = \frac{8gd^3SW^2}{Q^2} \tag{2.14}$$

where,

Q is the water discharge,

d is the depth,

S is the slope,

W is the width, and

g is the acceleration due to gravity.

2.8.4 Principle of Minimum Energy Dissipation Rate Yang et al. (1981) describe the principle as: A system is in an equilibrium condition when its rate of energy dissipation is at a minimum value. When a system is not in an equilibrium condition its rate of energy dissipation is not at its minimum value. However, the system will adjust in such a manner that the rate of energy dissipation can be reduced until it reaches the minimum and regains equilibrium.

The energy dissipation rate is defined as:

$$E_r = \left[\gamma Q + \gamma_s Q_s\right] lS \tag{2.15}$$

where,

γ is the specific weight of water,

γ_s is the specific weight of sediment,

Q_s is the sediment discharge,

l is the reach length.

The other variables are as defined before.

2.8.5 Principle of Maximum Sediment Transport White et al. (1982) hypothesized that for a particular water discharge and slope, the width of the river adjusts itself to maximize the sediment transport rate. Although they could not provide a physical explanation, they observed that the principle worked in practice. This is perhaps not surprising since the ability of the river to transport the sediment is crucial in the development of the morphology. This is especially the case for a river with an abundant availability of sediment for transportation. The following discussion further addresses this issue.

With regard to the relationship between the width and the transport capacity, three contradictory views are present in the literature. These are: (a) the sediment transport decreases as width increases, (b) the sediment transport increases as the width increases, and (c) the sediment transport increases and then decreases as width increases, with a peak transport capacity at some intermediate width. The view in (a) has been expressed by Henderson (1966) and can be shown to be dependent on an invalid premise; whereas, that in (b) (Bagnold, 1977; Parker, 1979) produces conclusions that are inconsistent with transport formulas or are restricted to channels at a near-threshold state (Carson & Griffiths, 1987). In both of these cases, the transport rate is shown as a monotonic function of width which does not seem right. The bed load capacity should approach zero as width tends to zero opposing the view in (a), and should approach zero as depth tends to zero opposing the view in (b), since infinite width corresponds to zero depth. Therefore, the view in (c) expressed by White et al. (1982) is more realistic and suggests that relationship function between transport rate and width has an optimum rather than increasing or decreasing monotonically. As width increases, transport rate per unit width of a strip of moving sediment decreases since the width of the strip itself increases. This decrease corresponds to the resulting decrease in depth and hence, the excess tractive stress on the bed. When the channel is wide, a further fractional increase in width results in a bigger fractional decrease in tractive stress and hence, the unit transport rate decreases. On the other hand, for a fractional increase in width in narrow channel results in a smaller decrease in unit transport rate and therefore, the total transport rate increases. This has been demonstrated by Carson & Griffiths (1987) and is given in *Appendix-A*.

The principle of maximum transport capacity is equivalent to that of minimum stream power concept (White et al., 1982). This is shown in *Appendix-B*. Onishi, Jain & Kennedy (1976) has suggested that a principle of maximum sediment transport capacity is involved in determining the plan shape of a river noting that a meandering channel can be more efficient than a straight one.

The variational principles have been widely used in explaining different aspects of flow in an alluvial river. Griffiths (1984), however, argued that their use represents an illusion of progress. To substantiate this he provides a demonstration that use of the principles lead to a conclusion of constant Einstein's sediment discharge function and Shield's entrainment function with the magnitude of the particular constant being dependent on the principle used. In fact both of these expressions are highly variable in natural channels. This demonstration has been criticized. Song & Yang (1986) disputed Griffiths' claim and points out the inconsistency in formulation in support of the use of these principles. In reply Griffiths (1986) provided a rebuttal. The issue is still unresolved.

The idea that a river must follow an extremum principle in order to be stable is not self-evident and no satisfactory theoretical explanation has yet been found. Nevertheless, the above described variational principles were used in this study to determine the dimensions of the individual branch channels because of their success in application to single channel natural rivers. The principle of maximum sediment transport capacity was found to be better suited in this study as will be shown later.

The application of this principle to the branch channels of a braided river requires that the discharge in each of the branches are known. This, however, is not the case for the Brahmaputra river. The water discharge in each branch channel under equilibrium condition has to be determined. As mentioned in Chapter One, this is done by using the relationship between depth and width (discussed in Chapter Five) and the principle of maximum entropy discussed below.

2.9 Principle of Maximum Entropy

Entropy has been used in the science of systems where there is a basic uncertainty as to what particular state will prevail among the many that are physically possible and compatible with the known constraints. It measures the degree of randomness of the system. The specific characteristics of entropy, which enables researchers to deal with uncertainty, found wide application in a variety of fields. Besides thermodynamics and communication (which are the major fields that contributed to the development of the concept), entropy has been successfully applied to statistics (e.g., Jaynes, 1957a; 1957b; Kullback, 1968), marketing research (e.g., Kumar, 1983), economics (e.g., Theil, 1967; Theil & Fiebig, 1984), transportation (e.g., Wilson, 1968), biological sciences (e.g., Chaitin, 1979), operational research (e.g., Guiasu, 1977), psychology (e.g., Attneave, 1959), physical processes (e.g., Bevensee, 1993), water distribution network (e.g., Awumah, 1990; Awumah, Goulter & Bhatt, 1989), contaminant transport (e.g., Woodbury & Ulrych, 1993), and many others. In each case the evaluation and subsequent utility of entropy will depend on the field of application and the constraints imposed by the specific problems.

The concept of entropy originated in the development stage of classical thermodynamics where it was defined in an abstract manner as a thermodynamic variable of the system. This function was at the time rather mysterious since the measure did not obey a conservation law (unlike mass, energy etc.) and was found to be always increasing (Tribus, 1979). A complete and satisfactory 'explanation' for this was not found until the concept was used in the information theory as popularised by Shannon (1948a: 1948b) and improved by Jaynes (1957a, 1957b).

In statistical mechanics, a second definition of entropy was developed. It is defined as a measure of the number of ways in which the elementary particles of the system may be arranged under given circumstances (Fast, 1962), and can be written as:

$$S = -kN \sum_i \left(\frac{N_i}{N}\right) ln\left(\frac{N_i}{N}\right) \tag{2.16}$$

or, alternatively,

$$S = -k' \sum_i p_i \, ln p_i \tag{2.17}$$

where,

S = entropy of the system,

N = total number of particles in the system,

N_i = number of particles in energy state i,

p_i = fraction of particles in energy state i, and

k' = Boltzmann's constant.

In statistical information theory, the concept of entropy was introduced by Shannon (1948a; 1948b). It is viewed as a measure of the degree of 'uninformativeness'. The following axioms are concerned with the information content $h(p)$ of a reliable message (event E) given that the probability of E prior to its arrival was p.

Axiom 1: The information content $h(p)$ depends only on the probability p of the message. The information does not depend on the importance of the event to which the message refers to.

Axiom 2: The function $h(p)$ is a continuous function of p in the interval $0 < p \leq 1$ and it is monotonically decreasing, i.e., $h(p_1) > h(p_2)$ if $0 < p_1 < p_2 \leq 1$.

Axiom 3: The message has zero information when E has unit probability, i.e., $h(1) = 0$.

Axiom 4: The information content of the message is additive for stochastically independent events. If the probability of occurrence of events E_1 and E_2 are p_1 and p_2 respectively, then the probability of their joint occurrence is p_1p_2. The information content of the message which states that both events occurred equals to the information of the message on E_1 only plus that on E_2 only, i.e., $h(p_1p_2) = h(p_1) + h(p_2)$ if $0 < p_1, p_2 \leq 1$.

It can be shown that the simplest function that is compatible with foregoing axioms is $h(p) = -\log p$. This is a decreasing function from ∞ at $p = 0$ to 0 at $p = 1$ suggesting that the more unlikely the event was before the message arrived, the more informative the message is.

If we consider a complete set of mutually exclusive events, E_1, ... , E_N with probabilities p_1, ..., p_N, then the message that E_i has occurred, increases the information content received by $h(p_i) = -log\, p_i$. We do not have any prior knowledge of which E_i will occur, except that the probability of occurrence of each E_i is p_i. Therefore, the expected information prior to the arrival of the message is:

$$S = \sum_{i=1}^{N} p_i h(p_i) = -\sum_{i=1}^{N} p_i\, log p_i \tag{2.18}$$

which is Shannon's entropy. It has the same form as the entropy definition used in statistical mechanics. It defines the expected or mean information content provided by the set of potential events.

This information content is evidently dependent on the values of p_i. For the unconstrained case, it reaches a maximum value when the values of all p_i are equal which implies maximum randomness. When a constraint applies there is a set of p_i values consistent with the constraint that maximizes S. This defines the maximum randomness consistent with the constraint.

Jaynes (1957a; 1957b) proposed that Shannon's entropy (measure of uncertainty) be used to define the values of the probabilities p_i that will maximize S thereby maximizing randomness or minimizing prior information about the chance of p_i. Thus, Jaynes converted entropy to a powerful instrument for the generation of statistical hypothesis where one wants to avoid building prior "chances" into the system (Tribus, 1979).

Jaynes' use of entropy is straightforward in its mathematical description, and can be written as follows. Let there be a system involving the variables x_i that can assume a large number of different states and that is subject to m constraints given as:

$$\sum_{i} p_i f_r(x_i) = \langle f_r \rangle, \qquad r = 1, 2, ..., m \tag{2.19}$$

where, p_i is the probability associated with x_i so that

$$\sum_i p_i = 1 \tag{2.20}$$

$f_r(x_i)$ is the rth constraint function, and $<f_r>$ are constants in the constraint equations.

The least presumptive probability distribution is a set of p_i which obeys the above $(m+1)$ equations, and maximizes the quantity given by:

$$S = -K\sum_i p_i ln p_i$$

This definition of probability through entropy paved the way to use Shannon's measures in virtually all fields of study where probability theory is used.

Since its introduction to information theory, other forms of information measures have been introduced by many (e.g., Kullback & Leibler, 1951; Tverberg, 1958) of which Shannon's entropy is a limiting case. Entropy has been viewed as system complexity (e.g., Ferdinand, 1974), and information without probability (Ingarden & Urbanik, 1962). Rényi (1960; 1967) introduced similar entropies for possibly incomplete probability distributions and has formulated the problem of characterizing all these new entropies. The problem was later solved by Aczél & Daróczy (see Aczél & Daróczy, 1975, for a complete mathematical treatment of entropy).

2.9.1 Entropy of a Continuous Distribution The expressions for entropy given above are all for discrete probability distributions for which entropy is defined as the negative expectation of the logarithm of the probability. In a similar manner, the definition is extended to a continuous distribution as the negative expectation of the logarithm of the density. For a distribution with a density function of $f(x)$, the entropy S is defined as:

$$S = -\int_{-\infty}^{\infty} f(x) ln f(x) dx \tag{2.21}$$

It is customary to use natural logarithms for a continuous distribution.

For a normal distribution with mean μ and variance σ^2, the entropy can be expressed as (see Theil & Fiebig, 1984):

$$S = \frac{1}{2} \log 2\pi e \sigma^2 \tag{2.23}$$

which shows that the entropy for a normal distribution depends on the variance and *not* on the mean. This reflects that the exclusive source of uncertainty of a random outcome from a normal distribution is the variance. For solutions of other distributions see Lazo & Rathier (1978).

Selection of a continuous distribution, which is as uninformative as possible, is sought by statisticians in a number of situations. For example, the challenge of formulating a prior distribution of a parameter that varies continuously is faced by Bayesian statisticians. The distribution with maximum entropy subject to the constraint(s) formed from a priori knowledge about the parameter may be selected for this purpose. This is allowed provided *all* the *known* information about the parameter are included, but *none* of the *unknown* information about the parameter is included in the constraint(s).

2.9.2 Entropy as a Measure of System Flexibility Kumar (1987) used the entropy of a system to measure its flexibility, that is: the ability of the system to overcome without any significant change in its capacity the failure of one or more of its component. He provided an objective theory of flexibility in manufacturing systems.

The concept of system flexibility has a direct application for a water transporting system where imposed constraint cause a redistribution of the transport in each component. Such is the case in a braided river where the existing share of water transport of one or more branch channels is changed and a redistribution of flow becomes necessary. The redistribution can take on many forms and is subject to the constraint that the total amount of transport remains the same.

The similarity of a goal-oriented system flexibility to the problem at hand is used in this study. A general axiomatic approach towards the development of a measure, H, is provided in Chapter Six.

2.9.3 Other Expressions of Entropy In addition to the communication and thermodynamic entropies, other mathematical expressions of entropy exist in the literature. These expressions were developed by different researchers working on different problems. Some of these expressions are given below.

Rényi (1961) provided an expression of entropy which can be written as:

$$S(x_1, x_2 ...., x_n) = \frac{1}{1-\alpha} ln\left(\sum_{i=1}^{n} x_i^{\alpha}\right), \qquad \alpha \neq 1 \tag{2.23}$$

This is a generalized expression of entropy for possibly incomplete distributions. This function satisfies all the properties of an entropy function. However, the optimizing values of x_i's do not automatically satisfy non-negativity constraint. Also, note that the proportions, x_i's, do not have to add up to unity as in the case of the entropy due to Shannon. This property is an added advantage of this expression. Note that this expression of entropy for possibly incomplete distribution approaches Shannon's function as $\alpha \rightarrow 1$. Bessat & Raviv (1978) discussed the concavity of this function and proved that for $0 < \alpha \leq 1$, S is strictly concave with respect to x.

Another expression which is due to Kapur (1986) has two parameters and is written as:

$$S\frac{(x_1, x_2, ..., x_n)}{(x_1, x_2, ..., x_n)} = \frac{1}{1-\alpha} ln\left[\frac{\sum_i x_i^{\alpha+\beta-1}}{\sum_i x_i^{\beta}}\right], \qquad \alpha \neq \beta \tag{2.24}$$

This expression reduces to Rényi's expression (Eqn. 2.23) for $\beta = 1$, and to Shannon's expression (Eqn. 2.18) for $\beta = 1$ *and* $\alpha \rightarrow 1$. The property of monotonicity and concavity of this function with respect to α *and* β are not known (Kumar, 1987).

Belis & Guiasu (1968) proposed a function of entropy and called it 'useful entropy'. This is written as:

$$S = \sum_i u_i x_i ln x_i \tag{2.25}$$

This is a modification of Shannon's entropy. The 'usefulness' or the 'effectiveness' of the received information is reflected by the weights u_i. The sum of the parameters is not required to be unity. The function obeys the monotonicity and concavity property of an entropy function.

There are other functions that are suitable as expressions of the measure of entropy of a system which can be used for the specific nature of the problem. In this study, however, the expression due to Shannon is adopted because of its simplicity and wide use.

2.10 Earlier Studies of the Brahmaputra

Most studies on the bankline movement of the Brahmaputra river that have been undertaken in the past concentrated on the social impact of the bank erosion. Some studies examined the magnitude of the movement and compared it with past records in an effort to find a general trend. As discussed in Chapter One, these studies led to different conclusions.

There have, however, not been many attempts to predict bankline movement in advance because the behaviour of the Brahmaputra river seemed too erratic and the researchers tended to look for clear trends in the movement. A few studies that did attempt to predict movement are briefly discussed in the following.

2.10.1 Statistical Model Khan (1988) used the Transfer Function Noise model suggested by Box & Jenkins (1976) in the space-time domain to predict the location of the banklines of the Brahmaputra river on a yearly basis. In that study, satellite imageries of 1973 and 1976-1983 were used. The location of banklines at 97 points were measured from a North-South reference line, and the yearly locations of 1976-1983 were related to the 1973 bankline position. It was assumed that the position of the bankline at any location in any given year is the output of a dynamic system in which bankline positions in the previous year are the input. All other variables were assumed to be lumped in the system.

The appropriate Transfer Function was found to be a first order Moving Average model with a noise term that may be modelled by a first order Autoregressive process. This model was able to predict the bankline position at the locations with a standard error of about 270 m. The model did not perform that well for the points where the changes were abrupt and large. Since the details of the cross-sections were not measured at most of the 97 locations considered in the study, it was not possible to investigate the reasons for the deviations. Nonetheless, this was a first attempt to predict the bankline movement quantitatively.

2.10.2 Conveyance Model Attempts have also been made to predict channel migration by means of conveyance models. In these models the location of centroid of the channel cross-sections was assumed to define the position of the stream. Its movement with time was assumed to show the direction of bank migration and its future trend.

Goulter & Dubois (1988) worked with the centroid of the cross-sectional area. Cross-sectional area however is not a correct representation of flow capacity which is determined by several variables. Burger et al. (1988) chose to work with the centroid of the channel conveyance. They divided the cross-section into vertical strips and computed the conveyance of each strip using Manning's equation and assuming the wetted perimeter of the strip to be the contact line with the solid boundary. As pointed out by Suharyanto (1992), the assumption that the flow in a strip is independent of the flow in adjacent strips induces an error. Suharyanto (1992) therefore used a finite element approach in the computation of the conveyance. He estimated the average velocity for the vertical strips, considering each strip to be an interconnected portion of the flow. RPT, NEDECO & BCL (1986; 1988) used the approach of Burger et al. (1988) and computed relative conveyance as the ratio of the conveyance of each strip to the conveyance of the entire river cross-section. In a similar attempt Coleman (1969) studied the thalweg (deepest point of the channel) movement to identify channel migration.

The studies in this category with the exception of the study of Coleman (1969) concluded that the Brahmaputra river in Bangladesh does not show any consistent migration pattern. The studies, however, were not able to account for the large movements that do occur and that do cause people to lose their land and livelihood.

Monitoring the movement of the centroid of conveyance of the cross-section would give an indication of the direction of movement of the river if the river had a single channel. The centroid of conveyance in a bend of a meandering river would show that the flow is concentrated close to the outer end of the bend. Monitoring the position of centroid over time would show the erosion of the outer bend where the erosion is most active. Erosion of the inner bend does not occur. The study of the thalweg would show a similar movement pattern.

Channel migration in a braided river is different. Here, erosion may occur on both banks simultaneously or even on the bank which is located furthest away from the thalweg or the centroid of conveyance. This situation was observed in several instances for the Brahmaputra river by BWDB (1978b) where erosion occurred in the bank furthest from the thalweg. It is clear that the movement of the centroid of conveyance does not necessarily show the bank movement for a braided river. For instance, in a section with major portion of the conveyance located near one bank, the centroid of conveyance would not show significant movement if erosion or deposition occurs on the shallower part of the cross-section. The conveyance models of these studies, therefore, arrived at the obvious conclusion that there is no significant movement of the banks of this river. The observed bank movements do not support this conclusion.

2.11 Summary

This chapter briefly described the classification of rivers into straight, meandering and braided patterns. Criteria that govern the occurrence of these patterns and threshold values for the controlling variables were discussed. The most widely accepted view of what initiates the braiding process seems to be the concept of a mid-channel bar formation due to the local incompetency of the channel that occurs due to the formation of opposing circulatory cells. Other mechanisms of braiding may, however, be observed under special circumstances. Analytical methods of predicting bank movement, even for a single channel river are not available. Researchers have therefore sought solutions through the construction of lumped mathematical models. For the Brahmaputra river, these models have had very little success.

Chapter Three

PHYSICAL SETTING OF THE BRAHMAPUTRA

3.1 Introduction

The Brahmaputra river in Bangladesh has not been affected by human activities to a great extent. Except for a partial flood protection embankment on the right bank, the river flows in an almost natural environment. While similar to other braided rivers in general terms, the Brahmaputra has some special features that must be pointed out for a better understanding of its behaviour relating to the bank movement. This chapter describes briefly the origin, recent developments, and the principal morphological and hydrological features of the Brahmaputra. The available morphological and hydrological data and their sources are listed and briefly discussed.

3.2 Course of the River

The name Brahmaputra means the son of *Brahma*, the ultimate creator of the universe. The river is in fact one of the greatest creations on earth because of its grandeur and vastness, and its distinctive albeit erratic behaviour. Thus, at least in a figurative sense, the river might be entitled to its name.

Originating in a great glacier mass in the *Kailas* range of the Himalayas south of lake Gunkyud in southwest Tibet (Fig. 3.1), the river under the name *Tsangpo* flows through Tibet, China, India and Bangladesh and drains into the Bay of Bengal. In Tibet it flows eastward for about 1125 km (700 miles) along the bottom of a longitudinal graben parallel to and about 150 km north of the Himalayas. The *Tsangpo* is braided at wider channel sections (about one kilometre at some places) and flows through a valley filled with gravel. At the extreme eastern end of the valley the river enters a deep narrow gorge at *Pe* which leads it around the *Namcha Barwa* Peak and in a southerly direction across the Eastern Himalayas.

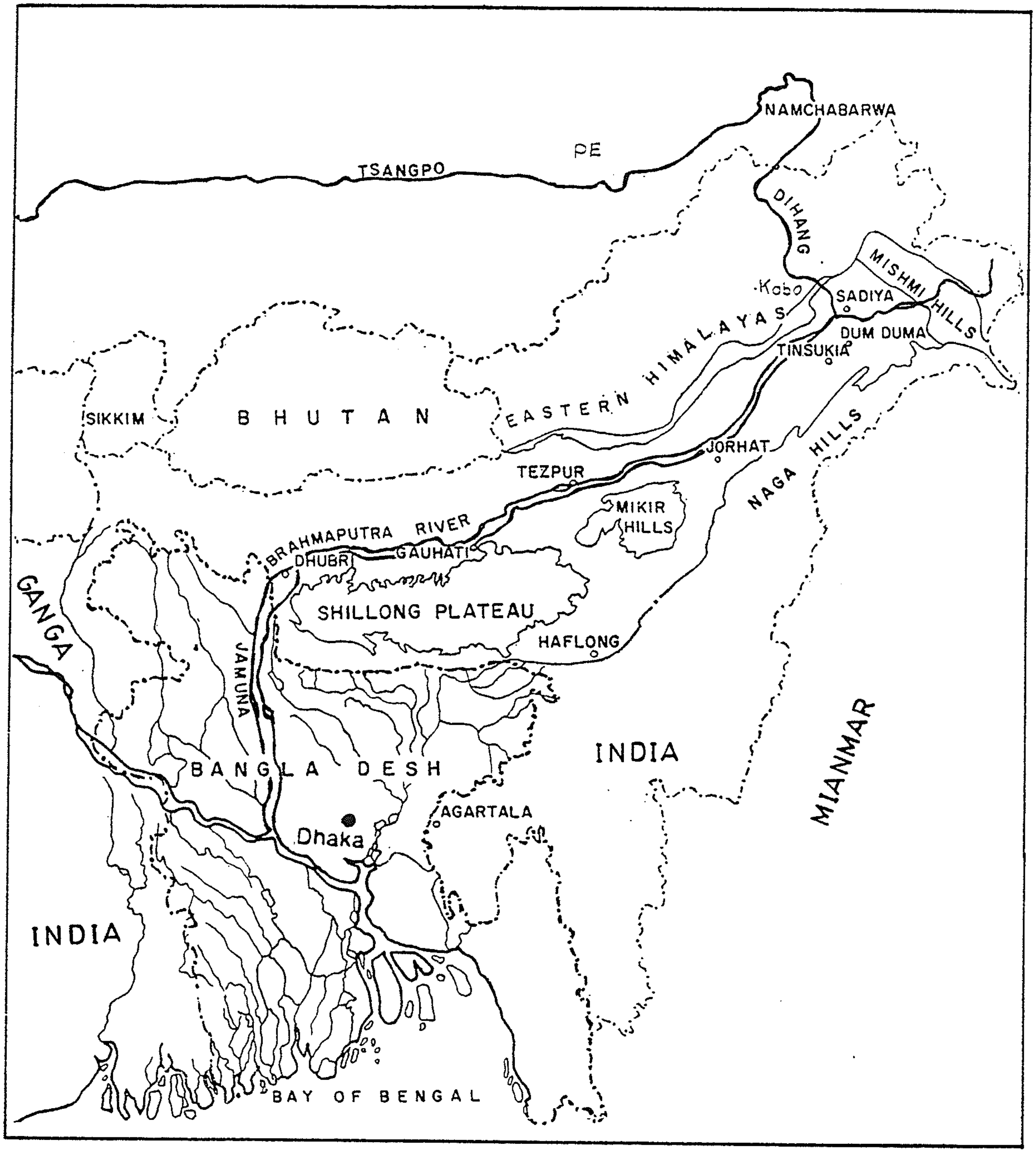

Fig. 3.1: Origin and Course of the Brahmaputra River

The *Tsangpo* enters India under the name *Dihang*. It flows for about 230 km through the gorge after which it enters the Assam plain near *Pashighat*. Near *Kobo* in Assam, two rivers, the *Dibang* and the *Lohit*, joins the *Dihang*. Downstream of the confluence, the river is called the *Brahmaputra*. It then curves west and flows for about 725 km (450 miles) through Assam. Near *Dhubri* the river takes a sharp southern turn before entering northern Bangladesh. In Assam, the river flows in a highly braided channel characterized by numerous lateral and midchannel bars and islands.

After entering northern Bangladesh the Brahmaputra river receives the flow from the *Dudkumar*, the *Dharla* and the *Teesta*. Downstream of the diversion with the *Old Brahmaputra* at *Bahadurabad*, the river is locally known as the *Jamuna*. Some government agencies refer to this part of the river as the *Brahmaputra-Jamuna*. Continuing along what is now the new course, it distributes flow to the *Dhaleswari* and receives flow from *Karotoa-Atrai-Hurashagar* before joining the *Ganga (Ganges)* near *Goalundo*. From its source to the *Ganga-Brahmaputra* confluence, the length of the river is 2900 km (1800 miles) of which 275 km (170 miles) is in Bangladesh. Downstream of the confluence with the *Ganges* the combined river takes the name *Padma* which later on meets the *Meghna* and discharges into the Bay of Bengal as the *Lower Meghna*.

In 1787 the river experienced a major change in course with devastating effects on the environment. Prior to 1787 the river followed the course now known as *"Old Brahmaputra"* in a south easterly direction. It then joined the *Meghna* near *Bhairab Bazar* (see Fig. 3.1). There were two small distributaries of the Brahmaputra in the direction and area of its present course, the *Jenai* and the *Konai*. After 1787 these two channels started receiving more and more discharge and by 1830 they had formed the present course of the Brahmaputra river which carries nearly all the flow. The old course became a mere spill channel. In the early years following this major change in course the new river was of the meandering type, perhaps due to the early meandering course of the *Jenai* and the *Konai* (Thomas, 1970). In time, however, the river became highly braided with multiple channels between widely spaced high banks separated by large sand shoals or islands locally known as chars. The width of the river varies between 1.5 km and 17 km (Khan, 1988).

The cause of this major change in river course is a matter of controversy. Some authors viewed this change to be geologic and some to be hydrologic in nature. Ferguson (1863) suggested that an uplift in the *Madhupur Tract*, a Pleistocene alluvial terrace in the Bengal Basin, may have caused the diversion. This view was disputed by La Touche (1919) according to whom the beheading of the *Tsangpo* river in Tibet by the *Dihang* tributary increased the flow in the Brahmaputra thereby initiating the change (Rashid, 1978). Hirst (1916) viewed the zone of subsidence between the *Madhupur Tract* and the *Barind Tract* (another Pleistocene alluvial terrace of the Bengal basin) as the cause of the diversion. Hayden & Pascoe (1919) opposed Hirst's (1916) argument and supported La Touche's (1919) theory. Later on, Morgan & McIntire (1959) supported Hirst's view of the existence of a zone of subsidence between the two Pleistocene blocks. According to them the change in the Brahmaputra river was in response to the steeper gradient in the direction of its present course and was initiated by a single flood event.

It is likely that more than one factor played a role in the shift in course. It is interesting in this connection to note that the water levels for a given discharge have been rising with time (Khan, 1988) suggesting a rise in bed elevation. If this trend persists, then a flood may cause the river to abandon its present course creating a new channel which may, in future, become the new course of the river. Especially, the *Bangali* river, which is very close to the *Brahmaputra Right Flood Embankment*, is a possible diversion channel. It has been observed that the distance between this river and the Brahmaputra is decreasing over the years (BWDB, 1978b). In a separate study Goswami (1983) found a similar trend in the Assam area upstream of Bangladesh. These observations suggest the need for a closer study of possible avulsions. In this study, however, changes of this nature will not be discussed.

3.3 Drainage Basin and Climate

The Brahmaputra river has a drainage basin of about 580,000 sq. km (224,00 sq. miles) in areal extent. About 50% of the basin is in Tibet, 42% is in India and about 8% (46,00 sq. km) in Bangladesh. The upper catchment of the river and its north bank tributaries lie in the east-west trending ranges of the eastern Himalayas that rise steeply to the north above the Brahmaputra plain. The Himalayan watershed of the Brahmaputra river can be divided into four sets of unique geologic and structural units. These are (a) Trans-Himalayas (average elevation 6500 m), (b) Greater Himalayas (average elevation 6000 m), (c) Middle Himalayas (average elevation 4000 m), and (d) Sub-Himalayas (average elevation 1000 m). The Trans-Himalayas of Tibet is in the north and consist of highly fossiliferous sedimentary formation of Palaeozoic to Eocene age (Wadia, 1968). The Greater Himalayas consist primarily of granites and gneiss with some sedimentary sequences in between; and are composed of a series of structural overthrust, nappes and recumbent folds. The Middle Himalayas consist of Palaeozoic deposits of shales, slates and phyllite. These are succeeded by a belt of highly metamorphosed schists, quartzite and dolomites to the north. The Sub-Himalayas consist of mainly Tertiary sandstones and have many raised and relatively young terraces which are composed of pebbles, cobbles and boulders of gneiss, quartzite, granites and sandstones.

The Brahmaputra valley at the extreme east is bordered by the Tertiary hills of the *Patkai-Naga* ranges consisting of various metamorphic rocks in the foothill regions overlain by higher metamorphic rocks. In the easternmost valley of the Brahmaputra river, a considerable portion of these rocks lies under the Recent alluvium deposits. Some of them are exposed to the northeast of the valley. The *Meghalaya* plateau and the *Mikir Hills* are highlands flanking the southern part of the valley and are primarily made up of gneiss and schists and some younger intrusive rocks.

The southernmost part of the valley is the Bengal basin that is bounded by the Lower Jurassic trap rocks in hills on the west, the Eocene sandstone and limestone *Shillong* hills on the northeast, the *Tripura* and *Chittagong* hills on the east and the Bay of Bengal on the south. The whole region consists predominantly of Recent alluvial and deltaic deposits except for the four major Pleistocene alluvial terraces. These are the *Barind* and *Madhupur* tracts and the two river flanking terraces in the east and the west. They have straight and distinct boundaries with the Recent sediments and are highly oxidized, reddish-brown in colour and more compacted and weathered (Coleman, 1969). The Recent sediments are loosely compacted, grey in colour, have a higher water content, and consist of sand and silt in the upper valley, and silt and clay in the lower valley.

Rao (1979) reported the investigations of the Geological Survey of India which noted a stepped sequence of three to four geomorphic surfaces in the Quaternary landscape of the Brahmaputra valley. The stratigraphic units are further identified into Piedmont Plain Facies composed of gravelly deposits, and Flood Plain Facies characterized by a fine sand-silt-clay complex. The floodplain exhibits alluvial features of natural levees, point bars, meander scrolls, ox-bow lakes, channel bars, and large number of depressions (locally known as *beels*) outside the natural levees.

The Brahmaputra valley and its highlands form an extremely unstable seismic region. The sharp changes in the river course have been studied in relation to the subsurface structure of the region and it was concluded that the tectonics of the area have controlled the main course of the river (Murthy & Sastri, 1981). The subduction faults in the north and the transform faults in the east are the causes of the active tectonic nature of the region. Chaudhury & Srivastava (1981) plotted the epicentres of the earthquakes of magnitude five or more on the Richter scale from historical and instrumental earthquake data up to 1971. Their map shows that the Brahmaputra river flows through a highly tectonic area. Devastating earthquakes have occurred in the vicinity of the river including Bangladesh.

The 1897 and 1950 earthquakes that occurred in Assam were both of 8.7 on the Richter scale and are the most severe ones in the recorded history. These caused landslides, rockfall on hillslopes, ground subsidence, and morphological and course changes in the tributary channels. The landslides caused by the 1950 earthquake temporarily blocked the courses of the *Subansiri* in Assam, and *Dihang* and *Dibang* in *Arunachal Pradesh* in India. The water ponded by the blockage was subsequently released by a sudden bursting of the obstructions causing devastating floods downstream (Poddar, 1952, reported in Goswami, 1983). This occurrence added an enormous amount of sediment to the river system, a portion of which, some believe, is still being transported by the river to the Bay of Bengal.

The thick pile of Recent sediment is supported by the Precambrian base which is criss-crossed with fractures. Also blocks exist which move vertically at different rates. This has been established by recent geophysical investigations. The Manas basin is controlled by tectonic and erosional scarps, the *Subansiri* basin is sinking and experiencing sedimentation. Some believe that the Brahmaputra basin is still sinking otherwise the sedimentation would have raised many of the tributaries and the lowlands on either side of the river to the flood level.

The Brahmaputra river carries snowmelt from the mountains of the Himalayas and the rainwater from the drainage basin. Most of the catchment area of the river lies in the monsoon region of southeast Asia. A major portion of the runoff of this river and its tributaries comes from the summer rainfall. A belt of depressions, known as the monsoon axis which extends from northwest India to the Bay of Bengal controls precipitation in the summer. Heavy rainfall in Assam caused by the position of the monsoon axis near the foothills of the Himalayas has a major effect on the discharge of the Brahmaputra river.

There is a substantial spatial and temporal variation of rainfall in the Brahmaputra catchment area. The variation is mainly due to the location of the area in relation to the monsoon axis and to the highlands. The average annual rainfall is about 230 cm with a variation of about 15% to 20%. The rainfall occurs mainly in summer from June to September. The pre-summer monsoon rainfall is about 20% - 25% of the total annual rainfall (Goswami, 1983). The pre-monsoon rainfall is primarily caused by depressions moving in from the west and by local convectional storms during March, April and May. The snowpack in the Himalayas melts from March to June and sometimes causes pre-monsoon floods.

The natural vegetation of the Brahmaputra basin varies with the variation of altitude from tropical vegetation in Bangladesh to alpine meadows in the higher ranges of Tibet. About 20% of the valley is forested. Large areas of subtropical forests are in the foothills and in the forest reserves in the region. The active floodplain, the river banks and the islands are grassed.

The Brahmaputra valley is primarily composed of alluvial soil formed on Recent river deposits. Soils in the upper part of the river consists of boulders, gravels, pebbles and coarse sand. The intermediate part consists of coarse and fine sand; and the lower part consists of mostly fine sand and clay. In Bangladesh, the river bed is composed of medium and fine sands. Similar material is found up to depths of more than 50 m below the river bed (Petrobangla, 1983).

The river carries a huge amount of sediment during the flood period. Different organizations have studied the variation of the size distribution of the transported sediment across the channel and at different locations along the channel. The values of D_{50} at *Bahadurabad* and at *Sirajganj* were observed to be slightly different. The values were found to be within 0.25 mm to 0.15 mm (FAO, 1966-67, reported in Petrobangla, 1983). The D_{65} did not vary during the period of observation between 1966 to 1969 across the channel. However, it dropped from 0.3 mm in February to 0.22 mm in October. From *Nagarbari* to *Sirajganj* D_{65} was found to be fairly constant at 0.18 mm. The bed material was found to become coarser during the flood season at all of the stations. BWDB (1978b) estimated Manning's roughness coefficient at different locations of the river. These are (from downstream to upstream): *Nagarbari* 0.022; *Sirajganj* 0.018; *Gabargaon* 0.021; and *Bahadurabad* 0.035.

The distance elevation plot of the Brahmaputra river in Bangladesh (Fig. 3.2) has been taken from Khan (1988). The bank elevation plot with distance for the entire river is taken from Goswami (1985) and plotted on the same figure. In Khan's plot, data of the low water months have been taken from 33 almost evenly spaced cross sections of the river between the India-Bangladesh border in the upstream to the *Ganga-Brahmaputra* confluence in the downstream within Bangladesh. It is to be noted that the water surface elevations in different channels of the river may be different at the same cross section even on the same date. The difference can be as high as 0.9 m. Also, the water surface elevation measured at the same channel only an hour later are sometimes different. The combined effect of superelevation due to the flow and the wind may have been the cause of such variation. Khan (1988) showed that the river can be divided into two reaches of different slope. The upper reach is much steeper. It has a slope of 0.000105 compared to the lower reach which has a slope of 0.000053. The average slope of the river in Bangladesh is 0.0000819 (0.432 ft/mile). During flood stages the water surface slope is 0.35 ft/mile (Coleman, 1969).

As noted by Burger & Smith (1985) the rivers in Bangladesh are almost completely unaffected by human activities except for some in the coastal region. This is also true for the Brahmaputra river. Existing embankment structures have practically no effect on the flow regime. Some concern has been

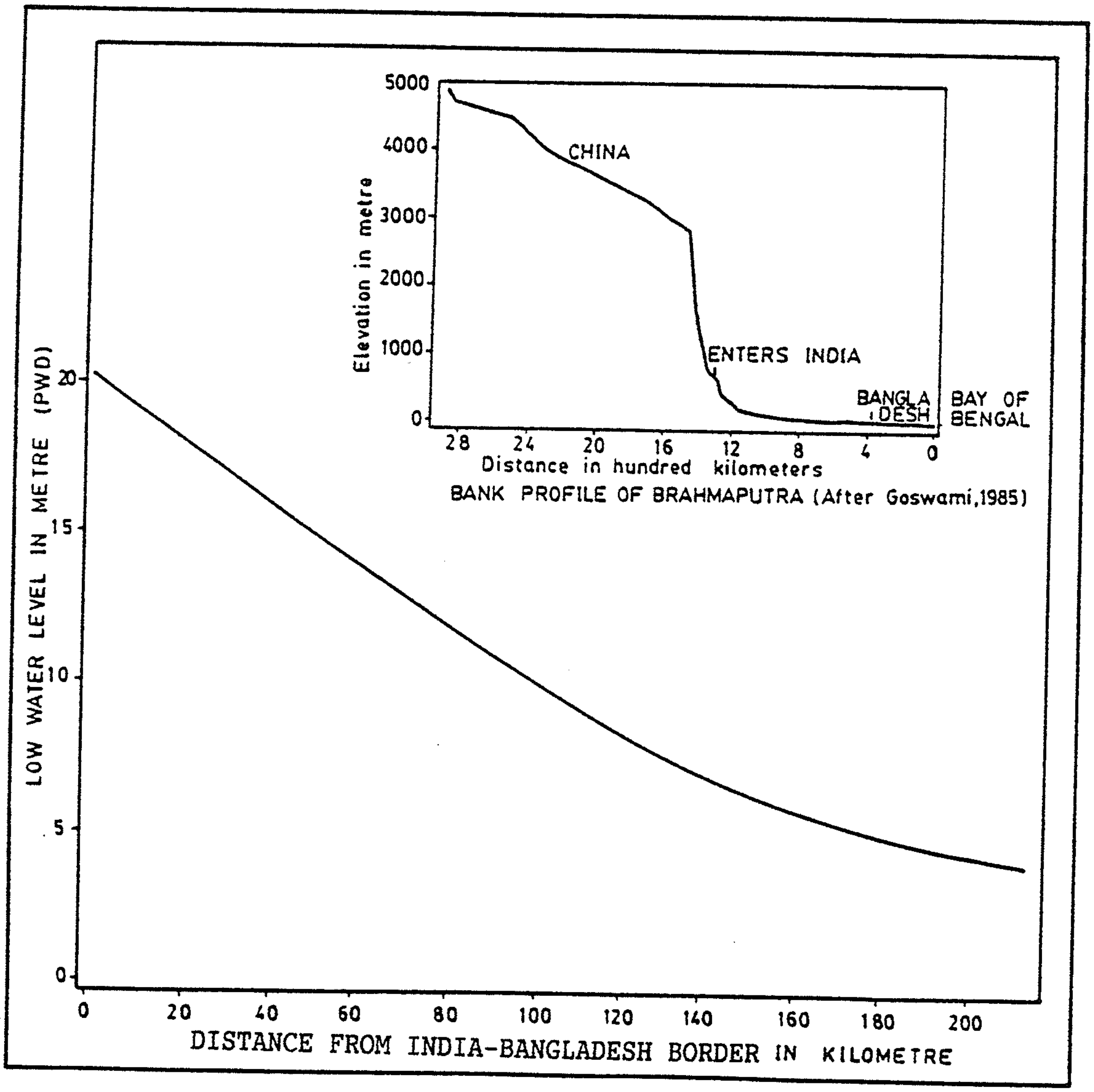

Fig. 3.2: Distance - Elevation Plot of the Brahmaputra River (after Khan, 1988)

expressed, however, about the effect of *Brahmaputra Right Flood Embankment* which is the main structure built along the west bank of the river. It was built in 1968 with eight T-headed groins. It was designed to protect the districts of *Rangpur, Bogra* and *Pabna* from flood waters. It was not intended for bank stabilization. The embankment, however, confines the flow on one side of the river eliminating the storage on the right bank and increasing the peak discharge downstream. Thomas (1970) expressed concern that the regime balance could be upset by such confinement. He argued that the confinement will prevent the river from changing back to the original meandering form which characterized the new course of the Brahmaputra river soon after the avulsion of 1830. Thomas' concern appears to be predicated on his view that the river will again start to meander which he believes to be the stable form of the river. There is, however, no clear evidence supporting this view. On the contrary, it appears that erosion threatens the embankment itself. Originally, it was built about 1500 metres west of the right bank. By 1978 a back up embankment had to be constructed further away from the river bank because the erosion threatened the embankment in ten locations (BWDB, 1977).

Another major structure across the river is the *East-West Interconnector* at *Nagarbari*. This is a powerline crossing designed to transmit hydropower generated in the eastern part of the country at a lower cost to the west part. This overhanging powerline is supported by towers located on 12 m diameter piers. Those piers can only have a minor effect on the flow regime (Hinch, McDowell, & Rowe, 1984).

In addition, the planning and design phase of the *Jamuna Bridge* near *Sirajganj* is almost finalized. If and when completed, this bridge will connect the east and the western part of Bangladesh. This bridge will also have only a minor effect on the flow regime downstream.

3.4 Hydrology

The seasonal variation of the monsoon and the freeze-thaw cycle of the Himalayan snow determines the hydrologic regime of the Brahmaputra river. It is one of the largest rivers of the world in terms of the water and sediment discharge. In the following the river stage, water and sediment discharge together with their temporal and spatial variation and observed relationships are briefly discussed.

3.4.1 Water Discharge As mentioned before, the water discharge of the Brahmaputra river comes from the snowmelt in the Himalayas and the monsoon rain in the catchment area. The river may carry huge amounts of water. In 1988, for example, the peak flow was estimated at 98,600 m^3/sec. The lowest recorded flow was 2,860 m^3/sec in 1970. The average annual discharge is about 20,000 m^3/sec at Bahadurabad. In terms of the average discharge at the mouth, the Brahmaputra river ranks fourth in the world. On the basis of the average discharge per unit of drainage area which amounts to 0.0345 $m^3/sec/km^2$, this river surpasses all other rivers in the world.

Usually, there is more than one major flow peak in the hydrograph (Fig. 3.3). The first peak is the result of snowmelt in the mountains between late May and early June. The second and any additional peaks are caused by the heavy monsoon rainfall in the catchment area. They occur in July, August, or sometimes in September. The maximum flow may occur as early as late June and as late as early October. The yearly maximum flow has varied over the period of record between 43,100 m^3/sec and 98,600 m^3/sec with an average of 65,600 m^3/sec. The minimum flow occurs between January and April, usually between February and March. The annual maximum, mean and minimum discharge are shown in Fig. 3.4. Note that no data for 1971 were recorded because of the liberation war. The annual range of discharge (the difference between the annual maximum discharge and the annual minimum discharge) varies substantially with time. The variation has been between 39,740 m^3/sec and 93,360 m^3/sec. The total annual runoff can vary between 425 billion m^3 and 735 billion m^3.

The contribution of the tributaries in Bangladesh, the *Dudkumar*, the *Dharla*, the *Teesta*, the *Korotoa-Atrai-Hurasagar* and other small rivers is small in relation to the total flow in the Brahmaputra. Any diversion of flow into the distributaries Old Brahmaputra, *Dhaleswari*, and other minor channels is not substantial either. Therefore, for all practical purposes the discharge at *Bahadurabad* may be considered as the discharge of the Brahmaputra river in Bangladesh.

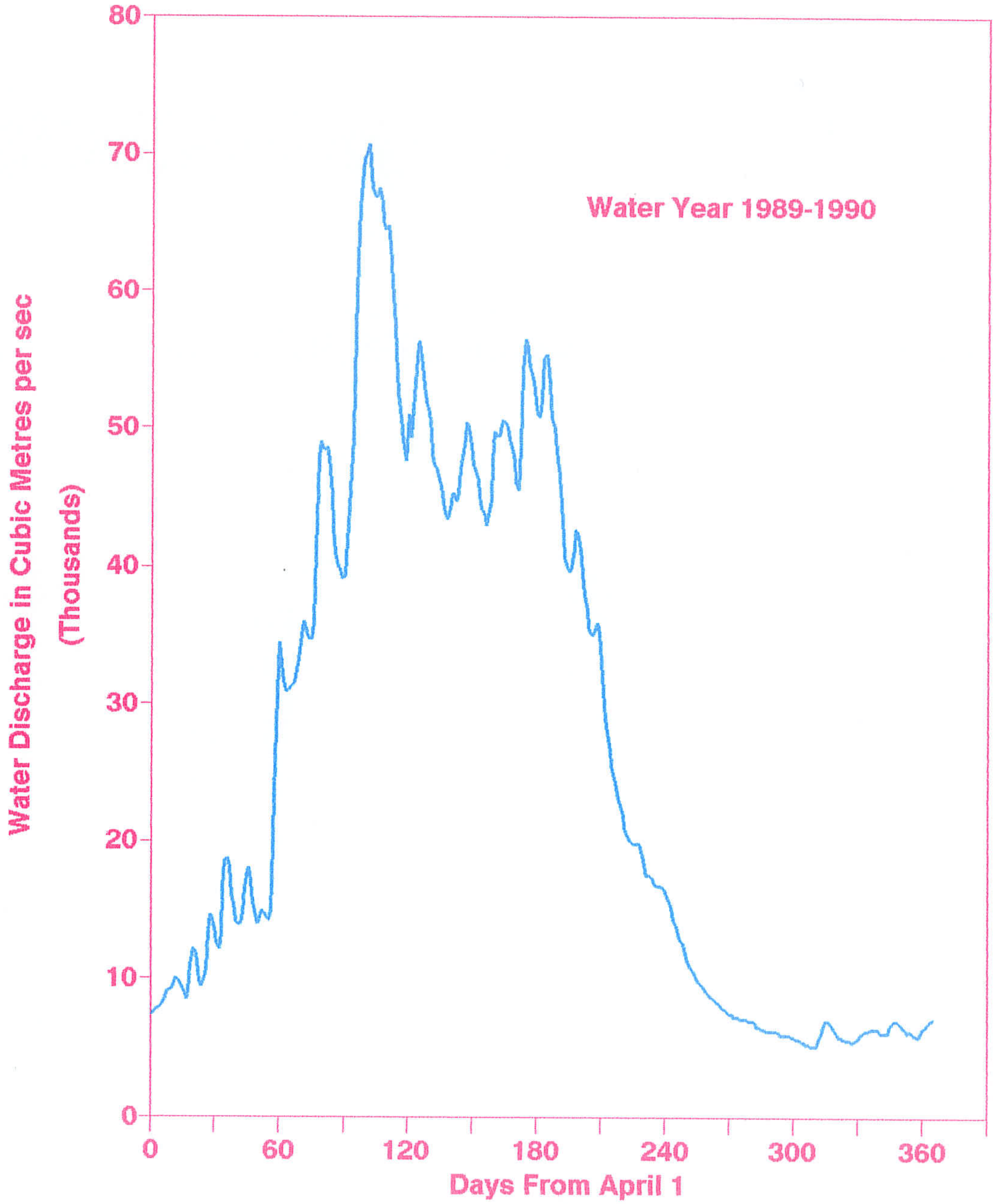

Fig. 3.3: A Typical Discharge Hydrograph of the Brahmaputra River

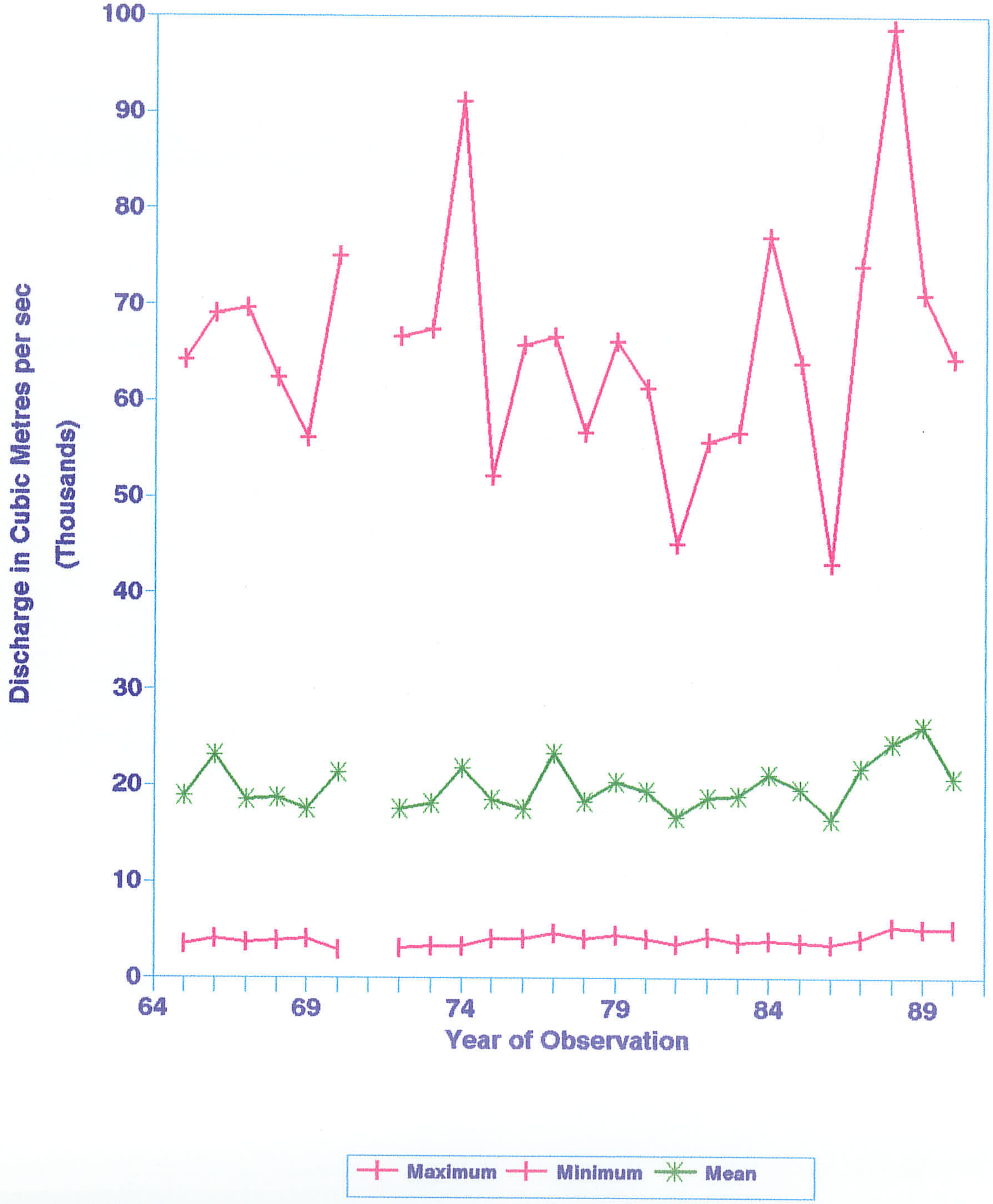

Fig. 3.4: Time Variation of Annual Maximum, Mean and Minimum Discharge

3.4.2 Water Level The maximum water level recorded at *Bahadurabad* is 20.62 m and the minimum water level is 12.88 m above PWD (Public Works Department) datum. Corresponding values at *Sirajganj* are 14.24 m and 6.40 m respectively. The danger level set by BWDB (Bangladesh Water Development Board) is 19.36 m at *Bahadurabad.*

The peak water level occurs between late June and mid October, more frequently, however, in July or August. The minimum water level occurs between January and April. It is interesting to note that the latter is noticeably out of phase with the minimum discharge. Annual maximum, mean and minimum water levels at *Bahadurabad* are shown in Fig. 3.5 for the period of record.

BWDB (1978b) noted that the lowest water level at *Bahadurabad* shows a rising trend. The minimum discharge does not show such a trend. It was therefore inferred that the river bed at that station is rising. The water surface slope for different reaches is given by BWDB (1978b). Reported slopes are: from *Nagarbari* to *Sirajganj* 0.29 ft/mile for high flow and 0.28 ft/mile for low flow; from *Sirajganj* to *Milanpur* 0.37 ft/mile for high flow and 0.34 fl/mile for low flow; from *Milanpur* to *Golna* 0.38 ft/mile for high flow and 0.35 ft/mile for low flow; and from *Golna* to *Porarchar* 0.49 ft/mile for high flow and 0.47 ft/mile for low flow. The water surface slope is higher during the high water season. The average slopes were found to be similar across the river. Also, as mentioned earlier, a transverse slope was observed at some cross sections.

3.4.3 Stage - Discharge Relationship Khan (1988) analyzed the stage and discharge relationship of the Brahmaputra river using seventeen years of water level and discharge data recorded at *Bahadurabad*. It was found that eleven out of seventeen rating curves are looped. Some rating curves have multiple loops while others have regular simple loop. An example of one such a looped curve is shown in Fig. 3.6.

A looped rating curve is caused by factors such as the acceleration of the flow in space and time, and changes in roughness and bed form at different water stages. This has been verified in State Hydrologic Institute of Leningrad experiments (Ivanova, 1967, reported in Cunge, Holly, & Verwey,

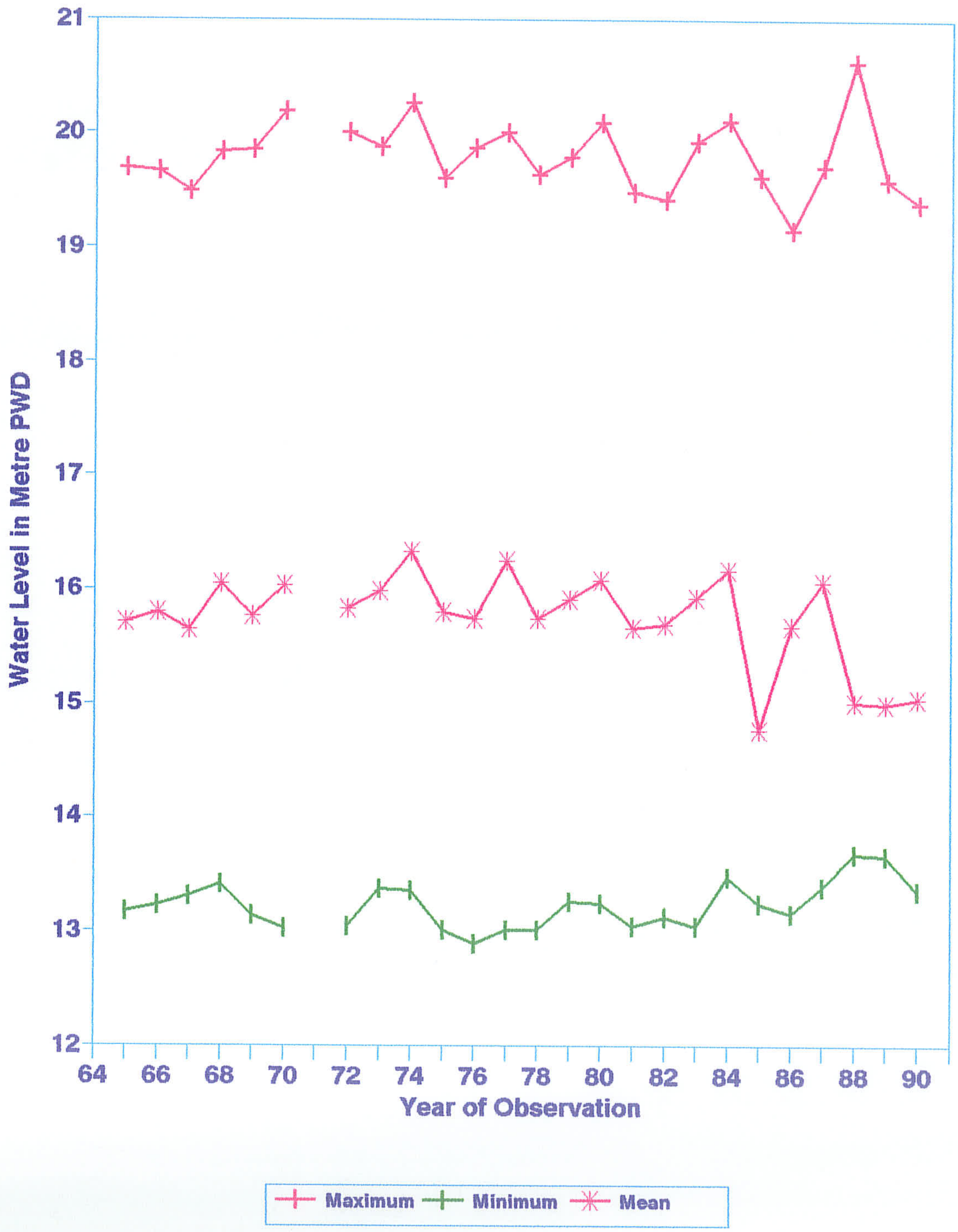

Fig. 3.5: Time Variation of Annual Maximum, Mean and Minimum Water Level

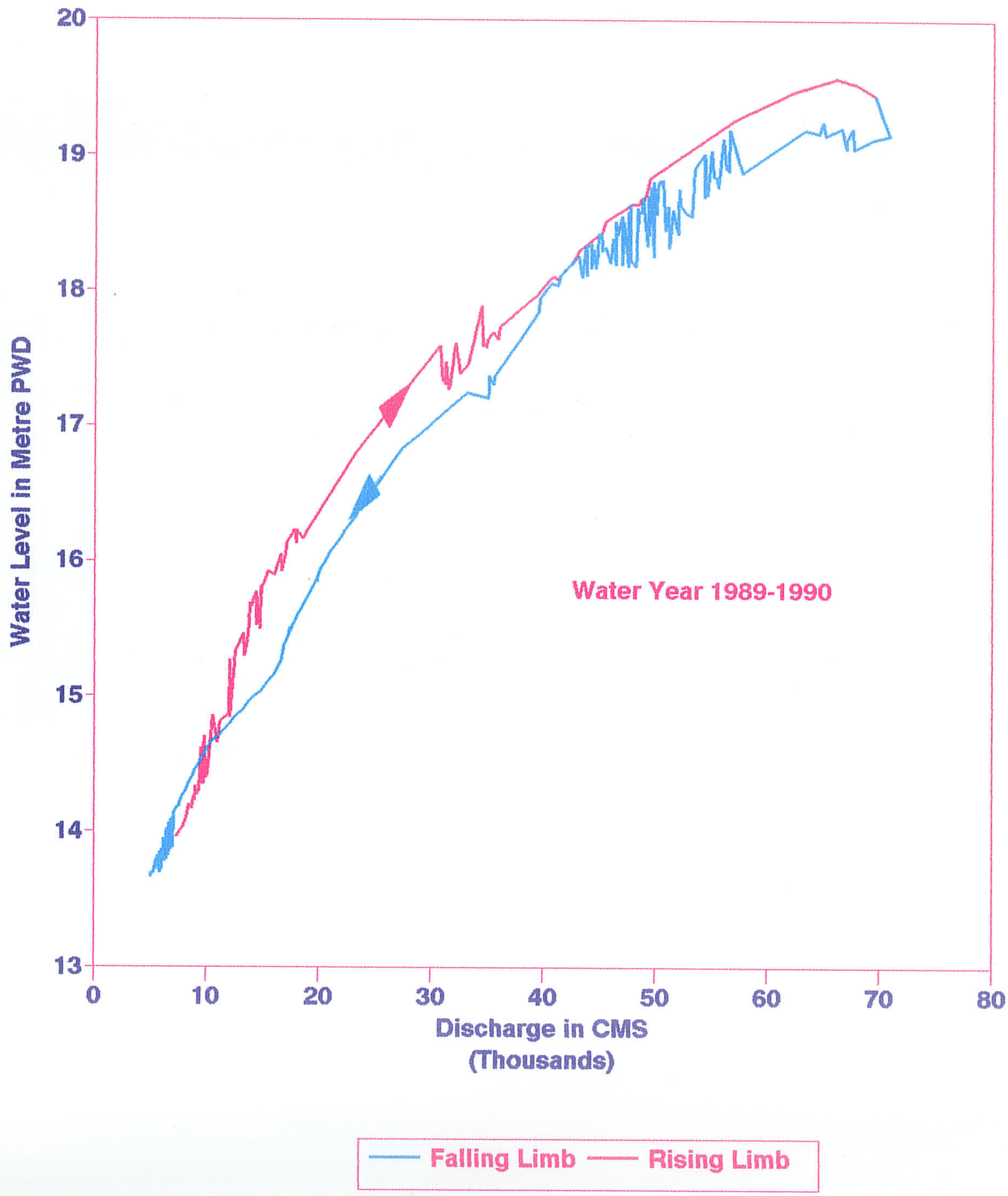

Fig. 3.6: A Loop Rating Curve of the Brahmaputra River

1980). The deviation of discharge from the steady state discharge, assuming relative difference to be small, can be written as (see Khan, 1988) :

$$(Q - Q_s) = -\frac{Q_s}{2S_b}\frac{\partial h}{\partial x} \approx \frac{Q_s}{2S_b c}\frac{\partial h}{\partial t} \tag{3.1}$$

in which

Q is the discharge,

Q_s is the steady state discharge,

S_b is the bottom slope,

c is the kinematic wave velocity,

h is the water depth, and

t is the time.

The right hand side of Eqn. (3.1) can be positive or negative suggesting that the rating curve is transformed into a loop with the deviation positive in the rising part and negative in the falling part. For slow natural floods the loop is more or less symmetric about the steady flow rating curve; deviations may be larger but are nearly equal in absolute value during rising and falling water level. This symmetry, however, may disappear for flood waves quickly varying with time (Cunge et al., 1980, p. 196).

The looped rating curve generally follows a counter clockwise direction from the rising to the falling stage. For a slow natural flood the loop is symmetric, but for flood waves that quickly vary with time, this symmetry may disappear (Cunge et. al., 1980). Also, a greater bed slope makes the deviation smaller. The increase in roughness which accompanies the passing of a flood decreases the flood peak discharge and opens up the loops more. The increase is caused by the formation of new islands from the remnants of the giant dunes formed during a major flood event (Coleman, 1969). Also, the creation of new and the closure of existing channels (Latif, 1969; Khan & Booy, 1989); and the division of a channel into two or more branches (Khan & Booy, 1990) cause the roughness coefficient to increase. The time dependency of the stage-discharge relationship is also due to the changes in bed level caused by scour or deposition during flood, and changes in energy grade line (Simons, Stevens, & Duke, 1973). Statistical relations between stage and discharge have been obtained both for the before peak and the after peak portions of the rating curve by Khan (1988) for all available records. It was found that the before peak and after peak portions of the analyzed rating curves are statistically different.

A plot of the rating curve for different years at *Bahadurabad* in Bangladesh (Khan, 1988), and at *Pandu* in Assam, India (Goswami, 1985) shows somewhat increased elevation for the same discharge implying a slight rise in the bed level of the river with time. Although this phenomenon has not been established with more studies, it should be observed carefully. BWDB (1978b) has also noted the similar phenomenon for the low water levels at *Bahadurabad*.

3.4.4 Sediment Discharge The Brahmaputra river is one of the world's rivers most heavily laden with sediment. It is second only to the Yellow river in China in terms of the sediment transported per unit drainage area (Holeman, 1968; Milliman & Meade, 1983). Goswami (1985) placed the Brahmaputra in fourth place on the basis of total load measurements at Pandu, Assam. He found that more than 95% of the total sediment is transported by the river during the rainy season (May through October) with the major portion of the load (95%) being carried by the moderate events of relatively frequent occurrence. This is in line with suggestions by Wolman & Miller (1960).

Sediment discharge observations in Bangladesh are not closely spaced with respect to time and location. Therefore, the available sediment discharge hydrographs (Fig. 3.7) are only approximate. The peak sediment discharge may well have been missed in these hydrographs simply because the measurements are weekly (sometimes fortnightly) and since the concentration of suspended sediments increases very rapidly with the increase of water discharge.

The sediment transported by the lower Brahmaputra (Assam and Bangladesh) is mainly medium to fine sand and silt. The banks are also primarily composed of the same sand and silt. They rarely contain any clay (less than 5%). Thus the banks are non-cohesive. This makes them very susceptible to erosion. The sediments through which the river flows have originated from the same

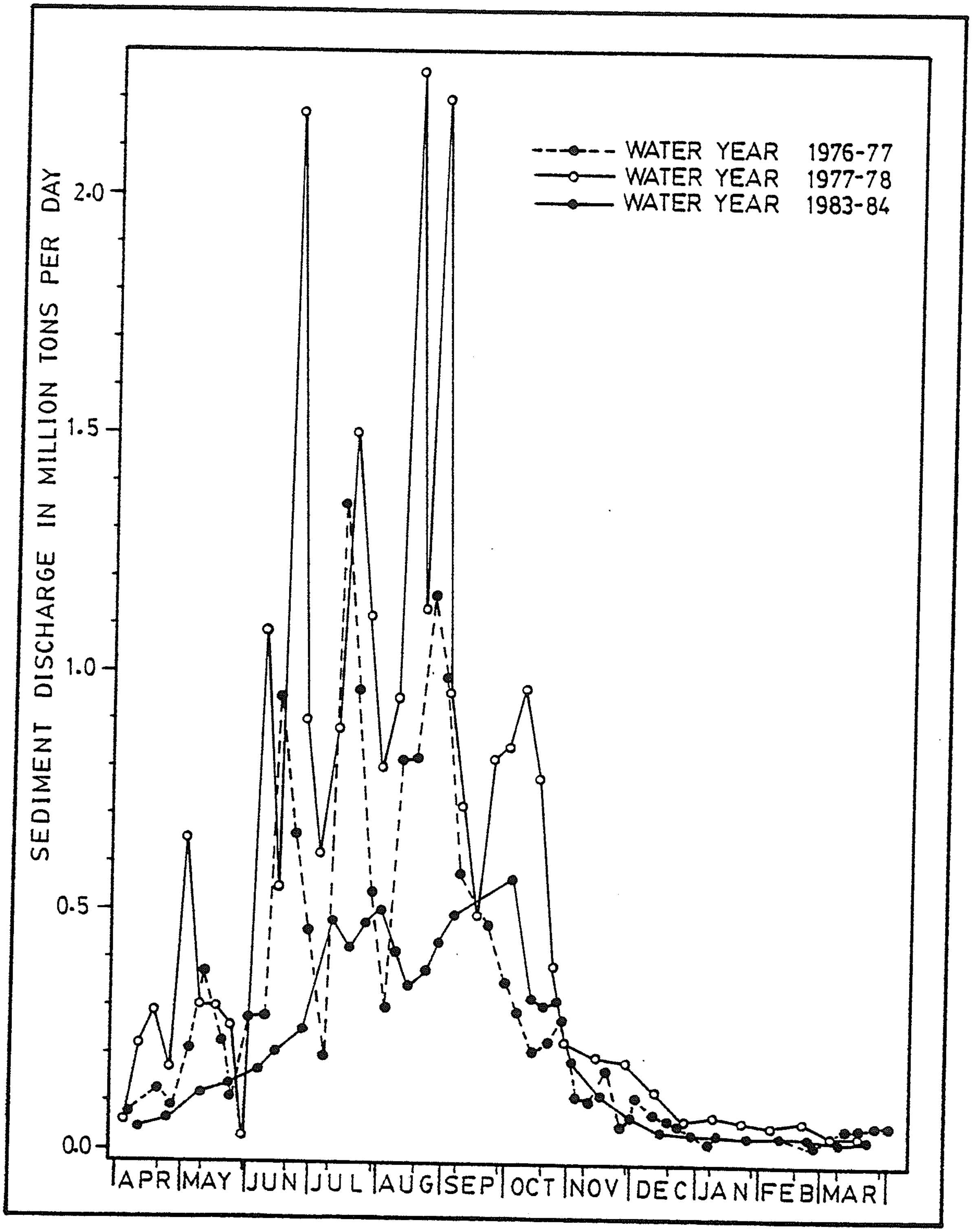

Fig. 3.7: Sediment Discharge Hydrograph (after Khan, 1988)

catchment area. As a result, bed and bank material is almost the same and there is little variation in material size in the lower Brahmaputra. For practical purposes, the bed load material can be taken as the same for the entire reach of the river in Bangladesh.

Nearly all sediment is carried by the Brahmaputra river as bedload and suspended load. There is very little wash load. The depth integrated samples that are collected for measuring sediment loads contain the suspended load and some of the bedload. The recorded daily sediment transport rate of the Brahmaputra river in Bangladesh varies between a high of 2.26 million tons/day to a low of 0.02 million tons/day. Coleman (1969), however, reported a transport rate of 5.0 million tons/day and a total annual transport of the order of 600 million tons from the measurements made in the mid-sixties. Based on the measurements in the upstream, at Pandu in Assam, Goswami (1983) reported an average transport rate of 2.12 million tons/day with a maximum of 26.5 million tons/day during the peak flow season. It is to be noted that the measurements of sediment concentration of the Brahmaputra river in Assam are more frequent in space and time than those in Bangladesh.

The sediment discharge increases rapidly with the water discharge as shown in Fig. 3.8 (after Khan, 1988). The logarithmic plot suggests that the regression line underestimates sediment discharge at higher flows. This observation in itself may not be statistically significant. However, a similar phenomenon has been reported by others (e.g., Richards, 1982). Khan (1988) suggested the relationship between the water and sediment discharge as:

$$Q_s = 0.38 Q_w^{1.36} \tag{3.2}$$

where,

Q_S is the sediment discharge in tons/day, and

Q_w is the water discharge in m^3/sec.

The increased agricultural activities in the catchment area of the Brahmaputra river and the deforestation upstream, especially in Nepal, have been a concern to both authorities and researchers in this area. The concern is that the increased human activities increases the sediment load in the river.

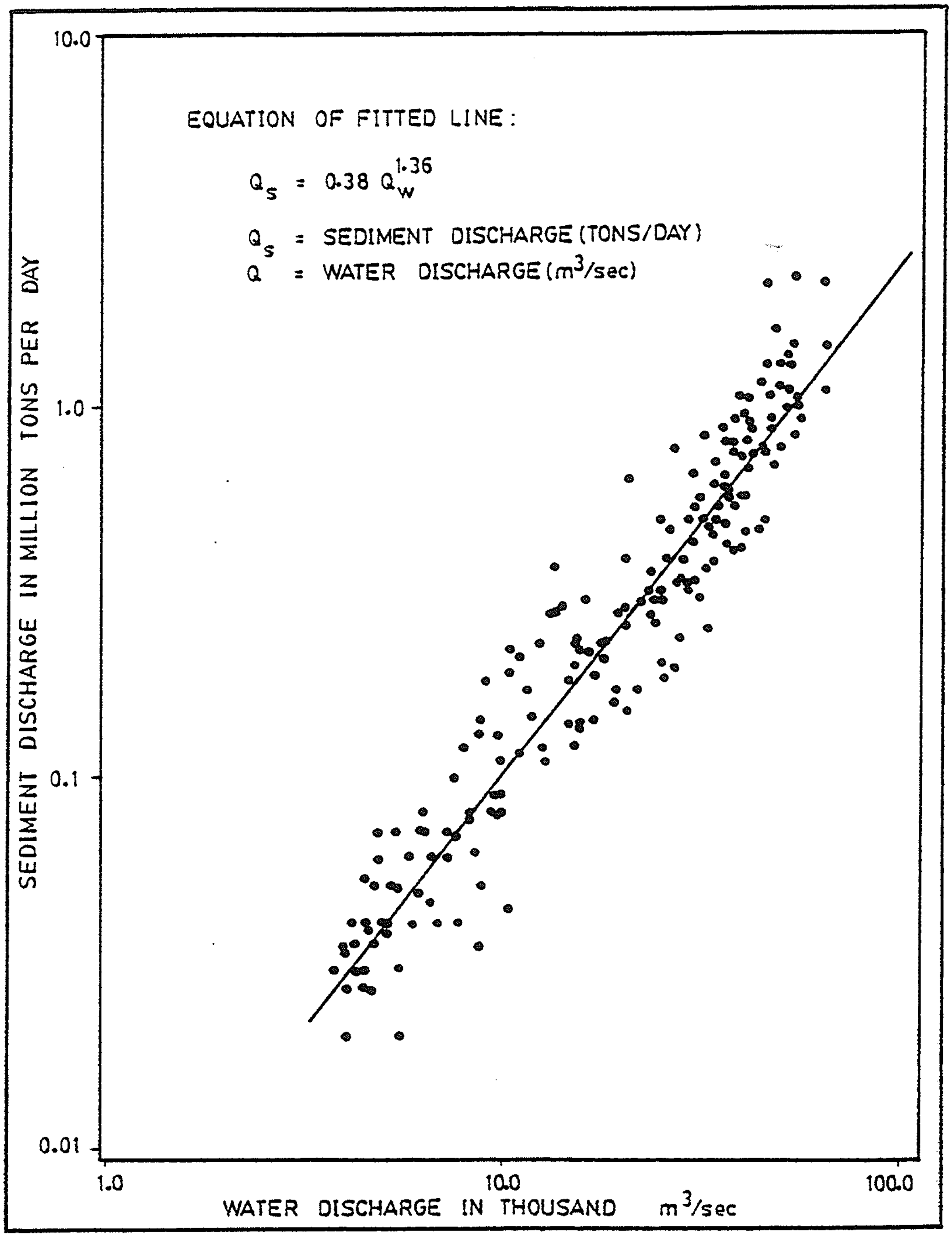

Fig. 3.8: Water and Sediment Discharge Relationship (after Khan, 1988)

Froehlich & Starkel (1993) reported that the earlier natural tendency of channel incision has changed to prevailing aggradation due to deforestation and extensive land use in the *Darjeeling Himalaya* area. These increased loads are ultimately carried to the Brahmaputra through the *Teesta* river. The belief prevails that these increased loads are reducing the water transport capacity of the river by choking the channels and thereby causing frequent and devastating floods. The 1988 flood, which was the worst in terms of damage and human sufferings in the recorded history, is often cited as an example of the phenomenon. Stene (1993), however, noted that the channel bed elevation of the Brahmaputra river in Bangladesh has decreased throughout 1976-83 period with an average degradation of 23 cm. He also noted alternating scour and fill from one year to the next. More detailed studies are needed to prove or disprove the assumption of increased sediment supply in this river due to increased human activities in upstream catchment areas.

3.5 Available Data

The temporal and spatial distribution of hydrologic and geomorphic data on the Brahmaputra river in Bangladesh is neither continuous nor extensive. This is characteristic of most developing countries of the world and is caused by economic limitations. A systematic and detailed data collection did not start in Bangladesh until the mid-sixties. Prior to that time only occasional land survey maps showing the banklines of the river are available. However, a continuous water level record at one station is available from 1949. Other data describing the location of banklines with time, hydrologic data of water and sediment discharge and water levels, cross section data, and geologic data describing the history and division of the area are available from the early seventies on with some data from the early sixties.

Between 1764 and 1773, *Major James Rennel* surveyed the *Old Brahmaputra* river for the first time. The earliest survey map of the new course was done by *Captain Wilcox* in 1830. Bangladesh Water Development Board (BWDB) land survey maps are available from 1956 (1956, 1963, 1966, 1974, 1976, 1977, 1982 and 1983). These are based on the measurements at 33 cross sections (almost evenly spaced over 206 km) from the India-Bangladesh border in the upstream to *Nagarbari* in the downstream near

Ganga-Brahmaputra confluence. Also, maps of 1867, 1875, 1935, 1944 and 1952 are available. IECO (1964) suggested that pre-1952 maps should not be used because of serious errors. Some of the maps were subsequently corrected using ground control and aerial photography.

The Multispectral Scanner (MSS) images from LANDSAT are available from 1972 in photographic and digital format. The reflectance values of bands 4, 5, 6 and 7 for each ground resolution of size 79 m X 79 m are contained in the Computer Compatible Tapes (CCT) in digital form and can provide important information about the land-water interface. The Thematic Mapper (TM) aboard Landsats 4 to 6 provide higher resolution of 30 m square from 1982 onwards (Richason, 1983).

Cross-sectional data measured by BWDB in the low flow period from 1976 (excluding 1982) are available at 33 sections along the river. These are available in map forms, and the recent ones in digital forms as the distances and water depths of points across the section. Stene (1993) noted that these low water cross-section data are a reflection of the dominant channel forming discharge of the previous year and can be used for bank movement studies. These data together with the discharge and water level records form the core of the data base used in this study.

The Bangladesh Water Development Board (BWDB) is the primary source of the hydrologic data. Their data collection stations on and around the Brahmaputra river are shown in Fig. 3.9. Mean daily water discharge and water level information are available at these stations. The longest record is available at *Bahadurabad* right below the diversion with the *Old Brahmaputra*. Daily mean discharge are available at that station since 1965. Water levels at other stations including *Chilmari* upstream of the junction with the *Teesta, Sirajganj*, and at *Nagarbari* upstream of the Ganga-Brahmaputra confluence are available since 1965. Water level and discharge records at other stations are available prior to 1965 but not continuously. Petrobangla (1983), however, reported that the pre-1965 data are incomplete and less reliable. Such data are not used in this study. Water level information on the tributaries and the distributaries are also available.

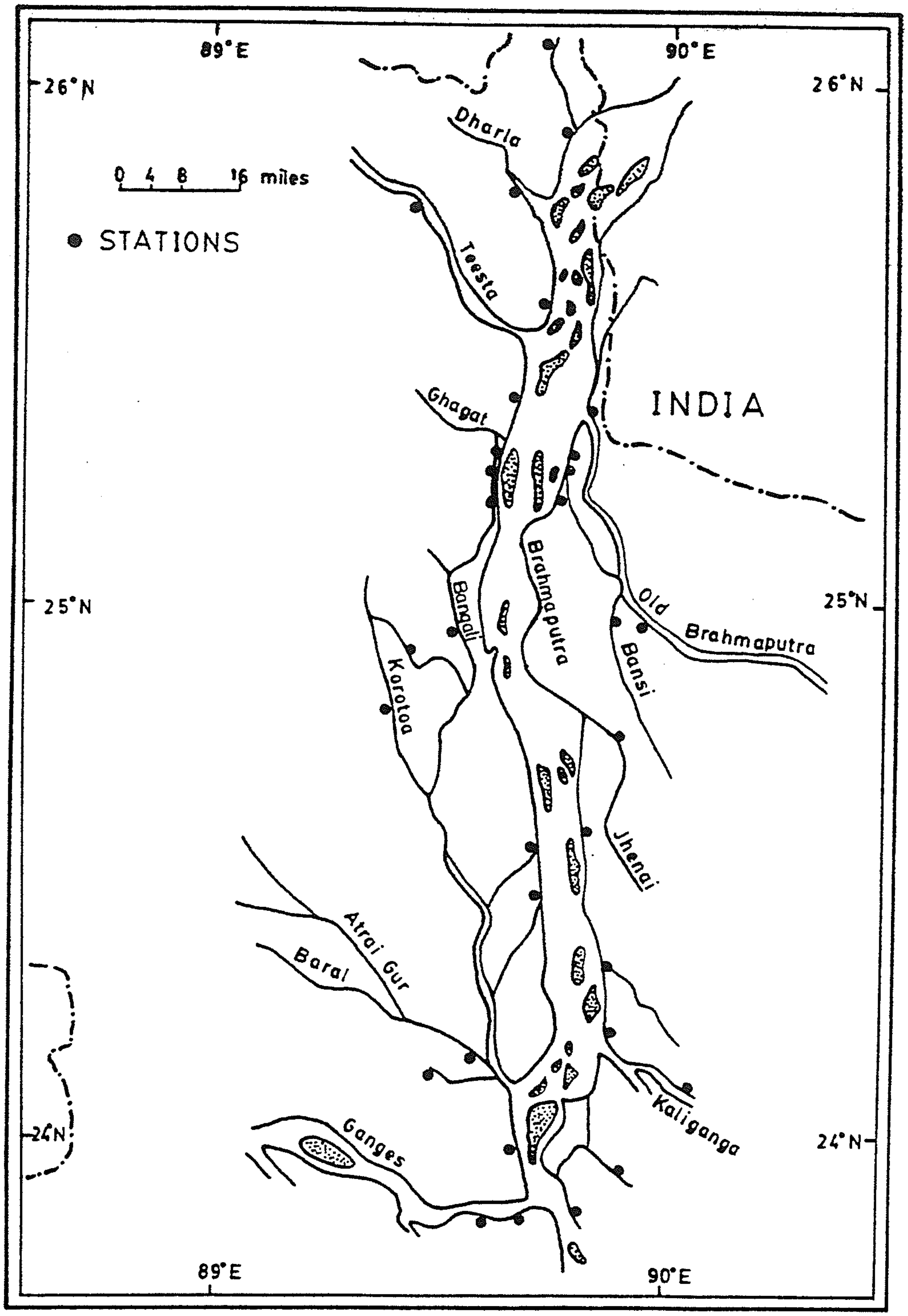

Fig. 3.9: Data Collecting Stations on the Brahmaputra River

The sediment discharge data that are available from the BWDB are not collected in the same systematic and continuous manner as those of the water level and discharge records. Only very recent records of suspended sediment loads are available at a few selected stations (e.g., *Nagarbari* and *Sirajganj*) during the flood events. These measurements are normally made on a weekly basis during the flood period and fortnightly during the lower flow period. Size distribution of these samples are also available. Bed load information is not available from any of the stations.

Geologic characteristics of the Bengal Basin and its subdivisions (Morgan & McIntire, 1959) are also available. Recently, the Geological Survey of Bangladesh modified the subdivisions and their characteristics. The data along with the tectonic activities in the basin are available from this organization.

Some data are available from other sources. The Bangladesh Inland Water Transport Authority (BIWTA) carries out soundings on a regular basis on specific reaches in order to monitor the water depth for the sake of managing the transport network. Some international development organizations working closely with the government of Bangladesh carry out measurements of their own. Secondary data from some research projects are also available.

The data that are available from different organizations are not free from errors. Before using any of the data in this study, they were checked manually for possible detectable errors. Errors of personal nature (e.g, printing mistakes) that were found were corrected as much as possible. For example, if recorded water level at a point on one day is 13.25 m, the following day 43.20 m, and the next day 13.17 m, then it was concluded that the second value of 43.20 m probably have been 13.20 m. Other forms of errors are not easily detectable.

3.6 Historical Notes on Bankline Movement

As mentioned before, the Brahmaputra river in Bangladesh experienced a major change in course that occurred in the early nineteenth century. Besides this major change, lateral movement of the banks throughout its length have been documented in many old books and reports (e.g., Martin, 1838; The Imperial Gazetteer of India, 1881; 1882). About the Brahmaputra, Martin (1838) wrote:

> Below the mouth of the Chhonnokosh, again, the right bank of the Brohmoputro has been gaining, and the channels on that side have been diminishing, so that many of the chars and islands have been united with the main, but I had no opportunity of being able to trace the alterations in particular manner. Near Chilmari, ... as I have mentioned before, the river threatens to carry away all the vicinity of Dewangunj, and perhaps, to force its way through the Konayi into the heart of Natore. (p. 304).

He also wrote:

> It is only when sudden changes take place that great evils arise, and none such has happened since the year of Bengal era 1194, or for 20 years before this year 1809. The change which then took place in the Tista, owing to a great storm, was accompanied with a deluge, by which one half of both people and cattle were swept from the whole of the country near the new course, which the river assumed. (p. 399)

Although these reports do not provide any information about the quantitative changes of the banklines, they provide an idea of stability of the river as it was almost two centuries ago. Some quantitative idea about these changes may be obtained from the plot of Coleman (1969) which shows the changes in widths of 1963 compared to those of 1830 (Fig. 3.10).

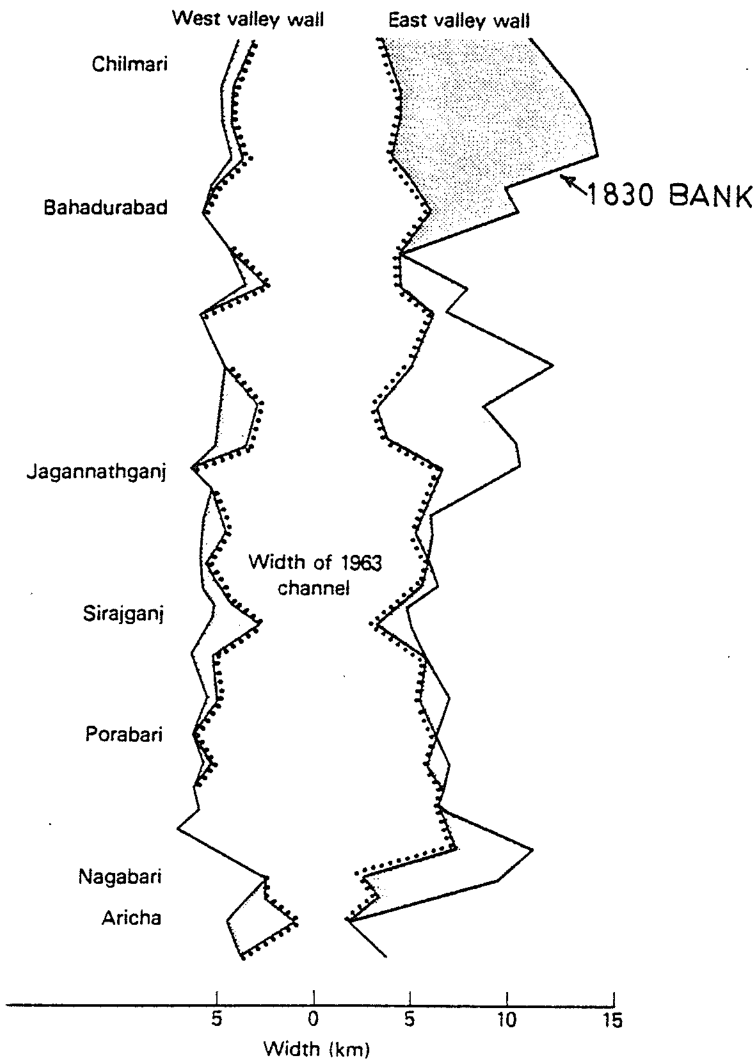

Fig. 3.10: Historical Width Changes of the Brahmaputra River (after Coleman, 1969)

Chapter Four

STABILITY OF THE RIVER AND ITS COMPONENTS

4.1 General

Bank movement is caused by two different aspects of morphological change in a river - changes in planform and changes in channel cross-section. The first aspect relates to the river as a whole, the second to the channels that form the braided river. Planform and cross-sections show not only continuing change but also a typical stability in the sense that certain features tend to remain relatively constant or tend to be re-established in time. Progressive or irreversible changes, however, do also occur.

An investigation of the dynamic stability of pertinent morphological features is of great importance in a study of bank movement. These features include the planform geometry, cross-section geometry, vertical and horizontal stability of the channels, the stability of the bifurcation points, and the bedforms. In this and the following chapter these features will be described qualitatively as well as quantitatively using descriptive parameters. This is followed by a discussion of observed changes in time of the parameters.

In this chapter the stability of the planform of the river will first be examined. The network configuration will be described both verbally and by means of topological parameters used in the analysis of network stability. Following this the vertical and horizontal stability of the river as a whole will be considered. Next, the stability of the individual channels will be discussed, in particular with regard to the depth and width of the channels. This is followed by an analysis of observed cross-sectional changes in the channels and a discussion of corresponding bankline movements. Following this, the relative stability of the bifurcation points of channels will be discussed. Finally, a brief description will be given of observed changes in bedform.

4.2 Planform Stability

The planform of an alluvial river is always in a state of flux because of erosion and sedimentation. The typical configuration, however, may be relatively stable. Significant features, such as width, average depth, sinuosity, etc. usually remain relatively constant. For example, the channel of a meandering river is in a continuing state of change. Yet, the pattern can be regarded as stable and bank movement is limited and, within limits, predictable. Large changes, however, may also occur. A river that causes delta formation, for example, may become unstable when it reaches a stage where a sudden shift in course is imminent. This shift may cause changes in some local channel features but may leave other features unaffected.

Planform stability is therefore a relative concept. It pertains to certain, but not all, morphological features. It is also relative to time. Some changes may be reversible so that earlier features are re-established, other changes may be progressive. Such progressive changes may be gradual or catastrophic. It is evident therefore that the *degree of stability* of pertinent features and the time relationship of any change is of fundamental importance in a study of bank movement.

4.2.1 General Network Shape There may be as many as 15 channels in a single cross-section of the Brahmaputra river at mean water level. These channels form a network that changes and reconfigures itself presumably in response to changes in the flow of water and sediment. The network configuration is different in different reaches of the river.

Of particular interest is the existence of nodal points along the river. There the river flows in a single channel that has experienced little or no movement at all in recorded time (Coleman, 1969; Khan, 1988; Khan & Booy, 1989). The nodal points are shown in Fig. 4.1. No satisfactory explanation of the occurrence of such nodal points has been provided to date. Coleman (1969) thought that their occurrence was related to differences in erodibility of the deposits through which the river flows. He suggested that the river bed is composed of sediments that were deposited by numerous rivers that

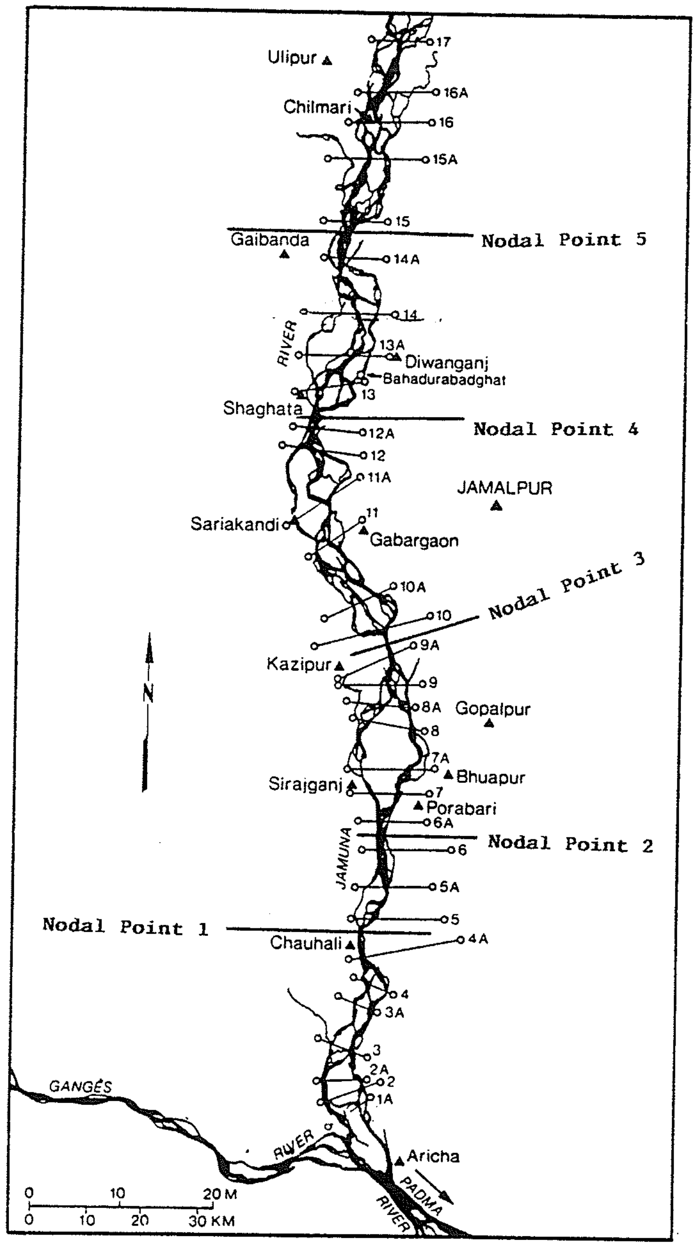

Fig. 4.1: Cross-section Monitoring Stations on the Brahmaputra River

previously existed in this region. He speculated that nodal points are located on previous depositions of less erodible material containing more clay. Such differences in bed material, resulting from earlier depositions, would then also explain why the braiding pattern is different in the different reaches between the nodal points. There is, however, no evidence that the bed and bank materials are indeed significantly different in terms of erodibility along the river throughout Bangladesh. Coleman's conjecture thus lacks confirmation.

Khan (1988) compared the distances between the nodal points with the theoretical wavelength of braids or meanders that according to Hayashi & Ozaki (1980) result from the bed instability. These investigators considered meandering and braiding of the river channels to be basically the same process. They argued that the instability of the planform of the river channel originates from bed instability. From these considerations, Hayashi & Ozaki calculated the wavelength associated with the instability process. The relationships pertaining to a braided river are given below:

$$\frac{L}{W} = \frac{6}{\sqrt{2\pi}} \frac{1}{m^{3/2}} \left(\frac{WS}{d_o}\right)^{\frac{1}{2}} \tag{4.1}$$

$$KC_1 = A_s S m^2 \pi^2 \left(\frac{WS}{d_o}\right)^{-2} \left[1 - \frac{4\sqrt{2\pi}}{3} m \left(\frac{WS}{d_o}\right)^{-1}\right] \tag{4.2}$$

$$A_s = \frac{\tau_o}{\sqrt{g d_o^3}} = \left[\frac{(s-1)D^3}{d_o^3}\right]^{\frac{1}{2}} F_f \phi_o \tag{4.3}$$

$$\phi_o = 40\,\tau_{*o}^3 \; ; \; \tau_{*o} = \frac{\tau_*}{[\rho(s-1)gD]} \tag{4.4}$$

$$F_f = \left[\frac{2}{3} + \frac{36\nu^2}{gD^3(s-1)}\right]^{\frac{1}{2}} - \left[\frac{36\nu^2}{gD^3(s-1)}\right]^{\frac{1}{2}} \tag{4.5}$$

where,

L is the dominant wave length;

W is the width of the whole river;

KC_I is the initial growth rate;

S is the mean channel slope;

d_o is the depth of undisturbed flow;

D is the characteristic sediment diameter;

s is the relative density of the sediment;

m is an integer describing the number of flow patterns associated with the two-dimensional sinusoidal bed waves;

ρ is the density of fluid;

g is the acceleration due to gravity;

ν is the kinematic viscosity of fluid;

τ_* is the dimensionless bed shear stress $= \tau/[\rho g(s-1)d]$; and

τ_o is the bed shear stress in undisturbed flow.

Note that the value of m is equivalent to the number of channels present in a cross-section. For example, $m = 1$ for a meandering channel. These relationships are valid for:

$$\left(\frac{WS}{m\pi d_o}\right)^2 >> 1 \tag{4.6}$$

Hayashi & Ozaki (1980) argued that the dominant wave length that prevails in nature corresponds to the condition for which the initial growth rate, KC_I, is positive and maximum. When this condition prevails Eqn. (4.2) reduces to:

$$m = \left(\frac{WS}{d_o \pi 2\sqrt{2}}\right) \tag{4.7}$$

This allows the solution of Eqn. (4.1) for the wave length L.

The wavelength predicted by Eqn. (4.1) corresponding to the average flow condition of the Brahmaputra river varies between 25 km to 35 km. It may be observed that the average distance between two nodal points along the Brahmaputra is in the order of 25 km except between one pair of nodes which are about 45 km apart.

The 45 km long reach shows more changes from year to year than any of the others. One might thus tentatively conclude that the wavelength in this reach is unstable and that a node could form there at some time in the future. In fact, a semi-nodal point did exist within this reach in 1973 as evidenced from the satellite imagery. The spatial distribution of the nodal points along the Brahmaputra river thus seems to follow roughly the wavelength predicted by Hayashi & Ozaki (1980).

It should be noted here that Hayashi & Ozaki (1980) only predict the wave *length* which simply means that, if a node exists, other nodes are expected at the distances of the wavelength. Why such a nodal point would occur at a particular location and why it would persist in an environment of continual change is not addressed by this analytical relationship. No study was found that has produced a prediction regarding the permanency of these nodal points.

4.2.2 Topological Parameters The characteristics of the network can be described quantitatively by several topological parameters such as braiding index (Brice, 1964) and parameters of degree of connection (Howard et al., 1970). The changes (or lack of change) in the values of these parameters over space and time are indicative of the degree of stability of the network.

The topographical maps prepared by various organizations are not detailed enough to be used in the study of planform characteristics. The only suitable data that are available for this purpose are the satellite imageries which are available for the period of 1973-1985. These data have been used in the study.

The braiding index and other topological parameters computed in this study are discussed in the following sections.

4.2.3 Braiding Index The braiding index of a river reach is defined as twice the sum of the lengths of bars and islands in the reach divided by the length of the reach measured midway between the banks

(Brice, 1964). This index is a measure of the degree of anastomosity of a river and can be used to quantify change in a network of channels in a reach with time. A river with a braiding index of 1.5 or less is considered non-braiding.

To compute the braiding index of the Brahmaputra river in Bangladesh, the river was divided into the six reaches that lie between two consecutive nodal points. These reaches are numbered as follows: Reach Number 1 is between section 1 and 4A (see Fig. 4.1); Reach Number 2 is between section 4A and 6A; Reach Number 3 is between section 6A and 9A; Reach Number 4 is between section 9A and 12A; Reach Number 5 is between section 12A and 15; and Reach Number 6 is between section 15 and 17. Note that the uppermost reach (Reach 6) is not included in this analysis because clear satellite imageries of this part of the river were unavailable for some of the years. For the sake of comparison, the entire river in Bangladesh up to the Ganga-Brahmaputra confluence is also considered to be a reach namely reach 0. Channel lengths and bar lengths were measured from the satellite images for each reach and for each year of record.

The computed braiding indices for different reaches are plotted against time and are shown in Fig. 4.2. A number of observations can be made from the figure.

First, curve C0 shows that the braiding index for the entire river reach within Bangladesh shows a small but consistent increase with time during the period of observation. This, however, is not the case for all reaches. Some reaches show a decreasing braiding index over time (Curves, C3, C2, and part of C1), while others show an increasing index (curves C4, C5, and part of C1). No satisfactory explanation of this difference in behaviour between the subreaches could be found. However, it may be noted that curves C4 and C5 (with increasing braiding index) represent upper reaches of the river where the distance between the nodal points is larger than other reaches.

Secondly, an attempt was made to find a relationship between trend in the braiding indices and the changes in discharge (average, bankfull, minimum) over the years. No significant relationship could be found.

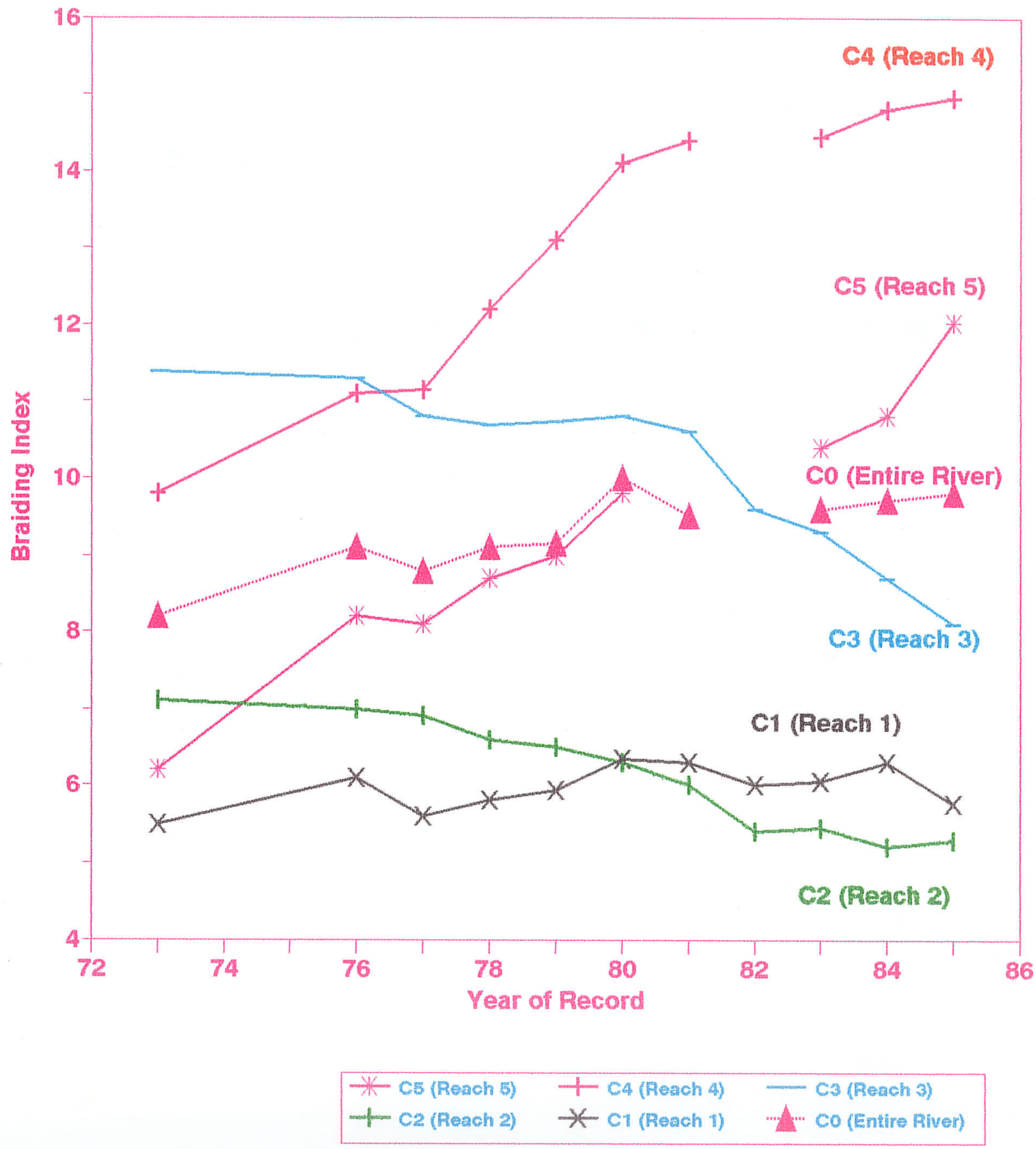

Fig. 4.2: Braiding Index of Different Reaches of the Brahmaputra River

Thirdly, for some reaches, the changes in braiding index (increase or decrease) were very small (Curves C1, and C2) while for others they were quite large (Curves C3, C4, and C5). Moreover, some changes were gradual while others were quite abrupt. No physical explanation for these properties could be found. The braiding indices for the two downstream reaches (C1 and C2) are lower than those of the three upstream reaches (C3, C4, and C5) although the upstream reaches did not follow this pattern within them. This may partly be due to the lesser downstream valley slope (Leopold et al., 1964).

In the absence of an obvious reason for the difference in braiding index between the reaches, one would have to conclude that the observed difference in braiding pattern reflect random changes in river behaviour in the sense that they do not lend themselves to predictions.

4.2.4 Other Topological Parameters As pointed out in Chapter Two, the definition of Braiding Index is dependent on the river stage and is therefore not a very good index to describe the degree of braiding. Howard et al. (1970) suggested a method in which features that are independent of the river stage are taken from a topographical map and are used to describe the braiding network. Some studies (e.g., RPT et al., 1988) have used these topological characteristics for the Brahmaputra river. They concluded that the configuration is relatively stable. In order to verify this conclusion, the topological characteristics of the Brahmaputra river were determined following the definitions of Howard et al. (1970) and compared with the Braiding Index.

The parameters proposed by Howard et al. (1970) are a combination of graph theory (Berge, 1972) and the theory used in transportation network (Garrison & Marble, 1962; Kansky, 1963). Howard et al. (1970) defined a reach of a braided river as the length between two adjacent cut lines, where the cutlines do not both cut the same branch (called segment), and do not pass directly through any joining point (called node) of branches. For such a reach of a braided river, the number of nodes (joining points of segments), the number of islands, and the number of segments (branches) are counted and used in numerical relationships. These relationships serve as three measures of degree of connectivity α, β and γ. These are defined by Howard et al. (1970) as:

$$\alpha = \frac{t-(n+e)+1}{2(n+e)-5} \tag{4.8}$$

$$\beta = \frac{t}{n+e} \tag{4.9}$$

$$\gamma = \frac{t}{3(n+e-2)} \tag{4.10}$$

where,

α is the ratio of the observed number of islands to the greatest possible number of islands for a given number of nodes,

γ is the ratio of the observed number of channel segments to the greatest possible number of segments for a given number of nodes,

t is the total number of segments including segments lying entirely within the section (entire segments) and those bisected by the lines bounding the section (bisected segments),

e is the number of bisected segments, and

n is the total number of nodes within the section.

For a long section $n >> e$, and $n \approx 2/3\, t$. Then, α, β and γ approach 0.25, 1.5, and 0.5 respectively. In all of the above, it has been assumed that not more than three segments join at a node. This would be a very rare event in natural rivers.

Other parameters that were defined are:

E = the average number of segments bisected by the crosslines at the ends and interior of the section,

N = the total number of segments entirely within the section and entering the section from the upstream section,

C = the average width of the stream between the outermost segments within the section, and

E_i = the excess segment index

$= E - 1$.

Note that for purely meandering stream, $E_i = 0$.

The satellite data of the period of 1973-1984 were used to measure these topological parameters for the six reaches of the Brahmaputra river defined earlier. These parameters for the entire length of the river within Bangladesh were also computed considering the entire length to be one reach (designated as Reach 0).

The measured parameters for the year 1977 are shown in Table 4.1. The values for the other years of measurement are given in *Appendix-C.* The tabulated values of parameters of degree of connection (α, β and γ) show that all the parameters have values below the approach values suggested by Howard et al. (1970) i.e., α is lower than 0.25; β is lower than 1.5; and γ is lower than 0.5 for all the reaches. This is true for the whole river as well.

Table 4.1 Topological Parameters for 1977

Reach No	t	n	e	E	N	C (km)	α	β	γ
1	48	36	8	3.50	44	6.56	0.06	1.091	0.381
2	35	21	8	4.25	31	6.24	0.13	1.207	0.432
3	62	38	11	5.75	58	10.42	0.15	1.265	0.440
4	53	35	14	6.67	46	11.29	0.05	1.082	0.376
5	28	12	12	5.67	21	10.16	0.12	1.167	0.424
6	71	41	9	4.75	69	11.43	0.23	1.420	0.493
0 (entire length)	273	183	8	4.88	269	9.33	0.22	1.429	0.481

The parameters were observed to vary with different reaches for the same year of record. An example is provided in Fig. 4.3 where the values of α are plotted for different reaches with time. The variation in their values did not follow a definite pattern. This is true for all of the parameters. Similarly, all of the parameters were observed to vary with time for the same reach. No definite pattern for such temporal variation of the parameters could be detected.

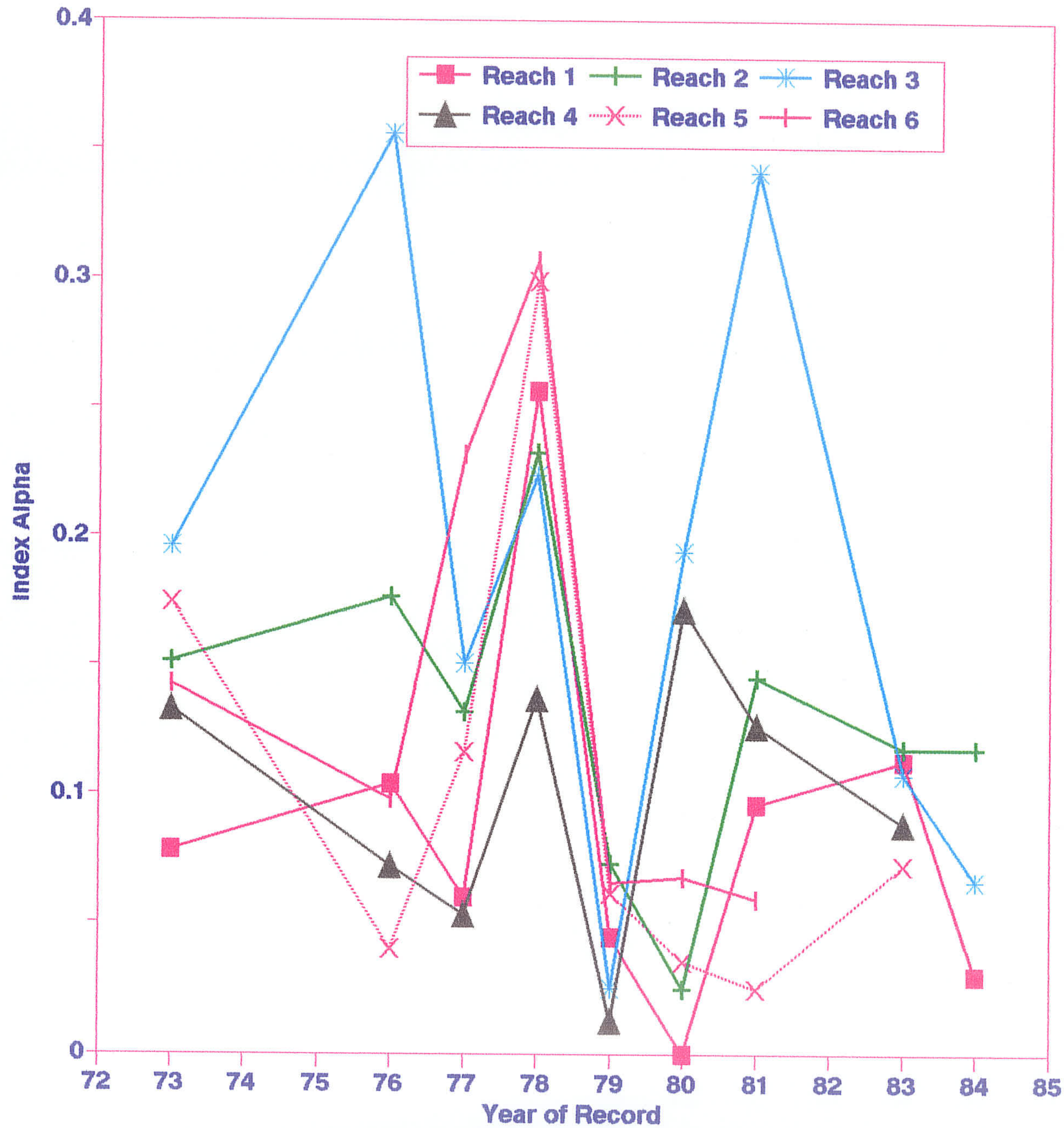

Fig. 4.3: Topological Parameter α for the Period of Record

It may be noted that Howard et al. (1970) related the excess segment index with hydraulic variables in order to have a priori prediction of the degree of braiding of a river. They reported that the found relationship supported Fahnestock's (1963) view of higher degree of braiding to be associated with higher variability of water discharge. Such an attempt to relate excess segment index with water discharge (average, bankfull or maximum) in this study resulted in very weak statistical relationships (the best one is significant at 7% level of significance, and has a value of R^2 of 0.39). This finding of this study is in agreement with that of Brice (1964) who found little association between degree of braiding and water discharge. Note that the best relationships between the excess segment index $\mathbf{E_i}$ and other hydraulic variables as reported by Howard et al. (1970) has a coefficient of correlation of 0.39 (see Table 3 of Howard et al., 1970) which amounts to an R^2 of 0.15.

4.2.5 Changes of Topological Parameters Over Space and Time Figures 4.2 and 4.3 show the variation of the computed topological parameters with time for different river reaches. No definite pattern of spatial or temporal variation of the parameter values is apparent. There also seems to be no relationship between the variation of the parameter values and the hydraulic variables.

It may be concluded from this that the configuration of the braiding network is not stable with time. The braiding index as well as the parameters of connection confirm that the planform of the river as a whole is subject to random variations, the occurrence and stability of the main nodes being the exception.

To further support the notion of network instability, plots of the river reach downstream of *Sirajganj*, and upstream of *Sirajganj* (Bristow, 1985, reported in RPT, NEDECO & BCL, 1986) are shown in Fig. 4.4 and 4.5 respectively. Both of these figures show substantial changes of the network of channels of the Brahmaputra river near *Sirajganj*. This conclusion, however, can easily be made qualitatively from a simple comparison of satellite imageries.

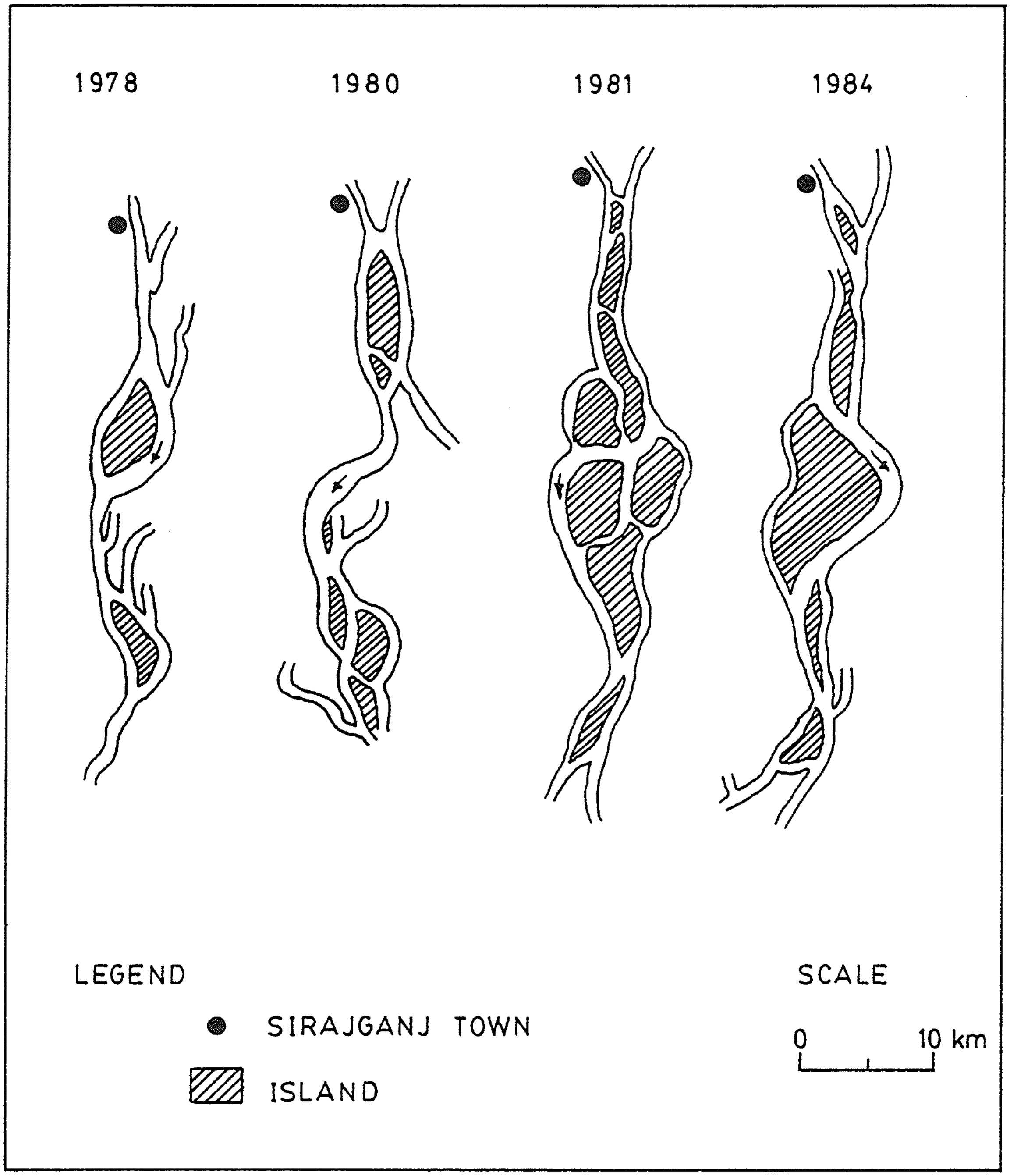

Fig. 4.4: Reach Downstream of Sirajganj Showing Channel Network Changes

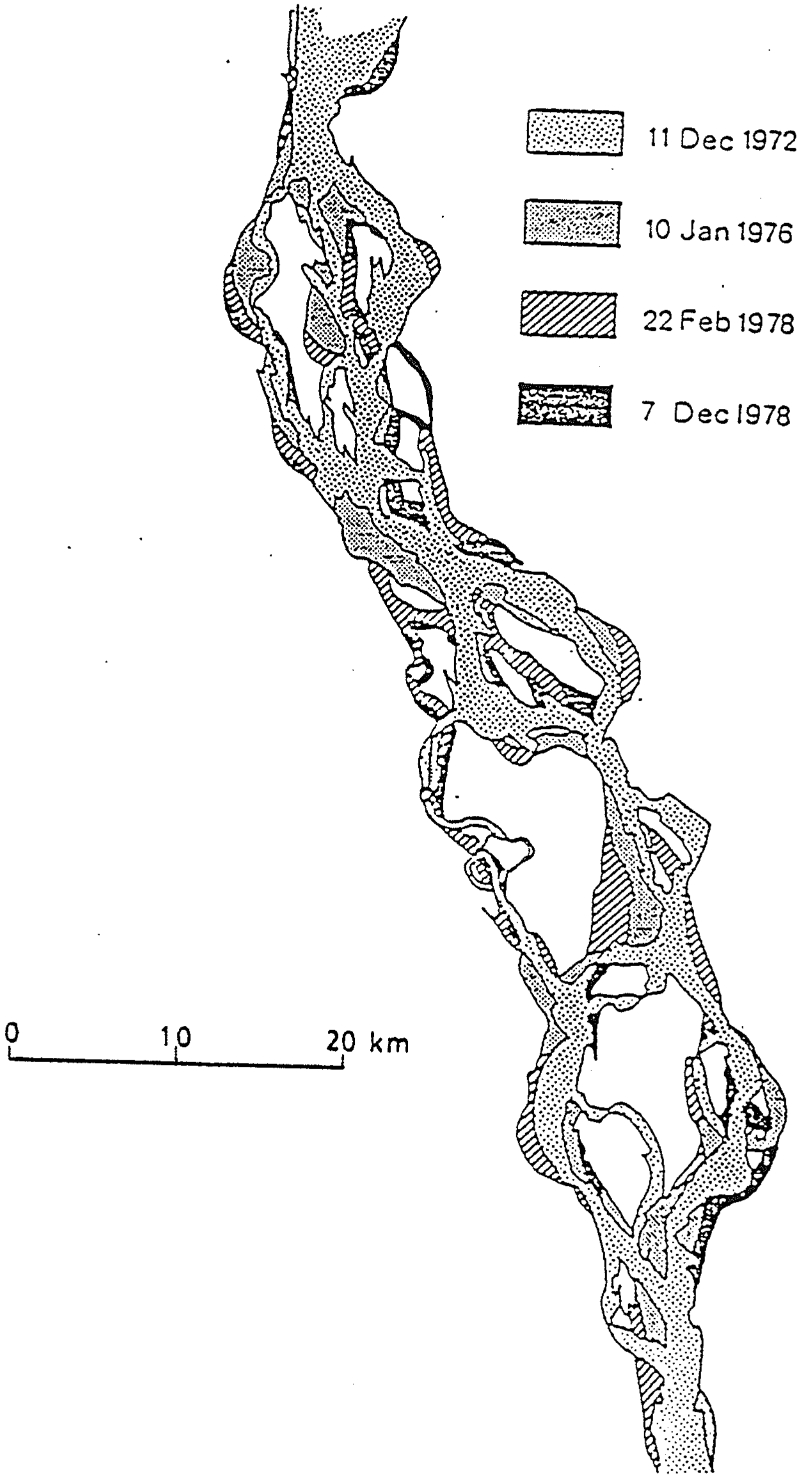

Fig. 4.5: Reach Upstream of Sirajganj Showing Bankline Erosion (Bristow, 1985)

4.3 Vertical Stability

The vertical stability of a river section relates to changes in the bed level over time. Such changes occur all the time since all alluvial rivers experience alternative episodes of erosion and deposition in response to the variability of the runoff. A river may nevertheless be regarded as vertically stable if it returns consistently to an equilibrium position over time.

In a braided river the situation is complicated by the fact that the number and the cross-sectional dimensions of the component channels may change locally while the river as a whole may not show persistent degradation or aggradation. A river that is raising its bed will have a greater tendency to shift its course than a river that is lowering its bed by erosion.

RPT et al. (1988) studied the vertical stability of the Brahmaputra river by considering the changes in average bed levels of the total river cross-section over time at selected locations. Some of the results are reproduced in Fig. 4.6. The location of the cross-sections is shown on Fig. 4.1. On the basis of this study RPT et al. came to the conclusion that the river is vertically stable.

This conclusion may be correct but is not well supported. Figure 4.6 includes a typical cross-section (section J-4) where the change in average bedlevel was 3.5 m over the period of observation. The change in this 8-km wide channel appeared to be consistent over time. One might therefore be tempted to conclude that it provides evidence of a trend toward deepening. One should realize, however, that the bed level was averaged over entire cross-section, which includes the islands in between the channels, where erosion and deposition presumably have very little effect on the elevation. Figure 4.7 shows the cross-section in a number of years. This demonstrates the difficulty in deducing from the average depth what happens to the river as a whole. It should be pointed out that Section J-2-1, which is a short distance downstream of Section J-4 shows a fairly consistent increase in average depth over the same period.

Perhaps all that can be said about the vertical stability of the river is that there is no evidence of a persistent trend in this respect. It seems likely that the changes in average bed elevation reflect the random changes in cross-section of the component branch channels.

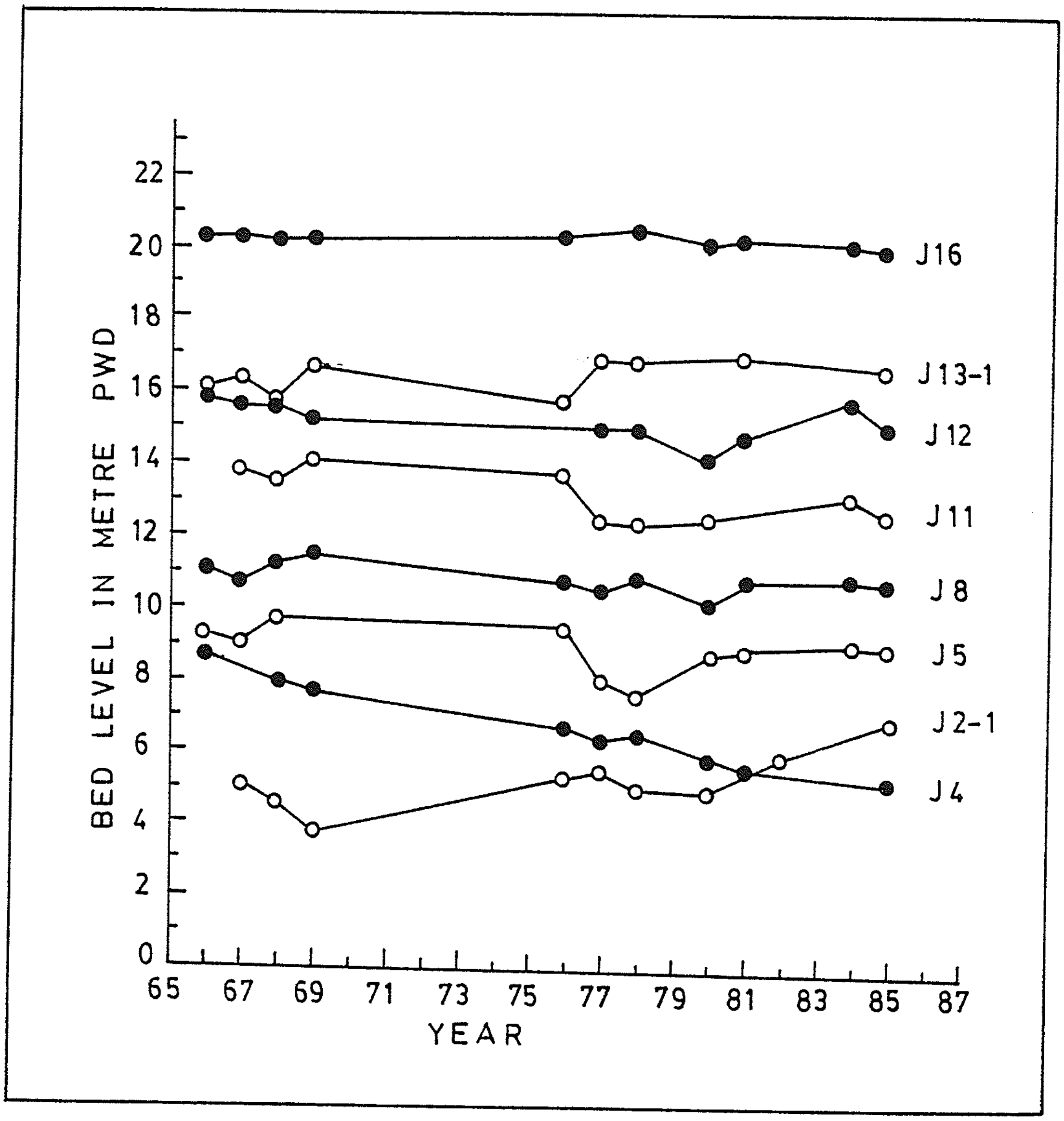

Fig. 4.6: Change in Average Bed Levels with Time (after RPT et al., 1986)

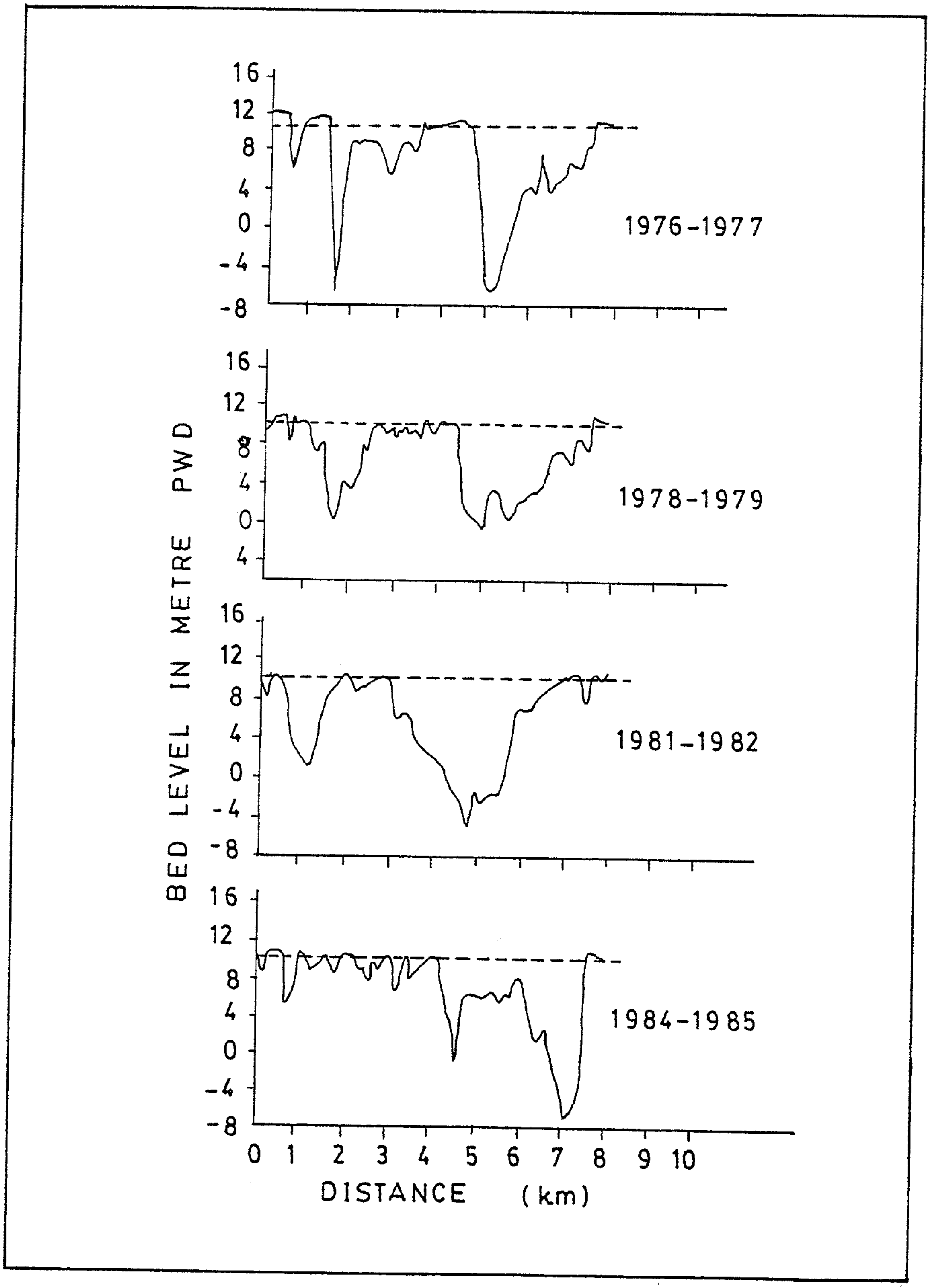

Fig. 4.7: A Typical Cross-section with time (Section J-4)

4.4 Horizontal Shifting of the River

Horizontal stability relates to the change of the river in the horizontal direction reflecting the movement of the outer river banks. A river, which is stable in a horizontal sense, will not significantly change its position over time although local and periodic changes do occur. The stability of the Brahmaputra river in this respect has also been assessed differently by different authors. Coleman (1969) found the river to be continually moving westward, whereas, RPT et al. (1988) concluded that over time the river appears to be stable in horizontal direction.

The movement of the inner banks of the Brahmaputra river (banks of the anabranches) is also important because of the existence of dwellings and farm lands on the islands. RPT et al. (1988) studied the bank stability by plotting the relative conveyance for a sequence of years, and concluded that over the last two decades there is no significant shifting of the river in either direction since there is no consistent shift in relative conveyance in either direction. This observation needs to be examined further.

Figure 4.8 is a reproduction of their plot of the distribution of the total conveyance over section J-13-1. A simple visual comparison of the cross-sections reveals that the river shifted about 3 km to the right and that the shift is fairly gradual. The shift in the line of conveyance distribution is neither gradual nor consistent. A consistent and gradual shift in the relative conveyance line simply reflects the unidirectional gradual change in a single channel river. It can easily be shown that the positions of the inner segments of the relative conveyance lines do not represent the movement of the outer banks of the river. Rather, it is important whether the beginning and the end of the relative conveyance lines show a gradual and consistent trend over time. These are the points that reflect the positions of the outer banks over time. Similarly, for the inner banks the shift (and trend of shift) of the relative conveyance lines at the location of the inner banks reflect the movement of these banks with time. Such a demonstration is shown in Fig. 4.9 where the bank positions of successive years are marked by 1, 2, 3 and 4. With this information, re-examination of the plot in Fig. 4.8 reveals that the right bank is shifting to the right which is evident from the shift of the right end-point of the conveyance lines to the right. Obviously, this observation does not support the claim made by RPT et al. (1988).

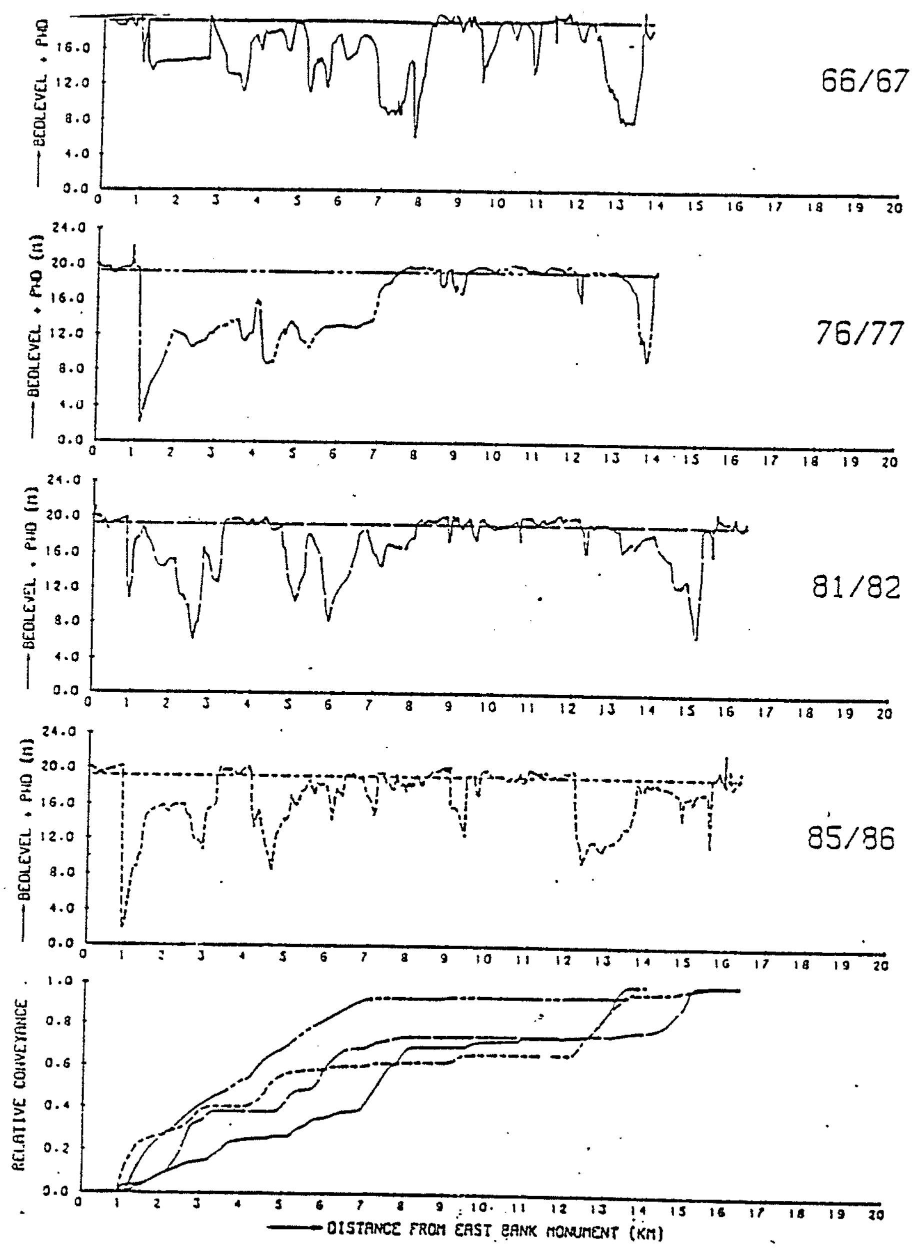

Fig. 4.8: Plot of Relative Conveyance of Section J-13-1 (RPT et al., 1986)

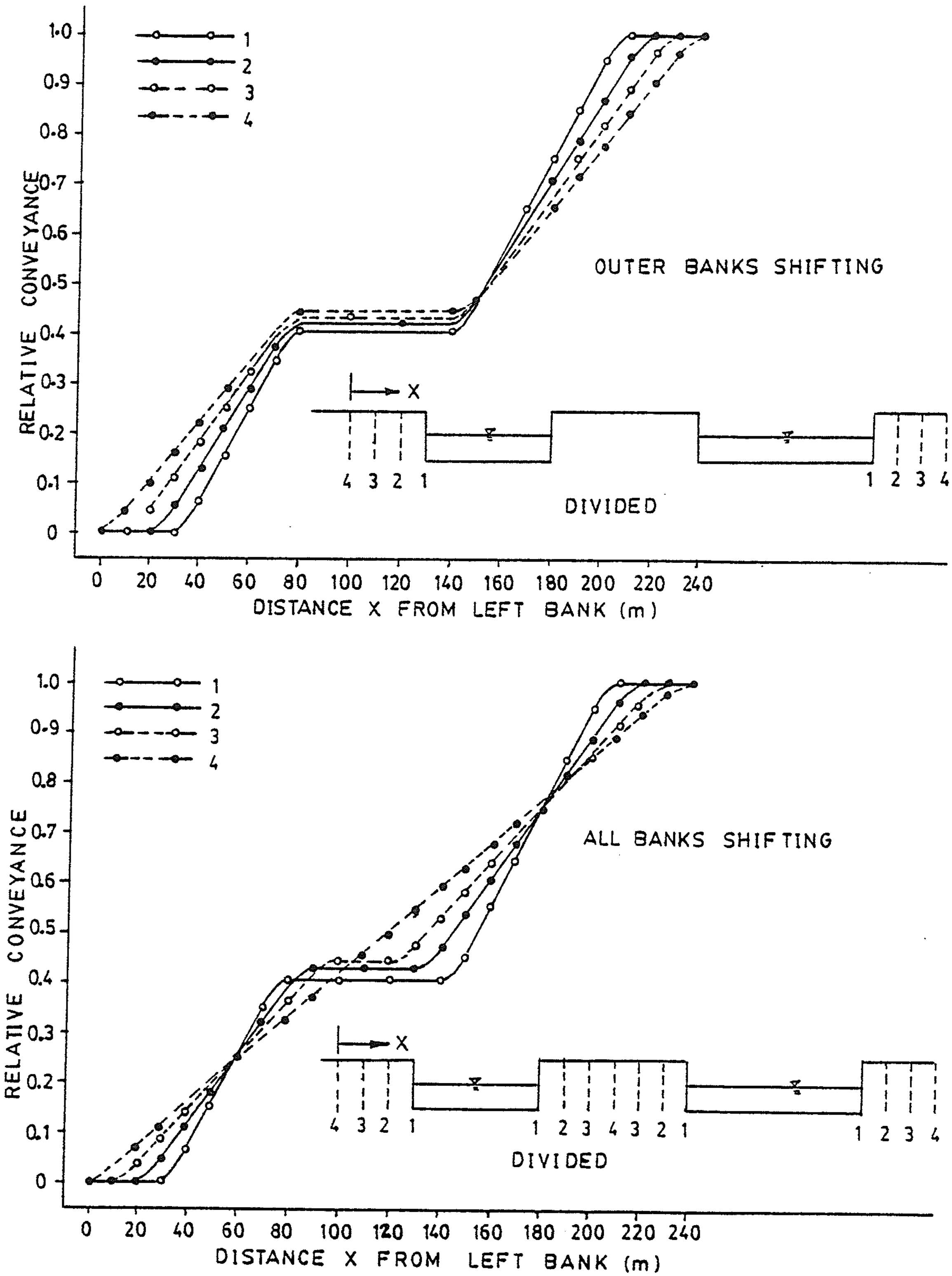

Fig. 4.9: Relative Conveyance of an Example Divided Channel with Bank Movement

It is possible that the movements of the outer banks are bounded within a given belt beyond which the river will not shift. Such a belt within which the river is observed to be moving back and forth has neither been established nor has it been proposed. Also, if such a belt existed, its extent would be so large that for all practical purposes the river can be regarded as horizontally unstable for it has been observed that the Brahmaputra river shifted as much as 5 km in a single year. It is true that there are small reaches (nodal points) where the river experienced very little or no bank movements in the recorded history. However, the record is not very long and it has not been established that those points will not move in foreseeable future.

The inner banks as well as the outer banks of the Brahmaputra river have been observed to experience movement in the horizontal direction resulting in movement in the vertical direction as well. These movements may not be gradual or unidirectional at all points but they are substantial at most of the points in time and space. Especially the changes in the horizontal direction has caused so much of a problem that it has drawn international attention. Recently The National Geographic published an article on the bank movement of the Brahmaputra river (Cobb, 1993).

It would appear that attempts at dealing with the problem of horizontal shifting of the Brahmaputra river as a whole are frustrated by the complexity of the changes in the branch channels. This problem will be re-examined after the qualitative analysis of branch channel changes has been discussed in a forthcoming chapter.

4.5 Stability of Planform of Individual Network Components

The characteristics of the overall network was found to be subject to erratic fluctuations in time and space. The network however is composed of single channel sections that also have specific planform characteristics such as radius of curvature, arc length and width. The stability of these features should also be investigated. Data for the investigation were taken from RPT et al. (1988).

It seemed appropriate to compare the geometry of the individual channel segments with the geometry of single channel rivers. Leopold et al. (1964) provided the following empirical relations for meanders in alluvial valleys.

$$\lambda = 10.9 W^{1.01} \tag{4.11}$$

$$\lambda = 4.7 R^{0.98} \tag{4.12}$$

where,

λ is the meander wavelength,

W is the channel width, and

R is the radius of curvature.

Combining Eqn. (4.11) and (4.12) the radius of curvature can be written as:

$$R = 2.36 W^{1.031} \tag{4.13}$$

RPT et al. (1986; 1988) plotted the width and radius of curvature of the individual channels measured from the satellite imageries for low water periods. This plot is reproduced in Fig. 4.10. (It is to be noted that the line representing Eqn. (4.13) on their figure shows an exponent 0.99 which is incorrect, the proper value being 1.03. However, the error is small and has little effect on the graph). It can be seen that the observations by RPT et al. deviate from Eqn. (4.13) by an appreciable amount. This, according to RPT et al. (1988), is caused in part by the use of the low flow width rather than the bankfull width. They claimed that this discrepancy could be corrected by multiplying the widths by a factor of 3.0 (since the ratio of bankfull to low flow is about 10, and according to regime theory the width of stable rivers varies approximately as the square root of the discharge) which of course is not correct. More importantly, the scatter cannot be reduced by this correction.

It appears that the individual channels of the Brahmaputra river have radii of curvature less than those of meandering rivers, and that the radius of curvature generally increases with the width of the channel.

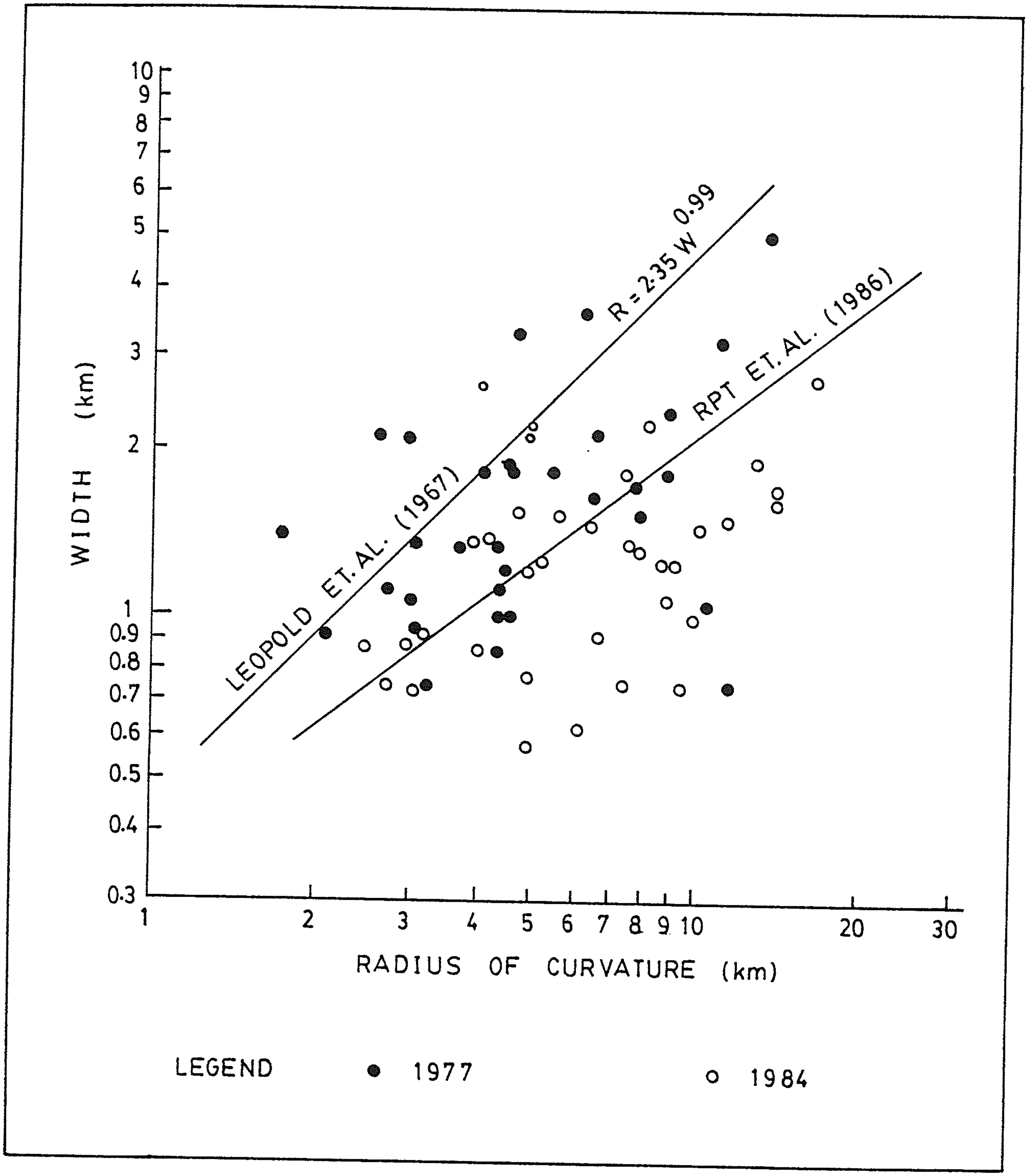

Fig. 4.10: Plot of Channel Radius of Curvature with Width (RPT et al., 1986)

A plot of the radius of curvature of the channel against distance of the measured section is shown in Fig. 4.11. No significant trend in the values of radius of curvature with distance is apparent from this figure. However, it can be said that the radii of curvature of the channels downstream of Sirajganj (section J-7) are somewhat larger than those of the upstream for 1977 data. Also, the radius of curvature of the channel was smaller in 1977 than in 1984, particularly for the sections upstream of Sirajganj. No conclusions regarding possible prediction of river behaviour could be drawn from this study.

4.6 Cross-Sectional Changes of Branch Channels

The changes in cross-sectional dimensions of the branch channels will be examined in this section, in particular the variation in width and depth of the individual channels. A comparison of the branch channel dimensions to those of single channel rivers as obtained from regime relations will also be provided. The analysis of this section will form the data base for later use in the study.

4.6.1 Data The Bangladesh Water Development Board (BWDB) records depth measurements on an annual basis since the 1966-1967 water year at 33 measuring sections shown in Fig. 4.1. From these data 33 cross-sections were obtained for each of the years 1973 to 1985. The cross sections were obtained each year during the low flow period between November and February. *Appendix-D* shows for illustration some of the cross sections together with their changes with time. Not all of the cross sections are perpendicular to the flow since the individual channels in the river do not always follow the direction of the valley slope. Moreover, the horizontal reference for measurement at some stations had to be relocated because of either channel erosion or other reasons. Fortunately, the cross-section plots contained this information making it possible to make the necessary corrections prior to the analysis.

The cross sectional plots were digitized using a computer-based digitizer and stored for further computation. The monument used by BWDB for measurements at a section was taken as the horizontal

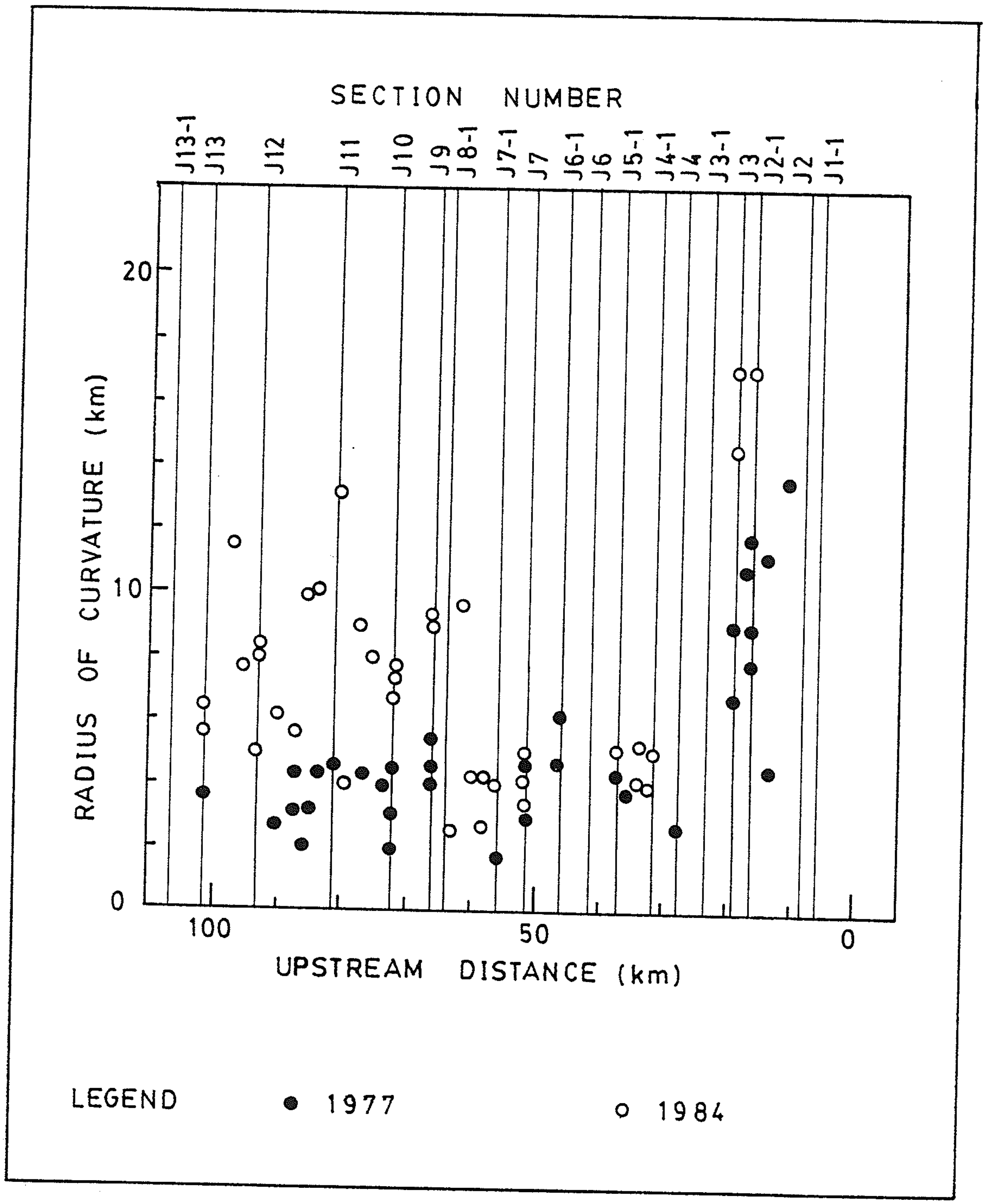

Fig. 4.11: Plot of Channel Radius of Curvature with Distance (after RPT et al., 1986)

reference, and the Department of Public Works datum was taken as the vertical reference. The horizontal distances between the points on the cross section where bottom elevations were measured were adjusted as to include breakpoints in the channel bottom configuration and to obtain more accurate values for cross sectional area and hydraulic radius. In this way a series of orthogonal coordinate pairs were obtained for each cross-section, X being the horizontal distance across the channel from a fixed reference point and Y being the vertical distance between the datum and the channel bottom.

A computer program was developed to process the information and to obtain for a given water level (either measured or computed from the program) the width, the average and the maximum depth, the area, the hydraulic radius, the location of the centroid of the area, the conveyance, the water surface slope, Manning's roughness coefficient, the location of the left and right bank of the branch channels of the section as well as of the total section; the number of channels in the section; and the distribution of water discharge in each branch channel. An estimation of the sediment discharge in each branch as well as in the whole section was obtained for all sections and for each year of record. The program description is given in *Appendix-E*.

The computation of the above properties was performed for six different discharges. These are: 10,000, 20,000, 30,000, 40,000, 50,000 and 60,000 m^3/sec. Of these values, 20,000 m^3/sec is the average; 40,000 m^3/sec is the bankfull; and 60,000 m^3/sec is average maximum discharge.

Some remarks about data management and computations are in order. In the first place, it should be mentioned that of the 33 cross sections that were measured each year, only seven have water level information. The water levels in the other sections were computed in the program by routing procedures.

In the second place, the water levels for the specific discharges mentioned above were obtained by interpolation between levels at two dates that bracketed the discharge for which the level was needed (see *Appendix-F* for more on this interpolation). In addition, the water level for a given discharge will normally have different values for the rising and for the falling stage of the hydrograph. At the sections with recorded levels, the water levels for the rising stage and for the falling stage for a given discharge were averaged. This was, of course, not needed for the sections where the water levels were determined by routing. The results are shown in *Appendix-F*.

In the third place, the geometric properties for a particular discharge were computed from the measured depth and the measured (or computed) water levels at a section. Note that some organizations use a formula describing the observed relationship between the water level and a particular property to compute the value of the property for a given water level. No such relationship was used in this study because: (a) the coefficients of such relationships were found to vary significantly for different sections and for different periods of record (see *Appendix-G*); and (b) the number of channels also vary for different water levels, and for different sections along the river.

In the fourth place, after all of the geometric properties were computed, the total discharge was divided into the branch channels assuming the same slope while taking the differences in frictional characteristics into account. This was, again, a trial and error procedure. Thus, a complete set of hydraulic properties of the branch channels was obtained for a range of discharges of both water and sediment. This information formed the input for the next phase of the study. A few observations on the computed depth, width and other variables are presented in the following sections. Most of the discussion focuses on the assumed bankfull discharge ($Q=40{,}000\ m^3/sec$) unless otherwise noted.

4.6.2 Depth The average depth of branch channels varied between one metre for shallow channels to 10 m for the deep ones. The maximum depth varied between 2 m to 30 m.

Variability:

When the whole river cross section at a point is considered, the maximum to average depth ratio is found to be less than 3.5 with a few exceptions (RPT et al., 1986). However, when the individual branch channels are considered, a substantial number of channels are found to have a ratio higher than 3.5; for some channels it is as high as 5.5. The variability in depth is shown in Fig. 4.12 where the dimensionless ratio of maximum to average depth (d) is plotted against the average depth of the branch channels.

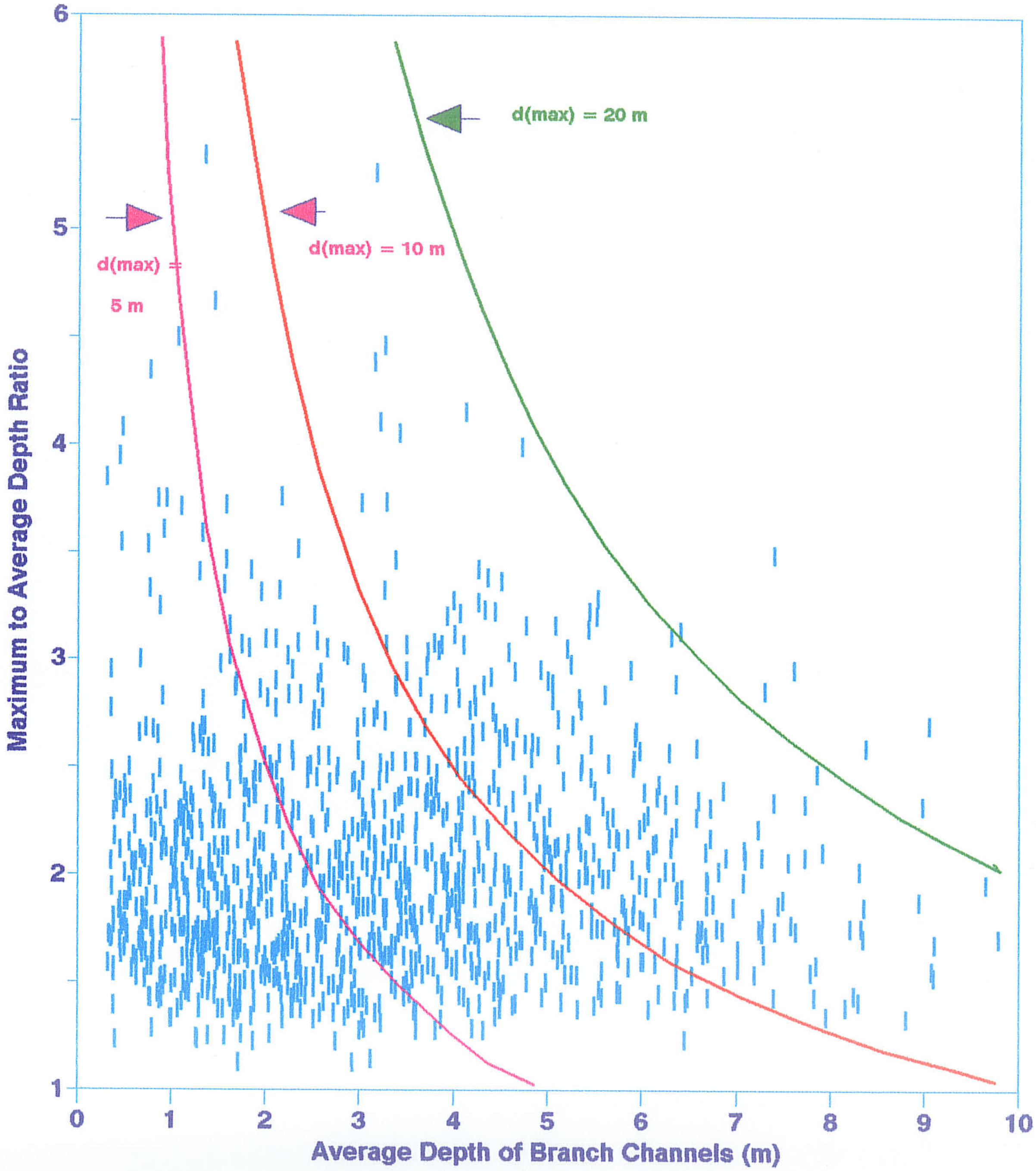

Fig. 4.12: Maximum to Average Depth Ratio of Branch Channels

Soundings taken from Bangladesh Inland Water Transport Authority (BIWTA) reveal that the maximum depth occurs at (a) outer bends of main channels; (b) downstream of the confluence with a river; (c) downstream of a confluence with another branch channel; and (d) in reaches where bank protection confines the flow. Fig. 4.13 shows minimum bed levels against latitude (after RPT et al., 1986). Similar data obtained by BWDB and BIWTA are shown in Fig. 4.14 (RPT et al., 1986). This shows a wide variation between observations from different sources which may reflect the mobility of the bedforms. However, the average depth computed from different sources does not vary substantially. It seems reasonable to assume that the average depth is a more stable parameter and therefore more suitable as a characteristic flow measure for the purpose of analysis.

Comparison with Single Channel River

The average depth of each branch channel was plotted against the estimated branch channel discharge. The results are shown in Fig. 4.15. In the same figure, Lacey's regime relationship is also plotted. The fitted regression line is given by:

$$d = 0.2669\ Q^{0.3253} \qquad (4.14)$$

where,

d = average depth of branch channel in metres, and

Q = discharge through the branch channel in m^3/sec.

The regression line is almost parallel to Lacey's line but is placed well below it. It is observed that all the channels except a very few had lower average depth than that predicted by Lacey's relationship. Evidently, the branch channels in a braided river tend to be shallower and wider than single channel rivers for the same dominant discharge.

A plot of the maximum depth against the discharge of the branch channels (Fig. 4.16) reveals an interesting feature. The regression line through the observed data is given by:

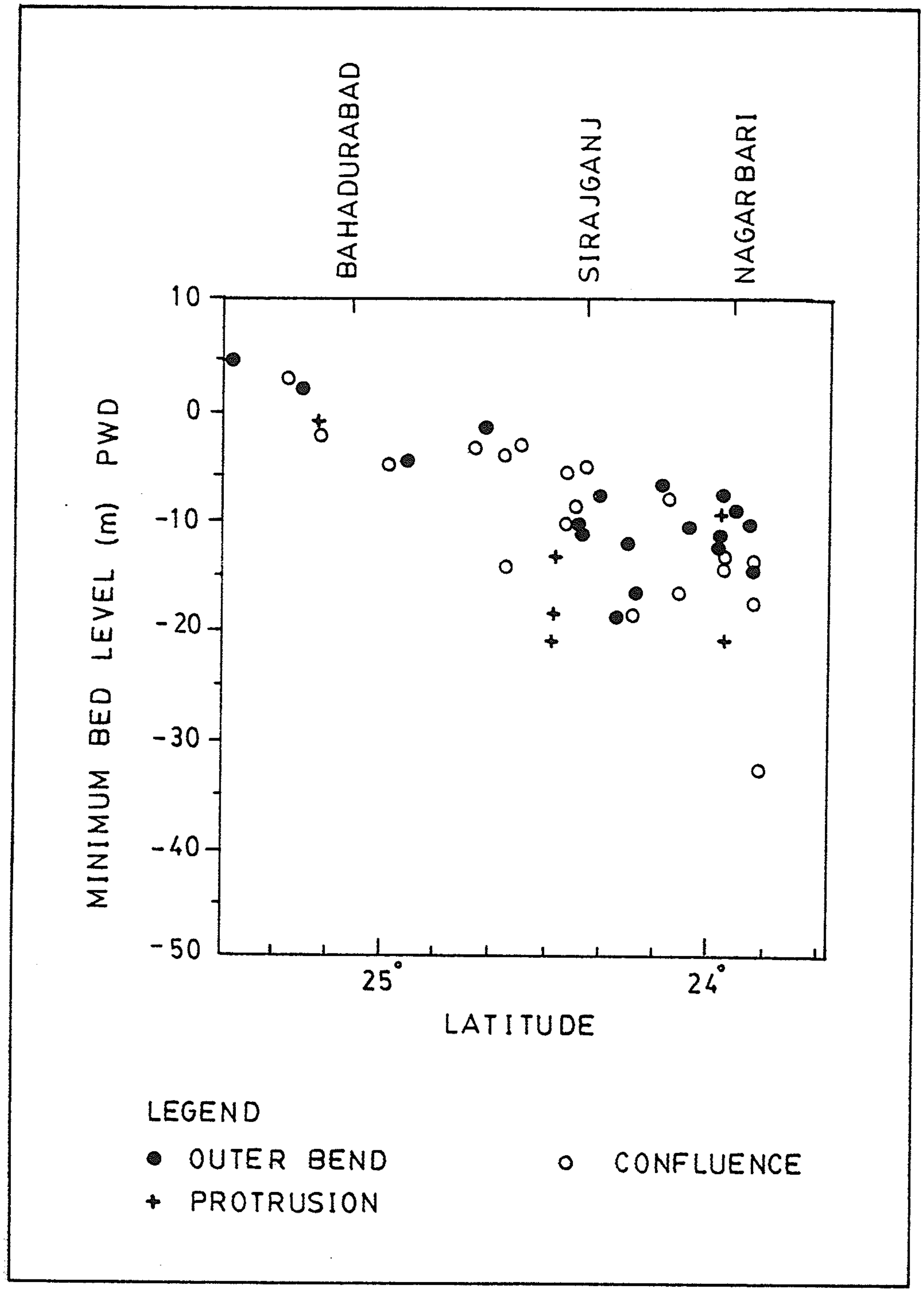

Fig. 4.13: Variation of Minimum Bed Level with Latitude (after RPT et al., 1986)

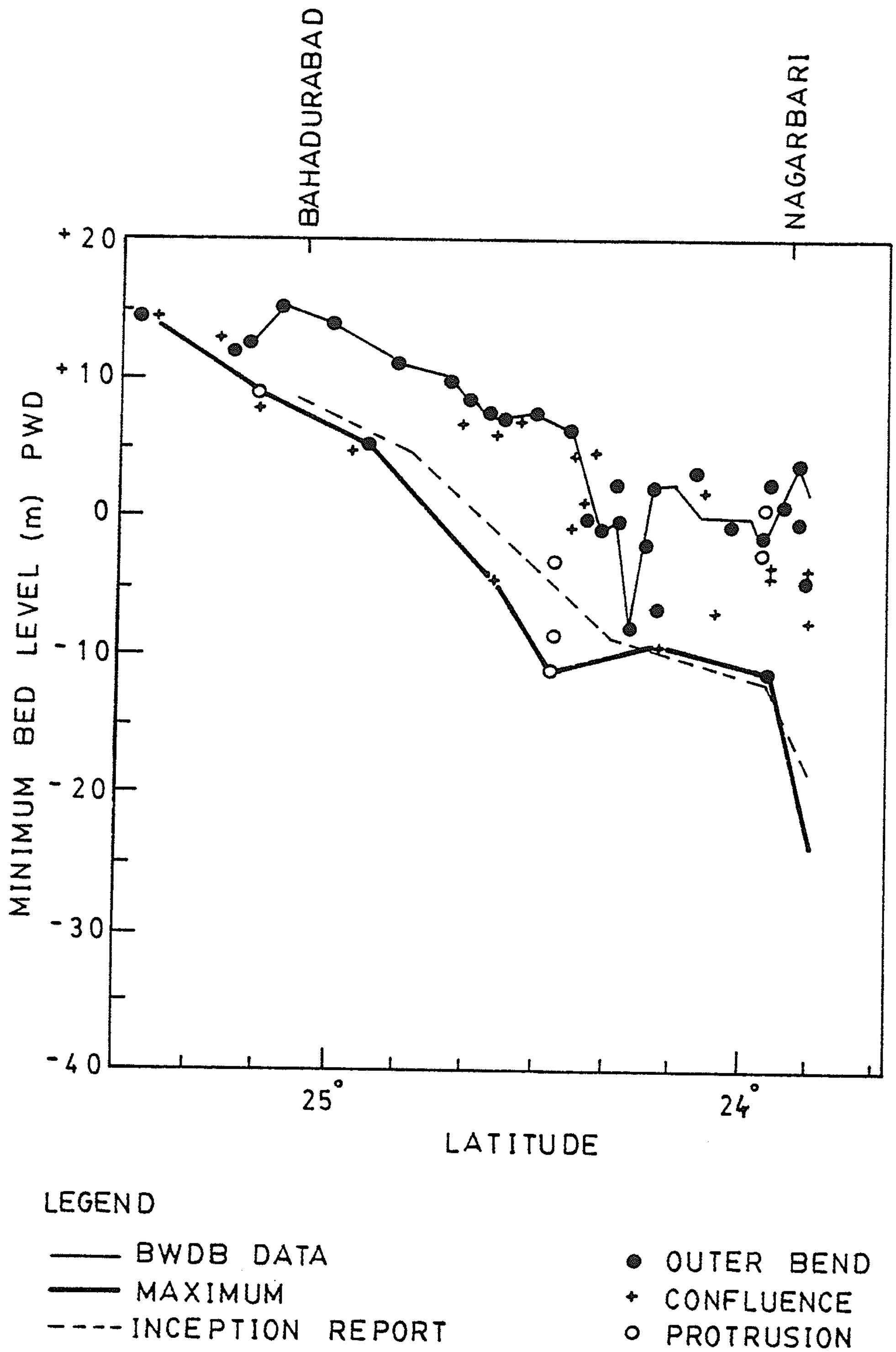

Fig. 4.14: Comparison of Bed Levels from Different Sources (after RPT et al., 1986)

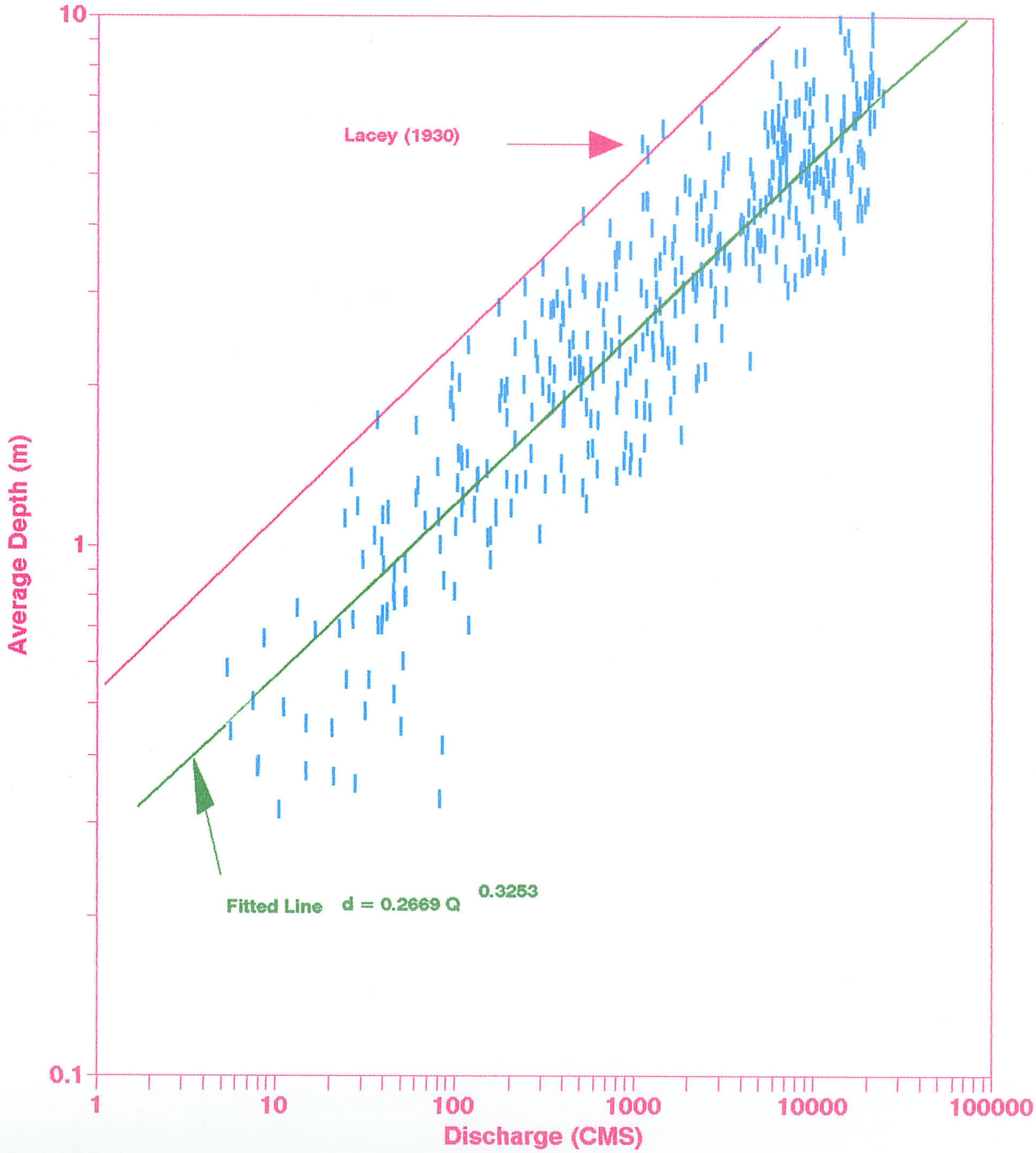

Fig. 4.15: Average Depth and Discharge of Branch Channels

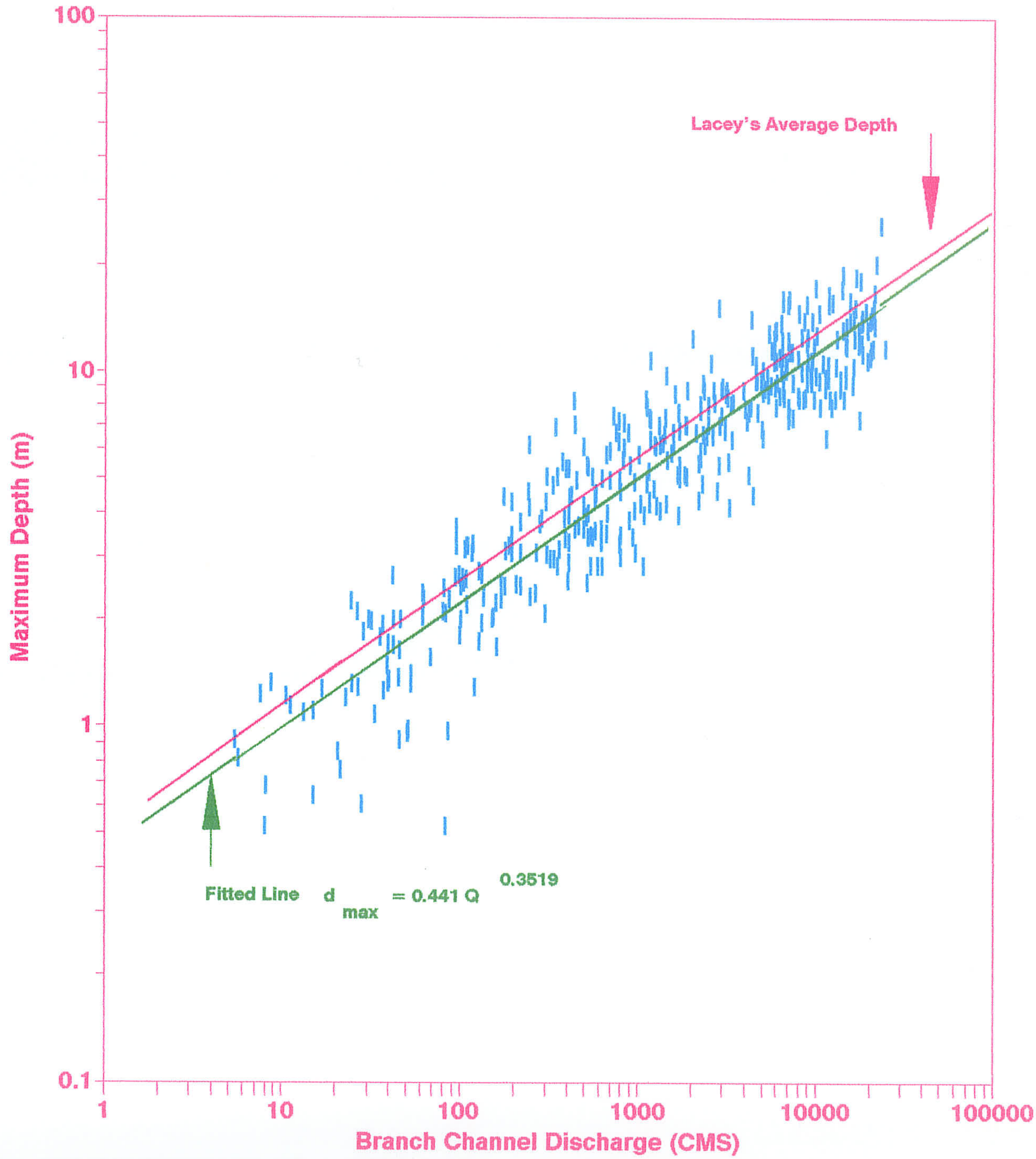

Fig. 4.16: Maximum Depth and Discharge of Branch Channels

$$d_{max} = 0.441\ Q^{0.3519} \tag{4.15}$$

where,

d_{max} = maximum depth of branch channels in metres, and

Q = the discharge of the branch channel in m^3/sec.

The regression line is almost the same as Lacey's average depth line. It shows that on average the ratio of the maximum to the average depth is the same for branches of widely different dominant discharge. It indicates that while the branches differ from each other hydraulically in scale their morphology is fundamentally the same, although it differs from single channel rivers in that the branch channels tend to be wider and shallower.

4.6.3 Width The width of the Brahmaputra river in Bangladesh varies between 1.6 and 15 km which is a factor of 10. The width of the branch channels, however, varies between 60 m to 6 km, which is a factor of 100. These widths are plotted against the branch channel discharge in Fig. 4.17. This figure also shows Lacey's regime relation. The regression line is given by:

$$W = 28.626\ Q^{0.4521} \tag{4.16}$$

where,

W = width of the branch channel in metres, and

Q = discharge through the branch channel in m^3/sec.

All branch channels are wider than Lacey's predicted width, as expected, and plot above Lacey's line in Fig. 4.17.

Wetted perimeter and discharge of branch channels are plotted in Fig. 4.18, also with Lacey relation and with Simons & Albertson's (1960) regime relation. All the channels have higher wetted perimeters than those predicted by Lacey or Simons & Albertson's regime relation. The fitted regression line is given by:

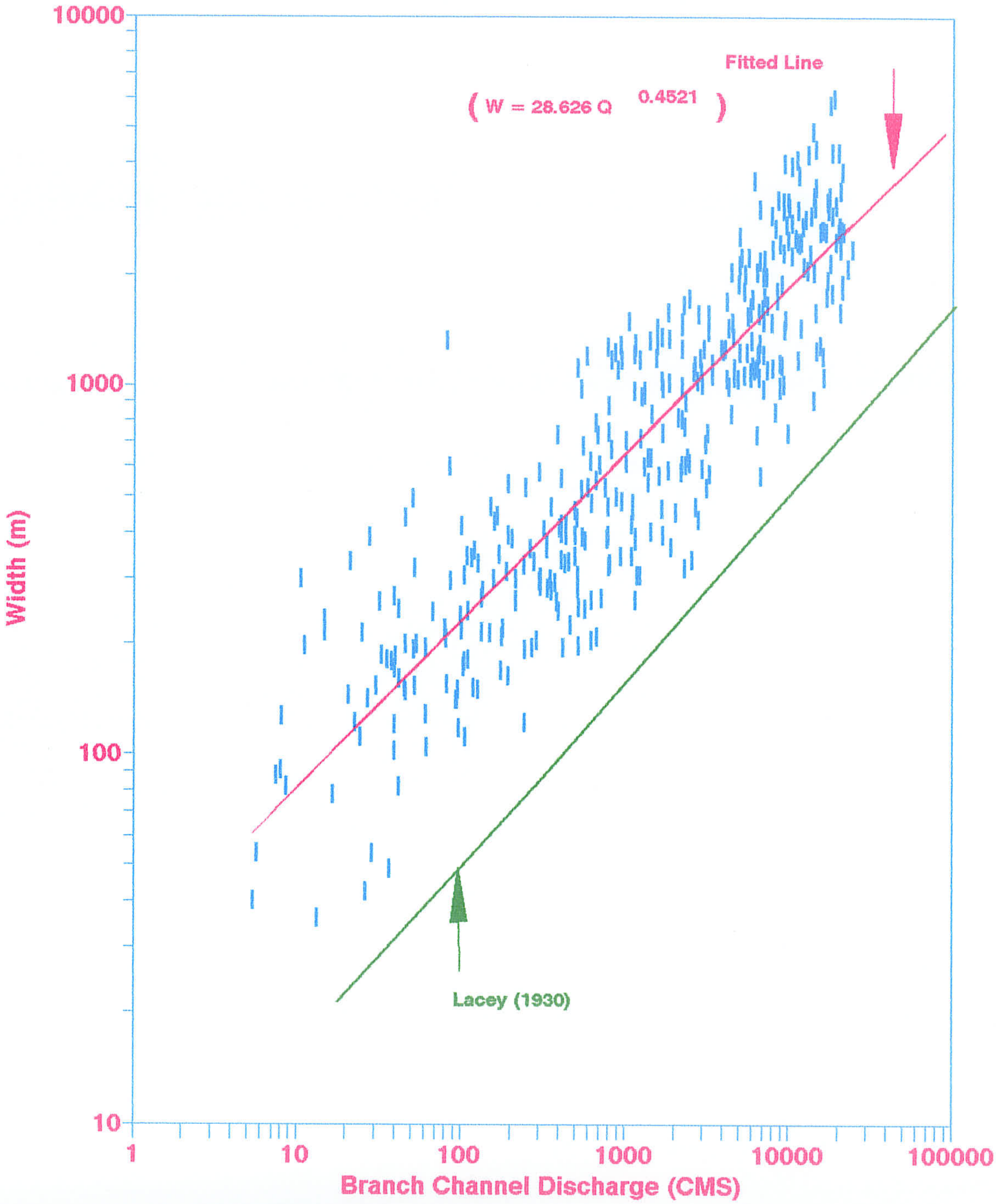

Fig. 4.17: Width and Discharge of Branch Channels

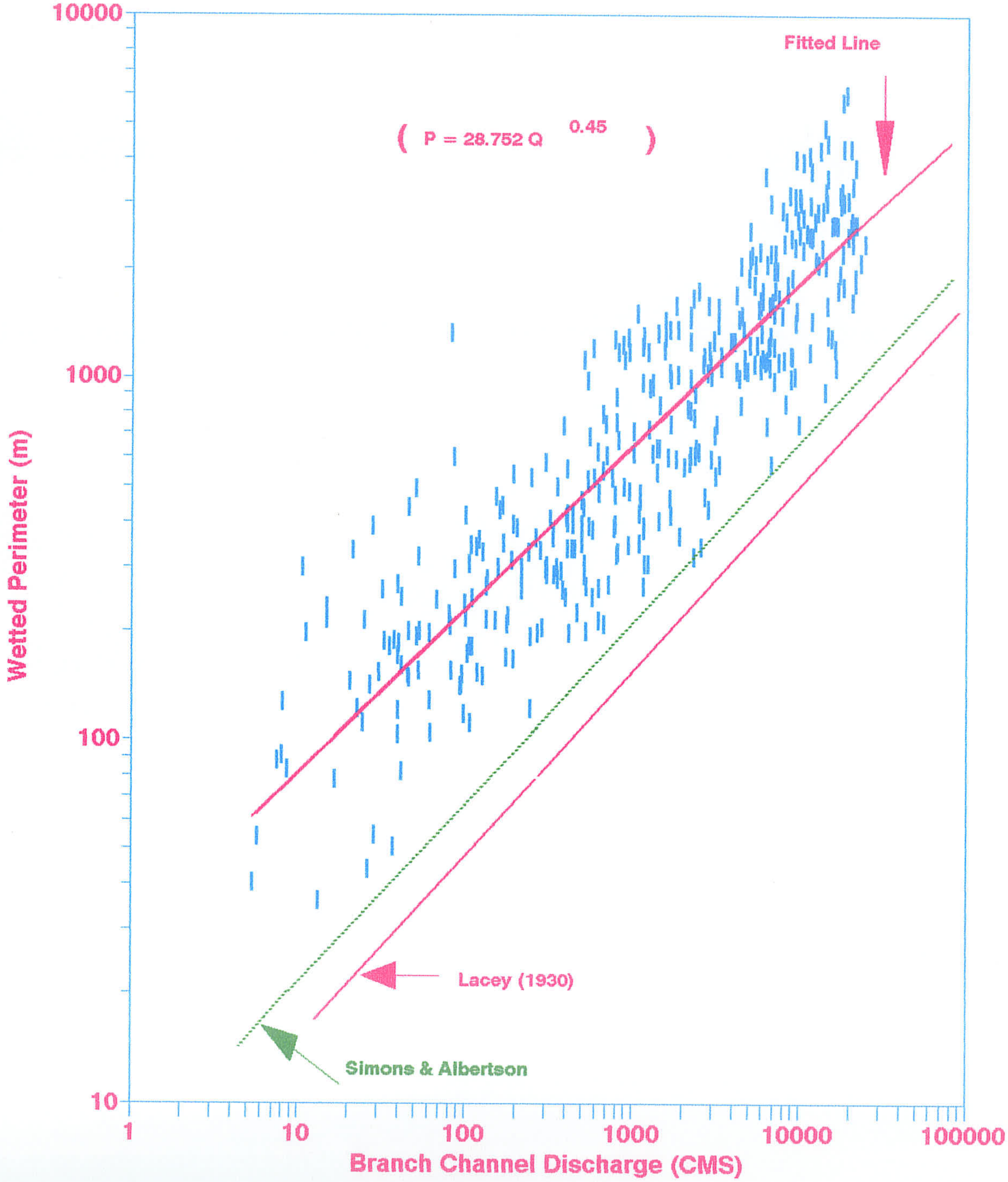

Fig. 4.18: Wetted Perimeter and Discharge of Branch Channels

$$P = 28.752\ Q^{0.45} \tag{4.17}$$

where,

P = wetted perimeter of the branch channel in metres, and

Q = discharge through the branch channel in m^3/sec.

A plot of cross sectional flow area against discharge of the branch channels is given in Fig. 4.19. It shows that the observed cross-sectional area is higher than that predicted by Lacey for single channel river. The fitted regression line is given by:

$$A = 7.64\ Q^{0.7755} \tag{4.18}$$

where,

A = cross-sectional area of the branch channel in square metres, and

Q = discharge of the branch channel in m^3/sec.

The graphs demonstrate clearly that the Brahmaputra river in Bangladesh forms braids of branch channels that are shallow and wide compared to stable single channels. The wider and shallower channels have high potential for change since a shallow channel silts up more readily than a deep one and a wide channel will sooner split into two branches than a narrow channel.

More discussion on width and its relationship with other variables particularly in the context of bankline movement will be given in the next chapter.

4.7 Stability of Bifurcations

Since at the bifurcation both sediment and water flow are divided over two channels in ways that depend on the geometry of the bifurcation, one may surmise that the channel characteristics at the bifurcations of the Brahmaputra river are of special interest. They may give a clue to the likelihood that either one of the channels will tend to become smaller and eventually disappear. It may also be surmised that a study of the behaviour of the branch channels at a bifurcation point may lead to a criterion that may

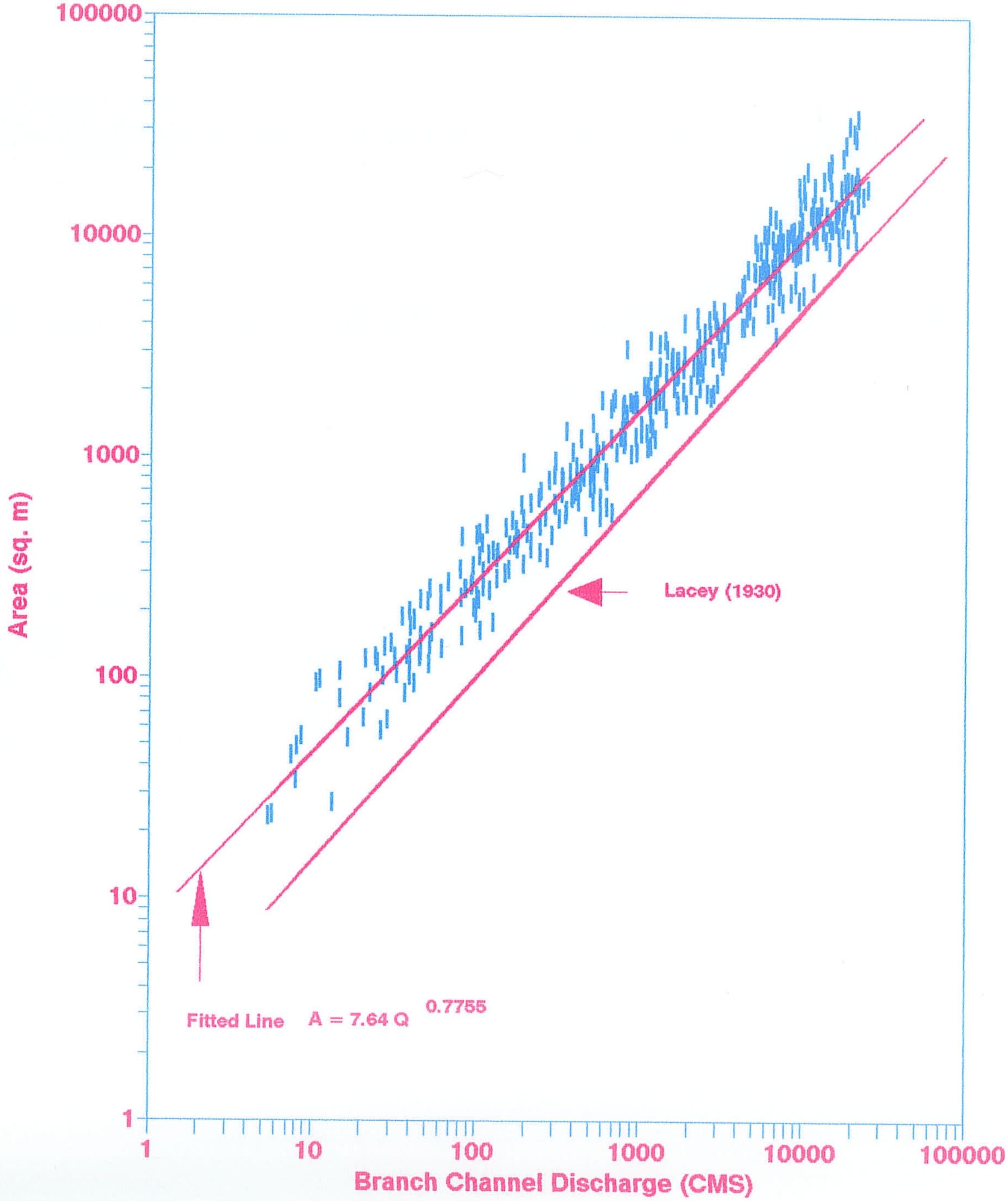

Fig. 4.19: Cross Sectional Area and Discharge of Branch Channels

indicate whether a channel is susceptible to splitting. Establishment of such a criterion would be useful in predicting the process of braiding in a river.

Dry period satellite imageries of the Brahmaputra river for the year 1973, 1976, 1977, 1978, 1979, 1980 and 1981 were used to measure different channel characteristics at bifurcations. Most of the bifurcations changed either moving in location or changing their size and shape over time. Some of them ceased to exist because of joining of the branches in the following year. This suggests that the bifurcations are also not stable. The changes appear to be random.

Nineteen bifurcation points were selected on each of the seven imageries that did not substantially change in shape over time and were easily identifiable on all imageries. This produced 113 observations on bifurcation points. Different characteristics of the incoming channel before bifurcation (called the main channel), and of the channels after bifurcation (called the east branch channel and the west branch channel) were measured. The measured characteristics were the width, the radius of curvature, the angle of inclination with respect to a north-south reference line, and the yearly change in width of the main channel as well as of the east and west branch channels. From these data, the angle of asymmetry (the deviation from symmetry of the downstream branch channels from the incoming main channel), the relative radius of curvature (the ratio of branch channel curvature to the main channel curvature) of the branch channels, and the relative width ratios are computed.

It was assumed that for a given bifurcation, a relationship exists between these geometrical variables which defines the stability of the bifurcation. Different statistical relationships (through both linear and nonlinear regression) were tried to relate these variables. The following equations were found to be significant:

$$W_1 = 0.834 W_m - 0.481 W_2; \quad [R^2 = 0.86] \tag{4.19}$$

$$W_2 = 0.846 W_m - 0.523 W_1; \quad [R^2 = 0.85] \tag{4.20}$$

where,

W_m is the width of the incoming main channel before bifurcation;

W_1 is the width of the east branch channel; and

W_2 is the width of the west branch channel, all in FPS units.

It is to be noted that Eqn. (4.19) and (4.20) are obtained through two-stage least square estimation which solves simultaneous system of equations. This is required because of the fact that the widths of the downstream branch channels depend on the upstream main channel as well as on each other. For example, the dimensions of the east channel will not only be dependent on the channel before bifurcation but also on the dimensions of the other channel after the bifurcation thereby forming a system of equations that are related to each other. Here, the dimensions of the main channel has been considered to be independent of the dimensions of the downstream channels. Strictly speaking, there is an inter-dependency (feedback) between the upstream and the downstream channels. This has been ignored because of the alluvial nature of the river in which the feedback is relatively small. Consideration of the feedback did not substantially change the system of equations anyway.

Comparison of Eqn. (4.19) and (4.20) reveals that the relationships of W_1 and W_2 are very similar as one would expect. Thus only one equation may be used for both of the branch channels.

It may be pointed out that the bifurcations with channels of significantly different width did not remain stable during the period of record. This is consistent with the Eqn. (4.19) and (4.20) which indicate that the sum of the two branch channel widths becomes less than the width of the main channel when one branch channel width is more than twice the other. The smaller channels at such bifurcations were observed to grow smaller until it completely clogged.

It would appear that the bifurcation points may remain relatively stable when the branch channels are approximately equal in size. Any condition that tends to reduce the one or increase the size of the other is likely to upset the equilibrium so that the difference in size becomes progressively more pronounced. Attempts at predicting this process from the geometry of the channel and bifurcation topography were not successful. For the time being the process is best regarded to be a random phenomenon.

4.8 Bed Forms

Information on the bed forms in the Brahmaputra river is very limited. Coleman (1969) observed four different kinds of bedforms. These are: ripples with height between 0.2 m to 0.5 m; mega ripples with height of 1.0 m and a celerity of 120 m/day; dunes with typical height of 5 m and celerity of 60 m/day; and sand waves of height of 10 m and celerity of 200 m/day.

More recently, RPT et al. (1988) conducted a survey during flood conditions in 1986 and 1987 to measure the bed forms in the Brahmaputra river. The plot of average dune height versus average depth is shown in Fig. 4.20. The plot also shows the predicted values by Allen (1968) and Yalin (1964) both of which under-predicted the dune heights during high flow conditions. They also observed that during peak flow large dunes with average height of 3.0 m and average length of 200 m are present with superimposed small dunes with average height of 1.1 m and average length of 24 m. After the peak flood, the superimposed small dunes disappeared and the large dunes were reduced in size. Afterwards, the large dunes disappear with the exception of a few. This survey found the maximum dune height to be 6 m as compared to Coleman (1969) who found it to be 15 m.

It is to be noted here that the entire river bed in a cross-section does not contain one single bed form at all points. Rather, a combination of different bedforms such as ripples, dunes and sandwaves may exist on the bed across this wide river. Care should be taken in using these measurements since it contradicts the resistance to flow during the peak discharge as observed by RPT et al. (1988).

4.9 Summary

This chapter described the network characteristics and its stability for the Brahmaputra river. The existing network dimensions is considered to be variable in space and time. Although the spacing of the nodal points approximately follows the wavelength predicted by Hayashi & Ozaki's (1980) analytical relations, no satisfactory answer as to why they do not change could be provided. The variables width

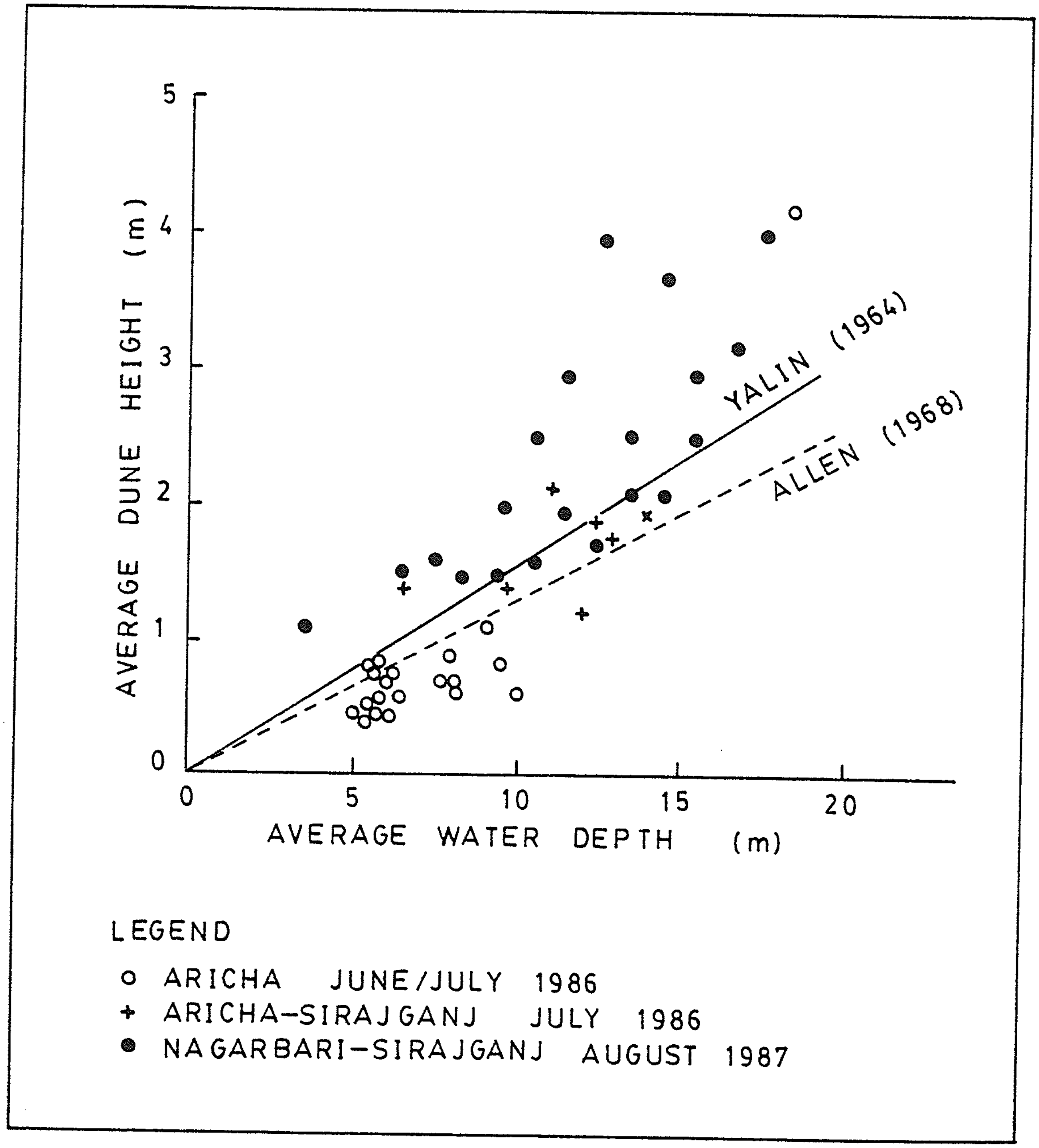

Fig. 4.20: Average Dune Height and Average Water Depth (RPT et al., 1988)

and depth have been studied in detail. The vertical and horizontal stability of the river as observed in this study and as reported by others have also been discussed. It appears that there is no evidence of a persistent trend in movement in either horizontal or vertical directions. The bifurcation points of the river were found to be changing in time with no definite pattern of change. A study of stable bifurcations did not provide any criterion based on which any particular bifurcation could be considered stable.

Chapter Five

QUALITATIVE PREDICTIONS BASED ON WIDTH - DEPTH RELATIONSHIP

5.1 Introduction

Single channel alluvial rivers show a characteristic relationship between the width, the depth and the formative discharge which represents equilibrium conditions in the erosion-deposition process. One cannot expect such a relationship for the branch channels in a braided river since some channels are in the process of widening while others are getting narrower. Indeed, the continuing change both in planform and channel characteristics show that the channels are as a rule *not* in equilibrium.

One would expect, however, that the *average* width-depth relation for the branch channels is not far removed from the equilibrium condition since there are probably as many channels in the process of widening as there are that are narrowing. Indeed, it was shown in the previous chapter that *on average* the branch channels show a relationship between width, depth and formative discharge that is quite similar to the relation observed for single channels.

This chapter explores the possibility of using the deviation of individual channels from the assumed equilibrium condition for the purpose of predicting incipient change. Two aspects of change will be considered.

The first part of the investigation is aimed at predicting changes in width. It will be shown that it is possible to predict whether a channel will become narrower or wider with over 90% accuracy.

The second part of the investigation aims at predicting the likelihood that two channels will join or that a single channel will split into two branches. This investigation also met with a fair degree of success.

The equilibrium conditions developed in this chapter will form the basis for a bank movement model which will be discussed in the next chapter.

5.2 Theoretical Considerations

5.2.1 Width-Depth Relations for the River as a Whole When a given reach in an alluvial river is in equilibrium then the formative discharge must be able to carry the incoming sediment load without further erosion or deposition. In other words, there must be continuity of both water and sediment transport. Discharge transport formulas show that for a given slope and roughness the required discharge capacity can be met with an infinite range of width-depth ratios. The requirement that the river must also provide a given sediment transport then imposes an additional constraint leading to a definite width-depth relationship. For a single channel river, the additional relationship is sufficient to determine the depth as a function of the width. This can be shown as follows.

This derivation is for a *discharge-defined* reach of a single channel river. A *discharge-defined* reach is one that has an insignificant local water and sediment inflow compared to what is coming from upstream. The width-depth relationship for such a reach can be arrived at in two ways: using observed regime relations and using hydraulic theory. Theoretical hydraulics considers the flow of water and sediment in a channel as governed by water discharge and a sediment transport function. Different authors used different sediment transport functions for this purpose. Straub (1935) uses DuBoys transport relation whereas Komura (1966; 1971) uses Einstein type transport function to derive such relationship. Tamburi (1976) and Vanoni (1975) reviewed the available relationship in the form of a power function and the variation of its exponent. Shewfelt (1989) attempted to establish the relationship as an addition to the regime equations.

The following derivation follows Straub (1935). It uses the Manning's formula for the discharge and the DuBoys transport formula for the sediment. The latter can be written as:

$$g_s = \psi_D \tau_0 (\tau_0 - \tau_c) \qquad (5.1)$$

where,

g_s is sediment discharge in weight per unit width,

ψ_D is a coefficient,

$\tau_0 = \gamma r_b S$ = bed shear stress,

γ is the specific weight of water,

τ_c is the critical bed shear stress at which sediment movement begins,

r_b is the bed hydraulic radius, and

S is the slope of the stream.

Equating the sediment transport in two cross-sections (1) and (2) leads to:

$$\left[\psi_D \tau_2(\tau_2 - \tau_c)\right] W_2 = \left[\psi_D \tau_1(\tau_1 - \tau_c)\right] W_1 \tag{5.2}$$

where,

$\tau = \gamma d S$ for wide channel,

d is the average depth,

W is the width, and subscripts 1 and 2 refer to the sections 1 and 2 respectively.

Since ψ_D depends on the sediment size which is considered to be the same in the two cross-sections, Eqn. (5.2) can be written as:

$$\left(\frac{d_2}{d_1}\right) = \left(\frac{S_1}{S_2}\right)\left(\frac{W_1}{W_2}\right)\left(\frac{\gamma d_1 S_1 - \tau_c}{\gamma d_2 S_2 - \tau_c}\right) \tag{5.3}$$

Equating the water transport in the two cross-sections leads to:

$$\left(\frac{S_1}{S_2}\right) = \left(\frac{W_2}{W_1}\right)^2 \left(\frac{d_2}{d_1}\right)^{\frac{10}{3}} \tag{5.4}$$

Combining Eqn. (5.3) and (5.4) one can write with a little algebraic manipulation:

$$\left[\left(\frac{d_2}{d_1}\right)^{\frac{7}{3}}\right]^2 \left(1 - \frac{\tau_c}{\tau_1}\right) + \left(\frac{W_1}{W_2}\right)\left(\frac{\tau_c}{\tau_1}\right)\left[\left(\frac{d_2}{d_1}\right)^{\frac{7}{3}}\right] - \left(\frac{W_1}{W_2}\right)^3 = 0 \tag{5.5}$$

which is a quadratic equation in $(d_2/d_1)^{7/3}$. Taking only the positive root of Eqn. (5.5) one can write:

$$\left(\frac{d_2}{d_1}\right) = \left(\frac{W_1}{W_2}\right)^{\frac{3}{7}} \left[\frac{-\frac{\tau_c}{\tau_1} + \left[\left(\frac{\tau_c}{\tau_1}\right)^2 + 4\left(1 - \frac{\tau_c}{\tau_1}\right)\left(\frac{W_1}{W_2}\right)\right]^{\frac{1}{2}}}{2\left(1 - \frac{\tau_c}{\tau_1}\right)}\right]^{\frac{3}{7}} \tag{5.6}$$

Two extreme conditions should now be considered. *If τ_c is small compared to τ_1*, the ratio becomes close to zero, and thus Eqn (5.6) can be written as:

$$\left(\frac{d_2}{d_1}\right) = \left(\frac{W_1}{W_2}\right)^{\frac{3}{7}} \left[\left(\frac{W_1}{W_2}\right)^{\frac{1}{2}}\right]^{\frac{3}{7}} = \left(\frac{W_1}{W_2}\right)^{\frac{9}{14}} = \left(\frac{W_1}{W_2}\right)^{0.643} \tag{5.7}$$

On the other hand, *if τ_c is very close or equal to τ_1*, then the stress ratio becomes unity. Substitution of this in Eqn. (5.5) yields:

$$\left(\frac{d_2}{d_1}\right) = \left(\frac{W_1}{W_2}\right)^{\frac{6}{7}} = \left(\frac{W_1}{W_2}\right)^{0.857} \tag{5.8}$$

Note that this cannot be obtained directly from Eqn. (5.6) since the right hand side under the condition of stress ratio 1, becomes indeterminate.

For a τ_c to τ_1 ratio of zero, the exponent is *9/14*; and for a ratio of *1*, the exponent is *6/7*. Thus the exponent varies between these limits depending on the value of the stress ratio. Therefore, Eqn (5.7) or (5.8) can be generalized to write:

$$\left(\frac{d_2}{d_1}\right) = \left(\frac{W_1}{W_2}\right)^{\theta}; \qquad \frac{9}{14} \le \theta \le \frac{6}{7} \tag{5.9}$$

Alternatively, one can write:

$$d = k\left(\frac{1}{W}\right)^{\theta} \tag{5.10}$$

or

$$d = kW^n; \qquad 0.643 \leq |n| \leq 0.857 \tag{5.11}$$

where k is a constant whose value depends on the river reach, and n is negative.

This relationship expresses that for a given reach of river with given sediment characteristics and a given formative discharge the depth is related to the width as shown.

5.2.2 Application to the Brahmaputra River A relationship in the form of Eqn. (5.11), was fitted to the cross-sections of the Brahmaputra River taking the total width and the depth, averaged over the entire cross section. Figure 5.1 shows the results. The least square fit of the function for the bankfull discharge is:

$$d = 103\ W^{-0.35}; \qquad [R^2 = 0.41] \tag{5.12}$$

in which W and d are in metres. Figure 5.2 shows the residuals. It is to be noted that the value of the exponent n of *-0.35* lies outside the range derived earlier for a single channel. In addition, the scatter is far too much to make the relationship useful.

The reason why relation (5.11) cannot be applied to relate width and average depth in a braided river as a whole is that the basic assumptions underlying both the Manning's equation and the sediment transport formula are violated. Both equations are based on the assumption that, the flow of water and sediment is governed by the average shear stress over the entire wetted perimeter. This is not true when the cross-section is composed of channels with widely varying cross-sectional dimensions.

The relationship of Eqn. (5.11) was derived for single channel rivers. It was investigated whether this relationship can also be applied to the branch channels of the Brahmaputra river. The average depth and the width of the branch channels of the Brahmaputra river are plotted in Fig. 5.3 with a fitted regression line in the form of Eqn. (5.11). The equation of the regression line is:

$$d = 0.30\ W^{0.327}; \qquad [R^2 = 0.36] \tag{5.13}$$

where d and W are in metres. The scatter is evidently very large. The residuals are plotted in Fig. 5.4.

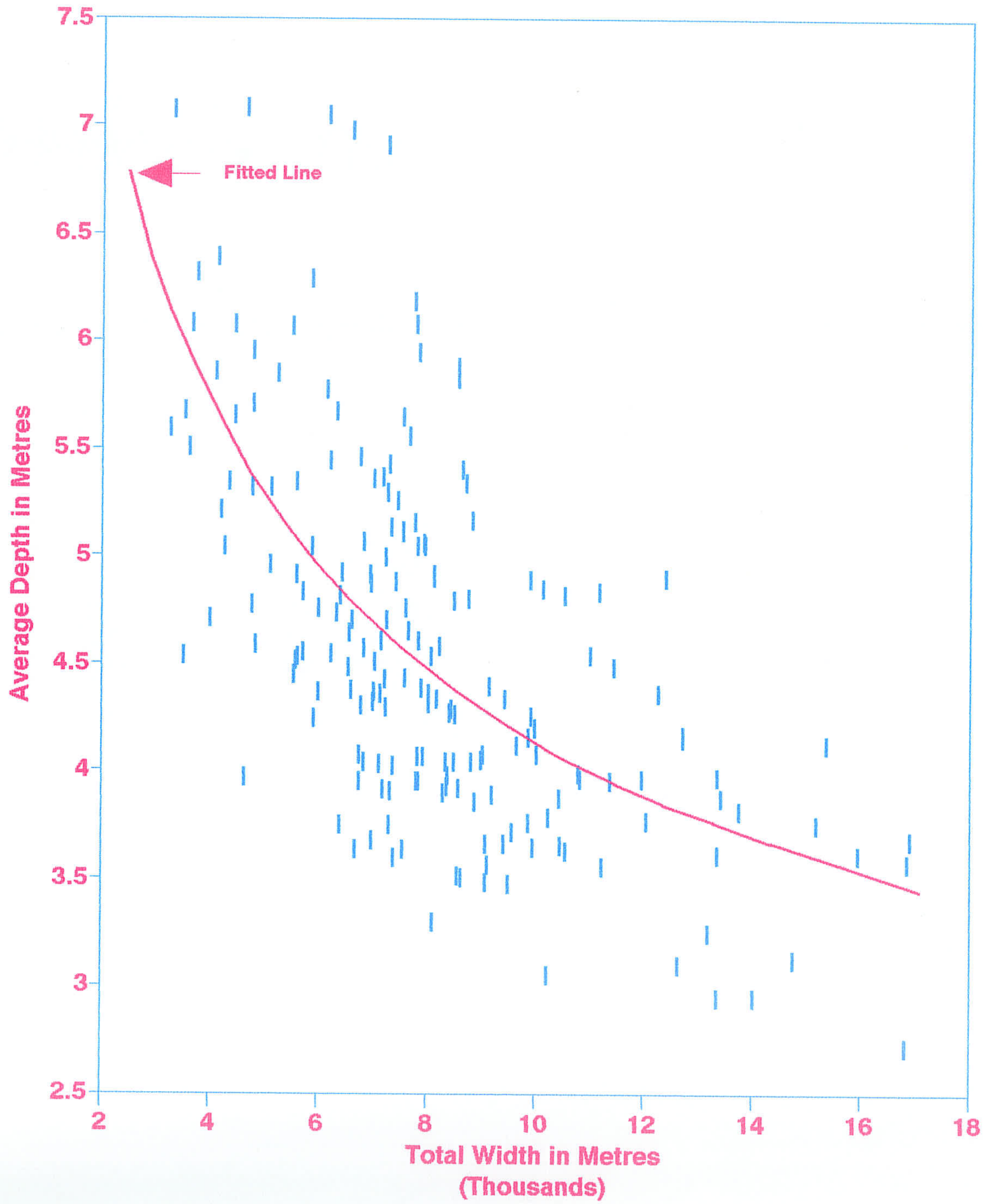

Fig. 5.1: Total Width and Depth at Bankfull Discharge

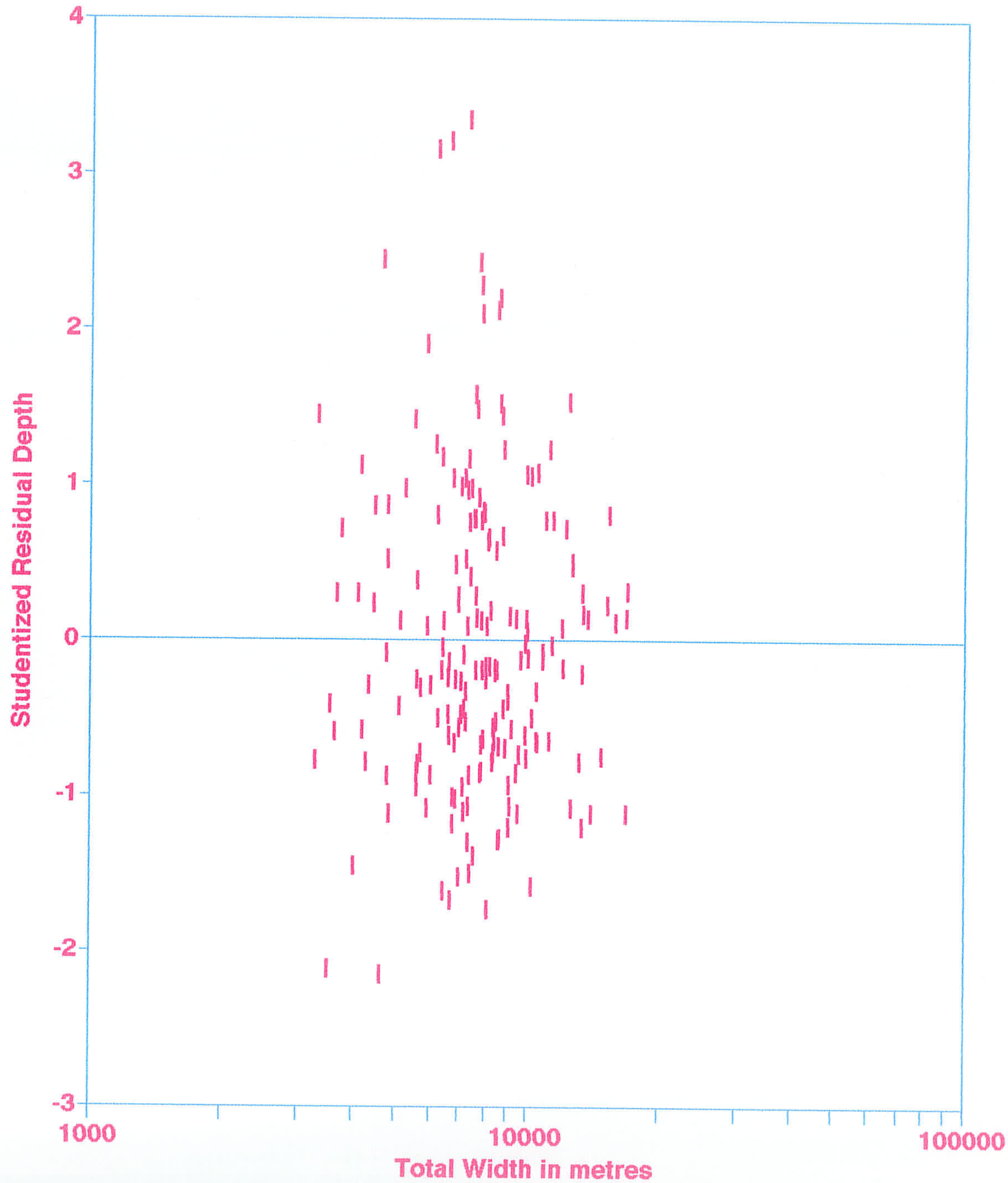

Fig. 5.2: Residuals of Total Width-Depth Relation at Bankfull Discharge

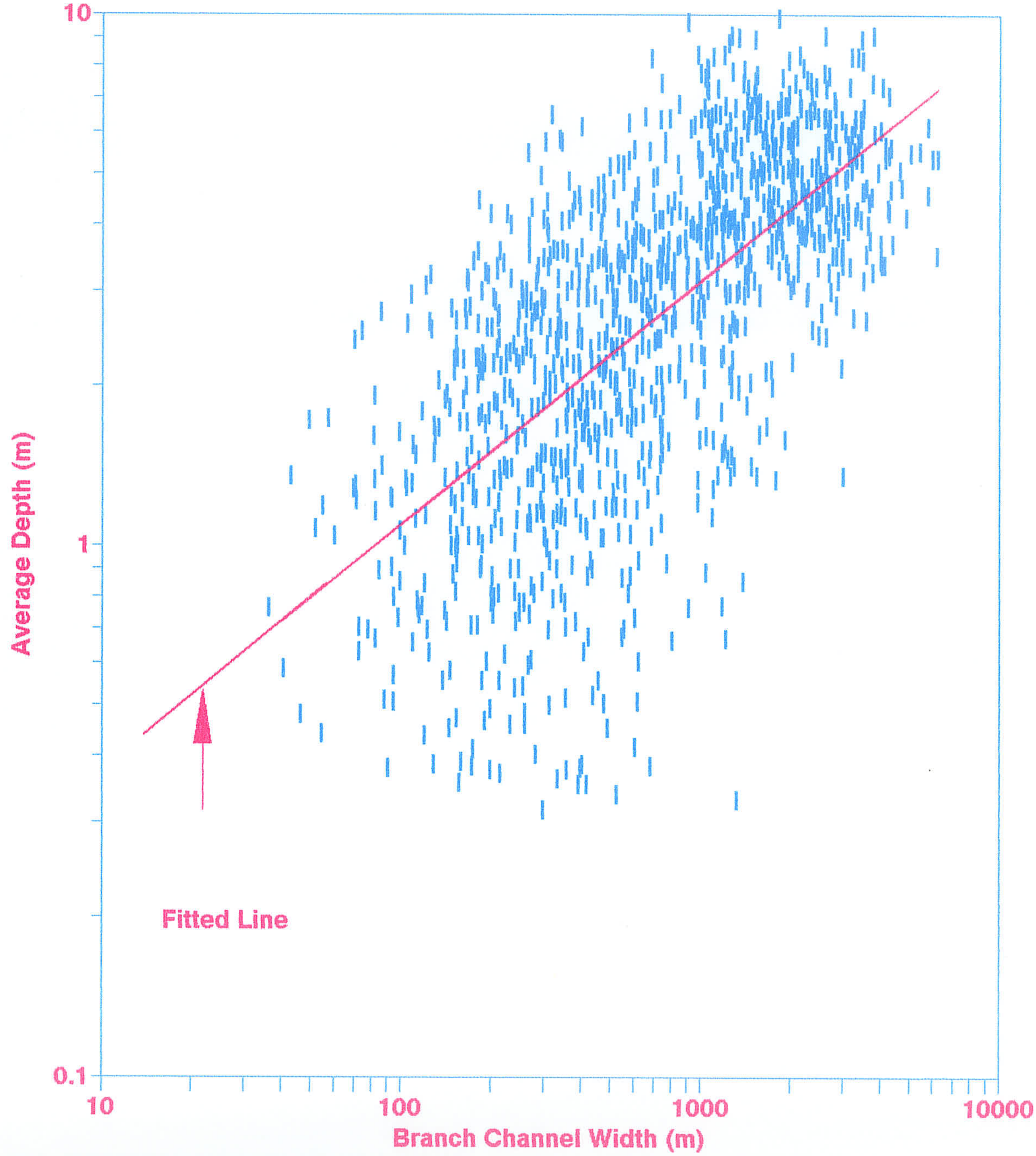

Fig. 5.3: Width and Average Depth of Branch Channels at Bankfull Discharge

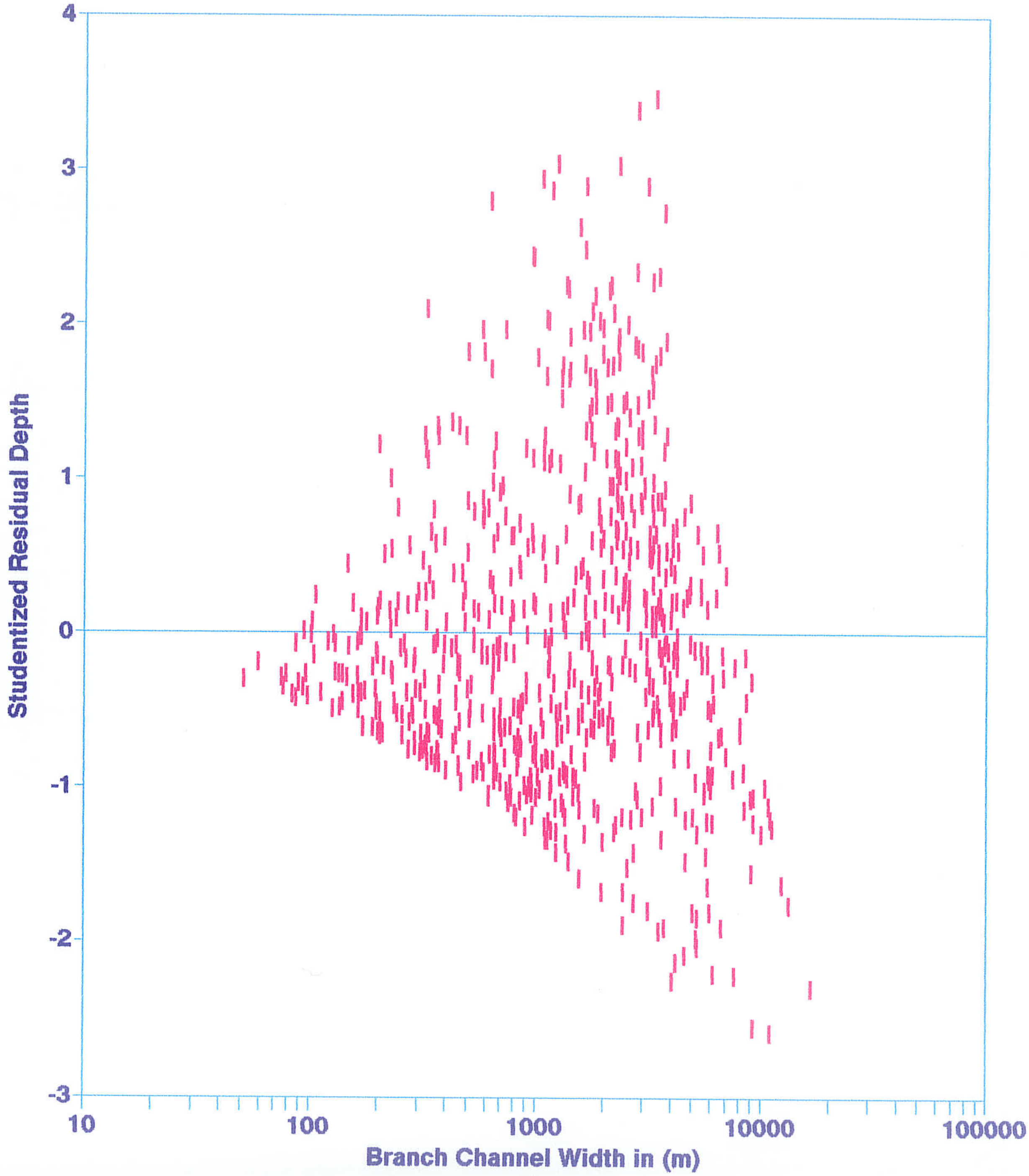

Fig. 5.4: Residuals from Width and Average Depth Relation of Eqn. (5.13)

The large scatter is caused by two factors. In the first place, the regime type relationship between width and depth holds for single channel rivers where the *constant* formative discharge causes an equilibrium between sediment deposition on the banks and bank erosion. With a braided river the discharge varies greatly over the branch channels.

In the second place, a braided river can respond to an incidental change in width or depth by a redistribution of the flow of water and sediment, which in turn will affect cross-sectional dimensions. In other words, the branch channels can be expected to be much further away from equilibrium conditions than a single channel river. For such branch channels, it is possible to have more than one value of average depth for the same width (see *Appendix-H*).

5.3 Width - Depth Relation of the Branch Channels

To derive the equilibrium condition for individual branch channels of the braided river the discharge in the branch channels must be taken into account. It may be assumed that each branch channel transports a share of the total discharge that is roughly proportional to its relative size as well as the portion of the sediment flow. Between the nodes the flow of water is constant. If a channel is not capable of transporting the incoming sediment load then the channel becomes smaller; if the transporting capacity is greater then the channel cross-section increases. A channel is in equilibrium for the given water and sediment flux when neither the width nor the depth change.

It may be argued that the average width-depth relations of the 727 branch channels investigated are probably not far removed from equilibrium conditions since one may expect that the number of channels that are widening is roughly the same as the number of channels that are narrowing. In the previous chapter the following relationships were found:

$$d = 0.2669 Q^{0.3253} \qquad (4.14)$$

$$W = 28.626 Q^{0.4521} \qquad (4.16)$$

where,

d = the average depth of the branch channel at bankfull discharge in metres,

W = the width of the branch channel at bankfull discharge in metres, and

Q = the bankfull discharge of the branch channel in m^3/sec.

Statistically these relationships are not significantly different from the regime type equations of single channel rivers:

$$d = c_1 Q^{\frac{1}{3}} \tag{5.14}$$

$$W = c_2 Q^{\frac{1}{2}} \tag{5.15}$$

where, c_1 and c_2 are constants. It follows that for a branch channel *with a given Q* one can write:

$$d = c_3 W^{\frac{2}{3}} \tag{5.16}$$

We must, however, take into account that the branch channels have widely varying discharges. The depth therefore depends not only on the width but also on the proportion of the flow the branch channel carries. The latter quantity can be expressed as a function of the ratio of the branch channel cross-section to the total river cross-section. We are therefore looking for a relationship of the form:

$$d = K W^n A_r^m \tag{5.17}$$

where,

$A_r = (A/A_T)$,

A is the area of the branch channel,

A_T is the area of the total section, and

K, n and m are constants. Eqn. (5.17) can be rearranged to write:

$$d = KW^{n}\left(\frac{dW}{A_T}\right)^{m}$$

Assuming that A_T is constant for a given number of channels, we can write:

$$d^{(1-m)} = C_1 W^{(n+m)}$$

where, C_1 is a constant. Rearranging the exponents, one can write:

$$d = CW^{\left(\frac{n+m}{1-m}\right)} \tag{5.18}$$

Comparing the exponents of Eqn. (5.16) and (5.18), one can write:

$$\frac{n+m}{1-m} = \frac{2}{3} \tag{5.19}$$

For a single channel, the Chezy formula can be written as:

$$Q = C(dW)d^{\frac{1}{2}}S^{\frac{1}{2}}$$

For a constant bankfull discharge and constant slope, Chezy formula for a single channel can be rearranged to write:

$$d = CW^{-\frac{2}{3}} \tag{5.20}$$

Note that in Section 5.2.1 we derived Eqn. (5.11) as:

$$d = kW^{n}; \qquad 0.643 \leq |n| \leq 0.857 \tag{5.11}$$

Equation (5.20) is consistent with this so that we can write:

$$n = -\frac{2}{3}$$

Substituting the value of n in Eqn. (5.19), we find:

$$m = \frac{4}{5}$$

Therefore, Eqn. (5.17) reduces to:

$$d = KW^{-\frac{2}{3}} A_r^{\frac{4}{5}} \tag{5.21}$$

It may be expected that Eqn. (5.21) represents the average of observed width-depth relationships of the branch channels of the Brahmaputra river since there are presumably as many channels that are widening and getting shallower as that are narrowing and deepening. Therefore, Eqn. (5.21) represents the equilibrium conditions for the given distribution of water and sediment. This does not necessarily mean that a channel that is out of equilibrium will attain the equilibrium condition predicted by this relationship in the next year. The frequently occurring changes in configuration cause changes in discharge and sediment distribution. Also, what happens to one channel affects the others in the cross-section.

Equation (5.21) is evidently only an approximate relationship. The values of the coefficients of Eqn. (5.17) must be estimated from the data. It is possible, however, to apply a width-depth relationship to branch channels that are in a state of relative equilibrium, that is, channels which do not change as long as the supply of water and sediment remains unchanged. The validity of the proposed form of the width-depth relation will be demonstrated in the following section.

5.3.1 Estimation of Coefficients of Width - Depth Relation A linear regression technique was employed to estimate the coefficients of Eqn. (5.17) for the bankfull discharge of the Brahmaputra river. A total of 727 data points were available for the entire river. The resulting equation is:

$$d = 1790\ W^{-0.656} A_r^{0.773}; \quad [R^2 = 0.90] \tag{5.22}$$

where,

W is the width of the branch channels in metres,

d is the average depth of the branch channels in metres, and

A_r is the ratio of each branch channel area to the area of the total cross-section.

The residual plot is given in Fig. 5.5. The residuals were found to be independent, normal and follow the homoscedasticity assumption. Similar results were found for the average discharge of the Brahmaputra river.

The relationship described by Eqn. (5.22) represents the average conditions of the entire river corresponding to the bankfull discharge. The coefficients, however, will be different for different reaches because the slope in the six reaches is not the same and the total cross-sectional area varies. For this reason the coefficients in the equation were determined for each reach separately. The resulting regression equations for each reach together with the value of R^2 are given below:

$$Reach\ 1:\ d = 3190\,W^{-0.729}A_r^{0.823}; \quad [R^2 = 0.90] \tag{5.23}$$

$$Reach\ 2:\ d = 2600\,W^{-0.711}A_r^{0.818}; \quad [R^2 = 0.90] \tag{5.24}$$

$$Reach\ 3:\ d = 7115\,W^{-0.806}A_r^{0.882}; \quad [R^2 = 0.92] \tag{5.25}$$

$$Reach\ 4:\ d = 1600\,W^{-0.634}A_r^{0.728}; \quad [R^2 = 0.88] \tag{5.26}$$

$$Reach\ 5:\ D = 660\,W^{-0.543}A_r^{0.688}; \quad [R^2 = 0.92] \tag{5.27}$$

$$Reach\ 6:\ d = 9710\,W^{-0.861}A_r^{0.909}; \quad [R^2 = 0.93] \tag{5.28}$$

the variables of which are described before. Note that these equations are for the bankfull discharge and the dimensions are in metric units. Similar results were found for the average discharge.

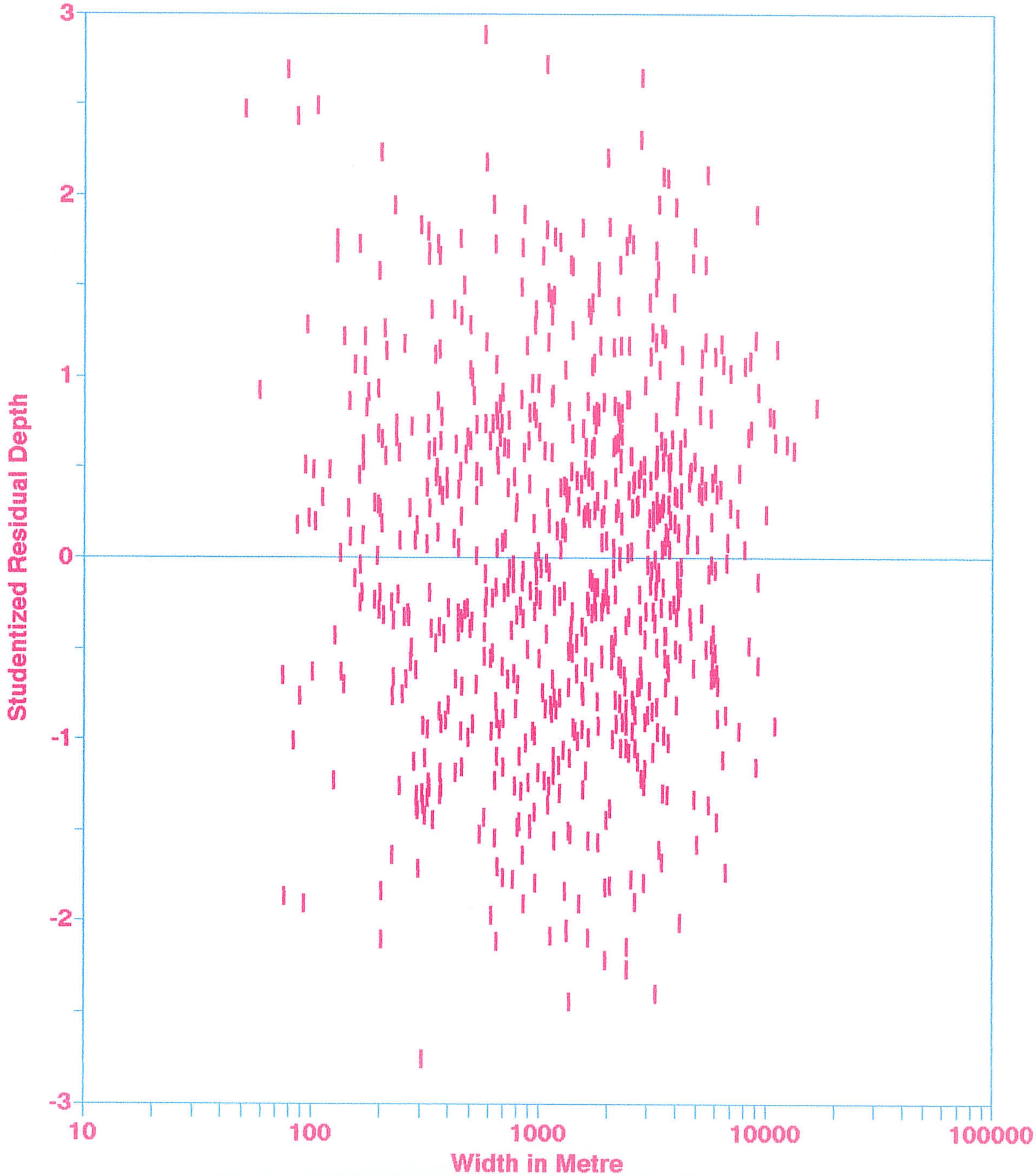

Fig. 5.5: Residuals from Width and Average Depth Relation of Eqn. (5.22)

The equations for different reaches of the river (Eqn. 5.23 to 5.28) were tested to determine whether they are significantly different from each other. The General Linear Test (see Neter, Wasserman & Kutner, 1985) was employed for this purpose. The differences between the equations were found to be statistically significant (at 95% confidence level). Therefore, it is concluded that separate equations should be used for different reach of the river.

Table 5.1 lists the values of the coefficients *K*, *m* and *n* of Eqn. (5.17) for different reaches as well as for different year of record. Note that the values of *n* for the entire reach as well as for different sub-reaches are negative and close to the constraint imposed on the equation for the single channel river (Eqn. 5.11). The Table also provides the values of R^2 and the number of observations used for developing these relationships.

A comparison of the coefficients for the reaches and the years shows considerable variation. With regard to the variation over the years, it should be noted that equilibrium conditions were disturbed during the years because of changes in discharge and sediment transport and as a consequence of channel splitting and joining. It would seem, therefore, that the average relationship over all years probably presents a better relationship of equilibrium conditions than the data for the individual years.

The differences between the sections, on the other hand, may reflect different conditions of river regime. The observed difference in behaviour of the river along different reaches and with time in terms of the anastomosity and the number of branch channels, as discussed before, suggest such variation. Reach 5 represents a distance between nodes that is almost twice the distance between the other nodes. Reach 6 is the furthest upstream where the river enters Bangladesh. It would seem advisable to use for each reach the depth width relationship derived for it. Note that the data from 1983 were not included in these relationships since they were not used for any prediction purposes as subsequent data were not available for this study.

From the above discussion it is evident that single channel equations should not be used for a braided river, particularly for the Brahmaputra river in Bangladesh. However, the equation for the braided river (Eqn. 5.17) is valid also for a single channel river. The value of the term A_r which is the ratio of the branch channel area to the total sectional area, is unity for a single channel river. Thus, putting $A_r = 1$ in Eqn. (5.17) yield a relationship of the form of Eqn. (5.11), which is the equation for a single channel river.

Table 5.1: Reach and year-wise coefficients of depth-width

Reach	Coeffi-cients	All Data	1976 Data	1977 Data	1978 Data	1979 Data	1980 Data	1981 Data
All	K	1790	703	2465	3390	1450	1290	3625
	m	0.7731	0.7010	0.7947	0.8205	0.7565	0.7419	0.8410
	n	-0.6560	-0.5483	-0.6943	-0.7265	-0.6363	-0.6145	-0.7397
	N	727	103	116	120	133	119	131
	R^2	0.89	0.89	0.90	0.90	0.88	0.88	0.91
1	K	3190	5350	7820	10820	1695	1390	7115
	m	0.8232	0.8748	0.8907	0.9229	0.7653	0.7451	0.8867
	n	-0.7293	-0.7871	-0.8343	-0.8674	-0.6713	-0.6299	-0.8172
	N	142	16	24	24	27	24	22
	R^2	0.90	0.91	0.96	0.95	0.85	0.86	0.95
2	K	2600	890	3985	24215	1527	9034	5375
	m	0.8180	0.7613	0.7885	0.9724	0.7971	0.9065	0.8967
	n	-0.7110	-0.6059	-0.7720	-0.9722	-0.6474	-0.8488	-0.7807
	N	87	10	15	8	12	21	16
	R^2	0.90	0.97	0.77	0.99	0.95	0.98	0.93
3	K	7115	1710	4410	13125	8507	60395	18810
	m	0.8815	0.7543	0.8497	0.9333	0.9143	1.0270	0.9534
	n	-0.8056	-0.6413	-0.7513	-0.8715	-0.8388	-1.0325	-0.9165
	N	155	24	26	25	33	16	26
	R^2	0.92	0.92	0.94	0.96	0.93	0.97	0.93
4	K	1600	1885	4490	8360	2143	1605	4475
	m	0.7278	0.7542	0.8124	0.8158	0.7449	0.7065	0.8493
	n	-0.6341	-0.6598	-0.7476	-0.8152	-0.6682	-0.6191	-0.7605
	N	100	16	11	16	14	18	20
	R^2	0.88	0.92	0.91	0.86	0.88	0.95	0.96
5	K	660	540	3700	1950	436	425	470
	m	0.6880	0.6795	0.8424	0.7518	0.6525	0.6819	0.6558
	n	-0.5427	-0.5062	-0.7503	-0.6725	-0.4869	-0.4880	-0.5100
	N	120	17	18	16	24	18	22
	R^2	0.92	0.91	0.95	0.91	0.91	0.96	0.93
6	K	9710	8415	14900	10990	23250	6865	14560
	m	0.9091	0.8852	0.9373	0.900	0.9676	0.8694	0.9686
	n	-0.8613	-0.8416	-0.9086	-0.8776	-0.9563	-0.8294	-0.9218
	N	118	15	17	26	18	17	20
	R^2	0.93	0.98	0.94	0.96	0.98	0.92	0.97

Note: N is the number of observation in the subgroup; R^2 is the coefficient of determination.

5.4 Width-Depth Relation as a Qualitative Predictor of Cross-sectional Change

The equilibrium width-depth relationships derived in the preceding section can be used to predict changes in branch channel width, at least qualitatively. It may be assumed that, while the branch channels are subject to frequent change they tend to return to dimensions for which they are in a state of equilibrium under the imposed water and sediment influx. One cannot expect them to always reach such equilibrium conditions because the equilibrium is continually upset by changes in planform geometry caused by splitting and joining of the channels, which changes the water and sediment distribution. One would expect, however, that the developed relations can be used to predict whether a branch channel tends to get narrower or wider in the following year.

In order to verify this the deviations of the equilibrium width from the observed width were plotted against the actual changes that occurred in the following year. The results are shown on Fig. 5.6. Similarly, the predicted changes in depths of the branch channels were plotted against the observed changes in depths in Fig. 5.7. Note that positive values indicate bank deposition or narrowing of the channel while negative values indicate bank erosion or widening of the channel.

It should be pointed out that the data that are plotted in Figs. 5.6 and 5.7 were obtained from 266 branch channels that neither split into two or more nor united with other branches in the following year. The phenomenon of splitting and joining, which evidently requires new equilibrium conditions, will be discussed in a following section.

Figures 5.6 and 5.7 show that more than 90% of the points are in the first and third quadrants. This shows that a comparison with the equilibrium dimensions enables one to predict the direction of changes in widths and depths of the branch channels with over 90% accuracy.

Figures 5.6 and 5.7 also show large deviations from the line of perfect agreement. This implies that the proposed comparison is not very effective in predicting the magnitude of the changes. This is not surprising since the changes in one branch channel affects the changes in the others. A method for quantitative prediction of change will be derived in the next chapter.

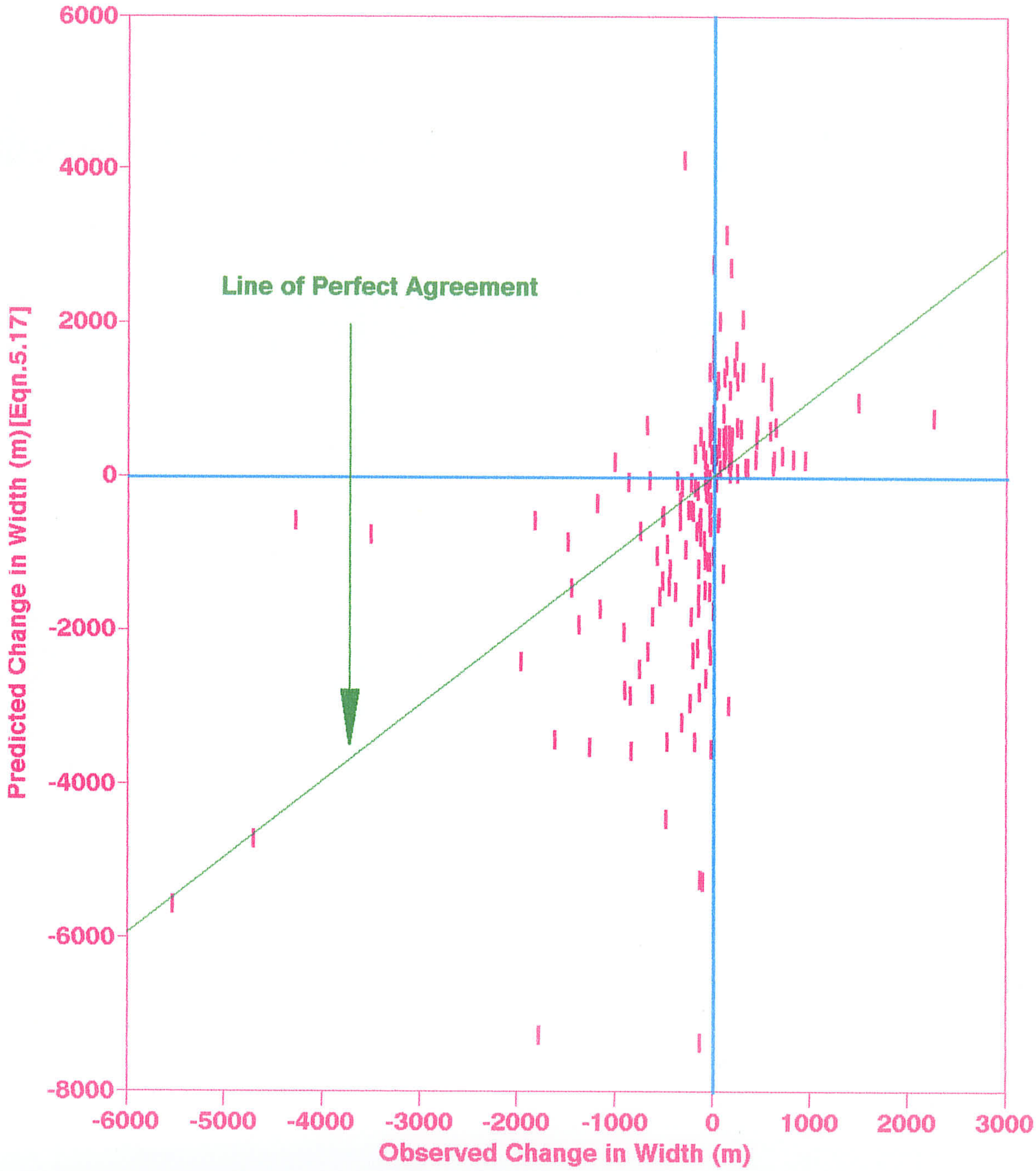

Fig. 5.6: Observed and Qualitatively Predicted Changes in Width

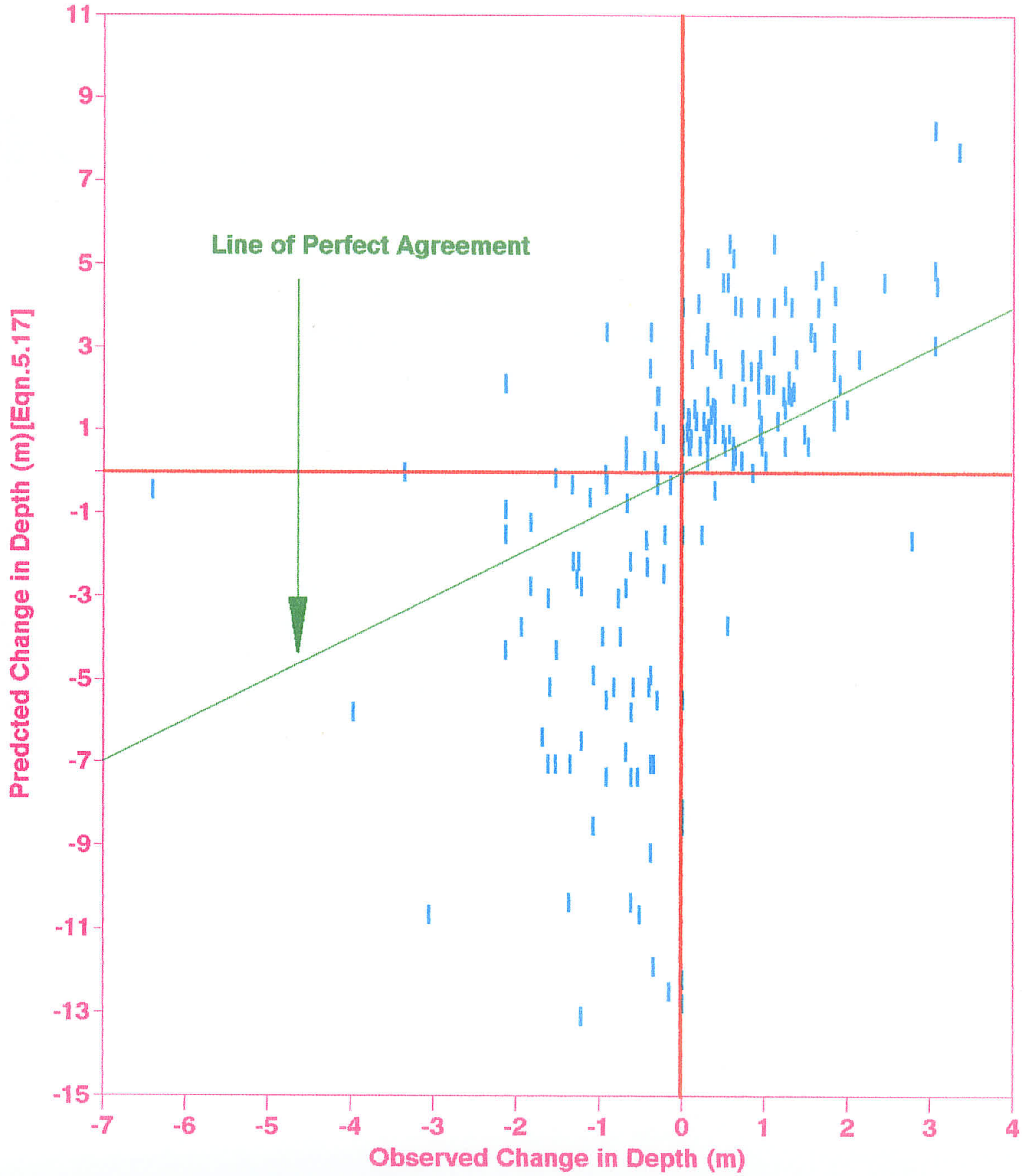

Fig. 5.7: Observed and Qualitatively Predicted Change in Depth

5.5 Effect of Width-Depth Changes on Planform Geometry

Changes in planform geometry occur when the width-depth changes cause (a) a wide branch channel to split in two or more narrower channels; (b) two or more channels unite to form a single branch channel; (c) an altogether new channel is created; and (d) an existing channel is closed resulting into a changed number of channels in a cross-section.

It may be assumed that the formation of a new channel and the disappearance of an old channel are special cases of splitting and joining of channels. Splitting and joining are both related to changes in channel width.

Theoretically, two channels could join by a lateral displacement of one or both without a change in width. As a rule, however, bank erosion is a result of mismatch between the channel dimensions and the flow of water and sediment imposed on the channel. One can therefore expect that joining is mostly associated with a widening process. This will occur when the width is smaller than the equilibrium width.

Channels are not likely to split when they are narrow and deep. This could suggest that splitting is also associated with a trend towards widening. Such a conclusion, however, would be premature since channels must already have attained a relatively large width in order to split.

In the following section methods for the prediction of channel joining and channel splitting will be developed.

5.5.1 Bank Movement Associated with Channel Widening The preceding section discussed cross-sectional change and it was argued that the joining of two channels is in practically all cases the result of channel widening. The direct cause of channel joining is bank movement. It is therefore necessary to relate bank erosion to channel widening. If the centre of gravity of the channel cross-section is in the middle, the flow may be taken to be symmetric about the centre. In this case, for a widening channel, bank erosion can be expected to occur evenly along both banks. If the centre of gravity is closer to one bank (thalweg closer to the bank), the bulk of flow will be concentrated to that bank and the flow would be asymmetric. The erosion of that bank can be expected to be higher than the other by an amount proportional to the deviation of the centroid from the mid-point. For a narrowing channel, deposition would occur in the reverse way i.e., deposition to the bank closer to the thalweg will be lesser than to the other bank.

It is therefore assumed that the distribution of the amount of erosion on both banks would be inversely proportional to the distance of the centroid from the banks. On the other hand, the amount of deposition on the banks would be directly proportional to the distance of the centroid from the banks. This assumption implies that the composition of bed and bank material is such that it is homogeneous across the channel. This is true for the Brahmaputra river in Bangladesh since the size distribution of the fine sand found on bed and bank of the river is practically the same for the entire river. The assumption also requires that the curvature of the channel is negligible so that any effect of local currents can be ignored. This implied assumption of velocity distribution is a simplified situation, and would introduce some errors.

Based on the assumption discussed above, the amount of bank movement can be computed in terms of the equilibrium width of the channel. The derivation of the expressions for movement are given in *Appendix-I*. In order to keep a common reference, all the distances are measured from the mid-point of the existing channel. For a widening channel where the equilibrium width is greater than the existing width, the amount by which a bank moves is given by:

$$[W_m]_L = \frac{(W_e - W)(W + 2x)}{2W} \tag{5.29}$$

$$[W_m]_R = \frac{(W_e - W)(W - 2x)}{2W} \tag{5.30}$$

where,

W_m is the amount by which the bank moves (in this case erodes),

W_e is the equilibrium width,

W is the width,

x is the distance of centre of gravity of the channel section from the centre (mid-width) of the section, positive to the left,

L is a subscript for the left bank, and

R is a subscript for the right bank.

For a narrowing channel where the equilibrium width is less than the existing width, these movements (depositions) are given by:

$$[W_m]_L = \frac{(W_e - W)(W - 2x)}{2W} \tag{5.31}$$

$$[W_m]_R = \frac{(W_e - W)(W + 2x)}{2W} \tag{5.32}$$

in which the variables are as described before. Note that Eqn. (5.29) and (5.30) are valid for widening channels where the bank movements are erosion, while Eqn. (5.31) and (5.32) are for narrowing channels where the bank movements are deposition.

The derivation of Eqn. (5.29) to (5.32) is based on a very simple rectangular cross-section, banks of which are either eroding or experiencing deposition at the same time. The total change in width is assumed to be linearly distributed over the total width with an inverse relationship to the distance of the bank from the centroid of the section. In other words, the bank closer to the centroid undergoes more erosion than the bank further away from the centroid for a widening channel. On the other hand, the bank closer to the centroid experiences lesser deposition than the bank further away from the centroid for a narrowing channel (see *Appendix-I*).

This principle can be used to predict the joining of two channels as described in the following section.

5.5.2 Channel Joining When the equilibrium width of the channel is larger than the observed width, the channel widens. In the process of widening a channel joins another channel if one exists (or moves in) within a distance of movement defined by Eqn. (5.29) or (5.30) from the bank.

All the channels that joined one another in the subsequent years were identified for the Brahmaputra river. It was verified that for all of these branches, the equilibrium width is larger and the depth shallower than the existing dimensions. Also, the combined erosion predicted by Eqn. (5.29) or (5.30) for the two adjacent channels either equals or exceeds the distance between the banks of the two channels. Therefore, it may be concluded that the criterion described above is a good predictor of channel unification. However, it is to be noted that not all the channels which followed the above condition did join together. This means that in some cases, the amount by which the channels eroded their banks were less than the amount expected according to the prediction. This discrepancy is, of course, expected because of the statistical nature of the equilibrium relation and also because of local conditions that have not been considered. On the whole, however, the ability of this method to predict channel joining is useful.

A more exact method requires a quantitative prediction of the amount of bank erosion. This will be discussed in the next chapter.

5.5.3 Channel Splitting Channel splitting is also important in the study of bank movement since it may cause severe erosion of the outer banklines. Also, a radical change in one of the channels may upset the equilibrium of the outer channels.

One hundred and seven channel splitting events were studied for the Brahmaputra river in an attempt to determine parameters that can be used to predict whether or not splitting will occur. Consideration of geometric properties in an attempt to estimate the potential of a channel to divide into two did not produce satisfactory results. As described in the previous chapter, the study of the stable bifurcation in an attempt to select a criterion for a threshold value beyond which the channel would bifurcate, was not able to identify any such criterion.

To investigate whether the deviation from the equilibrium width causes channels to split, these 107 channels that did split in the following year were studied further. For these channels, a deviation ratio was defined as:

$$\Delta W_r = \frac{(W_e - W)}{W_e} \tag{5.33}$$

where,

ΔW_r is the deviation ratio,

W_e is the equilibrium width of the channel predicted by Eqn. (5.17), and

W is the existing width of the channel.

This deviation ratio can be viewed to be analogous to the stress ratio (lateral to axial) which influences the crackline propagation and splitting in brittle materials as observed in solid mechanics (see Hori & Nemat-Nasser, 1986). It was not possible to find any threshold value of the relative deviation ratio beyond which the channels would divide. Seventy one of these channels (66%) that bifurcated followed the condition of shallowing-widening and the values were scattered all over. The bar graph in Fig. 5.8 shows the result of this analysis. Note that the groups in the figure are shown with the mid-point of the interval. It should be noted that a substantial number of channels (34%) that bifurcated were not in the process of shallowing and widening. Perhaps, the local conditions that have not been taken into account had a stronger influence on these channels. Fifty two percent of the channels that did split in the following year had the deviation ratio greater than 0.2. It is to be noted that not all the channels that had the deviation ratio greater than 0.2 did divide in the following year. The analysis was repeated by redefining the deviation ratio as the ratio of the deviation from the equilibrium width to the existing width (instead of equilibrium width as in Eqn. 5.33). Similar results were found with no definite threshold value of the ratio.

The width to depth ratio of these 107 channels were analyzed to find a threshold value of the ratio beyond which a channel would bifurcate. The result is shown in Fig. 5.9 as a bar graph. Also plotted on this graph are the results of 266 channels that did not bifurcate in the following year. Since the total number of channels in these two categories are different, the frequencies are expressed as percentages. This would allow a comparison of distribution between these two.

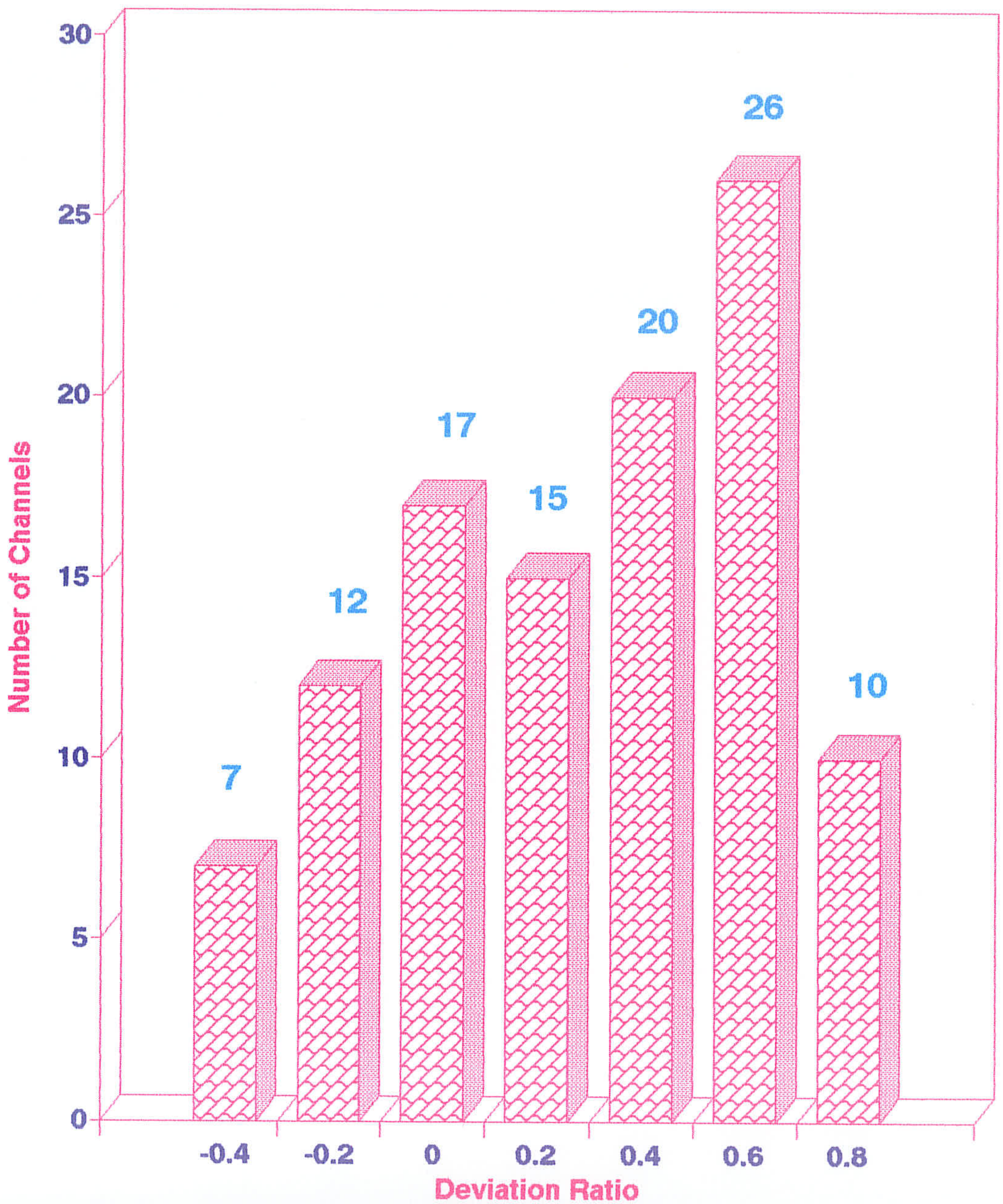

Fig. 5.8: Relative Deviation Ratios of the Bifurcated Channels

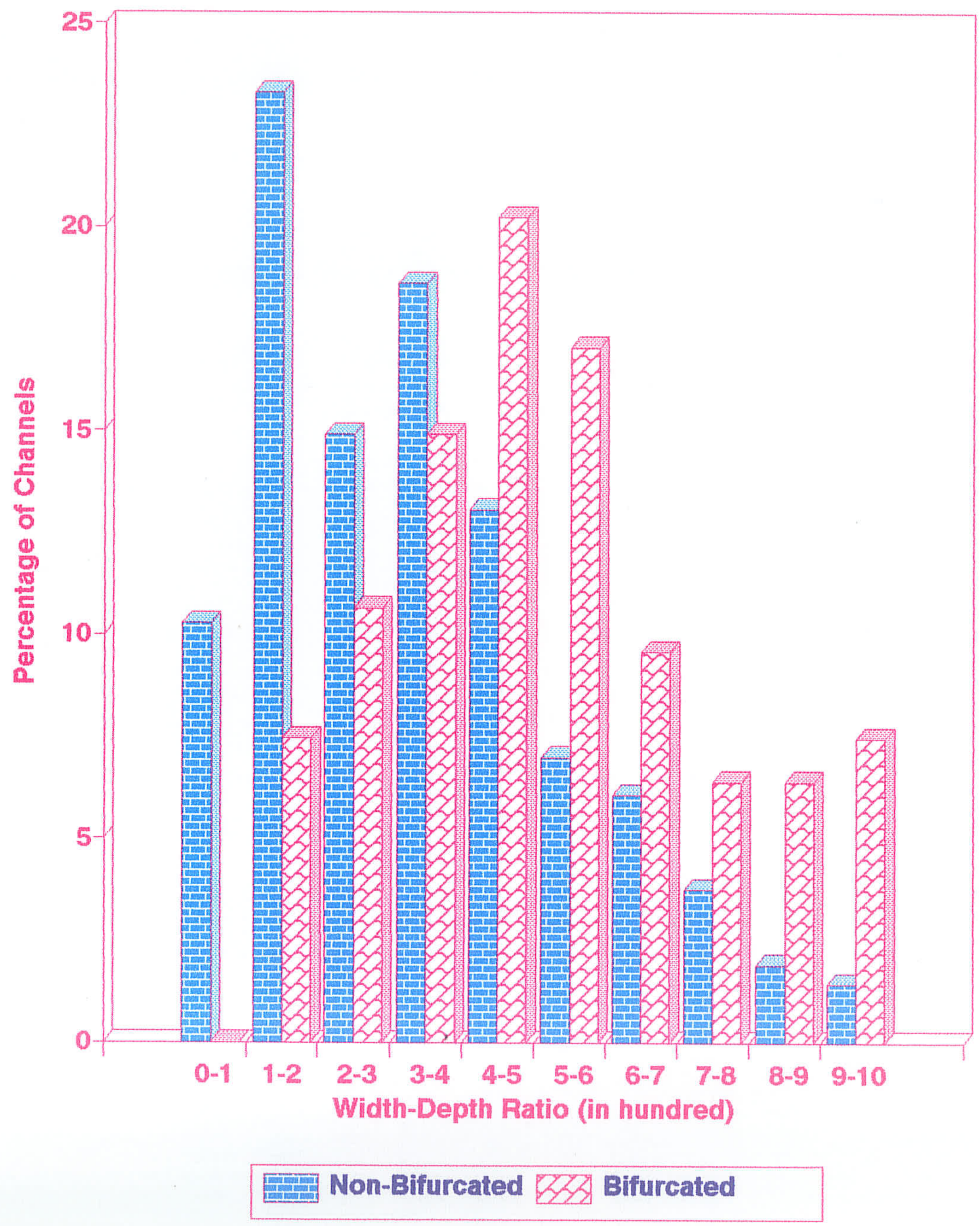

Fig. 5.9: Width-Depth Ratios of Bifurcated and Single Channels

About 87% of the channels that bifurcated in the following year had a width to depth ratio between 100 to 800, the majority being in the range of 300 to 600 (about 52%). None of the channels that bifurcated was found to have a width-depth ratio less than 100. However, it cannot be said that if a channel has a width to depth ratio greater than 100, it will split in the following year. This is because there are a large number of channels in this range that did not divide in the following year.

Figure 5.9 also shows that the width to depth ratios of the majority of the branch channels that did not bifurcate in the following year are lower than those of the bifurcated channels. The distributions of these two groups of measured channels are given in Fig. 5.10 in which the probability of having a width-depth ratio of certain magnitude or less of the bifurcated and the non-bifurcated channels are plotted separately. These probabilities of observing a channel with a width-depth ratio of less than or equal to certain magnitude were computed by using Hazen plotting position formula for each of the two groups (bifurcated and non-bifurcated). This figure also shows that the channels that were observed to bifurcate in the following year generally had higher width to depth ratio.

The observed branch channels of the two groups were combined together to determine the probability of bifurcation of a channel. The probability of bifurcation is defined as:

$$p(B) = \frac{n(x>X)}{N(X)} \qquad (5.34)$$

where,

$p(B)$ = probability of bifurcation of a branch channel with a width-depth ratio greater than X,

$n(x>X)$ = the number of bifurcated branch channels that were observed to have width-depth ratio greater than X,

x = width-depth ratio,

X = the width-depth ratio of the branch channel for which the probability of bifurcation is to be determined, and

$N(X)$ = the total number of observed channel with width-depth ratio greater than X.

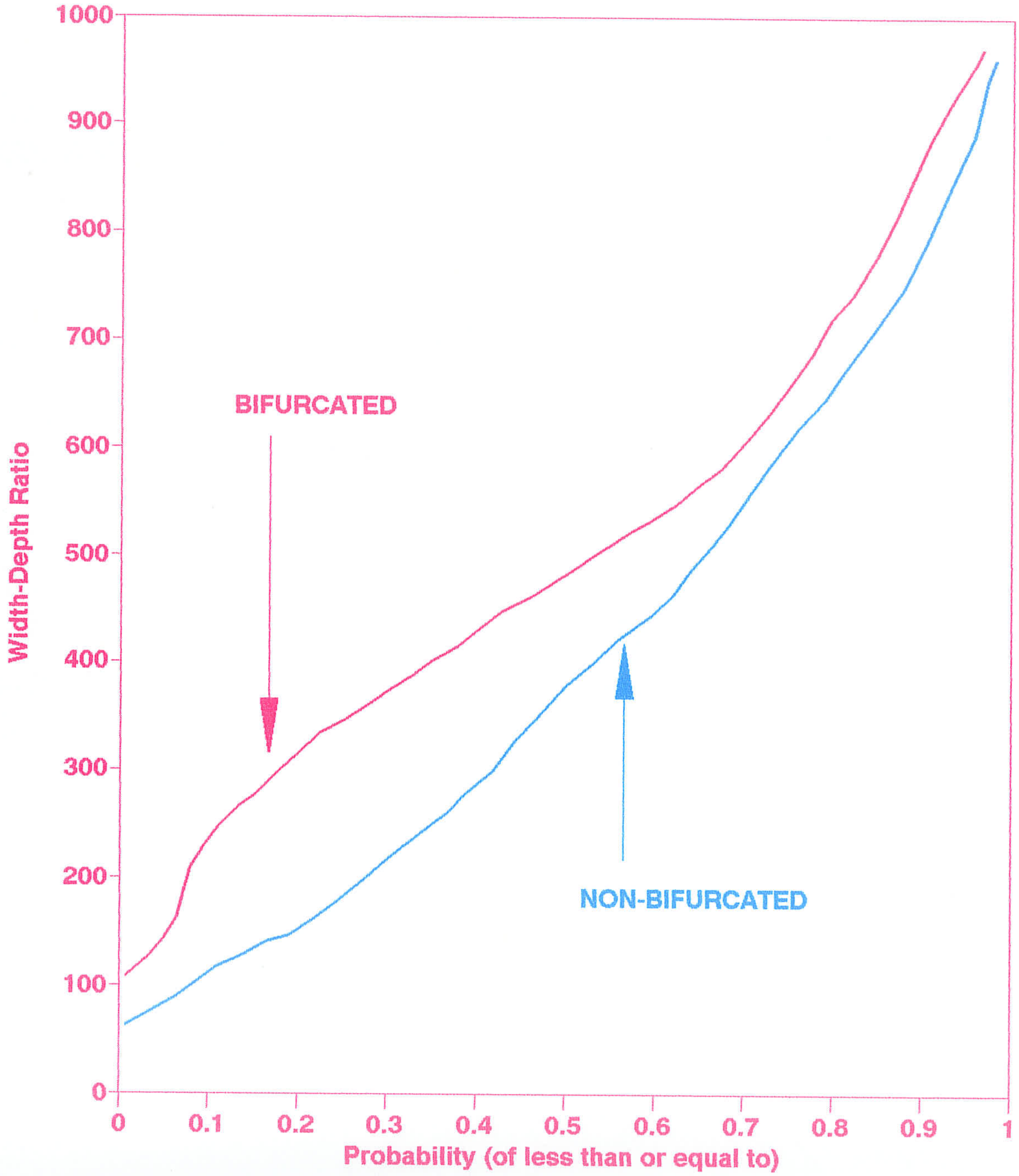

Fig. 5.10: Distribution of Width-Depth Ratios

Note that $N(X)$ contains both the bifurcated and the non-bifurcated channels. For this analysis, $N(X)$ was 373.

The computed probability of bifurcation is plotted in Fig. 5.11. The least square fitted line is also shown in this figure. The figure shows that branch channels with higher width-depth ratio have higher probability of dividing into two (or more). For example, a channel with a width to depth ratio of 350 or more has about 40% chance of dividing, whereas a channel with a width-depth ratio of 950 or more has about 70% chance of dividing in the following year. A higher value of width to depth ratio implies a wider and shallower channel. It seems plausible that such a wider and narrower channel will have a higher probability of bifurcating as indicated by the plot. The increase in probability of bifurcation with the increase in width to depth ratio is demonstrated by the positive slope of the line in Fig. 5.11.

It should be pointed out that the data plotted in Fig. 5.11 contains the branches that were either undivided or were divided into only two channels in the following year. Channels that were divided into more than two were not included in the figure. Identifying the parent channel in such a situation became too cumbersome, and was not done in this analysis. Note that all of the channels were manually checked to determine whether they remain undivided or not.

The width to depth ratios of the branch channels were plotted against the width of the branch channels in Fig. 5.12. Channels that were observed to bifurcate in the following year are shown by '+' and the undivided channels are shown by vertical lines. A closer look at the plot reveals that the majority of the bifurcated channels are located in the upper left half, while the majority of the undivided channels are in the lower half. There exists a zone in the middle of the plot where channels of both groups are located. This suggests a possible threshold band above which channels are bifurcated.

An arbitrary line is drawn on the plot in Fig. 5.12 above which the points correspond to the channels that were bifurcated in the following year. This line can be described as:

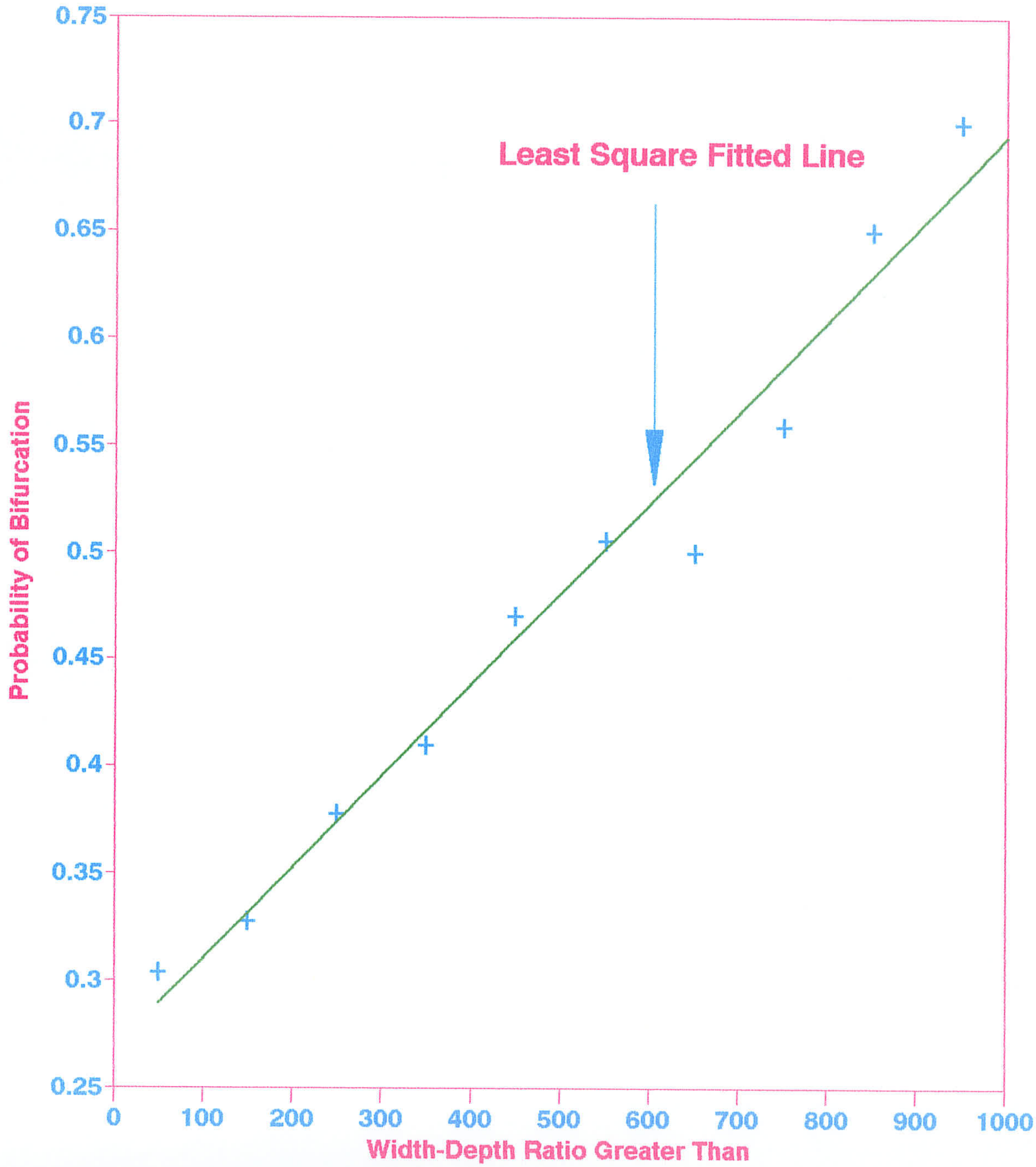

Fig. 5.11: Probability of Bifurcation

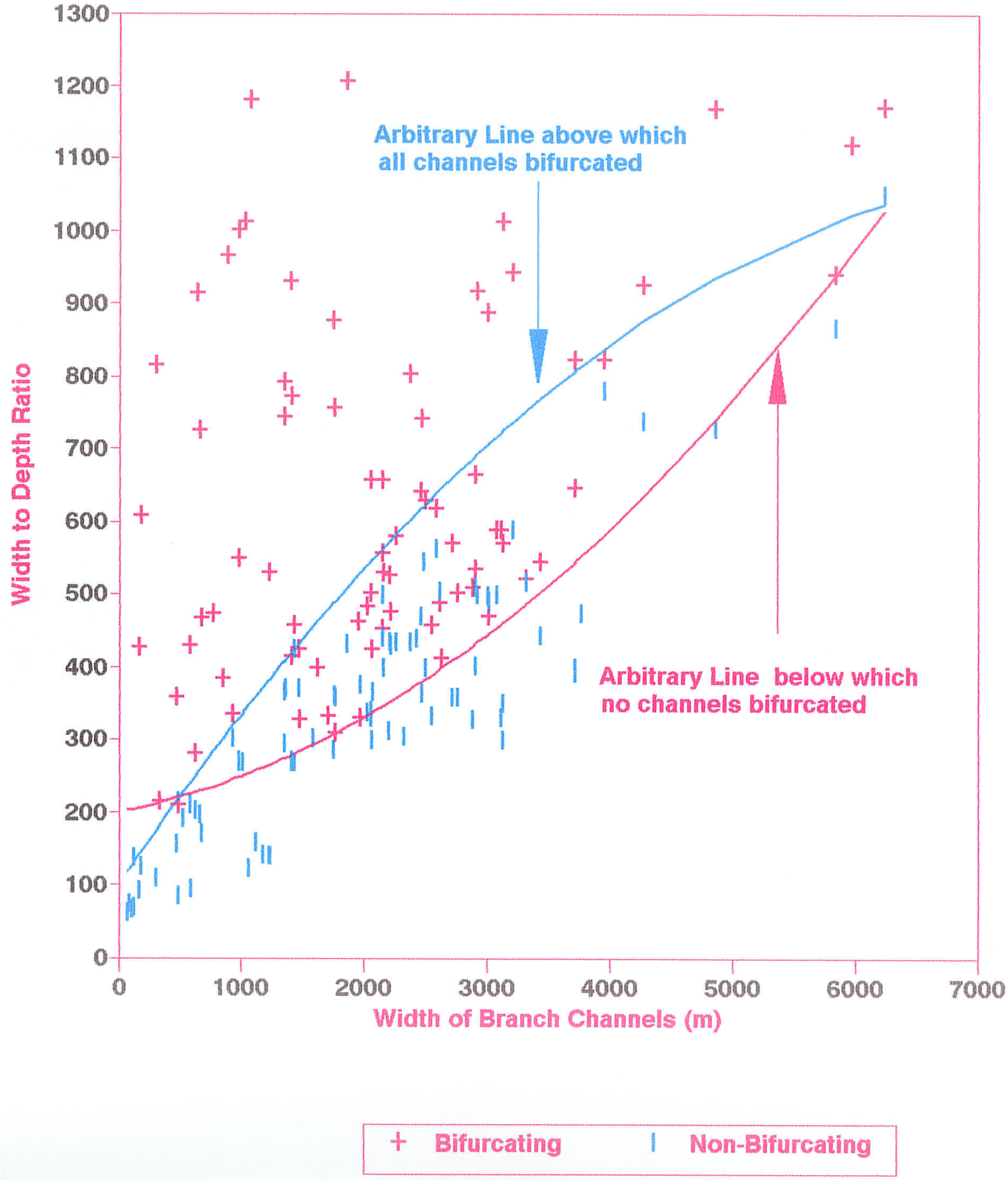

Fig. 5.12: Threshold Criterion of Bifurcation

$$\left(\frac{W}{d}\right) = -\left(\frac{W}{250}\right)^2 + \left(\frac{W}{4}\right) + 100 \tag{5.35}$$

where,

W = the width of the channel in metres, and

d = the depth of the channel in metres.

For a given channel, if the width to depth ratio plots above the line, it is likely that the channel will divide in the following year. In other words, if the computed width-depth ratio from Eqn. (5.35) is less than the observed width-depth ratio, the channel will split into two. The reverse, however, is not true. If the computed ratio is higher than the observed (i.e., the observed point plots below the line), the channel may or may not split. It is therefore required to draw a similar line of demarkation between the transition zone and the zone in which all the channels continue to remain undivided.

A second arbitrary demarkation line is drawn on Fig 5.12. Channels below this line were observed to remain undivided in the following year. This line is expressed as:

$$\left(\frac{W}{d}\right) = \left(\frac{W}{250}\right)^2 + \left(\frac{W}{30}\right) + 200 \tag{5.36}$$

in which the variables are as described before. As before, channels that plot above this line may or may not remain undivided. Although two lines of demarkation could be drawn on Fig. 5.12, a definite threshold value could not be found.

One source of error is that, in this study, the formation of a new by-pass channel, as has been reported to occur in the Brahmaputra river (e.g., BWDB, 1978b; Galay, 1980), has been considered to be the product of splitting of the nearby channel. The formation of a new channel may have been due to an entirely different process other than the shallowing-widening process. This could also happen due to some existing local condition. Whatever changes occur in a branch channel of a braided river may not only depend on the conditions prevailing at the branch but also on the other branches of the section. In that regard, splitting may not be completely explained by the conditions of the branch channel alone.

From the above analyses, it was not possible to establish a criterion which is able to completely preserve the splitting characteristics of the branch channels of a section of the Brahmaputra river. However, a fair degree of success was achieved in obtaining a width-depth threshold criterion to partly describe splitting. A more detailed study considering other local characteristics of sections is needed to identify a splitting criterion.

5.5.3 Bank Movement and Width - Depth Relationship Because the total section must transport total water discharge and total sediment load, there will be an interrelationship between the changes in the branch channels. In this study, an attempt was made to utilize the quantitative prediction capability of Eqn. (5.17) to find a simple relationship between the movement of the outer banklines of the Brahmaputra river and other variables as discussed in *Appendix-J*. Results showed that the behaviours of different reaches were different.

5.6 Summary

The width and depth relationship of the Brahmaputra river is discussed in this chapter. It was found that the relationship for a single channel river cannot adequately describe the conditions of a braided river. A new formulation of width and depth relationship of the branch channels of a braided river is proposed. More variables are needed for such relationships of a braided river. Use of this relationship for explaining bank movement was partially successful. A threshold band for width-depth ratio was found corresponding to which a channel would split. However, the transition zone that was found to exist is too wide and the channels within the zone could not be identified to possess (or not to possess) splitting characteristics.

Chapter Six

DEVELOPMENT OF A QUANTITATIVE PROBABILISTIC BANK MOVEMENT MODEL

6.1 Introduction

Qualitative models have been developed in Chapter Five to predict (a) the direction of change in channel width, and (b) the likelihood of splitting and joining. Bank movement was shown to be erratic and affected by changes in planform geometry. The process does not lend itself to deterministic analysis.

The purpose of the research, described in this chapter, is the development of a probabilistic bank movement model that is aimed at quantifying the risk of a given local bank movement for a given year, based on the information available at the beginning of that year. Given the configuration in a particular year, the most likely bank movement for the following year will be determined.

In general, the width-depth relationships of the branch channels differ from the equilibrium conditions. It is assumed that the changes of the individual channels will tend to restore equilibrium. It is further assumed that splitting and joining of channels, as well as other changes in water and sediment distribution impose a noise on this process, that will change the equilibrium and force new adjustments.

It is realized that one would like to have a much longer lead time than one year in order to cope effectively with the risk of major movements. This may be possible in the future when more and more reliable information is available and when a better understanding of the morphology of braiding is obtained. It is hoped, however, that the research presented here will make a contribution to this goal.

6.2 Model Design

The basic approach to the development of a probabilistic model is as follows. It is assumed that the existing branch channels will tend to change their cross-sectional dimensions towards equilibrium conditions that are compatible with the distribution of water flow and sediment discharge over these channels. It is further assumed that the equilibrium conditions for each channel are defined by the relationships that were developed in the previous chapter. It is finally assumed that the distribution of water and sediment over the channels is subject to continual change due to changes in discharge and because of the splitting and joining of branch channels. In other words, while there is a continuing movement towards equilibrium conditions there are also random events that will disturb the process. The entire channel forming process is thus seen as a trend towards equilibrium on which a large random noise is superimposed. A more sophisticated model could be constructed if one could also identify and quantify trends in the re-distribution of water and sediment flow, that cause channels to move away from equilibrium conditions. This did not appear possible at this time.

The aim is then to produce a computer based model that takes as input the channel geometry at the beginning of a given year for all 33 available sections and produces as output the bank movement at the outer banks of each section during that year.

It was shown in the previous chapter that a comparison between actual branch channel dimensions and equilibrium dimensions could be used to predict the direction of the changes but not their magnitude. The reason is that any changes in the cross-sectional dimensions must satisfy the continuity constraints of water and sediment flow. This means that the changes in branch channel dimensions are interdependent. There are, however, many ways in which these constraints can be met.

This leads to two areas in which the micro relations of fluid mechanics must be augmented with macro relationships that determine the probable outcome of the change. The first area of uncertainty is the determination of the branch channel dimensions that correspond to a changed but fixed flow distribution. While the friction slope can be assumed constant in the short term, a branch channel can adjust both width and depth in a multitude of ways to satisfy the discharge equation for the given or implied channel roughness. One needs therefore an additional equation to solve for both width and average depth. This problem, which also arises in the design of stable canals in erodible material, has been addressed by several researchers, who have proposed different variational principles, such as minimum stream power, maximum friction, minimum energy dissipation, or maximum sediment transport as additional relationship.

The second area of uncertainty is that there are numerous ways in which the flow distribution itself in a set of branch channels can change in response to out-of-equilibrium conditions for some or all of the channels. The maximum entropy principle was employed to determine the most probable revised flow distribution. This principle was selected because it provides the maximum variability that is consistent with the known constraints. With these two basic assumptions the model design may be outlined as follows.

6.2.1 Model Outline For a given channel configuration, the distribution of water and sediment over the branch channels is first calculated.

Next, the equilibrium width and depth are determined using Eqn. (5.17) given as:

$$d_e = K W_e^n A_r^m \tag{5.17}$$

This can be re-written as:

$$\begin{aligned} W_e &= K_1 d_e^{n_1} A_r^{m_1} \\ &= K_1 \left(\frac{A}{W_e}\right)^{n_1} \left(\frac{A}{A_T}\right)^{m_1} \\ W_e^{(1+n_1)} &= K_1 A^{(n_1+m_1)} A_T^{-m_1} \end{aligned}$$

where, K_1, n_1 and m_1 are constants. Thus, for a given A_T, the equilibrium width and depth can be determined when the assumption is made that the cross-sectional area A do not change in the first instance.

Changing the existing width and depth to the equilibrium width and depth, however, causes a change in branch channel discharge. The sum of these changes must be equal to zero. This imposes a constraint on the amount of change in the direction of equilibrium that can actually occur. As a result, the actual change in discharge in each channel is only a proportion, λ_i, of the change required to reach equilibrium conditions.

Evidently, the constraint imposed by the constant discharge can be met in an infinite number of ways, each involving a different set of λ_i. Given the available information about the process, the most probable set of values can be determined using the maximum entropy principle with the given constraint of constant discharge.

Once the probable changes in branch channel discharge are known the new dimensions of each channel are re-calculated using an appropriate extremum principle. The maximum sediment transport criterion was chosen for this purpose.

Following the calculation of changes in channel width, the bank movement is estimated. The total change in width is divided over the two banks in proportion to the relative distance between the bank and the centroid of the cross-section. This procedure is followed for all sections and all years for which information is available. The calculated and observed bank line movements are then compared for the river as a whole as well as for the six different reaches.

In order to construct the model, a number of preliminary calculations must be made as described in the following sections. Section 6.3 describes the determination of the formative discharge and the corresponding water levels. In Section 6.4 the sediment discharge is related to the flow of water and appropriate sediment transport formulas are discussed. The next section describes the fluid resistance formulas used. Then follows a section on the selection of the variational principle used to relate width and depth in alluvial channels with known discharge. The next section describes the use of the principle of maximum entropy to determine probable changes in channel discharge as a result of deviations from equilibrium conditions. The final section gives a brief description of the effect of the expected changes on branch channel width and the expected outer bank movement. The actual construction of the model and its application are discussed in Chapter Seven.

6.3 Formative Discharge

The development of the proposed model assumes a known and constant formative discharge. For this purpose the bankfull discharge was adopted. Chapter Two discusses the reasons for this choice.

There is only one station where the discharge is measured regularly. This is the station of *Bahadurabad* (Section J-13A, see Fig. 4.1 for location). At this location the bankfull discharge was determined and found to be 40,000 m^3/sec.

To determine the corresponding water levels at other cross-sections a distinction had to be made between the sections where water levels are measured daily and cross-sections where this is not the case. For the sections where daily water levels are available the levels for 40,000 m^3/sec was taken to be the level at the time the 40,000 m^3/sec discharge occurred at Bahadurabad. The water levels at the other sections were determined by means of a routing procedure. It was considered unnecessary to take the travel time of the flood wave into account since for bankfull stage the travel time between Bahadurabad and any other cross-section used in the analysis is less than one day.

6.4 Sediment Transport Formula

A large number of sediment transport formulas exist and the transport rates obtained from them varies widely. It has been said that their estimates are between half and double the correct value. Recent formulas, however, give better results than the older ones because they have used a large number of observations to evaluate the parameters. Considering this and taking also into account the complexity of some of the newer formulas, the Ackers & White (1973) sediment transport formula was chosen.

This formula calculates the sediment concentration in the flow. The sediment transport is described by three dimensionless parameters; the particle mobility F_{gr}, the dimensionless parameter size D_{gr}, and the sediment transport G_{gr}. Fine sediments are assumed to be suspended by turbulence and transported by the main body of water. As the intensity of turbulence is dependent on the total energy degradation, for fine sediments the particle mobility is expressed as a function of the total shear stress. The coarse sediment transport is considered to be travelling mainly as bed load and quantities were found to be a function of the grain shear stresses and would depend on whether or not the bed of the channel was plain. The particle mobility, in case of coarse sediment, is expressed in terms of net grain resistance. The fine and coarse material are defined in terms of dimensionless particle size D_{gr} given by:

$$D_{gr} = D\left[\frac{g(s-1)}{v^2}\right]^{\frac{1}{3}} \tag{6.1}$$

where,

g = acceleration due to gravity,

D = representative grain size = D_{35} for large rivers,

s = specific weight of sediment, and

v = kinematic viscosity of fluid at given temperature.

Based on extensive analysis of flume data, sediments with $D_{gr} > 60$ are classified as coarse, and those with $D_{gr} < 1.0$ as fine. For $1.0 \leq D_{gr} \leq 60$, the particular size can behave as both fine and coarse.

The particle mobility F_{gr}, sediment transport G_{gr}, and sediment concentration X are given by:

$$F_{gr} = \frac{V_*^N}{\sqrt{gd(s-1)}}\left[\frac{V}{\sqrt{32}\,\log(\frac{\alpha d}{D})}\right]^{1-N} \tag{6.2}$$

$$G_{gr} = c\left[\frac{F_{gr}}{A} - 1\right]^M \tag{6.3}$$

$$X = \frac{G_{gr} s D}{d\left(\frac{V_*}{V}\right)^N} \tag{6.4}$$

where,

V_* = shear velocity = $(gDS)^{0.5}$,

d = mean depth of flow,

S = slope,

V = mean velocity of flow,

α = coefficient in rough turbulent equation ≈ 10,

N = transition exponent defined as:

$$N = 1.00 - 0.56 \log(D_{gr}) \qquad (6.5)$$
$$= 0.10 \quad for \quad D_{gr} > 60$$

c = a coefficient defined as:

$$\log(c) = 2.86 \log(D_{gr}) - (\log D_{gr})^2 - 3.53 \qquad (6.6)$$
$$c = 0.025 \quad for \quad D_{gr} > 60$$

A = initial motion parameter i.e., the value of F_{gr} at the threshold of movement, and is defined as:

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14 \qquad (6.7)$$
$$= 0.17 \; for \; D_{gr} > 60$$

M = an exponent defined as:

$$M = \frac{9.66}{D_{gr}} + 1.34 \qquad (6.8)$$
$$= 1.50 \quad for \; D_{gr} > 60$$

X = concentration of sediment in water by mass.

For coarse sediments, the value of the transition exponent N is zero, while for fine sediments, $N = 1$. The value of N for transitional sizes of sediment may vary between 0 and 1 depending on the value of D_{gr}.

This transport equation has been tested for a wide variety of laboratory and field data. These include the data available in the literature, field data from small sand rivers, field data from very large sand rivers, and field data from major estuaries. The relations were found to perform well in all cases. They also noted that this transport equation when applied to a large river in Paraguay works better for total transport of bed sediment if D_{35} instead of D_{50} is taken to be the representative sediment size. This observation is supported by previous researchers' observations (see Ackers & White, 1973).

The performance of the Ackers & White (1973) sediment transport equations was tested in this study for the Brahmaputra river by comparing the results with the available sediment discharge measurements. The agreement was found to be satisfactory and better than for other transport functions. Einstein's transport formula, for example, was found to be way off compared to the measurement data. RPT et al. (1988) reached a similar conclusion when comparing the performance of the Ackers & White equations with other transport formulas for the Brahmaputra river. The validity of the relations in other situations has been tested and confirmed by many investigators (e.g., Bettess, 1984; Paris, 1984; White, Paris & Bettess, 1980; White, Bettess & Shiqiang, 1987).

6.5 Frictional Characteristics

The resistance to flow in an alluvial channel depends on the bedform and the particular regime the river is in. White, Paris & Bettess (1980) developed relations that describe the resistance in alluvial rivers. This method uses the same basic parameters as Ackers & White (1973). The particle mobility is defined in such a way that only the relevant shear forces are used. That is, total shear is used for fine sediments, grain shear is used for the coarse sediment, and an intermediate value depending on the dimensionless grain size is used for the transitional sediment sizes. White et al. (1980) found that there is a linear relationship between the mobilities related to the total shear stress, F_{fg}, and the mobility related to the effective shear stress, F_{gr}. This relationship is given as:

$$\frac{F_{gr}-A}{F_{fg}-A} = 1.0 - 0.76\left[1.0 - \frac{1.0}{exp(log D_{gr})^{1.7}}\right] \tag{6.9}$$

where,

$$F_{fg} = \frac{V_*}{\sqrt{gd(s-1)}} \tag{6.10}$$

and other variables are as described in the preceding section on sediment transport.

This method has been compared with the traditional methods such as Einstein & Barbarossa (1952), Engelund (1966) and Raudkivi (1967). The agreement was found to be satisfactory for sediment sizes in the range of 0.04 mm to 68 mm (White et al., 1980).

The frictional relationship of White et al. (1980) given by Eqn. (6.9) is applicable to data for which the Froude number is less than 0.8. This corresponds to the lower flow regime for which the bedforms are predominantly dunes, ripples or flat beds with no sediment movement.

White, Bettess & Shiqiang (1987) noted that for the available data two distinct relationship exists, one for ripple and dune beds (lower regime) and another for transition, anti-dunes and chute and pool beds (upper regime). The relationship expressed by Eqn. (6.9) is for the ripple and dune beds, while for the upper flow regime the relationship can be written as:

$$\frac{(F_{gr}-A)+0.07(F_{gr}-A)^4}{F_{fg}-A} = 1.07-0.18\log_{10}D_{gr} \qquad (6.11)$$

in which the variables are as described before.

The use of two separate relationships for the two regimes of flow poses two problems, namely (a) determining the proper regime to use in a given situation, and (b) determining the transition from one regime to another. It is reasonable to assume that the distinction between the two regimes can be based on a criterion which is related to those used to specify the occurrence of different bed forms. White et al. (1987) found the criterion for distinguishing upper and lower regimes to be dependent on their non-dimensionalized unit stream power, U_E defined as:

$$U_E = \frac{VS}{(g\,\nu)^{1/3}D_{gr}} \qquad (6.12)$$

in which the variables are as described before. This criterion was arrived at by following Simons & Richardson's (1963) distinction of different bed forms from the values of stream power and median fall diameter.

By plotting U_E against D_{gr} for a range of readily available data from the literature, White et al. (1987) found that for values of U_E less than 0.00035 plane bed occurs; between 0.00035 and 0.011 ripples occur if D_{gr} is less than 15, otherwise dunes occur; between 0.011 to 0.02 transition region exists; greater than 0.02 flat bed and anti-dunes occur. It was concluded that the lower regime curves are appropriate if U_E is less than 0.011, and the upper regime curves are appropriate if U_E is greater than 0.011.

White et al. (1987) also suggested the existence of a hysteresis effect. This means that if the flow is in lower regime, it will continue to be in the lower regime until the value of U_E rises to 0.011 when the flow will change into the upper regime. On the other hand, a flow in the upper regime will remain in the upper regime until the value of U_E falls to 0.011 when it will change into the lower regime. The information about the nature of the flow or its past history should be used to determine the regime conditions if they are available. If not, the criterion for the lower regime is:

$$[U_E]^L + [U_E]^U \leq 0.022 \tag{6.13}$$

and the criterion for the upper regime is:

$$[U_E]^L + [U_E]^U > 0.022 \tag{6.14}$$

where,

$[U_E]^L$ is the value of U_E obtained by using the relationship for the lower flow regime (i.e., Eqn. 6.9), and

$[U_E]^L$ is the value of U_E obtained by using the relationship for the upper flow regime (i.e., Eqn. 6.11).

6.6 Variational Principle

As explained in the introduction to this chapter an extra equation is needed to solve for the width and the average depth when the flow is given. A variational principle in some form is usually employed to obtain the extra relationship. This is justified with the argument that in a dynamic situation changes in cross-sectional shape are related to changes in specified functions such as stream power, energy dissipation, friction or sediment transport. If such a relationship shows an extremum for the given constraints then continuing changes in, say, width/depth ratio would cause a reversal in the function. If the change in cross-section is indeed related to the changes in, say, sediment transport then one would look for equilibrium conditions that cause the sediment transport to reach an extremum.

A summary of the different variational principles is given and discussed in Chapter Two. The variational principle used in this research is the principle of maximum sediment transport.

6.7 Principle of Maximum Entropy

To demonstrate the use of principle of maximum entropy let us consider a given cross-section of the river with n branch channels. The flow is assumed to be the bankfull flow, which was computed to be 40,000 m^3/sec. It is also assumed that the slope of the energy line is practically the same for all branch channels in the cross-section.

The first task is then to determine the prevailing discharge in each of the branch channels for bankfull conditions. Since the properties of the sediment and the cross-sectional dimensions of the channels are known, the flow in each branch channel can be determined by iteration, using the Ackers & White (1973) transport function and the frictional relationship proposed by White et al. (1987).

One can then write:

$$\sum_{i=1}^{i=n} Q_i = Q_b \qquad (6.15)$$

$$\sum_{i=1}^{i=n} Qs_i = Qs \qquad (6.16)$$

where,

Q_i is the water discharge in i-th branch at bankfull stage,

Q_{Si} is the sediment discharge in i-th branch at bankfull stage,

Q_b is the total discharge at the cross-section at bankfull stage, and

n is the number of branches at the cross-section at bankfull stage.

The observed channel cross-sections will not all be in equilibrium as far as the width/depth relationships are concerned. Some will be too wide and will have a tendency to narrow, while others may follow the opposite trend.

The relationships developed in Chapter Five can be used to determine for each of the branch channels the equilibrium width and depth for bankfull conditions. These can then be used to repeat the calculation of the discharge in each channel. The result is a new set of equilibrium channel discharges Q_{ei}.

If this set of "equilibrium discharges" would add up to the bankfull discharge then one could simply assume that each channel would take on the calculated equilibrium dimensions. This, however, is normally not the case. Consequently, the set of "equilibrium discharges" will not be attained either.

The basic assumption is now made that the changes in branch channel discharge from the calculated values Q_i that may occur are proportional to the difference between Q_i and Q_{ei}. These changes must evidently add up to zero so that the total discharge will remain equal to the bankfull discharge of 40,000 m^3/sec. It is to be expected that the coefficients of proportionality tend to be as close to unity as the other conditions will allow. In other words, any change that will move a coefficient closer to unity will occur and no coefficient will be in excess of unity.

It can easily be verified that for two branch channels the constraints uniquely determine the new flow distribution. For three or more branch channels, however, the problem is indeterminate and one may expect that the flow distribution that will be obtained is affected by factors about which we have no knowledge.

This is a typical problem in which the maximum entropy principle provides the most probable solution that is compatible with the given constraints and that does not require or imply any other assumption. It was therefore decided to use this principle. It should be emphasized, however, that the maximum entropy principle only provides a probability assessment for a specified random mechanism with given constraints. It can be shown (Jaynes, 1979) that the expectation of the frequency with which a particular solution will be observed coincides with the solution indicated by the maximum entropy principle. Some deviation between the expected and the subsequently observed bank movements due to chance must thus be anticipated. Of course, if the deviations are large then they will cast doubt on the correctness of the basic constraint assumptions.

6.7.1 Derivation of the Entropy Measure For each branch channel i of a cross-section with n branches, the discharge under existing condition (Q_i) and the discharge under predicted equilibrium condition (Q_{ei}) have been calculated. The problem is to find out the changes in discharge in each branch channel so that the movement of the banks can be estimated.

The deviation of each branch channel discharge from the equilibrium discharge can be written as:

$$\Delta Q_i = [Qe_i - Q_i], \qquad i=1,2,\ldots\ldots,n \tag{6.17}$$

where,

ΔQ_i is the deviation of discharge in branch channel i,

Q_{ei} is the discharge in the branch channel i corresponding to the equilibrium dimensions as computed from the width-depth relation, and

Q_i is the existing discharge in the branch channel i.

The quantity ΔQ_i can be positive or negative depending on whether the discharge under the equilibrium condition is greater or less than the existing discharge. In fact, in almost all cases some are positive and some are negative in a cross-section.

The summation of the deviations in the individual branch channels is given by:

$$Q_{eT} = \sum_{i=1}^{n} \Delta Q_i$$

This total deviation cannot occur because the total discharge over the section must remain the same. In other words, each of the values ΔQ_i must be adjusted so that the summation of the adjusted deviations becomes zero.

The problem here is two-fold. First, the change in discharge in branch channel i should depend on the value of ΔQ_i because it is assumed that the channel will tend to reach the equilibrium dimensions. There will be no need for any adjustment if the deviations are zero i.e., the existing and the equilibrium dimensions are the same. Secondly, the summation of all the equilibrium discharges over the cross-section is usually not equal to the total discharge, i.e., the total deviations over a cross-section is usually not equal to zero.

Let the correction of each branch channel discharge be written as:

$$x_i \, \Delta Q_i \tag{6.18}$$

where, x_i are the positive coefficients of proportionality which can take any value between 0 and 1.

The first constraint then becomes:

$$\sum_{i=1}^{i=n} x_i \, \Delta Q_i = 0 \tag{6.19}$$

For the coefficients of proportionality one may expect that they will not be less than zero, which would mean a movement further away from equilibrium and that they will tend to be close to unity. One can therefore write:

$$\sum_{i=1}^{i=n} x_i \leq n \tag{6.20}$$

Substituting $x_i = n\lambda_i$, Eqn. (6.19) and (6.20) can be re-written as:

$$\sum_{i=1}^{n} \lambda_i \Delta Q_i = 0$$

and

$$\sum_{i=1}^{n} \lambda_i \leq 1$$

The problem reduces to the redistribution of the excess discharge of which each branch carries a proportion. What would be the values of these proportions are not known. The system has the freedom of choosing any value of discharge for any branch. In this situation, the most likely solution is obtained by maximizing an entropy function subject to the constraints imposed by the system (see Section 2.9 in Chapter Two).

The choice of an entropy function should follow the necessary and desirable axioms of a system in general and a goal-oriented system in particular. These are provided in *Appendix-K*. From a first glance at the constraints, it may be argued that a Rényi (1960) type function of possibly incomplete distribution would be suitable as an entropy function. The following argument, however, shows that Shannon's maximum entropy function can be used.

The process of redistribution of the excess amount of discharge is controlled by the adjustment allowed by the system over all the branch channels. As far as the system is concerned, the process of redistribution is complete for the moment, since the equilibrium discharges in branch channels are predicted amounts and are *not* the amount dictated by the system. If we consider a hypothetical branch which will transport the difference between the total adjustment and ΔQ_{eT}, then we are left with a proportionality coefficient, λ_h, for the hypothetical channel which would be equal to $(1-\Sigma\lambda_i)$. Thus, consideration of a hypothetical branch would allow the summation of the coefficients of proportionality to add up to unity. However, the portion of adjustments in the hypothetical channel depends completely on the adjustments in discharge of the branch channels. In other words, this hypothetical channel is an addition with no freedom to the system. Addition of an option with no freedom does not change the entropy of the system. This can equivalently be viewed as a goal-oriented system of Belis & Gisau (1968) where the utilities of the branch channels are *equal* and *non-zero*, and the utility of the hypothetical channel is *zero*. This is so because the system goal is to distribute the discharge among the branch channels having *no* interest *at all* to what happens to the difference between the predicted total equilibrium discharge (predicted by the researcher) and the total discharge (followed by the system). Thus, the coefficient for the hypothetical channel should not be included in the maximization process, since the system is not free to choose this coefficient (see *Appendix-K*). The fact that the sum of the proportions is less than or equal to unity merely suggest the upper bound of the summation, and *not* the incompleteness of the process. Indeed, in a study of customer's brand (of commodities) switching problem, Kumar (1983) suggested that the balance of the probability, if they do not add up to unity, be assigned to a hypothetical brand for which data are not available.

The dimensions that the branches are most likely to assume would then be considered to follow the maximum entropy function as proposed by Shannon (1948) provided that the maximization process only include the coefficients of the branch channels. The problem, thus, is reduced to a maximization problem as given in the following set of equations:

$$Max \sum_{i=1}^{i=n} -\lambda_i \, ln(\lambda_i) \tag{6.21}$$

$$s.t. \sum_{i=1}^{i=n} \lambda_i \le 1 \tag{6.22}$$

$$\sum_{i=1}^{i=n} \lambda_i \Delta Q_i = 0 \tag{6.23}$$

$$\lambda_i \ge 0 \tag{6.24}$$

By observing the analogy of the classical optimization problem, we can form the Lagrangian as:

$$\Phi(\lambda, \mu) = \sum_{i=1}^{i=n} -\lambda_i ln(\lambda_i) - \mu_1 \sum_{i=1}^{i=n} \lambda_i [\Delta Q_i] - \mu_2 \left[\sum_{i=1}^{i=n} \lambda_i - 1 \right] \tag{6.25}$$

The necessary conditions for a relative maximum are:

$$\lambda_i \geq 0 \tag{6.26}$$

$$\frac{\partial \Phi}{\partial \lambda_i} \leq 0 \tag{6.27}$$

$$\lambda_i \left[\frac{\partial \Phi}{\partial \lambda_i} \right] = 0 \tag{6.28}$$

$$\mu_j \geq 0 \tag{6.29}$$

$$\frac{\partial \Phi}{\partial \mu_j} \leq 0 \tag{6.30}$$

$$\mu_j \left[\frac{\partial \Phi}{\partial \mu_j} \right] = 0 \tag{6.31}$$

In the above equations,

i = branch channel number = *1, 2, ...*, n, and

j = constraint number = *1, 2.*

For a cross section with more than two channels, analytical solution becomes too cumbersome (see *Appendix-L* for a solution of a three channel section), and a numerical optimization approach is adopted. This will be discussed in the next chapter.

6.8 Effect of Changes in Planform Geometry

The methodology established thus far aims at establishing equilibrium conditions for a channel system with a given planform geometry. It does not address the question of changes in the planform geometry itself. Such changes occur because channels may disappear, split or merge with others.

It is evident that these changes represents discontinuities in the process of cross-sectional change. In the development of the bank movement model the changes are therefore treated as a random noise.

This limitation is less serious than it would seem because the interest is not in the behaviour of each individual branch channel but in the annual erosion and sedimentation along the outer banks which is governed by the bank movement of the outer branch channels in a single year.

This leaves the problem then of how to relate the outer bank movement to the changes in the width of the outer channels. To solve this problem the assumption was made that the displacement of each bank of a branch channel is inversely proportional to the distance of that bank to the centre of gravity of the cross-section.

Chapter Seven

APPLICATION OF THE PROPOSED MODEL

7.1 Introduction

The probabilistic model described in the previous chapter aims at predicting the bank movements of the Brahmaputra river a year in advance. This chapter gives a detailed description of the data used and a step by step explanation of the procedure that was followed. Each major part of the process is explained with the aid of a flow diagram that highlights the computational procedure. This is followed by a detailed description of the procedure. The results are discussed in Chapter Eight.

7.2 Data Base

7.2.1 Source The input data base for the application of the model to the Brahmaputra river consists of:

1. Complete cross-sections of the river at 33 stations, approximately equally spaced along the river in Bangladesh. These records are available for 1976, 1977, 1978, 1979, 1980, 1981, 1983, 1985 and 1986.
2. The location of the cross-sections on a topographical map (see Fig. 4.1).
3. Daily water level and discharge measurements at one station (*Bahadurabad*). These records are available from 1965.
4. Daily water levels at *Chilmari, Bahadurabad, Jognaichar, Jagannathganj, Sirajganj, Porabari, Mathura* and *Teota*.
5. Results of sediment sample analysis reported by Petrobangla (1983) at various locations along the river.
6. Sediment transport measurements at *Bahadurabad* and *Nagarbari*. These records are not continuous and are available only for a few occasions.
7. Satellite imageries of the river for the year 1973, 1976, 1977, 1978, 1979, 1980, 1981, 1983 and 1985.

7.2.2 Processed Input Data The available information described above were processed to form the input data for the model. These include (a) the total bankfull discharge of the river, (b) the geometrical properties of the branch channels at the measurement stations for the water level corresponding to the bankfull discharge, and (c) the sediment properties. These are discussed in the following.

(a) The Bankfull Discharge

The previously mentioned bankfull discharge of 40,000 m^3/sec was determined from an analysis of the cross-sectional properties at the *Bahadurabad* station, the only station for which daily flow records are available. Of the available procedures discussed in Chapter Two, two procedures were selected. They are (1) the determination of the water elevation for which the width-depth ratio reaches a minimum, and (2) the determination of the water elevation for which the area-top width relation shows a significant change. The obtained bankfull discharge and corresponding bankfull level from both of these procedures were found to be in substantial agreement.

The bankfull water discharge at *Bahadurabad* has been taken to be constant throughout the study reach. The water levels at other cross-sections corresponding to this discharge are either interpolated from the recorded values or computed through routing depending on whether or not the water levels are recorded at the section. The bankfull discharge computed in this study was compared with the bankfull discharge of the Brahmaputra river determined by RPT et al. (1986), and was found to be very close.

(b) The Geometric Properties of the Branch Channels

The geometrical properties of the branch channels required in the analysis are the number of channels in the cross-section, the position of the left and right bank, the location of the centroid, the average depth, the maximum depth, the location of maximum depth, the width of each channel; and the width-depth relationship of the section developed in Chapter Five.

The geometric properties of the cross-sections were calculated from the observed records of Bangladesh Water Development Board. These records, which are available as maps, were digitized and the information was stored as coordinates from a horizontal reference on the ground and a vertical datum. The monument established at each station of measurement constructed by the Bangladesh Water Development Board was taken as the horizontal reference. The datum Zero established by the Public Works Department (referred to as 0 PWD) was taken as the vertical reference. Note that the water levels are also provided with respect to this vertical reference. A computer program was developed to compute all cross-sectional properties corresponding to the bankfull water elevation using these coordinate values. A discussion of this computing procedure is provided in Chapter Four (Section 4.3).

It should be mentioned that in the routing procedure, for each trial water elevation, all the sectional properties were computed from the coordinate values for each branch channel separately instead of using any relationship between the water level and the properties such as area, hydraulic radius etc. derived for the section as a whole (as was done in some previous documents). This was necessary because of the changes in cross-section that occur in each year of record as a result of changes in planform geometry.

(c) Sediment Grain Size

The sediment properties were taken from the sediment sample analysis reported by Petrobangla (1983). The grain size distributions at various locations along the river were also given in that report. The grain size distribution along the river is almost the same and an average value was taken. As suggested by Ackers & White (1973) transport formula, the value of D_{35} is the representative grain size for a large river. The average value of D_{35} was taken to be 0.15 mm. The specific weight of sediment and the average water temperature was also taken from the Petrobangla report.

7.3 Computational Procedure

A computer program was developed to describe the probabilistic model of Chapter Six. It solves the system of equations to estimate the yearly bank movement of the Brahmaputra river in Bangladesh. With the data described in the previous section of one year as input, the program produces the predicted bank movement of the following year as output.

The computations can be grouped into a number of primary steps. These are: (I) computations of parameters used in the program that are constant throughout the program, (II) distribution of water discharge into branch channels, (III) computation of equilibrium dimensions of the branch channels, (IV) distribution of deviation of discharge into the branch channels, (V) estimation of new dimensions of the branch channels for the adjusted discharge, and (VI) estimation of movement of each channel banks. These steps are shown in Fig. 7.1 as a flow diagram.

All of the primary steps are described by component steps. Except step I and step VI, all other steps are iterative. A sequential description of each of these steps are given in the following sections. While the flow diagram of each step provides an overview of the component steps, detail of the procedure of each step is described in the sections.

7.3.1 Input Parameters (Step I)

In this step, the initial input values are read and required parameters are computed. The flow diagram of this step is given in Fig.7.2. The procedures that are followed in this phase of computation can be summarized as:

1. The coordinates of the banks and the centre of gravity of each branch channel for the bankfull discharge, which are already computed and stored, are read from a file. Besides these sectional information, other data that do not change for the rest of the program are also read. These include: the total bankfull water discharge, D_{35} of sediment, specific gravity of sediment, dynamic viscosity of water, and energy coefficient. The energy coefficient, is taken as 1.25 as suggested by Richardson, Simons, Karaki, Mahmood & Stevens (1974).

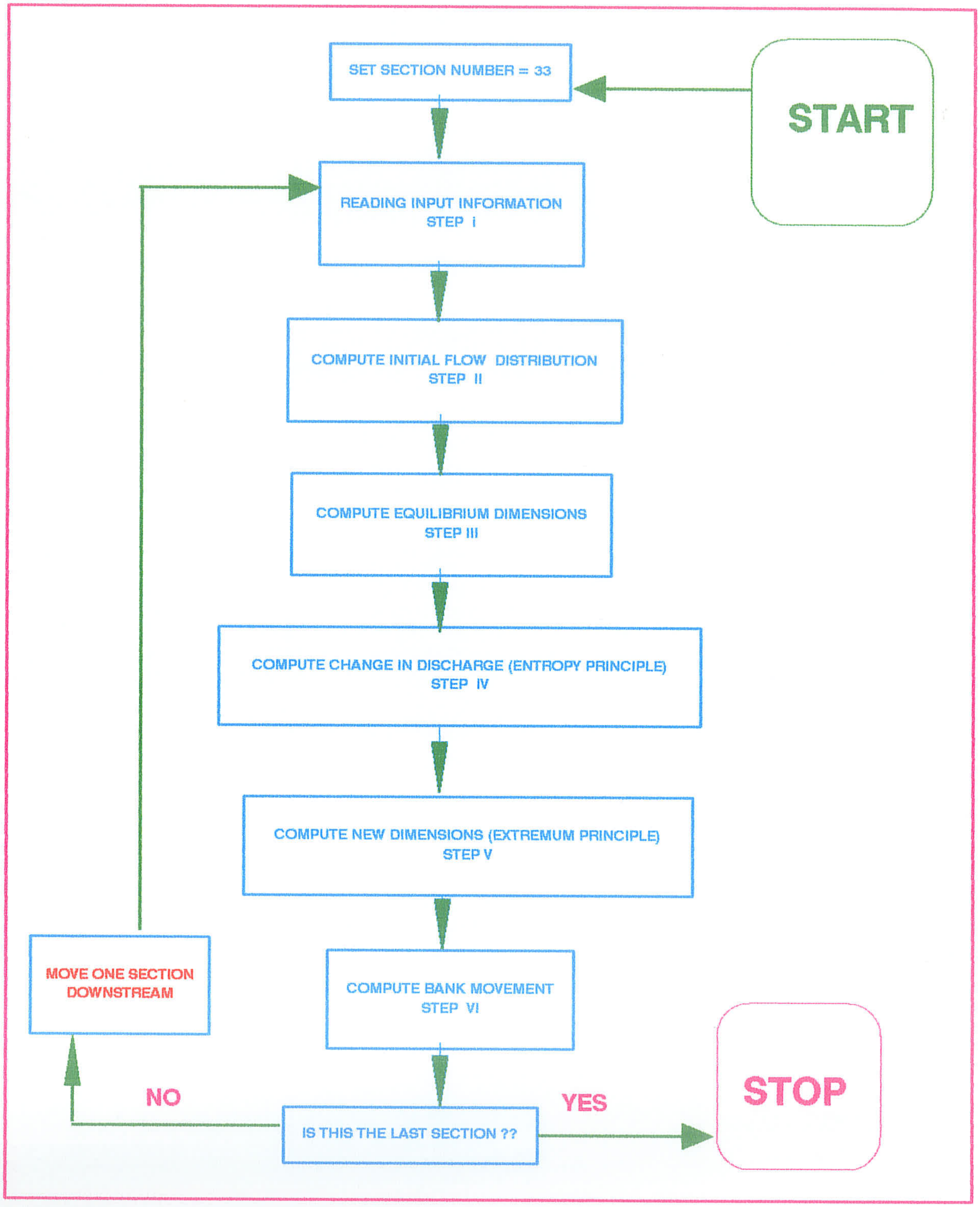

Fig. 7.1: Flow Diagram of Overall Computational Procedure

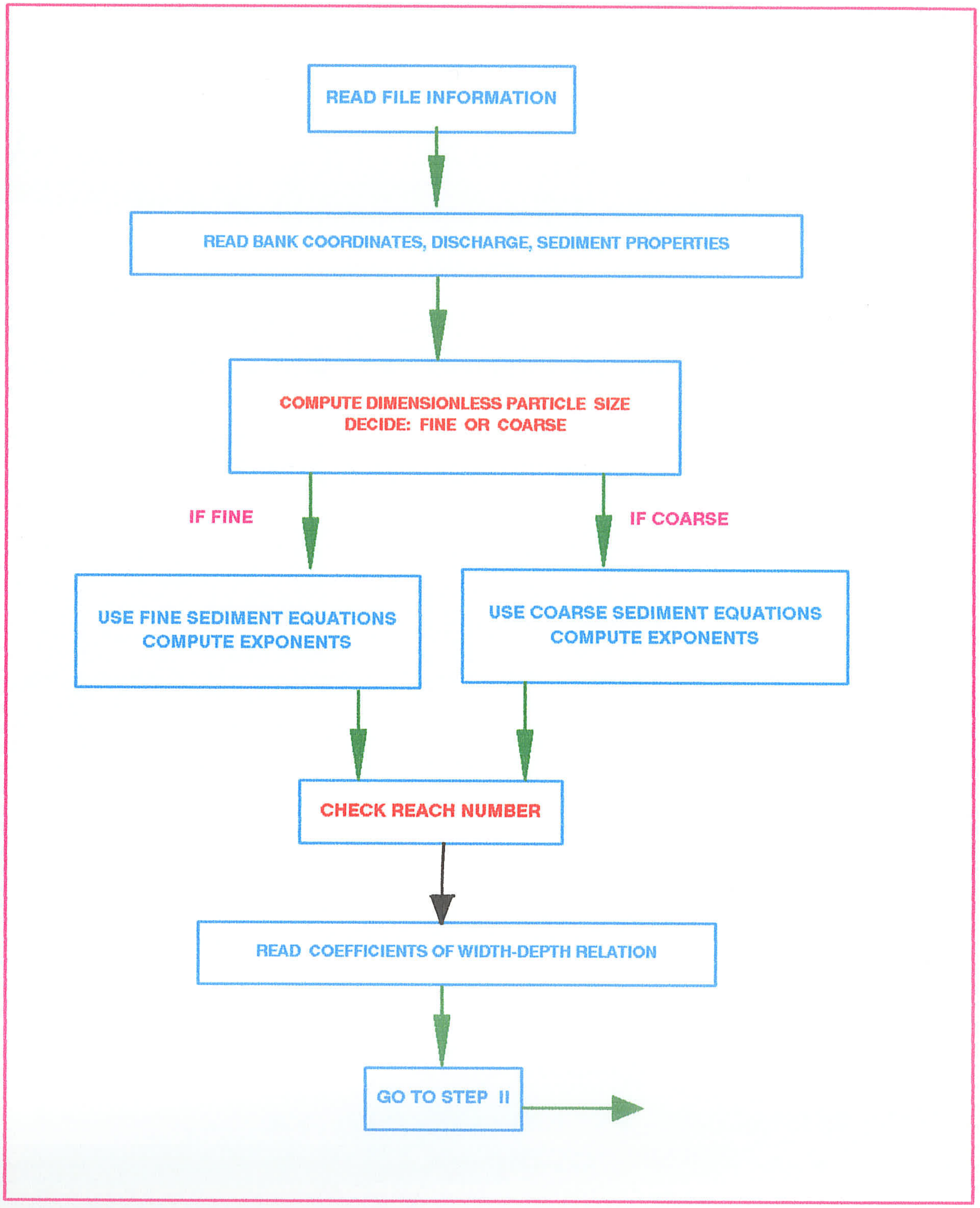

Fig. 7.2: Flow Diagram of Step I

2. The dimensionless particle size, D_{gr}, was computed using Eqn. (6.1) written as:

$$D_{gr} = D\left[\frac{g(s-1)}{\nu^2}\right]^{\frac{1}{3}} \tag{6.1}$$

The value of D_{gr} was found to be 4.18 which is in the fine particle range. The value of the dimensionless particle size is important since other parameters of the sediment and frictional relations depend on it. Appropriate relations are chosen according to its value. Note that if D_{gr} is less than 1.0, the sediment is classified as fine. If the value is greater than 60 then the particle size is classified as coarse. If the value of D_{gr} is between 1.0 and 60, the sediment may behave as both fine and coarse.

3. The transition exponent, N, was computed from Eqn. (6.5) given as:

$$\begin{aligned} N &= 1.00 - 0.56\ \log(D_{gr}) \\ &= 0.10 \quad for \quad D_{gr} > 60 \end{aligned} \tag{6.5}$$

using the value of D_{gr}. This exponent is needed to compute the dimensionless particle mobility F_{gr}.

4. The exponent M of Eqn. (6.3) for the dimensionless sediment transport G_{gr} was computed from Eqn. (6.8) using the value of D_{gr}. Equation (6.8) is given as:

$$\begin{aligned} M &= \frac{9.66}{D_{gr}} + 1.34 \\ &= 1.50 \quad for\ D_{gr} > 60 \end{aligned} \tag{6.8}$$

5. The coefficient A of the same equation for G_{gr} was computed from Eqn. (6.7):

$$\begin{aligned} A &= \frac{0.23}{\sqrt{D_{gr}}} + 0.14 \\ &= 0.17\ for\ D_{gr} > 60 \end{aligned} \tag{6.7}$$

6. The coefficient c was computed from Eqn. (6.6):

$$\begin{aligned} \log(c) &= 2.86\ \log(D_{gr}) - (\log D_{gr})^2 - 3.53 \\ c &= 0.025 \quad for \quad D_{gr} > 60 \end{aligned} \tag{6.6}$$

7. The coefficients and exponents of the width-depth relations, which were computed previously using a separate program, were taken as input. Note that separate relations were developed for different

reaches of the river. Accordingly, coefficients appropriate for the reach in which the section (for which the computations were in progress at a specific point) belongs were chosen.

The values of these coefficients, exponents and parameters were constant throughout the program, and were used in the next steps of the procedure described below.

7.3.2 Distribution of Water discharge (Step II)

The bankfull discharge that was computed previously from the measured water level, discharge and cross-sections, is the total discharge for the entire cross-section. The amount of water flowing through the individual branch channels are not known, and should be computed next.

The distribution of the known total discharge into the branch channels under the existing condition is an iterative process. This is so because the slope is unknown and the frictional characteristics of flow through the branches are not determined. Depending on whether the flow is in the lower or the upper regime, separate frictional relationships are used. Therefore, the procedure should check the existing flow regime corresponding to each step of iteration.

The flow diagram of this step is given in Fig. 7.3. Details of the procedure of each component step are:

1. The shear velocity V_* is computed from

$$V_* = \sqrt{gdS}$$

where,

g is the acceleration due to gravity,

d is the mean depth of flow, and

S is the slope.

The mean depth of flow d of the branch channels are known. An initial friction slope S is assumed.

2. The total shear stress mobility F_{fg} corresponding to this shear velocity is computed from Eqn. (6.10):

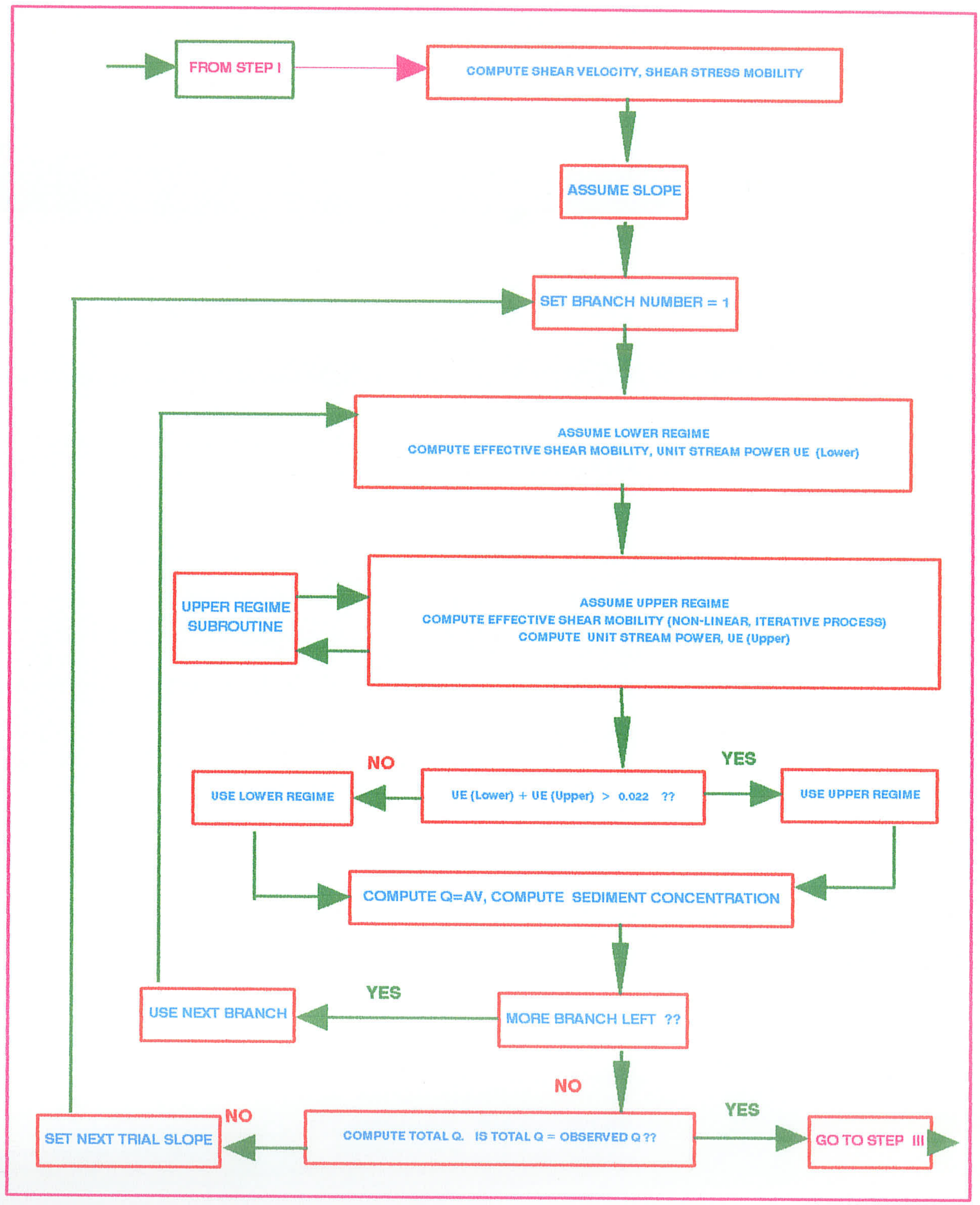

Fig. 7.3: Flow Diagram of Step II

$$F_{fg} = \frac{V_*}{\sqrt{gd(s-1)}} \tag{6.10}$$

3. The mean velocity of flow is computed next. For this purpose, algebraic manipulation of the equations described earlier is needed. Substituting Eqn. (6.10) for F_{fg} into Eqn. (6.2) for F_{gr}, one can write:

$$F_{gr} = F_{fg} V_*^{N-1} \left[\frac{V}{\sqrt{32}\, log\left(\frac{\alpha d}{D}\right)} \right]^{1-N} \tag{7.1}$$

where,

V is the mean velocity of flow,

D is the representative grain diameter,

α is the coefficient in rough turbulent equation, and

the exponents and the other variables are as described before.

Eqn. (7.1) can be rearranged to solve for the mean velocity of flow, V as:

$$V = \left[V_* \sqrt{32} log\left(\frac{\alpha d}{D}\right) \right] \left[\frac{F_{gr}}{F_{fg}} \right]^{\frac{1.0}{1.0-N}} \tag{7.2}$$

in which the variables are as described above. Solution of this equation is obtained according to the steps that follow.

4. For any trial value of S, first it is assumed that the flow is in the lower regime. The corresponding value of the effective shear mobility, F_{gr} can be computed from Eqn. (6.9). For this purpose, this equation can be rewritten as:

$$F_{gr} = \left(1.0 - 0.76 \left[1.0 - \frac{1.0}{exp\,(log\,D_{gr})^{1.7}} \right] \right) (F_{fg} - A) + A \tag{7.3}$$

With the known value of F_{gr} and, as suggested by Ackers & White (1973), with $\alpha = 10$, Eqn. (7.2) is then used to determine the mean velocity of flow V.

5. The non-dimensionalized unit stream power, U_E, corresponding to the lower flow regime is computed from Eqn. (6.12):

$$U_E = \frac{VS}{(g\,\nu)^{1/3} D_{gr}} \tag{6.12}$$

6. The flow is then assumed to be in the upper regime. The computation of F_{gr} in this case is not straightforward. The value of F_{gr} is given by Eqn. (6.11) which is non-linear in F_{gr} with an exponent of 4, and is given as:

$$\frac{(F_{gr}-A) + 0.07(F_{gr}-A)^4}{F_{fg}-A} = 1.07 - 0.18 \log_{10} D_{gr} \tag{6.11}$$

   Solution of this equation is an iterative process by itself for which the method of bisection is used. This method uses the mid-point of the pre-selected upper and lower bound of the root as the next trial value. The convergence is very fast.

7. With the value of F_{gr} as obtained from Eqn. (6.11), the mean velocity V for the assumed upper flow regime is computed from Eqn. (7.2), and the dimensionless stream power U_E is computed from Eqn. (6.12) as before.

8. If the summation of the dimensionless unit stream power U_E for the lower and the upper regime assumption is less than or equal to 0.022 (Eqn. 6.13), then the flow is considered to be in the lower regime. If the summation is greater than 0.022 (Eqn. 6.14), then the flow is assumed to be in the upper regime. The values of F_{gr} and V corresponding to the determined existing regime of flow estimated from the above are used for the rest of the computation.

9. The water discharge in the branch channel is computed by multiplying the mean velocity of flow by the cross-sectional area of the branch already taken as input to the program in the beginning.

10. The dimensionless sediment transport is calculated by using Eqn. (6.3):

$$G_{gr} = c\left[\frac{F_{gr}}{A} - 1\right]^M \tag{6.3}$$

11. The sediment concentration, X, per unit of water is computed from Eqn. (6.4) given as:

$$X = \frac{G_{gr} s D}{d\left(\frac{V_*}{V}\right)^N} \tag{6.4}$$

The sediment discharge is then calculated by multiplying the water discharge by the computed X.

12. Computation for all of the above steps were repeated for all of the branch channels of the cross-section.
13. The total computed water discharge is found by adding all of the computed water discharges of the individual branches.
14. If the computed total water discharge differs from the total observed water discharge by more than 0.1% (i.e., 40 m^3/sec for the bankfull discharge), the whole process is repeated with a different value of S. The new trial value of S is obtained by increasing the previous trial value of S by a percentage obtained from the algebraic ratio of the difference between the computed and observed total water discharge to the observed total water discharge. In other words:

$$S_{new} = S_{old}\left(1.0 + \frac{Q_d}{Q_T}\right) \tag{7.4}$$

where,

Q_d is the difference between the computed and the observed total water discharge, and

Q_T is the observed total water discharge.

Assuming the new trial value of S changed this way, the convergence was faster.

15. When the condition is met, there may still remain a small difference between the observed and the computed values which is within the tolerance limit. This error is adjusted by redistributing it in all of the branch channels by using the ratio of the branch channel top width to the total top width of

the cross-section. The sediment discharge is also adjusted according to the adjusted water discharge for each branch channel. The branch channel sediment discharges are added to obtain the total sediment discharge of the section.

All the branch channels of the cross-section are assumed to have the same friction slope in the above computation. The roughness coefficients are assumed to be different for different channels. The assumption of the same friction slope and different roughness coefficients is a reasonable one. This is because the data used are not those of near the peak flood. Also, the water levels are not observed to vary substantially for different channels. Values of Manning's roughness coefficient and friction slope for each channel is stored to be used as future trial value.

7.3.3 Computation of Equilibrium Dimensions (Step III) Once the water discharge of the individual branch channels are known from the above procedure, the equilibrium dimensions of the channels are obtained. The flow diagram is shown in Fig. 7.4. The details of this step are given below.

1. The observed width-depth relation for the corresponding reach in which the section is located is chosen from Eqns. (5.16) to (5.21). This is used to compute the equilibrium depth and width of the channels of the cross-section. Note that Eqn. (5.17) can be rearranged to write:

$$W_e = \left[\left(\frac{K}{A}\right)\left(\frac{A}{A_T}\right)^m\right]^{\frac{-1.0}{1.0+n}} \tag{7.5}$$

where,

A_T is the summation of areas of all branch channels,

A is the area of the branch channel,

K is a constant, and

m and n are the exponents.

The values of K, m and n are taken from the appropriate relation for the reach from Eqns. (5.16) to (5.21).

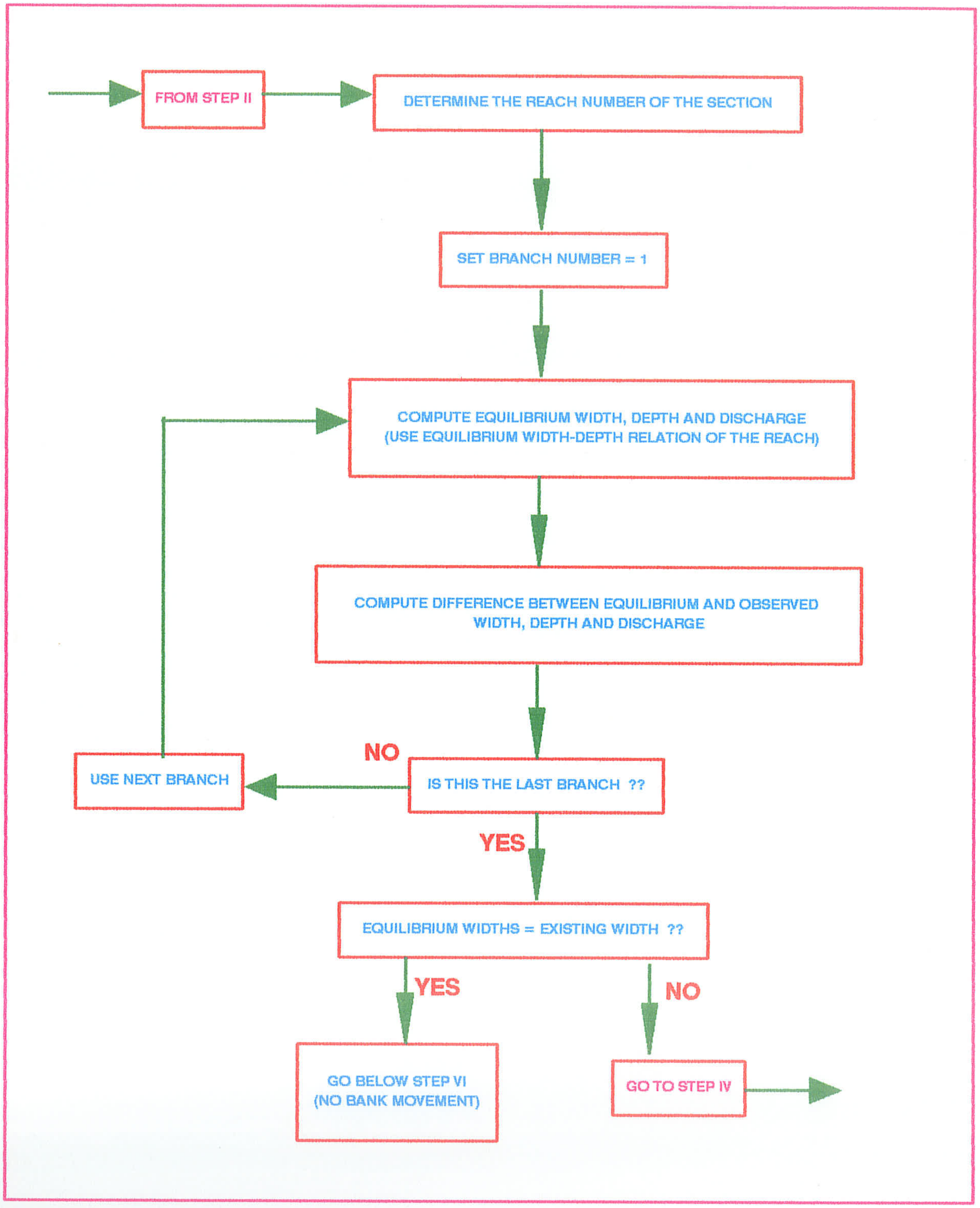

Fig. 7.4: Flow Diagram of Step III

2. The deviations in width, ΔW, is computed as the difference between the equilibrium width and the observed width as:

$$\Delta W = (W_e - W)$$

Similarly, the deviation in depth, Δd is computed as the difference between the equilibrium and the observed depth as:

$$\Delta d = (d_e - d)$$

These values of deviation would indicate the general direction in which the equilibrium section lies and the channel would want to change.

3. The total equilibrium width, W_{eT}, is computed as the algebraic summation of the equilibrium widths of the channels.

4. If the computed equilibrium widths are very close or equal to the existing width then the section is considered to remain with its existing dimensions in the following year and the movement of banklines would be zero or negligible. If the equilibrium widths are different from the existing ones, solution is obtained through an iterative procedure described in the steps to follow. The changed dimensions of the channels of the cross-section are assumed to preserve the total observed water discharge. Note that, in the case when an iterative solution is sought, the existing channel dimensions will preserve the water and sediment discharge but are not considered as the solution.

7.3.4 Distribution of Deviation of Discharge (Step IV) The dimensions computed above defines the equilibrium branch channels which will usually have a discharge potential different from the exiting discharge through the channels. While each branch channel will try to achieve the discharge according to its equilibrium dimensions, the total discharge through the whole section should be preserved. Therefore, the actual discharge, which a branch will ultimately achieve, depends on the discharges through the other branch channels. For each branch, the change in discharge (from the existing) is computed using the maximum entropy principle.

The flow diagram of this step is given in Fig. 7.5. The details of the procedure are as follows:

1. The equilibrium dimensions as computed from Eqn. (7.5) are used to determine the corresponding water discharge, Qe_i, for all branch channels using the existing slope.
2. The deviation (ΔQ_i) between this computed discharge and the observed discharge is computed using Eqn. (6.17):

$$\Delta Q_i = [Qe_i - Q_i], \qquad i = 1, 2, \ldots\ldots, n \tag{6.17}$$

3. The coefficients λ_i which determines the most likely amount of deviation of discharge in each branch channel is obtained through maximizing the objective function of Eqn. (6.21) subjected to the constraints given by Eqns. (6.22) to (6.24) :

$$Max \sum_{i=1}^{i=n} -\lambda_i \, ln(\lambda_i) \tag{6.21}$$

$$s.t. \sum_{i=1}^{i=n} \lambda_i \leq 1 \tag{6.22}$$

$$\sum_{i=1}^{i=n} \lambda_i \, \Delta Q_i = 0 \tag{6.23}$$

$$\lambda_i \geq 0 \tag{6.24}$$

Note that the number of terms in these equations are different for different sections. The number of terms in those equations depend on the number of channels which varies from section to section. Therefore, in the computer code used in this study, the expressions of the objective functions and of the constraints are formed first.

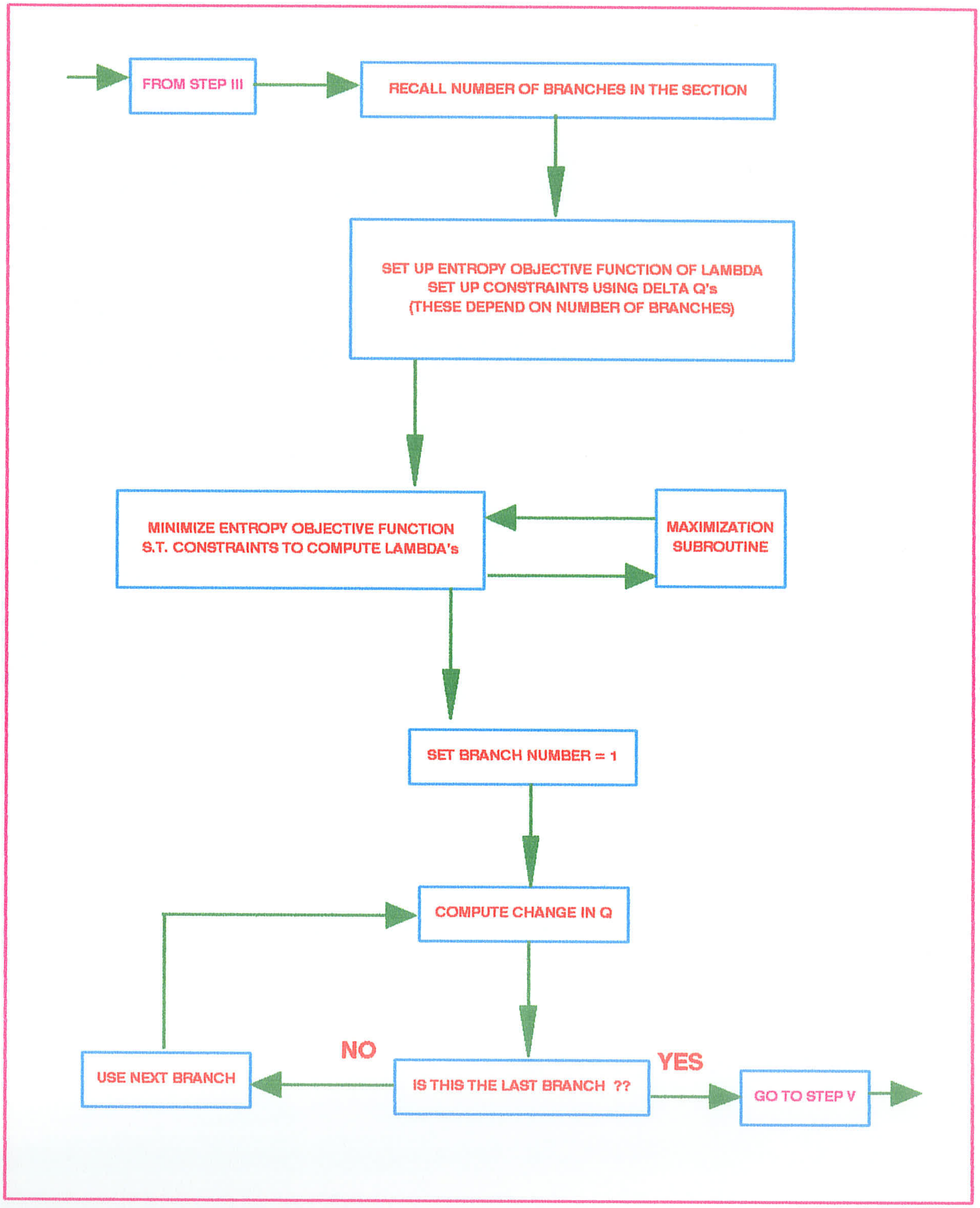

Fig. 7.5: Flow Diagram of Step IV

4. A numerical solution of the maximization problem is adopted because the analytical solution is too cumbersome. The procedure followed is based on the gradient projection algorithm first developed by Rosen (1960). This is briefly described below. Note that since the number of terms in the problem are not constant for all sections, a generalized discussion is presented.

(a) For a non-linear objective function F involving n variables (λ) and subjected to m number of linear constraints (G), the maximization process starts with the selection of a feasible starting point and an initial step size (ΔS). The derivatives of the objective function with respect to the independent variables are evaluated at this base point. The normalized direction vector components, M_i are computed as:

$$M_i = \frac{\dfrac{\partial F}{\partial \lambda_i}}{\sqrt{\sum_{j=1}^{n} \dfrac{\partial F^2}{\partial \lambda_j}}} \tag{7.6}$$

(b) If the derivatives are less than or equal to the specified limit then the procedure is stopped. The corresponding values of λ_i describes the required point.

(c) If the derivatives are greater than the specified limit, a new point is obtained as:

$$[\lambda_i]_{new} = [\lambda_i]_{old} + \Delta S M_i \qquad i = 1, 2, \ldots, n$$

The value of the objective function is computed with the new points. If an improvement of the objective function occurs without violating the constraints the step size is doubled and the direction vector components are evaluated at the improved new point. If the new point does not improve the objective function, then the step size is halved for the next trial from the last point.

(d) If the objective function is improved but one or more constraints are violated, a return to the last feasible point is made and the step size is determined which places the next point on the violated constraint. New direction vectors are computed as follows:

$$M_i = \frac{\left[\frac{\partial F}{\partial \lambda_i} + \sum_{k=1}^{l} \mu_k \frac{\partial G_k}{\partial \lambda_i}\right]}{\left[\sum_{j=1}^{n}\left(\frac{\partial F}{\partial \lambda_i} + \sum_{k=1}^{l}\left(\mu_k \frac{\partial G_k}{\partial \lambda_j}\right)\right)^2\right]^{0.5}} \tag{7.7}$$

where,

l is the number of violated constraints, and

μ_k for $k = 1 \; to \; l$ are determined from the following l equations:

$$\sum_{i=1}^{n}\sum_{j=1}^{l}\left(\mu_j \frac{\partial G_j}{\partial \lambda_i} \cdot \frac{\partial G_k}{\partial \lambda_i}\right) = -\sum_{i=1}^{n}\left(\frac{\partial G_k}{\partial \lambda_i}\frac{\partial F}{\partial \lambda_i}\right) \tag{7.8}$$

(e) Convergence of the procedure is assumed if

$$\frac{\partial F}{\partial \lambda_j} + \sum_{k=1}^{l}\left(\mu_k \frac{\partial G_k}{\partial \lambda_j}\right) \leq limit$$

Otherwise, the procedure is continued with new search directions.

The constraint of Eqn. (6.23) requires that some of the ΔQ values should be opposite in sign since the λ_i are all positive. However, it was observed that for a few sections all of the branches were expected to either widen or narrow implying that all of the ΔQ values are either positive or negative. In this situation, a solution to the maximization problem is not possible without violating the constraint. In this study, it was assumed that under this condition, equal weights are assigned to the deviations and the total discharge is conserved by a change in slope. This particular aspect of the problem should be studied in detail in future.

The values of λ_i obtained from the above procedure are used to compute the discharge in each branch channel under the equilibrium condition. These values of the share of discharge for each branch channel is used to determine the expected dimensions of the channels as described in the following.

7.3.5 Estimation of New Dimensions (Step V) The channels will assume new dimensions corresponding to the estimated new discharge. The flow diagram of this step is given in Fig. 7.6 and the procedure is described below.

1. Once the ΔQ values are known, they are added to the existing discharge to obtain the discharge under equilibrium condition. With the new discharge so obtained, the width and depth of the branch channels are determined following an extremum principle for which the principle of maximum sediment transport was chosen.
2. Obtaining the dimensions corresponding to the new discharge is an iterative process. For example, for the principle of maximum sediment discharge, the selected values of width and depth is such that the amount of sediment transport is maximized. The trial values are checked to see whether the objective function (in this case the sediment transport) is maximized. The procedure is similar for the minimum energy, maximum sediment transport or the maximum friction criteria. Note that for all trial values, the sediment transport and frictional equations given in Section 6.4 and 6.5 are solved following the procedure described in Section 7.3.2. That is, for each trial, the frictional properties are computed by checking whether the flow is in the upper or in the lower regime.

Note: More than one form of extremum principle was actually considered in this study in order to select the one producing the best result for the bank movement problem of the Brahmaputra river. During computation, another criteria called the Root Mean Square Deviation (RMSD) was used in this study. This is defined as:

$$RMSD = \sqrt{\left(\frac{Q_s - Q_s(tr)}{Q_s}\right)^2 + \left(\frac{W_e - W(tr)}{W}\right)^2}$$

where, (tr) stands for the trial value. The RMSD so defined is minimized and the corresponding dimensions are recorded. The solutions corresponding to the above extremum principle criteria are compared to find the best set of results. It was found that the maximum sediment transport criterion produces the best result. It was also observed that the RMSD criterion produced results similar to the maximum sediment transport criterion. The other principles mentioned above were also satisfactory with the exception of the minimum energy principle. The reasons of different principle performing differently is not addressed in this study.

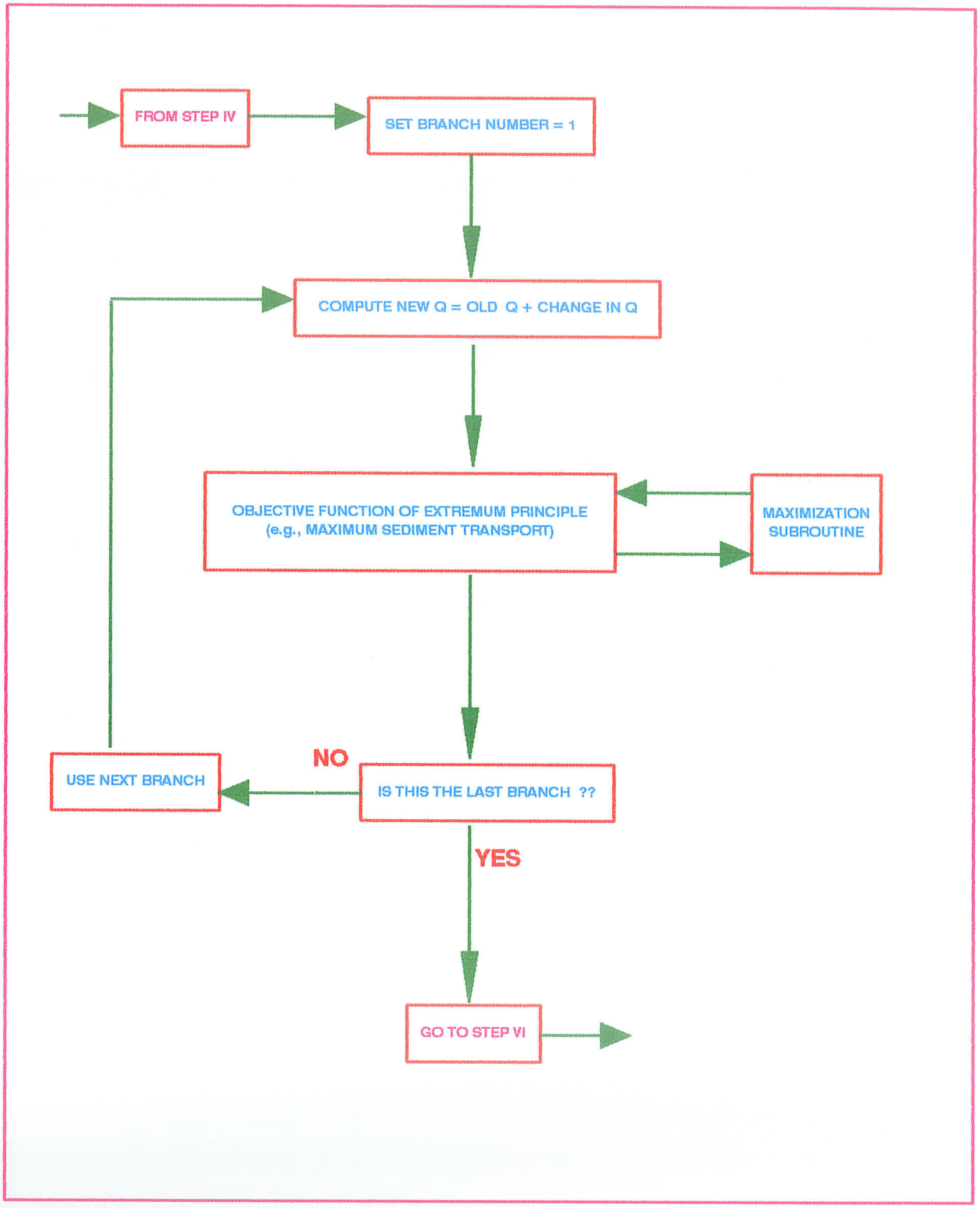

Fig. 7.6: Flow Diagram of Step V

7.3.6 Estimation of Bank Movement (Step VI) The new width of each branch channel subtracted from the existing width represents the total movement of the banks of the branch. This total bank movement was divided between left and right bank as shown in the flow diagram in Fig. 7.7. The procedure is described below.

1. The amount by which the left and the right bank are expected to move are determined by the Eqns. (5.29) to (5.32) in which emphasis is given on the existing shape of the cross-section of the branch by considering the location of the centroid of the branch. These are written for a widening channel as:

$$[W_m]_L = \frac{(W_p - W)(W + 2x)}{2W} \tag{5.29}$$

$$[W_m]_R = \frac{(W_p - W)(W - 2x)}{2W} \tag{5.30}$$

and for a narrowing channel as:

$$[W_m]_L = \frac{(W_p - W)(W - 2x)}{2W} \tag{5.31}$$

$$[W_m]_R = \frac{(W_p - W)(W + 2x)}{2W} \tag{5.32}$$

where,

W_m is the amount by which the bank moves,

W_p is the predicted width,

W is the existing width,

x is the distance of centroid of the channel section from the centre (mid-width) of the section, positive to the left,

L is the subscript representing left bank, and

R is the subscript representing right bank.

If the movement is erosion, the bank closer to the centroid will erode more compared to the other bank. The distribution is considered to be linear as discussed in Section 5.7.1 and in *Appendix-I*.

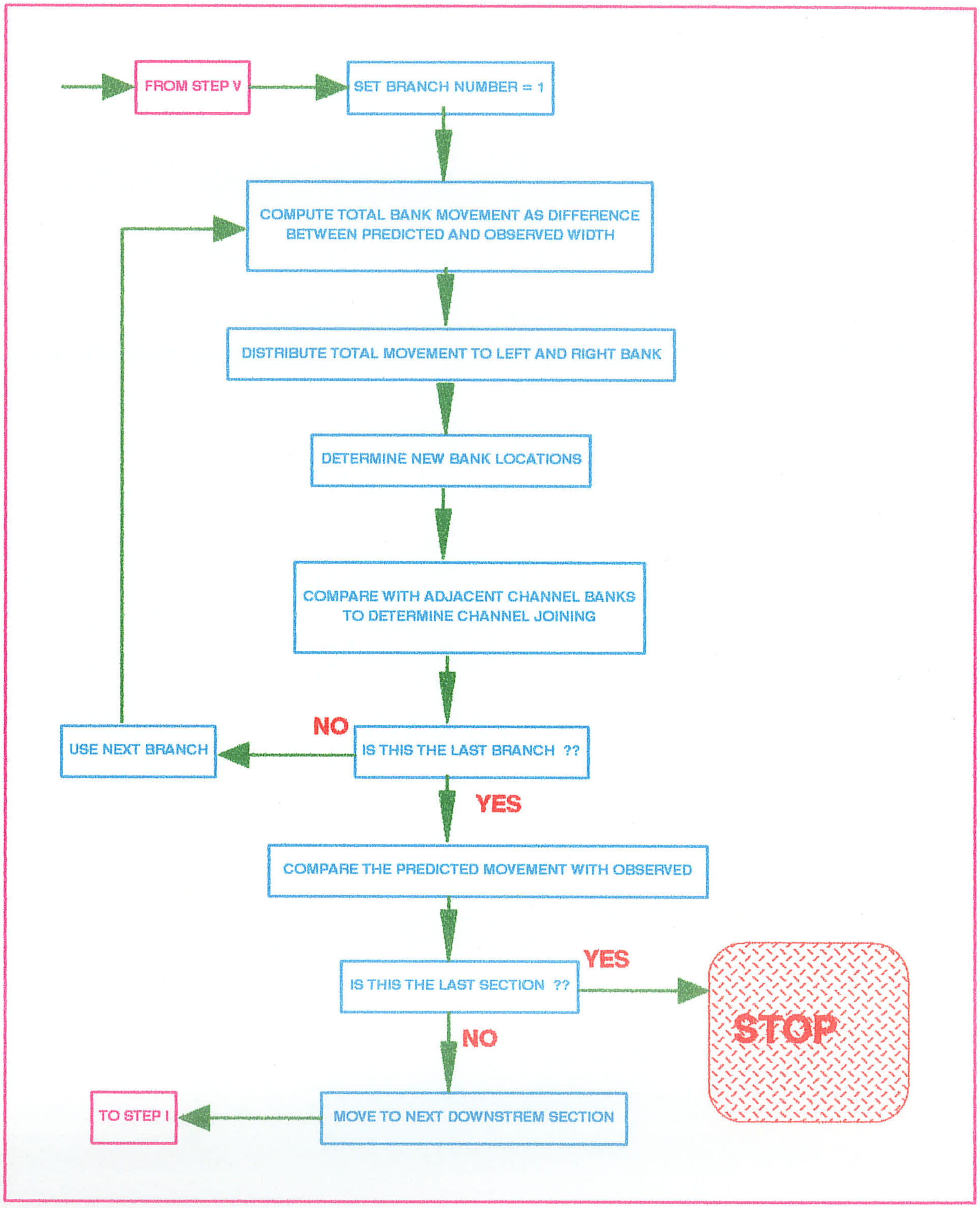

Fig. 7.7: Flow Diagram of Step VI

2. The amount of movement thus computed is used to determine the location of each bank. For the left bank, the location is determined by adding or subtracting this amount with the existing location of the bank depending on whether the movement is deposition or erosion. For the right bank, the amount of movement is added to the location of the bank if the movement is erosion, and subtracted if it is deposition.

3. The new location of banks thus computed for the channels within the river are checked to see if there is any union of channels.

4. The left bank location of the outer left channel and the right bank location of the outer right channel are the expected outer banklines of the river in the following year. The movements associated with these new locations are the expected movement of the river.

5. These predicted movements of the banklines are compared to the observed movements of the following year to examine the performance of the model.

The computing time required by the model was satisfactory. If the cross sectional data of the monitoring stations are pre-processed, then the solution for one year required about one hour of computing time on an 386 Personal Computer. However, if the cross sectional data have to be processed as well, then the solution for one year would require about three hours.

The results obtained from the procedures described above are discussed in the next chapter.

Chapter Eight

RESULTS OF QUANTITATIVE BANK MOVEMENT PREDICTION

8.1 Introduction

The probabilistic model described in the previous chapter was used to predict annual bank movements of the Brahmaputra at 33 locations along the river for five successive years. They were compared with observed movements. The results are shown in this chapter.

In order to assess the results, a suitable measure of model performance is first defined. This measure is then used to evaluate the predictions of the outer bank movements for the river as a whole. This is followed by a discussion of the separate movements of the left bank, the right bank, and the banks within different reaches and for different years.

The prediction of the inner bank movement was done only to the point that similarity in performance with the outer bank movement could be established. Quantitative prediction of inner bank movement was required only for the purpose of predicting channel joining.

The accuracy of a qualitative prediction of the change in branch channel dimensions is also discussed in this chapter. Finally, the results obtained from this study are compared with those from an earlier study.

8.2 Measure of Model Performance

A measure is needed to assess the magnitude of the error in the prediction of the bank movements. For this purpose the sum of squared errors in the bank movements was compared with the sum of squared observed movements in a manner similar to the well known R^2 value used in regression analysis.

$$R^2 = \frac{SS(M_o)_{t+1} - SS(\epsilon)_{t+1}}{SS(M_o)_{t+1}} \tag{8.1}$$

where,

SS stands for the sum of squares of the quantity in the parenthesis,

$(M_o)_{t+1}$ is the observed bank movement of year $(t+1)$ compared to year t, and

ϵ is the error in prediction.

It is evident that a value of $R^2=1$ corresponds to no error and that the larger the errors, the lower the value of R^2. One cannot, however, simply assert that the value of R^2 corresponds to the percentage of the total variance that can be explained by the model. The mean of the observed movements and the mean of the errors are not necessarily zero nor are they necessarily the same. It will be shown, however, that neither is far from zero. and that the deviation probably corresponds to a systematic trend of the river to move in a certain direction. More study is needed, however, to incorporate such a trend in the prediction model. Although some caution is needed in the interpretation of R^2, it would appear that the measure is a useful indicator of the relative performance of the model.

The observed bank movement is denoted as M_o and is defined as:

$$(M_o)_{t+1} = (x_o)_{t+1} - (x_o)_t \tag{8.2}$$

where,

$(x_o)_t$ is the observed distance of the bank of year t from a reference line on the left.

Note that the value of M_o for the left bank will be negative if the bank experiences erosion, and positive if it experiences deposition. For the right bank the sign is reversed.

The predicted bank movement is denoted by M_p and is defined as:

$$(M_p)_{t+1} = (x_p)_{t+1} - (x_o)_t \tag{8.3}$$

where,

x_p is the predicted distance of the bank in the year t from the same horizontal reference line on the left.

It is to be noted that the displacement for year $(t+1)$ is obtained from the model by using the input values of year t. No observation of year $(t+1)$ is used in the prediction.

The error in prediction is denoted by ϵ and is defined as:

$$(\epsilon)_{t+1} = (M_p)_{t+1} - (M_o)_{t+1} \tag{8.4}$$

The errors were further examined in order to detect any obvious trend for different levels of observed movement. For this purpose the studentized error Z_ϵ, was defined as follows:

$$Z_\epsilon = \left[\frac{\epsilon - \epsilon_m}{S_\epsilon}\right]_{t+1} \tag{8.5}$$

where,

ϵ_m is the observed mean of the errors, and

S_ϵ is the observed standard deviation of the errors.

8.3 Outer Bank Movement

The movements of the outer banks of the Brahmaputra river are the most important because of their effect on the population and on any structures along the river. These movements were therefore studied in detail. The performance of the model in predicting the outer bank movements at specified locations is discussed under four separate headings: (a) all of the left and the right bank movements together, (b) the movements of the left bank only, (c) the movements of the right bank only, and (d) the bank movements in different reaches and different years. In this way any possible difference in model performance between the left bank and the right bank, and between the six different reaches could be determined as well as any persistent trend over the period of observation.

8.3.1 All Movement Data Together In this group, the observed and the predicted movements of both the left and the right banks are lumped together. Over the recorded five year period, 33 sections were used giving a possible 330 observations for this group. However, thirteen sections had missing values (26 data points) leaving a total of 304 data points available for analysis.

Two downstream sections and two upstream sections had unusually high values of error (difference between predicted and observed movements). The downstream sections are near the confluence of the Ganges and the Brahmaputra rivers, which may explain their deviant behaviour. The two upstream sections are upstream of the diversion point of the Brahmaputra and the Old Brahmaputra. They showed splitting of branches. No other explanation for the very different behaviour of these sections has been identified in this study.

In order to judge whether a particular observed error can be regarded as an outlier one may determine the *t-value* that has a 5% probability of being exceeded in 304 trials. This value is 3.58. All of those 8 observations had *t-values* beyond this limit. They were considered to be outliers and were excluded from the results reported in the following. Since the largest *t-value* of the remaining data points is about 3 one may accept that all remaining prediction errors can be attributed to chance.

The observed bank movement closely follows a normal distribution with a mean of −66 m and a standard deviation of 382 m. It is to be noted that the mean bank erosion of -66 m is statistically significant at the 0.9987 level. This does imply that, at least over the period of five years, there was a small but significant trend of the river to move westward. This is in agreement with Coleman's (1969) observation of the river's westward shifting.

The observed and the predicted bank movements are plotted in Fig. 8.1 which shows the scatter of points around the line of perfect agreement. The value of R^2, as defined before in Eqn. (8.1), is 0.924. It confirms the visual conclusion of a reasonably good fit.

The studentized errors are plotted against the observed bank movement in Fig. 8.2. There does not seem to be an apparent trend in the errors, nor is there an evident difference in error variance.

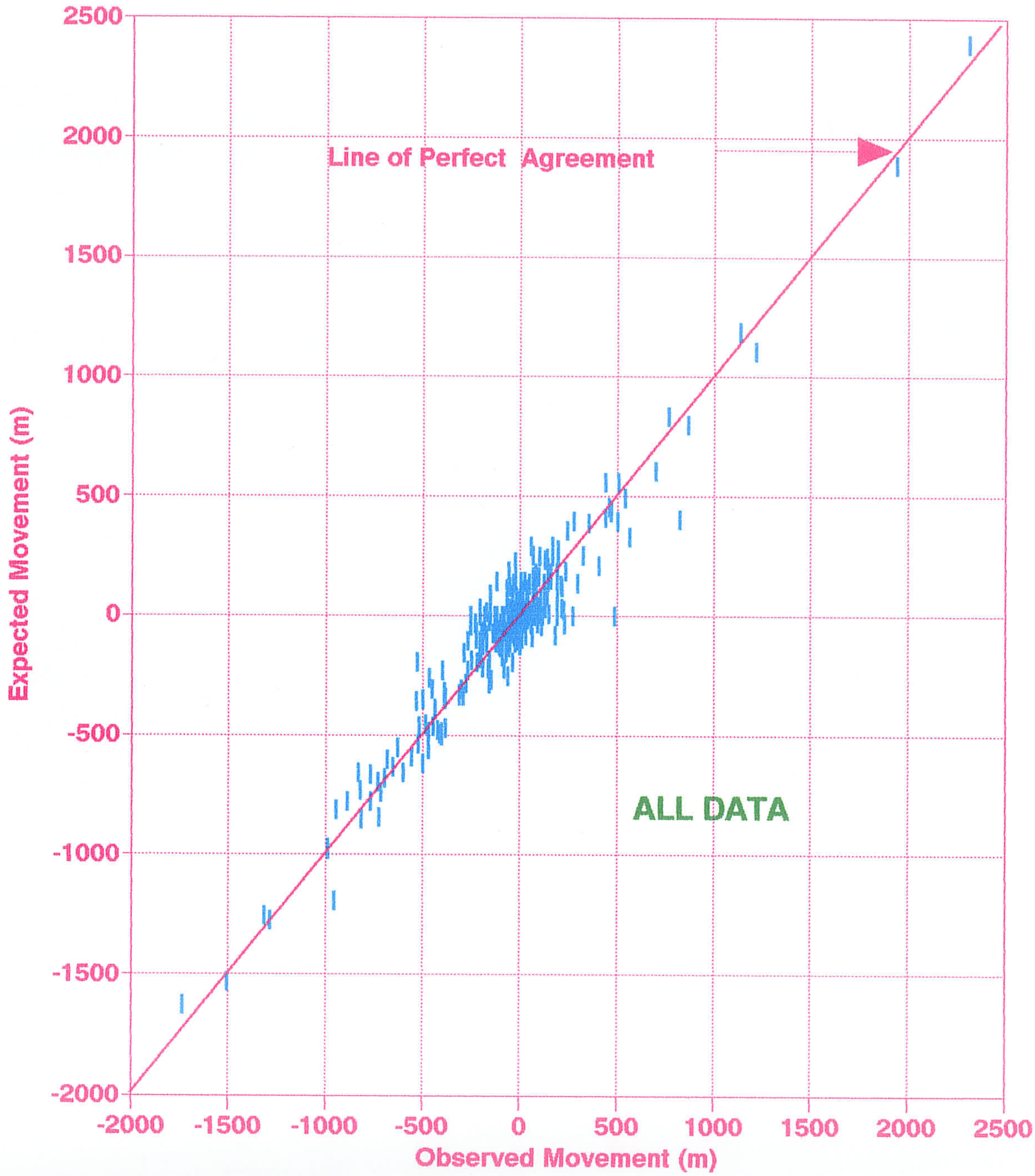

Fig. 8.1: Observed and Expected Bank Movement for All Data

Fig. 8.2: Residual and Observed Bank Movement (All Data)

The normal probability plot, the stem-leaf plot and the W-statistic for normality all show that the error can be taken as normally distributed with a mean of 15 m and a standard deviation of 106 m. The t-statistic for the mean error is 2.46. This gives a probability of exceedance of less than 1% so that this deviation from zero is also statistically significant. This deviation, however, is an annual deviation since the bank movements were related to the bank position of the previous year. The mean of 15 m is therefore not inconsistent with the observed westward movement of the entire river of -66 m in five years. If one would correct the observations for this trend one would find a calculated mean error very close to zero.

Figure 8.1 and 8.2 shows that the variance of the prediction error is fairly constant over the range of observed movements. This implies that the error in the movement is relatively independent of the movement. As a result the model is not very good at predicting the direction of small movements qualitatively. On the whole, the predicted values are in agreement with the observed direction of movement 77% of the time. That is, the values of the observed and the predicted movements had the same sign (erosion or deposition) 77% of the time. The bulk of the disagreements occur when the movements are small in magnitude. Note that the outliers are not included in these figures.

From the above discussion, it can be concluded that the overall performance of the model in predicting the movements of the banks in the following year from an input set of data of the previous year is quite satisfactory.

8.3.2 Left Bank Movement The performance of the model was also examined for the data points on the left bank of the entire river in Bangladesh so as to compare it with the bank movement of the river as a whole.

The observed movements of the left bank are normally distributed with a mean of -7 m and a standard deviation of 328 m. Both of these values are a little lower in absolute magnitude than those of the entire river. Evidently, more deposition and less erosion occurred on this bank compared to the right bank.

The plot of the predicted and the observed bank movements is shown in Fig. 8.3. The plot shows that the agreement between the model and the observation is satisfactory. The value of R^2 is 0.90 which is somewhat lower than that for the entire data set.

The errors in prediction are plotted in Fig. 8.4 as studentized values. From this figure, it appears that the error variance is constant and that there is no obvious trend of the errors.

A normal probability plot, a stem-leaf plot and the W-statistic showed that the errors for the left bank are normally distributed. The mean of the errors is 31 m and the standard deviation is 104 m. The mean is significantly different from zero but not inconsistent with the noted overall westward trend in the movement of the entire river.

8.3.3 Right Bank Movement The movement of the right bank was also studied separately to evaluate the performance of the model for this bank.

The observed movements of the right bank have a mean of -125 m and a standard deviation of 423 m. This is also a significant deviation from a mean of zero but consistent with the observed mean movement of the left bank and the river as a whole.

The predicted and the observed movements are plotted in Fig. 8.5. The plot shows that the scatter around the perfect agreement line is less than that for the left bank. This shows up in the value of R^2 of 0.94 which is higher than that for the left bank. The mean of the errors is zero and the standard deviation is 105 m. The values of errors in prediction are plotted in Fig. 8.6 as the studentized errors.

The plots show a constant variance and an absence of any substantial trends.

Although the performance of the model for the right bank is a little better than that for the left bank, the overall behaviour is similar. The predicted and the observed values of bank movement follows the same relationship (although better), and the behaviour of the errors are similar as well. Therefore, it can be concluded that the model performs the same way for both the left and the right banks of the Brahmaputra river.

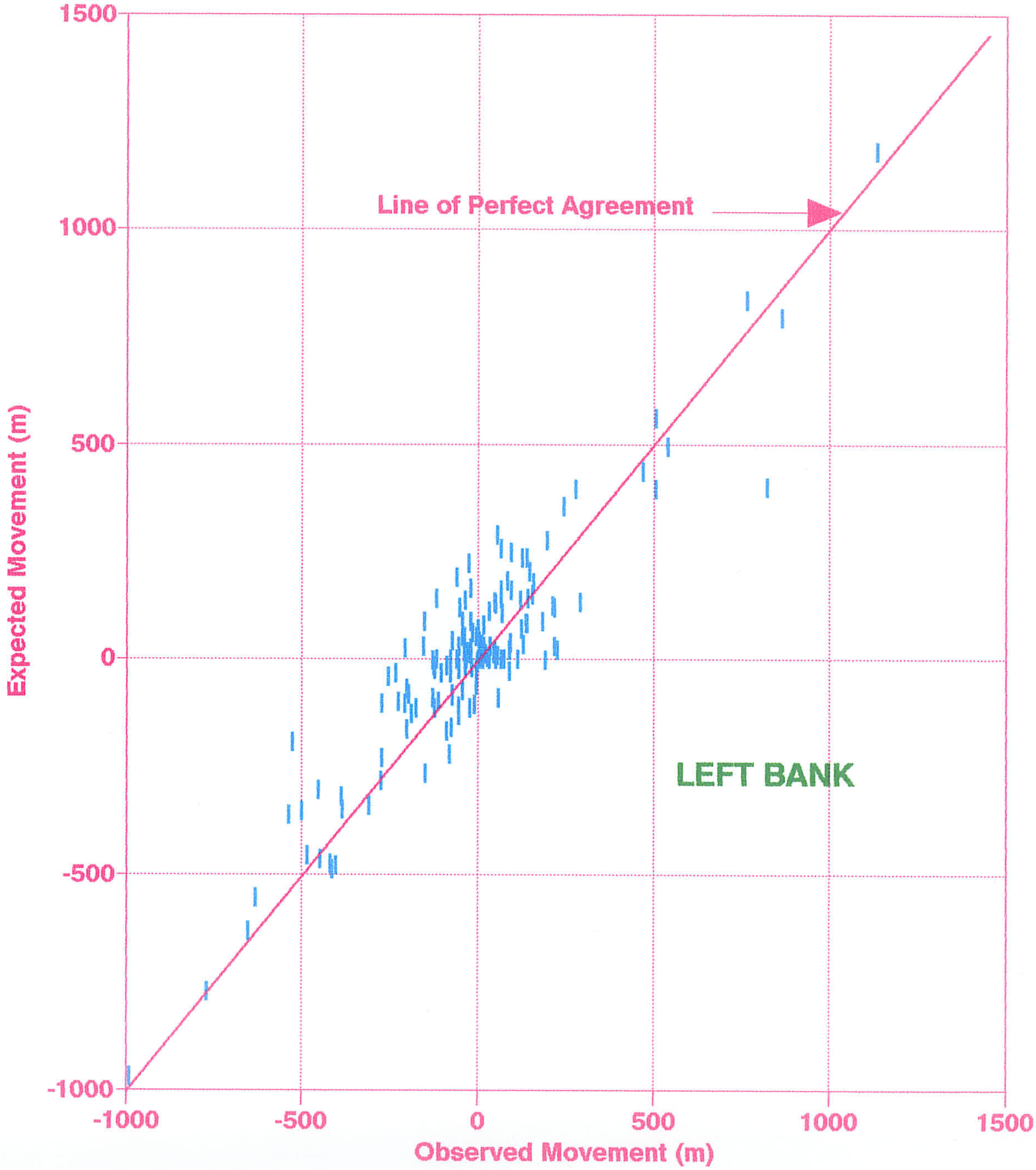

Fig. 8.3: Observed and Expected Movement of the Left Bank

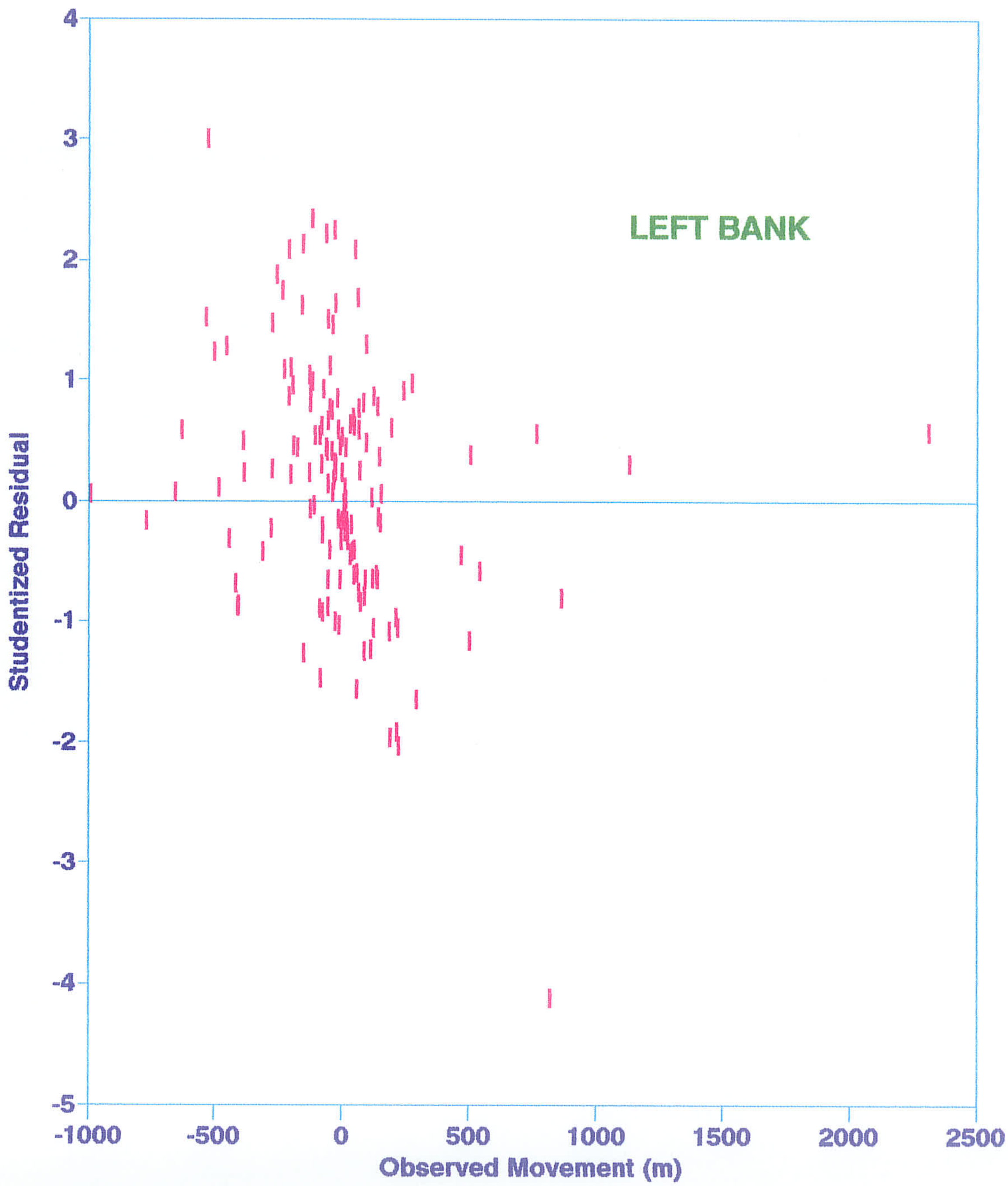

Fig. 8.4: Residual and Observed Movement of Left Bank

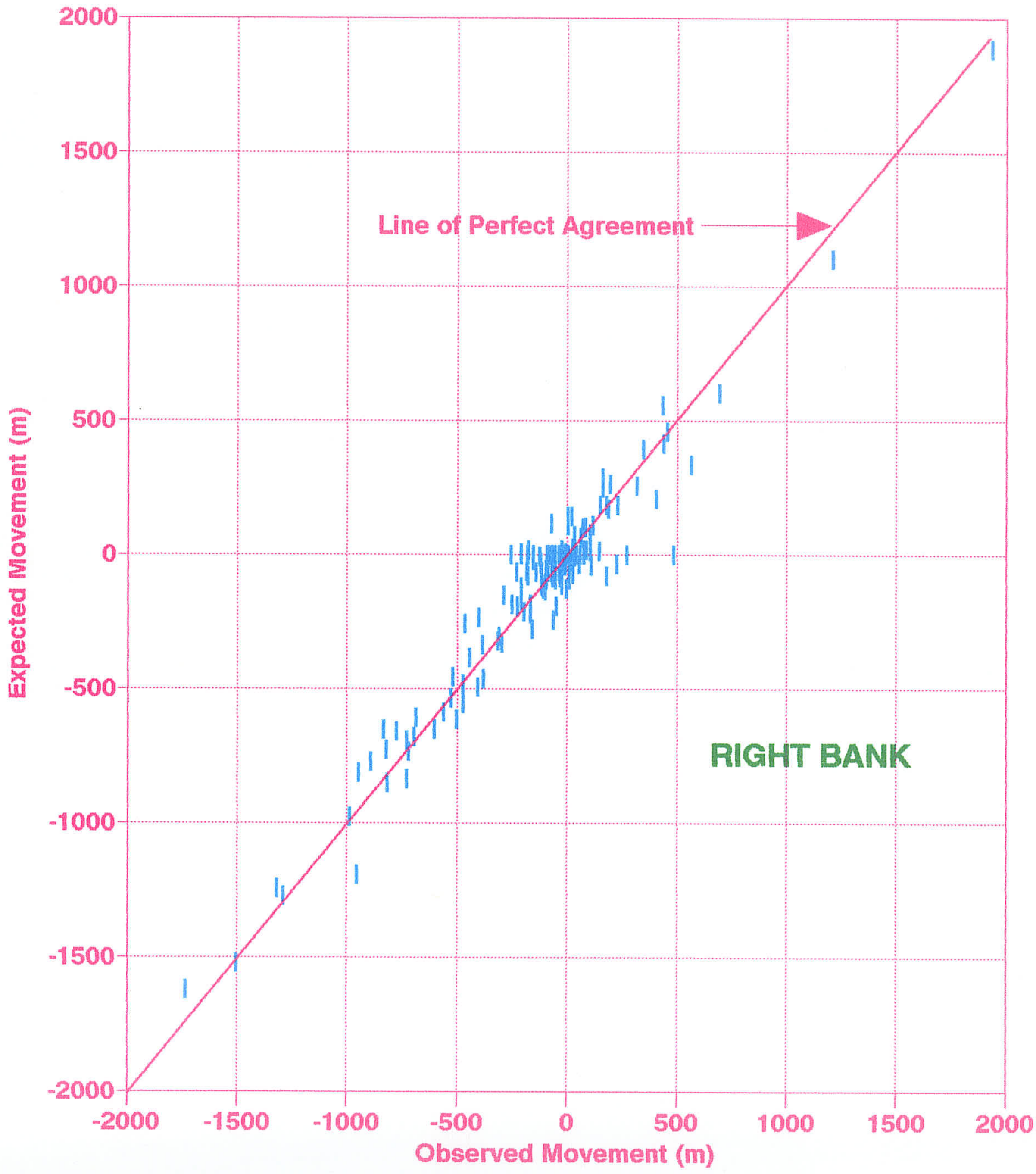

Fig. 8.5: Observed and Expected Movement of the Right Bank

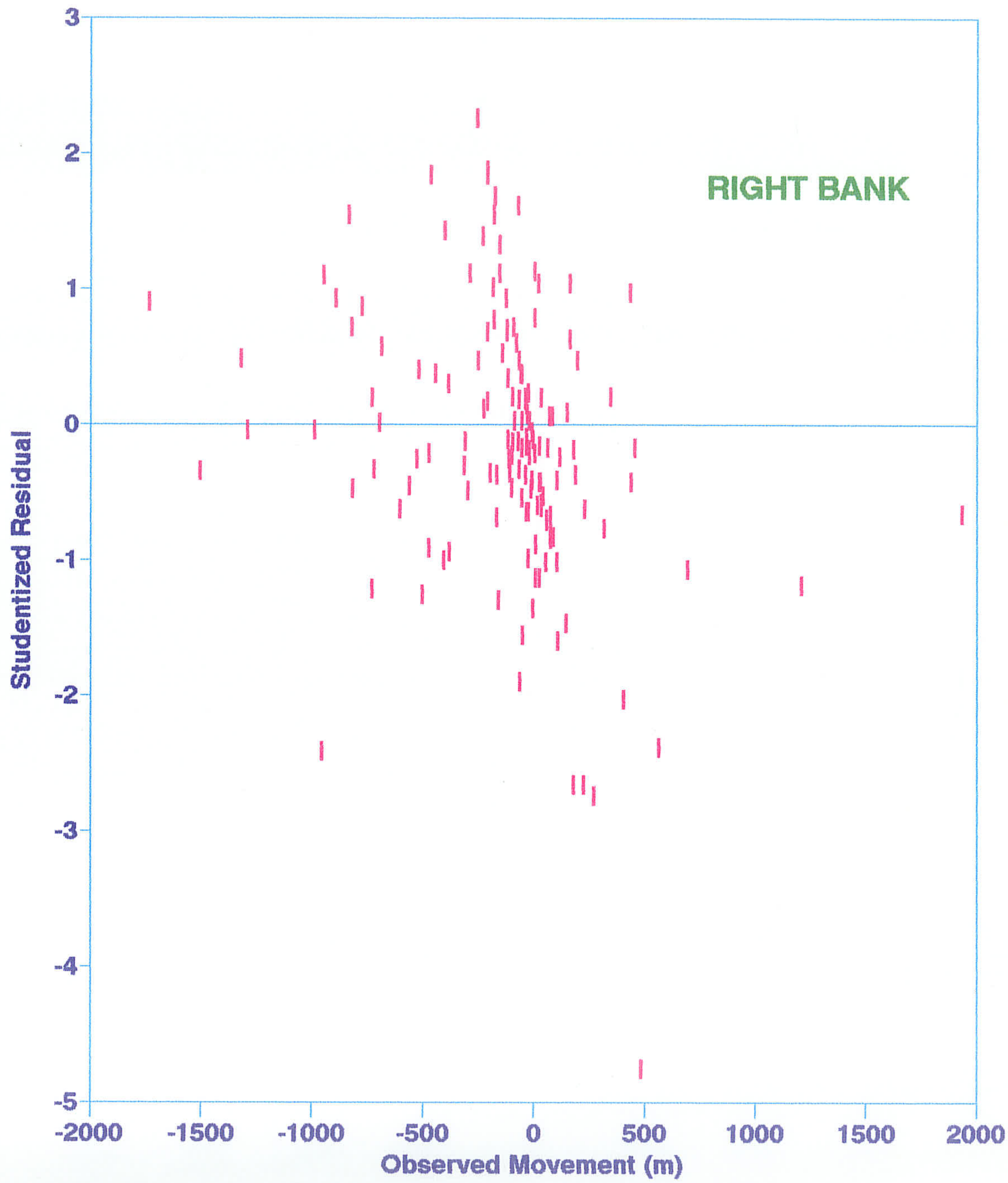

Fig. 8.6: Residual and Observed Movement of the Right Bank

8.3.4 Different Reaches of the River The performance of the model was studied separately for the six reaches of the Brahmaputra river so as to compare the behaviour of individual reaches. Since the prediction errors of the left and the right bank were not significantly different for the entire river, the movements for both banks were combined for each reach.

8.3.4.1 Bank Movement in Reach 1 The observed bank movements of reach 1 are normally distributed with a mean of 89 m and a standard deviation of 334 m. The predicted and observed movements are shown in Fig. 8.7 with the line of perfect agreement. The value of R^2 is 0.91 which is almost the same as for the entire river. The prediction errors are normally distributed with a mean of 8 m and a standard deviation of 101 m. The plot of studentized errors against observed movement on Fig. 8.8 shows relatively constant variance and no significant trend. in the error.

The result does not include the two downstream sections for which the model did not perform well. These show observed movements of -953, 820, -526, -729 m, errors of 761, -1221, 729, 587 m and studentized errors of 7.03, -11.38, 6.64 and 5.40. It may thus be argued that they represent outliers. As a result the analysis was based on the seventy six data points that remained available for this reach.

8.3.4.2 Bank Movement in Reach 2 The observed bank movements of reach 2 have a mean of -59 m and a standard deviation of 391 m. Fig. 8.9 shows the plot of the observed and the predicted bank movement for this reach. The scatter around the perfect agreement line is not substantial. This is reflected in an R^2 value of 0.93, which is about the same as was found for the entire data set. The prediction errors are normally distributed with a mean of 20 m and a standard deviation of 102 m. the studentized values are shown on Fig. 8.10. No trend in the errors is apparent. For this reach, 34 data points were available.

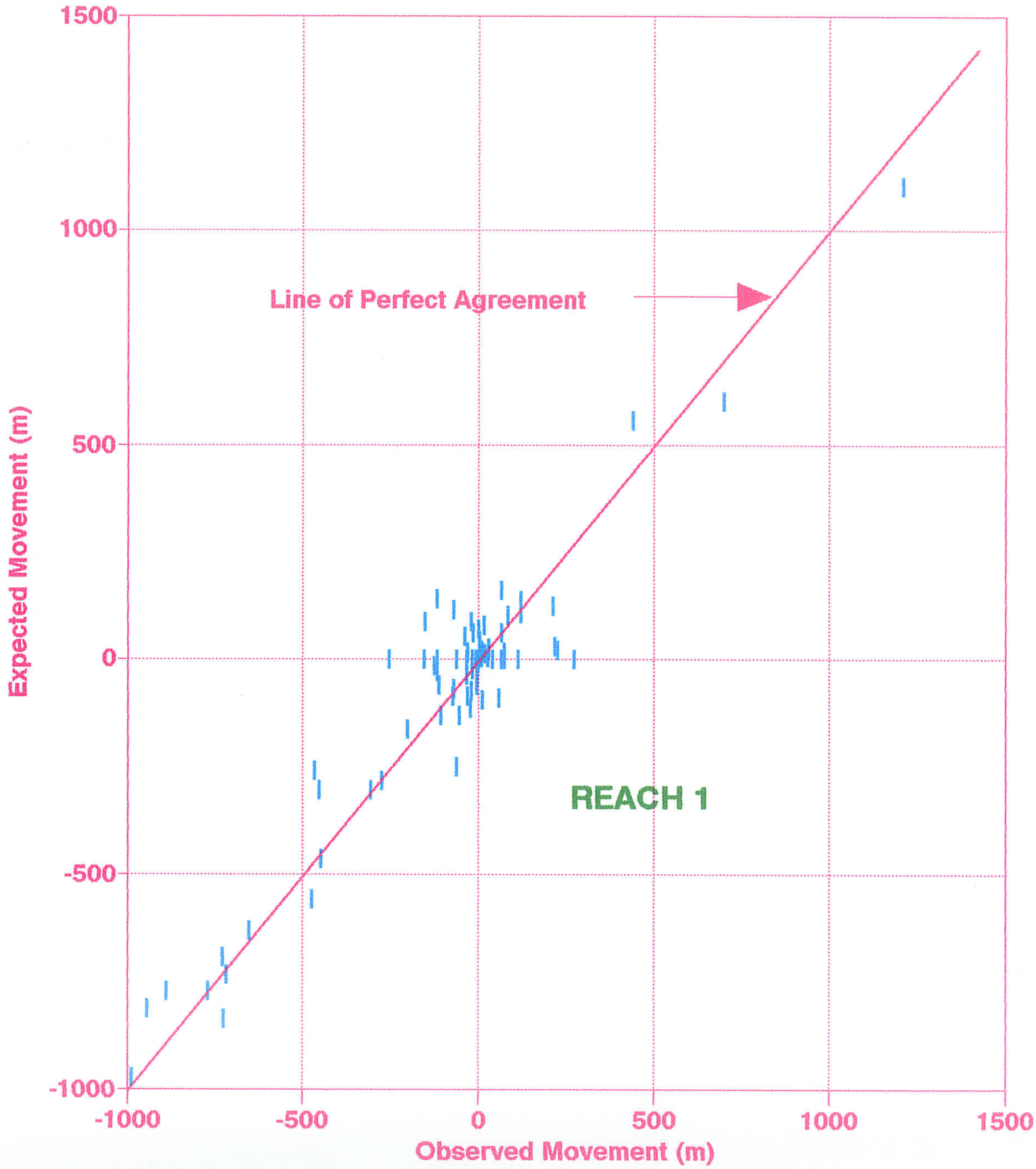

Fig. 8.7: Observed and Expected Bank Movement of Reach 1

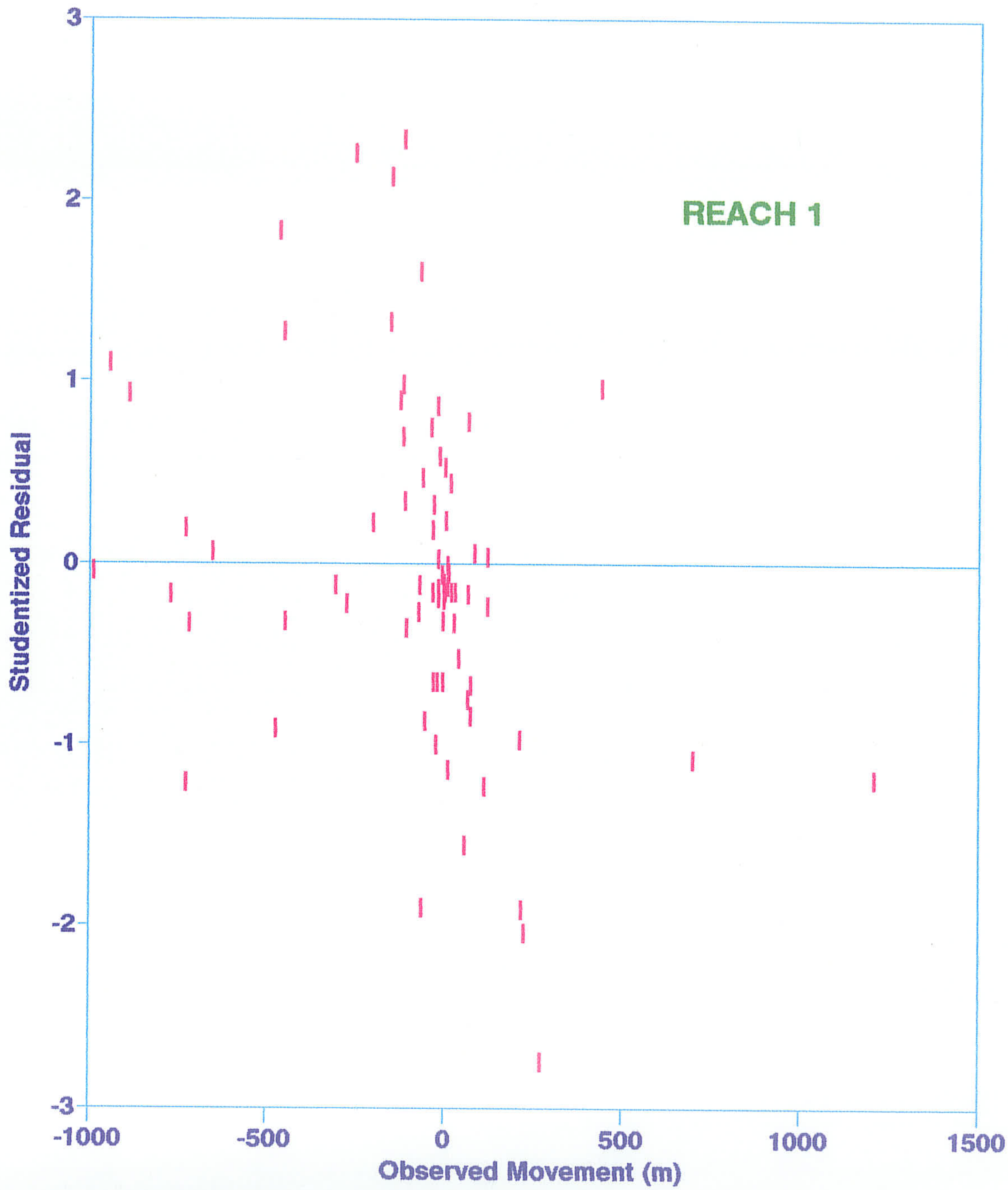

Fig. 8.8: Residual and Observed Bank Movement of Reach 1

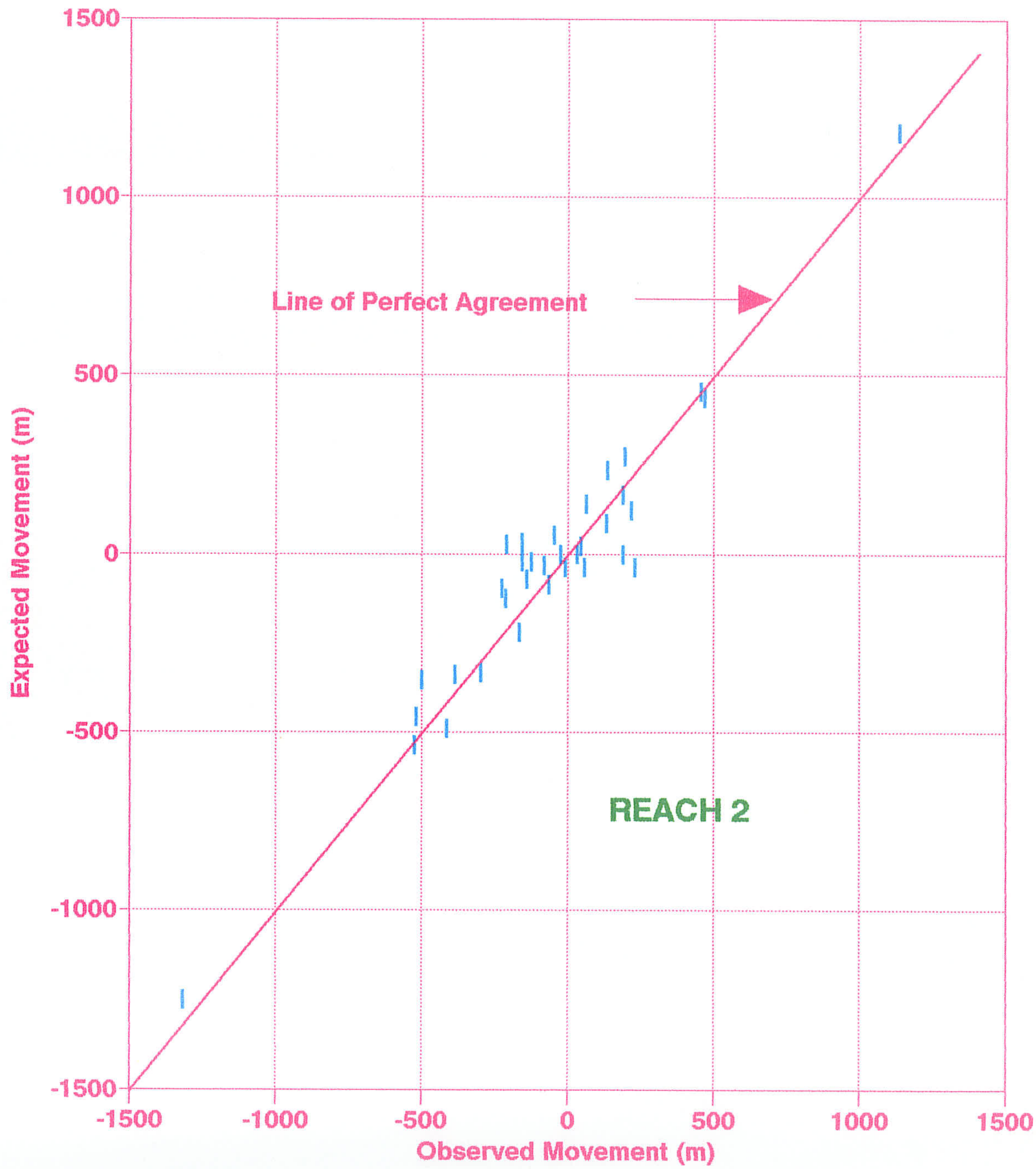

Fig. 8.9: Observed and Expected Bank Movement of Reach 2

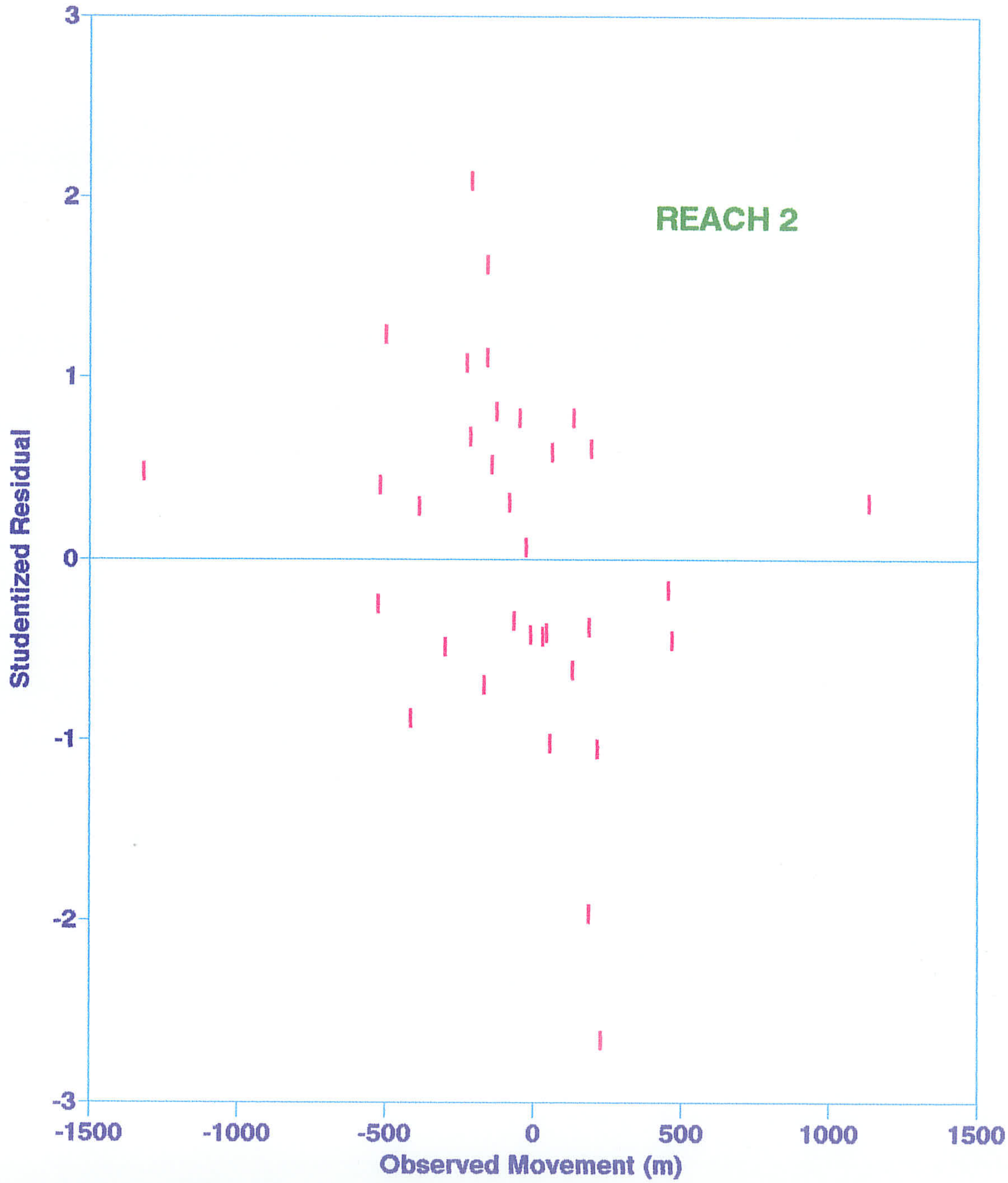

Fig. 8.10: Residual and Observed Bank Movement of Reach 2

8.3.4.3 Bank Movement in Reach 3 The observed bank movement of reach 3 has a mean of − 104 m and a standard deviation of 365 m. The distribution of these movements is skewed to the right. However, the model predicted the very large erosions that occurred rather well. The observed and the predicted bank movements are shown in Fig. 8.11. The value of R^2 is 0.91 which implies a satisfactory agreement between the observed and the predicted values. The observed errors are plotted against the actual bank movement values in Fig. 8.12. The mean of the errors is 5 m and the standard deviation is 111 m. Fifty four data points were available for this reach.

This figure shows that there may be a trend present in the errors with the level of bank movement. The behaviour of the errors for this reach is different from all the other reaches. (Note that this reach showed different behaviour in the prediction of bank movement from the deviation of observed equilibrium widths as given in *Appendix-J*).

8.3.4.4 Bank Movement in Reach 4 The actual bank movements of this reach has a mean of − 46 m and a standard deviation of 479 m. The distribution of these values is slightly skewed to the right. The plot of actual bank movements and those predicted by the model is shown in Fig. 8.13. The scatter in this plot is small which is also evident from the value of R^2 of 0.94. The standardized errors are shown in Fig. 8.14 as a plot with the observed bank movement. There is no evidence of a trend of the errors with levels of bank movement. A statistical analysis shows, as before, that the errors are normally distributed. They have a mean of 24 m and a standard deviation of 115 m. The performance of the model for this reach is good. Fifty six data points were available for this reach.

8.3.4.5 Bank Movement in Reach 5 The mean and the standard deviation of the observed bank movements of reach 5 are − 51 m and 249 m respectively. The distribution of these values is normal. The plot of observed and the predicted movements in Fig. 8.15 shows a considerable amount of scatter around the line of perfect agreement. This is also evident from the value of R^2 of 0.79. The performance of the model for this reach is not as good as for the other reaches. The errors in prediction for this

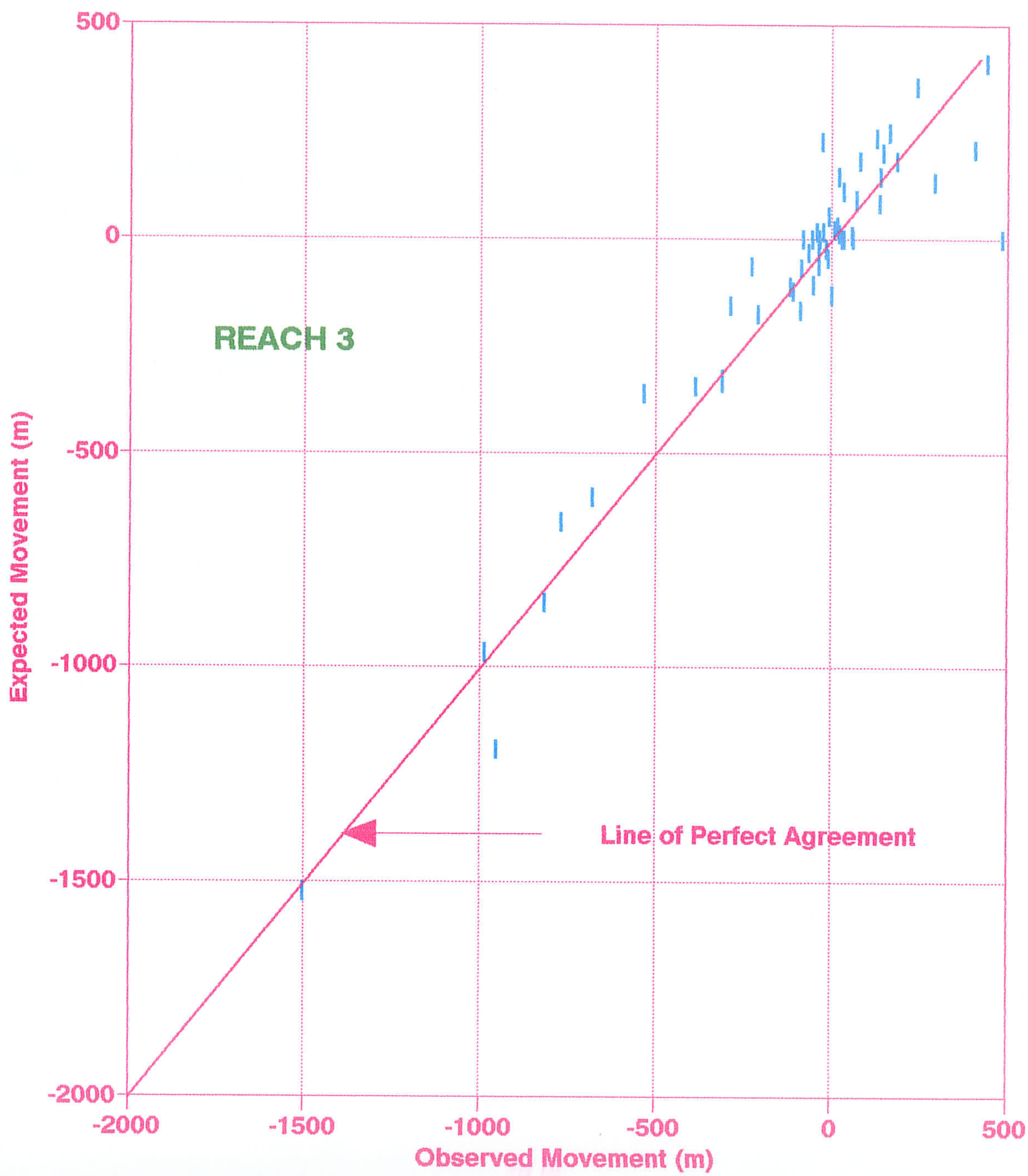

Fig. 8.11: Observed and Expected Bank Movement of Reach 3

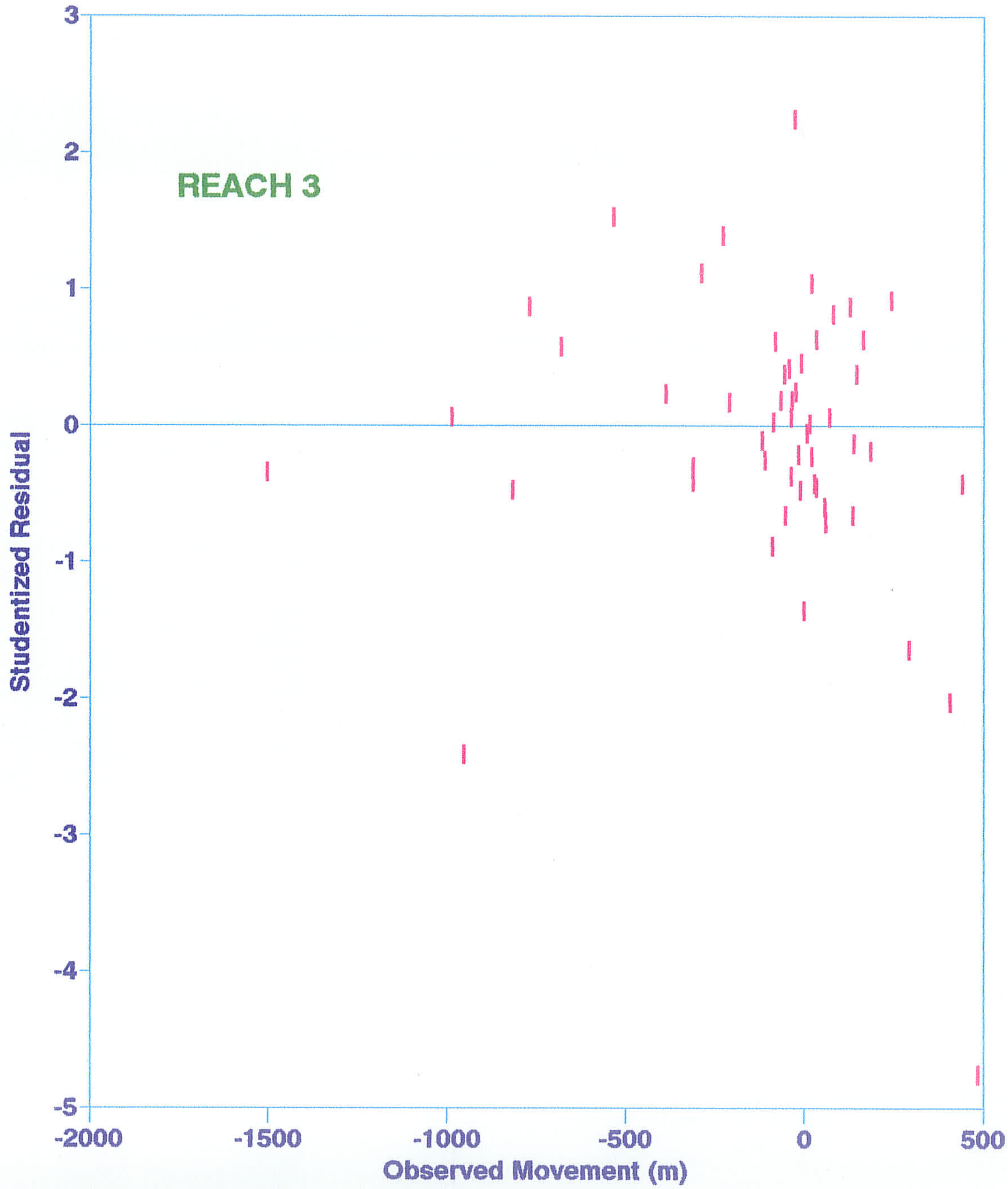

Fig. 8.12: Residual and Observed Bank Movement of Reach 3

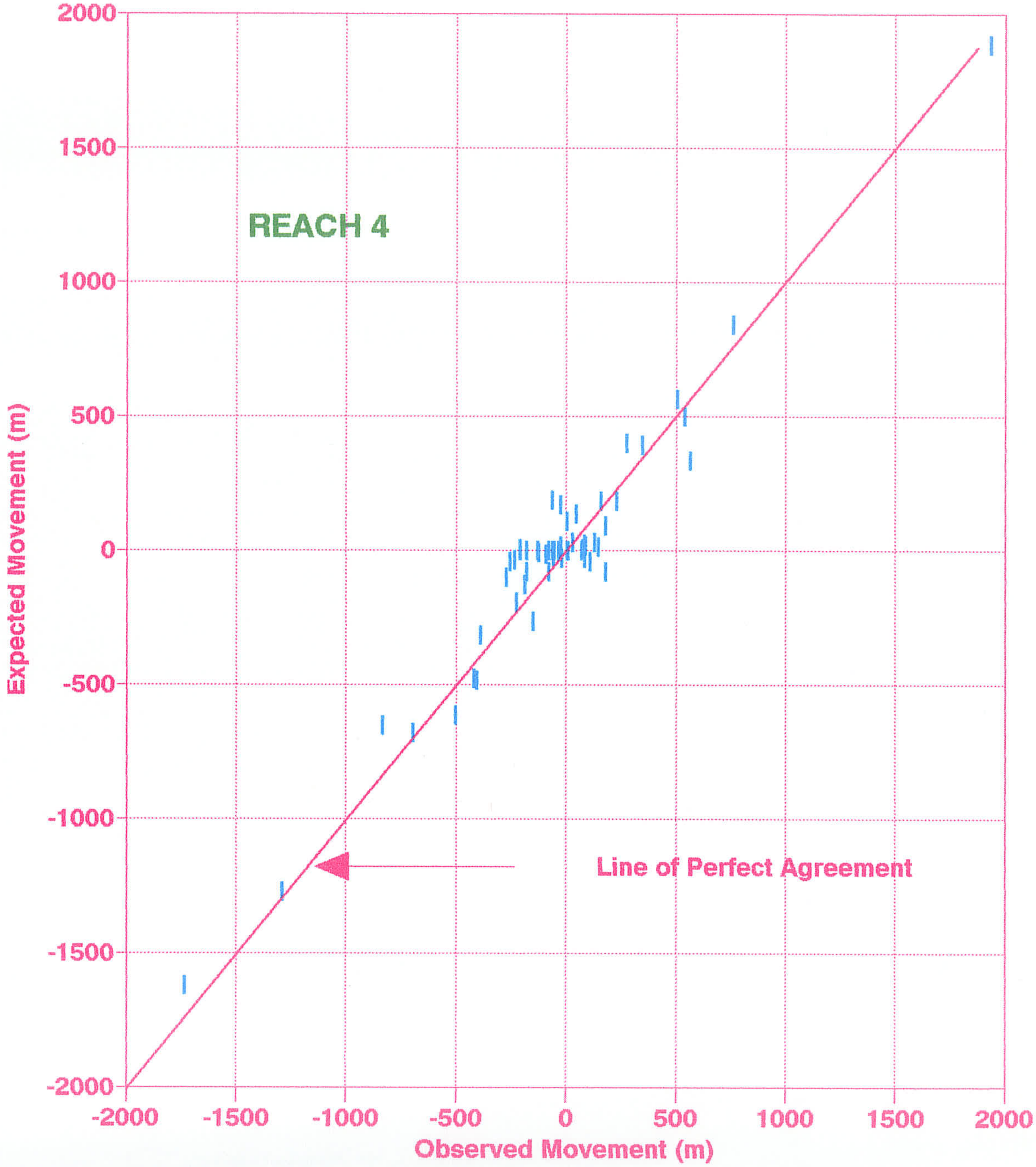

Fig. 8.13: Observed and Expected Bank Movement of Reach 4

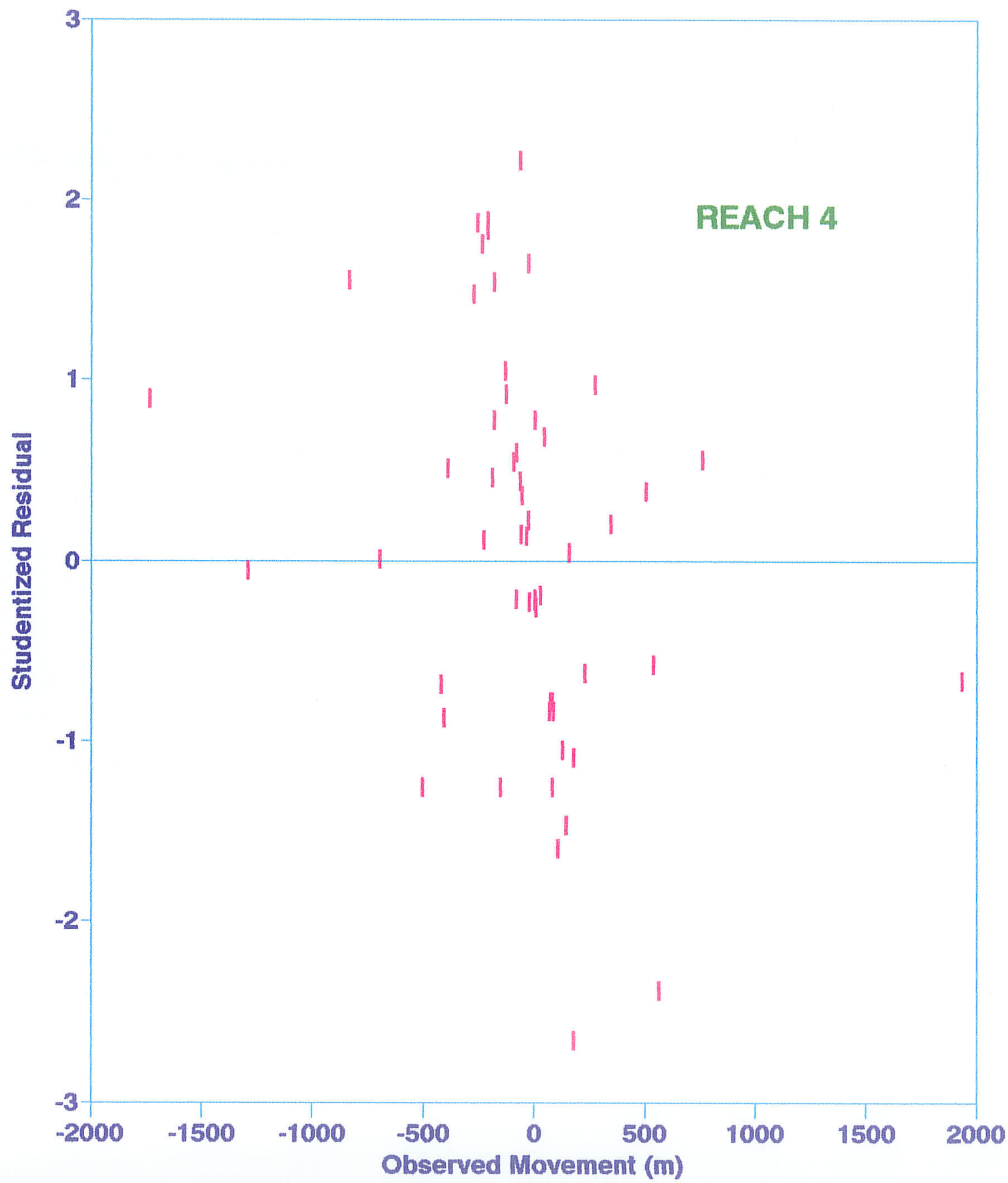

Fig. 8.14: Residual and Observed Bank Movement of Reach 4

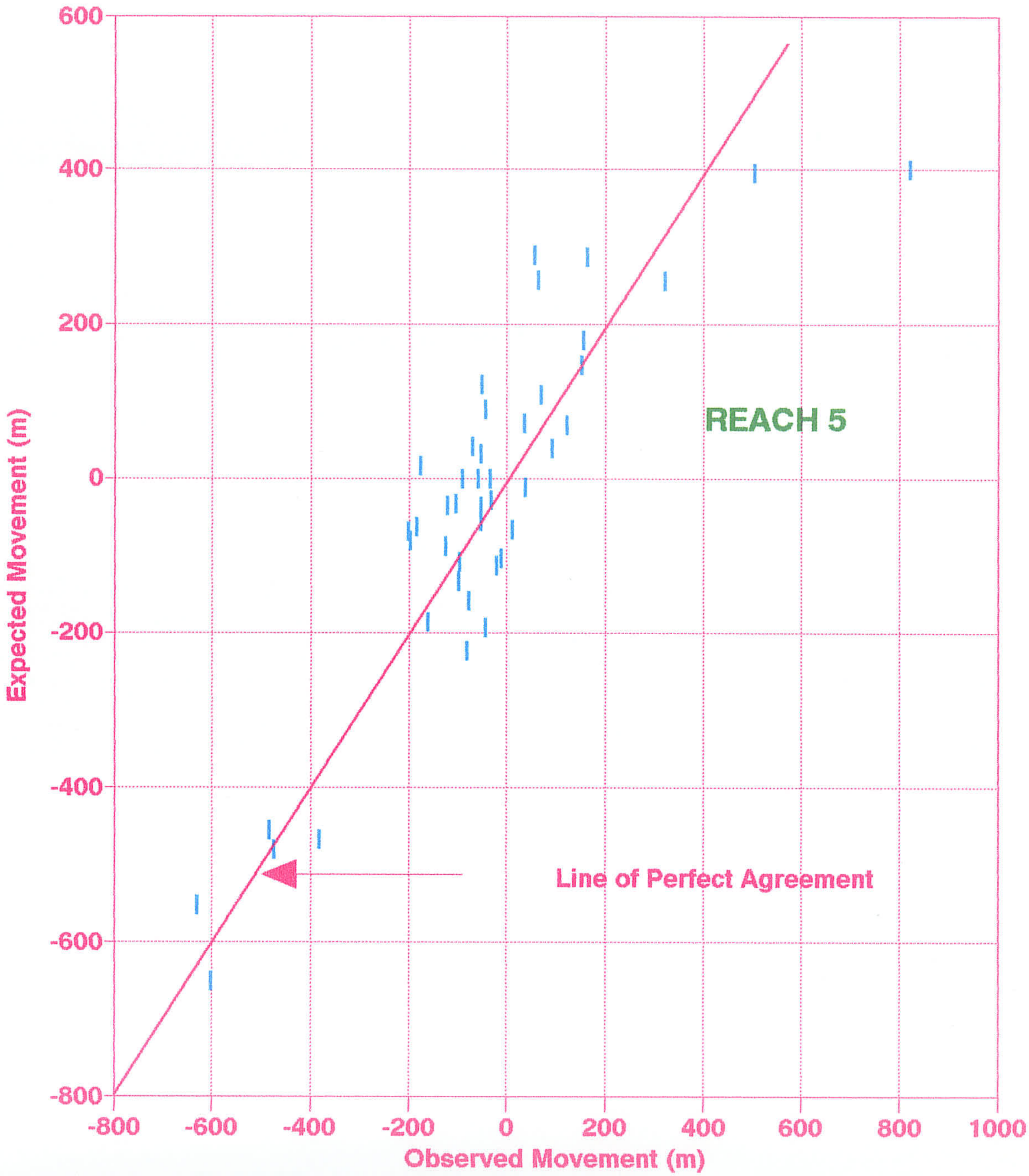

Fig. 8.15: Observed and Expected Bank Movement of Reach 5

reach are skewed to the right. They have a mean of 15 m and a standard deviation of 115 m. The plot of standardized errors with the observed bank movement is shown in Fig. 8.16. Forty four data points were available for this reach.

It can be argued from the figure that there is a trend in the error with different levels of observed movement. The trend is similar to that found in reach 3.

8.3.4.6 Bank Movement in Reach 6 The actual bank movements of reach 6 is skewed to the left with a mean of −26 m and a standard deviation of 461 m. The predicted bank movements from the model and that as observed are shown in Fig. 8.17. This plot has less scatter around the line of perfect agreement. The value of R^2 for this reach is 0.96 which is the best compared to the values for the total data and for other reaches. The errors in prediction is also skewed to the left with a mean of 27 m and a standard deviation of 88 m. The plot of standardized errors against the actual bank movement values is shown in Fig. 8.18. There were 40 data points available for the reach.

Except for reach 5, all the other reaches show that the performance of the model is either similar or better than that for the combined data. Although the model performance is poorer for reach 5, the value of R^2 of 0.79 or higher shows that the performance is acceptable. In conclusion it can be said that the developed model performs rather well for the entire data set as well as for different banks and different reaches of the length of the Brahmaputra river under study.

8.3.5 Bank Movement in Different Years The performance of the model for the entire river for different years was studied separately. It was found that, on the whole, the model performed as well for the separate years as for the entire period. The exception is the year 1980-1981 for which the value of R^2 is 0.752. The values of R^2, and the mean and standard deviations of the observed movements and the errors are given in Table 8.1.

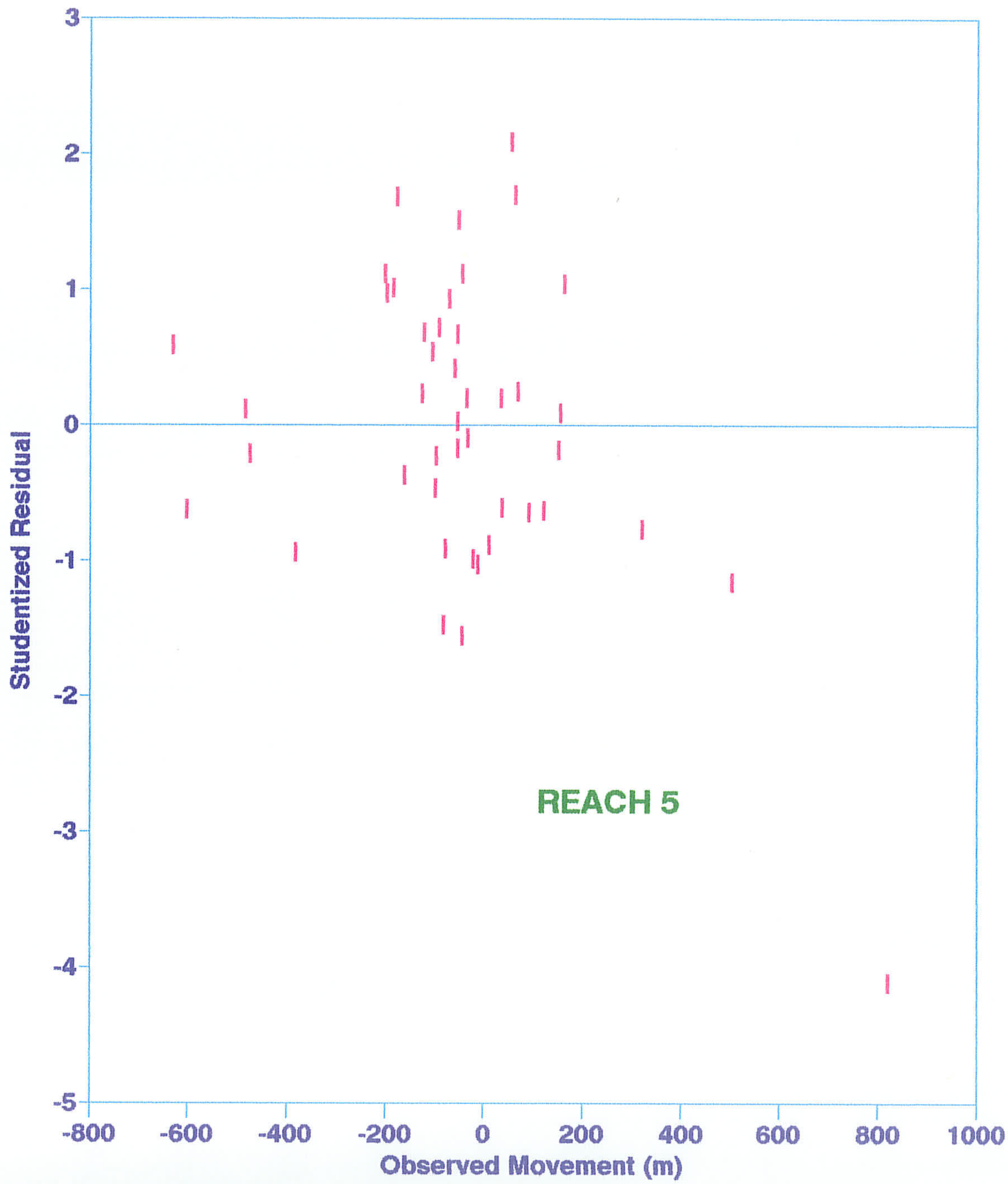

Fig. 8.16: Residual and Observed Bank Movement of Reach 5

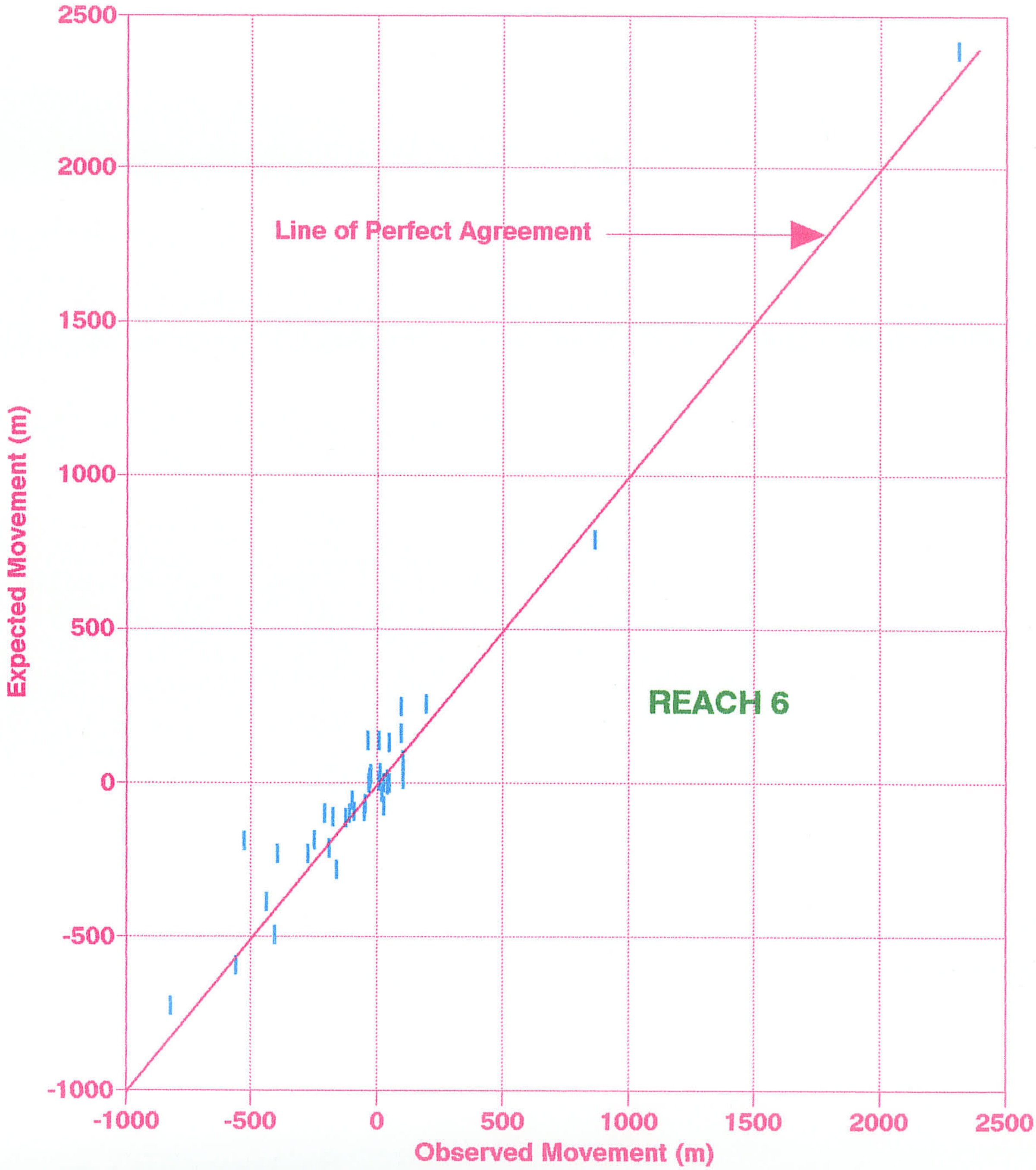

Fig. 8.17: Observed and Expected Bank Movement of Reach 6

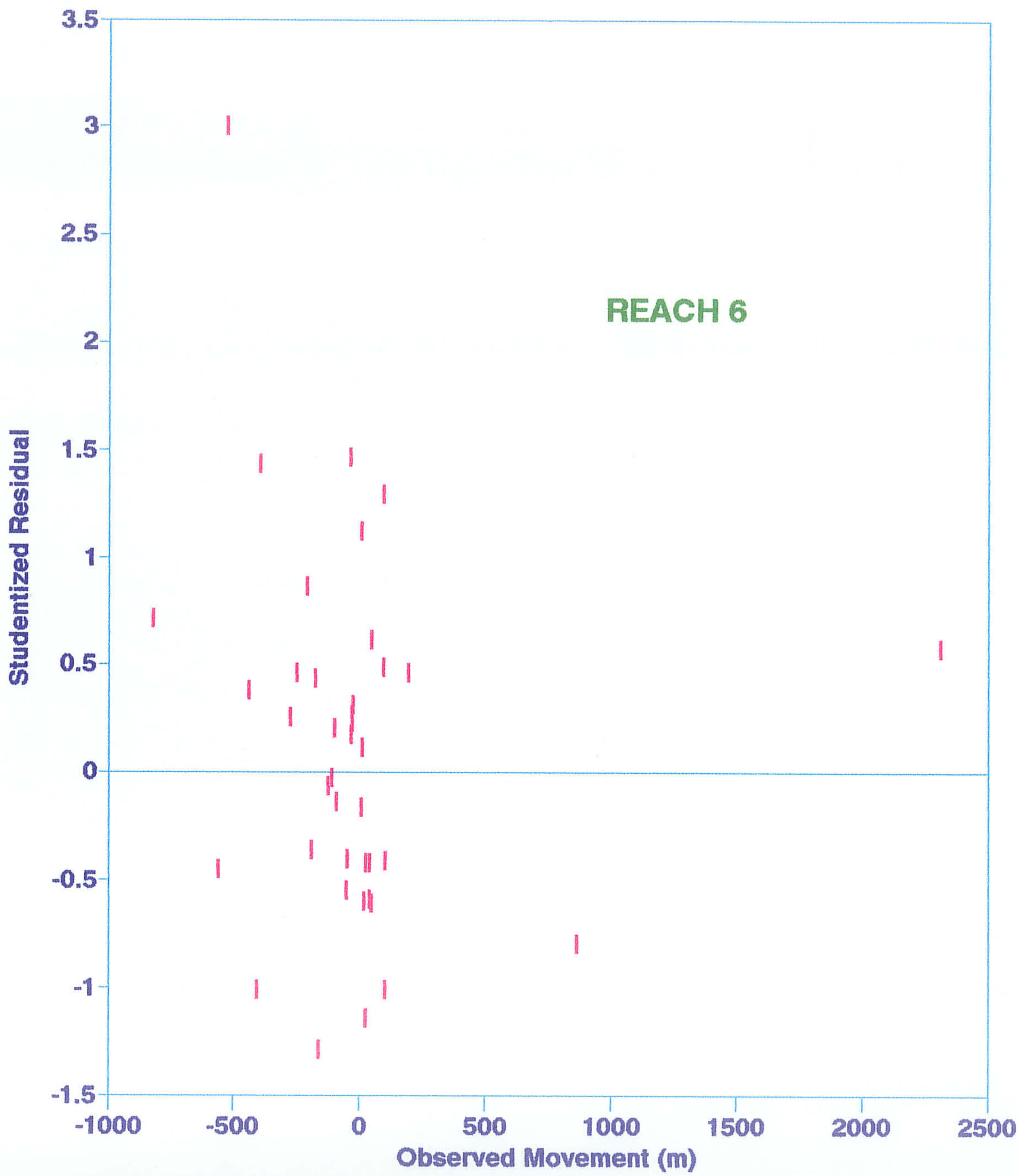

Fig. 8.18: Residual and Observed Bank Movement of Reach 6

Table 8.1: Performance of the Model for data of different years

Year of Data Used	No.of Data	Observed Movement		Error		R^2
		Mean	Std.Dev.	Mean	Std. Dev.	
1976 - 1977	60	-226	429	26	111	0.933
1977 - 1978	60	-61	362	21	83	0.947
1978 - 1979	60	-36	296	5	109	0.864
1979 - 1980	64	-9	484	25	95	0.961
1980 - 1981	60	-4	253	-0.7	126	0.752
All Data	304	-66	382	15	106	0.924

The computed and the observed movements of the left bank for 1977 are shown in Fig. 8.19. Data for the right bank are shown in Fig. 8.20. Note that in these figures the zero line represents the observed bankline of 1976 (which in the field is, of course, a curved line). Note that the right bank of 1977 (Fig. 8.20) experienced more erosion than the left bank (Fig. 8.19).

8.4 Inner Banks

The model is also capable of predicting the movements of the banks of the branch channels within the river. These movements were not studied in detail because they are less important. Also, the large number of data points, (1494) makes it necessary to rely entirely on computer analysis. Because of frequent channel splitting and joining it is not possible to rely on mechanical computer routines to decide in each case which bank must be associated with which channel in the preceding year.

It is difficult to compute the observed movements because comparison of the bank positions of two consecutive years cannot be easily done by simply recording the coordinate values of the banks with respect to some fixed reference. The combined effect of the change in number of channels either by joining two or more branches or by splitting the branch channel(s), and the movement of the banks within the river makes it very difficult to identify which bank corresponds to which channel of the previous year unless these were manually checked.

Fig. 8.19: Observed and Predicted Movement of the Left Bank (1977)

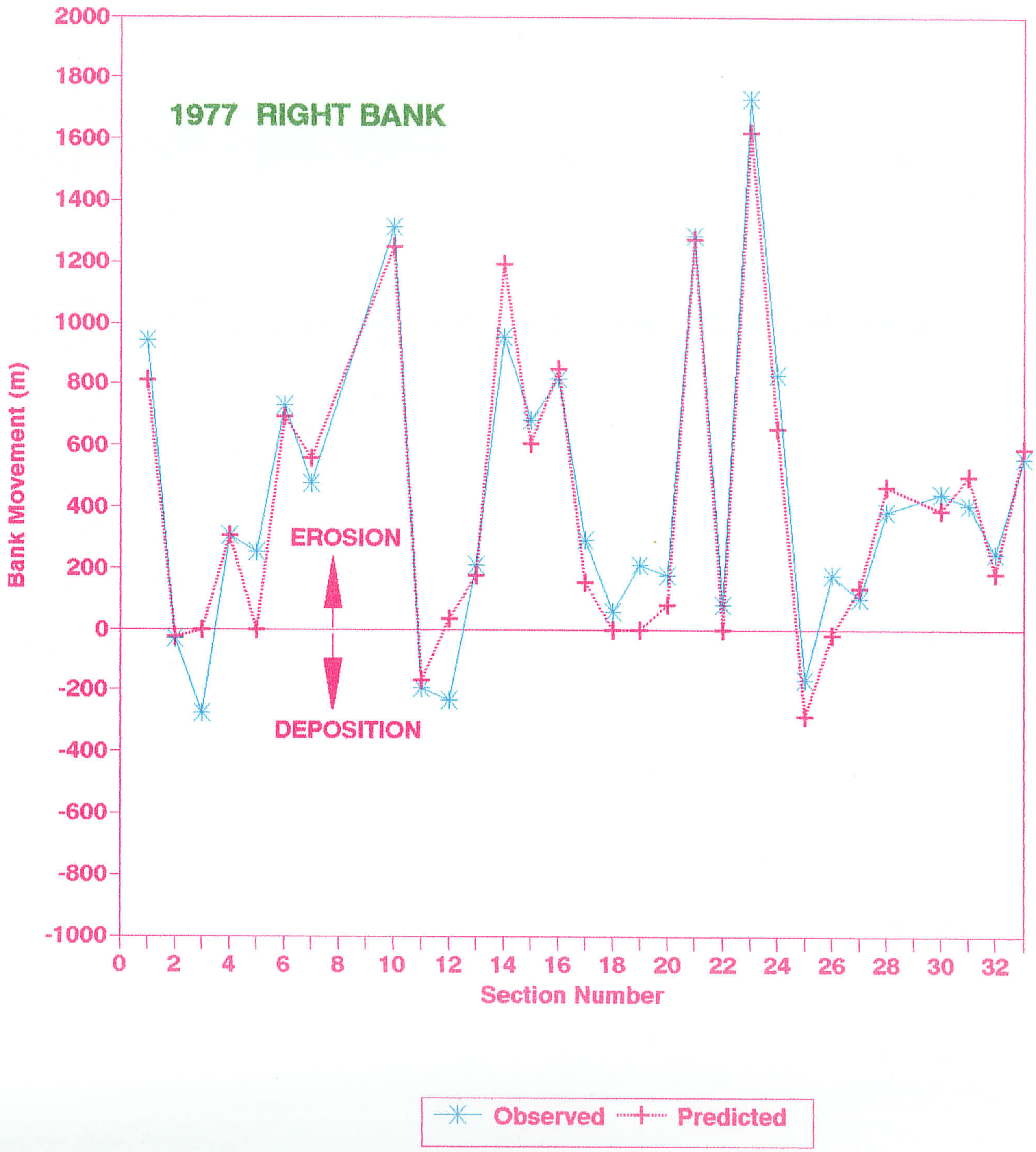

Fig. 8.20: Observed and Predicted Movement of the Right Bank (1977)

When banks of two adjacent branches are predicted to be overlapped, they are considered to form one channel, and the movement of these two banks are adjusted accordingly. As discussed in the previous chapter, the event of channel splitting is not included in the model because of the lack of an appropriate criterion to identify the event. Therefore, the new banks that are formed because of splitting are not considered in the prediction. Instead, the movement of the banks of the previous channels are predicted. Obviously, the predicted values for the movement becomes different from the observed values in these cases.

For the sake of comparison a set of 130 data points along the inner banks were examined for which the channel splitting were not dominant events. The analysis showed that the model performed the same way as it did for the outer banks. The coefficient of R^2 was found to be 0.82 for these points.

8.5 Prediction of Channel Joining

The quantitative model can be used for another useful purpose, namely, to predict whether two channels will join. The model cannot be used to predict channel splitting, as was explained in Chapter 5.

Changes in channel network configuration through channel joining are very frequent in the Brahmaputra river. By proportioning the expected bank movement over the two banks of each channel one can predict whether or not two channels may join.

The performance of the model in this type of prediction is satisfactory but not perfect. All the channels that were observed to join with other channel(s) were in the set of channels expected to join.. However, not all channels in this set did so. This is not surprising since the model does not always predict the amount of bank erosion accurately. In addition, the distribution of the erosion between the two banks of a channel probably depends on more factors than proximity of the banks to the centroid of the cross-section.

It is difficult to conclude whether two adjacent channels joined or not from a comparison of bank line coordinates. This is because the number of channels in a particular river cross section may change because of channel splitting. This makes it difficult to keep track of the individual channel number in a computer program. Furthermore, one (or both) of the channels may split, and the splitted part may join the other channel (or its splitted part). In this situation, it is not clear whether one should designate this event as joining. Therefore, branch channels were manually checked for joining. The results indicate that channel joining can be predicted with 90% accuracy. This result may be considered to be satisfactory.

8.6 Comparison of the Model with Earlier Studies

It is difficult to compare the performance of the model with the conclusions of earlier studies since most of the studies did not attempt to quantify the expected movements of the Brahmaputra river. Some studies claimed that the river is stable both in the horizontal and the vertical directions. Other studies tried to identify significant trend in movement but failed to do so because they did not quantify the expected movement. It is clear from the present study that the movements are neither monotonic nor unidirectional but rather erratic. The only other study that attempted to predict the movements of this river quantitatively is that of Khan (1988).

A comparison of the results obtained by Khan (1988) with those of this study shows the following. First, the standard error of predicted values in Khan's (1988) study was 270 m compared to 106 m (Fig. 8.21) of this study. The improvement in this regard is quite substantial. The error was also larger for the points with larger amount of movement in Khan (1988). The present study also has the advantage over Khan's earlier study in that inference about the movements of the inner banks is possible, at least for the channels that did not split in the following year. In the previous study, the inner branch channels were not considered at all. Moreover, the present study is based on an analysis of the physical processes that govern riverflow of water and sediment. In the previous study the variables describing these processes were lumped together into the system. There is no question that the present study marks an improvement.

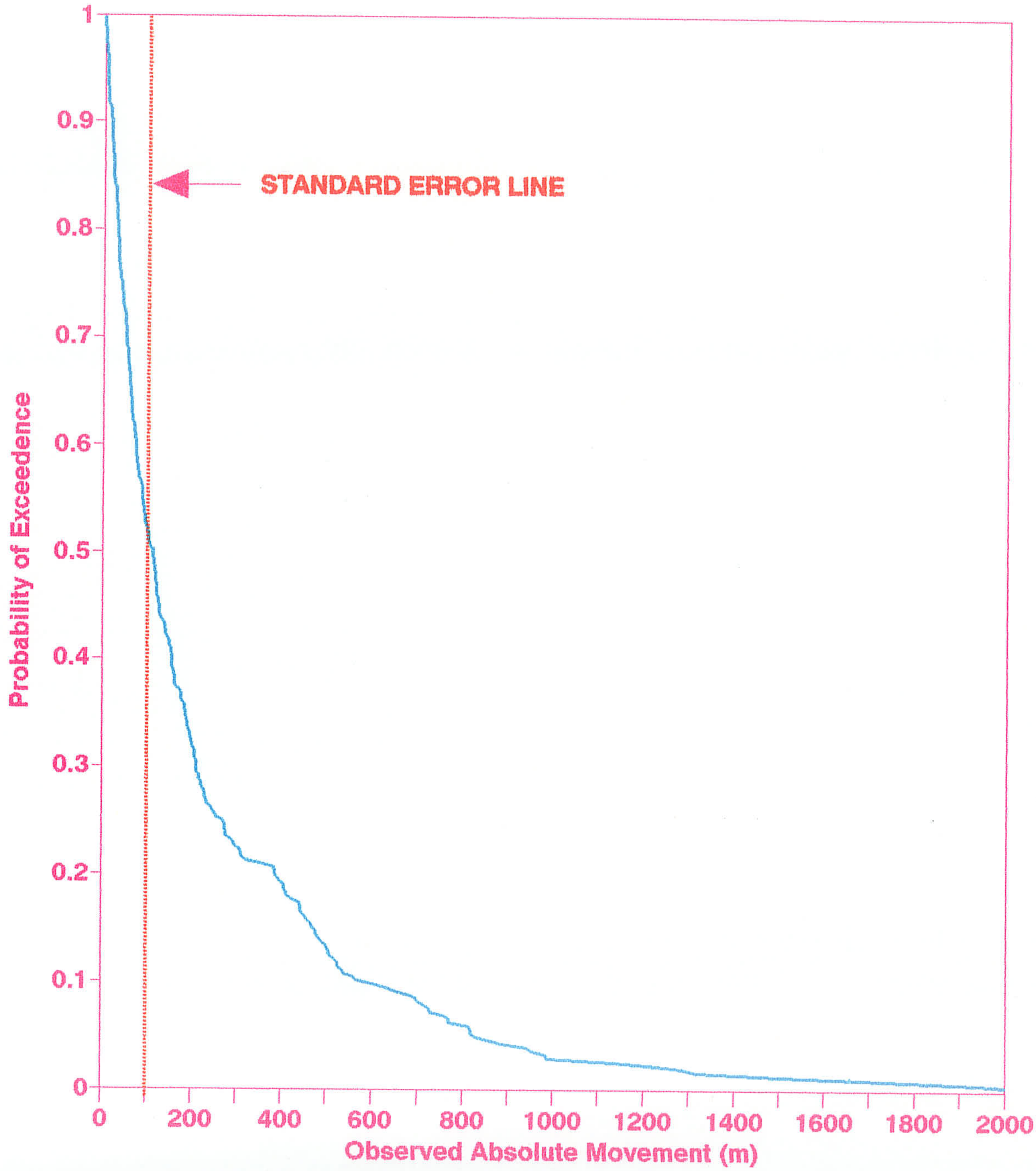

Fig. 8.21: Standard Error of Prediction

8.7 Summary

The bank movements estimated from the developed model are discussed in this chapter. It is found that the developed model works well in quantifying the expected movement of the following year for the Brahmaputra river in Bangladesh. The performance of the model is measured using a coefficient similar to the coefficient of determination of the linear regression models. The value of R^2 is found to be 0.93 which is satisfactory. The performance of the model for different data sets are studied separately. It is found that except for the data of reach 5, the model performed very well. For the data of reach 5 the R^2 is 0.79 which is also acceptable. When data were divided for different years of record, the model performed well for all the years except the year 1981 for which the R^2 was only 0.75. The behaviour of the model in terms of the performance for different reaches as well as for different years was found to be similar. Finally, the model was found to be superior to other available models of bank movement of the Brahmaputra river.

Chapter Nine

CONCLUSIONS AND RECOMMENDATIONS

The conclusions that can be drawn from this study, and the recommendations that are made for future studies are given in this chapter. The conclusions can be grouped in two categories: (a) conclusions from the behaviour of the river, and (b) conclusions from the development and the application of the methodology. The recommendations are listed in two groups: (a) recommendations for systematic data collection scheme needed for a more comprehensive study, and (b) recommendations for further studies.

9.1 Conclusions

1. In between apparently stable nodal points, the Brahmaputra river in Bangladesh is a highly unstable river in terms of its planform. The sections between the nodal points experience frequent channel splitting and joining. Individual branch channels also show relatively large changes in cross-sectional dimensions in successive years.
2. The study supports the basic assumption that the changes in river topography are dominated by two factors. The first is the sporadic changes (away from equilibrium conditions) in branch channel dimensions caused by splitting and joining or other related factors causing changes in water and sediment distribution over the branch channels. The second is the trend of the individual branch channels to return to equilibrium conditions.
3. Over the period of record examined, the river showed in some reaches a persistent westward movement.
4. The regime type width - depth relationship developed for a single channel river is not applicable to a braided river as a whole. Separate relationships for the branch channels, however, can be developed.

5. Little progress has been made in predicting the sporadic changes in water and sediment distribution (splitting, joining or substantial changes in the distribution of the flow of the sediment over the branch channels).
6. A fair degree of success was achieved in predicting the joining events of adjacent channels. The channels that joined together were all expected to widen and to become shallower.
7. The channels that split had a very high width to depth ratio. It was not possible to establish any definite threshold of width-depth ratio which separates the divided channels from the undivided ones. A large number of channels appear to lie in a transition region where they may or may not divide.
8. The divided channels that remained divided over the period of record were found to follow a regime type relationship between their widths developed in this study.
9. The model that was developed is capable of predicting outer bank movements for the following year with a standard error of approximately 100 m. The performance can be expressed by a coefficient R^2 similar to the coefficient of determination in linear regression. The value of overall R^2 was found to be 0.93.
10. There is no significant difference in the outer bank movement behaviour of different reaches along the river except that for reach 5. The prediction was not as good as for the other reaches.
11. There is no significant difference between the behaviour of movement of the left bank and the right bank.
12. The performance of the model is better than any other available model dealing with the problem of bank movement of the Brahmaputra river.

9.3 Recommendations for Data Collection

1. The cross sectional data should be collected more frequently as it is done for the water level and discharge. This will not increase the cost of data collection dramatically since the technique of echo-sounding (as in use) can automatically record the cross-section while measuring water discharge.

2. Sediment transport data should be collected more frequently and at more stations. The survey vessels available to the Bangladesh Water Development Board are sufficient to record more information about the sediment discharge.
3. The availability of such data would facilitate studies of bank movements using sediment budgeting, and micro-level investigations.

9.4 Recommendations for Future Studies

1. Upon availability of sufficient spatial and temporal data of water and sediment discharge and of the cross sections, the total hydrograph of water discharge (instead of a single value) should be included in the model. This will improve the performance of the model in terms of prediction.
2. Local variation of the variables may be included for better performance.
3. Nodal points should be studied in detail and should be included in the model.
4. Splitting and joining events should be included in the model so that the number of channels in a section in the following year can be predicted as well. This will facilitate any study to predict movements over a longer period of time.

REFERENCES

Aczél, J. & Daróczy, Z. (1975). *On Measures of Information and Their Characterizations. Vol. 115, Mathematics in Science and Engineering.* New York: Academic Press.

Adachi, S. A. (1967). Theory of stability of streams. *Proc. 12th Congress IAHR, Fort Collins, Colorado*, Vol.1, pp. 338-343.

Ackers, P. & Charlton, F. G. (1970). The slope and resistance of small meandering channels. *Proceedings of the Institution of Civil Engineers*, Vol.47, Supplementary Paper 7362-S, pp. 349-370.

Ackers, P. & White, W. R. (1973). Sediment transport: New approach and analysis. *Journal of Hydraulics Division, ASCE*, Vol.99, No. HY11, pp. 2041-2060.

Alexander, D. (1979). Simulation of channel morphology: Problems and prospects. *Progress in Physical Geography*, Vol.3, No.1, pp. 544-572.

Allen, J. R. L. (1968). *Current Ripples: Their Relation to Pattern of Water and Sediment Motion.* Amsterdam: North-Holland Publishing Co., 433 pp.

Antropovskiy, V. I. (1972). Critical relations of types of channel processes. *Soviet Hydrology: Selected Papers 5*, pp. 371-381.

Attneave, F. (1959). *Applications of Information Theory to Psychology.* New York: Henry Holt & Co.

Awumah, K. (1990). *Entropy Based Redundancy Measures in Water Distribution Network Design.* Unpublished Ph. D. Thesis. University of Manitoba, Winnipeg, Manitoba. 217 pages.

Awumah, K., Goulter, I., & Bhatt, S. (1989). Entropy Based Redundancy Measures for Water Distribution Networks. *Water Resources Research Report*, No. 012, Winnipeg: Department of Civil Engineering, University of Manitoba. 34 pages.

Bagnold, R. A. (1977). Bedload transport by natural rivers. *Water Resources Research*, Vol.13, No.2, pp. 303-312.

Bessat, M. B. & Raviv, J. (1978). Renyi's entropy and the probability of error. *IEEE Tansactions on Information Theory*, Vol. IT-24, No. 3, pp.324-331.

Belis, M. & Guiasu, S. (1968). A quantitative and qualitative measure of information in cybernatic systems. *IEEE Transactions on Information Theory*, Vol. I-14, No. 4, pp. 593-594.

Berge, C. (1962). *Theory of Graphs and Its Application*. New York: John Wiley.

Bettess, R. (1984). Initiation of sediment transport in gravel streams. *Proceedings of the Institution of Civil Engineers*, Part 2, Technical Note 407, Vol.77, pp.79-88.

Bevensee, R. M. (1993). *Maximum Entropy Solutions to Scientific Problems*. Englewood Cliffs, New Jersey: P T R Prentice Hall. 194 p.

Bhattacharya, S. K. (1973). Deltaic activity of Bhagirathi-Hoogly river system. *Journal of Waterways, Harbors and Coastal Engineering Division, ASCE*, Vol.99, No.WW1, pp.69-87.

Blench, T. (1952). Regime theory for self formed sediment bearing channels. *Transactions ASCE*, Vol.117, pp. 383-400.

Blench, T. (1969). *Mobile-bed Fluviology*. Edmonton: University of Alberta Press.

Blondeaux, P. & Seminara, G. (1985). A unified bar-bend theory of river meanders. *Journal of Fluid Mechanics*, Vol.157, pp. 449-470.

Box, G. E. P. & Jenkins, G. M. (1976). *Time Series Analysis: Forecasting and Control*. San Francisco: Holden Day.

Brice, J. C. (1964). Channel patterns and terraces of the Loup rivers in Nebraska. *U. S. Geological Survey Professional Paper 422-D*, p. 27.

Bridge, J. S. (1976). Mathematical model and Fortran IV program to predict flow, bed topography and grain size in open channel bends. *Computer and Geosciences*, Vol.2, No.4, pp. 407-416.

Bristow, C. (1987). Brahmaputra River: Channel migration and deposition. In F. G. Ethridge (ed.), *Recent Developments in Fluvial Sedimentology*, Society of Economic Paleontologists and Mineralogist, Special Publication No. 39, pp. 43-74.

Brotherton, D. I. (1979). On the origin and characteristics of river channel patterns. *Journal of Hydrology*, Vol.44, pp. 211-230.

Brown, D. A. (1971). Stream channels and flow relations. *Water Resources Research*, Vol.7, pp. 304-310.

Burger, J. W., Klassen, G. & Prins, A. (1988). Bank erosion and channel processes in the Jamuna river. *International Symposium on the Impact of Riverbank Erosion, Flood Hazard, and the Problem of Population Displacement*, Dhaka.

Burger, J. W., & Smith, E. C. (1985). Closure of tidal channels in Bangladesh. *Land + Water International*, Vol.55, pp. 29-36.

Burkham, D. E. (1972). Channel Changes of the Gila River in Safford Valley, Arizona, 1846-1970. *U. S. Geological Survey Professional Paper 655G*, 24p.

BWDB (Bangladesh Water Development Board) (1977). *Feasibility Report on Brahmaputra Right Flood Embankment Project (Rehabilitation and Drainage)*. Dhaka: BWDB.

BWDB (Bangladesh Water Development Board) (1978a). *Morphological Features of Major Rivers of Bangladesh*. Dhaka: Directorate of River Morphology, BWDB.

BWDB (Bangladesh Water Development Board) (1978b). *Report of the Expert Team on Effective Measures Against Erosion of Brahmaputra Right Flood Embankment*. Published by the Expert Team on Brahmaputra Erosion Investigation, Dhaka.

Cadle, A. B. & Cairncross, B. (1992). Lateral-accretion deposition in braided fluvial systems: A case-study from the Karoo sequence, South Africa. *Information Circular No. 254, Economic Geology Research Unit*, University of the Witwatersrand, Johannesburg, 29p.

Callander, R. A. (1969). Instability and river channels. *Journal of Fluid Mechanics*, Vol.36, Part 3, pp. 465-480.

Carson, M. A. & Griffiths, G. A. (1987). Influence of channel width on bed load transport capacity. *Journal of Hydraulic Engineering, ASCE*, Vol.113, No.12, pp. 1489-1509.

Chaitin, G. J. (1979). Toward a mathematical definition of "Life". In Levine, R. D. & Tribus, M. (eds.) *Maximum Entropy Formalism Conference, Massachusetts Institute of Technology*, 1978, (2nd Edition), pp. 477-498. Cambridge, MA: The MIT Press.

Chang, H. H. (1979) Minimum stream power and river channel patterns. *Journal of Hydrology*, Vol. 41, pp. 303-327.

Chang, H. H. (1980a). Stable alluvial canal design. *Journal of the Hydraulics Division, ASCE,* Vol.106, No.HY5, pp. 873-891.

Chang, H. H. (1980b). Geometry of gravel streams. *Journal of the Hydraulics Division, ASCE*, Vol.106, No. HY9, pp.1443-1456.

Chang, H. H., & Hill, J. C. (1977). Minimum stream power for rivers and deltas. *Journal of Hydraulics Division, ASCE*, Vol.103, No. HY12, pp.1375-1389.

Chaudhury, H. M., & Srivastava, H. N. (1981). Neotectonics in relation to plate theory in Brahmaputra basin and neighborhood. *Proc. of the Seminar on Fluvial Process and Geomorphology of the Brahmaputra Basin*, A collection of papers presented at the seminar held on October 5-6, 1972 in Calcutta, Geological Survey of India, Misc. Publ. No. 46, pp. 117-122.

Chien, N. (1961). The braided stream of the lower Yellow river. *Scientia Sinica*, Vol.X, No.6, pp. 734-754.

Chitale, S. V. (1973). Theories and relationships of river channel patterns. *Journal of Hydrology*, Vol. 19, pp. 285-308.

Church, M. A. (1972). Baffin Island sanders: a study of Arctic fluvial processes. *Geological Survey of Canada*, GSC Bulletin No. 216.

Cobb, E. Jr. (1993). Bangladesh: When the water comes. *National Geographic*, Vol.183, No.6, June, pp. 118-134.

Coleman, J. M. (1969). Brahmaputra river: Channel processes and sedimentation. *International Journal of Applied and Regional Sedimentology*, Vol.III, No.2/3, pp. 129-239.

Cunge, J. A., Holly, F. M., & Verwey, A. (1980). *Practical Aspects of Computational River Hydraulics*. Pitman Advanced Publishing Program, 420 p.

Davies, T. R. H., & Sutherland, A. J. (1980). Resistance to flow past deformable boundaries. *Earth Surface Processes*, Vol.5, pp.175-179.

Davies, T. R. H., & Sutherland, A. J. (1983). Extremal hypotheses for river behavior. *Water Resources Research*, Vol.19, No.1, pp. 141-148.

Diemer, (1979). *Isovels and Secondary Circulation in a High Mountain Braided Delta*. Unpublished M. Sc. Thesis, New York: SUNY - Binghamton.

Dury, G. H. (1976). Discharge prediction, present and former, from channel dimensions. *Journal of Hydrology*, Vol. 30, pp. 219-245.

Einstein, H. A. & Barbarossa, N. L. (1952). River channel roughness. *Transactions, ASCE*, Vol.117, pp. 1121-1132.

Emmett, W. W. (1972). The hydraulic geometry of some Alaskan streams south of the Yukon River. Open File Report, *U. S. Geological Survey*, Anchorage, Alaska, 102p.

Emmett, W. W. (1975). The channels and waters of the Upper Salmon River area, Idaho. *U. S. Geological Survey Professional Paper 870-A*, 116p.

Engelund, F. (1966). Hydraulic resistance of alluvial streams. *Journal of the Hydraulics Division, ASCE*, Vol. 92, No. HY2, pp. 315-326.

Engelund, F. & Skovgaard, O. (1973). On the origin of meandering and braiding in alluvial streams. *Journal of Fluid Mechanics*, Vol.57, Part 2, pp. 289-302.

Evans, R. (1979). A new approach for deciding upon constraints in the maximum entropy formalism. In Levine, R. D. & Tribus, M. (eds.) *Maximum Entropy Formalism Conference, Massachusetts Institute of Technology*, 1978, 2nd Edition, pp. 169-206. Cambridge, MA: The MIT Press.

Fahnestock, R. K. (1963). Morphology and hydrology of a glacial stream - White River, Mount Rainer, Washington, *U. S. Geological Survey Professional Paper 422-A*, pp.1-67.

Fast, J. D. (1962). *Entropy: The Significance of the Concept of Entropy and its Applications in Science and Technology*. New York: McGraw-Hill Book Company Inc., 313 pages.

Ferdinand, A. E. (1974). A theory of system complexity. *International Journal of General Systems*, Vol. 1, pp. 19-33.

Ferguson, J. (1863). Delta of Ganges: Recent changes. *Quarterly Journal of the Geological Society*, Vol.XIX, London.

Ferguson, R. I. (1981). Channel forms and channel changes. In J. Lewin (ed.), *British Rivers*. London: George Allen & Unwin, pp. 90-125.

Forrest, S. B., & Haff, P. K. (1992). Mechanics of wind ripple stratigraphy. *Science*, Vol. 255, pp. 1240-1243.

Froehlich, W., & Starkel, L. (1993). The effects of deforestation on slope and channel evolution in the tectonically active Darjeeling Himalayas. *Earth Surface Processes and Landforms*, Vol. 18, pp. 285-290.

Galay, V. J. (1980). River channel shifting on large rivers in Bangladesh. Prepared for the *International Symposium on River Sedimentation*, Beijing, pp. 543-564.

Garrison, W. L., & Marble, D. (1962). The structure of transportation networks. *U. S. Army Transportation Command Technical Report*, 62-11.

Goff, J. R., & Ashmore, P. (1994). Gravel transport and morphological change in braided Sunwapta River, Alberta, Canada. *Earth Surface Processes and Landforms*, Vol. 19, pp. 195-212.

Goswami, D. C. (1983). *Brahmaputra River, Assam (India): Suspended Sediment Transport, Valley Aggradation and Basin Denudation*. Unpublished Ph.D. Thesis, submitted to John Hopkins University, Baltimore, Maryland.

Goswami, D. C. (1985). Brahmaputra river: Assam, India: Physiography, basin denudation, and channel aggradation. *Water Resources Research*, Vol.21, No.7, pp. 959-978.

Goulter, I. & Dubois, D. (1988). Methods of presenting trends in channel capacity movement as an indicator of riverbank erosion risk in the Jamuna. *International Symposium on Impact of Riverbank Erosion, Flood Hazard, and the Problem of Population Displacement*, Dhaka.

Griffiths, G. A. (1984). Extremal hypotheses for river regime: An illusion of progress. *Water Resources Research*, Vol.20, No.1, pp.113-118.

Griffiths, G. A. (1986). Reply to comment of C. C. S. Song & C. T. Yang on "Extremal Hypothesis for river regime: An illusion of progress" by G. A. Griffiths. *Water Resources Research*, Vol.22. No.6, pp.995-996.

Guiasu, S. (1977). *Information Theory with Applications*. New York: McGraw Hill.

Harvey, A. M. (1969). Channel capacity and the adjustment of streams to hydrologic regime. *Journal of Hydrology*, Vol.8, pp.82-98.

Hasegawa, K. (1989). Universal bank erosion coefficient for meandering rivers. *Journal of Hydraulic Engineering, ASCE*, Vol.115, No.6, pp. 744-765.

Hayden, H. H., & Pascoe, E. H. (1919). *Report on the Hoogly River and its Headwaters*. Calcutta.

Hayashi, T. & Ozaki, S. (1980). Alluvial bedform analysis I: Formation of alternating bars and braids. In H. W. Shen & H. Kikkawa (eds.) *Application of Stochastic Processes in Sediment Transport*. Littleton, Colorado: Water Resources Publications.

Henderson, F. M. (1961). Stability of alluvial channels. *Journal of the Hydraulics Division, ASCE*, Vol. 87, No. HY 6, pp. 109-138.

Henderson, F. M. (1963). Stability of alluvial channels. *Transactions of the American Society of Civil Engineers*, Vol. 128, Part I, pp. 657-686.

Henderson, F. M. (1966). *Open Channel Flow*. New York: McMillan Publishing Co.

Hickin, E. J. & Nanson, G. C. (1975). The character of channel migration and incision on the Beatton river, northeast British Columbia, Canada. *Geological Society of America Bulletin*, Vol. 86, pp. 487-494.

Hinch, L. W., McDowell, D. M., & Rowe, P. W. (1984). Jamuna river 230KV crossing -Bangladesh, I. Design of foundations. *Proceedings of the Institution of Civil Engineers*, Part 1, Vol.76, pp. 927-949.

Hirst, F. C. (1916). *Report on the Nadia Rivers*. Calcutta: Bengal Secretariat Book Depot.

Hoey, T. B. (1992). Temporal variations in bedload transport rates and sediment storage in gravel-bed rivers. *Progress in Physical Geography*, Vol. 16, pp. 319-338.

Holeman, J. N. (1968). The sediment yield of major rivers of the world. *Water Resources Research*, Vol.4, No.4, pp. 737-747.

Hong, L. B. & Davies, T. R. H. (1979). A study of stream braiding: summary. *Geological Society of America Bulletin*, Vol. 90, pp. 1094-1095.

Hooke, J. M. (1980). Magnitude and distribution of rates of river bank erosion. *Earth Surface Processes*, 5, pp. 143-157.

Hori, H., & Nemat-Nasser, S. (1986). Brittle failure in compression: splitting, faulting and brittle-ductile transition, *Philosophical Transaction of Royal Society London*, A 319, pp. 337-374.

Howard, A. D., Keetch, M. E., & Vincent, C. L. (1970). Topological and geometrical properties of braided streams. *Water Resources Research*, Vol.6, No.6, pp. 1674-1688.

IECO (International Engineering Company). (1964). *Bankline Movement of the Brahmaputra-Jamuna River*. Prepared for East Pakistan Water and Power Development Authority, Dhaka.

Ikeda, S., Parker, G., & Sawai, K. (1981). Bend theory of river meanders, Part I. Linear development. *Journal of Fluid Mechanics*, Vol.112, pp. 363-377.

Ingarden, R. S., & Urbanik, K. (1962). Information without probability. *Colloquium Mathematicum*, Vol. IX, pp. 131-150.

Jansen, P. Ph., Van Bendegom, L., Van den Berg, J., de Vries, M., & Zanen, A. (1979). *Principles of River Engineering The Non-Tidal Alluvial River*. Pitman Publ. Ltd.

Japanese Reconnaissance Team (1972). *Note on River Crossing Project over Jamuna*. Dhaka.

Jaynes, E. T. (1979). Where do we stand on Maximum Entropy. In R. D. Levine & M. Tribus (eds.) *The Maximum Entropy Formalism Conference, Massachusetts Institute of Technology*, 1978, (2nd Edition), pp. 15-118. Massachusetts: The MIT Press.

Jaynes, E. T. (1957a). Information theory and statistical mechanics. *Physical Review*, Vol. 106, No. 4, pp. 620-630.

Jaynes, E. T. (1957b). Information theory and statistical mechanics. *Physical Review*, Vol. 108, No. 2, pp. 171-190.

JICA (Japanese International Cooperation Agency). (1974). *Feasibility Study for Jamuna River Bridge Construction*. Dhaka.

Kansky, K. J. (1963). Structure of transportation networks. *Department of Geography Research Paper 84*. University of Chicago, Chicago, Illinois, pp. 1-55.

Kapur, J. N. (1986). Four families of measures of entropy. *Indian Journal of Pure and Applied Mathematics*, Vol. 17, pp. 121-127.

Kellerhals, R. (1967). Stable channels with gravel paved beds. *Journal of Waterways and Harbors Division, ASCE*, Vol. 93, pp. 63-84.

Kellerhals, R., Church, M., & Bray, D. I. (1976). Classification of river processes. *Journal of Hydraulics Division, ASCE*, Vol. 102, No. HY7, pp. 813-829.

Kennedy, J. F. (1963). The mechanics of dunes and antidunes in erodible bed channels. *Journal of Fluid Mechanics*, Vol.16, Part 4, pp. 521-544.

Khan, M. A. O. A. (1988). *Statistical Analysis of Bank Erosion of the Brahmaputra River in Bangladesh*. Unpublished M.Sc. Thesis, University of Manitoba, Winnipeg.

Khan, M. A. O. A., & Booy, C. (1988). Statistical analysis of bank erosion of the Brahmaputra river: A Transfer Function approach. *International Symposium on the Impact of Riverbank Erosion, Flood Hazard, and the Problem of Population Displacement*, Dhaka.

Khan, M. A. O. A., & Booy, C. (1989). Statistical modelling of bank erosion of the Brahmaputra river in Bangladesh. *Proc. 9th Hydrotechnical Conference, CSCE*, St. John's, pp. 616-633.

Khan, M. A. O. A., & Booy, C. (1990). Width-depth relationship of a braided river. *Proc. 10th Hydrotechnical Conference, CSCE*, Hamilton.

Kitanidis, P. K., & Kennedy, J. F. (1984). Secondary current and river meander formation. *Journal of Fluid Mechanics*, Vol.114, pp. 217-229.

Knighton, A. D. (1972). Changes in a braided reach. *Geological Society of America Bulletin*, Vol. 85, pp. 3813-3822.

Knighton, A. D. (1984). *Fluvial Forms and Processes*. London: Edward Arnold Ltd. 218p.

Knighton, A. D., & Nanson, G. C. (1993). Anastomosis and the continuum of channel pattern. *Earth Surface Processes and Landforms*, Vol. 18, pp. 613-625.

Komura, S. (1966). Equilibrium depth of scour in long constrictions. *Journal of Hydraulics Division, ASCE*, Vol.92, No. HY5, pp.17-37.

Komura, S. (1971). River bed variations at long constrictions. *14th Congress, IAHR*, Paper C-14, Vol.3.

Kondrat'yev, N. Ye. & Popov, I. V. (1967). Methodological prerequisites for conducting network observations of the channel process. *Soviet Hydrology*, Vol. 6, pp. 273-297.

Krigström, A. (1962). Geomorphological studies of sandur plains and their braided rivers in Iceland. *Geografiska Annaler*, Vol. 44, pp. 328-346.

Krumbein, W. C. & Orme, A. R. (1972). Field mapping and computer simulation of braided stream networks. *Geological Society of America Bulletin*, Vol. 83, pp. 3369-3380.

Kullback, S. (1968). *Information Theory and Statistics*. New York: Dover Publications. 399p.

Kullback, S. & Leibler, R. A. (1951). On information and sufficiency. *The Annals of Mathematical Statistics*, Vol. 22, pp. 79-86.

Kumar, U. (1983). *Maximum Entropy Models for Brand Switching Behavior and Party Switching Behavior*. Unpublished M. Sc. Thesis. Winnipeg: University of Manitoba.

Kumar, V. (1987). Entropic measures of manufacturing flexibility. *International Journal of Production Research*, Vol. 25, No. 7, pp. 957-966.

La Touche, T. D. H. (1919). Relics of the great ice age on the plains in northern India. Reprinted in *Report on the Hoogly River and its Headwaters*. Calcutta: Bengal Secretariat Book Depot.

Lacey, G. (1930). Stable channels in alluvium. *Proceedings of the Institution of Civil Engineers*, 229, pp. 259-292.

Lacey, G. (1934). Uniform flow in alluvial rivers and canals. *Proceedings of the Institution of Civil Engineers*, 237, pp. 421-453.

Lane, E. W. (1957). *A study of the shape of channels formed by natural streams flowing in erodible material*. MRD Sediment Series 9, US Army Engineer Divn., Missouri River, Corps Engineers, Omaha, Nebraska, pp. 1-106.

Latif, A. (1969). Investigation on Brahmaputra river. *Journal of the Hydraulics Division, ASCE*, Vol.95, No.HY5, pp. 1687-1698.

Lavelle, J. W., & Mofjeld, H. O. (1987). Do critical stresses for incipient motion and erosion really exists?. *Journal of Hydraulic Engineering, ASCE*, Vol.113, No.3, pp. 370-385.

Lazo, A. C. G. V., & Rathie, P. N. (1978). On the entropy of continuous probability distributions. *IEEE Transactions on Information Theory*, Vol. IT-24, No. 1, pp. 120-122.

Leopold, L. B., & Wolman, M. G. (1957). River channel patterns: braided, meandering and straight. *U. S. Geological Survey Professional Paper 282-B*, pp.39-85.

Leopold, L. B., Wolman, M. G., & Miller, J. P. (1969). *Fluvial Processes in Geomorphology*. New Delhi: Eurasia Publ. House (Pvt.) Ltd. 522 p.

Mackin, J. H. (1956). Cause of braiding by a graded river. *Geological Society of America Bulletin*, Vol. 67, pp. 1717-1718.

Martin, M. (1838). *The History, Antiquities, Topography, and Statistics of Eastern India, Vol.V, Rangpur and Assam*. Reprint Edition. Delhi-110006: Cosmo Publications, 1976.

Maybeck, M. (1976). Total mineral dissolved transport by world major rivers. *Hydrological Science Bulletin*, Vol.21, pp. 265-284.

Miall, A. D. (1977). A review of the braided-river depositional environment. *Earth Science Reviews*, Vol.13, pp. 1-62.

Milliman, J. D., & Meade, R. H. (1983). World-wide delivery of river sediments to the oceans. *Journal of Geology*, Vol.91, No.1, pp. 1-21.

Morgan, J. P., & McIntire (1959). Quaternary geology of the Bengal basin, East Pakistan and India. *Geological Society of America Bulletin*, Vol.70, pp. 319-342.

Morisawa, M. (1985). *Rivers Form and Processes. Geomorphology Text 7.* New York: Longman Group Limited, 222p.

Mosley, M. P. (1982). Analysis of the effect of changing discharge on channel morphology and instream uses in a braided river, Ohau river, New Zealand. *Water Resources Research*, Vol.18, No.4, pp.800-812.

Murray, A. B., & Paola, C. (1994). A cellular model of braided rivers. *Nature*, Vol. 371, pp. 54-57.

Murthy, K. V. S., & Sastri (1981). Tectonic influence on the course of Brahmaputra. *Proc. of the Seminar on Fluvial Process and Geomorphology of the Brahmaputra Basin*, A Collection of Papers Presented at the Seminar Held on October 5-6, 1972 in Calcutta, Geological Survey of India, Misc. Publ. No.46, pp. 129-132.

Nanson, G.C., & Hickin, E. J. (1983). Channel migration and incision on the Beatton river. *Journal of Hydraulic Engineering, ASCE*, Vol.109, No.3, pp. 327-337.

NEDECO (1959). *River Studies and Recommendations on Improvements of Niger and Benue*. Amsterdam: North-Holland Publ.

Neter, J., Wasserman, W., & Kunter, M. H. (1985). *Applied Linear Statistical Models*. Second Edition, Richard D. Illinois: Irwin, 1127p.

Nevins, T. H. F. (1969). River training - the single thread channel. *New Zealand Engineering*, December, pp. 367-373.

Nixon, M. (1959). A study on bank-full discharges of rivers in England and Wales. *Proceedings of the Institution of Civil Engineers*, Vol. 12, pp. 157-174.

Odgaard, J. (1987). Streambank erosion along two rivers in Iowa. *Water Resources Research*, Vol.23, No.7, pp. 1225-1236.

Odgaard, J. (1989a). River-meander model I: Development. *Journal of Hydraulic Engineering, ASCE*, Vol.115, No.11, pp. 1433-1450.

Odgaard, J. (1989b). River-meander model II: Applications. *Journal of Hydraulic Engineering*, ASCE, Vol.115, No.11, pp. 1451-1464.

Onishi, Y., Jain, S. C., & Kennedy, J. F. (1976). Effects of meandering in alluvial streams. *Journal of the Hydraulics Division, ASCE*, Vol.102, No.HY7, pp. 899-917.

Osterkamp, W. R. (1978). Gradient, discharge, and particle size relations of alluvial channels in Kansas, with observations on Braiding. *American Journal of Science*, Vol. 278, pp.1253-1268.

Paris, E. (1984). River bed response to man-made changes. *Proceedings of the Institution of Civil Engineers*, Part 2, Research and Theory, Vol.77, pp. 67-77.

Parker, G. (1976). On the cause and characteristic scales of meandering and braiding in rivers. *Journal of Fluid Mechanics*, Vol. 76, pp. 457-480.

Parker, G. (1978). Self-formed rivers with equilibrium banks and mobile bed II, The gravel river. *Journal of Fluid Mechanics*, Vol.89, pp.127-146.

Parker, G. (1979). Hydraulic geometry of active gravel river. *Journal of the Hydraulics Division, ASCE*, Vol.105, No.HY9, pp. 1185-1201.

Petrobangla (1983). *Techno Economic Study for Gas Supply to Western Zone: Appendices*. Dhaka.

Pickup, G. & Rieger, W. A. (1979). A conceptual model of the relationship between channel characteristics and discharge. *Earth Surface Processes*, Vol. 4, pp. 37-42.

Pickup, G. & Warner, R. F. (1976). Effect of hydrologic regime on magnitude and frequency of dominant discharge. *Journal of Hydrology*, Vol.29, pp.51-75.

Poddar, M. C. (1952). Preliminary report of the Assam Earthquake, 15th August, 1950. *Bulletin of Geological Survey of India*, No.2, pp. 11-13.

Popov, I. V. (1964). Hydromorphological principles of the theory of channel processes and their use in hydrotechnical planning. *Soviet Hydrology* 1964, pp. 158-195.

Rao, K. L. (1979). *India's Water Wealth*. New Delhi: Orient Longman, pp. 75-79.

Rashid, H. E. (1978). *Geography of Bangladesh*. Westview Press, Boulder, Colorado.

Raudkivi, A. J. (1967). Analysis of resistance in fluvial channels. *Journal of the Hydraulics Division, ASCE*, Vol. 93, pp. 73-84.

Rényi, A. (1961). On measures of entropy and information. *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*. Vol. 1, pp. 547-561.

Rényi, A. (1967). On some basic problems of statistics from the point of view of information theory. *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability*. Vol. 1, pp. 531-543.

Reynolds, A. J. (1965). Waves on the erodible bed of an open channel. *Journal of Fluid Mechanics*, Vol.22, pp. 113-133.

Richards, K. (1982). *Rivers Form and Process in Alluvial Channels*. New York: Metheun, 358p.

Richardson, E. V., Simons, D. B., Karaki, S., Mahmood, K., & Stevens, M. A. (1974). *Highways in the River Environment Hydraulic and Environmental Design Considerations*. Training and design manual prepared for Federal Highway Administration. Fort Collins, Colorado: Department of Civil Engineering, Colorado State University.

Richason Jr., B. F. (ed.) (1983). *Introduction to Remote Sensing of the Environment*. 2nd Edition, Dubuque, Iowa: Kendal/Hunt Publishing Co.

Riggs, H. C. (1976). A simplified slope-area method for estimating flood discharges in natural channel. *U. S. Geological Survey*, J. Res., 4, pp. 285-291.

Riley, S. J. (1972). A comparison of morphometric measures of bankful discharge. *Journal of Hydrology*, Vol.17, pp.23-31.

RPT (Rendel Palmer & Tritton), NEDECO, & BCL (Bangladesh Consultants Ltd.) (1986). *Jamuna Bridge Appraisal Study: Characteristics and configuration - Phase I. Draft Final Report. Appendix - C: Hydrology, Morphology and River Engineering*, submitted to the Jamuna Multipurpose Bridge Authority of the Government of the People's Republic of Bangladesh.

RPT, NEDECO, & BCL. (1988). *Jamuna Bridge Study II: Draft Feasibility Report, Vol.II, Annexes A: Hydrology, B: River Morphology, D: River Training Works*, Report submitted to the Government of Bangladesh, United Nations Development Programme/World Bank.

Rundle, A. (1985a). The mechanism of braiding. *Zeitschrift für Geomorphologie*, Supplementband 55, Fluvial Geomorphology, In memorium J. N. Jennings, Edited by H. Bremmer, pp. 1-13.

Rundle, A. (1985b). Braid morphology and the formation of multiple channels: The Rakaia, New Zealand. *Zeitschrift für Geomorphologie*, Supplementband 55, In memorium J. N. Jennings, Edited by H. Bremmer, pp. 15-37.

Rundquist, L. A. (1975). *A classification and analysis of natural rivers*. Unpublished Ph. D. Thesis, Colorado State University, Fort Collins, 377 p.

Saleheen, M. (1991). The mobility characteristics of displacees: A case study from riverbank erosion hazard area. In K. M. Elahi, K. S. Ahmed & M. Mofizuddin (eds.), *Riverbank Erosion, Flood and Population Displacement in Bangladesh*. Dhaka: REIS-JU.

Schumm, S. A. (1963a). A tentative classification of alluvial river channels. *U. S. Geological Survey Circular*, 477.

Schumm, S. A. (1963b). Sinuosity of alluvial rivers on the Great Plains. *Geological Society of America Bulletin*, Vol. 74, pp. 1089-1100.

Schumm, S. A. (1968). River adjustment to altered hydrologic regimen, Murrumbidgee River, Australia. *US Geological Survey Professional Paper 598*, 65p.

Schumm, S. A. (1980). Some applications of the concept of geomorphic thresholds. In D. R. Coates & J. D. Vitek (eds.), *Thresholds in Geomorphology*. Boston: George Allen & Urwin, pp. 473-485.

Schumm, S. A. & Khan, H. R. (1972). Experimental study of channel patterns. *Geological Society of America Bulletin*, Vol. 83, pp. 1755-1770.

Schumm, S. A. & Lichty, R. W. (1963). Channel Widening and Flood-Plain Construction Along Cimarron River in South-Western Kansas. *U. S. Geological Survey Professional Paper 352D*, pp. 71-88.

Shannon, C. E. (1948a). A mathematical theory of communication. *The Bell System Technical Journal*, Vol. XXVII, No. 3, pp. 379-423.

Shannon, C. E. (1948b). A mathematical theory of communication. *The Bell System Technical Journal*, Vol. XXVII, pp. 623-656.

Shewfelt, P. B. (1989). *Width-Depth regime analysis for alluvial rivers*. Unpublished M. Sc. Thesis, Winnipeg, Manitoba: Department of Civil Engineering, University of Manitoba.

Simons, D. B. & Albertson, M. L. (1960). Uniform water conveyance channels in alluvial material. *Journal of the Hydraulics Division*, ASCE, Vol. 86, No. HY 5, pp. 33-71.

Simons, D. B. & Richardson, E. V. (1963). A study of variables affecting flow characteristics and sediment transport in alluvial channels. *Proc. of Federal Inter-Agency Sedimentation Conference*, Washington.

Simons, D. B., Stevens, M. A., & Duke, J. H. Jr. (1973). Predicting stages on sand bed rivers. *Journal of the Waterways, Harbors and Coastal Engineering Division, ASCE*, Vol. 99, No. WW2, pp. 231-243.

Smith, N. D. (1970). The braided stream depositional environment: comparison of the Platte River with some Silurian clastic rocks, north-central Appalachians. *Geological Society of America Bulletin*, Vol. 81, pp. 2993-3014.

Smith, N. D. (1971). Transverse bar and braiding in the lower Platte River, Nebraska. *Geological Society of America Bulletin*, Vol. 82, pp. 3407-3420.

Song, C. C. S., & Yang, C. T. (1980). Minimum stream power: Theory. *Journal of Hydraulics Divison, ASCE*, Vol.106, No.HY6, pp. 1477-1487.

Song, C. C. S., & Yang, C. T. (1986). Comment on "Extremal hypotheses for river regime: An illusion of progress" by George A. Griffiths. *Water Resources Research*, Vol.22, No.6, pp.993-994.

Stene, L. P. (1988). Jamuna bankline behaviour: 1972-1984. *International Symposium on Impact of Riverbank Erosion, Flood Hazard, and the Problem of Population Displacement*. Dhaka.

Stene, L. P. (1993). Brahmaputra-Jamuna channel behaviour, Bangladesh: 1976-1983. *Water Resources Research Report No. 028*. Winnipeg: Department of Civil Engineering, University of Manitoba, 51p.

Stevens, M. A. (1989). Width of straight alluvial channels. *Journal of Hydraulic Engineering, ASCE*, Vol.115, No.3, pp. 309-326.

Stevens, M. A. & Nordin, C. F. (1987). Critique of the regime theory for alluvial channels. *Journal of Hydraulic Engineering*, ASCE, Vol. 113, No. 11, pp. 1359-1380.

Straub, L. G. (1935). *Missouri river report*. House Document 238, Appendix XV, Corps of Engineers, United State Congress, 2nd Session.

Suharyanto (1992). *Analyzing movement of centroid of conveyance of a braided river: An attempt to identify the channel migration of the Brahmaputra river in Bangladesh*. Unpublished M.Sc. Thesis, Winnipeg: Department of Civil Engineering, University of Manitoba.

Tamburi, A. J. (1976). Regime studies of the lower Fraser. *3rd Annual Symposium of the Waterways, Harbors and Coastal Engineering Division of ASCE*, Fort Collins, Colorado, Vol.II, pp. 1680-1699.

The Imperial Gazetteer of India, Vol.I, Descriptive (1881). Published under the authority of His Majesty's Secretary of State for India in Council, (New Ed.). New Delhi-5: Today and Tomorrow's Printers and Publishers.

The Imperial Gazetteer of India, Vol.III, Bereilly to Berasia (1882). Published under the authority of His Majesty's Secretary of State for India in Council, (New Ed.). New Delhi-5: Today and Tomorrow's Printers and Publishers.

Theil, H. (1967). *Economics and Information Theory. Studies in Mathematical and Managerial Economics, Volume 7*. Chicago: Rand McNally & Company.

Theil, H. & Fiebig, D. G. (1984). *Exploiting Continuity Maximum Entropy Estimation of Continuous Distributions*. Vol. 1 of series on Econometrics and Management Sciences. Cambridge, MA: Ballinger Publishing Company.

Thomas, A. R. (1970). Discussion of "Investigation of Brahmaputra River, by Abdul Latif", *Journal of the Hydraulics Division, ASCE*, Vol.96, No.HY7, pp.1623-1624.

Tribus, M. (1981). Thirty years of information theory. In Levine, R. D. & Tribus, M. (eds.) *The Maximum Entropy Formalism Conference, Massachusetts Institute of Technology*, 1978, pp. 1-14. Cambridge, MA: The MIT Press.

Tverberge, H. (1958). A new derivation of information function. *Math. Scand.*, Vol. 6, pp. 297-298.

Vanoni, V. A. (ed.) (1975). *Sedimentation Engineering*. ASCE Manuals and Reports on Engineering Practice, No. 54.

Wadia, D. N. (1968). The Himalayas mountains: its origin and geographical relations. In B. C. Law (ed.) *Mountains and rivers of India*. 21st National Geological Congress, Calcutta, p. 42.

Warburton, J., & Davies, T. (1994). Variability of bedload transport and channel morphology in a braided river hydraulic model. *Earth Surface Processes and Landforms*, Vol. 19, pp. 403-421.

Werner, B. T., & Fink, T. M. (1993). Beach Cusps as self-organized patterns. *Science*, Vol. 260, pp. 968-971.

Werner, B. T., & Hallet, B. (1993). Numerical simulation of self-organized stone stripes. *Nature*, Vol. 361, pp. 142-145.

White, W. R., Bettess, R., & Paris, E. (1982). Analytical approach to river regime. *Journal of the Hydraulics Division, ASCE*, Vol.108, No.HY10, pp. 1179-1193.

White, W. R., Paris, E., & Bettess, R. (1980). The frictional characteristics of alluvial streams: A new approach. *Proceedings of the Institution of Civil Engineers*, Part 2, Vol.69, pp. 737-750.

White, W. R., Bettess, R., & Shiqiang, W. (1987). Frictional characteristics of alluvial streams in the lower and upper regimes. *Proceedings of the Institution of Civil Engineers*, Part 2, Vol. 83, Paper 9153, pp. 685-700.

Williams, G. P. (1978). Bankful discharge of rivers. *Water Resources Research*, Vol.14, No.6, pp.1141-1154.

Wilson, A. G. (1968). *The Use of Entropy Maximizing Models in the Theory of Trip Distribution, Mode Split and Route Split*. Working Paper, CESWP1, London: Centre for Environmental Studies, 35p.

Wolman, M. G. & Leopold, L. B. (1957). River flood plains: some observations on their formation. *U. S. Geological Survey Professional Paper 282C*, pp.87-109.

Wolman, M. G. & Miller, J. P. (1960). Magnitude and frequency of forces in geomorphic processes. *Journal of Geology*, Vol. 68, pp. 54-74.

Woodbury, A. D. & Ulrych, T. J. (1993). Minimum relative entropy: forward probabilistic modeling. *Water Resources Research*, Vol. 29, No. 8, pp. 2847-2860.

Yalin, M. S. (1964). Geometrical properties of sand waves. *Journal of the Hydraulics Division, ASCE*, Vol.90, No. HY5, pp.105-169.

Yang, C. T. (1976). Minimum unit stream power and fluvial hydraulics. *Journal of the Hydraulics Division, ASCE*, Vol.102, No.HY7, pp. 919-934.

Yang, C. T., & Song, C. C. S. (1979). Theory of minimum rate of energy dissipations. *Journal of the Hydraulics Division, ASCE*, Vol.105, No.HY7, pp. 769-784.

Yang, C. T., Song, C. C. S., & Woldenberg, M. J. (1981). Hydraulic geometry and minimum rate of energy dissipation. *Water Resources Research*, Vol.17, No.4, pp.1014-1018.

Appendix A

MAXIMUM TRANSPORT CAPACITY

A.1 Maximum Transport Capacity (MTC) Without Bank Resistance

The total bed load capacity for a channel of width W can be written as:

$$Q_s = Wq_s = KW(d-d_o)^m j^m \tag{A.1}$$

where,

$j = s/(G_s - 1)D$,

d_o = threshold depth for entrainment of bed material,

D = particle size,

d = depth of flow,

Q = the total discharge,

q = the unit discharge, and

subscript s stands for sediment.

Small changes in width ΔW and depth Δd produces a changed transport capacity which is given by:

$$Q_s + \Delta Q_s = (W+\Delta W)(q_s+\Delta q_s) = K(W+\Delta W)(d+\Delta d-d_o)^m j^m \tag{A.2}$$

The fractional change in capacity is obtained by dividing Eqn. (A.2) by Eqn. (A.1) as:

$$\frac{Q_s+\Delta Q_s}{Q_s} = \left(1+\frac{\Delta W}{W}\right)\left(1+\frac{\Delta q_s}{q_s}\right) \tag{A.3}$$

The magnitude of the two quantities of Eqn. (A.3) determines whether the capacity increases or decreases in response to a change in width. Combining Eqn. (A.1) and (A.2) one can write:

$$\left(1+\frac{\Delta Q_s}{q_s}\right)=\left[1+\frac{\Delta d}{(d-d_o)}\right]^m \qquad \text{(A.4)}$$

The changes in width and flow depth of a rectangular channel can be generally written as:

$$Wd^c = Z = \frac{nQ}{S^{0.5}} \qquad \text{(A.5)}$$

with c is usually greater than m. From Eqn. (A.5) one can write that:

$$\left(1+\frac{\Delta W}{W}\right)=\left(1+\frac{\Delta d}{d}\right)^{-c} \qquad \text{(A.6)}$$

From Eqn. (A.3) it can be seen that Q_S decreases with decrease in W if the fractional decrease in W outweigh fractional increase in q_S. This can only occur if:

$$\left(1+\frac{\Delta W}{W}\right)^{-1} > \left(1+\frac{\Delta q_s}{q_s}\right) \qquad \text{(A.7)}$$

Following Eqn. (A.4) and (A.5), one can write Eqn. (A.7) as:

$$\left(1+\frac{\Delta d}{d}\right)^c > \left[1+\frac{\Delta d}{(d-d_o)}\right]^m \qquad \text{(A.8)}$$

Since c and m are similar and $c > m$, Eqn. (A.8) can be satisfied if $d >> d_o$ i.e., if W is small. Thus, it is only at small width, Q_S decreases as W decreases. On the other hand, a decrease in W yields an increase in Q_S if left hand side of Eqn. (A.8) is less than the right hand side. This is only possible if $d << d_o$ i.e., at large W.

There must exist an intermediate width for which Q_S is a maximum. If there is no threshold depth there is no maximum Q_S, the direction of monotonic change being controlled by whether or not $c > m$. The existence of this stationary value for any value of m $(m < c)$ is easily verified from Eqn. (A.1):

$$dQ_s = kj^m(d-d_o)^{m-1}[(d-d_o)dW + Wm\,dd] \tag{A.9}$$

in which *dd* stands for the derivative of *d*. Using Eqn. (A.5) and substituting *dd* by

$$dd = -\left(\frac{d^{c+}}{Z^c}\right)dW \tag{A.10}$$

yields

$$\frac{dQ_s}{dW} = \frac{k}{c}j^m(d-d_o)^{m-1}[(c-m)d - cd_o] \tag{A.11}$$

At minimum Q_s, Eqn. (A.11) is equal to zero which yields

$$d = \frac{cd_o}{(c-m)} \tag{A.12}$$

which is positive for all $c > m$.

A.2 Maximum Transport Capacity (MTC) with Bank Resistance

Using Manning's equation to a rectangular channel one can write:

$$\frac{Qn}{S^{0.5}} = \left[\left(\frac{W}{d}\right)+2\right]^2 \frac{R^{2.67}}{\left(\frac{W}{d}\right)} \tag{A.13}$$

where the hydraulic mean radius R is numerically equal to the hydraulic radius of bed R_b when boundary roughness is uniform as is assumed here. The width and depth are not uniquely determined by R although R is a unique value for given W and d. If the value of R that maximizes Q_s can be determined, R in Eqn. (A.13) can be eliminated to give the width-depth ratio of the MTC condition in terms of known quantities.

Assuming that the Manning-Stickler equation holds (i.e., the boundary roughness is constant with flow depth and is determined by the grain size of the boundary), and Meyer-Peter-Mueller equation of bedload transport can be applied, the Lagrangian equation becomes:

$$Q_s = KW(jR - E_f)^{1.5} + L\left[WdR^{0.67} - \left(\frac{nQ}{S^{0.5}}\right)\right] \tag{A.14}$$

where,

E_f = the critical Shield's stress for entrainment, and

L = the undetermined multiplier.

The condition for which Q_s is a maximum is then given by

$$\frac{\partial Q_s}{\partial W} = 0; \qquad \frac{\partial Q_s}{\partial d} = 0$$

Eliminating L between these two conditions yields:

$$(jR - E_f)\left[1 + \left(\frac{2d}{3R}\right)\frac{\partial R}{\partial d}\right] = \frac{3j}{2}\left[d\frac{\partial R}{\partial d} - B\frac{\partial R}{\partial W}\right] \tag{A.15}$$

Eqn. (A.15) can be simplified to write

$$R = \frac{(10A + 12)E_f(s - 1)D}{S(A + 30)} \tag{A.16}$$

where, $A = W/d$. Substituting Eqn. (A.16) into Eqn. (A.13) one can write:

$$A^* = \frac{QnS^{2.17}}{[E_f(s - 1)D]^{2.67}} \tag{A.17}$$

where,

$$A^* = \frac{(A + 2)^2}{A}\left(\frac{10A + 12}{A + 30}\right)^{2.67} \tag{A.18}$$

and right hand side of Eqn. (A.17) is known. The depth of MTC channel can be determined from

$$d = \frac{R(A + 2)}{A}$$

and the optimum width is given by $W = A.d$

Appendix B

EQUIVALENCE BETWEEN MAXIMUM SEDIMENT TRANSPORT AND MINIMUM STREAM POWER

The six variables that are considered here are: the sediment concentration X, discharge Q, slope S, velocity x_1, depth x_2, and width x_3. These variables are related through a sediment transport equation, a resistance equation, and a continuity equation. The sediment transport equation is of the form:

$$X = G(S, x_1, x_2, x_3) \tag{B.1}$$

The resistance equation is of the form:

$$S = F(x_1, x_2, x_3) \tag{B.2}$$

The continuity equation is of the form:

$$\phi(Q, x_1, x_2, x_3) = 0 \tag{B.3}$$

Let us consider first the case when Q and X are fixed and S is minimized subject to satisfying Eqns. (B.1) and (B.3). Using Lagrange multiplier, this is equivalent to minimizing the expression:

$$F(x_1, x_2, x_3) + \lambda\phi + \mu[-X + G(S, x_1, x_2, x_3)] \tag{B.4}$$

The values of x_1, x_2, x_3, S, λ, and μ that provide the extremum are solutions of the equations:

$$\frac{\partial F}{\partial x_j} + \lambda\frac{\partial \phi}{\partial x_j} + \mu\frac{\partial G}{\partial x_j} = 0, \qquad j = 1, 2, 3 \tag{B.5}$$

for

$$\phi = 0 \tag{B.6}$$

and

$$G(S, x_1, x_2, x_3) = X \tag{B.7}$$

in which

$$S = F(x_1, x_2, x_3) \tag{B.8}$$

Now, consider the problem in which Q and S are fixed and X is maximized subject to Eqns. (B.2) and (B.3). This is equivalent to maximizing the expression

$$G(S, x_1, x_2, x_3) + \lambda'\phi + \mu'[-S + F(x_1, x_2, x_3)] \tag{B.9}$$

The values of x_1, x_2, x_3, S, λ', and μ' that provide the extremum are solutions of the equations

$$\frac{\partial G}{\partial x_j} + \lambda'\frac{\partial \phi}{\partial x_j} + \mu'\frac{\partial F}{\partial x_j} = 0 \tag{B.10}$$

and

$$\phi = 0 \tag{B.11}$$

and

$$F(x_1, x_2, x_3) = S \tag{B.12}$$

with the corresponding extremum value of X is given by

$$X = G(S, x_1, x_2, x_3) \tag{B.13}$$

Note that Eqns. (B.5) to (B.8) are identical to Eqns. (B.10) to (B.13) provided

$$\mu\mu' = 1 \quad and \quad \mu\lambda' = \lambda \tag{B.14}$$

and provided that the value of X used in Eqns. (B.5) to (B.8) is the same as determined from Eqns. (B.10) to (B.13).

Thus, if Q and X are given and the width, the velocity and the depth are calculated to give an extremum value of the slope, then for that slope and the given Q the same values of width, velocity and depth give an extremum value of sediment concentration. It should be noted that it has not been shown here that an extremum value exists. Also, if an extremum value exists it has not been shown whether the extremum is a maxima or a minima.

Appendix C

TOPOLOGICAL PARAMETERS

The topological parameters α, β, γ, E and C that were computed for different reaches and for different years of record are given in Table C.1. It is evident from the values listed in the table that there is a substantial variation of parameter values across time and between different reaches.

The values of the parameters α and β are plotted in Figs. C.1 and C.2 respectively. Parameters E and C are plotted in Figs. C.3 and C.4. These figures show the variation of these parameters with time as well as with river reach. Also, these parameters differ substantially from their approach values as suggested by Howard et al. (1970). Note that the approach values are reached for a well developed braid and as such may be considered stable.

Table C.1: Topological Parameters for Different Reaches of Different Years

REACH	YEAR	TOPOLOGICAL PARAMETERS				
		α	β	γ	E	C
1	1973	0.078	1.107	0.397	3.000	6.850
	1976	0.104	1.171	0.410	4.750	9.190
	1977	0.060	1.091	0.381	3.500	6.560
	1978	0.256	1.417	0.515	3.750	7.780
	1979	0.045	1.056	0.373	3.000	6.710
	1980	0.000	0.971	0.343	4.250	7.510
	1981	0.096	1.154	0.405	3.750	7.300
	1983	0.113	1.172	0.420	4.000	6.950
	1984	0.030	1.028	0.363	4.000	7.890

2	1973	0.152	1.211	0.451	3.670	6.460
	1976	0.176	1.286	0.462	5.000	7.970
	1977	0.132	1.207	0.432	4.250	6.240
	1978	0.233	1.375	0.500	3.750	5.980
	1979	0.073	1.087	0.397	3.670	7.170
	1980	0.024	1.000	0.365	4.250	6.750
	1981	0.145	1.233	0.440	3.750	6.300
	1983	0.118	1.179	0.423	3.500	6.590
	1984	0.118	1.179	0.423	3.670	6.000
3	1973	0.196	1.321	0.474	3.750	8.980
	1976	0.356	1.625	0.578	5.250	10.120
	1977	0.151	1.265	0.440	5.750	10.420
	1978	0.224	1.400	0.488	6.250	10.900
	1979	0.025	1.032	0.356	5.000	11.150
	1980	0.194	1.352	0.468	6.750	10.670
	1981	0.341	1.622	0.566	6.750	11.000
	1983	0.108	1.184	0.411	4.750	9.580
	1984	0.067	1.100	0.386	5.250	10.830
4	1973	0.133	1.200	0.435	4.000	10.190
	1976	0.072	1.108	0.390	5.000	10.820
	1977	0.054	1.082	0.376	6.670	11.290
	1978	0.137	1.214	0.436	6.330	11.710
	1979	0.012	1.000	0.350	5.330	11.190
	1980	0.172	1.308	0.453	7.500	12.380
	1983	0.126	1.217	0.424	6.330	12.300
	1984	0.089	1.143	0.400	6.000	12.300
5	1973	0.175	1.294	0.458	4.250	8.800
	1976	0.040	1.050	0.368	5.250	9.630
	1977	0.116	1.167	0.424	3.670	10.160
	1978	0.298	1.516	0.540	5.750	9.470

	1979	0.062	1.086	0.384	4.670	8.820
	1980	0.035	1.051	0.363	7.000	11.220
	1983	0.025	1.023	0.358	5.330	9.680
	1984	0.073	1.100	0.393	4.670	11.110
6	1973	0.143	1.244	0.436	4.750	9.600
	1976	0.098	1.152	0.409	6.330	11.480
	1977	0.232	1.420	0.493	4.750	11.430
	1978	0.307	1.550	0.544	4.750	9.790
	1979	0.066	1.104	0.384	5.000	9.990
	1983	0.068	1.111	0.385	4.750	11.360
	1984	0.060	1.083	0.382	4.000	12.480
ENTIR E RIVER	1973	0.218	1.421	0.481	4.060	8.830
	1976	0.221	1.429	0.482	5.380	10.160
	1977	0.220	1.429	0.481	4.880	9.330
	1978	0.423	1.824	0.617	5.050	9.470
	1979	0.095	1.183	0.398	4.500	9.610
	1980	0.166	1.322	0.446	6.000	9.870
	1981	0.251	1.480	0.503	4.900	8.750
	1983	0.150	1.291	0.434	4.710	9.630
	1984	0.136	1.262	0.426	4.670	10.160

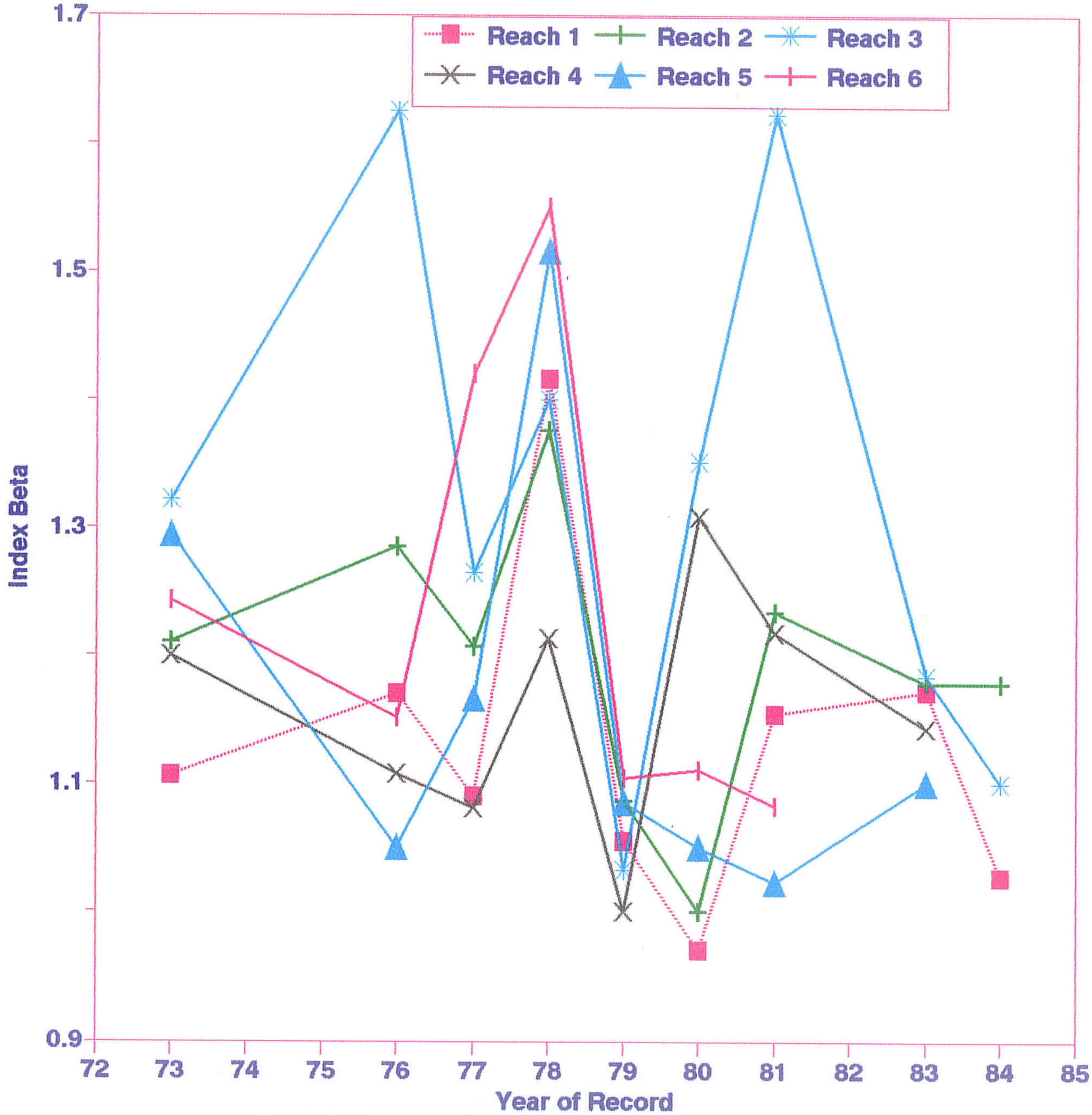

Fig. C.1: Variation of Topological Parameter β

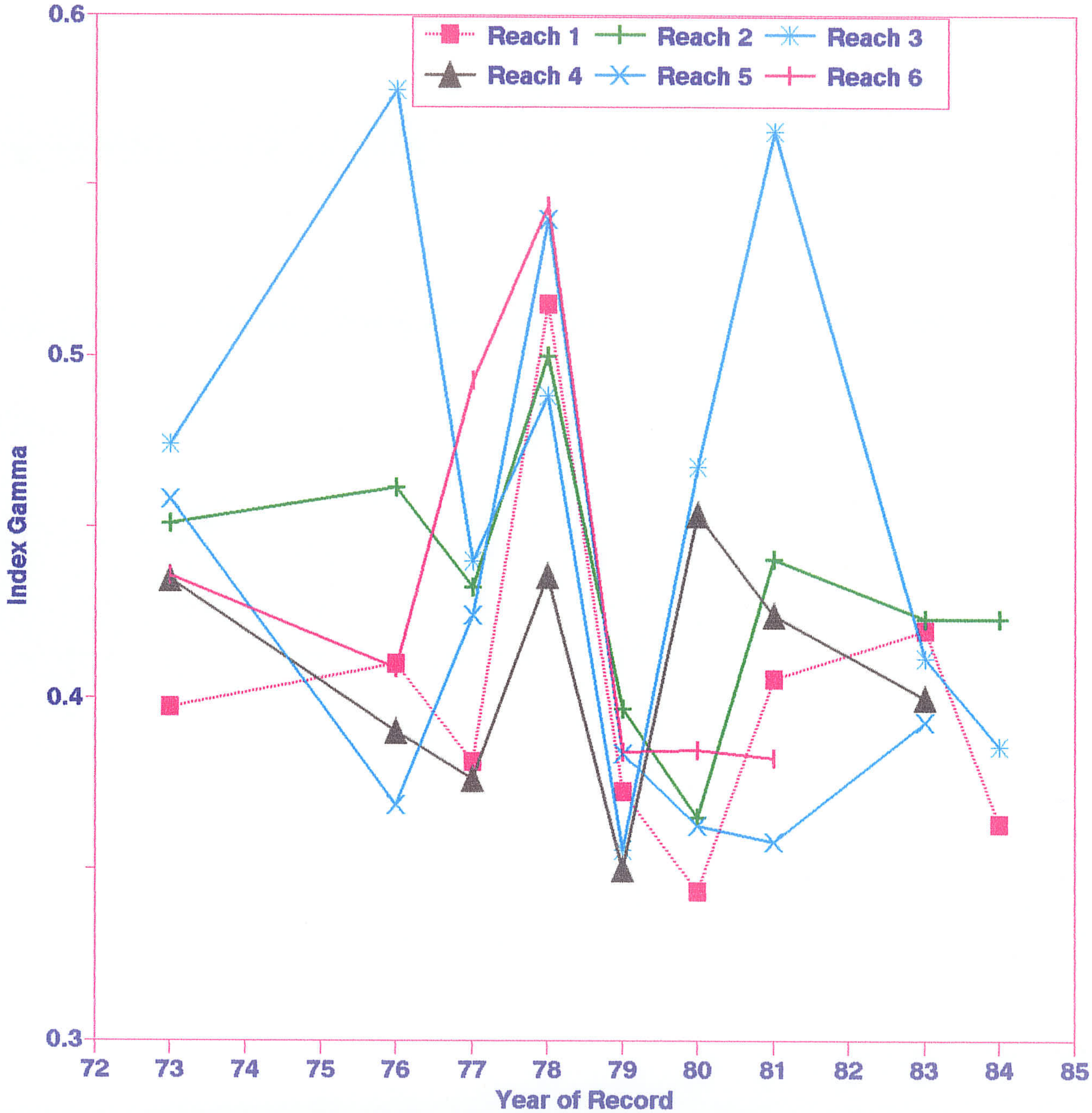

Fig. C.2: Variation of Topological Parameter γ

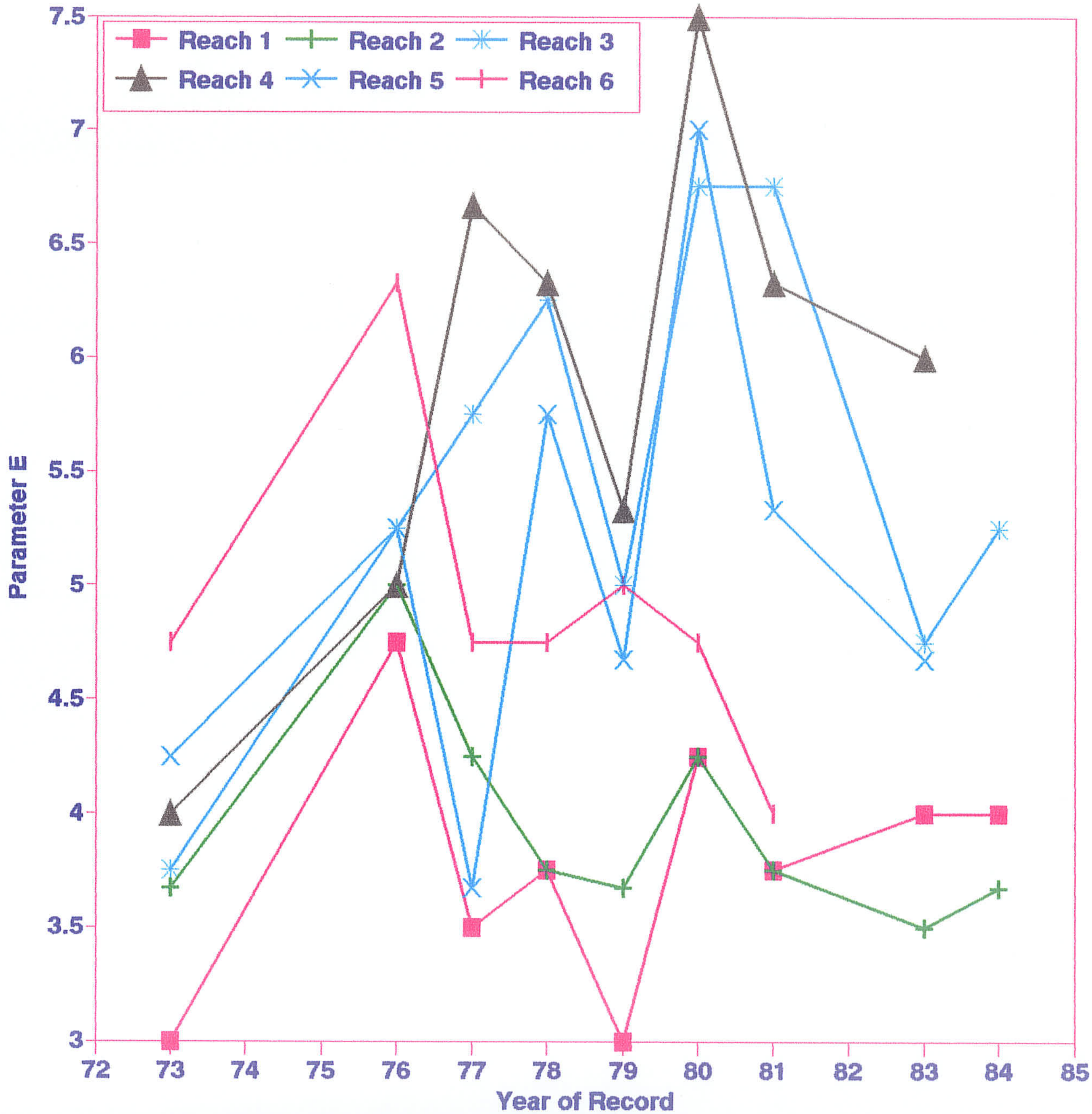

Fig. C.3: Variation of Topological Parameter E

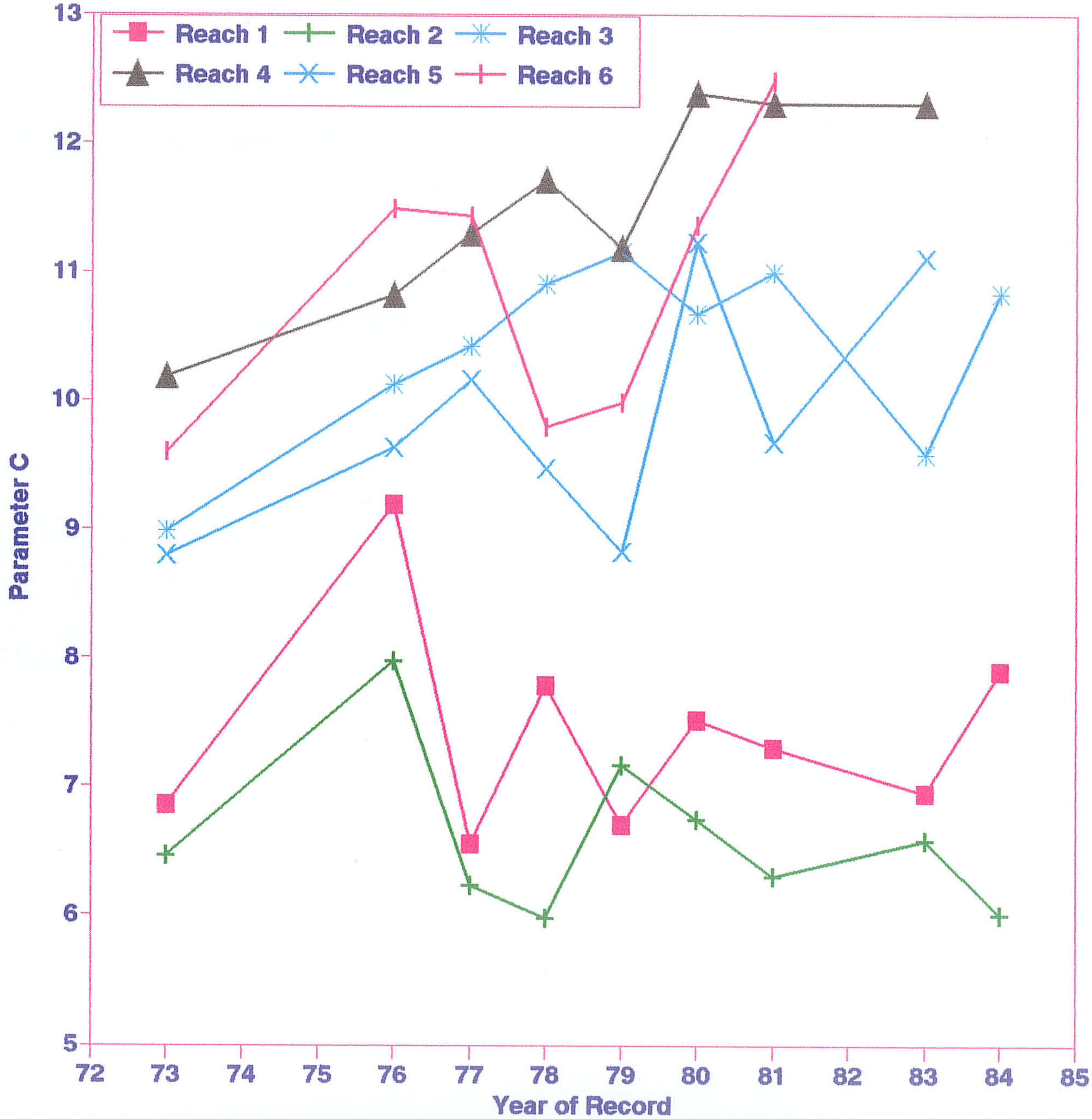

Fig. C.4: Variation of Topological Parameter C

Appendix D

CHANGES IN CROSS SECTIONS WITH TIME

Some of the cross sections of the Brahmaputra river in Bangladesh as recorded by Bangladesh Water Development Board are given in the following. These plots show the changes in location of the river with time. The changes of channels that occur within the river in a short period of time is also evident from these plots.

Note that the vertical and the horizontal scales are different in these plots. The approximate vertical scale is 1:800 and the approximate horizontal scale is 1:90000. Each frame in these figures are of equal height with the level of 0 m PWD being in the middle of the frames. The top frame shows the vertical and horizontal reference lines.

Each frame of these figures describes the cross section of a particular year of record. The cross sections that are shown on each figure are of the years as follows: from the top to the bottom of the figure, the frames show the cross sections of 1976, 1977, 1978, 1979, 1980, 1981 and 1983.

Note that these cross-sections are measured in the low-water period of the year. Although these cross-sections changes with the influx of flood, these cross-sections can be regarded as a reflection of the formative discharge of the previous year (Stene, 1994). The justification of this assumption can be attributed to the arguments put forward by previous researchers (e.g., Leopold & Wolman, 1963) that the major changes are caused by a discharge which is lower than the peak discharge. This discharge is considered to be responsible for carrying the major sediment load causing major changes in the cross-section.

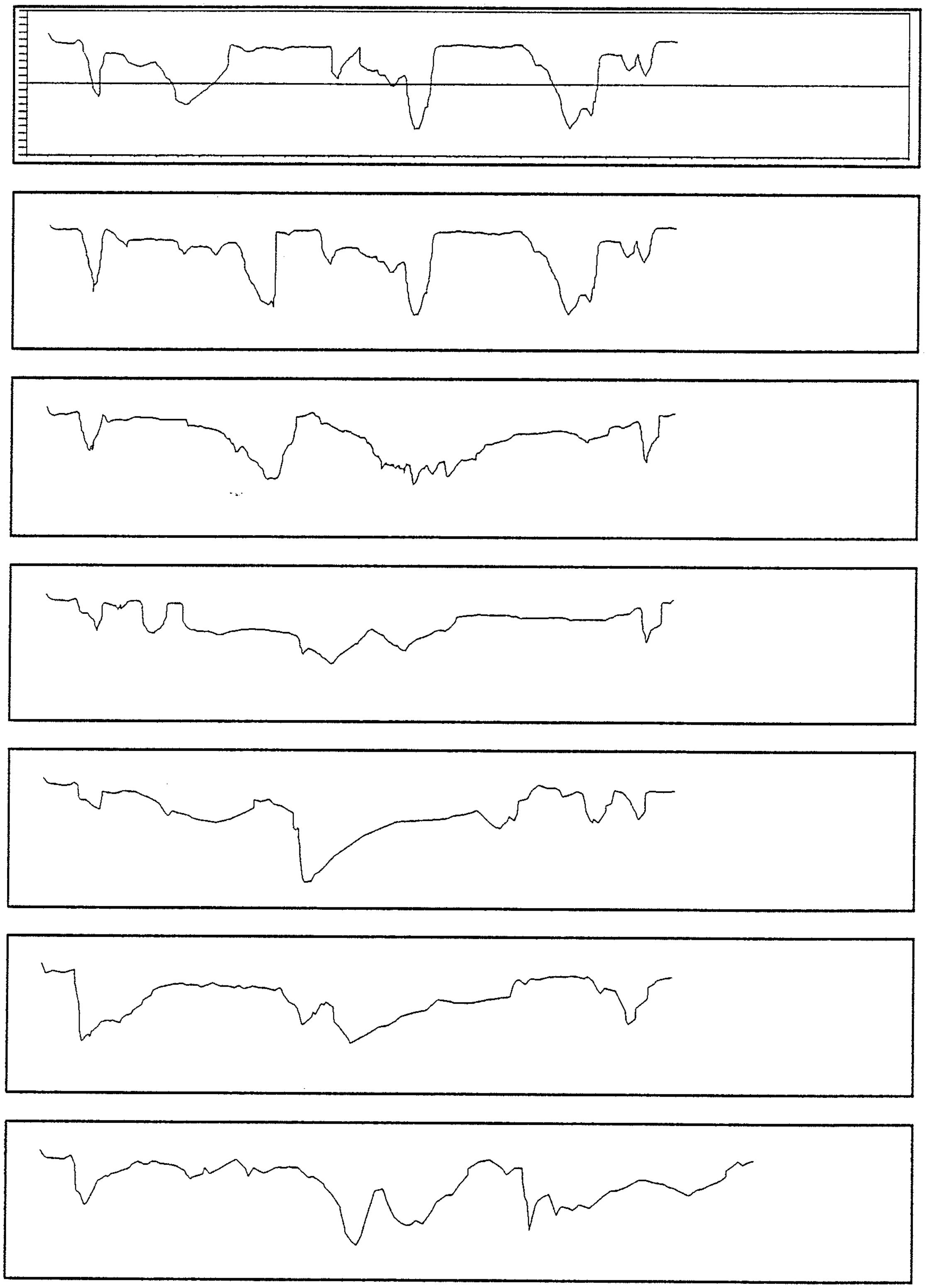

Fig. D.1: Section J-1-1 of the Brahmaputra River (1976-1983)

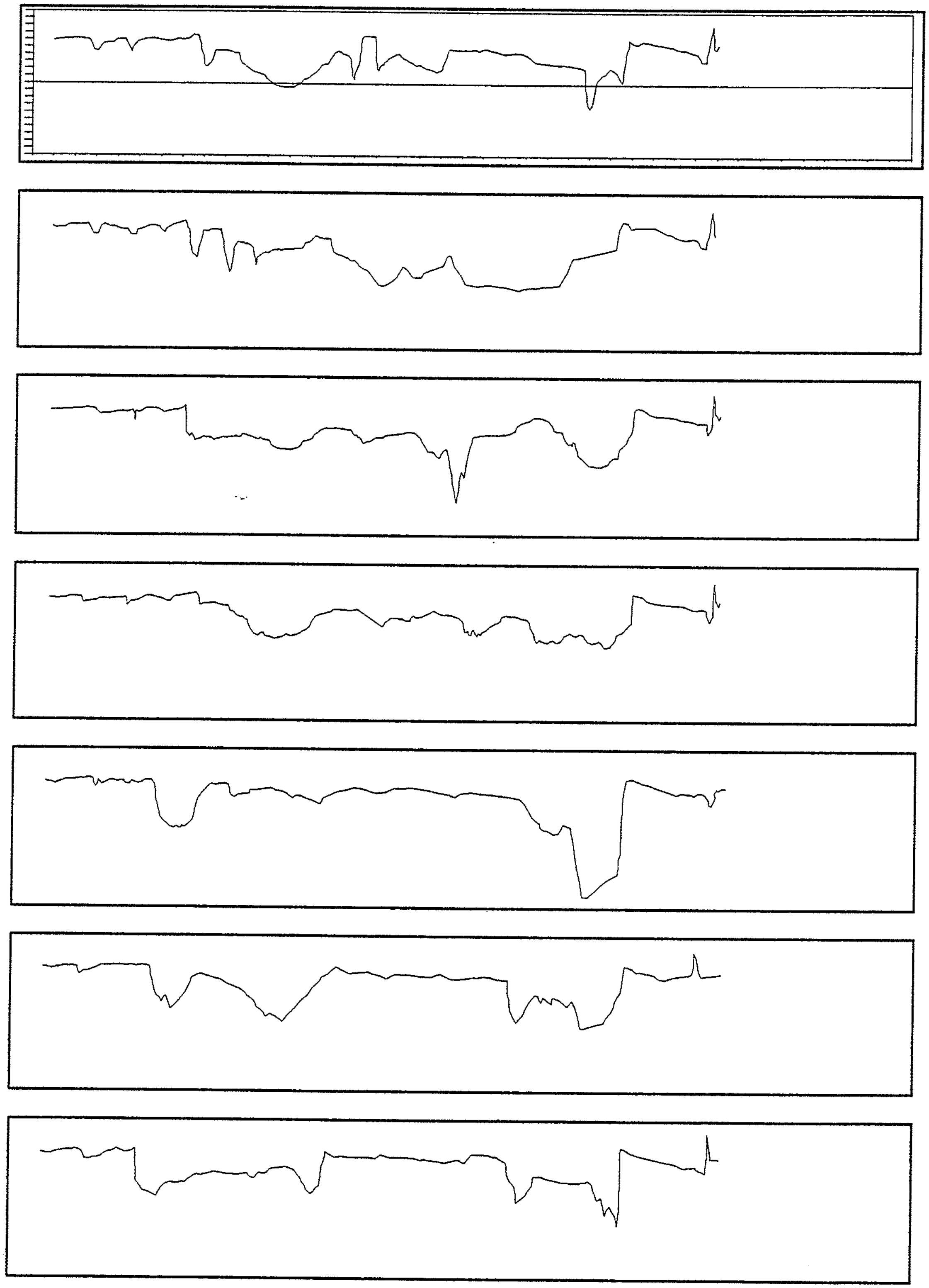

Fig. D.2: Section J-3 of the Brahmaputra River (1976-1983)

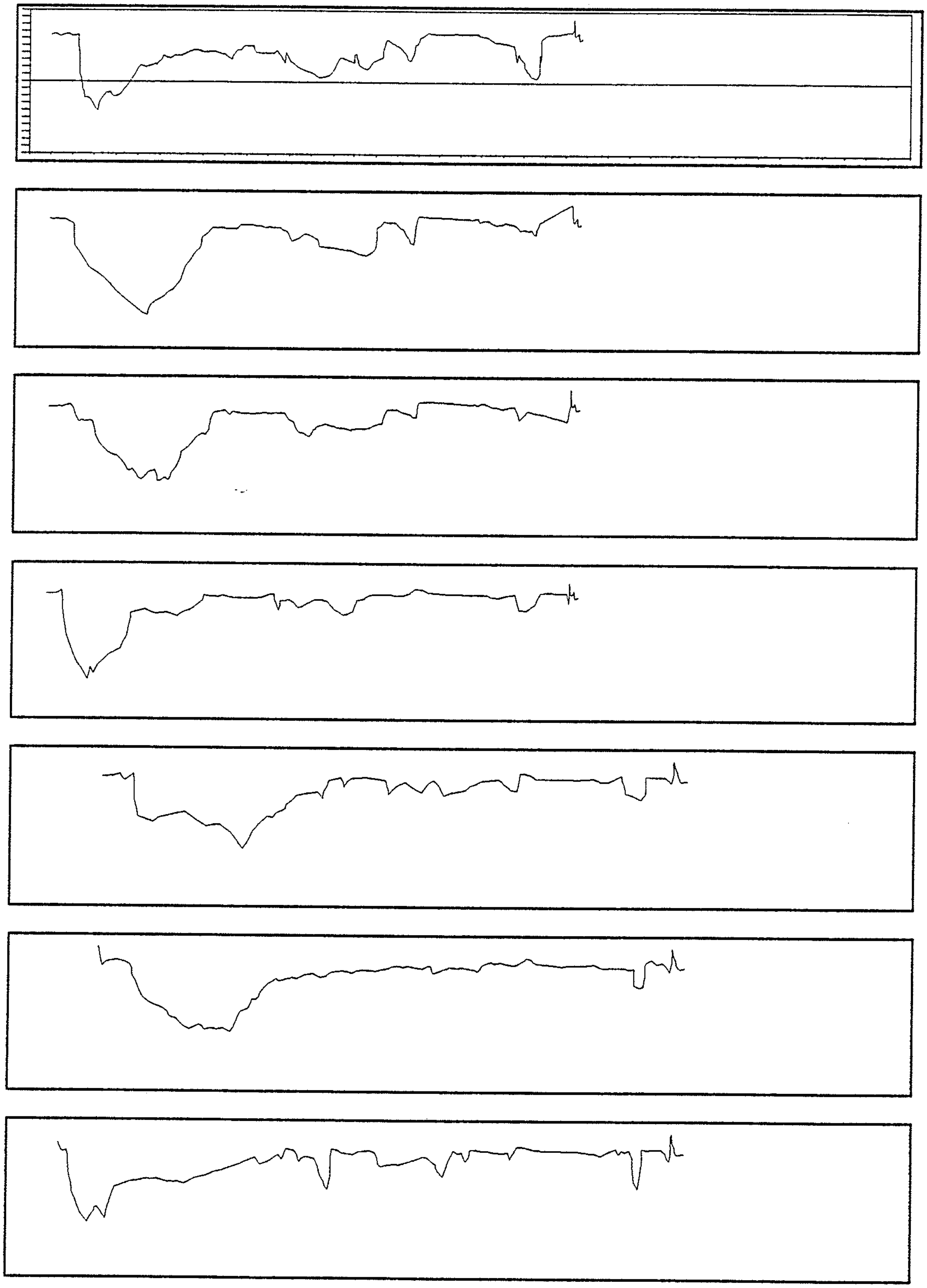

Fig. D.3: Section J-3-1 of the Brahmaputra River (1976-1983)

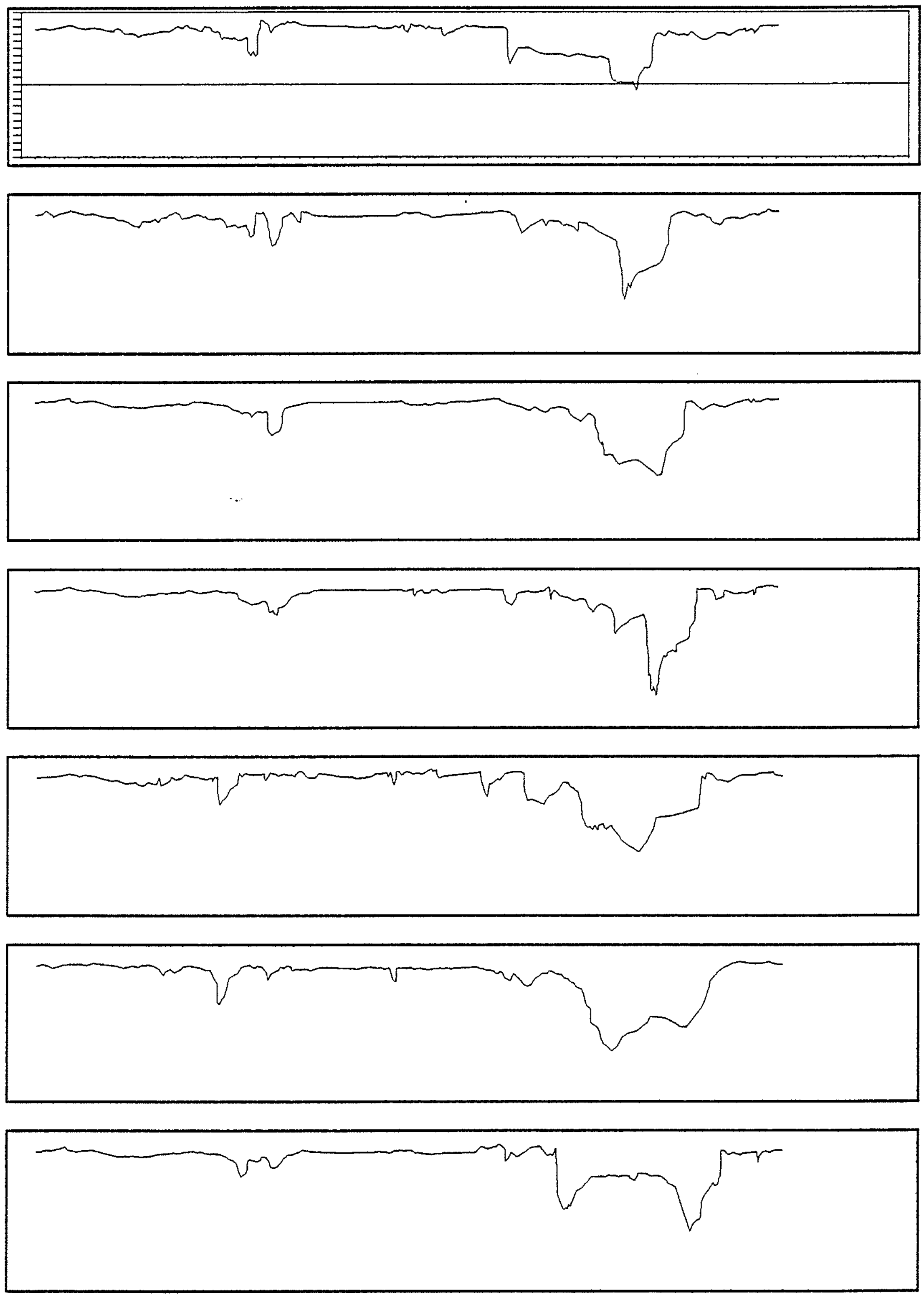

Fig. D.4: Section J-6 of the Brahmaputra River (1976-1983)

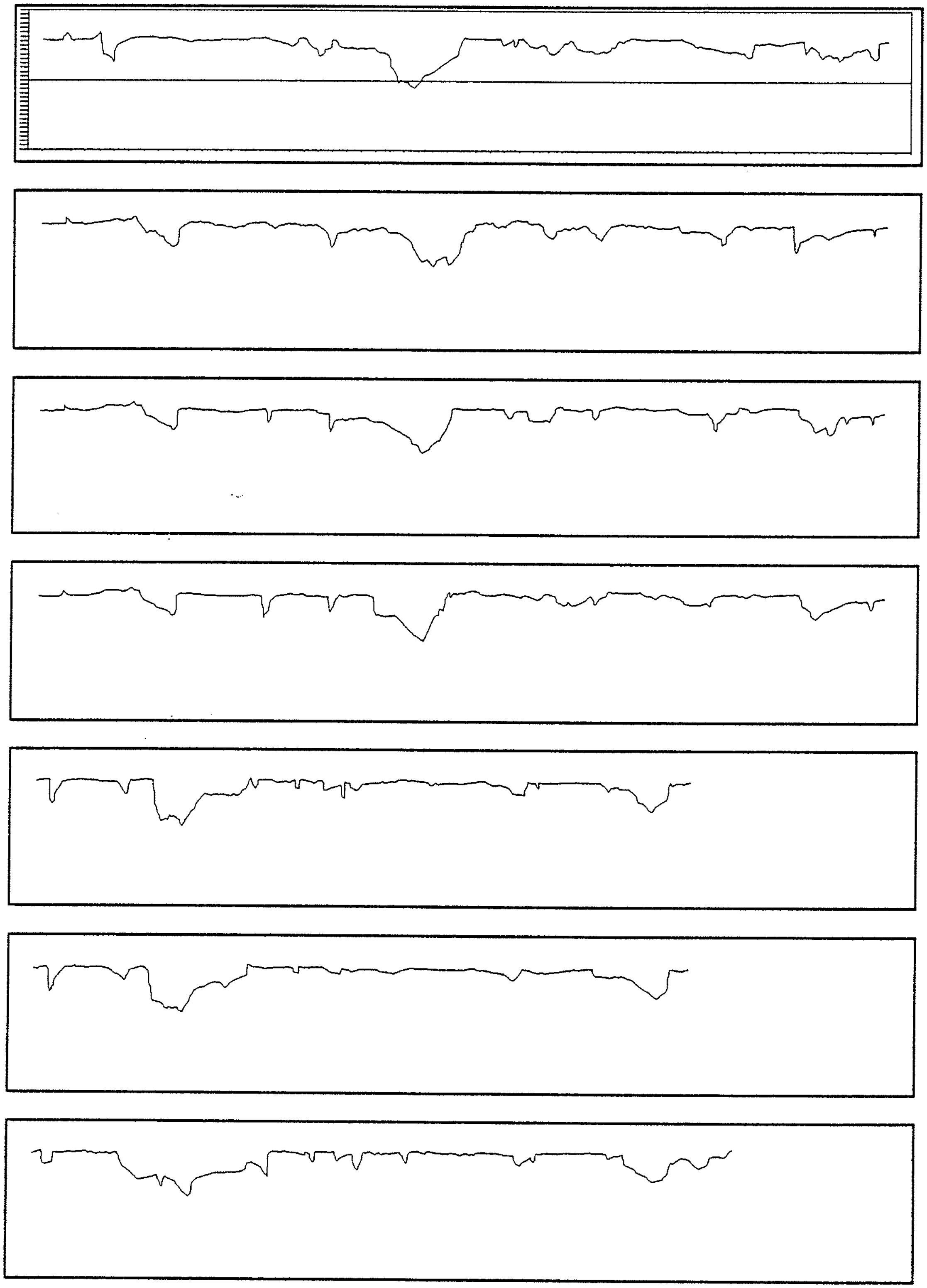

Fig. D.5: Section J-9-1 of the Brahmaputra River (1976-1983)

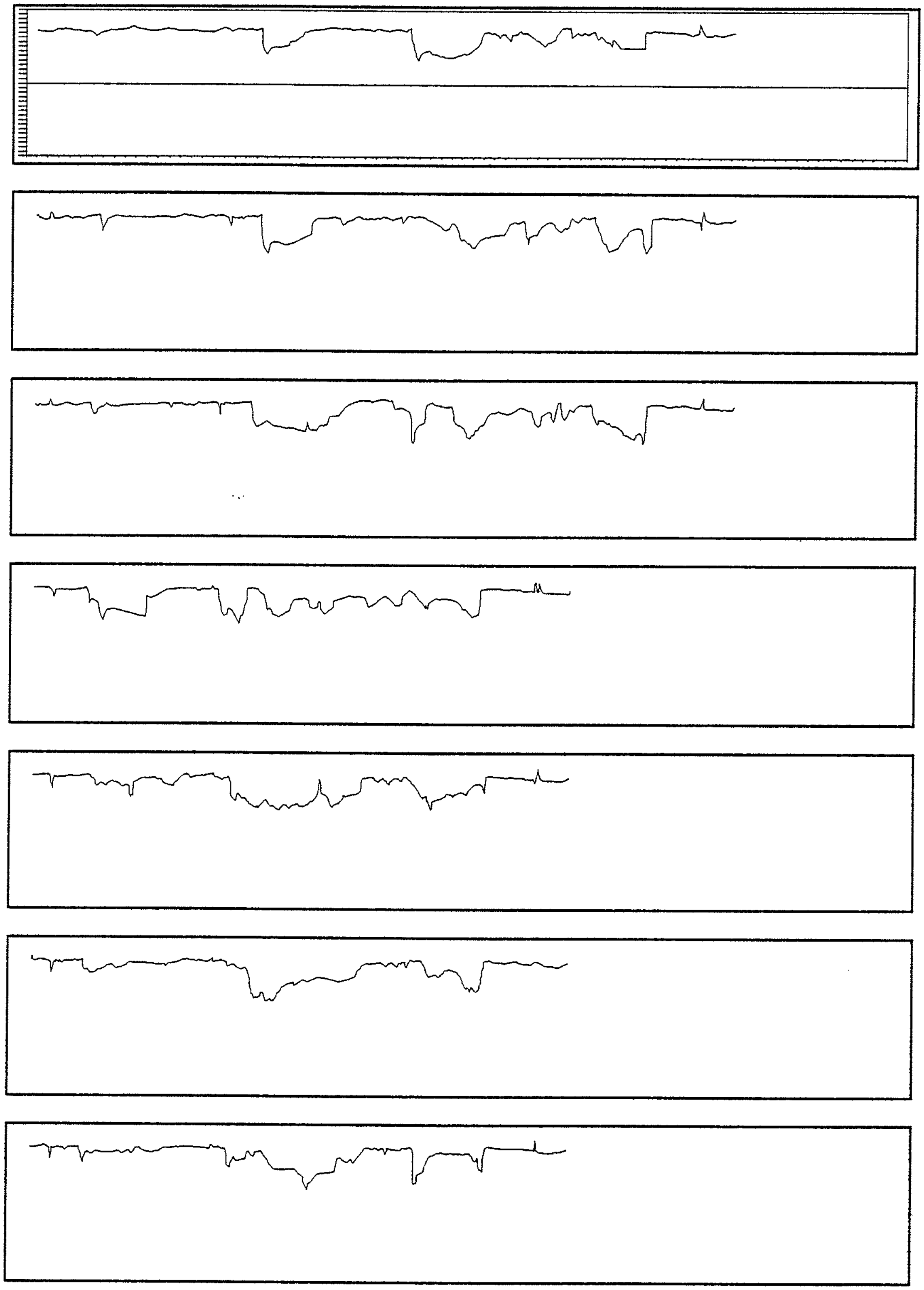

Fig. D.6: Section J-12 of the Brahmaputra River (1976-1983)

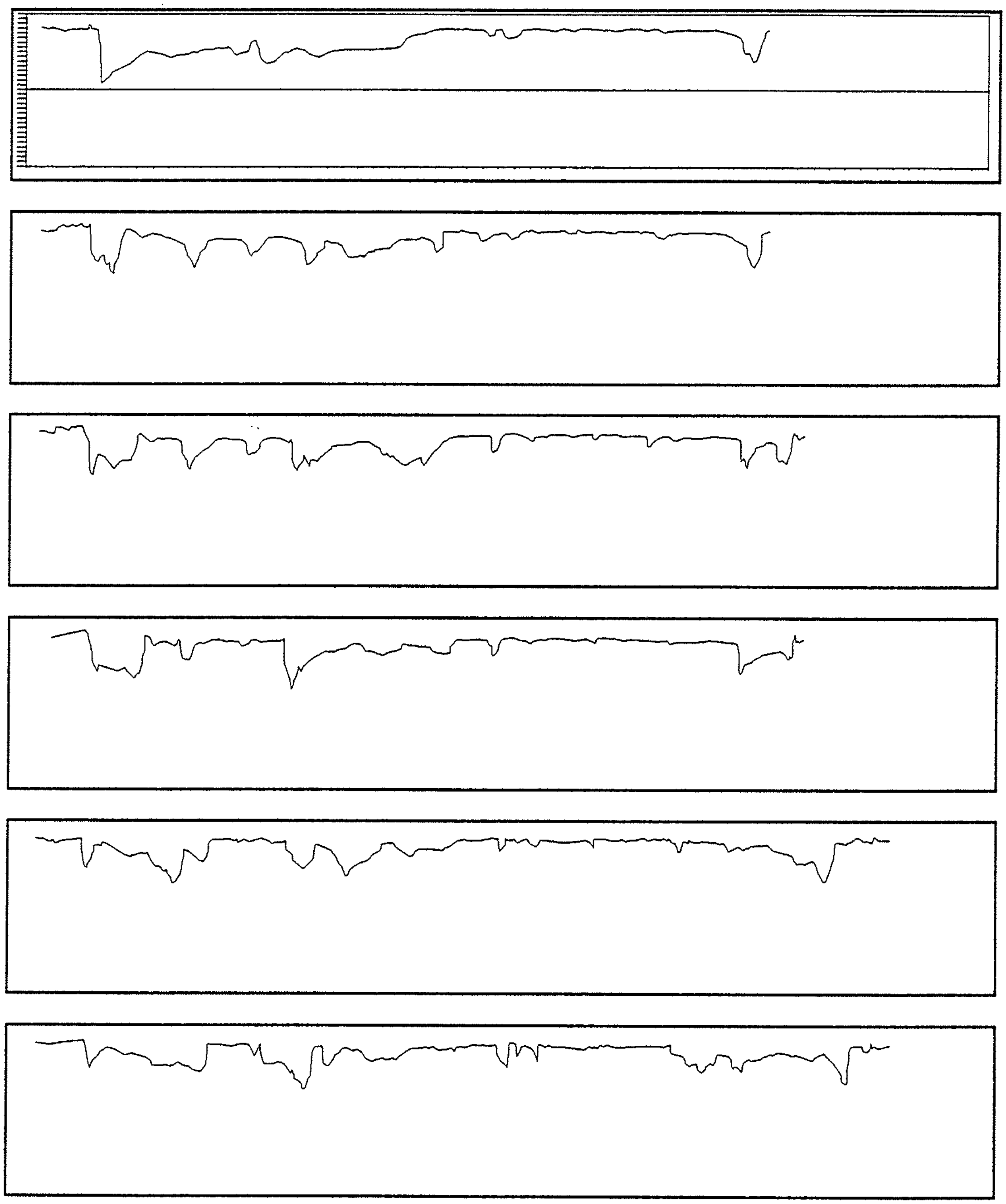

Fig. D.7: Section J-13-1 of the Brahmaputra River (1976-1981)

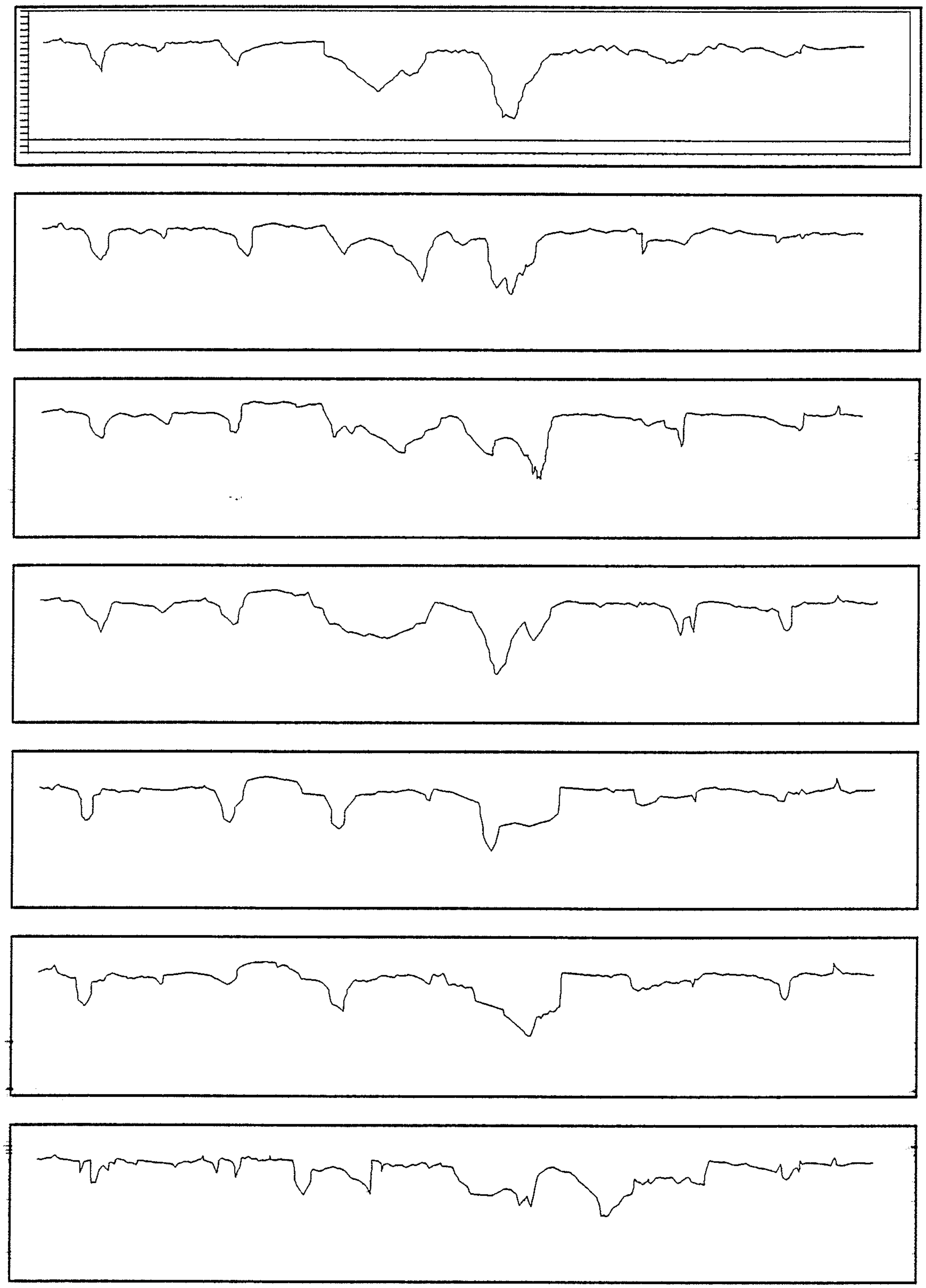

Fig. D.8: Section J-15-1 of the Brahmaputra River (1976-1983)

Appendix E

COMPUTATION OF SECTIONAL PROPERTIES

E.1 Description

The developed program computes the sectional properties of the branch channels as well as of the total section of a braided river. The input values are: *X* and *Y* coordinates of the section, the water discharge, water levels of the sections if it is known, the distance between the sections, section numbers for which the *X*, *Y* coordinates are missing. All of these values for different years of record can be supplied together. These are read from information stored in files.

The water levels at the sections where they are recorded regularly were processed before using in this program. This pre-processing was required for two reasons. First, the water levels at recording sections are reported as daily values instead of hourly values. Therefore, water level corresponding to a particular discharge had to be obtained by interpolating between values recorded at two consecutive days. Second, the before peak and after peak values of water levels for the same discharge are different due to the hysteresis effect present in the observed rating curve of the river (see *Appendix-F*). The average of these two values (before peak and after peak) are taken in this program.

The program is written in WATFOR-77 for Personal Computers and uses remote Blocks and Subroutines. The file information is supplied from the screen. The output contains all the values of the properties for the branch channels as well as for the total sections as described in Chapter Four.

The computational procedure is as follows. Starting from the downstream section the program first checks the information to determine which sections do not have water level records. For these sections water levels have to be computed. Also, the program checks the information to determine for which sections the coordinate information is missing. These sections with missing coordinates are excluded from computation of sectional properties.

To determine the unknown water level at a section the following procedure is adopted. First, two consecutive sections where the water levels are known are determined. These sections with known water levels are taken as the reference sections. The unknown water levels for the sections in between these two are computed by routing the discharge. The process is a trial-and-error one. When the difference between the computed and the recorded water level is within the tolerance limit, the procedure stops. Standard step method of computation is followed.

When convergence is obtained, the friction coefficient is recorded. The sections with missing coordinates are not considered in the computations, and the distances between the sections are adjusted accordingly.

The number of channels in a section is determined by the water level and the coordinates of the section alone. This number corresponding to a certain water level is computed for all the sections. Note that the number of channels is different for different sections. A channel is defined as the part of the section for which the bed elevations are lower than the water level. Also, a channel should lie between two consecutive portion of the section (or banks) for which the elevation is higher than the water level. The vertical and horizontal coordinates of the point of intersection of the water surface (for a given level) and the bank, in most cases, has to be interpolated since the coordinates are measured at discrete points. The program checks in both directions to determine whether the point is on an increasing or on a decreasing slope. The bed-line between two consecutive measurements points of the section is considered to be linear.

For every increment in the trial water level, all of the sectional properties are recalculated. The sectional properties of each branch channel as well as the total section corresponding to the condition for which convergence is obtained are recorded in a file.

Appendix F

WATER LEVEL AT DIFFERENT DISCHARGES

F.1 Introduction

Computation of the cross sectional properties is largely dependent on the value of water level corresponding to the water discharge of interest. The water levels do not hold a one to one relationship with the water discharge, rather a hysteresis effect is present. The before-peak and the after-peak values of water level for the same discharge are different. Also, the discharge of interest usually has to be interpolated between two dates of records, and the water level has to be interpolated accordingly.

The cross sectional measurements are not taken as regularly as the water level and discharge. The cross section data that are available are of the low water period (usually between October and May). The water year and the calendar year are not the same. The water year starts on the first day of April which is still during the low flow period. The cross sectional measurements are referenced to the calendar year. Therefore, to avoid confusion, a definition of the year of record is provided in the following section.

F.2 Definition of Year of Record

It is convenient to designate the year of record of cross sectional measurements to be the corresponding water year as explained below. The water year representing the low-water period in which the cross-sectional measurements are taken will be considered as the year of record for the cross-section if the measurements are taken before April 1. The previous water year will be taken as the year of record of the cross-section if the measurements are taken after April 1 but before the peak flood. The maximum discharge of 1984, for example, refers to the maximum value of discharge recorded between April 1, 1984 and March 31, 1985. The cross sectional measurements, on the other hand, made within the falling limb of the hydrograph of the water year 1984-1985 and the rising limb of the hydrograph of the water year 1985-1986 is referred to as the measurement of 1984. For example, cross section measured in the last week of April 1985 would still be referred to as the cross section of 1984 even though the period is within the 1985-1986 water year, assuming that the peak flood of 1985-1986 water year (which usually occurs during July-August period) has not passed by this time.

The cross sections are usually measured during the low water period (between two peak floods of two consecutive water year). This cross section is taken to be valid for the discharge of interest even though the measurement may not have been made for that particular value of discharge. This definition is showed in Fig. F.1 with the example water years of 1984-1985 and 1985-1986. Also on this figure, an arbitrary discharge line (30,000 m^3/sec) is shown. There are two different dates of interest for which this discharge occurred, one is the after-peak value and the other is the before-peak value. Since water levels corresponding to these dates are different (for the same discharge) because of the hysteresis effect, the average value is taken to be the water level for this discharge.

F.3 Interpolation of Water Level

The water level and the discharge data as reported by the Bangladesh Water Development Board are the daily mean values. Therefore, the discharge of interest would usually lie between the values of two consecutive days. Occurrence of observing the exact discharge (of interest) is a rare event. Water level corresponding to this discharge is, therefore, needed to be interpolated.

Let Q be the discharge of interest which lies between the observation of day 1 and day 2. Discharge recorded on day 1 and day 2 are Q_1 and Q_2 respectively. The corresponding water levels are H_1 and H_2. It is assumed that these values vary linearly within the 24 hour time period. Then, for a discharge change of $(Q_1 - Q_2)$, the water level changes by $(H_1 - H_2)$. If the first day discharge deviates from Q by ΔQ_1 i.e., if $\Delta Q_1 = (Q_1 - Q)$, then the deviation in water level corresponding to this deviation in discharge can be written as:

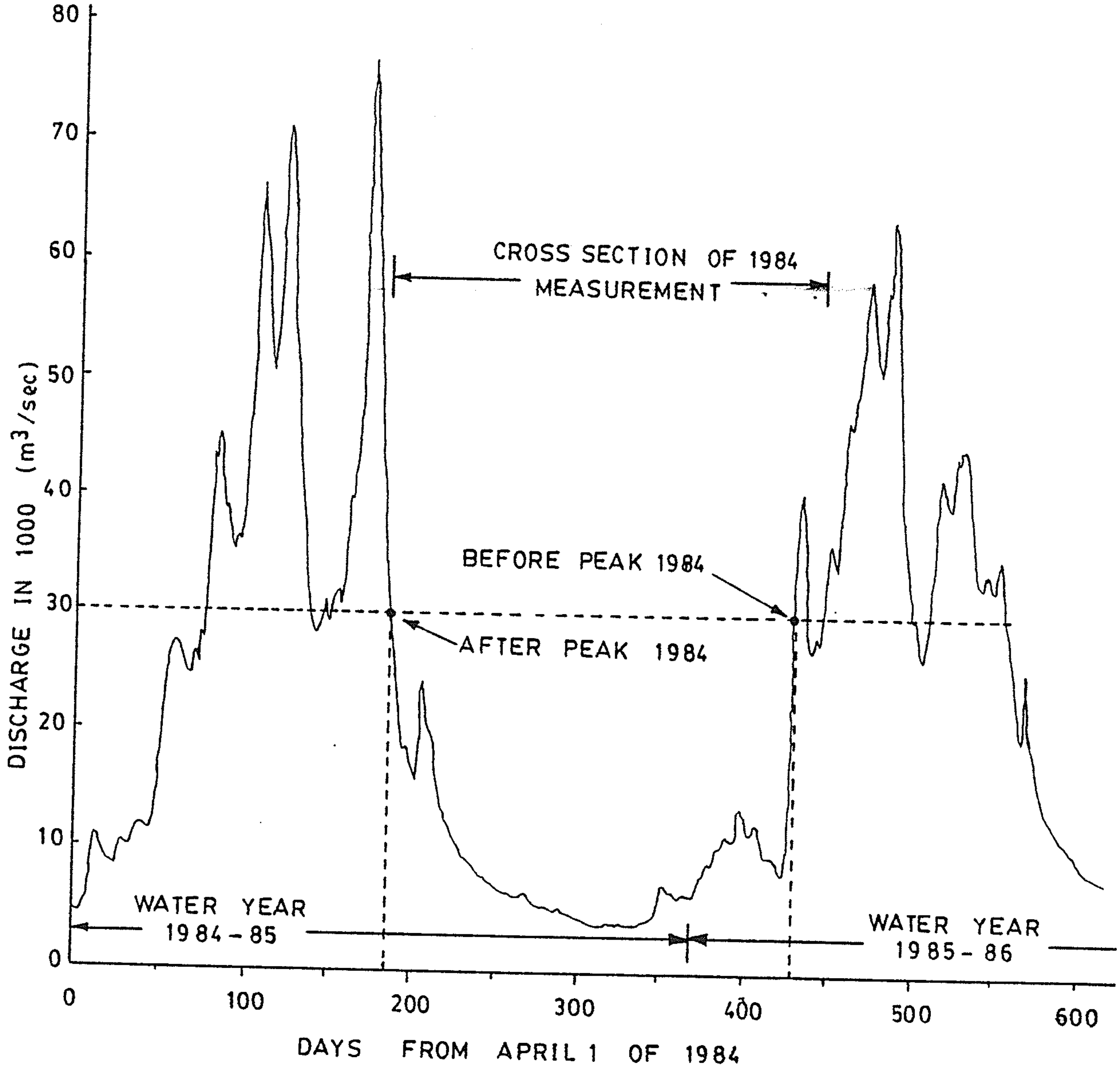

Fig. F.1: Definition Sketch of Year of Record

$$\left(\frac{H_1 - H_2}{Q_1 - Q_2}\right) \Delta Q_1 \tag{F.1}$$

The water level corresponding to Q is then given by:

$$H = H_1 - \left(\frac{H_1 - H_2}{Q_1 - Q_2}\right) \Delta Q_1 \tag{F.2}$$

Note that the quantity $(H_1 - H_2)/(Q_1 - Q_2)$ is always positive. The quantity ΔQ_1, however, may be positive or negative depending on whether the discharge increased or decreased between day 1 and day 2. Equation (F.2) is therefore valid for both cases.

To find the water level for a discharge of 30,000 m^3/sec for 1984, one has to look at the discharge and water level records of water year 1984-1985 and 1985-1986 (see Fig. F.1). The after-peak value of 30,000 m^3/sec occurred between day 185 and 186. The corresponding discharge and water level values are: Q_1 = 32,500 m^3/sec, H_1 = 18.068 m for day 185; Q_2 = 29,700 m^3/sec, H_2 = 17.870 m for day 186 respectively. Using Eqn. (F.2), the water level corresponding to Q = 30,000 m^3/sec is found to be 17.891 m.

The before-peak value of Q occurred between day 429 and 430. The corresponding values are: Q_1 = 26,800 m^3/sec, H_1 = 17.892 m; Q_2 = 33,900 m^3/sec, and H_2 = 18.478 m. Using Eqn. (F.2) the water level corresponding to Q = 30,000 m^3/sec is found to be 18.156 m. Note that the before- and the after-peak values are different. The average is 18.024 m. This value is taken to be the water level for a discharge of 30,000 m^3/sec.

The graphical solution of Eqn. (F.2) is given in Fig. F.2. This figure has two ordinates; the left one is for discharge and the right one is for the water level. The line for Q = 30,000 m^3/sec intersects with the discharge line at a point. Follow this point vertically to intersect the water level line. The water level corresponding to this point of intersection is the required value. The same procedure is followed to find both the after- and the before-peak values. An average of these two values are taken.

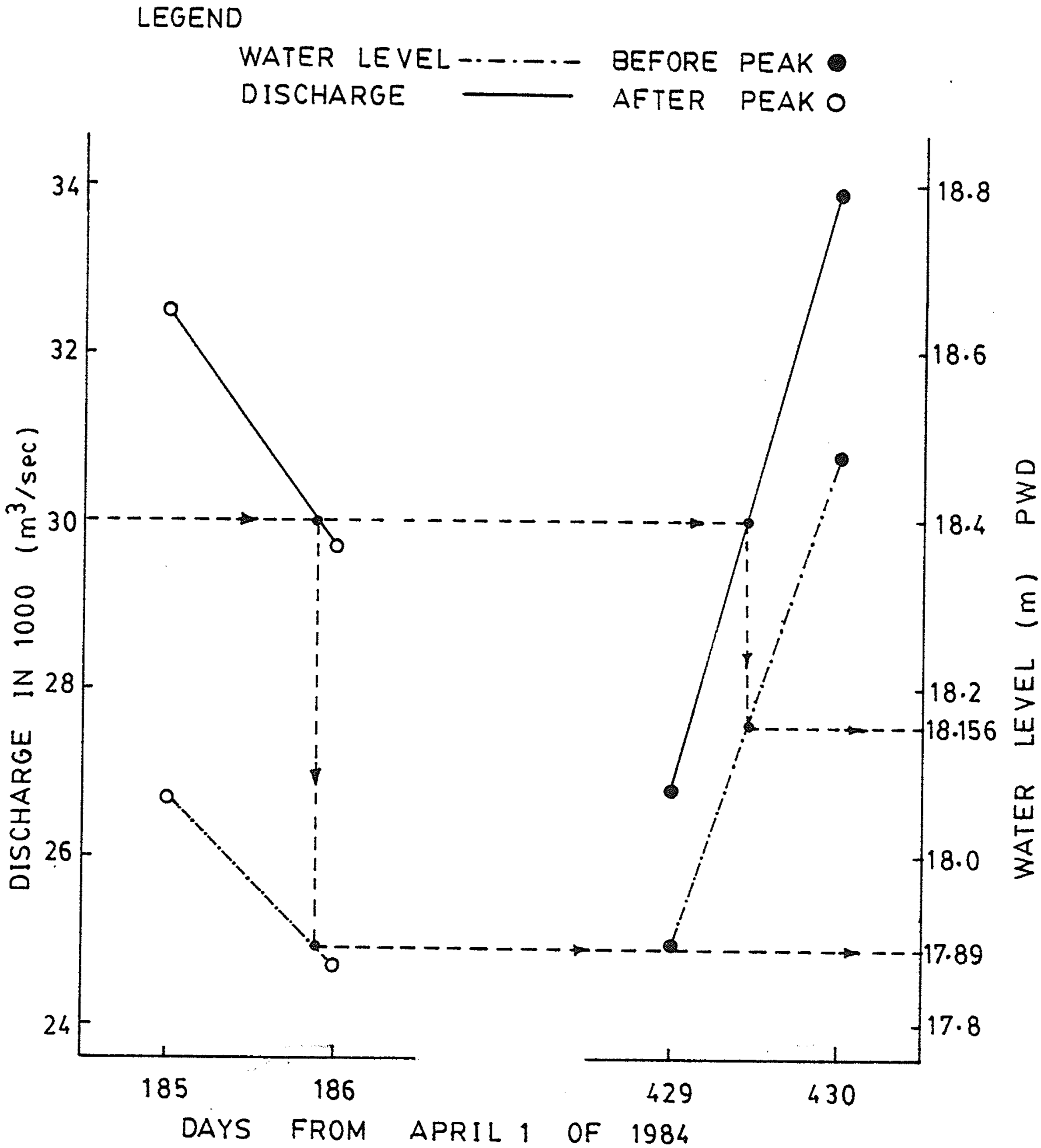

Fig. F.2: Interpolation of Water Level at Specific Discharge

F.4 Water Levels at Different Discharges

Following the procedure described above, the water levels at all the sections of the Brahmaputra river for different discharges of different years are computed. For the sections with recorded data, Eqn. (F.2) was followed. For others, the water level was computed by routing the flow using the known records of the measurement sections. Some of the resulting values are given in the following tables.

Table F.1: Water Levels at Different Sections for Q = 10,000 m^3/sec

Section	1976	1977	1978	1979	1980	1981	1983
J-1	4.563	4.432	4.225	3.603	4.249	4.435	4.252
J-1-1	4.695	4.571	4.501	4.32	4.569	4.834	4.34
J-2	4.878	4.827	4.871	4.592	5.114	5.325	4.624
J-2-1	4.943	4.846	5.048	4.618	5.26	5.757	4.872
J-3	5.15	5.063	5.343	4.735	5.399	6.365	5.325
J-3-1	5.572	5.338	5.782	4.93	5.756	7.009	6.181
J-4	5.753	5.52	6.229	5.17	6.223	7.404	6.527
J-4-1	6.38	6.838	7.037	5.722	7.1	7.816	6.757
J-5	6.492	6.965	7.298	5.914	7.575	8.21	6.915
J-5-1	6.559	7.161	7.361	6.415	7.728	8.522	7.327
J-6	6.974	7.452	7.506	6.925	7.873	8.628	7.658
J-6-1	7.709	7.772	7.72	7.146	8.134	8.798	7.97
J-7	8.469	8.099	8.121	7.611	8.669	9.067	8.332
J-7-1	8.666	8.431	8.49	8.126	8.923	9.254	8.679
J-8	9.452	9.314	9.884	8.764	9.864	10.338	9.294
J-8-1	9.993	9.75	10.179	9.315	10.211	10.724	9.522
J-9	10.568	10.133	10.398	9.907	10.643	11.145	10.144
J-9-1	11.094	11.356	11.209	10.92	11.619	11.356	10.777
J-10	11.31	11.969	11.938	11.856	12.35	11.597	10.963
J-10-1	11.611	12.457	12.566	12.465	12.775	11.979	11.159
J-11	12.917	13.536	13.931	13.459	13.71	13.033	12.002

J-11-1	13.662	14.111	14.321	13.893	14.309	13.347	12.605
J-12	14.521	14.69	14.717	14.295	14.789	13.743	13.243
J-12-1	14.824	14.819	14.85	14.456	14.994	14.187	14.005
J-13	15.029	14.912	14.991	14.619	15.257	14.73	14.4
J-13-1	15.157	15.023	15.101	14.716	15.883	15.176	14.955
J-14	15.518	15.926	15.508	15.527	16.162	15.806	16.047
J-14-1	16.62	16.664	16.102	16.207	16.953	16.547	16.961
J-15	17.603	17.046	16.691	17.04	17.578	17.381	17.82
J-15-1	18.07	17.562	17.457	17.833	18.219	18.067	18.39
J-16	18.835	18.51	18.593	18.762	18.766	18.494	18.883
J-16-1	19.554	19.235	19.33	19.568	19.163	18.948	19.197
J-17	20.978	19.668	19.8	20.498	19.745	19.974	19.877

Table F.2: Water Levels at Different Sections for Q = 30,000 m^3/sec

Section	1976	1977	1978	1979	1980	1981	1983
J-1	7.6353	6.9617	7.1781	6.3795	6.9526	6.855	6.9952
J-1-1	7.7954	7.1803	7.4006	6.9349	7.2117	7.5283	7.144
J-2	7.9005	7.4311	7.6262	7.2482	7.7237	8.4004	7.5015
J-2-1	7.9542	7.7664	7.7707	7.3574	7.7987	8.422	7.7509
J-3	8.3178	8.2538	8.0864	7.6801	7.999	8.6512	8.3833
J-3-1	8.8838	8.4662	8.5132	8.1608	8.333	8.8436	9.2779
J-4	9.2642	8.85	9.0216	8.8704	8.6653	9.0758	9.8775
J-4-1	9.8162	9.6623	9.8125	9.4523	9.0734	9.2118	10.375
J-5	9.9366	9.7933	10.062	9.7232	9.4703	9.3331	10.598
J-5-1	10.216	9.9445	10.319	10.106	10.199	10.629	11.005
J-6	10.621	10.377	10.688	10.564	10.534	10.874	11.256
J-6-1	11.136	10.962	11.017	11.083	11.089	11.174	11.604
J-7	11.878	11.284	11.495	11.525	11.967	11.642	12.102
J-7-1	12.067	11.726	11.86	11.72	12.354	11.851	12.616

J-8	12.652	12.46	12.765	12.082	13.34	13.031	12.887
J-8-1	13.128	12.863	13.134	12.436	13.655	13.652	13.089
J-9	13.704	13.341	13.439	13	14.183	14.042	13.478
J-9-1	14.136	14.308	14.401	13.967	14.784	14.228	13.766
J-10	14.417	14.889	15.062	14.737	15.475	14.517	14.013
J-10-1	14.668	15.252	15.552	15.247	15.841	14.738	14.215
J-11	15.84	16.288	16.839	16.486	16.619	15.34	14.839
J-11-1	16.803	16.91	17.257	16.953	17.189	15.702	15.401
J-12	17.471	17.456	17.633	17.365	17.673	16.119	15.976
J-12-1	17.699	17.581	17.68	17.524	17.881	16.577	16.8
J-13	17.96	17.79	17.839	17.825	18.184	17.194	17.283
J-13-1	18.054	17.871	17.858	17.965	18.593	17.822	17.755
J-14	18.591	18.855	18.595	18.705	18.935	18.403	18.401
J-14-1	19.882	19.541	19.463	19.443	19.639	19.072	19.199
J-15	20.929	20.106	20.331	20.547	20.442	20.019	20.082
J-15-1	21.483	20.9	21.079	21.217	21.115	20.708	20.735
J-16	22.205	21.717	21.845	21.982	21.857	21.314	21.258
J-16-1	22.89	22.188	22.512	22.614	22.171	21.808	21.691
J-17	23.848	22.663	23.147	23.593	22.827	22.695	22.321

Table F.3: Water Levels at different Sections for Q = 40,000 m^3/sec

Section	1976	1977	1978	1979	1980	1981	1983
J-1	8.117	7.638	7.931	7.321	8.085	7.385	7.797
J-1-1	8.343	7.816	8.142	7.806	8.23	8.122	7.922
J-2	8.513	8.044	8.364	8.025	8.839	9.004	8.203
J-2-1	8.569	8.319	8.453	8.084	8.992	9.03	8.364
J-3	9.01	8.792	8.77	8.36	9.327	9.346	8.992
J-3-1	9.457	9.006	9.263	8.635	9.571	9.571	9.887

J-4	9.753	9.425	9.785	9.389	9.754	9.855	10.47
J-4-1	10.307	10.256	10.513	9.879	9.937	10.043	10.946
J-5	10.516	10.479	10.766	10.315	10.063	10.229	11.215
J-5-1	10.77	10.628	11.006	10.679	11.339	11.352	11.649
J-6	11.119	11.107	11.471	11.151	11.704	11.703	11.853
J-6-1	11.592	11.755	11.979	11.69	12.04	12.02	12.161
J-7	12.379	12.083	12.494	12.157	12.558	12.456	12.61
J-7-1	12.692	12.433	12.75	12.39	13.296	12.646	13.323
J-8	13.284	13.206	13.54	12.804	13.686	13.323	13.625
J-8-1	13.731	13.663	13.958	13.154	14.387	13.745	13.843
J-9	14.311	14.195	14.311	13.749	14.873	14.183	14.31
J-9-1	14.708	14.939	14.844	14.867	15.21	14.418	14.541
J-10	15.031	15.491	15.606	15.604	15.393	14.85	14.794
J-10-1	15.301	15.912	16.033	16.039	16.002	15.119	14.995
J-11	16.532	17.028	17.313	17.302	17.069	15.892	15.608
J-11-1	17.489	17.545	17.801	17.804	17.679	16.319	16.156
J-12	18.145	18.133	18.224	18.136	18.531	16.81	16.726
J-12-1	18.405	18.306	18.288	18.217	18.593	17.332	17.627
J-13	18.672	18.596	18.502	18.421	18.715	17.992	18.081
J-13-1	18.781	18.652	18.551	18.425	18.835	18.544	18.55
J-14	19.172	19.142	19.203	19.267	19.172	19.155	19.229
J-14-1	19.507	19.507	19.965	20.02	19.507	19.811	20.093
J-15	20.727	20.696	21.184	21.255	20.727	20.842	21.07
J-15-1	21.946	21.885	21.946	21.955	21.946	21.499	21.785
J-16	22.915	22.591	22.55	22.559	22.872	22.212	22.306
J-16-1	23.165	23.165	23.165	23.184	23.165	22.657	22.772
J-17	23.47	23.47	23.47	24.191	23.47	23.554	23.345

Table F.4: Water Levels at Different Sections for Q = 50,000 m^3/sec

Section	1976	1977	1978	1979	1980	1981	1983
J-1	8.818	8.492	8.132	8.391	8.534	7.937	8.385
J-1-1	9.103	8.690	8.328	8.827	8.900	8.482	8.544
J-2	9.333	8.845	8.611	9.127	9.330	9.175	8.767
J-2-1	9.443	9.108	8.665	9.205	9.510	9.429	8.897
J-3	9.715	9.691	9.079	9.418	9.693	9.516	9.533
J-3-1	10.077	19.133	9.543	9.601	10.059	9.710	10.413
J-4	10.362	10.505	10.128	10.059	10.211	10.069	11.057
J-4-1	10.929	11.263	10.982	10.982	10.516	10.256	11.613
J-5	11.125	11.272	11.159	11.198	10.707	10.500	11.853
J-5-1	11.309	11.454	11.384	11.339	11.430	11.743	12.244
J-6	11.626	11.708	11.800	11.644	11.857	12.177	12.472
J-6-1	11.971	12.231	12.285	12.040	12.192	12.476	12.856
J-7	12.730	12.500	12.749	12.558	13.076	12.940	13.468
J-7-1	13.183	13.104	13.070	13.289	13.748	13.036	13.802
J-8	13.662	13.802	13.906	13.899	14.173	13.302	14.103
J-8-1	14.076	14.231	14.359	14.356	14.813	13.570	14.321
J-9	14.647	14.831	14.753	14.635	15.295	13.856	14.839
J-9-1	14.996	14.996	14.996	14.996	15.606	14.135	15.131
J-10	15.789	15.789	15.789	15.789	16.002	14.717	15.404
J-10-1	16.368	16.368	16.368	16.368	16.612	15.015	15.633
J-11	17.221	17.221	17.221	17.221	17.435	16.062	16.496
J-11-1	18.227	18.227	18.227	18.227	18.044	16.510	17.021
J-12	18.572	18.387	18.667	18.780	19.167	17.027	17.496
J-12-1	18.837	18.532	18.837	18.867	19.233	17.603	18.211
J-13	19.050	18.745	19.050	19.142	19.355	18.291	18.731
J-13-1	19.285	19.036	19.187	19.240	19.389	18.968	19.075
J-14	20.056	19.660	19.660	19.660	19.660	19.676	19.675
J-14-1	20.727	20.361	20.361	20.361	20.361	20.380	20.517

J-15	21.946	21.580	21.580	21.580	21.580	21.579	21.492
J-15-1	22.555	22.251	22.251	22.403	22.403	22.149	22.165
J-16	23.331	23.046	23.228	23.698	23.659	22.847	22.678
J-16-1	23.622	23.622	23.622	23.775	23.927	23.394	23.168
J-17	23.927	23.927	23.927	24.019	24.232	24.305	23.738

Appendix G

SECTIONAL PROPERTIES

G.1 Computation

This section explores the relationship between stage and other sectional properties. For this purpose, the cross-sectional area (A), hydraulic radius (R), and width (W) are calculated for different stage (H) for all sections and for all years of available record. The stage corresponding to the minimum discharge was taken as the lower limit and the stage corresponding to the bankfull discharge was taken as the upper limit of computation. Within this range, the above variables were computed at an interval of one foot. These values were used to find any statistical relationship of the following form:

$$A = C_1 H^b \tag{G.1}$$

$$R = C_2 H^d \tag{G.2}$$

$$W = C_3 H^e \tag{G.3}$$

where,

A is the cross-sectional area in ft^2,

R is the hydraulic radius in ft,

W is the total width in ft, and

H is the stage in ft.

The coefficients of Eqn. (G.1) to (G.3) for different year of record of different cross-sections are given in the following tables (Tables G.1 to G.6). Also, the coefficients obtained considering all data together is given in Table G.7.

Table G.1: Coefficients of Sectional Properties for Year 1976

Section Number	$A = C_1 H^b$		$R = C_2 H^d$		$W = C_3 H^e$	
	C_1	b	C_2	d	C_3	e
J-1	6532	1.28	5.16	0.44	1270	0.843
J-1-1	5606	1.33	3.30	0.56	1689	0.774
J-2	3889	1.47	1.54	0.75	2515	0.712
J-2-1	1568	1.61	4.13	0.44	380	1.17
J-3	292	2.08	0.47	1.02	620	1.06
J-3-1	669	1.80	0.896	0.87	734	0.938
J-4	124	2.213	0.34	1.09	365	1.119
J-4-1	445	2.03	2.35	0.55	190	1.477
J-5	-	-	-	-	-	-
J-5-1	52	2.50	2.75	0.47	19	2.029
J-6	5.9	3.01	1.62	0.57	3.63	2.447
J-6-1	2.97	3.12	1.09	0.68	2.71	2.44
J-7	9.34	2.85	0.917	0.74	10.14	2.11
J-7-1	2.90	3.22	0.113	1.29	25.82	1.93
J-8	3.64	3.10	0.067	1.43	54.6	1.67
J-8-1	1.01	3.38	0.075	1.38	13.42	2.00
J-9	0.466	3.59	0.183	1.13	2.56	2.46
J-9-1	0.675	3.51	-	-	0.339	2.99
J-10	0.995	3.43	0.311	0.993	3.198	2.437
J-10-1	0.220	3.71	0.048	1.46	4.56	2.25
J-11	0.121	3.73	0.339	0.94	0.357	2.79
J-11-1	0.143	3.74	0.181	1.103	0.790	2.64
J-12	3.67E-6	6.25	2.23E-3	2.091	1.64E-3	4.16
J-12-1	0.039	3.92	-	-	0.011	3.59
J-13	7.33E-3	4.37	0.022	1.599	0.336	2.77
J-13-1	0.011	4.29	0.019	1.64	0.588	2.65
J-14	4.34E-3	4.44	-	-	2.32E-4	4.54

J-14-1	8.69E-3	4.12	-	-	3.40E-4	4.26
J-15	3.34E-4	4.97	9.12E-4	2.315	0.368	2.66
J-15-1	2.97E-4	4.94	-	-	4.65E-5	4.76
J-16	4.52E-7	6.38	1.91E-3	2.05	2.36E-4	4.33
J-16-1	9.25E-5	5.11	-	-	1.16E-6	5.50
J-17	3.32E-10	7.91	4.93E-5	2.86	6.54E-6	5.06

Note: '-' denotes insignificant relationship

Table G.2: Coefficients of Sectional Properties for Year 1977

Section Number	$A = C_1 H^b$		$R = C_2 H^d$		$W = C_3 H^e$	
	C_1	b	C_2	d	C_3	e
J-1	7003	1.28	2.55	0.65	2748	0.626
J-1-1	4986	1.364	3.55	0.54	1399	0.827
J-2	1671	1.654	2.62	0.57	638	1.086
J-2-1	1544	1.617	5.39	0.37	285	1.252
J-3	7059	1.293	2.53	0.64	2785	0.648
J-3-1	3395	1.365	9.26	0.26	367	1.107
J-4	426	1.906	0.80	0.89	530	1.020
J-4-1	227	2.181	-	-	28.1	1.991
J-5	1236	1.744	-	-	127	1.549
J-5-1	30	2.642	1.16	0.70	25.8	1.943
J-6	11.7	2.808	-	-	0.498	2.960
J-6-1	74	2.330	3.95	0.39	18.53	1.942
J-7	6.63	2.932	1.45	0.61	4.53	2.326
J-7-1	2.34	3.274	0.20	1.14	11.87	2.132
J-8	3.44	3.115	0.107	1.31	32.17	1.808
J-8-1	5.58	2.945	0.182	1.17	30.61	1.777
J-9	0.66	3.489	0.452	0.89	1.45	2.598
J-9-1	0.31	3.70	-	-	0.11	3.265

J-10	0.24	3.77	0.144	1.17	1.64	2.60
J-10-1	0.50	3.47	0.470	0.88	1.06	2.60
J-11	1.18	3.25	0.076	1.37	15.59	1.882
J-11-1	0.06	3.95	0.05	1.41	1.09	2.562
J-12	0.01	4.40	0.098	1.21	0.10	3.183
J-12-1	0.20	3.53	-	-	0.059	3.17
J-13	0.04	3.94	0.36	0.913	0.115	3.03
J-13-1	6.25E-4	5.43	0.03	1.463	2.13E-3	3.97
J-14	1.6E-8	7.41	3.46E-4	2.50	4.59E-5	4.91
J-14-1	0.021	3.93	-	-	0.02	3.284
J-15	3.31E-5	5.46	4.79E-3	1.89	6.9E-3	3.571
J-15-1	3.26E-5	5.40	-	-	3.89E-6	5.31
J-16	5.23E-9	7.40	2.12E-3	2.00	2.46E-6	5.40
J-16-1	1.07E-5	5.62	0.11	1.121	9.84E-5	4.49
J-17	6.02E-7	6.25	2.26E-5	3.09	0.027	3.164

Note: '-' denotes insignificant relationship

Table G.3: Coefficients of Sectional Properties for Year 1978

Section Number	$A = C_1 H^b$		$R = C_2 H^d$		$W = C_3 H^e$	
	C_1	b	C_2	d	C_3	e
J-1	6024	1.335	1.302	0.844	4615	0.492
J-1-1	3997	1.436	1.541	0.773	2584	0.664
J-2	2810	1.543	1.615	0.729	1737	0.814
J-2-1	1757	1.606	3.291	0.510	572	1.070
J-3	1487	1.690	0.602	1.004	2466	0.687
J-3-1	979	1.678	4.881	0.396	200	1.281
J-4	306	1.984	0.712	0.909	430	1.075
J-4-1	44	2.597	3.192	0.410	13.48	2.192
J-5	1485	1.719	7.212	0.284	208	1.431

J-5-1	174	2.170	67.85	0.381	2.52	2.556
J-6	114	2.254	-	-	3.85	2.421
J-6-1	33.42	2.546	-	-	5.18	2.297
J-7	31.82	2.586	0.53	0.914	60.00	1.672
J-7-1	2.06	3.308	0.114	1.285	18.12	2.022
J-8	9.37	2.830	1.563	0.608	5.96	2.224
J-8-1	76.67	2.326	1.194	0.731	63.98	1.595
J-9	5.64	2.924	3.50	0.382	1.44	2.569
J-9-1	0.186	3.758	1.815	0.505	0.102	3.253
J-10	0.317	3.656	0.045	1.482	7.114	2.171
J-10-1	0.039	4.080	0.193	1.072	0.205	3.007
J-11	1.534	3.194	0.051	1.472	30.66	1.715
J-11-1	0.065	3.949	0.020	1.649	3.17	2.305
J-12	8.38E-3	4.454	0.029	1.527	0.291	2.929
J-12-1	0.124	3.634	-	-	0.013	3.530
J-13	1.32E-3	4.764	0.021	1.588	0.063	3.176
J-13-1	1.96E-4	5.218	5.15E-3	1.910	0.038	3.308
J-14	3.12E-4	5.069	0.108	1.150	2.91E-3	3.918
J-14-1	7.23E-3	4.182	0.489	0.813	0.012	3.408
J-15	7.68E-4	4.729	0.131	1.122	4.54E-3	3.623
J-15-1	1.09E-5	5.682	0.189	0.982	4.16E-5	4.774
J-16	2.01E-7	6.529	0.154	1.009	1.32E-6	5.517
J-16-1	5.7E-5	5.249	0.485	0.790	1.17E-4	4.459
J-17	4.93E-6	5.728	0.036	1.378	1.34E-4	4.352

Note: '-' denotes insignificant relationship

Table G.4: Coefficients of Sectional Properties for Year 1979

Section Number	$A = C_1 H^b$		$R = C_2 H^d$		$W = C_3 H^e$	
	C_1	b	C_2	d	C_3	e
J-1	1282	1.715	0.489	1.068	2620	0.647
J-1-1	2916	1.529	0.630	1.024	4631	0.505
J-2	2357	1.607	0.866	0.899	2380	0.737
J-2-1	1829	1.603	2.122	0.644	8603	0.958
J-3	645	1.888	0.633	0.959	1020	0.929
J-3-1	431	1.849	-	-	22.70	1.864
J-4	152	2.164	0.465	1.012	328	1.151
J-4-1	30	2.700	3.014	0.416	9.87	2.285
J-5	249	2.150	4.848	0.345	51.62	1.802
J-5-1	52	2.501	1.072	0.728	46.44	1.784
J-6	38	2.525	-	-	1.318	2.708
J-6-1	75	2.326	-	-	2.779	2.448
J-7	19	2.726	0.224	1.141	87.10	1.585
J-7-1	2.59	3.257	0.091	1.352	28.50	1.905
J-8	2.93	3.151	0.125	1.261	23.46	1.890
J-8-1	0.470	3.546	0.112	1.260	4.17	2.288
J-9	1.355	3.265	1.117	0.657	1.207	2.609
J-9-1	0.159	3.788	2.073	0.468	0.076	3.321
J-10	0.011	4.472	0.011	1.815	1.041	2.659
J-10-1	0.133	3.785	0.546	0.819	0.243	2.966
J-11	0.119	3.794	0.030	1.572	3.971	2.222
J-11-1	0.168	3.708	0.238	1.031	0.699	2.677
J-12	1.7E-3	4.819	2.80E-3	2.105	0.602	2.714
J-12-1	6.2E-3	4.368	0.908	0.656	6.85E-3	3.710
J-13	7.8E-6	6.000	2.02E-4	2.709	0.039	3.291
J-13-1	1.03E-3	4.783	0.268	0.938	3.94E-3	3.838
J-14	1.21E-4	5.358	7.60E-3	1.814	0.016	3.543

J-14-1	6.27E-3	4.196	0.925	0.655	6.80E-3	3.541
J-15	2.44E-4	4.995	0.014	1.648	0.017	3.346
J-15-1	9.26E-6	5.732	0.223	0.948	4.16E-5	4.783
J-16	2.02E-6	6.010	-	-	6.16E-7	5.698
J-16-1	7.02E-7	6.243	5.34E-3	1.808	1.31E-4	4.434
J-17	5.2E-7	6.216	-	-	8.30E-7	5.509

Note: '-' denotes insignificant relationship

Table G.5: Coefficients of Sectional Properties for Year 1980

Section Number	$A = C_1 H^b$		$R = C_2 H^d$		$W = C_3 H^e$	
	C_1	b	C_2	d	C_3	e
J-1	13333	1.127	5.15	0.478	2592	0.648
J-1-1	6778	1.314	1.31	0.841	5144	0.474
J-2	3777	1.470	3.34	0.529	1139	0.937
J-2-1	3790	1.400	9.99	0.222	377	1.180
J-3	3279	1.446	-	-	177	1.406
J-3-1	626	1.800	3.28	0.487	192	1.310
J-4	664	1.806	0.714	0.934	934	0.870
J-4-1	835	3.049	1.236	0.642	6.74	2.407
J-5	124	2.312	-	-	17.13	2.095
J-5-1	608	1.853	-	-	21.73	1.965
J-6	176	2.152	-	-	15.03	2.061
J-6-1	19024	2.677	0.768	0.805	25.01	1.872
J-7	18.76	2.718	1.504	0.628	12.35	2.093
J-7-1	6.00	3.020	0.574	0.865	10.76	2.147
J-8	171	2.143	2.154	0.583	79.38	1.559
J-8-1	-	-	-	-	-	-
J-9	2.85	3.149	0.195	1.135	14.82	2.010
J-9-1	0.955	3.332	3.160	0.395	0.302	2.937

J-10	0.919	3.396	0.133	1.219	6.92	2.177
J-10-1	0.264	3.681	0.023	1.652	11.36	2.027
J-11	1.313	3.218	0.153	1.187	8.56	2.031
J-11-1	0.064	3.975	0.023	1.617	2.78	2.358
J-12	0.053	3.985	0.032	1.530	1.675	2.454
J-12-1	0.040	3.931	0.553	0.802	0.073	3.128
J-13	6.42E-3	4.357	0.116	1.169	0.056	3.184
J-13-1	-	-	-	-	-	-
J-14	9.56E-10	8.116	9.93E-5	2.809	9.64E-6	5.307
J-14-1	4.02E-4	4.832	0.055	1.315	7.40E-3	3.515
J-15	2.12E-5	5.569	1.92E-3	2.107	0.011	3.462
J-15-1	1.69E-6	6.073	0.072	1.191	2.34E-5	4.883
J-16	2.63E-5	5.460	0.190	0.997	1.37E-4	4.464
J-16-1	7.73E-6	5.696	0.016	1.569	4.92E-4	4.127
J-17	4.19E-6	5.744	0.035	1.386	1.21E-4	4.359

Note: '-' denotes insignificant relationship

Table G.6: Coefficients of Sectional Properties for Year 1981

Section Number	$A = C_1 H^b$		$R = C_2 H^d$		$W = C_3 H^e$	
	C_1	b	C_2	d	C_3	e
J-1	5569	1.354	1.145	0.876	4871	0.477
J-1-1	6960	1.311	1.478	0.809	4708	0.502
J-2	1666	1.668	2.433	0.591	687	1.075
J-2-1	883	1.764	3.438	0.475	241	1.305
J-3	1170	1.713	1.939	0.648	603	1.065
J-3-1	1026	1.653	-	-	80	1.539
J-4	1596	1.582	2.514	0.609	622	0.978
J-4-1	110	2.395	-	-	9.16	2.331
J-5	24.20	2.742	1.822	0.561	13.316	2.161

J-5-1	708	1.820	-	-	21.200	1.976
J-6	637	1.887	-	-	18.234	2.032
J-6-1	46.34	2.458	1.578	0.628	29.370	1.830
J-7	410	2.016	0.579	0.962	707	1.055
J-7-1	7.575	2.963	0.289	1.051	26.23	1.912
J-8	16.71	2.686	4.560	0.336	3.650	2.350
J-8-1	67.76	2.347	1.573	0.652	43.82	1.688
J-9	2.729	3.148	0.238	1.079	11.40	2.070
J-9-1	1.271	3.263	3.124	0.402	0.406	2.861
J-10	0.682	3.463	0.142	1.192	4.787	2.272
J-10-1	3.10E-3	4.747	9.53E-3	1.832	0.328	2.914
J-11	1.257	3.219	0.208	1.107	6.05	2.112
J-11-1	0.120	3.779	0.462	0.859	0.256	2.922
J-12	0.017	4.213	0.336	0.916	0.050	3.297
J-12-1	0.005	4.448	0.082	1.263	0.062	3.182
J-13	2.36E-3	4.580	0.316	0.908	0.007	3.672
J-13-1	1.66E-4	5.225	0.125	1.108	1.33E-3	4.117
J-14	7.91E-6	5.959	5.70E-3	1.854	1.38E-3	4.106
J-14-1	6.80E-4	4.710	0.370	0.860	1.84E-3	3.849
J-15	1.97E-5	5.581	0.015	1.621	2.22E-3	3.960
J-15-1	2.10E-5	5.498	0.968	0.594	2.18E-5	4.902
J-16	1.07E-4	5.128	0.610	0.731	1.75E-4	4.398
J-16-1	1.07E-6	6.110	0.588	0.713	1.80E-6	5.400
J-17	7.30E-7	6.166	4.04E-4	2.410	1.80E-3	3.756

Note: '-' denotes insignificant relationship

Table G.7: Time Averaged Coefficients of Sectional Properties

Section Number	$A = C_1 H^b$		$R = C_2 H^d$		$W = C_3 H^e$	
	C_1	b	C_2	d	C_3	e
J-1	6624	1.349	2.633	0.726	3119	0.622
J-1-1	5207	1.381	1.968	0.758	3359	0.624
J-2	2695	1.569	2.069	0.678	1516	0.894
J-2-1	1895	1.600	3.674	0.488	1743	1.156
J-3	2322	1.685	1.235	0.854	1279	0.966
J-3-1	1182	1.691	4.579	0.503	266	1.340
J-4	545	1.943	0.924	0.907	535	1.036
J-4-1	282	2.492	2.448	0.505	42.89	2.114
J-5	624	2.130	4.627	0.397	23.41	1.808
J-5-1	271	2.248	1.661	0.633	22.78	2.042
J-6	164	2.439	1.620	0.566	7.09	2.438
J-6-1	41.82	2.576	1.847	0.625	13.93	2.138
J-7	82.59	2.638	0.867	0.832	46.90	1.807
J-7-1	3.91	3.174	0.230	1.164	20.21	2.008
J-8	34.50	2.838	1.429	0.921	33.20	1.917
J-8-1	30.30	2.910	0.627	1.039	31.20	1.870
J-9	2.28	3.260	0.948	0.879	5.48	2.386
J-9-1	0.593	3.559	2.543	0.443	0.223	3.105
J-10	0.527	3.698	0.131	1.312	4.12	2.386
J-10-1	0.193	3.912	0.215	1.286	2.96	2.627
J-11	0.921	3.400	0.143	1.275	10.86	2.125
J-11-1	0.103	3.850	0.162	1.278	1.46	2.577
J-12	0.015	4.687	0.083	1.563	0.453	3.123
J-12-1	0.069	3.971	0.511	0.907	0.037	3.385
J-13	9.57E-3	9.668	0.139	1.481	0.103	3.187
J-13-1	2.49E-3	4.990	0.089	1.412	0.127	3.577
J-14	7.97E-4	6.059	0.024	2.205	0.0034	4.387

J-14-1	7.38E-3	4.328	0.576	0.856	0.0081	3.643
J-15	2.37E-4	5.217	0.028	1.784	0.0684	3.437
J-15-1	6.21E-5	5.554	0.363	0.929	2.98E-5	4.902
J-16	2.26E-5	6.151	0.192	1.357	9.21E-5	4.968
J-16-1	2.83E-5	5.671	0.241	1.206	1.40E-4	4.735
J-17	1.83E-6	6.336	0.014	2.224	0.0048	4.367

Note: '-' denotes insignificant relationship

Appendix H

DERIVATION OF AVERAGE DEPTH RELATIONSHIP

The average depth and width of the branch channels of a braided river are used to obtain a relationship similar to the width-depth relationship of a single channel river. One of the difficulties that arises with this attempt is inherent in the definition of the average depth. It is possible to have more than one value of average depth for the same width of the branch channels of a braided river because of their constantly changing characteristics. This can be demonstrated even for a very simplified situation as shown in the following.

Let us take a triangular section divided into two parts by a vertical line located at a distance of mW from the left as shown in Fig. H.1 where, W is the width of the total section, and m is a proportion between 0 and 1. The shaded area of the triangle is one example division of the section, and is referred to as the branch. The average depth of the branch will vary depending on the position of the dividing line. It will also vary depending on the location of the deepest point of the original section. This is true even for the same area of the original cross section. Let the deepest point with height H be located at a distance of kW from left where, k is a proportion between 0 and 1. Depending on the position of the dividing line, there are two possible ways to divide the section. These are: (a) when m lies between 0 and k, and (b) when m lies between k and 1. These are discussed separately below.

Note that quantities that are compared are kept dimensionless in order to make the discussion more general. The depths are converted to depth ratios, the widths to width ratios, and the cross-sectional areas to area ratios. These ratios are obtained by dividing the branch channel dimensions by the original channel dimensions (the average depth and the top width of the total section are constants for a given section).

Case 1: $0 \leq m \leq k$

The definition sketch for $m < k$ is given in Fig. H.1. The altitude, h of the shaded area can be found by comparing similar triangles. This can be written as:

$$\frac{H}{h} = \frac{kW}{mW} \qquad i.e., \quad h = \frac{mH}{k} \tag{H.1}$$

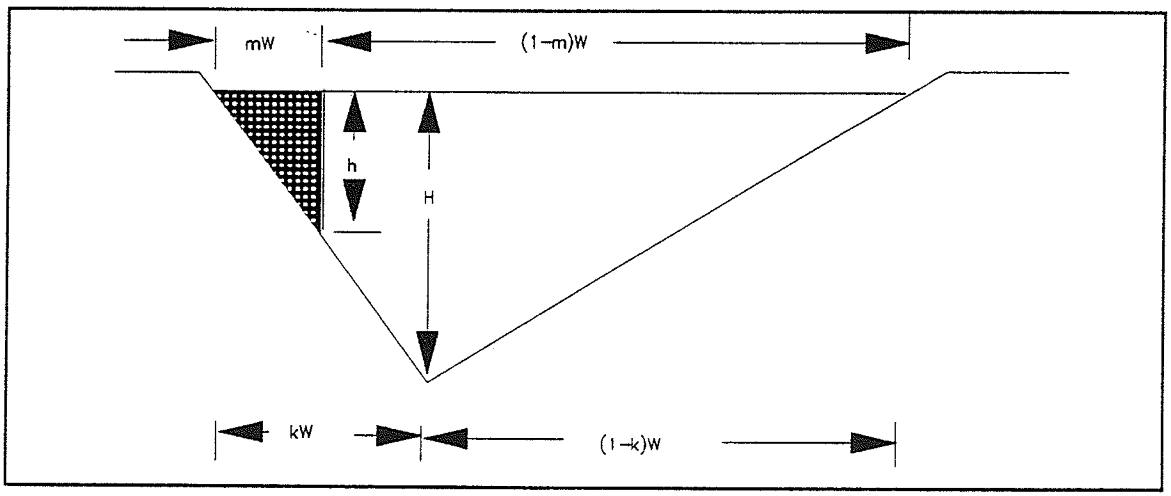

Fig. H.1: Definition sketch for $0 \leq m \leq k$

The area A of the shaded branch can then be written as:

$$A = \frac{1}{2}(mW)h = \frac{1}{2k}(m^2 WH) \tag{H.2}$$

The average depth, d, of the shaded branch is:

$$d = \frac{A}{mW} = \frac{1}{2k}(mH), \qquad for\ 0 \le m \le k \tag{H.3}$$

The average depth of the total section is given by:

$$d_o = \frac{0.5WH}{W} = \frac{H}{2} \tag{H.4}$$

The ratio of the average depth of the branch to the average depth of the total section is obtained by dividing Eqn. (H.3) by Eqn. (H.4) as:

$$\frac{d}{d_o} = \frac{m}{k}, \qquad for\ \ 0 \le m \le k \tag{H.5}$$

Case 2: k ≤ m ≤ 1

The definition sketch for this case is shown in Fig. H.2 in which the shaded area is the branch to be considered. The altitude h can be found from:

$$\frac{h}{H} = \frac{(1-m)W}{(1-k)W} \qquad i.e., \ \ h = \left(\frac{1-m}{1-k}\right)H \tag{H.6}$$

The trapezoidal area A then becomes:

$$A = \frac{1}{2}kWH + \frac{1}{2}H(mW-kW) + \frac{1}{2}h(mW-kW) \tag{H.7}$$

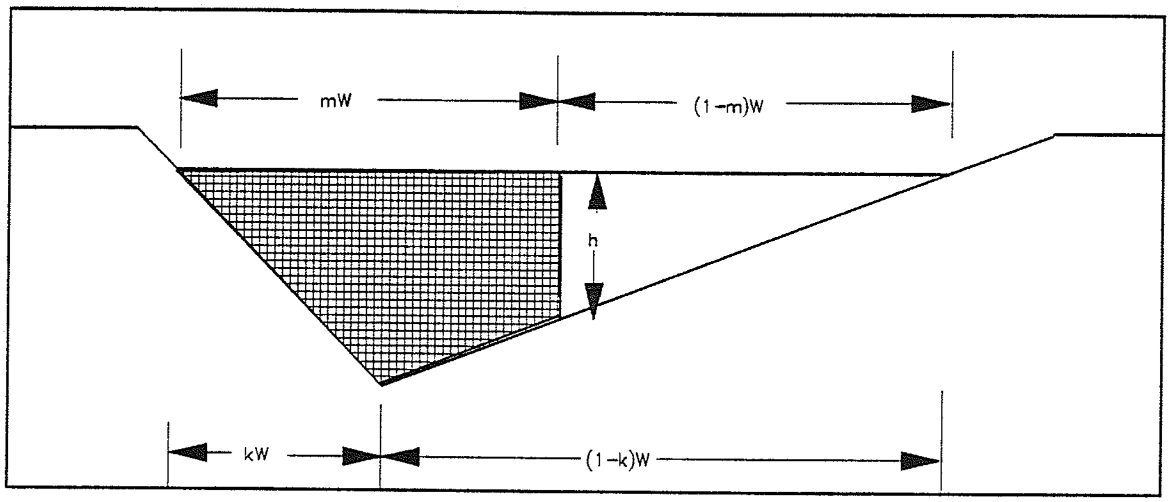

Fig. H.2: Definition sketch for $k \leq m \leq 1$

Substitution of the value of H in Eqn. (H.7) yields:

$$A = \frac{1}{2}WH\left[k + (m-k) + (m-k)\frac{1-m}{1-k}\right] \tag{H.8}$$

which can be reduced to:

$$A = \frac{WH}{2(1-k)}\left[2m - m^2 - k\right] \tag{H.9}$$

The average depth d is obtained by dividing the area in Eqn. (H.9) by the width mW as:

$$d = \frac{H}{2m(1-k)}\left[2m - m^2 - k\right] \tag{H.10}$$

The ratio (d/d_o) is obtained by dividing Eqn. (H.10) by d_o $(= H/2)$ as follows:

$$\frac{d}{d_o} = \frac{2m - m^2 - k}{m(1-k)} \tag{H.11}$$

which can be rearranged to write:

$$m^2 - \left[2 - (1-k)\frac{d}{d_o}\right]m + k = 0 \qquad \text{(H.12)}$$

Eqn. (H.12) is quadratic in m, two values of which are given by:

$$m = \left[1 - (1-k)\frac{d}{2d_o}\right] \pm \left[\left(1 - (1-k)\frac{d}{2d_o}\right)^2 - k\right]^{0.5} \qquad \text{(H.13)}$$

Eqn. (H.13) shows that there are two possible widths of the branch for the same average depth for a given value of k in the range $k \leq m \leq 1$.

From Eqn. (H.3) and (H.10) it is evident that the value of the average depth d depends on the values of k for given values of m and H implying multiple values of d. Eqn. (H.5) implies a one-to-one correspondence between d/d_o and m for the same value of k in the specified interval with the upper limit of $m = k$. On the other hand, Eqn. (H.11) is quadratic in m which suggests that for the same value of d there are two values of m for the same k. These are given by Eqn. (H.13).

The solution of Eqn. (H.5) and (H.11) are shown as plots of the ratio of the average depth of a branch to the average depth of the total section (d/d_o) versus the top width ratio of the branch to the total section (m) in Fig. H.3. Points on a single line represent these ratios for different positions of the vertical dividing line for the same deepest point of the whole section. Different lines on the plot correspond to different positions of deepest point. This figure shows that for the same average depth ratio, multiple values of width ratios are possible to occur for different locations of the deepest point. Even, it is possible to have two widths for the same average depth in all cases (and within the limit of $k \leq m \leq 1$) except when the deepest point is at the end of the section (i.e., on the line for which $k=1$ and $k \geq m$) in which case the values vary linearly. Also note that for a different position of the deepest point, in this example, the height of the triangular section is kept the same in order to have the same area.

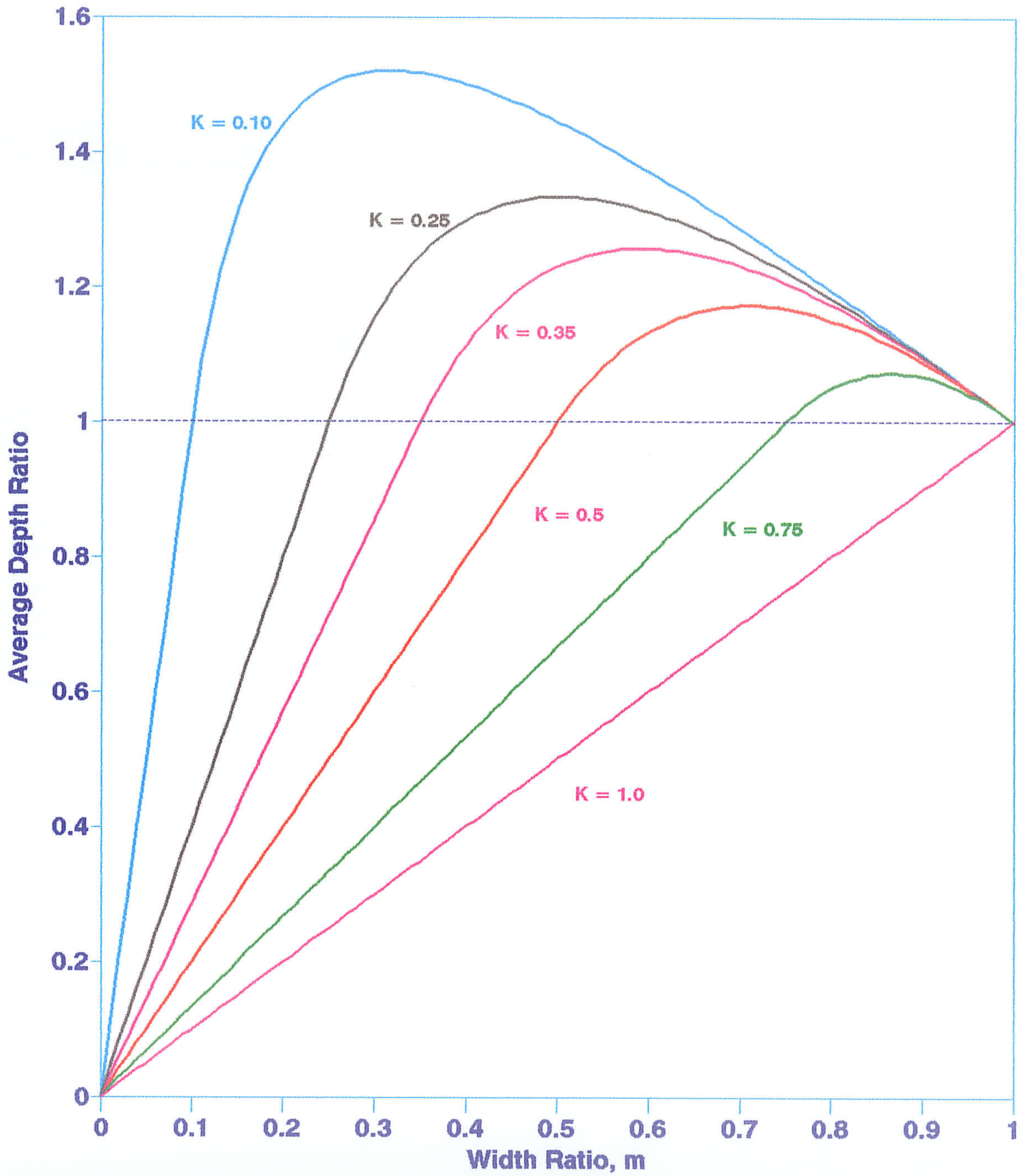

Fig. H.3: Multiple Width for the Same Average Depth in an Example Section

The channel side slopes changes frequently in a changing environment of a braided river. The vertical separating line is not an exact representation of the branch channels in a braided river. However, it is similar. Since the branch channels do change frequently, some of them may not have sufficient time to establish a shape smoother than an almost triangular one. In real situations, the curves of Fig. H.3 will be flatter, but there will be channels for which observing multiple widths for the same depth would still be possible. In other words, the width-depth relations of the form of Eqn. (5.11) will have too much scatter around it, and may not be statistically significant at all. The width of the branch channel for the same average depth, in the preceding example, is dependent on the relative area of the branch. For the same value of k and same average depth, one width (smaller) occurs for a deep and narrow branch (smaller relative area) and another width (larger) occurs for a shallow and wider branch (larger relative area).

Appendix I

DERIVATION OF MOVEMENT RELATIONSHIP

I.1 Widening Channel

Let us consider a section with width W and an average depth d. Let the equilibrium width of the section be W_e and the equilibrium depth be d_e (see Fig. I.1). The figure shows a situation in which the equilibrium channel is wider and shallower than the original channel.

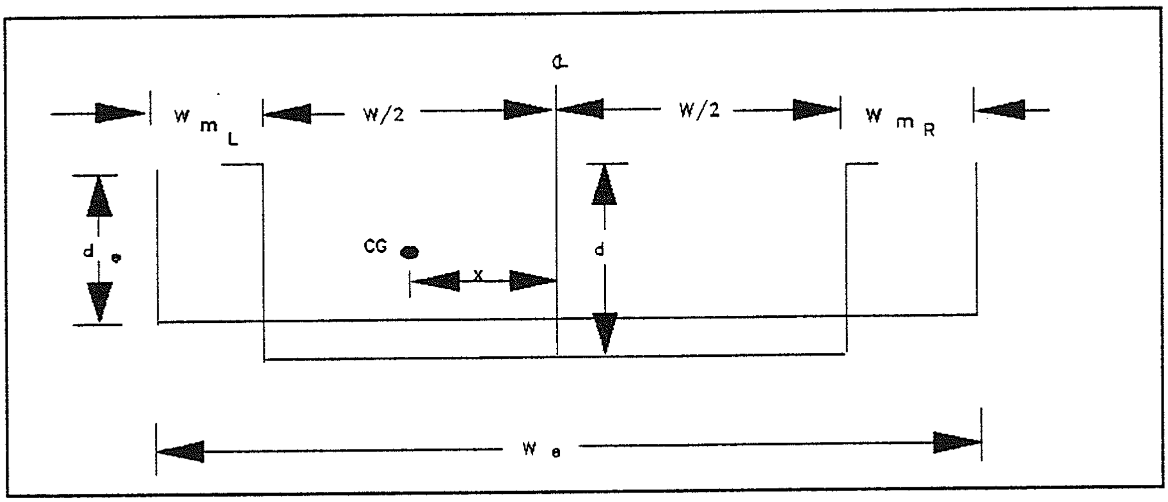

Fig. I.1: Definition Sketch of Existing and Equilibrium Dimensions

Let x be the distance of the Centre of Gravity (CG) from the centreline of the original section. The movements of the banks are assumed to be linear with the distance of the bank from the CG i.e., the closer is the bank to the CG, the higher is the movement. The movement of the right bank corresponding to different positions of the CG is graphically shown in Fig. I.2.

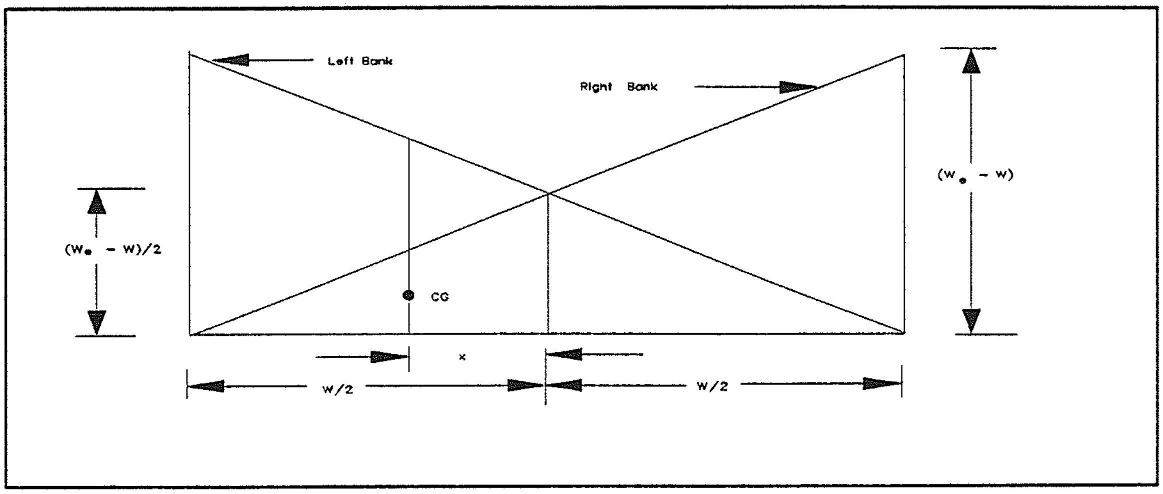

Fig. I.2: Distribution of Erosion for the Banks

The slope of the change line is given by:

$$Slope = \frac{W_e - W}{W} \qquad (I.1)$$

For a position of CG at any distance x from the mid-channel, the movement of the right bank is given by:

$$[W_m]_R = \frac{W_e - W}{W}\left(\frac{W}{2} - x\right) = \frac{(W_e - W)(W - 2x)}{2W} \qquad (I.2)$$

Note that the sign of $(W_e\text{-}W)$ determines whether the movement is erosion or deposition. Also, note that the summation of movements of the left and the right banks at any location is equal to $(W_e\text{-}W)$. Therefore,

$$[W_m]_L + [W_m]_R = W_e - W \qquad (I.3)$$

Substituting the value of $[W_m]_R$ and simplifying,

$$[W_m]_L = \frac{1}{2W}[WW_e - W^2 + 2xW_e - 2xW] \qquad (I.4)$$

which can be re-written as:

$$[W_m]_L = \frac{(W_e - W)(W + 2x)}{2W} \tag{I.5}$$

I.2 Narrowing Channel

Let the equilibrium channel width W_e be narrower than the original width W, and the equilibrium depth d_e be deeper than the original depth d. The sections are shown in Fig. I.3 where the original channel is wider and shallower than the equilibrium channel.

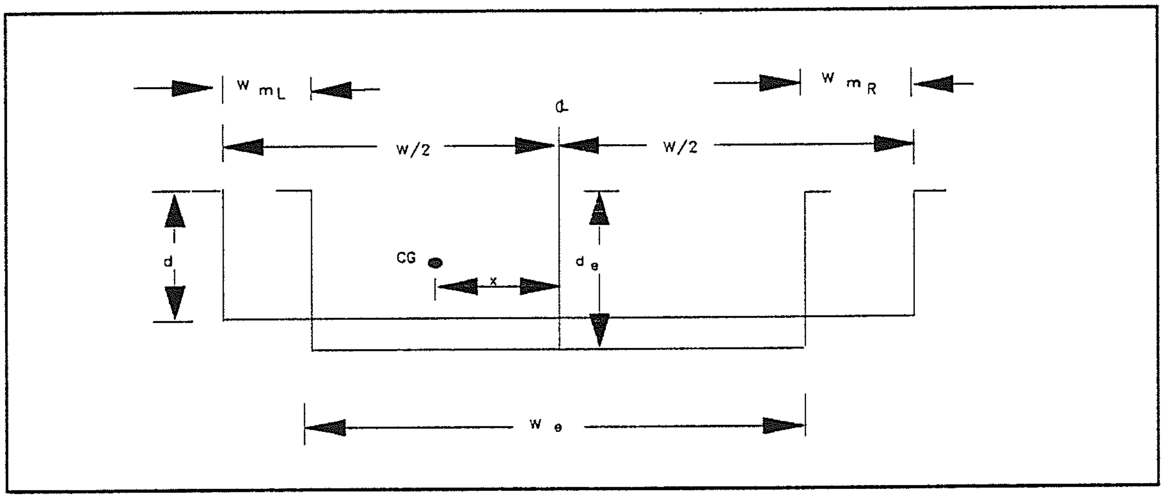

Fig. I.3: Definition Sketch of a Narrowing Channel

In this case, it is assumed that the bank closer to the centre of gravity will experience less deposition than the bank further away from it. The movement of the banks are graphically given in Fig. I.4. Following the definitions of the above, the deposition of the left bank is given by:

$$[W_m]_L = \frac{(W_e - W)(W - 2x)}{2W} \tag{I.6}$$

The deposition experienced by the right bank is then given by:

$$[W_m]_R = \frac{(W_e - W)(W + 2x)}{2W} \tag{I.7}$$

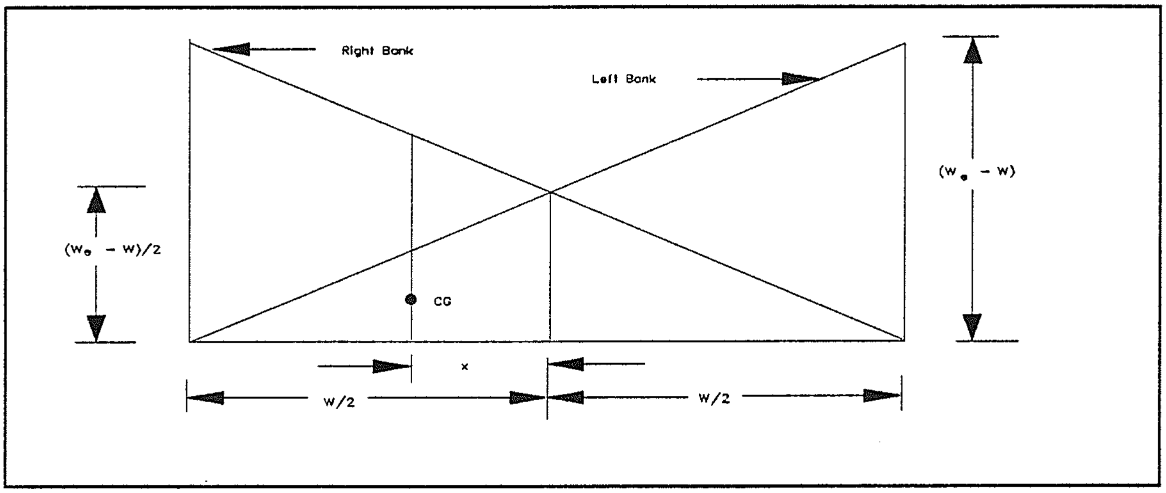

Fig. I.4: The Definition Sketch for Variation of Deposition

Appendix J

BANK MOVEMENT AND WIDTH-DEPTH RELATIONSHIP

J.1 Introduction

An attempt was made in this study to quantitatively predict the bank movement by obtaining a relationship between bank movement and the deviations of the branch channels from equilibrium, and the geometric conditions of the branches only. The central argument of this attempt was that the changes in the individual branch channels of a braided river depends on the changes that occur in other branches of the section in addition to the magnitude and distribution of the incoming water and sediment load. Movement of the outer banks is of the greatest interest. Since the movement of the outer banks is the reflection of what is happening in the entire cross-section of the river, the deviation of widths of individual channels from their equilibrium widths are an important factor in determining this movement. The weights of these deviations by which they influence the outer bank displacement are thought to be dependent on a number of factors describing the relative condition of the branches. These are: the relative size, relative position, and relative shape of the branch channels and the distribution of the incoming flux across the section. The definitions of these variables are given in the following.

J.2 Deviation from Equilibrium Width

From the observed width-depth relationship (Eqn. 5.17), the equilibrium widths of all the channels in a cross-section were computed for the average and bankfull discharge of the section. For the n-th branch, the deviation of width from the equilibrium width is computed as:

$$\Delta W_n = (W_e - W)_n \tag{J.1}$$

where,

ΔW_n = deviation from the equilibrium width of channel n,

W_e = the equilibrium width determined from Eqn. (5.17),

W = the observed width, and

n = *1, 2, 3* etc. = channel number of the branch channel, the left-most branch channel being channel number 1.

A positive value of ΔW signifies a tendency for channel widening while a negative value signifies a tendency of channel shallowing. It is assumed that ΔW_n will have a non-linear relationship with the bank displacement and the exponent of ΔW will be same for each branch channel.

J.3 Relative Size

The relative size of branch channels is assumed to have an influence on the magnitude of outer bank displacement. Larger channels carry major portion of water and sediment and, therefore, the deviation from equilibrium position of such channels will have more influence on the outer bank displacement. The ratio of the area of a branch channel to the total area of the cross-section can be a measure of relative size since the surface slopes and Manning's roughness coefficient can be practically considered to be constant for all branches of a cross-section. The relative size of a branch channel is defined as:

$$(A_r)_n = \frac{A_n}{A_T} \tag{J.2}$$

where,

A_r = the relative size of channel n,

A_n = the area of the branch channel n, and

A_T = total area of the section.

J.4 Relative Position

Even if the relative size of a branch channel is larger, its deviation from equilibrium may not have a greater influence on the outer bank displacement if its relative position is far from the outer banks. In other words, channels closer to the outer banks will have more influence in the magnitude of the movement than the other channels. The ratio of the distance of a branch channel to the total width of the section can be a measure of the relative position of the branch and can be written as:

$$(X_r)_n = \frac{x_n}{W_T} \tag{J.3}$$

where,

X_r = the relative position of branch channel n,

x_n = the distance of channel n from the bank, and

W_T = the total width at the cross-section.

J.5 Relative Shape

In this study, computations are done with the assumption that the channels are rectangular with the given width and an average depth. Larger deviation of the branch channels from this assumption will introduce larger errors which should be accounted for. The ratio of the maximum depth to the average depth of the branch can be a measure of the relative shape and can be written as:

$$[S_r]_n = \left[\frac{d_{max}}{d}\right]_n \tag{J.4}$$

where,

S_r = the relative shape of channel n,

d_{max} = maximum depth of the branch, and

d = average depth of the branch.

Note that for a rectangular section, $S_r = 1$; for a triangular section, $S_r = 2$; and for a semicircular section, $S_r = 4/\pi = 1.27$. The higher the relative shape factor the further is the deviation from the rectangular assumption.

J.6 Channel Radius of Curvature

The distribution of water and sediment coming into the section will not only depend on the relative size of the branch but also on the curvature the branch may have at the point of distribution. The radius of curvature (or its rate of change) may be a measure of the distribution of incoming water and sediment. For measurement, the radius of curvature has been taken as the radius of an average circle, and is computed as:

$$R = \frac{h}{2} + \frac{a^2}{8h} \tag{J.5}$$

where,

R = radius of curvature,

a = the length of the chord joining the end points of the curve, and

h = the distance of the crown of the curve from the chord.

Note that the radius of curvature R was calculated from measured h and a.

J.7 Proposed Relationship

It seems logical, from the above discussion, that a combination of the above variables together with the predicted deviations from Eqn. (5.17) should partially explain the bank movement of the Brahmaputra river. The following functional relationship is proposed:

$$\Delta Y = K + \sum_{n=1}^{NC} f(\Delta W_n, A_{r_n}, X_{r_n}, S_{r_n}, R_n) \tag{J.6}$$

where,

ΔY = the change in bankline position at a point of cross-section measurement in the following year $(T+1)$,

K = a constant,

NC = the total number of branches at the section, and

f is a function to be determined from the observed data.

The other terms are for the current year, T, and are defined before. For example, if the effect of deviation in width and relative size of three branch channels are significant, the relationship may be of the form:

$$\Delta Y = K + C_1 \Delta W_1^p A_{r_1} + C_2 \Delta W_2^p A_{r_2} + C_3 \Delta W_3^p A_{r_3} \tag{J.7}$$

where, K, C and p are constants determined from data, and subscripts denote the branch channel numbers.

The data for these relationships were obtained from the observed cross-sections at 33 points along the river for the period 1976-1983. The observed movement of the left and the right bank were obtained by comparing the records of the bank position with that of the following year. For each branch channel of each cross-section of each year of record, the other variables were computed. Eqn. (5.17) was used to compute ΔW values. The resulting data formed the basis of this exercise.

The objective of this attempt is to explore the relationship between the observed bank movements and other variables that are easy to obtain and need less expertise to extract. A great number of relationships combining the variables given in Eqn. (J.6) were explored. Both linear and non-linear (with exponents on different variables) relationships were tried for each section, for all sections together, for the left bank and for the right bank together and separately. All of these computations were repeated for the average discharge (20,000 m^3/sec), for the bankfull discharge (40,000 m^3/sec), and for a discharge corresponding to the danger level at Bahadurabad (50,000 m^3/sec). Also, the whole set of relations were tried separately for the six different reaches (see definitions of reach in Section 4.2.3). Marquardt method was used for the computation of non-linear relationship. A substantial amount of computer time (5 hours on the average on an IBM 386 machine for each relation of different reaches) was needed for convergence.

The relationships of the reaches computed separately were better compared to those for the whole reach. Also, the left and the right banks produced different relationships and were better when computed separately. The linear relationships of different reaches for the left bank are given below:

$$Reach\ 1:\ \Delta Y = 317-0.47\Delta W_2+0.42\Delta W_4+0.09\Delta W_5 \quad [R^2=0.74] \tag{J.8}$$

$$Reach\ 2:\ \Delta W = 4937-3.26\Delta W_1+7.15\Delta W_2 \quad [R^2=0.75] \tag{J.9}$$

$$Reach\ 3:\ \Delta Y = -452-0.50\Delta W_1+0.31\Delta W_2 \quad [R^2=0.08] \tag{J.10}$$

$$Reach\ 4:\ \Delta Y = -5740-0.71\Delta W_3-0.69\Delta W_4 \quad [R^2=0.36] \tag{J.11}$$

$$Reach\ 5:\ \Delta Y = 628-0.84\Delta W_3+0.54\Delta W_4+0.92\Delta W_5 \quad [R^2=0.77] \tag{J.12}$$

$$Reach\ 6:\ \Delta Y = 2309-3.9\Delta W_1-5.7\Delta W_2+1.6\Delta W_4+2.8\Delta W_5 \quad [R^2=0.26] \tag{J.13}$$

where,

ΔY = the change in bankline position at a point of cross-section measurement in the following year,

ΔW_n = the deviation from the equilibrium width of channel n, and

n = channel number, the left-most channel being channel number 1.

In all of the above equations, the displacements are expressed in FPS units. Note that a negative value of ΔY means erosion and a positive value means deposition in the left bank.

Comparison of Eqns. (J.8) to (J.13) shows that the equations are different not only in the value of the coefficients but also in the variables. For example, channel number 1 and 2 have major influence on the bank movement of reach 2 [Eqn. (J.9)] whereas, channel number 3 and 4 are important for reach 4 [Eqn. (J.11)]. No two reaches have the same channels as important (note that Eqn. (J.10) is statistically insignificant). It is not possible to generalize these equations with only differing the values of the constants for different reaches. This may mean that due to local conditions, different channels are important for different reaches. Any changes in these conditions may altogether change the equations in future. Therefore, it may not be a good idea to use these equations even though the values of R^2 are not discouraging (except for one). Also, the equations for the right bank are different than those for the left bank. The idea of developing simple generalized relations using variables that are easy to obtain is, thus, not a very attractive one.

Other non-linear relations that were tried did not improve this situation very much. Variables that were found to be important in these non-linear relations were still different for different reaches. Also, the improvement in R^2 value over the linear relations is not so much as to justify the use of these complicated non-linear equations.

Appendix K

DERIVATION OF AN ENTROPY FUNCTION

K.1 General Systems

Let us assume that a statistical population adopted an option out of n options and thus dispersed its choices over to a set of n cells. Let the proportion of the population in these cells are denoted by x_i which represent the circumstances of selecting various options and reflect the freedom of the population. Thus, x_1, ..., x_n are individual shares and represent the freedom of choosing the option. These shares are all positive and they (or the normalized values) add up to unity, i.e.,

$$\sum_{i=1}^{n} x_i = 1 \tag{K.1}$$

The axioms to develop an entropy measure for such a situation can be divided into two groups: (a) the necessary axioms, and (b) the desirable axioms. The first group of axioms relates to the system and are essential. The second group depend solely on mathematical issues regarding the possible use of the measure. The two groups of axioms are given below.

K.1.1 Necessary Axioms

The measure, H, must have the necessary properties given below.

1. It should be a function of x_1, x_2, ..., x_n of n choices such that these choices are represented by fractions.
2. H should be zero if and only if there is no freedom of choosing any other option but one. In other words, $H=0$ if all the x_i's are zero except one.
3. It should have its maximum value when all the x_i's are equal for a given number of options.
4. The maximum value of H should monotonically increase (or at least should not decrease) with n.
5. H should be a continuous function of x_i's.

6. H should be a symmetric function of x_i's.

7. The function H should not change when an additional option with no freedom is allowed.

K.1.2 Desirable Axioms Since the measure will be used for design which may involve optimization, the following properties of the function are desirable.

1. The function H should be a differentiable function of x_i's. This is needed if the function is to be maximized.

2. It should be a concave, pseudo-concave or quasi-concave function of x_i's. This property is useful when the function is to be maximized subject to constraints (which are usually linear). This property ensures that the local maximum is also the global maximum.

3. The optimizing values of x_i's, which are usually obtained by Lagrange's method, should automatically satisfy non-negativity constraints. This is desirable since we have the constraints $x_i \geq 0$, and in general, Lagrange's method of constrained optimization is easier to apply.

4. The measure should increase if x_i's are brought closer to each other. In a more generalized sense, H should increase if any averaging operation on the x'_j's is done such that

$$x_i' = \sum_j a_{ij} x_j \tag{K.2}$$

where,

$$\sum_i a_{ij} = \sum_j a_{ij} = 1, \qquad all \quad a_{ij} \geq 0 \tag{K.3}$$

If this transformation is simply a permutation of x_i's, the value of H should remain unchanged.

5. The measure may contain a parameter to allow us to consider the factors that may not otherwise be taken into account.

K.2 Goal-Oriented System

For a goal-oriented system, the dispersion of its choices over to a set of n cells is more or less relevant depending upon the goal to be reached. In other words, the choices have different utilities for the system. The utility of an option is independent of its proportion. For instance, an option with a greater proportion can have a utility equal to zero with respect to the goal of the system.

Let us denote the importance of the option with respect to the goal of the system by attaching a non-negative number $u_i \geq 0$, called the utility of the option. An option irrelevant to the goal of the system will have an utility equal to zero. Different options of same magnitude may have different utilities with respect to the goal of the system.

The occurrence of a single option of proportion x and utility u brings about an information H which is a function of both variables, $H = H(u, x)$. First, if we consider the options to be indistinguishable in terms of their utility, i.e., if we consider same utility for all, the information supplied by two independent options of the system must be equal to the sum of the information supplied by each option (*Axiom 4 of Section 2.9*). In other words,

$$H(u, x_1 x_2) = H(u, x_1) + H(u, x_2) \tag{K.4}$$

for every $u \geq 0$, $0 \leq x_1 \leq 1$, and $0 \leq x_2 \leq 1$.

Secondly, the information supplied by an option having a proportion of x must be directly proportional to the utility of that option. In other words, the supplied information should equally increase with the increase of the utility. This can be written as:

$$H(\rho u, x) = \rho H(u, x) \tag{K.5}$$

for every utility $u \geq 0$, $0 \leq x \leq 1$, and non-negative number ρ.

A function that satisfies conditions given by Eqn. (K.4) and (K.5) can be written as:

$$H(u,x) = -ku\log x \tag{K.6}$$

where, k is an arbitrary constant. (For proof, see Belis & Gisau, 1968). For the complete set of options of the system, we can write:

$$H = -k\sum_{i=1}^{n} u_i x_i \log x_i \tag{K.7}$$

This function obeys all the necessary and desirable axioms of Section K.1.1 and K.1.2 as well.

If all the options have the same utility (all u_i's are unity), then Eqn. (K.7) can be written as:

$$H = -k\sum_{i=1}^{n} x_i \log x_i \tag{K.8}$$

which is exactly Shannon's entropy.

K.3 Distribution of Branch Channel Equilibrium Discharge

Returning to the problem of redistributing equilibrium discharges among the branch channels, let us assume that the corrected discharge in the channels follow:

$$\sum_{i=1}^{n} \lambda_i \Delta Q_i = 0 \tag{K.9}$$

where, ΔQ_i are the difference between the equilibrium discharge and the existing discharge of the branch channel i, and λ_i are the constant of proportionality which follow

$$\sum_{i=1}^{n} \lambda_i \leq 1 \tag{K.10}$$

Let us add a hypothetical branch channel in the cross-section which carries the rest of the discharge to be adjusted such that the positive coefficient of proportionality λ_h, after normalization, is equal to $(1 - \Sigma\ \lambda_i)$. We now have a system for which Eqn. (K.1) is obeyed, i.e., the proportions add up to unity.

The process of redistribution of the excess amount of discharge is controlled by the adjustment allowed by the system over all the branch channels. As far as the system is concerned, the process of redistribution is complete for the moment, since the equilibrium discharges in branch channels are predicted amounts and are *not* the amount dictated by the system. What happens to this hypothetical channel is not at all important to the system. Thus, the system can now be viewed as a goal-oriented system described in Section K.1.3 where the distribution of the discharge among the branch channels and the hypothetical channel will have different utilities. The utility of the hypothetical channel is *zero*. The entropy of the system can now be expressed as:

$$H = -k\sum_{i=1}^{n} u_i \lambda_i \log \lambda_i - k u_h \lambda_h \log \lambda_h \tag{K.11}$$

for which

$$\sum_{i=1}^{n+1} \lambda_i = 1 \tag{K.12}$$

Putting $u_h = 0$ in Eqn. (K.11) one can write:

$$H = -k\sum_{i=1}^{n} u_i \lambda_i \log \lambda_i \tag{K.13}$$

Now, the utilities of the branch channels are equal and can be assumed to be unity. This reduces Eqn. (K.13) to Shannon's expression of entropy.

The dimensions that the branches are most likely to assume would then be considered to follow the maximum entropy function provided that the maximization process only include the coefficients of the branch channels. The problem, thus, is reduced to a maximization problem as given in the following set of equations:

$$Max \;\; H = \sum_{i=1}^{i=n} -\lambda_i \; ln(\lambda_i) \tag{K.14}$$

$$s.t.\ \sum_{i=1}^{i=n} \lambda_i \le 1 \tag{K.15}$$

$$\sum_{i=1}^{i=n} \lambda_i \Delta Q_i = 0 \tag{K.16}$$

$$\lambda_i \ge 0 \tag{K.17}$$

Appendix L

ENTROPY PRINCIPLE IN A THREE CHANNEL SECTION

The entropy principle is applied to a section with three branch channels. In the existing condition, the discharge in branches 1, 2, and 3 are Q_1, Q_2 and Q_3 respectively. The summation of these discharges is equal to the total discharge Q_T of the section, i.e., $Q_T = Q_1 + Q_2 + Q_3$. The discharges of the branches corresponding to the equilibrium condition are Qe_1, Qe_2 and Qe_3. Note that $Qe_1 + Qe_2 + Qe_3 \neq Q_T$. The corresponding deviations from the discharge $\Delta Q_1 = (Qe_1 - Q_1)$, $\Delta Q_2 = (Qe_2 - Q_2)$ and $\Delta Q_3 = (Qe_3 - Q_3)$. While the river adjusts its dimensions so as to attain these changes, the actual changes will be different depending on the existing conditions and the time involved in the changes. It is assumed that the actual changes achieved by the branches in one year may be written as $\lambda_1 \Delta Q_1$, $\lambda_2 \Delta Q_2$ and $\lambda_3 \Delta Q_3$ respectively. The following constraints hold:

$$\lambda_1 \Delta Q_1 + \lambda_2 \Delta Q_2 + \lambda_3 \Delta Q_3 = 0$$

where:

$$\lambda_1 \geq 0; \ \lambda_2 \geq 0; \ \lambda_3 \geq 0$$

Therefore, the problem reduces to:

$$\begin{array}{ll} Max. & -\lambda_1 ln \lambda_1 - \lambda_2 ln \lambda_2 - \lambda_3 ln \lambda_3 = f(\bar{\lambda}) \\ s.t. & \lambda_1 \Delta Q_1 + \lambda_2 \Delta Q_2 + \lambda_2 \Delta Q_3 = 0 \\ & \lambda_1 + \lambda_2 + \lambda_3 = 1 \\ & \lambda_1 > 0 \\ & \lambda_2 > 0 \\ & \lambda_3 > 0 \end{array}$$

Using the classical optimization approach, the Lagrangian is formed and Kuhn-Tucker conditions are used. The Lagrangian is:

$$\phi(\bar{\lambda}, \bar{\mu}) = -\lambda_1 ln \lambda_1 - \lambda_2 ln \lambda_2 - \lambda_3 ln \lambda_3 - \mu_1(\lambda_1 \Delta Q_1 + \lambda_2 \Delta Q_2 + \lambda_3 \Delta Q_3) - \mu_2(\lambda_1 + \lambda_2 + \lambda_3 - 1)$$

The Kuhn-Tucker conditions are:

$$\frac{\partial\phi}{\partial\lambda_1} \leq 0 \quad \frac{\partial\phi}{\partial\lambda_2} \leq 0 \quad \frac{\partial\phi}{\partial\lambda_3} \quad \frac{\partial\phi}{\partial\mu_1} \leq 0 \quad \frac{\partial\phi}{\partial\mu_2} \leq 0$$

$$\lambda_1\frac{\partial\phi}{\partial\lambda_1} = 0 \quad \lambda_2\frac{\partial\phi}{\partial\lambda_2} = 0 \quad \lambda_3\frac{\partial\phi}{\partial\lambda_3} \quad \mu_1\frac{\partial\phi}{\partial\mu_1} = 0 \quad \mu_2\frac{\partial\phi}{\partial\mu_2} = 0$$

The partial derivatives can be found as:

$$\frac{\partial\phi}{\partial\lambda_1} = -(1 + ln \lambda_1) - \mu_1 \Delta Q_1 - \mu_2 = -(1 + ln \lambda_1 + \mu_1 \Delta Q_1 + \mu_2)$$

$$\frac{\partial\phi}{\partial\lambda_2} = -(1 + ln \lambda_2) - \mu_1 \Delta Q_2 - \mu_2 = -(1 + ln \lambda_2 + \mu_1 \Delta Q_2 + \mu - 2)$$

$$\frac{\partial\phi}{\partial\lambda_3} = -(1 + ln \lambda_3) - \mu_1 \Delta Q_3 - \mu_2 = -(1 + ln \lambda_3 + \mu_1 \Delta Q_3 + \mu_2)$$

$$\frac{\partial\phi}{\partial\mu_1} = -(\lambda_1 \Delta Q_1 + \lambda_2 \Delta Q_2 + \lambda_3 \Delta Q_3)$$

$$\frac{\partial\phi}{\partial\mu_2} = -(\lambda_1 + \lambda_2 + \lambda_3 - 1)$$

The Kuhn-Tucker conditions then become:

$$-(1 + ln \lambda_1 + \mu_1 \Delta Q + \mu_2) \leq 0$$

$$-(1 + ln \lambda_2 + \mu_1 \Delta Q_2 + \mu_2) \leq 0$$

$$-(1 + ln \lambda_3 + \mu_1 \Delta Q_3 + \mu_2) \leq 0$$

$$-(\lambda_1 \Delta Q_1 + \lambda_2 \Delta Q_2 + \lambda_3 \Delta Q_3) \leq 0$$

$$-(\lambda_1 + \lambda_2 + \lambda_3 - 1) \leq 0$$

$$-\lambda_1(1+\ln\lambda_1+\mu_1\Delta Q_1+\mu_2) = 0$$

$$-\lambda_2(1+\ln\lambda_2+\mu_1\Delta Q_2+\mu_2) = 0$$

$$-\lambda_3(1+\ln\lambda_3+\mu_1\Delta Q_3+\mu_2) = 0$$

$$-\mu_1(\lambda_1\Delta Q_1+\lambda_2\Delta Q_2+\lambda_3\Delta Q_3) = 0$$

$$-\mu_2(\lambda_1+\lambda_2+\lambda_3-1) = 0$$

Using the equality conditions of Kuhn-Tucker and recognizing that:

$$\lambda_1 \neq 0; \quad \lambda_2 \neq 0; \quad \lambda_3 \neq 0; \quad \mu_1 \neq 0; \quad \mu_2 \neq 0$$

one can construct five equations to solve for the five unknowns $(\lambda_1, \lambda_2, \lambda_3, \mu_1$ and $\mu_2)$. These equations can be written as:

$$1 + \ln\lambda_1 + \mu_1\Delta Q_1 + \mu_2 = 0 \qquad \text{(L.1)}$$

$$1 + \ln\lambda_2 + \mu_1\Delta Q_2 + \mu_2 = 0 \qquad \text{(L.2)}$$

$$1 + \ln\lambda_3 + \mu_1\Delta Q_3 + \mu_3 = 0 \qquad \text{(L.3)}$$

$$\lambda_1\Delta Q_1 + \lambda_2\Delta Q_2 + \lambda_3\Delta Q_3 = 0 \qquad \text{(L.4)}$$

$$\lambda_1 + \lambda_2 + \lambda_3 - 1 = 0 \qquad \text{(L.6)}$$

Subtracting Eqn. (L.2) from Eqn. (L.1) one can write:

$$\ln\lambda_1 - \ln\lambda_2 + \mu_1(\Delta Q_1 - \Delta Q_2) = 0$$

using which μ_1 can be solved as:

$$\mu_1 = \frac{\ln\lambda_2 - \ln\lambda_1}{\Delta Q_1 - \Delta Q_2} \tag{L.6}$$

Multiplying Eqn. (L.1) by ΔQ_2, and Eqn. (L.2) by ΔQ_1 and subtracting one of these products from the other, one can write:

$$(\Delta Q_2 - \Delta Q_1) + \Delta Q_2 \ln\lambda_1 - \Delta Q_1 \ln\lambda_2 + \mu_2(\Delta Q_2 - \Delta Q_1) = 0$$

which can be used to solve for μ_2 as:

$$\mu_2 = \frac{\Delta Q_1 \ln\lambda_2 - \Delta Q_2 \ln\lambda_1}{\Delta Q_2 - \Delta Q_1} - 1 \tag{K.7}$$

Substituting the values of μ_1 and μ_2 from Eqn. (L.6) and (L.7) in Eqn. (L.3) one can write after a little algebraic manipulation:

$$(\Delta Q_1 - \Delta Q_2)\ln\lambda_3 + (\Delta Q_3 - \Delta Q_1)\ln\lambda_2 + (\Delta Q_2 - \Delta Q_3)\ln\lambda_1 = 0 \tag{L.8}$$

Multiplying Eqn. (L.5) by ΔQ_2 and then subtracting from Eqn. (L.4) one can write:

$$(\Delta Q_1 - \Delta Q_2)\lambda_1 + (\Delta Q_3 - \Delta Q_2)\lambda_3 + \Delta Q_2 = 0$$

from which one can obtain:

$$\lambda_1 = \frac{(\Delta Q_2 - \Delta Q_3)\lambda_3 - \Delta Q_2}{(\Delta Q_1 - \Delta Q_2)} \tag{L.9}$$

Again, multiplying Eqn. (L.5) by ΔQ_1 and then subtracting from Eqn. (L.4) we can write:

$$(\Delta Q_1 - \Delta Q_2)\lambda_2 + (\Delta Q_1 - \Delta Q_3)\lambda_3 - \Delta Q_1 = 0$$

which can be solved to obtain:

$$\lambda_2 = \frac{(\Delta Q_3 - \Delta Q_1)\lambda_3 + \Delta Q_1}{(\Delta Q_1 - \Delta Q_2)} \tag{L.10}$$

Substituting the values of λ_1 and λ_2 in terms of λ_3 from Eqn. (L.9) and (L.10) into Eqn. (L.8), one can obtain after some algebraic manipulation:

$$(\Delta Q_1 - \Delta Q_2) \ln[(\Delta Q_1 - \Delta Q_2)\lambda_3] + (\Delta Q_3 - \Delta Q_1) \ln[(\Delta Q_3 - \Delta Q_1)\lambda_3 + \Delta Q_1] + (\Delta Q_2 - \Delta Q_3) \ln[(\Delta Q_2 - \Delta Q_3)\lambda_3 - \Delta Q_2] = 0 \tag{L.11}$$

Equation (L.11) is a non-linear equation with only one variable λ_3. This equation can be re-written as:

$$[(\Delta Q_1 - \Delta Q_2)\lambda_3]^{(\Delta Q_1 - \Delta Q_2)} [(\Delta Q_3 - \Delta Q_1)\lambda_3 + \Delta Q_1]^{(\Delta Q_3 - \Delta Q_1)} * [(\Delta Q_2 - \Delta Q_3)\lambda_3 - \Delta Q_2]^{(\Delta Q_2 - \Delta Q_3)} = 1 \tag{L.12}$$

This non-linear equation can be solved for λ_3. Similar relations can be obtained for λ_1 and λ_2. Note that the analytical solution results in non-linear equations from which the values should be obtained by numerical methods. Therefore, in this study the analytical solution is not used. Note that the example is for a three-branch channel. Obtaining solutions for more branches would be even more complicated.

Once the values of λ's are known, they can be used to find the distribution of deviation of discharge for the branch channels.