LIQUID LEVEL DETECTION BY MICROWAVE TELEMETRY

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Abstract

The feasibility of using methods of microwave telemetry, in particular a phase measurement procedure for determining relatively short absolute distances, that are commonly encountered in liquid level detection, is discussed. A theoretical analysis and an experimental model technique is presented, using a multiple frequency method, thus eliminating ambiguous distance measurements. This multiple frequency technique is first employed in conjunction with slotted line measurement techniques to verify the method and to determine the possible limitations of accuracy. Since the slotted line technique is not readily applicable in practice, the multiple frequency method is implemented in conjunction with data obtained from a phase meter. Employing such a procedure enables the measurement of phase differences between a reference signal and the signal reflected from the termination; whether it be a liquid or a solid terminating material. The resulting measured phase-shift is subsequently used for calculating the distances to be measured.

Potential applications of the developed measurement model technique are the monitoring of [1]: Water levels in piezometers, liquid levels in large storage tanks (e.g. oil, liquid chemicals, etc.), solid level in blast furnaces and levels of molten metal in electrolytic tanks.

TO MY WIFE

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CHAPTER I

INTRODUCTION

A large variety of liquids (water, petroleum, chemicals, molten metals, etc.) are used in industrial processes. It is very important for proper process operations, logistics and cost accounting purposes to know what amount of material is being processed, how much raw material is available in storage, and how much has been consumed [2]. Liquid level measurements can provide this necessary and important information.

Many methods for determining liquid levels are employed in industry, such as [2]: Direct methods using a calibrated stick, float, or more indirect methods using pressure, capacitive, ultrasonic, nuclear or other systems. The method employed is mainly dependent on: the required accuracy, the necessity of continuous or non-continuous level determination, and the requirement of control systems for maintaining a proper level.

There is an apparent lack of an accurate and continuous method for monitoring and controlling liquid levels in severe environments. Since microwaves are, by nature, electrical signals they are ideally suited for automatic control systems. Therefore, it was decided to perform a feasibility study using microwave techniques to determine whether the already proven method of microwave telemetric measurements could provide the required accuracy. Microwave techniques, for measuring long distances accurately, have been used extensively for measuring the altitudes of flying aircraft and spacecraft [3,4], and for surveying in cartography and geodesy [4,5]. It remains to be shown that microwave telemetric techniques can, in fact, also be used for measuring relatively shorter distances which are commonly encountered in liquid level measurements.

Liquid levels can be determined, in practice, with microwave techniques by either partially immersing an open-ended waveguide into a liquid or by using an antenna to radiate electromagnetic energy towards the liquid. In both cases reflections from the liquid surface are utilized to provide the necessary data for liquid level calculations. This thesis expounds only the theory to be used with a partially immersed waveguide; therefore, neither theoretical evaluations nor techniques for antenna applications have been given. However, only a slight change in theory and experimental set-up would be necessary to allow for the use of antennas.

The liquid level measurements pursued during the course of this thesis are based on the measurement of the phase difference between a normally incident reference signal and the signal reflected from the surface of the liquid. This phase difference (i.e. the electrical length) is subsequently converted to its corresponding physical distance. Since phase measurements are made and the accuracy of the measured distances depends on these phase measurements, it is very important to know what the phase shift (argument of the reflection coefficient) due to the air liquid boundary is, so that adequate corrections can be made. For this reason, the concept of the reflection coefficient is developed and its effect on liquid level measurements is discussed in chapter II.

Chapter III introduces the concept of a multiple frequency technique, which is used for measuring distances unambiguously i.e. physical distances that have an equivalent electrical distance less than 2π. Distances to a short circuit and to a water surface are determined using a slotted line, and the results are tabulated. The necessity for measuring the argument of the reflection coefficient and hence, the relevance of a correction factor is also discussed.

The slotted line is a laboratory tool which can not be adapted, very readily, for industrial use. For this reason the presentation that is given in chapter IV depends on phase measurements resulting from the application of a HP vector voltmeter. These phase measurements are used to calculate the required distances which are compared with those measured mechanically. The accuracy of this phase measurement method is also discussed and a comparison is made with the slotted line technique.

Finally, the usefulness and accuracy of this method are discussed, and its applicability to practical problems is illustrated in chapter V.

CHAPTER II

PHASE CHANGE AT AIR LIQUID BOUNDARY

An electromagnetic wave normally incident on an interface, separating two different materials, will be either partially or totally reflected, with the reflected wave at the interface incurring a phase shift relative to the incident wave at the interface. This phase shift can be conveniently determined by making use of the reflection coefficient Γ (see Appendix A) which is defined, for a wave traveling from medium (1) towards medium (2), as follows

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{1}$$

where Z_1 is the characteristic impedance of medium (1) and Z_2 is that of medium (2). Medium (2) is considered to be of infinite extent or to be match-terminated.

The modulus of the reflection coefficient, on the other hand, can be expressed as [6]

$$|\Gamma| = \frac{S - 1}{S + 1} \tag{2}$$

where S is the voltage standing wave ratio.

Equation (1) is used for calculating the complex value of the reflection coefficient while equation (2) is used for determining the modulus of the reflection coefficient from experimental data. The argument of the reflection coefficient is determined experimentally by, simply, comparing the position of a minimum due to the material under test with that due to a short circuit.

2.1 Theoretical Determination of Reflection Coefficient

2.1.1. Free-Space Reflection Coefficient

The equation for the reflection coefficient for a wave in free space incident normally on another material can be expressed as

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \tag{3}$$

Where η_1 and η_2, the intrinsic wave impedances of the respective media are, in effect, the characteristic impedances Z_1 and Z_2 used in equation (1).

The intrinsic wave impedances, in turn, can be expressed in the following convenient form (see Appendix B).

$$\eta = \sqrt{\frac{\omega(\mu' - j\mu'')}{\omega(\varepsilon' - j\varepsilon'') - j\sigma}} \tag{4}$$

For non-magnetic and low conductivity material, $\mu'' \approx 0$, $\mu' \approx \mu_o$ and $\sigma \approx 0$, equation (4) becomes

$$\eta = \sqrt{\frac{\mu_o}{\varepsilon' - j\varepsilon''}} \tag{5}$$

which can also be expressed as

$$\eta = \frac{\eta_o}{\sqrt{\varepsilon_r' - j\varepsilon_r''}} \tag{6}$$

where η_o is the intrinsic wave impedance of free space and $\varepsilon_r' = \varepsilon'/\varepsilon_o$, $\varepsilon_r'' = \varepsilon''/\varepsilon_o$ are the relative permittivity components.

By substituting equation (6) into (3), the reflection coefficient, for a wave in free space normally incident on a boundary, can be expressed as follows

$$\Gamma = \frac{1 - \sqrt{\varepsilon_r' - j\varepsilon_r''}}{1 + \sqrt{\varepsilon_r' - j\varepsilon_r''}} \qquad (7)$$

This final result is used later for reflection coefficient calculations of water.

2.1.2 Reflection Coefficient in Waveguide

The reflection coefficient resulting from the immersion of rectangular or circular waveguides into a liquid, operated in the TE modes normal to a liquid surface, can be determined by using equation (1)

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where $Z_1 + Z_2$ are the characteristic impedances of the waveguide in air and the liquid respectively. These impedances, can be expressed as [6]

$$Z = \frac{\eta_m}{\sqrt{1 - (\lambda_m/\lambda_c)^2}} \qquad (8)$$

where η_m is the intrinsic wave impedance of the medium, λ_m is the wavelength in the medium, and λ_c is the cutoff wavelength of the particular waveguide used.

Equation (8) can be written in a more useful form by making the following substitution

$$\lambda_m = \frac{v_m}{f} = \frac{1}{f\sqrt{\hat{\varepsilon}\hat{\mu}}} = \frac{v_o}{f}\frac{1}{\sqrt{\hat{\varepsilon}_r\hat{\mu}_r}} = \frac{\lambda_o}{\sqrt{\hat{\varepsilon}_r\hat{\mu}_r}}$$

where $\hat{\varepsilon}$ and $\hat{\mu}$ are the complex permittivity and permeability respectively. The result, after simplification, becomes

$$Z = \frac{\eta_o \mu_r}{\sqrt{\hat{\varepsilon}_r \hat{\mu}_r - (\lambda_o/\lambda_c)^2}} \tag{9}$$

where $\eta_o = \eta_m \sqrt{\hat{\varepsilon}_r / \hat{\mu}_r}$

For non-magnetic materials $\hat{\mu}_r \simeq 1$, which when substituted into equation (9), yields

$$Z = \frac{\eta_o}{\sqrt{\varepsilon_r' - (\lambda_o/\lambda_c)^2 - j\varepsilon_r''}} \tag{10}$$

The wave impedances in the air and the liquid can be expressed, respectively, as

$$Z_1 = \frac{\eta_o}{\sqrt{1 - (\lambda_o/\lambda_c)^2}}$$

$$Z_2 = \frac{\eta_o}{\sqrt{\varepsilon_r' - (\lambda_o/\lambda_c)^2 - j\varepsilon_r''}}$$

which, when used in equation (1), yield the equation of the reflection coefficient in its most useful form, namely,

$$\Gamma = \frac{\sqrt{1 - (\lambda_o/\lambda_c)^2} - \sqrt{\varepsilon_r' - (\lambda_o/\lambda_c)^2 - j\varepsilon_r''}}{\sqrt{1 - (\lambda_o/\lambda_c)^2} + \sqrt{\varepsilon_r' - (\lambda_o/\lambda_c)^2 - j\varepsilon_r''}} \tag{11}$$

Tables of permittivities for water at various temperatures [7] and X-band equipment are readily available, therefore, a frequency of 10 GHz was chosen for both the theoretical and experimental determination of the reflection coefficient. Equation (11) can be simplified by making the following substitution

$$\lambda_c/\lambda_o = f/f_c = 1.525$$

which in turn yields the final result

$$\Gamma = \frac{0.755 - \sqrt{\varepsilon_r' - 0.430 - j\varepsilon_r''}}{0.755 + \sqrt{\varepsilon_r' - 0.430 - j\varepsilon_r''}} \tag{12}$$

2.1.3 Results

Computer calculations were used to make the lengthy computations required for solving equations (7) and (12), and the resulting values for the modulus and argument of the reflection coefficient are entered in Table I. It can be seen from Table I that sufficient signal is reflected ($|\Gamma| \simeq 0.8$) to allow for comparison measurements. Also, the argument of the reflection coefficient for the free space case is almost the same as that for the waveguide case. In fact, the maximum difference in angle of 1.5° represents a distance of only 0.00827 cm for x-band. For most case, therefore, where extreme accuracy is not required, the argument for the reflection coefficient in the free space case could also be used for the waveguide case or vice versa. The maximum variation

TABLE I

REFLECTION COEFFICIENT OF WATER, OF A NORMALLY INCIDENT WAVE, IN FREE SPACE AND IN A WAVEGUIDE FOR A FREQUENCY OF 10 GHz

	FREE SPACE		WAVEGUIDE	
T°C	Modulus of Γ	Argument of Γ	Modulus of Γ	Argument of Γ
-12	0.281	180.0	0.374	180.0
1.5	0.779	173.9	0.828	175.4
5	0.781	174.3	0.829	175.7
15	0.781	175.5	0.829	176.6
25	0.782	176.3	0.830	177.3
35	0.781	177.0	0.830	177.7
45	0.781	177.2	0.829	177.9
55	0.781	177.4	0.829	178.1
65	0.777	177.7	0.826	178.3
75	0.771	177.9	0.822	178.4
85	0.765	178.0	0.817	178.5

of the argument of Γ with respect to the temperature range of 1.5° C to 85° C, corresponds to a distance of 0.0171 cm, which can in general also be ignored.

It can be seen from equation (7) and (12) that the argument of Γ is 180° for lossless media. For slightly lossy materials the argument of Γ can also be considered to be 180° for all practical purposes. Even for water, which is very lossy ($\varepsilon_r' = 38$, $\varepsilon_r'' = 39$ at 1.5°), the argument of Γ is 175°. Since 5° at 10GHz represents an electrical distance of approximately 0.01 cm, it can be seen that the argument of Γ for water can be chosen to be 180°.

This information leads us to the conclusion, that for many liquids the argument of Γ need not be measured, which in turn, signifies that liquid level measurements will be greatly simplified.

For carrier frequencies lower than microwave frequencies, however, the argument of the reflection coefficient assumes increased importance, since the lower the frequency the greater is the wavelength and hence, the distance per degree of phase angle. In this case the argument of Γ must be known very precisely and its difference from that of a short circuit could no longer be ignored.

2.2 Experimental Determination of Reflection Coefficient

The experimental set-up used for determining the reflection coefficient is shown in Figure 1. A X-13 Klystron followed by a ferrite isolator, a pin

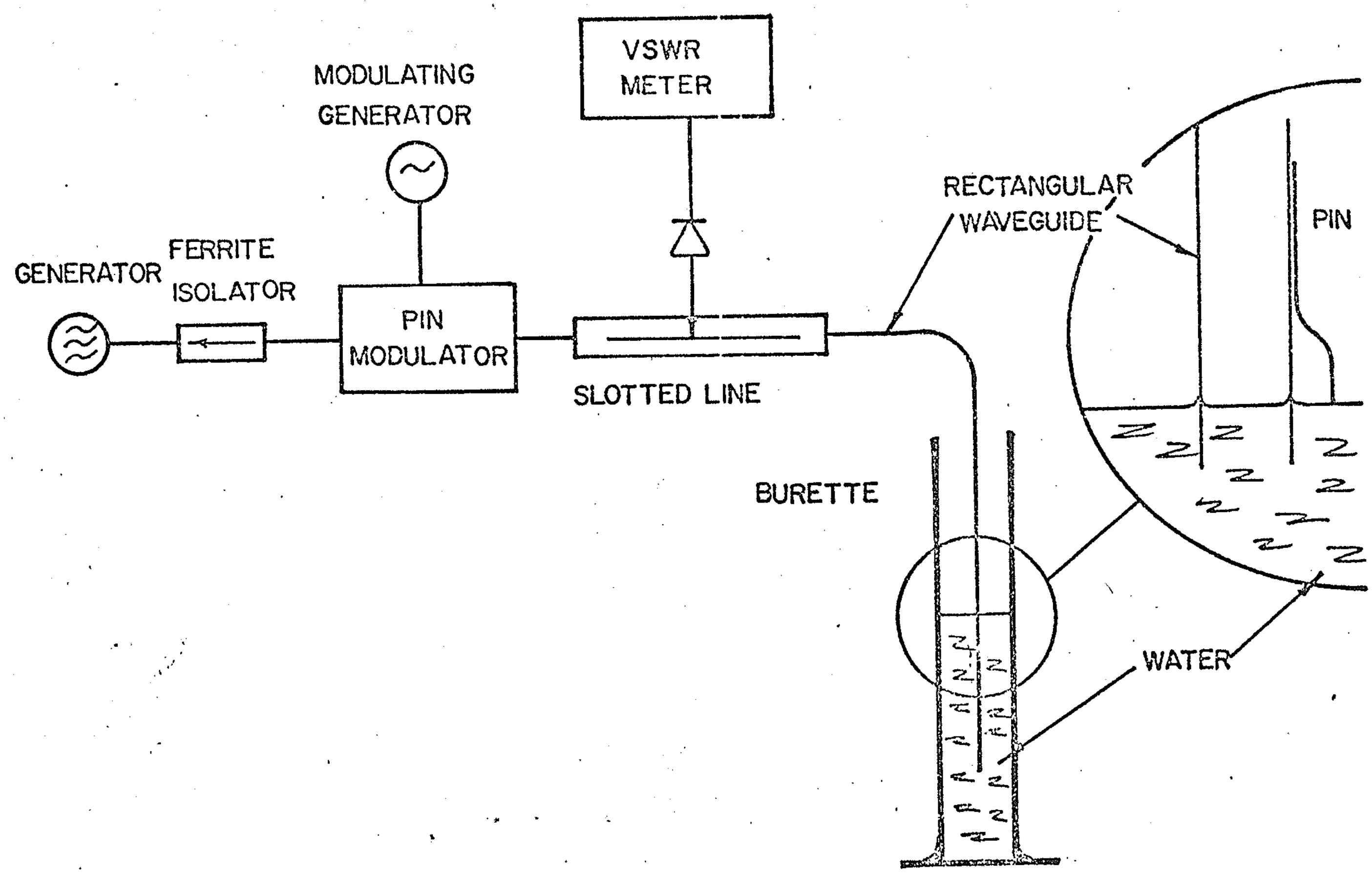

FIGURE 1

SET–UP FOR MEASURING REFLECTION COEFFICIENT IN WAVEGUIDE modulator, a slotted line and a section of waveguide partly immersed in a burette filled with water were used. It was not necessary to match-terminate the section of waveguide immersed in the water, since secondary reflections are sufficiently attenuated by water (see Appendix C), so that no erroneous results are obtained.

The modulus of the reflection coefficient was simply found by using equation (2) in conjunction with the voltage standing wave ratio (VSWR), that was obtained from the use of a precision attenuator and a VSWR meter.

The argument of the reflection coefficient was found using a comparison technique, which was performed as follows:

1. A section of waveguide machined to a length of 12.705 cm $\pm$ 0.005 cm was connected to a 90° bend which in turn was connected to the slotted line. The other end of the machined section of waveguide was short circuited and the position of a voltage minimum on the slotted line, was recorded. This section of the waveguide was then disconnected from the 90° bend.

2. A needle was soldered to an outside wall of a longer section of waveguide, and the distance from the upper flange to the tip of the needle was adjusted to 12.705 cm. This section of the waveguide was connected to the 90° bend and the water level was adjusted so that the tip of the needle contacted the water surface. Finally, the minimum position was recorded.

A scribed line on the waveguide (12.705 cm from the top of the flange) was not used since surface tension effects cause the water to hang onto the line as the waveguide is moved. Due to this hang-up, it is not possible to know the distance from the upper flange to the water surface accurately. With a needle connected to calipers, however, the water surface was detected to an approximate accuracy of ± 0.0025 cm. For this reason a needle was used in the above procedure i.e. to insure that the distance from the upper flange of the waveguide to the water surface is 12.705 cm.

If a waveguide is terminated with a short circuit, voltage minima positions can be detected on the slotted line. The subsequent loading of the waveguide with water (see steps 1 and 2 above) results in a shift of the minima positions towards the load. It is known that the argument of Γ of a short circuit is 180°, therefore, the argument of Γ due to water will be less than 180° by an amount which is proportional to the difference in minima positions.

The angle of the reflection coefficient can be determined from the distance between minima by noting that

$$\psi' = \beta x$$

where β is the propagation constant and x is the distance between minima. Since the distance between minima is traversed twice, the previous equations must be written as

$$\psi = 2\beta x$$

Finally, since $\beta = 2\pi/\lambda g$

$$\psi = \frac{4\pi x}{\lambda g} \tag{13}$$

and the argument of Γ of water is $180° - \psi$

2.2.1 Results

It was found (see Table II) that the experimental results of the modules of $\Gamma (0.824)$ agreed very well with the calculated value (0.831). There was not, however, such a close agreement between the experimental and calculated value for the argument of Γ. In fact, the experimental value for the argument of Γ depended on whether the water approached the reference level from the top or from the bottom. Actually, any possible values for the argument of Γ between the extremes of 169° and 176° (see Table II) can be obtained.

The occurrence of a range of values, instead of only one value, for the argument of the reflection coefficient can be attributed to the existence of surface effects. Due to the very low contact angle of the waveguide in water (approximately 0°) [8], a meniscus is present on the inside of the waveguide. A meniscus, of course,is also present on the outside of the waveguide; however, there is no concern regarding this effect as long as the tip of the needle is far enough from the waveguide so that the actual liquid surface in the burette is detected. The shape of the meniscus changes slightly whenever there is a reversal of the direction in which the waveguide moves relative to the water (or the

TABLE II

COMPARISON OF CALCULATED AND EXPERIMENTALLY DETERMINED VALUES OF THE REFLECTION COEFFICIENT OF WATER AT 25° C IN A RECTANGULAR WAVEGUIDE

	CALCULATED	EXPERIMENTAL	
		Water approaches from top	Water approaches from bottom
Modulus of Γ	0.831	0.824	0.824
Argument of Γ	177.4	176.60	169°

water level moves relative to the waveguide). This change of meniscus causes a concurrent shift in the minimum position; hence, the result is a 7.6° range for the argument of Γ as determined from Table II.

If a greater degree of accuracy is required the effects of the méniscus can be minimized by covering the inside of the waveguide with teflon or any other material with a high contact angle (equal to or greater than 90°) [8,9].

If the waveguide is tilted the argument of the reflection coefficient also changes, as can be seen from Figure 2. It can be seen that tilting the waveguide in the E plane (H parallel to the water surface) causes the greatest change in the argument of Γ. In this case, for a tilt angle of 30° the argument of the reflection coefficient can vary over a range of 13°, which represents a distance of 0.0717 cm.

After a few degrees of tilt, however, the change in the angle of the reflection coefficient becomes overshadowed by the direct effect that tilting has on liquid level measurements. As the angle of tilt increases the length of the waveguide, to the water surface, will still be measured accurately (affected only by the change in the argument of Γ) but this length will no longer reflect the true water level. The water level can be expressed as

$$D = L \cos \theta$$

where D is the water level, L the measured length of waveguide to the surface and θ the angle of tilt. It can be seen that D is directly

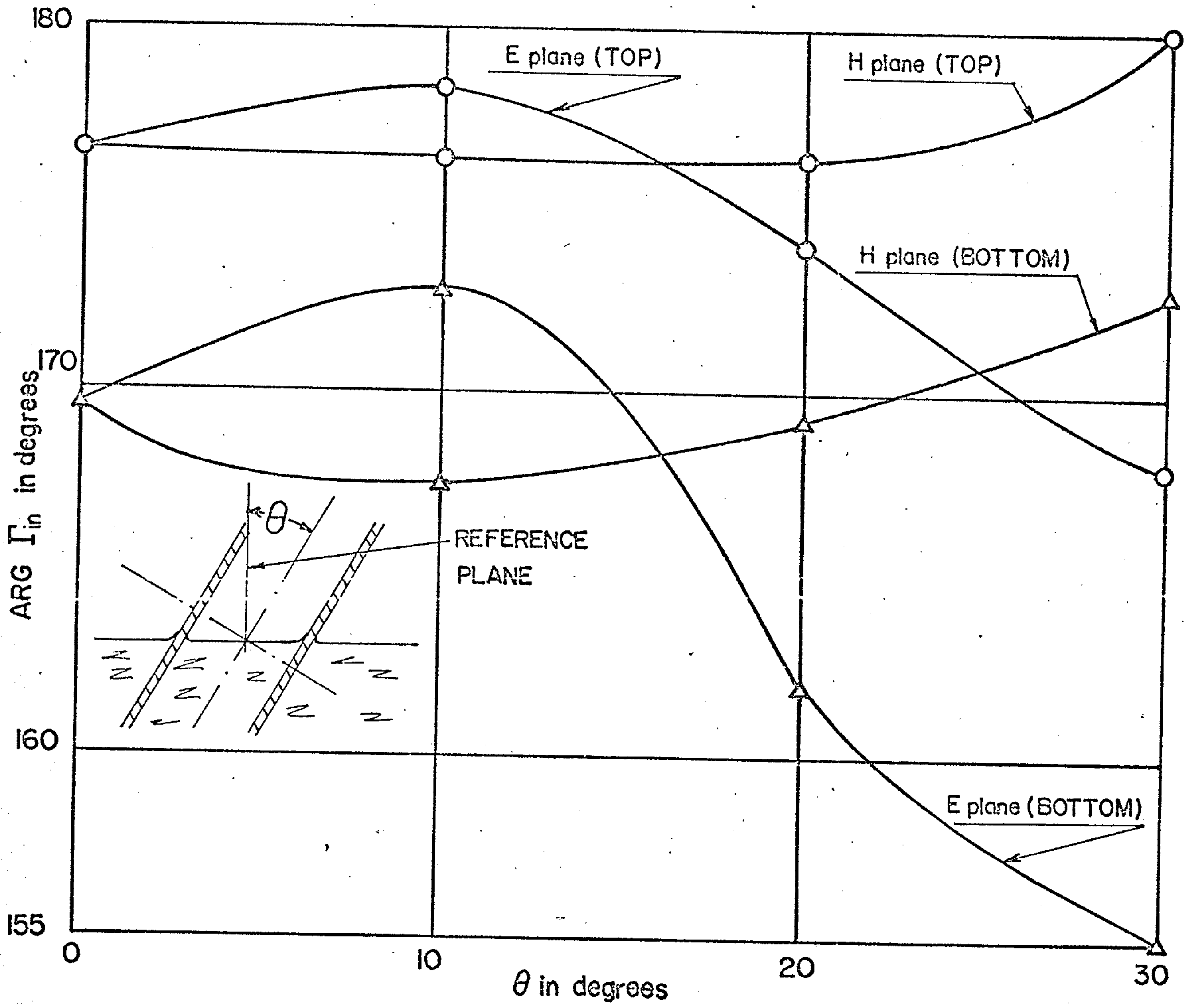

FIGURE 2

ARGUMENT OF Γ OF WATER IN WAVEGUIDE AS A FUNCTION OF TILT ANGLE proportional to the cosine of the tilt angle, therefore, if the water level is to be measured accurately any random tilting of the waveguide must be confined to a certain range which is dependent on the accuracy requirements, (i.e. for a measured length of 200 cm and an angle of tilt of 5° the actual level is 199.2 cm and the error is 0.8 cm).

CHAPTER III

MEASUREMENT OF DISTANCES AND LIQUID LEVELS USING A SLOTTED LINE

The slotted line, a very versatile laboratory tool, is in essence, a phase measuring device. It is used here primarily to prove the feasibility of measuring distances by using phase measurements and also as a means of comparison with the results obtained in chapter IV, where a vector voltmeter was used to measure the phase.

It is shown below how the slotted line, in conjunction with a multiple frequency technique, can be used for determining distances.

3.1 Theoretical Analysis of Multiple Frequency Concept

The length of a waveguide terminated by a short circuit can be expressed as

$$D = n_o \frac{\lambda_{go}}{2} + x_o \tag{14}$$

where n_o is the integral number of half wavelengths from the short circuit to the other end of the waveguide, λ_{go} the wave-guide wavelength, and x_o is the distance from the flange of the wave-guide to the last minimum (see Figures 3). Since λ_{go} is known and x_o can be determined quite readily, it is only necessary to solve for n_o. This is done by choosing the proper frequencies, as shown below.

If a frequency is chosen so that the corresponding wavelength is

$\lambda_{g1} = 10/9\ \lambda_{go}$

the minima due to λ_{g1} will be separated from the minima due to λ_{go} by $\lambda_{g1}/20$, $2\lambda_{g1}/20$, $3\lambda_{g1}/20$, etc., depending on how many half wavelengths from the short circuit a measurement is made (see Figure 4). This fact can be used to determine n_o, by simply measuring the difference in the position of minima and dividing this distance by $0.1\lambda_{g1}/2$ (see Figure 4).However, as soon as the difference between minima is equal to or greater than $\lambda_{go}/2$, n_o becomes ambiguous. A distance up to $10\lambda_{go}/2$ can be measured without ambiguity, which at 10 GHz is only about 19.8 cm. Greater distances can be measured by subsequently choosing frequencies such that the distance between the first minima of λ_{go} and λ_{gn} become smaller and smaller.

By choosing the next frequency such that the corresponding wavelength is

$\lambda_{g2} = 100/99\ \lambda_{go}$

n_o can be determined up to 99 before it becomes ambiguous. An unambiguous distance of approximately 198 cm can therefore be obtained.

Similarly, another frequency can be chosen so that its corresponding wavelength is

$\lambda_{g3} = 1000/999\ \lambda_{go}$

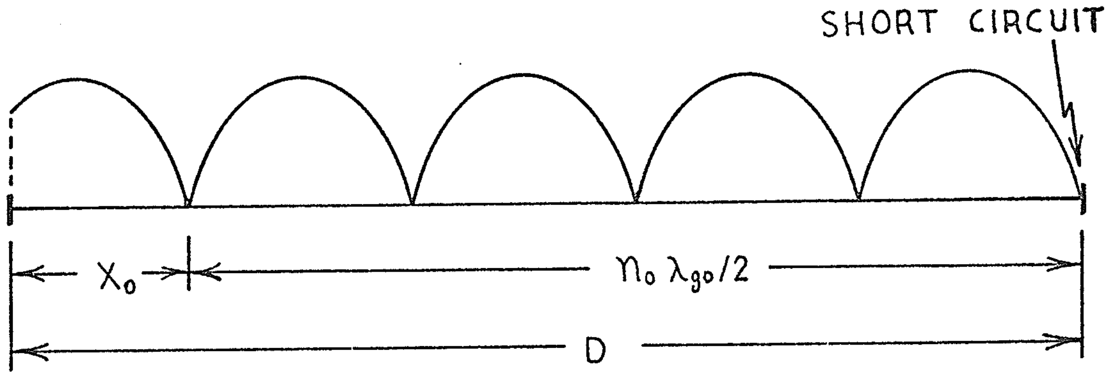

FIGURE 3

LENGTH OF WAVEGUIDE EXPRESSED IN TERMS OF NO. OF HALF WAVELENGTHS PLUS A FRACTIONAL HALF WAVELENGTH

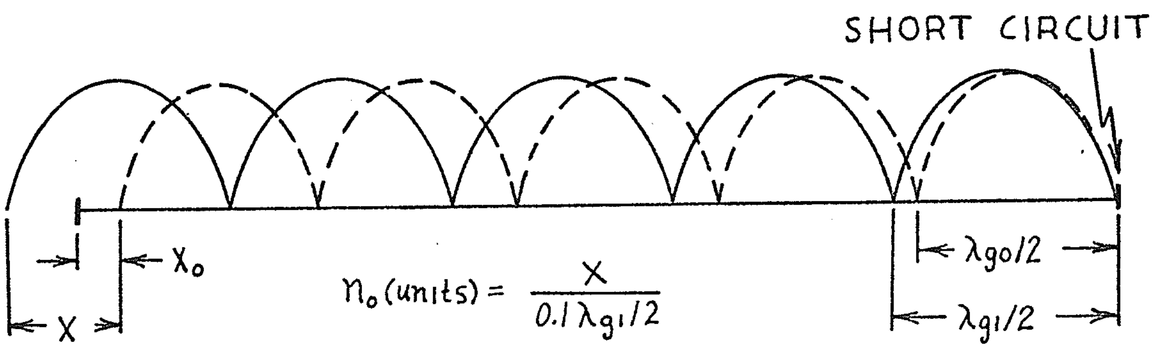

FIGURE 4

DETERMINATION OF n_o BY MEASURING THE DISTANCE BETWEEN MINIMA OF λ_{go} AND λ_{g1}

which, in turn, will allow an unambiguous distance measurement of approximately 1985 cm to be made. This step is essentially the last one that can be taken, in general, due to the lack of frequency resolution and stability. Therefore, it can be seen that the slotted line can be used only for measuring relatively small distances.

The values of λ_{g1}, λ_{g2} and λ_{g3} have been chosen so that a decimal readout can be used. This choice of wavelengths allows the use of λ_{g3} to give only the number of hundreds digits of n_o, λ_{g2} to give the number of tens, and λ_{g1} to give the number of units digits of n_o. In this way better accuracy can be obtained than would be possible if only λ_{g3} or λ_{g2} were used in conjunction with λ_{g0} for determining n_o.

If n_{01}, n_{02} and n_{03} are used to represent the units, tens and hundreds digits respectively, it can be seen from figure 4 that

$$n_{01} = \frac{X}{\lambda_{g1}/2 - \lambda_{g0}/2} \tag{15}$$

where $X = \lambda_{g1}/2\ (\text{min.}) - \lambda_{g0}/2\ (\text{min.})$

and $\lambda_{g1}/2 - \lambda_{g0}/2 = \lambda_{g1}/2 - 9/10\ \lambda_{g1}/2 = 0.1\ \lambda_{g1}/2$

using these values, equation (15) can be rewritten as

$$n_{01} = \frac{\lambda_{g1}/2\ (\text{min.}) - \lambda_{g0}/2\ (\text{min.})}{0.1\lambda_{g1}/2} \tag{16}$$

The following equations can be determined in a similar manner, namely

$$n_{02} = \frac{\lambda_{g2}/2\ (\text{min.}) - \lambda_{g0}/2\ (\text{min.})}{0.01\ \lambda_{g2}/2} \tag{17}$$

$$n_{03} = \frac{\lambda_{g3}/2\ (\text{min.}) - \lambda_{g0}/2\ (\text{min.})}{0.001\ \lambda_{g3}/2} \qquad (18)$$

Equations (16), (17) and (18) are used with the appropriate experimental data, in conjunction with, equation (14) to determine the length of a waveguide or the distance to a liquid surface.

3.2 Experimental Results Using Multiple Frequency Concept

3.2.1 Measurement of Waveguide Length

The feasibility of the multiple frequency concept was proven by using the set-up shown in Figure 1. Instead of a section of waveguide being immersed in water, however, a straight section of waveguide was connected to one end of a slotted line and the other end of the waveguide was short circuited.

The multiple frequency measurement technique, that was employed, can be explained as follows:

1. A primary frequency (such as 9 GHz) is chosen and the corresponding wavelength (λ_{g0}) is determined from tables or calculations.

   A. The minimum position with a short circuit at the slotted line is recorded.

   B. The short is removed from the slotted line, a section of waveguide (whose length is to be determined) is connected to the slotted line, the other end of the waveguide is shorted and the resulting minimum position is recorded.

2. A second wavelength is chosen such that $\lambda_{g1} = 10/9 \ \lambda_{go}$ and the corresponding frequency is calculated as shown in appendix D.

A. The Klystron is set for the new frequency.

B. The minima positions, with the slotted line shorted and subsequently the waveguide shorted (as above), are recorded.

3. A third and fourth wavelength are chosen, if necessary, so that $\lambda_{g2} = 100/99 \ \lambda_{g0}$ and $\lambda_{g3} = 1000/999 \ \lambda_{g0}$, and the corresponding frequencies are calculated

A. The klystron is set for the new frequencies (one at a time as above).

B. The minima positions are noted as in steps 1 and 2 above, and recorded in Table D-1.

In order to clarify the use of this multiple frequency method, for determining distances, a sample calculation is included in Appendix D.

The waveguide section, that was tested, is an accurately machined component whose length was measured mechanically (with calipers) to be 60.986 cm $\pm$ 0.00245 cm. This mechanical measurement was chosen as the reference or comparison length, that is, the accuracy of the electrical measurements was determined by comparison with this mechanically measured length.

Measurements of the length of this waveguide were made for three primary frequencies, the results of which are found in Table III. An accuracy of 0.032 cm (0.053%) or better was found. It can be seen that this method is very precise. It is possible, however, to obtain greater accuracy by stabilizing the frequency and by using a dial indicator for locating the minima positions.

3.2.2. Measurement of Water Level

Water level measurements were next made, using the same multiple frequency concept expounded above. The set-up used for making these measurements was the same as that shown in Figure 1. The mechanical distance measurement was carried out in the same manner as was previously done for determining the reflection coefficient, i.e. the length of a section of waveguide from the upper flange to the tip of a needle soldered to a wall of the waveguide was measured. This mechanically measured distance of 73.691 cm $\pm$ 0.00254 cm was used as the reference and the electrically determined distances were compared to this reference.

The water level was adjusted so that the water surface contacted the tip of the needle, and the minima positions were recorded (see Table D-II) for various primary and secondary frequencies, as previously.

3.2.3 Correction Factor

It can be noted that the difference in the argument of the reflection coefficient between the short circuit and that due to the water must be considered at this point. If a short circuit is used initially and finally, the argument of Γ cancels. If, however, the

TABLE III

WAVEGUIDE LENGTH MEASUREMENT FOR VARIOUS PRIMARY FREQUENCIES USING A SLOTTED LINE

f-GHz	Mechanically measured length cm.	Electrically measured length cm.	Error cm.	Error %
9	60.986	60.986	0	0
10	60.986	61.018	0.032	0.053
11	60.986	60.992	0.006	0.0098

waveguide is loaded with some other material, the phase of the short circuit doesn't cancel the phase due to the material; therefore, the difference in phase must be used to obtain accurate results. This difference in phase, converted to a distance, can be considered a correction factor.

Since the argument of Γ, for water, has a range of possible values, an average was used. The approximate correction factor, found to be 0.04 cm, was used for determining some of the values found in Table IV. This same correction factor was used for 9, 10 and 11 GHz, even though the correction factor was only determined for 10 GHz. The reason for this is that the permittivities are essentially the same for these frequencies; therefore, the argument of Γ will be almost identical for the three cases.

The measured water level is the distance from the initial short circuit, at the 90° bend, to the water surface. These electrically measured water levels, with and without correction being applied, are found in Table IV. It can be seen that the maximum error, using the correction factor, is 0.045 cm or 0.061 %. If, however, the correction factor is not used, the maximum error is 0.084 cm or 0.114%. An error of this magnitude can still be acceptable for most practical liquid level measuring purposes.

This last experimental result compares favorably with the theory expounded in chapter II, namely, that the argument of the reflection coef-

TABLE IV

WATER LEVEL MEASUREMENTS FOR VARIOUS PRIMARY FREQUENCIES, WITH SLOTTED LINE.

MECHANICALLY MEASURED DISTANCE OF 73.691 CM USED AS REFERENCE

f-GHz	Water Approaches from bottom				Water Approaches from top			
	Water Level cm		Error		Water Level cm		Error	
	Using corr. Factor	not using corr. Factor	cm	%	using corr. Factor	not using corr. Factor	cm	%
9	73.736	73.775	0.045 0.084	0.061 0.114	73.706	73.745	0.015 0.054	0.020 0.073
10	73.735	73.774	0.044 0.083	0.060 0.113	73.690	73.729	0.001 0.038	0.001 0.052
11	73.734	73.773	0.043 0.082	0.058 0.111	73.704	73.743	0.013 0.052	0.018 0.071

ficient can be considered to be 180° for most practical purposes. There is, however, an additional element which was not considered previously, namely, surface tension. The effects of surface tension cause a greater error than was previously anticipated. If there are very strong surface tension effects between a very lossy liquid and the waveguide, it may no longer be possible to disregard the correction factor. Ultimately, the necessity for determining the argument of the reflection coefficient and hence, the correction factor for a particular liquid, will depend on the final accuracy requirement.

CHAPTER IV

MEASUREMENT OF DISTANCES AND LIQUID LEVELS USING A PHASE METER

The intent of this chapter is to show that direct phase measurements (using a phase meter) can be used for determining distances or liquid levels in waveguides. Although the theory is quite similar to that which was used in the previous chapter, there are some differences which warrant a separate treatment for the sake of clarity.

4.1 Theoretical Analysis of Multiple Frequency Concept

A H.P. vector voltmeter which measures phase differences on the modulation envelope was used in the experiments that follow. Since phase measurements are made using the modulation envelope, it is no longer possible to use the guide wavelength but it is, rather, necessary to use the modulation wavelength. Therefore equation (14) must be re-written, as follows, to reflect this change

$$D = \frac{1}{2}(\eta_o\lambda_o + x_o) \tag{19}$$

Since phase measurements are used in this chapter, it is necessary to convert x_o into its equivalent phase, as follows

$$x_o = \frac{\psi_o \lambda_o}{2\pi}$$

The result from substituting this value into equation (19) is

$$D = \frac{\lambda_o}{2} (n_o + \frac{\psi_o}{2\pi}) \qquad (20)$$

If this same distance is measured by using a different frequency, a similar equation results, namely,

$$D = \frac{\lambda_1}{2} (n_1 + \frac{\psi_1}{2\pi}) \qquad (21)$$

If ψ_o and ψ_1 are calculated from equations (20) and (21) and their difference is rearranged, the following equation results

$$D = \frac{\lambda_o}{2} \frac{\lambda_1}{(\lambda_1 - \lambda_o)} [(n_o - n_1) + (\frac{\psi_o - \psi_1}{2\pi})] \qquad (22)$$

It can be seen from equation (22) that D will be unambiguous only if $\psi_o - \psi_1 < 2\pi$ and hence, $(n_o - n_1) = 0$

If this second frequency is chosen such that

$$\lambda_1 = 10/9\lambda_o$$

(as was done previously in the slotted line case) it can be noted that for an unambigous distance measurement, equation (22) reduces to

$$D = \frac{10\lambda_o}{2} (\frac{\psi_o - \psi_1}{2\pi}) \qquad (23)$$

It can be seen from equation (23) that $(\psi_o - \psi_1)$ must be less than 2π

for an unambiguous distance measurement; therefore, the maximum distance that can be measured without ambiguity is less than $10\lambda_o/2$.

If a third and fourth frequency are chosen so that the respective wavelengths are

$\lambda_2 = 100/99\ \lambda_o$ and $\lambda_3 = 1000/999\ \lambda_o$

equation (22) reduces to

$$D = \frac{100\ \lambda_o}{2}\ \left(\frac{\psi_o - \psi_2}{2\pi}\right) \qquad (24)$$

and

$$D = \frac{1000\ \lambda_o}{2}\ \left(\frac{\psi_o - \psi_3}{2\pi}\right) \qquad (25)$$

The maximum unambiguous distances that can be determined employing (24) and (25) are less than $100\ \lambda_o/2$ and less than $1000\ \lambda_o/2$ respectively. If a modulation frequency of 100 MHz is used in conjunction with equation (25), an unambiguous distance of approximately 993 meters can be obtained. Thus, it can be seen that much greater distances can be measured than were possible with the slotted line.

It is not generally possible to obtain accurate results by using equations (23), (24) or (25), since any inaccuracies of the phase measurements will be multiplied by 10, 100 and 1000 respectively. If, however, $(\psi_o - \psi_3)$ is used only to determine the number of hundred digits of n_o, $(\psi_o - \psi_2)$ the number of tens digits, and $(\psi_o - \psi_1)$ the number of units digits of n_o (as was done in a similar manner with the slotted line case), it is possible to obtain good results by using equation (20). It can be seen from equation (20) that the accuracy of the distance measurements will depend on the precision of the phase measurements (there is no multiplicative constant such as is found in equations (23), (24) or (25))and the stability of the modulation frequency from which λ_o is calculated.

The value of n_o can be found (in a similar manner as was used in the slotted line case) by first converting $(\psi_o - \psi_n)$ to a distance and then dividing this value by the difference in wavelength between λ_o and λ_n, namely,

$$n_{01} = \frac{(\psi_o - \psi_1)\lambda_o}{2\pi(0.1)\lambda_1} \tag{26}$$

$$n_{02} = \frac{(\psi_o - \psi_2)\lambda_o}{2\pi(0.01)\lambda_2} \tag{27}$$

$$n_{03} = \frac{(\psi_o - \psi_3)\lambda_o}{2\pi(0.001)\lambda_3} \tag{28}$$

The length that can be measured at a certain frequency is dependent on the stability of the modulation source. If a very stable modulation source is available, it may be possible to calculate n_{04}, but this would require for a 200 MHz modulation that the frequency would have to be set and maintained at 199.98 MHz. This, indeed, is difficult if not impossible for most ordinary distance measurements therefore, equation (28) would usually be considered to provide the final step.

Distances and water levels can be determined by using either equations (26), (27) or (28), or all three of them in conjunction with equation (20), depending on the distance to be measured.

4.2 Experimental Results Using Multiple Frequency Concept

4.2.1 Measurement of Waveguide Length

The experimental set-up used for the measurement of distances with a H.P. vector voltmeter is shown in Figure 5. The rf signal generated by a X-13 Klystron is modulated by a sine wave generator via a Pin modulator. The reference signal for the HP vector voltmeter is obtained from a 10db directional coupler while the main portion of the signal proceeds through a precision attenuator to a circulator. The energy incident on the circulator proceeds through a shorting switch, through the waveguide (whose distance is to be measured) and then is reflected by the short circuit at the end of the waveguide. This reflected signal is finally transmitted to the comparison probe of the vector voltmeter, and a phase measurement is made.

Phase measurements are made as follows: The shorting switch is initially closed and the phase indicator needle set to zero; the shorting switch is opened and the phase reading taken. This phase measurement represents the distance traveled from the shorting switch to the short circuit at the other end of the waveguide and back again to the

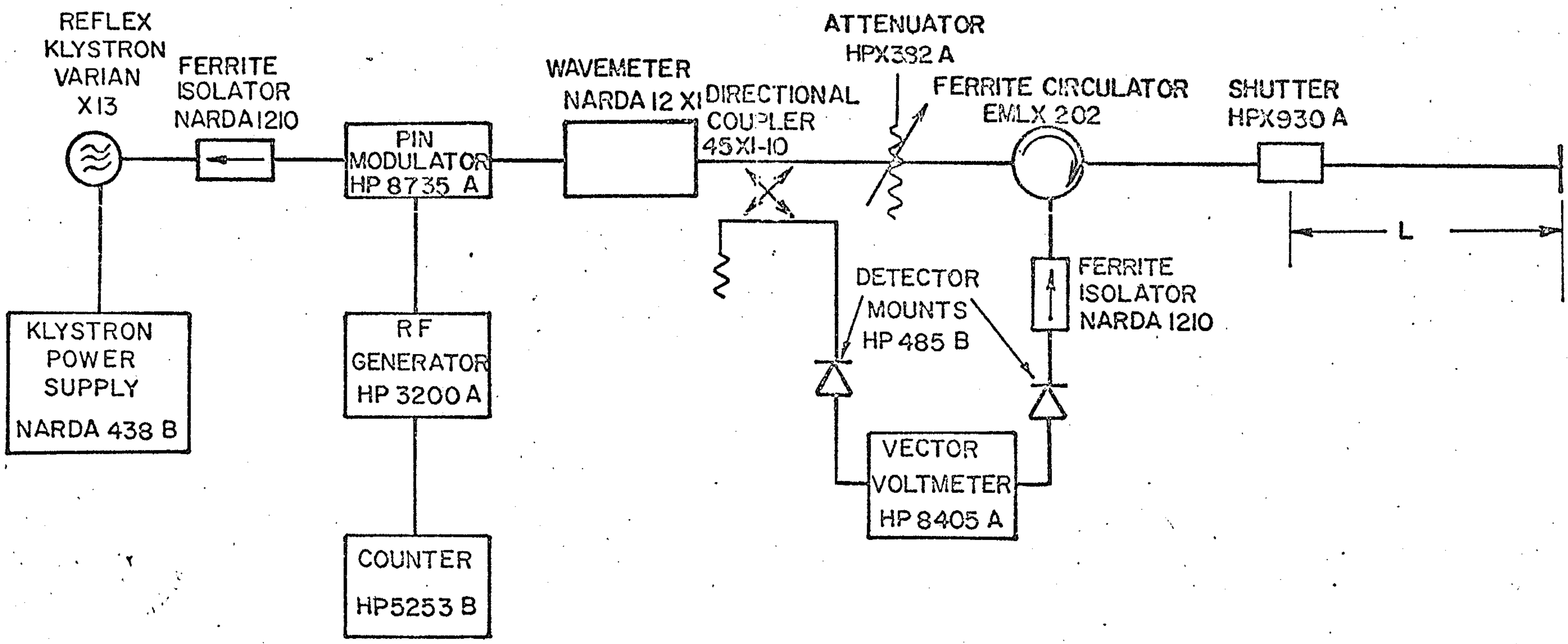

FIGURE 5

PHASE MEASURING SET-UP USING
VECTOR VOLTMETER

shorting switch. The shorting switch (shutter) provides the reference short circuit from which all distances are determined.

The attenuation in the reverse direction to the normal power flow between the circulator ports was determined experimentally; since the maximum isolation was obtained at 8.75 GHz, the following experiments were performed using this carrier frequency.

The limiting accuracy of this distance measurement system is due to the vector voltmeter which has a phase accuracy of $\pm$ 1.5^o. It can be seen from Table V that the accuracy increases as the modulation frequency increases. This is primarily due to the fact that at higher frequencies a phase of 1.5^o represents a smaller distance.

Although, the values of accuracy that can be obtained using a vector voltmeter can't match those obtained with a slotted line; the higher modulation frequencies, usually, give results that are adequate for many purposes. As an example, consider a modulation frequency of 200 MHz: It can be seen from Table V that in the worst case, an error of 0.3 cm for a distance of 35.4 cm or 0.85% was ascertained; however, in the best case an error of only 0.2 cm for a distance of 381.3 cm or 0.05% was determined.

Since the greatest part of the error is contributed by the HP vector voltmeter (the error is constant at $\pm$ 1.5^o over its frequency range) the percentage error decreases with increasing lengths; therefore, this method would be most suitable for measuring large distances to liquid surfaces.

TABLE V

WAVEGUIDE LENGTH MEASUREMENTS FOR VARIOUS PRIMARY FREQUENCIES USING A PHASE METER

Mod. Freq. MHz	Mech. Meas. cm	Elect. Meas. cm	Fractional error cm	Percentage error %	Estimated accuracy cm
	35.4	34.9	0.5	1.41	
	65.9	67.5	1.6	2.43	
30	111.4	111.8	0.4	0.36	± 2.76
	163.8	164.5	0.7	0.43	
	270.3	271.0	0.7	0.26	
	381.3	283.0	1.7	0.45	
	35.4	34.8	0.6	1.7	
	65.9	65.4	0.5	0.76	
100	111.4	111.5	0.1	0.09	± 0.877
	163.8	163.5	0.3	0.18	
	270.3	271.0	0.7	0.26	
	381.3	382.0	0.7	0.18	
	35.4	35.7	0.3	0.85	
	65.9	66.3	0.4	0.61	
	111.4	111.8	0.4	0.36	
200	163.8				± 0.413
	270.3	270.7	0.4	0.15	
	381.3	381.5	0.2	0.05	

4.2.2 Measurement of Water Level

Due to the problems encountered in making liquid level measurements for relatively large distances to the water surface, only one measurement was carried out for a liquid level of 78.574 cm. The same set-up, as shown in Figure 5, was used; however, the waveguide was immersed in a burette filled with water (see Photograph 1). The distance from the upper flange of the waveguide to the tip of the needle was measured mechanically (as previously), and the water level was adjusted so that the tip of the needle contacted the water surface.

The measured distance for a modulation frequency of 100 MHz was 78.2 cm; whereas, the reference level was determined to be 78.574 cm. This error of 0.374 cm or 0.476% is well within the given accuracy of ± 0.877 cm for the vector voltmeter. It was not necessary to apply the correction factor for water (0.04 cm) since it is minimal when compared to the accuracy of the vector voltmeter.

It can be seen here, as has been pointed out previously, that liquid level measurements using a H.P. vector voltmeter are not very accurate for measuring small distances. When larger distances to liquid surfaces are measured, however, the percentage error decreases; hence, the measurements have a greater potential value.

CHAPTER V

SUMMARY AND CONCLUSIONS

Relatively short distances, such as are encountered in liquid level detection, can be determined from phase measurements. Phase measurements utilizing a slotted line give an accuracy of 0.05 cm; whereas, the use of a H.P. vector voltmeter results in an accuracy of approximately 0.5 cm or better for modulation frequencies of 100 MHz or greater. Although an error of $\pm$ 0.5 cm would not in general be acceptable for short distance measurements, it would be acceptable in most cases where distances greater than two meters are measured. The accuracy of liquid level measurements utilizing a H.P. vector voltmeter is primarily dependent on the limitations of this meter, which has an accuracy of $\pm$ 1.5°. At higher modulation frequencies, the error introduced by the phase measurement becomes smaller, i.e. at a frequency of 1 GHz an accuracy of $\pm$ 0.125 cm can be expected.

The slotted line can be used in the laboratory for measuring accurate distances such as the length of waveguides and liquid levels. Liquid levels can be determined very accurately by first determining the argument of the reflection coefficient and then by applying the resulting correction factor. Unfortunately, the slotted line cannot be applied to ordinary industrial uses since a technician must operate the equipment and then calculate the distances measured.

The use of a vector voltmeter eliminates the requirement for a correction factor since the value of the correction factor (0.04 cm for water) is much less than the accuracy of the vector voltmeter

($\pm$ 0.887 cm at 100 MHz). Also, the introduction of more automated equipment, used in conjunction with a vector voltmeter, would eliminate the need for a technician.

The next obvious step that must be taken is to automate this system such that it can be applied to control industrial processes. Therefore, it is necessary to develop a system that will generate the primary and required secondary modulation frequencies either simultaneously or at short intervals, such that the phase difference between the primary and secondary waves can be measured and the liquid level displayed on a meter for convenient read-out. This measured liquid level could also be automatically compared to a preset level and a control system actuated whenever more or less liquid is required.

Where simple liquid level measurements are to be made, this system would not be required nor justified. The greatest potential use would be in an area where it is not possible to use other less-expensive systems, such as, for measuring the liquid level of molten aluminum or other metals. It may also prove important in measuring liquid levels of very corrosive liquids. In these cases, of course, a waveguide could not be immersed in the liquid but, rather, an antenna must be used to transmit the electromagnetic energy to the surface of the liquid and to receive the reflected energy, and thus could be applied to determine the level.

In general, the method of microwave telemetric techniques for detecting and controlling relatively large differences in liquid levels (greater than 2 meters) would be useful wherever accuracy, reliability and signal for automatic control systems are of prime importance and where expense is only of a secondary nature.

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APPENDICES

APPENDIX A

GENERAL SOLUTION FOR REFLECTION COEFFICIENT

Consider two different materials to be in contact with each other and bounded only at their interface. An electromagnetic wave generated in medium (1) will be either partially or totally reflected at the interface. The total field in medium (1) (the sum of the incident and reflected waves), for harmonic time dependence, can be expressed as

$$E_{x(1)} = E_o^+ e^{-jk_1 z} + E_o^- e^{jk_1 z} \tag{A-1}$$

$$H_{y(1)} = \frac{E_o^+}{Z_1} e^{-jk_1 z} - \frac{E_o^-}{Z_1} e^{jk_1 z} \tag{A-2}$$

where Z_1 is the characteristic impedance of medium (1).

If the electric type reflection coefficient is defined as

$$\Gamma = \left.\frac{E^-_{x(1)}}{E^+_{x(1)}}\right|_{z=0} = \frac{E_o^-}{E_o^+} \tag{A-3}$$

where the interface occurs at $z = 0$, it can be seen that Γ can, in fact, be used to determine the relative phase shift between the incident and reflected field at the boundary.

By substituting (A-3) into (A-1) and (A-2) the following results are obtained

$$E_{x(1)} = E_o^+ (e^{-jk_1 z} + \Gamma e^{jk_1 z}) \tag{A-4}$$

$$H_{y(1)} = \frac{E_o^+}{Z_1} (e^{-jk_1 z} - \Gamma e^{jk_1 z}) \tag{A-5}$$

The field in medium (2) contains only a transmitted $+Z$ traveling wave which can be expressed as

$$E_{x(2)} = E_o^+ T e^{-jk_2 z} \quad . \tag{A-6}$$

$$H_{y(2)} = \frac{E_o^+}{Z_2} T e^{-jk_2 z} \tag{A-7}$$

The tangential electric and magnetic field must be continuous across the interface, therefore, the impedance at the interface can be expressed as

$$Z_z \Big|_{z=0} = \frac{E_{x(1)}}{H_{y(1)}} \Bigg|_{z=0} = \frac{E_{x(2)}}{H_{y(2)}} \Bigg|_{z=0} \tag{A-8}$$

The substitution of (A-4, 5, 6, 7) into (A-8) yield the following

$$Z_1 \frac{(1 + \Gamma)}{(1 - \Gamma)} = Z_2$$

or

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{A-9}$$

This final result holds in the transmission line case, the waveguide case and the free space case. The only difference is that Z_1 and Z_2 denote the characteristic impedances of the transmission lines and the waveguides, and the intrinsic wave impedances in free space, respectively.

APPENDIX B

SOLUTION FOR INTRINSIC WAVE IMPEDANCE USING MAXWELL'S EQUATIONS

Maxwell's equations with steady state, sinusoidal time dependence can be expressed as [10]

$$\nabla \times \vec{E} = -j\omega\vec{B} \tag{B-1}$$

$$\nabla \times \vec{H} = j\omega\vec{D} + \vec{J} \tag{B-2}$$

$$\nabla \cdot \vec{D} = \rho \tag{B-3}$$

$$\nabla \cdot \vec{B} = 0 \tag{B-4}$$

The intrinsic wave impedance is determined from equations (B-1) and (B-2), which can also be expressed as,

$$\nabla \times \vec{E} = -j\omega\hat{\mu}\vec{H} \tag{B-5}$$

$$\nabla \times \vec{H} = j\omega\hat{\varepsilon}\vec{E} + \sigma\,\vec{E} = (j\omega\varepsilon + \sigma)\vec{E} \tag{B-6}$$

where $\hat{\mu}$ and $\hat{\varepsilon}$ are the complex permeability and permitvity, respectively, Equations (B-5) and (B-6) can be simplified by making the following substitutions

$$\hat{z} = j\omega\hat{\mu} \tag{B-7}$$

$$\hat{y} = j\omega\hat{\varepsilon} + \sigma \tag{B-8}$$

which yield

$$\nabla \times \vec{E} = - \hat{z}\vec{H} \tag{B-9}$$

$$\nabla \times \vec{H} = \hat{y}\vec{E} \tag{B-10}$$

It can be shown that equation (B-9) can be written (for homogeneous, isotropic media and rectangular coordinates) in the following form, namely

$$\nabla^2 \vec{E} + k^2\vec{E} = 0 \tag{B-11}$$

where $k^2 = - \hat{z}\hat{y}$ (B-12)

If a wave travels in the +z direction and the electric field is linearly polarized in the +z direction, the vector wave equation (B-11) reduces to the following one-dimensional Helmholtz equation

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0 \tag{B-13}$$

which has a solution

$$E_x = E_o e^{-jkz} \tag{B-14}$$

The associated magnetic field can be found by using equation (B-19), namely

$$-\vec{u}_y \, jk \, E_o \, e^{-jkz} = -\hat{z}\vec{H}$$

which reduces to the following

$$H_y = \frac{jk}{\hat{z}} E_x \qquad \text{(B-15)}$$

Ratios of components of E to components of H have the dimensions of impedance, therefore equation (B-15) can be rearranged to yield

$$\eta = \frac{E_x}{H_y} = \frac{\hat{z}}{jk} \qquad \text{(B-16)}$$

If equations (B-7), (B-8) and (B-12) are substituted into equation (B-16) η becomes

$$\eta = \sqrt{\frac{\omega \hat{\mu}}{\omega \hat{\varepsilon} - j\sigma}} \qquad \text{(B-17)}$$

For isotropic media, $\hat{\mu}$ and $\hat{\varepsilon}$ can be spearated into its real and imaginary parts, namely, $\hat{\mu} = \mu' - j\mu''$ and $\hat{\varepsilon} = \varepsilon' - j\varepsilon_r''$, and equation (B-17) is expressed in the final form

$$\eta = \sqrt{\frac{\omega(\mu' - j\mu'')}{\omega(\varepsilon' - j\varepsilon'') - j\sigma}} \qquad \text{(B-18)}$$

APPENDIX C

ATTENUATION OF MICROWAVE ENERGY BY WATER

The attenuation constant for water can be found from the following expression [11]

$$\alpha_D = \frac{830\ \varepsilon_r''}{\lambda} \left(\frac{\lambda_g}{\lambda} \right) \text{ db/ft} \tag{C-1}$$

where

$$\frac{\lambda_g}{\lambda} = \frac{1}{\sqrt{\varepsilon_r' - (\lambda/\lambda_c)^2}} \times \frac{1}{\sqrt{\frac{1}{2} + \frac{1}{2}\left[\frac{\varepsilon_r''^2}{[\varepsilon_r' - (\lambda/\lambda_c)^2]^2} + 1\right]}}$$

and λ is the free space wavelength

For water at 25° C and $f = 10^{10}$Hz $\varepsilon_r' = 55$, $\varepsilon_r'' = 30$

$$\frac{\lambda_g}{\lambda} - 0.148$$

Therefore using (C-1) α_D becomes

$\alpha_D = 123$ db/ft

The water was maintained in the waveguide, throughout the experiment, at an approximate level of 0.15 meters. Part of the electromagnetic energy that is incident on the water surface (0.8) is reflected, and the rest (0.2) is transmitted. Since the transmitted wave must travel at least 0.3 m in the water before its egress from the water to the air, the attenuation due to the water is approximately 123 db. Therefore it is seen that secondary reflections are of no consequence.

APPENDIX D

SAMPLE CALCULATION OF WAVEGUIDE LENGTH
USING MULTIPLE FREQUENCY CONCEPT WITH SLOTTED LINE

For a primary frequency and wavelength of

$f_o = 9.000$ GHz $\qquad \lambda_{go} = 4.8625$

the secondary wavelengths can be calculated to be

$\lambda_{g1} = \frac{10}{9}\ \lambda_{go} = 5.4028$

$\lambda_{g2} = \frac{100}{99}\ \lambda_{go} = 4.9116$

The corresponding frequencies for λ_{g1} and λ_{g2} can be found by making use of the following two formulas

$$\lambda_g = \frac{\lambda_m}{\sqrt{1 - (f_c/f)^2}} \tag{D-1}$$

and

$$\lambda_g = \frac{\lambda_m}{\sqrt{1 - (\lambda_m/\lambda_c)^2}} \tag{D-2}$$

Solving for λ_m in equation (D-2), substituting this value into equation (D-1), and finally solving for the frequency, the following is obtained

$$f = \frac{f_c \lambda_c}{\lambda_g} \sqrt{1 + (\lambda_g/\lambda_c)^2} \tag{D-3}$$

Equation (D-3) was solved by using computer calculations since the frequency must be known to an accuracy of 5 digits if accurate measurements are to be obtained. The calculated frequencies and the experimental data are found in Table D-I. The use of the minima values from Table D-I and equations (16), (17) and (18) yield

$$\lambda_{g0min} = 11.93 - 9.71 = 2.22$$

$$\lambda_{g1min} = 11.93 - 10.79 + 2.7014 = 3.84$$

$$\lambda_{g2min} = 10.285 - 9.82 + 2.4558 = 2.921$$

$$n_{01} = \frac{3.84 - 2.22}{0.27014} = \frac{1.62}{0.27014} = 6$$

$$n_{02} = \frac{2.921 - 2.22}{0.0246} = \frac{0.701}{0.0246} = 28.5 = 20$$

$$n_0 = n_{01} + n_{02} = 26$$

$$D = (26)(2.4313) - 2.22 = 60.986 \text{ cm}$$

TABLE D-I

EXPERIMENTAL VALUES FOR WAVEGUIDE LENGTH

MEASUREMENTS USING A SLOTTED LINE

f - GHz λ - cm	Slotted Line Shorted min.	Waveguide Shorted min.
$f_o = 9.000$ $\lambda_{go}/2 = 2.4313$	9.71	11.93
$f_1 = 8.589$ $\lambda_{g1}/2 = 2.7014$	10.79	11.93
$f_2 = 8.958$ $\lambda_{g2}/2 = 2.4558$	9.82	10.285
$f_o = 10.000$ $\lambda_{g0}/2 = 1.9851$	9.91	10.43
$f_1 = 9.443$ $\lambda_{g1}/2 = 2.2057$	11.01	11.74
$f_2 = 9.943$ $\lambda_{g2}/2 = 2.0052$	10.01	11.19
$f_0 = 11.000$ $\lambda_{g0}/2 = 1.6970$	10.16	10.26
$f_1 = 10.304$ $\lambda_{g1}/2 = 1.8856$	9.41	10.66
$f_2 = 10.929$ $\lambda_{g2}/2 = 1.7142$	8.55	9.27

TABLE D-II

EXPERIMENTAL VALUES FOR WATER LEVEL

MEASUREMENTS USING A SLOTTED LINE

f - GHz	Short Circuit at Elbow min	Waveguide Section Terminated with water min
f_o = 9.000	9.0	10.595 B*
		10.625 T$
f_1 = 8.589	10.925	12.805
		12.835
f_2 = 8.958	9.20	11.66
		11.69
f_o = 10.000	9.825	11.465
		11.51
f_1 = 9.443	9.61	10.765
		10.835
f_2 = 9.943	10.015	10.43
		10.485
f_0 = 11.000	8.91	9.805
		9.835
f_1 = 10.304	8.94	10.58
		10.61
f_2 = 10.929	9.075	10.71
		10.73

B* - water approaching final level from bottom
T$ - water approaching final level from top

APPENDIX E

SAMPLE CALCULATION OF WAVEGUIDE LENGTH USING MULTIPLE FREQUENCY CONCEPT WITH VECTOR VOLTMETER

The vector voltmeter measures the phase between the reference and reflected modulation envelopes; therefore, the modulation wave length must be known before any calculations can be made. It can be seen from [12] that the modulation envelope travels at the group velocity, which can be expressed as

$$v_m = v\sqrt{1 - (f_c/f)^2} \tag{E-1}$$

where v_m is the velocity of the modulation envelope, v is the free space velocity, f_c is the cutoff frequency of the particular waveguide used, and f is the carrier frequency.

The wavelength of the modulation envelope can be expressed as

$$\lambda_o = \frac{v_m}{f_m} \tag{E-2}$$

where f_m is the frequency of modulation.

For a carrier frequency of 8.75 GHz, equation (E-1) yields the following result

$$v_m = 1.986 \times 10^{10} \text{ cm/sec}$$

and for a modulation frequency of 200 MHz equation (E-2) yields

$\lambda_0 = 99.3$ cm

Since a maximum distance of 380 cm must be measured, it is necessary only to choose a second wavelength, namely

$$\lambda_1 = \frac{10}{9}\lambda_0 = 110.33 \text{ cm}$$

which allows the measurement of an unambiguous distance of approximately 500 cm.

Since there is an inverse proportionality between the wavelength and the frequency, f_1 can be found, simply by making it nine tenths as large as f_0, namely

$$f_1 = \frac{9}{10} f_0 = \frac{9}{10} (200 \text{ MHz}) = 180 \text{ MHz}$$

A sample calculation is made using the data in Table E-1 for a distance of 270.3 cm and a fundamental modulation frequency of 200 MHz. By using equation (26), n_0 is determined quite readily

$$n_o = \frac{197 \ (99.3)}{0.1 \ (110.3) \ 360} = 4.92 = 5$$

Due to the lack of accuracy in the phase measurement ($\pm$ 1.5°), n_0 will not be an integer; however, n_0 must be an integer, therefore, it is necessary to simply choose the integer which is closest to the calculated value, as above.

The distance can be calculated by using n_0 in equation (20), namely,

$$D = 49.65\ (5 + \frac{165}{360})$$

$$D = 270.7$$

If a much longer section of waveguide had been used, it would have been necessary to, also, use ψ_2 and possibly ψ_3 to determine n_0.

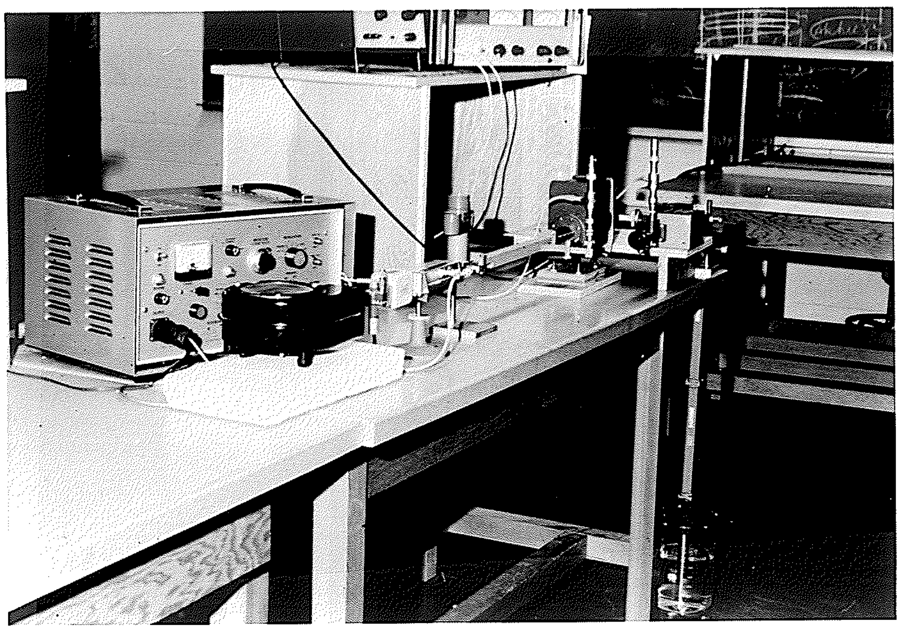
TABLE E-I

EXPERIMENTAL DATA FROM PHASE MEASUREMENTS

AT VARIOUS MODULATION FREQUENCIES

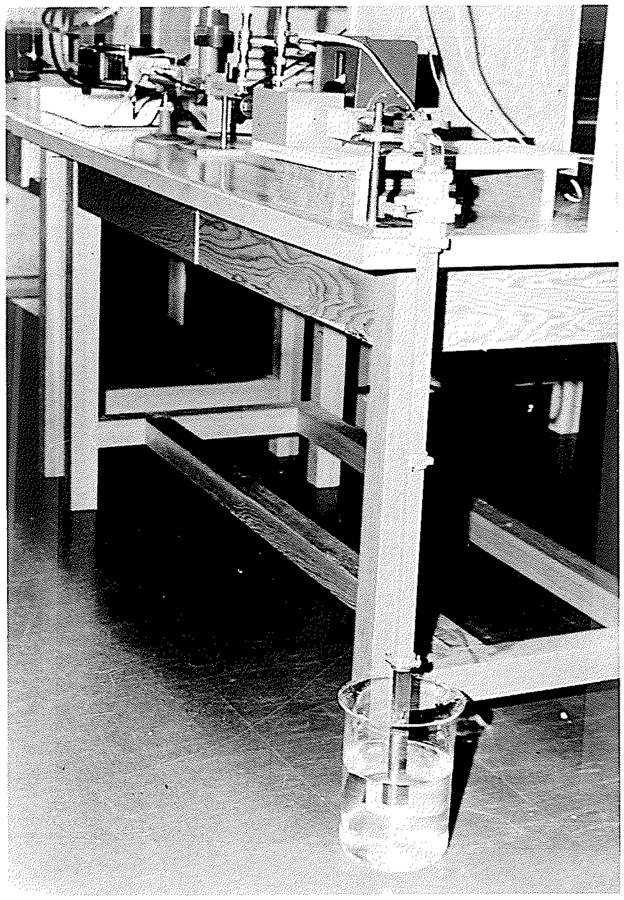
Mech. Meas. of distance cm.	30 MHz Modulation	100 MHz Modulation	200 MHz Modulation	
	ψ_o-degrees	ψ_o	ψ_o	ψ_1
35.4	38	126	259.4	228
65.9	73	237	122	71
111.4	121.7	44	90	8
163.8	178.5	173	114	349
270.3	295	263	165	328
381.3	57	302	253	332

PHOTOGRAPHS



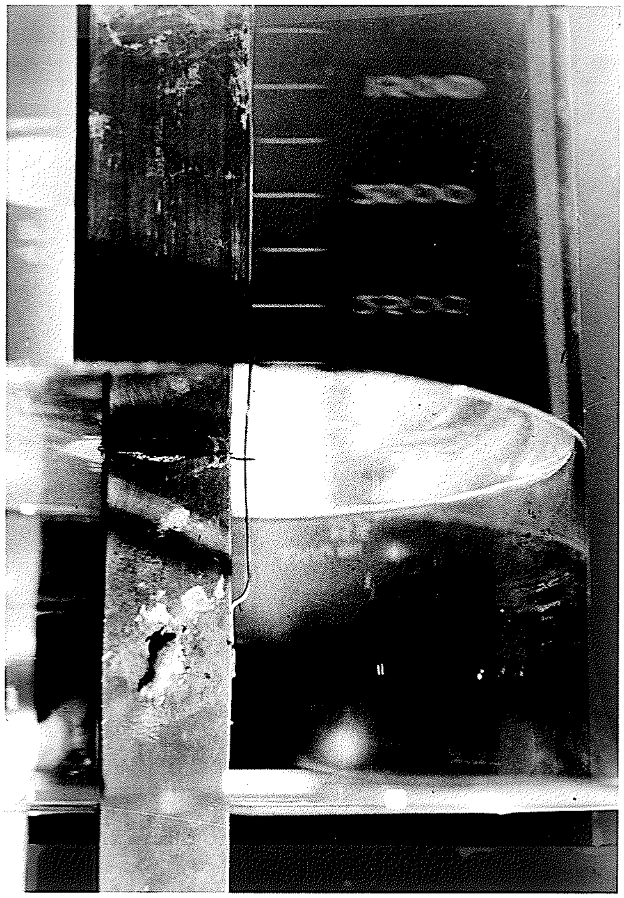
PHOTOGRAPH 1

SET-UP USING A HP VECTOR VOLTMETER FOR DETERMINING WATER LEVEL



PHOTOGRAPH 2

CLOSE-UP VIEW OF REFERENCE WAVEGUIDE IMMERSED IN WATER



PHOTOGRAPH 3

CLOSE-UP VIEW, SHOWING USE OF A NEEDLE TO SET WATER LEVEL AT A CERTAIN WAVEGUIDE REFERENCE POINT