

CHARGE SYMMETRY BREAKING IN NEUTRON-PROTON SCATTERING
——CSB ISOSPIN MIXING PARAMETER

by

Lei Ge

A thesis
presented to the University of Manitoba
in partial fulfillment of the
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Master of Science
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ABSTRACT

Charge symmetry breaking (CSB) in neutron-proton elastic scattering is examined. The isospin singlet-triplet mixing parameter $\bar{\gamma}_1$ is calculated in a new way; namely, making use of an operator method to calculate the CSB matrix element, for the center of mass energy from 10 Mev to 500 Mev by using the Livermore phase shift phenomenological data for different angular momentum states. The distorted wave Born approximation is used in the investigation of CSB due to certain class IV forces: electromagnetic interaction (EM) with nucleon form factors, ρ^0 - ω mesonic mixing and nucleon mass-difference effects (MD). The Paris potential is used for the computation of distorted wave functions.

The results show that the isospin mixing is essentially a small quantity and of the order of 1.6×10^{-2} for the MD effect on D-states. On the average, EM and MD give stronger CSB than ρ^0 - ω effect. With the opposite sign, MD leads to the larger violation (at center of mass energy ≥ 100 Mev) than EM does. Despite the traditional idea of the magnitude of any scattering quantity decreasing with increasing angular momentum, our calculation shows that the isospin mixing has an "anomalous" J-dependence; namely, the strongest mixing occurs on D- states at c.m. energy below

350 Mev. As the energy further increases, the maximum in $\bar{\gamma}_T$ due to EM effect shifts to the G-state. This tells us that DWBA provides more information about tensor force and therefore is a better physical approximation than the Born approximation. Our calculation concludes that CS has the largest violation at around c.m. energy 300 Mev (on D-states), which implies that the Lab energy 600 Mev will be a good experimental energy.

For comparison with ρ^0 - ω mixing, π^0 - η mixing is considered as well. This gives rise to a class III force, and so gives no contribution in the neutron-proton system.

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SYMBOLS

1. e : Absolute value of electronic charge
- S : Spin
- $\vec{\sigma}$: Pauli matrix
- ν : The component of spin
- I : Isospin
- ν_t : The component of isospin
- L : Orbital angular momentum
- J : Total angular momentum
- $\overline{\gamma}_J$: Isospin mixing parameter
- ϵ_J : Angular momentum mixing parameter
2. $\hat{A}$: Unit vector of A
- $\hat{A}$: Operator A
- $\vec{A}$: Vector or Operator A
- A : Scalar A
3. m_a : Mass of particle a (in Mev)
- M : Nucleon mass, 938.2592 for $I=1$, 938.9055 for $I=0$
- $m_\pi=137.27$, $m_\eta=548.8$, $m_\rho=770.0$, $m_\omega=782.6$
- $g^2_{ab}/4\pi$: Coupling constant of a,b
- $g^2_Y/4\pi=1/137$, $g^2_{\rho\omega}/4\pi=6.6445$, $g^2_{\pi NN}/4\pi=14.5$
4. Ψ : Nucleon wave function
- Φ : Pion wave function
- Y^m_ℓ : Spherical harmonics
- P^m_ℓ : Associated Legendre function
- j_ℓ : Spherical Bessel function

5. K_n : Neutron anomalous magnetic moment, $K_n = -1.91$
 K_p : Proton anomalous magnetic moment, $K_p = 1.79$
 μ_p : Proton magnetic moment, $\mu_p = 1 + K_p$
 K^V : Isovector magnetic moment, $K^V = 3.7$
6. The natural unit system is used. ($\hbar = c = 1$)

CONTENTS

ABSTRACT	iv
ACKNOWLEDGEMENTS	vi
SYMBOLS	vii
 <u>Chapter</u>	 <u>page</u>
CHAPTER I. INTRODUCTION	1
CHAPTER II. CSB IN N-P SCATTERING	9
2.1. Charge symmetry breaking and class III, IV forces	9
2.2. Characteristic scattering quantities of CSB	10
2.3. Formal scattering theory generalized to CSB	14
2.4. Calculation of CSB mixing parameter	19
CHAPTER III. CSB CLASS IV FORCE MECHANISMS	24
3.1. The direct electromagnetic effect	24
3.2. $\rho^0 - \omega$ mixing	28
3.3. Nucleon-mass difference	32
CHAPTER IV. COMPUTATION ,RESULTS AND DISCUSSION	35
4.1. Computation	35
4.2. Results and discussions	39
CHAPTER V. SUMMARY AND CONCLUSION	46
APPENDIX A. DETAILED CALCULATION OF T_{S1}	47
A.1. The calculation of the CSB amplitude f_B	47
A.2. The detailed calculation of CSB T-matrix element T_{S1}	49
A.3. Proof of the identity (A2.2)	54
A.4. Behaviours of the asymptotic wave function at $r=0.1$ fm	56
APPENDIX B. DETAILED CALCULATION OF CLASS III FORCE OF $\pi^0 - \eta$ MIXING EFFECT	58

APPENDIX C. PROGRAMME LISTING 64

REFERENCES 80

CHAPTER I. INTRODUCTION

The study of charge symmetry breaking (CSB) in nucleon-nucleon scattering has particular importance for several reasons. First of all, in neutron-proton scattering, one would like to verify that isospin is not conserved, giving rise to transition between isospin singlet and triplet states. In proton-proton or the neutron-neutron scatterings, isospin is conserved but the charge symmetry is broken. Secondly, it will provide some fundamental information about nuclear structure, charge-dependent nuclear forces and the internal symmetry properties of elementary particle physics. In addition, because of the unambiguous discrepancy between results from two recent theoretical calculations of the neutron-proton polarization difference [1], this topic seems still to require further study.

The properties of nuclear force have been studied for a long time. Since neutron and proton have so many like properties except their charge, they are considered as two states of the same particle, the nucleon [2]. Such a two-state system is mathematically identical to the formalism of a spin-1/2 particle, and is now called isospin. In this isospin formalism, proton and neutron states are the

eigenstates of a two-by-two Pauli matrix τ_3 with eigenvalues ± 1 , i.e.

$$|p\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |n\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (1.1)$$

$$\tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (1.2)$$

then the charge is related to isospin ($I_3 = 1/2 \tau_3$) in the nucleon case as

$$Q = (I_3 + 1/2)e \quad (1.3)$$

where I_3 has the values $\pm 1/2$ and e is the electronic charge. The isospin formalism has been considered as a basic tool to study the nucleon interaction.

Early in 1932, the hypothesis by Heisenberg, Condon and Cassen [3] stated that the specific nuclear forces should be considered charge independent (CI), which means that the forces between n-p, n-n, p-p without taking into account the Coulomb electromagnetic effect, are assumed to be the same. The interaction of a system is charge independent if the Hamiltonian of the system H commutes with the total isospin operator $\hat{T}$, i.e.

$$[H, \hat{T}] = 0 \quad (1.4)$$

which implies that H is an isoscalar and is invariant under arbitrary rotations in isospin space. Those postulates had been in agreement with some low energy scattering experiments until in 1960's, when it was found that CI of the internucleon potential is only an approximate rather than an exact symmetry [3,4,5]. For CI it must be assumed that the masses of the different members of the meson isospin multiplets (e.g. π^+, π^0, π^-) are the same and the nucleon coupling constants (e.g. G_{π^+np}, G_{π^0nn} etc.) as well. However, this is obviously an approximation since [5]

$$\frac{\Delta m}{m} = \frac{m_{\pi^+} - m_{\pi^0}}{\frac{1}{2}(m_{\pi^+} + m_{\pi^0})} \approx 3.4 \% \quad (1.5)$$

The nucleon forces are considered as charge symmetric (CS) if one only examines the equality of the n-n and p-p force but says nothing concerning about the n-p force.

By definition, charge symmetry requires the same behaviour for a charge-reflected system as for the original one, or the invariance under reflection about 1-2 plane of charge space, where the charge corresponds to 3-axis. By defining the CS operator, or charge parity operator, [6]

$$\hat{P}_{CS} = \exp(i\pi\hat{T}_2) = \prod_{i=1}^A \exp(i\pi\hat{T}_2(i)) \quad (1.6)$$

where $\hat{T}_2$ is the component of $\hat{T}$ along the 2-axis, i.e.

$$\hat{T}_2 = \sum_{i=1}^A \hat{T}_2(i) \quad (1.7)$$

with the baryon number (or nucleon number) A . An isospinor function Ψ therefore can be transformed as follows,

$$\hat{P}_{CS} \Psi = \exp(i\pi\hat{\tau}_2/2)\Psi = i\tau_2\Psi \quad (1.8)$$

where $\hat{\tau}_2 = 2\hat{T}_2$ is the isospin Pauli matrix and τ_2 is the corresponding quantum number. In particular, for neutron and proton, the wave function is transformed as

$$\Psi_p \rightarrow \Psi_n, \quad \Psi_n \rightarrow -\Psi_p \quad (1.9)$$

Similarly, an isovector function Φ can be transformed as

$$\hat{P}_{CS} \Phi = \exp(i\pi\hat{T}_2)\Phi \quad (1.10)$$

and for neutral and charged pions, the wave functions are transformed as

$$\Phi^+ \rightarrow -\Phi^-, \quad \Phi^- \rightarrow -\Phi^+, \quad \Phi^0 \rightarrow -\Phi^0 \quad (1.11)$$

If CS is a valid symmetry, then

$$[\hat{P}_{CS}, H] = 0 \quad (1.12)$$

On the other hand, equality of n-n and p-p forces is only a necessary but not a sufficient test of charge symmetry. For two nucleon (i, j) system, P_{CS} does not only commute with the isospin scalar operator, (which corresponds to CI)

$$[\hat{P}_{CS}, \vec{T}(i) \cdot \vec{T}(j)] = 0 \quad (1.13)$$

but also commutes with the third component of isospin (which corresponds to CS)

$$[P, T_3(i)T_3(j)] = 0 \quad (1.14)$$

Some experimental results of the reactions in few nucleon systems, such as $^2\text{H}(n,p)nn$, $^3\text{He}(^3\text{He}, ^4\text{He})pp$, $^3\text{He}(^3\text{H}, ^4\text{He})np$

TABLE 1

Low energy NN scattering parameters: scattering lengths a and effective ranges r

	$a(\text{fm})$	$r(\text{fm})$
np(I=1)	-23.715 ± 0.015	2.73 ± 0.03
nn	-16.9 ± 0.6	2.65 ± 0.18
pp	-7.828 ± 0.008	2.80 ± 0.02
(corrected for EM effects)	-17.1 ± 0.2	2.84 ± 0.03

are summarized in table 1 [7,1]. In the absence of the Coulomb interaction, the difference of scattering parameters implies that the nuclear force is not charge independent but may still be charge symmetric since the scattering lengths a after the Coulomb correction are matched within the error bars, which implies that CSB is a weaker effect than CIB. In the 1970's, it was realized that the evidence for charge symmetry breaking (CSB) came from the few-body reactions with two nucleons in states of mixed isospin, then the systematic investigations of Coulomb energy differences in mirror nuclei and isobaric analog states were carried out [6]. Besides the 'Nolen-Schiffer' Coulomb energy anomaly [8], which is the most convincing example for the CS

violation, perhaps the best case for observing CSB comes from the detailed study of the binding energy difference of the mirror nuclei ${}^3\text{H}$ - ${}^3\text{He}$ because of the simplicity of the wave function. Brandenburg, Coon and Sauer (1978) [9] studied the static coulomb potential contribution by considering not only n-p mass difference, but also the effect of exchange currents on the electromagnetic form factors. They computed the total electromagnetic contribution to the binding energy difference between the mirror pair, and found that the difference between the theoretical calculation (683 ± 29 kev) and the experimental value (764 kev) is 81 ± 29 kev. For the two like nucleons, the ground state wave function is more than 90% in state ${}^1\text{S}_0$, the small binding energy difference directly reflects charge asymmetry in the isospin $I=1$ N-N system.

Theoretically, there are two kinds of definitions of CSB in use. One is stronger than the other. The weak statement says that the n-n interaction is not the same as the nuclear part of the p-p interaction. This statement is tested by comparing n-n scattering with p-p scattering. The stronger one states that exchanging neutrons with protons simultaneously in any experiment obtains different result, which somehow implies that there is a term $\frac{\vec{\sigma}_1^{(n)} \cdot \vec{\sigma}_2^{(p)}}{(\sigma_1 - \sigma_2)} \cdot \hat{n}$ present (see eq.(2.9)). This can be tested either in finite nuclei (e.g. mirror nuclei) or in nucleon-nucleon scattering. So far, there has been no direct measurement of CSB obtained.

CSB has been studied mainly through the comparison of the differences between neutron-proton scattering quantities.

Recently, E.M. Henley, G.A. Miller and C.Y. Cheung [10] (1979) computed the difference of neutron-proton polarization caused by class IV CSB forces [11] in n-p scattering. In the center of mass frame, CSB effects due to one-photon exchange, meson mixing and nucleon mass difference were examined. On the other hand, A.Gersten [12,13] calculated those effects in the helicity representation and with the Born approximation (BA), obtaining a phase parametrization, $\bar{\gamma}_J$, of CSB, as well as the difference of neutron-proton polarization. The results from these two calculations are dramatically different, hence this seems to raise an interesting question to see the relation and difference between them, and more importantly, to find a better way for describing CSB phenomena.

In this work, the CSB phase parametrisation is calculated in a new way, namely by use of an operator method to calculate the $\bar{\gamma}_J$ in terms of the scattering matrix CSB element, hence one can get a series of NN scattering quantities. In Chapter 2, a basic CSB theory is presented mathematically. Formal scattering theory is generalized to CSB case. In terms of the distorted wave Born approximation (DWBA), a CSB isospin mixing parameter formula is derived. In Chapter 3, in the n-p system, CSB mechanisms, i.e. one photon exchange, ρ^0 - ω mesonic mixing, neutron-proton mass

difference are studied for different energies and various angular momentum states. In Chapter 4, the Paris NN interaction potential is cited parametrically and used for the computation of distorted wave functions. The asymptotic behaviour of the wave functions is determined in Appendix A(4). Results for different center of mass energies from 10 to 500 Mev are discussed. Physically, since the DWBA includes more information about the tensor forces, the isospin mixing will have a more complicated angular momentum dependence than that from the Born approximation [12]. In some sense, the DWBA is a better approximation than the BA for the n-p scattering. From the calculation of this characteristic quantity, we expect to obtain one particular energy at which the CS has the largest breaking. At such an optimum energy, the CSB effect can best be observed experimentally. Finally, a summary and conclusion is made in Chapter 5. In Appendix A(1-3), the detailed calculation of CSB scattering matrix is provided. As a comparison with the class IV force mesonic mixing, a theoretical derivation for the class III force due to π^0 - η mixing is given in Appendix B. The computer programme listing is attached in Appendix C.

CHAPTER II. CSB IN N-P SCATTERING

2.1. CHARGE SYMMETRY BREAKING AND CLASS III, IV FORCES

Henley and Miller have classified four types of nuclear forces according to their isospin (or charge) dependence [6]. Class I force is proportional to $\vec{\tau}(i) \cdot \vec{\tau}(j)$ where $\vec{\tau}(i)$ is the Pauli isospin matrix of the i th particle. Because this commutes with total isospin, it is charge independent. Class II force is proportional to $\tau_3(i)\tau_3(j)$ where $\tau_3(i)$ is the third component of $\vec{\tau}$. Obviously it maintains CS since it satisfies the equ.(1.14) but breaks CI only. Class III and Class IV forces can break both CI and CS. They are both isovectors and are defined as

$$V_{\text{III}} = d (\tau_3(i) + \tau_3(j)) \quad (2.1)$$

for class III, where d is a coefficient independent of isospin and

$$V_{\text{IV}} = e (\tau_3(i) - \tau_3(j)) + f (\vec{\tau}(i) \times \vec{\tau}(j))_3 \quad (2.2)$$

for class IV and e, f are different functions of space and spin variables.

It is important to note that a class III force does not give rise to isospin mixing in the two-body system since

$$[V_{\text{III}}, T^2] \propto [T_3, T^2] = 0 \quad (2.3)$$

hence it only affects n-n and p-p systems and is symmetric in isospin under particle exchange. However, a Class IV force does cause isospin mixing, so it is antisymmetric under the exchange of $1 \leftrightarrow 2$ nucleons. The simplest form of class IV force is

$$[\tau_3(1) - \tau_3(2)][\vec{\sigma}(1) - \vec{\sigma}(2)] \cdot \vec{L} F_e(\vec{r}^2, \vec{p}^2, \vec{L}^2) \quad (2.4)$$

$$[\vec{\tau}(1) \times \vec{\tau}(2)]_3 [\vec{\sigma}(1) \times \vec{\sigma}(2)] \cdot \vec{L} F_f(\vec{r}^2, \vec{p}^2, \vec{L}^2)$$

where $\vec{L}$ is the orbital angular momentum operator, $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$ are the relative coordinate and momentum, respectively, of the two-nucleon system. This leads to the transition between 3L_J and 1L_J states. In this work, we will study the class IV force to calculate the CSB mixing parameter and only examine the class III force comparatively.

2.2. CHARACTERISTIC SCATTERING QUANTITIES OF CSB

It is known that CS is a consequence of isospin invariance, which is the first "internal symmetry" postulated in fundamental particle physics. In view of its practical and principal importance, it would be highly desirable to have a detailed as well as accurate test to investigate not only the degree of isospin non-conservation, but the more important characteristic CSB parameters, i.e. charge symmetry breaking isospin mixing parameter, the CSB

amplitude as a function of kinematical variables and the polarization difference in two-body scattering as well as other spin observables.

Because charge symmetry breaking makes transition between spin singlet and triplet states possible, this means that the transition amplitude is nonvanishing. For a given n-p scattering angle θ , as shown in Fig.1, the polarization difference between proton and neutron is non-zero,

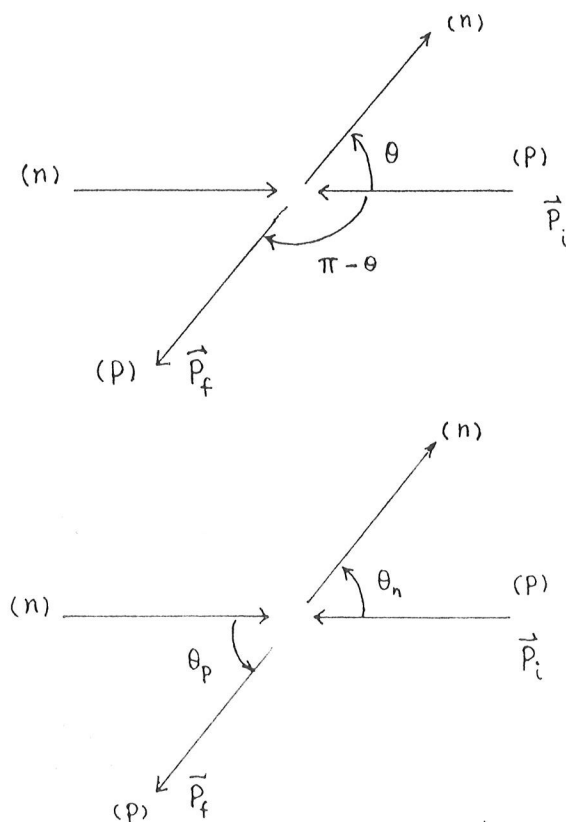


Fig 1. n-p scattering in center of mass frame

$$\Delta P(\theta) = P_n(\theta) - P_p(\pi - \theta) \quad (2.5)$$

and similarly for the analyzing power difference

$$\Delta A(\theta) = A_n(\theta) - A_p(\pi - \theta) \quad (2.6)$$

Considering that neutrons and protons are Fermions, the generalized Pauli exclusion principle requires that the two-nucleon state $|a, I, S, L\rangle$ be antisymmetric under space, spin and isospin exchange. Since,

$$\hat{P}_I \hat{P}_S \hat{P}_L |a, I, S, L\rangle = (-1)^{I+S+L} |a, I, S, L\rangle \quad (2.7)$$

this means

$$(-1)^{I+S+L} = -1 \quad (2.8)$$

that is, the constraint $I+S+L=\text{odd}$ must hold, where P, P_I, P_S are the parity, isospin, spin exchange operators and I, S , and L are isospin, spin and orbital angular momentum quantum numbers respectively.

In the n - p system, a class IV forces requires $\Delta I = I' - I = 1$, and since parity conservation requires $\Delta L = \text{even}$, one can only have $\Delta S = s' - s = 1$. As a consequence, spin (isospin) singlets and triplets can mix. On the other hand, in the case of class III forces, $\Delta I = 0$, then parity conservation implies that $\Delta S = 0$ as well.

According to scattering theory [14], the scattering amplitude including CSB, which is usually called six-amplitude form, can be written as

$$T(\vec{k}, \vec{k}) = a + \beta \vec{\sigma}_1 \cdot \hat{n} \vec{\sigma}_2 \cdot \hat{n} + i \gamma (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{n} \\ + i f_B (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n} + \delta \vec{\sigma}_1 \cdot \hat{m} \vec{\sigma}_2 \cdot \hat{m} + \epsilon \vec{\sigma}_1 \cdot \hat{l} \vec{\sigma}_2 \cdot \hat{l} \quad (2.9)$$

where the coefficients are calculated by equ.(2.10) and in particular, the CSB coefficient is f_B (see Appendix A(1)).

$$a = \frac{1}{4} \text{Tr } T \\ \beta = \frac{1}{4} \text{Tr} [T \vec{\sigma}_1 \cdot \hat{n} \vec{\sigma}_2 \cdot \hat{n}] \\ \gamma = \frac{1}{8i} \text{Tr} [T (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{n}] \\ \delta = \frac{1}{4} \text{Tr} [T \vec{\sigma}_1 \cdot \hat{m} \vec{\sigma}_2 \cdot \hat{m}] \\ \epsilon = \frac{1}{4} \text{Tr} [T \vec{\sigma}_1 \cdot \hat{l} \vec{\sigma}_2 \cdot \hat{l}] \\ f_B = \frac{1}{8i} \text{Tr} [T (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n}] \quad (2.10)$$

Equivalently, in the center of mass frame, the scattering T-matrix is

$$T(\vec{k}_f, \vec{k}) = \frac{1}{2} \{ (a+b) + (a-b) (\vec{\sigma}_1 \cdot \hat{n}) (\vec{\sigma}_2 \cdot \hat{n}) + (c+d) (\vec{\sigma}_1 \cdot \hat{m}) (\vec{\sigma}_2 \cdot \hat{m}) \\ + (c-d) (\vec{\sigma}_1 \cdot \hat{l}) (\vec{\sigma}_2 \cdot \hat{l}) + e (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{n} + f (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n} \} \quad (2.11)$$

where use has been made of three mutually orthogonal unit vectors

$$\hat{l} = \frac{\vec{k}_f + \vec{k}_i}{|\vec{k}_f + \vec{k}_i|} \quad \hat{m} = \frac{\vec{k}_f - \vec{k}_i}{|\vec{k}_f - \vec{k}_i|} \quad \hat{n} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_f \times \vec{k}_i|} \quad (2.12)$$

The relation between these two frames is obvious, for example,

$$\frac{1}{2}(a+b) = a \quad , \quad \frac{1}{2}(a-b) = \beta \quad (2.13)$$

Therefore, the non-zero analysing power difference can be computed for the polarized nucleons as [15]

$$\Delta A = \frac{2}{\sigma_0} \text{Re}(b^* f) = \frac{2}{\sigma_0} \text{Re}[(a-\beta)^* f_B] \quad (2.14)$$

where a, β, f_B are defined as before and σ_0 is the differential cross section for the scattering of unpolarised particles.

The polarization difference, in terms of a standard formula, [16] is

$$\Delta(\theta) = \frac{1}{2} \text{Re} \text{Tr}(T_S^+ (\vec{\sigma}_n - \vec{\sigma}_p) \cdot \hat{n} \delta T) / \sigma_0 \quad (2.15)$$

where the transition operator T has been considered as the dominant charge symmetric part T_S plus the symmetry-breaking part δT , defined by

$$T = T_S + \delta T \quad (2.16)$$

where T_S is determined by a phase shift analysis such as that of reference [17,18]. As long as one knows the transition matrix, all the scattering quantities can be calculated.

2.3. FORMAL SCATTERING THEORY GENERALIZED TO CSB

In this section, we describe scattering theory in the CSB case such that the isospin transition can be studied explicitly. Formal scattering theory tells us that the S-matrix in the barycentric system has the general form [19]

$$\langle b|S|a\rangle = \delta(\vec{P}_b - 0) \langle \hat{k}_f, s', \nu' | S | \hat{k}, s, \nu \rangle \quad (2.17)$$

where we have already chosen the total initial momentum to be zero, and $\langle \hat{k}_f, s', \nu' | S | \hat{k}, s, \nu \rangle$ represents an element of S-matrix in the barycentric subspace, which means Lorentz transformation invariant in the scattering system. On the energy shell, we have

$$\langle \hat{k}_f, s', \nu' | S | \hat{k}, s, \nu \rangle = \frac{1}{\rho_\epsilon} \delta[\epsilon(k_f) - \epsilon(k)] \langle \hat{k}_f, s', \nu' | S(k) | \hat{k}, s, \nu \rangle \quad (2.18)$$

where

$$\rho_\epsilon = k^2 / \left(\frac{d\epsilon(k)}{dk} \right) \quad (2.19)$$

ρ_ϵ is the density of energy and is equal to $kM/2$ for neutron, proton system. M is the nucleon mass. $\epsilon(k)$ is the energy of the two particles when their respective momenta are $\vec{k}$ and $-\vec{k}$. Also the unitarity of S-matrix gives

$$\langle \hat{k}' | S^\dagger S | \hat{k} \rangle = \delta_{\hat{k}', \hat{k}} \quad (2.20)$$

where

$$\delta_{\hat{k}', \hat{k}} = \sum_{l=0}^{\infty} \sum_{m=-l}^l Y_l^m(\hat{k}') Y_l^{m*}(\hat{k}) \quad (2.21)$$

The relation between $S(k)$ and $T(k)$ is

$$\langle \hat{k}_f, s', \nu' | S(k) | \hat{k}, s, \nu \rangle = \delta_{s', s} \delta_{\nu', \nu} \delta_{\hat{k}_f, \hat{k}} - 2\pi i \rho_\epsilon \langle \hat{k}_f, s', \nu' | T(k) | \hat{k}, s, \nu \rangle \quad (2.22)$$

if spin singlet and triplet are mixed ($s' \neq s$), equ.(2.22) is simplified to give S and T proportional to each other,

$$\langle \hat{k}_f, s', \nu' | T(k) | \hat{k}, s, \nu \rangle = -\frac{1}{2\pi i \rho_e} \langle \hat{k}_f, s', \nu' | S(k) | \hat{k}, s, \nu \rangle \quad (2.23)$$

To solve for the matrix $\langle \hat{k}_f, s', \nu' | S(k) | \hat{k}, s, \nu \rangle$, first of all, remember the notation $\langle \hat{k}_f, s', \nu' | l', m', s', \nu' \rangle = Y_l^{m'}(\hat{k}_f)$, then making use of the Wigner coefficient $\langle l', s', m', \nu' | J, M \rangle$, one has a very useful formula

$$\langle \hat{k}_f, s', \nu' | S(k) | \hat{k}, s, \nu \rangle = \sum_{l'l'} \langle \hat{k}_f, l' s' \nu' | Y^J | \hat{k}, l s \nu \rangle S_{l's', l s}^J(k) \quad (2.24)$$

where Y^J is defined as

$$\langle \hat{k}', l' s' \nu' | Y^J | \hat{k}, l s \nu \rangle = \sum_{m' m} Y_l^{m'}(\hat{k}') Y_l^{m*}(\hat{k}) \cdot \langle l' s' m' \nu' | J, M \rangle \langle l s m \nu | J, M \rangle \quad (2.25)$$

Finally, the T-matrix in (LSJM) representation is

$$\langle \hat{k}', s', \nu' | T(k) | \hat{k}, s, \nu \rangle = \sum_{l'l'} \langle \hat{k}', l' s' \nu' | Y^J | \hat{k}, l s \nu \rangle T_{l's', l s}^J(k', k) \quad (2.26)$$

on the energy shell, $k'=k$, write

$$T_{l's', l s}^J(k) = T_{l's', l s}^J(k, k) \quad (2.27)$$

It is clear that equ.(2.22) or equ.(2.23) has the symmetry of rotation invariance. Also, if parity is conserved, the orbital angular momenta L and L' must be both even or odd. Moreover, time reversal invariance implies that $S_{l's', l s}^J$ is symmetric, i.e.

$$S_{l's',ls}^J = S_{ls,l's'}^J \quad (2.28)$$

Application of group theory [20] guarantees that the symmetric, unitary matrix $S_{l's',ls}^J$ can be diagonalized by a real, orthogonal matrix-U with

$$S_{l's',ls}^J = \sum_{\alpha} U_{l's',\alpha} e^{2i\delta_{\alpha}^J} U_{\alpha,ls} \quad (2.29)$$

where the exponential term is the diagonal element of the S-matrix and δ_{α}^J is the real eigenphase shift indexed by α corresponding to spin 1 and 0. Thus the scattering matrix can be expressed explicitly in terms of the eigenphase shift δ_{α}^J and some mixing parameters. To characterize the submatrix $S_{l's',ls}^J$, one needs $n(n+1)/2$ real parameters, where n is the number of combinations of (L,S) . Generally, for a system of two spin-1/2 particles, the four different states of L and S which can couple to a given value of $|JM\rangle$ are

$$\begin{aligned} &|k L=J S=0 I J M > \\ &|k L=J S=1 I J M > \\ &|k L=J-1 S=1 I J M > \\ &|k L=J+1 S=1 I J M > \end{aligned} \quad (2.30)$$

Recalling the fact that neutrons and protons are Fermions and the isospin is determined by the condition that $L+S+I$ be odd, there are all together thirty two independent terms $\langle k'L'S'I'JM | T | kLSIJM \rangle$ contributing to the transition. The general constraints of conservation of angular momentum, time-reversal invariance, parity conservation and charge

asymmetry allow us to reduce these thirty two terms. Namely, parity conservation implies that the NN interaction can only result in the T-matrix elements with

$$\begin{aligned}
 & \langle k' L'=J S' I' | T | k L=J S I \rangle \\
 & \langle k' L'=J+1 S' I' | T | k L=J \pm 1 S I \rangle \\
 & \langle k' L'=J-1 S' I' | T | k L=J \pm 1 S I \rangle
 \end{aligned} \tag{2.31}$$

In n-p scattering, CSB requires $\Delta I = 1$, then $\Delta S = 1$, the possible T-matrix elements are

$$\begin{aligned}
 & \langle k' L'=J S'=1 I'=1 | T | k L=J S=0 I=0 \rangle \\
 & \langle k' L'=J S'=0 I'=0 | T | k L=J S=1 I=1 \rangle \\
 & \langle k' L'=J S'=0 I'=1 | T | k L=J S=1 I=0 \rangle \\
 & \langle k' L'=J S'=1 I'=0 | T | k L=J S=0 I=1 \rangle
 \end{aligned} \tag{2.32}$$

and time-reversal invariance says that $\langle A, I'=1 | T | B, I=0 \rangle = \langle B, I'=0 | T | A, I=1 \rangle$ and similarly $\langle A, S'=1 | T | B, S=0 \rangle = \langle B, S'=0 | T | A, S=1 \rangle$ such that the only possible cases that concern us are

$$\begin{aligned}
 & \langle k' L'=J S'=1 I'=1 | T | k L=J S=0 I=0 \rangle \\
 & \langle k' L'=J S'=0 I'=1 | T | k L=J S=1 I=0 \rangle
 \end{aligned} \tag{2.33}$$

which are related each other by some phase factors. In the two-body interaction, $n(n+1)/2 = 3$, hence one can find a 2 by 2 matrix with three independent parameters to characterize all the properties of $S_{l's', l s}^T$.

2.4. CALCULATION OF CSB MIXING PARAMETER

To investigate the n-p CSB phenomenon, one can calculate some scattering quantities, such as scattering amplitude, polarization, analysing power etc. One of the most important characteristic quantities is the isospin mixing parameter. From the equ.(2.29), recalling that $L=J$ here, we define an unitary matrix in terms of an isospin (hence spin) mixing parameter $\bar{\gamma}_J$.

$$U_{JS,\alpha}^J = \begin{bmatrix} \cos \bar{\gamma}_J & \sin \bar{\gamma}_J \\ -\sin \bar{\gamma}_J & \cos \bar{\gamma}_J \end{bmatrix} \quad (2.34)$$

Here the 4 by 4 scattering matrix S is reduced to a 2 by 2 CSB one.

$$\begin{array}{cc} s=0 & s=1 \\ \nu=0 & \nu=1 & \nu=0 & \nu=-1 \end{array} \quad \begin{array}{c} s'=0, \nu'=0 \\ \nu'=1 \\ s'=1, \nu'=0 \\ \nu'=-1 \end{array} \left[\begin{array}{cc} \left(\begin{array}{cc} s & \\ s & \end{array} \right)_{2 \times 2} & s \\ s & \end{array} \right]_{4 \times 4} \rightarrow S_{2 \times 2} \quad (2.35)$$

and the form of the S-matrix for $\nu'=0,1$; $\nu=0,1$ is

$$S_{J's',JS}^J = \begin{bmatrix} \cos(2\bar{\gamma}_J) \exp(2i\bar{\delta}_{JJo}) & i \sin(2\bar{\gamma}_J) \exp[i(\bar{\delta}_{JJo} + \bar{\delta}_{J11})] \\ i \sin(2\bar{\gamma}_J) \exp[i(\bar{\delta}_{JJo} + \bar{\delta}_{J11})] & \cos(2\bar{\gamma}_J) \exp(2i\bar{\delta}_{J11}) \end{bmatrix} \quad (2.36)$$

where use has been made of the "Stapp" or "bar" phase shift parametrisation [21]. Another parametrisation is proposed by Blatt and Biedenharn [22] and the CSB S-matrix has the form

$$S_{J'S',JS}^J = \begin{bmatrix} \cos^2 \gamma_T e^{2i\delta_{JT0}} + \sin^2 \gamma_T e^{2i\delta_{JT1}} & \frac{1}{2} \sin 2\gamma_T (e^{2i\delta_{JT0}} - e^{2i\delta_{JT1}}) \\ \frac{1}{2} \sin 2\gamma_T (e^{2i\delta_{JT0}} - e^{2i\delta_{JT1}}) & \cos^2 \gamma_T e^{2i\delta_{JT0}} + \sin^2 \gamma_T e^{2i\delta_{JT1}} \end{bmatrix} \quad (2.37)$$

However, it is quite easy to show that [23]

$$\begin{aligned} \bar{\delta}_{JT0} + \bar{\delta}_{JT1} &= \delta_{JT0} + \delta_{JT1} \\ \sin(\bar{\delta}_{JT0} - \bar{\delta}_{JT1}) &= \tan 2\bar{\gamma}_T / \tan 2\gamma_T \\ \sin(\delta_{JT0} - \delta_{JT1}) &= \sin 2\bar{\gamma}_T / \sin 2\gamma_T \end{aligned} \quad (2.38)$$

It is more convenient to use "bar" parametrisation in the n-p one-photon exchange interaction.

Now as $J=L'=L$, $\nu=1$, $\nu'=0$ we have the sub-matrix element

$$\begin{aligned} S_{T0,T1}^J &= i \sin(2\bar{\gamma}_T) \exp[i(\bar{\delta}_{JT0} + \bar{\delta}_{JT1})] \\ &= i \sin(2\bar{\gamma}_T) \exp[i(\delta_{JT0} + \delta_{JT1})] \end{aligned} \quad (2.39)$$

where the different phase shifts correspond to the spin singlet and triplet transition state (JLS). Making use of equ.(2.23), we have the CSB scattering T-matrix element in terms of phase shift and the "bar" mixing parameter

$$T_{T0,T1}^J = -\frac{1}{2\pi\rho_e} \sin(2\bar{\gamma}_T) \exp[i(\delta_{JT0} + \delta_{JT1})] \quad (2.40)$$

Since CSB is a weak breaking of symmetry, $\overline{\gamma}_J$ is a small quantity, then equ.(2.40) becomes

$$\overline{\gamma}_J = -\pi \rho_\epsilon T_{J0,J1}^J \exp[-i(\delta_{JJ0} + \delta_{JJ1})] \quad (2.41)$$

As long as the transition matrix is known, $\overline{\gamma}_J$ can be calculated directly.

There are many ways to calculate the T-matrix. A.Gersten used the Born approximation in the helicity representation [12,24] and C.Y.Cheung computed it with the distorted wave Born approximation [25]. The result of Cheung is [26]

$$\delta T_{fi} = \frac{1}{2\pi} \langle \Phi_f^- | V^I | \Phi_i^+ \rangle \quad (2.42)$$

and for $S_f=0$, $S_i=1$

$$\delta T_{S1} = \sum_J \frac{8}{\pi} (2J+1) G_J \exp[i(\delta_{JJ0} + \delta_{JJ1})] \cdot \exp(im_i \phi) P_J^1(\theta) (-m_i) \langle J1; Jm_i | J01m_i \rangle \quad (2.43)$$

where

$$G_J = \int dr r^2 f(r) R_{JJ0}(r) R_{JJ1}(r) \quad (2.44)$$

as $m_i=1$,

$$\delta T_{S1} = \sum_J \frac{8}{\sqrt{2}\pi} (2J+1) G_J \exp[i(\delta_{JJ0} + \delta_{JJ1})] e^{i\phi} P_J^1(\theta) \quad (2.45)$$

where R_{JJ0} and R_{JJ1} are radial wave functions for $s=0$ and $s=1$, and $f(r)$ is the radial dependence for various CSB mechanisms.

Before starting to work out T_{S_1} by an operator procedure, the spherical coordinate frame with $(\hat{k}_f, \hat{\theta}, \hat{\phi})$ is suggested to be used here. $\hat{k}_f, \hat{\theta}, \hat{\phi}$ are as shown in Fig.2, with

$$\begin{aligned}\cos \theta &= \hat{k}_f \cdot \hat{k} \\ \sin \theta \hat{\phi}_f &= \hat{k} \times \hat{k}_f \\ \hat{\theta}_f &= \hat{\phi}_f \times \hat{k}_f\end{aligned}\tag{2.46}$$

then

$$\nabla_{k_f} = \hat{k}_f \frac{\partial}{\partial k_f} + \hat{\theta}_f \frac{1}{k_f} \frac{\partial}{\partial \theta} + \hat{\phi}_f \frac{1}{k_f \sin \theta} \frac{\partial}{\partial \phi}\tag{2.47}$$

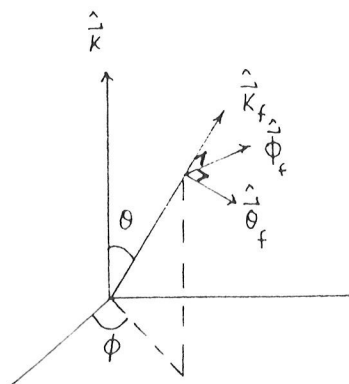


Fig 2. Illustration of the unit vectors

In terms of equ.(2.26), the matrix element of $S_f = \nu' = 0$, $S_i = 1$, $\nu = 1$ can be calculated as

$$\delta T_{S1} = \langle \hat{k}', 00 | T(k) | \hat{k}, 11 \rangle = \sum_{l'l'} \langle \hat{k}', 1'00 | y^J | \hat{k}, 111 \rangle T_{l'l}^J \quad (2.48)$$

and for $l=l'=J$,

$$\delta T_{S1} = \sum_J \langle \hat{k}', J00 | y^J | \hat{k}, J11 \rangle T_{J0,J1}^J \quad (2.49)$$

After some calculation (see Appendix A(2)), we have the CSB scattering matrix element

$$\delta T_{S1} = \frac{1}{J^2} e^{i\phi} \sum_J \frac{(2J+1)a_J}{\sqrt{J(J+1)}4\pi} P_J^1(\theta) T_{J0,J1}^J \quad (2.50)$$

where a_J is a coefficient which is equal to 1 [19]. Combining equ.(2.50) with equ.(2.19,44,45), eventually we obtain the "bar" mixing parameter in the DWBA as the following,

$$\bar{\gamma}_J = -16\pi kM \sqrt{J(J+1)} G_J \quad (2.51)$$

where k is the momentum, M is the nucleon mass and G_J is computed with the help of the Paris potential [27] and the 1983 phase shift analysis by Arndt et al[18]. In principle, after δT is known, in terms of equ.(2.14,15), the difference of polarization and analyzing power etc. can be computed as well.

CHAPTER III. CSB CLASS IV FORCE MECHANISMS

In general, charge asymmetry and charge dependent effects can be broken up into direct and indirect interactions [3]: The direct interaction includes a) the coulomb force with or without nucleon form factor, b) the vacuum polarization, c) nucleon mass difference, d) the magnetic forces. These effects are present even in the absence of hadronic forces. The indirect interaction includes a) the mass differences of neutral and charged mesons which are exchanged between nucleons, b) mass differences of baryons on meson exchange, c) radiative correction to meson-nucleon coupling constants, d) meson mixing caused by electromagnetic forces, e) a force which arises from meson and photon interaction, f) life-time of the meson exchanged. These forces cannot occur when there is no hadronic force. In this work, only three effects are discussed.

3.1. THE DIRECT ELECTROMAGNETIC EFFECT

The considerable success of quantum electrodynamics in accounting for the properties of the electron-photon interaction has been developed to deal with the electromagnetic interaction of elementary particles. The general electromagnetic Lagrangian density can be written in the form [5]

$$L_{em}(x) = e \Gamma_{\lambda}^{(em)}(x) A_{\lambda}(x) \quad (3.1)$$

where A_{λ} is the four-vector potential describing the electromagnetic field and $\Gamma_{\lambda}^{(em)}$ is the total electromagnetic current which satisfies the conservation relation $\partial_{\lambda} \Gamma_{\lambda}^{(em)} = 0$. Further, $\Gamma_{\lambda}^{(em)}$ can be decomposed into hadronic and leptonic parts each separately conserved, thus

$$\Gamma_{\lambda}^{(em)} = \Gamma_{\lambda} + \ell_{\lambda}^{(em)} \quad (3.2)$$

The leptonic part has contributions from both electrons and muons, which will not be discussed here. For Γ_{λ} , the hadronic part, it is generally assumed that it consists of isovector and isoscalar parts and is expressed with Lagrangian field theory. However, of more importance is the use of Lorentz invariance arguments to express the matrix element of Γ_{λ} between hadronic states in terms of form factors. In the n-p system and isospin notation, the form factors are defined as

$$\begin{aligned} F_1^S &= \frac{1}{2} (F_1^{(p)} + F_1^{(n)}) & F_2^S &= \frac{1}{2} (F_2^{(p)} + F_2^{(n)}) \\ F_1^V &= \frac{1}{2} (F_1^{(p)} - F_1^{(n)}) & F_2^V &= \frac{1}{2} (F_2^{(p)} - F_2^{(n)}) \end{aligned} \quad (3.3a)$$

$$\begin{aligned} F_1^{(p)}(0) &= 1 & F_2^{(p)}(0) &= K_p \\ F_1^{(n)}(0) &= 0 & F_2^{(n)}(0) &= K_n \end{aligned} \quad (3.3b)$$

where the superscripts (s) and (v) indicate isoscalar and isovector respectively. In the limit of zero momentum transfer (non-relativistic case), $K^2=0$, one has

$$\begin{aligned} F_1^S(0) &= 1/2 & F_2^S(0) &= (K_p + K_n)/2 \\ F_1^V(0) &= 1/2 & F_2^V(0) &= (K_p - K_n)/2 \end{aligned} \quad (3.3c)$$

and a form factor relation [26]

$$F_1^V F_2^S - F_1^S F_2^V = \frac{K_n}{2} [G_p^E(q)]^2 \left[\mu_p - \frac{K_n}{1 + \frac{q^2}{4M^2}} \right] \quad (3.3d)$$

where $G_p^E(q) = (1 + q^2/Q^2)^{-2}$, with $Q^2 = 18.23 \text{ fm}^{-2}$, and $K_p = \mu_p - 1$, $K_n = \mu_n$ are the anomalous magnetic moments of the proton and neutron respectively. μ_p is the magnetic moment.

Thus, the direct electromagnetic effect with one photon exchange is shown as Fig.3

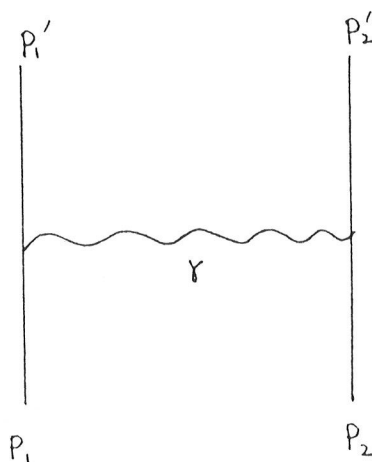


Fig 3. EM effect of CSB

The nonrelativistic photon-nucleon (OPE) interaction Hamiltonian is [28]

$$H_{\gamma NN} = \bar{\Psi} e \{ [F_1^S + F_1^V \tau_3] \gamma^\mu A_\mu - (F_2^S + F_2^V \tau_3) \frac{1}{2M} \sigma^{\mu\nu} (\nabla_\nu A_\mu) \} \Psi \quad (3.4)$$

where $F_1^S, F_1^V, F_2^S, F_2^V$ are defined as in equ.(3.3c). Through the application of Feynman Rules and transformation of scattering amplitude to nuclear potential, i.e.

$$V = \frac{1}{8\pi^3} \int \exp(i\vec{q} \cdot \vec{r}) T d^3q \quad (3.5)$$

the class IV part of the interaction potential is [26]

$$V_{\gamma}^N = -\frac{e^2}{4\pi} \frac{K_n}{4M^2} [\tau_3(1) - \tau_3(2)] (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} \left\{ \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} \right) + \frac{Q^8}{3!} \left(\frac{d^2}{dQ^2} \right)^3 \right. \\ \left. \left[\left(\frac{1}{Q^2} - \frac{K_p}{4M^2 - Q^2} \right) \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} e^{-Qr} \right) + \frac{K_p}{4M^2 - Q^2} \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} e^{-2Mr} \right) \right] \right\} \quad (3.6)$$

If we write down the class IV potential in the form

$$V^N = f(r) [\tau_3(1) - \tau_3(2)] (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} \quad (3.7)$$

we have for one-photon exchange force

$$f(r) = -\frac{e^2}{4\pi} \frac{K_n}{4M^2} \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} \right) + \text{form-factor contribution} \quad (3.8)$$

where the form-factor contribution is of the form in equ.(3.6), and is to be considered in our computation.

3.2. $\rho^0 - \omega$ MIXING

Another source of CSB is the electromagnetic mixing of neutral mesons of the same spin and parity, but of different isospin. Mixing occurs when a perturbing Hamiltonian H' violates some quantum number; thus when the total Hamiltonian ($H_0 + H'$) is diagonalized, one has eigenstates with definite quantum numbers. One effect is $\rho^0 - \omega$ mixing, which can be experimentally observed and the mixing fraction of electromagnetic is of the order of 4% [6]. From the Feynman graph shown in Fig 4,

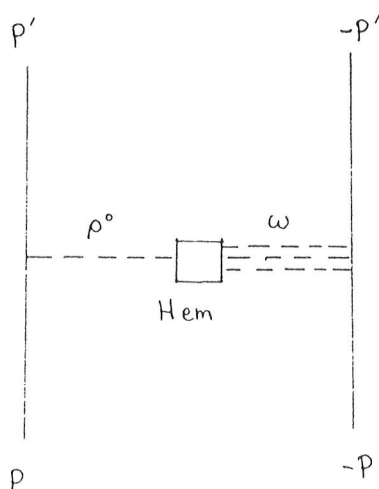


Fig 4. $\rho^0 - \omega$ mixing effect of CSB

the neutron-proton scattering matrix in the non-relativistic limit, with the Born approximation is given by [6]

$$\begin{aligned}
 T_{NN}^{\rho^0\omega} = & - \frac{g_\rho g_\omega \langle \omega | H_{em} | \rho^0 \rangle}{(q^2 + m_\rho^2)(q^2 + m_\omega^2)} \left\{ [\tau_3(1) + \tau_3(2)] \cdot \right. \\
 & \left\{ 1 - \frac{K^V q^2}{4M^2} + \frac{(1+K^V)}{4M^2} \left[\vec{S}_T q^2 - \frac{2}{3} \vec{\sigma}(1) \cdot \vec{\sigma}(2) q^2 \right] \right. \\
 & + \frac{3/2 + K^V}{4M^2} i (\vec{\sigma}(1) + \vec{\sigma}(2)) \cdot \vec{q} \times \vec{P} \left. \right\} \\
 & + [\tau_3(1) - \tau_3(2)] \frac{K^V}{4M^2} i [\vec{\sigma}(1) - \vec{\sigma}(2)] \cdot \vec{q} \times \vec{P} \left. \right\}
 \end{aligned}
 \tag{3.9}$$

where $\vec{q} = \vec{p}' - \vec{p}$, $\vec{P} = \vec{p}' + \vec{p}$, $q_0 = 0$ and $\vec{S} = \vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$ is the tensor operator. K^V is the isovector magnetic moment = 3.7. Here we have ignored the term of order P^2/M^2 and the normalization factor obtained from the conversion from the Bethe-Salpeter equation to the Blankenbecler-Sugar equation. Similarly to use equ.(3.5), one can obtain the class IV potential part of the ρ^0 - ω mixing

$$\begin{aligned}
 V_{\rho\omega}^{IV} = & - \frac{g_\rho g_\omega K_n}{4\pi 4M^2} \frac{\langle \omega | H_{em} | \rho^0 \rangle}{m_\omega^2 - m_\rho^2} [\tau_3(1) - \tau_3(2)] (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} \\
 & \cdot \frac{1}{r} \frac{d}{dr} \frac{1}{r} (e^{-m_\rho r} - e^{-m_\omega r})
 \end{aligned}
 \tag{3.10}$$

where the mixing matrix element $\langle \omega | H_{em} | \rho^0 \rangle$ is predicted by several methods. For example, Henley and Keliher [6] give

$$\begin{aligned}
 \langle \omega | H_{em} | \rho^0 \rangle & \approx 2m_\rho (m_\omega - m_\rho) (0.0045 - 0.1894 q^2/m_\rho^2) \\
 & \approx -3.7 \times 10^3 \text{ Mev}^2 \quad \text{at the pole } q^2 = m_\rho^2
 \end{aligned}
 \tag{3.11a}$$

In our calculation, the experimental value is adopted,

$$\langle \omega | H_{em} | \rho^0 \rangle = -3.4 \times 10^3 \text{ Mev}^2 \quad (3.11b)$$

Some numerical results of nucleon-nucleon phase shift have been calculated by taking into account the appropriate superposition of one-boson exchange amplitude with a form factor. In principle, having replaced the integral numerically by discretized weighted power series while solving the Lippmann-Schwinger equation, a phase shift can be obtained. However, extensive analysis gives good agreement with experimental data [18]. Also, the force constants can be chosen quite differently [29]. In terms of the Vector Dominance Model (VDM) [30], based on the hypothesis of virtual photons interaction with hadrons through an intermediate neutral vector meson, one obtains the constants as

$$f_{\rho\pi\pi}^2/4\pi = 2.43, \quad g_\rho^2/4\pi = 2.56. \quad (3.12)$$

In experimental measurements of $\pi^+\pi^-$ production by e^+e^- annihilation in the range of the ρ -energy, the coupling constants are

$$f_{\rho\pi\pi}^2/4\pi = 2.84, \quad g_\rho^2/4\pi = 2.26 \quad (3.13a)$$

similarly in the ω -energy range, the ω -production by e^+e^- annihilation gives

$$g_\omega^2/4\pi = 18.4 \quad (3.13b)$$

In our computation, the meson coupling constants from equ.(3.13b) and from the experimental data,

$$g_{\omega}^2/4\pi = 2.4 \quad (3.14)$$

are used.

Unlike the class IV forces, the class III forces do not give isospin mixing. Then, because of parity conservation, there is no spin mixing. An example of this is the π^0 - η mixing (see Appendix B). The scattering matrix is calculated with the help of the DWBA. Similarly to the class IV force, the scattering angular momentum coupling parameter can be computed from the known ϵ_J -T matrix elements relations by Goldberger and Watson [19].

3.3. NUCLEON-MASS DIFFERENCE

It is well known that the mass of the neutron is greater than that of the proton ($\Delta m/m = 0.14\%$). Especially in the low-energy case, it has a non-negligible effect on CSB. The neutron-proton mass difference can introduce CSB into the pion-nucleon interaction shown in Fig.5 ,

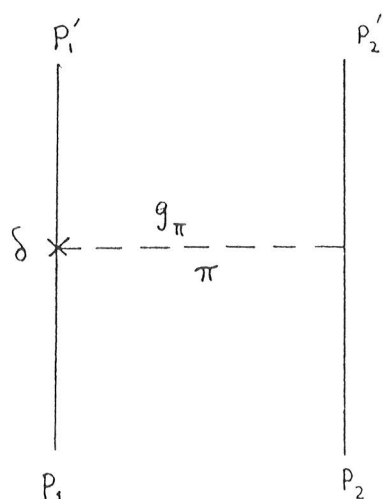


Fig 5. MD effect of CSB

There are two types of pion-nucleon couplings. One is pseudovector, i.e.

$$H_{PV} = \frac{f}{\mu} \sum_{i=1}^3 \bar{\Psi} \gamma_{\mu} \gamma_5 \tau_i \Psi \nabla^{\mu} \Phi_i \quad (3.15)$$

and the other is pseudoscalar,

$$H_{PS} = -g i \sum_{i=1}^3 \bar{\Psi} \tau_i \gamma_5 \Psi \Phi_i \quad (3.16)$$

Since the spin of π^0 is zero, and experiment shows π^0 is pseudoscalar, π^0 -N interaction requires parity conservation so that we only need consider pseudoscalar πNN coupling. In the nonrelativistic limit, the total interaction Hamiltonian can be written as [26]

$$H_{\pi NN} = H_0 + H_1 + H_2 \quad (3.17)$$

where

$$H_0 = -\frac{g_{\pi}}{2M} \vec{\sigma} \cdot \nabla \tau \cdot \vec{\Phi} \quad (3.18)$$

$$H_1 = -\delta \frac{g_{\pi}}{2M} \vec{\sigma} \cdot \nabla \Phi_3 \quad (3.19)$$

$$H_2 = -\delta \frac{g_{\pi}}{2M} \vec{\sigma} \cdot (\vec{p}' + \vec{p}) (\vec{\tau} \times \vec{\Phi})_3 \quad (3.20)$$

$$\delta = \frac{m_n - m_p}{m_n + m_p} \quad (3.21)$$

It is clear that H_0 is the unperturbed Hamiltonian which gives the CS part, and H_1 and H_2 are both CI and CS

violating. Because H_1 is not proportional to either $\tau_3(1)-\tau_3(2)$ or $(\vec{\tau}(1) \times \vec{\tau}(2))_3$, we will not discuss it here, only the term H_2 gives rise to the class IV force. A direct application of the Hamiltonian H_2 to the Feynman diagram (Fig.5) gives the invariant scattering amplitude in the center of mass frame as

$$T_{fi}^{IV} = 2\delta\left(\frac{g_\pi}{2M}\right)^2 V^2(q) \frac{-1}{\vec{q}^2 + m_\pi^2} (\vec{\tau}_1 \times \vec{\tau}_2)_3 i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot (\vec{p}_1 \times \vec{p}_2) \quad (3.22)$$

By using equ.(3.5), the class IV potential part is

$$V_\delta^{IV} = -\frac{g_\pi^2}{4\pi} \frac{\delta}{2M^2} (\vec{\tau}_1 \times \vec{\tau}_2)_3 (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{L} \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} e^{-m_\pi r} \right) \quad (3.23)$$

where the pion-nucleon form factor $V^2(q)$ in equ.(3.22) has been set to unity. In our computation, the coupling constant $g_\pi^2/4\pi=14.5$ will be used.

CHAPTER IV. COMPUTATION ,RESULTS AND DISCUSSION

4.1. COMPUTATION

To calculate the distorted wave function, we need a model for the (charge-symmetric) strong interaction in the two-body system. In the short-range part of the nucleon-nucleon interaction, because of heavier system and subhadronic structure effects, boson-exchange models have no validity for the potential in the range of $r \leq 0.8$ fm, therefore the short range ($r \rightarrow 0$) can only be given phenomenologically. Here the Paris potential [31,32] is used.

From ordinary quantum mechanics, the radial Schrödinger equation is

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2\mu r^2} + V(r) \right] \chi_{K,L,S}(r) = E_{K,L,S} \chi_{K,L,S}(r) \quad (4.1)$$

where μ is the reduced mass of neutron-proton system, and $\chi_{K,L,S}$ depends on spin and satisfies the boundary condition

$$\chi_{K,L,S}(r=0) = 0 \quad (4.2)$$

The Paris potential consists of various terms, i.e. singlet, triplet, orbital-spin coupling, tensor, quadratic LS coupling, and is of the form [32]

$$V(\vec{r}, \vec{p}^2) = V_0(r, p^2)\Omega_0 + V_1(r, p^2)\Omega_1 + V_{LS}(r)\Omega_{LS} + V_T(r)\Omega_T + V_{S02}(r)\Omega_{S02} \quad (4.3)$$

where

$$\begin{aligned} \Omega_0 &= \frac{1}{4}(1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) & \rightarrow 1 & \quad S=0 \\ \Omega_1 &= \frac{1}{4}(3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2) & \rightarrow 1 & \quad S=1 \\ \Omega_{LS} &= \vec{L} \cdot \vec{S} & \begin{cases} -1 & S=1 \\ 0 & S=0 \end{cases} & \quad (4.4) \\ \Omega_T &= 3 \frac{\vec{\sigma}_1 \cdot \vec{r} \vec{\sigma}_2 \cdot \vec{r}}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 & \begin{cases} 2 & S=1 \\ 0 & S=0 \end{cases} \\ \Omega_{S02} &= \frac{1}{2}(\vec{\sigma}_1 \cdot \vec{L} \vec{\sigma}_2 \cdot \vec{L} + \vec{\sigma}_2 \cdot \vec{L} \vec{\sigma}_1 \cdot \vec{L}) & \begin{cases} 1-L(L+1) & S=1 \\ -L(L+1) & S=0 \end{cases} \end{aligned}$$

and their corresponding quantum numbers are listed for $J=L$ [33]. Each $V(r, p^2)$ has the form

$$V(r, p^2) = V^a(r) + \frac{p^2}{m} V^b(r) + V^b(r) \frac{p^2}{m} \quad (4.5)$$

with

$$p^2 = -\hbar^2 \left[\frac{1}{r} \frac{d^2}{dr^2} r - \frac{\vec{L}^2}{r^2} \right] \quad (4.6)$$

here $\vec{L}$ is angular momentum operator, and m is the nucleon mass which is taken as 938.2592 Mev for $I=1$, and 938.9055

MeV for $I=0$. For each $V(r)$, a set of parameters g_j and m_j are introduced phenomenologically, i.e.

$$V(r) = \sum_{j=1}^{n=12} g_j F(m_j r) \frac{e^{-m_j r}}{m_j r} \quad (4.7)$$

where

$$\begin{aligned} F(m_j r) &= 1 \quad \text{for } V_o^a, V_o^b, V_i^a, V_i^b \\ F(m_j r) &= \frac{1}{m_j r} + \frac{1}{(m_j r)^2} \quad \text{for } V_{LS} \\ F(m_j r) &= 1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2} \quad \text{for } V_T \\ F(m_j r) &= \frac{1}{(m_j r)^2} \left[1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2} \right] \quad \text{for } V_{S02} \end{aligned} \quad (4.8)$$

g_j , m_j are provided in [32] and some missing g -factors are computed by making use of the formulae,

$$\begin{aligned} g_n &= -m_n \sum_{j=1}^{n-1} \frac{g_j}{m_j} \quad \text{for } V_o^a, \text{ and } V_o^b \\ \left. \begin{aligned} g_{n-1} &= \frac{m_{n-1}^3}{m_{n-1}^2 - m_n^2} \left(m_n^2 \sum_{j=1}^{n-2} \frac{g_j}{m_j^3} - \sum_{j=1}^{n-2} \frac{g_j}{m_j} \right) \\ g_n &= \frac{m_n^3}{m_n^2 - m_{n-1}^2} \left(m_{n-1}^2 \sum_{j=1}^{n-2} \frac{g_j}{m_j^3} - \sum_{j=1}^{n-2} \frac{g_j}{m_j} \right) \end{aligned} \right\} \quad \text{for } V_{LS}, V_T \\ g_{n-2} &= \frac{m_{n-2}^5}{(m_n^2 - m_{n-2}^2)(m_{n-1}^2 - m_{n-2}^2)} \left[-m_{n-1}^2 m_n^2 \sum_{j=1}^{n-3} \frac{g_j}{m_j^5} \right. \\ &\quad \left. + (m_{n-1}^2 + m_n^2) \sum_{j=1}^{n-3} \frac{g_j}{m_j^3} - \sum_{j=1}^{n-3} \frac{g_j}{m_j} \right] \quad \text{for } V_{S02} \end{aligned} \quad (4.9)$$

and two other relations which can be deduced by circular permutation of $n-2$, $n-1$, and n .

Substituting Equ.(4.3-6) into Equ.(4.1), the Schrödinger equation with the Paris potential can be written as

$$\begin{aligned} (1+2V^b\Omega) \frac{d^2X_l}{dr^2} = \frac{m}{\hbar^2} \left[-E + V^a\Omega + V_{LS}\Omega_{LS} + V_T\Omega_T \right. \\ \left. + V_{S02}\Omega_{S02} \right] X_l + \left[(1+2V^b\Omega) \frac{l(l+1)}{r^2} - \frac{d^2V^b}{dr^2}\Omega \right] X_l \\ - 2 \frac{dV^b}{dr}\Omega \frac{dX_l}{dr} \end{aligned} \quad (4.10)$$

where $m \approx 2\mu$, $\Omega = \Omega_0$ for $I=0$ and $=\Omega_1$ for $I=1$.

For angular momentum greater than 0, at $r \leq 0.1$ fm and $r \geq 10$ fm, the asymptotic behaviours of the potential are studied. Solving the asymptotic form of the Equ.(4.10) by power series expansion (see Appendix A(4)), we calculated the initial wave function at $r \leq 0.1$ fm as

$$X_l \propto r^{1+\sqrt{2l(l+1)}} \quad (4.11)$$

It is well known that the strong interaction is a short range interaction while the electromagnetic force is rather long range force, so the numerical integration is carried out to infinity. However, for large r , the strong interaction is essentially zero, so we use the asymptotic wave function which oscillates as a sine function for $r > 17$ fm, i.e.

$$X_l \propto \sin(kr - \frac{\pi}{2}l + \delta_{lS}) \quad (4.12)$$

where k is determined by the c.m. energy E and δ_{lS} has different for $S=0$ and $S=1$. Here $r=17$ fm is assumed to be the effective range after which the scattering potential has no significant contribution, therefore the out-going wave function becomes a plane wave.

The wave function is normalized with the help of the experimental phase shift [18], for angular momentum smaller and equal to 6, and by theoretical formula [34] when L is greater than 6. It is important to choose the effective range properly such that the normalization reference wave function (equ.(4.12)) avoids a zero value at the effective range for all the angular momentum states.

4.2. RESULTS AND DISCUSSIONS

The results of $\overline{\gamma}_j$ for different effects, different energies, and various angular momenta are shown in tables 2 to 9. At the highest c.m. energies the phase shifts have an imaginary part which is neglected in this work. It is not known at which energy this neglect begins to become significant. (Form factor is considered only in the electromagnetic effect.)

TABLE 2

Mixing parameter $\bar{\gamma}_J$ when CM energy=10 Mev

	orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
			ρ^0 - ω mixing	Mass differ.
1		0.4871×10^{-2}	0.1820×10^{-3}	-0.2020×10^{-2}
2		0.3831×10^{-2}	0.2268×10^{-4}	-0.7051×10^{-3}
3		0.2528×10^{-2}	0.2026×10^{-6}	-0.1252×10^{-3}
4		0.2139×10^{-2}	0.4896×10^{-8}	-0.2976×10^{-4}
5		0.1634×10^{-2}	0.6887×10^{-10}	-0.6992×10^{-5}
6		0.1595×10^{-2}	0.1245×10^{-11}	-0.1708×10^{-5}

TABLE 3

Mixing parameter $\bar{\gamma}_J$ when CM energy=50 Mev

	orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
			ρ^0 - ω mixing	Mass differ.
1		0.7722×10^{-2}	0.1543×10^{-2}	-0.6981×10^{-2}
2		0.9550×10^{-2}	0.9233×10^{-3}	-0.6865×10^{-2}
3		0.5585×10^{-2}	0.3474×10^{-4}	-0.2003×10^{-2}
4		0.4621×10^{-2}	0.4184×10^{-5}	-0.1020×10^{-2}
5		0.3544×10^{-2}	0.2400×10^{-6}	-0.4482×10^{-3}
6		0.3061×10^{-2}	0.2020×10^{-7}	-0.2234×10^{-3}

TABLE 4

Mixing parameter $\bar{\gamma}_J$ when CM energy=150 Mev

	orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
			ρ^0 - ω mixing	Mass differ.
1		0.8262×10^{-2}	0.4716×10^{-2}	-0.1122×10^{-1}
2		0.1291×10^{-1}	0.4977×10^{-2}	-0.1521×10^{-1}
3		0.9111×10^{-2}	0.6531×10^{-3}	-0.6686×10^{-2}
4		0.8093×10^{-2}	0.2017×10^{-3}	-0.4676×10^{-2}
5		0.6247×10^{-2}	0.2756×10^{-4}	-0.2556×10^{-2}
6		0.5423×10^{-2}	0.5837×10^{-5}	-0.1679×10^{-2}

TABLE 5

Mixing parameter $\bar{\gamma}_J$ when CM energy=250 Mev

	orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
			ρ^0 - ω mixing	Mass differ.
1		0.7637×10^{-2}	0.6658×10^{-2}	-0.1250×10^{-1}
2		0.1166×10^{-1}	0.7182×10^{-2}	-0.1639×10^{-1}
3		0.1120×10^{-1}	0.1798×10^{-2}	-0.9566×10^{-2}
4		0.1006×10^{-1}	0.8333×10^{-3}	-0.7835×10^{-2}
5		0.8027×10^{-2}	0.1697×10^{-3}	-0.4697×10^{-2}
6		0.7022×10^{-2}	0.5118×10^{-4}	-0.3344×10^{-2}

TABLE 6

Mixing parameter $\bar{\gamma}_J$ when CM energy=300 Mev

	orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
			ρ^0 - ω mixing	Mass differ.
1		0.7248×10^{-2}	0.7252×10^{-2}	-0.1268×10^{-1}
2		0.1112×10^{-1}	0.7894×10^{-2}	-0.1657×10^{-1}
3		0.1030×10^{-1}	0.2410×10^{-2}	-0.1049×10^{-1}
4		0.1078×10^{-1}	0.1294×10^{-2}	-0.9235×10^{-2}
5		0.8593×10^{-2}	0.2976×10^{-3}	-0.5617×10^{-2}
6		0.7704×10^{-2}	0.1024×10^{-3}	-0.4177×10^{-2}

TABLE 7

Mixing parameter $\bar{\gamma}_J$ when CM energy=350 Mev

	orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
			ρ^0 - ω mixing	Mass differ.
1		0.6982×10^{-2}	0.7798×10^{-2}	-0.1291×10^{-1}
2		0.1043×10^{-1}	0.8263×10^{-2}	-0.1627×10^{-1}
3		0.1063×10^{-1}	0.3101×10^{-2}	-0.1157×10^{-1}
4		0.1134×10^{-1}	0.1811×10^{-2}	-0.1047×10^{-1}
5		0.9113×10^{-2}	0.4654×10^{-3}	-0.6492×10^{-2}
6		0.8165×10^{-2}	0.1746×10^{-3}	-0.4893×10^{-2}

TABLE 8

Mixing parameter $\bar{\gamma}_J$ when CM energy=400 Mev

orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
		ρ^0 - ω mixing	Mass differ.
1	0.6454×10^{-2}	0.7899×10^{-2}	-0.1252×10^{-1}
2	0.9080×10^{-2}	0.7866×10^{-2}	-0.1473×10^{-1}
3	0.1055×10^{-1}	0.3682×10^{-2}	-0.1216×10^{-1}
4	0.1168×10^{-1}	0.2343×10^{-2}	-0.1148×10^{-1}
5	0.9588×10^{-2}	0.6717×10^{-3}	-0.7337×10^{-2}
6	0.8612×10^{-2}	0.2710×10^{-3}	-0.5605×10^{-2}

TABLE 9

Mixing parameter $\bar{\gamma}_J$ when CM energy=500 Mev

orbital angular momentum L=J	EM effect	mixing parameter $\bar{\gamma}_J$	
		ρ^0 - ω mixing	Mass differ.
1	0.6022×10^{-2}	0.8513×10^{-2}	-0.1266×10^{-1}
2	0.9404×10^{-2}	0.9347×10^{-2}	-0.1625×10^{-1}
3	0.1099×10^{-1}	0.5043×10^{-2}	-0.1388×10^{-1}
4	0.1200×10^{-1}	0.3397×10^{-2}	-0.1302×10^{-1}
5	0.1044×10^{-1}	0.1188×10^{-2}	-0.8950×10^{-2}
6	0.9745×10^{-2}	0.5546×10^{-3}	-0.7189×10^{-2}

From tables 2 to 9, we can see that $\bar{\gamma}_J$ is essentially a small quantity, which means that CSB is a weak violation. EM

and MD give larger isospin mixing than ρ^0 - ω does. Also EM, ρ^0 - ω have the same sign of mixing but MD has the opposite sign, which is due to the sign of the class IV forces. When the c.m. energy is larger than (about) 100 Mev, MD gives a larger CSB contribution than EM does. Our calculation shows that $\bar{\gamma}_J$ has a quite interesting J-dependence. According to traditional ideas, the magnitude of the scattering quantities would have a monotonic decreasing trend as the angular momentum increases. However, from our results, for the c.m. energy below 350 Mev, $\bar{\gamma}_J$ has a maximum value at $\bar{\gamma}_2$, which means that the strongest isospin mixing occurs in D-states. For the EM effect, as the energy increases, the maximum point shifts to G-states but the magnitude of $\bar{\gamma}_J$ decreases again. The physical reason needs to be further studied. Perhaps it is due to the weak behaviour of the tensor force on P,F-states, or caused by the dominant L, L' coupling of the Paris potential on D (and G) angular momentum states. As the c.m. energy reaches about 300 Mev, CSB has a most significant maximum value. This suggests that the Lab energy around 600 Mev would be a good experimental energy.

To compare with the result given by the Born approximation [12], we tested $\bar{\gamma}_J$ only for EM at c.m. energy 100 Mev. The results and the differences are listed in table 10.

TABLE 10

Mixing parameter from BA and DWBA at c.m.E=100 Mev

J=L	$\overline{\gamma}_J$ (EM with BA)	$\overline{\gamma}_J$ (EM with DWBA)	$\Delta\overline{\gamma}_J / \overline{\gamma}_J$
1	0.2024×10^{-1}	0.8354×10^{-2}	58.7%
2	0.1169×10^{-1}	0.1230×10^{-1}	5.2%
3	0.8265×10^{-2}	0.7640×10^{-2}	7.6%
4	0.6402×10^{-2}	0.6565×10^{-2}	2.5%
5	0.5227×10^{-2}	0.5104×10^{-2}	2.4%
6	0.4417×10^{-2}	0.4396×10^{-2}	0.5%

The extreme differences at $J=1$ indicate that the distorted wave has a large influence on P-states, it implies that the DWBA provides more information about orbital angular momentum coupling affecting the isospin coupling than the BA does, hence the DWBA is a better approximation for the n-p scattering procedure. On the other hand, replacing the spin dependent radial wave functions $R_{J\gamma 0}$ and $R_{J\gamma 1}$ by the spherical Bessel function j_l , our $\overline{\gamma}_J$ goes back exactly to the expression for $\overline{\gamma}_J$ given by the BA [35].

CHAPTER V. SUMMARY AND CONCLUSION

We have examined the CSB in the neutron-proton scattering caused by the Class IV type of forces. A formula for the isospin mixing parameter $\bar{\gamma}_T$ is derived in a new way with the distorted wave Born approximation. The results show that $\bar{\gamma}_T$ is a small quantity of the order of 1.6×10^{-2} for the largest MD effect. EM and MD give larger CSB than ρ^0 - ω does, and MD has a stronger CSB than EM when the c.m.energy is greater than 100 Mev. For c.m.energy above 50 Mev, the results give a quite untraditional angular momentum dependence. CSB does not occur on S-states and has a maximum distribution about D-states. For the EM effect, the higher the energy, the larger the "anomaly" is. Because of the tensor force, D,G-states have the stronger isospin mixing, a result which cannot be obtained by the simple Born approximation. According to our calculation, around c.m.energy 300 Mev, (Lab 600 Mev), the largest breaking of charge symmetry occurs.

APPENDIX A. DETAILED CALCULATION OF T_{S1}

A.1. THE CALCULATION OF THE CSB AMPLITUDE f_B

Since the total scattering matrix is given by equ.(2.9), the CSB amplitude can be calculated in the spherical coordinate frame, i.e.

$$\begin{aligned}\hat{n} &= \hat{n}(-\sin\phi, \cos\phi, 0) \\ \hat{l} &= \hat{l}(\sin\frac{\theta}{2}\cos\phi, \sin\frac{\theta}{2}\sin\phi, \cos\frac{\theta}{2}) \\ \hat{m} &= \hat{m}(-\cos\frac{\theta}{2}\cos\phi, -\cos\frac{\theta}{2}\sin\phi, \sin\frac{\theta}{2})\end{aligned}$$

then immediately one can calculate the CSB amplitude f_B as

$$\begin{aligned}\text{Tr} [T(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n}] \\ = \text{Tr} \{if_B [(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n}]^2\} \\ = if_B \text{Tr}[2(\vec{\sigma}_1 \cdot \hat{n})^2] \\ = 8i f_B\end{aligned}\tag{A1.1}$$

since it is quite easy to see that other terms vanish except

$$f_B = \frac{1}{8i} \text{Tr}[T(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n}]\tag{A1.2}$$

Now to evaluate the matrix element,

$$\begin{aligned}\langle s' \nu' | (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n} | s \nu \rangle \\ = \frac{1}{2i} \langle s' \nu' | e^{-i\phi} (\sigma_{+1} - \sigma_{+2}) - e^{i\phi} (\sigma_{-1} - \sigma_{-2}) | s \nu \rangle\end{aligned}\tag{A1.3}$$

Directly, we get the elements as in table 11.

TABLE 11. Matrix element $\langle s' \nu' | (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{n} | s \nu \rangle$

$\nu' \backslash \nu$		s=1			s=0
		1	0	-1	0
s'=1	1	0	0	0	$\sqrt{2}i\exp(-i\phi)$
	0	0	0	0	0
	-1	0	0	0	$\sqrt{2}i\exp(i\phi)$
s'=0	0	$-\sqrt{2}i\exp(i\phi)$	0	$-\sqrt{2}i\exp(-i\phi)$	0

Note that all the CS terms have been calculated [19] before. By combining all the CSB terms with the CS matrix elements, one gets the matrix form of equ.(2.9) as

$$T = \begin{bmatrix} T_{11} & T_{10} & T_{1-1} & T_{1S} \\ T_{01} & T_{00} & T_{0-1} & T_{0S} \\ T_{-11} & T_{-10} & T_{-1-1} & T_{-1S} \\ T_{S1} & T_{S0} & T_{S-1} & T_{SS} \end{bmatrix} \quad (A1.4)$$

where

$$\begin{aligned} T_{11} &= T_{-1-1} = a + \delta \sin^2 \frac{\theta}{2} + \epsilon \cos^2 \frac{\theta}{2} \\ T_{10} &= -T_{-10} = \sqrt{2}\gamma - \frac{1}{\sqrt{2}}\delta \sin \theta + \frac{1}{\sqrt{2}}\epsilon \sin \theta \\ T_{1-1} &= T_{-11} = -\beta + \cos^2 \frac{\theta}{2} \delta + \epsilon \sin^2 \frac{\theta}{2} \\ T_{01} &= -T_{0-1} = -\sqrt{2}\gamma - \frac{\delta}{\sqrt{2}} \sin \theta + \frac{\epsilon}{\sqrt{2}} \sin \theta \\ T_{00} &= a + \beta + \delta \cos \theta - \epsilon \cos \theta \end{aligned} \quad (A1.5)$$

$$T_{1S} = -T_{S-1} = f_B \sqrt{2} i \exp(-i\phi)$$

$$T_{-1S} = -T_{S1} = f_B \sqrt{2} i \exp(i\phi)$$

$$T_{0S} = T_{S0} = 0 \quad T_{SS} = \alpha - \beta - \delta - \epsilon$$

A.2. THE DETAILED CALCULATION OF CSB T-MATRIX ELEMENT T_{S1}

$$\begin{aligned}
 T_{S1} &= \langle \hat{k}', 00 | T | \hat{k}, 11 \rangle \\
 &= \sum_{JL'L} \langle \hat{k}', 1'00 | Y^J | \hat{k}, 111 \rangle T_{L'0,L1}^J \Big|_{J=L'=L} \\
 &= \sum_{\substack{JL'L \\ Mm'm}} Y_{L'}^{m'}(\hat{k}') Y_L^{m*}(\hat{k}) \langle 1'0, m'0 | 1'0; JM \rangle \langle 11, m1 | 11; JM \rangle T_{L'0,L1}^J \Big|_{J=L'=L} \\
 &= \sum_{\substack{JL'L \\ Mm'm}} Y_{L'}^{m'}(\hat{k}') Y_L^{m*}(\hat{k}) \langle 1'0, m'0 | \frac{1}{\sqrt{J(J+1)}} (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} | 1'1; JM \rangle \\
 &\quad \cdot \langle 11; JM | 11, m1 \rangle T_{L'0,L1}^J \Big|_{J=L'=L} \\
 &= \sum_{\substack{JL'L \\ m'm}} Y_{L'}^{m'}(\hat{k}') Y_L^{m*}(\hat{k}) \langle 1'0, m'0 | (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} \Lambda_{J,J} | 11, m1 \rangle \\
 &\quad \cdot \frac{1}{\sqrt{J(J+1)}} T_{L'0,L1}^J \Big|_{J=L'=L} \tag{A2.1}
 \end{aligned}$$

where use has been made of (see Appendix A(3))

$$\frac{1}{\sqrt{J(J+1)}} (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} | J1; JM \rangle = | J0; JM \rangle \tag{A2.2}$$

and $\Lambda_{J,J} = \sum_M | J1; JM \rangle \langle J1; JM |$ is a projection operator for states with $J=L$. Then, in the specific angular momentum representation,

$$\begin{aligned}
 T_{S1} &= \sum_{JL'Lm'm} \left(\int d\Omega_q d\Omega_s \right) Y_{L'}^{m'}(\hat{k}') Y_L^{m*}(\hat{q}) u_{00}^* \\
 &\quad \cdot \frac{1}{\sqrt{J(J+1)}} (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} \Lambda_{J,J} Y_L^{m*}(\hat{k}) Y_L^m(\hat{q}) u_{11} T_{L'0,L1}^J \Big|_{J=L'=L} \tag{A2.3}
 \end{aligned}$$

where $d^3k = \rho_\epsilon d\epsilon d\Omega_q$, making use of two identities,

$$\delta_{\hat{k}', \hat{k}} = \sum_{l=0}^{\infty} \sum_{m=-l}^l Y_l^m(\hat{k}') Y_l^{m*}(\hat{k}) \quad (\text{A2.4})$$

$$\sum_{m=-l}^l Y_l^m(\hat{q}) Y_l^{m*}(\hat{k}) = \frac{2l+1}{4\pi} P_l(\hat{q} \cdot \hat{k}) \quad (\text{A2.5})$$

the equ.(A2.3) becomes

$$\begin{aligned} T_{S1} &= \sum_J (u_{00}, \frac{1}{\sqrt{J(J+1)}} \frac{2J+1}{4\pi} (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} \Lambda_{J,J} u_{11}) P_J(\hat{k}' \cdot \hat{k}) T_{J0, J1}^J \\ &= \sum_J \frac{2J+1}{4\pi \sqrt{J(J+1)}} (u_{00}, (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} \Lambda_{J,J} u_{11}) P_J(\cos\theta) T_{J0, J1}^J \end{aligned} \quad (\text{A2.6})$$

where $\cos\theta = \cos(\hat{k}' \cdot \hat{k})$ is defined as before. To express $\Lambda_{J,J}$ in terms of $(\vec{\sigma} \cdot \vec{L})$, we write it as

$$\Lambda_{J,J} = a_J + b_J (\vec{\sigma} \cdot \vec{L}) + c_J (\vec{\sigma} \cdot \vec{L})^2 \quad (\text{A2.7})$$

Writing the matrix element in the operator form, and separating it into three terms,

$$\begin{aligned} T_{S1} &= \sum_J (u_{00}, (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} [a_J + b_J (\vec{\sigma} \cdot \vec{L}) + c_J (\vec{\sigma} \cdot \vec{L})^2] u_{11}) P_J(\cos\theta) \\ &\quad \cdot \frac{1}{\sqrt{J(J+1)}} \frac{2J+1}{4\pi} T_{J0, J1}^J \\ &= \text{I} + \text{II} + \text{III} \end{aligned} \quad (\text{A2.8})$$

where

$$\begin{aligned} \text{first term} &= (u_{00}, \sum_J (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} a_J P_J(\cos\theta) u_{11}) T_{J0, J1}^J \frac{2J+1}{4\pi \sqrt{J(J+1)}} \\ &\quad (\text{A2.9}) \end{aligned}$$

$$\begin{aligned} \text{second term} = & (u_{00}, \sum_j (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} b_j (\vec{\sigma} \cdot \vec{L}) P_j(\cos\theta) u_{11}) \\ & \cdot \frac{1}{\sqrt{j(j+1)}} \frac{2j+1}{4\pi} T_{j0,j1}^j \end{aligned} \quad (\text{A2.10})$$

$$\begin{aligned} \text{third term} = & (u_{00}, \sum_j (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} c_j (\vec{\sigma} \cdot \vec{L})^2 P_j(\cos\theta) u_{11}) \\ & \cdot \frac{1}{\sqrt{j(j+1)}} \frac{2j+1}{4\pi} T_{j0,j1}^j \end{aligned} \quad (\text{A2.11})$$

Now let us see term I first,

$$\begin{aligned} I = & (u_{00}, \sum_j (\vec{s}_1 - \vec{s}_2) \cdot [\vec{k}_f \times \frac{1}{i} \nabla_{\vec{k}_f}] a_j P_j(\cos\theta) u_{11}) \\ & \cdot T_{j0,j1}^j \frac{1}{\sqrt{j(j+1)}} \frac{2j+1}{4\pi} \\ = & (u_{00}, (\vec{s}_1 - \vec{s}_2) u_{11}) \cdot \sum_j a_j \left[-\frac{1}{i} \frac{\partial}{\partial \phi} P_j^1(\cos\theta) \right] T_{j0,j1}^j \frac{2j+1}{4\pi \sqrt{j(j+1)}} \end{aligned} \quad (\text{A2.12})$$

here use has been made of the equ.(2.47) with the fact that P_j is only a function of θ , and of the property of the Legendre function,

$$P_l^m = \sin\theta \frac{d^m}{(d\cos\theta)^m} P_l(\cos\theta) \quad (\text{A2.13})$$

Making use of the identity, which is easily verified,

$$(\vec{s}_1 - \vec{s}_2) u_{1\nu} = u_{00} \hat{e}_\nu \quad (\text{A2.14})$$

where the unit vector $\hat{e}_\nu$ is defined as

$$\begin{aligned} \hat{e}_{+1} &= -\frac{1}{\sqrt{2}} (\hat{e}_x + i \hat{e}_y) \\ \hat{e}_0 &= \hat{e}_z \end{aligned} \quad (\text{A2.15})$$

$$\hat{e}_{-1} = \frac{1}{\sqrt{2}} (\hat{e}_x - i \hat{e}_y)$$

and $\hat{e}_x, \hat{e}_y, \hat{e}_z$ are ordinary Cartesian coordinates. Therefore the equ.(A2.12) becomes

$$\begin{aligned} I &= (u_{00}, \hat{e}_{+1} u_{00}) \cdot \hat{\phi} \sum_J a_J P_J^1(\cos\theta) T_{J0,J1}^J \frac{i}{4\pi} \frac{2J+1}{J(J+1)} \\ &= (\hat{e}_{+1} \cdot \hat{\phi}) \sum_J a_J P_J^1(\cos\theta) T_{J0,J1}^J \frac{i}{4\pi} \frac{2J+1}{J(J+1)} \end{aligned} \quad (A2.16)$$

Thus, we have the first term

$$I = \frac{1}{\sqrt{2}} \exp(i\phi) \sum_J a_J P_J^1(\cos\theta) T_{J0,J1}^J \frac{1}{4\pi} \frac{2J+1}{J(J+1)} \quad (A2.17)$$

where $\hat{e}_{+1} \cdot \hat{\phi} = -\frac{1}{\sqrt{2}} i \exp(i\phi)$ has been used.

For the second term II, we only need calculate the part $(\vec{s}_1 - \vec{s}_2) \cdot \vec{L} (\vec{\sigma} \cdot \vec{L}) P_J(\cos\theta) u_{11}$, i.e.

$$\begin{aligned} & (\vec{s}_1 - \vec{s}_2) \cdot (\vec{k}_f \times \frac{1}{i} \nabla_{\vec{k}_f}) (\vec{\sigma} \cdot \vec{k}_f \times \frac{1}{i} \nabla_{\vec{k}_f}) P_J u_{11} \\ &= (\vec{s}_1 - \vec{s}_2) (\vec{k}_f \times \frac{1}{i} \nabla_{\vec{k}_f}) \vec{\sigma} \cdot (-\frac{1}{i} \hat{\phi} P_J^1) u_{11} \\ &= (\hat{k}_f \times \nabla_{\vec{k}_f} P_J^1) \cdot (\vec{s}_1 - \vec{s}_2) 2 (\vec{s} \cdot \hat{\phi}) u_{11} \\ &= (\hat{k}_f \times \nabla_{\vec{k}_f} P_J^1) \cdot (\vec{s}_1 - \vec{s}_2) i \sqrt{2} \exp(i\phi) u_{10} \end{aligned} \quad (A2.18)$$

here use has been made of the identity

$$2 (\vec{s} \cdot \hat{\phi}) u_{11} = i \sqrt{2} \exp(i\phi) u_{10} \quad (A2.19)$$

therefore, one has

$$(\vec{s}_1 - \vec{s}_2) \cdot \vec{L} (\vec{\sigma} \cdot \vec{L}) P_J u_{11} = i \sqrt{2} \exp(i\phi) \hat{\phi} \cdot \frac{\partial}{\partial \theta} P_J^1(\cos\theta) u_{00} \hat{e}_0$$

$$=0$$

$$(A2.20)$$

since $\hat{\vec{e}}_0$ is perpendicular to $\hat{\vec{\phi}}$. Hence the second term gives no contribution to T_{S1} .

For the third term, recall that

$$(\vec{\sigma} \cdot \vec{L})^2 = 2\vec{L}^2 + (\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L}) + (\vec{\sigma}_2 \cdot \vec{L})(\vec{\sigma}_1 \cdot \vec{L}) \quad (A2.21)$$

and

$$(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L})u_{11} = (\vec{\sigma}_2 \cdot \vec{L})(\vec{\sigma}_1 \cdot \vec{L})u_{11} \quad (A2.22)$$

$$(\vec{s}_1 - \vec{s}_2) \cdot \vec{L} (\vec{\sigma} \cdot \vec{L})^2 P_J(\cos\theta) u_{11}$$

$$= (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} (2\vec{L}^2 + 2(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L})) P_J(\cos\theta) u_{11} \quad (A2.23)$$

here the first part $2\vec{L}^3(\vec{s}_1 - \vec{s}_2) P_J(\cos\theta) u_{11}$ is

$$\begin{aligned} & 2(\vec{s}_1 - \vec{s}_2) \left(\hat{k}_f \times \frac{\hat{\vec{\phi}}}{\theta_f} \frac{1}{i} \right) \frac{\partial}{\partial \theta} \left(-\frac{\partial^2}{\partial \theta^2} \right) P_J(\cos\theta) u_{11} \\ &= -2u_{00} (\hat{\vec{e}}_{+1} \cdot \hat{\vec{\phi}}) \frac{1}{i} \frac{\partial^3}{\partial \theta^3} P_J(\cos\theta) \\ &= \sqrt{2} u_{00} \exp(i\phi) \frac{\partial^3}{\partial \theta^3} P_J(\cos\theta) \end{aligned} \quad (A2.24)$$

the second part $2(\vec{s}_1 - \vec{s}_2) \cdot \vec{L} (\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L}) P_J(\cos\theta) u_{11}$ is

$$\begin{aligned} & 2(\vec{s}_1 - \vec{s}_2) \cdot \vec{L} (\vec{\sigma}_1 \cdot \vec{L}) (2\vec{s}_2 \cdot \frac{\hat{\vec{\phi}}}{i}) \frac{\partial}{\partial \theta} P_J(\cos\theta) u_{11} \\ &= -2(\vec{s}_1 - \vec{s}_2) \cdot \frac{\hat{\vec{\phi}}}{i} (2\vec{s}_1 \cdot \frac{\hat{\vec{\phi}}}{i}) (2\vec{s}_2 \cdot \frac{\hat{\vec{\phi}}}{i}) \frac{\partial^3}{\partial \theta^3} P_J(\cos\theta) u_{11} \\ &= 2(\vec{s}_1 - \vec{s}_2) \cdot \frac{\hat{\vec{\phi}}}{i} \exp(i2\phi) u_{1-1} \frac{\partial^3}{\partial \theta^3} P_J(\cos\theta) \end{aligned} \quad (A2.25)$$

since

$$2(\vec{s}_2 \cdot \frac{\hat{\vec{\phi}}}{i}) u_{11} = i \exp(i\phi) |\uparrow\downarrow\rangle \quad (A2.26)$$

and

$$-4(\vec{s}_1 \cdot \hat{\phi})(\vec{s}_2 \cdot \hat{\phi})u_{11} = \exp(i2\phi)u_{1-1} \quad (\text{A2.27})$$

thus the second part becomes

$$\begin{aligned} & 2(\vec{s}_1 - \vec{s}_2) \cdot \vec{L} (\vec{\sigma}_1 \cdot \vec{L}) (\vec{\sigma}_2 \cdot \vec{L}) P_T(\cos\theta) u_{11} \\ &= 2 \frac{1}{i} u_{00} \hat{e}_{-1} \cdot \hat{\phi} \exp(2i\phi) \frac{\partial^3}{\partial \theta^3} P_T(\cos\theta) \\ &= \sqrt{2} u_{00} \exp(i\phi) \frac{\partial^3}{\partial \theta^3} P_T(\cos\theta) \end{aligned} \quad (\text{A2.28})$$

where use has been made of $\hat{e}_{-1} \cdot \hat{\phi} = -\frac{1}{\sqrt{2}} i \exp(-i\phi)$. Obviously, the third term also vanishes, equ.(A2.24) cancels equ.(A2.28) each other. Substituting all three terms, one gets equ.(2.50).

A.3. PROOF OF THE IDENTITY (A2.2)

The Wigner 3-j symbol [36] is here used in angular momentum coupling.

$$\begin{aligned} & (\vec{s}_1 - \vec{s}_2) \cdot \vec{L} |J1, JM\rangle \\ &= \sum_{Mm\nu} \langle J1, m\nu | JM \rangle u_{00} \hat{e}_{\nu} \cdot \left[\frac{1}{\sqrt{2}} \hat{e}_{-1} \sqrt{J(J+1)-m(m+1)} Y_{J,m+1} \right. \\ & \quad \left. - \frac{1}{\sqrt{2}} \hat{e}_{+1} \sqrt{J(J+1)-m(m-1)} Y_{J,m-1} + m Y_{J,m} \hat{e}_0 \right] \end{aligned} \quad (\text{A3.1})$$

To calculate these terms separately, the first term of equ.(A3.1) is

$$\begin{aligned} & \sum_{Mm\nu} \langle J1, m\nu | JM \rangle u_{00} \frac{1}{\sqrt{2}} \hat{e}_{\nu} \cdot \hat{e}_{-1} \sqrt{J(J+1)-m(m+1)} Y_{J,m+1} \\ &= \sum_{Mm} -\langle J1, m1 | JM \rangle u_{00} \sqrt{J(J+1)} \langle J1, m-1 | JM \rangle Y_{J,m+1} \end{aligned} \quad (\text{A3.2})$$

where use has been made of an identity

$$\hat{e}_q \cdot \hat{e}_{-q'} (-1)^q = \delta_{q,q'} \quad (\text{A3.3})$$

with $q, q' = 1, 0, -1$. Making use of some identities,

$$\begin{aligned} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} &= (-1)^{j_1 - j_2 - m_3} (2j_3 + 1)^{-\frac{1}{2}} \langle j_1 m_1, j_2 m_2 | j_1 j_2, j_3 - m_3 \rangle \\ \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} &= (-1)^{j_1 + j_2 + j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ -m_1 & -m_2 & -m_3 \end{pmatrix} \\ \sum_{j_3 m_3} (2j_3 + 1) \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ m'_1 & m'_2 & m_3 \end{pmatrix} &= \delta_{m_1 m'_1} \delta_{m_2 m'_2} \\ \sum_{m_1 m_2} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j'_3 \\ m_1 & m_2 & m'_3 \end{pmatrix} &= (2j_3 + 1)^{-1} \delta_{j'_3, j_3} \delta_{m'_3, m_3} \delta(j_1 j_2 j_3) \quad (\text{A3.4}) \end{aligned}$$

where $\delta(j_1 j_2 j_3) = 1$ if the three j 's satisfy the triangular condition and is zero otherwise. Then the equ.(A3.2) can be easily calculated as

$$\text{The first term} = \sqrt{J(J+1)} |J0; JM\rangle \quad (\text{A3.5})$$

Similarly, the second term of the equ.(A3.1) is

$$\sum_{m m'} \langle J1, m\nu | JM \rangle u_{00} \hat{e}_\nu \cdot \left(-\frac{1}{\sqrt{2}}\right) \hat{e}_{+1} \sqrt{1(1+1) - m(m+1)} Y_{J, m-1} = -\sqrt{J(J+1)} u_{00} Y_{J, M} \quad (\text{A3.6})$$

and the third term has the same value as the first term. therefore, the sum of the three terms gives the equation

$$(\vec{s}_1 - \vec{s}_2) \cdot \vec{L} |J1, JM\rangle = \sqrt{J(J+1)} |J0; JM\rangle \quad (A3.7)$$

A.4. BEHAVIOURS OF THE ASYMPTOTIC WAVE FUNCTION AT $R=0.1$ FM

At $r \leq 0.1$ fm, the central force has the dominant effect while using the Paris potential model. The Schrödinger equation (4.10) can be written in an asymptotic form for c.m. energy smaller than the order of 1 Gev.

$$\frac{d^2 X_l}{dr^2} \approx \frac{-1}{1+2V^b \Omega} \left[\frac{d^2 V^b}{dr^2} \Omega X_l + 2 \frac{dV^b}{dr} \Omega \frac{dX_l}{dr} \right] + \frac{l(l+1)}{r^2} X_l \quad (A4.1)$$

in terms of the Paris parametrisation [32],

$$\left. \frac{dV^b}{dr} \right|_{r=0.1} = \sum_{j=1}^{12} g_j \frac{e^{-m_j r}}{m_j r} \left(-\frac{1}{r} - m_j \right)_{r=0.1} = -\left(\frac{1}{r} + \bar{m} \right) V^b(r=0.1)$$

$$\left. \frac{d^2 V^b}{dr^2} \right|_{r=0.1} = \frac{V^b}{r^2} \left[(1 + \bar{m} r)^2 + 1 \right]_{r=0.1} \quad (A4.2)$$

where $\bar{m}$ is a certain mean-value which makes the equations valid. Then the equ.(A4.1) becomes

$$\frac{d^2 X_l}{dr^2} = \left[\frac{l(l+1)-1}{r^2} - \frac{\bar{m}}{r} - \frac{\bar{m}^2}{2} \right] X_l + \left(\frac{1}{r} + \bar{m} \right) \frac{dX_l}{dr} \quad (A4.3)$$

To solve this differential equation, the power series expansion has been used, i.e. let $X_l \propto r^\rho$. Substituting this into equ.(A4.3) and remembering that r must be a real, unique variable, quite easily one has a relation for ρ ,

$$(\rho-1)^2 - 2[\rho(\rho-1) - \rho - 1(1+1) + 1] = 0 \quad (\text{A4.4})$$

due to the restriction of the convergence of the wave function at the origin, the only root accepted physically is

$$\rho = 1 + \sqrt{2l(l+1)} \quad (\text{A4.5})$$

hence, the asymptotic wave function for $r \leq 0.1$ is

$$x_l \propto r^{1 + \sqrt{2l(l+1)}} \quad (\text{A4.6})$$

APPENDIX B. DETAILED CALCULATION OF CLASS III
FORCE OF π^0 - η MIXING EFFECT

Unlike ρ^0 - ω mixing, π^0 - η mixing is expected to be smaller and has not been observed by direct experimental measurement. The matrix element $\langle \pi^0 | H_{em} | \eta \rangle$ can be inferred from the well-measured mass splitting of the SU(3) octet of pseudoscalar mesons [37]. Fig 6. shows the π^0 - η mixing,

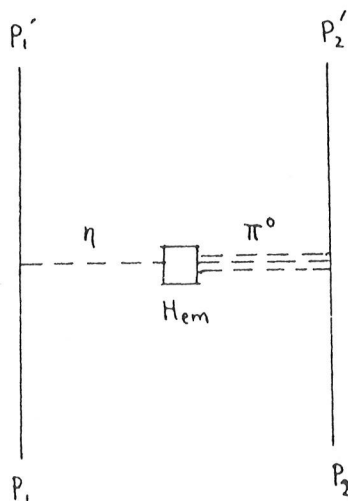


Fig 6. π^0 - η mixing effect of CSB

The interaction Hamiltonian through the pion-nucleon interaction is

$$H_I = ig_\eta \bar{\Psi} \eta \gamma_5 \Psi \quad (B.1)$$

With the Born approximation and in the non-relativistic limit, the class III part of the potential is

$$V_{\pi\eta} = -\frac{g_\pi g_\eta}{4\pi} \frac{\langle \eta | H_{em} | \pi^0 \rangle}{m_\eta^2 - m_\pi^2} \frac{\tau_3(1) + \tau_3(2)}{4M^2} \cdot \left\{ \vec{S}_T W_{\pi\eta}(r) + \frac{1}{3} \vec{\sigma}(1) \cdot \vec{\sigma}(2) V_{\pi\eta}(r) \right\} \quad (B.2)$$

where $w_{\pi\eta}$ and $v_{\pi\eta}$ are of the form

$$\begin{aligned} r V_{\pi\eta}(r) &= m_\pi^2 e^{-m_\pi r} - m_\eta^2 e^{-m_\eta r} \\ r W_{\pi\eta}(r) &= m_\pi^2 \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right) e^{-m_\pi r} \\ &\quad - m_\eta^2 \left(1 + \frac{3}{m_\eta r} + \frac{3}{(m_\eta r)^2} \right) e^{-m_\eta r} \end{aligned} \quad (B.3)$$

and $\vec{S}_T$ is defined as equ.(3.9). Because η - π^0 mixing is a class III force, it satisfies the isospin equation

$$\langle I=0 | \tau_3(1) + \tau_3(2) | I=1 \rangle = 0 \quad (B.4)$$

hence it has no contribution to n-p scattering, but only to n-n or p-p interactions.

The fact that a class III force does not give isospin mixing implies that $\Delta I=0$, considering the conservation of parity, angular momentum etc., $\Delta s=0$, the only possible transition matrix elements are

$$\langle IS'L' | T | ISL \rangle = \langle ISL' | T | ISL \rangle \quad (B.5)$$

where $I=S=0,1$ for odd L, L' and $I \neq S; I, S=0,1$ for even L, L' . Obviously, the $I'=I=0 \quad \nu'=\nu=0$ case has no contribution to CSB since one can easily verify that

$$\langle I=0 | \tau_3(1) + \tau_3(2) | I=0 \rangle = 0 \quad (B.6)$$

and the contribution of $I=1$ is

$$\langle I=1, \nu_t' | \tau_3(1) + \tau_3(2) | I=1, \nu_t \rangle = \delta_{\nu_t', \nu_t} \cdot \nu_t \quad (B.7)$$

where the $\nu_t', \nu_t = 1, 0, -1$ are the components of the coupled isospin. Hence one can evaluate the mixing parameter by calculating one matrix element, say, T_{11} .

In the distorted wave Born approximation [25], the initial scattering wave function is

$$\Phi = \sqrt{\frac{2}{\pi}} \sum_{JML} \sqrt{4\pi(2l+1)} i^l j_l(kr) |ls; JM\rangle \langle ls; JM | ls, 0\nu\rangle \quad (B.8)$$

where the incident direction is chosen properly such that $m=0$. The distorted initial wave function is

$$\Phi_i^+ = \sqrt{\frac{2}{\pi}} \sum_{JLM} \sqrt{4\pi(2l+1)} i^l e^{i\delta(JLS)} R_{JLS}(r) |ls; JM\rangle \langle ls; JM | 10s\nu\rangle \quad (B.9)$$

and the final state is

$$\Phi_f^- = \sqrt{\frac{2}{\pi}} \sum_{JLMm} 4\pi i^L e^{-i\delta(JLS)} R_{JLS}(r) Y_{Lm}^*(\hat{k}') |1s; JM\rangle \langle 1s; JM | 1ms\nu\rangle \quad (B.10)$$

It is quite easy to see that if $S=0$, the singlet state does not have angular momentum mixing, hence no CSB occurs, and for the $S=1$ case,

$$\delta T_{fi} = \frac{1}{2\pi} \langle \phi_f^- | V^{\text{III}} | \phi_i^+ \rangle \quad (B.11)$$

$$\begin{aligned} \delta T_{fi} = \frac{1}{2\pi} \int \left\{ \sqrt{\frac{2}{\pi}} \sum_{JL'Mm} 4\pi i^{L'} e^{i\delta(J'L'S)} R_{J'L'S}(r) (-1)^{L'} Y_{L'm}(\hat{k}') \langle 1'1; JM | \right. \\ \cdot \langle 1'1; JM | 1m, 1\nu \rangle \} f(r) \langle I=1, \nu_t'=1 | [\tau_3(1) + \tau_3(2)] | I=1, \nu_t=1 \rangle \\ \cdot \left\{ \sqrt{\frac{2}{\pi}} \sum_{JLM} \sqrt{4\pi(2L+1)} i^L e^{i\delta(JLS)} R_{JLS}(r) | 11; JM \rangle \langle 11; JM | 11, 0\nu \rangle \right\} r^2 dr \end{aligned} \quad (B.12)$$

where $f(r)$ is the class III potential, i.e.

$$V^{\text{III}} = f(r) [\tau_3(1) + \tau_3(2)] \quad (B.13)$$

with

$$\begin{aligned} f_{\pi\eta}(r) = -\frac{g_\pi g_\eta}{4\pi} \frac{1}{4M^2} \frac{\langle \eta | H_{em} | \pi^0 \rangle}{m_\eta^2 - m_\pi^2} \\ \cdot \left\{ \vec{S}_\tau W_{\pi\eta}(r) + \frac{1}{3} \vec{\sigma}(1) \cdot \vec{\sigma}(2) V_{\pi\eta}(r) \right\} \end{aligned} \quad (B.14)$$

and $w_{\pi\eta}$, $v_{\pi\eta}$, $\vec{S}_\tau$ are defined as before. It is easy to work out some matrix elements and coefficients as [38]

$$3 \langle 1'1; JM | \vec{S}_\tau | 11; JM \rangle = \begin{cases} -\frac{2(J-1)}{2J+1} & 1'=1=J-1 \\ 2 & 1'=1=J \end{cases}$$

$$\left(\begin{array}{c} -\frac{2(J+2)}{2J+1} \quad l'=l=J+1 \end{array} \right)$$

$$\langle J+1, 1; JM | \vec{S}_T | J-1, 1; JM \rangle = 2\sqrt{J(J+1)} / (2J+1) \quad (\text{B.15.a})$$

and

$$\langle l' 1; JM | \vec{\sigma}_1 \cdot \vec{\sigma}_2 | l 1; JM \rangle = \sum_{\nu} \langle l' 1, 0 \nu | JM \rangle \langle l 1, 0 \nu | JM \rangle \delta_{l'l} \quad (\text{B.15.b})$$

for all $\nu' = \nu$. To calculate the two C-G coefficients in (B.12), Wigner 3-j symbols are used and the result is

$$\begin{aligned} & \sum_{Mm} \langle l' 1; JM | l' m, 1 \nu \rangle \langle l 1; JM | l 1, 0 \nu \rangle \\ &= \sum_{Mm} (-1)^{l'+l} (2J+1) \begin{pmatrix} 1 & J & l' \\ \nu' & -M & m \end{pmatrix} \begin{pmatrix} 1 & J & l \\ \nu & -M & 0 \end{pmatrix} \end{aligned} \quad (\text{B.16.a})$$

as the spin components $\nu' = \nu = 1$

$$\sum_{Mm} \langle l' 1; JM | l' m 11 \rangle \langle l 1; JM | l 1 01 \rangle = \frac{2J+1}{2l+1} \delta_{m0} \delta_{l'l} \quad (\text{B.16.b})$$

finally one obtains the CSB matrix δT_{11} as

$$\begin{aligned} \delta T_{11} = & \frac{8c}{3\pi} \nu_t \sum_J [a_J^{J+1} e^{2i\delta_{T,T+1}} P_{J+1}(\cos\theta) \\ & + a_J^J e^{2i\delta_{T,T,1}} P_J(\cos\theta) + a_J^{J-1} e^{2i\delta_{T,T+1}} P_{J-1}(\cos\theta)] \end{aligned} \quad (\text{B.17})$$

where

$$a_J^{J+1} = \int r^2 dr \sqrt{\frac{2J+1}{2J+3}} R_{T,T+1}^2(r) \left[-\frac{2(J+2)}{2J+1} w_{\pi\eta}(r) + \frac{J}{2(2J+3)} v_{\pi\eta}(r) \right] \quad (\text{B.18.a})$$

$$a_J^J = \int r^2 dr R_{T,T}^2(r) \left[2w_{\pi\eta}(r) + \frac{1}{2} v_{\pi\eta}(r) \right] \quad (\text{B.18.b})$$

$$a_J^{J-1} = \int r^2 dr \frac{2J+1}{\sqrt{2J-1}} R_{J,J-1}^2(r) \left[-\frac{2(J-1)}{2J+1} w_{\pi\eta}(r) + \frac{J+1}{2(2J-1)} v_{\pi\eta}(r) \right] \quad (\text{B.18.c})$$

where $w_{\pi\eta}$ and $v_{\pi\eta}$ are defined as before and

$$C = -\frac{g_\pi g_\eta}{4\pi} \frac{1}{4M^2} \frac{\langle \eta | H_{em} | \pi^0 \rangle}{m_\eta^2 - m_\pi^2} \quad (\text{B.19})$$

On the other hand, similar to equ.(2.34-40), the T-matrix can be expressed in terms of the mixing parameter ϵ_J [19] for $\nu' = \nu = 1$,

$$\langle k', 11 | T | k, 11 \rangle = \frac{-1}{8\pi^2 \rho_\epsilon} \sum_L [(L+2)a_L^{L+1} + (2L+1)a_L^L + (L-1)a_L^{L-1}] P_L \quad (\text{B.20})$$

where a_L^J are the functions of ϵ_J with $L=J\pm 1, J$ and have been calculated in [19]. From the equ.(B.17) and (B.20), the class III force mixing parameter ϵ_J can be calculated in principle.

APPENDIX C. PROGRAMME LISTING

```

//CSBPAR JOB ',, ,T=1M,L=3,I=10,C=0',GE,CLASS=1
// EXEC WATFIV,REGION=512K
//SYSIN DD *
$JOB WATFIV GE,NOEXT
C  PROGRAM CSB FOR SPIN MIXING PARAMETER
  DIMENSION XRL1(1000),XRL2(1000)
  DIMENSION GOA0(12),GOA1(12),G1A0(12),G1A1(12),GOB0(12),
*GOB1(12),G1B0(12),G1B1(12),GLS0(12),GLS1(12),GT0(12),GT1(12),
*GSO20(12),GSO21(12),ALM(12)
  EXTERNAL FEM,FPW,FMD
  INTEGER S,E
  DIMENSION GEM(10),GPW(10),GMD(10)
  DIMENSION GEM2(10),GPW2(10),GMD2(10)
  DIMENSION GEM1(10),GPW1(10),GMD1(10)
  DIMENSION D1(10),D2(10)
  COMMON /FCT/ CEM,CPW,CMD
  COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
  COMMON /ASY/ HCL,RE1,RE3,REF,REI
  COMMON /ZP/ ZPP,S,XER1,XER2
  COMMON /PASS/ DLM1
  DIMENSION W(32),ZPP(32)

  C
  C  WHOLE INTEGRATION IS DIVIDED INTO 3 REGIONS. N == THE NUMBER
  C  OF SUB-REGION IN THE MIDDLE REGION. MM == ANGULAR MOMENTUM.
  C  REF == THE UP BOUND OF ONE REGION AND REI == THE LOW BOUND.
  C  W == THE WEIGHT AND ZPP == THE ZERO POINTS OF LEGENDRE
  C  FUNCTION USED IN GAUSSIAN INTERGRATION.
  C  LL == ODD MOMENTUM ONLY, L == ISOSPIN AND SPIN LOOP INDEX.
  C  S == TOTAL SPIN. I == TOTAL ISOSPIN. E == CENTER OF MASS
  C  ENERGY. LM == MAXIMUM ORBITAL MOMENTUM. C == HBAR SQUARE
  C  OVER 2M, M == NUCLEON MASS. HC == LOW LIMIT. ER == EFFEC-
  C  TIVE RANGE. GEM IS THE ISOSPIN MIXING PARAMETER DEFINDED
  C  BY 'STAPP' OR 'BAR' PHASE SHIFT PARAMETRISATION GAMMA-BAR
  C  FOR EM EFFECT; GPW IS THAT FOR RHO-OMEGA MIXING;
  C  GMD IS FOR MASS DIFFERENCE; FEM, FPW, FMD ARE F FUNCTIONS
  C  FROM POTENTIAL 4. XER1,XER2 == XRL AT POINT ER.
  C

  READ (5,*) E,LM,CEM,CPW,CMD
  WRITE (6,10) E,CEM,CPW,CMD,LM
10  FORMAT(10X,'E=',I4,5X,'CEM=',F8.5,5X,'CPW=',F8.5,5X,'CMD=',F8.5,
*5X,'LM=',I2)
  DLM1=4.321
  CALL LIVERM (E,D1,D2)
  IF (DLM1.EQ.1.234) GO TO 15
  C  HERE 1.234 ,4.321 ARE ONLY THE PASS SYMBOLS.
  DO 152 I=1,6
  WRITE (6,*) 'D1=',D1(I),'D2=',D2(I),'MM=',I

```

```

152 CONTINUE
15  READ (5,*) N,ER,HC,HCL
    WRITE (6,*) N,ER,HC,HCL
    IF (N.EQ.0) N=12
    IF (ER.EQ.0) ER=17.0
    IF (HCL.EQ.0) HCL=0.000001

C
DO 68 I=1,12
  READ(5,*) G0A0(I),G0A1(I),G1A0(I),G1A1(I)
  READ(5,*) G0B0(I),G0B1(I),G1B0(I),G1B1(I)
  READ(5,*) GLS0(I),GLS1(I),GT0(I),GT1(I)
  READ(5,*) GSO20(I),GSO21(I),ALM(I)
C  WRITE(6,*) G0A0(I),G0A1(I),G1A0(I),G1A1(I)
C  WRITE(6,*) G0B0(I),G0B1(I),G1B0(I),G1B1(I)
C  WRITE(6,*) GLS0(I),GLS1(I),GT0(I),GT1(I)
C  WRITE(6,*) GSO20(I),GSO21(I),ALM(I)
68 CONTINUE
DO 72 I=1,16
  READ (5,*) ZPP(I),W(I)
C  WRITE (6,*) ZPP(I),W(I)
72 CONTINUE
DO 723 I=1,16
  ZPP(16+I)=-ZPP(17-I)
  W(16+I)=W(17-I)
C  WRITE (6,*) ZPP(16+I),W(16+I)
723 CONTINUE
C  CALCULATION OF XRJ IS DIVIDED INTO N DIVISIONS IN THE MIDDLE
C  REGION WHERE GAUSSIAN METHOD (32 POINTS) IS USED. ALL
C  THESE FUNCTION VALUES ARE SUMMED OVER MESH POINTS FROM THE
C  'STORAGE' ARRAY XRL1,XRL2
C  RE IS THE MIDDLE REGION LENGTH. RE1,RE3 ARE THE LENGTH OF THE
C  ASYMPTOTIC REGIONS. TRAPS IS A STANDARD TOUTINE WHICH MAKES
C  ANY 'UNDERFLOW' VALUES TO ZERO.
  CALL TRAPS (0,0,1000,0,0)
  RE=(ER-HC)/N
  RE1=(HC-HCL)/2
C  C=41.47
  DO 50 LL=1,LM,2
C  CHANGE LL FROM 1 TO 6 (ONE BY ONE), TO GET ALL THE W.F.
C  NORMALIZED EXACTLY AT THE MAXIMUM AMPLITUDE POSITIONS.
  MM=LL
C  TEST L=ONE EVEN NUMBER, 2 OR 4 OR ...
  IF (MOD(LL,2).EQ.0) GO TO 57
  I=0
  S=0
  BM=938.9055
  C=197.3**2/BM
C  THE POTENTIAL FOR SCHRODINGER'S EQUATION IS CALLED PARIS
C  OM0=(1-S1S2*4)/4
C  OM1=(3+S1S2*4)/4
C  OMLS=LS
C  OMT=3*(S1R)(S2R)*4/R**2-4*S1S2
C  OMSO2=(4*S1LS2L+4*S2LS1L)/2
  OM0=1.0
  OMLS0=0

```



```

OMT0=0
OMSO20=-MM*(MM+1)
DO 119 J1=1,3
R1=HCL+(J1-1)*RE1
XRL1(J1)=R1** (1+SQRT(-OMSO20*2+1.0))
XRL2(J1)=XRL1(J1)
119 CONTINUE
DO 85 LR=1,N
REI=RE*(LR-1)+HC
REF=REI+RE
CALL XRJ(V0A,V0B,ALM,G0A0,G0B0,GLS0,GT0,
*GSO20,OM0,OMLS0,OMT0,OMSO20,DXRL,MM,XRL1,XRL2)
85 CONTINUE
IF (DLM1.EQ.1.234) GO TO 45
IF (MM-6) 44,44,45
45 CALL PSH(D1,MM)
44 I=1
S=1
BM=938.2592
C=197.3**2/BM
ALM(1)=ALM(1)-0.01551
OM1=1.0
OMLS1=-1.0
OMT1=2.0
OMSO21=1-MM*(MM+1)
DO 88 LR=1,N
REI=RE*(LR-1)+HC
REF=REI+RE
CALL XRJ(V1A,V1B,ALM,G1A1,G1B1,GLS1,GT1,
*GSO21,OM1,OMLS1,OMT1,OMSO21,DXRL,MM,XRL1,XRL2)
88 CONTINUE
IF (DLM1.EQ.1.234) GO TO 83
IF (MM-6) 84,84,83
83 CALL PSH(D2,MM)
84 CALL ASSM(MM,XRL1,XRL2,D1,D2,W)
IF (LL.GE.LM) GO TO 50
MM=LL+1
C TEST L=ONE EVEN NUMBER, 2 OR 4 OR ...
57 OM0=1.0
OMLS0=0
OMT0=0
OMSO20=-MM*(MM+1)
OM1=1.0
OMLS1=-1.0
OMT1=2.0
OMSO21=1-MM*(MM+1)
I=1
S=0
BM=938.2592
C=197.3**2/BM
OMSO20=-MM*(MM+1)
REAL=MM*MM+MM
DO 117 J1=1,3
R1=HCL+(J1-1)*RE1
XRL1(J1)=R1** (1+SQRT(REAL*2+1.0))

```

```

117 XRL2(J1)=XRL1(J1)
CONTINUE
DO 92 LR=1,N
REI=RE*(LR-1)+HC
REF=REI+RE
CALL XRJ(V0A,V0B,ALM,G1A0,G1B0,GLS1,GT1,
*GSO21,OM0,OMLS0,OMT0,OMSO20,DXRL,MM,XRL1,XRL2)
92 CONTINUE
IF (DLM1.EQ.1.234) GO TO 67
IF (MM-6) 66,66,67
67 CALL PSH(D1,MM)
66 I=0
S=1
BM=938.9055
C=197.3**2/BM
ALM(1)=ALM(1)+0.0155106
OMSO21=1-MM*(MM+1)
DO 87 LR=1,N
REI=RE*(LR-1)+HC
REF=REI+RE
CALL XRJ(V1A,V1B,ALM,G0A1,G0B1,GLS0,GT0,
*GSO20,OM1,OMLS1,OMT1,OMSO21,DXRL,MM,XRL1,XRL2)
87 CONTINUE
IF (DLM1.EQ.1.234) GO TO 89
IF (MM-6) 81,81,89
89 CALL PSH(D2,MM)
81 CALL ASSM(MM,XRL1,XRL2,D1,D2,W)
50 CONTINUE
STOP
END

```

```

C
SUBROUTINE ASSM(MM,XRL1,XRL2,D1,D2,W)
DIMENSION XRL1(1000),XRL2(1000)
DIMENSION G0A0(12),G0A1(12),G1A0(12),G1A1(12),G0B0(12),G0B1(12),
*G1B0(12),G1B1(12),GLS0(12),GLS1(12),GT0(12),GT1(12),GSO20(12),
*GSO21(12),ALM(12)
EXTERNAL FEM,FPW,FMD
INTEGER S,E
DIMENSION GEM(10),GPW(10),GMD(10)
DIMENSION GEM2(10),GPW2(10),GMD2(10)
DIMENSION GEM1(10),GPW1(10),GMD1(10)
DIMENSION GEM3(10),GPW3(10),GMD3(10)
DIMENSION D1(10),D2(10)
COMMON /FCT/ CEM,CPW,CMD
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
COMMON /ASY/ HCL,RE1,RE3,REF,REI
COMMON /ZP/ ZPP,S,XER1,XER2
DIMENSION W(32),ZPP(32)
CALL NORM(XRL1,XRL2,D1,D2,MM)
CALL ASYMP1(GEM1,FEM,MM,XRL1,XRL2)
CALL ASYMP1(GPW1,FPW,MM,XRL1,XRL2)
CALL ASYMP1(GMD1,FMD,MM,XRL1,XRL2)
CALL GAUSSI(GEM2,FEM,XRL1,XRL2,W,MM,ZPP)
CALL GAUSSI(GPW2,FPW,XRL1,XRL2,W,MM,ZPP)
CALL GAUSSI(GMD2,FMD,XRL1,XRL2,W,MM,ZPP)

```

```

CALL GAS(GEM3,FEM,W,MM,XRL1,XRL2,D1,D2)
CALL GAS(GPW3,FPW,W,MM,XRL1,XRL2,D1,D2)
CALL GAS(GMD3,FMD,W,MM,XRL1,XRL2,D1,D2)
WRITE(6,44) GEM1(MM),GPW1(MM),GMD1(MM),MM
44  FORMAT(10X,'GEM1=',E13.6,5X,'GPW1=',E13.6,5X,'GMD1=',E13.6,
*      5X,'G IN REGION 1;L=',I2)
WRITE(6,55) GEM2(MM),GPW2(MM),GMD2(MM),MM
55  FORMAT(10X,'GEM2=',E13.6,5X,'GPW2=',E13.6,5X,'GMD2=',E13.6,
*      5X,'G IN REGION 2;L=',I2)
WRITE(6,66) GEM3(MM),GPW3(MM),GMD3(MM),MM
66  FORMAT(10X,'GEM3=',E13.6,5X,'GPW3=',E13.6,5X,'GMD3=',E13.6,
*      5X,'G IN REGION 3;L=',I2)
REAL=MM*(MM+1)
C   GAMMA-BAR=-SQRT(MM*(MM+1))*(16*PI*M*K)*INTEGRATION
GEM(MM)=(GEM1(MM)+GEM2(MM)+GEM3(MM))*SQRT(REAL*E/C)*(-239.1)
GPW(MM)=(GPW1(MM)+GPW2(MM)+GPW3(MM))*SQRT(REAL*E/C)*(-239.1)
GMD(MM)=(GMD1(MM)+GMD2(MM)+GMD3(MM))*SQRT(REAL*E/C)*(-239.1)
WRITE(6,67) GEM(MM),GPW(MM),GMD(MM),MM
67  FORMAT(10X,'GEM=',E13.6,5X,'GPW=',E13.6,5X,'GMD=',
*E13.6,5X,'MM=',I2)
C   THESE G'S ARE GAMMA-BAR PARAMETERS
RETURN
END

C
C   SUBROUTINE FOR ASYMPTOTIC FORM IN REGION ONE
SUBROUTINE ASYMP1(GL,FLL,MM,XRL1,XRL2)
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
COMMON /ASY/ HCL,RE1,RE3,REF,REI
DIMENSION GFU(3),GL(10)
DIMENSION XRL1(3),XRL2(3)
EXTERNAL FLL
DO 111 J1=1,3
R1=HCL+(J1-1)*RE1
GFU(J1)=FLL(R1)*XRL1(J1)*XRL2(J1)
111 CONTINUE
GL(MM)=((GFU(1)+GFU(3))/2+GFU(2))*RE1
RETURN
END

C
C   SUBROUTINE FOR RADIUS WAVE FUNCTION      (XRL=RADI.*R)
SUBROUTINE XRJ(VA,VB,AM,GA,GB,GLS,GT,
*GSO2,OM,OMLS,OMT,OMSO2,DXRL,MM,XRL1,XRL2)
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
COMMON /ASY/ HCL,RE1,RE3,REF,REI
COMMON /ZP/ ZPP,S,XER1,XER2
INTEGER S,E
DIMENSION AM(12),GA(12),GB(12),GLS(12),GT(12),GSO2(12)
DIMENSION XRL1(1000),XRL2(1000),ZPP(32)
GG=REI
REAL=MM*MM+MM
DO 200 L=1,32
G=(REF-REI)/2*ZPP(L)+(REF+REI)/2
IF (GG.NE.HC) GO TO 91
XRL=HC*(1+SQRT(REAL*2+1.0))
DXRL=(1+SQRT(REAL*2+1.0))*XRL/HC

```

```

91 CALL RUNGE(G,GG,XRL,DXRL,VA,VB,AM,GA,GB,GLS,GT,GSO2,OM,OMLS,
*OMT,OMSO2,MM)
C M IS MESH POINTS' POSITION
M=L+32*(LR-1)+3
IF (S.EQ.1) GO TO 121
XRL1(M)=XRL
GO TO 122
121 XRL2(M)=XRL
122 GG=G
IF (LR.NE.N) GO TO 200
IF (L.NE.32) GO TO 200
C TO CALCULATE XRL AT ER POINT
G=ER
CALL RUNGE(G,GG,XRL,DXRL,VA,VB,AM,GA,GB,GLS,GT,GSO2,OM,OMLS,
*OMT,OMSO2,MM)
IF (S.EQ.1) GO TO 77
XER1=XRL
DLXR=DXRL/XRL
GO TO 200
77 XER2=XRL
DLXR=DXRL/XRL
200 CONTINUE
RETURN
END

```

```

C
C ROUNGE-KUTTA METHOD
SUBROUTINE RUNGE(G,GG,XRL,DXRL,VA,VB,AM,GA,GB,GLS,GT,GSO2,
*OM,OMLS,OMT,OMSO2,MM)
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
COMMON /ASY/ HCL,RE1,RE3,REF,REI
DIMENSION AM(12),GA(12),GB(12),GLS(12),GT(12),GSO2(12)
DIMENSION XRL1(1000),XRL2(1000),ZPP(32)
IF (N.GT.6) GO TO 13
IR=10
GO TO 14
13 IR=2
14 ST=(G-GG)/IR
DO 210 I=1,IR
R=(I-1)*ST+GG
CALL V(R,VA,VB,VLS,VT,VSO2,DVB,DDVB,GA,GB,GLS,GT,GSO2,AM)
AK1=ST*Q(R,XRL,DXRL,VA,VB,VLS,VT,VSO2,DVB,DDVB,
*OM,OMLS,OMT,OMSO2,MM)
R=R+ST/2
CALL V(R,VA,VB,VLS,VT,VSO2,DVB,DDVB,GA,GB,GLS,GT,GSO2,AM)
AK2=ST*Q(R,XRL+ST*DXRL/2+ST/8*AK1,DXRL+AK1/2,
*VA,VB,VLS,VT,VSO2,DVB,DDVB,OM,OMLS,OMT,OMSO2,MM)
AK3=ST*Q(R,XRL+ST*DXRL/2+ST*AK2/8,DXRL+AK2/2,
*VA,VB,VLS,VT,VSO2,DVB,DDVB,OM,OMLS,OMT,OMSO2,MM)
R=R+ST/2
CALL V(R,VA,VB,VLS,VT,VSO2,DVB,DDVB,GA,GB,GLS,GT,GSO2,AM)
AK4=ST*Q(R,XRL+ST*DXRL+AK3*ST/2,DXRL+AK3,
*VA,VB,VLS,VT,VSO2,DVB,DDVB,OM,OMLS,OMT,OMSO2,MM)
XRL=XRL+ST*(DXRL+(AK1+AK2+AK3)/6)
DXRL=DXRL+(AK1+2*AK2+2*AK3+AK4)/6
210 CONTINUE

```

RETURN
END

C SUBROUTINE V(R,VA,VB,VLS,VT,VSO2,DVB,DDVB,GA,GB,GLS,GT,GSO2,AM)
DIMENSION AM(12),GA(12),GB(12),GLS(12),GT(12),GSO2(12)
CALL TRAPS(0,0,1000,0,0)
VA=0
VB=0
VLS=0
VT=0
VSO2=0
DVB=0
DDVB=0
DO 700 J=1,12
VVA=GA(J)*EXP(-AM(J)*R)/(AM(J)*R)
VVB=GB(J)*EXP(-AM(J)*R)/(AM(J)*R)
VVLS=GLS(J)*EXP(-AM(J)*R)/(AM(J)*R)*(1/(AM(J)*R)+1/(
*AM(J)*R)**2)
VVT=GT(J)*EXP(-AM(J)*R)/(AM(J)*R)*(1+3/(AM(J)*R)+3/(AM(
*J)*R)**2)
VVSO2=GSO2(J)*EXP(-AM(J)*R)/(AM(J)*R)**4
*(AM(J)*R+3+3/(AM(J)*R))
DVB1=-GB(J)*EXP(-AM(J)*R)/R*(1+1/(R*AM(J)))
DDVB1=GB(J)*EXP(-AM(J)*R)/R*(AM(J)+2/R+2/(AM(J)*R*R))
VA=VA+VVA
VB=VB+VVB
VLS=VLS+VVLS
VT=VT+VVT
VSO2=VSO2+VVSO2
DVB=DVB+DVB1
DDVB=DDVB+DDVB1
700 CONTINUE
RETURN
END

C SCHRODINGER'S EQUATION BECOMES $XDD = (2M/HB^{**2}/(1+2*VB*OM) * (-E+VLS*OMLS+VT*OMT+VSO2*OMSO2+VA*OM-HB^{**2}/M*VBDD*OM) + L * (L+1)/R^{**2}) * XRL - 2*VBD*XRLD*OM / (1+2*VB*OM)$
C
C
C FUNCTION Q(R,XRLI,DXRLI,VA,VB,VLS,VT,VSO2,DVB,DDVB,
*OM,OMLS,OMT,OMSO2,L)
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
INTEGER E
Q=((-E+VLS*OMLS+VT*OMT+VSO2*OMSO2+(VA*OM)
-C(DDVB*OM))/C/(1+2*VB*OM)+L*(L+1)/R**2)*XRLI-2*DVB*
*DXRLI*OM/(1+2*VB*OM)
RETURN
END

C SUBROUTINE FOR PHASE SHIFT DJLS
C SUBROUTINE PHSH(DL,L)
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
COMMON /ASY/ HCL,RE1,RE3,REF,REI
DIMENSION BJ(12),BN(12),DJ(11),DA(11)
C BJ,BN,DJ,DA ARE SPHERERICAL BESSEL AND NEUMAN FUNCTIONS
C AND THEIR DERIVATIVES

```
DIMENSION DL(10)
```

```
INTEGER E
```

```
R=ER
```

```
R=(REF-REI)/2*0.9972638+(REF+REI)/2
```

```
YK=SQRT(E/C)
```

```
Y=R*YK
```

```
BJ(1)=SIN(Y)/Y
```

```
BN(1)=-COS(Y)/Y
```

```
BJ(2)=(SIN(Y)/Y-COS(Y))/Y
```

```
BN(2)=-(COS(Y)/Y+SIN(Y))/Y
```

```
DO 10 M=1,L
```

```
BJ(M+2)=(2*M+1)/Y*BJ(M+1)-BJ(M)
```

```
BN(M+2)=(2*M+1)/Y*BN(M+1)-BN(M)
```

```
DJ(1)=-BJ(2)
```

```
DA(1)=-BN(2)
```

```
DO 20 M=1,L
```

```
DJ(M+1)=(M*BJ(M)-(M+1)*BJ(M+2))/(2*M+1)
```

```
DA(M+1)=(M*BN(M)-(M+1)*BN(M+2))/(2*M+1)
```

```
YJ=R*BJ(L+1)
```

```
YN=R*BN(L+1)
```

```
YDJ=BJ(L+1)+Y*DJ(L+1)
```

```
YDN=BN(L+1)+Y*DA(L+1)
```

```
TANDL=(YDJ-DLXR*YJ)/(YDN-DLXR*YN)
```

```
DL(L)=ATAN(TANDL)
```

```
WRITE (6,41) DL(L),L
```

```
FORMAT (10X,'DL=',F13.8,5X,'MM=',I2)
```

```
RETURN
```

```
END
```

```
SUBROUTINE NORM(XRL1,XRL2,D1,D2,L)
```

```
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
```

```
COMMON /ZP/ ZPP,S,XER1,XER2
```

```
COMMON /ASY/ HCL,RE1,RE3,REF,REI
```

```
INTEGER S,E
```

```
BJ,BN ARE SPHERICAL BESSEL AND NEUMAN FUNCTION VALUES AT ER
```

```
DIMENSION XRL1(1000),XRL2(1000),ZPP(32)
```

```
DIMENSION BJ(11),BN(11),D1(10),D2(10)
```

```
BJ AND BN ARE SPHERICAL BESSEL AND NEUMAN FUNCTIONS
```

```
YK=SQRT(E/C)
```

```
YK IS A FREE WAVE NUMBER
```

```
R=ER
```

```
Y=R*YK
```

```
BJ(1)=SIN(Y)/Y
```

```
BN(1)=-COS(Y)/Y
```

```
BJ(2)=(SIN(Y)/Y-COS(Y))/Y
```

```
BN(2)=-(COS(Y)/Y+SIN(Y))/Y
```

```
L1=L-1
```

```
IF (L1.EQ.0) GO TO 100
```

```
DO 10 M=1,L1
```

```
BJ(M+2)=(2*M+1)/Y*BJ(M+1)-BJ(M)
```

```
BN(M+2)=(2*M+1)/Y*BN(M+1)-BN(M)
```

```
YJ=R*BJ(L+1)
```

```
YN=R*BN(L+1)
```

```
YF1=COS(D1(L))*YJ-SIN(D1(L))*YN
```

```
YF2=COS(D2(L))*YJ-SIN(D2(L))*YN
```

```

M=32*N+3
FN1=YF1/XRL1(M)
FN2=YF2/XRL2(M)
C  FN IS THE FACTOR OF NORMALIZATION
DO 139 I=1,M
XRL1(I)=XRL1(I)*FN1
139 XRL2(I)=XRL2(I)*FN2
CONTINUE
RETURN
END

```

```

C
C  FUNCTION FEM(R)
COMMON /FCT/ CEM,CPW,CMD
REAL KP
C  GO TO 37
PM=938.9/197.33
KP=1.79
Q=SQRT(18.2)
F1=1/Q/Q
F2=1/(Q*Q-4*PM*PM)
F1D=-F1**2
F2D=-F2**2*KP
F1DD=2*F1**3
F2DD=2*F2**3*KP
F1DDD=-6*F1**4
F2DDD=-6*F2**4*KP
F2=F2*KP
H1=F1+F2
H1D=F1D+F2D
H1DD=F1DD+F2DD
H1DDD=F1DDD+F2DDD
H2=(1+Q*R)*EXP(-Q*R)
H2D=-R*R/2*EXP(-Q*R)
H2DD=R**3/4*EXP(-Q*R)/Q
H2DDD=-(R/2/Q)**3*H2
FEM=CEM*(-0.0210846)/R**3*(1+Q**8/6*(H1DDD*H2+3*H1DD*H2D
C7 *+3*H1D*H2DD+H1*H2DDD)-F2DDD*(1+2*PM*R)*EXP(-2*PM*R))
FEM=CEM*(-0.0210846)/R**3
RETURN
END

```

```

C
C  FUNCTION FPW(R)
COMMON /FCT/ CEM,CPW,CMD
C  CO=CPW*3.6907/1000*(-1)
UP=3.903287
UW=3.966025
FPW=((UP+1/R)*EXP(-UP*R)-(UW+1/R)*
*EXP(-UW*R))/R**2*CPW*(-0.0036907)
RETURN
END

```

```

C
C  FUNCTION FMD(R)
COMMON /FCT/ CEM,CPW,CMD
REAL MU

```



```

C CO=CMD*1.5275/100000
MU=137.26475/197.32858
FMD=(1+MU*R)*EXP(-MU*R)/R**3*CMD*1.5275/100000
RETURN
END

```

```

C USING GAUSSIAN METHOD FOR THE INTEGRATION (FOR N REGION)
C SUBROUTINE GAUSSI(GL,FLL,XRL1,XRL2,W,MM,ZPP)
EXTERNAL FLL
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
DIMENSION W(32),ZPP(32),XRL1(1000),XRL2(1000)
DIMENSION GL(LM)
COMMON /FCT/ CEM,CPW,CMD
GL(MM)=0.0
DO 143 LR=1,N
REI=RE*(LR-1)+HC
REF=REI+RE
GFU=0.0
DO 136 L=1,32
R=(REF-REI)/2*ZPP(L)+(REF+REI)/2
M=L+32*(LR-1)+3
GLL=FLL(R)*XRL1(M)*XRL2(M)
IF (LR.NE.N) GO TO 999
IF (FLL(R).NE.FEM(R)) GO TO 999
999 GFU=GFU+GLL*W(L)
136 CONTINUE
GL(MM)=GL(MM)+GFU*RE/2
143 CONTINUE
RETURN
END

```

```

C GAUSSIAN INTEGRATION FOR THE ASYMPTOTIC REGION 3
C SUBROUTINE GAS(GL,FLL,W,MM,XRL1,XRL2,D1,D2)
COMMON /PAR/ HC,ER,E,N,C,LM,DLXR,RE,LR
DIMENSION W(32),ZPP(32),XRL1(1000),XRL2(1000)
COMMON /ZP/ ZPP,S,XER1,XER2
DIMENSION D1(10),D2(10),GL(LM)
INTEGER E
YK=SQRT(E/C)
M=32*N+3
GL(MM)=0.0
ANG1=YK*ER-1.5707963*MM+D1(MM)
A=XRL1(M)/SIN(ANG1)
ANG2=YK*ER-1.5707963*MM+D2(MM)
B=XRL2(M)/SIN(ANG2)
DO 300 I3=1,32
R=ER+5*(1+ZPP(I3))/(1-ZPP(I3))
C HERE GAUSSIAN POINTS ARE DISTRIBUTED AS [ER,ER+5]+[ER+5,INF.]
ANG3=YK*R-1.5707963*MM+D1(MM)
XRL1(I3)=A*SIN(ANG3)
ANG4=YK*R-1.5707963*MM+D2(MM)
XRL2(I3)=B*SIN(ANG4)
GL3=FLL(R)*XRL1(I3)*XRL2(I3)
GF3=-GL3*W(I3)*10*ZPP(MM)/(1-ZPP(MM))**2
GL(MM)=GF3+GL(MM)

```



```

300 CONTINUE
    RETURN
    END

C SUBROUTINE LIVERM(E,D1,D2)
C LIVERMORE PHASE SHIFT FOR LAB ENERGY 20 TO 1000 MEV
C E IS THE CENTRE OF MASS ENERGY WHICH EQUALS TO HALF OF
C LAB ENERGY (FOR NUCLEON-NUCLEON INTERACTION, MU=M/2)
C DIMENSION D1(10),D2(10)
    INTEGER E
    COMMON /PASS/ DLM
    R=57.29578
    R IS RADIAN WHICH EQUALS 180/PI
    IF (E.EQ.10) GO TO 20
    IF (E.EQ.50) GO TO 100
    IF (E.EQ.75) GO TO 150
    IF (E.EQ.100) GO TO 200
    IF (E.EQ.150) GO TO 300
    IF (E.EQ.200) GO TO 400
    IF (E.EQ.250) GO TO 500
    IF (E.EQ.275) GO TO 550
    IF (E.EQ.300) GO TO 600
    IF (E.EQ.325) GO TO 650
    IF (E.EQ.350) GO TO 700
    IF (E.EQ.400) GO TO 800
    IF (E.EQ.500) GO TO 1000

C PRINT,'LIVERMORE PHASE SHIFT IS NOT USED'
    PRINT,'PHASE SHIFT IS CALCULATED'
    DLM=1.234
C A SYMBOL FOR CALLING THE ROUTINE PHASE SHIFT LATTER
    RETURN

C LIVERMORE PHASE SHIFT FOR LAB ENERGY=20 MEV
C 20 D1(1)=-3.23/R
    D1(2)=0.56/R
    D1(3)=-0.32/R
    D1(4)=0.03/R
    D1(5)=-0.02/R
    D1(6)=0.00/R
    D2(1)=-4.49/R
    D2(2)=2.79/R
    D2(3)=-0.17/R
    D2(4)=0.11/R
    D2(5)=-0.01/R
    D2(6)=0.01/R
    RETURN
C LIVERMORE PHASE SHIFT FOR LAB ENERGY=100 MEV
C 100 D1(1)=-12.23/R
    D1(2)=3.83/R
    D1(3)=-2.66/R
    D1(4)=0.44/R
    D1(5)=-0.47/R
    D1(6)=0.10/R
    D2(1)=-13.92/R

```

D2(2)=15.78/R
 D2(3)=-1.85/R
 D2(4)=2.34/R
 D2(5)=-0.34/R
 D2(6)=0.46/R

RETURN

C LIVERMORE PHASE SHIFT FOR LAB ENERGY=1000 MEV
 1000

D1(1)=-37.19/R
 D1(2)=-4.13/R
 D1(3)=-4.28/R
 D1(4)=5.35/R
 D1(5)=-11.23/R
 D1(6)=2.08/R
 D2(1)=-59.85/R
 D2(2)=8.45/R
 D2(3)=-15.96/R
 D2(4)=6.94/R
 D2(5)=0.03/R
 D2(6)=7.87/R

RETURN

C LIVERMORE PHASE SHIFT FOR LAB ENERGY=300 MEV
 300

D1(1)=-31.05/R
 D1(2)=9.65/R
 D1(3)=-5.04/R
 D1(4)=1.60/R
 D1(5)=-1.20/R
 D1(6)=0.53/R
 D2(1)=-28.72/R
 D2(2)=23.02/R
 D2(3)=-3.31/R
 D2(4)=7.30/R
 D2(5)=-1.35/R
 D2(6)=2.47/R

RETURN

C LIVERMORE PHASE SHIFT FOR LAB ENERGY=150 MEV
 150

D1(1)=-18.64/R
 D1(2)=5.65/R
 D1(3)=-3.60/R
 D1(4)=0.72/R
 D1(5)=-0.67/R
 D1(6)=0.20/R
 D2(1)=-18.22/R
 D2(2)=19.76/R
 D2(3)=-2.60/R
 D2(4)=3.87/R
 D2(5)=-0.62/R
 D2(6)=0.94/R

RETURN

C LIVERMORE PHASE SHIFT FOR LAB ENERGY=400 MEV
 400

D1(1)=-35.00/R
 D1(2)=9.65/R
 D1(3)=-5.04/R
 D1(4)=2.20/R
 D1(5)=-1.86/R
 D1(6)=0.76/R

D2(1)=-34.05/R
 D2(2)=22.16/R
 D2(3)=-2.99/R
 D2(4)=8.58/R
 D2(5)=-1.72/R
 D2(6)=3.43/R
 RETURN

C
 200 LIVERMORE PHASE SHIFT FOR LAB ENERGY=200 MEV

D1(1)=-23.93/R
 D1(2)=7.22/R
 D1(3)=-4.26/R
 D1(4)=1.01/R
 D1(5)=-0.82/R
 D1(6)=0.31/R
 D2(1)=-22.10/R
 D2(2)=21.85/R
 D2(3)=-3.05/R
 D2(4)=5.23/R
 D2(5)=-0.89/R
 D2(6)=1.45/R
 RETURN

C
 500 LIVERMORE PHASE SHIFT FOR LAB ENERGY=500 MEV

D1(1)=-37.10/R
 D1(2)=12.98/R
 D1(3)=-5.46/R
 D1(4)=2.81/R
 D1(5)=-2.87/R
 D1(6)=0.99/R
 D2(1)=-39.16/R
 D2(2)=20.36/R
 D2(3)=-2.17/R
 D2(4)=9.22/R
 D2(5)=-1.99/R
 D2(6)=4.32/R
 RETURN

C
 550 LIVERMORE PHASE SHIFT FOR LAB ENERGY=550 MEV

D1(1)=-37.70/R
 D1(2)=12.39/R
 D1(3)=-5.43/R
 D1(4)=3.10/R
 D1(5)=-3.49/R
 D1(6)=1.11/R
 D2(1)=-39.16/R
 D2(2)=19.27/R
 D2(3)=-1.46/R
 D2(4)=9.34/R
 D2(5)=-2.09/R
 D2(6)=1.15/R
 RETURN

C
 600 LIVERMORE PHASE SHIFT FOR LAB ENERGY=600 MEV

D1(1)=-38.10/R
 D1(2)=10.71/R
 D1(3)=-5.37/R
 D1(4)=3.40/R
 D1(5)=-4.18/R

D1(6)=1.23/R
 D2(1)=-45.07/R
 D2(2)=18.11/R
 D2(3)=-0.71/R
 D2(4)=9.36/R
 D2(5)=-2.13/R
 D2(6)=1.32/R

RETURN

C
 650. LIVERMORE PHASE SHIFT FOR LAB ENERGY=650 MEV

D1(1)=-38.32/R
 D1(2)=8.52/R
 D1(3)=-5.28/R
 D1(4)=3.68/R
 D1(5)=-4.93/R
 D1(6)=1.34/R
 D2(1)=-47.68/R
 D2(2)=16.91/R
 D2(3)=-0.53/R
 D2(4)=9.25/R
 D2(5)=-2.04/R
 D2(6)=1.48/R

RETURN

C
 700 LIVERMORE PHASE SHIFT FOR LAB ENERGY=700 MEV

D1(1)=-38.41/R
 D1(2)=6.33/R
 D1(3)=-5.18/R
 D1(4)=3.94/R
 D1(5)=-5.74/R
 D1(6)=1.45/R
 D2(1)=-49.98/R
 D2(2)=15.68/R
 D2(3)=-1.74/R
 D2(4)=9.13/R
 D2(5)=-1.82/R
 D2(6)=5.91/R

RETURN

C
 800 LIVERMORE PHASE SHIFT FOR LAB ENERGY=800 MEV

D1(1)=-38.28/R
 D1(2)=2.48/R
 D1(3)=-4.92/R
 D1(4)=4.47/R
 D1(5)=-7.46/R
 D1(6)=1.67/R
 D2(1)=-53.87/R
 D2(2)=13.20/R
 D2(3)=-7.58/R
 D2(4)=8.60/R
 D2(5)=-1.16/R
 D2(6)=6.61/R

RETURN

C
 C
 C
 C
 C
 ER HAS TO BE CHOSEN SUCH THAT AT THAT POINT, THE
 NORMALIZATION REFERENCE WAVE FUNCTION (SINE FORM)
 IS AT THE MAXIMUM VALUE FOR ALL J=L.

C FOR PARAMETER
C DATA OF G AND M FOR THE DIFFERENT TERMS OF THE POTENTIAL PARIS

\$ENTRY

150,6,0.007299,6.6445,14.5,

12,20.36,0.1,0.000001

0.32290874E+02,	-0.10763625E+02,	-0.10077427E+02,	0.33591422E+01,
-0.85980096E-02,	0.28660032E-02,	0.26851393E-02,	-0.89504644E-03,
0.0E+00,	0.0E+00,	-0.10763625E+02,	0.33591422E+01,
0.0E+00,	0.0E+00,	0.69953600E+00,	
-0.82465631E+02,	-0.42973669E+02,	-0.12049564E+03,	-0.86479568E+02,
0.26814385E-01,	-0.81798046E-03,	0.51092455E-01,	0.37488481E-01,
-0.66176421E+02,	-0.42600359E+03,	-0.46818029E+00,	-0.85945824E+00,
-0.62851020E+00,	-0.52218640E+00,	0.16000000E+01,	
0.12329384E+04,	-0.71856844E+03,	-0.21236460E+03,	-0.46593111E+03,
-0.13280693E+01,	-0.53314560E+00,	-0.84264258E+00,	-0.89373089E+00,
0.28903688E+04,	0.26279517E+05,	0.60147739E+02,	-0.10476340E+03,
-0.76290197E+02,	0.18644558E+03,	0.23000000E+01,	
-0.16859879E+05,	0.42469120E+04,	-0.87174198E+04,	0.18673085E+04,
0.10324289E+02,	0.83162030E+00,	0.14736312E+02,	0.14123475E+02,
-0.62592400E+05,	-0.57557033E+06,	0.35256941E+03,	0.12629465E+04,
-0.78827581E+03,	-0.37091115E+04,	0.30000000E+01,	
0.17292683E+06,	-0.34574024E+05,	0.54383377E+05,	0.38509213E+04,
-0.11527060E+03,	-0.31192395E+02,	-0.14521993E+03,	-0.14660152E+03,
0.69146141E+06,	0.60033934E+07,	0.51432170E+03,	-0.18881061E+05,
-0.64904798E+04,	0.55913117E+05,	0.37000000E+01,	
-0.76835277E+06,	0.12671169E+06,	-0.21342147E+06,	-0.19674338E+05,
0.69456175E+03,	0.30041384E+03,	0.84158389E+03,	0.84191462E+03,
-0.40969146E+07,	-0.34519443E+08,	0.11637302E+05,	0.10613246E+06,
0.54734378E+04,	-0.36998560E+06,	0.44000000E+01,	
0.21890475E+07,	-0.27416841E+06,	0.49458357E+06,	0.12323140E+06,
-0.23879335E+04,	-0.12415067E+04,	-0.27861170E+04,	-0.28394273E+04,
0.14032093E+08,	0.11355459E+09,	-0.44595415E+05,	-0.33211910E+06,
-0.32941912E+05,	0.14537543E+07,	0.51000000E+01,	
-0.38447287E+07,	0.52960724E+06,	-0.66715334E+06,	-0.31449361E+06,
0.42388011E+04,	0.24762241E+04,	0.50564510E+04,	0.52653427E+04,
-0.26827468E+08,	-0.20729209E+09,	0.69211738E+05,	0.55585762E+06,
0.24949132E+06,	-0.31352471E+07,	0.58000000E+01,	
0.27990559E+07,	-0.36606713E+06,	0.52957598E+06,	0.24242440E+06,
-0.24521604E+04,	-0.13043030E+04,	-0.33674205E+04,	-0.35000430E+04,
0.23511442E+08,	0.17131548E+09,	-0.48127668E+05,	-0.34916664E+06,
-0.16012956E+05,	0.24339081E+07,	0.65000000E+01,	
0.50251828E+06,	-0.22303673E+06,	-0.13703412E+06,	0.16690404E+06,
-0.19512821E+04,	-0.21496577E+04,	-0.17845529E+04,	-0.24879479E+04,
-0.14688461E+08,	-0.86418222E+08,	0.70514008E+04,	-0.11945013E+06,
-0.86736260E+06,	0.14589740E+07,	0.82000000E+01,	
-0.26006124E+07,	0.40683833E+06,	-0.34697194E+06,	-0.48534364E+06,
0.41801160E+04,	0.40996917E+04,	0.53548266E+04,	0.73068121E+04,
0.12206590E+08,	0.56269610E+08,	0.30187530E+05,	0.32952990E+06,
0.55456280E+06,	-0.52669870E+07,	0.99000000E+01,	
0.15528290E+07,	-0.17678810E+06,	0.28509970E+06,	0.27678610E+06,
-0.22508700E+04,	-0.22000160E+04,	-0.32465480E+04,	-0.45574760E+04,
-0.47570340E+07,	-0.18348350E+08,	-0.29442820E+05,	-0.17078810E+06,
0.18614580E+06,	0.34497880E+07,	0.11300000E+02,	
-0.9972638618,	0.0070186100,		
-0.9856115115,	0.0162743947,		

-0.9647622556, 0.0253920653,
-0.9349060759, 0.0342738629,
-0.8963211557, 0.0428358980,
-0.8493676137, 0.0509980592,
-0.7944837959, 0.0586840934,
-0.7321821187, 0.0658222227,
-0.6630442669, 0.0723457941,
-0.5877157572, 0.0781938958,
-0.5068999089, 0.0833119242,
-0.4213512761, 0.0876520930,
-0.3318686022, 0.0911738787,
-0.2392873622, 0.0938443991,
-0.1444719616, 0.0956387201,
-0.0483076657, 0.0965400885,

/*

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