THE UNIVERSITY OF MANITOBA

ANALYSIS OF FRAMES WITH FLEXIBLE CONNECTIONS

by

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NOTATION

A = cross-sectional area of member

A_2 = "shear area" in direction 2

a_j = dimensionless exponent

C_i = constant

C_A = bending deformation component for connection A

C_B = bending deformation component for connection B

D_I = joint displacement vector for joint I

E = matrix defined by Eq. (4.18)

$\overline{E}$ = modulus of elasticity

F_{BB} = flexibility matrix at B for member AB

F_K = diagonal flexibility matrix for connection K

$\overline{G}$ = modulus of rigidity

G^t = matrix defined by Eq. (4.16)

H = rotation transformation matrix

I = identity matrix

$\overline{I}$ = moment of inertia

K_{BB} = stiffness matrix at B for continuous elastic member AB

K_S = structure stiffness matrix

$K_{BB}{}^M$ = modified stiffness matrix at B

L = length of member

λ = semi-rigid connection factor defined by Montforton and Wu

M = applied moment at a particular connection

m = number of size parameters that influence the moment-rotation relationships

P = joint force vector for all joints in the structure

$\bar{P}_I$ = joint force vector at joint I

P_J = member load vector

P_j = numerical values of connection parameters

Q = matrix defined by Eq. (4.17)

R_{BA} = member force vector at B for member AB

S = matrix defined by Eq. (4.29)

T_{AB} = force transformation matrix from B to A

$T_{AB}{}^t$ = displacement transformation matrix from B to A

U_{BA} = cantilever deflection at B for member AB

$U_{BA}{}^C$ = connection distortion

$V_{BA}{}^M$ = member distortion

$V_K{}^C$ = distortion vector for connection K

$V_{BA}{}^E$ = elastic distortion of member AB referred to B

W = uniform distributed load

ϕ = connectional rotational deformation

CHAPTER I

INTRODUCTION

1.1 Object of Study

The high cost of structural steel framing connections, and their significant contribution to structural displacements have made the subject of framing connections a source of interest in recent times. While the connections constitute a small percentage of the total weight of a structure, they have a relatively high labour content and hence represent a substantial percentage of the total framing cost. Connection deformation is often responsible for a large proportion of the overall deflection of a structure. It is generally much more significant than axial deformation of members, which has been considered in most structural analysis computer programs for some time, or member shearing deformation which has often been considered.

In the conventional analysis of steel structures, beam to column connections have normally been assumed to be either completely flexible or completely fixed. These assumptions are not consistent with actual structural behaviour, but have been used to simplify analysis. Methods have been proposed for incorporating the effects of structural connections into the analysis procedure. However, because of the large amount of calculation and time involved, these methods remained unattractive until the digital computer removed the burden of lengthy computations and allowed a return to the basic principles of structural analysis.

To incorporate the effects of connection deformation into a structural analysis computer program, it is first necessary to have available force-deformation information for the different types of connections in use. Secondly, this information must be put into a form which requires a minimum of computer storage. Finally, the connection characteristics must be incorporated into the member force-displacement relationships.

Based on these requirements, this study consists of three distinct phases.

(a) The assembly of all available experimentally obtained force-deformation information on the most commonly used connection types. The majority of the test data available are in the form of moment-rotation (M-ϕ) curves which relate the applied moment, M, at a particular connection to the corresponding rotational deformation, ϕ, which occurs at the connection.

(b) The standardization of the M-ϕ relationship for each connection type to minimize the amount of connection information that must be stored.

(c) The generation of a structural analysis program which incorporates effects of connection deformation. Because of the non-linearity of the connection moment-rotation curve, the program must employ an iterative analysis procedure.

1.2 Relationship To Previous Studies

Since 1930, there has been considerable research aimed at determining the behaviour of structural connections. The original work was carried out simultaneously in Great Britain by C. Batho and H. C. Rowan (3)* and in the United States by J.C. Rathbun.(27) These tests were conducted to find the relationship between the moment applied to a connection, and the corresponding rotation between the elastic lines of the connected members. Since this original work, there has been extensive research on many of the connection types in use today. This work is summarized in Chapter 2. The availability of an increasing volume of connection information has made it possible to include the effects of flexible connections in the analysis of a structural framework.

J. F. Baker (1) and J. C. Rathbun applied slope deflection and moment distribution methods to analyze frames with flexible or semi-rigid connections. C. Batho and H. C. Rowan presented a beam line method for analyzing semi-rigid

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* Numbers in parentheses refer to entries in List of References.

frames. G. R. Montforton and T. S. Wu[22] incorporated the effects of flexible or semi-rigid connections into a stiffness analysis program. They assumed the approximate linear relationship of the form,

$$\phi = M\lambda \ , \tag{1.1}$$

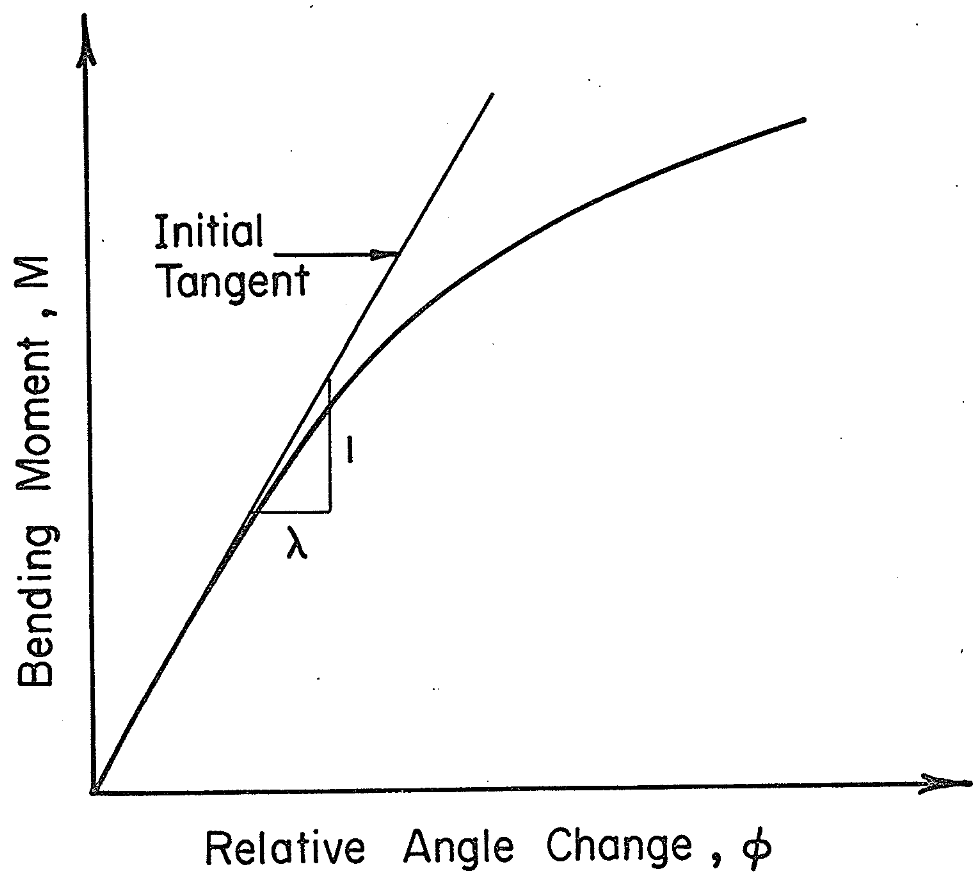
to adequately represent the connection moment-rotation behaviour in a frame with semi-rigid connections. In Eq. (1.1),

M = applied moment

λ = relative rotation of elastic lines of connected members

λ is defined as a semi-rigid connection factor and represents the inverse of the slope of the assumed straight line portion of the moment-rotation curve shown in Fig. 1.1. Its magnitude depends on the type of connection. Montforton and Wu used the semi-rigid connection factor to modify the member stiffness matrix and the member fixed-end-forces. Their method, which depends on the approximate linear relationship between moment and rotation, is acceptable in the range where applied moment is proportional to the relative rotation of the beam and column. However, many connection types exhibit a non-linear behaviour even at working loads, and the procedure would give misleading results if applied to these connections.

In this study, the non-linear connection effects are considered by employing an iterative procedure involving repeated cycles of linear analysis. After each cycle, the



MOMENT-ROTATION CURVE FOR SEMI-RIGID CONNECTIONS AFTER MONTFORTON AND WU

FIG. 1.1

flexibilities are modified and the new connection flexibilities used to modify the member stiffness matrices and the member fixed-end-forces for the next analysis. The procedure continues until the rotation and moment calculated for each connection, in the linear analysis, satisfy the equation of the non-linear moment-rotation curve for the connection.

While in general the analysis procedure is applicable to any type of structure, in this study it has been implemented only for planer frames in which only the rotational deformation of the connection is considered.

1.3 Assumptions and Limitations

The assumptions employed, and the limitations of the analysis program developed in this study are:

(a) The effects of shear and axial load on connection deformation are ignored.

(b) The program is limited to the analysis of planer frames.

(c) All members are assumed to be prismatic and straight.

(d) Only statical loading in the form of concentrated or uniformly distributed loads can be accomodated.

(e) The program uses an "in-core" equation solver. Hence the size of the structure that can be analyzed may be limited by computer primary storage capacity. Appendix D gives an indication of the size of structure that can be analyzed for a given core capacity.

(f) Possible buckling of individual members or portions of the structure is ignored.

(g) The effects of strain hardening are neglected.

(h) The material in the members is linearly elastic.

(i) It is assumed that the structural deflections are sufficiently small that they do not affect the geometry of the structure.

(j) The only cause of non-linear structural behaviour is the non-linear force-deformation characteristics of the connections.

1.4 Conventions Used

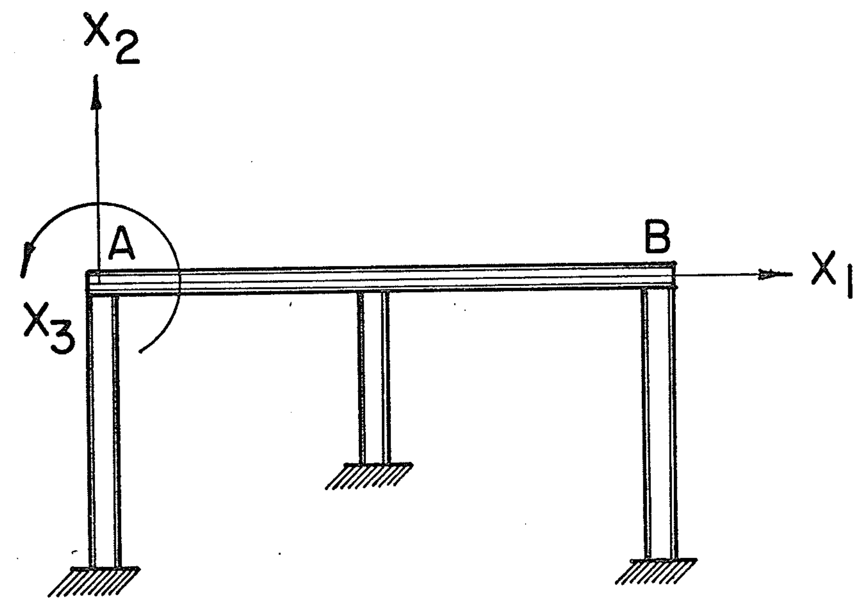
Matrix algebra techniques[20] are employed throughout this study for all structural analysis formulations. As illustrated in Fig. 1.2, the two types of coordinate systems used are:

(1) Global system - A single right hand coordinate system for the whole structure. All loads, joint displacements, reactions, and joint coordinates are expressed in the global system.

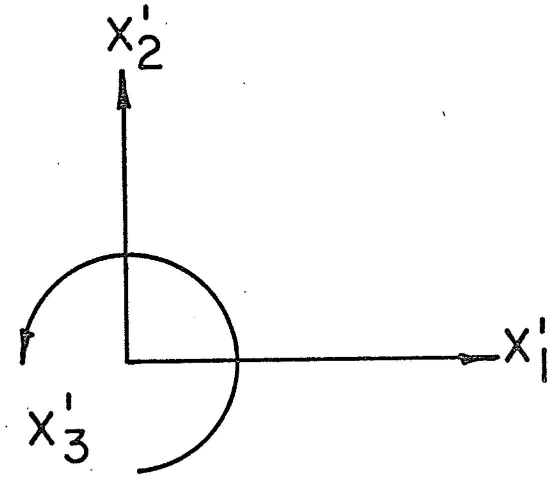
(2) Local system - Each member has associated with it a right hand local coordinate system whose X_1 axis has the same direction as that assumed for the member, as illustrated for member AB shown in Fig. 1.2. Member forces

and distortions are expressed in the local system.

Each member is assumed to have a direction from its A end to its B end. The X_1 axis of the local system lies along the member axis, and has its positive direction from A to B.



Local Coordinate System



Global Coordinate System

COORDINATE SYSTEMS

FIG. 1.2

CHAPTER II

CLASSIFICATION AND BEHAVIOUR OF STRUCTURAL CONNECTIONS

In this chapter, structural connections are classified as to their behaviour. The moment-rotation diagram is discussed, and practical working definitions of rigid, semi-rigid, and simple connections are presented. The behaviour of the most commonly used structural connections is discussed.

2.1 Introduction

At one time, riveting predominated as the most common connecting medium in steel structures. Present trends, however, are to an increased use of welding and high strength bolting. While these terms reveal the method of connecting, they shed little light on the behaviour of the connection.

The CSA Standard S-16 1965 [36] and the AISC Specification of Steel Construction 1967[37] recognized three types of connection behaviour:

(a) rigid framing

required for a theoretically flexible connection.

Conventional rigid and simple framing analysis procedures are not unduly difficult. However, as Ostrander[25] has pointed out, the practical problems encountered in the manual analysis of frames with semi-rigid connections are numerous. Research is continually required to determine the degree of rigidity of each new type and size of connection. Methods are required to extrapolate test results and to develop simplified linearly elastic design procedures for connections which generally act inelastically even in the range of working loads. The recently released CSA Standard S-16 1969[38] omits reference to semi-rigid framing as a standard construction method, although semi-rigid connections may still be used under this standard.

The increasing volume of experimental connection data coupled with computer analysis procedures now makes it possible to consider the actual connection behaviour in the design and analysis of steel frames.

2.2 The Moment-Rotation Curve

The primary distortion of a connection is the rotational deformation caused by moment. Methods have been proposed for calculating the M-ϕ relationship for semi-rigid connections, but most M-ϕ curves must be determined experimentally. Appendix A contains a series of experimentally obtained M-ϕ curves for a large number of

(b) simple framing

(c) semi-rigid framing

An ideally rigid connection is one whose M-ϕ curve is a straight vertical line. Regardless of the moment acting on the joint, there will be no relative rotation between the two elastic lines. Likewise, an ideally simple connection is one with a horizontal M-ϕ curve. Regardless of the relative rotation imposed on the two members, the connection will exert no resistance. Any intermediate condition is semi-rigid. It is easily appreciated that full rigidity and full flexibility are extreme conditions, never actually obtained. Practical working definitions of rigid, simple, and semi-rigid connections are given by Brandes and Mains [5] as follows:

(a) Any connection which develops beam restraint of less than 20% of the fixed-end-moment, thereby permitting 80% or more of the beam rotation required for a theoretically flexible connection, will be called a flexible connection.

(b) Any connection which develops 90% or more of the full fixed-end-moment, thereby permitting no more than 10% of the beam rotation required for a theoretically flexible connection, will be called a rigid connection.

(c) The semi-rigid connection is one capable of carrying from 20% to 90% of the full fixed-end-moment, thereby permitting from 10% to 80% of the beam rotation connection types. The moment-rotation curve for a typical semi-rigid connection is illustrated in Fig. 2.1. Observation of this figure and curves in Appendix A reveals that almost all connections behave inelastically. The flexible connection types exhibit non-linear behaviour almost from the start of loading, and the rigid connections at a later stage.

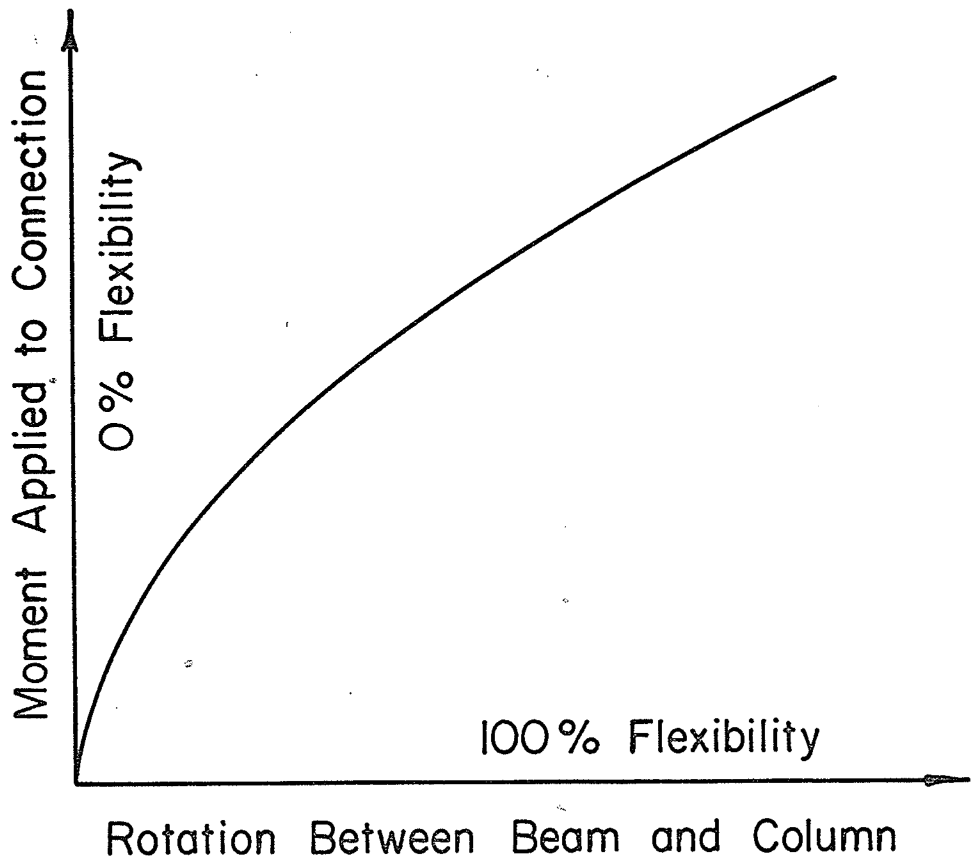
2.3 Connection Description and Behaviour

There are many different types of connections in use today. There follows a description of the most commonly used connection types and a discussion of their behaviour.

2.3.1 Double Web Angle Connections

Web framing angles, as illustrated in Fig. 2.2, constitute one of the most commonly used beam connection types. This type of connection is often termed a simple or flexible connection since it is designed to resist only vertical loads. Because of its frequent use, it has been standardized in most codes and manuals of steel construction. Although assumed to be simple, it does provide some moment restraint, and under normal conditions is subjected to both shear and moment.

Moment-rotation experiments have been performed on double web angle connections by J. C. Rathbun,[27] H. S. Somner,[30] and by Munse, Lewitt, and Chesson.[18] These



TYPICAL MOMENT-ROTATION CURVE FOR SEMI-RIGID CONNECTIONS

FIG. 2.1

experiments showed that in double web angle connections, flexibility was largely the result of bending and twisting of the angle legs. It was found that angles of the order of 3/8 inch thickness conformed to the end slope of the beam while offering little resistance. With thicker angles, an appreciable moment resistance was developed.

Framing angles, however, are inefficient in developing flexural resistance since most of the moment in a wide flange or I-beam is developed by flange forces. To develop the flange forces by web angles necessitates funneling the forces through the beam web. This results in early local web yielding under the stress concentrations that occur. This limits the end moment developed.

Tests have also shown that the end moment developed by a particular pair of connection angles depends on the length of the angles, which in turn is a function of the beam depth. Other factors which have been shown to affect the connection moment developed are the gage or gages of the connection angles, the fastener type and size, whether the connection is to a column flange or to a column web, and the physical properties of the angle material.

2.3.2 Single Web Angle Connections

Single web angle connections, illustrated in Fig. 2.3, are very similar in behaviour to double web angle connections. They offer some advantages over double web

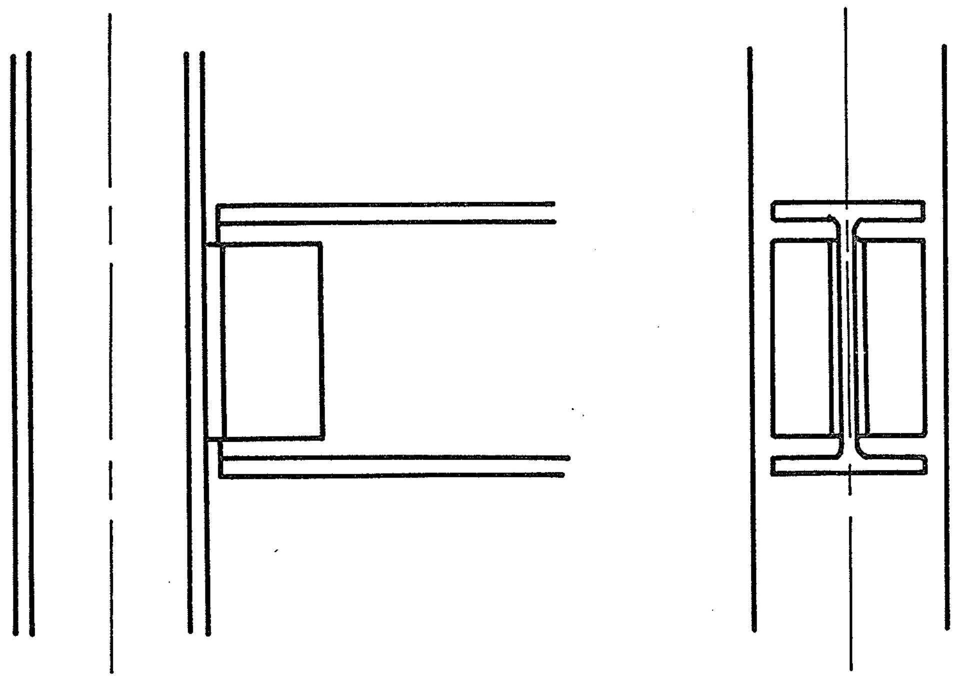


FIG. 2.2 DOUBLE WEB ANGLE CONNECTION

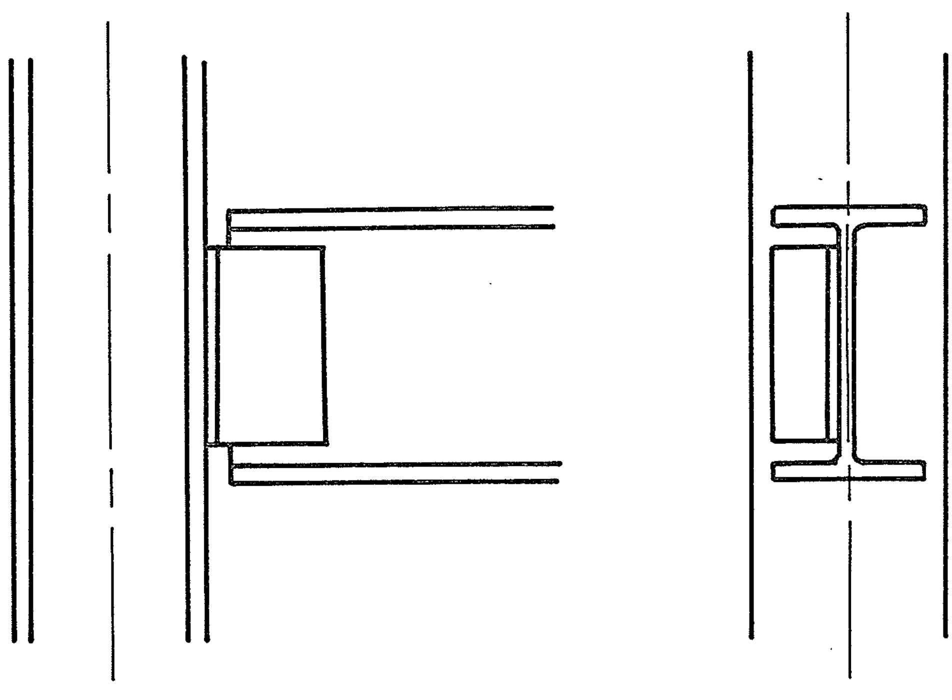


FIG. 2.3 SINGLE WEB ANGLE CONNECTION

angle connections, in that they are more economical and easier to erect.

S. L. Lipson[19] performed a series of tests on single web angle connections. These tests showed that the relatively small moment developed by the connections was a function of the length and size of angle, size of fasteners, and connection gage.

2.3.3 Header Plate Connections

Welded header plate connections, illustrated in Fig. 2.4, are similar to single and double angle framing connections in that they are intended to be simple beam connections. They are designed on the basis of shear, and like web angle connections, the moment transfer is small.

H. S. Somner conducted a series of experiments to determine the moment-rotation characteristics of different header plate connections. These tests showed that at low loads, the connection behaviour was essentially elastic. With increasing loads, there was considerable yielding in the plate adjacent to the welds and bolts. The large inelastic deformation in the header plate resulted in large rotations at the column. With progressive yielding of the beam web, the header plate was pushed into the beam web with the result that the bottom flange approached and finally came into contact with the column. This resulted in an increased rotational stiffness since all subsequent rotation

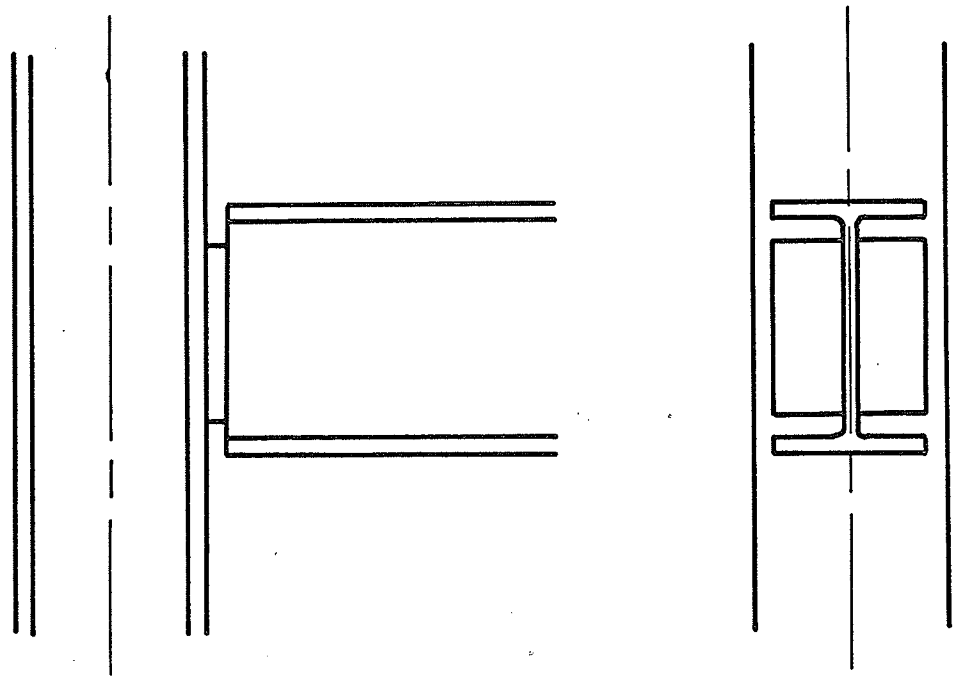


FIG. 2.4 HEADER PLATE CONNECTION

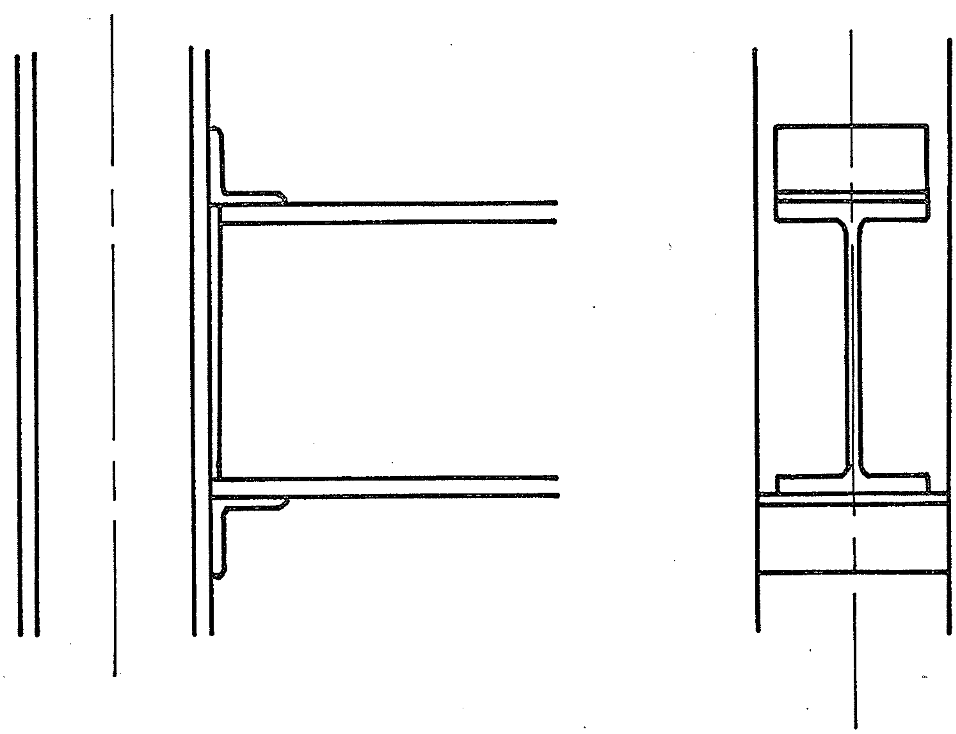


FIG. 2.5 TOP AND SEAT ANGLE CONNECTION

occurred about the the bottom flange as a pivot. This rotation was obtained from further deformation of the header plates, varying from a maximum at the top of the plate to a minimum at the bottom.

The behaviour of header plate connections depends on the length of plate, the thickness of the plate, and the connection gage. Differences in behaviour between header plate and web angle connections may be attributed to differences in geometry of the two connections.

2.3.4 Top and Seat Angle Connections

This type of connection, which is generally regarded as being of the semi-rigid variety, is illustrated in Fig. 2.5. Unlike web angle and header plate connections, the top and seat angle connection is designed to carry vertical load and to resist a significant amount of end moment.

Research on the behaviour of top and seat angle connections has been carried out by C. Batho and H. C. Rowan, R. A. Hechtman and B. G. Johnston,[13] and J. C. Rathbun. Test results from the experiments conducted by Hechtman and Johnston showed that the main factors contributing to rotation were bending of the top angle and column flange, extension of the tension fasteners, and slip of the rivets in the top flange of the beam. This type of connection passed through three stages, beginning with an initial stage with moment approximately proportional to rotation, a second stage in which yielding spreads within the connection, and a final stage characterized by accelerated rotation resulting in either fracture or excessive deformation.

Tests also revealed that in the case of light beam flanges, the top angle rotated as a whole and caused considerable deformation of the beam flange at high moments. The greatest deformation of the beam flanges occurred in the connection with the greatest thickness of top angle. In addition, considerable slip occurred in the rivets fastening the top angle to the beam flanges. It was also observed that a column with very heavy flanges increased the stiffness of a top angle to column connection, as compared with lighter weight column sizes.

2.3.5 End Plate Connections

End plate connections, illustrated in Fig. 2.6, may be flexible, semi-rigid, or rigid, depending on the thickness of the end plate, the size,number and distribution of the bolts or rivets, and whether the end plate is welded to the beam flanges or not. The connection between the beam and its end plate is usually a butt weld or a double fillet weld.

The most significant research on end plate connections has been carried out by R. Douty,[7] J. R. Ostrander, and A. N. Sherbourne.[29] Douty and Ostrander found that plate flexibility, together with bolt elongation, had an effect on connection rotation. Tests by Ostrander showed that column flange distortion also contributed to end rotation if no column stiffners were provided. In the majority of cases, column web and beam deformation made only minor contributions to total rotation.

The end plate connection does not stiffen the beam, but because of the plate flexing action and bolt elongation, may permit a much larger amount of rotation than would a butt weld in a welded connection. To develop the full potential of the fasteners, rather thick plates are required. By locating some of the fastener group outside the tension flange, the flexure arm of the fastener group is increased, and the bending of the end plate is reduced.

The research on end plate connections has shown that column stiffners increase the rigidity of an end plate connection by restraining the column web adjacent to the beam compression flanges and by confining and restraining the deformation of the column flanges adjacent to the beam tension flanges. The deformation of an unstiffened column is not confined as effectively to the immediate region of the connection as is the deformation of a stiffened column.

2.3.6 Welded Top Plate and Seat Connections

Welded top plate and seat connections, illustrated in Fig. 2.7, can be designed either to develop the full moment

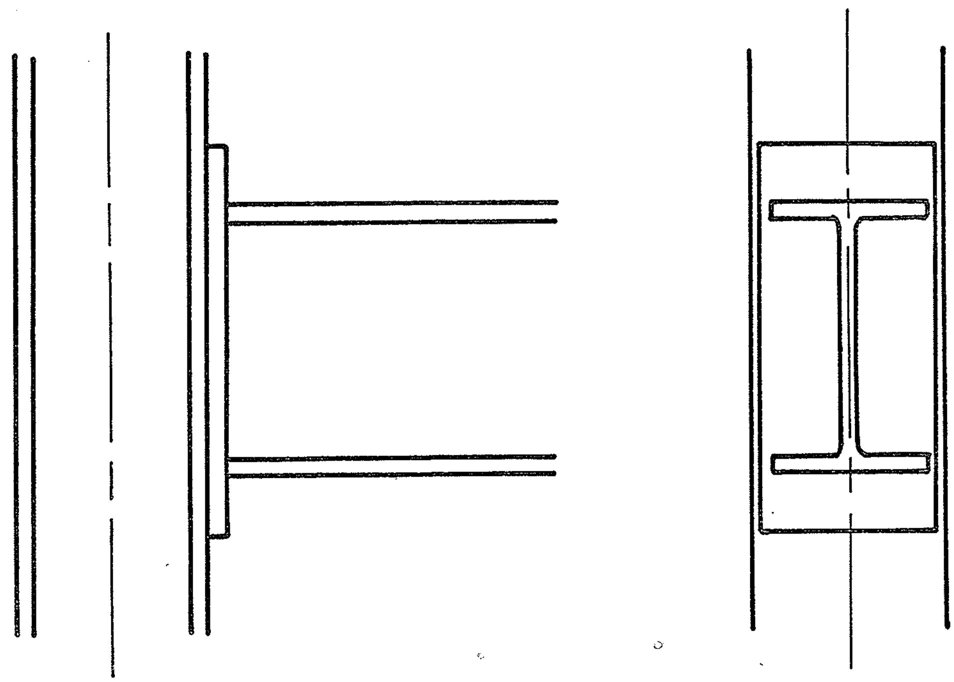


FIG. 2.6 END PLATE CONNECTION

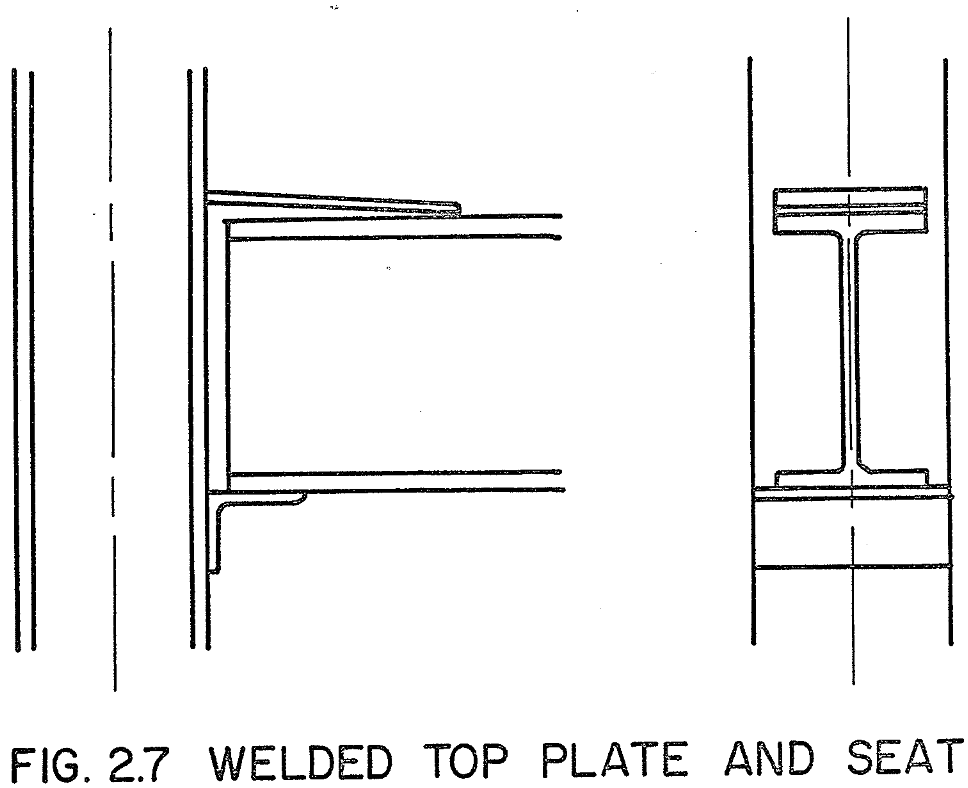


FIG. 2.7 WELDED TOP PLATE AND SEAT CONNECTION

capacity of the beam, or to restrain the beam by some lesser amount. The size of the top plate is based on the moment that the connection must develop. The smaller the top plate, the smaller the moment transmitted from the column into the beam. The connection must be capable of resisting definite moments without overstressing the welds.

In a moment connection of this type, some means must be provided to carry the thrust of the bottom flange. This is usually accomplished by specifying a square butt weld between the end of the beam flange and the column. To prevent stress concentrations in the top plate, a curved transition from the widened end of the basic plate is often used. The vertical beam reaction is carried by the bottom seat, and selection of the seat is based on the vertical reaction to be carried.

Several research programs have been carried out on welded top plate and seat connections. J. L. Brandes and R. M. Mains performed an extensive series of experiments on connections that were intended to be of the semi-rigid and flexible type. These experiments determined the behaviour of several top plate and seat details. L. G. Johnson, J. C. Cannon, and L. A. Spooner[15] tested several welded top plate and seat connections that were designed as rigid connections. R. F. Pray and C. Jensen[26] conducted a short test program to check a proposed design procedure for this type of connection.

2.3.7 T-Stub Connections

The T-stub connection, illustrated in Fig. 2.8, is one of the most commonly used connections for transmitting moments between beams and columns in bolted and riveted construction. As usually designed, the T-stub connection is sufficiently stiff to be classified as rigid. However, it is relatively simple to control the flexibility by varying the flexibility of the T-stub flange.

In the T-stub type of moment connection, the fasteners in the top stub flange are in tension. An additional tension in these bolts is caused by a prying action of the flange flexing. The greater the flexibility of either the column flange or T-stub flange, the greater will be the prying action on the bolts. The subsequent bolt elongation and deflection at the centre of the T-stub flange contribute to the rotational deformation of the connection.

The principal research on T-stub connections has been conducted by C. Batho and H. C. Rowan, and R. Douty. Experimental work by Douty showed that bolt elongation and flange flexure were the primary cause of stub deformations on the tension side of the connection. It is thus possible to control the rotational flexibility of the connection by varying the thickness of the T-stub flange. Tests by Douty also showed that high shear had negligible effect on the overall performance of the connection.

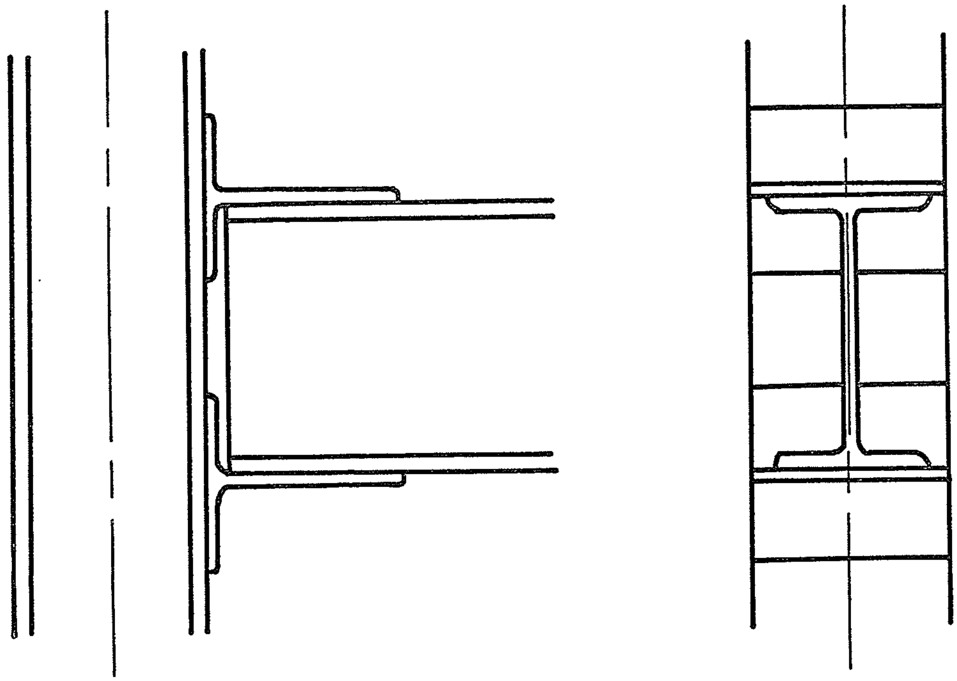


FIG. 2.8 T-STUB CONNECTION

CHAPTER III

STANDARDIZATION OF MOMENT-ROTATION CURVES

In this chapter, the method of standardization of the moment-rotation relationship for various connection types is presented. The procedure is illustrated, using as an example a double web angle connection.

3.1 Introduction

The constitutive relationships between moment and rotation for various connections is important in the determination of the force deformation relationships for a member with flexible connections. In order for a structural analysis computer program to incorporate the effects of connection deformation, the moment-rotation relationships for the connections used must be available. There are two ways that these relationships can be incorporated into such a program.

(a) The moment-curvature information for every connection of every type can be stored. Since for any given type of connection, there are a number of "size parameters" such as connection depth, angle thickness, etc., this requires the storing of the force-deformation information for an extremely large number of connections, many of which may be identical except for one size parameter.

(b) Since the moment-rotation characteristics for all connections of a given type are similar, a "standardized" moment-curvature relationship for that connection type can be derived. This standardized relationship is a function of the size parameters for that connection type. The moment-rotation characteristics for a particular connection can then be generated by substituting its size parameters into the standardized relationship.

The latter procedure has the obvious advantage over the former, that it drastically reduces the amount of connection information that must be stored. Using the standardization procedure, the description of only a single moment-rotation function for each connection type is necessary to be stored. Consequently, the standardization procedure has been used in this study. It makes use of experimentally obtained moment-rotation curves for connections of a particular type and involves isolating the effects of the various size parameters and incorporating them into the standardized moment-rotation function.

The procedure was derived by H. S. Somner[30] and applied to header plate connections. In this study, it has been extended to the following connection types:

(a) double web angle connections

(b) single web angle connections

(c) header plate connections

(d) top and seat angle connections

(e) T-stub connections

(f) end plate connections without column stiffeners

(g) end plate connections with column stiffeners.

3.2 Standardization Procedure

The standardization procedure employed in this study involves the representation of the moment-rotation curves for all connections of a given type by a single function of the form:

$$\phi = \sum_{i=1}^{\infty} C_i \, (KM)^i \tag{3.1}$$

where

ϕ = rotational deformation of connection, radians,

C = constant,

K = dimensionless factor whose value depends on the size parameters for the particular connection considered,

M = moment applied to the connection.

The factor K is assumed to have the form,

$$K = \prod_{j=1}^{m} P_j^{\,a_j} \tag{3.2}$$

where

P_j = numerical value of jth size parameter,

a_j = a dimensionless exponent which indicates the effect of the jth size parameter on the moment-rotation relationship,

m = total number of size parameters which are assumed to influence the moment-rotation relationships for the connection types considered.

The evaluation of the exponents a in Eq. (3.2) can be illustrated by considering a_j family of experimentally obtained moment-rotation curves for connections which are identical except for parameter P_j, as illustrated in Fig. 3.1.

A pair of curves, say curves 1 and 2, is considered and the relationship between moments M_1 and M_2 at a particular rotation, ϕ, is assumed to have the form:

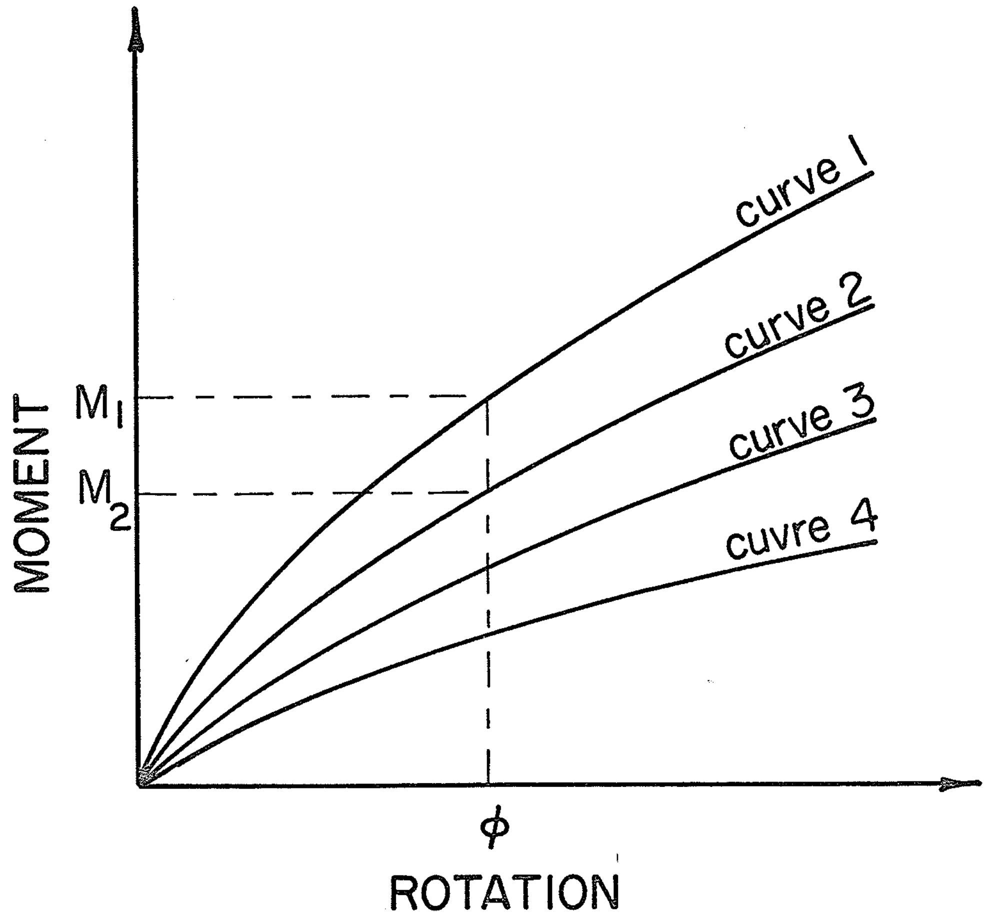
$$\frac{M_1}{M_2} = \left(\frac{P_{j2}}{P_{j1}}\right)^{a_j} \qquad (3.3)$$

where P_{j1} and P_{j2} are the numerical values of parameter P for connections 1 and 2 (corresponding to curves 1 and 2) respectively. M_1 and M_2 are the moment values, at rotation ϕ, for curves 1 and 2 respectively.

Eq. (3.3) is then rewritten and solved for a_j as follows:

$$a_j = \frac{\log\ (M_1/M_2)}{\log\ (P_{j2}/P_{j1})} \qquad (3.4)$$

Eq. (3.4) is used to calculate a_j values corresponding to several different rotations for each combination of experimental curves, such as 1 and 2, 1 and 3, 1 and 4, 2 and 3, 2 and 4, etc. The mean of the a_j values thus obtained is used in Eq. (3.2)



FAMILY OF MOMENT-ROTATION CURVES FOR CONNECTIONS WITH DIFFERENT VALUES P_j

FIG. 3.1

When average values have been calculated for all m exponents a_j in Eq. (3.2) they are used to obtain a series of points on a standardized moment-rotation (KM vs ϕ) diagram. Finally a least squares curve fitting procedure is used to derive the standardized moment-rotation relationship in the form of Eq. (3.1).

Since the moment-rotation function is an odd function, only terms involving odd powers i in Eq. (3.1) appear in the standardized functions. Furthermore, for the functions derived in this study, it was assumed that sufficient accuracy was obtained by including only three non-zero terms.

3.3 Standardized Moment-Rotation Relationship for Double Web Angle Connections

The standardization procedure is illustrated below for double web angle connections.

3.3.1 Parameters Affecting Moment-Rotation Characteristics of Double Web Angle Connections

For double web angle connections, the parameters which most strongly affect the moment-rotation characteristics are:

(a) depth of connection - d (in)

(b) gage of column connection - g (in)

(c) thickness of angles - t (in) as illustrated in Fig.

3.2.

3.3.2 Calculation of Factor K

The moment-rotation curves used to calculate the values of exponents a_j for the various parameters are shown in Fig. A.1 to Fig. A.3 in Appendix A. The necessary calculations are shown below.

(a) Parameter 1 - Depth of Connection

The moment-rotation curves used to calculate the value of exponent a_1 for the depth parameter were obtained by Munse, Lewitt, and Chesson Jr.,[18] and are illustrated in Fig. A.1. Consider firstly the curves for their test 4 and test 6 as reproduced in Fig. 3.3. The curves were obtained for connections which were identical in every respect except for the depth parameter. For convenience, the curves for tests 4 and 6 have been labelled curve 1 and curve 2 respectively. For a rotation value of $\phi = 2.5 \times 10^{-3}$ radians, the moment values obtained were

M_1 (curve 1) = 300 in. kips

M_2 (curve 2) = 600 in. kips

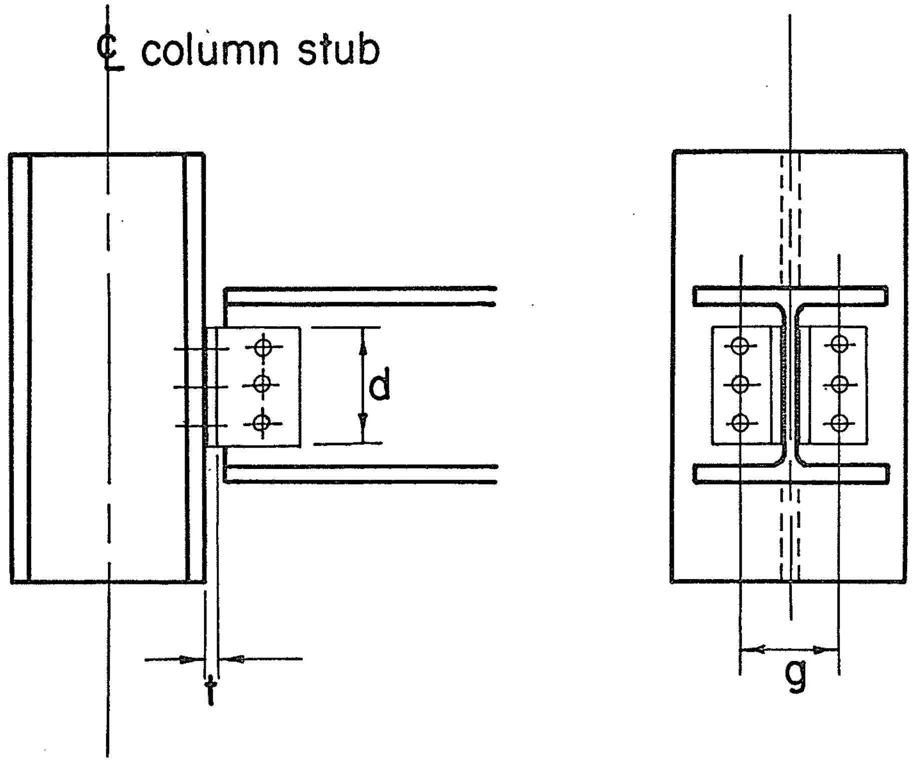
The corresponding depth parameters were

d_1 (curve 1) = 17.5 in.

d_2 (curve 2) = 23.5 in.

Thus from example 3.2,

$$a_1 = \frac{\log\left(\frac{300}{600}\right)}{\log\left(\frac{23.5}{17.5}\right)} = \frac{-.3010}{.1271} = -2.36$$



SIZE PARAMETERS FOR DOUBLE WEB ANGLE CONNECTIONS

FIG. 3.2

Repeating the above procedure for different values of the depth parameter and for other rotation values and averaging the resultant exponents, a mean value of $a_1 = -2.3$ is obtained.

(b) Parameter 2 - Thickness of Web Angles

The value of exponent a_2 for the angle thickness parameter is determined by a similar procedure to that used for the depth parameter. The moment-rotation curves used to calculate the value of exponent a were obtained by C. Batho [3] and are reproduced in Fig. 3.4. For convenience they have been labelled curve curve 2 and curve 3. These tests were performed on double web angle connections which were identical in every respect except for the thickness of the web angles For a rotation value of 7.0 x 10^{-3} radians, the following moment values were obtained.

M_1 (curve 1) = 188 in. kips.
M_2 (curve 2) = 200 in. kips.
M_3 (curve 3) = 212 in. kips.

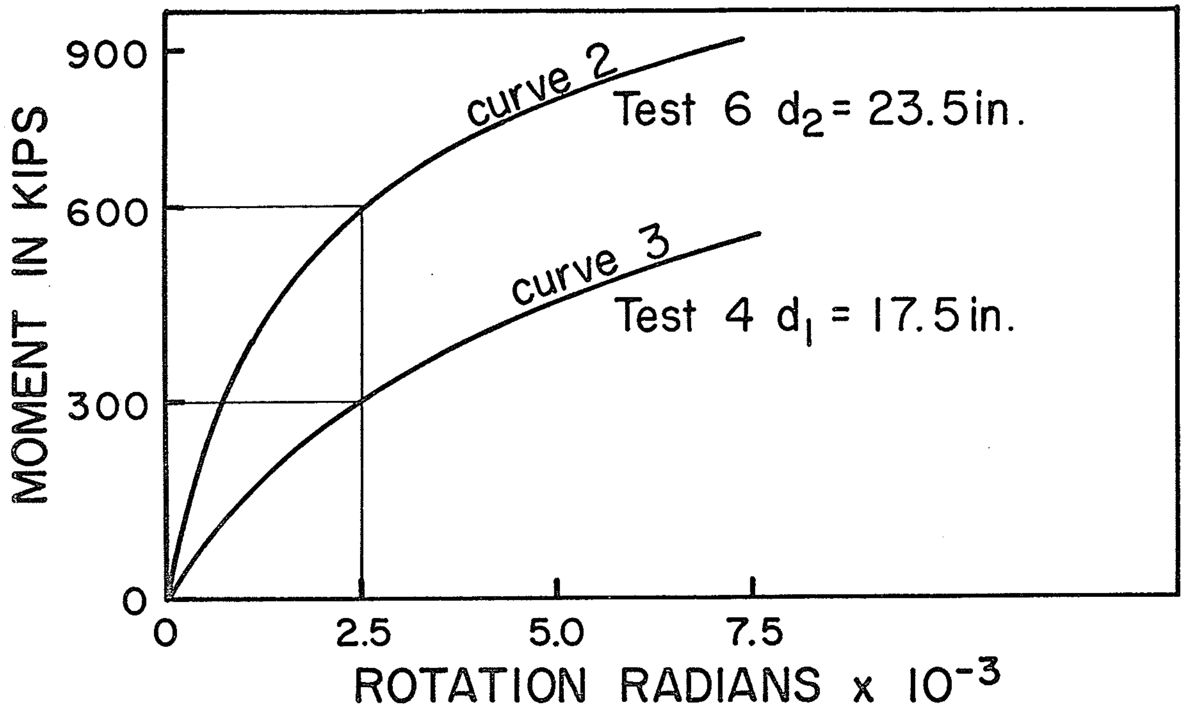
The corresponding angle thickness parameters were

t_1 (curve 1) = 3/8 in. (6 x 3 1/2 x 3/8 angle).
t_2 (curve 2) = 1/2 in. (6 x 3 1/2 x 1/2 angle).
t_3 (curve 3) = 5/8 in. (6 x 3 1/2 x 5/8 angle).

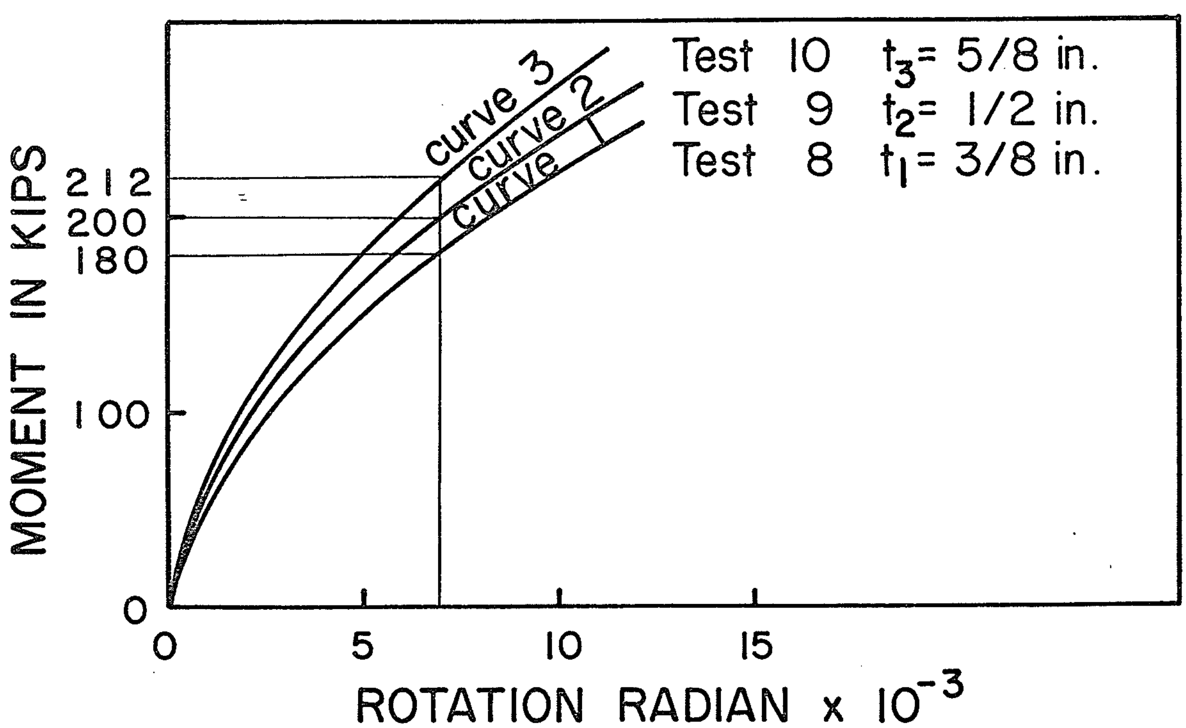
Again, substituting the values for curves 1 and 2 into Eq. (3.2),

$$a_2 = \frac{\log\left(\frac{188}{200}\right)}{\log\left(\frac{12}{38}\right)} = -0.216$$

Similarly, using the values for curves 1 and 3,



MOMENT-ROTATION CURVES AFTER MUNSE LEWITT, CHESSON JR. FIG. 3.3



MOMENT-ROTATION CURVE AFTER BATHO

FIG. 3.4

$$a_2 = \frac{\log\left(\frac{188}{212}\right)}{\log\left(\frac{58}{38}\right)} = -\ 0.235$$

The final value of exponent a_2 for web angle thickness, again found by averaging the exponents calculated for several values and several pairs of curves, is $a_2 = -0.23$.

(c) Parameter 3 - Connection Gage

Insufficient data are available to obtain an accurate value of exponent a for connection gage. Therefore, the value $a_3 = +1.6$, obtained for header plate connections, is used.

The K factor for the double web angle connection, obtained by substituting exponents a_1, a_2, and a_3 into Eq. (3.2) is thus:

$$K = d^{-2.3}\ t^{-.23}\ g^{1.6}$$

3.3.3 Calculation of Standard Moment-Rotation Curves

The final step in the standardization procedure is to obtain a standardized moment-rotation curve. For each double web angle connection of Appendix A, a unique factor K can be calculated. Each moment-rotation curve is multiplied by its corresponding K value, and a mean curve is drawn through the band of test results as illustrated in Fig. 3.5. The least squares curve fitting program, which was used to obtain equations of standardized moment-rotation curves for

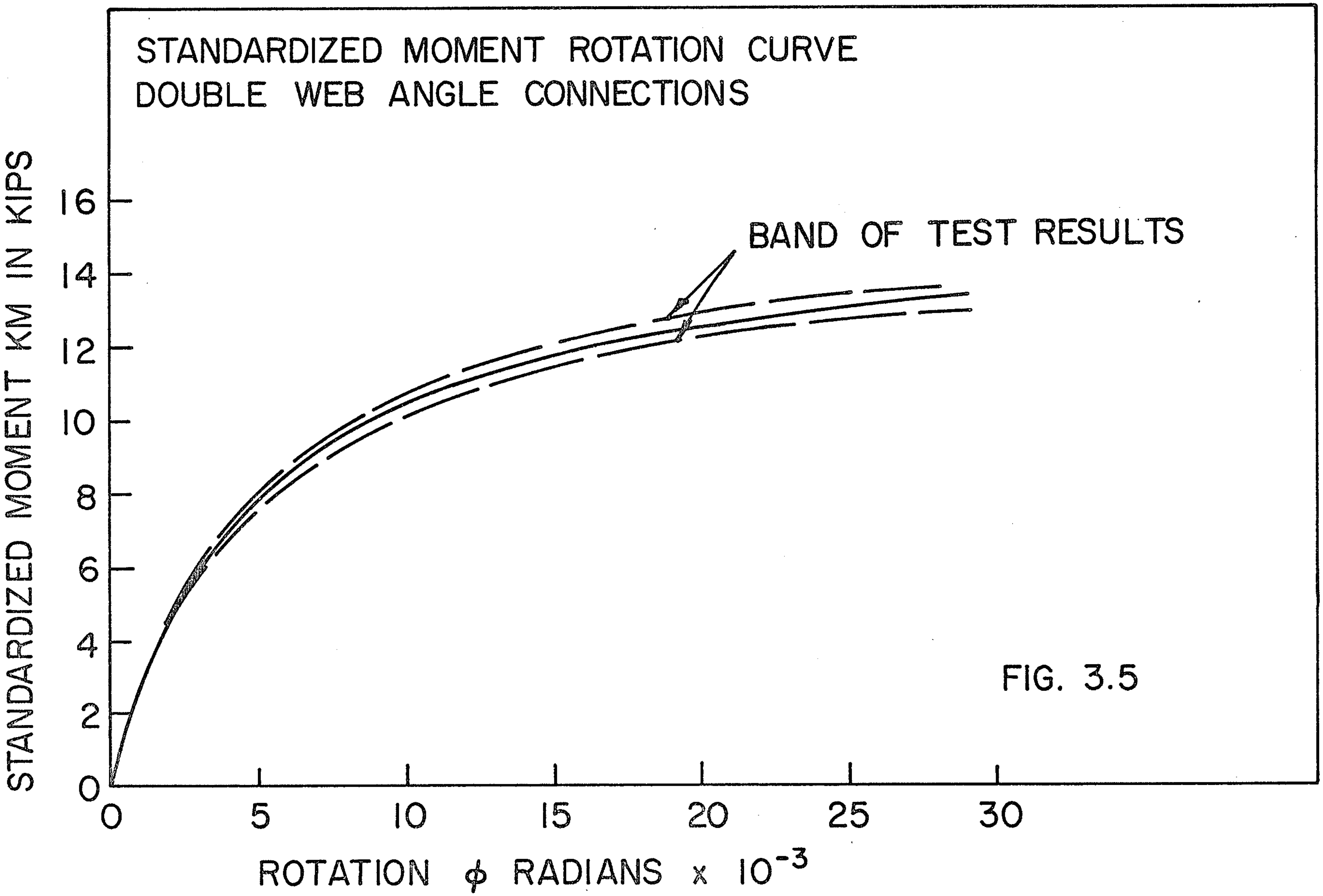

STANDARDIZED MOMENT ROTATION CURVE
DOUBLE WEB ANGLE CONNECTIONS
BAND OF TEST RESULTS
STANDARDIZED MOMENT KM IN KIPS
0
2
4
6
8
10
12
14
16
0
5
10
15
20
25
30
ROTATION ϕ RADIANS x 10^{-3}

FIG. 3.5

all of the connection types considered in this study, yields the following fifth order equation for the standardized moment-rotation curve for the double web angle connection:

$$\phi = 3.66\ (KM)\ 10^{-4} + 1.15\ (KM)^3\ 10^{-6} + 4.57\ (KM)^5\ 10^{-8}$$

This equation can be used to reproduce the moment-rotation curves for double web angle connections within the range of test results. Appendix B lists the standardized moment-rotation functions for each of the connection types considered.

3.3.4 Accuracy of Standardization Procedure

The accuracy of the standardization procedure can be checked by comparing the moment-rotation curves generated by the standardized equation with actual experimentally obtained curves. Fig. 3.6 shows a typical moment-rotation curve comparison for two double web angle connections. Additional comparisons have been made for other types of connections and these are included in Appendix B.

With few exceptions, the procedure was found to produce an accurate moment-rotation curve for a connection within the range of test results available. Table 3-1 gives an approximate indication of maximum percentage deviation from experimental curves for each connection type.

TABLE 3-1 COMPARISON OF STANDARDIZED AND EXPERIMENTAL CONNECTION MOMENT-ROTATION CURVES

Connection Type	% Deviation of Standardized Curve from Experimental Curve
Double Web Angle Connection	6 %
Single Web Angle Connection	10 %
Header Plate Connection	4 %
Top and Seat Angle Connection	11 %
End Plate Connection Without Stiffeners	3 %
End Plate Connection With Stiffeners	6 %
T-stub Connection	12 %

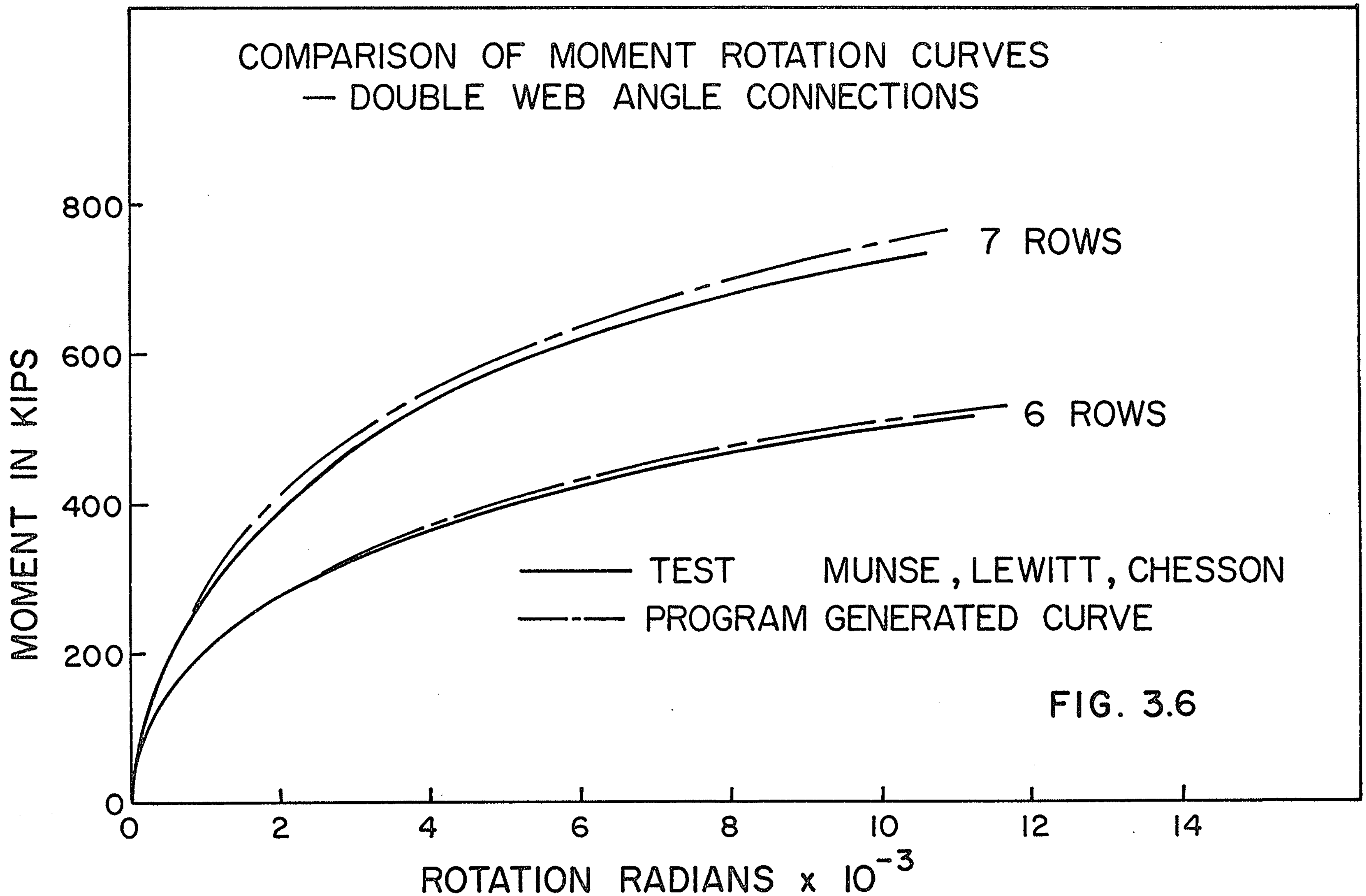

COMPARISON OF MOMENT ROTATION CURVES
— DOUBLE WEB ANGLE CONNECTIONS
MOMENT IN KIPS
800
600
400
200
0
0
2
4
6
8
10
12
14
ROTATION RADIANS x 10^{-3}
7 ROWS
6 ROWS
TEST MUNSE, LEWITT, CHESSON
PROGRAM GENERATED CURVE

FIG. 3.6

CHAPTER IV

FORCE DEFORMATION RELATIONSHIPS FOR A MEMBER

In this chapter, the force-deformation relationships for a continuous elastic member are summarized. These relationships are then modified to include the effects of flexible connections. As an illustration of the procedure, the force-deformation relationships are calculated for a member with rigid end connections and one with pinned ends.

4.1 Introduction

The force-deformation relationships for a typical member in a frame can be adequately represented by two sets of quantities, the stiffness matrix referred to one end of the member, and the fixed-end-force vector that would occur at that end if the member were loaded along its length with its ends rigidly fixed. Once these quantities have been rigidly determined, the stiffness matrix and the fixed-end-force vector at the other end of the member can be calculated using only statics and geometry.

The force-deformation relationships for a continuous elastic member can be derived by considering only statics, member geometry, and the stress-strain characteristics of the material. For a member with flexible connections, the member distortion consists of elastic member distortion and distortion due to connection deformation. The effect of connection deformation can be incorporated into a structural analysis by modifying the member stiffness matrix and fixed-end-forces.

4.2 Force-Deformation Relationships for a Continuous Elastic Member

Consider member AB shown in Fig. 4.1. The member is loaded by concentrated loads P_J and by forces R_{AB} and R_{BA} at ends A and B respectively. If end A is displaced by an amount u_{AB}, the displacement u_{BA} at end B is:

$$u_{BA} = T_{AB}{}^{t}u_{AB} + F_{BB}R_{BA} + \sum_{j=1}^{NL} T_{JB}{}^{t}F_{JJ}P_J \tag{4.1}$$

where:

T_{AB} = force transformation matrix from B to A

F_{BB} = flexibility matrix at B for member AB

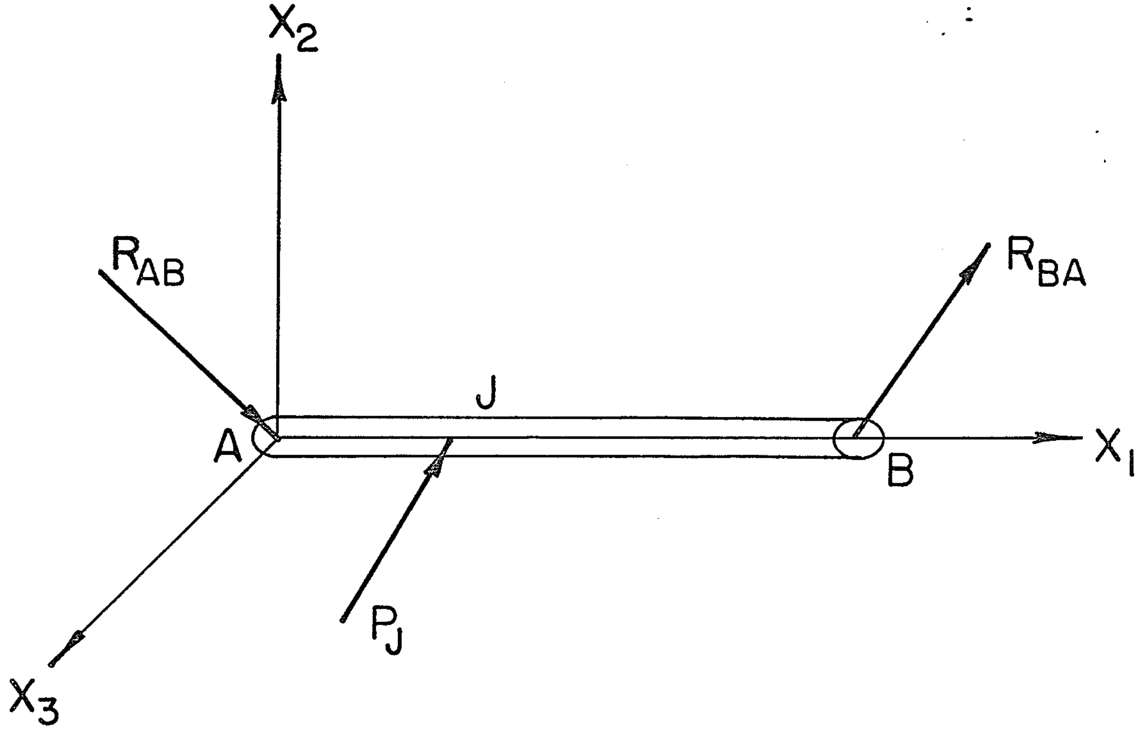
F_{JJ} = flexibility matrix at J for segment AJ

NL = total number of loads P_J on member.

The matrix $T_{AB}{}^{t}$ translates displacements from A to B. It is convenient to define:

$$U_{BA} = \sum_{j=1}^{NL} T_{JB}{}^{t}F_{JJ}P_J \tag{4.2}$$

where U_{BA} = cantilever deflection at B due to loads P_J.



CONTINUOUS ELASTIC MEMBER

FIG. 4.1

Premultiplication of Eq. (4.1) by $F_{BB}^{-1} = K_{BB}$, and substitution of Eq. (4.2) gives:

$$R_{BA} = K_{BB}(u_{BA} - T_{AB}^{t}\, u_{AB}) - K_{BB} U_{BA} \qquad (4.3)$$

where K_{BB} = stiffness matrix at B for member AB.

The elastic distortion of member AB referred to B is the distortional displacement of B relative to A. It is defined by:

$$V_{BA}^{E} = u_{BA} - T_{AB}^{t}\, u_{AB} \qquad (4.4)$$

Substitution of this definition into Eq. (4.3) gives the following force deformation relationship for member AB, referred to B:

$$R_{BA} = K_{BB} V_{BA}^{E} - K_{BB} U_{BA} \qquad (4.5)$$

The fixed-end-force at B is found by setting the member distortion to zero in Eq. (4.5), and solving for the member force at B. Thus:

$$R_{BA}^{F} = -K_{BB}\, U_{BA} \qquad (4.6)$$

where R_{BA}^{F} = fixed-end-force vector at B.

4.3 Force Deformation Relationship for Members With Flexible Connections

To formulate the force-displacement relationships for a member which has any number of flexible connections at cross-sections K, consider member AB shown in Fig. 4.2. The member is loaded by concentrated loads P_J applied at cross-sections J, and by end forces R_{AB} and R_{BA}. While connections would normally be used only at the ends of members, the method is applicable for connections located anywhere along the member:

Assume a flexibility matrix, F_K, for connection K, such that:

$$V_K^C = F_K R_K \tag{4.7}$$

where:

V_K^C = distortion vector for connection K. It gives the relative displacements at the two sides of the connection.

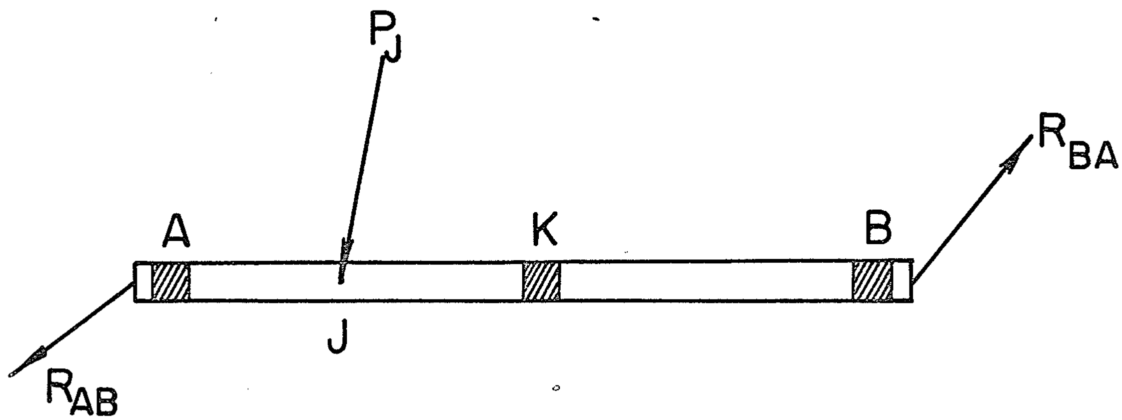
F_K = a diagonal flexibility matrix for connection K. The main diagonal element F_{II} gives the inverse slope of the force deformation curve for the Ith force component of the connection.

R_K = force vector at connection K.

Under the action of the applied loads, the total distortion, V_{BA}, of the member and its connections, consists of member distortion V_{BA}^M, and distortion V_{BA}^C due to the deformation of the connections. Compatibility requires that:

$$V_{BA} = V_{BA}^M + V_{BA}^C \tag{4.8}$$

Distortion V_{BA}^C can be expressed in terms of the



FLEXIBLE CONNECTIONS AT A, B, AND K

MEMBER WITH FLEXIBLE CONNECTIONS

FIG. 4.2

deformations of the connections as follows:

$$V_{BA}^{C} = \sum_{K=1}^{NC} T_{KB}^{t} V_{K}^{C} \tag{4.9}$$

where:

NC = number of connections

T_{KB}^{t} = matrix which translates displacements from K to B.

The member distortion is thus:

$$\begin{aligned} V_{BA}^{M} &= V_{BA} - V_{BA}^{C} \\ &= V_{BA} - \sum_{K=1}^{NC} T_{KB}^{t} V_{K}^{C} \end{aligned} \tag{4.10}$$

Substituting Eq. (4.9), Eq. (4.10) becomes:

$$V_{BA}^{M} = V_{BA} - \sum_{K=1}^{NC} T_{KB}^{t} F_{K} R_{K} \tag{4.11}$$

The force vector at B, defined in terms of the member distortion and the cantilever deflection U_{BA}, is:

$$R_{BA} = K_{BB} V_{BA}^{M} - K_{BB} U_{BA} \tag{4.12}$$

Substituting Eq. (4.11), Eq. (4.12) becomes:

$$R_{BA} = K_{BB}(V_{BA} - U_{BA}) - K_{BB} \sum_{K=1}^{NC} T_{KB}^{t} F_{K} R_{K} \tag{4.13}$$

The force vector at any connection K is:

$$R_{K} = T_{KB} R_{BA} + \sum_{j=1}^{NK} T_{KJ} P_{J} \tag{4.14}$$

where NK = number of loads on segment KB of the member.

Substituting Eq. (4.14) into Eq. (4.13):

$$R_{BA} = K_{BB}(V_{BA} - U_{BA}) - K_{BB}\sum_{K=1}^{NC} T_{KB}^{t} F_K T_{KB} R_{BA} - K_{BB}\sum_{K=1}^{NC} T_{KB}^{t} F_K \sum_{J=1}^{NK} T_{KJ} P_J \tag{4.15}$$

For convenience define:

$$G^t = \begin{bmatrix} T_{1B} \\ \vdots \\ T_{KB} \\ \vdots \\ T_{NC,B} \end{bmatrix} \tag{4.16}$$

$$Q = \begin{bmatrix} \sum_{J=1}^{NK} T_{1J} P_J \\ \vdots \\ \sum_{J=1}^{NK} T_{KJ} P_J \\ \vdots \\ \sum_{J=1}^{NK} T_{NC,J} P_J \end{bmatrix} \tag{4.17}$$

and,

$$E = \begin{bmatrix} F_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & F_2 & 0 & 0 & 0 & 0 & 0 \\ & & \ddots & & & & \\ 0 & 0 & 0 & 0 & F_k & 0 & 0 \\ & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{NC,NC} \end{bmatrix} \tag{4.18}$$

where:

G^t = an NC x 1 vector whose Kth element is the translation matrix for connection K.

Q = an NC x 1 vector, the Kth element of which gives the statical effect at connection K, of the loads on portion

KB of the member.

E = a diagonal matrix for which the Kth diagonal term is the diagonal matrix F_K for connection K.

Then:

$$R_{BA} = K_{BB}(V_{BA} - U_{BA}) - K_{BB}GEG^{t}R_{BA} - K_{BB}GEQ$$

or:

$$(I + K_{BB}GEG^{t})R_{BA} = K_{BB}(V_{BA} - U_{BA}) - K_{BB}GEQ \qquad (4.19)$$

where here, I represents a unit matrix.

Next, define:

$$S = (I + K_{BB}GEG^{t})^{-1} \qquad (4.20)$$

Then:

$$R_{BA} = SK_{BB}(V_{BA}) - SK_{BB}(U_{BA} + GEQ) \qquad (4.21)$$

Eq. (4.21) gives the force-deformation relationships for a member with any number of flexible connections located anywhere along its length. By comparing Eq. (4.21) with Eq. (4.5), it can be seen that:

(a) The modified stiffness matrix for a member with flexible connections is

$$K_{BB}^{M} = SK_{BB} \qquad (4.22)$$

(b) The fixed-end-force vector at B is

$$P_B^{F} = -K_{BB}^{M}(U_{BA} + GEQ) \qquad (4.23)$$

4.4 Modified Stiffness Matrix for Plane Frame Members With Connections at Ends Only

To illustrate the effects of connection deformations, consider a plane frame member AB with flexible connections at its ends. The E matrix for the member is:

$$E = \begin{bmatrix} F_A & 0 \\ 0 & F_B \end{bmatrix} \tag{4.24}$$

where:

F_A = flexibility matrix for connection A.

F_B = flexibility matrix for connection B.

The flexibility matrices F_A and F_B are diagonal matrices which represent axial, shear, and moment deformations produced by unit loads. Axial and shear effects can be considered negligible and hence the flexibilities of the connections may be represented by the bending flexibility component only. This component is represented by the inverse of the slope of the moment-rotation curve and is naturally dependent on the connection type. The E matrix can therefore be written as:

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_B \end{bmatrix} \tag{4.25}$$

where:

C_A = bending deformation component for connection A

C_B = bending deformation component for connection B.

The G^t matrix can also be written as:

$$G^t = \begin{bmatrix} T_{AB} \\ \\ I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & L & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (4.26)$$

and the product GEG^t becomes:

$$GEG^t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & L^2C_A & LC_A \\ 0 & LC_A & (C_A+C_B) \end{bmatrix} \qquad (4.27)$$

From Eq. (4.20)

$$S = (I+K_{BB}GEG^t)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (1+\frac{6\bar{E}\bar{I}}{L}C_A) & \frac{6\bar{E}\bar{I}}{L^2}(C_A-C_B) \\ 0 & -2\bar{E}\bar{I}C_A & 1+\frac{2\bar{E}\bar{I}}{L}(2C_B-C_A) \end{bmatrix}$$

where: $\bar{E}$ = modulus of elasticity

$\bar{I}$ = moment of inertia (4.28)

L = length of member

Thus:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \dfrac{1+\frac{2\overline{EI}}{L}(2C_B - C_A)}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} & \dfrac{-\frac{6\overline{EI}}{L}(C_A - C_B)}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} \\ 0 & \dfrac{\frac{2\overline{EI}C_A}{L}}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} & \dfrac{\frac{6\overline{EI}C_A}{L}}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} \end{bmatrix} \tag{4.29}$$

The modified stiffness matrix for the member can then be generated from

$$K_{BB}^{M} = SK_{BB} \tag{4.22}$$

and is

$$K_{BB}^{M} = \begin{bmatrix} \frac{A\overline{E}}{L} & 0 & 0 \\ 0 & \dfrac{\frac{12\overline{EI}}{L^2}(1+\frac{\overline{EI}C_A}{L}+\frac{\overline{EI}C_B}{L})}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} & \dfrac{-\frac{6\overline{EI}}{L^2}(1+\frac{2\overline{EI}}{L}C_A)}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} \\ 0 & \dfrac{-\frac{6\overline{EI}}{L^2}(1+\frac{2\overline{EI}}{L}C_A)}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} & \dfrac{\frac{4\overline{EI}}{L}(1+\frac{3\overline{EI}}{L}C_A)}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L}C_A C_B)} \end{bmatrix} \tag{4.30}$$

4.4.1 <u>Modified Stiffness Matrix for Member With Rigid End Connections</u>

For a member AB with rigid connections at ends A and B, $C_A = C_B = 0$ and the modified stiffness matrix in Eq. (4.30) degenerates to:

$$K_{BB}^{M} = \begin{bmatrix} \frac{A\overline{E}}{L} & 0 & 0 \\ 0 & \frac{12\overline{EI}}{L^3} & \frac{-6\overline{EI}}{L^2} \\ 0 & \frac{-6\overline{EI}}{L^2} & \frac{4\overline{EI}}{L} \end{bmatrix} \tag{4.31}$$

which is identical to the stiffness matrix K_{BB}, at the end of a continuous member.

4.4.2 Modified Stiffness Matrix for Member With Pinned Connections at A and B Ends

For a member AB with pinned connections at A and B, $C_A = C_B = \infty$. Dividing the numerator and denominator of each component of the modified stiffness matrix in Eq. (4.30) by $C_A = C_B$, the matrix becomes:

$$K_{BB}^{M} = \begin{bmatrix} \frac{A\overline{E}}{L} & 0 & 0 \\ 0 & \dfrac{\frac{12\overline{EI}}{L^3}\left[\frac{1}{C_A}+\frac{2\overline{EI}}{L}\right]}{\frac{1}{C_A}+\frac{4\overline{EI}}{L}\left(2+\frac{3\overline{EI}C_A}{L}\right)} & \dfrac{\frac{-6\overline{EI}}{L}\left[\frac{1}{C_A}+\frac{2\overline{EI}}{L}\right]}{\frac{1}{C_A}+\frac{4\overline{EI}}{L}\left(2+\frac{3\overline{EI}C_A}{L}\right)} \\ 0 & \dfrac{\frac{-6\overline{EI}}{L^2C_A}-12\frac{(\overline{EI})^2}{L^3}}{\frac{1}{C_A}+\frac{4\overline{EI}}{L}\left(2+\frac{3\overline{EI}C_A}{L}\right)} & \dfrac{\frac{4\overline{EI}}{LC_A}+12\left(\frac{\overline{EI}}{L}\right)^2}{\frac{1}{C_A}+\frac{4\overline{EI}}{L}\left(2+\frac{3\overline{EI}C_A}{L}\right)} \end{bmatrix}$$

Then setting $C_A = \infty$,

$$K_{BB}^{M} = \begin{bmatrix} \frac{A\overline{E}}{L} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.33}$$

which is the B-end stiffness matrix for a member with pins at both ends.

4.5 Fixed-End-Forces for Member With Flexible Connections

The fixed-end-forces for member AB of Fig. 4.2 can be calculated from Eq. (4.23)

$$P_B^{\,F} = -SK_{BB}U_{BA} - SK_{BB}GEQ \tag{4.23}$$

where all terms have been previously defined.

The fixed-end-force vector at B is calculated below for a member that is continuous at end B and pinned at end A, firstly for a single concentrated load at midspan, and then for a uniformly distributed load covering whole span.

4.5.1 Fixed-End-Forces for Member With Concentrated Load

Consider a single concentrated load placed at the midspan of member AB as shown in Fig. 4.3. Member AB is continuous at end B and pinned at end A.

The modified stiffness matrix for the member can be generated from Eq. (4.30), and is

$$K_{BB}^{M}=SK_{BB}=\begin{bmatrix} \frac{\overline{AE}}{L} & 0 & 0 \\ 0 & \dfrac{\frac{12\overline{EI}}{L^3}\left(1+\frac{\overline{EI}}{L}C_A+\frac{\overline{EI}}{L}C_B\right)}{1+\frac{4\overline{EI}}{L}\left(C_A+C_B+\frac{3\overline{EI}}{L}C_AC_B\right)} & \dfrac{-\frac{6\overline{EI}}{L^2}\left(1+\frac{2\overline{EI}}{L}C_A\right)}{1+\frac{4\overline{EI}}{L}\left(C_A+C_B+\frac{3\overline{EI}}{L}C_AC_B\right)} \\ 0 & \dfrac{-\frac{6\overline{EI}}{L^2}\left(1+\frac{2\overline{EI}}{L}C_A\right)}{1+\frac{4\overline{EI}}{L}\left(C_A+C_B+\frac{3\overline{EI}}{L}C_AC_B\right)} & \dfrac{\frac{4\overline{EI}}{L}\left(1+\frac{3\overline{EI}}{L}C_A\right)}{1+\frac{4\overline{EI}}{L}\left(C_A+C_B+\frac{3\overline{EI}}{L}C_AC_B\right)} \end{bmatrix}$$

The cantilever deflection, U_{BA}, is given by

$$U_{BA} = T_{JB}{}^{t}U_{JA} = T_{JB}{}^{t}F_{JJ}P_J \tag{4.34}$$

and is

$$U_{BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{L}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{L}{2\overline{AE}} & 0 & 0 \\ 0 & \frac{L^3}{24\overline{EI}} & \frac{L^2}{8\overline{EI}} \\ 0 & \frac{L^2}{8\overline{EI}} & \frac{L}{2\overline{EI}} \end{bmatrix} \begin{bmatrix} 0 \\ -P \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-5PL^3}{48\overline{EI}} \\ \frac{-PL^2}{8\overline{EI}} \end{bmatrix} \tag{4.35}$$

Therefore, the first term of Eq. (4.23) becomes

$$-SK_{BB}U_{BA} = \begin{bmatrix} 0 \\ \dfrac{\frac{P}{2}-\frac{P\overline{EI}}{4L}C_A+\frac{5P\overline{EI}}{4L}C_B}{1+\frac{4\overline{EI}}{L}\left(C_A+C_B+\frac{3\overline{EI}}{L}C_AC_B\right)} \\ \dfrac{-\frac{PL}{8}+\frac{P\overline{EI}}{4}C_A}{1+\frac{4\overline{EI}}{L}\left(C_A+C_B+\frac{3\overline{EI}}{L}C_AC_B\right)} \end{bmatrix} \tag{4.36}$$

and GEQ is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & L & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_B \end{bmatrix} \begin{bmatrix} 0 \\ -P \\ -\frac{PL}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.37}$$

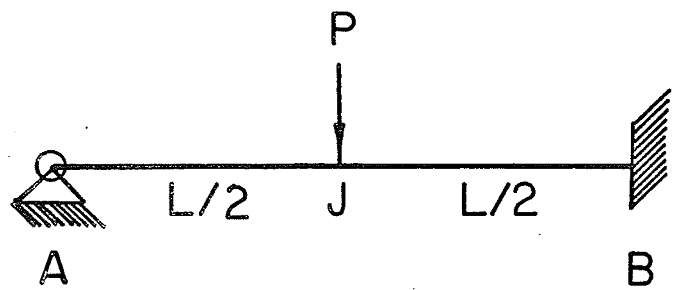
The product $-SK_{BB}GEQ$ becomes

$$\begin{bmatrix} 0 \\ \dfrac{\frac{3\overline{EI}C_A P}{L}+6\left[\frac{\overline{EI}}{L}\right]^2 C_A C_B P}{1+\frac{4\overline{EI}}{L}(C_A+C_B+\frac{3\overline{EI}}{L}C_A C_B)} \\ \\ \dfrac{-\overline{EI}C_A P}{1+\frac{4\overline{EI}}{L}(C_A+C_B+\frac{3\overline{EI}}{L}C_A C_B)} \end{bmatrix} \tag{4.38}$$

Therefore, adding Eq. (4.36) and Eq. (4.38), the B end fixed-end-forces are:

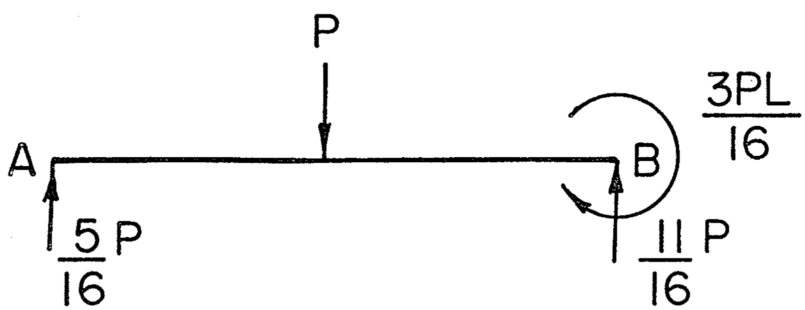
$$P_B{}^F = \begin{bmatrix} 0 \\ \dfrac{\frac{P}{2}+\frac{11\overline{EI}C_A P}{4L}+\frac{5P\overline{EI}C_B}{4L}+6\left[\frac{\overline{EI}}{L}\right]^2 C_A C_B P}{1+\frac{4\overline{EI}}{L}(C_A+C_B+\frac{3\overline{EI}}{L}C_A C_B)} \\ \\ \dfrac{-\frac{PL}{8}-\frac{3\overline{EI}C_A P}{4}}{1+\frac{4\overline{EI}}{L}(C_A+C_B+\frac{3\overline{EI}}{L}C_A C_B)} \end{bmatrix} \tag{4.39}$$

For member AB, $C = \infty$ and $C = 0$. Therefore



MEMBER AB WITH MIDSPAN LOAD

FIG. 4.3



FREE BODY DIAGRAM FOR MEMBER AB

FIG. 4.4

$$P_B^F = \begin{bmatrix} 0 \\ \frac{11P}{16} \\ \frac{-3PL}{16} \end{bmatrix} \quad (4.40)$$

By statics

$$P_A^F = \begin{bmatrix} 0 \\ \frac{5P}{16} \\ 0 \end{bmatrix} \quad (4.41)$$

Fig. 4.4 illustrates the free-body diagram for member AB

4.5.2 Fixed-End-Forces for Member With Uniformly Distributed Load

Consider member AB which carries a uniformly distributed load over its entire span, as shown in Fig. 4.5. The member has a rigid connection at end B and a and pinned connection at end A.

The cantilever deflection at B corresponding to the uniformly distributed load shown in Fig. 4.6 is:

$$U_{BA} = \int_0^L T_{CB}^t F_C P_C \, ds$$

$$= \int_0^L T_{CB}^t F_C \left(\int_0^S T_{CD} W_D dz \right) ds \quad (4.42)$$

where: F_C = unit flexibility matrix

P_C = force vector at cross section C

$$\int_0^S T_{CD} W_D dz = \int_0^S \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & z & 1 \end{bmatrix} \begin{bmatrix} 0 \\ W_z \\ 0 \end{bmatrix} dz = \begin{bmatrix} 0 \\ W_2 s \\ \frac{s^2 W_2}{2} \end{bmatrix} \tag{4.43}$$

Then:

$$U_{BA} = \int_0^L \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & S \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{A\overline{E}} & 1 & 0 \\ 0 & \frac{1}{A_2\overline{G}} & 0 \\ 0 & 0 & \frac{1}{\overline{EI}} \end{bmatrix} \begin{bmatrix} 0 \\ W_2 S \\ \frac{S^2 W_2}{2} \end{bmatrix}$$

where: A = cross sectional area

A_2 = "shear area" in direction 2

$\overline{G}$ = modulus of rigidity

$$U_{BA} = \begin{bmatrix} 0 \\ \frac{W_2 L^4}{8\overline{EI}} \\ \frac{W_2 L^3}{6\overline{EI}} \end{bmatrix} \tag{4.44}$$

Therefore, the first term of Eq. (4.23) becomes

$$SK_{BB}\, U_{BA} = \begin{bmatrix} 0 \\ \dfrac{\frac{W_2 L}{2} - \frac{W_2 \overline{EI} C_A}{2} + \frac{3W_2 \overline{EI} C_B}{2}}{1 + \frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L} C_A C_B)} \\ \dfrac{\frac{-W_2 L^2}{12} + \frac{\overline{EI} W_2 C_A L}{2}}{1 + \frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L} C_A C_B)} \end{bmatrix} \tag{4.45}$$

GEQ is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & L & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_B \end{bmatrix} \begin{bmatrix} 0 \\ -W_2 L \\ \frac{-W_2 L^2}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \frac{-W_2 L^3 C_A}{2} \\ \frac{-W_2 L^2 C_A}{2} \end{bmatrix} \tag{4.46}$$

The product - $SK_{BB}GEQ$ becomes

$$\begin{bmatrix} 0 \\ \dfrac{3\overline{EI}C_A + 3(\overline{EI})^2 W_2 C_A C_B}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L} C_A C_B)} \\ \dfrac{-\overline{EI}LW\ C}{1+\frac{4\overline{EI}}{L}(C_A + C_B + \frac{3\overline{EI}}{L} C_A C_B)} \end{bmatrix} \tag{4.47}$$

Therefore, adding Eq. (4.45) and Eq. (4.47), the B end fixed-end-forces are:

$$P_B^F = \begin{bmatrix} 0 \\ \dfrac{\dfrac{W_2}{2}L + \dfrac{5\overline{EI}W_2C_A}{2} + \dfrac{3\overline{EI}W_2C_B}{2} + 3(\overline{EI})^2W_2C_AC_B}{1+\dfrac{4\overline{EI}}{L}(C_A+C_B+\dfrac{3\overline{EI}}{L}C_AC_B)} \\ \dfrac{\dfrac{W_2L^2}{12} - \dfrac{\overline{EI}W_2C_AL}{2}}{1+\dfrac{4\overline{EI}}{L}(C_A+C_B+\dfrac{3\overline{EI}}{L}C_AC_B)} \end{bmatrix} \quad (4.48)$$

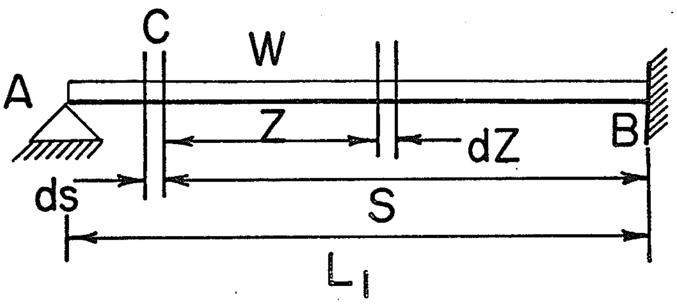
For member AB, C = ∞ and C = 0. Therefore

$$P_B^F = \begin{bmatrix} 0 \\ \dfrac{5W_2L}{8} \\ \dfrac{-W_2L^2}{8} \end{bmatrix} \quad (4.49)$$

By statics

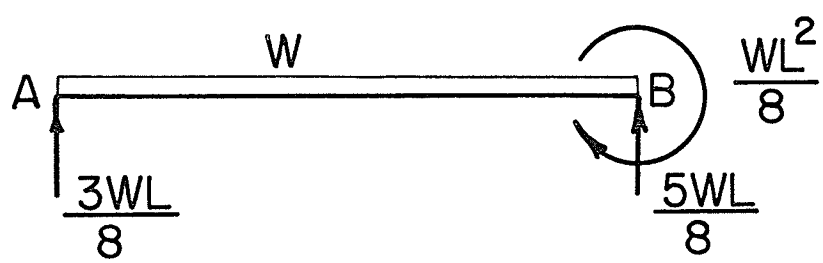
$$P_A^F = \begin{bmatrix} 0 \\ \dfrac{3W_2L}{8} \\ 0 \end{bmatrix} \quad (4.50)$$

Fig. 4.6 illustrates the free-body diagram for member AB.



MEMBER AB WITH UNIFORMLY DISTRIBUTED LOAD

FIG. 4.5



FREE BODY DIAGRAM FOR MEMBER AB

FIG. 4.6

CHAPTER V

LINEAR AND NON-LINEAR ANALYSIS PROCEDURES

In this chapter, the stiffness method of analysis for linear structures is reviewed. An iterative procedure which has been implemented in this study for the analysis of plane frames with non-linear effects is presented.

5.1 Introduction

A linear structure is one in which all displacements and internal forces are linear functions of the applied loads. Most practical structures behave in an approximately linear manner at working loads. The assumption of linearity has two important advantages. In the first place, it greatly simplifies the actual task of analysing a structure under a particular loading system. In the second place, it allows the superposition of solutions, with a consequent saving of effort when many different loading systems have to be considered.

The two basic linear structural analysis methods are the flexibility (force) method and the stiffness (displacement) method. Both methods are based on the fact that a structure must simultaneously satisfy the equilibrium and compatibility conditions, while the material in the structure satisfies known stress-strain relationships. The difference between the two methods is the order of application of the equilibrium and compatibility conditions. The flexibility method assumes equilibrium at the outset, but violates compatibility. Compatibility is then re-established by writing compatibility equations.

The stiffness method assumes compatibility at the outset, but violates equilibrium. Equilibrium is then re-established by writing equilibrium equations. The stiffness method is well suited for use of the digital computer. While it generally involves more computation than the flexibility method, the computations are much more systematic and therefore more easily programmed. For this reason, the stiffness method has been employed in this study.

There are three important causes of non-linear behaviour in structures. The first is non-linear behaviour of the material from which the structure is made. This normally affects the behaviour of the structure only at loads beyond the working range.

The second cause is usually referred to as "gross deformation". In linear analysis, it is necessary to assume that the deformations of a structure are small compared to its dimensions, so that the overall geometry of the structure is not significantly altered by the process of loading it.

The third cause of non-linear behaviour is essentially a particular case of the second, but is of sufficient practical importance to be mentioned separately. This is the effect which axial forces have on bending stiffness of members in rigid-jointed frames and trusses. If the axial force in a member is compressive, the bending stiffness is reduced, while, if it is tensile, the stiffness is increased. This effect may, in extreme cases, cause a structure to become unstable while still remaining elastic.

For structures with flexible connections, joint displacements at working loads are normally sufficiently small to preclude non-linearity due to "large displacements". Furthermore, the effects of axial forces on member stiffness can generally be neglected. However, while the members are generally linearly elastic, the connections often behave non-linearly at working loads. Therefore, a non-linear analysis procedure is required for flexibly connected structures.

Non-linear analysis procedures generally involve the linearization of structural behaviour. They employ repeated cycles of linear analysis to arrive at a set of displacements and internal forces that satisfy compatibility, equilibrium, and the force-displacement relationships for the structural members and connections.

5.2 Linear Stiffness Formulation of Structural Analysis

The stiffness analysis procedure involves the systematic application of the following three types of conditions to a structure:

(a) Equilibrium - the forces exerted on a joint by all members framing into it must exactly balance the external load applied to the joint.

(b) Force Displacement Relationships - these equations relate the member end forces to the corresponding displacements.

(c) Compatability - the displacement of the end of each member framing into a joint must be the same as the displacement of the joint.

Consider a typical joint I in a structure as shown in Fig. 5.1. The equilibrium equation at the joint can be written:

$$P_I = \sum_{K=1}^{n_K} H_K R_{B_K} - \sum_{J=1}^{n_J} H_J T_J R_{B_J} \tag{5.1}$$

where:

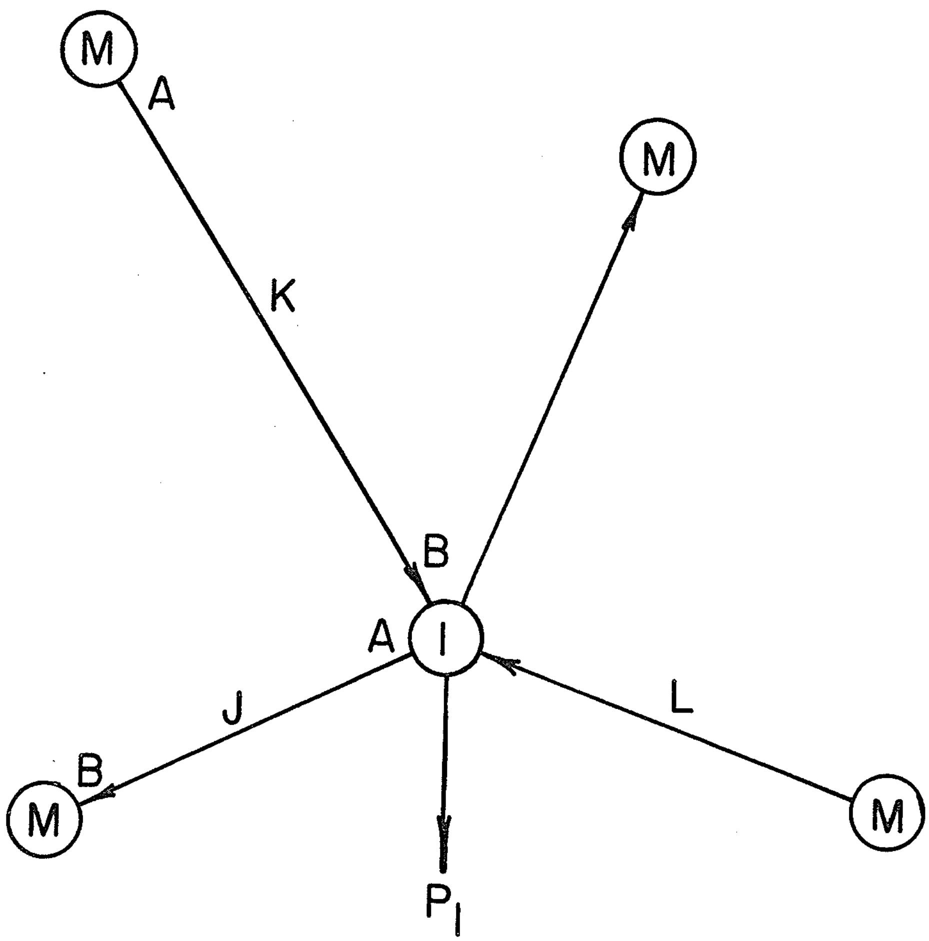
P_I = total external load on joint I

n_K = number of members whose B ends frame into joint I

n_J = number of members whose A ends frame into Joint I

H = rotation transformation matrix which transforms force and displacement vectors from local coordinate systems to the global system.

T = translation matrix



JOINT I, WITH MEMBER LOADS
AND EXTERNAL JOINT LOADS

FIG. 5.1

R_{B_K} = force acting on B end of any member K whose B end frames into joint I.

R_{B_J} = force acting on B end of any member J whose A end frames into joint I.

The force-displacement equation for any member L which frames into joint I is expressed by Eq. (4.21),

$$R_{B_L} = S_L K_{BB_L}(u_{BA_L} - T_K^t u_{AB_L}) + R_{B_L}^F .$$

Consider joint I which has a displacement D expressed in the global system. The compatibility conditions at the joint can be expressed as follows:

For any member K whose B end frames into joint I,

$$u_{BA} = H_K^t D_I \tag{5.2}$$

and

$$u_{AB} = H_K^t D_M \tag{5.3}$$

where:

H_K^t = rotation transformation matrix which converts the displacement vector from the global to the local system.

M is a generic symbol used to represent the joint at the opposite end of any member from joint I.

Similarly, for any member J whose A end frames into joint I,

$$u_{BA_J} = H_J^t D_M \tag{5.4}$$

$$u_{BA_J} = H_J{}^t D_I \tag{5.5}$$

Substituting the compatability equations and force-displacement equations into the joint equilibrium equation, (4.21), and for simplicity, dropping the subscripts, B, from the stiffness matrix and fixed-end-force vectors:

$$\begin{aligned} P_I = & \sum_{K=1}^{n_K} H_K \left[SK(H_K{}^t D_I - T_K{}^t H_K{}^t D_M) + R_K{}^F \right] \\ & - \sum_{J=1}^{n_J} H_J T_J \left[SK(T_J{}^t H_J{}^t D_I - H_J{}^t D_M) + R_J{}^F \right] \end{aligned} \tag{5.6}$$

Eq. (5.6) can be re-written

$$\begin{aligned} \overline{P}_I = P_I - \sum_{K=1}^{n_K} H_K R_K{}^F + \sum_{J=1}^{n_J} H_J T_J R_J{}^F = & \sum_{K=1}^{n_K} H_K \left[SKH_K{}^t D_I - SKT_K{}^t H_K{}^t D_K \right] \\ & + \sum_{J=1}^{n_J} H_J \left[T_J SKT_J{}^t H_J{}^t D_I - T_J SKH_J{}^t D_M \right] \end{aligned} \tag{5.7}$$

where:

$\overline{P}_I$ is the joint force vector at joint I. It is a force vector which includes the external load at joint I and the negatives of the fixed-end-forces for all members framing into joint I.

Equilibrium Eq. (5.7) relates the external load at joint I to the displacements of at least two joints in the structure. One such equilibrium equation is written at each joint in the structure. The resulting set of equations can be expressed in the form:

$$P = K_s D \tag{5.8}$$

where:

P = vector of joint forces for all joints in the structure.

K_s = structure stiffness matrix, which is assembled from the member stiffness matrices transformed to the global system as in Eq. (5.7). K_s relates joint forces and the resulting joint displacements for all joints in the structure.

D = vector of all unknown joint displacement components.

Eqs. (5.8) can be solved for the joint displacements of the structure. The resulting joint displacements can then be substituted into the force-displacement equations, Eq. (4.21), to determine the member end forces.

5.3 Non-Linear Structural Analysis Procedure

Non-linear structural analysis procedures are generally iterative in nature. They generally involve linearizing the load-displacement characteristics of the structure over finite loading increments. Non-linear analysis methods can be classified as "successive correction methods" and "successive approximation methods".

The successive correction methods, of which the Newton-Raphson approach is the most widely used, involve applying proportional increments of loading, and performing a linear analysis for each loading increment. Proportional increments of load are applied and a linear analysis is performed for each increment to determine the incremental displacements and internal forces. Cumulative joint displacements and member end forces are calculated by accumulating the appropriate incremental values.

Incremental analysis procedures permit the tracing of the approximate load-displacement behaviour of the structure over the loading range considered. However, it is usual in most practical analysis problems to require only the final structural deflections and internal forces.

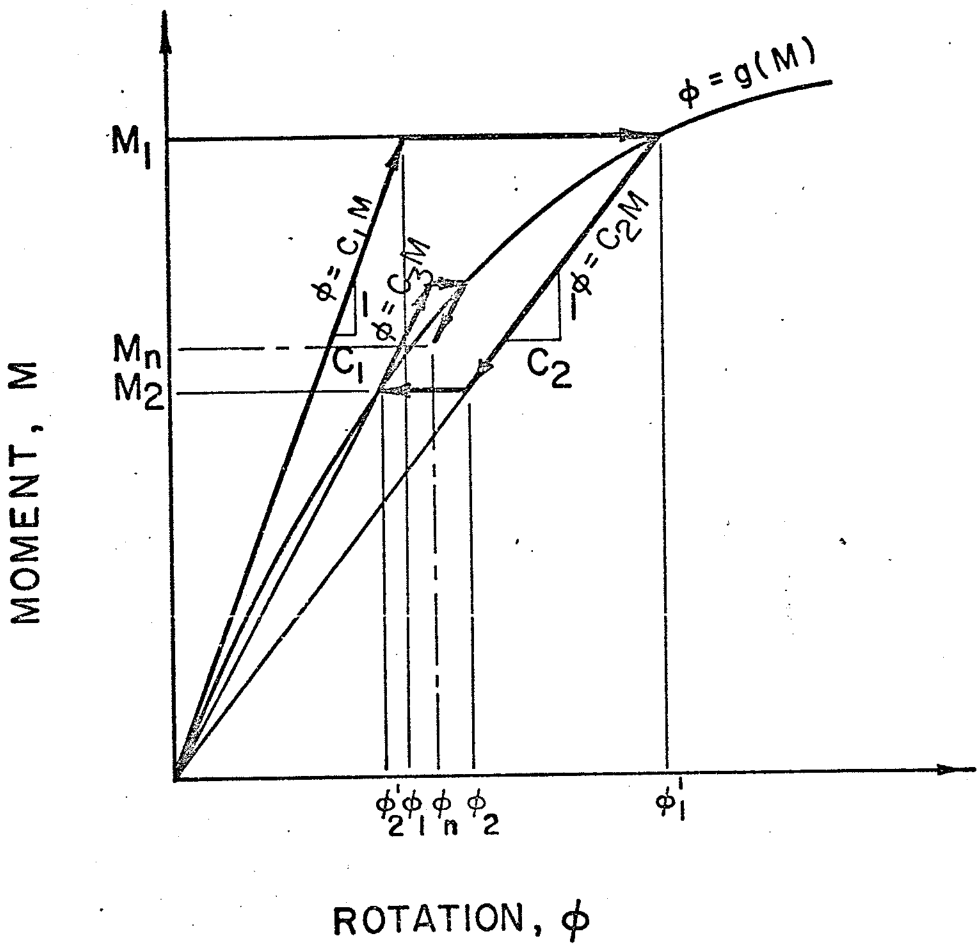
Furthermore, incremental analysis procedures require the storing of both incremental and cumulative displacements and internal forces. In addition, it is necessary to calculate the remaining loads to be applied after each loading increment. To avoid these disadvantages, the following successive approximation method was developed for this study.

To describe the method, consider a structure whose connections have non-linear moment-rotation characteristics. The moment-rotation function for a typical connection is illustrated in Fig. 5.2, and has the form

$$\phi = g(M) \tag{5.9}$$

where $g(M)$ is a non-linear function of the moment acting on the connection.

The analysis procedure is begun by replacing the



MODIFICATION CYCLE

FIG. 5.2

non-linear moment-rotation function for the connection considered, by a linear relationship of the form:

$$\phi = C_1 M \tag{5.10}$$

The moment-rotation relationships for all other connections considered in the structure are similarly linearized. As illustrated in Fig. 5.2 , Eq. (5.10) describes the initial tangent to the M-ϕ curve.

Corresponding to the linearized M-ϕ relationships for the connections at the ends of a given member AB, the member force-displacement relationships can be written:

$$R_B = S_1 K(u_{BA} - T^t u_{AB}) + R^F_{BA_1} \tag{5.11}$$

where:

$S_1 K$ and $R^F_{BA_1}$ are the modified stiffness matrix and the B-end fixed-end-force vector corresponding to the assumed connection flexibilities.

Assuming member force-displacement relationships as given by Eq. (5.11) , a linear analysis is performed and the member end forces are calculated. The corresponding connection rotation is

$$\phi_1 = C_1 M_1 \tag{5.12}$$

However, the rotation calculated from the correct non-linear relationship of Eq. (5.9), is:

$$\phi_1' = g(M_1) \tag{5.13}$$

A better approximation to the connection moment-rotation function is thus seen to be

$$\phi = C_2 M \tag{5.14}$$

where

$$C_2 = \frac{\phi_1'}{M_1} \tag{5.15}$$

as illustrated in Fig. 5.2.

Eq. (5.14) and similar relationships for all other connections are then used to calculate the new member force displacement relationships and a second linear analysis is performed.

A new moment, M_2, is found to occur at the typical connection and the corresponding connection rotation, as shown in Fig. 5.2, is

$$\phi_2 = C_2 M_2 \tag{5.16}$$

Again the connection rotation as calculated from the non-linear relationship is:

$$\phi_2' = g(M_2) \tag{5.17}$$

Hence, a third linear relationship, which will lead to better approximations to the correct moment and rotation at the connection, is

$$\phi = C_3 M \tag{5.18}$$

where

$$C_3 = \phi_2' / M_2 \tag{5.19}$$

The above procedure is repeated until, as illustrated in Fig. 5.2 , the rotation at each connection, calculated from the linear relationship for the current cycle, is sufficiently close to that given by the appropriate non-linear relationship of the form of Eq. 5.9.

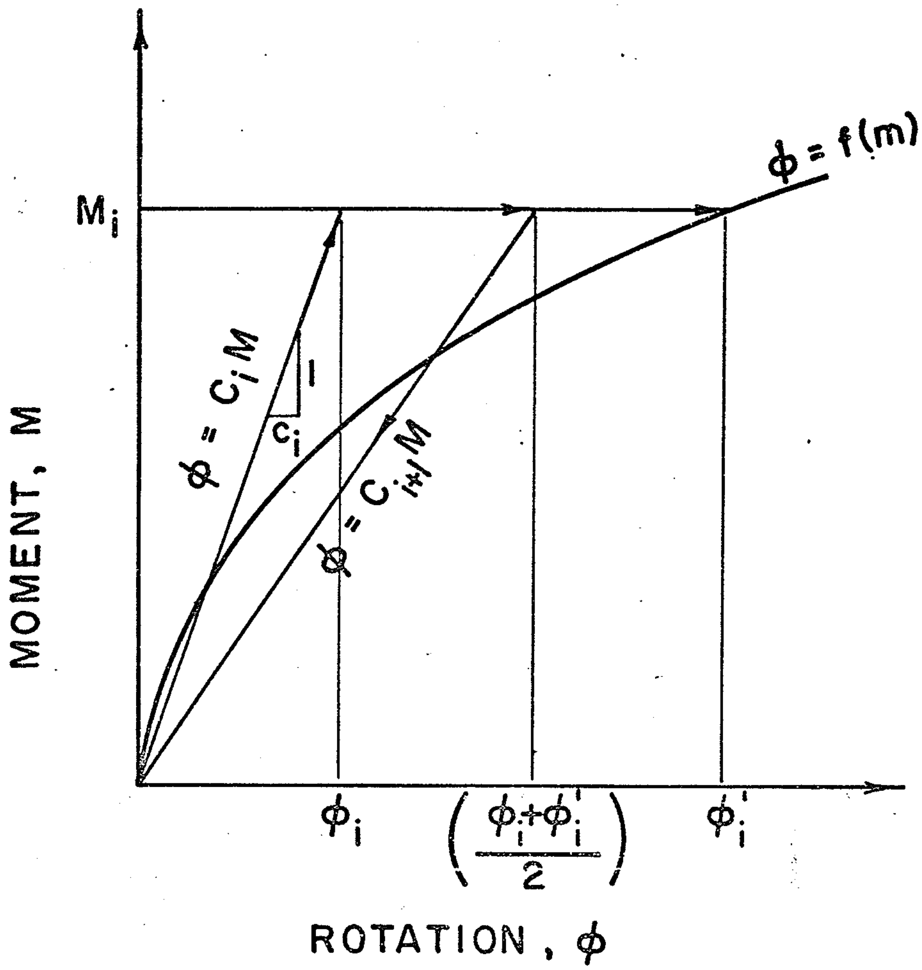
Assuming convergence of the procedure after n cycles of iteration, the final moment and rotation at the typical connection would thus be M_n and ϕ_n, as illustrated in Fig. 5.2.

The rate of convergence of the above procedure can be increased by employing an "under correction" in each cycle, as illustrated in Fig.5.3.

The figure illustrates the i th modification of the flexibility, for a typical connection. The "under correction" is accomplished by arbitrarily using only one half of the difference between ϕ_i' and ϕ_i, rather than the total difference, when modifying the connection flexibility.

Thus the flexibility to be used in the i+1st cycle is

$$C_{i+1} = \frac{\phi_i' - \phi_i}{2M_i} \tag{5.20}$$



"UNDER CORRECTION" PROCEDURE

FIG. 5.3

CHAPTER VI

ANALYSIS PROCESS

In this chapter, the specification of acceptable loading arrangements and connection types is outlined. The major steps in the analysis procedure are described.

6.1 Definition of Problem

While the analysis procedure outlined in this study is applicable to any type of structure, it has been implemented in a form that is applicable to planar structures only. The members of the structure can be pin connected, rigidly connected, or joined together by connections with any desired flexibility characteristics.

The structure loading may consist of any number of concentrated joint loads, concentrated member loads, or uniformly distributed member loads. Because of the non-linearity of the moment-rotation characteristics of the connections, the principle of superposition cannot be used to combine the results of one analysis with those of another. Therefore, the structure must be analyzed separately for each loading system.

For each flexible connection type used, the associated size parameters must be specified. These size parameters allow the analysis program to generate the moment-rotation relationship for the connection. The permissible connection types and required size parameters are listed in Appendix D.

6.2 Analysis Procedure

The analysis procedure, in general, consists of initialization, followed by repeated cycles of the following steps:

(a) linear analysis

(b) tests for termination

(c) modification of the connection flexibility characteristics

For frames with only pinned and rigid connections, the iterative procedure is not required, and the solution is obtained from the first linear analysis.

The steps of the analysis procedure are described with reference to the flow diagram in Appendix C. A user's manual for the program is included as Appendix D.

6.2.1 Initialization

The initialization consists of specifying the characteristics of the structure and then setting to zero all loads and member end forces.

The characteristics of the structure are described by means of a member incidence table which establishes the topology, a table of joint coordinates which establishes the geometry, and a table of member cross section properties. The member end connection types must also be specified along with any necessary size parameters.

Unless otherwise specified, the modulus of elasticity is taken as 30,000 k.s.i. All loads are in kips and dimensions are in feet, except for member cross section properties and connection parameters which are expressed in units of inches.

If no connection type is specified, the connection is assumed to be rigid.

6.2.2 Linear Analysis

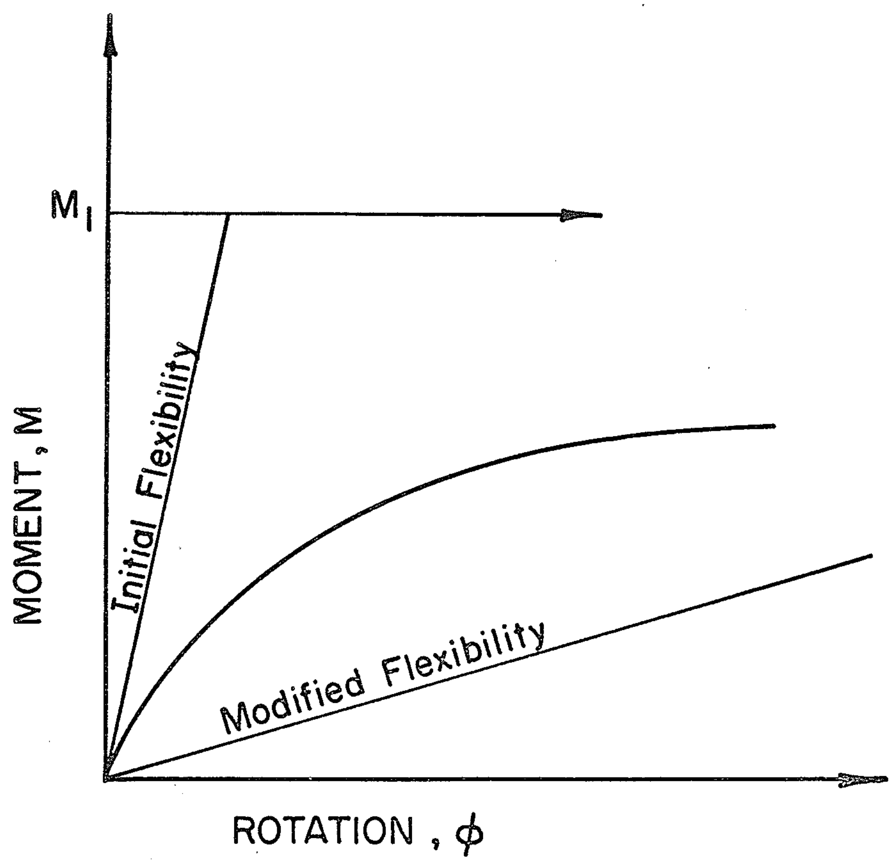
The stiffness method previously discussed is used to perform the linear analysis. The program employs an in-core Gaussian elimination, variable band width equation solver. Because of symmetry of the structure stiffness matrix, only the elements above the main diagonal are stored. The non-zero band is stored as a series of 3 x 3 submatrices. The structure stiffness matrix is generated one row at a time, and the previously generated rows are used in performing the elimination on the current row, before proceeding to the generation of the next row.

Each member stiffness matrix, which incorporates the effects of flexible connections at the ends of the member is regenerated each time it is used. The member fixed-end-forces are also dependent on the connection characteristics and must be recalculated for each linear analysis.

6.2.3 Termination Criteria

The primary criterion for the termination of the analysis is the convergence of the iterative procedure to the suitable connection flexibility values. Convergence is indicated when the rotation for each connection, as obtained from the linear analysis, is approximately equal to the rotation for that connection, as calculated from the non-linear moment-rotation function. When this condition has been achieved, each connection has undergone the appropriate deformation corresponding to the applied end moment.

For frames with very flexible connections and high loading, it is possible that the iterative procedure will not converge on a value of connection flexibility. In this event, the connection end moments obtained from the first linear analysis exceed the maximum capacity of the connection by a considerable amount as illustrated for a typical connection shown in Fig. 6.1. Increasing the connection flexibility reduces the moment carried by the connection and redistributes the moment to other connections and other points in the structure. The other connections, however, have already exceeded their maximum capacity and



MOMENT-ROTATION CURVE FOR NON-CONVERGING ITERATIVE PROCEDURE

FIG. 6.1

hence the analysis procedure will fail to converge. Therefore, a counter has been incorporated in the program and the analysis is automatically terminated with an appropriate message after m cycles of iteration.

6.2.4 Connection Flexibility Modification

After each analysis, the assumed flexibility of each connection is modified if the connection rotation predicted by the linear analysis differs from that predicted by the non-linear moment-rotation curve by more than an acceptable amount. The flexibility modification procedure has been described in Sec. 5.3.

6.2.5 Program Output

The program output consists of a detailed listing of the following items:

(a) all program input (for checking purposes)

(b) final connection flexibilities

(c) joint displacements

(d) member end forces

(e) joint support reactions

(f) volumn and total weight of steel in structure

The rotation and deflections of each joint, and the joint support reactions are expressed in the global coordinate system, while the forces (axial, shear, and bending moment) at both ends of the member are expressed in the local coordinate system.

CHAPTER VII

APPLICATIONS OF THE ANALYSIS PROCESS

7.1 Introduction

In this chapter, several examples are presented to illustrate the analysis process. For the sake of simplicity, only selected results are presented and discussed, and these are illustrated by means of deflection and bending moment diagrams. Examples have been chosen which best demonstrate the effects of connection deformations, and the capabilities of the analysis program.

In all examples, loads and forces are expressed in kips, and moments are in inch-kips. Linear displacement and distortion components are expressed in inches, and rotational displacements and distortions are expressed in radians.

7.2 Effect of Connection Deformations on Displacements and Internal Forces

While the connections in a structure generally represent a small percentage of the total material weight, they have a high labour content, and consequently, often represent a substantial percentage of the total framing cost.

Furthermore, the deformations that occur in structural steel framing connections may be responsible for the major proportion of the displacements of the structure, and may have a very strong influence on the internal force distribution.

It is highly desirable to know the effects of connection deformations so that:

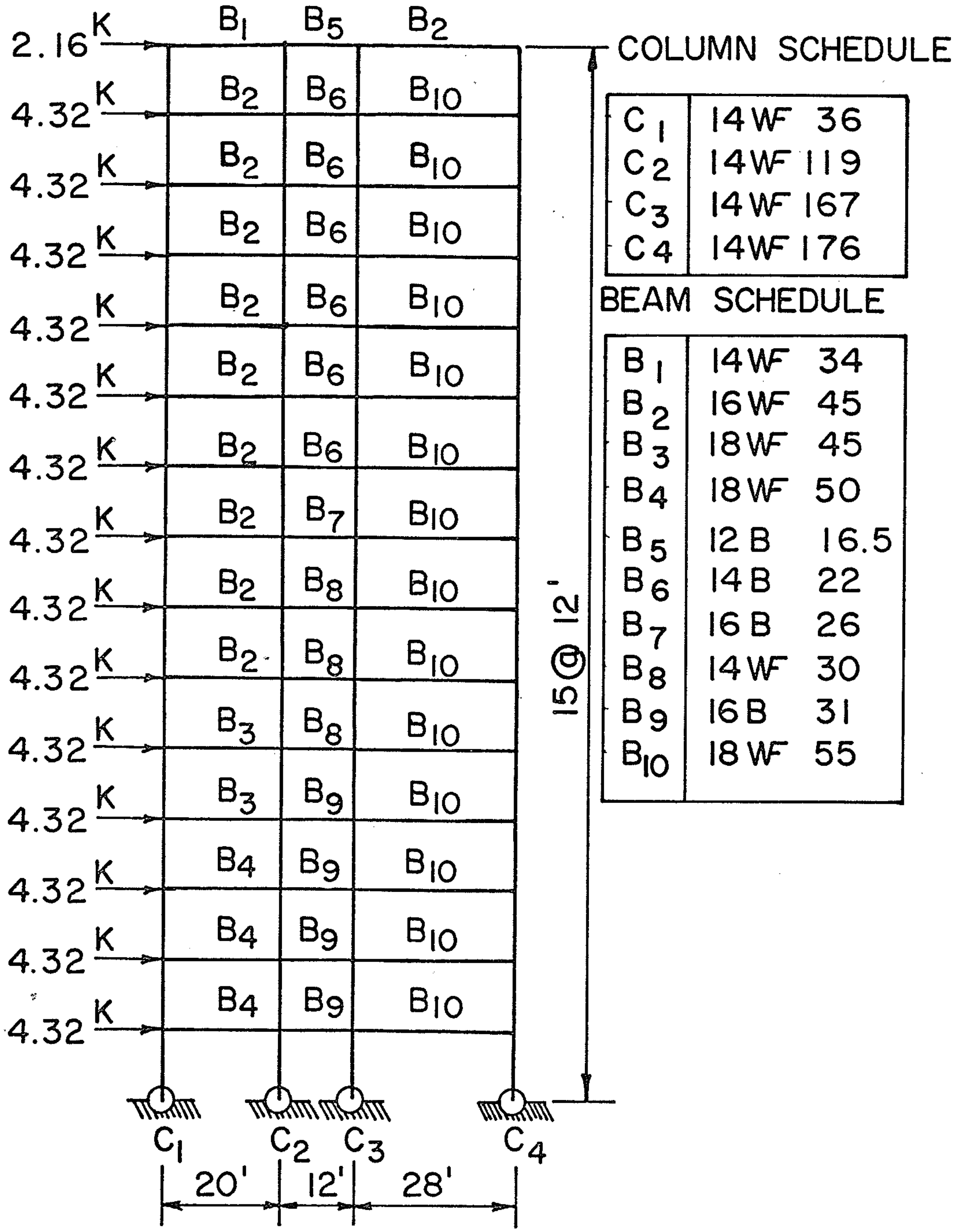
(a) connection types that would lead to unacceptably large deflections or undesirable internal force distributions can be replaced by more suitable connection types.

(b) where possible, expensive connection types can be replaced by less expensive (probably more flexible) types without adverse effects on the structural behaviour.

Several examples have been included to illustrate the effects of connection deformation on structural behaviour. All of the connection types used in the illustrative examples are illustrated in Figs. 2.2 to 2.8 inclusive.

<u>Example 1</u>

The first example involves the analysis of a 15 storey, 3-bay frame with lateral wind loading applied at each floor level, as illustrated in Fig. 7.1. To determine the effect of connection deformation on the lateral deflection of the



EXAMPLE 1

15 STOREY THREE BAY FRAME AND LOADING

FIG. 7.1

structure, it was first analyzed with all connections assumed to be completely rigid. The identical frame was subsequently analyzed with the rigid beam-column connections replaced by the following connection types in turn:

(a) connections with numerically specified flexibility characteristics

(b) T-stub connections

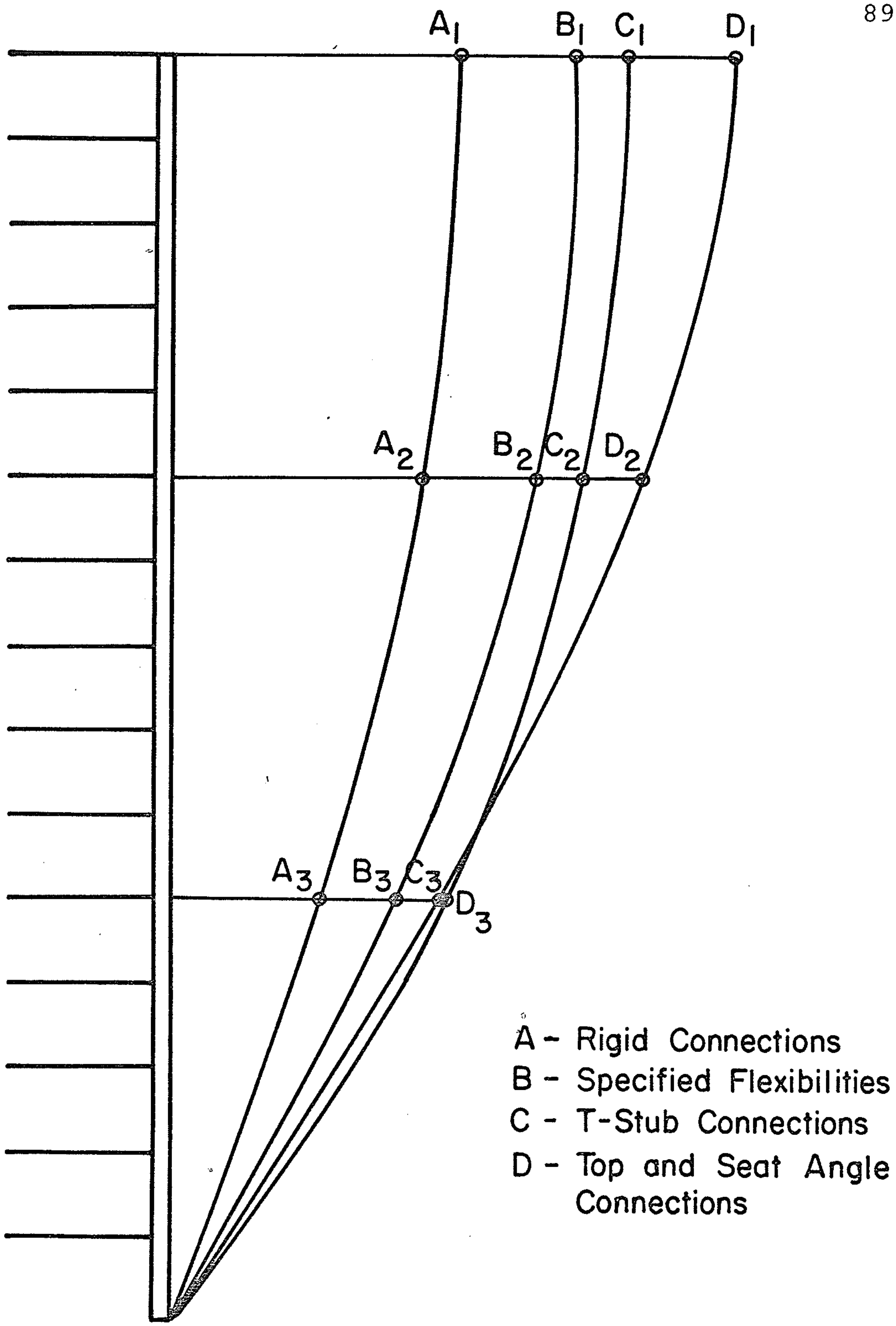
(c) top and seat angle connections

The connection flexibilities specified in (a) corresponded to relatively rigid connections that would likely be used in a tall frame of this type. The T-stub connections consisted of two structural tees (ST 12 WF 47). Two 5x5x$\frac{3}{4}$- 14 inch angles were used for the top and seat angle connections. The fasteners used were $\frac{7}{8}$ inch diameter H.T. bolts.

The lateral deflections for each of the four structures are plotted in Fig. 7.2. In Table 7-1, the lateral deflections at the 5th, 10th, and 15th floor levels are listed and also expressed in terms of percentages of the corresponding deflections for the rigidly connected structure. This example illustrates that connection deformation contributes very substantially to overall deformation of a structure. It can be seen from the table that top and seat angle connections have contributed to an increase of approximately 100% of that for the frame with rigid connections.

Table 7-1 Lateral Deflection of 15 Storey Frame

CONNECTION TYPE	15th LEVEL		10th LEVEL		5th LEVEL	
	Deflection	% of rigid	Deflection	% of rigid	Deflection	% of rigid
Rigid	5.058	100%	4.375	100%	2.757	100%
Specified	7.380	146%	6.370	146%	4.004	146%
T-Stub	8.196	162%	7.266	166%	4.817	175%
Top and Seat	9.985	198%	8.593	197%	4.566	166%



LATERAL DEFLECTION PLOT OF 15 STOREY FRAME

FIG. 7.2

Example 2

As a second illustration of the influence of connection deformation, a skewed Vierendeel truss was analyzed for the vertical loading shown in Fig. 7.3. The connections of the vertical members to the chords were assumed to be rigid for an initial analysis, and were replaced by each of the following progressively more flexible connection types in turn:

(a) T-stub connections

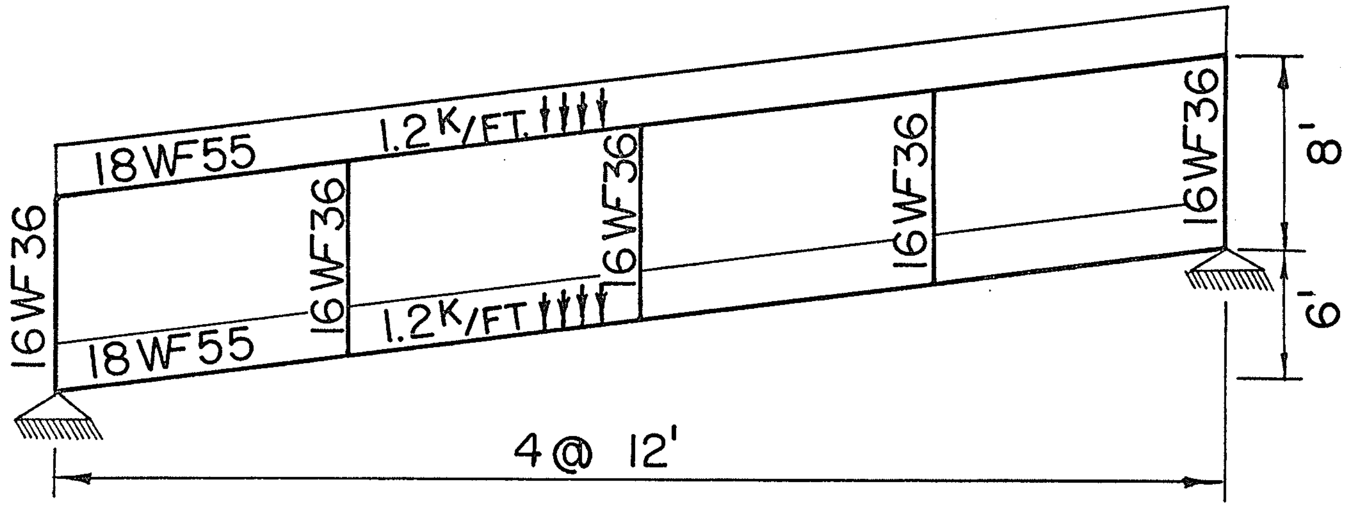
(b) end plate connections with no stiffeners

(c) top and seat angle connections

(d) double web angle connections

For the T-stub connections two ST 12 WF 47 were used. The end plates used for connection type (b) were 16x6x½ inch plates welded to the ends of the vertical struts and bolted to the top and bottom chords of the truss. The top and seat angle connections consisted of two 4x4x½ - 14 inch angles Two 3½x3½x$^3{}_8$ inch angles were used for the double web angle connections. Four lines of inch diameter H.T. bolts were used on either side of the strut web for both the double web angle and the end plate connections.

Fig. 7.4 is a plot of the bottom chord deflections obtained in the four analyses. It can be seen that there is an increase in the deflection of the structure as the strut connections become progressively more flexible. Table 7-2 compares the bottom chord deflection for the rigidly

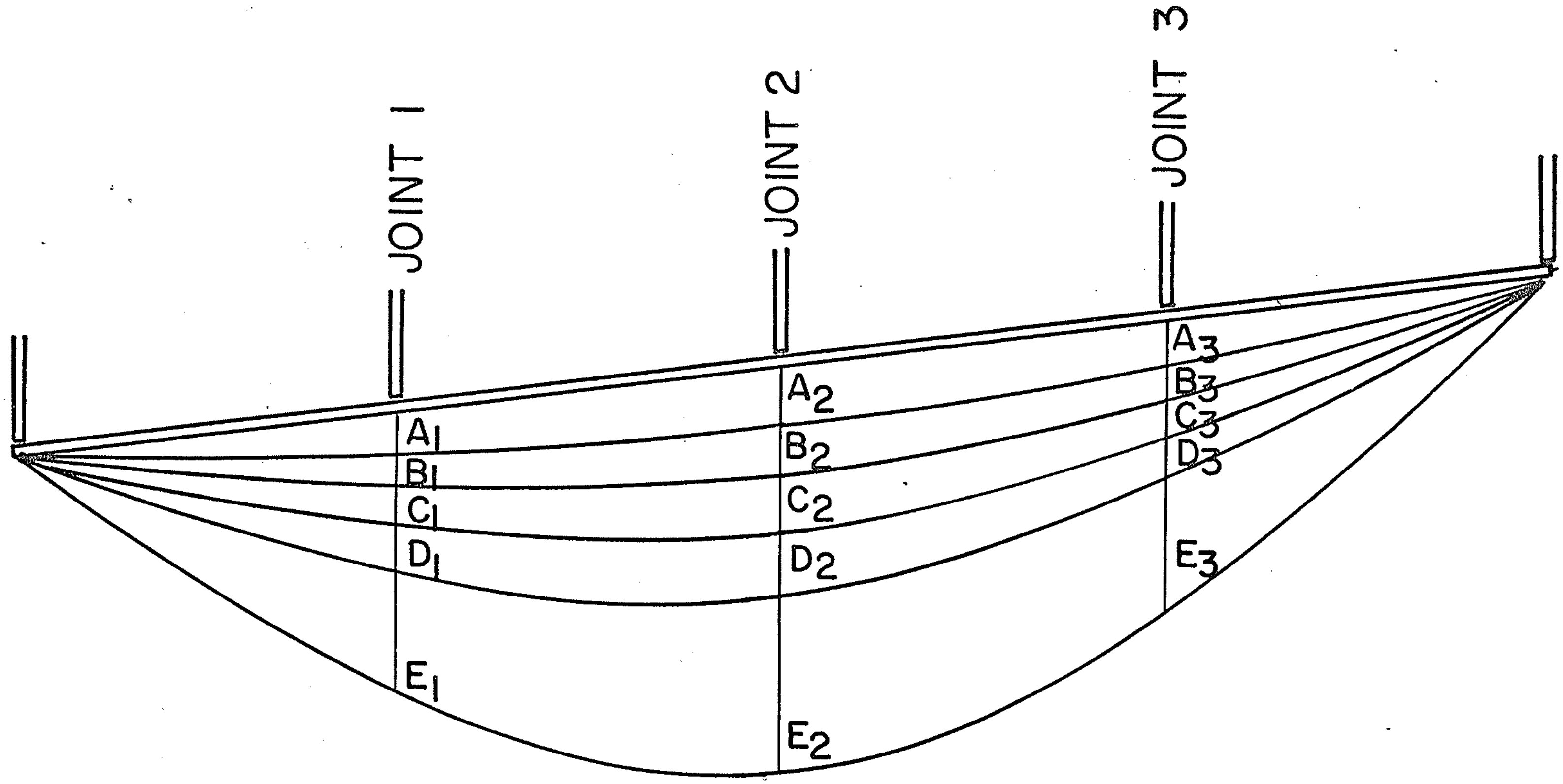


EXAMPLE 2 VIERENDEEL TRUSS AND LOADING

FIG. 7.3

Table 7-2 Bottom Chord Deflection

CONNECTION TYPE	JOINT 1		JOINT 2		JOINT 3	
	Deflection	% of rigid	Deflection	% of rigid	Deflection	% of rigid
Rigid	.479	100%	.682	100%	.479	100%
T-Stub	.813	170%	1.160	170%	.813	170%
End Plate	1.291	249%	1.833	269%	1.291	249%
Top and Seat	1.734	362%	2.454	360%	1.734	362%
Double Web	3.089	647%	4.337	635%	3.089	647%



BOTTOM CHORD DEFLECTIONS VIERENDEEL TRUSS

FIG. 7.4

connected structure with that obtained with each of the other connection types. This example demonstrates again that connection deformation accounts for a high percentage of overall frame displacement.

Example 3

To illustrate the influence of connection flexibility on the distribution of internal moments, and to compare the flexibilities of various commonly used connection types, an unsymmetrical 2-bay frame was analyzed for the loading shown in Fig. 7.5. For this example the following connection types were used:

(a) rigid connections

(b) T-stub connections

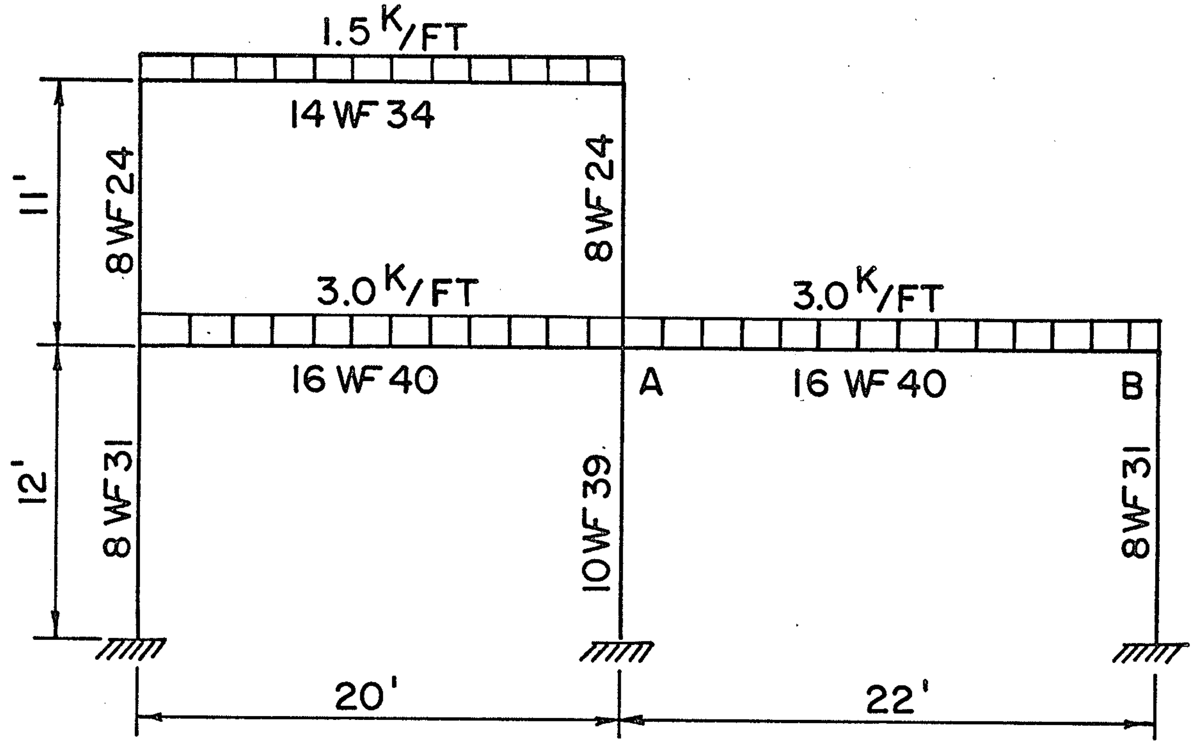
(c) top and seat angle connections

(d) header plate connections

(e) double web angle connections

(f) single web angle connections

The structural tees used for the T-stub connections were ST 12 WF 47. For the top and seat angle connections, 2 - $4 \times 4 \times \frac{1}{2}$ angles were used. The header plates were $11\frac{1}{2} \times 5 \times \frac{3}{8}$-14 inch plates welded to the ends of the 16 inch beams, and $9\frac{1}{2} \times 5 \times \frac{3}{8}$ inch plates welded to the ends of the 14 inch beam. The double web angle connections employed 2 - $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ inch angles. A single $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ angle was used for the single web angle connections. The double web angles and single web



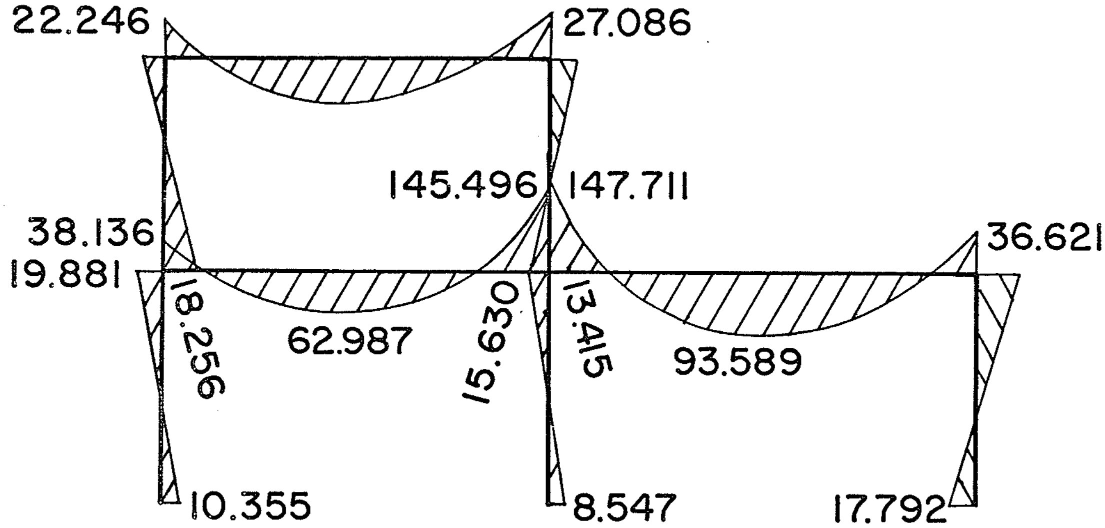
EXAMPLE 3

2 STOREY 2 BAY FRAME AND LOADING

FIG. 7.5

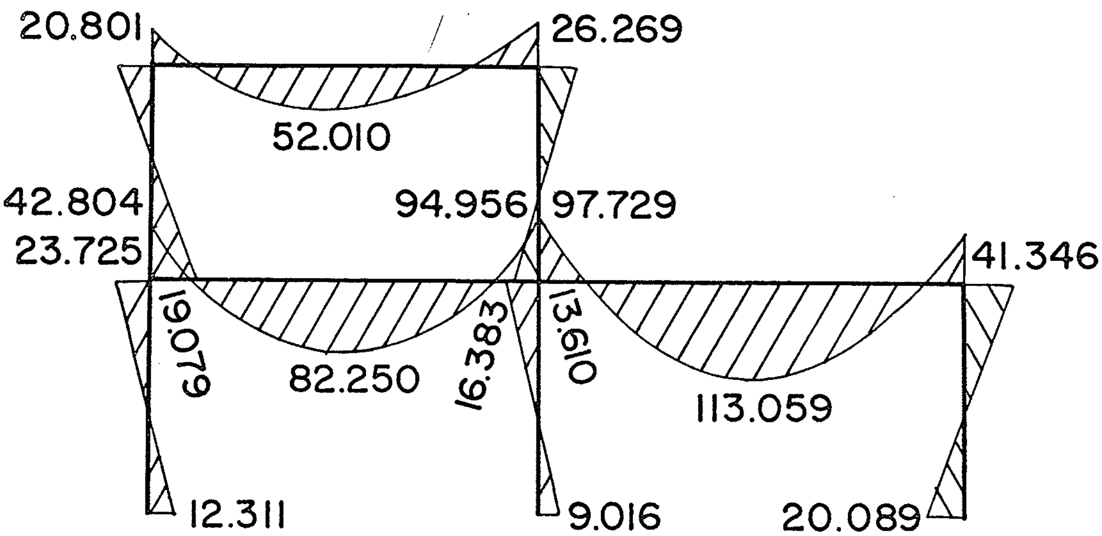
Table 7-3 Comparison of Connection Flexibilities and End Moments

CONNECTION TYPE	FLEXIBILITY	MOMENT	% OF RIGID MOMENT
Rigid	0	147.711	100%
T-Stub	.00000275	97.727	67%
Top and Seat Angle	.00001123	50.601	35%
Double Web Angle	.00002878	26.584	18%
Header Plate	.00003764	21.599	14%
Single Web Angle	.00004656	18.132	12%



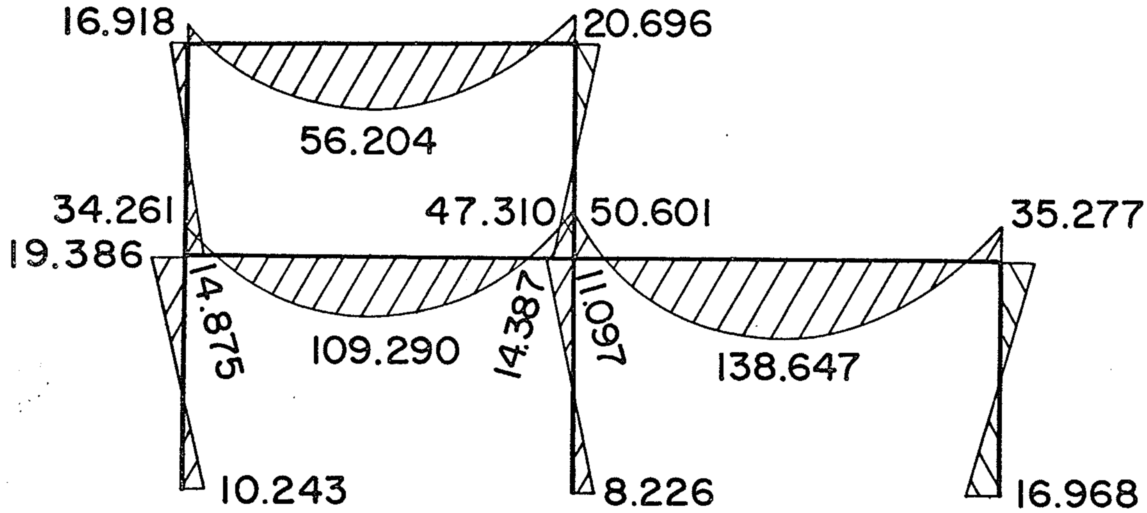
BENDING MOMENT DIAGRAM FOR 2 STOREY 2 BAY FRAME RIGID CONNECTIONS

FIG. 7.6



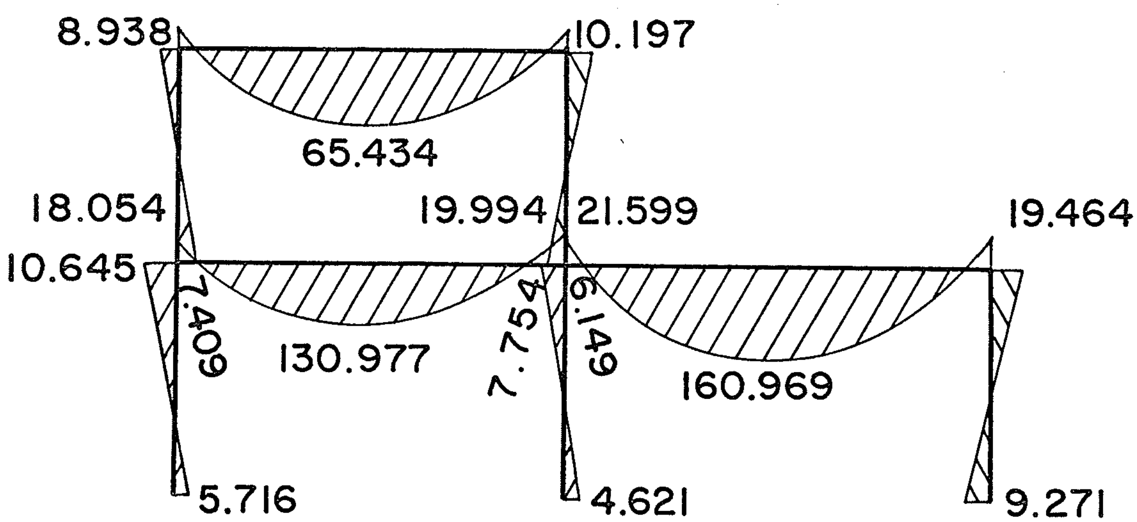
BENDING MOMENT DIAGRAM FOR 2 STOREY 2 BAY FRAME T-STUB CONNECTIONS

FIG. 7.7



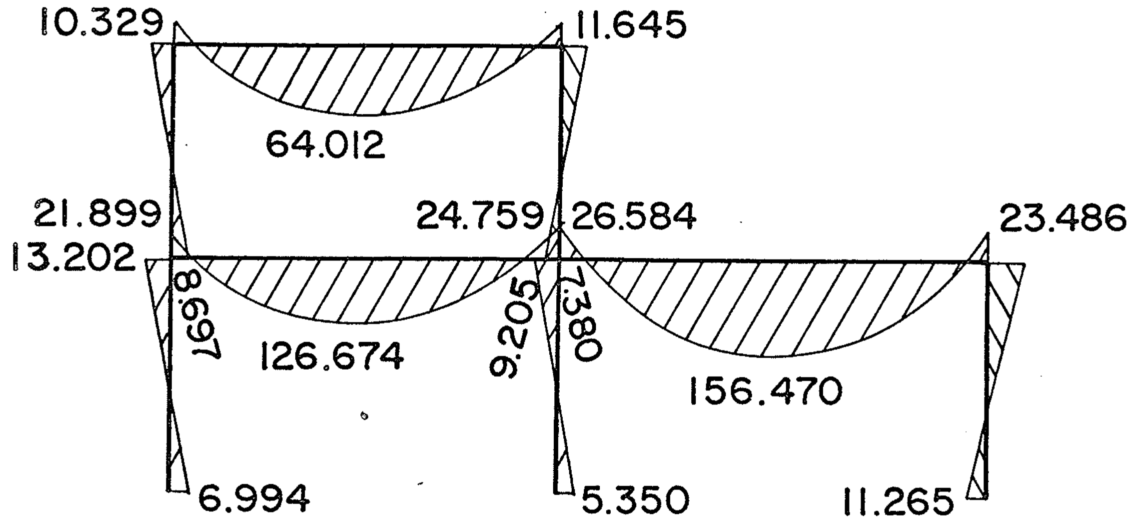
BENDING MOMENT DIAGRAM FOR 2 STOREY 2 BAY FRAME TOP AND SEAT ANGLE CONNECTIONS

FIG. 7.8



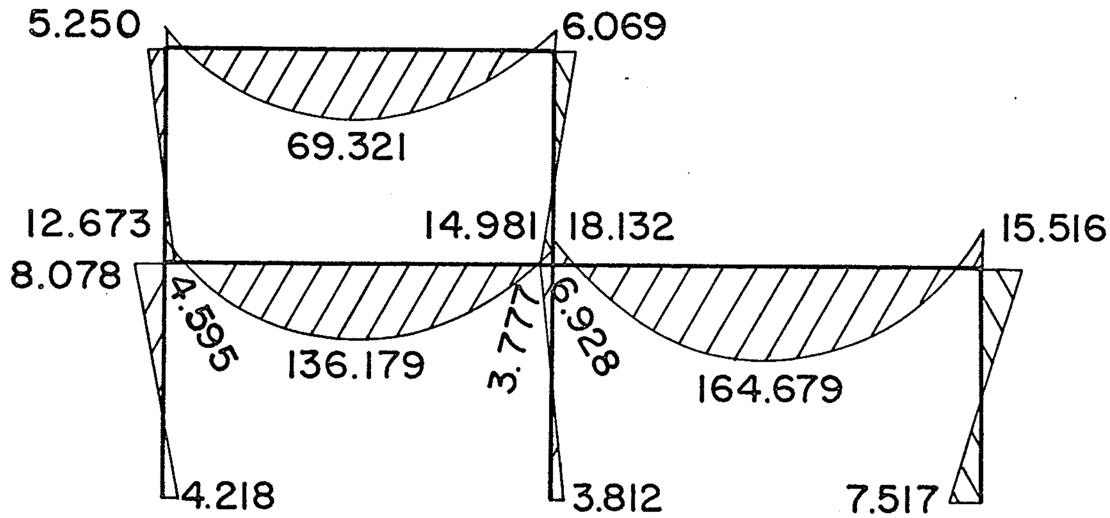
BENDING MOMENT DIAGRAM FOR 2 STOREY 2 BAY FRAME HEADER PLATE CONNECTIONS

FIG. 7.9



BENDING MOMENT DIAGRAM FOR 2 STOREY 2 BAY FRAME DOUBLE WEB ANGLE CONNECTIONS

FIG. 7.10



BENDING MOMENT DIAGRAM FOR 2 STOREY 2 BAY FRAME SINGLE WEB ANGLE CONNECTIONS

FIG. 7.11

angles were $14\frac{1}{2}$ inches in length for the 16 inch beam, and $11\frac{1}{2}$ inches for the 14 inch beam. Three lines of inch diameter H.T. bolts were used for the $11\frac{1}{2}$ inch angles and header plates, and four lines of bolts were used for the $14\frac{1}{2}$ inch angles and plates.

Bending moment diagrams for the frame have been plotted in Fig. 7.6 to Fig. 7.11. Examination of the frame bending moment diagrams reveals that the connection type has a marked effect on the distribution of internal moments. Table 7-3 compares the flexibilities and the end moments at joint A of the 22' member AB shown in Fig. 7.5, for the different types of connections. The tabulated flexibilities are the inverse slopes of the linearized M-ϕ relationships used in the final linear analysis. That is the M-ϕ lines that intersect the non-linear M-ϕ curve at very nearly the correct moment and rotation. The end moments are also expressed as a percentage of the rigid frame end moments.

7.3 Accuracy of Successive Approximation Method

The basic premise of the successive approximation procedure developed and employed in this study, is that the correct deflections and internal forces for a structure with non-linear connections can be obtained from a single linear analysis, provided the correct flexibility is assumed for each connection.

To illustrate, assume that the moment-rotation curve for a typical connection in a structure is as shown in Fig.

7.12. Assume further that, for a given loading, the correct moment and rotation at the connection are M_1 and ϕ_1 respectively. The appropriate connection flexibility (the connection flexibility that would yield the correct results for the loading considered), is thus C, the inverse slope of line OA in the figure.

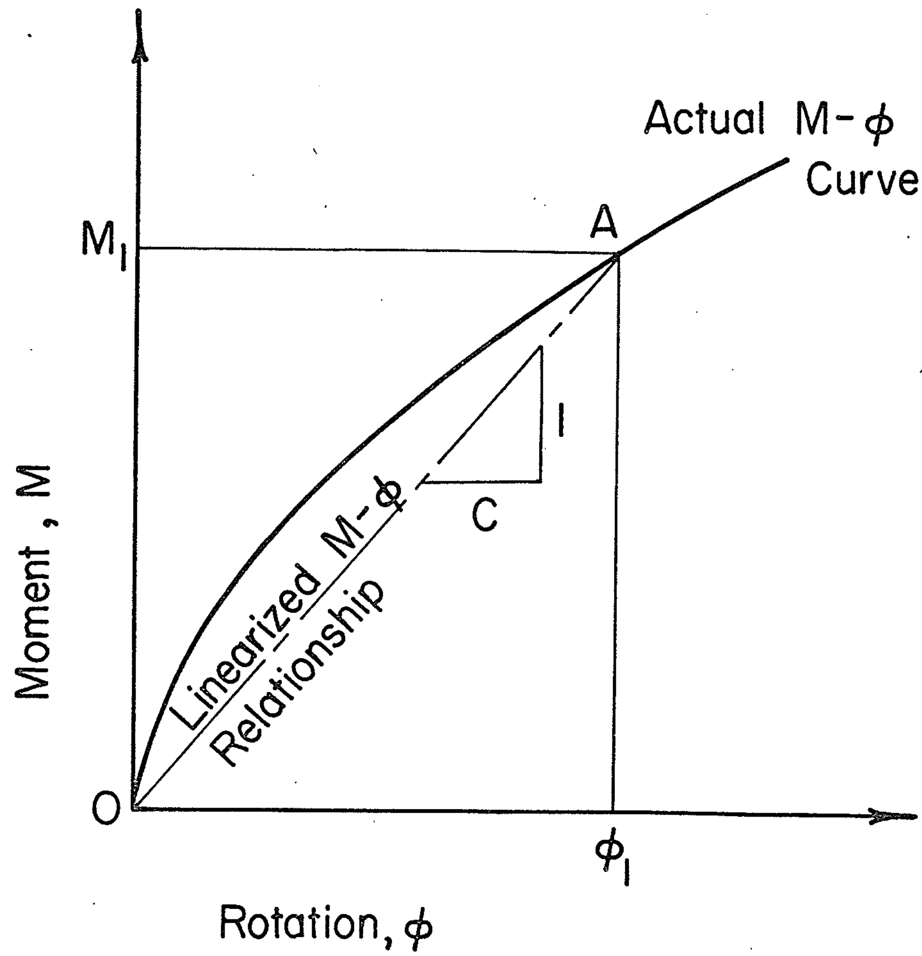
Furthermore, if flexibility C happens to be assumed for the connection under consideration, and if appropriate flexibilities happen to be similarly assumed for all other connections in the structure, a single linear analysis will yield the correct final forces and deflections for the non-linear structure.

The successive approximation method thus involves repeated cycles of an iterative procedure, whose purpose is to determine appropriate flexibilities for the various connections in a structure. When the appropriate flexibilities have been determined with sufficient accuracy, they are employed in a linear analysis to calculate the correct structural displacements and forces.

Two examples were employed to illustrate the validity of the procedure and to give an indication of its accuracy.

Example 1

The first of these examples involved the analysis of the fixed-ended beam shown in Fig. 7.13(a), loaded by the 40 kip load shown. To permit a relatively simple check of the results, the beam end connections were assumed to have



LINEARIZATION OF CONNECTION MOMENT-ROTATION CURVE

FIG. 7.12

rigid-perfectly plastic moment rotation characteristics, as illustrated by the moment-rotation curves in Fig. 7.13(b).

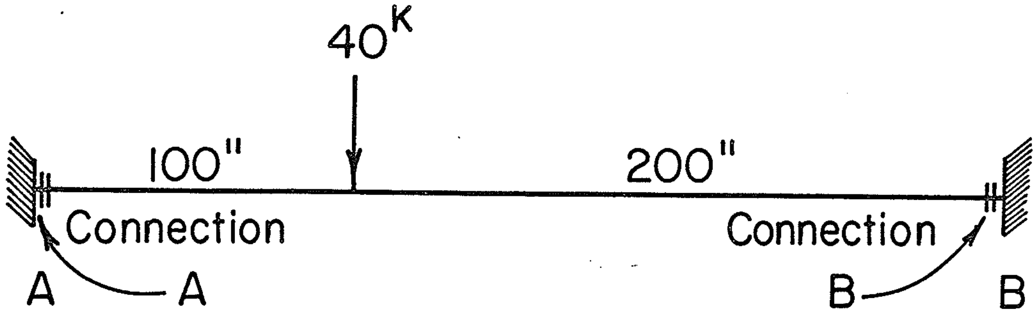
The structure was first analyzed by hand computation, applying the loading in three increments. The structure behaved linearly over each increment. It was initially treated as a fixed-end beam, and the load required to produce a moment of 1000 in kips at connections A, calculated. The corresponding moment at connection B was also calculated.

Because connection A had become perfectly plastic, the structure was analyzed as a propped cantilever, pinned at connection A, for the second loading increment. The loading required to increase the total moment at connection B was calculated, along with the rotation produced at connection A.

Finally, because both connections had become perfectly plastic, the structure was analyzed as a simply supported beam, for the remainder of the 40 kip load. The rotations at both connections A and B were calculated for this final loading.

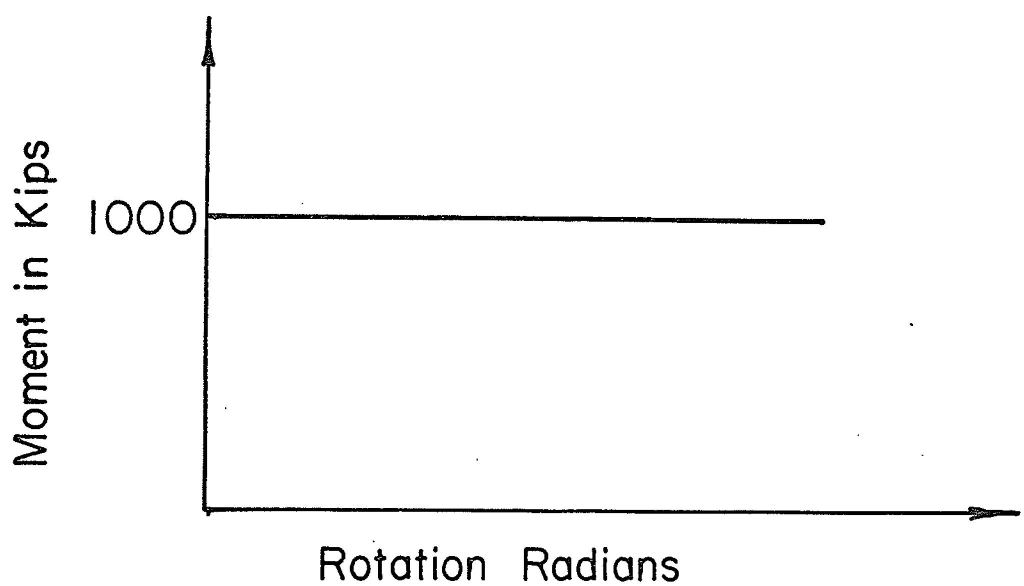
The total moments and rotations at the connection were then obtained by summing the results for the three loading increments. The pertinent quantities are illustrated in Fig. 7.14.

Because the analysis program is not able to accomodate



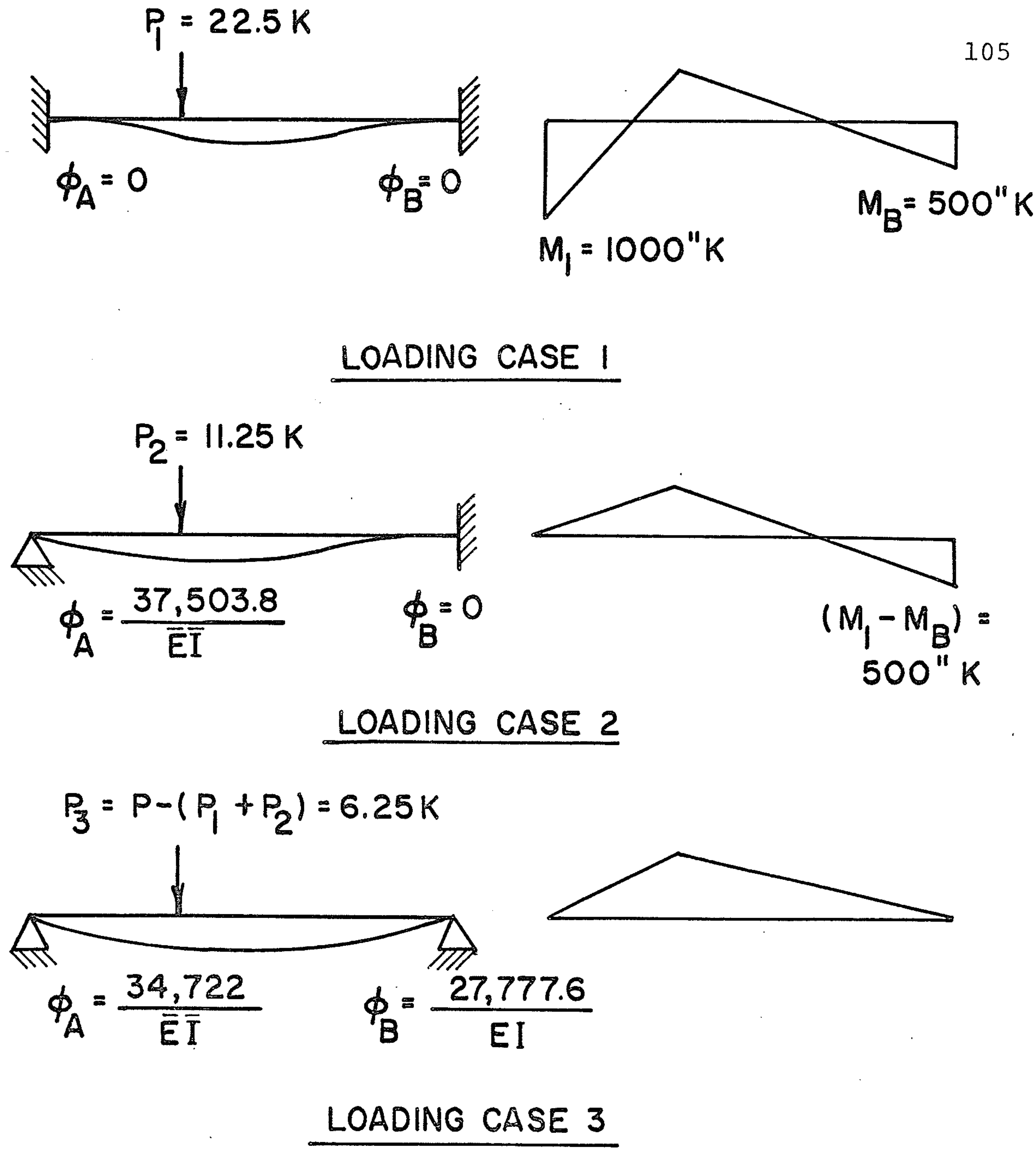
EXAMPLE I -
UNSYMETRICALLY LOADED FIXED BEAM

FIG. 7.13(a)



MOMENT-ROTATION CURVES FOR BEAM END CONNECTIONS

FIG. 7.13(b)



TOTALS : $M_A = M_B =$ 1000 in. kips

$$\phi_A = \frac{72,225.8}{EI} \qquad \phi_B = \frac{27,777.6}{EI}$$

LOAD DIAGRAMS AND BENDING MOMENT DIAGRAMS FOR EXAMPLE I

FIG. 7.14

the idealized, rigid-perfectly plastic connection characteristics assumed in this example, "appropriate" connection flexibility values were calculated by dividing the above calculated connection rotations by the corresponding connection moments. These flexibility values were then input, and the analysis program used to calculate the beam end forces.

As can be seem from Table 7-4, the two analyses yielded identical results.

The connection properties assumed in the preceeding example are a special case of those illustrated in Fig. 7.15 (a) and (b). Hence, the incremental (successive correction) analysis procedure, for which the structure is piecewise linearized over finite loading increments, could be used to verify the validity of the successive approximation procedure for structures whose connections have the characteristics illustrated.

In addition, the moment-rotation diagram illustrated in Fig. 7.15(c) is a generalization of that shown in Fig. 7.15(b), where the former diagram is assumed to have an infinite number of infinitesimal segments. Hence, the validity of the successive approximation procedure can be verified for a structure with continuously non-linear connections.

<u>Example 2</u>

To further illustrate the successive approximation procedure, and compare it with a successive correction procedure, the frame shown in Fig. 7.16 was analyzed by both methods. The beam to column connections for the frame were double web angle connections, as illustrated in Fig. 7.17, employing $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ inch angles with 6 - $\frac{7}{8}$ inch diameter H.T. bolts per angle leg. The moment-rotation curve for the connection is also shown in the figure.

For the successive correction procedure, the connection moment-rotation curves were piecewise linearized over three intervals, as illustrated in Fig. 7.17. The analysis procedure then involved applying successive loading increments of such magnitude that each increment was terminated when the moment at one of the connections reached the end of one of the linear segments.

The entire load was applied initially, and a linear analysis performed. A load factor was then calculated for each connection and the minimum value retained. The load factor for a given connection was assumed to be the ratio of the load required to increase the moment at the connection to the limit of the current linear segment, to the total applied load.

The member end forces, joint displacements and support reactions were then multiplied by the minimum load factor and the factored values retained. The initial loading was then decremented by the product of the initial loading and the minimum load factor.

Table 7-4 Results of Fixed Beam Analysis

LOADING CASE	P	M_A	M_B	ϕ_A	ϕ_B
1	22.5 K	1000 in.k	500 in.k	-	-
2	11.25 K	-	500 in.k	37,503.8/EI	-
3	6.25 K	-	-	34,722.0/EI	27,777.6/EI
TOTALS	40.00K	1000 in.k	1000 in.k	72,225.8/EI	27,777.6/EI

For EI = 72,225,800 $\phi_A = .001$ $\phi_B = .00038$

Therefore:

$$\text{Flexibility A end} = \frac{.001}{1000} = .000001$$

$$\text{Flexibility B end} = \frac{.00038}{1000} = .0000038$$

Program analysis results:

For $C_A = .000001$; $C_B = .0000038$

$M_A = 1000$ in.kips $M_B = 1000$ in.kips

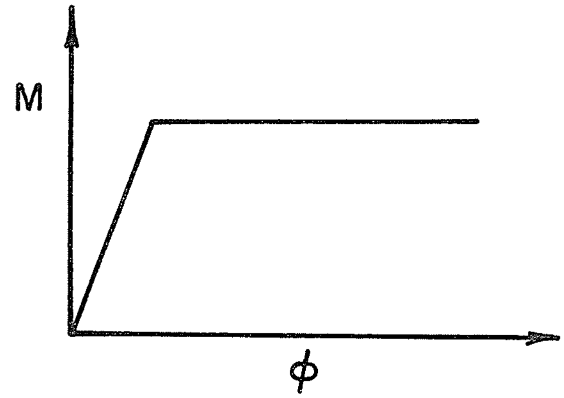


FIG. 7.15(a) ELASTIC PERFECTLY PLASTIC M-ϕ BEHAVIOUR

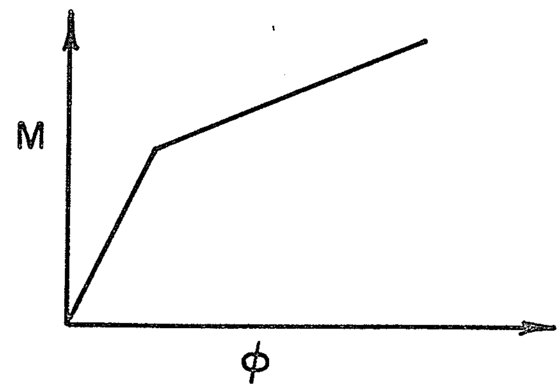


FIG. 7.15(b) ELASTIC-PLASTIC WITH STRAIN HARDENING M-ϕ BEHAVIOUR

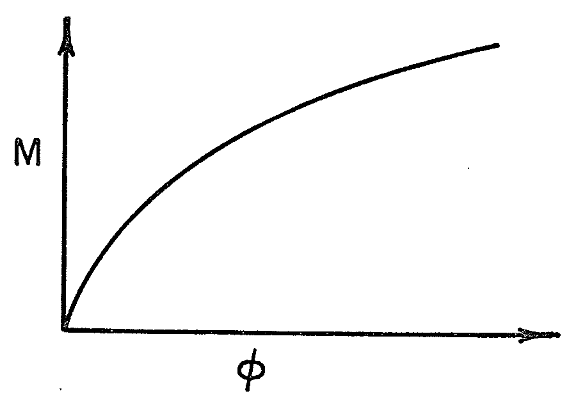
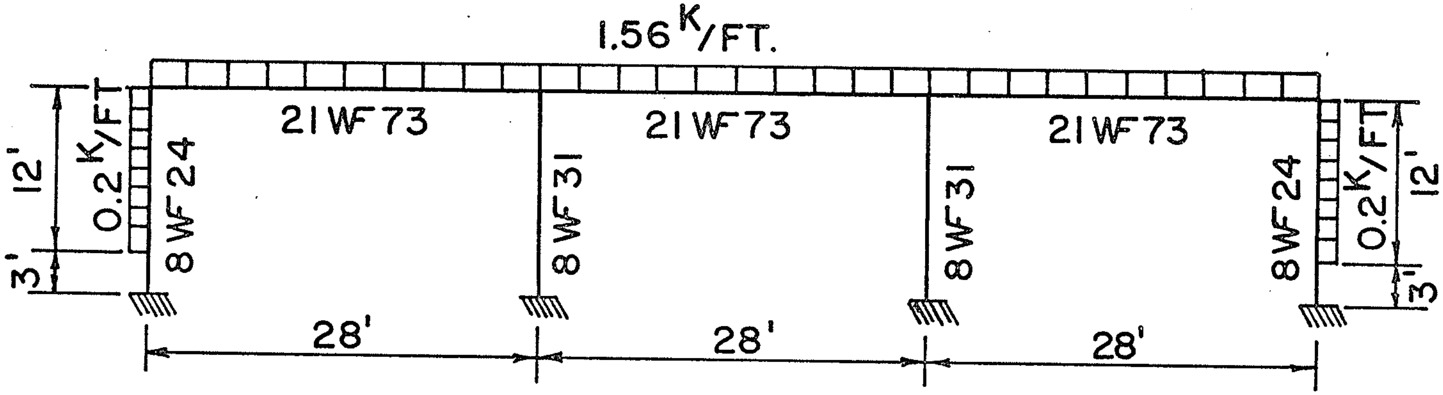


FIG. 7.15(c) CONTINUOUSLY NON-LINEAR M-ϕ BEHAVIOUR



I STOREY 3 BAY FRAME AND LOADING

FIG. 7.16

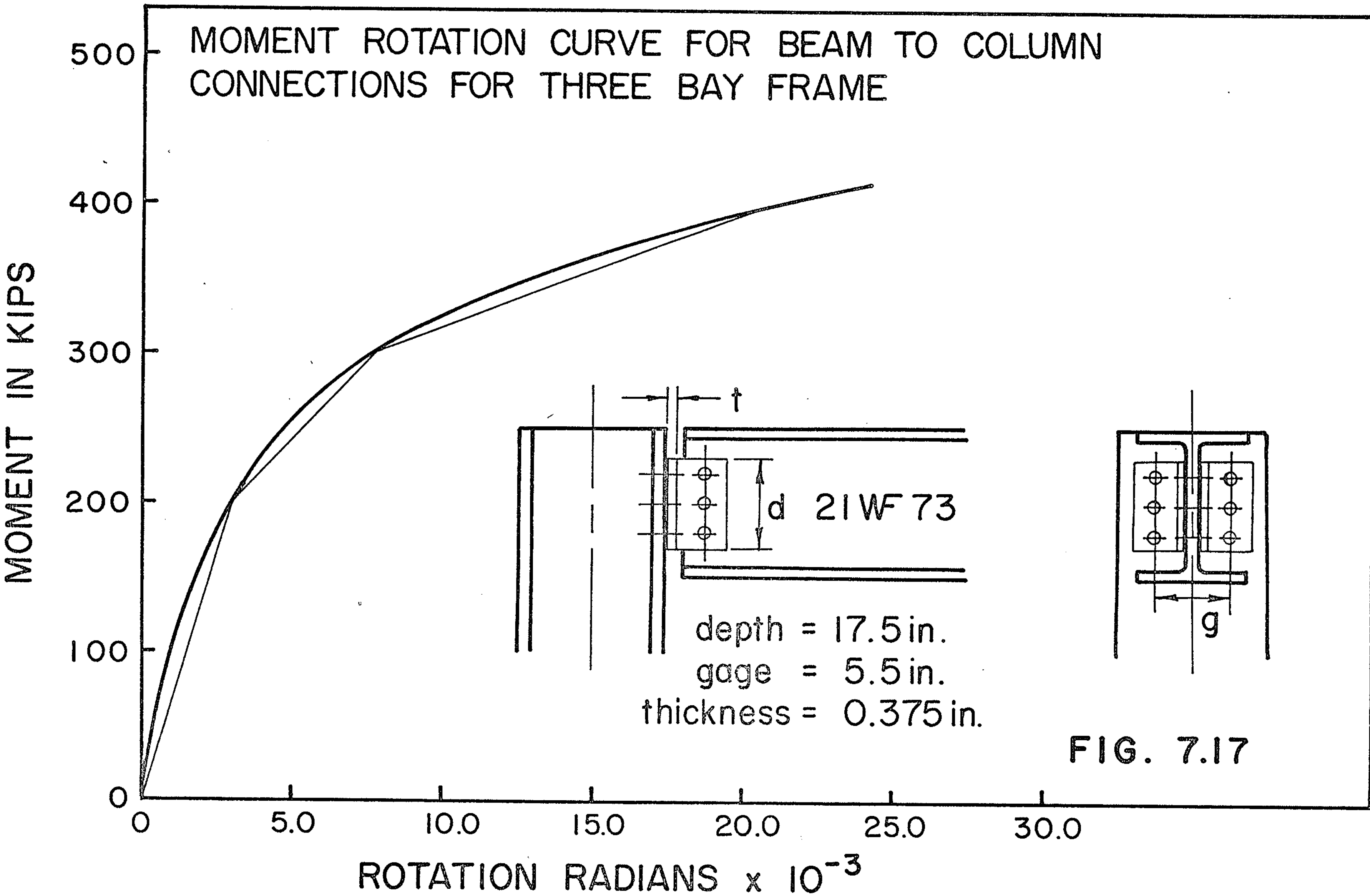

MOMENT ROTATION CURVE FOR BEAM TO COLUMN CONNECTIONS FOR THREE BAY FRAME
MOMENT IN KIPS
0
100
200
300
400
500
0
5.0
10.0
15.0
20.0
25.0
30.0
ROTATION RADIANS x 10^{-3}
t
d
21WF 73
g
depth = 17.5 in.
gage = 5.5 in.
thickness = 0.375 in.

FIG. 7.17

This reduced loading was then applied and a second linear analysis performed. Load factors were again calculated for all connections, and the minimum load factor retained. New factored member end forces, etc. were again calculated and added to the previous values, and the load was again decremented using the minimum load factor.

The procedure was repeated until the total loading had been applied, and the cumulative structural quantities retained.

Table 7-5 shows the loading that remained to be applied at the beginning of each of the seven loading increments that were used. The connection flexibilities, which correspond to the inverse slope of the segments of the piecewise linearized moment-rotation curve, are also tabulated.

The results of each linear analysis are given in Table 7-6 along with the cumulative end moments for the three beams in the structure. The structure was analyzed by the computer program developed in this study and the resulting beam end moments obtained from the latter analysis are also included in Table 7-6.

In general, the results obtained by the successive correction procedure and successive approximation procedure were in close agreement. To complete the successive correction procedure it was necessary to calculate all load factors, loading increments, cumulative totals, and new

Table 7-5 Variation of Loading and Connection Flexibility

Load Increment	LOADING			CONNECTION FLEXIBILITY					
	Vertical Load Remaining	Horizontal Load Remaining	% Total	Member 5		Member 6		Member 7	
				A end	B end	A end	B end	A end	B end
1	1.5600	.2000	100%	.00001500	.00001500	.00001500	.00001500	.00001500	.00001500
2	1.0778	.1382	69%	do	.00004750	do	do	do	do
3	1.0748	.1378	68.8%	do	do	do	.00004750	do	do
4	.6558	.0841	42%	do	do	do	do	do	.00004750
5	.6065	.0778	38.8%	do	.00013250	do	do	do	do
6	.6006	.0770	38.5%	do	do	do	.00013250	do	do
7	.1013	.0130	6.5%	do	do	do	do	do	.00013250

Table 7-6 Beam End Moments By Successive Corrections and Successive Approximations

		MEMBER 5		MEMBER 6		MEMBER 7	
		A END	B END	A END	B END	A END	B END
Analysis 1	Results	22.7400	-647.0759	305.8318	-642.8640	314.2080	-311.8198
	%	7.0285	-200.0000	94.5273	-198.6981	97.1162	-96.3781
	Cumulation	7.0285	-200.0000	94.5273	-198.6981	97.1162	-96.3781
Analysis 2	Results	1.2600	-215.8079	52.4400	-467.9158	204.6000	-237.0840
	%	.0035	-.6005	.1459	-1.3019	.5693	-.6596
	Cumulation	7.0230	-200.6005	94.6732	-200.0000	97.6855	-97.0377
Analysis 3	Results	-21.8280	-227.1599	40.5360	-225.3120	37.5240	-264.0840
	%	-8.5104	-88.5662	15.8044	-87.8457	14.6300	-102.9623
	Cumulation	-1.4784	-289.1667	110.4776	-287.8457	112.3155	-200.0000
Analysis 4	Results	-28.3440	-144.1320	9.9600	-143.8440	10.7160	-107.1840
	%	-2.1304	-10.8333	.7486	-10.8116	.8054	-8.0562
	Cumulation	-3.6088	-300.0000	111.2262	-298.6573	113.2209	-208.0562
Analysis 5	Results	-32.2200	-56.8680	-44.6280	-137.1840	2.2680	-104.2800
	%	-.3154	-.5566	-.4368	-1.3427	.0222	-1.0206
	Cumulation	-3.9252	-300.5566	110.7894	-300.0000	113.2431	-209.0768
Analysis 6	Results	-40.3920	-58.0920	-50.4720	-57.9480	-52.7040	-109.3800
	%	-33.5762	-48.2895	-41.9553	-48.1968	-43.8107	-90.9232
	Cumulation	-37.5014	-348.8461	68.8341	-348.1698	69.4324	-300.0000
Analysis 7	Results	-9.3480	-10.1880	-11.3160	-10.2240	-11.3400	-9.8800
	%	-9.3480	-10.1880	-11.3160	-10.2240	-11.3400	-9.8800
	Cumulation	-46.8494	-359.0341	57.5181	-358.3938	58.0924	-309.8800
Iteration		-45.9480	-366.5640	75.7440	-366.6479	76.9080	-310.1880

flexibilities by hand. Discrepancies between the two procedures can be attributed to error in these calculations. Additional error was also introduced by the crude approximation of the non-linear moment-rotation curve by only three linear secants. More accurate results would have been obtained for the successive correction analysis had smaller intervals been used.

CHAPTER VIII

CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

8.1 Conclusions

In this study, the experimental force-deformation information for the most commonly used structural steel framing connection types has been summarized. These experimental data which are in the form of moment-rotation curves, have been standardized to minimize the amount of connection information that must be stored in a structural analysis computer program.

A procedure has been outlined for incorporating the effects of flexible connections into a frame analysis program. The procedure involves modifying the stiffness matrix and the fixed-end-force vectors for any member to account for the effects of the connections at its ends.

Because of the non-linear nature of the force-deformation relationships for the majority of connection types encountered, an iterative procedure has been developed which involves repeated modifications to the assumed connection flexibilities until the structure satisfies equilibrium, compatability, and non-linear connection moment-rotation relationships.

A structural analysis computer program has been developed which is capable of analysing plane frames with any combination of rigid connections, pinned connections, any of seven common connection types, or connections with any specified bending flexibility.

Several examples have been included to illustrate that connection deformation may contribute to a significant percentage of overall structural displacement and may also substantially affect the internal force distribution in a structure. The iterative procedure has been compared with a piecewise linearization procedure for non-linear analysis. The results agreed very closely.

8.2 Suggestions For Further Study

The objectives of further investigation should be to supplement and extend the available connection test data, and to extend the capabilities of the analysis process.

Most of the available connection moment-rotation curves have been incorporated into this study. However, much of the information is for the now outdated riveted connections. Additional test data for high strength bolted connections should be obtained and incorporated into the analysis program. Since much of the available connection test data were obtained in the nineteen-thirties it would be desirable to verify them using the more accurate testing equipment now available.

Because of an increased use of new structural shapes such as hollow structural sections, it would be useful to incorporate connection data for these shapes into the analysis program. There are also several other conventional connection types [8, 9, 10, 11, 17, 34] that could be included in the analysis program when there is sufficient test data. The deformation of beam and column splices, and column bases should be included to provide a more complete picture of frame displacement caused by connection deformation.

In this study, only a single connection force component (moment) and the corresponding deformation component (rotation) have been considered. However, the analysis procedure could be extended to include the effects of shear and axial load on each connection.

At the present time, the analysis program is capable of treating only statically loaded structures. It would be highly desirable to extend the analysis procedure to dynamically loaded structures.

It would be of considerable practical value to adapt the analysis process developed in this study to a general structural steel floor system. The floor system could be analyzed as a planar grid of members connected by flexible connections, and loaded normal to the plane of the grid. The analysis program could then be combined with a member selection program to produce a computer program capable of designing steel floor systems.

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APPENDIX A

This appendix contains moment-rotation curves for the following connection types:

a) double web angle connections

b) single web angle connections

c) header plate connections

d) top and seat angle connections

e) welded top plate connections

f) end plate connections with column stiffeners

g) end plate connections without column stiffeners

h) T-stub connections.

The test numbers refer to the actual experimental test number. The following is a summary of the pertinent parameters for the above connection types:

<u>TABLE A-1 DOUBLE WEB ANGLE CONNECTIONS</u>

Investigator	Test No.	Beam Size	Column Size	Web Angle Size	Fastener Diameter	Rows	Gage
Munse, Lewitt, Chesson	1	12WF27	10WF49	4×3½×⅜×8½	¾	3	5½
	2	18WF50	12WF65	4×3½×⅜×11½	¾	4	5½
	3	21WF55	12WF65	4×3½×⅜×14½	¾	5	5½
	4	21WF55	12WF65	4×3½×⅜×17½	¾	6	5½
	5	24WF68	12WF65	4×3½×⅜×20½	¾	7	5½
	6	27WF84	12WF65	4×3½×⅜×23½	¾	8	5½
	7	33WF118	12WF65	4×3½×⅜×26½	¾	9	5½
	8	36WF135	12WF65	4×3½×⅜×29½	¾	10	5½
J. C. Rathbun	1	6I12.5	9×½×10 Pl.	6×4×⅜×2½	¾	1	5½
	2	8I18.4	9×½×1-0 Pl.	6×4×⅜×6	¾	2	5½
	3	8I18.4	11¾×½×1-1 Pl.	6×6×⅜×6	¾	3	5-10*
	4	12I31.8	9×½×1-4 Pl.	4×3½×⅜×9	¾	3	5½
	5	12I31.8	13×½×1-4 Pl.	6×6×⅜×9	¾	3	5-10*
	6	18I54.7	9×¾×2 Pl.	4×3½×⅜×1-3	¾	5	5½
	7	18I54.7	13×¾×2 Pl.	6×6×⅜×1-3	¾	5	5-10*
H. S. Somner	21	18WF45	14WF38	3½×3×⅜×9	¾	3	4½
	22	18WF45	14WF38	3½×3×⅜×12	¾	4	4½
	23	24WF76	14WF38	4×3×⅜×1-3	¾	5	5½
	24	24WF76	14WF38	4×3×⅜×1-8	¾	6	5½

* 2 rows of bolts

TABLE A-2 SINGLE WEB ANGLE CONNECTIONS

Investigator	Test No.	Beam Size	Column Size	Web Angle Size	Fastener Diameter	Rows	Gage
S. L. Lipson	BB4-1	21WF62	$\frac{3}{4}$ Pl.	$4 \times 3\frac{1}{2} \times \frac{5}{16} - 13\frac{1}{2}$	$\frac{3}{4}$	4	$2\frac{9}{16}$
	C4	21WF62	$\frac{3}{4}$ Pl.	$3\frac{1}{2} \times 5 \times \frac{5}{16} - 13\frac{1}{2}$	$\frac{3}{4}$	4	$1\frac{15}{16}$
	D4	21WF62	$\frac{3}{4}$ Pl.	$3\frac{1}{2} \times 5 \times \frac{5}{16} - 13\frac{1}{2}$	$\frac{3}{4}$	4	$1\frac{9}{16}$
	AA2-1	21WF62	$\frac{3}{4}$ Pl.	$4 \times 3\frac{1}{2} \times \frac{1}{4} - 7\frac{1}{2}$	$\frac{3}{4}$	2	$2\frac{9}{16}$
	AA3-1	21WF62	$\frac{3}{4}$ Pl.	$4 \times 3\frac{1}{2} \times \frac{1}{4} - 10\frac{1}{2}$	$\frac{3}{4}$	3	$2\frac{9}{16}$
	AA4-1	21WF62	$\frac{3}{4}$ Pl.	$4 \times 3\frac{1}{2} \times \frac{1}{4} - 13\frac{1}{2}$	$\frac{3}{4}$	4	$2\frac{9}{16}$
	AA5-1	21WF62	$\frac{3}{4}$ Pl.	$4 \times 3\frac{1}{2} \times \frac{1}{4} - 16\frac{1}{2}$	$\frac{3}{4}$	5	$2\frac{9}{16}$
	AA6-1	21WF62	$\frac{3}{4}$ Pl.	$4 \times 3\frac{1}{2} \times \frac{1}{4} - 19\frac{1}{2}$	$\frac{3}{4}$	6	$2\frac{9}{16}$

TABLE A-3 HEADER PLATE CONNECTIONS

Investigator	Test No.	Beam Size	Column Size	Plate Size	Fastener Diameter	Rows	Gage
H. S. Somner	5	18WF45	14WF38	$15\times6\times\frac{1}{4}$	$\frac{3}{4}$	5	4
	6	24WF76	14WF38	$9\times6\times\frac{1}{4}$	$\frac{3}{4}$	3	4
	7	24WF76	14WF38	$12\times6\times\frac{1}{4}$	$\frac{3}{4}$	4	4
	8	24WF76	14WF38	$15\times6\times\frac{1}{4}$	$\frac{3}{4}$	5	4
	9	24WF76	14WF38	$18\times6\times\frac{1}{4}$	$\frac{3}{4}$	6	4
	10	18WF45	14WF38	$9\times6\times\frac{3}{8}$	$\frac{3}{4}$	3	4
	11	18WF45	14WF38	$12\times6\times\frac{3}{8}$	$\frac{3}{4}$	4	4
	12	24WF76	14WF38	$15\times6\times\frac{3}{8}$	$\frac{3}{4}$	5	4
	13	24WF76	14WF38	$9\times6\times\frac{3}{8}$	$\frac{3}{4}$	3	4
	14	24WF76	14WF38	$12\times6\times\frac{3}{8}$	$\frac{3}{4}$	4	4
	15	24WF76	14WF38	$15\times7\frac{1}{2}\times\frac{3}{8}$	$\frac{3}{4}$	5	$5\frac{1}{2}$
	16	24WF76	14WF38	$18\times7\frac{1}{2}\times\frac{3}{8}$	$\frac{3}{4}$	6	$5\frac{1}{2}$
	17	24WF76	14WF38	$12\times7\frac{1}{2}\times\frac{1}{4}$	$\frac{3}{4}$	4	$5\frac{1}{2}$
	18	24WF76	14WF38	$15\times7\frac{1}{2}\times\frac{1}{4}$	$\frac{3}{4}$	5	$5\frac{1}{2}$
	19	24WF76	14WF38	$12\times7\frac{1}{2}\times\frac{1}{4}$	$\frac{3}{4}$	4	$5\frac{1}{2}$
	20	24WF76	14WF38	$15\times7\frac{1}{2}\times\frac{1}{4}$	$\frac{3}{4}$	5	$5\frac{1}{2}$

TABLE A-4 TOP AND SEAT ANGLE CONNECTIONS

Investigator	Test No.	Beam Size	Column Size	Top Angle Size	Seat Size	Fastener Diameter	
C. Batho, H. C. Rowan	1	12RSJ@30#	12RSJ@65#	$4\times4\times\frac{1}{2}\times5$	$4\times4\times\frac{1}{2}\times5$	$\frac{3}{4}$	
	2	12RSJ@30#	12RSJ@65#	$4\times4\times\frac{3}{4}\times5$	$4\times4\times\frac{3}{4}\times5$	$\frac{3}{4}$	
	3	12RSJ@30#	12RSJ@65#	$4\times4\times1\times5$	$4\times4\times1\times5$	$\frac{3}{4}$	
	4	12RSJ@30#	12RSJ@65#	$4\times4\times1\times5$	$4\times4\times1\times5$	$\frac{7}{8}$	
	5	12RSJ@30#	12RSJ@65#	$6\times6\times\frac{1}{2}\times5$	$6\times6\times\frac{1}{2}\times5$	$\frac{3}{4}$	
	6	12RSJ@30#	12RSJ@65#	$6\times6\times\frac{3}{4}\times5$	$6\times6\times\frac{3}{4}\times5$	$\frac{3}{4}$	
	7	12RSJ@30#	12RSJ@65#	$6\times6\times1\times5$	$6\times6\times1\times5$	$\frac{3}{4}$	
	11	12RSJ@30#	12RSJ@65#	$6\times6\times\frac{1}{2}\times5$	$6\times6\times\frac{1}{2}\times5$	$\frac{3}{4}$	Web angles
	12	12RSJ@30#	12RSJ@65#	$6\times6\times\frac{3}{4}\times5$	$6\times6\times\frac{3}{4}\times5$	$\frac{3}{4}$	
	16	12RSJ@30#	12RSJ@65#	$6\times6\times\frac{3}{8}\times5$	$6\times6\times\frac{3}{8}\times5$	$\frac{3}{4}$	High strength
	17	12RSJ@30#	12RSJ@65#	$6\times6\times\frac{1}{2}\times5$	$6\times6\times\frac{1}{2}\times5$	$\frac{3}{4}$	bolts
R. A. Hechtman, B. G. Johnston	2	12WF25	10WF49	$6\times4\times\frac{3}{8}\times6\frac{3}{4}$	$6\times6\times\frac{1}{2}\times6\frac{3}{4}$	$\frac{3}{4}$	
	9	18WF47	12WF65	$6\times4\times\frac{5}{8}\times1'$-0	$6\times6\times\frac{7}{8}\times7\frac{1}{2}$	$\frac{3}{4}$	
	10	18WF47	12WF65	$6\times4\times\frac{3}{4}\times1'$-0	$6\times6\times\frac{7}{8}\times7\frac{1}{2}$	$\frac{3}{4}$	
	11	18WF47	14WF58	$6\times4\times\frac{1}{2}\times10$	$6\times6\times\frac{7}{8}\times7\frac{1}{2}$	$\frac{3}{4}$	
	16	12WF25	10WF49	$6\times4\times\frac{1}{2}\times6\frac{3}{4}$	$6\times6\times\frac{1}{2}\times6\frac{3}{4}$	$\frac{3}{4}$	
	17	12WF25	10WF49	$6\times4\times\frac{1}{2}\times6\frac{3}{4}$	$6\times6\frac{1}{2}\times6\frac{3}{4}$	$\frac{3}{4}$	
	18	12WF50	10WF49	$6\times4\times\frac{1}{2}\times8$	$6\times6\times\frac{1}{2}\times8$	$\frac{3}{4}$	
	20	14WF34	12WF65	$6\times4\times\frac{5}{8}\times1'$-0	$6\times6\times\frac{5}{8}\times7\frac{1}{4}$	$\frac{3}{4}$	
	22	16WF40	12WF65	$6\times4\times\frac{5}{8}\times1'$-0	$6\times6\times\frac{3}{4}\times7\frac{1}{4}$	$\frac{3}{4}$	
	23	16WF40	14WF58	$6\times4\times\frac{5}{8}\times10''$	$6\times6\times\frac{3}{4}\times7\frac{1}{4}$	$\frac{3}{4}$	
	24	18WF47	12WF65	$6\times4\times\frac{5}{8}\times1'$-0'$\frac{1}{2}$	$6\times6\times\frac{7}{8}\times7\frac{1}{2}$	$\frac{7}{8}$	
	25	21WF59	14WF87	$6\times4\times\frac{3}{4}\times1$-2	stiff angle	$\frac{7}{8}$	
	26	24WF74	14WF87	$6\times4\times\frac{3}{4}\times1$-2	stiff angle	$\frac{7}{8}$	

(continued)

TABLE A-4 TOP AND SEAT ANGLE CONNECTIONS (continued)

Investigator	Test No.	Beam Size	Column Size	Top Angle Size	Seat Size	Fastener Diameter	
R. A. Hechtman, B. G. Johnston	31	24WF120	14WF87	$6\times4\times\frac{3}{4}\times1$-2	stiff angle	$\frac{7}{8}$	
	32	21WF103	14WF87	$6\times4\times\frac{3}{4}\times1$-2	stiff angle	$\frac{7}{8}$	
	35	12WF25	10WF49	$6\times4\times\frac{5}{8}\times8\frac{1}{2}$	$6\times6\times\frac{1}{2}\times8\frac{1}{2}$	$\frac{7}{8}$	
	36	18WF47	14WF58	$6\times4\times\frac{5}{8}\times11\frac{1}{4}$	$6\times6\times\frac{7}{8}\times11\frac{1}{4}$	$\frac{7}{8}$	— one side of col. web
	37	18WF47	14WF58	$6\times4\times\frac{5}{8}\times11\frac{1}{4}$	$6\times6\times\frac{7}{8}\times11\frac{1}{4}$	$\frac{7}{8}$	— both sides of col. web
J. C. Rathbun	8	12I31.8	$6\times1\times2$'-0 Pl.	$6\times4\times\frac{3}{8}\times6$	$6\times6\times\frac{3}{8}\times6$	$\frac{3}{4}$	
	9	12I31.8	$8\times1\times2$'-0 Pl.	$6\times4\times\frac{3}{8}\times8$"	$6\times6\times\frac{3}{8}\times8$	$\frac{3}{4}$	
	10	12I31.8	$14\times1\times2$'-0 Pl.	$6\times4\times\frac{3}{8}\times1$'-2	$6\times6\times\frac{3}{8}\times1$-2	$\frac{3}{4}$	
	11	12I31.8	$9\times1\times2$'0 Pl.	$6\times4\times\frac{3}{8}\times9$	$6\times6\times\frac{3}{8}\times9$	$\frac{3}{4}$	— web angles
	12	12I31.8	$14\times1\times2$'0 Pl.	$6\times4\times\frac{3}{8}\times1$-2	$6\times6\times\frac{3}{8}\times1$'-2	$\frac{3}{4}$	— web angles

TABLE A-5 WELDED TOP PLATE AND SEAT CONNECTIONS

Investigator	Test No.	Beam Size	Column Size	Top Plate Size	Seat Size
J. L. Brandes, R. M. Mains	2	12WF50	12WF65	$7\times\frac{5}{8}\times9$"	ST13WF45.5$\times$6
	3	12WF47	12WF65	$6\times\frac{5}{8}\times8$	ST9WF32$\times$9
	4	12WF85	12WF65	$7\frac{1}{2}\times\frac{7}{8}\times12$	ST15WF54$\times7\frac{1}{2}$
	5	12WF85	12WF65	$7\frac{1}{2}\times\frac{7}{8}\times12$	$8\times8\times\frac{7}{8}-10$
	6	12WF85	12WF65	$7\frac{1}{2}\times\frac{7}{8}\times12$	ST15WF54$\times7\frac{1}{2}$
	7	12WF85	12WF65	$7\frac{1}{2}\times\frac{7}{8}\times12$	ST15WF54$\times7\frac{1}{2}$
	9	12WF85	12WF65	$7\frac{1}{4}\times1\times12$	ST15WF54$\times8\frac{1}{2}$
	10	12WF85	12WF65	$7\frac{1}{4}\times1\times12$	$8\times8\times1-10\frac{1}{2}$
	11	12WF85	12WF65	6&3$\times\frac{5}{16}\times12$	Pl. Tee
	12	18WF85	12WF65	6&3$\times\frac{5}{16}\times12$	Pl. Tee
	13	18WF70	12WF65	6&3$\times\frac{5}{16}\times12$	$6\times3\frac{1}{2}\times\frac{3}{4}-10$
	14	18WF45	12WF65	6&3$\times\frac{5}{16}\times12$	$6\times3\frac{1}{2}\times\frac{5}{8}-9$
	15	24WF74	14WF61	6&3$\times\frac{3}{8}\times12$	Pl. Tee
	16	12WF85	12WF65	$10\frac{1}{2}\times\frac{3}{4}\times15$	ST15WF54$\times7\frac{1}{2}$
	17	12WF85	12WF65	$7\frac{1}{2}\times\frac{7}{8}\times12$	ST15WF54$\times7\frac{1}{2}$
	18	18WF85	12WF65	6&3$\times\frac{3}{8}\times12$	Pl. Tee
	3	10I25	8I45	$10\frac{1}{2}\times5\times\frac{1}{2}$	$10\frac{1}{2}\times5\times\frac{1}{2}$ Pl.
	6	10I25	8I45	$10\frac{1}{2}\times5\times\frac{1}{2}$	$6\times3\frac{1}{2}\times\frac{3}{8}\times5\frac{1}{2}$ Pl.
	9	18WF47	12WF65	$6\times\frac{5}{16}\times11$	$6\times3\frac{1}{2}\times\frac{5}{8}\times9$ L.
	10	18WF47	12WF65	$6\times\frac{5}{16}\times4$	$6\times3\frac{1}{2}\times\frac{5}{8}\times9$ L.
	21	18WF85	12WF92	$11\times\frac{5}{16}\times6$	Pl. Tee

TABLE A-6 END PLATE CONNECTIONS WITH NO COLUMN STIFFENERS

Investigator	Test No.	Beam Size	Column Size	End Plate Size
J. R. Ostran-der	1	10WF21	8WF28	$6\frac{1}{2} \times 11 \times \frac{1}{2}$
	3	10WF21	8WF28	$6\frac{1}{2} \times 11 \times \frac{3}{8}$
	4	10WF21	8WF28	$6\frac{1}{2} \times 11 \times \frac{1}{4}$
	9	10WF21	8WF28	$6\frac{1}{2} \times 11 \times \frac{3}{4}$
	11	12WF27	8WF40	$7\frac{1}{2} \times 13 \times \frac{3}{8}$
	12	12WF27	8WF40	$7\frac{1}{2} \times 13 \times \frac{1}{2}$
	13	12WF27	8WF40	$7\frac{1}{2} \times 13 \times \frac{5}{8}$
	17	12WF27	8WF24	$7\frac{1}{2} \times 13 \times \frac{3}{8}$
	18	12WF27	8WF24	$7\frac{1}{2} \times 13 \times \frac{1}{2}$
	19	12WF27	8WF24	$7\frac{1}{2} \times 13 \times \frac{5}{8}$
	23	12WF27	8WF48	$7\frac{1}{2} \times 13 \times \frac{5}{8}$
Sherbourne	A1	15×5×42#	8×8×35#	$7 \times 1 - 6\frac{1}{2} \times 1\frac{1}{4}$

TABLE A-7 END PLATE CONNECTIONS WITH COLUMN STIFFENERS

Investigator	Test No.	Beam Size	Column Size	End Plate	Stiffener Size
J. R. Ostrander	2	10WF21	8WF28	$6\frac{1}{2}\times11\times\frac{1}{2}$	$3\times7\frac{1}{8}\times\frac{3}{8}$
	5	10WF21	8WF28	$6\frac{1}{2}\times11\times\frac{1}{2}$	$3\times7\frac{1}{8}\times\frac{1}{4}$
	6	10WF21	8WF28	$6\frac{1}{2}\times11\times\frac{3}{8}$	$3\times7\frac{1}{8}\times\frac{1}{4}$
	7	10WF21	8WF28	$6\frac{1}{2}\times11\times\frac{1}{4}$	$3\times7\frac{1}{8}\times\frac{1}{4}$
	8	10WF21	8WF28	$6\frac{1}{2}\times11\times\frac{1}{4}$	$3\times7\frac{1}{8}\times\frac{3}{8}$
	10	12WF27	8WF28	$6\frac{1}{2}\times11\times\frac{3}{4}$	$3\times7\frac{1}{8}\times\frac{3}{8}$
	14	12WF27	8WF40	$7\frac{1}{2}\times13\times\frac{3}{8}$	$4\times7\times\frac{1}{4}$
	15	12WF27	8WF40	$7\frac{1}{2}\times13\times\frac{1}{2}$	$4\times7\times\frac{1}{4}$
	16	12WF27	8WF40	$7\frac{1}{2}\times13\times\frac{5}{8}$	$4\times7\times\frac{1}{4}$
	20	12WF27	8WF24	$7\frac{1}{2}\times13\times\frac{3}{8}$	$3\times7\frac{1}{8}\times\frac{1}{4}$
	21	12WF27	8WF24	$7\frac{1}{2}\times13\times\frac{1}{2}$	$3\times7\frac{1}{8}\times\frac{1}{4}$
	22	12WF27	8WF24	$7\frac{1}{2}\times13\times\frac{5}{8}$	$3\times7\frac{1}{8}\times\frac{1}{4}$
	24	12WF27	8WF48	$7\frac{1}{2}\times13\times\frac{5}{8}$	$4\times6\frac{3}{4}\times\frac{1}{4}$
A. N. Sherbourne	A2	15×5×42#	8×8×35#	$7\times1-6\frac{1}{2}\times1\frac{1}{4}$	$3\frac{1}{2}\times7\times\frac{5}{16}$
	A3	15×5×42#	8×8×35#	$7\times1-6\frac{1}{2}\times\frac{3}{4}$	$3\frac{1}{2}\times7\times\frac{5}{8}$
	B1	15×5×42#	8×8×35#	$7\times1-6\frac{1}{2}\times1$	$3\frac{1}{2}\times7\times\frac{5}{16}$
	B2	15×5×42#	8×8×35#	$7\times1-6\frac{1}{2}\times\frac{3}{4}$	$3\frac{1}{2}\times7\times\frac{1}{2}$
L. G. Johnson, J. C. Cannon, L. A. Spooner	5	10I25	8I45	$6\times1-1\frac{3}{4}\times\frac{1}{2}$	$3\times8\times\frac{1}{2}$

TABLE A-8 T-STUB CONNECTIONS

Investigator	Test No.	Beam Size	Column Size	T-Stub Size	
C. Batho, H. C. Rowan	13	12RSJ@30	12RSJ@65	15I45	
	14	12RSJ@30	12RSJ@65	15I45	Web angles
	15	12RSJ@30	12RSJ@65	15I45	Shear connections
A. Bannister	a	10×8 B.S.	12×8 B.S.	24×7½ B.S.	4 bolts
	b	10×8 B.S.	12×8 B.S.	24×7½ B.S.	6 bolts
	c	10×8 B.S.	12×8 B.S.	24×7½ B.S.	8 bolts
	d	10×8 B.S.	12×8 B.S.	24×7½ B.S.	10 bolts
	e	13×8 B.S.	12×8 B.S.	24×7½ B.S.	8 bolts
J. C. Rathbun	13	12I31.8	9×1×1'-10 Pl.	15G@99#×9"	
	14	12I31.8	14×1×1-10 Pl.	15G@99#×1'-2"	
	15	16G83	15×1×2'-3 Pl.	24I105.9×1'-3"	
	16	22G101	15×1×3-3'4 Pl.	30G240×1-3	
	17	22G101	15×1×3-3'4 Pl.	30G240×1-3	2 lines of bolts
	18	16G83	14H167	24I105.9×1-3	
R. Douty	D-1	14WF34	14WF150	18WF70	
	D-2	16WF40	14WF150	16WF40	
	D-3	21WF62	14WF150	21WF62	

B.S. -- British Standard Section

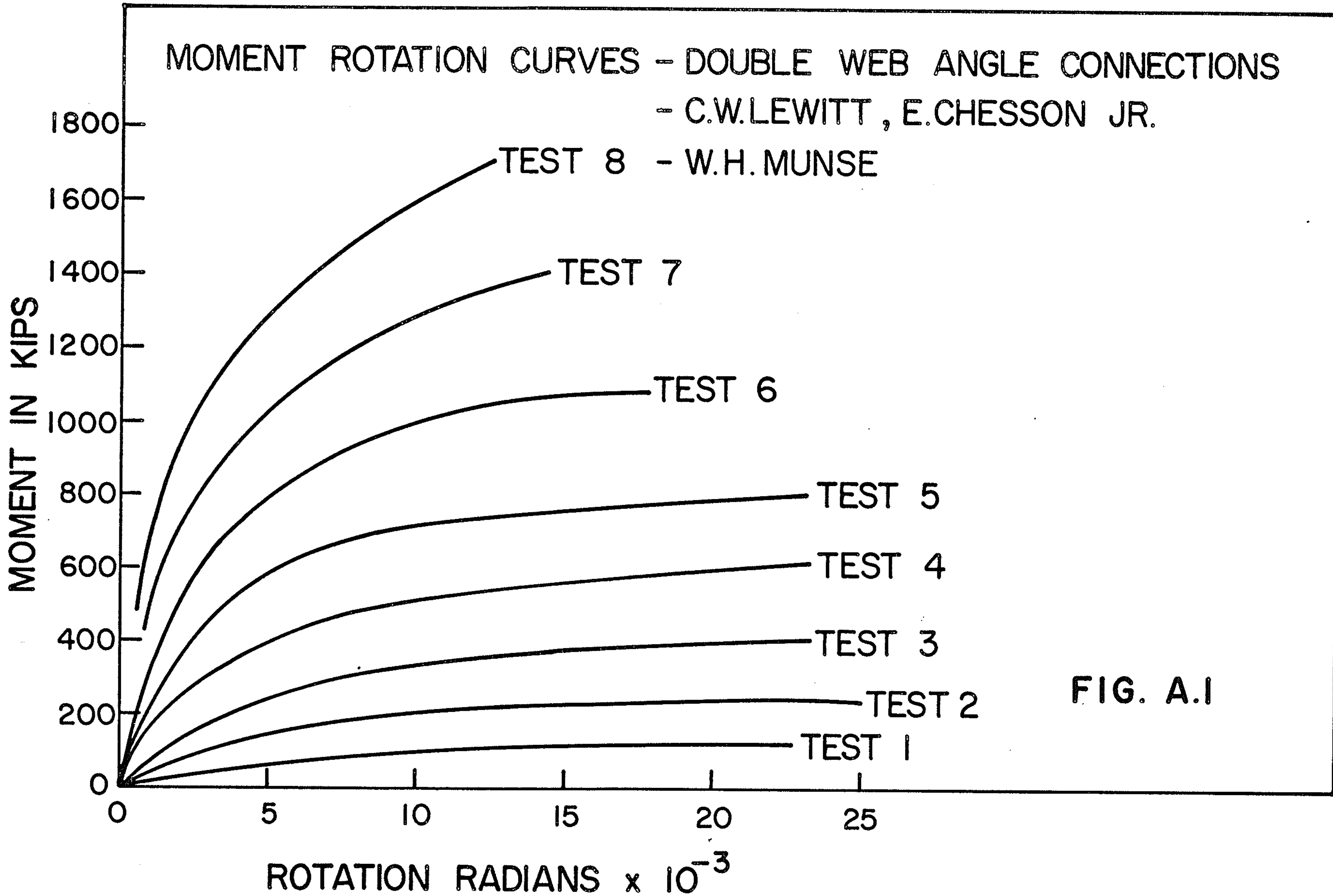

MOMENT ROTATION CURVES - DOUBLE WEB ANGLE CONNECTIONS
- C.W. LEWITT, E. CHESSON JR.
- W.H. MUNSE
TEST 8
TEST 7
TEST 6
TEST 5
TEST 4
TEST 3
TEST 2
TEST 1
MOMENT IN KIPS
0
200
400
600
800
1000
1200
1400
1600
1800
0
5
10
15
20
25
ROTATION RADIANS x 10^{-3}

FIG. A.1

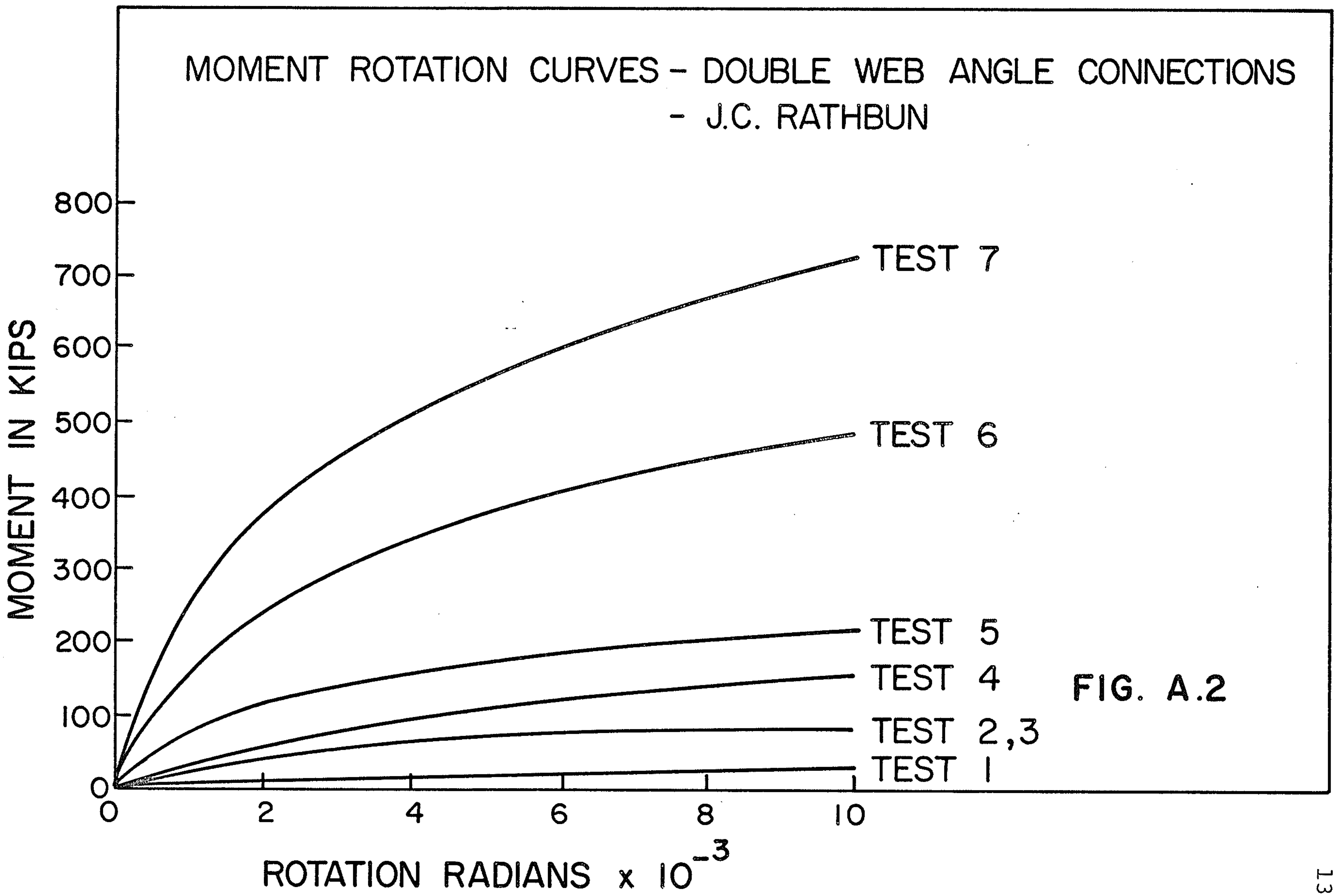

MOMENT ROTATION CURVES - DOUBLE WEB ANGLE CONNECTIONS
- J.C. RATHBUN
MOMENT IN KIPS
0
100
200
300
400
500
600
700
800
ROTATION RADIANS x 10^{-3}
0
2
4
6
8
10
TEST 7
TEST 6
TEST 5
TEST 4
TEST 2,3
TEST 1

FIG. A.2

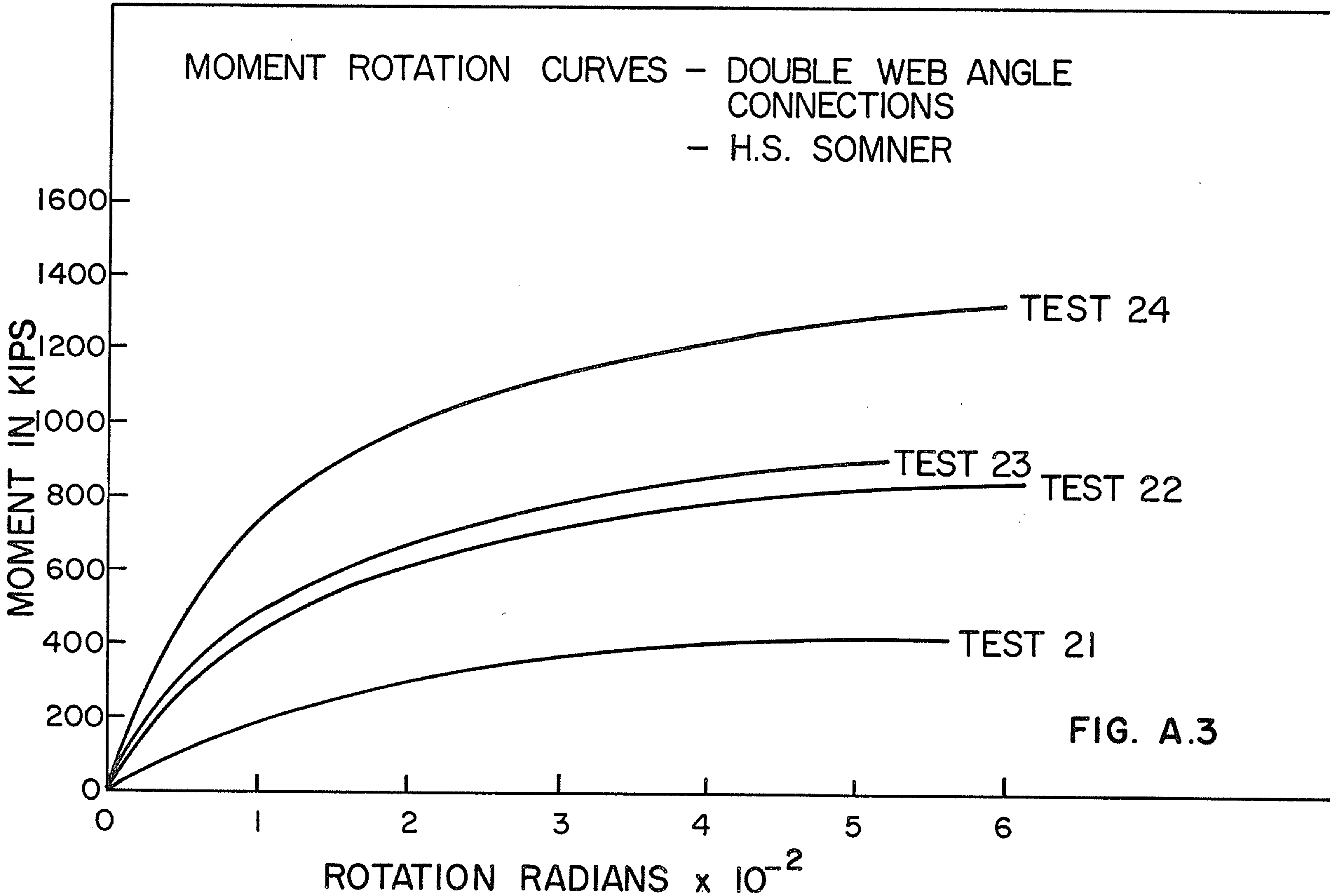

MOMENT ROTATION CURVES - DOUBLE WEB ANGLE CONNECTIONS
- H.S. SOMNER
MOMENT IN KIPS
0
200
400
600
800
1000
1200
1400
1600
TEST 24
TEST 23
TEST 22
TEST 21
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.3

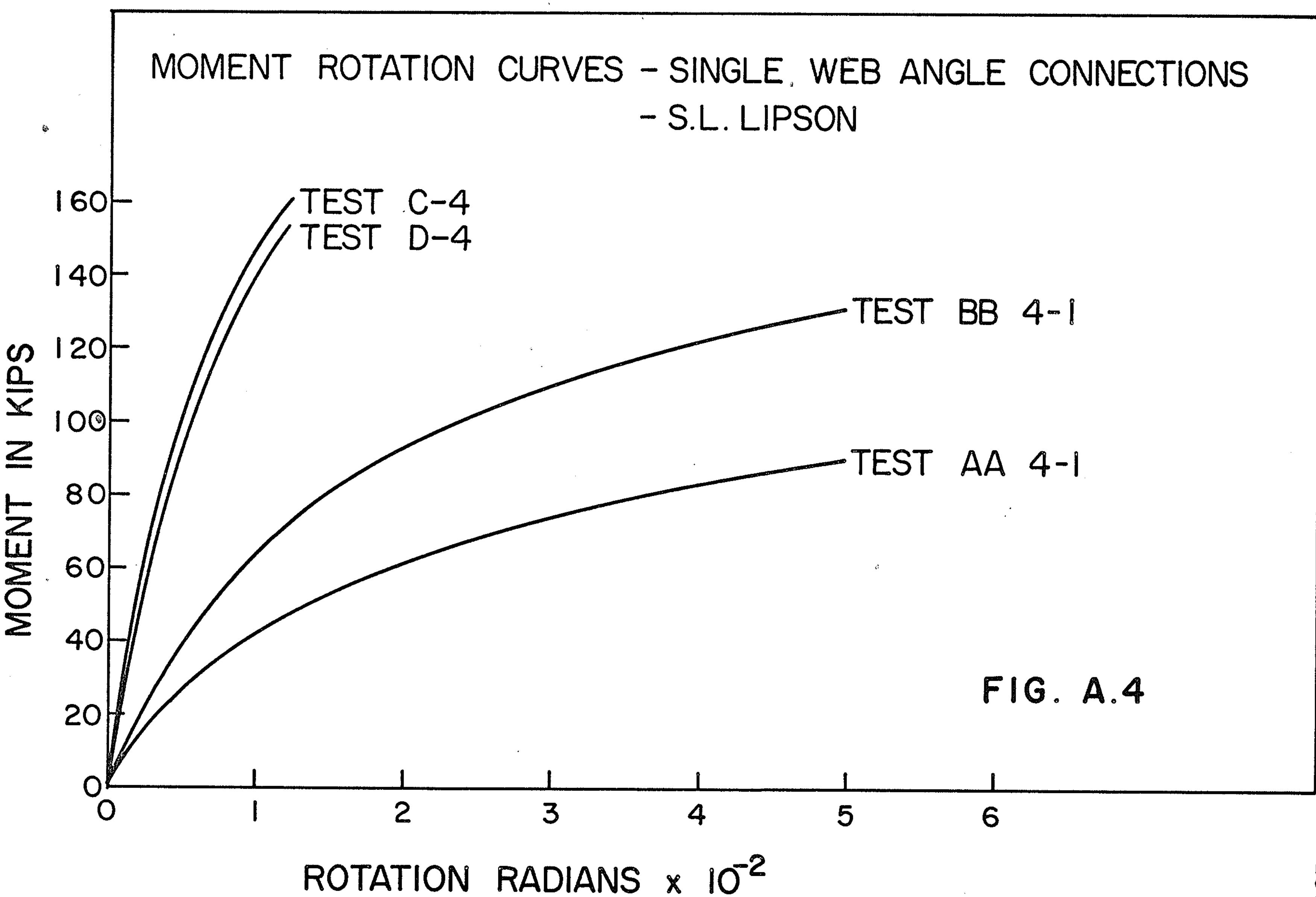

MOMENT ROTATION CURVES - SINGLE WEB ANGLE CONNECTIONS
- S.L. LIPSON
TEST C-4
TEST D-4
TEST BB 4-1
TEST AA 4-1
MOMENT IN KIPS
0
20
40
60
80
100
120
140
160
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.4

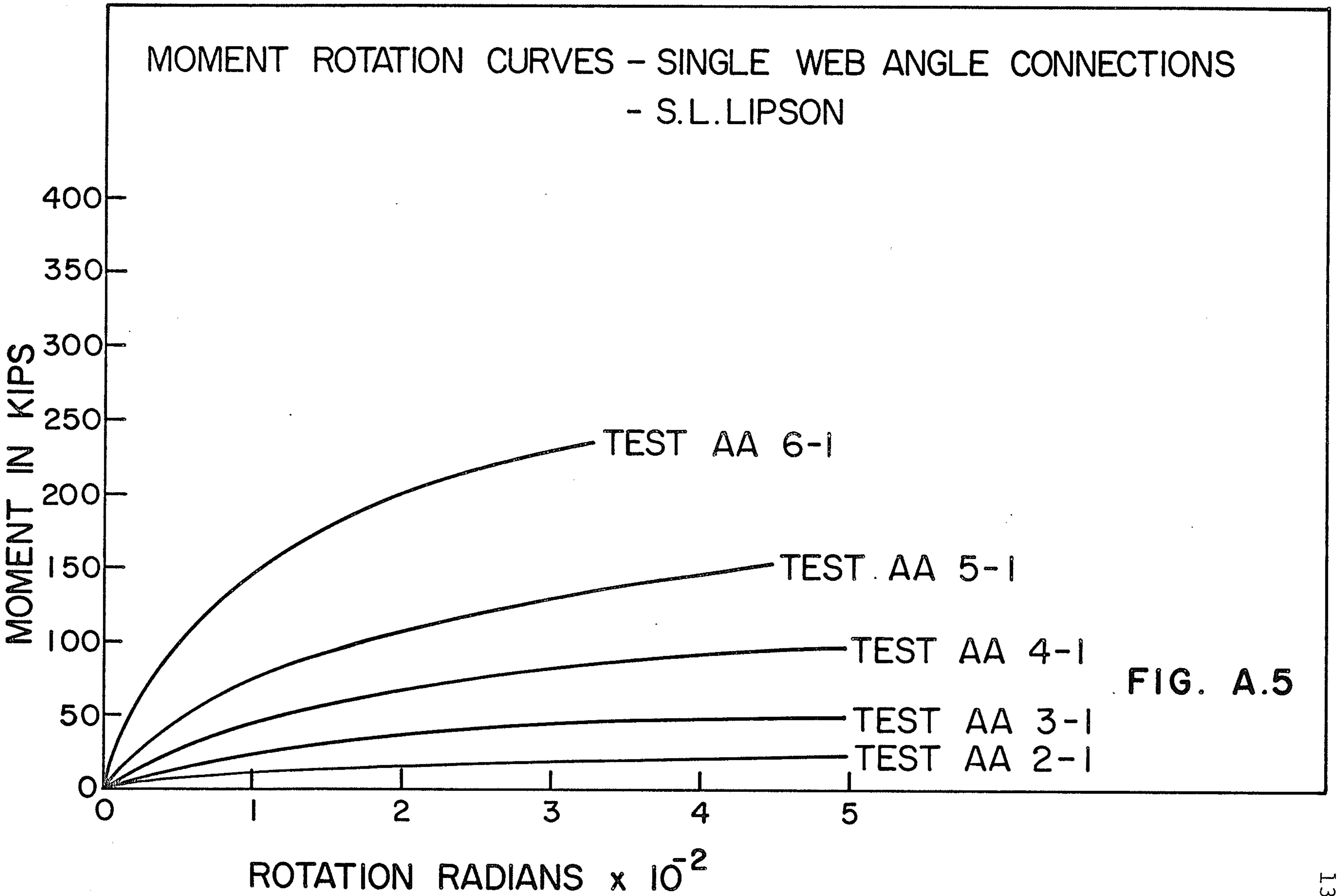

MOMENT ROTATION CURVES - SINGLE WEB ANGLE CONNECTIONS
- S.L. LIPSON
MOMENT IN KIPS
400
350
300
250
200
150
100
50
0
TEST AA 6-1
TEST AA 5-1
TEST AA 4-1
TEST AA 3-1
TEST AA 2-1
0
1
2
3
4
5
ROTATION RADIANS x 10^{-2}

FIG. A.5

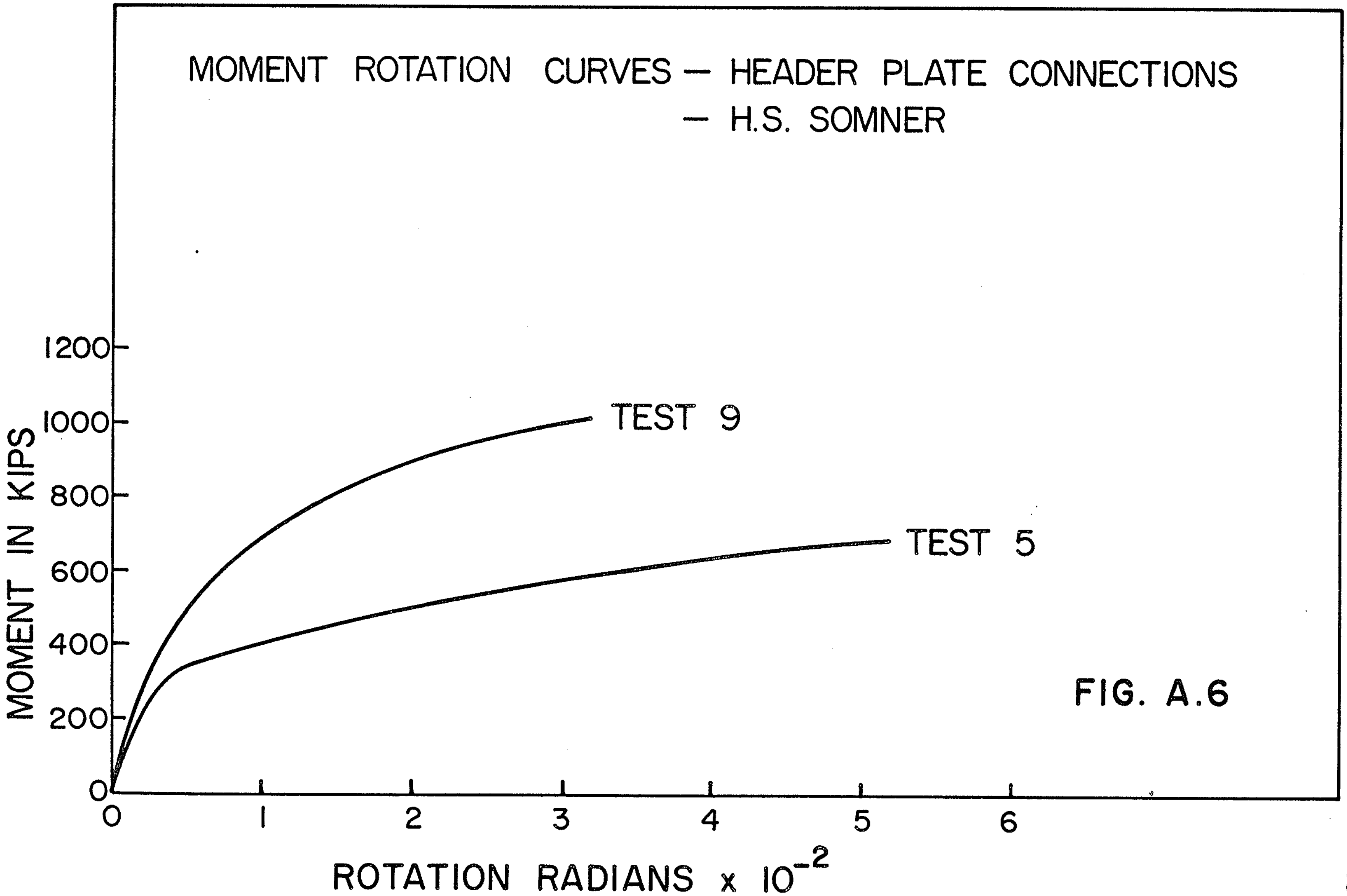

MOMENT ROTATION CURVES — HEADER PLATE CONNECTIONS
— H.S. SOMNER
TEST 9
TEST 5
MOMENT IN KIPS
0
200
400
600
800
1000
1200
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.6

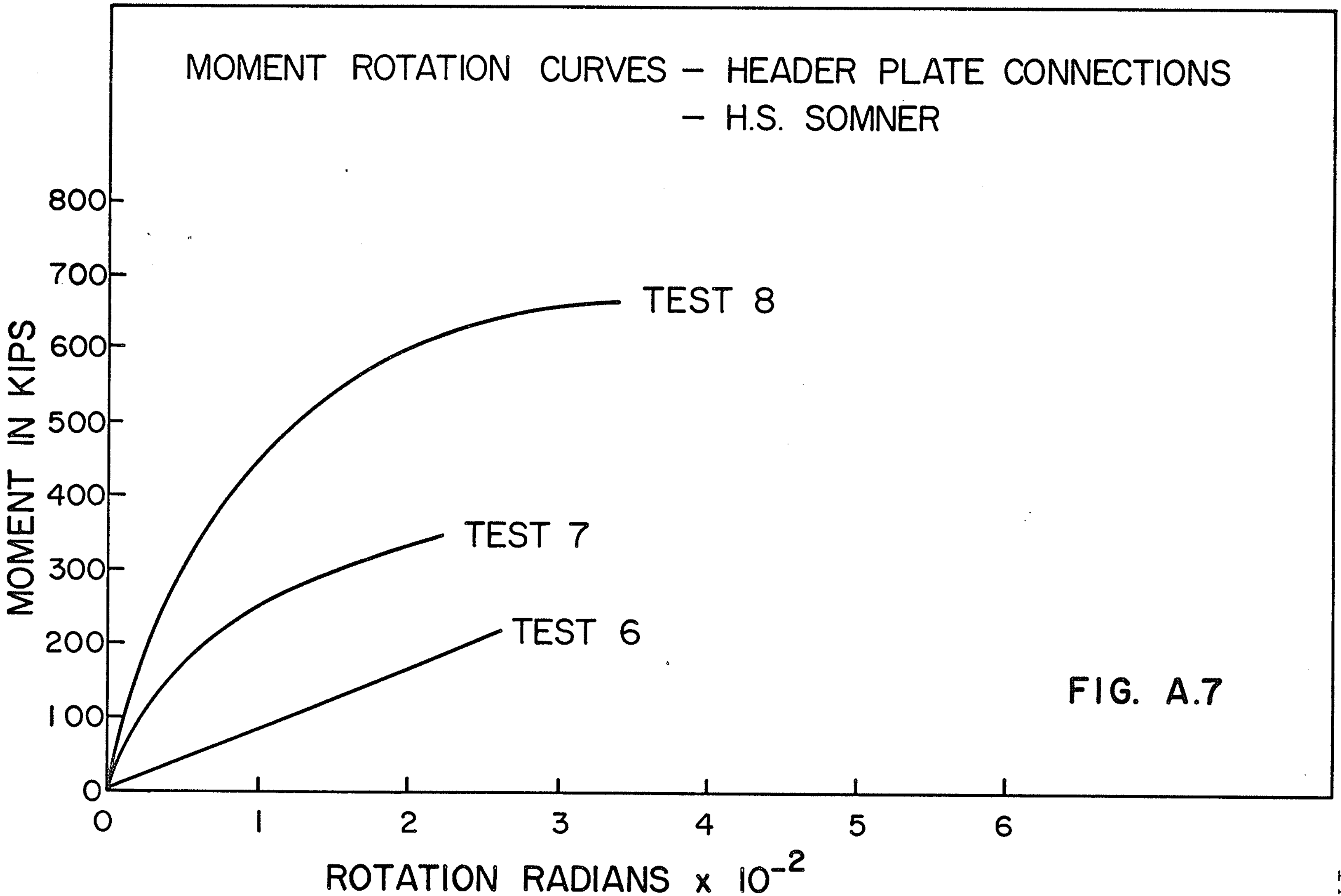
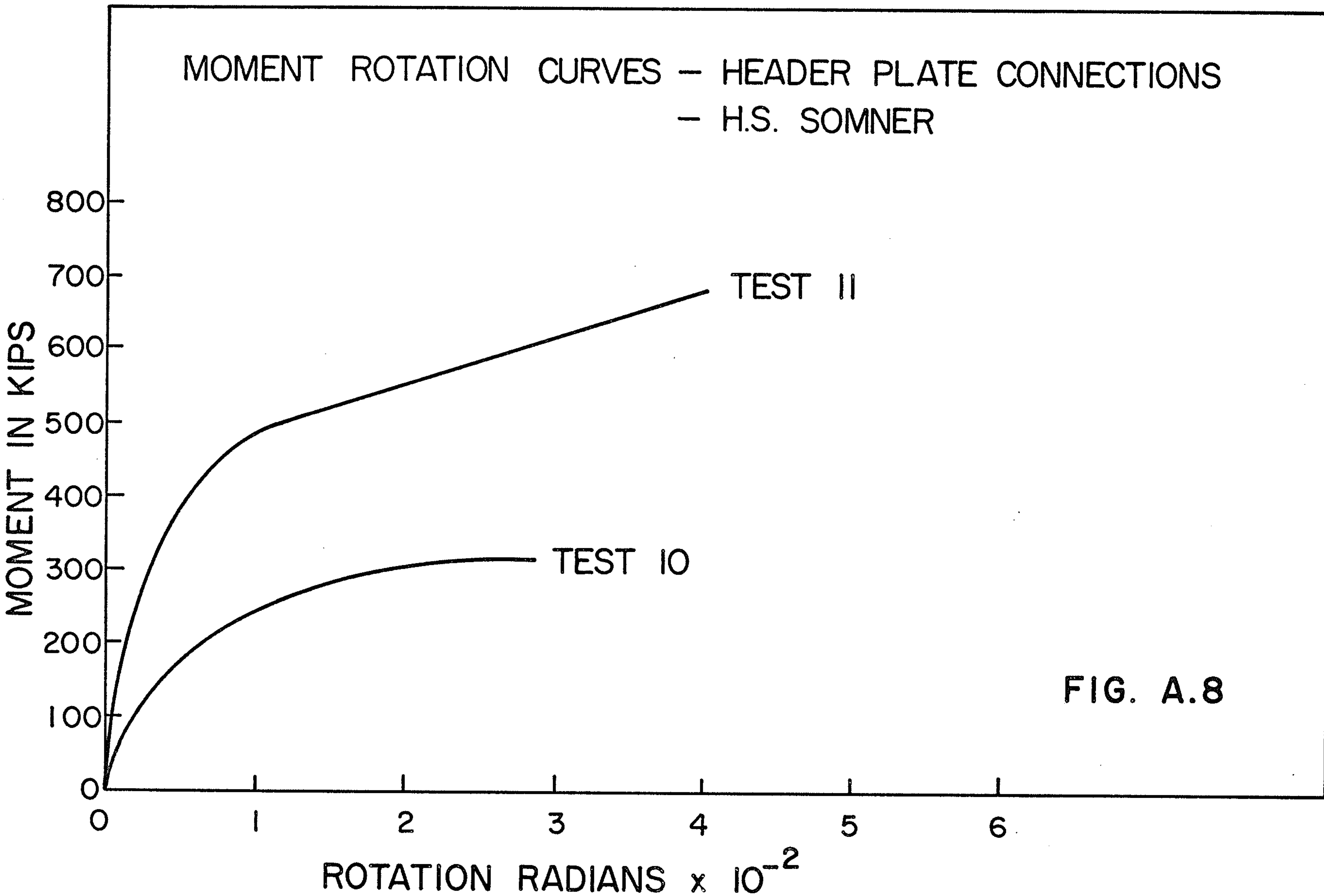

MOMENT ROTATION CURVES – HEADER PLATE CONNECTIONS
– H.S. SOMNER
MOMENT IN KIPS
800
700
600
500
400
300
200
100
0
TEST 8
TEST 7
TEST 6
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.7



MOMENT ROTATION CURVES – HEADER PLATE CONNECTIONS
– H.S. SOMNER
MOMENT IN KIPS
800
700
600
500
400
300
200
100
0
TEST 11
TEST 10
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.8

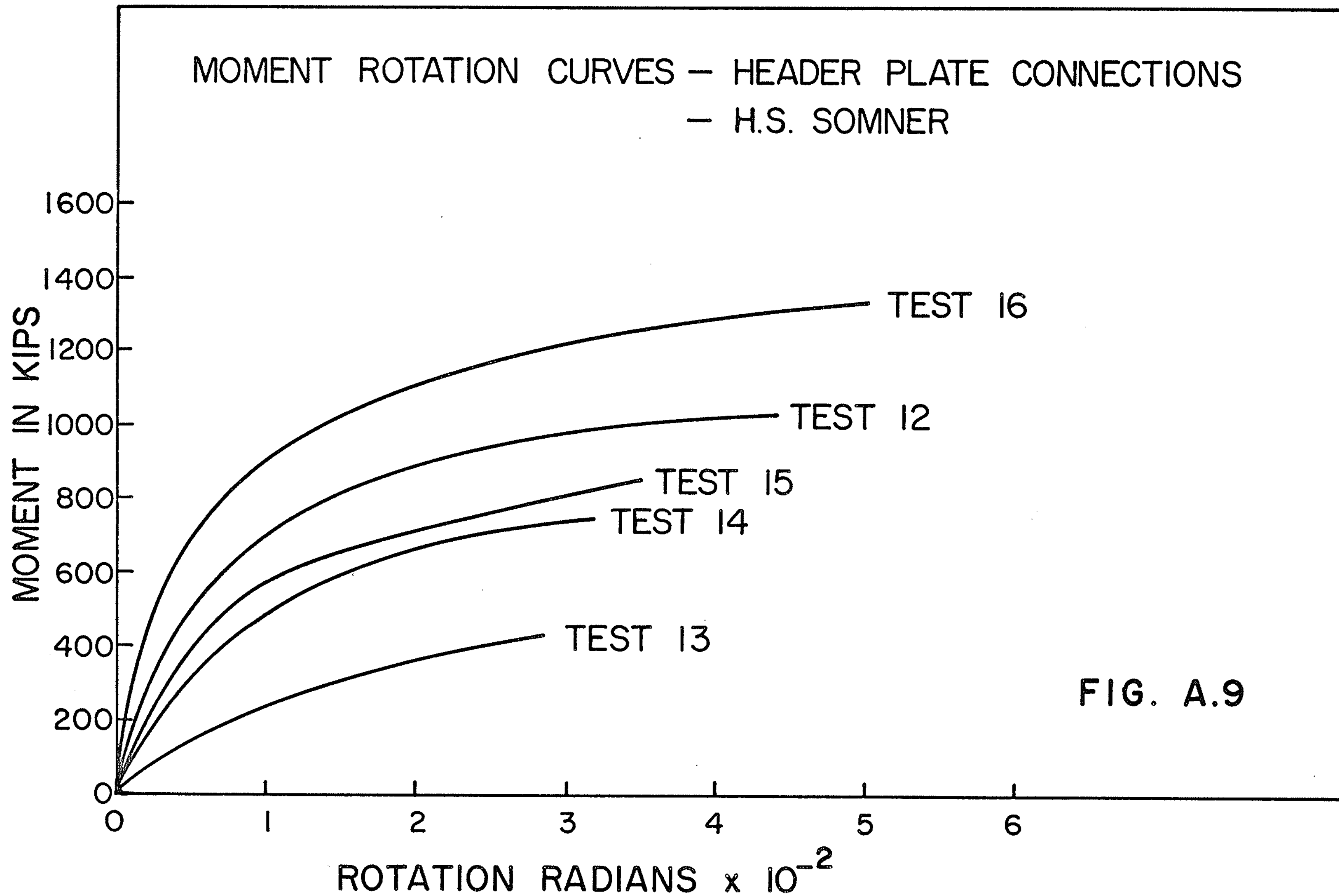

MOMENT ROTATION CURVES – HEADER PLATE CONNECTIONS
– H.S. SOMNER
MOMENT IN KIPS
0
200
400
600
800
1000
1200
1400
1600
ROTATION RADIANS x 10^{-2}
0
1
2
3
4
5
6
TEST 16
TEST 12
TEST 15
TEST 14
TEST 13

FIG. A.9

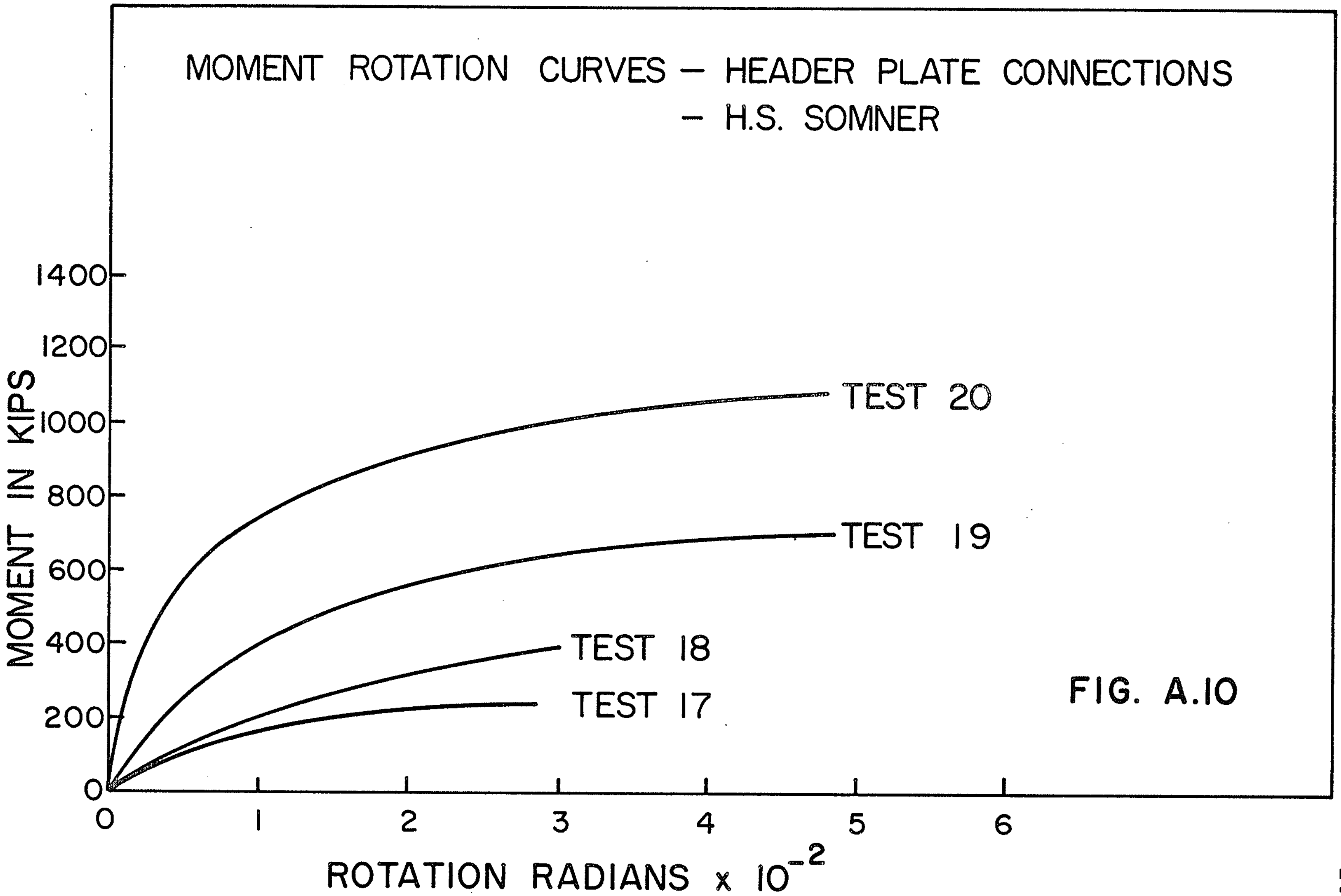

MOMENT ROTATION CURVES – HEADER PLATE CONNECTIONS
– H.S. SOMNER
MOMENT IN KIPS
1400
1200
1000
800
600
400
200
0
TEST 20
TEST 19
TEST 18
TEST 17
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.10

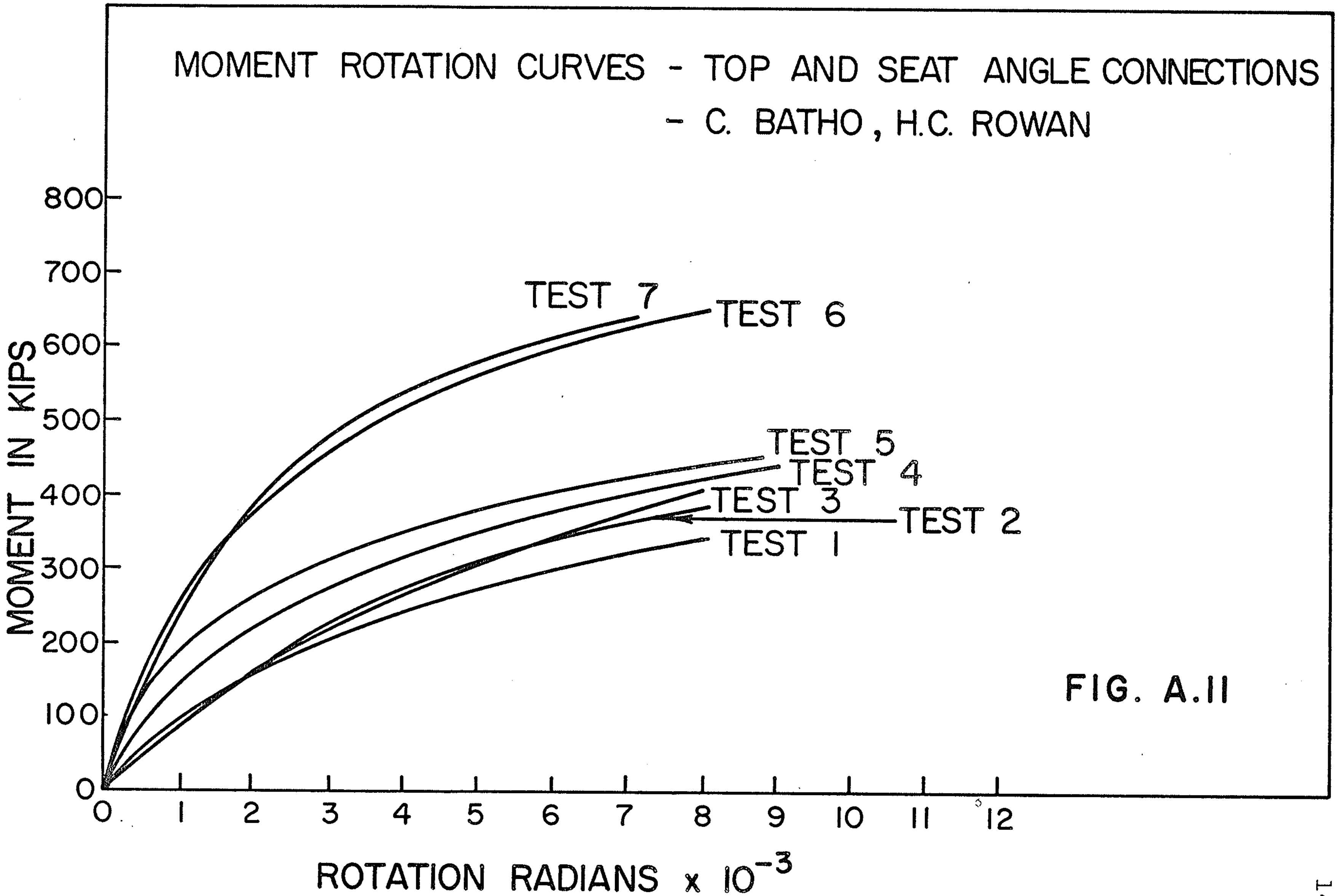

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- C. BATHO, H.C. ROWAN
MOMENT IN KIPS
0
100
200
300
400
500
600
700
800
TEST 7
TEST 6
TEST 5
TEST 4
TEST 3
TEST 2
TEST 1
0
1
2
3
4
5
6
7
8
9
10
11
12
ROTATION RADIANS x 10^{-3}

FIG. A.11

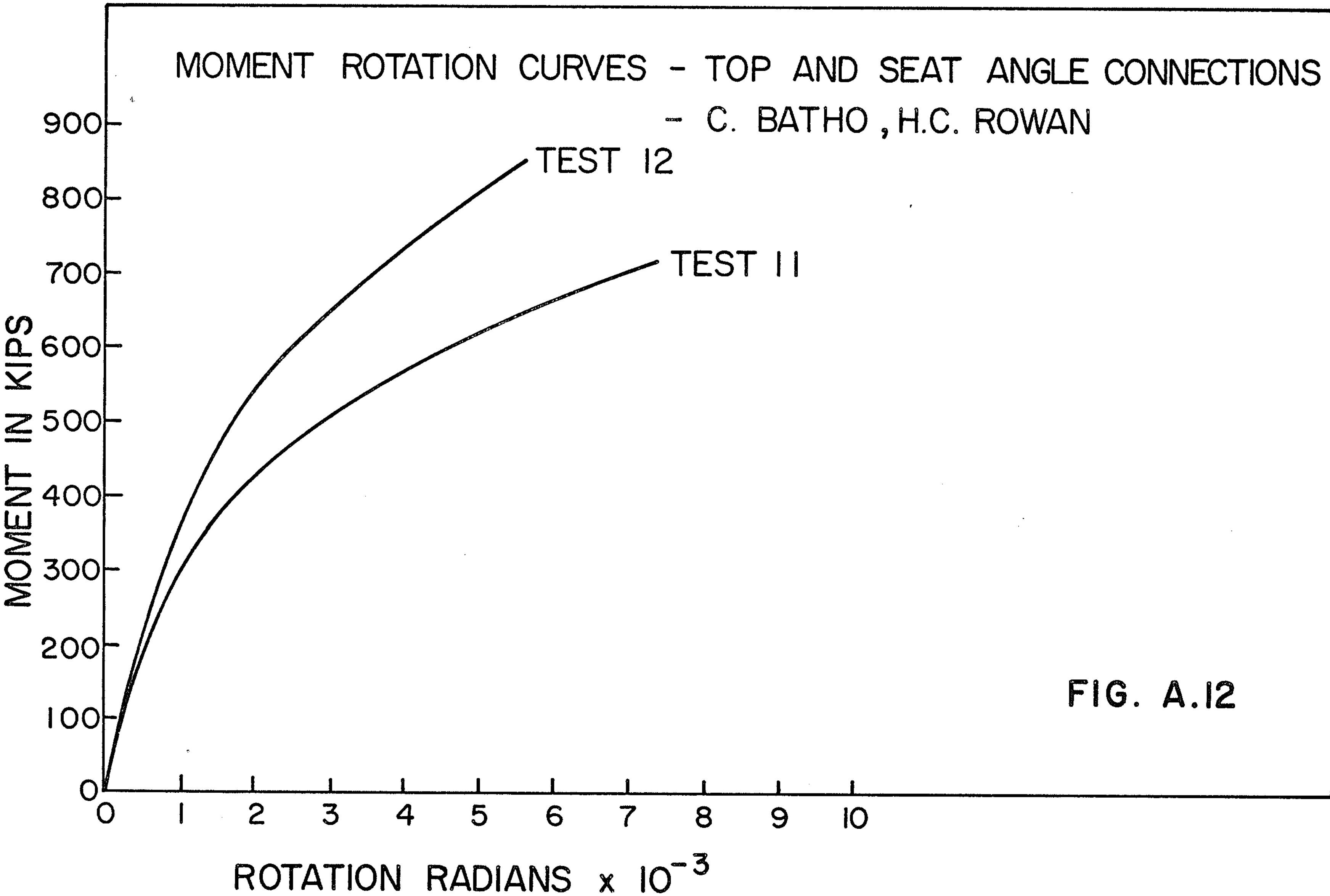
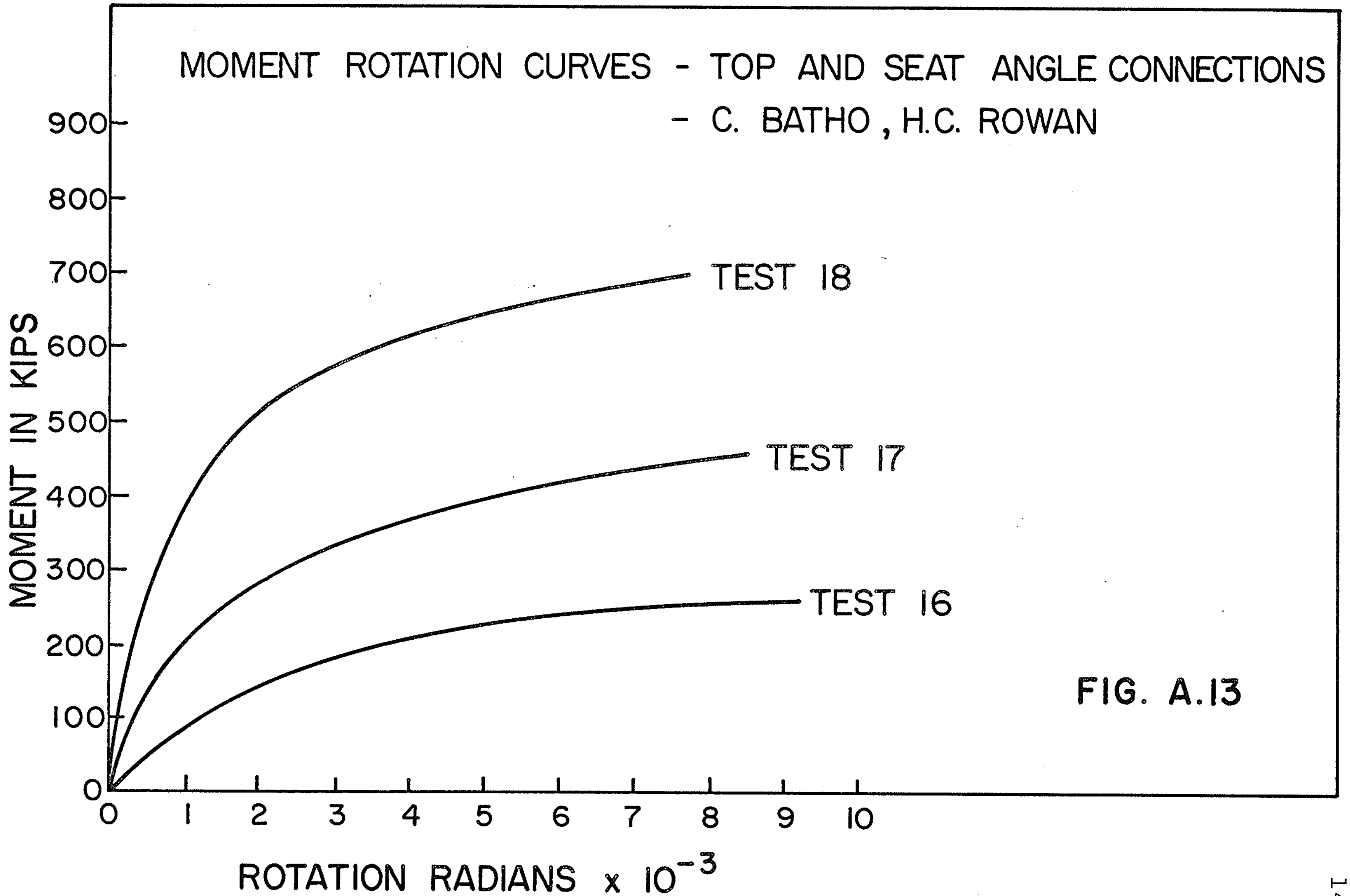

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- C. BATHO, H.C. ROWAN
TEST 12
TEST 11
MOMENT IN KIPS
0
100
200
300
400
500
600
700
800
900
0
1
2
3
4
5
6
7
8
9
10
ROTATION RADIANS x 10^{-3}

FIG. A.12


MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- C. BATHO , H.C. ROWAN
MOMENT IN KIPS
0
100
200
300
400
500
600
700
800
900
TEST 18
TEST 17
TEST 16
0
1
2
3
4
5
6
7
8
9
10
ROTATION RADIANS x 10^{-3}
FIG. A.13

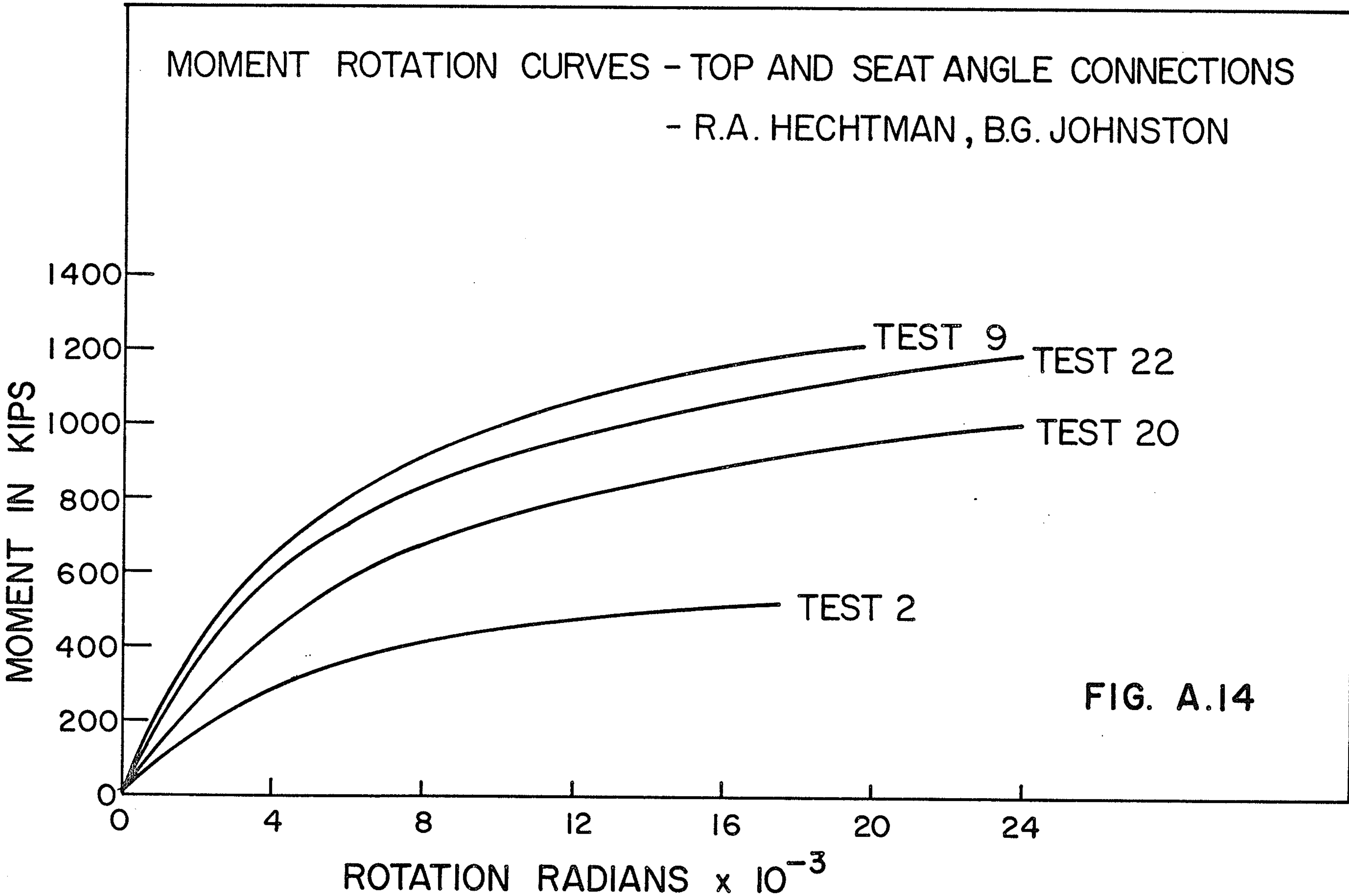

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- R.A. HECHTMAN, B.G. JOHNSTON
MOMENT IN KIPS
1400
1200
1000
800
600
400
200
0
0
4
8
12
16
20
24
ROTATION RADIANS x 10^{-3}
TEST 9
TEST 22
TEST 20
TEST 2

FIG. A.14

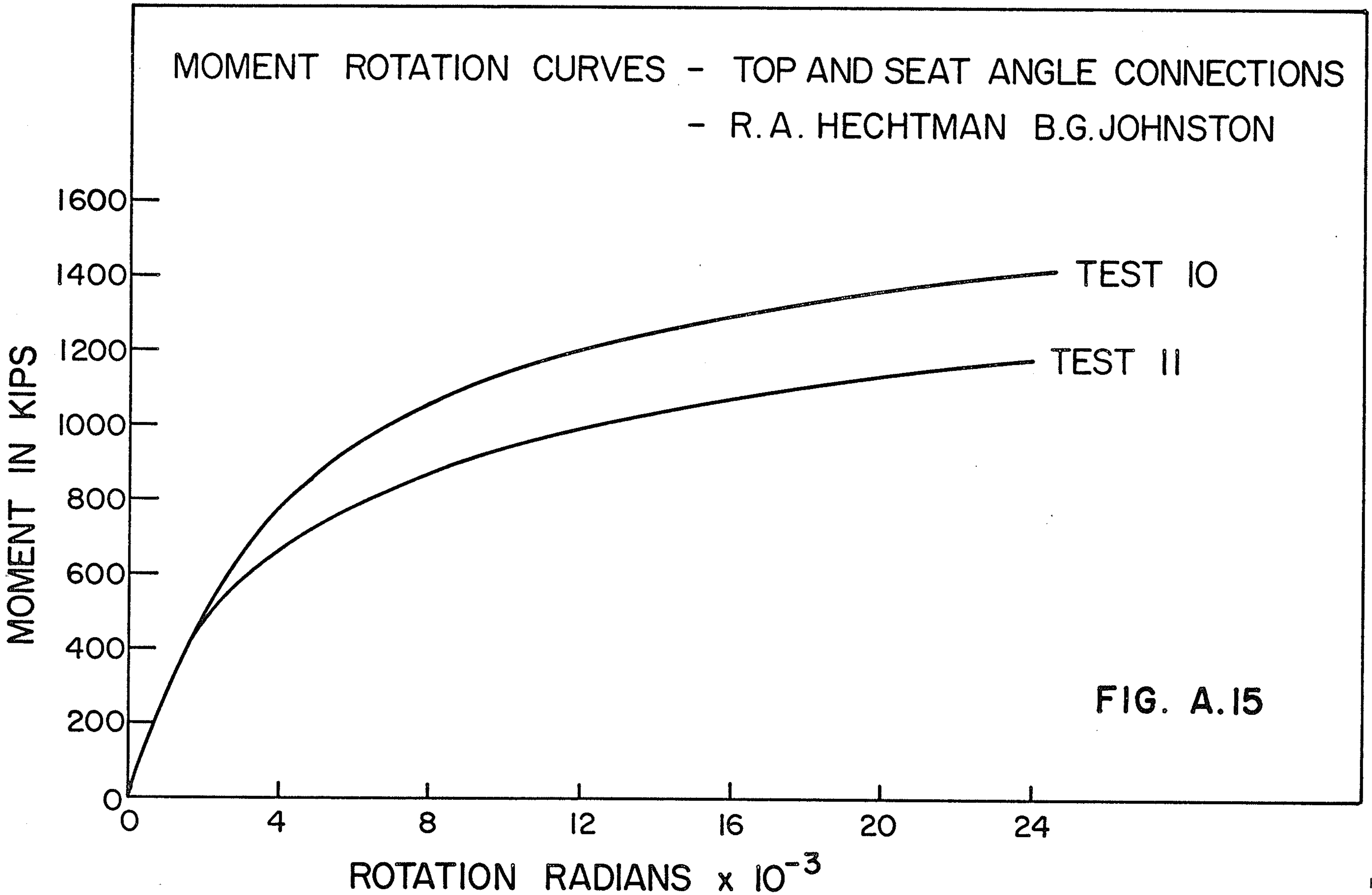

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- R.A. HECHTMAN B.G. JOHNSTON
MOMENT IN KIPS
0
200
400
600
800
1000
1200
1400
1600
TEST 10
TEST 11
0
4
8
12
16
20
24
ROTATION RADIANS x 10^{-3}

FIG. A.15

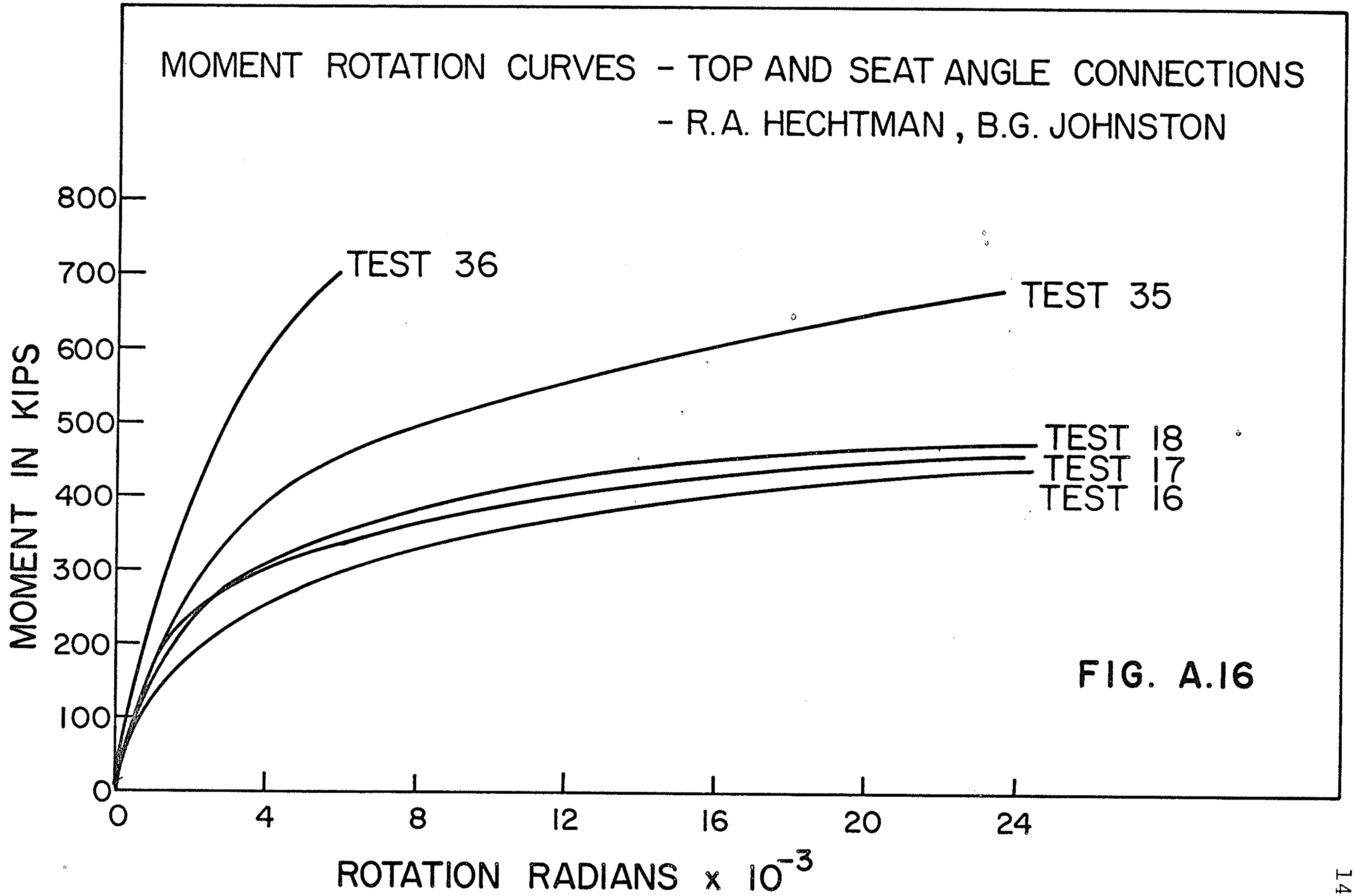

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- R.A. HECHTMAN, B.G. JOHNSTON
MOMENT IN KIPS
800
700
600
500
400
300
200
100
0
TEST 36
TEST 35
TEST 18
TEST 17
TEST 16
0
4
8
12
16
20
24
ROTATION RADIANS x 10^{-3}

FIG. A.16

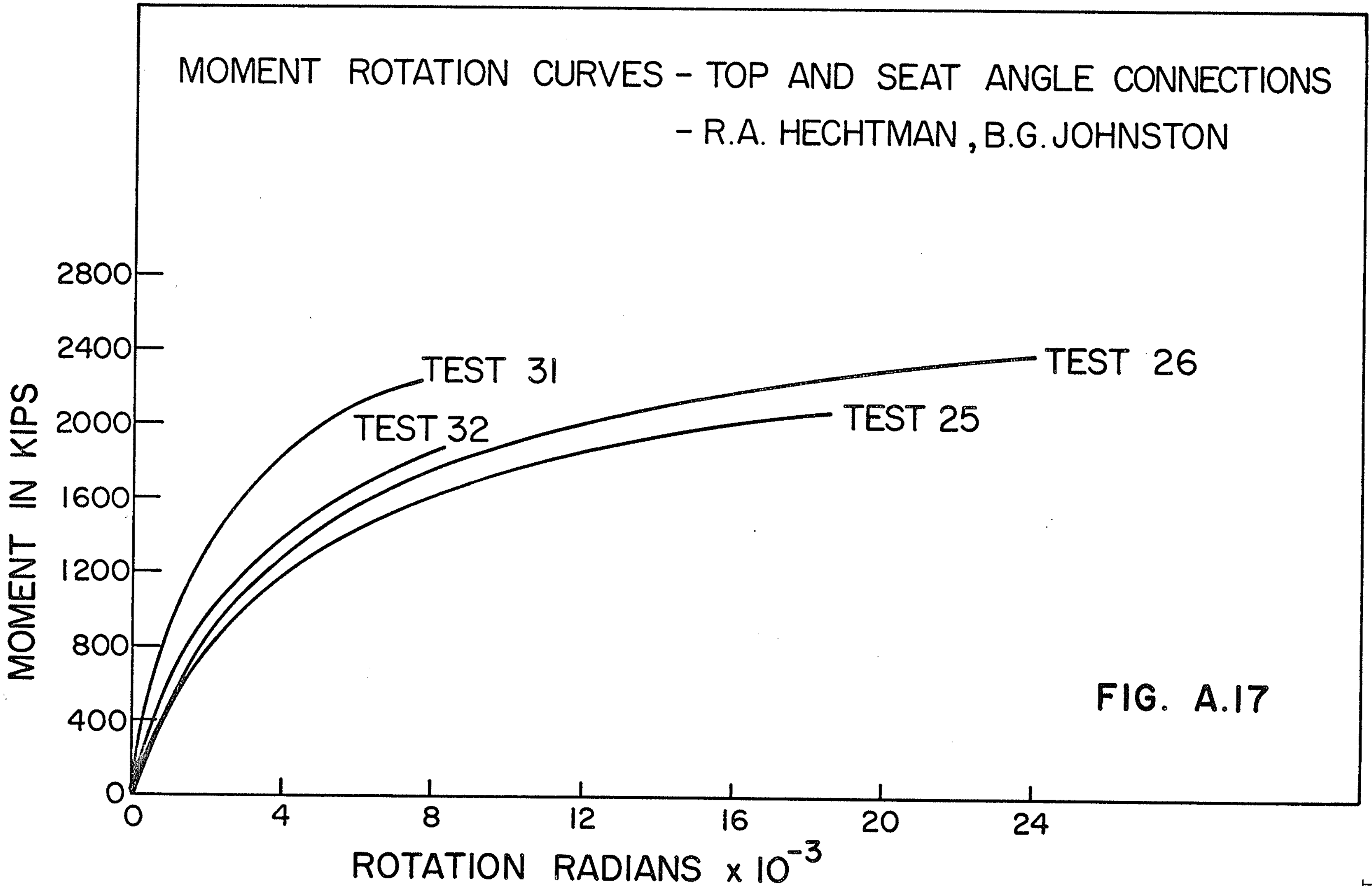

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- R.A. HECHTMAN, B.G. JOHNSTON
MOMENT IN KIPS
2800
2400
2000
1600
1200
800
400
0
0
4
8
12
16
20
24
ROTATION RADIANS x 10^{-3}
TEST 31
TEST 32
TEST 26
TEST 25

FIG. A.17

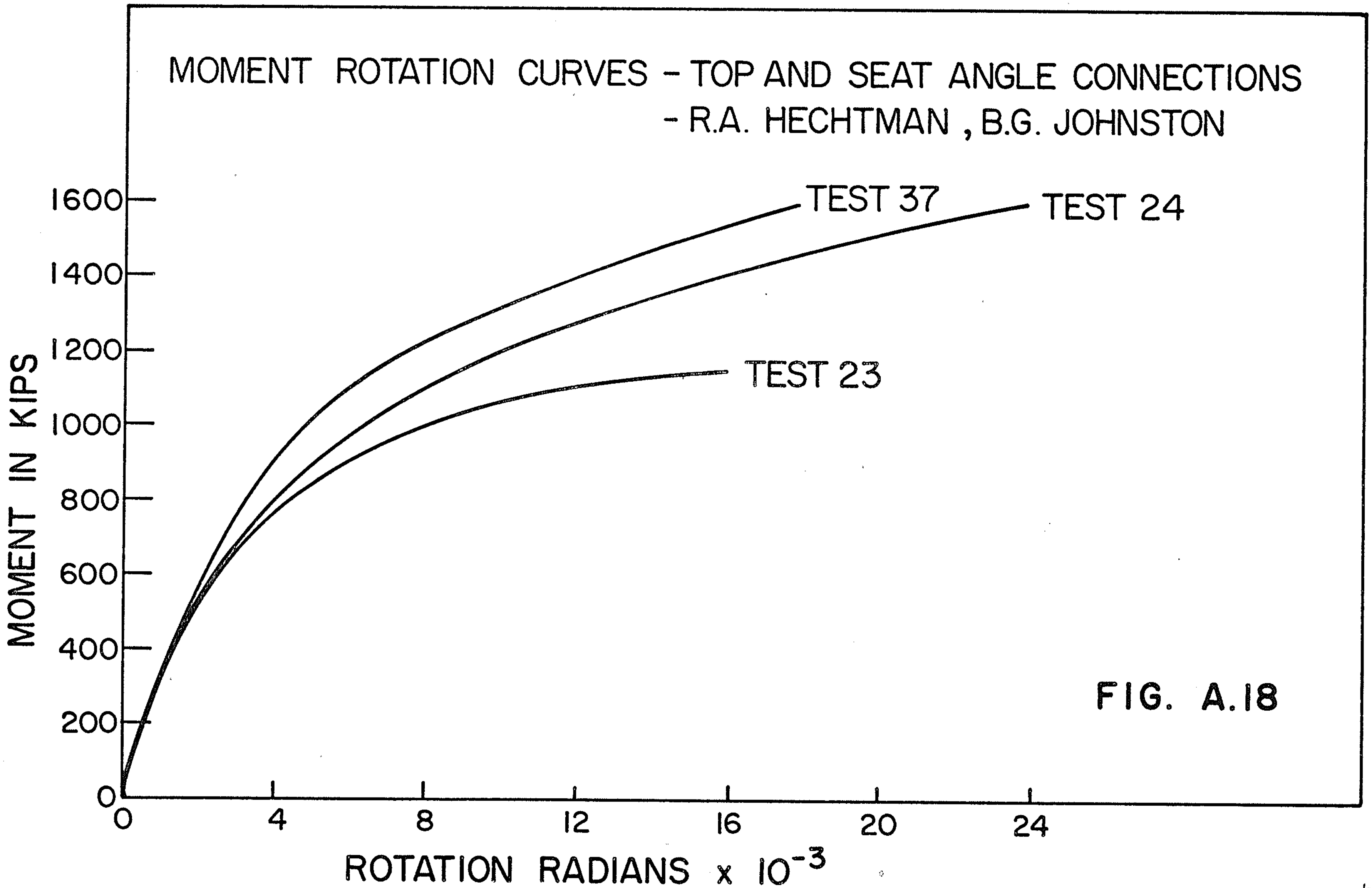

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- R.A. HECHTMAN , B.G. JOHNSTON
TEST 37
TEST 24
TEST 23
MOMENT IN KIPS
0
200
400
600
800
1000
1200
1400
1600
0
4
8
12
16
20
24
ROTATION RADIANS x 10^{-3}

FIG. A.18

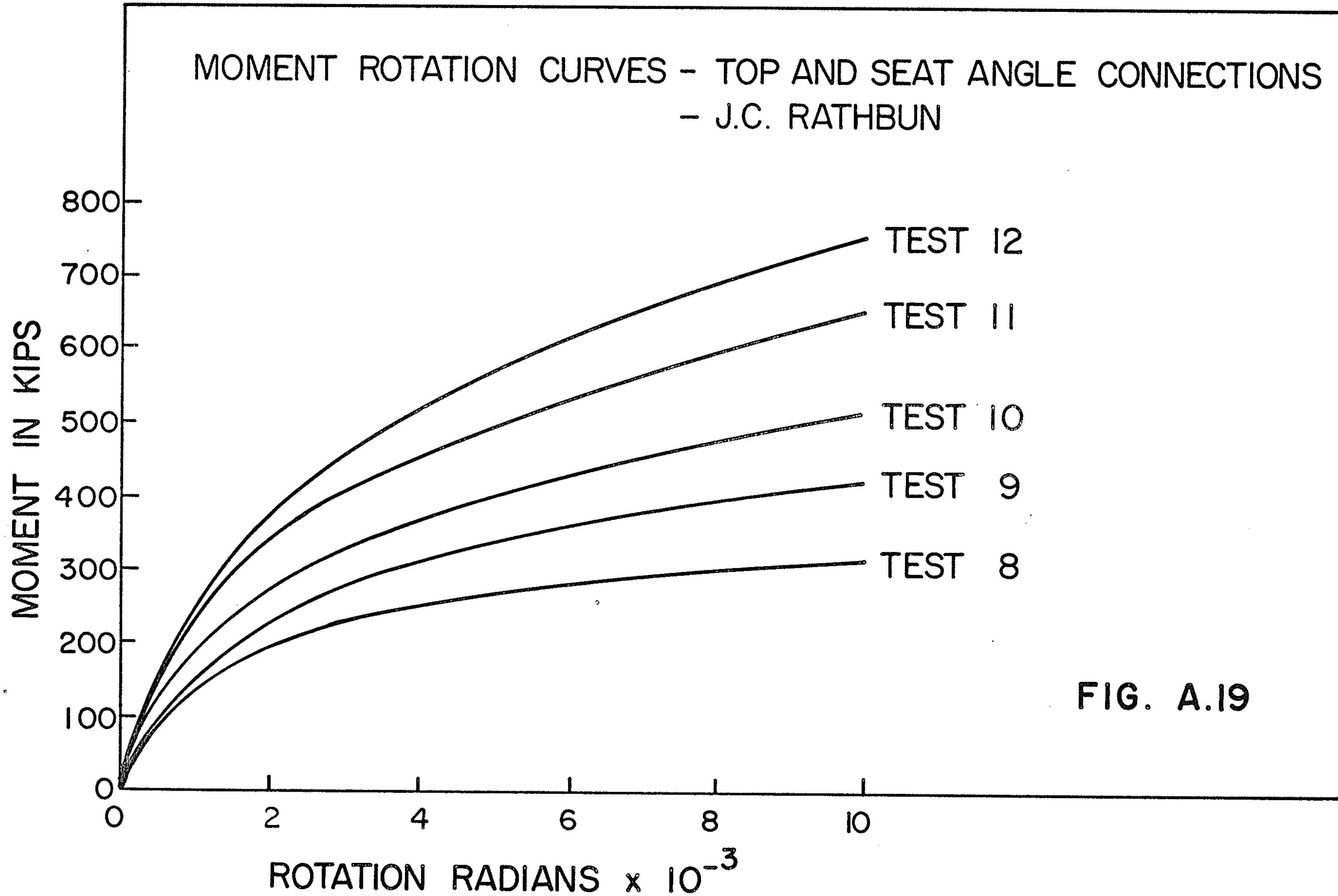

MOMENT ROTATION CURVES - TOP AND SEAT ANGLE CONNECTIONS
- J.C. RATHBUN
MOMENT IN KIPS
0
100
200
300
400
500
600
700
800
ROTATION RADIANS x 10^{-3}
0
2
4
6
8
10
TEST 12
TEST 11
TEST 10
TEST 9
TEST 8

FIG. A.19

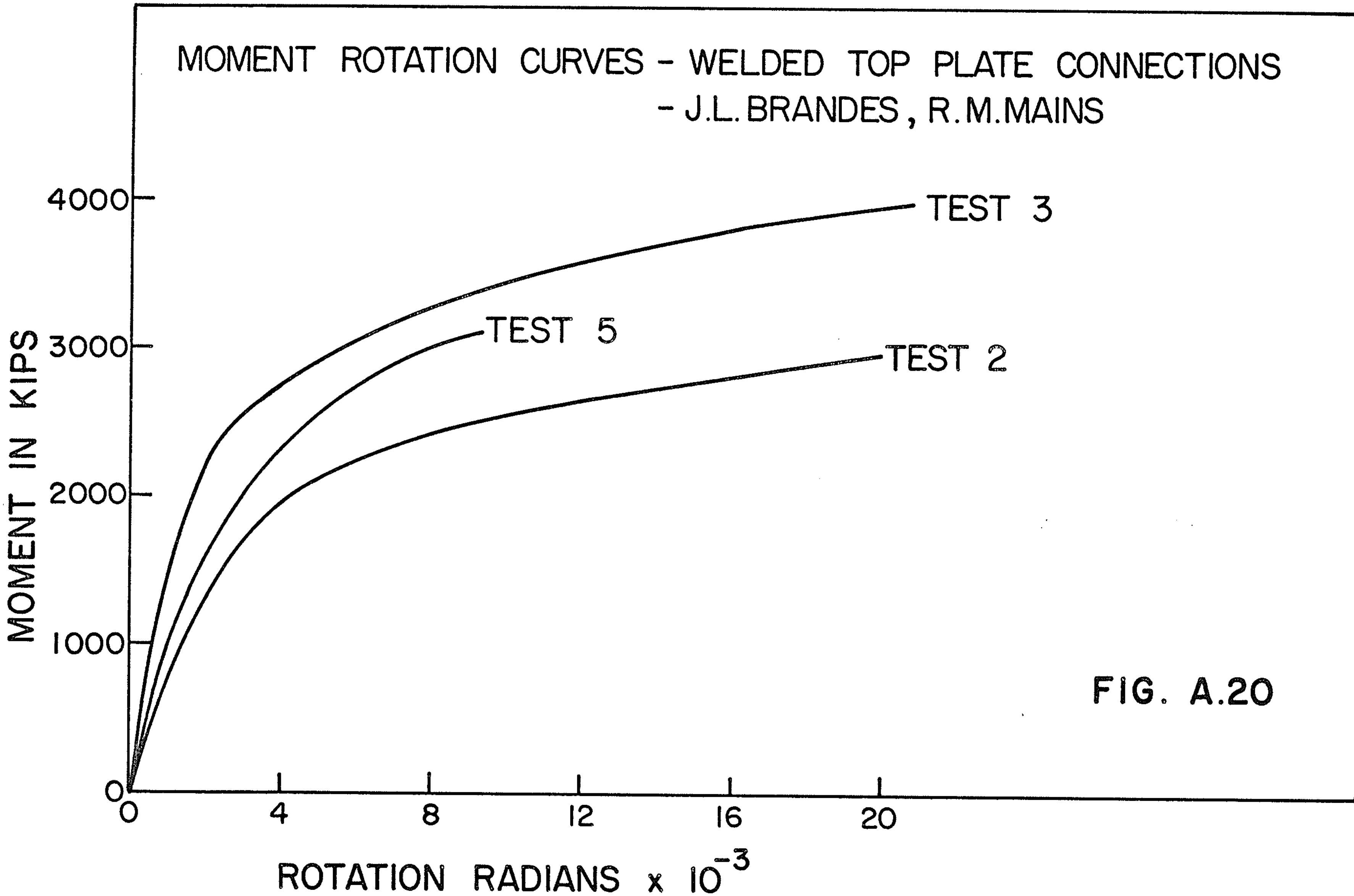

MOMENT ROTATION CURVES - WELDED TOP PLATE CONNECTIONS
- J.L. BRANDES, R.M. MAINS
MOMENT IN KIPS
4000
3000
2000
1000
0
ROTATION RADIANS x 10^{-3}
0
4
8
12
16
20
TEST 3
TEST 5
TEST 2

FIG. A.20

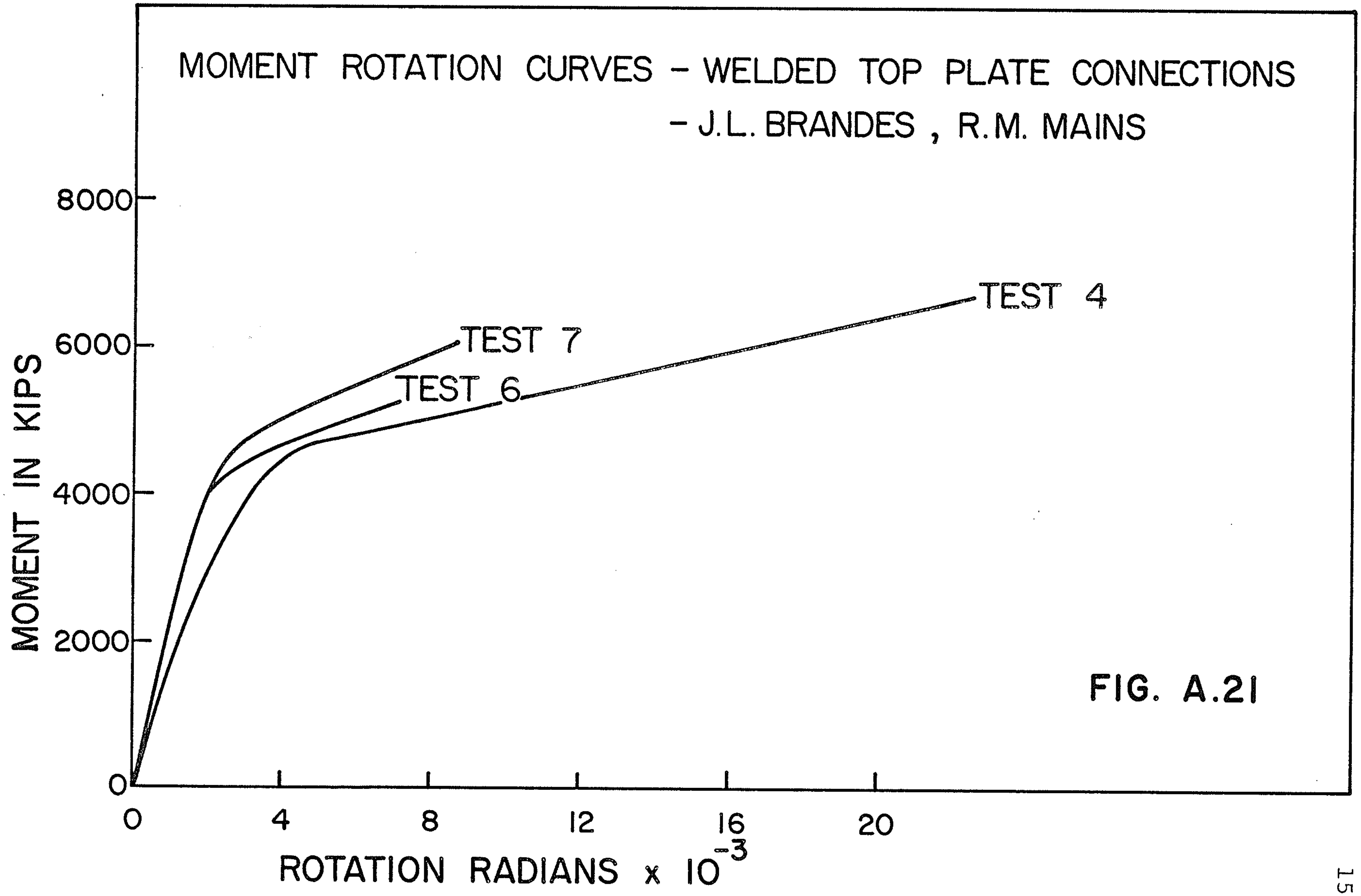

MOMENT ROTATION CURVES - WELDED TOP PLATE CONNECTIONS
- J.L. BRANDES , R.M. MAINS
TEST 7
TEST 6
TEST 4
MOMENT IN KIPS
8000
6000
4000
2000
0
0
4
8
12
16
20
ROTATION RADIANS x 10^-3

FIG. A.21

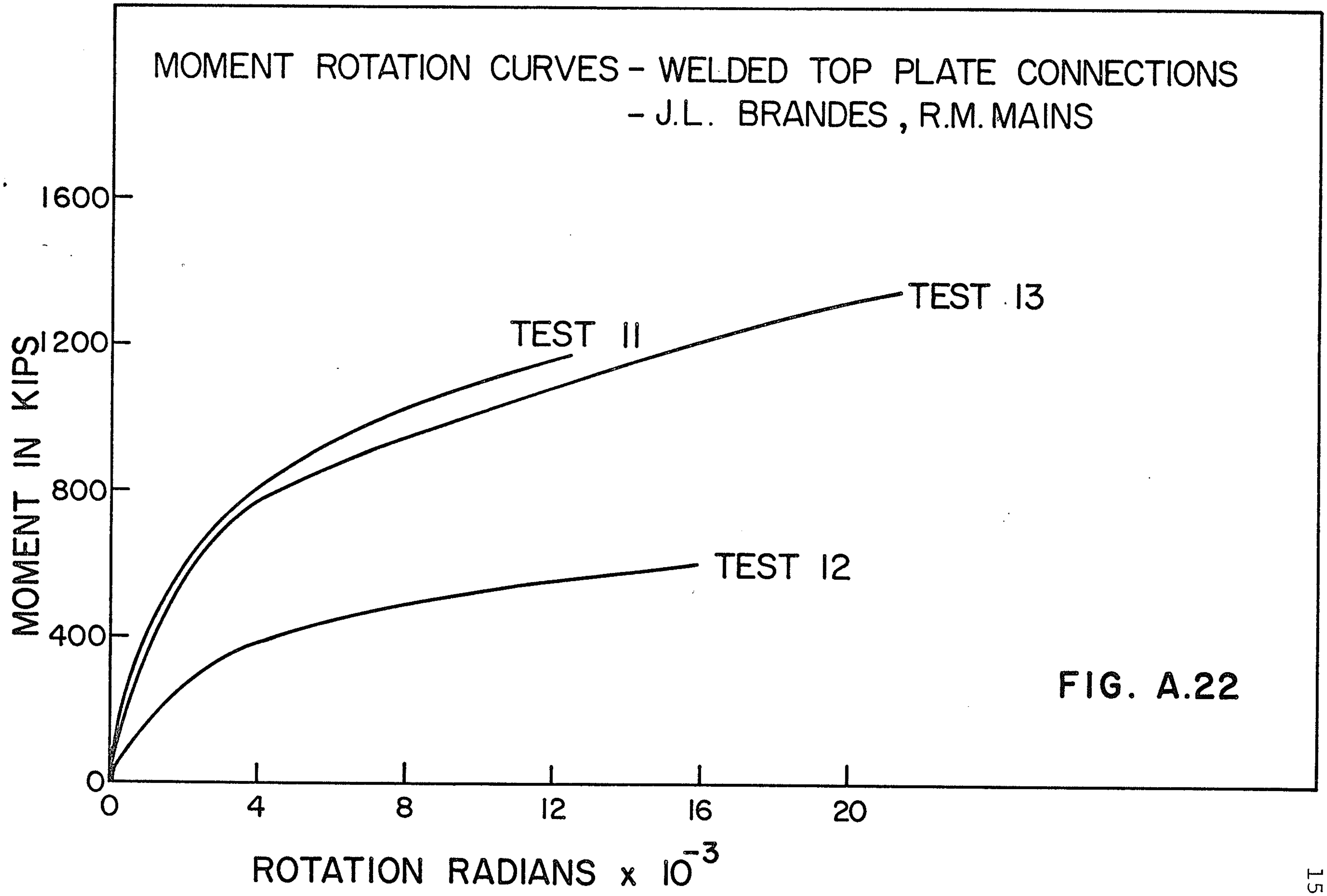

MOMENT ROTATION CURVES - WELDED TOP PLATE CONNECTIONS
- J.L. BRANDES, R.M. MAINS
TEST 11
TEST 13
TEST 12
1600
1200
800
400
0
0
4
8
12
16
20
MOMENT IN KIPS
ROTATION RADIANS x 10^{-3}

FIG. A.22

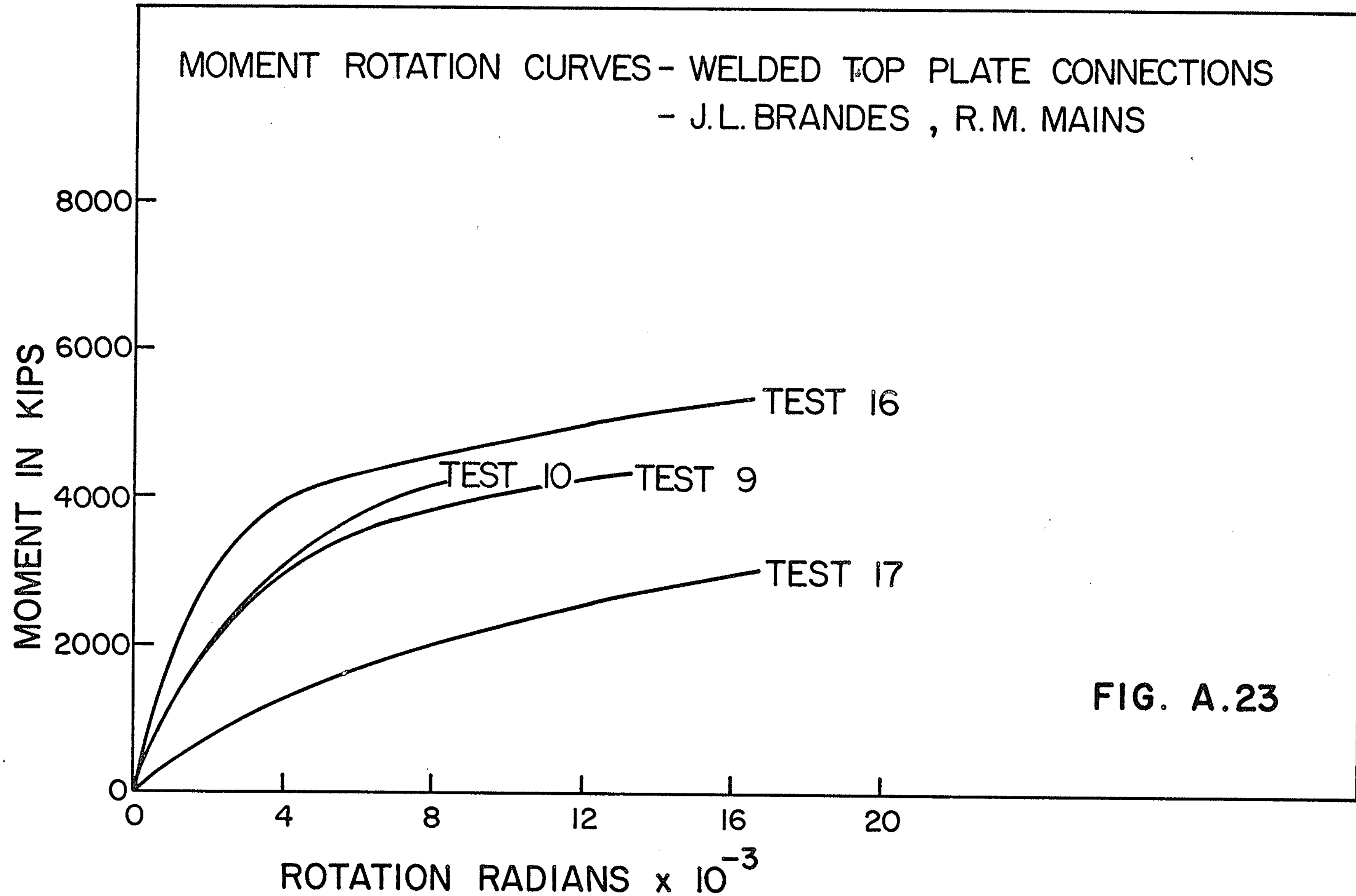
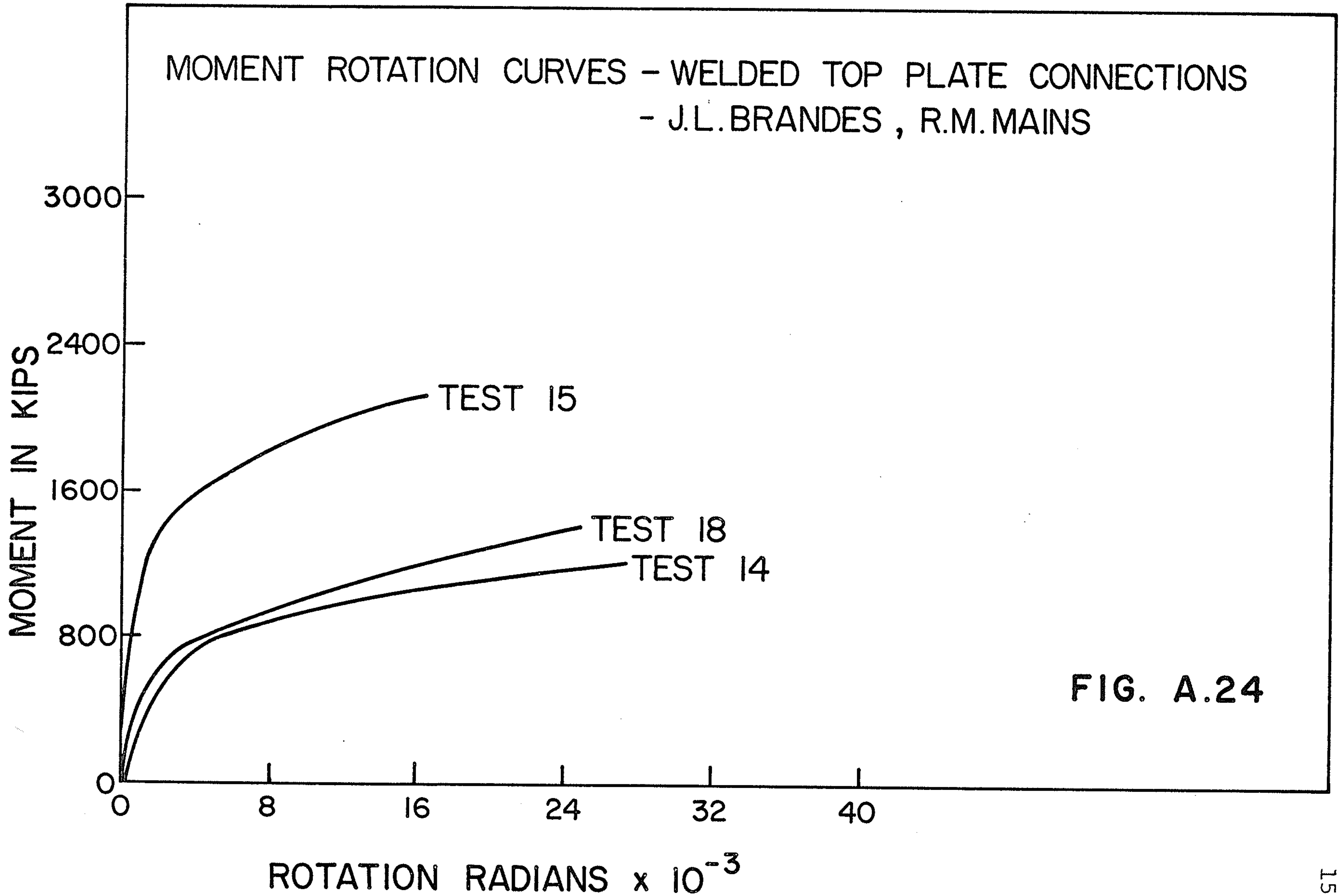

MOMENT ROTATION CURVES - WELDED TOP PLATE CONNECTIONS
- J.L. BRANDES , R.M. MAINS
8000
6000
4000
2000
0
MOMENT IN KIPS
TEST 16
TEST 10
TEST 9
TEST 17
0
4
8
12
16
20
ROTATION RADIANS x 10^{-3}

FIG. A.23



MOMENT ROTATION CURVES - WELDED TOP PLATE CONNECTIONS - J.L. BRANDES, R.M. MAINS

FIG. A.24

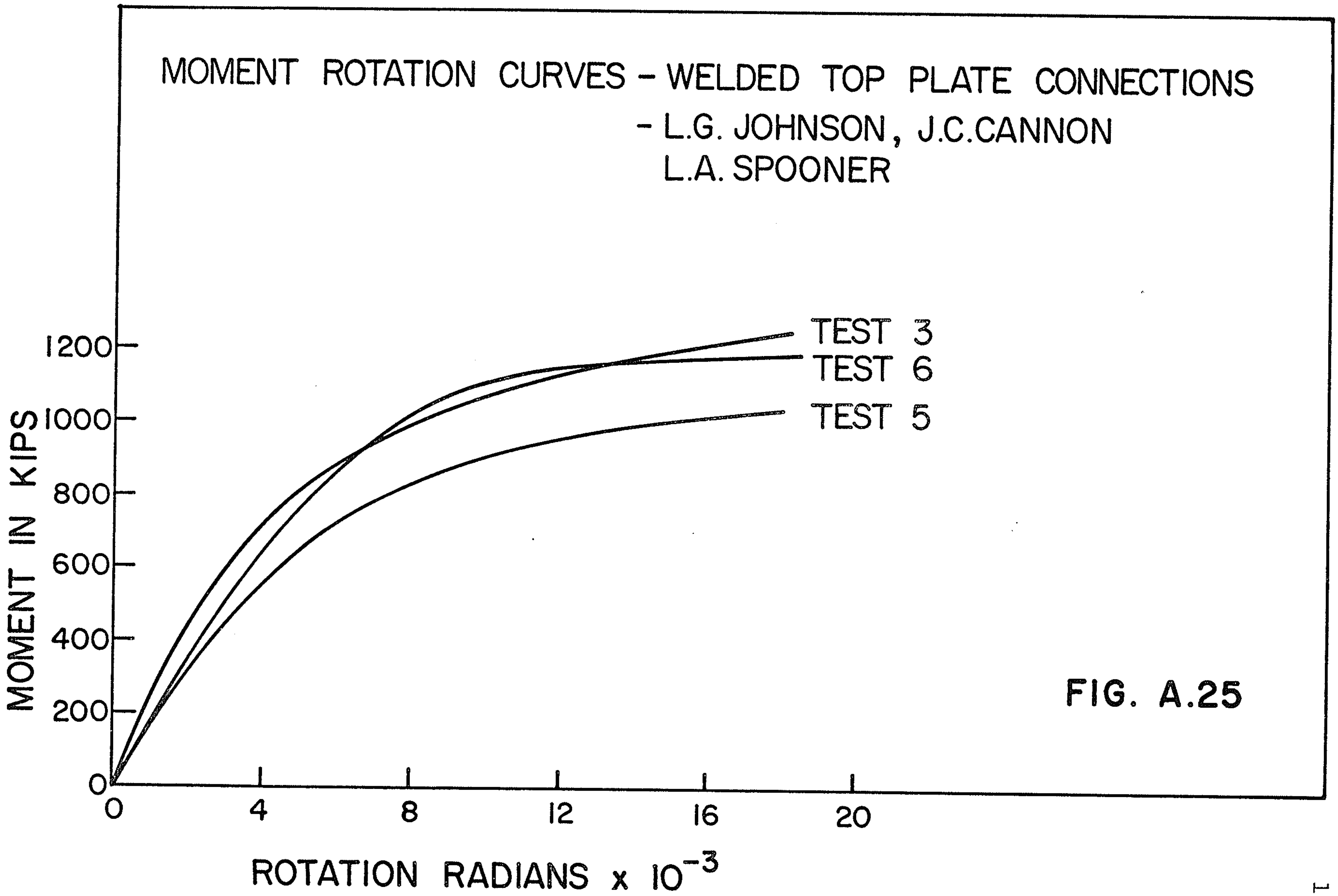

MOMENT ROTATION CURVES - WELDED TOP PLATE CONNECTIONS
- L.G. JOHNSON, J.C. CANNON
L.A. SPOONER
TEST 3
TEST 6
TEST 5
MOMENT IN KIPS
0
200
400
600
800
1000
1200
0
4
8
12
16
20
ROTATION RADIANS x 10^{-3}

FIG. A.25

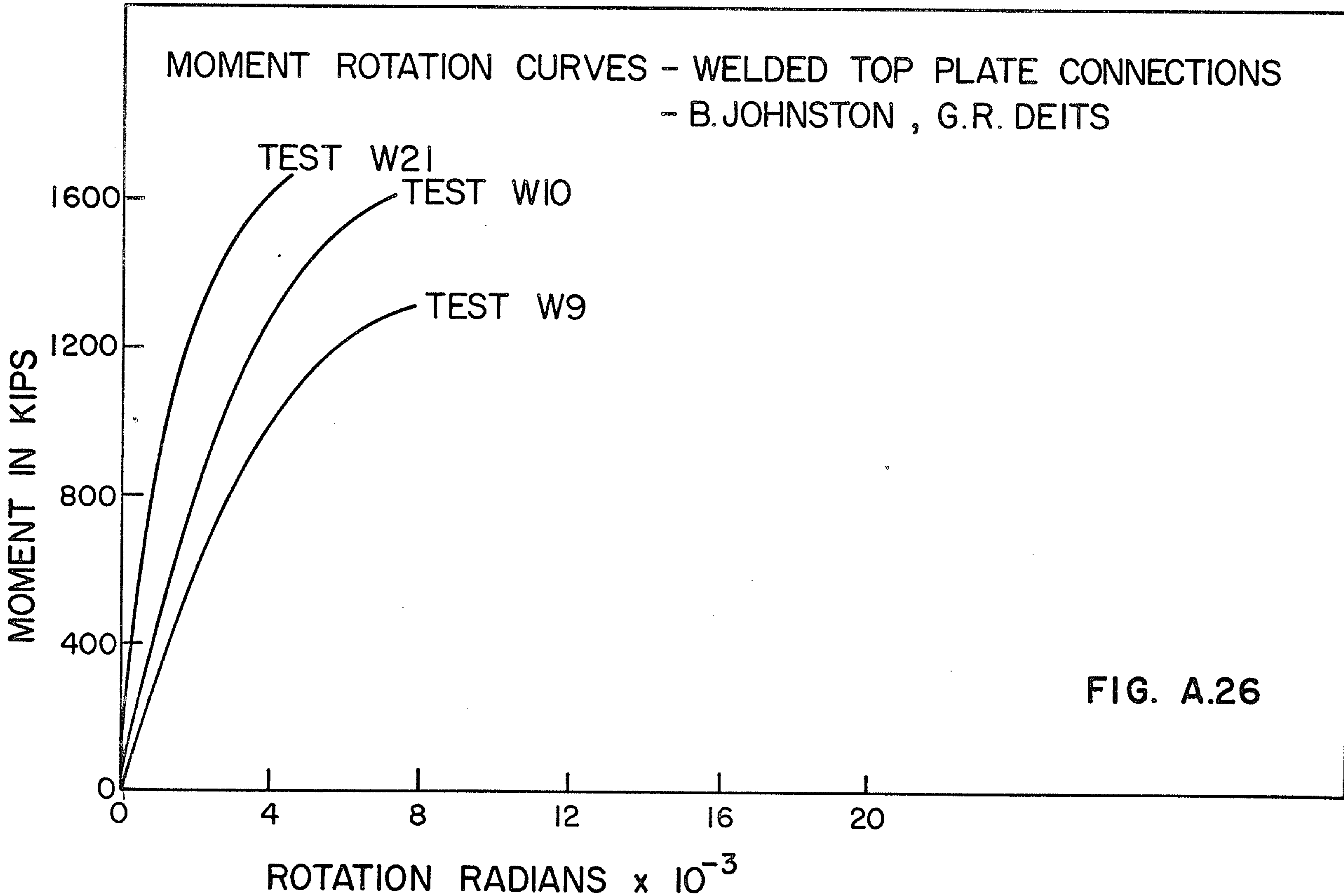

MOMENT ROTATION CURVES - WELDED TOP PLATE CONNECTIONS
- B. JOHNSTON , G.R. DEITS
TEST W21
TEST W10
TEST W9
MOMENT IN KIPS
1600
1200
800
400
0
0
4
8
12
16
20
ROTATION RADIANS x 10^{-3}

FIG. A.26

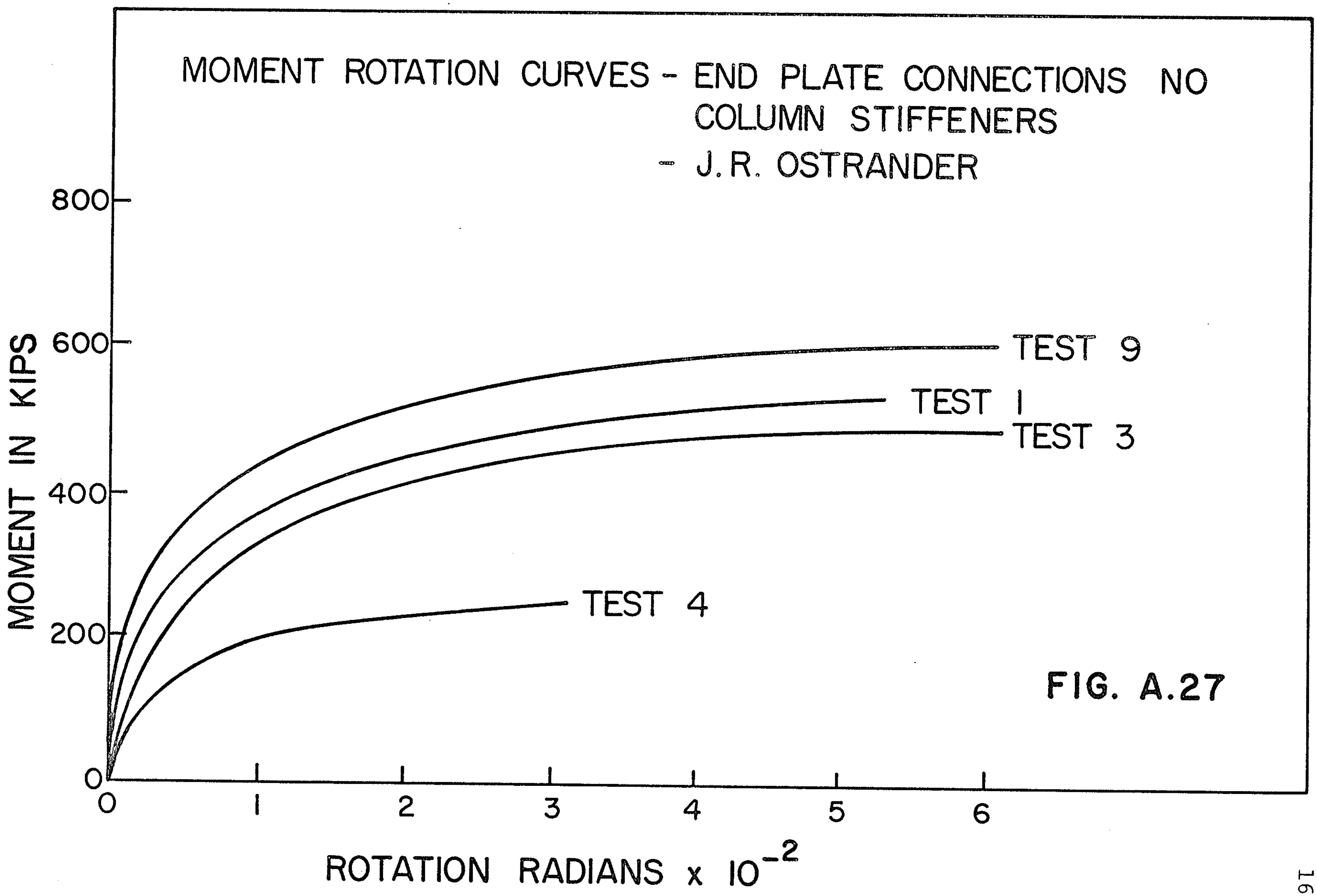

MOMENT ROTATION CURVES - END PLATE CONNECTIONS NO COLUMN STIFFENERS
- J.R. OSTRANDER
MOMENT IN KIPS
800
600
400
200
0
TEST 9
TEST 1
TEST 3
TEST 4
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.27

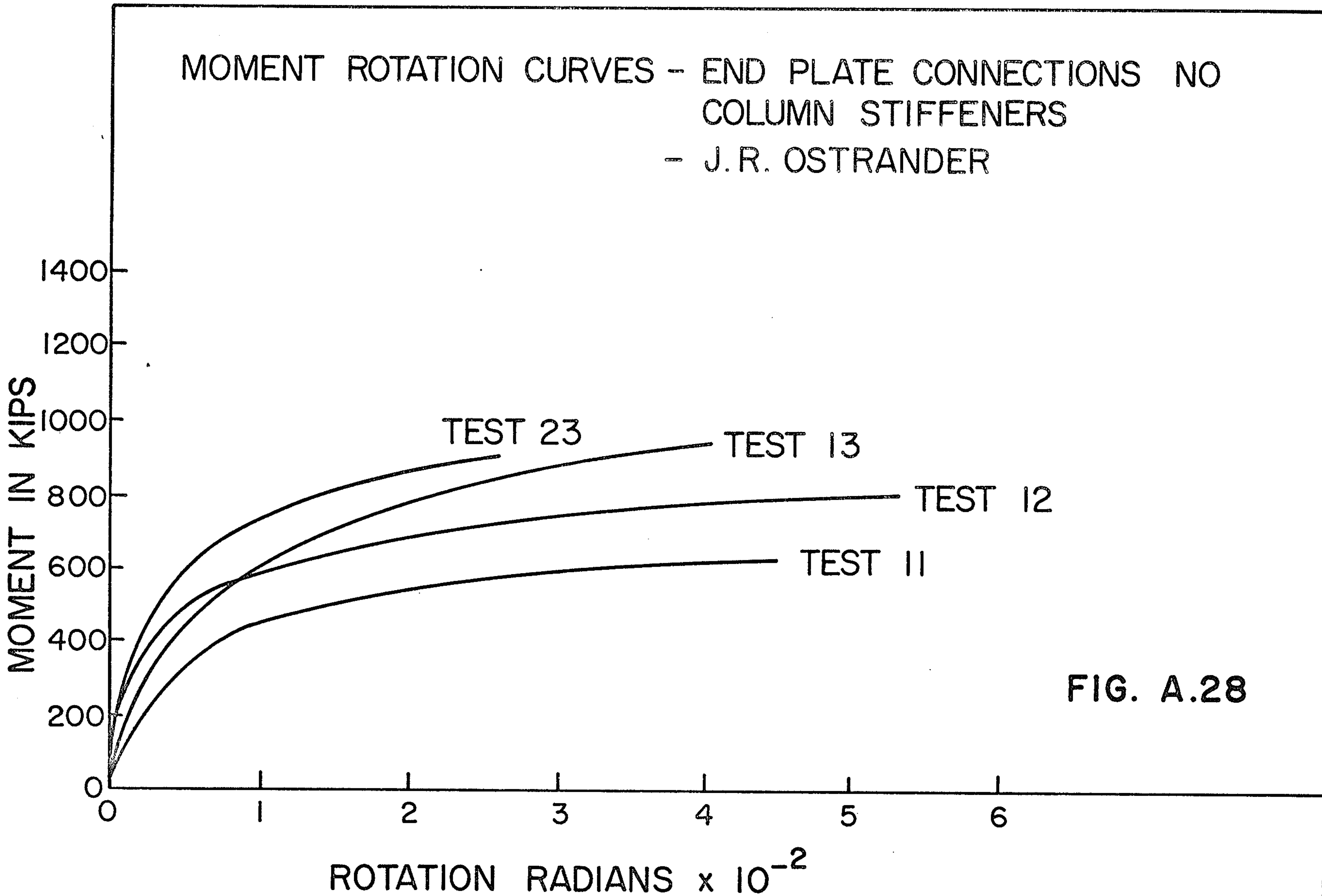

MOMENT ROTATION CURVES - END PLATE CONNECTIONS NO COLUMN STIFFENERS
- J.R. OSTRANDER
MOMENT IN KIPS
0
200
400
600
800
1000
1200
1400
ROTATION RADIANS x 10^{-2}
0
1
2
3
4
5
6
TEST 23
TEST 13
TEST 12
TEST 11

FIG. A.28

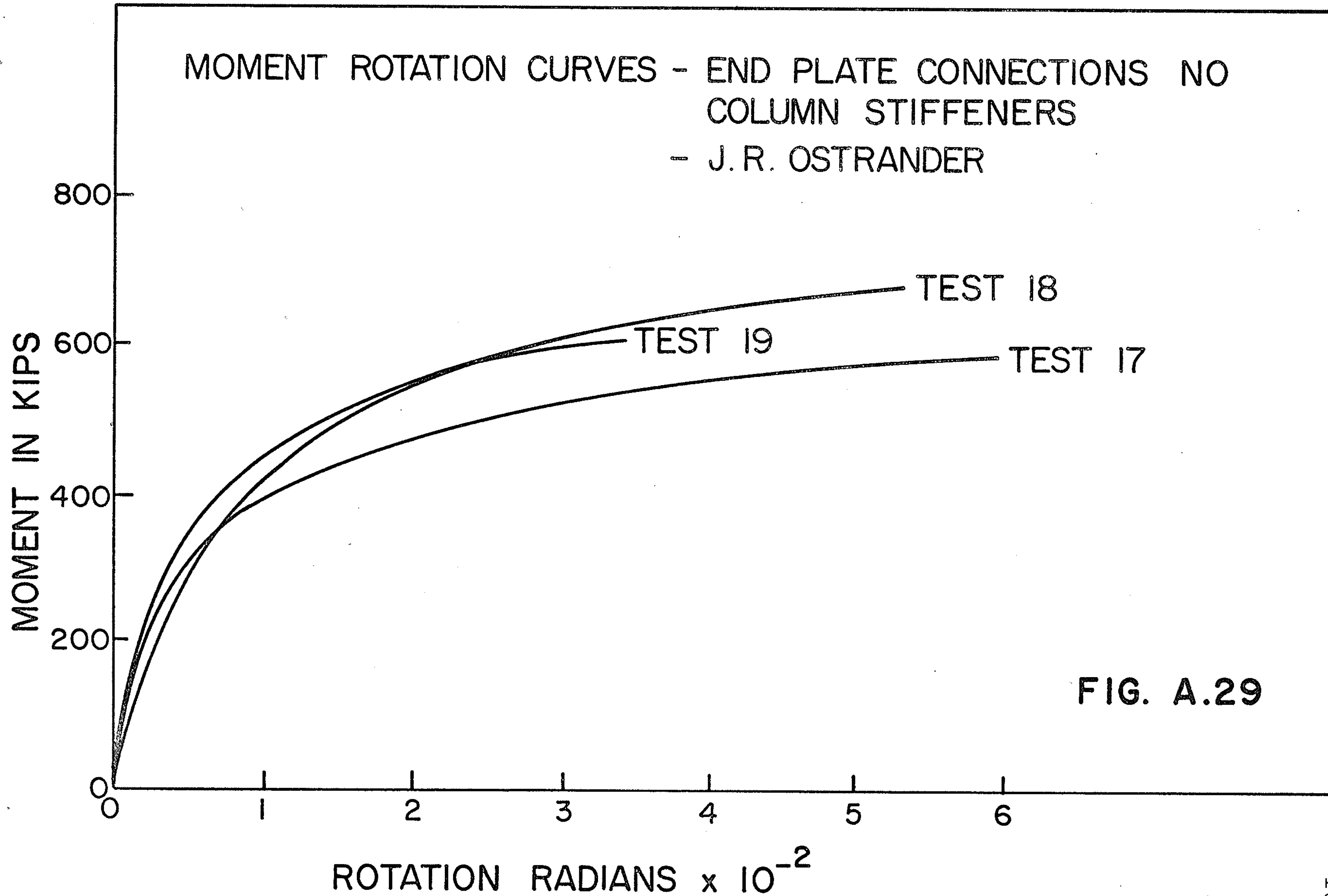

MOMENT ROTATION CURVES - END PLATE CONNECTIONS NO COLUMN STIFFENERS
- J. R. OSTRANDER
MOMENT IN KIPS
800
600
400
200
0
TEST 18
TEST 19
TEST 17
0
1
2
3
4
5
6
ROTATION RADIANS x 10^-2

FIG. A.29

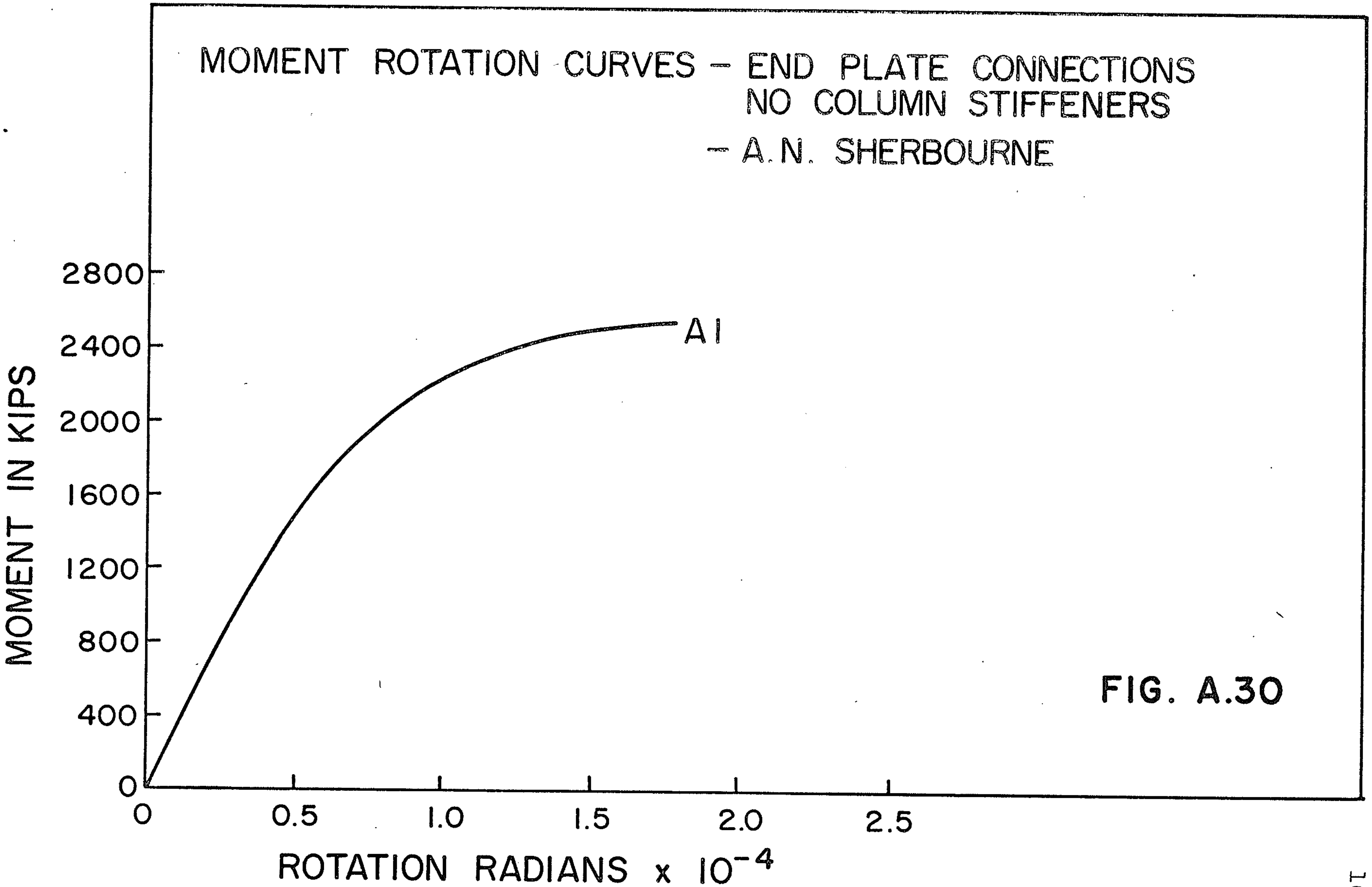

MOMENT ROTATION CURVES – END PLATE CONNECTIONS
NO COLUMN STIFFENERS
– A.N. SHERBOURNE
MOMENT IN KIPS
0
400
800
1200
1600
2000
2400
2800
A1
0
0.5
1.0
1.5
2.0
2.5
ROTATION RADIANS x 10^{-4}

FIG. A.30

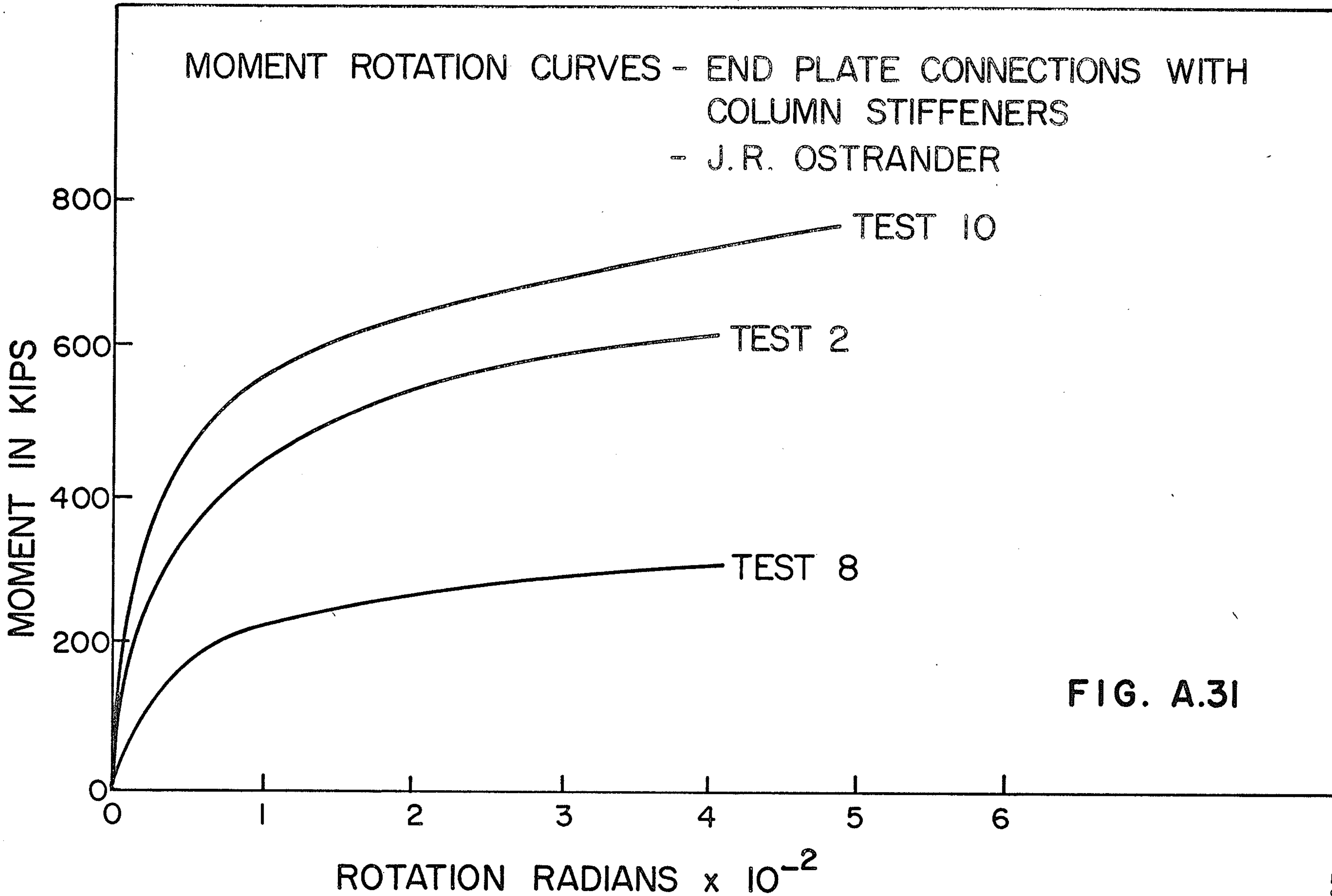

MOMENT ROTATION CURVES - END PLATE CONNECTIONS WITH COLUMN STIFFENERS
- J.R. OSTRANDER
MOMENT IN KIPS
800
600
400
200
0
TEST 10
TEST 2
TEST 8
0
1
2
3
4
5
6
ROTATION RADIANS x 10^-2

FIG. A.31

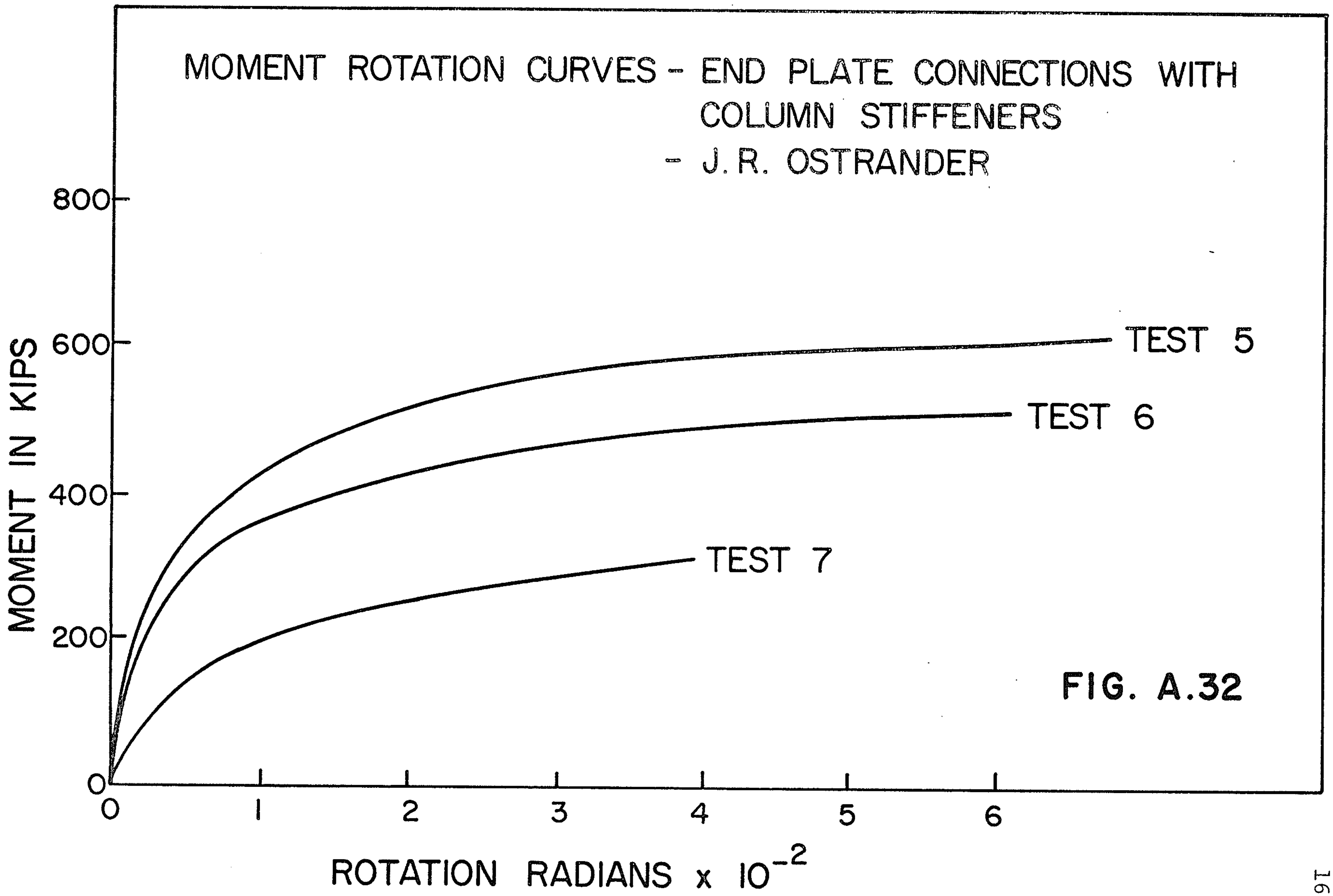

MOMENT ROTATION CURVES - END PLATE CONNECTIONS WITH COLUMN STIFFENERS
- J.R. OSTRANDER
MOMENT IN KIPS
800
600
400
200
0
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}
TEST 5
TEST 6
TEST 7

FIG. A.32

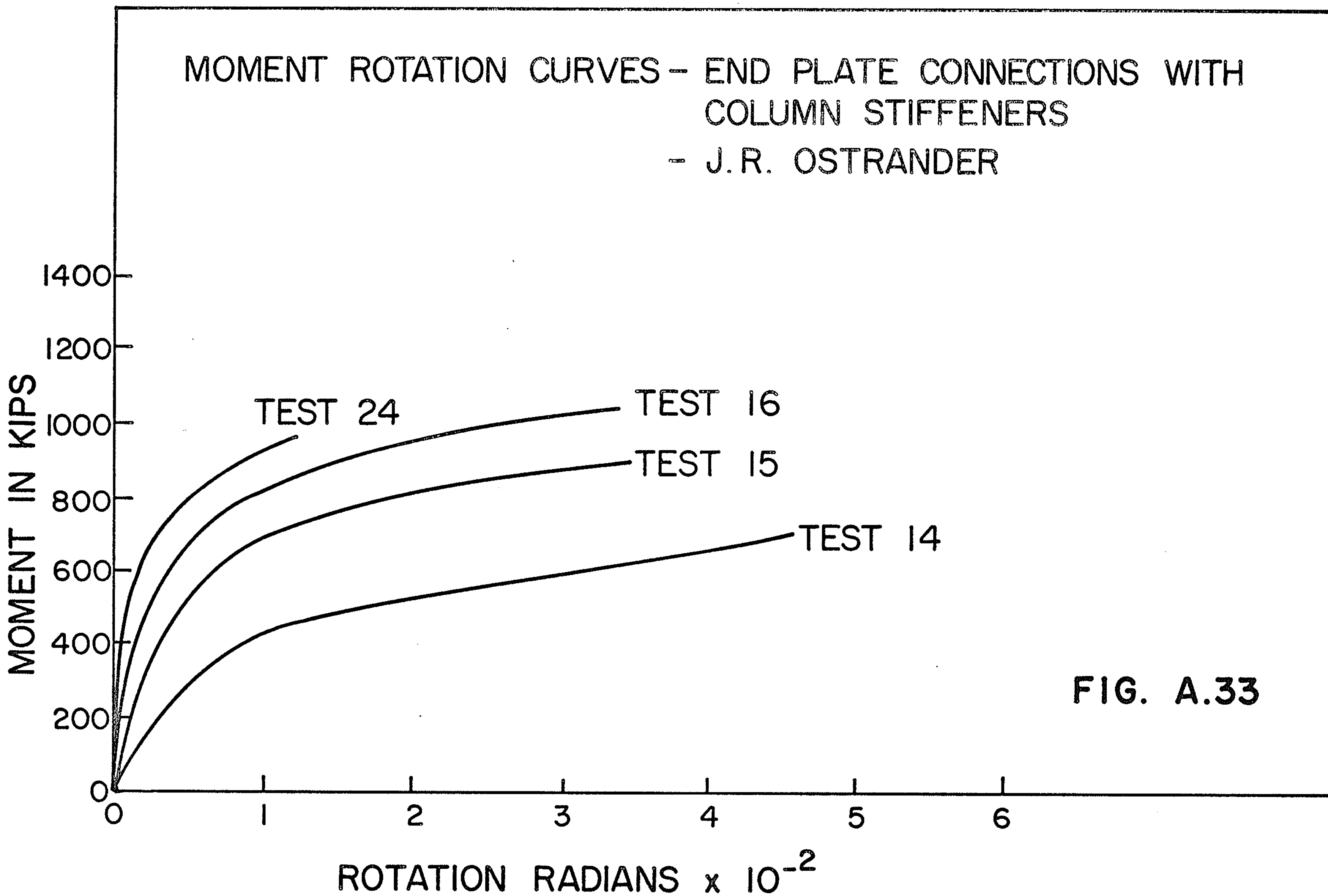
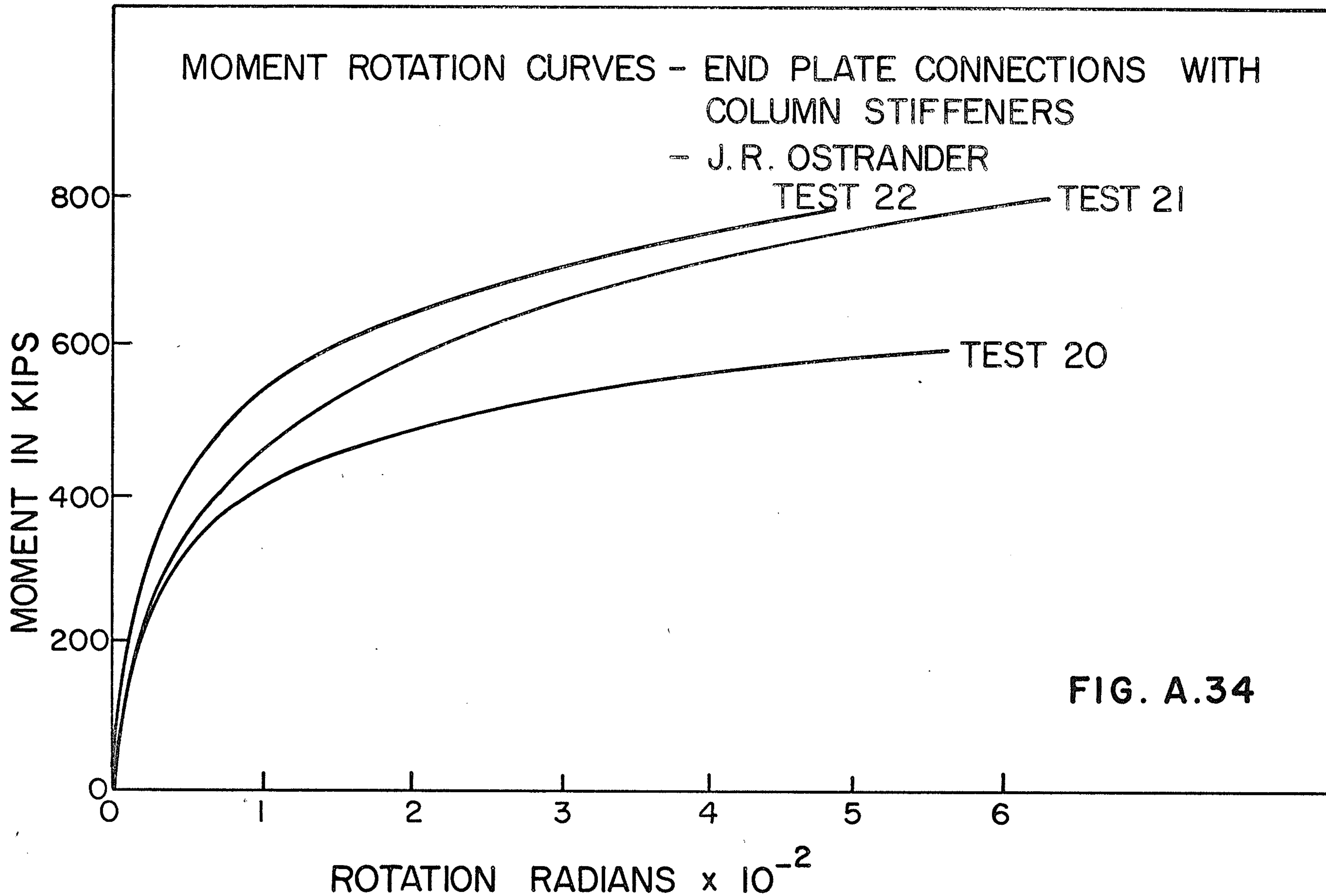

MOMENT ROTATION CURVES - END PLATE CONNECTIONS WITH COLUMN STIFFENERS
- J.R. OSTRANDER
MOMENT IN KIPS
1400
1200
1000
800
600
400
200
0
TEST 24
TEST 16
TEST 15
TEST 14
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. A.33


MOMENT ROTATION CURVES - END PLATE CONNECTIONS WITH COLUMN STIFFENERS
- J.R. OSTRANDER
TEST 22
TEST 21
TEST 20
FIG. A.34
MOMENT IN KIPS
0
200
400
600
800
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

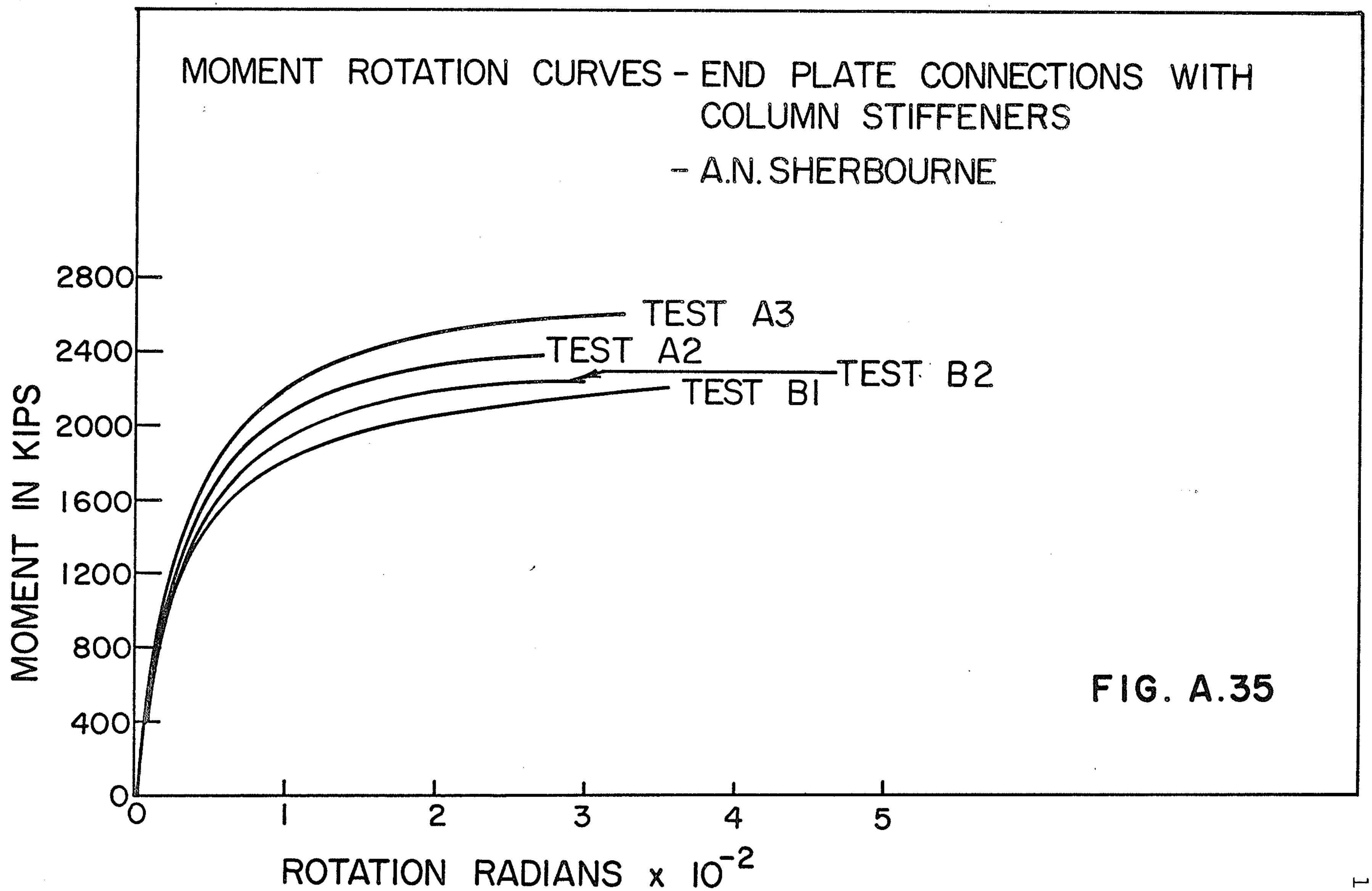

MOMENT ROTATION CURVES - END PLATE CONNECTIONS WITH COLUMN STIFFENERS
- A.N. SHERBOURNE
TEST A3
TEST A2
TEST B2
TEST B1
MOMENT IN KIPS
2800
2400
2000
1600
1200
800
400
0
0
1
2
3
4
5
ROTATION RADIANS x 10^{-2}

FIG. A.35

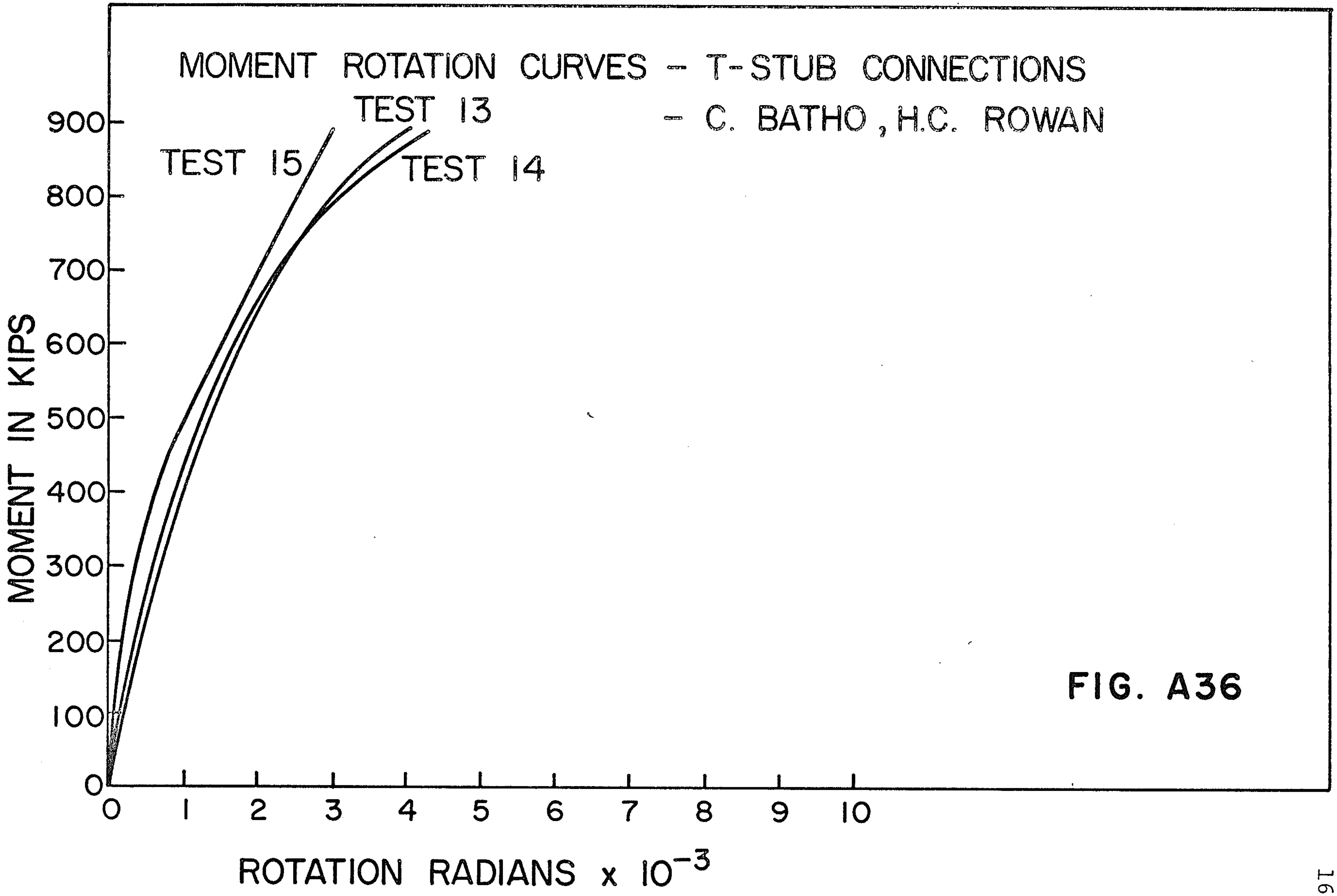
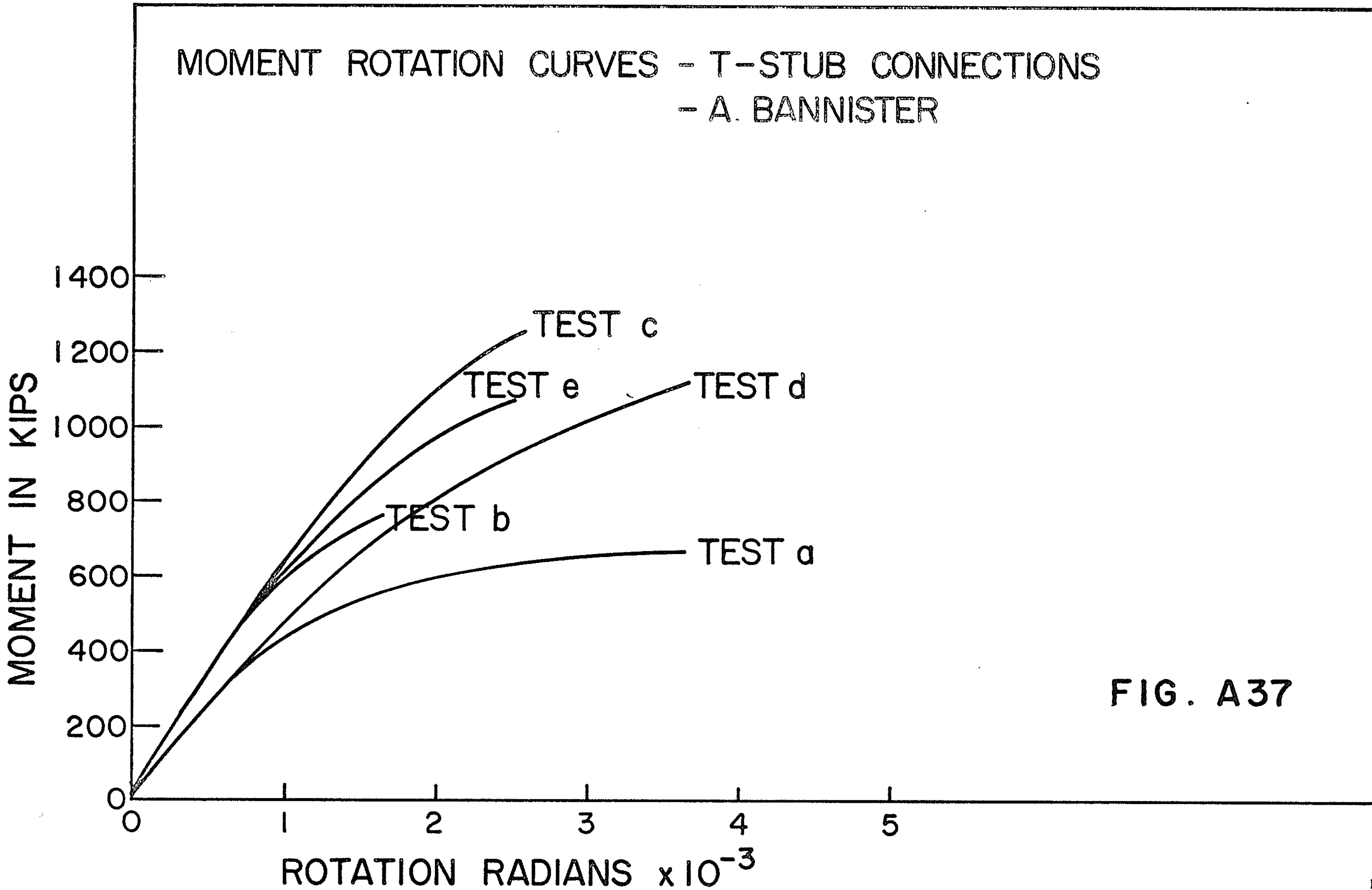
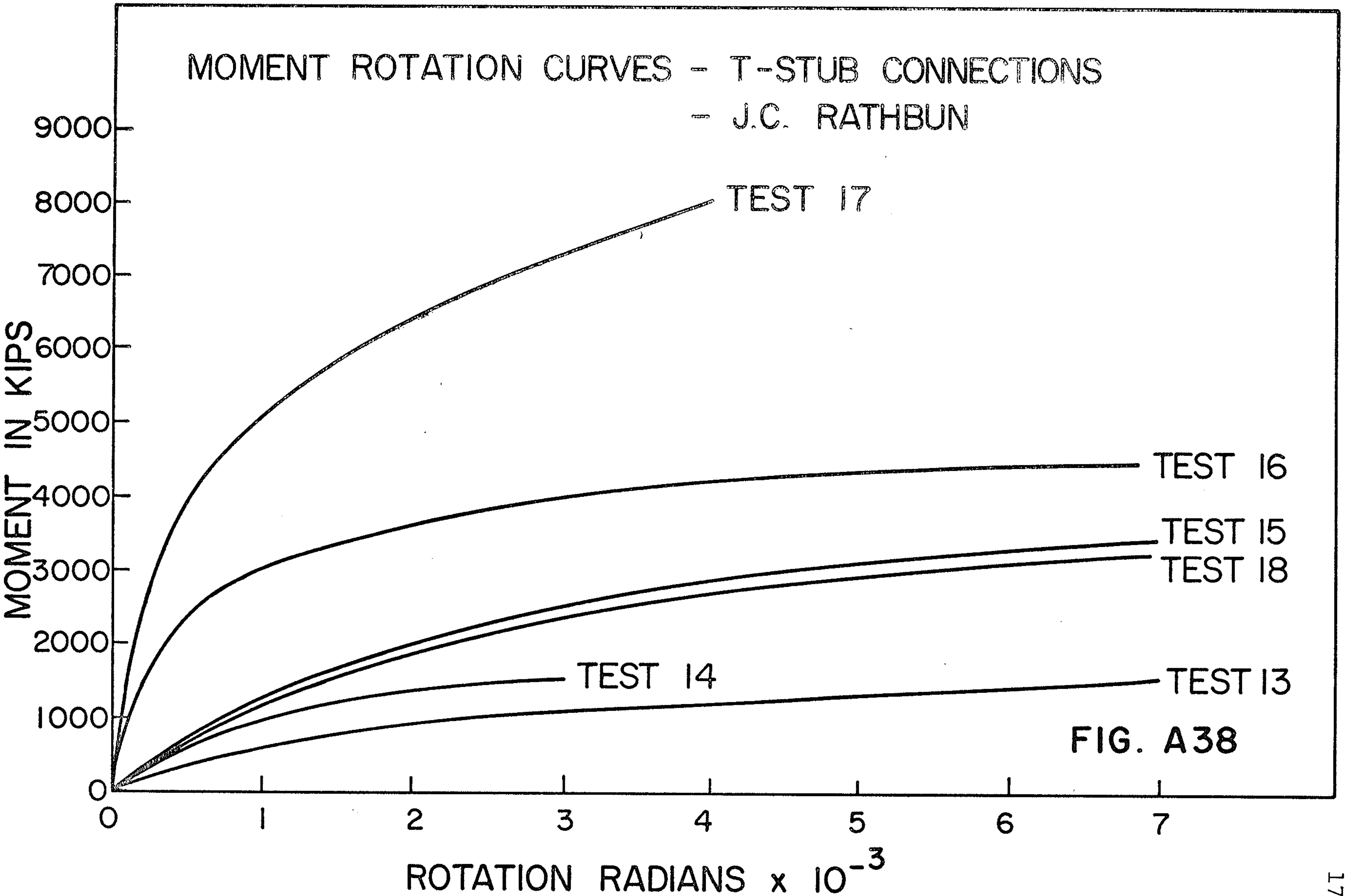

MOMENT ROTATION CURVES – T-STUB CONNECTIONS
– C. BATHO, H.C. ROWAN
TEST 13
TEST 15
TEST 14
MOMENT IN KIPS
0
100
200
300
400
500
600
700
800
900
ROTATION RADIANS x 10^{-3}
0
1
2
3
4
5
6
7
8
9
10

FIG. A36



MOMENT ROTATION CURVES - T-STUB CONNECTIONS
- A. BANNISTER
TEST c
TEST e
TEST d
TEST b
TEST a
MOMENT IN KIPS
1400
1200
1000
800
600
400
200
0
0
1
2
3
4
5
ROTATION RADIANS x 10^{-3}

FIG. A37


MOMENT ROTATION CURVES - T-STUB CONNECTIONS
- J.C. RATHBUN
MOMENT IN KIPS
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
TEST 17
TEST 16
TEST 15
TEST 18
TEST 14
TEST 13
0
1
2
3
4
5
6
7
ROTATION RADIANS x 10^{-3}
FIG. A38

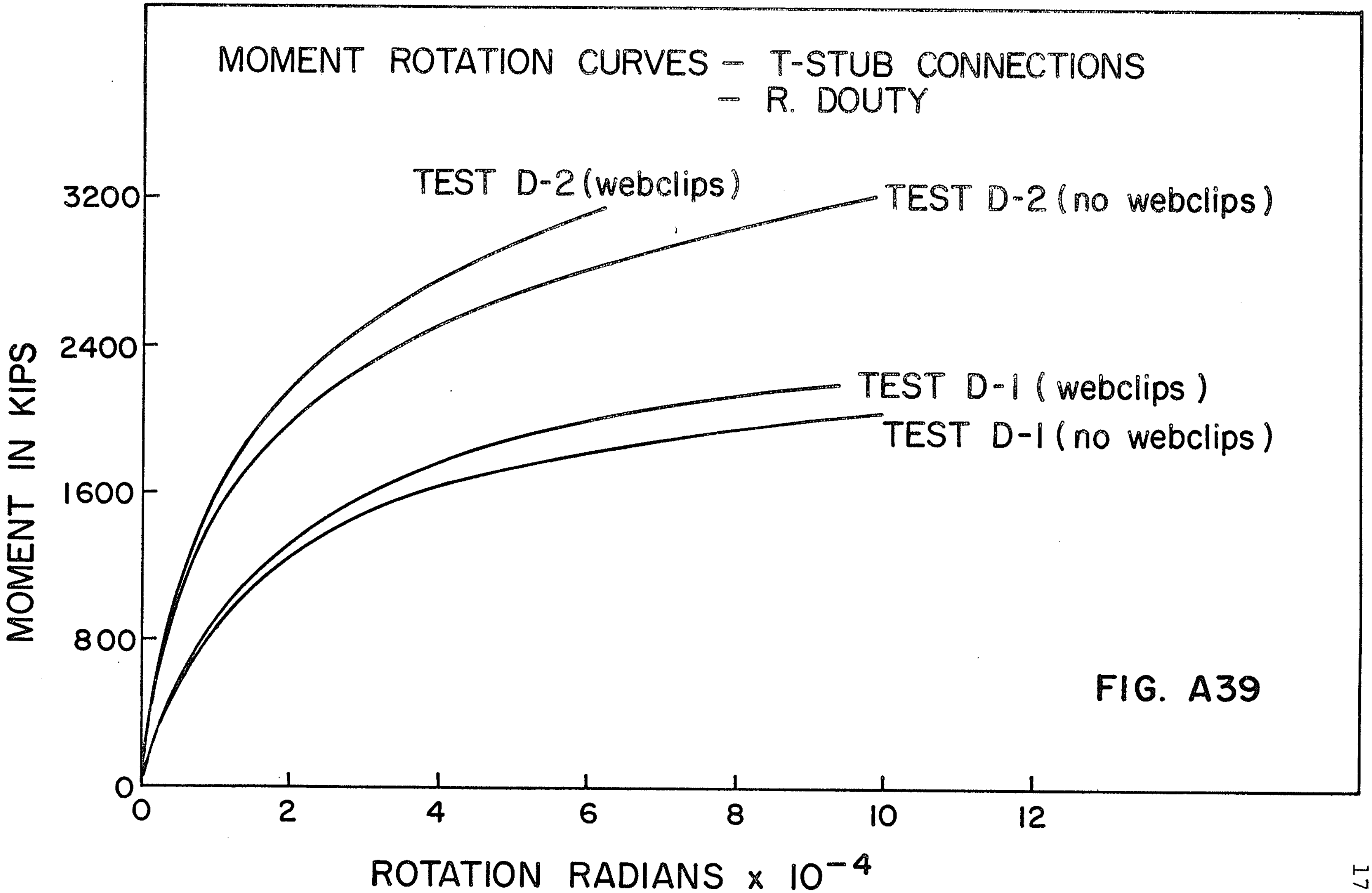
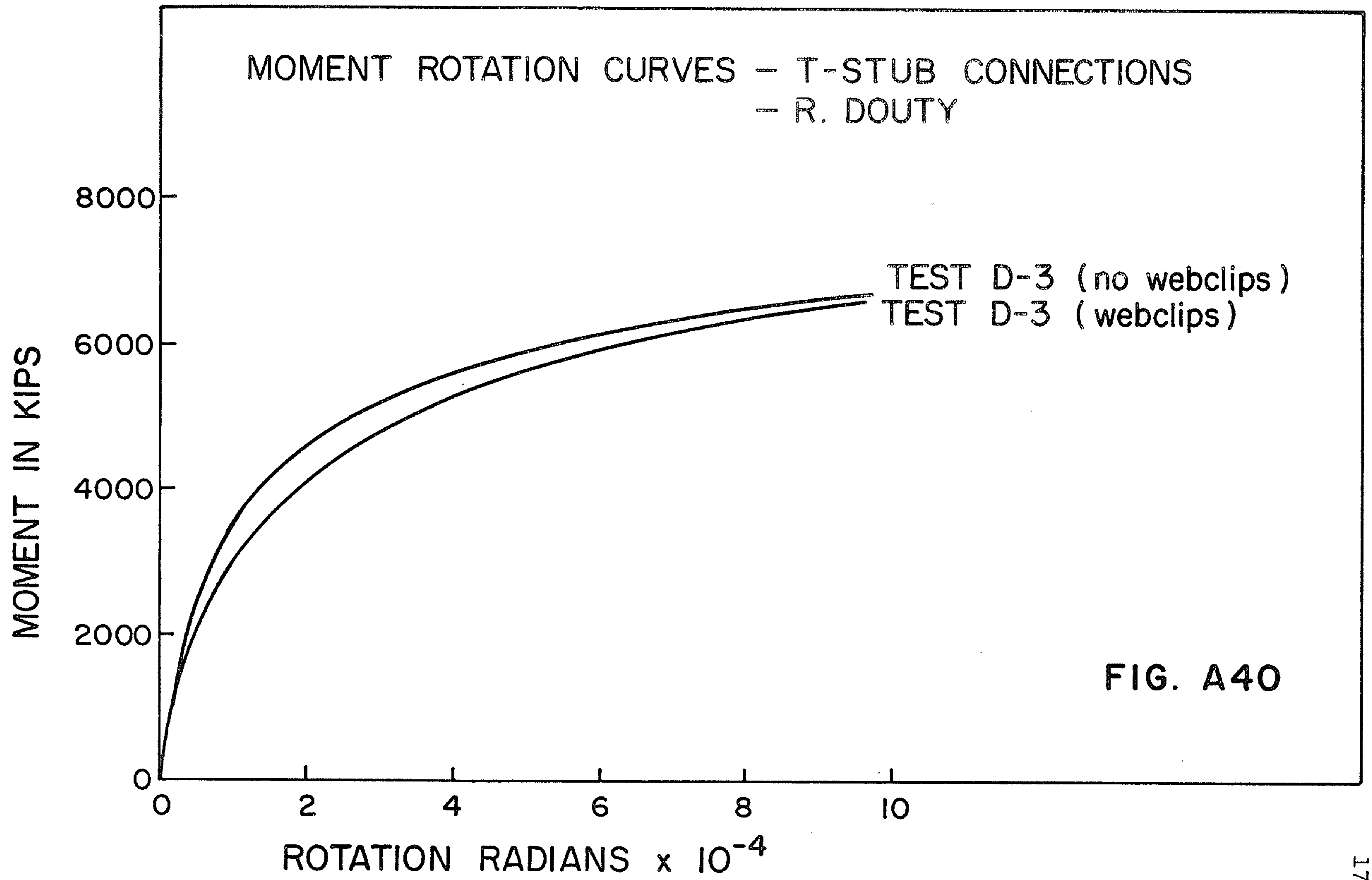

MOMENT ROTATION CURVES – T-STUB CONNECTIONS
– R. DOUTY
TEST D-2 (webclips)
TEST D-2 (no webclips)
TEST D-1 (webclips)
TEST D-1 (no webclips)
MOMENT IN KIPS
0
800
1600
2400
3200
0
2
4
6
8
10
12
ROTATION RADIANS x 10^{-4}

FIG. A39


MOMENT ROTATION CURVES – T-STUB CONNECTIONS
– R. DOUTY
TEST D-3 (no webclips)
TEST D-3 (webclips)
MOMENT IN KIPS
8000
6000
4000
2000
0
0
2
4
6
8
10
ROTATION RADIANS x 10^-4
FIG. A40

APPENDIX B

This Appendix contains the standardization constants and the standardized moment-rotation equations for the various connection types considered in this study. Figs B.1 to Fig. B.7 are the standardized curves, and Figs. B.8 to Fig. B.18 are comparisons of experimentally obtained moment-rotation curves with those obtained from the standardized equation.

B.1 Double Web Angle Connections

Standardization constant

$$K = d^{-2.4} \ t^{-0.23} \ g^{.16}$$

where:

d = depth of angle

t = angle thickness

g = connection gage

Standardized moment-rotation equation

$$\phi = 3.66 \ (KM) \ 10^{-4} + 1.15 \ (KM)^3 \ 10^{-6} + 4.57 \ (KM)^5 \ 10^{-8}$$

B.2 Single Web Angle Connections

Standardization constant

$$K = d^{-2.4} \ t^{-1.81} g^{0.15}$$

where:

d = depth of angle

t = angle thickness

g = connection gage

Standardized moment-rotation equation

$$\phi = 4.28\ (KM)\ 10^{-3} + 1.45\ (KM)\ 10^{-9} + 1.51\ (KM)\ 10^{-16}$$

B.3 <u>Header Plate Connections</u>

Standardization constant

$$K = t^{-1.6}\ g^{1.6}\ g^{-2.3}\ w^{-0.5}$$

where:

t = thickness of header plate

g = connection gage

d = depth of connection

w = web thickness

Standardized moment-rotation curve

$$\phi = 5.1\ (KM)\ 10^{-5} + 6.2\ (KM)^3\ 10^{-10} + 2.4\ (KM)^5\ 10^{-13}$$

B.4 <u>Top and Seat Angle Connections</u>

Standardization constant

$$K = t^{-0.5}\ d^{-1.5}\ f^{-1.1}\ l^{-.7}$$

where:

t = angle thickness

d = depth of connection

f = fastener diamenter

l = angle length

Standardized moment-rotation curve

$$\phi = 8.46\ (KM)\ 10^{-4} + 1.01\ (KM)^3\ 10^{-4} + 1.24\ (KM)^5\ 10^{-8}$$

B.5 End Plate Connectionsrs With No Column Stiffners

Standardization constant

$$K = d^{-2.4}\ t^{-.4}\ f^{-1.1}$$

where:

d = depth of connection

t = thickness of plate

f = fastener diameter

Standardized moment-rotation curve

$$\phi = 1.83\ (KM)\ 10^{-3} - 1.04\ (KM)^3\ 10^{-4} + 6.38\ (KM)^5\ 10^{-6}$$

B.6 End Plate Connections With Column Stiffners

Standardization constant

$$K = d^{-2.4}\ t^{-0.6}$$

where:

d = depth of connection

t = thickness of plate

Standardized moment-rotation curve

$$\phi = 1.79\ (KM)\ 10^{-3} + 1.76\ (KM)^3\ 10^{-4} + 2.04\ (KM)^5\ 10^{-4}$$

B.7 T-Stub Connections

Standardization constant

$$K = d^{-1.5}\ t^{-0.5}\ f^{-1.1}\ l^{-0.7}$$

where:

d = depth of connection

t = thickness of T-stub flange

f = fastener diameter

l = length of T-stub

Standardized moment-rotation curve

$$\phi = 2.1\ (KM)\ 10^{-4} + 6.2\ (KM)^3\ 10^{-6} - 7.6\ (KM)^5\ 10^{-9}$$

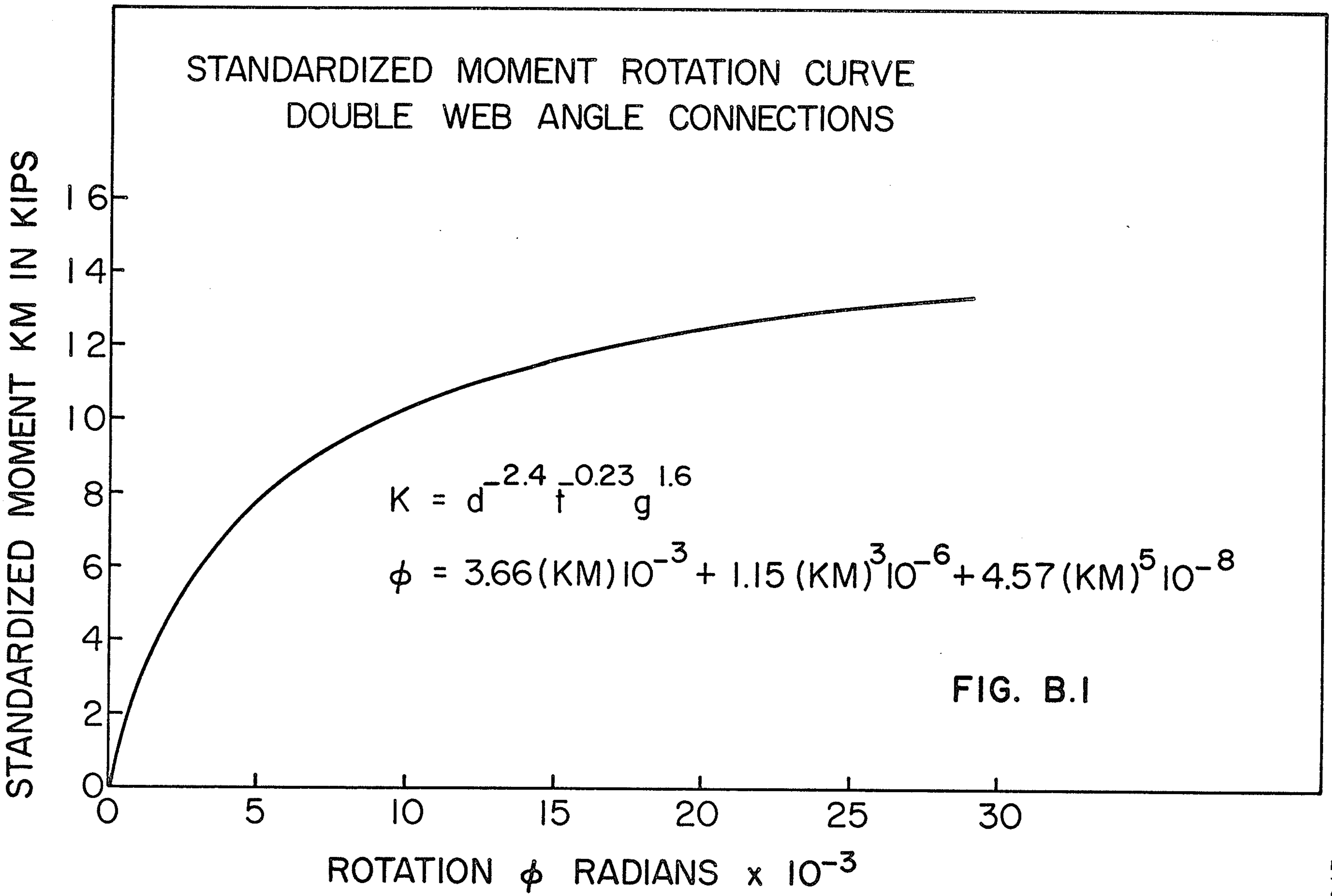

STANDARDIZED MOMENT ROTATION CURVE
DOUBLE WEB ANGLE CONNECTIONS
$K = d^{-2.4}\, t^{-0.23}\, g^{1.6}$
$\phi = 3.66\,(KM)\,10^{-3} + 1.15\,(KM)^{3}\,10^{-6} + 4.57\,(KM)^{5}\,10^{-8}$
STANDARDIZED MOMENT KM IN KIPS
0
2
4
6
8
10
12
14
16
0
5
10
15
20
25
30
ROTATION ϕ RADIANS x 10^{-3}

FIG. B.1

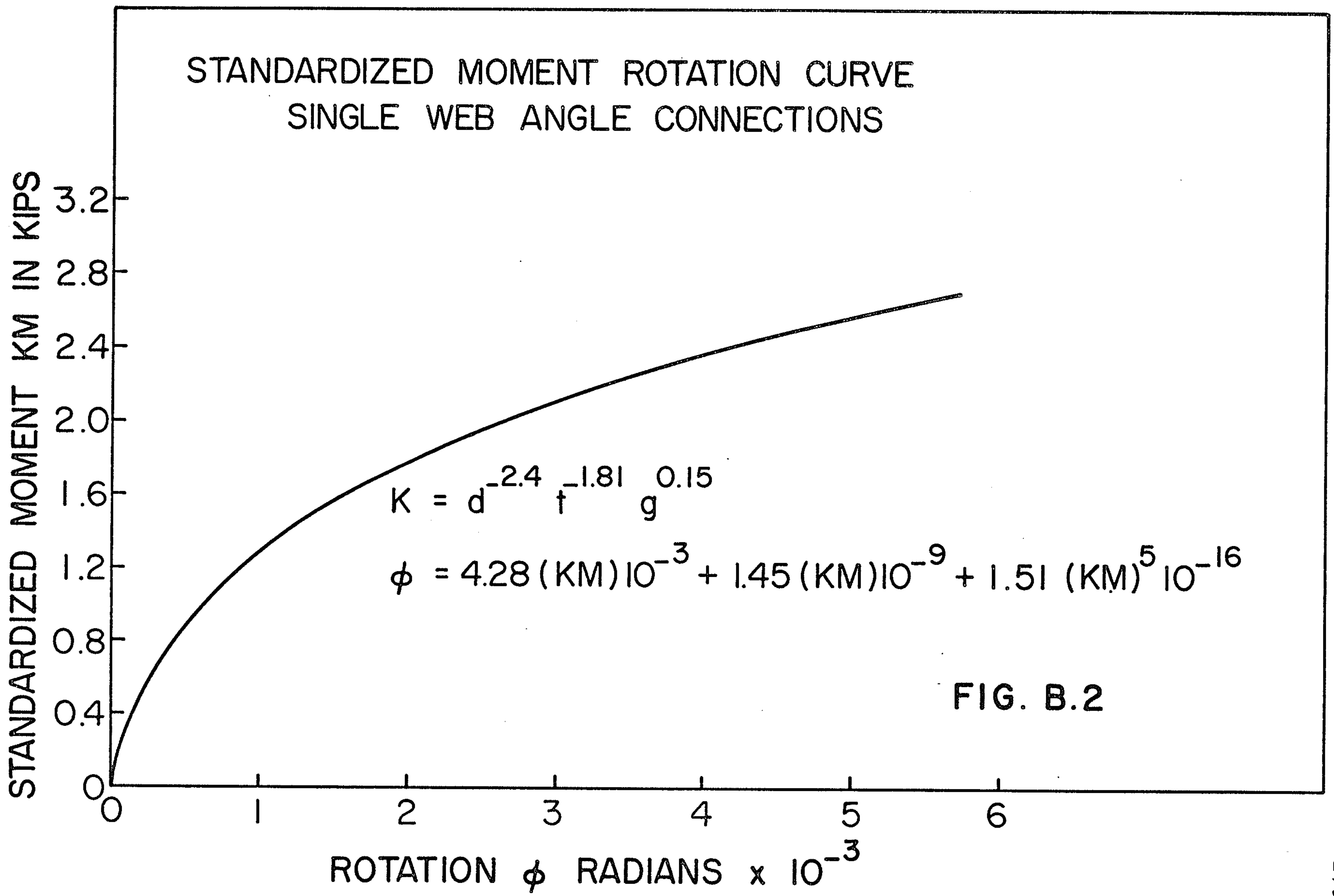

STANDARDIZED MOMENT ROTATION CURVE
SINGLE WEB ANGLE CONNECTIONS
STANDARDIZED MOMENT KM IN KIPS
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
0
1
2
3
4
5
6
ROTATION ϕ RADIANS x 10^{-3}
$K = d^{-2.4}\, t^{-1.81}\, g^{0.15}$
$\phi = 4.28\,(KM)\,10^{-3} + 1.45\,(KM)\,10^{-9} + 1.51\,(KM)^{5}\,10^{-16}$

FIG. B.2

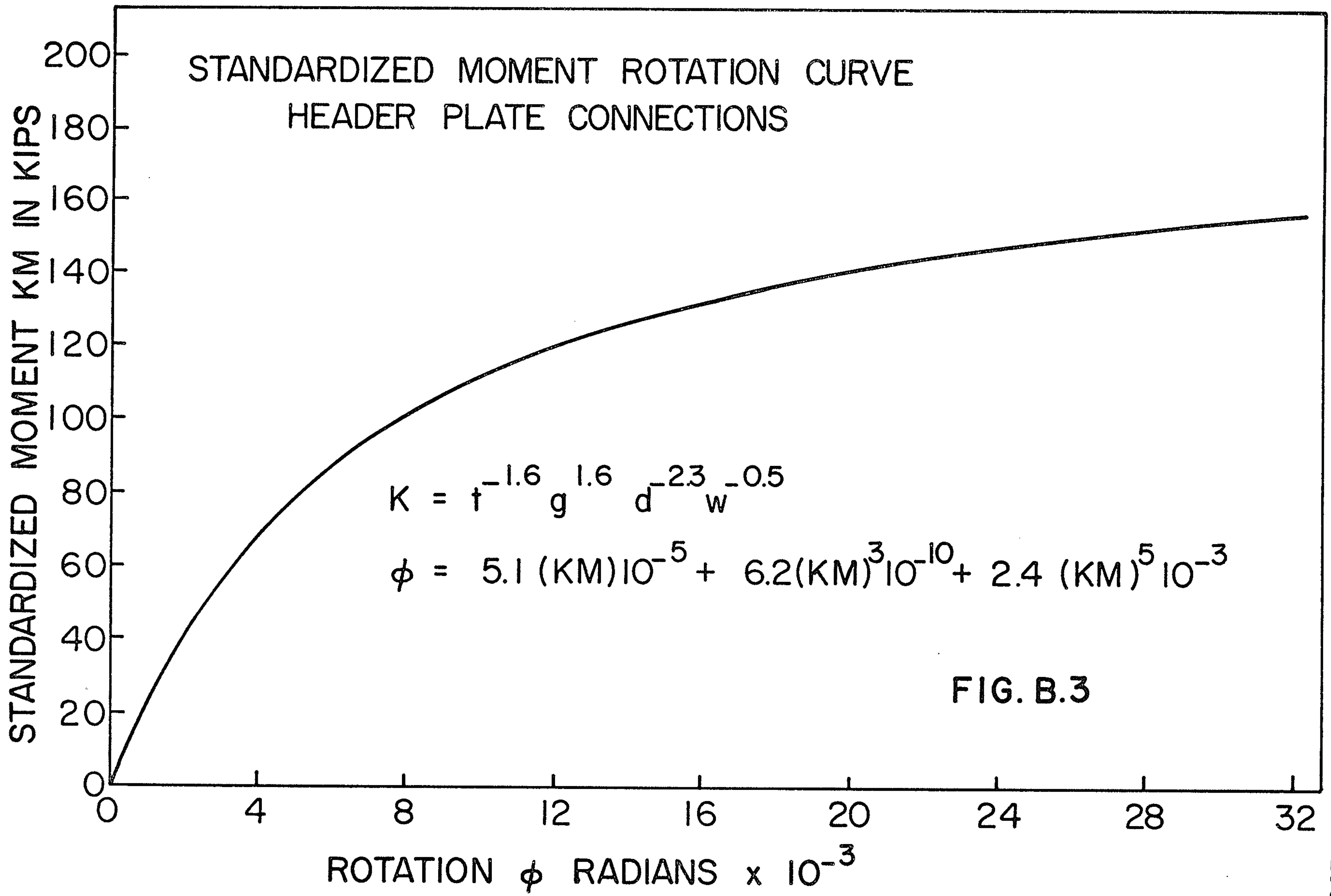

STANDARDIZED MOMENT ROTATION CURVE
HEADER PLATE CONNECTIONS
$K = t^{-1.6} g^{1.6} d^{-2.3} w^{-0.5}$
$\phi = 5.1\,(KM)10^{-5} + 6.2(KM)^{3}10^{-10} + 2.4\,(KM)^{5}10^{-3}$
STANDARDIZED MOMENT KM IN KIPS
ROTATION ϕ RADIANS $\times 10^{-3}$
0
20
40
60
80
100
120
140
160
180
200
4
8
12
16
20
24
28
32

FIG. B.3

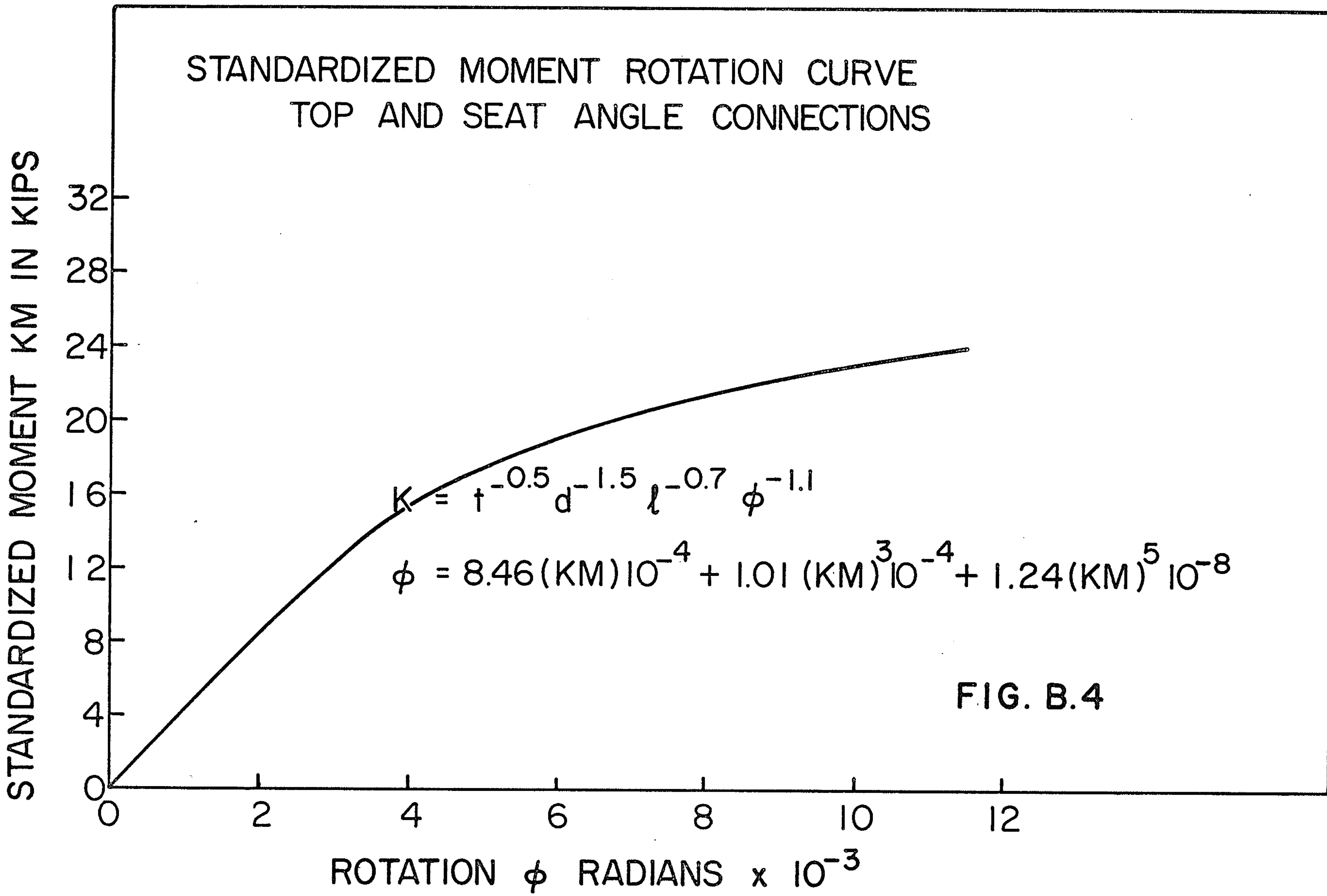

STANDARDIZED MOMENT ROTATION CURVE
TOP AND SEAT ANGLE CONNECTIONS
$K = t^{-0.5} d^{-1.5} \ell^{-0.7} \phi^{-1.1}$
$\phi = 8.46(KM)10^{-4} + 1.01(KM)^3 10^{-4} + 1.24(KM)^5 10^{-8}$
STANDARDIZED MOMENT KM IN KIPS
0
4
8
12
16
20
24
28
32
0
2
4
6
8
10
12
ROTATION ϕ RADIANS x 10^{-3}

FIG. B.4

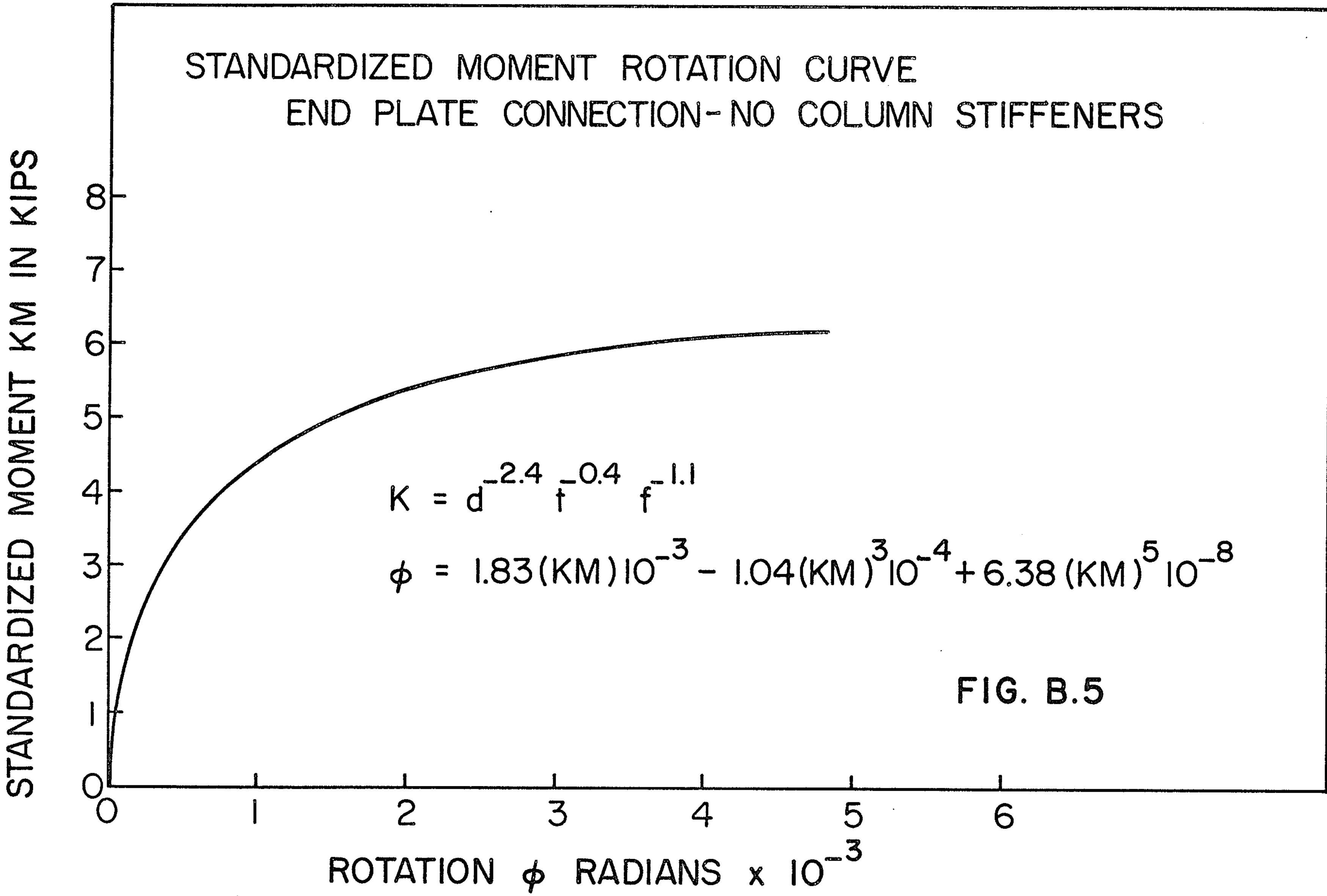

STANDARDIZED MOMENT ROTATION CURVE
END PLATE CONNECTION-NO COLUMN STIFFENERS
$K = d^{-2.4}\, t^{-0.4}\, f^{-1.1}$
$\phi = 1.83(KM)10^{-3} - 1.04(KM)^{3}10^{-4} + 6.38(KM)^{5}10^{-8}$
STANDARDIZED MOMENT KM IN KIPS
ROTATION ϕ RADIANS x 10^{-3}
0
1
2
3
4
5
6
7
8

FIG. B.5

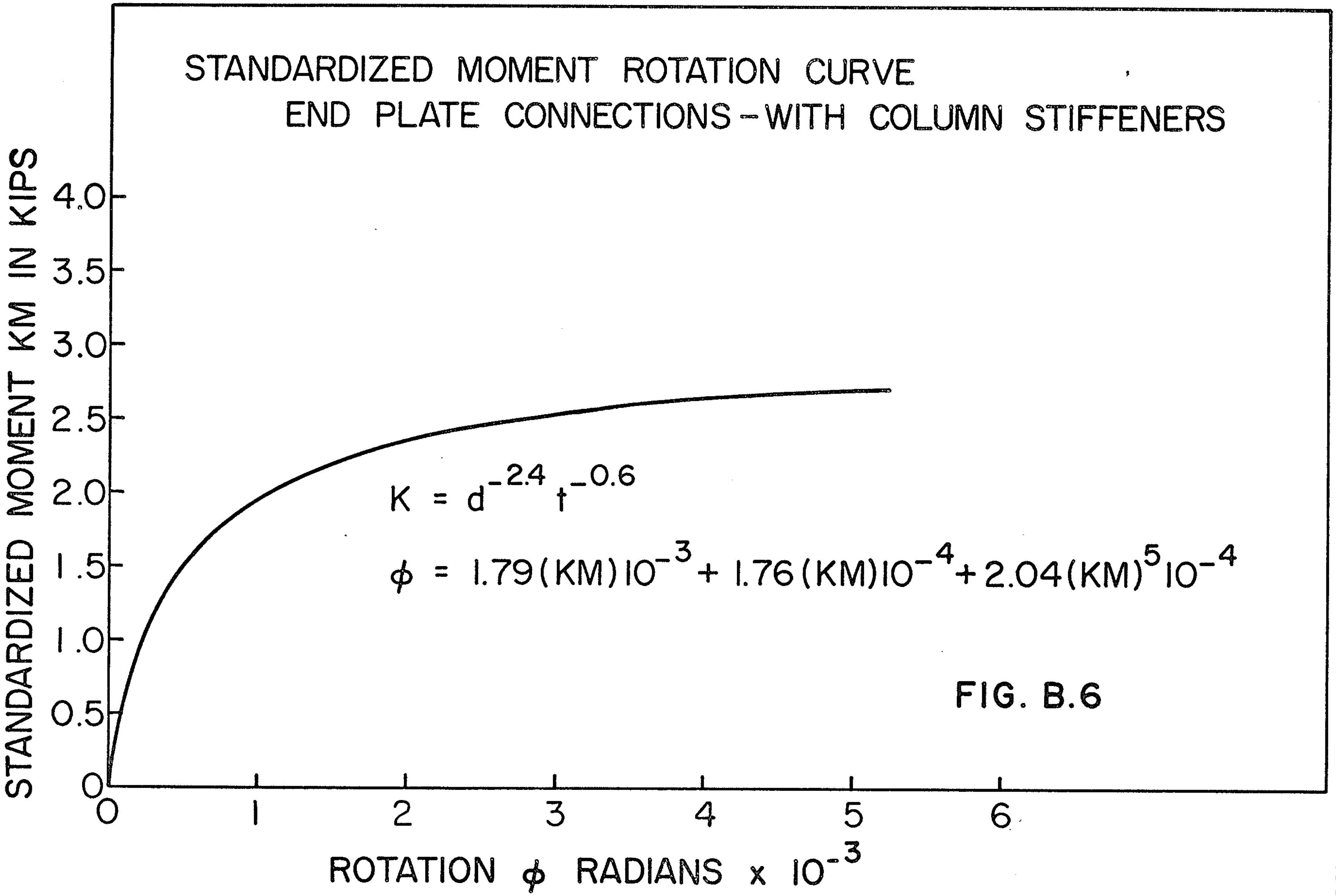

STANDARDIZED MOMENT ROTATION CURVE
END PLATE CONNECTIONS - WITH COLUMN STIFFENERS
$K = d^{-2.4} t^{-0.6}$
$\phi = 1.79(KM)10^{-3} + 1.76(KM)10^{-4} + 2.04(KM)^{5}10^{-4}$
STANDARDIZED MOMENT KM IN KIPS
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
1
2
3
4
5
6
ROTATION ϕ RADIANS x 10^{-3}

FIG. B.6

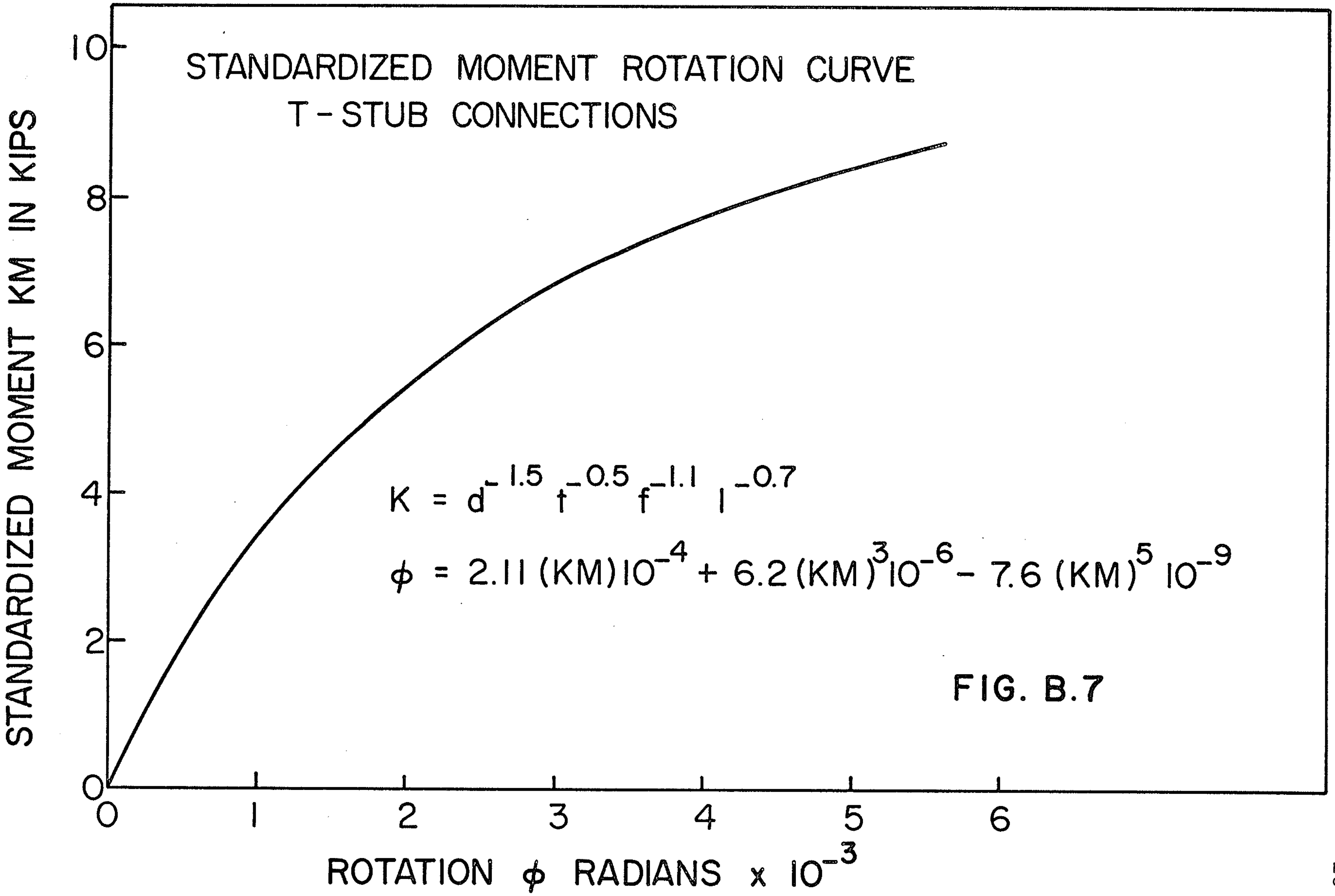

STANDARDIZED MOMENT ROTATION CURVE
T - STUB CONNECTIONS
$K = d^{-1.5} t^{-0.5} f^{-1.1} l^{-0.7}$
$\phi = 2.11\,(KM)\,10^{-4} + 6.2\,(KM)^3\,10^{-6} - 7.6\,(KM)^5\,10^{-9}$
STANDARDIZED MOMENT KM IN KIPS
ROTATION ϕ RADIANS x 10^{-3}
0
2
4
6
8
10
1
3
5

FIG. B.7

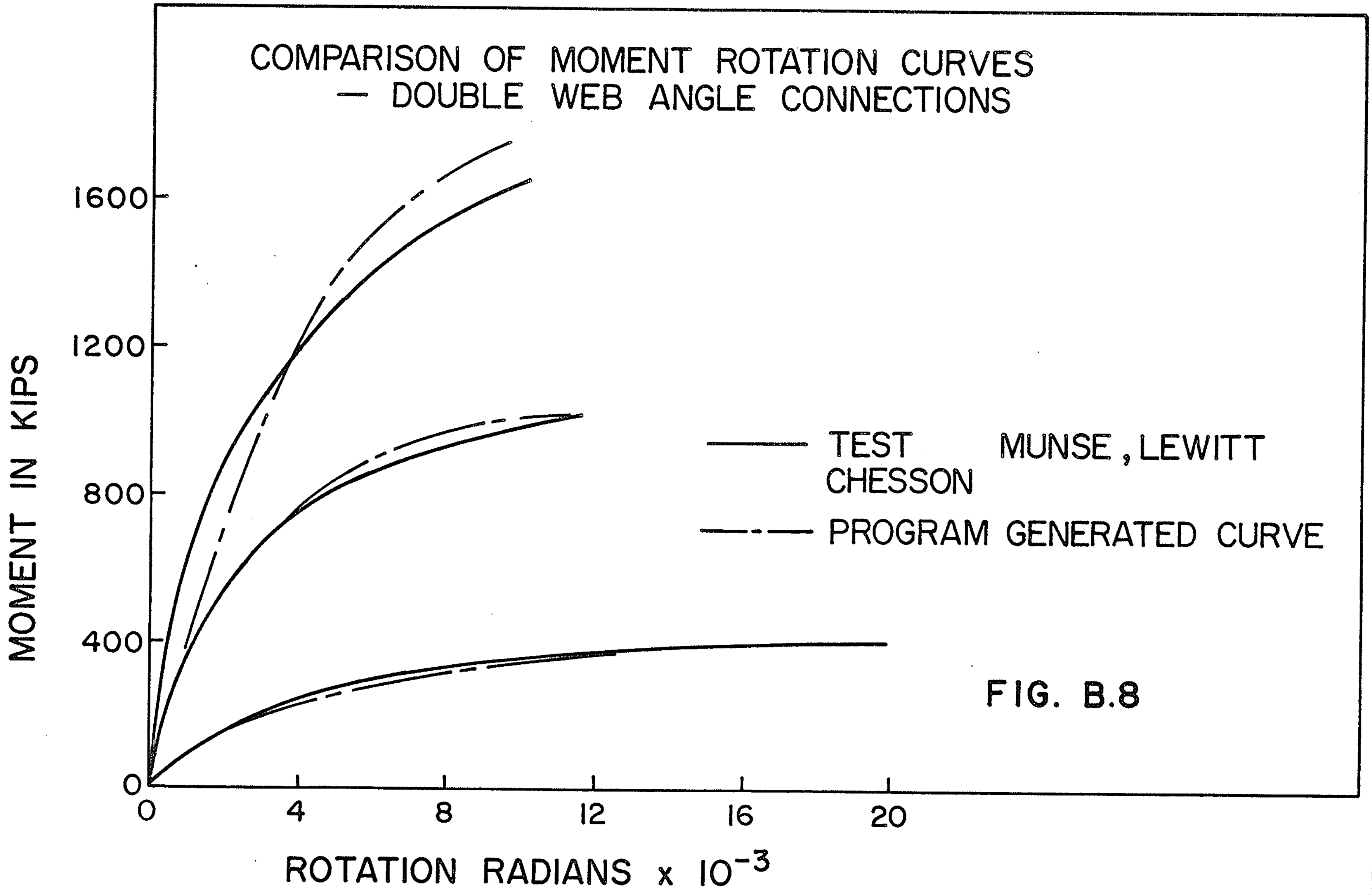

COMPARISON OF MOMENT ROTATION CURVES
— DOUBLE WEB ANGLE CONNECTIONS
MOMENT IN KIPS
0
400
800
1200
1600
0
4
8
12
16
20
ROTATION RADIANS x 10^{-3}
TEST MUNSE, LEWITT CHESSON
PROGRAM GENERATED CURVE

FIG. B.8

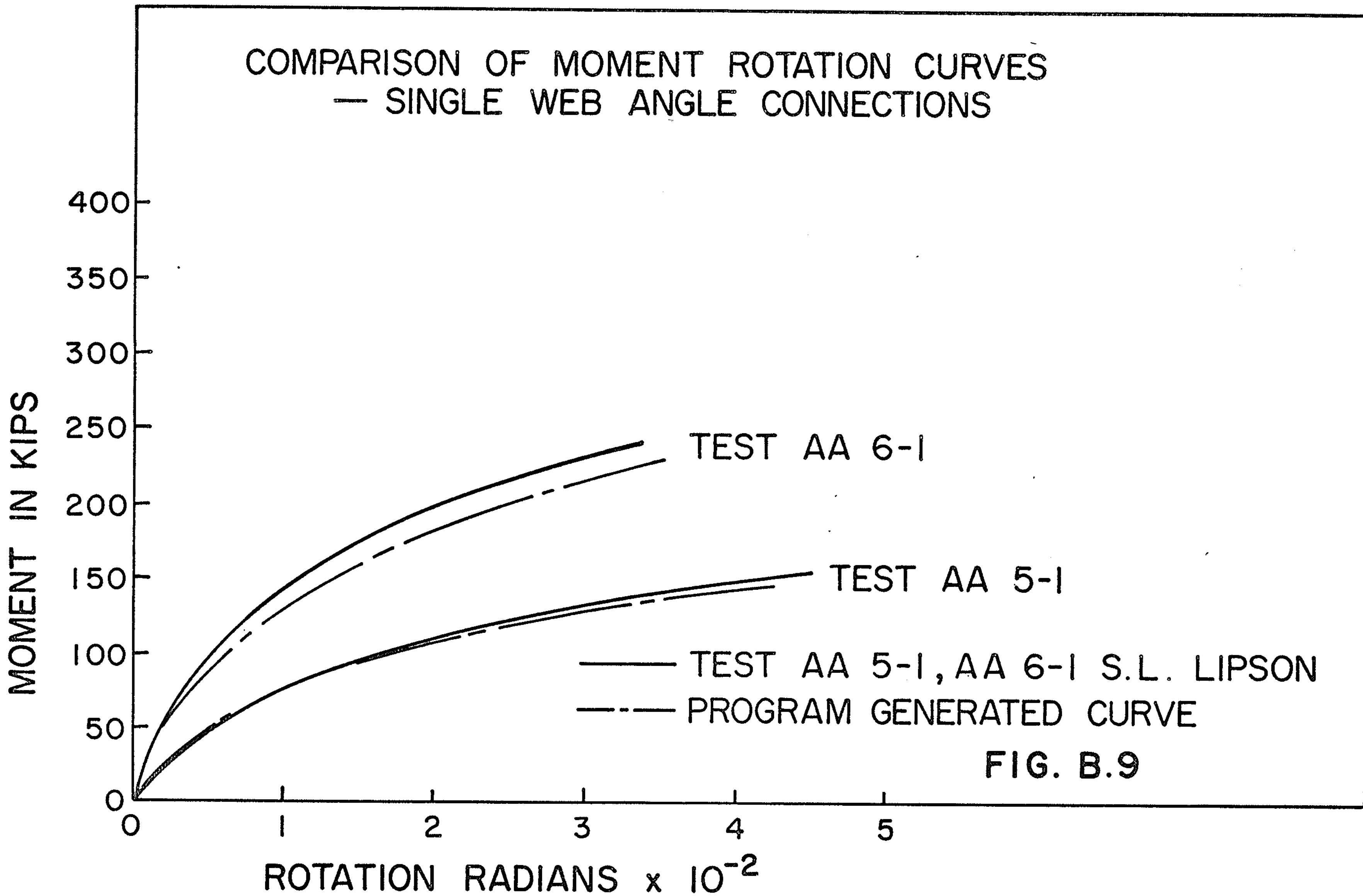
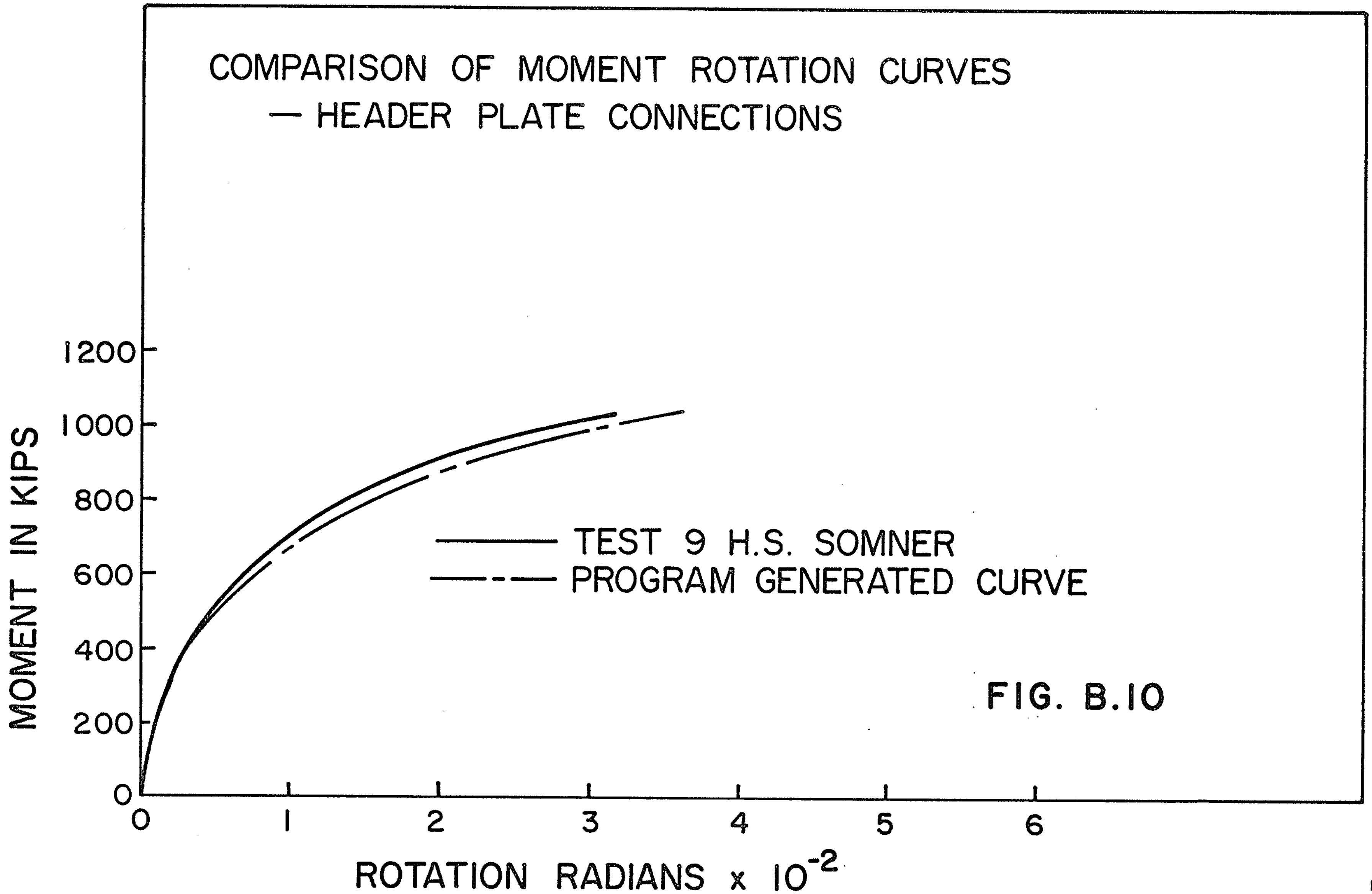

COMPARISON OF MOMENT ROTATION CURVES
— SINGLE WEB ANGLE CONNECTIONS
MOMENT IN KIPS
400
350
300
250
200
150
100
50
0
0
1
2
3
4
5
ROTATION RADIANS x 10^{-2}
TEST AA 6-1
TEST AA 5-1
TEST AA 5-1, AA 6-1 S.L. LIPSON
PROGRAM GENERATED CURVE

FIG. B.9



COMPARISON OF MOMENT ROTATION CURVES
— HEADER PLATE CONNECTIONS
MOMENT IN KIPS
1200
1000
800
600
400
200
0
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}
TEST 9 H.S. SOMNER
PROGRAM GENERATED CURVE

FIG. B.10

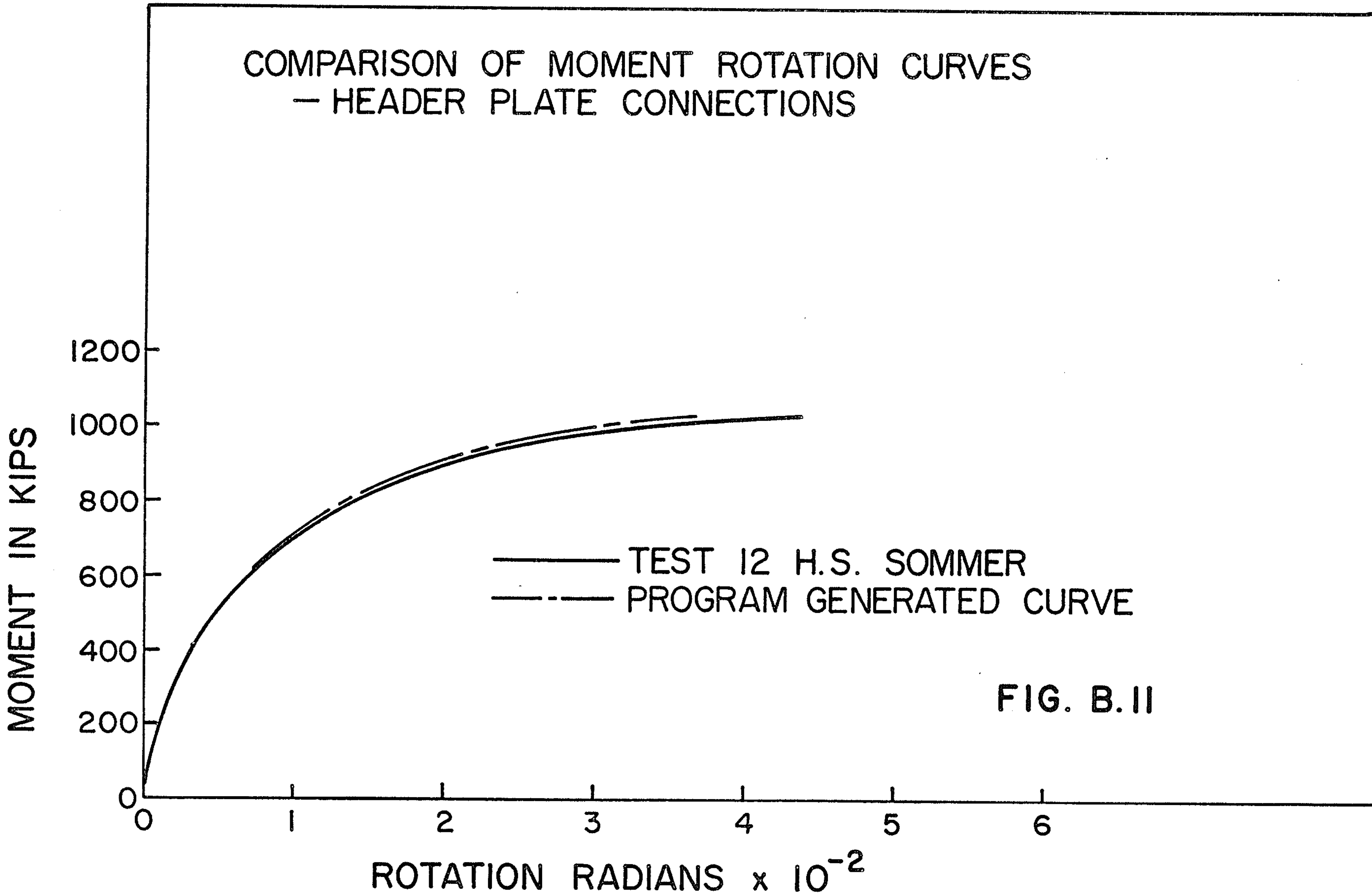

COMPARISON OF MOMENT ROTATION CURVES
— HEADER PLATE CONNECTIONS
MOMENT IN KIPS
1200
1000
800
600
400
200
0
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}
TEST 12 H.S. SOMMER
PROGRAM GENERATED CURVE

FIG. B.11

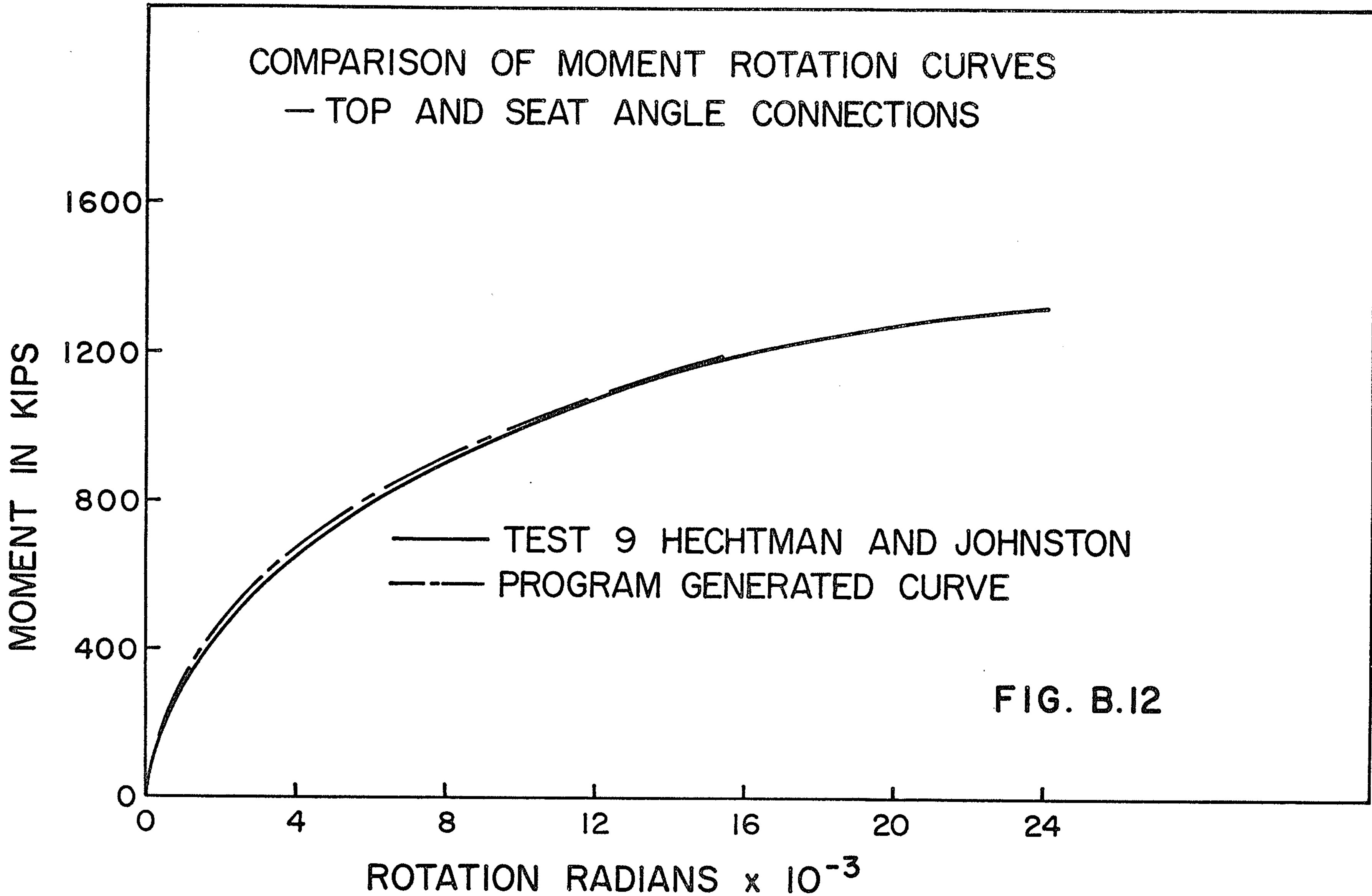

COMPARISON OF MOMENT ROTATION CURVES
— TOP AND SEAT ANGLE CONNECTIONS
MOMENT IN KIPS
1600
1200
800
400
0
0
4
8
12
16
20
24
ROTATION RADIANS x 10^{-3}
TEST 9 HECHTMAN AND JOHNSTON
PROGRAM GENERATED CURVE

FIG. B.12

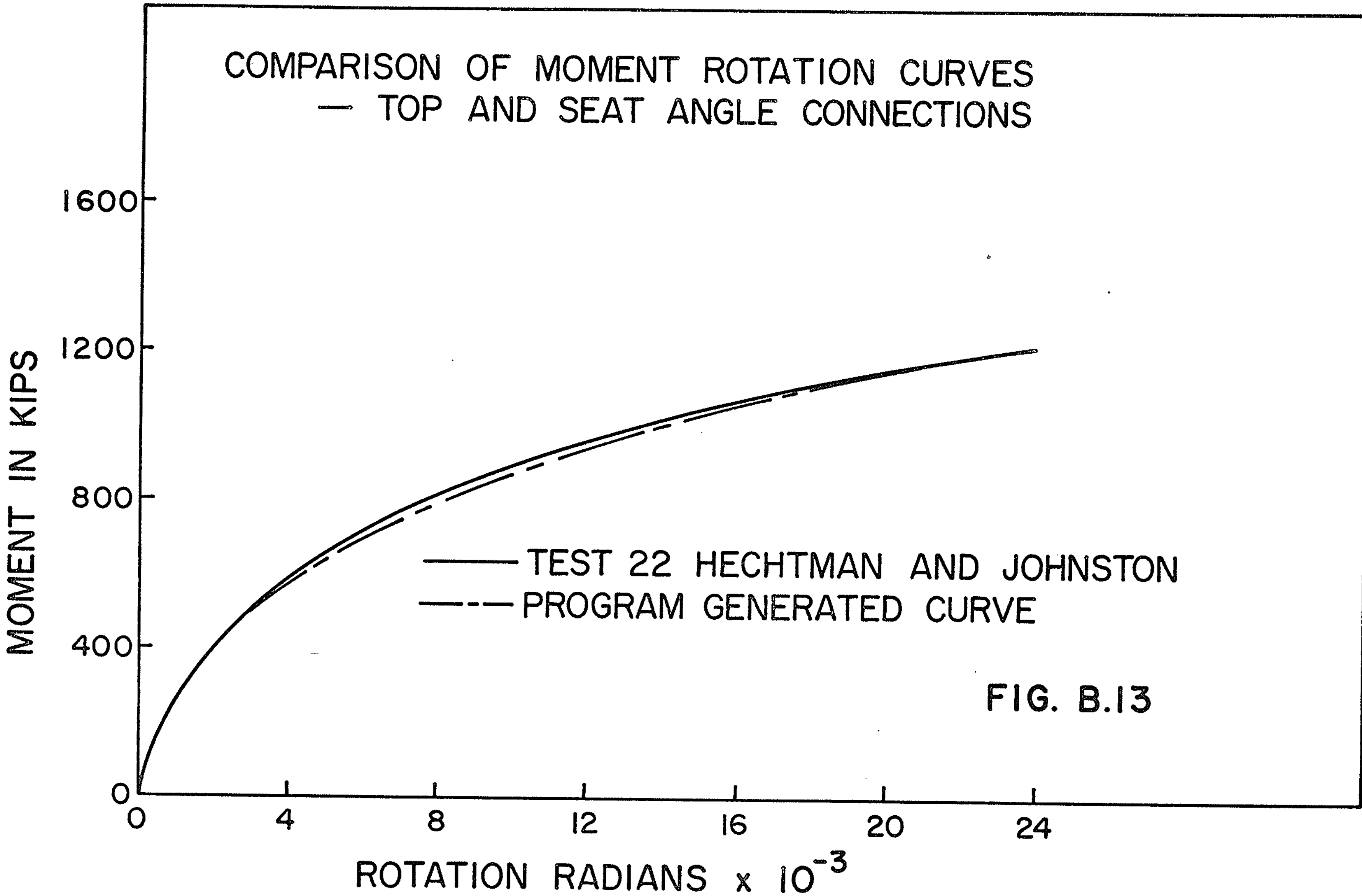

COMPARISON OF MOMENT ROTATION CURVES
— TOP AND SEAT ANGLE CONNECTIONS
MOMENT IN KIPS
0
400
800
1200
1600
ROTATION RADIANS x 10^{-3}
0
4
8
12
16
20
24
TEST 22 HECHTMAN AND JOHNSTON
PROGRAM GENERATED CURVE

FIG. B.13

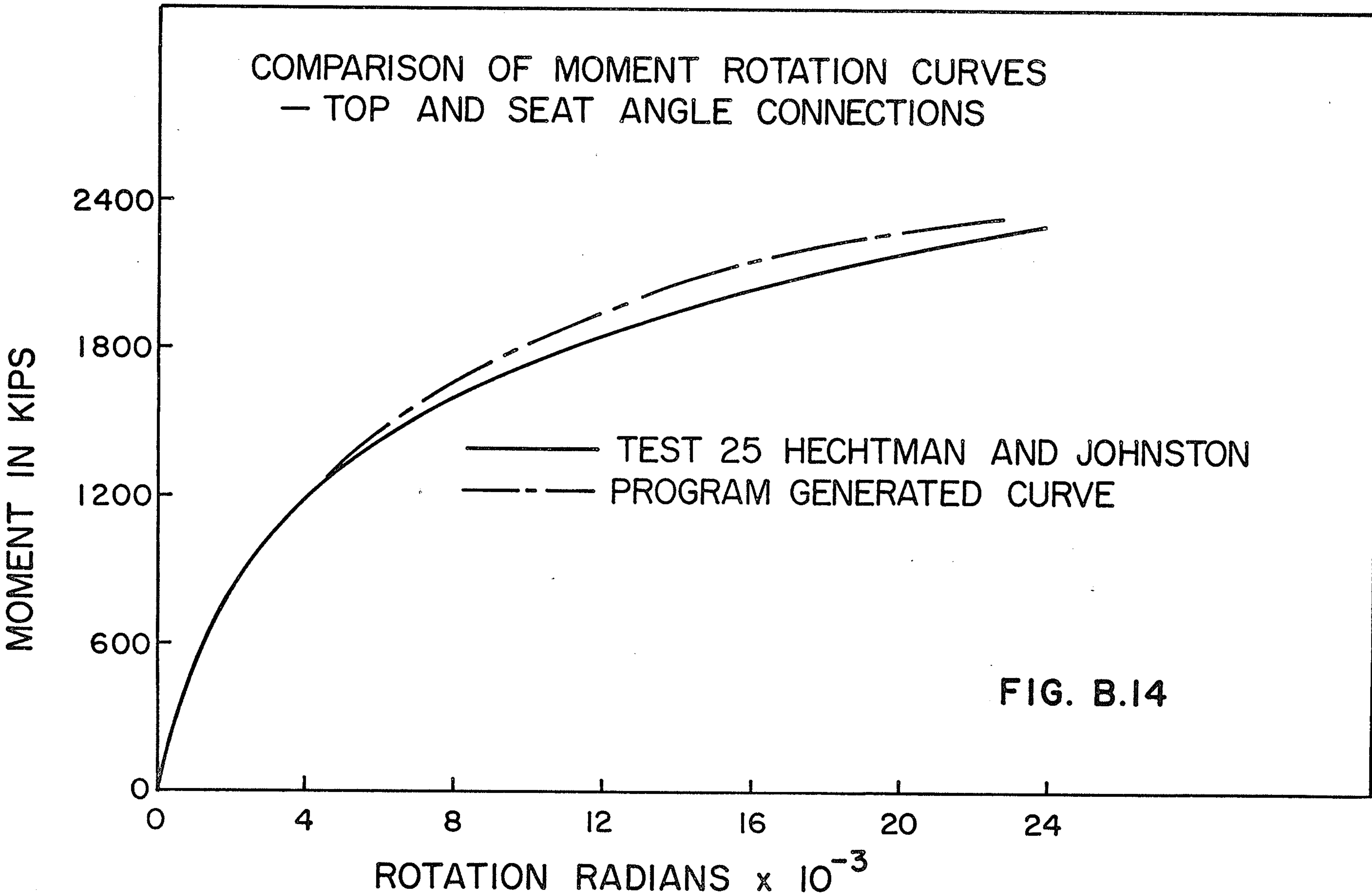

COMPARISON OF MOMENT ROTATION CURVES
— TOP AND SEAT ANGLE CONNECTIONS
2400
1800
1200
600
0
MOMENT IN KIPS
TEST 25 HECHTMAN AND JOHNSTON
PROGRAM GENERATED CURVE
0
4
8
12
16
20
24
ROTATION RADIANS x 10^{-3}

FIG. B.14

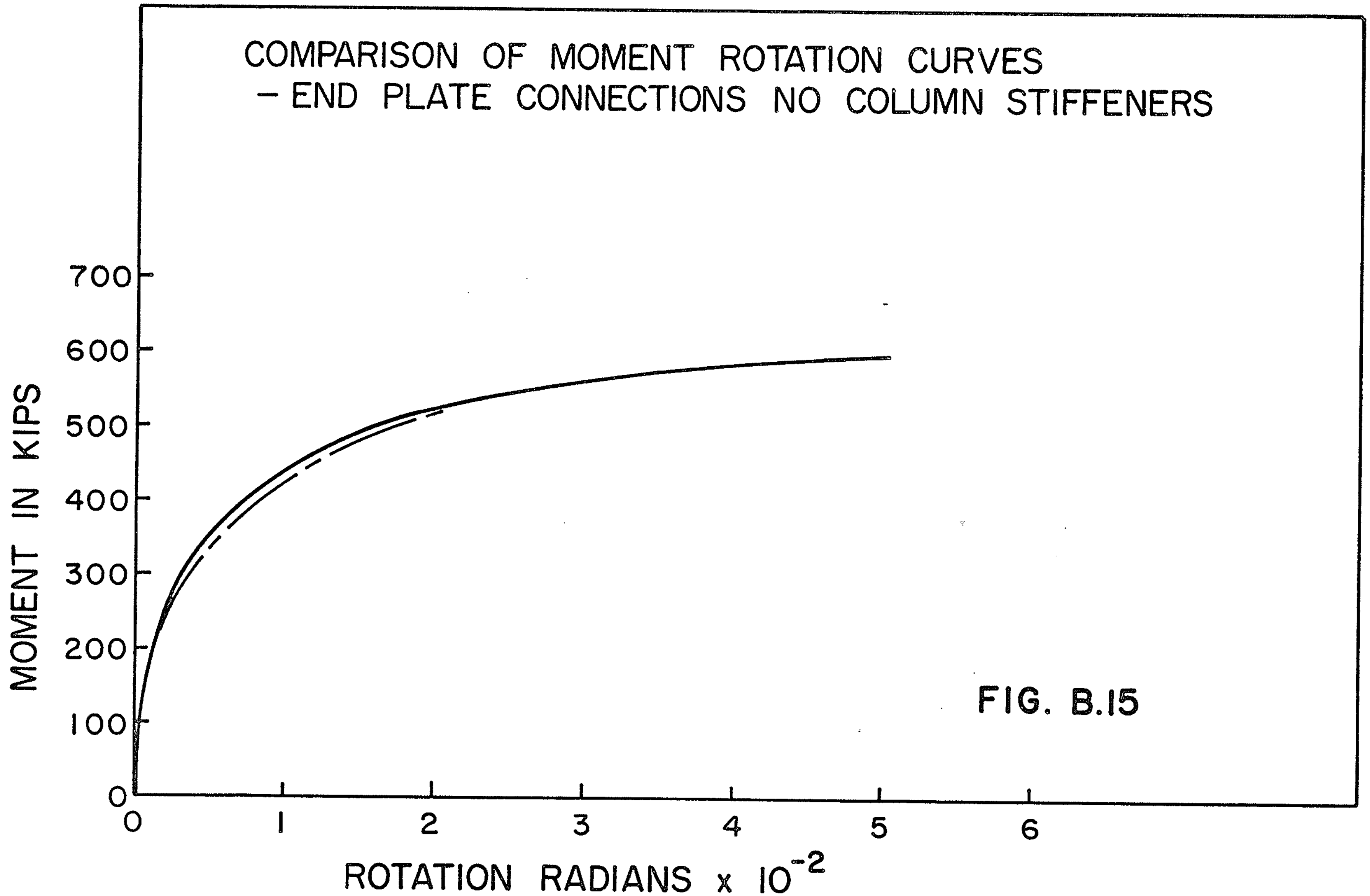

COMPARISON OF MOMENT ROTATION CURVES
– END PLATE CONNECTIONS NO COLUMN STIFFENERS
MOMENT IN KIPS
0
100
200
300
400
500
600
700
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. B.15

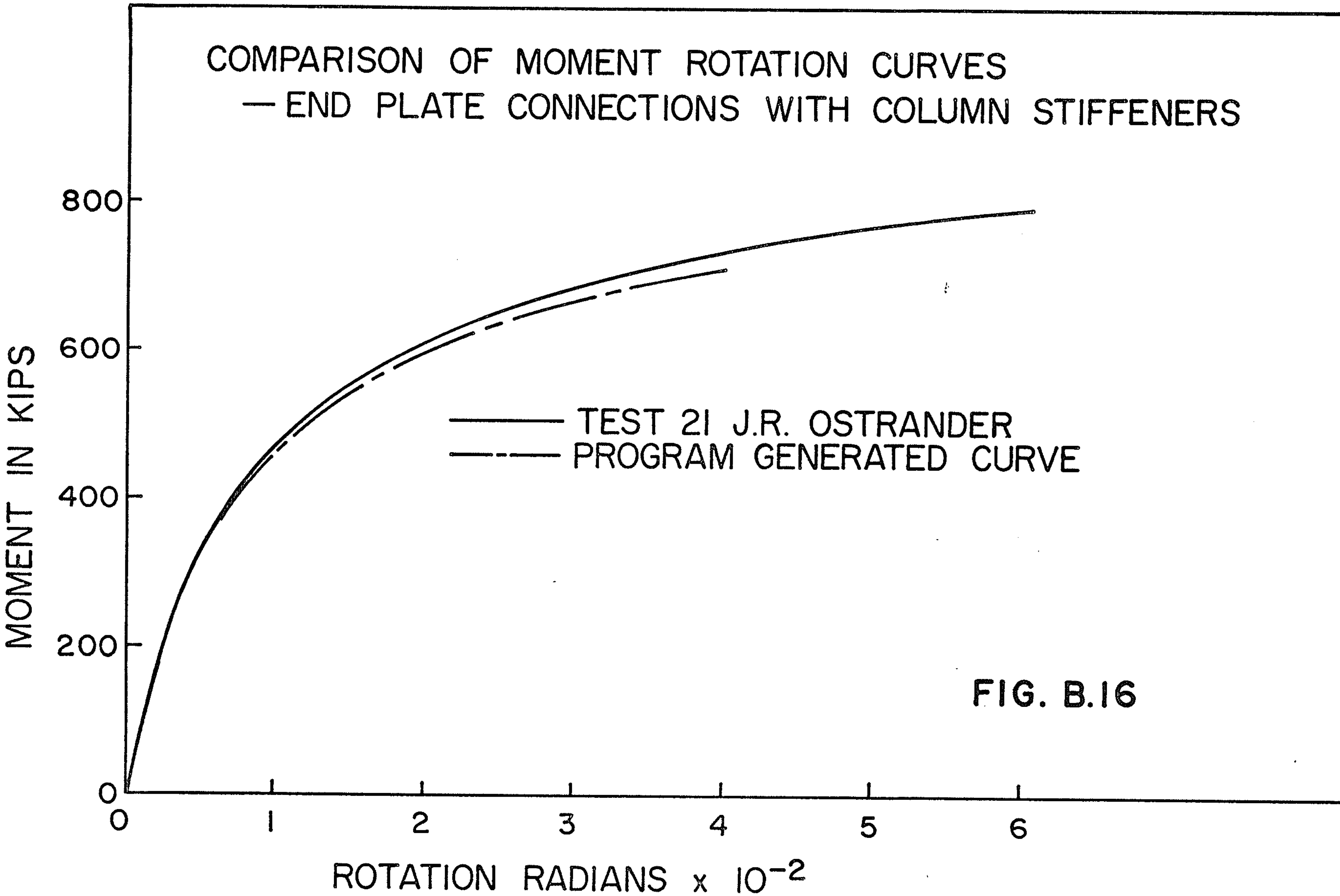

COMPARISON OF MOMENT ROTATION CURVES
— END PLATE CONNECTIONS WITH COLUMN STIFFENERS
TEST 21 J.R. OSTRANDER
PROGRAM GENERATED CURVE
MOMENT IN KIPS
0
200
400
600
800
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}

FIG. B.16

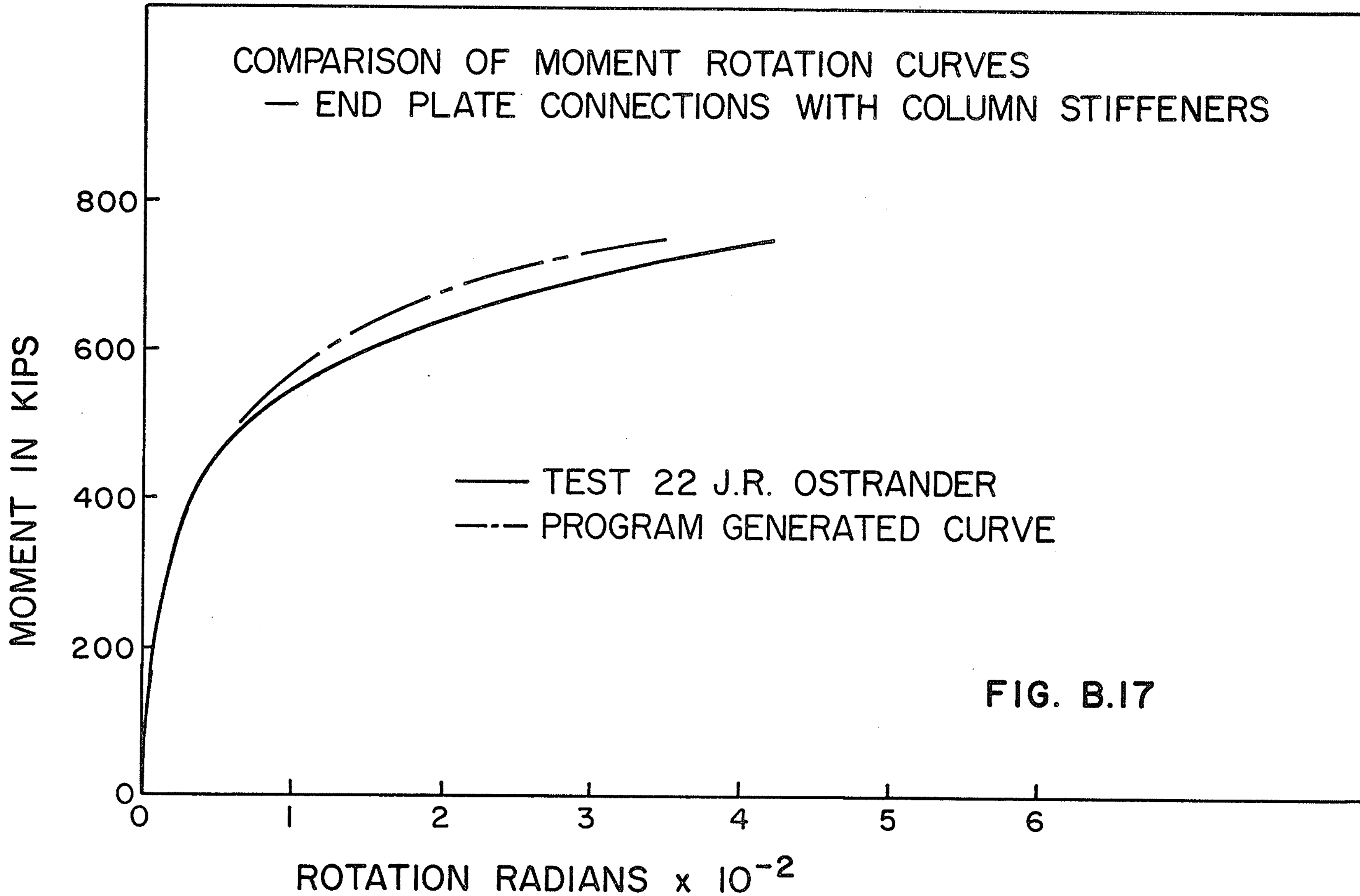

COMPARISON OF MOMENT ROTATION CURVES
— END PLATE CONNECTIONS WITH COLUMN STIFFENERS
MOMENT IN KIPS
800
600
400
200
0
0
1
2
3
4
5
6
ROTATION RADIANS x 10^{-2}
TEST 22 J.R. OSTRANDER
PROGRAM GENERATED CURVE

FIG. B.17

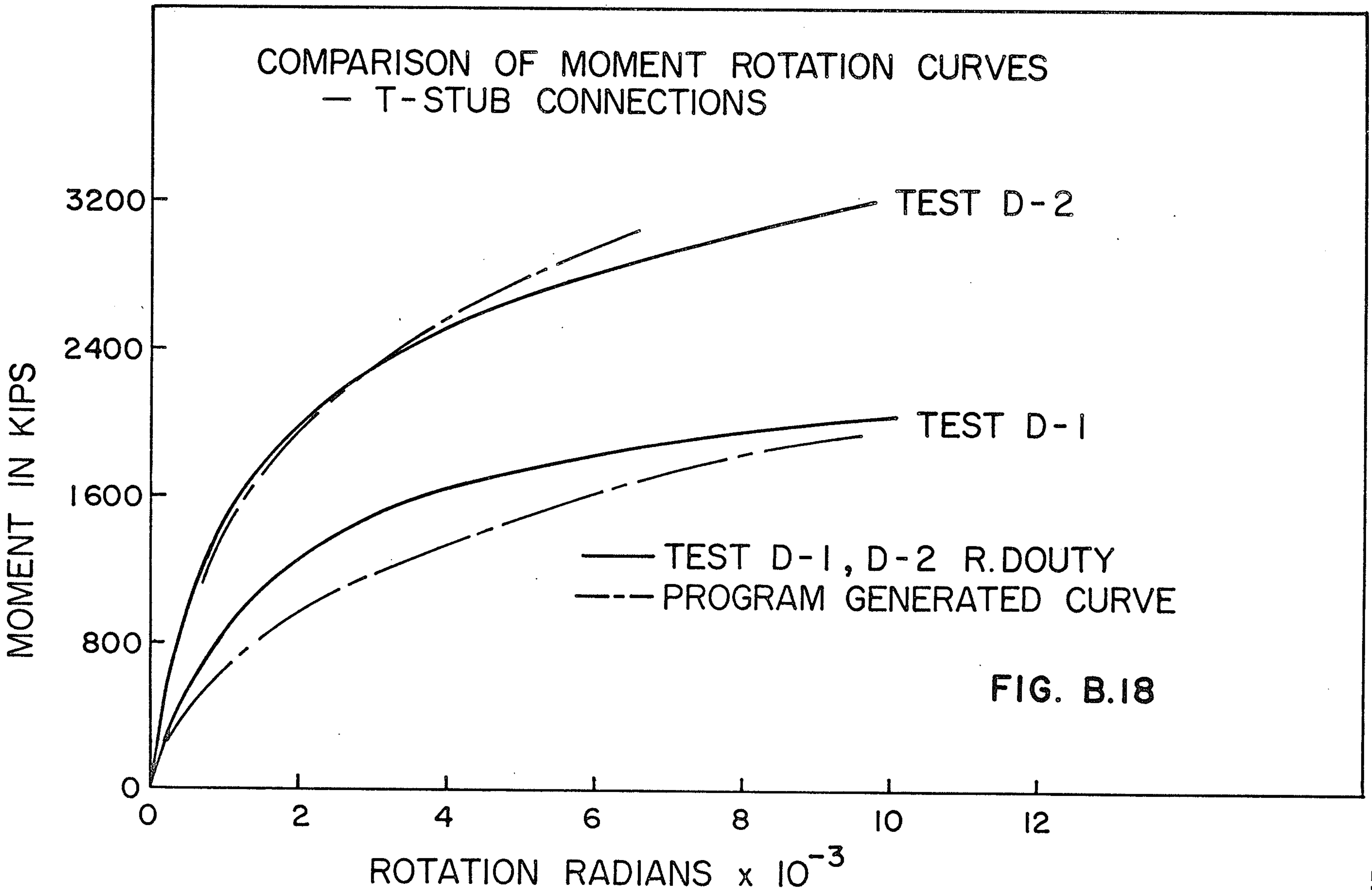
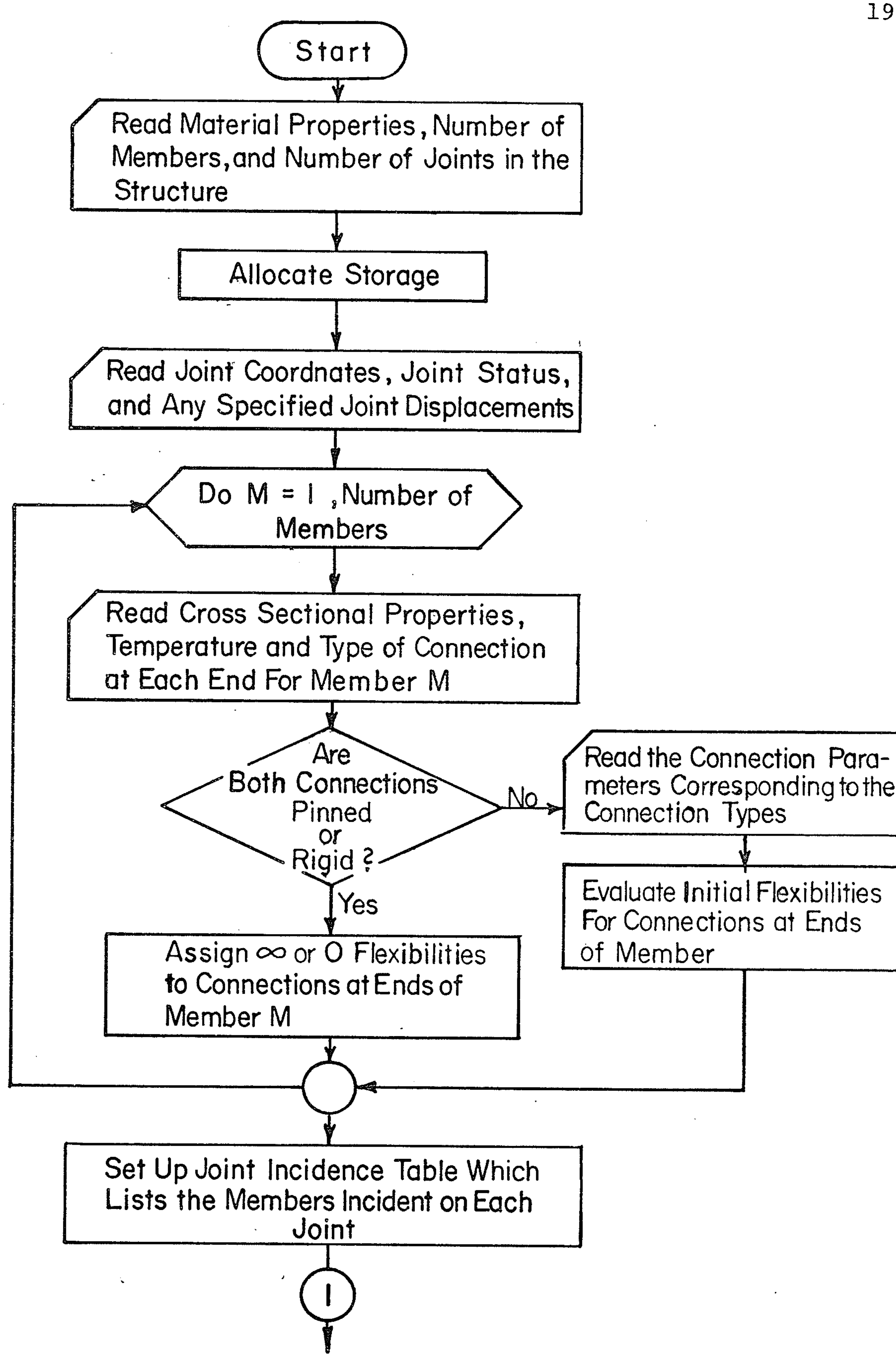
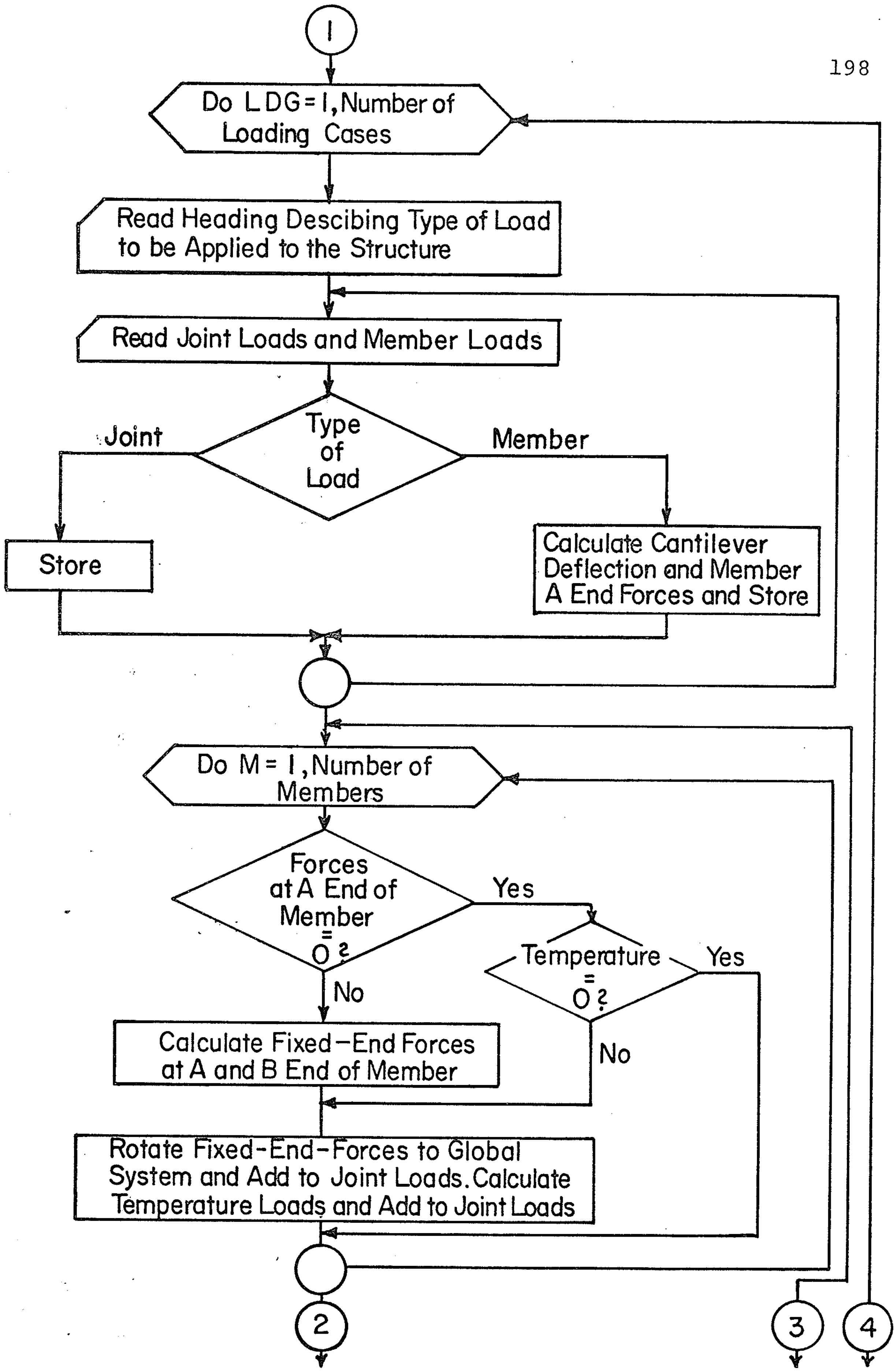

COMPARISON OF MOMENT ROTATION CURVES
— T-STUB CONNECTIONS
3200
2400
1600
800
0
MOMENT IN KIPS
TEST D-2
TEST D-1
TEST D-1, D-2 R.DOUTY
PROGRAM GENERATED CURVE
0
2
4
6
8
10
12
ROTATION RADIANS x 10^{-3}

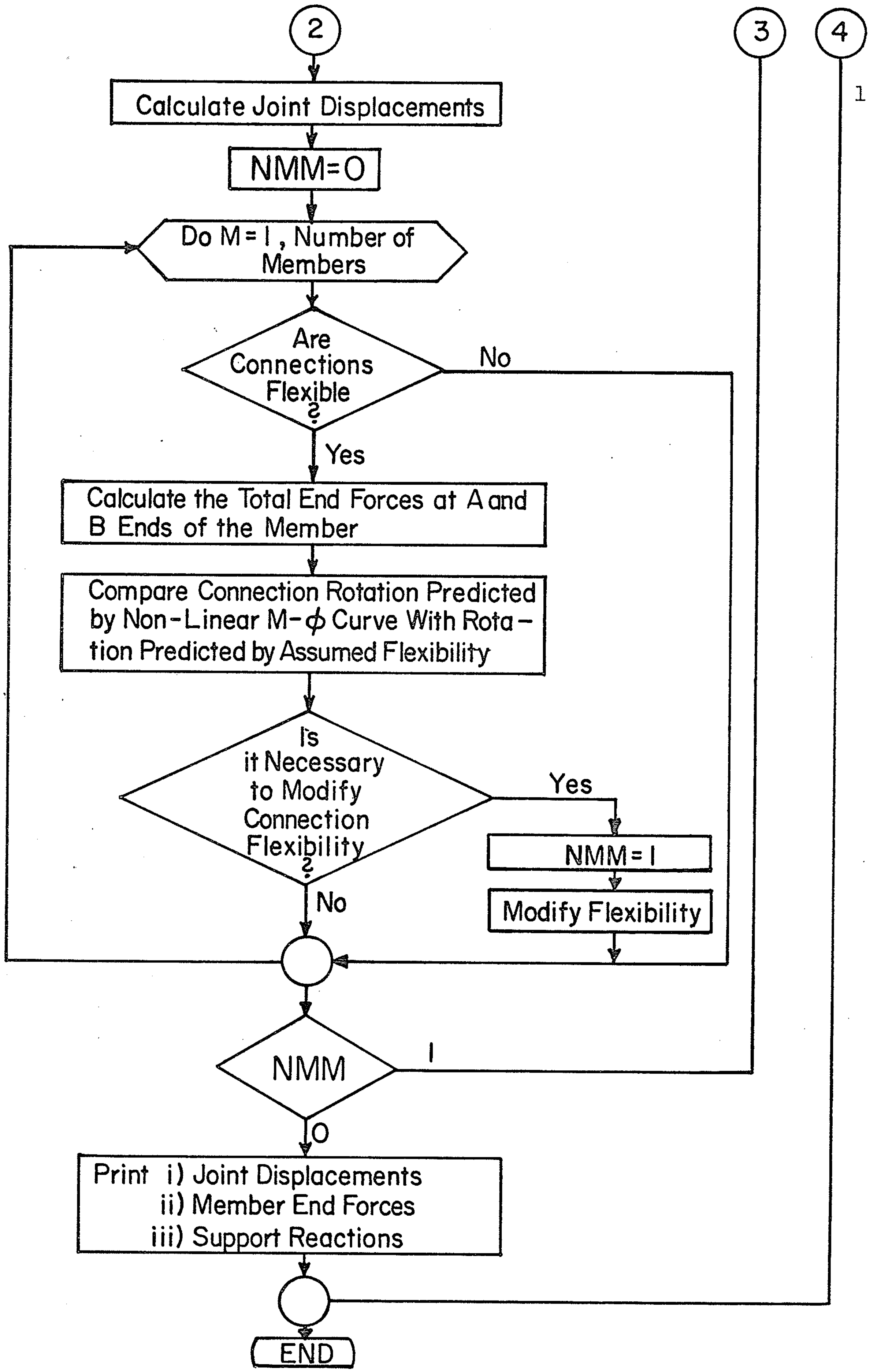
FIG. B.18

APPENDIX C

FLOW DIAGRAM


Start
Read Material Properties, Number of Members, and Number of Joints in the Structure
Allocate Storage
Read Joint Coordnates, Joint Status, and Any Specified Joint Displacements
Do M = 1, Number of Members
Read Cross Sectional Properties, Temperature and Type of Connection at Each End For Member M
Are Both Connections Pinned or Rigid ?
No
Read the Connection Parameters Corresponding to the Connection Types
Evaluate Initial Flexibilities For Connections at Ends of Member
Yes
Assign ∞ or 0 Flexibilities to Connections at Ends of Member M
Set Up Joint Incidence Table Which Lists the Members Incident on Each Joint
1


1
Do LDG = 1, Number of Loading Cases
Read Heading Descibing Type of Load to be Applied to the Structure
Read Joint Loads and Member Loads
Type of Load
Joint
Member
Store
Calculate Cantilever Deflection and Member A End Forces and Store
Do M = 1, Number of Members
Forces at A End of Member = 0?
Yes
No
Temperature = 0?
Yes
No
Calculate Fixed-End Forces at A and B End of Member
Rotate Fixed-End-Forces to Global System and Add to Joint Loads. Calculate Temperature Loads and Add to Joint Loads
2
3
4



2
3
4
Calculate Joint Displacements
NMM=O
Do M = I , Number of Members
Are Connections Flexible ?
No
Yes
Calculate the Total End Forces at A and B Ends of the Member
Compare Connection Rotation Predicted by Non-Linear M-ϕ Curve With Rotation Predicted by Assumed Flexibility
Is it Necessary to Modify Connection Flexibility ?
Yes
NMM = I
Modify Flexibility
No
NMM
I
O
Print i) Joint Displacements
ii) Member End Forces
iii) Support Reactions
END

APPENDIX D

USER'S MANUAL

1. IDENTIFICATION

SRFRAME - this program performs a linear structural analysis of planar frames whose connections have any degree of rotational flexibility. The flexibility of a connection may be specified in any of the following ways:

(a) connection may be rigid

(b) connection may be pinned

(c) connection flexibility may be specified numerically

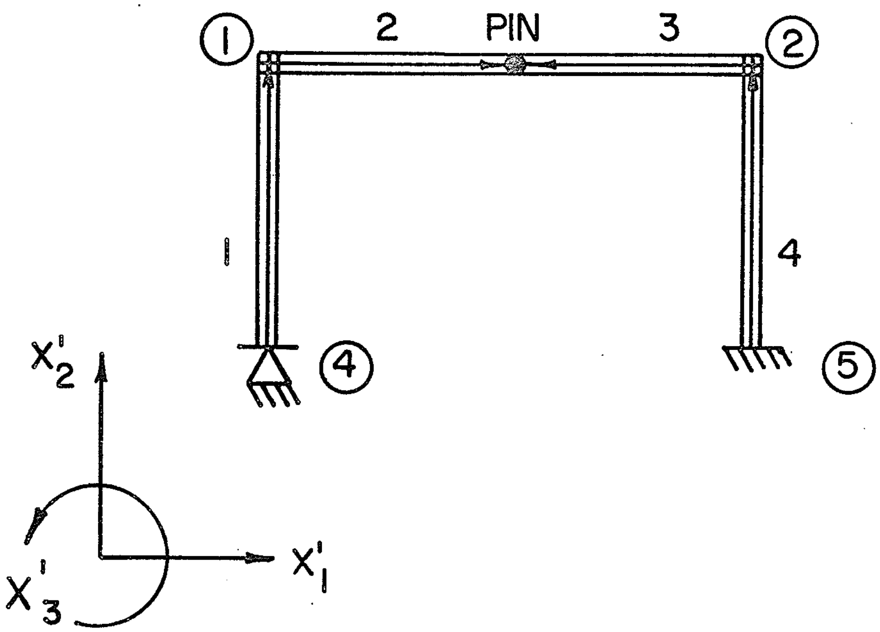
(d) one of eight standard orthogonal connection types may be specified along with its size parameters

Any number of consecutive loading systems can be considered but they cannot be superimposed.

2. DESCRIPTION OF STRUCTURE AND LOADING

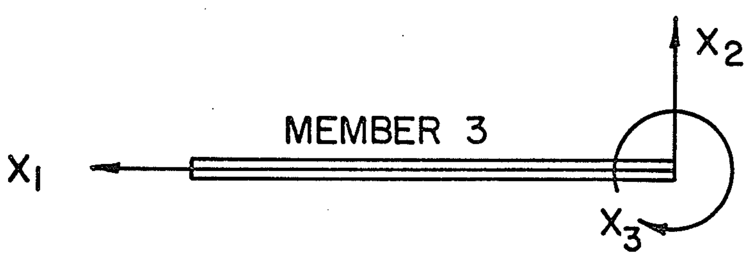
The input consists of a description of the structure and each loading system.

In describing the structure, all joints are numbered in an arbitrary sequence as illustrated in Fig. D.1(a). All members are numbered and each member is arbitrarily assigned



GLOBAL COORDINATE SYSTEM

(a)



MEMBER 3 LOCAL COORDINATE SYSTEM

(b)

IDENTIFICATION OF STRUCTURE

FIG. D.1

a direction, as illustrated by the arrows in the figure.

Two different types of coordinate systems are used:

(a) Global system - a single right handed coordinate system applicable to the whole structure. Its origin can be located anywhere and all loadings, joint coordinates, joint displacements, and support reactions are expressed in the global system.

(b) Member system - Each member has associated with it a right hand local coordinate system whose X_1 axis has the same direction as that assumed for the member, as illustrated for member 3 in Fig. D.1(b). The member is assumed to have a "start" and an "end" as shown, and the positive directions for the member end axial forces, shear forces, and moments are the positive X_1, X_2, and X_3 directions as shown. Regardless of the member orientation, axis X_2 is above or in the horizontal plane containing the origin, and direction X_3 is clockwise or counterclockwise depending on whether X_1 is directed to the right or to the left.

The size of structure that can be analyzed depends on the available storage. The number of words of memory required for data storage for a given structure is approximately:

$$(NJ)^2 + 22(NJ) + 17(NM) = Z \tag{D.1}$$

where:

NJ = number of joints in structure

NM = number of members in structure

Z = total number of data quantities to be stored.

The program employs a one-dimensional data storage pool, and Table D-1 lists the maximum permissible size of storage pool corresponding to several different core allocations for the IBM system 360/65 computer.

Table D-1 Storage Capacities

Available Core	Maximum Z Dimension
150 K	22,500
200 K	35,000
250 k	47,500
300 K	60,000

3. CONNECTION INFORMATION

The program is capable of analyzing structures which include any of the following connection types:

(a) double web angle connections

(b) single web angle connections

(c) header plate connections

(d) top and seat angle connections

(e) end plate connections with column stiffeners

(f) end plate connections without column stiffeners

(g) T-stub connections.

In addition, rigid and pinned connections and those with a numerically specified flexibility can be included.

Connection types (a) to (g) inclusive have their flexibilities generated by the program. For each connection in the structure, it is necessary to input one or more parameters which are used to generate the moment-rotation information for the connection.

4. INPUT

The program input is described with reference to the example below. Except for descriptive heading cards, each data card is divided into 10 column fields. Each data item can be placed anywhere in its field and decimal points are optional.

DATA CARDS:

(a) Program Name - SRFRAME

(b) Job Description - card to contain a job description which is printed as a heading over output.

(c) Structure Information

Field 1 - number of joints

Field 2 - number of members

Field 3 - modulus of elasticity E (ksi).

(d) Joint Information - (one card for each joint)

Field 1 - joint status:

blank = non-support joint

F or FIXED = fixed support

H or HORIZ = horizontal roller

V or Vert = vertical roller

R or ROTATION = pin

(combinations of H, V, and R may be used).

Field 2 - joint number

Field 3 - joint X (horizontal) coordinate (ft.)

Field 4 - joint Y (vertical) coordinate (ft.).

(e) <u>Member Information</u> - (One card is required for each member with any combination of rigid or pinned connections. Two cards are required for members with any of the standard connection types listed above or connections with numerically specified flexibility.)

First Card:

Field 1 - member number

Field 2 - number of joint at member "start"

Field 3 - number of joint at member "end"

Field 4, 5 - member area, A (sq. in.) and moment of inertia, I (in^4). If A or I is left blank, the value is assumed to be the same as for the preceding member; if no values are supplied, the following are assumed: A = 5.0 sq. in., I = 100.0 in^4.

Field 6 - member temperature. If member temperatures are provided, temperature displacements and forces are incorporated into the analysis; otherwise, temperature effects are ignored.

Field 7 - connection type at member "start"

Field 8 - connection type at member "end".

Continuation Card - An asterisk (*) in column 1 of a projection of member

Field 4 - vertical load (kips/ft.) on horizontal projection

Field 5 - distance (ft.) to start of load

Field 6 - distance (ft.) from end of member load to end of member.

(iii) Concentrated member load -

Field 1 - P

Field 2 - member number

Field 3 - horizontal load (kips)

Field 4 - vertical load (kips)

Field 5 - distance (ft.) from member "start".

(h) <u>Solve</u> - card to contain the word SOLVE. This instructs the computer to begin analysis.

5. <u>OUTPUT</u>

The output consists of the following:

(a) a listing of all input quantities

(b) connection flexibilities as generated

(c) joint displacements

(d) member end forces

(e) support reactions

(f) volume and weight of steel in structure.

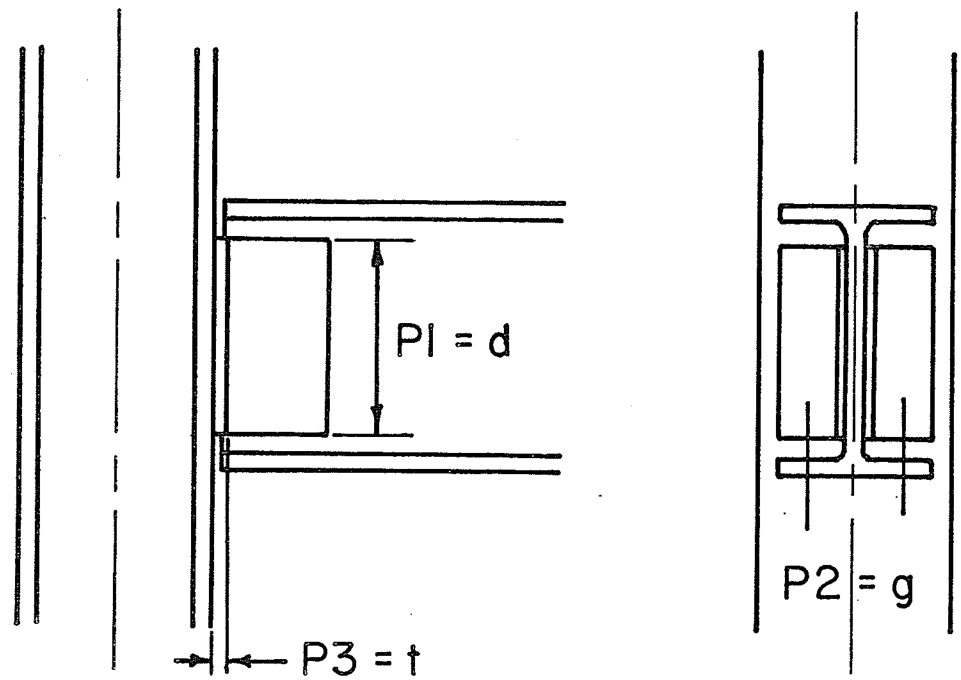


FIG. D.2 DOUBLE WEB ANGLE CONNECTIONS

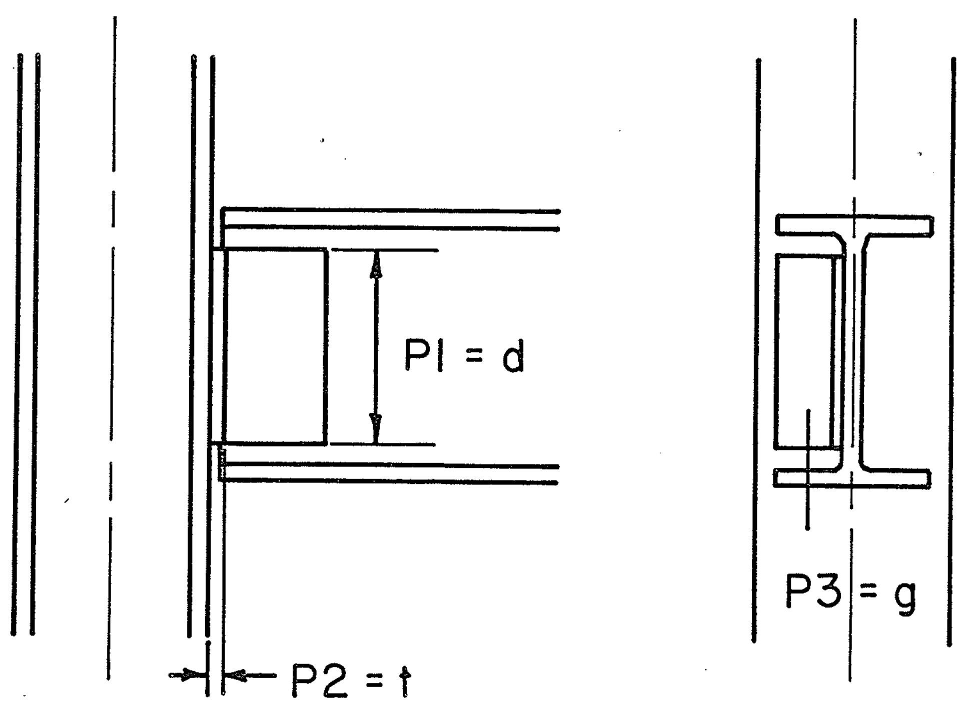


FIG. D.3 SINGLE WEB ANGLE CONNECTIONS

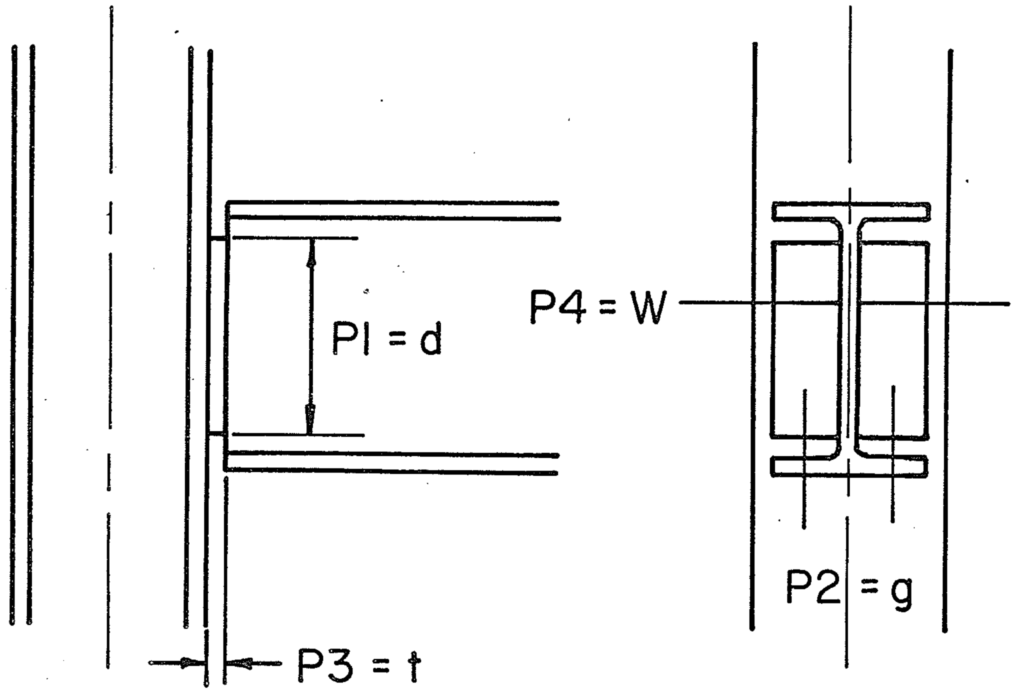


FIG. D.4 HEADER PLATE CONNECTIONS

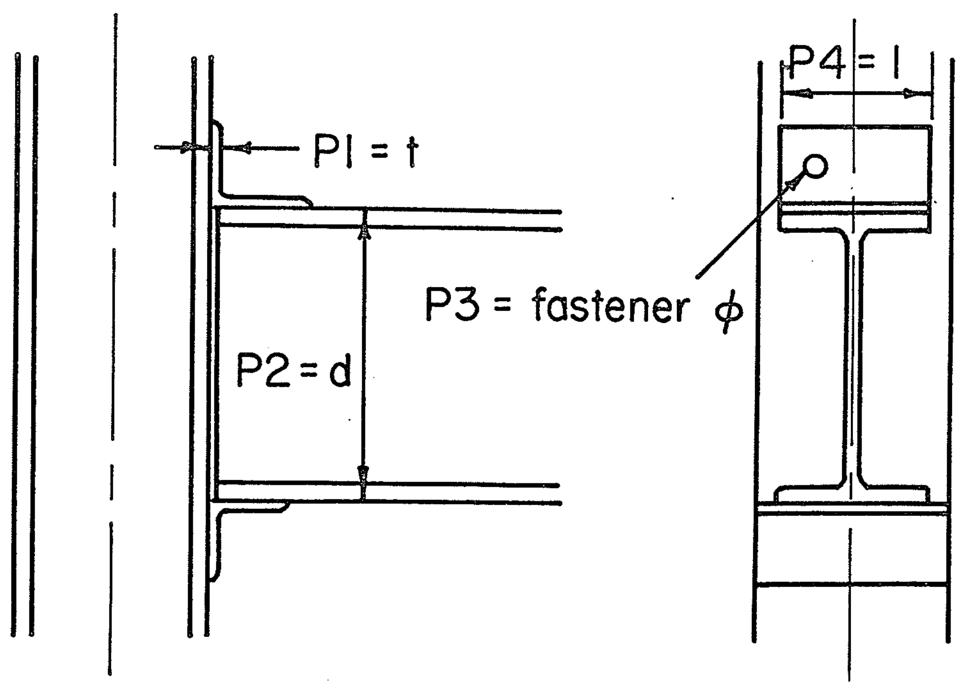


FIG. D.5 TOP AND SEAT ANGLE CONNECTIONS

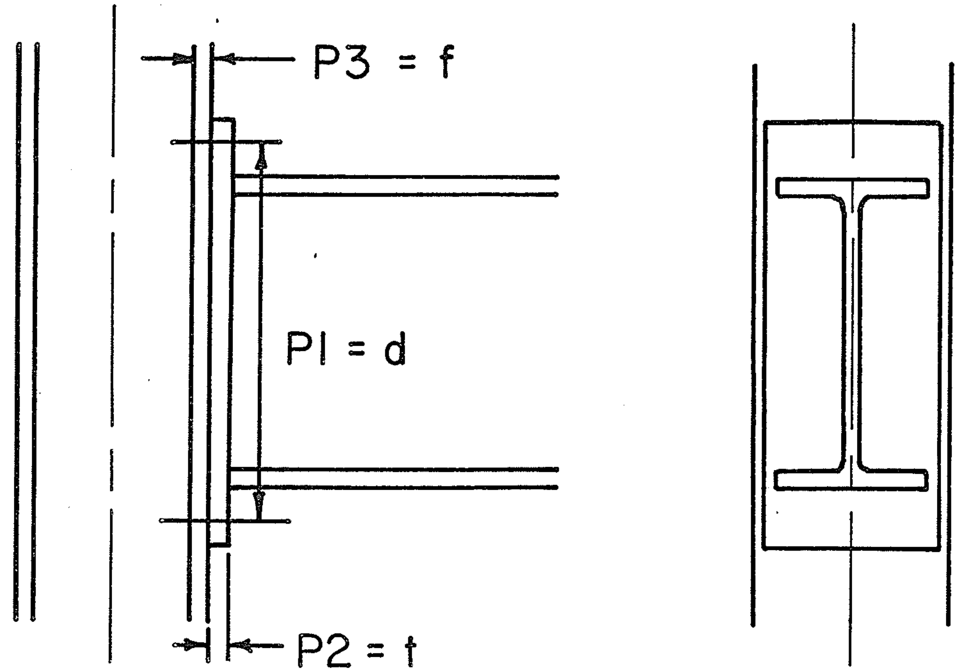


FIG. D.6 END PLATE CONNECTIONS
NO COLUMN STIFFENERS

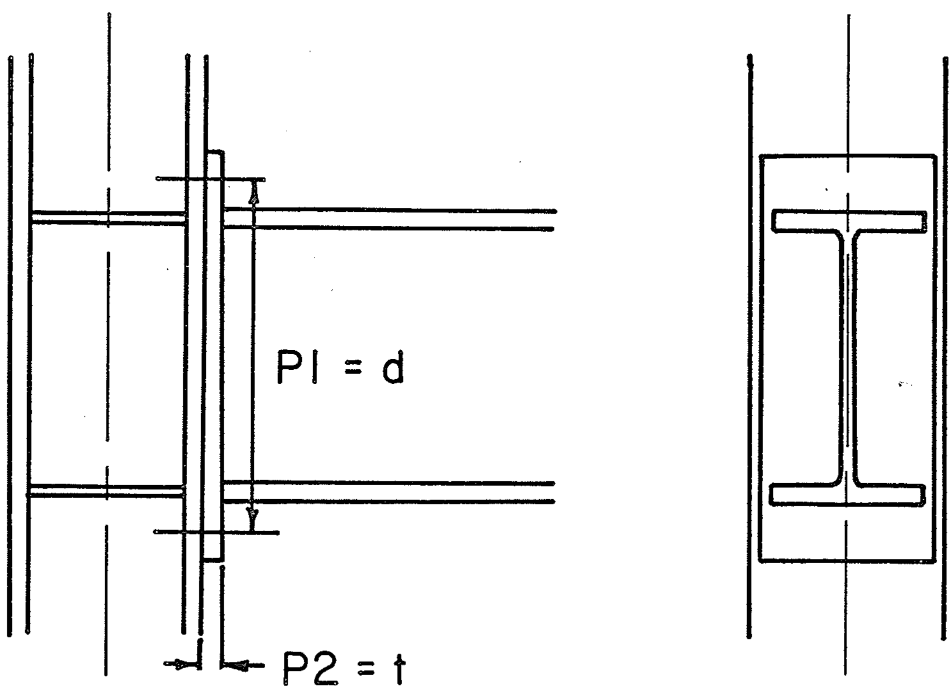


FIG. D.7 END PLATE CONNECTIONS WITH
COLUMN STIFFENERS

APPENDIX E

PROGRAM LISTING

```
C     ****************************************************************
C
C
C                         MAIN PROGRAM
C
C
C     ****************************************************************
      DIMENSION Z(20000)
      INTEGER*2 INP(80), LDTYP(4, 80)
      COMMON E, M, DL, JRD, JWT, FN(14), INP/MN/NJ, NM, LDTYP, HDG(20),
     &ALPHA
      JRD = 5
      JWT = 6
C     READ JOB TITLE
C     ********************READ************************************
   10 READ (JRD,40,END=30) HDG
      NC = NC+1
      WRITE (JWT,50) HDG
      WRITE (JWT,60)
C     ********************READ************************************
      READ (JRD,70) INP
      CALL CNVRT(1, 1, 3)
      NJ = FN(1)
      NM = FN(2)
      E = FN(3)
      ALPHA = .0000065
      NEE = (NM-1)/2+1
      NNN = (NJ-1)/2+1
      N1 = 1
      N2 = N1+2*NJ
      N3 = N2+3*NM
      N4 = N3+3*NM
      N5 = N4+NNN
      N6 = N5+6*NNN
      N7 = N6+3*NJ
```

```
      N8 = N7+2*NEE                                                     MAIN0360
      N9 = N8+NM                                                        MAIN0370
      N10 = N9+NM                                                       MAIN0380
      N11 = N10+NM                                                      MAIN0390
      N12 = N11+NEE                                                     MAIN0400
      N13 = N12+NEE                                                     MAIN0410
      N14 = N13+NNN                                                     MAIN0420
      N15 = N14+9*NJ                                                    MAIN0430
      N16 = N15+NNN+1                                                   MAIN0440
      N17 = N16+2*NM                                                    MAIN0450
      N18 = N17+3*NJ                                                    MAIN0460
      N19 = N18+NM                                                      MAIN0470
      N20 = N19+NM                                                      MAIN0480
      N21 = N20+NM                                                      MAIN0490
      N22 = N21+NM                                                      MAIN0500
      CALL PLFR(Z(N1), Z(N2), Z(N3), Z(N4), Z(N5), Z(N6), Z(N7), Z(N8), MAIN0510
     &Z(N9), Z(N10), Z(N11), Z(N12), Z(N13), Z(N14), Z(N15), Z(N16), Z( MAIN0520
     &N17), Z(N18), Z(N19), Z(N20), Z(N21), Z(N22))                     MAIN0530
      DO 20 I = 1, 80                                                   MAIN0540
   20 LDTYP(1, I) = INP(1)                                              MAIN0550
      GO TO 10                                                          MAIN0560
   30 STOP                                                              MAIN0570
   40 FORMAT (20A4)                                                     MAIN0580
   50 FORMAT ('1'///1X20A4)                                             MAIN0590
   60 FORMAT (//'  ANALYSIS OF PLANE FRAME WITH RIGID , SEMI-RIGID , OR MAIN0600
     &SIMPLE CONNECTIONS'//'  INPUT DATA'//)                            MAIN0610
   70 FORMAT (80A1)                                                     MAIN0620
      END                                                               MAIN0630
C      *****************************************************************PLFR0010
C                                                                       PLFR0020
C                           SUBROUTINE PLFR                             PLFR0030
C                                                                       PLFR0040
C      *****************************************************************PLFR0050
      SUBROUTINE PLFR(CJ, FA, FB, NMIJ, JI, JL, MI, AR, XI, TEM, MSRA,  PLFR0060
     &MSRB, ISR, A, LIST, C, PJ, SLPA, SLPB, CONA, CONB, STORE)         PLFR0070
```

```
      REAL KBB(3, 3), KBA(3, 3), JL                                     PLFR0080
      COMMON E, M, DL, JRD, JWT, FN(14), INP/MN/NJ, NM, LDTYP, HDG(20), PLFR0090
     &ALPHA/SPL/KBB, DSTIF/RT/COSA, SINA, R(3, 3), H(3, 3)              PLFR0100
      INTEGER*2 MI, ISR, MSRA, MSRB, LIST, NMIJ, JI, INP(80), LDTYP(4,  PLFR0110
     &80)                                                               PLFR0120
      DIMENSION CJ(2, 1), MI(2, 1), AR(1), XI(1), ISR(1), MSRA(1), MSRB(PLFR0130
     &1), TEM(1), NMIJ(1), JI(6, 1), JL(3, 1), SLPA(1), SLPB(1), CONA(1)PLFR0140
     &, CONB(1)                                                         PLFR0150
      DIMENSION PJ(3, 1)                                                PLFR0160
      DIMENSION FEFA(3), FEFB(3), PLME(3), D(3), FLBB(3, 3)             PLFR0170
      DIMENSION A(3, 3, 1), LIST(1), B(3, 2), BB(3, 3, 2)               PLFR0180
      DIMENSION C(2, 1)                                                 PLFR0190
      DIMENSION STORE(3, 3, 1)                                          PLFR0200
      DIMENSION FA(3, 1), FB(3, 1), TEMP(3, 3), TEMP1(3), TEMP11(3)     PLFR0210
      DIMENSION TEMP2(3, 3), TEMP3(3)                                   PLFR0220
      DIMENSION D4(6), D3(3), D1(3), D2(3)                              PLFR0230
      INTEGER*2 INPT(6)/' ', 'F', 'H', 'V', 'R', 'D'/                   PLFR0240
      INTEGER*2 INPTC(11)/' ', 'P', 'S', 'A', 'B', 'C', 'D', 'E', 'F', 'PLFR0250
     &G', 'H'/                                                          PLFR0260
      INTEGER*2 IU/'U'/, IP/'P'/                                        PLFR0270
      INTEGER*2 CONT/'*'/                                               PLFR0280
      INTEGER INSRC(11)/'PIN ', 'SPEC', 'DWEB', 'SWEB', 'HPLT', 'T&SE', PLFR0290
     &'EPLT', 'EPLT', 'TSTB', 'TPLT', 'RIGD'/                           PLFR0300
      INTEGER INSR(6)/'H   ', 'V   ', 'H V ', 'R   ', 'H,R ', 'V R '/   PLFR0310
      INTEGER*2 ILD(5)/'S', 'O', 'L', 'V', 'E'/                         PLFR0320
      EQUIVALENCE (NE, NJ)                                              PLFR0330
      DO 20 I = 1, 3                                                    PLFR0340
      DO 20 J = 1, 3                                                    PLFR0350
      R(I, J) = 0.                                                      PLFR0360
   10 H(I, J) = 0.                                                      PLFR0370
   20 H(I, I) = 1.                                                      PLFR0380
      J = 0                                                             PLFR0390
      DO 90 JJ = 1, NJ                                                  PLFR0400
      J = JJ                                                            PLFR0410
C       ****************   READ   ******************************        PLFR0420
```

```
C       NON-BLANK CHARACTER IN COL 80. CONVERT COORDINATES TO INCHES.      PLFR0430
C       READ JOINT STATUS AND COORDINATES OF JOINTS. LAST JOINT GIVEN      PLFR0440
        READ (JRD,1450,END=1440) INP                                       PLFR0450
        CALL CNVRT(1, 2, 8)                                                PLFR0460
        IF (FN(1) .NE. 0.) J = FN(1)                                       PLFR0470
        CJ(1, J) = FN(2)                                                   PLFR0480
        CJ(2, J) = FN(3)                                                   PLFR0490
        JL(1, J) = FN(4)                                                   PLFR0500
        JL(2, J) = FN(5)                                                   PLFR0510
        JL(3, J) = FN(6)                                                   PLFR0520
        IF (FN(7) .NE. 0.) ALPHA = FN(7)                                   PLFR0530
        ISR(J) = 0                                                         PLFR0540
        I = 1                                                              PLFR0550
     30 IF (INP(I) .EQ. INPT(1)) GO TO 80                                  PLFR0560
        IF (INP(I) .EQ. INPT(2) .OR. INP(I) .EQ. INPT(6)) GO TO 70         PLFR0570
     40 IF (INP(I) .NE. INPT(3)) GO TO 50                                  PLFR0580
        ISR(J) = ISR(J)+1                                                  PLFR0590
        GO TO 80                                                           PLFR0600
     50 IF (INP(I) .NE. INPT(4)) GO TO 60                                  PLFR0610
        ISR(J) = ISR(J)+2                                                  PLFR0620
        GO TO 80                                                           PLFR0630
     60 IF (INP(I) .NE. INPT(5)) GO TO 70                                  PLFR0640
        ISR(J) = ISR(J)+4                                                  PLFR0650
        GO TO 80                                                           PLFR0660
     70 ISR(J) = 8                                                         PLFR0670
        GO TO 90                                                           PLFR0680
     80 I = I+1                                                            PLFR0690
        IF (I .LE. 10) GO TO 30                                            PLFR0700
     90 CONTINUE                                                           PLFR0710
        M = 0                                                              PLFR0720
        AO = 5.                                                            PLFR0730
        XO = 100.                                                          PLFR0740
C        ****************   READ   *******************************         PLFR0750
    100 READ (JRD,1450,END=1440) INP                                       PLFR0760
    110 M = M+1                                                            PLFR0770
```

```
      CALL CNVRT(1, 1, 6)                                               PLFR0780
      IF (FN(1) .NE. 0.) M = FN(1)                                      PLFR0790
      MSRA(M) = 0                                                       PLFR0800
      MSRB(M) = 0                                                       PLFR0810
      TEM(M) = FN(6)                                                    PLFR0820
      IF (FN(4) .NE. 0.) AO = FN(4)                                     PLFR0830
      IF (FN(5) .NE. 0.) XO = FN(5)                                     PLFR0840
      AR(M) = AO                                                        PLFR0850
      XI(M) = XO                                                        PLFR0860
      MI(1, M) = FN(2)                                                  PLFR0870
      MI(2, M) = FN(3)                                                  PLFR0880
      DO 220 I = 61, 70                                                 PLFR0890
      IF (INP(I) .EQ. INPTC(1)) GO TO 220                               PLFR0900
      IF (INP(I) .NE. INPTC(2)) GO TO 120                               PLFR0910
      MSRA(M) = 1                                                       PLFR0920
      GO TO 230                                                         PLFR0930
  120 IF (INP(I) .NE. INPTC(3)) GO TO 130                               PLFR0940
      MSRA(M) = 2                                                       PLFR0950
      GO TO 230                                                         PLFR0960
  130 IF (INP(I) .NE. INPTC(4)) GO TO 140                               PLFR0970
      MSRA(M) = 3                                                       PLFR0980
      GO TO 230                                                         PLFR0990
  140 IF (INP(I) .NE. INPTC(5)) GO TO 150                               PLFR1000
      MSRA(M) = 4                                                       PLFR1010
      GO TO 230                                                         PLFR1020
  150 IF (INP(I) .NE. INPTC(6)) GO TO 160                               PLFR1030
      MSRA(M) = 5                                                       PLFR1040
      GO TO 230                                                         PLFR1050
  160 IF (INP(I) .NE. INPTC(7)) GO TO 170                               PLFR1060
      MSRA(M) = 6                                                       PLFR1070
      GO TO 230                                                         PLFR1080
  170 IF (INP(I) .NE. INPTC(8)) GO TO 180                               PLFR1090
      MSRA(M) = 7                                                       PLFR1100
      GO TO 230                                                         PLFR1110
  180 IF (INP(I) .NE. INPTC(9)) GO TO 190                               PLFR1120
```

```
      MSRA(M) = 8
      GO TO 230
  190 IF (INP(I) .NE. INPTC(10)) GO TO 200
      MSRA(M) = 9
      GO TO 230
  200 IF (INP(I) .EQ. INPTC(11)) GO TO 210
      GO TO 1430
  210 MSRA(M) = 10
      GO TO 230
  220 CONTINUE
  230 CONTINUE
      DO 340 I = 71, 80
      IF (INP(I) .EQ. INPTC(1)) GO TO 340
      IF (INP(I) .NE. INPTC(2)) GO TO 240
      MSRB(M) = 1
      GO TO 350
  240 IF (INP(I) .NE. INPTC(3)) GO TO 250
      MSRB(M) = 2
      GO TO 350
  250 IF (INP(I) .NE. INPTC(4)) GO TO 260
      MSRB(M) = 3
      GO TO 350
  260 IF (INP(I) .NE. INPTC(5)) GO TO 270
      MSRB(M) = 4
      GO TO 350
  270 IF (INP(I) .NE. INPTC(6)) GO TO 280
      MSRB(M) = 5
      GO TO 350
  280 IF (INP(I) .NE. INPTC(7)) GO TO 290
      MSRB(M) = 6
      GO TO 350
  290 IF (INP(I) .NE. INPTC(8)) GO TO 300
      MSRB(M) = 7
      GO TO 350
  300 IF (INP(I) .NE. INPTC(9)) GO TO 310
```

```
      MSRB(M) = 8
      GO TO 350
  310 IF (INP(I) .NE. INPTC(10)) GO TO 320
      MSRB(M) = 9
      GO TO 350
  320 IF (INP(I) .EQ. INPTC(11)) GO TO 330
      GO TO 1430
  330 MSRB(M) = 10
      GO TO 350
  340 CONTINUE
C     *****************************READ*********************************
  350 READ (JRD,1450,END=1440) INP
      IF (INP(1) .EQ. CONT) GO TO 420
      IF (MSRA(M) .EQ. 0) GO TO 360
      IF (MSRA(M) .EQ. 1) GO TO 370
      GO TO 1430
  360 C(1, M) = 0.
      GO TO 380
  370 C(1, M) = 10.*10.**25
      GO TO 380
  380 IF (MSRB(M) .EQ. 0) GO TO 390
      IF (MSRB(M) .EQ. 1) GO TO 400
      GO TO 1430
  390 C(2, M) = 0.
      GO TO 410
  400 C(2, M) = 10.*10.**25
      GO TO 410
  410 CONTINUE
      IF (M .NE. NM) GO TO 110
      GO TO 430
  420 INP(1) = INPT(1)
      CALL CNVRT(7, 1, 8)
      KK = MSRA(M)
      JJ = MSRB(M)
      IF (JJ .EQ. 0) JJ = 11
```

```
      IF (KK .EQ. 0) KK = 11                                            PLFR1830
      CALL GENCUR(KK, JJ, C, SLPA, SLPB, CONA, CONB)                    PLFR1840
      IF (M .NE. NM) GO TO 100                                          PLFR1850
C      *****************************READ*****************************PLFR1860
      READ (JRD,1450,END=1440) INP                                      PLFR1870
  430 CONTINUE                                                          PLFR1880
      WRITE (JWT,1460) NM, NJ, E, ALPHA                                 PLFR1890
      WRITE (JWT,1470)                                                  PLFR1900
      II = 1                                                            PLFR1910
      DO 470 J = 1, NJ                                                  PLFR1920
      IF (ISR(J) .NE. 0) GO TO 440                                      PLFR1930
      WRITE (JWT,1540) J, CJ(1, J), CJ(2, J)                            PLFR1940
      GO TO 470                                                         PLFR1950
  440 IF (ABS(JL(1, J))+ABS(JL(2, J))+ABS(JL(3, J)) .LT. .0001) GO TO  PLFR1960
     &450                                                               PLFR1970
      WRITE (JWT,1480) J, CJ(1, J), CJ(2, J), (JL(I, J), I = 1, 3)      PLFR1980
      GO TO 470                                                         PLFR1990
  450 IF (ISR(J) .LT. 8) GO TO 460                                      PLFR2000
      WRITE (JWT,1490) J, CJ(1, J), CJ(2, J)                            PLFR2010
      GO TO 470                                                         PLFR2020
  460 K = ISR(J)                                                        PLFR2030
      WRITE (JWT,1490) J, CJ(1, J), CJ(2, J), INSR(K)                   PLFR2040
  470 CONTINUE                                                          PLFR2050
  480 DO 490 J = 1, NJ                                                  PLFR2060
      DO 490 I = 1, 2                                                   PLFR2070
  490 CJ(I, J) = CJ(I, J)*12.                                           PLFR2080
C                                                                       PLFR2090
C     GENERATE JOINT INCIDENCE TABLE                                    PLFR2100
C                                                                       PLFR2110
      DO 500 J = 1, NJ                                                  PLFR2120
      NMIJ(J) = 0                                                       PLFR2130
      DO 500 M = 1, 6                                                   PLFR2140
  500 JI(M, J) = 0                                                      PLFR2150
      DO 510 M = 1, NM                                                  PLFR2160
      J = MI(1, M)                                                      PLFR2170
```

```
      NMIJ(J) = NMIJ(J)+1                                               PLFR2180
      K = NMIJ(J)                                                       PLFR2190
      JI(K, J) = -M                                                     PLFR2200
      J = MI(2, M)                                                      PLFR2210
      NMIJ(J) = NMIJ(J)+1                                               PLFR2220
      K = NMIJ(J)                                                       PLFR2230
  510 JI(K, J) = M                                                      PLFR2240
      WRITE (JWT,1500)                                                  PLFR2250
      DO 520 M = 1, NM                                                  PLFR2260
      K = MSRA(M)                                                       PLFR2270
      J = MSRB(M)                                                       PLFR2280
      IF (K .EQ. 0) K = 11                                              PLFR2290
      IF (J .EQ. 0) J = 11                                              PLFR2300
  520 WRITE (JWT,1530) M, MI(1, M), MI(2, M), AR(M), XI(M), INSRC(K),   PLFR2310
     &INSRC(J), TEM(M)                                                  PLFR2320
      WRITE (JWT,1510)                                                  PLFR2330
      DO 530 M = 1, NM                                                  PLFR2340
  530 WRITE (JWT,1520) M, C(1, M), C(2, M)                              PLFR2350
      LDG = 1                                                           PLFR2360
  540 DO 550 J = 1, NJ                                                  PLFR2370
      PJ(1, J) = 0.                                                     PLFR2380
      PJ(2, J) = 0.                                                     PLFR2390
  550 PJ(3, J) = 0.                                                     PLFR2400
      DO 570 M = 1, NM                                                  PLFR2410
      DO 560 III = 1, 3                                                 PLFR2420
      FA(III, M) = 0.                                                   PLFR2430
      FB(III, M) = 0.                                                   PLFR2440
  560 CONTINUE                                                          PLFR2450
  570 CONTINUE                                                          PLFR2460
      IF (LDG .GT. 1) GO TO 590                                         PLFR2470
      DO 580 I = 1, 80                                                  PLFR2480
  580 LDTYP(LDG, I) = INP(I)                                            PLFR2490
  590 DO 610 IL = 1, 80                                                 PLFR2500
      IF (LDTYP(LDG, IL) .EQ. INPT(1)) GO TO 610                        PLFR2510
      DO 600 I = 1, 5                                                   PLFR2520
```

```
      J = IL+I-1                                                        PLFR2530
      IF (LDTYP(LDG, J) .NE. ILD(I)) GO TO 620                          PLFR2540
  600 CONTINUE                                                          PLFR2550
      GO TO 880                                                         PLFR2560
  610 CONTINUE                                                          PLFR2570
  620 CONTINUE                                                          PLFR2580
      WRITE (JWT,1550) (LDTYP(LDG, I), I = 1, 80)                       PLFR2590
      KK = 0                                                            PLFR2600
      LL = 0                                                            PLFR2610
      ITER = 0                                                          PLFR2620
C      *******************************READ*****************************PLFR2630
  630 READ (JRD,1450,END=1440) INP                                      PLFR2640
      CALL CNVRT(1, 2, 6)                                               PLFR2650
      IF (FN(1) .EQ. 0.) GO TO 710                                      PLFR2660
      II = FN(1)                                                        PLFR2670
      DO 640 I = 1, 10                                                  PLFR2680
      IF (INP(I) .NE. INPT(1)) GO TO 650                                PLFR2690
  640 CONTINUE                                                          PLFR2700
      IF (KK .EQ. 0) WRITE (JWT,1560)                                   PLFR2710
      KK = KK+1                                                         PLFR2720
      PJ(1, II) = FN(2)                                                 PLFR2730
      PJ(2, II) = FN(3)                                                 PLFR2740
      WRITE (JWT,1540) II, FN(2), FN(3), FN(4)                          PLFR2750
      PJ(3, II) = FN(4)*12.                                             PLFR2760
      GO TO 630                                                         PLFR2770
  650 IF (LL .EQ. 0) WRITE (JWT,1680)                                   PLFR2780
      LL = LL+1                                                         PLFR2790
      IF (INP(I) .NE. IU) GO TO 660                                     PLFR2800
      IF (II .NE. 0) M = II                                             PLFR2810
      W1 = FN(2)                                                        PLFR2820
      W2 = FN(3)                                                        PLFR2830
      WRITE (JWT,1690) M, INP(I), W1, W2                                PLFR2840
      S = FN(4)*12.                                                     PLFR2850
      T = FN(5)*12.                                                     PLFR2860
      CALL ROT(CJ, MI)                                                  PLFR2870
```

```
      WW1 = W1/12.*ABS(SINA)                                            PLFR2880
      WW2 = W2/12.*ABS(COSA)                                            PLFR2890
      W1 = (R(1, 1)*WW1+R(2, 1)*WW2)                                    PLFR2900
      W2 = (R(1, 2)*WW1+R(2, 2)*WW2)                                    PLFR2910
      AA = DL-S-T                                                       PLFR2920
      FA(1, M) = FA(1, M)+W1*AA                                         PLFR2930
      FA(2, M) = FA(2, M)+W2*AA                                         PLFR2940
      FA(3, M) = FA(3, M)+W2*AA*(S+AA/2.)                               PLFR2950
      AA = DL-T                                                         PLFR2960
      BBB = AA*AA                                                       PLFR2970
      CC = BBB*AA                                                       PLFR2980
      DD = CC*AA                                                        PLFR2990
      EE = W2/6./E/XI(M)                                                PLFR3000
      FB(1, M) = FB(1, M)+(W1/2./AR(M)/E)*(BBB-S**2)                    PLFR3010
      FB(2, M) = FB(2, M)+.75*EE*(DD-S**4)+EE*(T*CC-(S**3)*(DL-S))      PLFR3020
      FB(3, M) = EE*(CC-S**3)                                           PLFR3030
      GO TO 630                                                         PLFR3040
  660 IF (INP(I) .EQ. IP) GO TO 670                                     PLFR3050
      GO TO 1430                                                        PLFR3060
  670 IF (II .NE. 0) M = II                                             PLFR3070
      D3(1) = FN(2)                                                     PLFR3080
      D3(2) = FN(3)                                                     PLFR3090
      D3(3) = 0.                                                        PLFR3100
      DA = FN(4)                                                        PLFR3110
      DA = DA*12.                                                       PLFR3120
      WRITE (JWT,1690) M, INP(I), D3(1), D3(2), DA                      PLFR3130
      CALL ROT(CJ, MI)                                                  PLFR3140
      CALL TRANSP(R, TEMP)                                              PLFR3150
C       ROTATE GLOBAL FORCE VECTPR TO MEMBER FORCE VECTOR               PLFR3160
      DO 680 III = 1, 3                                                 PLFR3170
      PLME(III) = 0.                                                    PLFR3180
      DO 680 KKK = 1, 3                                                 PLFR3190
  680 PLME(III) = TEMP(III, KKK)*D3(KKK)+PLME(III)                      PLFR3200
      FA(1, M) = FA(1, M)+PLME(1)                                       PLFR3210
      FA(2, M) = FA(2, M)+PLME(2)                                       PLFR3220
```

```
      FA(3, M) = FA(3, M)+PLME(2)*DA+PLME(3)
      DO 690 III = 1, 3
      DO 690 JJJ = 1, 3
  690 FLBB(III, JJJ) = 0.
C      FLBB=FLEXIBILITY MATRIX
      FLBB(1, 1) = DA/E/AR(M)
      FLBB(2, 2) = DA**3/3./E/XI(M)
      FLBB(2, 3) = 1.5/DA*FLBB(2, 2)
      FLBB(3, 2) = FLBB(2, 3)
      FLBB(3, 3) = 2.*FLBB(2, 3)/DA
      DO 700 III = 1, 3
      D2(III) = 0.
      DO 700 KKK = 1, 3
  700 D2(III) = FLBB(III, KKK)*PLME(KKK)+D2(III)
C     CALCULATE (H TRANSPOSE) * D2(III) TO GET CANTILEVER DEFLECTION AT
C      END
      FB(1, M) = FB(1, M)+D2(1)
      FB(2, M) = FB(2, M)+D2(2)+D2(3)*DL-D2(3)*DA
      FB(3, M) = FB(3, M)+D2(3)
      GO TO 630
  710 DO 720 N = 1, NJ
      JL(1, N) = PJ(1, N)
      JL(2, N) = PJ(2, N)
      JL(3, N) = PJ(3, N)
  720 CONTINUE
  730 DO 870 M = 1, NM
      DO 740 III = 1, 3
      IF (FA(III, M) .NE. 0.) GO TO 750
  740 CONTINUE
      IF (TEM(M) .EQ. 0.) GO TO 870
      GO TO 780
  750 CALL ROT(CJ, MI)
      CALL SEMPL(AR, XI, C)
      D(1) = 0.
      D(2) = (6.*E*XI(M)/DL**2*(C(1, M)+2.*E*XI(M)*C(1, M)*C(2, M)/DL))/
```

```
     &DSTIF*FA(3, M)
      D(3) = ((-2.*E*XI(M)*C(1, M)/DL)/DSTIF)*FA(3, M)
      DO 760 III = 1, 3
      D1(III) = 0.
      DO 760 KKK = 1, 3
  760 D1(III) = -KBB(III, KKK)*FB(KKK, M)+D1(III)
      DO 770 III = 1, 3
  770 FEFB(III) = D1(III)-D(III)
C     CALCULATE FEFA FROM STATICS
      FEFA(1) = -FA(1, M)-FEFB(1)
      FEFA(2) = -FA(2, M)-FEFB(2)
      FEFA(3) = -FEFB(3)-FEFB(2)*DL-FA(3, M)
  780 FEFA(1) = FEFA(1)+ALPHA*E*AR(M)*TEM(M)
      FEFB(1) = FEFB(1)+ALPHA*E*AR(M)*TEM(M)
      JF = MI(1, M)
      JN = MI(2, M)
  790 CONTINUE
C     ADD NEGATIVES OF FIXED END FORCES TO JOINT LOADS (ROTATED TO GLOBA
C     SYSTEM) - R * FA(M),   R * FB(M).
      CALL MLT1(B, 1, R, 1, FEFA, 1)
      IF (ISR(JF) .NE. 0) GO TO 810
      DO 800 I = 1, 3
  800 JL(I, JF) = JL(I, JF)-B(I, 1)
      GO TO 830
  810 K = ISR(JF)
C     OMIT FIXED-END-FORCES FOR RELEASED COMPONENTS.
      DO 820 I = 1, 3
      IF (K-2*(K/2) .NE. 0) JL(I, JF) = JL(I, JF)-B(I, 1)
  820 K = K/2
  830 CALL MLT1(B, 1, R, 1, FEFB, 1)
      IF (ISR(JN) .NE. 0) GO TO 850
      DO 840 I = 1, 3
  840 JL(I, JN) = JL(I, JN)-B(I, 1)
      GO TO 870
  850 K = ISR(JN)
```

```
      DO 860 I = 1, 3                                                   PLFR3930
      IF (K-2*(K/2) .NE. 0) JL(I, JN) = JL(I, JN)-B(I, 1)               PLFR3940
  860 K = K/2                                                           PLFR3950
  870 CONTINUE                                                          PLFR3960
C     GENERATION AND ELIMINATION OF JOINT EQUILIBRIUM EQUATIONS         PLFR3970
C     GENERATE I TH  ROW OF STIFFNESS MATRIX AND STORE IN A TEMPORARILY PLFR3980
  880 LIST(1) = 1                                                       PLFR3990
      DO 1160 I = 1, NJ                                                 PLFR4000
C     NON ZERO BAND OF ROW I IN STIFFNESS IS FROM KL TO KH. KL = LOWEST PLFR4010
C     JOINT NO FOR JOINTS INCIDENT ON MEMBERS FRAMING INTO JOINT I, KH=HPLFR4020
      KL = I                                                            PLFR4030
      KH = I                                                            PLFR4040
      IM = NMIJ(I)                                                      PLFR4050
      DO 940 J = 1, IM                                                  PLFR4060
      M = JI(J, I)                                                      PLFR4070
      K = 1                                                             PLFR4080
      IF (M) 890, 900, 900                                              PLFR4090
  890 M = -M                                                            PLFR4100
      K = 2                                                             PLFR4110
C     JF = FAR END JOINT FOR MEMBER M                                   PLFR4120
  900 JF = MI(K, M)                                                     PLFR4130
      IF (JF-KH) 920, 920, 910                                          PLFR4140
  910 KH = JF                                                           PLFR4150
      GO TO 940                                                         PLFR4160
  920 IF (JF-KL) 930, 940, 940                                          PLFR4170
  930 KL = JF                                                           PLFR4180
  940 CONTINUE                                                          PLFR4190
C     ZERO ALL A MATRICES IN NON - ZERO BAND                            PLFR4200
      K = KH-KL+1                                                       PLFR4210
      DO 950 J = 1, K                                                   PLFR4220
      DO 950 IND = 1, 3                                                 PLFR4230
      DO 950 IND1 = 1, 3                                                PLFR4240
  950 A(IND, IND1, J) = 0.                                              PLFR4250
C     INSERT STIFFNESS MATRICES INTO NON ZERO BAND                      PLFR4260
      DO 1000 J = 1, IM                                                 PLFR4270
```

```
      M = JI(J, I)
      K = 2
      IF (M .GE. 0) GO TO 960
      M = -M
      K = 1
  960 JN = MI(K, M)-KL+1
C     JN=NEAR END JOINT FOR MEMBER  M  -(POSITION IN ROW RELATIVE TO KL
C     =1)
      K = 3-K
      JF = MI(K, M)-KL+1
C     JF= FAR END JOINT FOR MEMBER M RELATIVE TO KL=1
      IK = MI(K, M)
C     GENERATE R, H AND KBB MATRICES FOR MEMBER M
      CALL ROT(CJ, MI)
      CALL SEMPL(AR, XI, C)
C     TEST WHETHER A OR B END INCIDENT ON JOINT I
      M = JI(J, I)
      IF (M .GE. 0) GO TO 970
C     A END - NEAR END STIFF = H*KBB*H TR, FAR END = H*KBB
      CALL MLT3(KBA, 1, H, 1, KBB, 1)
C     TRANSPOSE H
      H(3, 2) = 0.
      H(2, 3) = DL
      CALL MLT3(KBB, 1, KBA, 1, H, 1)
      H(2, 3) = 0.
      GO TO 980
C     B END - NEAR STIFF = KBB, FAR = KBB*H TR
C     TRANSPOSE H.
  970 H(3, 2) = 0.
      H(2, 3) = DL
      CALL MLT3(KBA, 1, KBB, 1, H, 1)
      H(2, 3) = 0.
C     ROTATE TO GLOBAL SYSTEM;  R * KBB * R TR,  R * KBA * R TR
  980 CALL MLT3(BB, 1, R, 1, KBB, 1)
      CALL MLT3(BB, 2, R, 1, KBA, 1)
```

```
C     TRANSPOSE R                                                       PLFR4630
      T = R(1, 2)                                                       PLFR4640
      R(1, 2) = R(2, 1)                                                 PLFR4650
      R(2, 1) = T                                                       PLFR4660
      CALL MLT3(KBB, 1, BB, 1, R, 1)                                    PLFR4670
      CALL MLT3(KBA, 1, BB, 2, R, 1)                                    PLFR4680
C     INSERT NEAR AND FAR END STIFFNESS MATRICES                        PLFR4690
      DO 990 IND = 1, 3                                                 PLFR4700
      DO 990 IND1 = 1, 3                                                PLFR4710
      A(IND, IND1, JN) = A(IND, IND1, JN)+KBB(IND, IND1)                PLFR4720
  990 A(IND, IND1, JF) = A(IND, IND1, JF)-KBA(IND, IND1)                PLFR4730
 1000 CONTINUE                                                          PLFR4740
C     MODIFY EQUATION IF ANY RELEASES AT JOINT I. INSERT LARGE NO. ON MAPLFR4750
C     DIAGONAL AND MULTIPLY JOINT DISPLACEMENT BY SAME LARGE NUMBER.    PLFR4760
      IF (ISR(I) .EQ. 0) GO TO 1050                                     PLFR4770
 1010 IJ = I+1-KL                                                       PLFR4780
      II = ISR(I)                                                       PLFR4790
      DO 1040 K = 1, 3                                                  PLFR4800
      IF (II-2*(II/2) .NE. 0) GO TO 1040                                PLFR4810
 1020 A(K, K, IJ) = 10.**25                                             PLFR4820
 1030 JL(K, I) = JL(K, I)*10.**25                                       PLFR4830
 1040 II = II/2                                                         PLFR4840
 1050 LINC = KH-I                                                       PLFR4850
C     FOR FIRST EQUATION, BYPASS ELIMINATION                            PLFR4860
      IF (I .LE. KL) GO TO 1120                                         PLFR4870
C     PERFORM ELIMINATION FOR ROW I TO ZERO BELOW MAIN DIAGONAL          PLFR4880
      KU = I-1                                                          PLFR4890
      DO 1110 K = KL, KU                                                PLFR4900
C     IK = PIVOTAL COLUMN RELATIVE TO  KL = 1                           PLFR4910
      IK = K+1-KL                                                       PLFR4920
      IM = LIST(K+1)-LIST(K)                                            PLFR4930
      IJ = K+IM-I-LINC                                                  PLFR4940
C     IF NON ZERO BAND FOR PIVOTAL EQ ENDS TO RIGHT OF THAT FOR EQ I,   PLFR4950
C     EXTEND FOR EQ I                                                   PLFR4960
      IF (IJ .LE. 0.) GO TO 1070                                        PLFR4970
```

```
      KK = LINC+I-KL+2                                                  PLFR4980
      LINC = LINC+IJ                                                    PLFR4990
      LL = IJ+KK-1                                                      PLFR5000
      DO 1060 L = KK, LL                                                PLFR5010
      DO 1060 IND = 1, 3                                                PLFR5020
      DO 1060 IND1 = 1, 3                                               PLFR5030
 1060 A(IND, IND1, L) = 0.                                              PLFR5040
 1070 IF (IM) 1100, 1100, 1080                                          PLFR5050
 1080 DO 1090 J = 1, IM                                                 PLFR5060
      IJ = IK+J                                                         PLFR5070
      KJ = LIST(K)+J-1                                                  PLFR5080
      CALL MLT3(BB, 1, A, IK, STORE, KJ)                                PLFR5090
      DO 1090 IND = 1, 3                                                PLFR5100
      DO 1090 IND1 = 1, 3                                               PLFR5110
 1090 A(IND, IND1, IJ) = A(IND, IND1, IJ)-BB(IND, IND1, 1)              PLFR5120
 1100 CONTINUE                                                          PLFR5130
      CALL MLT1(B, 1, A, IK, JL, K)                                     PLFR5140
      DO 1110 J = 1, 3                                                  PLFR5150
 1110 JL(J, I) = JL(J, I)-B(J, 1)                                       PLFR5160
C     NORMALIZE ROW I . MULTIPLY BY INVERSE OF MAIN DIAGONAL MATRIX.=IJVPLFR5170
C     PIVOTAL ELEMENT RELATIVE TO KL= 1.                                PLFR5180
 1120 IL = I+1-KL                                                       PLFR5190
      LIST(I+1) = LINC+LIST(I)                                          PLFR5200
      CALL INV(A, IL)                                                   PLFR5210
      IF (LINC .LE. 0) GO TO 1140                                       PLFR5220
      IJ = IL                                                           PLFR5230
      DO 1130 J = 1, LINC                                               PLFR5240
      IJ = IJ+1                                                         PLFR5250
      IK = LIST(I)+J-1                                                  PLFR5260
 1130 CALL MLT3(STORE, IK, A, IL, A, IJ)                                PLFR5270
C     NORMALIZE LOAD VECTOR I                                           PLFR5280
 1140 CONTINUE                                                          PLFR5290
      CALL MLT1(B, 2, A, IL, JL, I)                                     PLFR5300
      DO 1150 J = 1, 3                                                  PLFR5310
 1150 JL(J, I) = B(J, 2)                                                PLFR5320
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```
 1160 CONTINUE                                                          PLFR5330
C     START BACK SUBSTITUTION                                           PLFR5340
      N2 = NJ-1                                                         PLFR5350
      IF (N2 .LE. 0) GO TO 1290                                         PLFR5360
      DO 1180 K = 1, N2                                                 PLFR5370
      I = NJ-K                                                          PLFR5380
      KU = LIST(I+1)-LIST(I)                                            PLFR5390
      DO 1170 J = 1, KU                                                 PLFR5400
      IK = LIST(I)+J-1                                                  PLFR5410
      IJ = I+J                                                          PLFR5420
      CALL MLT1(B, 1, STORE, IK, JL, IJ)                                PLFR5430
      DO 1170 L = 1, 3                                                  PLFR5440
 1170 JL(L, I) = JL(L, I)-B(L, 1)                                       PLFR5450
 1180 CONTINUE                                                          PLFR5460
      NMM = 0                                                           PLFR5470
      DO 1240 M = 1, NM                                                 PLFR5480
      IF (MSRA(M) .GT. 2) GO TO 1190                                    PLFR5490
      IF (MSRB(M) .LT. 3) GO TO 1240                                    PLFR5500
C CALCULATE TOTAL END FORCES IF FLEXIBLE CONNECTIONS                    PLFR5510
 1190 CALL ROT(CJ, MI)                                                  PLFR5520
      CALL SEMPL(AR, XI, C)                                             PLFR5530
      D(1) = 0.                                                         PLFR5540
      D(2) = (6.*E*XI(M)/DL**2*(C(1, M)+2.*E*XI(M)*C(1, M)*C(2, M)/DL))/PLFR5550
     &DSTIF*FA(3, M)                                                    PLFR5560
      D(3) = ((-2.*E*XI(M)*C(1, M)/DL)/DSTIF)*FA(3, M)                  PLFR5570
      DO 1200 III = 1, 3                                                PLFR5580
      D1(III) = 0.                                                      PLFR5590
      DO 1200 KKK = 1, 3                                                PLFR5600
 1200 D1(III) = -KBB(III, KKK)*FB(KKK, M)+D1(III)                       PLFR5610
      DO 1210 III = 1, 3                                                PLFR5620
 1210 FEFB(III) = D1(III)-D(III)                                        PLFR5630
      FEFA(1) = -FA(1, M)-FEFB(1)                                       PLFR5640
      FEFA(2) = -FA(2, M)-FEFB(2)                                       PLFR5650
      FEFA(3) = -FEFB(3)-FEFB(2)*DL-FA(3, M)                            PLFR5660
      FEFA(1) = FEFA(1)+ALPHA*E*AR(M)*TEM(M)                            PLFR5670
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```
      FEFB(1) = FEFB(1)+ALPHA*E*AR(M)*TEM(M)                            PLFR5680
C     TRANSPOSE R                                                       PLFR5690
      T = R(2, 1)                                                       PLFR5700
      R(2, 1) = R(1, 2)                                                 PLFR5710
      R(1, 2) = T                                                       PLFR5720
      JN = MI(2, M)                                                     PLFR5730
      JF = MI(1, M)                                                     PLFR5740
C     TRANSPOSE H                                                       PLFR5750
      H(3, 2) = 0.                                                      PLFR5760
      H(2, 3) = DL                                                      PLFR5770
C     FA(M) = FA(M) + KBB * (R TR * JL(JN) - H TR * R TR * JL(JF))      PLFR5780
      CALL MLT3(BB, 1, H, 1, R, 1)                                      PLFR5790
      CALL MLT1(B, 2, BB, 1, JL, JF)                                    PLFR5800
      CALL MLT1(B, 1, R, 1, JL, JN)                                     PLFR5810
      DO 1220 I = 1, 3                                                  PLFR5820
 1220 KBA(I, 1) = B(I, 1)-B(I, 2)                                       PLFR5830
      CALL MLT1(B, 1, KBB, 1, KBA, 1)                                   PLFR5840
C     FB(M) = FB(M) + H * KBB * (R TR * -------------)                  PLFR5850
      H(3, 2) = DL                                                      PLFR5860
      H(2, 3) = 0.                                                      PLFR5870
      CALL MLT1(B, 2, H, 1, B, 1)                                       PLFR5880
      DO 1230 I = 1, 3                                                  PLFR5890
      FEFB(I) = FEFB(I)+B(I, 1)                                         PLFR5900
 1230 FEFA(I) = FEFA(I)-B(I, 2)                                         PLFR5910
      KK = MSRA(M)-2                                                    PLFR5920
      CALL ITER1(KK, FEFA, SLPA, CONA, NMM, C)                          PLFR5930
      JJ = MSRB(M)-2                                                    PLFR5940
      CALL ITER2(JJ, FEFB, SLPB, CONB, NMM, C)                          PLFR5950
 1240 CONTINUE                                                          PLFR5960
      ITER = ITER+1                                                     PLFR5970
      WRITE (JWT,1250) ITER                                             PLFR5980
 1250 FORMAT (///'  ITERATION  NO.', I8)                                PLFR5990
      DO 1260 M = 1, NM                                                 PLFR6000
      WRITE (JWT,1520) M, C(1, M), C(2, M)                              PLFR6010
 1260 CONTINUE                                                          PLFR6020
```

```
      IF (ITER .LT. 12) GO TO 1280                                      PLFR6030
      WRITE (JWT,1270)                                                  PLFR6040
 1270 FORMAT (//' ALLOWABLE ITERATIONS EXCEEDED ANALYSIS DOES NOT CONVERPLFR6050
     &GE , USE STIFFER CONNECTIONS')                                    PLFR6060
      GO TO 1410                                                        PLFR6070
 1280 IF (NMM .GT. 0) GO TO 710                                         PLFR6080
C      NMM=0 ITERATIONS COMPLETED                                       PLFR6090
C     CALCULATE FINAL MEMBER END FORCES                                 PLFR6100
 1290 VOL = 0.                                                          PLFR6110
      WRITE (JWT,1650) (LDTYP(LDG, I), I = 1, 80)                       PLFR6120
C     WRITE JOINT DISPLACEMENTS                                         PLFR6130
      WRITE (JWT,1700) HDG                                              PLFR6140
      WRITE (JWT,1710)                                                  PLFR6150
      DO 1300 L = 1, NJ                                                 PLFR6160
 1300 WRITE (JWT,1670) L, (JL(J, L), J = 1, 3)                          PLFR6170
C     CALCULATE MEMBER END FORCES                                       PLFR6180
      WRITE (JWT,1720)                                                  PLFR6190
      DO 1350 M = 1, NM                                                 PLFR6200
      CALL ROT(CJ, MI)                                                  PLFR6210
      CALL SEMPL(AR, XI, C)                                             PLFR6220
      D(1) = 0.                                                         PLFR6230
      D(2) = (6.*E*XI(M)/DL**2*(C(1, M)+2.*E*XI(M)*C(1, M)*C(2, M)/DL))/PLFR6240
     &DSTIF*FA(3, M)                                                    PLFR6250
      D(3) = ((-2.*E*XI(M)*C(1, M)/DL)/DSTIF)*FA(3, M)                  PLFR6260
      DO 1310 III = 1, 3                                                PLFR6270
      D1(III) = 0.                                                      PLFR6280
      DO 1310 KKK = 1, 3                                                PLFR6290
 1310 D1(III) = -KBB(III, KKK)*FB(KKK, M)+D1(III)                       PLFR6300
      DO 1320 III = 1, 3                                                PLFR6310
 1320 FEFB(III) = D1(III)-D(III)                                        PLFR6320
      FEFA(1) = -FA(1, M)-FEFB(1)                                       PLFR6330
      FEFA(2) = -FA(2, M)-FEFB(2)                                       PLFR6340
      FEFA(3) = -FEFB(3)-FEFB(2)*DL-FA(3, M)                            PLFR6350
      FEFA(1) = FEFA(1)+ALPHA*E*AR(M)*TEM(M)                            PLFR6360
      FEFB(1) = FEFB(1)+ALPHA*E*AR(M)*TEM(M)                            PLFR6370
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```
      T = R(2, 1)                                                       PLFR6380
      R(2, 1) = R(1, 2)                                                 PLFR6390
      R(1, 2) = T                                                       PLFR6400
      IF (LDG .EQ. 1) VOL = VOL+DL*AR(M)                                PLFR6410
      JN = MI(2, M)                                                     PLFR6420
      JF = MI(1, M)                                                     PLFR6430
C     TRANSPOSE H                                                       PLFR6440
      H(3, 2) = 0.                                                      PLFR6450
      H(2, 3) = DL                                                      PLFR6460
C     FA(M)=FA(M)+KBB*(R TR *JL(JN)- H TR * R TR * JL(JF))              PLFR6470
      CALL MLT3(BB, 1, H, 1, R, 1)                                      PLFR6480
      CALL MLT1(B, 2, BB, 1, JL, JF)                                    PLFR6490
      CALL MLT1(B, 1, R, 1, JL, JN)                                     PLFR6500
      DO 1330 I = 1, 3                                                  PLFR6510
 1330 KBA(I, 1) = B(I, 1)-B(I, 2)                                       PLFR6520
      CALL MLT1(B, 1, KBB, 1, KBA, 1)                                   PLFR6530
      H(3, 2) = DL                                                      PLFR6540
      H(2, 3) = 0.                                                      PLFR6550
      CALL MLT1(B, 2, H, 1, B, 1)                                       PLFR6560
      DO 1340 I = 1, 3                                                  PLFR6570
      FB(I, M) = FEFB(I)+B(I, 1)                                        PLFR6580
 1340 FA(I, M) = FEFA(I)-B(I, 2)                                        PLFR6590
      FA(3, M) = FA(3, M)/12.                                           PLFR6600
      FB(3, M) = FB(3, M)/12.                                           PLFR6610
 1350 WRITE (JWT,1730) M, (FA(I, M), I = 1, 3), (FB(I, M), I = 1, 3)    PLFR6620
C     CALCULATE AND PRINT SUPPORT REACTIONS                             PLFR6630
      WRITE (JWT,1740)                                                  PLFR6640
      DO 1400 J = 1, NJ                                                 PLFR6650
      IF (ISR(J) .EQ. 0) GO TO 1400                                     PLFR6660
      DO 1360 IND = 1, 3                                                PLFR6670
 1360 A(IND, 1, J) = 0.                                                 PLFR6680
      IM = NMIJ(J)                                                      PLFR6690
      DO 1390 I = 1, IM                                                 PLFR6700
      M = JI(I, J)                                                      PLFR6710
      IF (M .GE. 0) GO TO 1370                                          PLFR6720
```

```
      M = -M                                                            PLFR6730
C     REACTION + REACTION + R * FA(M)                                   PLFR6740
      CALL ROT(CJ, MI)                                                  PLFR6750
      CALL MLT1(B, 1, R, 1, FA, M)                                      PLFR6760
      GO TO 1380                                                        PLFR6770
C     REACTION = REACTION + R * FB(M)                                   PLFR6780
 1370 CALL ROT(CJ, MI)                                                  PLFR6790
      CALL MLT1(B, 1, R, 1, FB, M)                                      PLFR6800
 1380 DO 1390 IND = 1, 3                                                PLFR6810
 1390 A(IND, 1, J) = A(IND, 1, J)+B(IND, 1)                             PLFR6820
      WRITE (JWT,1670) J, (A(IND, 1, J), IND = 1, 3)                    PLFR6830
 1400 CONTINUE                                                          PLFR6840
 1410 CONTINUE                                                          PLFR6850
      LDG = LDG+1                                                       PLFR6860
      READ (JRD,1450,END=1440) (LDTYP(LDG, I), I = 1, 80)               PLFR6870
      DO 1420 I = 1, 80                                                 PLFR6880
      IF (LDTYP(LDG, I) .NE. INPT(1)) GO TO 540                         PLFR6890
 1420 CONTINUE                                                          PLFR6900
      WS = VOL*3.4/12000.                                               PLFR6910
      WRITE (JWT,1750) VOL, WS                                          PLFR6920
      WRITE (JWT,1760)                                                  PLFR6930
      RETURN                                                            PLFR6940
 1430 WRITE (JWT,1770)                                                  PLFR6950
 1440 CALL EXIT                                                         PLFR6960
 1450 FORMAT (80A1)                                                     PLFR6970
 1460 FORMAT (I6, ' MEMBERS', I4, ' JOINTS. MODULUS OF ELASTICITY =',   PLFR6980
     &F9.1, ' (KSI)'//'  THERMAL EXPANSION COEFFICIENT (FOR CALC OF TEMPPLFR6990
     & STRESS)- ', F10.7, //)                                           PLFR7000
 1470 FORMAT (///'  JOINT COORDINATES (FT)'//'  JOINT     X COORD      YPLFR7010
     & COORD            RELEASES       SPEC DISPL'//)                   PLFR7020
 1480 FORMAT (I6, 2F12.3, '    SUPPORT', 8X3F10.3)                      PLFR7030
 1490 FORMAT (I6, 2F12.3, '    SUPPORT', 6XA4)                          PLFR7040
 1500 FORMAT (//'  MEMBER INFORMATION'//'  MEMBER     START     END     PLFR7050
     &AREA (SQ IN)   IXX (IN**4)   CONNECTION A END   CONNECTION B END  PLFR7060
     & TEMPERATURE'//)                                                  PLFR7070
```

```
1510 FORMAT (//'  CONNECTION INFORMATION'//'  MEMBER  FLEXIBILITY A ENDPLFR7080
    &  FLEXIBILITY B END'//)                                          PLFR7090
1520 FORMAT (I8, F20.8, F20.8)                                         PLFR7100
1530 FORMAT (3I8, 2F14.2, 8XA10, 9XA10, 9X, F10.1)                     PLFR7110
1540 FORMAT (I6, 6F12.3)                                               PLFR7120
1550 FORMAT ('1'/80A1)                                                 PLFR7130
1560 FORMAT (///'NON-ZEROJOINTLOADS'//'  JOINT  PX(KIPS)  PY(KIPS)  MOMPLFR7140
    & ENTKIPS)'//)                                                    PLFR7150
1570 FORMAT (I10)                                                      PLFR7160
1580 FORMAT (80A1)                                                     PLFR7170
1590 FORMAT (I6, ' MEMBERS', I4, ' JOINTS. MODULUS OF ELASTICITY =',   PLFR7180
    &F9.1, ' (KSI)'//'  THERMAL EXPANSION COEFFICIENT (FOR CALC OF TEMPPLFR7190
    & STRESS)- ', F10.7, //)                                          PLFR7200
1600 FORMAT (///'  JOINT COORDINATES (FT)'//'  JOINT     X COORD     YPLFR7210
    & COORD            RELEASES      SPEC DISPL'//)                   PLFR7220
1610 FORMAT (I6, 2F12.3, '    SUPPORT', 8X3F10.3)                      PLFR7230
1620 FORMAT (I6, 2F12.3, '    SUPPORT', 6XA4)                          PLFR7240
1630 FORMAT (//'  MEMBER INFORMATION'//'  MEMBER     START    END    ARPLFR7250
    &EA (SQ IN)   IXX (IN**4)   PINNED ENDS   TEMPERATURE'//)         PLFR7260
1640 FORMAT (3I8, 2F14.2, 9XA4, F10.1)                                 PLFR7270
1650 FORMAT ('1'/80A1)                                                 PLFR7280
1660 FORMAT (///'  NON-ZERO JOINT LOADS'//'  JOINT   PX (KIPS)   PY (KIPLFR7290
    &PS)    MOM(FT KIPS)'//)                                          PLFR7300
1670 FORMAT (I6, 6F12.3)                                               PLFR7310
1680 FORMAT (///'  NON-ZERO MEMBER LOADS'//'  MEMBER   LOAD TYPE    HORPLFR7320
    &IZ     VERTICAL    DIST FROM START MEMBER'//)                    PLFR7330
1690 FORMAT (I5, 10XA4, 3F10.2)                                        PLFR7340
1700 FORMAT (//1X20A4, //'  RESULTS')                                  PLFR7350
1710 FORMAT (//'  JOINT DISPLACEMENTS'//'  JOINT         X          Y PLFR7360
    &      ROTATION'//)                                               PLFR7370
1720 FORMAT (///'  MEMBER END FORCES'//'  MEMBER', 16X' START', 25X' ENPLFR7380
    &D'//11X'AXIAL', 6X'SHEAR', 5X'MOMENT', 6X'AXIAL', 6X'SHEAR', 6X'MOPLFR7390
    &MENT'//)                                                         PLFR7400
1730 FORMAT (I6, 6F11.3)                                               PLFR7410
1740 FORMAT (////'  SUPPORT REACTIONS'//'  SUPPORT   HORIZONTAL   VERTIPLFR7420
```

```
     &CAL   MOMENT'//)
 1750 FORMAT (//'  TOTAL VOLUMN OF MEMBERS IN FRAME'/'  IS', F15.1, ' CU
     &BIC IN'/'  WEIGHT IF FRAME IS:'/'      STEEL  -  ', F10.3, ' KIPS'
     &)
 1760 FORMAT (//'  UNITS:   DISTANCES = FT, ROTATIONS = RADIANS, '//'
     &        FORCES = KIPS, MOMENTS = FT K, CROSS SECTIONAL DIMENSIONS'
     &//'          = INCHES'/'1')
 1770 FORMAT (///'1 INPUT ERROR ON DATA CARD', I4, ',CHECK INPUT')
      END
C      ****************************************************************
C
C                        SUBROUTINE SEMPL
C
C      ****************************************************************
      SUBROUTINE SEMPL(AR, XI, C)
      REAL KBB(3, 3), KBA(3, 3)
      COMMON E, M, DL/SPL/KBB, DSTIF
      DIMENSION AR(1), XI(1)
      DIMENSION C(2, 1)
      DSTIF = 0.
      DSTIF = DSTIF+1.+4.*E*XI(M)/DL*(C(1, M)+C(2, M)+3.*E*XI(M)/DL*(C(1
     &, M)*C(2, M)))
      DO 10 I = 1, 3
      DO 10 J = 1, 3
   10 KBB(I, J) = 0.
      KBB(1, 1) = AR(M)*E/DL
      KBB(2, 2) = 12.*E*XI(M)/DL**3
      KBB(3, 3) = 4.*E*XI(M)/DL
      KBB(3, 2) = -6.*E*XI(M)/DL**2
      KBB(2, 2) = KBB(2, 2)*(1.+E*XI(M)*C(1, M)/DL+E*XI(M)*C(2, M)/DL)/
     &DSTIF
      KBB(3, 3) = KBB(3, 3)*(1.+3.*E*XI(M)*C(1, M)/DL)/DSTIF
      KBB(3, 2) = KBB(3, 2)*(1.+2.*E*XI(M)*C(1, M)/DL)/DSTIF
      KBB(2, 3) = KBB(3, 2)
      RETURN
```

```
      END                                                               SEMP0270
C     ******************************************************************CVRT0010
C                                                                       CVRT0020
C                           SUBROUTINE CNVRT                            CVRT0030
C                                                                       CVRT0040
C     ******************************************************************CVRT0050
      SUBROUTINE CNVRT(NN, I1, I2)                                      CVRT0060
      INTEGER*2 INP(80), IN(5)/'0', '9', '.', ' ', '-'/                 CVRT0070
      COMMON E, M, DL, JRD, JWT, FN(14), INP                            CVRT0080
      J = 10*I2+1                                                       CVRT0090
      IFL = I2-I1+NN                                                    CVRT0100
   10 FK = 0.                                                           CVRT0110
      K = 1                                                             CVRT0120
      L = 0                                                             CVRT0130
      DO 50 I = 1, 10                                                   CVRT0140
      J = J-1                                                           CVRT0150
      IF (INP(J) .NE. IN(3)) GO TO 20                                   CVRT0160
      FK = FK/10.**L                                                    CVRT0170
      L = 0                                                             CVRT0180
      GO TO 50                                                          CVRT0190
   20 IF (INP(J) .LT. IN(1) .OR. INP(J) .GT. IN(2)) GO TO 30            CVRT0200
      IJ = INP(J)/256+15                                                CVRT0210
      L = L+1                                                           CVRT0220
      FK = FK+IJ*10.**L/10.                                             CVRT0230
      GO TO 50                                                          CVRT0240
   30 IF (INP(J) .NE. IN(5)) GO TO 40                                   CVRT0250
      FK = -FK                                                          CVRT0260
      GO TO 50                                                          CVRT0270
   40 IF (INP(J) .EQ. IN(4)) GO TO 50                                   CVRT0280
      K = 0                                                             CVRT0290
      GO TO 50                                                          CVRT0300
   50 CONTINUE                                                          CVRT0310
   60 FN(IFL) = FK*K                                                    CVRT0320
      IFL = IFL-1                                                       CVRT0330
      IF (NN .NE. 1) GO TO 70                                           CVRT0340
```

```
      IF (IFL .GT. 0) GO TO 10
      GO TO 80
   70 IF (IFL .GT. 6) GO TO 10
   80 CONTINUE
      RETURN
      END
C      ****************************************************************
C
C                            SUBROUTINE GENCUR
C
C      ****************************************************************
      SUBROUTINE GENCUR(KK, JJ, C, SLPA, SLPB, CONA, CONB)
      COMMON E, M, DL, JRD, JWT, FN(14)
      DIMENSION SLPA(1), SLPB(1), CONA(1), CONB(1)
      INTEGER*2 INP(80)
      DIMENSION C(2, 1)
      P1 = FN(7)
      P2 = FN(8)
      P3 = FN(9)
      P4 = FN(10)
      GO TO (10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110), KK
   10 C(1, M) = 10.*10.**25
      GO TO 120
   20 C(1, M) = FN(7)
      GO TO 120
C     DOUBLE WEB ANGLE CONNECTIONS
   30 CONA(M) = (1./P1**2.4)*(P2**1.6)*(1./P3**.23)
      SLPA(M) = 3.66*CONA(M)*.0001
      C(1, M) = SLPA(M)
      GO TO 120
C     SINGLE WEB ANGLE CONNECTIONS
   40 CONA(M) = (1./P1**2.4)*(1./P2**1.81)*(P3**.15)
      SLPA(M) = 4.28*CONA(M)*.001
      C(1, M) = SLPA(M)
      GO TO 120
```

```
C       HEADER PLATE CONNECTIONS                                              GENC0300
   50 CONA(M) = (1./P1**2.3)*(P2**1.6)*(1./P3**1.6)*(1./P4**.5)               GENC0310
      SLPA(M) = 5.1*CONA(M)*.00001                                            GENC0320
      C(1, M) = SLPA(M)                                                       GENC0330
      GO TO 120                                                               GENC0340
C       TOP AND SEAT ANGLE CONNECTIONS                                        GENC0350
   60 CONA(M) = (1./P1**.5)*(1./P2**1.5)*(1./P3**1.1)*(1./P4**.7)             GENC0360
      SLPA(M) = 8.46*CONA(M)*.0001                                            GENC0370
      C(1, M) = SLPA(M)                                                       GENC0380
      GO TO 120                                                               GENC0390
C       END PLATE CONNECTIONS WITH NO STIFFNERS                               GENC0400
   70 CONA(M) = (1./P1**2.4)*(1./P2**.4)*(1./P3**1.1)                         GENC0410
      SLPA(M) = 1.83*CONA(M)*.001                                             GENC0420
      C(1, M) = SLPA(M)                                                       GENC0430
      GO TO 120                                                               GENC0440
C       END PLATE CONNECTIONS WITH STIFFNERS                                  GENC0450
   80 CONA(M) = (1./P1**2.4)*(1./P2**.6)                                      GENC0460
      SLPA(M) = 1.79*CONA(M)*.001                                             GENC0470
      C(1, M) = SLPA(M)                                                       GENC0480
      GO TO 120                                                               GENC0490
C       T- STUB CONNECTIONS                                                   GENC0500
   90 CONA(M) = (1./P1**1.5)*(1./P2**.5)*(1./P3**1.1)*(1./P4**.7)             GENC0510
      SLPA(M) = 2.11*CONA(M)*.0001                                            GENC0520
      C(1, M) = SLPA(M)                                                       GENC0530
      GO TO 120                                                               GENC0540
C        INSERT WELDED TOP PLATE AND SEAT ANGLE CONNECTIONS HERE              GENC0550
  100 CONA(M) = 1.                                                            GENC0560
      SLPA(M) = 1.                                                            GENC0570
      C(1, M) = SLPA(M)                                                       GENC0580
      GO TO 120                                                               GENC0590
  110 C(1, M) = 0.                                                            GENC0600
  120 CONTINUE                                                                GENC0610
      P5 = FN(11)                                                             GENC0620
      P6 = FN(12)                                                             GENC0630
      P7 = FN(13)                                                             GENC0640
```

```
      P8 = FN(14)                                                       GENC0650
      GO TO (130, 140, 150, 160, 170, 180, 190, 200, 210, 220, 230), JJ GENC0660
  130 C(2, M) = 10.*10.**25                                             GENC0670
      GO TO 240                                                         GENC0680
  140 C(2, M) = FN(11)                                                  GENC0690
      GO TO 240                                                         GENC0700
  150 CONB(M) = (1./P5**2.4)*(P6**1.6)*(1./P7**.23)                     GENC0710
      SLPB(M) = 3.66*CONB(M)*.0001                                      GENC0720
      C(2, M) = SLPB(M)                                                 GENC0730
      GO TO 240                                                         GENC0740
  160 CONB(M) = (1./P5**2.4)*(1./P6**1.81)*(P7**.15)                    GENC0750
      SLPB(M) = 4.28*CONB(M)*.001                                       GENC0760
      C(2, M) = SLPB(M)                                                 GENC0770
      GO TO 240                                                         GENC0780
  170 CONB(M) = (1./P5**2.3)*(P6**1.6)*(1./P7**1.6)*(1./P8**.5)         GENC0790
      SLPB(M) = 5.1*CONB(M)*.00001                                      GENC0800
      C(2, M) = SLPB(M)                                                 GENC0810
      GO TO 240                                                         GENC0820
  180 CONB(M) = (1./P5**.5)*(1./P6**1.5)*(1./P7**1.1)*(1./P8**.7)       GENC0830
      SLPB(M) = 8.46*CONB(M)*.0001                                      GENC0840
      C(2, M) = SLPB(M)                                                 GENC0850
      GO TO 240                                                         GENC0860
  190 CONB(M) = (1./P5**2.4)*(1./P6**.4)*(1./P7**1.1)                   GENC0870
      SLPB(M) = 1.83*CONB(M)*.001                                       GENC0880
      C(2, M) = SLPB(M)                                                 GENC0890
      GO TO 240                                                         GENC0900
  200 CONB(M) = (1./P5**2.4)*(1./P6**.6)                                GENC0910
      SLPB(M) = 1.79*CONB(M)*.001                                       GENC0920
      C(2, M) = SLPB(M)                                                 GENC0930
      GO TO 240                                                         GENC0940
  210 CONB(M) = (1./P5**1.5)*(1./P6**.5)*(1./P7**1.1)*(1./P8**.7)       GENC0950
      SLPB(M) = 2.11*CONB(M)*.0001                                      GENC0960
      C(2, M) = SLPB(M)                                                 GENC0970
      GO TO 240                                                         GENC0980
C       INSERT WELDED TOP PLATE AND SEAT ANGLE CONNECTIONS HERE         GENC0990
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```
  220 CONB(M) = 1.                                                        GENC1000
      SLPB(M) = 1.                                                        GENC1010
      C(2, M) = SLPB(M)                                                   GENC1020
      GO TO 240                                                           GENC1030
  230 C(2, M) = 0.                                                        GENC1040
  240 CONTINUE                                                            GENC1050
      RETURN                                                              GENC1060
      END                                                                 GENC1070
C      ****************************************************************ITER0010
C                                                                         ITER0020
C                           SUBROUTINE ITERAT                             ITER0030
C                                                                         ITER0040
C      ****************************************************************ITER0050
      SUBROUTINE ITERAT                                                   ITER0060
      COMMON E, M                                                         ITER0070
      DIMENSION C(2, 1)                                                   ITER0080
      ENTRY ITER1(KK, FEFA, SLPA, CONA, NMM, C)                           ITER0090
      DIMENSION SLPA(1), CONA(1)                                          ITER0100
      DIMENSION FEFA(3)                                                   ITER0110
      ABFA = ABS(FEFA(3))                                                 ITER0120
      PHI0A = ABFA*SLPA(M)                                                ITER0130
      PHI1A = 0.                                                          ITER0140
      GO TO (10, 20, 30, 40, 50, 60, 70, 80), KK                          ITER0150
C     DOUBLE WEB ANGLE CONNECTIONS                                        ITER0160
   10 PHI1A = PHI1A+3.66*(CONA(M)*ABFA)*.0001+1.15*(CONA(M)*ABFA)**3*     ITER0170
     &.000001+4.57*(CONA(M)*ABFA)**5*.00000001                            ITER0180
      GO TO 90                                                            ITER0190
C     SINGLE WEB ANGLE CONNECTIONS                                        ITER0200
   20 PHI1A = PHI1A+4.28*(CONA(M)*ABFA)*.001+(1.45*(CONA(M)*ABFA)**3)*(   ITER0210
     &.1**9)+(1.51*(CONA(M)*ABFA)**5)*(.1**16)                            ITER0220
      GO TO 90                                                            ITER0230
C     HEADER PLATE CONNECTIONS                                            ITER0240
   30 PHI1A = PHI1A+5.1*(CONA(M)*ABFA)*.00001+6.2*(CONA(M)*ABFA)**3*      ITER0250
     &.000000001+2.4*(CONA(M)*ABFA)**5*.000000000001                      ITER0260
      GO TO 90                                                            ITER0270
```

```
C       TOP AND SEAT ANGLE CONNECTIONS                                          ITER0280
     40 PHI1A = PHI1A+8.46*(CONA(M)*ABFA)*.0001+1.01*(CONA(M)*ABFA)**3*         ITER0290
     &.0001+1.24*(CONA(M)*ABFA)**5*.00000001                                     ITER0300
        GO TO 90                                                                ITER0310
C       END PLATE CONNECTIONS WITH NO STIFFNERS                                 ITER0320
     50 PHI1A = PHI1A+1.83*(CONA(M)*ABFA)*.001-1.04*(CONA(M)*ABFA)**3*          ITER0330
     &.0001+6.38*(CONA(M)*ABFA)**5*.000001                                       ITER0340
        GO TO 90                                                                ITER0350
C       END PLATE CONNECTIONS WITH STIFFNERS                                    ITER0360
     60 PHI1A = PHI1A+1.79*(CONA(M)*ABFA)*.001+1.76*(CONA(M)*ABFA)**3*          ITER0370
     &.0001+2.04*(CONA(M)*ABFA)**5*.0001                                         ITER0380
        GO TO 90                                                                ITER0390
C       T-STUB CONNECTIONS                                                      ITER0400
     70 PHI1A = PHI1A+2.11*(CONA(M)*ABFA)*.0001+6.2*(CONA(M)*ABFA)**3*          ITER0410
     &.000001-7.6*(CONA(M)*ABFA)**5*.000000001                                   ITER0420
        GO TO 90                                                                ITER0430
C        INSERT WELDED TOP PLATE AND SEAT ANGLE CONNECTIONS HERE                ITER0440
     80 CONTINUE                                                                ITER0450
        GO TO 90                                                                ITER0460
     90 DELPHI = PHI1A-PHI0A                                                    ITER0470
        TERPHI = DELPHI/PHI1A                                                   ITER0480
        IF (ABS(TERPHI) .LT. .05) GO TO 190                                     ITER0490
        SLPA(M) = (PHI0A+.5*DELPHI)/ABFA                                        ITER0500
        C(1, M) = SLPA(M)                                                       ITER0510
        NMM = 1                                                                 ITER0520
        RETURN                                                                  ITER0530
        ENTRY ITER2(JJ, FEFB, SLPB, CONB, NMM, C)                               ITER0540
        DIMENSION SLPB(1), CONB(1)                                              ITER0550
        DIMENSION FEFB(3)                                                       ITER0560
        ABFB = ABS(FEFB(3))                                                     ITER0570
        PHI0B = ABFB*SLPB(M)                                                    ITER0580
        PHI1B = 0.                                                              ITER0590
        GO TO (100, 110, 120, 130, 140, 150, 160, 170), JJ                      ITER0600
    100 PHI1B = PHI1B+3.66*(CONB(M)*ABFB)*.0001+1.15*(CONB(M)*ABFB)**3*         ITER0610
     &.000001+4.57*(CONB(M)*ABFB)**5*.00000001                                   ITER0620
```

```
      GO TO 180                                                         ITER0630
  110 PHI1B = PHI1B+4.28*(CONB(M)*ABFB)*.001+(1.45*(CONB(M)*ABFB)**3)*( ITER0640
     &.1**9)+(1.51*(CONB(M)*ABFB)**5)*(.1**16)                          ITER0650
      GO TO 180                                                         ITER0660
  120 PHI1B = PHI1B+5.1*(CONB(M)*ABFB)*.00001+6.2*(CONB(M)*ABFB)**3*    ITER0670
     &.000000001+2.4*(CONB(M)*ABFB)**5*.000000000001                    ITER0680
      GO TO 180                                                         ITER0690
  130 PHI1B = PHI1B+8.46*(CONB(M)*ABFB)*.0001+1.01*(CONB(M)*ABFB)**3*   ITER0700
     &.0001+1.24*(CONB(M)*ABFB)**5*.00000001                            ITER0710
      GO TO 180                                                         ITER0720
  140 PHI1B = PHI1B+1.83*(CONB(M)*ABFB)*.001-1.04*(CONB(M)*ABFB)**3*    ITER0730
     &.0001+6.38*(CONB(M)*ABFB)**5*.000001                              ITER0740
      GO TO 180                                                         ITER0750
  150 PHI1B = PHI1B+1.79*(CONB(M)*ABFB)*.001+1.76*(CONB(M)*ABFB)**3*    ITER0760
     &.0001+2.04*(CONB(M)*ABFB)**5*.0001                                ITER0770
      GO TO 180                                                         ITER0780
  160 PHI1B = PHI1B+2.11*(CONB(M)*ABFB)*.0001+6.2*(CONB(M)*ABFB)**3*    ITER0790
     &.000001-7.6*(CONB(M)*ABFB)**5*.000000001                          ITER0800
      GO TO 180                                                         ITER0810
C       INSERT WELDED TOP PLATE AND SEAT ANGLE CONNECTIONS HERE         ITER0820
  170 CONTINUE                                                          ITER0830
      GO TO 180                                                         ITER0840
  180 DELPHI = PHI1B-PHI0B                                              ITER0850
      TERPHI = DELPHI/PHI1B                                             ITER0860
      IF (ABS(TERPHI) .LT. .05) GO TO 190                               ITER0870
      SLPB(M) = (PHI0B+.5*DELPHI)/ABFB                                  ITER0880
      C(2, M) = SLPB(M)                                                 ITER0890
      NMM = 1                                                           ITER0900
  190 CONTINUE                                                          ITER0910
      RETURN                                                            ITER0920
      END                                                               ITER0930
C      *****************************************************************MLT10010
C                                                                       MLT10020
C                        SUBROUTINE MLT1                                MLT10030
C                                                                       MLT10040
```

```
C      *****************************************************************MLT10050
       SUBROUTINE MLT1(C, NC, A, NA, B, NB)                             MLT10060
       DIMENSION A(3, 3, 1), B(3, 1), C(3, 1)                           MLT10070
       DO 20 I = 1, 3                                                   MLT10080
       SUM = 0.                                                         MLT10090
       DO 10 K = 1, 3                                                   MLT10100
    10 SUM = SUM+A(I, K, NA)*B(K, NB)                                   MLT10110
    20 C(I, NC) = SUM                                                   MLT10120
       RETURN                                                           MLT10130
       END                                                              MLT10140
C      *****************************************************************MLT30010
C                                                                       MLT30020
C                         SUBROUTINE MLT3                               MLT30030
C                                                                       MLT30040
C      *****************************************************************MLT30050
       SUBROUTINE MLT3(C, NC, A, NA, B, NB)                             MLT30060
       DIMENSION A(3, 3, 1), B(3, 3, 1), C(3, 3, 1)                     MLT30070
       DO 20 I = 1, 3                                                   MLT30080
       DO 20 J = 1, 3                                                   MLT30090
       SUM = 0.                                                         MLT30100
       DO 10 K = 1, 3                                                   MLT30110
    10 SUM = SUM+A(I, K, NA)*B(K, J, NB)                                MLT30120
    20 C(I, J, NC) = SUM                                                MLT30130
       RETURN                                                           MLT30140
       END                                                              MLT30150
C      *****************************************************************ROT 0010
C                                                                       ROT 0020
C                         SUBROUTINE ROT                                ROT 0030
C                                                                       ROT 0040
C      *****************************************************************ROT 0050
       SUBROUTINE ROT(CJ, MI)                                           ROT 0060
       REAL KBB(3, 3), KBA(3, 3)                                        ROT 0070
       COMMON E, M, DL/RT/COSA, SINA, R(3, 3), H(3, 3)                  ROT 0080
       INTEGER*2 MI                                                     ROT 0090
       DIMENSION CJ(2, 1), MI(2, 1)                                     ROT 0100
```

```
C     THIS SUBROUTINE BUILDS THE ROTATION MATRIX FOR MEMBER M           ROT 0110
      I = MI(1, M)                                                      ROT 0120
      J = MI(2, M)                                                      ROT 0130
      X = CJ(1, J)-CJ(1, I)                                             ROT 0140
      Y = CJ(2, J)-CJ(2, I)                                             ROT 0150
      DL = SQRT(X*X+Y*Y)                                                ROT 0160
      COSA = X/DL                                                       ROT 0170
      SINA = Y/DL                                                       ROT 0180
      R(1, 1) = COSA                                                    ROT 0190
      R(2, 1) = SINA                                                    ROT 0200
      X = ABS(COSA)                                                     ROT 0210
      R(2, 2) = X                                                       ROT 0220
      IF (X) 20, 10, 20                                                 ROT 0230
   10 R(1, 2) = -1.                                                     ROT 0240
      IF (SINA .LT. 0.) R(1, 2) = 1.                                    ROT 0250
      COSA = 1.                                                         ROT 0260
      R(3, 3) = 1.                                                      ROT 0270
      GO TO 30                                                          ROT 0280
   20 R(1, 2) = -COSA*SINA/X                                            ROT 0290
      R(3, 3) = COSA/X                                                  ROT 0300
   30 H(3, 2) = DL                                                      ROT 0310
      RETURN                                                            ROT 0320
      END                                                               ROT 0330
C      *****************************************************************TRSP0010
C                                                                       TRSP0020
C                           SUBROUTINE TRANSP                           TRSP0030
C                                                                       TRSP0040
C      *****************************************************************TRSP0050
      SUBROUTINE TRANSP(A, B)                                           TRSP0060
C     THIS SUBROUTINE INSERTS A TRANSPOSE INTO B(3*3)                   TRSP0070
      DIMENSION A(3, 3), B(3, 3)                                        TRSP0080
      DO 10 I = 1, 3                                                    TRSP0090
      DO 10 J = 1, 3                                                    TRSP0100
   10 B(J, I) = A(I, J)                                                 TRSP0110
      RETURN                                                            TRSP0120
```

```
      END
C      *****************************************************************
C
C                        SUBROUTINE INV
C
C      *****************************************************************
      SUBROUTINE INV(A, IJ)
      DIMENSION A(3, 3, 1)
      DO 50 N = 1, 3
      D = A(N, N, IJ)
      DO 20 J = 1, 3
      IF (D .GE. 1.E50 .AND. A(N, J, IJ) .LE. 1.E50) GO TO 10
      A(N, J, IJ) = -A(N, J, IJ)/D
      GO TO 20
   10 A(N, J, IJ) = 0.
   20 CONTINUE
      DO 40 I = 1, 3
      IF (N .EQ. I) GO TO 40
      DO 30 J = 1, 3
      IF (N .EQ. J) GO TO 30
      A(I, J, IJ) = A(I, J, IJ)+A(I, N, IJ)*A(N, J, IJ)
   30 CONTINUE
   40 A(I, N, IJ) = A(I, N, IJ)/D
      A(N, N, IJ) = 1./D
   50 CONTINUE
      RETURN
      END
**EOF**
```