AN ERROR SENSING UNIT
FOR THE
COMPLEX PLANE SCANNER

A Thesis

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and Research

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Master of Science

in

Electrical Engineering

by

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ABSTRACT

This thesis consists of the description of a unit to be used in conjunction with a complex plane scanner and a Mosley X-Y recorder with curve follower attachment. The unit is used to compute the instantaneous absolute difference, the average absolute difference and the peak absolute difference between the slowly varying voltage proportional to magnitude, log magnitude, phase or phase slope of a network response function simulated by the complex plane scanner and the voltage output from the curve follower, proportional to the corresponding desired response function plotted on the X-Y recorder sheet. Since the difference between the two responses may be considered to be the error, the unit used to compute these various differences is called the Error Sensing Unit. A discussion of the applications and performance tests of the Error Sensing Unit is included.

PREFACE

The complex plane scanner built by the Department of Electrical Engineering of the University of Manitoba in 1958 was the subject of E.P. Valstyn's M.Sc. Thesis (Va 1)[1]. Since that time, additional units have been added to the scanner by H.F.J. Wagerer and J.L. Woonsam and have been the subjects of their respective M.Sc. Theses. In this Thesis, another unit called the Error Sensing Unit is described and its implementation discussed.

The error sensing unit described in this thesis is designed for use as an integral part of the complex plane scanner in order that the pole-zero locations of a desired network response function might be quickly obtained. From the complex plane scanner it is possible to obtain voltages proportional to the magnitude, phase, logarithm of the magnitude, or phase slope versus frequency of a given immittance[2] function F(s) as represented by its pole-zero configuration in the complex "s" plane. It was considered desirable to be able to find the average absolute difference between the response curve resulting from a pole-zero configuration set up on the complex plane scanner, and a desired response curve. This difference shall henceforth be called the error. It was also thought advantageous to have an indication of the peak absolute error, the absolute error during the sweep of the $j\omega$ axis and to be able to weight the average absolute error over certain frequency

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[1] The letters in parentheses refer to the Bibliography.
[2] Immittance is a generic word, introduced by H.W. Bode for both impedance and admittance. (Bo 1, p. 15)

ranges.

To implement the above operations, an error sensing unit as described in this thesis was designed, constructed and tested.

This thesis consists of four chapters. In Chaper I, a general description of the error sensing unit, and its operation is given. Chapter II describes in detail the circuitry used in the unit, in its logical operating sequence. The third chapter consists of the tests run on the error sensing unit and their results. In the fourth chapter a discussion of the test results is presented. Also included are several appendices. The first of these consists of an outline of the alignment procedure which must be carried out periodically to assure optimum performance of the unit.

Grateful acknowledgment is made to the National Research Council for their financial assistance in this project, through both grants in support of the work, and a bursary to the author. The author also wishes to thank Professor R.A. Johnson for his competent guidance and supervision of this project. The help received from Mr. T.J. White and Mr. D. Johnson is greatly appreciated.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

DESCRIPTION OF OPERATION

The complex plane scanner generates a voltage proportional to frequency as the $j\omega$ axis is swept and scanned to yield the magnitude, logarithm of the magnitude, phase and phase slope of the immittance function represented by its pole-zero configuration in the complex "s" plane. That is, whenever the response of an immittance function to a sinusoidal excitation is generated by the scanner, a voltage proportional to frequency is developed. Since the $j\omega$ axis is swept linearly for 15 seconds from zero frequency to some high frequency, dependent upon the scale factor chosen for pole and zero positions, and then back to zero frequency, the voltage is a linear time function, rising and falling with a period of 30 seconds. This voltage is used to drive the X axis of an X-Y recorder which is fitted with a curve follower attachment. The curve follower allows the Y axis servo to follow a desired response function drawn on the recorder paper in conducting ink. The Y axis servo also drives the slider on a linear 20 kilohm potentiometer. Thus, with a known voltage applied across this potentiometer and with one end of the potentiometer taken as reference, the variation in voltage from the slider to reference is proportional to the amplitude of the response function drawn on the recorder sheet. Thus two voltages proportional to the amplitude of immittance response functions, one from the scanner and the other from the X-Y recorder, may be generated simultaneously. In normal design work using the complex plane scanner, the pole and zero positions on the complex plane are varied to obtain a desired response function drawn on the X-Y recorder sheet. The gain of the scanner output may be varied to match that of the recorder output through a step attenuator in the scanner output unit and an overlapping continuously variable attenuator on the scanner input of the error sensing unit.

In particular, the log magnitude response of an immittance function

$$F_1(s) = H\left[\frac{(s-z_1)(s-z_2)\text{------}(s-z_n)}{(s-p_1)(s-p_2)\text{------}(s-p_m)}\right] \quad (1)$$

as generated by the complex plane scanner may be represented by the equation

$$\ln\left|F_1(s)\right| = K_1V_1(s) + E_1 \quad \text{(Va 1, p. 11)} \quad (2)$$

where K_1 is a constant of the scanner in db/volt and E_1 is a d.c. offset voltage dependent upon both the scanner characteristics and the particular $F_1(s)$. When only the response of the immittance function to sinusoidal excitations is required, as is the normal case $s = j\omega$. The frequency response of the immittance function is thus obtained and equation (2) becomes

$$\ln\left|F_1(j\omega)\right| = K_1V_1(j\omega) + E_1 \quad (3)$$

If the log magnitude frequency response curve of a function $F_2(s)$ is drawn on the recorder sheet and scanned by the curve follower, the output of the recorder is

$$\ln\left|F_2(j\omega)\right| = K_2V_2(j\omega) + E_2 \quad (4)$$

where K_2 is a constant dependent on the scale to which the response curve is drawn on the recorder paper in db/volt and E_2 is a function of the position of the curve on the recorder paper and on the absolute level with respect to ground potential of the voltage placed across the 20 kilohm Y axis potentiometer. It is the function of the level setting circuit in the error sensing unit to set the average d.c. level of the recorder output equal to that of the scanner output. To implement this, on alternate 15 second sweeps of the $j\omega$ axis the difference in voltage between the two response curves is divided by 7.5 and integrated with a gain of 0.5 for 15 seconds and the integrated output at the end of the sweep sampled. This sampled voltage, which is then the average difference in d.c. level between the two inputs, is used to set the d.c. level of the constant voltage applied across the 20 kilohm potentiometer on the recorder Y axis servo, such that the average level of the recorder output is the same as the average level of the scanner output. Thus on the following sweep of the $j\omega$ axis the error voltage developed by the difference circuit is simply

$$\ln \left| F_1(j\omega) \right| - \ln \left| F_2(j\omega) \right| = K_1 V_1(j\omega) - K_2 V_2(j\omega)$$

As mentioned previously, the gain of the scanner output may be varied to equal that of the recorder output and thus K_1 may be set equal to K_2. Since K_1 is a function of the scanner and not of the particular immittance function under investigation, this gain adjustment is the same for all log magnitude frequency response investigations. The error voltage thus becomes

$$\ln \left| F_1(j\omega) \right| - \ln \left| F_2(j\omega) \right| = K_2 \left(V_1(j\omega) - V_2(j\omega) \right)$$

Similarly, the phase, phase slope and magnitude output voltages from the complex plane scanner consist of d.c. offset voltages plus slowly varying voltages proportional to the respective functions. What is more important, the proportionality constants between volts and phase, phase slope or magnitude ~~is a~~ are fixed values for each quantity dependent only on the scanner characteristics and not on the particular function under investigation. Thus, as in the case for the log magnitude response, the gain adjustment may be fixed for the particular scanner output to match that of the recorder for all immittance functions. The d.c. offset voltage of the particular scanner output and that of the recorder output are always set equal by the level setting circuit.

Since all the machine outputs which are to be used with the error sensing units are functions of ω and since ω, as represented in the complex plane scanner, is a linear time function, the machine outputs are also functions of time. In general, the error sensing unit has two inputs, A(t) and B(t), from the recorder and scanner respectively, and each has the same gain constant in db/volt, degrees/volt, $\frac{\text{degrees}}{\text{rad/sec}}$/volt or magnitude/volt as the case may be. These inputs are first differenced and divided by 7.5 to yield the error divided by 7.5. The absolute value of the error is then found and this is integrated with a gain of 0.5 for 15 seconds to yield the average absolute error and peak detected with unity gain to obtain the peak absolute error divided by 7.5. Provision is made for metering the average absolute error, the absolute error divided by 7.5 and the peak absolute error divided by 7.5.

Thus:

$$\frac{\text{Absolute Error}}{7.5} = \frac{|A(t)-B(t)|}{7.5}$$

$$\text{Average absolute error} = 0.5 \int_0^{15} \frac{|A(t)-B(t)|}{7.5} dt$$

$$\frac{\text{Peak absolute error}}{7.5} = \text{Max.} \frac{|A(t)-B(t)|}{7.5}$$

It must be noted that the error sensing sweep occurs alternately with the level setting sweep. At the end of the level setting sweep, the average d.c. level of the recorder output is set equal to that of the scanner output and this level is held constant during the following error sensing sweep and until the end of the level setting sweep when the average levels are again set equal. During the level setting sweep the integrator used to find the average absolute error and the peak detector are re-set to zero.

Provision is also made for weighting the average absolute and peak error by multiplying the output of the absolute value circuit by a weighting function, $w(t)$, generated by an external function generator and then integrating and peak detecting the multiplier output as shown in Figure (4). The multiplier output is thus

$$\frac{\text{Weighted absolute error}}{7.5} = w(t) \left| \frac{A(t)-B(t)}{7.5} \right|$$

The integrated output then becomes the

$$\text{Weighted average absolute error} = 0.5 \int_0^{15} w(t) \left| \frac{A(t)-B(t)}{7.5} \right| dt$$

and the peak detector output becomes the

$$\text{Weighted peak absolute error} = \text{Max. } w(t) \left| \frac{A(t)-B(t)}{7.5} \right|$$

The provision for weighting was included so that the error in a particular part of the response curve, such as a "break" region, could be more readily studied.

A block diagram of the error sensing unit is shown in Figure (1). and photographs of the unit are found on page (8).

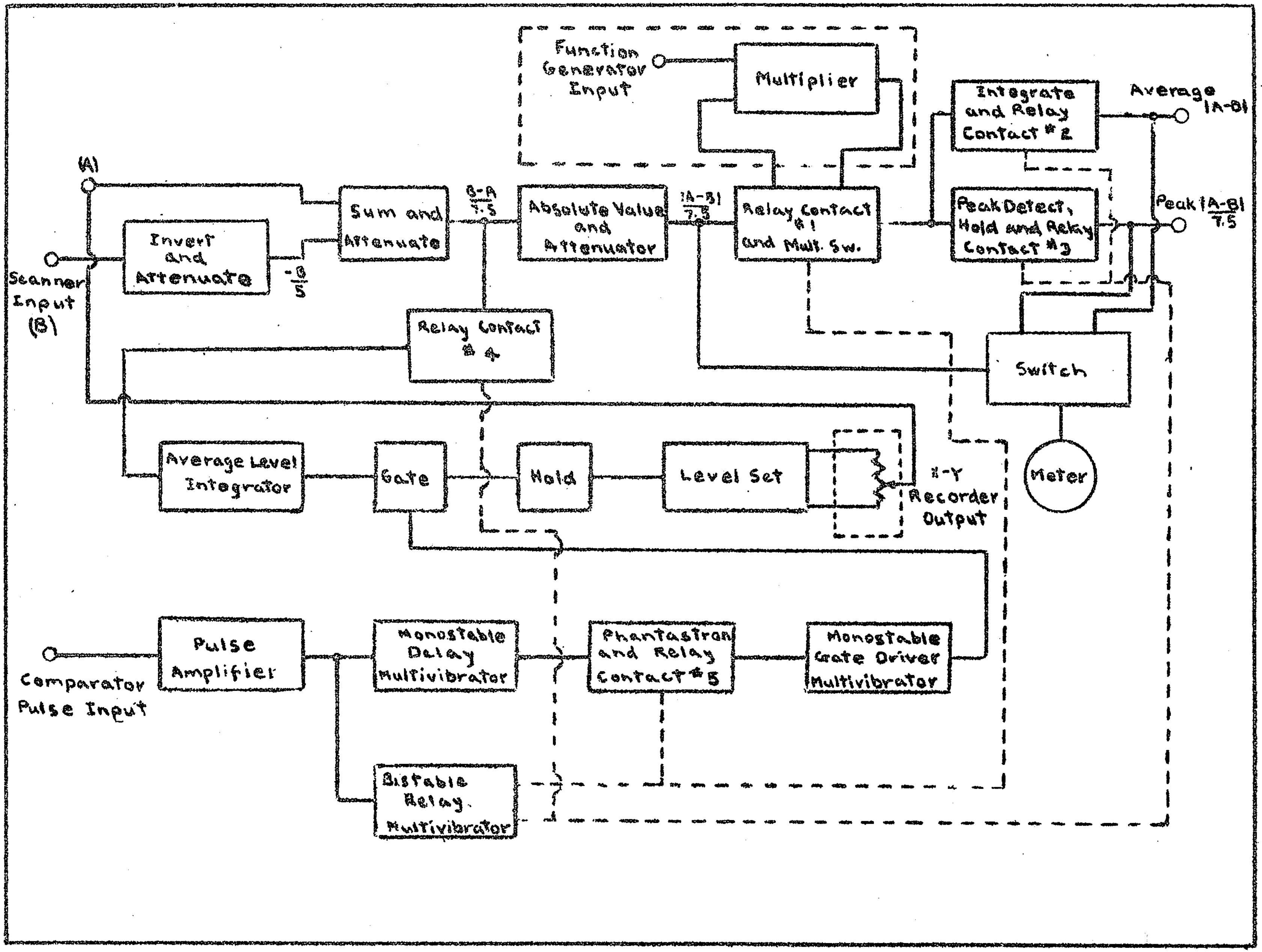
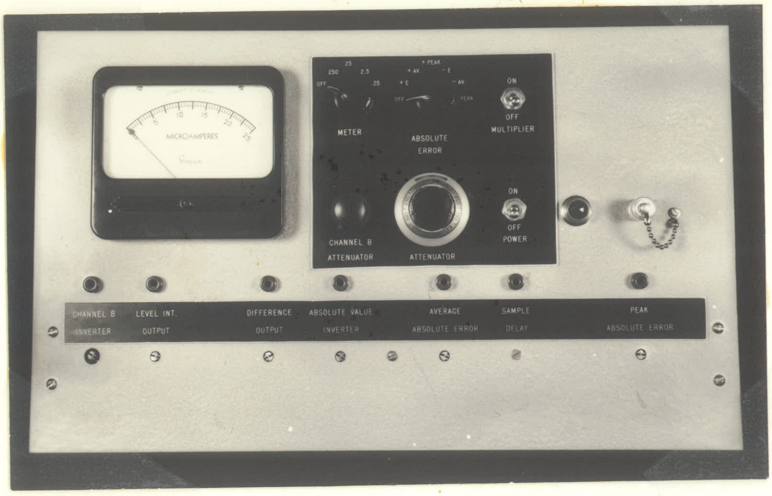
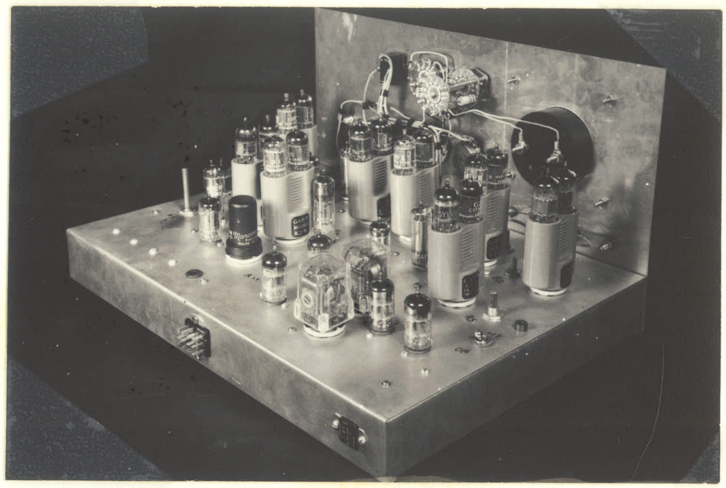


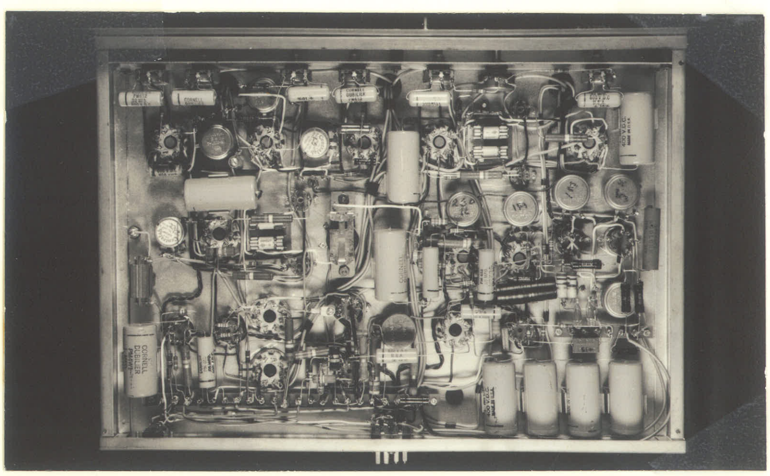
Figure 1: Block Diagram of Error Sensing Unit



Front Panel of the
Error Sensing Unit.



Rear View of the
Error Sensing Unit.



Underside of the
Error Sensing Unit.

CHAPTER II

CIRCUITRY

THE DIFFERENCE CIRCUIT

The difference circuit shown in Figure (2) consists of an inverting operational amplifier into which one input is fed and whose output is summed with the other input by an operational summer (Ko 1, p. 13) to yield the difference, with theoretically infinite common mode rejection ratio, between the two inputs. Input A is that from the X-Y recorder while input B is that from the complex plane scanner output. The inverter in channel B has a gain control used to match the gain of the scanner to that of the recorder. This control varies the gain of the B channel over a range of about three to one which overlaps the step attenuator on the scanner output to provide a continuously variable gain when used in conjunction with the latter.

The 10 kilohm variable resistance and 10 kilohm balancing potentiometer are included in the adder circuit so that the overall gain of the difference circuit may be adjusted to exactly two-fifteenths for both inputs. This attenuation is required so that the level setting integrator which operates on the adder output for the duration of the 15 second sweep, and which has an RC time constant of 2.0 seconds will yield an output which is the average voltage difference between the two inputs.

G.A. Philbrick K2-W and K2-X d.c. operational amplifiers are used extensively in the above and following circuits.

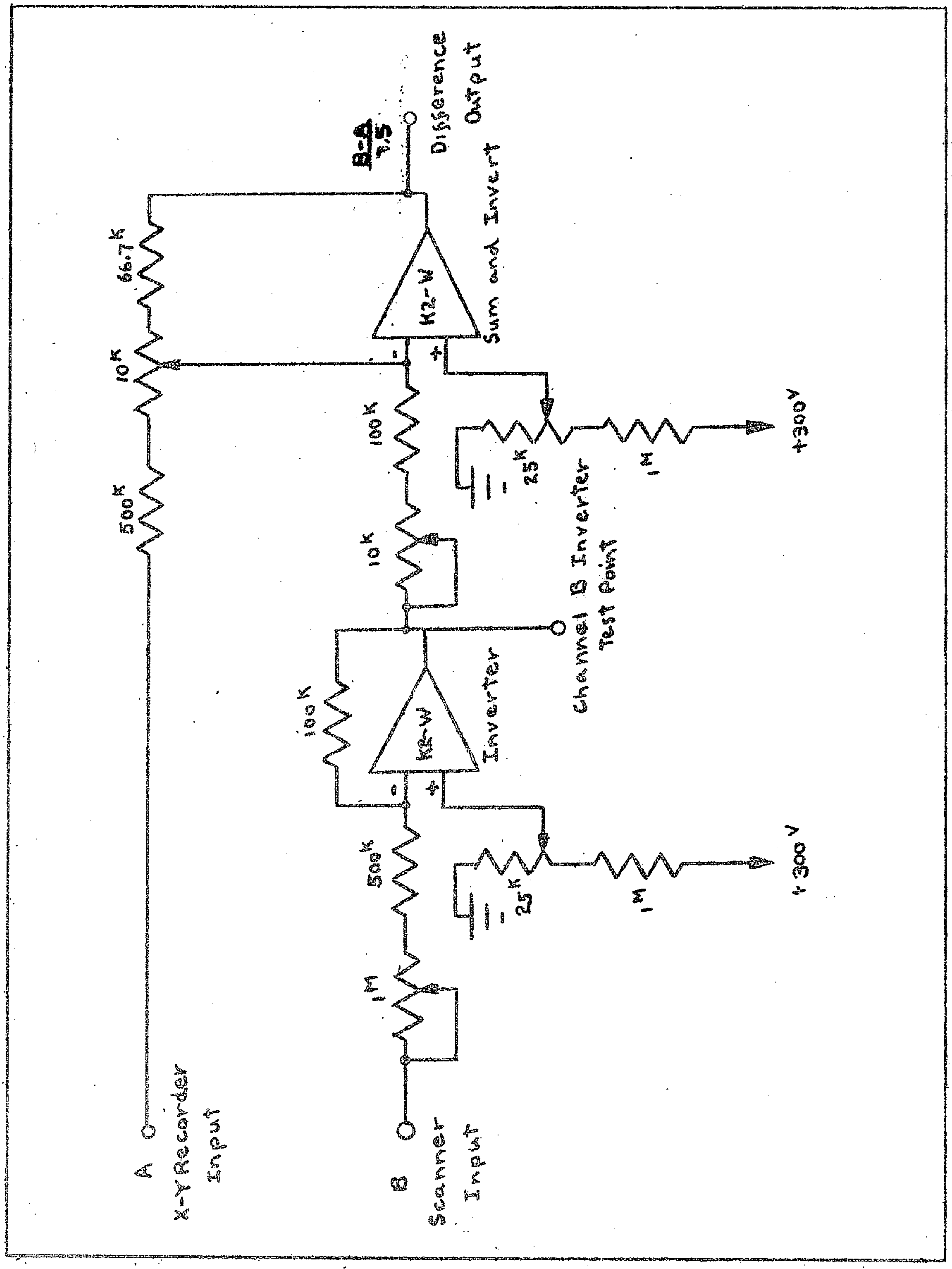


Figure 2: Difference Circuit

ABSOLUTE VALUE CIRCUIT

The output of the difference circuit is fed to a circuit whose output is always the absolute value of the difference. As shown in Figure (3), it consists of a unity gain inverting amplifier feeding one diode while the other diode is fed directly. This is simply an "OR" circuit in digital computer language. (Mi 1, p. 394) The diodes have 0.4 volts forward bias applied so that with zero error, and thus zero voltage applied, they are just on the verge of significant conduction. Thus the output of the absolute value circuit is always the magnitude of the error divided by 7.5, plus 0.4 volts offset. Since the maximum error expected is 200 volts, and the difference circuit has an attenuation of two-fifteenths, the diodes need only be able to withstand a reverse voltage of $2 \times \frac{200}{7.5} = 53.4$ volts. Since the 1N457 diodes used have a reverse voltage rating of 60 volts, they operate well within their rating. A 50 kilohm calibrated attenuator (Helipot) is included to adjust the output so that the following integrator does not saturate during the 15 second error sensing sweep.

ABSOLUTE ERROR INTEGRATOR, PEAK DETECTOR, METER SWITCHING CIRCUIT and WEIGHTING CIRCUIT

The output of the absolute value circuit is fed to both an operational integrator (Ko 1, p. 13) and peak holding circuit as shown in Figure (4).

The integrator provides real time integration multiplied by 0.5 in order that the integrated output of the difference circuit be equal to the value of the absolute average error. The integrator

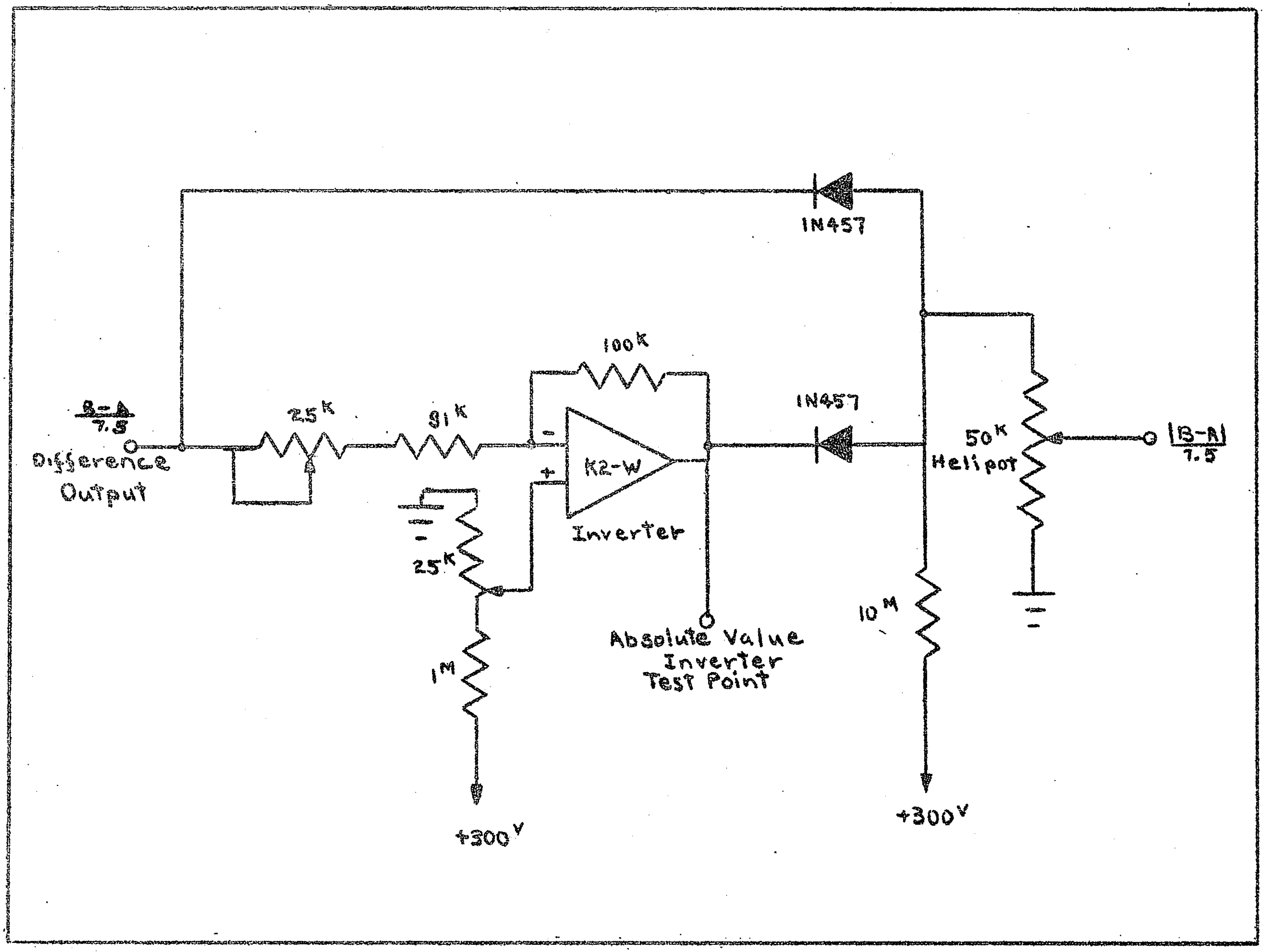


Figure 3: Absolute Value Circuit

bias control is adjusted to offset the integrator zero input so that the +0.4 volts bias from the absolute value circuit corresponds to zero error. However, this yields a d.c. offset at the integrator output of + 0.4 volts which must be subtracted from the average absolute error. This bias control must be re-adjusted if the 50 kilohm attenuator on the integrator input is not set for zero attenuation. The integrator output voltage is limited to + 105 volts by the voltage regulator tube and normally back biased diode connected to its output. This limiting circuit was deemed necessary to prevent saturation of the integrating amplifier. During the level setting sweeps, the integrator is re-set to zero output by a relay which discharges the integrator capacitor.

The peak detector output is equal to the highest input voltage (divided by 7.5) incurred during the error sensing sweep minus an initial offset voltage of −0.6 volts. The same relay which discharges the integrator capacitor is used to place an initial voltage of +0.8 volts on the peak detector capacitor such that the peak detector diode is biased to the verge of significant conduction. A variable bias control for the diode is provided to compensate for any supply voltage variation and to allow the bias to be set correctly if the 50 kilohm input attenuator is not set for zero attenuation.

The meter switching circuits for monitoring average absolute, peak absolute and absolute error, as well as the weighting circuit are also shown in Figure (4).

The meter switch has four voltage ranges, from 250 volts full scale to 0.25 volts full scale. The absolute error selector

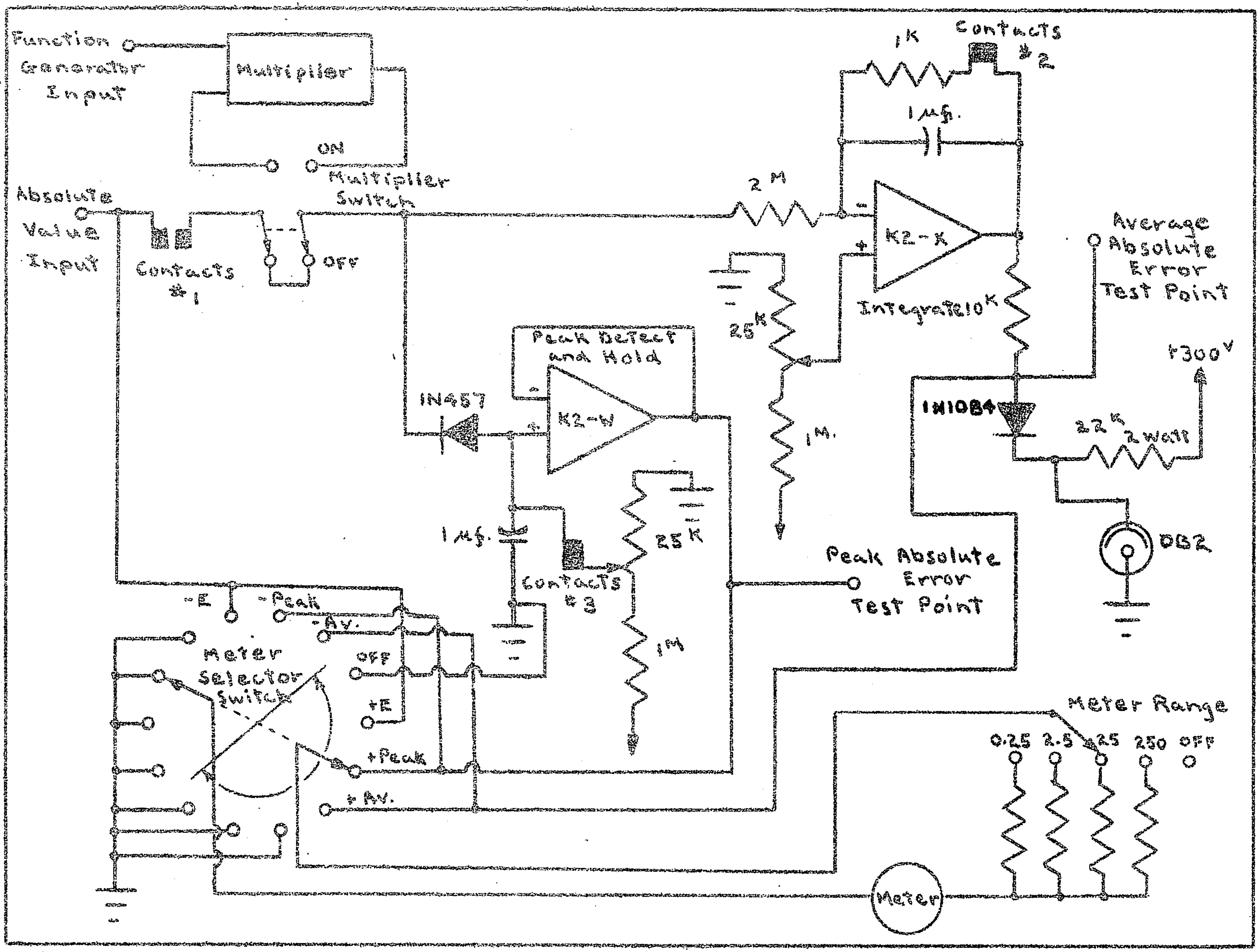


Figure 4: Weighting Circuit, Absolute Error Integrator, Peak Detector and Meter Switching Circuit

switch is arranged for both positive and negative detection and this may seem contradictory since it is absolute error which is being detected. Provision for detecting both polarities is included to facilitate alignment of the error sensing unit since if the biasing controls on the average error integrator or peak detector are not adjusted correctly, then the output polarity may be opposite to that found in normal operation. In normal operation, the peak absolute and absolute errors are negative voltages, while the average absolute error is a positive voltage.

One channel of a G.A. Philbrick Model MU/DV Duplex Multiplier/Divider is used in weighting the peak absolute and average absolute error. The multiplier switch, when placed in the "on" position connects the output of the absolute value circuit to one multiplier input and connects the multiplier output to the average absolute error integrator and peak detector. The other multiplier input is then connected to an external function generator which generates the weighting function. It is noted that 25 volts applied to the function generator input of the multiplier yields multiplication by one.

AVERAGE LEVEL INTEGRATOR, GATE AND HOLD CIRCUIT

As shown in Figure (5) the circuit used to compute the difference in average levels of the two inputs during the 15 second level setting sweep is simply an operational integrator with a time constant of 2.0 seconds operating on the output of the difference circuit. Thus, the integrator output is

$$0.5 \int_{0}^{15} \left(\frac{B(t)-A(t)}{7.5}\right) dt = \overline{B(t)-A(t)}$$

That is, the output of this integrator is, at the end of the 15 second level setting sweep the average difference in levels of the two response curves. This output is sampled at the end of the sweep and used to set the level of the recorder output exactly equal to that of the scanner output. The output is sampled by the closing of a relay gate which connects the integrator output to a hold circuit as shown in Figure (5).

The integrator output is limited to ± 105 volts by the OB2 voltage regulator tubes and normally reverse biased diodes to prevent saturation of the operational amplifier used in the integrator circuit.

The hold circuit consists of one-half of a low grid current tube operated at reduced filament voltage to further reduce the grid current. The tube type is an E80CC. The tube is operated as a cathode follower with a high cathode resistance and with a 1 μf mylar capacitor connected between the grid and **ground**. The leakage resistance of the capacitor is 10^7 megohms so that the major leakage path for the capacitor is through grid current of the cathode follower. The hold circuit output voltage decays less than 0.5 volts, in a period of 30 seconds with the hold capacitor charged to + 100 volts. This indicates that the leakage resistance is in excess of 6000 megohms. This calculation is found in Appendix (2).

PULSE AMPLIFIER AND RELAY MULTIVIBRATOR

Since the error sensing unit alternately measures the actual error, and then sets the average d.c. levels of the two inputs equal on each successive sweep of the response curve being examined, some

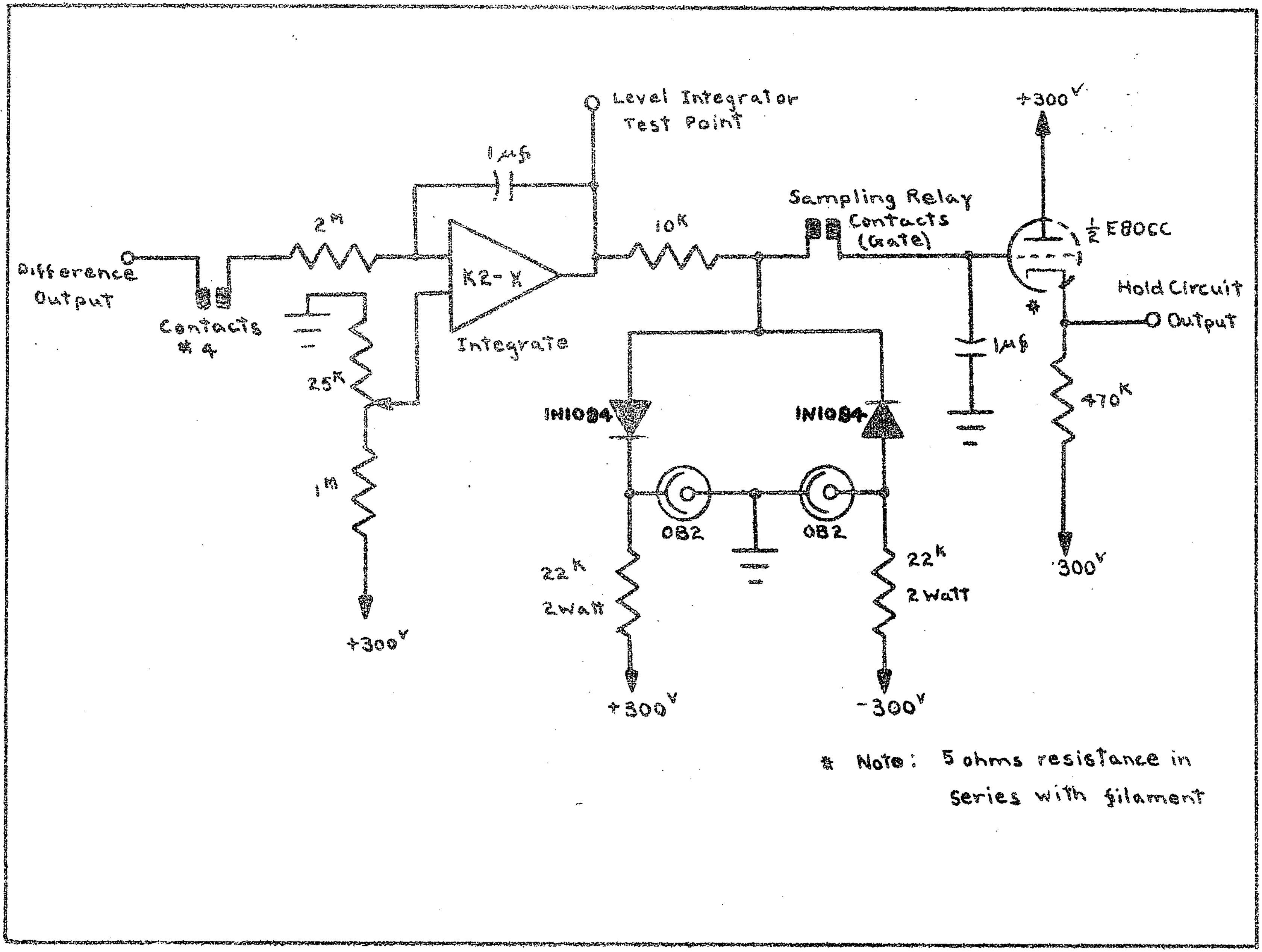


Figure 5: Average Level Integrator, Gate and Hold Circuit

method of switching from one operation to the other is required. At the end of each sweep of the $j\omega$ axis in the complex plane scanner a pulse is available from a comparator which reverses the slope direction of the sweep voltage. This pulse is amplified in the error sensing unit and used to trigger a bistable multivibrator with two relay plate loads as shown in Figure (6).

The pulse amplifier is direct coupled since the intention was to obtain the trigger pulse by amplifying and differentiating a sawtooth sweep voltage which was originally used. When the sweep was changed to a triangular waveshape it was decided to use a trigger pulse from the comparator. Since the pulse amplifier had already been constructed it was decided not to alter it.

The 1 μf coupling capacitor and large trigger pulse, (about 200 volts), are required to trigger the bistable multivibrator reliably because of the inductance of the relay coils in the plate circuit.

When relay KCP11 is energized relay contacts number 1, connecting the peak detector and integrator for the error sensing chain, and number 5 making the phantastron inoperative are closed. Relay contacts 2, 3 and 4 are closed and 1 and 5 open when relay KCP12 is energized making the level setting circuit operative. Figure (7) shows the operating sequence of the contacts.

PHANTASTRON DELAY CIRCUIT

A phantastron (Mi 1, p. 221) is used to provide the delayed pulse for triggering the relay driving multivibrator. Since the relay must be closed for a short sampling period

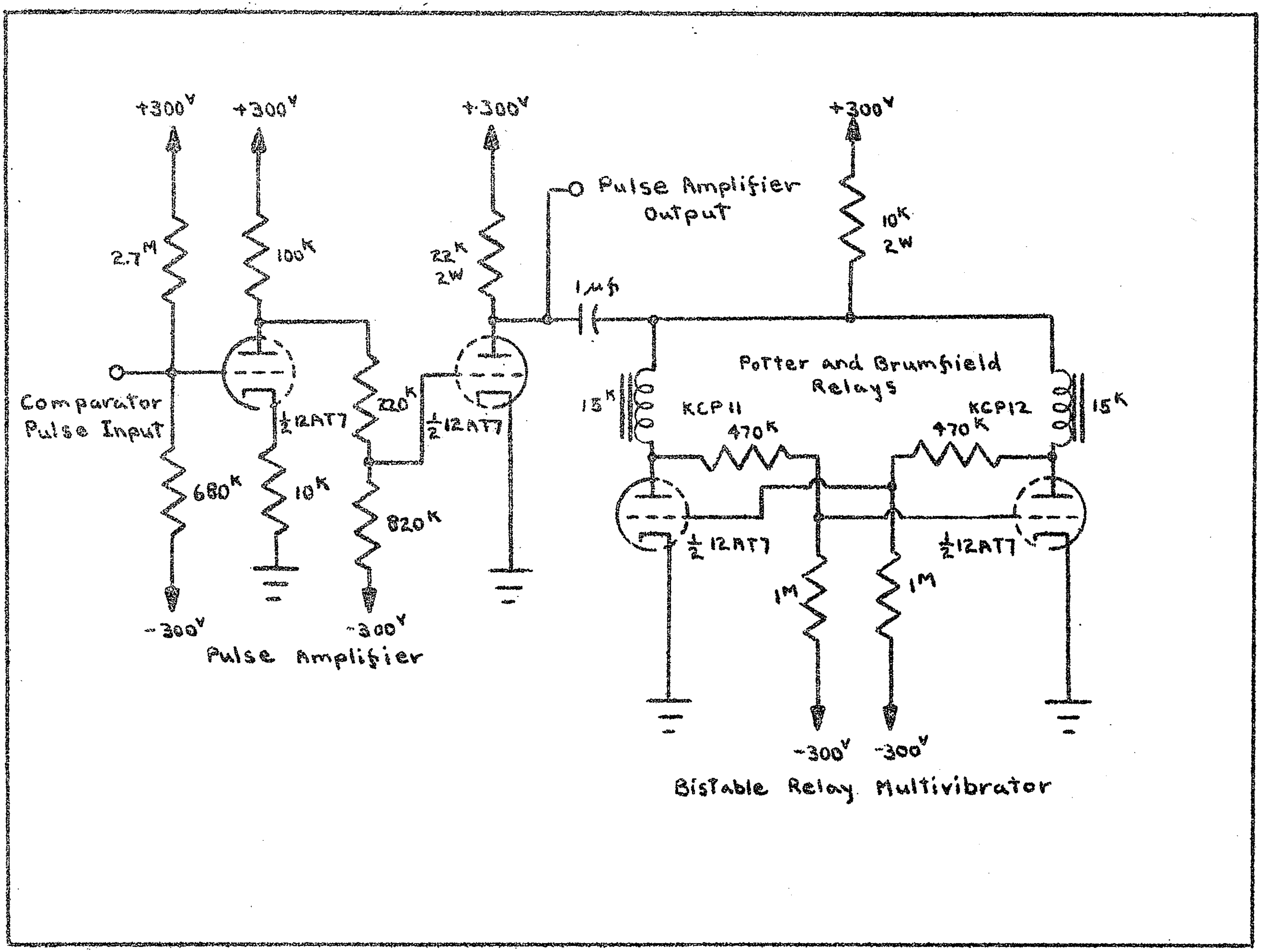


Figure 6: Pulse Amplifier and Relay Multivibrator

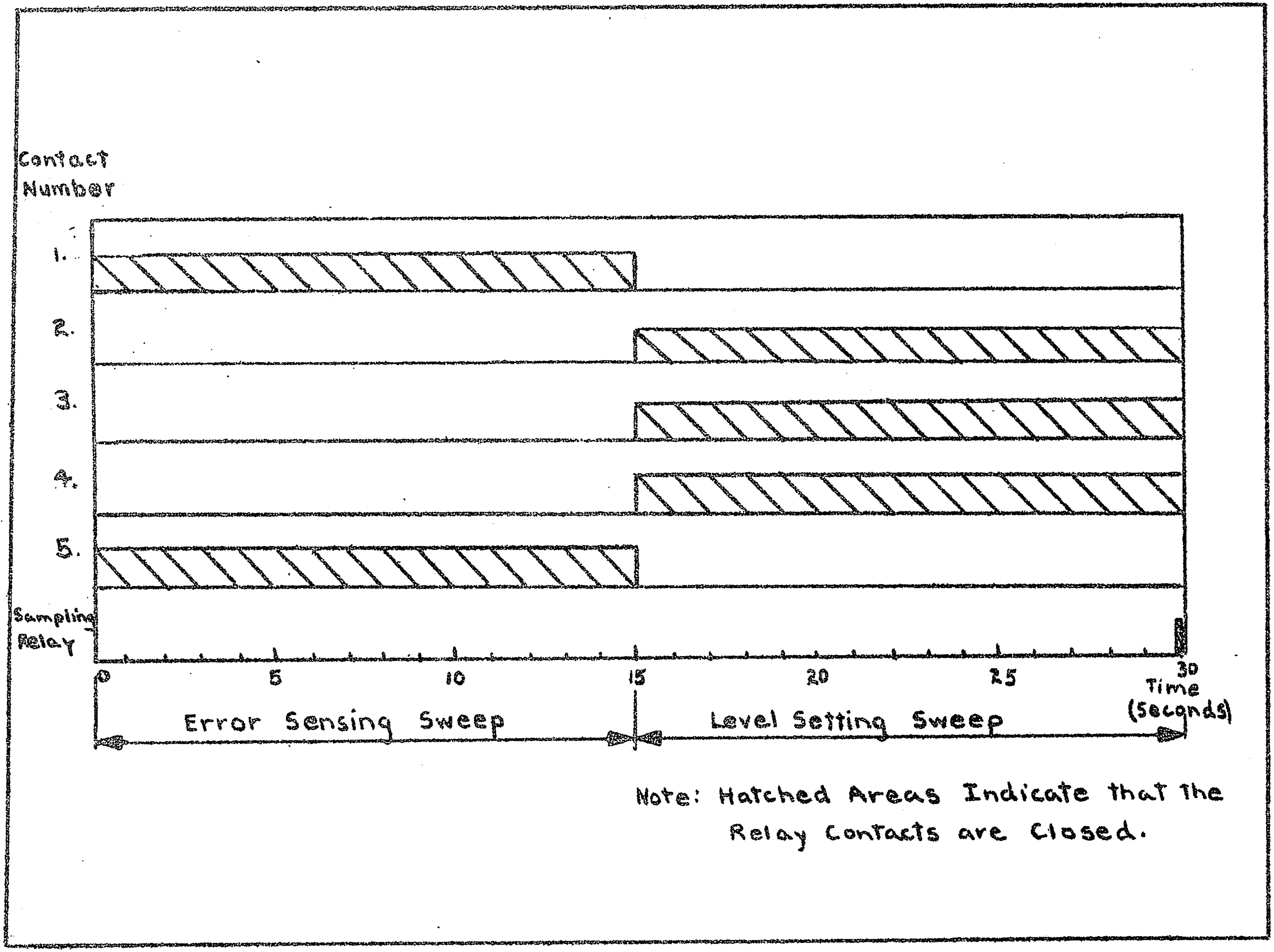


Figure 7. Operating Sequence of Relay Contacts

just prior to the end of the 15 second level setting sweep in order to sample the level setting integrator output, the phantastron must provide a delay pulse about 14.85 seconds after the commencement of the level setting sweep. This period involves a large time constant for the phantastron timing circuit as may be seen from the size of C_1 and R_1 in Figure (8).

The delay period may be changed by varying R_1 and should be adjusted so that the relay closes 130 milliseconds before the end of the sweep. This means that the relay is re-opened 30 milliseconds before the end of the sweep. This 30 millisecond period is maintained to allow for any drift in the phantastron delay or in the 15 second sweep time.

During the error sensing sweep the phantastron is made inoperative by the closing of relay contacts number 5 which set the screen potential to minus 300 volts.

It was found that if the phantastron was triggered directly from the pulse amplifier, it would trigger on every pulse due to the delay in the closure of relay contacts number 5. Thus, at the beginning of every second sweep the phantastron would trigger only to be cut off a few milliseconds later by the relay closing. This, of course, caused the relay to sample the level setting integrator output which was intolerable. In order to prevent this "misfiring", the phantastron is triggered from a monostable multivibrator which provides a pulse delayed about 20 milliseconds. This allows sufficient time for relay contacts number 5 to close and cut off the phantastron at the beginning of the error sensing sweep, before the trigger pulse reaches it from the monostable multivibrator.

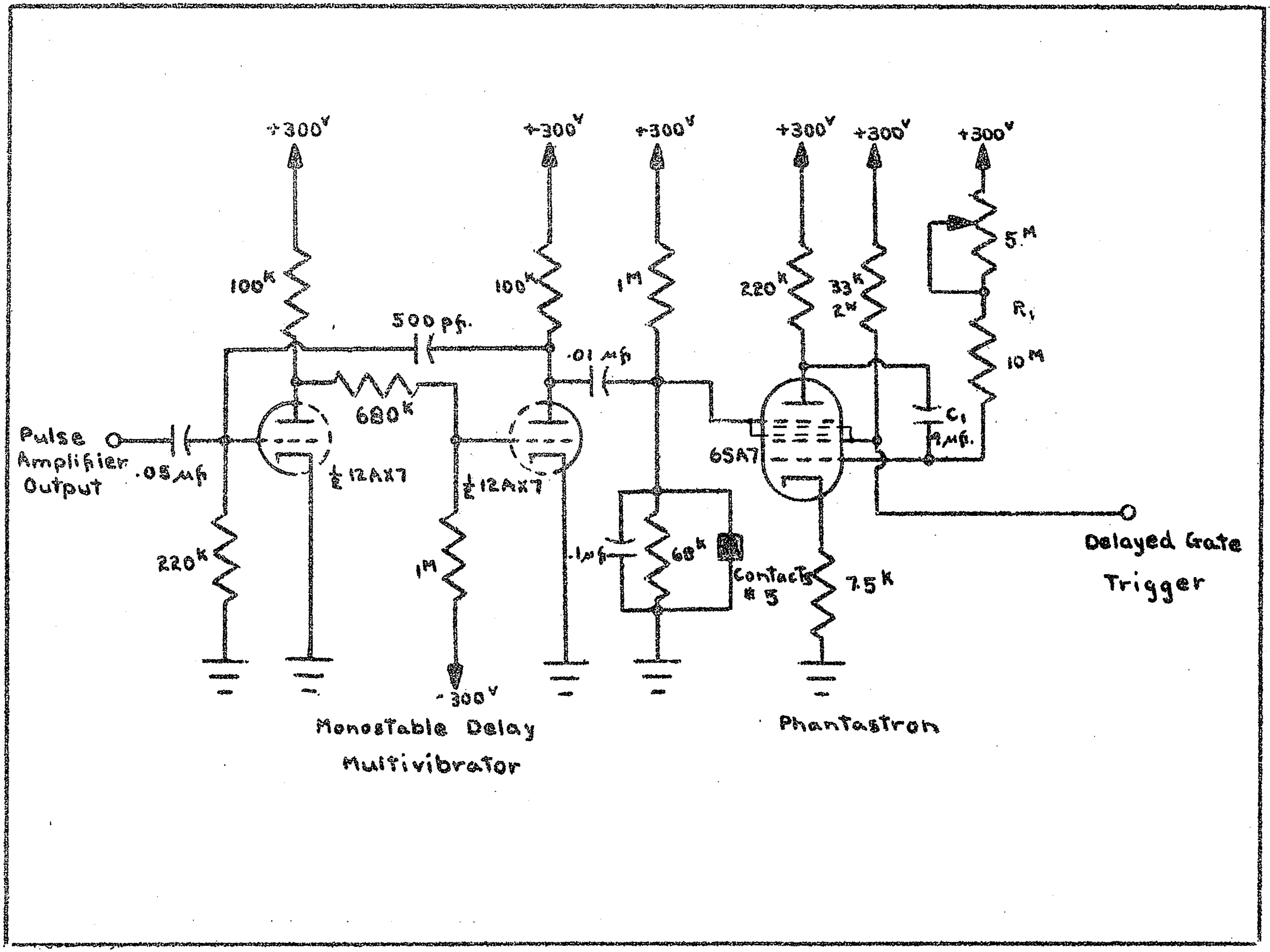


Figure 8: Phantastron Delay Circuit

DRIVER MULTIVIBRATOR FOR SAMPLING RELAY

The large drop in the screen potential at the end of the phantastron's charging cycle is used to trigger a monostable multivibrator having a period of 100 milliseconds and which in turn drives the relay in its plate circuit. The circuit involved is shown in Figure (9).

The period of 100 milliseconds was chosen to allow sufficient time for the integrator to charge the hold capacitor for the maximum possible swing of 210 volts. This calculation is found in Appendix (3).

Thus, at the end of the level setting sweep the average difference in level given by the level set integrator output is sampled and used to charge the hold circuit capacitor. The hold circuit capacitor must remain charged to this value for the following 29.85 seconds, that is during the following error sensing sweep, and ensuing level setting sweep up to the time of sampling.

LEVEL SETTING CIRCUIT

The necessity of being able to continuously vary the absolute voltage level of a 20 kilohm potentiometer over a range of 200 volts while maintaining a constant current flow of 10 milliamperes through the potentiometer resulted in a rather novel level setting circuit as shown in Figure (10).

Basically, the two tubes in the potentiometer chain act as variable resistances, the sum of which remains constant at 30 kilohms.

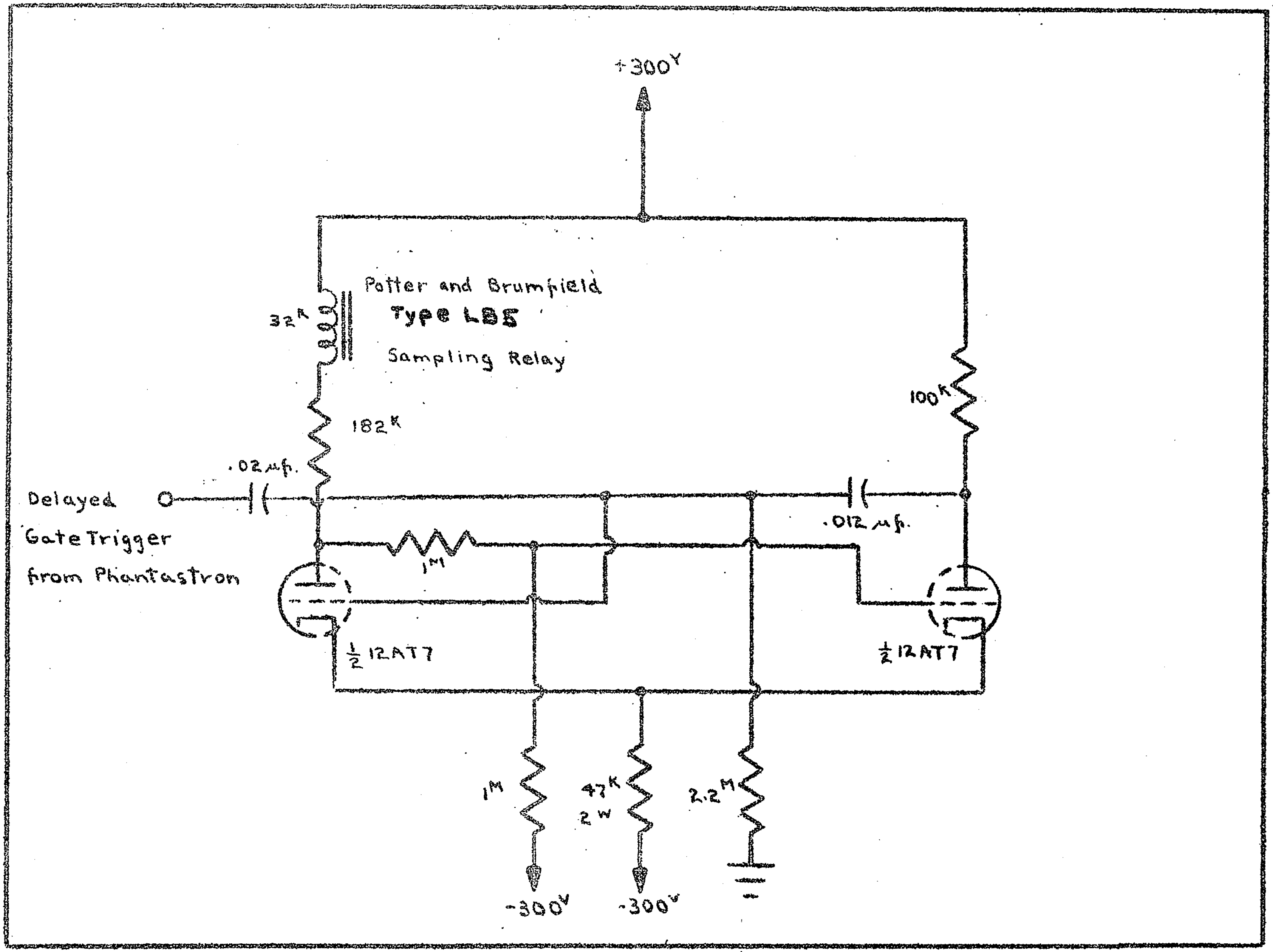


Figure 9: Driver Multivibrator for Sampling Relay

That is, their respective grids are driven in such a way that the sum of the beam resistances of T_1 and T_2 is always 30 kilohms. Since point "X" must vary from a potential of 0 to 200 volts with respect to ground, the plate to cathode potential E_{b_1} must vary from 250 to 50 volts while the plate current remains constant at 10 milliamperes. Concurrently, E_{b_2} must swing from 50 to 250 volts. The type 5687 dual triode was chosen for this application because of its very linear plate characteristics (see Figure (11)) over a wide plate voltage swing. At a plate current level of 10 milliamperes the grid bias must swing from -1 to -13 volts about a mean of -7 volts in order for the plate potential to swing from 50 to 250 volts. It is noted that the grid potential of T_1, that is E_{cc_1}, must change not only over the bias swing but must also change by the amount of the plate to cathode potential swing. A table of values for E_{cc_1} and E_{cc_2} is shown below.

E_m(v)	E_{c_1}(v)	E_{cc_1}(v)	E_{c_2}(v)	E_{cc_2}(v)
100	-1	249	-13	-263
0	-7	143	-7	-257
-100	-13	37	-1	-251

From this it is seen that

$$\frac{\Delta E_{cc_1}}{\Delta E_{cc_2}} = \frac{-212}{12} = -17.67$$

This gain ratio is realized by the phase splitting amplifier driving the grids of T_1 and T_2. It is seen that the

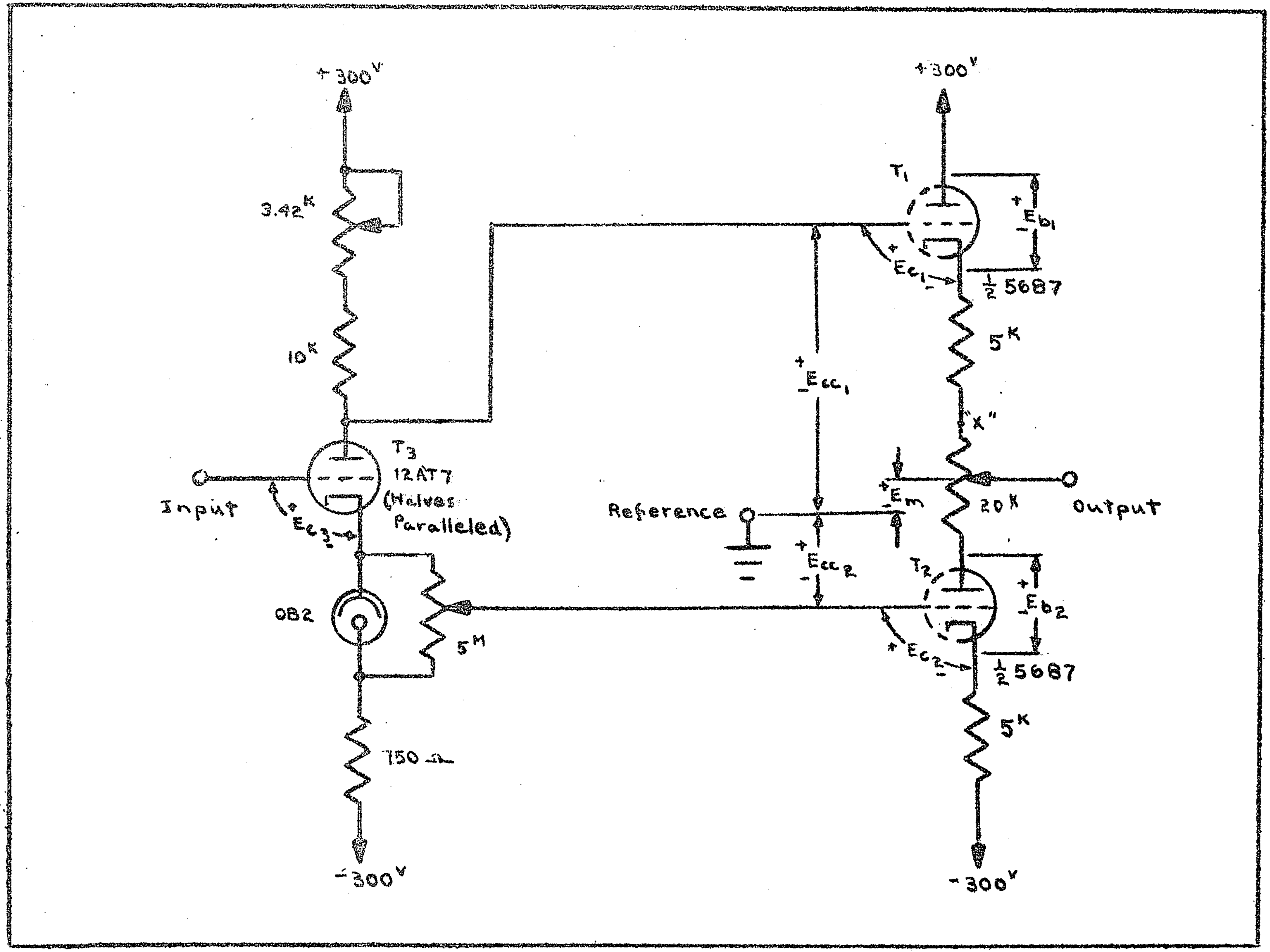


Figure 10: Level Setting Circuit

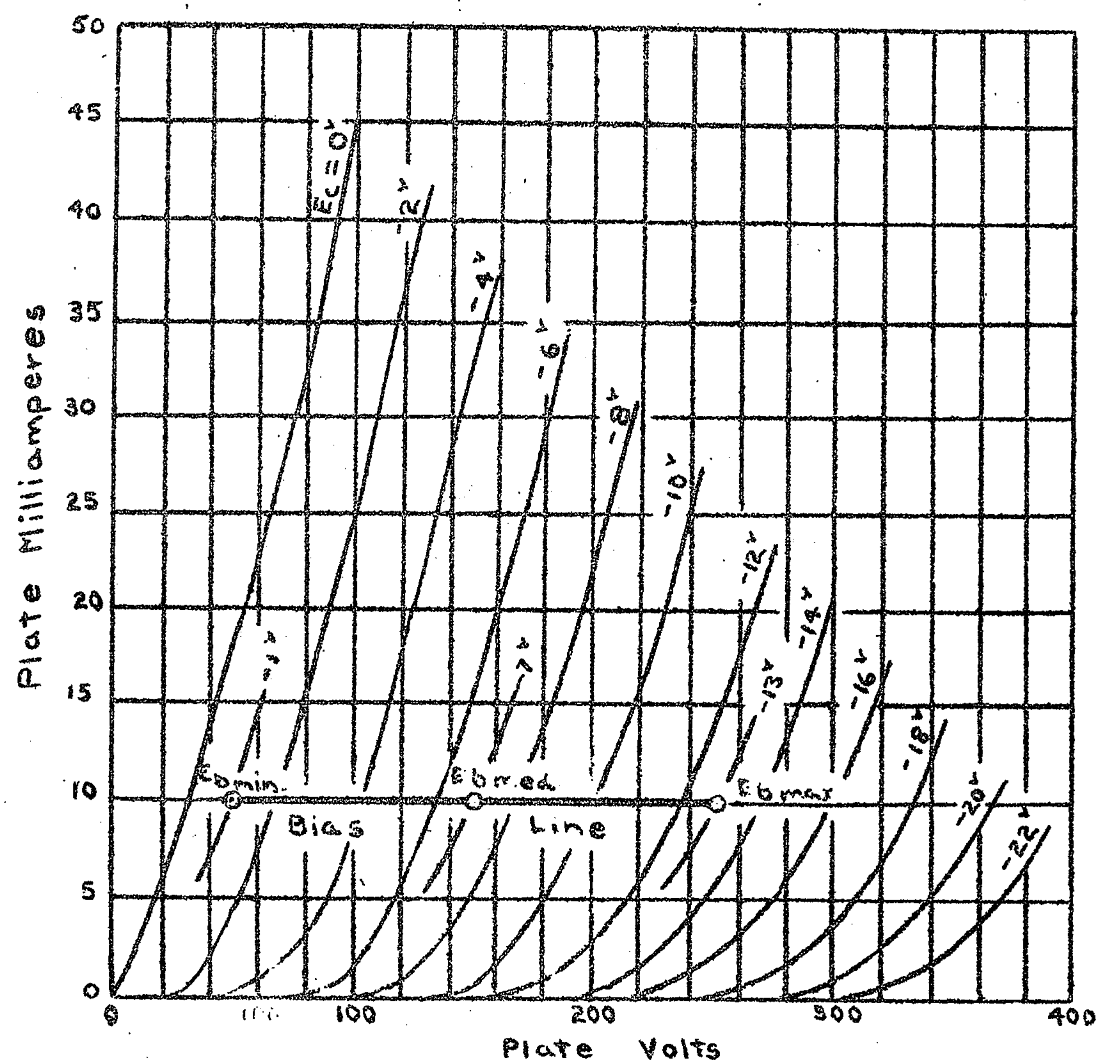


Figure 11: Tube Type 5687 Plate Characteristics

ratio of plate to cathode resistance of T_3 is

$$\frac{13420}{750} = 17.67$$

The regulator tube and 5 megohm potentiometer in the cathode circuit of T_3 are used to set the d.c. level for the bias of T_2 correctly.

Tests run on this level setting circuit indicate that the potential across the 20 kilohm potentiometer changes by ± 0.4 volts or by ± 0.2 percent from the nominal 200 volt value over the 200 volt operating range. The grid to cathode swing of the phase splitter for the full operating range of the circuit was 6.2 volts. Thus the average gain of the system is

$$\frac{-200}{6.2} = -32.3$$

From Figure (12) it is seen that the gain varies from -14.7 to -139 over the operating range.

The level set circuit must follow the output of the hold circuit with unity gain, since the hold circuit output is simply the difference in average level of the two inputs. When the relay samples the average level integrator output and the hold circuit capacitor is charged to the average difference in level of the two inputs, the level setting circuit must follow the hold circuit output in order to null out this average difference. In order that the level set system should have unity gain over its operating range, the level control circuit is preceded by an input operational amplifier and an operational amplifier with a gain of 100 and the

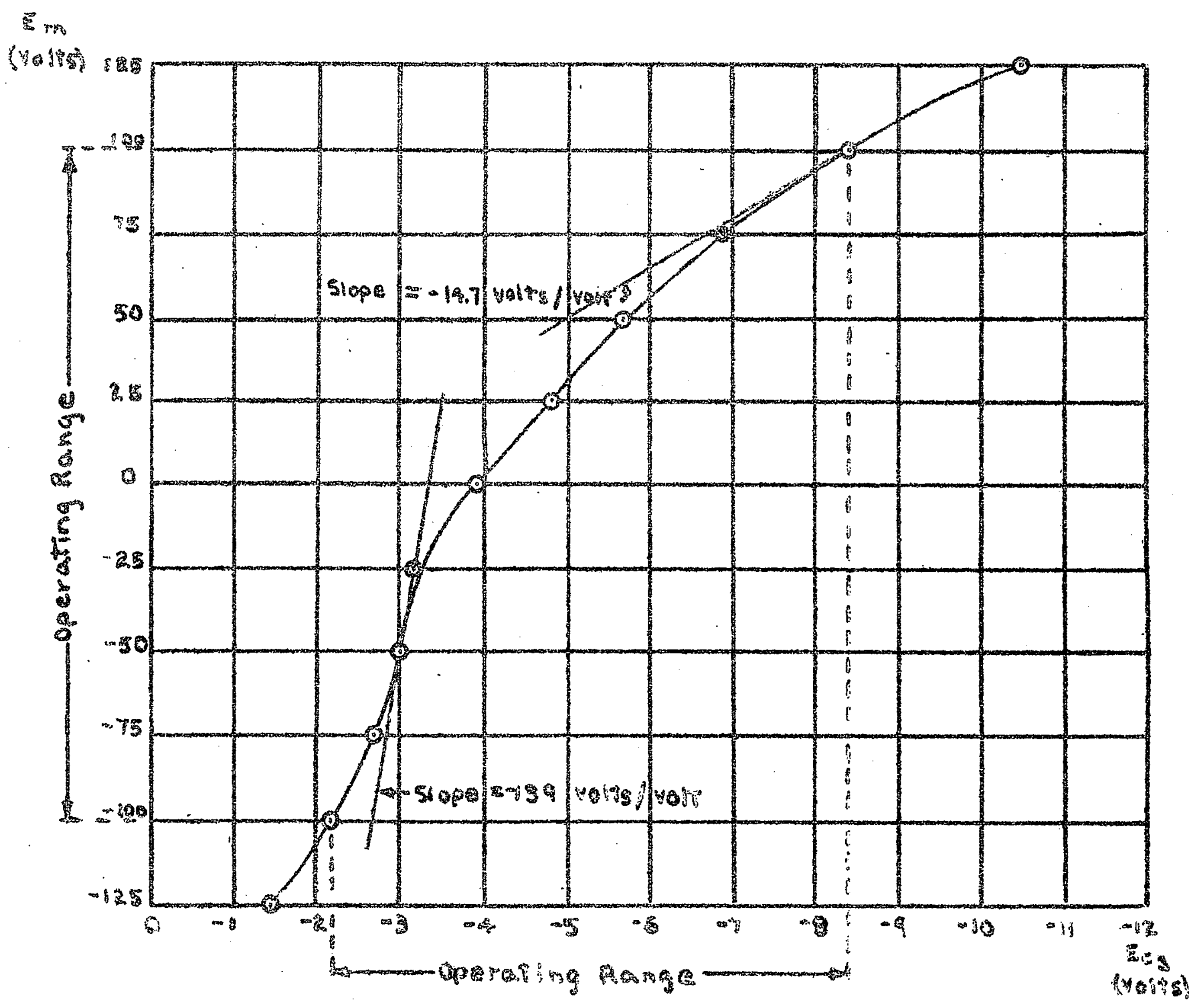


Figure 12: Gain Characteristic of Level Setting Circuit

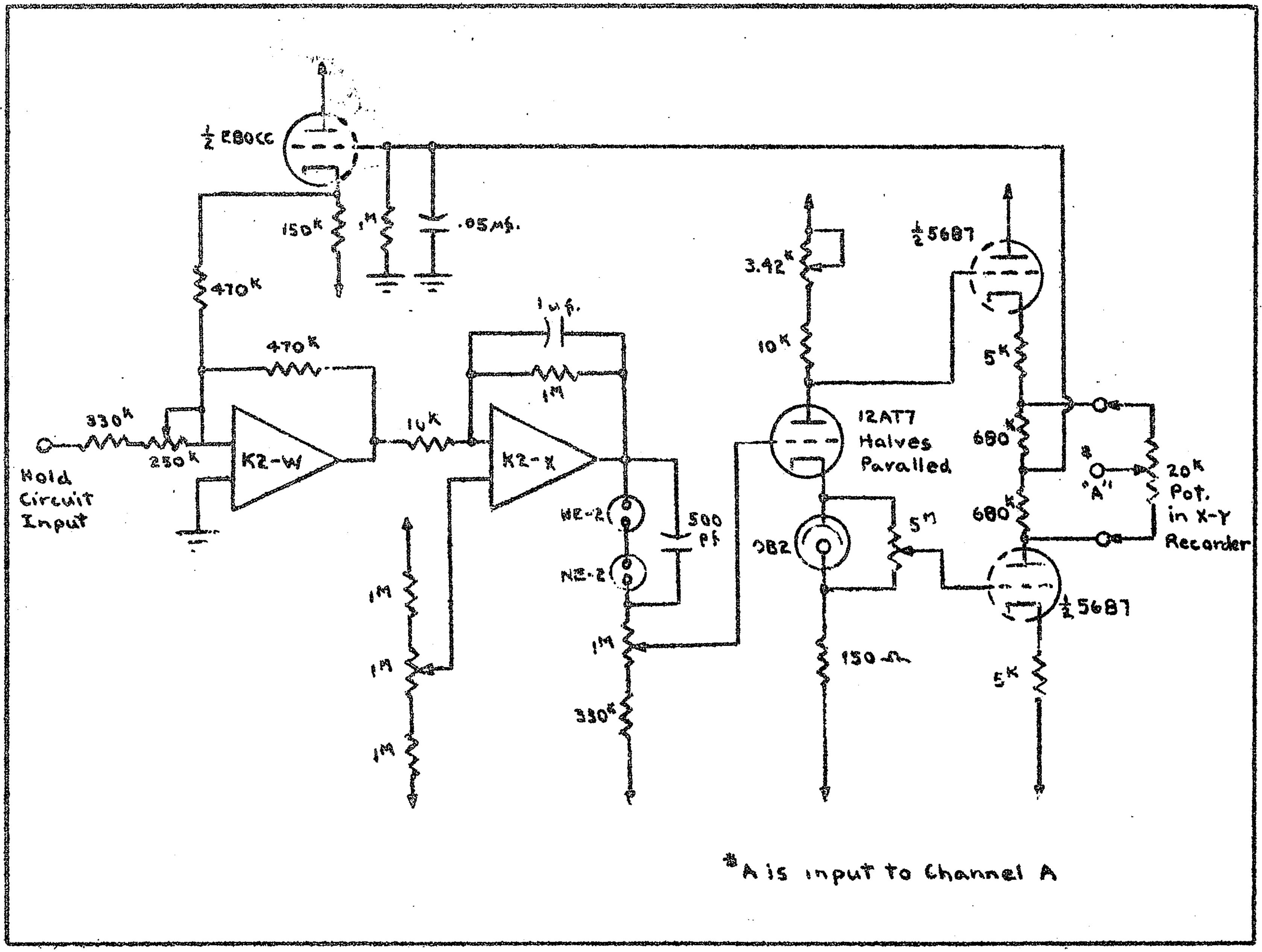


Figure 13: Level Setting Loop

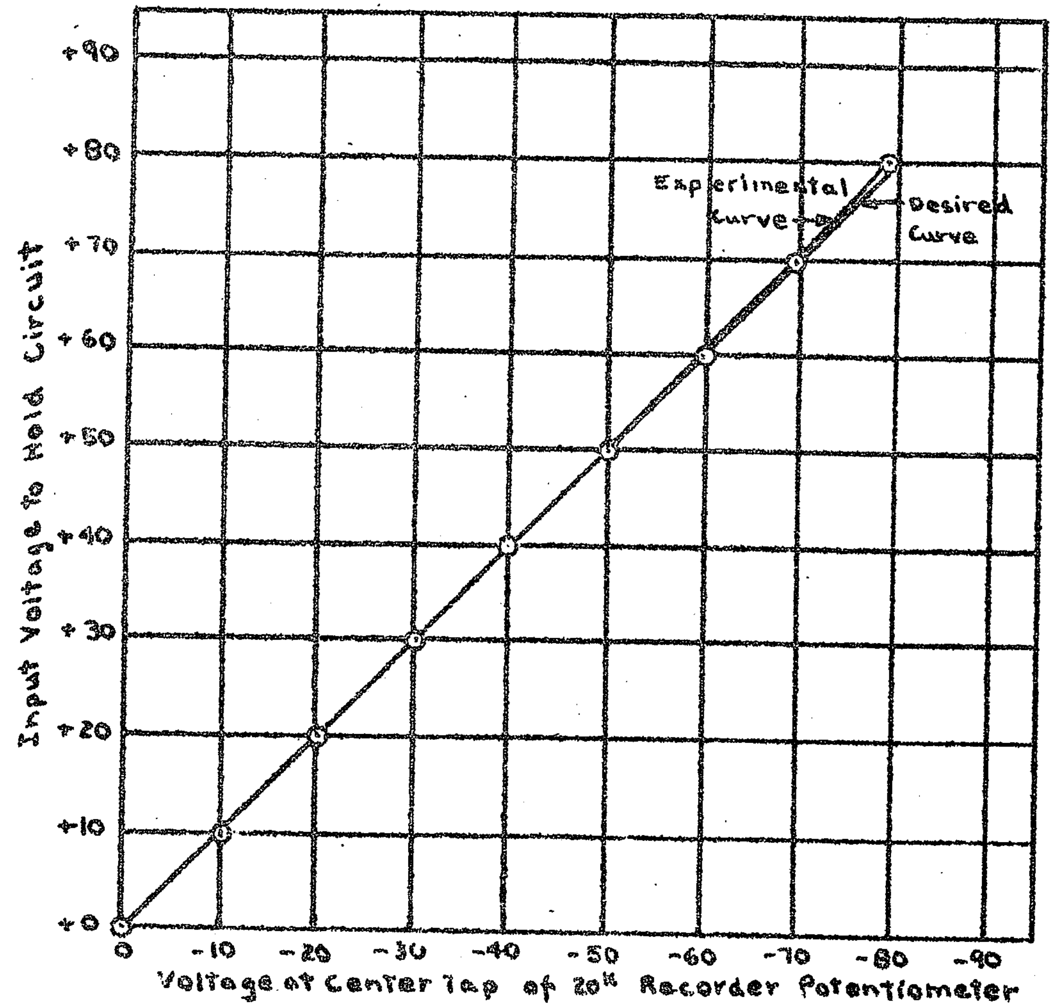


Figure 14: Linearity of Hold Circuit and Level Setting Loop (Positive Input)

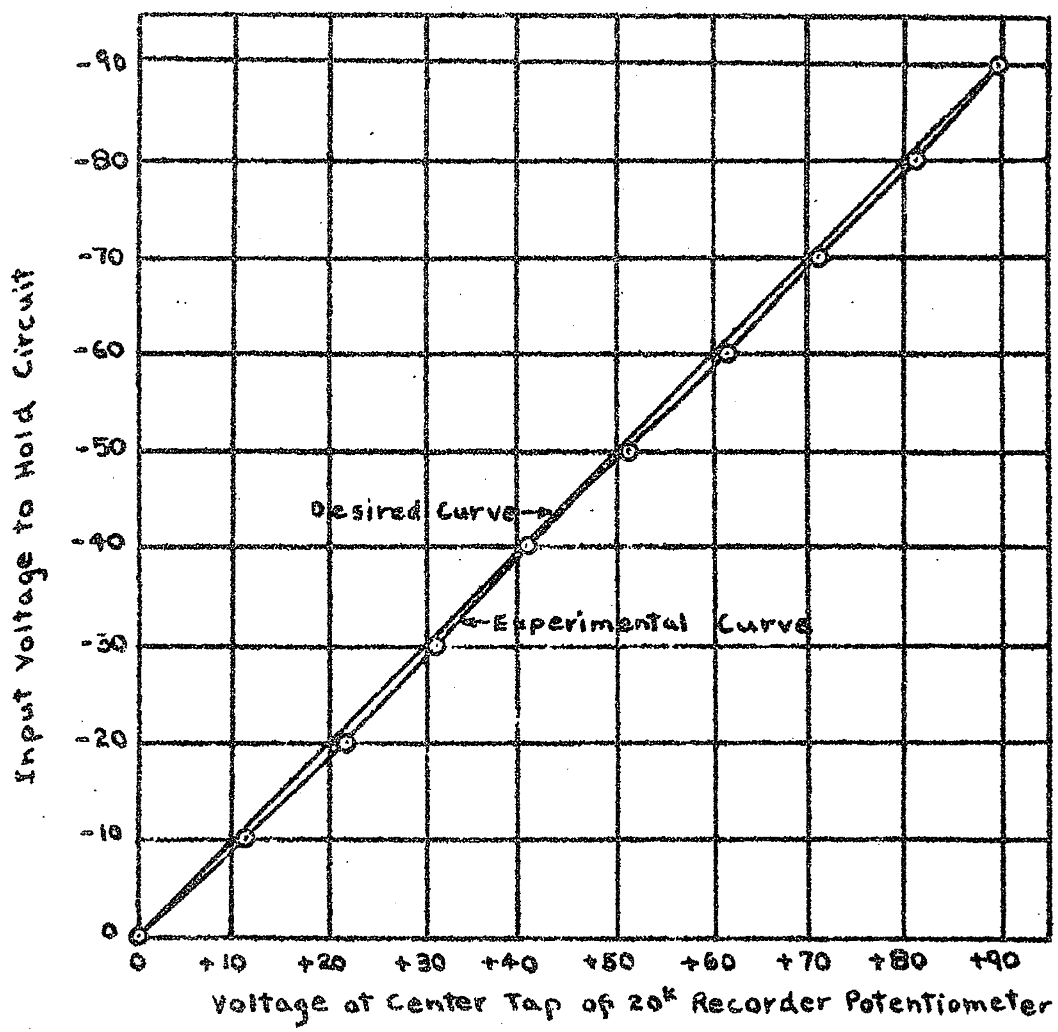


Figure 15: Linearity of Hold Circuit and Level Setting Loop (Negative Input)

whole system fully fed back through a cathode follower as shown in Figure (13). The loop gain of the system thus varies from 1470 to 13,900 depending on the operating point of the level control circuit. The gain of this level setting loop is unity to within less than ±1.5 volts error as may be seen from Figures (14) and (15).

CHAPTER III

TESTS OF ERROR SENSING UNIT

TEST 1: LINEARITY OF DIFFERENCE AND ABSOLUTE VALUE CIRCUIT

The difference circuit is theoretically perfect if the open loop gains of the operational amplifiers involved are considered infinite and drift is neglected.

The absolute value circuit is also theoretically exact if the diodes have an idealy sharp break characteristic between their conducting and nonconducting states, and if the inverting amplifier is perfect. These assumptions do not hold true in practice and therefore a test was performed to investigate the accuracy of both the difference circuit, and the absolute value circuit used in conjunction with the difference circuit.

The test was performed by applying constant but different voltages to the "A" and "B" channel inputs and measuring the output voltages of the difference and absolute value circuits.

The test data, along with the calculated outputs to the same number of significant figures as could be read from the test instruments, ~~is~~ are listed in Tables I and II.

The output of the absolute value circuit for zero input is 0.4 volts since the diodes are biased to the verge of significant conduction. Thus, the output of the absolute value circuit is corrected for this offset by subtracting 0.4 volts from all readings. The percentage error between the calculated and experimental values for the difference and absolute value circuits are also listed in Tables I and II respectively.

As may be seen from the results, the error in both cases is less than 1.2 percent and this may be attributed to drift in the operational amplifiers and to meter error.

TABLE I

Volts Input Channel A	B	Difference Output Volts Actual	Calculated	Error (Volts)	Percent Error
0	0	0.0	0.0	0.0	0.0
+ 50	0	-6.75	-6.67	+0.08	+1.2
+100	0	-13.4	-13.3	+0.1	+0.76
+150	0	-20.0	-20.0	0.0	0.0
+200	0	-26.8	-26.7	+0.1	+0.37
- 50	0	+ 6.75	+ 6.67	+0.08	+1.2
-100	0	+13.4	+13.3	+0.1	+0.76
-150	0	+20.0	+20.0	0.0	0.0
-200	0	+26.8	+26.7	+0.1	+0.37
+100	+100	0.0	0.0	0.0	0.0
+200	+100	-13.4	-13.3	+0.1	+0.76
0	+100	+13.4	+ 13.3	+0.1	+0.76
-100	-100	0.0	0.0	0.0	0.0
-200	-100	+13.4	+13.3	+0.1	+0.76
0	-100	-13.4	-13.3	+0.1	+0.76

TABLE II

Volts Input Channel A	Volts Input Channel B	Absolute Value Output (Volts) Actual	Absolute Value Output (Volts) Corrected	Absolute Value Output (Volts) Calculated	Error (Volts)	Percent Error
0	0	+0.4	0.0	0.0	0.0	0.0
+ 50	0	-6.2	-6.6	-6.67	-0.07	-1.05
+100	0	-12.8	-13.2	-13.3	-0.1	-0.75
+150	0	-19.4	-19.8	-20.0	-0.2	-1.00
+200	0	-26.2	-26.6	-26.7	-0.1	-0.37
- 50	0	-6.2	-6.6	-6.67	-0.07	-1.05
-100	0	-12.8	-13.2	-13.3	-0.1	-0.75
-150	0	-19.5	-19.9	-20.0	-0.1	-0.50
-200	0	-26.2	-26.6	-26.7	-0.1	-0.37
+100	+100	+0.4	0.0	0.0	0.0	0.0
+200	+100	-12.8	-13.2	-13.3	-0.1	-0.75
0	+100	-12.8	-13.2	-13.3	-0.1	-0.75
-100	-100	+0.4	0.0	0.0	0.0	0.0
-200	-100	-12.8	-13.2	-13.3	-0.1	-0.75
0	-100	-12.8	-13.2	-13.3	-0.1	-0.75

TEST 2: OVERALL GAIN OF ERROR SENSING CHAIN

This test was performed to calibrate the overall gain of the error sensing chain. That is, the overall gain of the difference circuit, the absolute value circuit and average error integrator or peak detector was found.

The test consisted of applying constant but different voltages to the "A" and "B" channel inputs and recording the output of the average level integrator and peak detector on a Sanborn recorder. Pulses from the comparator in the complex plane scanner were used to reset the error sensing unit after the 15 second error sensing sweep, as in normal operation of the unit; but the level setting chain was disconnected for this test. Thus, theoretically at the end of the 15 second error sensing sweep, the output of the average error integrator should have been the difference in voltage applied to the "A" and "B" channel inputs minus an offset voltage of 0.4 volts while the output of the peak detector should be the difference in applied voltage divided by 7.5 minus an offset voltage of 0.6 volts.

Tables III and IV list the test results as read from the Sanborn recorder paper, the results corrected for offset, the calculated outputs and gain constants for the average absolute error output and peak detector output. From these tables it is seen that the average absolute error output is only 0.96 of the actual average absolute error and all readings should be corrected by dividing them by 0.96 after correcting for the offset voltage. Similarly the peak absolute error output is slightly higher than calculated. Thus to obtain the actual peak absolute error the output of the peak detector should be multiplied by $\frac{7.5}{1.04} = 7.21$ after correcting for offset.

TABLE III

Volts Input Channel A	Volts Input Channel B	Average Absolute Error Voltage: Actual	Corrected	Calculated	Gain Constant
0	0	+ 0.4	0.0	0.0	1.0
+ 50	0	+48.1	+47.7	+ 50	.953
+100	0	+96.3	+95.9	+ 100	.959
-50	0	+47.7	+47.3	+ 50	.946
-100	0	+95.5	+95.1	+ 100	.951

Mean Gain Constant = 0.962

TABLE IV

Volts Input Channel A	Volts Input Channel B	Peak Absolute Error Voltage: Actual	7.5 Corrected	Calculated	Gain Constant
0	0	-0.6	0.0	0.0	1.0
+ 50	0	-7.9	-7.3	-6.7	1.09
+100	0	-14.7	-14.0	-13.3	1.05
-50	0	-7.5	-46.9	-6.7	1.03
-100	0	-14.4	-13.8	-13.3	1.04

Mean Gain Constant = 1.04

TEST 3: CURVE FOLLOWER ERROR

The accuracy of the curve follower in following a given response curve directly affects the accuracy of the error sensing unit in two ways. Obviously, as in any servomechanism, there must be some error in following the response curve in order to actuate the servo system. If this error is constant for all slopes of the curve being followed then it results in only a slight change in the average level of the Y axis potentiometer output and is compensated for by the level setting chain in the error sensing unit. If, however, the error varies with the slope of the response curve, large errors can be introduced in the peak absolute error sensing output of the error sensing unit although the accuracy of the average absolute error output may not be greatly affected. The second possible source of error results if the error in following the response curve is not identical for positive and negative slopes of the curve being followed. This is very important since the error sensing unit sets the average level of the two inputs equal and calculates the average absolute, peak absolute and absolute error on alternate sweeps in opposite directions along the response curve. Thus, the error sensing sweep might take place as the curve follower sweeps the response curve from right to left while the level setting sweep occurs as the curve follower sweeps from left to right. It now becomes obvious that if the error in following the curve is not the same for positive and negative slopes, the level setting circuit will not set the average level of recorder output exactly equal to that of the scanner input so far as the error sensing sweep is concerned. This, of course, would introduce a serious error into the accuracy of the average absolute error output and some error into the peak absolute error output. In order to investigate the curve follower accuracy a recorder pen was attached to the curve follower head and tracings of the recorder path made as the curve follower was run over several test curves with various slopes. The curve follower X axis servo was driven by the triangular sweep voltage with a period of 30 seconds, used in normal operation of the error sensing unit so that the test curves were swept alternately from right to left and left to right for 15 seconds. The displacement of the line traced by the pen on alternate sweeps gave the difference between negative and positive slopes. This difference in displacement was converted to a proportional voltage difference using the fact that 1 inch displacement in the Y direction of the curve follower head corresponds to 18.2 volts difference in the recorder Y axis potentiometer output. This difference voltage was then plotted against the slope of the test curve in volts per second and degrees as shown in Figure (16) to yield a useful indication of the curve follower accuracy for various slopes of response curve. The slopes in volts per second were easily computed since the curve was swept at the rate of 1 inch per second in the X direction, and as mentioned previously, the Y axis displacement may be calibrated in volts at the rate of 18.2 volts per inch.

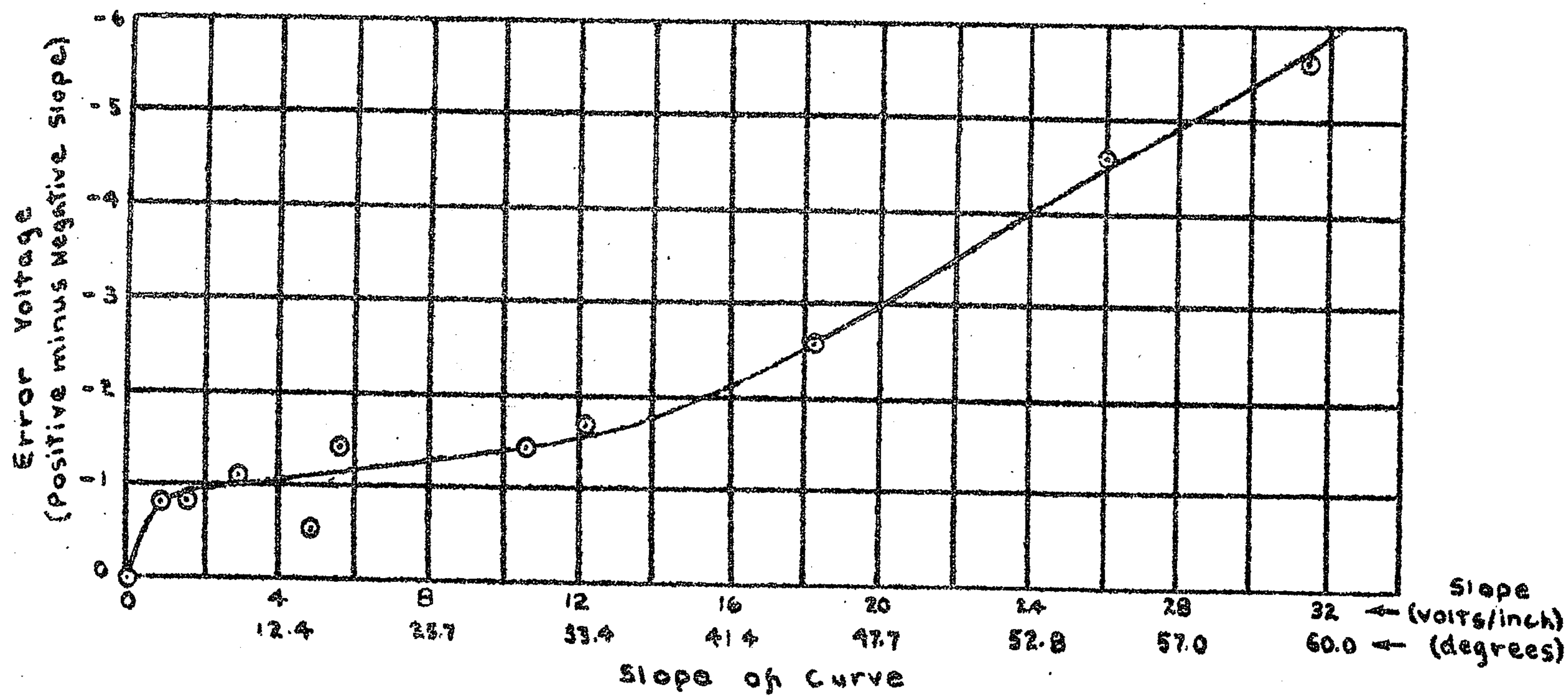


Figure 16: Error Voltage due to Curve Follower Between Sweeping Positively and Negatively

TEST 4: ACCURACY TEST OF ERROR SENSING UNIT

This test was run to give an indication of the overall accuracy of the error sensing unit and curve follower. A diagonal line was painted on the X-Y recorder in conducting ink from the lower left-hand to upper right-hand corner of the recorder sheet and the curve follower was used to follow this line. Thus the recorder output was a voltage varying linearly with time from -91 volts to 91 volts and back to -91 volts with a period of 30 seconds. The input voltage to the "B" channel of the error sensing unit was held constant for each test while the average absolute and peak absolute voltages were measured. The results of this test are shown in Tables V and VI.

The results of the absolute average error test indicate that the error sensing unit accuracy is about 4.5 percent low. Also, the average error due to the curve follower, though approximately the correct magnitude to account for this error, has no effect for this particular test because of the linear voltage input to channel "A" and constant voltage input to channel "B". In other words, the average absolute error is independent of the average level of the channel "A" input for the particular test voltages used. The reason for the apparently poor accuracy is the slow response of the meter used for monitoring the average absolute error. The output of the average error integrator was recorded on a Sanborn recorder and it was found that the average absolute error for 0 volts input to channel "B", as read from the recorder paper, was 44.5 volts, while the meter reading reached only 42 volts. The reading from the recorder, when corrected for offset and gain, yields an average absolute error of 45.9 volts or 0.88 percent high. It is thus concluded that the poor accuracy obtained using the meter readings was due almost entirely to the slow response of the meter and that the overall accuracy is within ±2.0 percent.

The peak absolute error readings appear to be about 1.6 percent low on the average.

TABLE V

Channel A input Volts	Average Absolute Error Voltage			
	Actual	Corrected for offset	Corrected for gain	Calculated
0	+42	+41.6	+43.4	+45.5
+20	42	41.6	43.4	45.5
+40	42	41.6	43.4	45.5
+60	42	41.6	43.4	45.5
+80	42	41.6	43.4	45.5
+100	42	41.6	43.4	45.5
-20	42	41.6	43.4	45.5
-40	42	41.6	43.4	45.5
-60	42	41.6	43.4	45.5
-80	42	41.6	43.4	45.5
-90	42	41.6	43.4	45.5
-95	42.5	42.1	43.9	45.5
-100	43	42.6	44.4	45.5

TABLE VI

Channel A input Volts	Peak Absolute Error Voltage			
	Actual	Corrected for offset	7.5 Corrected for gain	Calculated
0	-13.0	-12.4	-11.9	-12.1
+20	-13.0	-12.4	-11.9	-12.1
+40	-13.0	-12.4	-11.9	-12.1
+60	-13.0	-12.4	-11.9	-12.1
+80	-13.0	-12.4	-11.9	-12.1
+100	-13.0	-12.4	-11.9	-12.1
-20	-13.0	-12.4	-11.9	-12.1
-40	-13.0	-12.4	-11.9	-12.1
-60	-13.0	-12.4	-11.9	-12.1
-80	-13.0	-12.4	-11.9	-12.1
-90	-13.0	-12.4	-11.9	-12.1
-95	-13.2	-12.6	-12.1	-12.1
-100	-13.6	-13.0	-12.5	-12.1

TEST 5: WEIGHTING THE ERROR

This test was used to check the operation of the system used for weighting the average absolute and peak absolute error. The weighting function used is described by the equation.

$$w(t) = \left(1-\frac{t}{15}\right)$$

where t = 0 at the beginning of the error sensing sweep and t = 15 at the end of the sweep. The same test input for channel "A" was used for this test as for Test 4. The channel "B" input was held constant at zero volts for both the weighted and unweighted error tests. Providing the level setting chain is working properly, the absolute error may then be described by the equation

$$E = 91\left(1-\frac{t}{7.5}\right)u(t) + 182\left(\frac{t}{7.5}-1\right)u(t-7.5)$$

The average absolute weighted error is thus

$$\frac{91}{15}\int_{0}^{15}\left(1-\frac{t}{15}\right)\left(1-\frac{t}{7.5}\right)dt + \frac{182}{15}\int_{7.5}^{15}\left(1-\frac{t}{15}\right)\left(\frac{t}{7.5}-1\right)dt = 22.75$$

or exactly one-half of the unweighted average absolute error. The peak absolute error divided by 7.5 is theoretically the same as in the previous test, that is 12.1 volts, since the maximum occurs at the commencement of the error sensing sweep when the weighting function has a value of one.

The Sanborn recorder was used to record the output of the average absolute error integrator, while the peak detector output was monitored with a vacuum tube voltmeter.

As in Test 4 when the Sanborn recorder was used, it was found that for the unweighted error the average absolute error after corrections for offset and gain was 45.9 volts while the average absolute error for the weighted function was 23.0 volts after corrections for offset and gain. Thus the average absolute weighted error was 1.1 percent higher than the theoretical value.

The peak absolute error divided by 7.5, after corrections for offset and gain were identical for both the weighted and unweighted error and equal to the value of 11.9 volts found in Test 4. This value is 1.6 percent low as found previously.

TEST 6: APPLICATION OF THE ERROR SENSING UNIT TO AN IMMITTANCE FUNCTION

For this test an immittance function

$$F(s) = \frac{(s+20)(s+10+j40)(s+10-j40)}{(s+10+j20)(s+10-j20)}$$

was set up on the complex plane scanner and its log magnitude frequency response curve recorded on the X-Y recorder. This curve was then painted with conducting ink and used in conjunction with the curve follower. Care was taken in recording the scanner output to assure that the tip of the pen was displaced the same amount from the recorder carriage as is the curve follower head, in order that there would be no time displacement between the scanner output and curve follower Y axis potentiometer output when they were run simultaneously. The scanner log magnitude output was then connected to the "B" channel input of the error sensing unit and the error between the curve follower output and scanner output measured. The peak absolute error divided by 7.5 was found to be 0.46 volts while the average absolute error was 1.81 volts after corrections for gain and offset. Both the peak and average errors should, theoretically have been zero since identical response functions were being compared. It was assumed that the error was largely due to the inaccuracy of the curve follower. To verify this assumption, the recorder response curve was approximated by a number of straight line segments and the average error which the curve follower would contribute was calculated from Figure (16). This calculation yielded a value of .78 volts which would account for about 42 percent of the average error. The curve follower error was thus largely responsible for the average absolute error and in all likelihood for a large part of the peak absolute error.

An attempt was then made to find the sensitivity of the error sensing unit to variations of the pole and zero positions in both the σ and $j\omega$ directions in the complex plane for the F(s) used previously. The results were individually inconclusive due to the inaccurate operation of the scanner sweep unit which was in a rather experimental state at the time the tests were performed. However, by averaging the results it was found that a $\frac{\Delta\omega}{\omega_0}$ of .05 (where ω_0 is the original pole or zero ω co-ordinate and $\Delta\omega$ the change in the co-ordinate) should be easily detectable in the peak absolute error measurement, while a $\frac{\Delta\sigma}{\sigma_0}$ of 0.10 (where σ_0 is the original pole or zero σ co-ordinate and $\Delta\sigma$ the change in the co-ordinate) should be detectable also in the peak absolute error measurement. The change in average absolute error in response to a change in pole or zero position was not nearly as pronounced as that of the peak absolute error.

CHAPTER IV

DISCUSSION AND FUTURE MODIFICATIONS

The test results of Chapter III indicate that the error sensing unit is accurate to within less ~~and~~ than ± 2.0 percent of the correct value in the measurement of peak absolute and average absolute error after the corrections for gain and offset are made. However, this accuracy can only be obtained in the average absolute error measurement if a recorder or voltmeter with a frequency response in the order of 50 to 100 cps. is used. The meter on the error sensing unit and ordinary voltmeters are too highly damped to follow the average error integrator output. It is felt that a four channel recorder can be used in conjunction with the error sensing unit to excellent advantage. This allows the simultaneous recording of the $\frac{\text{error}}{7.5}$, the $\frac{\text{absolute error}}{7.5}$, the average absolute error and the scanner output. The recording may then be used to find at which points the scanner output is in greatest error with respect to the desired output from the curve follower. The pole-zero positions of the immittance function represented by the scanner may then be changed, a new recording made, and the new recording compared with the previous one to see how the response curve and errors have changed. In this way, an immittance function with the desired response curve can, in all probability, be more quickly obtained than by using only the error sensing unit.

The use of a weighting function to emphasize certain portions of a required response curve is desirable. An example of this might be the design of a filter for which the break frequency and rate of attenuation in a certain region of the log magnitude curve are quite critical. The critical region could then be emphasized by an appropriate weighting function. Weighting is also advantageous for investigations into the effects of changes in pole or zero positions of an immittance function in certain regions of its response curve.

Caution must be exercised in the use of the curve follower. The desired response curve should be drawn on the curve follower paper to a scale such that the maximum slope of the curve does not exceed 30 volts/sec. (about 60 degrees). Slopes greater than this value will cause excessive errors due to the inaccuracy of the curve follower in reproducing the curve. A rough estimate of the average absolute error to be expected from the curve follower inaccuracy may be made by approximating the response curve with a series of straight line segments, and calculating the average absolute error from Figure (16). It is difficult, however, to estimate the peak absolute error to be expected from the curve follower inaccuracy. Since the curve follower is the major source of inaccuracy in the error sensing system, a modification to reduce this error or the purchase of a more accurate curve follower is highly recommended.

In Test 6, it was found that the scanner output voltage for the log magnitude frequency response of the immittance function had a peak-to-peak variation of only about 45 volts. This, of course, necessitated recording the response over only about one-fourth of the available curve follower sheet since one inch of the 10 inch wide sheet corresponds to 18.2 volts. The curve follower error was then quite large in comparison to the overall swing of the response curve. If the level setting circuit were modified to supply a constant 5 instead of 10 milliamperes to the 20 kilohm curve follower Y axis potentiometer, twice as much of curve follower sheet could be utilized. This would reduce the relative size of the curve follower error and it is recommended that the modification be carried out. It must also be realized, however, that doubling the amplitude of the response curve on the curve follower sheet will also double the slopes which the curve follower head must follow, and if the slopes become too excessive the curve follower becomes inaccurate. In this case the overall accuracy may be poorer than if only a small portion of the curve follower sheet is used. The modification will, however, allow more versatility in the scale to which the response curve is plotted and with judicious use, the inaccuracy in error measurements due to the curve follower should be reduced.

Because the complex plane scanner was still in a rather experimental state with the phase and phase slope outputs inoperative and the sweep unit being modified, more complete tests of the error sensing unit could not be carried out. When the scanner is completed, however, the gain constants of the various outputs in magnitude/volt, db/volt, degrees/volt, and $\frac{\text{degrees}}{\text{rad/sec}}/\text{volt}$ may be determined and the scale to which the response curves for the curve follower are plotted and corresponding settings of the "B" channel input attenuator calculated. Several combinations of attenuator setting and scale of response curve should be computed in order that excessive slopes of the response curve on the curve follower paper may be avoided.

It is felt that upon completion of the complex plane scanner, further investigation into the sensitivity of the error sensing unit in detecting changes in the pole-zero locations of immittance functions should be carried out.

The tests performed with the error sensing unit indicate that the unit will perform successfully the operations for which it was designed. These operations are outlined in the preface and are valuable contributions to the usefulness of the complex plane scanner.

APPENDICES

APPENDIX 1: ALIGNMENT PROCEDURE

The alignment procedure for the error sensing unit consists of adjusting the bias voltages of the Philbrick d.c. operational amplifiers in the error sensing chain, the bias voltage of the level setting integrator and of adjusting the time delay of the sampling relay. All of these adjustments may be made from the front panel and should be carried out after at least a one-hour warm-up period of the error sensing unit and complex plane scanner. The scanner warm-up period is necessary to assure that the 15 second period between comparator pulses at the end of every sweep of the $j\omega$ axis in the scanner has stabilized, and thus so the sample delay may be set accurately.

Several screwdriver adjustments are also found on the top of the error sensing unit chassis but these should not need adjustment in normal usage of the unit.

The alignment procedure should be carried out as follows:

1. Place the shorting cap on the scanner input co-axial connector.

2. Move the toggle switch found on the top left side of the chassis to the "off" position and connect a ground to the test point found beside the switch.

3. Monitor the "Channel "B" Inverter" voltage at the test point on the front panel and adjust for zero volts.

4. Monitor the "Difference Output" voltage at the test point on the front panel and adjust for zero volts.

5. Monitor the "Absolute Value Inverter" voltage at the test point on the front panel and adjust for zero volts.

6. Monitor the "Level Integrator Output" voltage at the test point on the front panel and adjust for no change in output, indicating that the bias is correct and no integration is taking place. This is most accurately carried out by changing the bias control so that the integrator output approaches zero volts and then using a sensitive range on the monitor voltmeter, adjust the bias control until no integration takes place. It is noted that the "Difference Output" must be zero while this adjustment is made.

7. Monitor the "Average Absolute Error" and adjust for zero integration with the absolute error attenuator set at 1000. Note that this adjustment must be made on the error sensing sweep since on the level setting sweep the average absolute error integrator capacitor is shorted.

8. Using a dual trace oscilloscope and a sweep time of 0.1 seconds per centimeter, simultaneously monitor the comparator pulse found at pin number 1 of the 12 pin plug at the rear of the chassis and the "Sample Delay" found on the front panel. Adjust the sample delay so that the pulse which energizes the sample relay occurs 0.13 seconds before the comparator pulse.

9. The "Peak Absolute Error" test point is connected to the peak detector output while the adjustment beneath it controls the initial condition voltage on the peak detector hold capacitor. This adjustment should not be moved unless the "Absolute Error Attenuator" is not set at 1000. In the latter case, the initial condition voltage placed on the hold capacitor should be

$$0.4 \times \text{Attenuation} + 0.4 \text{ volts.}$$

To make this adjustment the error sensing unit must be removed from its rack and an extension cord used since the hold capacitor can only be reached from beneath the chassis.

10. Remove the ground connection from the test point on the top left side of the chassis and move the associated toggle switch to the "on" position.

APPENDIX 2: LEAKAGE OF THE LEVEL SET HOLD CAPACITOR

To check the leakage of the level set hold circuit the capacitor was charged first to +100 volts and then to -100 volts and the change in the cathode follower output voltage after 30 seconds measured for each case. The change was 0.5 volts when the capacitor was charged to +100 volts and negligible when charged to -100 volts. Thus for the capacitor charged to +100 volts the voltage after 30 seconds has fallen to 99.5 volts or

$$99.5 = 100\, e^{-\frac{t}{RC}}$$

$$= 100\, e^{-\frac{30}{R\times 10^{-6}}}$$

$$\frac{30 \times 10^{6}}{R} = .005$$

$$R = 6 \times 10^{9} = 6000 \text{ megohms}$$

Thus the leakage resistance of the hold circuit is in excess of 6000 megohms. ~~assuming that the cathode follower gain is very close to unity~~.

APPENDIX 3: CHARGING PERIOD OF LEVEL SET HOLD CAPACITOR

It is conceivable that when the sampling relay closes, the voltage to which the level set hold capacitor is charged would have to change by the maximum swing of the level integrator output. That is, the level set capacitor voltage would be required to change by 210 volts. Since the integrator is a G.A. Philbrick K2-X which is capable of supplying 3 milliamperes and

$$I(t) = C\frac{dV(t)}{dt} \approx C\frac{\Delta V}{\Delta t}$$

Then

$$\Delta t \approx \frac{C\Delta V}{I(t)}$$

$$= \frac{10^{-6}x210}{3x10^{-3}} = .07 \text{ seconds}$$

The sampling relay must therefore remain closed for at least 70 milliseconds to assure adequate time for the capacitor to charge fully. To allow for delay in the relay closing, the driver multivibrator for the sampling relay was designed for a switching period of 100 milliseconds. This causes the relay to remain closed for about 80 milliseconds which is quite adequate.

APPENDIX 4: NOTES ON OPERATION OF THE ERROR SENSING UNIT

The error sensing unit must under no circumstances be turned on unless the curve follower Y axis potentiometer is connected to the error sensing unit through the four pin receptacle at the rear of the chassis. The pin connections for this plug are as follows:

Pin 1 - Low voltage side of 20 kilohm potentiometer.
Pin 2 - Potentiometer slider.
Pin 3 - Not used.
Pin 4 - High voltage side of 20 kilohm potentiometer.

The connections to the main 12 pin plug at the rear of the chassis are listed below:

Pin 1 - Comparator pulse input.
Pin 2 - Multiplier input.
Pin 3 - Not used.
Pin 4 - Not used.
Pin 5 - Multiplier output.
Pin 6 - Not used.
Pin 7 - 6.3 volts a.c.
Pin 8 - Ground.
Pin 9 - 300 volts.
Pin 10 - 6.3 volts a.c.
Pin 11 - Chassis.
Pin 12 - 300 volts.

The power switch on the front panel controls only the positive and negative high voltage supplies. The filaments are turned on when the complex plane scanner is in use.

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