HYDROGEOLOGIC AND DIGITAL MODEL STUDIES OF A SHALLOW UNCONFINED AQUIFER IN THE SOURIS RIVER BASIN, MANITOBA

A Thesis

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by

HAMID BAKHTIARI

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ABSTRACT

A reconnaissance test drilling program was conducted in 1968 and 1969 in the southwestern portion of the Souris River basin to provide hydrogeologic information on several shallow unconfined aquifers in the area. One of the aquifers, referred to as the Broomhill Experimental Aquifer, was then chosen for detailed studies of its hydrologic response and yield characteristics. The water-table configurations, lateral boundaries, gross internal properties and thickness variations were determined using a relatively detailed network of water-table wells and drill holes.

The aquifer underlies an area of 25 square miles, generally ranges in thickness between 10 and 40 feet, and is bounded on its northern periphery by relatively impervious glacial till, and on its eastern periphery by low permeability silts and fine sands of lacustrine origin. The southern boundary is a groundwater divide. The aquifer is traversed from west to east by Stony Creek, a small ephemeral stream.

Maps of the aquifer showing the distribution of effective porosity and saturated hydraulic conductivity were prepared by combining and correlating drill logs, drill sample descriptions, and results of laboratory analyses of relatively undisturbed samples collected from a gravel quarry in the aquifer. The saturated hydraulic conductivities of the aquifer materials typically range between 1×10^{-3} and 3×10^{-3} feet per second, and effective porosity between 30 and 35 percent. Maps of the average water-table configuration based on the well network and drill results indicate the water table slopes from west-

southwest to northeast towards the Souris River.

The above information was incorporated into digital models of the aquifer system using the methods developed by Pinder and Bredehoft (1968), which solve a linear, parabolic, partial-differential equation approximating the non steady-state flow of a fluid through a nonhomogeneous fully-saturated porous medium.

One of the models was used to analyze data from a 24-hour pumping test conducted in a segment of the aquifer by the Manitoba Water Resources Branch. The simulation confirmed that the aquifer properties determined from the test hole and laboratory data were reasonable and also indicated that non-instantaneous release of water during the initial phase of the pumping test is a factor which significantly affects the time drawdown graphs.

Digital models were also used to obtain an understanding of the response and general yield capabilities of the eastern half of the aquifer in which the most detailed drilling results are available. Model simulations were conducted using up to 26 pumping centers located in the zones of maximum initial saturated thickness in the eastern half of the aquifer. Pumping periods of 99 days were used to evaluate the aquifer response to pumping from well fields during an entire irrigation season. It was found that 56 percent of the aquifer area is capable of yielding 2,310 acre-feet of water over the 99-day period.

The aquifer yield capabilities were also investigated using a hydrologic budget approach. Analysis of hydrologic data for the period 1965-1971 suggest that the average yield capability of the aquifer which could be realized by conjunctive use schemes under ideal conditions involving induced or artificial recharge from Stony Creek is 9330 acre-feet of water per year. Estimates of the annual developmental yield of the aquifer during the above period range between 2 and 13 inches and indicate that the aquifer yield capability is controlled by major effects of climatic conditions. It is, therefore, recommended that a more detailed yield analysis be performed using long-term regional climatic and stream flow data from adjacent watersheds. The yield analysis for the western portion of the aquifer could also be refined if more detailed test drilling data were available.

It is concluded that the Broomhill Experimental Aquifer is a significant source of high-quality water in the Souris River basin. The hydrologic controls on the aquifer system are such that optimal development of the groundwater resource could only be attained by instituting a detailed water management scheme.

ACKNOWLEDGEMENTS

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure		Page

LIST OF MAJOR SYMBOLS

Symbol	Definition	Dimension
A	Area	(L^2)
B	Aquifer base elevation	(L)
b_o	Saturated thickness of the aquifer	(L)
b_s	Thickness of the stream bed and lake bottom	(L)
C	Permeability to wetting fluid when the medium is occupied by more than one fluid phase	(L^2)
C_o	Saturated permeability to wetting fluid at complete saturation	(L^2)
c	Constant in time increment equation for computer program	(none)
ET	Actual evapotranspiration	(L)
G	Impirical constant	(none)
g	Acceleration due to gravity	(LT^{-2})
H	Hydraulic head in the streams and lakes	(L)
h	Hydraulic head in the aquifer	(L)
h_t	Hydraulic head in hydraulic conductivity experiment	(L)
I	Normalized iteration parameter	(none)
I_g	Groundwater inflow	(L)
I_s	Surface inflow	(L)
i	Index in the x direction	(none)
j	Index in the y direction	(none)

Symbol	Definition	Dimension
K	Hydraulic conductivity at any saturation	(LT^{-1})
K_o	Saturated hydraulic conductivity to wetting fluid at complete saturation	(LT^{-1})
$\overline{K}_o$	Saturated hydraulic conductivity assigned to the total test hole column	(LT^{-1})
K_s	Hydraulic conductivity of streams and lake bottoms	(LT^{-1})
k	Index in the t direction	(none)
L'	Mass flux into or out of a fluid system	$(ML^{-3}T^{-1})$
l	Distance between tensiometers in hydraulic conductivity experiment	(L)
M	Mass	(M)
n	Index indicating iteration number	(none)
O_g	Groundwater outflow	(L)
O_s	Surface outflow	(L)
P	Hydrostatic pressure	$(ML^{-1}T^{-2})$
P_a	Precipitation	(L)
P_b	Bubbling pressure	$(ML^{-1}T^{-2})$
P_c	Capillary pressure	$(ML^{-1}T^{-2})$
Q	Rate of discharge or recharge at any node	(L^3T^{-1})
q	Volume flux	(LT^{-1})
R_w	Mass of retention water per unit mass of dry medium	(none)
r	Distance between well and observation well	(L)
S	Storage coefficient	(none)
S_s	Specific storage	(L^{-1})
T	Transmissibility	(L^2T^{-1})

Symbol	Definition	Dimension
t	Time	(T)
V	Volume	(L^3)
v	Apparent velocity	(LT^{-1})
x, y, z	Rectangular co-ordinates	(none)
γ	Specific weight	$(ML^{-2}T^{-2})$
ΔS_g	Change in groundwater storage	(L)
ΔS_m	Change in moisture storage	(L)
Δt	Time increment	(T)
$\Delta x, \Delta y$	Space increments	(L)
λ	Pore-size distribution index	(none)
μ	Dynamic viscosity of wetting fluid	$(ML^{-1}T^{-1})$
ρ	Density of wetting fluid	(ML^{-3})
ρ_b	Bulk density of the porous medium	(ML^{-3})
ρ_w	Density of water	(ML^{-3})
ρ_s	Particle density of the medium solids	(ML^{-3})
ϕ	Total porosity	(none)
ϕ_e	Effective porosity	(none)
$\overline{\phi}_e$	Effective porosity assigned to the total test hole column	(none)
ϕ_r	Retention porosity	(none)
∇H	Hydraulic gradient	(none)

CHAPTER I

INTRODUCTION

The southwestern portion of the Souris River basin, Manitoba, is characterized by extensive surficial deposits of glacial outwash, mainly sands and gravels, which were laid down during the Pleistocene Epoch. These sediments form highly-permeable, unconfined aquifers which are underlain by a relatively impervious glacial till. The quality of the groundwater in the aquifers is excellent for domestic and irrigation purposes; however, very little development of this resource has occurred. In response to renewed interest in the subsurface water resources of the area, in 1962 the Southwest Manitoba Development Commission was founded in order to organize and promote the economic development of the aquifers. The first significant investigation to evaluate the groundwater availability in the region was conducted by the Manitoba Water Control and Conservation Branch (1968), during the period 1963 - 1968. This study indicated that the only major sources of high-quality groundwater in this region occur in the surficial outwash aquifers. The present study was undertaken to obtain an understanding of the hydrologic conditions and the yield capabilities of these aquifers. After completing a reconnaissance investigation of the geology of many of the outwash aquifers in this region, an aquifer near Broomhill, here referred to as the Broomhill Experimental Aquifer, was chosen for detailed studies. As the need for information on the other outwash aquifers in the region develops, data and conclusions from the Broomhill aquifer will be extrapolated or used as a guideline for further studies. The Broomhill aquifer was chosen for intensified studies because:

(a) its boundaries are reasonably well-defined,
(b) it has an area of relatively large thickness,
(c) it is traversed by a stream which could be used for induced or artificial recharge schemes,
(d) it is easily accessible.

The specific objectives of the present study are:

(i) to define the physical properties and the controlling boundary conditions of the Broomhill Experimental Aquifer,
(ii) to define the pattern of groundwater movement in the aquifer under natural conditions,
(iii) to develop a useful, two-dimensional, digital model of the aquifer system using field data and the modelling technique of Pinder (1969),
(iv) to conduct a preliminary appraisal of the yield capabilities of the aquifer using field data and

the digital model.

Data relevant to the above four objectives were derived from:

(a) description logs of several test holes drilled in the aquifer,

(b) water level measurements taken in 80 small-diameter water-table wells,

(c) interpretation of airphoto mosaics of the area,

(d) collection of seven relatively undisturbed samples of the aquifer,

(e) laboratory tests on the undisturbed samples to determine the saturated hydraulic conductivity and effective porosity of the aquifer, using the techniques of Laliberte *et al.* (1966, 1967, 1968).

A digital model of the aquifer system was developed to simulate the aquifer system with a natural water-table configuration and an idealized horizontal water table. The models were pumped for selected periods of time and a comparison was made between the results. Finally, the digital model was subjected to a pumping period sufficiently long to evaluate the aquifer as a potential source of groundwater for the purpose of irrigation. Another model was also designed to simulate a pumping test which was conducted in October, 1966, by the Manitoba Water Control and Conservation Branch (unpublished).

CHAPTER II

GENERAL PHYSIOGRAPHY AND GEOHYDROLOGIC SETTINGS

The Broomhill Experimental Aquifer is located in the southwest corner of the province of Manitoba, within the southwest portion of the Souris River basin (Fig. 1). The surface topography (Fig. 2) in this portion of the basin is typical of the prairie region of west central Canada, with gently rolling to flat terrain intersected by occasional shallow channels. The regional slope is approximately 1 to 2 feet per 1,000 feet from the northwest towards the Souris River.

The climate in the area is semi-arid with cold winters and warm to hot summers. The mean annual precipitation extrapolated from the mean monthly amount of precipitation, based on the period 1931 - 1960, is about 18 inches (Canada Department of Transport, Meteorological Branch, 1967). About 8 inches of the annual precipitation occurs during the growing season of June to August, inclusive.

The regional geologic setting consists of a sequence of glacial sediments between 200 and 300 feet thick which rest on the marine shale of the Upper Cretaceous, Riding Mountain Formation (Manitoba Water Control and Conservation Branch, 1968). The bedrock topography in the region is

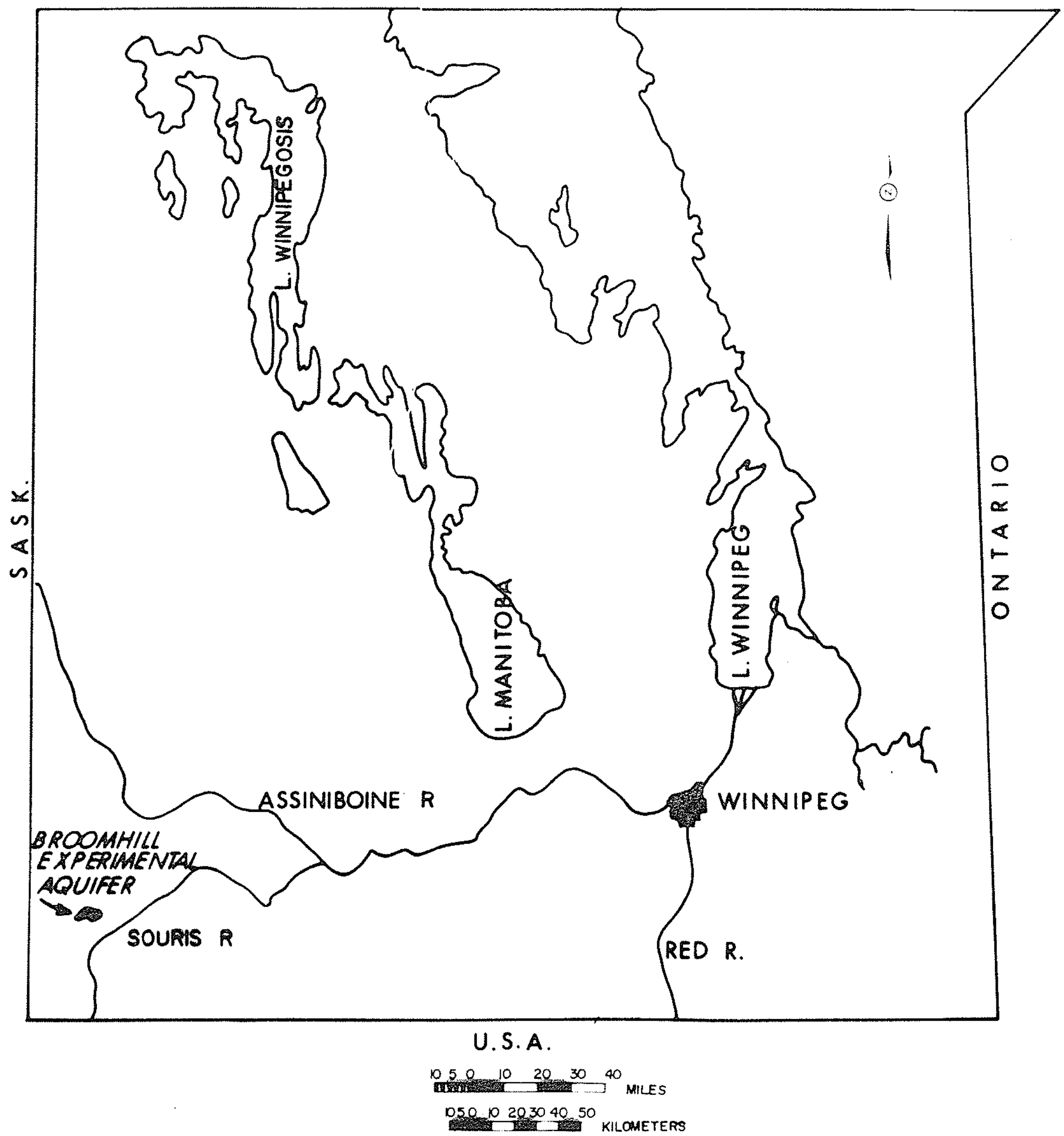

Fig. 1. General location of the Broomhill Experimental Aquifer.

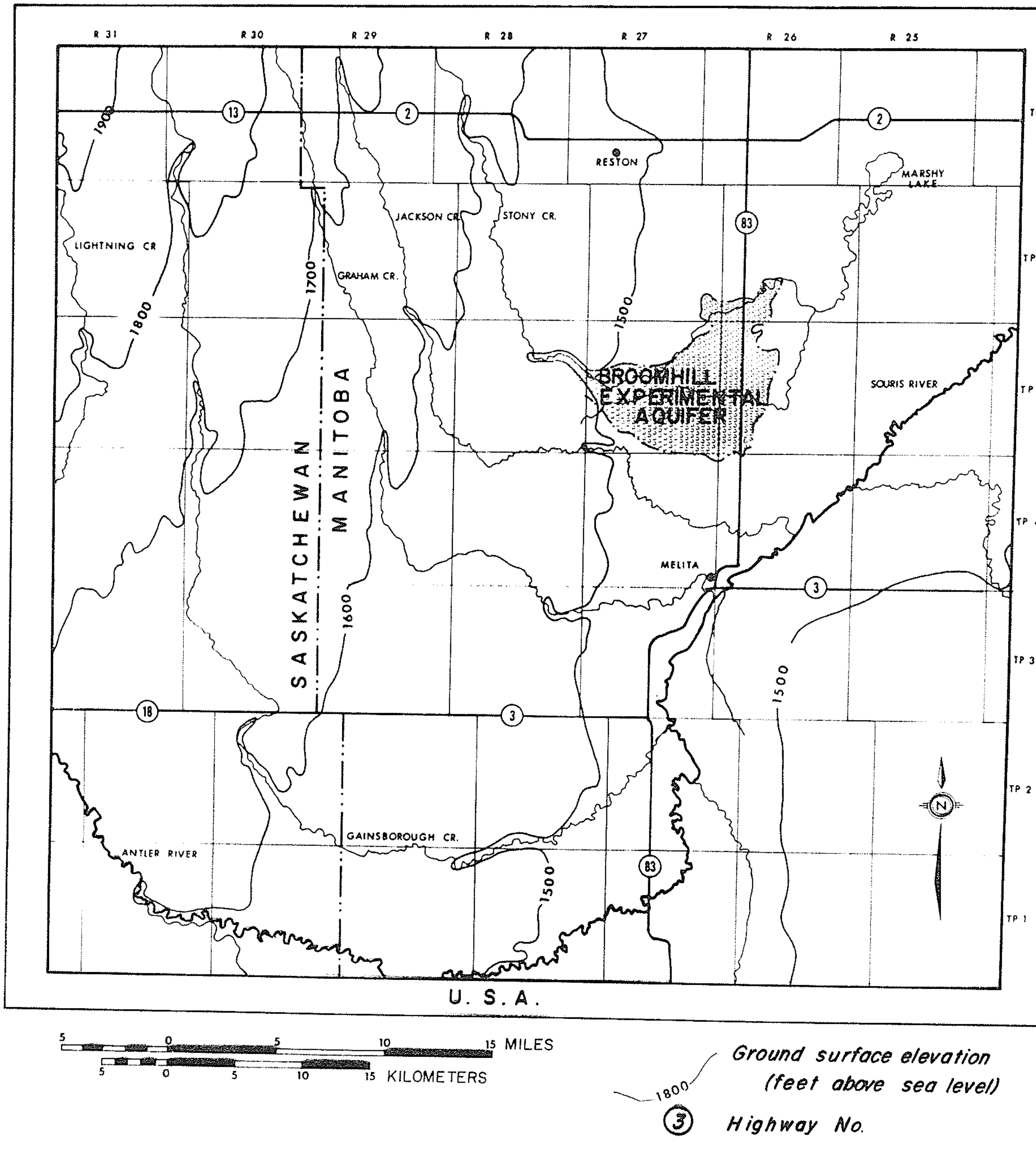

Fig. 2. General physiographic setting of the Broomhill Experimental Aquifer.

shown in Fig. 3. The glacial sediments consist primarily of clay-loam till which includes occasional interbeds of sand or gravel. The till sequence is overlain by glacio-lacustrine sediments in the eastern portion of the region and discontinuous deposits of glacio-fluvial sand and gravel in the western area (Fig. 4). The sand and gravel has been referred to as delta deposits by Elson (1958) and outwash deltas by the Manitoba Water Control and Conservation Branch (1968). In this thesis the term outwash fan will be used because the grain-size and bedding characteristics indicate a glacio-fluvial origin. The present stream channels in the area were presumably occupied by meltwater streams and are associated with minor deposits of sand and gravel. The channel deposits cover only a small portion of the investigated region, whereas the outwash fan deposits which form highly-permeable shallow aquifers cover some 120 square miles of the area, with thicknesses up to 50 feet. The glacio-lacustrine deposits consist of medium to fine sand, silt, clay, and a mixture of these sediments. The thickness of this unit varies up to 30 feet immediately east of the Broomhill aquifer.

From the viewpoint of groundwater utilization the outwash fan deposits constitute the most significant aquifers in this region. Investigations conducted by the Manitoba Water Control and Conservation Branch (1968) indicated that the saturated thickness of the outwash

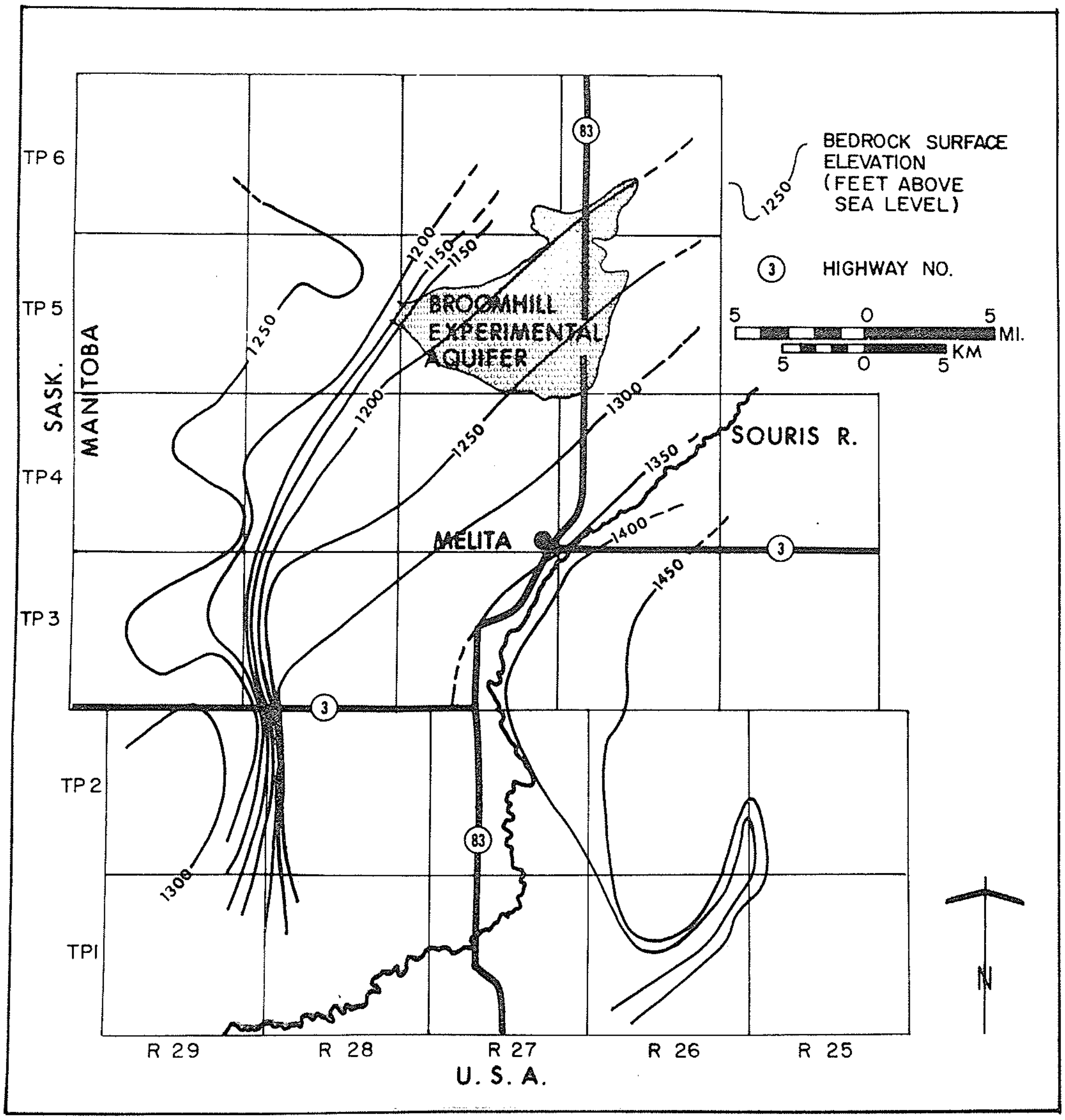

Fig. 3. Topography of the bedrock surface (adapted from Manitoba Water Control and Conservation Branch, 1968).

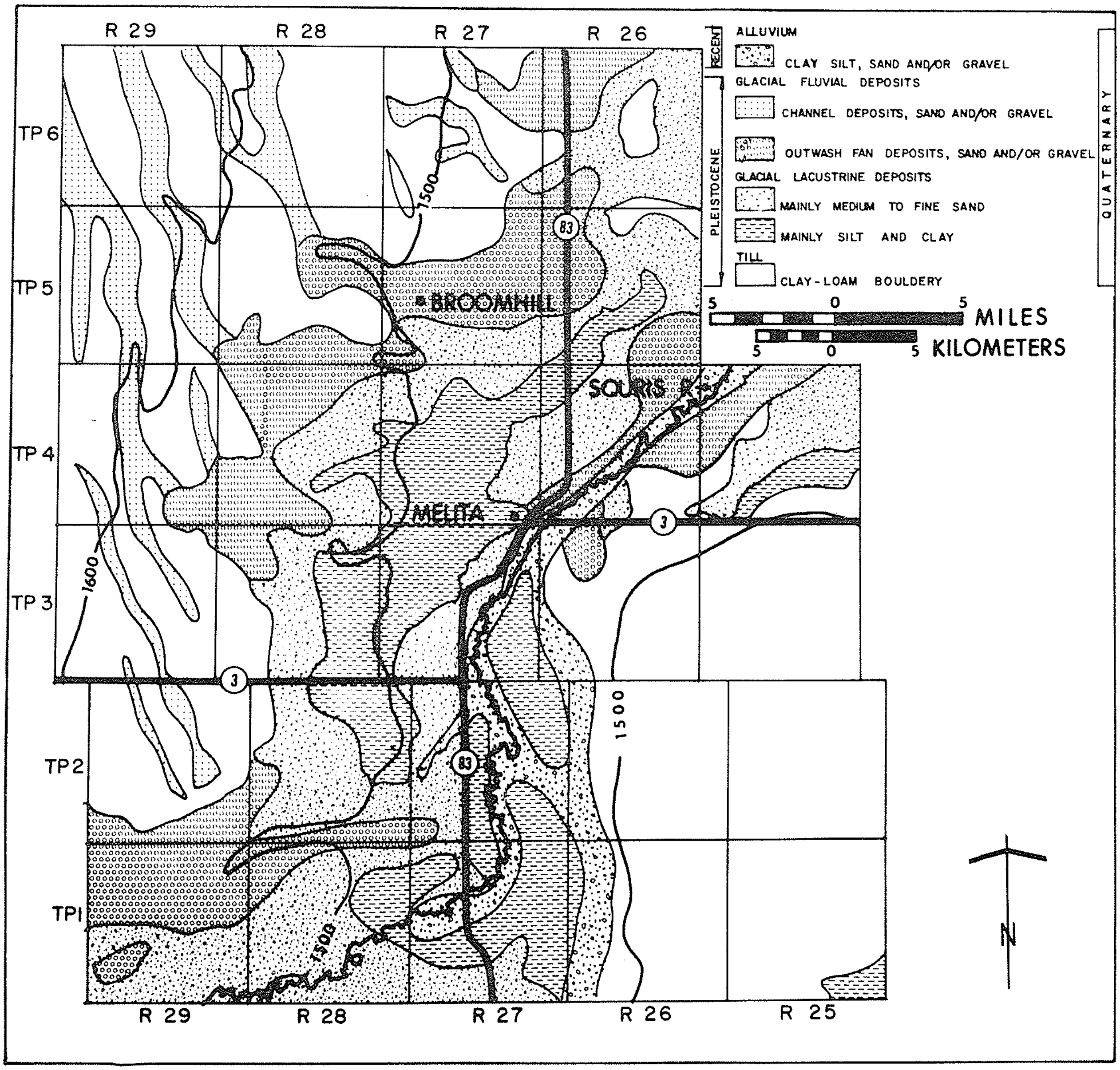

Fig. 4. Regional surficial geology of the region (adapted from Manitoba Water Control and Conservation Branch, 1968).

aquifers is about 20 feet in approximately one-quarter of the 120 square miles covered by the outwash fan deposits. The present study indicates that the saturated thickness occasionally attains a maximum of 40 feet. The sandy glacio-lacustrine deposits are relatively thin, generally fine-grained, and commonly contain significant percentages of silt and clay. Their yield capabilities are small compared to the outwash aquifers, and therefore, they are not considered further in this thesis.

CHAPTER III

METHODS OF INVESTIGATION

Aquifer Exploration

The Broomhill Experimental Aquifer has been studied in the field at various times during the period 1968 - 1970. The initial phase of the investigation consisted of a reconnaissance appraisal of the geologic and hydrologic properties of many of the outwash fans in the region. Test drilling was conducted in the fall of 1968 and spring of 1969 using a truck-mounted power auger to establish the thickness of the outwash at representative locations. It was concluded that at least several of the areas underlain by the outwash deposits were potentially significant aquifers, and that the investigation could proceed most efficiently by selecting one of the aquifers as a site for more detailed study. Additional test drilling was conducted in the late spring of 1969, and resulted in the selection of Broomhill Experimental Aquifer for further investigations. On completion of the test drilling phase of the investigation, 173 test holes had been drilled, 120 of which were located in the Broomhill aquifer (Fig. 5).

During the test drilling and the well installation

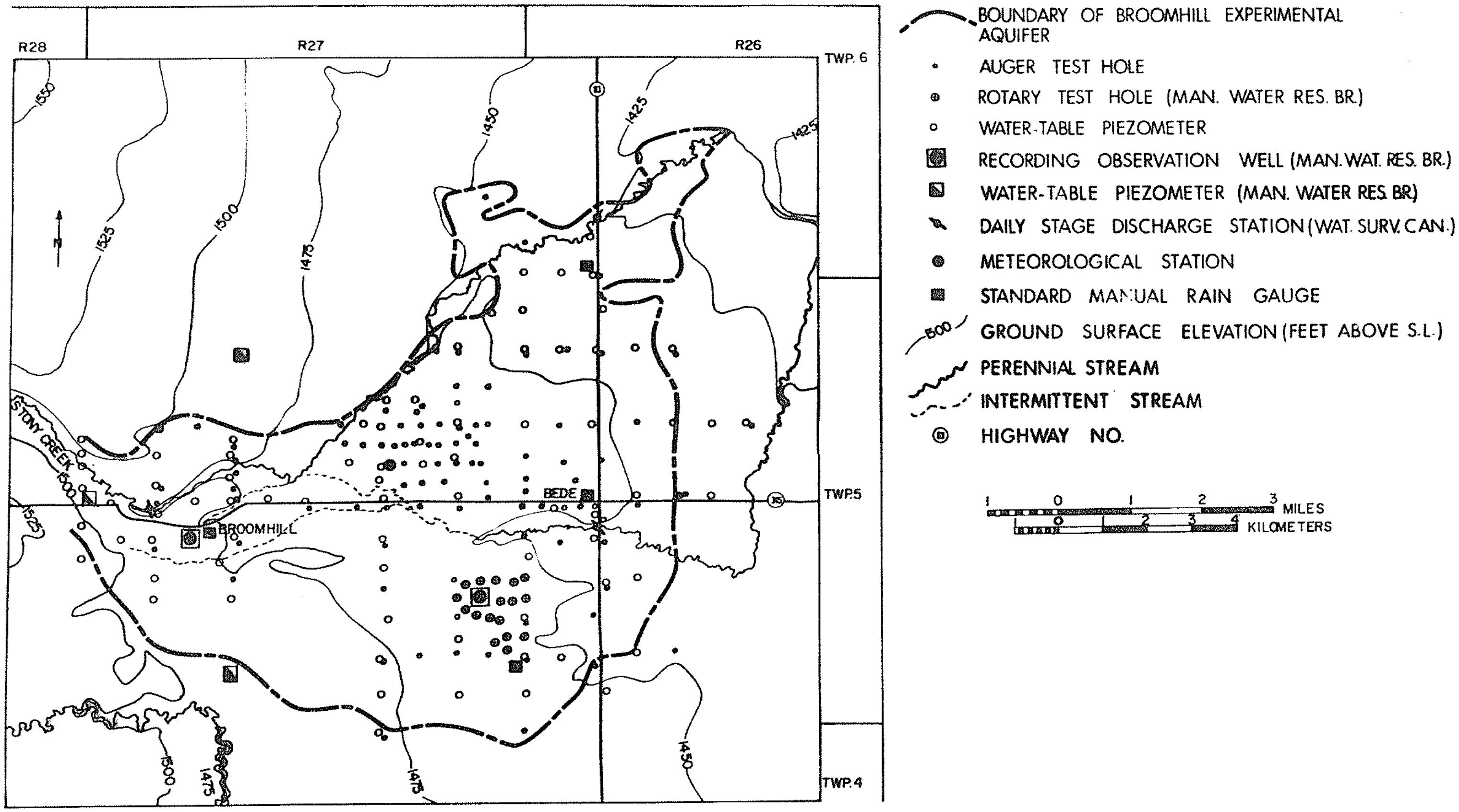

Fig. 5. Topography and location of instrumentation in the Broomhill Experimental Aquifer.

programs more than 300 disturbed samples were collected off the auger flights. During the drilling program it was possible to determine the sample depths to within 1 or 2 feet. These samples are representative of the various textural units in each test hole. In addition a total of seven relatively undisturbed core samples of the aquifer materials were collected from the wall of a deep gravel quarry excavation, located approximately 1 mile north of Broomhill, and from two other sites, one located approximately 3 miles north of Bede and the other in the eastern portion of the aquifer. These samples include a wide range of texture, from fine to very coarse sands, with depths varying between 1 and 16 feet, and were obtained by using a removable-sleeve sampler (7.50 inches long and 2 inches inside diameter) a photograph of which is shown in Fig. 6. In one location two samples were taken, one in the horizontal and one in the vertical direction. The cores were obtained by manually forcing the sampler including the plastic sleeve insert, approximately 8 inches into the sample medium. The inner plastic sleeve containing the sample was removed from the sampler in the field and packaged in a manner to prevent disturbance during transportation to the laboratory. Several bulk-density samples were also collected using the field volumetric method of Zwarich and Shaykewich (1969).

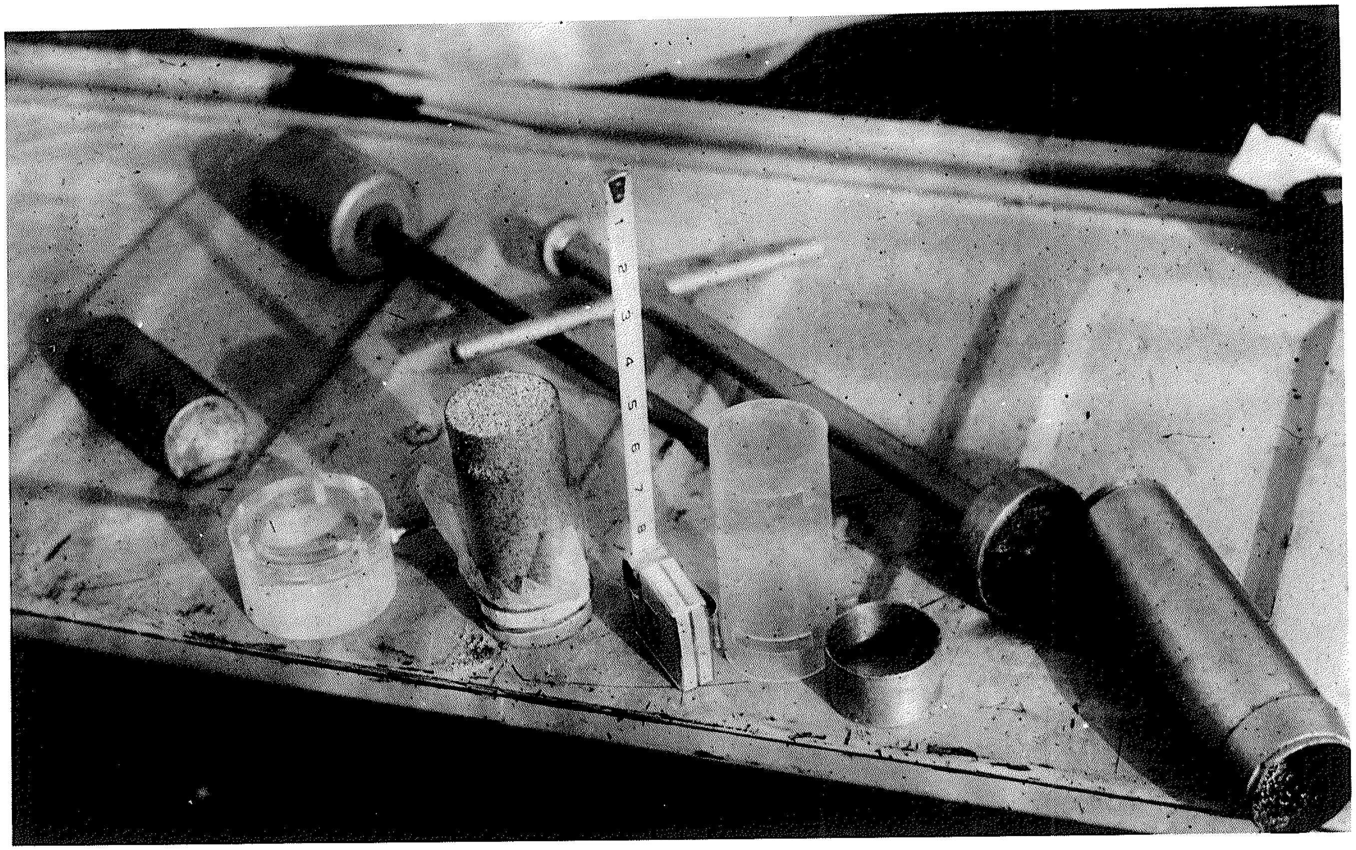

Fig. 6. Photograph of the removable-sleeve soil sampler.

Aquifer Instrumentation

In order to determine the water-table configuration and transient water-table characteristics of the aquifer, 80 narrow-diameter observation wells were installed in July, 1969. Water-level measurements have been obtained weekly, during the spring, summer and fall months since installation. Additional water-table data is being collected from the two large-diameter continuous recording wells (MN24 near Bede and MN18 near Broomhill) operated by the Manitoba Water Resources Branch since 1965. The recording instrument of these two observation wells is of type "F" (Leupold and Stevens), and the chart period is eight days.

Meteorological Instrumentation

The following meteorological instruments were installed in the aquifer area in late winter, 1970:

(i) one tipping bucket recording standard rain gauge manufactured by the Meteorological Service of Canada (MSC), chart period one week

(ii) four standard rain gauges (MSC)

(iii) one barograph (MSC), chart period 3 days

(iv) one hygrothermograph, manufactured by LAMBRECHT, chart period one week.

Operation of the instrumentation network (Fig. 5) on a continuous basis was started in the spring of 1970 prior to the snowmelt period and continued until November, 1970. Operation of the network recommenced in April, 1971, and will continue at least until fall, 1971.

The meteorological information, along with a study of the water-table configuration and its fluctuations, is directed towards defining the water budget of the aquifer system under natural (non-pumping) conditions.

Laboratory Investigations of Aquifer Properties

In this phase of the investigation the intention was to provide data which will enable the qualitative drill log descriptions of the aquifer material to be reformulated into a quantitative framework. This was accomplished by using a laboratory technique to determine the saturated hydraulic conductivity and the effective porosity of the seven relatively undisturbed samples collected within the aquifer area, and using these results to evaluate the hydraulic properties of the several disturbed aquifer samples collected during the test drilling and the well installation program.

The laboratory techniques used in this study were similar to that used by Laliberte *et al.* (1966, 1967, 1968). The laboratory procedure is described in Appendix A.

Digital Modelling Technique

A digital model which simulates the pumping from a two-dimensional unconfined aquifer, was used to obtain a preliminary appraisal of the yield capabilities of the aquifer system. The model solves the finite-difference approximations to the unsteady-state saturated flow equations, using an iterative alternating-direction implicit procedure, developed by Pinder (1969). The mathematical model can tolerate heterogeneous distribution of effective porosity and saturated hydraulic conductivity, vertical recharge, relatively complex boundary conditions and one or more pumping wells. In the mathematical modelling procedure the groundwater and its boundaries are replaced by a finite set of points, each of which is representative of a small two-dimensional nearby region. The partial differential equation of flow is then replaced by a finite set of simultaneous finite-difference equations (one equation for each point). These equations are then computed to determine the hydraulic head value at each point in the network for a given boundary condition.

CHAPTER IV

GEOHYDROLOGIC CHARACTERISTICS OF THE AQUIFER SYSTEM

Aquifer Properties Determined from Laboratory Tests

The hydraulic properties of the aquifer materials were determined by conducting laboratory tests on a representative group of undisturbed samples. The tests were conducted on the core samples without removing the soil from the plastic sleeve. The relatively undisturbed character of the samples was therefore maintained. To obtain an estimation of the effect of disturbance on the hydraulic conductivity, each sample was disturbed and the tests were repeated. In this study the disturbed materials were repacked to the same bulk density as their corresponding undisturbed samples. After completing the hydraulic conductivity tests, effective porosity was determined by measuring the total porosity and the retention porosity of the samples. A summary of the results along with a brief description of the tested samples is given in Table I.

The results indicate that the saturated hydraulic conductivities of the tested aquifer samples vary between 2.15×10^{-4} and 2.72×10^{-3} feet per second when undisturbed, and between 1.18×10^{-4} and 3.88×10^{-3} feet per second after

Table I

Bulk density, saturated hydraulic conductivity (disturbed and undisturbed), and effective porosity along with a brief description of the tested aquifer samples.

Sample No. and Orientation	Brief Description	Bulk Density gm/cm^3	Saturated Hydraulic Conductivity at 4°C ft/sec		Effective Porosity (dimensionless)
			Undisturbed	Disturbed	
SS1-Vertical	uniform fine gravel, no clay depth 5-6 feet	1.63	2.72×10^{-3}	3.88×10^{-3}	0.347
SS2-					
(a) Horizontal	very uniform coarse sand depth 7-8 feet	1.64	1.16×10^{-3}	1.33×10^{-3}	0.349
(b) Vertical		1.63	8.73×10^{-4}	1.26×10^{-3}	0.357
SS3-Vertical	coarse sand with pebbles up to 1 cm depth 15-16 feet	1.68	1.02×10^{-3}	1.09×10^{-3}	0.313
SS4-Vertical	coarse sand + medium to fine sands depth 15-16 feet	1.66	8.79×10^{-4}	1.07×10^{-3}	0.304
SS5-Vertical	medium clean sand depth 3-4 feet	1.59	4.89×10^{-4}	5.51×10^{-4}	0.336
SS6-Vertical	fine sand, very uniform depth 1-2 feet	1.63	2.15×10^{-4}	1.18×10^{-4}	0.307

being disturbed. The general range of the hydraulic conductivities correlates with that considered by Walton (1970) for sand and gravel. The effective porosity of the samples ranged between 30.4 and 35.7 percent. These values are somewhat higher than the range of common values reported by Todd (1964) and Walton (1970). However, the tested samples were clean and well-sorted, and the individual grains were generally subrounded to rounded. This would account for the high values of effective porosity. For comparison, the results of a theoretical study by Graton and Fraser (1935) are of interest. They found that the loosest packing of uniform-sized spheres produces an effective porosity of 47.6 percent, and that the tightest packing an effective porosity of 26 percent. The results listed in Table I indicate that for all tested samples but one, the saturated hydraulic conductivities of the disturbed samples is between 6.9 and 44.3 percent higher than the corresponding undisturbed samples. The results also indicate that the saturated hydraulic conductivity of the sample which was obtained in the horizontal direction is 32.9 percent higher than the one obtained in the vertical direction, when tested in the undisturbed state. These samples yielded approximately the same hydraulic conductivity in the disturbed state. The complete results of the laboratory experiments are described in detail in Appendix A.

In order to reformulate the qualitative drill log descriptions obtained by drilling into a quantitative framework suitable for aquifer analysis using modelling methods, more than 300 disturbed drill samples were visually compared with the tested samples for estimation of effective porosity and saturated hydraulic conductivity. A representative value was obtained for each test hole using the following relationships.

$$\bar{K}_o = \frac{\sum_{n=1}^{N} K_{o_n} b_n}{\sum_{n=1}^{N} b_n} \qquad [1]$$

and

$$\bar{\phi}_e = \frac{\sum_{n=1}^{N} \phi_{e_n} b_n}{\sum_{n=1}^{N} b_n} \qquad [2]$$

where $\bar{K}_o$ and $\bar{\phi}_e$ are representative values of the saturated hydraulic conductivity and effective porosity, assigned to the total test hole column which includes N strata; K_o and ϕ_e are the saturated hydraulic conductivity and effective porosity of each stratum and b is the thickness of each stratum.

The results were prepared in a matrix form for both the saturated hydraulic conductivity (Fig. 7) and the effective porosity (Fig. 8) distribution.

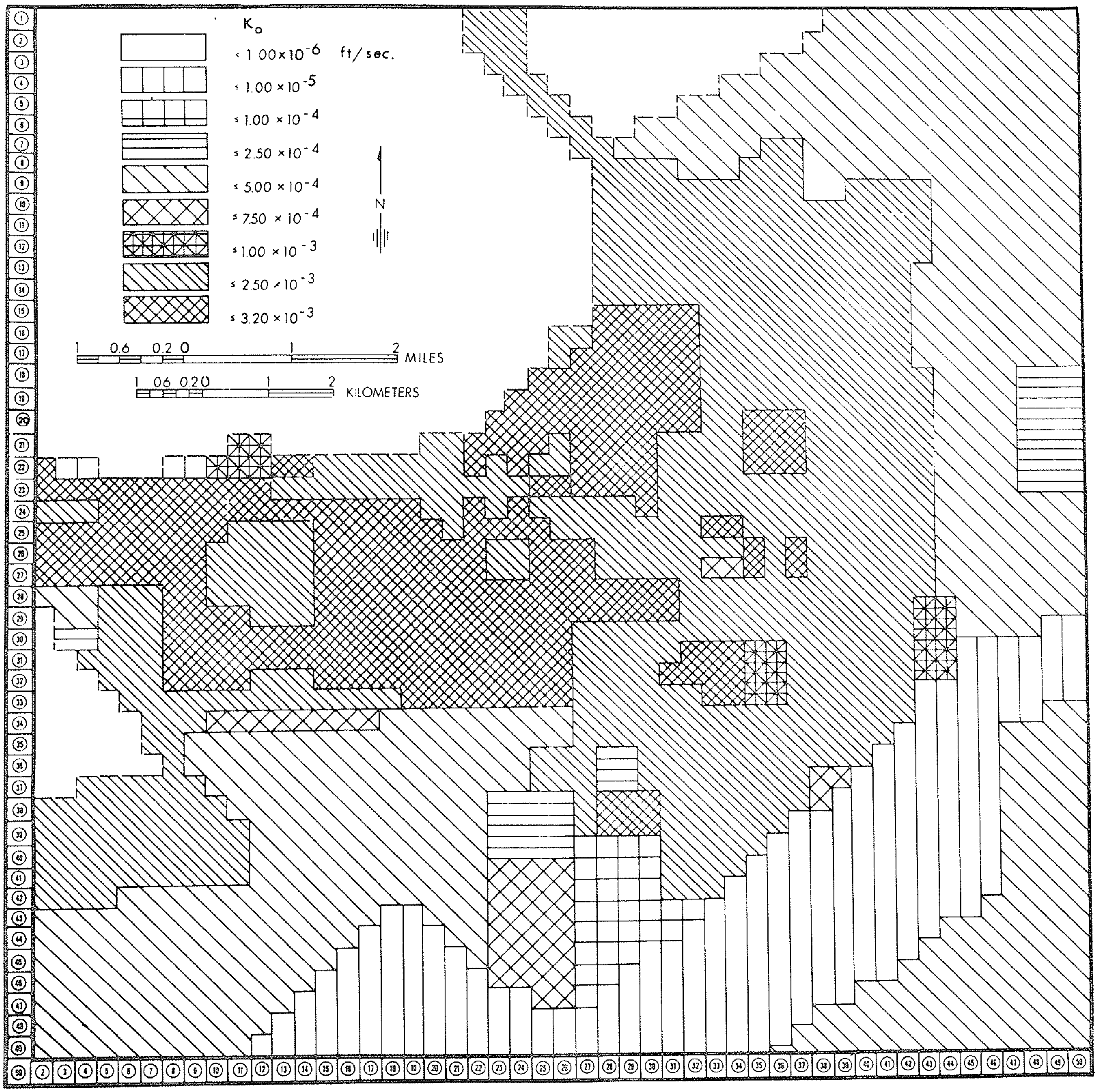

Fig. 7. Distribution of average saturated hydraulic conductivity in the Broomhill Experimental Aquifer.

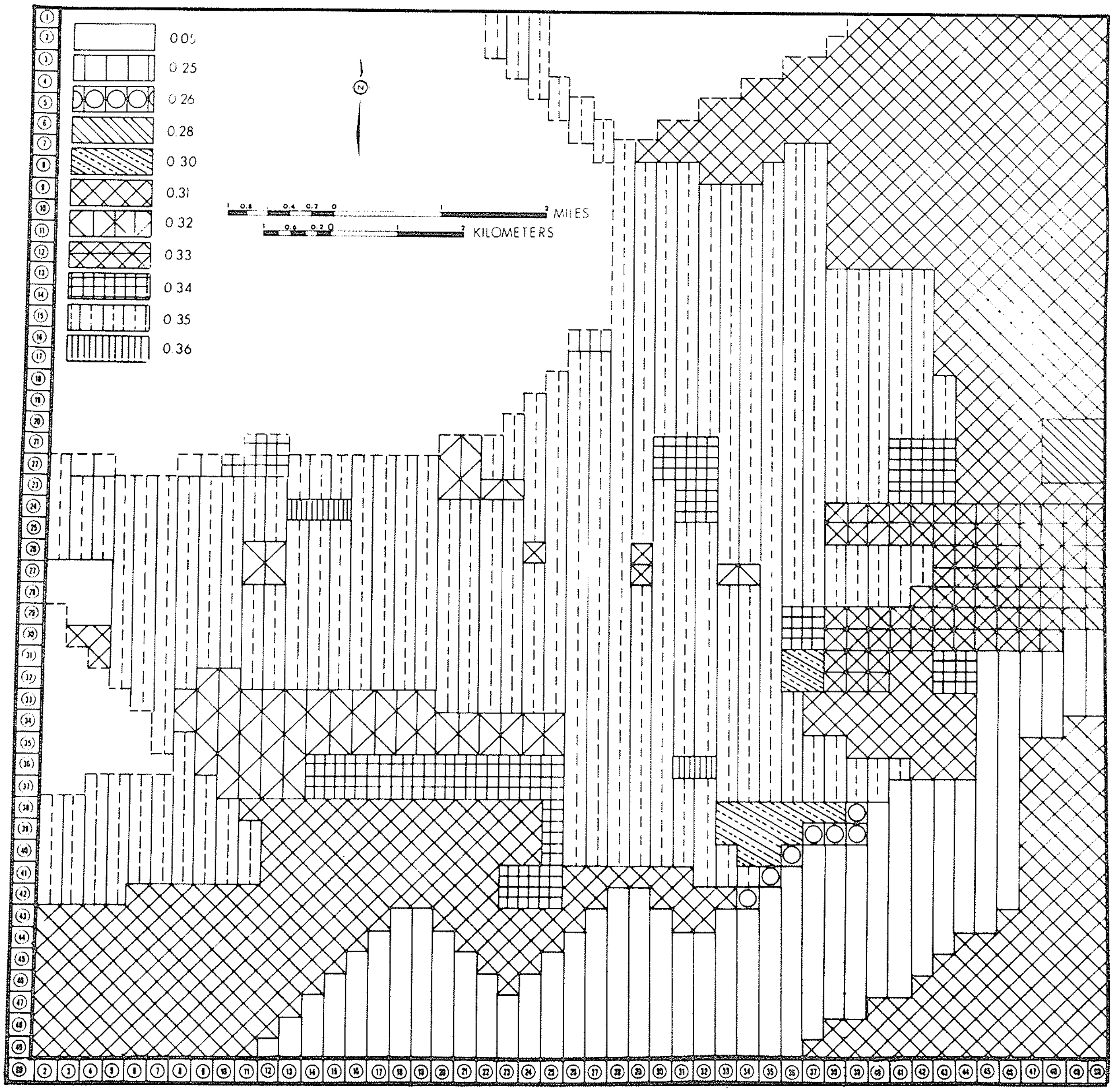

Fig. 8. Distribution of average effective porosity in the Broomhill Experimental Aquifer.

Hydrogeology of the Aquifer System

The results of the test drilling program indicate that the aquifer is composed of clean coarse sand and very fine to medium gravel which, at most locations, rests on a base of clay-loam till. The outwash deposits range in thickness up to 50 feet (Fig. 9). Although the surface topography is relatively flat, the bottom of the aquifer varies considerably in depth over relatively short horizontal distances (Fig. 10). The till surface appears to be a complex network of minor channels which were eroded during the period of active discharge of meltwater streams during the Pleistocene Epoch. The glacio-lacustrine deposits consist of medium to very fine sand, silt and clay. They occur along the southern and eastern boundaries of the aquifer (Fig. 11) and vary in thickness up to 30 feet.

The clay-loam till, which includes numerous boulders and pebbles, forms a boundary along the northern and western segments of the aquifer. The till plain is characterized by a lack of integrated drainage channels and numerous small hills and depressions (Fig. 12). It is typical of moderate-relief ablation moraine, as described by Cherry (1966), although it is referred to as ground moraine by Elson (1958).

The log descriptions of the 120 test holes drilled within the aquifer area and observations of several gravel quarries in the aquifer indicated that horizontal and near-horizontal bedding planes are frequent. Lateral and

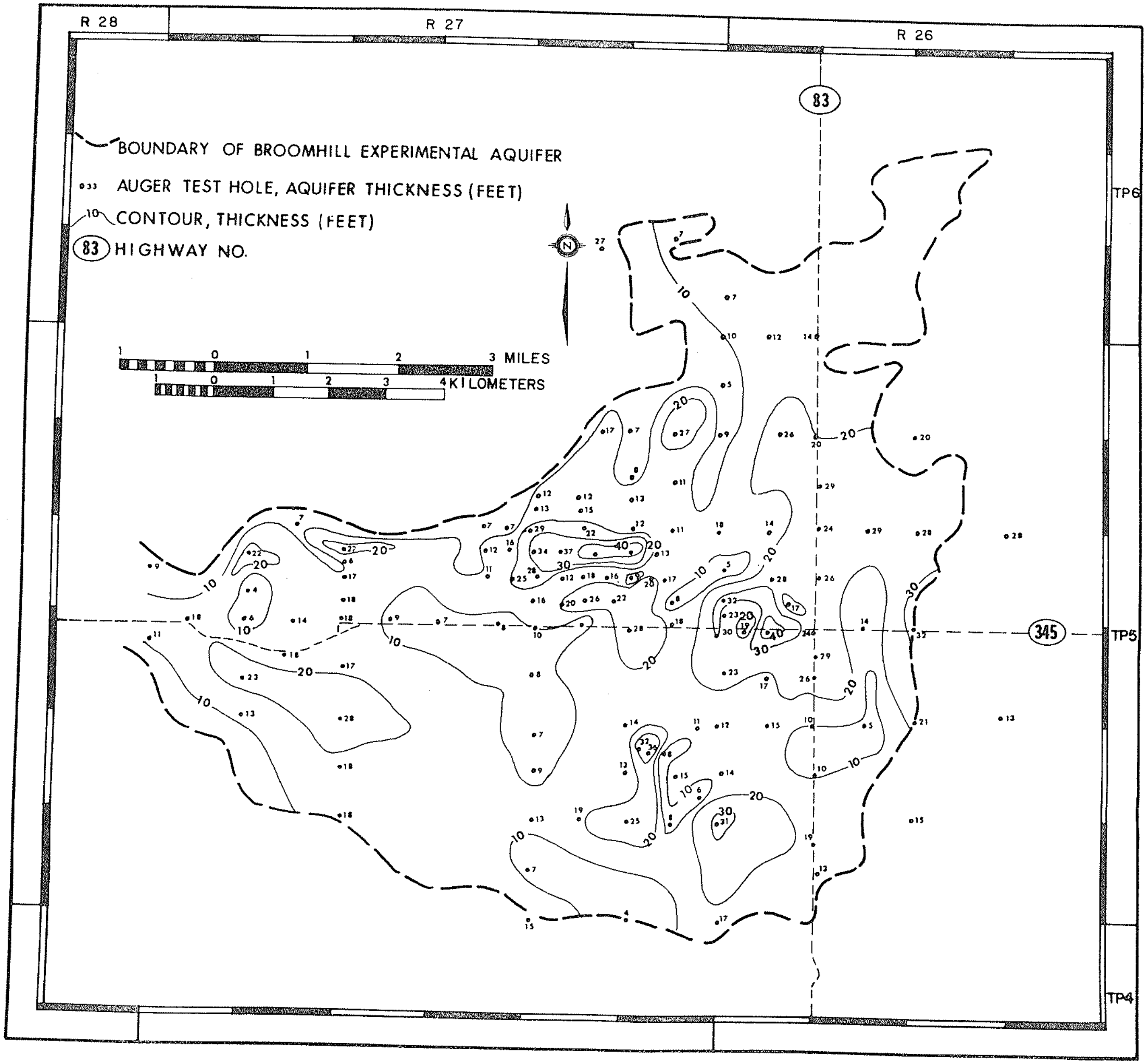

Fig. 9. Contour map of total aquifer thickness.

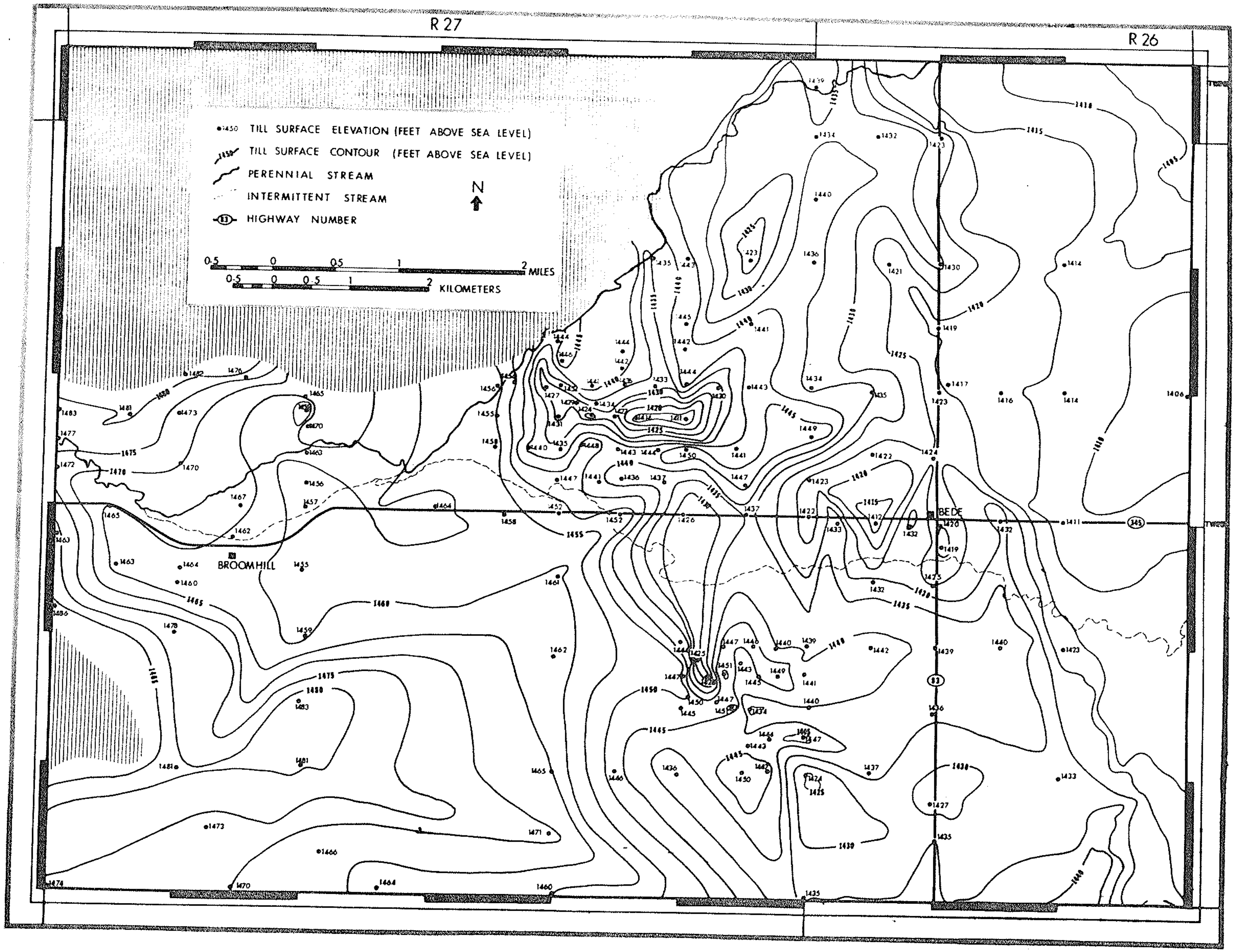

Fig. 10. Topography of the base of the aquifer deposits.

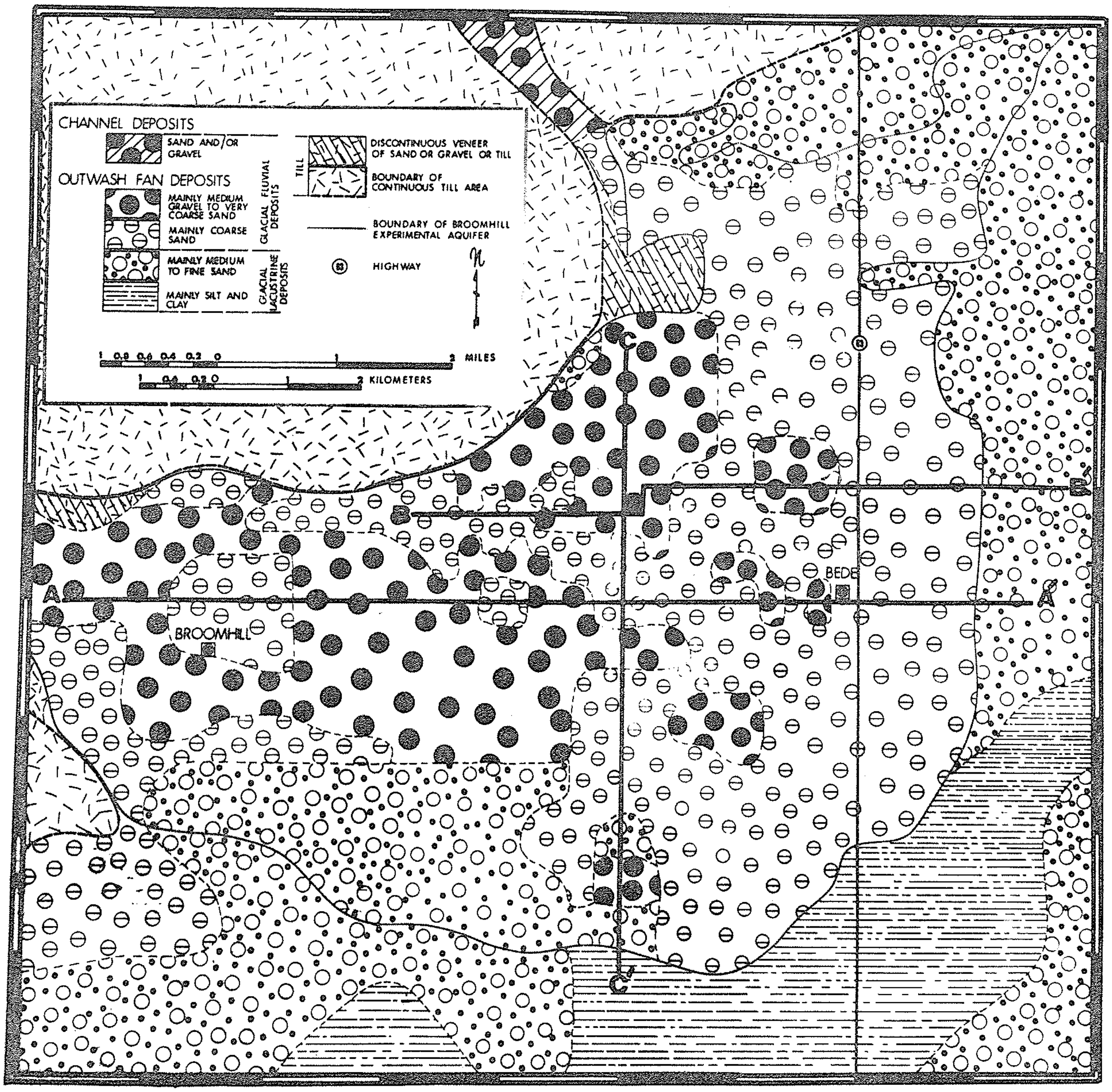

Fig. 11. Distribution of general grain-size properties of the Broomhill Experimental Aquifer and adjacent areas.

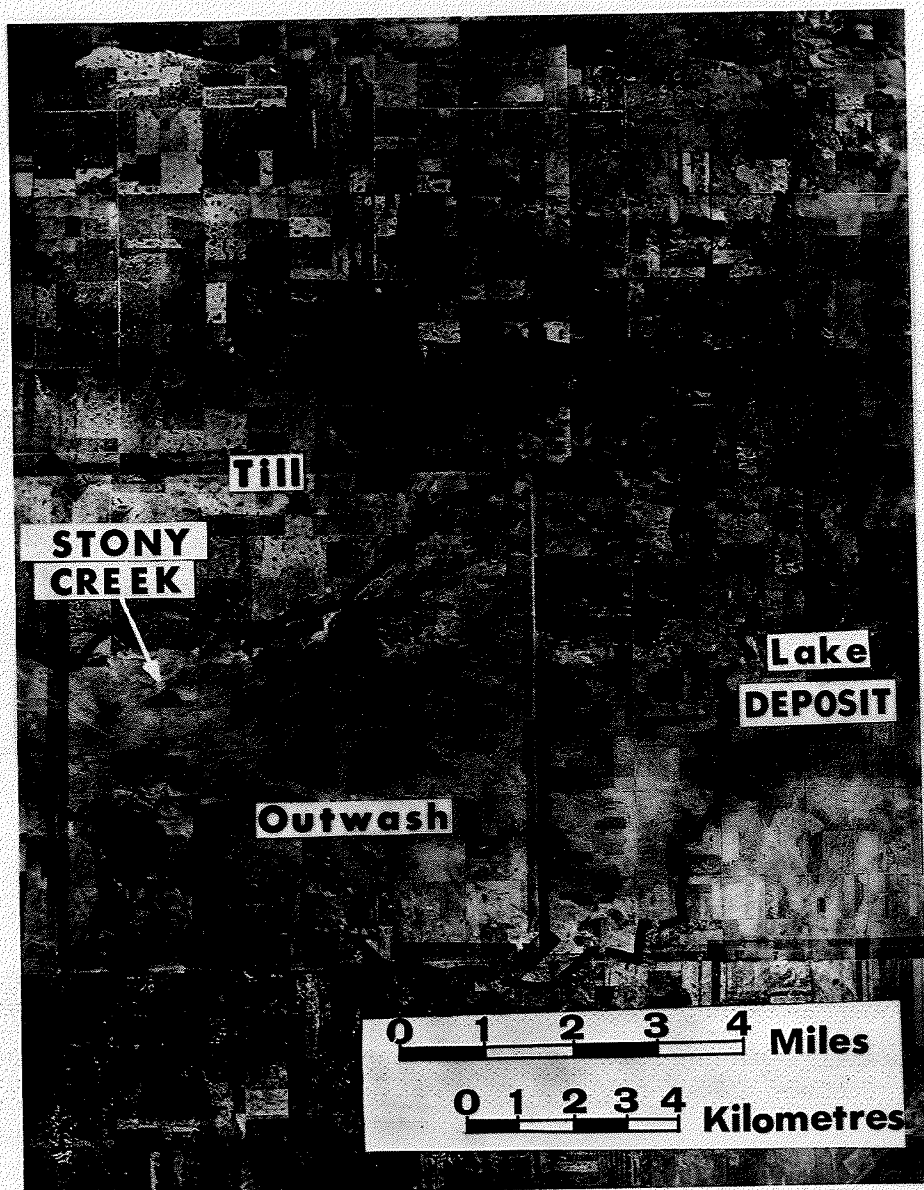
Till
STONY CREEK
Lake DEPOSIT
Outwash
0 1 2 3 4 Miles
0 1 2 3 4 Kilometres

Fig. 12. Vertical air photo mosaic of the Broomhill Experimental Aquifer and adjacent areas.

vertical heterogeneity is also common, as can be seen from the three selected geologic cross-sections shown in Figs. 13, 14 and 15. The location of the three cross-sections is shown in Fig. 11.

Groundwater movement in the aquifer interpreted from the natural water-table configuration (Fig. 16) is from west-southwest to north-northeast towards the Souris River. In the upper portion relatively smaller cross-sectional area resulted in higher hydraulic gradients than the middle and lower portions of the aquifer. The saturated thickness of the aquifer (Fig. 17) indicates a good correlation with the total thickness (Fig. 9), and results from the fact that the water table generally follows the topography.

Preliminary Calculations of the Hydrologic Budget of the Aquifer

Hydrologic Budget Equation

The hydrologic budget is defined as a quantitative statement of the balance between the total water gains and losses of a specified area or aquifer system. It can be used to estimate the actual evapotranspiration from an unconfined aquifer. In this method water entering an aquifer is equated to water leaving an aquifer during a specified time interval plus or minus the change in groundwater storage during the same period.

A relatively general hydrologic budget equation for

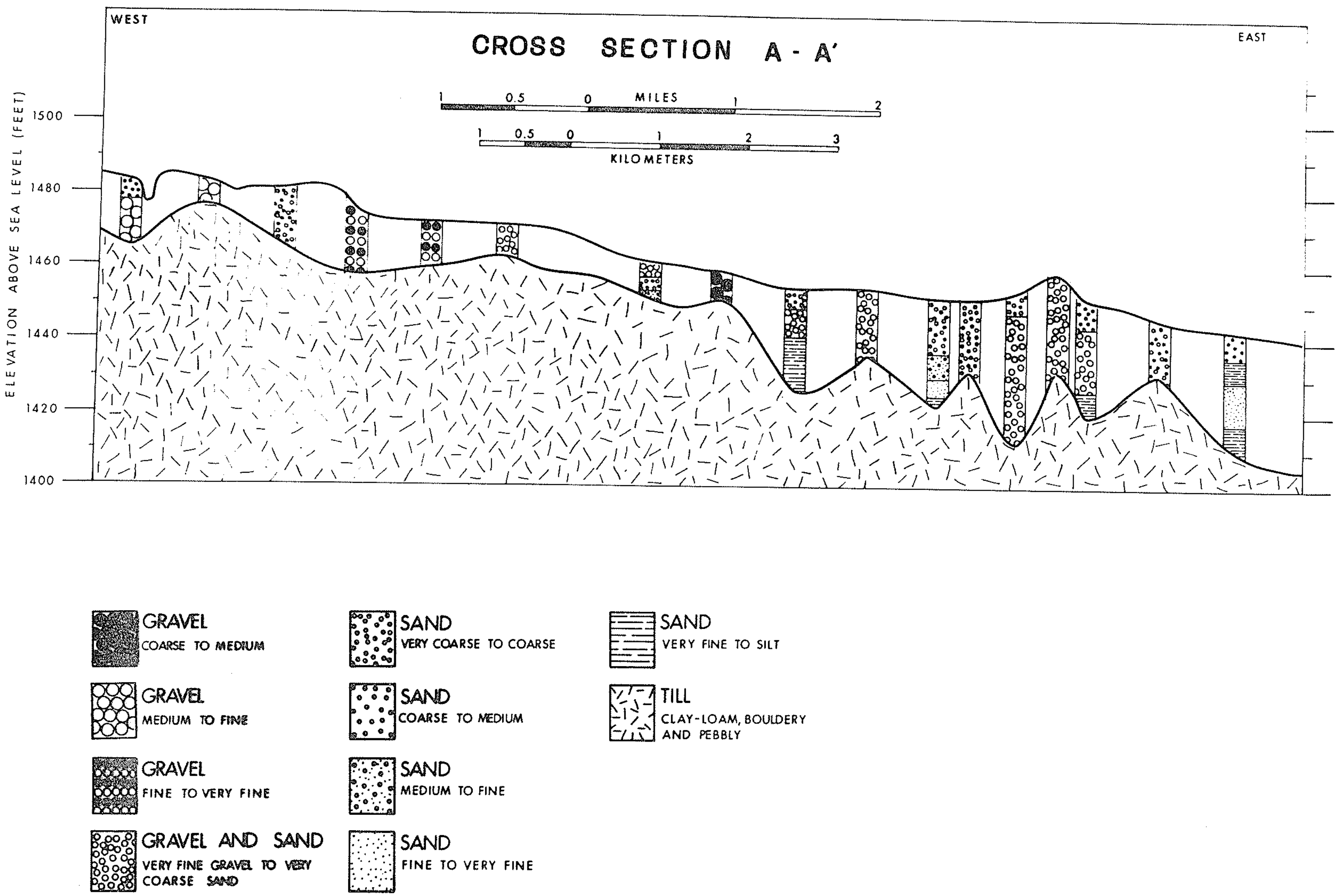

Fig. 13. Geologic cross-section A-A'.

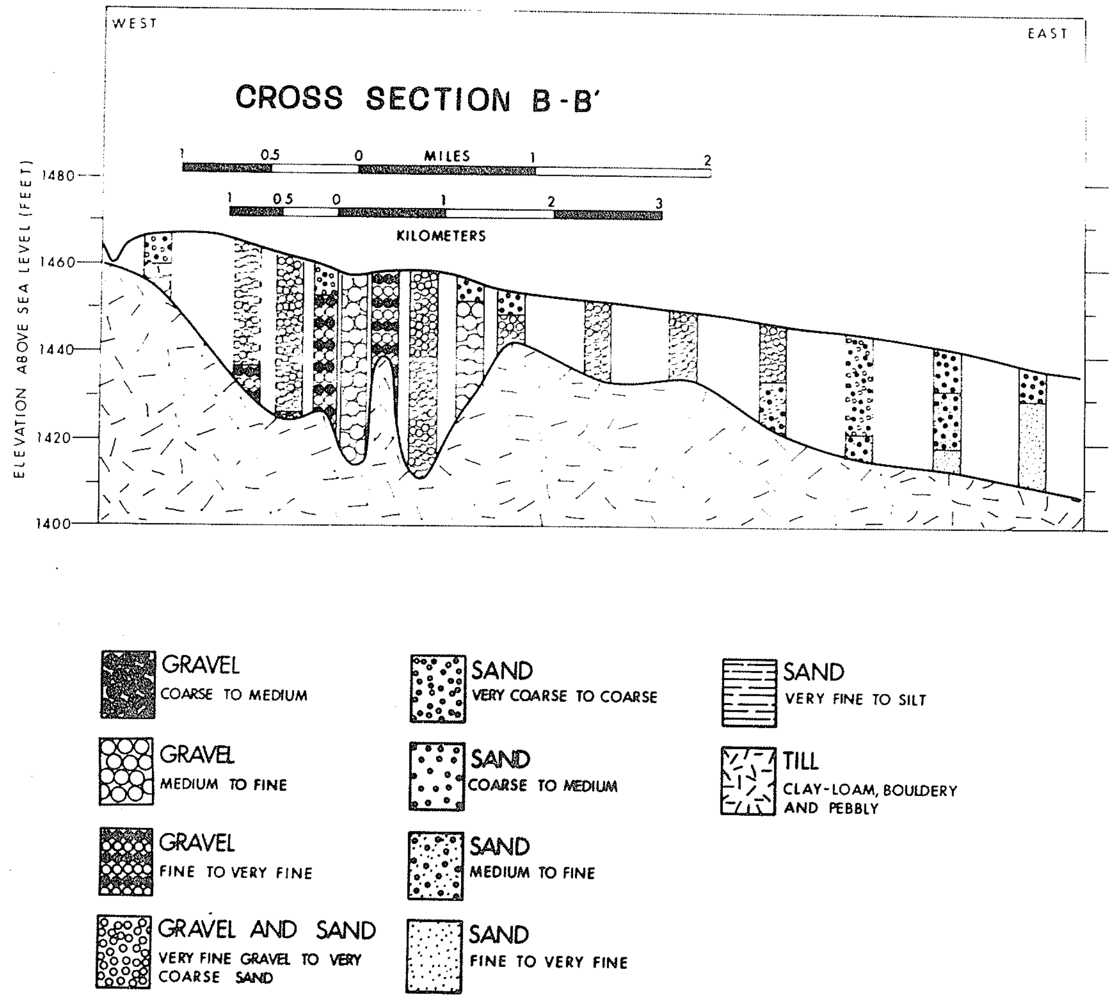

Fig. 14. Geologic cross-section B-B'.

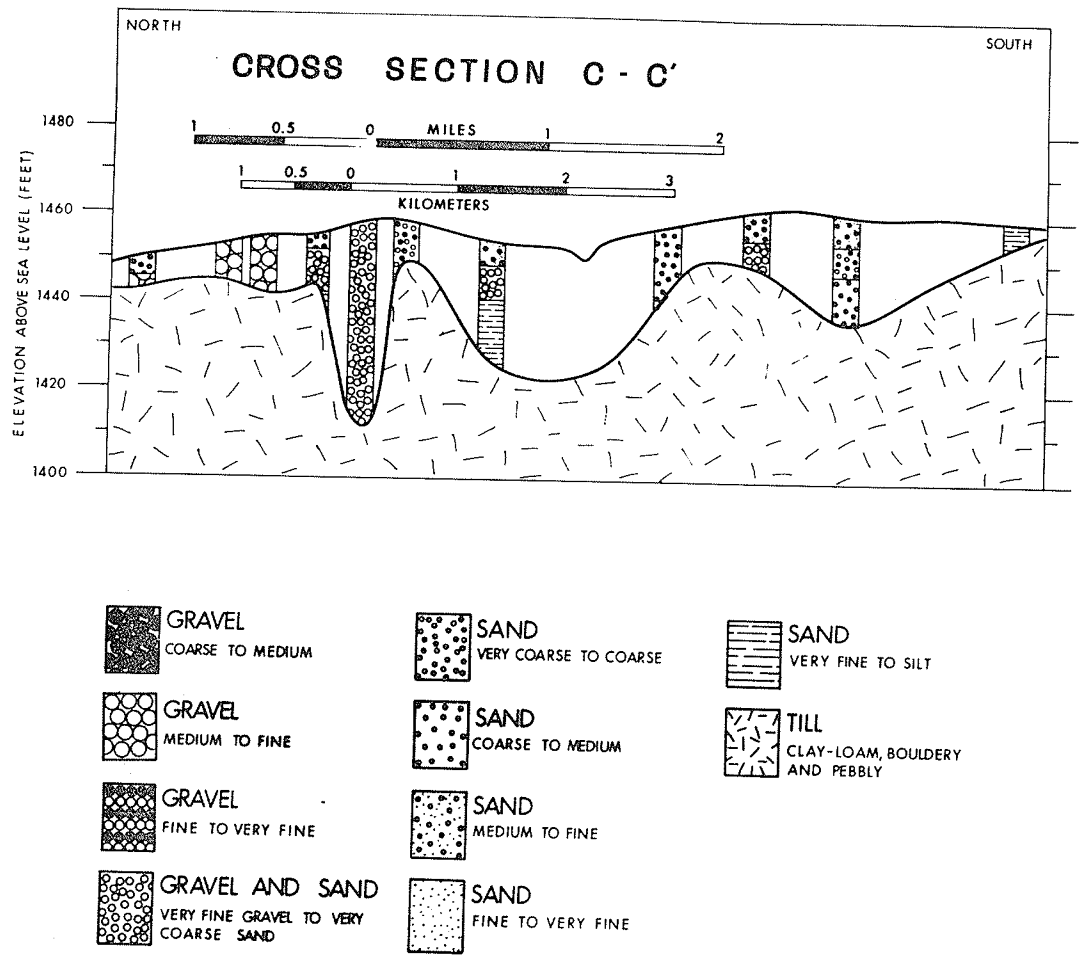

Fig. 15. Geologic cross-section C-C'.

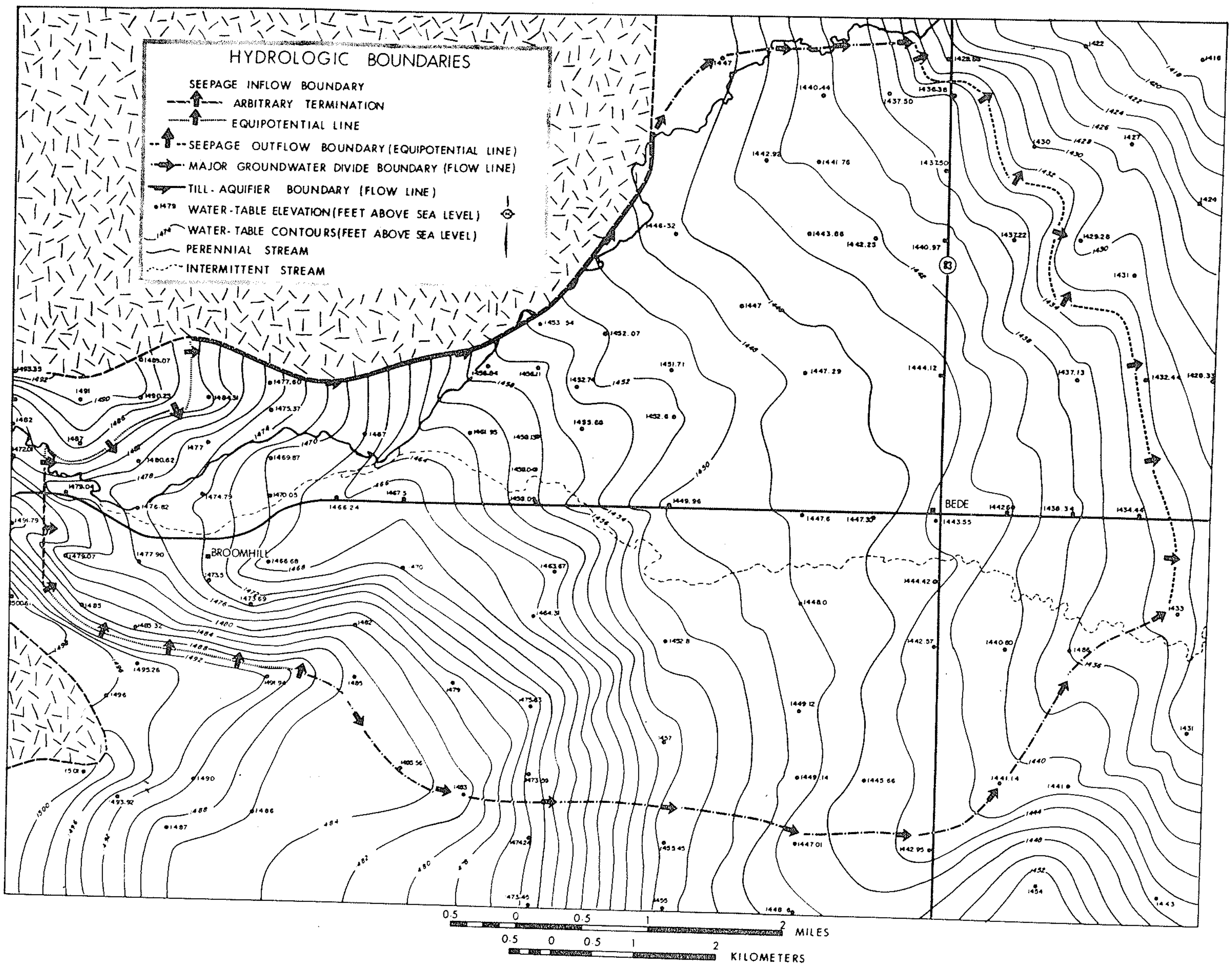

Fig. 16. Hydrologic boundaries and average natural water-table configuration of the Broomhill Experimental Aquifer in July, 1970.

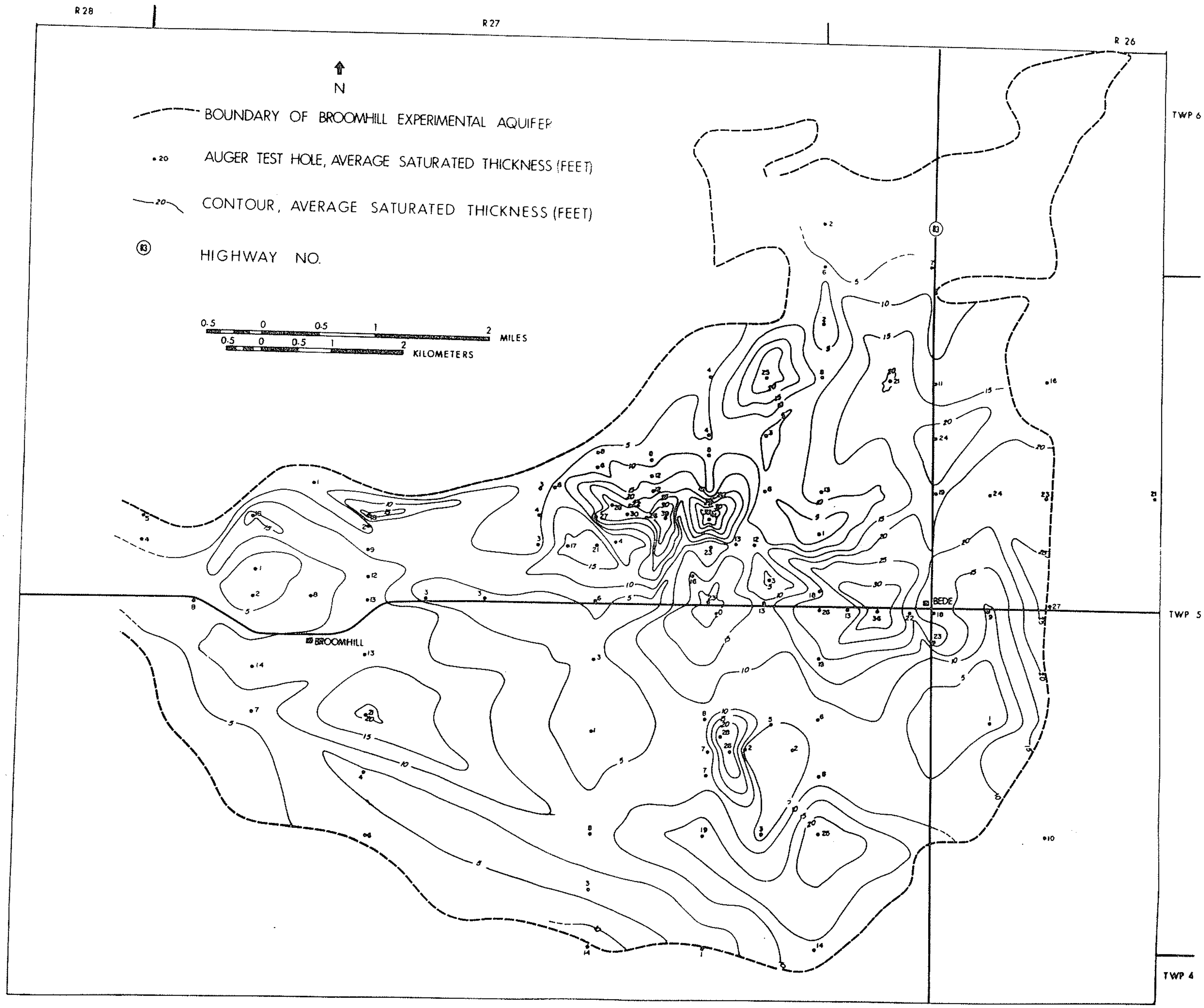

Fig. 17. Average saturated thickness of the Broomhill Experimental Aquifer in July, 1970.

the Broomhill aquifer during a time interval, Δt, is given as

$$P_a + I_s + I_g = O_s + O_g + ET \pm \Delta S_g \pm \Delta S_m \qquad [3]$$

where,

P_a = precipitation,

I_s = surface inflow,

I_g = groundwater inflow,

O_s = surface outflow,

O_g = groundwater outflow,

ET = actual evapotranspiration,

ΔS_g = change in groundwater storage, and

ΔS_m = change in soil moisture storage.

Preliminary calculation of some of the components of the above equation during time intervals within the period 1965 - 1971 has been made to obtain an estimate of the gross yield capabilities of the aquifer under natural pre-pumping conditions.

In the following discussion a time interval of one hydrologic year is used. The hydrologic year is defined as extending from April 1st to March 31st. The water quantities are expressed in acre-feet and inches of water over the area of the aquifer which is approximately 25 square miles or 16,000 acres.

In the following paragraphs each of the components of the hydrologic budget is assessed individually and then is included in budget calculations as a means of estimating the general magnitude of evapotranspiration and the natural recharge to the aquifer. Change in soil moisture storage during one hydrologic year is assumed to be negligible compared to the other variables and was therefore dropped from the budget equations.

Precipitation, P_a

The mean annual precipitation at Bede, measured during the period 1965 - 1970 by the Meteorological Branch using a manual rain gauge, is approximately 17 inches. The monthly distribution of precipitation during this period and the first three months of 1971 is listed in Table II. The long term mean annual precipitation for the general region is about 18 inches as calculated from the mean monthly precipitation during the period 1931 - 1960. On the average 7 inches of this total occurs during the winter and transition months of October to mid-May. The remaining 11 inches occurs during the summer months of mid-May to September (Canada Department of Transport, Meteorological Branch, 1967). The mean precipitation during the growing season of June to August inclusive is about 8 inches.

A comparison made between the total winter precipitation and the rise of water table in the aquifer during the spring period indicated that most of the winter precipitation recharges the aquifer during and immediately following the snowmelt period which normally occurs in April.

Table II

Monthly Precipitation at Bede, 1965-1971

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total	
													Inches	Acre-feet
1965	0.50	0.15	0.20	1.15	3.59	3.34	1.68	2.02	5.01	0.02	1.00	1.00	19.66	26200
1966	0.70	0.20	0.41	1.43	2.15	1.96	0.99	3.06	0.21	0.39	0.60	0.30	12.40	16500
1967	1.30	0.30	1.15	2.20	0.25	1.19	0.57	0.38	0.60	2.15	0.45	0.96	11.50	15300
1968	1.22	0.15	0.30	0.90	1.85	2.01	2.66	6.96	1.96	1.22	0.75	0.35	20.33	27100
1969	1.94	1.20	0.40	1.66	1.31	3.49	3.77	1.33	0.99	1.15	0.12	0.50	17.86	23800
1970	0.50	0.85	1.08	3.40	2.38	1.31	6.25	0.20	2.73	1.24	0.15	0.30	20.39	27200
1971	0.70	0.05	0.97					records not available						

The annual precipitation per hydrologic year during the period 1965 - 1971, obtained using the records listed in Table II, is tabulated in Table III. These tables indicate that during the tabulated period the annual precipitation has varied from a minimum of 11.50 inches during 1967 to a maximum of 20.39 inches during 1970. During the hydrologic years precipitation varied from a minimum of 10.42 inches during 1967 - 1968 to a maximum of 22.20 inches during 1968 - 1969. Higher variation of precipitation can be expected when a longer period is considered.

Streamflow, I_s and O_s

Surface water enters the aquifer area via Stony Creek from the west and discharges via Stony Creek and via an unnamed intermittent stream towards Marshy Lake located northeast of the Broomhill aquifer (Fig. 2). Additional surface water outflow from the aquifer is prevented by the embankment of Highway 83 which is located nearly coincident with the eastern boundary of the aquifer.

Streamflow records are available for Stony Creek during the seven-year period of 1965 - 1971. The inflow, I_s, is gauged by the Water Survey of Canada near Broomhill and outflow, O_s, near Bede at locations shown in Fig. 5.

The surface inflow and outflow records for Stony Creek per hydrologic year during the period 1965 - 1971, as tabulated in Table IV, indicate that stream outflow is

Table III

Precipitation Per Hydrologic Year, 1965-1971

Hydrologic Year	Inches	Acre-feet
1965-1966	20.12	26800
1966-1967	13.84	18500
1967-1968	10.42	13900
1968-1969	22.20	29600
1969-1970	16.75	22300
1970-1971	19.68	26200

Table IV

Surface Inflow and Outflow for Stony Creek, 1965-1971

Hydrologic Year	Surface Inflow (I_s)		Surface Outflow (O_s)		(O_s - I_s)	
	Acre-feet	Inches	Acre-feet	Inches	Acre-feet	Inches
1965-1966	3740	2.81	4180	3.14	440	0.33
1966-1967	3540	2.66	4450	3.34	910	0.68
1967-1968	850	0.64	747	0.56	-103	-0.08
1968-1969	226	0.17	331	0.25	105	0.08
1969-1970	6570	4.93	10400	7.80	3830	2.87
1970-1971	10600	7.95	13600	10.20	3000	2.25

generally higher than inflow. Since discharge measurements for the intermittent stream were not obtained during the above period the actual outflow-inflow differences, $O_s - I_s$, would be higher than that indicated in this table particularly during high-flow years such as 1969 and 1970 when active discharge in the intermittent stream was observed by local residents during April and early May. During moderate and low runoff years such as 1967 and 1968 this outlet was not active. It is probable that even during high runoff years discharge via Stony Creek accounts for more than 80 to 90 percent of the total surface outflow. Therefore, exclusion of the surface outflow of the intermittent stream from the calculations of the hydrologic budget should not generate serious errors in the results.

Change in Groundwater Storage, ΔS_g

Additions and losses of water stored in the aquifer are reflected by fluctuations of the water table. Changes in aquifer storage were calculated during the period 1965 - 1971 by multiplying the change in the water-table elevation ΔH_s, during an inventory period by the average effective porosity of deposits within the zone of the water-table fluctuations. This relationship is expressed as follows

$$\Delta S_g = \Delta H_s \times \phi_e \qquad [4]$$

To obtain estimations of the gross storage changes in the aquifer on an annual basis, during the above period, hydrographs from the two continuous-recording observation wells, MN 18 near Broomhill and MN 24 southwest of Bede (Fig. 5), were used. The water-table elevations on April 1st during the 1965 - 1971 period are listed in Table V. Calculated storage changes for hydrologic years during the 1966 - 1971 period are indicated in the same table and are based on an average effective porosity of 30 percent. As indicated previously this value was obtained by comparing tested laboratory samples to drill hole samples obtained from the well locations. The effective porosity may be somewhat higher, but the above value has been used as a conservative magnitude.

The highest water-table elevation on April 1st during the period 1966 - 1971 was recorded in 1966 and was caused by the above-average precipitation of 22 and 20 inches during 1964 and 1965. This period was followed by extremely dry years, 1966 and 1967, during which the two-year precipitation was only 24 inches. Consequently, on April 1st, 1968, the water table was approximately 33 inches lower than that of April 1st, 1966. The water-table elevation on April 1st, 1969, was the same as that of 1968, despite about 22 inches of precipitation which occurred during this period. However, more than 50 percent of this precipitation occurred during the summer months of June to August inclusive and therefore did not contribute greatly to groundwater recharge.

Table V

Change in groundwater storage (ΔS_g), 1965-1971

Hydrologic Year	MN 18 Near Broomhill					MN 24 Near Bede					Average ΔS_g	
	H_g feet		$\Delta H_g = H_2 - H_1$	$\Delta S_g = \Delta H_g x \phi_e$*		H_g feet		$\Delta H_g = H_2 - H_1$	$\Delta S_g = \Delta H_g x \phi_e$*			
	H_1	H_2	Inches	Inches	Acre-feet	H_1	H_2	Inches	Inches	Acre-feet	Inches	Acre-feet
1965-66	**	1475.8	-	-	-	**	**	-	-	-	-	-
1966-67	1475.8	1473.6	-26.4	-7.92	-10560	**	**	-	-7.92†	-10560†	-7.92	-10600
1967-68	1473.6	1473.2	- 4.8	-1.44	- 1920	1453.0	1452.3	-8.4	-2.52	- 3360	-1.98	- 2640
1968-69	1473.2	1473.2	0.0	0.00	0	1452.3	1452.3	0.0	0.00	0	0.00	0
1969-70	1473.2	1474.1	10.8	3.24	4320	1452.3	1452.7	4.8	1.44	1920	2.34	3120
1970-71	1474.1	1473.7	- 4.8	-1.44	- 1920	1452.7	1452.9	2.4	0.72	960	-0.36	- 480

* $\phi_e = 0.30$

** elevation was not recorded

† value assumed equal to MN 18 for same period

Water-table elevation on April 1st during 1966 - 1971 suggested that the largest decrease in groundwater storage occurred during the 1966 - 1967 hydrologic year. The water-table elevation on April 1st, 1971, was almost the same as that of 1967 and therefore the net change in groundwater storage during this period was negligible.

Groundwater Inflow, I_g

The approximate magnitude of the groundwater inflow to the aquifer across the western seepage inflow boundary (Fig. 16) was calculated using the D'Arcy equation. For this calculation an inflow width of four miles and an average hydraulic gradient of 0.005 feet per foot were estimated from the map of the natural water-table configurations (Fig. 16). Similarly, an average seepage cross-sectional depth of 7 feet was estimated from the map of the saturated aquifer thickness (Fig. 17).

A maximum magnitude of the annual groundwater subsurface inflow of 1.11 inches was obtained using the highest estimated hydraulic conductivity of 2.76×10^{-3} feet per second. A minimum value of 0.56 inches was obtained using the lowest estimated hydraulic conductivity of 1.39×10^{-3} feet per second. Since the highest and lowest values of hydraulic conductivity are representative of only a small portion of the inflow width, the most probable estimate of average annual groundwater inflow to the aquifer is considered to be about 0.80 inches, or 1,070 acre-feet. This value corresponds to an average hydraulic conductivity of 2.00 x 10^{-3} feet per second.

The hydraulic gradient across the seepage inflow boundary during the spring of 1971 was comparable in magnitude with the 1970 gradient and suggests that the hydraulic gradient across this boundary has been relatively constant during the past few years.

The aquifer is relatively thin in the inflow area and a lower water-table level would be expected to parallel the ground surface as a first approximation. It must be recognized, however, that the estimated inflow rate could vary significantly on a long-term basis as the cross-sectional depth varies with long-term climatic cycles.

Groundwater Outflow, O_g

The annual subsurface groundwater outflow from the aquifer across the seepage outflow boundary (Fig. 16) was estimated using two independent methods. The first was based on the water-table decline during the winter months, while the second utilized the D'Arcy equation.

The first method makes use of the observation that discharge via Stony Creek and evapotranspiration losses from the aquifer are negligible during the winter months. Groundwater inflow, I_g, is the only input to the aquifer during this period. Therefore, the average decrease in the groundwater storage during the winter months may be assumed to be equal to the net difference between the groundwater outflow and inflow. The annual groundwater outflow from the aquifer during the 1965 - 1971 period was estimated using the relationship

$$O_g = \Delta H_g \times \phi_e + I_g \qquad [5]$$

where ΔH_g is the average decline of the water table over the aquifer area during an inventory period, ϕ_e is the average effective porosity of deposits within the zone of the water-table decline, and the other quantities are as previously defined.

The average monthly water-table decline during the winter months was obtained for the 1965 - 1971 period from the hydrographs of the two continuous-recording observation wells. The results listed in Table VI were obtained using an average effective porosity of 30 percent and an estimated annual groundwater seepage inflow of 0.80 inches. This method of estimating aquifer outflow was used by the Manitoba Water Control and Conservation Branch (1968).

In the second method of estimating groundwater outflow an outflow width of 8 miles and an average hydraulic gradient of 1.19×10^{-3} feet per foot, estimated from Fig. 16, were used in the D'Arcy equation. Similarly, an average saturated hydraulic conductivity of 2.50×10^{-3} feet per second and an average cross-sectional depth of 15 feet were estimated from the maps of hydraulic conductivity and saturated thickness (Figs. 7 and 17).

Table VI

Groundwater subsurface outflow estimated using water-table decline during the winter months, 1965-1971

Hydrologic Year	MN 18 Near Broomhill: Period, Months	MN 18 Near Broomhill: Water-table Decline Per Period, Inches	MN 18 Near Broomhill: Average Per Month, Inches	MN 24 Near Bede: Period, Months	MN 24 Near Bede: Water-table Decline Per Period, Inches	MN 24 Near Bede: Average Per Month, Inches	Gross* Average Decline Per Month, Inches	Decline Per Year ΔH_g, Inches	$O_g = \Delta H_g \times \phi_e + I_g$ Inches	$O_g = \Delta H_g \times \phi_e + I_g$ Acre-feet
1965-1966	3	11.8	3.93	-	-	3.93†	3.93	47.16	14.95	19900
1966-1967	3	5.0	1.67	-	-	1.67†	1.67	20.04	6.81	9080
1967-1968	4	4.8	1.20	4	5.3	1.33	1.27	15.24	5.37	7160
1968-1969	3	7.2	2.40	3	7.2	2.40	2.40	28.80	9.44	12600
1969-1970	3	8.4	2.80	3	7.2	2.40	2.60	31.20	10.16	13500
1970-1971	2	6.0	3.00	2	7.2	3.60	3.30	39.60	12.68	16900

* Average of columns four and seven

† Records interrupted, valve assumed equal to MN 18 for the same period

Calculations based on the D'Arcy equation using the above data yield an average annual groundwater outflow of 1.02 inches. This quantity is very small compared to the results obtained from the water-table decline method, however, the D'Arcy results are very sensitive to the estimates of hydraulic conductivity and hydraulic gradient. The gradient value could reasonably vary between 0.001 to 0.002 feet per foot and the average hydraulic conductivity value could be within 2×10^{-3} to 5×10^{-3} feet per second. For example, pumping test data from a portion of the aquifer located 2 miles southwest of Bede suggest a conductivity range of 3×10^{-3} to 5×10^{-3} feet per second. However, comparison between the laboratory conductivity results and estimates from drill-hole samples obtained from the same portion of the aquifer indicated a conductivity value of 1.62×10^{-3} feet per second. It is apparent, therefore, that conductivity values cannot be approximated more accurately than half an order of magnitude. Assuming the upper limits of the gradient and conductivity, the D'Arcy method yields a maximum annual seepage outflow of about 4 inches. This quantity is still only 40 percent of the average annual seepage outflow during the 1965 - 1971 period obtained using the winter water-table data.

The water-table records obtained during 1970 - 1971 together with a review of the hydrologic characteristics of similar aquifer reported by Walton (1970), suggested that the gradient, hydraulic conductivity and effective porosity values as used previously are within a reasonable range. It was, therefore, concluded that the annual groundwater outflow from the aquifer is probably between 1 to 4 inches. The higher values obtained using the water-table decline method could be explained by redistribution of water within the aquifer rather than by outflow. For example, water could be moving from the segments of the aquifer in which the water-table elevation is high to the stream channels and marshy areas where the levels are lower. It is probable that data from the two recording wells are not representative of the average winter water-table response. The higher outflow values could also be explained if the average effective porosity of the deposits in the water-table zone is considerably lower than 0.30, the value used in the outflow calculations presented previously. This does not appear feasible, however, in light of the laboratory results and the results of a pumping test analysis discussed in Chapter VIII.

The groundwater estimations obtained using both methods are entered independently in the calculations of the annual evapotranspiration from the aquifer. The results of these calculations are then appraised in light of other information.

Evapotranspiration, ET

Equation 3 was used to calculate the annual evapotranspiration from the aquifer during the period 1966 - 1971. The calculations were performed using the inflow and outflow components of the aquifer system presented previously. Change in soil moisture storage was assumed negligible compared to the other budget components.

As previously indicated, the annual groundwater outflow from the aquifer was estimated using the D'Arcy equation and the winter water-table decline methods. During each cycle of calculation within the above period the budget equation was solved using the results of both methods (Tables VII, VIII, and IX). Tables VII and VIII are similar except that the results are expressed in different units.

The results indicate that evapotranspiration varies considerably depending on whether the water-table decline method (Tables VII and VIII) or the D'Arcy method (Table IX) was used to estimate the groundwater outflow from the aquifer. To evaluate the results the annual evapotranspiration was compared to annual distribution of precipitation and change in the groundwater storage as follows.

The annual evapotranspiration values calculated using the groundwater outflow values obtained from the winter water-table decline are unreasonably low. This is illustrated in the following examples.

The hydrographs of the two recording wells indicate

Table VII

Evapotranspiration estimation using aquifer water budget method, groundwater outflow estimated using water-table decline method, 1966-71 (*inches*)

Hydrologic Year	Inflow				Outflow and Change in Storage				Evapotranspiration
	Precipitation (P_a)	Surface Inflow (I_s)	Groundwater Inflow (I_g)	Total Inflow $I=P_a+I_s+I_g$	Surface Outflow (O_s)	Groundwater Outflow (O_g)	Change in Storage ($\pm\Delta S_g$)	Total Outflow + Change in Storage	
1966-67	13.84	2.66	0.80	17.30	3.34	6.81	-7.92	2.23	15.07
1967-68	10.42	0.64	0.80	11.86	0.56	5.37	-1.98	3.95	7.91
1968-69	22.20	0.17	0.80	23.17	0.25	9.44	0.00	9.69	13.48
1969-70	16.75	4.93	0.80	22.48	7.80	10.16	2.34	20.30	2.18
1970-71	19.68	7.95	0.80	28.43	10.20	12.68	-0.36	22.52	5.91

Table VIII

Evapotranspiration estimation using aquifer water budget method, groundwater outflow estimated using water-table decline method, 1966-1971 (*Acre-feet*)

Hydrologic Year	Inflow				Outflow and Change in Storage				Evapo-trans-piration
	Precipitation (P_a)	Surface Inflow (I_s)	Groundwater Inflow (I_g)	Total Inflow $I=P_a+I_s+I_g$	Surface Outflow (O_s)	Groundwater Outflow (O_g)	Change in Storage ($\pm\Delta S_g$)	Total Out-flow + change in Storage	
1966-1967	18500	3540	1070	23110	4450	9080	-10600	2930	20180
1967-1968	13900	850	1070	15820	747	7160	- 2640	5267	10553
1968-1969	29600	226	1070	30896	331	12600	0	12931	17965
1969-1970	22300	6570	1070	29940	10400	13500	3120	27020	2920
1970-1971	26200	10600	1070	37870	13600	16900	- 480	30020	7850

Table IX

Evapotranspiration estimation using aquifer water budget method, groundwater outflow estimated using D'Arcy equation, 1966-71

Hydrologic Year	Inflow inches	Outflow and Change in Storage inches						Evapotranspiration inches		Evapotranspiration acre-feet	
	Total Inflow* $I=P_a+I_s+I_g$	Surface Outflow (O_g)	Groundwater Outflow (O_g) min.	Groundwater Outflow (O_g) max.	Change in Storage $(\pm \Delta S_g)$	Outflow + Change in Storage min.	Outflow + Change in Storage max.	max.	min.	max.	min.
1966-67	17.30	3.34	1	4	-7.92	- 3.58	- 0.58	20.88	17.88	27800	23800
1967-68	11.86	0.56	1	4	-1.98	- 0.42	2.58	12.28	9.28	16400	12400
1968-69	23.17	0.25	1	4	0.00	1.25	4.25	21.92	18.92	29200	25200
1969-70	22.48	7.80	1	4	2.34	11.14	14.14	11.34	8.34	15100	11100
1970-71	28.43	10.20	1	4	-0.36	10.84	13.84	17.59	14.59	23500	19500

* from Table VII

that monthly precipitation of more than 2 inches during the summer period of June to August is normally required to cause groundwater recharge. On this basis the monthly precipitation data shown in Table II suggest that in the summer of 1969 at least 5.33 inches of water were lost by evapotranspiration; however, the annual evapotranspiration value for 1969 - 1970, calculated using the outflow estimates obtained from the water-table decline, is 2.18 inches. It follows that the outflow values are definitely not reasonable for this year and therefore do not appear to be realistic for the other years either.

The annual evapotranspiration values obtained using the groundwater outflow estimated from the D'Arcy method are within a more reasonable range. In these calculations a minimum and a maximum estimate of annual evapotranspiration were obtained using the maximum and minimum values for groundwater outflow.

In the 1966 - 1967 season, both the minimum and maximum values of evapotranspiration, obtained by using the D'Arcy results, exceeded the total inflow to the aquifer. The water loss from storage of about 8 inches is congruent with this relationship.

In the 1969 - 1970 season the evapotranspiration values calculated using the D'Arcy method did not exceed the annual precipitation and total inflow to the aquifer. The increase in aquifer storage of about 2 inches, therefore, is in agreement with the relationship.

The highest range of evapotranspiration during the tabulated period (Table IX) occurred in the 1968 - 1969 season. This is reasonable because precipitation during the growing season was also exceptionally high.

Finally, the average actual evapotranspiration obtained using the D'Arcy method of estimating groundwater ouflow compares favourably with the long-term average of 16 inches for the general area reported by Laycock (1967). From Table IX the average minimum and maximum annual evapotranspiration during the period 1966 - 1971 are about 14 and 17 inches.

The general magnitude of the evapotranspiration results was also appraised in light of calculated potential evapotranspiration and consumptive-use values when water is not a limiting factor. Preliminary estimations of potential evapotranspiration and consumptive-use requirement values were estimated for representative time intervals within the period 1966 - 1971 (Tables X, XI, and XII). Potential evapotranspiration results were obtained using the Thornthwaite Method, and the consumptive-use values were obtained using the Blaney-Criddle Method and Class "A" pan evaporation data. A summary of the calculation procedures is outlined by Gray *et al.* (1970). The estimations were based on climatic data including temperature and precipitation at Bede and Class "A" pan evaporation data from stations at Baldur and

Table X

Potential evapotranspiration estimated using Thornthwaite Method, 1966-1971

Hydrologic Year	Season	Evapotranspiration Inches	Evapotranspiration Acre-feet
1966-1967	May - Oct.	23.48	31300
1967-1968	April - Oct.	23.63	31500
1968-1969	April - Oct.	23.73	31600
1969-1970	April - Oct.	23.64	31500
1970-1971	April - Oct.	26.57	35400

Table XI

Seasonal consumptive use for grass when water is not a limiting factor estimated using Blaney-Criddle Method, 1966-1971

Hydrologic Year	Season	Consumptive Use	
		Inches	Acre-feet
1966-1967	May - Oct.	25.54	34100
1967-1968	April - Oct.	27.86	37100
1968-1969	April - Oct.	27.63	36800
1969-1970	April - Oct.	27.79	37100
1970-1971	April - Oct.	29.28	39000

Table XII

Seasonal consumptive use for grass when water is not a limiting factor estimated using Class "A" pan evaporation data, 1966-1971

Hydrologic Year	Season	Consumptive Use	
		Inches	Acre-feet
1966-1967	May - Sept.	25.43	33900
1967-1968	May - Sept.	27.50	36700
1968-1969	May - Sept.	20.10	26800
1969-1970	May - Sept.	22.10	29500
1970-1971	May - Sept.	25.14	33500

Rivers located within 80 miles of the aquifer. These data were made available by the Meteorological Branch, Canada Ministry of Transport.

Seasonal potential evapotranspiration values obtained using the Thornthwaite Method varied between 23 and 27 inches with an average value of 24 inches per season (Table X).

Values of seasonal consumptive use estimated from the Blaney-Criddle Method were obtained using the assumption that the aquifer is covered with grass. In this calculation a crop coefficient of 0.75 was applied over the entire length of the growing season which included the frost-free months of each hydrologic year. The Blaney-Criddle results varied between 25 and 30 inches with an average value of 28 inches per season (Table XI). These values are somewhat higher than the estimated values of potential evapotranspiration. Theoretically, consumptive use for any given crop must be lower than, or at best equal to, potential evapotranspiration. It should not, therefore, exceed the potential evapotranspiration in the same climatic conditions. It must be recognized that consumptive-use and potential evapotranspiration values, obtained using the above empirical methods, generally include at least 10 percent error.

Seasonal consumptive-use results obtained from the average pan evaporation at Baldur and Rivers were calculated using a crop coefficient of 0.68. This coefficient was suggested by Sonmor (1963) for estimating the consumptive-use requirement of grass or pasture from pan evaporation data in southern Alberta. The results of this method varied between 20 and 28 inches with an average value of 24 inches per season (Table XII). These values are lower than consumptive-use results obtained using the Blaney-Criddle Method partly because evaporation during April and October was not included in the calculations. Pan evaporation during these two months was not recorded at the field stations but is known to be relatively small.

The above results indicate that the actual evapotranspiration values, obtained using both the D'Arcy and water-table decline results are lower than the potential and consumptive-use estimates. It is reasonable to assume that if precipitation during a given period is extremely high, the actual evapotranspiration value would approach the potential and consumptive-use values for the case where water is not a limiting factor during the same period. In this respect, a more reasonable estimate of the actual evapotranspiration from the aquifer was obtained using the D'Arcy method of estimating the groundwater outflow. For example, average annual evapotranspiration for the 1968 - 1969 season, calculated using the D'Arcy results, is 20.4 inches. During this season, the annual precipitation and potential evapotranspiration values are 22.2 and 23.7 inches, respectively.

Yield Capabilities of the Aquifer

The gross yield capabilities of the aquifer have been estimated using the hydrological data presented in the preceeding sections. An initial estimate was made considering the aquifer independent of the recharge capabilities of Stony Creek. In this case the gross annual yield under natural conditions is taken as the sum of annual groundwater outflow plus the volume of water contributed from the aquifer area to Stony Creek, the latter being taken as the difference between the annual surface inflow and outflow via the creek. A second estimate was made in which the above quantities were augmented by the annual surface water inflow to the aquifer area via Stony Creek. In this situation it is assumed that the total yield from the aquifer would be realized by conjunctive use schemes involving induced or artificial recharge methods.

As indicated previously the groundwater outflow from the aquifer is in the range of 1 to 4 inches annually. The contribution of water from the aquifer to Stony Creek is indicated in Table IV. These data represent the annual difference between stream inflow to the aquifer at Broomhill and the outflow at Bede. The results range from a negligible difference in 1967 and 1968 to a maximum contribution of 2.87 inches in 1969. The gross annual yield of the aquifer during the 1965 - 1971 period, therefore, would be between 2 and 6 inches when calculated using an average D'Arcy outflow and the minimum and maximum estimates of groundwater contribution to Stony Creek. The average annual yield during the above period would be 4 inches or 5,330 acre-feet. It should be recognized, however, that these estimates could be smaller than that of the actual yield values if the representative field conductivities are in the upper portion of the range of probable values discussed previously.

The second estimate of the yield capabilities of the aquifer is based on the assumption that the total surface water and groundwater outflow from the aquifer area would be available annually from the aquifer under appropriate developmental conditions. The annual surface outflow from the aquifer area via Stony Creek during the 1965 - 1971 period is listed in Table IV. The results range from a minimum surface outflow of 0.25 inches in 1968 to a maximum outflow of 10.20 inches in 1970. The gross annual developmental yield of the aquifer during the above period would, therefore, be between 2 and 13 inches. These results were obtained using the minimum and maximum surface outflow and the average D'Arcy outflow. The average annual developmental yield during the tabulated period was 7 inches or 9,330 acre-feet. In other words, these volumes of water could have been obtained from the aquifer if aquifer withdrawals were made so that all of the outgoing streamflow was used to recharge the aquifer and that groundwater outflow was prevented by changing the water-table configuration. Recharge using stream flow from Stony

Creek could be accomplished by constructing a series of small catchment basins and distribution channels or possibly by installing a system of pumping wells along the stream. It should be recognized, however, that the actual developmental yield of the aquifer would be somewhat lower than the above values because it is unlikely that all of the stream and groundwater outflow could be effectively utilized.

The above conclusions are based primarily on data from the 1965 - 1971 period and indicate that the aquifer is subjected to major effects of climatic conditions. It is recommended that the data be extrapolated for a longer period by using regional precipitation and streamflow data from adjacent watersheds. It must be emphasized that it is not primarily the average conditions that control the appraisal of the aquifer system but rather is the frequency-duration relationships of the hydrologic processes.

CHAPTER V

THEORETICAL BASIS OF MATHEMATICAL MODELS

Historical Development of Methods of Aquifer Analysis

Quantitative methods of aquifer analysis were first investigated scientifically by Thiem (1906), who developed an equation which described the behaviour of the water table in an infinite, homogeneous, isotropic aquifer during pumping from one well under equilibrium conditions. A more general, second order, nonlinear, differential equation describing steady and unsteady flow of a fluid in both saturated and partially saturated porous media was developed by Richards (1931) by combining the D'Arcy equation and the continuity equation. Practical solutions of his equation for flow in the saturated zone were not obtained until many years later. A nonequilibrium formula derived by Theis (1935) describes the behaviour of the potentiometric surface of a confined, infinite, homogeneous, isotropic aquifer during pumping from a fully-penetrating well. During the past 30 years this equation has been widely used both to determine values of hydraulic conductivity and storage coefficient from pumping test data, and to evaluate the effects of proposed pumping schemes. Wenzel (1942) demonstrated the usefulness of this equation in appropriate cases for determining properties of confined aquifers. It was found, however, that the Theis equation yielded anomalous results when applied to pumping test data from unconfined aquifers. Boulton (1955) suggested that these anomalies were due to the delay in the yield of water from storage as the water table declines. Further limitations of the Theis equation in analyzing the results of short-period pumping tests of unconfined aquifers have been discussed by Walton (1960) and Boulton (1963). Numerous analytical solutions to the equations which describe the response of aquifer systems to pumping are now available in the literature. Nearly all of the solutions, however, relate to aquifers with relatively simple boundary conditions. More complex boundary conditions such as are encountered in the field inevitably involve more complex mathematical functions. Numerical methods of equation solving have been developed which circumvent the necessity of using complex analytical expressions and can be adapted to complex boundary-value problems.

The first significant attempt to solve the flow problems numerically was made by Terwilliger *et al.* (1951) in connection with a two-phase flow problem of gravity drainage of an oil reservoir. Stallman (1956(a) and 1956(b)) introduced a numerical approach in solving groundwater problems by using a finite-difference approximation technique.

He developed the procedure for determining the hydraulic conductivity and the storage coefficient of an aquifer by numerical analysis of the groundwater levels. The finite-difference forms of the flow equations were solved by many investigators using the electric analog technique (Skibitzke, 1961; Stallman, 1963) during the late 1950's and early 1960's, largely because digital computer availability and storage capabilities were limited. Freeze (1966) and Freeze and Witherspoon (1966 and 1967) investigated regional groundwater flow patterns in a nonhomogeneous anisotropic system using models solved by finite-difference methods programmed for a digital computer. They developed both the analytical and numerical solutions to the mathematical model of steady-state regional groundwater flow, and concluded that the numerical methods could be used for both two-dimensional and three-dimensional cases, whereas the analytical separation of variables technique was restricted to simple two-dimensional layered media. They further indicated that the numerical technique was mathematically simpler and that only one mathematical derivation was necessary in order to design a computer program which could handle any water-table and geologic configurations.

The next important application of the numerical approach to groundwater flow problems was made by Pinder (1968) and Pinder and Bredehoeft (1968), who described unsteady flow in a confined nonhomogeneous aquifer using linear, parabolic, partial-differential equations. They developed a mathematical model in which finite-difference approximations to these equations were solved using an alternating-direction scheme developed by Peaceman and Rachford (1955) and Douglas and Peaceman (1955). Pinder and Bredehoeft compared the digital model results with analytical solutions for problems in homogeneous media with a simple geometry, and with electric analog results for a realistic aquifer. The following conclusions were obtained:

(i) The digital model compared favourably with analytical and analog results.

(ii) Irregular boundaries as well as differing boundary conditions provided little difficulty.

(iii) The model was readily adjusted to produce historical variations in the water-table and potentiometric surface data.

Pinder (1969) developed a digital computer program which simulates the response of confined or unconfined aquifers with considerable geologic complexities to a wide range of hydrologic stresses. This model is identical in principal to the basic model of Pinder and Bredehoeft, although the finite-difference forms of the flow equations are solved using an iterative alternating-direction implicit procedure. An adaptation of this model is used in the present investigation. It is capable of describing the behaviour of two-dimensional, nonhomogeneous aquifers which may be irregular in shape, and may receive infiltration from one or more lakes and streams.

Taylor and Luthin (1969) suggested that in analyzing the response of an unconfined aquifer to pumping stress under transient conditions, some hydraulic properties of the unsaturated portion of the aquifer must also be incorporated. This can be accomplished by including parameters relating to water content, hydraulic conductivity, and capillary pressure head of the unsaturated zone of the aquifer. Although the Pinder (1969) model does not include all these complexities, it does meet the requirements of the present study.

Another numerical method of equation solving relies on the finite-element approach wherein the field is replaced by a system of finite elements. The method was originally developed in the aircraft industry and has been used recently for aquifer analysis by many investigators (Zienkiewicz and Cheung, 1965; Taylor and Brown, 1967; and Javandel and Witherspoon, 1969). This method has definite advantages over finite-difference procedures when analyzing steady-state flow problems. However, programs offering major advantages in modelling unsteady-state problems have not yet been developed.

Mathematical Formulation of the Model

Controlling Physical Parameters

Following Pinder (1968), for two-dimensional laminar flow in an anisotropic, nonhomogeneous, porous medium the D'Arcy equation may be expressed as

$$\begin{vmatrix} v_x \\ \\ v_y \end{vmatrix} = - \begin{vmatrix} K_{o\,(xx)} & K_{o\,(xy)} \\ \\ K_{o\,(yx)} & K_{o\,(yy)} \end{vmatrix} \begin{vmatrix} \frac{\partial h}{\partial x} \\ \\ \frac{\partial h}{\partial y} \end{vmatrix} \qquad [6]$$

or,

$$v_i = -K_{(i,j)} \frac{\partial h}{\partial x} \qquad [7]$$

In these equations: v_x and v_y or v_i are the components of the apparent velocity, LT^{-1}; $K_{o\,(xx)}$, $K_{o\,(xy)}$, $K_{o\,(yx)}$, and $K_{o\,(yy)}$ or $K_{o\,(i,j)}$ are the components of the saturated hydraulic conductivity, LT^{-1}, and are a function of the space co-ordinates x and y; and h is the hydraulic head, L.

On a microscopic scale, the hydraulic head and its gradient change continuously along any flow line due to frequent changes in the small cross-sectional area of the pores through which the flow occurs. On the macroscopic scale, however, the hydraulic head and its gradient may be assumed uniform and equal to the average of the head and the gradient at different points over small volumes containing a great number of pores. Consequently, the velocity may also be assumed uniform and equal to the average of the flows through many elemental cross-sections of the pores. As a result, the physical system of flow is replaced by a mathematical continuum in which the flow obeys the D'Arcy equation.

The hydraulic head, h, at a point within a partially-saturated or saturated medium is the sum of the pressure head at the point and the elevation of the point above an arbitrary datum. Thus, the hydraulic head $h(x,y,z,t)$ at the point (x,y,z) can be defined (Hantush, 1964) as

$$h(x,y,z,t) = \frac{P}{\gamma} + z + f \qquad [8]$$

where:

P = hydrostatic pressure at a point whose vertical co-ordinate is z $(ML^{-1}T^{-2})$

γ = specific weight $(ML^{-2}T^{-2})$

f = elevation of the x,y plane above an arbitrarily chosen datum of elevation (L)

x,y,z = rectangular co-ordinates

t = time of observation (T)

The quantity γh is known as hydraulic potential or piezometric potential and denotes the sum of pressure energy, P, and the gravitational energy potential, γz. Both the hydraulic head and the hydraulic potential are potential quantities and consequently obey all the laws of potential theory.

Derivation of the Basic Flow Equation

The law of conservation of matter is expressed by the equation of continuity, which for unsteady compressible flow through porous media, takes the form (Jacob, 1950; Pinder, 1968):

$$- \operatorname{div}(\rho v_i) = \frac{S_s}{g} \frac{\partial P}{\partial t} + L'(x,y,t) \qquad [9]$$

where:

ρ	= fluid density	(ML^{-3})
S_s	= specific storage	(L^{-1})
g	= acceleration due to gravity	(LT^{-2})
L'	= mass flux of fluid into or out of the system	$(ML^{-3}T^{-1})$

The coefficient S_s has been termed the specific storage of the aquifer (Hantush, 1964), and is defined as the volume of water which a unit volume of the aquifer releases from storage because of expansion of water and compression of the aquifer under a unit decline in the average head within the unit volume of the aquifer. The storage coefficient is related to the specific storage of the aquifer by the relation:

$$S = S_s \cdot b_o \qquad [10]$$

where:

S = storage coefficient (dimensionless)

b_o = saturated thickness of the aquifer (L)

Differentiating P, from equation [8] with respect to t we get

$$\frac{\partial P}{\partial t} = (h\frac{\partial \gamma}{\partial t} + \gamma\frac{\partial h}{\partial t}) \qquad [11]$$

or,

$$\frac{\partial P}{\partial t} = g(h\frac{\partial \rho}{\partial t} + \rho\frac{\partial h}{\partial t}) \qquad [12]$$

The change in density, ρ, over time is generally very small and [12] reduces to

$$\frac{\partial P}{\partial t} \approx g\rho\frac{\partial h}{\partial t} \qquad [13]$$

Substituting [13] into [9] we obtain

$$-\text{div}(\rho v_i) = S_s\rho\frac{\partial h}{\partial t} + L'(x,y,t) \qquad [14]$$

The left-hand side of [14] can be stated (Pinder, 1968) as

$$\operatorname{div}(\rho v_i) = -\rho\frac{\partial}{\partial x_i}\left(K_{o\,(i,j)}\frac{\partial h}{\partial x_j}\right) \qquad [15]$$

substituting [15] into [14] we have

$$\frac{\partial}{\partial x_i}\left(K_{o\,(i,j)}\frac{\partial h}{\partial x_j}\right) = S_s\,\frac{\partial h}{\partial t} + \frac{L'(x,y,t)}{\rho} \qquad [16]$$

Multiplying the saturated hydraulic conductivity tensor $K_{o\,(i,j)}$ by the saturated thickness of the aquifer, b_o, equation [16] becomes

$$\frac{\partial}{\partial x_i}\left(T_{(i,j)}\frac{\partial h}{\partial x_j}\right) = S\,\frac{\partial h}{\partial t} + \frac{b_o L'(x,y,t)}{\rho} \qquad [17]$$

where $T_{(i,j)}$ is the transmissibility tensor which in matrix notation can be written as

$$T_{(i,j)} = \begin{vmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{vmatrix} \qquad [18]$$

for the principal axes, x and y, equation [18] reduces to

$$T_{(i,j)} = \begin{vmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{vmatrix} \qquad [19]$$

and equation [17] becomes (Pinder, 1968)

$$T_{xx}\frac{\partial^2 h}{\partial x^2} + \frac{\partial T_{xx}}{\partial x}\frac{\partial h}{\partial x} + T_{yy}\frac{\partial^2 h}{\partial y^2} + \frac{\partial T_{yy}}{\partial y}\frac{\partial h}{\partial y} = S\frac{\partial h}{\partial t} + L(x,y,t) \qquad [20]$$

where $L = \frac{b_o L'}{\rho}$ (dimension LT^{-1}) accounts for the flux from vertical leakage, and also the net effect of recharge and discharge from the aquifer.

Equation [20] is the approximate differential equation describing nonsteady flow of a compressible fluid in an elastic nonhomogeneous porous medium.

Numerical Solution of the Flow Equation

The finite-difference technique consists of replacement of a derivative by values of difference quotients of the function at separate discrete points (Volynskii and Bukhman, 1965). Such a technique is based upon the application of the Taylor Series where the field is replaced by a finite number of N points, each of which represents a small two-dimensional region of the length Δx and Δy.

A finite-difference approximation to equation [20] for a rectangular-regular mesh of side lengths Δx and Δy is presented by Pinder and Bredehoeft (1968), which for a rectangular-irregular mesh is as follows. The nodal array used for the development of the finite-difference equations is shown in Fig. 18.

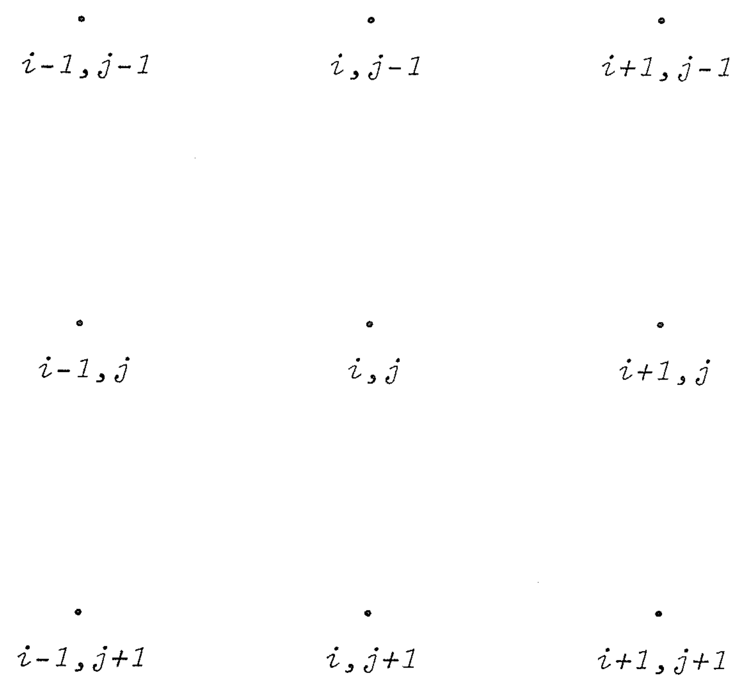

Fig. 18. Nodal array for the development of the finite-difference equations.

$$T'_{xx\,(i-\frac{1}{2},\,j)}\left(\frac{h_{(i-1,\,j,\,k)} - h_{(i,\,j,\,k)}}{\Delta x_i}\right)$$

$$+\ T'_{xx\,(i+\frac{1}{2},\,j)}\left(\frac{h_{(i+1,\,j,\,k)} - h_{(i,\,j,\,k)}}{\Delta x_i}\right)$$

$$+\ T'_{yy\,(i,\,j-\frac{1}{2})}\left(\frac{h_{(i,\,j-1,\,k)} - h_{(i,\,j,\,k)}}{\Delta y_j}\right)$$

$$+\ T'_{yy\,(i,\,j+\frac{1}{2})}\left(\frac{h_{(i,\,j+1,\,k)} - h_{(i,\,j,\,k)}}{\Delta y_j}\right)$$

$$=\ S\left(\frac{h_{(i,\,j,\,k)} - h_{(i,\,j,\,k-1)}}{\Delta t}\right) + \frac{Q_{(i,\,j)}}{\Delta x_i \Delta y_j}$$

$$-\ K_s\left(\frac{2H_{(i,\,j)} - h_{(i,\,j,\,k)} - h_{(i,\,j,\,k-1)}}{2b_{s\,(i,\,j)}}\right) \qquad [21]$$

where

$$T'_{xx\,(i+\frac{1}{2},\,j)} = \frac{2T_{xx\,(i,j)}T_{xx\,(i+1,j)}}{T_{xx\,(i,j)}\Delta x_{(i+1)} + T_{xx\,(i+1,j)}\Delta x_i}$$

Thus, the coefficient of the term $(h_{(i+1,j,k)} - h_{(i,j,k)})$ is given by the harmonic mean of

$$\frac{T_{xx\,(i,j)}}{\Delta x_i} \quad \text{and} \quad \frac{T_{xx\,(i+1,j)}}{\Delta x_{(i+1)}}$$

and the coefficient for the term $(h_{(i,j,k)} - h_{(i+1,j,k)})$ is also identical to that of the term $(h_{(i+1,j,k)} - h_{(i,j,k)})$,

i = index in the x-direction

j = index in the y-direction

k = index in the t-direction

Q = rate of discharge or recharge at any node (L^3T^{-1})

$\Delta x, \Delta y$ = space increments (L)

Δt = time increment (T)

K_s = hydraulic conductivity of streams and lakes bottoms (LT^{-1})

H = hydraulic head in the streams and lakes (L)

b_s = thickness of the stream bed and lake bottom (L)

For a more complete discussion of finite-difference techniques and applications to boundary-value problems the reader is directed to Crank (1956), McCracken and Dorn (1964), and Volynskii and Bukhman (1965).

Equation [21] describes the flow of groundwater in confined formations. To adapt the equation for application to unconfined aquifers several modifications are necessary. In an unconfined aquifer transmissibility, T, is a function of time as well as a function of space co-ordinates. A non-linear set of finite-difference equations are, therefore, required to describe the non-steady state behaviour of an unconfined aquifer. This condition can be approximated by replacing transmissibility in equation [21] with the product of the saturated hydraulic conductivity, K_o, and the saturated thickness of the aquifer, b_o, from the previous time step

For example,

$$T_{(i,j,k)} = K_{o(i,j)} b_{o(i,j,k-1)} \qquad [22]$$

Another modification necessary when applying the equation to unconfined aquifers relates to the storage term. In a confined aquifer the storage function is referred to as the storage coefficient and is defined as the volume of water that the aquifer releases from, or takes into, storage per unit surface area of the aquifer per unit decline, or rise, of the potentiometric surface of the aquifer (Walton, 1970). It has been shown by Jacob (1941) and Cooper (1966) that this storage coefficient is a function of the compressibility of water and the aquifer materials. The storage coefficient of an unconfined aquifer, however, can be represented by the effective porosity of the aquifer, which for coarse-grained granular formations of unconfined aquifers is approximately equal to the specific yield. Specific yield is normally defined as the volume of water released from storage in an unconfined aquifer per unit surface area of the aquifer per unit decline of the water-table or in terms of effective porosity, it can be referred to as the volume of water a fully saturated soil releases under gravity force per total volume of the soil.

Boundary Conditions

Equation [20] describes the flow of a compressible fluid in an aquifer; however, before useful solutions to the equation can be obtained, boundary values must be specified. To design a relatively general and useful mathematical model for nonhomogeneous aquifers, it is necessary to obtain a method of solution which is capable of taking into account the effect of recharge boundaries, major groundwater divides, geologic boundaries, and the boundary conditions along the piezometric surface or water table.

A solution to equation [20] is obtained when the head or the derivative of the head is specified. In this model impermeable barriers are simulated by specifying the hydraulic conductivity beyond the aquifer boundary to be zero. This provides the condition of zero flow across the boundary and results in a constant hydraulic head beyond the boundary. The hydraulic head in the aquifer adjacent to the boundary, however, is permitted to vary. A constant head boundary is considered when a river or lake is incorporated in the model. At this boundary the head is defined at each point in time.

Computational Procedures

In order to obtain a numerical solution for the head distribution in an unconfined aquifer, a nodal array similar to that shown in Fig. 18 is superimposed on a plan view of the groundwater reservoir. Then, the equation of flow, as described by equation [21], must be written for each node in the network. Thus, if n_c is the number of nodes in a column of the matrix and n_r is the number of nodes in a row of a matrix, the procedure generates $N = n_c \times n_r$ simultaneous equations, which may be solved using several mathematical procedures. For the problem considered here, the iterative alternating-direction implicit technique developed by Pinder (1969) was used. In this technique the hydraulic head values are calculated for the entire matrix by solving alternately the equations for rows and columns implicitly. The entire matrix is, therefore, solved twice during one complete cycle of 2N calculations. Equation [21] for the calculation of rows is;

$$T'_{xx\,(i-\frac{1}{2},j)}\left(\frac{h^{n}_{(i-1,j,k)} - h^{n}_{(i,j,k)}}{\Delta x_i}\right)$$

$$+\ T'_{xx\,(i+\frac{1}{2},j)}\left(\frac{h^{n}_{(i+1,j,k)} - h^{n}_{(i,j,k)}}{\Delta x_i}\right)$$

$$+\ T'_{yy\,(i,j-\frac{1}{2})}\left(\frac{h^{n-1}_{(i,j-1,k)} - h^{n-1}_{(i,j,k)}}{\Delta y_j}\right)$$

$$+\ T'_{yy\,(i,j+\frac{1}{2})}\left(\frac{h^{n-1}_{(i,j+1,k)} - h^{n-1}_{(i,j,k)}}{\Delta y_j}\right)$$

$$=\ S\left(\frac{h^{n}_{(i,j,k)} - h_{(i,j,k-1)}}{\Delta t}\right) + \frac{Q_{(i,j)}}{\Delta x_i \Delta y_j}$$

$$-\ K_s\left(\frac{2H_{(i,j)} - h^{n}_{(i,j,k)} - h_{(i,j,k-1)}}{2b_{s\,(i,j)}}\right)$$

$$+\ I\left(h^{n}_{(i,j,k)} - h^{n-1}_{(i,j,k)}\right) \qquad [23]$$

In this equation, n is the index indicating iteration number and I is a normalized iteration parameter. The equation for the calculation of columns is of the same form as described for the calculation of the rows.

Pinder (1969) developed a computer program to calculate the hydraulic head values for each node in the finite difference mesh after Δt seconds of pumping by solving equation [21] implicitly. The program computes alternately equation [23] for the calculation of the rows and a similar equation for the calculation of the columns at each time step until the greatest hydraulic head difference between a row and a column computation of any node is less than a prescribed error criterion. When the desired closure is achieved, the program proceeds to calculate the head values for the next time step. The procedure is continued until the prescribed duration of pumping has been simulated. The program uses the hydraulic head values calculated at each time step as the initial conditions for the calculation of the hydraulic head at the new time step.

Since the discharge from a well is assumed to occur over the area of the node allocated to the well, the hydraulic head values calculated within the well node do not accurately describe the head distribution in this zone. However, the accuracy of the solution near pumping wells can be improved by increasing the number of nodes in the pumping areas.

The computer program along with the detailed description of the parameters and the arrangement of data sets is presented in Appendix B.

CHAPTER VI

DIGITAL MODEL STUDIES OF AQUIFER RESPONSE AND YIELD CAPABILITIES

Objectives

The main objective of this part of the investigation is to obtain an understanding of the response behaviour and general yield capabilities of the Broomhill Experimental Aquifer under natural conditions, initial and boundary, when subjected to high pumping rates at numerous pumping centers. This was accomplished by using the digital modelling procedures described by Pinder (1969) and quantitative descriptions of the aquifer properties derived from geologic and hydrologic data.

To develop the model it was necessary to prepare quantitative approximations of peripheral boundary conditions, effective porosity, saturated hydraulic conductivity, recharge mechanisms and rates, and natural water-table configurations. It was also necessary to specify appropriate pumping schemes and evaluate effects of various grid dimensions. These aspects of the investigation and the results of operating the model are discussed in the following pages.

Hydrologic Boundary Conditions

Prior to development of the model it was necessary to define the hydrologic boundary conditions which would be incorporated into the model. Since the prime objective was to simulate the aquifer behaviour under natural conditions, the natural water-table configuration, as shown in Fig. 16, was used to determine the hydrologic boundaries of the aquifer system. The hydrologic boundaries as shown in the same figure, are described below.

(i) *Seepage inflow boundary*; which consists of two separate segments, an equipotential line and a somewhat arbitrary boundary terminating the aquifer to the west. Under natural conditions water enters the aquifer by lateral flow across this boundary.

(ii) *Seepage outflow boundary*; which corresponds to an equipotential line bounding the aquifer to the east. The equipotential line was chosen so that it coincides approximately with the transition zone between the coarse-grained outwash deposits of the aquifer and the finer-grained lacustrine sediments to the east. Under natural conditions water flows from the aquifer across this boundary.

(iii) *Groundwater divide boundary*; which corresponds to a major flow line. This boundary defines the southern periphery of the aquifer flow system. Coarse-grained outwash sediments extend south of this flow line, however, beyond this boundary groundwater is part of a different flow system.

(iv) *Till-aquifer boundary*; which corresponds to a flow line, but also represents the geologic boundary between coarse-grained aquifer deposits and the relatively impermeable clay-loam till plain to the north. Hydraulic conductivity of clay-loam till appears to be generally 2 or more orders of magnitude lower than the coarse-grained glacial deposits (Cherry *et al.*, 1970). The clay-loam till, therefore, is regarded as forming impermeable barrier under both natural and pumping conditions. Similarly, the bottom of the aquifer, which also consists primarily of the clay-loam till, is regarded as an impermeable boundary.

(v) *Water-table control boundary*; which consists of stream flow and marshy terrain, and defines a segment of the aquifer in which the water-table elevation is generally controlled by the surface water regime. Stony Creek generally controls the water-table levels in the aquifer adjacent to its channel. During periods of high stream flow, water seeps from the stream bed into the aquifer. During the low-flow periods water enters the stream channel from the aquifer. Seasonal variation of the water table adjacent to stream appears to be generally limited to less than 2 and 3 feet. To the south of Broomhill the water table is generally close to or above the ground surface and forms marshy terrain or sloughs. Water enters the marshy terrain from the adjacent areas where the water table elevation is relatively high. As a result the water-table levels in these segments of the aquifer remain relatively constant.

Model Boundary Conditions

To formulate the digital model the aquifer boundary conditions were described in terms of mathematical properties which approximate the hydrologic conditions. The digital model boundaries are shown in Fig. 19, and consist of the following segments:

(i) *Zero hydraulic conductivity boundaries*; which are imaginary impermeable (zero-flux) barriers.

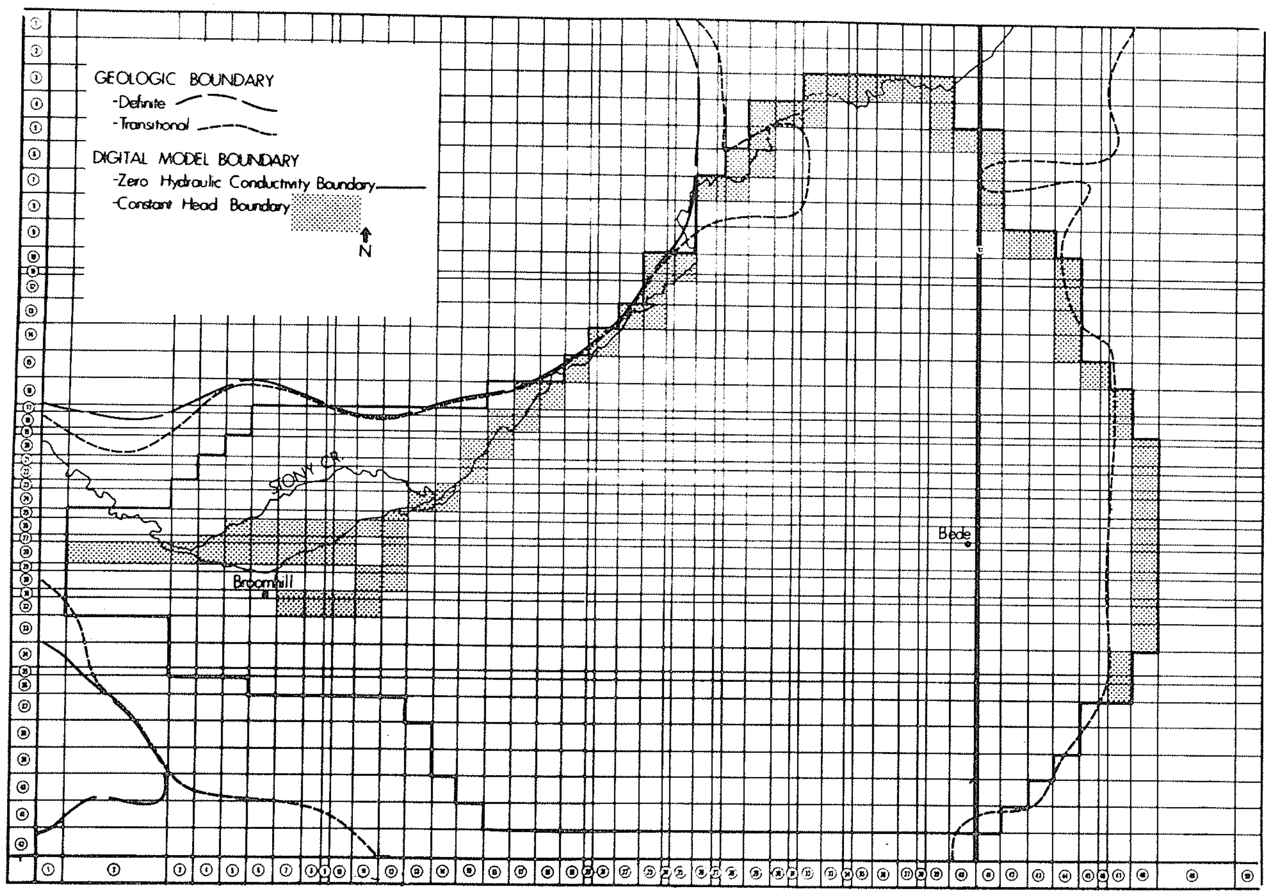

Fig. 19. Finite-difference grid and boundary conditions used in digital simulations of the aquifer system.

The hydraulic conductivities beyond the boundaries are zero. These boundaries are used physically to approximate the field conditions and to theoretically satisfy the mathematical solution which requires that the gradient across the aquifer boundary be zero. The zero hydraulic conductivity boundaries were used in the digital model of the Broomhill aquifer to approximate two types of hydrologic boundaries, the groundwater divide and the till-aquifer boundary.

(ii) *Constant head boundaries*; which represent zones with relatively constant water-table elevations. Water can move into or out of the aquifer across these boundaries. At the beginning of a simulation period two hydraulic head values are assigned to each node located at these boundaries, one representing the initial aquifer head and the other representing an imaginary constant head separated from the aquifer by an imaginary unit of a porous medium which is assigned a value of hydraulic conductivity. Flow of water will occur between the two head values through the porous medium, if the head values are different. The aquifer head is a function of time and is calculated for each time step and compared with the constant head value assigned to the node. If deviation occurs, inflow or outflow into or from each node of the aquifer is simulated using the third term in the right hand side of equation [21], in which the flux is proportional to the magnitude of the deviation. This mechanism maintains the water table in the aquifer at a relatively constant value at and near the constant head boundaries. Constant head boundaries were incorporated into the model to simulate several different hydraulic conditions of the aquifer system. These are summarized below:

(a) *Water-table control by stream*; as indicated on page 86 Stony Creek affects the water-table levels in the aquifer adjacent to its channel. This effect was incorporated into the aquifer model to allow vertical inflow and/or outflow along the stream channel at any node during the simulation periods. At any node where hydraulic head in the stream is higher than the aquifer, water seeps from the stream into the aquifer. At any node where the hydraulic head in the aquifer is higher than the stream, water enters the stream from the aquifer. Since the natural water-table configuration simulated by the digital model corresponded to periods of relatively low stream flow, the main objective of the arrangement considered here was to provide natural groundwater seepage to the stream. This condition would be reversed, however, when the water table drops below the hydraulic head in the stream due to pumping.

(b) *Water-table control by marshy terrain*; as indicated on page 86 the water table in marshy segments of the aquifer is at or above the land surface. These areas behave in a manner somewhat similar to the stream channels, in that the water table is relatively constant. In the digital simulation in which the natural water table was used as an initial condition, water enters the marsh segments from adjacent aquifer areas with high hydraulic head values. Under natural conditions water entering the marshes evaporates. In the digital simulation water leaves the aquifer through the constant head boundaries.

(c) *Water-table control by subsurface outflow;* which corresponds to the seepage outflow boundary and was incorporated into the model to simulate subsurface outflow from the aquifer under natural conditions.

Water entering the aquifer across the subsurface inflow boundary was not incorporated into the aquifer simulations. This boundary was approximated into the aquifer model by an imaginary zero hydraulic conductivity boundary.

Modelling of Internal Aquifer Characteristics

Prior to operating the model to simulate the effects of aquifer pumping shemes it was necessary to adjust the grid spacing at and near the proposed pumping centers, to improve the accuracy of the model. The adjustment was made by comparing time-drawdown and distance-drawdown graphs obtained from the model simulating a homogeneous, isotropic, semi-infinite aquifer with results obtained from the Theis equation. Using this result a rectangular grid composed of 42 rows and 50 columns (Fig. 19) was made to accommodate the pumping centers and aquifer boundaries, with the distance in the x and y directions indicated in Table XIII. A

Table XIII

Distance between nodes, Δx_i and Δy_j, used in the digital model of the aquifer system.

Grid Spacing in x Direction (feet)

$\Delta x_i \quad i = 1, 2, 3, \ldots 50$

(1)*	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1056.00	4224.00	1056.00	1056.00	1056.00	1056.00	1056.00	981.00	150.00	981.00
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
1056.00	1056.00	1056.00	1056.00	1056.00	1056.00	1056.00	1056.00	981.00	150.00
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
981.00	1056.00	981.00	150.00	981.00	981.00	150.00	981.00	1056.00	981.00
(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
150.00	981.00	981.00	150.00	981.00	1056.00	981.00	150.00	981.00	1056.00
(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)
1056.00	1056.00	1056.00	1056.00	981.00	150.00	981.00	1056.00	3168.00	1056.00

Grid Spacing in y Direction (feet)

$\Delta y_j \quad j = 1, 2, 3, \ldots 42$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1056.00	1056.00	1056.00	1056.00	1056.00	1056.00	1056.00	1056.00	1056.00	981.00
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
150.00	981.00	1056.00	1056.00	1056.00	981.00	150.00	906.00	150.00	906.00
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
150.00	906.00	150.00	906.00	150.00	906.00	150.00	906.00	150.00	906.00
(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
150.00	981.00	1056.00	981.00	150.00	981.00	1056.00	1056.00	1056.00	1056.00
(41)	(42)								
1056.00	1056.00								

* Numbers are indices in x and y directions

theoretical form of such a grid is shown in Fig. 20.

The internal characteristics of the aquifer were then modelled by superimposing the finite-difference grid (Fig. 19) over the aquifer area and assigning each node representative values of saturated hydraulic conductivity, effective porosity, aquifer base elevation, and initial hydraulic head (Figs. 7, 8, 10, and 16). Inflow or outflow across the constant head boundaries was permitted through imaginary one-foot thick beds having the same transmissibility as the aquifer in the boundary areas.

Operation of the Model

The aquifer model was subjected to pumping conditions for 8.55×10^6 seconds or, approximately 99 days. This pumping period was chosen because it approximates the three-month irrigation period which would be required for irrigation farming in the Souris River basin (Manitoba Water Control and Conservation Branch, 1968). Pumping centers were located at 26 locations, selected by combining the hydrologic and geologic data obtained from the saturated hydraulic conductivity, effective porosity, and saturated thickness maps (Figs. 7, 8, and 17). The discharge rates were adjusted so that the drawdown at the end of 99 days for each pumping center was about seven feet above the base of the aquifer. This drawdown, for example, would simulate the effect of wells with five-foot screens installed at the

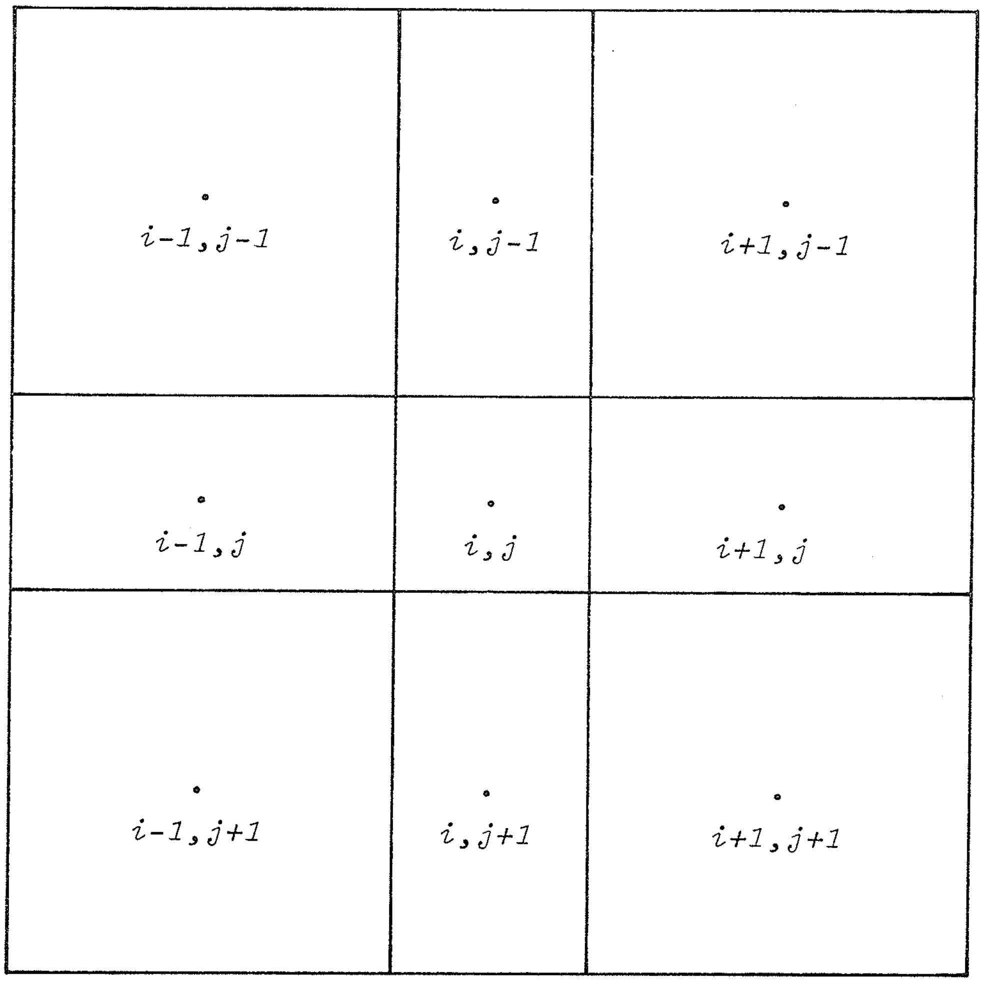

Fig. 20. A 3 x 3 nodal array for the digital model.

base of the aquifer and would allow for two feet of well loss. However, drawdown of this magnitude could be obtained by many other types of well installations. All pumping rates were greater than or equal to 75 gpm. Pumping occurred simultaneously at all pumping centers. The discharge rates along with nodal-locations of pumping centers are indicated in Table XIV.

An initial time step of 10 seconds was introduced at the beginning of simulation and increased logarithmically in the program according to the equation

$$\Delta t_k = \Delta t_{(k-1)} + c(\Delta t_{(k-1)}) \qquad [24]$$

where the coefficient, c, depends upon the dynamics of the system. More precise solutions can be obtained using smaller values of c. Values ranging between 0.25 and 1.0 have provided satisfactory results in most cases. In this investigation the final results were obtained using a c value of 0.25, which was sufficiently small to obtain relatively precise solutions.

A mass balance of the aquifer system was obtained at each time step using a subroutine program which computed the volume of water removed from the aquifer by integration over the cone of depression and compared it to the total discharge from the pumping centers.

The aquifer model was initially formulated to simulate response of the aquifer to multiple pumping centers

Table XIV

Discharge rates and nodal-locations of the pumping centers

Center No.	Node (i,j)*	Discharge Rate CFS	GPM
1	(11,31)	0.334	150.0
2	(11,38)	0.167	75.0
3	(17,20)	0.445	199.8
4	(17,24)	0.223	100.1
5	(19,20)	0.334	150.0
6	(19,24)	1.559	700.0
7	(19,27)	1.559	700.0
8	(21,20)	0.334	150.0
9	(21,27)	0.167	75.0
10	(21,38)	0.390	175.1
11	(23,20)	0.278	124.8
12	(23,34)	0.167	75.0
13	(23,38)	0.612	274.8
14	(25,34)	0.445	199.8
15	(25.38)	0.835	374.9
16	(27,27)	0.278	124.8
17	(27,34)	0.223	100.1
18	(27,38)	0.835	374.9
19	(29,24)	0.167	75.0
20	(29,27)	0.557	250.1
21	(29,34)	0.278	124.8
22	(29,38)	0.334	150.0
23	(31,24)	0.167	75.0
24	(31,27)	0.668	299.9
25	(31,34)	0.223	100.1
26	(31,38)	0.167	75.0
Total		11.746	5274.0

* i and j, row and column number respectively

when commencing with an initial water-table configuration approximating natural conditions. The model was then used to simulate the effect of an initially horizontal water table. The purpose of this phase of the study was to determine whether the natural water-table configuration significantly influenced the response and yield capabilities and whether the introduction of this complexity was necessary. To model the aquifer with an initial horizontal water-table configuration the base elevation of the aquifer was adjusted at each node to maintain the initial saturated thickness. The adjustment was made according to the expression

$$B_{(i,j)} = B'_{(i,j)} - (h'_{(i,j)} - h) \qquad [25]$$

where $B_{(i,j)}$ and $B'_{(i,j)}$ are the adjusted and initial elevation of the impermeable base respectively (L), $h'_{(i,j)}$ the initial hydraulic head (L), and h is the arbitrarily chosen elevation of the horizontal water table referred to the same datum as $B'_{(i,j)}$ and $h'_{(i,j)}$.

The saturated thickness, b_o, remains unchanged in both models, that is:

$$\begin{aligned} b_o &= h'_{(i,j)} - B'_{(i,j)} \\ &= h - B_{(i,j)} \end{aligned} \qquad [26]$$

The only modification, therefore, consisted of replacing $B'_{(i,j)}$ and $h'_{(i,j)}$ with $B_{(i,j)}$ and h respectively, otherwise the two water-table simulations were similar in all other aspects.

To test the validity of the natural water-table configuration obtained from field data and approximated in the model, the aquifer flow system (two-dimensional basin) was simulated for 99 days with no artificial pumping centers being operated. Only a small adjustment of the initial water-table data was necessary to obtain a reasonable simulation of natural water-table behaviour during the summer period.

To evaluate the aquifer response to pumping stress under no-recharge boundary conditions, the aquifer model with a horizontal water-table configuration was further simplified by excluding the constant head boundaries.

CHAPTER VII

ANALYSIS OF PUMPING TEST RESULTS

USING THE DIGITAL MODEL

This phase of the investigation is directed towards analysis of a pumping test conducted in a portion of the Broomhill Experimental Aquifer by the Manitoba Water Control and Conservation Branch in 1966 (Fig. 21). The purpose of the analysis is to obtain an independent estimate of the hydraulic conductivity and effective porosity of a segment of the aquifer from field drawdown and pumping rate data and to obtain a better understanding of the yield mechanisms of the aquifer during short pumping periods.

The pumping test was conducted in an area underlain by a narrow channel surrounded by a relatively thin aquifer segment (Fig. 22). The channel is approximately 2,500 feet long and up to 300 feet wide and has a saturated aquifer thickness of more than 20 feet. A well, located near the center of the channel, was pumped at a rate of 0.67 cfs (300 gpm) for 24 hours. The drawdown of the water table was measured in four observations wells located at distances of 47, 104, 149, and 350 feet from the pumping well (Fig. 22). The test was terminated when the water level in the pumping

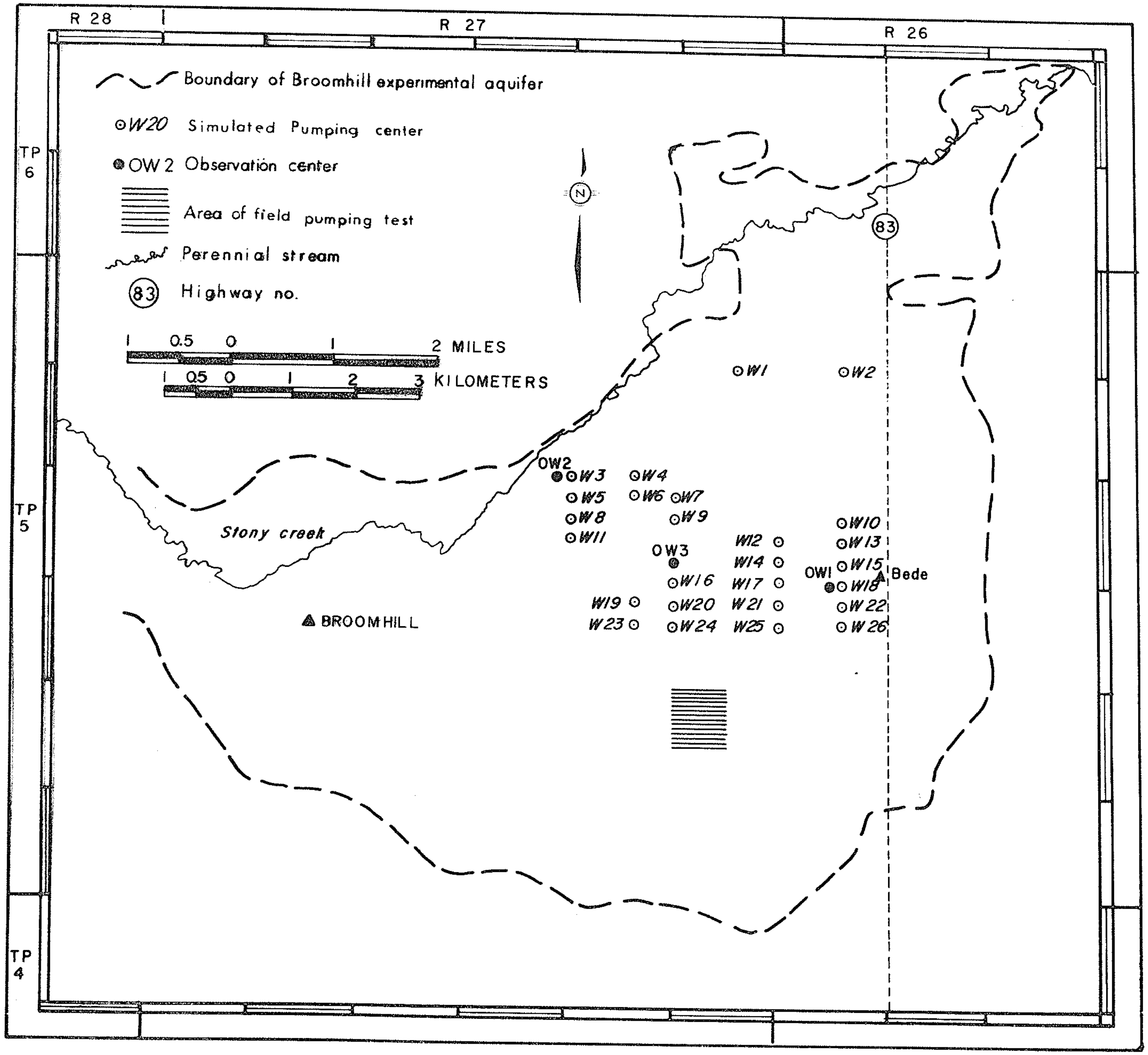

Fig. 21. Location of simulated pumping centers and area of field pumping test (time-drawdown responses at observation centers OW 1, 2, and 3, are shown in Fig. 27).

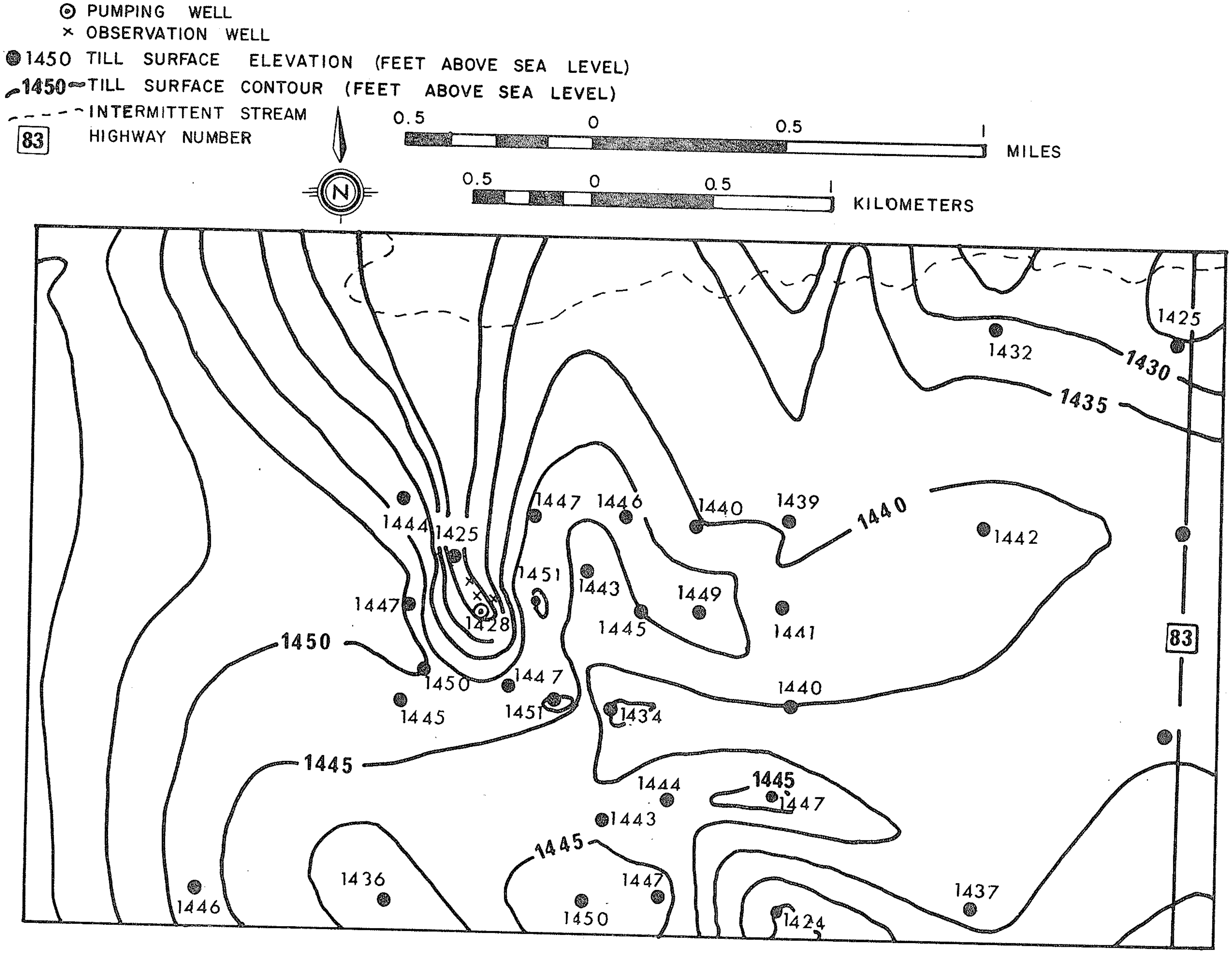

Fig. 22. Location of pumping and observation wells used in the aquifer pumping test.

well had approximately attained equilibrium, although the observation wells were still declining. Because the pumping well and observation wells were located in a narrow valley, effects of irregular lateral discontinuities rapidly influenced the drawdown response. It was, therefore, impossible to obtain a reasonable estimate of hydraulic conductivity and effective porosity from the time-drawdown data using conventional analytical methods such as those outlined by Walton (1970). To overcome this difficulty the digital model was applied in the pumping test analysis by superimposing a 50 x 50 rectangular grid over the portion of the aquifer affected by the test. The grid spacing was decreased in the area occupied by the pumping well and observation wells to provide more detailed results. The grid spacing is indicated in Table XV. The water table was assumed to be horizontal with an elevation of 53 feet above an arbitrary datum. The elevation of the impermeable base (Fig. 22) was referred to the same datum and was recorded at each node.

The model was initially used to study the time-drawdown response of the aquifer using values of effective porosity which were maintained constant with time. It was found that the field response of the aquifer could not be simulated accurately on this basis. Model simulations were then conducted using time-dependent porosity function, summarized as follows. The effect of variable storage

Table XV

Distance between nodes Δx_i and Δy_j, used in simulations of the pumping test.

Grid Spacing in x direction (feet)

$\Delta x_i \quad i = 1, 2, 3, \ldots 50$

(1)*	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
5280.00	5280.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	100.00
(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)
100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	5280.00	5280.00

Grid Spacing in y direction (feet)

$\Delta y_j \quad j = 1, 2, 3, \ldots 50$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
5280.00	5280.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	100.00
(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)
100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	5280.00	5280.00

* Numbers are indices in x and y directions

functions is often referred to as "delayed yield" to evaluate the response of unconfined aquifers.

Case 1: Effective porosity as a step-function of time.

An initial value of effective porosity was used at the beginning of the pumping period and increased in a step-by-step manner to a maximum value after a given period of time. During each step the effective porosity was constant.

Case 2: Effective porosity as a linear-function of time.

A prescribed value of effective porosity was introduced at the beginning of the pumping period and increased linearly to a maximum value for a given period of time, t', according to the expression

$$\phi_e = \phi_e' + (\phi_e'' - \phi_e')\left(\frac{t}{t'}\right) \qquad [27]$$

where ϕ_e is the effective porosity at time t, ϕ_e' represents the initial value of effective porosity, ϕ_e'' represents the value of porosity at time t', and t indicates the time since pumping started. The computer program was arranged so that

$$\phi_e = \phi_e'' \qquad \text{for } t \geq t'$$

Case 3: Effective porosity as an exponential function of time.

This case is similar to case 2, except that effective porosity is increased exponentially according to the expression

$$\phi_e = \phi_e' + (\phi_e'' - \phi_e')(1 - e^{-u}) \qquad [28]$$

where $u = \frac{Gt}{t'}$, and G is an empirical constant chosen so that

$$\phi_e = \phi_e'' \qquad \text{for } t \geq t'$$

The time dependency of effective porosity was introduced to simulate non-instantaneous yield of water from the unsaturated zone above the water table as the water table declines due to pumping. In other words, it recognizes the fact that water does not drain out of the pore spaces instantaneously in response to a variable hydraulic head regime. The effective porosity, as given by equation [27] and equation [28] has two components; the first term determines the instantaneous release of water and is independent of time, while the second term allows for non-instantaneous water yield from storage which is assumed to vary with time over the entire aquifer system. These methods are a rough approximation to the analytical solution presented by Boulton (1963, 1955), who incorporated time-dependent storage functions in the analysis of confined and unconfined aquifers.

CHAPTER VIII

RESULTS AND DISCUSSION

Effect of Multiple Pumping Centers

Results

As indicated in Chapter VI the digital model was used to simulate the following aquifer conditions:

Natural water-table conditions; simulation of the aquifer response under natural water-table configuration with no pumping centers. Constant head boundary conditions were used to represent the stream and marsh areas and the subsurface outflow periphery of the aquifer.

Multiple pumping center condition, using the following simulations:

Simulation (i):
At the beginning of the pumping period, the water-table configuration represented natural field conditions. Constant head boundary conditions were used to represent the stream and marsh areas and the subsurface outflow

periphery.

Simulation (ii):

At the beginning of the pumping period, the water-table configuration was horizontal and base elevation was adjusted accordingly. Other boundary conditions were similar to simulation (*i*).

Simulation (iii):

At the beginning of the pumping period the water-table configuration was horizontal and base elevation was adjusted accordingly. The effects of streams, marshes, and subsurface outflow were eliminated from the simulation by omitting the constant head boundaries.

The aquifer response and yield capabilities obtained from the digital model using the above simulations are described below. Emphasis is placed on determining the factors which affect the response of the aquifer during discharge from multiple pumping centers. A preliminary estimate of the yield capabilities of the entire aquifer is presented.

Contoured maps of water-table drawdown after 99 days of continuous pumping during simulations (*i*) to (*iii*) are shown in Figs. 23 to 25. The maps were prepared using a

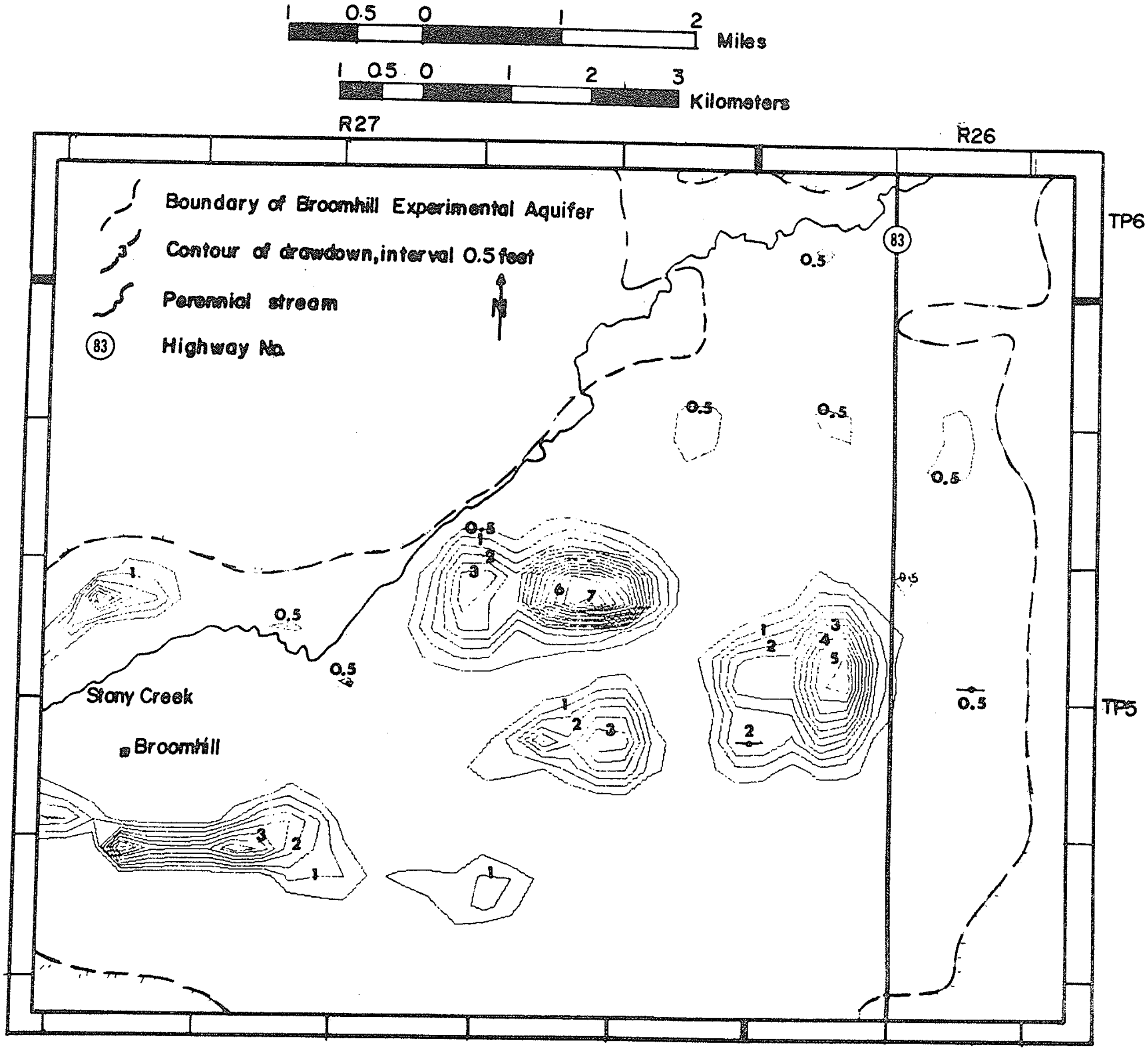

Fig. 23. Computer plot of digitally-simulated water-table response to 26 pumping centers; *natural water table used as initial condition.*

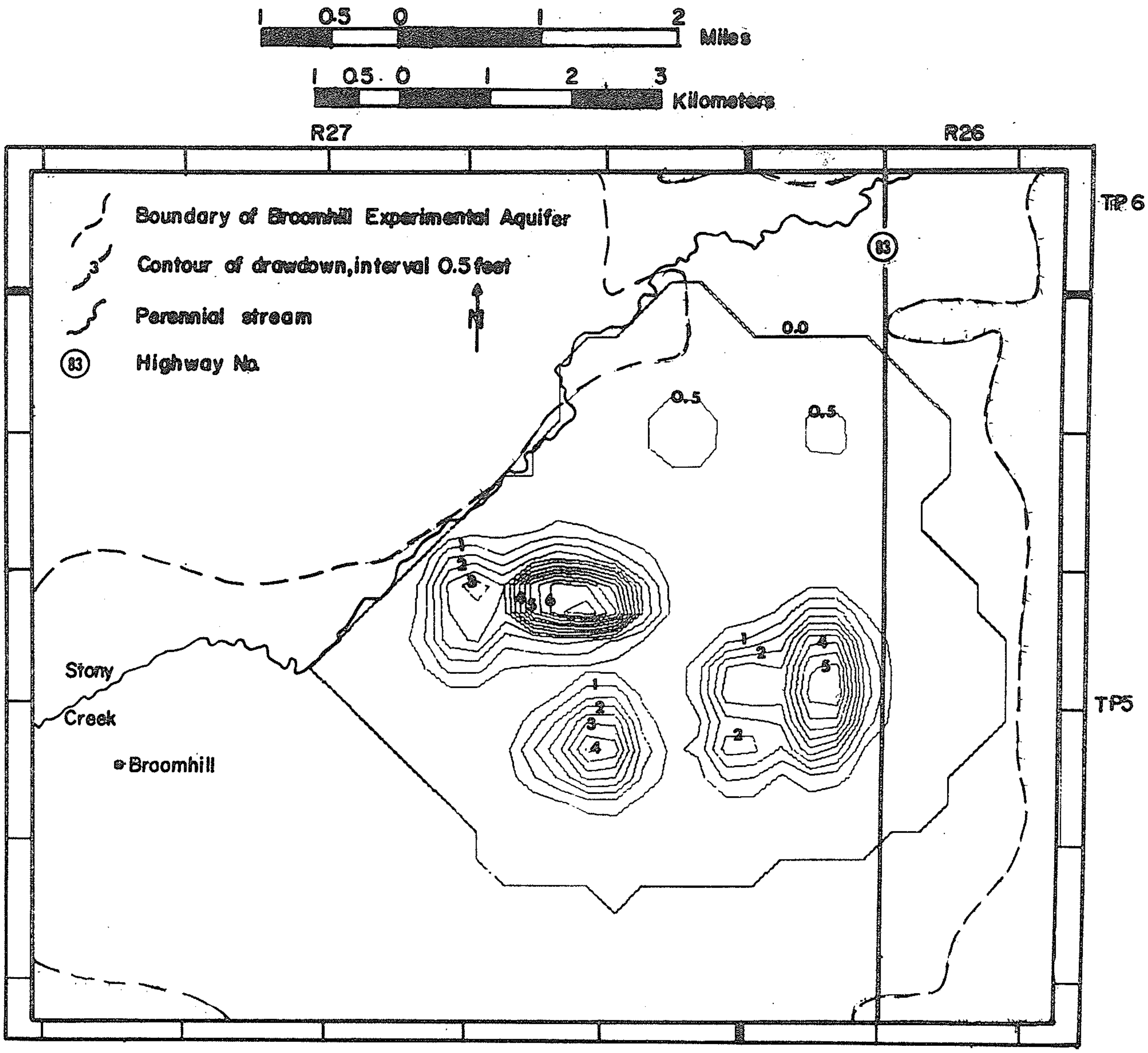

Fig. 24. Computer plot of digitally-simulated water-table response to 26 pumping centers; *horizontal water table used as initial condition, constant head boundaries included.*

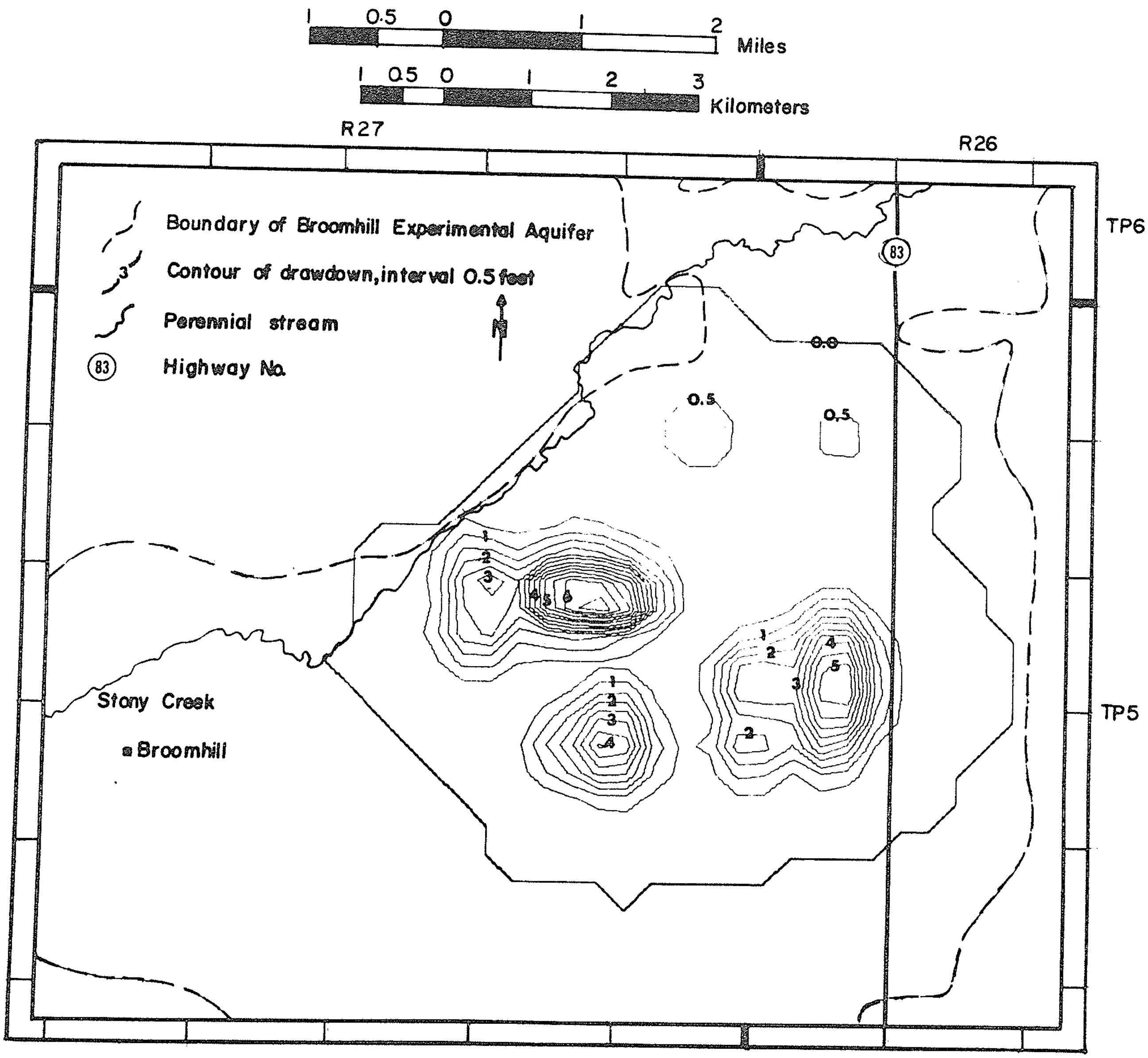

Fig. 25. Computer plot of digitally-simulated water-table response to 26 pumping centers; *horizontal water table used as initial condition, constant head boundaries excluded.*

computer plotting program on a Calcomp pen plotter. The drawdown resulting from simulation (*iii*) is also shown in alphanumeric form in Fig. 26, as an example of the output form that is normally used. The time-drawdown response at three arbitrarily located observation centers are shown in Fig. 27. Drawdowns at the pumping centers after 99 days are listed in Table XVI. The changes in total aquifer storage volume after 99 days of pumping as computed from the water-table decline were 125.5, 99.7 and 100.5 million cubic feet for the first, second, and third simulations respectively. The total yield from the pumping centers during the 99-day period was 100.5 million cubic feet. The volume of water discharged from the aquifer system to the stream and the other constant head boundaries totalled 25.0 million cubic feet using the first simulation. Using the second simulation 0.8 million cubic feet of water entered the aquifer system. For the third simulation the volumes balanced because the aquifer was defined as a closed system.

Discussion

The simulation of the field response of the aquifer system for 99 days with no artificial pumping operations resulted in a natural water-table behaviour reasonably similar to what normally occurs during the summer season. This result supports the interpretation of the natural water-table configuration as based on field data. Compared

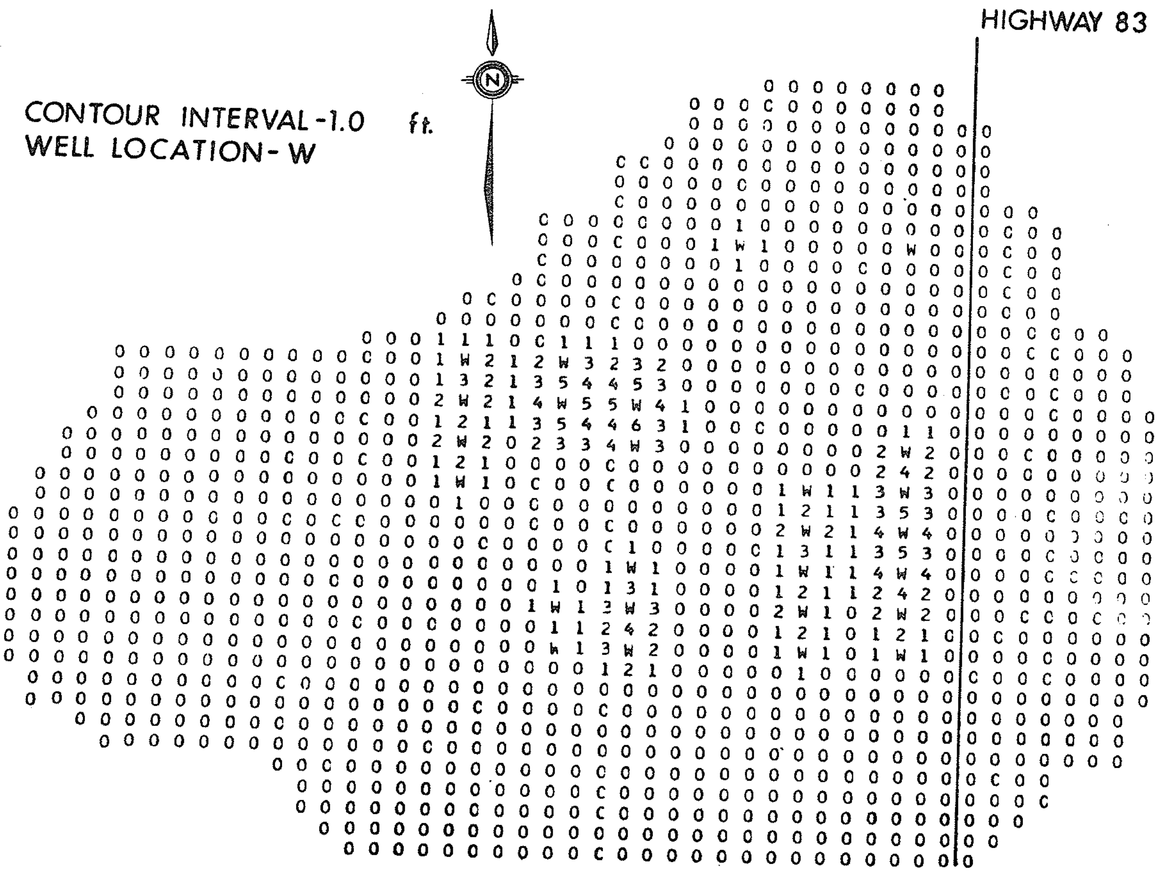

Fig. 26. Alphanumeric form of computer output for simulation *(iii)* after 63.4 days of pumping.

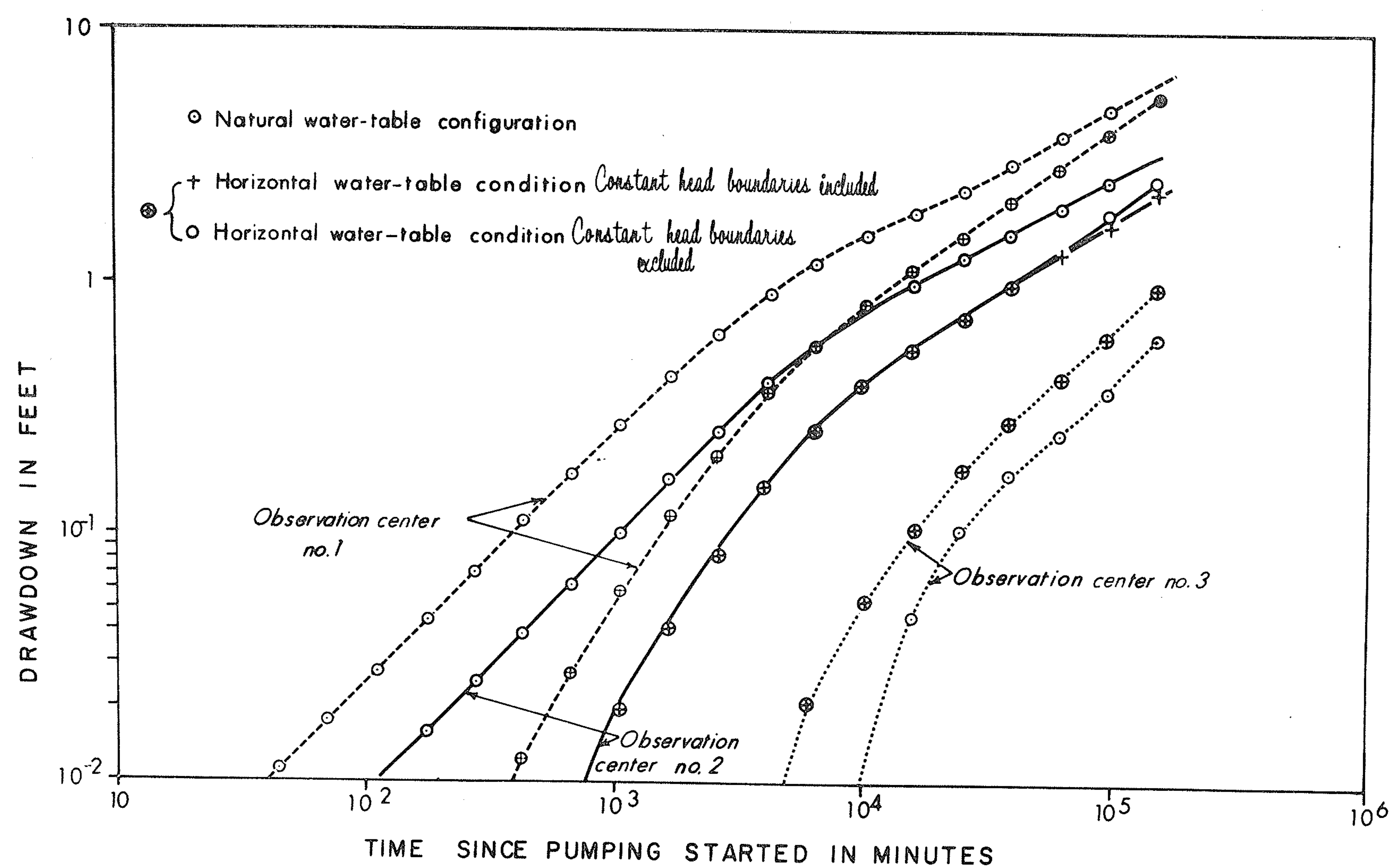

Fig. 27. Time-drawdown response of the water table at observation centers OW 1, 2, and 3, during simulations *(i)*, *(ii)*, and *(iii)*.

Table XVI

Drawdown at the pumping centers determined from the digital model of the entire aquifer system after 99 days of pumping

Center No.	Node (i,j)*	First Simulation	Second Simulation	Third Simulation
1	(11,31)	9.04	9.33	9.33
2	(11,38)	9.52	9.35	9.35
3	(17,20)	14.40	13.50	13.60
4	(17,24)	10.20	10.60	10.60
5	(19,20)	15.20	13.80	13.80
6	(19,24)	31.20	29.90	29.90
7	(19,27)	31.70	30.50	30.50
8	(21,20)	11.20	10.50	10.50
9	(21,27)	13.30	12.90	12.90
10	(21,38)	14.50	14.60	14.60
11	(23,20)	10.10	9.86	9.86
12	(23,34)	9.52	9.28	9.28
13	(23,38)	19.10	18.50	18.50
14	(25.34)	12.20	12.00	12.00
15	(25.38)	23.30	22.20	22.20
16	(27,27)	13.90	14.70	14.70
17	(27,34)	17.90	17.70	17.70
18	(27,38)	22.90	21.50	21.50
19	(29,24)	8.30	6.59	6.59
20	(29,27)	15.40	16.20	16.20
21	(29,34)	11.10	11.20	11.20
22	(29,38)	12.00	12.10	12.10
23	(31,24)	5.79	6.12	6.12
24	(31,27)	16.40	17.20	17.20
25	(31,34)	8.67	8.79	8.79
26	(31,38)	9.92	10.00	10.00

* i and j, row and column number respectively

to simulations (*ii*) and (*iii*), simulation (*i*), which used the natural water-table configuration at commencement of pumping, resulted in higher drawdowns during the pumping period in segments of the aquifer located near the streams and marshes, particularly south of Broomhill. These results were to be expected because with a sloping water table, water moves from these segments of the aquifer into the adjacent stream channels and marshy terrain. It should be noted, however, that drawdown of the water table in the aquifer area in which most of the pumping centers were located did not vary appreciably between simulations (*i*) and (*ii*). This reflects the fact that the pumping centers were located in the central portion of the aquifer in which the natural hydraulic gradient is very low. It should be noted, however, that digital modelling of the aquifer system under natural water-table conditions is a much more complex operation than the horizontal water-table cases. An overall comparison made between the water-table drawdown obtained from simulations (*i*), (*ii*) and (*iii*) indicated that simplifying the boundary conditions of the model has no major effect on the response and yield capabilities of the aquifer, when the pumping centers are located relatively far from the peripheral boundaries. Using the results of simulation (*ii*) the volume of water withdrawn from the aquifer computed from the decline of the water table after 99 days of pumping was 99 percent of the total yield of the pumping centers. This indicates that the cone of influence caused by the pumping was not influenced significantly by the constant head boundaries. In other words, essentially all of the water withdrawn was obtained from aquifer storage.

In simulations (*i*), (*ii*) and (*iii*), the area of water-table drawdown due to 99-day pumping from the 26 centers accounted for only 56 percent of the aquifer area. The pumping centers were located in areas in which greatest aquifer thickness occurs. These areas generally coincide with portions of the aquifer in which detailed test drilling data are available. The other areas of the aquifer may also have deep segments in which major pumping centers could be located. However, until more detailed drilling data are available it will not be possible to evaluate the yield capabilities of these areas on a definitive basis.

The total yield from the 26 pumping centers was approximately 3.1 inches of water over the 14 square miles of influence. This is equivalent to 2,310 acre-feet. As indicated in Chapter IV, the average annual developmental yield of the aquifer is estimated to be approximately 7 inches. This is equivalent to 5,230 acre-feet for the 14 square miles of influence and suggests that a quantity of water equal to 44 percent of the estimated annual developmental yield would be withdrawn during the 99-day pumping period. However, this volume of water represents only 25 percent of the annual average developmental yield if the entire aquifer is considered. It may be possible to increase the gross developmental yield of the aquifer if lateral recharge is induced by the drawdown cone across the aquifer boundaries. However, it appears that only small quantities of water would be obtained laterally by this mechanism, because much of the aquifer area is encompassed with clay-loam till and lacustrine sand and silt, which generally have hydraulic conductivities which are two orders of magnitude or more smaller than the aquifer conductivities.

Analysis of Pumping Test Data

Data obtained from the pumping test outlined in Chapter VII were analysed using the digital model. The methods of analysis are presented in the same chapter. As mentioned previously, the objective of this phase of the investigation is to simulate the response of the water table as observed in three observation wells during the pumping period in order to obtain a better understanding of the aquifer characteristics. This was accomplished by adjusting the aquifer parameters until reasonable simulations were obtained.

Effect of Hydraulic Conductivity

The time-drawdown data obtained from the pumping test were first analysed by simulating the response of the aquifer system using a uniform hydraulic conductivity of

1.62×10^{-3} feet per second, and an effective porosity of 35 percent. These values were initially estimated by correlating drill hole logs, field sample description and the results of the laboratory analyses on the undisturbed gravel quarry samples. For observation well 3, the simulated drawdown values obtained using the above data were smaller than the field data during the initial phase of pumping and greater than the field drawdown values during the terminal phase of the pumping test. For observation well 1, computed drawdown values were always smaller than field response. These relationships are illustrated in Fig. 28, which shows the actual and simulated drawdown at observation wells 1 and 3 located 350 and 104 feet from the pumping well. The model was reformulated using hydraulic conductivity values which were larger and then smaller than initial estimates. The simulated drawdown values obtained using a uniform hydraulic conductivity of 4×10^{-3} feet per second were smaller than the field response in both observation wells 1 and 3 (Fig. 28).

The results indicated that the smaller hydraulic conductivity values yield larger slopes of the time-drawdown graph. The results also indicate that a reasonable duplication of the time-drawdown graphs cannot be obtained by simply adjusting the hydraulic conductivity uniformly over the aquifer area. Drill logs in the aquifer area indicated that it is unlikely that major spatial variation in conduc-

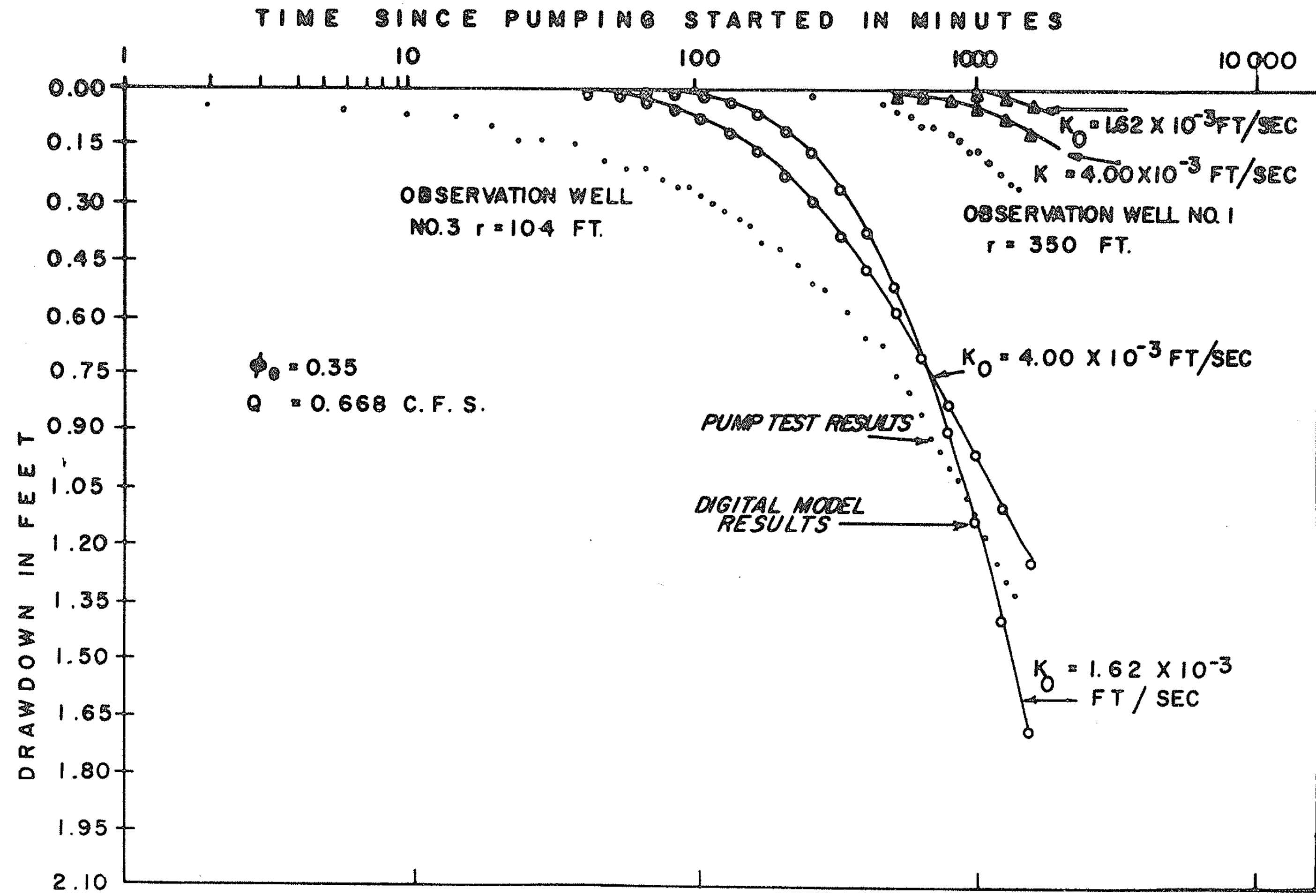

Fig. 28. Simulated effect of variations of assumed hydraulic conductivity on the time-drawdown response of the pumping test segment of the aquifer.

tivity strongly affected the pumping test data. It is concluded, therefore, that the pumping test response must be accounted for in terms of effective porosity or some other aquifer phenomena.

Simulations Using Time-Independent Effective Porosity Distributions

The results of time-drawdown simulations at observation wells 1 and 3 using several values of uniform effective porosity are compared to the field data in Fig. 29. The simulation graphs were obtained using a value of hydraulic conductivity of 4×10^{-3} feet per second in order to commence with simulated time-drawdown graphs which were not totally dissimilar from the field response.

Simulation Using Time-Dependent Effective Porosity Distributions

The time dependency of effective porosity was incorporated in the model to simulate non-instantaneous yield of water from storage.

The computed values of the time-drawdown curves for the three observation wells (Fig. 30) were obtained by increasing the effective porosity in a stepwise manner. Using this technique, it was possible to obtain a reasonable agreement between the digital models and the pumping test results, however, the disadvantages of this method were the lack of smoothness of the time-drawdown curves due to sudden

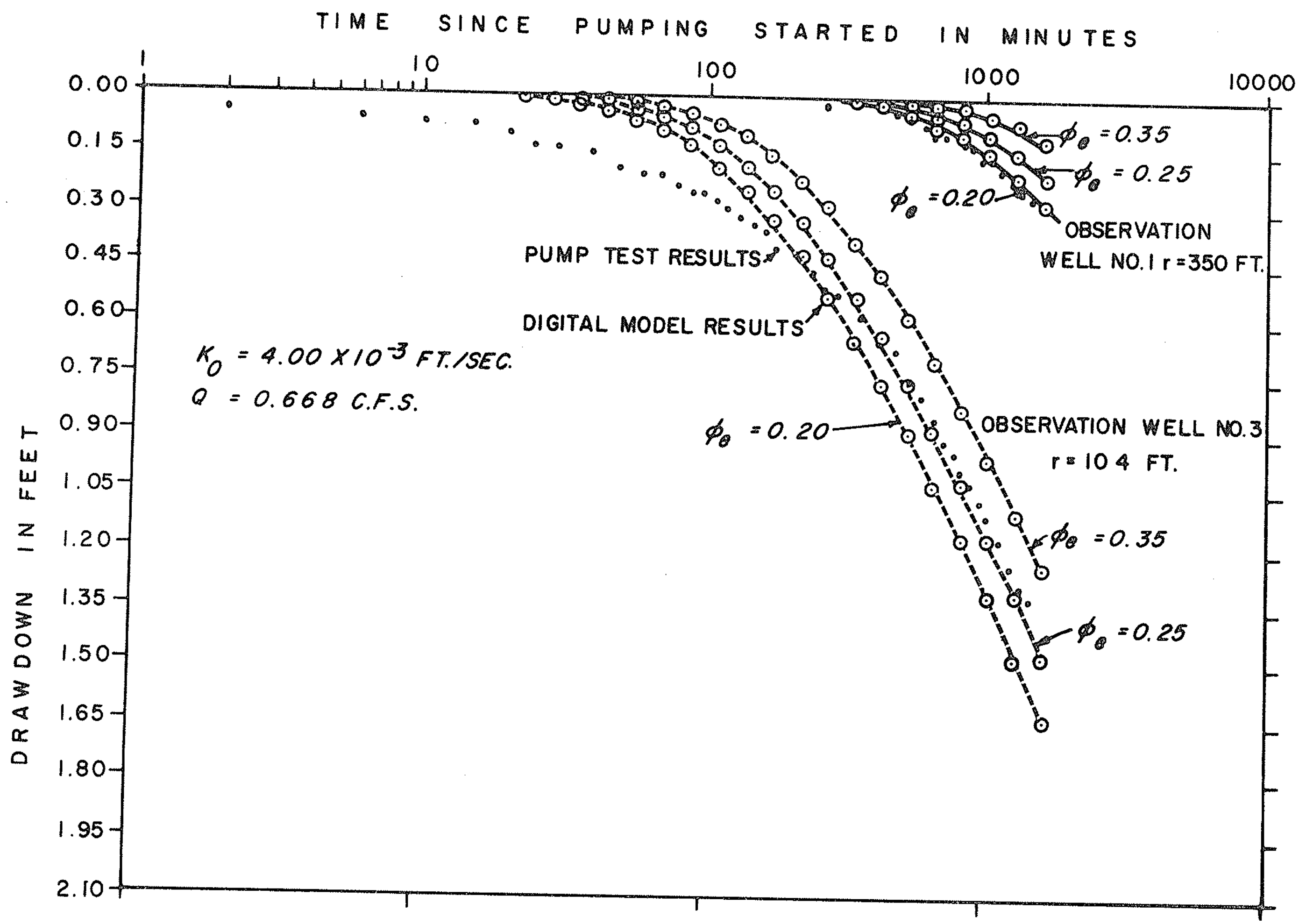

Fig. 29. Simulated effect of variations of effective porosity on the time-drawdown response of the pumping test segment of the aquifer.

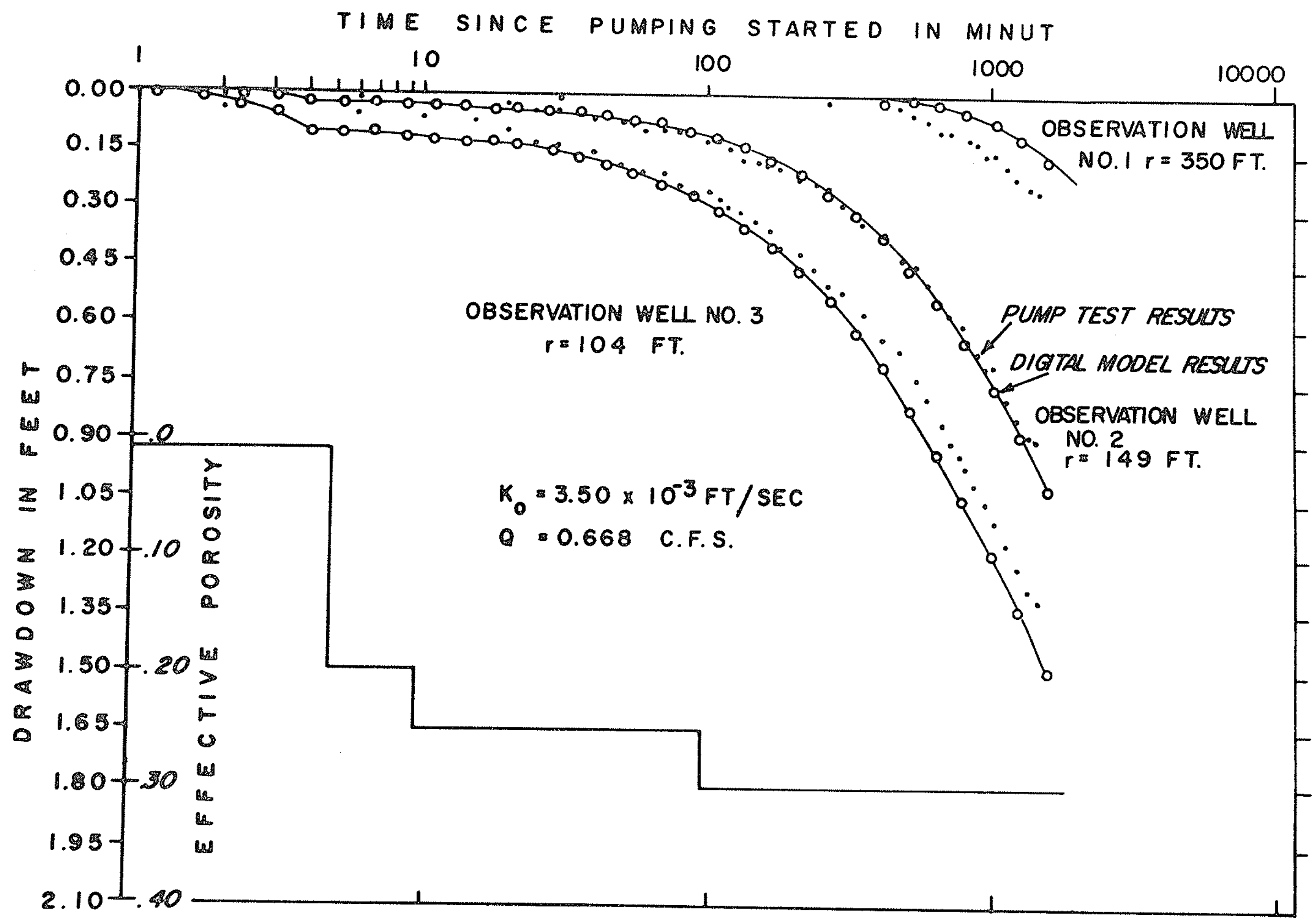

Fig. 30. Simulated effect of time-dependent effective porosity on the time-drawdown response of the pumping test segment of the aquifer; *effective porosity as a step-function of time.*

increases of ϕ_e and that increasing ϕ_e in this manner could not be duplicated in the field.

The digital model results were then compared with the field responses by increasing ϕ_e linearly with time according to equation [27]. The type "a" curves (Fig. 31) were obtained by introducing an initial value of effective porosity (ϕ_e') of 0.10 at the beginning of pumping and increasing the effective porosity linearly to a maximum of 0.35 after 672 minutes; the type "b" curves were obtained using $\phi_e' = 0.14$.

The final comparison was made between digital model and field responses by expressing ϕ_e as an exponential function of time according to equation [28]. The results shown in Fig. 32 were obtained using an effective porosity value of 0.02 at the beginning of the pumping and increased exponentially to a maximum of 0.30 after 538 minutes.

The shape of the time-drawdown curve (Fig. 33) was changed considerably by increasing the effective porosity exponentially rather than linearly. In this figure, the two time-drawdown curves for which ϕ_e is time-dependent were obtained by introducing a ϕ_e value of 0.02 at the beginning of the pumping and increasing to a maximum of 0.30 after 538 minutes. The third curve was obtained by assuming a value of 0.30 throughout the pumping period. If we assume that the irregular boundary conditions did not greatly influence the pump test results, the most reason-

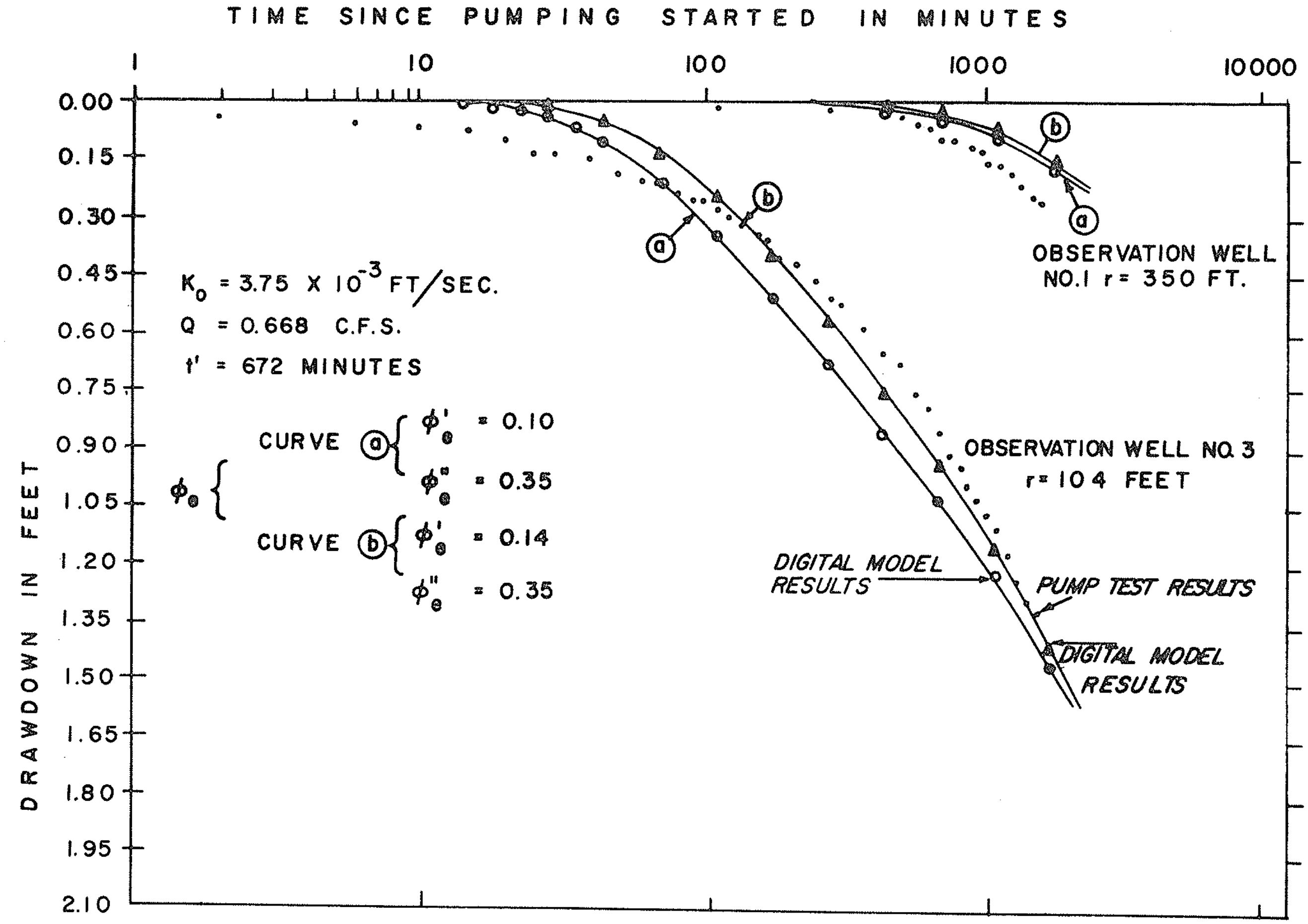

Fig. 31. Simulated effect of time-dependent effective porosity on the time-drawdown response of the pumping test segment of the aquifer; *effective porosity as a linear function of time.*

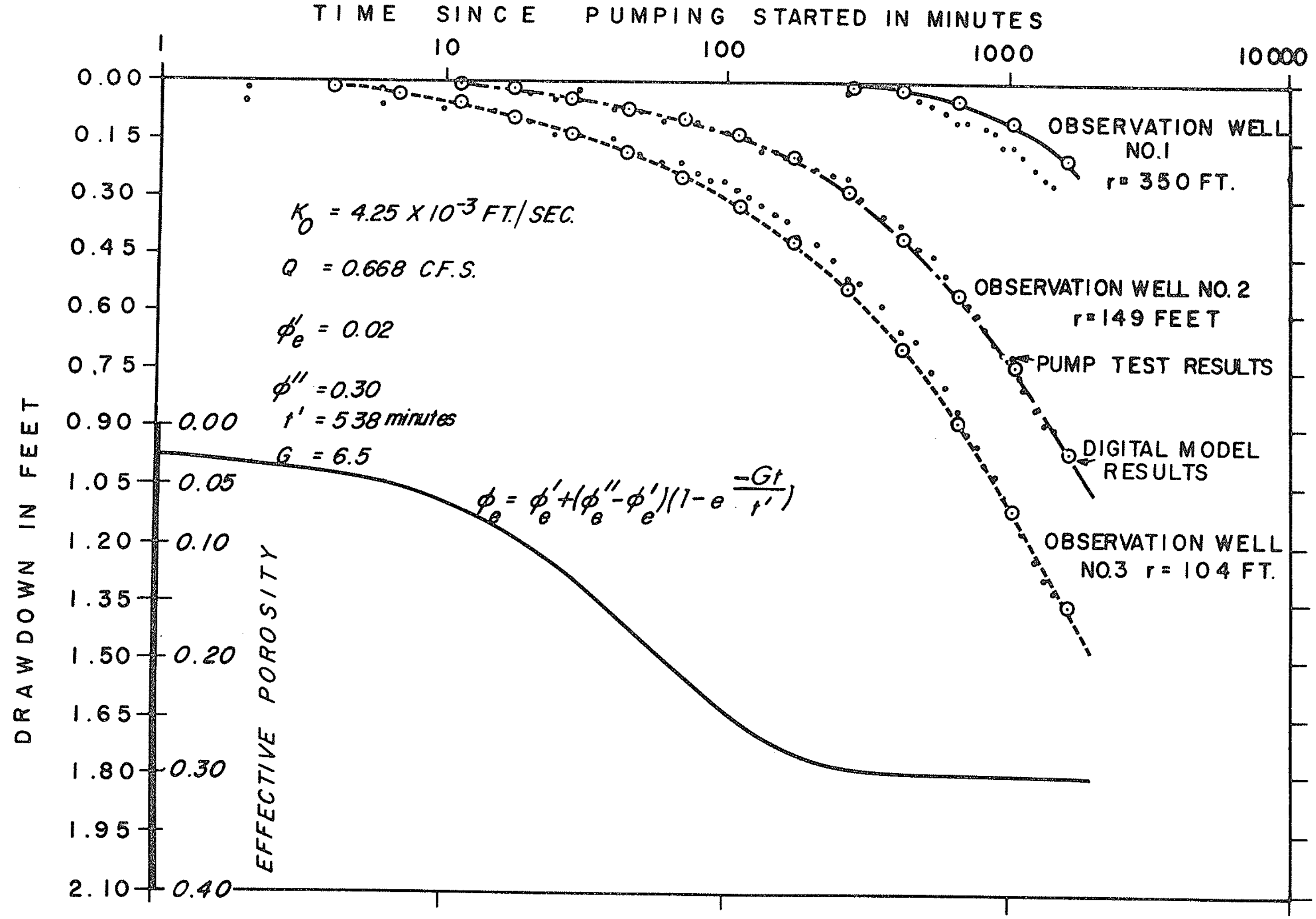

Fig. 32. Simulated effect of time-dependent effective porosity on the time-drawdown response of the pumping test segment of the aquifer; *effective porosity as an exponential function of time.*

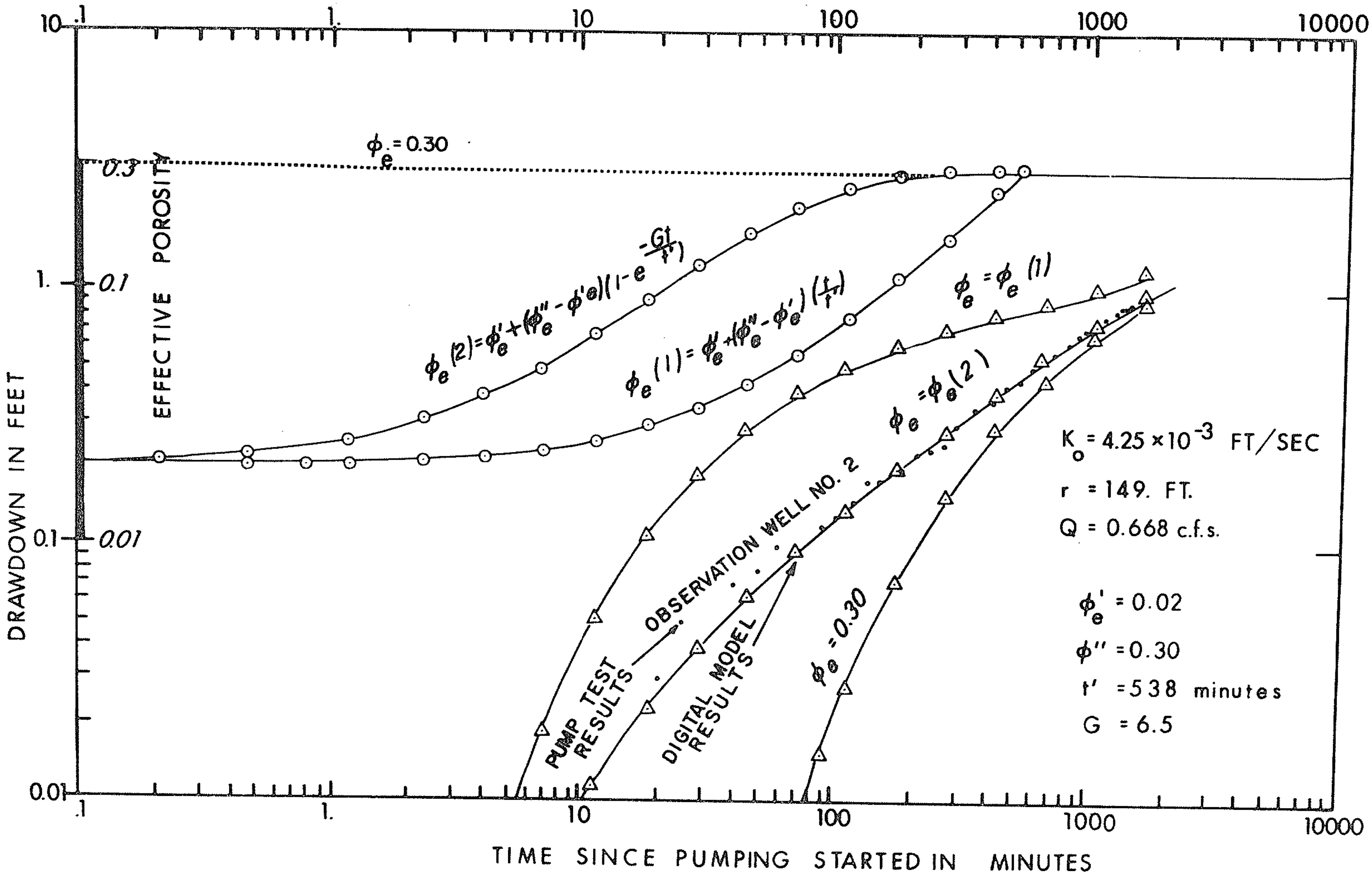

Fig. 33. Simulated effect of time-dependent and time-independent effective porosity on the time-drawdown response of the pumping test segment of the aquifer.

able model response was obtained by increasing the effective porosity exponentially. The above analysis indicates that non-instantaneous yield of water from storage had strongly affected aquifer response during the pumping period.

The simulation results indicate that by using a homogeneous distribution of the effective porosity and hydraulic conductivity, the simultaneous drawdown graphs of all 3 observation wells can not be totally matched with the model response. For example, drawdown in observation well 1 is generally greater than the model results when the drawdowns for wells 2 and 3 are similar to the model response. However, a minor spatial variation in porosity and conductivity would likely overcome this difficulty.

CHAPTER IX

SUMMARY AND CONCLUSIONS

Following a reconnaissance investigation of shallow outwash deposits in the southwestern portion of the Souris River basin, the Broomhill Experimental Aquifer was chosen for detailed investigation of its geologic characteristics and response and yield properties.

Test drilling and compilation of existing data indicated that the Broomhill Experimental Aquifer is composed of clean coarse sand and gravel varying in thickness up to 50 feet.

Hydraulic properties of a representative group of disturbed and relatively undisturbed samples from the aquifer deposits were determined from laboratory tests. The results indicate that

(i) Saturated hydraulic conductivities of the relatively undisturbed samples range between 2.15×10^{-4} and 2.72×10^{-3} feet per second at 4°C and effective porosity between 30.4 and 35.7 percent.

(ii) Retention porosity usually increased as saturated hydraulic conductivity decreased.

(iii) Effective porosity was only slightly correlated with saturated hydraulic conductivity.

The results of the laboratory tests were combined with the field data to obtain maps of the effective porosity and saturated hydraulic conductivity of the aquifer.

The mathematical model based on the method of Pinder and Bredehoft (1968), in which the finite-difference approximation to the flow equations was solved using an iterative alternating-direction technique, was used to analyze the results of the pumping test conducted by the Manitoba Water Resources Branch. The results indicated that the aquifer response during 24 hours of pumping from a single well was strongly dependent on non-instantaneous release of water from the aquifer during the initial phase of the pumping test. The digital analysis also indicated that the estimates of aquifer properties in the pumping test area based on correlations between test drilling samples, drill logs, and results of laboratory analyses conducted on the quarry samples were reasonable.

Digital models were also used to obtain an understanding of the response behaviour and general yield capabilities of the aquifer system. The aquifer model was initially formulated to simulate response of the aquifer to multiple pumping centers when commencing with the initial water-table configuration approximating natural conditions. The model was then used to simulate the effect of an initially horizontal water table. The results indicated that drawdown of the water table in the aquifer area in which the natural hydraulic gradient was low did not vary appreciably between the above simulations. Model simulations were conducted using up to 26 pumping centers located in the zone of maximum initial saturated thickness in the eastern half of the aquifer. It was found that 56 percent of the aquifer area is capable of yielding 2310 acre-feet of water over the 99-day period of pumping.

The aquifer yield capabilities were also investigated using the hydrologic-budget approach in which the annual evapotranspiration from the aquifer was estimated using the D'Arcy method and the winter water-table decline method of estimating the annual groundwater outflow. The general magnitude of the annual groundwater outflow was appraised by comparing the estimated evapotranspiration values with calculated potential evapotranspiration and consumptive-use values, as well as with the annual distribution of precipitation and change in the groundwater storage. The results indicate that a more reasonable estimate of the annual groundwater outflow from the aquifer was obtained using the D'Arcy method.

The average yield capability of the aquifer during the 1965-1971 period, which could be realized by conjunctive use schemes under ideal conditions involving induced or artificial recharge from Stony Creek, is estimated to be

9330 acre-feet of water per year. Estimates of the annual developmental yield of the aquifer during the above period range between 2 and 13 inches and indicate that the aquifer yield capability is controlled by major effects of climatic conditions. It is, therefore, recommended that a more detailed yield analysis be performed using long-term regional climatic and stream flow data from adjacent water-sheds. The yield analysis for the western portion of the aquifer could also be refined if more detailed test drilling data were available.

It is concluded that the Broomhill Experimental Aquifer is a significant source of high-quality water in the Souris River basin. The hydrologic controls on the aquifer system are such that optimal development of the groundwater resource could only be attained by instituting a detailed water management scheme.

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APPENDIX A

A LABORATORY METHOD TO DETERMINE THE HYDRAULIC PROPERTIES OF DISTURBED AND UNDISTURBED SOIL

Determination of Hydraulic Conductivity

The method described herein is similar to that used by Laliberte *et al.* (1966, 1967, 1968) and is based on the determination of soil permeability as a function of capillary pressure. Gardner (1958) represented the functional relationships between permeability and capillary pressure over the entire range of capillary pressures by

$$C = \frac{a}{b + P_c^n} \qquad [A-1]$$

where,

C = intrinsic permeability, cm^2

P_c = capillary pressure, $dynes/cm^2$

n = a positive constant

a,b = constants, units dependent on the units of permeability, capillary pressure, and on the value of n.

The Gardner equation demonstrates a continuous smooth relationship between permeability and capillary pressure. However, experimental research on porous rocks, as pointed out by Laliberte and Corey (1967), shows that this relationship is often not smooth. The permeability and capillary pressure relationship often has a sharp transition occurring at a capillary pressure very near bubbling pressure. Brooks and Corey (1966), therefore, have represented the relationship as

$$C = C_o \quad \text{for } P_c \leq P_b$$
$$C = C_o \left(\frac{P_b}{P_c}\right)^{\eta} \quad \text{for } P_c \geq P_b \qquad \text{[A-2]}$$

where

C_o = intrinsic permeability when the soil is fully saturated, cm^2

P_b = bubbling pressure, $dynes/cm^2$

η = positive dimensionless number

and the other quantities are as previously defined. They related η to a pore-size distribution index, λ, by the relation

$$\eta = 2 + 3\lambda$$

The pore-size distribution index is dimensionless and can be obtained from capillary pressure desaturation data.

The hydraulic conductivity, K (dimension LT^{-1}), is related to intrinsic permeability, C (dimension L^2), by the equation

$$K = C\frac{\rho g}{\mu}$$

where

ρ = density of the wetting fluid, ML^{-3}

g = acceleration due to gravity, LT^{-2}

μ = dynamic viscosity of the wetting fluid, $ML^{-1}T^{-1}$

Method of Test and Calculation

For this study a total of seven relatively undisturbed soil samples were collected at three locations in the aquifer area. All samples but two were obtained in the vertical direction. At one site two samples were collected; one in the horizontal and one in the vertical direction.

The samples included a wide range of textures, from fine to very coarse sands, taken from depths varying between 1 and 16 feet. A brief description of these samples is given in Table A-1.

All samples were obtained using a removable-sleeve sampler, a photograph of which is shown in Fig. 6. For a detailed description of this apparatus, the reader is referred to Laliberte and Corey (1967). Samples were obtained by manually forcing the sampler into the soil approximately 8 inches below the soil surface. Then the inner sleeve, containing the sample, was removed from the sampler at site and carefully packed to prevent any disturbance during transportation to the laboratory. In order to remove the inner sleeve containing the soil sample from the sampler it was desirable to lubricate its outer surface. For this purpose an ordinary type of oil was used. Care also was taken while pushing the sampler into the soil to protect the soil sample

Table A-1

Brief description of the tested samples

Sample Number	Bulk Density ρ_b, gm/cm^3	Orientation	Depth Feet	Description*
SS1	1.63	Vertical	5-6	A rather uniform fine gravel, no clay, grain size 0.2 to 0.5 cm.
SS2 (a)	1.64	Horizontal	7-8	very uniform coarse sands
SS2 (b)	1.63	Vertical		
SS3	1.68	Vertical	15-16	Coarse sands with pebbles up to 1 cm
SS4	1.66	Vertical	15-16	A mixture of coarse and medium to fine sands with pebbles up to 1 cm.
SS5	1.59	Vertical	3-4	Medium clean sands with some coarser materials
SS6	1.63	Vertical	1-2	Fine sands, very uniform

* All samples are from glacial outwash sediments collected from the Broomhill Experimental Aquifer.

against the possibility of compaction.

The samples were dried in a vacuum oven at 60°C, without removing the soil from its holding sleeve. At this stage the bulk density of each sample, ρ_b (gm/cm^3), was determined as follows:

$$\rho_b = \frac{M_T - M_s}{V_s} \qquad \text{[A-3]}$$

where

M_T = total mass of the dry soil plus sleeve, gm

M_s = sleeve mass, gm

V_s = sleeve volume in cm^3, which is the same as the volume of the undisturbed sample.

The computed values of ρ_b are shown in Table A-1. The bulk density of the samples was also determined on site using the field volumetric method of Zwarich and Shaykewich (1969). The two techniques gave satisfactory agreement.

Prior to vacuum saturation of each sample, two pressure controllers made from Porvic[1] filter material were mounted in plastic to fit the ends of the sleeve containing the sample. The tensiometers, also made from Porvic filter material, were installed in provided spaces and fastened with a waterproof plastic tape. The plastic tape was left attached throughout the vacuum saturation process which often lasted several hours depending upon the texture of the sample

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[1] Porvic Filter Material, Pritchett and Gold and E.P.S., Co., Dagenham Dock, Essex, England.

materials.

The wetting fluid used in this experiment was a solution of 0.1 percent formaldehyde in distilled water. Topp (1969) used a water solution of 0.01N $CaSO_4$ and 0.1 percent formaldehyde in his investigation. The saturation equipment was a vacuum pump connected to an acrylic plastic chamber large enough to accommodate the wetting fluid and the entire assembly (sample and barriers). After having fully saturated the sample, it was transported to the mounting board, and the tubes leading to the manometers, the supply, and the outflow siphons were connected to the tensiometers and capillary barriers. Figures A-1 and A-2 show a schematic diagram of the assembly and a photograph of the apparatus respectively. The elevation of the supply bottle and the outflow was adjusted to produce steady downward flow, with capillary pressure low enough to insure no desaturation ($P_c << P_b$).

The hydraulic conductivity in cm/sec was calculated as follows:

$$K = \frac{q}{\nabla H} \qquad [A-4]$$

where

q = volume flux in cm/sec

∇H = hydraulic gradient between the tensiometers, dimensionless

The hydraulic conductivity measured at complete saturation,

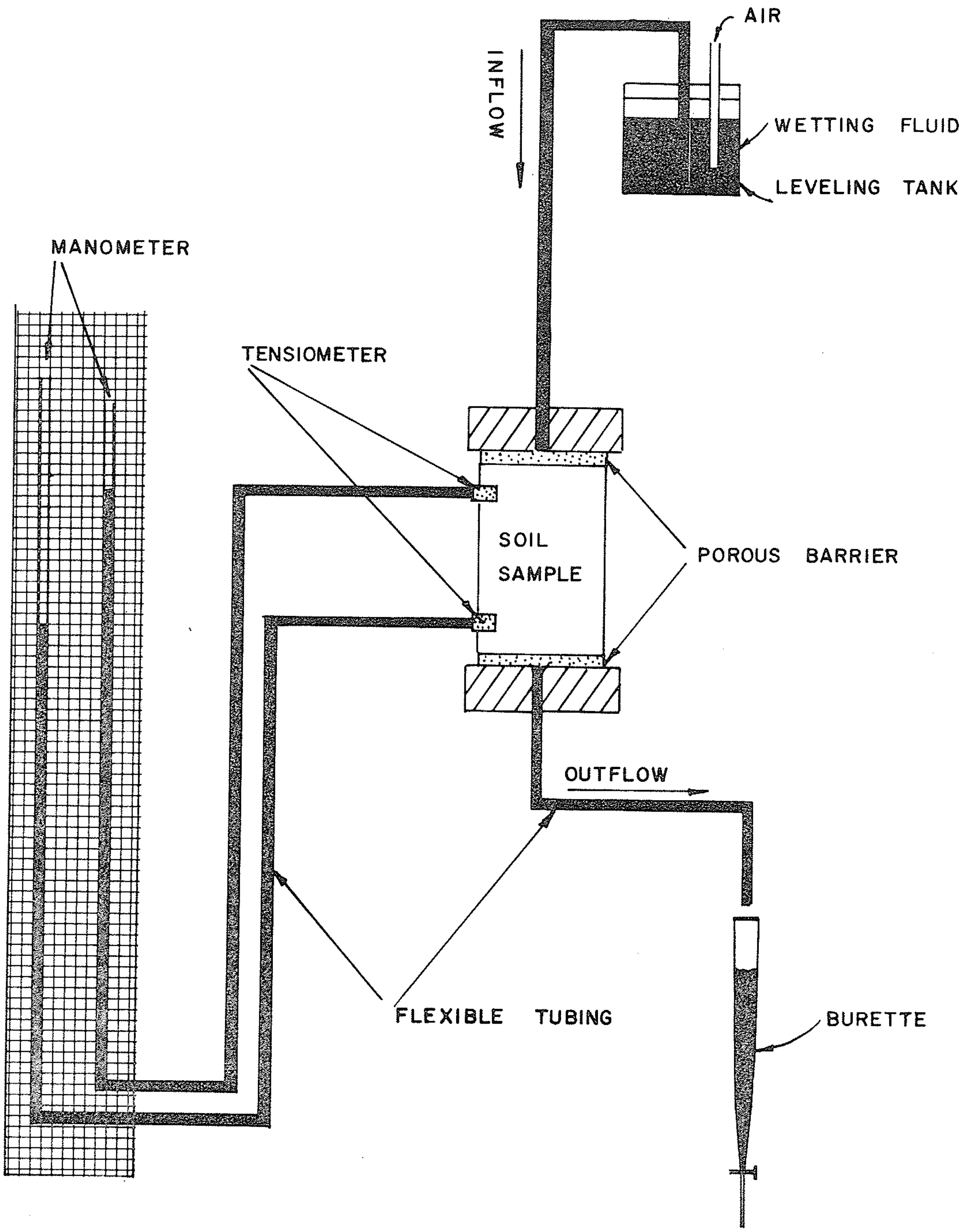

Fig. A-1. Schematic diagram of hydraulic conductivity apparatus.

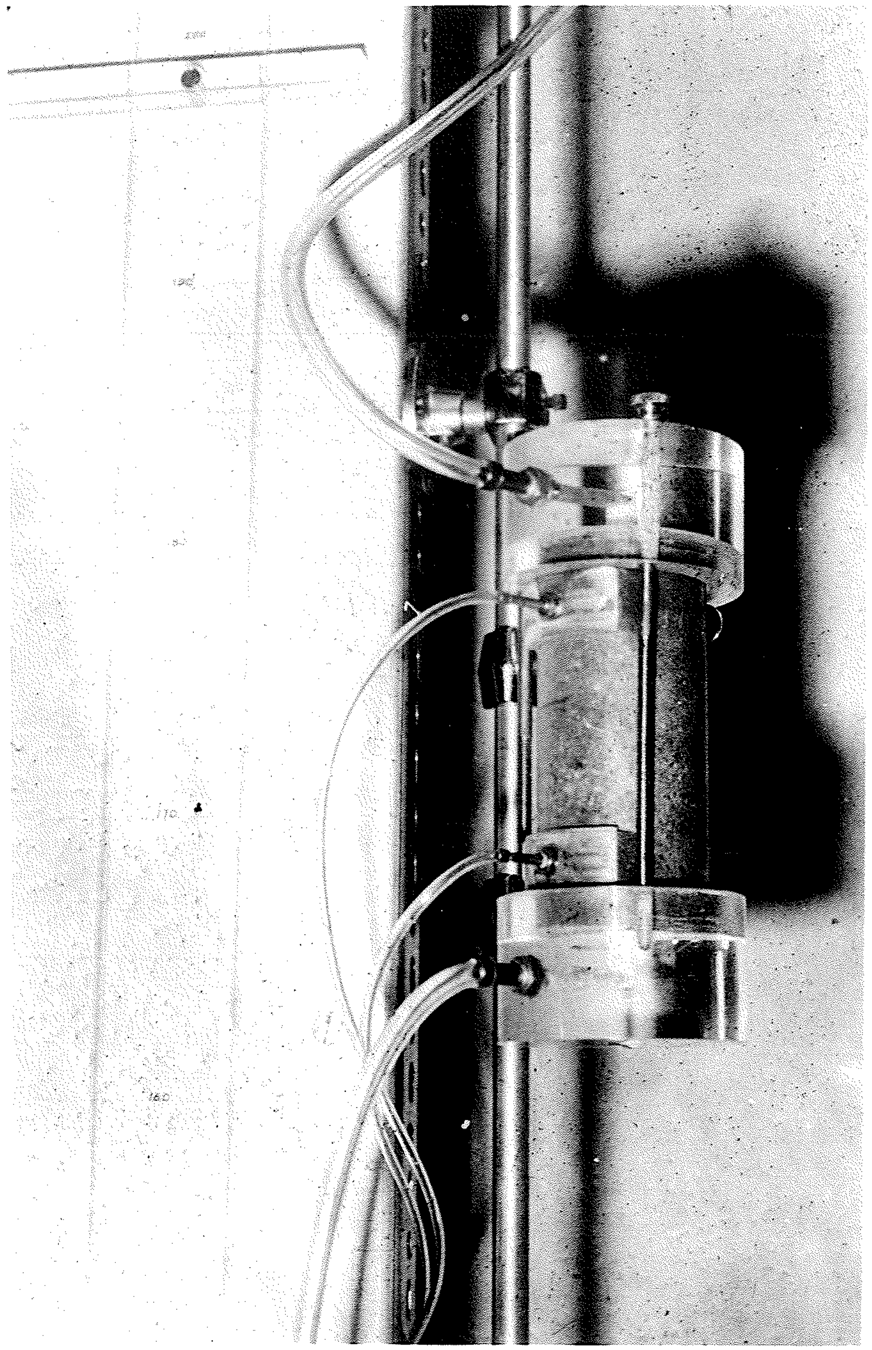

Fig. A-2. Photograph of an undisturbed aquifer sample during the hydraulic conductivity test, showing inflow, outflow, pressure controllers and tensiometers.

$P_c \leq P_b$, was designated K_o.

Volume flux was calculated using the relation

$$q = \frac{V_w}{t \cdot A} \qquad [A-5]$$

where

V_w = volume of water, cm^3, measured in an outflow burette

t = time, sec, corresponding to V_w

A = cross-section area of the sample, cm^2.

Hydraulic gradient was calculated as follows:

$$\nabla H = \frac{\Delta h_t}{l} \qquad [A-6]$$

where

Δh_t = hydraulic head difference between tensiometers in cm of wetting fluid

l = distance in cm between tensiometers.

A study was made to determine the effect of disturbance on the saturated hydraulic conductivity, K_o, for the materials of the aquifer. After measurement was completed on the undisturbed materials, they were disturbed, and the measurements were repeated. Each disturbed material was repacked to the same bulk density as its corresponding undisturbed sample. The repacking procedure was similar to that used by Laliberte (1966) in which a long-spout funnel was used to fill the container. The funnel was first filled with the soil, with the lower end of its spout positioned on the bottom of the container. As the funnel was slowly withdrawn, a random motion was imparted to its lower end in order to minimize the concentration of large particles at the wall of the column. The desired bulk density was attained by gently tapping the side and top of the column with a rubber mallet.

The method of determining the saturated hydraulic conductivity of disturbed samples was identical in principle to that used for undisturbed samples except that the apparatus had a somewhat different appearance (Fig. A-3).

When testing a sample, either disturbed or undisturbed, the readings were repeated several times and an average value was calculated.

Determination of Effective Porosity

The effective porosity ϕ_e (dimensionless) was calculated as follows:

$$\phi_e = \phi - \phi_r \qquad \text{[A-7]}$$

where

ϕ = total porosity of the porous medium, dimensionless

ϕ_r = retention porosity, also dimensionless.

The total porosity was determined as follows:

$$\phi = 1 - \frac{\rho_b}{\rho_s} \qquad \text{[A-8]}$$

where

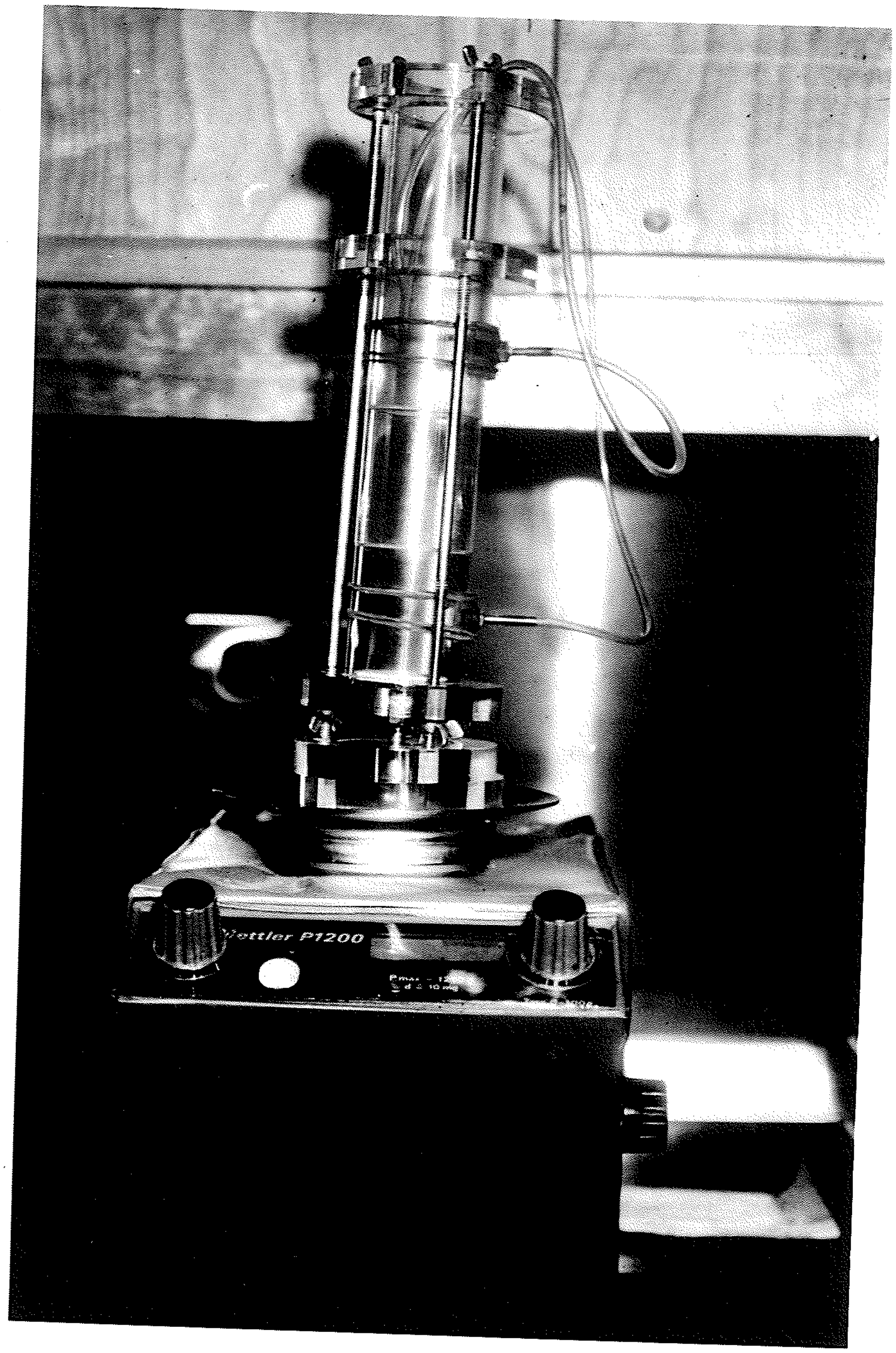

Fig. A-3. Photograph of apparatus for testing the hydraulic conductivity of disturbed materials.

ρ_b and ρ_s = bulk density and particle density respectively in gm/cm³.

The particle density, ρ_s, was computed from the expression:

$$\rho_s = \frac{M_s \times \rho_w}{M_{fw} + M_s - M_{fws}} \qquad \text{[A-9]}$$

where

M_s = mass of the solids, gm

ρ_w = density of water, gm/cm³, at temperature of test

M_{fw} = mass of the flask + water, gm

M_{fws} = mass of the flask + water + soil, gm.

The apparatus and procedure for determining ρ_s was similar to that described by Lambe (1951).

The retention porosity or specific retention of a rock is defined as a measure of the water-retaining capacity of a rock. It is expressed quantitatively as the percentage of the total volume of rock occupied by groundwater that will be retained in interstices against the force of gravity Walton (1970). This definition yields the expression:

$$\phi_r = R_w \frac{\rho_b}{\rho_w} \qquad \text{[A-10]}$$

with

$$R_w = \frac{M_w}{M_s} \qquad \text{[A-11]}$$

where

ρ_b and ρ_w = bulk density of soil and the density of water respectively, gm/cm^3

R_w = retention water expressed as the mass, M_w, of water that will be retained in porous soil against the force of gravity per unit mass, M_s, of dry soil (dimensionless).

Equation [A-11] can also be expressed as

$$R_w = \frac{M_{sw}}{M_s} - 1 \qquad \text{[A-12]}$$

with

$$M_{sw} = M_s + M_w \qquad \text{[A-13]}$$

After calculating the saturated hydraulic conductivity of a sample, the sample was used to measure M_{sw}. Thus, the inflow tube was disconnected and the sample was subjected to one meter suction for a period of 24 hours after which the sample was weighed.

Results and Discussion

Saturated Hydraulic Conductivity

Table A-2 shows the saturated hydraulic conductivity results of the representative group of aquifer samples described earlier in Table A-1, calculated for several measurements during each test. The disturbed samples were repacked to the same bulk density as that of their corresponding undisturbed materials.

Table A-2

Change in saturated hydraulic conductivity, K_o, values of the porous media during laboratory tests

Sample Number	Orientation	State	Room Temp. °C	Number of Measurements 1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
				Saturated hydraulic conductivity (mm/sec)					
SS1	Vertical	Undist.	24.0	1.224	1.415	1.427			
		Dist.	22.4	1.950					
SS2 (a)	Horizontal	Undist.	24.7	0.624	0.609	0.608	0.623		
		Dist.	23.0	0.659	0.677	0.679	0.684	0.688	
SS2 (b)	Vertical	Undist.	24.0	0.451	0.455	0.460			
		Dist.	25.9	0.682	0.689				
SS3	Vertical	Undist.	24.2	0.590	0.537	0.551	0.502	0.544	
		Dist.	25.0	0.581	0.578	0.581	0.581	0.581	
SS4	Vertical	Undist.	24.5	0.461	0.464	0.464	0.465	0.465	
		Dist.	24.9	0.566	0.568	0.568	0.568		
SS5	Vertical	Undist.	20.3	0.238	0.230	0.231			
		Dist.	24.8	0.291	0.292	0.290	0.293	0.295	0.295
SS6	Vertical	Undist.	24.0	0.116	0.108				
		Dist.	26.4	0.0647	0.0676	0.0651	0.0636	0.0651	

An average value of K_o was calculated for each sample using the results indicated in Table A-2 and corrected to 4°C taken as the standard temperature. The correction was obtained using the relationship:

$$K_2 = \frac{\mu_1 \times \rho_2}{\rho_1 \times \mu_2} K_1 \qquad \text{[A-14]}$$

where

K = hydraulic conductivity, cm/sec

μ = dynamic viscosity, centipoises

ρ = density of the wetting fluid, gm/cm^3

and subscripts $_1$ and $_2$ refer to the properties of the wetting fluid and the soil at test temperature and 4°C respectively. Computed values of K_2 along with the hydraulic properties used in these calculations are presented in Table A-3.

The saturated hydraulic conductivities of undisturbed samples, K_o, and of their corresponding disturbed samples, K_{od} are compared in Table A-4. The results disclose that for all tested samples but one, disturbed materials have a larger hydraulic conductivity when fully saturated than do the corresponding undisturbed samples. In order to gain an insight into the cause of the above phenomena it is useful to recall some factors affecting hydraulic conductivity of a porous material. Hydraulic conductivity of a porous solid, among other factors, is directly proportional to porosity and to the mean value of the square of the hydraulic radius,

Table A-3

Computed values of saturated hydraulic conductivity at 4°C with the hydraulic properties used in calculating the equation $K_2 = \frac{\mu_1 \rho_2}{\rho_1 \mu_2} K_1$ for several aquifer samples*

Sample	K_1, mm/sec at Test Temp.	Test Temp. °C	μ_1 Centipoises	ρ_1 gm/cm^3	K_2, mm/sec at 4°C	K_2, ft/sec at 4°C
SS1-Undisturbed	1.420	24.0	0.9142	0.99732	0.830	2.72×10^{-3}
Disturbed	1.950	22.4	0.9490	0.99756	1.184	3.88×10^{-3}
SS2 (a) Undisturbed	0.616	24.7	0.8999	0.99714	0.355	1.16×10^{-3}
SS2 (a) Disturbed	0.677	23.0	0.9358	0.99756	0.405	1.33×10^{-3}
SS2 (b) Undisturbed	0.455	24.0	0.9142	0.99732	0.266	8.73×10^{-4}
SS2 (b) Disturbed	0.686	25.9	0.8757	0.99684	0.384	1.26×10^{-3}
SS3-Undisturbed	0.534	24.2	0.9101	0.99727	0.311	1.02×10^{-3}
Disturbed	0.580	25.0	0.8937	0.99707	0.332	1.09×10^{-3}
SS4-Undisturbed	0.464	24.5	0.9040	0.99720	0.268	8.79×10^{-4}
Disturbed	0.568	24.9	0.8958	0.99710	0.326	1.07×10^{-3}
SS5-Undisturbed	0.233	20.3	0.9978	0.99817	0.149	4.89×10^{-4}
Disturbed	0.293	24.8	0.8978	0.99712	0.168	5.51×10^{-4}
SS6-Undisturbed	0.112	24.0	0.9142	0.99732	0.0655	2.15×10^{-4}
Disturbed	0.652	26.4	0.8660	0.99670	0.0361	1.18×10^{-4}

* $\rho_2/\mu_2 = 0.638$

Table A-4

Comparison between saturated hydraulic conductivity of disturbed and undisturbed samples of the aquifer formations

Sample Number	Orientation	Saturated hydraulic conductivity to water at 4°C, ft/sec		K_o/K_{od}
		Undisturbed K_o	Disturbed K_{od}	
SS1	Vertical	2.72×10^{-3}	3.88×10^{-3}	0.701
SS2 (a)	Horizontal	1.16×10^{-3}	1.33×10^{-3}	0.872
SS2 (b)	Vertical	8.73×10^{-4}	1.26×10^{-3}	0.693
SS3	Vertical	1.02×10^{-3}	1.09×10^{-3}	0.936
SS4	Vertical	8.79×10^{-4}	1.07×10^{-3}	0.821
SS5	Vertical	4.89×10^{-4}	5.51×10^{-4}	0.887
SS6	Vertical	2.15×10^{-4}	1.18×10^{-4}	1.822

and is inversely proportional to a factor called tortuosity. Therefore, a larger value of hydraulic conductivity will result by increasing porosity and hydraulic radius and/or decreasing tortuosity of a given soil. It is known that the hydraulic radius will be greater for porous solids having secondary porosity as well as primary porosity. Therefore, an increase of secondary porosity of the disturbed samples relative to the undisturbed samples due to disturbance, could have caused the hydraulic conductivity to increase.

Another factor which appeared to be more important in this regard, is the difference between the tortuosity value of disturbed and undisturbed samples. According to Carman (1937) the tortuosity of fully-saturated porous solids which are isotropic and granular in nature is about 2, but if the medium has bedding planes composed of clay particles oriented perpendicular to the flow direction, the tortuosity will be much larger than 2. If such bedding planes are oriented parallel to the flow, tortuosity will be smaller than 2. Since most of the undisturbed samples tested here were taken in the vertical direction, and therefore perpendicular to the aquifer bedding planes, and were subjected to downward flow during the saturated hydraulic conductivity test, it is reasonable to assume that bedding planes composed of clay particles, if any, could have produced a larger value of tortuosity for the undisturbed sample than in corresponding disturbed materials.

The only sample (out of seven tested samples) which did not satisfy the relation $K_o < K_{od}$ was a very fine sand in lake deposits. The accuracy of the result for this sample when undisturbed is dubious, however, because the sample was eroded while being tested and furthermore its holding sleeve was broken.

The results also show that for the tested samples SS2(a) and SS2(b) in their undisturbed states, the horizontal sample had larger hydraulic conductivity when fully saturated than the corresponding vertical sample. This also could have resulted from the bedding planes of clay materials being perpendicular to the flow direction for the vertical sample and parallel to the flow for the horizontal sample. This is supported by the fact that the saturated hydraulic conductivity of these samples in the disturbed state are almost the same (Table A-4).

Effective Porosity

A summary of the hydraulic properties of the representative group of aquifer materials such as bulk density (ρ_b), particle density (ρ_s), total porosity (ϕ), retention water (R_w), and retention porosity (ϕ_r) along with computed values of effective porosity (ϕ_e) is presented in Table A-5.

The results disclose that for the samples tested here, the lowest value of bulk density is about five percent smaller than its highest value, whereas for particle density

Table A-5

Effective porosity, total porosity, retention porosity, and particle density of the several aquifer formations

Sample Number	ρ_b gm/cm³	ρ_s gm/cm³	$\frac{\rho_b}{\rho_s}$	$\phi=1-\frac{\rho_b}{\rho_s}$	M_{sw} gm	M_s gm	$R_w=\frac{M_{sw}}{M_s}-1$	$\phi_r=R_w\frac{\rho_b}{\rho_w}$	$\phi_e=\phi-\phi_r$
SS1	1.63	2.712	0.601	0.399	272.31	263.75	0.032	0.052	0.347
SS2 (a)	1.64	2.679	0.612	0.388	336.10	328.30	0.024	0.039	0.349
SS2 (b)	1.63	2.685	0.607	0.393	275.00	269.02	0.022	0.036	0.357
SS3	1.68	2.677	0.628	0.372	309.95	299.49	0.035	0.059	0.313
SS4	1.66	2.658	0.625	0.375	326.90	313.46	0.043	0.071	0.304
SS5	1.59	2.684	0.592	0.408	285.45	273.10	0.045	0.072	0.336
SS6	1.63	2.669	0.611	0.389	287.20	273.50	0.050	0.082	0.307

it is only two percent. Total porosity, therefore, depends mainly upon the variation of bulk density, and consequently, particle density could be treated as constant. A comparison made between the results shown in Tables A-4 and A-5 disclosed that retention porosity usually increases as saturated hydraulic conductivity decreases. Effective porosity, however was only slightly correlated with saturated hydraulic conductivity.

APPENDIX B

A COMPUTER PROGRAM FOR AQUIFER EVALUATION

The digital computer program described here computes the hydraulic head distribution in a confined or unconfined aquifer during periods of water withdrawal or recharge at one or more locations. The program solves the finite-difference approximations of the unsteady-state saturated flow equation using an iterative alternating direction implicit technique described by Pinder (1969). The program can accommodate heterogeneous distributions of effective porosity and saturated hydraulic conductivity and relatively complex boundary conditions. The dimensions of the nodal array can be up to 50 x 50 nodes. The program is written in "Fortran IV" for the IBM-360 system. The computer program is basically similar to one developed by Pinder (1969), however, several changes have been made to meet the requirements of the present study.

The time required for the program to complete its computations depends mainly upon the nodal array dimensions and the number of time steps demanded during simulation. The larger the nodal array and the number of time steps, the more computer time is required. For example, a 50 x 50 nodal array and 35 time steps; the computer time required is approximately 6 minutes on an IBM-360/65 computer.

The message "EXCEEDED PERMITTED NUMBER OF ITERATIONS" is printed when the head difference between a row and column computation at any node is greater than a prescribed error criterion after 100 iterations. An error in the input data would generally result in this message being obtained. The message "WELL GOES DRY" is printed when the cone of depression in an unconfined aquifer drops below the impermeable bottom of the aquifer.

The value of the error criterion, ERR, and the number of the iteration, LENGTH, depends upon the specific problem. A value for ERR of 0.001 and number for LENGTH of 3 to 5 provided satisfactory results.

A mass balance of the groundwater reservoir is computed in the program by comparing the volume of water removed from the aquifer with the volume computed by integration over the cone of depression. The volume of water removed from the aquifer is calculated directly from the discharge rates. The mass balance results can be used to measure the model's precision if there is no flux boundaries and to calculate the volume of water entering or leaving (in addition to pumping discharge) in the cases where flux boundaries occur.

The hydraulic head matrix and the elapsed period of pumping may be punched on cards at the end of the computation. These cards can be used as input data, if it is necessary, to extend the simulation period.

The Arrangement and Definition of the Program Variables

Parameter Cards

Card 1

Column	Variable	
1-10	TMAX	period of pumping (hours)
11-20	DIML	number of nodes in a column of the matrix (no decimal point)
21-30	DIMW	number of nodes in a row of the matrix (no decimal point)
31-40	NUMT	maximum number of time steps (no decimal point)
41-50	QRE	vertical recharge into the aquifer (feet per second)
51-60	DELT	size of initial time increment (seconds)
61-70	M	thickness of porous medium through which water recharges into or discharges from the aquifer (feet). The parameter is normally referred to as thickness of the stream or lake bed.
71-80	KTH	number of time steps between printouts (no decimal point)

Card 2

1-10	LENGTH	number of iteration parameters (no decimal point)
11-20	ERR	error criterion for closure
21-30	SPACING	contour interval (feet)

Card 3

1-20	SUM	elapsed time at beginning of computation, usually 0, unless provided as punched output, (seconds)

Card 4

1-5	PUNCH	indicator for punched output, if punched output is desired at the end of the computations write PUNCH, otherwise leave the card blank

Card 5

1-10	WATER	indicator of water-table condition, if an unconfined aquifer is to be modelled write WATERTABLE, otherwise leave the card blank

Card 6

1-7	CONTR	indicator for contoured printout, if a contour map of drawdown is desired write CONTOUR, otherwise leave the card blank

Card 7

1-7	NUM	indicator for numerical printout, if the numerical drawdown values are desired write NUMERIC, otherwise leave the card blank

Card 8

1-5	CHCK	indicator for computational check, if a mass balance of the aquifer system is desired write CHECK, otherwise leave the card blank

Data Cards

DELX and DELY: distance between nodes in the prototype in the x and y directions in feet; the distance in the x direction, DELX, is recorded from left to right and is followed, beginning in a new card by the distance in the y direction, DELY, recorded from top to bottom. There would be DIMW values for DELX and DIML values for DELY. The values for DELX and DELY must remain constant for each column and row respectively. FORMAT:8G10.0.

The values of the following variables are recorded from left to right along the rows beginning with the first row from the top, under the FORMAT:8G10.0. The values of the first row are recorded first and are followed, beginning on a new card, by the values of the second row. The procedure is then continued until the last row in the matrix is reached. There would be, therefore, DIMW x DIML values for each variable.

STRT:	Initial hydraulic head in the aquifer in feet.
RIVER:	Hydraulic head in stream and lakes in feet. (This set of data is omitted if an unconfined aquifer problem is considered, in such cases the computer program equates the hydraulic head in stream and lake to the initial hydraulic head in the aquifer, STRT, at appropriate nodes).
RATE:	Pumping rates in cfs, and hydraulic conductivity of stream and lake bed in feet per second. At any node where a pumping well is located the discharge rate is recorded preceded by a negative sign. At any node where a stream or lake is located the hydraulic conductivity of the bed is recorded.
PERM:	Hydraulic conductivity of the aquifer in feet per second. (This set of data is omitted if a confined aquifer problem is considered).
T:	Transmissibility of the aquifer in square feet per second. (This set of data is omitted if an unconfined aquifer problem is considered).
BOTTOM:	Elevation of the impermeable bottom of the aquifer in feet. (This set of data is omitted

if a confined aquifer problem is considered).

S: Storage coefficient or effective porosity of the aquifer (dimensionless).

Computer Program

```
C        TO PROVIDE THE POTENTIAL DISTRIBUTION IN A CONFINED OR UNCONFINED
C       AQUIFER AFTER A DESIGNATED PERIOD OF TIME
C       ***************************************************************
C
C       DESCRIPTION OF PARAMETERS
C       A=COEFFICIENT IN FINITE DIFFERENCE EQUATION
C       B=COEFFICIENT IN FINITE DIFFERENCE EQUQTION
C       BE(I)=DUMMY VARIABLE DEFINED AS C/W
C       BOTTOM(I,J)=ELEVATION OF BOTTOM(FEET)
C       C=COEFFICIENT IN FINITE DIFFERENCE EQUATION
C       D=DUMMY VARIABLE REPRESENTING PARAMETERS OF EQUATION WHOSE VALUES
C       ARE KNOWN FROM PREVIOUS ITERATION
C       DAYS=NUMBER OF DAYS SINCE PUMPING STARTED
C       DELT= LENGTH OF INITIAL TIME STEP (SECONDS)
C       DELX=DISTANCE BETWEEN NODES IN THE PROTOTYPE IN THE X DIRECTION
C            (FEET)
C       DELY=DISTANCE BETWEEN NODES IN THE PROTOTYPE IN THE Y DIRECTION
C            (FEET)
C       DIML=NUMBER OF NODES IN COLUMN OF MATRIX
C       DIMW=NUMBER OF NODES IN ROW OF MATRIX
C       ERR=CLOSURE CRITERIA FOR ITERATION
C       G=DUMMY VARIABLE DEFINED AS (D-A*G(J-1))/W
C       HRS=NUMBER OF HOURS SINCE PUMPING STARTED
C       IMK=ITERATION PARAMETER
C       KEEP(I,J)=STORAGE MATRIX FOR PHI VALUES
C       KTH=NUMBER OF TIME STEPS BETWEEN PRINTOUTS
C       M=THICKNESS OF STREAM BED OR LAKE BOTTOM (FEET)
C       NUMT=MAXIMUM NUMBER OF TIME STEPS
C       PHI(I,J)=HEAD IN AQUIFER AT NODE (I,J) IN FEET
C       PRNT(I,J)=ARRAY FOR PRINTING OUT HEAD MATRIX IN CONTOUR FORM
C       QRE= RECHARGE FLUX FROM THE CONFINING LAYER (FEET PER SECOND)
C       RATE(I,J)=RATE OF PUMPING IN CFS IF NEGATIVE
C                 HYDRAULIC CONDUCTIVITY OF STREAM OR LAKE BOTTOM IF
C       POSITIVE (FEET PER SECOND)
C       RHO=DUMMY VARIABLE DEFINED AS S(I,J)/DELT
C       RIVER(I,J)=HYDRAULIC HEAD IN RIVER (FEET)
C       RR=VARIABLE ACCOUNTING FOR VERTICAL LEAKAGE
C       RW=VARIABLE ACCOUNTING FOR PUMPING
C       S(I,J)=STORAGE COEFFICIENT OR EFFECTIVE POROSITY (DIMENSIONLESS)
C       SATHI(I,J)=SATURATED THICKNESS IN AQUIFER (FEET)
C       STRT(I,J)=INITIAL HYDRAULIC HEAD IN AQUIFER(FEET)
C       SUM=DURATION OF PUMPING IN SECONDS
C       SYM(J)=LINEAR ARRAY CONTAINING SYMBOLS FOR PRINTING OUT HEAD
C            MATRIX
C       T(I,J)=TRANSMISSIBILITY (FEET**2/SECOND)
C       TEMP(I)=TEMPORY LOCATION OF CURRENTLY CALCULATED VALUES OF HEAD
C       TMAX=MAXIMUM ALLOTTED PERIOD OF PUMPING (HOURS)
C       W=DUMMY VARIABLE DEFINED AS B-A*BE(J-1)
C
C       ...............................................................
C       ...............................................................
C
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      DIMENSION S(50,50), PERM(50,50), BOTTOM(50,50), RATE(50,50), KEEP(
     150,50), G(50), TEMP(50), BE(50), RHOP(25), CHK(10), STRT(50,50), T
     2(50,50), PHI(50,50),RIVER(50,50),SATHI(50,50)
```

```
      DIMENSION DELX(50), DELY(50)
      REAL MINS,M,K
      REAL*8 KEEP,IMK,NUM
      INTEGER DIML,DIMW
      DOUBLE PRECISION PHI,KEEP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,RHOP,PARAM,IMK,RIVER,CHK,PNCH,WATER,CONTR,NUM,CHCK
C
C     ...............................................................
C
      DATA CHK(1)/5HPUNCH/,CHK(2)/8HWATERTAB/,CHK(3)/7HCONTOUR/,CHK(4)/7
     1HNUMERIC/,CHK(5)/5HCHECK/
C
      READ (5,520) TMAX,DIML,DIMW,NUMT,QRE,DELT,M,KTH,LENGTH,ERR,SPACNG
      READ (5,610) SUM
      READ (5,600) PNCH,WATER,CONTR,NUM,CHCK
      READ (5,680) (DELX(J),J=1,DIMW)
      READ (5,680) (DELY(I),I=1,DIML)
      DO 5  I=1,DIML
      READ (5,620) (STRT(I,J),J=1,DIMW)
      DO 5  J=1,DIMW
    5 PHI(I,J)=STRT(I,J)
      IF (WATER.NE.CHK(2)) GO TO 15
      DO 10 I=1,DIML
      DO 10 J=1,DIMW
   10 RIVER(I,J)=STRT(I,J)
      GO TO 20
   15 DO 20 I=1,DIML
      READ (5,620) (RIVER (I,J),J=1,DIMW)
   20 CONTINUE
      DO 25 I=1,DIML
   25 READ (5,620) (RATE(I,J),J=1,DIMW)
      IF (WATER.NE.CHK(2)) GO TO 50
      DO 30 I=1,DIML
   30 READ (5,620) (PERM(I,J),J=1,DIMW)
      DO 40 I=1,DIML
      READ (5,620) (BOTTOM(I,J),J=1,DIMW)
      DO 40 J=1,DIMW
   40 T(I,J)=PERM(I,J)*(STRT(I,J)-BOTTOM(I,J))
      GO TO 70
   50 DO 60 I=1,DIML
   60 READ (5,620) (T(I,J),J=1,DIMW)
   70 DO 80 I=1,DIML
   80 READ (5,620) (S(I,J),J=1,DIMW)
C
C     ...............................................................
C
C     PRINT PARAMETER VALUES
      WRITE (6,550) DELT,TMAX,NUMT,QRE,DIML,DIMW,LENGTH,ERR,M,KTH
      WRITE (6,690) (DELX(J),J=1,DIMW)
      WRITE (6,700) (DELY(I),I=1,DIML)
      TMAX=TMAX*3600.
      IF (WATER.EQ.CHK(2)) GO TO 100
      WRITE (6,560)
      DO 90 I=1,DIML
   90 WRITE (6,570) I,(T(I,J),J=1,DIMW)
      WRITE (6,575)
```

```
      DO 95 I=1,DIML
   95 WRITE (6,570)I,(RIVER(I,J),J=1,DIMW)
  100 WRITE (6,590)
      DO 110 I=1,DIML
  110 WRITE (6,570) I,(S(I,J),J=1,DIMW)
      WRITE (6,540)
      DO 120 I=1,DIML
  120 WRITE (6,570) I,(RATE(I,J),J=1,DIMW)
      IF (WATER.NE.CHK(2)) GO TO 150
      WRITE (6,630)
      DO 130 I=1,DIML
  130 WRITE (6,570) I,(PERM(I,J),J=1,DIMW)
      WRITE (6,640)
      DO 140 I=1,DIML
  140 WRITE (6,570) I,(BOTTOM(I,J),J=1,DIMW)
  150 JN01=DIMW-1
C
C     ...................................................................
C
C     COMPUTE ITERATION PARAMETERS
C     COMPUTE HMIN
      HMIN=2.
      XVAL=3.1415**2/(2.*DIMW**2)
      YVAL=3.1415**2/(2.*DIML**2)
      DO 160 I=2,DIML
      DO 160 J=2,DIMW
      IF (T(I,J).EQ.0.) GO TO 160
      XPART=XVAL*(1/(1+DELX(J)**2/DELY(I)**2))
      YPART=YVAL*(1/(1+DELY(I)**2/DELX(J)**2))
      HMIN=AMIN1(HMIN,XPART,YPART)
  160 CONTINUE
      ALPHA=EXP(ALOG(1/HMIN)/(LENGTH-1))
      RHOP(1)=HMIN
      DO 170 NTIME=2,LENGTH
  170 RHOP(NTIME)=RHOP(NTIME-1)*ALPHA
      PARAM=RHOP(1)
      WRITE (6,650) (RHOP(J),J=1,LENGTH)
      KT=0
      TEST=0
C
C     ...................................................................
C
C     IF TEST EQUALS 1 CONTINUE ITERATION, IF TEST EQUALS 0 GO TO NEXT
C     TIME STEP
  180 IF (TEST.EQ.1) GO TO 250
      IFINAL=0
      IF (KT.GT.NUMT.OR.SUM.GT.TMAX) IFINAL=1
      NTH=0
      IF (WATER.NE.CHK(2)) GO TO 190
      WRITE (6,185) KT,DELT,SUM
      DO 190 I=1,DIML
      DO 190 J=1,DIMW
      IF (RATE(I,J) .GE.0.0) GO TO 190
      SATHI(I,J)=PHI(I,J)-BOTTOM(I,J)
      WRITE(6,750) I,J,RATE(I,J),SATHI(I,J)
  190 CONTINUE
```

```
      IF (MOD(KT,KTH).NE.0.OR.KT.EQ.0.AND.IFINAL.NE.1) GO TO 210
      WRITE (6,580) KT,DELT,SUM,MINS,HRS,DAYS,KOUNT
      IF (CONTR.EQ.CHK(3)) CALL PRNTA
      IF (NUM.EQ.CHK(4)) CALL PRNT1
      IF (CHCK.EQ.CHK(5)) CALL CHECK
      IF (IFINAL.NE.1) GO TO 210
      IF (PNCH.NE.CHK(1)) STOP
      DO 200 I=1,DIML
  200 WRITE (7,620) (PHI(I,J),J=1,DIMW)
      WRITE (7,610) SUM
  205 STOP
  210 CONTINUE
      KT=KT+1
      KOUNT=0
      DO 220 I=1,DIML
      DO 220 J=1,DIMW
  220 KEEP(I,J)=PHI(I,J)
      IF (WATER.NE.CHK(2)) GO TO 240
      DO 230 I=1,DIML
      DO 230 J=1,DIMW
      T(I,J)=PERM(I,J)*(PHI(I,J)-BOTTOM(I,J))
      IF (T(I,J).GE.0.) GO TO 230
      SATHI(I,J)=PHI(I,J)-BOTTOM(I,J)
      WRITE(6,800)I,J,SATHI(I,J),PERM(I,J),T(I,J)
  230 CONTINUE
      DO 235 I=1,DIML
      DO 235 J=1,DIMW
      IF (T(I,J).GE.0.) GO TO 235
      WRITE (6,660)
      GO TO 205
  235 CONTINUE
  240 DELT=DELT+(.25*DELT)
      SUM=SUM+DELT
      HRS=SUM/3600.
      MINS=HRS*60.
      DAYS=HRS/24.
      GO TO 270
  250 IF (KOUNT.LT.100) GO TO 260
      WRITE (6,670)
      GO TO 205
  260 KOUNT=KOUNT+1
      IF (MOD(KOUNT,LENGTH)) 270,270,280
  270 NTH=0
  280 NTH=NTH+1
      PARAM=RHOP(NTH)
      TEST=0.
  290 DO 300 J=1,DIMW
  300 TEMP(J)=PHI(1,J)
      DO 400 I=2,DIML
      DO 360 J=2,JN01
C
C     ...............................................................
C
C     DETERMINE WHETHER NODE IS OUTSIDE AQUIFER BOUNDARY
      IF (T(I,J)) 310,360,310
C
```

```
C      ...............................................................
C
  310 RHO=S(I,J)/DELT
C
C      ...............................................................
C
C      CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
C      NODE CONFIGURATIONO  T1=LEFT, T2=RIGHT, T3=UPPER, T4=LOWER
       T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DELX
      1(J)
       T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DELX
      1(J)
       T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
      1(I)
       T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
      1(I)
       IMK=PARAM*(T1+T2+T3+T4)
       K=0.0
C
C      ...............................................................
C
C      CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE
       IF (RATE(I,J)) 330,330,320
  320  K=RATE(I,J)
C
C      ...............................................................
C
C      CALCULATE VALUES FOR PARAMETERS A,B,C, AND BE
  330  B=-T1-T2-RHO-K/(2.*M)-IMK
       A=T1
       C=T2
       W=B-A*BE(J-1)
       BE(J)=C/W
C
C      ...............................................................
C
C      CHECK NODE FOR POSSIBLE WELL LOCATION
       RR=QRE
       RW=0.0
       IF (RATE(I,J)) 340,350,350
  340  RW=-RATE(I,J)/(DELX(J)*DELY(I))
  350  D=-T3*PHI(I-1,J)+(T4+T3-IMK)*PHI(I,J)-T4*PHI(I+1,J)-RHO*KEEP(I,J)-
      1(K/M)*(RIVER(I,J)-KEEP(I,J)/2.)+RW-RR
       G(J)=(D-A*G(J-1))/W
  360  CONTINUE
C
C      ...............................................................
C
C      CALCULATE HEAD VALUES FOR ROWS OF MATRIX AND PLACE THEM IN
C      TEMPORARY LOCATION TEMP
       NO3=DIMW-2
       DO 390 KNO4=1,NO3
       NO4=DIMW-KNO4
       PHI(I-1,NO4)=TEMP(NO4)
       IF (T(I,NO4)) 380,370,380
  370  TEMP(NO4)=PHI(I,NO4)
```

```
      GO TO 390
  380 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
  390 CONTINUE
  400 CONTINUE
C
C     ******************************************************************
C
C     FOLLOW SIMILIAR PROCEDURE FOR COLUMNS OF MATRIX AS THAT CONSIDERE[
C     FOR ROWS
      DO 410 I=1,DIML
  410 TEMP(I)=PHI(I,1)
      INO1=DIML-1
      DO 510 J=2,DIMW
      DO 470 I=2,INO1
      IF (T(I,J)) 420,470,420
C
C     ..................................................................
C
  420 RHO=S(I,J)/DELT
C
C     ..................................................................
C
C     CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
      T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DEL>
     1(J)
      T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DEL>
     1(J)
      T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
     1(I)
      T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
     1(I)
      IMK=PARAM*(T1+T2+T3+T4)
      K=0.0
C
C     ..................................................................
C
C     CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE
      IF (RATE(I,J)) 440,440,430
  430 K=RATE(I,J)
C
C     ..................................................................
C
C     CALCULATE VALUES FOR PARAMETERS A,B,C,AND BE
  440 A=T3
      C=T4
      B=-T4-T3-RHO-K/(2.*M)-IMK
      W=B-A*BE(I-1)
      BE(I)=C/W
C
C     ..................................................................
C
C     CHECK NODE FOR POSSIBLE WELL LOCATION
      RR=QRE
      RW=0.0
      IF (RATE(I,J)) 450,460,460
  450 RW=-RATE(I,J)/(DELX(J)*DELY(I))
```

```
  460 D=-T1*PHI(I,J-1)+(T1+T2-IMK)*PHI(I,J)-T2*PHI(I,J+1)-RHO*KEEP(I,J)-
     1(K/M)*(RIVER(I,J)-KEEP(I,J)/2.)+RW-RR
      G(I)=(D-A*G(I-1))/W
  470 CONTINUE
C
C     .................................................................
C
C     CALCULATE HEAD VALUES FOR COLUMNS OF MATRIX AND PLACE IN TEMPORARY
C     LOCATION TEMP
      NO3=DIML-2
      DO 500 KNO4=1,NO3
      NO4=DIML-KNO4
      PHI(NO4,J-1)=TEMP(NO4)
      IF (T(NO4,J)) 490,480,490
  480 TEMP(NO4)=PHI(NO4,J)
      GO TO 500
  490 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
      IF (DABS(TEMP(NO4)-PHI(NO4,J)).GT.ERR) TEST=1.
  500 CONTINUE
  510 CONTINUE
      GO TO 180
C
C     ****************************************************************
C
C
C
  185 FORMAT (1H1,55X,17HTIME STEP NUMBER=,I10/50X,29HSIZE OF TIME STEP
     1IN SECONDS=,E10.3/40X,48HDURATION OF PUMPING AT THIS PRINTOUT IN S
     2ECONDS=,E10.3)
  520 FORMAT (8G10.0)
  540 FORMAT (1H1,61X,11HRATE MATRIX)
  550 FORMAT ('1',60X,' INPUT PARAMETERS',//,' LENGTH OF INITIAL TIME ST
     1EP IN SECOND=',E10.3//,' MAXIMUM PERMITTED PERIOD OF PUMPING IN HO
     2URS=',E10.3//,' MAXIMUM PERMITTED NUMBER OF TIME STEPS=',I4//,' FL
     3UX DUE TO VERTICAL LEAKAGE FROM A CONFINING LAYER IN FEET PER SECO
     4ND=',E10.3//,' NUMBER OF NODES IN COLUMN OF MATRIX=',I4//,' NUMBER
     5 OF NODES IN ROW OF MATRIX=',I4//,' NUMBER OF ITERATION PARAMETERS
     6=',I4//' ERROR CRITERIA FOR CLOSURE=',E10.3//,' THICKNESS OF STREA
     7M BED IN FEET=',E10.3//,' NUMBER OF TIME STEPS BETWEEN PRINTOUTS =
     8',I4)
  560 FORMAT (1H1,64X,23HTRANSMISSIBILITY MATRIX)
  570 FORMAT (1H0,I3,1X,1P10E12.3/(5X,1P10E12.3))
  575 FORMAT ('1',61X,' RIVER MATRIX')
  580 FORMAT (1H1,55X,17HTIME STEP NUMBER=,I10/50X,29HSIZE OF TIME STEP
     1IN SECONDS=,E10.3/40X,48HDURATION OF PUMPING AT THIS PRINTOUT IN S
     2ECONDS=,E10.3/86X,8HMINUTES=,E10.3/86X,6HHOURS=,E10.3/86X,5HDAYS=,
     3E10.3/55X,17HITERATION NUMBER=,I10)
  590 FORMAT (1H1,54X,26HSTORAGE COEFFICIENT MATRIX)
  600 FORMAT (A5/A8/A7/A7/A5)
  610 FORMAT (F20.3)
  620 FORMAT (8G10.0)
  630 FORMAT (1H1,52X,29HHYDRAULIC CONDUCTIVITY MATRIX)
  640 FORMAT (1H1,46X,40HELEVATION OF IMPERMEABLE BASE OF AQUIFER)
  650 FORMAT (1H1,56X,20HITERATION PARAMETERS///(1H ,10E12.3))
  660 FORMAT (1H0,13HWELL GOES DRY)
  670 FORMAT (1H0,39HEXCEEDED PERMITTED NUMBER OF ITERATIONS)
```

```
  680 FORMAT (8G10.0)
  690 FORMAT (1H1,40X,40HGRID SPACING IN PROTOTYPE IN X DIRECTION///(1H0
     1,10F12.2))
  700 FORMAT (1H0,40X,40HGRID SPACING IN PROTOTYPE IN Y DIRECTION///(1H0
     1,10F12.2))
  750 FORMAT(  /' GRIDE(',I3,',',I3,')' 5X,  ' DISCHARGE RATE IN CU.F/SEC
     1. ='1PE12.3, 20X,' SATURATED THICKNESS IN FEET ='1PE12.3)
  800 FORMAT (////7H GRIDE(,I3,1H,,I3,1H)//20X,30H SATURATED THICKNESS
     1IN FEET=, 1PE12.3//20X,30H HYDRAULIC CONDUCTIVITY F/S  =,1PE12.3
     2//20X,30H TRANSMISSIBILITY IN FT**2/S =,1PE12.3///1X)
      END
C     ******************************************************************
      SUBROUTINE PRNTA
C     ******************************************************************
C     THIS SUBROUTINE PRINTS OUT THE DRAWDOWN MATRIX AS ALPHABETIC
C     CONTOURS
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      INTEGER DIML,DIMW
      REAL K
      DIMENSION RATE(50,50), SYM(39), PRNT(60), PHI(50,50), STRT(50,50),
     1 BLANK(60), S(50,50)
      DIMENSION DELY(50), DELX(50)
      DOUBLE PRECISION PHI
      WRITE (6,50)
      DATA SYM/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1HA,1HB,1HC,1HD,1HE,1
     1HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,1HO,1HP,1HQ,1HR,1HS,1HT,1HU,1HV
     2,1HW,1HX,1HY,1HZ,1H0,1H ,1H*,1HG/,BLANK/60*1H /
      IND=(65-DIMW)/2
      DO 40 IB=1,DIML
      DO 30 JB=1,DIMW
      K=STRT(IB,JB)-PHI(IB,JB)
      K=K/SPACNG
      IF (K.LT.0) GO TO 10
      K=AMOD(K,36.)
      IF (K.LT.1.) PRNT(JB)=SYM(36)
   10 IF (K.LT.0) PRNT(JB)=SYM(39)
      IF (PHI(IB,JB).EQ.STRT(IB,JB)) PRNT(JB)=SYM(37)
      N=K
      IF (N.LT.1) GO TO 20
      PRNT(JB)=SYM(N)
   20 IF (RATE(IB,JB).GT.0.) PRNT(JB)=SYM(27)
      IF (RATE(IB,JB).LT.0.) PRNT(JB)=SYM(32)
   30 CONTINUE
   40 WRITE (6,60) (BLANK(I),I=1,IND),(PRNT(JB),JB=1,DIMW)
      WRITE (6,70) SPACNG
      RETURN
C
C     ******************************************************************
C
C
C
   50 FORMAT (1H0,50X,32HALPHABETIC CONTOURS FOR DRAWDOWN,////)
   60 FORMAT (1H ,65A2)
   70 FORMAT (10H0LEGEND***/18H0CONTOUR INTERVAL=,F10.3/32H0LOCATION OF
     1RECHARGE BOUNDARY=R/16H0WELL LOCATION=W/21H0CONE OF IMPRESSION=G)
      END
```

```
C     ******************************************************************
      SUBROUTINE PRNT1
C     ******************************************************************
C     THIS SUBROUTINE PRINTS OUT THE DRAWDOWN MATRIX IN NUMERICAL FORM
C     ..................................................................
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      DIMENSION RATE(50,50), DDN(60), PHI(50,50), S(50,50), STRT(50,50)
      DIMENSION DELY(50), DELX(50)
      INTEGER DIML,DIMW
      DOUBLE PRECISION PHI
      WRITE (6,30)
      DO 20 I=1,DIML
      DO 10 J=1,DIMW
   10 DDN(J)=STRT(I,J)-PHI(I,J)
   20 WRITE (6,40) I,(DDN(K),K=1,DIMW)
      RETURN
C
C     ******************************************************************
C
C
C
   30 FORMAT (1H1,58X,16HDRAWDOWN IN FEET//)
   40 FORMAT(1H0,I3,1X,10E12.3/(5X,10E12.3))
      END
C     ******************************************************************
      SUBROUTINE CHECK
C     ******************************************************************
C     THIS SUBROUTINE COMPUTES A MASS BALANCE OF THE AQUIFER SYSTEM
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      INTEGER DIML,DIMW
      DIMENSION RATE(50,50), S(50,50), PHI(50,50), STRT(50,50)
      DIMENSION DELY(50), DELX(50)
      DOUBLE PRECISION PHI
      TOTL=0.0
      PUMP=0.0
      DO 10 IC=2,DIML
      DO 10 JC=2,DIMW
      TOTL=TOTL+S(IC,JC)*DELX(JC)*DELY(IC)*(STRT(IC,JC)-PHI(IC,JC))
   10 IF (RATE(IC,JC).LT.0.0) PUMP=PUMP-RATE(IC,JC)*SUM
      DIFF=PUMP-TOTL
      PERCNT=(DIFF/PUMP)*100.
      WRITE (6,20) TOTL,PUMP,DIFF,PERCNT
      RETURN
C
C     ******************************************************************
C
C
C
   20 FORMAT (63H0QUANTITY PUMPED ACCORDING TO CONE OF DEPRESSION IN CUB
     1IC FEET=,E20.10//59H QUANTITY PUMPED ACCORDING TO WELL DISCHARGE I
     2N CUBIC FEET=,E20.10//51H ESTIMATE FROM PUMPING LESS ESTIMATE FROM
     3 DRAWDOWN=,E20.10//42H DIFFERENCE AS A PERCENT OF VOLUME PUMPED=,E
     420.10)
      END
```