THE ADVISABILITY OF INCLUDING A UNIT ON VECTORS IN THE MATHEMATICS 101 PROGRAM

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by

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A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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ABSTRACT

The purpose of this study was to investigate the advisability of including a unit of vectors in the Manitoba Mathematics 101 program. Because vectors have practical applications, it was hoped that students would appreciate the usefulness of vectors and accept positively a study of the topic. Attitude towards and achievement in vectors were considered in the study.

A group of fifty Manitoba Mathematics 101 students were taught a unit on vectors for a period of five weeks. Student attitude towards mathematics was measured before vectors were studied; attitude towards the study of vectors was measured after the completion of the unit. A paired t-test was employed to compare the pre-test and post-test attitude scores. In addition, a test was prepared to measure achievement in the unit on vectors. This test was administered after the completion of the unit. A t-test compared the achievement of the Manitoba students to a standard which was established by having the test written by 238 Ontario students who had studied the same material. The sets of data were subjected to a statistical analysis.

Analysis of the data provided descriptive information about the experimental group's attitude and achievement. It was found that scores representing positive attitude towards

vectors were significantly higher than scores representing positive attitude towards mathematics. There was no significant difference between the achievement of the Manitoba students and the achievement of the Ontario students.

On the basis of this evidence, it was concluded that the Mathematics 101 students had a favorable attitude towards the study of vectors and that they had achieved satisfactorily. It was therefore recommended that a unit on vectors be considered as a possible topic for the Mathematics 101 program.

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Chapter 1

INTRODUCTION

STATEMENT OF THE PROBLEM

The mathematics program in Manitoba has undergone changes in the past few years. New texts have replaced old texts at all levels of instruction. The introduction of the General Course in the early 1960's has provided an alternate curriculum for mathematics students in grades ten, eleven, and twelve. It may be that further changes should occur. Perhaps some units should be deleted and replaced by new units and perhaps some new topics should be introduced.

This study investigated a possible change in the mathematics program. Specifically it dealt with the advisability of including a unit on vectors in the Mathematics 101 program.

PURPOSE OF THE STUDY

This study was conducted to determine if vectors might be a suitable topic for inclusion in the Mathematics 101 program. Attitude towards, and achievement in a unit on vectors were considered in the investigation.

The changing curriculum and the need for further change suggested that specific topics for specific courses should be scientifically studied before change occurs.

Mathematics and indeed education in general has undergone significant changes over the past few decades. J. Shibli made an observation in a book published in 1932:

> The new philosophy of education has emphasized the changing character of our civilization and has demanded an education that is not static but dynamic. These developments have had a marked influence upon the curriculum of the secondary school. The various courses of study have been altered or modified to meet the new demands. Teachers of mathematics have been reorganizing the courses of study and changing the methods of instruction in harmony with the spirit of the age.[1]

The mathematics program in Manitoba has been modified in recent years. Curriculum planners need to continue to evaluate present programs so that appropriate additional modifications can be made. Ideally a change in curriculum should occur only if a positive effect is to be expected. If possible, subjective evaluation of programs should be supported by scientific research when an alteration is being considered. When scientific research precedes the implementation of a new unit or program there is a greater probability that a positive effect will result. Donovan Johnson recognizes the importance of scientific research in the following:

> The type of research accepted by many researchers as having the greatest promise for finding definitive answers is scientific experimentation.[2]

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[1] J. Shibli, Recent Developments in the Teaching of Geometry (State College, Pennsylvania: J. Shibli Publishing, 1932), p. 33.

[2] Donovan A. Johnson, "A Pattern for Research in the Mathematics Classroom," The Mathematics Teacher, 59:420, May 1966.

In this study, scientific research was employed to evaluate the unit on vectors as a possible addition to the Mathematics 101 program.

A review of the research entitled "Student Achievement and Attitude in a Modern and a Traditional Grade Ten Geometry Program," by Robert Walter Cross, provided some of the impetus for this study. The results of the study made by Cross are summarized briefly in the following excerpt from the abstract of his thesis:

> In all comparison groups, except one, equivalent results were obtained by both treatments. This supported and verified the findings of similar studies which have concluded that students taking a modern geometry course do as well as students taking a traditional geometry course. However, nowhere in the data of this study was there any indication of a strongly positive acceptance of geometry by either the pilot or the traditional groups. Consequently further research is recommended to study methods which will increase student enjoyment and acceptance of geometry.[3]

There may be doubt as to whether the change in the curriculum referred to in the above study was truly beneficial to the students. Perhaps more extensive research should have preceded the implementation of the program.

The finding that in neither the traditional nor the pilot group was there a positive acceptance of geometry is of concern. It would seem that for students to receive maximum benefits from a course they should have a positive attitude

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[3] Robert Walter Cross, "Student Achievement and Attitude in a Modern and a Traditional Grade Ten Geometry Program," (unpublished Master of Education thesis, University of Manitoba, 1968), pp. vii-viii.

toward the work. "The attitudes of students towards mathematics play a vital part in their learning."[4] A positive attitude towards a particular topic may develop whereas other topics may generate a negative attitude. A basic purpose of this study was to determine whether the study of vectors would be accepted favourably by the students.

The writer assumes that in addition to having a favourable attitude towards a subject the students should be expected to meet a certain standard of achievement. To this end achievement was measured and compared to a standard.

If a positive attitude towards vectors could be shown to exist and if the level of achievement could be shown to be satisfactory it might be advisable to consider including a unit on vectors in the program.

THE STUDY

The study was concerned with grade ten general mathematics commonly designated as Mathematics 101. It was designed to determine the advisability of including a unit on vectors in the Mathematics 101 program. This investigation was concerned primarily with the attitude towards mathematics as compared to the attitude towards vectors. Achievement in

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[4] Mary Corcoran and Glenadine E. Gibb, Evaluation in Mathematics, The Twenty-sixth Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1961), p. 105.

the topic was measured and compared to a standard established by other students writing the same test. The study began in April, 1972 and was completed in May, 1972. The duration of the experiment was approximately equal to the time allotment as is suggested in Mathematics 10 Teacher's Commentary for a similar unit on vectors.[5]

The experiment involved the teaching of the unit on vectors to two Mathematics 101 classes or a total of fifty students. Steps were taken to eliminate the "Hawthorne Effect."[6] The classes were not told that they were participating in an experiment. When students made inquiries they were given the impression that the unit on vectors was being studied as a normal option in the regular program. The researcher taught both classes and made every effort to teach the experimental topic with the same degree of enthusiasm, patience, and enterprise as he displayed in his other teaching.

Attitude Measurement

To determine whether the students had accepted the vector topic with a favourable attitude, a pre-test, post-test

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[5] J. J. Del Grande, J. C. Eggard, and H. A. Mulligan, Mathematics 10 Teacher's Commentary, Second Edition (Toronto: W. S. Gage Limited, 1968), p. vii.

[6] Stephen S. Willoughby, "Issues in the Teaching of Mathematics," Mathematics Education, The Sixty-ninth Yearbook of the National Society for the Study of Education (Chicago: The University of Chicago Press, 1970), p. 261.

statistical technique was employed. Immediately prior to the teaching of the unit on vectors both classes were given two attitude tests. The first of these was entitled Mathematics Attitude Scale (MAS). It was developed by Aikens of the University of North Carolina. The other was an instrument developed by the writer under the title Mathematics Attitude Measure (MAM). Immediately after the completion of the teaching of the unit on vectors the two tests were administered again. Each test, however, was modified by:

1. replacing the word "mathematics" with the word "vectors";
2. making appropriate grammatical corrections necessitated by the changes described in 1 above;
3. arranging the items in a different random order.

The modified tests were respectively entitled Mathematics Attitude Scale Vectors (MASV) and Mathematics Attitude Measure Vectors (MAMV).

Appropriate statistical methods were used to determine if the attitude towards vectors differed from the attitude towards mathematics.

Achievement Measurement

An achievement test was given to determine the level of achievement attained. The researcher developed an instrument under the title Vector Achievement Test (VAT). A jury of experienced and qualified teachers examined the test. The jury agreed that the test adequately tested the unit on vectors.

The Vector Achievement Test was given to 238 students in Grade X in Ontario who had recently completed a similar topic on vectors. The Ontario sample was selected randomly from the entire province; thus it was assumed that the results on VAT formed a good basis for the comparison. The achievement level of the Ontario sample was statistically compared to the achievement level of the Manitoba sample.

DEFINITIONS, ASSUMPTIONS, AND LIMITATIONS

For this study some definitions, assumptions, and limitations were recognized.

Definitions

Mathematics 101--Mathematics 101 is the course name for the mathematics program which was authorized for grade ten in the General Course in Manitoba. The General Course was designed for the average student. University Entrance Courses are maintained for those students desiring a stronger and more rigorous academic background. The majority of Manitoba students have traditionally taken either courses from the General Course or courses from the University Entrance Course. A smaller percentage of the students pursue the Commercial, Vocational, or Occupational Entrance Course. Recently there has been a trend away from students following a program which includes only General Course subjects or only University Entrance Course subjects. Students are able to choose subjects from each of the two courses. At present Mathematics 101 can be considered to be a course offered to those students who desire a general mathematics background but who do not wish to pursue the more rigorous Mathematics 100. A student's choice in mathematics need not be affected by his choice of the other subjects he wishes to study.

Unit on Vectors--The unit on vectors is descriptive of the topic on vectors taught to the experimental classes. It is essentially Chapter Twelve of Mathematics 10 (Second Edition), a textbook written by J. J. Del Grande, J. C. Egsgard, and H. A. Mulligan. Mathematics 10 is the text currently being used in Ontario at the tenth grade level.

Assumptions

It was assumed that the tests used in the study were both reliable and valid. It was also assumed that the results obtained were not affected by teacher variation. Thus any differences in the attitude or achievement measurements were dependent on the nature of the mathematical topic. It was further assumed that all students involved in the study had the necessary ability to handle multiple choice type questions and that none of the students had previous exposure to any of the test items. Finally, it was assumed that all test time limits were adhered to and generally that tests were properly administered.

Limitations

A limitation on the study may have been the length of time during which the experiment was conducted. Long range results may not agree with the results obtained in this study. A longer experimental period, however, was not considered as the topic was adequately covered in the five week period. It was not the intention of this study to examine the advisability of a longer, more thorough treatment of vectors in a Mathematics 101 program.

HYPOTHESES TESTED

This study was designed to compare attitude towards the study of vectors with attitude towards mathematics. To do this, the first and second hypotheses were formulated. Achievement in vectors was compared to a standard established in an out-of-province class. The third hypothesis was established to make this comparison. For the purpose of the statistical analysis, the hypotheses were stated in null form.

First Hypothesis

There is no significant difference between the attitude towards mathematics as measured by the pre-test, *Mathematics Attitude Scale*, and the attitude towards vectors as measured by the post-test, *Mathematics Attitude Scale Vectors*.

Second Hypothesis

For each of the following categories of attitude,

a) student attitude towards content of subject (content);

b) student attitude towards usefulness of subject (needs); and

c) student interest in subject (interest) no significant differences exist between the attitude towards mathematics as measured by the pre-test Mathematics Attitude Measure, and the attitude toward vectors as measured by the post-test Mathematics Attitude Measure Vectors.

Third Hypothesis

No significant difference exists between the achievement level of the Mathematics 101 students from Manitoba and the achievement level of the Grade X students from the Ontario sample as measured by the achievement test entitled Vector Achievement Test.

SUMMARY

Change in curriculum is continuous. There is a need to examine carefully all proposed program alterations in order to be certain that only appropriate changes are made. Student attitude towards and achievement in a subject were considered to be important criteria that should be examined before implementing curricular changes. This study examined the possibility that vectors might appropriately be included in the Mathematics 101 program. To do this, student attitudes towards and achievement in vectors were considered and the results used as a basis to formulate a recommendation to the Mathematics 101 curriculum committee.

The literature was reviewed to determine if there was support for vectors as a mathematical topic and to ascertain if there was support for the importance of attitude in the learning process.

Chapter 2

REVIEW OF THE LITERATURE

This chapter is presented under four main areas, each of which relates significantly to the study. The first area deals with geometry and its importance as a discipline within the mathematics program. Secondly, literature related to attitude was reviewed. Because a basic purpose of this thesis was to examine attitude, there was a need to determine whether or not there was support for the importance of attitude in the learning process. The third area concerns vectors both as an important mathematical topic and as a topic which lends itself to practical application. In addition, this section discusses the desirability of using applications to motivate learning. The last area deals with the Mathematics 101 course as it exists at present.

GEOMETRY IN THE PROGRAM

Mathematics 101 contains very little geometry. It may be that these students would profit from the studying of more geometry. Since vectors are a form of geometry, literature was reviewed to attempt to ascertain the prevailing sentiment regarding the inclusion of geometry into the program.

The relative importance of geometry in the mathematics program is viewed with varying degrees of enthusiasm.

H.S.M. Coxeter fears that geometry does not always receive sufficient emphasis in the Mathematics curriculum. He made the following comment at a joint session of the Mathematics Association of America and the National Council of the Teachers of Mathematics at Houston, Texas in 1967:

> In the prevalent desire for single courses in mathematics rather than separate courses in the various branches, there has been a tendency for geometry to be squeezed out. This tendency is not only regrettable but unreasonable.[1]

Coxeter believes that a study of geometry offers an opportunity that no other study can do as well. He suggests that, "In geometry, perhaps more than in any other subject, a student can exercise originality and ingenuity in devising a construction or seeking a proof."[2]

Shibli considers geometry to be important. He suggests that, "Many of the best teachers today look upon the study of geometry as an opportunity for the pupil to do his own thinking and to cultivate the spirit of investigation and discovery."[3]

Intuitive or Rigorous

There is disagreement on whether a geometry program

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[1] H.S.M. Coxeter, "The Ontario K-13 Geometry Report," Geometry in the Secondary School (Washington: The National Council of Teachers of Mathematics, January, 1967), p. 8.

[2] Ibid., p. 9.

[3] Shibli, op. cit., p. 240.

should be intuitive or rigorous. Bruce Meserve says that in the United States, informal geometry in elementary school and junior high school is a most promising approach. He suggests that until students reach higher and more sophisticated levels they should study geometry from an intuitive approach.[4]

Mathematics in general is subject to criticism in connection with the degree of rigor that is most appropriate. Morris Kline does not favor modern mathematics. He believes mathematics for the average student should be less rigorous. He would like mathematics to be presented as intuitively as possible and that a student should prove only what he thinks he should prove.[5]

The opinion of Kline is not shared by all. James H. Zant does not agree with Kline. He claims that Kline is making unsupported criticism of the program. He challenges Kline to write a book encompassing his beliefs so that his ideas can be tried and evaluated by teachers in the schools.[6]

Carl B. Allendoerfer questions the value of a great deal of rigor in geometry:

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[4]Bruce Meserve, "Geometry in the U.S.," Geometry in the Secondary School (Washington: The National Council of Teachers of Mathematics, January, 1967), p. 6.

[5]Morris Kline, "A Proposal for the High School Mathematics Curriculum," The Mathematics Teacher, 59:322-330, April, 1966.

[6]James H. Zant, "A Proposal for the High School Mathematics Curriculum--What Does It Mean?" The Mathematics Teacher, 59:331-334, April, 1966.

> Virtually every modern treatment seeks to find a set of axioms that will fix the weaknesses of Euclid, and then everything must be proved on this basis. This leads to enormous tedium, and there is no escape if rigor is to be preserved. Even so, most of these schemes are bogus, for they assume the completeness properties of the real numbers, which are seldom if ever taught in the schools. Good teachers know how to compromise at this stage, and I encourage them to do so. We must not bury the geometry under an avalanche of rigor.
>
> I think well of another suggestion that has been around a long time. This is that we abandon the goal of proving everything from the axioms. When intuition suffices, let us use it.[7]

Andrew Elliot describes the position of geometry in Great Britain. He says that geometry is offered to the majority of students. It is presented as a useful tool and for its inherent beauty. The approach is somewhat intuitive; however, it is designed to provide adequate background for higher mathematics.[8]

Coxeter is concerned about the degree of rigor that is appropriate in a geometry course. The following summary exemplifies his view:

> By dealing with geometry informally, by plausible reasoning rather than by strict proof, it is possible to reach interesting and surprising results much more quickly. The student does not spend a whole hour on a proof of something he regards as obvious. Thus several theorems can be covered in one lesson, and students can be given a bird's eye view of the subject as a whole.[9]

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[7] Carl B. Allendoerfer, "The Dilemma in Geometry," The Mathematics Teacher, 62: 165-169, March, 1969.

[8] Andrew Elliot, "Geometry in Great Britain," Geometry in the Secondary School (Washington: The National Council of Teachers of Mathematics, January, 1967), p. 17.

[9] Coxeter, op. cit., p. 8.

ATTITUDE IN THE LEARNING PROCESS

Mary Corcoran and Glenadine Gibb expressed their views on attitude in The Twenty-sixth Yearbook of the National Council of Teachers of Mathematics. They suggest that a student's attitude towards mathematics is dependent on factors such as intellectual appreciation of and emotional reaction to the subject. They believe that the attitude of students towards a subject greatly affects learning.[10]

Cross compared a modern and a traditional approach to geometry. When in neither group was there a strong acceptance of geometry, Cross suggested that, "It may be speculated that both pilot and traditional groups would most likely have achieved a higher standard if they had a more positive attitude towards geometry."[11]

Chapter II of The Twenty-first Yearbook of the National Council of Teachers of Mathematics was written by Maurice L. Hartung. It is a discussion of motivations in mathematics education. Included in his list of motives which are useful in teaching are purpose, interests and attitudes.[12]

Hartung makes the following comment:

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[10] Corcoran and Gibb, op. cit., pp. 105-106.

[11] Cross, op. cit., p. 142.

[12] Maurice L. Hartung, "Motivation for Education in Mathematics," The Learning of Mathematics, Its Theory and Practice. The Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1953), p. 44.

> Attitudes influence behavior and hence act as motives. They are learned and, in turn, they often make new learning easier or harder to acquire. One of the chief obstacles to the effective learning of mathematics is the unfavorable attitude toward the subject which has been acquired by many students.[13]

If a student's attitude toward a subject is recognized to be worthy of consideration, it would seem useful to look at factors affecting attitude. Hartung says,

> Prominent among the causes given for the unfavorable attitudes were lack of understanding, failure to provide enough application to life and social usage, poor teaching techniques, poor motivation, and feelings of inferiority and insecurity.[14]

He recognizes that "Learning is greatly influenced by the student's awareness of his need for what is to be learned, or of its potential usefulness and value to him."[15]

Importance of Application

Many educators believe that attitude towards mathematics is greatly affected by a student's perception of the topic as it relates to practical application in the real world.

Shibli suggests that, "One of the means of vitalizing geometry is the introduction of applied problems that are encountered in the various practical fields of human activity."[16]

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[13] Ibid., pp. 45-46.

[14] Ibid., p. 55.

[15] Ibid., p. 45.

[16] Shibli, op. cit., p. 159.

Morris Kline feels strongly that topics in mathematics should have application. He says that mathematics teachers should show students how mathematics is used in other fields.[17] He claims that ninety-eight percent of high school students find mathematics without application, meaningless and uninteresting.[18] His feelings are expressed in the following:

> Mathematics is not an isolated, self-sufficient body of knowledge. Mathematics exists primarily to help man understand and master the physical world and, to a slight extent, the economic and social worlds. Mathematics serves ends and purposes. It does something. We must constantly show what mathematics accomplishes in domains outside of mathematics.[19]

Stephen S. Willoughby describes the situation regarding theoretical versus applied aspects of mathematics:

> For many years, geometry was thought of as something people should study in order to improve their ability to reason. E. H. Moore, of the United States, criticized school geometry and other parts of school mathematics for not being sufficiently applied and suggested specific changes that they believed would be desirable. After these recommendations were made (in the late 1800's and early 1900's), the curriculum in England did begin to include more applied mathematics, but relatively little movement in that direction has occurred in the United States.[20]

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[17]Morris Kline, Mathematics in Western Culture (New York: Oxford University Press, 1953), p. 330.

[18]Ibid., p. 329.

[19]Ibid., p. 324.

[20]Stephen S. Willoughby, "Issues in the Teaching of Mathematics," Mathematics Education, The Sixty-ninth Yearbook of the National Society for the Study of Education (Chicago: University of Chicago Press, 1970), pp. 273-274.

Willoughby goes on to say that modern mathematics developed for the space age tends to be abstract. One of the reasons cited was that it is difficult to find good applied problems. However, at present two of the notable school mathematics projects (S.M.S.G. and the Cambridge Conference on School Mathematics) have made a genuine effort to make use of physical applications in the teaching of school mathematics.[21]

The literature reviewed generally suggests that a good attitude is important. Students are apt to have a better attitude towards a topic if they can see where the topic will be useful to them or at least if they can see a practical application in the real world.

VECTORS

> The relation of successful experiences to attitudes, and hence to motivation and learning, provides one of the strongest arguments for curriculum reorganizations. It suggests that content too difficult for a particular time should be relocated at a more favorable time, or in some cases omitted entirely.[22]

A unit on vectors may be a topic which should be included in the Mathematics 101 program. Vectors could provide students with an area of study which can readily be applied to physical situations.

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[21] Ibid.

[22] Hartung, op. cit., p. 58.

Vectors are being taught in the German Gymnasium.[23] A study in Germany beginning twenty years ago resulted in a decision that vectors should play a major role in the mathematics program. Athen makes the following comment:

> In the German Gymnasium it has been found that the teaching of vectors in mathematics has proved most advantageous by serving as a complement to the instruction in vectors given in physics. Thus the mathematical-tool idea of vectors is stressed as much as the theoretical structure of a vector space.[24]

Dan Smith supports the teaching of vectors in an article entitled "Vectors--An Aid to Mathematical Understanding."[25] He suggests that a study of vectors supplies an interesting way of practicing addition and subtraction of signed numbers. In addition, according to Smith, vectors provide another example for use in study of the properties of a group.

The Japanese introduced a study of vectors in Mathematics II-B (eleventh school year). The outline for the course indicates that the study is advanced and probably suited to top-calibre students.[26]

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[23] Dr. Herman Athen, "The Teaching of Vectors in the German Gymnasium I," The Mathematics Teacher, 59:485-495, April, 1966.

[24] Ibid., p. 485.

[25] Dan Smith, "Vectors--An Aid to Mathematical Understanding," The Mathematics Teacher, 52:608-613, October, 1966.

[26] Howard F. Fehr, "The Mathematics Program in Japanese Secondary Schools," The Mathematics Teacher, 59:577-583, October, 1966.

Stewart Moredock suggests that because vector quantities such as displacement, force, velocity and acceleration are being studied in science programs, there is a need for studying vectors in the mathematics program.[27] He also sees a study of vectors as an opportunity to help develop the concept of a mathematical group.

THE MATHEMATICS 101 SITUATION

Since the introduction of Mathematics 101 into the high school program, very little research has been done to examine the suitability of the topics for the type of student in the course. Recently a new text has been authorized for the program. However, without adequate research it is difficult to ascertain whether or not the text and the related curriculum outline are overcoming any shortcomings of the original Mathematics 101 program.

There is constant concern over the suitability of the mathematics curriculum. Planners are constantly seeking to improve the quality of the program.

After having taught Mathematics 101 the author felt that some of the program does not appeal to the students who opt into the course. It was difficult to make students cognizant of why they were studying some of the topics. Since

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[27] Stewart Moredock, "Geometry and Measurement," Mathematics Education, The Sixty-ninth Yearbook of the National Society for the Study of Education (Chicago: The University of Chicago Press, 1970), pp. 202-203.

many students could not see practical application for the topics, they were inclined to be unenthusiastic about their mathematics.

It could be that a unit with practical application where a student could see how the mathematics is useful would be more appealing to Mathematics 101 students. If students could see the practicality of the topic, attitude might be better, achievement levels higher, and, very importantly, the students would enjoy mathematics.

Chapter 3

THE DESIGN OF THE EXPERIMENT

THE SAMPLE POPULATION

The sample consisted of all students enrolled in Mathematics 101 in Charleswood Collegiate in Winnipeg, Manitoba in 1971-72. There was a total of fifty students in two classes of approximately equal size. Since all of the Mathematics 101 students in the school were participants in the study, there was no need for random selection of the group. All students were given attitude tests before and after the teaching of the unit on vectors. In addition, the students were given an achievement test on vectors at the completion of the unit.

Several Grade X students from Ontario were given the same achievement test. These students were selected at random. This involved randomly choosing the names of fourteen schools in Ontario from The Directory of Schools 1970/71, Province of Ontario. The schools were written to with the purpose of inviting them to participate in the study. Five schools responded to the request. These schools administered and returned answer sheets from a total of 238 students.

THE UNIT ON VECTORS

The topic on vectors was precisely Chapter Twelve of

Mathematics 10 (Second Edition), written by S. S. Del Grande, J. C. Egsgard, and H. A. Mulligan. This was the text used in Ontario schools at the Grade X level. The unit is an introduction to vectors which culminates in the study of some applications of vectors. The following description is found in the teacher's commentary designed to accompany the text.

> This chapter constitutes a non-rigorous and intuitive introduction to vectors. We begin (section 12.1) by studying the course of an airplane and show that the course can be represented by an ordered pair of real numbers or by a directed line segment. The meaning of addition of courses is discussed. Vectors in 2-space are defined (section 12.2) as any mathematical entities that can be representative either by ordered pairs of real numbers or by directed line segments and that have a rule of addition like that for courses. Addition and subtraction of vectors as well as multiplation of a vector by a scalar are discussed. The chapter ends with the application of vectors to geometry and to navigation, in 2-space.[1]

Students were taught the section in a traditional manner combining lecture, class discussion, and problem solving. Exercises were always of a nature that was similar to that given in the text. In fact, most exercises were assigned directly from the text.

The author of this thesis had previously taught Mathematics 10, including Chapter Twelve, the unit on vectors, under the Ontario curriculum for two years. He found that the unit adequately covered an introductory investigation of vectors with some applications.

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[1] S. S. Del Grande, J. C. Egsgard, and H. A. Mulligan, Mathematics 10 Teacher's Commentary, Second Edition (Toronto: W. S. Gage Limited, 1968), p. 55.

He also felt that students found the unit challenging and rewarding. The material covered seemed to appeal to the students. Because of the interest which the students displayed towards the topic, it was considered that it might be possible to successfully include a unit on vectors in the Mathematics 101 program.

ATTITUDE MEASUREMENT INSTRUMENTS

Before vectors were taught, attitude tests were given to the fifty students in the sample from Manitoba. The tests were designed to measure the attitude of students towards mathematics. They consisted of several items, each of which was a statement intended to measure the student's attitude towards mathematics. One-half of the items were positive in nature, such as item number three of MAS: "Mathematics is very interesting to me, and I enjoy math courses." The other one-half of the items were of a negative nature, such as item number two of MAS: "I do not like mathematics, and it scares me to have to take it." The attitude towards mathematics of each students was measured by using a Likert scale for marking.[2] Students were instructed to indicate their feeling towards each item by circling their choice of the following:

SD - Strongly Disagree
D - Disagree
N - Neither Agree nor Disagree
A - Agree
SA - Strongly Agree

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[2]Robert T. Hedley, "Student Attitudes and Achievements in Science Courses in Manitoba Secondary Schools," (unpublished Doctoral Thesis, Mighican State University, East Lansing, 1966), pp. 86-87.

Table 1

POINTS ASSIGNED TO ITEMS ON ATTITUDE INSTRUMENTS

Response	Type of Statement	Points
SD	positive	1
D	positive	2
N	positive	3
A	positive	4
SA	positive	5
SD	negative	5
D	negative	4
N	negative	3
A	negative	2
SA	negative	1

Thus if he agreed (A) with a positive statement, he was allotted 4 points, while if he strongly disagreed (SD) with a negative statement, he was allotted 5 points. The sum of the points for all items on each test was then considered to be representative of the student's attitude towards mathematics.

The preamble to the attitude tests was designed to ensure that students would respond to the items in an honest and frank manner. The front page of the test is reproduced on the following page:

Do Not Begin Until You Are Told To Do So.

This questionnaire contains a set of statements about the mathematics course you are taking. The statements are presented as opinions rather than as facts. As opinions, they are neither right nor wrong. This is not a test, so it is very important that you answer the questions according to your own feelings and judgments. After you have read each statement carefully, it is best to answer by giving your first impression or reaction and then go on to the next item. Feel free to answer honestly and frankly. Your answers will in no way affect your grades.

Indicate how you feel about each statement by circling your choice on the answer sheet provided. The choices that you will be given are the following:

SD - Strongly Disagree
D - Disagree
N - Neither Agree nor Disagree
A - Agree
SA - Strongly Agree

Do Not Begin Until You Are Told To Do So.

Immediately after the unit on vectors was taught, modified attitude tests were administered to each of the fifty students.

The modified tests were designed to measure the student's attitude towards vectors. Each test was similar to the original test. The only changes were:

1) the word "mathematics" was replaced by the word "vectors";

2) appropriate grammatical corrections due to changes described above were made;

3) the items were arranged in a different random order. (This rearrangement was recommended by the statistical advisors for this study.)

The changes made can be exemplified by considering item number 18 of MAS which read: "I am happier in math class than in any other class." This statement later appeared on MASV as item number 9 which read: "I am happier in a vectors class than in any other class."

The attitude towards "vectors" was represented numerically in the same manner as was the attitude towards mathematics.

Mathematics Attitude Scale

Mathematics Attitude Scale (MAS) was an instrument developed by Dr. Lewis R. Aiken and Ralph M. Dreger of the University of North Carolina. It contained twenty items which were designed to measure the student's attitude towards mathematics. Under the marking scale the maximum score possible was 100 (20 items times 5 points maximum) while the minimum score possible was 20 (20 items times 1 point minimum).

The modified test, administered after the unit on vectors, entitled Mathematics Attitude Scale (Vectors), (MASV), similarly had a maximum of 100 points and a minimum of 20 points.

In order to determine whether there was a difference in the attitude towards mathematics and the attitude towards vectors, the MAS and MASV scores of each student were compared. To do this, a paired t-test was employed. John E. Freund, in Modern Elementary Statistics, states the following regarding comparisons of this type of data:

> The samples are not independent; in fact, in each case the data are paired. To handle problems of this kind,

> we work with the differences of the paired data (retaining their signs) and test whether these differences may be looked upon as a sample from a population for which $\mu_o = 0$ . . . and if the sample is large, we use the test. . . .[3]

According to Freund, the sample is large if it contains 30 or more pairs.[4] Since the data for the sample under study had 50 pairs, the following statistic was applicable:

$$Z = \frac{\bar{\bar{X}} - \mu_o}{S/\sqrt{n}} \quad \text{where ,}$$

$\bar{X}$ = mean of sample differences.

μ_o = mean of population. Here $\mu_o = 0$ since the null hypothesis stated that there exists no difference between the before and after scores.

S = sample standard deviation.

n = number in the sample; that is, the number of sample differences.

The sample standard deviation was:

$$S = \sqrt{\frac{\sum (X_i - \bar{\bar{X}})^2}{n - 1}} \quad \text{where ,}$$

n = number in the sample.

$\bar{X}$ = mean of sample differences.

Xi = each sample difference.

Tables were established (see Appendix A) and the

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[3] John E. Freund, Modern Elementary Statistics (Englewood Cliffs: Prentice-Hall, Inc., 1967), p. 260.

[4] Ibid., p. 252.

value of the statistic determined. A level of significance of .05 was then used to investigate the first null hypothesis which stated that no significant difference exists between the attitude towards mathematics as measured by the pre-test MAS and attitude towards vectors as measured by the post-test MASV.

Mathematics Attitude Measure

A second instrument under the title Mathematics Attitude Measure (MAM) was a modified version of the test used by Robert Cross in his study of attitudes towards geometry.[5] MAM contained 18 items which were selected from Cross's Geometrical Attitude Scale. The items chosen were applicable to this study.

The test was designed to measure three areas of attitude:

a. student attitude towards content (content);

b. student attitude towards need (needs); and

c. student interest in the subject (interest).

Six items were devoted to each area. The 18 items were then arranged randomly and the test administered prior to the teaching of the unit on vectors. Three items in each area were of a negative nature while the other three were positive statements.

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[5] Robert Walter Cross, "Student Achievement and Attitude in a Modern and a Traditional Grade Ten Geometry Program," (unpublished Master of Education thesis, University of Manitoba, 1968), pp. 161-165.

ments.

A negative statement involving content is exemplified by item 5 of MAM which read:

> Item 5 I do only a few of the mathematics questions on my own. My mathematics teacher has to work out most of the solutions before I understand them.

This item is designed to provide the student with the opportunity to indicate his feeling towards the course content.

If the student agreed with this statement, he would indicate his feeling by circling A and thus be allotted two points, a score indicating less than an average attitude towards the content.

Item number 1 of MAM was designed to measure a student's attitude towards needs:

> Item 1 The topics I am studying this year in my mathematics course will be of little use to me in the future.

A student was able to indicate whether or not he thought the course was useful and fulfilled what he thought were his mathematical needs.

The third area was designed to measure a student's interest in the course. Item number 2 of MAM is a positive statement allowing a student to indicate his degree of interest in the course:

> Item 2 I pay more attention in mathematics classes then in other classes because I am interested in the topics we are studying.

After the completion of the teaching of the unit on vectors, the modified version of MAM was given. The modification made the items relative to vectors and the instrument was renamed Mathematics Attitude Measure Vectors (MASV).

For each student, each area of attitude was investigated by comparing scores on the pre-test MAM and the post-test MAMV. With the marking scale utilized, the maximum score for each area was 30 (6 items times 5 points maximum) while the minimum score possible was 6 (6 items times 1 point minimum).

Since the samples are not independent, but rather in each case the data are paired, a paired t-test statistic again was employed to test the hypothesis.

Tables were constructed (see Appendix B) and the value of the statistic calculated. A level of significance of .05 was then used to investigate the second null hypothesis which stated that no significant differences exist between attitudes towards mathematics as measured by the pre-test MAM and the attitudes towards vectors as measured by the post-test MAMV.

ACHIEVEMENT MEASUREMENT

Although this study was concerned mainly with comparisons of attitudes towards mathematics and attitudes towards vectors, it was felt that a certain standard of achievement should be expected. The writer developed an achievement test entitled Vector Achievement Test (VAT). The test was examined and edited by experienced and qualified teachers in Manitoba. It was felt that the test adequately and objectively tested knowledge and understanding of vectors.

In addition, teachers from Ontario involved in the study were invited to criticize the test. These teachers were teaching the same unit on vectors and were in a position to judge the test.

From the teachers who reacted to this invitation, all comments were favourable. One Hamilton, Ontario teacher stated, "Seems to be a very fair test." A Strathroy, Ontario teacher made the comment, "This is an advanced level grade 10 class. They found the test very comprehensive."

Although reactions were general in nature, they indicated the test adequately tested the unit.

VAT was a 20-item multiple choice test. Each question had five choices. The choices were designed so that only one answer was correct, but the other four choices were answers which one could arrive at if he made an error commonly made on that type of question.

Scores for the tests were determined by assigning one point for each correct answer. Thus the maximum score was 20. There was no penalty for wrong answers. In order to avoid confusion, the following statement was printed at the beginning of the test:

> This is a multiple-choice type test. For each item there is one correct answer. Place the letter corresponding to the correct answer in the appropriate space on the answer sheet provided. There will be no penalty for wrong answers thus it is to your advantage to answer each question even if you are not sure of an answer. You will be allowed 35 minutes to do the test.

Immediately after completing the unit on vectors, the test was given to the fifty Manitoba students. The appropriate number of test papers were sent to each of the schools in Ontario that had agreed to participate in the study. The teachers administering the test in Ontario were instructed to return the answer sheets for marking by the researcher.

All tests were scored by recording each item as either correct or incorrect. The total number of correct responses was the score assigned to each student.

The scores obtained by the 50 students from Manitoba were then compared to the scores obtained by the 238 students from Ontario. The independent, large samples (neither n_1 nor n_2 should be less than 30), were compared by employing the following statistics:[6]

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$ where

$\bar{X}_1$ = mean of Manitoba group

$\bar{X}_2$ = mean of Ontario group

S_1^2 = variance for Manitoba

S_2^2 = variance for Ontario group

n_1 = number in Manitoba group

n_2 = number in Ontario group

The following determined the variance for each group:

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1} \qquad \text{where}$$

X_i = each score in the group

$\bar{X}$ = mean score for the group

n = number of scores in the group.

Tables were established (see Appendix C) and the value of the statistic computed. A level of significance of .05 was used to test the third hypothesis which stated that no significant difference exists between the achievement levels of the Mathematics 101 students from Manitoba and the achievement levels of the students from the Ontario sample as measured by the achievement test entitled the Vector Achievement Test.

The researcher was to be content if there was no difference in achievement between the two groups. It was not expected that the Manitoba group would achieve higher on the tests. It was hoped, however, that the Manitoba group would not have a significantly lower achievement level than the Ontario group.

SUMMARY

After the completion of the administration of the pre-tests for attitude towards mathematics, MASV and MAMV, and the achievement test VAT to both the Ontario group and the Manitoba group, the data was analyzed. The analysis is presented in the following chapter.

Chapter 4

ANALYSIS OF THE DATA

The data accumulated from the scoring of the attitude and achievement tests was organized into Tables (appendixes A, B, and C). The appropriate statistics were applied and the results examined with a view to making conclusions which would be representative of the experimental evidence.

ATTITUDE

Since this study was mainly concerned with attitude, differences in attitude were investigated, using two pairs of instruments.

First Hypothesis Tested

Data generated from a pre-test and a post-test were used to test the first null hypothesis which is restated below:

> For the sample population no significant difference exists between the attitude towards mathematics as measured by the pre-test, Mathematics Attitude Scale (MAS) and the post-test Mathematics Attitude Scale Vectors (MASV).

Computation of the statistics involved using a table to list all students, who had been assigned numbers from one to fifty. The scores that students obtained on MAS were placed under column A_i while scores that students obtained on MASV were placed under B_i. Part of the table is reproduced on the next page.

Table 2

SAMPLE OF TABLE USED FOR RECORDING AND CALCULATING t-STATISTIC FOR MAS AND MASV

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
1	59	78	+19	+8.5	72.25
2	65	77	+12	+1.5	2.25
3	92	92	0	-10.5	110.25
4	59	70	+11	+ .5	.25
5	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
50	46	66	20	9.5	90.25
			525		8316.50

The following steps were involved in calculating the value of the statistic:

1. The differences in scores for each student were determined and listed under $X_i = B_i - A_i$.

2. The algebraic sum ($\sum X_i$) of the differences was determined and found to be 525.

3. The means of the differences was calculated by dividing the algebraic sum ($\sum X_i = 525$) by the number of elements in the sample (n = 50), i.e. mean of differences $= \overline{X} = \frac{\sum X_i}{n} = \frac{525}{50} = 10.5$

4. The difference between the difference in scores for each student (X_i) and the mean of the differences was determined and listed under $X_i - \overline{X}$. For example, for student number one:
$$X_i - \overline{X} = 19 - 10.5 = 8.5$$

5. The squares of $X_i - \overline{X}$ were determined and listed under $(X_i - \overline{X})^2$. For example $(X_1 - \overline{X})^2 = 72.25$.

6. The sum of the column $(X_i - \overline{X})^2$ was determined to be 8316.50.

7. The sample standard deviation (S) was calculated as follows:

$$S = \sqrt{\frac{\Sigma(X_i - \overline{X})^2}{n - 1}} = \sqrt{\frac{8316.50}{50 - 1}} = 13.03$$

8. The value of the statistic (Z) was calculated as follows:

$$Z = \frac{\overline{X} - \mu_o}{S/\sqrt{n}} = \frac{10.5 - 0}{13.03/\sqrt{50}} = 5.70$$

The mean of the population $(\mu_o) = 0$ because the statistic is testing the hypothesis that there is no difference in the scores. Thus the mean of the differences under the hypothesis would be zero.

This test had a two-sided alternative because it was possible for the post-test results to be either higher or lower than the pre-test results. Hence to test at a level of significance of .05 ($\alpha = .05$), $Z_{0.025}$ was used in the comparison. Since $Z_{0.025} = 1.96$ for 30 or more elements and the value of the statistic $Z = 5.70$ exceeded $Z_{0.025} = 1.96$, the hypothesis was rejected.

Rejecting the hypothesis implied that there was a difference between the attitude towards mathematics as measured by MAS and the attitude towards vectors as measured by MASV.

It was possible that the difference could be either favorable or unfavorable in connection with this experiment.

An examination of the mean of the differences ($\bar{X}$) resulted in the conclusion that the difference was favorable since $\bar{X}$ was positive in sign. Thus scores on MASV were significantly higher than the scores on MAS. It was concluded that attitude towards vectors was significantly better than student attitude towards mathematics as measured by the tests MAS and MASV.

Second Hypothesis Tested

Data generated from the pre-test and post-test were used to test the second hypothesis which is restated below:

> For the sample population and for each of the following categories of attitude:
>
> a. student attitude towards content (content);
>
> b. student attitude towards usefulness of subject (needs)
>
> c. student interest in subject (interest.
>
> no significant differences exist between the attitude towards mathematics as measured by the pre-test Mathematics Attitude Measure (MAM) and the attitude towards vectors as measured by the post-test Mathematics Attitude Measure Vectors (MAMV).

In each case the paired t-test for large samples was used. The nature of the calculations was identical to those done for MAS and MASV in testing the first hypothesis. Data (see Appendix B) was used to evaluate the statistics for each of the three cases. The results are tabulated on the following page:

Table 3

VALUES OF MEAN DIFFERENCES ($\bar{X}$), STANDARD DEVIATIONS (S), VALUE OF STATISTIC (Z), AND VALUE OF $Z_{0.025}$ FOR MAM AND MAMV

Category of Attitude	Mean of Differences ($\bar{X}$)	Standard Deviation (S)	Value of Statistic (Z)	Value of $Z_{0.025}$
a. content	+3.44	4.21	5.73	1.96
b. needs	- .04	4.10	- .07	1.96
c. interest	+2.30	4.32	3.77	1.96

A level of significance of .05 was used in the test. In each case, the test had a two-sided alternative as the post-test could either by higher or lower than the pre-test. Thus to test at a level of significance of .05, $Z_{0.025} = 1.96$ was used in the comparison.

In category a. (content), $5.73 > 1.96$, thus the null hypothesis had to be rejected. Similarly in category c. (interest), the null hypothesis had to be rejected as $3.77 > 1.96$. Rejecting these hypotheses implied that there was a significant difference for categories a. and c.. In both cases the means of the post-test MAMV were higher than the means of pre-test MAM. It was concluded that the scores on

a. student attitude towards content of subject, and

c. student interest in subject

were significantly higher on MAMV than on MAM. Thus as measured by MAM and MAMV student attitude towards content of vectors was significantly higher than student attitude towards content of mathematics. Also, student interest in vectors was significantly higher than student interest in mathematics.

For category b. (needs), −.04 did not exceed $Z_{0.025} = 1.96$; thus the null hypothesis could not be rejected. It was concluded that there was no significant difference between the attitude towards the usefulness of mathematics and the attitude towards the usefulness of vectors as measured by the tests MAM (needs) and MAMV (needs).

At this stage it was decided to test whether the total of MAMV was significantly higher than the total of MAM. To do this, an auxiliary hypothesis was proposed. The auxiliary hypothesis was stated in null form for statistical purposes.

Auxiliary Hypothesis

No significant difference exists between the attitude towards mathematics as measured by the pre-test Mathematics Attitude Measure (MAM) and the attitude towards vectors as measured by the post-test Mathematics Attitude Measure Vectors (MAMV).

The hypothesis was tested at a level of .05. The test had a two-sided alternative.

The value of the paired-t statistic was determined to be 3.77, which exceeded $Z_{0.025} = 1.96$. The null hypothesis was rejected. Since the mean of MAMV was higher than the mean of MAM, it was concluded that the attitude towards vectors as measured by MAMV was significantly higher than the attitude towards mathematics as measured by MAM.

ACHIEVEMENT

During the course of the study it was hoped that the achievement level would be as high for the Manitoba Mathematics 101 students as the achievement level for the Ontario sample. The achievement test entitled Vector Achievement Test (VAT) was used to compare the two groups for knowledge and understanding of vectors.

Third Hypothesis Tested

To make the comparison, the third null hypothesis was tested. It is restated below:

> No significant difference exists between the achievement level of the Mathematics 101 students from Manitoba and the achievement level of the Grade X students from the Ontario sample as measured by the achievement test entitled Vector Achievement Test (VAT).

Data was organized into tables (see Appendix C). The following calculations resulted in a value for the statistic:

1. The mean ($\bar{X}_2$) for the Manitoba group was determined to be 12.72.
2. The mean ($\bar{X}_1$) for the Ontario group was determined to be 12.56.
3. The variance $S_1^2 = \dfrac{\sum (X_i - \bar{X}_1)^2}{n - 1}$ for the Manitoba group was determined to be 11.00.

4. The variance $S_2^2 = \frac{\sum (X_i - \bar{X}_2)^2}{n - 1}$ for the Ontario group was determined to be 10.49.

5. The value of the statistic for large independent samples was determined:

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{12.72 - 12.56}{\sqrt{\frac{11.00}{50} + \frac{10.44}{238}}} = .31$$

The test had a two-sided alternative as it was possible for either group to have a higher mean. Thus to test at a level of significance of .05, $Z_{0.025} = 1.96$ was utilized. Since .31 did not exceed 1.96 the hypothesis could not be rejected. It was therefore concluded that there was no significant difference between the levels of achievement for the two groups. It was only hoped at the outset, however, that the Manitoba group would achieve as well as the Ontario group. Although there was no significant difference between the levels of achievement, it is noteworthy that the mean for the Manitoba group was slightly higher than the mean for the Ontario group.

SUMMARY

Analysis of the data revealed information about the experimental group's attitude and achievement. It was found that attitude towards the study of vectors was higher than attitude towards the study of mathematics in all categories but one. No significant difference between the achievement of the Manitoba students and the achievement of the Ontario students was evident from the analysis.

The results of the analysis provided opportunity to draw conclusions and to make some recommendations.

Chapter 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

SUMMARY

In this study, fifty Mathematics 101 students were given attitude tests Mathematics Attitude Scale and Mathematics Attitude Measure to measure attitude towards mathematics. The students were then taught a five-week unit on vectors and consequently given attitude tests Mathematics Attitude Scale Vectors and Mathematics Attitude Measure Vectors to measure attitude towards vectors.

Scores on these tests were statistically analyzed to determine if student attitude towards vectors was higher than student attitude towards mathematics.

The fifty students were given an achievement test, Vector Achievement Test, at the completion of the unit on vectors. The scores for this test were statistically compared to a standard established by having 238 Grade X students from Ontario write the same test.

It was originally expected that three hypotheses would be tested. However, the statistical analysis of these hypotheses suggested that an auxiliary hypothesis should be investigated. The three original hypotheses tested are restated here:

I. There is no significant difference between the attitude towards mathematics as measured by the pre-test MAS and the attitude towards vectors as measured by the post-test MASV.

II. For each of the following categories of attitude:

a. student attitude towards content (content);

b. student attitude towards usefulness of subject (needs); and

c. student interest in subject (interest),

no significant differences exist between the attitude towards mathematics as measured by the pre-test MAM and the attitude towards vectors as measured by the post-test MAMV.

III. No significant difference exists between the achievement level of the Mathematics 101 students from Manitoba and the achievement level of the Grade 10 students from the Ontario sample as measured by the achievement test entitled Vector Achievement Test.

The auxiliary hypothesis was:

No significant difference exists between the attitude towards mathematics as measured by the pre-test MAM and the attitude towards vectors as measured by the post-test MAMV.

The results of the statistical analysis of these hypotheses are summarized in Table 4 on the following page.

Generally, attitude towards vectors was better than attitude towards mathematics. Only under hypothesis II b was the attitude of vectors not significantly higher than the attitude towards mathematics. Under II b, however, the attitude towards vectors was not significantly lower than the attitude towards mathematics.

As there was no significant difference between the means of the achievement test for the two groups it was concluded that the Mathematics 101 students from Manitoba were able to cope with the material on vectors as effectively as did the Grade X students from Ontario.

Table 4

SUMMARY OF HYPOTHESES TESTED AND CONCLUSIONS REACHED

Hypothesis	Level of Significance	Action	Conclusion
I	.05	rejected	Attitude towards vectors is better than attitude towards mathematics.
II a.	.05	rejected	Attitude towards content of vectors is better than attitude towards content of mathematics.
II b.	.05	not rejected	There is no significant difference in attitude towards usefulness of vectors (needs) and the attitude towards usefulness of mathematics (needs).
II c.	.05	rejected	Interest in vectors is higher than interest in mathematics.
III	.05	not rejected	Mathematics 101 students achieved at least as highly as Grade X Ontario students.
Auxiliary	.05	rejected	Attitude towards vectors is better than attitude towards mathematics.

CONCLUSIONS

It has been shown that attitude towards vectors was better than attitude towards mathematics for Mathematics 101 students. In addition, Mathematics 101 students were able to achieve satisfactorily in the work on vectors.

In reviewing the literature the author found support for the importance of attitude in the learning process. Mary Corcoran and Glenadine Gibb suggest that, "The attitudes of students towards mathematics play a vital part in their learning."[1] Because attitude towards vectors is favorable and achievement in vectors is satisfactory, it may be feasible to include a unit on vectors in the Mathematics 101 program.

The author undertook this study as he believed that some of the Mathematics 101 program is irrelevant as far as students are concerned. Not only are some students unable to see specific uses for some of the topics in their mathematics in their own lives but they find difficulty in recognizing how some topics could be practically applied in the everyday world.

The unit on vectors offers some practical aspects. Towards the end of the unit, students are able to solve simple navigation problems. The unit could easily be expanded to include some other problems of a practical nature. For example,

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[1]Mary Corcoran and Glenadine E. Gibb, Evaluation in Mathematics, The Twenty-sixth Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1961), p. 105.

students could learn to solve problems which involve combining forces that are not in the same direction. They might learn to solve problems involving the resolution of vector quantities into components. Studying vectors in Mathematics 101 would surely prove profitable for students pursuing a course in physics.

It was also concluded that scientific research can be used to investigate student attitude towards a subject. Programs could and should be thoroughly evaluated before their introduction and curriculum planners should constantly seek to improve programs. If courses could be developed where student attitude is clearly favorable, there should be a positive acceptance of the subject and learning should flourish.

RECOMMENDATIONS

As attitude towards the study of vectors is favorable and achievement in vectors is satisfactory, the author wishes to recommend that curriculum planners dealing with Mathematics 101 should consider adopting a unit on vectors as part of the Mathematics 101 program.

In addition, several supplementary recommendations are suggested. Curriculum planners should use scientific research techniques to establish the advisability of altering courses or introducing new courses. When considering the advisability of altering courses or introducing new courses, emphasis should be placed on student attitude towards the material. New programs, particularly at the 01 level should contain practical material. Although the students may not use it personally, they should be able to perceive how the material can be applied in a practical way.

Finally, there are other areas which might profitably be studied. Further research dealing with vectors in Mathematics 101 could be undertaken. It might be profitable to try to extend the unit by offering a unit on vectors which places greater emphasis on applications. Also, other texts containing a unit on vectors could be investigated. This should be done with a view to finding the most acceptable material to use as a teaching basis for the topic. Last of all, other topics could be attempted and researched for the Mathematics 101 course.

BIBLIOGRAPHY

A. BOOKS

Carman, Robert A. A Programmed Introduction to Vectors. New York: John Wiley and Sons, Inc., 1963.

Campbell, William G. Form and Style in Thesis Writing. Boston: Houghton Mifflin Co., 1969.

Del Grande, J. J., J. C. Egsgard, and H. A. Mulligan. Mathematics Ten. Toronto: W. J. Gage Limited, 1967.

Johnson, Donovan A., and Gerald Rising. Guidelines for Teaching Mathematics. Belmont: Wadsworth Publishing Co., Inc., 1969.

Kline, Morris. Mathematics in Western Culture. New York: Oxford University Press, 1953.

Shibli, J. Recent Developments in the Teaching of Geometry. State College, Pennsylvania: J. Shibli Publishing, 1932.

Sommerville, D.M.Y. The Elements of Non-Euclidean Geometry. New York: Dover Publications, Inc., 1958.

Wolfe, Harold E. Non-Euclidean Geometry. New York: Rinehart and Winston, 1965.

B. OTHER PUBLICATIONS

Coxeter, H.S.M. "The Ontario K-13 Geometry Report," Geometry in the Secondary School, pp. 8-12. A Compendium of Papers Presented in Houston, Texas, January 29, 1967, at a Session of the NCTM. Washington: NCTM., 1968. (ERIC. ED. 035 544).

Elliot, Andrew. "Geometry in Great Britain," Geometry in the Secondary School, pp. 13-17. A Compendium of Papers Presented in Houston, Texas, January 29, 1967, at a Session of the NCTM. Washington: NCTM., 1968. (ERIC ED. 035 544).

Meserve, Bruce E. "Geometry in the United States," Geometry in the Secondary School, pp. 1-7. A Compendium of Papers Presented in Houston, Texas, January 29, 1967 at a Session of the NCTM. Washington: NCTM., 1968. (ERIC. ED. 035 544).

Scandura, Joseph M. (ed.) Research in Mathematics Education. Washington: National Council of Teachers of Mathematics, 1967. (ERIC. ED. 035 545).

C. PERIODICALS

Aiken, Lewis R., and Ralph Mason Dreger. "The Effect of Attitudes on Performance in Mathematics," Journal of Educational Psychology, 52: 19-24, February, 1961.

Allendoerfer, Carl B. "The Dilemma in Geometry," The Mathematics Teacher, 62: 165-169, March, 1969.

Athen, Dr. Herman. "The Teaching of Vectors in the German Gymnasium," The Mathematics Teacher, I, II, 59: 382-393, April, 1966, and 59: 485-495, May, 1966.

Corcoran, Mary, and Glenadine E. Gibb. "Appraising Attitudes in the Learning of Mathematics," Evaluation in Mathematics. The Twenty-sixth Yearbook of the National Council of Teachers of Mathematics. Washington: National Council of Teachers of Mathematics, 1961.

Fehr, Howard F. "The Mathematics Program in Japanese Secondary Schools," The Mathematics Teacher, 59: 577-583, October, 1966.

Glicksman, A. M. "Vectors in Algebra and Geometry," The Mathematics Teacher, 58: 327-332, April, 1965.

Hartung, Maurice L. "Motivation for Education in Mathematics," The Learning of Mathematics, Its Theory and Practice. The Twenty-first Yearbook of the National Council of Teachers of Mathematics. Washington: National Council of Teachers of Mathematics, 1953.

Johnson, Donovan A. "A Pattern for Research in the Mathematics Classroom," The Mathematics Teacher, 59: 418-425, May, 1966.

Kline, Morris. "A Proposal for the High School Mathematics Curriculum," The Mathematics Teacher, 59: 322-330, April, 1966.

Moredock, Stewart H. "Geometry and Measurement," Mathematics Education. The Sixty-ninth Yearbook of the National Society for the Study of Education. Chicago: The University of Chicago Press, 1970.

Pawlikowski, George J. "The Men Responsible for the Development of Vectors," The Mathematics Teacher, 60: 393-396, April, 1967.

Smith, Dan. "Vectors--An Aid to Mathematical Understanding," The Mathematics Teacher, 52: 608-613, December, 1959.

Szabo, Steven. "An Approach to Euclidean Geometry through Vectors," The Mathematics Teacher, 59: 218-235, March, 1966.

Willoughby, Steven S. "Issues in the Teaching of Mathematics," Mathematics Education. The Sixty-ninth Yearbook of the National Society for the Study of Education. Chicago: The University of Chicago Press, 1970.

Zant, James H. "A Proposal for the High School Mathematics Curriculum--What Does It Mean?" The Mathematics Teacher, 59: 331-334, April, 1966.

D. UNPUBLISHED MATERIAL

Cross, Robert Walter. "Student Achievement and Attitude in a Modern and a Traditional Grade Ten Geometry Program." Unpublished Master of Education thesis, University of Manitoba, 1968.

Hedley, Robert Lloyd. "Student Attitude and Achievement in Science Courses in Manitoba Secondary Schools," Unpublished Doctoral thesis, East Lansing, Michigan State University, 1966.

Shulte, Albert P. "Uses of Mathematics in Other Subject Areas." Ann Arbor, 1966. (ERIC ED. 036 436).

APPENDIX A

Mathematics Attitude Scale (MAS)

Mathematics Attitude Scale Vectors (MASV)

MAS - MASV Data

Do Not Begin Until You Are Told To Do So

This questionnaire contains a set of statements about the section on "vectors" that you have just taken. The statements are presented as opinions rather than as facts. As opinions they are neither right nor wrong. This is not a test, so it is very important that you answer the questions according to your own feelings and judgements. After you have read each statement carefully, it is best to answer by giving your first impression or reaction and then go on to the next item. Feel free to answer honestly and frankly. Your answers will in no way affect your grades.

Indicate how you feel about each statement by circling your choice on the answer sheet provided. The choices that you will be given are the following:

SD - Strongly Disagree
D - Disagree
N - Neither Agree Nor Disagree
A - Agree
SA - Strongly Agree

Do Not Begin Until You Are Told To Do So

1. I am always under a terrific strain in a math class.

2. I do not like mathematics, and it scares me to have to take it.

3. Mathematics is very interesting to me, and I enjoy math courses.

4. Mathematics is fascinating and fun.

5. Mathematics makes me feel secure, and at the same time it is stimulating.

6. My mind goes blank, and I am unable to think clearly when working math.

7. I feel a sense of insecurity when attempting mathematics.

8. Mathematics makes me feel uncomfortable, restless, irritable and impatient.

9. The feeling that I have toward mathematics is a good feeling.

10. Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way.

11. Mathematics is something which I enjoy a great deal.

12. When I hear the word math, I have a feeling of dislike.

13. I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.

14. I really like mathematics.

15. Mathematics is a course in school which I have always enjoyed studying.

16. It makes me nervous to even think about having to do a math problem.

17. I have never liked math, and it is my most dreaded subject.

18. I am happier in a math class than in any other class.

19. I feel at ease in mathematics, and I like it very much.

20. I feel a definite positive reaction to mathematics; it is enjoyable.

1. The feeling that I have towards "vectors" is a good feeling.
2. My mind goes blank, and I am unable to think clearly when working with "vectors".
3. It makes me nervous to even think about having to do a "vectors" problem.
4. "Vectors" is very interesting to me, and I enjoy it.
5. I feel at ease in "vectors" and I like it very much.
6. When I hear the word " vectors", I have a feeling of dislike.
7. I really like "vectors".
8. "Vectors" is fascinating and fun.
9. I am happier in a "vectors" class than in any other class.
10. "Vectors" makes me feel as though I'm lost in a jungle of numbers and can't find my way.
11. I am always under a terrific strain in a "vectors" class.
12. I feel a definite positive reaction to "vectors"; it is enjoyable.
13. "Vectors" is something which I enjoy a great deal.
14. "Vectors" make me feel uncomfortable, restless, irritable and impatient.
15. I approach "vectors" with a feeling of hesitation, resulting from a fear of not being able to do the work.
16. "Vectors" make me feel secure and at the same time it is stimulating.
17. I have never liked "vectors" and it is my most dreaded subject.
18. I feel a sense of insecurity when attempting "vectors".
19. "Vectors" is a course in school which I enjoy studying.
20. I do not like "vectors" and it scares me to have to take "vectors".

ANSWER SHEET

Name: ______________________

Indicate how much you agree or disagree with each of the following statements by circling the letter which best represents your feelings toward the statement.

SD - Strongly Disagree
D - Disagree
N - Neither Agree nor Disagree
A - Agree
SA - Strongly Agree

1.	SD	D	N	A	SA
2.	SD	D	N	A	SA
3.	SD	D	N	A	SA
4.	SD	D	N	A	SA
5.	SD	D	N	A	SA
6.	SD	D	N	A	SA
7.	SD	D	N	A	SA
8.	SD	D	N	A	SA
9.	SD	D	N	A	SA
10.	SD	D	N	A	SA
11.	SD	D	N	A	SA
12.	SD	D	N	A	SA
13.	SD	D	N	A	SA
14.	SD	D	N	A	SA
15.	SD	D	N	A	SA
16.	SD	D	N	A	SA
17.	SD	D	N	A	SA
18.	SD	D	N	A	SA
19.	SD	D	N	A	SA
20.	SD	D	N	A	SA

MAS SCORES

MAS – A_i MAS (V) – B_i

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
1	59	78	+19	+8.5	72.25
2	65	77	+12	+1.5	2.25
3	92	92	0	-10.5	110.25
4	59	70	+11	+.5	.25
5	73	83	+10	-.5	.25
6	53	75	22	+11.5	132.25
7	70	82	+12	+1.5	2.25
8	75	72	-3	-13.5	182.25
9	88	82	-6	-16.5	272.25
10	61	48	-13	-23.5	552.25
11	71	73	+2	-8.5	72.25
12	68	78	+10	-.5	.25
13	38	82	+44	+33.5	1122.25
14	75	68	-7	-17.5	306.25
15	58	72	+14	+3.5	12.25
16	56	75	+19	+8.5	72.25
17	52	93	+41	+30.5	930.25
18	47	73	+26	+15.5	240.25
19	58	86	+28	+17.5	306.25
20	48	76	+28	+17.5	306.25
21	66	75	+9	-1.5	2.25
22	71	72	+1	-9.5	90.25
23	70	81	+11	+.5	.25
24	64	87	+13	+2.5	6.25
25	90	79	-11	-21.5	462.25

(cont'd)

(cont'd)

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
26	73	88	+15	+4.5	20.25
27	44	61	+17	+6.5	42.25
28	60	56	-4	-14.5	210.25
29	82	78	-4	-14.5	210.25
30	51	69	+18	+7.5	56.25
31	51	77	+26	+15.5	240.25
32	73	79	+6	-4.5	20.25
33	69	65	-4	-14.5	210.25
34	57	69	+12	+1.5	2.25
35	63	73	+10	-.5	.25
36	63	94	+31	+20.5	420.25
37	73	74	+1	-9.5	90.25
38	71	64	-7	-17.5	306.25
39	50	65	+15	+4.5	20.25
40	82	78	-4	-14.5	210.25
41	67	68	+1	-9.5	90.25
42	62	74	+12	+1.5	2.25
43	45	64	+19	+8.5	72.25
44	67	67	0	-10.5	110.25
45	73	67	-6	-16.5	272.25
46	77	79	+2	-8.5	72.25
47	57	80	+23	+12.5	156.25
48	81	93	+12	+1.5	2.25
49	59	81	+22	+11.5	132.25
50	46	66	+20	+9.5	90.25
			525		8316.50

$$\overline{X} = \frac{525}{50} = 10.5$$

APPENDIX B

Mathematics Attitude Measurement (MAM)

Mathematics Attitude Measurement Vectors (MAMV)

MAM - MAMV Data

Areas of Attitude	MAM Items	MAMV Items
a) content	3, 5, 9, 12, 13, 16	1, 2, 9, 11, 14, 18
b) needs	1, 4, 7, 11, 15, 18	3, 5, 6, 7, 10, 15
c) interest	2, 6, 8, 10, 14, 17	4, 8, 12, 13, 16, 17

Do Not Begin Until You Are Told To Do So.

This questionnaire contains a set of statements about the mathematics course you are taking. The statements are presented as opinions rather than as facts. As opinions, they are neither right nor wrong. This is <u>not a test</u>, so it is very important that you answer the questions according to <u>your own</u> feelings and judgements. After you have read each statement carefully, it is best to answer by giving your <u>first</u> impression or reaction and then go on to the next item. Feel free to answer honestly and frankly. Your answers will in no way affect your grades.

Indicate how you feel about each statement by circling your choice on the answer sheet provided. The choices that you will be given are the following:

SD - Strongly Disagree
D - Disagree
N - Neither Agree nor Disagree
A - Agree
SA - Strongly Agree

Do Not Begin Until You Are Told To Do So.

1. The topics I am studying this year in my mathematics course will be of little use to me in the future.

2. I pay more attention in mathematics classes than in other classes because I am interested in the topics we are studying.

3. I would rather be taking another mathematics course this year than the course we are taking.

4. When I study a topic or section in my mathematics course, I
c can usually see why it is important for me to study it.

5. I do only a few of the mathematics questions and problems on my own. My mathematics teacher has to work out most of the solutions before I understand them.

6. I am not interested in taking a mathematics course like this one next year, but would rather take almost any other subject.

7. In general, I think I am learning things from my mathematics course that will be of value to me.

8. Because of my interest in mathematics, I normally spend more time on my mathematics homework than on other subjects.

9. I think the mathematics work that I have done this year has begun to make me think as I imagine a mathematician thinks.

10. I get little satisfaction from doing mathematics.

11. I see little reason for giving anyone the mathematics course we are studying.

12. I find the mathematics course easy.

13. Because of the difficulty of the mathematics course, I find that I have to spend more time on mathematics than on other subjects.

14. I usually look forward to mathematics classes.

15. Too much time is devoted to the study of mathematics and not enough time to the study of other subjects.

16. In my previous years of math, I have never had the opportunity to think out the answers for myself as I have had in mathematics this year.

17. I still hate math. Math is really a bore.

18. I often notice in things around me applications of some of the mathematical concepts I have studied this year.

1. I have done only a few of the "vectors" questions and problems on my own. My "vectors" teacher has had to work out most of the solutions before I understood them.

2. I found the "vectors" course easy.

3. Too much time is devoted to the study of "vectors" and not enough time to the study of other topics in mathematics.

4. I paid more attention in vectors classes than in other classes because I was interested in the topics we were studying.

5. I often notice in things around me applications of some of the concepts of "vectors" which I have studied this year.

6. When I studied a topic or section in my "vectors" course, I could usually see why it was important for me to study it.

7. In general, I think I learned things from my "vectors" course that will be of value to me.

8. Because of my interest in "vectors" I normally spent more time on my "vectors" homework than on homework in other subjects.

9. In my previous years of math, I have never had the opportunity to think out the answers for myself as I have had in "vectors" this year.

10. I see little reason for giving anyone the "vectors" course we had to study.

11. I would rather have taken another mathematics topic this year than the "vectors" course we had.

12. I got little satisfaction from doing "vectors".

13. I am not interested in taking a "vectors" course like this one next year, but would rather take almost any other topic of mathematics.

14. I think the work on "vectors" that I have done this year has begun to make me think as I imagine a mathematician thinks.

15. The topics I have studied in the "vectors" course will be of little use to me in the future.

16. I still hate math. "Vectors" is really a bore.

17. I usually looked forward to "vectors" classes.

18. Because of the difficulty of "vectors", I found I had to spend more time on "vectors" than on other subjects.

ANSWER SHEET

Name: ____________________

Indicate how much you agree or disagree with each of the following statements by circling the letter which best represents your feelings toward the statement.

SD - Strongly Disagree
D - Disagree
N - Neither Agree nor Disagree
A - Agree
SA - Strongly Agree

1.	SD	D	N	A	SA
2.	SD	D	N	A	SA
3.	SD	D	N	A	SA
4.	SD	D	N	A	SA
5.	SD	D	N	A	SA
6.	SD	D	N	A	SA
7.	SD	D	N	A	SA
8.	SD	D	N	A	SA
9.	SD	D	N	A	SA
10.	SD	D	N	A	SA
11.	SD	D	N	A	SA
12.	SD	D	N	A	SA
13.	SD	D	N	A	SA
14.	SD	D	N	A	SA
15.	SD	D	N	A	SA
16.	SD	D	N	A	SA
17.	SD	D	N	A	SA
18.	SD	D	N	A	SA
19.	SD	D	N	A	SA
20.	SD	D	N	A	SA

MAM - A_i MAM(V) - B_i

CONTENT

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
1	14	22	+8	4.56	20.79
2	19	22	+3	-.44	.19
3	25	25	0	-3.44	11.83
4	18	23	+5	1.56	2.43
5	17	22	+5	1.56	2.43
6	17	22	+5	1.56	2.43
7	19	22	+3	-.44	.19
8	18	21	+3	-.44	.19
9	19	23	+4	.56	.31
10	16	15	-1	-4.44	19.71
11	18	20	+2	-1.44	2.07
12	18	21	+3	-.44	.19
13	8	23	+15	11.56	133.63
14	19	15	-4	-7.44	55.35
15	15	20	+5	1.56	2.43
16	15	22	+7	3.56	12.67
17	15	25	+10	6.56	43.03
18	16	22	+6	2.56	6.55
19	15	22	+7	3.56	12.67
20	13	20	+7	3.56	12.67
21	17	18	+1	-2.44	5.95
22	21	22	+1	-2.44	5.95
23	22	22	0	-3.44	11.83
24	12	21	+9	5.56	30.91
25	20	23	+3	-.44	.19

(cont'd)

(cont'd)

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
26	20	26	+6	2.56	6.55
27	17	20	+3	-.44	.19
28	18	18	0	-3.44	11.83
29	18	21	+3	-.44	.19
30	17	21	+4	.56	.31
31	10	24	+14	10.56	111.51
32	21	22	+1	-2.44	5.95
33	18	21	+3	-.44	.19
34	19	20	+1	-2.44	5.95
35	18	21	+3	-.44	.19
36	15	24	+9	5.56	30.91
37	21	17	-4	-7.44	55.35
38	21	16	-5	-8.44	71.23
39	21	21	0	-3.44	11.83
40	23	23	0	-3.44	11.83
41	21	20	-1	-4.44	19.71
42	15	21	+6	2.56	6.55
43	19	19	0	-3.44	11.83
44	21	18	-3	-6.44	41.47
45	21	19	-2	-5.44	29.59
46	16	24	+8	4.56	20.79
47	16	23	+7	3.56	12.67
48	18	20	+2	-1.44	2.07
49	19	24	+5	1.56	2.43
50	16	21	+5	1.56	2.43
			172		870.14

$$\bar{X} = \frac{172}{50} = 3.44$$

MAM - A_i NEEDS   MAM(V) - B_i

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
1	22	20	-2	-1.96	3.84
2	16	19	3	3.04	9.24
3	30	20	-10	-9.96	99.01
4	17	21	4	4.04	16.32
5	23	23	0	.04	.00
6	20	20	0	.04	.00
7	20	21	1	1.04	1.08
8	25	26	1	1.04	1.08
9	25	24	-1	-.96	.92
10	23	15	-8	-7.96	63.36
11	16	19	3	3.04	9.24
12	25	21	-4	-3.96	15.68
13	10	17	7	7.04	49.56
14	21	18	-3	-2.96	8.76
15	18	21	3	3.04	9.24
16	16	24	8	8.04	64.64
17	19	22	3	3.04	9.24
18	16	19	3	3.04	9.24
19	20	21	1	1.04	1.08
20	20	17	-3	-2.96	8.76
21	23	22	-1	-.96	.92
22	21	23	2	2.04	4.16
23	23	21	-2	-1.96	3.84
24	24	18	-6	-5.96	35.52
25	22	23	1	1.04	1.08

(cont'd)

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
26	29	25	-4	-3.96	15.68
27	17	19	2	2.04	4.16
28	24	20	-4	-3.96	15.68
29	20	19	-1	-.96	.92
30	14	17	3	3.04	9.24
31	22	24	2	2.04	4.16
32	22	20	-2	-1.96	3.84
33	18	19	1	1.04	1.08
34	23	15	-8	-7.96	63.36
35	20	20	0	.04	.00
36	24	25	1	1.04	1.08
37	21	18	-3	-2.96	8.76
38	21	19	-2	-1.96	3.84
39	14	16	2	2.04	4.16
40	22	20	-2	-1.96	3.84
41	26	23	-3	-2.96	8.76
42	17	20	3	3.04	9.24
43	12	20	8	8.04	64.64
44	24	16	-8	-7.96	63.36
45	20	17	-3	-2.96	8.76
46	16	24	8	8.04	64.64
47	18	22	4	4.04	16.32
48	19	17	-2	-1.96	3.84
49	21	23	2	2.04	4.16
50	21	25	4	4.04	16.32
			-2		825.65

$$\overline{X} = \frac{-2}{50} = -.04$$

MAM - A_i MAM(V) - B_i

INTEREST

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
1	16	24	+8	5.70	32.49
2	15	20	+5	2.70	7.29
3	27	23	-4	-6.30	39.69
4	16	21	+5	2.70	7.29
5	20	25	+5	2.70	7.29
6	13	20	+7	4.70	22.09
7	21	25	+4	1.70	2.89
8	20	21	+1	-1.30	1.69
9	22	23	+1	-1.30	1.69
10	17	13	-4	-6.30	39.69
11	21	20	-1	-3.30	10.89
12	23	21	-2	-4.30	18.49
13	10	20	+10	7.70	59.29
14	21	17	-4	-6.30	39.69
15	16	20	+4	1.70	2.89
16	20	20	0	-2.30	5.29
17	15	27	+12	9.70	94.09
18	11	19	+8	5.70	32.49
19	10	20	+10	7.70	59.29
20	16	18	+2	-1.30	.09
21	20	21	+1	-1.30	1.69
22	20	20	0	-2.30	5.29
23	22	21	-1	-3.30	10.89
24	15	19	+4	1.70	2.89
25	18	21	+3	.70	.49

(cont'd)

(cont'd)

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
26	23	22	-1	-3.30	10.89
27	17	17	0	-2.30	5.29
28	15	14	-1	-3.30	10.89
29	21	19	-2	-4.30	18.49
30	13	17	+4	1.70	2.89
31	17	24	+7	4.70	22.09
32	22	20	-2	-4.30	18.49
33	21	18	-3	-5.30	28.09
34	20	19	-1	-3.30	10.89
35	14	20	+6	3.70	13.69
36	16	25	+9	6.70	44.89
37	19	18	-1	-3.30	10.89
38	21	16	-5	-7.30	53.29
39	10	17	+7	4.70	22.09
40	21	22	+1	-1.3	1.69
41	19	17	-2	-4.30	18.49
42	15	19	+4	1.70	2.89
43	12	16	+4	1.70	2.89
44	20	20	0	-2.30	5.29
45	20	21	+1	-1.3	1.69
46	18	22	+4	1.70	2.89
47	15	23	+8	5.70	32.49
48	26	21	-5	-7.30	53.29
49	16	22	+6	3.70	13.69
50	16	19	+3	.70	.49
			115		914.50

$$\overline{X} = \frac{115}{50} = 2.30$$

MAM - A_i MAMV - B_i

TOTAL

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
1	52	66	+14	8.30	68.89
2	50	61	+11	5.30	28.09
3	82	68	-14	-19.70	388.09
4	51	65	+14	8.30	68.89
5	60	70	+10	4.30	18.49
6	50	62	+12	6.30	39.69
7	60	68	+8	2.30	5.29
8	63	68	+5	-.70	.49
9	66	70	+4	-1.70	2.89
10	56	43	-13	-18.70	349.69
11	55	59	+4	-1.70	2.89
12	66	63	-3	-8.70	75.69
13	28	60	+32	26.30	691.69
14	61	50	-11	-16.70	278.89
15	49	61	+12	6.30	39.69
16	51	66	+15	9.30	86.49
17	49	74	+25	19.30	372.49
18	43	60	+17	11.30	127.69
19	45	63	+18	12.30	151.29
20	49	55	+6	.30	.09
21	60	61	+1	-4.70	22.09
22	62	65	+3	-2.70	7.29
23	67	64	-3	-8.70	75.69
24	51	58	+7	1.30	1.69
25	60	67	+7	1.30	1.69

(cont'd)

(cont'd)

Student	A_i	B_i	$X_i = B_i - A_i$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
26	72	73	+1	-4.70	22.09
27	51	56	+5	-.70	.49
28	57	52	-5	-10.70	114.49
29	59	59	0	-5.70	32.49
30	44	55	+11	5.30	28.09
31	49	72	+23	17.30	299.29
32	65	62	-3	-8.70	75.69
33	57	58	+1	-4.70	22.09
34	62	54	-8	-13.70	187.69
35	52	61	+9	3.30	10.89
36	55	74	+19	13.30	176.89
37	61	53	-8	-13.70	187.69
38	63	51	-12	-17.70	313.29
39	45	54	+9	3.30	10.89
40	66	65	-1	-6.70	44.89
41	66	60	-6	-11.70	136.89
42	47	60	+13	7.30	53.29
43	43	55	+12	6.30	39.69
44	65	54	-11	-16.70	278.89
45	61	57	-4	-9.70	94.09
46	50	70	+20	14.30	204.49
47	49	68	+19	13.30	176.89
48	63	58	-5	-10.70	114.49
49	56	69	+13	7.30	53.29
50	53	65	+12	6.30	39.69
			285		5624.50

$$\overline{X} = \frac{285}{50} = +5.70$$

APPENDIX C

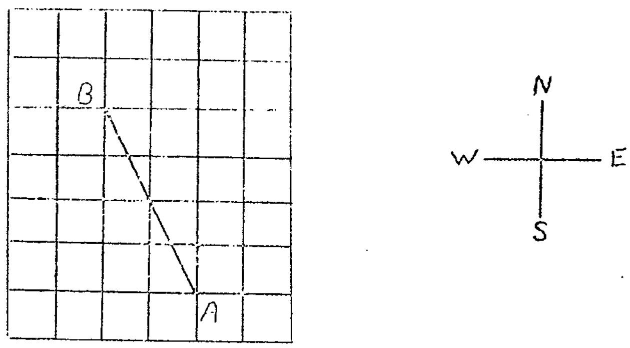
Vector Achievement Test (VAT)

VAT Data

VECTOR ACHIEVEMENT TEST

This is a multiple-choice type test. For each item there is ONE correct answer. Place the letter corresponding to the correct answer in the appropriate space on the answer sheet provided. There will be no penalty for wrong answers thus it is to your advantage to answer each question even if you are not sure of an answer. You will be allowed 35 minutes to do the test.
Do not begin until you are told to do so.

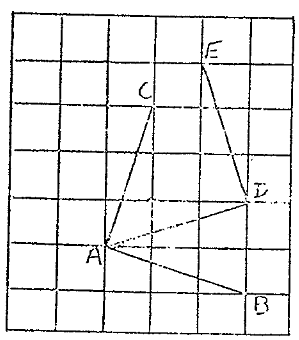
1.



In the diagram above $\overrightarrow{BA}$ can be represented by the ordered pair:

A. (2,4)
B. (-2,4)
C. (2,-4)
D. (-4,-2)
E. (-4,2)

2.



If $\vec{a}$ = (-3,-1) then $\vec{a}$ is illustrated on the above grid as:

A. $\overrightarrow{AB}$
B. $\overrightarrow{AC}$
C. $\overrightarrow{AD}$
D. $\overrightarrow{DA}$
E. $\overrightarrow{DE}$

3. If two vectors are equal and are represented by line segments with arrows to indicate direction then:

A. They must have the same starting point but a different end point.
B. They must have the same starting point and same ending point.
C. They must have same length but a different direction.
D. They must have the same direction but different length.
E. They must have the same length and same direction.

Vector Achievement Test - page 2

4. If $\vec{a} = (3,-2)$ and $\vec{b} = (4,-6)$ $\vec{a} + \vec{b}$ equals:

A. (1,-2)

B. (-3,2)

C. (7,-8)

D. (1) + (-2)

E. (7,8)

5. If $\overrightarrow{AB} = (3,4)$ then $\overrightarrow{BA}$ equals:

A. (4,3)

B. (-3,4)

C. (-4,-3)

D. (4,-3)

E. (-3,-4)

6. If $\vec{a} = (3,-6)$ and $\vec{b} = (-2,1)$ $\vec{a} + 2\vec{b}$ equals:

A. (1,-5)

B. (-1,-4)

C. (7,-4)

D. (7,-8)

E. (-1,-8)

7. If $\vec{a} = (c,d)$ then $\vec{a} + \vec{0}$ equals:

A. (c,d) + (-c,-d)

B. (0,0)

C. (-c,-d)

D. (c,d)

E. None of these

8. If $\vec{c} = (4,5)$ and $\vec{d} = (-3,1)$ $\vec{c} - \vec{d}$ equals:

A. (7,4)

B. (1,6)

C. (1,4)

D. (-1,-2)

E. (1,-4)

9. If $\vec{a} = (-7,5)$ and if $\vec{a} + \vec{b} = \vec{0}$ then $\vec{b}$ equals:

A. $\vec{a}$

B. (7,5)

C. (-7,5)

D. (7,-5)

E. (0,0)

10. If $\vec{x} = (-4,3)$ then $|\vec{x}|$ equals:

A. -1

B. +1

C. 5

D. -5

E. -7

Vector Achievement Test - page 3

11. If $\vec{m} = (3,-4)$ and $k = -2$ then $k\vec{m}$ equals:

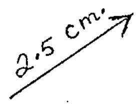
A. (3k,4k)

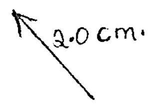
B. (6,-8)

C. (-6,-8)

D. (-3k,-4k)

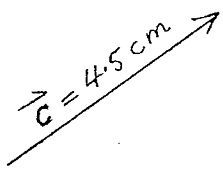
E. (-6,8)

12. If $\vec{a}$ = 

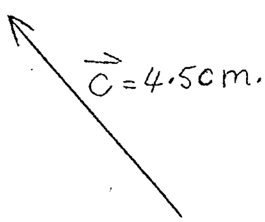
and $\vec{b}$ = 

and $\vec{c} = \vec{a} + \vec{b}$ then $\vec{c}$ is best represented by:

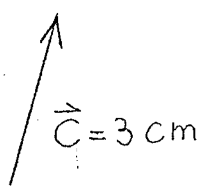
A.



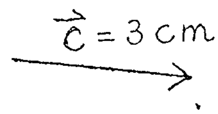
B.



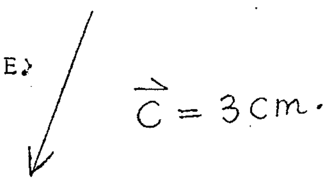
C.



D.



E.



13. If $\vec{a}$ = 15 miles North and $\vec{b}$ = 25 miles West
$\vec{a} + \vec{b}$ can be best represented by:

A. 29 miles N 59° W

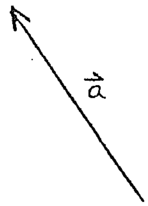
B. 40 miles North-West

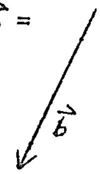
C. 10 miles West

D. 29 miles West

E. 29 miles N 31° W

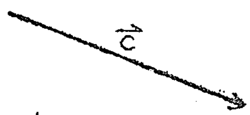
Vector Achievement Test - page 4

14. If $\vec{a}$ = 

and $\vec{b}$ = 

then $\vec{a} - \vec{b}$ can best be represented by $\vec{c}$ where $\vec{c}$ =

A. 

B. 

C. 

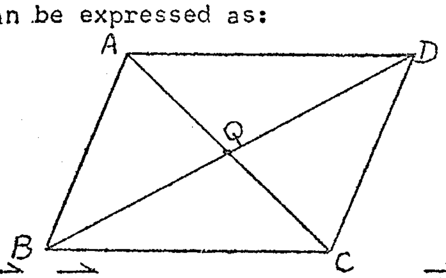
D. 

E. 

15. If $\vec{a}$ = 4cm North and $\vec{b}$ = 3cm South $\vec{a} - \vec{b}$ equals:

A. 7 cm North
B. 7 cm South
C. 1 cm North
D. 1 cm South
E. None of these

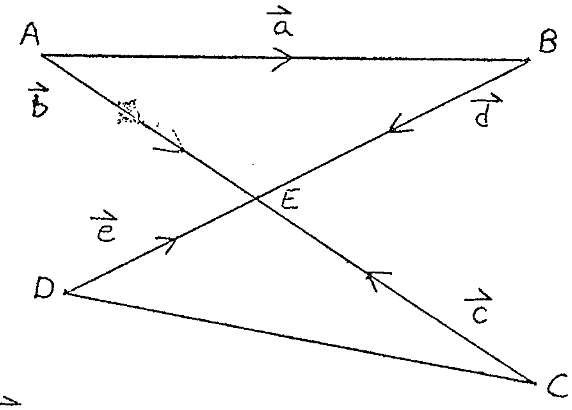
16. In the parallelogram ABCD
$\overrightarrow{AQ}$ can be expressed as:



A. $\overrightarrow{AB} + \overrightarrow{QB}$
B. $\overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CQ}$
C. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{QC}$
D. $\overrightarrow{QA}$
E. $\overrightarrow{AC} + \overrightarrow{QC}$

Vector Achievement Test - page 5

17.



$\overrightarrow{DC}$ can be expressed as:

A. $\vec{e} - \vec{b} + \vec{a} + \vec{d} + \vec{c}$

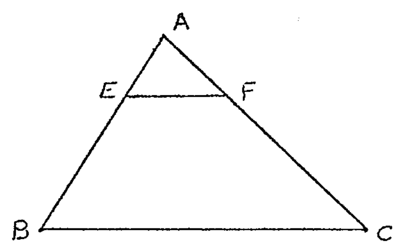
B. $\vec{e} - \vec{b} + \vec{a} + \vec{d} - \vec{c}$

C. $\vec{e} - \vec{d} - \vec{a} + \vec{b} - \vec{e}$

D. $\vec{e} + \vec{b} + \vec{a} + \vec{d} - \vec{c}$

E. $\vec{e} + \vec{c}$

18.



Given that $\overrightarrow{AE} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{AF} = \frac{1}{4}\overrightarrow{AC}$ we can prove that:

A. $\overrightarrow{EF} = \overrightarrow{BC}$

B. $\overrightarrow{EF} = 4\,\overrightarrow{BC}$

C. Four times the length of EF is equal to BC but EF is not parallel to BC.

D. The line joining E and F is parallel to BC and one-quarter of the length.

E. $4\left|\overrightarrow{EF}\right| = \left|\overrightarrow{BC}\right|$ and EF and BC are not parallel.

19. An airplane travels at 300 mph on a bearing of 45°. The plane encounters a 50 mph east wind. The resultant velocity is best represented by:

A. 350 mph with a bearing of 45°

B. 265 mph with a bearing of 52°

C. 345 mph with a bearing of 52°

D. 345 mph with a bearing of 37°

E. 265 mph with a bearing of 37°

20. If $|\vec{a}| = 2|\vec{c}|$ we can conclude:

A. $\vec{a}$ has same direction as $\vec{c}$ and has equal magnitude.

B. $\vec{a}$ is parallel to $\vec{c}$ but $\vec{a}$ has double the magnitude of $\vec{c}$

C. $\vec{a}$ has magnitude equal to twice the magnitude of $\vec{c}$ but directions of $\vec{a}$ and $\vec{c}$ may not be the same.

D. $\vec{a}$ cannot be parallel to $\vec{c}$

E. $\vec{a} = 2\vec{c}$

VECTOR ACHIEVEMENT TEST

ANSWER SHEET

PLEASE PRINT:

NAME: ______________________________

SCHOOL: ____________________________

DATE: _______________________

Place the letter corresponding to the correct answer in the appropriate blank. USE PRINTED CAPITAL LETTERS.

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
8. __________
9. __________
10. __________
11. __________
12. __________
13. __________
14. __________
15. __________
16. __________
17. __________
18. __________
19. __________
20. __________

VAT - MANITOBA

Student	X_i	$X_i - \overline{X}_1$	$(X_i - \overline{X}_1)^2$	Student	X_i	$X_i - \overline{X}_1$	$(X_i - \overline{X}_1)^2$
1	16	3.28	10.76	26	16	3.28	10.76
2	16	3.28	10.76	27	10	-2.72	7.40
3	14	1.28	1.64	28	11	-1.72	2.96
4	15	2.28	5.20	29	14	1.28	1.64
5	15	2.28	5.20	30	16	3.28	10.76
6	10	-2.72	7.40	31	7	-5.72	32.72
7	10	-2.72	7.40	32	17	4.28	18.32
8	8	-4.72	23.23	33	16	3.28	10.76
9	15	2.28	5.20	34	14	1.28	1.64
10	14	1.28	1.64	35	16	3.28	10.76
11	18	5.28	27.88	36	14	1.28	1.64
12	17	4.28	18.32	37	14	1.28	1.64
13	11	-1.72	2.96	38	11	-1.72	2.96
14	9	-3.72	13.84	39	12	-.72	.52
15	7	-5.72	32.72	40	18	5.28	27.88
16	11	-1.72	2.96	41	13	.28	.08
17	10	-2.72	7.40	42	13	.28	.08
18	4	-8.72	76.04	43	11	-1.72	2.96
19	14	1.28	1.64	44	9	-3.72	13.84
20	11	-1.72	2.96	45	12	-.72	.52
21	11	-1.72	2.96	46	16	3.28	10.76
22	16	3.28	10.76	47	9	-3.72	13.84
23	11	-1.72	2.96	48	12	-.72	.52
24	7	-5.72	32.72	49	19	6.28	39.44
25	13	.28	.08	50	13	.28	.08
					636		539.11

$$\overline{X}_1 = \frac{636}{50} = 12.72$$

VAT - ONTARIO

(X) Mark	$f_{(z)}$	$X.f_{(z)}$	$X_i - \bar{X}_2$	$(X_i - \bar{X}_2)^2$	$f_{(z)}.(X_i - \bar{X}_2)^2$
0					
1					
2					
3	1	3	-9.56	91.39	91.39
4					
5	1	5	-7.56	57.15	57.15
6	1	6	-6.56	43.03	43.03
7	7	49	-5.56	30.91	216.37
8	17	136	-4.56	20.79	353.43
9	18	162	-3.56	12.67	228.06
10	21	210	-2.56	6.55	137.55
11	25	275	-1.56	2.43	60.75
12	30	360	-.56	.31	9.30
13	30	390	.44	.19	5.70
14	20	280	1.44	2.07	41.40
15	18	270	2.44	5.95	107.10
16	16	256	3.44	11.83	189.28
17	18	306	4.44	19.71	354.78
18	5	90	5.44	29.59	147.95
19	8	152	6.44	41.47	331.76
20	2	40	7.44	55.35	110.70
	238	2990			2485.70

$$\bar{X}_2 = \frac{2990}{238} = 12.56$$

APPENDIX D

Letters, Forms, and Information Sent to Participating Ontario Schools.

[illegible]
Winnipeg, Manitoba
R3R 1R9
January 15, 1972

Teachers of Grade 10 Mathematics using the text:

Mathematics 10, by Del Grande, Egsgard, and Mulligan, published by W. J. Gage Ltd., Toronto.

Dear Colleague:

I am conducting a study to determine the advisability of teaching a unit on vectors to Manitoba mathematics students at the tenth grade level. I will be using Chapter 12, "Vectors" of <u>Mathematics 10</u>.

For part of the study I have prepared an achievement test entitled, <u>Vector Achievement Test</u> which the students involved in the study will write. In order to assess the results I would like to compare scores obtained by the Manitoba students to scores obtained by Ontario students who have studied the same chapter.

I would be much obliged if you would give the test to your students after having completed the chapter.

The test consists of twenty multiple choice type questions. The duration of the test is 35 minutes. After the tests have been written please return the sheets to me for marking and I will send the results to you as soon as possible.

If you wish to assist in this endeavour please complete and return the enclosed form.

Thank you for your consideration.

Yours truly,

enc.

Roy Johnson

VECTOR ACHIEVEMENT TEST

Based on Chapter 12, "Vectors" of

Mathematics 10

If interested in participating in the study please complete this form and return to:

Westgrove Way
Winnipeg, Manitoba
R3R 1R9

Teacher in Charge ______________________________

School ______________________________

Address ______________________________

Number of Papers Required ______________________________

Approximate date that test will be administered

____

To Teachers Administering Vector Achievement Test

Students are allowed thirty-five (35) minutes to write this test.

Each student should have a protractor and ruler.

Please return answer sheets to:

Westrove Way
Winnipeg, Manitoba
R3R 1R9

You may retain the question papers for class discussion or for future use. For your convenience the answers are listed below:

1.	C	6.	B	11.	E	16.	B
2.	D	7.	D	12.	C	17.	B
3.	E	8.	A	13.	A	18.	D
4.	C	9.	D	14.	D	19.	E
5.	E	10.	C	15.	A	20.	C

Results will be forwarded to you as soon as possible.

Thank you for your assistance in this study.