THE UNIVERSITY OF MANITOBA

CROP YIELD RESPONSES AND ECONOMIC OPTIMA IN FERTILIZER USE AT VARIOUS LOCATIONS IN THE PRAIRIES, 1959-68

by

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ABSTRACT

CROP YIELD RESPONSES AND ECONOMIC OPTIMA IN FERTILIZER USE AT VARIOUS LOCATIONS IN THE PRAIRIES, 1959-68

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In the period 1945 to 1969, fertilizer sales in Canada increased from 575,107 tons to 1,909,496 tons, an increase of 332 per cent for the twenty-five year period. Because of this large increase in the use of fertilizer, there is an increasing need for more information on the physical and economic relationships involved in the optimum use of fertilizer. With this kind of information, the farmer can then decide how much fertilizer to use.

With this goal in mind, the objectives of this study were:

1. To determine the yield response to varying levels and combinations of N and P_2O_5 for the crops, wheat, oats, barley and rye seeded on summerfallow and stubble, at various locations in the Prairie Provinces, for the years 1959 to 1968.

2. To derive the least cost combinations for specified crop yields and the maximum profit positions for selected crop prices, and to compare these economic optima among locations, among years at given locations and among the different crops in a given year and location.

The data used in this study were the results of fertilizer experiments in cereals that were conducted in the Prairie Provinces and the Peace River area of British Columbia in the ten year period, 1959 to 1968. Of the 4,385 fertilizer experiments whose results were tabulated, 3,458 of these experiments were excluded from further analysis in the study. Most

of the experiments were excluded because they did not have a sufficient number of observations to estimate a yield response function to fertilizer inputs.

Four different production functions were fitted by least squares regression to the observations in each of the remaining 927 experiments. Although regression equations were obtained for the 927 experiments, there were only 93 experiments in which the signs of the regression coefficients for the four derived equations were the same as hypothesized. The hypothesized signs of the regression coefficients for the linear, Cobb Douglas, quadratic and square root functions are shown in the following equations:

$$\hat{Y} = b_o + b_1N + b_2P$$

$$\hat{Y} = b_oN^{b_1} N^{b_2}$$

$$\hat{Y} = b_o + b_1N + b_2P - b_3N^2 - b_4P^2 + b_5NP$$

$$\hat{Y} = b_o - b_1N - b_2P + b_3\sqrt{N} + b_4\sqrt{P} + b_5\sqrt{NP}$$

The quadratic form of the production function was selected as the "best estimate" of the physical relationship between the fertilizer inputs and the crop yields. The selection was made on the basis of certain regression statistics obtained for each function. Because there were 93 experiments involved, the selection was based on the average values for these statistics for each function.

The quadratic production equations derived for each of the 93 experiments were then used to obtain the least cost nutrient combinations for specified crop yields and the maximum profit positions for selected crop prices for each of these experiments. However, the least cost nutrient combinations and the maximum profit positions for only 56 experiments are presented in the thesis. The economic optima for 36 experiments are not presented, because in most of these experiments the most profitable application of N and/or P_2O_5 was negative.

The maximum profit positions of N, P_2O_5 and yield were compared among years at given locations and among locations for a given price. These comparisons could not be made for every crop, because in some cases there were only one or two fertilizer experiments for the crop. There was considerable variation in the maximum profit yields for both wheat on summerfallow and barley on stubble, while in the case of wheat on stubble, most of the experiments had optimal yields which fell within the range of 25 to 35 bushels per acre. The combinations of N and P_2O_5 for both the maximum profit positions and the least cost combinations for specified yields were also quite variable among years for given locations and among locations. Comparisons of the economic optima for different crops could only be made for two locations.

The regression-equation approach not only expresses the physical relationships between fertilizer levels and yields, but it also permits one to determine the economic optima in fertilizer use. The fertilizer-input crop -output relationships, however, apply to particular soils for certain years; production surfaces obtained under other rainfall and soil conditions can be expected to differ from those obtained in the experiments reported.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

INTRODUCTION

THE PROBLEM

Fertilizer use in agriculture in Canada and in the Prairie Provinces has increased significantly. Between 1945 and 1969 sales of fertilizer in Canada increased from 575,107 tons to 1,909, 496 tons, an increase of 332 per cent for the twenty-five year period (Table A.1 of Appendix A). Fertilizer sales in the Prairie Provinces increased by 2037 per cent for the same period. The increase in the Prairie Provinces has not been continuous, however. Fertilizer consumption fell off sharply in the period 1954 to 1958 and again 1969. In 1970 in the Prairie Provinces, it was estimated that farmers intended to use 30 per cent less fertilizer than was used in 1969.[1] The reduction in fertilizer use in these two periods is due in part to the reduced crop acres and the decline in wheat exports as compared with the immediately preceding periods (Table A.2 and Table A.3).

One of the reasons why fertilizer use has increased in the period 1945 to 1969, particularly in the Prairie Provinces, is that farmers are moving towards more intensive, rather than extensive, production. The disappearance of the land frontier has tended to place greater premiums on the existing land resources. Consequently, as land prices increase, farmers often find it more profitable to invest their limited capital in more intensive production through the use of fertilizer than in additional land. The expansion in livestock production in this period has also

[1] Dominion Bureau of Statistics, Agriculture Division, Crops Section, "Intended Acreage of Principal Field Crops in Canada, 1970", Field Crop Reporting Series, (No. 2, 1970).

increased the use of fertilizer in the Prairie Provinces. In the last ten years an increasing proportion of the coarse grains, oats and barley, were grown on stubble land which require higher rates of application of fertilizer than crops grown on summerfallow.[2] Also, fertilizer use increased as a result of the increase in the acreage of the major field crops (all wheat, oats, barley, all rye, mixed grains, flaxseed, rapeseed and tame hay) in this period (see Table A.2). In 1945-46 in the Prairie Provinces, the acreage of these crops was 41,732,000 acres. In 1969-70, there were 50,126,000 acres of these crops. Although the 1969-70 acreage was down from that of the previous crop year, nevertheless, it is still substantially larger than the 1945-46 acreage. Because of the large amount of money presently being expanded on fertilizer, there is an urgent need for expanded research into the physical and economic relationships involved in determining the optimum combinations and levels of fertilizer use.

Initial research in fertilizer in agriculture has been concerned with determining whether or not there is a significant response in crop yield from the application of fertilizer. However, once responses have been found to exist, the farmer needs to consider fertilizer along with other farm inputs and practices in his farm management decisions. First, he must decide whether or not to use any fertilizer. While yield responses to fertilizer may be certain, he must decide whether or not a dollar put in fertilizer will return more than the same dollar invested in livestock, seed, chemicals, land or other investment alternatives. If he decides to use fertilizer, he must then decide: (1) where to use fertilizer in

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[2] Dominion Bureau of Statistics, Agriculture Division, Crops Section, "Summerfallow and Stubble, Acreage and Yield of Specified Crops Prairie Provinces", Field Crop Reporting Series, (No. 1, 1963 - 1970).

terms of which soils and crops will give the highest return for each one dollar invested; (2) what combination of fertilizer nutrients to use; and (3) how much of a given nutrient combination to apply on a given crop. These decisions can be made only if fertilizer information is provided in the form of incremental response data, that is, data which show the successive additions to yield resulting from successive fertilizer applications. Accordingly, if the initial research shows that crop yields do respond to fertilizer, the next steps in fertilizer research are to derive (1) the incremental yields forthcoming from various rates of fertilizer application under specified crop and soil conditions, and (2) the economic optimum quantity of fertilizer, considering crop and fertilizer prices.

The economic optimum quantity of fertilizer and the corresponding optimum yield will not be the same for every year in a particular area or location. The yield responses to the various rates of fertilizer application are affected by such factors such as weather, soil moisture, and soil fertility which change from one year to the next. In some years, the optimum rate of fertilizer application may be quite high; yet the optimum yield may be quite low because of the low soil fertility, rainfall and soil moisture. In other years, the exact opposite may be the case. If the optimum yields and quantities of fertilizer were obtained for a number of years, the farmer would then be in a better position to decide how much he should invest in fertilizer. The farmer with the limited capital will tend to fertilize at a lower rate than the farmer with the unlimited capital, because of the greater risk and uncertainty associated with the heavier rates of fertilizer application. The farmer whose capital is limited is not only concerned with the risk involved in a large investment in fertilizer, but also whether or not a higher return could be obtained if the capital was invested elsewhere in the farm business. The farmer with the unlimited capital can afford to take a greater risk and is therefore able to apply the heavier rates of fertilizer.

The economic optimum in fertilizer use will also differ among locations. Differences in soil moisture, soil fertility and weather as well as differences in soil type will affect the optimum use of fertilizer among the various locations. Because these differences in the optimum use of fertilizer do exist among locations, it is not advisable for a farmer in one area to fertilize on the basis of the results obtained in another. He may be applying too much fertilizer, particularly if, he is located in a drier area and in a different soil zone. For example, if a farmer in the Swift Current area of Saskatchewan were to apply the optimum quantity of fertilizer derived for the Melfort area, then he would be applying in most years too much fertilizer. On the other hand, a farmer may be applying not enough fertilizer if the area in which he is located has a wetter climate and a more productive soil. If the economic optimum in fertilizer use were derived for a number of locations, then the farmer would be able to choose the optimum quantity of fertilizer which is the most appropriate for his farm.

This, in part, summarizes the problem facing the farmer in determining what is the optimum quantity of fertilizer to apply, given certain crop and fertilizer prices. The problem is further complicated by the fact that the optimum quantity of fertilizer changes from one year to the next and from one location to another.

OBJECTIVES OF THE STUDY

The major objectives of this study are:

1. To determine the yield response to varying levels and combinations of the fertilizer nutrients, N and P_2O_5, for the selected crops, wheat, oats, barley, and rye seeded on summerfallow and on stubble, at various locations in the Prairie Provinces, for the years 1959 to 1968.

2. To determine the least cost combination of N and P_2O_5 in producing specified yields for each of these crops for the various locations and years.

3. To determine the most profitable application of N and P_2O_5 for five selected prices of each crop for all locations and years.

Because the optimum application of fertilizer and the corresponding yield are not the same for every year at a particular location, and because they also differ among locations, two further objectives of the study are:

1. To compare the maximum profit position of N, P_2O_5 and yield among years at given locations.

2. To compare the maximum profit position of N, P_2O_5 and yield among locations.

The least cost combination of fertilizer nutrients for specified yields will also vary among years at each location and among locations. Therefore, two additional objectives of the study are to compare the least cost combinations of N and P_2O_5 for the same crop yield among years and among locations.

Another objective is to compare the return per dollar invested in fertilizer for the different crops in a given year and location.

Since several of the objectives involve economic production principles, these principles will be discussed in the next chapter. A description of the nature and source of the fertilizer-yield data and an evaluation of the different types of functions that can be used to estimate the yield responses to fertilizer inputs will be presented in Chapter 3. The methodological procedures and the equations involved in determining the economic optima in fertilizer use will be examined in the first section of Chapter 4. In the second section, the economic optima for fertilizer experiments conducted at various locations in the Prairies will be presented and compared. Chapter 5 will be a summary and conclusion.

Chapter 2

THEORETICAL CONSIDERATIONS INVOLVED IN ALLOCATION OF FACTORS OF PRODUCTION

There are three basic relationships in production economics. These are the factor-product, the factor-factor, and the product-product relationship. They provide the framework within which economic efficiency is determined, whether the choices relate to an individual farm, the agricultural industry, or the nation as a whole.[1] In the factor-product type of relationship, one is concerned with the transformation of a single variable factor of production into a single product. In the second type, one is looking at the relation between two variable factors of production and a single product. In the product-product type of relationship, one is concerned with the substitution between two products for various levels of input. In this chapter, only the first two will be dealt with in detail.

THE SINGLE VARIABLE FACTOR-PRODUCT CASE

This first section will examine the production principles associated with the input of a single variable factor of production, with all other factors held constant at some level, to yield a single product. Many farm decisions fall within the framework of a single variable factor-product type of relationship. The problem involved is usually one of intensity of production. For example, how much fertilizer to apply per acre is a decision that has to be made by many farmers. Similarly,

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[1] Earl O. Heady, Economics of Agricultural Production and Resource Use (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965), p. 26.

decisions have to be made on the amount of feed to be fed per animal, the amount of labor or capital to be applied per acre or to a farm of a given size. The problem of intensity can also apply to output. The decision as to what level of production per acre, per animal, or for the farm is most profitable, is dependent upon a factor-product relationship.

The term production function refers to the physical relationship between the input of a factor or factors and the output of a product. It can be expressed in mathematical terminology as:

$$Y = f(X_1 \mid X_2, X_3, \quad . \quad . \quad . \quad , X_n).$$

This expression states that the output of product Y is a function of, or is dependent on, the input of the variable factor X_1, with factors X_2, X_3 to X_n fixed at some level. For example, the yield of wheat per acre is a function of the input of fertilizer. As the use of fertilizer is varied, the seeding rate, land, labour and machinery inputs are likely to remain constant.

A particular level of output may be produced from a number of different combinations of inputs (non-variable inputs included). Alternatively, the same combination of production inputs may give different amounts of output, depending upon how efficiently the productive inputs are organized. For instance, the hay yield per acre obtained when fertilizer is broadcasted in the fall may be higher than that obtained when fertilizer is broadcasted in the spring. Therefore, if the production function is to give only one value for the output from a given input combination, the function must be so defined that it expresses "the maximum product attainable from the combination at the existing state of technical knowledge".[2]

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[2] Sune Carlson, A Study of the Pure Theory of Production (New York, New York: Sentry Press, 1965), pp. 14-15.

The production function can be plotted on a graph in which the total physical product curve, $TPPx_1$, represents the total output of the product for various levels of input of the variable factor, with the other factors held constant at some specified level. The total physical product for variable input X_1, $TPPx_1$, can: (1) increase at an increasing rate, (2) increase at a constant rate, (3) increase at a decreasing rate, and (4) decrease with increases in the variable factor X_1. These relationships are shown in Figure 2.1. It is not necessary that any particular production function should exhibit all situations described above.

The average and marginal physical products can be derived from the total physical product. The average physical product curve, $APPx_1$, denotes the amount of product per unit of the variable input. The marginal physical product curve, $MPPx_1$, represents the addition or reduction in the output of the product resulting from an additional unit of variable factor X_1. In other words, it denotes the changes in the slope of the total physical product curve.

Production functions can be divided into three segments called stages of production, which are distinguished by whether their marginal physical products are increasing, decreasing or negative. The classic production function which is characterized by all three stages is illustrated in Figure 2.1. Stage 1 extends from the origin to the level of input where the average physical product curve reaches a maximum. The marginal physical product first increases and then decreases in this stage. Stage 2 extends from the input level denoting maximum average physical product to the one where the marginal physical product becomes zero. The marginal physical product becomes zero when the total physical product reaches a maximum. Stage 3 includes all input levels which have

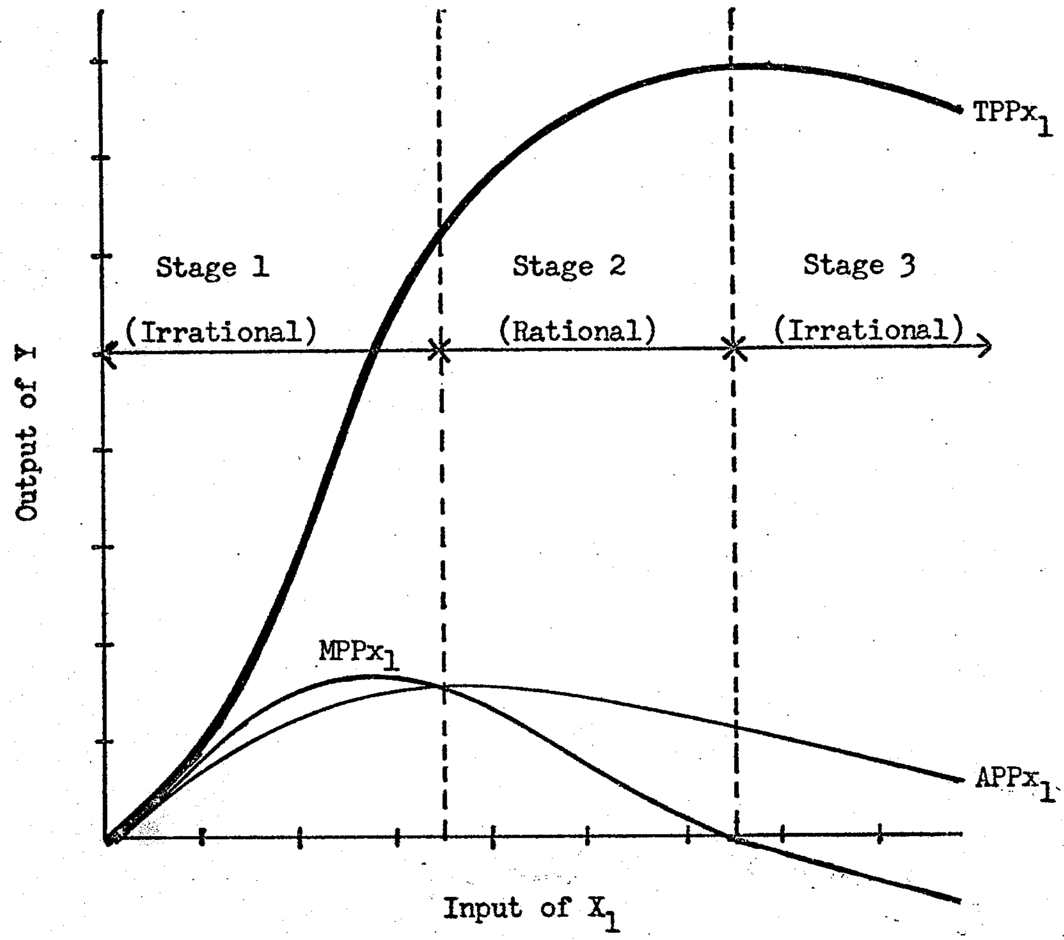

Figure 2.1. An Input-Output Relationship Showing Total, Average and Marginal Physical Products and the Three Stages of Production.

a negative marginal physical product and a declining total physical product.

In stage 1, the average physical productivity of the variable factor increases continuously as additional units of the factor are applied to the fixed factors, since in this stage of production the marginal physical product is always greater than the average physical product. Any level of resource use falling in stage 1 is considered to be uneconomic or irrational because an additional unit of input yields a higher output than the previous input. As production is pushed to the limits of stage 1, a greater product is also forthcoming from the fixed factors as well from each unit of the variable factor. Therefore, if it is economical to produce any output, it is economical to produce at least up to the limit of stage 1, where the average physical product is equal to the marginal physical product.

Stage 3 is also an area of irrational or uneconomic production because each additional unit of input of the variable factor reduces the total physical product. In this stage of production, the marginal physical product is less than zero. The third stage signifies conditions of resource waste because the same amount of product could be produced with a smaller number of units of the variable input. Therefore, if the entrepeneur is rational he will never intend to produce in the third stage. Even if the variable input is free he will only go to the end of the second stage.

In stage 2, the marginal physical product decreases continuously and is always less than the average physical product, but greater than, or equal to, zero. Stage 2 is considered to be the rational area of production because each additional unit of input of the variable factor results in a smaller and smaller addition to total output. Therefore, if it is profitable to produce any output, the maximum profit position will be somewhere in stage 2. The level of input of the variable factor which will maximize profits, depends on the productivity of the variable factor as well as its price and the price of the product.

The maximum profit position in a single variable factor-product relationship can be determined by equating the marginal physical product to the factor/product price ratio. The maximum profit position occurs where the price ratio line is tangent to the total physical product curve (Figure 2.2). At this point, the slope of the price ratio line, Px_1/Py is equal to the slope of the total physical product curve, dY/dX_1. The slope of the total physical product curve is the marginal physical product.

The discussion to this point has involved the relationship known in production economics as the factor-product relationship. The concept of a single factor-product relationship has been used to set forth the basic principles of resource allocation. The discussion which follows in the next section deals with the factor-factor relationship. This economic concept is concerned not only with the transformation of resources into products, but also the substitution of one resource for another.

THE TWO VARIABLE FACTOR-PRODUCT CASE

This section will examine the production principles associated with the input of two variable factors of production, with all other factors at some fixed level, to yield a single product. There are many farm decisions which fall within the framework of a factor-factor or resource substitution relationship. For example, what combinations of forage

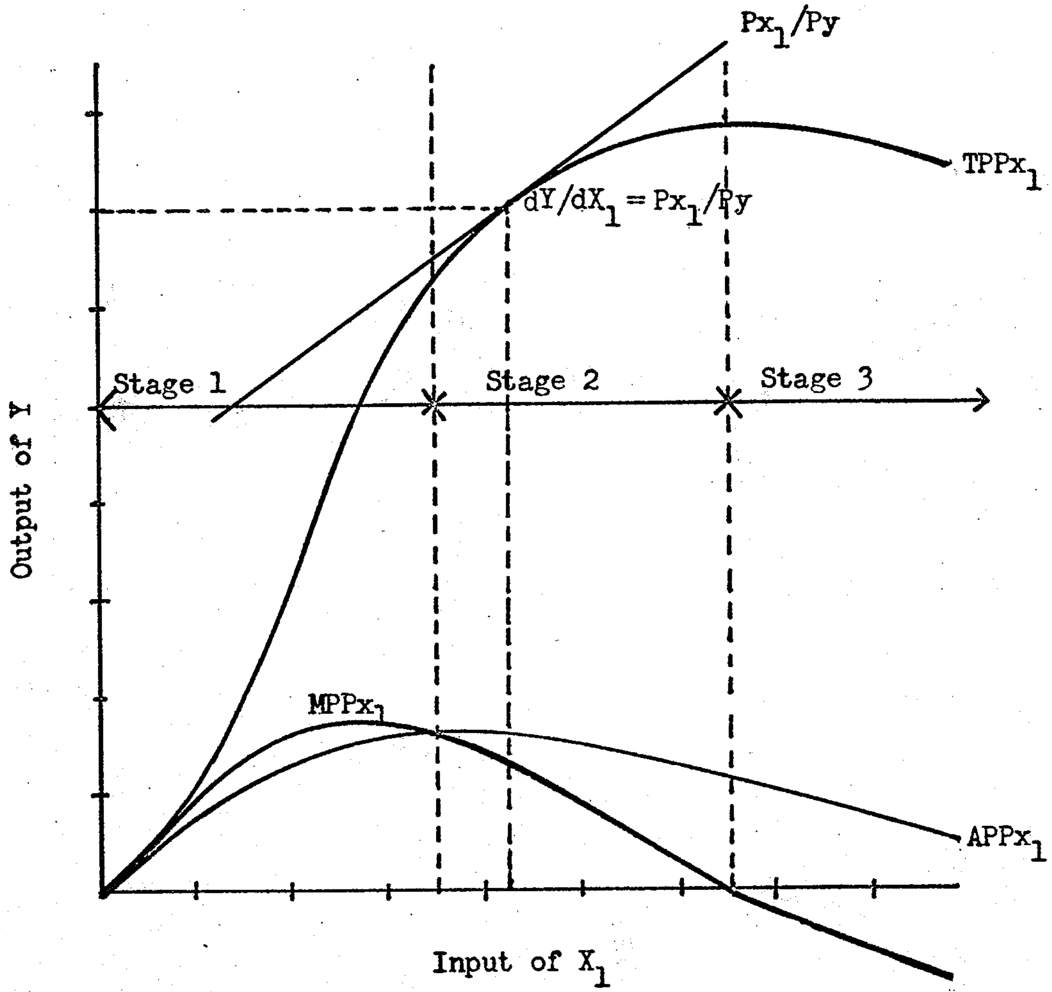

Figure 2.2. A Total Physical Product Curve Showing the Equality of the Marginal Physical Product and the Factor/Product Price Ratio in the Maximum Profit Position.

and grain should be used in a feed ration for a feeder steer; or what levels of nitrogen and phosphate should be applied per acre for a specific crop; or what should be the substitution of labour and machine capital for all animals or acres of a farm which is fixed in terms of acres and other resources? These are all decisions which involve substitution between two factors of production.

In a factor-factor type of relationship, the adjustments in the variable factors can be divided into three distinct types.[3] These are: (1) both factors can be increased in the same proportion, (2) one factor can be held constant while the other is increased, and (3) output can be held constant while one factor is increased and the other is decreased in quantity. All three types of adjustments have important implications in resource use. The first type of adjustment can be used to determine whether there is increasing or decreasing returns (marginal physical productivity) to both factors. The second type can be used to determine whether high levels of application of one factor will have adverse effects on output. The third type of adjustment is essential in determining the optimum combination of two factors in producing a given output or yield.

The production function for a single output produced by two variable factors, with other factors held constant, can be expressed algebraically as:

$$Y = f(X_1, X_2 \mid X_3, X_4, \ldots, X_n)$$

where output Y is a function of the variable factors X_1 and X_2, with the inputs X_3, X_4 to X_n held constant at a given level. The production function presupposes technical efficiency, that is, the factors (variable and fixed) are combined in such a manner that they cannot be rearranged

3 Heady, op.cit., pp. 133-34.

to give: (1) a greater physical product with the same level of inputs, or (2) the same physical product with less of one or more factors. Therefore, the function states the maximum output obtainable from every possible input combination.

The geometric form of a production function with two variable factors and a single product is called a production surface. Many types of production surfaces are possible, depending upon the underlying production function. Some production surfaces will have increasing and decreasing returns (marginal physical productivity) to both factors. Others, such as in Figure 2.3, will have only decreasing returns to both factors.

In Figure 2.3, if a vertical slice is made through the production surface, parallel to the X_1 axis, then the resulting curve ab expresses the input-output relationship between the variable factor X_1 and the product Y, when the variable factor X_2 is fixed at a level of 20 units. Many such vertical slices are possible. In fact, a different input-output curve, showing the output for every level of X_1, exists for each constant level of X_2. These input-output curves can be represented in two dimensions as in Figure 2.4. Alternatively, vertical slices can be made through the production surface parallel to the X_2 axis to express the input-output relationships for X_2 and Y with X_1 held constant. The slopes of the individual input-output curves indicate the marginal physical products of the variable input. The marginal physical products indicate the amount added to total product by each successive unit of the variable resource. These marginal physical products will be used later in applying economic principles in specifying optimum resource use.

The contour line, cd, on the production surface in Figure 2.3 is

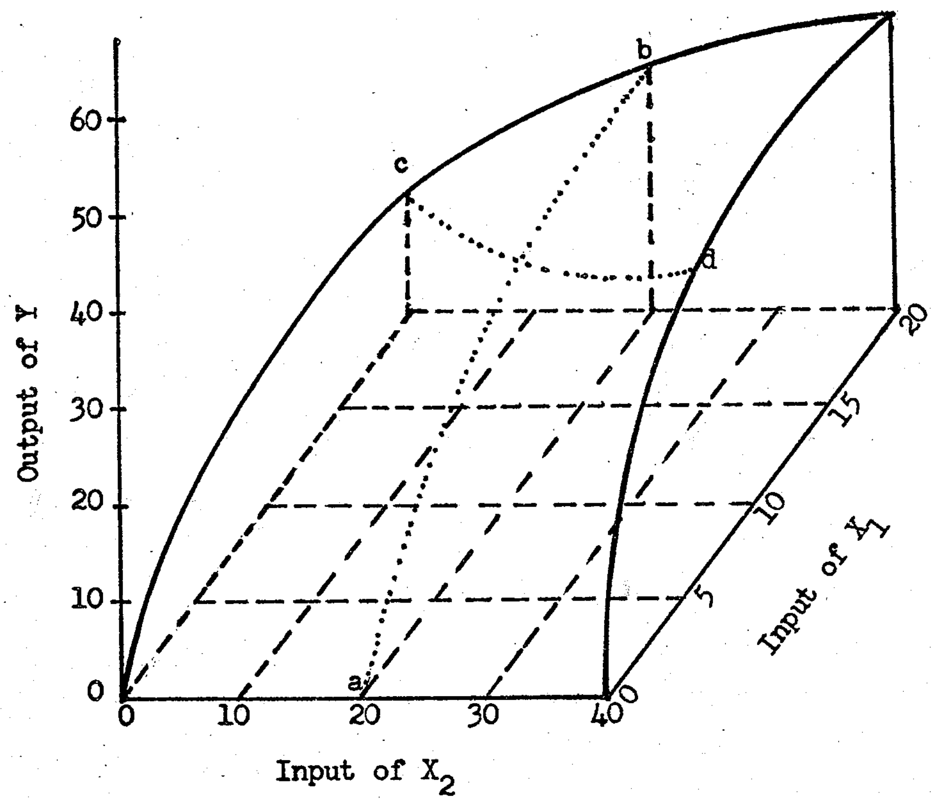

Figure 2.3. Production Surface Relating Output of Product Y to Input of Factors X_1 and X_2.

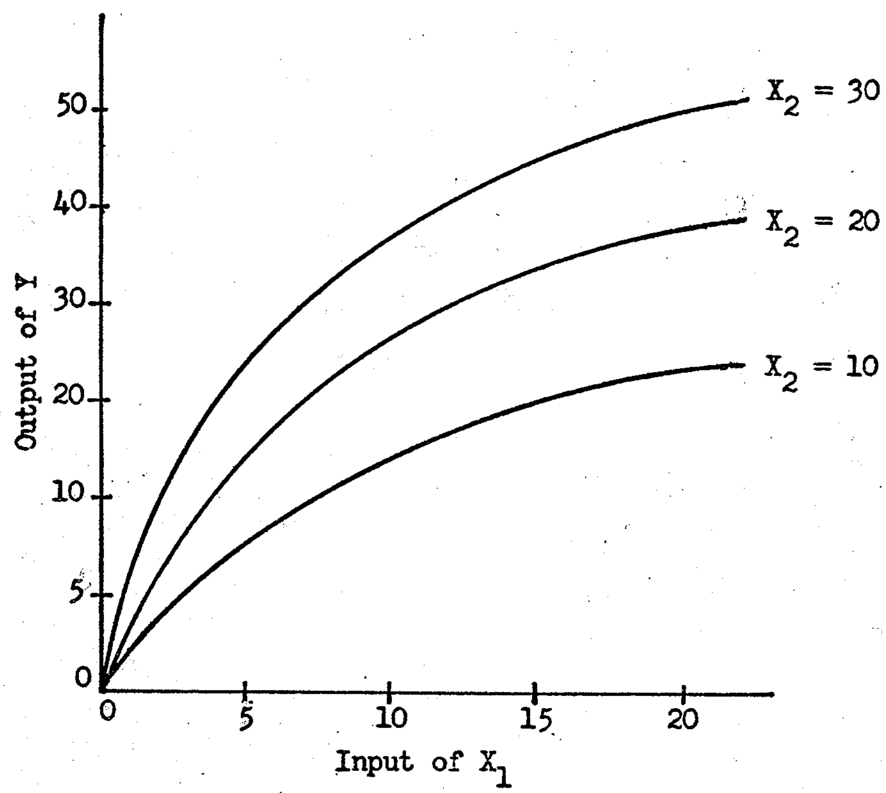

Figure 2.4. Input-Output Relationship Between Y and X_1 with X_2 Constant.

called an iso-product line or isoquant since it indicates all of the possible combinations of the two variable factors which will produce the same level of output. Many such isoquants can be derived for a particular production surface. These isoquants can be represented in two dimensions as in Figure 2.5. The curves y_1, y_2, and y_3 are isoquants for three different levels of output and various quantities of the two variable factors.

The slopes of each isoquant indicate the rate of which one factor substitutes for, or replaces the other, if output is to be maintained at a specific level. When the isoquants are curved such as in Figure 2.5, the rate at which one factor substitutes for the other, declines as the given output is produced with more of the former and less of the latter. Indeed, these slopes are crucial in specifying the optimum combinations of labour and capital in farming, the optimum proportions of feeds in livestock production, the optimum proportions of nutrients in crop fertilization,etc.[4] If the isoquants are straight lines with a constant slope, then generally only one factor should be used in producing the specified output. If the input-output curves are linear, there is no limit to the level of input and output which is profitable, if it is profitable at all.

The lines indicated by k_1, k_2, k_3, and k_4 in Figure 2.5 are isoclines in the sense that they connect points of equal slope on successively higher isoquants. Hence, they connect points on the isoquants which denote equal marginal rates of substitution between variable factors. Isoclines are also called expansion paths, since they show the path which the mix of inputs should follow if output is to be expanded. If the

[4] Earl O. Heady, and John L. Dillon, Agricultural Production Functions (Ames, Iowa: Iowa State University Press, 1961). p. 36.

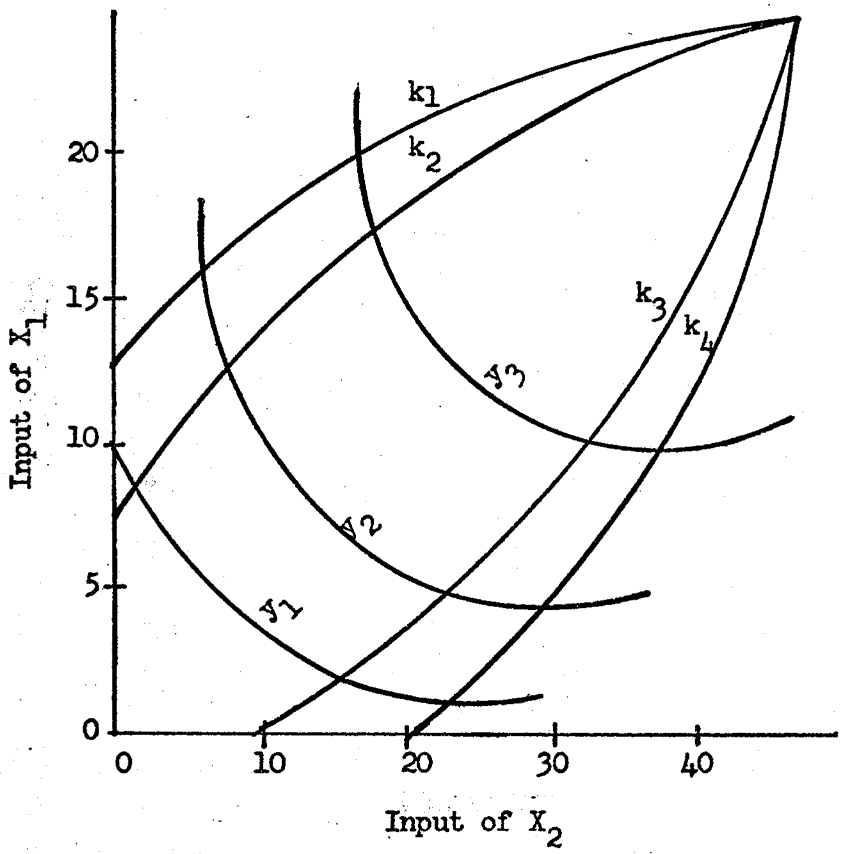

Figure 2.5. Isoquants and Isoclines.

isoclines are linear and intersect the origin, the proportion of the two factors inputs will remain the same as output is expanded. If the isoclines are curved or if they are linear and do not pass through the origin, then the factor mix will change as output is expanded.

The isoclines represented by k_1 and k_4 in Figure 2.5 are also termed ridge lines since they denote infinite and zero rates of substitution between factors. The ridge line k_1 connects points on the isoquants which have an infinite slope while the ridge line k_4 joins points on the isoquants which have a zero slope. Production in the areas outside the ridge lines is irrational because the same level of output can be produced in the area inside the ridge lines with less variables resources. In Figure 2.6, an output of 30 units can be produced in the rational area of resource combination with 5 units of X_1 and 15 units of X_2 or in the irrational area with 5 units of X_1 and 40 units of X_2.

If the production surface rises to a distinct peak, then the isoclines will converge to a single point as in Figure 2.5. The marginal rates of substitution and the marginal physical productivities are zero at this point. Alternatively, if the production surface slopes to a plateau then the isoclines will not converge to a single point.

The optimum or least cost combination of the variable factors for a given level of output is determined by equating the marginal rate of substitution to the inverse price ratio of the two factors. The least cost combination can be illustrated geometrically. Just as the isoquant indicates all possible combinations of the two factors which produce a given level of output, an iso-cost or iso-outlay line indicates all possible combinations of the two factors which can be purchased with a given outlay of funds. The iso-cost lines are linear for a farm or other

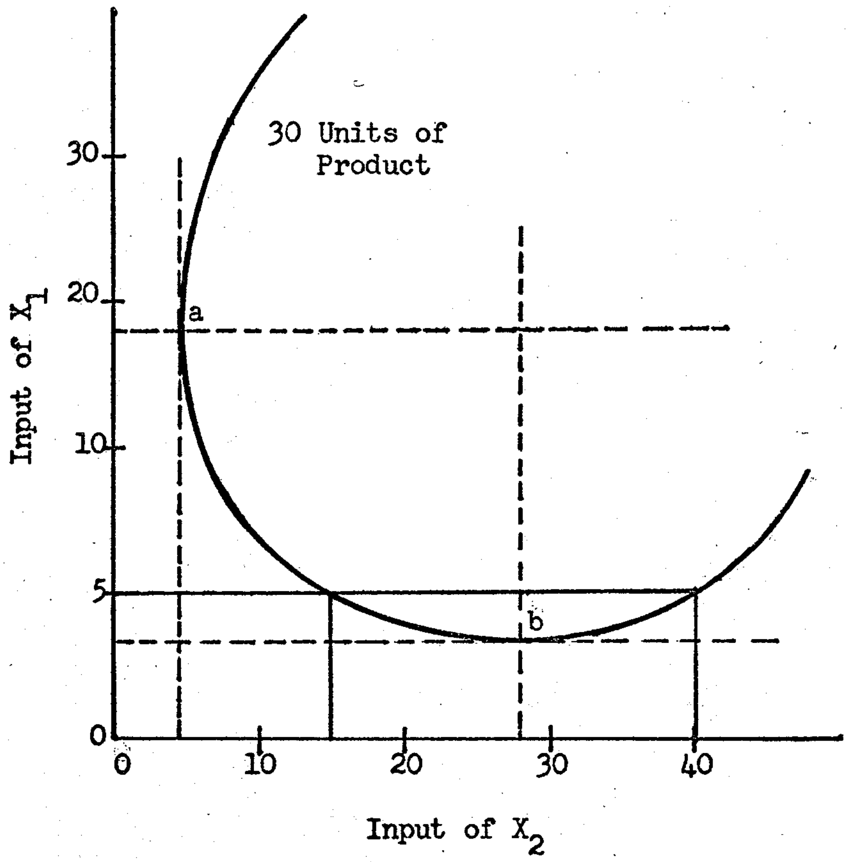

Figure 2.6. An Isoquant Showing Areas of Rational and Irrational Resource Combination.

competitive firms which do not purchase enough of the factor to affect the market price. The slope of the isoquant represents the marginal rate of substitution of X_2 for X_1, dX_1/dX_2. The slope of the iso-cost line indicates the rate at which one factor substitutes for the other, if the cost outlay is to remain constant. The rate at which one factor substitutes for the other depends on the relative prices of the factors. Hence, the slope of the iso-cost line is indicative of the ratio of factor prices. For example, if the prices of X_1 and X_2 are \$1.00 and \$.50 per unit, respectively, then 2 units of X_2 can be exchanged for 1 unit of X_1, without changing the amount spent. Therefore, the slope of the iso-cost line is 0.5, which is equivalent to the inverse price ratio of the two factors, .50/1.00. When the slope of the isoquant is equal to the slope of the iso-cost line, cost is at a minimum. In Figure 2.7, the least cost combinations of X_1 and X_2 for three specified levels of output occur where the isoquants are just tangent to the iso-cost lines, c_1, c_2 and c_3. At the points of tangency, the marginal rate of substitution of X_2 for X_1 is equal to the slope of the iso-cost line, Px_2/Px_1.

In Figure 2.7, the expansion path, E, has been drawn through the points of tangency and indicates the change in the proportion of the two factors as output is increased at least cost. In this particular case, since the expansion path has an upward curvature, a greater proportion of X_1 should be used as output is expanded. If the expansion path is linear and passes through the origin, then the proportion of the two factors will remain constant as output is increased. Output should be expanded as long as the marginal value of the product (price of the product times the marginal physical product) is greater than or equal to the marginal cost of the resources added.

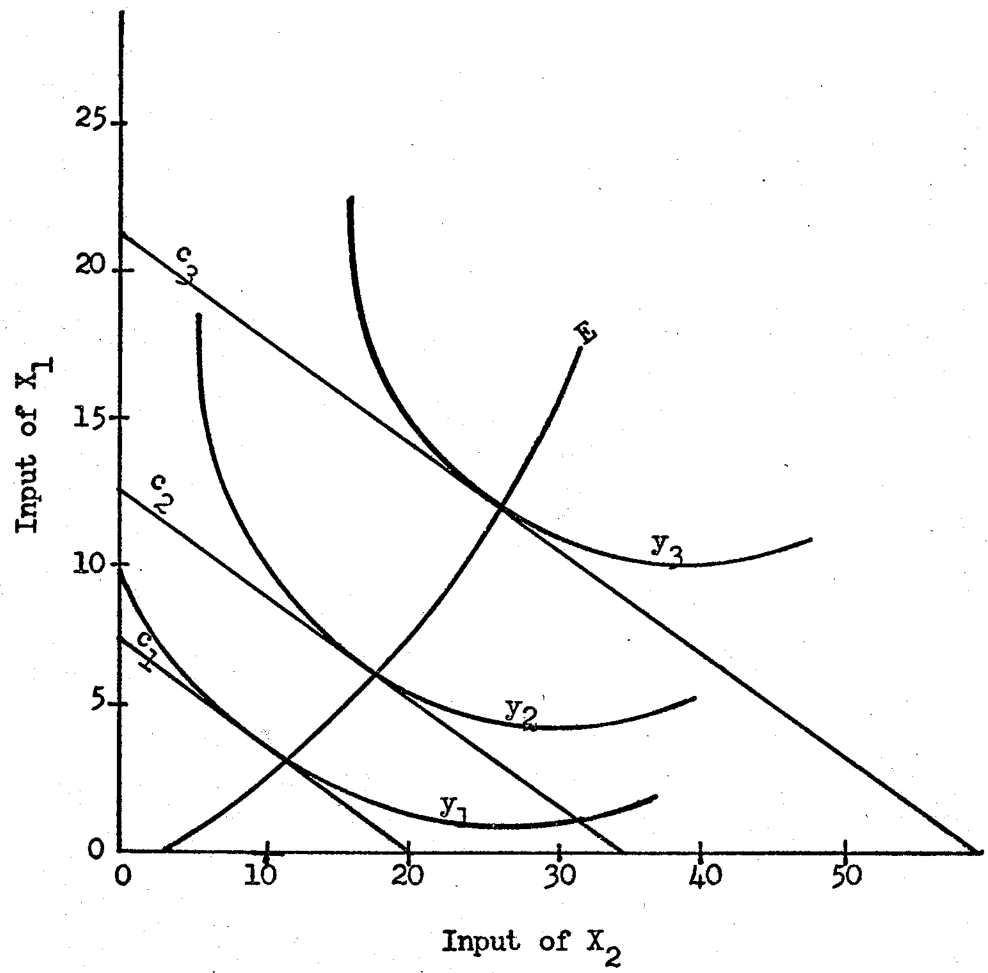

Figure 2.7. The Use of Iso-Cost Curves and Isoquants to Indicate Least Costs.

Having determined the least cost combinations for a number of output levels, the next step is to obtain the maximum profit position. In the single factor-product case, the maximum profit position occurred where the marginal physical product of the variable factor was equal to the factor/product price ratio. In other words, it is profitable to expand output up to the point where the value of the increment in output is equal to the increment cost of the factor. In the factor-factor case, the maximum profit position is found by setting the marginal physical products of the individual factors equal to their respective factor/product price ratios, and then solving the relationships $MPPx_1 = Px_1/Py$ and $MPPx_2 = Px_2/Py$ simultaneously to obtain the optimum combination of factors and output. This combination of the factors is also the least cost combination for the associated level of output.

In this section, only the production principles associated with two variable factors were examined. The principles are basically the same, however, for relationships involving three or more variable factors with a single product being produced.

Chapter 3

METHODOLOGY

THE NATURE OF THE EXPERIMENTS

The data used in this study are the results of fertilizer experiments in cereals that have been conducted in the three Prairie Provinces in the ten year period, 1959 to 1968. In addition, fertilizer experiments conducted in the Peace River area of British Columbia were also included in the study since the type of agriculture in this area is very similar to that of the Prairies. These experiments are located in all crop districts of the Prairies although not necessarily for every year. Most of the experiments were designed to illustrate the yield response to various applications of nitrogen and phosphate. In others, the yield responses to various rates of application of potash were also examined. The latter experiments were generally located on the lighter textured soils found in the Prairie Provinces.

The major sources of the fertilizer data were the annual reports compiled by the fertilizer committees in each province.[1] These reports contain the results of fertilizer trials conducted by various agencies in each province. The fertilizer data for the Peace River area of British Columbia were collected from the Alberta annual reports (see Footnote 1). In 1968, however, the format of presentation for fertilizer test results in Alberta was changed. Instead of the actual results being

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[1] Manitoba Soil Science Society, Papers Presented at the Annual Manitoba Soil Science Meeting, Annual Reports for 1959 - 1968 (Assembled and Distributed by the Publications Branch, Manitoba Department of Agriculture); Saskatchewan Advisory Fertilizer Council, "Results of Fertilizer Experiments in Saskatchewan, Part I, Cereals", Annual Reports for 1959 - 1968; and Alberta Soils Advisory Committee, "Soils and Fertilizer Test Results in Alberta", Annual Reports for 1959 - 1967.

compiled into a single report, only a summary of the results was presented. However, some fertilizer data were obtained for Alberta in 1968 from a report put out by the Alberta Department of Agriculture.[2] Fertilizer data for the Prairie Provinces were also obtained from a report published by a fertilizer company on its 1968 experiments.[3]

In this study, the experimental results were tabulated for wheat, oats, barley and rye seeded on summerfallow and stubble. The data were collected for locations throughout the Prairie Provinces and the Peace River area of British Columbia for the years 1959 to 1968. In one year, there may be several experiments for a particular crop and location. In the next year, there may not be any experiments for that location. Some locations may have fertilizer test results for several crops for all ten years, while others may have test results for only one crop for one or two years. These data were recorded on computer cards to facilitate the handling of a large quantity of data. The kind of data that were recorded on each card are given in Appendix A.

In some fertilizer trials, the results of certain plots were omitted. These plots received special treatments which the other plots in the trial did not receive. For example, the results of plots which were sprayed with wild oat sprays were not included in the study. The yield responses obtained in these plots were due to more than just the application of fertilizer. The results of plots which received

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[2] Alberta Department of Agriculture, "1968 Fertilizer Demonstration Results".

[3] W. E. Janke, Crop Responses to Fertilizer in Western Canada, (Fort Saskatchewan, Alberta: Research and Development Division, Sherritt-Gordon Mines Limited, December, 1968).

applications of lime or where the fertilizer was placed in bands six and twelve inches below the seed were also excluded from the study for the same reason.

In those experiments where the individual replicate yields were given, the average of these yields were used for each level or combination of nitrogen and phosphate. This was done in order to maintain consistency, since in most experiments only the average of the individual replicate yields was reported.

In this study, the results of 4385 fertilizer experiments were collected (Table 3.1). Of the 4385 experiments, 12 per cent were located in Manitoba, 54 per cent in Saskatchewan, and 34 per cent in Alberta and the Peace River area of British Columbia. The 4385 experiments had a total of 29,883 observations for an average of 6.8 per experiment. The range in the number of observations in these experiments was from one to thirty-five.

DERIVATION OF FERTILIZER PRODUCTION FUNCTIONS

A major objective of this study is to derive mathematical equations (production functions) which express the physical relationships between varying levels and combinations of N and P_2O_5 and the corresponding crop yields. Such an equation defines the production surface and the relevant quantities of fertilizer nutrients associated with it. The derivation of the production function equation is itself complex, because a number of forms of production functions can be used to estimate the physical relationships between the fertilizer nutrients and yields. In this study, four functions with two variable nutrients were fitted to the observations in each experiment. These were the linear, the logarithmic or Cobb Douglas,

Table 3.1

Number of Fertilizer Experiments Tabulated in the Prairie Provinces and Number of Observations in These Experiments.

Year	Number of Experiments				Number of Observations			
	Man.	Sask.	Alta. and B.C.	Total	Man.	Sask.	Alta. and B.C.	Total
1959	59	275	133	467	415	1801	1315	3531
1960	78	257	121	456	496	1821	830	3147
1961	41	144	63	248	260	1125	619	2004
1962	12	221	99	332	46	1753	772	2571
1963	18	228	128	374	94	1646	838	2578
1964	82	279	102	463	249	1533	678	2460
1965	26	266	208	500	216	1415	1347	2978
1966	31	205	253	489	231	1336	1535	3102
1967	94	259	293	646	491	1988	2184	4663
1968	85	234	91	410	521	1685	643	2849
Total	526	2368	1491	4385	3019	16103	10761	29883

the quadratic and the square root functions. The respective algebraic forms of these functions are as follows:

$$\hat{Y} = b_0 + b_1N + b_2P$$

$$\hat{Y} = b_0N^{b_1} P^{b_2}$$

$$\hat{Y} = b_0 + b_1N + b_2P + b_3N^2 + b_4P^2 + b_5NP$$

$$\hat{Y} = b_0 + b_1N + b_2P + b_3\sqrt{N} + b_4\sqrt{P} + b_5\sqrt{NP}$$

In the functions, P refers to P_2O_5 in pounds per acre, N refers to nitrogen in pounds per acre and Y refers to total yield in bushels per acre.

The linear production function cannot be applied to experiments in which there is a diminishing total yield because the function has no maximum. Also, the function assumes a constant marginal physical product for each nutrient for all levels of input. Hence, the isoquants for this function will be linear.

The Cobb Douglas function also cannot be applied to data indicating a diminishing total yield because the maximum output for the function is undefined. The isoquants of this production function are asymptotic to the input axes. Therefore, there are no ridgelines in the normal sense. Instead, the ridgelines, denoting zero and infinite marginal rates of substitution between factors, are identical with the input axes of a two dimensional isoquant map (Figure 2.5). Also, the Cobb Douglas function forces the yield isoquants to have a constant slope along a fixed nutrient ratio line, that is, it does not permit the marginal rate of substitution to change along the scale line as higher yields are attained. Therefore, the same nutrient combination should be used for all yield levels. In addition, the function assumes each nutrient to be limitational and that the yield is zero when the input of either N or P_2O_5 is zero. However, in the actual experiments, there were positive yields even when the application of both nutrients was zero.

The quadratic and square root functions can be used for fertilizer experiments which have a diminishing total yield because the functions allow both declining and negative marginal products. Although both the quadratic and the square root functions permit a diminishing total yield, the quadratic function has marginal products which decline at a constant rate, whereas, the square root function has marginal products which decline at a diminishing rate. Unlike the Cobb Douglas function, these two functions allow the yield isoquants to change in slope along a fixed nutrient ratio line. Therefore, the quadratic and the square root functions permit the ratios of N and P_2O_5 to change as higher yields are obtained. Also, they permit the interaction of the two fertilizer nutrients in the production process. In addition, the two variable inputs in the quadratic and the square root functions are not assumed to be limitational. Certain yields can therefore be obtained even when the input of N and/or P_2O_5 is zero.

The four functions were fitted to the experimental fertilizer data, utilizing least squares regression analysis.[4] Functions were not fitted for all of the 4385 experiments. Data were utilized only from those experiments which had (1) nine or more observations, (2) at least three

[4] Bernard Ostle, Statistics in Research, (Ames, Iowa: Iowa State University Press, 1964), pp. 161-62. Least squares regression is one of a number of methods that can be used to fit a line (function) to the data. The method of least squares, however, assures us that the sum of the squares of the vertical deviations from the points to the fitted line will be less than the sum of the squares of the vertical deviations from the points to any other line, no matter how computed. That is, $S = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$ is a minimum, where S is the sum of squares, Y_i is the actual yield and $\hat{Y}_i$ is the estimated yield.

different levels of application for each fertilizer nutrient, N and P_2O_5, and (3) at least seven different combinations of N and P_2O_5. These conditions were necessitated in order to obtain meaningful parameter estimates when fitting the quadratic and the square root functions. A minimum of nine observations was chosen to allow for a minimum of three degrees of freedom for the sum of squares of deviations for these two functions.[5] The second and third conditions are necessary because the two functions have five independent variables. In one of the fertilizer experiments, there were twenty-six observations, but there were only six different combinations of N and P_2O_5. Therefore, there are only six actual observations in the experiment. Although there may be five observations with the same combination of fertilizer nutrients, in regression analysis they are treated as one observation. Hence, with only six actual observations there are zero degrees of freedom associated with the sum of squares of deviations for this experiment.

In a number of experiments, potash was applied along with N and P_2O_5. For these experiments, the functions were fitted only to those observations in which potash was not applied, because this study was not designed to estimate the effect of potash on yields. In addition, there were 35 experiments in Alberta in which potash, K_2O, was applied at the same level to all plots including the check plot. Since the level of K_2O was the same for all plots, the yield increases over the check were due to the applications of N and/or P_2O_5. The 35 experiments, however, were excluded from further analysis in this study because the yields obtained

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5 The number of degrees of freedom for the sum of squares of deviations is $n-k-1$ where n is the total number of observations and k is the number of independent variables in the function.

in these experiments are not comparable to those obtained when potash is not applied. In a number of experiments, sulphur was also applied. These observations were included in the estimation of the regression equations, since sulphur was assumed not to have an appreciable effect on yields.

Out of a total of 4385 fertilizer experiments, there were only 927, or one in five, which were acceptable for regression analysis. Of the 927 experiments, 7 per cent were in Manitoba, 66 per cent were in Saskatchewan, and 27 per cent in Alberta and the Peace River area of British Columbia (Table 3.2). The corresponding distribution for the total number of fertilizer experiments was 12, 54 and 34 per cent for Manitoba, Saskatchewan, and the Alberta area, respectively. Excluding those observations in which potash was applied, the 927 experiments had a total of 13,239 observations. The average number of observations per experiment utilized was 14.3, as compared to average of 6.8 for all experiments.

The four functions, linear, Cobb Douglas, quadratic and square root, were fitted to the observations in each of the 927 fertilizer experiments. Mathematical (regression) equations were obtained for each of the four functions for the 927 experiments.

SELECTION OF THE FERTILIZER PRODUCTION FUNCTION

Although regression equations were obtained for the 927 experiments, there were only 93 experiments in which the signs of the regression coefficients for the four derived equations were the same as hypothesized. The hypothesized signs of the regression coefficients for the linear, Cobb Douglas, quadratic and square root functions are shown in the following equations:

$$\hat{Y} = b_0 + b_1 N + b_2 P$$

Table 3.2

Number of Fertilizer Experiments Acceptable for Regression Analysis in the Prairie Provinces and Number of Observations Acceptable in These Experiments

Year	Number of Experiments				Number of Observations			
	Man.	Sask.	Alta. and B.C.	Total	Man.	Sask.	Alta. and B.C.	Total
1959	10	68	52	130	150	1136	814	2100
1960	19	62	31	112	232	1028	480	1740
1961	9	35	11	55	126	542	164	832
1962	-	53	14	67	-	867	212	1079
1963	-	58	15	73	-	798	187	985
1964	-	59	15	74	-	710	231	941
1965	4	47	36	87	76	627	487	1190
1966	4	63	24	91	84	792	379	1255
1967	7	87	41	135	91	1005	661	1757
1968	10	80	13	103	142	918	300	1360
Total	63	612	252	927	901	8423	3915	13239

$$\hat{Y} = b_0 N^{b_1} P^{b_2}$$

$$\hat{Y} = b_0 + b_1 N + b_2 P - b_3 N^2 - b_4 P^2 + b_5 NP$$

$$\hat{Y} = b_0 - b_1 N - b_2 P + b_3 \sqrt{N} + b_4 \sqrt{P} + b_5 \sqrt{NP}$$

If the regression coefficients, b_i's, for the quadratic and the square root equations had only positive signs, then there would only be increasing marginal physical productivity. However, since some of the factors are fixed such as land and seeding rate, it is logical to expect a declining marginal physical productivity. Hence, the hypothesized signs are negative for b_3 and b_4 of the quadratic equation, and for b_1 and b_2 of the square root and either positive or negative for b_5 of both equations. The negative signs for these regression coefficients also permit a diminishing total yield at high rates of fertilization. If the signs for the regression coefficients were negative for the linear equation, then the highest yield would occur where the rate of application of fertilizer is zero. If the signs of the regression coefficients, b_1 and b_2, for the Cobb Douglas equation were negative, then the yields would decline for all rates of fertilizer application greater than zero. Alternatively, if one coefficient is positive and the other is negative, but the sum of the two is less than zero, then yields will decline as the rate of fertilization is increased. Therefore, the signs of the two regression coefficients should be positive. The sum of the two coefficients should be less than one. If it is greater than one, then there is increasing marginal physical productivity as the level of application of N and P_2O_5 is increased.

In most of the experiments that were rejected, the signs of the regression coefficients for the derived quadratic and/or square root equations did not correspond to their hypothesized values. Likewise, a number of experiments were rejected because of conflicting signs for the regression coefficients of the derived linear and Cobb Douglas equations. There were a number of experiments rejected in which none of the derived equations corresponded with the expected signs for the four functions.

Of the 93 experiments whose regression equations were not rejected, 9 were located in Manitoba, 35 in Saskatchewan, and 49 in Alberta and the Peace River area of British Columbia (Table 3.3). The percentage distribution of these 93 experiments among the three regions was 10, 37 and 53 per cent, respectively. For the 93 experiments giving acceptable results, the number of experiments in each year for the Prairies as a whole ranged from a low of one in 1963, to a high of 20 in 1967 (Table 3.3).

After selecting those experiments which were acceptable for further analysis, the next step was to determine which of the four functions provided the "best fit" to the observations for the 93 experiments. The function selected is assumed to give the best estimate of the fertilizer-crop, input-output relationship. The selection of one such equation is difficult to make, however, because there is no direct or objective test for comparing the "goodness of fit" among the four functions.

Heady, Pesek and Brown fitted three functions with two variable nutrients to the observations in each fertilizer experiment.[6] These were the Cobb Douglas, the quadratic and the square root functions. The function which they selected as having the best fit was the square root. The selection of this function was made by (1) comparing the "regression statistics" for each function, (2) comparing single-nutrient response

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[6] Earl O. Heady, John T. Pesek, and William G. Brown, in co-operation with Tennessee Valley Authority, Crop Response Surfaces and Economic Optima in Fertilizer Use, (Ames, Iowa: Agricultural Experiment Station, Iowa State College, Research Bulletin 424, March, 1955).

Table 3.3

Number of Fertilizer Experiments in the Prairie Provinces with Acceptable Regression Coefficients and Number of Observations in These Experiments

	Number of Experiments				Number of Observations			
Year	Man.	Sask.	Alta. and B.C.	Total	Man.	Sask.	Alta. and B.C.	Total
1959	1	3	5	9	17	48	74	139
1960	2	5	9	16	20	91	144	255
1961	-	5	-	5	-	84	-	84
1962	-	8	2	10	-	132	32	164
1963	-	1	-	1	-	16	-	16
1964	-	5	5	10	-	89	80	169
1965	-	-	4	4	-	-	51	51
1966	1	2	8	11	21	27	128	176
1967	3	4	13	20	47	41	208	296
1968	2	2	3	7	22	24	81	127
Total	9	35	49	93	127	552	798	1477

curves derived by the three two-variable functions with a similar curve estimated from the best fitting single-variable function, and (3) comparing the response curves and yield isoquants estimated from the two-variable functions with scatter diagrams of the observations. Gilson and Bjarnarson, in 1958, fitted these three functions to Manitoba data.[7] They concluded that the quadratic function provided the best fit for their data. Their selection was made on the basis of the coefficient of determination and the t-tests for the regression coefficients. Tolton, Gilson and Hedlin, in a study to determine the yield response of barley on stubble land to application of fertilizer, fitted the Cobb Douglas and quadratic functions to their data.[8] The quadratic function was again selected to express the logical physical relationship between the fertilizer nutrients and the crop yields, because it provided a better fit to the data than the Cobb Douglas. The selection was made on the basis of the coefficient of determination and the t-values for the regression coefficients for each function.

In this study, the selection of the fertilizer production function was made on the basis of the coefficient of determination, the t-tests for the regression coefficients, and the F-ratio for each function.[9] The average values of the various statistics for each function for the 93 experiments are given in Table 3.4.[10] The regression statistics for the

[7] J. C. Gilson and V. W. Bjarnarson, "Effects of Fertilizer Use on Barley in Northern Manitoba", Journal of Farm Economics, XL (No. 4, November, 1958), 932-41.

[8] H. E. Tolton, J. C. Gilson, and R. A. Hedlin, "Physical and Economic Relationships Involved in Fertilizer Use", Canadian Journal of Agricultural Economics, V (No. 2, 1957), 1-8.

[9] For an explanation of these terms see George W. Snedecor and William G. Cochran, Statistical Methods (Ames, Iowa: Iowa State University Press, 1967), pp. 381-418.

[10] The averages were found by adding up the values for each function in each experiment and then dividing the sums by 93, the number of experiments.

individual experiments are given in Table A.5.

Table 3.4

Average Values of Regression Statistics by Function for the 93 Experiments

Function	Value of R^2	Value of t-test for the regression coefficients of the following terms: N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	F-ratio
Linear	0.528	2.37*	2.61*				9.38**
Cobb Douglas	0.736	4.40**	4.86**				41.62**
Quadratic	0.803	2.83*	3.18**	2.62*	2.77*	1.32	16.21**
Square Root	0.816	1.67+	1.76+	2.02++	2.09++	1.81++	24.36**

Note: The average number of observations for the 93 experiments was 15.9.

** Significant at the 1 per cent level.

* Significant at the 5 per cent level.

++ Significant at the 10 per cent level.

\+ Significant at the 20 per cent level.

The coefficients of determination (R^2) show considerable differences in percentages of yield explained by the variable nutrients. For example, in the linear 53 per cent is explained by differences in the application of N and P_2O_5, while utilizing the square root they explain 82 per cent. The t values show that over one-half of the regression coefficients are significant at the 5 per cent level of probability. This level of significance indicates a probability of .05 of rejecting the hypothesis under test.[11] At the 20 per cent level of probability, all regression coefficients but one are significant. The F-ratio, which is the test of significance for the coefficient of determination, was significant at the one per cent level for all four functions.

[11] For additional information on tests of significance, see Snedecor and Cochran, op.cit., pp. 26-31, 391-92.

The Cobb Douglas and the linear functions were not selected as the best representation of the physical relationship between the fertilizer inputs and the crop yields. Although on average the t values for the regression coefficients in the Cobb Douglas and the linear functions were significant at the one per cent and five per cent levels, respectively, the R^2 values were somewhat lower for these two functions than for the quadratic and square root functions. In addition, since these two functions do not allow diminishing total yields, they can be rejected on logical grounds for experiments with high fertilization. The linear isoquants of the linear function and the constant marginal rate of substitution along a scale line of the Cobb Douglas function, make these two functions less desireable than the quadratic and square root functions, in determining the economic relationships involved in fertilizer use.

There was little basis to distinguish between the quadratic and square root functions. The t values for the regression coefficients in the quadratic function were on average higher than those for the square root function; whereas, the square root function had slightly higher coefficients of determination. However, on the basis of the higher t values, the quadratic function was considered to provide a "better estimate" of the yield responses to fertilizer inputs. The quadratic function is therefore used as the basis for discussion of fertilizer use in the remainder of this thesis.

In the first section of the next chapter, we will examine the economic optima in the use of fertilizer for a particular location and year. In the second section, the economic optima in fertilizer use among locations and among years for various crops will be explored.

Chapter 4

PHYSICAL AND ECONOMIC RELATIONSHIPS INVOLVED IN FERTILIZER USE

YIELD RESPONSE SURFACE AND ECONOMIC OPTIMA IN FERTILIZER USE FOR BARLEY GROWN ON PEACE HILLS FINE SANDY LOAM

The purpose of this section is to illustrate, through the use of a particular fertilizer experiment, the methodological procedures and the equations involved in determining the yield response surface and economic optima in fertilizer use, using the quadratic form of the production function. The economic optima in fertilizer use for the various fertilizer experiments are presented in the next section and in Appendices A and B. However, the equations used to derive the economic optima are not included, since the equations are the same for every experiment except for the regression coefficients and the crop prices.

The experiment was barley seeded on stubble on Peace Hills fine sandy loam at Lacombe in 1967. There were four levels of application of each nutrient, N and P_2O_5, at 0, 30, 60 and 90 pounds per acre for a total of 16 treatments, with three replicates for each treatment. The different combinations of N and P_2O_5 and the corresponding average yields of barley from three replicates are presented in Table 4.1

Table 4.1

Actual Yields per Acre of Barley for Specified Levels of N and P_2O_5, Lacombe, 1967

Pounds of P_2O_5 Per Acre	Pounds of N Per Acre 0	30	60	90
		-bushels-		
0	33.1	40.4	45.2	41.5
30	44.4	55.4	50.4	46.7
60	40.6	56.7	58.1	50.6
90	41.0	51.0	56.5	52.1

The Production Function

A quadratic function of the general form:

$$\hat{Y} = b_0 + b_1N + b_2P + b_3N^2 + b_4P^2 + b_5NP$$

was fitted by least squares regression to the fertilizer-yield observations in Table 4.1.[1] The resulting equation was:

$$\hat{Y} = 33.6616 + 0.4553N + 0.3438P - 0.0044N^2 - 0.0029P^2 + 0.0006NP$$

The regression statistics for the quadratic function and those for three other functions, that were fitted to the data, are given in Table 4.2. In this particular experiment, the t values and the coefficient of determination were both higher for the quadratic function than for the square root function.

Table 4.2

The Coefficient of Determination, the t-Tests for the Regression Coefficients and the F-Ratio of the Four Equations

Equation	Value of R^2	Value of t-test for the regression coefficients of the following terms: N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	F-ratio
Linear	0.433	1.94++	2.48*				4.97*
Cobb Douglas	0.798	5.03**	5.12**				25.74**
Quadratic	0.904	5.93**	4.48**	5.80**	3.83**	1.02	18.95**
Square Root	0.884	3.91**	2.00++	4.44**	2.90*	0.67	15.23**

** Significant at the 1 per cent level.

* Significant at the 5 per cent level.

++ Significant at the 10 per cent level.

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[1] The linear, Cobb Douglas and square root functions were also fitted to the data given in Table 4.1. The resulting equations were:

$$\hat{Y} = 39.0163 + 0.0851N + 0.1086P$$

$$\hat{Y} = 34.043 \; N^{0.0540} \; P^{0.0549}$$

$$\hat{Y} = 33.2074 - 0.3212N - 0.1647P + 3.7416\sqrt{N} + 2.4432\sqrt{P} + 0.0396\sqrt{NP}$$

Production Surface Estimates

The barley yields estimated from the quadratic equation are shown in Table 4.3 for different applications of N and P_2O_5.[2] It will be observed that the estimated yields in Table 4.3 conformed very closely to the actual yields in Table 4.1. Both diminishing marginal physical product and diminishing total physical product can be observed from the data in Table 4.3. For example, with P_2O_5 held constant at 30 pounds per acre and N increasing from 0 to 90 pounds per acre, barley yield increases at a diminishing rate from 41.3 bushels to a maximum yield of 53.9 bushels. The yield drops back to 48.2 bushels with N at 90 pounds per acre. Similarly, with N held constant at 30 pounds per acre and P_2O_5 varied from 0 to 90 pounds per acre, barley yield increases at a diminishing rate.

These estimated yields are the tabular counterpart of a production surface. They represent distinct points on the surface corresponding to the P_2O_5 and N quantities in the rows and columns. The nature of the

Table 4.3

Yields per Acre of Barley Predicted by the Quadratic Function for Specified Levels of N and P_2O_5, Lacombe, 1967

Pounds of P_2O_5 Per Acre	Pounds of N Per Acre 0	30	60	90
		-bushels-		
0	33.7	43.3	45.1	38.8
30	41.3	51.6	53.9	48.2
60	43.8	54.6	57.4	52.3
90	40.9	52.3	55.7	51.1

[2] Yields can also be estimated for fertilizer treatments which were not included in the actual experiment.

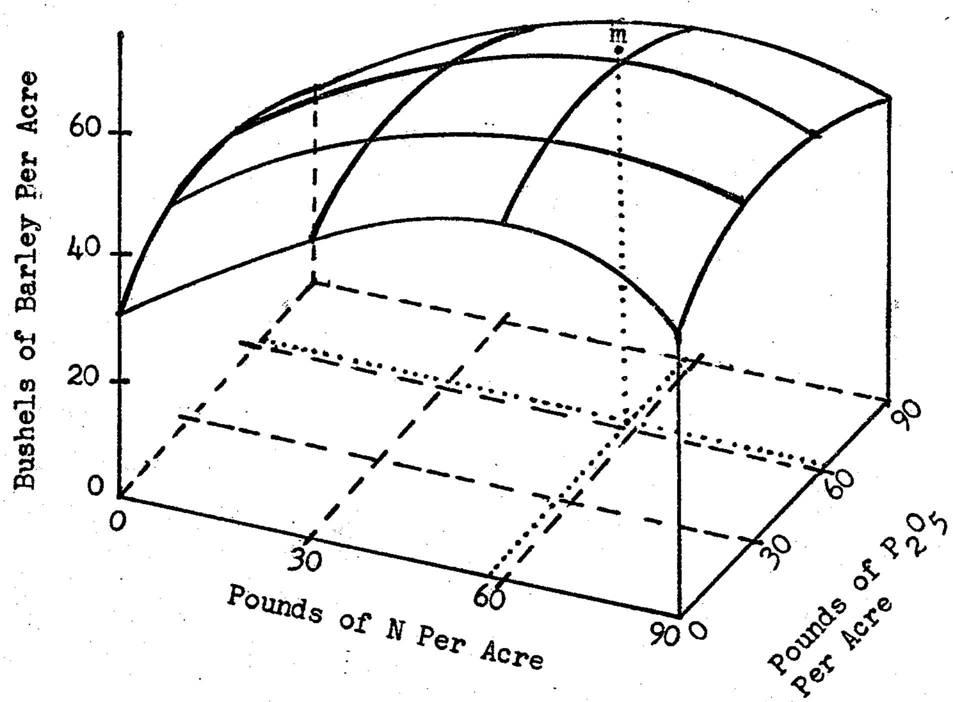

Figure 4.1. Yield Response Surface for Barley Predicted by Quadratic Function.

production surface is expressed in Figure 4.1. The estimated maximum yield which corresponds to the highest point on the production surface is 57.6 bushels with 56.2 pounds of N and 65.1 pounds of P_2O_5.[3] This maximum yield occurs at point m in Figure 4.1.

Single Variable Input-Output Curves

Each column in Table 4.3 is the counterpart of a vertical slice through the production surface, parallel to the P_2O_5 axis. The yields correspond to points on a single variable input-output curve with P_2O_5 as the variable input while N is fixed in the amount shown at the top of the column. The rows on the other hand represent a vertical slice through the surface, parallel to the N axis, with N variable and P_2O_5 fixed. Figures 4.2 and 4.3 are input-output curves or yield response curves with one fertilizer nutrient variable and the other fixed at some specified level. Figure 4.2 shows the input-output curves for nitrogen fixed at 0, 30, 60, and 90 pounds per acre while Figure 4.3 shows the yield response to N with P_2O_5 fixed at 0, 30, 60 and 90 pounds per acre.

The input-output curves in both figures show diminishing total yield for all four levels of the fixed nutrient. Also, the curves are steeper between the 0 and 30 pound levels of application than between the 30 and 60 pound levels of application of the variable nutrient, indicating that there is a declining marginal physical product. In both figures, the

[3] The estimated maximum yield was obtained in the following manner. The marginal physical product equations for both nutrients were set equal to zero and solved simultaneously for the quantities of each nutrient which would maximize yield. The marginal physical product equations are the partial derivatives of Y with respect to N and P.

$$\partial Y/\partial N = 0.4553 - 0.0088N + 0.0006P = 0$$

$$\partial Y/\partial P = 0.3438 - 0.0058P + 0.0006N = 0$$

These quantities were substituted back into the original equation and the maximum yield was estimated accordingly.

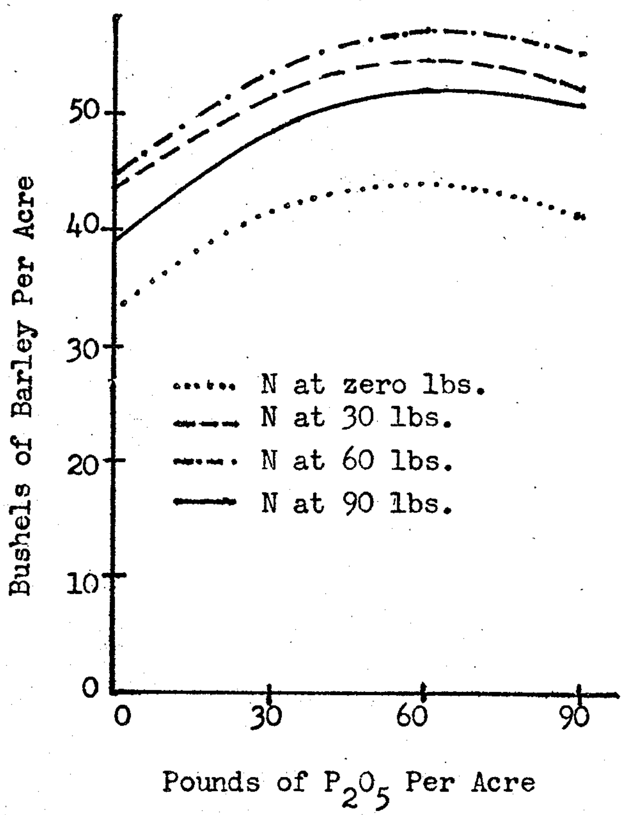

Figure 4.2. Yield Response of Barley to P_2O_5 with N Fixed at 0, 30, 60 and 90 Pounds, Predicted by Quadratic Function.

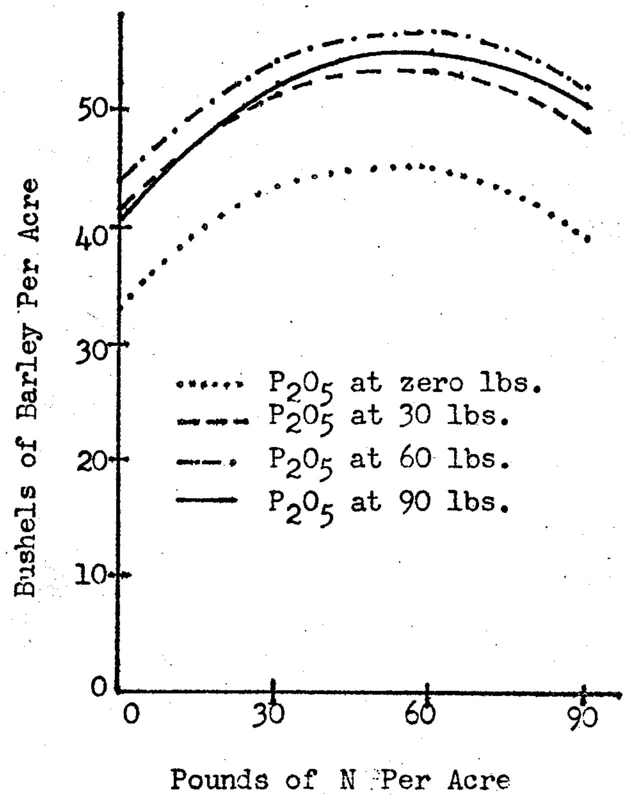

Figure 4.3. Yield Response of Barley to N with P_2O_5 Fixed at 0, 30, 60 and 90 Pounds, Predicted by Quadratic Function.

curve for the 90 pound application of the fixed nutrient is below the 60 pound application curve. In the case of nitrogen, higher levels of nitrogen tend to result in excess leaf growth, delayed maturity and lodging, thus reducing yields. As for P_2O_5, the reduced yields are more likely the result of an imbalance between the levels of the two nutrients.

Yield Isoquants

The yield isoquant expresses all combinations of N and P_2O_5 which will produce a specified yield. The various combinations of N and P_2O_5 which may be used in producing yields of 40 and 50 bushels of barley are given in Table 4.4.[4] These yield isoquants are presented in Figure 4.4.

The marginal rates of substitution of N for P_2O_5 at the various combinations of the two fertilizer elements are given in columns 3 and 6

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[4] These results were derived from the following isoquant equation:

$$P = \frac{-(0.3438+0.0006N)\pm\sqrt{(0.3438+0.0006N)^2+0.0116(33.6616+0.4553N-0.0044N^2-\hat{Y})}}{-0.0058}$$

This equation was derived from the quadratic regression equation:

$$\hat{Y} = 33.6616 + 0.4553N + 0.3438P - 0.0044N^2 - 0.0029P^2 + 0.0006NP$$

quadratic formula: $X = \frac{-B\pm\sqrt{B^2 - 4AC}}{2A}$

Converting the above equation to the standard form of $AX^2 + BX + C = 0$, we have $-0.0029P^2 + 0.3438P + 0.0006NP + 36.6616 + 0.4553N - 0.0044N^2 - \hat{Y} = 0$

A B C

The values for A, B, and C were substituted into the formula to obtain the isoquant equation.

Table 4.4

Fertilizer Combinations and Corresponding Marginal Rates of Nutrient Substitution (N for P_2O_5) for Barley Yields of 40 and 50 Bushels Predicted by the Quadratic Function, Lacombe, 1967

40 Bushels of Barley			50 Bushels of Barley		
N	P_2O_5	M.R.S. of N for P_2O_5*	N	P_2O_5	M.R.S. of N for P_2O_5*
-pounds per acre-					
(1)	(2)	(3)	(4)	(5)	(6)
0.0	22.9	-2.24	15.0	53.1	-8.35
1.0	20.8	-2.06	16.0	47.3	-4.44
2.0	18.8	-1.91	17.0	43.5	-3.31
3.0	16.9	-1.78	18.0	40.5	-2.72
4.0	15.2	-1.67	19.0	38.0	-2.33
5.0	13.6	-1.57	20.0	35.8	-2.05
6.0	12.1	-1.48	21.0	33.9	-1.83
7.0	10.6	-1.40	22.0	32.1	-1.65
8.0	9.3	-1.32	23.0	30.5	-1.51
9.0	8.0	-1.26	24.0	29.1	-1.38
10.0	6.7	-1.19	25.0	27.8	-1.28
11.0	5.6	-1.14	26.0	26.5	-1.18
12.0	4.5	-1.08	27.0	25.4	-1.10
13.0	3.4	-1.03	28.0	24.3	-1.02
14.0	2.4	-0.98	29.0	23.4	-0.95
15.0	1.4	-0.94	30.0	22.4	-0.88
16.0	0.5	-0.90	31.0	21.6	-0.82
			32.0	20.8	-0.76
			33.0	20.1	-0.71
			34.0	19.4	-0.66

* Negative numbers indicate the pounds of P_2O_5 replaced by one pound of N at each of the fertilizer combinations listed.

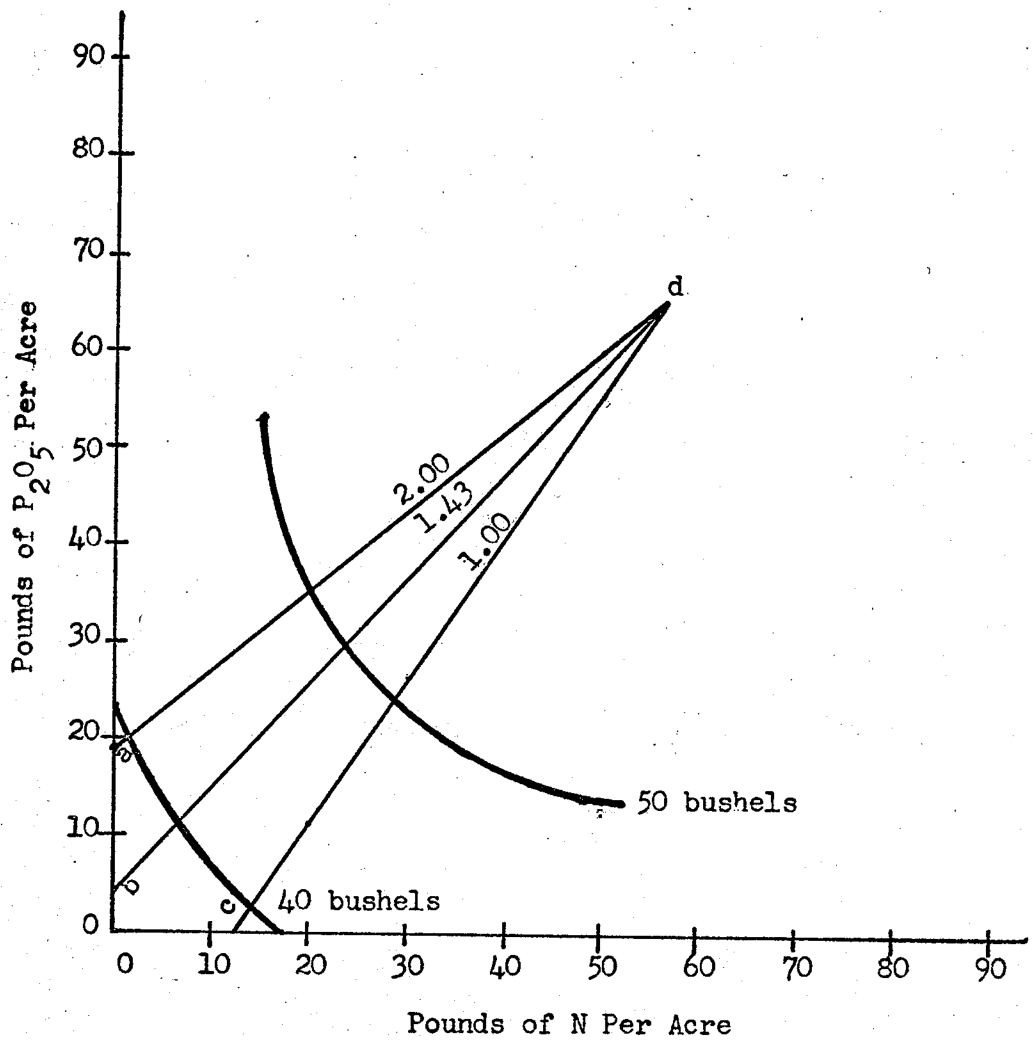

Figure 4.4. Isoquants and Isoclines for Barley Predicted by Quadratic Function for Lacombe, 1967.

of Table 4.4.[5] The marginal rate of substitution designates the slope of a given yield isoquant at a particular point. It will be noted that a diminishing marginal rate of substitution existed between the two fertilizer elements. In column 3, for example, at the point where zero pounds of N and 22.9 pounds of P_2O_5 are used to produce 40 bushels of barley, one additional pound of N will replace 2.24 pounds of P_2O_5. At the point where 14 pounds of N and 2.4 pounds of P_2O_5 produce 40 bushels of barley, one pound of N will replace only 0.98 pounds of P_2O_5.

The isocline, bd, in Figure 4.4, illustrates all combinations where the marginal rate of substitution of N for P_2O_5 (dP/dN) is -1.43.[6] On the 50 bushel isoquant, for example, where N is 23.6 pounds and P_2O_5 is 29.7 pounds, one pound of N will substitute for 1.43 pounds of P_2O_5. The marginal rates of substitution at all points on isoclines ad and cd

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[5] The marginal rate of substitution of N for P_2O_5 was derived from the following equation:

$$\frac{dP}{dN} = -\frac{0.4553 - 0.0088N + 0.0006P}{0.3438 - 0.0058P + 0.0006N}$$

This equation can be obtained by taking the derivative, dP/dN, of the isoquant equation, since the slope of the isoquant is the marginal rate of substitution. It can also be obtained by differentiating implicitly the production function. The marginal rate of substitution is also equivalent to the inverse ratio of the two marginal physical products, that is, $M.R.S. = \frac{dP}{dN} = \frac{\partial Y/\partial N}{\partial Y/\partial P}$ where $\partial Y/\partial N = 0.4553 - 0.0088N + 0.0006P$ and $\partial Y/\partial P = 0.3438 - 0.0058P + 0.0006N$.
The values obtained by the marginal rate of substitution equation are the substitution or replacement rates at the exact nutrient combinations shown; they are not averages between nutrient combinations.

[6] An isocline indicates combinations of N and P_2O_5 which have the same marginal rate of substitution. The combinations were determined from the following isocline equation:

$$P = \frac{k(0.3438) - 0.4553}{0.0006 + k(0.0058)} + \left[\frac{k(0.0006) + 0.0088}{0.0006 + k(0.0058)}\right] N$$

This equation was derived by setting the marginal rate of substitution equal to a constant k, to represent a given marginal rate of substitution and solving for one input in terms of the other.

are -2.00 and -1.00, respectively.[7] Table 4.5 gives the various combinations of N and P_2O_5 which have marginal rates of substitution of -2.00, -1.43 and -1.00. The three isoclines converge to a single point which corresponds to the point of maximum yield on the production surface.

Table 4.5

Combinations of N and P_2O_5 for Specified Isoclines Predicted by Quadratic Function, Lacombe, 1967

k = -2.00		k = -1.43		k = -1.00	
N	P_2O_5	N	P_2O_5	N	P_2O_5
		-pounds per acre-			
0.0	19.0	0.0	4.1	0.0	-18.0
10.0	27.2	10.0	15.0	10.0	- 3.5
20.0	35.4	20.0	25.9	20.0	11.4
30.0	43.6	30.0	36.8	30.0	26.1
40.0	51.8	40.0	47.7	40.0	40.8
50.0	60.0	50.0	58.6	50.0	55.5

Least Cost Nutrient Combination for a Specified Yield

The change in the slopes of the yield isoquants (along a scale line) suggests that the combination of the two nutrients which will give the lowest cost, for a stated yield, changes with the level of yield. The nutrient combination which is optimum for a 50 bushel yield is not also optimum for a 40 bushel yield. This point also is illustrated with the isoquant and substitution data of Table 4.4. The least cost nutrient combination for a given yield is attained when the marginal rate of

[7] The values, -2.00, -1.43 and -1.00 for k, are the inverse price ratios, Pn/Pp, of the fertilizer nutrients when N is \$0.1712, \$0.1226 and \$0.0856 per pound and P_2O_5 is \$0.0856 per pound.

substitution of the two nutrients is equal to their inverse price ratio, that is, M.R.S. = dP/dN = Pn/Pp. With the price of N at $0.1226 per pound and the price of P_2O_5 at $0.0856 per pound, the inverse price ratio Pn/Pp, is -1.43. The least cost combination of these two elements therefore lies along the isocline bd in Figure 4.4.

The least cost combination of N and P_2O_5 for the 40 and 50 bushel yields of barley are given in Table 4.6.[8] For the 40 bushel yield, the least cost combination occurs where 6.6 pounds of N and 11.2 pounds of P_2O_5 are used. The marginal rate of substitution at this point on the yield isoquant is -1.43. For the 50 bushel yield, the least cost nutrient combination is 23.6 pounds of N and 29.7 pounds of P_2O_5 with a M.R.S. of -1.43.[9]

From Table 4.6, it can be seen that as yields increase nitrogen

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[8] In this study, the least cost combination for a given yield was determined by first solving for P in terms of N in the isocline equation where k is set equal to -1.43.

$$P = \underbrace{\frac{1.43\ (0.3438) - 0.4553}{0.0006 + (1.43)(0.0058)}}_{A} + \underbrace{\left[\frac{1.43(0.0006) + 0.0088}{0.0006 + 1.43\ (0.0058)}\right]}_{B} N$$

This value for P was then substituted for P in the production equation to obtain the value for N

$$\hat{Y} = 33.6616 + 0.4553N + 0.3438(A + BN) - 0.0044N^2 - 0.3029(A + BN)^2 + 0.0006N(A+BN)$$

The value for N was then substituted back into the isocline equation to obtain the value for P. The values obtained for N and P are the least cost nutrient combinations for the specified yield.

The method first used in this study gave only an approximation of the least cost nutrient combination. The marginal rate of substitution was calculated for each one pound increment in N starting with N=0, for each specified yield. The combination of N and P_2O_5 which had a marginal rate of substitution closest to 1.43 was assumed to be the least cost combination for that yield. If, however, the N had been increased by an increment of 0.1 pound, then this iterative method would be almost as accurate as the exact method given above.

[9] The values obtained for N and P_2O_5 by the iterative method for the 40 bushel yield are 7.0 and 10.6 pounds per acre, respectively. For the 50 bushel yield, the values were 24.0 and 29.1 pounds for N and P_2O_5, respectively. The marginal rates of substitution were -1.40 and -1.38.

Table 4.6

Combinations of N and P_2O_5 to Minimize Fertilizer Costs per Specified Yield Level for a Constant Price Ratio Predicted by Quadratic Function, Lacombe, 1967.

Yield level	N	P_2O_5	M.R.S. dP / dN	$\frac{N}{P_2O_5}$ Ratio
bushels	-pounds-			
40.0	6.6	11.2	-1.43	0.59
50.0	23.6	29.7	-1.43	0.79

becomes relatively more important. The ratio of N to P_2O_5 in the least cost combination increases from .59 for the 40 bushel yield to .79 for the 50 bushel yield.

The significance of finding the least cost combination of fertilizer nutrients may be illustrated by comparing the cost of two different combinations of N and P_2O_5 in producing 50 bushels of barley. In one case, 15.0 pounds of N and 53.1 pounds of P_2O_5 could be used. The total cost of fertilizer in this instance, with the price of N at $0.1226 per pound and the price of P_2O_5 at $0.0856 per pound, would be $6.38. On the other hand, the least cost combination of 23.6 pounds of N and 29.7 pounds of P_2O_5, with the same nutrient prices, would have a cost of $5.44. Thus, the difference in cost per acre between the two fertilizer combinations is $0.94. Similar cost calculations could be made for any other combinations of N and P_2O_5 in producing a given yield of barley.

A farmer with limited capital, may be more interested in knowing the maximum yield that can be obtained from a given capital outlay, rather than the combination of fertilizer nutrients which will produce a specified yield at least cost. The $5.38 expenditure per acre for the least cost combination of nutrients for the 50 bushel yield may be more than the farmer can afford. He may want to know what is the highest yield that can be attained with an expenditure of $2.50 per acre for fertilizer. Assuming the same nutrient prices, the highest yield that can be attained with an outlay of $2.50 for fertilizer is 42.3 bushels of barley per acre. The combination of fertilizer nutrients with a total cost of $2.50 that will produce this yield is 10.0 pounds of N and 14.9 pounds of P_2O_5.[10] This nutrient combination is also the least cost combination for that particular yield. The combinations of N and P_2O_5 and the corresponding yields for outlays of $0.00, $2.50, $5.00, $7.50 and $10.00 for fertilizer per acre are given in Table 4.7.

It can be observed from Table 4.7 that each $2.50 increase in fertilizer expenditure results in a smaller and smaller increase in yield.

Table 4.7

Barley Yields, and N and P_2O_5 Combinations Predicted for Limited Capital Situations by the Quadratic Function, Lacombe, 1967

Capital	Barley yield	N	P_2O_5	M.R.S. dP / dN	$\frac{N}{P_2O_5}$ Ratio
	bushels	-pounds-			
$ 0.00	33.7	0.0	0.0	-	-
2.50	42.3	10.0	14.9	1.43	0.67
5.00	49.0	21.6	27.5	1.43	0.79
7.50	53.8	33.2	40.1	1.43	0.83
10.00	56.6	44.8	52.6	1.43	0.85

[10] The fertilizer nutrient combinations were derived from the following isocline and cost equations:

$$P = \frac{1.43(0.3438) - 0.4553}{0.0006 + 1.43\,(0.0058)} + \left[\frac{1.43(0.0006) + 0.0088}{0.0006 + 1.43\,(0.0058)}\right] N$$

$$C = 0.1224N + 0.0856P$$

where C is the expenditure level per acre. The values derived for N and P_2O_5 were substituted into the original production function to obtain the corresponding yield for each expenditure level.

An outlay of \$2.50 on fertilizer results in a yield increase of 8.6 bushels of barley per acre. An additional \$2.50 expenditure on fertilizer results in a yield increase of only 6.7 bushels. As yields increase from 42.3 to 56.6 bushels per acre, the ratio of N to P_2O_5 increases from .67 to .85. Hence, the fertilizer mixture should contain relatively more nitrogen for higher yield levels, if costs are to be minimized. Traditionally, this distinction has not been made in fertilizer recommendations; the same fertilizer mix has, for a given soil and productivity situation, usually been recommended for numerous yield levels.

Profit Maximization

The next step is to determine simultaneously: (1) the least cost combination of N and P_2O_5 to produce a given yield, and (2) the amount of the two nutrients to apply to produce the profit maximizing yield for a given price of barley. These economic optima are attained by setting the partial derivatives (the marginal physical products) for both nutrients equal to their respective nutrient/barley price ratio and solving simultaneously for the quantities of N and P_2O_5 which will maximize profits. These quantities are then substituted back into the original production equation to obtain the most profitable yield.

The maximum profit position was calculated for five different prices of barley (Table 4.8). For example, when the price of barley is \$0.65 per bushel and the price of N and P_2O_5 is \$0.1226 and \$0.0856 per pound, respectively, the most profitable use of fertilizer occurs with an application of 32.9 pounds of N and 39.8 pounds of P_2O_5 per acre.[11] The

11 The most profitable application of the two nutrients was found by simultaneously solving the following two equations:

$$\partial Y/\partial N = 0.4553 - 0.0088N + 0.0006P = Pn/Pb$$
$$\partial Y/\partial P = 0.3438 - 0.0058P + 0.0006N = Pp/Pb$$

where $\partial Y/\partial N$ is the marginal productivity of N and $\partial Y/\partial P$ is the marginal productivity of P_2O_5, and Pb, Pn and Pp are the respective prices for barley, N and P_2O_5.

corresponding yield was 53.7 bushels of barley.

The most profitable application of the two fertilizer nutrients is important to farmers with limited funds as well as to farmers with unlimited capital. However, for a farmer who has limited capital, a higher return may be realized if he invested the capital elsewhere in the business. Suppose a farmer with limited capital can earn \$2.00 return per dollar invested on funds for other lines of his business such as seed, chemical sprays, livestock or machinery. If he invests in fertilizer to produce a yield of 53.7 bushels of barley per acre, his return per dollar invested is only \$1.75. Therefore, the farmer will allocate his scarce funds where he can get \$2.00. If, however, the farmer invests only \$2.50 in fertilizer rather than \$7.44, then his return per dollar invested in fertilizer is \$2.24.

From Table 4.8, it can be seen that the ratio of N to P_2O_5 is virtually unchanged as the price of barley increases by \$0.40 from \$0.65 to \$1.05 per bushel. It would appear that at these yield levels both N and P_2O_5 should be increased in the same proportion, if profit is to be at a maximum. However, this is contrary to the situation for the lower yields where proportionally more nitrogen should be used as yields increase. Although the price of barley increases by 62 per cent, from \$0.65 to \$1.05, the increase in the maximum profit yield is only 4 per cent, indicating that the response surface is comparatively flat at these yield levels.

Two aspects of the preceeding analysis should be noted at this time. It would not be practical for farmers to attempt to apply the exact amounts of N and P_2O_5 as indicated by the optimum. Yields are never that certain. In addition, it is difficult for a farmer to obtain the exact ratio of N to P_2O_5 using a mixture of pre specified fertilizer formulations

Table 4.8

Optimum Combinations of N and P_2O_5 for Specified Price Relationships Predicted by the Quadratic Function, Lacombe, 1967*

Price relationships	Barley yield	N	P_2O_5	$\frac{N}{P_2O_5}$ Ratio
	bushels	-pounds-		
Barley at $0.65	53.7	32.9	39.8	0.83
Barley at 0.75	54.7	36.0	43.1	0.84
Barley at 0.85	55.3	38.4	45.6	0.84
Barley at 0.95	55.7	40.2	47.7	0.84
Barley at 1.05	56.1	41.7	49.3	0.85

* The prices for N and P_2O_5, respectively, were $0.1226 and $0.0856 per pound.

such as 16-20-0 and 11-48-0, unless he has facilities for bulk blending of fertilizer. Secondly, the least cost combination of fertilizer nutrients does not necessarily follow a fixed ratio for all possible yield levels. Therefore, a different fertilizer mix would be required for each yield level.

The objective of the analysis in this section was to examine the methodological procedures involved in determining the economic optima in fertilizer use, using the quadratic form of the production function. In the next section, the economic optima for a number of selected experiments will be examined and discussed. Only the economic optima for each experiment will be presented. The procedures that are involved in determining the economic optima will not be presented, since they are the same for every experiment. Also, the form of the equations used to obtain the various economic optima will be the same for each experiment. Obviously, the values for the regression coefficients will differ among experiments.

ECONOMIC OPTIMA IN FERTILIZER USE FOR VARIOUS LOCATIONS FOR DIFFERENT YEARS AND CROPS.

The purpose of this section is to examine the least cost combinations and the maximum profit positions for 57 fertilizer experiments conducted in the Prairies for locations, years and crops.[12]

Least cost combinations were derived for specific yields for wheat, oats, barley and rye seeded on summerfallow and stubble. Table 4.9

Table 4.9

Specified Yields for Wheat, Oats, Barley, and Rye

Wheat	Oats	Barley	Rye
	-bushels per acre-		
10	30	10	20
15	40	20	25
20	50	30	30
25	60	40	35
30	70	50	40
35	80	60	
40	90	70	
45		80	
50		90	
55			
60			

[12] The other 36 experiments were not included in the thesis for a number of reasons. Twenty-five experiments were excluded because the level of N and P_2O_5 in the maximum profit position was negative for one or more of the price levels of the crop. The level of application of the nutrient or nutrients was negative because there was no positive level of application which had a marginal physical product equal to the nutrient/crop price ratio. The equating of marginal physical product and the price ratio is one of the necessary conditions for profit maximization. Six experiments were omitted because they were duplicates of other experiments included in the thesis. Where there was more than one experiment for a particular location, crop and year, the experiment which had the best fitting quadratic equation was used. The remaining five experiments were excluded because the level of nutrient application estimated for the maximum profit position was much higher than the highest application of the nutrients in the actual experiment.

gives the specified yields for each crop. These yields were the same for all locations, in order to facilitate comparisons among regions.

The same fertilizer nutrient prices of $0.1226 per pound for N and $0.0856 per pound for P_2O_5 were used for all locations in the Prairies in order to facilitate regional comparisons. The prices for the nutrients were obtained from a 1967-68 fertilizer retail price list for the Prairies.[13] Fertilizer pricing in the Prairies is divided into five zones, with zone 1 having the lowest prices and zone 5 the highest. The price differential between the zones is $1.00 per ton. The prices used were those for zone 3.

The maximum profit position was calculated for five separate prices for each grain. The prices that were selected represent the range in prices received by farmers in the Prairies for the various crops in the period 1959 to 1968.[14] The prices for the various crops, which were the same for all Prairie locations, are given in Table 4.10.

Table 4.10
Selected Prices per Bushel of Wheat, Oats, Barley and Rye

Wheat	Oats	Barley	Rye
	-dollars-		
1.40	0.50	0.65	0.95
1.50	0.55	0.75	1.00
1.60	0.60	0.85	1.05
1.70	0.65	0.95	1.10
1.80	0.70	1.05	1.15

[13] National Grain Company Limited, "1967-68 Confidential Dealer Fertilizer Price List", (August, 1967).

[14] The selected prices for the various crops were based upon the farm prices reported in Table 2 of the Quarterly Bulletin of Agricultural Statistics for January-March, 1964, 1966, and 1969.

<u>Economic Optima for Wheat</u>

Economic optima were obtained for 22 experiments for wheat seeded on summerfallow. The least cost nutrient combinations for specified yields and the maximum profit yields for the five selected crop prices are presented in Table B.1 of Appendix B. The maximum profit yields for the lowest and highest prices of wheat for the 22 experiments are illustrated in Figure 4.5. The optimum fertilizer nutrient levels for these yields are shown in Figure 4.6. From Figure 4.5, it can be seen that, although the price of wheat increases by 40 cents, the increases in the maximum profit yields of wheat are quite small. The largest yield increase was 2.1 bushels at Melfort in 1964. The smallest increase was 0.1 bushels at Glaslyn in 1962 and Bonnyville in 1960. These small yield increases indicate that the marginal physical productivities of the individual fertilizer nutrients are almost zero at these rates of nutrient application, and that the maximum profit yields are approaching the maximum yield in each experiment. They also indicate that the application of fertilizer on wheat seeded on summerfallow is almost as profitable when the price of the grain is $1.40 per bushel as when the price is considerably higher.

The maximum profit positions for wheat seeded on summerfallow in Table B.1 and Figure 4.5, indicate that for a given location, the maximum profit yields will vary considerably among years. At Melfort, for example, with the price of wheat at $1.40 per bushel, the most profitable yields of wheat on summerfallow are 33.9 bushels in 1960, 55.3 bushels in 1962, 26.2 bushels in 1964, and 62.1 bushels in 1966. Similarly, for wheat seeded on summerfallow at Glaslyn, there is considerable variation in the yields between the years 1962 and 1963. The maximum profit yield for wheat at $1.40 per bushel was 29.5 bushels per acre in 1962 and 57.7

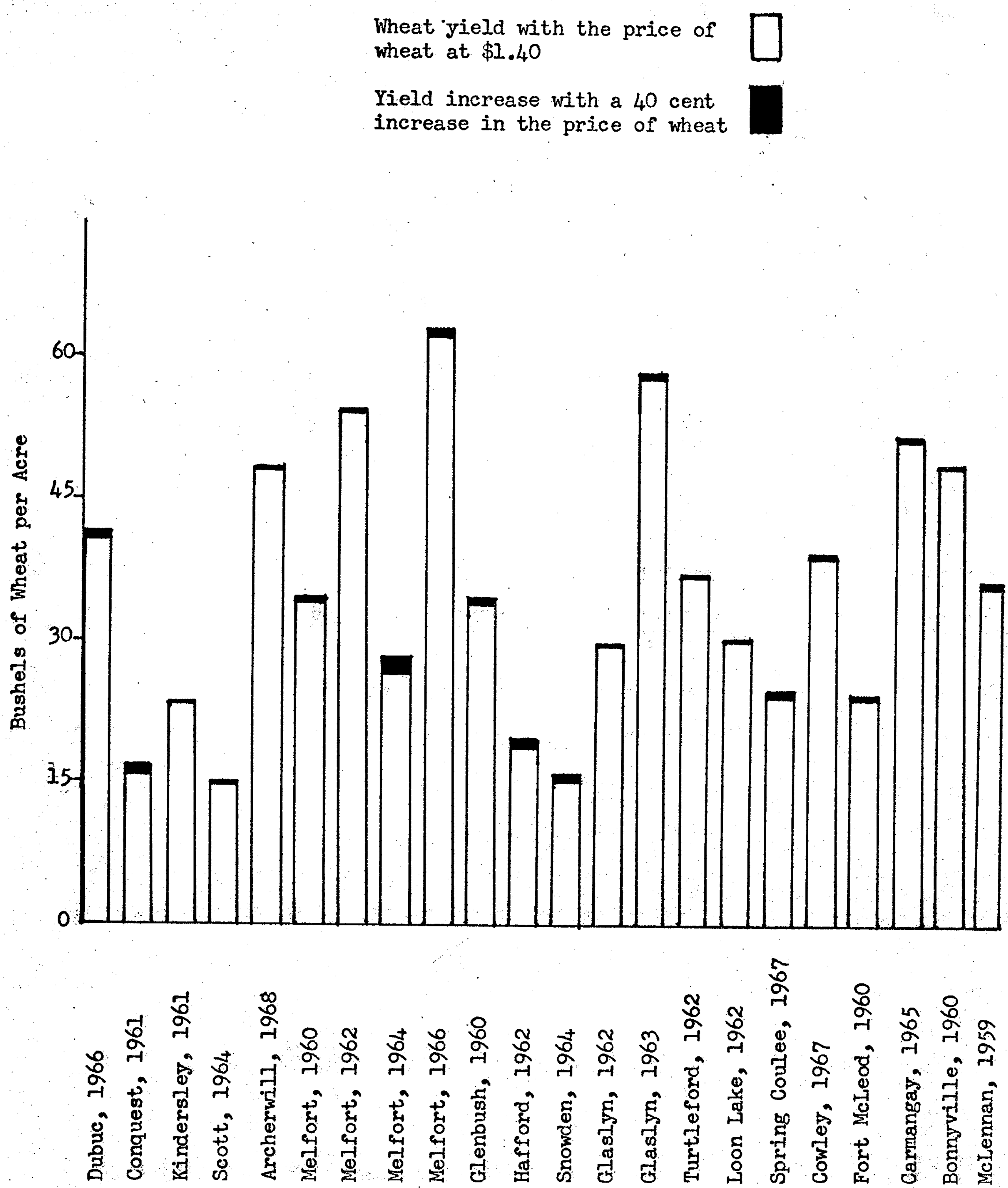

Figure 4.5. Maximum Profit Yields for Wheat on Summerfallow Predicted by the Quadratic Function for Wheat Priced at $1.40 and $1.80.

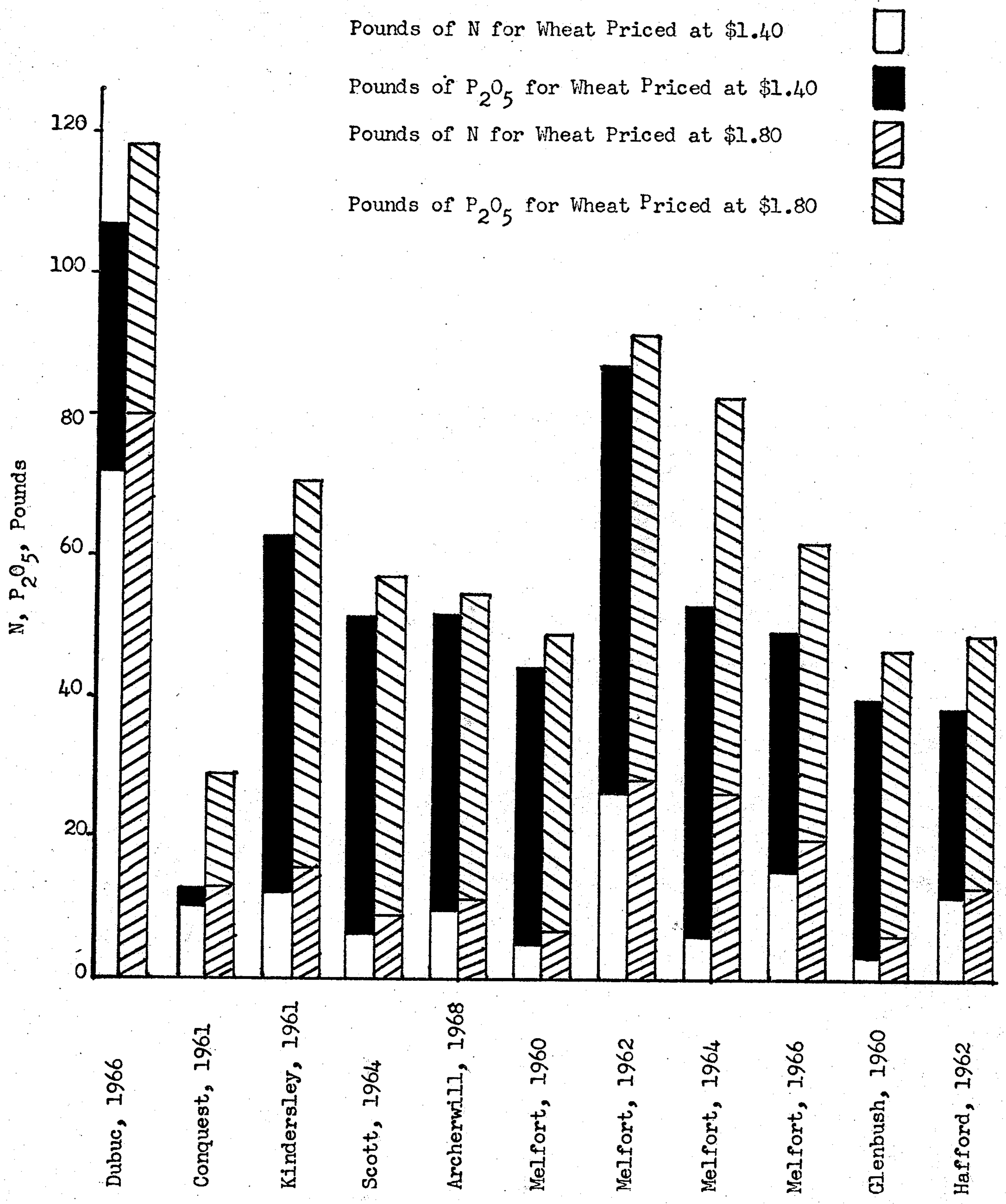

Figure 4.6. Optimum Levels and Combinations of N and P_2O_5 for Wheat on Summerfallow Predicted by the Quadratic Function for Wheat Priced at \$1.40 and \$1.80.

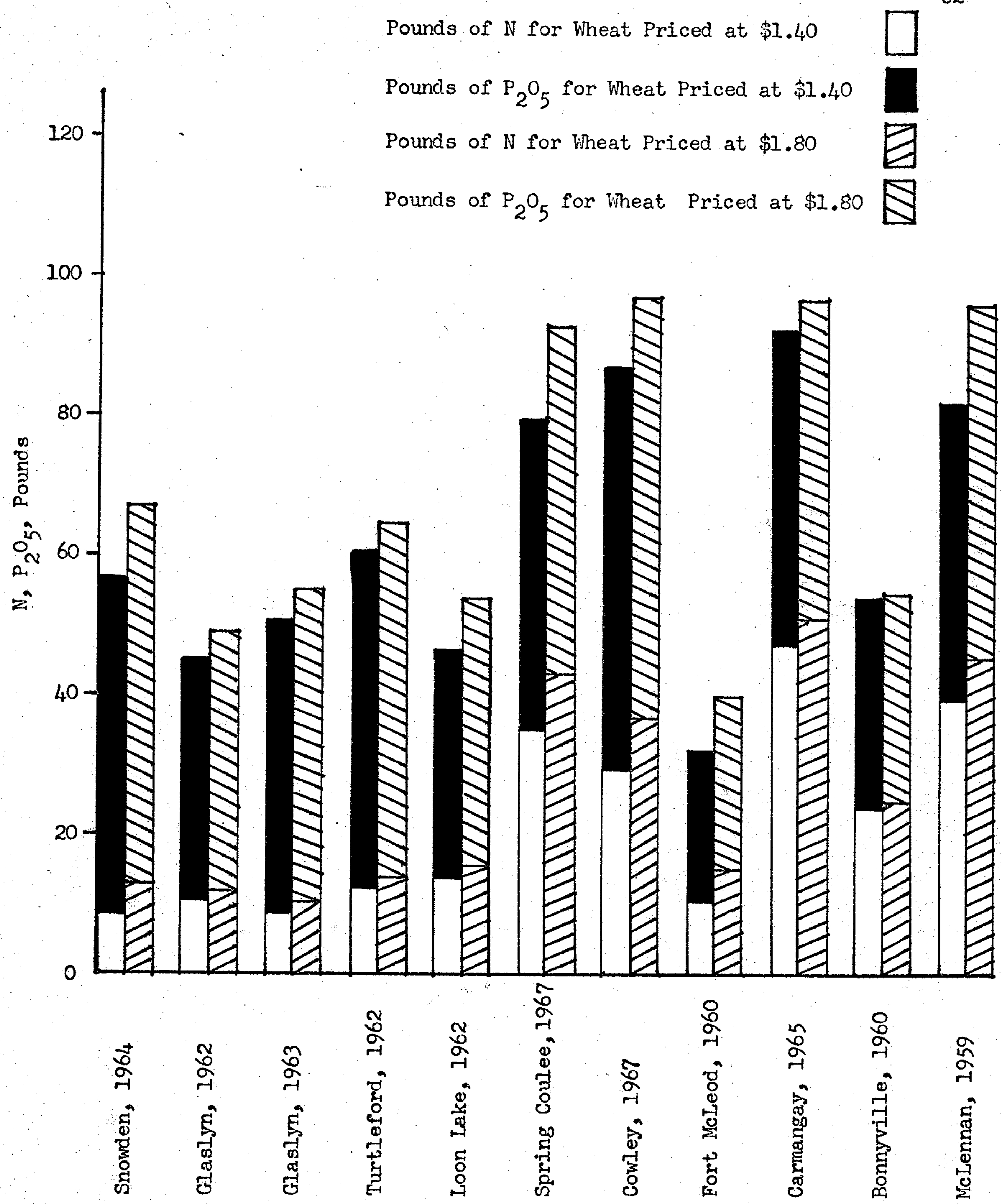

Figure 4.6 continued.

bushels per acre in 1963. There are a number of factors which will affect the yield response to fertilizer. Some of the more important of these factors are rainfall during the growing season, the soil fertility and the amount of soil moisture at the time of seeding, and the tillage and weed control practices followed. In a study done in Kansas, researchers examined the effects of the depth of soil moisture stored at planting time, the rainfall during the growing season, and the applications of N and P_2O_5 on the yields of sorghum.[15] They found that the depth of soil moisture stored at planting time had the most significant effect on yields. Also, the greater the depth of soil moisture, the greater were the effects of rainfall and nitrogen on yields.

In looking at the maximum profit positions for these experiments, it will be observed that P_2O_5 is applied at a much higher rate than N. This is consistent with the general fertilizer recommendations for cereals seeded on summerfallow, where the recommended rate of application is much higher for P_2O_5 than for N. However, as yields increase, the ratio of N to P_2O_5 does increase slightly, indicating that nitrogen becomes relatively more important at the higher yields. This is the case in most of the experiments for wheat seeded on summerfallow. There are, however, one one or two experiments for summerfallow wheat, notably at Dubuc and Conquest, where the application of N is higher than it is for P_2O_5. The soil at these two locations may have been low in nitrogen, hence a higher application of N than P_2O_5 was needed to maximize profits.

There are large differences in the most profitable yields of

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[15] Frank Orazem and Roy B. Herring, "Economic Aspects of the Effects of Fertilizer, Soil Moisture, and Rainfall on the Yields of Sorghum in the 'Sandy Lands' of South West Kansas", Journal of Farm Economics, XL (No. 3, August, 1958), 698, 708.

wheat on summerfallow among locations for the same year, even when the locations were found in the same crop district. The most profitable yields in 1967 for Spring Coulee and Cowley, in Crop District number 3 in Alberta, were 23.9 and 38.3 bushels per acre with the price of wheat at $1.40 per bushel (see experiment 19 and 20, Table B.1). The difference in yields could be attributed to one or more of the factors mentioned previously. They are, however, two additional factors which may account for the difference in yields. First, the soil at Spring Coulee is a dark brown, whereas, at Cowley it is a thin black. Secondly, the wheat grown at Spring Coulee is the spring variety, while at Cowley it is a winter wheat.[16]

There are also differences in the maximum profit yields for the same year for experiments located in the same soil zone. Dubuc and Melfort, which are both located in the black soil zone of Saskatchewan, had in 1966 maximum profit yields of 40.3 and 62.1 bushels per acre, respectively, (experiment 1 and 11, Table B.1). On the other hand, at Glaslyn and Loon Lake, which are both located on grey wooded soils, the most profitable yields for wheat in 1962 were 29.5 and 29.6 bushels per acre, respectively, (experiment 15 and 18, Table B.1). Even the optimum applications of the fertilizer nutrients were very similar for the two locations.

The maximum profit yields for wheat seeded on stubble (Table B.2) are not nearly as variable as they were for wheat seeded on summerfallow. The maximum profit yields for wheat, priced at $1.40 and $1.80 are illustrated in Figure 4.7 for the 10 stubble wheat experiments. The optimum

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[16] The fertilizer experiment conducted at Carmangay in 1965 was the only other experiment which involved winter wheat rather than spring wheat. The years 1965 and 1967 refer to the year in which the crop was harvested.

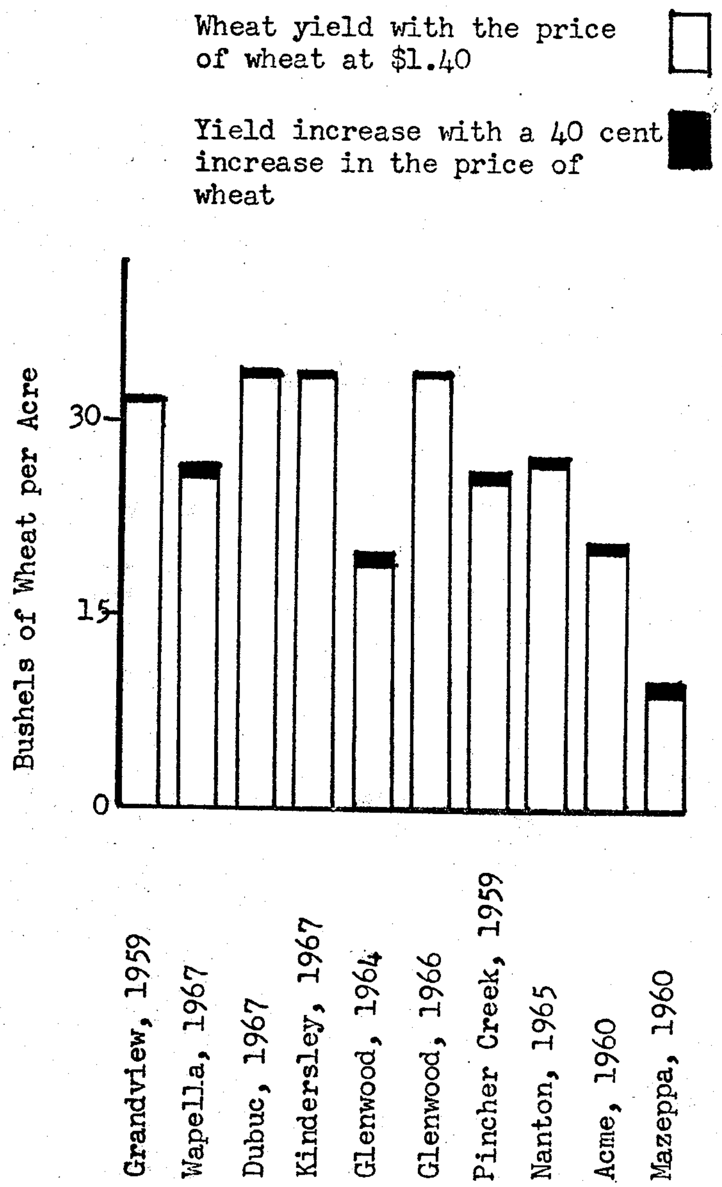

Figure 4.7. Maximum Profit Yields for Wheat on Stubble Predicted by the Quadratic Function for Wheat Priced at $1.40 and $1.80.

combination of N and P_2O_5 for these yields are shown in Figure 4.8. In most of the experiments the maximum profit yields fall into the 25.0 to 35.0 bushel range. The most profitable applications of N and P_2O_5 also tend to be less variable than they were for summerfallow wheat.

There were, however, significant differences in the most profitable yields for 1964 and 1966 at Glenwood (experiment 33 and 34, Table B.1). For wheat at $1.40 per bushel, the most profitable yield for 1964 was 18.8 bushels per acre and for 1966 it was 33.3 bushels per acre. The difference in yields may be due in part to the fact that the 1964 experiment was located on a clay textured soil, whereas, the 1966 experiment was conducted on a silt loam textured soil.

Economic Optima for Oats

There are only three fertilizer experiments for oats, two on summerfallow and one on stubble. The least cost nutrient combinations for specified yields and the maximum profit positions for various price relationships for these three experiments are given in Table B.3 and Table B.4. In one of the experiments for oats on summerfallow, (experiment 44), the least cost combination of N and P_2O_5 for the various specified yields required only the application of P_2O_5 for yields as high as 80 bushels per acre. Both of the summerfallow oats experiments were at Melfort, one in 1961 and the other in 1962. Although 1961 was considered to be a dry year, the 3.56 inches of rainfall during the growing season is only slightly below the 4.16 inches for the same time period in 1962.[17] Nevertheless, the maximum profit yields for oats at $0.50 per bushel was 28.4 bushels

[17] Dominion Bureau of Statistics, Agriculture Division, Crops Section, "Telegraphic Crop Report, Canada", Field Crop Reporting Series, (No. 17, 1961); and Dominion Bureau of Statistics, Agricultural Division, Quarterly Bulletin of Agricultural Statistics, (July- September, 1962).

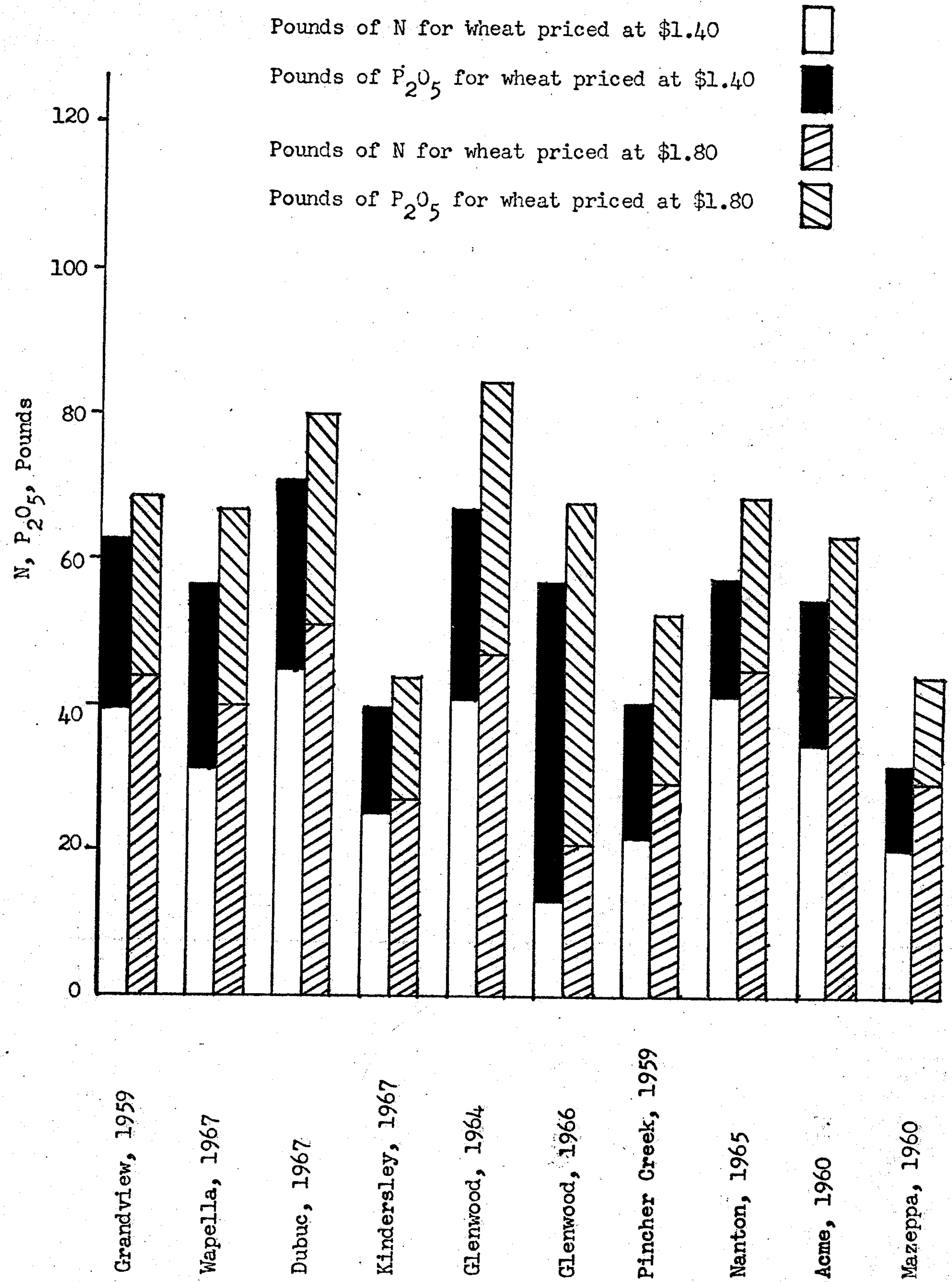

Figure 4.8. Optimum Levels and Combinations of N and P_2O_5 for Wheat on Stubble Predicted by the Quadratic Function for Wheat Priced at $1.40 and $1.80.

per acre in 1961 and 92.5 bushels per acre in 1962. It would appear that the level of soil moisture and/or the level of soil fertility at the time of seeding were not the same for both years. The timeliness of precipitation will also affect yields. In 1962, it would appear that the crops did receive rain during the critical periods of the growing season.

The economic responses of fertilizer for wheat and oats seeded on summerfallow, in 1962 at Melfort will be compared.[18] A comparison of the returns per dollar invested in fertilizer for wheat and oats will enable the farmer with limited capital to decide which crop he should fertilize. The return per dollar invested in fertilizer for the least cost combinations of the various specified yields of the two crops are presented in Table 4.11. The price of N and P_2O_5 is $0.1226 and $0.0886 per pound,

Table 4.11

Returns per Dollar Invested in Fertilizer for Wheat and Oats Seeded on Summerfallow, Melfort, 1962[19]

Wheat on Summerfallow				Oats on Summerfallow			
Yield	Fertilizer cost	Wheat at $1.40	Wheat at $1.80	Yield	Fertilizer cost	Oats at $0.50	Oats at $0.70
bushels	-dollars-			bushels	-dollars-		
35.0	0.41	7.85	10.10	50.0	0.59	8.31	11.63
40.0	1.49	6.86	8.82	60.0	1.28	7.73	10.83
45.0	2.94	5.86	7.53	70.0	2.15	6.93	9.70
50.0	4.86	4.98	6.41	80.0	3.41	5.84	8.17
55.0	8.07	3.87	4.97	90.0	6.24	3.99	5.59

18 The economic responses of fertilizer or the returns per dollar invested in fertilizer are determined by dividing the value of the yield increase by the cost of the fertilizer required to produce this increase in yield.

19 The optimum combinations of N and P_2O_5 used in the calculations of the return per dollar invested in fertilizer for the various specified yields in this table were taken from experiment 9, Table B.1 and experiment 44, Table B.3. The yields for wheat and oats when no fertilizer is applied were 32.7 and 40.2 bushels per acre, respectively.

respectively, with the price of wheat at $1.40 and $1.80 per bushel, and the price of oats at $0.50 and $0.70 per bushel. For example, with oats at $0.50 per bushel and wheat at $1.40 per bushel, the return per dollar invested in fertilizer to produce 50.0 bushels of oats is $8.22, while the return per dollar invested to produce 35.0 bushels of wheat is only $7.85.

In these two particular experiments, the yield response to the application of fertilizer is quite high. Consequently, the return per dollar invested in fertilizer will also be high.

Since there was only one experiment for oats on stubble, no direct comparisons could be made with regard to differences in yield levels for different locations and years.

Economic Optima for Barley

There were three experiments for barley seeded on summerfallow. The least cost nutrient combinations and the maximum profit positions for each experiment are given in Table B.5. The three experiments were conducted at Scott, Saskatchewan, and Fort McLeod and Lacombe, Alberta, in 1962, 1960, and 1967, respectively. The maximum profit yields are approximately the same at all three locations. In the experiments at Scott and Fort McLeod, P_2O_5 is applied at a much higher rate than N for the maximum profit yields. In the experiment at Lacombe, N is applied at a higher level than P_2O_5, with proportionately more N being applied as the price of barley increases. The high application of N indicates that the soil was very low in nitrogen.

The economic optima of fertilizer use for barley on stubble are presented in Table B.6. The maximum profit yields for the 17 experiments for barley seeded on stubble are illustrated in Figure 4.9 for barley priced at $0.65 and $1.05 per bushel. The optimum nutrient combinations for these yields are shown in Figure 4.10. From Figure 4.9, it can be observed that there is tremendous variation in the maximum profit yields of barley among the various locations in the Prairie Provinces. With barley at $0.65 per bushel, the maximum profit yields vary from 22.2 bushels per acre at New Dayton, Alberta, in 1960, to 94.1 bushels per acre at Red Deer, Alberta, in 1967. The yield increases for barley on stubble for a 40 cent increase in price ranged from a low of 1.4 bushels per acre at Dorintosh in 1960 to a high of 11.3 bushels per acre at Roland in 1967. The average yield increase was 3.8 bushels per acre. For wheat seeded on stubble, the average yield increase was 0.7 bushels per acre for a 40 cent increase in the price.

The returns per dollar invested in fertilizer for stubble wheat and barley can also be compared. The comparison is made between wheat and barley both seeded on stubble in 1964 at Glenwood. The returns per dollar invested in fertilizer for the various specified yields of each crop are presented in Table 4.12. For example, with barley at $0.65 per bushel and wheat at $1.40 per bushel, the return per dollar invested in fertilizer to produce 20.0 bushels of barley, is $2.47; whereas, the return per dollar invested in fertilizer to produce 15.0 bushels of wheat is only $2.30. Therefore, at these yield levels and prices, barley gives a higher return for each dollar invested in fertilizer.

The returns per dollar invested in fertilizer for the two fertilizer experiments are not as high as they were for wheat and oats on summerfallow at Melfort in 1962. This is to be expected because the level of fertilizer nutrients in the soil, particularly nitrogen, is generally lower for land that has been in crop than it is for land that

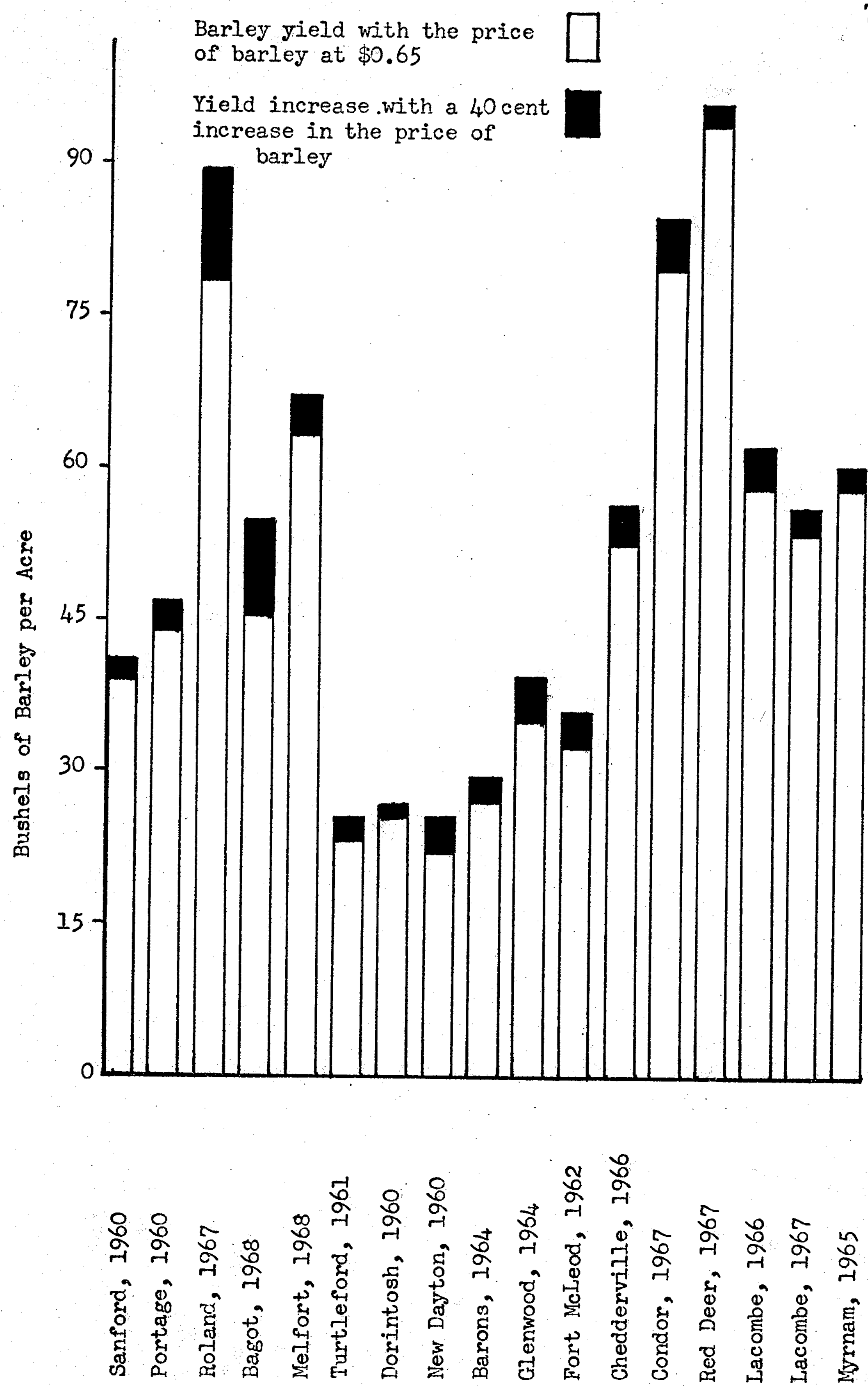

Figure 4.9. Maximum Profit Yields for Barley on Stubble Predicted by the Quadratic Function for Barley Priced at $0.65 and $1.05.

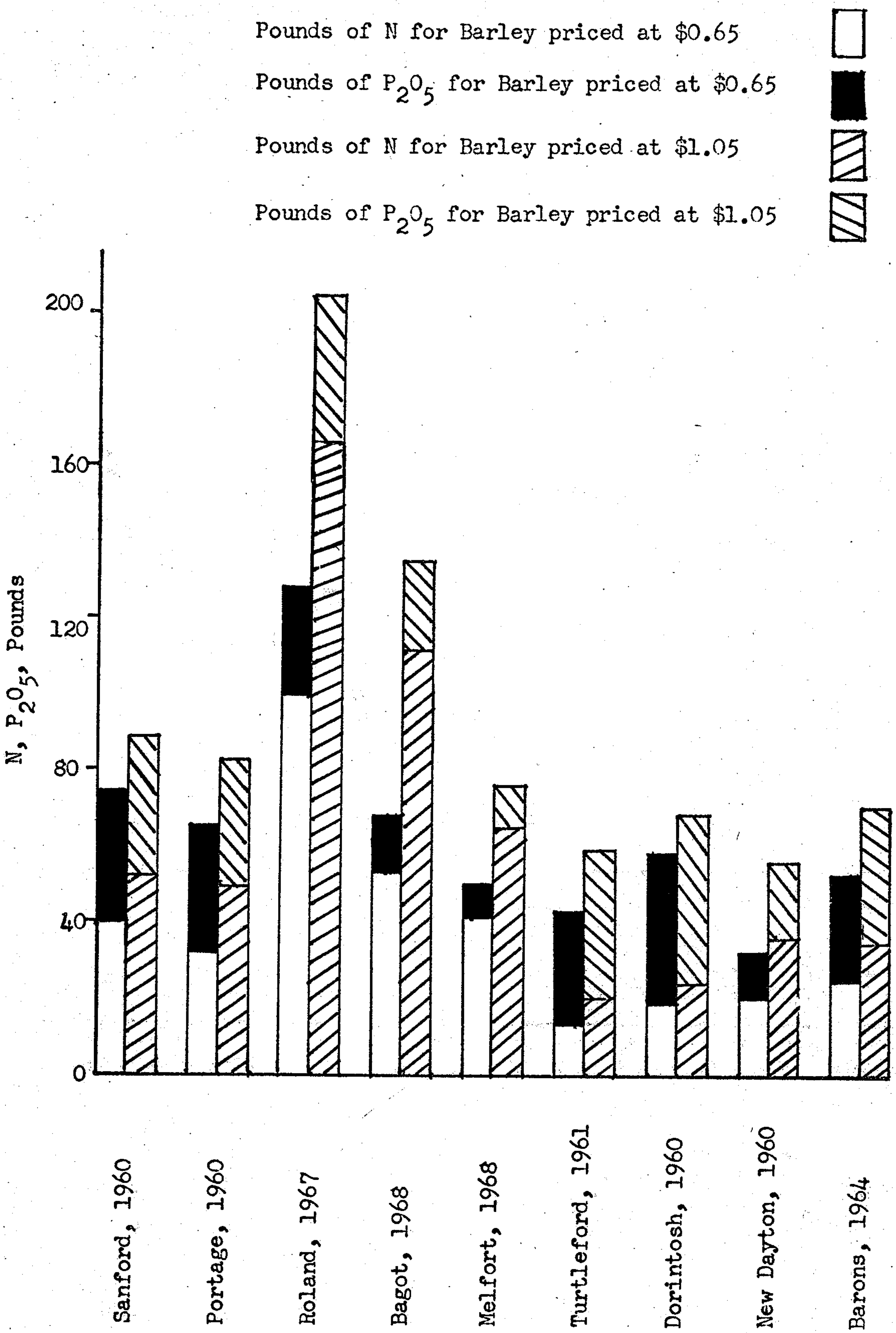

Figure 4.10. Optimum Levels and Combinations of N and P_2O_5 for Barley on Stubble Predicted by the Quadratic Function for Barley Priced at $0.65 and $1.05.

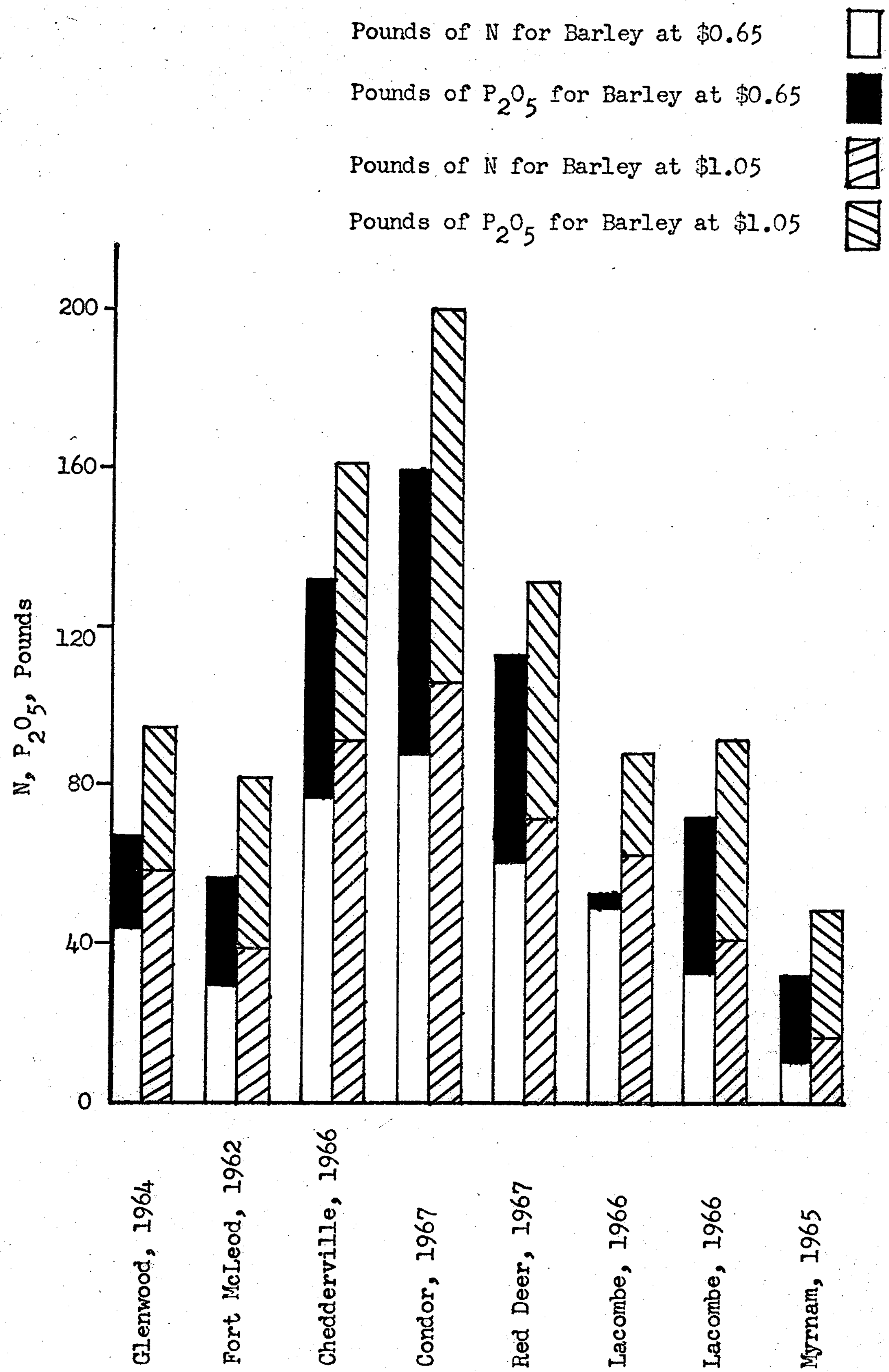

Figure 4.10 continued

Table 4.12

Returns per Dollar Invested in Fertilizer for Wheat and Barley Seeded on Stubble, Glenwood, 1964[20]

Wheat on stubble				Barley on stubble			
Yield	Fertilizer cost	Wheat $1.40	Wheat $1.80	Yield	Fertilizer cost	Barley $0.65	Barley $1.05
bushels	-dollars-			bushels	-dollars-		
15.0	3.16	2.30	2.96	20.0	0.74	2.47	4.00
20.0	9.30	1.54	1.97	30.0	4.34	1.92	3.10
				40.0	11.24	1.32	2.13

has been in summerfallow. Consequently, the amount of fertilizer that has to be applied, 'ceteris paribus', to achieve the same yield response for stubble crops as for summerfallow crops is much higher. Hence, the return for each dollar invested in fertilizer will be lower.

Economic Optima for Rye

There were only two experiments for rye. In both experiments the crop was fall rye seeded on summerfallow. These experiments were conducted in 1960 and 1964 at Indian Head. The economic relationships of fertilizer use for these two experiments are presented in Table B.7. For rye at $0.95 per bushel, the estimated maximum profit yield in 1960 was 25.2 bushels per acre and in 1964 it was 42.4 bushels per acre, 1.7 times higher than the 1960 yield. In both experiments, there are only slight increases in the maximum profit yields as the price increases from $0.95 to $1.15 per bushel, thus indicating that in each year the yields

[20] The optimum combinations of N and P_2O_5 used in the calculation of the return per dollar invested in fertilizer for the various specified yields in this table were taken from experiment 33, Table B.7 and experiment 69, Table B.6. The yields for wheat and barley, when zero fertilizer is applied, were 9.8 and 17.2 bushels per acre, respectively.

are approaching their maximum. The ratio of N to P_2O_5 is less than 1.0 in both experiments. In one experiment, it remains constant as the most profitable yields increase. Therefore, in this particular case, the two nutrients N and P_2O_5 should be increased in the same proportion as yields increase.

In summary, it is noted that the least cost combination of N and P_2O_5 for producing a given yield of a particular crop varied considerably among locations for the same year and among years for the same location. Similarly, the maximum profit position of N, P_2O_5 and yield changed considerably from one location to the next and from one year to the next for each of the crops. Consequently, recommendations of fertilizer use made on the basis of the results of any one experiment will have a high degree of uncertainty associated with them.

Chapter 5

SUMMARY AND CONCLUSIONS

In the period 1945 to 1969, fertilizer sales in Canada increased from 575,107 tons to 1,909,496 tons, an increase of 332 per cent for the twenty-five year period. Because of this large increase in the use of fertilizer, there is an increasing need for more information on the physical and the economic relationships involved in determining the optimum combinations and levels of fertilizer use. With this information, the farmer can then decide whether or not a dollar invested in fertilizer will return more than the same dollar invested elsewhere in his business. If he decides to use fertilizer, he must then decide: (1) where to use fertilizer in terms of which soils and crops will give the highest return for each one dollar invested; (2) what combination of fertilizer nutrients to use; and (3) how much of a given nutrient combination to apply on a given crop.

With this goal in mind, the objectives of this study were as follows:

1. To determine the yield response to varying levels and combinations of N and P_2O_5 for wheat, oats, barley and rye seeded on summerfallow and stubble, at various locations in the Prairie Provinces, for the years 1959 to 1968.

2. To determine the least cost nutrient combinations in producing specified yields for each of these crops for the various locations and years.

3. To determine the most profitable application of N and P_2O_5 for five selected prices of each crop for all locations and years.

4. To compare the least cost nutrient combinations for the same

yield and the maximum profit positions of N, P_2O_5 and yield among years at given locations.

5. To compare the least cost nutrient combinations for the same yield and the maximum profit position of N, P_2O_5 and yield among locations.

6. To compare the return per dollar invested in fertilizer for the different crops in a given year and location.

The data for this study were obtained from the results of fertilizer experiments or trials in cereals that were conducted in the three Prairie Provinces in the ten year period, 1959 to 1968. The results of fertilizer trials conducted in the Peace River area of British Columbia were also included in the study. Of the 4,385 fertilizer experiments whose results were tabulated, 3,458 of these experiments were excluded from further analysis in the study. Most of the experiments were excluded because they did not have a sufficient number of observations to estimate a yield response function to fertilizer inputs.

Four different production functions were fitted by least squares regression to the observations in each of the remaining 927 experiments. Although regression equations were obtained for the 927 experiments, there were only 93 experiments in which the signs of the regression coefficients for the four derived equations were the same as hypothesized. The hypothesized signs of the regression coefficients for the linear, Cobb Douglas, quadratic and square root functions are shown in the following equations:

$$\hat{Y} = b_0 + b_1 N + b_2 P$$

$$\hat{Y} = b_0 N^{b_1} P^{b_2}$$

$$\hat{Y} = b_0 + b_1 N + b_2 P - b_3 N^2 - b_4 P^2 + b_5 NP$$

$$\hat{Y} = b_0 - b_1 N - b_2 P + b_3 \sqrt{N} + b_4 \sqrt{P} + b_5 \sqrt{NP}$$

The negative signs of the regression coefficients for the quadratic and square root equations allow for a total diminishing yield and a negative marginal physical product for the individual fertilizer nutrients, N and P_2O_5.

In this study, the selection of the fertilizer production function was made on the basis of the coefficient of determination, the t tests for the regression coefficients, and F-ratio for each function. Because there were 93 experiments involved, the selection was based on the average value for these statistics for each function. The quadratic form of the production function was selected as the "best estimate" of the physical relationship between the fertilizer inputs and the crop yields. The Cobb Douglas and the linear forms of the production function were not selected because the R^2 values were somewhat lower for these two functions than for the quadratic and the square root functions. Also, the linear and Cobb Douglas functions do not permit diminishing total yield. There was little basis to distinguish between the quadratic and square root functions. The t-values for the regression coefficients in the quadratic function were on average higher than those for the square root function; whereas, the square root function had a slightly higher average coefficient of determination. However, on the basis of the higher t-values, the quadratic function was considered to provide a "better estimate" of the fertilizer input crop-output relationship. The quadratic function was therefore used as the basis for determining the yield response to various levels and combinations of N and P_2O_5 for wheat, oats, barley and rye seeded on summerfallow and stubble, at various locations in the Prairies, for the years 1959 to 1968.

The production surface and the single variable input-output curves with one nutrient variable and the other constant at specified levels were derived for a particular experiment. This experiment was barley seeded on stubble on Peace Hills fine sandy loam at Lacombe in 1967. These relationships were estimated from the following production function equation:

$$\hat{Y} = 33.6616 + 0.4553N + 0.3438P - 0.0044N^2 - 0.0029P^2 + 0.0006NP.$$

The yield isoquants for specified yields, the marginal rate of substitution of N for P_2O_5 at the various combinations of the two nutrients to produce these specified yields, and the isoclines for different constant values were also derived for this experiment. These relationships were estimated from equations that were derived from the preceding equation. The least cost combination of N and P_2O_5 for two specified yields of the crops and the maximum profit position with respect to N, P_2O_5 and yield for five different prices of barley were also determined for the experiment.

The purpose for doing the preceding analysis for only one experiment was to illustrate the methodological procedures and the equations involved in determining the economic optima in fertilizer use, using the quadratic form of the production function. The least cost nutrient combinations and the maximum profit positions for 56 fertilizer experiments are presented in the thesis. However, the equations used to derive the values for these two relationships are not included, since the equations are the same for every experiment except for the regression coefficients and the crop prices. The economic optima for 36 experiments are not presented. In most of these experiments the most profitable application of N and P_2O_5 involved negative levels of the two nutrients.

The maximum profit positions of N, P_2O_5 and yield were compared among years at given locations and among locations for a given price.

These comparisons could not be made for every crop, because in some cases there were only one or two fertilizer experiments for the crop. There was considerable variation in the maximum profit yields for both wheat on summerfallow and barley on stubble, while in the case of wheat on stubble, most of the experiments had optimal yields which fell within the range of 25 to 35 bushels per acre. The combinations N and P_2O_5 for both the maximum profit positions and the least cost combinations for specified yields were also quite variable among years for given locations and among locations. In comparing the return per dollar invested in fertilizer for specified yields of different crops in a given year and location, it was found that the return was higher for oats on summerfallow than for wheat on summerfallow at Melfort in 1962, and higher for barley on stubble than for wheat on stubble at Glenwood in 1964, when the lowest price was used for each crop. However, the relative position of the crop, with respect to the return per dollar invested in fertilizer, changes if the price of one crop increases relative to the other.

The overall objective of this study was to determine the physical and economic relationships involved in the use of fertilizer in the Prairie Provinces and at the same time to examine the methodological procedures involved in the analysis of fertilizer data. The regression-equation approach to the analysis of fertilizer-yield data is an extremely useful device. The regression equation not only expresses the physical relationships between fertilizer levels and yields, but it also permits one to determine the least cost combination of fertilizer nutrients, as well as to determine the most profitable application of fertilizer. The fertilizer-input crop-output relationships, however, apply to particular soils for certain years; production surfaces obtained under other rainfall and soil conditions can be expected to differ from those obtained in the experiments reported. Therefore, the crop yield responses to fertilizer inputs obtained in this study should not be used to make general recommendations in the optimum use of fertilizer either at a particular location or for the Province or Prairies as a whole. There are, however, a number of inferences which can be drawn from these experiments. For example, one can determine whether or not a particular soil or area does respond to the application of fertilizer. From these experiments, one can also get an indication of the yields that can be obtained in an area with the application of fertilizer. Also, the experiments show the effect of a change in the price of the crop on the optimum levels and combinations of the fertilizer nutrients, N and P_2O_5. In addition, they indicate the change in the level of fertilizer needed to increase yields by a specified number of bushels. The experiments also point out that the least cost combination of fertilizer nutrients does not necessarily follow a fixed ratio for all possible yield levels.

There are a number of problems which arise when using the regression-equation approach. One of the biggest problems is finding experiments with a sufficient number of observations to estimate the yield responses to fertilizer inputs. Out of the 4,385 fertilizer experiments tabulated, only 927 experiments were found suitable for this method of analysis. A second problem is selecting the type of function to use, that is, whether a function such as the Cobb Douglas or functions such as the quadratic and square root more adequately describe the fertilizer-input crop-output relationships.

The sign of the regression coefficients for the function is also important. For example, if the quadratic function has the following form:

$$\hat{Y} = b_0 + b_1N + b_2P + b_3N^2 + b_4P^2 + b_5NP$$

then there is only increasing marginal physical productivity. Hence, the maximum profit position is undefined. If the function is of the form:

$$\hat{Y} = b_0 + b_1N + b_2P - b_3N^2 - b_4P^2 + b_5NP$$

then there is both declining and negative marginal physical productivity. Thus, it is possible to determine the maximum profit position in fertilizer use.

In this study, there were no variables for seasonal rainfall and soil fertility and soil moisture at the time of seeding. In future studies these variables should be included in the estimation equation. Data for seasonal rainfall could be obtained from other sources, if it is not presented along with the fertilizer-yield results. Although data on precipitation is not available for every location, the closest location for which precipitation is recorded could be used. Data on soil fertility has become available in the last few years with the advent of soil testing facilities in each province. However, data on soil moisture is available for only a few experiments in each province and only for recent years. Although the three additional variables would increase the minimum number of observations needed to estimate the fertilizer-input crop-output relationships, it may be possible to combine several experiments for the same soil type for different years with different amounts of rainfall and levels of soil fertility and soil moisture. In addition, the inclusion of these three variables would place the farmer in a better position to decide whether or not to apply a certain level of fertilizer, because at the time of seeding he knows what the soil fertility and the soil moisture levels are. In the case of rainfall, he could assume the average for the growing season. If his capital is fairly limited, he could assume a lower rainfall; but which has a higher probability of being obtained.

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APPENDIX A

Table A.1

Sales of Fertilizer Materials and Mixed Fertilizers in Canada and the Prairie Provinces During the Years Ended June 30, 1945 to 1969.

Year	Prairie Provinces	Canada	Sales in Prairie Provinces as per cent of total Canadian sales
		-short tons-	per cent
1945	28,698	575,107	5.0
1946	31,202	632,943	4.9
1947	41,489	660,721	6.3
1948	51,211	672,171	7.6
1949	67,902	741,726	9.2
1950	85,451	764,581	11.2
1951	90,020	770,507	11.7
1952	86,496	768,545	11.3
1953	101,476	819,803	12.4
1954	75,122	811,641	9.3
1955	54,880	790,774	6.9
1956	62,711	800,680	7.8
1957	74,786	808,251	9.3
1958	85,677	870,539	9.8
1959	113,911	908,214	12.5
1960	137,243	935,428	14.7
1961	173,480	1,077,412	16.1
1962	209,804	1,144,000	18.3
1963	289,834	1,256,841	23.1
1964	403,371	1,454,332	27.7
1965	463,864	1,593,593	29.1
1966	684,211	1,917,864	35.7
1967	845,924	2,183,444	38.7
1968	953,319	2,292,723	41.6
1969	584,702	1,909,496	30.6

Source: Quarterly Bulletin of Agricultural Statistics, January-March, 1947-1948, Dominion Bureau of Statistics, Agriculture Division, and Fertilizer Trade, 1947-48 to 1968-69, Dominion Bureau of Statistics, Manufacturing and Primary Industries Division, formerly Industry and Merchandising Division.

Table A.2

Prairie Acreages of All Wheat, Oats, Barley, All Rye, Mixed Grains, Flaxseed, Rapeseed, Tame Hay and Summerfallow, for Crop Years 1945-46 to 1970-71

Crop	Wheat	Oats	Barley	All Rye	Mixed Grain	Flax-seed	Rape-seed	Tame Hay	Total	Summer-fallow
					-thousand acres-					
1945-46	22,430	9785	6516	422	64	848	13	1654	41,732	20,640
1946-47	23,731	8470	5788	643	48	865	24	1650	41,219	20,398
1947-48	23,357	7818	7035	1124	42	1724	58	1774	42,932	19,744
1948-49	22,820	7516	6082	2225	78	1880	80	1775	42,456	20,704
1949-50	26,524	7355	5617	1095	91	290	20	1874	42,866	21,763
1950-51	26,382	7520	6205	1041	116	541	*	2071	43,876	21,606
1951-52	24,385	8312	7530	1047	142	1086	7	2177	44,685	21,569
1952-53	25,372	7560	8145	1153	132	1027	19	2281	45,689	21,460
1953-54	25,517	6490	8599	1431	137	908	30	2265	45,366	22,960
1954-55	24,707	6715	7568	687	177	1148	40	2586	43,628	25,630
1955-56	21,964	7788	9638	665	228	1809	138	2777	45,007	24,514
1956-57	22,064	7422	8181	452	305	3010	352	2895	44,681	24,113
1957-58	20,881	5633	9209	455	350	3462	618	3429	44,037	25,084
1958-59	21,480	5810	9104	431	411	2526	626	3680	44,068	26,399
1959-60	23,970	5626	7700	458	493	2026	214	3853	44,339	26,594
1960-61	23,900	6344	6680	490	520	2481	763	4144	45,322	26,893
1961-62	24,639	5122	5361	493	667	2051	710	4533	43,566	27,860
1962-63	26,237	7152	5097	556	616	1396	371	4691	46,117	27,495
1963-64	26,996	6260	5922	583	535	1629	478	4664	47,067	27,211
1964-65	29.080	5054	5217	620	548	1916	791	4816	48,042	26,375
1965-66	27,790	5645	5741	691	606	2265	1435	5032	49,205	26,580
1966-67	29,166	5450	7010	671	747	1883	1525	5249	51,701	25,224
1967-68	28,570	5090	7600	628	667	998	1620	5027	51,200	25,950
1968-69	28,860	5340	8330	619	676	1502	1052	4780	51,159	26,660
1969-70	24,400	5630	9000	859	705	2320	2012	5100	50,126	28,800
1970-71p	12,000	5390	9500	944	875	3350	3950	6160	42,169	36,900

Source: Handbook of Agricultural Statistics, Part I, Field Crops, 1908-63, (March, 1964); "Preliminary Estimates of Crops and Summerfallow Acreages, Canada", "September Forecast of Production of Principal Field Crops, Canada, 1970", and "November Estimate of Production of Principal Field Crops, Canada", Field Crop Reporting Series, (No. 13, 1963-1970, No. 14, 1962, No. 19, 1970, No. 20, 1963-1969, No. 23, 1962), Dominion Bureau of Statistics, Agriculture Division, Crops Section.

* Less than 500 acres

p Preliminary estimates

Table A.3

Canadian Wheat Supplies and Disposition for Crop Years 1945-46 to 1969-70.

Crop Year	Total Stocks at Beginning of Crop Year	Stocks Farms	Production	Total Supply	Export of Wheat and Flour	Domestic Disposition of Wheat	Total Disposition of Wheat
			-million bushels-				
1945-46	258	29	316	574	340	161	501
1946-47	74	27	412	485	243	156	399
1947-48	86	26	339	425	195	152	347
1948-49	78	39	381	459	232	125	357
1949-50	102	43	366	468	225	131	356
1950-51	112	12	466	579	241	148	389
1951-52	189	22	554	743	356	170	526
1952-53	217	19	702	919	386	150	536
1953-54	383	94	634	1,017	255	144	399
1954-55	619	232	332	951	252	162	414
1955-56	537	138	519	1,056	312	164	476
1956-57	580	204	573	1,153	264	155	419
1957-58	734	323	393	1,126	320	158	478
1958-59	648	241	398	1,046	294	164	458
1959-60	588	169	445	1,033	277	156	433
1960-61	600	144	518	1,118	353	156	509
1961-62	608	171	283	892	358	143	501
1962-63	391	59	566	957	331	138	469
1963-64	487	65	724	1,211	594	157	751
1964-65	459	121	601	1,060	400	147	547
1965-66	513	109	649	1,162	585	157	742
1966-67	420	100	827	1,247	515	156	671
1967-68	577	205	593	1,170	336	168	504
1968-69	666	236	650	1,315	306	158	464
1969-70	852	372	684	1,536	347[p]	178[p]	525[p]
1970-71[p]	1,011	543	330	1,341	-	-	-

Source: _1968-69 Annual Report of the Canadian Wheat Board_, Canadian Wheat Board; _The Wheat Review_, August, 1970, September, 1970), and "September Forecast of Production of Principal Field Crops, Canada, 1970, _Field Crop Reporting Series_ (No. 19, 1970), Dominion Bureau of Statistics, Agriculture, Crops Section.

p Preliminary estimates.

The types of data coded on each card were as follows:

Columns	Item
1, 2, . . . 9	FERT RESP
10, 11, 12	Region Number
13, 14, . . 32	Location
33	Crop (wheat = 1, oats = 3) (barley = 4, rye = 5)
35	Summerfallow = 1, Stubble = 2
37, 38, 39	Soil Type
40, 41	Soil Texture
43	Soil Texture Group
45, 46, 47, 48	Year
50, 51, 52	Nutrients Applied in Pounds per Acre N
53, 54, 55	" " " " P_2O_5
56, 57, 58	" " " " K_2O
59, 60	" " " " S
61, 62, 63, 64	Yields in Tenths of Bushels per Acre
66	Control = 1, Noncontrol = 0
68, 69, 70, 71	Experiment Number
73, 74	Total Number of Cards for Experiment
75, 76	Number of this Card

Table A.4

Data Pertaining to the 93 Fertilizer Experiments Included in the Study

Exp. no.	Location	Crop district	Province	Soil zone	Soil texture	Crop	Year	No. of the exp.
1	Dubuc	5 A	Sask.	black	loam	wht on smf	1966	2462
2	Conquest	6 B	"	dark brown	sandy loam	"	1961	4027
3	Kindersley	7 A	"	brown	clay	"	1961	4029
4	Scott	7 B	"	dark brown	loam	"	1964	3221
5	Archerwill	8 A	"	thick black	"	"	1968	2089
6	Melfort	8 B	"	black	clay	"	1959	4858
7	Melfort	8 B	"	"	"	"	1960	4327
8	Melfort	8 B	"	"	"	"	1960	4470
9	Melfort	8 B	"	"	"	"	1962	3759
10	Melfort	8 B	"	"	"	"	1964	3127
11	Melfort	8 B	"	"	"	"	1966	2504
12	Glenbush	9 A	"	thick black	loam	"	1960	4528
13	Hafford	9 A	"	black	"	"	1962	3821
14	Snowden	9 A	"	grey wooded	"	"	1964	3144
15	Glaslyn	9 B	"	"	"	"	1962	3823
16	Glaslyn	9 B	"	"	"	"	1963	3472
17	Turtleford	9 B	"	thick black	"	"	1962	3822
18	Loon Lake	9 B	"	grey wooded	"	"	1962	3824
19	Spring Coulee	3	Alta.	dark brown	"	"	1967	5445
20	Cowley	3	"	thin black	clay loam	"	1967	5453
21	Ft. McLeod	3	"	dark brown	"	"	1960	7198
22	Carmangay	2	"	"	sand	"	1965	6023
23	Bonnyville	6	"	black	loam	"	1960	7185
24	McLennan	7	"	dk.gr.wooded	clay loam	"	1959	7313
25	Grandview	11	Man.	black	"	wht on stb	1959	955
26	Moosomin	1 B	Sask.	"	loam	"	1964	3097
27	Wapella	1 B	"	"	"	"	1967	1766
28	Wapella	1 B	"	"	"	"	1967	1767

Table A.4 (continued)

Exp. no.	Location	Crop district	Province	Soil zone	Soil texture	Crop	Year	No. of the exp.
29	Dubuc	5A	Sask.	black	loam	wht on stb	1967	1769
30	Kindersley	7A	"	brown	clay	"	1967	1999
31	Scott	7B	"	dark brown	loam	"	1961	4039
32	Milk River	2	Alta.	brown	"	"	1960	7200
33	Glenwood	3	"	thin black	clay	"	1964	6388
34	Glenwood	3	"	"	loam	"	1966	5767
35	Pincher Creek	3	"	"	clay	"	1959	7431
36	Nanton	3	"	"	loam	"	1965	6015
37	Acme	2	"	"	"	"	1960	7191
38	Mazeppa	3	"	"	"	"	1960	7201
39	Ft. Saskatchewan	5	"	black	sandy loam	"	1968	5003
40	Bonnyville	6	"	"	loam	"	1959	7411
41	Blueberry Mtn.	7	"	dk.gr.wooded	clay loam	"	1959	7322
42	Melfort	8B	Sask.	black	clay	oats on smf	1961	3992
43	Melfort	8B	"	"	"	"	1962	3766
44	Melfort	8B	"	"	"	"	1962	3767
45	La Corey	6	Alta.	dk.gr.wooded	loam	oats on stb	1965	6041
46	Minnedosa	9	Man.	black	clay loam	bly on smf	1968	61
47	Scott	7B	Sask.	dark brown	loam	"	1962	3816
48	Ft. McLeod	3	Alta.	"	clay loam	"	1960	7199
49	Harmatton	3	"	dk.gr.wooded	loam	"	1960	7153
50	Lacombe	5	"	black	"	"	1966	5749
51	Lacombe	5	"	"	"	"	1967	5429
52	High Level	7	"	dk.gr.wooded	sandy loam	"	1959	7433
53	Sanford	3	Man.	black	clay	bly on stb	1960	804
54	Sanford	3	"	"	"	"	1966	204
55	Portage	3	"	"	loam	"	1960	807
56	Roland	3	"	"	sandy loam	"	1967	101
57	Elie	3	"	"	clay	"	1967	107

Table A.4 (continued)

Exp. no.	Location	Crop district	Province	Soil zone	Soil texture	Crop	Year	No. of the exp.
58	Rossendale	8	Man.	black	sand	bly on stb	1967	111
59	Bagot	8	"	"	"	"	1968	51
60	Loverna	7A	Sask.	brown	clay loam	"	1959	4849
61	Melfort	8B	"	black	clay	"	1968	2098
62	Turtleford	9B	"	thick black	loam	"	1959	4850
63	Turtleford	9B	"	"	"	"	1961	4043
64	Dorintosh	9B	"	grey wooded	"	"	1960	4541
65	Walsh	1	Alta.	dark brown	"	"	1964	6377
66	New Dayton	2	"	brown	"	"	1960	7204
67	Barons	2	"	dark brown	"	"	1964	6375
68	Welling	3	"	"	"	"	1962	6772
69	Glenwood	3	"	thin black	clay	"	1964	6389
70	Glenwood	3	"	"	loam	"	1967	5447
71	Ft. McLeod	3	"	dark brown	clay loam	"	1962	6776
72	Nanton	3	"	thin black	loam	"	1964	6391
73	Chedderville	5	"	dk.gr.wooded	"	"	1966	5717
74	Chedderville	5	"	"	"	"	1966	5718
75	Chedderville	5	"	"	"	"	1966	5719
76	Chedderville	5	"	"	"	"	1966	5720
77	Chedderville	5	"	"	"	"	1967	5409
78	Chedderville	5	"	"	"	"	1967	5411
79	Condor	5	"	"	"	"	1967	5402
80	Condor	5	"	"	"	"	1967	5403
81	Condor	5	"	"	"	"	1967	5405
82	Red Deer	5	"	black	"	"	1967	5395
83	Lacombe	5	"	"	sandy loam	"	1966	5721
84	Lacombe	5	"	"	"	"	1967	5389
85	Lacombe	5	"	"	"	"	1967	5390
86	Lacombe	5	"	"	"	"	1967	5391

Table A.4 (continued)

Exp. no.	Location	Crop district	Province	Soil zone	Soil texture	Crop	Year	No. of the exp.
87	Ponoka	5	Alta.	black	loam	bly on stb	1968	5005
88	Myrnam	4 B	"	dk.gr.wooded	"	"	1965	6009
89	Lamont	4 B	"	black	"	"	1960	7155
90	Bluesky	7	"	dk.gr.wooded	clay loam	"	1968	5007
91	Indian Head	2 A	Sask.	black	clay	rye on smf	1960	4275
92	Indian Head	2 A	"	"	"	"	1964	3082
93	Carmangay	2	Alta.	dark brown	sand	"	1966	5771

Table A.5

The Regression Coefficients, the t-Test for the Regression Coefficients, the Coefficient of Determination, and the F-ratio of the Linear (A), Quadratic (B), Cobb-Douglas (C), and Square Root (D) Equations in each of the 93 Experiments

Exp. no.	Equation	The regression coefficients and the value of t-test for the regression coefficients of the following terms						Value of R^2	F-ratio	No. of obs.
		b_o	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$			
1	A	22.7213	0.1316	0.1791						9
			3.00*	2.13++				0.785	10.96**	
	B	21.7625	0.2102	0.3819	-0.0029	-0.0131	0.0084			
			0.97	1.62	1.20	1.03	0.62	0.874	4.16+	
	C	20.214	0.0915	0.0469						
			3.18*	1.74+				0.829	14.53**	
	D	21.6350	-0.2677	-0.5121	1.9047	1.1196	0.5919			
			1.05	1.20	1.34	0.71	1.32	0.898	5.30+	
2	A	14.2925	0.1477	0.0036						16
			3.20**	0.32				0.443	5.17	
	B	13.8994	0.2590	0.0445	-0.0087	-0.0007	0.0020			
			1.66+	1.14	1.26	1.63+	1.41+	0.657	3.84*	
	C	13.379	0.0692	0.0128						
			3.62**	1.00				0.520	7.06**	
	D	14.8076	-0.0121	-0.0539	0.0766	0.2061	0.1389			
			0.12	2.18++	0.16	0.83	3.27**	0.812	8.62**	
3	A	16.7600	0.0994	0.0777						16
			1.97++	6.15**				0.762	20.84**	
	B	15.7342	0.1632	0.1896	-0.0056	-0.0015	0.0015			
			1.43+	6.63**	1.12	4.80**	1.48+	0.935	28.66**	
	C	15.307	0.0324	0.0792						
			3.87**	14.25**				0.944	109.04**	
	D	16.0434	-0.0042	-0.0390	0.0873	0.9095	0.0824			
			0.06	2.07++	0.23	4.82**	2.54*	0.961	49.37**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
4	A	9.0025	0.0534	0.0734						16
			1.06	5.80**				0.728	17.38**	
	B	7.4105	0.1311	0.2213	-0.0040	-0.0018	0.0002			
			2.10++	14.17**	1.47+	10.49**	0.31	0.978	87.85**	
	C	7.5640	0.0310	0.1390						
			3.07**	20.72**				0.971	219.25**	
	D	7.5578	-0.0212	-0.0554	0.2205	1.1605	0.0274			
			0.32	3.39**	0.67	7.08**	0.97	0.966	57.49**	
5	A	38.5511	0.1311	0.1675						11
			1.65+	5.10**				0.818	17.94**	
	B	36.8434	0.2185	0.4152	-0.0103	-0.0045	0.0019			
			1.16	5.73**	0.98	3.38*	0.54	0.951	19.37**	
	C	36.355	0.0102	0.0628						
			2.64*	18.76**				0.980	197.39**	
	D	36.9721	-0.0285	-0.0863	0.0235	1.9068	0.0876			
			0.17	1.98+	0.04	6.00**	1.44	0.985	65.72**	
6	A	43.4548	0.2959	0.4434						20
			1.69+	6.84**				0.745	24.78**	
	B	36.4939	1.0433	1.0824	-0.0382	-0.0109	0.0014			
			3.05**	10.07**	2.61*	6.86**	0.33	0.947	50.41**	
	C	37.362	0.0399	0.1322						
			5.27**	22.97**				0.970	277.75**	
	D	36.7612	-0.3759	-0.2997	2.5979	5.6117	0.1260			
			1.67+	3.49**	2.21*	7.83**	1.05	0.967	81.35**	
7	A	28.7217	0.0085	0.1081						20
			0.10	4.04**				0.491	8.19**	
	B	27.3484	0.1213	0.2520	-0.0069	-0.0026	0.0011			
			0.42	2.76*	0.56	1.89++	0.30	0.614	4.45*	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
7	C	26.915	0.0057	0.0562						
			0.41	5.83**				0.669	17.17**	
	D	27.2409	-0.0812	-0.0720	0.2556	1.3422	0.0466			
			0.36	0.86	0.22	1.90++	0.40	0.644	5.07**	
8	A	33.0604	0.0676	0.0491						20
			0.92	2.01++				0.229	2.52+	
	B	33.3477	0.0660	0.0914	-0.0089	-0.0016	0.0062			
			0.28	1.19	0.86	1.42+	2.10++	0.508	2.89++	
	C	31.566	0.0188	0.0250						
			1.51+	2.89*				0.391	5.47*	
	D	33.3493	-0.1792	-0.0679	0.2825	0.4536	0.2003			
			0.95	0.99	0.29	0.78	2.07++	0.563	3.60*	
9	A	36.1400	0.1254	0.1961						16
			1.74+	5.44**				0.715	16.30**	
	B	32.7425	0.3571	0.4848	-0.0072	-0.0039	0.0018			
			1.98++	5.36**	1.80+	3.89**	1.14	0.904	18.81**	
	C	32.129	0.0331	0.0916						
			3.30**	11.00**				0.910	65.97**	
	D	33.0620	-0.0979	-0.1021	0.9707	2.4662	0.1084			
			0.65	1.36	0.92	3.29**	1.19	0.909	20.04**	
10	A	20.6260	0.0233	0.1032						28
			1.17	5.40**				0.551	15.33**	
	B	19.2314	0.0533	0.2191	-0.0007	-0.0018	0.0009			
			0.76	3.67**	0.85	2.63*	1.48+	0.689	9.76**	
	C	17.956	0.0287	0.0809						
			2.15*	6.86**				0.675	25.96**	
	D	19.1623	-0.0353	-0.0161	0.3187	0.8290	0.0678			
			0.64	0.30	0.54	1.50+	1.24	0.680	9.37**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
11	A	57.2129	0.0697	0.0746						18
			1.34	2.81*				0.396	4.91*	
	B	57.1488	0.1193	0.1383	-0.0043	-0.0018	0.0029			
			0.62	1.48+	0.76	1.30	1.27	0.547	2.90++	
	C	55.817	0.0125	0.0186						
			1.78++	3.26**				0.478	6.88**	
	D	57.0117	-0.0843	-0.0151	0.5258	0.4435	0.0848			
			0.48	0.16	0.48	0.56	0.85	0.512	2.52++	
12	A	30.6911	0.0345	0.0468						17
			0.60	3.25**				0.440	5.49*	
	B	29.3472	0.1149	0.1586	-0.0037	-0.0013	0.0000			
			0.58	3.22**	0.43	2.50*	0.00	0.647	4.02*	
	C	29.369	0.0090	0.0310						
			0.90	4.66**				0.617	11.27**	
	D	29.7220	-0.0376	-0.0402	0.1750	0.7430	0.0396			
			0.23	0.97	0.21	1.76+	0.53	0.629	3.73*	
13	A	15.1178	0.1395	0.0312						17
			2.63*	2.36*				0.472	6.26*	
	B	14.0575	0.3824	0.0896	-0.0140	-0.0008	0.0014			
			2.29*	2.15++	1.92++	1.87++	0.94	0.703	5.21*	
	C	13.875	0.0627	0.0397						
			3.96**	3.78**				0.682	15.05**	
	D	14.1547	-0.0905	-0.0332	0.8507	0.4908	0.0521			
			0.63	0.92	1.17	1.35	0.81	0.687	4.83*	
14	A	10.2576	0.0162	0.0802						20
			0.42	4.08**				0.500	8.51**	
	B	8.6993	0.1133	0.1751	-0.0025	-0.0012	0.0004			
			0.80	2.53*	0.80	1.63+	0.27	0.604	4.28*	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
14	C	8.2124	0.0237	0.1361						
			0.74	5.09**				0.609	13.24**	
	D	8.8824	-0.0559	-0.0197	0.3124	0.8013	0.0483			
			0.50	0.35	0.38	1.40+	0.65	0.635	4.87**	
15	A	23.2359	0.1503	0.0066						
			1.34	0.24				0.117	0.93	17
	B	19.4512	0.7007	0.2603	-0.0300	-0.0033	0.0022			
			4.05**	6.03**	3.97**	6.96**	1.42+	0.880	16.13**	
	C	19.970	0.0646	0.0336						
			2.55*	2.00++				0.429	5.26*	
	D	19.5130	-0.4408	-0.2230	2.3585	1.9573	0.1003			
			2.95*	5.96**	3.13**	5.19**	1.51+	0.874	15.23**	
16	A	51.9093	0.0535	0.0607						
			0.50	2.37*				0.324	2.88++	15
	B	49.8974	0.3054	0.2373	-0.0150	-0.0023	0.0015			
			0.86	2.93*	1.00	2.54*	0.52	0.643	3.24++	
	C	49.703	0.0080	0.0281						
			0.80	4.60**				0.646	10.97**	
	D	49.6566	-0.1211	-0.1471	0.6114	1.8999	0.0246			
			0.47	2.22++	0.46	2.94*	0.23	0.708	4.37*	
17	A	26.8325	0.1869	0.0553						
			1.32	1.56+				0.230	2.09+	17
	B	25.0419	0.6730	0.1983	-0.0362	-0.0025	0.0077			
			1.80++	2.13++	2.22*	2.48*	2.27*	0.698	5.07*	
	C	23.488	0.0624	0.0489						
			2.26*	2.67*				0.468	6.15*	
	D	26.5452	-0.5472	-0.1467	1.6052	0.9758	0.3913			
			2.43*	2.60*	1.41+	1.72+	3.91**	0.844	11.93**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
18	A	24.4826	0.2003	0.0155						17
			2.81*	0.87				0.383	4.34*	
	B	23.3572	0.3854	0.1155	-0.0141	-0.0015	0.0032			
			1.98++	2.38*	1.66+	2.89*	1.79+	0.739	6.23**	
	C	22.718	0.0574	0.0211						
			3.62**	2.01++				0.551	8.60**	
	D	24.1233	-0.0502	-0.0980	0.4214	0.6945	0.1600			
			0.33	2.60*	0.56	1.83++	2.40*	0.780	7.80**	
19	A	15.0212	0.0568	0.0738						16
			2.34*	3.04**				0.530	7.34**	
	B	13.3737	0.1461	0.2274	-0.0016	-0.0024	0.0013			
			3.16*	4.91**	3.16*	4.68**	3.07*	0.909	19.89**	
	C	12.614	0.0611	0.0910						
			4.65**	6.92**				0.843	34.78**	
	D	14.2124	-0.0638	-0.1144	0.5589	1.2042	0.1197			
			2.74*	4.92**	2.40*	5.16**	5.97**	0.968	60.90**	
20	A	19.2600	0.0186	0.2411						16
			0.45	5.79**				0.722	16.87**	
	B	15.4650	0.0965	0.6389	-0.0016	-0.0054	0.0014			
			1.62+	10.76**	2.38*	8.28**	2.74*	0.970	64.00**	
	C	14.751	0.0139	0.2026						
			1.12	16.26**				0.953	132.75**	
	D	16.1218	-0.0945	-0.1526	0.4757	3.1515	0.1222			
			1.75+	2.82*	0.88	5.82**	2.63*	0.965	55.86**	
21	A	20.4987	0.0218	0.0772						16
			0.65	2.31*				0.307	2.88++	
	B	19.7870	0.1149	0.1800	-0.0030	-0.0032	0.0017			
			0.91	1.42+	1.06	1.15	0.75	0.467	1.75+	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
21	C	19.271	0.0218	0.0397						
			1.52+	2.78*				0.435	5.01*	
	D	19.1609	-0.1520	-0.0129	1.0994	0.5350	0.0208			
			1.46+	0.12	1.49+	0.72	0.23	0.503	2.02+	
22	A	34.4012	0.1094	0.0682						16
			1.95++	1.22				0.290	2.65+	
	B	29.8725	0.3132	0.4569	-0.0034	-0.0056	0.0022			
			2.38*	3.48**	2.33*	3.88**	1.85++	0.790	7.54**	
	C	29.081	0.0567	0.0586						
			3.07**	3.18**				0.601	9.77**	
	D	31.2368	-0.1153	-0.3644	1.1769	3.1606	0.1956			
			1.10	3.47**	1.12	3.00*	2.16++	0.815	8.83**	
23	A	29.2000	0.0909	0.3720						16
			0.90	3.68**				0.524	7.16**	
	B	24.7613	0.4320	1.1074	-0.0106	-0.0201	0.0055			
			1.58+	4.04**	1.75+	3.33**	1.13	0.813	8.68**	
	C	23.384	0.0444	0.1512						
			1.91++	6.50**				0.779	22.97**	
	D	23.4500	-0.3440	-0.4675	2.3780	5.1061	0.1588			
			1.80+	2.45*	1.76+	3.77**	0.96	0.875	13.97**	
24	A	26.6697	0.0722	0.0498						19
			2.20*	1.52+				0.331	3.97*	
	B	24.9189	0.2206	0.1280	-0.0025	-0.0014	0.0014			
			2.45*	1.42+	2.09++	1.19	1.76+	0.646	4.75*	
	C	23.542	0.0538	0.0385						
			3.88**	2.79*				0.563	10.30**	
	D	24.8969	-0.0961	-0.0081	1.1717	0.2859	0.0936			
			1.06	0.09	1.47+	0.36	1.40+	0.622	4.27*	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
25	A	23.7288	0.0661	0.0960						17
			2.93*	2.32*				0.522	7.64**	
	B	20.6849	0.2635	0.2661	-0.0023	-0.0046	0.0002			
			3.15**	2.08++	2.46*	1.61+	0.14	0.748	6.53**	
	C	19.572	0.0838	0.0348						
			5.59**	3.08**				0.793	26.79**	
	D	18.5971	-0.1466	-0.0640	2.2822	0.6437	0.0457			
			1.87++	0.61	2.95*	0.78	0.51	0.783	7.92**	
26	A	21.5722	0.0548	0.1692						9
			1.55+	2.42++				0.657	5.74*	
	B'	21.6145	0.0750	0.1621	-0.0010	-0.0020	0.0030			
			0.20	0.40	0.39	0.18	0.26	0.691	1.34	
	C	20.929	0.0670	0.0218						
			1.83+	0.50				0.682	6.42*	
	D	21.4582	-0.1407	-0.0753	0.5861	0.0106	0.3599			
			0.57	0.07	0.09	0.09	0.30	0.728	1.61	
27	A	18.4523	0.0523	0.1410						10
			1.74+	2.79*				0.681	7.48*	
	B	17.0552	0.1220	0.3407	-0.0017	-0.0074	0.0029			
			1.11	2.77++	1.61+	2.05+	0.68	0.874	5.54++	
	C	16.759	0.0475	0.0642						
			2.15++	3.23*				0.828	16.84**	
	D	17.4285	-0.2552	-0.3444	1.3726	0.9041	0.4465			
			2.29++	2.30++	2.07+	1.24	2.40++	0.922	9.43*	
28	A	15.4953	0.0734	0.1780						10
			3.78**	5.44**				0.897	30.62**	
	B	15.5303	0.0584	0.2303	-0.0008	-0.0038	0.0039			
			0.64	2.26++	0.93	1.26	1.13	0.933	11.22*	

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
28	C	14.419	0.0584	0.0804						
			2.62*	4.00**				0.879	25.48**	
	D	15.5056	-0.0617	-0.0341	0.4400	0.2388	0.2392			
			0.52	0.21	0.62	0.31	1.20	0.932	10.93*	
29	A	24.3099	0.0946	0.1073						10
			3.54**	2.38*				0.784	12.69**	
	B	22.9584	0.2013	0.2424	-0.0021	-0.0061	0.0031			
			2.57++	2.75++	2.85*	2.38++	1.02	0.945	13.68*	
	C	22.107	0.0708	0.0267						
			4.96**	2.08++				0.895	29.75**	
	D	22.8799	-0.2168	-0.2657	1.7312	0.3963	0.3780			
			2.38++	2.17++	3.20*	0.67	2.49++	0.955	17.16**	
30	A	28.0646	0.0222	0.1241						11
			0.35	0.81				0.132	0.61	
	B	26.8000	0.3545	0.1163	-0.0064	-0.0051	0.0037			
			1.02	0.19	1.94+	0.24	0.30	0.529	1.12	
	C	26.977	0.0314	0.0191						
			0.71	0.33				0.211	1.07	
	D	26.8000	-0.5289	-0.6967	1.2779	1.7897	0.8878			
			1.41	0.45	0.19	0.22	0.65	0.431	0.76	
31	A	17.8874	0.0019	0.0526						18
			0.06	3.36**				0.429	5.64*	
	B	16.6517	0.0514	0.1604	-0.0018	-0.0015	0.0008			
			0.52	3.37**	0.81	2.84*	0.97	0.684	5.19**	
	C	16.089	0.0077	0.0635						
			0.56	5.56**				0.675	15.56**	
	D	17.3668	-0.0685	-0.0613	0.0541	0.7613	0.0960			
			1.05	1.87++	0.12	2.30*	2.30*	0.793	9.19**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
32	A	9.3362	0.0301	0.0159						16
			2.68*	0.71				0.371	3.83*	
	B	8.5121	0.1169	0.0402	-0.0011	-0.0010	0.0005			
			3.44**	0.59	3.07*	0.66	0.79	0.693	4.52*	
	C	8.2235	0.0749	0.0187						
			5.56**	1.15				0.713	16.13**	
	D	8.6370	-0.0642	-0.0321	0.7519	0.1154	0.0423			
			2.75*	0.69	3.22**	0.35	1.49+	0.800	8.01**	
33	A	11.2625	0.0936	0.0271						16
			4.33**	1.25				0.610	10.17**	
	B	9.7811	0.2682	0.0671	-0.0026	-0.0009	0.0011			
			7.98**	2.00++	6.92**	2.53*	3.58**	0.949	37.52**	
	C	9.1977	0.1429	0.0338						
			9.43**	2.23+				0.878	46.94**	
	D	10.7358	-0.0914	-0.0479	1.2358	0.1859	0.1071			
			4.72**	2.47*	6.37**	0.96	6.42**	0.977	84.56**	
34	A	26.4625	0.0052	0.0919						16
			0.19	3.41**				0.473	5.84*	
	B	23.6711	0.0981	0.3190	-0.0014	-0.0030	0.0005			
			1.48+	4.81++	1.85++	4.07**	0.92	0.829	9.69**	
	C	23.363	0.0087	0.0766						
			0.92	8.06**				0.835	32.88**	
	D	24.1807	-0.0803	-0.1539	0.4675	1.9971	0.0704			
			1.96++	3.76**	1.14	4.87**	2.00++	0.910	20.27**	
35	A	22.4103	0.0188	0.0710						17
			1.26	2.60*				0.391	4.49*	
	B	20.9108	0.1390	0.1176	-0.0015	-0.0020	0.0008			
			2.34*	1.30	2.28*	0.99	0.89	0.627	3.70*	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
35	C	20.276	0.0355	0.0300						
			2.76*	3.09**				0.624	11.64**	
	D	19.3613	-0.1291	-0.0200	1.5012	0.1533	0.0624			
			2.37*	0.27	2.80*	0.27	1.00	0.694	4.99*	
36	A	18.7950	0.0953	0.0113						16
			3.67**	0.44				0.512	6.83**	
	B	15.5784	0.3812	0.0773	-0.0036	-0.0009	0.0003			
			11.64**	2.36*	9.95**	2.59*	1.13	0.958	46.00**	
	C	15.931	0.1049	0.0142						
			9.42**	1.27				0.874	45.19**	
	D	16.0623	-0.1489	-0.0457	2.1461	0.3607	0.0369			
			3.34**	1.02	4.80**	0.81	0.96	0.894	16.81**	
37	A	13.9500	0.0656	0.0574						16
			3.92**	1.72+				0.585	9.16**	
	B	12.0762	0.1687	0.2827	-0.0014	-0.0061	0.0008			
			4.35**	3.65**	3.29**	3.56**	1.10	0.880	14.73**	
	C	11.911	0.0840	0.0552						
			8.85**	4.82**				0.886	50.76**	
	D	11.7634	-0.0545	-0.1725	1.0135	1.3646	0.0376			
			2.15++	3.41**	4.00**	3.81**	1.22	0.930	26.56**	
38	A	6.7163	0.0407	0.0102						16
			3.04**	0.38				0.420	4.71*	
	B	5.7209	0.1276	0.1127	-0.0013	-0.0035	0.0012			
			4.51**	1.99++	4.20**	2.82*	2.44*	0.861	12.35**	
	C	5.3414	0.1238	0.0323						
			6.16**	1.33				0.753	19.85**	
	D	5.9146	-0.0635	-0.1138	0.7499	0.4913	0.0699			
			2.95*	2.64*	3.48**	1.61+	2.66*	0.889	15.99**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
39	A	17.7844	0.0149	0.0808						27
			1.85++	2.25*				0.340	6.18**	
	B	13.6892	0.0586	0.3299	-0.0002	-0.0043	0.0004			
			1.41+	2.63+	1.59+	2.54*	0.96	0.562	5.38**	
	C	14.077	0.0695	0.0511						
			2.98**	1.42+				0.563	15.46**	
	D	13.7070	-0.0625	-0.1415	0.9331	1.0880	0.1002			
			1.67+	0.85	0.98	0.76	0.97	0.547	5.07**	
40	A	30.9375	0.0225	0.3336						16
			0.62	4.61**				0.625	10.83**	
	B	28.4294	0.0141	1.0042	-0.0004	-0.0182	0.0024			
			0.19	6.79**	0.49	5.58**	1.79+	0.916	21.79**	
	C	27.263	0.0123	0.1207						
			1.16	9.51**				0.876	45.92**	
	D	27.6799	-0.0056	-0.3109	0.0723	4.0679	0.0561			
			0.07	2.02++	0.09	3.73**	0.60	0.874	13.91**	
41	A	21.4559	0.0260	0.0081						10
			1.36	0.42				0.231	1.05	
	B	20.4468	0.1556	0.0223	-0.0018	-0.0001	0.0003			
			2.83*	0.41	2.69++	0.21	0.75	0.761	2.55+	
	C	20.021	0.0333	0.0163						
			3.35*	1.64+				0.634	6.06*	
	D	19.8334	-0.1215	-0.0078	1.2930	0.2485	0.0079			
			2.62++	0.17	3.20*	0.61	0.30	0.835	4.06++	
42	A	22.6800	0.0089	0.1660						16
			0.12	6.59**				0.770	21.74**	
	B	21.2051	0.3066	0.2957	-0.0156	-0.0023	0.0008			
			1.12	3.39**	1.31	1.73+	0.26	0.844	10.82**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
42	C	21.050	0.0084	0.0923						
			0.52	8.21**				0.839	33.84**	
	D	21.5069	-0.2409	-0.0043	0.8812	1.1928	0.0668			
			1.11	0.05	0.80	1.77+	0.64	0.862	12.47**	
43	A	55.3875	0.0099	0.3551						16
			0.06	4.36**				0.594	9.50**	
	B	50.2772	0.2359	0.9662	-0.0112	-0.0088	0.0069			
			0.60	4.93**	1.30	4.09**	1.96++	0.874	13.89**	
	C	46.664	0.0060	0.1225						
			0.30	7.28**				0.803	26.57**	
	D	50.9984	-0.5117	-0.3124	1.5659	4.9857	0.3953			
			1.70+	2.08++	0.74	3.31**	2.16++	0.898	17.61**	
44	A	47.9550	0.0817	0.5654						16
			0.41	5.67**				0.713	16.17**	
	B	40.2536	0.1702	1.5046	-0.0076	-0.0127	0.0065			
			0.45	7.96**	0.92	6.11**	1.94++	0.945	34.22**	
	C	37.870	0.0090	0.1922						
			0.50	12.82**				0.927	82.25**	
	D	41.9448	-0.2912	-0.3466	0.4397	7.2300	0.4204			
			0.89	2.11++	0.19	4.40**	2.10++	0.943	32.90**	
45	A	41.1300	0.3593	0.2891						10
			2.80*	1.74+				0.638	6.17*	
	B	39.2569	0.5376	1.4290	-0.0079	-0.0483	0.0112			
			2.36++	4.42*	1.62+	4.76**	1.97+	0.952	15.71**	
	C	37.968	0.0816	0.0552						
			2.79*	2.35++				0.738	9.86**	
	D	40.2200	-0.2476	-1.0728	2.6767	4.0388	0.7135			
			0.57	2.79*	0.98	1.38	1.77+	0.930	10.65*	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
46	A	38.0995	0.0482	0.1186						11
			1.99++	1.27				0.497	3.96++	
	B	34.6479	0.1339	0.7048	-0.0006	-0.0158	0.0016			
			2.28++	2.31++	2.33++	2.23++	0.81	0.845	5.45*	
	C	33.391	0.0544	0.0337						
			2.74*	1.76+				0.657	7.66*	
	D	34.0519	-0.1043	-0.6834	1.6675	3.9109	0.1895			
			1.17	1.74+	1.79+	1.52+	1.04	0.798	3.96++	
47	A	35.3225	0.0266	0.0952						16
			0.31	4.50**				0.610	10.18**	
	B	32.4054	0.2977	0.3313	-0.0133	-0.0029	0.0001			
			2.33*	10.38**	2.36*	8.14**	0.11	0.952	40.05**	
	C	32.623	0.0086	0.0565						
			1.58+	15.72**				0.950	124.77**	
	D	32.4550	-0.1972	-0.1241	0.8834	1.9995	0.0372			
			2.64*	6.64**	2.36*	10.69**	1.16	0.978	87.09**	
48	A	32.3600	0.0174	0.2134						16
			0.34	4.14**				0.571	8.65**	
	B	30.5955	0.2251	0.4886	-0.0068	-0.0084	0.0041			
			1.58+	3.44**	2.16++	2.68*	1.63+	0.825	9.41**	
	C	29.744	0.0160	0.0709						
			1.37+	6.06**				0.748	19.33**	
	D	30.6117	-0.2697	-0.0863	1.4228	1.5081	0.1470			
			2.28*	0.73	1.70+	1.80+	1.44+	0.833	9.99**	
49	A	45.6725	0.2167	0.4484						16
			2.09++	4.33**				0.640	11.55**	
	B	41.0088	0.8117	0.9568	-0.0164	-0.0143	0.0048			
			2.63*	3.10*	2.41*	2.10++	0.88	0.829	9.68**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
			The regression coefficients and the value of t-test for the regression coefficients of the following terms							
49	C	40.713	0.0555	0.0908						
			3.69**	6.04**				0.794	25.09**	
	D	41.9139	-0.4179	-0.0307	3.4317	2.3833	0.2428			
			1.61+	0.12	1.86++	1.29	1.08	0.833	9.95**	
50	A	18.1163	0.1685	0.4344						16
			1.99++	5.13**				0.700	15.13**	
	B	10.1750	0.3408	1.1155	-0.0028	-0.0077	0.0027			
			1.92++	6.28**	1.80+	4.94**	2.17++	0.929	26.21**	
	C	11.635	0.0716	0.3322						
			3.05**	14.16**				0.942	104.91**	
	D	13.5630	-0.1154	-0.3244	1.0032	6.0657	0.3668			
			1.44+	4.04**	1.12	6.76**	5.31**	0.980	98.61**	
51	A	6.8588	0.1073	0.6092						16
			2.33*	5.83**				0.752	19.69**	
	B	2.3040	0.1890	1.5293	-0.0014	-0.0235	0.0035			
			2.56*	9.12**	2.21++	7.01**	2.91*	0.966	56.48**	
	C	3.9811	0.1006	0.5315						
			3.50**	14.94**				0.948	117.75**	
	D	3.8551	-0.0232	-0.3761	0.3944	5.4650	0.2724			
			0.58	4.14**	0.88	8.10**	5.25**	0.986	142.77**	
52	A	22.0642	0.0125	0.0932						12
			0.11	2.99*				0.499	4.48*	
	B	22.2857	0.0210	0.1420	-0.0175	-0.0023	0.0095			
			0.05	1.16	0.43	0.86	1.20	0.641	2.14+	
	C	21.210	0.0031	0.0472						
			0.17	3.82**				0.619	7.32*	
	D	21.7265	-0.1987	-0.0375	0.3363	0.7108	0.0950			
			0.39	0.36	0.20	0.97	0.60	0.636	2.10	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
53	A	15.6320	0.1879	0.2915						10
			3.33*	3.97**				0.819	15.87**	
	B	14.1000	0.4196	0.6201	-0.0045	-0.0104	0.0045			
			2.52++	3.60*	1.71+	2.99*	1.50	0.949	15.00*	
	C	13.365	0.1245	0.1511						
			5.50**	7.66**				0.959	81.24**	
	D	14.1002	-0.3124	-0.4350	3.1967	2.4219	0.4481			
			3.09*	8.23**	4.03*	3.85*	5.95**	0.997	248.39**	
54	A	5.8553	0.0677	0.2916						21
			5.53**	3.78**				0.763	28.93**	
	B	5.9001	0.0694	0.6612	-0.0004	-0.0166	0.0035			
			1.06	1.88++	2.48*	1.53+	1.44+	0.843	16.10**	
	C	4.7038	0.2029	0.1690						
			4.99**	3.66**				0.799	35.86**	
	D	5.9004	-0.0547	-0.4530	0.7591	2.2248	0.3005			
			1.00	0.94	1.15	0.84	2.06++	0.838	15.57**	
55	A	28.0269	0.1418	0.2367						10
			2.80*	3.59**				0.777	12.22**	
	B	25.5000	0.3522	0.4782	-0.0032	-0.0060	0.0013			
			1.67+	2.20++	0.96	1.36	0.36	0.876	5.67++	
	C	24.935	0.0748	0.0839						
			4.55**	5.86**				0.935	50.53**	
	D	25.5004	-0.3904	-0.2804	3.9390	1.3273	0.3499			
			1.14	1.57+	1.47	0.63	1.38	0.944	13.57*	
56	A	41.4150	0.1494	0.5775						25
			7.81**	4.77**				0.817	49.14**	
	B	41.1999	0.2012	1.0560	-0.0008	-0.0238	0.0051			
			2.82*	2.16*	3.15**	1.65+	1.92++	0.886	29.57**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
56	C	35.235	0.1225	0.0507						
			7.18**	2.58*				0.802	44.52**	
	D	41.2004	-0.1387	-0.5482	2.5072	1.8784	0.5729			
			1.90++	0.99	2.80*	0.60	3.46**	0.909	37.85**	
57	A	53.0503	0.0495	0.0979						11
			2.84*	1.46+				0.637	7.02*	
	B	52.4222	0.0873	0.2630	-0.0006	-0.0067	0.0027			
			1.99+	1.15	2.94*	1.27	1.79+	0.880	7.31*	
	C	49.152	0.0380	0.0175						
			3.19*	1.52+				0.686	8.75**	
	D	52.5714	-0.1368	-0.3017	1.4282	0.4499	0.3353			
			3.18*	1.59+	3.18*	0.36	3.81*	0.935	14.39**	
58	A	26.0394	0.0167	0.3391						11
			0.83	4.39**				0.754	12.25**	
	B	23.7936	0.0962	0.5806	-0.0006	-0.0078	0.0020			
			2.04++	2.37++	3.08*	1.37	1.23	0.929	13.07**	
	C	23.251	0.0389	0.1077						
			2.00++	5.76**				0.862	24.96**	
	D	24.1302	-0.1755	-0.1126	1.7268	0.9867	0.2986			
			2.70*	0.39	2.54++	0.52	2.24++	0.924	12.13**	
59	A	34.9716	0.1175	0.0707						11
			5.67**	0.89				0.833	19.92**	
	B	30.8089	0.2520	0.2354	-0.0006	-0.0041	0.0003			
			6.29**	1.13	3.52*	0.84	0.21	0.967	29.54**	
	C	27.232	0.1294	0.0099						
			10.75**	0.85				0.945	68.22**	
	D	29.5266	-0.0964	-0.1938	2.6585	0.3729	0.1888			
			3.34*	1.52+	8.82**	0.45	3.20*	0.990	103.51**	

Table A.5 (continued)

Exp. no.	Equation	The regression coefficients and the value of t-test for the regression coefficients of the following terms: b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
60	A	18.0157	0.0342	0.0200						14
			0.56	0.69				0.072	0.43	
	B	16.6236	0.1437	0.1419	-0.0054	-0.0017	0.0019			
			0.61	1.14	1.05	1.35	0.68	0.290	0.65	
	C	16.499	0.0236	0.0325						
			0.81	1.31				0.157	1.02	
	D	17.8572	-0.1627	-0.0428	0.6831	0.4060	0.0818			
			0.60	0.42	0.33	0.26	0.34	0.193	0.38	
61	A	55.4118	0.1041	0.0958						13
			3.03*	0.71				0.522	5.46*	
	B	50.1289	0.3115	0.4694	-0.0015	-0.0179	0.0002			
			2.26++	0.86	1.46+	0.98	0.04	0.706	3.36++	
	C	49.068	0.0632	0.0076						
			4.38**	0.57				0.716	12.58**	
	D	49.3278	-0.0792	-0.5048	2.3571	2.2807	0.0516			
			0.51	1.07	1.37	0.62	0.16	0.724	3.67++	
62	A	4.6818	0.0050	0.0091						14
			0.16	0.58				0.037	0.21	
	B	3.8890	0.0491	0.1210	-0.0040	-0.0016	0.0020			
			0.79	3.25*	2.39*	3.71**	2.28++	0.686	3.49++	
	C	3.5846	0.0083	0.1024						
			0.18	2.31*				0.333	2.75+	
	D	3.9414	-0.1525	-0.0862	0.4928	0.7752	0.0680			
			1.55+	2.46*	0.95	2.26++	1.36	0.605	2.45+	
63	A	17.0644	0.0867	0.0508						18
			1.37+	1.56+				0.226	2.19+	
	B	13.3591	0.2661	0.3535	-0.0058	-0.0041	0.0022			
			2.19*	6.03**	2.15++	6.42**	2.04++	0.850	13.57**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
63	C	13.465	0.0562	0.0927						
			2.20*	4.35**				0.609	11.70**	
	D	14.6561	-0.0856	-0.2372	0.4215	2.1534	0.1773			
			0.92	5.06**	0.62	4.55**	2.97*	0.867	15.67**	
64	A	12.8258	0.1420	0.0957						18
			1.98++	2.58*				0.417	5.37*	
	B	8.2633	0.6124	0.3596	-0.0132	-0.0037	0.0026			
			4.23**	5.15**	4.11**	4.89**	2.01++	0.876	16.98**	
	C	8.6190	0.1300	0.1510						
			4.68**	6.53**				0.809	31.71**	
	D	8.5868	-0.3097	-0.1794	2.3538	2.0408	0.1759			
			3.67**	4.24**	3.85**	4.77**	3.26**	0.937	35.57**	
65	A	17.8787	0.1202	0.0320						16
			4.02**	1.07				0.571	8.66**	
	B	16.8476	0.2794	0.0820	-0.0027	-0.0014	0.0018			
			3.90**	1.14	3.40**	1.73+	2.82*	0.868	13.15**	
	C	15.389	0.1127	0.0271						
			6.00**	1.44+				0.745	19.04**	
	D	17.8508	-0.0700	-0.0685	1.1086	0.2539	0.1428			
			1.08	1.05	1.70+	0.39	2.55*	0.850	11.34**	
66	A	17.2112	0.0790	0.0725						16
			3.00*	1.38+				0.455	5.43*	
	B	15.2370	0.2757	0.2273	-0.0029	-0.0058	0.0025			
			5.59**	2.30*	5.32**	2.67*	2.78*	0.898	17.53**	
	C	14.783	0.0915	0.0413						
			6.23**	2.33*				0.773	22.12**	
	D	15.9454	-0.1280	-0.0881	1.5280	0.4526	0.1277			
			2.39*	0.82	2.85*	0.60	1.96++	0.834	10.08**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
67	A	19.5350	0.0583	0.0493						16
			1.53+	1.30				0.236	2.01+	
	B	14.0379	0.3824	0.3009	-0.0040	-0.0031	0.0001			
			4.85**	3.81**	4.56**	3.35**	0.16	0.824	9.36**	
	C	15.337	0.0766	0.0668						
			4.20**	3.66**				0.704	15.49**	
	D	13.9741	-0.2159	-0.1952	2.4738	2.1903	0.0286			
			3.19**	2.88*	3.65**	2.23**	0.49	0.822	9.22**	
68	A	18.5525	0.0171	0.0411						16
			0.97	2.34*				0.331	3.21++	
	B	16.1577	0.1335	0.1781	-0.0014	-0.0017	0.0001			
			3.29**	4.38**	3.23**	3.79**	0.26	0.808	8.41**	
	C	16.463	0.0277	0.0508						
			2.82*	5.16**				0.726	17.26**	
	D	16.0813	-0.0871	-0.0869	0.9268	1.1534	0.0135			
			2.58*	2.58*	2.74*	3.42**	0.46	0.818	8.96**	
69	A	19.9737	0.2427	0.0153						16
			6.49**	0.41				0.765	21.14**	
	B	17.2012	0.4868	0.1533	-0.0036	-0.0024	0.0017			
			5.82**	1.83++	3.96**	2.57*	2.23*	0.937	29.71**	
	C	15.900	0.1816	0.0190						
			13.64**	1.42+				0.935	94.01**	
	D	17.8998	-0.0561	-0.1644	2.1003	0.9641	0.1518			
			1.14	3.36**	4.28**	1.97++	3.60**	0.970	65.20**	
70	A	31.8662	0.3552	0.0970						16
			7.14**	1.95++				0.808	27.40**	
	B	29.6371	0.6442	0.1625	-0.0044	-0.0017	0.0022			
			4.36**	1.10	2.71*	1.05	1.65+	0.909	20.07**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
70	C	27.142	0.1513	0.0398						
			12.72**	3.35**				0.930	86.48**	
	D	29.8022	-0.0148	-0.0864	2.5947	0.8139	0.1893			
			0.15	0.87	2.62*	0.82	2.22++	0.944	33.62**	
71	A	22.6325	0.0806	0.0845						16
			1.98++	2.07++				0.387	4.10*	
	B	18.6316	0.4667	0.1891	-0.0052	-0.0018	0.0013			
			7.87**	3.19**	8.00**	2.80*	2.54*	0.930	26.79**	
	C	18.541	0.0880	0.0591						
			5.35**	3.60**				0.762	20.78**	
	D	19.7397	-0.2959	-0.0484	2.9317	0.6081	0.1335			
			5.90**	0.96	5.84**	1.21	3.09*	0.932	27.22**	
72	A	15.2350	0.1467	0.0187						16
			3.54**	0.45				0.495	6.36*	
	B	14.4954	0.3169	0.0299	-0.0028	-0.0009	0.0018			
			2.21++	0.21	1.80+	0.58	1.44+	0.677	4.19*	
	C	12.223	0.1546	0.0251						
			5.94**	0.96				0.736	18.13**	
	D	13.7586	-0.0840	-0.0936	1.6612	0.5316	0.1093			
			0.70	0.78	1.38+	0.44	1.06	0.688	4.41*	
73	A	21.2738	0.2034	0.0827						16
			5.84**	1.90++				0.743	18.82**	
	B	16.0606	0.5388	0.1419	-0.0024	-0.0008	0.0005			
			8.13**	1.71+	6.13**	1.27	1.18	0.949	37.39**	
	C	17.263	0.1920	0.0486						
			17.09**	4.12**				0.960	154.49**	
	D	16.9654	-0.1569	-0.0150	4.0897	0.5963	0.0690			
			2.11++	0.16	4.16**	0.54	1.15	0.932	27.57**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
74	A	23.2500	0.1944	0.0489						16
			7.26**	1.46+				0.808	27.39**	
	B	20.5770	0.3980	0.0897	-0.0016	-0.0007	0.0005			
			5.60**	1.01	3.68**	0.99	1.19	0.926	25.03**	
	C	19.552	0.1725	0.0293						
			15.69**	2.54+				0.951	126.29**	
	D	21.2581	-0.0664	-0.0685	2.7175	0.6864	0.0868			
			1.23	1.02	3.81**	0.86	2.00++	0.955	42.34**	
75	A	19.2500	0.1984	0.1114						16
			4.75**	2.13++				0.676	13.53**	
	B	11.7986	0.5722	0.2983	-0.0027	-0.0019	0.0005			
			6.97**	2.91*	5.50**	2.45*	1.04	0.932	27.20**	
	C	14.603	0.2034	0.0792						
			17.03**	6.32**				0.962	165.02**	
	D	13.5041	-0.2236	-0.1527	4.6012	2.1743	0.1095			
			3.45**	1.88++	5.37**	2.27*	2.09++	0.955	42.20**	
76	A	27.9425	0.1558	0.0722						16
			3.64**	1.35				0.536	7.52**	
	B	22.3971	0.4565	0.2531	-0.0023	-0.0020	0.0008			
			4.08**	1.81+	3.45**	1.89++	1.12	0.827	9.53**	
	C	22.255	0.1367	0.0535						
			8.28**	3.09**				0.857	39.00**	
	D	23.4523	-0.1993	-0.2138	3.7032	2.3617	0.1178			
			1.97++	1.69+	2.77*	1.58+	1.44+	0.850	11.33**	
77	A	41.7688	0.0297	0.0816						16
			1.25	2.74*				0.411	4.54*	
	B	39.8746	0.1352	0.2134	-0.0010	-0.0015	0.0006			
			2.15++	2.72*	2.57*	2.57*	1.73+	0.775	6.89**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
77	C	37.395	0.0281	0.0511						
			2.72*	4.70**				0.694	14.75**	
	D	39.2788	-0.1237	-0.1336	1.5014	1.9423	0.0659			
			2.14++	1.85++	1.96++	2.27*	1.42+	0.799	7.93**	
78	A	40.8763	0.0311	0.0684						
			2.12++	3.73**				0.586	9.22**	
	B	38.0766	0.1586	0.1415	-0.0009	-0.0007	0.0001			
			4.80**	3.43**	4.56**	2.22++	0.61	0.885	15.44**	
	C	38.188	0.0284	0.0353						
			4.62**	5.48**				0.798	25.69**	
	D	38.8999	-0.1129	-0.0034	1.6074	0.5770	0.0318			
			3.65**	0.09	3.94**	1.26	1.28	0.894	16.80**	
79	A	32.4150	0.2613	0.0756						16
			5.37**	1.24				0.700	15.20**	
	B	24.6336	0.7215	0.1905	-0.0033	-0.0013	0.0006			
			7.27**	1.54+	5.57**	1.42+	0.96	0.932	27.34**	
	C	24.432	0.1916	0.0383						
			15.09**	2.87*				0.948	118.00**	
	D	23.6179	-0.2756	-0.0736	6.4114	1.2838	0.0543			
			3.15*	0.67	5.55**	0.99	0.77	0.944	33.67**	
80	A	31.7200	0.2578	0.1426						16
			4.38**	1.94++				0.638	11.46**	
	B	30.6621	0.6552	0.1988	-0.0037	-0.0020	0.0025			
			12.09**	2.94*	11.32**	4.05**	7.82**	0.983	117.24**	
	C	26.171	0.1722	0.0545						
			9.77**	2.95*				0.889	52.05**	
	D	34.6048	-0.2967	-0.1045	4.8113	0.3517	0.3321			
			4.06**	1.14	4.97**	0.32	5.63**	0.968	59.72**	

Table A.5 (continued)

Exp. no.	Equation	The regression coefficients and the value of t-test for the regression coefficients of the following terms						Value of R^2	F-ratio	No. of obs.
		b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$			
81	A	38.3713	0.2395	0.0197						16
			4.54**	0.30				0.614	10.33**	
	B	32.3395	0.6750	0.0728	-0.0032	-0.0009	0.0008			
			4.86**	0.42	3.88**	0.72	0.93	0.854	11.69**	
	C	29.829	0.1705	0.0070						
			9.63**	0.38				0.877	46.47**	
	D	34.6676	-0.2821	-0.1754	5.4736	0.9669	0.1681			
			2.61*	1.30	3.83**	0.61	1.93++	0.906	19.34**	
82	A	58.3188	0.3156	0.1744						16
			6.02**	3.33**				0.785	23.67**	
	B	52.4962	0.6436	0.4787	-0.0039	-0.0036	0.0004			
			4.38**	3.26**	2.65*	2.47*	0.38	0.907	19.58**	
	C	53.112	0.0817	0.0512						
			9.62**	6.03**				0.908	64.45**	
	D	52.4241	-0.0278	-0.1999	3.0725	3.3747	0.0493			
			0.20	1.45+	2.18++	2.39*	0.50	0.914	21.14**	
83	A	44.4762	0.2257	0.0291						16
			6.15**	0.79				0.747	19.22**	
	B	41.4300	0.4784	0.1230	-0.0030	-0.0012	0.0004			
			4.07**	1.05	2.56*	1.05	0.40	0.858	12.11**	
	C	40.920	0.0851	0.0147						
			9.10**	1.58+				0.868	42.63**	
	D	41.2302	-0.0433	-0.1133	2.4425	1.2060	0.0324			
			0.38	0.99	2.09++	1.03	0.39	0.858	12.12**	
84	A	39.0163	0.0851	0.1086						16
			1.94++	2.48*				0.433	4.97*	
	B	33.6616	0.4553	0.3438	-0.0044	-0.0029	0.0006			
			5.93**	4.48**	5.80**	3.83**	1.02	0.904	18.95**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
84	C	34.043	0.0540	0.0549						
			5.03**	5.12**				0.798	25.74**	
	D	33.2074	-0.3212	-0.1647	3.7416	2.4432	0.0396			
			3.91**	2.09++	4.44**	2.90*	0.67	0.884	15.23**	
85	A	42.4875	0.0663	0.0387						16
			1.55+	0.90				0.198	1.60	
	B	37.4741	0.3341	0.2965	-0.0031	-0.0030	0.0002			
			2.73*	2.42*	2.54*	2.44*	0.22	0.643	3.60*	
	C	38.285	0.0422	0.0249						
			2.81*	1.66+				0.451	5.33*	
	D	37.8445	-0.2276	-0.1736	2.7577	1.9607	0.0197			
			1.72+	1.32	2.04++	1.45+	0.21	0.559	2.54++	
86	A	57.2700	0.0292	0.0240						16
			0.70	0.58				0.059	0.41	
	B	53.7116	0.1605	0.3390	-0.0019	-0.0039	0.0008			
			1.47+	3.11*	1.72+	3.60**	0.93	0.648	3.69*	
	C	53.088	0.0178	0.0199						
			1.59+	1.78++				0.305	2.86++	
	D	55.0509	-0.1944	-0.3106	1.5486	2.6325	0.1117			
			1.93++	3.08*	1.50+	2.55*	1.54+	0.683	4.30*	
87	A	37.0583	0.0124	0.0529						27
			1.33+	1.27				0.170	2.46+	
	B	28.9673	0.0867	0.5342	-0.0003	-0.0076	0.0003			
			2.86**	5.83**	3.12**	6.16**	1.00	0.783	15.14**	
	C	31.731	0.0394	0.0237						
			2.76*	1.08				0.498	11.90**	
	D	29.7408	-0.0946	-0.2859	1.4970	2.0017	0.1045			
			2.69*	1.83++	1.67+	1.49+	1.08	0.629	7.13**	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
88	A	52.0055	0.0207	0.1387						9
			0.41	2.73*				0.559	3.80++	
	B	51.1527	0.2710	0.2810	-0.0059	-0.0038	0.0019			
			2.03+	2.11+	2.45++	1.56	1.10	0.895	5.13+	
	C	50.512	0.0101	0.0323						
			1.17	3.74**				0.719	7.67*	
	D	51.5259	-0.3382	-0.0898	2.1529	1.2311	0.0956			
			2.32+	0.62	2.05+	1.17	1.22	0.902	5.51++	
89	A	23.5575	0.3444	0.2687						16
			5.22**	4.07**				0.771	21.93**	
	B	23.8635	0.5076	0.3603	-0.0087	-0.0069	0.0112			
			3.96**	2.81*	3.06*	2.45*	4.88**	0.954	41.10**	
	C	21.431	0.1140	0.0853						
			8.26**	6.17**				0.891	53.14**	
	D	25.2931	-0.0101	-0.0019	0.8131	0.2469	0.4521			
			0.10	0.02	1.18	0.36	5.42**	0.964	52.99**	
90	A	43.4752	0.0224	0.0743						27
			2.14*	1.59+				0.300	5.15*	
	B	39.7625	0.1086	0.2778	-0.0005	-0.0047	0.0009			
			2.14*	1.82++	2.66*	2.25*	1.68+	0.591	6.06**	
	C	39.663	0.0489	0.0017						
			3.19**	0.07				0.448	9.75**	
	D	39.6258	-0.0895	-0.2736	0.7854	1.3641	0.2376			
			1.90++	1.31	0.65	0.76	1.84++	0.551	5.15**	
91	A	23.7798	0.0544	0.0497						16
			1.90++	1.89++				0.365	3.74++	
	B	23.0429	0.1984	0.1260	-0.0059	-0.0021	0.0007			
			1.77+	1.24	1.90++	0.92	0.28	0.569	2.64++	

Table A.5 (continued)

Exp. no.	Equation	b_0	N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Value of R^2	F-ratio	No. of obs.
		The regression coefficients and the value of t-test for the regression coefficients of the following terms								
91	C	22.700	0.0175	0.0296						
			2.10++	2.30*				0.504	6.60*	
	D	22.6179	-0.0843	-0.0790	0.4454	0.8626	0.0525			
			0.78	0.65	0.51	1.09	0.41	0.520	2.17+	
92	A	37.3419	0.1077	0.0993						16
			2.15++	2.96*				0.530	7.33**	
	B	35.9480	0.3050	0.2073	-0.0073	-0.0045	0.0033			
			1.73+	1.78+	1.37	2.60*	0.80	0.767	6.60**	
	C	35.498	0.0287	0.0286						
			2.95*	4.88**				0.791	24.58**	
	D	35.3240	-0.2182	-0.1458	1.7001	0.7470	0.1759			
			1.20	1.98++	1.70+	0.82	1.00	0.826	9.47**	
93	A	33.2575	0.0588	0.0378						16
			1.74+	1.12				0.247	2.14+	
	B	31.8990	0.1713	0.0787	-0.0014	-0.0006	0.0002			
			1.24	0.57	0.95	0.38	0.18	0.321	0.95	
	C	31.974	0.0295	0.0175						
			1.94++	1.15				0.280	2.53+	
	D	33.7192	-0.0204	-0.0073	0.3578	0.0323	0.0805			
			0.17	0.06	0.30	0.03	0.80	0.335	1.01	
Average	A		2.37*	2.61*				0.528	9.98**	16
	B		2.83*	3.18**	2.62*	2.77*	1.32	0.803	16.21**	
	C		4.40**	4.86**				0.736	41.62**	
	D		1.67+	1.76+	2.02++	2.09++	1.81++	0.816	24.36**	

** Significant at the 1 per cent level
* Significant at the 5 per cent level
++ Significant at the 10 per cent level
+ Significant at the 20 per cent level

Table A.6

The Number of Regression Statistics Significant at the Different Levels of Significance for the 93 Experiments for Each Type of Equation

Equation	Per cent level of significance	The number of significant t tests for the regression coefficients of the following terms: N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$	Number of significant F-ratios
Linear	1	24	32				49
	5	39	48				73
	10	51	57				79
	20	61	66				86
Quadratic	1	30	32	23	27	3	59
	5	43	50	46	50	12	76
	10	56	64	56	57	21	84
	20	69	72	69	67	36	89
Cobb Douglas	1	48	52				77
	5	65	65				88
	10	71	70				89
	20	77	79				91
Square Root	1	9	14	19	21	12	66
	5	26	25	33	30	22	78
	10	38	39	39	34	37	84
	20	49	46	52	45	45	88

APPENDIX B

Table B.1

Economic Optima in Fertilizer Use for Wheat on Summerfallow Predicted by the Quadratic Function

Experiment Number: 1 Year: 1966
Location: Dubuc, District 5A, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
25.0	3.3	8.6	-1.43
30.0	19.5	15.0	-1.43
35.0	39.6	22.8	-1.43
40.0	69.1	34.3	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	40.3	72.0	35.4	2.03
1.50	40.6	74.3	36.3	2.05
1.60	40.8	76.3	37.1	2.06
1.70	41.0	78.1	37.8	2.07
1.80	41.1	79.6	38.4	2.07

Experiment Number: 2 Year: 1961
Location: Conquest, District 6B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
15.0	5.1	0.0	-3.10

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	15.8	10.1	2.4	4.25
1.50	16.1	10.9	6.4	1.71
1.60	16.3	11.6	9.9	1.17
1.70	16.5	12.2	13.0	0.94
1.80	16.7	12.8	15.8	0.81

Table B.1 (continued)

Experiment Number: 3 Year: 1961
Location: Kindersley, District 7A, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	2.7	24.7	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	23.6	13.3	49.2	0.27
1.50	23.7	14.1	50.9	0.28
1.60	23.9	14.7	52.4	0.28
1.70	24.0	15.3	53.8	0.28
1.80	24.1	15.8	55.0	0.29

Experiment Number: 4 Year: 1964
Location: Scott, District 7B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
10.0	0.0	13.1	-0.77
15.0	10.2	50.7	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	14.4	6.3	44.7	0.14
1.50	14.5	7.1	45.9	0.15
1.60	14.7	7.7	46.9	0.17
1.70	14.7	8.3	47.8	0.17
1.80	14.8	8.8	48.6	0.18

Table B.1 (continued)

Experiment Number: 5 Year: 1968
Location: Archerwill, District 8 A, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
40.0	0.0	8.4	-0.69
45.0	0.9	26.9	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	48.3	10.1	41.6	0.24
1.50	48.4	10.5	42.1	0.25
1.60	48.4	10.7	42.6	0.25
1.70	48.4	11.0	43.0	0.26
1.80	48.5	11.2	43.3	0.26

Experiment Number: 7 Year: 1960
Location: Melfort, District 8 B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	0.0	12.0	-0.70

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	33.9	5.4	38.5	0.14
1.50	34.0	5.9	39.4	0.15
1.60	34.1	6.3	40.2	0.16
1.70	34.2	6.7	40.9	0.16
1.80	34.2	7.0	41.5	0.17

Table B.1 (continued)

Experiment Number: 9 Year: 1962
Location: Melfort, District 8B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
35.0	0.0	4.8	-0.82
40.0	0.0	17.4	-1.11
45.0	3.0	30.0	-1.43
50.0	11.2	40.7	-1.43
55.0	24.9	58.6	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	55.3	26.7	61.1	0.44
1.50	55.4	27.2	61.7	0.44
1.60	55.5	27.6	62.3	0.44
1.70	55.5	28.0	62.8	0.45
1.80	55.6	28.4	63.2	0.45

Experiment Number: 10 Year: 1964
Location: Melfort, District 8B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	0.0	3.6	-0.27
25.0	0.0	37.8	-1.02
30.0	49.2	65.8	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	26.2	6.4	46.6	0.14
1.50	26.9	12.6	49.4	0.26
1.60	27.5	18.0	51.8	0.35
1.70	27.9	22.8	54.0	0.42
1.80	28.3	27.1	55.9	0.48

Table B.1 (continued)

Experiment Number: 11 Year: 1966
Location: Melfort, District 8 B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
60.0	5.8	19.0	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	62.1	15.1	34.0	0.44
1.50	62.4	16.6	36.3	0.46
1.60	62.6	17.8	38.3	0.47
1.70	62.8	19.0	40.1	0.47
1.80	62.9	20.0	41.7	0.48

Experiment Number: 12 Year: 1960
Location: Glenbush, District 9 A, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	0.0	4.3	-0.78

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	33.7	3.7	36.3	0.10
1.50	33.9	4.4	37.8	0.12
1.60	34.0	5.1	39.2	0.13
1.70	34.1	5.7	40.3	0.14
1.80	34.2	6.3	41.4	0.15

Table B.1 (continued)

Experiment Number: 13 Year: 1962 Location: Hafford, District 9A, Saskatchewan			
Least Cost Combinations for Specified Yields			
Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
15.0	2.7	0.0	-3.27
20.0	14.6	48.1	-1.43

Profit Maximization				
Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	18.9	11.9	26.7	0.44
1.50	19.1	12.2	29.4	0.42
1.60	19.2	12.5	31.8	0.39
1.70	19.3	12.8	33.8	0.38
1.80	19.4	13.0	35.7	0.36

Experiment Number: 14 Year: 1964 Location: Snowden, District 9A, Saskatchewan			
Least Cost Combinations for Specified Yields			
Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
10.0	0.0	7.9	-0.74
15.0	7.3	46.2	-1.43

Profit Maximization				
Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	15.2	8.5	48.0	0.18
1.50	15.4	9.8	49.9	0.20
1.60	15.6	10.9	51.5	0.21
1.70	15.8	11.9	52.9	0.23
1.80	15.9	12.8	54.2	0.24

Table B.1 (continued)

Experiment Number: 15 Year: 1962 Location: Glaslyn, District 9 B, Saskatchewan			
Least Cost Combinations for Specified Yields			
Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	0.8	0.0	-2.49
25.0	6.8	8.8	-1.43

Profit Maximization				
Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	29.5	11.5	34.3	0.34
1.50	29.5	11.6	35.0	0.33
1.60	29.6	11.7	35.5	0.33
1.70	29.6	11.8	36.0	0.33
1.80	29.6	11.9	36.5	0.33

Experiment Number: 16 Year: 1963 Location: Glaslyn, District 9 B, Saskatchewan			
Least Cost Combinations for Specified Yields			
Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
55.0	4.2	20.9	-1.43

Profit Maximization				
Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	57.9	9.3	41.4	0.23
1.50	58.0	9.6	42.4	0.23
1.60	58.1	9.8	43.2	0.23
1.70	58.1	10.0	43.9	0.23
1.80	58.1	10.2	44.6	0.23

Table B.1 (continued)

Experiment Number: 17 Year: 1962
Location: Turtleford, District 9 B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	6.4	9.6	-1.43
35.0	11.1	35.9	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	36.2	13.1	47.3	0.28
1.50	36.2	13.3	48.4	0.28
1.60	36.3	13.5	49.3	0.27
1.70	36.4	13.7	50.2	0.27
1.80	36.4	13.8	51.0	0.27

Experiment Number: 18 Year: 1962
Location: Loon Lake, District 9 B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
25.0	5.3	0.0	-1.79
30.0	15.5	38.3	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	29.6	14.2	32.4	0.44
1.50	29.7	14.6	34.1	0.43
1.60	29.8	14.9	35.7	0.42
1.70	29.9	15.2	37.0	0.41
1.80	30.0	15.5	38.2	0.41

Table B.1 (continued)

Experiment Number: 19 Year: 1967
Location: Spring Coulee, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
15.0	0.0	7.8	-0.82
20.0	12.1	29.6	-1.43
25.0	45.7	50.5	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	23.9	35.5	44.2	0.80
1.50	24.2	37.9	45.7	0.83
1.60	24.4	40.0	47.0	0.85
1.70	24.6	41.9	48.2	0.87
1.80	24.8	43.5	49.2	0.88

Experiment Number: 20 Year: 1967
Location: Cowley, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	0.0	7.6	-0.19
25.0	0.0	17.5	-0.27
30.0	0.0	30.8	-0.46
35.0	6.9	50.4	-1.43
40.0	59.9	66.6	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	38.3	29.7	57.3	0.52
1.50	38.5	31.9	58.0	0.55
1.60	38.7	33.8	58.6	0.58
1.70	38.8	35.5	59.1	0.60
1.80	39.0	37.0	59.6	0.62

Table B.1 (continued)

Experiment Number: 21 Year: 1960
Location: Fort McLeod, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	0.0	1.2	-0.68

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	23.5	10.8	21.4	0.50
1.50	23.6	12.0	22.4	0.54
1.60	23.8	13.1	23.3	0.56
1.70	23.9	14.1	24.0	0.59
1.80	24.0	15.0	24.7	0.61

Experiment Number: 22 Year: 1965
Location: Carmangay, District 2, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
35.0	0.0	13.4	-1.12
40.0	7.7	22.9	-1.43
45.0	21.0	30.1	-1.43
50.0	41.1	40.9	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	51.0	47.7	44.5	1.07
1.50	51.1	48.8	45.1	1.08
1.60	51.2	49.7	45.6	1.09
1.70	51.3	50.5	46.0	1.10
1.80	51.3	51.2	46.4	1.10

Table B.1 (continued)

Experiment Number: 23 Year: 1960
Location: Bonnyville, District 6, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	0.0	5.2	-0.51
35.0	0.0	11.7	0.78
40.0	2.5	19.5	-1.43
45.0	12.0	23.8	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	48.1	24.0	29.3	0.82
1.50	48.2	24.3	29.5	0.82
1.60	48.2	24.6	29.6	0.83
1.70	48.2	24.8	29.7	0.83
1.80	48.2	25.0	29.8	0.84

Experiment Number: 24 Year: 1959
Location: McLennan, District 7, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	17.4	15.3	-1.43
35.0	39.1	43.0	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	35.0	39.0	42.9	0.91
1.50	35.3	40.9	45.3	0.90
1.60	35.5	42.5	47.3	0.90
1.70	35.7	43.9	49.1	0.89
1.80	35.9	45.2	50.8	0.89

Table B.2

Economic Optima in Fertilizer Use for Wheat on Stubble Predicted by the Quadratic Function

Experiment Number: 25 Year: 1959
Location: Grandview, District 11, Manitoba

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
25.0	7.6	11.4	-1.43
30.0	29.6	19.3	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	31.4	39.6	22.9	1.73
1.50	31.5	40.9	23.3	1.75
1.60	31.6	42.0	23.7	1.77
1.70	31.7	43.0	24.1	1.79
1.80	31.8	43.9	24.4	1.80

Experiment Number: 27 Year: 1967
Location: Wapella, District 1B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	0.0	11.5	-0.90
25.0	27.8	24.0	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	25.4	31.6	25.2	1.26
1.50	25.7	34.0	25.9	1.31
1.60	25.9	36.1	26.6	1.36
1.70	26.0	37.9	27.2	1.40
1.80	26.2	39.5	27.7	1.43

Table B.2 (continued)

Experiment Number: 29 Year: 1967
Location: Dubuc, District 5A, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
25.0	2.2	8.0	-1.43
30.0	23.1	16.8	-1.43
35.0	67.8	35.6	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	33.4	45.1	26.1	1.73
1.50	33.6	47.0	26.9	1.75
1.60	33.8	48.7	27.6	1.77
1.70	34.0	50.2	28.2	1.78
1.80	34.1	51.6	28.8	1.79

Experiment Number: 30 Year: 1967
Location: Kindersley, District 7A, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	11.0	0.6	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
1.40	33.6	24.9	14.5	1.72
1.50	33.7	25.5	15.1	1.69
1.60	33.8	26.1	15.7	1.67
1.70	33.8	26.6	16.1	1.65
1.80	33.9	27.0	16.6	1.63

Table B.2 (continued)

Experiment Number: 33 Year: 1964
Location: Glenwood, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
15.0	25.8	0.0	-1.43
20.0	48.2	39.6	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	18.8	40.9	26.6	1.53
1.50	19.1	42.7	29.8	1.43
1.60	19.4	44.3	32.6	1.36
1.70	19.6	45.7	35.1	1.30
1.80	19.8	46.9	37.3	1.26

Experiment Number: 34 Year: 1966
Location: Glenwood, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
25.0	0.0	4.3	-0.34
30.0	0.0	26.3	-0.69
35.0	34.4	52.9	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	33.3	12.9	44.6	0.29
1.50	33.6	15.2	45.5	0.33
1.60	33.8	17.3	46.3	0.37
1.70	34.0	19.1	47.0	0.41
1.80	34.1	20.7	47.6	0.43

Table B.2 (continued)

Experiment Number: 35 Year: 1959
Location: Pincher Creek, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
25.0	21.7	18.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	25.0	22.0	18.2	1.20
1.50	25.3	24.3	19.7	1.23
1.60	25.5	26.3	21.0	1.25
1.70	25.7	28.1	22.1	1.27
1.80	25.9	29.7	23.1	1.29

Experiment Number: 36 Year: 1965
Location: Nanton, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	13.3	0.0	-3.50
25.0	36.6	3.2	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	26.4	41.6	16.0	2.61
1.50	26.7	42.5	18.3	2.32
1.60	26.8	43.4	20.4	2.13
1.70	27.0	44.1	22.2	1.99
1.80	27.1	44.7	23.8	1.88

Table B.2 (continued)

Experiment Number: 37 Year: 1960
Location: Acme, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
15.0	1.4	13.3	-1.43
20.0	34.6	20.4	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	20.0	34.4	20.4	1.69
1.50	20.2	36.6	20.8	1.76
1.60	20.4	38.5	21.3	1.81
1.70	20.5	40.2	21.6	1.86
1.80	20.6	41.7	21.9	1.90

Experiment Number: 38 Year: 1960
Location: Mazeppa, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
10.0	32.3	15.5	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
1.40	8.9	20.4	10.9	1.87
1.50	9.2	23.2	12.0	1.93
1.60	9.4	25.5	12.9	1.98
1.70	9.6	27.6	13.7	2.02
1.80	9.8	29.5	14.4	2.04

Table B.3

Economic Optima in Fertilizer Use for Oats on Summerfallow Predicted by the Quadratic Function

Experiment Number: 42 Year: 1961
Location: Melfort, District 8B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	4.4	35.4	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.50	28.4	2.7	27.8	0.10
0.55	29.2	3.5	31.3	0.11
0.60	29.8	4.2	34.3	0.12
0.65	30.2	4.8	36.8	0.13
0.70	30.6	5.3	39.0	0.13

Experiment Number: 44 Year: 1962
Location: Melfort, District 8B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
50.0	0.0	6.9	-0.16
60.0	0.0	15.0	-0.24
70.0	0.0	25.1	-0.39
80.0	0.0	39.8	-0.87
90.0	13.3	53.9	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.50	92.5	19.6	57.5	0.34
0.55	93.1	21.6	58.6	0.37
0.60	93.6	23.2	59.5	0.39
0.65	94.0	24.5	60.3	0.41
0.70	94.3	25.7	61.0	0.42

Table B.4

Economic Optima in Fertilizer Use for Oats on Stubble Predicted by the Quadratic Function

Experiment Number: 45 Year: 1965
Location: La Corey, District 6, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP / dN
bushels	-pounds-		
40.0	0.0	0.5	-0.39
50.0	1.7	10.5	-1.43
60.0	18.6	14.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.50	64.3	30.2	16.5	1.83
0.55	64.7	31.8	16.9	1.89
0.60	65.1	33.2	17.2	1.93
0.65	65.4	34.4	17.4	1.97
0.70	65.6	35.4	17.6	2.01

Table B.5

Economic Optima in Fertilizer Use for Barley on Summerfallow Predicted by the Quadratic Function

Experiment Number: 47 Year: 1962
Location: Scott, District 7B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
40.0	2.1	28.0	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	41.5	4.3	35.0	0.12
0.75	42.1	5.2	38.0	0.14
0.85	42.5	6.0	40.4	0.15
0.95	42.7	6.5	42.3	0.15
1.05	42.9	7.0	43.8	0.16

Experiment Number: 48 Year: 1960
Location: Fort McLeod, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
40.0	9.9	23.7	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	40.0	9.9	23.7	0.42
0.75	40.6	12.3	25.4	0.49
0.85	41.0	14.1	26.6	0.53
0.95	41.3	15.5	27.6	0.56
1.05	41.5	16.7	28.4	0.59

Table B.5 (continued)

Experiment Number: 51 Year: 1967
Location: Lacombe, District 5, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP / dN
bushels	-pounds-		
10.0	0.0	5.5	-0.16
20.0	0.0	15.1	-0.29
30.0	11.1	29.5	-1.43
40.0	56.9	34.5	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.65	36.9	39.7	32.6	1.22
0.75	38.8	49.9	33.8	1.48
0.85	40.1	57.6	34.6	1.66
0.95	41.0	63.7	35.3	1.81
1.05	41.7	68.6	35.8	1.92

Table B.6

Economic Optima in Fertilizer Use for Barley on Stubble Predicted by the Quadratic Function

Experiment Number: 53 Year: 1960
Location: Sanford, District 3, Manitoba

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	0.0	11.9	-1.27
30.0	16.9	21.3	-1.43
40.0	46.4	34.6	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.65	38.9	41.5	32.4	1.28
0.75	39.7	45.0	34.0	1.33
0.85	40.3	47.8	35.2	1.36
0.95	40.6	49.9	36.2	1.38
1.05	40.9	51.7	37.0	1.40

Experiment Number: 55 Year: 1960
Location: Portage la Prairie, District 3, Manitoba.

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
30.0	0.0	10.9	-1.06
40.0	18.5	26.3	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.65	44.1	32.1	32.5	0.99
0.75	45.1	36.4	34.4	1.06
0.85	45.8	39.7	35.9	1.11
0.95	46.2	42.3	37.1	1.14
1.05	46.6	44.4	38.0	1.17

Table B.6 (continued)

Experiment Number: 56 Year: 1967
Location: Roland, District 3, Manitoba

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
50.0	0.0	11.1	-0.49
60.0	24.8	20.9	-1.43
70.0	62.1	25.5	-1.43
80.0	107.4	31.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	78.3	98.7	30.0	3.29
0.75	82.8	122.6	32.9	3.72
0.85	85.9	140.8	35.2	4.00
0.95	88.0	155.2	37.0	4.20
1.05	89.6	166.9	38.4	4.35

Experiment Number: 59 Year: 1968
Location: Bagot, District 8, Manitoba.

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
40.0	29.9	11.3	-1.43
50.0	78.4	18.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	45.2	53.3	14.5	3.67
0.75	49.2	73.8	17.4	4.24
0.85	51.8	89.5	19.6	4.56
0.95	53.7	101.9	21.3	4.77
1.05	55.0	111.9	22.7	4.92

Table B.6 (continued)

Experiment Number: 61 Year: 1968
Location: Melfort, District 8B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
60.0	26.2	8.7	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	63.3	41.1	9.6	4.28
0.75	64.8	49.4	10.1	4.88
0.85	65.8	55.8	10.5	5.30
0.95	66.6	60.8	10.8	5.61
1.05	67.1	64.9	11.1	5.85

Experiment Number: 63 Year: 1961
Location: Turtleford, District 9B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	3.6	20.9	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	23.6	12.5	30.3	0.41
0.75	24.4	15.2	33.2	0.46
0.85	24.9	17.3	35.4	0.49
0.95	25.3	18.9	37.1	0.51
1.05	25.6	20.2	38.5	0.53

Table B.6 (continued)

Experiment Number: 64 Year: 1960
Location: Dorintosh, District 9B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
10.0	3.0	0.0	-1.45
20.0	12.0	19.9	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.65	25.4	19.7	37.5	0.53
0.75	25.9	21.0	40.3	0.52
0.85	26.3	21.9	42.4	0.52
0.95	26.6	22.7	44.1	0.51
1.05	26.8	23.3	45.4	0.51

Experiment Number: 66 Year: 1960
Location: New Dayton, District 2, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	12.4	8.6	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.65	22.2	20.4	12.5	1.63
0.75	23.5	25.8	15.2	1.70
0.85	24.4	30.0	17.2	1.74
0.95	25.0	33.3	18.8	1.77
1.05	25.4	36.0	20.2	1.79

Table B.6 (continued)

Experiment Number: 67 Year: 1964
Location: Barons, District 2, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	7.7	12.4	-1.43
30.0	37.3	39.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	27.2	24.9	27.9	0.89
0.75	28.1	28.1	30.8	0.91
0.85	28.7	30.5	33.0	0.93
0.95	29.1	32.5	34.8	0.93
1.05	29.5	34.1	36.2	0.94

Experiment Number: 69 Year: 1964
Location: Glenwood, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	6.0	0.0	-2.71
30.0	31.9	5.0	-1.43
40.0	63.2	40.8	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	35.5	45.6	20.6	2.21
0.75	37.0	50.2	25.9	1.94
0.85	38.0	53.8	30.0	1.79
0.95	38.7	56.6	33.2	1.70
1.05	39.2	58.9	35.9	1.64

Table B.6 (continued)

Experiment Number: 71 Year: 1962
Location: Fort McLeod, District 3, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	3.0	0.0	-2.25
30.0	24.4	16.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	32.8	30.0	26.7	1.12
0.75	34.0	33.2	32.7	1.02
0.85	34.9	35.6	37.2	0.96
0.95	35.5	37.5	40.8	0.92
1.05	36.0	39.1	43.7	0.89

Experiment Number: 75 Year: 1966
Location: Chedderville, District 5, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
20.0	15.5	0.0	-1.60
30.0	32.1	8.8	-1.43
40.0	48.0	25.3	-1.43
50.0	69.0	47.2	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	52.6	76.3	54.8	1.39
0.75	54.2	81.5	60.1	1.35
0.85	55.2	85.5	64.3	1.33
0.95	56.0	88.6	67.5	1.31
1.05	56.5	91.1	70.1	1.30

Table B.6 (continued)

Experiment Number: 80 Year: 1967
Location: Condor, District 5, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
40.0	15.6	0.0	-2.27
50.0	35.6	2.3	-1.43
60.0	49.7	20.6	-1.43
70.0	66.6	42.7	-1.43
80.0	89.4	72.4	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	79.6	88.2	70.8	1.25
0.75	81.7	94.4	78.9	1.20
0.85	83.1	99.2	85.2	1.16
0.95	84.1	103.0	90.1	1.14
1.05	84.8	106.0	94.1	1.13

Experiment Number: 82 Year: 1967
Location: Red Deer, District 5, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
60.0	5.9	8.5	-1.43
70.0	17.5	17.5	-1.43
80.0	31.5	28.4	-1.43
90.0	50.4	43.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	94.1	61.8	52.0	1.19
0.75	95.0	65.2	54.6	1.19
0.85	95.6	67.8	56.6	1.20
0.95	96.1	69.9	58.2	1.20
1.05	96.4	71.5	59.5	1.20

Table B.6 (continued).

Experiment Number: 83 Year: 1966
Location: Lacombe, District 5, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
50.0	20.6	0.0	-2.72
60.0	53.5	12.0	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	58.1	48.6	3.8	12.76
0.75	59.9	53.3	11.7	4.57
0.85	61.1	56.9	17.7	3.22
0.95	62.0	59.7	22.4	2.67
1.05	62.6	62.0	26.2	2.36

Experiment Number: 84 Year: 1967
Location: Lacombe, District 5, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
40.0	6.6	11.2	-1.43
50.0	23.6	29.7	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	53.7	32.9	39.8	0.83
0.75	54.7	36.0	43.1	0.84
0.85	55.3	38.4	45.6	0.84
0.95	55.7	40.2	47.7	0.84
1.05	56.1	41.7	49.3	0.85

Table B.6 (continued)

Experiment Number: 88 Year: 1965
Location: Myrnam, District 4 B, Alberta

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP / dN
bushels	-pounds-		
60.0	17.0	29.8	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels		-pounds-	
0.65	58.2	10.5	22.3	0.47
0.75	59.0	13.0	25.3	0.52
0.85	59.5	15.0	27.5	0.55
0.95	59.9	16.6	29.3	0.57
1.05	60.2	17.8	30.8	0.58

Table B.7

Economic Optima in Fertilizer Use for Rye on Summerfallow Predicted by the Quadratic Function

Experiment Number: 91 Year: 1960
Location: Indian Head, District 2B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
25.0	6.0	8.7	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.95	25.2	6.5	9.7	0.67
1.00	25.3	7.1	10.9	0.66
1.05	25.5	7.7	11.9	0.64
1.10	25.6	8.2	12.9	0.64
1.15	25.7	8.7	13.8	0.63

Experiment Number: 92 Year: 1964
Location: Indian Head, District 2B, Saskatchewan

Least Cost Combinations for Specified Yields

Yield	N	P_2O_5	M.R.S. dP/dN
bushels	-pounds-		
40.0	8.8	10.1	-1.43

Profit Maximization

Price	Yield	N	P_2O_5	$\frac{N}{P_2O_5}$
dollars	bushels	-pounds-		
0.95	42.4	16.4	19.2	0.85
1.00	42.5	17.0	20.0	0.85
1.05	42.6	17.5	20.6	0.85
1.10	42.7	18.0	21.2	0.85
1.15	42.8	18.5	21.8	0.85