Modeling Spectrum Handoff in Overlay Cognitive Radio Networks - A Queueing Theoretic Approach

by

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Abstract

In the overlay Cognitive Radio (CR) networks, the low priority Secondary Users (SUs) must constantly monitor the occupied spectrum to detect the possible appearances of the high priority Primary Users (PUs) within the same spectrum portion. On detection, the SUs must vacate the occupied spectrum portion without interfering with the PUs beyond a certain threshold duration and must opportunistically access another idle spectrum portion to guarantee their seamless communication. This mechanism is known as the spectrum handoff process.

In this thesis, we first introduce a novel approach to model the CR channel which is capable of capturing a more realistic behavior of the spectrum occupancy by both user types and that is more suitable for modeling the spectrum handoff process as opposed to the existing approaches. Then using that as a base we focus on building analytical models to capture the various aspects of the spectrum handoff process in a realistic manner.

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Chapter 1

Introduction

With regard to the effective management of the radio spectrum, several valuable findings and recommendations were discussed in the report presented in [1] that was issued by the spectrum policy task force of the Federal Communications Commission (FCC). Among the issues, the spectrum access was identified as a more significant problem than the actual physical scarcity of the spectrum. In the report, the main reason for this was identified as the legacy command and control spectrum regulation that limits the ability of the potential spectrum users to obtain access resulting in heavy under utilization of the radio spectrum in most of the frequency bands. Alternatively, corresponding legacy scheme resulted in an overwhelming spectrum occupation in some specific bands. In the traditional command and control spectrum allocation approach, a regulatory body assigns license to the spectrum users giving exclusive operational rights within the corresponding spectrum bands for longer terms. This is achieved through mechanisms such as comparative hearings or spectrum auctions. As opposed to the problematic command and control approach, in the corresponding report, it was suggested to balance assigning spectrum usage rights among two newly proposed models, namely “Exclusive Use” and “Commons” or “Open Access” models. Among the two models, the spectrum “Commons” model is identified as the most flexible approach. In the “Commons” model, the spectrum is available to all users such that the operation of the unlicensed users is executed within the established standards and the specific criteria to mitigate the anticipated interference [2]. As opposed to the "Exclusive Use" model, it is identified that the "Commons" approach will lead to a higher spectral efficiency and the emergence of innovative technology. In addition, since no spectrum is exclusively assigned, the corresponding users can effectively switch between frequencies as opposed to the legacy command and control or "Exclusive Use" approaches. In the legacy command and control or "Exclusive Use" models, the licensed spectrum remains unutilized or idle when the licensed operator is not using it, leading to a deliberative spectrum scarcity scenario which is not the case for the "Commons" model. One weakness of the spectrum the "Commons" model is, no one would be given a priority and every one is considered as unlicensed users. Hence, it would discourage the sustainable investments by the users [2]. As a result, a balance between the two newly suggested models is required. In that regard, Cognitive Radio (CR) that was first proposed in [3] was identified as a mechanism to effectively execute and realize the flexible spectrum "Commons" model aspects. In CR, the concept of protecting the incumbent licensed users which have primary rights to operate within their spectrum is taken into account. In CR, a set of unlicensed users may effectively identify the idle or under utilized spectrum portions and opportunistically occupy them while mitigating interference.

The idea is to deploy a set of low priority unlicensed users called the Secondary Users (SUs) that execute the cognitive cycle in the Figure 1.0.1 that was first presented in [4] to identify and opportunistically access the unutilized or underutilized spectrum portions from the high priority licensed users known as the Primary Users (PUs). The SUs are required to effectively reconfigure the channel access parameters such as modulation schemes, access schemes, etc., with regard to the potential spectrum opportunity while mitigating the interference to the PUs. While exercising the cognitive capabilities defined in the cognitive cycle, the SUs mainly rely on Software Defined Radio (SDR) architectures for their parameter reconfiguration purposes [5]. Various forms of cognitive cycles are illustrated in the existing literature. In [3], main attention is given to the SDR perspective, while [5] presents it with respect to the signaling processing or communication theoretic and machine learning perspectives. In [4,6], the cognitive cycle introduced in [5] is further elaborated clearly identifying four spectrum management functions, namely, spectrum sensing, spectrum decision, spectrum mobility and spectrum sharing.

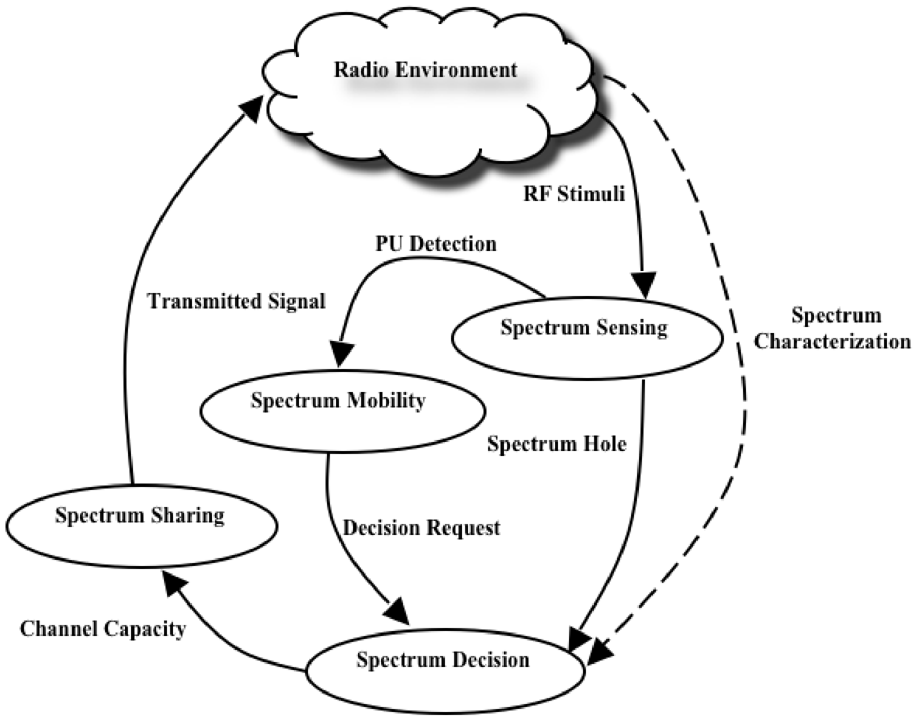

Figure 1.0.1: Cognitive Cycle

1.1 Spectrum Sensing

This is the most crucial function in the cognitive cycle. The SU is required to analyze the radio environment and identify the unutilized or underutilized spectrum portions also known as spectrum holes [1,2] or white spaces [7] in the spectrum. The idea is to determine the presence or absence of the PU signal within the intended spectrum portion by executing the following binary hypothesis test [8] defined by

$$
\begin{aligned}
&H_0 : \text{ PU signal is absent in the spectrum band i.e. } r(t) = n(t) \\
&\text{and} \\
&H_1 : \text{ PU signal is present in the spectrum band i.e. } r(t) = s(t) + n(t)\ ,
\end{aligned} \tag{1.1.1}
$$

where $r(t),\ s(t)$ and $n(t)$ are the received signal by the SU, transmitted PU signal and noise, respectively.

The effectiveness of the sensing mechanism can be evaluated with respect to two probabilities, i.e., the probability of mis-detection, P_m, and the probability of false alarms, P_f, [8] which are defined by

$$
P_m = Prob\,\{\text{Decision} = H_0|H_1\} \tag{1.1.2}
$$

and

$$
P_f = Prob\,\{\text{Decision} = H_1|H_0\}\,. \tag{1.1.3}
$$

The effectiveness of the detection strategy can be increased by several SUs cooperating with each other to make the final decision determining the absence or the presence of the PU signal giving rise to the cooperative spectrum sensing schemes, see [8, 9] for more details. There are several mechanisms that can be utilized to detect the presence of the PU signal in a spectrum band. Here onwards, we summarize the main spectrum sensing techniques in different subsections, but for a comprehensive survey on spectrum sensing, it is advised to refer to [10] for more details. The main spectrum sensing techniques can be categorized as energy detection, matched filter detection, cyclostationary feature detection and wavelet based detection.

1.1.1 Energy Detection

Detecting the energy of the received signals by an SU is the fastest and the simplest approach used to determine the presence or the absence of the PU signal in a desired spectrum portion without any knowledge about the PU signal. For example, in Wireless Regional Area Networks (WRAN), where the cognitive radio standard 802.22h is adapted, the channel sensing is periodically scheduled utilizing two sensing methods. Among the two methods, the method known as “Fast Sensing” is entirely based on energy detection as it takes less than 1 ms to detect the presence of the PU signal as opposed to the other method known as “Fine Sensing” which is based on feature detection of the PU signal [11]. In this approach, the received signal $r(t)$ is squared and integrated over an observation time window. Then the output of the integrator $\mathbf{Y}$ is utilized to perform the binary hypothesis test defined in (1.1.1) with respect to a threshold λ as illustrated in the Figure 1.1.1 [4]. The threshold λ is mainly dependent on the corresponding spectrum bandwidth, sampling rate, noise power spectral density, power spectral density of the received signal. See [11] for details.

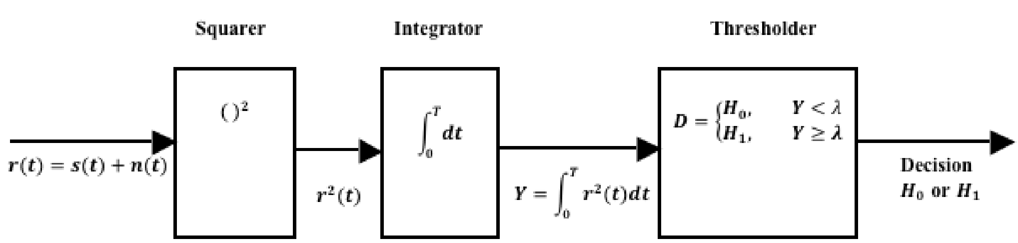

Figure 1.1.1: Energy Detector

The main drawback of the energy detection is the difficulty of selecting an appropriate threshold that leads to the effective detection of PUs minimizing the probabilities defined in (1.1.2) and (1.1.3), especially when the noise levels and the interference levels are high in the targeted spectrum portion. Also, it is unable to differentiate between a PU or another SU that is occupying the spectrum when a potential incoming SU is sensing the channel.

1.1.2 Matched Filter Detection

In this approach, the correlation between the received signal $r(t)$ and the original PU transmitted signal $s(t)$ is taken over the PU symbol duration T_s. Hence, a prior knowledge about the PU transmitted signal is required. For practical purposes, instead of the original PU transmitted signal, the pilot signals that demonstrate a stronger presence within the PU signal can be utilized too [12]. Strict synchronization between the PU transmitter and the SU is assumed. Then the matched filter output $y(t)$, i.e., the correlation outcome of $r(t)$ and $s(t)$ is also sampled at $t = T_s$ intervals. Next, for the corresponding samples $\mathbf{Y}$, the binary hypothesis test defined in (1.1.1) is performed as indicated in the Figure 1.1.2 [4].

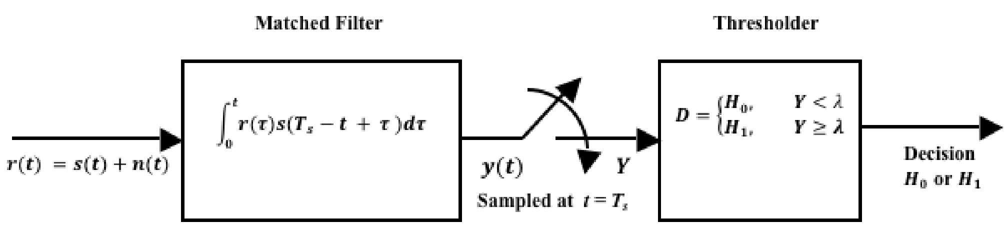

Figure 1.1.2: Matched Filter Detection

Even though matched filter optimizes the Signal to Noise Ratio (SNR) in an Additive White Gaussian Noise (AWGN) channel [8], the requirement of prior knowledge of the PU signal and the requirement of strict synchronization as highlighted in [4] are seen as major drawbacks of this approach. Also, as indicated in [13], for different PU signal types different matched filter implementations are required at the SU side.

1.1.3 Cyclostationary Feature Detection

Many modulated signals have one or more periodicities underlying their random fluctuations and these features can be effectively exploited by utilizing cyclostationary feature detection approach [14]. Hence, as utilized in [15, 16], the method is adapted to detect the presence of the PU signals. Here, periodic nature of the autocorrelation function of the received signal is exploited. The approach can be summarized as follows. First, as mentioned, cyclic autocorrelation function need to be derived from the received signal. Then by taking the Fast Fourier Transformation (FFT) of the resultant, the spectral correlation function which is a function of frequency and cyclic frequency is derived. Then the spectral correlation function is searched to find the unique cyclic frequencies corresponding to the peaks of it as indicated in [4, 8]. This method does not require a prior knowledge of the PU signal. Since the peaks corresponding to the random noise is situated at the zero cyclic frequency and the different modulated signals have unique cyclic frequencies, this method is robust to random noise and interference [4, 8]. However, feature detection is very complex and takes more time. For example, "Fine Sensing" method in 801.22h adapts feature detection and it takes 24.2 ms to detect the digital TV signal [11]. Another disadvantage would be the degradation of the cyclostationary signatures due to multi path fading making the detection hard [15].

1.1.4 Wavelet Based Detection

This is a wide band spectrum sensing approach. Consider a scenario that the PU signal is occupying several consecutive frequency sub bands over a wide frequency band. Then the Power Spectral Density (PSD) of the corresponding PU signal shows sudden abrupt changes at the boundaries of any two consecutive sub bands while it shows smooth variation within the corresponding sub bands [8]. Hence, the idea is to detect this sudden abrupt variation at the boundaries [8, 17]. In that regard, first, PSD of the received signal is obtained. Then the continuous wavelet transformation of the PSD is obtained by utilizing an appropriate wavelet smoothing function. Then sharp variation points are identified by tracking the local maximas of the modulus of the first order derivative of the wavelet transformation function obtained above. See [17] for details. The process is complex and the PUs may occupy sub bands in a non contiguous manner as well making the detection difficult.

1.2 Spectrum Decision

Once the potential spectrum opportunities or the spectrum holes are identified through an appropriate spectrum sensing method, it is essential to take the decision of selecting the spectrum holes that best fit to the requirements of the SUs. For this purpose, several essential steps have been identified in [4]. First, the spectrum holes need to be characterized based on the statistics obtained with regard to the dynamic nature of the CR environment and the spectrum parameters. As for understanding the dynamic nature of the CR environment, it is essential to analyze the PU activities of the spectrum portions. Hence, appropriate modeling of the CR channels plays a vital part in this regard. Also, spectrum parameters such as permissible interference levels, maximum allowed transmission powers, wireless link errors, etc., need to be taken into account. Then based on the QoS requirements and other required criteria of the SUs, suitable spectrum holes needed are selected and finally parameter reconfiguration is carried out by exercising the available features under the SDR architecture. In [18], based on the QoS requirements of the best-effort or real time SU application layer functionalities, it is identified that the spectrum decision process need to be initiated in three separate occasions. It is essential to initiate spectrum decision in situations such as 1) when an SU decides to come into the CR network, 2) when a PU appears in the current spectrum hole occupied by an SU, and 3) when the quality of the exiting channel degrades below the expected level. In here, dynamic nature of the CR environment with respect to the PU activity is modeled by assuming the CR channels as two state ON-OFF processes. In Chapter 2, a detailed version of various CR channel models utilized to represent the PU activity is presented. In [19], it is suggested to implement the spectrum decision process through Regional Spectrum Brokers (RSBs), where each RSB maintains bandwidth, delay, FPUA (Frequency of Primary User Appearance) statistics for each spectrum channel for a predefined area that the RSB assumes authority. In [20], for spectrum decision, a weighted set of parameters such as channel capacity, acceptable error rate, delay, jitter and the probability of the PU arrivals that is derived based on the Poisson arrivals are utilized.

1.3 Spectrum Mobility

When an SU occupies a spectrum hole, it should keep on monitoring the channel by executing in-band sensing process to detect the arrival of a PU. On detection, the SU should vacate the current spectrum portion and should find another spectrum hole to continue communication [4]. This process is known as spectrum mobility and it gives rise to the concept of spectrum handoff [4]. In general, SUs' spectrum handoff process is modeled either as a reactive scheme or as a proactive scheme [4]. Such a distinction is done based on the approach taken to vacate the currently occupied spectrum portion and the approach taken to decide on the next spectrum opportunity with respect to the appearance of a PU in the current channel. If the next spectrum opportunity is identified by executing out of band spectrum sensing procedure and vacate the current channel at the moment the SU got interrupted due to the appearance of a PU, then it is identified as a reactive approach. In proactive methods such decisions are formulated in advance of the actual appearance of the PUs based on various predictive methods. The idea is to identify the potential idle spectrum portions in advance, so they can be directly considered at the time of an arrival of a PU or even before the arrival of a PU. A considerable latency is associated with spectrum handoff due to the delay associated with discovering the new spectrum portions and due to the requirement of parameter reconfiguration before using the discovered spectrum [4]. In proactive methods, the associated delay is much less than the reactive methods, but proactive methods are dependent on the effective modeling of the CR channels.

In addition to the PU appearances, the spectrum handoff may be required due to the mobility of the SUs and due to the quality degradation of the current spectrum portion [4,6]. In those cases, an SU may require to switch to another spectrum portion too. See [4, 6] for more details and in Chapter 2 we have summarized the existing spectrum handoff modeling techniques in detail.

1.4 Spectrum Sharing

As implied, spectrum sharing focuses on effective spectrum resource allocation or spectrum resource sharing among multiple SUs in a CR network facilitating opportunistic access of the spectrum while mitigating interference to the PUs [4]. That is, it mainly focuses on effective power allocation and channel allocation aspects in a CR network. Even though a strict demarcation may not be always possible due to the overlapping nature of the approaches, in [6], spectrum sharing approaches are broadly categorized as 1) centralized and decentralized approaches based on the network architecture, 2) cooperative and non cooperative approaches based on the spectrum allocation behavior and 3) overlay and underlay approaches based on the spectrum access technique.

In the centralized spectrum sharing schemes, a central entity is assumed as the spectrum arbitrator or the spectrum enforcer facilitating the spectrum access of the SUs. In [21], a CR Base Station (CR BS) is assumed as the central entity and both CR BS and the SUs participate in the spectrum hole detection procedure. As for the SUs, they are required to update the information about the potential spectrum opportunities to the CR BS, so the CR BS can come up with the optimum approach to allocate available channels. Even though [21] assumes the communication between the SUs and the CR BS takes place via an opportunistically accessed channel, many centralized approaches assume of a dedicated Common Control Channel (CCC) in that regard. In [22], it assumes a dedicated control channel for the communication between the centralized entity and the rest of the nodes. In a cognitive radio environment, the practicality of this assumption is at question since the CCC may also need to be opportunistically obtained as in the case of other channels. This will lead to the challenge of carefully selecting a uniformly acceptable channel throughout a large portion of the network for a considerable portion of time. In addition, it should not lower the spectrum utilization efficiency due to the possible overheads and the dedicated channel allocation.

In decentralized schemes, supporting infrastructure such as central base stations are not assumed. Instead, SUs are required to come up with proper spectrum sharing strategies in a distributed manner. Instead of SUs cooperating with a central entity as in centralized schemes, in decentralized approaches, SUs can cooperate with each other by exchanging required information such as sensing details, transmitted power levels, observed interference levels, etc., to come up with a proper spectrum allocation strategy. Alternatively, SUs can work as standalone systems occupying spectrum in a competitive manner giving rise to decentralized cooperative and non cooperative spectrum sharing schemes respectively.

Motivated by non cooperative games in game theory, authors of [23] have proposed Non cooperative Power Control Game with Pricing (NPGP), which is derived from Non cooperative Power Control Game (NGP) used in traditional CDMA systems for realizing the non cooperative spectrum resource (power) allocation. In [23], each terminal maximizes its net utility which is given by the difference between the defined utility function and the pricing function. The class of pricing functions studied is linear in transmit power, where the pricing function is simply the product of a pricing factor and the transmit power. The power control algorithm is realized by a base station announcing the pricing factor to all the users which is the only involvement of the base station as opposed to centralized schemes. The announcement is followed by each terminal choosing the transmit power from its strategy space that maximizes its net utility. It has shown that the approach is capable of converging towards Nash Equilibrium and it is indeed Pareto improved which is a plus point. However, it assumes of a linear pricing function and the utility function is dependent on modulation schemes. In [24], authors have suggested Price based Iterative Water Filling (PIWF) for the resource allocation problem. The proposed method adapts one of the resource allocation algorithms known as Iterative Water Filling (IWF) coupled with pricing. In this approach, each user maximizes its own utility function by performing single user price based water filling. In here, each user first adjusts its pricing factor over all channels and then determines its best response, i.e., the optimal channel / power combination based on the measured total noise-plus-interference level over each channel. Then each user tries to achieve the best response, which is to maximize its individual utility function subject to the given constraints. The same procedure is iteratively applied for every user and eventually converging to the Nash Equilibrium (NE). In the suggested approach, certain information that is required to effectively configure the transmit power is exchanged embedded within the channel reservation messages such as RTS / CTS assuming a certain degree of cooperation between the SUs within the local neighborhood. Here, the sequential iterative execution may need a longer convergence time for larger networks. In [25], a distributed non cooperative power allocation mechanism based on a classic decentralized optimization technique is proposed. It is based on the dual decomposition and sub gradient descending approach as opposed to the game theoretic approach. The proposed scheme allows CR users to intelligently control their access parameters based on inference from observed link control signals of the PU communication. This allows the SUs to achieve a higher overall spectrum utilization while limiting their interference to the PU network. In this specific implementation, SUs are allowed to listen to the PUs' feedback channels to assess their own interference on the PU receiver, and adjust radio power accordingly to satisfy the PUs' outage probability constraints. The proposed approach introduces a discounted distributed power control algorithm to achieve non intrusive secondary spectrum access without either a centralized controller or active SU cooperation.

Above decentralized methods fall under the category of spectrum underlay technique as an SU is trying to coexist with the PUs as well as with the other SUs by effectively controlling its own transmitting power to satisfy the established constraints and to mitigate the interference caused to the PUs. Since the transmitted power is always controlled, this technique suffers from short transmission ranges. As opposed to spectrum underlay, spectrum overlay concentrates on opportunistic access of the spectrum portions that are free from the PUs and vacate the occupied spectrum portions on detection of the PU arrivals. In this technique, the SUs must vacate the channel within a certain time duration without further interfering the PUs beyond that. This is an interference avoidance mechanism, where spectrum handoff is severely exercised. Since SUs are allowed to transmit at their full potential higher transmission ranges can be achieved.

1.5 Main Contributions and Outline of the Thesis

In this thesis, we investigate an important problem in current CR channel modeling approaches. That is, the current CR channel models consider the busy (ON) and idle (OFF) states of a channel with respect to the PU activity only. However, the channel states are also affected by the SUs' activity as well. We mainly focus on two main aspects, 1) modeling the CR channel with respect to both PU and SU activity, 2) utilizing the proposed CR channel model to capture the spectrum handoff process with respect to both traffic components.

The outline of the thesis is as follows. In Chapter 2, existing CR channel models and existing handoff modeling approaches are summarized. In Chapter 3, the proposed channel models are introduced. In Chapter 4, proposed channel models are utilized to model the spectrum handoff process. In Chapter 5, numerical results derived based on the models are presented. Finally, concluding remarks are addressed.

Chapter 2

Existing CR Channel Models and Handoff Modeling Techniques

In this chapter, we first focus on studying the characteristics of the existing CR channel models. In here, we identify their strong and weak aspects. Then we study the existing handoff models.

2.1 Existing CR Channel Models

2.1.1 Two State Markov and Semi Markov Channel Model

This is the most common channel representation that can be observed in the existing literature. For example, in [26–31], the underlying CR channel model is the two state Markov model. This model assumes the channel ON-OFF periods are geometrically distributed in discrete time domain as indicated in (2.1.1), or in continuous time space they are exponentially distributed. The interpretation of (2.1.1) is, the channel may remain busy for $k_r - 1$ time slots with the probability $1 - r$ and it will turn from busy to idle with the probability r. Similarly, the channel may remain idle for $k_q - 1$ time slots with the probability $1 - q$ and it will turn from idle to busy with the probability q.

$$
\begin{aligned}
f_{on}^{k_r} &= (1-r)^{k_r-1} r \\
f_{off}^{k_q} &= (1-q)^{k_q-1} q
\end{aligned} \tag{2.1.1}
$$

Different discrete time domain representations of the same channel model can be identified in the existing literature. For example, the Discrete Time Markov Chain (DTMC) representation of the channel would be as follows.

$$
\mathbf{P} = \begin{array}{c c} & \begin{array}{cc} OFF & ON \end{array} \\ \begin{array}{c} OFF \\ ON \end{array} & \begin{pmatrix} 1-q & q \\ r & 1-r \end{pmatrix} \end{array}
$$

This form of ON-OFF representation was widely used in teletraffic models to represent the bursty sources. When modeling the bursty sources, the sending of information is modeled as a consecutive set of active ON periods separated by silent OFF periods [32]. When cognitive radio emerged, researchers attempted on relating the channel occupancy by the PUs in a similar manner based on the two state Markov model. In CR context, the ON state means, the channel is occupied by a PU. Hence, the channel is identified as busy by an in coming SU to the channel. The OFF state means, the channel is not occupied by a PU. The channel is thus seen as idle by an incoming SU. The major drawback in this approach is, it only assumes of the PU traffic component as observed by an incoming SU. In reality, in a CR channel, the channel may contain either PU or other SU traffic components as observed by an incoming SU to the channel. The actual channel busy or idle periods depend on either traffic components not only on the PU traffic component. Also, the traffic generated in networks does not closely resemble or align with this type of representation as this type of representation is unable to capture the longterm dependence of the network traffic or the self similarity nature of the traffic as highlighted by [33]. In the CR network context, it is also appropriate to assume such correlation within and between the ON and OFF periods, but it is not captured in the discussed representation.

In addition to the above approach, without adhering to the more specific geometric or exponential distributions, [34] and [35] represent the above two state ON-OFF process as a more generic semi Markov processes. In here, the model is formed such that it represents the sojourn times of the ON and OFF periods with random variables of arbitrary probability density functions, but still the ON periods are assumed to be independent and identically distributed and same holds for the OFF periods. Also, the ON and OFF periods are assumed to be independent of each other as well.

Slightly deviating from the above approaches, modeling the PU activity as an M/G/1 system is commonly observed too. In this case, the arrival process is still assumed to be Poisson and the PU packet length is assumed to take an arbitrary distribution. Then based on those assumptions, authors derive the probabilities that the channel being busy (ON) or idle (OFF) to formulate the model [36–38].

2.1.2 Higher Order Markov Channel Model

This is an extension of the above mentioned first order ON-OFF process. This was first proposed in [39] and further elaborated in [40] to capture the self similarity and the long range dependence aspects of teletraffic. It was adapted for CR networks in [41] to capture the correlation aspects of the CR channel evolution due to the PU traffic component. According to [40], an n^{th} order or n level hierarchical ON-OFF process $Y(t)$ can be represented by independent first order ON-OFF processes $X_i(t)$ such that

$$Y(t) = \prod_{i=1}^{n} X_i(t). \tag{2.1.2}$$

For example, if $n = 2$, then the resulting higher order ON-OFF process is called a 2^{nd} order or two level ON-OFF process which can be graphically represented with first order processes as illustrated in the Figure 2.1.1 similar to the representation in [41].

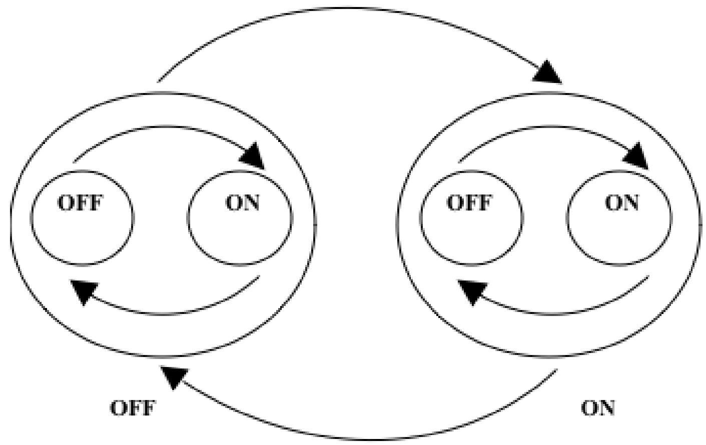

Figure 2.1.1: The 2^{nd} order ON-OFF process

In the Figure 2.1.1, if we denote the OFF state as 0 and the ON state as 1, the system will have an output of 1 when it is in the right most state, where all the ON-OFF processes are ON. In addition to the above form of representation, we can represent the corresponding 2^{nd} order ON-OFF process as a Markov process with an augmented state [40]. The resulting Markov process is illustrated in the Figure 2.1.2.

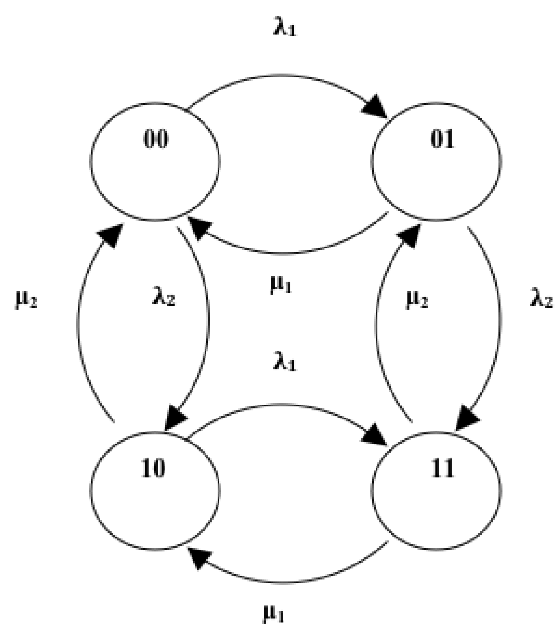

Figure 2.1.2: The 2^{nd} order Markov process

Note: We use continuous time representation here and self transitions are ignored.

In this case, the output of the process is 1 only when the system is in the state 11. That is, the channel is in the ON state. Rest of the 3 states 00, 01 and 10 are mapped into the OFF state. This model only represents the channel evolution with respect to the PU traffic component. Also, the transition within the ON period contains a single state while the OFF period has many states. Hence, the ON state does not contain long term memory as it is not heavy tailed [42]. Also, it was pointed out in [43], having a heavy tailed OFF period as opposed to having a heavy tailed ON period is less practical in the networking context. Hence, in CR context, it is also logical to follow the same, i.e., to develop models with a heavy tailed ON period.

2.1.3 Multi Phase Semi Markov Channel Model

In this approach, the sojourn times of the ON and OFF states are modeled as Phase Type (PH) distributions with multiple phases as opposed to the above geometric or exponential distributions [44]. Since most of the distributions can be represented as a PH distribution [45], this is a more generic form of a channel representation. In here, the sojourn time of the OFF (idle) state of the channel is represented by the parameters $(\boldsymbol{\alpha_q},\ \mathbf{Q})$ of order m, where m is the number of phases relevant to the OFF (idle) state with $\mathbf{Q^0} = \mathbf{e} - \mathbf{Q}\mathbf{e}$. The sojourn time of the ON (busy) state of the channel is represented by the parameters $(\boldsymbol{\alpha_r},\ \mathbf{R})$ of order n, where n is the number of phases relevant to the ON (busy) state with $\mathbf{R^0} = \mathbf{e} - \mathbf{R}\mathbf{e}$. In both cases, **e** is a column vector of ones with the same order as m or n. In addition, in both **Q** and **R**, at least one of the rows should be strictly less than one. Also, $\boldsymbol{\alpha_r}$ and $\boldsymbol{\alpha_q}$ are the initial state vectors such that $\boldsymbol{\alpha_r}\mathbf{e} = 1$ and $\boldsymbol{\alpha_q}\mathbf{e} = 1$.

As indicated in [44], if we denote the ON state as 1 and the OFF state as 0, we can form a discrete time semi Markov chain to capture the joint representation of the two states and the different phases of their sojourns by modulating the CR channel behavior as follows with the state space being $(0, u) \cup (1, v)$. In here, $u = 1,\ \ldots,\ m$ and $v = 1,\ \ldots,\ n$.

$$
\begin{array}{cc}
 & \begin{array}{cc} OFF & ON \end{array} \\
\mathbf{P} = \begin{array}{c} OFF \\ ON \end{array} & \begin{pmatrix} \mathbf{Q} & \mathbf{Q^0}\boldsymbol{\alpha}_r \\ \mathbf{R^0}\boldsymbol{\alpha}_q & \mathbf{R} \end{pmatrix}
\end{array}
$$

However, in both occasions [44] and [46] that this channel model is utilized, the authors only captured the sojourn of the two states with respect to the PU traffic payload. Also, this representation is unable to capture the correlation of these two states as well.

2.1.4 Multi State Alternating Markov Renewal Channel Model

In CR context, authors of [7] first proposed this method to capture the behavior of a CR channel. Based on [7], we can further elaborate on the approach. As Markov renewal process is capable of capturing broader range of distributions as well as the correlation aspects of the ON and OFF periods, this is a better way of representing the CR channel compared to the above mentioned methods. In here, the OFF (idle) state of the channel is represented by a set of states $1,\ \ldots,\ n_i$ and the ON (busy) state of the channel is represented by a set of states $n_i + 1,\ \ldots,\ n_i + n_b$. The sub stochastic matrices $\mathbf{D_i}$ and $\mathbf{D_b}$ capture the transitions within the OFF and ON states respectively. The matrices $\mathbf{d_{bi}}$ and $\mathbf{d_{ib}}$ capture the transitions from ON (busy) to OFF (idle) and OFF (idle) to ON (busy) respectively such that $\mathbf{D_b}.\mathbf{1} + \mathbf{d_{bi}}.\mathbf{1} = \mathbf{1}$ and $\mathbf{D_i}.\mathbf{1} + \mathbf{d_{ib}}.\mathbf{1} = \mathbf{1}$. Based on the above matrices, the channel evolution can be represented as follows.

$$
\begin{array}{cc}
 & \begin{array}{cc} OFF & ON \end{array} \\
\mathbf{P} = \begin{array}{c} OFF \\ ON \end{array} & \begin{pmatrix} \mathbf{D_i} & \mathbf{d_{ib}} \\ \mathbf{d_{bi}} & \mathbf{D_b} \end{pmatrix}
\end{array}
$$

The authors have suggested to adapt the transition matrix introduced in [47] that captures the self similar nature of the channel occupancy as follows.

$$
\mathbf{P} = \begin{array}{c} \\ OFF \\ \\ \\ \\ ON \end{array}
\begin{array}{c}
\begin{array}{ccccc} OFF & & & & ON \end{array} \\
\begin{pmatrix}
1-\frac{b}{a} & 0 & \dots & 0 & \frac{b}{a} \\
0 & 1-\left(\frac{b}{a}\right)^2 & \dots & 0 & \left(\frac{b}{a}\right)^2 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \dots & 1-\left(\frac{b}{a}\right)^{n-1} & \left(\frac{b}{a}\right)^{n-1} \\
 & & & & \\
\frac{1}{a} & \frac{1}{a^2} & \dots & \frac{1}{a^{n-1}} & 1-\sum_{k=1}^{n-1}\frac{1}{a^k}
\end{pmatrix}
\end{array}
$$

For a given n, the channel parameters a and b are determined as follows.

$$
E[B] = \left(\sum_{k=1}^{n-1} a^{-k}\right)^{-1} \tag{2.1.3}
$$

$$
\rho_{pu} = \frac{1-\frac{1}{b}}{1-\frac{1}{b^n}} \tag{2.1.4}
$$

In above, $E[B]$ is the average number of consecutive slots that the channel stays busy and ρ_{pu} is the utilization of the CR channel with respect to the PU traffic component. As a result, the model is formulated ignoring the SU traffic component. Also, the transition within the ON period only contains a single state while the OFF period has many states. Hence, the ON state does not contain long term memory as it is not heavy tailed. Also, it is more practical that the ON period to be heavy tailed as explained in the Subsection 2.1.2.

2.1.5 Channel Representation with M/G/1 Priority Queues

In this approach, the focus is not given to model the CR channel as a ON-OFF system. Instead, two M/G/1 queues with different priorities are utilized to represent the CR channel [48–52]. The high priority queue is utilized to represent the PU activity while the low priority queue is used to represent the SU activity. In here, the arrival processes of both user types are assumed to follow a Poisson process. The service times are assumed to follow an arbitrary distribution. The traffic loads of the PUs enter the high priority queue while the traffic loads of the SUs enter the low priority queue. Within the same queue First Come First Served (FCFS) policy is utilized.

2.2 Existing Spectrum Handoff Modeling Techniques

2.2.1 Modeling Handoff with M-Dimensional Markov Chains

In the simplest form of this approach, the behavior of the CR network is captured with respect to the total number of PUs and SUs in the system. Also, the approach is not based on a specific underlying CR channel model as in our case. In the simplest form of the approach, a two dimensional Markov chain is utilized such that one of the dimensions capturing the total number of PUs in the network at a given instance while the other dimension is capturing the total number of SUs in the network. In [53–55], the authors introduced the basic structure of the proposed model based on the Poisson arrivals and the exponential service times. In [56], the authors have further extended the model by introducing the Markovian Arrival Process (MAP) to capture the arrivals which is a more generic approach, but still the service times are exponentially distributed. By further extending the prior models, in [57], the authors have brought in the preemptive resume of the SU service with respect to a multi phase service process in the presence of high priority PUs as opposed to considering the preemptive resume of service under the exponential service times. In here, the service processes is represented with PH distributions and the behavior is modulated by a continuous time Markov chain as in rest of the cases. However, this approach requires expanding the corresponding Markov chain up to five dimensions and it is not based on a specific underlying channel model. In [58], the authors have suggested a three dimensional Markov chain such that the first dimension capturing the actual number of PUs in the system, the second dimension capturing the number of SUs in the system and the third dimension capturing the interrupted number of channels, but still the authors adapt the Poisson arrivals and the exponential service times to build their model. These approaches mainly model the reactive spectrum handoff scheme.

2.2.2 Modeling Spectrum Handoff Using ON-OFF CR Channels

In this approach, the CR channel is represented using one of the prior described channel representations in the Section 2.1. Throughout the sojourn time of the ON state, the channel is taken to be occupied only by a PU. The SU can occupy the channel during the OFF period, i.e., during the sojourn time that the channel is not being occupied by a PU. Based on the appearances of the PUs, the SU has to initiate handoff and need to select another spectrum opportunity to continue communication. In that regard, the ON-OFF channel models can be effectively utilized to develop various channel evacuation criteria and channel selection criteria.

For example, in [36–38], assuming Poisson arrivals and arbitrary distributions for the packet lengths of the PUs, the authors have derived the probabilities of a channel i being in the ON state or being in the OFF state at any given time t. The PUs are assumed as M/G/1 systems. Then the handoff is assumed to be required whenever the probability of the channel being in the OFF state is less than a certain threshold. Each SU predicts the current channel availability at the end of the frame according to the above approach. If the condition is met, the SU needs to perform a spectrum handoff at the end of the frame to avoid harmful interference to a PU who may use the current channel. In this case, the approach is formulated as a proactive method.

Also, in the case of selecting the target channels, the strategies can be formulated by adapting the underlying channel models described in the previous subsections. Based on the probabilities of a channel being in the ON or OFF states, one can develop the corresponding target channel selection criterion in a handoff scenario. The channel selection criteria are developed based on the characteristics observed among the channels assuming their behavior as an ON-OFF system. This is a situation that the SU needs to take a spectrum decision as mentioned in the Section 1.2. For example, in [59], based on the probability P_i that a channel i will be idle in the next time slot, one can choose the channel with the highest P_i or choose the channel with the longest expected remaining idle period $E(T_i)$ such that $E(T_i) = \frac{P_i}{\lambda_{xi}}$. In here, λ_{xi} is the mean of the exponentially distributed OFF period.

Especially in the situations such as selecting the channel that has a highest probability of becoming idle in the next time slot, modeling the channel with respect to PU activity by neglecting the other SU traffic component is not a better approach. Also, when modeling a situation that the next spectrum opportunity is identified reactively initiating spectrum sensing through energy detection, considering other SU activity is vital as energy detection is unable to differentiate between the PU and SU signals. When addressing handoff related aspects, the way that the CR channel is modeled plays a special importance, but the CR channel is always modeled with respect to the PU traffic only. Based on the channel model described in the Subsection 2.1.3, the authors of [44] derive several probabilities that are useful to formulate different channel selection criteria. For example, the authors derive the probabilities such as 1) the probability that a channel is idle when it was sensed, but gets busy before the end of M time slots, 2) the probability that the channel was idle when it was sensed and it remained idle for next $M - 1$ time slots, etc. Then the channel selection criteria such as 1) choosing the channel that minimizes the interference caused from the SU to the PUs, 2) choosing the channel that maximizes the SU's throughput, etc., are formulated based on those probabilities.

In addition, these ON-OFF channel models can be utilized in combination with various prediction mechanisms when determining the corresponding status of the channels and determining the next channel such that guarantying seamless communication in handoff scenarios. For example, the handoff models that are based on the Partially Observable Markov Decision Process (POMDP) tend to adapt both two state Markov ON-OFF channel representation and higher order Markov channel representation as the underlying channel model when representing the channel evolution [26, 41, 60]. However, what lacking in these approaches is, they do not focus on the analytical modeling of the preemptive resume nature of the SU service in a handoff situation with respect to both PU and other SU traffic components.

2.2.3 Modeling Spectrum Handoff Using M/G/1 Priority Queues

As mentioned in the Subsection 2.1.5, in this approach, the CR channel is modeled utilizing two M/G/1 queues with different priorities such that the high priority queue representing the PU traffic load while the low priority queue representing the SU traffic load. The SUs can utilize the channel when the channel is not utilized by a PU. On arrival of a PU, it preempts the service of the SU that is already in service. Then the SU is placed ahead of the low priority queue of the current channel or is placed at the end of a low priority queue of another channel. Such placement is performed based on the SU's decision to remain in the current channel and resume service once it turns idle or switch to another channel. See [48–50] for details.

Alternatively, deviating from representing the CR channel as a system of two priority queues, in [61], authors model the event of missing or vacating the channel due to the appearance of a PU as a failure or an interruption to the SU communication and denote the operating and recovery periods as the time to failure and time to recovery respectively. That is, the operation of the SU is modeled with alternating renewal processes. From the queueing point of view, the output buffer of the SU is modeled as a queue with random service interruptions assuming that the arrival process follows a Poisson distribution with a general service time distribution. Then the operation of the node is modeled as an M/G/1 queue with random service interruptions.

Chapter 3

Proposed Channel Models

In this chapter, we present the underlying channel models we developed to capture the spectrum handoff process. In the Section 3.1, we will first explain the main attributes of the proposed channel models and their key differences compared to the already existing channel models. Then in the Section 3.2, we will illustrate the system model for the proposed channel representations in detail utilizing the background theory illustrated in the previous chapters. Also, we present several variations of the same channel model that are capable of capturing more SU channel occupancy scenarios at the cost of increased complexity. The purpose is to check the possibility of adhering to the simplest form of the channel representation if all models behave without major differences when their behavioral results are analyzed. First, we form the corresponding channel models with respect to the preemptive resume service regime. Next, in the Section 3.3, we will exploit the possibility of integrating the preemptive repeat service regime to the same channel model without major modifications. Finally, in the Section 3.4, we will illustrate the derivation of useful performance metrics.

3.1 Key Attributes of the Proposed Channel Models

In the proposed models, we mainly consider both PU and other SU traffic components in a target channel as observed by an incoming SU. Our model facilitates tracking the service phase, where the SU got interrupted and the phase at which the service will resume or repeat. When comparing with the existing channel models described in the previous chapter, this is a more realistic way to represent the service process of the SUs with respect to the high priority PUs. In addition, we capture the joint behavior of the PU and SU activities. As a result, our model is capable of capturing the correlation between the idle and busy periods due to both types of user activities not limiting to the activities of the PUs. We represent the arrival process and the service times of the users in a more general manner that allows capturing broader range of distributions. In addition, the proposed model can be considered as an ON-OFF system, where the sojourns of the ON and OFF periods consisted of multiple phases representing the channel evolution as observed by an incoming SU.

3.2 System Model

We first consider a single channel scenario and develop our models as a base to model the spectrum handoff process later on. We can observe several states of the CR channel if we consider it with respect to both user activities as seen by an incoming SU. The channel may be: 1) idle due to absence of either a PU or an SU, 2) occupied by an SU in absence of PUs, 3) occupied by a PU with no SU waiting and 4) occupied by a PU while an SU remains interrupted in the channel. We denote those four states together with the appropriate supplementary phases as $(\mathbf{0}, \mathbf{0})$, $(\mathbf{0}, \mathbf{S})$, $(\mathbf{P}, \mathbf{0})$ and $(\mathbf{P}, \mathbf{S})$ in a Discrete Time Markov Chain (DTMC). We will assume initially the channel is idle. As the channel state evolves, the system can enter to any of the above four states. Also, in a spectrum handoff scenario due to the channel switching delays, an SU may find the channel turned out to be busy at the time it is actually going to occupy the channel even though the SU sensed the channel to be in its idle state before.

Considering the above mentioned situations, we allow the channel occupancy of the users to assume a general form modulated by a DTMC. We define an $n \times n$ matrix $\mathbf{D}_{u,\,v}$ to represent the arrival process that consists of u ($u = 0, 1$) PUs and v ($v = 0, 1$) SUs. In CR networks, once a PU accesses a channel it holds the channel for service until it finishes transmitting since it has higher priority over SUs. This process could be effectively modeled as a more generic PH distribution with the representation $(\boldsymbol{\beta_p},\ \mathbf{S_p})$ and the corresponding $\mathbf{S_p^0}$ which is given as

$$\mathbf{S_p^0} = \mathbf{e} - \mathbf{S_p}\mathbf{e}, \tag{3.2.1}$$

where **e** is a column vector of all ones with appropriate order. For details, see [45].

Similarly, if an SU could have uninterrupted access to a channel, then its channel holding time is also assumed to follow a PH distribution with the representation $(\boldsymbol{\beta_s},\ \mathbf{S_s})$ and the corresponding $\mathbf{S_s^0}$. However, an SU's service may be interrupted by the PUs. Therefore, it is required for an SU to keep track of the phase that the service was interrupted and resume it. Alternatively, may need to repeat its service. In that regard, we could effectively adapt the concepts of the discrete time domain time limited vacation queues and discrete time domain preemptive priority queues which were addressed in [62] and [63] respectively. We define a matrix $\mathbf{Q}$ which captures this process. For example, $\mathbf{Q}_{i,\,j}$ represents the probability that the service was interrupted in the phase i and starts in the phase j, when it resumes or repeats its service [62]. We will assume if the channel is already occupied, the arriving SU will not get into the channel and will not wait in the channel until it becomes idle. Instead, the SU will get bumped out of the channel. An SU will only wait in the channel until the channel turns free if it successfully gets into the channel when the channel was free and later if it got interrupted while in service. In that case, such SUs can resume service when the channel turns idle. An SU that just arrived and got into the channel as the channel was idle, but got interrupted without commencing any service due to the arrival of a PU at the very next instance is treated as follows. We can assume it will also get bumped out since it was unable to commence any service. Alternatively, as it got the opportunity to get into the channel, we can assume it will wait in the channel until the channel becomes idle and then commences service as the channel turns idle. We propose two single channel models based on these two assumptions in latter sections. We name them as Single Channel Model 1 and Single Channel Model 2 respectively.

3.2.1 DTMC Representation of the Single Channel Model 1

We develop this model based on the first assumption. That is, if an SU arrived to the channel and if a PU comes at the very next instance before the SU commences any service, the SU will be bumped out of the channel without further waiting in the channel. In this subsection as well as in the Subsection 3.2.2, we form the models adhering to preemptive resume service regime. Then in the Section 3.3, we address the required modification to capture the preemptive repeat service regime. For preemptive resume of service, $\mathbf{Q} = \mathbf{I}$, where $\mathbf{I}$ is an identity matrix that represents the phase at which an interrupted service of an SU resumes given that it was interrupted at a specific phase. In here, $\mathbf{I_s}$ is an identity matrix of order n_s, where n_s is the order of $\mathbf{S_s}$ and it captures the scenario of a partially service completed SU remains interrupted in a particular service phase without proceeding to the next as a PU is currently occupying the channel. For details, see [62] and [63]. Then the corresponding single channel model takes following form.

$$\mathbf{P} = \begin{array}{c} \\ (\mathbf{0},\,\mathbf{0}) \\ \cdots \\ (\mathbf{0},\,\mathbf{S}) \\ (\mathbf{P},\,\mathbf{0}) \\ (\mathbf{P},\,\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{ccccc} (\mathbf{0},\,\mathbf{0}) & \vdots & (\mathbf{0},\,\mathbf{S}) & (\mathbf{P},\,\mathbf{0}) & (\mathbf{P},\,\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P_0} & \vdots & \mathbf{P_1} & \mathbf{P_2} & \mathbf{0} & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{P_3} & \vdots & \mathbf{P_4} & \mathbf{P_5} & \mathbf{P_6} & \\ \mathbf{P_7} & \vdots & \mathbf{P_8} & \mathbf{P_9} & \mathbf{0} & \\ \mathbf{0} & \vdots & \mathbf{P_{10}} & \mathbf{0} & \mathbf{P_{11}} & \end{pmatrix} \end{array}$$

where,

$$
\begin{aligned}
\mathbf{P_0} &= \mathbf{D_{00}} \\
\mathbf{P_1} &= \mathbf{D_{01}} \otimes \boldsymbol{\beta_s} \\
\mathbf{P_2} &= (\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta_p} \\
\mathbf{P_3} &= \mathbf{D_{00}} \otimes \mathbf{S_s^0} \\
\mathbf{P_4} &= \mathbf{D_{00}} \otimes \mathbf{S_s} + \mathbf{D_{01}} \otimes \mathbf{S_s^0}\boldsymbol{\beta_s} + \ \mathbf{D_{01}} \otimes \mathbf{S_s} \\
\mathbf{P_5} &= (\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_s^0}\boldsymbol{\beta_p} \\
\mathbf{P_6} &= \mathbf{D_{10}} \otimes \boldsymbol{\beta_p} \otimes \mathbf{S_s^*} + \mathbf{D_{11}} \otimes \boldsymbol{\beta_p} \otimes \mathbf{S_s^*} \\
\mathbf{P_7} &= \mathbf{D_{00}} \otimes \mathbf{S_p^0} \\
\mathbf{P_8} &= \mathbf{D_{01}} \otimes \mathbf{S_p^0}\boldsymbol{\beta_s} \\
\mathbf{P_9} &= \mathbf{D_{10}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \\
\mathbf{P_{10}} &= (\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \\
\mathbf{P_{11}} &= \mathbf{D_{10}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \ \mathbf{D_{11}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \ \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s} \\
&\quad \ldots + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}
\end{aligned}
$$

Note: The symbol $\otimes$ is the Kronecker product. The symbols $\mathbf{Q}^*$ and $\mathbf{S}^*$ are used to keep the notation consistent with the Subsection 3.2.2, where $\mathbf{Q}^* = \mathbf{Q}$ and $\mathbf{S}^* = \mathbf{S}$ in this particular scenario.

3.2.2 DTMC Representation of the Single Channel Model 2

We develop this model based on the second assumption. That is, if an SU arrived to the channel and if a PU comes at the very next instance before commencing any service by the corresponding SU, the SU will wait in the channel until the channel becomes free and then commences service. We define the parameters $\mathbf{Q}^*$, $\mathbf{I_s}$, $e_{\mathbf{1}}{}^t(\mathbf{s})$ and $\mathbf{S}^*$ such that

$$\mathbf{Q}^* = \begin{bmatrix} \boldsymbol{\beta_s} \\ \mathbf{Q} \end{bmatrix}, \; \mathbf{I_s} = \mathbf{I}(n_s+1), \; e_{\mathbf{1}}{}^t(\mathbf{s}) = \mathbf{I_s}(\text{column } 1)^t$$

and

$$\mathbf{S}^* = [\mathbf{0} \;\; \mathbf{S_s}]. \tag{3.2.2}$$

In here, n_s is the order of $\mathbf{S_s}$.

The parameters hold a similar form as in [62], but in our case the purpose is to effectively address the various channel occupancy situations of an SU as opposed to capture the behavior of a time limited vacation model. We elaborate further on these parameters. For preemptive resume of service, $\mathbf{Q} = \mathbf{I}$, where $\mathbf{I}$ is an identity matrix that represents the phase at which an interrupted service of an SU resumes given that it was interrupted at a specific phase. The first row of $\mathbf{Q}^*$, i.e., $\boldsymbol{\beta_s}$ represents the beginning of the service for the SU that just entered into the channel as the channel was idle and got interrupted before entering into any service phase as the PU arrived at the very next instance. The identity matrix $\mathbf{I_s}$ is used to capture the scenario that an SU remains interrupted at a particular service phase within the state (**P**, **S**) without proceeding to the next phase of service as a PU is already getting the service. This captures the scenario of a partially service completed SU remains interrupted in a particular service phase without proceeding to the next or waiting of an SU that interrupted before commencing service and trying to begin its service as the channel turns idle. Hence, the order of the parameter $\mathbf{I_s}$ is $n_s + 1$ where n_s is the order of $\mathbf{S_s}$. The parameter $e_{\mathbf{1}}{}^t(\mathbf{s})$ is the transpose of the first column of $\mathbf{I_s}$ that is used to capture the behavior of the SU who just entered the channel as the channel was idle and got interrupted due to the arrival of a PU before commencing any service. The parameter $\mathbf{S}^*$ is used to represent an SU getting interrupted while in service due to the arrival of a PU. For details, see [62] and [63].

Based on the above mentioned concepts we can represent the activities of a channel as a DTMC with transition probability matrix, $\mathbf{P}$, as illustrated below.

$$\mathbf{P} = \begin{array}{c} \\ (\mathbf{0},\ \mathbf{0}) \\ \cdots \\ (\mathbf{0},\ \mathbf{S}) \\ (\mathbf{P},\ \mathbf{0}) \\ (\mathbf{P},\ \mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccccc} (\mathbf{0},\ \mathbf{0}) & \vdots & (\mathbf{0},\ \mathbf{S}) & (\mathbf{P},\ \mathbf{0}) & (\mathbf{P},\ \mathbf{S}) & \end{array} \\ \begin{pmatrix} \mathbf{P_0} & \vdots & \mathbf{P_1} & \mathbf{P_2} & \mathbf{P_3} & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{P_4} & \vdots & \mathbf{P_5} & \mathbf{P_6} & \mathbf{P_7} & \\ \mathbf{P_8} & \vdots & \mathbf{P_9} & \mathbf{P_{10}} & \mathbf{P_{11}} & \\ \mathbf{0} & \vdots & \mathbf{P_{12}} & \mathbf{0} & \mathbf{P_{13}} & \end{pmatrix} \end{array}$$

where,

$$\mathbf{P_0} = \mathbf{D_{00}}$$

$$\mathbf{P_1} = \mathbf{D_{01}} \otimes \boldsymbol{\beta_s}$$

$$\mathbf{P_2} = \mathbf{D_{10}} \otimes \boldsymbol{\beta_p}$$

$$\mathbf{P_3} = \mathbf{D_{11}} \otimes \boldsymbol{\beta_p} \otimes \boldsymbol{e_1}^t(\mathbf{s})$$

$$\mathbf{P_4} = \mathbf{D_{00}} \otimes \mathbf{S^0_s}$$

$$\mathbf{P_5} = \mathbf{D_{00}} \otimes \mathbf{S_s} + \mathbf{D_{01}} \otimes \mathbf{S^0_s}\boldsymbol{\beta_s} + \ \mathbf{D_{01}} \otimes \mathbf{S_s}$$

$$\mathbf{P_6} = \mathbf{D_{10}} \otimes \mathbf{S^0_s}\boldsymbol{\beta_p}$$

$$\mathbf{P_7} = \mathbf{D_{10}} \otimes \boldsymbol{\beta_p} \otimes \mathbf{S^*_s} + \mathbf{D_{11}} \otimes \boldsymbol{\beta_p} \otimes \mathbf{S^*_s} + \ \mathbf{D_{11}} \otimes \mathbf{S^0_s}\boldsymbol{\beta_p} \otimes \boldsymbol{e_1}^t(\mathbf{s})$$

$$\mathbf{P_8} = \mathbf{D_{00}} \otimes \mathbf{S^0_p}$$

$$\mathbf{P_9} = \mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}$$

$$\mathbf{P_{10}} = \mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes \mathbf{S_p}$$

$$\mathbf{P_{11}} = \mathbf{D_{11}} \otimes \mathbf{S^0_p}\beta_p \otimes \boldsymbol{e_1}^t(\mathbf{s})$$

$$\mathbf{P_{12}} = (\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_p} \otimes \mathbf{Q}^*$$

$$\mathbf{P_{13}} = \mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \ \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \ \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}$$
$$\ldots + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}$$

Note: The symbol $\otimes$ is the Kronecker product.

3.3 Preemptive Resume vs Preemptive Repeat of Service

All the above described models assumed preemptive resume of service by an interrupted SU. One could easily adapt the model for preemptive repeat of service as well. That is, an interrupted SU always tends to repeat the service from the beginning. In that case, the only required modification comes with the parameter $\mathbf{Q}^*$ such that $\mathbf{Q}^* = \mathbf{e} * \boldsymbol{\beta_s}$, where **e** is a column vector of ones with appropriate order [62]. For the Single Channel Model 1, the order of **e** would be the same order as $\mathbf{S_s}$. For the Single Channel Model 2, the order of **e** would be $n_s + 1$, where n_s is the order of $\mathbf{S_s}$. For details, see [62] and [63]. In the Section 5.5, we will verify the application of these two types of service schemes in the proposed models with appropriate parameters with respect to the total time need to spend in the channel to complete service.

3.4 Channel State Probabilities and Performance Metrics

In here, we elaborate on important channel state probabilities and related performance metrics. In the Subsections 3.4.1, 3.4.2, 3.4.3 and 3.4.4, the corresponding results are formulated in a general manner, i.e., they can be used under both service regimes that we described earlier with appropriate arrangement of the inner elements of the parameter $\mathbf{Q}^*$.

3.4.1 Channel State Probabilities

Using $\mathbf{P}$, we can indeed come up with an ON-OFF system as observed by an incoming SU, where $\mathbf{P'_i}$ matrix captures the channel being idle, $\mathbf{P'_{ib}}$ captures the transition from idle to busy, $\mathbf{P'_b}$ captures the channel being busy and $\mathbf{P'_{bi}}$ captures the transition from busy to idle.

$$\mathbf{P} = \begin{array}{c} \\ (\mathbf{0},\ \mathbf{0}) \\ \cdots \\ (\mathbf{0},\ \mathbf{S}) \\ (\mathbf{P},\ \mathbf{0}) \\ (\mathbf{P},\ \mathbf{S}) \end{array} \begin{array}{c} \begin{array}{ccccc} (\mathbf{0},\ \mathbf{0}) & \vdots & (\mathbf{0},\ \mathbf{S}) & (\mathbf{P},\ \mathbf{0}) & (\mathbf{P},\ \mathbf{S}) \end{array} \\ \left(\begin{array}{ccccccc} \mathbf{P}'_{\mathbf{i}} & \vdots & & \mathbf{P}'_{\mathbf{ib}} & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \vdots & & & & \\ \mathbf{P}'_{\mathbf{bi}} & \vdots & & \mathbf{P}'_{\mathbf{b}} & & \\ & \vdots & & & & \end{array} \right) \end{array}$$

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P},\ \boldsymbol{\pi}\mathbf{e} = 1. \tag{3.4.1}$$

With the above stationary distribution we can obtain the following important probabilities that give a better understanding of the channel behavior. As observed by an arriving SU, the probability that the channel being in the idle state P_i and the probability that the channel being in the busy state P_b are obtained as follows.

$$\begin{aligned} P_i &= \boldsymbol{\pi}_{(\mathbf{0},\ \mathbf{0})}\mathbf{e} \\ P_b &= [\boldsymbol{\pi}_{(\mathbf{0},\ \boldsymbol{s})}\ \boldsymbol{\pi}_{(\boldsymbol{p},\ \mathbf{0})}\ \boldsymbol{\pi}_{(\boldsymbol{p},\ \boldsymbol{s})}]\mathbf{e} \end{aligned} \tag{3.4.2}$$

Then the probabilities that the channel is in the states (**0**, **S**), (**P**, **0**) and (**P**, **S**) can be obtained in the following manner.

$$\begin{aligned} P_{(0,\ s)} &= \boldsymbol{\pi}_{(\mathbf{0},\ \boldsymbol{s})}\mathbf{e} \\ P_{(p,\ 0)} &= \boldsymbol{\pi}_{(\boldsymbol{p},\ \mathbf{0})}\mathbf{e} \\ P_{(p,\ s)} &= \boldsymbol{\pi}_{(\boldsymbol{p},\ \boldsymbol{s})}\mathbf{e} \end{aligned} \tag{3.4.3}$$

3.4.2 Average Time an SU Stays Interrupted Until Service is Resumed

Obtaining the average time that an SU stays interrupted until it resumes service can be obtained in the following manner. As for the Single Channel Model 1, the interest is regarding an SU that was interrupted while it was in service. As for the Single Channel Model 2, this can be either an SU that was interrupted just before entering the service or an SU that was interrupted while it was in service. First, the probability of an SU staying interrupted for k discrete time units (P_{int}^{k}) can be obtained as follows.

$$P_{int}^{k} = \frac{\boldsymbol{\pi}_{(p,\,s)}}{\boldsymbol{\pi}_{(p,\,s)}\mathbf{e}}\mathbf{P_{13}}^{k-1}\mathbf{P_{12}}\mathbf{e} \tag{3.4.4}$$

where,

$$\mathbf{P_{13}} = \mathbf{D_{10}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{11}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s} + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}$$

and

$$\mathbf{P_{12}} = (\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^{*} \tag{3.4.5}$$

From (3.4.4), the average time stays interrupted before resuming service (INT_{avg}) can be obtained as

$$INT_{avg} = \frac{\boldsymbol{\pi}_{(p,\,s)}}{\boldsymbol{\pi}_{(p,\,s)}\mathbf{e}}(\mathbf{I} - \mathbf{P_{13}})^{-2}\mathbf{P_{12}}\mathbf{e}. \tag{3.4.6}$$

3.4.3 Average Total Time Required to Complete Service

Obtaining the average total time required to complete service is not as straightforward as obtaining the average time an SU stays interrupted until its service is resumed. We need to create an absorbing Markov chain, $\mathbf{P_{ab1^*}}$, from the transition matrix $\mathbf{P}$ in order to obtain this result. We are mainly interested in an SU that entered to the channel at the initial stage of the channel evolution and is already in service. That is, an SU that got into the channel when the channel was in the state (**0**, **0**) and commenced service evolving the channel to the state (**0**, **S**). In this case, both models ultimately narrow down to the same absorbing Markov chain. Also, the states (**0**, **0**) and (**P**, **0**) will not be in this Markov chain. We are only interested in the states (**0**, **S**) and (**P**, **S**).

Let

$$\mathbf{Q}^* = \mathbf{Q}, \ \mathbf{I_s} = \mathbf{I}(n_s) \text{ and } \mathbf{S}^* = \mathbf{S_s} \tag{3.4.7}$$

$$\mathbf{D_0^*} = \mathbf{D_{00}} + \mathbf{D_{01}} \text{ and } \mathbf{D_1^*} = \mathbf{D_{10}} + \mathbf{D_{11}}. \tag{3.4.8}$$

Then the transition matrix $\mathbf{P_{ab1^*}}$ is given as

$$\mathbf{P_{ab1^*}} = \begin{array}{c} \\ \mathbf{ab1^*} \\ \cdots \\ (\mathbf{0},\mathbf{S})/\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{ab1^*} & \vdots & (\mathbf{0},\mathbf{S})\ \ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{I} & \vdots & \mathbf{0} & \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{L_1^0} & \vdots & \mathbf{L_1} & \end{pmatrix} \end{array},$$

where

$$\mathbf{L_1} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}) \\ \\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cc} (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{D_0^*} \otimes \mathbf{S_s} & \mathbf{D_1^*} \otimes \boldsymbol{\beta_p} \otimes \mathbf{S_s^*} \\ & \\ \mathbf{D_0^*} \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* & \left(\mathbf{D_1^*} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \ \mathbf{D_0^*} \otimes \mathbf{S_p} \otimes \mathbf{I_s}\right) \end{pmatrix} \end{array}$$

and

$$\mathbf{L_1^0} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}) \\ \\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \mathbf{ab1^*} \\ \begin{pmatrix} (\mathbf{D_0^*} + \mathbf{D_1^*}) \otimes \mathbf{S_s^0} \\ \\ \mathbf{0} \end{pmatrix} \end{array}.$$

Since an SU starts its service at the state (**0**, **S**), the initial state vector could be derived from

$\boldsymbol{\gamma}_{\mathbf{1}}$, where

$$\boldsymbol{\gamma}_{\mathbf{1}} = \frac{1}{\mathbf{e^t}(\mathbf{D_{01}} \otimes \boldsymbol{\beta_s})\mathbf{e}}[\mathbf{D_{01}} \otimes \boldsymbol{\beta_s} \quad \mathbf{0}]. \tag{3.4.9}$$

Similar to the previous subsection, the probability of completing service in k discrete time units can be written as

$$P_{srvt}^{k} = \mathbf{e^t}\boldsymbol{\gamma}_{\mathbf{1}}\mathbf{L_1}^{k-1}\mathbf{L_1^0}\mathbf{e}. \tag{3.4.10}$$

Then similar to the previous subsection we can derive the average total time to complete service ($SRVT_{avg}$) by using the probability of service being completed in k time slots in the following manner.

$$SRVT_{avg} = \mathbf{e^t}\boldsymbol{\gamma}(\mathbf{I} - \mathbf{L_1})^{-2}\mathbf{L_1^0}\mathbf{e} \tag{3.4.11}$$

3.4.4 Average Transmission Time without an Interruption

We are interested in finding the average time an SU can transmit successfully without an interruption after it gets into the channel when the channel is in the state (**0, 0**). Once it does, the SU could successfully commence service evolving the channel into the state (**0, S**) and either it could finish transmitting its payload without any interruption or it could transmit partially until it faces with an interruption. We develop following absorbing Markov chain for this purpose. It will consists of the following states. A service completion state, which is the state $\mathbf{ab1}^*$ that was mentioned in the previous subsection, an interruption state, i.e., the state (**P, S**) and the state (**0, S**). In here, both $\mathbf{ab1}^*$ and (**P, S**) are absorbing states such that

$$\mathbf{P_{ab2^*}} = \begin{array}{l} \\ \mathbf{ab1}^*/\,(\mathbf{P},\mathbf{S}) \\ \dots \\ (\mathbf{0},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{ab1}^*\ (\mathbf{P},\mathbf{S}) & & (\mathbf{0},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{I} & \vdots & \mathbf{0} & \\ \dots & \dots & \dots & \dots \\ \mathbf{L_2^0} & \vdots & \mathbf{L_2} & \end{pmatrix} \end{array},$$

where

$$\mathbf{L_2} = \begin{array}{cc} & (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}) & \begin{pmatrix} \mathbf{D_0^*} \otimes \mathbf{S_s} \end{pmatrix} \end{array}$$

and

$$\mathbf{L_2^0} = \begin{array}{ccc} & \mathbf{ab1^*} & \mathbf{P},\mathbf{S} \\ \mathbf{0},\mathbf{S} & \Big((\mathbf{D_0^*} + \mathbf{D_1^*}) \otimes \mathbf{S_s^0} & \mathbf{D_1^*} \otimes \boldsymbol{\beta_p} \otimes \mathbf{S_s^*} \Big) \end{array}.$$

Since an SU starts its service at the state (**0**, **S**), the initial state vector could be derived from $\boldsymbol{\gamma_2}$, where

$$\boldsymbol{\gamma_2} = \frac{1}{\mathbf{e^t}(\mathbf{D_{01}} \otimes \boldsymbol{\beta_s})\mathbf{e}}[\mathbf{D_{01}} \otimes \boldsymbol{\beta_s}]. \tag{3.4.12}$$

Then similar to the previous subsections we can derive the Average Transmission Time without an Interruption ($TWOI_{avg}$) as follows.

$$TWOI_{avg} = \mathbf{e^t}\boldsymbol{\gamma_2}(\mathbf{I} - \mathbf{L_2})^{-2}\mathbf{L_2^0}\mathbf{e} \tag{3.4.13}$$

Chapter 4

Modeling Spectrum Handoff

In this chapter, the focus is given to explain the way that the single channel models are utilized to capture the spectrum handoff process. In here, we will expand the single channel models into two channel systems in order to model the spectrum handoff process. The specific structures of the corresponding transition matrices that capture the entire behavior of the resulting systems are illustrated in the relevant appendices. The readers are advised to refer to them for details as indicated in the latter sections accordingly. The chapter is organized as follows. First, we elaborate on the specific rules that we should adhere when modeling the spectrum handoff process. We discuss two different handoff strategies here and the preemptive resume service regime is assumed. Handoff Strategy 1: if interrupted while in the current channel, always switch into the other channel at the next epoch if the other channel is idle rather than waiting in the current channel to be idle and resume service in it. The SU may remain and resume service in the current channel if the other channel is not available at the next epoch. Handoff Strategy 2: If interrupted in the current channel, wait and try to resume service in the current channel at the next epoch. The SU may switch into the other channel if the current channel is not available at the next epoch. We will utilize the two single channel models we proposed in previous chapter to capture these two handoff strategies separately. As mentioned, corresponding transition matrices that capture the behavior relevant to the above two strategies are illustrated in detail in the corresponding appendices. Next, we will derive the probability of being in the handoff initiation states that is used to analyze the variation of the requirement for handoff with respect to both types of user activities in the Section 5.6. Finally, we will justify the reason that we cannot rely on the trivial approach of solely taking the Kronecker product of the single channel models to capture the spectrum handoff process in a two channel or multi channel system.

4.1 Model Assumptions and Rules for the Handoff Strategy 1

- We will assume, if a channel is already occupied either by a PU or another SU, the arriving SU will not get into the channel and will not wait in the channel until the channel becomes idle. However, an arriving PU can interrupt the service of an SU and get into the channel as the PU has higher priority.
- If an SU is interrupted due to the arrival of a PU in the current channel and if the other channel is idle or becomes idle at the next epoch, the interrupted SU will not wait in the current channel until it turns idle. Handoff is initiated to the other channel immediately.
- When the SU is interrupted in the current channel and if the other channel is not idle at the next epoch, then the SU will wait in the current channel until the current channel turns idle and resumes service in it.
- If there is a conflict between the SUs, the SU already waits in the channel until it becomes idle is given the first priority to resume service. In addition, the SU that arrives due to a handoff is given priority over a fresh SU that comes to the channel.
- If an SU is about to initiate a handoff from the current channel, we assume the order of events for the current channel within the corresponding epoch as follows, i.e.,

handoffs, service completions and then the arrivals.

- As for the target channel, the order of the events is service completions and then the arrivals.
- As the SU arrives to the target channel if a PU comes at the very next instance within the considered epoch, the SU will get interrupted and the SU will remain interrupted in the target channel without being able to resume service. This situation is captured by the corresponding parameter $\mathbf{I_s}$.
- As the SU arrives to the target channel if another SU arrives at the very next instance or neither a PU nor another SU arrivals happen within the considered epoch, the SU that initiated the handoff will resume service in the target channel in either case. This situation is captured by the corresponding parameter $\mathbf{Q}^*$.
- Also, in the case that an SU decided to stay in the current channel, it may or may not be able to resume service based on whether a PU arrives to the current channel at the very next instance or not within the considered epoch. In that case, resuming service or staying interrupted in the current channel is captured using the parameters $\mathbf{Q}^*$ and $\mathbf{I_s}$ as well.
- The SU in service goes to the interrupted state on arrival of a PU, i.e., one among the following $\Big(\mathbf{(0,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,0)}\Big)$, $\Big(\mathbf{(0,S),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,S)}\Big)$, $\Big(\mathbf{(P,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(P,0)}\Big)$ and $\Big(\mathbf{(P,S),(P,S)}\Big)$ states depending on the current state.
- The handoffs are possible from the states $\Big(\mathbf{(0,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,0)}\Big)$, $\Big(\mathbf{(0,S),(P,S)}\Big)$, $\Big(\mathbf{(P,S), (0,S)}\Big)$, $\Big(\mathbf{(P,0),(P,S)}\Big)$ and $\Big(\mathbf{(P,S),(P,0)}\Big)$ only.
- The state space is maintained in the following order, i.e., Channel 1 Arrival Process, Channel 1 Service process, Channel 2 Arrival Process and Channel 2 Service process throughout the two channel DTMC models.

- Within the service process, the order is maintained such that the service relevant to the user from the channel 1 is captured first and the service relevant to the user from the channel 2 is captured next.

4.1.1 Handoff Strategy 1 with the Single Channel Models 1 and 2

In the transition matrix that captures the behavior for Handoff Strategy 1, all the terms corresponding to the state transitions originated from the non-handoff states can be directly inferred from the Kronecker product between the two replicas of the same single channel model. That is, the corresponding terms can be directly inferred from the Kronecker product between the two replicas of the Single Channel Model 1 or Single Channel Model 2. See Appendix A for further clarifications. As a result, we are only interested in illustrating the terms relevant to the state transitions originated from the above described handoff states. We will only express those terms while we illustrate the structure of the resulting DTMCs in the Appendices B and C respectively. Also, in the two channel system we use the $\hat{.}$ sign for the parameters relevant to the channel two while we write the parameters relevant to the channel one without such for clarity.

4.2 Model Assumptions and Rules for the Handoff Strategy 2

- We will assume, if a channel is already occupied either by a PU or another SU, the arriving SU will not get into the channel and will not wait in the channel until the channel becomes idle. However, an arriving PU can interrupt the service of an SU and get into the channel as the PU has higher priority.

- If an SU is interrupted due to the arrival of a PU in the current channel and even though the other channel is idle or becomes idle at the next epoch, the interrupted SU will not immediately initiate a handoff. Instead, it will wait in the current channel until the channel turns idle at the next epoch and resumes service in the current channel.

- The SU initiates a handoff to the other channel if the current channel is not becoming idle and if the other channel is idle at the next epoch.

- If there is a conflict between the SUs, the SU already waits in the channel until it becomes idle is given the first priority to resume service. In addition, the SU that arrives due to a handoff is given priority over a fresh SU that comes to the channel.

- If an SU is about to initiate a handoff from the current channel, we assume the order of events for the current channel as follows, i.e., service completions, handoffs and then the arrivals within the considered epoch.

- As for the target channel, the order of events would be service completions and then the arrivals.

- In the case of a handoff, as the SU arrives to the target channel if a PU comes at the very next instance within the considered epoch, the SU will get interrupted and the SU will remain interrupted in the target channel without being able to resume service. This situation is captured by the corresponding parameter $\mathbf{I_s}$.

- In the case of a handoff, as the SU arrives to the target channel if another SU arrives at the very next instance or neither a PU nor another SU arrivals happen within the considered epoch, the handoffing SU will resume service in the target channel in either case. This situation is captured by the corresponding parameter $\mathbf{Q}^*$.

- After getting interrupted even though the SU waits in the current channel until the PU finishes service in the next epoch, the SU may or may not be able to resume service depending on another PU arrives to the channel at the very next instance or not within the considered next epoch. If no PU arrivals happen, the SU can successfully resume its service and this situation is captured by the parameter $\mathbf{Q}^*$. If a PU arrival happens then the SU again will get interrupted and the corresponding situation is captured by the parameter $\mathbf{I_s}$.

- The SU in service goes to the interrupted state on arrival of a PU, i.e, one among the following $\Big(\mathbf{(0,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,0)}\Big)$, $\Big(\mathbf{(0,S),(P,S)}\Big)$, $\Big(\mathbf{(P,S), (0,S)}\Big)$, $\Big(\mathbf{(P,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(P,0)}\Big)$ and $\Big(\mathbf{(P,S),(P,S)}\Big)$ states depending on the current state.

- The handoffs are possible from the states $\Big(\mathbf{(0,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,0)}\Big)$, $\Big(\mathbf{(0,S),(P,S)}\Big)$, $\Big(\mathbf{(P,S), (0,S)}\Big)$, $\Big(\mathbf{(P,0),(P,S)}\Big)$ and $\Big(\mathbf{(P,S),(P,0)}\Big)$ only.

- The state space is maintained in the following order, i.e., Channel 1 Arrival Process, Channel 1 Service process, Channel 2 Arrival Process and Channel 2 Service process throughout the two channel DTMC models.

- Within the service process, the order is maintained such that the service relevant to the user from the channel 1 and the user waited without going to the channel 1 is captured first. Then the service relevant to the user from the channel 2 and the user waited without going to the channel 2 captured next.

4.2.1 Handoff Strategy 2 with the Single Channel Models 1 and 2

In the transition matrix that captures the behavior for Handoff Strategy 2, all the terms corresponding to the state transitions originated from the non-handoff states can be directly inferred from the Kronecker product between the two replicas of the same single channel model. That is, the corresponding terms can be directly inferred from the Kronecker product between the two replicas of the Single Channel Model 1 or Single Channel Model 2. See Appendix A for further clarifications. As a result, we are only interested in illustrating the terms relevant to the state transitions originated from the above described handoff states. We will only express those terms while we illustrate the structure of the resulting DTMCs in the Appendices D and E respectively. Also, in the two channel system we use the $\hat{.}$ sign for the parameters relevant to the channel two while we write the parameters relevant to the channel one without such for clarity.

4.3 The Probability of Being in the Handoff States

From the stationary distributions of the corresponding transition matrices illustrated in the relevant appendices, the probability of being in the handoff initiation states (P_h), i.e, the probability of being in the states $\Big(\mathbf{(0,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,0)}\Big)$, $\Big(\mathbf{(0,S),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,S)}\Big)$, $\Big(\mathbf{(P,0),(P,S)}\Big)$ and $\Big(\mathbf{(P,S),(P,0)}\Big)$ can be obtained as follows.

$$P_h = [\boldsymbol{\pi}_{(0,0),(p,s)}\ \boldsymbol{\pi}_{(p,s),(0,0)}\ \boldsymbol{\pi}_{(0,s),(p,s)}\ \boldsymbol{\pi}_{((p,s),(0,s))}\ \boldsymbol{\pi}_{(p,0),(p,s)}\ \boldsymbol{\pi}_{(p,s),(p,0)}]\mathbf{e} \quad (4.3.1)$$

In the Section 5.6, we have analyzed the variation of P_h with respect to the PU and SU activity for each model.

4.4 The Effect of not Embedding Handoff into the Kronecker Product of the Single Channel Models

We need to take a complex approach to capture the handoffs as illustrated in the corresponding appendices because the Kronecker product between the two replicas of the same single channel model will not capture the handoff scenarios successfully for a two channel system. Such an approach is only capable of capturing the independent behavior of the two channels. Hence, we can only infer the terms related to the non-handoff states from their Kronecker product. An analysis of the stationary distributions of the two cases reveals the difference.

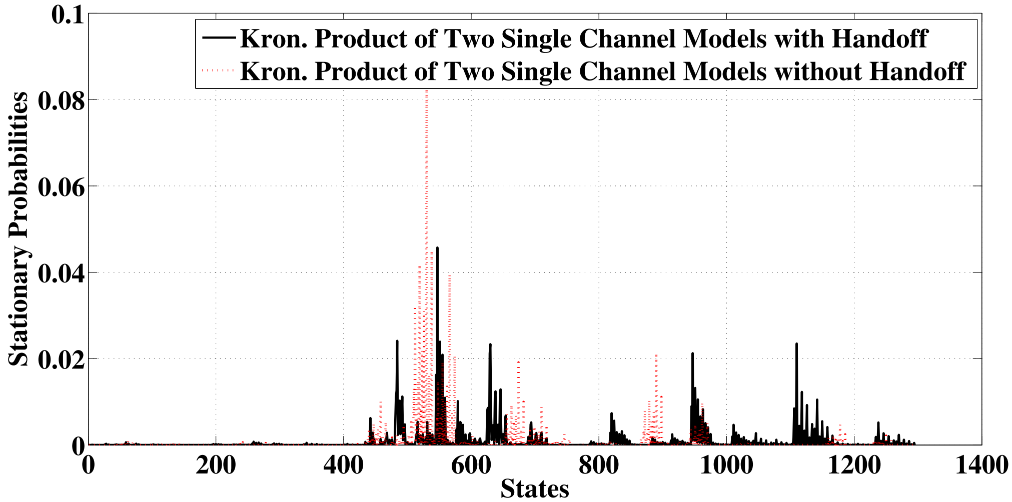

Figure 4.4.1: Stationary Distributions with and without Handoff States

4.5 Chapter Summary

In this chapter, the focus was given to explain the way that the single channel models are utilized to capture various spectrum handoff strategies. The specific structures of the corresponding transition matrices are illustrated in the relevant appendices. Then, based on the stationary distributions of the corresponding transition matrices, the probability of being in the handoff initiation states was derived. Finally, the stationary distributions of the transition matrices of the Kronecker product between two single channel models were compared with and without handoff states embedded into them.

Chapter 5

Numerical Results

In this chapter, we present the numerical results based on the derivations obtained in the Chapter 3 and Chapter 4 as a special case, i.e., with special parameters. Our aim is to study the system behavior. We assume PH distributions to represent the arrival processes of the PUs and SUs. We assume the PU arrival process is represented by $(\boldsymbol{\alpha_p},\ \mathbf{T_p})$ and the SU arrival process is represented by $(\boldsymbol{\alpha_s},\ \mathbf{T_s})$, with $\mathbf{T_p^0} = \mathbf{e} - \mathbf{T_p}\mathbf{e}$ and $\mathbf{T_s^0} = \mathbf{e} - \mathbf{T_s}\mathbf{e}$. Then we have,

$$\begin{aligned}
\mathbf{D_{00}} &= \mathbf{T_p} \otimes \mathbf{T_s} \\
\mathbf{D_{01}} &= \mathbf{T_p} \otimes \mathbf{T_s^0}\boldsymbol{\alpha_s} \\
\mathbf{D_{10}} &= \mathbf{T_p^0}\boldsymbol{\alpha_p} \otimes \mathbf{T_s} \\
\mathbf{D_{11}} &= \mathbf{T_p^0}\boldsymbol{\alpha_p} \otimes \mathbf{T_s^0}\boldsymbol{\alpha_s}
\end{aligned} \tag{5.0.1}$$

5.1 Variation of the Channel Probabilities

In here, we present the variation of the channel probabilities, i.e., the variation of the probabilities P_i, P_b, $P_{(0,\, s)}$, $P_{(p,\, 0)}$ and $P_{(p,\, s)}$ that were derived in the Subsection 3.4.1 with respect to the single channel models developed.

We use the following specific form of PH distribution parameters illustrated in (5.1.1) to represent the user arrival process and the service times in a similar way as in [44]. In each matrix, the elements are randomly generated such that the random values are drawn from the standard uniform distribution on the open interval (0,1).

$$\mathbf{T_p} = \begin{bmatrix} t_{p1} & 0 \\ 0 & t_{p2} \end{bmatrix}, \; \boldsymbol{\alpha_p} = [\alpha_{p1} \; (1 - \alpha_{p1})]$$

$$\mathbf{S_p} = \begin{bmatrix} s_{p1} & 0 \\ 0 & s_{p2} \end{bmatrix}, \; \boldsymbol{\beta_p} = [\beta_{p1} \; (1 - \beta_{p1})]$$

$$\mathbf{T_s} = \begin{bmatrix} t_{s1} & 0 \\ 0 & t_{s2} \end{bmatrix}, \; \boldsymbol{\alpha_s} = [\alpha_{s1} \; (1 - \alpha_{s1})] \tag{5.1.1}$$

$$\mathbf{S_s} = \begin{bmatrix} s_{s1} & 0 \\ 0 & s_{s2} \end{bmatrix}, \; \boldsymbol{\beta_s} = [\beta_{s1} \; (1 - \beta_{s1})]$$

From Subsection 5.1.1 to the Section 5.4, we will use the above representation in all the numerical examples we present for the single channel models that adapt the preemptive resume service regime.

5.1.1 Variation of the Channel Probabilities with respect to the Mean PU Arrival Rate

The Figures 5.1.1 and 5.1.2 show the variation of the channel probabilities derived in (3.4.2) and (3.4.3) with respect to the mean arrival rate of the PUs. As the mean PU arrival rate increases, it is obvious that the probabilities P_i and $P_{(0,\, s)}$ must reduce gradually as more PUs tend to occupy the channel. Also, P_b, $P_{(p,\, 0)}$ and $P_{(p,\, s)}$ must increase gradually as well. This is exactly reflected in the proposed models. In the Single Channel Model 2, as the arrival rate of the PUs increases, the probability of being in the state (**P**, **S**) is increased as opposed to the Single Channel Model 1. This is because the Single Channel Model 2 has more tendency to being in the state (**P**, **S**). Hence, a reduction in $P_{(p,\,0)}$ could be observed for higher PU arrival rates. Refer to the main assumptions we made in the Section 3.2 when formulating the two distinct single channel models for better clarification.

Table 5.1: Parameters for Analyzing the Variation of the Channel Probabilities with respect to the Mean PU Arrival Rate

Parameters	Values
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.6490 & 0 \\ 0 & 0.9829 \end{bmatrix}$, $[0.1059\ 0.8941]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.8848 & 0 \\ 0 & 0.4625 \end{bmatrix}$, $[0.9309\ 0.0691]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.7217 & 0 \\ 0 & 0.6598 \end{bmatrix}$, $[0.8225\ 0.1775]$

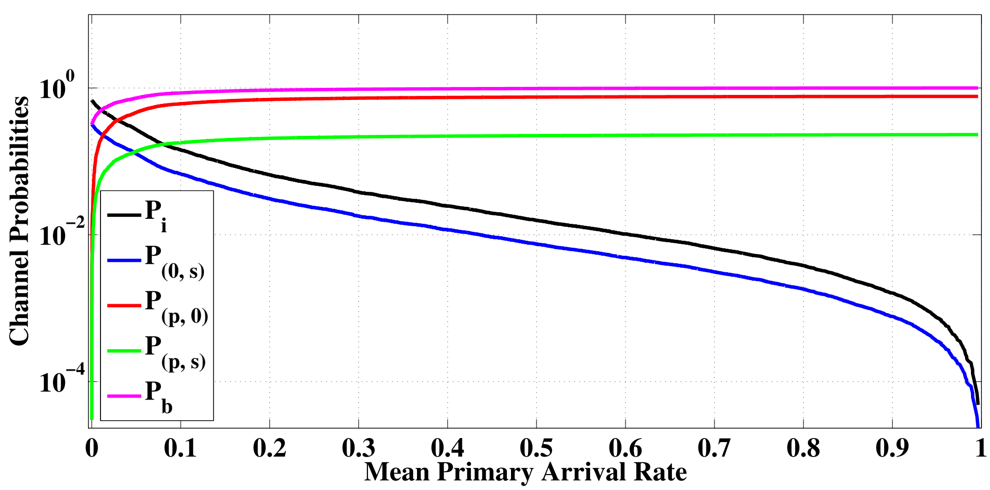

Figure 5.1.1: Channel Probabilities vs PU Arrival Rate - Single Channel Model 1

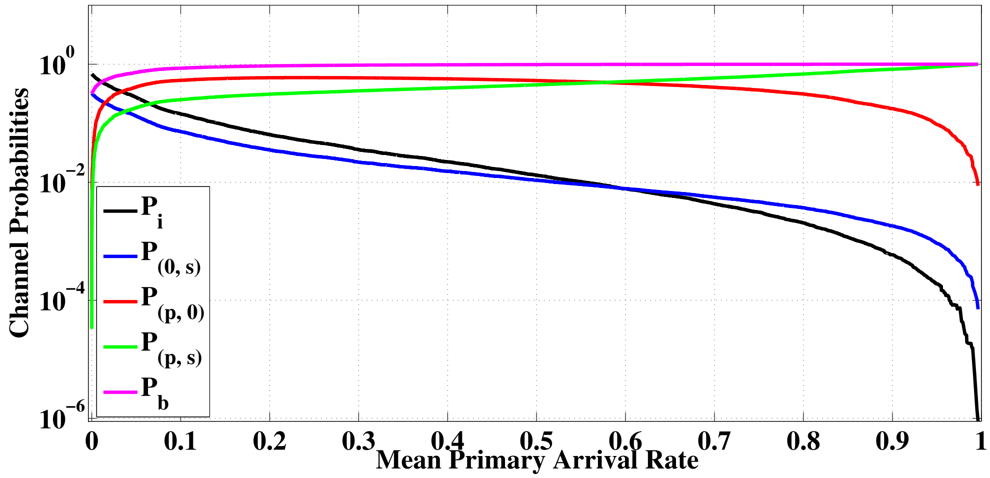

Figure 5.1.2: Channel Probabilities vs PU Arrival Rate - Single Channel Model 2

5.1.2 Variation of the Channel Probabilities with respect to the Mean PU Service Rate

The Figures 5.1.3 and 5.1.4 illustrate the variation of the corresponding probabilities along with the increase of the PU service rate. Since the PUs tend to finish their service much faster, the channel being in the idle state will get increased as reflected in the increase of P_i and decrease of P_b. SUs find more opportunities to get into the state (**0**, **S**), where they can commence or resume service. Hence, the increase of $P_{(0,\ s)}$ is clear. As the PUs finish service quickly, the probability that an SU stays interrupted is also reduced. Obviously, the PU remaining in the state (**P**, **0**) is being reduced as it quickly finishes service evolving the channel into the state (**0**, **0**) or (**0**, **S**) with higher probability.

Table 5.2: Parameters for Analyzing the Variation of the Channel Probabilities with respect to the Mean PU Service Rate

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.6052 & 0 \\ 0 & 0.6003 \end{bmatrix}$, [0.2843 0.7157]
$\mathbf{T_S},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.1145 & 0 \\ 0 & 0.9645 \end{bmatrix}$, [0.2808 0.7192]
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.8263 & 0 \\ 0 & 0.1852 \end{bmatrix}$, [0.9240 0.0760]

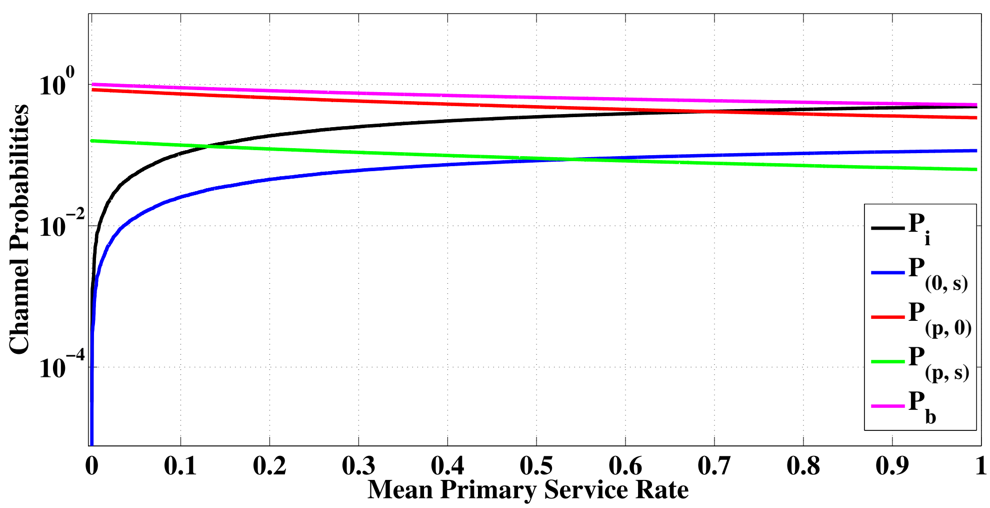

Figure 5.1.3: Channel Probabilities vs PU Service Rate - Single Channel Model 1

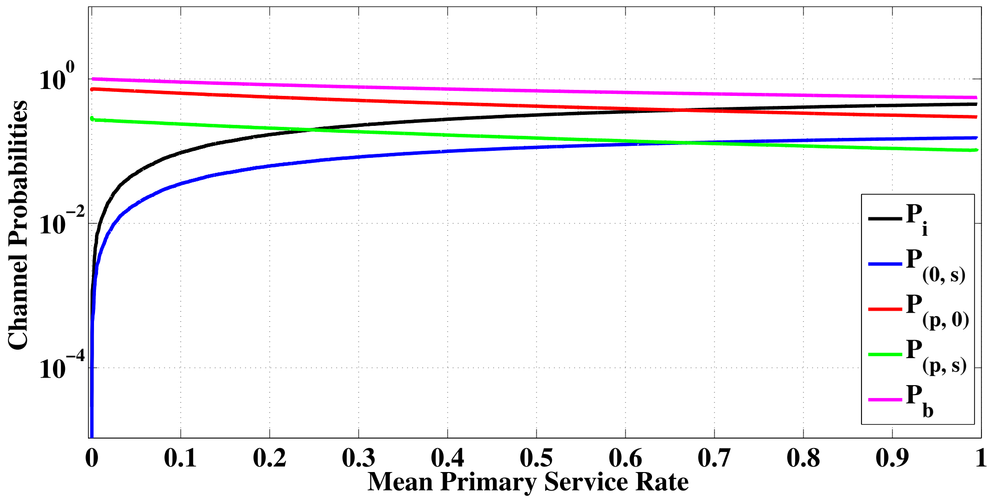

Figure 5.1.4: Channel Probabilities vs PU Service Rate - Single Channel Model 2

5.1.3 Variation of the Channel Probabilities with respect to the Mean SU Arrival Rate

In the Figures 5.1.5 and 5.1.6, as the arrival rate of the SUs increases, $P_{(p,\,0)}$ is reduced, but $P_{(p,\,s)}$ is increased. As more SUs come to the channel, the PUs are more likely to get encounter with the situation that they have to commence their service while interrupting an existing SU rather than the scenario they can start their service, where no SUs in the channel. In this situation, the variation of the other probabilities are obvious and as expected. Note that P_i reduces while P_b increases. Also, as mentioned earlier, in the Single Channel Model 2 there exists more tendency to being end up in the state (**P, S**) with the increase of SU arrival rate as well. Hence, a significant decrease in $P_{(p,\,0)}$ could be observed compared to the model 1.

Table 5.3: Parameters for Analyzing the Variation of the Channel Probabilities with respect to the Mean SU Arrival Rate

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.6174 & 0 \\ 0 & 0.8702 \end{bmatrix},\ [0.6353\ 0.3647]$
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.2728 & 0 \\ 0 & 0.9635 \end{bmatrix},\ [0.7016\ 0.2984]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.1278 & 0 \\ 0 & 0.6903 \end{bmatrix},\ [0.4810\ 0.5190]$

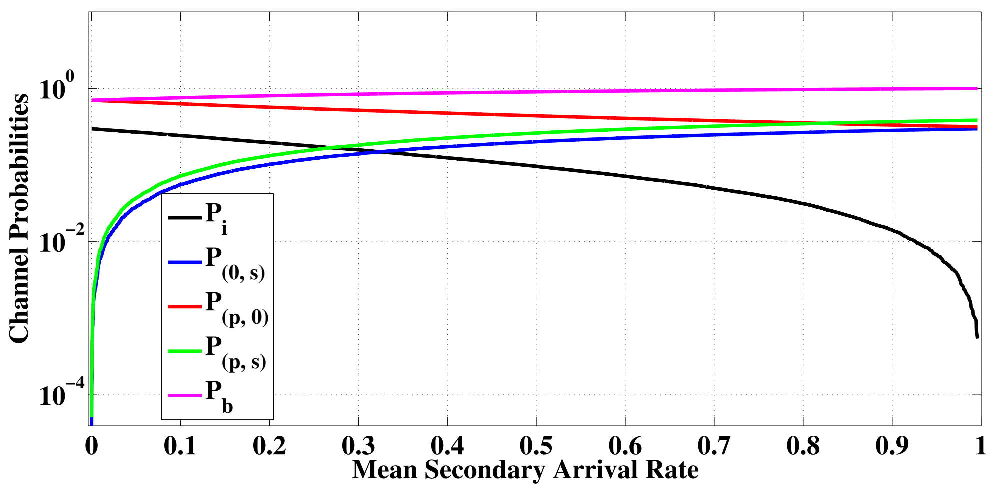

Figure 5.1.5: Channel Probabilities vs SU Arrival Rate - Single Channel Model 1

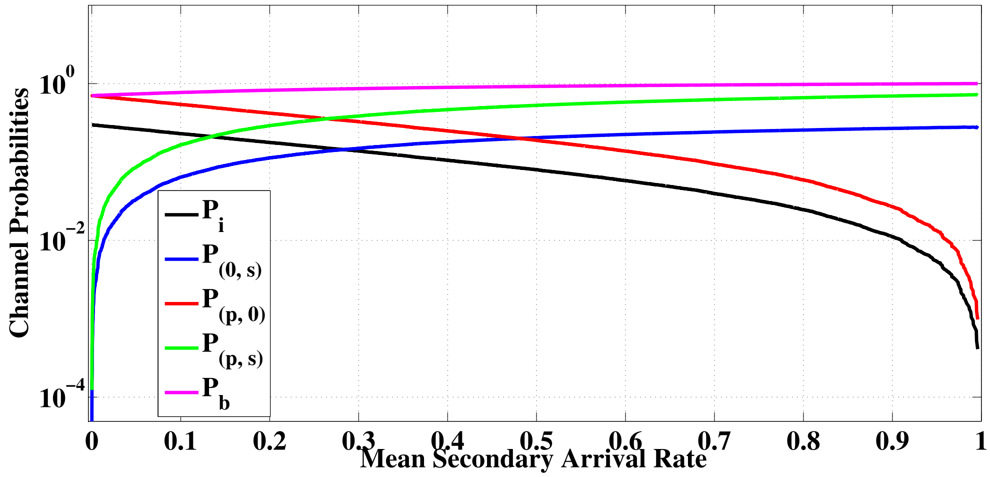

Figure 5.1.6: Channel Probabilities vs SU Arrival Rate - Single Channel Model 2

5.1.4 Variation of the Channel Probabilities with respect to the Mean SU Service Rate

In the Figures 5.1.7 and 5.1.8, as the secondary service rate increases, the SUs tend to finish service quickly and the tendency they stay in the channel to get interrupted is reduced. Hence, both $P_{(0,\ s)}$ and $P_{(p,\ s)}$ are being reduced. In addition, the possibility of the PUs come across with the SUs is also reduced. As a result, $P_{(p,\ 0)}$ is increased. As SUs complete service quickly, the probability of the channel being in the idle state increases imposing a reduction in the probability of the channel being in the busy state. Since the Single Channel Model 2 possesses a higher tendency of being end up in the state (**P, S**), even though the SU service rate increases the reduction of the probability $P_{(p,\ s)}$ is less compared to the channel model 1.

Table 5.4: Parameters for Analyzing the Variation of the Channel Probabilities with respect to the Mean SU Service Rate

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.9147 & 0 \\ 0 & 0.3591 \end{bmatrix}$, [0.9912 0.0088]
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.5750 & 0 \\ 0 & 0.5222 \end{bmatrix}$, [0.6723 0.3277]
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.5709 & 0 \\ 0 & 0.6127 \end{bmatrix}$, [0.9150 0.0850]

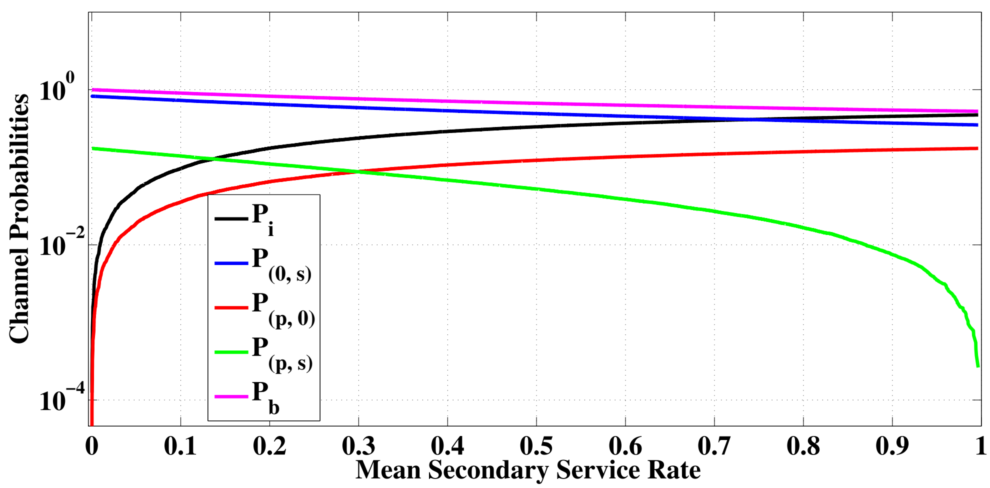

Figure 5.1.7: Channel Probabilities vs SU Service Rate - Single Channel Model 1

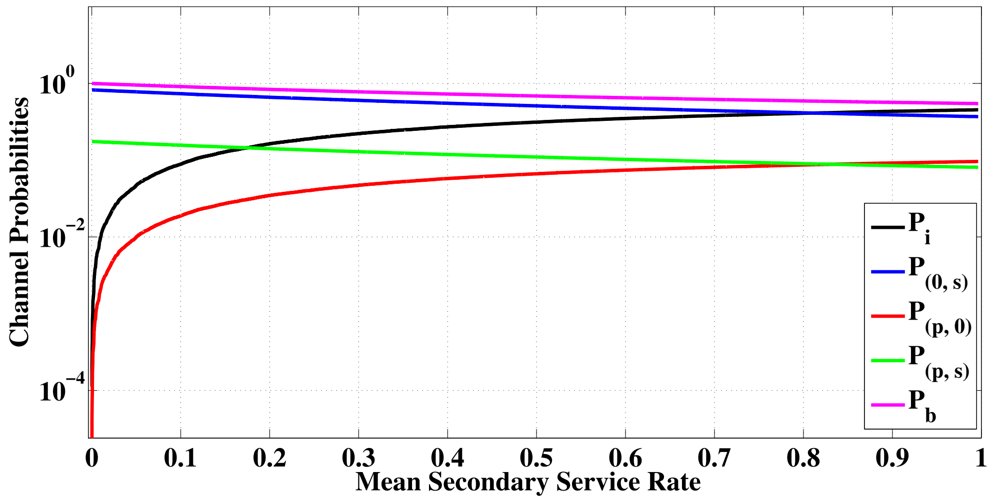

Figure 5.1.8: Channel Probabilities vs SU Service Rate - Single Channel Model 2

5.2 Average Time an SU Stays Interrupted Until Service is Resumed

The Figure 5.2.1 illustrates the variation of INT_{avg} obtained in (3.4.6). As the PU arrival rate increases, the SUs are more probable of ending up in the state (**P**, **S**) and remaining there because the PUs always get the priority over the SUs. Hence, the time that an SU needs to stay interrupted is increased. The average time an SU has to stay interrupted reduces as the service rate of the PU increases because now the SU can make a transition from the state (**P**, **S**) to the state (**0**, **S**) often and resumes service.

Table 5.5: Parameters for Analyzing INT_{avg} vs Mean PU Arrival Rate

Parameters	Values
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.0460 & 0 \\ 0 & 0.2216 \end{bmatrix}$, $[0.5819\ 0.4181]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.6735 & 0 \\ 0 & 0.5501 \end{bmatrix}$, $[0.2811\ 0.7189]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.2453 & 0 \\ 0 & 0.2519 \end{bmatrix}$, $[0.1257\ 0.8743]$

Table 5.6: Parameters for Analyzing INT_{avg} vs Mean PU Service Rate

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.0460 & 0 \\ 0 & 0.2216 \end{bmatrix}$, $[0.5819\ 0.4181]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.6735 & 0 \\ 0 & 0.5501 \end{bmatrix}$, $[0.2811\ 0.7189]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.2453 & 0 \\ 0 & 0.2519 \end{bmatrix}$, $[0.1257\ 0.8743]$

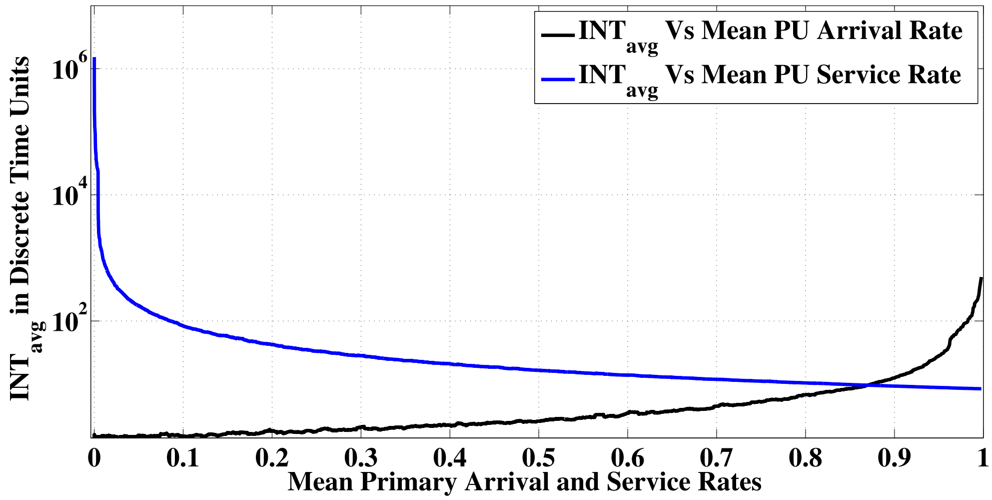

Figure 5.2.1: Average Time an SU Stays Interrupted Until Service is Resumed

5.3 Average Total Time Required to Complete Service

The Figure 5.3.1 illustrates the variation of $SRVT_{avg}$ obtained in (3.4.11). When the arrival rate of the PU increases it is more likely that the SU need to stay in the state (**P**, **S**) interrupted. Hence, $SRVT_{avg}$ is increased. According to the Figure 5.3.1, the average total time that an SU takes to complete its service reduces as the service rate of the PU is increased. This is because now SUs have frequent opportunities to get into the state (**0**, **S**) and continue service as the PUs leave quickly. As the SU service rate increases, the average total time need to complete service is reduced.

Table 5.7: Parameters for Analyzing $SRVT_{avg}$ vs Mean PU Arrival Rate

Parameters	Values
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.2154 & 0 \\ 0 & 0.7848 \end{bmatrix}$, $[0.7309\ 0.2691]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.1011 & 0 \\ 0 & 0.3479 \end{bmatrix}$, $[0.3744\ 0.6256]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.9379 & 0 \\ 0 & 0.6872 \end{bmatrix}$, $[0.7398\ 0.2602]$

Table 5.8: Parameters for Analyzing $SRVT_{avg}$ vs Mean PU Service Rate

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.2154 & 0 \\ 0 & 0.7848 \end{bmatrix}$, $[0.7309\ 0.2691]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.1011 & 0 \\ 0 & 0.3479 \end{bmatrix}$, $[0.3744\ 0.6256]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.9379 & 0 \\ 0 & 0.6872 \end{bmatrix}$, $[0.7398\ 0.2602]$

Table 5.9: Parameters for Analyzing $SRVT_{avg}$ vs Mean SU Service Rate

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.2154 & 0 \\ 0 & 0.7848 \end{bmatrix}$, [0.7309 0.2691]
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.1011 & 0 \\ 0 & 0.3479 \end{bmatrix}$, [0.3744 0.6256]
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.9379 & 0 \\ 0 & 0.6872 \end{bmatrix}$, [0.7398 0.2602]

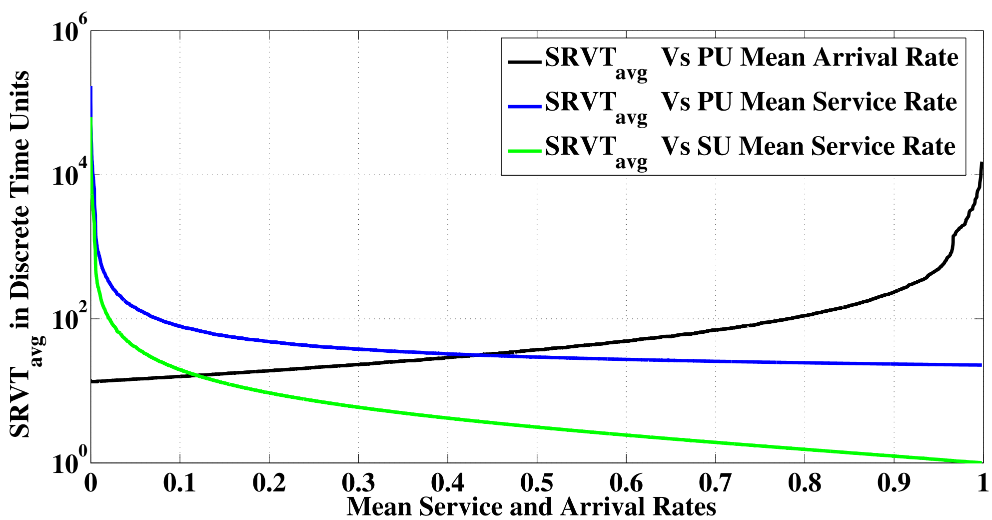

Figure 5.3.1: Average Total Time Required to Complete Service

5.4 Average Transmission Time without an Interruption

The Figure 5.4.1 illustrates the variation of $TWOI_{avg}$ obtained in (3.4.13). As the PU arrival rate increases, the time that an SU could transmit without an interruption is getting reduced as expected. Also, as the SU service rate increases, the SU can quickly complete service without an interruption. Hence, the transmission time without an interruption is also reduced.

Table 5.10: Parameters for Analyzing $TWOI_{avg}$ vs Mean PU Arrival Rate

Parameters	Values
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.9901 & 0 \\ 0 & 0.8710 \end{bmatrix}$, $[0.0600\ 0.9400]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.1479 & 0 \\ 0 & 0.5733 \end{bmatrix}$, $[0.6563\ 0.3437]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.7465 & 0 \\ 0 & 0.7954 \end{bmatrix}$, $[0.2981\ 0.7019]$

Table 5.11: Parameters for Analyzing $TWOI_{avg}$ vs Mean SU Service Rate

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.9901 & 0 \\ 0 & 0.8710 \end{bmatrix}$, $[0.0600\ 0.9400]$
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.1479 & 0 \\ 0 & 0.5733 \end{bmatrix}$, $[0.6563\ 0.3437]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.7465 & 0 \\ 0 & 0.7954 \end{bmatrix}$, $[0.2981\ 0.7019]$

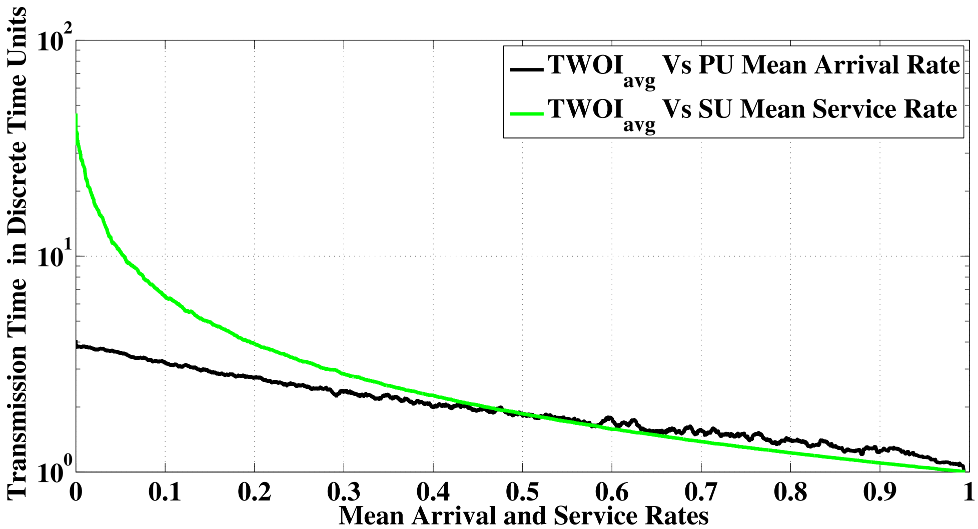

Figure 5.4.1: Average Transmission Time without an Interruption

5.5 Effects of Preemptive Resume and Preemptive Repeat of Service

In here, we analyze the effect of the preemptive repeat service regime with respect to the preemptive resume service regime in a single channel system. We analyze their effects with respect to the total time required to complete transmission. That is, we will separately observe the variation of $SRVT_{avg}$ with respect to the PU and SU activity when SU adapts preemptive resume and preemptive repeat service schemes. We use the following specific form of PH distribution parameters in (5.5.1) to define the arrival and service processes of the users in a similar way as in [64], where the corresponding authors have adapted to compare the above two service regimes. In each matrix, the elements are randomly generated such that the random values are drawn from the standard uniform distribution on the open interval (0,1). Obviously, when SU adapts preemptive repeat, it should take more time to complete its service, which is indeed reflected in the analysis.

$$\mathbf{T_p} = \begin{bmatrix} t_{p1} & 1-t_{p1} \\ 0 & t_{p2} \end{bmatrix}, \ \boldsymbol{\alpha_p} = [1 \ \ 0]$$

$$\mathbf{S_p} = \begin{bmatrix} s_{p1} & 1-s_{p1} \\ 0 & s_{p2} \end{bmatrix}, \ \boldsymbol{\beta_p} = [1 \ \ 0]$$

$$\mathbf{T_s} = \begin{bmatrix} t_{s1} & 1-t_{s1} \\ 0 & t_{s2} \end{bmatrix}, \ \boldsymbol{\alpha_s} = [1 \ \ 0] \tag{5.5.1}$$

$$\mathbf{S_s} = \begin{bmatrix} s_{s1} & 1-s_{s1} \\ 0 & s_{s2} \end{bmatrix}, \ \boldsymbol{\beta_s} = [1 \ \ 0]$$

Table 5.12: Parameters for Analyzing the Effects of the Preemptive Resume and Repeat Service Regimes with respect to the Mean PU Service Time

Parameters	Values
$\mathbf{T_p}, \ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.0958 & 0.9042 \\ 0 & 0.1394 \end{bmatrix}, \ [1 \ 0]$
$\mathbf{T_s}, \ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.1471 & 0.8529 \\ 0 & 0.4408 \end{bmatrix}, \ [1 \ 0]$
$\mathbf{S_s}, \ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.0289 & 0.9711 \\ 0 & 0.9269 \end{bmatrix}, \ [1 \ 0]$

Table 5.13: Parameters for Analyzing the Effects of the Preemptive Resume and Repeat Service Regimes with respect to the Mean PU Inter Arrival Time

Parameters	Values
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.0958 & 0.9042 \\ 0 & 0.1394 \end{bmatrix},\ [1\ 0]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.1471 & 0.8529 \\ 0 & 0.4408 \end{bmatrix},\ [1\ 0]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.0289 & 0.9711 \\ 0 & 0.9269 \end{bmatrix},\ [1\ 0]$

Table 5.14: Parameters for Analyzing the Effects of the Preemptive Resume and Repeat Service Regimes with respect to the Mean SU Service Time

Parameters	Values
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.0958 & 0.9042 \\ 0 & 0.1394 \end{bmatrix},\ [1\ 0]$
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.1471 & 0.8529 \\ 0 & 0.4408 \end{bmatrix},\ [1\ 0]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.0289 & 0.9711 \\ 0 & 0.9269 \end{bmatrix},\ [1\ 0]$

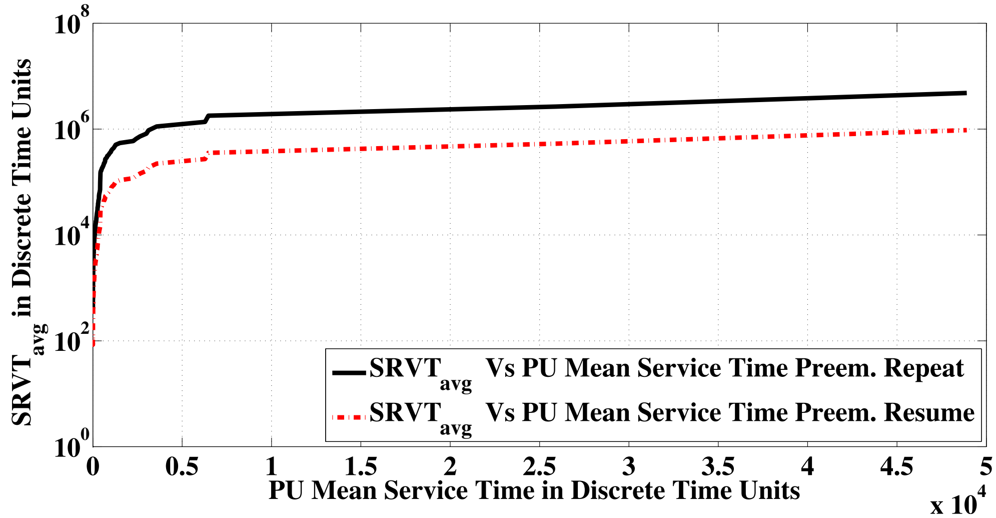

Figure 5.5.1: Analysis of the Preemptive Resume and Repeat Service Regimes with respect to the Mean PU Service Time

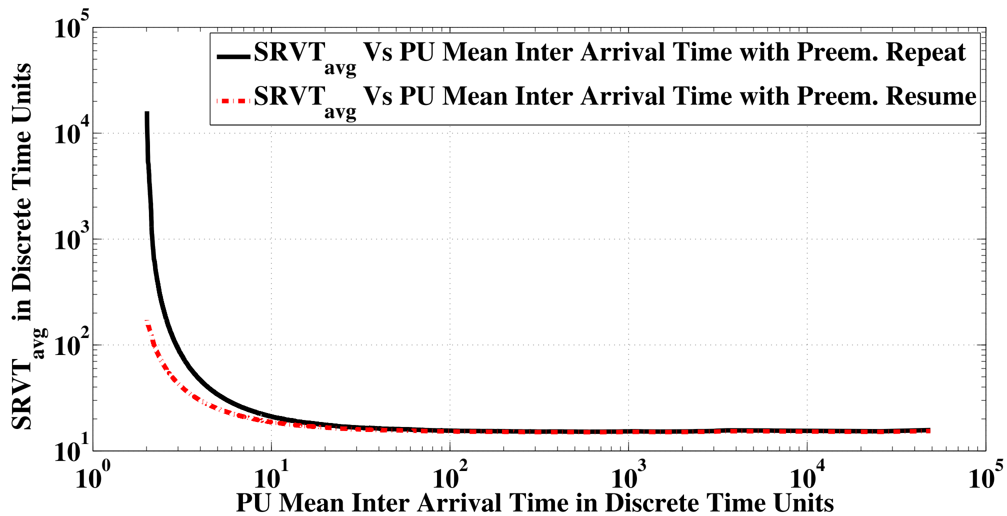

Figure 5.5.2: Analysis of the Preemptive Resume and Repeat Service Regimes with respect to the Mean PU Inter Arrival Time

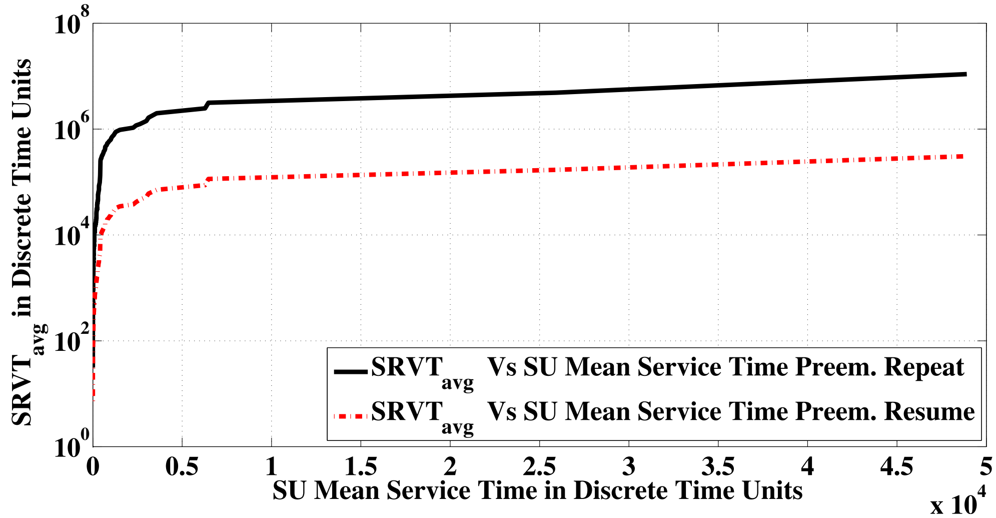

Figure 5.5.3: Analysis of the Preemptive Resume and Repeat Service Regimes with respect to the Mean SU Service Time

5.6 Variation of the Probability of being in the Handoff States

We analyze the variation of the probability of being in the handoff initiation states (P_h) which we obtained in (4.3.1). That is, the probability of being in the states $\Big(\mathbf{(0,0),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,0)}\Big)$, $\Big(\mathbf{(0,S),(P,S)}\Big)$, $\Big(\mathbf{(P,S),(0,S)}\Big)$, $\Big(\mathbf{(P,0),(P,S)}\Big)$ and $\Big(\mathbf{(P,S),(P,0)}\Big)$ of a two channel system. We use the matrices introduced in (5.5.1) to capture the arrival and service processes. We use the preemptive resume service regime in the following results.

Table 5.15: Parameters for Analyzing P_h vs Mean PU Arrival Rate

Parameters	Values for Channel 1	Values for Channel 2
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.4945 & 0.5055 \\ 0 & 0.8976 \end{bmatrix},\ [1\ 0]$	$\begin{bmatrix} 0.9382 & 0.0618 \\ 0 & 0.2828 \end{bmatrix},\ [1\ 0]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.2886 & 0.7114 \\ 0 & 0.5942 \end{bmatrix},\ [1\ 0]$	$\begin{bmatrix} 0.2052 & 0.7948 \\ 0 & 0.0273 \end{bmatrix},\ [1\ 0]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.4759 & 0.5241 \\ 0 & 0.6556 \end{bmatrix},\ [1\ 0]$	$\begin{bmatrix} 0.8762 & 0.1238 \\ 0 & 0.2036 \end{bmatrix},\ [1\ 0]$

Table 5.16: Parameters for Analyzing P_h vs Mean PU Service Rate

Parameters	Values for Channel 1	Values for Channel 2
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.4945 & 0.5055 \\ 0 & 0.8976 \end{bmatrix},\ [1\ 0]$	$\begin{bmatrix} 0.9382 & 0.0618 \\ 0 & 0.2828 \end{bmatrix},\ [1\ 0]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.2886 & 0.7114 \\ 0 & 0.5942 \end{bmatrix},\ [1\ 0]$	$\begin{bmatrix} 0.2052 & 0.7948 \\ 0 & 0.0273 \end{bmatrix},\ [1\ 0]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.4759 & 0.5241 \\ 0 & 0.6556 \end{bmatrix},\ [1\ 0]$	$\begin{bmatrix} 0.8762 & 0.1238 \\ 0 & 0.2036 \end{bmatrix},\ [1\ 0]$

Table 5.17: Parameters for Analyzing P_h vs Mean SU Arrival Rate

Parameters	Values for Channel 1	Values for Channel 2
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.2886 & 0.7114 \\ 0 & 0.5942 \end{bmatrix}$, $[1\ 0]$	$\begin{bmatrix} 0.9382 & 0.0618 \\ 0 & 0.2828 \end{bmatrix}$, $[1\ 0]$
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.4945 & 0.5055 \\ 0 & 0.8976 \end{bmatrix}$, $[1\ 0]$	$\begin{bmatrix} 0.2052 & 0.7948 \\ 0 & 0.0273 \end{bmatrix}$, $[1\ 0]$
$\mathbf{S_s},\ \boldsymbol{\beta_s}$	$\begin{bmatrix} 0.4759 & 0.5241 \\ 0 & 0.6556 \end{bmatrix}$, $[1\ 0]$	$\begin{bmatrix} 0.8762 & 0.1238 \\ 0 & 0.2036 \end{bmatrix}$, $[1\ 0]$

Table 5.18: Parameters for Analyzing P_h vs Mean SU Service Rate

Parameters	Values for Channel 1	Values for Channel 2
$\mathbf{T_p},\ \boldsymbol{\alpha_p}$	$\begin{bmatrix} 0.2886 & 0.7114 \\ 0 & 0.5942 \end{bmatrix}$, $[1\ 0]$	$\begin{bmatrix} 0.9382 & 0.0618 \\ 0 & 0.2828 \end{bmatrix}$, $[1\ 0]$
$\mathbf{S_p},\ \boldsymbol{\beta_p}$	$\begin{bmatrix} 0.4945 & 0.5055 \\ 0 & 0.8976 \end{bmatrix}$, $[1\ 0]$	$\begin{bmatrix} 0.2052 & 0.7948 \\ 0 & 0.0273 \end{bmatrix}$, $[1\ 0]$
$\mathbf{T_s},\ \boldsymbol{\alpha_s}$	$\begin{bmatrix} 0.4759 & 0.5241 \\ 0 & 0.6556 \end{bmatrix}$, $[1\ 0]$	$\begin{bmatrix} 0.8762 & 0.1238 \\ 0 & 0.2036 \end{bmatrix}$, $[1\ 0]$

5.6.1 The Probability of being in the Handoff States with respect to the PU Activity

We analyze the variation of the above probability with respect to the PU activity. As the PU arrival rate increases more SUs will tend to initiate handoffs as their service is getting interrupted in the current channel by the PUs. As a result, the SUs will get pushed into the handoff initiation states with a higher tendency. Hence, P_h will increase along with the PU arrival rate. As the PU service rate increases P_h is reduced. Even though the SUs will get end up in the handoff initiation states with respect to the considered arrival rate of the PUs, the tendency they have to remain in this states is reduced with the increase of PU service rate.

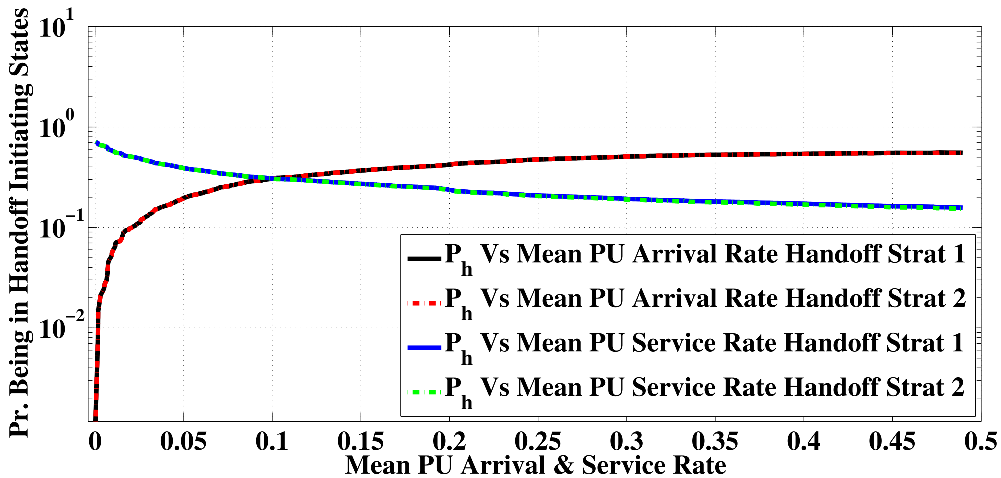

Figure 5.6.1: P_h vs PU Activity in a Two Channel System Using Single Channel Model 1

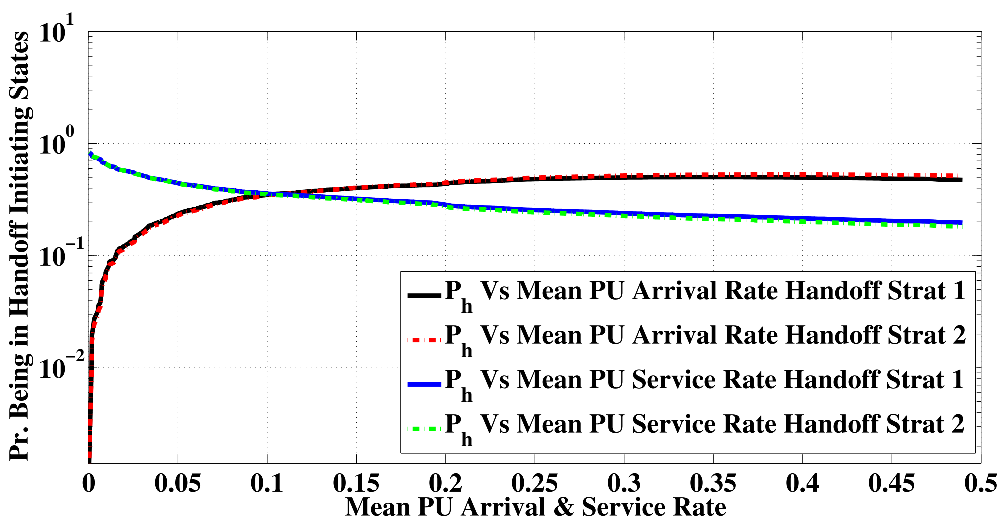

Figure 5.6.2: P_h vs PU Activity in a Two Channel System Using Single Channel Model 2

5.6.2 The Probability of being in the Handoff States with respect to the SU Activity

As the SU arrival rate increases, the tendency of the PUs come across with an SU will get increased pushing the SUs more towards the handoff initiating states. Hence, the increase of corresponding probability is expected. Also, as the service rate of the SUs increases, the corresponding probability is reduced because the SUs tend to complete their service quickly. As a result, the chance of getting interrupted within the current channel is less as now the possibility of the PUs come across with an SU that is already in service is less. Hence, the requirement for handoff is less too.

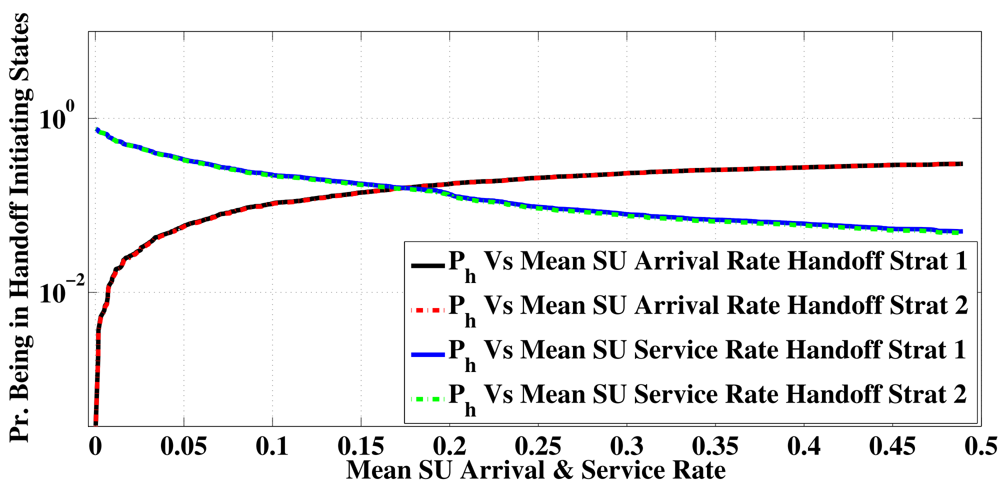

Figure 5.6.3: P_h vs SU Activity in a Two Channel System Using Single Channel Model 1

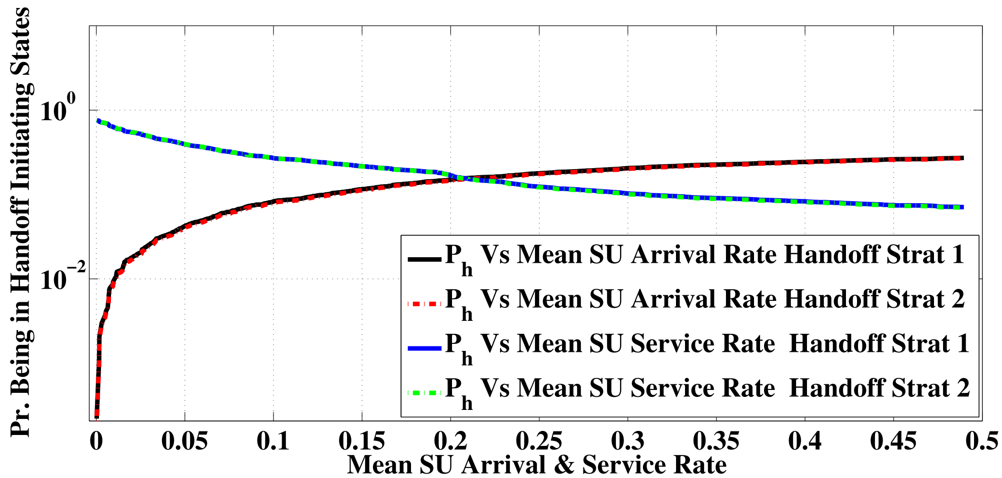

Figure 5.6.4: P_h vs SU Activity in a Two Channel System Using Single Channel Model 2

Chapter 6

Conclusion and Future Work

In this thesis, an improved CR channel model is presented. The model considers both primary and other secondary user activities of a CR channel as observed by a potential incoming secondary user to the channel. The model defines the channel behavior realistically such that it can be adapted to model scenarios like spectrum decision and spectrum handoff. The proposed model is capable of tracking the service phase of an SU, where it was interrupted and resume service in the appropriate phase or repeat service from the beginning based on the requirement. The proposed model is capable of capturing the joint behavior of both PU and SU activities. The model can be observed as an ON-OFF system. The ON and OFF periods are correlated not only on the primary user activity, but also on other secondary user activity as observed by an incoming SU. Due to the usage of PH distributions any other type of arrivals and service distributions can be easily modeled. Derived numerical results suggest that the proposed model is behaving as expected and the channel states depend on both activities. Based on the channel models, two handoff strategies were modeled and the corresponding numerical results were presented as well.

As for the future work, based on this channel model, one can expand our work to capture the handoff scenarios in a N channel CR network. Also, the proposed channel model can be utilized as a base to develop various strategies similar to the strategies addressed in the Subsection 2.2.2. Although the attention is not given to embed imperfect sensing into the model with respect to the false alarms and mis-detections as it is a well researched area, one can form the model by taking those facts into account.

Appendix A

Example on Inferring Non-Handoff Terms

Assume we want to infer the non-handoff terms for the following block of the corresponding transition matrix from the Appendix B that captures the behavior of the Handoff Strategy 1 based on the Single Channel Model 1.

$$
\mathbf{P_{00}} = \begin{array}{r c}
 & \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{r} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} &
\begin{pmatrix}
\mathbf{P^0_{00}} & \mathbf{P^1_{00}} & \mathbf{P^2_{00}} & \mathbf{0} \\
\mathbf{P^3_{00}} & \mathbf{P^4_{00}} & \mathbf{P^5_{00}} & \mathbf{P^6_{00}} \\
\mathbf{P^7_{00}} & \mathbf{P^8_{00}} & \mathbf{P^9_{00}} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{pmatrix}
\end{array}
$$

Recall the Single Channel Model 1.

$$
\mathbf{P} = \begin{array}{r c}
 & \begin{array}{ccccc} (\mathbf{0},\ \mathbf{0}) & \vdots & (\mathbf{0},\ \mathbf{S}) & (\mathbf{P},\ \mathbf{0}) & (\mathbf{P},\ \mathbf{S}) \end{array} \\
\begin{array}{r} (\mathbf{0},\ \mathbf{0}) \\ \cdots \\ (\mathbf{0},\ \mathbf{S}) \\ (\mathbf{P},\ \mathbf{0}) \\ (\mathbf{P},\ \mathbf{S}) \end{array} &
\begin{pmatrix}
\mathbf{P_0} & \vdots & \mathbf{P_1} & \mathbf{P_2} & \mathbf{0} & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\mathbf{P_3} & \vdots & \mathbf{P_4} & \mathbf{P_5} & \mathbf{P_6} & \\
\mathbf{P_7} & \vdots & \mathbf{P_8} & \mathbf{P_9} & \mathbf{0} & \\
\mathbf{0} & \vdots & \mathbf{P_{10}} & \mathbf{0} & \mathbf{P_{11}} &
\end{pmatrix}
\end{array}
$$

When considering the Kronecker product of the two replicas of the Single Channel Model 1, corresponding block $\mathbf{P_{00}}$ of the resulting matrix can be derived in the following manner with terms corresponding to the handoff initiation state $\Big($(**0,0**), (**P,S**)$\Big)$ adjusted accordingly.

$$\mathbf{P_{00}} = \mathbf{P_0} \otimes \begin{pmatrix} \mathbf{P_0} & \mathbf{P_1} & \mathbf{P_2} & \mathbf{0} \\ \mathbf{P_3} & \mathbf{P_4} & \mathbf{P_5} & \mathbf{P_6} \\ \mathbf{P_7} & \mathbf{P_8} & \mathbf{P_9} & \mathbf{0} \\ - & - & - & - \end{pmatrix}$$

Then the non-handoff terms of $\mathbf{P_{00}}$ would be as follows.

$$\begin{aligned}
\mathbf{P^0_{00}} &= \mathbf{P_0} \otimes \mathbf{P_0} \\
\mathbf{P^1_{00}} &= \mathbf{P_0} \otimes \mathbf{P_1} \\
\mathbf{P^2_{00}} &= \mathbf{P_0} \otimes \mathbf{P_2} \\
\mathbf{P^3_{00}} &= \mathbf{P_0} \otimes \mathbf{P_3} \\
\mathbf{P^4_{00}} &= \mathbf{P_0} \otimes \mathbf{P_4} \\
\mathbf{P^5_{00}} &= \mathbf{P_0} \otimes \mathbf{P_5} \\
\mathbf{P^6_{00}} &= \mathbf{P_0} \otimes \mathbf{P_6} \\
\mathbf{P^7_{00}} &= \mathbf{P_0} \otimes \mathbf{P_7} \\
\mathbf{P^8_{00}} &= \mathbf{P_0} \otimes \mathbf{P_8} \\
\mathbf{P^9_{00}} &= \mathbf{P_0} \otimes \mathbf{P_9}
\end{aligned} \tag{A.0.1}$$

Appendix B

Handoff Strategy 1 with the Single Channel Model 1

$$
\mathbf{P} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) \\ \vdots \\ \vdots \\ (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c}
\begin{array}{cccccc} (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) & \dots & \dots & \dots & \dots & (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix}
\mathbf{P_{00}} & \mathbf{P_{01}} & \mathbf{P_{02}} & \mathbf{P_{03}} \\
\mathbf{P_{10}} & \mathbf{P_{11}} & \mathbf{P_{12}} & \mathbf{P_{13}} \\
\mathbf{P_{20}} & \mathbf{P_{21}} & \mathbf{P_{22}} & \mathbf{P_{23}} \\
\mathbf{P_{30}} & \mathbf{P_{31}} & \mathbf{P_{32}} & \mathbf{P_{33}}
\end{pmatrix}
\end{array}
$$

where,

$$
\mathbf{P_{00}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c}
\begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix}
\mathbf{P^0_{00}} & \mathbf{P^1_{00}} & \mathbf{P^2_{00}} & \mathbf{0} \\
\mathbf{P^3_{00}} & \mathbf{P^4_{00}} & \mathbf{P^5_{00}} & \mathbf{P^6_{00}} \\
\mathbf{P^7_{00}} & \mathbf{P^8_{00}} & \mathbf{P^9_{00}} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{pmatrix}
\end{array}
$$

$$\mathbf{P_{01}} = \begin{array}{rc}
 & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{r} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \left(\begin{array}{cccc}
\mathbf{P}_{\mathbf{01}}^{\mathbf{0}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{1}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{2}} & \mathbf{0} \\
\mathbf{P}_{\mathbf{01}}^{\mathbf{3}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{4}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{5}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{6}} \\
\mathbf{P}_{\mathbf{01}}^{\mathbf{7}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{8}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{9}} & \mathbf{0} \\
\mathbf{P}_{\mathbf{01}}^{\mathbf{10}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{11}} & \mathbf{P}_{\mathbf{01}}^{\mathbf{12}} & \mathbf{0}
\end{array}\right)
\end{array}$$

$$\mathbf{P_{02}} = \begin{array}{rc}
 & \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{r} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \left(\begin{array}{cccc}
\mathbf{P}_{\mathbf{02}}^{\mathbf{0}} & \mathbf{P}_{\mathbf{02}}^{\mathbf{1}} & \mathbf{P}_{\mathbf{02}}^{\mathbf{2}} & \mathbf{0} \\
\mathbf{P}_{\mathbf{02}}^{\mathbf{3}} & \mathbf{P}_{\mathbf{02}}^{\mathbf{4}} & \mathbf{P}_{\mathbf{02}}^{\mathbf{5}} & \mathbf{P}_{\mathbf{02}}^{\mathbf{6}} \\
\mathbf{P}_{\mathbf{02}}^{\mathbf{7}} & \mathbf{P}_{\mathbf{02}}^{\mathbf{8}} & \mathbf{P}_{\mathbf{02}}^{\mathbf{9}} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array}\right)
\end{array}$$

$$\mathbf{P_{03}} = \begin{array}{rc}
 & \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{r} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \left(\begin{array}{cccc}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{P}_{\mathbf{03}}^{\mathbf{0}} & \mathbf{P}_{\mathbf{03}}^{\mathbf{1}} & \mathbf{P}_{\mathbf{03}}^{\mathbf{2}} & \mathbf{0}
\end{array}\right)
\end{array}$$

$$\mathbf{P_{10}} = \begin{array}{rc}
 & \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{r} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \left(\begin{array}{cccc}
\mathbf{P}_{\mathbf{10}}^{\mathbf{0}} & \mathbf{P}_{\mathbf{10}}^{\mathbf{1}} & \mathbf{P}_{\mathbf{10}}^{\mathbf{2}} & \mathbf{0} \\
\mathbf{P}_{\mathbf{10}}^{\mathbf{3}} & \mathbf{P}_{\mathbf{10}}^{\mathbf{4}} & \mathbf{P}_{\mathbf{10}}^{\mathbf{5}} & \mathbf{P}_{\mathbf{10}}^{\mathbf{6}} \\
\mathbf{P}_{\mathbf{10}}^{\mathbf{7}} & \mathbf{P}_{\mathbf{10}}^{\mathbf{8}} & \mathbf{P}_{\mathbf{10}}^{\mathbf{9}} & 0 \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array}\right)
\end{array}$$

$$\mathbf{P_{11}} = \begin{array}{rc}
 & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{r} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \left(\begin{array}{cccc}
\mathbf{P}_{\mathbf{11}}^{\mathbf{0}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{1}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{2}} & \mathbf{0} \\
\mathbf{P}_{\mathbf{11}}^{\mathbf{3}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{4}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{5}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{6}} \\
\mathbf{P}_{\mathbf{11}}^{\mathbf{7}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{8}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{9}} & \mathbf{0} \\
\mathbf{P}_{\mathbf{11}}^{\mathbf{10}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{11}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{12}} & \mathbf{P}_{\mathbf{11}}^{\mathbf{13}}
\end{array}\right)
\end{array}$$

$$\mathbf{P_{12}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{12}} & \mathbf{P^1_{12}} & \mathbf{P^2_{12}} & \mathbf{0} \\ \mathbf{P^3_{12}} & \mathbf{P^4_{12}} & \mathbf{P^5_{12}} & \mathbf{P^6_{12}} \\ \mathbf{P^7_{12}} & \mathbf{P^8_{12}} & \mathbf{P^9_{12}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{13}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{13}} & \mathbf{P^1_{13}} & \mathbf{P^2_{13}} & \mathbf{0} \\ \mathbf{P^3_{13}} & \mathbf{P^4_{13}} & \mathbf{P^5_{13}} & \mathbf{P^6_{13}} \\ \mathbf{P^7_{13}} & \mathbf{P^8_{13}} & \mathbf{P^9_{13}} & \mathbf{0} \\ \mathbf{P^{10}_{13}} & \mathbf{P^{11}_{13}} & \mathbf{P^{12}_{13}} & \mathbf{P^{13}_{13}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{20}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{20}} & \mathbf{P^1_{20}} & \mathbf{P^2_{20}} & \mathbf{0} \\ \mathbf{P^3_{20}} & \mathbf{P^4_{20}} & \mathbf{P^5_{20}} & \mathbf{P^6_{20}} \\ \mathbf{P^7_{20}} & \mathbf{P^8_{20}} & \mathbf{P^9_{20}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{21}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{21}} & \mathbf{P^1_{21}} & \mathbf{P^2_{21}} & \mathbf{0} \\ \mathbf{P^3_{21}} & \mathbf{P^4_{21}} & \mathbf{P^5_{21}} & \mathbf{P^6_{21}} \\ \mathbf{P^7_{21}} & \mathbf{P^8_{21}} & \mathbf{P^9_{21}} & \mathbf{0} \\ \mathbf{P^{10}_{21}} & \mathbf{P^{11}_{21}} & \mathbf{P^{12}_{21}} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{22}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{22}} & \mathbf{P^1_{22}} & \mathbf{P^2_{22}} & \mathbf{0} \\ \mathbf{P^3_{22}} & \mathbf{P^4_{22}} & \mathbf{P^5_{22}} & \mathbf{P^6_{22}} \\ \mathbf{P^7_{22}} & \mathbf{P^8_{22}} & \mathbf{P^9_{22}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{22}} & \mathbf{0} & \mathbf{P^{11}_{22}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{23}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{P^0_{23}} & \mathbf{P^1_{23}} & \mathbf{P^2_{23}} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{30}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{P^0_{30}} & \mathbf{0} & \mathbf{P^1_{30}} \\ \mathbf{0} & \mathbf{P^2_{30}} & \mathbf{0} & \mathbf{P^3_{30}} \\ \mathbf{0} & \mathbf{P^4_{30}} & \mathbf{0} & \mathbf{P^5_{30}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{31}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{P^0_{31}} & \mathbf{0} & \mathbf{P^1_{31}} \\ \mathbf{0} & \mathbf{P^2_{31}} & \mathbf{0} & \mathbf{P^3_{31}} \\ \mathbf{0} & \mathbf{P^4_{31}} & \mathbf{P^5_{31}} & \mathbf{P^6_{31}} \\ \mathbf{0} & \mathbf{P^7_{31}} & \mathbf{0} & \mathbf{P^8_{31}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{32}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{P^0_{32}} & \mathbf{0} & \mathbf{P^1_{32}} \\ \mathbf{0} & \mathbf{P^2_{32}} & \mathbf{0} & \mathbf{P^3_{32}} \\ \mathbf{0} & \mathbf{P^4_{32}} & \mathbf{0} & \mathbf{P^5_{32}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{33}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^0_{33}} & \mathbf{0} & \mathbf{P^1_{33}} \\ \mathbf{0} & \mathbf{0} & \mathbf{P^2_{33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^3_{33}} & \mathbf{0} & \mathbf{P^4_{33}} \end{pmatrix} \end{array}$$

such that,

$$\mathbf{P_{01}^{10}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P_{01}^{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$
$$\mathbf{P_{01}^{12}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) + \ \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}$$
$$\ldots + \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}})\Big)$$

$$\mathbf{P_{03}^{0}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P_{03}^{1}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$
$$\mathbf{P_{03}^{2}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) + \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}$$
$$\ldots + \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}})\Big)$$

$$\mathbf{P_{11}^{10}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s^0} \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P_{11}^{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s^0} \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big) + \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s}\Big)$$
$$\ldots \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \otimes \hat{\mathbf{Q}}^*\Big)$$
$$\mathbf{P_{11}^{12}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s^0} \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) + \ \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}$$
$$\ldots + \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}})\Big)$$
$$\mathbf{P_{11}^{13}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} + \ \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}}$$
$$\ldots + \ \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{s}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{s}}\Big)$$

$$\mathbf{P_{13}^{10}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_s^0}\boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P_{13}^{11}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_s^0}\boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\beta}_{s}\Big) + \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{S_s^*}\Big)$$
$$\ldots \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \otimes \hat{\mathbf{Q}}^*\Big)$$
$$\mathbf{P_{13}^{12}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_s^0}\boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) + \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}$$
$$\ldots + \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}})\Big)$$
$$\mathbf{P_{13}^{13}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{S_s^*}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} + \ \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}}$$
$$\ldots + \ \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{s}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{s}}\Big)$$

$$\mathbf{P_{21}^{10}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{21}^{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$

$$\mathbf{P_{21}^{12}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^*\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) + \ \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}$$

$$\ldots + \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}})\Big)$$

$$\mathbf{P_{22}^{10}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}} + \mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_p}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \otimes \hat{\mathbf{Q}}^*\Big)$$

$$\mathbf{P_{22}^{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}} + \mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_p}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} + \ \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}$$

$$\ldots + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} + \ \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{s}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{s}}\Big)$$

$$\mathbf{P_{23}^{0}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{23}^{1}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\beta}_{s}\Big)$$

$$\mathbf{P_{23}^{2}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s}\Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{10}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) + \hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}} + \hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}$$

$$\ldots + \hat{\mathbf{D}}_{\mathbf{11}} \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}})\Big)$$

$$\mathbf{P_{30}^{0}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^*\Big)$$

$$\mathbf{P_{30}^{1}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{30}^{2}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{30}^{3}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{30}^{4}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{30}^{5}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{31}^{0}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S_p^0}\boldsymbol{\beta}_{\boldsymbol{s}}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^*\Big)$$

$$\mathbf{P_{31}^{1}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S_p^0}\boldsymbol{\beta}_{\boldsymbol{s}}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{31}^{2}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S_p^0}\boldsymbol{\beta}_{\boldsymbol{s}}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\Big) + \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^*\Big)$$

$$\ldots \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{s}}\Big)$$

$$\mathbf{P^3_{31}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\beta_s\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big) + \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_p} \otimes \mathbf{Q}^*\Big)$$

$$\ldots \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \otimes \mathbf{\hat{S}^*_s}\Big)$$

$$\mathbf{P^4_{31}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big)$$

$$\mathbf{P^5_{31}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_p} \otimes \mathbf{Q}^*\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{S}_p}\Big)$$

$$\mathbf{P^6_{31}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^0_{32}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p})\Big)$$

$$\ldots \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^*\Big)$$

$$\mathbf{P^1_{32}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p})\Big)$$

$$\ldots \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^2_{32}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p})\Big)$$

$$\ldots \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_s}\Big)$$

$$\mathbf{P^3_{32}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p})\Big)$$

$$\ldots \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^4_{32}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p})\Big)$$

$$\ldots \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big)$$

$$\mathbf{P^5_{32}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p})\Big)$$

$$\ldots \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^0_{33}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}$$

$$\ldots + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{S}_s}\Big)$$

$$\mathbf{P^1_{33}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}$$

$$\ldots + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \otimes \mathbf{\hat{S}^*_s}\Big)$$

$$\mathbf{P^2_{33}} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{11}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}$$

$$\ldots + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{S}_p}\Big)$$

Note: As mentioned, rest of the terms can be directly inferred from the Kronecker product between the two replicas of the Single Channel Model 1 with different channel parameters.

Appendix C

Handoff Strategy 1 with the Single Channel Model 2

$$
\mathbf{P} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) \\ \vdots \\ \vdots \\ (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \\ \\ \end{array}
\begin{array}{c}
\begin{array}{cccccc} (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) & \cdots & \cdots & \cdots & \cdots & (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix}
\mathbf{P_{00}} & \mathbf{P_{01}} & \mathbf{P_{02}} & \mathbf{P_{03}} \\
\mathbf{P_{10}} & \mathbf{P_{11}} & \mathbf{P_{12}} & \mathbf{P_{13}} \\
\mathbf{P_{20}} & \mathbf{P_{21}} & \mathbf{P_{22}} & \mathbf{P_{23}} \\
\mathbf{P_{30}} & \mathbf{P_{31}} & \mathbf{P_{32}} & \mathbf{P_{33}} \\
 & & &
\end{pmatrix}
\end{array}
$$

where,

$$
\mathbf{P_{00}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c}
\begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix}
\mathbf{P^0_{00}} & \mathbf{P^1_{00}} & \mathbf{P^2_{00}} & \mathbf{P^3_{00}} \\
\mathbf{P^4_{00}} & \mathbf{P^5_{00}} & \mathbf{P^6_{00}} & \mathbf{P^7_{00}} \\
\mathbf{P^8_{00}} & \mathbf{P^9_{00}} & \mathbf{P^{10}_{00}} & \mathbf{P^{11}_{00}} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{pmatrix}
\end{array}
$$

$$\mathbf{P_{01}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^{0}_{01}} & \mathbf{P^{1}_{01}} & \mathbf{P^{2}_{01}} & \mathbf{P^{3}_{01}} \\ \mathbf{P^{4}_{01}} & \mathbf{P^{5}_{01}} & \mathbf{P^{6}_{01}} & \mathbf{P^{7}_{01}} \\ \mathbf{P^{8}_{01}} & \mathbf{P^{9}_{01}} & \mathbf{P^{10}_{01}} & \mathbf{P^{11}_{01}} \\ \mathbf{P^{12}_{01}} & \mathbf{P^{13}_{01}} & \mathbf{P^{14}_{01}} & \mathbf{P^{15}_{01}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{02}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^{0}_{02}} & \mathbf{P^{1}_{02}} & \mathbf{P^{2}_{02}} & \mathbf{P^{3}_{02}} \\ \mathbf{P^{4}_{02}} & \mathbf{P^{5}_{02}} & \mathbf{P^{6}_{02}} & \mathbf{P^{7}_{02}} \\ \mathbf{P^{8}_{02}} & \mathbf{P^{9}_{02}} & \mathbf{P^{10}_{02}} & \mathbf{P^{11}_{02}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{03}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^{0}_{03}} & \mathbf{P^{1}_{03}} & \mathbf{P^{2}_{03}} & \mathbf{P^{3}_{03}} \\ \mathbf{P^{4}_{03}} & \mathbf{P^{5}_{03}} & \mathbf{P^{6}_{03}} & \mathbf{P^{7}_{03}} \\ \mathbf{P^{8}_{03}} & \mathbf{P^{9}_{03}} & \mathbf{P^{10}_{03}} & \mathbf{P^{11}_{03}} \\ \mathbf{P^{12}_{03}} & \mathbf{P^{13}_{03}} & \mathbf{P^{14}_{03}} & \mathbf{P^{15}_{03}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{10}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^{0}_{10}} & \mathbf{P^{1}_{10}} & \mathbf{P^{2}_{10}} & \mathbf{P^{3}_{10}} \\ \mathbf{P^{4}_{10}} & \mathbf{P^{5}_{10}} & \mathbf{P^{6}_{10}} & \mathbf{P^{7}_{10}} \\ \mathbf{P^{8}_{10}} & \mathbf{P^{9}_{10}} & \mathbf{P^{10}_{10}} & \mathbf{P^{11}_{10}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{11}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^{0}_{11}} & \mathbf{P^{1}_{11}} & \mathbf{P^{2}_{11}} & \mathbf{P^{3}_{11}} \\ \mathbf{P^{4}_{11}} & \mathbf{P^{5}_{11}} & \mathbf{P^{6}_{11}} & \mathbf{P^{7}_{11}} \\ \mathbf{P^{8}_{11}} & \mathbf{P^{9}_{11}} & \mathbf{P^{10}_{11}} & \mathbf{P^{11}_{11}} \\ \mathbf{P^{12}_{11}} & \mathbf{P^{13}_{11}} & \mathbf{P^{14}_{11}} & \mathbf{P^{15}_{11}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{12}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{12}} & \mathbf{P^1_{12}} & \mathbf{P^2_{12}} & \mathbf{P^3_{12}} \\ \mathbf{P^4_{12}} & \mathbf{P^5_{12}} & \mathbf{P^6_{12}} & \mathbf{P^7_{12}} \\ \mathbf{P^8_{12}} & \mathbf{P^9_{12}} & \mathbf{P^{10}_{12}} & \mathbf{P^{11}_{12}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{13}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{13}} & \mathbf{P^1_{13}} & \mathbf{P^2_{13}} & \mathbf{P^3_{13}} \\ \mathbf{P^4_{13}} & \mathbf{P^5_{13}} & \mathbf{P^6_{13}} & \mathbf{P^7_{13}} \\ \mathbf{P^8_{13}} & \mathbf{P^9_{13}} & \mathbf{P^{10}_{13}} & \mathbf{P^{11}_{13}} \\ \mathbf{P^{12}_{13}} & \mathbf{P^{13}_{13}} & \mathbf{P^{14}_{13}} & \mathbf{P^{15}_{13}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{20}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{20}} & \mathbf{P^1_{20}} & \mathbf{P^2_{20}} & \mathbf{P^3_{20}} \\ \mathbf{P^4_{20}} & \mathbf{P^5_{20}} & \mathbf{P^6_{20}} & \mathbf{P^7_{20}} \\ \mathbf{P^8_{20}} & \mathbf{P^9_{20}} & \mathbf{P^{10}_{20}} & \mathbf{P^{11}_{20}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{21}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{21}} & \mathbf{P^1_{21}} & \mathbf{P^2_{21}} & \mathbf{P^3_{21}} \\ \mathbf{P^4_{21}} & \mathbf{P^5_{21}} & \mathbf{P^6_{21}} & \mathbf{P^7_{21}} \\ \mathbf{P^8_{21}} & \mathbf{P^9_{21}} & \mathbf{P^{10}_{21}} & \mathbf{P^{11}_{21}} \\ \mathbf{P^{12}_{21}} & \mathbf{P^{13}_{21}} & \mathbf{P^{14}_{21}} & \mathbf{P^{15}_{21}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{22}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{22}} & \mathbf{P^1_{22}} & \mathbf{P^2_{22}} & \mathbf{P^3_{22}} \\ \mathbf{P^4_{22}} & \mathbf{P^5_{22}} & \mathbf{P^6_{22}} & \mathbf{P^7_{22}} \\ \mathbf{P^8_{22}} & \mathbf{P^9_{22}} & \mathbf{P^{10}_{22}} & \mathbf{P^{11}_{22}} \\ \mathbf{0} & \mathbf{P^{12}_{22}} & \mathbf{0} & \mathbf{P^{13}_{22}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{23}} = \begin{array}{c|cccc|} & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & \mathbf{P^0_{23}} & \mathbf{P^1_{23}} & \mathbf{P^2_{23}} & \mathbf{P^3_{23}} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & \mathbf{P^4_{23}} & \mathbf{P^5_{23}} & \mathbf{P^6_{23}} & \mathbf{P^7_{23}} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & \mathbf{P^8_{23}} & \mathbf{P^9_{23}} & \mathbf{P^{10}_{23}} & \mathbf{P^{11}_{23}} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) & \mathbf{P^{12}_{23}} & \mathbf{P^{13}_{23}} & \mathbf{P^{14}_{23}} & \mathbf{P^{15}_{23}} \end{array}$$

$$\mathbf{P_{30}} = \begin{array}{c|cccc|} & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & \mathbf{0} & \mathbf{P^0_{30}} & \mathbf{0} & \mathbf{P^1_{30}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & \mathbf{0} & \mathbf{P^2_{30}} & \mathbf{0} & \mathbf{P^3_{30}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & \mathbf{0} & \mathbf{P^4_{30}} & \mathbf{0} & \mathbf{P^5_{30}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}$$

$$\mathbf{P_{31}} = \begin{array}{c|cccc|} & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & \mathbf{0} & \mathbf{P^0_{31}} & \mathbf{0} & \mathbf{P^1_{31}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & \mathbf{0} & \mathbf{P^2_{31}} & \mathbf{0} & \mathbf{P^3_{31}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & \mathbf{0} & \mathbf{P^4_{31}} & \mathbf{P^5_{31}} & \mathbf{P^6_{31}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) & \mathbf{0} & \mathbf{P^7_{31}} & \mathbf{0} & \mathbf{P^8_{31}} \end{array}$$

$$\mathbf{P_{32}} = \begin{array}{c|cccc|} & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & \mathbf{0} & \mathbf{P^0_{32}} & \mathbf{0} & \mathbf{P^1_{32}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & \mathbf{0} & \mathbf{P^2_{32}} & \mathbf{0} & \mathbf{P^3_{32}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & \mathbf{0} & \mathbf{P^4_{32}} & \mathbf{0} & \mathbf{P^5_{32}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}$$

$$\mathbf{P_{33}} = \begin{array}{c|cccc|} & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{00}) & \mathbf{0} & \mathbf{P^0_{33}} & \mathbf{0} & \mathbf{P^1_{33}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0S}) & \mathbf{0} & \mathbf{P^2_{33}} & \mathbf{0} & \mathbf{P^3_{33}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P0}) & \mathbf{0} & \mathbf{P^4_{33}} & \mathbf{P^5_{33}} & \mathbf{P^6_{33}} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{PS}) & \mathbf{0} & \mathbf{P^7_{33}} & \mathbf{0} & \mathbf{P^8_{33}} \end{array}$$

such that,

$$\mathbf{P^{12}_{01}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}^0_p} \Big)$$
$$\mathbf{P^{13}_{01}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{s}} \Big)$$
$$\mathbf{P^{14}_{01}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{10}} \otimes (\mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \mathbf{\hat{S}_p}) + \ \mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}_p} + \mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}_p} + \mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}_p} \Big)$$
$$\mathbf{P^{15}_{01}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \otimes \hat{e_1}^t(\mathbf{s}) \Big)$$

$$\mathbf{P^{12}_{03}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s} \Big) \otimes \Big(\mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}^0_p} \Big)$$
$$\mathbf{P^{13}_{03}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s} \Big) \otimes \Big(\mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{s}} \Big)$$
$$\mathbf{P^{14}_{03}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s} \Big) \otimes \Big(\mathbf{\hat{D}_{10}} \otimes (\mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \mathbf{\hat{S}_p}) + \mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}_p} + \mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}_p}$$
$$\ldots + \mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}_p} \Big)$$
$$\mathbf{P^{15}_{03}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s} \Big) \otimes \Big(\mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \otimes \hat{e_1}^t(\mathbf{s}) \Big)$$

$$\mathbf{P^{12}_{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_s} \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}^0_p} \Big)$$
$$\mathbf{P^{13}_{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_s} \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{s}} \Big) + \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s} \Big)$$
$$\ldots \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{S}^0_p} \otimes \mathbf{\hat{Q}}^* \Big)$$
$$\mathbf{P^{14}_{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_s} \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{10}} \otimes (\mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \mathbf{\hat{S}_p}) + \ \mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}_p} + \mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}_p}$$
$$\ldots + \mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}_p} \Big)$$
$$\mathbf{P^{15}_{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_s} \otimes \mathbf{Q}^* \Big) \otimes \Big(\mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \otimes \hat{e_1}^t(\mathbf{s}) \Big) \ + \ \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s} \Big)$$
$$\ldots \otimes \Big(\mathbf{\hat{D}_{10}} \otimes (\mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \mathbf{\hat{S}_p}) \otimes \mathbf{\hat{I}_s} + \mathbf{\hat{D}_{11}} \otimes (\mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \mathbf{\hat{S}_p}) \otimes \mathbf{\hat{I}_s} + \ \mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}_p} \otimes \mathbf{\hat{I}_s}$$
$$\ldots + \mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}_p} \otimes \mathbf{\hat{I}_s} \Big)$$

$$\mathbf{P^{12}_{13}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S^0_s} \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s} \Big) \otimes \Big(\mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}^0_p} \Big)$$
$$\mathbf{P^{13}_{13}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S^0_s} \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s} \Big) \otimes \Big(\mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}^0_p} \hat{\beta}_s \Big) + \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{S^*_s} \Big)$$
$$\ldots \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{S}^0_p} \otimes \mathbf{\hat{Q}}^* \Big)$$
$$\mathbf{P^{14}_{13}} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S^0_s} \boldsymbol{\beta}_{\boldsymbol{p}} \otimes \mathbf{I_s} \Big) \otimes \Big(\mathbf{\hat{D}_{10}} \otimes (\mathbf{\hat{S}^0_p} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \mathbf{\hat{S}_p}) + \mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}_p} + \mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}_p}$$
$$\ldots + \mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}_p} \Big)$$

$$\mathbf{P_{13}^{15}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes \mathbf{S_s^0}\boldsymbol{\beta_p}\otimes \mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{11}}\otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes \hat{e_1}^t(\mathbf{s})\Big) \;+\; \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes \boldsymbol{\beta_p}\otimes \mathbf{S_s^*}\Big)$$

$$\ldots\otimes\Big(\hat{\mathbf{D}}_{\mathbf{10}}\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}+\hat{\mathbf{D}}_{\mathbf{11}}\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}+\;\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}\otimes\hat{\mathbf{I}}_{\mathbf{s}}$$

$$\ldots+\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}\otimes\hat{\mathbf{I}}_{\mathbf{s}}\Big)$$

$$\mathbf{P_{21}^{12}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{21}^{13}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$

$$\mathbf{P_{21}^{14}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{10}}\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})+\;\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}+\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}$$

$$\ldots+\hat{\mathbf{D}}_{\mathbf{11}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P_{21}^{15}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{11}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes\hat{e_1}^t(\mathbf{s})\Big)$$

$$\mathbf{P_{22}^{12}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\otimes\hat{\mathbf{Q}}^*\Big)$$

$$\mathbf{P_{22}^{13}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{10}}\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}$$

$$\ldots+\;\hat{\mathbf{D}}_{\mathbf{11}}\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}+\;\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}\otimes\hat{\mathbf{I}}_{\mathbf{s}}+\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}\otimes\hat{\mathbf{I}}_{\mathbf{s}}\Big)$$

$$\mathbf{P_{23}^{12}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{23}^{13}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\beta}_s\Big)$$

$$\mathbf{P_{23}^{14}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{10}}\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})+\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}+\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}$$

$$\ldots+\hat{\mathbf{D}}_{\mathbf{11}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P_{23}^{15}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{11}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes\hat{e_1}^t(\mathbf{s})\Big)$$

$$\mathbf{P_{30}^{0}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\Big)$$

$$\mathbf{P_{30}^{1}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{30}^{2}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{30}^{3}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{30}^{4}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{30}^{5}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^0_{31}} = \left(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}\right) \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^*\right)$$

$$\mathbf{P^1_{31}} = \left(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}\right) \otimes \left((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \boldsymbol{\hat{\beta}_p}\right)$$

$$\mathbf{P^2_{31}} = \left(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}\right) \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_s}\right) + \left((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_p} \otimes \mathbf{Q}^*\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{S}_s}\right)$$

$$\mathbf{P^3_{31}} = \left(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\beta_s\right) \otimes \left((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_s}\boldsymbol{\hat{\beta}_p}\right) + \left((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_p} \otimes \mathbf{Q}^*\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \boldsymbol{\hat{\beta}_p} \otimes \mathbf{\hat{S}^*_s}\right)$$

$$\mathbf{P^4_{31}} = \left(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}\right) \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\right)$$

$$\mathbf{P^5_{31}} = \left((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_p} \otimes \mathbf{Q}^*\right) \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{S}_p}\right)$$

$$\mathbf{P^6_{31}} = \left(\mathbf{D_{01}} \otimes \mathbf{S^0_p}\boldsymbol{\beta_s}\right) \otimes \left((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\boldsymbol{\hat{\beta}_p}\right)$$

$$\mathbf{P^0_{32}} = \left(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes \mathbf{S_p}\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^*\right)$$

$$\mathbf{P^1_{32}} = \left(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes \mathbf{S_p}\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \boldsymbol{\hat{\beta}_p}\right)$$

$$\mathbf{P^2_{32}} = \left(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes \mathbf{S_p}\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_s}\right)$$

$$\mathbf{P^3_{32}} = \left(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes \mathbf{S_p}\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_s}\boldsymbol{\hat{\beta}_p}\right)$$

$$\mathbf{P^4_{32}} = \left(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes \mathbf{S_p}\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\right)$$

$$\mathbf{P^5_{32}} = \left(\mathbf{D_{10}} \otimes (\mathbf{S^0_p}\boldsymbol{\beta_p} + \mathbf{S_p}) + \mathbf{D_{00}} \otimes \mathbf{S_p} + \mathbf{D_{01}} \otimes \mathbf{S_p} + \mathbf{D_{11}} \otimes \mathbf{S_p}\right)$$

$$\ldots \otimes \left((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\boldsymbol{\hat{\beta}_p}\right)$$

$$\mathbf{P_{33}^0} = \Big(\mathbf{D_{11}} \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes e_1{}^t(\mathbf{s})\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^*\Big)$$

$$\mathbf{P_{33}^1} = \Big(\mathbf{D_{11}} \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes e_1{}^t(\mathbf{s})\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{33}^2} = \Big(\mathbf{D_{11}} \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes e_1{}^t(\mathbf{s})\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\Big) \ + \ \Big(\mathbf{D_{10}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s}$$

$$\ldots + \mathbf{D_{11}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s} + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{s}}\Big)$$

$$\mathbf{P_{33}^3} = \Big(\mathbf{D_{11}} \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes e_1{}^t(\mathbf{s})\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big) \ + \ \Big(\mathbf{D_{10}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s}$$

$$\ldots + \mathbf{D_{11}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s} + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \otimes \hat{\mathbf{S}}_{\mathbf{s}}^*\Big)$$

$$\mathbf{P_{33}^4} = \Big(\mathbf{D_{11}} \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes e_1{}^t(\mathbf{s})\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{33}^5} = \Big(\mathbf{D_{10}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \ \mathbf{D_{11}} \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + \ \mathbf{D_{00}} \otimes \mathbf{S_p} \otimes \mathbf{I_s} + \mathbf{D_{01}} \otimes \mathbf{S_p} \otimes \mathbf{I_s}\Big)$$

$$\ldots \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}} + \hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P_{33}^6} = \Big(\mathbf{D_{11}} \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes e_1{}^t(\mathbf{s})\Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

Note: As mentioned, rest of the terms can be directly inferred from the Kronecker product between the two replicas of the Single Channel Model 2 with different channel parameters.

Appendix D

Handoff Strategy 2 with the Single Channel Model 1

$$\mathbf{P} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) \\ \vdots \\ \vdots \\ (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccccc} (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) & \cdots & \cdots & \cdots & \cdots & (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P_{00}} & \mathbf{P_{01}} & \mathbf{P_{02}} & \mathbf{P_{03}} \\ \mathbf{P_{10}} & \mathbf{P_{11}} & \mathbf{P_{12}} & \mathbf{P_{13}} \\ \mathbf{P_{20}} & \mathbf{P_{21}} & \mathbf{P_{22}} & \mathbf{P_{23}} \\ \mathbf{P_{30}} & \mathbf{P_{31}} & \mathbf{P_{32}} & \mathbf{P_{33}} \end{pmatrix} \end{array}$$

where,

$$\mathbf{P_{00}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{00}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{PS}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{00}} & \mathbf{P^1_{00}} & \mathbf{P^2_{00}} & \mathbf{0} \\ \mathbf{P^3_{00}} & \mathbf{P^4_{00}} & \mathbf{P^5_{00}} & \mathbf{P^6_{00}} \\ \mathbf{P^7_{00}} & \mathbf{P^8_{00}} & \mathbf{P^9_{00}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{00}} & \mathbf{0} & \mathbf{P^{11}_{00}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{01}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{01}} & \mathbf{P^1_{01}} & \mathbf{P^2_{01}} & \mathbf{0} \\ \mathbf{P^3_{01}} & \mathbf{P^4_{01}} & \mathbf{P^5_{01}} & \mathbf{P^6_{01}} \\ \mathbf{P^7_{01}} & \mathbf{P^8_{01}} & \mathbf{P^9_{01}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{01}} & \mathbf{P^{11}_{01}} & \mathbf{P^{12}_{01}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{02}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{02}} & \mathbf{P^1_{02}} & \mathbf{P^2_{02}} & \mathbf{0} \\ \mathbf{P^3_{02}} & \mathbf{P^4_{02}} & \mathbf{P^5_{02}} & \mathbf{P^6_{02}} \\ \mathbf{P^7_{02}} & \mathbf{P^8_{02}} & \mathbf{P^9_{02}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{02}} & \mathbf{0} & \mathbf{P^{11}_{02}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{03}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{0}),\ (\mathbf{00}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{PS}) \end{array} & \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P^0_{03}} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{10}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{10}} & \mathbf{P^1_{10}} & \mathbf{P^2_{10}} & \mathbf{0} \\ \mathbf{P^3_{10}} & \mathbf{P^4_{10}} & \mathbf{P^5_{10}} & \mathbf{P^6_{10}} \\ \mathbf{P^7_{10}} & \mathbf{P^8_{10}} & \mathbf{P^9_{10}} & 0 \\ \mathbf{0} & \mathbf{P^{10}_{10}} & \mathbf{0} & \mathbf{P^{11}_{10}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{11}} = \begin{array}{cc} & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{11}} & \mathbf{P^1_{11}} & \mathbf{P^2_{11}} & \mathbf{0} \\ \mathbf{P^3_{11}} & \mathbf{P^4_{11}} & \mathbf{P^5_{11}} & \mathbf{P^6_{11}} \\ \mathbf{P^7_{11}} & \mathbf{P^8_{11}} & \mathbf{P^9_{11}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{11}} & \mathbf{P^{11}_{11}} & \mathbf{P^{12}_{11}} \end{pmatrix} \end{array}$$

$$
\mathbf{P_{12}} = \begin{array}{c c}
 & \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} &
\begin{pmatrix} \mathbf{P^0_{12}} & \mathbf{P^1_{12}} & \mathbf{P^2_{12}} & \mathbf{0} \\ \mathbf{P^3_{12}} & \mathbf{P^4_{12}} & \mathbf{P^5_{12}} & \mathbf{P^6_{12}} \\ \mathbf{P^7_{12}} & \mathbf{P^8_{12}} & \mathbf{P^9_{12}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{12}} & \mathbf{0} & \mathbf{P^{11}_{12}} \end{pmatrix}
\end{array}
$$

$$
\mathbf{P_{13}} = \begin{array}{c c}
 & \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} &
\begin{pmatrix} \mathbf{P^0_{13}} & \mathbf{P^1_{13}} & \mathbf{P^2_{13}} & \mathbf{0} \\ \mathbf{P^3_{13}} & \mathbf{P^4_{13}} & \mathbf{P^5_{13}} & \mathbf{P^6_{13}} \\ \mathbf{P^7_{13}} & \mathbf{P^8_{13}} & \mathbf{P^9_{13}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{13}} & \mathbf{P^{11}_{13}} & \mathbf{P^{12}_{13}} \end{pmatrix}
\end{array}
$$

$$
\mathbf{P_{20}} = \begin{array}{c c}
 & \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{c} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} &
\begin{pmatrix} \mathbf{P^0_{20}} & \mathbf{P^1_{20}} & \mathbf{P^2_{20}} & \mathbf{0} \\ \mathbf{P^3_{20}} & \mathbf{P^4_{20}} & \mathbf{P^5_{20}} & \mathbf{P^6_{20}} \\ \mathbf{P^7_{20}} & \mathbf{P^8_{20}} & \mathbf{P^9_{20}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{20}} & \mathbf{0} & \mathbf{P^{11}_{20}} \end{pmatrix}
\end{array}
$$

$$
\mathbf{P_{21}} = \begin{array}{c c}
 & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{c} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} &
\begin{pmatrix} \mathbf{P^0_{21}} & \mathbf{P^1_{21}} & \mathbf{P^2_{21}} & \mathbf{0} \\ \mathbf{P^3_{21}} & \mathbf{P^4_{21}} & \mathbf{P^5_{21}} & \mathbf{P^6_{21}} \\ \mathbf{P^7_{21}} & \mathbf{P^8_{21}} & \mathbf{P^9_{21}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{21}} & \mathbf{P^{11}_{21}} & \mathbf{P^{12}_{21}} \end{pmatrix}
\end{array}
$$

$$
\mathbf{P_{22}} = \begin{array}{c c}
 & \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{array}{c} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} &
\begin{pmatrix} \mathbf{P^0_{22}} & \mathbf{P^1_{22}} & \mathbf{P^2_{22}} & \mathbf{0} \\ \mathbf{P^3_{22}} & \mathbf{P^4_{22}} & \mathbf{P^5_{22}} & \mathbf{P^6_{22}} \\ \mathbf{P^7_{22}} & \mathbf{P^8_{22}} & \mathbf{P^9_{22}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{22}} & \mathbf{0} & \mathbf{P^{11}_{22}} \end{pmatrix}
\end{array}
$$

$$\mathbf{P_{23}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P^0_{23}} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{30}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{31}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{31}} & \mathbf{P^1_{31}} & \mathbf{P^2_{31}} & \mathbf{0} \\ \mathbf{P^3_{31}} & \mathbf{P^4_{31}} & \mathbf{P^5_{31}} & \mathbf{P^6_{31}} \\ \mathbf{P^7_{31}} & \mathbf{P^8_{31}} & \mathbf{P^9_{31}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{31}} & \mathbf{0} & \mathbf{P^{11}_{31}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{32}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{P^0_{32}} & \mathbf{0} & \mathbf{P^1_{32}} \\ \mathbf{0} & \mathbf{P^2_{32}} & \mathbf{0} & \mathbf{P^3_{32}} \\ \mathbf{0} & \mathbf{P^4_{32}} & \mathbf{0} & \mathbf{P^5_{32}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

$$\mathbf{P_{33}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{33}} & \mathbf{P^1_{33}} & \mathbf{P^2_{33}} & \mathbf{0} \\ \mathbf{P^3_{33}} & \mathbf{P^4_{33}} & \mathbf{P^5_{33}} & \mathbf{P^6_{33}} \\ \mathbf{P^7_{33}} & \mathbf{P^8_{33}} & \mathbf{P^9_{33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P^{10}_{33}} & \mathbf{0} & \mathbf{P^{11}_{33}} \end{pmatrix} \end{array}$$

such that,

$$\mathbf{P}_{00}^{10} = \Big(\mathbf{D_{00}}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01}) \otimes \hat{\mathbf{Q}}^{*} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P}_{00}^{11} = \Big(\mathbf{D_{00}}\Big) \otimes \Big((\hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P}_{01}^{10} = \Big(\mathbf{D_{01}} \otimes \boldsymbol{\beta_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{01} + \hat{\mathbf{D}}_{00}) \otimes \hat{\mathbf{Q}}^{*} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P}_{01}^{11} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^{*}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01} + \hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}\Big)$$
$$\mathbf{P}_{01}^{12} = \Big(\mathbf{D_{01}} \otimes \boldsymbol{\beta_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P}_{02}^{10} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \boldsymbol{\beta_p}\Big) \otimes \Big((\hat{\mathbf{D}}_{01} + \hat{\mathbf{D}}_{00}) \otimes \hat{\mathbf{Q}}^{*} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P}_{02}^{11} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \boldsymbol{\beta_p}\Big) \otimes \Big((\hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\boldsymbol{\beta_p}\Big)$$

$$\mathbf{P}_{03}^{0} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01} + \hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P}_{10}^{10} = \Big(\mathbf{D_{00}} \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01}) \otimes \hat{\mathbf{Q}}^{*} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P}_{10}^{11} = \Big(\mathbf{D_{00}} \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}}\Big) \otimes \Big((\hat{\mathbf{D}}_{11} + \hat{\mathbf{D}}_{10}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P}_{11}^{10} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \otimes \hat{\mathbf{Q}}^{*}\Big) + \Big(\mathbf{D_{01}} \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}}\boldsymbol{\beta_s}\Big)$$
$$\ldots \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01}) \otimes \hat{\mathbf{Q}}^{*} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P}_{11}^{11} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}} \otimes \mathbf{Q}^{*}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01} + \hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}\Big)$$
$$\mathbf{P}_{11}^{12} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{11} + \hat{\mathbf{D}}_{10}) \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} + (\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01}) \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{S}}\Big)$$
$$\ldots + \Big(\mathbf{D_{01}} \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}}\boldsymbol{\beta_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P}_{12}^{10} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}}\boldsymbol{\beta_p}\Big) \otimes \Big((\hat{\mathbf{D}}_{01} + \hat{\mathbf{D}}_{00}) \otimes \hat{\mathbf{Q}}^{*} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$
$$\mathbf{P}_{12}^{11} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}}\boldsymbol{\beta_p}\Big) \otimes \Big((\hat{\mathbf{D}}_{11} + \hat{\mathbf{D}}_{10}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P}_{13}^{10} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \boldsymbol{\beta_p} \otimes \mathbf{S}_{\mathbf{s}}^{*}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \otimes \hat{\mathbf{Q}}^{*}\Big)$$
$$\mathbf{P}_{13}^{11} = \Big((\mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S}_{\mathbf{s}}^{\mathbf{0}}\boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\hat{\mathbf{D}}_{00} + \hat{\mathbf{D}}_{01} + \hat{\mathbf{D}}_{10} + \hat{\mathbf{D}}_{11}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P_{13}^{12}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \boldsymbol{\beta_p} \otimes \mathbf{S_s^*} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{11}} + \hat{\mathbf{D}}_{\mathbf{10}}) \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} + (\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{S}} \Big)$$

$$\mathbf{P_{20}^{10}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \Big)$$
$$\mathbf{P_{20}^{11}} = \Big(\mathbf{D_{00}} \otimes \mathbf{S_p^0} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{11}} + \hat{\mathbf{D}}_{\mathbf{10}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \Big)$$

$$\mathbf{P_{21}^{10}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S_p^0} \boldsymbol{\beta_s} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \Big)$$
$$\mathbf{P_{21}^{11}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}} + \hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}} \Big)$$
$$\mathbf{P_{21}^{12}} = \Big(\mathbf{D_{01}} \otimes \mathbf{S_p^0} \boldsymbol{\beta_s} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{11}} + \hat{\mathbf{D}}_{\mathbf{10}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \Big)$$

$$\mathbf{P_{22}^{10}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0} \boldsymbol{\beta_p} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{Q}}^* \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \Big) + \Big((\mathbf{D_{00}} + \mathbf{D_{01}} + \mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_p} \Big)$$
$$\ldots \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \otimes \hat{\mathbf{Q}}^* \Big)$$
$$\mathbf{P_{22}^{11}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0} \boldsymbol{\beta_p} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{11}} + \hat{\mathbf{D}}_{\mathbf{10}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \Big) + \Big((\mathbf{D_{00}} + \mathbf{D_{01}} + \mathbf{D_{10}} + \mathbf{D_{11}}) \otimes \mathbf{S_p} \Big)$$
$$\ldots \otimes \Big((\hat{\mathbf{D}}_{\mathbf{11}} + \hat{\mathbf{D}}_{\mathbf{10}}) \otimes (\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\mathbf{S}}_{\mathbf{p}}) \otimes \hat{\mathbf{I}}_{\mathbf{s}} + (\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}} \otimes \hat{\mathbf{I}}_{\mathbf{S}} \Big)$$

$$\mathbf{P_{23}^{0}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0} \boldsymbol{\beta_p} \otimes \mathbf{I_s} \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}} + \hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}} \Big)$$

$$\mathbf{P_{31}^{0}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \Big)$$
$$\mathbf{P_{31}^{1}} = \Big((\mathbf{D_{01}} + \mathbf{D_{00}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{s}} \Big)$$
$$\mathbf{P_{31}^{2}} = \Big((\mathbf{D_{01}} + \mathbf{D_{00}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{11}} + \hat{\mathbf{D}}_{\mathbf{10}}) \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \Big)$$
$$\mathbf{P_{31}^{3}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}} \Big)$$
$$\mathbf{P_{31}^{4}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{00}} + \hat{\mathbf{D}}_{\mathbf{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{s}} \Big) + \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big)$$
$$\ldots \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{s}} \Big)$$
$$\mathbf{P_{31}^{5}} = \Big((\mathbf{D_{01}} + \mathbf{D_{00}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{10}} + \hat{\mathbf{D}}_{\mathbf{11}}) \otimes \hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \Big)$$
$$\mathbf{P_{31}^{6}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big((\hat{\mathbf{D}}_{\mathbf{11}} + \hat{\mathbf{D}}_{\mathbf{10}}) \otimes \hat{\boldsymbol{\beta}}_{\boldsymbol{p}} \otimes \hat{\mathbf{S}}_{\mathbf{s}}^* \Big)$$
$$\mathbf{P_{31}^{7}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \otimes \hat{\mathbf{Q}}^* \Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{00}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \Big)$$
$$\mathbf{P_{31}^{8}} = \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p^0} \otimes \mathbf{Q}^* \Big) \otimes \Big(\hat{\mathbf{D}}_{\mathbf{01}} \otimes \hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}} \hat{\boldsymbol{\beta}}_{\boldsymbol{s}} \Big)$$

$$\mathbf{P^9_{31}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{p}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)+\Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)$$
$$\ldots\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}}+\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P^0_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\Big)$$
$$\mathbf{P^1_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$
$$\mathbf{P^2_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{s}}\Big)$$
$$\mathbf{P^3_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{s}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$
$$\mathbf{P^4_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{p}}\Big)$$
$$\mathbf{P^5_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{p}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^0_{33}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{00}}\Big)$$
$$\mathbf{P^1_{33}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$
$$\mathbf{P^2_{33}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_S}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$
$$\mathbf{P^3_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{s}}\Big)$$
$$\mathbf{P^4_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes(\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}+\mathbf{S_p})\otimes\mathbf{I_s}+(\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p}\otimes\mathbf{I_S}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{S}}_{\mathbf{s}}\Big)$$
$$\ldots+\Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{s}}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$
$$\mathbf{P^5_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{s}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$
$$\mathbf{P^6_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes(\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}+\mathbf{S_p})\otimes\mathbf{I_s}+(\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p}\otimes\mathbf{I_s}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes\hat{\mathbf{S}}^*_{\mathbf{s}}\Big)$$
$$\mathbf{P^7_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{p}}\Big)$$
$$\mathbf{P^8_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{p}}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$
$$\mathbf{P^9_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\mathbf{S}}^{\mathbf{0}}_{\mathbf{p}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)+\Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes(\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}+\mathbf{S_p})\otimes\mathbf{I_s}$$
$$\ldots+(\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p}\otimes\mathbf{I_S}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}}+\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

Note: As mentioned, rest of the terms can be directly inferred from the Kronecker product between the two replicas of the Single Channel Model 1 with different channel parameters.

Appendix E

Handoff Strategy 2 with the Single Channel Model 2

$$
\mathbf{P} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) \\ \vdots \\ \vdots \\ (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c} \begin{array}{ccccc} (\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0}) & \cdots & \cdots & \cdots\ \ \cdots & (\mathbf{P},\mathbf{S}),(\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix} \mathbf{P_{00}} & \mathbf{P_{01}} & \mathbf{P_{02}} & \mathbf{P_{03}} \\ \mathbf{P_{10}} & \mathbf{P_{11}} & \mathbf{P_{12}} & \mathbf{P_{13}} \\ \mathbf{P_{20}} & \mathbf{P_{21}} & \mathbf{P_{22}} & \mathbf{P_{23}} \\ \mathbf{P_{30}} & \mathbf{P_{31}} & \mathbf{P_{32}} & \mathbf{P_{33}} \end{pmatrix} \end{array}
$$

where,

$$
\mathbf{P_{00}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix} \mathbf{P_{00}^{0}} & \mathbf{P_{00}^{1}} & \mathbf{P_{00}^{2}} & \mathbf{P_{00}^{3}} \\ \mathbf{P_{00}^{4}} & \mathbf{P_{00}^{5}} & \mathbf{P_{00}^{6}} & \mathbf{P_{00}^{7}} \\ \mathbf{P_{00}^{8}} & \mathbf{P_{00}^{9}} & \mathbf{P_{00}^{10}} & \mathbf{P_{00}^{11}} \\ \mathbf{0} & \mathbf{P_{00}^{12}} & \mathbf{0} & \mathbf{P_{00}^{13}} \end{pmatrix} \end{array}
$$

$$\mathbf{P_{01}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{01}} & \mathbf{P^1_{01}} & \mathbf{P^2_{01}} & \mathbf{P^3_{01}} \\ \mathbf{P^4_{01}} & \mathbf{P^5_{01}} & \mathbf{P^6_{01}} & \mathbf{P^7_{01}} \\ \mathbf{P^8_{01}} & \mathbf{P^9_{01}} & \mathbf{P^{10}_{01}} & \mathbf{P^{11}_{01}} \\ \mathbf{0} & \mathbf{P^{12}_{01}} & \mathbf{P^{13}_{01}} & \mathbf{P^{14}_{01}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{02}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{02}} & \mathbf{P^1_{02}} & \mathbf{P^2_{02}} & \mathbf{P^3_{02}} \\ \mathbf{P^4_{02}} & \mathbf{P^5_{02}} & \mathbf{P^6_{02}} & \mathbf{P^7_{02}} \\ \mathbf{P^8_{02}} & \mathbf{P^9_{02}} & \mathbf{P^{10}_{02}} & \mathbf{P^{11}_{02}} \\ \mathbf{0} & \mathbf{P^{12}_{02}} & \mathbf{0} & \mathbf{P^{13}_{02}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{03}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{03}} & \mathbf{P^1_{03}} & \mathbf{P^2_{03}} & \mathbf{P^3_{03}} \\ \mathbf{P^4_{03}} & \mathbf{P^5_{03}} & \mathbf{P^6_{03}} & \mathbf{P^7_{03}} \\ \mathbf{P^8_{03}} & \mathbf{P^9_{03}} & \mathbf{P^{10}_{03}} & \mathbf{P^{11}_{03}} \\ \mathbf{0} & \mathbf{P^{12}_{03}} & \mathbf{P^{13}_{03}} & \mathbf{P^{14}_{03}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{10}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{10}} & \mathbf{P^1_{10}} & \mathbf{P^2_{10}} & \mathbf{P^3_{10}} \\ \mathbf{P^4_{10}} & \mathbf{P^5_{10}} & \mathbf{P^6_{10}} & \mathbf{P^7_{10}} \\ \mathbf{P^8_{10}} & \mathbf{P^9_{10}} & \mathbf{P^{10}_{10}} & \mathbf{P^{11}_{10}} \\ \mathbf{0} & \mathbf{P^{12}_{10}} & \mathbf{0} & \mathbf{P^{13}_{10}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{11}} = \begin{array}{c} \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{pmatrix} \mathbf{P^0_{11}} & \mathbf{P^1_{11}} & \mathbf{P^2_{11}} & \mathbf{P^3_{11}} \\ \mathbf{P^4_{11}} & \mathbf{P^5_{11}} & \mathbf{P^6_{11}} & \mathbf{P^7_{11}} \\ \mathbf{P^8_{11}} & \mathbf{P^9_{11}} & \mathbf{P^{10}_{11}} & \mathbf{P^{11}_{11}} \\ \mathbf{0} & \mathbf{P^{12}_{11}} & \mathbf{P^{13}_{11}} & \mathbf{P^{14}_{11}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{12}} = \begin{array}{c c} & \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{12}} & \mathbf{P^1_{12}} & \mathbf{P^2_{12}} & \mathbf{P^3_{12}} \\ \mathbf{P^4_{12}} & \mathbf{P^5_{12}} & \mathbf{P^6_{12}} & \mathbf{P^7_{12}} \\ \mathbf{P^8_{12}} & \mathbf{P^9_{12}} & \mathbf{P^{10}_{12}} & \mathbf{P^{11}_{12}} \\ \mathbf{0} & \mathbf{P^{12}_{12}} & \mathbf{0} & \mathbf{P^{13}_{12}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{13}} = \begin{array}{c c} & \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{13}} & \mathbf{P^1_{13}} & \mathbf{P^2_{13}} & \mathbf{P^3_{13}} \\ \mathbf{P^4_{13}} & \mathbf{P^5_{13}} & \mathbf{P^6_{13}} & \mathbf{P^7_{13}} \\ \mathbf{P^8_{13}} & \mathbf{P^9_{13}} & \mathbf{P^{10}_{13}} & \mathbf{P^{11}_{13}} \\ \mathbf{0} & \mathbf{P^{12}_{13}} & \mathbf{P^{13}_{13}} & \mathbf{P^{14}_{13}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{20}} = \begin{array}{c c} & \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{20}} & \mathbf{P^1_{20}} & \mathbf{P^2_{20}} & \mathbf{P^3_{20}} \\ \mathbf{P^4_{20}} & \mathbf{P^5_{20}} & \mathbf{P^6_{20}} & \mathbf{P^7_{20}} \\ \mathbf{P^8_{20}} & \mathbf{P^9_{20}} & \mathbf{P^{10}_{20}} & \mathbf{P^{11}_{20}} \\ \mathbf{0} & \mathbf{P^{12}_{20}} & \mathbf{0} & \mathbf{P^{13}_{20}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{21}} = \begin{array}{c c} & \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{21}} & \mathbf{P^1_{21}} & \mathbf{P^2_{21}} & \mathbf{P^3_{21}} \\ \mathbf{P^4_{21}} & \mathbf{P^5_{21}} & \mathbf{P^6_{21}} & \mathbf{P^7_{21}} \\ \mathbf{P^8_{21}} & \mathbf{P^9_{21}} & \mathbf{P^{10}_{21}} & \mathbf{P^{11}_{21}} \\ \mathbf{0} & \mathbf{P^{12}_{21}} & \mathbf{P^{13}_{21}} & \mathbf{P^{14}_{21}} \end{pmatrix} \end{array}$$

$$\mathbf{P_{22}} = \begin{array}{c c} & \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\ \begin{array}{c} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} & \begin{pmatrix} \mathbf{P^0_{22}} & \mathbf{P^1_{22}} & \mathbf{P^2_{22}} & \mathbf{P^3_{22}} \\ \mathbf{P^4_{22}} & \mathbf{P^5_{22}} & \mathbf{P^6_{22}} & \mathbf{P^7_{22}} \\ \mathbf{P^8_{22}} & \mathbf{P^9_{22}} & \mathbf{P^{10}_{22}} & \mathbf{P^{11}_{22}} \\ \mathbf{0} & \mathbf{P^{12}_{22}} & \mathbf{0} & \mathbf{P^{13}_{22}} \end{pmatrix} \end{array}$$

$$
\mathbf{P_{23}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix} \mathbf{P^0_{23}} & \mathbf{P^1_{23}} & \mathbf{P^2_{23}} & \mathbf{P^3_{23}} \\ \mathbf{P^4_{23}} & \mathbf{P^5_{23}} & \mathbf{P^6_{23}} & \mathbf{P^7_{23}} \\ \mathbf{P^8_{23}} & \mathbf{P^9_{23}} & \mathbf{P^{10}_{23}} & \mathbf{P^{11}_{23}} \\ \mathbf{0} & \mathbf{P^{12}_{23}} & \mathbf{P^{13}_{23}} & \mathbf{P^{14}_{23}} \end{pmatrix} \end{array}
$$

$$
\mathbf{P_{30}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}
$$

$$
\mathbf{P_{31}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c} \begin{array}{cccc} (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{0},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix} \mathbf{P^0_{31}} & \mathbf{P^1_{31}} & \mathbf{P^2_{31}} & \mathbf{P^3_{31}} \\ \mathbf{P^4_{31}} & \mathbf{P^5_{31}} & \mathbf{P^6_{31}} & \mathbf{P^7_{31}} \\ \mathbf{P^8_{31}} & \mathbf{P^9_{31}} & \mathbf{P^{10}_{31}} & \mathbf{P^{11}_{31}} \\ \mathbf{0} & \mathbf{P^{12}_{31}} & \mathbf{0} & \mathbf{P^{13}_{31}} \end{pmatrix} \end{array}
$$

$$
\mathbf{P_{32}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{0}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix} \mathbf{0} & \mathbf{P^0_{32}} & \mathbf{0} & \mathbf{P^1_{32}} \\ \mathbf{0} & \mathbf{P^2_{32}} & \mathbf{0} & \mathbf{P^3_{32}} \\ \mathbf{0} & \mathbf{P^4_{32}} & \mathbf{0} & \mathbf{P^5_{32}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}
$$

$$
\mathbf{P_{33}} = \begin{array}{c} \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) \\ (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array}
\begin{array}{c} \begin{array}{cccc} (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{0},\mathbf{S}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{0}) & (\mathbf{P},\mathbf{S}),\ (\mathbf{P},\mathbf{S}) \end{array} \\
\begin{pmatrix} \mathbf{P^0_{33}} & \mathbf{P^1_{33}} & \mathbf{P^2_{33}} & \mathbf{P^3_{33}} \\ \mathbf{P^4_{33}} & \mathbf{P^5_{33}} & \mathbf{P^6_{33}} & \mathbf{P^7_{33}} \\ \mathbf{P^8_{33}} & \mathbf{P^9_{33}} & \mathbf{P^{10}_{33}} & \mathbf{P^{11}_{33}} \\ \mathbf{0} & \mathbf{P^{12}_{33}} & \mathbf{0} & \mathbf{P^{13}_{33}} \end{pmatrix} \end{array}
$$

such that,

$$
\begin{aligned}
\mathbf{P^{12}_{00}} &= \Big(\mathbf{D_{00}}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big) \\
\mathbf{P^{13}_{00}} &= \Big(\mathbf{D_{00}}\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P^{12}_{01}} &= \Big(\mathbf{D_{01}} \otimes \boldsymbol{\beta_s}\Big) \otimes \Big((\mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{00}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big) \\
\mathbf{P^{13}_{01}} &= \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{Q}^*\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{S}_p}\Big) \\
\mathbf{P^{14}_{01}} &= \Big(\mathbf{D_{01}} \otimes \boldsymbol{\beta_s}\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P^{12}_{02}} &= \Big(\mathbf{D_{10}} \otimes \boldsymbol{\beta_p}\Big) \otimes \Big((\mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{00}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big) \\
\mathbf{P^{13}_{02}} &= \Big(\mathbf{D_{10}} \otimes \boldsymbol{\beta_p}\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\boldsymbol{\beta_p}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P^{12}_{03}} &= \Big(\mathbf{D_{11}} \otimes \boldsymbol{\beta_p} \otimes e_{\mathbf{1}}{}^t(\mathbf{s})\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big) \\
\mathbf{P^{13}_{03}} &= \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{S}_p}\Big) \\
\mathbf{P^{14}_{03}} &= \Big(\mathbf{D_{11}} \otimes \boldsymbol{\beta_p} \otimes e_{\mathbf{1}}{}^t(\mathbf{s})\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P^{12}_{10}} &= \Big(\mathbf{D_{00}} \otimes \mathbf{S^0_s}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big) \\
\mathbf{P^{13}_{10}} &= \Big(\mathbf{D_{00}} \otimes \mathbf{S^0_s}\Big) \otimes \Big((\mathbf{\hat{D}_{11}} + \mathbf{\hat{D}_{10}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P^{12}_{11}} &= \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{S}^0_p} \otimes \mathbf{\hat{Q}}^*\Big) + \Big(\mathbf{D_{01}} \otimes \mathbf{S^0_s}\boldsymbol{\beta_s}\Big) \\
&\quad \ldots \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big) \\
\mathbf{P^{13}_{11}} &= \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S^0_s} \otimes \mathbf{Q}^*\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{S}_p}\Big) \\
\mathbf{P^{14}_{11}} &= \Big((\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_s}\Big) \otimes \Big((\mathbf{\hat{D}_{11}} + \mathbf{\hat{D}_{10}}) \otimes (\mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \mathbf{\hat{S}_p}) \otimes \mathbf{\hat{I}_s} + (\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}}) \otimes \mathbf{\hat{S}_p} \otimes \mathbf{\hat{I}_S}\Big) \\
&\quad \ldots + \Big(\mathbf{D_{01}} \otimes \mathbf{S^0_s}\boldsymbol{\beta_s}\Big) \otimes \Big((\mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P^{12}_{12}} &= \Big(\mathbf{D_{10}} \otimes \mathbf{S^0_s}\boldsymbol{\beta_p}\Big) \otimes \Big((\mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{00}}) \otimes \mathbf{\hat{Q}}^* \otimes \mathbf{\hat{S}^0_p}\Big) \\
\mathbf{P^{13}_{12}} &= \Big(\mathbf{D_{10}} \otimes \mathbf{S^0_s}\boldsymbol{\beta_p}\Big) \otimes \Big((\mathbf{\hat{D}_{11}} + \mathbf{\hat{D}_{10}}) \otimes \mathbf{\hat{I}_s} \otimes \mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)
\end{aligned}
$$

$$\mathbf{P_{13}^{12}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\boldsymbol{\beta_p}\otimes\mathbf{S_s^*}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\otimes\hat{\mathbf{Q}}^*\Big)+\Big(\mathbf{D_{11}}\otimes\mathbf{S_s^0}\boldsymbol{\beta_p}\otimes {e_1}^t(\mathbf{s})\Big)$$

$$\ldots\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{13}^{13}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_s^0}\boldsymbol{\beta_p}\otimes\mathbf{I_s}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}}+\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P_{13}^{14}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\boldsymbol{\beta_p}\otimes\mathbf{S_s^*}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}+(\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}\otimes\hat{\mathbf{I}}_{\mathbf{S}}\Big)$$

$$\ldots+\Big(\mathbf{D_{11}}\otimes\mathbf{S_s^0}\boldsymbol{\beta_p}\otimes {e_1}^t(\mathbf{s})\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{20}^{12}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{20}^{13}} = \Big(\mathbf{D_{00}}\otimes\mathbf{S_p^0}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{21}^{12}} = \Big(\mathbf{D_{01}}\otimes\mathbf{S_p^0}\boldsymbol{\beta_s}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{21}^{13}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}}+\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P_{21}^{14}} = \Big(\mathbf{D_{01}}\otimes\mathbf{S_p^0}\boldsymbol{\beta_s}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{22}^{12}} = \Big(\mathbf{D_{10}}\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)+\Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)$$

$$\ldots\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\otimes\hat{\mathbf{Q}}^*\Big)$$

$$\mathbf{P_{22}^{13}} = \Big(\mathbf{D_{10}}\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)+\Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)$$

$$\ldots\otimes\Big((\hat{\mathbf{D}}_{\mathbf{11}}+\hat{\mathbf{D}}_{\mathbf{10}})\otimes(\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}+\hat{\mathbf{S}}_{\mathbf{p}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}+(\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}\otimes\hat{\mathbf{I}}_{\mathbf{S}}\Big)$$

$$\mathbf{P_{23}^{12}} = \Big(\mathbf{D_{11}}\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\otimes {e_1}^t(\mathbf{s})\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}})\otimes\hat{\mathbf{Q}}^*\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\Big)$$

$$\mathbf{P_{23}^{13}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\otimes\mathbf{I_s}\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{00}}+\hat{\mathbf{D}}_{\mathbf{01}}+\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{S}}_{\mathbf{p}}\Big)$$

$$\mathbf{P_{23}^{14}} = \Big(\mathbf{D_{11}}\otimes\mathbf{S_p^0}\boldsymbol{\beta_p}\otimes {e_1}^t(\mathbf{s})\Big)\otimes\Big((\hat{\mathbf{D}}_{\mathbf{10}}+\hat{\mathbf{D}}_{\mathbf{11}})\otimes\hat{\mathbf{I}}_{\mathbf{s}}\otimes\hat{\mathbf{S}}_{\mathbf{p}}^{\mathbf{0}}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{31}^{0}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{00}}\Big)$$

$$\mathbf{P_{31}^{1}} = \Big((\mathbf{D_{01}}+\mathbf{D_{00}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{01}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$

$$\mathbf{P_{31}^{2}} = \Big((\mathbf{D_{01}}+\mathbf{D_{00}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{10}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{31}^{3}} = \Big((\mathbf{D_{01}}+\mathbf{D_{00}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{11}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes \hat{e_1}^t(\mathbf{s})\Big)$$

$$\mathbf{P_{31}^{4}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p^0}\otimes\mathbf{Q}^*\Big)\otimes\Big(\hat{\mathbf{D}}_{\mathbf{00}}\otimes\hat{\mathbf{S}}_{\mathbf{s}}^{\mathbf{0}}\Big)$$

$$\mathbf{P^5_{31}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)\otimes\Big((\mathbf{\hat{D}_{00}}+\mathbf{\hat{D}_{01}})\otimes\mathbf{\hat{S}_s}\Big)+\Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)$$

$$\ldots\otimes\Big(\mathbf{\hat{D}_{01}}\otimes\mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$

$$\mathbf{P^6_{31}} = \Big((\mathbf{D_{01}}+\mathbf{D_{00}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)\otimes\Big(\mathbf{\hat{D}_{10}}\otimes\mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^7_{31}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)\otimes\Big((\mathbf{\hat{D}_{11}}+\mathbf{\hat{D}_{10}})\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes\mathbf{\hat{S}^*_s}\Big)+\Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)$$

$$\ldots\otimes\Big(\mathbf{\hat{D}_{11}}\otimes\mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes\hat{e_1}^t(\mathbf{s})\Big)$$

$$\mathbf{P^8_{31}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{\hat{S}^0_p}\otimes\mathbf{\hat{Q}}^*\Big)\otimes\Big(\mathbf{\hat{D}_{00}}\otimes\mathbf{\hat{S}^0_p}\Big)$$

$$\mathbf{P^9_{31}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)\otimes\Big(\mathbf{\hat{D}_{01}}\otimes\mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$

$$\mathbf{P^{10}_{31}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)\otimes\Big(\mathbf{\hat{D}_{10}}\otimes\mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)+\Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)$$

$$\ldots\otimes\Big((\mathbf{\hat{D}_{00}}+\mathbf{\hat{D}_{01}}+\mathbf{\hat{D}_{10}}+\mathbf{\hat{D}_{11}})\otimes\mathbf{\hat{S}_p}\Big)$$

$$\mathbf{P^{11}_{31}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S^0_p}\otimes\mathbf{Q}^*\Big)\otimes\Big(\mathbf{\hat{D}_{11}}\otimes\mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes\hat{e_1}^t(\mathbf{s})\Big)$$

$$\mathbf{P^0_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\mathbf{\hat{D}_{00}}+\mathbf{\hat{D}_{01}})\otimes\mathbf{\hat{Q}}^*\Big)$$

$$\mathbf{P^1_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\mathbf{\hat{D}_{11}}+\mathbf{\hat{D}_{10}})\otimes\mathbf{\hat{I}_s}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^2_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\mathbf{\hat{D}_{00}}+\mathbf{\hat{D}_{01}})\otimes\mathbf{\hat{Q}}^*\otimes\mathbf{\hat{S}^0_s}\Big)$$

$$\mathbf{P^3_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\mathbf{\hat{D}_{10}}+\mathbf{\hat{D}_{11}})\otimes\mathbf{\hat{I}_s}\otimes\mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^4_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\mathbf{\hat{D}_{00}}+\mathbf{\hat{D}_{01}})\otimes\mathbf{\hat{Q}}^*\otimes\mathbf{\hat{S}^0_p}\Big)$$

$$\mathbf{P^5_{32}} = \Big((\mathbf{D_{00}}+\mathbf{D_{01}}+\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S_p}\Big)\otimes\Big((\mathbf{\hat{D}_{11}}+\mathbf{\hat{D}_{10}})\otimes\mathbf{\hat{I}_s}\otimes\mathbf{\hat{S}^0_p}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^0_{33}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\mathbf{\hat{D}_{00}}\Big)$$

$$\mathbf{P^1_{33}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\mathbf{\hat{D}_{01}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$

$$\mathbf{P^2_{33}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_S}\Big)\otimes\Big(\mathbf{\hat{D}_{10}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P^3_{33}} = \Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_S}\Big)\otimes\Big(\mathbf{\hat{D}_{11}}\otimes\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\otimes\hat{e_1}^t(\mathbf{s})\Big)$$

$$\mathbf{P^4_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\mathbf{\hat{D}_{00}}\otimes\mathbf{\hat{S}^0_s}\Big)$$

$$\mathbf{P^5_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes(\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}+\mathbf{S_p})\otimes\mathbf{I_s}+(\mathbf{D_{00}}+\mathbf{D_{01}})\otimes\mathbf{S_p}\otimes\mathbf{I_S}\Big)\otimes\Big((\mathbf{\hat{D}_{00}}+\mathbf{\hat{D}_{01}})\otimes\mathbf{\hat{S}_s}\Big)$$

$$\ldots+\Big((\mathbf{D_{10}}+\mathbf{D_{11}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\mathbf{\hat{D}_{01}}\otimes\mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{s}}\Big)$$

$$\mathbf{P^6_{33}} = \Big((\mathbf{D_{11}}+\mathbf{D_{10}})\otimes\mathbf{S^0_p}\boldsymbol{\beta}_{\boldsymbol{p}}\otimes\mathbf{I_s}\Big)\otimes\Big(\mathbf{\hat{D}_{10}}\otimes\mathbf{\hat{S}^0_s}\hat{\boldsymbol{\beta}}_{\boldsymbol{p}}\Big)$$

$$\mathbf{P_{33}^{7}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s} + (\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p} \otimes \mathbf{I_s}\Big) \otimes \Big((\mathbf{\hat{D}_{11}} + \mathbf{\hat{D}_{10}}) \otimes \boldsymbol{\hat{\beta}_p} \otimes \mathbf{\hat{S}_s^{*}}\Big)$$

$$\ldots + \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big(\mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}_s^0}\boldsymbol{\hat{\beta}_p} \otimes \hat{e_1}^t(\mathbf{s})\Big)$$

$$\mathbf{P_{33}^{8}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big(\mathbf{\hat{D}_{00}} \otimes \mathbf{\hat{S}_p^0}\Big)$$

$$\mathbf{P_{33}^{9}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big(\mathbf{\hat{D}_{01}} \otimes \mathbf{\hat{S}_p^0}\boldsymbol{\hat{\beta}_s}\Big)$$

$$\mathbf{P_{33}^{10}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big(\mathbf{\hat{D}_{10}} \otimes \mathbf{\hat{S}_p^0}\boldsymbol{\hat{\beta}_p}\Big) + \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes (\mathbf{S_p^0}\boldsymbol{\beta_p} + \mathbf{S_p}) \otimes \mathbf{I_s}$$

$$\ldots + (\mathbf{D_{00}} + \mathbf{D_{01}}) \otimes \mathbf{S_p} \otimes \mathbf{I_S}\Big) \otimes \Big((\mathbf{\hat{D}_{00}} + \mathbf{\hat{D}_{01}} + \mathbf{\hat{D}_{10}} + \mathbf{\hat{D}_{11}}) \otimes \mathbf{\hat{S}_p}\Big)$$

$$\mathbf{P_{33}^{11}} = \Big((\mathbf{D_{11}} + \mathbf{D_{10}}) \otimes \mathbf{S_p^0}\boldsymbol{\beta_p} \otimes \mathbf{I_s}\Big) \otimes \Big(\mathbf{\hat{D}_{11}} \otimes \mathbf{\hat{S}_p^0}\boldsymbol{\beta_p} \otimes \hat{e_1}^t(\mathbf{s})\Big)$$

Note: As mentioned, rest of the terms can be directly inferred from the Kronecker product between the two replicas of the Single Channel Model 2 with different channel parameters.

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