# The University of Manitoba 

Optimal Operation of a Flood Control System
by

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# A Thesis <br> Submitted to the Faculty of Graduate Studies in Partial Fulfilment of the Requirements for the Degree of Master of Science 

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> MASTER OF SCIENCE

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Monique
Page
Acknowledgements ..... i
List of Figures ..... vi
1.0 INTRODUCTION ..... 2
1.1 A Flood Control System ..... 2
1.2 Simulation Analysis ..... 3
1.3 Optimization Analysis ..... 4
1.4 Optimization-Simulation Comparison ..... 5
1.5 The Optimization Model Selected in this Thesis ..... 6
2.0 THE ASSINIBOINE RIVER SYSTEM ..... 10
2.1 History ..... 10
2.2 Flood Control Investigations ..... 10
2.3 Shellmouth Reservoir ..... 12
2.4 Assiniboine River Diversion ..... 13
3.0 THE GENERAL LINEAR PROGRAMMING MODEL FOR A FLOOD CONTROL SYSTEM ..... 22
3.1 The Linear Programming Technique ..... 22
3.2 The Model Description ..... 25
3.3 Linear Programming Model Constraints ..... 26
3.3.1 Physical Constraints ..... 27
3.3.1.1 Storage Constraints ..... 27
3.3.1.2 Reservoir Mass Balance Constraint ..... 28
Page
3.3.1.3 Reservoir Release Constraint ..... 28
3.3.1.4 Channel Continuity of Storage Constraint ..... 29
3.3.1.5 Riparian Constraint ..... 29
3.3.1.6 Diversion Capacity Constraint ..... 30
3.3.1.7 Upper Limit on Diversion Flow Values ..... 30
3.3.2 Computational Constraints ..... 31
3.3.2.1 Peak Discharge Constraint ..... 31
3.3.2.2 Non-Negativity Constraint ..... 32
3.4 Objective Function ..... 32
3.5 Time Dependency of Objective Function at Damage Centres ..... 33
3.6 Nonlinearity ..... 35
3.7 Size of the Linear Programming Problem ..... 38
3.7.1 Elimination of Variable $Q_{i}^{t}$ ..... 38
3.7.2 Elimination of Fixed Valued Variables ..... 39
4.0 THE LINEAR PROGRAMMING MODEL APPLIED TO THE ASSINIBOINE RIVER SYSTEM ..... 46
4.1 The Assiniboine River Model ..... 46
4.2 Study Operational Rules ..... 47
4.3 Flood Damage Functions ..... 48
4.4 Specific Linear Progranming Model for the Assiniboine River System ..... 53
4.4.1 Physical Constraints ..... 53
4.4.1.1 Storage Constraints ..... 53
4.4.1.2 Reservoir Mass Balance Equation ..... 53
4.4.1.3 Reservoir Release Constraint ..... 54
Page
4.4.1.4 Riparian Constraint ..... 54
4.4.1.5 Diversion Capacity Constraint ..... 55
4.4.1.6 Upper Limit on Diversion Flow Values ..... 55
4.4.2 Computational Constraints ..... 56
4.4.2.1 Peak Discharge Constraint ..... 56
4.5 Sumnary of Problem Constraints ..... 57
5.0 COMPUTATIONAL TECHNIQUES ..... 77
5.1 The Computer Model ..... 77
5.2 Matrix Generator ..... 77
5.3 The Linear Programming Solution Algorithm ..... 79
5.4 The Report Writer ..... 79
6.0 RESULTS ..... 82
6.1 The Optimization Mode1 ..... 82
6.2 Discussion of Results ..... 84
6.3 Model Run Cost ..... 86
6.4 Sunmary Conclusions ..... 87
APPENDIX A ..... 100
APPENDIX B ..... 146
BIBLIOGRAPHY ..... 158

## Figure

Page

II-1 Assiniboine Riyer Drainage Basin 17
II -2 Shellmouth Reservoir, Location 18
II-3 Assiniboine River Diversion, Location 19
II-4 Assiniboine River Diversion, Layout Details 20
III-I Sample Linear Programming Problem 41
III-2 Hypothetical River Basin System 42
III-3 Definition of Time periods used in Analysis 43
III-4 Separable Progranming Example Figure 44
IV-1 Assiniboine River Drainage Basin 63
IV-2 Water Survey of Canada Streamflow Gauging Locations
in the Study Area 64
$\begin{array}{ll}\text { IV-3 Discharge Damage Relationship - Assiniboine River at } \\ & \text { Russell }\end{array}$
IV-4 Discharge Damage Relationship - Assiniboine River at Miniota

IV-5 Discharge Damage Relationship - Assiniboine River at
Brandon ..... 67

IV-6 Discharge Damage Relationship - Assiniboine River at
Portage la Prairie ..... 68
IV-7 Discharge Damage Relationship - Assiniboine River Diversion ..... 69
IV-8 Discharge Damage Relationship - Assiniboine River at Headingley ..... 70

## List of Figures cont.

Page
IV-9 Discharge Damage Relationship - Assiniboine River at Winnipeg ..... 71
IV-10 Elevation-Discharge Relationship Shellmouth Reservoir ..... 72
IV-11 Elevation-Storage Relationship Shellmouth Reservoir ..... 73
IV-12 Outflow-Storage Relationship Shellmouth Reservoir ..... 74
IV-13 Schematic Representation of Assiniboine River Basin ..... 75
VI-1 Natural and Optima1 Flow Values, Russell ..... 88
VI-2 Natural and Optimal Flow Values, Miniota ..... 89
VI-3 Natural and Optimal Flow Values, Brandon ..... 90
VI-4 Natural and Optimal Flow Values, Portage 1a Prairie ..... 91
VI-5 Natural and Optimal Flow Values, Headingley ..... 92
VI-6 Natural and Optimal Flow Values, Wimnipeg ..... 93
VI-7 Optima1 Shellmouth Reservoir Operating Schedule ..... 94
VI-8 Optimal Assiniboine River Diversion Operating Schedule ..... 95
VI-9 Output from Report Writer ..... 96
VI-10 Output from Report Writer ..... 97
VI-11 Output from Report Writer ..... 98

INTRODUCTION

OPTIMAL OPERATION OF A FLOOD CONTROL SYSTEM

### 1.0 INTRODUCTION

### 1.1 A Flood Control System

Two kinds of flood control works may be actively operated in order to alleviate flood damage, reservoirs and diversions. Reservoirs reduce flood damage by storing flood waters and releasing them at a later date so as to reduce the peak of the flood wave and lengthen its duration. Diversions simply divert flood waters away to an area less prone to flood damage. An operable flood control system as discussed in this thesis is composed of reservoirs and diversions in addition to passive flood control works such as dykes.

The object of operation of any flood control system is to minimize flood damage. This requirement defines the optimal operating schedule for each of the system components. The degree of complexity of the optimal operation of a flood control system is determined by the number of flood control reservoirs and/or diversions which comprise the system for each flood control component requires its own operating schedule. Since the operating schedules of the components of the system may be highly inter-related it may be appreciated that the difficulty in arriving at optimal operating schedules increases very rapidly as the number of components of the flood control system increases.

The optimal operating procedure is based on anticipated flows. The uncertainty in forecasting flows therefore adds a degree of uncertainty to the optimal operating procedure. It need not add to the complexity of the analysis if the uncertainty itself is not a factor in the objective of the system operation.

In this thesis all flow values have been treated as being deterministic and no allowance for stochastic variation has been included. Stochastic variation of flow values could be addressed by means of new estimates of flow values which would then be subject to another analysis. In an operational flood control context this might take the form of a high, best and low estimate of system in flows with each estimate requiring a separate analysis. In this manner any stochastic variation in flow estimates can be partially addressed within the deterministic framework of the model constructed in a manner acceptable for operational flood control purposes.

### 1.2 Simulation Analysis

Simulation analysis is a commonly accepted method of arriving at an operating schedule for a flood control system. In a simulation analysis, the river system is modelled so that movement in time and space of river flows throughout the river basin may be simulated. This is generally done by mathematical models of the river basin rather than by analog models and is carried out on an electronic computer.

Forecast inflows to the river basin are input to the simulation model of the system and an operating schedule is postulated for each system component. The output from the simulation model consists of discharges and water levels at various locations of interest in the river basin. Each run results in a calculated value of flood damage. Operating schedules for each system component are changed between runs so as to improve the performance and the simulation analysis is repeated. The results of individual simulation runs are compared and revised operating schedules are postulated until a best operating schedule for the system
components is arrived at. No systematic search for optimal operating schedules of system components is necessarily implied in a simulation analysis although it can be incorporated therein.

The main difficulty with this technique lies in the revision of the operating schedules for individual system components so as to improve the performance. If the variables are changed one by one by small amounts, the amount of work required to achieve the optimum becomes prohibitive. If larger steps are taken or several variables changed at once, the analysis may well miss the optimum altogether. Therefore, simulation analysis works well to refine initial postulates of system component operating schedules. If the initial operation postulates are far from optimal simulation may give problems.

When the flood control system is not overly complex the limitations of simulation analysis are not severe. However, as the complexity of the system increases it may prove very difficult, if not impossible, to find the optimal operating schedule through simulation analysis.

### 1.3 Optimization Analysis

Optimization techniques differ from simulation in that a systematic search for the optimum operating schedule is incorporated in the technique. Theoretically, this would solve the problem.

However, in practice it is often found that the large number of variables involved makes an optimization technique much too laborious even with the aid of large electronic computers. It is possible in many cases to set the target in optimization somewhat lower and to aim for a solution that is not too far away from the optimum. The so called fine tuning can then be accomplished by means of simulation analysis.

In optimization, all the components of the flood control system must be analyzed simultaneously over their entire range of possible operating schedules. In addition, the technique must realistically reflect any operating schedule inter-relationships between individual system components. The time interval, however, can be increased substantially to reduce the number of variables.

An optimization technique known as linear programming was employed in this thesis to provide initial operating schedule postulates. It was decided to use a time interval of seven days in the optimization technique rather than the daily time interval generally used in flood control simulation analysis. This is acceptable when one keeps in mind that the primary function of the optimization technique is to provide initial system component operating schedule postulates, that are near optimal, for input to a simulation analysis which will then fine tune these postulates. The use of the seven day time period greatly reduces the computational burden of the optimization process.

### 1.4 Optimization-Simulation Comparison

A comparison of the optimization and simulation processes is in order. Optimization tends to look at the overall picture with respect to operating schedules of individual flood control system components. In an optimization analysis the entire physically feasible range of operating schedules for operating system components is analyzed. Generally, the flood control system is not modelled in great detail with respect to either the time step employed in the analysis or the degree of exact modelling of the system under consideration.

Simulation analysis, on the other hand, tends to look at a very limited picture with respect to operating schedules of the individual flood
control system components. In simulation analysis a narrow range of operating schedules for individual system components is analysed. Generally, the system is modelled in considerable detail with respect to the time step employed and the degree of exact modelling of the system.

It may be appreciated that a trade off in terms of computational burden and exactness of results is involved in simulation and optimization analysis.

A point may be made that the optimization and simulation analysis processes are always carried out together in the determining of optimal operating schedule for flood control components. For the case of less complex flood control systems the optimization analysis takes the form of intuitive reasoning by the person responsible for the analysis. For more complex flood control systems, a more formal optimization process, together with the all important intuitive reasoning of the person responsible for the analysis, is felt to be more appropriate.
1.5 The Optimization Model Selected in this Thesis

A linear programming technique was selected as the optimization model in this thesis. It is intended to optimize operation of flood control works so as to minimize damage caused by flooding. Flood damage is represented by flooded area in this context. Implicit in this statement is the assumption that all flooded areas in the model have the same dollar value per acre with regard to flood damage for a given time of year. This assumption may be relaxed quite easily should a particular river system require different dollar per acre values of flood damage. The linear programming model constructed herein produces optimal mean seven day operating schedules for each of a flood control system's control
works on the basis of minimizing total flooded area as a function of peak mean seven day discharges at selected damage centres throughout the flood control system. Peak mean seven day discharges at each damage centre in the analysis, together with their corresponding damage coefficients, form the objective function in the analysis. The total value of this objective function is to be minimized.

No limitation as to the number of flood control reservoirs or diversions in the flood control system is implied by the model although in reality the computational capacity of the electronic computer employed in the analysis or the cost factor of the analysis itself would be the limiting factor as to the size of the system which could be optimized and/or the detail to which the system could be modelled in the optimization.

The optimization model is formulated to address river basins in which the predominant damages due to flooding are agricultural in nature. In this respect the phenomenon that agricultural flood damages are time dependent is addressed in the model. This feature is an extension rather than a restriction to the use of the model as non-agricultural damages, such as flooding of buildings or loss of bridges are non time dependent and may be addressed directly by the model constructed.

The general optimization model constructed in this thesis was applied to the Assiniboine River in Manitoba to assess its applicability. It should be noted that the flood control system in existence on the Assiniboine River system may be treated as one in which acceptable postulates for a simulation analysis may be obtained by intuitive reasoning rather than requiring the use of an optimization model. However, it should also be noted that it is this very non-complexity of the flood control system in terms of the number of operable system components that allows a
reasonable check of the applicability of the optimization model as compared to intuitive reasoning.

The linear programming optimization model constructed in this thesis effectively reflects the inter-relationships of the flood control system on the Assiniboine River in Manitoba and produces acceptable starting postulates for a simulation analysis.
2.0

THE ASSINIBOINE RIVER SYSTEM

### 2.0 THE ASSINIBOINE RIVER SYSTEM

### 2.1 History

Since the beginning of the settlement of the west the Assiniboine River has had a record of destructive floods. On several occasions during the $1800^{\prime} \mathrm{s}$ floods did occur but much of this information is very sketchy. Detailed records have been kept since 1913. In the latter period there were seventeen years in which major flooding occurred.

The Assiniboine River flows in a valley from Kamsack to Portage la Prairie. Flooding in this reach would generally affect the river valley bottem land only. Downstream of Portage la Prairie the Assiniboine enters the flat prairie which once comprised the bottem of Lake Agassis. In this reach the adjacent land falls away from the river banks which have been raised by silt deposits. Once the banks are overtopped the water cannot return to the river channel when the flood peak has passed. The flood waters eventually drain via old creeks and channels to Lake Manitoba to the north and into the La Salle River to the south. However, drainage is generally poor and flood waters may remain on the land for several weeks.

### 2.2 Flood Control Investigations

Following the 1950 flood, measures were studied for both the Assiniboine River and the Red River. In 1958 the Royal Commission on Flood Cost Benefit recommended specific flood control works.

With respect to the Assiniboine River, the Royal Conmission investigated storage as well as diversion proposals. For every principal tributary investigations were conducted to assess the feasibility of
flood control storage; each analysis, however, showed that tributary contribution was insufficient to warrant the construction of a major flood control dam.

It was finally decided that a substantial storage reservoir on the main stem of the Assiniboine was required in order to achieve the desired level of flood control. Three sites for a major dam were thoroughly investigated; the first at St. Lazare, the second west of Russell and the third below the confluence of the Shell and Assiniboine Rivers. All three sites were found to be economically feasible. The Shellmouth site was finally chosen because of more satisfactory foundation conditions, a smaller acreage of agricultural land in the reservoir area and a better source of construction materials.

Extensive investigations to divert flood waters into Lake Manitoba were conducted. Several different locations and diversion capacities were examined. Several alternative diversion routes were found which is not surprising since areal photographs clearly show that the Assiniboine at one time flowed north to Lake Manitoba. It followed several different routes to the lake before it broke through to its present course eastward to the Red River.

The diversion route finally selected has its beginning two miles west of Portage la Prairie. From there the channel runs almost due north to Lake Manitoba.

The length of river between Portage la Prairie and Winnipeg has been dyked.

The drainage basin of the Assiniboine River in Manitoba together with the location of the above noted flood control works are shown on Figure II-1. Both the Shellmouth Reservoir and the Portage Diversion will be discussed in more detail below.

### 2.3 Shellmouth Reservoir

The Shellmouth Reservoir, on the Assiniboine River, is located approximately two miles north and two miles east of the Village of Shellmouth in an area where the Assiniboine River Valley is wide, with high banks.

The earth dam is 75 feet high and 4000 feet long. It is equipped with a concrete conduit to control releases from the reservoir. An uncontrolled concrete spillway passes flows in excess of the conduit capacity. The storage capacity is used for water conservation as well as flood control. The conservation pool is up to an elevation of 1391.0 , representing 165,000 acre-feet of storage, while combined conservation and flood control capacity is between elevations 1391.0 and 1402.5 resulting in a storage of 136,000 acre-feet. A further 87,000 acre-feet of flood storage capacity is available between elevation 1402.5 and 1408.5 , the elevation of the uncontrolled spillway.

The location of the dam is shown on Figure II-2.
The prime purpose of Shellmouth Reservoir is to reduce the flood damage along the Assiniboine River and in Wimnipeg by storing the majority of the flood runoff that originates upstream from the Shellmouth Reservoir in the Assiniboine River Basin. A secondary benefit that can be achieved by operation of the reservoir is that of augmenting low.flows on the Assiniboine River during dry periods.

The Water Resources Division of the Department of Mines, Resources and Environmental Management of the Government of Manitoba has determined the following principles of operation.

In order that the full capacity of the reservoir for flood control purposes be available in the spring the reservoir is lowered from a summer level of 1402.5, to 1391.0 over the period November 1 to March 31.

The lowering is carried out at as uniform rate of release as is possible based on an early winter forecast of the inflow to the reservoir during the winter period and recognizing the amount of water in storage that must be released. In years with evidence of a high spring runoff the reservoir may be drawn down below 1391.0 by March 31 .

In order that storage spece be available in the reservoir subsequent to the spring runoff to reduce flood damage from summer floods on the Assiniboine River, the water level in the reservoir is lowered to 1402.50 immediately after the spring runoff at a rate which does not cause downstream flooding. The reservoir is then maintained at 1402.50 throughout the summer and early fall until November 1 when releases begin to lower the reservoir for spring flood control as noted earlier. If drought conditions plus high water supply demands prevail, these elevations may be impossible to obtain.

This system of allocating the available storage to the various purposes makes it possible to provide sufficient storage to reduce peak flows along the Assiniboine River downstream of the Shellmouth Reservoir and at the same time make it possible to maintain a flow of 250 c.f.s. in the Assiniboine River at Brandon compared to the recorded minimum of 7 c.f.s.

### 2.4 Assiniboine River Diversion

The Assiniboine River Diversion channel begins two miles west of Portage la Prairie and runs almost due north to Lake Manitoba. It is 18 miles long and is designed to carry up to 25,000 c.f.s. away from the Assiniboine River. The removal of water through the Diversion gives flood protection to the cities of Portage la Prairie and Winnipeg and the areas between them.

An earthfill dam across the Assiniboine River with a concrete spillway control structure creates a small reservoir with a storage capacity of 14,600 acre-feet. North and west of the dam at the upper end of the diversion channel an inlet control structure regulates flow to Lake Manitoba. The diversion channel has three drop structure along its route so as to keep water velocities below those which would cause erosion.

For economic reasons approximately the last 3 miles of the diversion, which are located in the Delta Marsh, have been designed to carry only 15,000 c.f.s. The excess flow, which at full design discharge could be up to 10,000 c.f.s. is spilled over into the west Delta Marsh. A section of the west dyke of the diversion channel in the vicinity of Cram Creek was designed and constructed at a lower elevation than the remaining portion of the dykes through the marsh. This particular reach of lower designed dyking concentrates the overflow and reduces the probability of an extended failure along the dyke.

The location of the diversion and details of its layout are shown on Figure II-3 and Figure II-4.

The purpose of the Assiniboine River Diversion is to provide flood protection to Winnipeg and the area from Portage la Prairie to Winnipeg. The diversion will accommodate and regulate the Assiniboine River flow up to a maximum of 45,000 c.f.s. At this design flood flow, 25,000 c.f.s. is diverted into Lake Manitoba, while the remaining 20,000 c.f.s. passes downstream into the Assiniboine River. At Assiniboine River flows of greater than 45,000 c.f.s. flood damage will occur either along the Diversion or the Assiniboine River depending on whether the flow is diverted or allowed to flow down the Assiniboine. The following points illustrate the procedures now being followed by the Water Resources Division in the operation of the Diversion.

1) While there is ice on the Assiniboine River downstream of the Portage Diversion it is desirable to maintain flows less than 5,000 c.f.s. in the river because of the possibility of ice jams.
2) After the ice has gone from the Assiniboine River downstream of the Portage Diversion it is desirable to maintain flows less than $10,000 \mathrm{c} . \mathrm{f} . \mathrm{s}$. in the river. Flows greater than 10,000 c.f.s. are above the natural bank stage of the river and backup of local streams which outlet into the Assiniboine may occur at this level. There also may be seepage problems through dykes, leakage through gated through-dyke culverts and flooding of cultivated land between dykes.
3) The bankfull capacity of the Assiniboine River downstream of the Assiniboine River Diversion is 20,000 c.f.s.
4) The overflow section of the west dyke of the Portage Diversion which allows flows in excess of 15,000 c.f.s. to spill out into the west marsh of Delta Marsh should only be overtopped when dictated by an extreme condition on the main stem of the Assiniboine.
5) The design capacity of the Portage Diversion is 25,000 c.f.s.
6) If possible flows on the Assiniboine River downstream of the Diversion while ice is still present should only exceed 5,000 c.f.s. if the Winnipeg James Avenue stage is below 745.57. The level of Lake Manitoba should not be taken into account while there is ice on the Assiniboine River, as the period during which there is ice on the river during the sprin runoff is only a few days, and diverted flows for this short period of time would have a negligible effect on the level of Lake Manitoba.
7) For Assiniboine River inflows to the Portage Reservoir in the 25,000 to 35,000 c.f.s. range, the diverted flow may in some instances be limited to 15,000 c.f.s. to prevent overtopping of the west dyke overflow section. Thus flows in the 10,000 to 20,000 c.f.s. range would occur on the river. In this instance the James Avenue stage would be the deciding factor as to how much water should be sent down the river and how much diverted. Flows in excess of 10,000 c.f.s. in the river should only be permitted if Lake Manitoba is high and the James Avenue level is low. If both the lake and the James Avenue levels are high, presumably flows would be diverted to the lake rather than down the river.
8) The 20,000 c.f.s. limit to flows down the Assiniboine River should only be exceeded in the event of inflows greater than 45,000 c.f.s.
9) The 15,000 c.f.s. limit to flows down the Diversion should only be exceeded when,
a) there is the possibility of exceeding a flow of 20,000 c.f.s. in an ice-free Assiniboine River.
b) there is the possibility of exceeding a flow of 5,000 c.f.s. in an ice-bound Assiniboine River.
c) the stage of James Avenue is at such a level that flows down the Assiniboine River must be reduced. This condition may be reached at all inflows as conditions at James Avenue may require that the entire inflow or a very large portion of it be diverted.


Figure II-1


Figure $\Pi$-2


Figure II-3
ASSINIBOINE RIVER DIVERSION LOCATION


Figure II-4
ASSINIBOINE RIVER DIVERSION INLET CONTROL STRUCTURE

## 3.0

THE GENERAL LINEAR PROGRAMMING
MODEL FOR A FLOOD CONTROL SYSTEM

### 3.0 THE GENERAL LINEAR PROGRAMMING MODEL FOR A FLOOD CONTROL SYSTEM

### 3.1 The Linear Programming Technique

In the development of an optimization model to determine optimal operating schedules for flood control system components it was decided in this thesis to employ a linear programming analysis. Linear programming has decided advantages in that if the problem at hand can be adapted to the linear programming technique an optimal solution can be obtained very quickly at a reasonable cost. Most non-linear optimization techniques on the other hand, such as a pattern or gradient search may arrive at a local optimum solution which could be quite different from the true global or overall optimal solution to the problem at hand. Many times the cost of the non-1inear technique may be prohibitive. This problem does not exist with the linear programming technique.

Linear programming is a numerical technique that generates a solution to the optimization problem at hand by means of a iterative procedure. The linear programing technique involved in optimization manipulates all the variables in the analysis simultaneously on each iteration subject to the constraints of the problem in the quest for an optimum solution.

A linear programming problem is one in which a linear function is the criterion to be minimized or maximized. This linear criterion to be minimized or maximized is subject to constraints that are also linear functions. A combination of variables denoted in general by $X$ is said to be linear if the variables can be assembled in the form:

$$
C_{1} x_{1}+C_{2} x_{2}+\cdots C_{n} x_{n}
$$

where the C's are constants. For example the function:

$$
4 X_{1}+3 X_{2}+5 X_{3}+2
$$

is linear in the variable $X_{1}, X_{2}, X_{3}$, whereas the function:

$$
2\left(X_{1}\right)+X_{1} X_{2}+3 \exp \left(X_{3}\right)
$$

is non-linear in the same variables. The linear programming technique solves equations of the following form:

$$
\mathrm{U}=\mathrm{C}_{1} \mathrm{X}_{1}+\mathrm{C}_{2} \mathrm{X}_{2}+\ldots+\mathrm{C}_{\mathrm{n}} \mathrm{X}_{\mathrm{n}}=\text { minimum (or maximum) }
$$

Subject to:

$$
\begin{aligned}
& \mathrm{A}_{11} \mathrm{X}_{1}+\mathrm{A}_{12} \mathrm{X}_{2}+\ldots+\mathrm{A}_{1 n} \mathrm{X}_{\mathrm{n}} \stackrel{\rightharpoonup}{<} \mathrm{b}_{1} \\
& \mathrm{~A}_{21} \mathrm{X}_{1}+\mathrm{A}_{22} \mathrm{X}_{2}+\ldots+\mathrm{A}_{2 n} \mathrm{X}_{\mathrm{n}} \stackrel{\geq}{<} \mathrm{b}_{2}
\end{aligned}
$$

$$
A_{m 1} X_{1}+A_{m 2} X_{2}+\ldots+A_{m n} X_{n} \geq b_{m}
$$

The above equations may be written in a much more concise mathematical form.

$$
\underset{(\text { or Maximize })}{\operatorname{Minimize}} \quad U=\sum_{j=1}^{n} C_{j} X_{j}
$$

Subject to:

$$
\begin{gathered}
\sum_{j=1}^{n} A_{i j} x_{j} \underset{<}{>} b_{i} ; \text { for } i=1, m \\
x_{j} \geq 0 \quad ; \text { for } j=1, n
\end{gathered}
$$

The a's, b's and c's in the above equations are constants and the x's are variables whose values are sought. A simple linear programing problem will serve to illustrate the technique.

Let us consider a graphical solution of a simple linear programming problem. The problem may be formulated as follows:

Objective Function: $\quad U=4 X_{1}+X_{2}$
Subject to: $\quad 2 X_{1}+X_{2} \geq 2$
$4 x_{1}-3 x_{2} \geq-3$
$2 X_{1}+3 X_{2} \leq 21$
$4 X_{1}-X_{2} \leq 16$
$X_{1} \geq 0$
$x_{2} \geq 0$
It is desired to maximize the objective function in this problem. This problem is shown graphically on Figure III-1.

It is to be noted that all lines on Figure III-1 are straight and the optimization surface is a plane. It may readily be deduced that the limit equations are satisfied by any solution inside the shaded area. If it proves difficult to determine the direction of feasibility graphically for a limit equation this may be determined quickly by taking any point, usually the origin, and evaluating the function at hand. This inmediately tells where the origin is on the feasible or infeasible side of the line.

Let us rewrite the objective function $U$ in the form:

$$
X_{2}=U-4 X_{1}
$$

The dashed lines on Figure III-1 represent this equation for various values of the objective function $U$. It is apparent that point $D$ is the optimum solution with objective function $U$ having a value of
3.43. Variable $X_{1}$ having the value 4.93 and variable $X_{2}$ having the value of 3.71 .

Essentially the procedure in linear programming is to start in any one corner of the shaded polygon shown and jump to an adjacent corner having a higher value of the objective function $U$. This is continued until there is no adjacent corner having a higher value. The general problem has more than two variables and cannot be thought of in geometrical terms. However, the mathematical method is essentially similar. Various computational algorithms have been developed to solve the linear programming problem.

The linear programming solution algorithm employed in this thesis is based upon the mutual primal dual simplex method of Michael L. Polinsky and Ralph E. Gomery. The algorithm in question was supplied through the courtesy of Charles Howard and Associates, Professional Engineers, Winnipeg, Manitoba. It is recognized that different algorithms are available for the solutions of linear programming problems. However, the algorithm used in this thesis proved extremely efficient. Moreso, valuable assistance was available from Charles Howard and Associates regarding the use of the algorithm in particular and the linear programming technique in general.

### 3.2 The Model Description

The linear programming model is comprised of two components. The first of these is a linear objective function and the second is a linear system of constraints. These two components are as described in the preceding section.

Some general conments regarding the linear progranming model developed
in this thesis are in order. It is assumed in the formulation of the model that all reservoirs and/or diversions have gated or controllable outflows and that decisions regarding the size and location of individual flood control system components have already been made. It is not intended, in its present form, to size reservoirs and/or diversions in a planning study. In other words, the linear programming model constructed herein is an operative model rather than a planning model. The optimal operating schedules for a flood control system proposed in this thesis are based upon the object of minimizing the total flooded area, with respect to the system as a whole, based on the peak mean seven day discharges at selected damage centres within the system. The use of a seven day time period in the formulation of the model allows adequate starting postulates for a simulation analysis and reduces the computational burden of the linear programming solution.

For the purpose of illustrating the linear programming model developed in this thesis a hypothetical river system is shown on Figure III-2. While this system, in general, is much simpler than an actual flood control system, it serves to illustrate the linear programing model for the optimal operation of a flood control system as developed in this thesis.

### 3.3 Linear Programming Model Constraints

The constraints, which together with the objective function comprise the linear programming model, may be classified in two types. These are physical constraints and computational constraints. Physical constraints force the linear programming model to follow the physical realities of the flood control system being modelled. These would include upper storage limits for reservoirs in the flood control system, diversion
capacity of diversions within the flood control system, and minimum flow requirements within the system. Computational constraints adapt the mathematical linear programming technique to the determining of optimal operating schedules for individual components of the flood control system. An example of this type of constraint is that which is required to determine peak discharges at individual damage centers in a form suitable for the linear programning technique.

### 3.3.1 Physical Constraints

### 3.3.1.1 Storage Constraints

Each reservoir in the flood control system is limited to its maximum storage capacity. That is, at the start of any time period in the analysis the quantity of water stored in a given reservoir must be equal to or less than the maximum available flood storage volume of that reservoir. This may be represented mathematically as

$$
S_{i}^{t} \leq V_{i}
$$

where $S_{i}$ is the volume of water stored in reservoir $i$ at the start of a given time period $t$, and $V$ is the maximum available flood storage volume at the reservoir in question. It should be noted that the maximum available storage for a given reservoir may be set in the model at any preselected magnitude less than or equal to the absolute physical reservoir capacity in order to address, for example, recreation use constraints on the reservoir in question.

### 3.3.1.2 Reservoir Mass Balance Constraint

This constraint is based on the principle of continuity and states that during any time period the inflow volume minus the outflow volume for a given reservoir must equal the change in reservoir storage. Evaporation and seepage losses from reservoirs during flood periods are generally an insignificant portion of the total flow and are therefore not included in this model. Because of the seven day time period used in this study all storage terms are in units of c.f.s.-weeks. This constraint may be represented mathematically as

$$
S_{i}^{t}=S_{i}^{t-1}+I_{i}^{t-1}-0_{i}^{t-1}
$$

t-1
where I is the inflow to a given reservoir over a given time period $t-1 ; \quad{ }_{0}^{i} \quad{ }_{i}^{t-1}$ is the corresponding outflow from the reservoir in question over the same time period and $S$ and $S$ are storage volumes for a given reservoir at the beginning of the time period in question and the beginning of the preceding time period. All terms in this constraint are expressed in a consistent set of units, in this case c.f.s.-weeks. Figure III-3 shows the definition of time periods.

### 3.3.1.3 Reservoir Release Constraint

Reservoir releases are limited by the conduit discharge capacity which is a function of the reservoir storage volume. In general this is a non-linear function. The reservoir release constraint may be expressed mathematically by the equation

$$
{ }_{i} \leq f\left(S_{i}\right)
$$

where $i$ denotes Reservoir' $i$ and $t$ denotes time period $t . ~ f\left(S_{i}\right)$ is the outflow limiting function for reservoir i.

### 3.3.1.4 Channel Continuity of Storage Constraint

Inflow must equal outflow for any river channel reach over each week in the model as changes in channel storage are neglected. Channel inflows are comprised of the outflow from the immediately upstream channel or reservoir, local inflow to the reach, expressed as end of reach inflow, and as a negative term, any flow lost from the reach by diversion in the reach in question. This may be expressed mathematically as follows:

$$
Q_{i}^{t}=I_{i}^{t}+\operatorname{LOC}_{i}^{t}-\operatorname{DIVR}_{i}^{t}
$$

With reference to Figure III-2 this would be expressed as

$$
Q_{i}^{t}=0_{i}^{t}+\operatorname{LOC}_{i}^{t}-\operatorname{DIVR}_{i}^{t}
$$

t
where $Q_{i}$ is the mean seven day discharge at the end of reach $i$ over time period $t ; 0$ is the mean seven day outflow from reservoir $i$ over time period $t$, that is, the mean seven day inflow to reach $i$ over time period $t$; LOC ${ }_{i}^{t}$ is the mean seven day local inflow to reach i over time period t; DIVR is the mean seven day diversion flow diverted from reach i over time period $t$.

### 3.3.1.5 Riparian Constraint

to meet downstream riparian rights. This may be expressed mathematically as follows:

$$
0_{i}^{t} \geq C_{i}
$$

t
where 0 is the mean seven day outflow from reservoir i over time i period $t$; and $C$ is the minimum mean seven day release allowed for reservoir i.

### 3.3.1.6 Diversion Capacity Constraint

Any diversion in the flood control system will have an upper limit conceptionally similar to that specified for reservoir storage. In the case of a diversion it is the diversion capacity. This may be expressed mathematically as follows:

$$
\operatorname{Divr}_{i}^{t} \leq D_{i}
$$

t
where Divr is the mean seven day diversion flow diverted from channel reach i over time period $t$; $D$ is the absolute diversion capacity for i the diversion in channel reach i.

### 3.3.1.7 Upper Limit on Diversion Flow Values

As well as the physical diversion capacity described in the preceding section, any diversion within the flood control system must be constrained so that its flow is less than or equal to the inflow above it in the river system. That is, it is impossible for a diversion to divert more flow away from a damage centre than is available to the diversion. With
reference to Figure III-2 this may be expressed mathematically as

$$
\operatorname{Divr}_{i}^{t} \leq 0_{i}^{t}+\operatorname{LOC}_{i}^{t}
$$

or in general

$$
\operatorname{Divr}_{i}^{t} \leq I_{i}^{t}+\operatorname{LOC}_{i}^{t}
$$

where $\operatorname{Divr}_{i}^{t}$ is the mean seven day diversion flow diverted from reach $i$ over time period $t ; I_{i}^{t}$ is the mean seven day inflow to reach $i$ over time period $t$; $L O C_{i}^{t}$ is the mean seven day local inflow to reach $i$, expressed as end of reach inflow over time period $t$. All terms are in units of mean seven day c.f.s.

### 3.3.2 Computational Constraints

3.3.2.1 Peak Discharge Constraint

This constraint is a computational constraint used simply to identify the peak mean seven day discharge as the largest of all mean seven day discharges during the time periods in the analysis at each damage centre. It may be represented mathematically as

$$
Q_{i} \geq Q_{i}^{t}
$$

where $Q_{i}^{t}$ is the mean seven day discharge at the end of channel reach $i$, that is, at damage centre $i$ over the time interval $t ; Q P_{i}$ is the peak mean seven day discharge at damage centre i. It should be noted that damage centres are assumed to be located at the end of a given reach.

The peak mean seven day discharge at the respective damage centres are used in the objective function.

### 3.3.2.2 Non-negativity Constraint

This constraint is fundamental to the linear progranming technique and states that all variables in the linear programing analysis must be greater than or equal to zero. No variables may take on negative values.

### 3.4 Objective Function

The linear programing technique requires a linear relationship of the variables involved in the analysis to be minimized. This relationship is known as the objective function. In this thesis the objective function is the damage cost of flooding at selected damage centres expressed in arbitrary damage units. For simplicity these damage units are taken as weighted magnitudes of flooded areas. For the purposes of this thesis, that is the obtaining of preliminary optimal operating schedules for the flood control system components it is not necessary to express the damage in dollars. The weighted magnitudes of the flooded areas serve equally well provided the weights make the dollar values and the areas roughly proportional.

Thus the objective of the linear programming model constructed in this thesis is to obtain optimal operating schedules for the flood control system components so as to minimize the total flooded area, considering all locations being protected, given the run-off hydrographs at all significant points in the system. The flood damage to be minimized, or objective function, is assumed to be some function of only peak mean
weekly discharge. The problem therefore requires finding the minimum of a non-linear objective function subject to a large number of linear and/or non-1inear constraints.

### 3.5 Time Dependency Of Objective Function at Damage Centres

In the application of the linear programming technique to the determination of optimal operating schedules for the components of the Assiniboine River flood control system it must be recognized that a major portion of the flood prone area in the Assiniboine River Basin is agricultural in nature. Agricultural flood damages are decidedly time dependent in that flooding early in a crop year, before the crop is planted, may cause little or no reduction to the crop yield that year and therefore little or no damage. However, flooding later during the year will cause steadily increasing damage as the crop yield is reduced due to a shortened growing season until, at some point, it is too late to plant a crop at all during the year in question and harvest the crop during the same year. At this point the entire value of the crop in question is lost.

It is necessary that this time dependency of agricultural flood damages be adequately reflected in the linear programming model. It has been noted earlier that the basis of optimization in this thesis is to minimize flooded area within the river basin. By this technique all flooded area would be assumed to have the same value regardless of the time of flooding. What is required to reflect the time variability of agricultural damage is that flooded areas in the river basin may take on different values of flood damage, dependent on the time of flooding.

This may be accomplished by including in the objective function
several flood peak values for each damage centre each for a different time period.

By means of introducing this family of time dependent peak mean seven day dishcarges denoted as $Q P_{L i}$ where $i$ is the damage centre in question with each $Q P_{\text {Li }}$ defined for a different time dependent family component 1 over a different time period $t, t=t_{1}, t_{2}$, it is possible to model the phenomenon that agricultural flood damages are time dependent. Each $\mathrm{QP}_{\mathrm{Li}}$ will be the peak discharge at damage centre $i$ for the time dependent family component $I$ over time period $t$ to $t$.

Each variable $Q_{L i}$ has a different coefficient in the objective function reflecting the relative time variability of agricultural damages. All individual time periods $t$ must be included once in the analysis and in only one time dependent family component.

This concept may be illustrated as follows with reference to equation III-10.

$$
Q P \geq Q_{i}^{t}
$$

For the purpose of this discussion let us assume ten time periods. Therefore $t$ varies from one to ten. Also let us assume that the damage relationship for time periods one to ten is as follows:

Time Period Damage Relationship
One to three $\quad$ Damage $=0.25 \mathrm{QP}_{1 i}$
Four to Six $\quad$ Damage $=0.50 \mathrm{QP}_{2 i}$
Seven to ten $\quad$ Damage $=0.90 \mathrm{QP}_{3 i}$
There are three time dependent family components in this example. Equation III-10 may be modified to represent this situation as follows:


Each variable $\underset{L_{i}}{ }, L=1,3$ is given a different value in the objective function as follows:
0.25 QP

1i
0.50 QP
$2 i$
0.90 QP
$3 i$
As shown, equation III-10 has been replaced with three separate equations, each modelling one segment of a time dependent family relationship for flood damage.

### 3.6 Nonlinearity

In the application of the linear programming algorithm to the flood control problem in this study non-linear relations occur in the objective function. These nonlinear relations must be replaced by piecewise linear approximations so that the problem may be solved by a standard linear programming solution procedure employing a separable programming analysis.

The application of the separable programming technique will be illustrated in the case of the damage curves for the damage centers in the model. These non-linear damage curves provide the information needed to derive the optimal release schedules for the system. The form of these functions is important since the total damage is to be minimized in
the optimal solution. If the curves are convex in shape, the problem may be solved in a straightforward manner. Concave curves or a combination of concave and convex curves, on the other hand, tend to create difficulties as a linear programming technique may, in this case, arrive at a local optimum rather than a global optimum for the solution. Concave curves must either be approximated in a convex manner, or if this involves an unacceptable loss of accuracy in the analysis, dealt with through another form of optimization rather than linear programming. This thesis deals only with convex functions.

The method of solution, used in this study, involving a modification of linear programming known as separable programming will be illustrated with reference to Figure III-4, which is assumed to represent the peak mean seven day discharge-damage function for a given damage centre.

A convex cost curve for the damage centre is first approximated by piecewise linear approximation, as shown in Figure III-4. This is accomplished by selecting a number of breakpoints, or points at which the slope of the piecewise linear function changes. Let us define $M$ as the number of segments in the piecewise linear function approximating $U_{j}\left(Q_{j}\right)$ and let

be the ascending values of $Q$ at which the slopes of the piecewise linear segments change value. Next let us subdivide the peak discharge at the damage centre $Q$, into a set of auxiliary variable such that

$$
Q_{j}=\sum_{m=1}^{M} Q_{m j}
$$

where $M=1,2, \ldots M$ and each $Q_{m j}$ is bounded as follows:

$$
Q_{m j} \leq U_{m j}-U_{m-1 j}
$$

The nonlinear cost function $U_{j}\left(Q_{j}\right)$ can now be defined in terms of its piecewise linear approximations:

$$
U_{j}(Q) \sum_{\mathrm{IIF}=1}^{\mathrm{M}} K_{\mathrm{mj}}^{Q_{\mathrm{mj}}}
$$

where $K_{m j}$ is the partial difference of the objective function with respect to $Q_{m j}$ and is defined as:

$$
\underset{\mathrm{mj}}{\mathrm{~K}}=\stackrel{\Delta C / \Delta Q_{\mathrm{mj}}}{ }
$$

When the nonlinear terms are replaced by the piecewise linear terms, the objective function becomes:

$$
\operatorname{Min}\left(\sum_{j=1}^{N} \sum_{m=1}^{M} k_{m j} Q_{m j}\right)
$$

where M is the number of components for a given cost function approximation and N is the number of damage centres in the analysis.

The problem has now been transformed into an ordinary linear programming problem. Since the variables $K_{m j}$ are increasing in character, in a cost minimization problem the linear programming algorithm will select the cost variables in their correct sequential order, and no additional constraints are required. That is, the linear programning algorithm will naturally exhaust the lower valued $K_{m j}$ variables first, which is computationally correct.

### 3.7 Size of the Linear Programming Problem

The computational burden of solving a linear programming application is directly reflected in the cost of doing so on an electronic computer. In order for any model constructed to be operationally applicable it is extremely important that this computational cost be minimized. It should be noted that computational costs involved in the solution of linear progranming applications rise as a positively increasing exponential function of the number of variables in the analysis. It may be readily appreciated that is is imperative to minimize the number of variables in the analysis while at the same time constructing an adequate model of the flood control system under analysis. Two techniques were applied in this thesis to minimize the number of variables in the analysis.

### 3.7.1 Elimination of Variable $Q_{i}^{t}$

The variable $Q_{i}^{t}$ has been defined previously in this thesis as the mean seven day flow at damage centre $i$ over time period $t$. The variables $Q_{i}^{t}$ do not appear directly in the linear programing model formulated in this thesis. Rather, they are calculated as the sumation of all system inflows, positive and negative, at the damage centre in question. Equations III-4 and III-10 are repeated at this point for reference.

$$
\begin{align*}
& Q_{i}^{t}=I_{i}^{t}+L O C_{i}^{t}-\text { Divr }_{i}^{t} \\
& Q P{ }_{i} \geq Q_{i}^{t}
\end{align*}
$$

With reference to equation III-5 and III-10 the following equation can be arrived

$$
\mathrm{QP}_{i} \geq I_{i}^{t}+\operatorname{LOC}_{i}^{t}-\operatorname{Divr}_{i}^{t}
$$

Variable $\mathrm{QP}_{\mathrm{i}}$ is the peak mean seven day discharge at damage centre $i$, which is located at the end of channel reach $i$, over time period $t$. This variable is as defined earlier in this thesis. $I_{i}^{t}$ is the mean seven day inflow to channel reach $i$ over time period $t . ~ L O C C_{i}^{t}$ is the mean seven day local inflow to channel reach $i$ over time period $t$, and $\operatorname{Divr}_{i}^{t}$ is the mean seven day diversion flow diverted from channel reach i over time period $t$.

It is to be noted that no new variables have been introduced in the formulation of III-16, rather the variable $Q_{i}^{t}$ has been eliminated. With reference to equation III-16 it may be seen that the peak mean seven day discharge at any damage centre in question, which is noted as $\mathrm{QP}_{\mathrm{i}}$ is defined as being greater than or equal to the sum of all inflow arriving at a damage centre in question for a given time period. The given time period varies over the entire range of time periods analysed in the model. It must be noted that both positive and negative flows to the damage centre in question are considered in the formulation of equation III-16.

### 3.7.2 Elimination of Fixed Valued Variables

Certain variables in the analysis are fixed in value by their very nature and thus cannot be allowed to assume different values at the optimal solution. Examples of variables of this type are system inflows or initial reservoir storage levels. In the interest of computational efficiency a transformation is carried out with respect to these noted fixed valued variables. For any constraints containing fixed valued variables the variables in question are first multiplied by their corresponding
coefficient in the constraint equation and then subtracted from the right hand side of the constraint in question. In this manner the number of variables that the linear programming algorithm must address is significantly reduced.


Figure III-1
SAMPLE LINEAR PROGRAMMING PROBLEM


Figure III-2
HYPOTHETICAL RIVER SYSTEM



Figure III-3
DEFINITION OF TIME PERIODS USED IN ANALYSIS


Figure III-4
SEPARABLE PROGRAMMING EXAMPLE FIGURE
4.0

THE LINEAR PROGRAMMING MODEL APPLIED TO THE ASSINIBOINE RIVER SYSTEM

### 4.0 THE LINEAR PROGRAMMING MODEL APPLIED TO THE ASSINIBOINE RIVER SYSTEM

### 4.1 The Assiniboine River Model

The Assiniboine River basin in Manitoba between Shellmouth Reservoir and Winnipeg is the river basin or physical system modelled in this thesis. It is broken down into six river reaches on the basis of the location of Water Survey of Canada streamflow gauging stations on the Assiniboine River. These river reaches are from Shellmouth Reservoir to Russell, Russell to Miniota, Miniota to Brandon, Brandon to Portage la Prairie, Portage la Priarie to Headingley and Headingley to Winnipeg. The system thus described is shown on Figure IV-1, and the locations of the above noted Water Survey of Canada streamflow gauging stations are shown on Figure IV-2.

There are seven points of inflow in this model. These are: the inflow to Shellmouth Reservoir, inflow from Shellmouth Reservoir to Russell, inflow from Russell to Miniota, inflow from Miniota to Brandon, inflow from Brandon to Portage la Prairie, inflow from Portage la Prairie to Headingley, and the inflow from Headingley to Winnipeg. The inflow from Headingley to Winnipeg includes flow on the Red River in Manitoba not diverted down the Red River Floodway.

The model constructed in this thesis was tested using 1974 streamflow data over the time period April 15 to August 11. The year 1974 was selected because it was a high flow year and hydrometric flow data was readily available. The inflows to Shellmouth Reservoir were calculated by the Water Resources Division, Department of Mines, Resources and Environmental Management, Government of Manitoba. The remaining inflow values were calculated on a mean seven day basis as the difference in mean seven
day flow values between successive Water Survey of Canada streamflow gauging stations. The exception to the above noted calculation technique involves the inflow at Winnipeg, which includes flow from Assiniboine River and the Red River minus whatever flow was diverted through the Red River Floodway.

### 4.2 Study Operationa1 Rules

For the purposes of this thesis the operational guidelines for the two major flood control works on the Assiniboine River, that is Shellmouth Reservoir and the Assiniboine River Diversion, have been simplified somewhat from those outlined in Chapter II. Shellmouth Reservoir is regulated so that the spillway will not be used. In other words there is no use of live storage above the spillway in flood control operations. This assumption is in accordance with actual operating criteria for the Shellmouth Reservoir in that reservoir live storage is generally not relied on for flood control purposes by the Water Resources Division. As well, a mean seven day release from the reservoir of 100 c.f.s. has been assumed for riparian purposes. That is, during no week during the analysis will the discharge from the Shellmouth Reservoir be allowed to fall below 100 c.f.s.

The operation of the Assiniboine River Diversion has been constrained to follow the Assiniboine River Diversion operation guideline outlined in Chapter II. The pattern of operation assumed for the purpose of this thesis is as follows:
Point

| First 10000 c.f.s. down Assiniboine River | 10000 | c.f.s. |
| :--- | :--- | :--- |
| Next 15000 c.f.s. down Assiniboine River Diversion | 25000 | c.f.s. |
| Next 10000 c.f.s. down Assiniboine River | 35000 | c.f.s. |
| Next 10000 c.f.s. down Assiniboine River Diversion | 45000 | c.f.s. |
| Remainder down the Assiniboine River | $45000+$ c.f.s. |  |

The flood stage at Winnipeg is taken into account at all times with regard to the operation of the Assiniboine River Diversion by means of placing a high damage value on flood stages greater than bank capacity in Winnipeg. It is to be noted that Winnipeg flows include flow of the Red River not diverted by the Winnipeg Floodway. The high value of possible damage at the Winnipeg damage centre may at any time overrule the assumed general pattern of operation of the Assiniboine River Diversion. This in accord with the present actual operating practices with respect to the Assiniboine River Diversion in which flood protection at Winnipeg assumes paramount importance.

### 4.3 Flood Damage Functions

As noted previously the linear programming algorithm requires a linear relationship of the variables in the problem to be minimized. This is known as the objective function. In this thesis the objective function is comprised of damage functions at each damage centre in the analysis. These damage functions are described earlier in Chapter 3.0 in section 3.4, Objective Function.

There are seven damage centres in the analysis carried out in this thesis. These damage centres are located at Water Survey of Canada streamflow gauging stations. The locations are as follows:

Location

Water Survey of Canada
Station Number

Assiniboine River near Russell 05ME001
Assiniboine River near Miniota 05ME006
Assiniboine River near Brandon 05MH001
Assiniboine River near Portage 1a Prairie 05MJ003
Assiniboine River near Headingley 05MJ001
Assiniboine River Diversion near Portage la Prairie 05LL019
Red River at Winnipeg 050J001

Each damage centre is assumed to be representative of the reach of river from the damage centre in question upstream to the next damage centre. The location of the damage centre at Water Survey of Canada streamflow gauging stations provided stage-discharge relationships at each damage centre. The only exception to this is at Winnipeg where the Water Survey of Canada does not calculate a stage-discharge relationship.

Flooded area or damage versus discharge relationships were obtained from the Water Resources Division, the Department of Mines, Resources and Environmental Management, Government of Manitoba, for the first four damage centres, Russell, Miniota, Brandon and Portage la Prairie. These relationships are based on air photo analysis from previous flood years. They are shown on Figures IV-3, IV-4, IV-5 and IV-6. From inspection of the above noted figures it may readily be seen that all of these functions
are convex linear approximations to distinctly non-linear functions. This non-1inearity is to be expected in functions of this type.

All of the functions have an initial segment of zero slope which infers zero damage. This represents the channel capacity for the damage centre in question and is in actual fact the range of flow values that may occur in the river reach between the proceeding damage centre and the centre in question before any flood damage occurs in the reach. It should be noted that this range may be quite different from the range at the actual site of the Water Survey of Canada gauging station. This phenomenon results from the fact that Water Survey of Canada streamflow gauging stations are generally located at constrictions in the river with high channel capacities so as to catch all flow in the river in easily metered channels.

The flooded area versus discharge relationships are converted to damage relationships by multiplying each curve segment slope by an agricultural damage factor as discussed in Chapter III to yield damage, in agricultural damage units, versus discharge relationships. The agricultural damage factors used were as follows:

Date
Week of Arialysis
Agricultural Damage
$\qquad$

| Apri1 15 - May 26 | $1-6$ | 0.333 |
| :--- | :---: | :--- |
| May 27 - June 23 | $7-10$ | 0.5 |
| June 24 - onwards | $11-17$ | 1.0 |

The selection of these weights is rather arbitrary since the purpose of the analysis is to test the model capability rather than to devise practical operating rules.

These weights imply that up to May 26 there is a linear agricultural damage relationship of 0.333 agricultural damage units per acre of land flooded. For the period of May 26 to June 23 there is a linear agricultural damage relationship of 0.5 agricultural units per acre of land flooded. For the period of June 24 onward there is a linear agricultural damage relationship of 1.0 agricultural damage units per acre of land flooded.

It is to be noted that in the application of the linear programming model to the Assiniboine River basin all flooded area was assumed to have the same unit value per acre. Agricultural damage factors were then applied against these unit values. This assumption may be easily relaxed to permit any differention in land values deemed necessary in a given analysis.

The final three discharge-damage relationships, namely, at Assiniboine River Diversion, at Headinley and in Winnipeg were not based on flooded area versus discharge relationships. Rather, they are what is termed in this thesis as surrogate damage functions in which preselected flood control component operating schedule characteristics are forced on the linear programming analysis by selected formulation of the above noted relationships. Operating rules for the Assiniboine River Diversion have been established as a matter of policy. To achieve compliance with these rules surrogate damage functions have been introduced. These relationships are shown on Figure IV-7, IV-8 and IV-9.

The pre-selected component operating characteristics forced on the linear programming analysis by these damage relationships is the previously discussed operation pattern of the Assiniboine River Diversion. The operation pattern of the Diversion desired in the analysis is controlled by
the stage discharge relationships for the Assiniboine River Diversion, the Assiniboine River at Headingley and the combined Assiniboine River and non-diverted Red River $£ 10 w$ at Winnipeg.

The last damage relationship, for the Assiniboine River at Winnipeg, is not formulated to control the operation characteristics of the Assiniboine River Diversion directly but rather to reflect the channel capacity of the Assiniboine-Red River complex in Winnipeg and the very high potential damage should flooding occur in the City. As noted previously the damage discharge relationship at Winnipeg may at any time overrule the preselected operation pattern of the Assiniboine River Diversion so as to relieve flood damage at Winnipeg. This is in accord to the present operating practices with respect to the Assiniboine River Diversion and is to be expected when one considers the extremely high potential damage of flooding in Winnipeg.

All flood damage relationships, with the exception of Winnipeg, reflect the time dependency of agricultural damages as noted previously in this thesis. In the case of Winnipeg the damage weights are assumed to be unity for all three time periods. That is, it is assumed that flood damage in Winnipeg is not time dependent and would be equally severe regardless of the time of year in which flooding would occur.

As noted previously in Chapter 3.0 the damage functions which comprise the objective functions for the linear program optimization technique are non-linear. This situation is addressed by the previously defined technique of separable programming. As well the previously defined technique allowing time variability in the objective function through a time dependent family of damage functions is employed in the application of the model to the Assiniboine River basin.
4.4 Specific Linear Progranming Model for the Assiniboine River System

### 4.4.1 Physical Constraints

### 4.4.1.1 Storage Constraints

The maximum storage available at Shellmouth Reservoir at the spillway elevation is 388,000 acre feet or approximately 27572 c.f.s. - weeks. With reference to equation (III-1) the storage variables for Shellmouth Reservoir in the linear programming model must be upper bounded at a value of 27572 that is
t

$$
\mathrm{S} \leq 27572 \quad \text { IV-1 }
$$

where $S^{t}$ is the volume of water stored in Shellmouth Reservoir at the start of time period $t$ and is in units of c.f.s. - weeks.

### 4.4.1.2 Reservoir Mass Balance Equation

With reference to equation (III-2) the reservoir mass balance equation for Shellmouth Reservoir may be stated as follows:

$$
S^{t}=s^{t-1}+T^{t-1} 0^{t-1}
$$

where $S^{t}$ and $S^{t-1}$ are the Shellmouth Reservoir storage volumes, in units of c.f.s. - weeks, at the beginning of time intervals $t$ and $t-1$. $I^{t-1}$ and $O^{t-1}$ are the mean seven day inflow and outflow from Shellmouth Reservoir over time period t-1. The storage units of c.f.s. - weeks are required in order to maintain dimensional homogenity in the problem in which all flow values are mean seven day values.

### 4.4.1.3 Reservoir Release Constraint

For the purpose of this study releases from Shellmouth Reservoir are only allowed via the reservoir conduit. That is, no flow over the spillway is allowed since the problem is constrained so that the water level on the reservoir may not exceed the spillway crest elevation.

An elevation-discharge relationship for the conduit and spillway flow is shown on Figure IV-10. The conduit rating curve on this figure is a maximum flow curve and based on a Prairie Farm Rehabilitation Authority memorandum of August 25, 1966.

An elevation storage relationship for Shellmouth Reservoir is shown on Figure IV-11.

Figures IV-10 and IV-11 may be combined to yield on outflow-storage relationship for conduit flow only or conduit and spillway combined. An outflow-storage relationship for conduit flow only is shown on Figure IV-12.

For the purposes of this thesis the conduit flow curve may be approximated as shown on Figure IV-12. All reservoir releases must be less than or equal to this limiting function defined by the reservoir storage for the time period in question.

With reference to equation (III-3) this may be represented as

$$
0^{t} \leq 0.066 \mathrm{~S}^{\mathrm{t}}+1930 \quad \text { IV }-3
$$

where $0^{t}$ is the mean seven day outflow from Shellmouth Reservoir over time period $t$; $S^{t}$ is the storage volume in Shellmouth Reservoir at the beginning of time period $t$.

### 4.4.1.4 Riparian Constraint

With reference to equation (III-6) the riparian constraint for

Shellmouth Reservoir for the purposes of this study may be stated as:


IV-4
where $0^{t}$ is the mean seven day outflow from Shellmouth Reservoir over time period t. This outflow is constrained to be at least $100 \mathrm{c} . \mathrm{f} . \mathrm{s}$. for all time periods.

### 4.4.1.5 Diversion Capacity Constraint

With reference to equation (II-8) the upper limit for flow dow the Assiniboine Diversion may be stated as:

$$
\text { DIVR }^{t} \leq 25000 \quad \text { IV-5 }
$$

where DIVR $^{t}$ is the mean seven day Assiniboine River Diversion $f 1$ wo over time period $t$ and is constrained to be less than 25000 c.f.s. for all time periods in the analysis.

### 4.4.1.6 Upper Limit on Diversion Flow Values

With reference to equation (II-10) the Assiniboine River Diversion flow must be constrained so that at all times it is less than or equal to the flow available to be diverted down the Diversion. This may be represented as:

$$
\operatorname{DIVR}^{t} \leq 0^{t}+\text { LOCA }^{t}+\text { LOCB }^{t}+\text { LOCC }^{t}+\text { LOCD }^{t}
$$

where all terms are as defined earlier.

### 4.4.2 Computational Constraints

### 4.4.2.1 Peak Discharge Constraint

The channel flow constraints are defined for each damage centre in the format shown in equation (II-6). With reference to Figure IV-13 they are defined as follows:

Damage Centre A - Russell

$$
\mathrm{QPA} \geq 0^{t}+L O C A^{t}
$$

Damage Centre B - Miniota

$$
\mathrm{QPB} \geq O^{t}+\operatorname{LOCA}^{t}+\operatorname{LOCB}^{t}
$$

Damage Centre C - Brandon

$$
Q P C \geq O^{t}+\text { LOCA }^{t}+\text { LOCB }^{t}+\operatorname{LOCC}^{t}
$$

Damage Centre D - Portage la Prairie

$$
Q P D \geq O^{t}+L O C A^{t}+L O C B^{t}+\text { LOCC }^{t}+L O C D{ }^{t}-\operatorname{DIVR}^{t} \quad \text { III-7 }
$$

Damage Centre E - Headingley

$$
\begin{aligned}
Q P E \geq O^{t} & +\operatorname{LOCA}^{t}+L O C B^{t}+\operatorname{LOCC}^{t}+\operatorname{LOCD}^{t}+\text { LOCE }^{t} \quad \text { III-8 } \\
& - \text { DIVR }^{t}
\end{aligned}
$$

Damage Centre F - Winnipeg

$$
\begin{aligned}
Q P F P^{\geq} O^{t} & + \text { LOCA }^{t}+\text { LOCB }^{t}+\text { LOCC }^{t}+\text { LOCD }^{t}+\text { LOCE }^{t} \quad \text { III-9 } \\
& + \text { LOCF }^{t}-\text { DIVR }^{t}
\end{aligned}
$$

Damage Centre G - Assiniboine River Diversion

$$
\mathrm{QPG} \geq \mathrm{DIVR}^{\mathrm{t}}
$$

where:
$0^{t} \quad$ is the mean seven day outflow from Shellmouth Reservoir over time period $t$
LOCA ${ }^{t}$ is the mean seven day inflow to reach A, Shellmouth Reservoir to Russell, over time period $t$

LOCB ${ }^{t}$ is the mean seven day inflow to reach $B$, Russell to Miniota, over time period $t$
LOCC ${ }^{t}$ is the mean seven day inflow to reach $C$, Miniota to Brandon; over time period $t$

LOCD ${ }^{t}$ is the mean seven day inflow to reach $D$, Brandon to Portage 1 a Prairie over time period $t$
LOCE ${ }^{t}$ is the mean seven day inflow to reach $E$, Portage la Prairie to Headingley, over time period $t$
LOCF ${ }^{t}$ is the mean seven day inflow to reach $F$, Headingley to Winnipeg, over time period $t$

QPA is the peak mean seven day flow at damage centre A, Russell
QPB is the peak mean seven day flow at damage centre $B$, Miniota
QPC is the peak mean seven day flow at damage centre $C$, Brandon
QPD is the peak mean seven day flow at damage centre D, Portage la Prairie

QPE is the peak mean seven day flow at damage centre E, Headingley
QPF is the peak mean seven day flow at damage centre F, Winnipeg
QPG is the peak mean seven day flow at damage centre G, Assiniboine River Diversion

DIVR is the mean seven day Assiniboine River Diversion flow

### 4.5 Summary of Problem Contraints

A summary of all constraints involved in the linear progranming model of the Assiniboine River flood control system is provided herein. This summary includes the previously discussed techniques of time dependency and separable programning employed to address the time dependence and non-1inearity situations inherent in the problem under analysis.

Storage Constraint

$$
S^{t} \leq 27572
$$

$$
\text { for } t=1,17
$$

Reservoir Mass Balance Constraint

$$
S^{t}=S^{t-1}+I^{t-1}-0^{t-1} \quad \text { for } t=2,17
$$

Reservoir Release Constraint

$$
O^{t} \leq 0.066 \mathrm{~S}^{t}+1930
$$

for $t=1,17$

Peak Flow Constraint at Damage Centre A - Russell

$$
\begin{array}{ll}
\text { PA1L }+ \text { PA2L }+ \text { PA3L } \geq 0^{t}+\text { LOCA }^{t} & \text { for } t=1,6 \\
\text { PA1M }+ \text { PA2M }+ \text { PA3M } \geq 0^{t}+L O C A ~ & \text { for } t=7,10 \\
P A 1 H+P A 2 H+P A 3 H \geq 0^{t}+L O C A
\end{array}
$$

Peak Flow Constraint at Damage Centre B - Miniota

$$
\begin{array}{ll}
\text { PB1L }+ \text { PB2L }+ \text { PB3L } \geq 0^{t}+L O C A^{t}+\text { LOCB }^{t} & \text { for } t=1,6 \\
\text { PB1M }+ \text { PB2M }+ \text { PB3M } \geq 0^{t}+L O C A^{t}+\text { LOCB }^{t} & \text { for } t=7,10 \\
\text { PB1H }+P B 2 H+P B 3 H \geq 0^{t}+L O C A^{t}+L O C B & \text { for } t=11,17
\end{array}
$$

Peak Flow Constraint at Damage Centre C - Brandon

PC1L + PC2L $\geq O^{t}+$ LOCA $^{t}+\operatorname{LOCB}^{t}+\operatorname{LOCC}^{t} \quad$ for $t=1,6$

$$
\begin{aligned}
& P C 1 M+P C 2 M \geq O^{t}+L O C A^{t}+\text { LOCB }^{t}+\text { LOCC }^{t} \quad \text { for } t=7,10 \\
& \text { PC1H }+ \text { PC2H } \geq 0^{t}+\text { LOCA }^{t}+\operatorname{LOCB}^{t}+\operatorname{LOCC}^{t} \quad \text { for } t=11,17
\end{aligned}
$$

Peak Flow Constraint at Damage Centre D - Portage la Prairie

$$
\begin{aligned}
& \text { PD1L }+ \text { PD2L } \geq O^{t}+\text { LOCA }^{t}+\text { LOCB }^{t}+\text { LOCC }^{t} \\
& + \text { LOCD }^{t}-\text { DIVR }^{t} \quad \text { for } t=1,6 \\
& \text { PD1M }+ \text { PD2M } \geq O^{t}+\text { LOCA }^{t}+\text { LOCB }^{t}+\text { LOCC }^{t} \\
& + \text { LOCD }^{t} \text { - DIVR }{ }^{t} \quad \text { for } t=7,10 \\
& \text { PD1H }+ \text { PD2H } \geq 0^{t}+\text { LOCA }^{t}+\text { LOCB }^{t}+\text { LOCC }^{t} \\
& + \text { LOCD }^{t} \text { - DIVR }{ }^{t} \text { for } t=11,17
\end{aligned}
$$

Peak Flow Constraint at Damage Centre E - Headingley

PE1L + PE2L + PE3L $\geq O^{t}+$ LOCA $^{t}+$ LOCB $^{t}+$ LOCC $^{t}$

$$
+\operatorname{LOCD}^{t}+\operatorname{LOCE}^{t}-\operatorname{DIVR}^{t} \text { for } t=1,6
$$

PE1M + PE2M + PESM $\geq O^{t}+$ LOCA $^{t}+$ LOCB $^{t}+$ LOCC $^{t}$

+ LOCD $^{t}+$ LOCE $^{t}-$ DIVR $^{t}$ for $t=7,10$
$P E 1 H+P E 2 H+P E 3 H \geq O^{t}+L_{O C A}{ }^{t}+$ LOCB $^{t}+$ LOCC $^{t}$

$$
+ \text { LOCD }^{t}+\text { LOCE }^{t}-\text { DIVR }^{t} \quad \text { for } t=11,17
$$

Peak Flow Constraint at Damage Centre F - Winnipeg

$$
\begin{aligned}
P F 1+P F 2+P F 3 \geq O^{t} & +L O C A^{t}+L O C B^{t}+L^{2} O C C^{t}+\operatorname{LOCD}^{t} \\
& +\operatorname{LOCE}^{t}+\operatorname{LOCF}^{t}=\operatorname{DIVR}^{t} \quad \text { for } t=1,17
\end{aligned}
$$

$$
\begin{array}{ll}
\text { PV1L }+ \text { PV2L } \geq \text { DIVR }^{t} & \text { for } t=1,6 \\
\text { PV1M }+ \text { PV2M } \geq \text { DIVR }^{t} & \text { for } t=7,10 \\
\text { PV1H }+ \text { PV2H } \geq \text { DIVR }^{t} & \text { for } t=11,17
\end{array}
$$

Riparian Constraint
$0^{t} \geq 100$ for $t=1,77$

Diversion Capacity Constraint
$\operatorname{DIVR}^{\mathrm{t}} \leq 25000$
for to $=1,17$

Upper limit on Diversion Flow Constraint
$\operatorname{DIVR}^{t} \leq O^{t}+$ LOCA $^{t}+\operatorname{LOCB}^{t}+\operatorname{LOCC}^{t}+\operatorname{LOCD}^{t} \quad$ for to $=1,17$
where:
$S^{t} \quad$ is the storage volume, in c.f.s. - weeks, in She11mouth Reservoir at the beginning of time period $t$
$I^{t}$ is the mean seven day inflow to Shellmouth Reservoir over time period $t$
$0^{t} \quad$ is the mean seven day outflow from Shellmouth Reservoir over time period $t$

PA1L is the peak mean seven day flow at damage centre $A$, the Assiniboine River near Russell. It is section 1 of the linear approximation to the non-linear damage function and is selected
over time period $L$ which varies from $t=1$ to $t=6$
PA2L is the peak mean seven day flow at damage centre A, the Assiniboine River near Russell. It is section 2 of the linear approximation to the non-linear damage function and is selected over time period $L$ which varies from $t=1$ to $t=6$. Other peak variables are identified in a similar manner. The first character of the variable name, $P$, indicates a peak mean seven day flow; the second character, $A, B, C, D, E, F$ or $V$ indicates a damage centre as follows:

A - Russel1
B - Miniota
C - Brandon
D - Portage la Prairie
E - Headingley
F - Winnipeg
V - Assiniboine River Diversion
The next character is numeric and represents the section of the linear approximation to the non-linear damage function the variable represents.

The last character represents the family of time dependent variables to which the variable in question belongs. This is defined as follows:

L - Apri1 15, to May 26
M - May 27 to June 23
H - June 24 onwards
$0^{t} \quad$ is the mean seven day outflow from Shellmouth Reservoir over time period $t$
LOCA ${ }^{t}$ is the mean seven day local inflow to reach A, over time period $t$ and is applied to the model at Russell

LOCB ${ }^{t}$ is the mean seven day local inflow to reach $B$, over time period $t$ and is applied to the model at Miniota
LOCC ${ }^{t}$ is the mean seven day local inflow to reach $C$, over time period $t$ and is applied to the model at Brandon
LOCD ${ }^{t}$ is the mean seven day local inflow to reach $D$, over time period $t$ and is applied to the model at Portage la Prairie
LOCE ${ }^{t}$ is the mean seven day local inflow to reach $E$, over time period $t$ and is applied to the model at Headingley
LOCF ${ }^{t}$ is the mean seven day local inflow to reach $F$, over time period $t$ and is applied to the model at Winnipeg
DIVR ${ }^{t}$ is the mean seven day flow down the Assiniboine River Diversion over time period t












Figure IV-13
SCHEMATIC REPRESENTATION OF ASSINIBOINE RIVER BASIN

## 5.0

COMPUTATIONAL TECHNIQUES

### 5.0 COMPUTATIONAL TECHNIQUES

### 5.1 The Computer Model

The computer model constructed in this thesis consists of three separate programs which are run sequentially but individually in the analysis. That is, each program is run and the output from the program in question is inspected and accepted before the next program is run. The three programs are:

1. the Matrix Generator
2. the Linear Programming Sollution Algorithm
3. the Report Writer

These individual programs will not be discussed separately.

### 5.2 Matrix Generator

The purpose of the matrix generator program is to transpose the problem under analysis from an algebraic representation of constraints and an objective function to the input format required by the linear progranming solution algorithm.

Each individual problem under analysis could be keypunched separately in the correct input format for the linear programming solution algorithm. However, one must realize the volume of input that is required. In the case of this study approximately 250 computer cards are required for every single problem analyzed. It should be noted that this means that for every single minor change made to a problem under analysis, a new deck of approximately 250 cards would be required. A minor change would involve such things as a different reservoir storage starting level or changes in any one of the inflows to the system. These changes would occur frequently on an operational basis.

It must also be noted that in the development of the working linear programming model there were literally hundreds of problem formulations attempted before the final linear programming model was determined. Without a matrix generator to generate this input for the linear programming solution algorithm, it may readily be seen that the volume of work required to develop the working linear programming model would be high enough to have been infeasible.

The matrix generator written for this thesis is a general matrix generator and is not limited to the problem analyzed in the thesis. The matrix generator will handle any linear programing problem as long as the problem is expressed in a simple algebraic notation. As well, the matrix generator has the flexibility of designating any variables so desired as constants. This type of variable would include local inflows or initial reservoir storage conditions. With the matrix generator developed in this thesis it is a simple matter to investigate different linear programming models or the effect of different inflow hydrographs on optimal operation of a given flood control system that has already been modelled.

As noted previously in this thesis, in the interest of computational efficiency, it is desirable to limit the number of variables actually in the analysis to as small a number as possible. The matrix generator developed in this thesis employs a transformation with regard to the previously noted fixed-valued variables. For any constraint containing constants, these constants are first multiplied by their corresponding coefficient in the constraint and then subtracted from the right-hand side of the constraint in question. In this manner the number of variables that the linear programming algorithm must address is reduced.

As noted previously the number of variables in the problem has a direct effect on computational efficiency and will be reflected in the cost of the computer resources required to solve the problem. It should be noted that if the number of variables exceeds an upper bound determined by the storage capability of the computer installation used for the analysis the particular linear programming solution algorithm employed in this thesis is inapplicable. A computer source code listing of the matrix generator developed in this thesis is provided in Appendix A. As we11, Appendix A contains documentation as to the format of input required by the matrix generator program and sample output from the computer program.

### 5.3 The Linear Programming Solution Algorithm

The linear programming solution algorithm employed in this thesis is based upon the mutual primal dual simplex method of Michael L. Bolinski and Ralph E Gomery. The algorithm in question was supplied through the courtesy of Charles D.D. Howard and Associates, Consulting Civil Engineers, Winnipeg, Manitoba.

The algorithm proved to be quite efficient in the solution of the linear programming problems involved in this thesis. Several different algorithms were tried out in the analysis and computational difficulties were encountered with them. On the other hand the mutual primal dual simplex method proved to be computationally practicable and quite efficient.

### 5.4 The Report Writer

An important point in an optimization model is that the model output must be presented in a readily understandable concise format. It is not
necessary that this output is meaningful from a linear programming point of view. Rather, it is important that the output related to the physical system being modelled especially since the object of the optimization model is to assist in the formulation of subsequent simulation analysis.

The report writer shows firstly all system inflow values. Under operational conditions these would be forecast values for all damage centres. Secondly, the natural streamflows are presented at all damage centres. Thirdly, optimal operating schedules for Shellmouth Reservoir and the Assiniboine River Diversion resulting from the optimization analysis are presented in a table together with streamflow conditions arising from the optimal operation of the flood control system. All terms are in units of mean c.f.s. - weeks.

The output from the report writer is shown in Appendix B. Also shown is a listing of the computer source code and user information to describe input to the report writer program.

## 6.0

RESULTS

### 6.0 RESULTS

### 6.1 The Optimization Mode1

The linear programming optimization model constructed in this thesis is not intended to provide actual operational decisions with respect to the operation of the Assiniboine River flood control system. Rather, it is intended to provide reasonable starting postulates for a more detailed simulation analysis of the flood control system in question. It is important nevertheless that the optimization model produce starting postulates which are near optimal with respect to the relative timing of the individual operations. The values of the reservoir and diversion releases should also be fairly close to optimal values. In the case of the Assiniboine River flood control system it is important that periods of high and low Shellmouth Reservoir release be accurately timed. The timing of the high and low values of the flood control system operating components is more important to the postulating of input to a simulation analysis than the actual values of the individual flood control system components obtained from the optimization analysis.

The results from the optimization model, that is operating schedules for the flood control system, may be input to a more detailed simulation model to fine tune the operating schedules. This will allow the determination of acceptably accurate estimates of the operating schedules with a minimum of simulation model computer runs. This in turn minimizes the cost of determining the operating schedules and, possibly from a flood control viewpoint, more important the time required for the analysis.

The linear programming optimization model constructed in this thesis was evaluated by applying the model spring 1974 conditions on the Assiniboine River in Manitoba. This application was described earlier in this thesis.

As noted previously there are six damage centers located along the Assiniboine River; Russell, Miniota, Portage la Prairie, Headingly and Winnipeg. For each of the above noted damage centers natural flow values and resultant optimal flow values from the optimization model constructed in the thesis are shown on Figures VI-1 to VI-6 inclusive.

The optimal operating schedules resulting from the linear progranming optimization model for both Shellmouth Reservoir and the Assiniboine River Diversion of the Assiniboine River flood control system are shown on Figures VI-7 and VI-8 respectively.

The computer printout from the report writer constructed in this thesis is shown on Figures VI-9 to VI-11 inclusive.

Inspection of the above noted figures will give a complete graphic and numeric summary of the results of the linear programming optimization model constructed in this thesis for a given hydrologic event. In this case the hydrologic event is the spring 1974 streamflow conditions on the Assiniboine River as noted earlier.

### 6.2 Discussion of Results

As noted previously in this thesis the objective function with respect to agricultural damage is decidedly time dependent. The time dependency of the objective function is as follows:

Agricultural
Date $\quad$ Week of Analysis
Damage Factor

| Apri1 $15-$ May 26 | $1-6$ | 0.333 |
| :--- | ---: | :--- |
| May $27-$ June 23 | $7=10$ | 0.5 |
| June 24 onwards | $11=17$ | 1.0 |

With reference to the above table it may be seen that least damage is caused in time periods 1 to 6 . More severe damage is caused in time periods 7 to 10 , and the most severe damage is caused in time periods 11 to 17 .

With reference to Figure VI-11, it may be seen that for each damage centre in the analysis the peak mean seven day discharge is greatest during time period 1 to 6 , less in time periods 7 to 10 and least in time periods 11 to 17 . As noted previously, flood damage in terms of agricultural damage units has been assumed to be a simple function of peak mean seven day discharge. Thus, for each damage centre in the analysis, flood damage is greatest in time periods 1 to 6 , less in time periods 7 to 10 and least in time periods 11 to 17 . This result is as expected because of the previously noted time dependency of agricultural damages in the objective function.

With reference to Figure VI-11 it may be seen that the first week in the analysis involves a spilling of Shellmouth Reservoir. This is
realistic when one considers that the second week of the analysis is the peak mean seven day inflow to Shellmouth Reservoir. With respect to the formulation of input for a subsequent simulation analysis it should be noted that the model indicates that a postulated generating schedule for Shellmouth Reservoir that spills early in the spring period is in order. The balance of time periods 1 to 6 indicate that the first time frame in the analysis, that is time periods 1 to 6 , generally be one of Shellmouth Reservoir spilling. This is in accord with the time dependency of agricultural damage functions used in the model developed in this thesis.

With reference to Figure VI-11, it may be noted that the mean seven day Assiniboine River Diversion flow rate for time period 2 is 16,100 c.f.s. This is greater than the previously noted primary limit for flow down the Assiniboine River Diversion of $15,000 \mathrm{c} . f . \mathrm{s}$. It is anticipated that investigations through the use of subsequent simulation analysis would ascertain whether or not the Assiniboine River Diversion discharge could be held to 15,000 c.f.s. without undue flood damage downstream on the Assiniboine River.

In overview Figure VI-11, the output from the report writer constructed in this thesis, provides the necessary timing relationships between the operation of individual flood control system components of the Assiniboine River system and the relative magnitude of the same operating schedules to formulate reasonable starting postulates for a subsequent simulation analysis. With reference to Figure VI-11 it may be seen that the operation of Shellmouth Reservoir has been determined by the optimization model to be one of generally early spilling of the reservoir followed by a period of moderate releases and finally by a period of minimal release as determined by riparian flow constraints.

The operation of the Assiniboine River Diversion may be seen to be one of initially high diverted flow values followed by a period of steadily decreasing diversion flow values and finally by a period of non use of the diversion.

The above noted timing of the operation of the individual flood control components together with approximate magnitudes of the operating schedules for the same flood control system components is what is required to formulate initial postulates for a subsequent simulation analysis. This is the objective of the optimization model constructed in this thesis.

### 6.3 Model Ruin Cost

The three components of the model constructed in this thesis, that is, the matrix generator, the linear programming solution algorithm and the report writer are run on a CDC Cyber, 170 computer located in Winnipeg at Cybershare Ltd.

Run costs were as follows:

| Matrix generator | $\$ 1.04$ |
| :--- | ---: |
| Linear progranming solution algorithm | 1.15 |
| Report writer | .36 |
| Total run cost | 2.55 |

From the above it may be seen that the total cost for the analysis of one set of forecast inflows for the system as a whole is $\$ 2.55$. It is argued that at this price rate it is quite feasible to evaluate the effect on system component operating schedules of different inflow values
to the system. In this manner it is thus possible to deal with the stochastic aspect of system inflow values in a manner which is acceptable from an operational if not theoretically elequant point of view.

### 6.4 Summary Conclusions

In summary it is concluded that the linear programming optimization model constructed in this thesis successfully provides feasible operating schedule postulates for a subsequent simulation analysis. The model constructed in this thesis has been successfully applied to a rather simple flood control network on the Assiniboine River in Manitoba. It is recognized that the application of the model in this specific instance may well be a form of overkill in that the system is in fact simple enough that acceptable starting postulates for a subsequent simulation analysis may be determined without the optimization model. However, it is argued that this would not be the case in a more complicated system. Moreover, it is the simplicity of the Assiniboine River flood control system that allows one to properly assess the applicability of the model.

The model can be used at a reasonable cost. This factor augers well for the feasibility of using this model on an operational basis for more complex flood control systems.









|  |  |
| :---: | :---: |




RPANEON
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RUSSELL




SHELL YOU~4


TTME PERIOD QUSSELL MINIOTA ROQNOCH

| $\begin{aligned} & \text { TIME } \\ & \text { PEDIOD } \end{aligned}$ | SHELLMOUTH EESERVOI？ |  |  | SYSTEM HYOFOEPCPHS |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | INFLOW | DUTFLOW | STOJAG： | PUSSELL | miniota | gaanden | poptage oiversion | POGTAGELA PRAICIE | hisaincly | hinNIfeg |
| 1 | 2000. | 2731. | 12143. | 3931. | 5431. | 9031. | 1031. | 13030. | 1020 ． | 3540 ． |
| 2 | 9300. | 100. | 12312. | 1300. | 7403. | 13500. | 15100. | 10030. | 13700. | 4890 － |
| 3 | 5920. | 1600. | 295： 2 。 | 1700. | 4503： | 10507. | 11530. | 10050. | 13700. | 4910 ， |
| 4 | 429n． | 3574. | 24912. | 3874. | 6274. | 3574. | 7974. | 10030. | 19205. | 4690こ。 |
| 5 | 2930． | 3515. | 25537. | 3725 | 7113. | 9615． | 5715. | 10000. | 12400 | 4730 － |
| 6 | 250\％． | 3558. | 249？2． | 3859． | 7253． | 8562. | 6558. | 9530. | 17100. | 49103. |
| 7 | 2cca． | 1251. | 23754. | 1361. | 6061. | 5251. | 2651. | 9230. | 12000. | $39200^{\circ}$ |
| 8 | 2－60． | 851. | 25333. | 351. | $4 \in E 1$. | 6261. | 1551. | 94.30. | 1300 c． | 33903. |
| 9 | 1970. | 1461. | 27233． | 1551. | 4561 ． | 5251. | 1561. | 8790． | 17000. | 263uc． |
| 17 | วก\％． | 1417. | 27572． | 1517. | 4117. | 3817． | 917. | 7730. | 10ccc． | 2193：。 |
| 11 | $4 \times 3$. | 10 O ． | 2735． | 100. | 1713． | 2976. | 0. | 5325. | 7693. | 16227． |
| 12 | 241. | 10 C ． | 27398． | 170. | 14と3． | 2012. | $\Delta$. | 36J3． | 5445. | 11525. |
| 13 | 14\％． | 10 C ． | 275？9． | 10 C ． | 1375． | 1773. | 0. | 3739. | 4140. | －765． |
| 14 | 77. | 100. | 27572． | 103. | 1171. | 1514. | 0. | 3299． | 200． | 6735. |
| 15 | 54. | 100. | 27549. | 103. | 1154. | 1817. | 0. | 2374. | 2374． | 5734. |
| 15 | ce． | 100. | 27513. | 100. | 1293． | 1484. | 0. | 2297. | 2297． | 6155. |
| 17 | 35. | 1．J． | 27453． | 100. | 115\％． | 1887. | 0. | 2696. | ？ 685. | E173． |

APPENDIX A
MATRIX GENERATOR

## APPENDIX A - MATRIX GENERATOR

## A-1 Introduction

The matrix generator program transfers the problem under analysis from an algebraic representation of constraints and an objective function to the input format required by the linear programing solution algorithm. The matrix generator allows the flexibility of designating any variables as constants. In this manner different variables in a given analysis may easily be held to constant values in the analysis.

## A-2 Program Logic

The matrix generator transforms time related families of constraints to the input format required by the linear programming solution algorithm. A time related family constraint is a constraint which has the same form with respect to the variables involved and the same coefficients over a finite number of time periods. Only the variables change in that a new variable is defined for each time period. This is illustrated as follows:

| $\quad$ Constraint Family Name | - |
| :--- | :--- |
| $\quad$ LESS |  |
| $\quad$ time periods defined over | -1 to 5 |
| Constraint Name | Constraint |
| LESS 1 | Q1 $\leq 5000$ |
| LESS 2 | Q2 $\leq 5000$ |
| LESS 3 | Q3 $\leq 5000$ |
| LESS 4 | Q4 $\leq 5000$ |
| LESS 5 | Q5 $\leq 5000$ |

In this example it may be seen that the same form of the constraint is maintained over the five time periods involved but a new variable is introduced for each time period.

The matrix generator reduces constraints of the family related type to the input matrix format required by the linear programming solution algorithm.

An internal function of the matrix generator program is to transfer each constraint name and each variable in the analysis to a unique integer value in the interests of programming efficiency. A cross reference table is built for both constraint names and variables in the analysis.

In the interest of computational efficiency the matrix generator employs a transformation with regard to the previously noted constants. For any constraint containing constants, these variables are first multiplied by their corresponding coefficient and then subtracted from the right hand side of the constraint in question. In this manner the number of variables that the linear programming solution algorithm must address is reduced. The number of variables in the problem under analysis has a direct effect on computational efficiency. A listing of the program is provided on Figures A-1 to A-7 inclusive.

## A-3 Capacity and Limitations

The following limitations are imposed:

1. A maximum of two hundred unique variables in the problem under analysis.
2. A maximum of sixty unique variables in any one constraint.

Problems involving either a greater number of equations or a greater number of variables in any given constraint than noted above will require 101 minor program modifications.

## A-4 Input

Sample Input is shown on Figures A-8 to A-14 inclusive. An input deck consists of the following:

1. System Initial Condition Cards

These cards define initial system conditions in the analysis, one constant per card.

| Columns | Description | Example |
| :---: | :--- | :--- |
| $2-5$ | Variable Family Name | S |
| $7-10$ | Variable Time Period | 1 |
| $11-20$ | Variable Constant Value | 12143 |

2. Constant Value Cards

These cards define constants in the analysis, one constant per card.

| Columns |  | Description | Example |
| :---: | :--- | :--- | :--- |
| $2-5$ |  | Variable Family Name | I |
| $7-10$ |  | Variable Time Period | $I$ |
| $11-20$ |  | Variable Constant Value | 2900 |

3. Run Title Card

This card contains the alpha-numeric description of the problem under analysis.

| Columns | Description <br> $1-80$ |
| :---: | :---: |
|  | Run Title |
|  | Assiniboine River L/P |
|  |  |
|  | Flood Control Optimization |
|  |  |
|  |  |

## 4. Objective Function and Constraints

The objective function and the constraints are input to the matrix generator in the same format. The objective function is input first, prior to the problem constraints.

All constraints are identified by a constraint type as follows:

Type
N
E -
L -
LE -
GE -
G

Description
Objective Function Equality Contraint: = Less than constraint; < Less than or equal to constraint; $\leq$ Greater than or equal to constraint; $\geq$ Greater than constraint; >

### 4.1 Constraint Name Card

| Columns | Description | Example |
| :---: | :--- | :--- |
| $2-5$ | Constraint family name | OBJT |
| $9-10$ | Constraint type | N |
| $11-20$ | Right Hand Side Value | Blank |
|  | of constraint |  |

### 4.2 Row Definition Card

| Columns | Description | Example |
| :---: | :--- | :---: |
| $1-10$ | Starting time value | 1 |
| for row names |  |  |
|  |  | Number of time periods |
|  | constraint is to be | 1 |

4.3 Variable Counter Card

| Columns | Description <br> $1-10$ | Number of variables in <br> constraint |
| :---: | :--- | :---: |

4.4 Variable Family Name Cards

Eight variable family names may be input per card.

| Columns | Description | Example |
| :---: | :---: | :---: |
| $2-5$ | Variable family name | PA1L |
| $7-10$ | Variable family name | PA2L |
| $37-40$ | Variable family name | PA2H |

### 4.5 Coefficient Card

Eight coefficients may be input per card to correspond to the variable family names on the preceding Variable Family Name Card.

| Colums |  | Description | Example |
| ---: | :---: | :---: | :---: |
| $1-10$ | Coefficient value | 0.0 |  |
| $11-20$ | Coefficient value | 0.06 |  |
| $71-80$ | Coefficient value | 0.18 |  |

4.6 Variable Lower Time Qualifier Card

This card defines the lower time qualifier for each variable input on the preceding Variable Family Name Card.

| Columns | $\begin{array}{c}\text { Description } \\ 1-10\end{array}$ | $\begin{array}{l}\text { Variable lower time } \\ 21-30\end{array}$ |
| :---: | :--- | :---: | $\left.\begin{array}{l}\text { qualifier }\end{array}\right]$| Variable lower time |
| :--- |
| $71-80$ | | qualifier |
| :--- |

### 4.7 Variable Upper Time Qualifier Card

This card defines the upper time qualifier for each variable input on the preceding Variable Family Name Card.

| Columns | Description | Example |
| :---: | :---: | :---: |
| 1-10 | Variable upper time | 1 |
|  | qualifier |  |
| 21-30 | Variable upper time | 1 |
|  | qualifier |  |
| 71-80 | Variable upper time | 1 |
|  | qualifier |  |

### 4.8 Fixed Value Card

This card defines whether or not the variables input on the preceding Variable Family Name Card are constants. Two types of constants are allowed as follows:

F - Fixed value variable or constant for all time periods.
F1 - Fixed value variable or constant for only the first time period.

| Columns | Description | Example |
| ---: | :---: | :---: |
| $1-10$ | Constant indicator |  |
| $11-20$ | Constant indicator |  |
| $71-80$ | Constant indicator |  |

### 4.9 Variable Upper Bound Card

This card is used to input an upper bound on all variables introduced on the preceding Variable Family Name Card.

| Columns | Description | Example |
| :---: | :---: | :---: |
| $1-10$ | Upper bound value | 1500 |
| $11-20$ | Upper bound value | 1300 |
| $71-80$ | Upper bound value | 1300 |

Card types 4.1 to 4.9 are repeated as necessary to introduce and define all the variables for a given family of constraints.

Output
The resultant output from the Matrix generator program is shown on Figures $\mathrm{A}-16$ to $\mathrm{A}-24 \mathrm{~b}$ inclusive.

The printed output is in two parts. The first part is a listing of each constraint family in the analysis. Each constraint family is identified by a one to four character alphameric name. To this name an integer is added to denote which time period the constraint applies to. For example with reference to Figure A-17:

MXAL 1-6
This corresponds to the following constraints:

MXAL1
MXAL2
MXAL3
MXAL4
MXAL5
MXAL6

Each variable in the constraint is identified by a one to four character alphameric name and an integer denoted time period. With reference to Figure A-17 it may be seen that a range is given for the time period. For example:

PA1L
1-1
Corresponds to: PA1L1
while:
0 1-6

Corresponds to: 01 02

For each constraint defined by the time period indicator the variables in the constraint are defined according to their family name and their time period qualifier. With reference to Figure A-17 for constraint MXAL 1-6 the following constraints are implied:

$$
\begin{array}{ll}
\text { MXAL1: } & -1.0 \text { PA1L }-1.0 \text { PA21 }-1.0 \text { PA3L }+01+\text { LOCA1 } \leq 0.0 \\
\text { MXAL2: } & -1.0 \text { PA1L }-1.0 \text { PA2L }-1.0 \text { PA3L }+02+\text { LOCA2 } \leq 0.0 \\
\text { MXAL3: } & -1.0 \text { PA11 -1.0 PA2L }-1.0 \text { PA3L }+03+\text { LOCA3 } \leq 0.0 \\
\text { MXAL4: } & -1.0 \text { PA1L }-1.0 \text { PA2L }-1.0 \text { PA3L }+04+\text { LOCA4 } \leq 0.0 \\
\text { MXAL5: } & -1.0 \text { PA1L -1.0 PA2L }-1.0 \text { PA3L }+05+\text { LOCA5 } \leq 0.0 \\
\text { MXAL6: } & -1.0 \text { PA1L -1.0 PA2L -1.0 PA3L }+06+\text { LOCA6 } \leq 0.0
\end{array}
$$

As shown above each variable in the constraint is multiplied by its corresponding coefficient as shown in the row labelled "coefficient".

As noted earlier any variable in the analysis may be held constant. Any variables held constant are shown in the next row labelled "Fixed Value". A ' $F$ ' indicates that the variable in question is fixed over the entire range
of the constraint in question while a "Fl" indicates that the variable in question is only held constant for the first time period in the analysis.

All variables are upper bounded in this analysis. The upper bound specified for each variable is shown in the next row of the output labelled "Upper Bounds".

As noted earlier all variable names and all row names are converted to integer values to facilitate the computer programing required. A cross-reference of variable names and integer representation and row names and integer representation is shown on Figures $\mathrm{A}-25$ to $\mathrm{A}-31$ inclusive.

The output from the matrix generator in the format required for the linear programming solution algorithm is shown on Figures A-32 to A-35 inclusive.


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Figure A-28
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APPENDIX B
REPORT WRITER

APPENDIX B - REPORT WRITER

## B-1 Introduction

This program translates the output from the linear programming solution algorithm into a readily readable and understandable format.

## B-2 Program Logic

The report writer program assembles and reduces the output from the linear programming solution algorithm into a concise meaningful format. Optimal operating schedules for the components of the Assiniboine River flood control system are presented. As well, the program calculates flows that would occur without regulation or diversion by the Assiniboine River Flood Control System. These flow values are referred to as natural flows in the report writer program. All flow values are in terms of mean c.f.s. - weeks. A listing of the program is provided on Figures B-1 to $\mathrm{B}-4$ inclusive.

B-3 Capacity and Limitations

1. A maximm of twenty time periods in the analysis.
2. A maximum of one hundred twenty five unique variables in the analysis.

Problems involving either a greater number of time periods or a greater number of unique variables than noted above will require minor program modifications.

## B-4 Input

Sample input to the program is shown on Figure B5.
An input deck consists of the following:

1. Date Card

This card identifies the first calendar date of the analysis. It is printed on each page of output as a sub-heading.

| Columns | Description | Example |
| :--- | :--- | :--- |
| $1-80$ | First date of analysis | April 17, 1974 |

2. Number of Timer Periods Card

This card inputs the number of time periods in the analysis.
$\frac{\text { Columns }}{1-10}$

Description
Number of time periods
Example
17
3. Required Variables Card(s)

This card inputs the variables required for output. One family of variables is described per card. Only non-constant variables may be requested for output.

| Columns |
| :---: |
| $2-5$ |
| $6-10$ |
| $11-15$ |

Description
Variable family name
Initial time period
Final time period

Example
S

17

## B-5 Output

The resultant output from the report writer program is shown on Figures B-6 to B-8 inclusive.

The first section of output, shown on Figure B-6, shows all inflows to the Assiniboine River System for the problem under analysis.

The second section of the output, shown on Figure B-7 shows the natural flow values throughout the river system for the problem under analysis.

The third section of the output, shown on Figure B-8, shows the optimal operating schedules determined for the individual flood control system components, Shellmouth Reservoir and the Assiniboine River Diversion, and the resultant flow throughout the river system.


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