AN ANALYSIS OF THE RELATIONSHIP BETWEEN TOTAL DAILY SOLAR RADIATION RECEIPT AND TOTAL DAILY DURATION OF SUNSHINE AT WINNIPEG, 1950 to 1967

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SYMBOLS--Continued

a	....	y intercept of the linear prediction equation $y = a + bx$
b	....	slope of the line in $y = a + bx$
r	....	correlation coefficient
r^2	....	coefficient of determination
F	....	tabulated test statistic
a'	....	estimated a
b'	....	estimated b

ABSTRACT

Variations in the relationship between total daily solar radiation and sunshine duration, as affected by changes in atmospheric turbidity, and displayed by the parameters a and b of the linear equation

$$Q = Q_o \ (a + b \ n/N)$$

are studied as a function of time. Daily data are used in the analysis. It is found that a and b are inversely related and vary in a curve that can be described as parabolic over the period of the year. It is found that

$$a' = 0.50187 - 0.0103760 \ x + 0.00012084 \ x^2, \text{ and}$$

$$b' = 0.35526 + 0.0162589 \ x - 0.000199034 \ x^2,$$

where a' and b' are the least squares estimators of the parameters a and b, and x is any 5-day period.

CHAPTER I

BACKGROUND TO THE PROBLEM

The rate at which solar radiant energy is received outside of the atmosphere on a surface normal to the incident radiation, at the earth's mean distance from the sun, is constant at 2.0 $gm.cal.cm.^{-2}min.^{-1}$* (Johnson 1954, p. 431) and depends wholly upon the properties of the sun.** The rate at which direct solar radiant energy is received on a horizontal surface at ground level depends on:

1) variation in the earth's distance from the sun,
2) the inclination of the incident rays to the horizontal — which is determined by latitude, time of year and time of day,
3) atmospheric depletion — which is small in pure air, but increases with the amount of pollution or turbidity associated with variable components such as water vapour, dust, haze, etc..

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* One $gm.cal.cm.^{-2}min.^{-1}$ is equivalent to one langley.

** The solar constant has been measured by various observers with results ranging from 1.89 to 2.05 langleys. Recent research (Stair and Ellis 1968) has suggested the solar constant is 1.95 ly. with a probable error of 5 per cent. However, this study will use Johnson's 1954 estimate in keeping with other recent research (Pochop, Shanklin and Herner 1968).

The total receipt of solar radiation at the earth's surface also depends upon the receipt of diffuse radiation. This is a function of the non-selective scattering of the solar beam by particles smaller in diameter than the wavelength of the incoming beam; particles such as atmospheric dust and fine water droplets.

Routine measurements of total daily solar radiation are made with pyrheliometers which are uniformly sensitive to the entire thermally effective spectrum, 0.3 - 3.0 microns (Robinson 1966, p. 94). These recording instruments however, are not densely or equitably distributed (Figure 1.1). Attempts to determine the distribution of solar radiation over the earth's surface have thus relied heavily upon generalizations from widely disparate observations. In order to make such generalizations more objective, researchers have resorted to estimating solar radiation from known climatological data, usually recorded by a more densely distributed network (Figure 1.2).

The chief difficulty in the application of these estimated observations is obtaining representative values of the various parameters used in the estimation; parameters such as cloud data and sunshine data. These are known reliably only for the stations at which the measurements are made. Thus, in the extension of the geographical area of application using the observed and the estimated values of solar radiation, it becomes necessary to assume that there is predictable spatial variation in atmospheric turbidity.

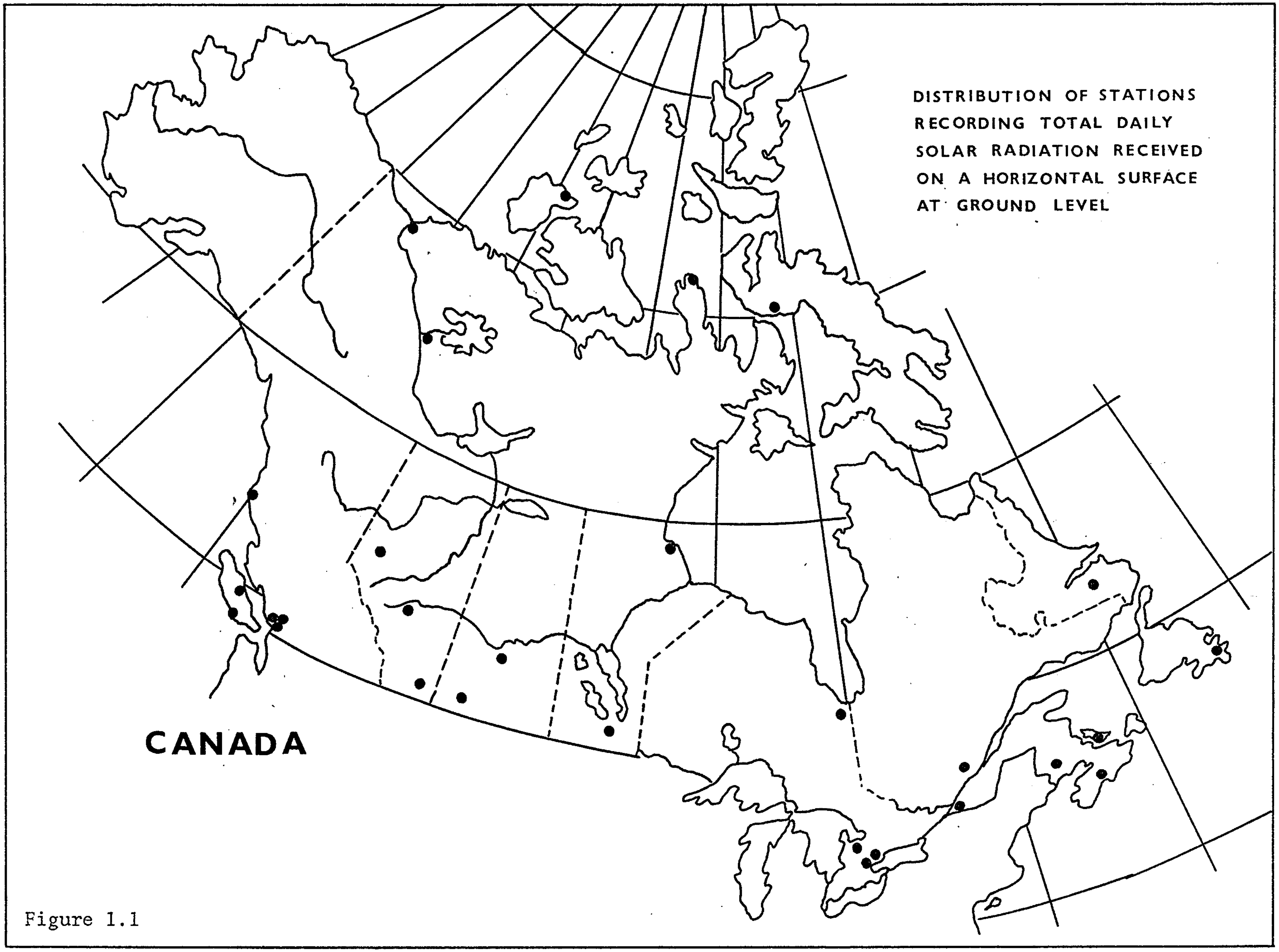
DISTRIBUTION OF STATIONS RECORDING TOTAL DAILY SOLAR RADIATION RECEIVED ON A HORIZONTAL SURFACE AT GROUND LEVEL
CANADA

Figure 1.1

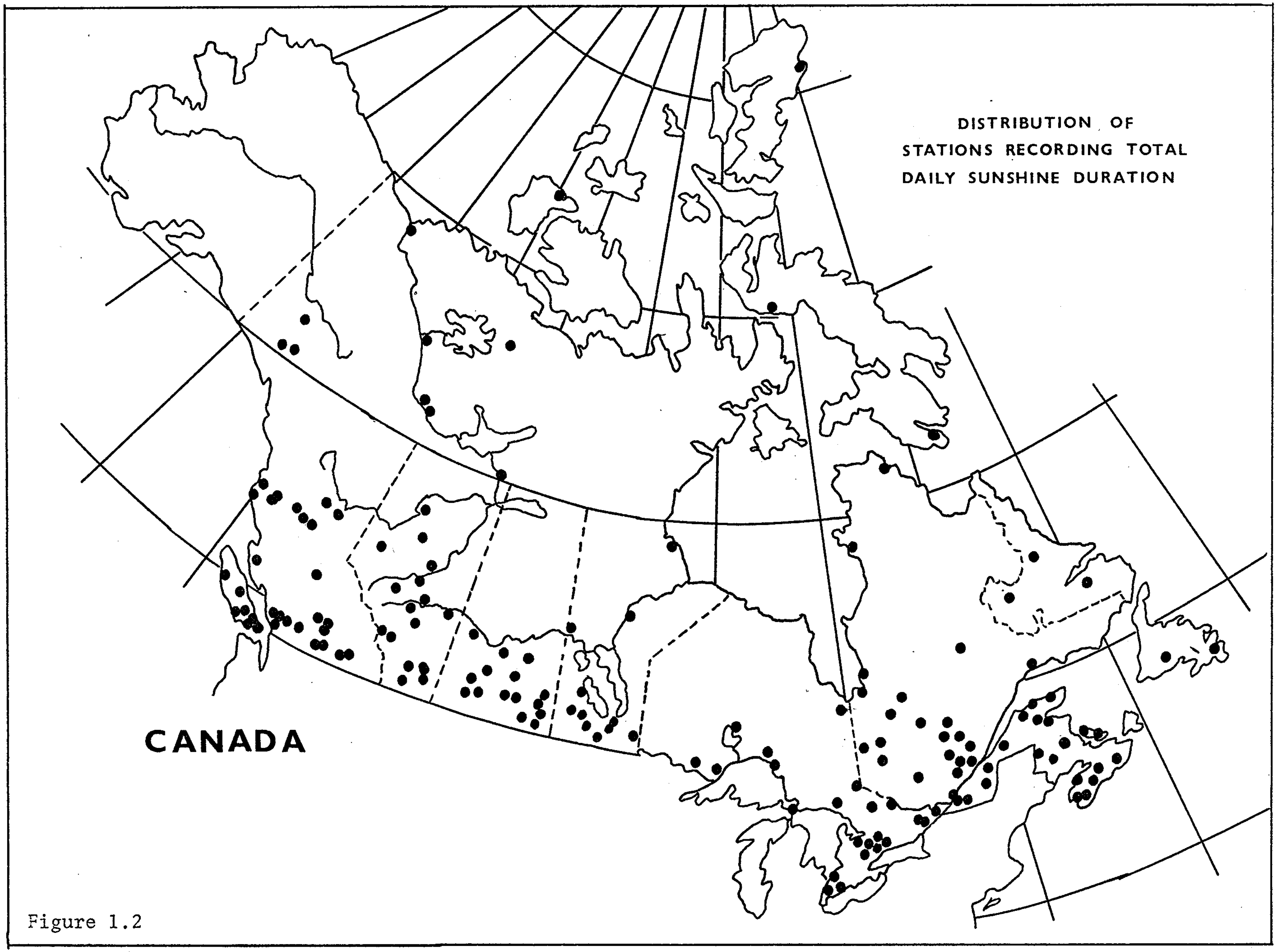
DISTRIBUTION OF
STATIONS RECORDING TOTAL
DAILY SUNSHINE DURATION
CANADA

Figure 1.2

Earlier Work

Much research has been devoted to the solution of this problem. This research can generally be allocated to one of two approaches. One of them involves measurements by a network of closely spaced stations, and the other is based on the use of physical formulae and constants. The former is essentially empirical, the latter is essentially theoretical. Apropos of this, the discussion of the literature will be divided into two sections.

The Theoretical Approach

The intensity of solar radiation received on a horizontal surface at the top of the atmosphere can be computed from the following fundamental relationship (Klein 1948, p. 119):

$$Q_a' = 2.00\ (\sin\phi\ \sin\delta + \cos\phi\ \cos\delta\ \cos\eta)\ /R^2 \qquad [1]$$

where 2.00 is the solar constant in langleys.

Passing through the earth's atmosphere the solar radiation suffers attenuation due to the effects of (Drummond *et al*. 1958, p. 375):

1) scattering by molecules of air and particles much smaller than the wave-length of light (Rayleigh scattering),
2) selective absorption by atmospheric gases, particularily oxygen, ozone, carbon dioxide and water vapour,
3) scattering and absorption by cloud masses, and
4) scattering and diffuse reflection from particles (e.g. dust and smoke) of size comparable with, or larger than, the wave-length of light.

The problem thus becomes one of determining what proportion of the solar radiation incident at the top of the atmosphere is actually received at the ground. Early research by Kimball (1927) demonstrated the importance of precipitatable water* in depleting solar radiation, and Ångström (1930 p. 131) determined a formula whereby the depletion of solar radiation of wave-length λ by scattering alone is given by

$$Q_\lambda = Q_a{'}_\lambda \, e^{-(a_1 + a_2)\, m}. \qquad [2]$$

This led to work by Neiburger (1949), Fritz (1949), Kaplan (1952), Plass (1952), Sheppard (1958), McDonald (1960), and Monteith (1962), among others, which was devoted to determining the physical relationships between solar radiation and the various constituents of the atmosphere.

This research is probably best summed up by Robinson (1966) who stated that the depletion of incident solar radiation may be characterized by the extinction coefficient c_λ which is a function of the wave-length λ and which represents both scattering and absorption.

"Consider a parallel beam of light incident on a single particle. If only scattering occurs then all the energy in the original beam will be present in the radiation field surrounding the particle, but it will be dispersed, so that it will travel in

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* Precipitable water is defined as the depth of liquid water which would result if all the water vapour in a vertical column of air extending to the top of the atmosphere were condensed and collected at the bottom.

all directions. In effect, the particle becomes a new source of light. For large particles this change in direction may be due to a number of causes, for example, diffraction, reflection, refraction, or a combination of these effects.

Absorption is said to occur if the sum of the scattered and transmitted energies integrated over all angles is less than the incident energy.

The change dQ_λ in the intensity Q_λ of a parallel monochromatic beam is given by Beer's law

$$dQ_\lambda = -Q_\lambda \, c'_\lambda \, dx = -Q_\lambda \, c''_\lambda \zeta \, dx \qquad [3]$$

where dx is the path length. Thus, in a homogeneous medium, equal increments of length dx will deplete equal fractions dQ_λ / Q_λ of the incident solar radiation. Integration of [3] yields

$$Q_\lambda = Q_{c\lambda} \exp\left[-\int_0^x c'_\lambda \, dx\right] = Q_{c\lambda} \exp\left[-\int_0^x c''_\lambda \zeta \, dx\right] . \qquad [4]$$

The absorbing and scattering material in the atmosphere varies along the path of the solar beam because the gaseous composition of the atmosphere is a function of height and time. This also applies to particulate matter, including clouds. Finally, each atmospheric constituent has its own scattering and absorption coefficients which also vary with wave-length" (Robinson 1955, p. 47-48).

Because suspended atmospheric particles vary over a wide range in both size and number (Hand 1937, p. 434) and because each particle has its own scattering and absorption coefficients, it is difficult to express depletion from them in terms of one equation. This difficulty severely complicates the problem of estimating solar radiation, particularily so since none of the above equations considers diffuse radiation. This must be included in estimates of total or global radiation at the earth's surface.

Thus, because their aim was to establish certain formulae for climatological purposes rather than to determine instantaneous values of solar radiation, some researchers have turned to empirical approaches to solve the problem of establishing the distribution of solar radiation.

The Empirical Approach

Clouds are the major factor affecting the receipt of solar radiation at the earth's surface (Houghton 1954, p. 2). They are complex phenomena, varying from thin, transparent cirrus which exert little influence on global radiation, to thick cumulo-nimbus clouds which may reduce global radiation to one per cent of its normal value (Robinson 1966, p. 125). The brightness of cumulus clouds may vary greatly, depending on their position with respect to the observer. Reflection by the sides of such clouds may substantially increase the total radiation received at the earth's surface.

Unfortunately, there are but few stations at which the type and extent of each cloud layer are noted. At most stations the estimations of cloud cover are of total cloudiness only.

In 1945 Haurwitz (p. 154) asserted that, since cloud observations are made at a relatively large number of stations, spatial distributions of solar radiation might be derived from investigations of the empirical relationships between solar radiation and cloud amount. Haurwitz examined these empirical relationships at Blue Hill Observatory (Massachusetts) but his results were not widely adopted for the evaluation of radiation receipts. This is mainly because cloud data are not continuous and are subjective since they cannot be measured by instruments. Thus, the duration of sunshine is taken as a measure of cloudiness since this type of information does take into account the optical density of the clouds at a certain stage (Robinson 1966, p. 125). Furthermore, because the sunshine record is both continuous and objective, it lends itself more easily to the development of empirical techniques for calculating the distribution of solar radiation receipts.

Ångström (1924, p. 121) developed the following empirical formula for estimating solar radiation receipts in Sweden:

$$Q = Q_o \, (a' + b'n/N) \qquad [5]$$

where a' is the ratio Q/Q_o on a cloudless day and b' is 1 - a', since Q equals Q_o on a cloudless day. Data for overcast days at Stockholm gave a value of a' = 0.25, from which b' = 0.75.

Fritz and McDonald (1949, p. 61) obtained the relation

$$Q = Q_o \, (0.35 + 0.61 \, n/N) \qquad [6]$$

for the contiguous United States by statistical methods. The failure of the regression coefficients to total to unity was attributed to the use of monthly means of Q and n/N, where n/N ranged from .35 to .97 only, so that extrapolation to n/N = 0 might not be valid. Page (1961, p. 378), however, attributed this failure of a + b to equal one to the nonlinear nature of the relationship between sunshine duration and solar radiation receipt. He also asserted that if the rule 1 - a = b is relaxed*, all the variation in Q will be found in n and so can be systematically examined.

Mateer (1955, p. 579), using Canadian data for the summer months only, obtained

$$Q = Q_o \ (0.35 + 0.68 \ n/N) \qquad [7]$$

and attributed the difference between [7] and [6] to the different methods of observation of sunshine duration in the United States and Canada (U.K., Met. Office 1956, p. 301). However, the mild disagreement between [6] and [7] could be due to the different latitudinal locations of the two study regions and also to the fact that [6] is based upon data for all months of the year, whereas [7] is based upon data for the summer months only. Factors that affect n/N and Q vary both seasonally and latitudinally.

Several workers have modified the Ångström equation by replacing Q_o with Q_a, which is the radiation received on a horizontal surface at the top of the atmosphere. This considerably facilitates spatial comparisons because Q_a can be predicted with ease, whereas Q_o, a function of atmospheric attenuation, suffers fluctuation due

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* Ångström's constants totalled to unity by definition. Subsequent research, because a and b in the equation $Q/Q_o = a + b \ n/N$ should theoretically equal one when actual sunshine duration equals the possible, has retained this rule.

to spatial and temporal variation in the composition of the atmosphere.

Analysing previous research Black, Bonython and Prescott (1954, p. 233), hypothesized that b should be approximately constant irrespective of latitude and that a should be inversely related to latitude. This hypothesis requires the assumption of a smoothly varying atmosphere since, statistically, b is the slope of the regression line that measures the amount of change in Q/Q_a for each unit change in n/N. An equation capable of general application can then be determined be making a latitudinal adjustment for a.

In testing their hypothesis Black _et al_. selected stations unevenly distributed over a range of 50^o of latitude. They discovered that b varies from 0.28 to 0.63 and that a varies from 0.19 to 0.4. However, there was no evidence of a consistent trend, and, since the correlation coefficients were high in each case, the data were pooled and a single equation calculated on the assumption that any loss in precision resulting from the inclusion of data not strictly comparable was justified by the extension of geographical coverage.

Glover and McCulloch (1958b, p. 173) supported the contention that b is independent of latitude, although they found (1958a, p. 58) that, at Kenya both a and b suffer irregular temporal variations. They concluded that the extreme variations are associated mainly with unusually large or small amounts of cloud randomly occurring in particular months.

This leads one to suspect that variations in a and b attributable to a real seasonal trend (or, by implication, latitudinal variation) may not be apparent until research into the problem is detailed enough. Thus, what has been dismissed by Glover and McCulloch as the effects of random cloudiness, and by Black *et al*. as the effects of the lack of a consistent trend, may really be a predictable phenomenon.

Page (1961, p. 380) has suggested that a and b vary in certain ways. If a low percentage of possible sunshine duration coincides with the season of high turbidity, a is low and b is high. If the season of high turbidity coincides with the period of greatest percentage of possible sunshine, this relationship will be reversed. Thus, in analysing a and b, the most important factors to be considered are the seasonal and regional variations in the transmission characteristics of the atmosphere.

Summary and Aims

The problem of estimating solar radiation from known climatological data rests upon a foundation of theoretical and empirical relationships. The instantaneous values of the important parameters are too explicit and variable to permit the general discussion of the distribution of solar radiation over the surface of the earth. Therefore, the empirical approaches are required for the climatological study of general spatial and temporal distributions of solar radiation receipt.

This research applies the empirical method of investigation of the relationship between daily solar radiation and daily duration of sunshine to determine the nature of, and the predictability of, seasonal variations in a and b. Daily data recorded at Winnipeg over the years 1950 to 1967 are used, and these data are analysed by periods of varying length. In this way both the short-term and long-term fluctuations in a and b may be revealed. Winnipeg is a good centre for a seasonal study of this nature since it suffers pronounced and predictable changes of surface albedo such as have been shown by Mateer (1955, p. 582) to effect this empirical relationship. Because this research is restricted to Winnipeg data, spatial variations in a and b are ignored.

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CHAPTER II

INSTRUMENTS AND DATA

The data were observed at Winnipeg International Airport during an eighteen year period extending from January 1950 to December 1967. The solar radiation data were obtained from an Eppley 180° pyrheliometer which records the total daily receipt of direct and diffuse radiation falling on a horizontal surface. The sunshine duration data were recorded by a Campbell-Stokes sunshine recorder.

These instruments are part of the Winnipeg synoptic meteorological station which is located to the northwest of the airport on an open plain on a non-industrial site. The station has not been moved during the years under investigation.

The Eppley 180° Pyrheliometer

In this instrument the receiver consists of two concentric rings about a central disk, the whole being enclosed inside a glass bulb. The outer ring is white and the middle ring is black; the inner circle is not active. A thermal insulator is placed between the two rings. The black surface absorbs almost all the radiation incident upon it, while the white surface reflects visible and near infra-red radiation. As a result a temperature differential exists between the two rings. The difference in temperature is measured by thermo-couples and can be recorded by a suitable potentiometer or galvanometer recorder. The upper surface of the receiver is mounted in the plane passing through the centre of the glass bulb and the whole instrument, for the purpose of measuring diffuse and direct radiation, is mounted on a horizontal plane.

On receipt of advice from the Meteorological Branch the following corrections were applied to the radiation data. The present pyrheliometer was installed in July 1956. The calibration factor of the older instrument is thought to have been incorrect by five per cent. Therefore, all data observed prior to July 7, 1956 were reduced by this percentage. The data observed prior to January 1958 were corrected for ambient air temperature effects since this had not been done in the published records. The data observed between February 5, 1957 and March 31, 1957 were adjusted by factors incorporating corrections for both ambient air temperature and for the change in radiation scale from the Smithsonian Scale (1913), to the International Pyrheliometric Scale (1956).

The Campbell-Stokes Sunshine Recorder

This instrument consists of a solid glass sphere which records sunshine duration by focusing the sun's rays onto printed cards. The total length of the burn is compared with the time scale on the card to obtain the duration of sunshine.

The card is not burned, however, until the intensity of the direct radiation from the sun reaches a certain minimum value (about 0.2 to 0.4 cal.$cm^{-2}min^{-1}$), and thus, on clear mornings it does not begin recording until some period after the sun has risen, and it ceases recording on clear evenings about the same period before sunset. Consequently, the apparent length of day is less than the actual length of day. The Meteorological Office (1956, p. 308) regards 3^{o} above the horizon as the critical elevation in this respect.

Gallagher (1967) has devised a method based on the relationship

$$\sin\alpha = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\eta \qquad [8]$$

whereby it is possible to correct the maximum possible duration of sunshine, as determined by the Smithsonian Institute (List 1949, p. 507), to conform with the length of day, as measured by the Campbell-Stokes sunshine recorder, by subtracting the time the sun is less than 3^{o} above the horizon from the possible sunshine duration. (Figure 2.1)

Thus the length of day is defined as that period during which the sun is at least 3^{o} above the horizon.

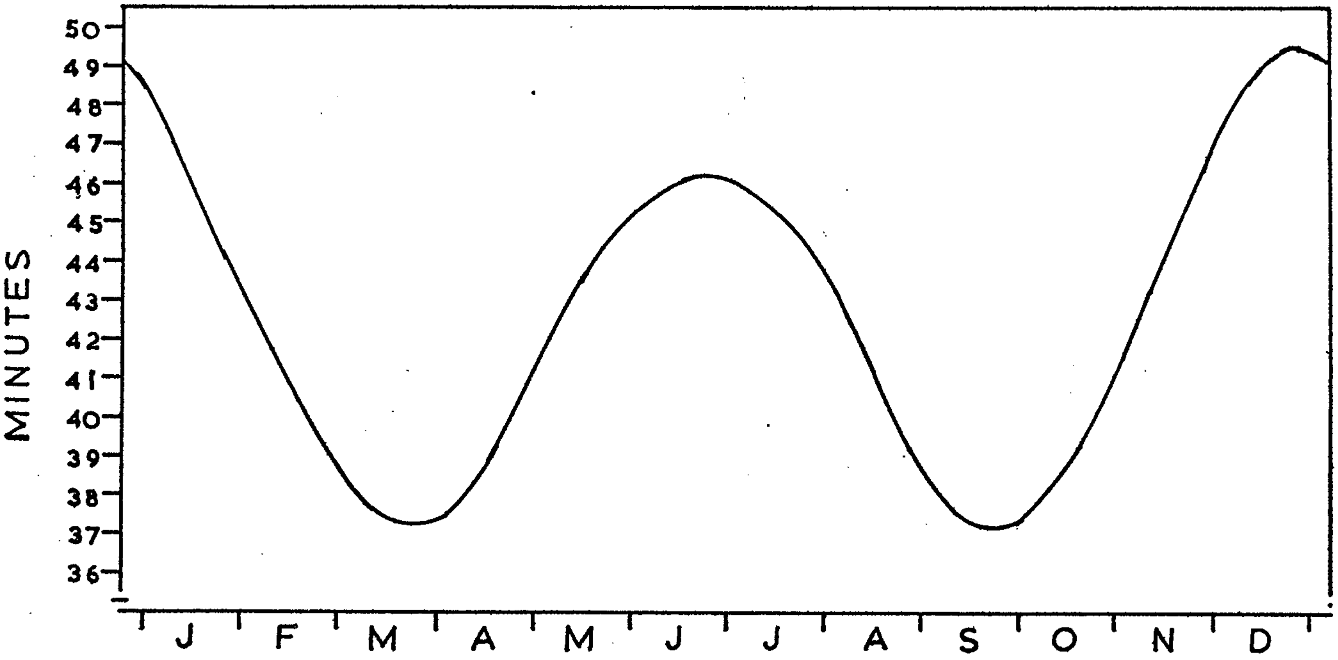

Figure 2.1

Length of time during which the sun is less than 3° above the horizon at 50° N. latitude. After Gallagher (1967). See Appendix A Table 4.

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CHAPTER III

THE METHOD

The Mathematical Model

In keeping with the empirical approach to the study of the relationship between solar radiation and sunshine duration used by Black *et al.* (1954), Hounam (1963), and Glover and McCulloch (1958a), a linear model has been chosen.

Implicit within the choice of a linear model are the following statistical assumptions (Snedecor and Cochran 1967, p. 141), where x is the independent variable and y is the dependent variable. These correspond to n and Q respectively.

1) For each selected x there is a normal distribution of y from which the sample value of y is drawn at random. These conditions are satisfied in this research since for any selected period of sunshine duration, the amount of solar radiation reaching the earth's surface is entirely a function of atmospheric attenuation over which the researcher has no control, and which varies according to the law of errors.
2) The population values of y corresponding to a selected x have a mean μ that lies on a straight line $\mu = \alpha + \beta x$ where α and β are parameters. In this study it is

assumed that the amount of solar radiation received on any one day is a random sample of size one from a population of values normally distributed with a mean μ. Each day also has such a population of sunshine durations. Robinson (1966, p. 127) states that if the sunshine cards of the Campbell-Stokes recorder are evaluated carefully following the World Meteorological Organizations' instructions, which include reasonable reductions for narrowing of the trace, a curvilinear relationship will always be the case between sunshine duration and total solar radiation. Page (1961, p. 378) makes the same observation regarding linearity. However, theoretically, the mean values of solar radiation should fall on a straight line if the factors of location and climate were to remain constant, and thus, the assumption of linearity is not precluded. It is the test of the strength of the linear relationship which determines whether the initial assumption is substantiated.

3) In each population, the standard deviation of y about its mean $\alpha + \beta x$ has the same value. The mathematical model is specified concisely by $y = \alpha + \beta x + \varepsilon$ where ε is a random error term.

The parameter α is the mean of the population that corresponds to $x = 0$. β is the slope of the regression line, the change in the mean of y per unit increase in x. Because the likelihood of receiving a value of solar radiation greater than the mean for any particular period of sunshine duration is as great as the likelihood of obtaining a value less than the mean for that particular sunshine duration, it is assumed that the mean of the $\varepsilon = 0$.

The prediction equation specifying the relationship between solar radiation and sunshine duration, therefore becomes $\hat{Q} = a + b\ n$.*

The Analysis

Elimination of Seasonal Effects

A meaningful analysis, with a view to discovering predictable trends in the parameters a and b of the estimation equation as these vary in response to fluctuations in atmospheric turbidity, cannot be performed on raw data that varies seasonally. Daily radiation receipt and the length of day in high latitudes suffer rapid seasonal change (Figures 3.1 and 3.2).

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* The symbol ^ over the Q specifies that Q is an estimated value. For convenience, this symbol will be omitted henceforth in this paper, but Q is still to be recognized as an estimated value only.

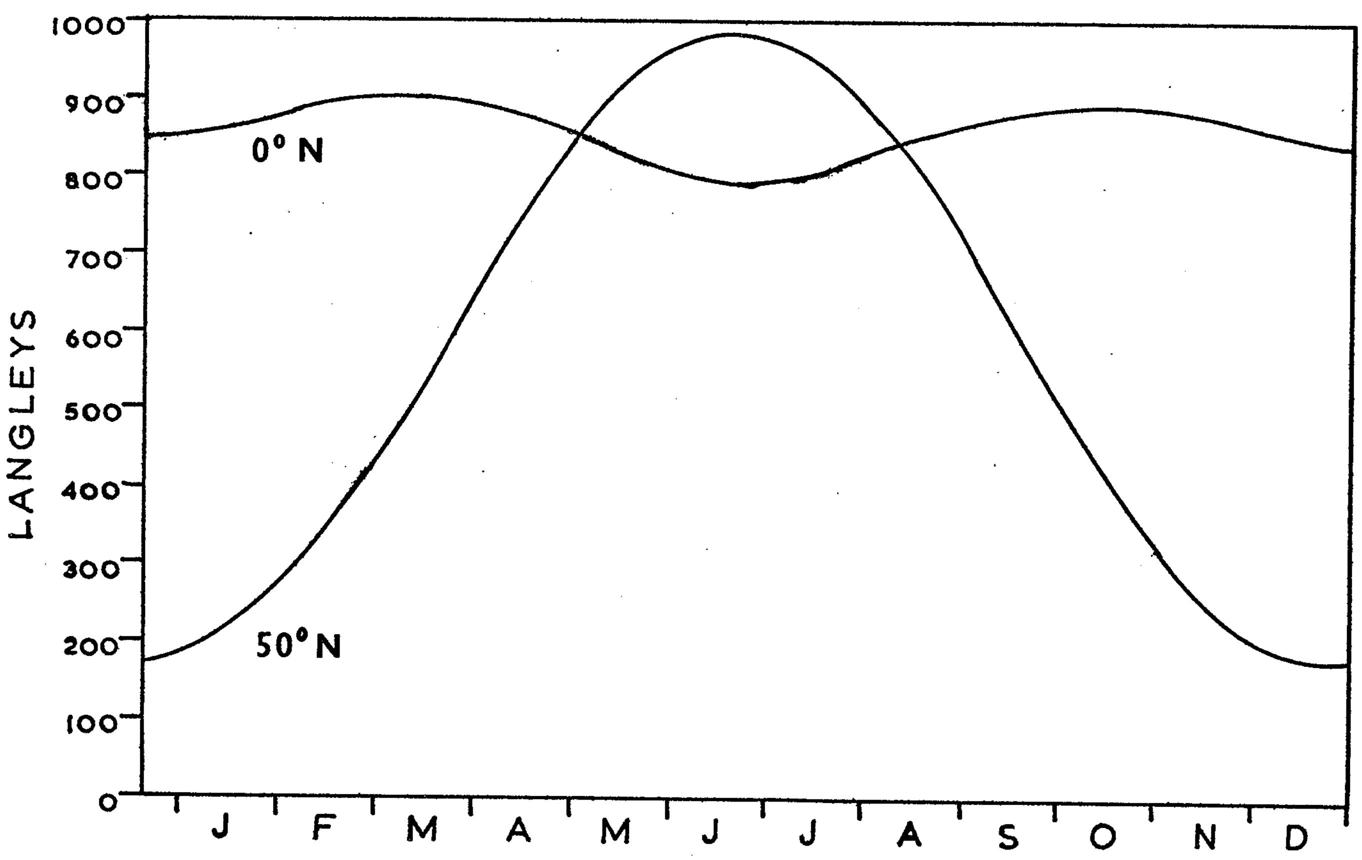

Figure 3.1

Receipt of solar radiation on a horizontal surface at 0° and 50° N. latitude at the top of the atmosphere (List 1949, p. 418). Appendix A Table 2.

To eliminate the obvious effects of seasonal variation in the raw data, both radiation receipt and sunshine duration recorded at the earth's surface are considered as percentages of the possible. Thus, the prediction equation becomes

$$Q = Q_{o_i} (a + b\ n/N_i)$$

where i refers to the day when Q and n were recorded.

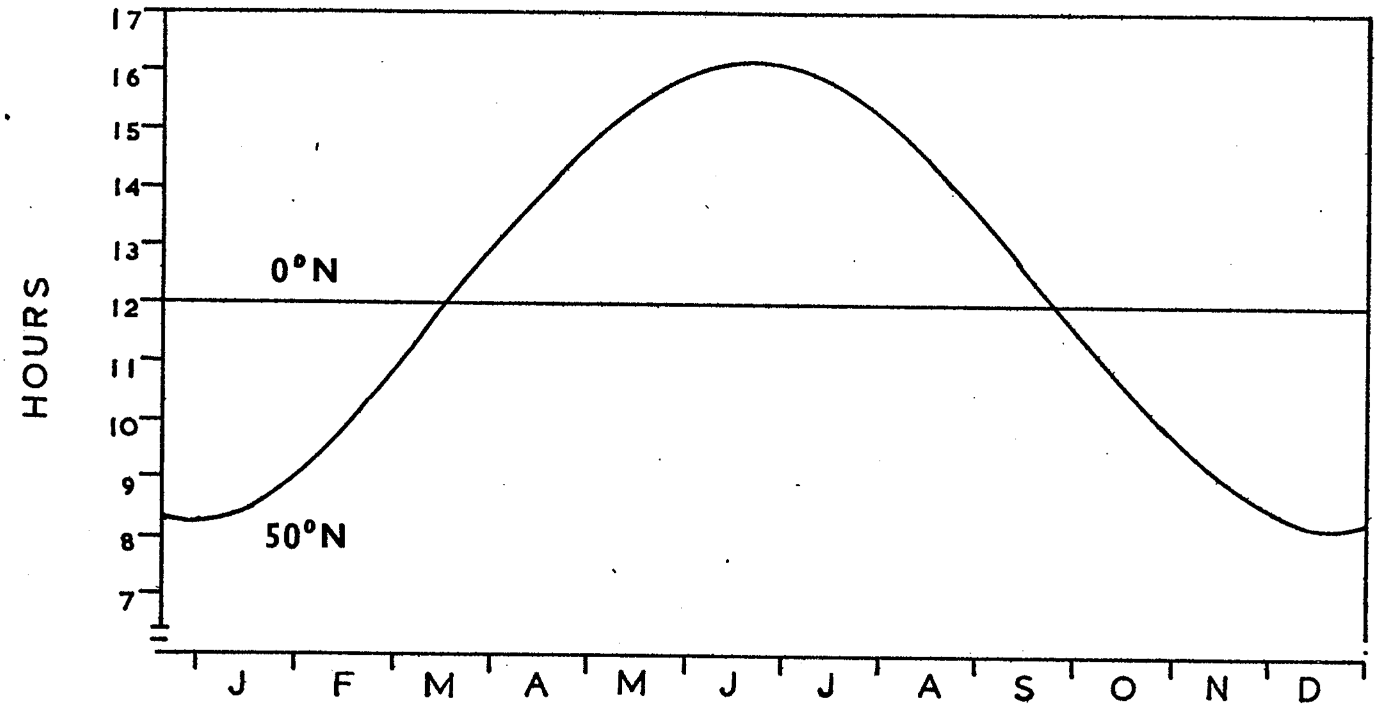

Figure 3.2

Length of day at 0° and 50° N. latitude (List 1949, p. 507).
See Appendix A Table 3.

Choice of Period Length

Page (1961, p. 378) stated that, in view of the rapid seasonal changes in radiation receipts in high latitudes, use of a short period length is indicated. To test this statement, the data will be analysed using 5, 7, 11, 15, 19, 25, and 29-day periods.

The choice of period length is purely arbitrary. For convenience, the month is defined as a period of 29-days duration, and all periods are chosen to extend over an odd number of days so that a mid-period value can easily be determined.

The Constants

Three constants are involved in this research. These are N_i, the possible duration of sunshine; Q_{oi}, the maximum receipt of solar radiation on a horizontal surface at ground level, and Q_{ai}, the receipt of solar radiation on a horizontal surface at the top of the atmosphere, where i refers to the middle day of any period.

N has already been defined above (See p. 19, and Appendix A, Tables 3 and 4).

Q_a for latitude 50° N. is calculated by the Smithsonian Institute (See Appendix A Table 2).

Q_o for Winnipeg is obtained by plotting the highest recorded value of total daily solar radiation for each day between the years 1950 and 1967 on a graph and drawing a smooth curve through the highest of these values by hand (Figure 3.3). This method is slightly modified from that of Sellers' (1965, p. 28) who used only one year's data. It is assumed that this curve most closely represents the maximum possible solar radiation receivable on a horizontal surface at ground level at Winnipeg on cloudless days. Presumably it is the solar radiation that exists after "normal" atmospheric attenuation.

The Procedure

Although Q_a can be predicted with ease, and lends itself well to spatial comparisons, Q_o will be used instead. A preliminary investigation (Appendix B) has shown that there is at least a 27 per cent increase in accountable variation when using Q_o, instead

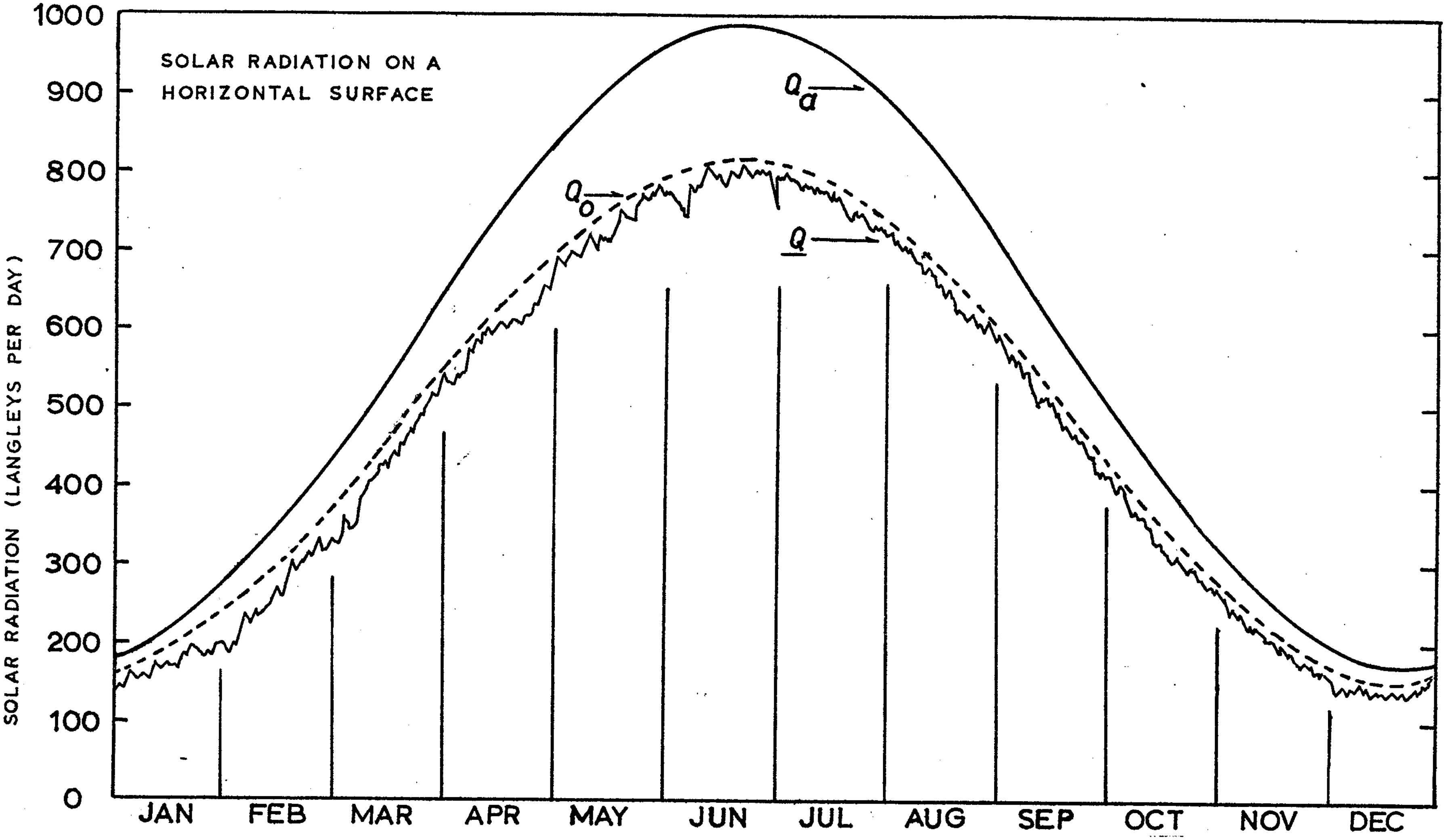

Figure 3.3

Daily total solar radiation on a horizontal surface at the top of the atmosphere (Qa), daily total solar radiation on a horizontal surface at the ground with clear skies (Qo), and the highest daily total solar radiation recorded on a horizontal surface at ground level (Q) over an eighteen year period at Winnipeg, Manitoba (1950 - 1967). See Appendix A, Table 1.

of Q_a, in the prediction equation

$$Q = Q_c\ (a + b\ n/N)$$

where Q_c is Q_a or Q_o. Because this study intends to determine the nature of, and the seasonal variation in, the predictability of the parameters a and b, it is desirable to maximize accountable variation.

To emphasize temporal changes, only mid-period values of Q_o and N will be used; that is, all the values of n and Q in any one period are considered to have the same possible maximum value. This reduces within-period variation since the daily values of Q_o and N in any period are the same. Thus, the differences between adjacent time periods are emphasized because Q_o and N, instead of being smoothly varying over the year, change by discrete amounts each period. This procedure makes the prediction of Q less precise, especially in the longer periods, but this research is primarily interested in the nature of the variations in a and b. Hence, at this stage, a precise estimate of Q is not required. To further emphasize trends, the periods will be overlapping, thus subduing the effects of any, possibly non-representative, extremes.

Consider the 5-day period. Period I consists of days 1 - 5; period II consists of days 2 - 6; period III consists of days 3 - 7; and so on. However, this use of overlapping periods means that period II is in the region of values defined by the mid-period constant applying to days 6 - 10.

Utilizing the fact of a common increment of 5,* a smoothing technique can be devised so that the constant K (Q_o or N), of any

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* or some other constant period length.

overlapping period p, where p = 1 to 366,** is as follows:

Period I $$K = \left[\frac{5}{5}\right] K_1 = K_1$$

II $$K = \left[\frac{5-1}{5}\right] K_1 + \left[\frac{1}{5}\right] K_2$$

III $$K = \left[\frac{5-2}{5}\right] K_1 + \left[\frac{2}{5}\right] K_2$$

until VI $$K = \left[\frac{5-5}{5}\right] K_1 + \left[\frac{5}{5}\right] K_2 = K_2$$

where K = 1 to 73.

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** Because February 29th is included in the analysis the year has 366 days. Furthermore, to obtain 366 periods of the same length, the first days of January are grouped with the last days of December.

CHAPTER IV

THE RESULTS AND THE STATISTICAL ANALYSIS

The results of the analysis are shown graphically in Figures 4.1 to 4.4, and are summarized in Appendix D. In all instances the linear dependence of Q/Q_o upon n/N over the duration of each time period is shown by a high correlation coefficient, and under the null hypothesis of no linear relationship, i.e., Ho: $\beta = 0$, each F-test is highly significant, thereby rejecting the null hypothesis (Appendix D).

Explanation of the Curves

The regression coefficients and corresponding y intercepts have been plotted for all periods. Each pair of a and b represents the linear prediction equation which summarizes the relationship between daily solar radiation and duration of sunshine that exists during a particular period. Because of the use of overlapping periods this produces 366 equations illustrating the seasonal trend of the parameters a and b.

All seven curves show that a is inversely related to b. There also appears to be a cyclic relationship. During the winter months, December through the middle of March, using periods of 5, 7, 11, and 15 days duration, a appears to be relatively high (between 0.4 and 0.5)

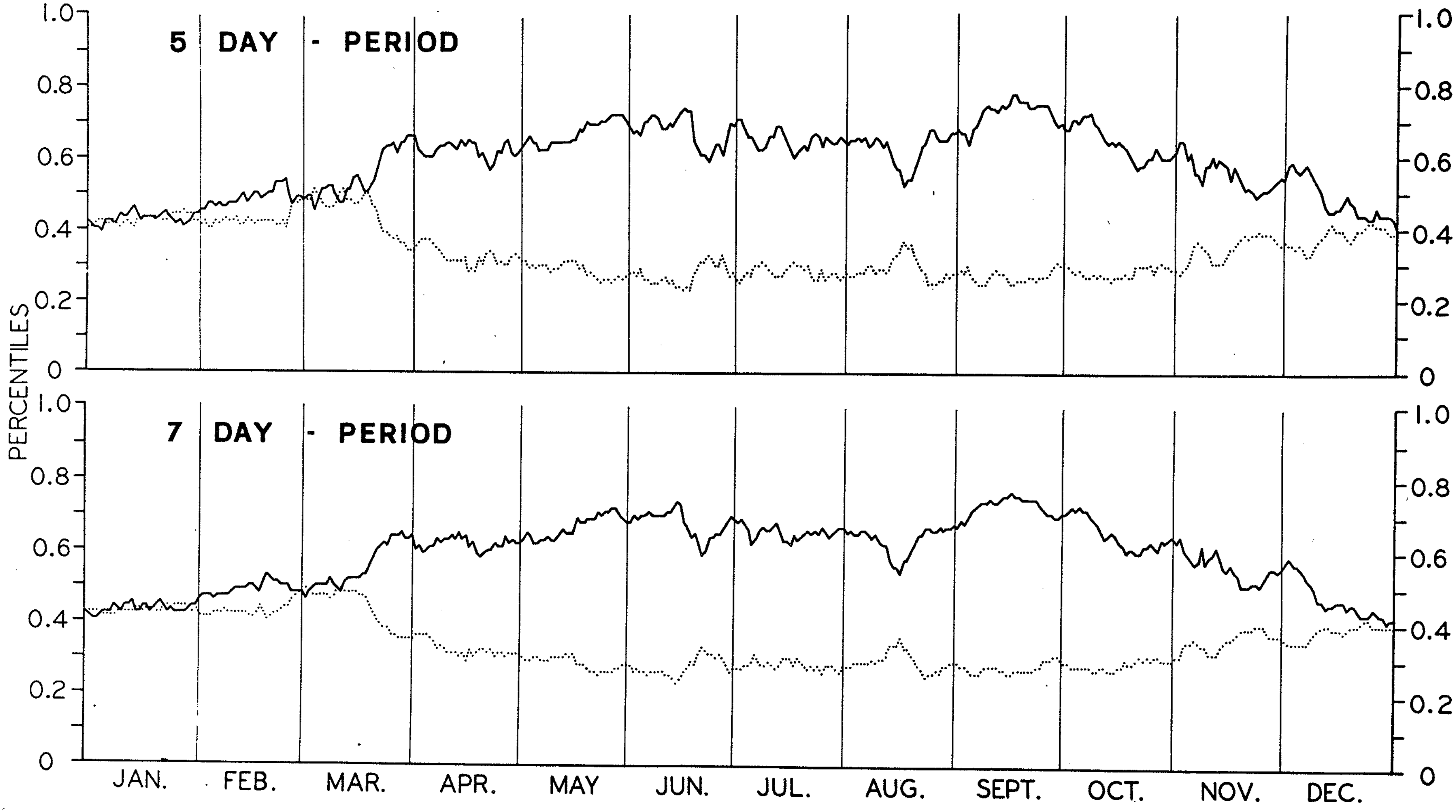

Figure 4.1
The parameters a and b for the regression of Q/Q_o upon n/N at Winnipeg 1950-67 for the 5 and 7-day periods are plotted for the entire year. The dotted line represents a and the solid line represents b.

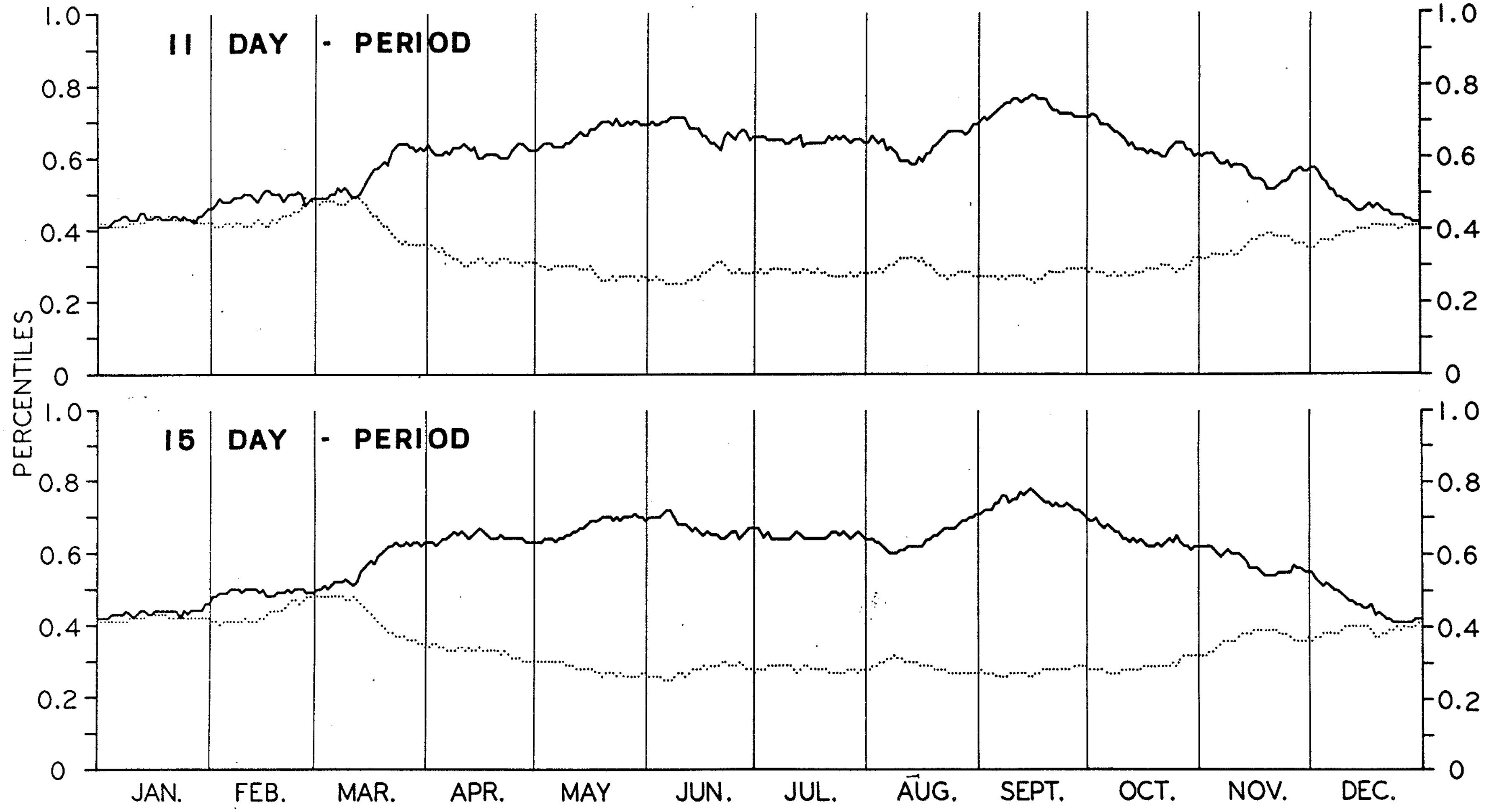

Figure 4.2
The parameters a and b for the regression of Q/Qo upon n/N at Winnipeg 1950-67 for the 11 and 15-day periods are plotted for the entire year. The dotted line represents a and the solid line represents b.

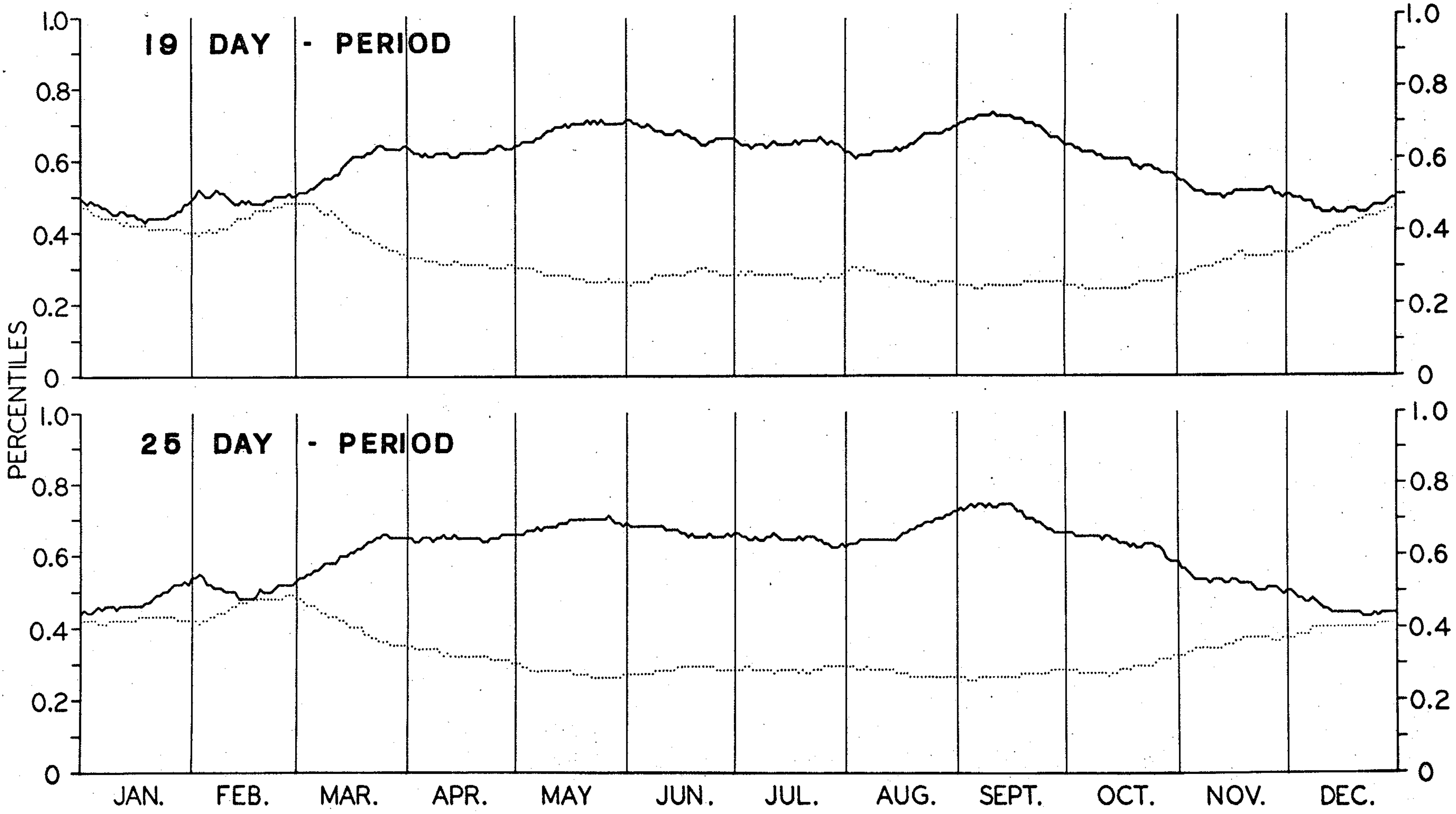

Figure 4.3
The parameters a and b for the regression of Q/Qo upon n/N at Winnipeg 1950-67 for the 19 and 25-day periods are plotted for the entire year. The dotted line represents a and the solid line represents b.

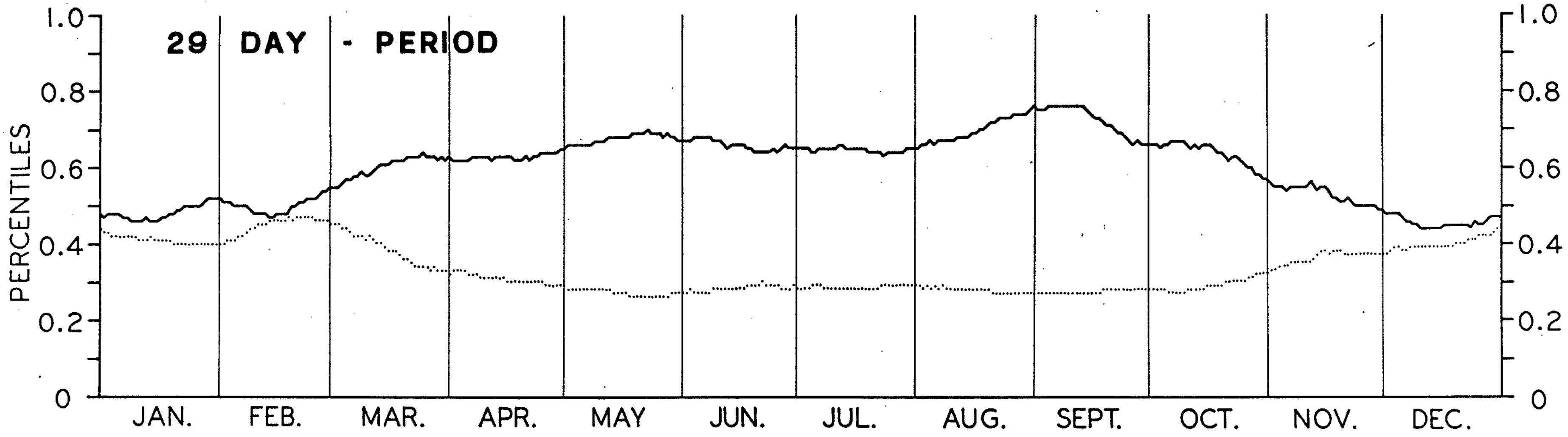

Figure 4.4
The parameters a and b for the regression of Q/Qo upon n/N at Winnipeg 1950-67 for the 29-day period are plotted for the entire year. The dotted line represents a and the solid line represents b.

and b appears to be relatively low (also between 0.4 and 0.5) when compared to the rest of the year when a decreases in value until it is approximately less than 0.3, and b increases to about 0.6. When periods of 19, 25, and 29-days are used, the high a and low b extend from December through February only.

The curves based upon the shorter periods exhibit marked variations in the values of a and b throughout the year but these tend to be depressed as the period length increases.

The curves based upon the shorter periods also exhibit a sudden divergence of a and b in the middle of March. This abrupt change does not have a comparably sharp convergence in fall. When longer periods (19, 25, and 29-days) are used the divergence of a and b in spring is about as gradual as their convergence in fall.

Two features, one occuring in March and the other in September, are noteworthy. These are the sudden divergence of a and b in spring and the peak in the values of b in September. However, the sudden divergence of a and b in March is evident only in the shorter periods, while the September peak in the values of b seems apparent only in the longer periods. This suggests that the durations of the causes of these phenomena are comparable to the length of the period of analysis which detects them. That is, the September peak is associated with a cause of a month or more in duration, while the March divergence is associated with an event of comparatively short duration.

The general appearance of the curves, in all seven cases, suggests that there are two distinct seasonal relationships between solar radiation and sunshine duration. The winter season ends when a and b diverge in March, and the summer season begins at this time.

Because of the lack of a similarly abrupt convergence in fall, the end of summer and the beginning of winter appears to be less easy to identify in these terms.

In order to establish the statistical significance of these seasonal relationships it is necessary to test the homogeneity of the regression coefficients.

Tests on the Regression Coefficients

The comparison of the regression coefficients is done by the analysis of covariance method (Snedecor and Cochran 1967, p. 419). Because statistical tests require the assumption of independent samples, only independent periods are tested. Furthermore, only the b's will be tested since they measure the dependence of Q/Q_o upon n/N and, therefore, best describe the relationship.

Test 1

This test determines whether any single regression coefficient is significantly different from any other regression coefficient; that is, it tests the homogeneity of the regression coefficients.

Under the null hypothesis

$$Ho: \beta_1 = \beta_2 = \beta_3 = \ldots\beta$$

where the b_i are the best estimates of the respective β_i and the alternate hypothesis,

$$Ha: \text{at least one of the } \beta_i \neq \beta$$

the results are as summarized in Table 4.1.

TABLE 4.1

COMPARISON OF REGRESSION COEFFICIENTS: ALL PERIODS

Period Length	Number of Periods	Pooled Residuals	Error df	Dif.btwn slopes	Slopes df	F*
5-Day	73	52.06	6328	7.21	72	12.18
7 "	52	53.17	6352	7.22	51	16.91
11 "	34	56.57	6563	7.00	33	24.60
15 "	25	59.57	6599	7.59	24	35.25
19 "	20	58.36	6699	5.61	19	33.89
25 "	15	68.75	6619	6.20	14	42.92
29 "	13	71.03	6659	6.02	12	47.04

Because each test statistic falls into the critical region, the null hypothesis is rejected for every period. This means that some of the regression coefficients are not alike.

The visual evidence suggests that there may be a two-season relationship between total daily solar radiation and sunshine duration. Apriori knowledge regarding the establishment of seasons indicates that in mid-continental North America, winter begins in early November and ends in late March (Bryson and Lahey 1968, p. 36). The criteria used in this de-limitation of winter are major changes in the atmospheric circulatory system, in response to changes in the atmospheric heat balance. Although the external parameters of the atmospheric heat balance are fixed, as in the rate of the earth's rotation, or are tied to the steady sinusoidal variation of the solar radiation, we know that the latter, while geometrically predictable and smoothly varying at the top of the atmosphere, affects the atmosphere largely

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* All F are significant at the one per cent level.

via the earth's surface (Bryson and Lahey 1958, p. 5). The disappearance of the snow cover in spring causes a lowering of the temperature differential which drives the circulatory system. Hence, spring is a period of intermediate storminess with diminishing frontal intensity during which cyclonic conditions tend to prevail. The end of autumn, although not necessarily defined by the presence of permanent snow cover, does respond to a stronger temperature differential because the northern regions receive a much lower intensity of solar radiation. The polar front moves far to the south (Northern Hemisphere) and a period of high storminess begins. Thereafter, continental highs tend to be well developed.

The regression coefficients are, therefore, divided into a "winter" group and a "summer" group, conforming as well as possible (within the limitations of the independent period requirement) to their limits as defined by Bryson and Lahey.*

Tests 2 and 3

Using the same null hypothesis and alternate hypothesis set down in the first test, the regression coefficients are analysed for a two-season relationship. Tables 4.2 and 4.3 summarize the results.

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* For convenience, winter will be arbitrarily defined as extending from November 1 to March 21. The first permanent snowfall generally occurs during the first 10 days of November, and, although the spring thaw occurs a week after the 21st of March, by this time the snow cover is fragmentary and quite dirty. In this division of the year into two seasons, this research agrees in substance with that of Mateer (1955, p. 582) wherein the year was divided into three groups; months with snow cover, months without snow cover, and transitional months.

TABLE 4.2

COMPARISON OF REGRESSION COEFFICIENTS; NOVEMBER 1 TO MARCH 21

Period Length	Number of Periods	Pooled Residuals	Error df	Dif.btwn slopes	Slopes df	F*
5-Day	28	22.83	2409	1.20	27	4.69
7 "	21	24.33	2547	1.39	20	7.27
11 "	12	23.74	2297	0.90	11	7.90
15 "	9	25.26	2356	1.47	8	17.09
19 "	7	24.48	2325	0.34	6	5.39
25 "	5	27.64	2180	0.30	4	5.69
29 "	5	33.05	2540	0.46	4	8.87

TABLE 4.3

COMPARISON OF REGRESSION COEFFICIENTS; MARCH 22 TO OCTOBER 31

Period Length	Number of Periods	Pooled Residuals	Error df	Dif.btwn slopes	Slopes df	F*
5-Day	45	29.21	3919	1.06	44	3.24
7 "	31	28.83	3805	0.81	30	3.57
11 "	21	30.96	4075	0.84	20	5.52
15 "	15	31.42	3891	0.83	14	7.47
19 "	12	30.03	4040	0.58	11	7.03
25 "	10	40.70	4439	0.56	9	6.78
29 "	8	37.97	4119	0.52	7	8.12

Again the null hypothesis is rejected. In both tests the conclusion is that some of the b's are significantly different from the others in each season.

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* All F are significant at the one per cent level.

Conclusions to the Tests on the Regression Coefficients

The results of these tests show that, despite appearances, statistically there is no support for the contention that the relationship between solar radiation and sunshine duration can be divided into seasonally unique periods based upon the present arbitrary division of the year into the two seasons defined. There is no evidence to support grouping the data in this manner in order to derive an empirical relationship that will estimate solar radiation from known sunshine data.

It might be possible to re-define the lengths of the seasons used in the test. But these results indicate that in this research, based upon the present sample data, this would not be advisable. A redefinition of the seasons might feasably adopt the astronomically defined winter which extends from December 21 to March 21. However, this would merely reduce the error degrees of freedom in the F-test. Even if a non-significant F is then obtained for "winter", one is still left with the significantly different regression coefficients of "summer".

Furthermore, the tests of the homogeneity of regression coefficients indicate that there does not appear to be any statistical advantage in analysing the data by a system of multiple period lengths. The results from the shortest period, that of the 5-day period, already yield a significant F. Increasing the period length has the effect of increasing the error degrees of freedom which makes the F-test more significant, thereby ensuring the rejection of the null hypothesis.

In the test statistic

$$F = \frac{\text{difference between slopes / slopes df}}{\text{pooled residuals / error df}} ,$$

the error degrees of freedom remain roughly constant since the total number of observations (73 × 5 for the 5-day period, or 13 × 29 for the 29-day period) are the same. However, because the degrees of freedom for slopes decrease as the period length increases, the effect is to increase F.

Page's (1961) suggestion that a shorter period of analysis in the high latitudes should be used when studying solar radiation by empirical methods is neither supported nor refuted. Climatologically, however, it is comforting to know that it is possible to study both the short term and long term effects within the relationship between total daily solar radiation and sunshine duration because both the short periods and the long periods yield significant F's.

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CHAPTER V

AN EMPIRICAL METHOD TO PREDICT a AND b

The statistical analysis of the regression coefficients depicting the day to day relationship between mean daily sunshine duration and mean daily solar radiation indicates that unique seasons are not identifiable; yet, inspection of the various curves of a and b suggests that some kind of periodicity does exist.

The march of the seasons is a cyclic phenomenon governed by the sinusoidal receipt of amounts of solar energy. This may, despite the elimination of seasonality in the raw data, be responsible for the periodic changes in a and b. An empirical method can be devised so that any estimates of solar radiation from sunshine data can be corrected for temporal variation by predicting the coefficients a and b as these vary throughout the year.

Independent time periods must be used in the prediction equation where a and b are the dependent variables and time is the independent variable. To ensure that the regression line fitted to the a and b curves will be based upon the maximum error degrees of freedom, the 5-day period will be used, since it has the greatest number of independent periods. Furthermore, the use of the 5-day period allows a sufficiently precise estimate of Q to be made from the equation $Q = Q_o\ (a + b\ n/N)$ because of the relatively small daily variation of the mid-period constants Q_o or N in each period. See

Appendix C Table 1. This precision of the estimate decreases as the period length increases, and, therefore, use of the shortest period length is advised.

The Linear Fit

Experimental results plotted on graphs sometimes display fictitious trends because of the exaggeration of one of the scales. For this reason, despite the apparent cyclic nature of the a and b curves, a straight line is initially fitted to the regression coefficients.

TABLE 5.1

ANALYSIS OF VARIANCE OF a AND b IN FIGURE 4.1
(5-DAY PERIOD) USING A LINEAR MODEL

Curve	Source of Variation	Error df	Sum of Squares	Mean Square	F*
a	regression	1	0.0667	0.067	16.94
	error	71	0.2794	0.004	
	total	72	0.3461		
b	regression	1	0.0759	0.076	8.88
	error	71	0.6065	0.009	
	total	72	0.6828		

Table 5.1 shows that, statistically, a linear estimation equation may be used to predict the values of a and b. The resultant equations are:

$$a' = 0.39 + 0.00143\ x$$

$$b' = 0.54 + 0.00153\ x,$$

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* F is significant at the one per cent level.

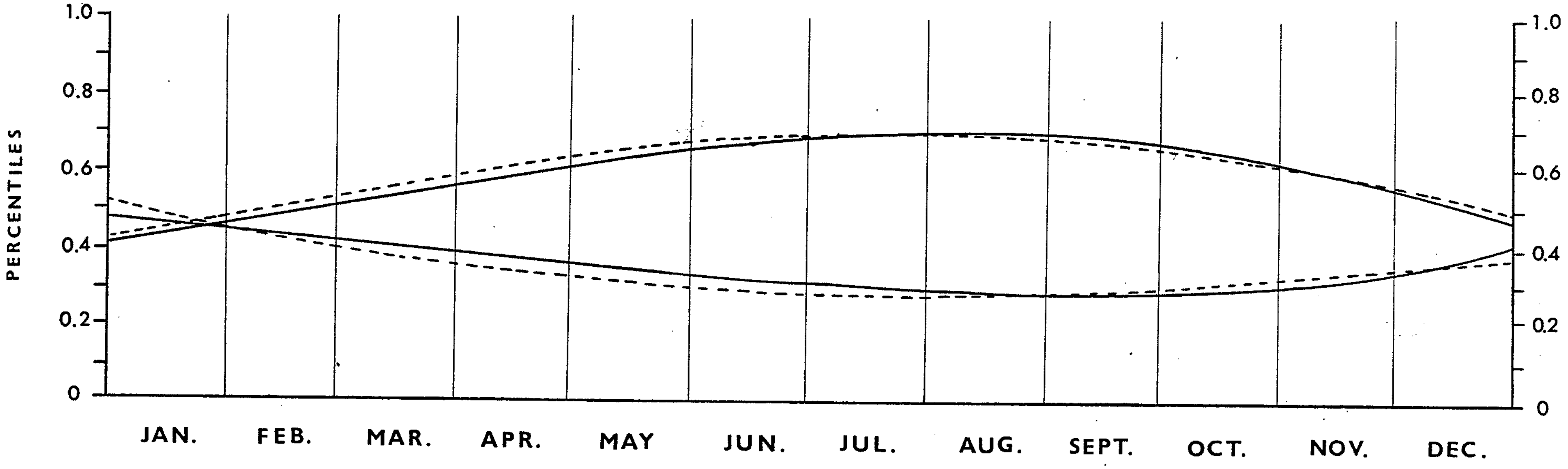

Figure 5.1
The second and third degree curves superposed upon each other and upon an implied distribution of the parameters a and b (Figure 4.1).
The second degree polynomial is the solid line and the third degree polynomial is the hatched line. Graphically the curves appear almost identical.

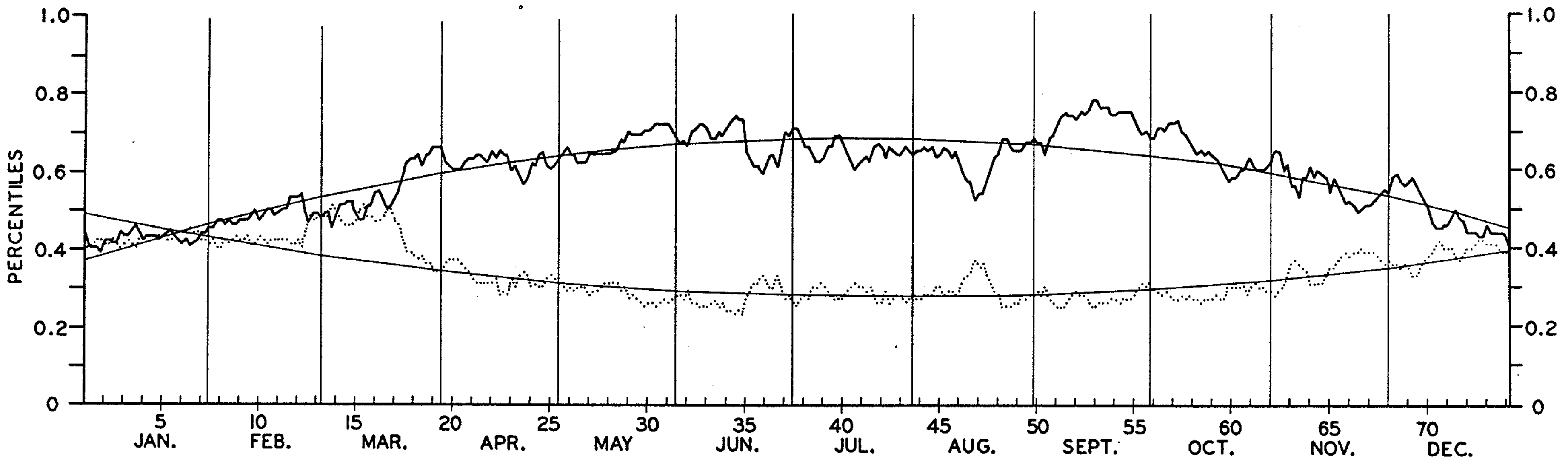

Figure 5.2
The second degree polynomial curves superposed upon the 5-day distribution of the parameters a and b.

where a' and b' are the predicted values of the dependent variable, and x is any 5-day period varying from 1 to 73. However, although the F-test is significant, thereby rejecting the null hypothesis of no linear relationship, the correlation coefficients are low (-0.44 and 0.33 respectively).

The Polynomial Fit

If the relation between a dependent variable and the independent variable is more curved than linear (as suggested by the visual evidence of Figures 4.1 to 4.4) a polynomial in the independent variable is often used as a descriptive equation (Snedecor and Cochran 1967, p. 349). Both a second degree and a third degree polynomial are used in order to ascertain a regression line of best fit.

TABLE 5.2

ANALYSIS OF VARIANCE OF a AND b IN FIGURE 4.1
(5-DAY PERIOD) USING A SECOND DEGREE POLYNOMIAL

Curve	Source of Variation	Error df	Sum of Squares	Mean Square	F*
a	regression	2	0.2347	0.117	73.88
	error	70	0.1112	0.002	
	total	72	0.3458		
b	regression	2	0.5317	0.266	123.78
	error	70	0.1504	0.002	
	total	72	0.6821		

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* F is significant at the one per cent level.

TABLE 5.3

ANALYSIS OF VARIANCE OF a AND b IN FIGURE 4.1 (5-DAY PERIOD) USING A THIRD DEGREE POLYNOMIAL

Curve	Source of Variation	Error df	Sum of Squares	Mean Square	F*
a	regression	3	0.2547	0.085	64.34
	error	69	0.0911	0.001	
	total	72	0.3458		
b	regression	3	0.5390	0.179	86.65
	error	69	0.1431	0.002	
	total	72	0.6821		

Both Table 5.2 and 5.3 show conclusively that the null hypothesis of no curvilinear relationship must be rejected.

The regression equations for the second degree polynomial are:

$$a' = 0.50187 - 0.0103760\ x + 0.00012084\ x^2$$

$$b' = 0.35526 + 0.0162589\ x - 0.00019903\ x^2,$$

with correlation coefficients of 0.82 and 0.88 respectively. The third degree polynomial yields:

$$a' = 0.45419 = 0.0028967\ x - 0.00013012\ x^2 + 0.00000226085\ x^3$$

$$b' = 0.38393 + 0.0117537\ x + 0.00004787\ x^2 - 0.00000136183\ x^3,$$

with correlation coefficients of 0.86 and 0.89 respectively.

Figure 5.1 shows the comparison between the second and third degree polynomial curves.

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* F is significant at the one per cent level.

The Prediction Equation

Statistically, all three descriptive equations can be used to predict the parameters a and b as these vary throughout the year. However, the correlation coefficients indicate that the polynomial regression equations are considerably better predictors of a and b than the linear equation (Table 5.4).

TABLE 5.4

COMPARISON OF THE COEFFICIENTS OF DETERMINATION

Curve		r	r^2 in %
a'	linear	- 0.44	19.36
	second	0.82	67.24
	third	0.86	73.96
b'	linear	0.33	10.89
	second	0.88	77.44
	third	0.89	79.21

The increased precision of the polynomial equations over the linear equation is shown by the coefficient of determination (r^2), which is a measure of the variation within the relationship attributable to the dependence of a or b upon x. There is a 48 per cent increase in the accountable variation in a' when using the second degree polynomial, and a 54 per cent increase when using the third degree polynomial. For b', these increases are 66 per cent and 68 per cent respectively. Therefore, since it is desirable to maximize the accountable variation when estimating total daily solar radiation from sunshine duration data, the choice of the polynomial equations to estimate the parameters a and b is definitely indicated.

This can also be shown statistically by the means of the extra sums of squares principle (Draper and Smith 1966, p. 67) which tests whether the use of a more sophisticated prediction equation is justified.

TABLE 5.5

COMPARISON OF THE PREDICTION EQUATIONS

Parameter	Equation	Sum of Squares Regression	Regression df	Error Mean Square	F
	linear	.0667	1		
a'	second deg.	.2347	2	.0016	105.0000*
	third deg.	.2547	3	.0021	9.5238*
	linear	.0759	1		
b'	second deg.	.5317	2	.0013	350.6154*
	third deg.	.5390	3	.0021	3.4762 N.S.

Table 5.5 shows that the added information explained by the second degree prediction equation, when compared to the information explained by the linear prediction equation, fully justifies its use. However, this conclusion is only partially valid for the additional information obtained by the use of the third degree prediction equation. The result of the F-test reveals that the additional information obtained when using the third degree equation instead of the second

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* The critical F at 1 per cent significance and at 69 and 70 degrees of freedom is approximately 8.35.

degree equation, in the case of the b' estimator is not significant statistically, but this is not so in the case of the a' estimator. Thus the optimum equations to use in order to predict a and b, are the second degree polynomial for b, and the third degree polynomial for a.

However, visual comparison of the a' estimates resulting when the second and third degree equations are plotted (Figure 5.1) reveals that the prediction curves are very similar. Furthermore, an examination of r^2 indicates that there is only a 6 per cent increase in the accountable variation in a' when using the third degree equation to predict a instead of the second degree equation.

Thus, the decision to use, or not to use, the third degree polynomial to predict a must be made on climatological grounds — depending on the purpose the estimates are required to serve. In most cases where estimates of solar radiation are required, time or effort may be important factors, and, since the a' polynomial equations are so similar, the second degree equation is probably adequate. Squaring x is a simple procedure on small desk calculators, but cubing x is a much more difficult and time consuming. Thus, for these reasons, and for the sake of conformity, the second degree polynomial equations will be recommended for use in Winnipeg (Figure 5.2).

References

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CHAPTER VI

CONCLUSION

Summary

Accounts of the distribution of solar radiation over the surface of the earth have to rely on estimated values derived from climatological data, as well as on measured values. Much research has been devoted to the determination of the optimum universal equation which will estimate solar radiation at any place and time of year. Such an equation, however, assumes that there is no temporal or spatial variation in atmospheric turbidity other than latitude. The universal equations derived in the past have often failed to detect the effects of such variations largely because they are based upon monthly and yearly data, usually derived from groups of stations, thereby losing accuracy through generalization.

The present research at Winnipeg has, by using daily data, been designed to detect and predict any effects of temporal variation in atmospheric turbidity, as displayed in the parameters a and b of the linear prediction equation

$$Q = Q_o (a + b\ n/N),$$

as this fluctuates throught the year in response to the changing planetary albedo.

Conclusion

It is found that a and b are inversely related and, using a sub-division of the year into 5-day periods, can be accurately estimated by

$$a' = 0.50187 - 0.0103760\ x + 0.00012084\ x^2$$

$$b' = 0.35526 + 0.0162589\ x - 0.00019903\ x^2$$

where x is the 5-day period under investigation. No restriction is placed upon the choice of any period x provided that successive periods are independent of each other.

Thus one of the limitations of a universal equation for estimating solar radiation is eliminated by providing a method to account for temporal variations in atmospheric turbidity.

Suggestions for Further Research

The most obvious extension of this research is to investigate the spatial variations in the relationship between a and b by applying the analysis to the data observed within a network of stations. Tentative steps have already been undertaken with respect to examining the data recorded at Great Falls, Mont., Bismark, N.D., Lethbridge, Alta., and Saskatoon, Sask. These stations all have a history of long, reliable records; and, all have a similar mid-continental, prairie location. Differences occur primarily in the general climatic events (beginning of spring, summer evapo-transpiration rate, etc.) due to positional factors such as latitude and longtitude. Consequently, these stations lend themselves well to spatial comparisons.

If predictable trends can be discovered, a method might profitably be devised whereby Q_a can be used instead of Q_o in the analysis so that spatial comparisons can be more readily made since Q_a can be predicted with ease while Q_o must be determined at each station. This procedure, however, is likely to seriously modify the results of the analysis because Q_o is a measure of the response of solar radiation to atmospheric attenuation, whereas, Q_a is a measure of solar radiation before attenuation, and may therefore be incapable of detecting similar variations in a and b as can Q_o.

Several of the behavioral characteristics of the curves a and b in Figures 4.1 to 4.4 require investigation. The sudden divergence of a and b in spring suggests an equally sudden change in atmospheric turbidity. In 1955 Mateer noted that his equation for Canada, based on monthly data, usually underestimated Q/Q_o in winter and overestimated Q/Q_o in the summer. He proposed two explanations for the low estimate in winter.

When snow cover is present along with broken or overcast cloud, some solar radiation may be trapped between the highly reflecting snow surface and the cloud. The resultant multiple reflections yield a high recording of solar radiation, while low sunshine duration is recorded beneath the cloudy skies.

He also put forward the possibility that rime deposition on the spherical glass lens of the sunshine recorder might interfere with the sunshine record in winter. Judging by the consistency of the high a and low b values in winter, rime interference on the recorder might possibly be ruled out as an important factor in this respect because the instrument is attended daily. Because a is the mean value of radiation received when sunshine is zero, the multiple reflection effect may explain the high value of a in winter. In spring the earth is at its vernal equinox and at this time the snow cover of the Northern Hemisphere disappears rapidly. This is a sudden and drastic change in albedo. Since the atmospheric heat budget responds to changes in the earth's surface cover, the sudden change in the relationship between daily solar radiation and sunshine duration may be attributed to variations in cloud amount and type as a result of fluctuations in the heat budget in response to the lower albedo.

The increase in the value of b during spring may be due to increased amounts of diffuse radiation (reflection from the sides of cumuliform clouds) reaching the surface of the earth, thus making the receipt of total daily solar radiation apparently more dependent on sunshine duration. The lower value of a probably shows the effect of a decreased albedo on overcast days.

The primary maximum of b in September is also worthy of investigation. At this time of the year most of the moisture of spring and early summer has been dissipated, plants are drying up, the albedo changes from that of a green surface cover to one of yellow and brown. This may cause a readjustment in the atmospheric heat budget or a further change in the rate of multiple reflection between cloud and ground. Diffuse radiation may be increased again through refraction and diffraction of the solar beam by hygroscopic dust nuclea.

Furthermore, the apparent constancy of the levels of a and b in December through March (Figures 4.1 to 4.4) may be real. This possibility should be investigated by the application of more sensitive tests than those discussed in this thesis.

References

Mateer, C.L., 1955: A Preliminary Estimate of the Average Insolation in Canada. Canad. J. Agric. Sci., V. 35 pp. 579 - 594.

APPENDIX A

TABLE 1

Highest values of total daily solar radiation recorded on each day of the year on a horizontal surface at Winnipeg, 1950-1967, in langleys.

TABLE 2

Total daily solar radiation on a horizontal surface at the top of the atmosphere at 50° N latitude, in langleys (List, 1959).

TABLE 3

Duration of daylight at 50° N latitude in hours and minutes (List, 1949).

TABLE 4

Duration of time during which the sun is less than 3° above the horizon at 50° N latitude (Gallagher, 1967).

TABLE 1

HIGHEST VALUES OF TOTAL DAILY SOLAR RADIATION RECORDED ON EACH DAY OF THE YEAR ON A HORIZONTAL SURFACE AT WINNIPEG, 1950-1967, IN LANGLEYS

Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys
Jan 1	142	Feb 1	209	Mar 1	362	Apr 1	533
2	149	2	231	2	367	2	540
3	146	3	231	3	375	3	536
4	140	4	211	4	397	4	561
5	163	5	228	5	383	5	529
6	140	6	246	6	398	6	575
7	157	7	250	7	387	7	586
8	138	8	246	8	393	8	560
9	147	9	259	9	399	9	612
10	150	10	257	10	380	10	568
11	159	11	257	11	431	11	621
12	155	12	261	12	425	12	604
13	158	13	274	13	430	13	608
14	188	14	300	14	438	14	635
15	168	15	286	15	456	15	631
16	170	16	296	16	464	16	604
17	171	17	300	17	445	17	631
18	180	18	309	18	428	18	641
19	173	19	303	19	479	19	632
20	189	20	322	20	479	20	599
21	184	21	319	21	477	21	565
22	201	22	321	22	489	22	627
23	183	23	330	23	491	23	648
24	191	24	348	24	494	24	628
25	192	25	339	25	518	25	635
26	204	26	338	26	507	26	630
27	198	27	357	27	515	27	646
28	204	28	369	28	491	28	664
29	210	29	349	29	530	29	650
30	210			30	521	30	666
31	221			31	507		

TABLE 1--Continued

Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys
May 1	685	June 1	767	July 1	725	Aug 1	681
2	669	2	753	2	766	2	691
3	675	3	793	3	742	3	675
4	706	4	754	4	738	4	663
5	691	5	735	5	780	5	678
6	686	6	755	6	660	6	669
7	695	7	734	7	746	7	656
8	700	8	723	8	738	8	636
9	672	9	744	9	750	9	670
10	705	10	769	10	769	10	644
11	682	11	746	11	717	11	631
12	737	12	762	12	740	12	639
13	714	13	745	13	748	13	636
14	733	14	782	14	721	14	636
15	743	15	791	15	692	15	623
16	732	16	784	16	742	16	614
17	701	17	759	17	732	17	597
18	722	18	742	18	705	18	601
19	727	19	774	19	708	19	618
20	740	20	807	20	736	20	588
21	736	21	780	21	716	21	582
22	765	22	766	22	702	22	577
23	747	23	772	23	690	23	600
24	749	24	744	24	703	24	589
25	772	25	736	25	698	25	572
26	754	26	767	26	712	26	571
27	758	27	768	27	721	27	549
28	785	28	724	28	678	28	572
29	734	29	749	29	694	29	513
30	721	30	797	30	679	30	558
31	781			31	682	31	540

TABLE 1--Continued

Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys
Sept 1	545	Oct 1	372	Nov 1	227	Dec 1	145
2	533	2	367	2	216	2	148
3	516	3	358	3	221	3	146
4	532	4	354	4	210	4	153
5	504	5	354	5	218	5	152
6	491	6	361	6	201	6	151
7	515	7	349	7	191	7	148
8	519	8	352	8	210	8	145
9	505	9	343	9	236	9	136
10	514	10	332	10	184	10	142
11	486	11	338	11	211	11	135
12	482	12	310	12	244	12	145
13	477	13	317	13	226	13	138
14	480	14	317	14	183	14	145
15	471	15	304	15	285	15	142
16	461	16	305	16	172	16	147
17	478	17	299	17	178	17	137
18	428	18	294	18	198	18	129
19	460	19	282	19	175	19	140
20	444	20	297	20	171	20	139
21	427	21	267	21	153	21	142
22	427	22	256	22	172	22	131
23	409	23	292	23	183	23	127
24	395	24	259	24	159	24	135
25	406	25	246	25	134	25	136
26	419	26	259	26	163	26	150
27	412	27	247	27	161	27	150
28	385	28	241	28	149	28	142
29	400	29	230	29	167	29	142
30	401	30	236	30	148	30	128
		31	233			31	135

TABLE 2

TOTAL DAILY SOLAR RADIATION ON A HORIZONTAL SURFACE AT THE TOP OF THE ATMOSHPHERE AT 50° N LATITUDE, IN LANGLEYS

(List, 1949)

The solar constant is assumed to be 1.94 ly. To obtain the value for a solar constant of 2.00 ly, a correction factor of 1.03093 must be applied.

Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys	Day of year	Q in lys
Jan 13	205	Apr 13	732	Jul 15	954	Oct 16	414
Feb 4	289	May 6	867	Aug 8	859	Nov 8	286
Feb 26	419	May 29	958	Aug 31	725	Nov 30	204
Mar 21	474	Jun 22	989	Sep 23	568	Dec 22	176

TABLE 3

DURATION OF DAYLIGHT AT 50° N LATITUDE IN HOURS AND MINUTES

(List, 1949)

	Day of the month							
	1	5	9	13	17	21	25	29
Month	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
Jan	08 10	08 15	08 21	08 30	08 38	08 48	08 59	09 11
Feb	09 20	09 33	09 46	10 00	10 15	10 28	10 43	10 58
Mar	10 58	11 12	11 28	11 42	11 58	12 13	12 28	12 43
Apr	12 55	13 09	13 24	13 38	13 53	14 07	14 21	14 34
May	14 41	14 54	15 07	15 19	15 30	15 40	15 50	15 59
Jun	16 04	16 11	16 16	16 20	16 22	16 23	16 21	16 20
Jul	16 18	16 13	16 08	16 01	15 53	15 44	15 34	15 22
Aug	15 14	15 03	14 50	14 37	14 23	14 09	13 55	13 41
Sep	13 31	13 16	13 01	12 47	12 32	12 17	12 02	11 47
Oct	11 29	11 24	11 10	10 56	10 40	10 26	10 12	09 59
Nov	09 48	09 35	09 22	09 10	08 58	08 47	08 38	08 29
Dec	08 24	08 17	08 12	08 08	08 06	08 04	08 05	08 07

TABLE 4

DURATION OF TIME DURING WHICH THE SUN IS LESS THAN 3° ABOVE THE HORIZON AT 50° N LATITUDE (Gallagher, 1967)

Day of year	Daily correction	Day of year	Daily correction	Day of year	Daily correction
Jan 1	48.72	May 1	40.40	Sept 1	38.08
10	47.52	10	42.48	10	37.60
22	45.28	22	43.20	22	37.36
Feb 3	42.48	Jun 3	44.96	Oct 1	38.40
12	40.88	12	45.84	10	38.96
24	39.12	21	46.24	22	39.52
Mar 2	38.48	Jul 3	45.84	Nov 3	41.36
11	37.76	12	45.92	12	43.04
24	37.47	21	43.76	24	46.40
Apr 1	37.60	Aug 2	41.76	Dec 3	46.80
10	37.92	11	40.48	12	47.92
22	39.76	23	39.04	21	49.50
				30	48.96

APPENDIX B

COMPARISON OF Q_o AND Q_a

A preliminary investigation was designed to compare the relative advantages of Q_o and Q_a. The data were grouped into independent 5 and 29-day periods so that one mean value for the eighteen years of observation of Q/Q_a, Q/Q_o and n/N for each period of the year could be obtained. A linear regression line was then fitted to the data.

TABLE 1

COMPARISON OF THE 5-DAY AND 29-DAY PERIODS SHOWING THE DIFFERENCES OBTAINED BY THE USE OF Q_o AND Q_a AT WINNIPEG.

Period	Constant	a	b	r	r^2	F
5	Q_a	0.33	0.42	0.66	0.44	55.24
29	Q_a	.35	.37	.59	.35	5.78 N.S.
5	Q_o	.39	.50	.84	.71	173.72
29	Q_o	.41	.48	.82	.66	21.88

Table 1 shows that there is a considerable difference in the results obtained by the use of Q_a and Q_o. This is particularily noticeable in r and r^2. The differences in r shows that a closer degree of linear association between Q and n is obtained when Q_o is used; and the corresponding differences in r^2 (27 per cent in the 5-day period and 31 per cent in the 29-day period) indicates that there is a greater amount of accountable variation in the relationship when using Q_o instead of Q_a.

The F-test reveals some interesting results in this respect. Under the null hypothesis that no linear relationship exists over the year, at the 1 per cent level of significance all null hypotheses except that of the 29-day (Q_a) period are rejected. We can be 99 per cent confident that a linear relationship exists over the year between solar radiation and sunshine duration when using Q_o, and when using Q_a for the 5-day period.

However, the failure to reject the null hypothesis when Q_a is used with a period size of 29-days suggests that past research which used monthly means to derive equations for universal application, is subject to question.

APPENDIX C

TABLES 1 - 7

The mid-period constants used in the equation

$$Q = Q_c\ (a + b\ n/N)$$

for the multiple-period analysis at Winnipeg where:

Q_c = Q_o or Q_a,

P = the period number,

N = maximum duration of sunshine (corrected length of day),

Q_o = total daily solar radiation received on a horizontal surface at the bottom of the atmosphere on a cloudless day,

Q_a = total daily solar radiation received on a horizontal surface at the top of the atmosphere.

TABLE 1

MID-PERIOD CONSTANTS FOR THE 5-DAY PERIOD

P	N	Q_o	Q_a	P	N	Q_o	Q_a
1	7.4	167	185	39	15.3	777	955
2	7.5	174	195	40	15.1	765	950
3	7.7	185	205	41	15.0	753	930
4	7.9	198	218	42	14.7	738	910
5	8.1	215	235	43	14.5	723	890
6	8.4	233	255	44	14.3	704	865
7	8.7	255	280	45	14.1	680	840
8	8.9	276	305	46	13.7	658	810
9	9.3	300	330	47	13.5	635	785
10	9.6	324	365	48	13.2	610	755
11	9.9	350	395	49	12.9	583	725
12	10.2	374	430	50	12.6	554	690
13	10.5	399	465	51	12.4	524	665
14	10.8	427	495	52	12.1	494	630
15	11.1	455	530	53	11.6	467	590
16	11.5	485	565	54	11.4	438	555
17	11.8	512	595	55	11.1	410	515
18	12.1	543	630	56	10.7	383	485
19	12.4	573	660	57	10.4	357	450
20	12.7	600	695	58	10.1	334	420
21	13.0	625	730	59	9.8	311	390
22	13.3	652	760	60	9.5	288	360
23	13.6	675	790	61	9.3	264	335
24	13.9	697	820	62	8.8	242	310
25	14.1	716	850	63	8.7	221	285
26	14.4	734	875	64	8.4	203	260
27	14.6	748	900	65	8.1	187	240
28	14.8	760	920	66	7.9	177	225
29	15.1	773	935	67	7.7	169	210
30	15.2	783	955	68	7.5	163	195
31	15.3	793	970	69	7.4	157	190
32	15.5	800	980	70	7.3	154	180
33	15.5	805	990	71	7.3	150	175
34	15.6	809	990	72	7.3	154	175
35	15.6	807	990	73	7.3	157	180
36	15.6	803	985				
37	15.5	795	980				
38	15.4	787	970				

TABLE 2

MID-PERIOD CONSTANTS FOR THE 7-DAY PERIOD

P	N	Q_o	Q_a	P	N	Q_o	Q_a
1	7.4	167	186	27	15.5	790	978
2	7.6	180	200	28	15.3	775	963
3	7.9	199	220	29	15.1	757	942
4	8.3	222	245	30	14.8	738	913
5	8.6	250	275	31	14.5	715	887
6	9.0	280	309	32	14.2	685	885
7	9.5	313	350	33	13.8	654	815
8	9.9	349	394	34	13.4	621	767
9	10.3	388	444	35	12.9	582	730
10	10.8	427	490	36	12.5	540	678
11	11.2	467	535	37	12.1	500	635
12	11.7	507	580	38	11.5	460	587
13	12.1	548	628	39	11.1	422	540
14	12.5	590	675	40	10.8	383	493
15	12.9	625	723	41	10.4	347	448
16	13.3	652	767	42	9.9	315	401
17	13.8	693	810	43	9.5	283	355
18	14.1	720	855	44	9.1	250	318
19	14.5	742	887	45	8.7	220	285
20	14.8	760	913	46	8.4	197	255
21	15.0	776	942	47	8.0	180	230
22	15.3	790	963	48	7.7	166	208
23	15.5	810	978	49	7.5	160	194
24	15.6	808	987	50	7.3	155	182
25	15.6	806	990	51	7.2	151	176
26	15.6	800	985	52	7.3	156	180

TABLE 3

MID-PERIOD CONSTANTS FOR THE 29-DAY PERIOD

P	N	Q_o	Q_a	P	N	Q_o	Q_a
1	7.8	183	211	8	14.3	700	872
2	9.3	305	340	9	12.7	543	701
3	11.1	455	521	10	10.9	395	505
4	12.9	618	715	11	9.2	255	322
5	14.4	745	888	12	7.7	167	209
6	15.3	803	980	13	7.3	155	183
7	15.3	785	972				

TABLE 4

MID-PERIOD CONSTANTS FOR THE 11-DAY PERIOD

P	N	Q_o	Q_a	P	N	Q_o	Q_a
1	7.4	170	190	18	15.3	177	964
2	7.9	195	218	19	14.9	750	932
3	8.4	233	257	20	14.4	715	875
4	9.1	281	309	21	13.9	667	833
5	9.7	334	375	22	13.3	615	765
6	10.3	388	445	23	12.7	552	695
7	11.0	450	515	24	12.0	488	622
8	11.7	513	587	25	11.4	432	547
9	12.4	577	612	26	10.6	366	472
10	13.0	635	736	27	9.9	315	400
11	13.5	689	805	28	9.3	264	332
12	14.3	730	867	29	8.7	217	280
13	14.8	760	910	30	8.1	183	236
14	15.1	785	949	31	7.7	165	203
15	15.5	803	980	32	7.4	155	183
16	15.6	807	989	33	7.2	149	176
17	15.6	797	985	34	7.4	156	185

TABLE 5

MID-PERIOD CONSTANTS FOR THE 25-DAY PERIOD

P	N	Q_o	Q_a	P	N	Q_o	Q_a
1	7.7	185	205	9	14.6	728	804
2	8.9	263	300	10	13.4	620	772
3	10.3	388	457	11	11.9	485	607
4	11.9	530	610	12	10.3	340	440
5	13.4	635	775	13	8.8	228	295
6	14.8	755	878	14	7.5	165	203
7	15.5	804	980	15	7.3	152	177
8	15.5	790	977				

TABLE 6

MID-PERIOD CONSTANTS FOR THE 15-DAY PERIOD

P	N	Q_o	Q_a
1	7.5	174	193
2	8.1	215	242
3	8.9	276	304
4	9.8	349	394
5	10.7	422	490
6	11.7	513	587
7	12.6	600	688
8	13.5	625	788
9	14.3	734	872
10	14.9	772	939
11	15.5	800	975
12	15.6	806	990
13	15.4	787	975
14	14.9	753	935
15	14.3	703	822
16	13.6	635	790
17	12.7	552	695
18	11.8	466	595
19	10.9	383	495
20	9.9	310	395
21	9.0	240	310
22	8.2	177	244
23	7.6	162	198
24	7.3	150	176
25	7.4	165	183

TABLE 7

MID-PERIOD CONSTANTS FOR THE 19-DAY PERIOD

P	N	Q_o	Q_a
1	7.6	155	199
2	8.5	243	260
3	9.6	325	365
4	10.7	420	469
5	11.9	525	610
6	13.0	633	735
7	14.1	718	850
8	15.1	770	935
9	15.6	804	980
10	15.6	800	985
11	15.0	766	955
12	14.5	715	875
13	13.6	635	790
14	12.4	536	776
15	11.4	432	553
16	10.7	347	434
17	9.2	255	322
18	8.2	190	243
19	7.5	160	190
20	7.2	152	177

APPENDIX D

TABLES 1 - 7

The generated data obtained from the statistical analysis of the linear relationship between total daily solar radiation falling upon a horizontal surface at ground level and daily bright sunshine at Winnipeg (1950-1967). All the data are for independent periods, where:

P = the period number,

df = the degrees of freedom upon which the tests of significance of the regression coefficients are based,

b = the estimated regression coefficient,

a = the estimated y-intercept,

r = the correlation coefficient,

SD = the standard deviation,

SE = the standard error of the estimate,

F = the tabulated value in the F-test.*

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* All F are significant at the 1 per cent level. Therefore, the null hypothesis that no linear relationship exists between total daily solar radiation and sunshine duration is conclusively rejected for all periods of each period length.

TABLE 1

GENERATED DATA FOR THE 5-DAY PERIOD

P	df	b	a	r	SD	SE	F
1	85	0.4101	0.4053	0.8720	0.0872	0.0249	271.38
2	87	.4251	.4065	.8734	.0906	.0254	280.77
3	87	.4274	.4249	.8860	.0837	.0239	320.56
4	80	.4291	.4338	.9166	.0748	.0208	423.45
5	82	.4450	.4179	.9307	.0671	.0192	539.94
6	87	.4259	.4403	.9258	.0632	.0185	531.05
7	84	.4360	.4254	.9006	.0800	.0228	364.19
8	84	.4624	.4155	.9047	.0837	.0237	381.60
9	88	.4747	.4240	.9065	.0781	.0236	405.37
10	86	.4861	.4234	.9100	.0721	.0238	418.47
11	84	.4988	.4252	.9086	.0819	.0249	401.34
12	73	.5407	.4051	.9164	.0843	.0276	384.95
13	88	.4842	.4792	.9070	.0721	.0239	411.91
14	86	.5122	.4680	.8899	.0877	.0282	328.53
15	87	.4676	.5086	.8745	.0843	.0278	283.68
16	88	.5475	.4712	.9087	.0812	.0266	422.52
17	88	.5379	.4577	.8894	.0812	.0293	336.10
18	88	.6428	.3737	.9018	.1020	.0328	383.48
19	86	.6658	.3458	.9043	.1020	.0339	386.42
20	88	.5975	.3681	.9107	.0949	.0288	431.48
21	88	.6412	.3139	.9060	.1015	.0318	406.37
22	88	.6347	.3183	.9265	.0800	.0274	538.58
23	86	.6149	.3057	.8823	.1175	.0353	303.14
24	88	.6196	.3087	.8820	.1175	.0353	308.47
25	86	.6154	.3211	.9482	.0714	.0222	771.16
26	88	.6408	.2942	.9209	.0985	.0289	491.69
27	87	.6402	.2921	.9102	.0949	.0312	422.52
28	87	.6493	.3077	.8933	.1122	.0350	344.36
29	88	.6929	.2764	.9272	.0889	.0298	540.33
30	88	.7128	.2556	.9548	.0831	.0236	916.53
31	86	.7026	.2715	.9501	.0775	.0249	799.26
32	88	.7012	.2582	.9542	.0678	.0234	899.09
33	88	.6849	.2691	.9570	.0686	.0219	975.21
34	87	.7322	.2371	.9300	.0849	.0309	563.48
35	88	.6338	.2878	.8321	.1241	.0451	197.94

TABLE 1--Continued

P	df	b	a	r	SD	SE	F
36	87	0.6420	0.3054	0.8810	0.1030	0.0369	303.70
37	87	.6897	.2792	.9494	.0656	.0247	778.35
38	87	.6655	.2740	.9548	.0608	.0222	898.29
39	88	.6557	.2860	.9085	.0854	.0320	419.35
40	86	.6490	.2876	.9262	.0707	.0283	524.01
41	88	.6422	.2871	.9535	.0557	.0215	892.09
42	88	.6333	.2879	.9473	.0608	.0227	774.59
43	86	.6663	.2591	.9523	.0557	.0227	859.87
44	86	.6553	.2769	.9639	.0548	.0193	1153.04
45	88	.6622	.2791	.9529	.0566	.0224	837.64
46	88	.5891	.3162	.8756	.0938	.0345	290.70
47	88	.5440	.3580	.8498	.1039	.0359	228.96
48	83	.6752	.2472	.9612	.0632	.0211	1019.99
49	84	.6473	.2783	.9264	.0900	.0286	513.80
50	88	.6678	.2823	.9149	.0922	.0314	415.72
51	88	.7439	.2496	.9513	.0787	.0256	841.02
52	88	.7482	.2802	.9630	.0671	.0221	1146.27
53	88	.7592	.2609	.9522	.0775	.0259	860.19
54	86	.7589	.2637	.9652	.0640	.0222	1168.02
55	85	.7011	.3058	.9439	.0812	.0265	697.53
56	84	.7104	.2891	.9473	.0843	.0263	738.03
57	88	.7276	.2698	.9628	.0728	.0217	1119.80
58	88	.6443	.2722	.9404	.0872	.0245	678.09
59	87	.6278	.2793	.9457	.0762	.0230	746.41
60	88	.5824	.2976	.8569	.1311	.0373	244.02
61	86	.6025	.3085	.9492	.0735	.0214	790.53
62	88	.6469	.2816	.9258	.0975	.0281	531.60
63	88	.5555	.3670	.8201	.1378	.0412	181.52
64	88	.5771	.3113	.8243	.1476	.0422	186.69
65	88	.5794	.3546	.8775	.1127	.0337	295.68
66	88	.5094	.3886	.8745	.0995	.0300	288.43
67	86	.5176	.3899	.8620	.1158	.0328	249.58
68	88	.5750	.3572	.8694	.1149	.0348	273.65
69	88	.5868	.3348	.8915	.1077	.0318	341.37
70	88	.4724	.3936	.8658	.1054	.0290	265.40
71	88	.4771	.4018	.8700	.1058	.0288	274.04
72	88	.4433	.3997	.8159	.1149	.0335	175.28
73	87	.4426	.4065	.8325	.1149	.0315	197.67

TABLE 2

GENERATED DATA FOR THE 7-DAY PERIOD

P	df	b	a	r	SD	SE	F
1	121	0.4089	0.4155	0.8537	0.0943	0.0226	326.31
2	123	.4373	.4088	.8870	.0872	.0204	457.44
3	113	.4242	.4307	.9184	.0721	.0171	614.31
4	119	.4373	.4350	.9091	.0728	.0182	547.30
5	120	.4283	.4275	.9029	.0785	.0185	533.02
6	120	.4659	.4203	.9029	.0831	.0201	536.54
7	123	.4934	.4185	.9050	.0781	.0209	559.96
8	119	.5094	.4178	.9111	.0812	.0210	587.37
9	109	.4988	.4449	.9032	.0794	.0227	484.77
10	122	.4942	.4678	.8856	.0854	.0233	449.24
11	123	.4883	.4860	.8824	.0860	.0234	433.49
12	124	.5265	.4671	.9017	.0775	.0226	543.05
13	123	.6145	.3820	.8967	.0964	.0273	506.61
14	123	.6367	.3463	.9030	.1039	.0272	546.58
15	124	.6286	.3216	.9131	.0949	.0252	624.14
16	122	.6347	.3067	.9144	.0933	.0254	625.01
17	124	.6057	.3130	.8800	.1162	.0294	425.85
18	122	.6307	.3070	.9418	.0781	.0204	960.31
19	124	.6219	.3006	.9060	.1024	.0261	586.49
20	122	.6561	.3010	.9035	.1030	.0281	543.16
21	124	.6868	.2684	.9372	.0872	.0229	900.18
22	122	.7218	.2610	.9550	.0794	.0202	1276.10
23	124	.6872	.2688	.9561	.0663	.0188	1335.32
24	123	.6957	.2621	.9370	.0787	.0233	892.83
25	124	.6704	.2691	.8652	.1140	.0348	370.49
26	122	.6399	.3096	.8905	.0954	.0295	470.76
27	123	.6902	.2665	.9533	.0632	.0197	1228.00
28	123	.6666	.2806	.9264	.0794	.0244	748.25
29	123	.6273	.3049	.9247	.0656	.0232	730.04
30	124	.6497	.2820	.9534	.0583	.0183	1264.08
31	122	.6581	.2734	.9513	.0574	.0192	1177.89
32	122	.6578	.2867	.9616	.0529	.0170	1503.21
33	124	.6169	.3032	.8904	.0917	.0282	479.95
34	121	.6037	.3122	.8939	.0949	.0275	481.91
35	118	.6609	.2727	.9364	.0848	.0227	849.95

TABLE 2--Continued

P	df	b	a	r	SD	SE	F
36	124	0.6911	0.2764	0.9199	0.0922	0.0264	687.24
37	124	.7403	.2810	.9630	.0671	.0185	1609.65
38	123	.7736	.2565	.9538	.0781	.0219	1253.35
39	120	.7471	.2788	.9628	.0671	.0191	1527.35
40	120	.7062	.3043	.9402	.0889	.0234	914.95
41	124	.7181	.2795	.9563	.0794	.0196	1339.67
42	123	.6564	.2693	.9366	.0860	.0221	882.02
43	124	.5987	.3044	.8823	.1192	.0286	437.51
44	122	.6354	.2994	.9334	.0917	.0221	825.26
45	124	.6009	.3504	.8543	.1304	.0328	334.95
46	124	.6037	.3162	.8408	.1421	.0349	299.22
47	124	.5402	.3831	.8813	.1044	.0259	434.50
48	122	.5158	.3987	.8587	.1136	.0278	343.83
49	124	.5880	.3519	.8653	.1183	.0305	370.33
50	124	.5019	.3760	.8795	.1035	.0234	425.51
51	124	.4704	.3873	.8714	.1044	.0238	391.01
52	123	.4356	.4220	.8172	.1166	.0277	247.22

TABLE 3

GENERATED DATA FOR THE 29-DAY PERIOD

P	df	b	a	r	SD	SE	F
1	500	0.4766	0.4365	0.8419	0.1166	0.0136	1219.55
2	506	.5247	.3973	.8482	.1192	.0145	1313.35
3	502	.5456	.4592	.8583	.1058	.0145	1412.64
4	516	.6267	.3350	.9054	.1010	.0129	2352.65
5	516	.6424	.2943	.9136	.0980	.0126	2615.73
6	517	.6871	.2602	.9337	.0866	.0116	2840.22
7	515	.6444	.2922	.9200	.0781	.0121	3526.27
8	516	.6431	.2852	.9340	.0693	.0108	3351.15
9	511	.7086	.2770	.9314	.0911	.0122	2728.02
10	511	.7285	.2744	.9174	.1068	.0140	2047.61
11	517	.6596	.2913	.8933	.1245	.0146	1430.06
12	518	.5462	.3738	.8567	.1187	.0144	1381.59

TABLE 4

GENERATED DATA FOR THE 11-DAY PERIOD

P	df	b	a	r	SD	SE	F
1	192	0.4116	0.4151	0.8658	0.0906	0.0172	574.81
2	184	.4505	.4222	.8980	.0862	.0154	770.26
3	190	.4337	.4348	.9113	.0728	.0142	941.03
4	190	.4880	.4085	.8993	.0877	.0171	811.79
5	190	.4856	.4297	.8941	.0825	.0176	758.74
6	179	.5132	.4509	.8876	.0894	.0198	668.60
7	195	.5181	.4761	.8818	.0933	.0198	687.59
8	196	.5732	.4351	.9040	.0854	.0193	885.45
9	194	.6251	.3620	.8942	.1039	.0224	776.65
10	196	.6330	.3185	.9186	.0894	.0194	1061.22
11	194	.6137	.3066	.8881	.1122	.0227	728.46
12	194	.6227	.3075	.9232	.0917	.0186	1125.55
13	194	.6577	.3008	.9072	.0995	.0219	905.04
14	194	.7082	.2558	.9516	.0825	.0164	1862.80
15	196	.6861	.2687	.9542	.0678	.0152	2043.52
16	195	.6811	.2674	.8799	.1058	.0263	670.29
17	193	.6741	.2841	.9203	.0837	.0206	1074.28
18	194	.6532	.2894	.9290	.0748	.0186	1237.81
19	196	.6448	.2802	.9469	.0583	.0155	1733.39
20	192	.6516	.2744	.9594	.0548	.0138	2230.36
21	196	.6171	.3053	.9070	.0800	.0205	909.01
22	189	.6279	.2860	.8981	.0964	.0223	793.76
23	194	.6881	.2747	.9375	.0843	.0183	1409.61
24	196	.7611	.2684	.9576	.0728	.0163	2182.07
25	191	.7328	.2803	.9522	.0742	.0170	1867.53
26	192	.7166	.2863	.9509	.0819	.0168	1815.00
27	195	.6405	.2664	.9143	.1020	.0202	1001.26
28	194	.6325	.2947	.9270	.0964	.0183	1188.18
29	196	.6130	.3266	.8462	.1386	.0276	494.65
30	196	.5385	.3716	.8627	.1114	.0225	571.07
31	194	.5611	.3738	.8625	.1175	.0236	566.64
32	196	.5112	.3736	.8792	.1030	.0198	669.97
33	195	.4679	.4095	.8305	.1208	.0224	434.52
34	191	.4156	.4100	.8616	.0933	.0177	551.46

TABLE 5

GENERATED DATA FOR THE 15-DAY PERIOD

P	df	b	a	r	SD	SE	F
1	263	0.4253	0.4147	0.8634	0.0943	0.0153	770.74
2	253	.4425	.4280	.9062	.0794	.0130	1165.45
3	260	.4639	.4206	.8804	.0933	.0155	898.61
4	247	.4965	.4188	.8880	.0894	.0163	930.58
5	265	.5032	.4837	.8728	.0933	.0173	851.19
6	268	.5792	.4290	.8957	.0917	.0176	1088.68
7	266	.6277	.3423	.9097	.0995	.0176	1276.66
8	266	.6664	.3362	.8930	.1153	.0206	1047.26
9	265	.6289	.3012	.9257	.0889	.0158	1591.14
10	267	.6808	.2793	.9280	.0949	.0167	1668.00
11	266	.6972	.2666	.9536	.0707	.0134	2714.67
12	266	.6698	.2769	.8807	.1058	.0219	920.40
13	266	.6661	.2814	.9355	.0721	.0154	1864.10
14	266	.6368	.2885	.9408	.0632	.0140	2058.86
15	264	.6606	.2714	.9564	.0557	.0123	2882.06
16	263	.6163	.3017	.8963	.0933	.0188	1073.49
17	264	.6920	.2702	.9303	.0883	.0168	1704.99
18	266	.7674	.2663	.9544	.0748	.0146	2751.38
19	261	.7229	.2893	.9416	.0873	.0158	2059.98
20	267	.6336	.2807	.9093	.1054	.0178	1274.07
21	266	.6254	.3168	.9043	.1077	.0181	1195.34
22	268	.6042	.3727	.8519	.1330	.0227	709.74
23	266	.5664	.3635	.8687	.1162	.0198	818.75
24	268	.4732	.4055	.8523	.1100	.0177	715.14
25	262	.4096	.3955	.8521	.0959	.0155	694.78

TABLE 6

GENERATED DATA FOR THE 19-DAY PERIOD

P	df	b	a	r	SD	SE	F
1	329	0.4989	0.4790	0.8622	0.1118	0.0161	975.52
2	324	.4426	.4131	.8831	.0889	.0131	1148.59
3	333	.5167	.4057	.8867	.0949	.0147	1231.93
4	322	.5039	.4797	.8600	.0995	.0166	917.18
5	338	.6140	.4051	.8939	.0980	.0167	1350.39
6	338	.6130	.3333	.9003	.1005	.0165	1448.02
7	338	.6288	.3007	.9157	.0985	.0150	1760.20
8	337	.6949	.2787	.9323	.0917	.0147	2240.20
9	338	.7070	.2590	.9461	.0755	.0131	2897.92
10	337	.6509	.2903	.8971	.0927	.0174	1393.01
11	338	.6409	.2827	.9288	.0707	.0139	2131.44
12	336	.6462	.2722	.9479	.0600	.0118	2986.86
13	331	.6255	.2768	.9103	.0894	.0156	1605.82
14	339	.7068	.2484	.9449	.0762	.0133	2842.48
15	332	.6942	.2581	.9419	.0787	.0135	2649.24
16	338	.6043	.2444	.9131	.0894	.0146	1707.25
17	339	.5610	.2714	.8621	.1127	.0179	982.86
18	338	.5078	.3384	.8667	.0990	.0158	1028.65
19	340	.4763	.3721	.8544	.1068	.1057	921.95
20	334	.4730	.4499	.8630	.1058	.0151	973.63

TABLE 7

GENERATED DATA FOR THE 25-DAY PERIOD

P	df	b	a	r	SD	SE	F
1	437	0.4652	0.4305	0.8570	0.1063	0.0135	1194.55
2	426	.5074	.4774	.8226	.1208	.0169	860.67
3	446	.6144	.4037	.8996	.0990	.0141	1895.24
4	444	.6549	.3314	.9054	.1063	.0146	2025.72
5	446	.6737	.2855	.9226	.0970	.0133	2564.51
6	445	.6884	.2649	.9272	.0872	.0132	2735.55
7	443	.6574	.2872	.9197	.0794	.0133	2435.70
8	444	.6488	.2741	.9521	.0574	.0098	4365.53
9	439	.6436	.2773	.9108	.0921	.0138	2159.70
10	443	.7392	.2554	.9476	.0181	.0118	3946.60
11	443	.6630	.2754	.8959	.1158	.0156	1812.04
12	446	.6160	.3085	.8756	.1229	.0161	1471.77
13	446	.5179	.3732	.8605	.1127	.0145	1277.11
14	442	.4353	.3961	.8488	.1025	.0125	1141.47

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