ADAPTIVE CONTROL BY MEANS

OF PULSE-FREQUENCY MODULATION

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ABSTRACT

by

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A simple adaptive control system was sought which would be suitable for a plant with two or more variable parameters which were inaccessible. Several possibilities were investigated including a system suggested by Murphy and West which made use of pulse-frequency modulation. The Murphy and West system seemed the most promising of the possibilities, but it proved to be quite difficult to design such a system.

A simpler system was suggested in which reasonable predictions could be made concerning the response by means of a quasi-describing function devised by Pavlidis and Jury. A system was designed for a plant with the transfer function $\frac{K}{S(s + \omega_1)}$.

ACKNOWLEDGEMENTS

I would like to thank my faculty advisor Professor W. H. Lehn and also Professor R. A. Johnson for their many useful suggestions.

Thanks are also due to the National Research Council who provided the finances for the work under Grant No. A-2171.

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CHAPTER I

INTRODUCTION

I. THE PROBLEM

The purpose of this work was to find a method of control for a plant with one or more parameters which vary over a wide range. A system which provides adequate control in spite of large parameter variations is usually designated as an "adaptive system".

A control scheme was sought in which (unlike many adaptive systems) no access to the plant parameters would be required.

II. ADAPTIVE SYSTEMS DEFINED

In the conventional design of a feedback control system the engineer determines the dynamics of the system to be controlled, and then designs compensation which causes the system to respond in the desired manner to certain deterministic inputs. Such a procedure is quite straight forward for linear time-invariant systems. One could visualize that the design procedure outlined above, including the compensation, might possibly be carried out by some sort of machine.[1] Then all one would have to do would be to connect a system to the machine, and a satisfactory control would result. Thus the engineer would become obsolete. Now, if the machine were permitted to repeat the design procedure at a rate which is rapid compared to the rate of system parameter variations, a method of control would be obtained which would

1. R.E. Kalman, "Design of a Self-Optimizing Control System," ASME Transactions, vol. 80 pp. 468-478, 1958.

be satisfactory for linear time-varying systems. Thus, if the machine described above could be realized, an adaptive system would result. It was not necessarily the purpose of this work to construct such a machine, but it is mentioned in order to give an intuitive idea of just what adaptive control is.

Just how complicated a system has to be in order to qualify as an adaptive system has been a matter of considerable controversy.[2] Truxal has taken the point of view that an adaptive system is any system designed from the adaptive point of view.[3] This definition, although trite sounding seems preferable to any of the many definitions given. Most other definitions are quite lengthy (see Whitaker's definition for example[4]); and generally the reason for this length is the authors' attempts to exclude the ordinary feedback system from the class of adaptive systems. One of the most well known advantages of feedback systems, however, is that the effect of parameter variations is reduced to a certain extent. So, if the feedback is included in order to minimize the effect of parameter variations, there is no reason why it could not be called adaptive control. Using Truxal's definition, one could think of the addition of a feedback loop as being the crudest form of adaptation.

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2. F.P. Caruthers and H. Levenstein, "Adaptive Control Systems," pp. 279-290, Pergamon Press, Oxford, England, 1963.

3. J.G. Truxal, "Trends in Adaptive Control Systems," Proceedings of the National Electronics Conference, vol. 15, pp. 1-16, 1959.

4. H.P. Whitaker, "Design Capabilities of Model Reference Adaptive Systems," Proceedings of the National Electronics Conference, vol. 18, pp. 241-249, 1962.

III. THE MECHANICS OF ADAPTIVE SYSTEMS

As mentioned above, an adaptive system consists of three processes which are exactly those carried out by the engineer in conventional system design:

1. The dynamics of the system are identified.
2. The response is compared to the desired response by means of some performance index, and a decision is made as to whether modification is required or not.
3. The required modification is carried out.

Gibson identifies the above as the three functions of adaptive control - namely identification, decision and modification.[5] A few remarks will be made concerning each of these functions.

Identification

This is one of the main problems in adaptive control. Increasing the speed of identification is an extremely important field of research, since, if the system characteristics cannot be identified in a time which is short compared to the rate of parameter variations, no useful adaptation can be carried out. Because previous knowledge of the form of the system allows a speedier identification, the adaptation problem is greatly simplified if one has some previous knowledge of system dynamics.

Methods of system identification fall into two rather obvious divisions: those that require an external identification signal and those

5. J.E. Gibson, "Nonlinear Automatic Control," p. 500, The McGraw-Hill Book Co. Inc., New York, 1963.

which do not.[6] If there is no external signal, the identification is carried out by means of a periodic sampling of the input and output signals. In general this method requires more sophisticated equipment and takes longer than the other method, but it does have the advantage that a more complete identification can be made.[7] Thus, if an identification signal is tolerable and the amount of information required is not too great, the use of an external signal is dictated. The most commonly used identification signals are white noise[8] and high frequency sinusoids.[9]

It is interesting to note that the human being - an extremely adaptable system - often makes use of small perturbations for the purpose of identification. Wiener gives the example of an automobile being driven down an icy road.[10] By means of small movements of the steering wheel and light applications of the brakes, the driver is able to judge the response of the vehicle to his commands, and thus choose a safe driving speed.

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6. R.A. Haddad and L. Braun Jr., "Automatic Methods of Process Identification," pp. 293-322 in "Adaptive Control Systems," edited by E. Mishkin and L. Braun Jr., The McGraw-Hill Book Co. Inc., New York, 1961.

7. R.A. Haddad and L. Braun Jr., Op. cit.

8. J.G. Truxal, "Control System Synthesis," pp. 435-439, The McGraw-Hill Book Co. Inc., New York, 1955.

9. H.V. Gururaja and B.L. Deekshatulu, "Process Identification and Adaptation by High Frequency Sinusoidal Test Signal," Int. J. Control, vol. 4, no. 6, pp. 567-584, Dec. 1966.

10. N. Wiener, "Cybernetics," second edition, p. 113, The M.I.T. Press, Cambridge, Mass., 1961.

Decision

Before a decision can be made to modify the system, there must be some index of system performance which is to be maximized or minimized. Commonly used performance indices fall into three broad classes:[11]

1. those which are only useful for second order systems, for example the impulse response area ratio (IRAR),
2. those which are not very selective but are mathematically convenient, for example the integrated squared error (ISE),
3. those which are highly selective but mathematically inconvenient, for example the integrated time-multiplied absolute value of error (ITAE).

All of the above indices have one thing in common: they take no account of the control effort required. An index which includes derivatives of the error necessarily puts a penalty on large control efforts. Thus a more realistic performance index might be:

$$I = \int_0^\infty \left[e^2 + T^2 \left(\frac{de}{dt} \right)^2 \right] dt \qquad 1$$

in which e is the deviation between the actual and the desired output, T is a constant and the system is assumed to have started at t equal to zero. Gibson shows that the minimization of the above performance index is

11. C.W. Sarture and J.A. Aseltine, "Performance Criteria in Adaptive Control," pp. 1-10 in Caruthers and Levenstein, Op. cit.

identical to minimizing the deviation (in a mean square sense) between the system output and the output of a model with the transfer function $\frac{1}{1 + Ts}$.[12] Naturally, if I contains higher order derivatives, a higher order model is obtained.

This suggests the well known model - reference adaptive control system. In this method an analog model is subjected to the same input as the system and the difference between the two outputs is found. The error signal thus obtained must then be minimized in some manner. The model-reference method is then seen to have the advantage that all the desired characteristics of the system can be built into an analog model. This might be difficult if a mathematical performance index were used. In addition it is generally a far simpler matter to construct an analog model than it is to calculate most performance indices.

Modification

The problem has now been reduced to the following. A plant with an input m and an output x is given. The dynamics of the plant have been identified and a decision has been made that the output x has deviated from the optimum. It is apparent that there are two ways x can be changed. Either m must be modified (control signal synthesis) or else the parameters of the plant must be changed (parameter adjustment).[13] If x could be modified satisfactorily, then the adaptive process is complete.

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12. J.E. Gibson, Op. cit., pp. 304-315

13. J.E. Gibson, Op. cit., pp. 536-541.

CHAPTER II

PRELIMINARY CONSIDERATIONS IN THE CHOICE OF A SYSTEM

Two Possible Solutions

Since the plant parameters have been assumed to be inaccessible, the modification function must be carried out by means of control signal synthesis. Figures 1 and 2 show two rather general schemes which could provide an adequate solution to the problem. Both systems are of the model-reference type, as it was decided to make use of a reference model because of the advantages mentioned in Chapter I.

In Figure 1 the adaptive controller causes one or more of the controller parameters to be changed in such a manner that e is always moved towards its minimum. It is apparent that the mechanization of the process is not necessarily simple, but if the plant is linear and of a known form, the controller might be nothing more complicated than a simple linear compensator with adjustable parameters.

In Figure 2, the adaptive controller (a digital computer) synthesizes the plant input necessary to produce the desired output. If the plant is linear, the desired input signal can be synthesized if the impulse response of the plant is known. The impulse response can be obtained by the well known technique of injecting a white noise signal into the plant. Then the value of the impulse response at time t, g(t), can be obtained by calculating the cross correlation function between the white noise input delayed by a time t and the plant output.[1]

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1. R.A. Haddad and L. Braun Jr., Op. cit.

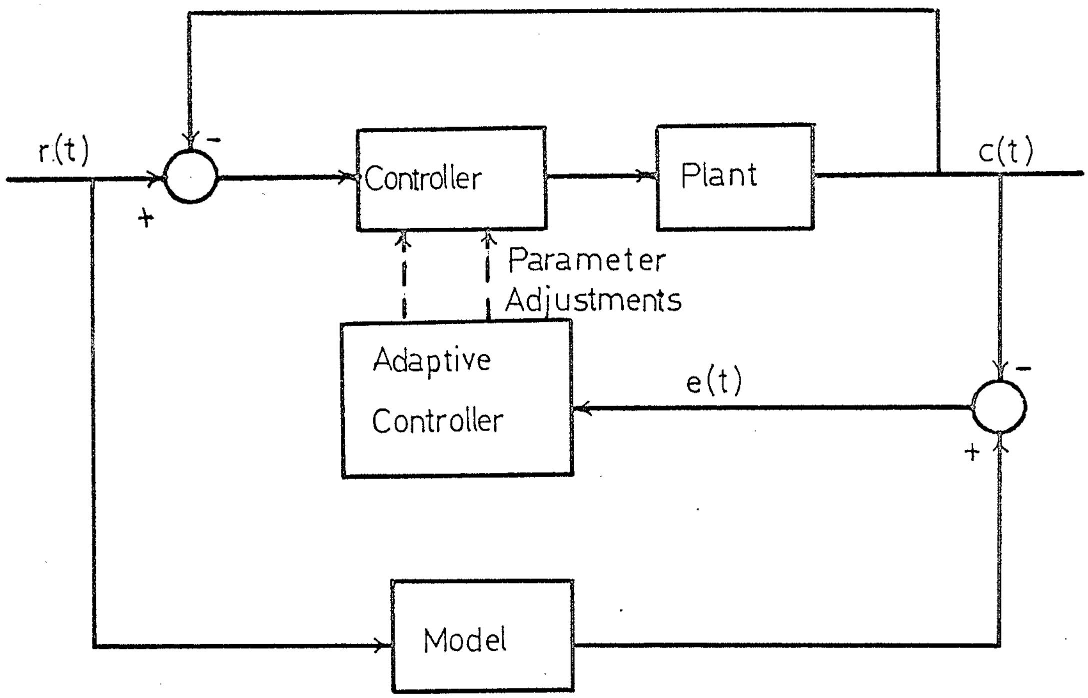

FIGURE 1

AN ADAPTIVE SCHEME WHICH DOES NOT REQUIRE A DIGITAL COMPUTER

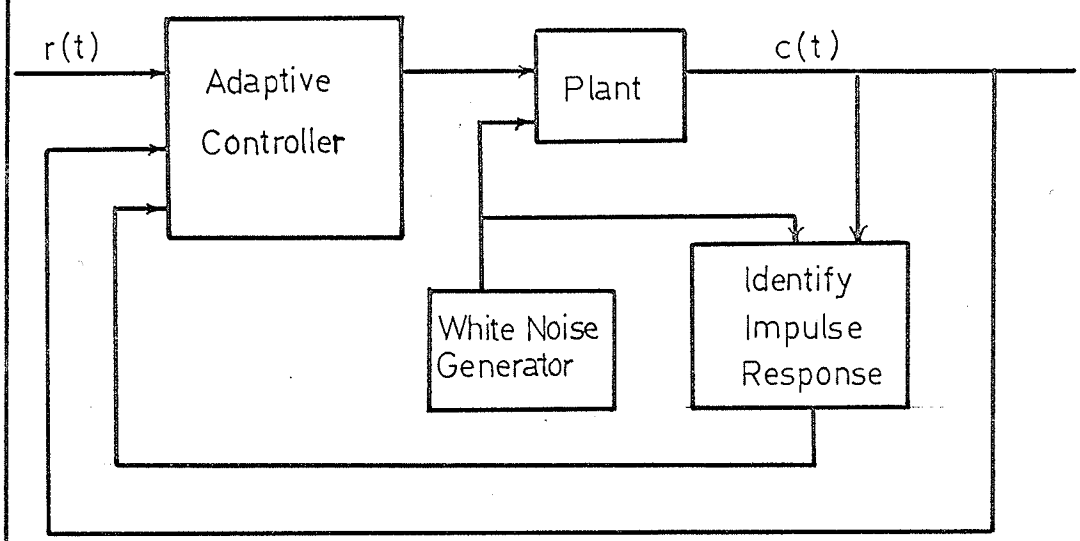

FIGURE 2

AN ADAPTIVE SCHEME REQUIRING A DIGITAL COMPUTER

Neither of the above systems presents an ideal solution. The system in Figure 1 does not make an identification directly, but the rate of plant parameter variations which can be handled will depend upon the means of varying the controller parameters. If servos are used, the rate of variation could be relatively slow. In addition, great care must be taken in the design to ensure that the system settles on an absolute minimum of e and not a relative minimum. Also, solution of the stability problem may be by no means simple.

The main disadvantage of the system in Figure 2 is that a reasonably complex digital computer is required. As long as the system is linear and the rate of plant variations is slow compared to the time required for identification, the system will operate satisfactorily.

The above two systems certainly are not the only possible solutions to the problem, but they are representative in that they bring out the problem of most commonly used adaptive schemes - that is either the system is complex and expensive (Figure 2), or else it is simpler but has stability problems or other undesirable characteristics (Figure 1).

A system was sought which would be relatively simple to implement so that it could be simulated on an analog computer. Hopefully the analysis of the system would not be difficult so that an accurate prediction could be made concerning stability. It seems likely that the response of such a system would be somewhat inferior to that of a system which makes use of a high speed digital computer.

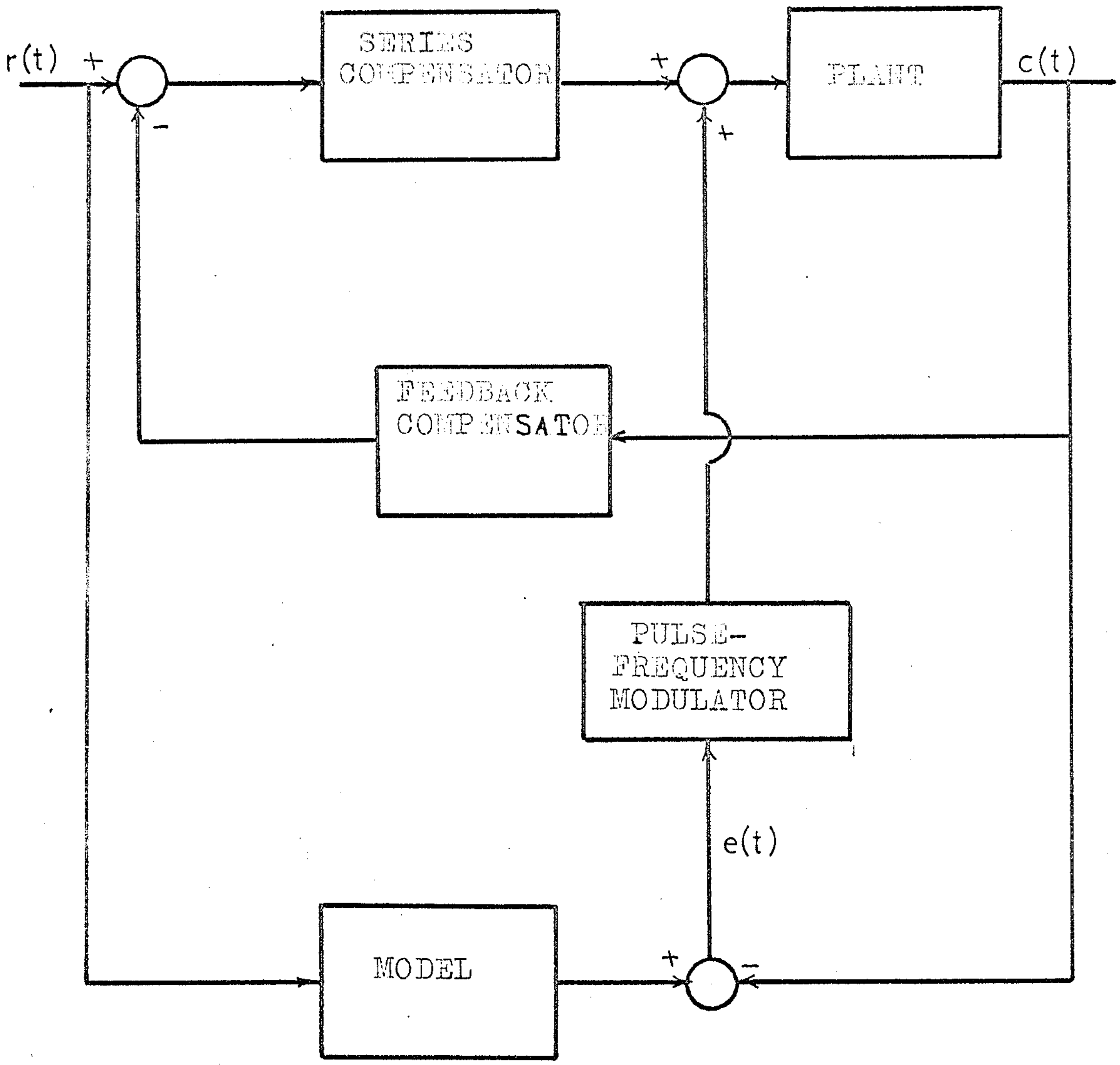

FIGURE 3

THE ADAPTIVE SCHEME DESIGNED BY MURPHY AND WEST

The Murphy and West System

One interesting possibility is the method proposed by Murphy and West.[2] This system makes use of pulse-frequency modulation as an additive signal to improve the system response. The system is shown in Figure 3.

Basically a pulse-frequency modulator is a device which emits a pulse whenever the absolute value of some function, f, of its input exceeds a given threshold level. After the emission of a pulse f is reset to zero and the process repeats itself. The pulses emitted are all of identical amplitude and width, but it can be seen that, if the input to the modulator is increased, the pulses will occur relatively more often if f is a monotonically increasing function.

The philosophy underlying the Murphy and West system is that linear compensators are designed to give the best possible response over the anticipated range of parameter variations. These compensators (which of course can be made very reliable) provide the primary control. Then an error signal, which is the difference between the plant output and that of an analog model, is used as the input to a pulse-frequency modulator. The output of the modulator is added to the plant input and is intended to act as a "supervisory control".

In Chapter III the analysis of pulse-frequency modulated systems will be discussed.

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2. G.J. Murphy and R.L. West, "The use of Pulse-Frequency Modulation for Adaptive Control," Proceedings of the National Electronics Conference, vol. 18, pp. 271-278, 1962.

CHAPTER III

ANALYSIS OF PULSE-FREQUENCY MODULATED SYSTEMS

Pulse-frequency modulated systems are of considerable interest because of the fact that the signal carried by the human nerve is a pulse-frequency modulated one.[1] It was this resemblance which suggested the use of pulse-frequency modulator to Murphy and West.

The only readily available work on the analysis of pulse-frequency modulated systems such as in Figure 4 is that of Pavlidis and Jury.[2] This chapter will summarize their results which are applicable to this work.

State Equations for a Pulse-Frequency Modulator

It is a common practice to assume that the pulses emitted can be represented by impulses of appropriate strength. There will certainly be little error in this approximation if the time constants of the system following the modulator are long compared to the duration of the pulses.

The simplest form of modulator is the integral pulse-frequency modulator (IPFM) which emits a pulse whenever the absolute value of the integral, p, of its input reaches a threshold level, r, and then resets the integral to zero. Then if x represents the input to the modulator, and y the output, the state equations can be written:

$$\frac{dp}{dt} = x - \left[r\,\mathrm{sgn}\,(p)\right]\delta\,(|p| - r) \qquad 2$$

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1. Dean E. Wooldridge, "The Machinery of the Brain," pp. 1-5, The McGraw-Hill Book Co. Inc., New York, 1963.

2. T. Pavlidis and E.I. Jury, "Analysis of a New Class of Pulse-Frequency Modulated Feedback Systems," IEEE Trans. Automatic Control, vol. PGAC-10, no. 1, pp. 35-43, Jan. 1965.

$$y = \left[M \operatorname{sgn}(p)\right] \delta(|p| - r) \qquad 3$$

in which $\delta(t)$ denotes a unit impulse occuring at t equal to zero, and M is the magnitude of the pulse emitted. Thus Equation 2 represents the resetting of p to zero after the emission of a pulse, and Equation 3 represents the emission of the pulse (See Figure 5).

A generalization of the above is to add a term to the left side of Equation 2. (Note that p is no longer the integral of x, but the integral of the difference between x and g(p)).

$$\frac{dp}{dt} + g(p) = x - \left[r \operatorname{sgn}(p)\right] \delta(|p| - r) \qquad 4$$

$$y = \left[M \operatorname{sgn}(p)\right] \delta(|p| - r) \qquad 5.$$

If g(p) is a nondecreasing, continuous, odd function of p, then the modulator described by Equations 4 and 5 is designated by Pavlidis and Jury as a sigma pulse-frequency modulator (ΣPFM). If g(p) is linear (that is equal to cp), the modulator has been called a neural pulse-frequency modulator (NPFM) because it has been used in the construction of models for neural nets.[3]

If x is a constant, x_o, then the time between the emission of pulses (firing time) is easily obtained:

$$t_o = \int_0^r \frac{dp}{|x_o| - g(p)} \qquad 6$$

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3. T. Pavlidis and E. I. Jury, Op. cit.

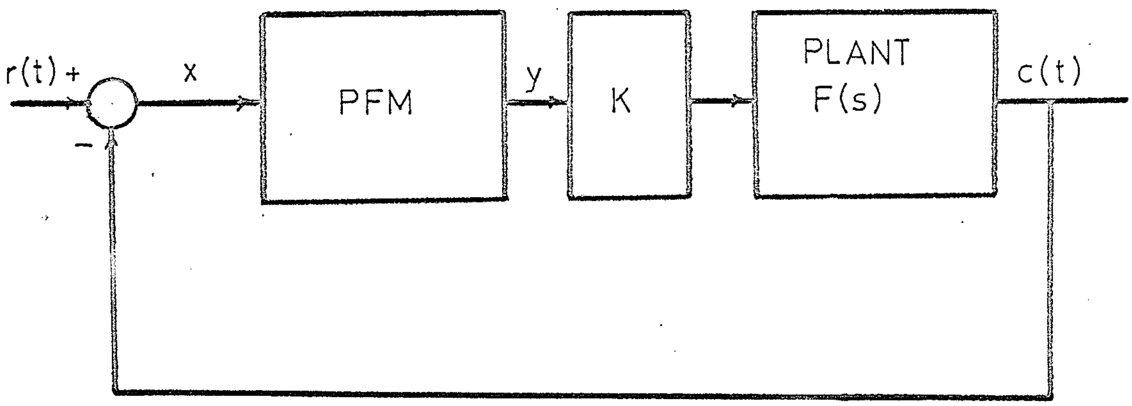

FIGURE 4

A FEEDBACK SYSTEM CONTAINING A PULSE-FREQUENCY MODULATOR

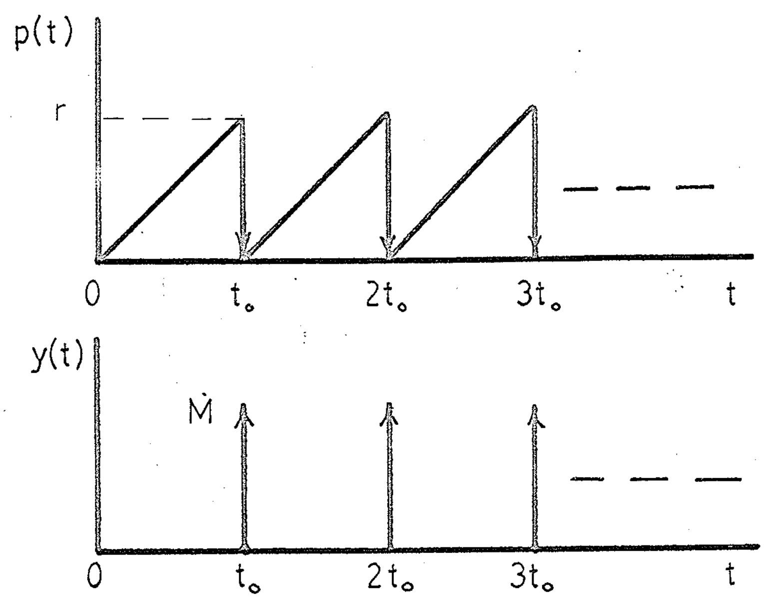

FIGURE 5

RESPONSE OF AN IPFM WITH A CONSTANT INPUT in which t_o represents the firing time. Equation 6 can obviously have no solution (That is no firing can occur.) if $|x_o|$ is less than g(r).

Feedback Systems Containing a Pulse-Frequency Modulator

A pulse-frequency modulator can be used in a feedback system as shown in Figure 4. The system is no more complex if the gain, K, of the portion following the modulator is a memoryless, odd nonlinearity since the pulses are of constant amplitude. Since the plant input is approximated by impulses, it is possible to find the exact response for a given plant and deterministic input. However, it would be difficult to obtain general results. It is possible, though, to determine the existence of an equilibrium position and the frequency and magnitude of any sustained oscillations which might exist.

A Theorem on the Existence of an Equilibrium Position

Equilibrium, as defined by Pavlidis and Jury, is the condition in which no firing occurs, and $\frac{dx}{dt}$ and $\frac{dp}{dt}$ are both equal to zero. The following theorem can be proved in a straightforward manner:[4]

> A sigma pulse-frequency modulator unity feedback system with a linear plant (as in Figure 4) has an equilibrium position (as defined above) for all inputs and any initial condition if and only if the limit as s approaches zero of sF(s) exists, is different from zero, and satisfies the inequality,

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4. T. Pavlidis and E.I. Jury, Op. cit.

$$2g(r) \geqslant MK \lim_{s \to o} sF(s) \qquad 7$$

in which F(s) is the transfer function of the linear plant.

It is immediately apparent that a system containing an integral pulse-frequency modulator cannot reach an equilibrium since the inequality cannot be satisfied.

Sustained Oscillations in Pulse-Frequency Modulated Systems

If the theorem above cannot be satisfied, then the system in Figure 5 would be expected to undergo sustained oscillations. Such systems do not exhibit a true limit cycle. Since the number of pulses required per half period for periodic oscillations is unlikely to be an integer, the system will move in a range of various numbers of pulses per half cycle. Thus the resultant oscillations will be random in nature between an inner and outer boundary. Such oscillations are known as a "limit annulus". Normally it is sufficient to know the inner and outer boundaries of the oscillations.

The Quasi-Describing Function

It would be an extremely difficult matter to obtain a normal describing function for a pulse-frequency modulator because of the difficulty in writing an expression describing the response of the modulator to a sinusoidal input. If, however, the input to the modulator is a square wave, this difficulty vanishes. Then if the fundamental of the input and output are compared, one obtains a "quasi-describing function" describing the modulator.

If the input to the modulator is a square wave of amplitude, S_o, and angular frequency, ω, then the quasi-describing function, $Q(S_o, \omega, \tau)$, of a sigma pulse-frequency modulator with firing time, t_o, (as defined in Equation 6) is found to be:[5]

$$Q(S_o,\omega,\tau) = \frac{M\omega}{4S_o} \frac{1+\cos(\omega\tau)}{\sin\left(\frac{\omega t_o}{2}\right)} \exp\left(\frac{-j\omega t_o}{2}\right) \qquad 8.$$

The parameter, τ, represents the final portion of the half period during which p has built up to some value less than r, and thus $\omega\tau$ can take on any value between zero and $\frac{\pi}{2}$. The maximum oscillation will occur when τ is equal to zero since this condition allows the maximum number of pulses to occur in the half-period. Obviously Equation 8 is valid only if ωt_o is less than π since no firing will occur otherwise.

Application of the Quasi-Describing Function

The existence of oscillations is determined in the normal manner from the intersection of the negative inverse of the quasi-describing function, $-\frac{1}{Q}$, and the Nyquist plot of the linear plant. Pavlidis and Jury make use of normalized variables to plot the quasi-describing function, but in this work it was found to be more convenient to plot curves of $-\frac{1}{Q}$ for various values of S_o with ω as a running parameter. Then the Nyquist plot for the linear plant could be plotted and the magnitude of the oscillations, S_o, could be obtained from the curve of $-\frac{1}{Q}$ on which ω coincided with the value of ω on the Nyquist plot.

5. T. Pavlidis and E.I. Jury, Op. cit.

The quasi-describing function has two limitations. The first is the obvious one that the input to the modulator might differ greatly from a square wave. If there is doubt, the actual response must be evaluated. The second is that, if the plant contains an integrator, the quasi-describing function will always give points of oscillation even though none may exist. Pavlidis has devised the following test to overcome this difficulty:[6]

"If the amplitude V and frequency ω of the oscillations indicated by the Q.D. function are "small", we compute the quantity $\frac{1}{2}\, g\left(\frac{\pi}{\omega}\right)$. If V is less than this no oscillations can exist."

It should be noted that, unlike most systems, small amplitude oscillations occur at a low frequency. This occurs naturally enough since the firing time will be long if the input to the modulator is small.

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6. T. Pavlidis and E.I. Jury, Op. cit.

CHAPTER IV

DESIGN OF AN ADAPTIVE SYSTEM

Drawbacks of the Murphy and West System

There are certain difficulties in the design of a system like that of Murphy and West. Figure 6 shows the block diagram of the autonomous system. It is seen that the pulse-frequency modulator and the plant form an inner loop. This inner loop could well be capable of sustained oscillations. Thus the choice of the linear compensators is far more difficult than is suggested by Murphy and West since the system requiring compensation (the plant-modulator combination) is a nonlinear one.[1] In fact, the choice of the compensators may be virtually impossible in many cases.

Another disadvantage of the Murphy and West system is that the response of the system is difficult to predict due to the mixture of pulses and a continuous signal at the input to the plant. In fact Murphy and West were unable to explain the instability which resulted in their system for one set of conditions. This difficulty could be overcome if a system were designed in which the input to the plant consists entirely of pulses.

A Simpler System

The system shown in Figure 7 presents a possible solution to the problem. An error signal (which is the difference between the model

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1. The system is, of course, only nonlinear if the modulator fires. If any useful adaption is to be carried out, though, the modulator must fire relatively often.

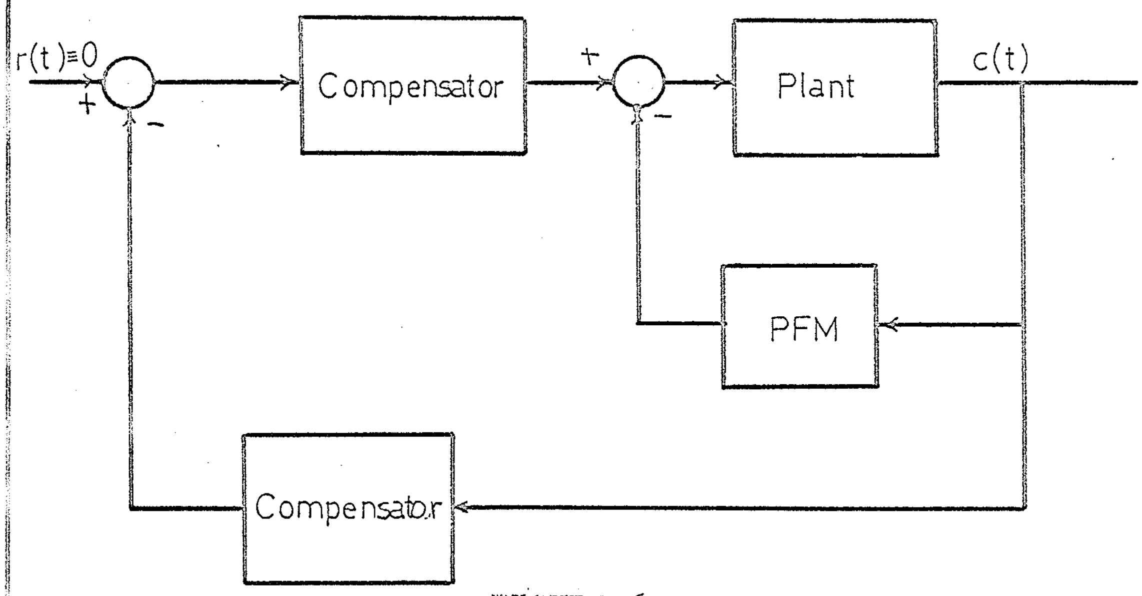

FIGURE 6
THE AUTONOMOUS MURPHY AND WEST SYSTEM

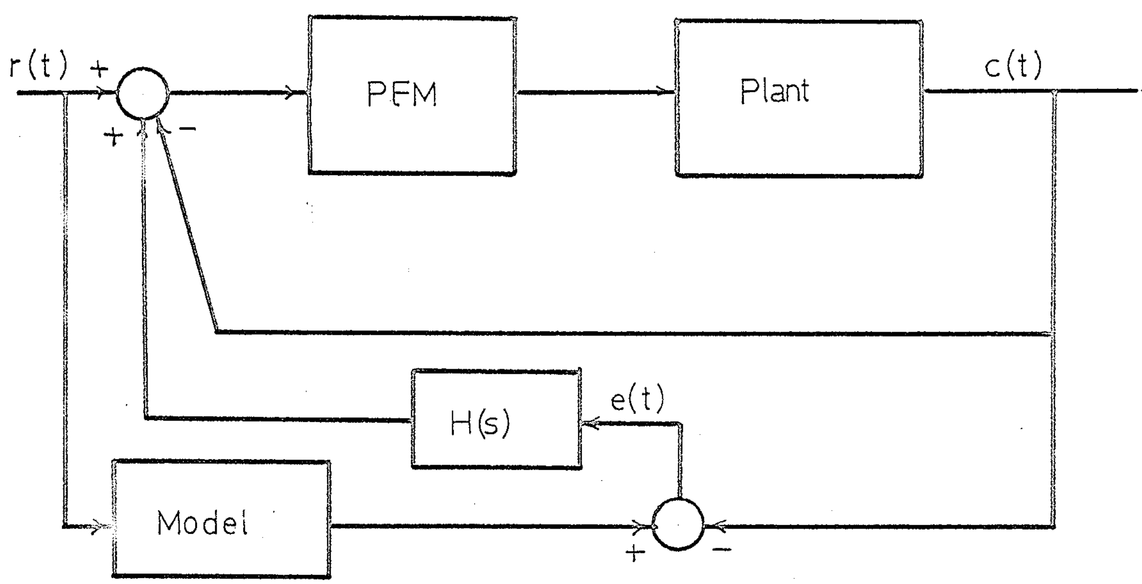

FIGURE 7
THE ADAPTIVE SCHEME USED

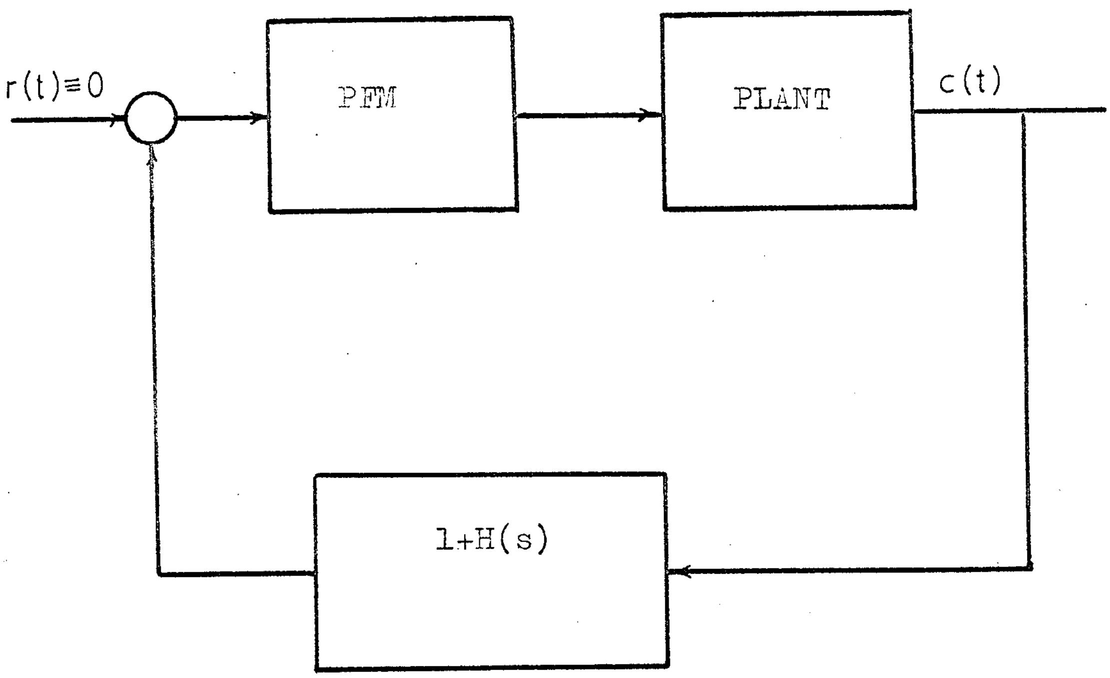

FIGURE 8
THE AUTONOMOUS SYSTEM

and plant outputs) is passed through a compensator, H(s), and the compensator output used to modify the input to the modulator. In this way, it was hoped to influence the dynamics of the system towards the model response and, at the same time, by a proper choice of H(s), ensure stability and a lack of objectionable oscillations. Figure 8 shows the autonomous system which is identical to the feedback system discussed in Chapter III. Thus the magnitude and frequency of any oscillations in the autonomous system can be found by means of the quasi-describing function. If no objectionable oscillations occur, one can expect that the system will respond in a relatively stable manner to an input. To ensure that the transient response is suitable, an exact calculation could be made for any given parameter values, but an analog simulation is certainly less complicated. Thus, the design procedure used was as follows:

1. Making use of the quasi-describing function, H(s) and the parameters of the modulator were chosen so that no objectionable oscillations would occur in the autonomous system.

2. The transient response of the system was obtained by means of an analog simulation.[2] If the response was not satisfactory a further change in H(s) and the modulator parameters had to be made.[3]

Actually the analog simulation may not be required to accept or reject a design since in many cases a reasonable idea of how the system will behave can be obtained intuitively.

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2. See Appendix A for the analog simulation of a pulse-frequency modulator.

3. It can be seen that this design could involve a certain amount of trial and error.

A Particular Example

An adaptive control system was constructed for the plant $\frac{K}{s(s + \omega_1)}$ in which both K and ω_1 were expected to vary between 0.1 and 10. The model transfer function was chosen to be $\frac{1}{1 + S}$. The design was based on a unit step input.

It was desired that the system should reach an equilibrium position (as defined in Chapter III) for as many operating conditions as possible. For this reason a sigma pulse-frequency modulator was used. In particular a neural pulse-frequency modulator was chosen because of its simplicity. In the preliminary part of the design, the compensator, H(s), was set equal to unity with additional compensation to be chosen when the need for it arose.

It was decided that the maximum tolerable steady state error (or average error if no steady state was possible) would be 0.05. In addition, if oscillations should occur, the maximum allowable deviation from the average was a further 0.05. Thus the maximum possible error in an acceptable system was 0.10.

With H(s) equal to unity, an error of 0.05 appears as 0.10 at the modulator input (See Figure 7). Thus the product, cr, was set equal to 0.10 so that the modulator would just fire when the input was 0.10. Since an increase in c decreases the firing time, it was decided to make c relatively large. Since setting r to anything less than 0.02 would be impractical in an analog simulation, c was chosen to be 5. This choice of c and r is somewhat arbitrary. The determination of optimum values would be an interesting research problem.

S_o is the amplitude of the input to the modulator. To keep the error within 0.10, it was therefore necessary that S_o never exceed 0.20 due to the double feedback of the error.

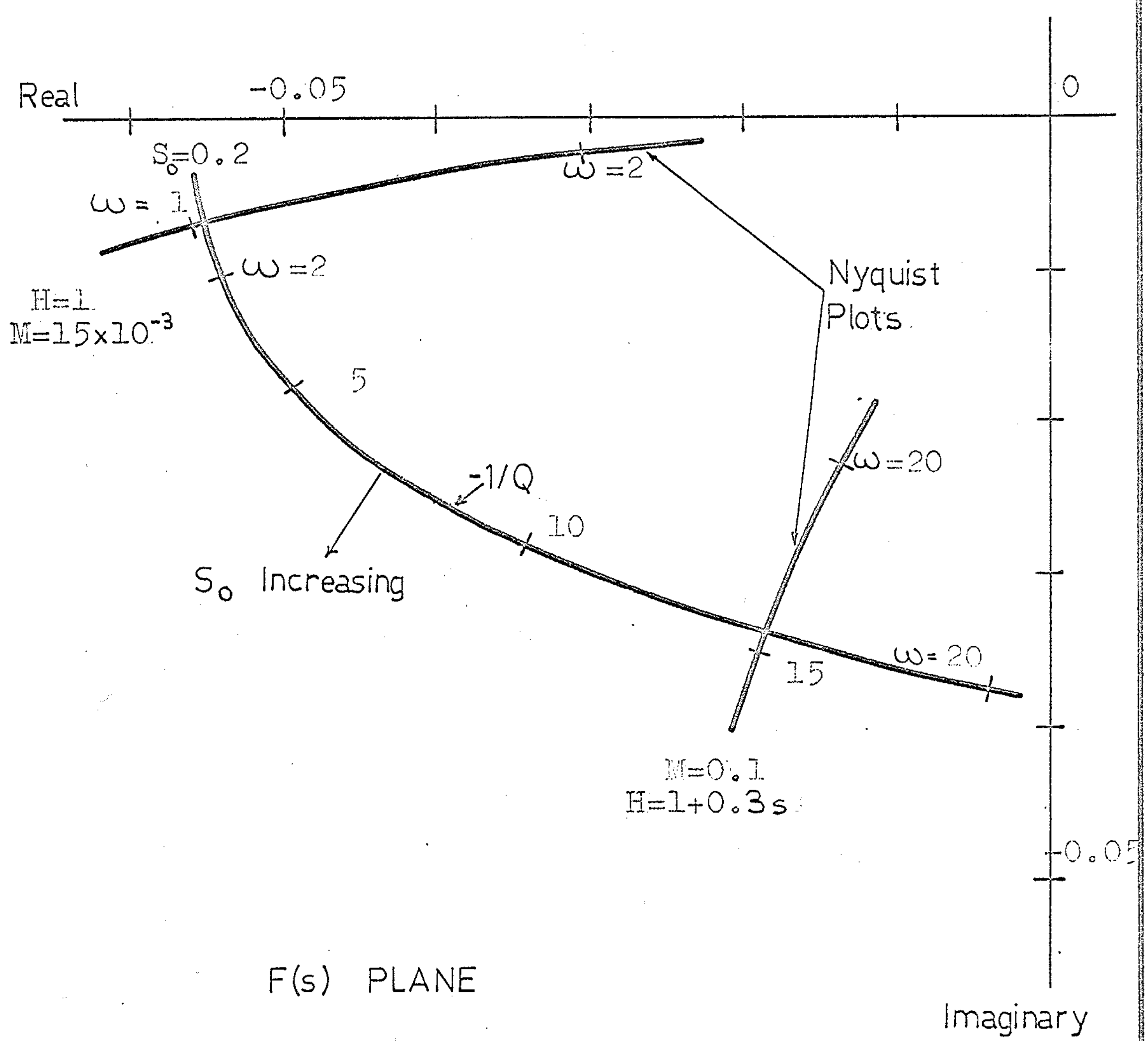

FIGURE 9

POINTS OF OSCILLATION OBTAINED BY MEANS OF THE QUASI-DESCRIBING FUNCTION

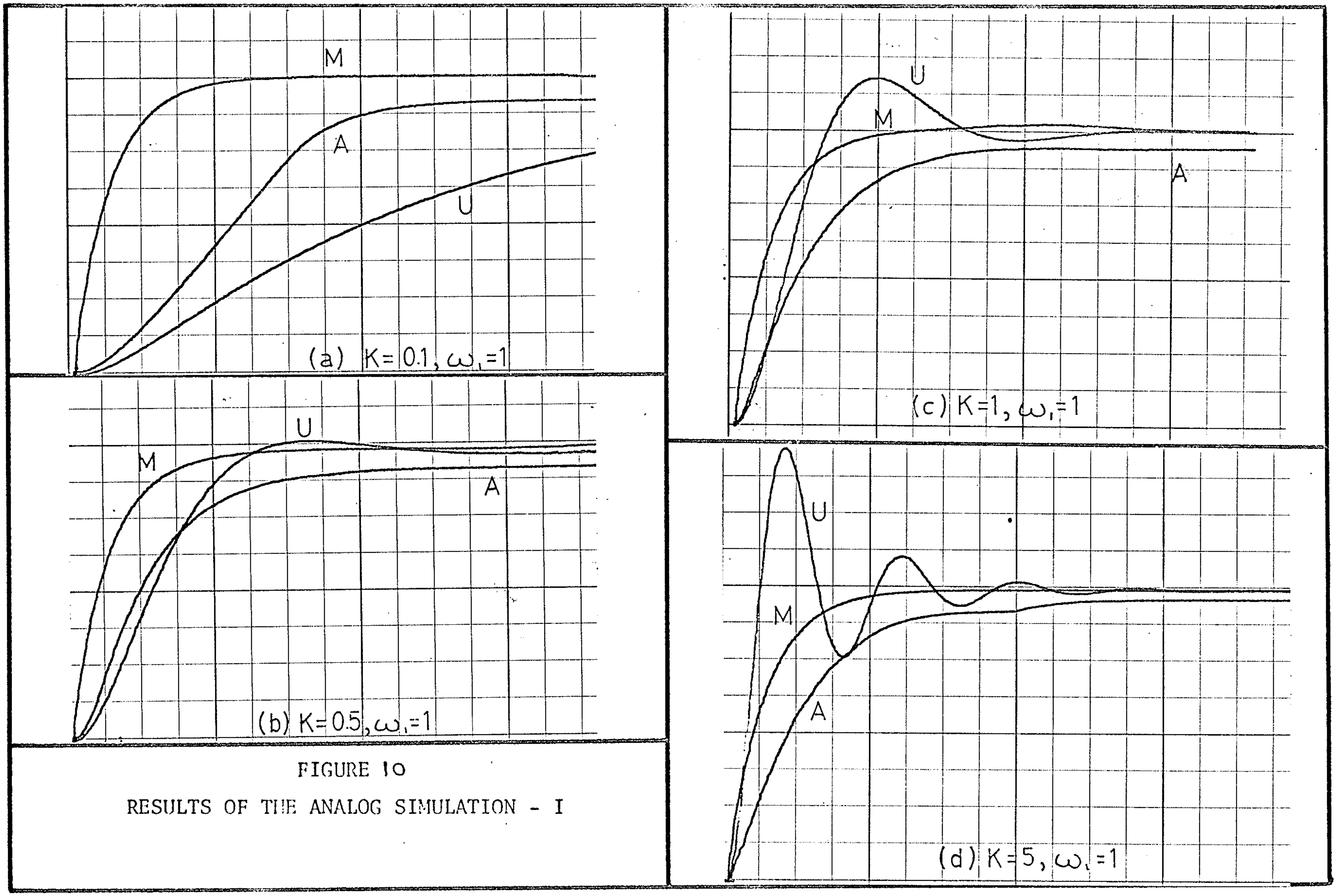

FIGURE 10

RESULTS OF THE ANALOG SIMULATION - I

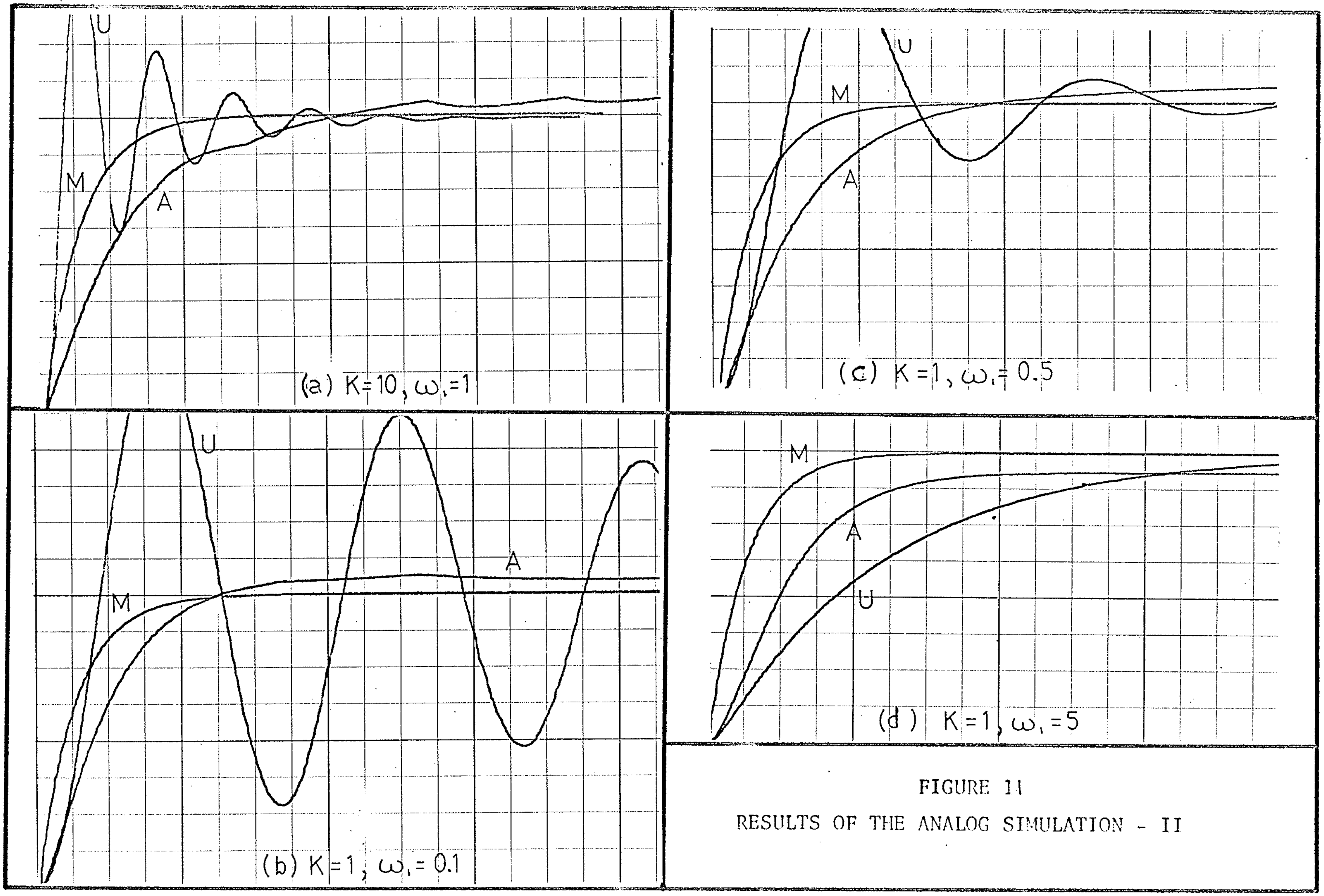

FIGURE 11

RESULTS OF THE ANALOG SIMULATION - II

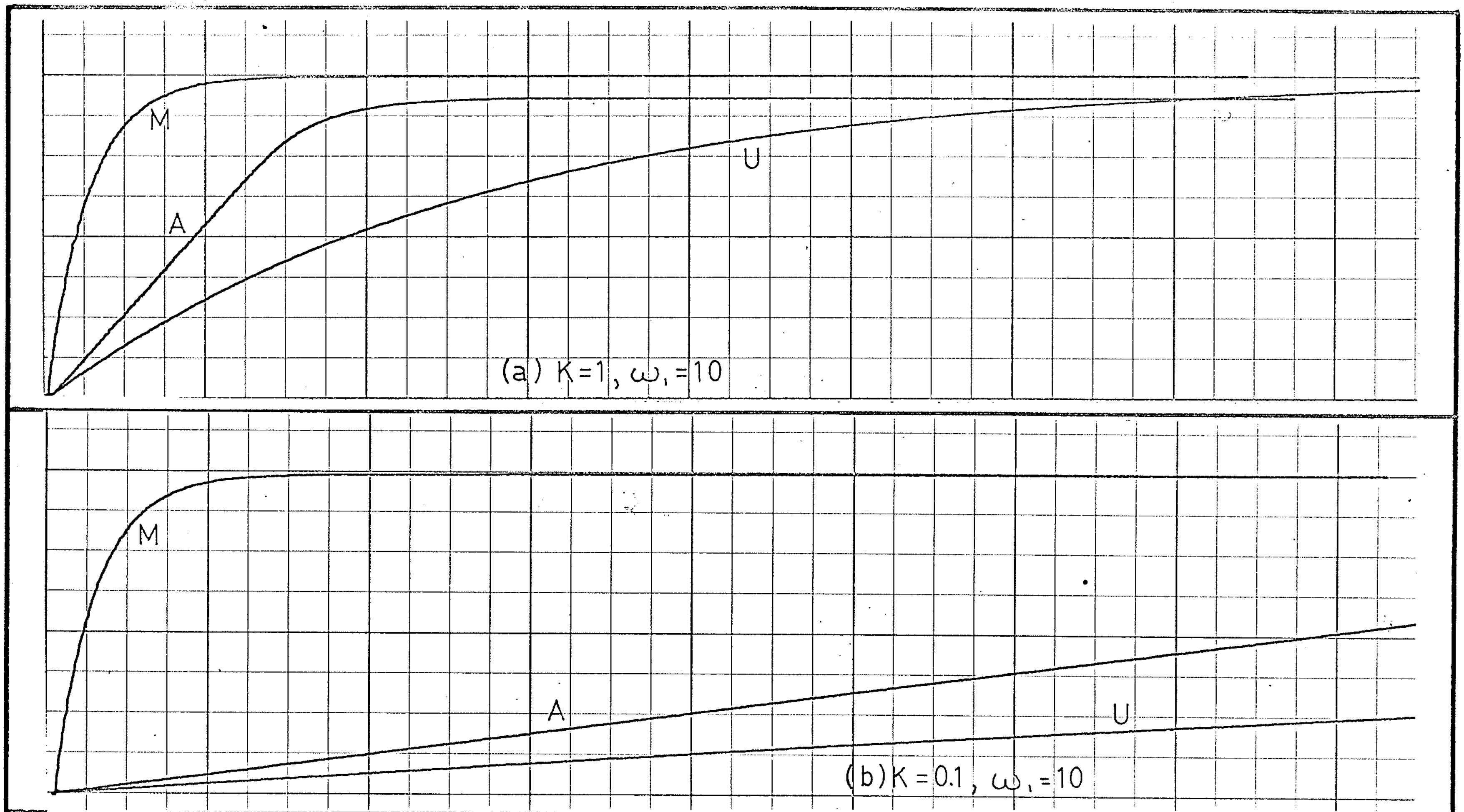

FIGURE 12

RESULTS OF THE ANALOG SIMULATION - III

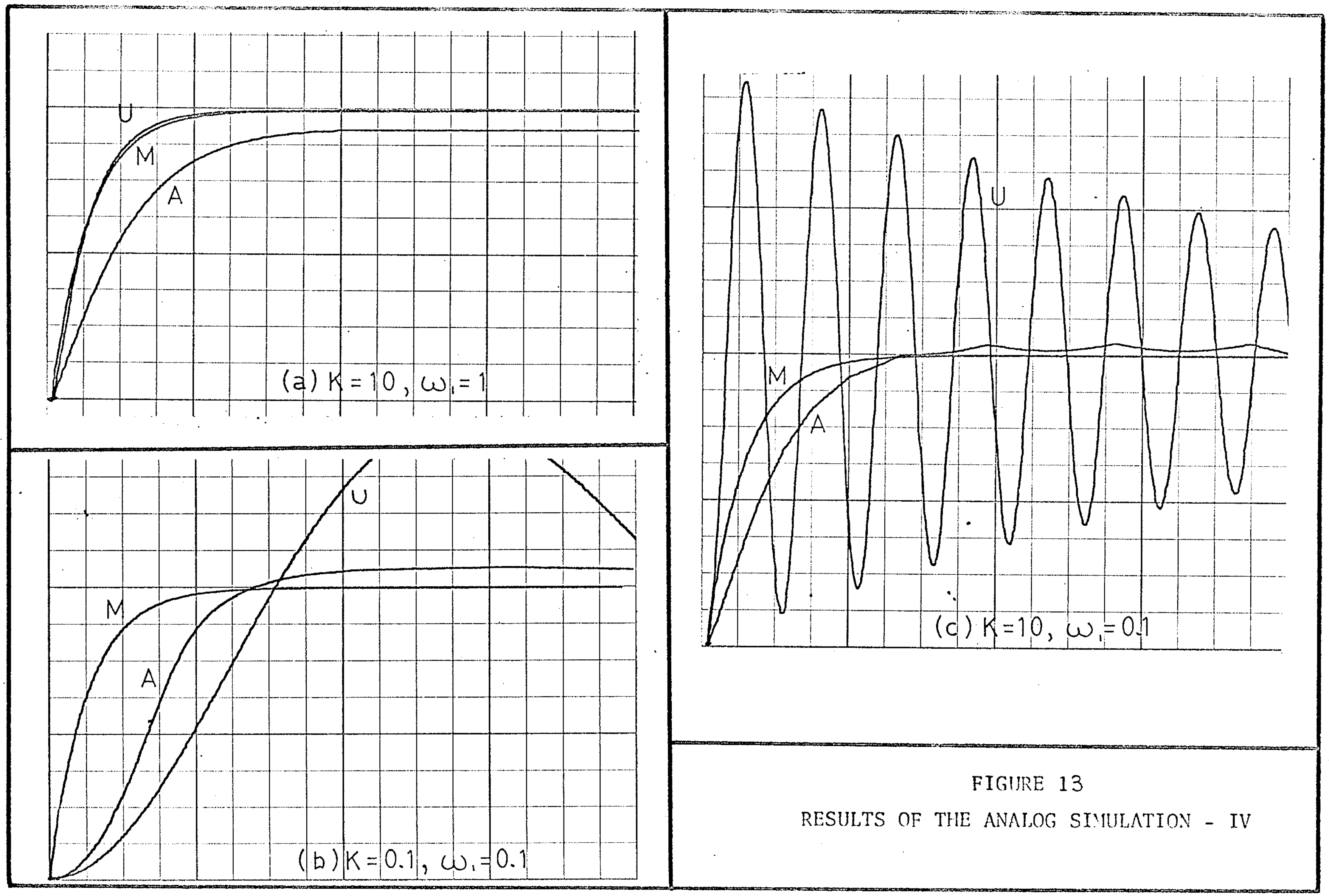

FIGURE 13

RESULTS OF THE ANALOG SIMULATION - IV

to 1 + 0.3s and M equal to 0.1. Figures 10, 11, 12 and 13 show the results of the simulation. On each recording the plant parameter values have been written and three responses shown, the model response, the adaptive system response and the response of the plant with only a unity feedback loop around it (and no model). These three responses have been labelled M, A and U respectively.

In all cases the adaptive system response was satisfactory in that the final error was within the five per cent specified in the design and no objectionable oscillations arose.[7] It is of interest to note that long term observations of the response for K and ω_1 equal to 10 and 0.1 respectively showed occasional errors equal to the maximum allowable but none greater. Thus the quasi-describing function gave a correct answer in this case even though the input to the modulator differed greatly from a square wave in that it had a d.c. component.

Due to the tachometer feedback the system response was somewhat slower than the model in all cases. This problem could be overcome by choosing a model which is slightly faster than the desired response. With the exception of three cases (Figures 10(a), 12(a) and 12(b) this would produce an extremely satisfactory series of responses since all the other responses are remarkably similar considering the wide range of parameter variations and the relatively simple adaptation used. As can be seen from the recordings the three unsatisfactory conditions arose when the system was bery heavily damped.[8]

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7. The adaptive system response in Figure 12(b) did settle at 0.95 although it had not yet reached its final value on the portion of the recording shown.

8. Perhaps "unsatisfactory" is a little strong for the description of these responses since the adaptive system response was a considerable improvement over that of the unadapted system in all three cases.

CHAPTER V

DISCUSSION

The original aim of this work was to devise a simple scheme to control a system with inaccessible parameters which vary over a wide range. The system designed in Chapter IV would be suitable for applications in which a steady state error and possibly small sustained oscillations are not objectionable. Obviously the method would be of no use for a plant of unknown form. The system was designed for a unit step input only but the modification required for other inputs would not be too great. If M and r were made proportional to the input magnitude, one could expect a satisfactory series of responses.

The use of pulse-frequency modulation is most beneficial for heavily damped systems since the pulses tend to speed up the response and generally oscillations are not a problem. In the system designed in Chapter IV the responses for the heavily damped conditions were unsatisfactory because the pulse size permitted was rather small. A more satisfactory series of response could be obtained if an identification device were introduced which identified only the relative damping of the system. Then, if the system were heavily damped, the pulse size could be increased by means of some switching mechanism.

The design procedure is not changed by the introduction of a higher order plant. It may be difficult, however, to chose a suitable H(s) in some cases. In fact one can visualize a situation in which several different compensators may be required, each being suitable for some range of parameter values. Then a means would be required for identifying the plant and selecting the appropriate compensator. At this point it might be well to look for another method of adaptive control.

If one compensator is suitable for the whole range of parameter variations expected, the system described in Chapter IV should handle any rate of parameter variations since no identification is carried out.[1] The problem of nonlinear gain similar to the problem of rapidly varying parameters since, if the input is varying, a nonlinear gain will appear to be a time varying one. Thus the pulse-frequency modulated system described in Chapter IV has one great advantage. Due to the constant amplitude of the pulses, any memoryless, odd nonlinearity can follow the modulator.

A great deal of research needs to be done yet on pulse-frequency modulated systems. One reason for the present interest in this type of system is that space satellites necessarily are controlled by means of pulse-frequency modulation.[2]

It has been mentioned that a pulse-frequency modulated signal is used by the human being for sensing and control. It would be of great interest to discover how the human system overcomes the difficulties inherent in pulse-frequency modulated systems. It is, of course, well known that the human is a learning, time-varying, nonlinear system, but it is difficult to visualize even a learning, time-varying, nonlinear mechanical system employing pulse-frequency modulated signals which would have neither a steady state error nor sustained oscillations.

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1. In fact in the analog simulation, it was observed that rapid variations of K and ω_1 produced no problems in the transient response.

2. T. Pavlidis, "Optimal Control of Pulse-Frequency Modulated Systems", IEEE Trans. Automatic Control, vol PGAC-11, no. 4, pp.676-684, Oct. 1966.

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APPENDIX A

ANALOG SIMULATION OF A PULSE-FREQUENCY MODULATOR

The circuit used for the simulation of the pulse-frequency modulator is shown in Figure 15.

Some explanation is required of the symbol which was used to represent a comparator-electronic switch combination. Figure 14 shows the symbol. If the algebraic sum of the voltages at the comparator inputs is positive, then the signal at the plus (+) appears at the output (with a reversal of sign). If a voltage which is negative or equal to zero appears at the comparator inputs, the signal at the minus (-) input appears at the output (again with a sign reversal).

The operation is then quite straightforward. Initially the integrator has an input equal to the absolute value of the modulator input ($|x|$) and a feedback factor equal to c. The output of Comparator 1 then is equal to zero until the output of the integrator, p, exceeds the threshold, r. As soon as p exceeds r, a negative voltage (equal to the pulse amplitude) appears at the output of Comparator 1. This voltage is fed through a time delay (equal to the pulse width).[1] Once the time delay has elapsed, the input to the integrator becomes zero and the feedback factor 50. This in effect resets p to zero and produces a pulse at the output of Comparator 1. The process can then repeat itself. Comparator 2 is required to make the polarity of the pulses correspond to that of x.

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1. The time delay was accomplished by means of a first order approximation.

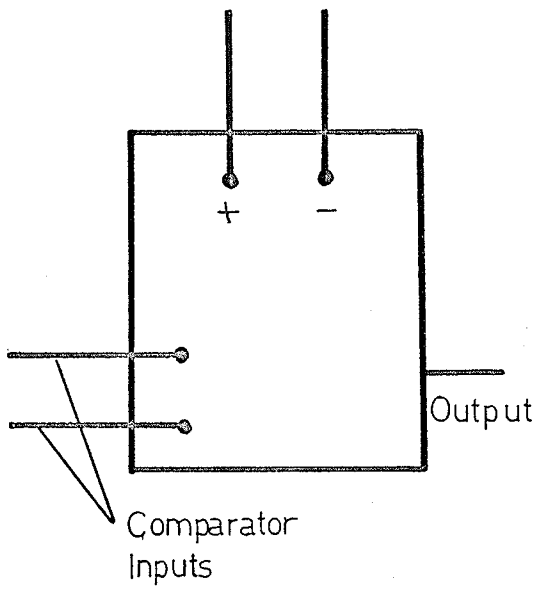

FIGURE 14

SYMBOL USED TO DENOTE COMPARATOR-ELECTRONIC SWITCH

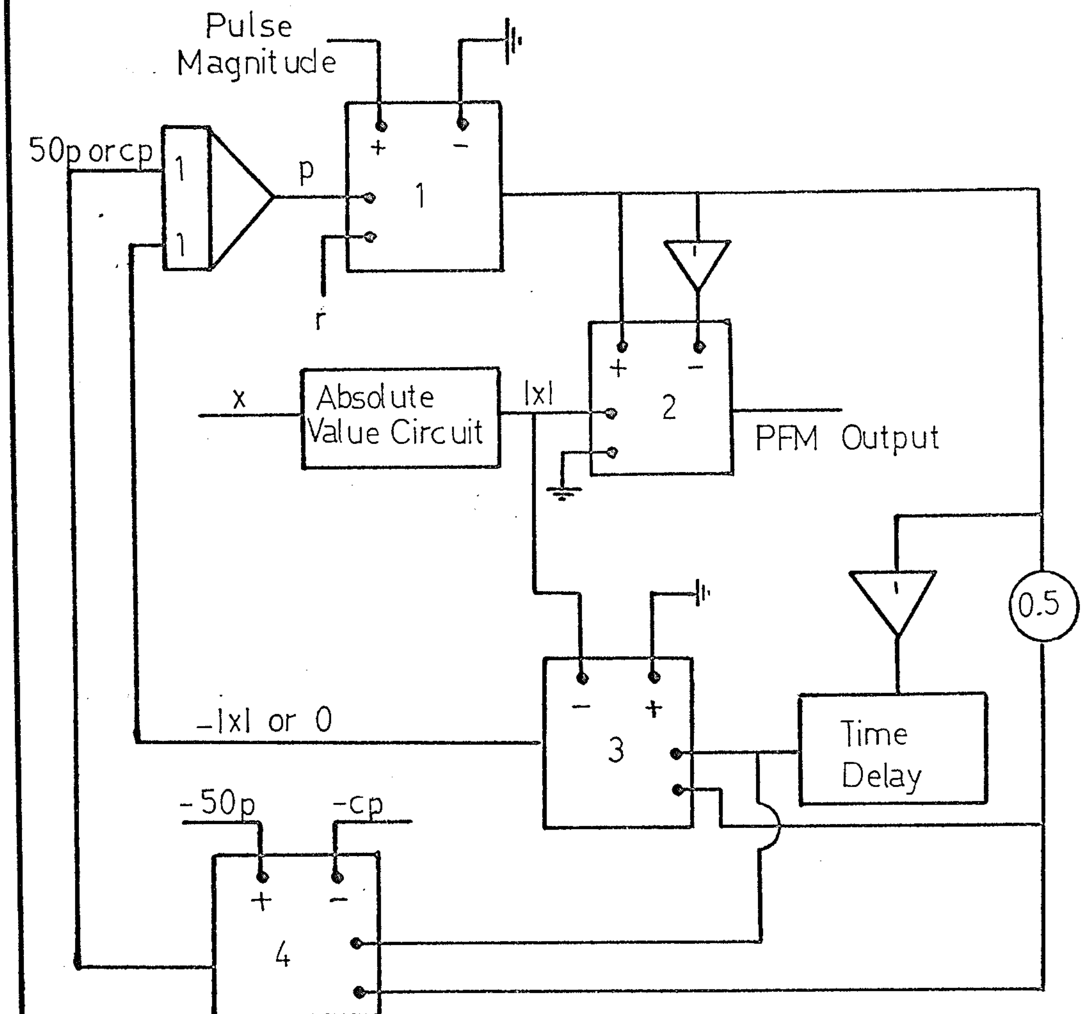

FIGURE 15

CIRCUIT FOR THE ANALOG SIMULATION OF A PULSE-FREQUENCY MODULATOR