THE UNIVERSITY OF MANITOBA

DIRECT STIFFNESS ANALYSIS OF ARCHED-FOLDED PLATES OF TRANSLATION

by

David Pathmaseelan Thambiratnam

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

WINNIPEG, MANITOBA

February 1975

DIRECT STIFFNESS ANALYSIS OF ARCHED-FOLDED PLATES OF TRANSLATION

by

David Pathmaseelan Thambiratnam

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

TO MY WIFE SULOGINI

ABSTRACT

The work presented in this thesis deals with the analysis of arched folded plates (of translation) utilising the finite strip method. A translational arched folded plate structure is obtained by translating the folds along a curve.

Based on this analysis a computer programme titled "Archfold" has been written incorporating the direct stiffness technique. This programme can analyse both arched and conventional (straight) folded plate structures subjected to any type of loading. Archfold is also capable of dealing with structures having intermediate supports and structures with rigid end supports.

The results of various tests carried out with the programme compared very satisfactorily with other existing solutions. Some of these results together with the results of sample anlyses are presented in the thesis.

ACKNOWLEDGEMENTS

This work would not have been possible, if it had not been for the help and guidance given to me at all times by my adviser Dr. Shah. While, appreciating his superior knowledge on the subject, I acknowledge very gratefully his sincere interest in my work.

I wish to thank Professor R.B. Pinkney and Mr. David Young for the invaluable help given to me pertaining to the computer programming. Professor Lansdown and Professor Glanville who read through my work gave me very valuable ideas and comments. Their interest and help are sincerely acknowledged.

Last but not least I wish to say a big "thank you" to my wife, Sulogini and my little son Preshan for their encouragement and support. I appreciate their patience and understanding at all times; especially when I was away from home for long hours on account of my work.

TABLE OF CONTENTS

LIST OF FIGURES AND TABLES

CHAPTER I

INTRODUCTION

1.1 Description

A prismatic folded plate structure is an assemblage of thin plate elements interconnected along longitudinal edges and framing into transverse end diaphragms. These types of structures can have various cross-section as shown in Fig. 1.1 and often give a pleasing appearance. Folded plate structures combine the strength characteristics of shell structures with the simplicity of plate type structures. Extensive use has been made of them in roofs, box girder bridges, floors, and foundations.

The conventional folded plate structures carry the superimposed load to their supports through considerable bending action in the longitudinal direction. This often results in large values of longitudinal (tensile) stresses. It is in this context that an arched folded plate structure becomes of interest (Fig. 1.2). An arched construction may be obtained in one of two ways, viz. (a) Rotation of the folds about an axis and (b) translation of the folds along a curve which is usually shallow, as shown in Fig. 1.3. The former is referred to as an arched folded plate of revolution (or rotation) while the latter earns the name arched folded plate of translation. The distinction between the two types is shown in detail in Fig. 1.4. It is of interest to know that an arched folded plate structure possesses the strength of a doubly-curved shell and yet it is easy to form due to straight edges in one direction.

Note: Numbers in the brackets [ ] designate the References given at the end of the thesis.

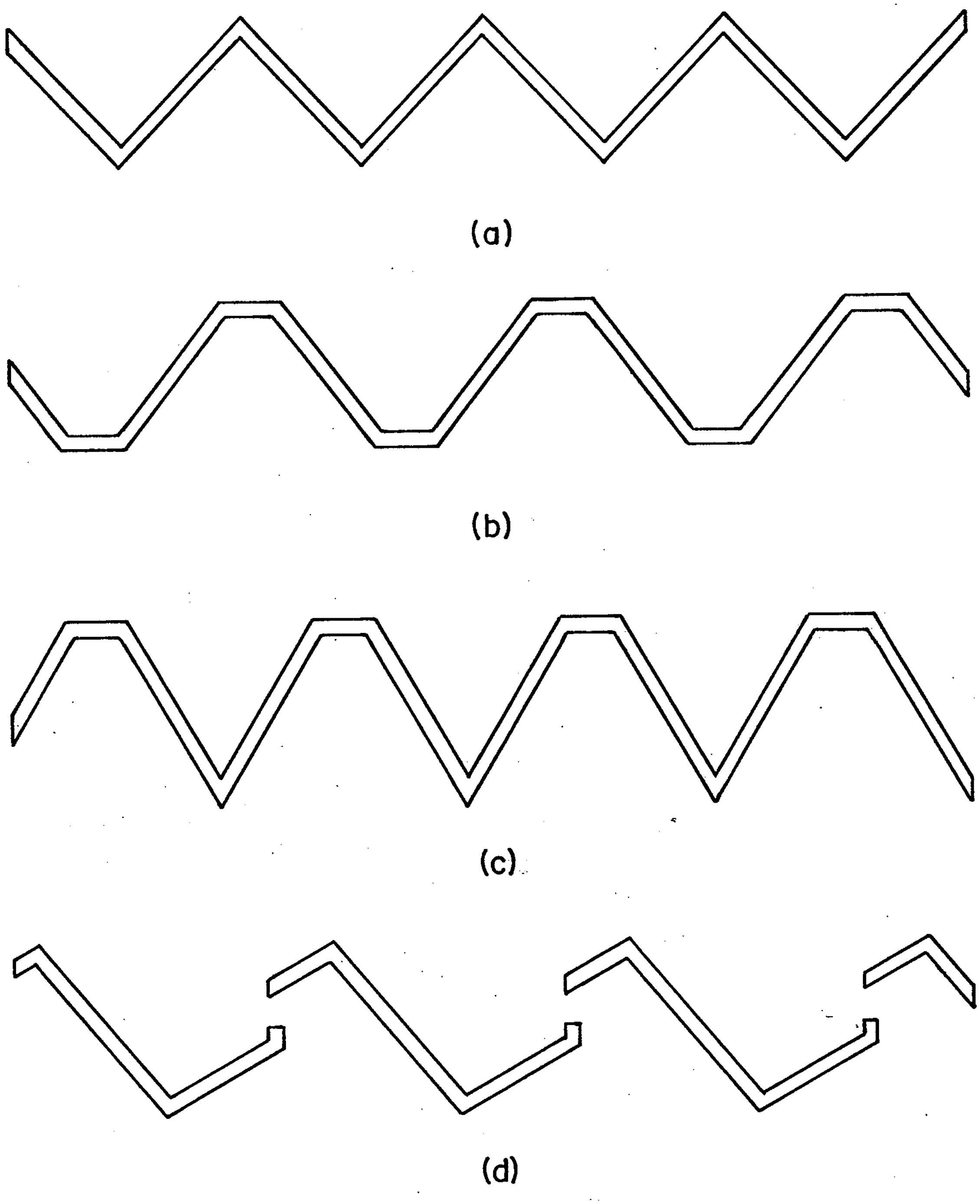
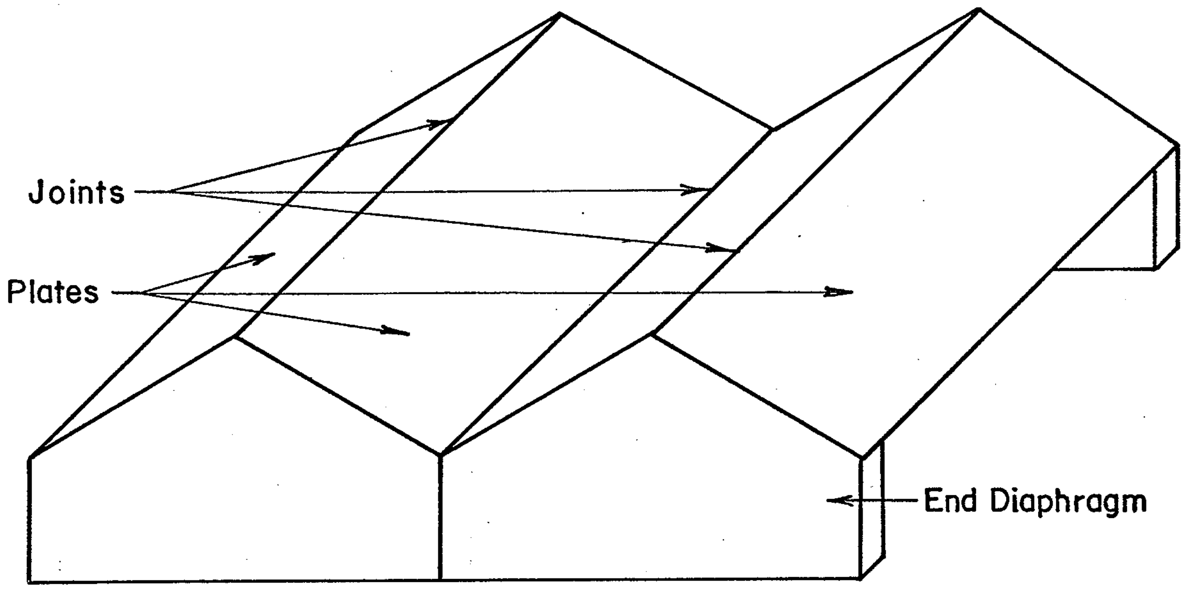
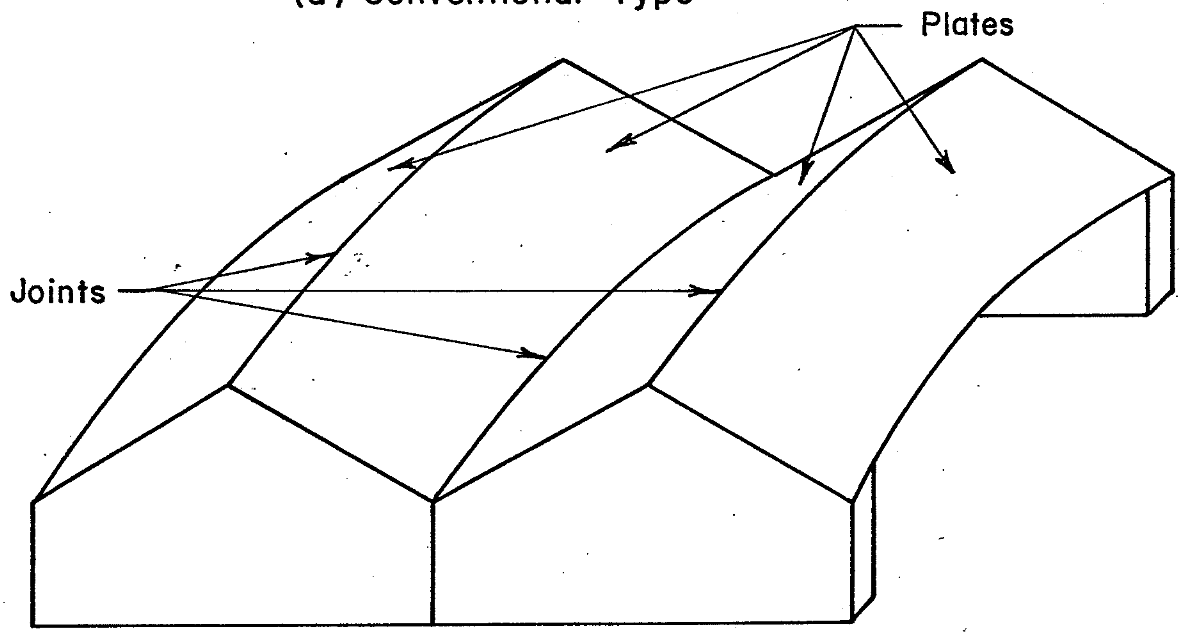


Figure 1.1 SOME CROSS-SECTIONS OF FOLDED PLATE STRUCTURES

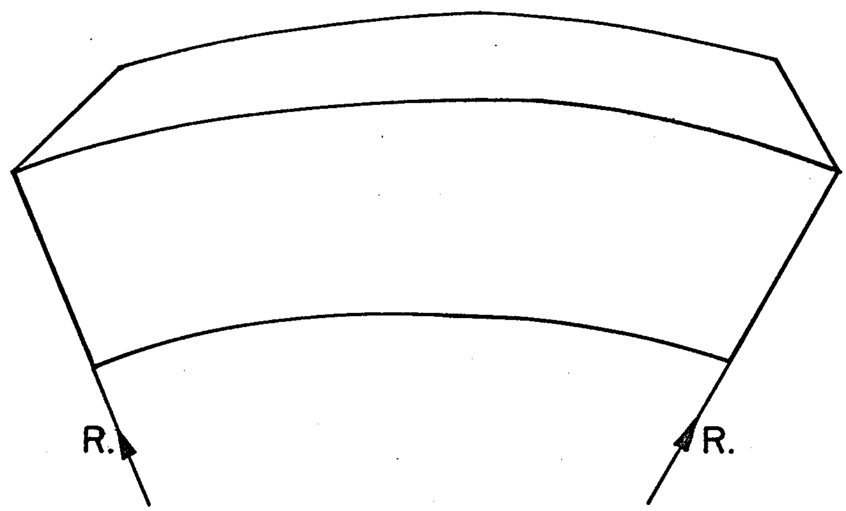


(a) Conventional Type

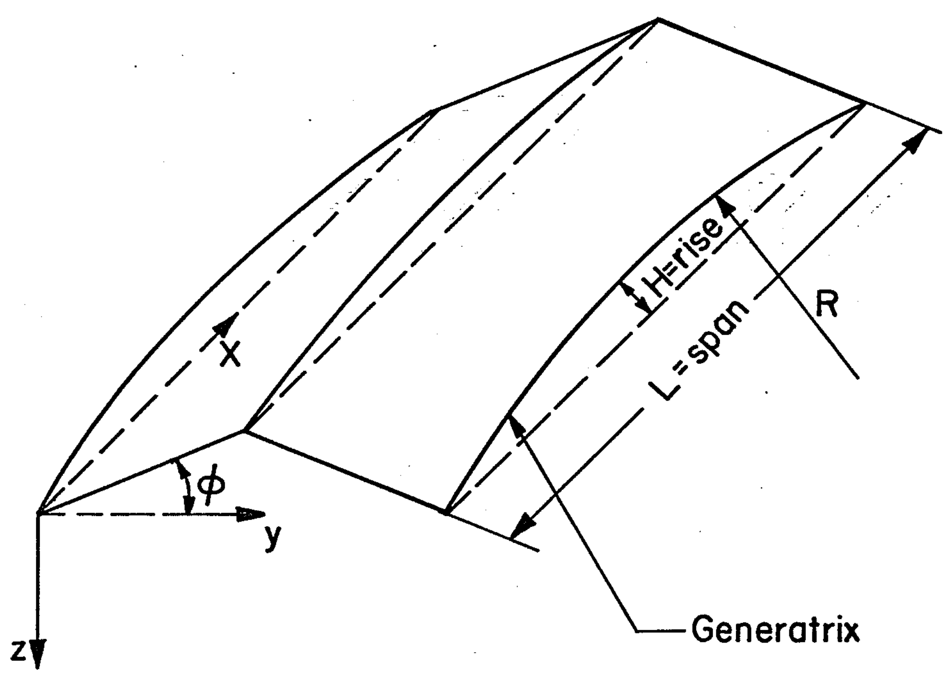


(b) Arched (translational) Type

Figure 1.2. FOLDED PLATE STRUCTURES.



(a). Rotational type.



(b). Translational Type

Figure 1.3. ARCHED FOLDED PLATES.

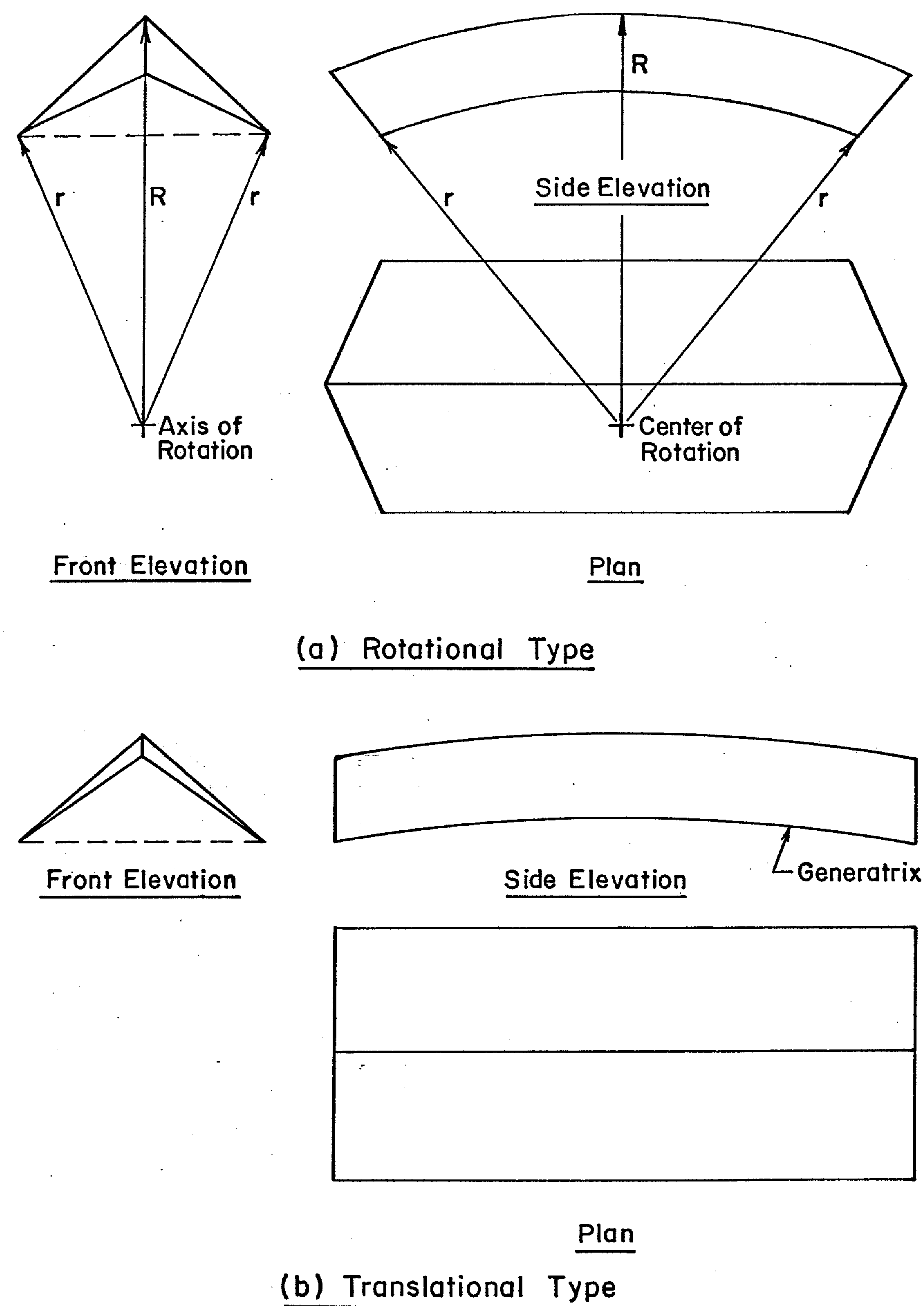


Figure I.4. DETAILS OF ARCHED FOLDED PLATES OF ROTATION AND TRANSLATION.

1.2 Background of Analyses

Considerable work has been carried out in the analysis of folded plate structures. The methods of analysis varied through the years. It is of interest to trace back some of the important methods used to date.

Originally the classical theory was employed to analyse folded plate structures making use of equilibrium and compatibility conditions between adjacent plates in turn. The works of Goldberg and Leve [1] and that of De Fries and Scordelis [2] are noteworthy examples of this. However, this method becomes rather tedious for large structures and certainly not suited for complicated goemetries. It was also not possible to analyse the structures subjected to any arbitary type of loading. A comprehensive report on the analyses of folded plate structures has been given by the ASCE task committee in 1963 [3].

The next phase was the use of the direct stiffness method to analyse folded plate structures. The stresses and displacements in each plate element being computed by classical thin plate bending theory and two-dimensional plane-stress élasticity theory. The use of the Direct Stiffness method facilitated programming the analysis. Much work has been done by Scordelis in this context. The programme "MUPDI3" is an example of such work by Scordelis and Lin [4].

The use of the finite element method made it possible to analyse a great many structures subjected to arbitary loading conditions. However, for this type of analysis a large computer programme and a fair-sized computer were necessary.

The finite strip method which closely resembles the finite element method was first used by Cheung [5] to analyse straight folded plate structures. The programme size was considerably reduced by the use of this method which proved very suitable for the analysis of plate structures. Meyer and Scordelis utilized this technique to analyse folded plate structures curved in plan [6]. On the basis of their analysis Meyer and Scordelis presented the programme "CURSTR" which essentially deals with shells of revolution.

Work on arched folded plates of translation has been quite limited. Shah and Lansdown [7] analysed the above type using the classical theory of shallow shells.

1.3 Object of Study

The object of this thesis is to extend the finite strip method for the analysis of arched folded plate structures of translation, utilizing the direct stiffness technique. An attempt is made to provide a general computer programme which is capable of analysing the majority of arched folded plate structures subject to any type of loading. The analysis requires the structure to be simply supported at its two transverse ends. Provision is made to accomodate structures with intermediate supports in the form of plane frames and/or rigid diaphragms.

Chapter II of the thesis deals with the geometry of arched folded plates, the assumptions made in the analysis and a description of the finite strip method.

Chapter III presents the analysis proper, while Chapter IV deals with the computer programme. The various tests carried out with the programme to show its validity and possible applications are described in Chapter V.

Results in this context are presented in graphical form in this chapter.

The last chapter concludes the thesis with a discussion on the results obtained.

CHAPTER II

PRELIMINARIES

2.1 Geometry of an Arched Folded Plate of Translation

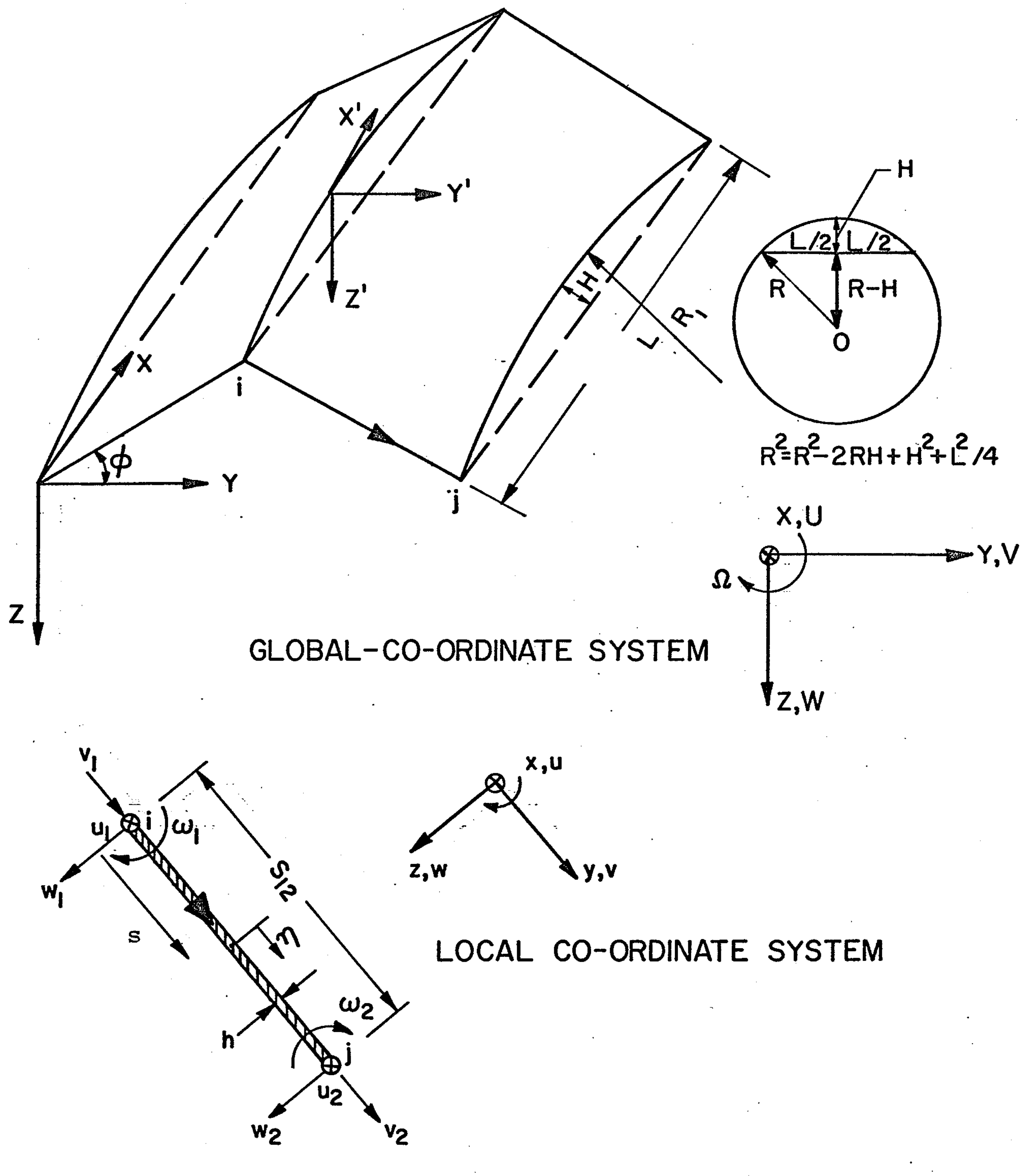
As shown in Figs. 1.2b and 1.3b, an arched folded plate structure (of translation) is obtained by translating the folds along a curve which is shallow for the case presented in this thesis. For the purpose of analysis, a global co-ordinate system, X, Y, Z for the entire structure and a local co-ordinate system, x, y, z for each element (or plate) of the structure are defined. These are shown in Fig. 2.1 where the global X axis and the local x axis are coincident and in the longitudinal direction. The equation of the shallow translational shell is given by

$$Z' = \frac{4H}{L^2} X'^2 + \tan\phi \, Y' \tag{2.1}$$

where H = maximum rise, L = span and $H/L \leq 1/5$, ϕ = inclination angle of plate. Also curvature in $Z'X'$ plane $= 1/R_1 \simeq \partial^2 Z'/\partial X'^2 = \partial^2 Z/\partial X^2 = 8H/L^2$ while curvature in $Z'Y'$ plane $= 1/R_2 \simeq \partial^2 Z'/\partial Y'^2 = \partial^2 Z/\partial Y^2 = 0$. Since there is only a curvature in ZX plane, the subscript is omitted and the curvature denoted by $\frac{1}{R}$.

Each element (plate) of the structure which in the present analysis will be finite strip, has two nodes or joints i and j; these joints extending in the full longitudinal direction.

The positive direction of the y (transverse) axis is from node i to node j, while the positive direction of the z axis is normal to the plate as shown. The transverse co-ordinate and the plate width are designated s and S_{12} respectively. For convenience a natural co-ordinate system η is defined with origin at the centre of the plate width and having



GEOMETRY OF STRUCTURE

Figure. 2.1

values -1 and +1 at nodes i and j respectively. The plate thickness is designated by h.

2.2 Displacements and Transformation Matrix

Associated with each co-ordinate direction in both the global and local systems we have the following displacement components at each node.

global system	local system
X - U	x - u
Y - V	y(s) - v
Z - W	z - w

In addition to the above, there is a rotation about the longitudinal axis, which is designated by Ω in the global system and ω in the local system. It is seen that $\omega = \frac{dw}{ds}$. During the analysis procedure it will become necessary to transform values from the local co-ordinate system to the global and vice-versa. Therefore, it becomes convenient to define a transformation matrix pertaining to the above displacements (or forces in the same directions). This matrix is given by

$$\{u_i\} = [A]\ \{U_i\}^{\dagger} \qquad (2.2)$$

for the transformation of the displacement components from the global to the local system as shown in Fig. 2.2.

† NOTATION: { } = column vector, < > = row vector, [ ] = rectangular matrix.

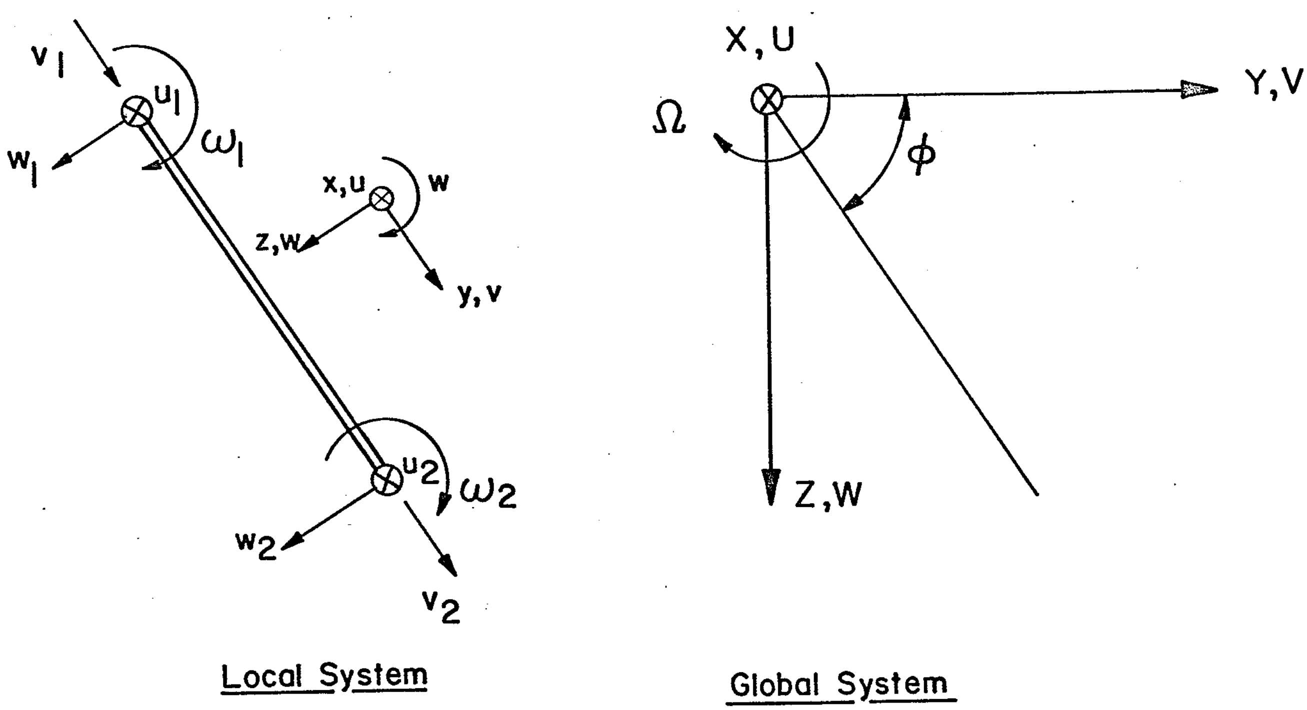


Figure 2.2 LOCAL AND GLOBAL SYSTEM

where $\{u_i\}^T = \langle \; u_1 \;\; u_2 \;\; v_1 \;\; v_2 \;\; w_1 \;\; w_2 \;\; \omega_1 \;\; \omega_2 \; \rangle$

$\{U_i\}^T = \langle \; V_1 \;\; W_1 \;\; \Omega_1 \;\; U_1 V_2 \;\; W_2 \;\; \Omega_2 \;\; U_2 \; \rangle$

and

$$[A] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cos\phi & \sin\phi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The matrix [A] is called the "Transformation Matrix". This transformation matrix will later be used to transform the element stiffness matrix and the load matrix from the local to the global system.

2.3 Interpolation Functions

In the analysis it becomes necessary to relate the internal displacement of a point in an element to the nodal displacements. Interpolation functions are used for this purpose. The functions used in this work assume a linear variation of the in-plane displacement components and a cubic variation of the normal displacement component between the nodes in the transverse direction. These functions which are derived in detail in Appendix A are given by

$$\langle\phi_u\rangle = \frac{1}{2} \langle(1-\eta) \quad (1+\eta)\rangle \qquad \text{for the longitudinal component}$$

$$\langle\phi_v\rangle = \frac{1}{2} \langle(1-\eta) \quad (1+\eta)\rangle \qquad \text{for the transverse component}$$

$$\langle\phi_w\rangle = \frac{1}{4} \langle(2-3\eta+\eta^3) \quad (2+3\eta-\eta^3) \quad \frac{S_{12}}{2}(1-\eta-\eta^2+\eta^3) \quad \frac{S_{12}}{2}(-1-\eta+\eta^2-\eta^3)\rangle$$

$$\text{for the normal component} \qquad (2.3)$$

2.4 Assumptions

The following assumptions are made in the theory:

1. The structure is made up of an assembly of strip elements and is simply supported by diaphragms at its two ends. The diaphragms are infinitely stiff in their own plane but perfectly flexible normal to their own plane.

2. The thickness of each strip element is constant and small compared with the other strip dimensions.

3. Straight lines which are perpendicular to the middle surface of the underformed element remain straight and perpendicular to the deformed middle surface.

4. The material is homogeneous and linearly elastic, with orthotropic properties which are constant throughout any element.

5. Deflections and deformations are small.

6. A second-degree symmetrical curve is assumed for the generatrix.

7. The shell is assumed shallow in the longitudinal direction.

The above-mentioned assumptions limit the application of the theory as follows:

(1) Due to the last assumption a restriction on the maximum rise-to-span ratio (H/L) is imposed. This ratio is approximately $\frac{1}{5}$.

(2) Even though the shell is not assumed to be shallow in the transverse direction, <u>the parametric curves are assumed to be orthogonal</u>. This assumption restricts the angle of inclination ϕ of the straight line generater. This angle was found to be approximately 40° for the results of the analysis to compare satsifactorly with those of other existing solutions.

Despite these two limitations, it is possible to apply the analysis to the majority of arched folded plate structures (of translation) found in practice.

2.5 The Finte Strip Method

In this method the structure which is simply supported at the two longitudinal ends by diaphragms is divided into a number of strips as shown in Fig. 2.3. It is not necessary for the strips to have equal widths.

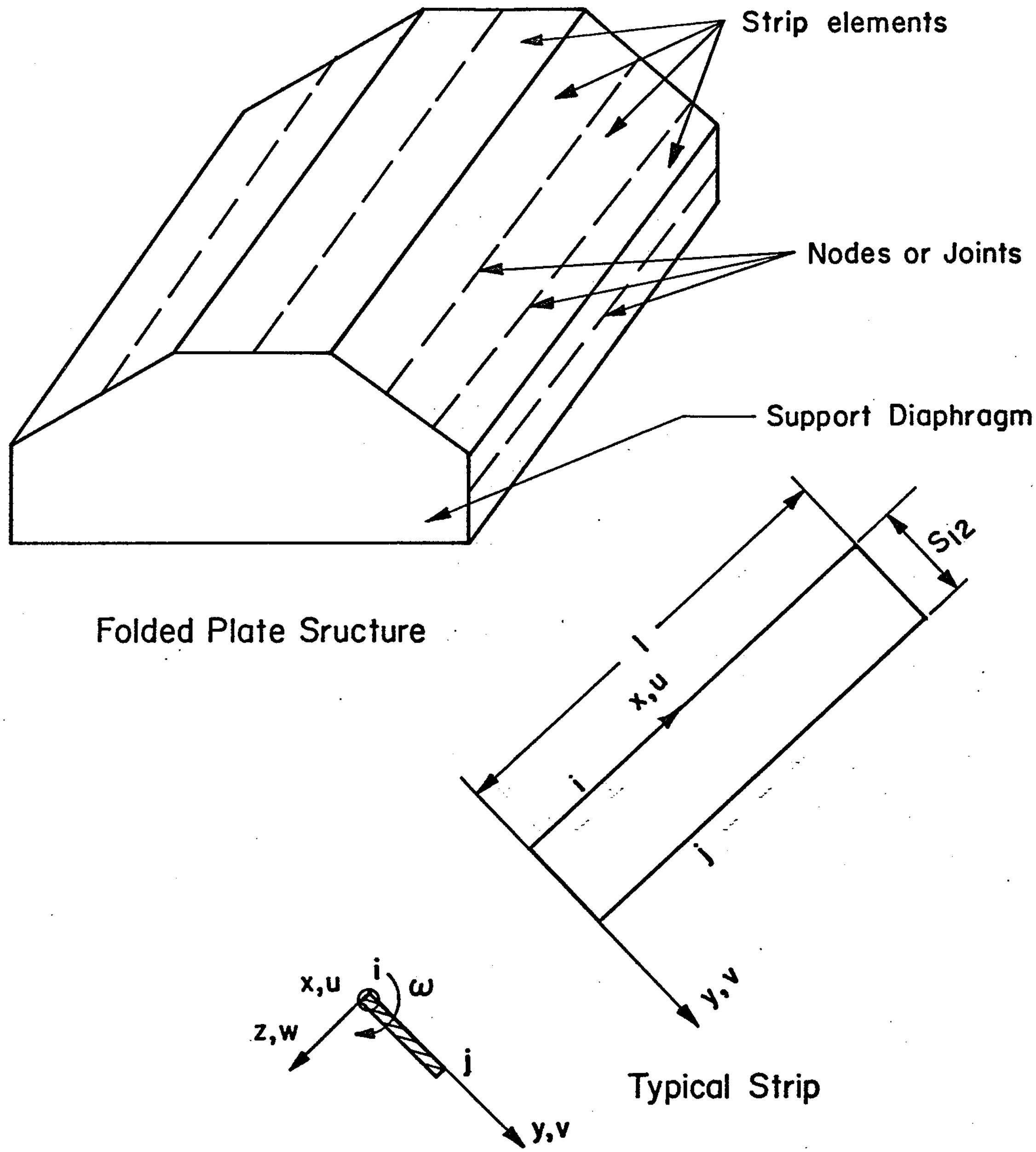


Figure 2.3 THE FINITE STRIP

The longitudinal sides of the strip are denoted by i and j and called joints corresponding to nodes in a bar type element (or in a finite element). The properties of the strip are regarded as constant in its own strip but can differ from strip to strip so as to approximate non-homogeneous cases. Thus the finite strip becomes the "element" in the folded plate structure and as described in section 2.1 will have four degrees of freedom at every joint or node. These are the three displacement components u, v, and w and the rotation ω as shown in Fig. 2.2.

The finite strip method resembles the finite element method in assuming displacement patterns. Trigonometric displacement patterns are assumed in the longitudinal direction, while displacement functions are used in the transverse direction to relate the displacements of interior points to the edge (nodal) displacements. These functions ϕ_u, ϕ_v, and ϕ_w were given in section 2.3. However, the number of elements is very much smaller in the finite strip method resulting in a much simpler computer programme. The method utilizes the direct stiffness technique.

The essence of the method is to compute the stiffness matrix of a typical strip element and then to assemble the overall structure stiffness matrix. The loading can be arbitary. All loads, displacements and forces are developed into Fourier Series. The whole process is very smooth and involves only a series of matrix operations. Due to the incorporation of the direct stiffness solution and the small number of elements, the finte strip method lends itself very suitably for programming.

CHAPTER III

STIFFNESS ANALYSIS FOR FINITE STRIP ELEMENTS

3.1 Stiffness Matrix for an Element

3.1.1 Displacement Functions

We assume a pattern for the transverse variation of the displacements by means of displacement functions, which relate the displacements of the internal points to the nodal values. In the longitudinal direction the displacements are formed into Fourier Series, making use of the simple support conditions at the ends. We, therefore, have for any point (η, x)

$$\begin{Bmatrix} u(\eta,x) \\ v(\eta,x) \\ w(\eta,x) \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} u_n(\eta)\ \cos\frac{n\pi x}{\ell} \\ v_n(\eta)\ \sin\frac{n\pi x}{\ell} \\ w_n(\eta)\ \sin\frac{n\pi x}{\ell} \end{Bmatrix} \tag{3.1}$$

$$= \sum_{n=1}^{\infty} \begin{bmatrix} <\phi_u(\eta)>\ \cos\frac{n\pi x}{\ell} & 0 & 0 \\ 0 & <\phi_v(\eta)>\ \sin\frac{n\pi x}{\ell} & 0 \\ 0 & 0 & <\phi_w(\eta)>\sin\frac{n\pi x}{\ell} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix}_n$$

where $<\phi_u(\eta)> = <\phi_v(\eta)> = \frac{1}{2} <\phi(1-\eta)\ \ (1+\eta)>$

and $<\phi_w(\eta)> = \frac{1}{4}\ <(2-3\eta+\eta^3)\ \ (2+3\eta-\eta^3)\ \ \frac{S_{12}}{2}\ (1-\eta-\eta^2+\eta^3)\ \ \frac{S_{12}}{2}\ (-1-\eta+\eta^2+\eta^3)>$

$$\{u_i\}_n = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}_n ; \quad \{v_i\}_n = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}_n ; \quad \{w_i\}_n = \begin{Bmatrix} w_1 \\ w_2 \\ \omega_1 \\ \omega_2 \end{Bmatrix}_n$$

are the nodal displacement components for a typical harmonic η.

$$\omega = \frac{\partial w}{\partial s} = \frac{\partial w}{\partial \eta} \cdot \frac{\partial \eta}{\partial s} = \frac{2}{S_{12}}\ \frac{\partial w}{\partial \eta}\ .$$

3.1.2 Strain Displacement Relationships

The equations given by Munroe [8] or Novozhilov [9] are used

$$
\left\{ \begin{matrix} \varepsilon_x \\ \varepsilon_s \\ \gamma_{xs} \\ K_x \\ K_s \\ K_{xs} \end{matrix} \right\} = \begin{bmatrix} \dfrac{\partial u}{\partial x} - \dfrac{w}{R} \\ \dfrac{\partial v}{\partial s} \\ \dfrac{\partial u}{\partial s} + \dfrac{\partial v}{\partial x} \\ - \dfrac{\partial^2 w}{\partial x^2} \\ - \dfrac{\partial^2 w}{\partial s^2} \\ - 2 \dfrac{\partial^2 w}{\partial x \partial s} \end{bmatrix}
$$

$$
= \sum_{n=1}^{\infty} \begin{bmatrix} - \dfrac{n\pi}{\ell} < \phi_u > \sin \dfrac{n\pi x}{\ell} & 0 & - \dfrac{1}{R} < \phi_w > \sin \dfrac{n\pi x}{\ell} \\ 0 & < \dfrac{\partial \phi v}{\partial s} > \sin \dfrac{n\pi x}{\ell} & 0 \\ < \dfrac{\partial \phi u}{\partial s} > \cos \dfrac{n\pi x}{\ell} & \dfrac{n\pi}{\ell} < \phi_v > \cos \dfrac{n\pi x}{\ell} & 0 \\ 0 & 0 & (\dfrac{n\pi}{\ell})^2 < \phi_w > \sin \dfrac{n\pi x}{\ell} \\ 0 & 0 & - < \dfrac{\partial^2 \phi w}{\partial s^2} > \sin \dfrac{n\pi x}{\ell} \\ 0 & 0 & - \dfrac{2n\pi}{\ell} < \dfrac{\partial \phi w}{\partial s} > \cos \dfrac{n\pi x}{\ell} \end{bmatrix} \left\{ \begin{matrix} u_i \\ v_i \\ w_i \end{matrix} \right\}_n
$$

or

$$
\{\varepsilon\} = [T] \, \{u_i\} = \sum^{\infty} [T]_n \, \{u_i\}_n \qquad (3.2)
$$

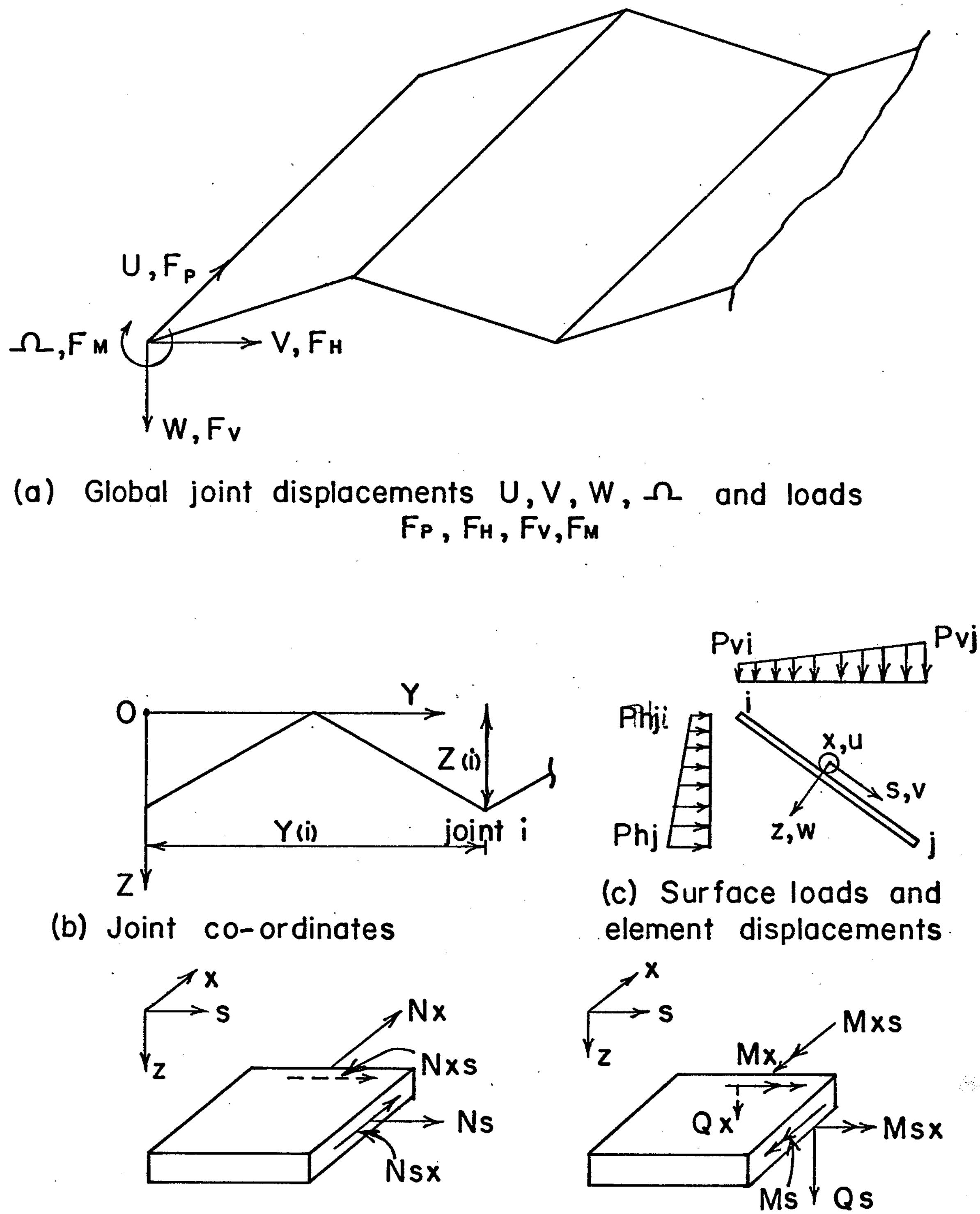
where

$$
\{\varepsilon\}^T = < \varepsilon_x \quad \varepsilon_s \quad \gamma_{xs} \quad K_x \quad K_s \quad K_{xs} >
$$

3.1.3 Stress (Force/Unit Length) - Strain Relationships

At each point in an element we have the in-plane forces N_x, N_s, N_{xs}; moments M_x, M_s, M_{xs}; and the normal shears, Q_x, Q_s as shown in Fig. 3.1(d). N_x and N_s are in the longitudinal (x) and transverse (local y) directions respectively, while N_{xs} is the in-plane shear force. M_x and M_s are moments about the transverse (local y) and longitudinal (x) axes respectively, while M_{xs} is the twisting moment. The shear forces Q_x and Q_s act normal to the plane of the element on planes perpendicular to the x and y (local) axes respectively. These shear forces being derivatives of the moments are not involved in the stiffness analysis. They are obtained later from the values of the moments. In classical theory the other internal forces (in-plane forces/unit length and moments per unit length) are related to the strains by

$$\begin{Bmatrix} N_x \\ N_s \\ N_{xs} \\ M_x \\ M_s \\ M_{xs} \end{Bmatrix} \doteq \begin{bmatrix} \dfrac{h_m E_{xm}}{1-\nu^m_{sx}\nu^m_{xs}} & \dfrac{\nu^m_{sx} h_m E_{sm}}{1-\nu^m_{sx}\nu^m_{sx}} & 0 & 0 & 0 & 0 \\ \dfrac{\nu^m_{xs} h_m E_{xm}}{1-\nu^m_{sx}\nu^m_{xs}} & \dfrac{h_m E_{sm}}{1-\nu^m_{sx}\nu^m_{xs}} & 0 & 0 & 0 & 0 \\ 0 & 0 & h_m G_m & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{h_b^3 E_{xb}}{12(1-\nu^b_{sx}\nu^b_{xs})} & \dfrac{\nu_{sx} h_b^3 E_{sb}}{12(1-\nu^b_{sx}\nu^b_{xs})} & 0 \\ 0 & 0 & 0 & \dfrac{\nu^b_{sx} h_b^3 E_{xb}}{12(1-\nu^b_{sx}\nu^b_{sx})} & \dfrac{h_b^3 E_{sb}}{12(1-\nu^b_{sx}\nu^b_{xs})} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{h_b^3 G_b}{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_s \\ \varepsilon_{xs} \\ K_x \\ K_s \\ K_{xs} \end{Bmatrix}$$



(a) Global joint displacements U, V, W, Ω and loads F_P, F_H, F_V, F_M

(b) Joint co-ordinates

(c) Surface loads and element displacements

(d) Internal forces and moments

Figure 3.1 <u>SIGN CONVENTION FOR STIFFNESS ANALYSIS</u>

or

$$\{\sigma\} = [D]\ \{\varepsilon\} = [D]\ [T]\ \{u_i\} = \sum_{n=1}^{\infty}\ [D]\ [T]_n\ \{u_i\}_n \tag{3.3}$$

where the subscript m denotes membrane or in-plane characteristics while subscript b stands for bending properties. E, G, and ν are elastic modulus, shear modulus, and Poisson's ratio respectively. The members of the 6 x 6 [D] matrix will later be referred to by D_{ij}. For an isotropic, homogeneous material.

$$E_{sm} = E_{sb} = E_{xm} = E_{xb} = E$$

$$\nu_{sx} = \nu_{xs} = \nu\ , \quad G_m = G_b = G = \frac{E}{2(1+\nu)}$$

3.1.4 Potential Energy Due to Straining

As a result of straining the potential energy $\overline{V}$ in an element is given by

$$\overline{V} = \frac{1}{2}\int_A \{\varepsilon\}^T\ \{\sigma\}dA \tag{3.4}$$

From equations (3.2) and (3.3)

$$\overline{V} = \frac{1}{2}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\{u_i\}_n^T\int_{s=o}^{S_{12}}\int_{x=o}^{\ell}[T]_n^T\ [D]\ [T]_m\ ds\ dx\ \{u_i\}_m. \tag{3.5}$$

Because of the orthogonality relationships the cross product term $[T]_n^T\ [T]_m$ will yield zero upon integration for $n \neq m$ and $\frac{\ell}{2}$ for $n = m$, since

$$\int_o^{\ell} \sin\frac{n\pi x}{\ell}\ \sin\frac{m\pi x}{\ell}\ dx = \int_o^{\ell}\cos\frac{n\pi x}{\ell}\ \cos\frac{m\pi x}{\ell}\ dx = \begin{cases} o & \text{when } n \neq m \\ \frac{\ell}{2} & \text{when } n = m \end{cases} \tag{3.6}$$

Using (3.6), equation (3.5) becomes

$$\overline{V} = \frac{1}{2} \sum_{n=1}^{\infty} \{u_i\}_n^T \frac{\ell}{2} \int_o^{S_{12}} [\bar{T}]_n^T [D] [\bar{T}]_n \, ds \cdot \{u_i\}_n \tag{3.7}$$

in which $[\bar{T}]_n = [T]_n$ with all $\sin \frac{n\pi x}{\ell}$ and $\cos \frac{n\pi x}{\ell}$ terms deleted.

Since potential energy $= \frac{1}{2} \{u_i\}^T [K] \{u_i\}$ the element stiffness for a typical harmonic n is given by

$$[k]_n = \frac{\ell}{2} \int_o^{S_{12}} [\bar{T}]_n^T [D] [\bar{T}]_n \, ds \tag{3.8}$$

Since $\frac{d\eta}{ds} = \frac{2}{S_{12}}$, equation (3.8) will become

$$[k]_n = \frac{\ell S_{12}}{4} \int_{-1}^{+1} [\bar{T}]_n^T [D] [\bar{T}]_n \, d\eta \tag{3.9}$$

therefore $[k]_n = \frac{\ell S_{12}}{4} \int_{-1}^{+1} [\bar{T}]_{n_{8x6}}^T [D]_{6x6} [\bar{T}]_{n_{6x8}} \, d\eta$ will be an 8x8 matrix and can be partitioned as

$$[k]_n = \frac{\ell S_{12}}{4} \begin{bmatrix} k_{uu} & k_{uv} & k_{uw} \\ & k_{vv} & k_{vw} \\ (\text{symm}) & & k_{ww} \end{bmatrix}_{8x8} \tag{3.10}$$

where k_{uu} is $[\ ]_{2x2}$, k_{uv} is $[\ ]_{2x2}$, k_{uw} is $[\ ]_{2x4}$, k_{vv} is $[\ ]_{2x2}$, k_{vw} is $[\ ]_{2x4}$ while k_{ww} is $[\ ]_{4x4}$.

$[k]_n$ is derived in detail in Appendix B, using a closed form solution for the integrals. Finally the stiffness matrix is transformed to the global system according to $[\bar{k}]_n = [A]^T [k]_n [A]$ where $[A]$ is the transformation matrix.

3.2 Load Analysis

It was mentioned earlier that the arched folded plate structure could be subjected to any type of loading, viz. uniformly distributed loads, line loads, point loads, etc. Point loads and line loads in the longitudinal direction are treated as joint loads by forming a node or joint at the points of application.

For distributed loads over the whole width of a plate element with an arbitary variation in the longitudinal direction the components in the plane of the element (x and y directions) and the components normal to the element (z direction) (as shown in Fig. 2.1) are considered. As in the case of displacements, we use Fourier Series representation in the longitudinal direction and the interpolation functions $< \phi_p > = < \phi_u >$ in the transverse direction to represent the loads. In all three directions we utilize the same interpolation function $< \phi_p >$, thereby assuming a linear variation of all components of the distributed loads between the nodes. i.e., $< \phi_p (\eta) > = < \phi_p > = \frac{1}{2} < (1-\eta) \; (1+\eta) >$

The value of the distributed load $p_i(\eta,x)$ at any point is given by

$$\{p_i(\eta,x)\} = [\phi_p] \{p_i(x)\} \tag{3.11}$$

where

$$[\phi_p] = \begin{bmatrix} < \phi_p > & 0 & 0 \\ 0 & < \phi_p > & 0 \\ 0 & 0 & < \phi_p > \end{bmatrix}$$

and

$$\{p_i(x)\} = \sum_{n=1}^{\infty} \begin{bmatrix} \cos\frac{n\pi x}{\ell} & 0 & 0 \\ 0 & \sin\frac{n\pi x}{\ell} & 0 \\ 0 & 0 & \sin\frac{n\pi x}{\ell} \end{bmatrix} \begin{Bmatrix} \{p_u\} \\ \{p_v\} \\ \{p_w\} \end{Bmatrix}_n$$

n denoting the particular harmonic. Thus $\{p_i(x)\}$ gives the intensities of the distributed load components at the nodes. The Fourier Series representation of the load vector $\{p_i(\eta,x)\}$ is applied to its nodal values only as $[\phi_p]$ is a function of η only. Therefore,

$$\{p_i(\eta,x)\} = \sum_{n=1}^{\infty} \begin{bmatrix} <\phi_p> \cos\frac{n\pi x}{\ell} & 0 & 0 \\ 0 & <\phi_p> \sin\frac{n\pi x}{\ell} & 0 \\ 0 & 0 & <\phi_p> \sin\frac{n\pi x}{\ell} \end{bmatrix} \begin{Bmatrix} \{p_u\} \\ \{p_v\} \\ \{p_w\} \end{Bmatrix}_n \tag{3.12}$$

In consistent load analysis, the above mentioned distributed loads are replaced by equivalent nodal loads using the principle of virtual work (variation principle). If the nodal loads are $\{R_i(x)\}$ where

$$\{R_i(x)\} = \sum_{n=1}^{\infty} \begin{bmatrix} \cos\frac{n\pi x}{\ell} & 0 & 0 \\ 0 & \sin\frac{n\pi x}{\ell} & 0 \\ 0 & 0 & \sin\frac{n\pi x}{\ell} \end{bmatrix} \begin{Bmatrix} \{R_u\} \\ \{R_v\} \\ \{R_w\} \end{Bmatrix} \tag{3.13}$$

we have by the principle of virtual work

$$\{u_i\}_n^T \{R_i\}_n = \frac{S_{12}}{2} \int_\eta \int_x \{u_i(\eta,x)\}^T \{p_i(\eta,x)\} \, d\eta \, dx \tag{3.14}$$

where

$$\{R_i\}_n = \begin{Bmatrix} \{R_u\} \\ \{R_v\} \\ \{R_w\} \end{Bmatrix}_{8x1}$$

The above equation (3.14) is solved for $\{R_i\}_n$ in Appendix C to give the load vector.

Since $\{R_i\}_n$ is in terms of the nodal values of the distributed loads $\{p_i\}_n$ in the u, v and w directions, the horizontal and vertical intensities ($P_h i$ and $P_v i$) of the load at a typical joint i as shown in Fig. 3.1(c) are resolved along and perpendicular to the plate element to give Pvl and Pwl respectively. (Fig. 3.2).

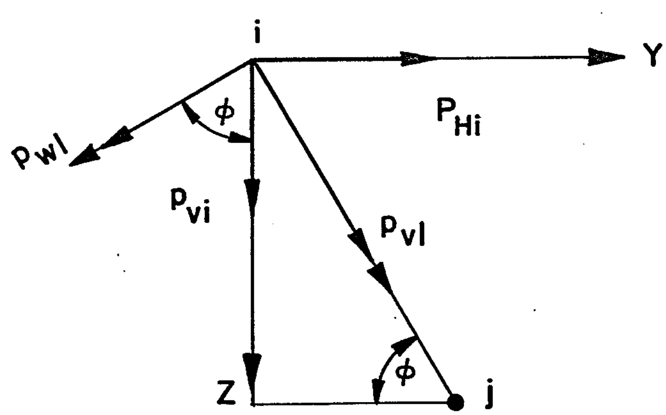


Figure 3.2 RESOLVING NODAL LOADS.

Finally, $\{R_i\}_n$ has to be transformed to the global co-ordinate system. This is accomplished by utilizing the transformation matrix [A]. The global components of the joint loads F_P, F_H, F_V and F_M are shown in Fig. 3.1(a).

3.3 Direct Stiffness Technique

The stiffness matrices and the load vectors developed for all the strip elements are assembled together after transformation to the global system to give the structure stiffness matrix [K] and the structure load vector {P}. These constitute the required set of equilibrium equations of the form $\{P\} = [K]\ \{\Delta\}$ where the joint displacements $\{\Delta\}$ are solved for as the unknowns.

At each node or joint there will be four global displacement components (U, V, Ω and W) giving eight displacement components for each element. These global joint displacement components are first transformed to the local system by utilizing the transformation matrix [A] to give eight local displacement components $\{u_i\}$. Thus, the end displacements $\{u_i\}$ of an element are given by $\{u_i\} = [A]\ \{U_i\}$ where $\{U_i\}$ is the global joint displacement vector.

3.4 Calculation of Internal Forces and Displacements

Once the end displacements of an element are known, the internal forces in the element are calculated using equation (3.3)

we had $\{\sigma\} = [D]\ \{\varepsilon\} = [D]\ [T]\ \{u_i\}$ (3.3)

where

$$\{\sigma\}^T = \langle N_x \ N_s \ N_{xs} \ M_x \ M_s \ M_{xs} \rangle$$

$$\{u_i\}^T = < u_i \ v_i \ w_i >$$

$$[D] = \begin{bmatrix} D_{11} & D_{12} & 0 & 0 & 0 & 0 \\ D_{12} & D_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & D_{45} & 0 \\ 0 & 0 & 0 & D_{45} & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \text{ is the constitutive matrix}$$

and

$$T = \sum_{n=1}^{\infty} [T]_n$$

where

$[T]_n$ is given in Section 3.1.2.

From the above the stress resultants can be written as

$$N_x = \sum_{n=1}^{\infty} - D_{11} \frac{n\pi}{\ell} < \phi_u > \sin \frac{n\pi x}{\ell} \{u_i\}_n + D_{12} < \frac{\partial \phi_v}{\partial s} > \sin \frac{n\pi x}{\ell} \{v_i\}_n -$$

$$- \frac{D_{11}}{R} < \phi_w > \sin \frac{n\pi x}{\ell} \{w_i\}_n$$

$$N_s = \sum_{n=1}^{\infty} - D_{12} \frac{n\pi}{\ell} < \phi_u > \sin \frac{n\pi x}{\ell} \{u_i\}_n + D_{22} < \frac{\partial \phi_v}{\partial s} > \sin \frac{n\pi x}{\ell} \{v_i\}_n -$$

$$- \frac{D_{12}}{R} < \phi_w > \sin \frac{n\pi x}{\pi} \{w_i\}_n$$

$$N_{xs} = \sum_{n=1}^{\infty} D_{33} < \frac{\partial \phi_u}{\partial s} > \cos \frac{n\pi x}{\ell} \{u_i\} + D_{33} \frac{n\pi}{\ell} < \phi_v > \cos \frac{n\pi x}{\ell} \{v_i\}_n$$

$$M_x = \sum_{n=1}^{\infty} D_{44} (\frac{n\pi}{\ell})^2 < \phi_w > \sin \frac{n\pi x}{\ell} \{w_i\}_n - D_{45} < \frac{\partial^2 \phi_w}{\partial s^2} > \sin \frac{n\pi x}{\ell} \{w_i\}_n$$

$$M_s = \sum_{n=1}^{\infty} D_{45} (\frac{n\pi}{\ell})^2 < \phi_w > \sin \frac{n\pi x}{\ell} \{w_i\}_n - D_{55} < \frac{\partial^2 \phi_w}{\partial s^2} > \sin \frac{n\pi x}{\ell} \{w_i\}_n$$

$$M_{xs} = \sum_{n=1}^{\infty} - D_{66} \frac{2n\pi}{\ell} < \frac{\partial \phi_w}{\partial s} > \cos \frac{n\pi x}{\ell} \{w_i\}_n \qquad (3.15)$$

From equation (3.1) the displacement components at any internal point can be written as

$$u = \sum_{n=1}^{\infty} \{\frac{1}{2} (1 - \eta) u_1 + \frac{1}{2} (1 + \eta) u_2\} \cos \frac{n\pi x}{\ell} \qquad (3.16)$$

$$v = \sum_{n=1}^{\infty} \{\frac{1}{2} (1 - \eta) v_1 + \frac{1}{2} (1 + \eta) v_2\} \sin \frac{n\pi x}{\ell} \qquad (3.17)$$

$$w = \sum_{n=1}^{\infty} \frac{1}{4} (2 - 3\eta + \eta^3) w_1 + \frac{1}{4} (2 + 3\eta - \eta^3) w_2 + \frac{S_{12}}{8} (1 - \eta - \eta^2 - \eta^3) \omega_1 +$$

$$+ \frac{S_{12}}{8} (-1 - \eta + \eta^2 + \eta^3) \omega_2\} \sin \frac{n\pi x}{\ell} \qquad (3.18)$$

where subscripts 1 and 2 denote values at the joints i and j of an element, respectively.

Also

$$\omega = \frac{2}{S_{12}} \frac{\partial w}{\partial \eta} = \sum_{n=1}^{\infty} \{\frac{3(\eta^2-1)}{2S_{12}} w_1 - \frac{3(\eta^2-1)}{2S_{12}} w_2 + \frac{1}{4} (3\eta^2-2\eta-1)\omega_1 +$$

$$+ \frac{1}{4} (3\eta^2+2\eta-1)\omega_2\} \sin \frac{n\pi x}{\ell} \tag{3.19}$$

using the notations

$$P_1 = \frac{1}{2} (1 - \eta)u_1 + \frac{1}{2} (1 + \eta)u_2$$

$$P_2 = \frac{1}{2} (1 - \eta)v_1 + \frac{1}{2} (1 - \eta)v_2$$

$$P_3 = (- \frac{u_1}{S_{12}} + \frac{u_2}{S_{12}})$$

$$P_4 = (- \frac{v_1}{S_{12}} + \frac{v_2}{S_{12}})$$

$$P_5 = \frac{1}{4} (2 - 3\eta + \eta^3)w_1 + \frac{1}{4} (2 + 3\eta - \eta^3)w_2 + \frac{S_{12}}{8} (1-\eta-\eta^2+\eta^3)\omega_1 +$$

$$+ \frac{S_{12}}{8} (-1-\eta + \eta^2 + \eta^3)\omega_2$$

we have

$$u = \sum_{n=1}^{\infty} P_1 \cos \frac{n\pi x}{\ell}$$

$$v = \sum_{n=1}^{\infty} P_2 \sin \frac{n\pi x}{\ell}$$

$$w = \sum_{n=1}^{\infty} P_5 \sin \frac{n\pi x}{\ell} \tag{3.20}$$

From equation (3.15) we get

$$N_x = \sum_{n=1}^{\infty} - D_{11} \frac{n\pi}{\ell} P_1 \sin \frac{n\pi x}{\ell} - D_{11} P_5 \sin \frac{n\pi x}{\ell} + D_{12} P_4 \sin \frac{n\pi x}{\ell} \tag{3.21}$$

as

$$< \frac{\partial \phi_v}{\partial s} > \{v_i\} = \frac{2}{S_{12}} \frac{\partial}{\partial \eta} \frac{1}{2} < (1 - \eta) \ (1 + \eta) > \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

$$= (- \frac{v_1}{S_{12}} + \frac{v_2}{S_{12}}) = P_4$$

$$N_s = \sum_{n=1}^{\infty} - D_{12} \frac{n\pi}{\ell} P_1 \sin \frac{n\pi x}{\ell} - \frac{D_{12}}{R} P_5 \sin \frac{n\pi x}{\ell} + D_{22} P_4 \sin \frac{n\pi x}{\ell} \tag{3.22}$$

$$N_{xs} = \sum_{n=1}^{\infty} D_{33} P_3 \cos \frac{n\pi x}{\ell} + D_{33} \frac{n\pi}{\ell} P_2 \cos \frac{n\pi x}{\ell} \tag{3.23}$$

From equation (3.19)

$$\frac{\partial \phi_w}{\partial s} = \frac{2}{S_{12}} \frac{\partial \phi_w}{\partial \eta} = < \frac{1.5}{S_{12}} (\eta^2-1) \quad - \frac{1.5}{S_{12}} (\eta^2-1) \quad \frac{1}{4} (3\eta^2-2\eta-1) \quad \frac{1}{4}(3\eta^2+2\eta-1) >$$

therefore

$$< \frac{\partial^2 \phi_w}{\partial s^2} > = \frac{4}{S_{12}^{\ 2}} < \frac{6\eta}{4} \quad - \frac{6\eta}{4} \quad \frac{(3\eta-1)}{S_{12}} \quad \frac{(3\eta+1)}{S_{12}} >$$

If

$$P_6 = \frac{1.5}{S_{12}} (\eta^2-1) w_1 - \frac{1.5}{S_{12}} (\eta^2-1) w_2 + \frac{1}{4} (3\eta^2-2\eta-1)\omega_1 + \frac{1}{4} (3\eta^2+2\eta-1)\omega_2$$

and

$$P_7 = \frac{6\eta}{S_{12}^{\ 2}} w_1 - \frac{6\eta}{S_{12}^{\ 2}} w_2 + \frac{(3\eta-1)}{S_{12}} \omega_1 + \frac{(3\eta-1)}{S_{12}} \omega_2$$

From equation (3.15) we have

$$M_x = \sum_{n=1}^{\infty} D_{44} \left(\frac{n\pi}{\ell}\right)^2 P_5 \sin\frac{n\pi x}{\ell} - D_{45} P_7 \sin\frac{n\pi x}{\ell} \tag{3.24}$$

$$M_s = \sum_{n=1}^{\infty} D_{45} \left(\frac{n\pi}{\ell}\right)^2 P_5 \sin\frac{n\pi x}{\ell} - D_{55} P_7 \sin\frac{n\pi x}{\ell} \tag{3.25}$$

$$M_{xs} = \sum_{n=1}^{\infty} - D_{66} \frac{2n\pi}{\ell} P_6 \cos\frac{n\pi x}{\ell} \tag{3.26}$$

Shear Forces

The shear forces per unit length given by Munroe [8] or Novozhilov [9] are

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xs}}{\partial s}$$

and

$$Q_s = \frac{\partial M_{xs}}{\partial x} + \frac{\partial M_s}{\partial s}$$

From equations (3.24) and (3.26)

$$\frac{\partial M_x}{\partial x} = \sum_{n=1}^{\infty} D_{44} \left(\frac{n\pi}{\ell}\right)^3 P_5 \cos\frac{n\pi x}{\ell} - D_{45} \left(\frac{n\pi}{\ell}\right) P_7 \cos\frac{n\pi x}{\ell}$$

$$\frac{\partial M_{xs}}{\partial s} = \sum_{n=1}^{\infty} - D_{66} \frac{2n\pi}{\ell} \frac{\partial P_6}{\partial s} \cos\frac{n\pi x}{\ell} = \sum_{n=1}^{\infty} - D_{66} \frac{2n\pi}{\ell} P_7 \cos\frac{n\pi x}{\ell}$$

therefore

$$Q_x = \sum_{n=1}^{\infty} \left\{\left(\frac{n\pi}{\ell}\right)^3 P_5 D_{44} - \left(\frac{n\pi}{\ell}\right) P_7 D_{45} - 2\left(\frac{n\pi}{\ell}\right) P_7 D_{66}\right\} \cos\frac{n\pi x}{\ell} \tag{3.27}$$

From equations (3.25) and (3.26)

$$\frac{\partial M_s}{\partial s} = \sum_{n=1}^{\infty} D_{45} \left(\frac{n\pi}{\ell}\right)^2 P_6 \sin\frac{n\pi x}{\ell} - D_{55} \frac{\partial P_7}{\partial s} \sin\frac{n\pi x}{\ell}$$

$$\frac{\partial M_{xs}}{\partial x} = \sum_{n=1}^{\infty} D_{66} \left(\frac{n\pi}{\ell}\right)^2 P_6 \sin \frac{n\pi x}{\ell}$$

$$\frac{\partial P_7}{\partial s} = \frac{12w_1}{S_{12}^3} - \frac{12w_2}{S_{12}^3} + \frac{6\omega_1}{S_{12}^2} + \frac{6\omega_2}{S_{12}^2}$$

If

$$P_8 = \frac{\partial P_7}{\partial s} = \frac{12w_1}{S_{12}^3} - \frac{12w_2}{S_{12}^3} + \frac{6\omega_1}{S_{12}^2} + \frac{6\omega_2}{S_{12}^2}$$

we have

$$Q_s = \sum_{n=1}^{\infty} \left\{ D_{66}\, 2\left(\frac{n\pi}{\ell}\right)^2 P_6 + D_{45} \left(\frac{n\pi}{\ell}\right)^2 P_6 - D_{55} P_8 \right\} \sin \frac{n\pi x}{\ell} \qquad (3.28)$$

3.5 Outline of Analysis Procedure

Due to the simple supports at the ends of the structure it was possible to have a Fourier Series representation in the longitudinal (x) direction for the loads and displacements and to perform a direct stiffness harmonic anlaysis. In this manner the problem was reduced to a truly two-dimensional one (at the transverse cross-section).

The solution technique is established for one particular harmonic n, and the super-position principle is used. In this way the method lends itself well for programming.

The entire longitudinal joint may be treated as a single nodal point, since the analysis for each harmonic load will produce displacements of the same variation. For example, a displacement pattern $\gamma(x) = \gamma_o \sin \frac{n\pi x}{\ell}$ gives the displacements at every point in the longitudinal direction for different values of x. Thus, the pattern could well be described merely by the parameters γ_o which is the amplitude of that particular displacement. In this way we could focus our attention on the nodes at one cross-section, instead of dealing with the full lengths of all the longitudinal joints.

If the conditions of equilibrium and compatibility are satisfied at a nodal point in a transverse section, they will also be satisfied along the entire longitudinal joint.

The steps in the analysis procedure in this thesis using the finite strip method and the direct stiffness technique may be summarized as below.

a. All surface or line loads distributed across the width of a strip element are replaced by a set of equivalent nodal loads. This is equivalent to finding fixed end forces. These nodal components are then transformed to the global system X, Y, Z as shown in Fig. 2.1.

b. The joint or nodal loads thus formed are resolved into Fourier Series in the longitudinal direction and the load vector $\{P\}_n$ is formed from all components for a typical term of the series. The dimension m of this vector will equal four times the number of joints (nodes) in the structure (section 3.2).

c. The 8 x 8 stiffness matrix $[k]_n$ is calculated for each strip for a typical term of the Fourier Series (section 3.1).

d. The stiffness matrix of each element is next transformed to the global co-ordinate system so that the structure stiffness matrix $[K]_n$ may be assembled, using the direct stiffness technique. This m x m matrix (where m = 4 x number of joints) together with the load vector constitute the set of equilibrium equations for a typical term of the Fourier series expansion.

e. The equations in (d) are solved for the unknown joint displacements $\{\Delta\}_n$.

f. The joint displacements are transformed back to the relevant element co-ordinate systems to determine the edge displacements of the strips for this particular harmonic.

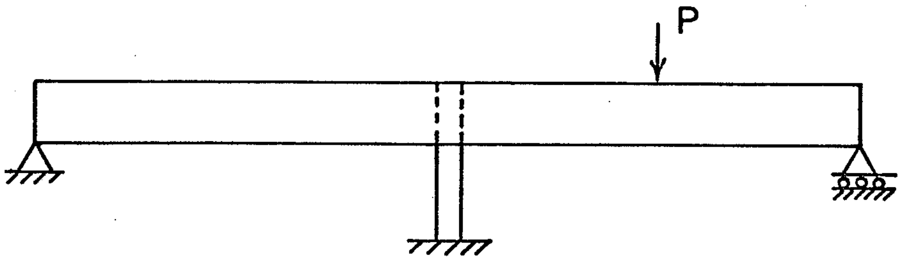
g. The strip (element) internal forces are calculated for the same harmonic.

h. All of the above are repeated for each harmonic of the Fourier Series and the contributions of each term are added up to obtain the final displacements and internal stress resultants throughout the structure.

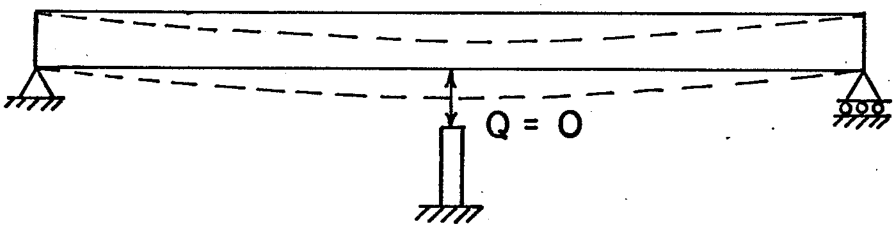
3.6 Provision for Intermediate Supports

The analysis procedure is extended to analyse structures that have intermediate supports. The supports could be in the form of plane frames and/or rigid diaphragms which are externally supported. A force method of analysis is used in which the interaction forces between the folded plates and the intermediate supports are treated as redundants. The interaction forces are assumed to act only at the common joints of connection as shown in Fig. 3.3. At each such joint there are three components of the interaction forces; viz. horizontal, vertical and rotational components. Longitudinal restraint between the folded plates and the intermediate support is neglected. The method is described in the following steps.

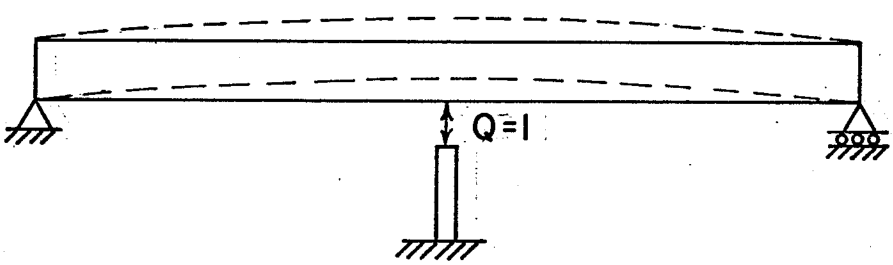
a. First the folded plate structure is analysed for the given external loading, with the redundants set to zero. This is the primary structure (Fig. 3.3(b).



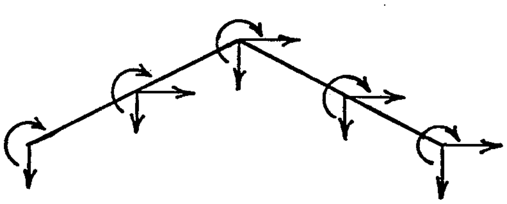
(a) Structure with intermediate support (elevation)



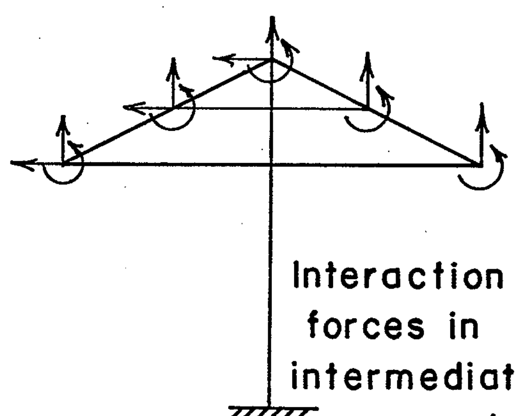
(b) Primary structure with redundants Q = O



(c) Structure under unit redundant force



Interaction forces in folded plate



Interaction forces in intermediate support

Figure 3.3 PROVISION OF INTERMEDIATE SUPPORTS

b. The displacements $\{\delta\}_o$ at the points where the redundants are to act are determined.

$$\{\delta\}_o^T = < \delta_1 \;\; \delta_2 \;\; \ldots\ldots.. \;\; \delta_c >_o \tag{3.29}$$

= displacements of joints connected to the intermediate support

c. The folded plate structures is next anlaysed for unit values of each of the redundant forces (Q) and the corresponding displacements of the joints $\{\delta\}_1$ determined (Fig. 3.3(c)) i.e.,

$$\{\delta\}_1 = [F]_1 \{Q\} \tag{3.30}$$

where

$$\{\delta\}_1^T = < \delta_1 \;\; \delta_2 \;\; \ldots.. \;\; \delta_c >_1$$

and

$[F]_1$ = flexibility matrix of the folded plate structure.

d. If the intermediate support is a plane frame the total structure stiffness matrix of the frame is formed using a plane frame analysis (programme) and then a static condensation is carried out to eliminate the degrees of freedom not corresponding to redundant forces. Next the stiffness matrix is inverted to give the displacements $\{\delta\}_2$ at the required points of the frame, i.e.,

$$\{\delta\}_2 = [F]_2 \{Q\} \tag{3.31}$$

where

$[F]_2$ = flexibility matrix of the frame.

e. Finally the compatibility condition at each point of redundancy requires that

$$\{\delta\}_o + [F]_1 \{Q\} + [F]_2 \{Q\} = 0 \tag{3.32}$$

or

$$\{\delta\}_o + [F] \{Q\} = 0$$

where

$$[F] = [F]_1 + [F]_2$$

$$\{Q\} = -[F]^{-1} \{\delta\}_o \tag{3.33}$$

giving the redundant interaction forces. It is to be noted that $[F]_2 = 0$ for a rigid diaphragm.

f. When the interaction forces between the folded plate structure and the intermediate support are known, the simply-supported folded-plate structure could be analysed subjected to the total load of external loads plus interaction forces to give the true internal forces and displacements. If the intermediate support is a plane frame, it is analysed for the interaction forces $\{Q\}$ plus any external load to which it is subjected to. If the intermediate support is a rigid diaphragm $[F]_2 = 0$ in (d) above and that step is by-passed.

If there are more than one intermediate support (frames or diaphragms), the analysis procedure is essentially the same with the redundants at each intermediate support.

It must be noted that conventional folded plate structures can be analysed by the theory presented in this work, by letting $\frac{1}{R} = 0$. For this case it is also possible to analyse structures having intermediate flexible movable diaphragms. The method of analysis with these type of diaphragms is presented in Appendix D. Flexible movable diaphragms are used mainly with box girder bridges.

3.7 Simulation of Fixed-end Conditions

In the theory presented here arched (or conventional) folded plate structures having rigid diaphragms as intermediate supports could be analysed. However, the structure has to be simply supported at the two ends. An attempt is made to simulate fixed-end support conditions by having rigid diaphragms very close to the end supports. This technique is illustrated in Chapter IV under "Applications of ARCHFOLD".

CHAPTER IV

COMPUTER PROGRAMME "ARCHFOLD"

4.1 Programme Description

On the basis of the theory presented earlier, a computer programme titled "Archfold" has been written in Fortran IV language for the IBM 360 computer at the University of Manitoba. It consists of one main programme and several subroutines that are called in. "Archfold" provides a rapid solution to arched folded plate structures simply supported at the ends and subjected to any arbitary type of loading. The structure can have intermediate supports in the form of rigid diaphragms or flexible plane frames. Straight conventional folded plate structures can also be analysed by inputting the parameter $\frac{1}{R}$ (= X R in programme) = 0.

Uniform or partial surface loads may be applied anywhere in the folded plate structure.

The restrictions on the number of joints, number of intermediate supports, number of elements, terms of Fourier Series etc., are given in Appendix E under the subtitle "Form of Input".

The computer solution based on the finite analysis utilizes the direct stiffness technique. Compatibility at the interior supports is accomplished by a force (flexibility) method of analysis. A harmonic analysis with up to 100 non-zero terms of the appropriate Fourier Series is used for the loads. A special moment integration option permits the evaluation of moments and the percentage of the total moment of a cross-section taken by each member. This can be used only for the case $\frac{1}{R} = 0$, when the angle of inclination ϕ of the elements is not restricted. This moment integration is specially applicable in analysing box girder bridges.

As mentioned earlier a plane frame programme (P FRAME) is incorporated to analyse any intermediate support that may be in the form of a planar frame.

4.2 Input and Output

The sign convention used in the programme is the same as that in the analysis. For inputting the co-ordinate of the intermediate plane frame supports any arbitary origin and any rectangular co-ordinate system may be chosen. A detailed description of the input which has been designed to require a minimum of effect in preparation is given under "Form of Input" in Appendix E.

A brief description of the input requirements is given below.

1. Span of structure.
2. Curvature of the arch in terms of $XR = \frac{1}{R} = \frac{8H}{L^2}$.
3. Typical transverse section properties in terms of number of plates, number of joints, number of intermediate supports and co-ordinates of all joints with respect to the chosen origin.
4. Material properties of the elements as required for the constitutive matrix [D].
5. Desciption of the loading which may consist of surface loads, varying linearly across the width of an element and constant over a specified portion in the longitudinal direction. The structure may also be subjected to joint loads extending either uniformly over the whole length of a joint or over a particular portion of it.
6. Specification of transverse section at which output is desired.
7. Maximum number of terms to be used in the Fourier Series analysis.

The output consists of the following information.

(1) The complete input data, properly labelled for checking purposes.

(2) The interaction forces between the folded plates and the intermediate supports if any.

(3) The displacements of all joints in the global system.

(4) The internal forces and displacements for all elements for each longitudinal section along a plate width and at the transverse sections specified.

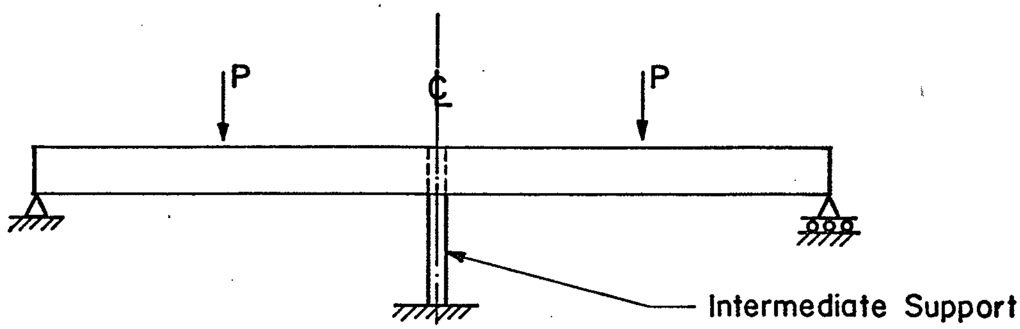
(5) For XR = 0, when analysing structures with girders, the moments taken by each girder at the specified cross-sections.

(6) For plane frame intermediate supports the complete analysis in the form of joint displacements, member end forces, etc.

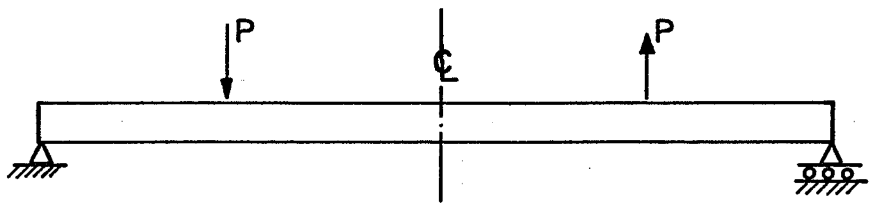
4.3 Special Considerations

If there is longitudinal symmetry of the structure about a transverse plane, a saving in the computing effort may be obtained by making use of the symmetry or anti-symmetry of the loading with respect to the transverse section of symmetry. For a symmetrical structure subjected to a symmetrical loading, only the odd terms of the Fourier Series have to be used (Fig. 4.1(a)). For anti-symmetric loading only the even terms are required. This could be achieved in the programme by specifying it in the "control card".

Advantage of symmetrical loading can be taken only in the cases of one centre support or no intermediate support, as the loading (which has to be symmetric) includes the external loads and the interaction forces. Advantage of anti-symmetry can be taken only in cases without any intermediate supports (Fig. 4.1(b)).

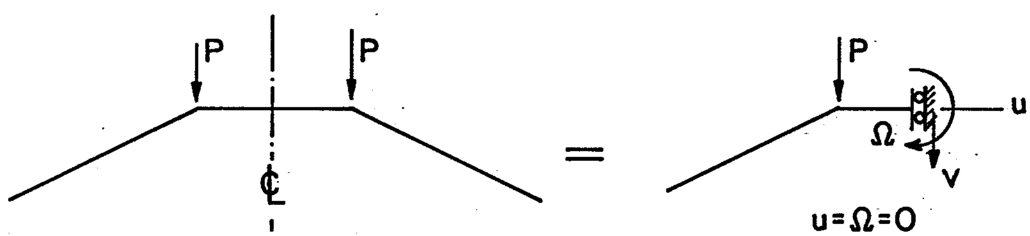


(a) Symmetrical Loading

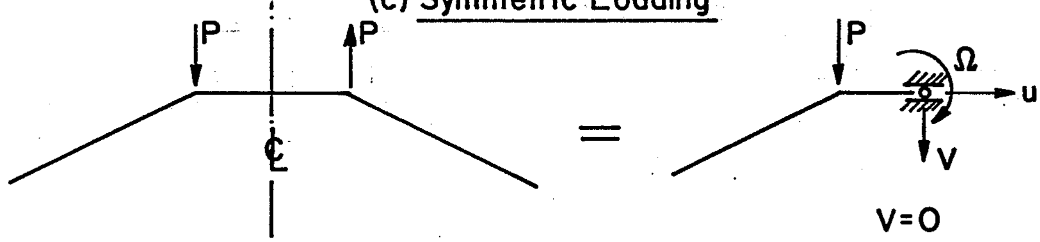


(b) Anti-Symmetrical Loading

LONGITUDINAL SYMMETRY AND ANTI-SYMMETRY



(c) Symmetric Loading



(d) Anti-Symmetric Loading

TRANSVERSE SYMMETRY AND ANTI-SYMMETRY

Figure 4.1 LONGITUDINAL AND TRANSVERSE SYMMETRY

If there is symmetry in the transverse section (i.e., about a longitudinal plane) of the structure, advantage of symmetry or anti-symmetry may be taken by analysing only one half of the cross-section and imposing proper boundary conditions at the longitudinal plane of symmetry (Figs. 4.1(c) and Fig. 4.1(d)).

When there are intermediate supports, it is necessary to specifiy a relatively large number for the maximum Fourier Series limit. This is because the analysis will involve expanding the interaction forces acting over narrow widths into Fourier Series and the convergence of the output quantities such as moments in the vicinity of the force will be very slow. A satisfactory output could be obtained by studying the convergence for an increasing number of harmonics. This can be accomplished in one run of the programme as it has the option to print out the results after a required number of harmonics. Generally 80 terms or 40 non-zero terms in symmetrical cases are recommended.

The connection between the folded plate structure and the intermediate support is only at discrete points. This may be a limitation in some cases where there is a continuous connection in reality. In such cases some averaging process has to be used to obtain meaningful values for the internal forces and moments if a plane frame is used as the intermediate support.

CHAPTER V

APPLICATIONS OF "ARCHFOLD"

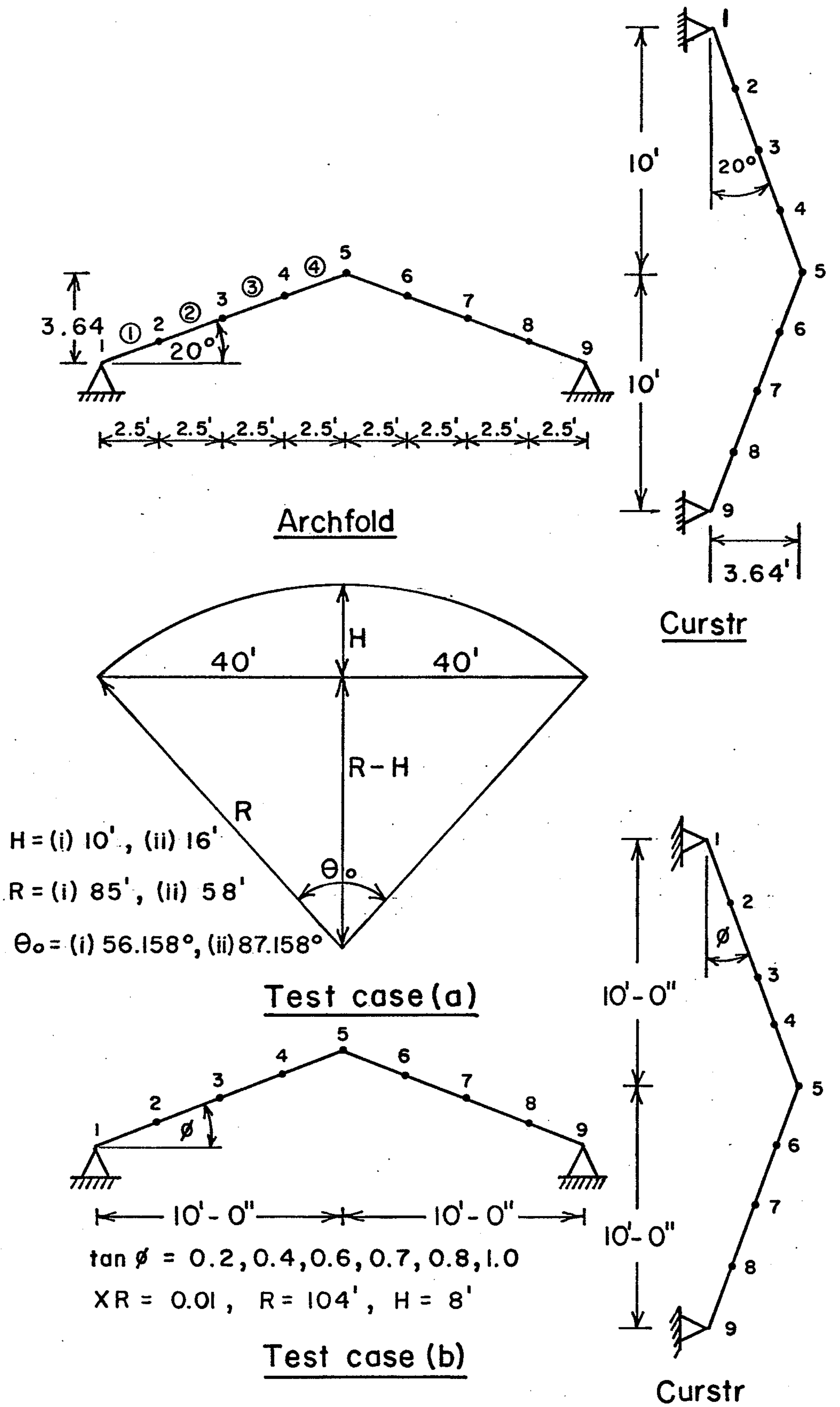
5.1 Introduction

The computer programme "Archfold" was specially written for the analysis of arched folded plate structures of translation utilizing the finite strip and direct stiffness techniques. But as stated earlier, it could very well be used for the analysis of conventional (straight) folded plate structures by inputting XR = 0. The structure could also be subjected to any type of loading. Thus, a great variety of problems, hitherto difficult and impossible to solve, could easily be solved. The various parameters that influence the results such as span, curvature of arch, thickness of plate, inclination angle of element, strip width etc., could be studied. The results of "Archfold" are compared with analysis by classical theory [7], "CURSTR" [6] (which is a programme for analysing folded plate structures of rotation) and with "MUPDI 3" [4] when XR = 0. Presented below are some of the studies mentioned.

5.2 Test Cases

5.2.1 Test Case (a): - Comparison With Classical Solution and "CURSTR"

The geometry of the cross-section is shown in Fig. 5.1a. This simple two-fold structure simply supported at the extreme longitudinal edges was chosen to test the validity of the programme. For increased accuracy the structure was divided into eight strips. The other details and dimensions were so chosen to match the classical analysis of Shah and Lansdown [7]. Thus, we have the following details.



NOTE: All plots are for the left hand plate (ie 1st 4 elements

Figure 5.1

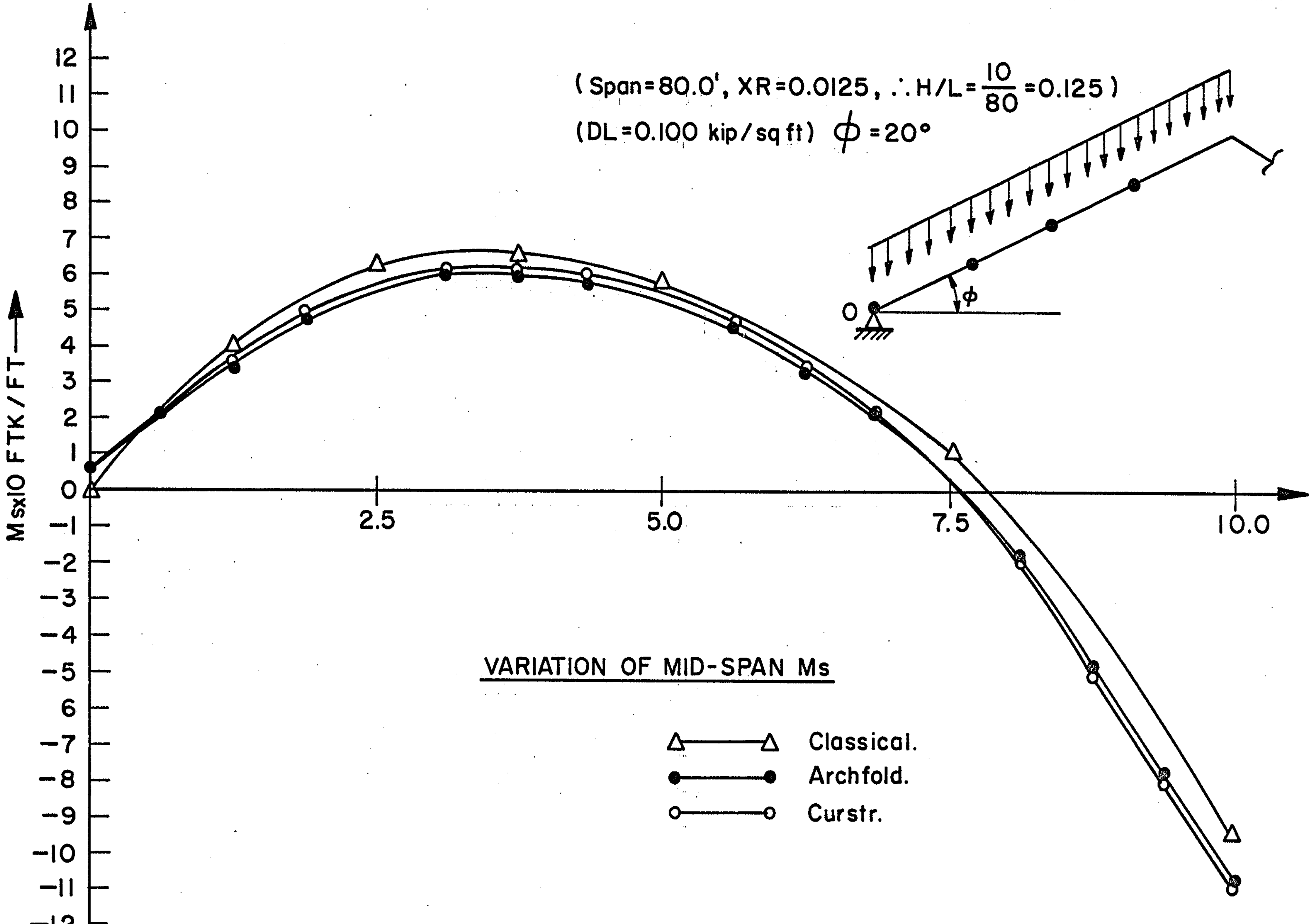


Figure 5.2 COMPARISON WITH CLASSICAL SOLUTION AND "CURSTR"

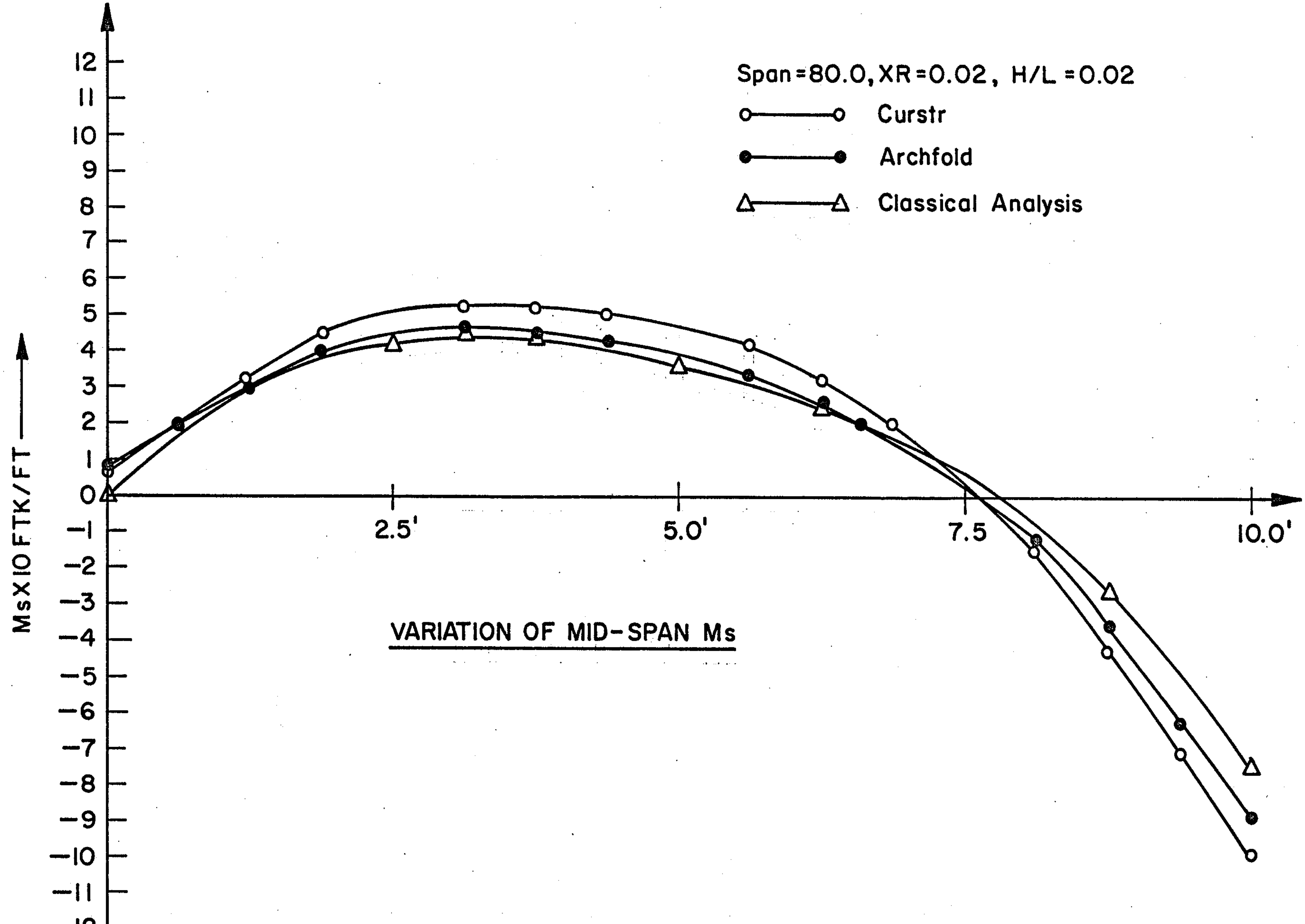


Figure 5.3 COMPARISON WITH CLASSICAL SOLUTION AND "CURSTR"

span of structure = 80.0 feet

curvature $(XR = \frac{1}{R}) =$ (i) 0.0125 (ii) 0.02

these correspond to $\frac{H}{L} =$ (i) $\frac{1}{8}$ (ii) $\frac{1}{5}$

inclination angle $\phi = 20°$

ratio of plate width to span = 0.133

load on structure = 100 lbs/ft^2 (inclined area)

thickness of plate = 4 inches = 0.333 feet

To utilize "CURSTR" the structure has to be rotated through 90° about a horizontal axis to obtain the curvature in plan. The radius of curvature in the horizontal plane and the angle θ_o are obtained using the values of H as shown in Fig. 5.1(a).

Since the longitudinal bending moments were quite small a comparison was made of the mid-span transverse bending moments M_s. These are shown in Figs. 5.2 and 5.3 where the maximum deviation is only about 10%.

5.2.2 Test Case (b): - Effects of Variation of Inclination Angle ϕ of the Plates

It was mentioned earlier that a limitation on the inclination angle would arise. The simple structure as in (a) is chosen to study the effect of ϕ on the validity of the results. (Fig. 5.1(b)). Comparison is made with "CURSTR". Tests were carried out for $\tan \phi = 0.2, 0.4, 0.6, 0.7, 0.8$ and 1.0. In all cases the span was 80.0 feet and the curvature 0.01 giving H = 8' (R = 104' for "CURSTR"). The load was again taken to be 100 lbs/sq ft (inclined area), while the thickness of plate was 4 inches.

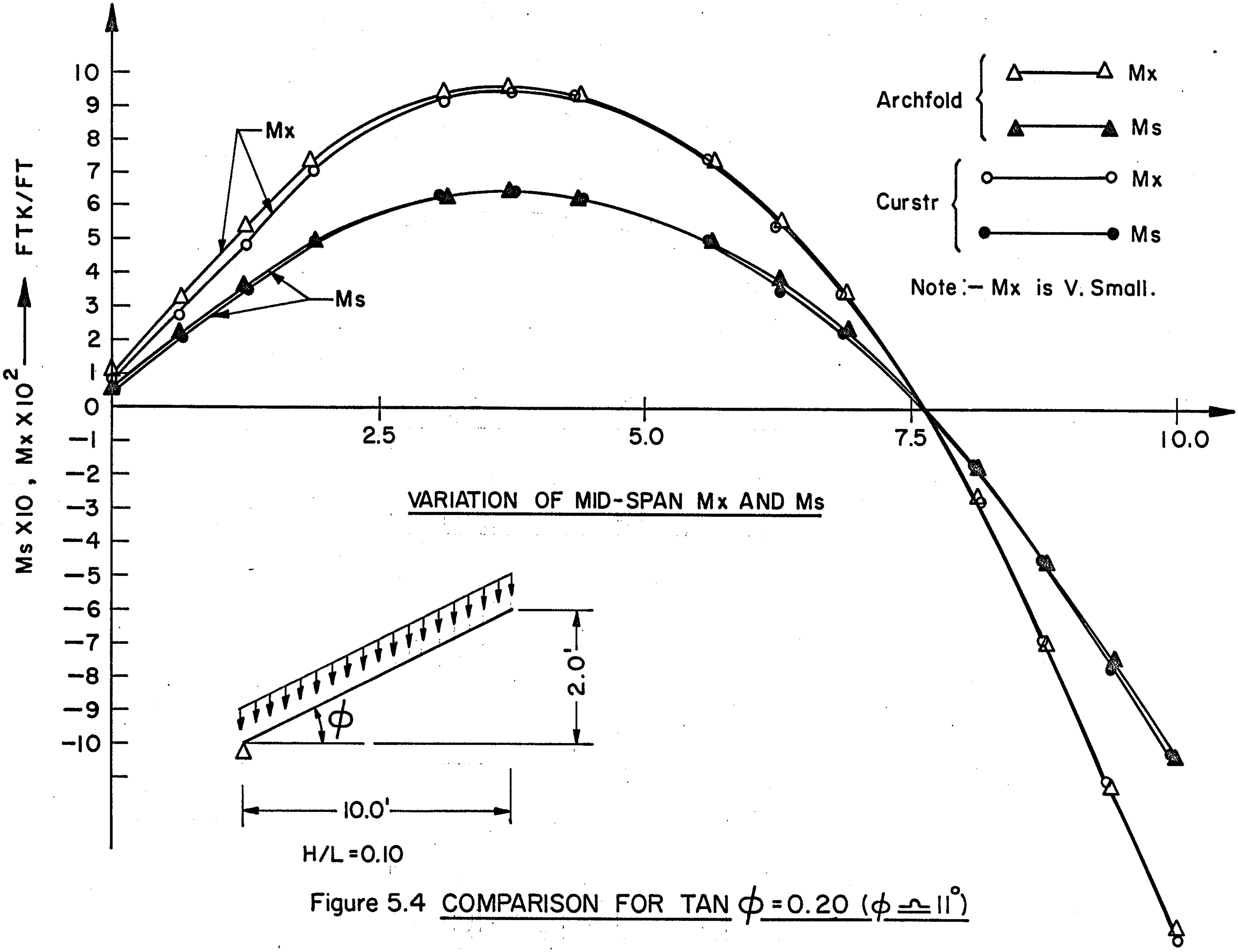


Figure 5.4 COMPARISON FOR TAN $\phi = 0.20$ ($\phi \simeq 11°$)

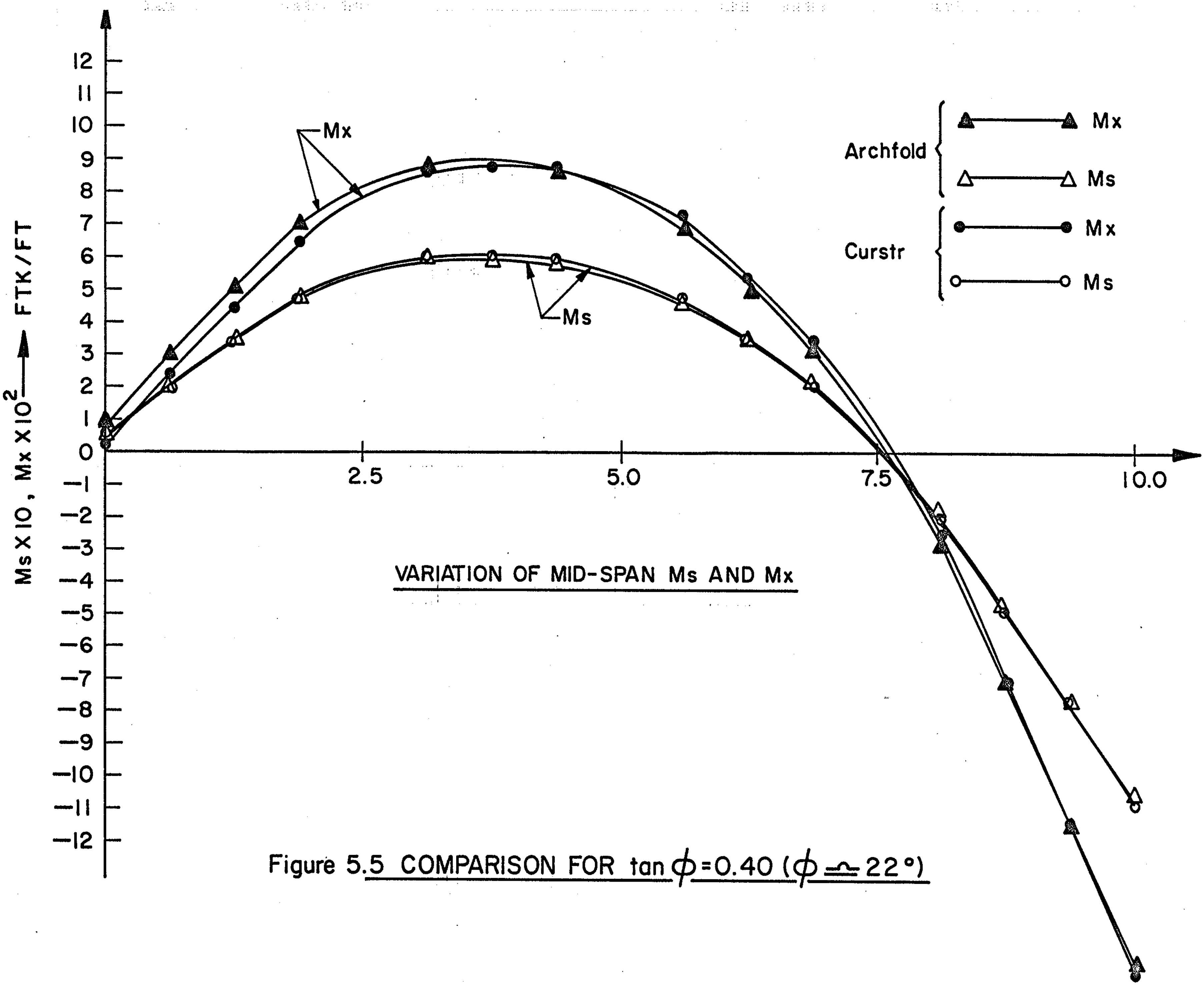


Figure 5.5 COMPARISON FOR $\tan\phi = 0.40$ ($\phi \simeq 22°$)

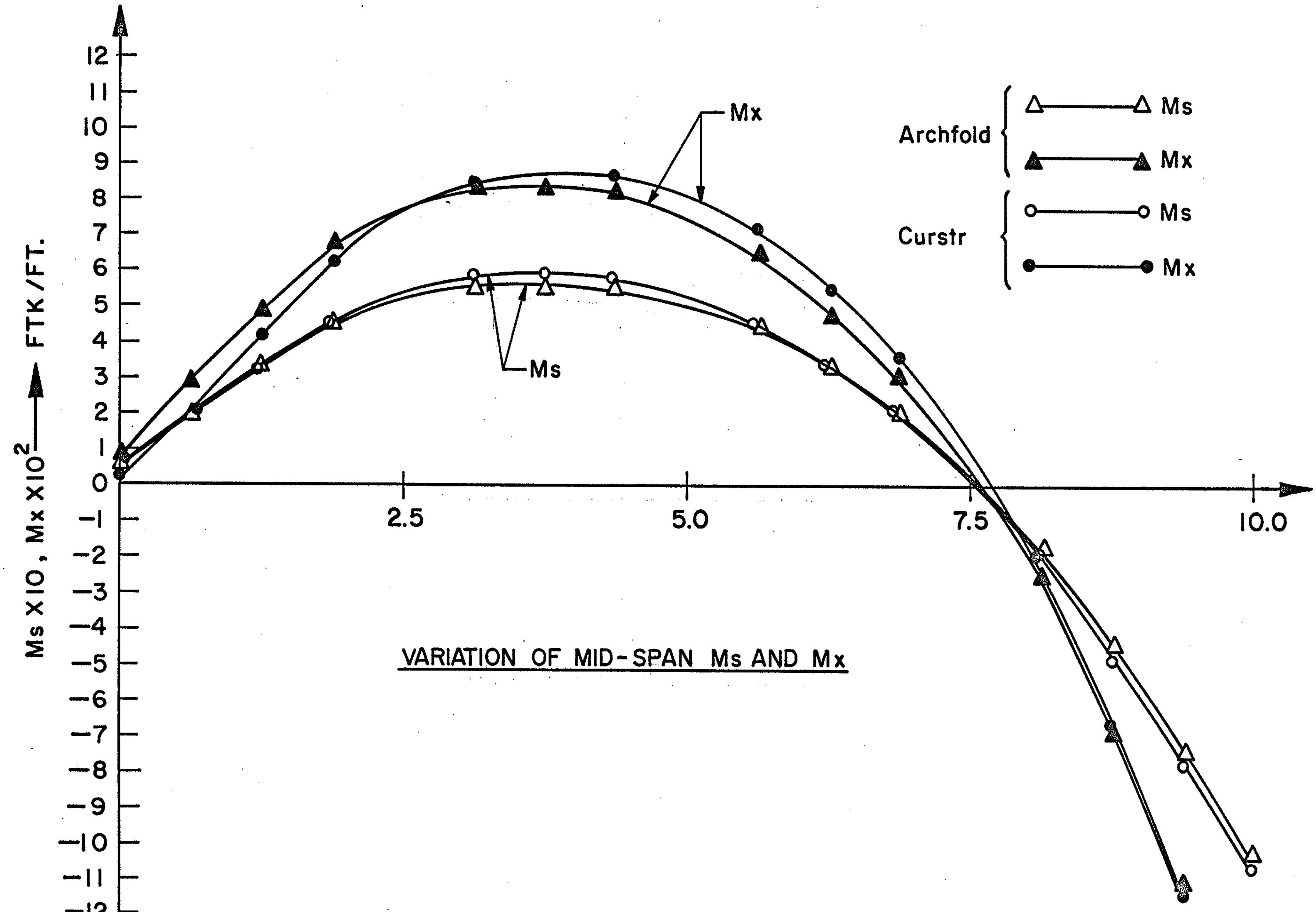


Figure 5.6 COMPARISON FOR $\tan\phi = 0.60$ ($\phi \simeq 31°$)

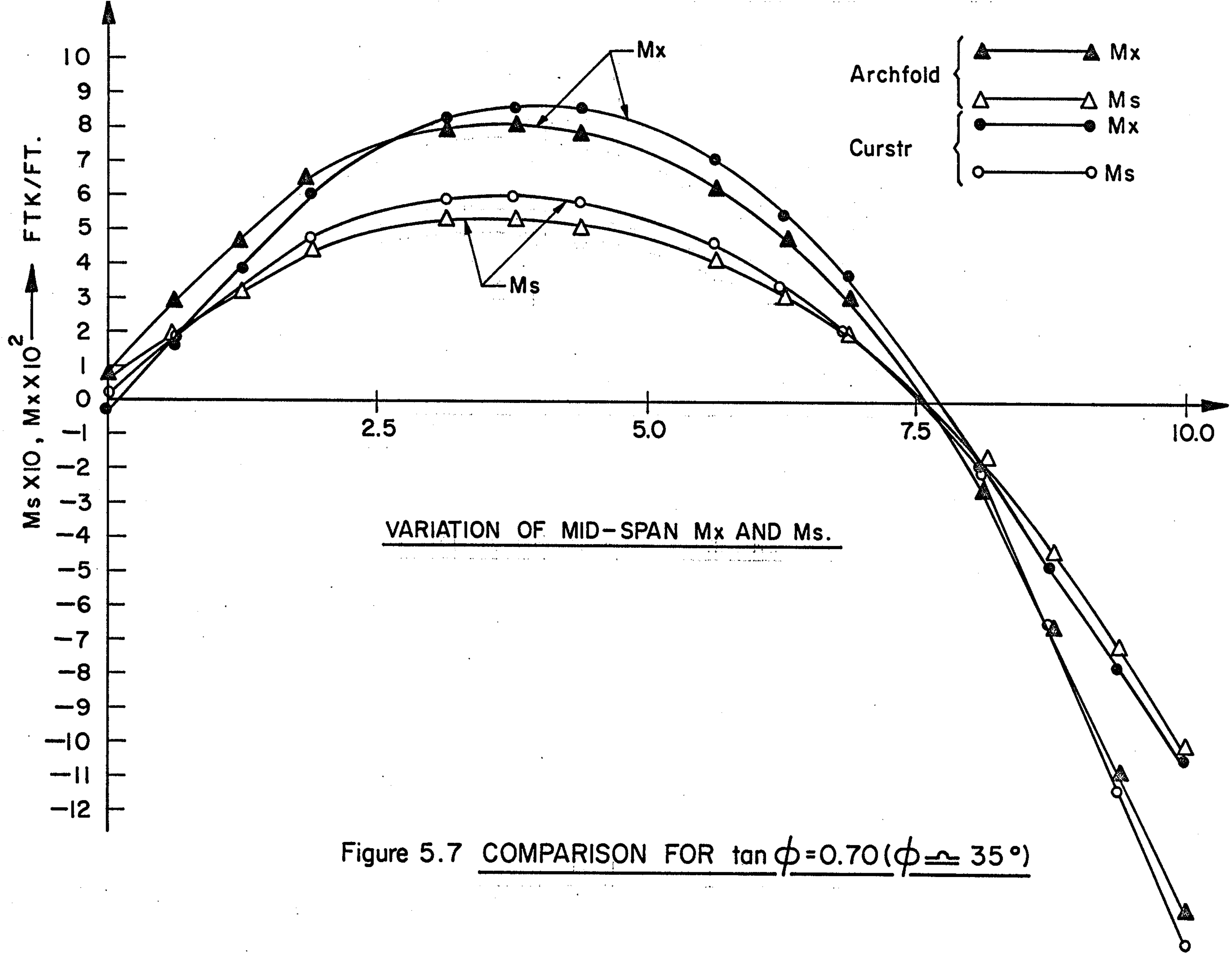


Figure 5.7 COMPARISON FOR $\tan\phi = 0.70 \, (\phi \simeq 35°)$

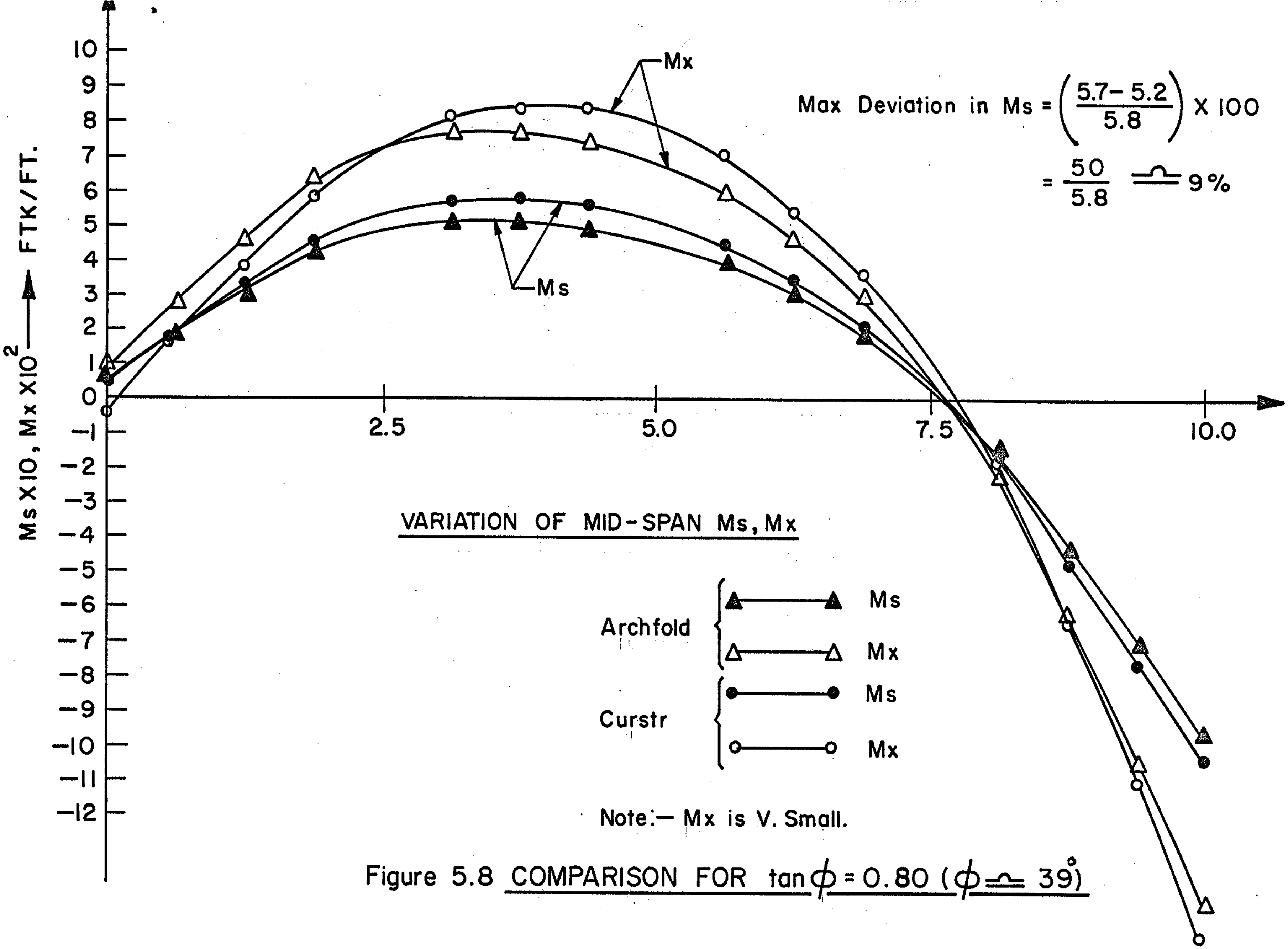


Figure 5.8 COMPARISON FOR $\tan\phi = 0.80$ ($\phi \simeq 39^\circ$)

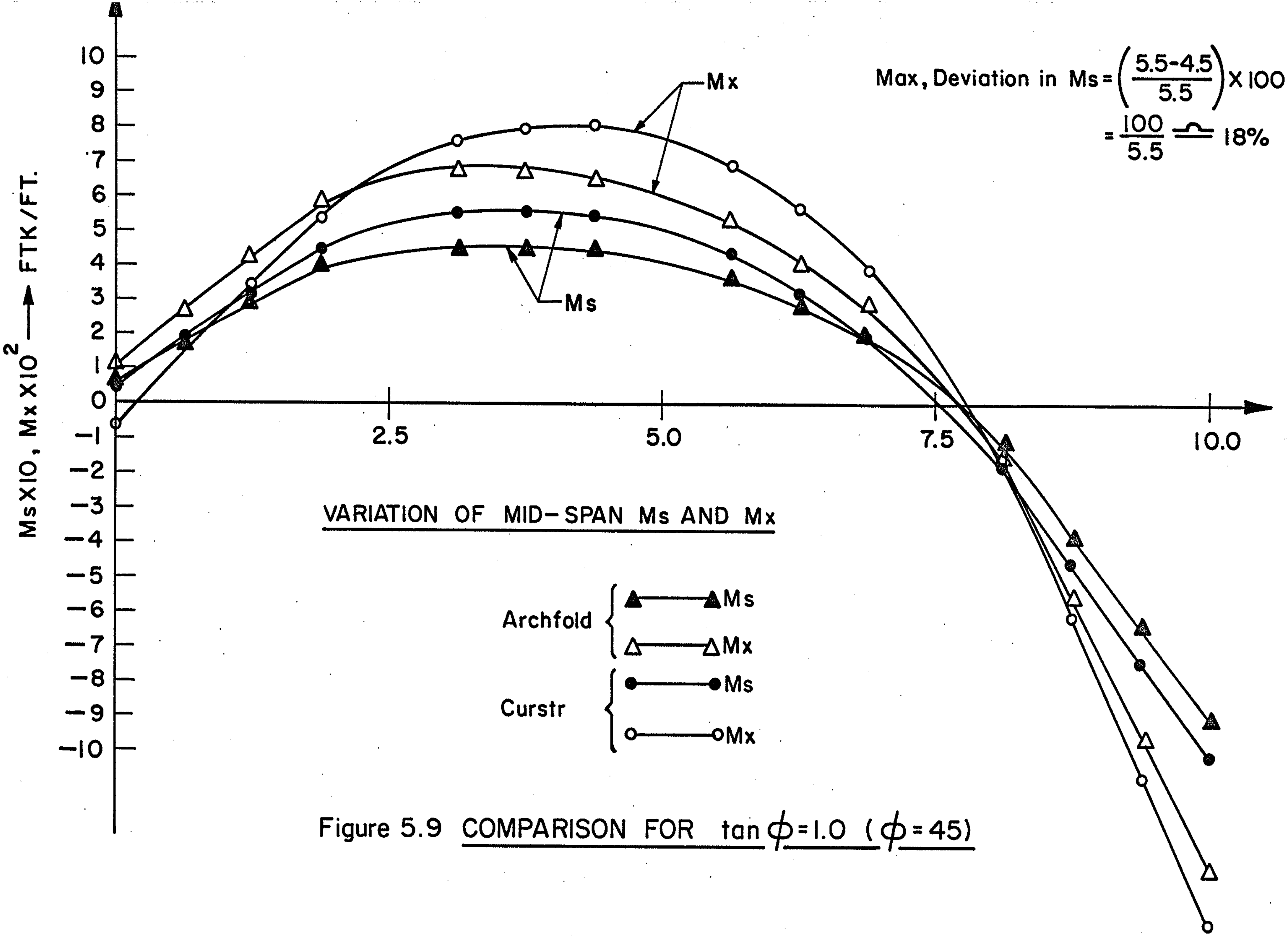


Figure 5.9 COMPARISON FOR tan ϕ = 1.0 (ϕ = 45)

The values of the longitudinal bending moment M_x and the transverse bending moment M_s are plotted as shown in Figs. 5.4 to 5.9. It is seen that $\tan \phi = 0.2$ and 0.4 give almost coincident values, while $\tan \phi = 0.6$ and 0.7 give very good agreement. The results for $\tan \phi = 0.8$ and 1.00 are also not very far apart. It could be concluded that the analysis presented in this work is of good accuracy for about $\tan \phi = 0.8$, i.e., $\phi \simeq 39°$.

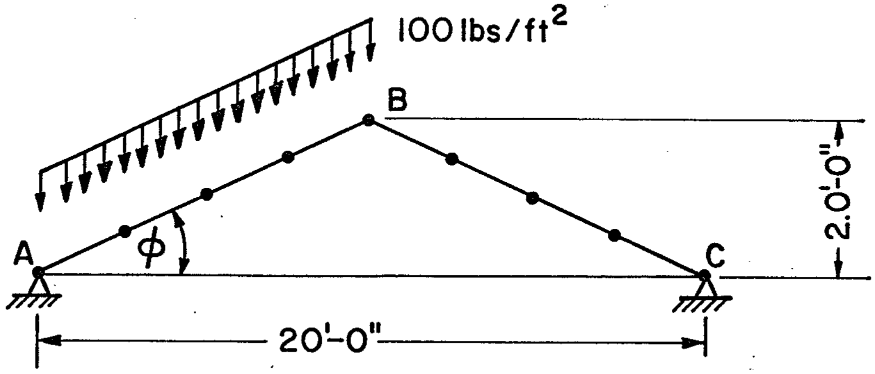
5.2.3 Test Case (c): - Effects of Arching (Parameter XR or H/L Study)

Once the validity of the programme is confirmed, we proceed to study the effects of arching. In this test the simple structure presented earlier is chosen with the same span of 80.0 feet and load of 100 lbs/sq ft. (inclined area). The thickness of the plates and the inclination angle ϕ were kept constant ($h = 4$ inches, $\tan \phi = 0.20$), while the curvature XR was given values 0.00, 0.005, 0.01 and 0.02. The transverse width of the structure was again kept at 20.0 feet (Fig. 5.10). Values of the mid-span longitudinal bending moment M_x and the transverse bending moment M_s, in-plane forces N_x, N_s and the normal deflection w are plotted for the cases where we have

XR = 0.00	central rise H = 0.00
XR = 0.005	central rise H = 4.00'
XR = 0.010	central rise H = 8.00'
XR = 0.020	central rise H =16.00'

(Figs. 5.11 to 5.14)

The reduction in M_x, M_s and w could be seen while N_x has increased with no appreciable change in N_s. It is to be noted that the



Span = 80.0'-0"   8 Equal Strips.

Thickness = 4"

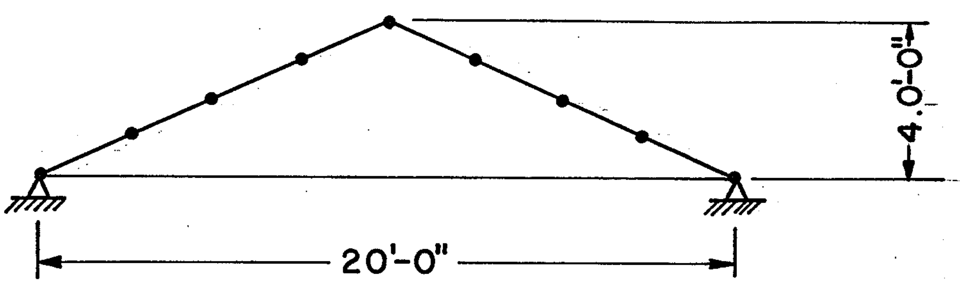
Tan ϕ = 0.20

Dead load = 100 lbs/ft^2 (Inclined Area)

XR = 0.0 , 0.005, 0.01, 0.02

Plots Given are for Plate-A B

TEST CASE (c)



Span = 80.0'–0   8 Equal Strips.

Thikness = 4"

Tan ϕ = 0.4

Dead load = 100 lbs/ft^2 (Inclined Area)

XR = 0.005, 0.01, 0.015, 0.02

TEST CASE (d)

Figure 5.10

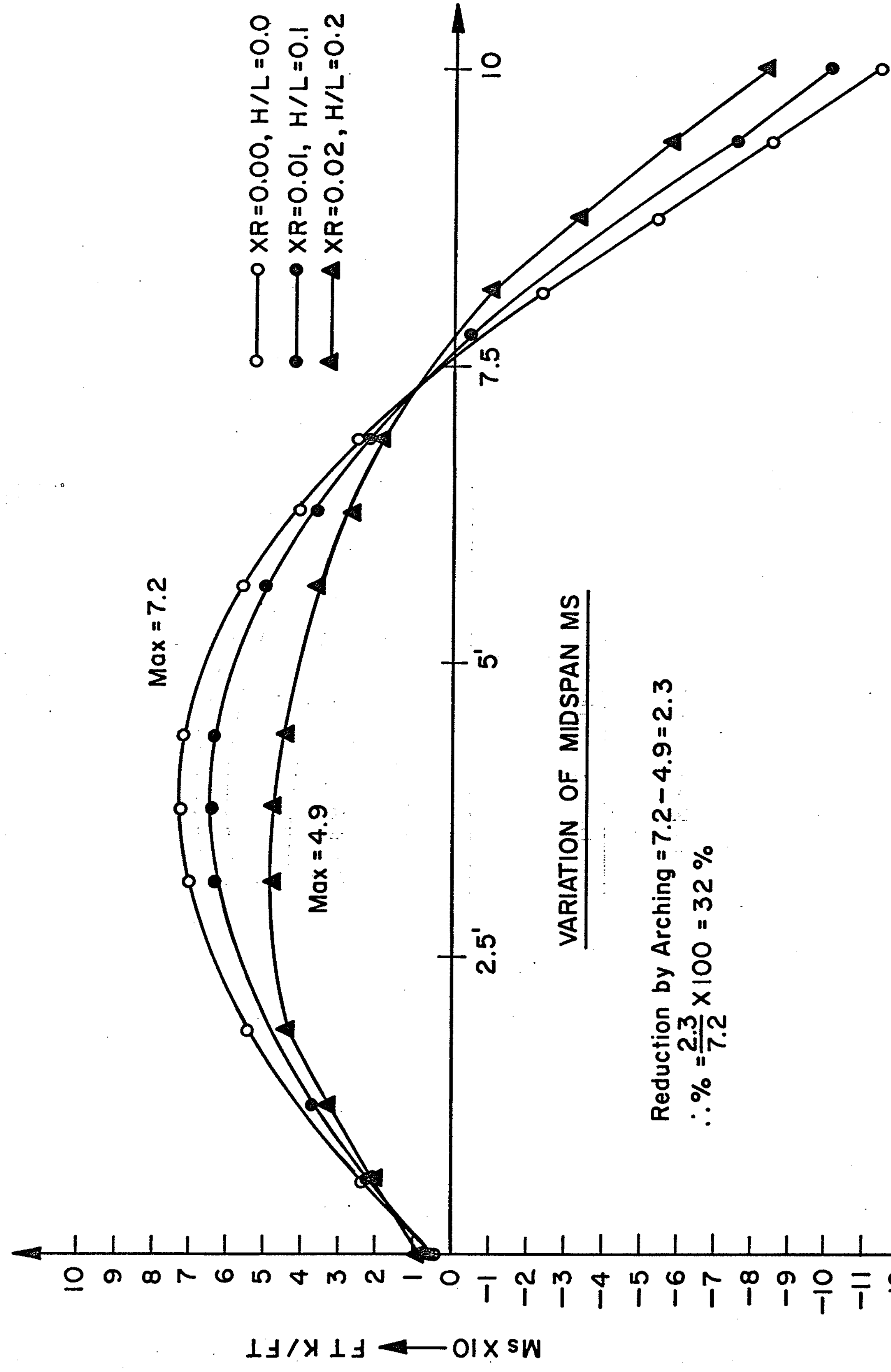


Figure 5.11 EFFECTS OF ARCHING

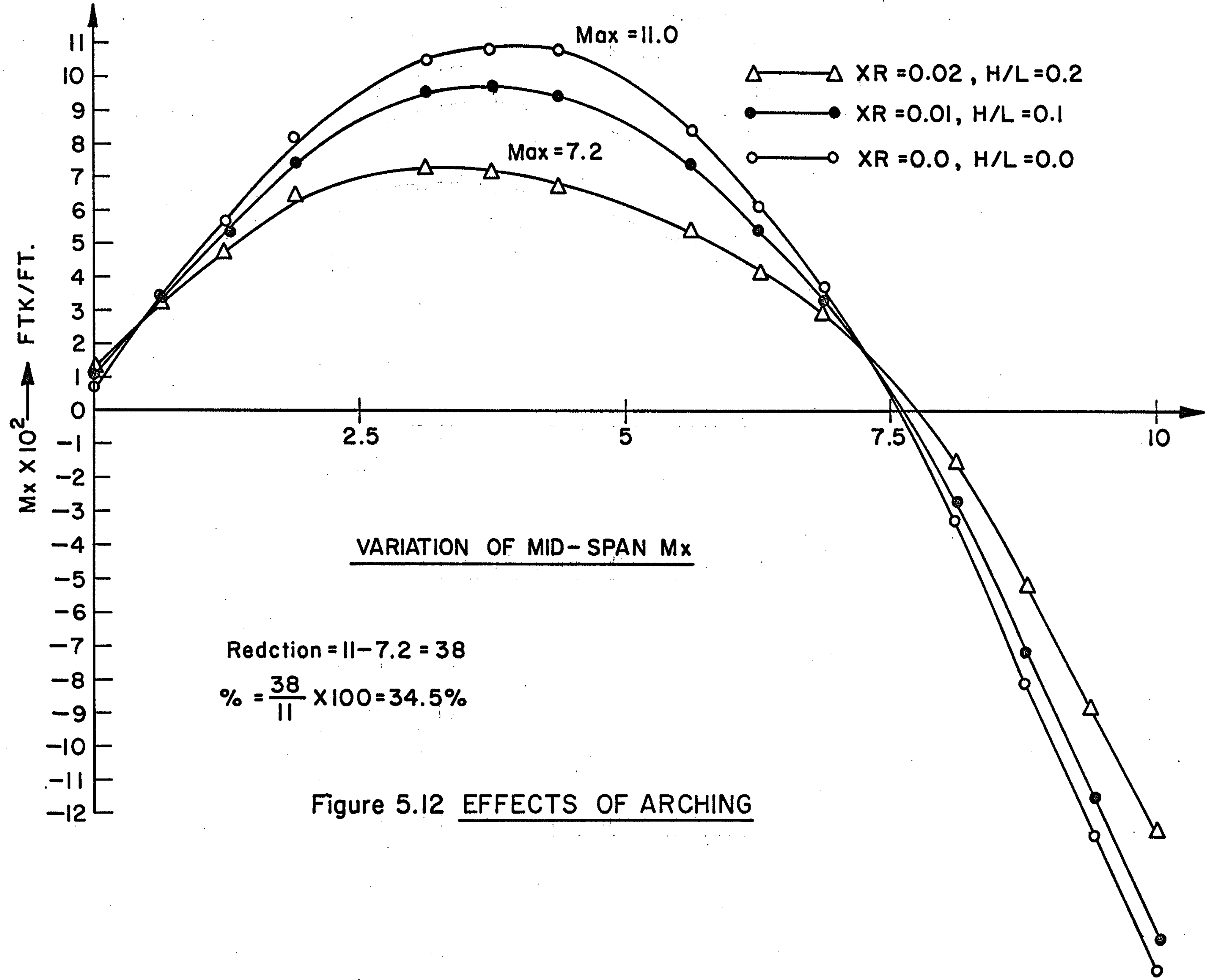


Figure 5.12 EFFECTS OF ARCHING

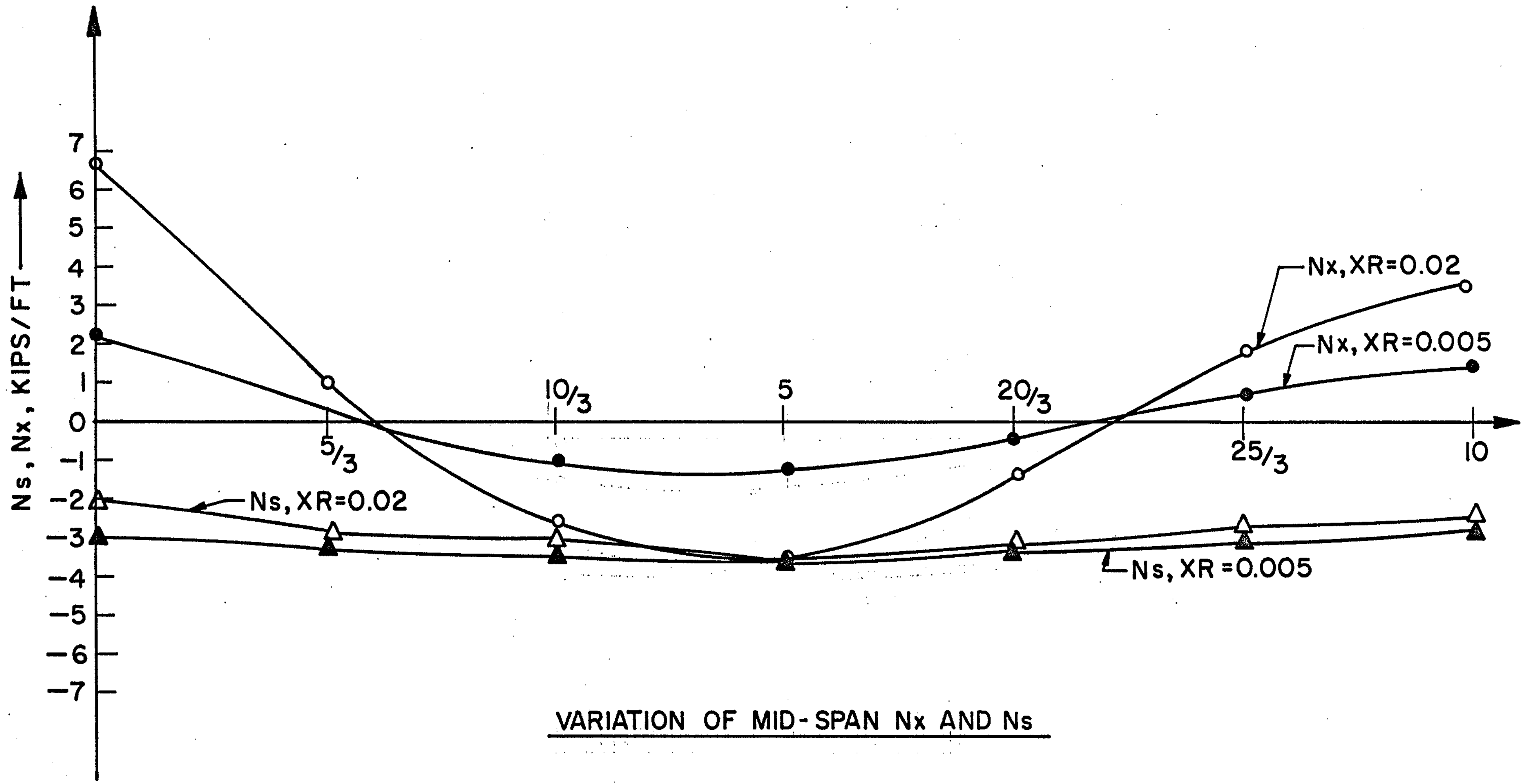


Figure 5.13 EFFECTS OF ARCHING

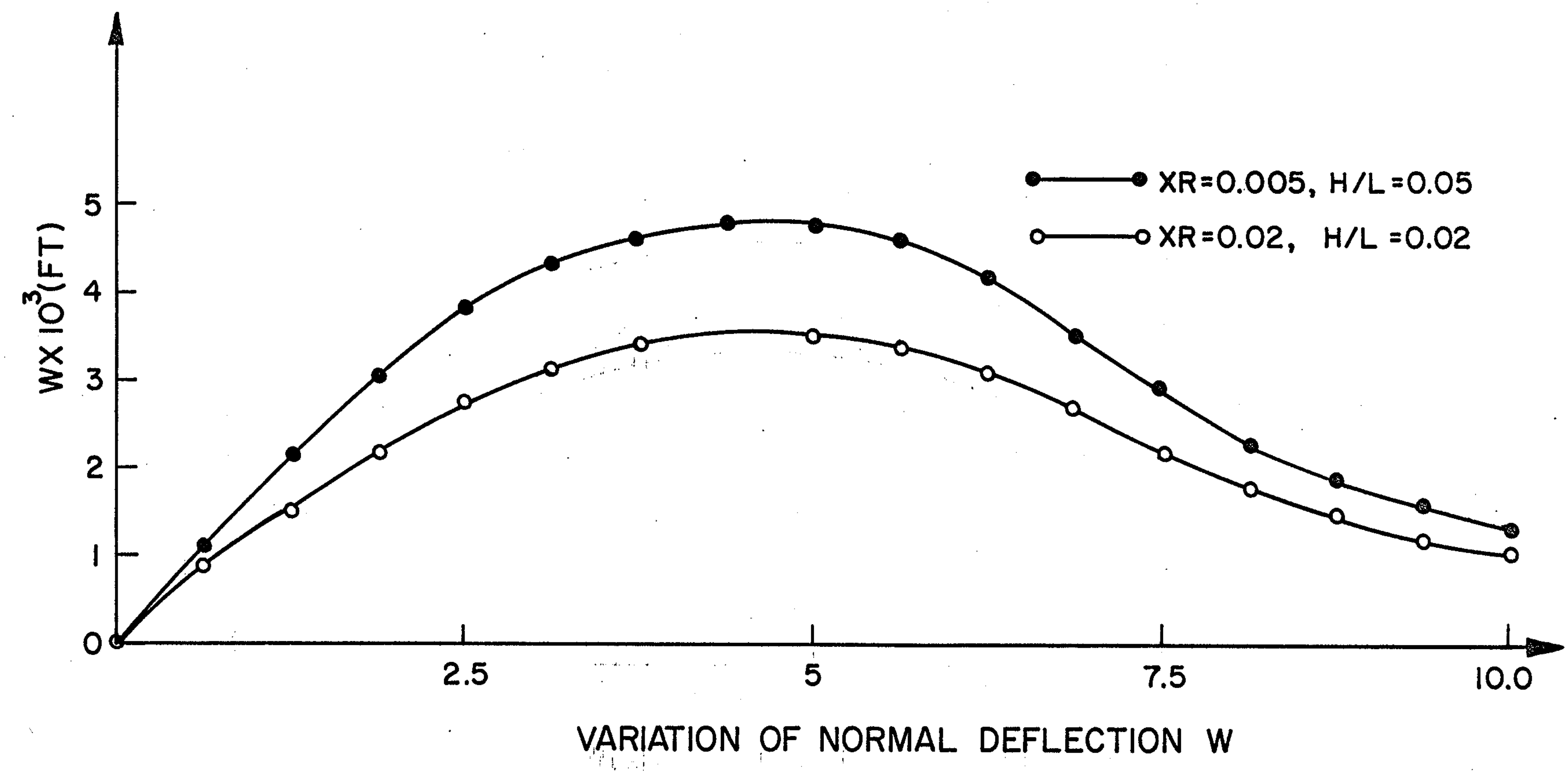


Figure 5.14 EFFECTS OF ARCHING

increase in N_x is small and still N_x is within the value for which nominal reinforcement is sufficient. The results are discussed in more detail in the following chapter.

5.2.4 Test Case (d): - Comparison Study "Archfold vs CURSTR" for Varying XR or H/L

In test case (a) the maximum premissible value of XR viz. 0.02 for shallow shells was used and very good agreement was obtained with "CURSTR" for the values of M_s. Tan ϕ was 0.2 in that case. In this test the same structure subjected to the same load is analysed but with tan ϕ = 0.4 (Fig. 5.10). XR was given the values 0;05, 0.01, 0.015 and 0.02 and the values of the M_x, M_s, N_x and N_s obtained are compared with "CURSTR" (Figs. 5.15 and 5.16). It can be seen that for N_x and N_s the agreement is excellent. In the case of M_x and M_s the agreement is very good as well except for a small deviation of 7% and 10% respectively for the case XR = 0.02.

5.2.5 Test Case (e): - Parameter Study Variation of M_s with h/L

The same simple structure as in (a) is analysed with everything remaining the same but for spans of 80.0', 60.0' and 40.0'. The values of M_s are plotted for the three cases. It is clearly seen that M_s decreases with increasing h/L.

span = 80.0'	h/L = 0.004
span = 60.0'	h/L = 0.006
span = 40.0'	h/L = 0.008

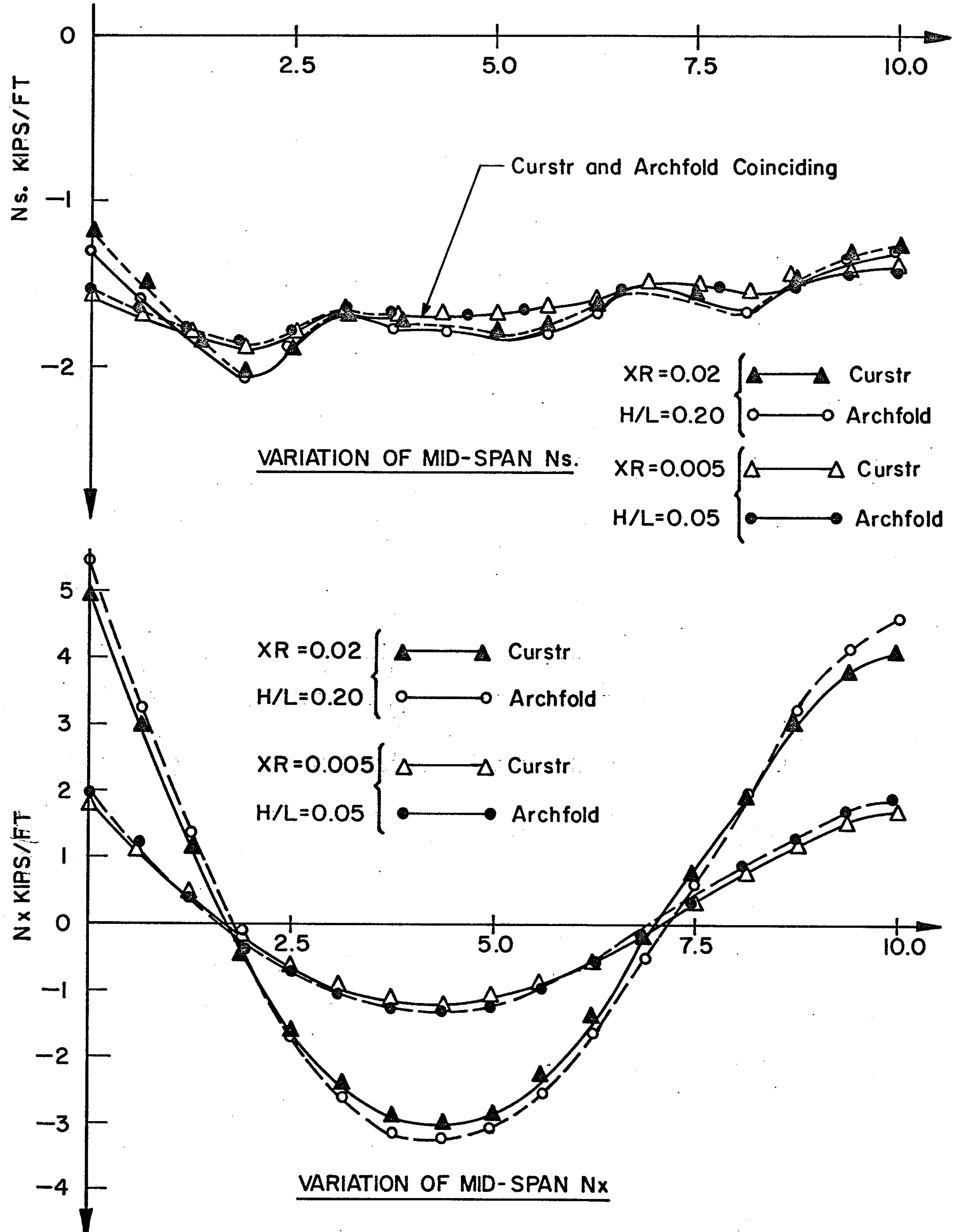


Figure 5.15 COMPARISON WITH "CURSTR"

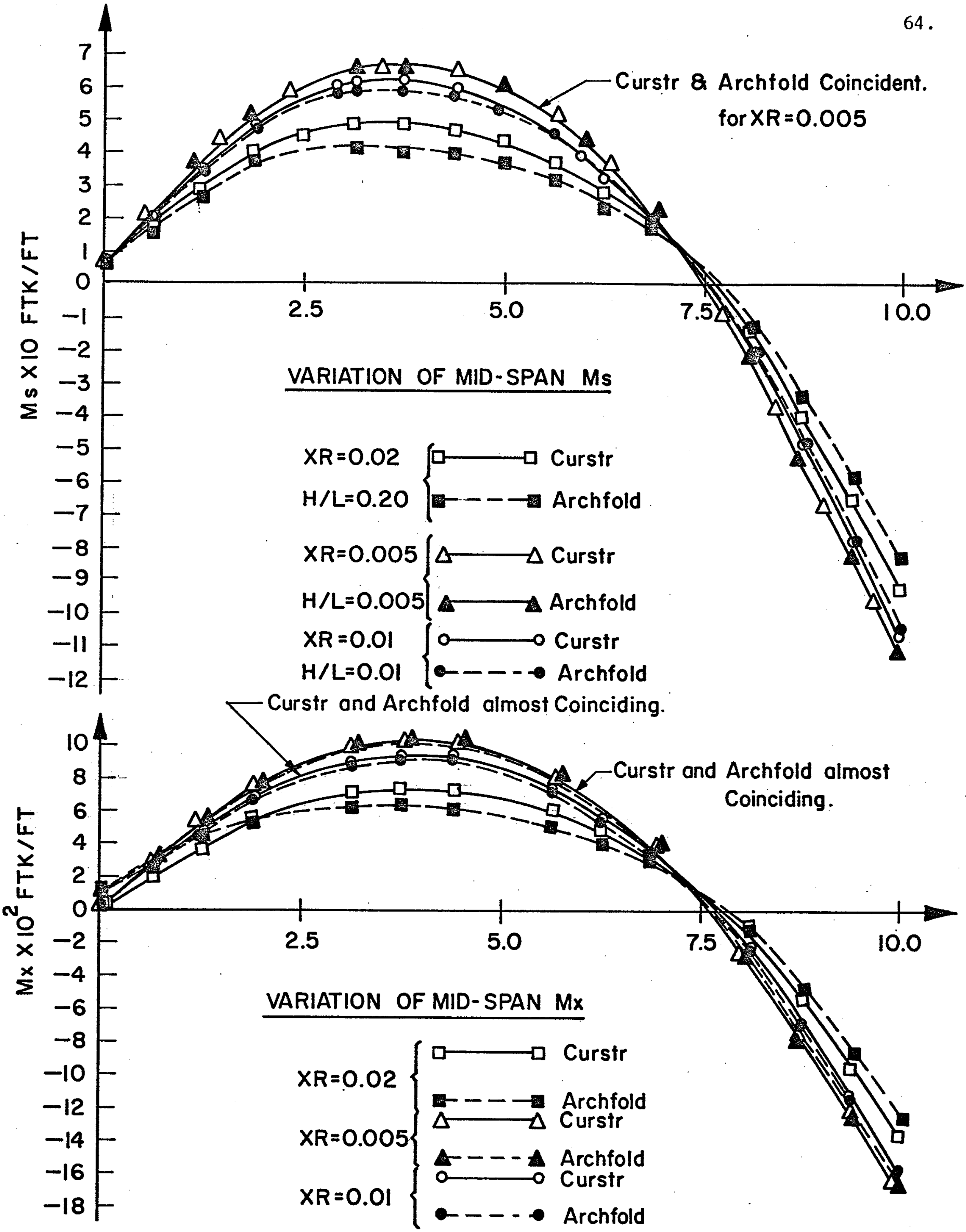


Figure 5.16 COMPARISON WITH "CURSTR"

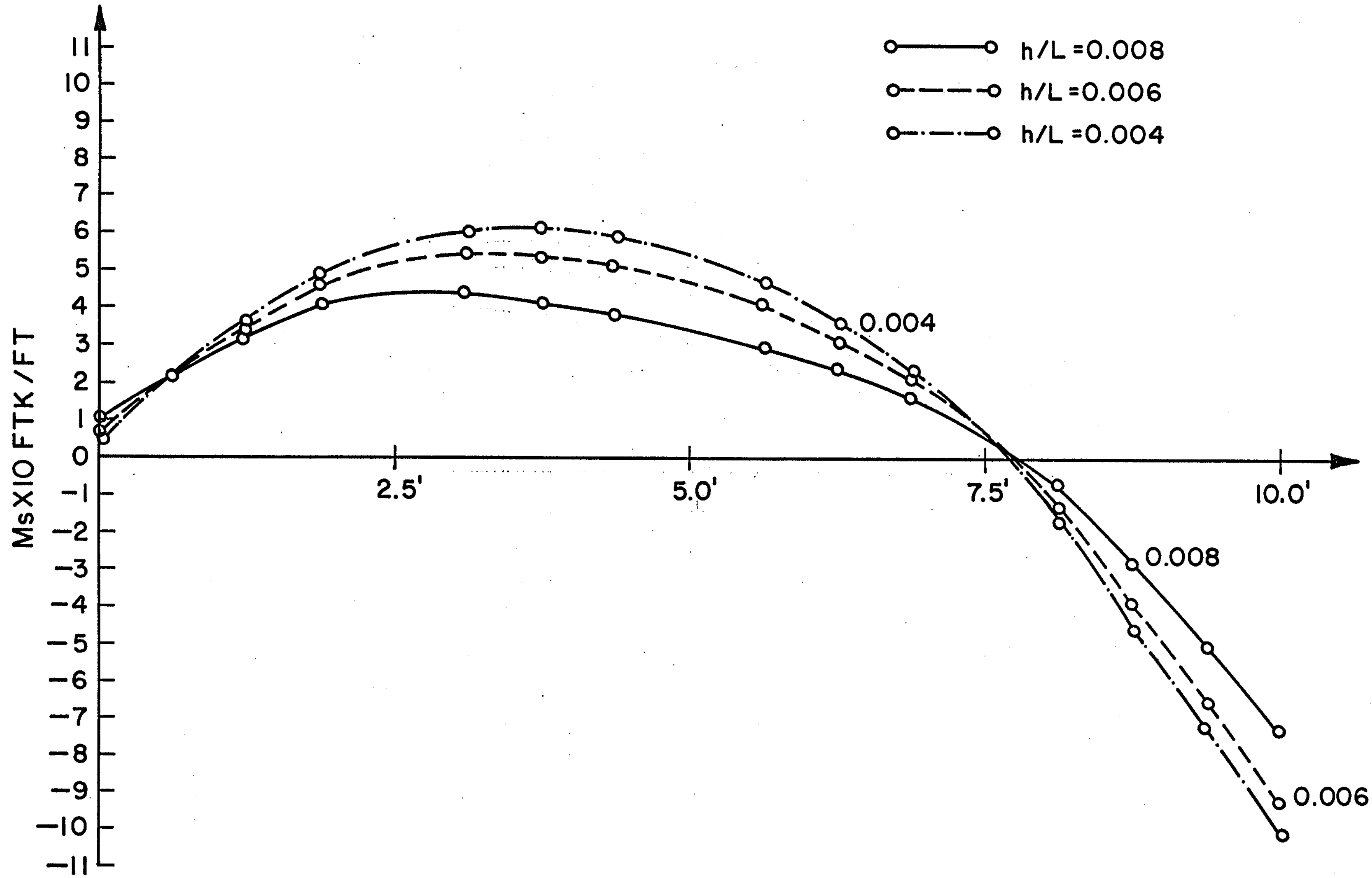
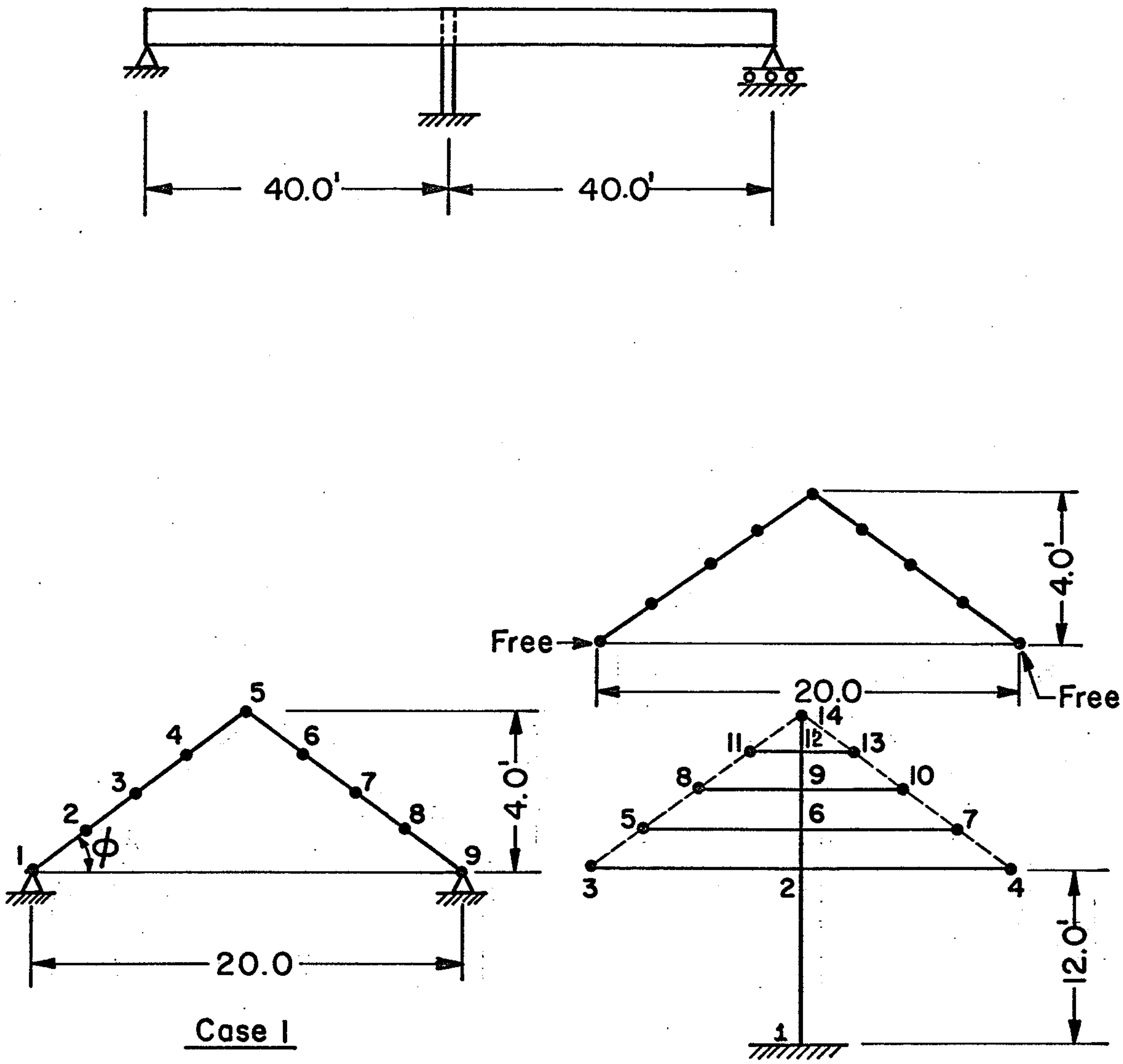


Figure 5.17 VARIATION OF Ms WITH h/L

5.3 Effect of Intermediate Supports

1. It was mentioned in Chapter III that structures having intermediate supports could be analysed by means of "Archfold". The two fold simple structures with simply supported longitudinal edges was considered first with one central diaphragm (rigid) and then with one central plane frame support as shown in Fig. 5.18. It was found that the effect of the central support was felt only at mid-span. Elsewhere the moments and forces still had their previous values. This is attributed to the very large rigidity of the structure that was considered with the transverse edges simply supported (Figs. 5.19 and 5.20).

2. The same structure was considered with the two types of central supports but this time with the edges (joints 1 and 9) free. In this case, as expected, there were large values of deflections, especially vertical, without the central support. The effects of the central support almost nullified the vertical deflections and reduced the others considerably. (Table. 5.21).The effects of the central support are also shown in the considerable reduction in the mid-span and quarter-span M_s, M_x and N_x (Figs. 5.22 to 5.24)..



Plane Frame Support

Case 2

Equal Strips = 8

Span = 80.0'

Thickness = 4"

Dead Load = 100 lbs/ft (Inclined Area)

Tan ϕ = 0.4

XR = 0.01

Figure 5.18 EFFECTS OF A CENTRAL SUPPORT

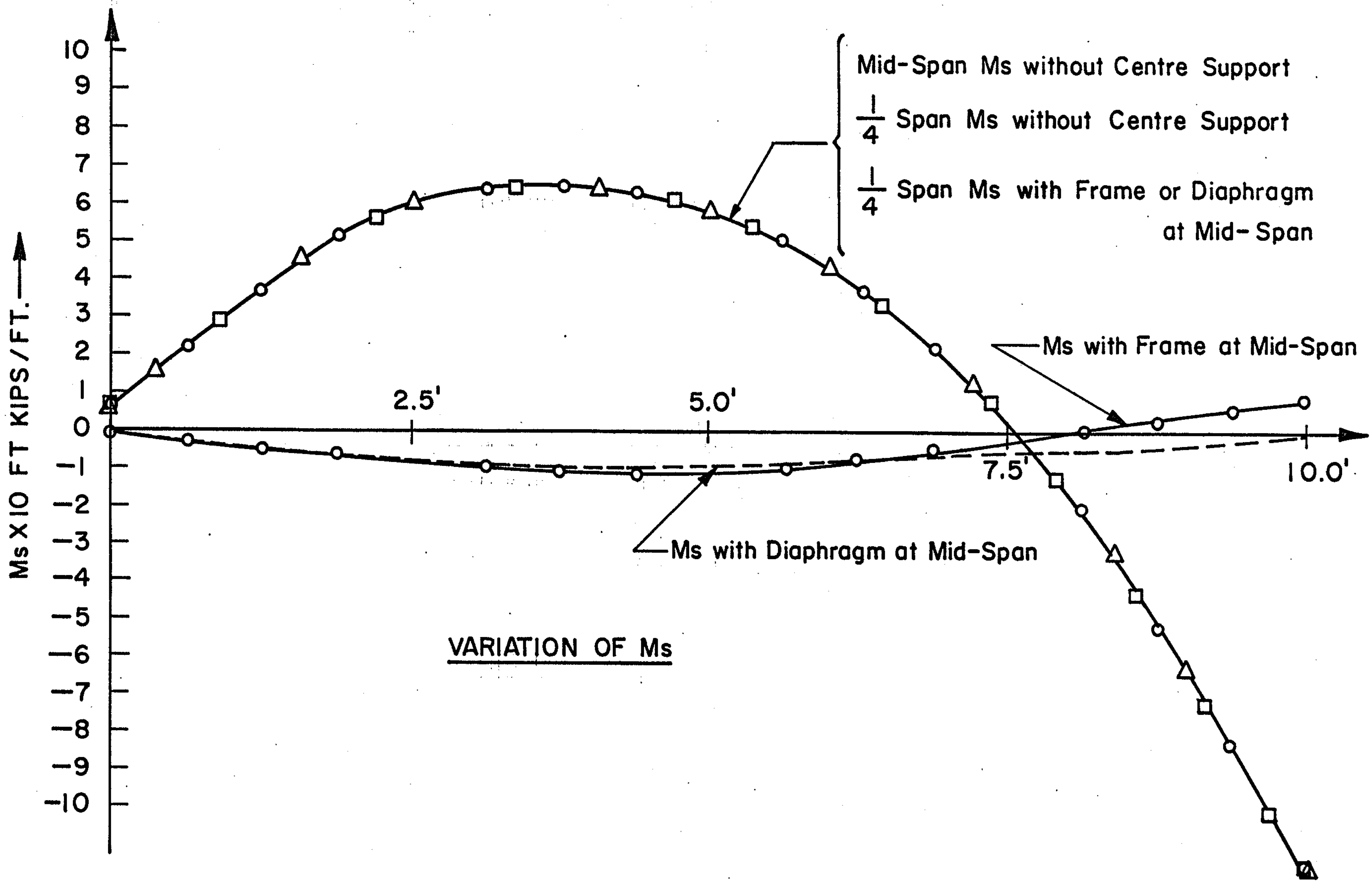


Figure 5.19 EFFECT OF MID-SPAN SUPPORT ON Ms.

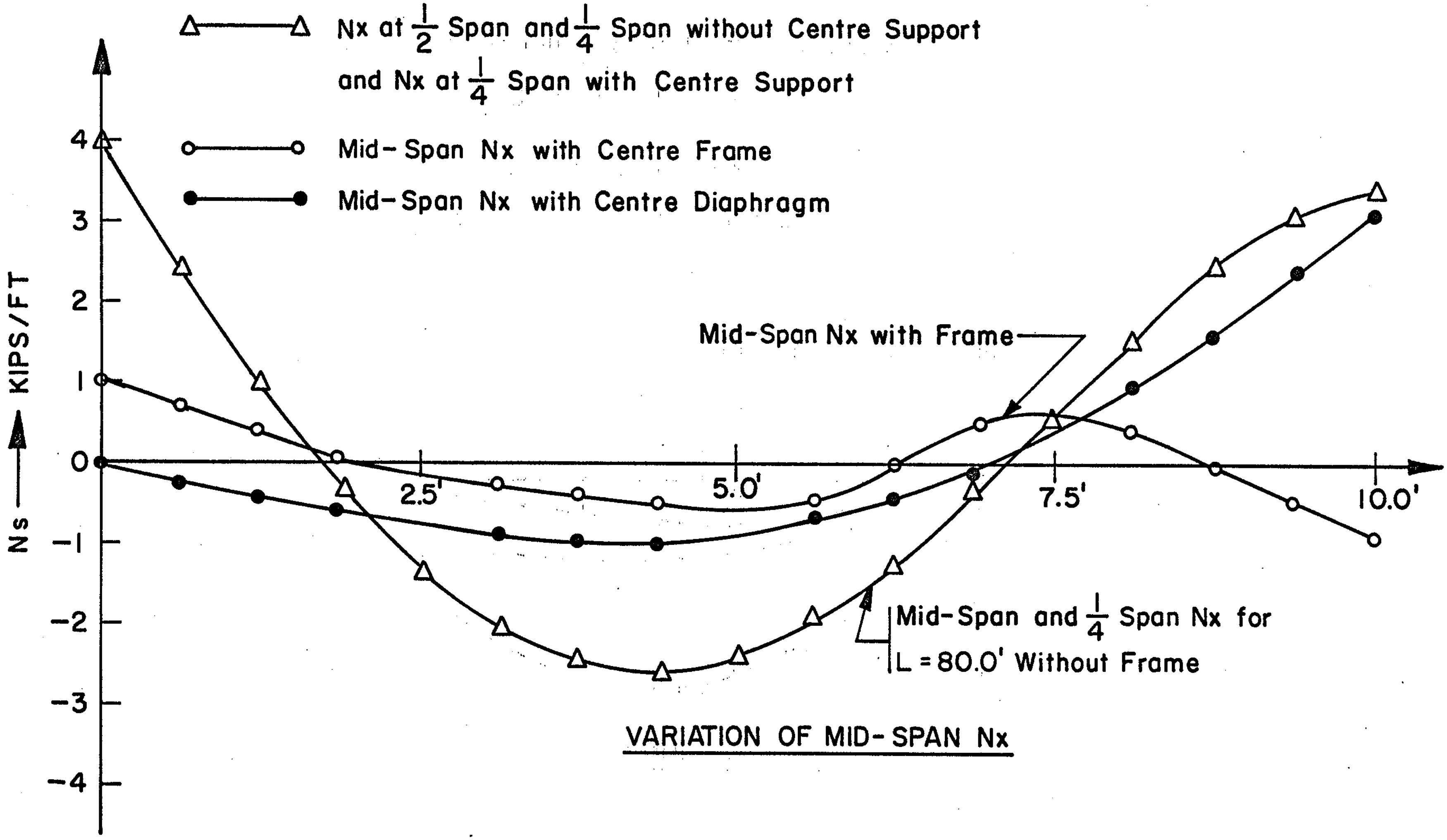


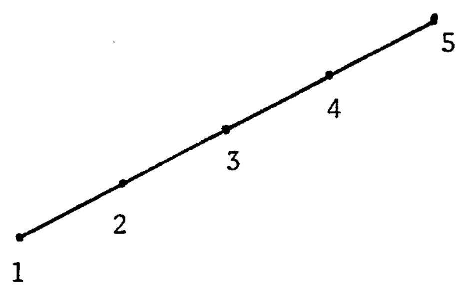
Figure 5.20 EFFECT OF MID-SPAN SUPPORT ON Nx

TABLE 5.21

REDUCTION IN VERTICAL DISPLACEMENTS OF JOINTS DUE TO CENTRE SUPPORT

(2 transverse edges free)

span = 80.0', XR = 0.01, tan ϕ = 0.4



Joints of structure

I. At Mid-span

(a) Without Centre Support	(b) With Centre Support	
1. 2.158	1. 1.865×10^{-5}	
2. 1.943	2. 2.028×10^{-5}	
3. 1.741	3. 1.624×10^{-5}	$\simeq 0$
4. 1.585	4. 1.823×10^{-5}	
5. 1.523	5. 1.397×10^{-5}	

II. At Quarter Span

1. 1.568	1. 8.428×10^{-2}	
2. 1.406	2. 6.651×10^{-2}	
3. 1.253	3. 4.869×10^{-2}	$\simeq 0$
4. 1.134	4. 3.287×10^{-2}	
5. 1.087	5. 2.537×10^{-2}	

Similarly horizontal displacements and rotations of joints are also reduced considerably.

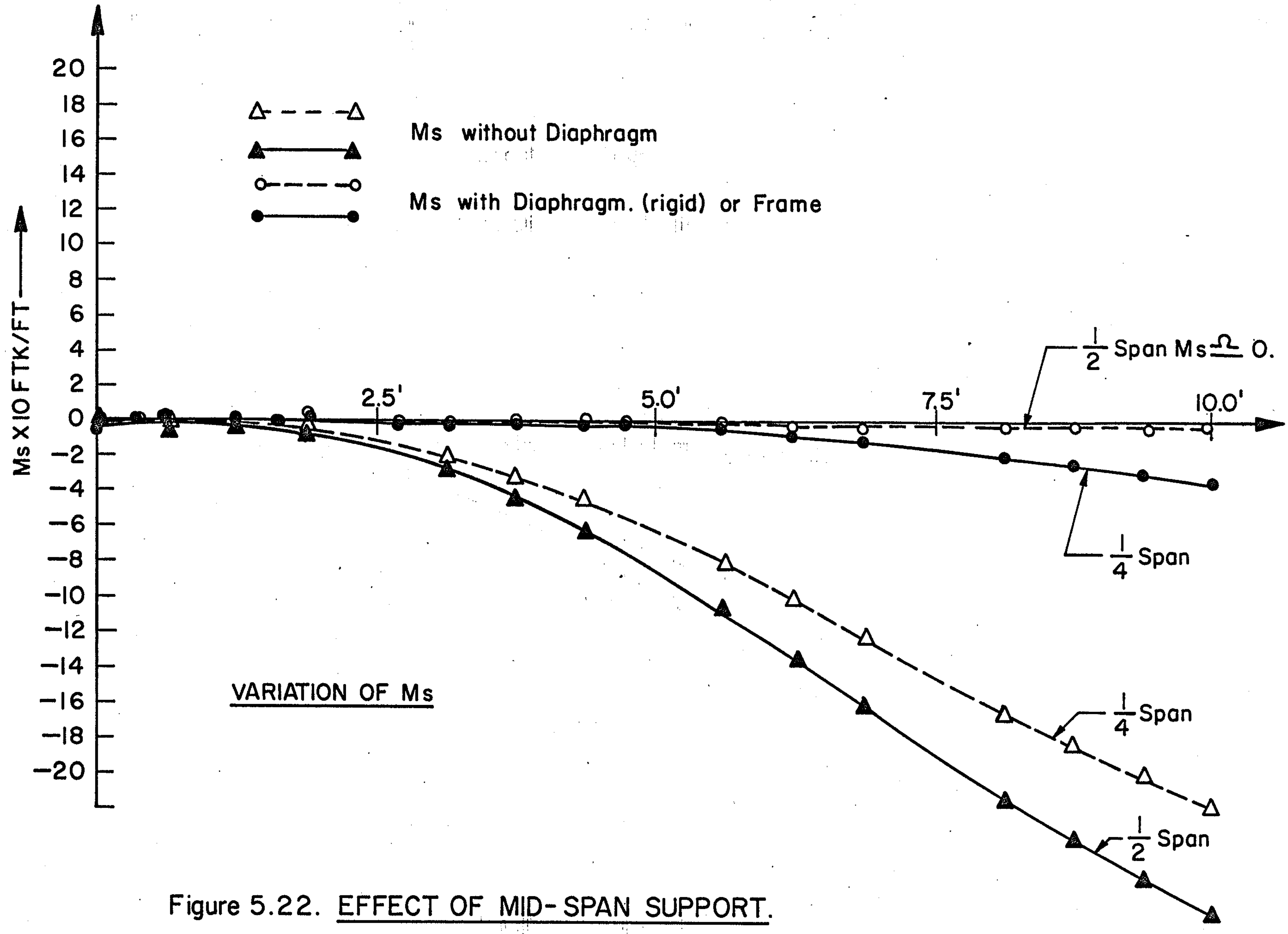


Figure 5.22. EFFECT OF MID-SPAN SUPPORT.

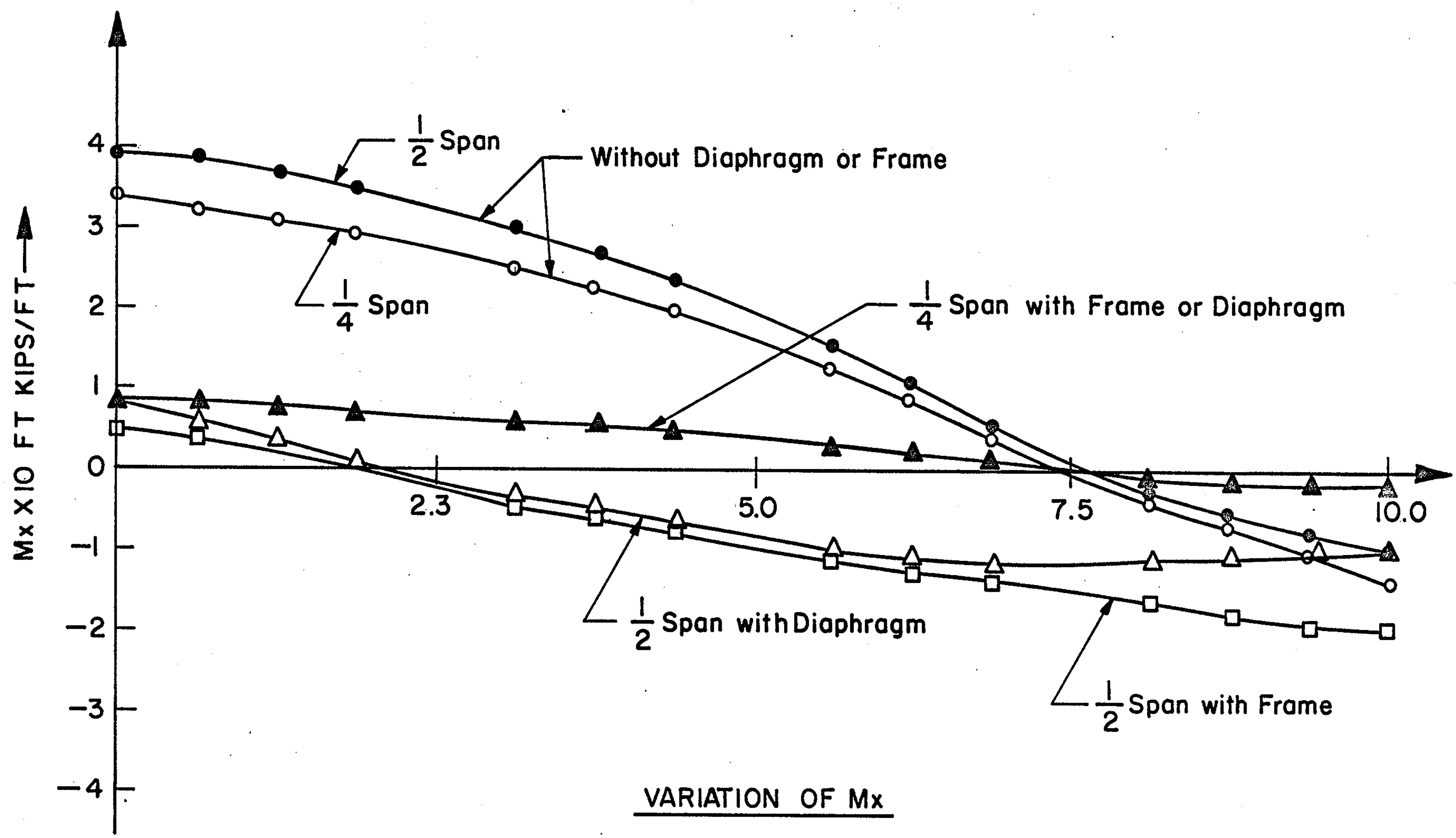


Figure. 5. 23 EFFECT OF MID-SPAN SUPPORT

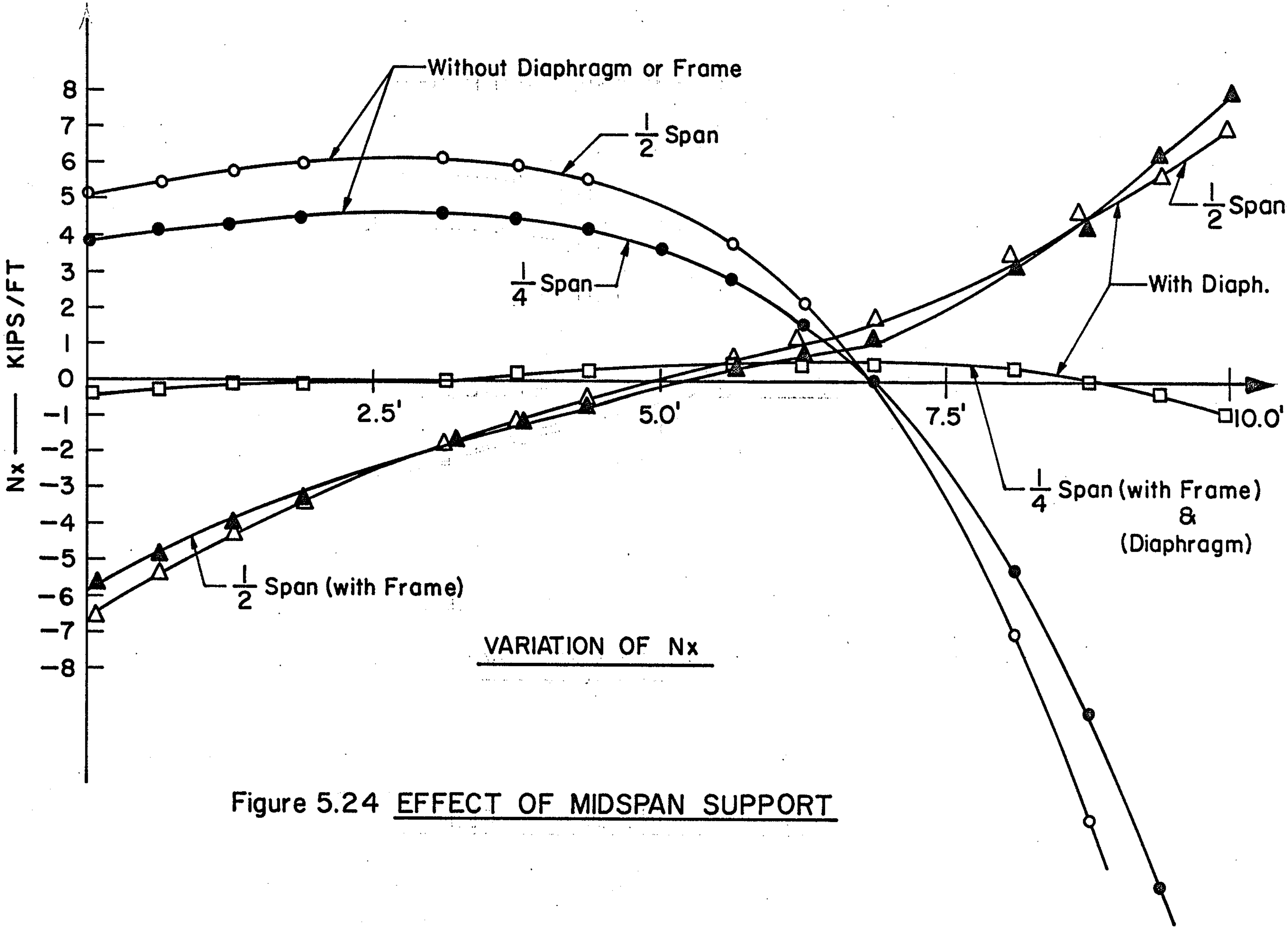


Figure 5.24 EFFECT OF MIDSPAN SUPPORT

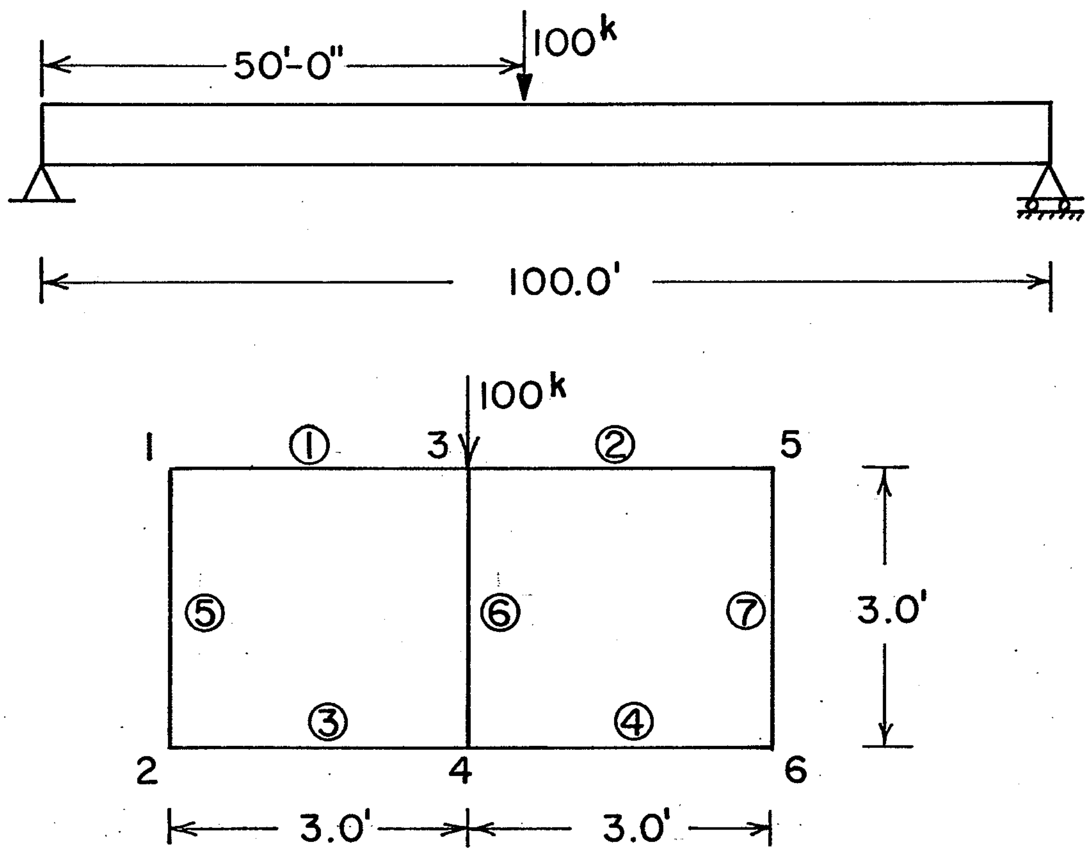
5.4 Analysis of Conventional Folded Plate Structures (XR = 0) and Comparison with "MUPDI 3"

The generality of the programme is shown in its ability to analyse any conventional (straight) folded plate structure. The example chosen is a two-cell box girder bridge subjected to a single concentrated central load of 100 kips as shown in Fig. 5.25. In this case it is possible to consider the effects of intermediate flexible movable diaphragms and also to obtain girder moment integration. For the sake of comparison, the latter was requested by specifying MCHECK = 1 in the control card. The results were compared with "MUPDI 3" [4] and excellent agreement was obtained for the values of in-plane forces and bending moments. In Figure 5.26 the comparison of the longitudinal bending moment taken up by the girders is shown. These were obtained using the moment-integration option mentioned above. It can be seen that the agreement is excellent.

The input and output for this case (with output at a reduced number of sections) are presented in section 5.6 to illustrate the input and output forms of the programme.

5.5 Example of the Analysis of a Typical Arched Folded Plate Structure

In sections 5.2 to 5.4, various test cases were considered where results from "Archfold" were compared with other existing solutions. Parameter studies were also carried out. A typical structure is now analysed using the programme. The cross-section of the structure and relevant details are shown in Fig. 5.27. A uniform vertical load of 200 lbs per sq. ft is used. The variation of transverse bending moments M_s along the cross-section and the variation of longitudinal bending moments



Cross-section or box girder bridge

Figure 5.25 ANALYSIS FOR XR = 0, COMPARISON WITH "MUPDI 3"

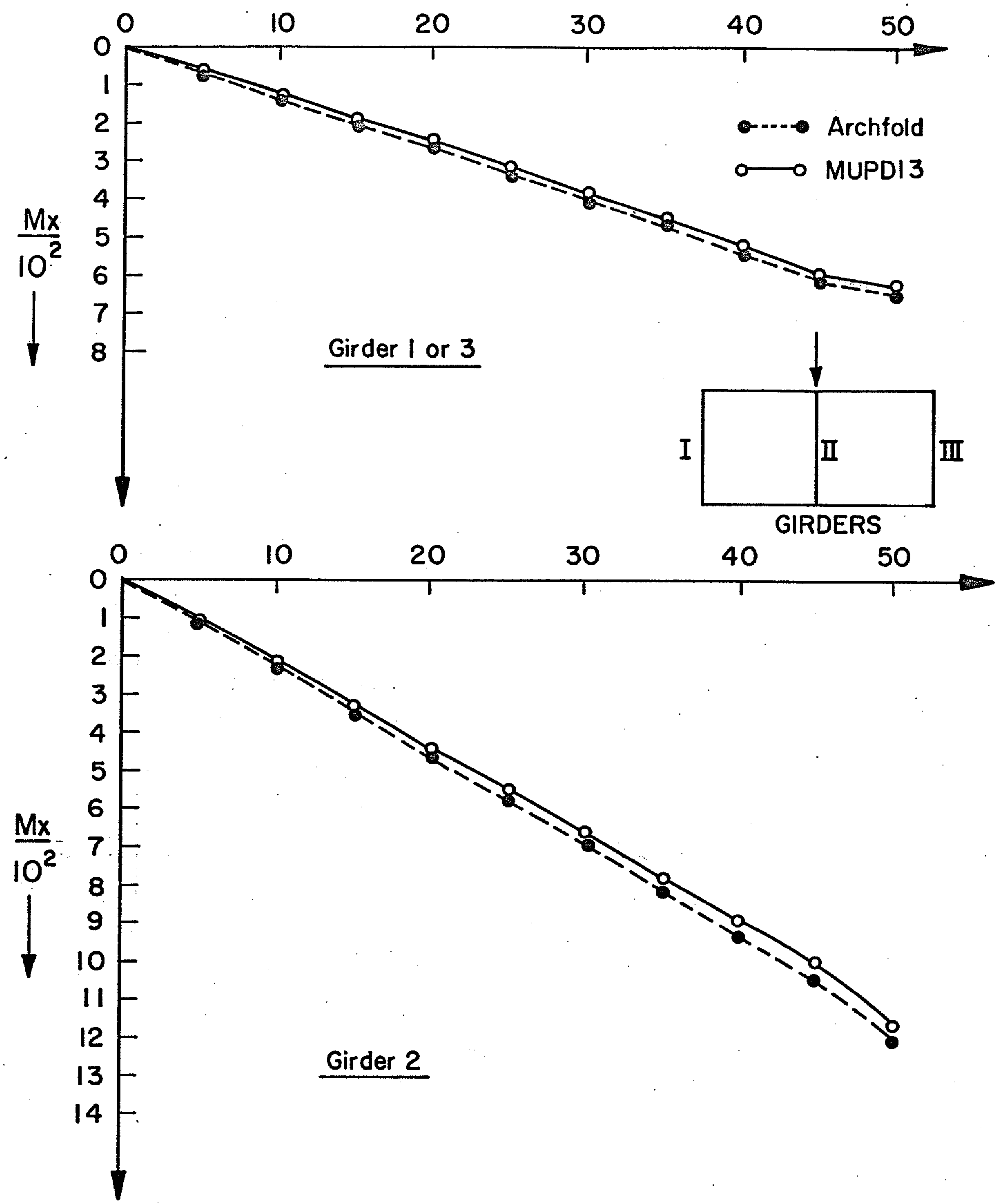
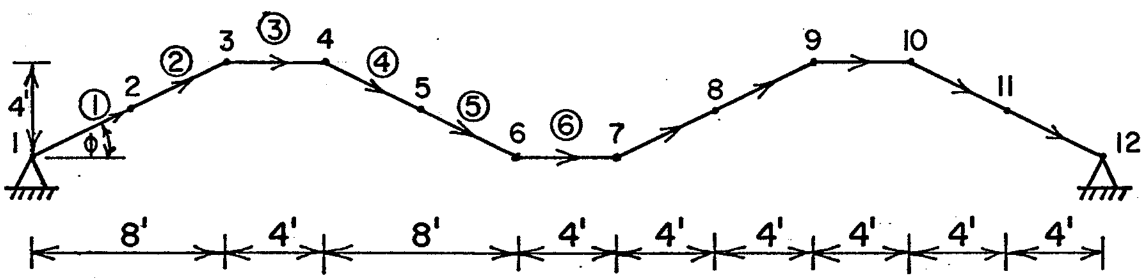


Figure 5.26 LONGITUDINAL VARIATION OF MOMENTS (FT-KIPS) TAKEN BY EACH GIRDER.



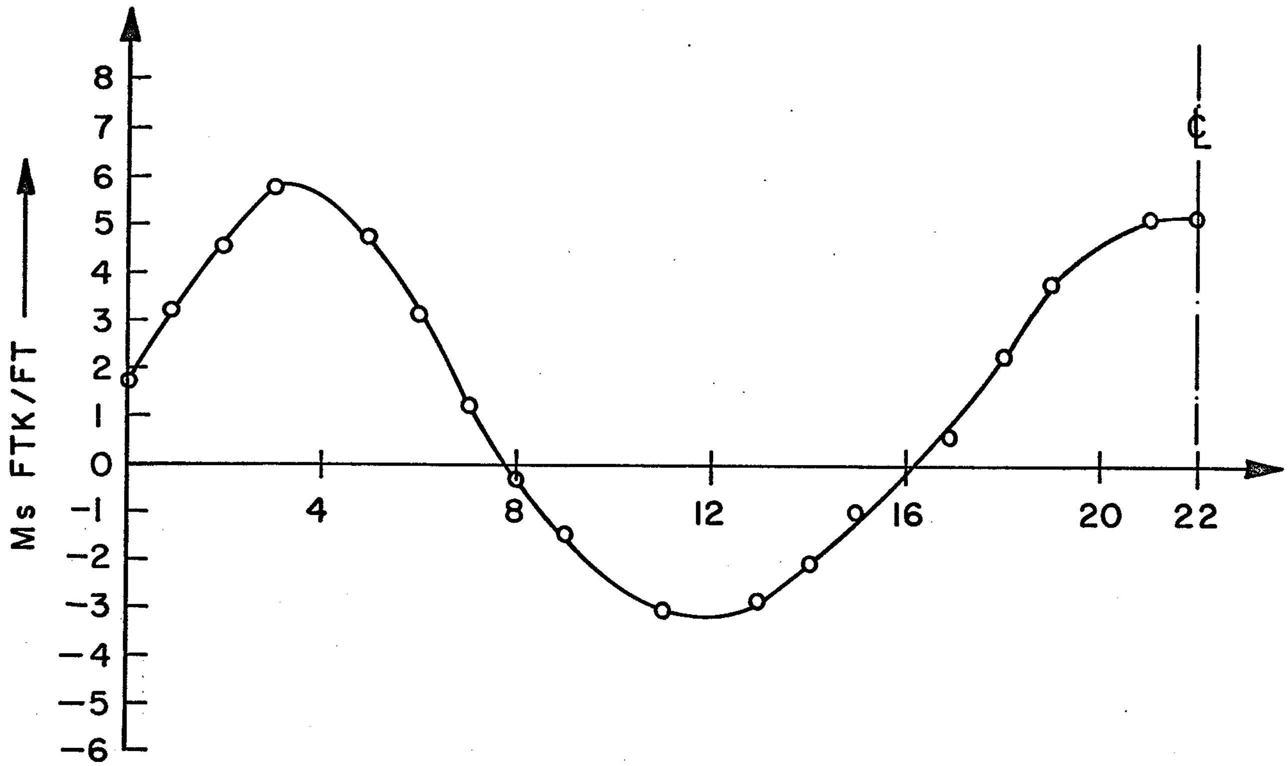
Cross-section

Span = 200.0'
Tan ϕ = 0.50
X R = 0.008

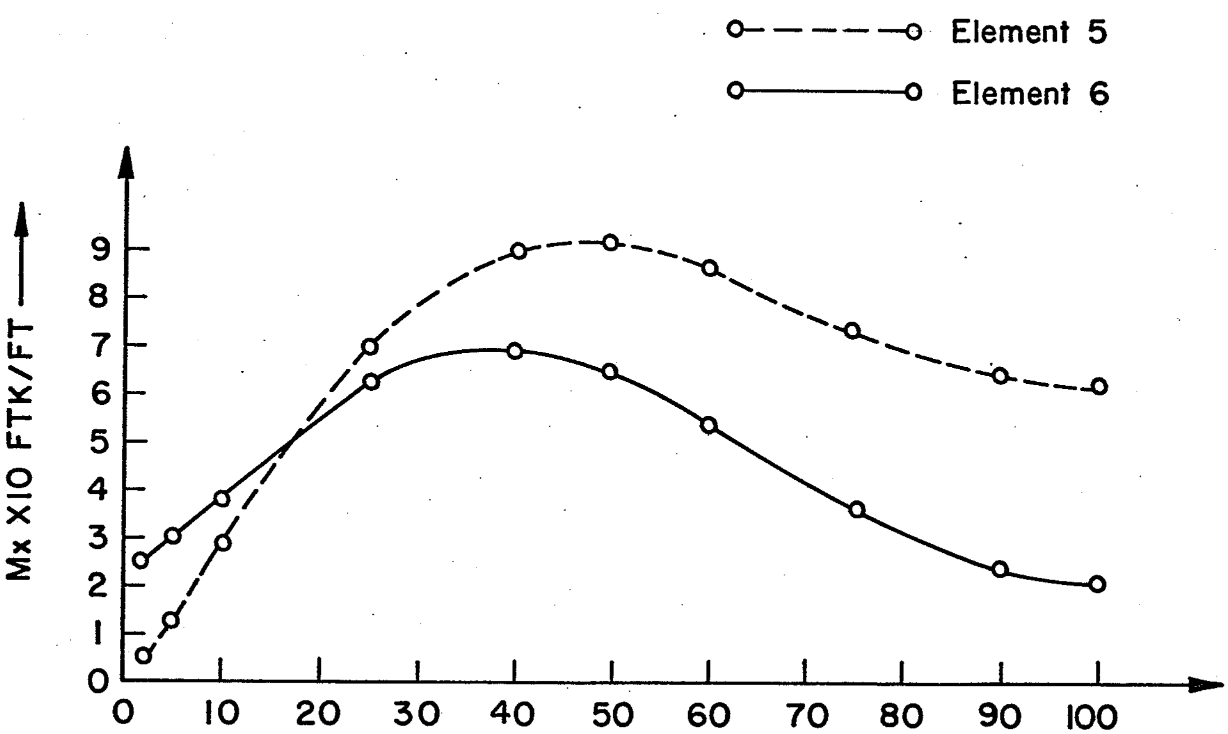
Thickness of plate = 4"
E = 0.432×10^6 k.s.f.
ν = 0.15

Total load = 200 lbs. / ft.^{2} (uniform)

Figure 5.27 ANALISIS OF A TYPICAL STRUCTURE



(a) Variation of Ms (Mid-Span)



(b) Variation of Mx (in Longitudinal Direction)

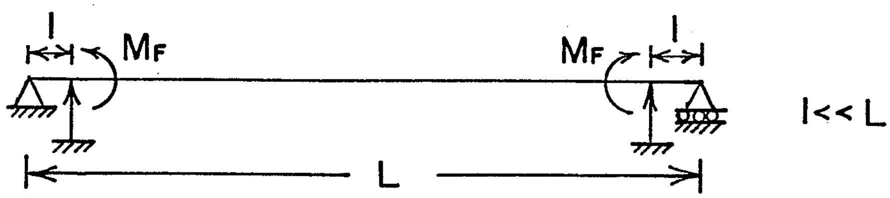
Figure 5.28 ANALYSIS OF A TYPICAL STRUCTURE

M_x along the span length at mid-points of elements no. 5 and 6 are shown in Fig. 5.28. The variation of the longitudinal in-plane force N_x at the same two points is shown in Fig. 5.30 (together with N_x for a fixed-ended structure).

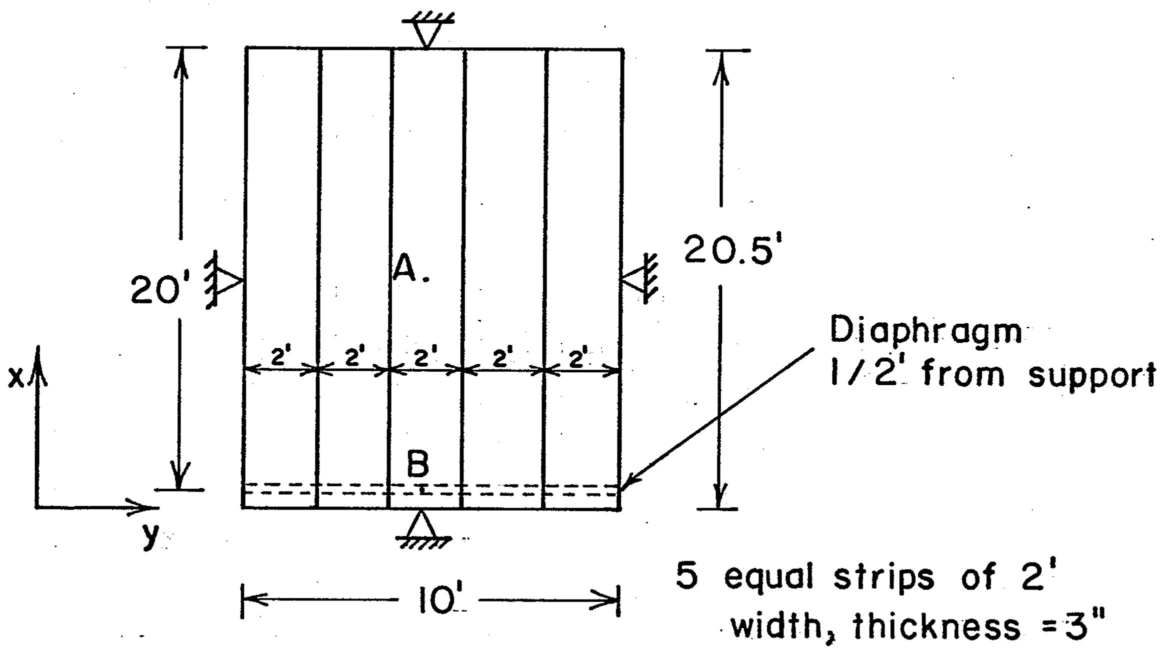
5.6 Simulation of Fixed-End Conditions

An attempt is made with "Archfold" to simulate fixed-end conditions in a structure. The basis of this method is illustrated in Fig. 5.29(a). The interior supports placed very near the simple supports of the structure will simulate fixed-end conditions. In "Archfold" this is achieved by having two diaphragms very close to the simple supports at the longitudinal ends of the structure. The analysis of a flat rectangular plate of length-to-width ratio two, having one fixed longitudinal end and the other three sides simply supported given by Timoshenko [10] was chosen for comparison. To simulate these conditions a flat plate 20.5' x 10' simply supported at all four edges was taken and the diaphragm was placed $\frac{1}{2}$ foot from one of the longitudinal ends as shown in Fig. 5.29(b). The plate was subjected to a uniformly distributed vertical load. The values of the bending moments M_x, M_y and the deflection (vertical) W at the point A and the bending moment M_x at B are compared with those given by Timoshenko in Fig. 5.28. It can be seen that there is very good agreement.

The application of this technique is extended to simulate fixed end-conditions in the structure considered in section 5.5. Fixed-end conditions were simulated at both longitudinal ends by placing diaphragms one half foot from the simple supports. The variation of the in-plane longitudinal



(a) Basis of the method



(b) Test problem

RESULTS;

	Archfold	Timoshenko
W at A	0.151	0.151
Mx at A	0.464	0.474
My at A	0.936	0.940
Mx at B	1.04	1.22

Figure 5.29 SIMULATION OF FIXED END CONDITIONS

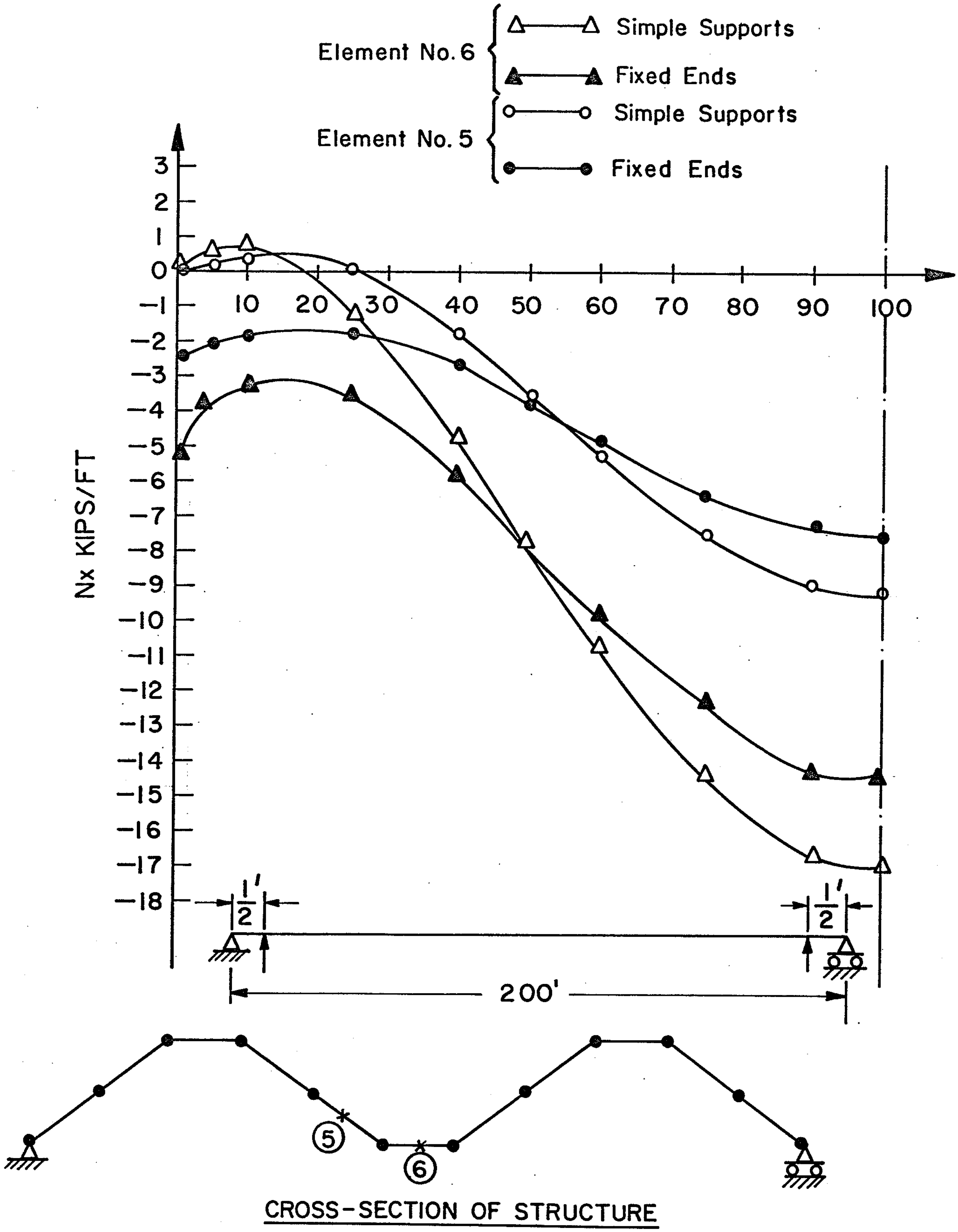


Figure. 5.30 VARIATION OF IN PLANE FORCE Nx

force N_x for the mid-points of elements (5) and (6) is shown and compared with N_x obtained in the case of simple supports in Fig. 5.30.

CHAPTER VI

CONCLUSIONS

6.1 Discussion of the Results

From the various tests and comparisons in Chapter V the validity of the programme was confirmed subject to certain limitations. In this chapter it is proposed to intepret the results of the various tests and form conclusions regarding usuage, accuracy and limitations of the programme. An important feature to note in the various results was the good degree of compatibility obtained between inter-element joint forces. It had been much more than expected. Use of finer strips will improve the continuity of output. For structures subject to uniformly distributed loads a width of 3 to 4 feet is recommended as a maximum. For concentrated loads, the strip width should be made smaller near the load but could be up to 5 feet away from it.

(a) In test case (a) the mid-span bending moment in the transverse direction Ms was chosen for comparison of results of "Archfold" with the classical analysis of Shah and Lansdown [7] and with "Curstr" [6]. Ms was chosen as it was much larger than Mx and sensitive to changes. The assumption of a shallow shell imposed a limitation of overall rise-to-span ratio of $\frac{1}{5}$ giving 0.02 as the maximum value for $\frac{1}{R}$. Comparisons were made for $\frac{1}{R} = 0.0125$ and 0.02. It could be seen that within the permissible range of $\frac{1}{R}$ very good agreement is obtained. Thus the maximum curvature of the structures that could be analysed by "Archfold" is given by $\frac{1}{R} = XR = 0.02$. It is necessary to emphasize that the classical analysis apart from being tedious has only limited applications while "CURSTR" is essentially dealing with shells of revolution.

(b) The test performed to determine the maximum limit in the angle of inclination of elements yielded very useful information. The maximum limit on ϕ was found to be approximately 40° for the results to agree well with "CURSTR". There is justification on this conclusion as "CURSTR" had been tested experimentally and by comparison with classical analysis. Thus, "Archfold" (utilizing shallow shell theory) can analyse elements having an inclination of about 40° quite accurately.

(c) It was mentioned in Chapter I that an arched construction is able to resist the external loads more effectively. This is shown in Figs. 5.11, 5.12, 5.13, and 5.14. The four cases considered had $\frac{H}{L}$ ratios 0, $\frac{1}{20}$, $\frac{1}{10}$ and $\frac{1}{5}$. In Fig. 5.11 the reduction in midspan M_s is clearly seen due to the effect of arching. The reduction in the maximum value is about 32%. The maximum values of M_x were at midspan. In Fig. 5.12 the reduction in M_x is seen which is about 35% at the highest value. Fig. 5.13 shows that while there is no appreciable change in N_s, N_x has increased due to arching. However, the increase is sufficiently small and does not exceed the value needed to provide reinforcement in excess of the nominal amount. Fig. 5.14 shows the normal deflections across the structure. Since the transverse deflections are very small, these deflections can also represent the vertical deflections of the structures at midspan to a good degree of accuracy. The reduction in the deflections is also reflected in a reduction in the bending moments. The maximum reduction is about 27%.

(d) In tests (d) the accuracy of the analysis is compared with "CURSTR" for the variation in "arching" (i.e., the maximum rise H). The cases considered had H = 16' and H = 8' and H = 4'. Comparison of mid-span values of N_s and N_x are shown in Fig. 5.15 and it can be seen that for the maximum permissible value of XR = 0.02 (H = 16') the agreement is excellent. In Fig. 5.16 the very good agreement in mid-span M_s and M_x are seen except for a 10% deviation in the maximum values of M_s for the case XR = 0.02 (H = 16').

(e) The effect of $\frac{h}{L}$ on M_s is studied in tests (e). It is very clearly seen that for constant h, an increase in span will result in an increase in M_s. The increase is true for all other internal forces also.

(f) It is possible to analyse structures with very long spans having intermediate supports. This gives an increased use for "Archfold". In the simple case with free transverse edges, the large reductions in M_s, M_x, and N_x are noteworthy.

(g) By inputting XR = 0, it is possible to utilize "Archfold" for analysing straight conventional folded plate structures. In the simple example of the box girder bridge excellent agreement of results were obtained when compared with "MUPDI 3"[4]. Thus, a single programme can be used to analyse both arched and straight folded plate structures.

(h) The application of "Archfold" to analyse a typical arched folded plate structure is seen in section 5.5. The cross-section can be quite arbitary within the limit for the inclination angle of the plates. The output of all the necessary in-plane forces and bending moments facilitate the design of the structure.

(i) The success of the simulation of fixed end conditions in a structure can be confirmed by the agreement of the results with Timoshenko [10]. Hence, "Archfold" can be utilized to analyse folded plate structures having simple or fixed end supports. In Fig. 5.30 it can be clearly seen that there is a reduction in the range of variation of N_x due to fixed edges.

6.2 Suggestions For Further Work

Within the limits of the shallow shell conditions, "Archfold" is able to analyse a great majority of structures that can be found in practice. Since the structure can also be analysed for any type of loading, it can be seen that further work on static analysis, would serve little purpose. However, work on dynamic analysis of (Arched) folded plate structures would be done using the finite strip method. The derivation of the mass matrix by this method will be quite straight forward. Arched folded plates subjected to seismic loads will form a useful study as an extension to that already presented in this thesis.

6.3 Conclusions

The results of "Archfold" were compared with classical analysis [7], "CURSTR" [6] and "MUPDI 3" [4]. The very good agreement obtained in all cases indicate the validity of the analysis procedure and the programme. This is conclusive due to the fact that the classical analysis is the closest to the exact solution, while "CURSTR" [6] had been tested both with experiment and by comparison with various classical analyses.

"MUPDI 3" [4] also utilizes the classical approach of Goldberg and Leve [1]. Of the three programmes "Archfold", CURSTR" and "MUPDI 3", only "Archfold" could analyse both straight and arched folded plate structures.

The subroutine generating the stiffness matrix in "Archfold" is very much smaller than that in "MUPDI 3" and in "CURSTR". CURSTR deals with a shell of revolution and utilizes numerical integration to generate the stiffness matrix. In the analyses used in programming "Archfold" closed form solutions of all the integrals were obtained for the derivation of the stiffness matrix.

It is emphasized that as in any finite strip or finite element solution, the nature of the method of analysis requires caution in the intepretation of the results. In particular it should be noted that differential equilibrium and force boundary conditions are not satisfied, resulting in such inconsistencies as the small unbalanced edge moments or in stress discontinuities at inter-element faces. These discontinuities can be reduced by making the element widths smaller. However, the continuity of inter-element joint forces has been quite satisfactory. Inter-element discontinuities may be treated either by averaging over adjacent elements or choosing mid-element output quantities as representative values. The results presented in this work utilize the latter option.

In addition to being able to analyse both arched and straight folded plate structures, the programme "Archfold" presented in this thesis has the following features:

(1) ability to deal with any type of loading on the structure,

(2) capability of analysing structures with intermediate supports, and

(3) possibility of simulating fixed end conditions.

It is hoped that the work presented in this thesis will be a useful addition to the field of folded plate and shell structures.

REFERENCES

1. Goldberg, J.E. and Leve, H.L. "Theory of Prismatic Folded Plate Structures", IABSE Vol. 17, 1957, pp. 59-86.

2. De Fries, A. and Scordelis, A.C. "Direct Stiffness Solution of Folded Plates", Journal of the Structural Division, A.S.C.E., Volume 90, No. ST4, Proc. Paper 3994, August, 1964, pp. 15-46.

3. Phase I report of the task committee on folded plate construction. Journal of the Structural Division, A.S.C.E., Volume 89, No. ST6, Proc. Paper 3741, December, 1963, pp. 365-406.

4. Lin, C.S. and Scordelis, A.C. "Computer Programme for Bridges on Flexible Bents", Report of structural engineering and structural mechanics, University of California, Report No. UC SESM 71-24, December, 1971.

5. Cheung, Y.K. "Folded Plate Structures by Finite Strip Method", Journal of the Structural Division, A.S.C.E., Volume 95, No. ST12, Proc. Paper 6985, December, 1969, pp. 2963-2979.

6. Meyer, C. and Scordelis, A.C. "Analysis of Curved Folded Plate Structures", Journal of the Structural Division, A.S.C.E., Volume 97, No. ST10, Proc. Paper 8434, October, 1971, pp. 2459-2480.

7. Shah, A.H. and Lansdown, A.M. "Shell Structures and Climatic Influences", I.A.S.S., July, 1972, pp. 227-234.

8. Munro, J. "The Linear Analysis of Thin Shallow Shells", Institution of Civil Engineers, Volume 19, Paper No. 6517, July, 1961, pp. 291-306.

9. Novozhilov, V.V. "Theory of Thin Shells" 2nd edition, P. Noovdhoff, Ltd., Groninsen, Netherlands, 1964.

10. Timoshenko, S and Krieger, S.W. "Theory of Plates and Shells" 2nd edition, McGraw-Hill Book Company, 1959.

APPENDIX A

Interpolation functions

Linear variation of the in-plane displacement components (u and v) and cubic variation of the normal displacement component (w) were assumed in section 2.3.

The figure A.1 below shows the variation of the longitudinal displacement component u.

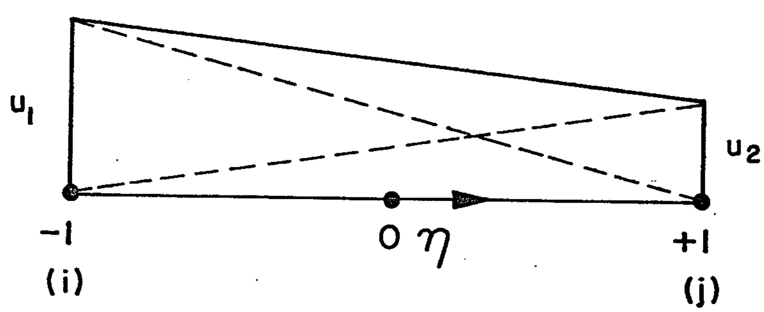


Figure A.1 VARIATION OF u WITH η.

If u_1, u_2 are the longitudinal displacements at nodes i and j of an element, we have at any point

$$u(\eta) = a + b\eta. \tag{A.1}$$

where a and b are constants since $u = u_1$ when $\eta = -1$ and $u = u_2$ when $\eta = +1$.

$$u_1 = a - b \qquad a = \frac{u_1 + u_2}{2}, \quad b = \frac{u_2 - u_1}{2}$$

$$u_2 = a + b$$

Therefore

$$u(\eta) = \frac{u_1 + u_2}{2} + \frac{u_2 - u_1}{2}\,\eta = \frac{1}{2}\,(1-\eta)\;u_1 + \frac{1}{2}\,(1+\eta)\;u_2 \qquad \text{(A.2)}$$

For displacement compent v in the transverse direction, the same function is chosen. i.e.,

$$v(\eta) = \frac{1}{2}\,(1-\eta)\;v_1 + \frac{1}{2}\,(1+\eta)\;v_2 \qquad \text{(A.3)}$$

For displacements normal to the plane of the element a cubic variation is assumed so as to include the rotation components $\omega = (\frac{dw}{ds})$ also. Thus

$$w = a + b\eta + c\eta^2 + d\eta^3 \qquad \text{(A.4)}$$

where a, b, c, and d are constants.

This variation is shown below in Fig. A.2.

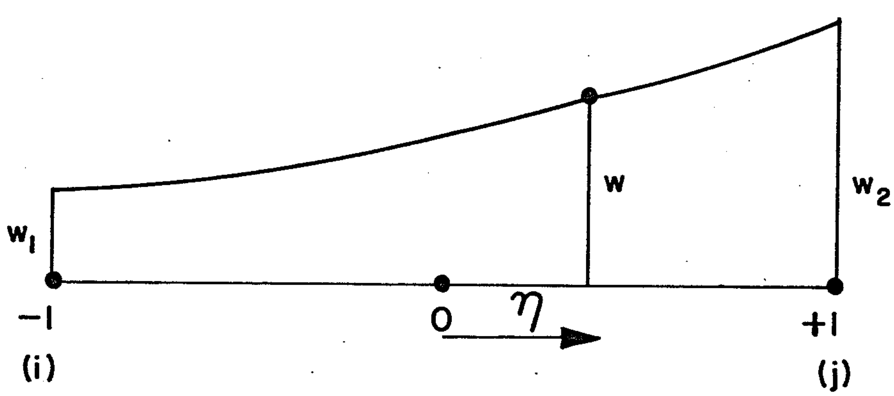


Figure A.2 VARIATION OF w WITH η.

Substituting the nodal values of w in (A.4) for $\eta = -1$ and $\eta = +1$

$$w_1 = a - b + c - d \tag{A.5}$$

$$w_2 = a + b + c + d \tag{A.6}$$

$$\omega = \frac{dw}{ds} = \frac{dw}{d\eta} \cdot \frac{d\eta}{ds} = \frac{2}{S_{12}} \frac{dw}{d\eta} \tag{A.7}$$

where S_{12} = plate width.

From equations (A.4) and (A.7)

$$\frac{S_{12}}{2} \omega = \frac{dw}{d\eta} = b + 2c\eta + 3d\eta^2 \tag{A.8}$$

Substituting nodal values of the rotation ω in equation (A.8), we have

$$\frac{S_{12}}{2} \omega_1 = b - 2c + 3d \tag{A.9}$$

$$\frac{S_{12}}{2} \omega_2 = b + 2c + 3d \tag{A.10}$$

Equations (A.5) and (A.6) gives

$$w_1 - w_2 = -2(b + d) \tag{A.11}$$

Equations (A.9) and (A.10) give

$$\frac{S_{12}}{2} (\omega_1 + \omega_2) = 2(b + 3d) \tag{A.12}$$

From equations (A.11) and (A.12)

$$\frac{S_{12}}{2} (\omega_1 + \omega_2) + (w_1 + w_2) = 4d \tag{A.13}$$

therefore

$$d = \frac{1}{4} [w_1 - w_2 + \frac{S_{12}}{2} \omega_1 + \frac{S_{12}}{2} \omega_2] \tag{A.14}$$

From equation (A.11)

$$b = \frac{w_2 - w_1}{2} - d = \frac{1}{4}\left[-3w_1 + 3w_2 - \frac{S_{12}}{2}\omega_1 - \frac{S_{12}}{2}\omega_2\right] \tag{A.15}$$

and from equation (A.9)

$$2c = b + 3d - \frac{S_{12}}{2}\omega_1 = \frac{S_{12}}{4}(\omega_1 + \omega_2) - \frac{S_{12}}{2}\omega_1$$

from equation (A.12). Therefore

$$c = \frac{S_{12}}{8}(-\omega_1 + \omega_2) \tag{A.16}$$

From equation (A.5),

$$a = w_1 + b + d - c$$

$$= w_1 + \left(\frac{w_2 - w_1}{2}\right) - \frac{S_{12}}{8}(-\omega_1 + \omega_2)$$

from equations (A.12), (A.14) and (A.15). Therefore

$$a = \frac{w_1 + w_2}{2} + \frac{S_{12}}{8}(\omega_1 - \omega_2)$$

therefore

$$w = \left[\frac{w_1 + w_2}{2} + \frac{S_{12}}{8}(\omega_1 - \omega_2)\right] + \frac{\eta}{4}\left[-3w_1 + 3w_2 - \frac{S_{12}}{2}\omega_1 - \frac{S_{12}}{2}\omega_2\right] + \\ + \frac{S_{12}\eta^2}{8}(-\omega_1+\omega_2) \\ + \frac{\eta^3}{4}\left[w_1 - w_2 + \frac{S_{12}}{2}\omega_1 + \frac{S_{12}}{2}\omega_2\right]$$

$$w = \frac{w_1}{4}(2-3\eta+\eta^3) + \frac{w_2}{4}(2+3\eta-\eta^3) + \omega_1\frac{S_{12}}{8}(1-\eta-\eta^2+\eta^3)$$

$$+ \omega_2\frac{S_{12}}{8}(-1-\eta+\eta^2-\eta^3) \tag{A.16}$$

That is

$$w = \frac{1}{4} \left< (2-3\eta+\eta^3) \quad (2+3\eta-\eta^3) \quad \frac{S_{12}}{2} (1-\eta-\eta^2+\eta^3) \quad \frac{S_{12}}{8} (-1-\eta+\eta^2-\eta^3) \right> \begin{Bmatrix} w_1 \\ w_2 \\ \omega_1 \\ \omega_2 \end{Bmatrix}$$

The interpolation functions for the displacement components u, v and w are therefore given by

$$< \phi_u > = \frac{1}{2} < (1 - \eta) \quad (1 + \eta) >$$

$$< \phi_v > = \frac{1}{2} < (1 - \eta) \quad (1 + \eta) >$$

$$< \phi_w > = \frac{1}{4} < (2-3\eta+\eta^3) \quad (2+3\eta-\eta^3) \quad \frac{S_{12}}{2} (1-\eta-\eta^2+\eta^3) \quad \frac{S_{12}}{2} (-1-\eta+\eta^2-\eta^3) >$$

In the load analysis similar functions are assumed for variation of load components between the nodes in the transverse direction.

APPENDIX B

<u>Stifness matrix for an element of a shell of translation</u>

The stiffness matrix is given by

$$[k]_n = \frac{\ell S_{12}}{4} \int_{-1}^{+1} [\bar{T}]_n^T \, [D] \, [\bar{T}]_n \, d\eta \;, \tag{B.1}$$

$$= \frac{\ell S_{12}}{4} \begin{bmatrix} [k_{uu}] & [k_{uv}] & [k_{uw}] \\ & [k_{vv}] & [k_{vw}] \\ \text{symm} & & [k_{ww}] \end{bmatrix} \tag{B.2}$$

k_{uu} is $[\;\;]_{2x2}$ k_{uv} is $[\;\;]_{2x2}$ k_{uw} is $[\;\;]_{2x4}$ k_{vv} is $[\;\;]_{2x2}$ k_{vw} is $[\;\;]_{2x4}$ and k_{ww} is $[\;\;]_{4x4}$.

$$[\bar{T}]_n = \begin{bmatrix} -\frac{n\pi}{\ell} <\phi_u> & 0 & -\frac{1}{R} <\phi_w> \\ 0 & <\frac{\partial\phi_v}{\partial s}> & 0 \\ <\frac{\partial\phi_u}{\partial s}> & \frac{n\pi}{\ell} <\phi_v> & 0 \\ 0 & 0 & (\frac{n\pi}{\ell})^2 <\phi_w> \\ 0 & 0 & -<\frac{\partial^2\phi}{\partial s^2}> \\ 0 & 0 & -\frac{2n\pi}{\ell} <\frac{\partial\phi_w}{\partial s}> \end{bmatrix}_{6x8}$$

$$[D] = \begin{bmatrix} d_{11} & d_{12} & 0 & 0 & 0 & 0 \\ d_{21} & d_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & d_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{44} & d_{45} & 0 \\ 0 & 0 & 0 & d_{54} & d_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{66} \end{bmatrix}_{6x6}$$

d_{ij} corresponds to D_{ij} of Chapter III ie $d_{ij} = D_{ij}$

therefore

$$[\bar{T}]_n^T\ [D]$$

$$= \begin{bmatrix} -\frac{n\pi}{\ell}\langle\phi_u\rangle^T d_{11} & -\frac{n\pi}{\ell}\langle\phi_u\rangle^T d_{12} & \langle\phi_u'\rangle^T d_{33} \\ -\langle\phi_v'\rangle^T d_{21} & \langle\phi_v'\rangle^T d_{22} & \frac{n\pi}{\ell}\langle\phi_v\rangle^T d_{33} \\ -\frac{1}{R}\langle\phi_w\rangle^T d_{11} & -\frac{1}{R}\langle\phi_w\rangle^T d_{12} & 0 \end{bmatrix.$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \begin{Bmatrix} (\frac{n\pi}{\ell})^2\langle\phi_w\rangle^T d_{44} \\ - \\ \langle\phi_w''\rangle^T d_{54} \end{Bmatrix} & \begin{Bmatrix} (\frac{n\pi}{\ell})^2\langle\phi_w\rangle^T d_{45} \\ - \\ \langle\phi_w''\rangle^T d_{55} \end{Bmatrix} & -\frac{2n\pi}{\ell}\langle\phi'\rangle^T d_{66} \end{bmatrix}_{8\mathrm{x}6}$$

and

$$[k]_n = \frac{\ell S_{12}}{4}\int_{-1}^{+1} [\bar{T}]_n^T\ [D]\ [\bar{T}]_n\ d\eta$$

$$= \frac{\ell S_{12}}{4}\int_{-1}^{+1} \begin{bmatrix} (\frac{n\pi}{\ell})^2\phi_u^T\phi_u d_{11} + \phi_u'^T\phi_u' d_{33} & -\frac{n\pi}{\ell}\phi_u^T\phi_v' d_{12} + \frac{n\pi}{\ell}\phi_u'^T\phi_v d_{33} \\ -\frac{n\pi}{\ell}\phi_v'^T\phi_u d_{21} + \frac{n\pi}{\ell}\phi_v^T\phi_v' d_{33} & \phi_v'^T\phi_v' d_{22} + (\frac{n\pi}{\ell})^2\phi_v^T\phi_v d_{33} \\ \frac{n\pi}{\ell}\frac{d_{11}}{R}\phi_w^T\phi_u & \end{bmatrix.$$

$$\left.\begin{array}{l}
\frac{n\pi}{\ell}\frac{d_{11}}{R}\phi_u^T\phi_w \\
-\frac{d_{21}}{R}\phi_v^{'T}\phi_w \\
\frac{d_{11}}{R^2}\phi_w^T\phi_w + (\frac{n\pi}{\ell})^4 d_{44}\phi_w^T\phi_w \\
-(\frac{n\pi}{\ell})^2 d_{54}\phi_w^{''T}\phi_w - (\frac{n\pi}{\ell})^2 d_{45}\phi_w^T\phi_w^{''} \\
+ d_{55}\phi_w^{''T}\phi_w^{''} + 4(\frac{n\pi}{\ell})^4 d_{66}\phi_w^{''T}\phi_w^{'}
\end{array}\right]_{8x8} \tag{B.3}$$

omitting the brackets for the displacement function vectors.

(1) From equation (B.2)

$$[k_{uu}]_n = \frac{\ell S_{12}}{4}\int_{-1}^{+1}(\frac{n\pi}{\ell})^2 d_{11}\phi_u^T\phi_u^{'T}\phi_u^{'}\,d\eta \tag{B.4}$$

since

$$\phi_u = \frac{1}{2}<(1-\eta)(1+\eta)>, \quad \phi_u^{'} = \frac{d\phi_u}{ds} = \frac{1}{S_{12}}<-1 \quad 1>$$

therefore

$$\phi_u^T\phi_u = \frac{1}{4}\begin{bmatrix}(1-2\eta+\eta^2) & (1-\eta^2) \\ (1-\eta^2) & (1+2\eta+\eta^2)\end{bmatrix}$$

and

$$\int_{-1}^{+1}\phi_u^T\phi_u d\eta = \frac{1}{4}\begin{bmatrix}\eta+\frac{\eta^3}{3} & \eta-\frac{\eta^3}{3} \\ \eta-\frac{\eta^3}{3} & \eta+\frac{\eta^3}{3}\end{bmatrix}_{-1}^{+1} = \frac{1}{3}\begin{bmatrix}2 & 1 \\ 1 & 2\end{bmatrix} \tag{B.5}$$

(the even powers of η give zero in the limits $-1 \rightarrow +1$).

$$\phi_u'^T\phi_u' = \frac{1}{S_{12}^2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

therefore

$$\int_{-1}^{+1} \phi_u'^T\phi_u d\eta = \frac{1}{S_{12}^2}\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix} \quad \text{(B.6)}$$

From equations (B.3), (B.4) and (B.5) we get

$$[k_{uu}]_n = \frac{\ell S_{12}}{4}\begin{bmatrix}(\frac{n\pi}{\ell})^2 d_{11}\ \frac{2}{3} + \frac{2d_{33}}{S_{12}^2} & (\frac{n\pi}{\ell})^2 d_{11}\ \frac{1}{3} - \frac{2}{S_{12}^2}\, d_{33}\\ (\frac{n\pi}{\ell})^2 d_{11}\ \frac{1}{3} - \frac{2}{S_{12}^2}\, d_{33} & (\frac{n\pi}{\ell})^2 d_{11}\ \frac{2}{3} + \frac{2}{S_{12}^2}\, d_{33}\end{bmatrix}_{2x2} \quad \text{(B.7)}$$

From equations (B.2) and (B.3) we have

$$(2) \qquad [k_{uv}]_n = \frac{\ell S_{12}}{4}\int_{-1}^{+1}\{-\frac{n\pi}{\ell}\,\phi_u^T\phi_v' d_{12} + \frac{n\pi}{\ell}\phi_u'^T\phi_v d_{33}\}d\eta \quad \text{(B.8)}$$

since

$$\phi_u = \phi_v\ ,\ \phi_u^T\phi_v' = \phi_u^T\phi_u' = \frac{1}{2}\begin{Bmatrix}1-\eta\\ 1+\eta\end{Bmatrix}\frac{1}{S_{12}} \langle -1 \quad 1 \rangle$$

$$= \frac{1}{2S_{12}}\begin{bmatrix}-1+\eta & 1-\eta\\ -1-\eta & 1+\eta\end{bmatrix}$$

therefore

$$\int_{-1}^{+1} \phi_u^T \phi_v' d\eta = \frac{1}{2S_{12}} \int_{-1}^{+1} \begin{bmatrix} -1+\eta & 1-\eta \\ -1-\eta & 1+\eta \end{bmatrix} d\eta = \frac{1}{S_{12}} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \tag{B.9}$$

$$\phi_u'^T \phi_v = \frac{1}{2S_{12}} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \langle (1-\eta)\,(1+\eta) \rangle = \frac{1}{2S_{12}} \begin{bmatrix} -1+\eta & -1-\eta \\ 1-\eta & 1+\eta \end{bmatrix}$$

therefore

$$\int_{-1}^{+1} \phi_u'^T \phi_v d\eta = \frac{1}{2S_{12}} \begin{bmatrix} -\eta & -\eta \\ \eta & \eta \end{bmatrix}_{-1}^{+1} = \frac{1}{S_{12}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \tag{B.10}$$

therefore from equations (B.8), (B.9) and (B.10) we get

$$[k_{uv}]_n = \frac{n\pi}{4} \begin{bmatrix} (d_{12} - d_{33}) & (-d_{12} - d_{33}) \\ (d_{12} + d_{33}) & (-d_{12} + d_{33}) \end{bmatrix}_{2x2} \tag{B.11}$$

(3) From equations (B.2) and (B.3) we get

$$[k_{uw}]_n = \frac{n\pi\ S_{12}\ d_{11}}{4R} \int_{-1}^{+1} \phi_u^T \phi_w\ d\eta \tag{B.12}$$

$$\phi_u^T \phi_w = \frac{1}{2} \begin{Bmatrix} (1-\eta) \\ (1+\eta) \end{Bmatrix} \frac{1}{4} \langle (2-3\eta + \eta^3)(2-3\eta - \eta^3)\frac{S_{12}}{2}(1-\eta-\eta^2+\eta^3)\frac{S_{12}}{2}(-1-\eta+\eta^2+\eta^3) \rangle$$

$$= \frac{1}{8} \begin{bmatrix} (2-5\eta+3\eta^2+\eta^3-\eta^4) & (2+\eta-3\eta^2-\eta^3+\eta^4) & \frac{S_{12}}{2}(1-2\eta+2\eta^3-\eta^4) & \frac{S_{12}}{2}(-1+2\eta^2-\eta^4) \\ (2-\eta-3\eta^2+\eta^3+\eta^4) & (2+5\eta+3\eta^2-\eta^3-\eta^4) & \frac{S_{12}}{2}(1-2\eta^2+\eta^4) & \frac{S_{12}}{2}(-1-2\eta+2\eta^3+\eta^4) \end{bmatrix}$$

therefore

$$\int_{-1}^{+1} \phi_u^T \phi_w d\eta = \frac{1}{8}\begin{bmatrix} (2\eta+\eta^3-\frac{\eta^3}{3}) & (2\eta-\eta^3+\frac{\eta^3}{3}) & \frac{S_{12}}{2}(\eta-\frac{\eta^3}{3}) & \frac{S_{12}}{2}(-\eta+\eta\frac{2\eta^3}{3}-\frac{\eta^3}{3}) \\ (2\eta-\eta^3+\frac{\eta^3}{3}) & (2\eta+\eta^3-\frac{\eta^3}{3}) & \frac{S_{12}}{2}(\eta-\eta\frac{2\eta^3}{3}+\frac{\eta^5}{5}) & \frac{S_{12}}{2}(-\eta+\frac{\eta^3}{3}) \end{bmatrix}_{-1}$$

$$= \frac{1}{10}\begin{bmatrix} 7 & 3 & S_{12} & -\frac{2}{3}S_{12} \\ 3 & 7 & \frac{2S_{12}}{3} & -S_{12} \end{bmatrix} \tag{B.13}$$

From equations (B.12) and (B.13)

$$[k_{uw}]_n = \frac{n\pi\ S_{12}\ d_{11}}{40R}\begin{bmatrix} 7 & 3 & S_{12} & -\frac{2}{3}S_{12} \\ 3 & 7 & \frac{2}{3}S_{12} & -S_{12} \end{bmatrix}_{2x4} \tag{B.14}$$

(4) From equations (B.2) and (B.3),

$$[k_{vv}]_n = \frac{\ell S_{12}}{4}\int_{-1}^{+1}\{\phi_v'^T\phi_v' d_{22} + (\frac{n\pi}{\ell})^2\phi_v^T\phi_v d_{33}\}d\eta \tag{B.15}$$

From equations (B.5) and (B.6)

$$\int_{-1}^{+1}\phi_v'^T\phi_v' d\eta = \frac{1}{S_{12}^2}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \quad \text{and} \quad \int_{-1}^{+1}\phi_v^T\phi_v d\eta = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

therefore

$$[k_{vv}]_n = \frac{\ell S_{12}}{4}\begin{bmatrix} \frac{2d_{22}}{S_{12}^2} + \frac{2}{3}(\frac{n\pi}{\ell})^2 d_{33} & -\frac{2d_{22}}{S_{12}^2} + \frac{1}{3}(\frac{n\pi}{\ell})^{1} d_{33} \\ -\frac{2d_{22}}{S_{12}^2} + \frac{1}{3}(\frac{n\pi}{\ell})^2 d_{33} & \frac{2d_{22}}{S_{12}^2} + \frac{2}{3}(\frac{n\pi}{\ell})^2 d_{33} \end{bmatrix} \tag{B.16}$$

(5) From equations (B.2) and (B.3)

$$[k_{vw}]_n = \frac{\ell S_{12}}{4}\int_{-1}^{+1} - \frac{d_{21}}{R}\,\phi_v'^T\phi_w d\eta \qquad (B.17)$$

$$\phi_v'^T\phi_w = \frac{1}{4S_{12}}\begin{bmatrix} (-2+3\eta-\eta^3)(-2-3\eta+\eta^3) & \frac{S_{12}}{2}(-1+\eta+\eta^2-\eta^3) & \frac{S_{12}}{2}(1+\eta-\eta^2-\eta^3) \\ (2-3\eta+\eta^3)(2+3\eta-\eta^3) & \frac{S_{12}}{2}(1-\eta-\eta^2+\eta^3) & \frac{S_{12}}{2}(-1-\eta+\eta^2+\eta^3) \end{bmatrix}$$

therefore

$$\int_{-1}^{+1}\phi_v'^T\phi_w d\eta = \frac{1}{4S_{12}}\left[\begin{matrix} -2\eta & -2\eta & \frac{S_{12}}{2}(-\eta+\frac{\eta^3}{3}) & \frac{S_{12}}{2}(\eta-\frac{\eta^3}{3}) \\ 2\eta & 2\eta & \frac{S_{12}}{2}(\eta-\frac{\eta^3}{3}) & \frac{S_{12}}{2}(-\eta+\frac{\eta^3}{3}) \end{matrix}\right| =$$

$$= -\frac{1}{2S_{12}}\begin{bmatrix} 2\eta & -2\eta & \frac{S_{12}}{3} & -\frac{S_{12}}{3} \\ -2\eta & -2\eta & -\frac{S_{12}}{3} & \frac{S_{12}}{3} \end{bmatrix} \qquad (B.18)$$

From equations (B.7) and (B.18) we get

$$[k_{vw}]_n = \frac{\ell d_{12}}{8R}\begin{bmatrix} 2\eta & 2\eta & \frac{S_{12}}{3} & -\frac{S_{12}}{3} \\ -2\eta & -2\eta & -\frac{S_{12}}{3} & \frac{S_{12}}{3} \end{bmatrix} \qquad (B.19)$$

(6) From equations (B.2) and (B.3) we have

$$[k_{ww}]_n^1 = \frac{\ell S_{12}}{4}\int_{-1}^{+1}\{\frac{d_{11}}{R^2}\,\phi_w^T\phi_w + (\frac{n\pi}{\ell})^2 d_{44}\phi_w^T\phi_w\}d\eta \qquad (B.20)$$

$$\phi_w^T\phi_w = \frac{1}{16}\begin{Bmatrix} (2-3\eta+\eta^3) \\ (2+3\eta-\eta^3) \\ \frac{S_{12}}{2}(1-\eta-\eta^2+\eta^3) \\ \frac{S_{12}}{2}(-1-\eta+\eta^2+\eta^3) \end{Bmatrix} \left\langle (2-3\eta+\eta^3)\ (2+3\eta-\eta^3)\ \ \frac{S_{12}}{2}(1-\eta-\eta^2+\eta^3)\ \frac{S_{12}}{2}(-1-\eta^2+\eta^3) \right\rangle$$

$$= \frac{1}{16}\left[\begin{array}{ll} (4-12\eta+9\eta^2+4\eta^3-6\eta^4+\eta^6) & (4-9\eta^2+6\eta^4-\eta^6) \ \bullet \\ (4-9\eta^2+6\eta^4-\eta^6) & (4+12\eta^3+9\eta^2-6\eta^4+\eta^6) \ \bullet \\ \frac{S_{12}}{2}(2-5\eta+\eta^6+6\eta^4-4\eta^4-\eta^5+\eta^6) & \frac{S_{12}}{2}(2+\eta-5\eta^2-2\eta^3+4\eta^4+\eta^5-\eta^6) \ \bullet \\ \frac{S_{12}}{2}(-2+\eta+5\eta^2-2\eta^3-4\eta^4+\eta^5+\eta^6) & \frac{S_{12}}{2}(-2-5\eta-\eta^2+6\eta^3+4\eta^4-\eta^5-\eta^6) \ \bullet \end{array}\right.$$

$$\left.\begin{array}{ll} \bullet\ \frac{S_{12}}{2}(2-5\eta+\eta^2+6\eta^3-4\eta^4-\eta^5+\eta^6) & \frac{S_{12}}{2}(-2+\eta+5\eta^2-2\eta^3-4\eta^4+\eta^5+\eta^6) \\ \bullet\ \frac{S_{12}}{2}(2+\eta-5\eta^2-2\eta^3+4\eta^4+\eta^5-\eta^6) & \frac{S_{12}}{2}(-2-5\eta-\eta^6+6\eta^3+4\eta^4-\eta^5-\eta^6) \\ \bullet\ \frac{S_{12}^2}{4}(1-2\eta-\eta^2+4\eta^3-\eta^4-2\eta^5+\eta^6) & \frac{S_{12}}{2}(-1+3\eta^2-3\eta^4+\eta^6) \\ \bullet\ \frac{S_{12}}{4}(-1+3\eta^2-3\eta^4+\eta^6) & \frac{S_{12}^2}{4}(1+2\eta-\eta^2-4\eta^3-\eta^4+2\eta^5+\eta^6) \end{array}\right]$$

therefore

$$\int_{-1}^{+1}\phi_w^T\phi_w d\eta = \frac{1}{16}\left[\begin{array}{ll} (4\eta+3\eta^3-\frac{6\eta^5}{5}+\frac{\eta^7}{7}) & (4\eta-3\eta^3+\frac{6\eta^5}{5}-\frac{\eta^7}{7}) \ \bullet \\ (4\eta-3\eta^3+\frac{6\eta^5}{5}-\frac{\eta^7}{7}) & (4\eta+\eta 3\eta^2\eta-\frac{6\eta^5}{5}+\frac{\eta^7}{7}) \ \bullet \\ \frac{S_{12}}{2}(2\eta+\frac{\eta^3}{3}-\frac{4\eta^5}{5}+\frac{\eta^7}{7}) & \frac{S_{12}}{2}(2\eta-\frac{5\eta^5}{5}+\frac{4\eta^5}{5}-\frac{\eta^7}{7}) \ \bullet \\ \frac{S_{12}}{2}(-2\eta+\frac{5\eta^3}{3}+\frac{5\eta^5}{5}+\frac{\eta^7}{7}) & \frac{S_{12}}{2}(-2\eta-\frac{\eta^3}{3}+\frac{4\eta^5}{5}-\frac{\eta^7}{7}) \ \bullet \end{array}\right.$$

$$\left.\begin{array}{ll} \bullet\ \frac{S_{12}}{2}(2\eta + \frac{\eta^3}{3} - \frac{4\eta^5}{5} + \frac{\eta^7}{7}) & \frac{S_{12}}{2}(-2\eta + \frac{5\eta^3}{3} - \frac{4\eta^5}{5} + \frac{\eta^7}{7}) \\ \bullet\ \frac{S_{12}}{2}(2\eta - \frac{5\eta^3}{3} + \frac{4\eta^5}{5} + \frac{\eta^7}{7}) & \frac{S_{12}}{2}(-2\eta - \frac{\eta^3}{3} + \frac{4\eta^5}{5} - \frac{\eta^7}{7}) \\ \bullet\ \frac{S^2_{12}}{4}(\eta - \frac{\eta^3}{3} - \frac{\eta^5}{5} + \frac{\eta^7}{7}) & \frac{S^2_{12}}{4}(-\eta + \eta^3\eta - \frac{3\eta^5}{5} + \frac{\eta^7}{7}) \\ \bullet\ \frac{S^2_{12}}{4}(-\eta + \eta^3 - \frac{3\eta^5}{5} + \frac{\eta^7}{7}) & \frac{S^2_{12}}{4}(\eta - \frac{\eta^3}{3} - \frac{\eta^5}{5} + \frac{\eta^7}{7}) \end{array}\right]_{-1}^{+1}$$

$$= \frac{1}{8}\begin{bmatrix} \frac{208}{35} & \frac{72}{35} & \frac{88S_{12}}{105} & -\frac{52S_{12}}{105} \\ \frac{72}{35} & \frac{208}{35} & \frac{52S_{12}}{105} & -\frac{88S_{12}}{105} \\ \frac{88S_{12}}{105} & \frac{52S_{12}}{105} & \frac{16^2_{12}}{105} & -\frac{4S^2_{12}}{35} \\ -\frac{52S_{12}}{105} & -\frac{88S_{12}}{105} & -\frac{4S^2_{12}}{35} & \frac{16S^2_{12}}{105} \end{bmatrix}$$

$$= \frac{1}{70}\begin{bmatrix} 52 & 18 & \frac{22S_{12}}{3} & -\frac{13S_{12}}{3} \\ 18 & 52 & \frac{13S_{12}}{3} & -\frac{22S_{12}}{3} \\ \frac{22S_{12}}{3} & \frac{13S_{12}}{3} & \frac{4S^2_{12}}{3} & -S^2_{12} \\ -\frac{13S_{12}}{105} & -\frac{22S_{12}}{3} & -S^2_{12} & \frac{4S^2_{12}}{3} \end{bmatrix} \tag{B.21}$$

From equations (B.20) and (B.21) we get

$$[k_{ww}]^1_n = A1\begin{bmatrix} 52 & 18 & \frac{22S_{12}}{3} & -\frac{13S_{12}}{3} \\ 18 & 52 & \frac{13S_{12}}{3} & -\frac{22S_{12}}{3} \\ \frac{22S_{12}}{3} & \frac{13S_{12}}{3} & \frac{4S^2_{12}}{3} & -S^2_{12} \\ -\frac{13S_{12}}{3} & -\frac{22S_{12}}{3} & -S^2_{12} & \frac{4}{3}S^2_{12} \end{bmatrix} \tag{B.22}$$

where

$$A \;=\; \frac{\ell S_{12}}{280}\{\frac{d_{11}}{R^2} + (\frac{n\pi}{\ell})^4 \; d_{44}\}$$

(6.2) From equations (B.2) and (B.3) we have

$$[k_{ww}]_n^2 = \frac{\ell S_{12}}{4}\int_{-1}^{+1} -(\frac{n\pi}{\ell})^2 d_{45}\{\phi_w''^T\phi_w + \phi_w^T\phi_w''\}d\eta \tag{B.23}$$

$$\phi_w''^T\phi_w = \frac{1}{4S_{12}^2}\begin{Bmatrix} 6\eta \\ -6\eta \\ S_{12}(3\eta-1) \\ S_{12}(3\eta+1) \end{Bmatrix} \left\langle (2-3\eta+\eta^3)\;(2+3\eta-\eta^3)\;\frac{S_{12}}{2}(1-\eta-\eta^2+\eta^3)\;\frac{S_{12}}{2}(-1-\eta+\eta^2+\eta^3) \right\rangle$$

$$= \frac{1}{4S_{12}^2}\begin{bmatrix} (12\eta-18\eta^2+6\eta^4) & (12\eta+18\eta^2-6\eta^4) & 3S_{12}(\eta-\eta^2-\eta^3+\eta^4) & 3S_{12}(-\eta-\eta^2+\eta^3+\eta^4) \\ (-12\eta+18\eta^2-6\eta^4) & (-12\eta-18\eta^2+6\eta^4) & 3S_{12}(-\eta+\eta^2+\eta^3-\eta^4) & 3S_{12}(\eta+\eta^2-\eta^3-\eta^4) \\ S_{12}(-2+9\eta-9\eta^2-\eta^3+3\eta^4) & S_{12}(-2+9\eta+9\eta^2-\eta^3-3\eta^4) & \frac{S_{12}^2}{2}(-1+4\eta-2\eta^2-4\eta^3+3\eta^4) & \frac{S_{12}^2}{2}(1-2\eta-4\eta^2+2\eta^3+3\eta^4) \\ S_{12}(2+3\eta-9\eta^2+\eta^3+3\eta^4) & S_{12}(2+9\eta+9\eta^2-\eta^3-3\eta^4) & \frac{S_{12}^2}{2}(1+2\eta-4\eta^2-2\eta^3+3\eta^4) & \frac{S_{12}^2}{2}(-1-4\eta-2\eta^2+4\eta^3+3\eta^4) \end{bmatrix}$$

therefore

$$\int_{-1}^{+1} \phi_w''^T \phi_w \, d\eta = \frac{1}{4S_{12}^2} \left[\begin{array}{cccc} (-6\eta^3 + \frac{6\eta^5}{5}) & (6\eta^3 - \frac{6\eta^5}{5}) & 3S_{12}(-\frac{\eta^3}{3} + \frac{\eta^5}{5}) & 3S_{12}(-\frac{\eta^3}{3} + \frac{\eta^5}{5}) \\ (6\eta^3 - \frac{6\eta^5}{5}) & (-6\eta^3 + \frac{6\eta^5}{5}) & 3S_{12}(\frac{\eta^3}{3} - \frac{\eta^5}{5}) & 3S_{12}(\frac{\eta^3}{3} - \frac{\eta^5}{5} \\ S_{12}(-2\eta-3\eta^3 + \frac{3\eta^5}{5}) & S_{12}(-2\eta+3\eta^3 - \frac{3\eta^5}{5}) & \frac{S_{12}^2}{2}(-\eta-\eta\frac{2\eta^3}{3} + \frac{3\eta^5}{5}) & \frac{S_{12}}{2}(\eta - \frac{4\eta^3}{3} + \frac{3\eta^5}{5}) \\ S_{12}(2\eta - 3\eta^3 + \frac{3\eta^5}{5}) & S_{12}(2\eta + 3\eta^3 - \frac{3\eta^5}{5}) & \frac{S_{12}^2}{2}(\eta - \frac{4\eta^3}{3} + \frac{3\eta^5}{5}) & \frac{S_{12}^2}{2}(-\eta - \frac{\eta^3}{3} + \frac{3\eta^5}{5}) \end{array} \right]_{-1}^{+1}$$

$$= \frac{1}{2S_{12}^2} \begin{bmatrix} -\frac{24}{5} & \frac{24}{5} & -\frac{2S_{12}}{5} & -\frac{2S_{12}}{5} \\ \frac{24}{5} & -\frac{24}{5} & \frac{2S_{12}}{5} & \frac{2S_{12}}{5} \\ -\frac{22S_{12}}{5} & \frac{2S_{12}}{5} & -\frac{8S_{12}^2}{5} & \frac{2S_{12}^2}{5} \\ -\frac{2S_{12}}{5} & \frac{22S_{12}}{5} & \frac{2S_{12}^2}{15} & -\frac{8S_{12}^2}{15} \end{bmatrix}$$

$$= \frac{1}{5S_{12}^2} \begin{bmatrix} -12 & 12 & -S_{12} & -S_{12} \\ 12 & -12 & S_{12} & S_{12} \\ -11S_{12} & S_{12} & -\frac{4S_{12}^2}{3} & \frac{S_{12}^2}{3} \\ -S_{12} & 11S_{12} & \frac{S_{12}^2}{3} & -\frac{4S_{12}^2}{3} \end{bmatrix} \qquad \text{(B.24)}$$

From equations (B.23) and (B.24) we get

$$[k_{ww}]_n^2 = -\left(\frac{n\pi}{\ell}\right)^2 d_{45}\,\frac{\ell S_{12}}{4}\,\frac{1}{5S_{12}^2}\left\{\begin{bmatrix} -12 & 12 & -S_{12} & -S_{12} \\ 12 & -12 & S_{12} & S_{12} \\ -11S_{12} & S_{12} & -\frac{4S_{12}^2}{3} & \frac{S_{12}^2}{3} \\ -S_{12} & 11S_{12} & \frac{S_{12}^2}{3} & -\frac{4S_{12}^2}{3} \end{bmatrix} + \begin{bmatrix} -12 & 12 & -11S_{12} & -S_{12} \\ 12 & -12 & S_{12} & 11S_{12} \\ -S_{12} & S_{12} & -\frac{4S_{12}^2}{3} & \frac{S_{12}^2}{3} \\ -S_{12} & S_{12} & \frac{S_{12}^2}{3} & -\frac{4S_{12}^2}{3} \end{bmatrix}\right\}$$

$$= A2 \begin{bmatrix} -12 & 12 & -6S_{12} & -S_{12} \\ 12 & -12 & S_{12} & 6S_{12} \\ -6S_{12} & S_{12} & -\frac{4S_{12}^2}{3} & \frac{S_{12}^2}{3} \\ -S_{12} & 6S_{12} & \frac{S_{12}^2}{3} & -\frac{4S_{12}^2}{3} \end{bmatrix} \qquad \text{(B.25)}$$

where

$$A2 = -\left(\frac{n\pi}{\ell}\right)^2 \frac{\ell d_{45}}{10S_{12}}$$

From equations (B.2) and (B.3) we have

$$[k_{ww}]_n^3 = \frac{\ell S_{12}}{4}\int_{-1}^{+1} d_{55}\ \phi_w''^T\phi_w'' d\eta \tag{B.26}$$

$$\phi_w''^T\phi_w'' = \frac{1}{S_{12}^4}\begin{Bmatrix} 6\eta \\ -6\eta \\ S_{12}(3\eta-1) \\ S_{12}(3\eta+1)\end{Bmatrix} \langle 6\eta\ \ -6\eta\ \ S_{12}(3\eta-1)\ \ S_{12}(3\eta+1)\rangle$$

$$= \frac{1}{S_{12}^4}\begin{bmatrix} 36\eta^2 & -36\eta^2 & 6S_{12}(3\eta^2-\eta) & 6S_{12}(3\eta^2+\eta) \\ -36\eta^2 & 36\eta^2 & 6S_{12}(-3\eta^2+\eta) & 6S_{12}(-3\eta^2-\eta) \\ 6S_{12}(3\eta^2-\eta) & -6S_{12}(3\eta^2-\eta) & S_{12}^2(9\eta^3-6\eta+1) & S_{12}^2(9\eta^2-1) \\ 6S_{12}(3\eta^3+\eta) & -6S_{12}(3\eta^2+\eta) & S_{12}^2(9\eta^2-1) & S_{12}^2(9\eta^2+6\eta+1)\end{bmatrix} \tag{B.27}$$

therefore from equation (B.26)

$$[k_{ww}]_n^3 = \frac{\ell d_{55}}{4S_{12}^3}\begin{bmatrix} 12\eta^3 & -12\eta^3 & 6S_{12}\eta^3 & 6S_{12}\eta^3 \\ -12\eta^3 & 12\eta^3 & -6S_{12}\eta^3 & -6S_{12}\eta^3 \\ 6S_{12}\eta^3 & -6S_{12}\eta^3 & S_{12}^2(3\eta^3+\eta) & S_{12}^2(3\eta^3-\eta) \\ 6S_{12}\eta^3 & -6S_{12}\eta^3 & S_{12}^2(3\eta-\eta) & S_{12}^2(3\eta^3+\eta)\end{bmatrix}_{-1}^{+1}$$

ie

$$[k_{ww}]_n^3 = A3\begin{bmatrix} 6 & -6 & 3S_{12} & 3S_{12} \\ -6 & 6 & -3S_{12} & -3S_{12} \\ 3S_{12} & -3S_{12} & 2S_{12}^2 & S_{12}^2 \\ 3S_{12} & -3S_{12} & S_{12}^2 & 2S_{12}^2\end{bmatrix} \tag{B.28}$$

where

$$A3 = \frac{\ell d_{55}}{S_{12}^3}$$

Again from equations (B.2) and (B.3) we have

$$[k_{ww}]_n^4 = \ell\, S_{12}\, d_{66} \left(\frac{n\pi}{\ell}\right)^2 \int_{-1}^{+1} \phi_w'^T \phi_w d\eta \qquad \text{(B.29)}$$

$$\phi_w'^T \phi_w = \frac{1}{4S_{12}^2} \begin{Bmatrix} (3\eta^2-3) \\ (-3\eta^2+3) \\ \frac{S_{12}}{2}(3\eta^2-2\eta-1) \\ \frac{S_{12}}{2}(3\eta^2+2\eta-1) \end{Bmatrix} \left\langle (3\eta^2-3)\ (-3\eta^2+3)\ \ \frac{S_{12}}{2}(3\eta^2-2\eta-1)\ \frac{S_{12}}{2}(3\eta^2+2\eta-1) \right\rangle$$

$$= \frac{1}{4S_{12}^2} \begin{bmatrix} (9\eta^4-18\eta^2+9) & (-9\eta^2+18\eta^2-9) & \cdot \\ (-9\eta^4+18\eta^2-9) & (9\eta^4-18\eta^2+9) & \cdot \\ \frac{S_{12}}{2}(9\eta^4-6\eta^3-12\eta^2+6\eta+3) & \frac{S_{12}}{2}(-9\eta^4+6\eta^3+12\eta^2-6\eta-3) & \cdot \\ \frac{S_{12}}{2}(9\eta^4+6\eta^3-12\eta^2-6\eta+3) & \frac{S_{12}}{2}(-9\eta^4-6\eta^3+12\eta^2+6\eta-3) & \cdot \end{bmatrix}$$

$$\begin{bmatrix} \cdot & \frac{S_{12}}{2}(9\eta^4-6\eta^3-12\eta^2+6\eta) & \frac{S_{12}}{2}(9\eta^4+6\eta^2-12\eta^2-6\eta+3) \\ \cdot & \frac{S_{12}}{2}(-9\eta^4+6\eta^3+12\eta^2-6\eta-3) & \frac{S_{12}}{2}(-9\eta^4-6\eta^3+12\eta^2+6\eta-3) \\ \cdot & \frac{S_{12}^2}{2}(9\eta^4-12\eta^3-2\eta^2+4\eta+1) & \frac{S_{12}^2}{2}(9\eta^4-10\eta^2+1) \\ \cdot & \frac{S_{12}^2}{2}(9\eta^4-10\eta^2+1) & \frac{S_{12}^2}{2}(9\eta^4+12\eta^3-2\eta^2-4\eta+1) \end{bmatrix}$$

therefore

$$\int_{-1}^{+1} \phi_w^{'T} \phi_w \, d\eta = \frac{1}{4S_{12}^2} \left[\begin{matrix} (\frac{9\eta^5}{5} - 6\eta^3 + 9\eta) & (-\frac{9\eta^5}{5} - 6\eta^3 - 9\eta) & \frac{S_{12}}{2}(\frac{9\eta^5}{5} - 4\eta^3 + 3\eta) & \frac{S_{12}}{2}(\frac{9\eta^5}{5} - 4\eta^3 + 3\eta) \\ (-\frac{9\eta^5}{5} + 6\eta^3 - 9\eta) & (\frac{9\eta^5}{5} - 6\eta^3 + 9\eta) & \frac{S_{12}}{2}(-\frac{9\eta^5}{5} + 4\eta^3 - 3\eta) & \frac{S_{12}}{2}(-\frac{9\eta^5}{5} + 4\eta^3 - 3\eta) \\ \frac{S_{12}}{2}(\frac{9\eta^5}{5} - 4\eta^3 + 3\eta) & \frac{S_{12}}{2}(-\frac{9\eta^5}{5} + 4\eta^3 - 3\eta) & \frac{S_{12}^2}{4}(-\frac{9\eta^5}{5} - \frac{2\eta^3}{3} + \eta) & \frac{S_{12}^2}{4}(\frac{9\eta^5}{5} - \frac{10\eta^3}{3} + \eta) \\ \frac{S_{12}}{2}(\frac{9\eta^5}{5} - 4\eta^3 + 3\eta) & \frac{S_{12}}{2}(-\frac{9\eta^5}{5} + 4\eta^3 - 3\eta) & \frac{S_{12}^3}{4}(\frac{9\eta^5}{5} - \frac{10\eta^3}{3} + \eta) & \frac{S_{12}^2}{4}(\frac{9\eta^5}{5} - \frac{2\eta^3}{3} + \eta) \end{matrix} \right]_{-1}^{+1}$$

$$= \frac{1}{2S_{12}^2} \begin{bmatrix} \frac{24}{5} & -\frac{24}{5} & \frac{2S_{12}}{5} & \frac{2S_{12}}{5} \\ -\frac{24}{5} & \frac{24}{5} & -\frac{2S_{12}}{5} & -\frac{2S_{12}}{5} \\ \frac{2S_{12}}{5} & -\frac{2S_{12}}{5} & \frac{8S_{12}^2}{5} & -\frac{2S_{12}^2}{5} \\ \frac{2S_{12}}{5} & -\frac{2S_{12}}{5} & -\frac{2S_{12}^2}{5} & \frac{8S_{12}^2}{5} \end{bmatrix} \qquad (B.30)$$

Equations (B.28) and (B.29) give

$$[k_{ww}]_n^4 = A4 \begin{bmatrix} 12 & -12 & S_{12} & S_{12} \\ -12 & 12 & -S_{12} & -S_{12} \\ S_{12} & -S_{12} & \frac{4S_{12}^2}{3} & -\frac{S_{12}^2}{3} \\ S_{12} & -S_{12} & -\frac{S_{12}^2}{3} & \frac{4S_{12}^2}{3} \end{bmatrix} \qquad (B.31)$$

where

$$A4 = \frac{\ell d_{66}}{5S_{12}} \left(\frac{n\pi}{\ell}\right)^2$$

From equations (B.22), B.25), (B.28) and (B.31) we compute

$$[k_{ww}]_n$$

where

$$[k_{ww}]_n = [k_{ww}]_n^1 + [k_{ww}]_n^2 + [k_{ww}]_n^3 + [k_{ww}]_n^4$$

Thus

$$[k_{ww}]_n = \begin{bmatrix} \begin{matrix}52A1 - 12A2 \\ +6A3 + 12A4\end{matrix} & \begin{matrix}18A1 + 12A2 \\ -6A3 - 12A4\end{matrix} & \begin{matrix}\frac{22S_{12}}{3} A1 - 6S_{12}A_2 \\ + 3S_{12}A3 \\ + S_{12}A4\end{matrix} & \begin{matrix}-\frac{13S_{12}}{3} A1 - S_{12}A_2 \\ + 3S_{12} A3 \\ + S_{12}A4\end{matrix} \\ \begin{matrix}18A1 + 12A2 \\ -6A3 - 12A4\end{matrix} & \begin{matrix}52A1 - 12A2 \\ +6A3 + 12A4\end{matrix} & \begin{matrix}\frac{13S_{12}}{3} A1 + S_{12}A2 \\ -3S_{12}A_3 - S_{12}A4\end{matrix} & \begin{matrix}-\frac{22S_{12}}{3} A1 + 6S_{12}A2 \\ -3S_{12}A3 - S_{12}A4\end{matrix} \\ & & & \end{bmatrix}$$

$$\left.\begin{array}{cccc}
\begin{array}{l}\frac{22S_{12}}{3}A1 - 6S_{12}A2\\ +3S_{12}A3 + S_{12}A4\end{array} &
\begin{array}{l}\frac{13S_{12}}{3}A1 + S_{12}A2\\ -3S_{12}A3 - S_{12}Ar\end{array} &
\begin{array}{l}\frac{4}{3}S_{12}^2A1 - \frac{4}{3}S_{12}^2A2\\ +2S_{12}^2A3 + \frac{4}{3}S_{12}^2A4\end{array} &
\begin{array}{l}-S_{12}^2A1 + \frac{S_{12}^2}{2}A2\\ +S_{12}^2A3 - \frac{S_{12}^2}{3}A4\end{array} \\[3em]
\begin{array}{l}-\frac{13S_{12}}{3}A1 - S_{12}A2\\ +3S_{12}A3\\ +S_{12}A4\end{array} &
\begin{array}{l}-\frac{22S_{12}}{3}A1 + 6S_{12}A2\\ -3S_{12}A3\\ -S_{12}A4\end{array} &
\begin{array}{l}-S_{12}^2A1 + \frac{S_{12}^2}{2}A2\\ +S_{12}^2A3\\ -\frac{S_{12}^2}{3}A4\end{array} &
\begin{array}{l}\frac{4}{3}S_{12}^2A1 - \frac{4}{3}S_{12}^2A2\\ +2S_{12}^2A3\\ +\frac{4}{3}S_{12}^2A4\end{array}
\end{array}\right]$$

where

$$A1 = \frac{\ell S_{12}}{280}\{d_{11}(XR)^2 + (\frac{n\pi}{\ell})^4 d_{44}\}$$

$$A2 = -(\frac{n\pi}{\ell})^2 (\frac{\ell d_{45}}{10S_{12}})$$

$$A3 = \frac{\ell d_{55}}{S_{12}^3} \quad , \quad A4 = \frac{\ell d_{66}}{5S_{12}} (\frac{n\pi}{\ell})^2$$

The (8x8) stiffness matrix for an element is the assembled as shown overleaf

8 x 8 Stiffness Matrix for an Element

$$
\begin{bmatrix}
((\frac{n\pi}{\ell})^2 \frac{2d_{11}}{3} + \frac{2d_{33}}{S_{12}^2}) & \frac{\ell S_{12}}{4}((\frac{n\pi}{\ell})^2 \frac{d_{11}}{3} - \frac{2d_{33}}{S_{12}^2}) & \frac{n\pi}{4}(d_{12} - d_{33}) & \frac{n\pi}{4}(-d_{12} - d_{33}) & \frac{n\pi S_{12}\ XR\ d_{11}}{40}(7) & \frac{n\pi S_{12}\ XR\ d_{11}}{40}(3) & \frac{n\pi S_{12}\ XR\ d_{11}}{40}(S_{12}) & \frac{n\pi S_{12}\ XR\ d_{11}}{40}(-\frac{2S_{12}}{3}) \\
((\frac{n\pi}{\ell})^2 \frac{d_{11}}{3} - \frac{2d_{33}}{S_{12}^2}) & \frac{\ell S_{12}}{4}((\frac{n\pi}{\ell})^2 \frac{2d_{11}}{3} + \frac{2d_{33}}{S_{12}^2}) & \frac{n\pi}{4}(d_{12} + d_{33}) & \frac{n\pi}{4}(-d_{12} + d_{33}) & \frac{n\pi S_{12}\ XR\ d_{11}}{40}(3) & \frac{n\pi S_{12}\ XR\ d_{11}}{40}(7) & n\pi S_{12}\ XR\ d_{11}(\frac{2S_{12}}{3}) & \frac{n\pi S_{12}\ XR\ d_{11}}{40}(-S_{12}) \\
[k_{vu}]_n & \frac{\ell S_{12}}{4}(\frac{2d_{22}}{S_{12}^2} + (\frac{n\pi}{\ell})^2 \frac{2d_{33}}{3}) & \frac{\ell S_{12}}{4}(\frac{2d_{22}}{S_{12}^2} + (\frac{n\pi}{\ell})^2 \frac{d_{33}}{3}) & \frac{\ell S_{12}}{4}(-\frac{2d_{22}}{S_{12}^2} + (\frac{n\pi}{\ell})^2 \frac{d_{33}}{3}) & \frac{\ell d_{12}\ XR}{8}(2) & \frac{\ell d_{12}\ XR}{8}(2) & \frac{\ell d_{12}\ XR}{8}(\frac{S_{12}}{3}) & \frac{\ell d_{12}\ XR}{8}(-\frac{S_{12}}{3}) \\
 & & \frac{\ell S_{12}}{4}(-\frac{2d_{22}}{S_{12}^2} + (\frac{n\pi}{\ell})^2 \frac{d_{33}}{3}) & \frac{\ell S_{12}}{4}(\frac{2d_{22}}{S_{12}^2} + (\frac{n\pi}{\ell})^2 \frac{2d_{33}}{3}) & \frac{\ell d_{12}\ XR}{8}(-2) & \frac{\ell d_{12}\ XR}{8}(-2) & \frac{\ell d_{12}\ XR}{8}(-\frac{S_{12}}{3}) & \frac{\ell d_{12}\ XR}{8}(\frac{S_{12}}{3}) \\
[k_{wu}]_n & & & & 52A1 - 12A2 + 6A3 + 12A4 & 18A1 + 12A2 - 6A3 - 12A4 & \frac{22}{3}S_{12}A1 - 6S_{12}A2 + 3S_{12}A3 + S_{12}A4 & -\frac{13}{3}S_{12}A1 - S_{12}A2 + 3S_{12}A3 + S_{12}A4 \\
 & & [k_{wv}]_n\ (symm.) & & & 52A1 - 12A2 + 6A3 + 12A4 & \frac{13}{3}S_{12}A1 + S_{12}A2 - 3S_{12}A3 - S_{12}A4 & -\frac{22}{3}S_{12}A1 + 6S_{12}A2 - 3S_{12}A3 - S_{12}A4 \\
 & & & & & & \frac{4}{3}S_{12}^2A1 - \frac{4}{3}S_{12}^2A2 + 2S_{12}^2A3 + \frac{4}{3}S_{12}^2A4 & -S_{12}^2A1 + \frac{S_{12}^2}{3}A2 + S_{12}^2A3 - \frac{S_{12}^2}{3}A4 \\
 & & & & & & & \frac{4}{3}S_{12}^2A1 - \frac{4}{3}S_{12}^2A2 + 2S_{12}^2A3 + \frac{4}{3}S_{12}^2A4
\end{bmatrix}
$$

A1, A2, A3, A4, are element constants

$$A1 = \frac{\ell S_{12}}{280}\ [d_{11}\ XR^2 + (\frac{n\pi}{\ell})^4 d_{44}], \quad A2 = -(\frac{n\pi}{\ell})^2 \cdot \frac{\ell d_{45}}{10S_{12}}, \quad A3 = \frac{\ell S_{55}}{S_{12}}, \quad A4 = \frac{d_{66}}{5S_{12}}(\frac{n\pi}{\ell})^2$$

APPENDIX C

Consistent load analysis

By the principal of virtual work we have

$$\{u_i\}_n^T \{R_i\}_n = \frac{S_{12}}{2} \int_\eta \int_x \{u_1 \ (\eta,x)\}^T \{p_i \ (\eta,x)\} \, d\eta \ \ dx \qquad \text{(C.1)}$$

relating the distributed loads $p_i(\eta,x)$ to the consistent nodal loads $\{R_i\}_n$ where

$$\{R_i\}_n = \begin{Bmatrix} \{R_u\} \\ \{R_v\} \\ \{R_w\} \end{Bmatrix}_{8 \times 1}$$

and

$$\{u_i\}_n = \begin{Bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ w_1 \\ w_2 \\ \omega_1 \\ \omega_2 \end{Bmatrix}$$

Now

$$\{u_i(\eta,x)\} = \{\phi(\eta)\} \ \ \{u_i(x)\}$$

where

$$\{u_i(x)\} = \sum_{n=1}^{\infty} \begin{bmatrix} \cos\frac{n\pi x}{\ell} & 0 & 0 \\ 0 & \sin\frac{n\pi x}{\ell} & 0 \\ 0 & 0 & \sin\frac{n\pi x}{\ell} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ w_1 \\ w_2 \\ \omega_1 \\ \omega_2 \end{Bmatrix}$$

and

$$\{\phi(\eta)\} = \begin{bmatrix} \phi_u(\eta) & 0 & 0 \\ 0 & \phi_v(\eta) & 0 \\ 0 & 0 & \phi_w(\eta) \end{bmatrix}$$

$$\phi_u(\eta) = \phi_v(\eta) = \frac{1}{2} < (1 - \eta) \quad (1 + \eta) >$$

and

$$\phi_w(\eta) = \frac{1}{4} \left\langle (2-3\eta+\eta^3) \quad (2+3\eta-\eta^3) \quad \frac{S_{12}}{2} (1-\eta-\eta^2+\eta^3) \quad \frac{S_{12}}{2} (-1-\eta+\eta^2-\eta^3) \right\rangle$$

also

$$p_i(\eta,x) = [\phi_p(\eta)] \quad \{p_i(x)\}$$

where

$$[\phi_p(\eta)] = \begin{bmatrix} \phi_p(\eta) & 0 & 0 \\ 0 & \phi_p(\eta) & 0 \\ 0 & 0 & \phi_p(\eta) \end{bmatrix}$$

$$\phi_p(\eta) = \frac{1}{2} < (1 - \eta) \quad (1 + \eta) >$$

and

$$\{p_i(x)\} = \sum_{n=1}^{\infty} \begin{bmatrix} \cos\frac{n\pi x}{\ell} & 0 & 0 \\ 0 & \sin\frac{n\pi x}{\ell} & 0 \\ 0 & 0 & \sin\frac{n\pi x}{\ell} \end{bmatrix} \begin{Bmatrix} \{p_u\} \\ \{p_v\} \\ \{p_w\} \end{Bmatrix}_n$$

therefore in equation (C.1) on the right hand side we get a product of the 3 x 3 square matrices in the trigonometrical terms. Since

$$\int_0^{\ell} \sin\frac{n\pi x}{\ell} \cdot \sin\frac{m\pi x}{\ell}\,dx = \int_0^{\ell} \cos\frac{n\pi x}{\ell} \cdot \cos\frac{n\pi x}{\ell}\,dx = 0 \quad \text{for } n \neq m$$

and $\frac{\ell}{2}$ for $n=m$. We have

$$\{u_i\}_n^T \{R_i\}_n = \frac{S_{12}}{2} \cdot \frac{\ell}{2} \int_{\eta} \{u_i\}_n^T [\phi(\eta)]^T [\phi_p(\eta)] \{p_i\}_n \,d\eta \tag{C.2}$$

or

$$\{R_i\}_n = \frac{S_{12}\ell}{4} \int_{\eta} [\phi(\eta)]^T [\phi_p(\eta)] \{p_i\}_n \,d\eta \tag{C.3}$$

let

$$I = \int_{-1}^{+1} [\phi(\eta)]^T [\phi_p(\eta)]\,d\eta \tag{C.4}$$

$$[\phi(\eta)]^T[\phi_p(\eta)] = \begin{bmatrix} \phi_u(\eta) & 0 & 0 \\ 0 & \phi_v(\eta) & 0 \\ 0 & 0 & \phi_w(\eta) \end{bmatrix}^T \begin{bmatrix} \phi_p(\eta) & 0 & 0 \\ 0 & \phi_p(\eta) & 0 \\ 0 & 0 & \phi_p(\eta) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} (1-\eta) & 0 & 0 \\ (1+\eta) & 0 & 0 \\ 0 & (1-\eta) & 0 \\ 0 & (1+\eta) & 0 \\ 0 & 0 & \frac{1}{2}(2-3\eta+\eta^2) \\ 0 & 0 & \frac{1}{2}(2+3\eta-\eta^2) \\ 0 & 0 & \frac{S_{12}}{4}\,(1-\eta-\eta^2+\eta^3) \\ 0 & 0 & \frac{S_{12}}{4}\,(-1-\eta+\eta^2-\eta^3) \end{bmatrix}_{8x3} \begin{bmatrix} (1-\ ) & (1-\ ) & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\ ) & (1+\ ) & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\ ) & (1+\ ) \end{bmatrix}_{3x6}$$

$$= \frac{1}{4}\begin{bmatrix} (1-2\eta+\eta^2) & (1-\eta^2) & 0 & 0 & 0 & 0 \\ (1-\eta^2) & (1+2\eta+\eta^2) & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-2\ \ +\eta^2) & (1-\eta^2) & 0 & 0 \\ 0 & 0 & (1-\eta^2) & (1+2\eta+\eta^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(2-5\eta+3\eta^2+\eta^3-\eta^4) & \frac{1}{2}(2-\eta-3\eta^2+\eta^3+\eta^4) \\ 0 & 0 & 0 & 0 & \frac{1}{2}(2+\eta-3\eta^2-\eta^3+\eta^4) & \frac{1}{2}(2+5\eta+3\eta^2-\eta^3-\eta^4) \\ 0 & 0 & 0 & 0 & \frac{S_{12}}{4}(1-2\eta+2\eta^3-\eta^4) & \frac{S_{12}}{4}(1-2\eta^2+\eta^4) \\ 0 & 0 & 0 & 0 & \frac{S_{12}}{4}(-1+2\eta^2-\eta^4) & \frac{S_{12}}{4}(-1-2\eta+2\eta^3+\eta^4) \end{bmatrix}$$

$$I=\int_{-1}^{1}[\phi(\eta)]^T[\phi_p(\eta)]d\eta=\frac{1}{2}\begin{bmatrix} \frac{4}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{4}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{7}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 & \frac{3}{5} & \frac{7}{5} \\ 0 & 0 & 0 & 0 & \frac{S_{12}}{5} & \frac{S_{12}}{5} \\ 0 & 0 & 0 & 0 & \frac{-2S_{12}}{5} & \frac{-S_{12}}{5} \end{bmatrix} = \frac{1}{30}\begin{bmatrix} 20 & 10 & 0 & 0 & 0 & 0 \\ 10 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 10 & 0 & 0 \\ 0 & 0 & 10 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 21 & 9 \\ 0 & 0 & 0 & 0 & 9 & 21 \\ 0 & 0 & 0 & 0 & 3S_{12} & 2S_{12} \\ 0 & 0 & 0 & 0 & -2S_{12} & -3S_{12} \end{bmatrix} \tag{C.5}$$

therefore

$$\begin{Bmatrix} R_{u1} \\ R_{u2} \\ R_{v1} \\ R_{v2} \\ R_{w1} \\ R_{w2} \\ R\omega_1 \\ R\omega_2 \end{Bmatrix}_n = \frac{S_{12}\ell}{120}\begin{bmatrix} 20 & 10 & 0 & 0 & 0 & 0 \\ 10 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 10 & 0 & 0 \\ 0 & 0 & 10 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 21 & 9 \\ 0 & 0 & 0 & 0 & 9 & 21 \\ 0 & 0 & 0 & 0 & 3S_{12} & 2S_{12} \\ 0 & 0 & 0 & 0 & -2S_{12} & -3S_{12} \end{bmatrix}_{8x6} \begin{Bmatrix} p_{u1} \\ p_{u2} \\ p_{v1} \\ p_{v2} \\ p_{w1} \\ p_{w2} \end{Bmatrix}_n \tag{C.6}$$

giving the vector of equivalent nodal loads

$$\{p_i\}_n = \begin{Bmatrix} p_{u1} \\ p_{u2} \\ p_{v1} \\ p_{v2} \\ p_{w1} \\ p_{w2} \end{Bmatrix}$$ is a vector depending on type of loading

Some typical values of $\{p_i\}_n$ are given below

(a) For a uniformly distributed load along the longitudinal direction

$\bar{p}^j_{i_n} = \frac{4}{n\pi}\ \bar{p}^j_i$ where $\bar{p}^j_i$ = load intensity at joint j.

(b) For a load intensity $\bar{p}^j_i$ between $(\ell_p - \frac{\delta}{2})$ and $(\ell_p + \frac{\delta}{2})$ and zero outside this range

$\bar{p}^j_{i_n} = \frac{4}{n\pi}\ \sin \frac{n\pi\ell p}{\ell}\ \sin \frac{n\pi\delta}{2\ell}\ p_i$ where ℓ = span

(c) For a transverse line load at $x = \ell_p$ of intensity $\bar{p}^j_i$ at joint j,

$\bar{p}^j_{i_n} = \frac{2}{\ell}\ \sin \frac{n\pi\ell p}{\ell}\ \bar{p}^j_i$

APPENDIX D

Analysis for structure with intermediate flexible movable diaphragms (only for XR = 0)

As in the case of analysis of structures having intermediate supports (dealt with in section 3.6), the interaction forces between the folded plate structure and the diaphragm are taken as the redundants. The diaphragms are treated as transverse beams in their own plane without any stiffness normal to their planes. It is assumed that the diaphragms are connected to the folded plate structures only at the joints. Since flexible movable diaphragms find application, essentially in box girder bridges, a simple box girder bridge is chosen for illustration (Fig. D.1). The procedure for the solution is summarized below.

1. The primary folded plate structure without the diaphragms is analysed for the external loading. The joint displacements $\{\delta\}_o$ at the location of the diaphragms are calculated. This displacement vector is given by

$$\{\delta\}_o = < \delta_1 \quad \delta_2 \cdots\cdots\cdots\cdots\cdots \delta_c >_o^T$$

2. Unit values of the redundant forces $\{Q\}$ are applied and the displacements $\{\delta\}_1$ at the same locations are calculated. Where

$$\{\delta\}_1 = < \delta_1 \quad \delta_2 \cdots\cdots\cdots\cdots\cdots \delta_c >_1^T$$

and

$$\{\delta\}_1 = [F]_1 \{Q\} \tag{D.1}$$

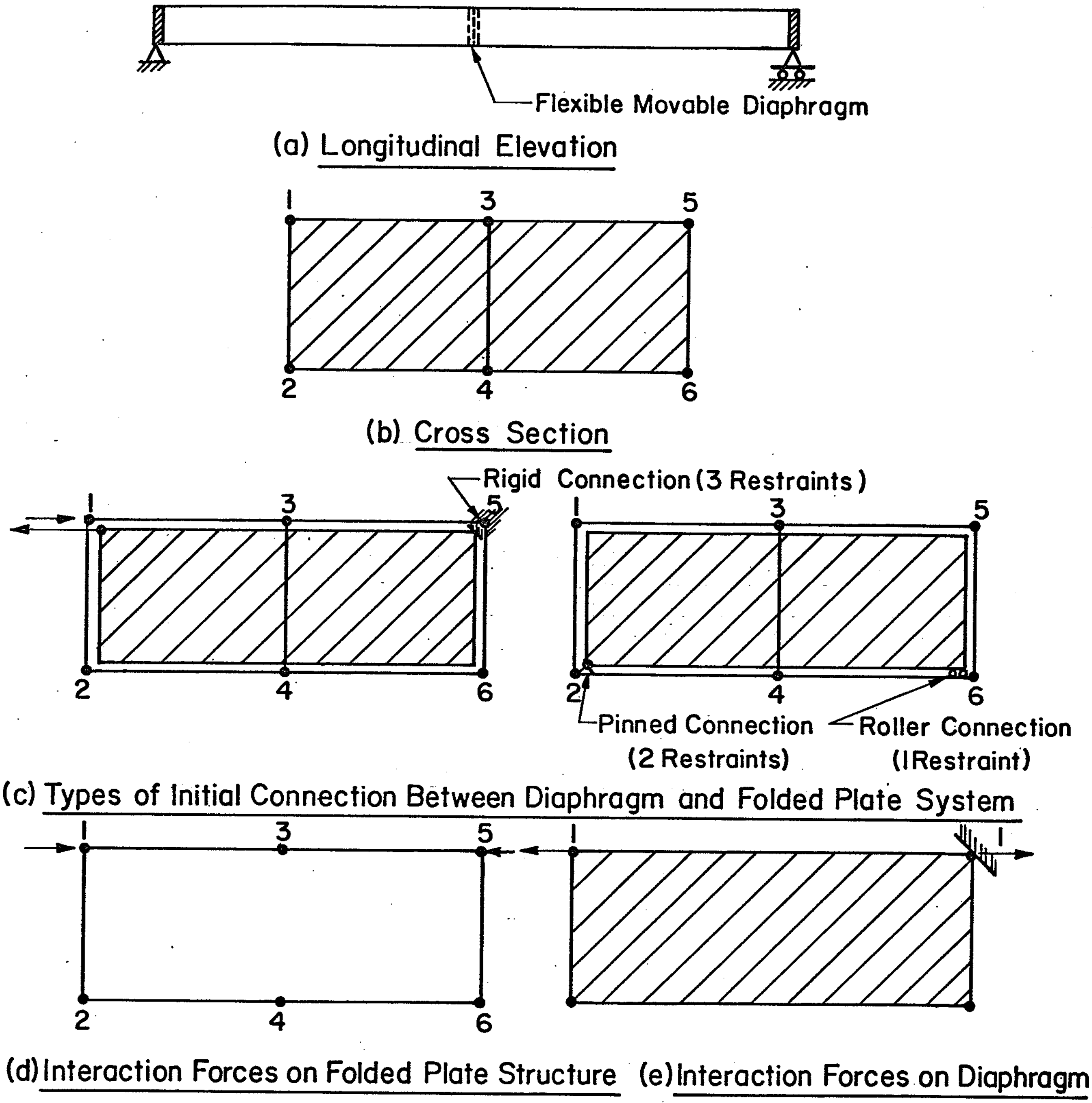


Figure D.1 INTERACTION BETWEEN DIAPHRAGM AND FOLDED PLATE STRUCTURE

$[F]_1$ being the flexibility matrix of the folded plate structure.

3. The primary structure required for the force method of analysis is formed by assuming the diaphragm to be connected in a statically determinate manner to the folded plate system. This is shown in Figs. D.1(c) and D.1(d). In the former case one rigid connection (3 restraints) is assumed at joint 5 while in the latter one pinned connection (2 restraints) at joint 2 and one roller connection (1 restraint) at joint 6 are assumed. The remaining interaction forces form the new set of redundants $\{\overline{Q}\}$ given by

$$\{Q\} = [B]\ \{\overline{Q}\} \tag{D.2}$$

where $[B]$ is the force transformation matrix. To illustrate the formation of the matrix $[B]$ we assume the initial rigid connection at joint 5 between the diaphragm and the folded plates (Fig. D.1(c)). Unit values of the remaining redundant forces $\{\overline{Q}\}$ are applied in turn to the diaphragm alone and the restraining forces at joint 5 determined. The original set of redundants $\{Q\}$ are then found by considering the folded plate system and determining the resulting joint forces on it. This is done in turn for each of the forces in $\{\overline{Q}\}$ and the $[B]$ matrix formed by using the principle of superposition.

It is to be noted that the vector $\{\overline{Q}\}$ and the matrix $[B]$ depend on the type of initial connection assumed, i.e., on the primary structure chosen.

4. Using the principle of virtual work the relative displacements $\{\overline{\delta}\}$ between the diaphragm and the folded plate system and the absolute displacements $\{\delta\}$ of the folded plate system are found to be related by

$$\{\bar{\delta}\} = [B]^T \{\delta\} \quad \text{and} \quad \{\bar{\delta}\}_o = [B]^T \{\delta\} \tag{D.3}$$

$$\{\bar{\delta}\}_1 = [B]^T \{\delta\}_1 \tag{D.4}$$

to correspond to equation (D.1)

From equations (D.4), (D.2) and (D.1) we get

$$\{\bar{\delta}\}_1 = [B]^T [F]_1 [B] \{\bar{Q}\}$$

or

$$\{\bar{\delta}\}_1 = [\bar{F}]_1 \{\bar{Q}\} \tag{D.5}$$

where

$$[F]_1 = [B]^T [F]_1 [B]$$

is the flexibility matrix excluding the contribution due to the deformation of the diaphragm.

5. The contribution to the flexibility matrix $[F]_1$ due to the deformation of the diaphragm is found by analysing the diaphragm by the force method with the already assumed initial connection. The diaphragm is considered as an assemblage of simple beam elements (Fig. D.2).

Each beam element is defined by the properties along the elastic axis of the diaphragm. It is assumed that plane sections remain plane in defining displacements at the interaction points of the diaphragm.

For each beam element (A) or (B) the flexibility matrix is given by

$$\{v\}_e = [f]_e \{s\}_e \tag{D.6}$$

where

$$\{v\}_e = < v_1 \quad v_2 \quad v_3 >^T$$

$$\{s\}_e = < s_1 \quad s_2 \quad s_3 >^T$$

$$[f]_e = \begin{bmatrix} \frac{L}{AE} & 0 & 0 \\ 0 & \frac{(4+\phi)L}{12EI} & \frac{(2-\phi)L}{12EI} \\ 0 & \frac{(2-\phi)L}{12EI} & \frac{(4+\phi)L}{12EI} \end{bmatrix}$$

and

$$\phi = \frac{12EI}{G\ A_s\ L^2}$$

with the usual notations.

Assembling all the element flexibility matrices we get

$$\{v\} = [f]\ \{s\} \tag{D.7}$$

where

$$\{v\} = < v_A \quad v_B \ \cdots\cdots\ v_I >^T$$

$$\{s\} = < s_A \quad s_B \ \cdots\cdots\ s_I >^T$$

and

$$[f] = \begin{bmatrix} f_A & \cdot & \cdot & \cdot & \cdot \\ \cdot & f_B & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & f_I \end{bmatrix} = \text{flexibility matrix of the diaphragm}$$

The relationship between the end forces $\{s\}$ of the beam elements and the interaction forces $\{\overline{Q}\}$ is found by applying unit values of the interaction forces in turn, therefore

$$\{s\} = [b]\ \{\overline{Q}\} \tag{D.8}$$

where $[b]$ = force transformation matrix.

The relative displacements $\{\overline{\delta}\}_2$ between the diaphragm and the folded plate system are related to the internal displacements of the beam elements by

$$\{\overline{\delta}\}_2 = [b]^T\ \{v\} \tag{D.9}$$

where $[b]$ = force transformation matrix.

From equations (D.9), (D.8) and (D.7) we get

$$\{\overline{\delta}\}_2 = [b]^T\ [f]\ [b]\ \{\overline{Q}\}$$

or

$$\{\overline{\delta}\}_2 = [\overline{F}]_2\ \{\overline{Q}\} \tag{D.10}$$

where

$$[\overline{F}]_2 = [b]^T\ [f]\ [b]$$

is the contribution to the overall flexibility matrix due to the deformation of the diaphragms.

6. Compatibility requires that the relative displacement between the diaphragm and the folded plate system at the interaction points be zero. Therefore

$$\{\overline{\delta}\} = \{\overline{\delta}\}_0 + \{\overline{\delta}\}_1 + \{\overline{\delta}\}_2 = 0 \tag{D.11}$$

From equations (D.11), (D.10) and (D.5) we have

$$\{\delta\}_0 + [\overline{F}]_1 \{\overline{Q}\} + [\overline{F}]_2 \{\overline{Q}\} = 0$$

or

$$\{\delta_0\} + [\overline{F}] \{\overline{Q}\} = 0 \tag{D.12}$$

where

$$[\overline{F}] = [\overline{F}]_1 + [\overline{F}]_2$$

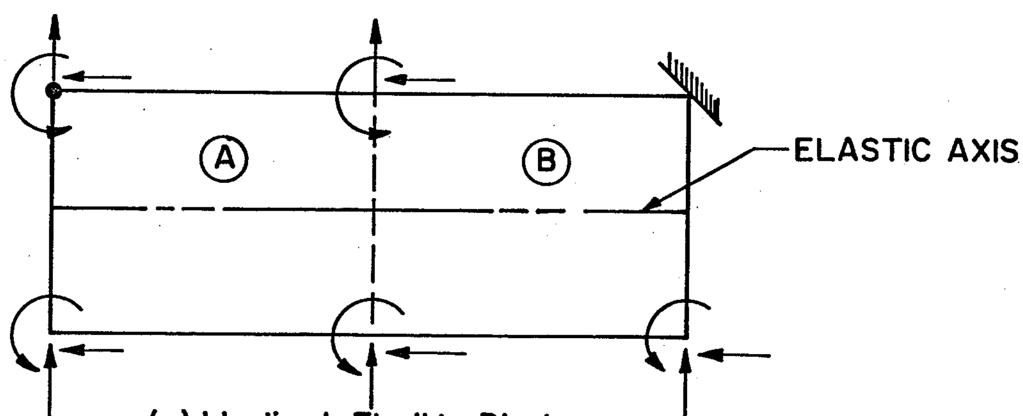
From equation (D.12) we get

$$\{\overline{Q}\} = -[\overline{F}]^{-1} \{\delta\}_0 \tag{D.13}$$

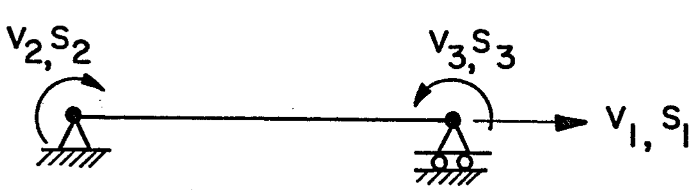
Using equation (D.2) the unknown interaction joint forces $\{Q\}$ are given by

$$\{Q\} = -[B] \; [\overline{F}]^{-1} \{\overline{\delta}\}_0 \tag{D.14}$$

when $\{Q\}$ is known the structure can be analysed subjected to $\{Q\}$ plus the given external loading.



(a) Idealised Flexible Diaphragm



(b) Typical Beam Element (A) or (B)

Figure D.2

APPENDIX E

Forms of Input, Output and Sample Print-Out for "Archfold"

E.1. Form of input

Input data are keypunched on cards as specified below. The order of the cards and the use of consistent units are two important considerations.

1. TITLE CARD (12A4). Title of the problem.

2. CONTROL CARD (2F 10.0, 14 I 4).

   Col. 1 to 10 - SPAN = span of the structure in feet

   Col. 11 to 20 - XRISE = curvature of arch in ft^{-1}

   Col. 21 to 24 - NPL = number of plate types. maximum = 15

   Col. 25 to 28 - NEL = number of elements. maximum = 30

   Col. 29 to 32 - NJT = number of joints. maximum = 20

   Col. 33 to 36 - NDIAPH = number of intermediate supports. maximum = 12

   Col. 37 to 40 - NXP = number of transverse sections for output of results. maximum = 14

   Col. 41 to 44 - MHARM = maximum Fourier Series Limit. maximum = 100

   Col. 45 to 48 - NCHECK = harmonic type indicator.
   +1 to work on odd terms only (symmetry)
   0 to include all terms
   -1 to work on even terms only (anti-symmetry)

   Col. 49 to 52 - MCHECK = girder moment integration option. Use only when XR = 0.
   0 No moment integration (XR≠0)
   1 moment integration desired (XR=0)

   Col. 53 to 56 - NBT = number of flexible plane frame intermediate supports. maximum = 8

   Col. 57 to 60 - NFMD = number of flexible movable (intermediate) diaphragms maximum = 8. Use only with XR=0. If not, leave blank

   Col. 61 to 64 - NSURL = number of partial surface loads

   Col. 65 to 68 - NCONL = number of concentrated loads

   Col. 69 to 72 - INTRES = number of harmonic series limits for which intermediate results are required

3. TRANSVERSE SECTIONS CARD (10 F7.2)

   Col. 1 to 70 - XP(I) = x Co-indinates along span

(longitude) for output of results
use 2nd card if NXP > 10, maximum = 14

4. INTERMEDIATE RESULT CARD (20 I 4)

Col. 1 to 80 - IRES(I) = harmonic series limits for which intermediate results are required
Omit this card if INTRES = 0.

5. INTERMEDIATE SUPPORT CARD (I10, 2F 10.0, 2 I 4)

Col. 1 to 10 - I = intermediate support number

Col. 11 to 20 - DIAPHX(I) = location of intermediate support (x - co-ordinate)

Col. 21 to 30 - DIADEL(I) = intermediate support thickness in longitudinal direction

Col. 31 to 34 - KODIA(I) = classification of support type.
1. Externally supported rigid diaphragm
2. Movable rigid diaphragm
3. Flexible plane frame
4. Flexible movable diaphragm (only for XR=0)

Col. 35 to 38 - KDTP(I) = type number of flexible plane frame or flexible movable diaphragm. leave blank if diaphragm is rigid (ie 1 or 2 above)

6. PLATE TYPE CARD

Two cards (I10, 5F 10.0/10X, 5F 10.0) are required for each type

FIRST CARD - membrane or inplane characteristics

Col. 1 to 10 - I = type number

Col. 11 to 20 - THM(I) = effective thickness

Col. 21 to 30 - ETM(I) = modules of elasticity in longitudinal direction

Col. 31 to 40 - ESM(I) = modules of elasticity in transverse direction

Col. 41 to 50 - GM(I) = shear modulus

Col. 51 to 60 - PRM(I) = Poisson's ratio

SECOND CARD - bending characteristcs

Col. 11 to 20 - THB(I) = effective thickness

Col, 21 to 30 - ETB(I) = modulus of elasticity in longitudinal direction

Col, 31 to 40 - ESB(I) = modulus of elasticity in transverse direction

Col, 41 to 50 - GB(I) = shear modulus

Col, 51 to 60 - PRB(I) = Poisson's ratio

7. ELEMENT CARDS (5I4, 5 F 10.0)

Each element or strip requires one card

Col. 1 to 4 - I = element number

Col. 5 to 8 - NPI(I) = joint i of element I

Col. 9 to 12 - NPJ(I) = joint j of element I

Col. 13 to 16 - KPL(I) = plate type number

Col. 16 to 20 - NSEC(I) = number of element subdivisions for output of internal forces and displacements. maximum = 4. If NSEC(I)=0, no output for element I

Col. 21 to 30 - DL(I) = dead load. (force per unit element area)

Col. 31 to 40 - HLI(I) = horizontal load intensity at joint i (force per unit vertically projected area)

Col. 41 to 50 - HLJ(I) = horizontal load intensity at joint j

Col. 51 to 60 - VLI(I) = vertical load intensity at joint i (force per unit horizontally projected area).

Col. 51 to 70 - VLJ(I) = vertical load intensity at joint j

8. PARTIAL SURFACE LOAD CARDS (I10, 6 F 10.0)

Each partial surface load requires one card

Skip if NSURL=0 in control card

Col. 1 to 10:- LEL(I) = element number

Col. 11 to 20:- PHLI(I) = horizontal load intensity at joint i

Col. 21 to 30:- PHLJ(I) - horizontal load intensity at joint j

Col. 31 to 40:- PVLI(I) = vertical load intensity at joint i

Col. 41 to 50 - PVLJ(I) = vertical load intensity at joint j

Col. 51 to 60 - SURT(I) = location of centroid of load in the longitudinal direction (ie x - co-ordinate).

Col. 61 to 70 - SURL(I) = longitudinal width of the load

9. JOINT CARDS (I10, 6F 10.0, 4 I 2)

each joint requires one card

Col. 1 to 10 - I = joint number

Col. 11 to 20 - Y(I) = Y co-ordinate of joint i
Col. 21 to 30 - Z(I) = Z co-ordinate of joint i
} arbitarily fixed origin

Col. 31 to 40 - AJFOR(1,I) = Applied horizontal joint force/displacement intensity

Col. 41 to 50 - AJFOR(3,I) = Applied vertical joint force/displacement intensity

Col. 51 to 60 - AJFOR(3,I) = Applied joint moment/rotation intensity

Col. 61 to 70 - AJFOR(4,I) = Applied longitudinal joint force/displacement intensity

Col. 71 to 72 - LCASE(1,I) = horizontal force/displacement index

Col. 73 to 74 - LCASE(2,I) = vertical force/displacement index

Col. 75 to 76 - LCASE(3,I) = moment/rotation index

Col. 77 to 78 - LCASE(4,I) = longitudinal force/displacement index

Force/displacement indices are as below

0 for given zero force or moment

1 for uniformly distributed force or moment (input uniform force per unit length for AJFOR)

2 for concentrated force or moment at mid-span (input total force for AJFOR)

3 for given zero displacement or rotation

4 for prestress force P at each end (input total force P for AJFOR, positive towards mid-span)

10. CONCENTRATED LOAD CARDS (I10, 6F 10.0)

Each concentrated load requires one card, although two loads may act at the same joint at different points on the span. Also SKIP if NCONL = 0 in control card

Col. 1 to 10 - LJT(I) = joint number

Col. 11 to 20 - FH(I) = total horizontal force

Col. 21 to 30 - FV(I) = total vertical force

Col. 31 to 40 - FM(I) = total moment

Col. 41 to 50 - FP(I) - total longitudinal force (for equilibrium there must be another force FP(I) along the same joint)

Col. 51 to 60 - FTL(I) = location of load along longitudinal direction (ie., x - direction)

Col. 61 to 70 - FTT(I) = width of load in longitudinal direction

11. GIRDER MOMENT INTEGRATION DATA Skip if MCHECK=0

(this is possible only when XR=0 in which case there will be no limitation for the inclination angle ϕ of the strip elements). When XR=0 and, girder moment integration is requested by specifying - MCHECK=1 in the control card. The following cards become necessary.

(a) FIRST CARD (2I 4)

Col. 1 to 4 - NOXMP = number of transverse sections (ie sections along x axis) at which girder moments are desired (maximum = 14)

Col. 5 to 8 - NBOX = number of girders (maximum = 10)

(b) SECOND CARD (10F 7.3)

Col. 1 to 70 - X(I) = x co-ordinates of sections at which girder moments are desired. This must be a subset of XP(I) in transverse section cards

(c) NEXT CARDS (3I4, 3F 10.0) - one card for each element

Col. 1 to 4 - I = number of elements in folded plate system

Col. 5 to 8 - NGIEL(I,1) = first girder number to which this element belongs. If it belongs to two list that which is nearest to node i first. Girders have to be numbered from left to right.

Col. 9 to 12 - NGIEL(I,2) = second girder number to which this element belongs. Punch zero or leave blank if there is no second girder.

Col. 13 to 22- DNAI(I) = vertical distance from assumed section neutral axis to node i (Downward is positive)

Col. 23 to 32 - DNAJ(I) = vertical distance from assumed section neutral axis to node j

Col. 33 to 42 - XDIV(I) - horizontal distance from node i to the dividing point if the element belongs to two girders. (Rightward is positive)

12. FLEXIBLE PLANE FRAME CARDS

No cards are required if no frames were input in the control card. If frames were specified (NBT>0), one set of the following cards will be required for each type of intermediate support frame. (Fig. E.1)

(a) CONTROL CARD (6I5)

Col. 1 to 5 - NFT = frame type number

Col. 6 to 10 - NUMEL = number of elements

Col. 11 to 15 - NUMNP = number of nodal points. maximum = 30

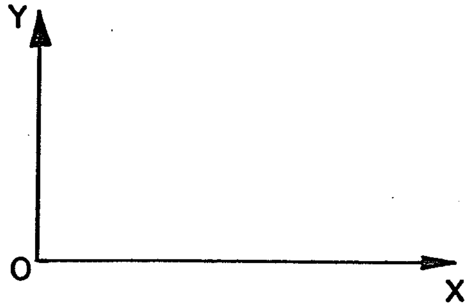
Col. 16 to 20 - NUMMAT = number of materials. maximum = 10

Col. 21 to 25 - NUMETP = number of element section property cards

Col. 26 to 30 - NUMSPR = number of elastic supports in the plane frame. maximum=40

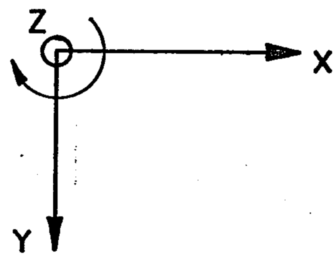
(b) MATERIAL PROPERTY CARDS (I5, E10.0, F 10.0) (Mattyp)

Col. 1 to 5 - N = material identification number; (any number from 1 to 10)

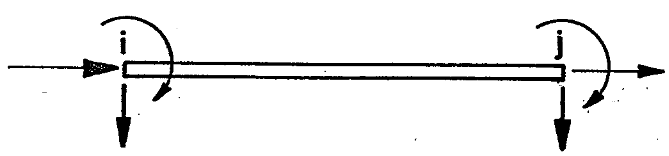


The origin O can be arbitary.

(a) Co-ordinate System for Geometry of Frames. (Independant of the Folded Plate Co-ordinate System)

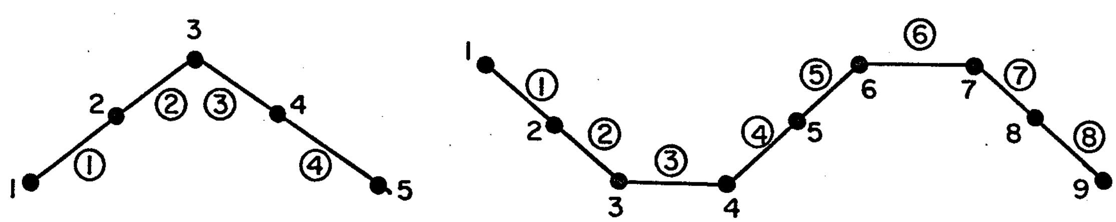


(b) Positive Directions for Joint Forces and Displacements. (Global System)
(Joint Forces include Interaction Forces and Reactions)



(c) Positive Member end Forces (Local System)

SIGN CONVENTION FOR INTERMEDIATE SUPPORT FRAME
(Looking from left end of the Structure)



4 Elements, 5Joints

8 Elements, 9 Joints

EXAMPLE OF NUMBERING IN THE FOLDED PLATE STRUCTURE

Figure E.I

Col. 6 to 15 - EFM(N) = Young's modulus

Col. 16 to 25 - G(N) = Poissen's ratio
(number of material properties restricted to a maximum of 10)

(c) ELASTIC SUPPORT CARDS (I5, 3F 10.0) Skip if no elastic supports in (a)

Col. 1 to 5 - N = identification number (any number from 1 to 40)

Col. 6 to 15 - SP(N,1) = X component of spring stiffness

Col. 16 to 25 - SP(N,2) = Y component of spring stiffness

Col. 26 to 35 - SP(N,3) = rotational spring stiffness

(d) SECTION PROPERTY CARDS (I5, 3F 10.0)

Col. 1 to 5 - N = identification number (any number from 1 to 200)

Col. 6 to 15 - COAX(N) = axial area

Col. 16 to 25 - COAY(N) = shear area (leave blank if shear deformations are to be neglected)

Col. 26 to 35 - COAAZ(N) = moment of inertia

(e) NODAL POINT DATA CARDS (2I5, 2F 10.0, 2I5) - one for each joint of the frame

Col. 1 to 5 - N = nodal point number

Col. 6 to 10 - KODE(N) = joint boundary condition code, a 3 digit number in Cols. 8, 9, 10. Use 1 for zero displacement, otherwise 0. (Cols 8 - X displacement, Col. 9 - Y displacement, Col. 10 - Z rotation).

Col. 11 to 20 - X(N) = global X co-ordinate

Col. 21 to 30 - Y(N) = global Y co-ordinate

Col. 31 to 35 - NPSTP(N) = elastic support identification number (leave blank if no elastic support)

Col. 36 to 40 - NFP(N) = corresponding node number in folded plate structure. (leave blank if not connected to folded plate system).

(f) ELEMENT DATA CARDS (5I5, I10) one for each element of the frame.

Col. 1 to 5 - NEL = identification number

Col. 6 to 10 - NI = node i

Col. 11 to 15 - NJ = node j

Col. 16 to 20 - NATTYP = material identification number

Col. 21 to 25 - MELTYP = section property identification number

Col. 26 to 35 - NELKOD = element code, which is a 6 digit number in Cols. 30 to 35 permitting member end releases (ex pin ends). Use 1 for zero member end force, otherwise use zero or leave blank. The 1st 3 digits correspond to node i and the last 3 to node j in the order horizontal, vertical and rotational components resp. as shown in the figure E.2

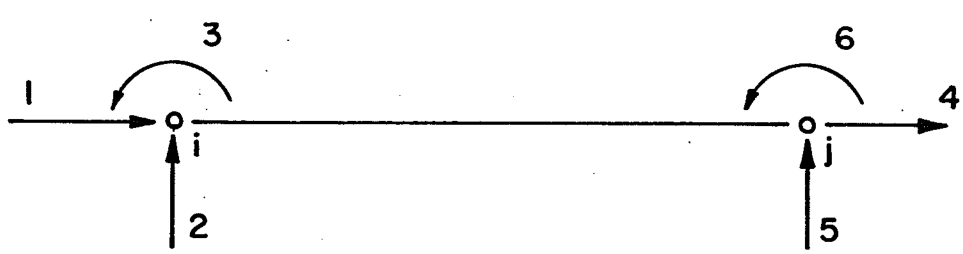


Figure E.2 MEMBER END RELEASE CODE

The above deck has to be repeated for each fram type intermediate support.

13. FLEXIBLE MOVABLE DIAPHRAGM CARDS Skip if NFMD=0

When XR=0, intermediate flexible movable diaphragms could be utilised. Two cards are required for each type of flexible movable diaphragm.

(a) FIRST CARD (2I4)

Col. 1 to 4 - type number

Col. 5 to 8 - option code (option for two ways of inputting data)
1. option one
2. option two

(b) SECOND CARD Use either option one or two

(1) option one (5 F10.0) diaphragm assumed to have rectangular cross-section

Col. 1 to 10 - DITH = diaphragm thickness

Col. 11 to 20 - DIDP = diaphragm depth (neutral axis is assumed at mid-depth)

Col. 21 to 30 - CODE = code for vertical location of neutral axis of diaphragm with respect to joint 1 of the folded plate system
= +1.0 if neutral axis is above joint 1
= -1.0 if neutral axis is below joint 1

Col. 31 to 40 - DIE = modulus of elasticity

Col. 41 to 50 - DINU = Poisson's ration

(2) Option two (6 F10.0)

Col. 1 to 10 - DIPHI = moment of inertia of diaphragm cross-section

Col. 11 to 20 - DIPHA = area of cross section

Col. 21 to 30 - DIAS = Shear area of cross-section (leave blank if shear deformations are to be neglected).

Col. 31 to 40 - CC = vertical location of diaphragm neutral axis with respect to joint 1 of folded plate system
\+ if neutral axis is above joint 1
\- if neutral axis is below joint 1

Col. 41 to 50 - DIE = modulus of elasticity

Col. 51 to 60 = DINU = Poisson's ration

The above card deck has to be repeated for each type of flexible movable diaphragm.

Two blank cards are added at the end of the data deck

REMARKS

(1) It is preferable to number all similar strips (elements) in consecutive order to save computing time.

(2) The joint numbering should be selected to minimize "band-width" by making the absolute joint difference in the strips as small as possible fig. (E.1).

(3) Total number of interaction forces between the intermediate supports and the folded plate structure should be less than 120. This is usually quite adequate in practical cases.

E.2. Form of output

The output will consist of two separate parts
a) Printout of Input
b) Results

a) PRINTOUT OF INPUT

The complete data that was input will be properly labelled and printed out. This will be useful in checking the data and for diagnosing errors in punching, input format and order of the cards.

b) RESULTS

The results that will be output will consist of the following in the order given below

1. If there are intermediate supports, the interaction forces between the supports and the folded plate system.

2. The global (horizontal, vertical and rotational) components of the final displacements of all the joints of the structure.

3. For each strip element, the following internal forces and displacements.
   (1) Longitudinal moment per unit length Mx
   (2) Transverse moment per unit length Ms
   (3) Torsinal moment per unit length Mxs
   (4) Longitudinal in-plane force per unit length Nx
   (5) Transverse in-plane force per unit length Ns
   (6) In-plane shear force per unit length Nxs

(7) Normal shear in transverse section per unit length Qx
(8) Normal shear in longitudinal section per unit length Qs
(9) Longitudinal displacement u
(10) Transverse displacement v
(11) Normal displacement w

All the above quantities are output for each transverse section specified at every point along the plate width specified by the user.

(4) When XR=0, and MCHECK=1, the following quantities at the specified cross-sections
1. Moment taken by each girder
2. Percentage of the cross-section moment taken by each girder
3. The resultant longitudinal tensile or compressive force taken by each girder

(5) If there are plane frames as intermediate supports and KFOR=1, the frame is analysed giving the following
1. Joint displacements
2. Member end forces
3. Applied joint loads (ie the interaction forces acting on the frame) and reactions

E.3. Sample printout of "Archfold"

For the sake of illustration, the input and output of the analysis of the box girder bridge dealt with in Section 5.4 are presented. A total of 28 cards are required to input the data. A check in the input data is obtained from the output which prints out the data properly labelled. The simplicity of the input is evident from this illustration. It is to be noted that the output prints all the internal forces and displacements of the elements at the specified locations in a very systematic manner. In each element results were requested only at three locations along the element width to restrict the length of the output. Girder moment integration was requested and the longitudinal bending moments taken by each girder are printed out at the specified sections. Output was requested at x = 10.0', x = 25.0' (quarter-span) and x = 50.0' (mid-span)

```
INPUT  DATA  FOR  EXAMPLE  5.4

EXAMPLE  PROBLEM

    100.0                 1    7    6        3 100      1                    1

   10.0   25.0   50.0

            1      .3333   432000.   432000.   187826.        .15
                   .3333   432000.   432000.   187826.        .15

   1    1    3    1    2
   2    3    5    1    2
   3    2    4    1    2
   4    4    6    1    2
   5    2    1    1    2
   6    4    3    1    2
   7    6    5    1    2

            1                                                                  111
            2                3.0                                               111
            3        3.0                                                       111
            4        3.0     3.0                                               111
            5        6.0                                                       111
            6        6.0     3.0                                               111

            3               100.0                               50.0    1.0

   3    3

   10.0   25.0   50.0

   1    1    2       -1.5      -1.5      1.5
   2    2    3       -1.5      -1.5      1.5
   3    1    2        1.5       1.5      1.5
   4    2    3        1.5       1.5      1.5
   5    1             1.5      -1.5
   6    2             1.5      -1.5
   7    3             1.5      -1.5

/*
//
```

Form of Output for Example 5.4

EXAMPLE PROBLEM

SPAN OF STRUCTURE = 100.000
CURVATURE OF ARCH = 0.0

NUMBER OF TYPES OF PLATE = 1

NUMBER OF ELEMENTS = 7

NUMBER OF JOINTS = 6

NUMBER OF DIAPHRAGMS = 0

NUMBER OF X-COORDINATES AT WHICH RESULTS ARE DESIRED = 3

MAXIMUM HARMONIC NUMBER = 100

NUMBER OF TYPES OF FLEXIBLE SUPPORTING FRAME BENT = 0

NUMBER OF TYPES OF FLEXIBLE MOVABLE DIAPHRAGM = 0

INTEGRATION OF GIRDER MOMENTS IS DESIRED

PRINT RESULTS AT SECTIONS WITH X EQUAL TO

10.00000 25.00000 50.00000

PLATE ELEMENT TYPES

NO.	MEMBRANE PROPERTIES TH	E-T	E-S	G	NU	PLATE BENDING PROPERTIES TH	E-T	E-S	G	NU
1	0.3333E 00	0.4320E 06	0.4320E 06	0.1878E 06	0.1500E 00	0.3333E 00	0.4320E 06	0.4320E 06	0.1878E 06	0.1500E 00

PLATE ELEMENTS

ELE NO	NODE I	NODE J	PLATE TYPE	NSEC	DL	HLI	HLJ	VLI	VLJ
1	1	3	1	2	0.0	0.0	0.0	0.0	0.0
2	3	5	1	2	0.0	0.0	0.0	0.0	0.0
3	2	4	1	2	0.0	0.0	0.0	0.0	0.0
4	4	6	1	2	0.0	0.0	0.0	0.0	0.0
5	2	1	1	2	0.0	0.0	0.0	0.0	0.0
6	4	3	1	2	0.0	0.0	0.0	0.0	0.0
7	6	5	1	2	0.0	0.0	0.0	0.0	0.0

TIAL JOINT LOADS

NT	H-LOAD	V-LOAD	MOMENT	LONG. FORCE	CENTER COORD	LOAD WIDTH
	0.0	100.000	0.0	0.0	50.000	1.000

UT LOADS OR DISPLACEMENTS AT JOINTS

NT	Y-COORD	Z-COORD	HORIZONTAL	IH	VERTICAL	IV	ROTATIONAL	IM	LONGITUDINAL	IS	RH	RV	RM
1	0.0	0.0	0.0	0	0.0	0	0.0	0	0.0	0	1	1	1
2	0.0	3.00	0.0	0	0.0	0	0.0	0	0.0	0	1	1	1
3	3.00	0.0	0.0	0	0.0	0	0.0	0	0.0	0	1	1	1
4	3.00	3.00	0.0	0	0.0	0	0.0	0	0.0	0	1	1	1
5	6.00	0.0	0.0	0	0.0	0	0.0	0	0.0	0	1	1	1
6	6.00	3.00	0.0	0	0.0	0	0.0	0	0.0	0	1	1	1

IV,IM,IS = 0 FOR GIVEN ZERO FORCE
1 FOR UNIF. DISTRIBUTED FORCE
2 MEANS CONC. FORCE AT MIDSPAN FOR IH, IV, IM AND PRESTRESS FOR IS
3 FOR GIVEN ZERO DISPLACEMENT

RV,RM = 0 - TO CONSIDER RESTRAINT FROM DIAPHRAGMS
NON-ZERO - TO NEGLECT RESTRAINT FROM DIAPHRAGMS

DITIONAL INFORMATION FOR DETERMINATION OF GIRDER MOMENT PERCENTAGES

OF SECTIONS FOR RESULTS = 3
OF GIRDERS = 3

LTS ARE DESIRED AT X =
0.000 25.000 50.000

NO.	BELONGS TO GIRDERS		DNAI	DNAJ	XDIV
1	1	2	-1.500	-1.500	1.500
2	2	3	-1.500	-1.500	1.500
3	1	2	1.500	1.500	1.500
4	2	3	1.500	1.500	1.500
5	1	0	1.500	-1.500	0.0
6	2	0	1.500	-1.500	0.0
7	3	0	1.500	-1.500	0.0

FINAL JOINT DISPLACEMENTS

HORIZONTAL DISPLACEMENTS

JOINT	X= 10.000	X= 25.000	X= 50.000	X=
1	-6.7989458E-05	-1.6927345E-04	-3.4513930E-04	
2	6.9333313E-05	1.7234522E-04	3.5563274E-04	
3	1.0941785E-06	2.4992587E-06	3.5234607E-06	
4	2.5543477E-07	5.8642939E-07	8.2909878E-07	
5	7.0178226E-05	1.7427333E-04	3.5218871E-04	
6	-6.8824069E-05	-1.7117651E-04	-3.5398151E-04	

VERTICAL DISPLACEMENTS

JOINT	X= 10.000	X= 25.000	X= 50.000	X=
1	1.2725157E-01	2.9568088E-01	4.3034023E-01	
2	1.2725204E-01	2.9568100E-01	4.3033957E-01	
3	1.2719321E-01	2.9558688E-01	4.3275058E-01	
4	1.2719315E-01	2.9558712E-01	4.3224227E-01	
5	1.2725383E-01	2.9568619E-01	4.3034810E-01	
6	1.2725419E-01	2.9568619E-01	4.3034720E-01	

ROTATIONS

JOINT	X= 10.000	X= 25.000	X= 50.000	X=
1	-4.3868145E-05	-9.8766133E-05	2.6638946E-04	
2	-4.3868946E-05	-9.8749559E-05	2.0224642E-04	
3	3.6239140E-07	8.4648030E-07	1.2258042E-06	
4	3.2586809E-07	7.6505745E-07	1.1138682E-06	
5	4.4510874E-05	1.0025605E-04	-2.6425114E-04	
6	4.4523738E-05	1.0026942E-04	-2.0006075E-04	

LONGITUDINAL DISPLACEMENTS

JOINT	X= 10.000	X= 25.000	X= 50.000	X=
1	1.8445652E-02	1.4405921E-02	1.9314950E-08	
2	-1.8451568E-02	-1.4412239E-02	-1.9310150E-08	
3	1.8416651E-02	1.4382686E-02	1.9268633E-08	
4	-1.8422894E-02	-1.4389277E-02	-1.9279199E-08	
5	1.8445425E-02	1.4405768E-02	1.9314129E-08	
6	-1.8452071E-02	-1.4412627E-02	-1.9310576E-08	

INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. 1 BETWEEN JOINTS 1 AND 3 AFTER 100 HARMONICS

N(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-2.2156418E 01	-5.5419174E 01	-1.0449664E 02	
2	-2.2148880E 01	-5.5349762E 01	-1.0952806E 02	
3	-2.2141251E 01	-5.5280243E 01	-1.1455952E 02	

N(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-7.8231357E-03	-6.8669796E-02	1.0598068E 00	
2	-6.6980273E-03	-5.8255941E-02	3.0510473E-01	
3	-5.5729598E-03	-4.7842529E-02	-4.4960701E-01	

N(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.0258598E 00	-9.0841746E-01	-8.3909060E-07	
2	-8.1234956E-01	-6.9423807E-01	-8.9885623E-07	
3	-5.9880364E-01	-4.8005784E-01	-9.5864834E-07	

M(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.3601565E-01	3.3623773E-01	4.1648662E-01	
2	1.3675749E-01	3.4233123E-01	1.2334337E 00	
3	1.3753009E-01	3.4876800E-01	2.1175308E 00	

M(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-5.0513074E-03	-4.0854983E-02	-1.6051197E 00	
2	8.6162402E-04	7.0907213E-03	3.0283231E-01	
3	6.7795068E-03	5.5088516E-02	2.2208567E 00	

M(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	3.9586797E-03	4.1054226E-03	-1.9442672E-07	
2	3.5584103E-03	-5.1768944E-03	5.1921006E-08	
3	-4.1048246E-05	-3.2292752E-05	1.4952095E-11	

Q(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.5776634E-02	1.7252212E-05	4.2247439E-07	
2	1.2630667E-03	2.7268488E-02	-1.3006356E-06	
3	-2.2065818E-02	6.6436827E-02	-4.5064126E-06	

Q(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	3.9127730E-03	3.1616289E-02	1.1281471E 00	
2	3.8298906E-03	3.1234935E-02	1.8165998E 00	
3	3.9533526E-03	3.2074079E-02	1.2176743E 00	

U

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.8445652E-02	1.4405921E-02	1.9314950E-08	
2	1.8431138E-02	1.4394287E-02	1.9291779E-08	
3	1.8416651E-02	1.4382686E-02	1.9268633E-08	

V

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-6.7989458E-05	-1.6927345E-04	-3.4513930E-04	
2	-3.3447301E-05	-8.3386782E-05	-1.7081022E-04	
3	1.0941785E-06	2.4992587E-06	3.5234607E-06	

W

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.2725157E-01	2.9568088E-01	4.3034023E-01	
2	1.2720591E-01	2.9559666E-01	4.3164480E-01	
3	1.2719321E-01	2.9558688E-01	4.3275058E-01	

INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. 2 BETWEEN JOINTS 3 AND 5 AFTER 100 HARMONICS

N(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-2.2141235E 01	-5.5280228E 01	-1.1455949E 02	
2	-2.2148590E 01	-5.5349289E 01	-1.0952762E 02	
3	-2.2155899E 01	-5.5418243E 01	-1.0449582E 02	

N(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-5.5533499E-03	-4.7774438E-02	-4.4947201E-01	
2	-6.6396259E-03	-5.8119055E-02	3.0530083E-01	
3	-7.7260956E-03	-6.8464041E-02	1.0600634E 00	

N(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	6.0718137E-01	4.8661363E-01	9.5705946E-07	
2	8.2072908E-01	7.0079648E-01	8.9723807E-07	
3	1.0342388E 00	9.1497737E-01	8.3743646E-07	

M(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.3750732E-01	3.4870481E-01	2.1174231E 00	
2	1.3676399E-01	3.4234804E-01	1.2334614E 00	
3	1.3605148E-01	3.3633465E-01	4.1664898E-01	

M(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	6.6273548E-03	5.4667711E-02	2.2201395E 00	
2	8.9910231E-04	7.1835220E-03	3.0297565E-01	
3	-4.8243813E-03	-4.0249016E-02	-1.6041164E 00	

M(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-4.1048245E-05	-3.2292752E-05	1.4952095E-11	
2	-3.6494916E-03	5.1018447E-03	-5.1840289E-08	
3	-4.0311063E-03	-4.1615369E-03	1.9444138E-07	

O(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-2.2082042E-02	6.6418350E-02	-4.5063289E-06	
2	1.2668814E-03	2.7272046E-02	-1.3006193E-06	
3	1.5800752E-02	4.2748099E-05	4.2236900E-07	

Q(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-3.8359114E-03	-3.1750344E-02	-1.2171230E 00	
2	-3.7126222E-03	-3.0911192E-02	-1.3160486E 00	
3	-3.7953868E-03	-3.1292792E-02	-1.1275988E 00	

U

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.8416651E-02	1.4382686E-02	1.9268633E-08	
2	1.8431023E-02	1.4394209E-02	1.9291367E-08	
3	1.8445425E-02	1.4405768E-02	1.9314129E-08	

V

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.0941785E-06	2.4992587E-06	3.5234607E-06	
2	3.5635865E-05	8.8385990E-05	1.7785850E-04	
3	7.0178226E-05	1.7427333E-04	3.5218871E-04	

W

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.2719321E-01	2.9558688E-01	4.3275058E-01	
2	1.2720710E-01	2.9559940E-01	4.3164885E-01	
3	1.2725383E-01	2.9568619E-01	4.3034810E-01	

INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. 3 BETWEEN

JOINTS 2 AND 4 AFTER 100 HARMONICS

N(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.2154007E 01	5.5413940E 01	1.0430016E 02	
2	2.2146820E 01	5.5345230E 01	1.1078241E 02	
3	2.2139389E 01	5.5276566E 01	1.1726471E 02	

N(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	7.7653378E-03	6.8555653E-02	-1.3840628E 00	
2	6.6752844E-03	5.8258399E-02	-4.1173482E-01	
3	5.5849403E-03	4.7960915E-02	5.6060749E-01	

N(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.0328751E 00	9.0092665E-01	2.0711886E-06	
2	8.1648302E-01	6.9065762E-01	1.3615236E-06	
3	6.0006046E-01	4.8038757E-01	6.5187254E-07	

M(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.3601279E-01	3.3623582E-01	4.7837400E-01	
2	1.3678789E-01	3.4221947E-01	7.1971160E-01	
3	1.3759905E-01	3.4852415E-01	9.4807261E-01	

M(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-5.0789900E-03	-4.0832199E-02	-1.2587729E 00	
2	8.8200718E-04	7.1175434E-03	1.9732112E-01	
3	6.8487786E-03	5.5115934E-02	1.6514740E 00	

M(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	4.9344413E-03	2.7921693E-03	3.3569663E-08	
2	1.5187638E-03	-2.4326567E-03	-2.1551557E-08	
3	-3.7002057E-05	-2.9524323E-05	1.9455132E-11	

Q(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	9.9456832E-03	7.8565180E-03	-1.7075376E-07	
2	1.6019266E-02	7.4208938E-03	7.2268369E-07	
3	2.1779403E-02	7.4619874E-03	1.6021240E-06	

Q(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	3.9286464E-03	3.1642381E-02	9.8565584E-01	
2	3.8838100E-03	3.1099755E-02	9.8100495E-01	
3	3.9762594E-03	3.2070369E-02	9.6837449E-01	

U

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.8451568E-02	-1.4412239E-02	-1.9310150E-08	
2	-1.8437229E-02	-1.4400754E-02	-1.9294671E-08	
3	-1.8422894E-02	-1.4389277E-02	-1.9279199E-08	

V

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	6.9333313E-05	1.7234522E-04	3.5563274E-04	
2	3.4794051E-05	8.6465545E-05	1.7823372E-04	
3	2.5543477E-07	5.8642939E-07	8.2909878E-07	

W

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.2725204E-01	2.9568100E-01	4.3033957E-01	
2	1.2720603E-01	2.9559642E-01	4.3136644E-01	
3	1.2719315E-01	2.9558712E-01	4.3224227E-01	

INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. 4 BETWEEN JOINTS 4 AND 6 AFTER 100 HARMONICS

N(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.2139389E 01	5.5276535E 01	1.1726465E 02	
2	2.2147095E 01	5.5346039E 01	1.1078368E 02	
3	2.2154633E 01	5.5415543E 01	1.0430273E 02	

N(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	5.5067167E-03	4.7757566E-02	5.6028259E-01	
2	6.6430382E-03	5.8178357E-02	-4.1186166E-01	
3	7.7801421E-03	6.8601131E-02	-1.3839903E 00	

N(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-6.0746980E-01	-4.8616368E-01	-6.4984908E-07	
2	-8.2389760E-01	-5.9643873E-01	-1.3594799E-06	
3	-1.0402937E 00	-9.0671217E-01	-2.0691377E-06	

M(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.3756883E-01	3.4844625E-01	9.4794673E-01	
2	1.3678843E-01	3.4222323E-01	7.1972036E-01	
3	1.3604420E-01	3.3632100E-01	4.7851676E-01	

M(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	6.6475160E-03	5.4596778E-02	1.6506395E 00	
2	8.8075199E-04	7.1226172E-03	1.9734102E-01	
3	-4.8805103E-03	-4.0303137E-02	-1.2578983E 00	

M(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-3.7002057E-05	-2.9524323E-05	1.9455132E-11	
2	-1.6075107E-03	2.3597511E-03	2.1632161E-08	
3	-5.0083287E-03	-2.8493968E-03	-3.3559097E-08	

Q(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.1758635E-02	7.4413233E-03	1.6022022E-06	
2	1.6019419E-02	7.4217655E-03	7.2267005E-07	
3	9.9667050E-03	7.8789182E-03	-1.7085841E-07	

Q(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-3.8263064E-03	-3.1681810E-02	-9.6774673E-01	
2	-3.7337739E-03	-3.0711014E-02	-9.8037648E-01	
3	-3.7784439E-03	-3.1253759E-02	-9.8502862E-01	

U

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.8422894E-02	-1.4389277E-02	-1.9279199E-08	
2	-1.8437482E-02	-1.4400948E-02	-1.9294880E-08	
3	-1.8452071E-02	-1.4412627E-02	-1.9310576E-08	

V

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.5543477E-07	5.8642939E-07	8.2909878E-07	
2	-3.4284007E-05	-8.5294785E-05	-1.7657883E-04	
3	-6.8824069E-05	-1.7117651E-04	-3.5398151E-04	

W

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.2719315E-01	2.9558712E-01	4.3224227E-01	
2	1.2720710E-01	2.9559898E-01	4.3137020E-01	
3	1.2725419E-01	2.9568619E-01	4.3034720E-01	

INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. 5 BETWEEN JOINTS 2 AND 1 AFTER 100 HARMONICS

N(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.2653086E 01	5.6679550E 01	1.0690898E 02	
2	-9.2237187E-04	-2.0441257E-03	-8.0049694E-02	
3	-2.2664978E 01	-5.6683685E 01	-1.0706905E 02	

N(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	3.4010239E 00	8.5060253E 00	1.6007477E 01	
2	1.3785849E-03	3.7337078E-03	-4.0718321E-02	
3	-3.3982677E 00	-8.4985561E 00	-1.6088821E 01	

N(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-5.2670631E 00	-5.1542330E 00	-5.5168510E-07	
2	-5.2704163E 00	-5.1494904E 00	-1.2524142E-06	
3	-5.2737370E 00	-5.1447458E 00	-1.9529598E-06	

M(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	7.8271818E-04	6.2253326E-03	2.1358007E-01	
2	-2.6738235E-06	1.8013394E-05	3.4859445E-02	
3	-7.8277220E-04	-6.2115937E-03	-2.2396773E-01	

M(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	5.1967315E-03	4.1235041E-02	1.2509117E 00	
2	-7.6398919E-07	1.0105546E-05	-2.3272425E-02	
3	-5.1975138E-03	-4.1218348E-02	-1.3094759E 00	

M(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	4.9344413E-03	2.7921693E-03	3.3569663E-08	
2	5.6903437E-03	6.0994998E-03	5.2225694E-08	
3	3.9536797E-03	4.1054226E-03	-1.9442672E-07	

Q(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.7732217E-03	4.4031851E-03	1.7881973E-07	
2	-4.0113851E-03	5.3982139E-03	-7.3275629E-07	
3	-1.2941768E-03	-5.0471649E-03	-1.7563264E-07	

Q(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-3.5118696E-03	-2.7908146E-02	-8.3345407E-01	
2	-3.4458148E-03	-2.7295392E-02	-8.6003762E-01	
3	-3.5047696E-03	-2.7937378E-02	-9.4026250E-01	

U

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.8451568E-02	-1.4412239E-02	-1.9310150E-08	
2	-2.9611665E-06	-3.1555664E-06	2.3972673E-12	
3	1.8445652E-02	1.4405921E-02	1.9314950E-08	

V

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.2725204E-01	-2.9568100E-01	-4.3033957E-01	
2	-1.2725193E-01	-2.9568112E-01	-4.3033916E-01	
3	-1.2725157E-01	-2.9568088E-01	-4.3034023E-01	

W

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	6.9333313E-05	1.7234522E-04	3.5563274E-04	
2	6.7153655E-07	1.5420792E-06	-1.8807797E-05	
3	-6.7989458E-05	-1.6927345E-04	-3.4513930E-04	

INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. 6 BETWEEN JOINTS 4 AND 3 AFTER 100 HARMONICS

N(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.2648376E 01	5.6541718E 01	1.1613414E 02	
2	-7.2430400E-04	-1.8003730E-03	-2.3635217E 00	
3	-2.2649887E 01	-5.6545303E 01	-1.2087122E 02	

N(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	3.3986397E 00	8.4819078E 00	-6.9767399E 00	
2	1.2102237E-03	3.3032917E-04	-2.4751953E 01	
3	-3.3962193E 00	-8.4812441E 00	-4.2527267E 01	

N(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-6.1747017E 00	-6.3065968E 00	-3.5856083E-06	
2	-6.0811958E 00	-6.4324455E 00	-3.3966380E-06	
3	-5.9874544E 00	-6.5581179E 00	-3.2078678E-06	

M(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.4236680E-05	6.4040301E-05	1.0463274E-04	
2	-1.5987980E-06	-3.4952664E-06	-4.8221918E-06	
3	-2.7383241E-05	-7.0939001E-05	-1.1413603E-04	

M(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.5948294E-04	4.2171148E-04	6.9033680E-04	
2	-1.6467122E-05	-3.6700920E-05	-5.0457355E-05	
3	-1.9241025E-04	-4.9510016E-04	-7.9122954E-04	

M(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-3.7002057E-05	-2.9524323E-05	1.9455132E-11	
2	-2.7380258E-05	-1.9151965E-05	-3.0136726E-11	
3	-4.1048246E-05	-3.2292752E-05	1.4952095E-11	

Q(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	1.6713777E-05	1.7380429E-05	-7.5584400E-11	
2	-1.5011983E-06	-1.0219110E-06	-1.9654191E-12	
3	-1.9709812E-05	-1.9419836E-05	7.2772316E-11	

Q(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.1707543E-04	-3.0482234E-04	-4.9235928E-04	
2	-1.1692068E-04	-3.0491897E-04	-4.9314462E-04	
3	-1.1700732E-04	-3.0469848E-04	-4.9217045E-04	

U

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.8422894E-02	-1.4389277E-02	-1.9279199E-08	
2	-3.1266409E-06	-3.2927464E-06	-5.2731803E-12	
3	1.8416651E-02	1.4382686E-02	1.9268633E-08	

V

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.2719315E-01	-2.9558712E-01	-4.3224227E-01	
2	-1.2719315E-01	-2.9558700E-01	-4.3249649E-01	
3	-1.2719321E-01	-2.9558688E-01	-4.3275058E-01	

W

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.5543477E-07	5.8642939E-07	8.2909878E-07	
2	6.6111340E-07	1.5123032E-06	2.1342839E-06	
3	1.0941785E-06	2.4992587E-06	3.5234607E-06	

INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. 7 BETWEEN

JOINTS 6 AND 5 AFTER 100 HARMONICS

N(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	2.2663330E 01	5.6680283E 01	1.0691028E 02	
2	-7.3891389E-04	-1.6694972E-03	-7.9671621E-02	
3	-2.2664810E 01	-5.6683685E 01	-1.0706956E 02	

N(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	3.3985710E 00	8.5002069E 00	1.5998919E 01	
2	-1.0829561E-03	-2.1404168E-03	-4.9452111E-02	
3	-3.4007368E 00	-8.5044832E 00	-1.6097702E 01	

N(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-5.2748327E 00	-5.1602583E 00	-5.4970957E-07	
2	-5.2784081E 00	-5.1556988E 00	-1.2505006E-06	
3	-5.2819624E 00	-5.1511269E 00	-1.9511153E-06	

M(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-7.4437017E-04	-6.1284676E-03	-2.1342623E-01	
2	5.3381045E-06	-1.1748623E-05	-3.4850452E-02	
3	7.5023319E-04	6.1272234E-03	2.2383183E-01	

M(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-4.9486868E-03	-4.0599786E-02	-1.2498999E 00	
2	6.5570747E-06	4.1776821E-06	2.3294598E-02	
3	4.9609020E-03	4.0611476E-02	1.3085089E 00	

M(XS)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-5.0033287E-03	-2.8493968E-03	-3.3559097E-08	
2	-5.7476945E-03	-5.1405301E-03	-5.2273986E-08	
3	-4.0311068E-03	-4.1615969E-03	1.9444138E-07	

Q(X)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.7477563E-03	-4.3781102E-03	-1.7891290E-07	
2	4.0121078E-03	-5.3976588E-03	7.3276846E-07	
3	1.2701680E-03	5.0233081E-03	1.7572802E-07	

Q(S)

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	3.3508891E-03	2.7495742E-02	8.3279729E-01	
2	3.2848925E-03	2.6882753E-02	8.5937989E-01	
3	3.3437773E-03	2.7525447E-02	9.3960559E-01	

U

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.8452071E-02	-1.4412627E-02	-1.9310576E-08	
2	-3.3273818E-06	-3.4257037E-06	1.7715681E-12	
3	1.8445425E-02	1.4405768E-02	1.9314129E-08	

V

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-1.2725419E-01	-2.9568619E-01	-4.3034720E-01	
2	-1.2725425E-01	-2.9568642E-01	-4.3034703E-01	
3	-1.2725383E-01	-2.9568619E-01	-4.3034810E-01	

W

SECT.	X= 10.000	X= 25.000	X= 50.000	X=
1	-6.8824069E-05	-1.7117651E-04	-3.5398151E-04	
2	6.8200154E-07	1.5534097E-06	2.3175962E-05	
3	7.0178226E-05	1.7427333E-04	3.5218871E-04	

MOMENTS TAKEN BY EACH GIRDER AT X = 10.000

GIRDER NO.	MOMENT	PERCENTAGE	TENSION	COMPRESSION
1	0.134087E 03	26.70	0.502222E 02	-0.502284E 02
2	0.234093E 03	46.61	0.334152E 02	-0.334229E 02
3	0.134087E 03	26.70	0.502232E 02	-0.502275E 02
TOTAL	0.502267E 03	100.00	0.183861E 03	-0.183879E 03

MOMENTS TAKEN BY EACH GIRDER AT X = 25.000

GIRDER NO.	MOMENT	PERCENTAGE	TENSION	COMPRESSION
1	0.335259E 03	26.71	0.125577E 03	-0.125591E 03
2	0.584704E 03	46.58	0.208338E 03	-0.208355E 03
3	0.335261E 03	26.71	0.125580E 03	-0.125590E 03
TOTAL	0.125522E 04	100.00	0.459496E 03	-0.459535E 03

MOMENTS TAKEN BY EACH GIRDER AT X = 50.000

GIRDER NO.	MOMENT	PERCENTAGE	TENSION	COMPRESSION
1	0.645365E 03	25.88	0.241434E 03	-0.240880E 03
2	0.120253E 04	48.23	0.427396E 03	-0.428561E 03
3	0.645369E 03	25.88	0.241438E 03	-0.240879E 03
TOTAL	0.249332E 04	100.00	0.910267E 03	-0.910320E 03