COMPARISON OF PARAMETRIC AND NONPARAMETRIC STREAMFLOW RECORD EXTENSION TECHNIQUES

A Thesis

Submitted to the Faculty of Graduate Studies in Partial

Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

by

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KEVIN SYDOR

A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University

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ABSTRACT

The extension of short data records, based on information from long term data records, is a common procedure used in the planning and operation of many water resources systems. Alternative methods for extending the available streamflow data record at locations where the period of recorded data is considered too short are presented. Various deficiencies in existing regression-based parametric techniques related to the assumption of normally distributed and random residuals are identified. An alternate nonparametric approach which is not subject to the above assumptions is presented. The nonparametric method utilizes the relationship between the index and base record to identify similar flow patterns that can be used to generate streamflow data.

The extension techniques were evaluated, and the results of the evaluation were verified, using monthly streamflow data from gauging stations in Manitoba and Ontario. The techniques are evaluated based on their relative performance in reproducing statistical features of the historic data. The parametric and nonparametric methods displayed comparable performance. The residual series of the parametric models did not follow the normal distribution, even though a data transformation was performed. Residual series from both techniques displayed autocorrelation, indicating the inability of the models in taking into account time varying relationships in the data. Model performance generally increased with common period of record. The nonparametric methods tended to improve as the available data increased.

Recommendations are made as to the preferred approach under varying data availability conditions. The nonparametric techniques are recommended as a viable alternative in cases where the residual series obtained from the parametric models are not normally distributed. Using a nonparametric model as an alternative to a parametric models may involve a trade-off in terms of statistical performance under certain conditions. A procedure for implementing the record extension techniques is presented.

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"*He who ignores the water under the bridge will soon find the bridge under water.*"

- Anonymous

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Chapter 1

INTRODUCTION

1.1 BACKGROUND

Many types of water resources projects are dependent on the principles and data of hydrology (Meinzer, 1942). Examples include reservoir design, irrigation and water supply systems. Additionally, there is a growing concern for anthropogenic effects on the environment and water quality with ever-increasing demands for water and conflicting water uses. Reliable streamflow records are required to determine the environmental impacts, on receiving water bodies for licensing proposed wastewater treatment facilities. Water is an extremely valuable resource.

Water resources systems undergo systematic analysis for optimization studies and decision support for development activities. Hydrologic time series are currently used in practice to simulate the outcome of hydropower production scheduling, systems operating policies, wastewater treatment plants and water rights agreements, for example. Estimating the frequency and duration of domestic, industrial and agricultural water supply shortfalls, as well as dependable reservoir yield for hydropower generation during peak demand are other examples of uses for streamflow records. The latter examples are commonly used to determine a project's economic feasibility and environmental impacts.

Optimal planning and management decisions require reliable data. Raman et al.(1995) state that in developing countries, the expansion of hydrologic data often accompanies or lags development and planning rather than preceding it. Raman et al.(1985) also state that reliable results can be obtained from a systems study when data are available for a minimum of 35 years. Locally, Manitoba Hydro has used streamflow data reconstruction techniques to generate flows for the Nelson River basin (Report No.: T.M. No.93-3). Here, available data are ranked as good or poor in terms of availability of observed natural flows.

Many streamflow recording gauges have operated for only a short time where long-term records are desired. Records available for many streams are too short to contain a sufficient range of hydrological conditions. Additionally, users of streamflow databases containing many streamflow records may desire the database be consistent in terms of record length. Jackson (1975) notes that hydrologic record lengths, in general, tend to be very short, and typically span 25 to 50 years. One difficulty with short streamflow records lies in the large variances resulting from small sample estimates of population parameters. This difficulty is compounded with unreliable or missing portions of data. Providing missing data is the justification for synthetic hydrology.

Streamflow record extension techniques are used to synthesize portions of a record where a longer period of record is desired. Two broad categories of streamflow generation models are referred to as deterministic and stochastic models. Deterministic models are based on physical characteristics of the drainage basin and the hydrologic concepts translating historical meteorological inputs, such as rainfall, into basin outflow. Stochastic models are statistical in nature. However, Clarke(1973) makes a distinction between stochastic and statistical models to emphasize the time-dependence of the hydrological variables related by the model. The difference between a deterministic and stochastic model is clearer in that a deterministic model's parameters are free from random variation and are not associated with a probability distribution.

Models within the stochastic group include parametric models such as linear regression, for example, which transpose information from longer records to shorter records by exploiting the interstation correlation between the short and the long record station. Parametric methods, however, involve estimating various statistical parameters (such as sample mean, variance and cross correlation) of the short and long record stations, which are subsequently used for the extensions.

Some assumptions related to parametric methods are the observations are independent across time, the residuals are normally distributed with zero mean, and are independent. Concurrent observations of the base record and the short record are assumed to have a bivariate normal probability distribution and exhibit stationarity and serial independence. Another assumption inherent in the parametric techniques is the validity of the estimated regression parameters after an appropriate data transformation.

1.2 OBJECTIVES

The objective of the study is to review the parametric models based on linear regression and develop a variation to existing nonparametric methods of streamflow record extension. Models are developed to carry out the extension techniques. The capability of the models are evaluated by a statistical performance comparison in generating flows.

Nonparametric streamflow record extension is presented as an alternative type of record synthesis technique. The nonparametric method recognizes flow patterns in the historical streamflow record. The flow patterns are utilized to generate synthetic flows based upon the relationship between a longer flow record and a (presumably) shorter flow record undergoing extension. Nonparametric record extension techniques do not require assumptions with regard to the probability distribution of the data or sample-estimated parameters.

Pattern recognition seeks to categorize input data, such as a streamflow record, into identifiable classes by extracting significant features, then associate that particular feature with one or more features from the past (memory). Examples of applications of pattern recognition are such automated systems as character recognition, weather forecasting, voice type-writing, medical diagnosis, data classification, target identification, and fingerprint identification (Tou, 1967). In each of these examples an abstract pattern is used as a basis for recognition, based upon the extraction of a set of recognized features associated with the subject.

Hydrologic phenomena measured in daily, weekly, or monthly intervals may be considered to occur in well-defined groups (Panu and Unny, 1978). Variation in hydrological data is influenced by environmental factors such as weather patterns which either cause it or affect it in some way. An example of this is the periodic nature of monthly streamflow data sequences which vary according to season, due to factors including spring snowmelt or precipitation, for example.

1.3 SCOPE OF WORK

The scope of the study will involve presenting the theoretical background and applying both parametric and nonparametric models for evaluation based on pre-selected adequacy measures on selected streamflow records from Manitoba and Ontario. The models are evaluated based on their ability to reproduce various features of historical monthly streamflow records. A method of implementing the most appropriate model to generate streamflows, based on a given set of data conditions, is presented.

Chapter 2 presents a review of selected literature on streamflow record extension techniques. Chapter 3 describes the current parametric record extension methods found in the literature and the theory and formulation of the nonparametric methods. Chapter 4 presents an evaluation and application of the parametric and nonparametric methods formulated in Chapter 3, as well as a discussion of the results. Chapter 5 summarizes the methodology analysis and results, and provides a recommendation for selection of the appropriate extension model, based upon the study findings. Future work on streamflow record extension techniques is recommended.

Chapter 2

LITERATURE REVIEW

There are many variations in the methods of transferring data from one location to another with stochastic models. Some variations utilize physical basin parameters to help improve the reliability of the estimates of synthetic streamflows. Hirsch (1979) utilized drainage area ratios, regional basin characteristics, and cross-correlation of flow records. Faucher (1994) utilized physiographic and climatic data as independent variables to develop regional flow duration curves and proration on drainage area and mean annual runoff to generate synthetic flows at an ungauged location. Parret and Cartier (1990) also used basin characteristic and climatic variables to estimate average monthly flows in Western Montana.

Simonovic (1995) developed three mathematical models for data interpolation, extrapolation and transfer to ungauged sites within the Red-Assiniboine and Interlake districts of Manitoba. Various physical parameters were incorporated into the models to enhance monthly streamflow synthesis. The data was transformed by logarithms, then replaced by their empirical probabilities, and finally standardized according to the inverse functions of the normal distributions. Including physical parameters improved the extension models, in terms of reducing the standard error of estimate. The most beneficial parameters were snowmelt, elevation, precipitation and elevation, however, different parameters provided better results when used in different regions.

Hirsch (1982) presented four extension methods (described in detail in Chapter 3), and evaluated them to determine their suitability in terms of reproducing historical sample statistics and comparing the bias and error in estimating sample statistics. In Hirsch's work, the parameters of the regression extension method are modified to maintain the sample mean and variance, rather than to minimize squared errors, for the case where the two streamflows are similar in distribution shape, serial correlation and seasonality. Alley and Burns (1983) utilized

Hirsch's maintenance of variance extension (MOVE) techniques using different stations as the base station to predict different flows for the same short (index) record. A decision rule was presented to determine whether to use only flow values from the same month or sequential flow values in the parametric methods. Vogel and Stedinger (1985) recommended improved, equal or lower-variance unbiased estimators of the mean and variance of the flows of the short record due to the small sample sizes often encountered in record extension practices.

Parret and Cartier (1990) estimated mean monthly discharge based on multiple regression of basin characteristics, climate variables, channel width and a maintenance of variance extension technique developed by Hirsch (1982). The evaluations of the techniques were based on standard error of estimate. The standard errors were reduced by calculating weighted average estimates from all three extension methods. Parret and Cartier (1990) state that "regression equations based on basin characteristics are generally not applicable to streams that receive their water from springs or that lose substantial flows because of permeable streambeds or other localized geologic features". This is assumed to not be a significant factor for the streamflows utilized in this study.

Beauchamp et al. (1989) compared regression and time-series methods for synthesizing daily streamflow records. Significant autocorrelation of error terms occurred with the regression method, but did not affect the estimated flows. The time series model was able to adequately model the autocorrelation existing in the data thus eliminating autocorrelation of errors. Vogel and Kroll (1991) evaluated the maintenance of variance extension techniques for low-flow and flood frequency analysis.

According to Vogel and Stedinger (1985), the relationship between the flows at the two sites is independent of the month in which the flows occurred, and the MOVE procedures are intended for situations where the two streamflow records do not differ substantially in terms of distribution shape, serial correlation, or seasonality. If the relationship between flows at the two sites exhibits seasonal differences, one could develop different models for flows occurring in different months within distinct seasons.

The regression technique cannot be expected to provide a record with the appropriate variability (Hirsch, 1982; Alley and Burns, 1983). A regression technique which incorporates an independent random noise component cannot be expected to provide records with the appropriate distribution shape or serial correlation (Hirsch, 1982). Furthermore there is no single unique record obtained from a regression model which incorporates added noise. Maintenance of variance procedures have been shown to reduce bias in the estimated mean and variance (Hirsch, 1982; Alley and Burns, 1983; Raman et al., 1995).

Pattern recognition involves identifying features in an object and relating similar features of one object to other objects, in order to determine their similarity. Pattern recognition concepts are used in a variety of applications, and are not necessarily restricted to the classical applications described in Chapter 1. Biquan et al. (1986) used pattern recognition techniques to predict the occurrence of large earthquakes based on intermediate earthquake activity and sunspot occurrences. This analysis determined that occurrences of large earthquakes does not correlate well with sunspot activity, but are related to acceleration or deceleration of the rotation of the earth. Henley and Hand (1996) utilized "nearest neighbor" techniques for assessing consumer credit risk in determining creditworthiness of consumer loan applicants. The nearest neighbor method classified the applicants into good or bad risk groups. The Euclidean distance metric was utilized in the decision function for classification.

Karlsson and Yakowitz (1987) utilized nearest neighbor methods for nonparametric rainfall-runoff forecasting. Karlsson and Yakowitz state that the nearest neighbor method has powerful theoretical properties, but at the same time is disarming in its simplicity and intuitiveness. They concluded that the nearest neighbor method is well suited to large-sample time series problems, and the method is applicable to virtually any decision problem..

Andrews (1972) presents mathematical techniques used in pattern recognition. Andrews states that the criterion for selecting appropriate dimensions or features of the data is one of maintaining those features which have the largest variance across the sample means of the classes, and presents techniques for ranking data in order to group patterns with similar features.

Cooper and Clarke (1980) state that parametric flood frequency estimation methods, as well as parametric streamflow record extension techniques make assumptions about the joint probability distribution of the flow records used. In flood frequency estimation, serial correlation in the streamflows used provides a complication in that the maximum likelihood estimates of the parameters must be adjusted in some way.

Kavvas and Delleur (1975) state the yearly rotation of the earth around the sun causes a yearly periodicity in the monthly hydrologic time series, which is manifested in the autocorrelation function with a 12 month period. Nonseasonal differencing and seasonal differencing are presented as means of removing the periodic component of the time series. Nonseasonal differencing is defined as taking the numerical differences of the first-lag flow values. Seasonal differencing takes the differences between flows separated by 12 months. Their evaluation was based on examination of the autocorrelation function. Their results on 15 watersheds in Indiana showed that both seasonal and nonseasonal differencing reduce the periodicity in the data.

Panu and et al. (1978) present a procedure for extracting information in hydrologic time series, based on pattern recognition principles. In this case, feature vectors are synthesized based on an assumed normal distribution of the elements of an associated reference vector of a particular pattern class. This procedure was used to generate flows for the South Saskatchewan River with good results. Panu and Unny (1980) extended the previous work of 1978 for small and medium sized catchment areas, namely for the Fraser and Black Rivers. The feature prediction model reproduced various statistics of the historic record adequately.

Yakowitz (1987) utilized nearest neighbor methods for runoff prediction in the Bird Creek Ohio watershed. In his study, the feature vector was arbitrarily composed of three sequential daily flows and two rainfall observations. The nearness of the feature vectors was calculated using a weighted Euclidean distance (Euclidean distance is described in Chapter 3). The evaluation was based on comparing the sum of squared errors. The 'best' number of nearest neighbors was searched for in the range of 3 to 6 only. The results of the nearest neighbor model were compared to a deterministic model and a second-order auto regressive moving average model (ARMA). The nearest neighbor and ARMA models produced comparable sum of squared error, while the physically based deterministic model performed less satisfactorily.

Tou and Gonzalez (1974) provide a comprehensive background to the basic concepts of pattern recognition, including feature vectors, and pattern space. Tou and Gonzalez suggest utilizing distance functions which measure the separation of feature vectors in pattern space. Andrews (1972) states that every human being is a pattern recognition expert, but few people can accurately describe the processes they use to classify or discern patterns. Tou (1967) states that a set of features suitable for pattern recognition reflect certain properties of the pattern class, and provides methods of associating statistical features of a particular pattern class, called kernels, which are determined from the observed data.

Chapter 3

PARAMETRIC AND NONPARAMETRIC STREAMFLOW RECORD EXTENSION TECHNIQUES

3.1 INTRODUCTION TO PARAMETRIC RECORD EXTENSION

While most records are available up to the present, the starting year of many records are different, resulting in different lengths of available common record period. In this study, the extension procedures are developed to address a need to obtain streamflow records which are consistent in terms of their common period of record. The extension methods can be used for either forecasting, for example, where the index record station was discontinued, or back-casting where the index and base station records do not begin in the same year, but may both have available record to the present. The parametric extension methods presented below assume there is a short record station, referred to as the index station, which is to be extended back in time using a longer record, referred to as the base station, for which data collection began at an earlier date than the index station.

The parametric extension methods transfer information from the base station to the index station by exploiting the inter-station correlation between the base and index station. Normally the records from the base and index station come either from within the same drainage basin or from nearby drainage basins having similar topography and geology, which result in significant cross correlation of streamflow characteristics.

There are deficiencies in using a single base station, as stated by Alley and Burns (1983). These include the potential to use other potentially important base stations to fill-in a portion of missing records in different time periods. The parametric methods described herein do not remove trend from the data, if it exists, to be modeled separately from residuals, then combined at the end to form the completed series. If a linear trend is not properly modeled, it can produce autocorrelation in error terms. Autocorrelation in error terms has been defined to be a departure from the model assumptions. Additionally, autocorrelation in error terms can occur if an independent variable has been left out of the analysis. In the case of streamflow record extension, the information left out is oftentimes the time varying effects, such as weather patterns that affect the relationship between the streamflow records of two gauges. These effects usually cannot be incorporated because they are often unknown or are not quantifiable. In this case Neter et al. (1989) propose a correlation transformation of the independent and dependent variables, or a first-differencing transformation of the data. Neter et al. (1989) note that the first-differencing procedure can overcorrect, resulting in negative autocorrelation in error terms.

A statistical procedure which is insensitive to departures from the assumptions which underlie it is called robust. There are departures from the regression assumptions which will be discussed in the formulation of the procedures. The regression model assumes that the error terms are normally distributed and have constant variance, and therefore the dependent variable has the same constant variance, regardless of the level of the independent variable. The regression model also assumes the error terms are uncorrelated such that the outcome in a particular trial is independent of the outcome of other trials, ie. the observations are independent.

The extension and common periods of record are defined as follows. The log-transformed base station flows are denoted x_i, where i is a time index corresponding to months of record. The log-transformed index station flows are thus denoted y_i. The extension period is defined as N_1 years, and the common period of record is defined as N_2 years. The observed and extension period events for the two flow sequences are represented in Table 3.1.

Table 3.1 Definition of Extension and Common Record Periods

Record	Extension Period	Common Period
Base Station, x_i	$x_1, \ldots, x_{N1}$	$x_{N1+1}, \ldots, x_{N1+N2}$
Index Station, y_i	$\hat{y}_1, \ldots, \hat{y}_{N1}$	$y_{N1+1}, \ldots, y_{N1+N2}$

3.2 PARAMETRIC METHODS

Parametric streamflow record extension techniques, as presented by Hirsch (1979; 1982), Alley and Burns (1983), Vogel and Stedinger (1985), and Grygier and Stedinger (1989), are based upon an equation of the following form

$$\hat{y}_i = \alpha + \beta \cdot x_i \qquad 3.1$$

where

$\hat{y}_i$ = synthesized log-streamflows at time i,

α and β = regression parameters,

x_i = log of base station flow.

Equation 3.1 can be described as a simple linear regression model. The model is said to be simple in that there is only a single independent variable. The model is linear in the regression parameters because none of the parameters appear as a nonlinear function, such as an exponent. The model is linear in the independent variable because x_i appears in the first power. Equation 3.1 could also be correctly referred to as a first order-model.

Assumptions related to parametric extension are that the time series are stationary and serially independent. Time series x_i and y_i are also assumed to have a bivariate normal distribution, with parameters μ_x, μ_y, σ_x, σ_y, and ρ, where μ_x and σ_x^2 represent the population mean and variance for series x_i, and μ_y and σ_y^2 represent the population mean and variance for y_i, and the product moment correlation coefficient is ρ.

The streamflows are first log-transformed to remove the skewness from the data. Alley and Burns (1983), Beauchamp et al. (1989), and Vogel and Kroll (1991), also utilized log-transformed flows for the parametric regression models. The parametric techniques are conducted upon the log-transformed data to generate synthetic streamflows. The synthesized streamflows $\hat{y}_i$ are reverse-transformed to achieve the flow value in m^3/s. If the estimated flow value, $\hat{y}_i$, exceeds or falls short of the historical value, y_i ,we define the error term amount, e_i, by

$$e_i = y_i - \hat{y}_i.$$

It will be shown that the error terms, e_i, are assumed to have constant variance, σ^2, thus y_i are assumed to have the same variance regardless of the level of x_i. The error terms are also assumed to be uncorrelated, so that one value of x_i has no effect on the error term for any other value of i. Thus, if the error terms are uncorrelated, so are the responses y_i (i.e. y_i is assumed to be a random variable).

There are departures from the model assumptions when equation 3.1 is used to extend streamflow data. An attempt is made to overcome these departures by taking log-transforms of y_i. The success of this transformation is a factor upon which the appropriateness and ultimately the acceptance or rejection of the parametric models will be based in Chapter 4.

Five parametric extension models are presented below. Regression, (REG), regression plus independent noise (RPN), and three maintenance of variation extension techniques, (MOVE.1, MOVE.2, and MOVE.3). All five parametric models utilize the form of equation 3.1, with the differences in the way in which the parameters α and β are developed. REG, RPN, MOVE.1 and MOVE.2 were given by Hirsch (1982), and compared to one another based on a 2000 trial Monte Carlo simulation. REG, MOVE.1, and MOVE.2 were utilized by Alley and Burns (1983), in simulating monthly streamflows with or without separate monthly relationships. MOVE.3 was introduced by Vogel and Stedinger (1985) as an improvement to small sample estimates of the mean and variance of the streamflow series.

A single regression model developed using the data from all concurrent flow periods is referred to as the noncyclic approach. Separate regression equations developed for each month or season is referred to as the cyclic approach. The noncyclic approach assumes that the variability in flows is random rather than partly cyclic and partly random (Alley and Burns, 1983). However, more parameters are required for the cyclic approach (by a factor of 12). Alley and Burns (1983) present a methodology for selecting which approach to use. Both approaches are evaluated in this study.

Parret and Cartier (1990) provide the following two considerations when attempting to fit a regression line to time series flows.

1. If the relationship between the concurrent high flows for the base and index records are different, a single straight line may not be appropriate.

2. If differences exist between the timing of runoff at the base and index records, a plot of concurrent discharges will resemble a loop, which is also not modeled well by a single regression line.

Utilization of log-transformed streamflow records which are similar in terms of geographic location and which exhibit a high cross correlation is hoped to minimize the deviation from a single straight line fit. Thus, the models are developed in a similar manner to that found in the literature, using a single regression line.

3.2.1 Linear Regression (REG)

The marginal distributions of f(x,y) are assumed to be univariate normal and with conditional distribution defined as $f(y|x) = f(x,y) / f(x)$ (Viessman et al., 1977). The subscript i denoting time have been omitted within portions of this section for convenience of discussion. Viessman et al. (1977) also state the conditional distribution of y, given x, has the form

$$f(y \mid x) = K' \exp\{M'\} \qquad 3.2$$

where

$$K' = \left[\sqrt{2\pi}\sigma_y \sqrt{1-\rho^2}\right]^{-1} \qquad 3.3$$

and

$$M' = \left\{-\left[\frac{1}{2\sigma_y^2(1-\rho^2)}\right]\left[(y-\mu_y) - \rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)\right]^2\right\}. \qquad 3.4$$

The distribution is normal with mean

$$\mu_{y|x} = \mu_y + \rho\frac{\sigma_y}{\sigma_x}(x-\mu_x) \qquad 3.5$$

and variance

$$\sigma_e^2 = \sigma_y^2(1-\rho) \qquad 3.6$$

where

σ_e^2 = the error variance.

Equation 3.2 expresses the probability distribution of a value of y for any given x. Equation 3.5 is a linear equation that expresses the linear dependence between y and x. The mean, or expected value of y can be calculated using equation 3.5 for fixed values of x. The fraction of the variance in the data which is explained by the regression line is obtained by rearranging equation 3.6 as follows

$$\rho^2 = 1 - \frac{\sigma_e^2}{\sigma_y^2}. \qquad 3.7$$

From equation 3.5, the slope of the regression line is

$$\rho\frac{\sigma_y}{\sigma_x}. \tag{3.8}$$

The parameters α and β in equation 3.1 are found by fitting the regression line described by equation 3.1 to the data such that the squared error is minimized. Re-introducing the time subscript, i, the squared error criterion, Q is given by the squared deviation of y_i from its expected value

$$Q=\sum(y_i-\alpha-\beta\cdot x_i)^2. \tag{3.9}$$

Point estimators of α and β are those values a and b, respectively, which minimize Q for a given set of sample observations (x_i,y_i) (Neter et al., 1989). The point estimators a and b may be found by partially differentiating Q with respect to α and β. Taking partial derivatives, using a and b to denote the respective values of α and β, and setting to zero, we get

$$\frac{\partial}{\partial\alpha}=-2\sum(y_i-a-b\cdot x_i)=0 \tag{3.10a}$$

$$\frac{\partial}{\partial\beta}=-2\sum x_i(y_i-a-b\cdot x_i)=0 \tag{3.10b}$$

It can be shown that the simultaneous solution of equations 3.10a and 3.10b yields

$$a=\frac{\sum y_i}{n}-\frac{b\sum x_i}{n} \text{ and} \tag{3.11}$$

$$b=\frac{\sum x_i y_i-\frac{1}{n}\sum x_i\sum y_i}{\sum x_i^2-\frac{1}{n}\left(\sum x_i\right)^2}, \tag{3.12a}$$

or, from equation 3.8,

$$b = r\frac{s_y}{s_x}, \tag{3.12b}$$

where

r = the product moment correlation coefficient estimated from the data,

S_y = the sample standard deviation of the y series,

S_x = the sample standard deviation of the x series,

n = sample size.

The above parameters are calculated as follows

$$S_x = \sqrt{\frac{\sum_i \left((x_i - \bar{x})^2\right)}{n-1}} \tag{3.13a}$$

$$S_y = \sqrt{\frac{\sum_i \left((y_i - \bar{y})^2\right)}{n-1}} \tag{3.13b}$$

$$r = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]^{0.5}}. \tag{3.13c}$$

and

$$\bar{x} = \frac{\sum_i x_i}{n} \tag{3.13d}$$

$$\bar{y} = \frac{\sum_i y_i}{n}. \qquad 3.13e$$

The REG equation is found by substituting parameters a, b and r into equation 3.1

$$\hat{y}_i = m(y_2) + r\frac{S(y_2)}{S(x_2)}(x_i - m(x_2)) \qquad 3.14$$

where the sample statistics are defined in Table 3.2. The nomenclature in Table 3.2 follows that of Hirsch (1982), and Alley and Burns (1983), except that the extension period, N_1, and common period, N_2, are reversed as explained previously. No numerical subscript is used with those estimates based on the entire record period, $N_1 + N_2$.

If the probability distribution of y is not normally distributed around a mean value for a given level of x, then the probability distribution of β is not normally distributed. Then inferences regarding the mean value of β become less valid, because β is a linear function of the observations of y, and a linear combination of normally distributed variables is itself normally distributed.

The regression model relates the means of the probability distribution of y_i for a given x_i to the level of x_i (Neter et al., 1989). Hirsch (1982) states that m(y) is an unbiased estimate of μ_y, and shows that $S^2(y)$ is a downward-biased estimate of σ_y^2 for $\rho < 1$. Stated another way, REG, by design, has the property of estimating the mean quite well, but provides a record with a lower variability than that which would be normally associated with streamflow records. One property of equation 3.14 is for r = 0, REG would provide a zero variance extended series. Alley and Burns (1983) state "the usual intent of record extension is to produce a time series that possesses statistical properties like those of an actual record for the station".

Table 3.2 Definition of Sample Statistics

Statistic	Definition
	Sample Mean of
$m(x_1)$	$x_1,...,x_{N1}$
$m(x_2)$	$x_{N1+1},...,x_{N1+N2}$
$m(x)$	$x_1,...,x_{N1}, x_{N1+1},...,x_{N1+N2}$
$m(y_1)$	$y_1,...,y_{N1}$
$m(y)$	$\hat{y}_1,...,\hat{y}_{N1} y_{N1+1},...,y_{N1+N2}$
	Sample Variance of
$S^2(x_1)$	$x_1,...,x_{N1}$
$S^2(x_2)$	$x_{N1+1},...,x_{N1+N2}$
$S^2(x)$	$x_1,...,x_{N1}, x_{N1+1},...,x_{N1+N2}$
$S^2(y_1)$	$y_1,...,y_{N1}$
$S^2(y)$	$\hat{y}_1,...,\hat{y}_{N1} y_{N1+1},...,y_{N1+N2}$

3.2.2 Linear Regression Plus Independent Noise (RPN)

A normally distributed random noise component, e_i, with zero mean and unit variance is added to the REG model as follows

$$\hat{y}_i = m(y_2) + r\frac{S(y_2)}{S(x_2)}(x_i - m(x_2)) + \alpha\sqrt{(1-r^2)}S(y_2)\cdot e_i \qquad 3.15$$

where

$$\alpha^2 = \frac{N_1(N_2-4)(N_2-1)}{(N_1-1)(N_2-3)(N_1-2)} \qquad 3.16$$

The estimates y_i are a weighted combination of the historical x_i series and an unrelated random noise component. The parameter α is a constant used to make the expected sample variance of y_i equal to its population value, which is the purpose of the noise component in equation 3.15 (Vogel and Stedinger, 1985).

Due to the random noise component which is presumably unique, there will be no single unique record obtained by using this method. The latter property may make the RPN method undesirable for use in management decisions regarding reservoir design and operation. An advantage with RPN, however, is that it provides unbiased estimates of mean and variance of the historical record (Hirsch, 1982). RPN has also proved useful for preserving interstation correlation between the index and base record (Alley and Burns, 1983).

3.2.3 Maintenance of Variance Extension Type 1 - MOVE.1

The importance of accurately estimating hydrologic extremes, as well as reducing bias from variance estimates served as one purpose to develop the maintenance of variance extension (MOVE) procedures. The MOVE procedures are appropriate to use when the index and base record do not differ substantially in distribution shape, serial correlation or seasonality (Vogel and Stedinger, 1985). However, where the cross correlation is relatively high, it is reasonable to assume that the serial correlation and seasonality do not differ substantially. A departure from this assumption would result in a reduction in the cross correlation between the index and base records.

The four sample estimates used in MOVE.1 are the sample means and variances of the x and y series estimated from the common record period, $N_{1+1},\ldots N_1+N_2$. For MOVE.1, a and b are chosen such that the sample mean and variance of the estimates equal the sample mean and variance of the index station during the common period of record. Hirsch (1982) states the above is accomplished by finding a and b such that the following two equalities are satisfied

$$\sum y_i = \sum \hat{y}_i$$

$$\sum (y_i - m(y_2))^2 = \sum (\hat{y}_i - m(y_2))^2,$$

to which a solution is given as

$$\hat{y}_i = m(y_2) + \frac{S(y_2)}{S(x_2)}(x_i - m(x_2)) \qquad 3.17$$

Parret and Cartier (1990) state that the MOVE.1 technique minimizes the areas of the right triangles formed by the horizontal and vertical deviations from the regression line, and that MOVE.1 provides an unbiased estimate of low flows.

A time series y_i generated by MOVE.1 using equation 3.17 for $i = 1, \ldots, N_1 + N_2$ would reproduce the historical sample moments $m(y_2)$ and $S^2(y_2)$ (Vogel and Stedinger, 1985). MOVE.1 has been found in practice to over-estimate the variance (Hirsch, 1982). Although MOVE.1 results in preservation of the index-record sample estimates of mean and variance, the estimated variance for short records may be unreliable (Alley and Burns, 1983). However, the estimates of mean and variance are asymptotically unbiased as the common record period approaches infinity (Alley and Burns, 1983).

3.2.4 Maintenance of Variance Extension Type 2 - MOVE.2

In MOVE.2, μ_y and σ_y^2 are set to the unbiased estimates developed originally by Matalas and Jacobs (1964), as reported by Hirsch (1982), Alley and Burns (1983), Vogel and Stedinger (1985), and Grygier et al. (1989). In contrast to MOVE.1, the sample estimates of mean and variance for x are based on the entire record, i.e. $m(x)$ and $S^2(x)$ are used in place of $m(x_1)$ and $S^2(x_1)$. The sample estimates of mean and variance of y are based on the historical y record, and more information from the base station record. The above estimates also make use of the correlation between the base and index station record to improve the estimate of the index record's mean and variance (Grygier et al., 1989).

The MOVE.2 equation is

$$\hat{y}_i = \hat{m}(y) + \frac{\hat{S}(y)}{S(x)}(x_i - m(x)) \qquad 3.18$$

where

$$\hat{m}(y) = \frac{N_2}{(N_1 + N_2)} r \frac{S(y_2)}{S(x_2)}(m(x_1) - m(x_2)) \qquad 3.19$$

$$\hat{S}^2(y) = \frac{1}{N_1 + N_2 - 1}\{(N_2 - 1)S^2(y_2)$$

$$+(N_1 - 1)r^2 \frac{S^2(y_2)}{S^2(x_2)} S^2(x_1) + (N_1 - 1)\alpha^2(1 - r^2)S^2(y_2)$$

$$+\frac{N_1 N_2}{(N_1 + N_2)} r^2 \frac{S^2(y_2)}{S^2(x_2)}(m(x_1) - m(x_2))^2\Big\}. \qquad 3.20$$

If MOVE.2 were used to generate an entire sequence $\hat{y}_i$, for i = 1,... N_1 + N_2, unbiased estimates of the mean and variance of the complete extended record would be reproduced. Alley and Burns (1983) state that improvement in the estimate of variance using MOVE.2 is achieved when the correlation coefficient exceeds about 0.65.

3.2.5 Maintenance of Variance Extension Type 3 - MOVE.3

In MOVE.3, μ_y and σ_y^2 are set to the conditional means and variances for the y_i series over the period of record for which the extension $\hat{y}_i$ is to be developed (Stedinger and Vogel, 1985; Grygier et al., 1989). The conditional mean and variance may be interpreted as the sample size weighted difference between the augmented mean and variance estimators and the mean and variance of the short record y_i series (Grygier et al., 1989).

The MOVE.3 equation is

$$\hat{y}_i = a' + b(x_i - m(x_1)). \qquad 3.21$$

The MOVE.3 estimates of a' and b are obtained from

$$a' = \frac{(N_1 + N_2)\hat{m}_y - N_2 m(y_2)}{N_1} \qquad 3.22$$

$$b^2 = \left((N_1 + N_2 - 1)\hat{S}^2(y) - (N_2 - 1)S^2(y_2) - N_2(m(y_2) - \hat{m}(y))\right.$$

$$\left. - N_1(a' - \hat{m}(y))^2\right)\left[(N_1 - 1)S^2(x_1)\right]^{-1}. \qquad 3.23$$

Grygier et al. (1989) found little difference between MOVE.2 and MOVE.3 when estimating μ_y and σ_y from a sample of 30-50 years of available record. Vogel and Stedinger (1985) state that MOVE.2 and MOVE.3 are "nearly indistinguishable" in the mean squared error (MSE) of the estimators of the mean and variance of the completed extended record.

3.3 THE CORRELATION COEFFICIENT

The correlation coefficient plays a significant role in the parametric record extension procedures. However, the existence of a high correlation coefficient does not establish a causal relationship between two time series. A correlation arises through a concurrent variation in time of the two time series. Thus, causation cannot be deduced by co-variation. Yule (1926) addressed the fact that quite high correlation may be obtained between time series to which no physical explanation can be made. Thus in the present work, the candidate streamflows used for applying the record extension methods will not only require a high correlation value, but will also be required to be relatively near in terms of location, such that the basin characteristics affecting the runoff would not be drastically dissimilar.

Kendall and Stewart (1961) defined a statistical relationship arising from the existence of a joint distribution between a pair of random variables. The statistical relationship is the basis of the parametric methods developed in Section 3.2. A functional relationship arises when the level of one variable is a deterministic function of one or more other variables.

If the base station lies either upstream or downstream from the index station, there is likely to be a high degree of correlation between the records. The functional relationship between the index and base record becomes apparent in that the index record flows would be directly caused by the base record flows if the index record is downstream of the base record.

However the occurrence of this situation does not invalidate the parametric models since no assumptions are made of the independence of the two time series being used in the analysis. In fact, as was shown in Section 3.2.1, the methodology on which the parametric methods are based, relies upon the strength of the conditional distribution between y and x.

One problem associated with utilizing the correlation coefficient as a determination of interdependence is that the coefficient is a measure of linear independence, and as such, cannot distinguish interdependence for more complex forms of interdependence. Thus, if x and y are independent, $\rho=0$, but the converse does not necessarily apply. Therefore, in a strict sense, ρ is recommended by Kendall and Stuart (1961) as an indicator of interdependence rather than a measure of independence, unless we are faced with normal or near normal variation between x and y.

It is useful at this point to make a distinction between the coefficient of determination, r^2, and the coefficient of correlation, r. Mathematically the interrelation is expressed as

$$r = \pm\sqrt{r^2}\,. \qquad 3.24$$

The degree of linear association between x and y is measured by r^2 as the ratio of the variance of the fitted line to the overall variance. Stated another way, r^2 is a measure of the effect of x in reducing the variation in y. The range of r^2 is

$$0 \le r^2 \le 1\,. \qquad 3.25$$

The range of r is

$$-1 \leq r \leq 1. \qquad 3.26$$

The nonparametric methods introduced in Section 3.4 do not utilize the correlation coefficient in generating streamflow data, although a significant correlation between the index and base records is a good indication that the streamflows exhibit similar patterns (or time variation characteristics).

3.4 INTRODUCTION TO NONPARAMETRIC RECORD EXTENSION

The nonparametric method of streamflow record extension utilizes pattern recognition concepts in order to identify flow patterns or analogues within the data. This information is used to develop synthetic flow sequences. As is the case with parametric extension methods, the interstation correlation between two streams is exploited, but in a different way. There is no requirement to use the sample estimate of ρ in the calculation of flow sequences, rather the existence of a significant ρ between the base and index station is used to indicate similar characteristics of the flow records.

The parametric methods described previously utilize data samples to estimate population parameters such as mean, variance, correlation, and parameters a and b. Additionally, there are certain assumptions regarding the necessity for normally distributed variables. The pattern recognition principles used herein do not rely on the data following the form of any parent distribution. No parameters are required from the samples to estimate population statistics. However, one premise is that the relationship between the occurrences of flow patterns is distinct, and that those combinations of flow patterns which occur at the same time (season, for instance) will tend to repeat themselves. Thus the co-variation existing between the base and index records is relied upon such that the relationship between the patterns which occur in the base and index record are relatively consistent throughout the duration of the records.

A pattern is defined by Tou and Gonzalez (1974) as the description of an object. An object could be a group of data on hydrologic phenomena observed at regular time intervals, such as a series of flows within a streamflow record. The concept of patterns within hydrologic phenomena is not new. Panu et al. (1978) state that sequences of hydrologic data occur in well defined groups which possess the collective properties of the data forming them. Evidence of recurring patterns in hydrologic data is provided locally with the high flows which occur in the Red River each spring. A hydrologic pattern is a collection of properties describing the groups formed within the data.

The act of pattern recognition can be viewed as two major types: the recognition of concrete items and the recognition of abstract items, as outlined by Tou and Gonzalez (1974). Examples of the first type include recognizing pictures, music, and the objects around us, also referred to as sensory recognition. Examples of recognizing abstract items include recognizing an old argument or solution to a problem, also termed conceptual recognition. The present work involves utilizing pattern recognition concepts associated with the sensory recognition application.

The process of sensory recognition involves identification and classification of patterns in the data presented. Tou and Gonzalez define pattern recognition as "categorization of input data into identifiable classes via the extraction of significant features or attributes of the data". Before the procedure is formalized, some terminology is presented.

A classical pattern recognition problem, such as optical character recognition or fingerprint analysis, presents three basic problems. The first is the sensing problem, where the input data measured from the objects are represented in some fashion. This problem does not present a difficulty in dealing with streamflows, since a graph of streamflows which would be analyzed is obtained in a table of discrete values corresponding to equal time intervals in the first place.

In pattern recognition (PR) terms as defined above, the objects in question correspond to portions of a given flow record. A group of flows, or section of the flow record corresponding to n measurements of the flow sequence is an object, (where n is less than the number of measurements in the flow record). Thus there may be many objects within an entire record. A pattern vector is a description of an object through a set of observed measurements. A pattern vector of n measurements are arranged in the form of a vector:

$$\mathbf{x} = [x_1, x_2, \ldots, x_n]^T . \qquad 3.27$$

Where x_n denotes the measured values, and $\mathbf{x}$ denotes the pattern vector. The pattern vector $\mathbf{x}$ thus contains all the measured information about the object. If the values x_n were flow, for instance, the pattern vector would represent a quantitative description of the underlying hydrologic processes resulting in a basin's runoff during a particular period of time. A hydrologic time wave form (HTW) is defined as a plot of the flow values versus time.

When pattern vector measurements are in the form of real numbers, it is appropriate to consider the pattern vector as a point in n-dimensional Euclidean space (Tou and Gonzalez, 1974). A set of patterns which belong to the same class correspond to a collection of points scattered within some region of the measurement space. Consider two pattern classes, ω_1 and ω_2 each containing two measurements, x_1 and x_2, as shown in Figure 3.1.

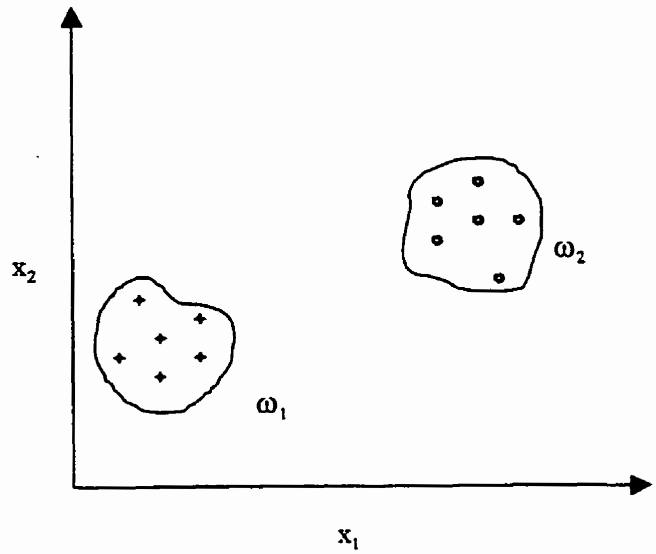

Figure 3.1 Two Hypothetical Pattern Classes

Each pattern class forms separate sets because the measurements of x_1 and x_2 are such that they do not fall in close vicinity in the measurement space.

The second problem lies in extracting characteristic features from the pattern vectors. The collective properties of an object are represented by features of the data contained within the object. It is desirable to utilize features from the data which represent the phenomenon and which can be preserved in the generated record. The contents of the feature vector may be either the flow values themselves, or other attributes identifying changes in flow, for example. This results in the formation of an m-dimensional feature space, where $m \leq n$, referred to as a feature vector

$$\mathbf{f} = [f_1, f_2, \ldots, f_m]^T. \tag{3.28}$$

The third problem involves determining the decision procedure to determine the "closeness" of a given feature to another. This is a modification from the classical PR sense, since the decision procedures are normally developed for the purpose of identifying and classifying an unknown object. However, it will be explained in the following sections that the concepts are modified somewhat to provide a solution to a specific problem. In the present study, the objects within the flow record will be classified based on examination of the HTW. The grouping of the feature vectors will involve determining those feature vectors within a same pattern class which are "near" to each other in the pattern space described above, referred to as "nearest neighbors".

The determination of nearest neighbors is accomplished through a variety of clustering techniques. Flow patterns from the base record during the extension period are compared with a number of nearest neighbors during the common record period to determine points in time (time indexes) where similar flow patterns occurred. This information is combined with the observation that the co-variation between two correlated streams is, by definition, relatively consistent throughout the record. Flow patterns are extracted from the index record corresponding to the identified time indexes, and used to generate synthetic flows during the extension period where the base station flow patterns were compared.

The following sections provide the theory and formulation of the nonparametric extension procedure for a general case. The clustering techniques for determining nearest neighbors are also described in detail. A methodology for determining the appropriate number of nearest neighbors is dealt with in the applications section, since it is not central to the theory.

3.5 NONPARAMETRIC EXTENSION METHOD

3.5.1 Flow Record Segmentation

The first step in formulating the nonparametric model is the segmentation of the base and index flow records into pattern classes. For a given record, let there be N years, or segments, of data, corresponding to N*12 monthly flow values. Following the nomenclature used by Panu et al. (1978), each segment, ζ_i, i = 1, . . .,N, is divided into K sections, or seasons. Thus we have $\zeta_i = \{\xi_i^k\}$ for k = 1,... ,K. The season, k, is constant for all years of a particular record.

In Chapter 4 a methodology is given for determining the segmentation of the streamflow series into seasons. The method of segmenting the flow record essentially utilizes a user-input classification of the known records, combined with a quantitative decision function to aid in the definition of the seasons.

In a classical PR problem, the pattern inputs are not known a priori, and so the classification must be determined from the separation in model space analogous to that shown in Figure 3.1. Then the "distance" (Section 3.5.3) between the feature vector and a (possibly arbitrary) reference vector is calculated and subsequently input into a decision function which determines the class membership of the feature vector. Thus, one may question the use of the classification, since the pattern classes are pre-defined. In the present study, classification is done with a different motivation. The feature vectors in a given class (season) are compared to all other feature vectors in the historical record of the same class, then ranked according to their similarity, in terms of a calculated distance metric which quantifies their separation in model space.

The next step is obtaining the pattern vector, **x**, from the flow measurements contained within ξ_i^k, for each season k, such that

$$\mathbf{x}_i^k = \left[x_{i1}^{\ k}, x_{i2}^{\ k}, \ldots x_{ij}^{\ k}\right]^T \qquad 3.29$$

where j is the number of flow measurements, q(t), within a given season, and t is the time interval. For example, let there be a given monthly flow record which has two seasons, namely a dry season and a wet season, and the wet season lasts from June to October. In this example, let us also define the wet season as k = 1, and the dry season as k = 2. For k = 1, j = {1,... ,5}, and for k = 2, j = {1,... ,7}. Note that the given values of j used in this example do not correspond with the month's sequence of occurrence from the beginning of the calendar year i.e. Jan ≠ 1, Feb ≠ 2, etc. Thus, $x_{ij}^{\ k}$ = q(t) for the discrete time intervals given.

3.5.2 Feature Discrimination

The objective of feature selection is to obtain discriminatory properties of the pattern. This process can be very complicated in many PR problems, since the most important features are not easily measurable, or their measurement is inhibited by the costs of obtaining the data. One example of the former problem occurs in recognition of handwritten features, where the most important discriminatory features are the sequence, direction, and arrangement of strokes, as well as the interrelationship between them (Tou and Gonzalez, 1974). An example of the latter problem occurs in oil prospecting where geographical regions are classified as either containing or lacking a quantity of oil sufficient to warrant exploration. The drilling of a great number of wells would provide the most significant features to correctly classify the region, but at a prohibitive cost. Thus in many cases exploration engineers must use information which conveys features of the data which convey less information.

A second objective of feature selection is to reduce the dimensionality of the pattern space for a simpler discriminatory analysis. In many cases, the given stimulus undergoes significant "pre-processing" through a highly complex series of nonlinear operations (Andrews, 1974). An example would be to convert the handwritten features mentioned above into an n-dimensional measurement vector through the use of a scanning device, then reducing the dimensionality of the pattern space through various transformations. It is a requirement that the attributes be in a form which can be utilized by digital computers, since the mathematical techniques described here would be difficult for a human to duplicate in a reasonable amount of time.

In utilizing streamflow records, we are fortunate that the streamflow measurements themselves are effectively a pre-processed form of data which are suitable for input to a computer program.

The feature vectors are defined by

$$\mathbf{f}_i^k = \left[f_{i1}^k, f_{i2}^k, \ldots, f_{ij}^k\right]^T \qquad 3.30$$

where

$\mathbf{f}_i^k$ = a feature vector,

f_{ij}^{k} = a feature vector element,

i = year index,

j = measurement index (month),

k = class index (season).

The notation $\mathbf{f}_i^k$ and $\mathbf{f}(i)^k$ are used interchangeably for convenience of presentation within certain equations.

One selection of the feature vector is the actual flow values within the pattern vector such that

$$\mathbf{x}_i^k = \mathbf{f}_i^k = q(t) \qquad 3.31$$

where

q(t) = measured flow at time t [m^3/s].

The feature vectors defined by 3.31 contain measurements of the flow patterns representing the underlying hydrologic process.

A second type of feature vector element is defined by

$$f_{ij}^{k} = \{q(t) - q(t\text{-}1)\} \qquad 3.32$$

where

f_{ij}^{k} = a feature vector element,

q(t) = measured flow at time t [m^3/s],

q(t-1) = measured flow at time t-1 [m^3/s].

The second feature vector type is arrived at by a process known as first-differencing. First differencing is utilized to distinguish the feature of increasing or decreasing flow in any two adjacent time periods, i.e. the slope of the hydrograph. These first-difference feature vectors are used in ranking the associated nearest neighbors, but not the actual flow synthesis. In this way the specific feature is utilized in feature discrimination, but a reverse-transformation is not required.

A third type of feature vector element may contain a transformation on the flows, such as standardization

$$f_{ij}^{\prime k} = \frac{f_{ij}^{k} - m(j)}{S(j)} \tag{3.33}$$

where

$f_{ij}^{\prime k}$ = standardized feature vector element,

f_{ij}^{k} = a feature vector element,

$m(j)$ = sample mean of feature vector elements j,

$S(j)$ = sample standard deviation of feature vector elements j.

The standardization transformation is used in the nonparametric model to determine the effects of removing the scale difference in the patterns features being compared. Subtracting the mean flow removes the scale difference between the flows in the transformed time series. Dividing each flow by the standard deviation of the respective month yields a constant variance throughout the time series (Kavvas and Delleur, 1975). Standardizing is done on data which have not undergone any other transformation.

3.5.3 Clustering and Feature Ordering

Feature extraction is performed on both the base and index records to obtain a set of feature vectors for each record. The feature vectors are ranked according to their similarity. The technique is nonparametric in that the data is not described in terms of sample statistics representing moments of a presumed underlying distribution, and also in that the feature vector similarity is determined in terms of the ordered sequential ranks of a calculated distance metric, described below.

A "nearest neighbor" is defined as a feature vector, $\mathbf{f_y}$, which has a small distance in relation to $\mathbf{f_x}$, within the measurement space. The principal purpose of determining nearest neighbors is quantifying the "closeness" of one feature vector to another, within a given pattern class. This is also known as clustering of data in PR terms. Burn (1993) and Tou and Gonzalez (1974), define "closeness" in terms of the Euclidean distance between the two arbitrary feature vectors, $\mathbf{f_x}$ and $\mathbf{f_y}$ as follows

$$D_{E\mathbf{f_x},\mathbf{f_y}}{}^{k} = \left\| \mathbf{f_x}^{k} - \mathbf{f_y}^{k} \right\| \qquad \text{Euclidean Operator} \qquad 3.34$$

and

$$D_{E\mathbf{f_x},\mathbf{f_y}}{}^{k} = \sqrt{\sum_{j=1}^{m} \left(\mathbf{f_x}{}_j{}^{k} - \mathbf{f_y}{}_j{}^{k} \right)^2} \qquad \text{Euclidean Distance} \qquad 3.35$$

where

$D_{E\mathbf{fx},\mathbf{fy}}{}^{k}$ = Euclidean distance between the feature vectors,

$\mathbf{f_x}$ and $\mathbf{f_y}$ = any two m – dimensional vectors,

j = element number, j = 1, . . . , m.

m = the number of elements contained in the feature vectors for a pattern class,

k = pattern class, or season, for 1,... ,K seasons.

A variation of equation 3.35 is to define the distance in terms of the absolute values of the distance between the elements of the feature vector. This is known as the "Manhattan", or "City Block" distance, because of the analogy to the distance one point would be from another point if their separation were defined in terms of x and y coordinates on a city block grid

$$D_{M\,\mathbf{f}_x,\mathbf{f}_y}{}^{k} = \sum_{j=1}^{m} \left| \mathbf{f}_{x\,j}{}^{k} - \mathbf{f}_{y\,j}{}^{k} \right|. \qquad \text{Manhattan Distance} \qquad 3.36$$

A second variation results from a variation in a distance measure proposed by Anderberg (1973), where the Manhattan distance is divided by a measure of the gross magnitude of the two data points, referred to as the Lance and Williams measure, D_{LW}. The equation for D_{LW} is given as

$$D_{LW\,\mathbf{f}_x,\mathbf{f}_y}{}^{k} = \sum_{j=1}^{m} \frac{\left| \mathbf{f}_{x\,j}{}^{k} - \mathbf{f}_{y\,j}{}^{k} \right|}{\left(\mathbf{f}_{x\,j}{}^{k} - \mathbf{f}_{y\,j}{}^{k} \right)}. \qquad \text{L.W. Distance} \qquad 3.37$$

However, equation 3.37 is not appropriate to use with negative values since substantial cancellations may occur in the denominator and possibly give negative distances. A modification was considered to 3.37 which would still take into account the magnitude of the data points in the form of an interaction term

$$D_{MLW\,\mathbf{f}_x,\mathbf{f}_y}{}^{k} = \sum_{j=1}^{m} \frac{\left| \mathbf{f}_{x\,j}{}^{k} - \mathbf{f}_{y\,j}{}^{k} \right|}{\left| \mathbf{f}_{x\,j}{}^{k} \cdot \mathbf{f}_{y\,j}{}^{k} \right|}. \qquad \text{Modified L.W. Distance} \qquad 3.38$$

Equation 3.38 gives more penalty to higher flow differences than equation 3.36. We would expect in advance that there are nearly as many negative first-difference feature vectors as positive, since each rising hydrograph limb is accompanied by a falling limb. Thus equation 3.37 is not appropriate to use since many negative values are likely to arise in the first-difference feature vectors.

Equation 3.38 provides a benefit in that if one flow is substantially smaller, the product in the denominator will decrease, and the calculated distance will increase. However, a disadvantage associated with equation 3.38 is as follows. The interaction denominator term in equation 3.38 would increase D_{MLW} when both flows are very small, and decrease D_{MLW}, when the flows are large. Equation 3.38 essentially calculates a greater distance for flow patterns exhibiting extreme flow events, which may tend to reduce the number of feature vectors of high flow periods being chosen as nearest neighbors. However, the chance of these occurrences depend partly on the accuracy of the classification scheme, as well as the nature of the relationship between the present and past flow patterns, which are not known before hand. Equation 3.38 was included in the analysis for interest since it cannot be determined in advance if the above characteristics are fatal to the method.

The nearest neighbors are selected on the basis of the ranked order (smallest to largest) of the distances calculated between each feature vector of the base station record during the extension period, i, and each other feature vector during the common period, p. Thus, $\mathbf{D}_\xi^k(i,p)$ is calculated between every $\mathbf{f_x}^k(i)$ for $i = 1, \ldots, N_1$, and every other $\mathbf{f_x}^k(p)$ during the common period for $p = N_{1+1}, \ldots N_1+N_2$, for each season, k. This is repeated for each distance metric from equations 3.35, 3.36, and 3.38.

We can define a sequential rank matrix which contains the index of the year of extension record being generated and the year of the r^{th} nearest neighbor corresponding to increasing distances, $\mathbf{D}_\xi^k(i,p)$, with rank, r, as follows

$$\mathbf{R}^k(i,r) = p_r \qquad 3.39$$

where

$\mathbf{R}^k(i,r)$ = a matrix of year indexes, r, sorted in increasing order of $\mathbf{D}_\xi^k(i,p)$,

r = rank of the nearest neighbor, r = 1 corresponds to the lowest $\mathbf{D}_\xi^k(i,p)$, and r = 2 corresponding to the next highest etc., for all $r = 1, \ldots, N_2$,

i = a year during the index period to be extended, $i = 1, \ldots, N_1$,

p_r = a year during the common record period, corresponding to the r^{th} rank as defined above, for all $p = N_{1+1}, \ldots, N_1+N_2$.

The values of $\mathbf{R}^k(i,r)$ are illustrated in Table 3.3, for a hypothetical example.

Table 3.3 Illustration of Sequential Rank Matrix

Year i	Year p_r	Distance, $\mathbf{D}_\xi^k(i,p)$	Rank, r	Rank Matrix Value, $\mathbf{R}^k(i,r)$
1943	1975	201	1	1975
1943	1990	250	2	1990
1943	1987	398	3	1987

The first column in Table 3.3 shows the year which is being generated. The distance between a feature vector from 1943 has been calculated between the corresponding feature vectors of every other year in the common record period. The second column lists the year (from the common period) which corresponds to the r^{th} lowest calculated distance metric, $\mathbf{D}_\xi^k(i,p)$, shown in column three. Column four lists the rank, r, of the distance metric from smallest to largest. The first three nearest neighbors are shown, out of a possible number, N_2, corresponding to the number of years in the common period. Column five contains the values of the rank vector, $\mathbf{R}^k(i,r)$, which contains the value of p_n.

Feature vectors of the index record corresponding to the first n nearest neighbors are identified as $\mathbf{f_y}^k(p_r)$, $r = 1, \ldots n$, for each season $k = 1, \ldots, K$. Each element of the synthetic feature vector is calculated as the average of the corresponding n nearest feature vector elements as follows

$$sfv_j^{\,k}(i) = \frac{1}{n}\sum_{r=1}^{n} f_j^{\,k}\left(\mathbf{R}^k(i,r)\right) \qquad 3.40$$

where

$sfv_j^k(i)$ = synthetic feature vector for year i, element j, season k,

n = number of nearest neighbors selected to include in the synthesis,

r = rank of the nearest neighbor feature vectors used in the synthesis,

$f_j^k()$ = feature vector element of the index record during the common period,

$\mathbf{R}^k(i,r)$ = rank matrix which gives the year of the r^{th} nearest neighbor.

The synthetic feature vectors are reverse transformed, if required, to obtain individual flow values. Standardized flow values of each element of the synthetic feature vector is reverse transformed as follows

$$sfv_j^{\,k}(i) = \left(sfv_j'^{\,k}(i) \cdot S(j)\right) + m(j) \qquad 3.41$$

where

$sfv_j^k(i)$ = reverse-transformed synthetic feature vector element,

$sfv'^k_j(i)$ = transformed synthetic feature vector element,

i = year of extension period under synthesis, i = 1,... ,N_1,

j = element number, j = 1,... ,m,

k = pattern class, k = 1,... ,K,

S(j) = sample standard deviation of feature vector elements j of index record,

m(j) = sample mean of feature vector elements j of index record.

There are no other reverse-transformations required. First difference feature vectors are used only for the nearest neighbor decision analysis. The actual nearest neighbors are taken from the untransformed feature vectors, thus no reverse-transformation is required for this case.

The complete synthetic feature vector for class k is comprised of the set of synthetic feature vector elements generated for each element j,

$$\mathbf{sfv}_i^k = \{sfv_j^k(i)\}\ j = 1,\ldots m. \qquad 3.42$$

The synthetic feature vectors are now constructed into a sequential flow sequence, then appended to the beginning of the index record to obtain the extended streamflow record.

The nonparametric streamflow record extension is also applied to generating separate monthly flow series. For this simple case, the method is run 12 separate times for a given record extension process, once for each month in the year. The dimensionality of the feature vectors is reduced to j = 1, and neither first differencing, nor deseasonalization transformations are made.

When the nonparametric model is used to generate 12 separate flow sequences (one for each month), each feature vector element simply contains the average monthly flow value occurring at that timestep.

Considering equation 3.40, there are other possibilities for the method of generating the synthetic flow other than the average of the elements of the nearest neighbors. Such adjustments include applying weight to nearest neighbors which occur closer in time to the extension period, as suggested by Burn (1993), or applying weights to individual feature vector elements depending on how close they are to the other elements occurring in the same pattern class. This may prove especially useful in the case where first-differencing is used to accentuate the importance of different characteristics of the data, such as rising or falling limbs of the hydrograph, or low flow periods, for example. These modifications are appropriate to use when information concerning the streamflow records being utilized is known in advance, i.e. where one has a specific reasoning to introduce a bias in determining nearest neighbors. No such prior information is known about the records used in the analysis, and so no such modifications are made to the clustering algorithm.

Chapter 4

MODEL EVALUATION AND APPLICATION

4.1 METHODOLOGY

Many trials of the streamflow record extension procedures presented herein are carried out on various long-length index records in order to evaluate their capability of reproducing important statistical characteristics in the extended record. The first portion of the index record, although known, is assumed "unknown" on a temporary basis. The record extension procedures are carried out to extend the index record during the "unknown" extension period. Important characteristics of the historical record and generated record during the extension period are compared, for each model evaluated. Various tests on the generated data and model adequacy measures are also used to compare and evaluate the extension techniques.

There are certain data availability conditions which occur depending upon the lengths of the base and index records, and their cross-correlation. Extension periods vary depending on the difference in length of the index and base record. The evaluation is carried out for various combinations of extension period, common period and cross correlation to determine the relative model performance for each case. A trial is defined herein as a record extension conducted for a single N_1, N_2 combination. Thus there is a finite number of trials associated with each run. Table 4.1 illustrates the combinations of extension period, N_1, and common period, N_2, used to evaluate the extension techniques.

Common periods less than the extension period were not used, since not enough information would be included in the common record to provide a reliable synthetic flow in that case. A similar reasoning is used in flood frequency analysis as it is not desirable to estimate, say, a 50 year event from a record less than 50 years long. A five year increment was arbitrarily chosen as long enough that different characteristics of the extension methods are realized, but short enough to include ranges where differences in model adequacy may occur.

Table 4.1 Record Periods Used in Evaluation

Extension Period, N_1, (Years)	Common Period, N_2 (Years)
5	5, 10, 15, . . . , to available data
10	10, 15, 20, . . . , to available data
15	15, 20, 25. . . , to available data
20	20, 25, 30, . . . , to available data

The number of trials for each N_1 decreases as N_1 increases, and there are many trials conducted for each run.

Different correlation between the base and index record are used to examine the effect of cross correlation on the merits of an extension technique. Candidate sets of base and index record are grouped according to the level of cross correlation exhibited. The first group includes base and index records with correlation greater than 0.666, and the second group have correlation less than that value. Although there are tests based on sample size which can be used to determine if correlation is statistically significant, the selection of a correlation of 0.666 to divide the analysis is arbitrary, since the sample sizes encountered in this study varied greatly.

The broad correlation categories are due to the nature of the data being used in the analysis. The actual streamflow records provide time series which exhibit a range of correlation. Synthetic data having specific statistical features could be used to run a Monte Carlo experiment, but the purpose of the research was defined as examining the techniques using actual data. Since generating a synthetic record using a low correlation is not likely to yield reliable results, no further division for correlation below 0.666 was used.

The streamflow record extension techniques are evaluated based on a list of criteria described below. The evaluation criteria include such considerations as checking for parametric model assumptions, capability of reproducing important statistical characteristics (Section 4.2.2) of the historical data, and other metrics which provide a quantitative way of determining which model performed better over all the trials conducted, taking all the adequacy measures into account. Other evaluation metrics are used to compare the capabilities of the model separately for each of the adequacy measures.

The stations used in the analysis are introduced, and general statistical properties of the streamflow records are discussed. Subsequently the development of the computer models used to carry out the evaluations is presented. Next, the evaluation is carried out conforming to the methodology outlined above. The model evaluation stage leads to the development of a decision rule which can be used to select an appropriate extension model based on given input data conditions, (i.e. extension period, common period, and correlation).

Two candidate sets of base and index records from the available data are excluded from the development/evaluation phase. These same records are used in application phase to provide a fair comparison of the computer models because the data are not used in the model development. The results of the application are compared to the expected behavior based upon the development/evaluation phase. The application phase also provides an opportunity to examine behavior of the techniques separately from the group of data on which their development is based.

4.2 EVALUATION CRITERIA

Evaluation of the synthetic records generated by the above methodology is comprised of three components. The first component is testing of the residual series to determine normality and serial correlation of residuals. One assumption associated with the development of the parametric streamflow models is the residual series must be random, and the joint probability distribution must approximate a bivariate normal distribution. Therefore, the parametric models are subject to tests on the normality of the residual series. If the residual series is found to be normally distributed, the parent series used in the regression are likely normally distributed as well. The nonparametric models are not subject to the test on normality of residuals because the formulation of the model makes no assumption regarding the underlying distribution of the time series. Both the parametric and nonparametric methods are tested for serial correlation of residuals as a measure of model adequacy.

The second evaluation criterion involves determining the ability of the methods to reproduce various statistical properties of the historical flows, as well as the cross correlation between the generated and historic flows, and mean percentage error. The adequacy measures listed in Section 4.2.2 are compared to target historical values during the extension period, rather than the entire y series so that the results of the comparison do not depend on the length of the extension or common period, since they are evaluated separately for each combination of extension and common period.

Direct comparison of the statistical measures, such as means and variances, for instance, would be cumbersome for the large amount of trials conducted in this study. Furthermore, the target value (i.e. the mean, variance, etc. of the historical record) changes depending on the record being "extended". The deviation of a calculated adequacy statistic from a respective target value, as a fraction of the target value, is calculated to remove the magnitude of the adequacy measure between comparisons, in order that the results may be compared directly. The general approach is to determine how often a particular model had better capability than all the other models in terms of the adequacy measures listed in Section 4.2.2.

The third evaluation criterion takes into account each model's combined ability in reproducing all of the statistical adequacy measures, in terms of a calculated objective function value. The objective function value depends upon the individual model's capability relative to the best and worst of the remaining models over all the statistical adequacy measures utilized. The objective function measure relates each model's adequacy in terms of the percentage of trials for which the model was "better" overall in reproducing the statistical measures.

4.2.1 Normality and Serial Correlation of Error Terms

The skewness and kurtosis of the residual series generated by the parametric methods are calculated to test for normality of errors. Departures from the normality assumption causes increased variance and bias in the parameter estimates, resulting in less reliability of the values generated by the regression model (Neter et al., 1989).

The skewness coefficient of a sample x_i of size n elements (Burn, 1993), is calculated by

$$g_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)^3}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)^2\right]^{3/2}} \qquad 4.1$$

where

g_1 = skewness coefficient,

n = sample size,

x_i = log transformed flow value,

μ_x = mean of log transformed flows.

The skewness coefficient is normally distributed with zero expected mean and variance, σ_g^2 = 6/n, that is, $g_1 \sim N(0, \sigma_{g1}^2)$. The expected value of g_1, $E\{g_1\} = 0$. The null hypothesis and alternate hypothesis are:

$$Ho: g_1 = 0 \qquad 4.2a$$

$$Ha: g_1 \neq 0. \qquad 4.2b$$

As the skewness can be positive or negative, we conduct a two sided test with 95% confidence limits, or 5% level of significance, α. We accept Ho if

$$- z(1-\alpha/2)^* \sigma_{g1} \leq g_1 \leq +z(1-\alpha/2)^* \sigma_{g1} \qquad 4.3$$

where

$z(1-\alpha/2)$ = the area under the standard normal curve.

The kurtosis coefficient of a sample x_i of size n elements (Burn, 1993), is calculated by

$$g_2 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)^4}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)^2\right]^2} \qquad 4.4$$

where

g_2 = kurtosis coefficient,

n = sample size,

x_i = log transformed flow value,

μ_x = mean of log transformed flows.

The distribution of g_2 is N(3, 24/n), where σ_{g2}^2 = 24/n. The null hypothesis and alternate hypothesis are:

$$Ho: g_2 = 3 \qquad 4.5a$$

$$Ha: g_2 \neq 3. \qquad 4.5b$$

As above, the decision rule with 95% confidence limits, is accept Ho if

$$3 - z(1-\alpha/2)^* \sigma_{g2} \leq g_2 \leq 3 + z(1-\alpha/2)^* \sigma_{g2}. \qquad 4.6$$

The serial correlation coefficient, r_e, of the adjacent error terms is calculated to test for autocorrelation of residuals. Autocorrelation in the residual series may indicate that one or more independent variables which have time-ordered effects on the independent variable is omitted from the model. In the case of time series, the missing information is likely the time dependent nature of the flows between the two gauges (Beauchamp et al., 1989), or seasonality effects (Neter et al., 1989). Models displaying autocorrelation of residuals explain a large percentage of the variation in the data, but do not account for the autocorrelation that exists among the original observations, which would be addressed by a time-series type of model (Beauchamp et al., 1989).

A test on the lag 1 serial correlation on the null hypothesis

$$Ho(r_{e1}) = 0 \qquad 4.7a$$

is suggested by Dahmen and Hall (1990). The alternate hypothesis is

$$Ha(r_{e1}) \neq 0. \qquad 4.7b$$

The critical region, U, at the 5% level of significance is defined by Dahmen and Hall (1990), and Anderson (1942), as

$$\{-1,(-1-1.96(n-2)^{0.5})/(n-1)\}\ U\ \{(-1+1.96(n-2)^{0.5})/(n-1)\}. \qquad 4.8$$

The log-transformed time series were tested for trend in the monthly series in a manner similar to skewness and kurtosis above, at the 95% confidence limit. The slope of the regression line describing average monthly flow versus time was tested to determine if the null hypothesis (that the slope = 0) could not be rejected with 95% confidence. No flow files were found to exhibit a trend by this method of analysis. Note however, that the latter only tests for linear trend, and is not useful in identifying trends which are not linear. A test which would account for nonlinear trend is found in the Spearman Rank Correlation Coefficient test. However, this test is also unreliable if half of the time series displays an opposite trend to the other half, (ie. if the trend is increasing in the first half, and decreasing in the second half).

4.2.2 Statistical Model Adequacy Measures

The following model adequacy measures are calculated for each trial conducted. The values from the generated series are compared to the historical or target values, for each trial conducted:

a) cross-correlation between the generated and historical flows during the extension period. The target value is 1.0, which would correspond to a perfect correlation between the synthetic and historical flows. The sample product moment correlation coefficient between two time series x_i and y_i is

$$r_{xy} = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i y_i - \frac{1}{n^2}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \mu_x^2} \cdot \sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2 - \mu_y^2}} \qquad 4.9$$

where

r_{xy} = cross-correlation between x_i and y_i time series,

x_i = generated flow series [m^3/s],

y_i = historic flow series [m^3/s],

n = number of data for which x_i and y_i have flows

μ_x = sample mean of x_i series,

μ_y = sample mean of y_i series.

Noting that equation 4.9 is equivalent to equation 3.13;

b) lag 1 serial correlation of the generated flows, r_1. The target is the historical serial correlation of y series during extension period. The equation for serial correlation (Burn, 1993), is as follows

$$r_1 = \frac{\frac{1}{n-1}\sum_{i=1}^{n-1} x_i x_{i-1} - \frac{1}{(n-1)^2}\sum_{i=1}^{n-1} x_i \sum_{i=1}^{n-1} x_{i-1}}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1} x_i^2 - \frac{1}{(n-1)^2}\left(\sum_{i=1}^{n-1} x_i\right)^2} \cdot \sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1} x_{i-1}^2 - \frac{1}{(n-1)^2}\left(\sum_{i=1}^{n-1} x_{i-1}\right)^2}}; \quad 4.10$$

c) variance of the generated flows. The target is the historical variance of y series during extension period;

d) mean of the generated flows. The target is the historical mean flow of y series during extension period;

e) flow duration curve value showing the percentage of time a particular flow is equaled or exceeded (Faucher, 1994). The target is the historical flow duration curve value of y series during extension period. The flow duration curve is developed by ranking the monthly flow values and sorting, and assigning empirical probabilities according to the Weibull formula (Burn, 1993)

$$p_i = \frac{m_i}{n+1} \quad 4.11$$

where

p_i = probability of exceedence for data point i,

m_i = the rank for data point i.

A flow duration curve value corresponding to 0.8 probability of exceedence was selected to compare the ability of the models in reproducing low flows.

The flow duration curve for God's River Below Allen Rapids is shown in Figure 4.1 as an illustrative example.

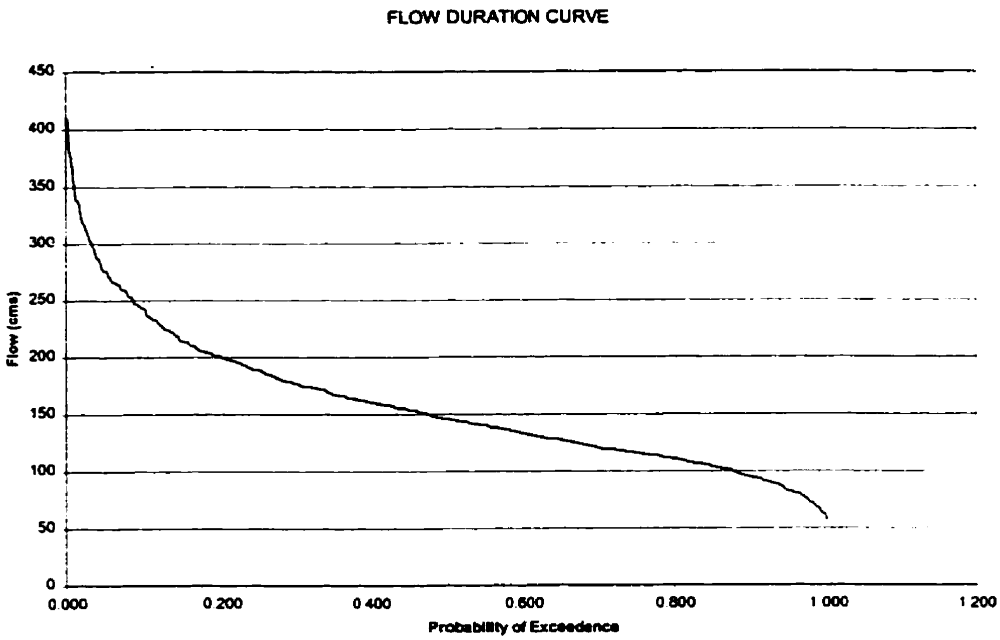

Figure 4.1 Flow Duration Curve for God's River Below Allen Rapids

f) mean percentage error, MPE, (Raman et al., 1995). This adequacy measure indicates the average percentage difference between the generated and historic flows. The target is 0. The formula for MPE is

$$MPE = \left[\frac{1}{n}\sum_{i=1}^{n}\frac{(y_i - \hat{y}_i)}{y_i}\right]\cdot 100; \qquad 4.12$$

g) sum of squared deviations between the generated and historical values, also called sum of squared error, SSE. The target is 0. The formula for SSE is

$$SSE = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 . \qquad 4.13$$

Recall that the variance of each y_i is assumed to be the same as each error term, e_i. SSE recognizes that each y_i comes from different probability distributions with different means, depending on the level of x_i. Thus the appropriate estimate of squared error for the regression model is the deviation of an observation y_i from its own estimated mean, $\hat{y}_i$. SSE is used extensively in analysis of variance approach to regression analysis (Neter et al., 1989).

The deviation of a statistical adequacy measure from the target value, expressed as a fraction of the target value, is calculated for every trial, t, as

$$f_{ij}(t) = \frac{v_{ij}(t) - tar_j(t)}{tar_j(t)} \qquad 4.14$$

where

$f_{ij}(t)$ = the fractional deviation of a calculated statistic from the target value for trial t,

i = model index,

j = adequacy measure index,

t = trial index,

v_{ij} = value of the calculated statistic from the generated data,

tar_j = target value for adequacy measure j, defined previously.

There are N_t trials conducted for each combination of extension and common period. The average fractional deviation is calculated for adequacy measure, j, as follows

$$\bar{f}_{ij} = \frac{1}{N_t}\sum_{t=1}^{N_t} f_{ij}(t) \qquad 4.15$$

where

$\bar{f}_{ij}$ = average fractional deviation of calculated adequacy measure j, for model i, from the target value over all trials,

N_t = the total number of trials,

$f_{ij}(t)$ = fractional deviation as before,

t = trial index.

The fractional deviation indicator is used to quantify the differences between model adequacy, as well as the expected amount of deviation which would occur for a given model. This measure determines how well a particular model may be expected to reproduce a certain statistic based on all the trials conducted.

4.2.3 Objective Function Model Adequacy Measure

A value which determines each model's overall deviation from the (historical) target values throughout all the adequacy measures in comparison to the "best" and "worst" model is calculated for each trial. The "best" model for a particular adequacy measure is that model which has the minimum deviation from it's target value. The "worst" model for a particular adequacy measure is that which has the maximum deviation from the respective target value. The objective for any model is to attain the minimum value of the above metric. This value is referred to herein as the objective value, (OBJ) for a particular model. Omitting the trial index, the formula for obtaining OBJ for any particular trial is

$$OBJ_i = \sum_{j=1}^{J} w_j \left[\frac{\left| f_{ij} - f_j^{best} \right|}{\left| f_j^{worst} - f_j^{best} \right|} \right] \qquad 4.16$$

where

OBJ_i = objective value for model i,

i = extension model index,

j = adequacy measure index,

f_{ij} = deviation of adequacy measure j from target value for model i,

f_j^{best} = smallest deviation of adequacy measure j from respective target value,

f_j^{worst} = largest deviation of adequacy measure j from respective target value,

w_j = weight for adequacy measure j.

Note that $w_j = 1$ for all adequacy measures, j, in this study, to indicate that a weight can be applied to a particular measure to increase its "importance" in calculation of the objective function. As an example, consider model i = 1 which exhibits variance (measure j) very close to the target (historical) value, for a particular trial. The calculated deviation of adequacy measure j from the target value, f_{1j}, would be small. Consider model i = 2 which exhibits a larger deviation of adequacy measure j, f_{2j}, from the same target. Applying $w_j > 1.0$ to f_{1j} will increase the value of $| OBJ_1 - OBJ_2 |$ over what it would be if $w_j = 1.0$, thus increasing the chance that model i = 1 will have the lowest overall objective function.

Over all the trials conducted, the fraction of trials in which a particular model obtains the lowest value of OBJ_i shall be referred to as OBJ_i^*. As a hypothetical example, consider model i, which obtains the minimum value of OBJ_i, for any arbitrary combination of N_1, N_2, and r_{xy}, 6 times out of a total of 15 extension trials conducted. Let $N_{OBJi(min)}$ represent the number of trials model i obtains the minimum objective function value; in this case, $N_{OBJi(min)} = 6$. Now, OBJ_i^* is calculated as

$$OBJ_i^* = \frac{N_{OBJ_i(\min)}}{N_t} = \frac{6}{15} = 0.4 \,. \tag{4.17}$$

In the above example, model i achieved the minimum objective function for 40 percent of the trials conducted.

The percentage of time a particular model has the lowest value of OBJ_i for each combination of N_1, N_2 and cross correlation, r_{xy} is used to determine which model is most appropriate for different N_1, N_2, r_{xy} combinations.

4.3 DATA COMPILATION

Unregulated (natural) monthly streamflow data from Manitoba and Ontario gauging stations obtained from Water Survey of Canada, a division of Environment Canada is used in this study. Missing data was not infilled to ensure each model's relative success in reproducing statistics is not dependent on the data infilling method.

Long records were generally selected for model evaluation so the same record extension could be made with as many common periods from the same record as possible. The location of the regions containing the Manitoba gauging station used in this study are shown in Figure 4.2. The location of the regions containing the Ontario gauging station used in this study are shown in Figure 4.3.

4.3.1 Gauging Stations Used in Study

Table 4.2 and Table 4.3 list the individual flow records used in this study for Manitoba and Ontario gauging stations, respectively.

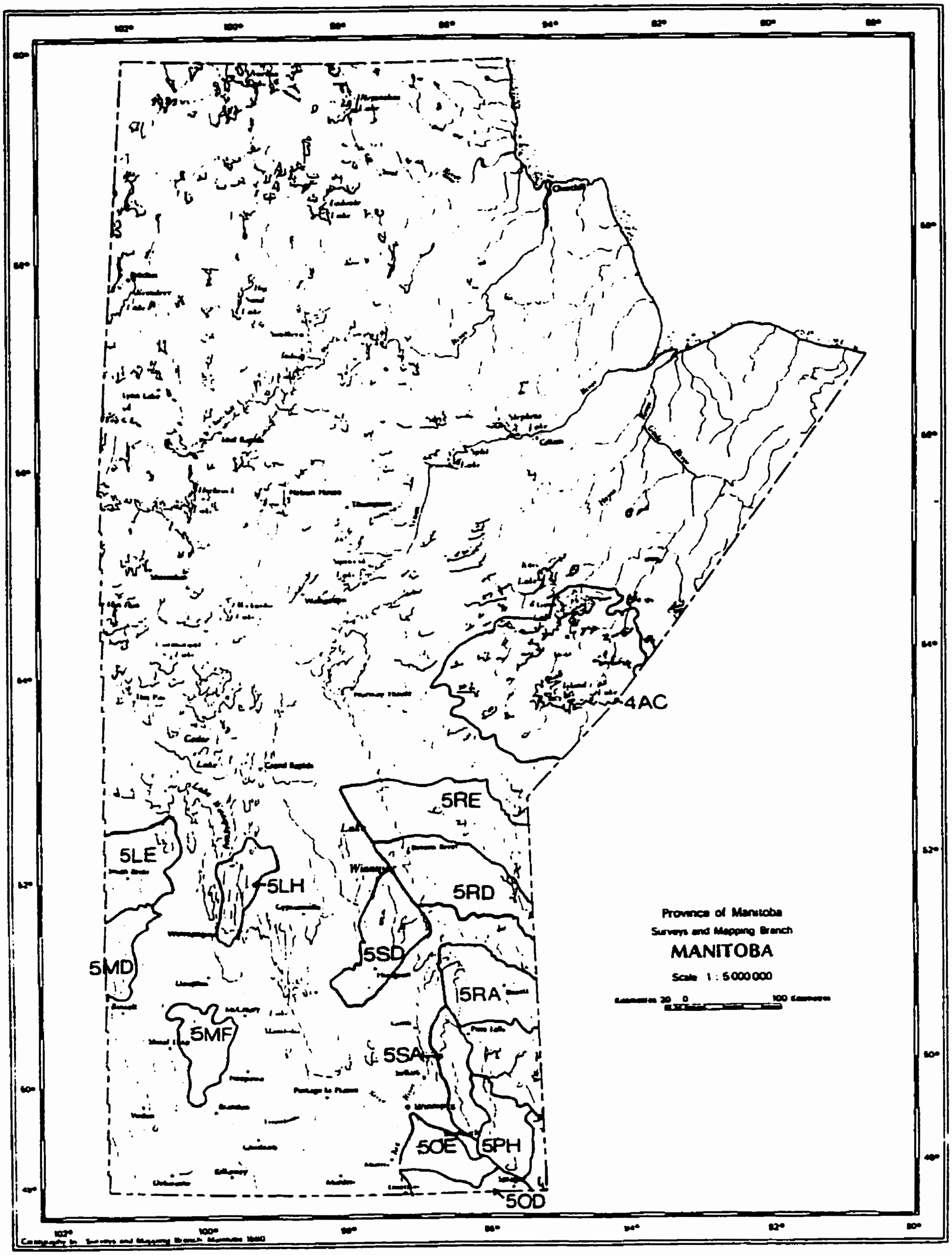

Figure 4.2 Major Watershed Divisions in Manitoba

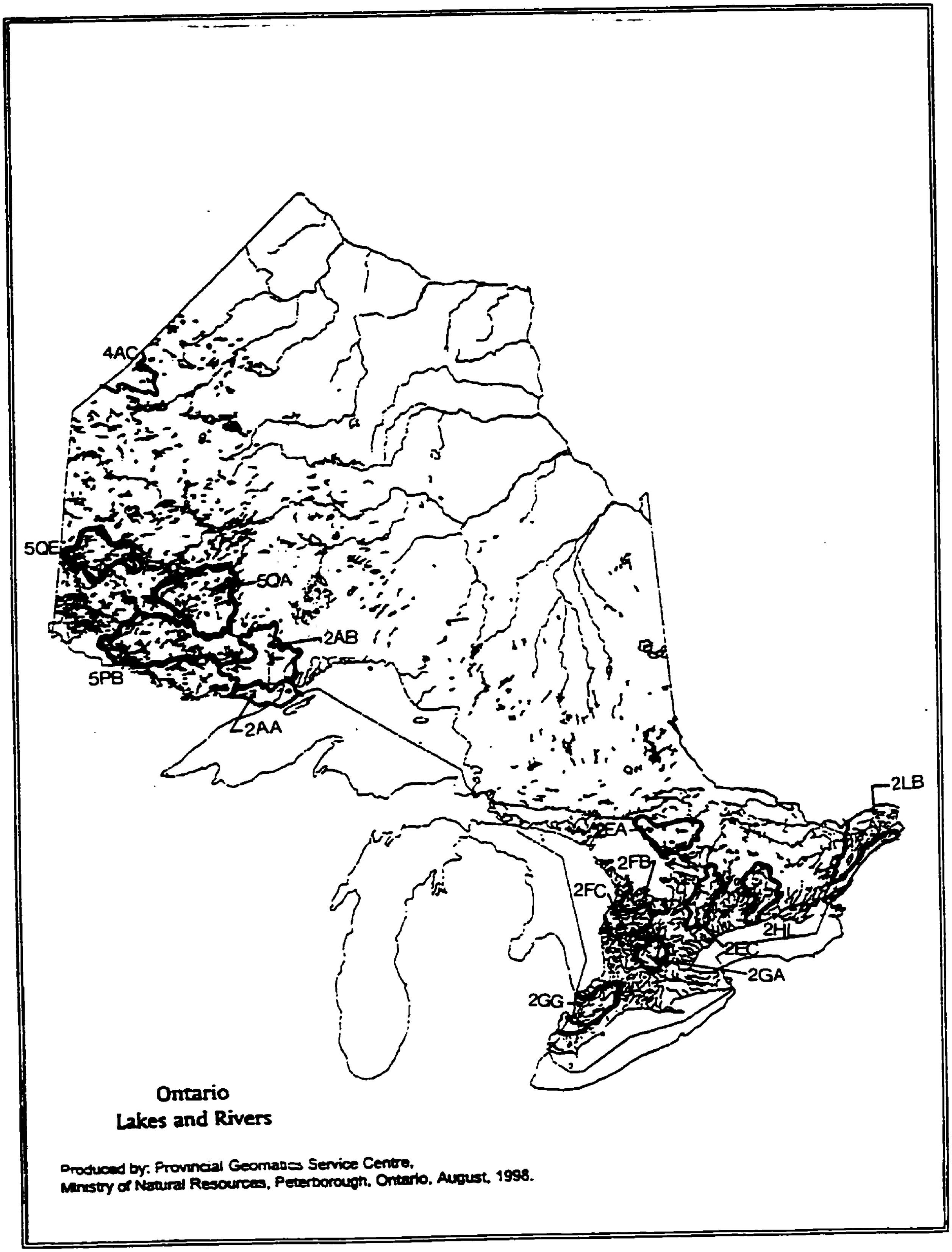

Figure 4.3 Major Watershed Divisions in Ontario

Table 4.2 Manitoba Unregulated Streamflow Data

No.	Station ID.	Station Name	Begin Year	End Year	Total Years	Drainage Area (km^2)
1	04AC005	GODS RIVER BELOW ALLEN RAPIDS	1933	1994	62	25,900
2	04AC007	ISLAND LAKE RIVER NEAR ISLAND LAKE	1933	1994	62	14,000
3	05LE006	SWAN RIVER NEAR MINITONAS	1961	1995	35	4,230
4	05LH005	WATERHEN RIVER NEAR WATERHEN	1953	1995	43	55,000
5	05MD005	SHELL RIVER NEAR INGLIS	1957	1995	39	2,000
6	05MF018	LITTLE SASKATCHEWAN RIVER NEAR RIVERS	1956	1995	40	3,910
7	05OD001	ROSEAU RIVER NEAR DOMINION CITY	1949	1995	47	5,260
8	05OE001	RAT RIVER NEAR OTTERBURNE	1912	1995	84	1,340
9	05OE004	RAT RIVER NEAR SUNDOWN	1960	1995	36	398
10	05PH003	WHITEMOUTH RIVER NEAR WHITEMOUTH	1957	1995	39	3,750
11	05RA001	MANIGOTAGAN RIVER NEAR MANIGOTAGAN	1960	1995	36	1,830
12	05RD008	PIGEON RIVER AT OUTLET OF ROUND LAKE	1958	1995	38	Unknown
13	05RE001	POPLAR RIVER AT OUTLET OF WEAVER LAKE	1968	1995	28	6,850
14	05SA002	BROKENHEAD RIVER NEAR BEAUSEJOUR	1958	1995	38	1,610
15	05SD003	FISHER RIVER NEAR DALLAS	1961	1995	35	1,720

Table 4.3 Ontario Unregulated Streamflow Data

No.	Station ID.	Station Name	Begin Year	End Year	Total Years	Drainage Area (km^2)
16	05PB014	TURTLE RIVER NEAR MINE CENTRE	1921	1995	75	4,870
17	05QA002	ENGLISH RIVER AT UMFREVILLE	1922	1995	74	6,230
18	05QE009	STURGEON RIVER AT OUTLET OF SALVESEN LAKE	1965	1995	31	1,530
19	02AA001	PIGEON RIVER AT MIDDLE FALLS	1924	1995	72	1,550
20	02AB008	NEEBING RIVER NEAR THUNDER BAY	1954	1995	42	187
21	02EA005	NORTH MAGNETAWAN RIVER NEAR BURK'S FALLS	1916	1995	80	321
22	02EA010	NORTH MAGNETAWAN RIVER ABOVE PICKEREL LAKE	1968	1995	28	149
23	02EC002	BLACK RIVER NEAR WASHAGO	1916	1995	80	1,520
24	02FB007	SYDENHAM RIVER NEAR OWEN SOUND	1949	1995	47	181
25	02FC001	SAUGEEN RIVER NEAR PORT ELGIN	1915	1995	81	3,960
26	02FC002	SAUGEEN RIVER NEAR WALKERTON	1915	1995	81	2,150
27	02GA010	NITH RIVER NEAR CANNING	1948	1995	48	1,030
28	02GG002	SYDENHAM RIVER NEAR ALVINSTON	1949	1995	47	730
29	02HL004	SKOOTAMATTA RIVER NEAR ACTINOLITE	1956	1995	40	712
30	02LB007	SOUTH NATION RIVER AT SPENCERVILLE	1949	1995	47	246

Table 4.2 and Table 4.3 show there are generally longer records of data available from Ontario gauging stations. Ontario data generally has less gaps than the Manitoba data. The Ontario data is included to supplement the Manitoba data.

The years of record where flows were collected only during the summer months, usually in an early portion of the record, were discarded so the amount of data used in the parametric and nonparametric methods would be as equal as possible. In the nonparametric model, a feature vector cannot be obtained for a season which does not contain all the monthly data available for calculating all the feature vector elements. Missing data for the parametric models does not have the effect of excluding other data which falls within the same season. Tables 4.2 and 4.3 list the years of data actually used in the study, not the total number of years of data which may contain only partially completed records.

4.3.2 Base and Index Stations Used for Evaluation and Verification

The combinations of base and index record stations used for the evaluation of the streamflow record extension techniques are shown in Table 4.4 and Table 4.5. Table 4.4 shows the base and index records used to evaluate the techniques under the condition of $r_{xy} > 0.666$. Table 4.5 shows the base and index records used to evaluate the techniques under the condition of $r_{xy} < 0.666$. The total number of trials conducted using a given base and index record is referred to as a "run" of trials. A single trial is conducted for a given combination of extension and common period. There are several trials associated with each run because the extension period and common period varied a number of times for each combination of base and index record used. Certain base and index records are omitted from the evaluation phase, in order to be used as independent data for verification. The number of trials associated with each "run", as well as which runs were selected for the verification exercises are also shown in Table 4.4 and Table 4.5.

Table 4.4 Base and Index Records Used for Evaluation and Verification, $r_{xy} > 0.666$

No.	Base Station	Index Station	r_{xy}	Comment
1	04AC005	04AC007	0.731	N_1=5 yrs not used due to missing flows. Used for verification phase. 21 trials.
2	05OE001	05OD001	0.852	20 trials.
3	05MD005	05MF018	0.788	12 trials.
4	05MF018	05LE006	0.685	12 trials.
5	05OE001	05OE004	0.908	12 trials.
6	05OE001	05PH003	0.836	12 trials.
7	05PH003	05OE004	0.896	12 trials.
8	05PH003	05RA001	0.749	12 trials.
9	05PB014	05QA002	0.868	Used for verification phase. 40 trials.
10	02AA001	02AB008	0.827	16 trials.
11	02EA005	02EC002	0.887	48 trials.
12	02EA005	02FC002	0.718	48 trials.
13	02EA005	02HL004	0.768	16 trials.
14	02EC002	02FC001	0.843	N_1=5 yrs not used due to missing flows. 48 trials.
15	02EC002	02FC002	0.875	48 trials.
16	02EC002	02GA010	0.740	20 trials.
17	02FC001	02FC002	0.984	48 trials.
18	02FC002	02GA010	0.901	20 trials.
19	02FC002	02GG002	0.695	20 trials.
20	02FC002	02HL004	0.884	16 trials.
21	02GA010	02GG002	0.828	20 trials.
22	02GA010	02HL004	0.805	16 trials.
23	02LB007	02HL004	0.864	16 trials.

Table 4.5 Base and Index Records Used for Evaluation and Verification, $r_{xy} < 0.666$

No.	Base Station	Index Station	r_{xy}	Comment
1	05LH005	05LE006	0.391	12 trials.
2	05OD001	05LE006	0.555	12 trials.
3	05LH005	05MD005	0.391	12 trials.
4	05OD001	05MD005	0.553	12 trials.
5	05OE001	05SD003	0.555	12 trials.
6	05RD008	05RE001	0.620	6 trials.
7	02EA005	02AA001	0.548	40 trials.
8	02AA001	02EA010	0.478	6 trials.
10	02EC002	02AA001	0.405	40 trials.
11	02EA005	02FB007	0.632	20 trials.
12	02EA005	02FC001	0.664	48 trials.
13	02EA005	02GA010	0.507	20 trials.
14	02EA005	02GG002	0.336	20 trials.
15	02EC002	02GG002	0.560	20 trials.
16	02GG002	02HL004	0.610	16 trials.
17	05SA002	05RA001	0.642	N_1=5 yrs, N_2=5 yrs trial not used due to missing flows. Used for verification phase. 11 trials.
18	05QA002	05QE009	0.626	Used for verification phase. 9 trials.

4.4 MODEL DEVELOPMENT

This section describes the development of the streamflow record extension models used to carry out the experimental trials described above. The evaluation of the computer models is based on the results of the trials. Streamflow record extension may be done using one relationship for all concurrent monthly flows or 12 separate relationships for flows from each month. The concurrent monthly flow method is referred to as the non-cyclic approach. Using 12 separate relationships for monthly flows is referred to as the cyclic approach. A trade-off exists between greater sample size for estimating statistical parameters versus the ability to preserve real month-to-month differences which may exist in the base station to index station relationship (Alley and Burns, 1983). However, no differences exist in the comparison of the flow sequences resulting from either the cyclic or non-cyclic approaches. Both the cyclic and non-cyclic approaches are investigated for each of the parametric and nonparametric extension methods.

The parametric and nonparametric models handle missing flow values. No synthetic flows are generated in time periods corresponding to missing data in the base station record. In the case of the parametric models and cyclic nonparametric model, a single missing flow value in the historical base record results in only one missing flow value in the generated series. For the non-cyclic nonparametric extension models, a missing flow within a particular season results in an incomplete feature vector of pattern attributes for that season. Incomplete feature vectors are not associated with any near neighbors, since a distance calculated with an incomplete feature vector would not be consistent with distances calculated between complete feature vectors. Thus, for the non-cyclic nonparametric models, the season of flows in the synthetic record corresponding to the incomplete season in the base record is not generated.

4.4.1 Definition of Record Extension Models

The nonparametric method is divided into 3 main types depending on the definition of the feature vector. Recall that the feature vectors were determined based on either standardized flows, raw flow values, or first differencing of monthly flows. Nonparametric Type 1 utilizes standardized flows for feature vector elements. Nonparametric Type 2 utilizes the historical monthly streamflow data for the feature vector elements. Nonparametric Type 3 utilizes first differences of consecutive flows for feature vector elements.

There are 3 further sub-divisions of each type of nonparametric model corresponding to the manner in which the nearest neighbors are determined. Sub-types A, B, and C correspond to nonparametric models which utilize Euclidean, Manhattan, and Modified Lance-Williams nearest neighbor distances, respectively. The streamflow record extension models are listed in Table 4.6. There are 10 parametric and 10 nonparametric models. The models are numbered and abbreviated for ease of reference within the discussions. Model numbers 1 to 14 use the

Table 4.6 Models for Streamflow Record Extension

Model	Description
1. REG	Simple linear regression
2. RPN	Linear regression plus independent noise
3. MV1	Maintenance of variance extension Type 1
4. MV2	Maintenance of variance extension Type 2
5. MV3	Maintenance of variance extension Type 3
6. NP1A	Nonparametric Type 1A– standardized flows, Euclidean distances
7. NP1B	Nonparametric Type 1B – standardized flows, Manhattan distances
8. NP1C	Nonparametric Type 1C – standardized flows, Mod. Lance/Williams distances
9. NP2A	Nonparametric Type 2A – monthly flows, Euclidean distances
10. NP2B	Nonparametric Type 2B – monthly flows, Manhattan distances
11. NP2C	Nonparametric Type 2C – monthly flows, Mod. Lance/Williams distances
12. NP3A	Nonparametric Type 3A – first differencing, Euclidean distances
13. NP3B	Nonparametric Type 3B – first differencing, Manhattan distances
14. NP3C	Nonparametric Type 3C – first differencing, mod. Lance/Williams distances
15. REGM	Simple linear regression – cyclic approach
16. RPNM	Regression plus independent noise – cyclic approach
17. MV1M	Maintenance of variance extension Type 1 – cyclic approach
18. MV2M	Maintenance of variance extension Type 2 – cyclic approach
19. MV3M	Maintenance of variance extension Type 3 – cyclic approach
20. NPM	Nonparametric – monthly flows, cyclic approach

non-cyclic approach and models 15 to 20 use the cyclic approach. The abbreviations for models 15 to 20 in Table 4.4 end with an "M" to signify they utilize 12 separate monthly relationships (cyclic approach).

4.4.2 Definition of Seasons

The pattern recognition process involves segmenting the flow record into pattern classes to obtain feature vectors by the various methods described in Chapter 3. The feature vectors are clustered into nearest neighbors, which are subsequently used to generate synthetic flows. The methodology for partitioning the streamflow data into pattern classes is described in this section.

Hydrologic data corresponding to monthly measurements occur in well-defined groups (Panu et al., 1978). Monthly flow sequences within a year are divided into pattern classes, or groups, corresponding to seasons. This allows different portions of the flow record for a particular year to be associated with a separate set of nearest neighbors, ensuring that the selection of nearest neighbors is not biased towards the patterns defined by any other season with the same year or other years. Nearest neighbors associated with a feature vector obtained from the flows during a given season are independent of the nearest neighbors chosen for all other feature vectors.

Panu et al. (1978) recommend the division of seasons based upon an examination of the time series of flows. The year is divided into seasons based upon an examination of the mean monthly hydrograph and systematically determining which segmentation best represents the historical pattern. The method presented herein is flexible and provides an opportunity for the hydrologist to utilize particular knowledge of the record's drainage basin or other factors. In the case of the present study, an intimate knowledge of the circumstances related to the particular flow records is not available. The classification is based upon both review of the monthly hydrographs as well as a quantitative decision function which relates the ability of the nonparametric technique to find similar patterns in the historic record using the chosen division of seasons.

The procedure for the division of seasons begins by determining the number of seasons to divide the year into. The average monthly hydrographs are shown in Appendix A. Panu et al. (1978) divided the flow year into two seasons corresponding to 6 month sections beginning in November and May, respectively, for the South Saskatchewan River. The group of monthly flows generally higher than average is known as the wet season, and the remaining lower flows are known as the dry season. An examination of the average monthly hydrographs show that a division of two seasons is also appropriate for the streamflows used in the present study, but the beginning and lengths of the seasons of different flow records vary from record to record.

An initial first guess of the months corresponding to the beginning and end of the dry and wet seasons is made. Note that the end of the dry season is equivalent to the beginning of the wet season, and the end of the wet season is the beginning of the dry season. The beginning and end of a particular season need not necessarily fall within the same year. The flow record is defined as both the index and base station. The extension period is defined as the first third of the historic record. The nonparametric model is utilized to generate synthetic flows for the extension period, using the initial guess of seasons using the same flow record as the base and index station. The resulting SSE from the initial guess of seasons is recorded. The duration of the wet season is then varied by a small amount (normally one month). The nonparametric model is used again to generate the same sequence of flows, but this time with the new seasons. The SSE from the subsequent run is again recorded. This process is repeated with different plausible season lengths. The division of seasons which minimized SSE indicates increased nonparametric model performance, in terms of increased ability of the nonparametric models to recognize the flow patterns. Therefore, the division of seasons which minimized SSE was used to segment the flow record into pattern classes.

The extension period is taken as one third of the total record to strike a balance between the amount of common record available to recognize similar feature vectors, and the confidence placed on the results of the extension. The chosen extension period is a factor in the level of SSE. The length of common period affects the probability of a similar pattern being utilized as a nearest neighbor, which directly impacts which seasonal division becomes the optimum, and ultimately the overall model performance.

The flow records were classified into their respective seasons at an early stage of the model development when the nearest neighbor optimization procedure was not yet developed. As such the division of seasons was based on feature vectors using only standardized feature vector elements and 3 nearest neighbors. The nonparametric model performance has likely not been impaired by the division of seasons using only 3 nearest neighbors and standardized feature vector elements because the method generally resulted in seasons consistent with that determined by visual examination of the mean monthly hydrographs. However, an area for future work lies in refining the methodology for segmentation of the yearly flows into seasons. Note that the above methodology is not a replacement for judgement in determining the seasons based on examining the average monthly hydrograph and utilizing knowledge of individual circumstances. Rather, this is a tool for confirming a preliminary selection or to aid in the selection where the discrimination is unclear from visual examination of the hydrograph.

Table 4.7 shows the seasonal flow record segmentation of the records used in this study. Only the months corresponding to the start and end of the wet season for each year are shown, since the beginning and end of the dry season is implied.

4.4.3 Optimum Number of Nearest Neighbors

For each trial conducted, the number of nearest neighbors, n, is increased from 1 to the maximum allowable with the given data (limited by the number of years of common record), and the SSE is denoted SSE(n). The optimum number of nearest neighbors, n_{opt}, is that n which corresponds to the minimum SSE, ie. SSE($n = n_{opt}$). Each nonparametric model determines n_{opt} for every trail conducted, since the optimum number of nearest neighbors is not necessarily equal for all nonparametric models. The results are used to recommend the appropriate number of nearest neighbors to use for a given N_1, N_2, r_{xy} combination.

The average number of nearest neighbors utilized by each type of nonparametric model for various combinations of extension period and common period were calculated based on all the evaluation trials conducted. The standard deviation of the optimum number of nearest neighbors selected by the nonparametric models was also calculated. The results for $r_{xy} > 0.666$ are shown in Tables B-1 to B-4, and the results for $r_{xy} < 0.666$ are shown in Tables B-5 to B-8, for all values of N_1. Tables B-1 to B-8 show that the optimum number of nearest neighbors chosen by the nonparametric models, in general, does not vary appreciably as the extension period increased. Additionally, the variability of the number of nearest neighbors selected increases as N_2 increased, but also did not vary appreciably with the extension period. The variation of n_{opt} is greater for $r_{xy} < 0.666$ than for $r_{xy} > 0.666$.

Tables 4.8 and 4.9 show the number of nearest neighbors utilized by each model averaged over all extension periods. Recall the extension period varied from 5 to 20 years, in 5 year increments. For $r_{xy} > 0.666$, the average number of nearest neighbors increases from a minimum of 2 for a 5 year common period to a maximum of 21 corresponding to a 75 year common period. For $r_{xy} < 0.666$, the average number of nearest neighbors increases from a minimum of 3 for a 5 year common period to a maximum of 48 corresponding to a 75 year common period. In general, the number of nearest neighbors chosen by the nonparametric models is less with higher cross correlation between the base and index station. Additionally, the spread between the number of nearest neighbors chosen by the nonparametric models is shown to be quite narrow for a common period less than 40 years, while the number of neighbors chosen by the models diverges quite rapidly at common periods greater than approximately 40 years.

Table 4.7 Division of Seasons

No.	Station ID.	Station Name	Start of Wet Season	End of Wet Season
1	04AC005	GODS RIVER BELOW ALLEN RAPIDS	June	November
2	04AC007	ISLAND LAKE RIVER NEAR ISLAND LAKE	June	November
3	05LE006	SWAN RIVER NEAR MINITONAS	April	May
4	05LH005	WATERHEN RIVER NEAR WATERHEN	May	December
5	05MD005	SHELL RIVER NEAR INGLIS	April	July
6	05MF018	LITTLE SASKATCHEWAN RIVER NEAR RIVERS	April	July
7	05OD001	ROSEAU RIVER NEAR DOMINION CITY	April	July
8	05OE001	RAT RIVER NEAR OTTERBURNE	April	May
9	05OE004	RAT RIVER NEAR SUNDOWN	April	May
10	05PH003	WHITEMOUTH RIVER NEAR WHITEMOUTH	April	June
11	05RA001	MANIGOTAGAN RIVER NEAR MANIGOTAGAN	May	July
12	05RD008	PIGEON RIVER AT OUTLET OF ROUND LAKE	June	September
13	05RE001	POPLAR RIVER AT OUTLET OF WEAVER LAKE	May	July
14	05SA002	BROKENHEAD RIVER NEAR BEAUSEJOUR	April	June
15	05SD003	FISHER RIVER NEAR DALLAS	April	May
16	05PB014	TURTLE RIVER NEAR MINE CENTRE	May	July
17	05QA002	ENGLISH RIVER AT UMFREVILLE	May	July
18	05QE009	STURGEON RIVER AT OUTLET OF SALVESEN LAKE	May	July
19	02AA001	PIGEON RIVER AT MIDDLE FALLS	April	June
20	02AB008	NEEBING RIVER NEAR THUNDER BAY	April	June
21	02EA005	NORTH MAGNETAWAN RIVER NEAR BURK'S FALLS	April	May
22	02EA010	NORTH MAGNETAWAN RIVER ABOVE PICKEREL LAKE	April	May
23	02EC002	BLACK RIVER NEAR WASHAGO	March	May
24	02FB007	SYDENHAM RIVER NEAR OWEN SOUND	March	April
25	02FC001	SAUGEEN RIVER NEAR PORT ELGIN	March	April
26	02FC002	SAUGEEN RIVER NEAR WALKERTON	March	May
27	02GA010	NITH RIVER NEAR CANNING	March	April
28	02GG002	SYDENHAM RIVER NEAR ALVINSTON	March	April
29	02HL004	SKOOTAMATTA RIVER NEAR ACTINOLITE	March	May
30	02LB007	SOUTH NATION RIVER AT SPENCERVILLE	March	April

Table 4.8 Average Number of Nearest Neighbors for Nonparametric Models, $r_{xy} > 0.666$

MODEL	Common Period, Years														
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
NP1A	3	3	4	5	6	7	7	9	10	9	12	12	14	12	12
NP1B	3	3	4	5	6	7	8	8	11	11	12	12	15	16	14
NP1C	3	5	5	5	5	6	6	6	7	7	8	8	9	9	8
NP2A	3	4	4	5	6	7	8	9	10	11	12	12	15	12	13
NP2B	3	4	4	5	6	6	7	9	10	10	11	11	12	13	11
NP2C	3	4	4	5	6	7	7	9	10	11	12	12	14	17	22
NP3A	3	4	5	6	6	7	7	8	10	10	9	11	11	13	11
NP3B	3	5	5	6	7	8	8	10	12	11	11	12	13	16	10
NP3C	3	5	5	5	6	6	7	8	9	7	9	9	10	10	10
NPM	2	3	4	5	6	7	8	7	9	9	11	12	14	16	14
Mean	2.9	4.0	4.4	5.2	6.0	6.8	7.3	8.3	9.8	9.6	10.7	11.1	12.7	13.4	12.5
Rounded	3	4	4	5	6	7	7	8	10	10	11	11	13	13	13
Stdev.	0.3	0.8	0.5	0.4	0.5	0.6	0.7	1.2	1.3	1.6	1.5	1.4	2.1	2.8	3.8

The number of nearest neighbors used by the nonparametric models averaged over all trials and for all extension periods for $r_{xy} > 0.666$ and $r_{xy} < 0.666$ are shown in Figure 4.4 and Figure 4.5, respectively. Figure 4.4 and Figure 4.5 show that the number of nearest neighbors, n_{opt}, used by all models is relatively uniform until the common period reaches approximately 40 years. In Figure 4.4, n_{opt} varies from an average of 58 to 17 percent of N_2 for $r_{xy} > 0.666$. In Figure 4.5, n_{opt} varies from an average of 72 to 37 percent of N_2 for $r_{xy} < 0.666$. Thus the number of nearest neighbors utilized is greater for small r_{xy}. Also the rate at which the nearest neighbors increases with N_2 is greater for $r_{xy} < 0.666$.

Table 4.9 Average Number of Nearest Neighbors for Nonparametric Models, $r_{xy} < 0.666$

MODEL	Common Period, Years														
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
NP1A	4	6	9	10	13	16	20	24	32	36	38	35	35	35	36
NP1B	4	6	8	10	12	15	18	25	31	35	38	35	38	39	48
NP1C	4	7	10	11	13	15	19	20	25	21	23	19	24	21	10
NP2A	3	6	9	10	12	16	20	25	30	36	36	36	36	41	45
NP2B	3	7	9	11	13	16	20	27	31	34	35	36	34	37	40
NP2C	3	7	9	12	12	15	19	24	29	36	32	33	36	34	41
NP3A	4	6	8	10	12	14	16	19	19	25	23	25	29	24	13
NP3B	4	6	8	10	12	14	17	21	26	31	27	29	30	24	12
NP3C	4	8	11	13	15	17	17	16	21	23	23	27	25	9	9
NPM	3	6	8	10	12	15	18	21	27	34	28	24	20	20	20
Mean	3.6	6.5	8.9	10.7	12.6	15.3	18.4	22.2	27.1	31.1	30.3	29.9	30.7	28.4	27.4
Rounded	4	7	9	11	13	15	18	22	27	31	30	30	31	28	27
Stdev	0.5	0.7	1.0	1.1	1.0	0.9	1.4	3.4	4.4	5.9	6.3	6.0	6.1	10.3	16.0

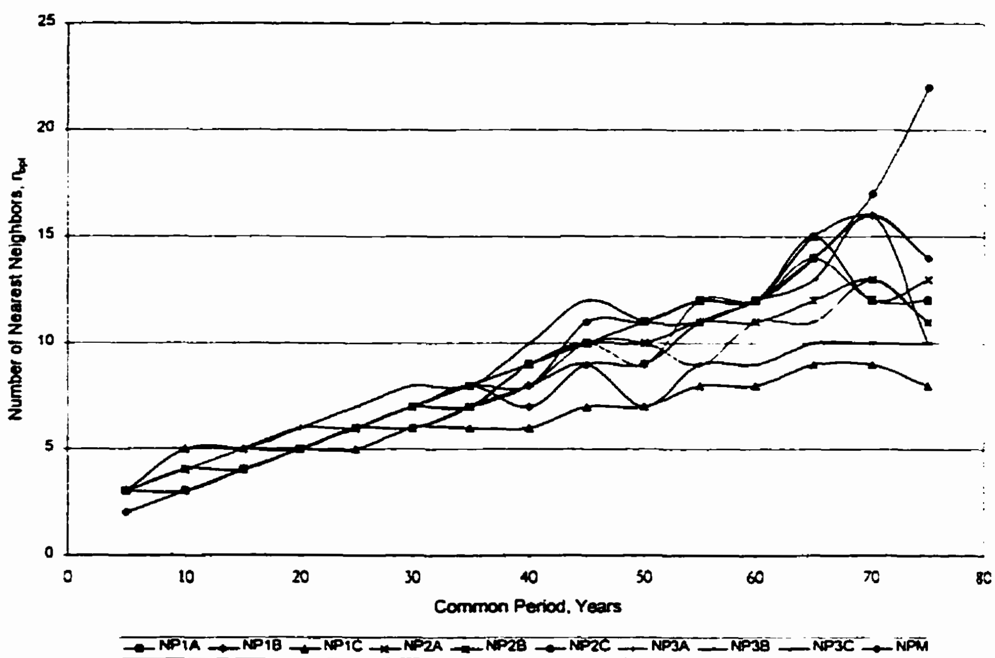

Figure 4.4 Average Optimum Number of Nearest Neighbors, n_{opt}, $r_{xy} > 0.666$

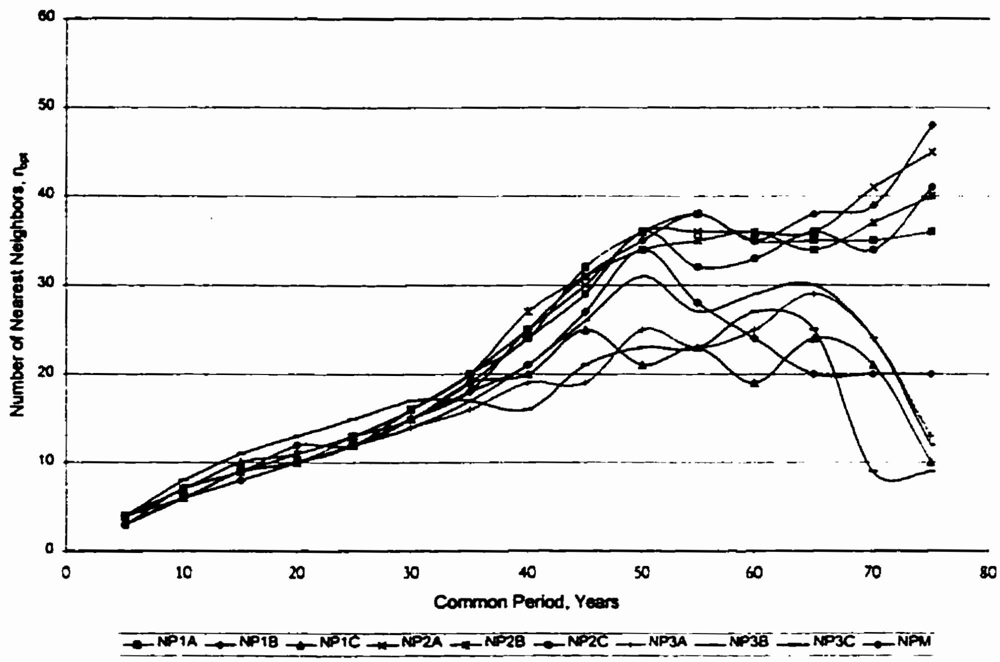

Figure 4.5 Average Optimum Number of Nearest Neighbors, n_{opt} r_{xy} <0.666

4.5 EVALUATION OF EXTENSION TECHNIQUES

In accordance with Section 3.2, log transforms of the data are used in the parametric models to help correct the skewness in the distribution of error terms, unequal error variances and non-linearity of regression function. This type of transformation was used by Hirsch (1982), Alley and Burns (1983), Beauchamp et al. (1989), and Parrett and Cartier (1990). From the log transformation of the data, it follows that the sample mean of the extended record of the logarithms is an unbiased estimate of the mean of the logarithms, but the sample mean of the extended record of flows is not an unbiased estimate of the mean of the flows. Hirsch (1982) states that the above observation is not detrimental to the techniques, because the parametric techniques reproduce cumulative distribution functions (CDF's) which are close approximations to the historical CDF's, particularly in the tails. A log transformation has less effect on the tails of the distributions.

4.5.1 Number of Trials Evaluated

A breakdown of the number of trials conducted for the combinations of extension period, N_1, and common period, N_2, for $r_{xy} > 0.666$ and $r_{xy} < 0.666$ are given in Table 4.10 and Table 4.11, respectively.

For $r_{xy} > 0.666$, there are a total of 492 trials, of which 61 trials are used for model verification. In the case of $r_{xy} < 0.666$, there are a total of 296 trials, of which 20 are used for model verification. There are comparatively fewer trials involving common periods greater than 30 to 40 years, due to a general lack of data records exhibiting very long flow records. Furthermore, as the extension period increases, the number of trials with longer common period decreases because the minimum common period is equal to the extension period.

Table 4.10 Number of Evaluation Trials, $r_{xy} > 0.666$

N_2 Years	N_1 = 5 years	N_1 = 10 years	N_1 = 15 years	N_1 = 20 years	Total Trials
5	21	0	0	0	21
10	21	21	0	0	42
15	21	21	21	0	63
20	21	21	21	15	78
25	21	21	15	10	67
30	21	15	10	5	51
35	15	10	5	5	35
40	10	5	5	5	25
45	5	5	5	5	20
50	5	5	5	5	20
55	5	5	5	5	20
60	5	5	5	5	20
65	5	5	5	0	15
70	5	5	0	0	10
75	5	0	0	0	5
Total	186	144	102	60	492

Table 4.11 Number of Evaluation Trials, r_{xy} < 0.666

N_2 Years	N_1 = 5 years	N_1 = 10 years	N_1 = 15 years	N_1 = 20 years	Total Trials
5	15	0	0	0	15
10	15	15	0	0	30
15	15	15	13	0	43
20	15	13	13	8	49
25	13	13	8	7	41
30	13	8	7	3	31
35	8	7	3	3	21
40	7	3	3	3	16
45	3	3	3	3	12
50	3	3	3	3	12
55	3	3	3	1	10
60	3	3	1	1	8
65	3	1	1	0	5
70	1	1	0	0	2
75	1	0	0	0	1
Total	118	88	58	32	296

There are fewer trials overall for $r_{xy} < 0.666$ in comparison to $r_{xy} > 0.666$, because the selection of candidate base and index records required that the two streams chosen be within a reasonable proximity of each other. In the case of $r_{xy} > 0.666$, this condition is easily met. However, in the case of $r_{xy} < 0.666$, there are many instances where a moderate cross correlation between two flow records can be realized, but the gauge locations are separated by greater than 4 or 5 watersheds. Generally, the cross correlation was found to decrease quite rapidly when the associated watersheds were separated by greater than 2 or 3 subcatchments. The number of candidate streamflow records displaying a low $r_{xy} < 0.666$ while simultaneously remaining within 4 subcatchment proximity was low. However, no strict decision rule was used to select the candidate base and extension records in terms of physical proximity, but the candidate records generally were required to be within 2 or 3 subcatchment proximity.

4.5.2 Tests for Skewness, Kurtosis, and Serial Correlation of Residuals

The skewness and kurtosis of residuals was tested on the log-transformed synthetic time series generated by the parametric models to determine if the parent series were normally distributed. The tests are conducted on the synthetic series before reverse transformation to determine if the log-transformed parent time series used in the parametric models conform to the model assumptions. The serial correlation test is done on the reverse transformed synthetic flow values. If the parent series are normally distributed, the residuals generated from the extension will be normally distributed.

Autocorrelation of residuals is an indicator of model inadequacy which may arise due to the model not accounting for time varying factors. The terms autocorrelation and serial correlation are used interchangeably. In linear regression, autocorrelation of residuals increases the uncertainty associated with the estimated parameters, the mean square error may underestimate the variance of the error terms, and the confidence intervals and tests on the model parameters are no longer strictly applicable (Neter et al., 1989). However, the estimated parameters a and b (Equations 3.11 and 3.12) are still unbiased estimators, but there is increased uncertainty in their estimation (Booy, 1992). Table 4.12 and Table 4.13 show the average skewness, kurtosis and serial correlation of residuals obtained for all the trials conducted for the parametric and nonparametric models, for $r_{xy} > 0.666$ and $r_{xy} < 0.666$, respectively. The 95% confidence limits are shown in the columns entitled ±95%.

Table 4.12 Average Test Parameters on Residual Series, $r_{xy} > 0.666$

MODEL	-95%	SKEW	+95%	-95%	KURT	+95%	-95%	SER	+95%
REG	-0.48	0.07	0.48	2.04	3.66	3.96	-0.21	0.22	0.18
σ:	*0.12*	*0.7*	*0.12*	*0.25*	*1.97*	*0.25*	*0.06*	*0.14*	*0.05*
RPN	-0.48	0.02	0.48	2.04	3.45	3.96	-0.21	0.16	0.18
σ:	*0.12*	*0.55*	*0.12*	*0.25*	*1.66*	*0.25*	*0.06*	*0.13*	*0.05*
MV1	-0.48	0.13	0.48	2.04	3.91	3.96	-0.21	0.13	0.18
σ:	*0.12*	*0.82*	*0.12*	*0.25*	*2.85*	*0.25*	*0.06*	*0.15*	*0.05*
MV2	-0.48	0.12	0.48	2.04	3.86	3.96	-0.21	0.13	0.18
σ:	*0.12*	*0.77*	*0.12*	*0.25*	*2.54*	*0.25*	*0.06*	*0.15*	*0.05*
MV3	-0.48	0.12	0.48	2.04	3.86	3.96	-0.21	0.12	0.18
σ:	*0.12*	*0.75*	*0.12*	*0.25*	*2.35*	*0.25*	*0.06*	*0.13*	*0.05*
NP1A							-0.21	0.11	0.18
σ:							*0.06*	*0.15*	*0.05*
NP1B							-0.21	0.10	0.18
σ:							*0.06*	*0.15*	*0.05*
NP1C							-0.21	0.11	0.18
σ:							*0.06*	*0.15*	*0.05*
NP2A							-0.21	0.11	0.18
σ:							*0.06*	*0.15*	*0.05*
NP2B							-0.21	0.11	0.18
σ:							*0.06*	*0.14*	*0.05*
NP2C							-0.21	0.11	0.18
σ:							*0.06*	*0.15*	*0.05*
NP3A							-0.21	0.22	0.18
σ:							*0.06*	*0.16*	*0.05*
NP3B							-0.21	0.21	0.18
σ:							*0.06*	*0.16*	*0.05*
NP3C							-0.21	0.20	0.18
σ:							*0.06*	*0.16*	*0.05*
REGM	-0.48	0.11	0.48	2.04	4.03	3.96	-0.21	0.14	0.18
σ:	*0.12*	*0.64*	*0.12*	*0.25*	*1.56*	*0.25*	*0.06*	*0.13*	*0.05*
RPNM	-0.48	0.05	0.48	2.04	3.75	3.96	-0.21	0.11	0.18
σ:	*0.12*	*0.53*	*0.12*	*0.25*	*1.38*	*0.25*	*0.06*	*0.13*	*0.05*
MV1M	-0.48	0.25	0.48	2.04	4.44	3.96	-0.21	0.11	0.18
σ:	*0.12*	*0.81*	*0.12*	*0.25*	*3.41*	*0.25*	*0.06*	*0.12*	*0.05*
MV2M	-0.48	0.13	0.48	2.04	3.95	3.96	-0.21	0.12	0.18
σ:	*0.12*	*0.55*	*0.12*	*0.25*	*1.24*	*0.25*	*0.06*	*0.12*	*0.05*
MV3M	-0.48	0.08	0.48	2.04	3.85	3.96	-0.21	0.12	0.18
σ:	*0.12*	*0.48*	*0.12*	*0.25*	*1.35*	*0.25*	*0.06*	*0.12*	*0.05*
NPM							-0.21	0.14	0.18
σ:							*0.06*	*0.14*	*0.05*

The nonparametric models are not subject to skewness and kurtosis tests, as explained previously. Thus skewness and kurtosis values are not reported for the nonparametric models.

The values given in Table 4.12 and Table 4.13 are average values obtained from a large number of trials. The standard error of prediction are shown in italics below the average value. The skewness, kurtosis and serial correlation, abbreviated SKEW, KURT, and SER, respectively, are located in between the associated average 95% confidence limits, for all the trials conducted. The confidence intervals depend upon the level of significance of the test as well as the sample size. The level of significance is constant for all the trials, but the sample size varies quite a bit. Strictly speaking, the confidence limits shown have no specific meaning in association with the averaged test statistics because of different sample sizes used in obtaining both the test statistics and the confidence limits. However, one may note that the standard error associated with the confidence limits is quite small, indicating the confidence limits did not vary to a great extent over the range of sample sizes encountered. Thus the average confidence limits, with the associated standard error, provide a general indication as to the range of confidence limits encountered during all the trials. Similarly, the average test statistic values, along with the associated standard deviation, provides a general indication as to the range of skewness, kurtosis and serial correlation of residuals encountered over all the trials conducted.

Referring to Table 4.12 and Table 4.13, the test statistics displayed large variance. This indicates that the values of the test statistics obtained from the trials were dispersed greatly about the mean value. The standard deviation of the skewness is as much as 5 times as great as the mean value, but the mean value over all the trials falls within the average 95% confidence limits. Thus, even though the average test statistic falls within the confidence limits, one would expect a large portion of the trials to "fail" the test, ie. the null hypothesis that the skewness of the residuals is zero would be rejected, supporting the conclusion that the residuals are not normally distributed. Similarly, the standard deviation of the kurtosis of residual series varied between approximately 30 and 80 percent, and that of the serial correlation between 40 and 120 percent of the average.

Table 4.13 Average Test Parameters on Residual Series, $r_{xy} < 0.666$

MODEL	-95%	SKEW	+95%	-95%	KURT	+95%	-95%	SER	+95%
REG	-0.48	0.27	0.48	2.03	3.26	3.97	-0.21	0.35	0.19
σ:	*0.12*	*0.62*	*0.12*	*0.24*	*2.16*	*0.24*	*0.05*	*0.15*	*0.04*
RPN	-0.48	0.14	0.48	2.03	3.22	3.97	-0.21	0.28	0.19
σ:	*0.12*	*0.48*	*0.12*	*0.24*	*1.8*	*0.24*	*0.05*	*0.15*	*0.04*
MV1	-0.48	0.35	0.48	2.03	3.69	3.97	-0.21	0.21	0.19
σ:	*0.12*	*0.76*	*0.12*	*0.24*	*2.99*	*0.24*	*0.05*	*0.16*	*0.04*
MV2	-0.48	0.3	0.48	2.03	3.53	3.97	-0.21	0.21	0.19
σ:	*0.12*	*0.64*	*0.12*	*0.24*	*2.35*	*0.24*	*0.05*	*0.16*	*0.04*
MV3	-0.48	0.29	0.48	2.03	3.53	3.97	-0.21	0.21	0.19
σ:	*0.12*	*0.63*	*0.12*	*0.24*	*2.36*	*0.24*	*0.05*	*0.16*	*0.04*
NP1A							-0.21	0.21	0.19
σ:							*0.05*	*0.16*	*0.04*
NP1B							-0.21	0.21	0.19
σ:							*0.05*	*0.16*	*0.04*
NP1C							-0.21	0.22	0.19
σ:							*0.05*	*0.17*	*0.04*
NP2A							-0.21	0.20	0.19
σ:							*0.05*	*0.16*	*0.04*
NP2B							-0.21	0.21	0.19
σ:							*0.05*	*0.16*	*0.04*
NP2C							-0.21	0.21	0.19
σ:							*0.05*	*0.17*	*0.04*
NP3A							-0.21	0.22	0.19
σ:							*0.05*	*0.16*	*0.04*
NP3B							-0.21	0.22	0.19
σ:							*0.05*	*0.16*	*0.04*
NP3C							-0.21	0.23	0.19
σ:							*0.05*	*0.17*	*0.04*
REGM	-0.48	0.14	0.48	2.03	3.83	3.97	-0.21	0.21	0.19
σ:	*0.12*	*0.6*	*0.12*	*0.24*	*2.28*	*0.24*	*0.05*	*0.15*	*0.04*
RPNM	-0.48	0.09	0.48	2.03	3.62	3.97	-0.21	0.15	0.19
σ:	*0.12*	*0.56*	*0.12*	*0.24*	*1.92*	*0.24*	*0.05*	*0.15*	*0.04*
MV1M	-0.48	0.27	0.48	2.03	4.18	3.97	-0.21	0.18	0.19
σ:	*0.12*	*0.61*	*0.12*	*0.24*	*1.69*	*0.24*	*0.05*	*0.17*	*0.04*
MV2M	-0.48	0.21	0.48	2.03	4.12	3.97	-0.21	0.18	0.19
σ:	*0.12*	*0.59*	*0.12*	*0.24*	*1.77*	*0.24*	*0.05*	*0.16*	*0.04*
MV3M	-0.48	0.18	0.48	2.03	4.08	3.97	-0.21	0.19	0.19
σ:	*0.12*	*0.63*	*0.12*	*0.24*	*1.99*	*0.24*	*0.05*	*0.15*	*0.04*
NPM							-0.21	0.25	0.19
σ:							*0.05*	*0.15*	*0.04*

The average skewness of residuals increased, for all models, when r_{xy} decreased. The average kurtosis of residuals decreased when r_{xy} decreased for all models except MV2M and MV3M. In all cases, the average serial correlation of the residual series increased with decreased cross correlation between the base and index records.

Generally, the average serial correlation obtained over all trials fell within the average 95% confidence limits, except for NP3A, with was marginally outside the confidence limits. However, given the high standard deviation of the serial correlation of errors, we would again expect a large percentage of trials to "fail" the test, supporting the conclusion that the models may not explain time-ordered effects, such as potential trends in the time series, or time varying relationships between the base and index record.

Histograms are frequently used in statistical analysis to graphically summarize the nature of the data. Histograms of the skewness, kurtosis and serial correlation were constructed in order to visually examine the empirical frequency distributions of the parameter values. The procedure for obtaining the histograms follows. The data is grouped into class intervals of 0.1 width (no units). The choice of class interval is determined somewhat arbitrarily, involving judgement to balance computational complexity and the desire to provide an adequate picture of the distribution. Small samples may also affect the size of class interval. The class frequency is determined as the number of data occurring within a particular class interval. The proportion of the set of observations in each class is obtained by dividing each class frequency by the total number of observations.

From Table 4.12, REGM has fairly high average skewness and kurtosis at 0.11, and 4.03, respectively. The standard deviations of the skewness and kurtosis are 0.64 and 1.56, respectively, indicating a large spread about the mean value. The skewness histogram for REGM is shown in Figure 4.6. The kurtosis histogram for REGM is shown in Figure 4.7.

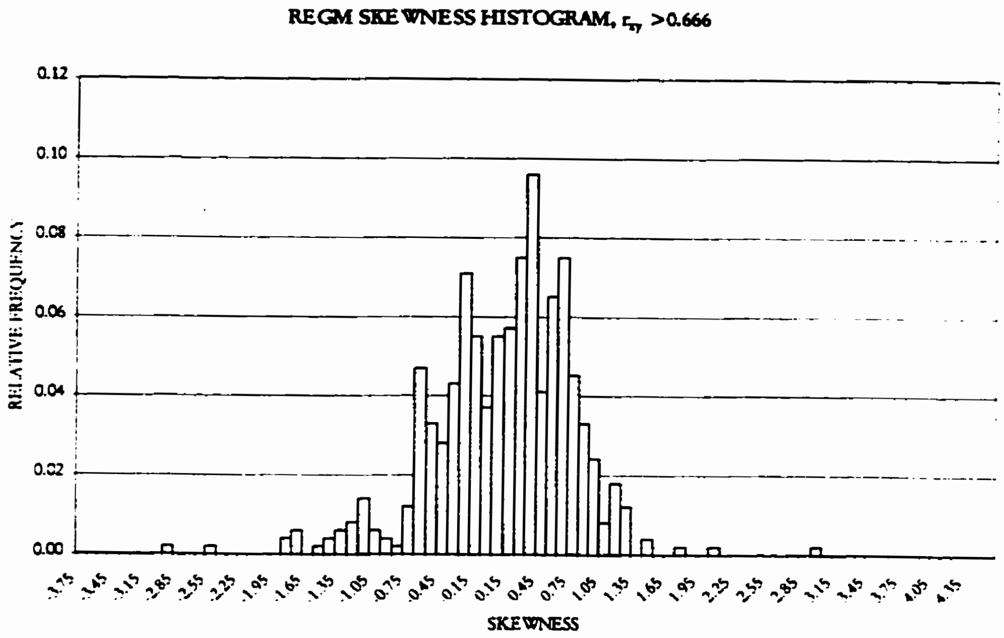

Figure 4.6 Histogram of Skewness of REGM Residual Series, r_{xy} >0.666

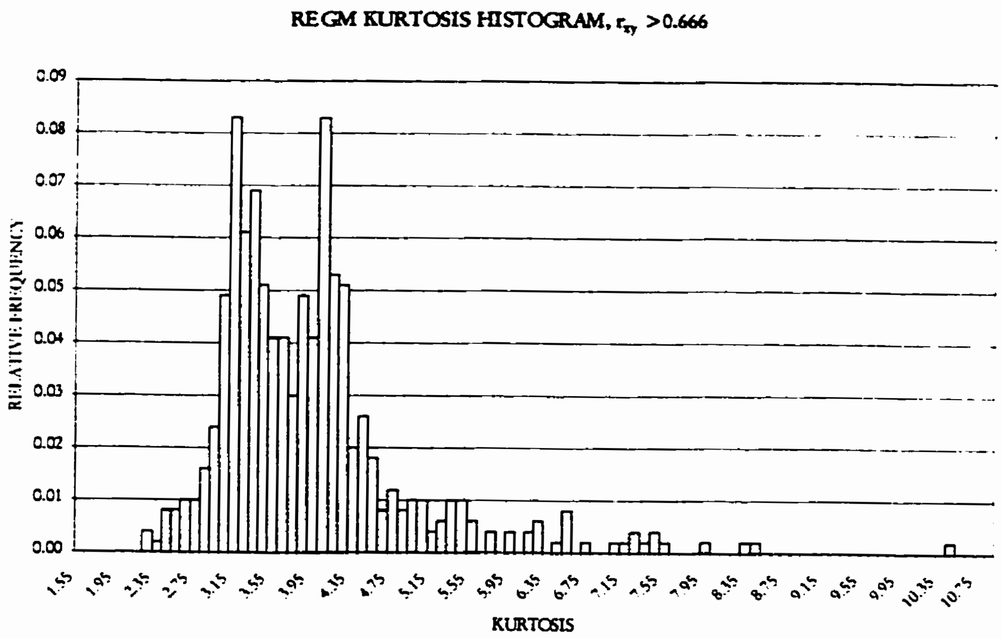

Figure 4.7 Histogram of Kurtosis of REGM Residual Series, r_{xy} >0.666

Figure 4.6 shows the distribution of skewness slightly skewed to the left, favoring higher values of skewness and a large variability. A skewness of 0.45 has the highest relative frequency of 0.096. Thus, the skewness histogram is shifted to the right of the expected value under normality. The area within the bars represents the probability of the skewness occurring, ie. $P(g_1)$. Roughly half the area on Figure 4.6 falls to the left and right of the "average confidence limits" described previously, therefore one would estimate approximately 50 percent of the trials involving REGM would fail the skewness test. Figure 4.7 shows the distribution of kurtosis is also skewed to the left, with a split into two peak frequencies centered around approximately 3.2 and 4.3. A large variability of kurtosis is also displayed. The left peak of the histogram occurs at 3.2 kurtosis with a relative frequency of 0.083. The right histogram peak occurs at 4.3 kurtosis also with a relative frequency of 0.083. Given the occurrence of the right peak outside the "average confidence limits" for this statistic, one would expect nearly half of the trials for REGM to fail the kurtosis test.

Figure 4.8 shows the histogram of serial correlation of the residual series, r_ϵ, for REG, which obtained an average $r_\epsilon = 0.22$. The standard deviation of r_ϵ was 0.14, indicating high variability.

Figure 4.8 shows the distribution of serial correlation has a slight positive skewness, centered around $r_\epsilon = 0.25$ with a corresponding relative frequency of 0.317. The five bars on the right side of the histogram fall outside the average confidence limits, leading one to expect a high percentage of trials will result in rejection of the null hypothesis that the residual series is not serially correlated.

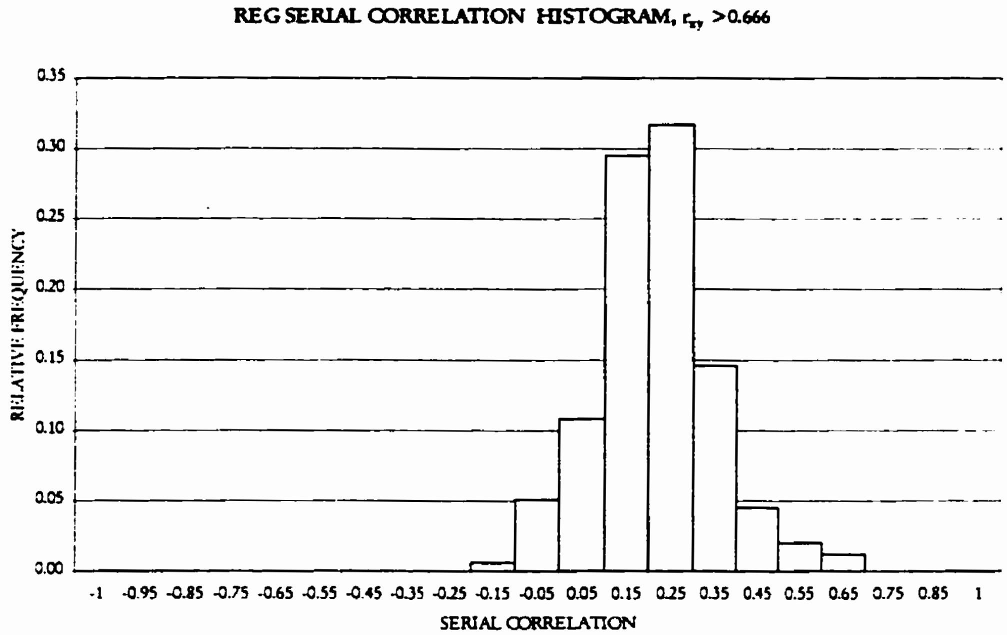

Figure 4.8 Histogram of Serial Correlation of REG Residual Series, r_{xy} >0.666

From Table 4.13, MV1 has high skewness, MV1M has high kurtosis and NPM has high r_e, at 0.35, 4.18, and 0.25, respectively. The standard deviations of the skewness, kurtosis and r_e are 0.76, 1.69 and 0.15, respectively, indicating large variability. The histogram of skewness of the residual series for MV1 is shown in Figure 4.9. Similarly, the corresponding kurtosis and serial correlation histograms are shown in Figure 4.10 and Figure 4.11 for MV1M and NPM, respectively.

The histogram in Figure 4.9 is slightly skewed to the right and shows a large variability. A skewness of 0.25 has the peak relative frequency of 0.122. Roughly one third of the histogram area falls outside the average confidence limits for skewness ±0.48, listed in Table 4.13.

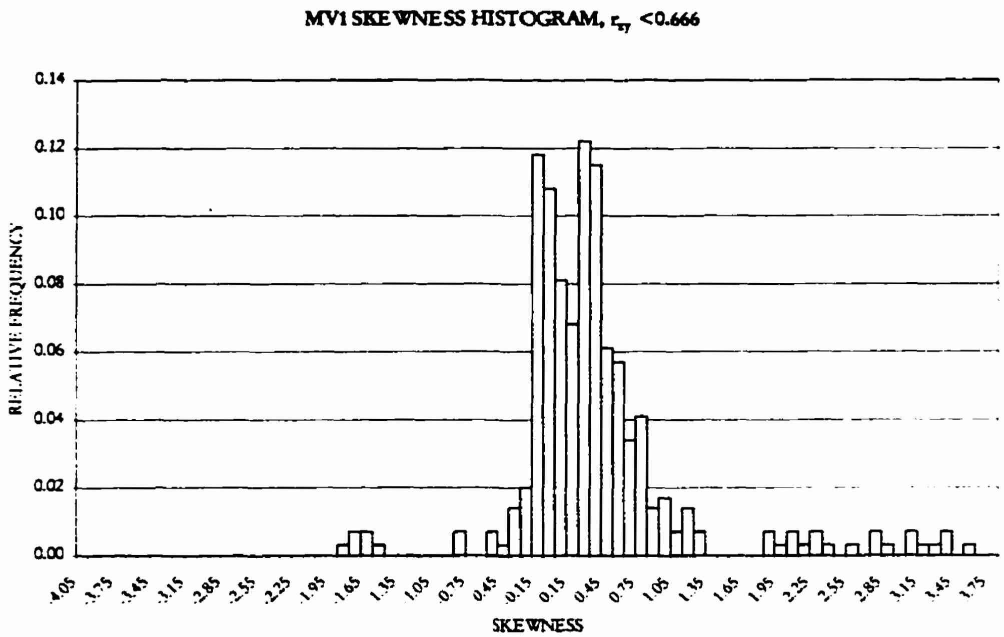

Figure 4.9 Histogram of Serial Correlation of MV1 Residual Series, r_{xy} <0.666

The kurtosis histogram in Figure 4.10 is skewed to the right (negative skew). The peak frequency is 0.071 at a kurtosis of 3.15 and the variability is again large. Again, a significant portion of the histogram lies outside the average 95% confidence limits, indicating a large portion of the residuals series would fail the skewness test. This would lead to the conclusion that the error terms are not normally distributed.

Figure 4.11 shows the serial correlation of the residual series is very slightly skewed to the right (positive skewness), but nearly symmetrical, centered around r_ϵ = 0.25, with a corresponding relative frequency of 0.27. This histogram is shifted far to the right of zero serial correlation leading to suspicion that a significant portion of the results display autocorrelated residuals.

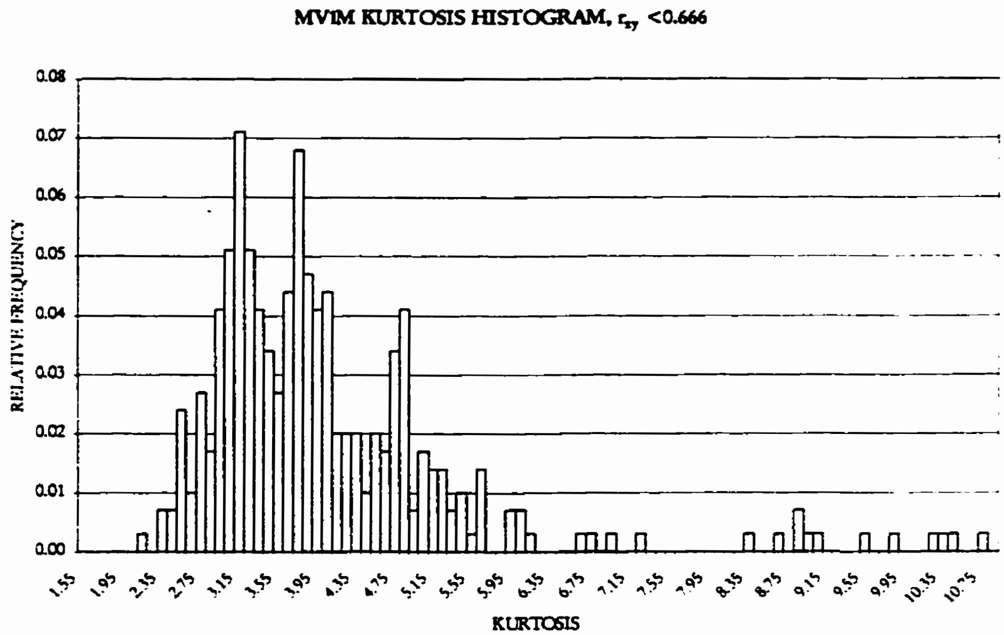

Figure 4.10 Histogram of Kurtosis of MV1M Residual Series, r_{xy} <0.666

Table 4.14 and Table 4.15 list the fraction of trials for which a particular model "fails" the error diagnostic tests, as described above. Again the nonparametric models are not subject to tests of skewness and kurtosis of residuals because the formulation of the nonparametric models includes no assumptions on the underlying distribution of the data.

A fairly high fraction of the tests conducted resulted in skewness, kurtosis and autocorrelation in the residual series. A fraction times an event occurred out the total number of trials may be interpreted as the percentage of time an event occurred, and is referenced as such interchangeably in the discussion. Between 18 and 54 percent of all the trials resulted in significant residual skewness. Between 16 and 44 percent of trials displayed kurtosis in the residual series that is significantly different from 3. Autocorrelation of residuals was found in the residual series of between 24 and 86 percent of the trials conducted.

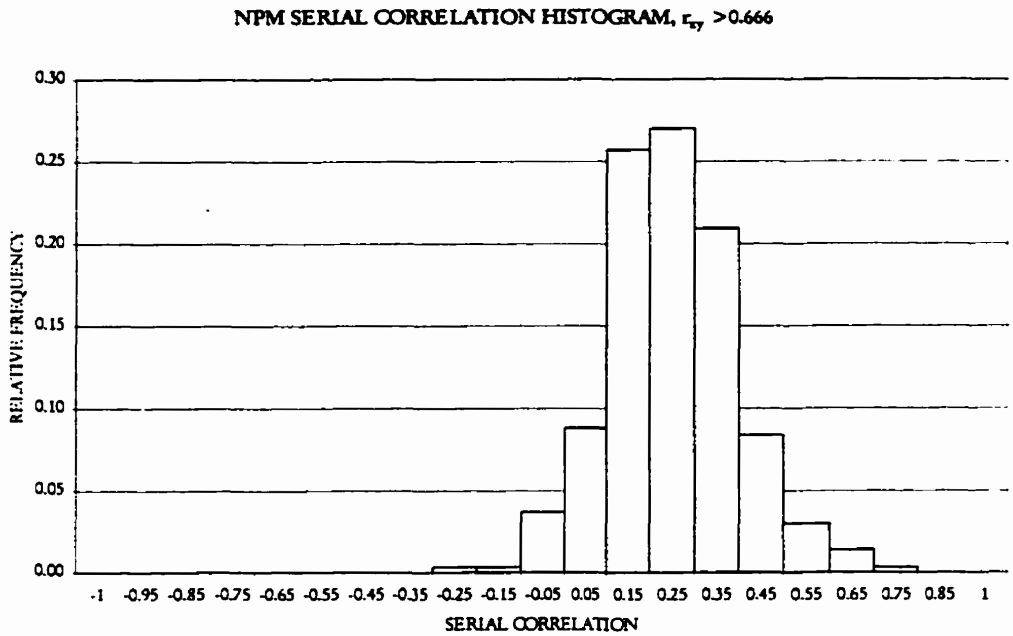

Figure 4.11 Histogram of Serial Correlation of NPM Residual Series, r_{xy} <0.666

For $r_{xy} > 0.666$, the average fraction of trials failing the autocorrelation test was 34% for the parametric models and 41% for the nonparametric models. For $r_{xy} < 0.666$, the average fraction of trials failing the autocorrelation test was 56% for the parametric models and 62% for the nonparametric models. As the cross correlation between the base and index record decreases, the autocorrelation in the residual series increases, indicating an overall reduction in model performance. Based on the above, we would expect, on average, the residual series of the parametric models to display less serial correlation than the nonparametric models.

Table 4.14 Fraction of Trials "Failing" Error Tests, $r_{xy} > 0.666$

MODEL	SKEW	KURT	SER
REG	0.54	0.21	0.63
RPN	0.27	0.16	0.41
MV1	0.43	0.25	0.31
MV2	0.42	0.26	0.31
MV3	0.45	0.28	0.30
NP1A			0.34
NP1B			0.31
NP1C			0.34
NP2A			0.32
NP2B			0.33
NP2C			0.32
NP3A			0.61
NP3B			0.58
NP3C			0.59
REGM	0.49	0.42	0.40
RPNM	0.33	0.28	0.30
MV1M	0.44	0.44	0.25
MV2M	0.39	0.41	0.26
MV3M	0.25	0.38	0.24
NPM			0.44

The fraction of skewness test failures decreased in relation to decreased r_{xy}. One would reasonably expect that the fraction of failures would increase with skewness. However, the standard deviation of the skewnesses obtained generally was greater for $r_{xy} > 0.666$ than for $r_{xy} < 0.666$, resulting in more trials falling outside the confidence limits.

In the case of the kurtosis tests, both the average kurtosis values and the standard deviations decreased with the smaller r_{xy} value. Accordingly, a general decrease in fraction of kurtosis test failures was realized. All of the parametric models, except REG, RPN and MV1M displayed a reduction in the fraction of trials failing the kurtosis test with decreased r_{xy}.

Table 4.15 Fraction of Trials "Failing" Error Tests, $r_{xy} < 0.666$

MODEL	SKEW	KURT	SER
REG	0.44	0.31	0.86
RPN	0.18	0.22	0.74
MV1	0.37	0.22	0.51
MV2	0.37	0.21	0.51
MV3	0.35	0.19	0.50
NP1A			0.58
NP1B			0.58
NP1C			0.65
NP2A			0.59
NP2B			0.58
NP2C			0.60
NP3A			0.65
NP3B			0.64
NP3C			0.65
REGM	0.25	0.17	0.61
RPNM	0.23	0.17	0.46
MV1M	0.43	0.44	0.42
MV2M	0.36	0.39	0.46
MV3M	0.27	0.36	0.52
NPM			0.67

The fraction of serial correlation test failures increased substantially for $r_{xy} < 0.666$ in comparison to $r_{xy} > 0.666$, but the standard deviations of the serial correlation values obtained did not differ substantially. There is no appreciable difference in either the average serial correlation or percent failure between the parametric and nonparametric models. However, REG and NP3A failed the serial correlation test approximately 20% more than the other models.

The regression parameter, b, is a linear function of the observations x_i and y_i, and a linear combination of normally distributed variables is itself normally distributed. Inferences regarding the variance of b are thus drawn from estimating the mean squared error, MSE, from the sample, rather than the true variance. The difficulty with variables that are not normally distributed is that MSE may underestimate the variance of b. The consequence is that MSE is no longer an unbiased estimator of the variance of b, and indeed may actually underestimate the uncertainty in b, resulting in more confidence being placed on the regression model parameters than is warranted. However, the results obtained usually remain reasonably valid unless the deviation from normality is quite pronounced (Booy, 1992).

Correlation in error terms also increases the uncertainty in b, due to the conditions of additive property of variances of terms as described above. The latter rule cannot be applied if y_i are correlated. In the case of autocorrelated error terms, the regression parameters a and b are still unbiased, but no longer have minimum variance, ie. there is greater uncertainty in their estimation. Persistence in autocorrelated error terms may lead to regression parameters which are quite different from the true parameters when ordinary least squares regression is used (Neter et al., 1989). Therefore, in least squares regression, greater precision in the regression coefficients may be indicated than is actually the case in the presence of autocorrelated error terms.

In the present study, we are not so much concerned with the slope of the regression line in drawing inferences on the functional response of a random variable to the level of an independent (and not necessarily random) variable. Rather, we are interested in the relationship between two random variables, and the use of the conditional distribution in making inferences regarding one variable given the level of another variable.

If the parent variables in the regression are not random, utilizing a bivariate normal surface to describe the joint probability distribution is not strictly valid. The conditional distributions used to make inferences regarding one variable given the level of another variable, is also not strictly valid when fitting a distribution to a set of data which may not strictly follow the assumed distribution. Thus, if the conditional distribution of x_i is biased, then the results of y_i obtained will be biased in relation to the bias of x_i.

Autocorrelation of residuals does not cause the parametric models to violate any of the model assumptions, but indicates model inadequacy. The error diagnostic tests do not give any information on the consequences in terms of generated flows. The calculation of the model adequacy statistics are used to aid in determining if the biases introduced from departures from model assumptions deteriorate the model adequacy.

4.5.3 Statistical Adequacy Measures

The average fractional deviation from the various target statistical values, and the associated standard deviations, for $r_{xy} > 0.666$ and $r_{xy} < 0.666$ are presented separately for each adequacy measure in Appendix C, and referenced below. For the tables referenced in the following paragraphs, there are several common periods for each extension period. Each table shows the average fractional deviation from the associated target value calculated based on all the trials conducted for a given extension period and all associated common periods. Note that the Tables in Appendix C show the models which achieved the minimum deviation from the target value, expressed as a fraction of the target value, and averaged over all the trials conducted.

4.5.3.1 Cross Correlation Between Generated and Historical Flows

The first adequacy measure presented is the cross correlation between the generated and historical flows, $r_{\hat{y}y}$. The goal is to obtain a generated record which has a cross correlation of 1.0 with the historical flows. All models achieved a cross correlation less than 1.0.

The model associated with the smallest average fractional deviation from the target value, may be expected to generate flow sequences having the highest degree of similarity with the historical flows. Tables C-1a to C-1d and Tables C-2a to C-2d show the models achieving the smallest average fractional deviation from the target, and the associated standard deviation of the results, for $r_{xy} > 0.666$ and $r_{xy} < 0.666$, respectively. In some cases more than one model obtained the same minimum average fractional deviation, in which case both are reported.

4.5.3.2 Serial Correlation of Generated Flows

Tables C-3a to C-3d and Tables C-4a to C-4d show the models achieving the minimum average deviation from the historic serial correlation. The models' objective is to generate a synthetic sequence of flows which displays the same serial correlation as the historic flows during the extension period, ie. the target value. The models showing the minimum average deviation from the target value more adequately reproduce the serial correlation statistic over all trials conducted.

4.5.3.3 Variance of Generated Flows

Tables C-5a to C-5d and Tables C-6a to C-6d show the models achieving the minimum average deviation from the historic variance. The models' objective is to generate a synthetic sequence of flows which displays the same variance as the historic flows during the extension period, ie. the target value. The models showing the minimum average deviation from the target value more adequately reproduce the variance over all trials conducted.

4.5.3.4 Mean of Generated Flows

Tables C-7a to C-7d and Tables C-8a to C-8d show the models achieving the minimum average deviation from the historic mean flows. The models' objective is to generate a synthetic sequence of flows which displays the same mean flow as the historic flows during the extension period, ie. the target value. The models showing the minimum average deviation from the target value more adequately reproduces the mean flows over all trials conducted.

4.5.3.5 Generated Low Flows

Tables C-9a to C-9d and Tables C-10a to C-10d show the models achieving the minimum average deviation from the historic low flows. The models' objective is to generate a synthetic sequence of flows which displays the same low flow as the historic flows during the extension period, ie. the target value. The models showing the minimum average deviation from the target value more adequately reproduces the low flows over all trials conducted.

4.5.3.6 Mean Percentage Error of Generated Flows

Tables C-11a to C-11d and Tables C-12a to C-12d show the models achieving the minimum average mean percentage error. The models' objective is to generate a synthetic sequence of flows which displays zero mean percentage error, ie. the target value. The models showing the minimum mean percentage error more adequately reproduces individual flow values over all trials conducted.

4.5.4 Objective Function Adequacy Measure

The objective function provides a measure of how accurately a particular model reproduces all the adequacy measures in relation to the performance of the other models. Recall that an objective function value, OBJ_i, is calculated for each model, i, for each trial conducted. The goal is to achieve the minimum OBJ_i for each trial conducted. The number of times a particular model achieves the minimum OBJ_i out of all the trials conducted indicates the relative adequacy of a given model in comparison to the other models over the range of extension period, common period and cross correlation between the index and base records presented.

The minimum objective function values for each model were determined. The values in the tables referenced in the following paragraphs represent the fraction of trials for which a particular model minimized the objective function calculation, thus taking into account the relative adequacy for all measures combined. Thus, the minimum objective function adequacy measure is not separately calculated for each statistical adequacy measure.

The models associated with the largest fraction of minimum objective functions may be expected to provide generated flow sequences which reproduce the adequacy measures in a general sense, taking all adequacy measures into account. Tables C-13a to C-13d and Tables C-14a to C-14d show the models achieving the largest fraction of trials in which OBJ_i was smallest, for $r_{xy} > 0.666$ and $r_{xy} < 0.666$, respectively. In some cases more than one model obtains the maximum percent-best objective function value, which is determined as a tie.

4.6 DISCUSSION OF EVALUATION RESULTS

4.6.1 Statistical Adequacy Measures

The ability of the extension models to reproduce statistical characteristics of the historical flows is discussed in terms of a comparison of the "best" models, separately for each adequacy measure. The model adequacy in reproducing the statistical adequacy measures are presented in terms of the fractional deviation from the target value, $\bar{f}_{ij}$, averaged over all the evaluation trials conduced, as described in Section 4.2.2. Note $\bar{f}_{ij}$ was not calculated for SSE, since the fractional deviation from zero is indeterminate, and a direct comparison of SSE between the large number of evaluation trials would not be useful.

The fractional deviations of cross correlation between the generated and historical flows can be directly converted into average correlation values obtained, because the target value is 1.0. Similarly, the MPE is averaged over all the trials conducted, and represents an average value of the percentage errors obtained for the streamflow extension trials conducted. However, the serial correlation, variance, mean and low flow measures, are expressed in fractional deviations because the target value is not the same for all trials.

4.6.1.1 Cross Correlation Between Generated and Historical Flows

$r_{xy} > 0.666$ - Tables C1(a to d)

The average correlation between the generated and historical flows for the best models varied between 0.84 and 0.92 as N_2 increased from 5 to 75 years. No significant variation in cross correlation was observed for different extension periods. The standard deviation of the cross correlation values remained relatively consistent, varying between approximately 35 to 50 percent of the mean value. The cyclic models generally achieved the highest cross correlation between generated and target flows. For extension periods of 5 and 10 years, REGM and MV2M performed quite well for $N_2 < 45$ years, while REGM and noncyclic nonparametric models performed better for $N_2 > 45$ years. For extension periods of 15 and 20 years, REGM and NPM achieved the best results.

$r_{xy} < 0.666$ - Tables C-2(a to d)

The average correlation between the generated and historical flows varied between 0.74 for and 0.89 as N_2 increased from 5 to 75 years. The standard deviation of the cross correlation again remained consistently lower than for $r_{xy} > 0.666$, varying between approximately 20 to 40 percent of the mean value. Both the cyclic and non-cyclic nonparametric models achieved the highest cross correlation between generated and target flows. Notably, there was only one instance where REG performed better than the nonparametric models, on average, namely for $N_1 = 5$ years and $N_2 = 70$ years. However, for this case, the nonparametric models achieved cross correlation with the historical record only marginally less than REG. There was very little difference in performance between the cyclic and non-cyclic nonparametric model performance, which is evident by the list of nonparametric models achieving the same average cross correlation with the historical record in Tables C-2(a to d). However, the noncyclic nonparametric models performed best overall.

4.6.1.2 Serial Correlation of Generated Flows

$r_{xy} > 0.666$ - Tables C-3(a to d)

The average serial correlation varied between -5 and 2 percent of the target value. The models more adequately reproducing the serial correlation of the historic flows varied with different extension periods. The percent deviations did not necessarily decrease with increasing common period for extension periods less than 10 years. However, for extension periods greater than 10 years, greater common periods generally resulted in increased model performance for both parametric and nonparametric models.

For $N_1 = 5$ years, the cyclic parametric models performed best, especially MV3M. There was no appreciable difference in model performance with changes in either N_1 or N_2. For $N_1 = 10$ years, the non-cyclic parametric models also performed well. For $N_1 = 15$ years, the best models varied with common period, beginning with REGM for common periods less than 50 years, then NP3A and NP3B up to $N_2 = 60$ years, then MV2 and MV3 for a 65 year common period. For $N_1 = 20$ years, however, various non-cyclic nonparametric models performed best, achieving between 0 and 2 percent deviation from the target value on average.

Generally, the parametric models tended to underestimate the serial correlation while the nonparametric models tended to overestimate the serial correlation, however, the nonparametric models performed better as the common period increased.

$r_{xy} < 0.666$ - Tables C-4(a to d)

In most cases the cyclic parametric models performed best, with the mean deviation varying from -6 to 8 percent. Decreasing r_{xy} marginally decreases the parametric model performance and seriously decreases the nonparametric model performance, for this adequacy statistic. The parametric and nonparametric models tended to underestimate and overestimate the serial correlation, respectively. The models generally performed better as N_2 increased. REG and RPNM did quite well in comparison to the other models.

4.6.1.3 Variance of Generated Flows

$r_{xy} > 0.666$ - Tables C-5(a to d)

On average, the best techniques in this adequacy measure produced variances generally within -22 to 9 percent of the historical series. The extension models generally did not reproduce variance well when the extension and common periods were within 5 years. The ability to reproduce variance increased with longer common periods. However, the parametric models tended to improve more than the nonparametric models as common period increased. The maintenance of variance (MOVE) techniques tended to overestimate the variance. The nonparametric techniques, REG and RPN tended to underestimate the variance.

For $N_1 = 5$ years, NP1C, MV1M and MV2M generally performed the best, yielding variances within 5 percent of the target. For $N_1 = 10$ years, REGM and NP1C performed best, averaging between 1 and 3 percent deviation in estimated variance. For $N_1 = 15$ years, the best models were REGM and RPNM with up to 9 percent deviation in estimated variance. No models was consistently the best for $N_1 = 20$ years. However, RPNM did obtain the lowest average percent deviations. Notably for N_2 between 40 and 50 years, the standard deviation of variance was only 2 percent from the target value for RPNM, indicating that RPNM consistently reproduced variance.

$r_{xy} < 0.666$ - Tables C-6(a to d)

For this case, the best models reproduced variance between −22 and 9 percent on average, generally decreasing in overall performance with decreased cross correlation between the base and index record. Similarly, poor performance was generally displayed when the extension and common periods were within 10 years, with the exception of a few models. Notably, the noncyclic nonparametric methods seemed to do quite well, for $N_1 = 10$ years, but not for any other extension period investigated. Again, the MOVE techniques overestimated variance and the nonparametric, REG and RPN techniques underestimated variance.

For $N_1 = 5$ years, MV2 and MV3 performed best, averaging between 1 and 12 percent deviation on average, up to $N_2 = 35$ years. For N_2 greater than 45 years, REGM performed best with 3 to 6 percent deviation on average. For N_2 greater than 70 years, MV3M was best, with 2 to 4 average percent deviation from the historical target value. For $N_1 = 10$ years, various non-cyclic nonparametric models generally performed best, achieving percent deviations between 0 and 5 percent on average, although this value may vary by between 20 to 60 percent, as indicated by the standard deviation column in Table C-6b. For $N_1 = 15$ years the noncyclic parametric models, in particular MV1 and MV3 generally performed best with the variance between 0 and 11 percent of the target, on average. For $N_1 = 20$ years, MV3 performed best overall, achieving between 1 and 4 percent deviation, except for N_2 between 35 and 50 years, where REGM performed better, also showing deviations in variance between 1 and 4 percent on average.

4.6.1.4 Mean of Generated Flows

$r_{xy} > 0.666$ - Tables C-7(a to d)

Overall, all extension techniques displayed generally acceptable ability in reproducing mean flows. Notably, REG consistently underestimated the mean flow, and the MOVE techniques generally overestimated the means. The remaining models obtained mean flows both higher and lower than the historical means.

The best models displayed mean flows which were within −1 to 2 percent, on average. The standard deviation of the mean flows varied up to 29 percent different from the target values. However, the standard deviations of the mean flows did not exceed 19 percent difference of the target values for common periods greater than 20 years. Both increase in extension and common period tended to decrease the deviation from target mean flows.

For N_1 = 5 years, REGM and NPM performed best. For N_1 = 10 years, a large variety of models performed well. For N_1 = 15 years, NPM generally performed the best. The non-cyclic nonparametric models generally outperformed the other models for N_2 = 20 years.

$r_{xy} < 0.666$ - Tables C-8(a to d)

Again, the best models reproduced the mean flows quite well, between −2 and 7 percent on average. However, the models' overall performance decreased with smaller r_{xy}. This is partly indicated by the increase in standard deviation of the difference in means from the target values, which increased to a maximum of 41 percent. REG, RPN, REGM and RPNM tended to underestimate the mean flow, while the remaining models tended to overestimate the mean flow, with the exception of the nonparametric models for common periods greater than approximately 70 years.

The standard deviation of the mean flows varied from 1 to 49 percent difference from the target values. As a comparison to $r_{xy} > 0.666$, the standard deviations of the mean flows was less than or equal to 31 percent difference from the target values for common periods greater than 20 years, but this value decreased to 13 percent for N_2 greater 35 years. There is no appreciable change in model performance with changes in either N_1 or N_2.

For N_1 = 5 years, REGM performed best up to N_2 = 30 years, followed by NPM up to N_2 = 65 years, then NP1A, NP1B and NP2B for N_2 = 70 to 75 years. For N_1 = 10 years, MV3 performed best to approximately N_2 = 25 years, followed by REGM and RPNM to N_2 = 60 years, and MP3A and MP3B for N_2 = 65 to 70 years. For N_1 = 15 years, a large variety of extension models performed best for different combinations of N_1 and N_2, with MV3 and RPNM displaying the best performance. For N_1 = 20 years, REGM performed best up to N_2 = 45 years, followed by NP1C, NP1B and NP2B for N_2 = 50 to 60 years.

4.6.1.5 Generated Low Flows

r_{xy} > 0.666 - Tables C-9(a to d)

Recall that the low flow analyzed is that flow which is expected to be equaled or exceeded 80 percent of the time. The cyclic parametric models more adequately reproduced the historical low flows than the other extension models. On average, the best extension techniques produced low flows -1 and 31 percent of the historical series. The ability of the models in reproducing low flows generally increased with both longer extension and common periods. Most extension models tended to overestimate the low flows, with the exception of the cyclic parametric models where N_1 is greater than or equal to 10 years, and N_2 is approximately between 30 and 50 years.

For N_1 = 5 years, MV1M and MV3M performed best for N_2 up to 45 years, achieving an average percent deviation from low flows within approximately 11 to 32 percent of the historical value. RPNM generally performed best for N_2 greater than 45 years, achieving between 1 and 5 average percent deviation from historical low flows. For N_1 = 10 years, MV1M performed best, achieving between 1 and 17 average percent deviation from historical low flows up to 35 years common period. This was followed by MV1 and MV2 for N_2 = 40 years, and MV1M and MV2M for N_2 greater than 55 years, all achieving less than or equal to 1 percent average deviation from target low flows. For $N_1 = 15$ and $N_1 = 20$ years, the cyclic MOVE techniques generally performed best, achieving between 0 and 9 average percent deviation from historical low flows. The variation in the above results was quite high. Therefore, even though the average results showed good agreement, there were many instances where the deviation from the target value was quite pronounced.

$r_{xy} < 0.666$ - Tables C-10(a to d)

Generally, the extension models' ability to reproduce low flows decreased slightly with smaller cross correlation between the base and index records. However, the best models actually obtained smaller deviations from target values in comparison to $r_{xy} > 0.666$. On average, the best extension techniques produced low flow frequencies between -3 and 17 percent of the historical series. Again, the overall ability of the models to reproduce low flows tended to increase with both extension and common period, but not necessarily with the best models. Generally, the extension techniques overestimated low flows with the exception of the cyclic MOVE techniques for common periods greater than approximately 40 years.

For $N_1 = 5$ years, MV1M performed best up to $N_2 = 35$ years, achieving between 0 and 17 percent deviation from the target low flows. For a common period of 40 years, MV2M performed best with 13 percent deviation from target low flows. For N_1 greater than or equal to 45 years, MV3, MV3M, RPNM and NPM performed best with percent deviations between 0 and 15 percent from the target low flows. For $N_2 = 10$ years to $N_1 = 20$ years, MV1M, MV2M and MV3M generally performed best, achieving between 0 and 11 percent deviation from target low flows.

4.6.1.6 Mean Percentage Error of Generated Flows

Recall the mean percentage error, MPE, is the numerical mean of the percentage difference between individual generated and historical flow values, over the time series generated. The average MPE is the sum of the MPE's calculated over all trials conducted, divided by the number of trials conducted. Therefore, the average MPE indicates the central tendency or the expected value of MPE for the sample of experimental trials conducted.

MPE is not necessarily correspondingly positive or negative with the percent difference in mean flow. This may occur in the case where a model overestimates the low flows, but underestimates the high flows because the underestimated high flows may have more impact on the calculation of the mean than the low flows.

An example of the above is presented in Table 4.16 and Figure 4.12. Table 4.16 contains the historical and REG-generated flows for index station 05OD001 Roseau River Near Dominion City and base station 05OE001 Rat River Near Otterburne for the period from 1949 to 1950. The cross correlation coefficient was 0.852, and the extension and common periods were both equal to 5 years.

In Table 4.16, the mean flow generated by REG is smaller than the historical flows for 05OD001, which results in (5.48-5.97)*100/5.48 = - 8.9 percent deviation from the target mean (for this sample), as defined in Section 4.2.2. However, the MPE (for this sample) in Table 4.16 is 41%. Extending the above example to a complete series of flows (ie. multiple years) shows that the MPE may be positive when the percent deviation from the historical mean flow is negative.

Table 4.16 Example Comparison of Mean Flow to MPE

Year	REG (m^3/s)	05OD001 (m^3/s)	$\frac{REG - 05OD001}{05OD001} \times 100$
1949.00	0.74	0.50	48
1949.08	0.63	0.44	43
1949.17	0.58	0.41	43
1949.25	18.58	28.00	-34
1949.33	12.99	23.40	-45
1949.42	4.88	6.68	-27
1949.50	0.86	2.06	-58
1949.58	5.21	3.75	39
1949.67	1.15	0.52	119
1949.75	10.63	3.52	202
1949.83	11.50	6.05	90
1949.92	2.41	1.65	46
1950.00	1.11	0.66	68
Average	5.48	5.97	41

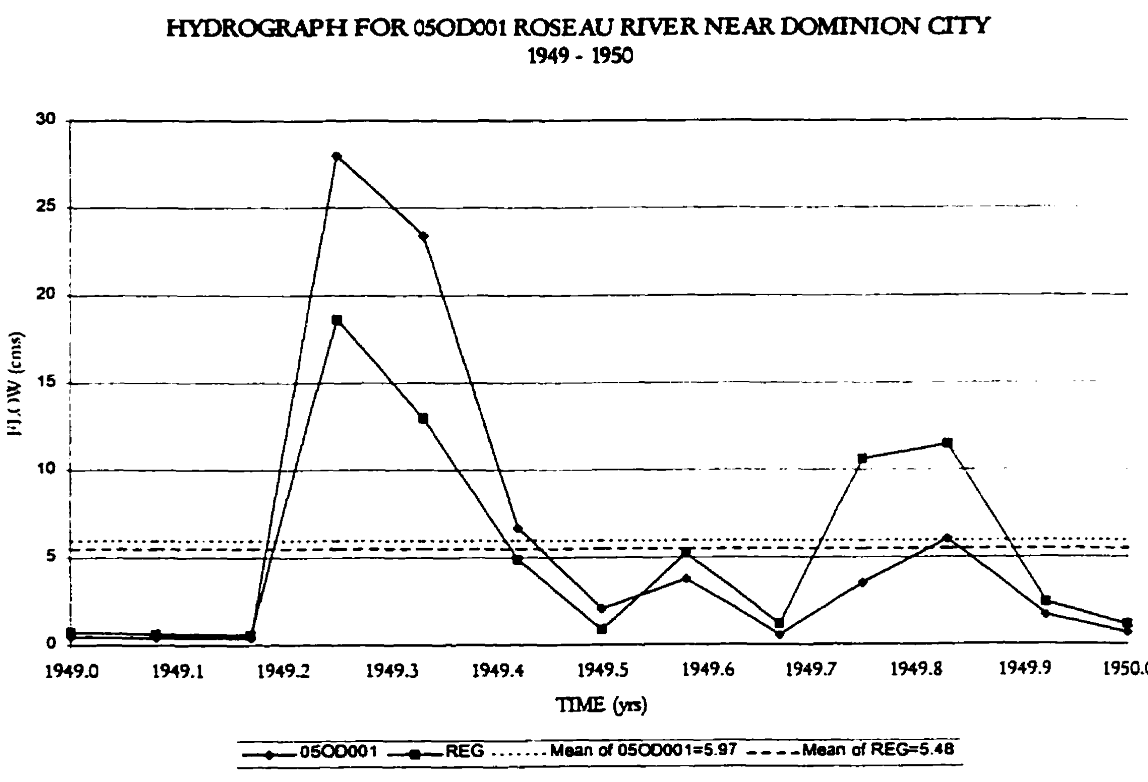

Figure 4.12 Hydrograph of 05OE001 and REG Flows for 1949 to 1950

$r_{xy} > 0.666$ - Tables C-11(a to d)

The average MPE of generated flows from the best models varied from 1 to 110 percent of historical flows, likely due to over-predicting low flows. The standard deviation of MPE varied from 3 to 149 percent, and decreased with greater N_2, indicating the high variability in results over the trials conducted, but an increase in model adequacy as more information is made available to the extension models. The MPE generally decreased as N_1 and N_2 increased, but the minimum average MPE did not necessarily occur at the highest value of N_2.

The minimum average MPE generally occurred for common periods of 45 years, 40 years, 35 years, and 30 years, for extension periods of 5 years, 10 years, 15 years, and 20 years, respectively. Note that each combination of total record period (N_1 + N_2) where the minimum average MPE generally occurs is equal to 50 years.

This phenomenon may have occurred strictly due to chance. However, increased common period and similar flow conditions in the extension and common period provide more information to the extension models for generating synthetic flows, and aids in model performance. Since the beginning of record for the stations used in the trials are not the same, this phenomenon may possibly be attributed to a time period corresponding to the occurrence of similar meteorological or physical factors affecting runoff.

For N_1 = 5 years, NP3B generally performed best for N_2 between 30 and 60 years, achieving between 10 and 85 MPE on average, followed by RPNM for N_2 between 60 and 75 years, achieving between 12 and 14 MPE. For N_1 = 10 years, MV1M performed best up to N_2 = 30 years, achieving between 29 and 65 average MPE, followed by REGM for N_2 greater than or equal to 35 years, achieving between 10 and 13 MPE on average. For N_1 = 15 years, various cyclic parametric models performed best for N_2 less than 35 years, followed by NP3B which achieved between 1 and 8 average MPE. Notably, the standard deviation of the MPE is relatively low, only between 3 and 7 percent, indicating good model performance in this case. For N_1 = 20 years, REGM generally performed best achieving between 11 and 29 average MPE, except for NP3B for N_2 between 30 and 35 years achieving between 8 and 11 MPE.

The results varied greatly throughout the trials. The standard deviation of MPE varied by up to 192 percent for $N_1 = 5$ years and $N_2 = 10$ years, decreasing to a minimum of 3 percent standard deviation for $N_1 = 15$ years and $N_2 = 65$ years.

$r_{xy} < 0.666$ - Tables C-12(a to d)

The average MPE of the best models varied from 11 to 125 percent of historical flows. MPE generally decreased as common period, N_2, increased. Again, the minimum average MPE did not necessarily occur at the highest value of N_2. However, there was no indication of a particular period of record which improved model performance consistently, which is reasonable since smaller cross correlation indicates the runoff response is dissimilar for the base and index record.

For $N_1 = 5$ years, various models performed best for N_2 up to 35 years, achieving 83 to 102 MPE on average, followed by REGM for N_2 greater than or equal to 35 years, achieving between 15 and 27 average MPE. For $N_1 = 10$ years, MV1M and MV3M performed best, achieving between 100 and 109 average MPE, followed by REGM and RPNM for N_2 greater than or equal to 25 years, achieving between 16 and 110 MPE, on average. For $N_1 = 15$ years, MV3M performed best up to $N_2 = 20$ years, achieving between 118 and 125 percent average MPE, followed by REGM, achieving 34 to 11 average MPE as N_2 increased from 25 to 65 years, respectively. For $N_1 = 20$ years, REGM performed best, achieving between 25 and 14 average MPE as N_2 increased from 20 to 60 years.

4.6.2 Objective Function Adequacy Measure

Recall OBJ_i^* represents the fraction of trials in which model i obtains the minimum value of OBJ_i, for a given combination of N_1, N_2, and r_{xy}. The model achieving the maximum OBJ_i^* more adequately reproduced the historical statistics, on average, over all the statistical adequacy measures presented previously.

$r_{xy} > 0.666$ - Tables C-13(a to d)

In general, REGM performed best up to approximately $N_2 = 40$ years, achieving between 20 and 40 percent OBJ_i^*. For common periods greater than 40 years, the cyclic and non-cyclic nonparametric models performed comparatively well. However, for $N_1 = 20$ years, REGM achieved between 40 and 100 percent OBJ_i^* for all N_2, except NPM which achieved 40 percent OBJ_i^* for $N_2 = 30$ years.

$r_{xy} < 0.666$ - Tables C-14(a to d)

REGM performed best for nearly all combinations of N_1 and N_2, achieving between 13 and 100 percent OBJ_i^*. Notably, the nonparametric techniques only performed well for $N_1 = 5$ years, with $N_2 = 5$, 55, 60, and 65 years, achieving between 13 and 67 percent OBJ_i^*. MV3M performed well at values of N_2 ranging from 70 to 75 years, for $N_1 = 5$, 15, and 20 years, in each case achieving 100 percent OBJ_i^*.

4.7 VERIFICATION TRIALS

The verification analysis was conducted for four separate sets of index and base records in order to form the basis for the conclusions and recommendations with regard to which of the models are most appropriate for a given combination of N_1, N_2 and r_{xy}. Two sets of base and index records were used for each of $r_{xy} > 0.666$ and $r_{xy} < 0.666$. The base and index records used for verification were not utilized in the evaluation, (ie. calibration) phase of the analysis presented in Section 4.6. The verification records were selected based upon providing a reasonable number of trials involving a wide range of extension and common periods.

The number of nearest neighbors used by the nonparametric methods for the verification trials are those developed in the evaluation phase, in Table 4.8 and Table 4.9, so that the model verification results are done with data separate from the evaluation results.

Descriptions of the verification trial sets are given in the following paragraphs. The fractional deviation from the various target statistical values, for each verification set are presented separately for each adequacy measure in Appendix D, and referenced below. The statistical adequacy measure results discussed in the following sections present the models achieving the minimum fractional deviation from the target values, f_{ij}, as described in Section 4.2.2.

The model(s) achieving the smallest fractional deviation from the target value, f_{ij}, are reported in a similar manner to the same in Section 4.6. In the verification phase there is no standard deviation associated with the calculated statistics, since the minimum deviation is associated with only a single trial. SSE_i is reported for the verification trials, but not as a fraction of the target (zero). Rather SSE_i is compared directly for the four verification trials.

Recall that in some instances, more than one model may achieve similar results. Therefore the tables in Appendix D may show more than one model achieving the minimum f_{ij} for a given N_1, N_2, and r_{xy} combination. If a particular model achieved minimum f_{ij} but failed either error diagnostic tests, an alternate model which did not fail the error diagnostic tests is reported. The alternate model selected is the model which had either equivalent or second lowest f_{ij} for that particular trial, which also did not fail the error diagnostics. Note that the alternate models are not necessarily nonparametric models.

4.7.1 Verification Trials, $r_{xy} > 0.666$ – Set 1

The base station used for the first set of verification trials is 04AC005 Gods River Below Allen Rapids, drainage area approximately 25,900 km^2. The corresponding index station is 04AC007 Island Lake River Near Island Lake, drainage area approximately 14,000 km^2. The Gods River and Island Lake River gauging stations were discontinued in 1994. The distance between the gauging stations on Gods River and Island Lake River is approximately 100 km. The relative locations are shown in Figure 4.2. The cross correlation, $r_{xy} = 0.731$. The total record period for both streams is 62 years, beginning in 1933. The wet season for Gods and Island Lake Rivers were defined in Section 4.4.2 as the months between June and November, inclusive. The corresponding dry season is defined as December to May.

Trials corresponding to $N_1 = 5$ years for this verification set were not successful due to missing flows in the base and index records. For the relatively small sample size of 5 years, missing data in the cyclic techniques produces some indeterminate results in the calculation of some parameters for the parametric models. However, this is not considered a major drawback since there are very few situations where extension of a streamflow record would be attempted when the extension and common periods are equal. For N_1 = 10 years, the common period varied from N_2 = 10 to 50 years, corresponding to 9 trials. For N_1 = 15 years, the common period varied from N_2 = 15 to 45 years, for 7 trials. For N_1 = 20 years, N_2 varied from 20 to 40 years, for an additional 5 trials. A total of 21 trials were conducted for this verification set.

The error diagnostics on the residual series are shown in Table 4.17. As in Section 4.5.2, these statistics are averaged over all the trials conducted for this verification set, (ie. for all combinations of N_1 and N_2, so the same considerations in terms of their interpretation apply in this case. Table 4.18 shows the fraction of trials in which the models fail the error diagnostic tests as described in Section 4.2.1.

Tables D-1(a to c) show the statistical adequacy measure results for cross correlation between the generated and historic flows. The latter for serial correlation, variance, mean flow, low flow, MPE, and SSE are shown in Tables D-2(a to c), Tables D-3(a to c), Tables D-4(a to c), Tables D-5(a to c), Tables D-6(a to c), and Tables D-7(a to c), respectively.

4.7.2 Verification Trials, $r_{xy} > 0.666$ - Set 2

The base station used for the second set of verification trials is 05PB014 Turtle River Near Mine Centre, drainage area approximately 4,870 km^2. The corresponding index station is 05QA002 English River at Umfreville, drainage area approximately 6,230 km^2. The gauging stations on the Turtle and English Rivers are separated by approximately 150 km. The relative locations are shown in Figure 4.3. The cross correlation, r_{xy} = 0.868. The total record period for 05PB014 is 75 years, commencing in 1921. The total record period for 05QA002 is 74 years, commencing in 1922. The wet season for the Turtle and English Rivers were defined in Section 4.4.2 as the months between May and July, inclusive. The corresponding dry season is defined as August to April. Both gauging stations are currently operational.

For $N_1 = 5$ years, the common period varied from 5 to 65 years, for 13 trials. For $N_1 = 10$ years, the common period varied from $N_2 = 10$ to 60 years, corresponding to 11 trials. For $N_1 = 15$ years, the common period varied from $N_2 = 15$ to 55 years, for 9 trials. For $N_1 = 20$ years, N_2 varied from 20 to 50 years, for an additional 7 trials. A total of 40 trials were conducted for this verification set.

The error diagnostics on the residual series are shown in Table 4.19. Table 4.20 shows the fraction of trials in which the models fail the error diagnostic tests. The statistical adequacy measure results for cross correlation, serial correlation, variance, mean flow, low flow, MPE, and SSE, are shown in Tables D-8(a to d), Tables D-9(a to d), Tables D-10(a to d), Tables D-11(a to d), Tables D-12(a to d), Tables D-13(a to d), and Tables D-14(a to d), respectively.

4.7.3 Verification Trials, $r_{xy} < 0.666$ - Set 3

The base station used for the third set of verification trials is 05SA002 Brokenhead River Near Beausejour, drainage area approximately 1,610 km^2. The corresponding index station is 05RA001 Manigotagan River Near Manigotagan, drainage area approximately 1,830 km^2. The gauging stations on the Brokenhead and Manigotagan Rivers are separated by approximately 120 km. The relative locations are shown in Figure 4.2. The cross correlation, $r_{xy} = 0.642$. The total record period for 05SA002 is 38 years, commencing in 1958. The total record period for 05RA001 is 36 years, commencing in 1960. The wet season for the Brokenhead River occurred between April and June, inclusive, with a corresponding dry season between July and March. The wet season for the Manigotagan River occurred between May and July, inclusive, with a corresponding dry season between August and April. Both gauging stations are currently operational.

Similar to verification set 1, results for $N_1 = 5$ years and $N_2 = 5$ years could not be obtained due to missing data in portions of the early record. However, results were obtained for N_2 from 10 to 30 years for 5 trials. For $N_1 = 10$ years, the common period varied from 10 to 25 years, for 4 trials. For $N_1 = 15$ years, the common period varied from $N_2 = 15$ to 20 years, for 2 trials. The lengths of the record precluded any trials involving extension periods greater than 15 years. A total of 11 trials were conducted for this verification set.

The error diagnostics on the residual series are shown in Table 4.21. Table 4.22 shows the fraction of trials in which the models fail the error diagnostic tests. The statistical adequacy measure results for cross correlation, serial correlation, variance, mean flow, low flow, MPE, and SSE, are shown in Tables D-15(a to c), Tables D-16(a to c), Tables D-17(a to c), Tables D-18(a to c), Tables D-19(a to c), Tables D-20(a to c), and Tables D-21(a to c), respectively.

4.7.4 Verification Trials, $r_{xy} < 0.666$ - Set 4

The base station used for the fourth set of verification trials is 05QA002 English River at Umfreville, drainage area approximately 6,230 km^2. The corresponding index station is 05QE009 Sturgeon River at Outlet of Salvensen Lake, drainage area approximately 1,530 km^2. The gauging stations on the English and Sturgeon Rivers are separated by approximately 220 km. The relative locations are shown in Figure 4.3. The cross correlation, r_{xy} = 0.626. The total record period for 05QA002 is 74 years, commencing in 1922. The total record period for 05QE009 is 31 years, commencing in 1965. The wet and dry seasons for the English River was defined previously. The wet season for the Sturgeon River occurred between May and July, inclusive, with a corresponding dry season between August and April. Both gauging stations are currently operational.

For N_1 = 5 years, the common period varied from 5 to 25 years, for 5 trials. For N_1 = 10 years, the common period varied from N_2 = 10 to 20 years, corresponding to 3 trials. For N_1 = 15 years, the only common period available due to the length of the index station was 15 years. A total of 9 trials were conducted for this verification set.

The error diagnostics on the residual series are shown in Table 4.23. Table 4.24 shows the fraction of trials in which the models fail the error diagnostic tests. The statistical adequacy measure results for cross correlation, serial correlation, variance, mean flow, low flow, MPE, and SSE are shown in Tables D-22(a to c), Tables D-23(a to c), Tables D-24(a to c), Tables D-25(a to c), Tables D-26(a to c), Tables D-27(a to c), and Tables D-28(a to c), respectively.

Table 4.17 Error Diagnostics on Residual Series, Verification Set 1, $r_{xy} > 0.666$

MODEL	-95%	SKEW	+95%	-95%	KURT	+95%	-95%	SER	+95%
REG	-0.41	0.37	0.41	2.18	3.55	3.82	-0.17	0.64	0.16
σ:	*0.04*	*0.04*	*0.04*	*0.09*	*0.21*	*0.09*	*0.02*	*0.03*	*0.02*
RPN	-0.41	0.29	0.41	2.18	3.25	3.82	-0.17	0.55	0.16
σ:	*0.04*	*0.04*	*0.04*	*0.09*	*0.16*	*0.09*	*0.02*	*0.02*	*0.02*
MV1	-0.41	0.54	0.41	2.18	3.08	3.82	-0.17	0.65	0.16
σ:	*0.04*	*0.07*	*0.04*	*0.09*	*0.12*	*0.09*	*0.02*	*0.04*	*0.02*
MV2	-0.41	0.54	0.41	2.18	3.04	3.82	-0.17	0.65	0.16
σ:	*0.04*	*0.07*	*0.04*	*0.09*	*0.13*	*0.09*	*0.02*	*0.03*	*0.02*
MV3	-0.41	0.54	0.41	2.18	2.91	3.82	-0.17	0.66	0.16
σ:	*0.04*	*0.06*	*0.04*	*0.09*	*0.19*	*0.09*	*0.02*	*0.03*	*0.02*
NP1A							-0.17	0.62	0.16
σ:							*0.02*	*0.06*	*0.02*
NP1B							-0.17	0.61	0.16
σ:							*0.02*	*0.07*	*0.02*
NP1C							-0.17	0.64	0.16
σ:							*0.02*	*0.06*	*0.02*
NP2A							-0.17	0.62	0.16
σ:							*0.02*	*0.06*	*0.02*
NP2B							-0.17	0.61	0.16
σ:							*0.02*	*0.07*	*0.02*
NP2C							-0.17	0.63	0.16
σ:							*0.02*	*0.06*	*0.02*
NP3A							-0.17	0.71	0.16
σ:							*0.02*	*0.07*	*0.02*
NP3B							-0.17	0.71	0.16
σ:							*0.02*	*0.06*	*0.02*
NP3C							-0.17	0.74	0.16
σ:							*0.02*	*0.04*	*0.02*
REGM	-0.41	0.21	0.41	2.18	5.28	3.82	-0.17	0.6	0.16
σ:	*0.04*	*0.20*	*0.04*	*0.09*	*0.63*	*0.09*	*0.02*	*0.07*	*0.02*
RPNM	-0.41	0.22	0.41	2.18	4.35	3.82	-0.17	0.43	0.16
σ:	*0.04*	*0.21*	*0.04*	*0.09*	*0.48*	*0.09*	*0.02*	*0.09*	*0.02*
MV1M	-0.41	0.53	0.41	2.18	5.35	3.82	-0.17	0.63	0.16
σ:	*0.04*	*0.20*	*0.04*	*0.09*	*0.58*	*0.09*	*0.02*	*0.05*	*0.02*
MV2M	-0.41	0.53	0.41	2.18	5.26	3.82	-0.17	0.64	0.16
σ:	*0.04*	*0.20*	*0.04*	*0.09*	*0.70*	*0.09*	*0.02*	*0.05*	*0.02*
MV3M	-0.41	0.58	0.41	2.18	4.99	3.82	-0.17	0.67	0.16
σ:	*0.04*	*0.16*	*0.04*	*0.09*	*0.68*	*0.09*	*0.02*	*0.03*	*0.02*
NPM							-0.17	0.56	0.16
σ:							*0.02*	*0.1*	*0.02*

Table 4.18 Fraction of Error Test Failures, Verification Set 1, $r_{xy} > 0.666$

MODEL	SKEW	KURT	SER
REG	0.24	0.00	1.00
RPN	0.00	0.00	1.00
MV1	1.00	0.00	1.00
MV2	1.00	0.00	1.00
MV3	1.00	0.00	1.00
NP1A			1.00
NP1B			1.00
NP1C			1.00
NP2A			1.00
NP2B			1.00
NP2C			1.00
NP3A			1.00
NP3B			1.00
NP3C			1.00
REGM	0.24	0.95	1.00
RPNM	0.05	0.86	1.00
MV1M	0.62	0.95	1.00
MV2M	0.57	0.95	1.00
MV3M	0.76	0.90	1.00
NPM			1.00

Table 4.19 Error Diagnostics on Residual Series, Verification Set 2, $r_{xy} > 0.666$

MODEL	-95%	SKEW	+95%	-95%	KURT	+95%	-95%	SER	+95%
REG	-0.46	-0.51	0.46	2.08	3.63	3.92	-0.20	0.47	0.18
σ:	*0.12*	*0.24*	*0.12*	*0.24*	*0.52*	*0.24*	*0.06*	*0.06*	*0.04*
RPN	-0.46	-0.17	0.46	2.08	3.24	3.92	-0.20	0.37	0.18
σ:	*0.12*	*0.12*	*0.12*	*0.24*	*0.33*	*0.24*	*0.06*	*0.09*	*0.04*
MV1	-0.46	-0.58	0.46	2.08	3.79	3.92	-0.20	0.46	0.18
σ:	*0.12*	*0.21*	*0.12*	*0.24*	*0.44*	*0.24*	*0.06*	*0.06*	*0.04*
MV2	-0.46	-0.58	0.46	2.08	3.79	3.92	-0.20	0.46	0.18
σ:	*0.12*	*0.21*	*0.12*	*0.24*	*0.43*	*0.24*	*0.06*	*0.06*	*0.04*
MV3	-0.46	-0.58	0.46	2.08	3.75	3.92	-0.20	0.47	0.18
σ:	*0.12*	*0.2*	*0.12*	*0.24*	*0.39*	*0.24*	*0.06*	*0.06*	*0.04*
NP1A							-0.20	0.58	0.18
σ:							*0.06*	*0.07*	*0.04*
NP1B							-0.20	0.58	0.18
σ:							*0.06*	*0.06*	*0.04*
NP1C							-0.20	0.58	0.18
σ:							*0.06*	*0.12*	*0.04*
NP2A							-0.20	0.57	0.18
σ:							*0.06*	*0.08*	*0.04*
NP2B							-0.20	0.57	0.18
σ:							*0.06*	*0.08*	*0.04*
NP2C							-0.20	0.52	0.18
σ:							*0.06*	*0.08*	*0.04*
NP3A							-0.20	0.67	0.18
σ:							*0.06*	*0.04*	*0.04*
NP3B							-0.20	0.65	0.18
σ:							*0.06*	*0.05*	*0.04*
NP3C							-0.20	0.64	0.18
σ:							*0.06*	*0.04*	*0.04*
REGM	-0.46	0.21	0.46	2.08	3.07	3.92	-0.20	0.54	0.18
σ:	*0.12*	*0.27*	*0.12*	*0.24*	*0.73*	*0.24*	*0.06*	*0.08*	*0.04*
RPNM	-0.46	0.19	0.46	2.08	3.07	3.92	-0.20	0.33	0.18
σ:	*0.12*	*0.22*	*0.12*	*0.24*	*0.58*	*0.24*	*0.06*	*0.09*	*0.04*
MV1M	-0.46	0.34	0.46	2.08	2.88	3.92	-0.20	0.54	0.18
σ:	*0.12*	*0.15*	*0.12*	*0.24*	*0.54*	*0.24*	*0.06*	*0.07*	*0.04*
MV2M	-0.46	0.27	0.46	2.08	2.95	3.92	-0.20	0.55	0.18
σ:	*0.12*	*0.22*	*0.12*	*0.24*	*0.67*	*0.24*	*0.06*	*0.08*	*0.04*
MV3M	-0.46	0.25	0.46	2.08	2.96	3.92	-0.20	0.57	0.18
σ:	*0.12*	*0.19*	*0.12*	*0.24*	*0.69*	*0.24*	*0.06*	*0.07*	*0.04*
NPM							-0.20	0.54	0.18
σ:							*0.06*	*0.07*	*0.04*

Table 4.20 Fraction of Error Test Failures, Verification Set 2, $r_{xy} > 0.666$

MODEL	SKEW	KURT	SER
REG	0.5	0.38	1
RPN	0	0	1
MV1	0.73	0.5	1
MV2	0.73	0.47	1
MV3	0.73	0.38	1
NP1A			1
NP1B			1
NP1C			1
NP2A			1
NP2B			1
NP2C			1
NP3A			1
NP3B			1
NP3C			1
REGM	0.17	0.17	1
RPNM	0.08	0.15	0.98
MV1M	0.43	0.15	1
MV2M	0.43	0.17	1
MV3M	0.3	0.17	1
NPM			1

Table 4.21 Error Diagnostics on Residual Series, Verification Set 3, $r_{xy} < 0.666$

MODEL	-95%	SKEW	+95%	-95%	KURT	+95%	-95%	SER	+95%
REG	-0.52	-1.14	0.52	1.96	4.28	4.04	-0.22	0.34	0.2
σ:	*0.11*	*0.22*	*0.11*	*0.22*	*1.13*	*0.22*	*0.05*	*0.01*	*0.04*
RPN	-0.52	-1.11	0.52	1.96	4.11	4.04	-0.22	0.28	0.2
σ:	*0.11*	*0.2*	*0.11*	*0.22*	*1.03*	*0.22*	*0.05*	*0.02*	*0.04*
MV1	-0.52	-0.97	0.52	1.96	3.68	4.04	-0.22	0.18	0.2
σ:	*0.11*	*0.17*	*0.11*	*0.22*	*0.78*	*0.22*	*0.05*	*0.03*	*0.04*
MV2	-0.52	-0.99	0.52	1.96	3.74	4.04	-0.22	0.20	0.2
σ:	*0.11*	*0.18*	*0.11*	*0.22*	*0.82*	*0.22*	*0.05*	*0.01*	*0.04*
MV3	-0.52	-1.02	0.52	1.96	3.8	4.04	-0.22	0.22	0.2
σ:	*0.11*	*0.17*	*0.11*	*0.22*	*0.82*	*0.22*	*0.05*	*0.01*	*0.04*
NP1A							-0.22	0.44	0.2
σ:							*0.05*	*0.03*	*0.04*
NP1B							-0.22	0.44	0.2
σ:							*0.05*	*0.03*	*0.04*
NP1C							-0.22	0.45	0.2
σ:							*0.05*	*0.03*	*0.04*
NP2A							-0.22	0.44	0.2
σ:							*0.05*	*0.03*	*0.04*
NP2B							-0.22	0.45	0.2
σ:							*0.05*	*0.03*	*0.04*
NP2C							-0.22	0.45	0.2
σ:							*0.05*	*0.02*	*0.04*
NP3A							-0.22	0.49	0.2
σ:							*0.05*	*0.02*	*0.04*
NP3B							-0.22	0.50	0.2
σ:							*0.05*	*0.02*	*0.04*
NP3C							-0.22	0.49	0.2
σ:							*0.05*	*0.03*	*0.04*
REGM	-0.52	-1.44	0.52	1.96	5.44	4.04	-0.22	0.46	0.2
σ:	*0.11*	*0.28*	*0.11*	*0.22*	*1.57*	*0.22*	*0.05*	*0.02*	*0.04*
RPNM	-0.52	-1.3	0.52	1.96	5.15	4.04	-0.22	0.35	0.2
σ:	*0.11*	*0.32*	*0.11*	*0.22*	*1.47*	*0.22*	*0.05*	*0.06*	*0.04*
MV1M	-0.52	-0.97	0.52	1.96	4.17	4.04	-0.22	0.40	0.2
σ:	*0.11*	*0.22*	*0.11*	*0.22*	*1.01*	*0.22*	*0.05*	*0.02*	*0.04*
MV2M	-0.52	-1.09	0.52	1.96	4.55	4.04	-0.22	0.36	0.2
σ:	*0.11*	*0.17*	*0.11*	*0.22*	*1.05*	*0.22*	*0.05*	*0.07*	*0.04*
MV3M	-0.52	-1.17	0.52	1.96	4.99	4.04	-0.22	0.41	0.2
σ:	*0.11*	*0.14*	*0.11*	*0.22*	*0.92*	*0.22*	*0.05*	*0.03*	*0.04*
NPM							-0.22	0.48	0.2
σ:							*0.05*	*0.03*	*0.04*

Table 4.22 Fraction of Error Test Failures, Verification Set 3, $r_{xy} < 0.666$

MODEL	SKEW	KURT	SER
REG	1	0.55	1
RPN	1	0.55	1
MV1	1	0.55	0.18
MV2	1	0.55	0.55
MV3	1	0.55	0.64
NP1A			1
NP1B			1
NP1C			1
NP2A			1
NP2B			1
NP2C			1
NP3A			1
NP3B			1
NP3C			1
REGM	1	0.55	1
RPNM	1	0.64	1
MV1M	0.91	0.55	1
MV2M	1	0.55	1
MV3M	1	0.64	1
NPM			1

Table 4.23 Error Diagnostics on Residual Series, Verification Set 4, $r_{xy} < 0.666$

MODEL	-95%	SKEW	+95%	-95%	KURT	+95%	-95%	SER	+95%
REG	-0.53	0.35	0.53	1.94	3.89	4.06	-0.23	0.46	0.2
σ:	*0.11*	*0.33*	*0.11*	*0.22*	*0.49*	*0.22*	*0.05*	*0.04*	*0.04*
RPN	-0.53	0.11	0.53	1.94	3.04	4.06	-0.23	0.32	0.2
σ:	*0.11*	*0.15*	*0.11*	*0.22*	*0.49*	*0.22*	*0.05*	*0.14*	*0.04*
MV1	-0.53	-0.27	0.53	1.94	3.53	4.06	-0.23	0.50	0.2
σ:	*0.11*	*0.24*	*0.11*	*0.22*	*1.12*	*0.22*	*0.05*	*0.02*	*0.04*
MV2	-0.53	-0.24	0.53	1.94	3.57	4.06	-0.23	0.49	0.2
σ:	*0.11*	*0.24*	*0.11*	*0.22*	*1.12*	*0.22*	*0.05*	*0.02*	*0.04*
MV3	-0.53	-0.23	0.53	1.94	3.57	4.06	-0.23	0.48	0.2
σ:	*0.11*	*0.27*	*0.11*	*0.22*	*1.08*	*0.22*	*0.05*	*0.02*	*0.04*
NP1A							-0.23	0.35	0.2
σ:							*0.05*	*0.19*	*0.04*
NP1B							-0.23	0.36	0.2
σ:							*0.05*	*0.18*	*0.04*
NP1C							-0.23	0.34	0.2
σ:							*0.05*	*0.19*	*0.04*
NP2A							-0.23	0.35	0.2
σ:							*0.05*	*0.19*	*0.04*
NP2B							-0.23	0.35	0.2
σ:							*0.05*	*0.20*	*0.04*
NP2C							-0.23	0.34	0.2
σ:							*0.05*	*0.19*	*0.04*
NP3A							-0.23	0.38	0.2
σ:							*0.05*	*0.16*	*0.04*
NP3B							-0.23	0.37	0.2
σ:							*0.05*	*0.18*	*0.04*
NP3C							-0.23	0.33	0.2
σ:							*0.05*	*0.21*	*0.04*
REGM	-0.53	0.16	0.53	1.94	3.89	4.06	-0.23	0.39	0.2
σ:	*0.11*	*0.53*	*0.11*	*0.22*	*0.45*	*0.22*	*0.05*	*0.06*	*0.04*
RPNM	-0.53	0.08	0.53	1.94	3.68	4.06	-0.23	0.32	0.2
σ:	*0.11*	*0.48*	*0.11*	*0.22*	*0.35*	*0.22*	*0.05*	*0.07*	*0.04*
MV1M	-0.53	-0.26	0.53	1.94	3.89	4.06	-0.23	0.45	0.2
σ:	*0.11*	*0.39*	*0.11*	*0.22*	*1.49*	*0.22*	*0.05*	*0.14*	*0.04*
MV2M	-0.53	-0.18	0.53	1.94	3.54	4.06	-0.23	0.51	0.2
σ:	*0.11*	*0.32*	*0.11*	*0.22*	*0.83*	*0.22*	*0.05*	*0.10*	*0.04*
MV3M	-0.53	-0.02	0.53	1.94	3.37	4.06	-0.23	0.46	0.2
σ:	*0.11*	*0.25*	*0.11*	*0.22*	*0.39*	*0.22*	*0.05*	*0.09*	*0.04*
NPM							-0.23	0.37	0.2
σ:							*0.05*	*0.15*	*0.04*

Table 4.24 Fraction of Error Test Failures, Verification Set 4, $r_{xy} < 0.666$

MODEL	SKEW	KURT	SER
REG	0.33	0.33	1
RPN	0	0	0.56
MV1	0.44	0.44	1
MV2	0.44	0.44	1
MV3	0.44	0.44	1
NP1A			0.56
NP1B			0.56
NP1C			0.44
NP2A			0.56
NP2B			0.56
NP2C			0.56
NP3A			0.67
NP3B			0.67
NP3C			0.44
REGM	0.44	0.33	1
RPNM	0.33	0.11	1
MV1M	0.22	0.44	0.89
MV2M	0.22	0.33	1
MV3M	0	0.11	1
NPM			0.78

4.8 DISCUSSION OF VERIFICATION TRIALS

Model adequacy in terms of the statistical adequacy measures is discussed and compared to the evaluation results in the following paragraphs.

4.8.1 Optimum Number of Nearest Neighbors

The verification trials are performed using the number of nearest neighbors selected from the evaluation trials in order to provide a fair comparison with all the other methods.

4.8.2 Diagnostics on Residual Series

The results of the verification trials show that the skewness, kurtosis and serial correlation of the residual series cannot be expected to mirror the average values from the evaluation trials, but does confirm the high variability in the results for the evaluation trials, as indicated by the large variance of the error diagnostic statistics shown in Tables 4.18 and 4.24. A detailed comparison of the results from the verification trials and evaluation trials is given below.

r_{xy} > 0.666 – Verification Set 1 and Set 2

Table 4.17 and Table 4.19 show the average skewness coefficient of the residual series for verification sets 1 and 2, respectively, are higher than for the evaluation trials in Table 4.13. The fraction of skewness test failures for verification sets 1 and 2 were higher than the evaluation trials. With the exception of RPN and RPNM, the parametric models failed between 24 and 100 percent of the skewness tests for the verification trials. However, the null hypothesis that the skewness was not different from zero was accepted for all RPN residual series, with 95% confidence in verification sets 1 and 2.

The noncyclic parametric models failed no kurtosis tests on the residuals series for set 1, however, the cyclic parametric models failed between 86 and 95 percent of the kurtosis tests, between 40 and 50 percent higher than the evaluation trials. In contrast to verification set 1, the noncyclic parametric models failed a greater portion of kurtosis tests, and the cyclic parametric models failed a smaller portion of kurtosis tests than the evaluation trials. This may be due to the nature of the streamflow files used in the extension. The variability of the kurtosis coefficients obtained was much lower for sets 1 and 2, than for the evaluation trials.

Therefore, a residual series from a parametric model may pass the skewness test and fail the kurtosis test or vice verse. The only model which passed both skewness and kurtosis tests in verification sets 1 and 2 was RPN.

The average serial correlation of residuals was significantly higher for all models. The variability of the r_ϵ was less for the verification trials than the evaluation trials. Accordingly, nearly all trials conducted for verification sets 1 and 2 produced residual series which displayed high serial correlation. Essentially 100 percent of all trials and all models failed the serial correlation test in verification sets 1 and 2.

The error diagnostic analysis on the verification trials for $r_{xy} > 0.666$ show that the noncyclic parametric models may be expected to fail over 50 percent of the skewness tests, less than 50 percent of the kurtosis tests, and also a large portion of the serial correlation tests. Less variability in the kurtosis coefficients was obtained in the verification trials than the evaluation trials. However, the fraction of test failures between set 1 and set 2 was quite different. The cyclic parametric models would be expected to fail up to approximately 50 percent of the skewness tests, and again, the expected kurtosis results are uncertain. Two exceptions to the above are RPN and RPNM, which would be expected to fail a significantly smaller portion of the tests for normality of error terms. All models were found to fail the serial correlation tests on the residual series, indicating general model inadequacy in taking into account the time dependent variation between the index and base record flows.

$r_{xy} < 0.666$ - Verification Set 3 and Set 4

Table 4.21 shows large negative skewness of the residual series obtained for set 3, in contrast to the positive skewness coefficients obtained for the evaluation trials shown in Table 4.13. Notably Table 4.22 shows a very significant fraction (between 91 and 100 percent) of the trials failed the residual skewness coefficient test, in comparison to between 20 and 45 percent failure for the evaluation trials.

Table 4.23 shows a combination of negative and positive skewness coefficients for set 4. From Table 4.24 with the exception of RPN and MV3M, the parametric models failed between 22 and 44 percent of the skewness tests. RPN and MV3M failed none. The variability of the skewness coefficients was less for sets 3 and 4 than the evaluation trials.

While the percentage of models failing the kurtosis tests for set 3 in Table 4.25 was much higher, at 55 to 64 percent of the verification trials conducted, that for set 4 in Table 4.28 was approximately equivalent to the calibration trials, at 11 to 44 percent. Note again the exception of RPN which failed no kurtosis tests, but only in verification set 4.

Overall, the average r_e obtained in sets 3 and 4 were slightly higher than the evaluation trials. In set 3, nearly all models failed between 55 and 100 percent of the serial correlation tests, except MV1 which failed only 18 percent. In set 4, however, the parametric models failed between 56 and 100 percent of the tests, while the noncyclic nonparametric performance was better, failing between 44 and 78 percent of the tests. The latter indicates that for smaller values of cross correlation between the base and index record, noncyclic parametric models may produce a residual series with lower serial correlation.

The error diagnostic analysis on the verification trials for $r_{xy} < 0.666$ show that the noncyclic parametric models may be again expected to fail over 50 percent of the skewness tests, approximately 50 percent of the kurtosis tests, and nearly all serial correlation tests. Again, less variability in the kurtosis coefficients was obtained. The cyclic parametric models would be expected to fail over 50 percent of the skewness tests, and marginally less than 50 percent of the kurtosis tests. RPN passed the skewness and kurtosis tests in set 4, and MV3M passed the skewness tests in set 4. Again, nearly all models were found to fail the serial correlation tests on the residual series, indicating general model inadequacy in taking into account the time dependent variation between the index and base record flows.

4.8.3 Statistical Adequacy Measures

4.8.3.1 Cross Correlation Between Generated and Historical Flows

$r_{xy} > 0.666$ – Verification Set 1 - Tables D-1(a to c) and Set 2 - Tables D-8(a to d)

The range of cross correlation between the generated and historical flows for the best models in set 1 varied between 0.86 and 0.94. The cross correlation does not vary appreciably with extension period, but tends to increase as N_2 increases. The model performance is generally consistent with the evaluation results. However, the models which performed best for the verification trials were not necessarily the same models which performed best, on the average of the evaluation trials.

Verification sets 1 and 2 provide many instances where the noncyclic nonparametric models are suggested as alternatives to the cyclic parametric models. In Tables D-1a to D-1c, for N_1 = 10, 15, and 20 years, respectively, noncyclic nonparametric models either performed best or were proposed as alternates to cyclic parametric models and performed within 7 percentage points. In Tables D-8a to D-8c, for N_1 = 5, 10, and 15 years, respectively, the cyclic parametric models performed best. However, for N_1 =20 years, noncyclic nonparametric models were suggested as alternates to the parametric models, as shown in Table D-8d.

r_{xy} < 0.666 - Verification Set 3 - Tables D-15(a to c) and Set 4 - Tables D-22(a to c)

The range of cross correlation for sets 3 and 4 varied from 0.75 to 0.89. The model performance was marginally better than the evaluation trials for set 3, and equivalent for set 4. Again, the cross correlation did not vary appreciably with extension period, but did not always decrease as N_2 increased. The noncyclic nonparametric models seemed to perform well, or nearly as well as the parametric models. Nonparametric models suggested as alternatives to parametric models which failed error diagnostics performed within 7 percentage points, as shown in Tables D-15a, D-15-c. However, in many instances the nonparametric models achieved the best cross correlation, as shown in Tables D-15b, and D-22(a to c).

4.8.3.2 Serial Correlation of Generated Flows

r_{xy} > 0.666 - Verification Set 1 - Tables D-2(a to c) and Set 2 - Tables D-9(a to d)

The best models deviated between -6 and 3 percent of the historical target value. The verification results generally agree with the evaluation results.

Overall, the noncyclic nonparametric techniques performed best up to N_2 = 20 years, after which the cyclic MOVE techniques performed better. Nonparametric models suggested as alternatives to parametric models which failed error diagnostics performed within 5 percentage points.

r_{xy} < 0.666 - Verification Set 3 - Tables D-16(a to c) and Set 4 - Tables D-23(a to c)

The deviation from target serial correlation varied between -3 and 1 percent, showing marginally better performance than the evaluation trials which varied up to 8 percent. For these trials, the nonparametric methods generally performed best, or were suggested as alternates within one or two percentage points of the rejected model. Model performance in this regard increased as N_2 increased. Nonparametric models suggested as alternatives to parametric models which failed error diagnostics performed within 1 percentage point.

4.8.3.3 Variance of Generated Flows

$r_{xy} > 0.666$ – Verification Set 1 - Tables D-3(a to c) and Set 2 - Tables D-10(a to d)

The best models achieved between -19 and 8 percent of the target variance, again, roughly corresponding to the range of results from the evaluation trials. In Tables D-3a to D-3c, the nonparametric models are suggested as alternates, achieving within 10 percentage points deviation. For $N_1 = 5$ and 10 in Tables D-10a to D-10b, NP2A and NP3C perform well, and are suggested as alternates for $N_1 = 15$ and 20 in Tables D-10c and D-10d. Model performance again increased as more common period was available. Nonparametric models suggested as alternatives to parametric models which failed error diagnostics generally performed within 21 percentage points.

$r_{xy} < 0.666$ – Verification Set 3 - Tables D-16(a to c) and Set 4 - Tables D-24(a to c)

The best models achieved between -74 to 6 percent of the target variance, displaying much worse performance than the evaluation trials. However, the results are comparable because of the high variability in results which was shown in the evaluation trials. The cyclic parametric models performed best in set 3 for $N_1 = 10$ to 15 years, in Tables D-24b and D-24c. Nonparametric models suggested as alternatives to parametric models which failed error diagnostics generally performed within 73 percentage points. The above indicates generally poor model performance with small r_{xy}.

4.8.3.4 Mean of Generated Flows

$r_{xy} > 0.666$ – Verification Set 1 - Tables D-4(a to c) and Set 2 - Tables D-11(a to d)

Mean flow was reproduced quite well, with the best models achieving between -3 and 9 percent deviation of the historical target. These results roughly agree with the evaluation trials. For N_1 =5 to 15 years, the noncyclic nonparametric models performed best, in some cases, being suggested as alternates to parametric models. No change in model performance was noted with changes in either N_1 or N_2.

Nonparametric models suggested as alternatives to parametric models which failed error diagnostics generally performed within 1 percentage point.

r_{xy} < 0.666 – Verification Set 3 - Tables D-17(a to c) and Set 4 - Tables D-25(a to c)

The models achieved between –24 to 5 percent deviation from the historical mean flow, slightly underestimating mean flow in comparison to the evaluation trials. NP2C generally performed well. Noncyclic nonparametric models suggested as alternates were within 4 percentage points.

4.8.3.5 Generated Low Flows

r_{xy} > 0.666 – Verification Set 1 - Tables D-5(a to c) and Set 2 - Tables D-12(a to d)

The models achieved between -3 and 34 percent deviation from the historical low flow, (which is expected to be equaled or exceeded 80 percent of the time). This is approximately equal to the results from the evaluation trials. Generally the parametric models reproduced the low flow best for shorter common periods, and the noncyclic parametric models performed better for longer common periods. Where nonparametric models were suggested as alternates, they fell within 14 percentage points of the corresponding parametric model which failed the error diagnostics.

r_{xy} < 0.666 – Verification Set 3 - Tables D-18(a to c) and Set 4 - Tables D-26(a to c)

For set 3, the models performed quite poorly, with between -1 and 174 percent deviation from the historical low flow, but for set 4, the results were much worse than the evaluation trials at between –3 and 17 percent deviation. For N_1 =5 years, in Table D-26a, the cyclic nonparametric models performed well. For N_1 =10 years, RPN performed best, reproducing low flows equivalent to the historical target. For N_1 =15 years, NP3B performed best. Alternate nonparametric models did not reproduce low flows quite as well, varying up to 158 percentage points different than the parametric counterpart.

4.8.3.6 Mean Percentage Error of Generated Flows

$r_{xy} > 0.666$ – Verification Set 1 - Tables D-6(a to c) and Set 2 - Tables D-13(a to d)

The models achieved between −2.1 and 28.4 MPE, indicating far better performance than the evaluation trials. MPE did not necessarily increase or decrease with N_1 or N_2 in the verification trials. In contrast, MPE had consistently decreased with both N_1 and N_2 in the evaluation trials, as shown in Tables C-11a to C-11d.

Where nonparametric models were suggested as alternates, they fell within 3 percentage points of the corresponding parametric model which failed the error diagnostics.

$r_{xy} < 0.666$ – Verification Set 3 - Tables D-19(a to c) and Set 4 - Tables D-28(a to c)

In this case, the models achieved extremely high MPE varying between −1.2 and 362 percent. Note, however that the variability of the results was shown to be quite high for the evaluation trials as well (Tables C-12a to C-12d). The verification trials seem to confirm this. Again, in contrast to the evaluation trials, MPE showed no tendency to decrease with either N_1 or N_2. The MPE was highest when the extension and common periods were similar.

4.8.3.7 SSE of Generated Flows

$r_{xy} > 0.666$ – Verification Set 1 - Tables D-7(a to c) and Set 2 - Tables D-14(a to d)

For N_1 =5 years, REGM generally obtained minimum SSE for set 2. For N_1 = 10 years, NP1A obtained minimum SSE for set 1 and REG and REGM obtained minimum SSE for set 2. For N_1 =15 years, NP3A performed best, for set 1 and REGM and NPM for set 2. For N_1 =20 years, REGM performed best for N_2 up to 25 years, then NP2B for N_2 =30 to 40 years, for set 1. For $N_1 = 20$ REGM again generally performed best. Generally, SSE decreased as common period increased. It is obvious that SSE increases as extension period increases.

Where nonparametric models were suggested as alternates, they produced SSE which ranged up to 40 percent higher than the parametric models which failed the error diagnostics.

$r_{xy} < 0.666$ – Verification Set 3 - Tables D-21(a to c) and Set 4 - Tables D-28(a to c)

For sets 3 and 4, NP2C either achieved minimum SSE, or provided alternate minimum SSE for many cases of N_1 and N_2. This indicates that for small r_{xy}, the noncyclic nonparametric models generate flows with a small variance in error terms, indicating good model performance. Again, SSE generally decreased with increasing N_2.

Where nonparametric models were suggested as alternates, they produced SSE up to 63 percent higher than the parametric model which failed the error diagnostics.

4.9 SUMMARY OF EVALUATION AND VERIFICATION RESULTS

Table 4.25 summarizes and compares the results of the evaluation and verification trials for $r_{xy} > 0.666$. Table 4.26 summarizes and compares the results of the evaluation and verification trials for $r_{xy} < 0.666$. The results of serial correlation, variance, mean flow, and low flow, are expressed in terms of fractional deviation from the target historical value, as was done previously. The fraction deviations shown for the evaluation trials were averaged over all evaluation trials, ($\bar{f}_{ij}$), while the values from the verification results apply to a single trial, (f_{ij}). SSE was not reported for evaluation trials because comparison of such a large number of SSE values was not deemed useful.

Comparison of Table 4.25 and Table 4.26 shows that the performance of the models decreases substantially when r_{xy} decreases. However, similar behavior such as increased model performance with increased available common period holds true. The verification results generally confirm the range of the best model performance one may expect to encounter, as previously found with the evaluation results, in the context of the streamflow stations used in the present study. Note, however, that the high variability in the results which may be obtained was also verified. As such, the values in Table 4.31 and Table 4.32 represent an approximate range of performance one may expect in using the extension models, however the results for some of the adequacy measures are associated with a high degree of variability.

The evaluation trials showed that inputting the number of nearest neighbors which the nonparametrics used in the verification trials, rather than allowing the nonparametric methods to optimize for the number nearest neighbors decreases the nonparametric model performance marginally.

The models performing best in the verification trials are not necessarily the same models which perform best in the evaluation trials. The verification results show there are alternative nonparametric models that perform comparably to the parametric models which failed the error diagnostic tests. In many cases, the alternate models are nonparametric. There are also many cases where the nonparametric models performed best.

In the case where only one statistical measure is used to determine the best model, it would be straightforward to compare a table of SSE values, then choose the model which yielded the minimum SSE. However, in many cases, other statistical features are of interest, which is the purpose of presenting the many adequacy statistics in this study. Section 4.10 provides a methodology and recommendation for selecting the best model taking into account all the adequacy measures which were described in Section 4.8.

Table 4.25 Summary of Evaluation and Verification Results, $r_{xy} > 0.666$

Adequacy Measure	Range of $\bar{f}_{ij}$ from Evaluation Trials	Range of f_{ij} from Verification Trials	Comment
Cross Correlation	0.84 – 0.92	0.86 – 0.94	Better model performance as N_2 increased. Cyclic parametric models performed well. Alternate nonparametric models within 6 percentage points of parametric.
Serial Correlation	-0.05 – 0.02	-0.06 – 0.03	Model performance does not vary with N_1 or N_2. Noncyclic nonparametric and cyclic MOVE procedures performed well. Alternate nonparametric models within 5 percentage points of parametric.
Variance	-0.22 – 0.09	-0.19 – 0.08	Better model performance as N_2 increased. High variability in evaluation results. Cyclic MOVE models performed well. Nonparametric models performed better as N_2 increased. Alternate nonparametric models within 15 percentage points of parametric.
Mean Flow	-0.01 – 0.02	-0.03 – 0.09	Model performance did not change with N_1 or N_2. Low variability in results. REG, RPN and noncyclic nonparametric models performed well. Alternate nonparametric models within 1 percentage point of parametric.
Low Flow	-0.01 – 0.31	-0.03 – 0.34	High variability in results. Results better as N_2 increased for evaluation trials, but increased and decreased in verification trials. Alternate nonparametric models within 14 percentage points of parametric.
MPE	-0.9 – 110.1	-2.1 – 28.4	High variability in results. Alternate nonparametric models within 3 percentage points of parametric.
SSE	n/a	n/a	NP1A, REG, and REGM performed well. SSE decreased as N_2 increased. Alternate nonparametric models up to 40% higher SSE.

Table 4.26 Summary of Evaluation and Verification Results, $r_{xy} < 0.666$

Adequacy Measure	Range of $\bar{f}_{ij}$ from Evaluation Trials	Range of f_{ij} from Verification Trials	Comment
Cross Correlation	0.74 – 0.89	0.75 – 0.89	Better model performance as N_2 increased. Noncyclic nonparametric models performed well. Alternate nonparametric models within 7 percentage points of parametric.
Serial Correlation	-0.06 – 0.08	-0.03 – 0.01	Model performance does not vary with N_1 or N_2. High variability in results. Noncyclic nonparametric procedures performed well as N_2 increased. Alternate nonparametric models within 1 percentage point of parametric.
Variance	-0.05 – 0.13	-0.74 – 0.06	Better model performance as N_2 increased. High variability in results. Cyclic MOVE models performed well. No change in model performance with either N_1 or N_2. Alternate nonparametric models did not perform well.
Mean Flow	-0.02 – 0.07	-0.24 – 0.05	Model performance did not change with N_1 or N_2. High variability in results. MOVE, and noncyclic nonparametric models performed well. Alternate nonparametric models within 11 percentage points of parametric.
Low Flow	-0.03 – 0.17	-0.01 – 1.70	High variability in results. Results better as N_2 increased for evaluation trials, but increased and decreased in verification trials. Models over-predict low flows. Alternate nonparametric models within 158 percentage points of parametric.
MPE	11.7 - 125.06	-1.2 - 362.	High variability in results, as above. Highest MPE when N_1 and N_2 are similar. Alternate nonparametric models within 267 percent of parametric
SSE	n/a	n/a	NP2C and REGM performed well. SSE decreased as N_2 increased. Alternate nonparametric models up to 63% higher SSE.

4.10 RECOMMENDED MODELS

Section 4.8 provides a detailed account of the best model performance separately for each statistical adequacy measure. The purpose of this section is to determine which model achieved the minimum total deviation from the target statistical adequacy measures, in relation to the performance of the best and worst models, while at the same time not failing the error diagnostics tests for nonnormality of residuals. The recommended models in a given case of extension period, N_1, common period, N_2, and r_{xy} are those models achieving the minimum value of OBJ_i in the verification trials. In this section the minimum value of OBJ_i is referred to as min(OBJ_i) for discussion purposes.

The following procedure was used to arrive at the recommendations. The models which obtained min(OBJ_i), and at the same time did not fail the error diagnostic tests for skewness and kurtosis, is determined. Again, the number of nearest neighbors used by the nonparametric models follows those determined by the evaluation trials. The results from the verification trials showed that nearly all models failed the serial correlation test on residuals, indicating all models were generally inadequate in taking into account the time varying relationship between the base and index record. Since the performance of all models in that regard are equivalently poor, the models were not censored due to serial correlation of residuals in the verification trials.

For the verification trials conducted, the same model did not obtain min(OBJ_i) in both sets of verification trials. It is reasonable that two sets of verification sets (for each case of r_{xy}) would suggest different models, because the data used in each verification set is different. The characteristics of one technique may lend itself as the most appropriate depending on different features of the data. For instance, in the case of the nonparametric techniques, if the flows tend to rise or fall sharply on the hydrograph, the first difference feature vectors may provide an edge, or if the scale differences between monthly flows interferes with the pattern recognition, the standardized feature vectors may be more useful. The investigation into these types of questions is suggested for further research. The purpose of the present study was to develop the alternate techniques and evaluate them in comparison to the currently available parametric techniques.

The addition of OBJ_i from two trials equates to the cumulative deviation from all the adequacy statistics from both sets of trials. In this way, both verification trials may be utilized to determine the overall best model. Calculating the objective function value using both verification trials essentially amounts to adding up all the deviations from the adequacy statistics twice for each model (once for each verification set). Note, however, that the value of OBJ_i cannot be compared to the value of OBJ_i from another trial to determine the best model, only the additive property is relevant. One could not, for instance say that model i performed better in trial p, than in trial q, because $OBJ_i(p) < OBJ_i(q)$.

Thus, the OBJ_i values for both verification sets were added together to provide a cumulative indication of the model adequacy taking into account all the adequacy measures for both verification trials. The recommended model is the model which achived the minimum value of the sum of objective functions from the respective evaluation trials. Recall that verification sets 1 and 2 apply to $r_{xy} > 0.666$, and verification sets 3 and 4 apply to $r_{xy} < 0.666$. If we let $OBJ_i(s)$ = the value of OBJ_i obtained for model i in verification set s, the recommended model for $r_{xy} > 0.666$ is the model which achieved $\min[OBJ_i(1) + OBJ_i(2)]$. The recommended model for $r_{xy} < 0.666$ is the model which achieved $\min[OBJ_i(3) + OBJ_i(4)]$.

A shortcoming of the RPN and RPNM techniques is that there is no single unique record obtained from these techniques, rather a whole sequence of different records may be obtained because of the random error component. Since the purpose of streamflow record extension is to establish a consistent database of flow records, this approach is not recommended. Therefore, RPN and RPNM are rejected when they happen to achieve minimum OBJ_i, in the verification trials, in favor of the next best model which does not fail the error diagnostics explained previously.

4.10.1 $r_{xy} > 0.666$

The models achieving the minimum objective function value combined for verification sets 1 and 2, min[$OBJ_i(1)+ OBJ_i(2)$], for each combination of N_1 and N_2 for r_{xy} >0.666 are shown in Table 4.27. The models shown in Table 4.27 are recommended to use for streamflow record extension for the respective extension period, common period, and cross correlation between the base and index record greater than 0.666. Note that verification set 1 unfortunately did not include N_1 =5 years due to missing data. All recommended nonparametric models, except NP1C for N_1 = 5 years, replaced other parametric models which achieved lower OBJ_i, but failed the error diagnostics.

Figure 4.13 shows the historic, NP1C, and REG generated monthly flows for English River at Umfreville. The extension and common periods are 5 and 10 years, respectively. NP1C is the recommended model, and REG is plotted for comparison. Neither NP1C nor REG tended to consistently underestimate or overestimate. NP1C and REG underestimate and overestimate high flows in the same years. Note the beginning and end months of the generated record are missing for the noncyclic nonparametric models because the dry season extends into the previous year, thus a feature vector could not be determined for that year. The gaps in the hydrographs correspond to missing data.

In Figure 4.13, the peak and low flows seem to correspond fairly well. Neither model reproduced the dry year in 1924, where the peak flow reached only 53 m^3/s. NP1C reproduced the peak flow in 1925 quite well.

An interesting feature of the nonparametric models is seen for the period between approximately September and February 1924 in Figure 4.13. NP1C generated a small rise in the flow during this period. Note during the beginning of 1923 and 1925, a similar rise in the flow occurs, but in other years it does not. What has occurred in this case is the flow patterns on which the nonparametric model based its generated flows (nearest neighbors) had a similar pattern during this period. Recall that the nearest neighbors are selected based on "distances" in the pattern space calculated between feature vectors for the base record during the extension and common periods. If the base record did not contain information regarding the flow conditions at the index record site which either caused or prohibited this pattern of flow from occurring, then the nonparametric method would select inappropriate nearest neighbors for that time period. Fortunately, the wet seasons and dry seasons do not utilize the same nearest neighbors, so the nonparametric method can "recover" quickly within the same flow year.

Figure 4.14 shows the historic, NP1C and REGM generated flows for English River Near Umfreville. The extension period is 15 years. The common period is 25 years. NP1C replaced REGM, which had a lower OBJ_i', but whose residuals were not normally distributed. The longer common period enhances model performance. The timing of the peak and low flows tend to correspond better with longer common period. Notably, a very high flow in 1927 was reproduced well by REGM.

Figure 4.15 shows the historic, NP1C and REGM generated flows for Island Lake River Near Island Lake. The extension period is 15 years and the common period is 25 years. NP1C was recommended, replacing REGM. NP1C modeled the peak and low flows quite well. Both models had some difficulty corresponding to the peaks between 1942 and 1944.

Table 4.27 Recommended Extension Models, r_{xy} > 0.666

N_2	N_1 =5 Years	N_1 =10 Years	N_1 = 15 Years	N_1 = 20 Years
5	MV3M	-	-	-
10	NP1C	REGM	-	-
15	REGM	NP1B	NPM	-
20	REGM	NPM	NP3B	NP2A
25	REGM	NP1A	NP1C*	NP2A
30	REGM	NP2B	NP1C	NP1C
35	REGM	NP2B	NP3B	NP2B
40	REGM	NP1C	NPM	NPM
45	REGM	NP1C	NP1C	NP2B
50	REGM	NPM	REGM	NP2B*
55	REGM	REG	REGM	-
60	REGM	REGM	-	-
65	REGM	-	-	-

* Recommended over random noise model.

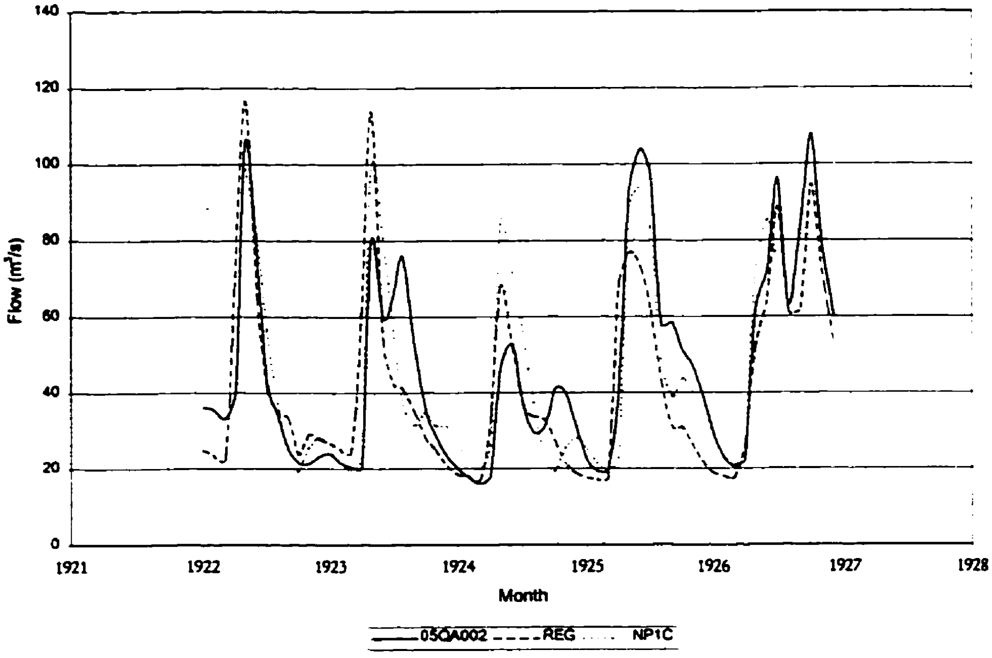

Figure 4.13 Generated and Historical Flows for English River at Umfreville
$N_1 = 5$ Years, $N_2 = 10$ Years, $r_{xy} > 0.666$

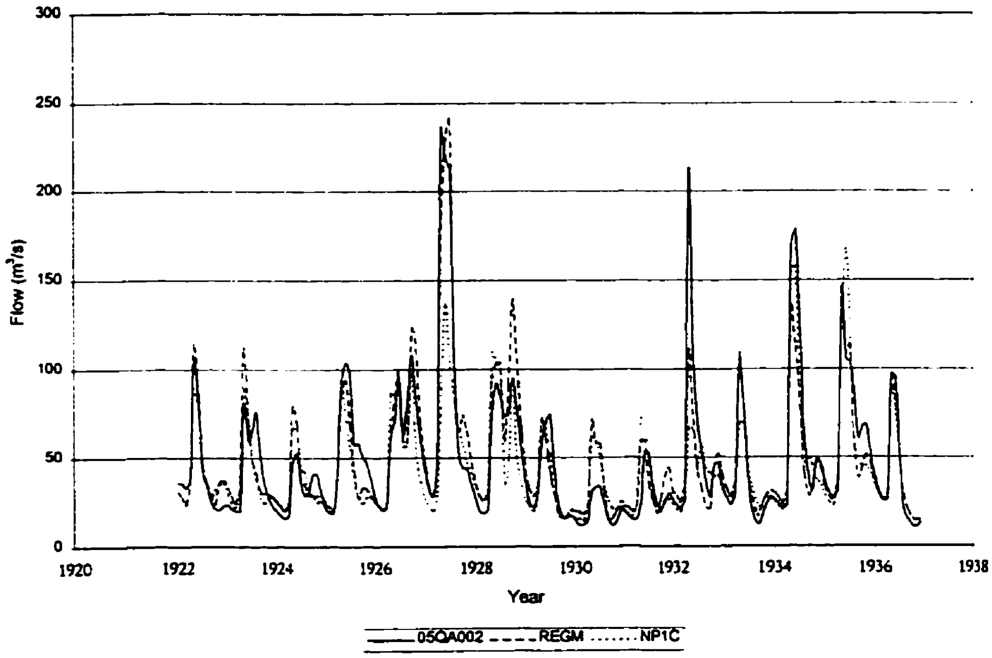

Figure 4.14 Generated and Historical Flows for English River at Umfreville
$N_1 = 15$ Years, $N_2 = 25$ Years, $r_{xy} > 0.666$

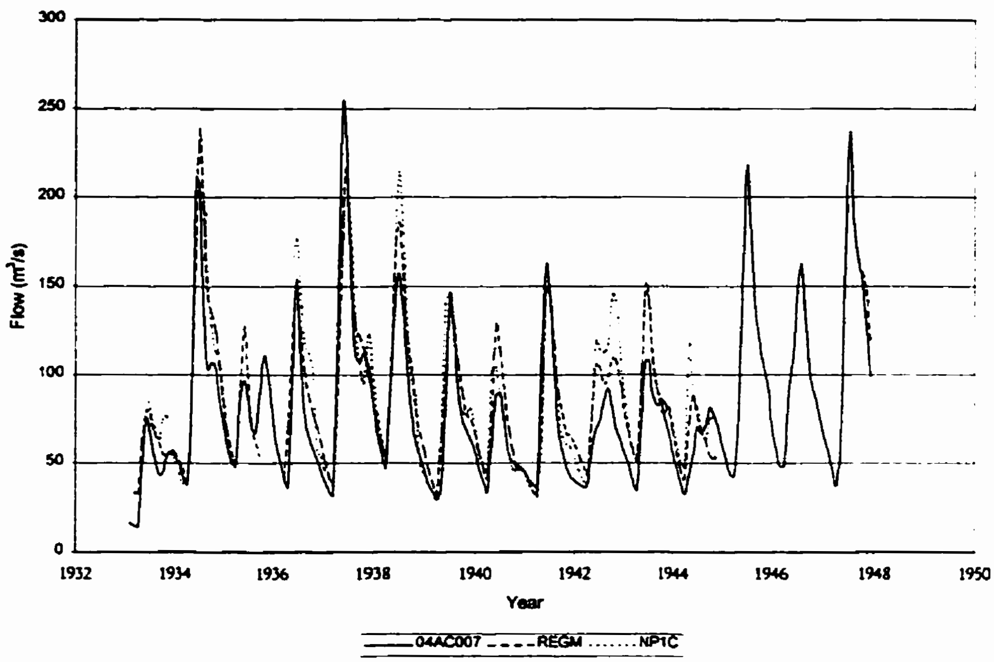

Figure 4.15 Generated and Historical Flows for Island Lake River at Island Lake
N_1 = 15 Years, N_2 = 25 Years, $r_{xy} > 0.666$

4.10.2 $r_{xy} < 0.666$

The models achieving the minimum objective function value combined for verification sets 3 and 4, min[$OBJ_i(3)+ OBJ_i(4)$], for each combination of N_1 and N_2 for r_{xy} <0.666 are shown in Table 4.28. The models shown in Table 4.28 are recommended to use for streamflow record extension for the respective extension period, common period, and cross correlation between the base and index record less than 0.666.

A point should be made at this point regarding the feasibility of using a set of base and index records where $r_{xy} < 0.666$. The analysis of the behavior of the various models for the case of r_{xy} < 0.666 was done more for investigative purposes rather than to make the statement about performing extensions using stations where the cross correlation is low. In fact, it is not the opinion of the author that performing an extension based upon low cross correlation provides any benefit other than the hydrologist will get a longer record out of the exercise. When r_{xy} is small, there may be no physical basis upon which the extension can be based, and factors affecting the runoff are different. The factors affecting runoff include subbasin geometry, such as drainage area, shape, slope and topography; geologic variables such as soil type, porosity, sediment characteristics and groundwater regime, for example. In the case of small r_{xy}, then, it is unlikely that a useful extension can be made with the parametric or nonparametric techniques, and an alternate form of record generation, such as using determininstic models, could be investigated. Even though r_{xy} can be tested to be statistically significant, there is no guarantee that there is a physical basis of high correlation. Such correlation may occur by chance. In the present study r_{xy} could not be generalized as to whether it is statistically significant or not, due to the large variation in sample sizes encountered in the data sets.

Figure 4.16 shows the historic, MV3M and NP2C generated flows for the Manigotagan River Near Manigotagan, verification set 3. The extension and common periods are 10 and 20 years, respectively. NP2C is the recommended model, as an alternate to MV3M, which failed the error diagnostics. In Figure 4.16, some of the peaks correspond well, but generally not for moderate flows. Both models provide variable flows during dry periods, when clearly the historical flows are quite uniform during this period. Note the extremely high flow in 1966 produced by MV3M, showing the high variability that this model is capable of producing. The general poor model performance is a reasonable occurrence in light of the low r_{xy}, and supports the assertion that relying on record extensions for low r_{xy} is not a preferred alternative. Even though some of the rising limbs are modeled well, the general shape of the hydrographs near the peaks, falling limbs and low flows are not reproduced well.

Figure 4.17 shows the historic, MV3M and NP2C generated flows for the Sturgeon River at Outlet of Salvensen Lake, verification set 4. The extension and common periods are 10 and 20 years, respectively. The peaks and valleys do not correspond well. This means the timing of the high and low flows in the generated model are different from the historic model. The models do not perform well in this case. This provides a visual confirmation of the evaluation and verification results which showed the cross correlation between the generated and historic flows, and the performance in reproducing the serial correlation of the historic flows was much less for $r_{xy} < 0.666$.

Table 4.28 Recommended Extension Models, $r_{xy} < 0.666$

N_2	N_1 =5 Years	N_1 =10 Years	N_1 = 15 Years
5	MV1	-	-
10	NP1C	NP1B	-
15	NP3C	NP2C	NPM
20	NPM	NP2C	NPM
25	NP2C	NP3C	-
30	NP2C	-	-

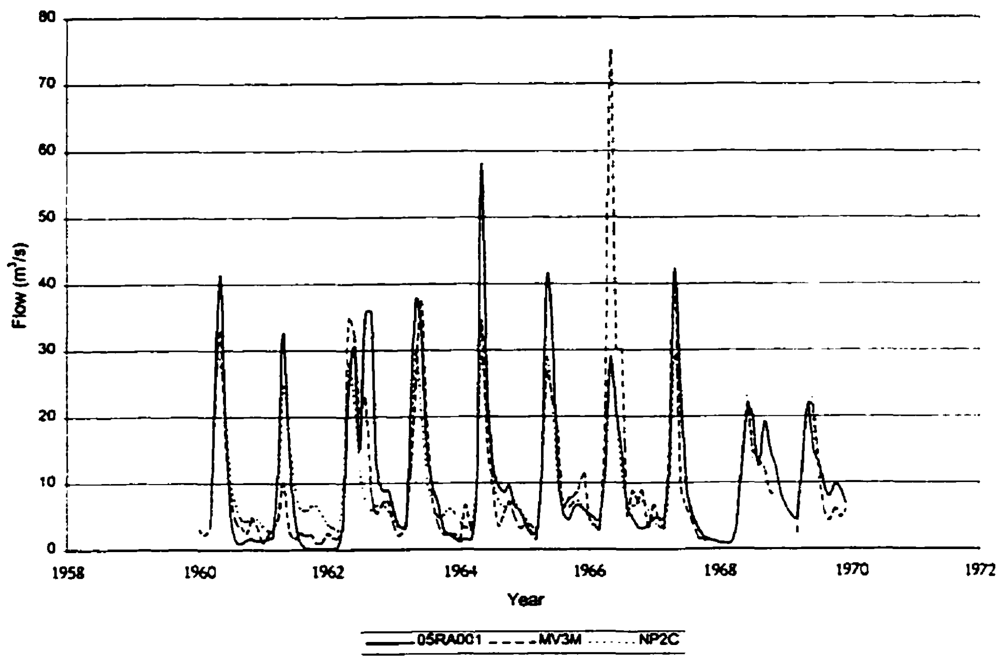

Figure 4.16 Generated and Historical Flows for Manigotagan River Near Manigotagan
N_1 = 10 Years, N_2 = 20 Years, $r_{xy} < 0.666$

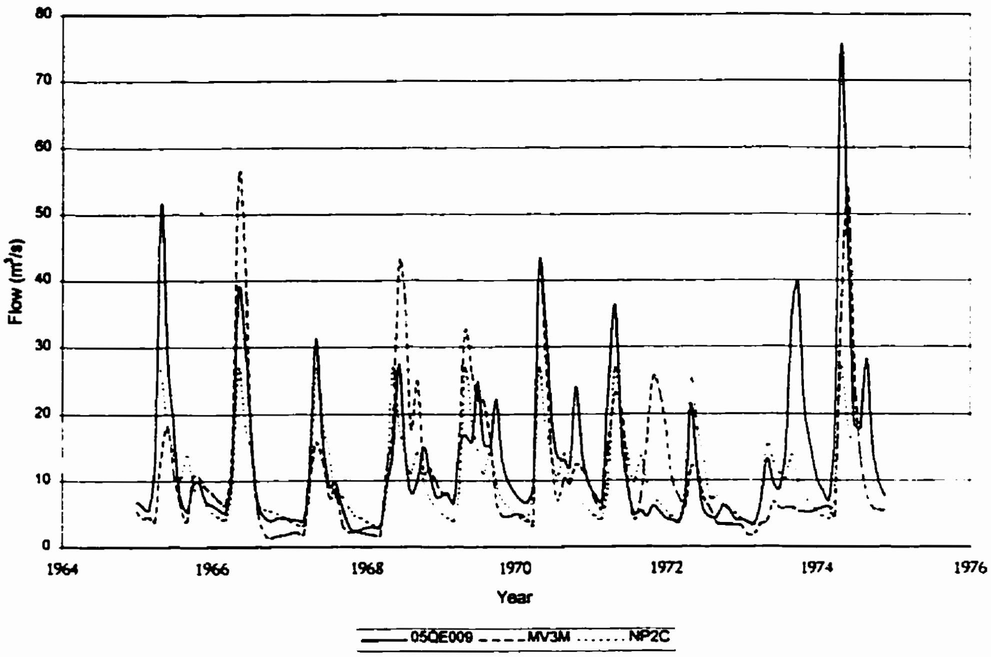

Figure 4.17 Generated and Historical Flows for Sturgeon River at Outlet of Salvensen Lake
N_1 = 10 Years, N_2 = 20 Years, $r_{xy} < 0.666$

4.10.3 Recommended Procedure to Determine Appropriate Model and Perform Record Extension

Based on the results of the verification data, the recommended extension model and general procedure for carrying out streamflow record extension for a given combination of extension period, N_1, common period, N_2, and cross correlation between the base and index record, r_{xy}, is summarized as follows.

Step 1 Select base and index streamflow stations with coinciding measurements. Determine the required extension period and product moment cross correlation by either equation 3.13c or 4.9. The common period used for selecting the model shall be the closest multiple of 5 years.

Step 2 If $r_{xy} > 0.666$, determine the recommended extension model from Table 4.27.

If $r_{xy} < 0.666$, determine the recommended extension model from Table 4.28.

Step 3 If the selected model is parametric, transform the monthly streamflows either with logarithms, or another power transformation of choice, then perform the extension using the method as outlined in Section 3.2. Go to Step 8.

If the selected model is noncyclic nonparametric, examine the monthly mean hydrograph, to classify the data into wet and dry seasons. If the classification is not visually apparent, perform the noncyclic nonparametric extension on a portion of the record, approximately 1/3 the length of the total record to determine the proper classification.

Step 4 Classify the flow data into feature vectors, utilizing the appropriate transformation in accordance with Section 3.5.2.

Step 5 If $r_{xy} > 0.666$, determine the appropriate number of nearest neighbors, n_{opt}, from Table 4.8.

If $r_{xy} < 0.666$, determine the appropriate number of nearest neighbors, n_{opt}, from Table 4.9.

Step 6 Rank the flow years within the same pattern class in accordance with Section 3.5.3. Choose the first n_{opt} feature vectors as the nearest neighbors to utilize for flow generation.

Step 7 Generate the synthetic feature vectors in accordance with equation 3.40.

Step 8 Reverse transform the data, if required to obtain the generated flow values.

Guidance as to the range of statistical performance which may be expected from a particular model can be obtained from Table 4.25 and Table 4.26 for $r_{xy} > 0.666$ and $r_{xy} < 0.666$, respectively.

Chapter 5

SUMMARY AND RECOMMENDATIONS

5.1 SUMMARY OF RESULTS

This study compared the performance of existing parametric streamflow record extension techniques and nonparametric method of streamflow record extension which was developed as a variation of existing nonparametric techniques. The nonparametric method utilizes the relationship between the index and base record to identify similar flow patterns that are used to generate streamflow data. The record extension techniques were evaluated based on a statistical comparison of the generated flows to the historic flows. Both cyclic and noncyclic forms of the techniques were examined. The performance of the techniques were examined for different cross correlation between the index and base station, $r_{xy} > 0.666$, and $r_{xy} < 0.666$, as well as for different combinations of extension period and available common period. The evaluation also consisted of examining the number of nearest neighbors used by the nonparametric techniques, and testing the validity of some theoretical assumptions made in applying the parametric techniques.

A set of evaluation data incorporating 21 sets of base and index streams for $r_{xy} > 0.666$ and 16 sets of base and index streams for $r_{xy} < 0.666$ from Manitoba and Ontario were used to examine the statistical properties of the techniques. The data was log-transformed prior to use in the parametric models to remove skewness. The extension techniques were then carried out on four separate sets of base and index streams (2 for each r_{xy}) for the verification of the results. The verification trials form the basis of the conclusions regarding which models are most appropriate.

The analysis was conducted on streamflow records of long length such that a portion of the record is temporarily assumed as "missing", then the extension is carried out in order to generate the "missing" historical flows. The generated flows are compared to the historical flows. The results of the verification trials generally confirmed the findings of the evaluation trials, and are summarized below.

Although the data were log-transformed, in many cases the parametric models displayed residual series which were not normally distributed, indicating that the parametric models did not follow the assumption of normality of error terms, upon which parametric class of models is based. The results also showed that both the parametric and nonparametric models yielded residual series which were serially correlated. This indicates that the variables used in the parametric models were not random, another contradiction to the parametric model assumptions, and that all the models generally did not adequately take into account time varying relationship between the base and index records. Where the residual series from a parametric model failed the tests for normality of residuals, an alternate model is proposed, which did not fail the tests for normality of residuals. The alternate model is not necessarily, but often was, a nonparametric model. In many cases, the nonparametric models either performed best or provided a relatively close alternative to a parametric model which did not conform to the model assumptions.

The optimum number of nearest neighbors used by the nonparametric models, n_{opt}, did not vary appreciably with extension period, but did vary with common period, r_{xy}, and the type of nonparametric model used. The spread between the number of nearest neighbors used by different nonparametric models was quite narrow for a common period less than 40 years, but diverges rapidly for common periods greater than 40 years. The optimum number of nearest neighbors increased with common period. The ratio of n_{opt} to common period decreased as common period increased, ie. the rate of increase in n_{opt} did not vary consistently with the number of years in the common period. Utilizing the recommended number of nearest neighbors decreases the performance of the nonparametric techniques, in comparison to the case where the nonparametric models utilizes the optimum number of nearest neighbors determined from extending portions of a long-length record.

In general, all the extension models generated flows which displayed fairly high cross correlation with the historic flows. The cross correlation increased as the record length increased, and decreased as r_{xy} decreased.

The serial correlation of the historic flows was reproduced best by noncyclic nonparametric and cyclic MOVE procedures. The serial correlation of the generated flows tended to be slightly smaller than the historic flows, and model performance neither increased nor decreased with extension or common period. The ability to reproduce serial correlation did not seem to be affected by variation in r_{xy}.

The cyclic parametric and noncyclic nonparametric models also reproduced variance better than the other models, however, the variance tended to be slightly over-estimated. The nonparametric models tended to perform better as common period increased. A decrease in r_{xy} resulted in underestimation of variance.

All models reproduced mean flow quite well. REG, RPN, and the noncyclic nonparametric models reproduced the mean flow the best overall. The nonparametric model performance increased with common period. The generated mean flows tended to be lower than the historical means when r_{xy} decreased.

There was a high variability in the models' capability to reproduce low flows. The noncyclic nonparametric techniques, and cyclic and noncyclic parametric MOVE techniques performed best. All models tended to overestimate the low flows. Model performance for this parameter increased as the common period increased, and decreased quite dramatically as r_{xy} decreased.

The noncyclic nonparametric, REG, and cyclic MOVE techniques provided the lowest MPE overall. Again, the results of mean percentage error, MPE, were quite variable, and performance seriously decreased as r_{xy} decreased.

REGM and noncyclic nonparametric models displayed the lowest sum of squared deviations from the historic flows, SSE. Performance in terms of SSE increased significantly as available common period increased, and decreased with r_{xy}.

5.2 RECOMMENDATIONS

The nonparametric techniques are recommended as a viable alternative in the cases where the parametric models displayed nonnormal residual series. The residuals of the flows generated by the parametric models were not normally distributed in many cases, showing that the parametric models do not follow the theoretical assumptions upon which they are based. However, the use of the nonparametric models as alternatives to the parametric models may involve a trade-off in terms of statistical performance in some cases.

The model which obtained the minimum objective function value, which also did not fail the test for normality of error terms is recommended as the best overall model to use in a given case of N_1 , N_2 , and r_{xy}. The recommended procedure for generating streamflow records is given in Section 4.10.3.

Great care must be taken when considering record extensions based on low cross correlation between the base and index record. There are likely factors which make record extension infeasible in such a case. If the extension or common period falls outside the range shown in Table 4.27 or Table 4.28, one may utilize the model recommended for the next closest value of extension or common period, or choose from the table the model which occurs most often out of all common periods.

In any case, judgement must be utilized in selecting the appropriate model in relation to many factors, including the statistical performance of the model. The initial evaluation should include examining features of the data being utilized relative to the features of the data on which these recommendations are based, since the best models based on the results from this study may not necessarily be the best models in every case. If the available common period is long enough, an initial trial extension could be performed on a portion of the index record currently known, then the performance of the models evaluated on that basis. The results from the initial evaluation should be factored into the final model selection.

The cyclic nonparametric model, as presented, is relatively simplistic in comparison to the noncyclic nonparametric models, in that only a single monthly flow is included in the feature vector. Expanding and refining the NPM model is suggested as future work. One consideration is including adjacent months as part of the feature vector.

Other refinements to consider for the nonparametric models, in general, include:

- including physical data such as precipitation or snowmelt, for example, in the feature vectors,
- applying weight to nearest neighbors closer in time to the extension period to determine the effect on feature vector selection,
- applying weights to adequacy measures in the objective function calculation to determine the effect on model selection,
- determine the optimum season lengths determined for each model separately, rather than using the results from one model,
- determine the effect of data infilling on model performance.

The question as to the overall effect of the parametric models not following the theoretical assumptions of normally distributed residuals is framed in terms of a theoretical standpoint in this study. The nonparametric models are proposed as an alternative to models which have problems following the theoretical assumptions upon which they are based. The effect of nonnormal residuals on the generated flows is also a suggested topic of further research. The results may provide some insight as to the seriousness of the failure of the parametric models to meet model assumptions.

As well, the applicability of the models should be investigated for streamflow records outside the study area, and the analysis would benefit if updated as new data become available.

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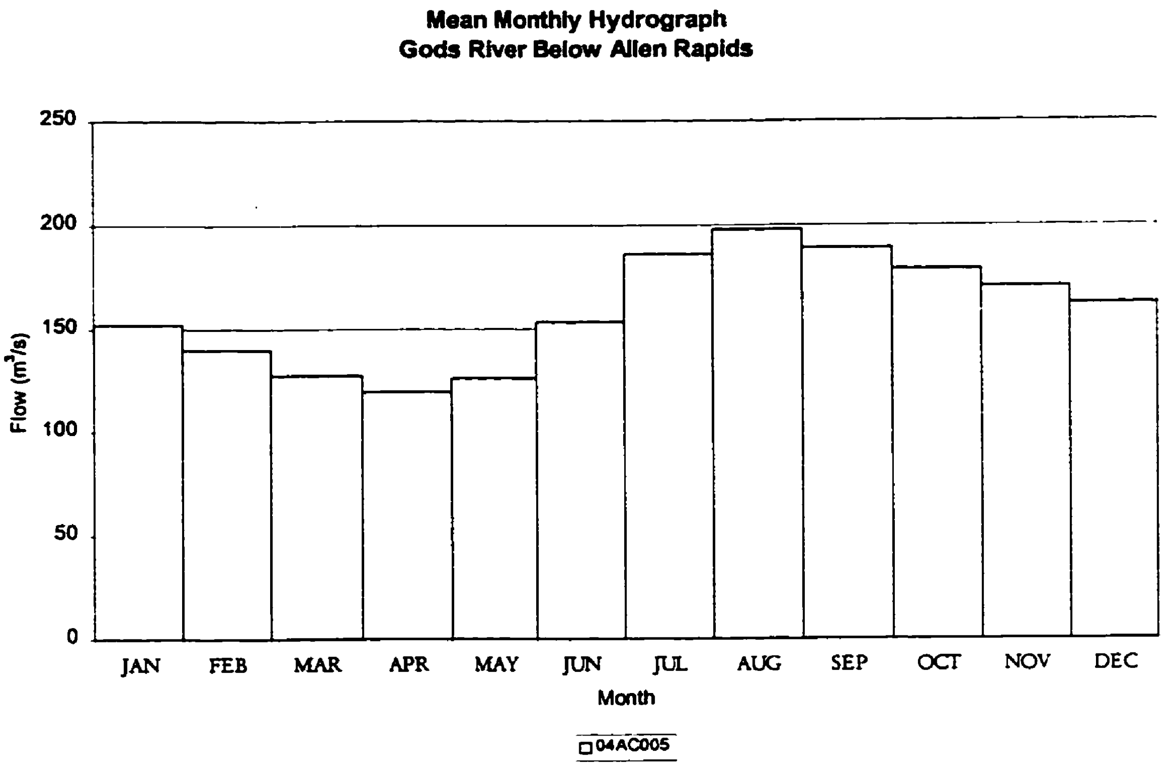
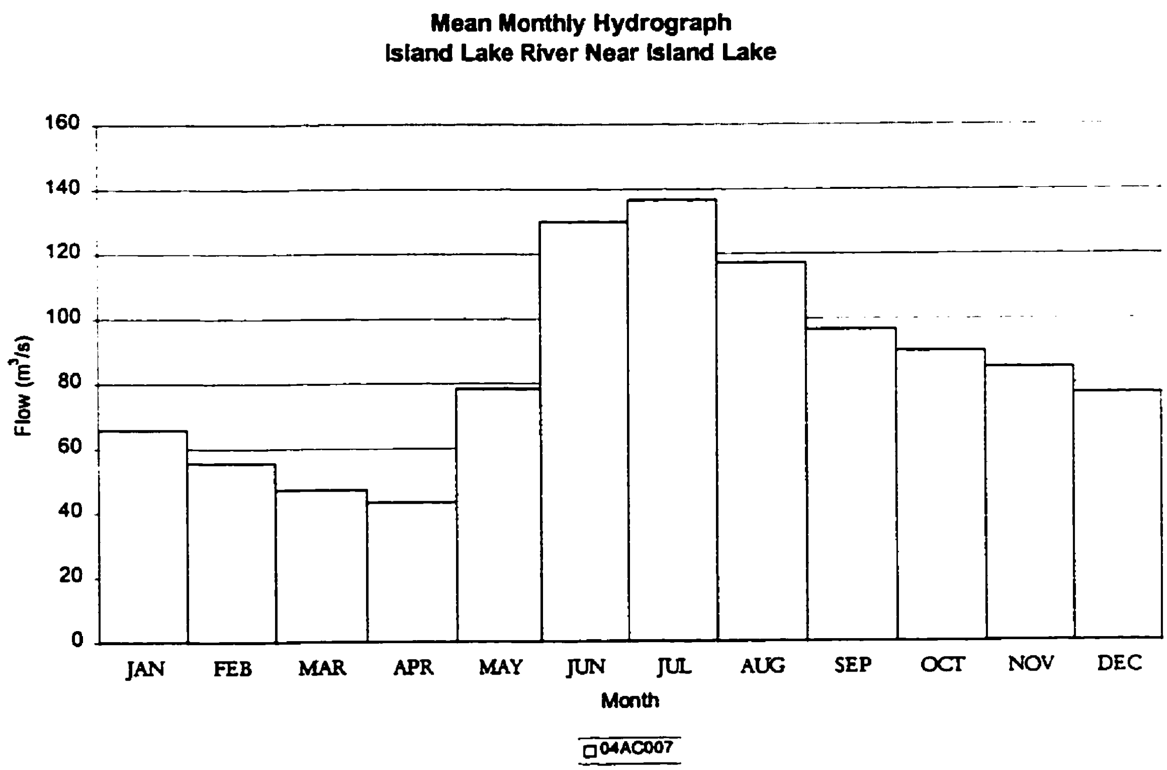
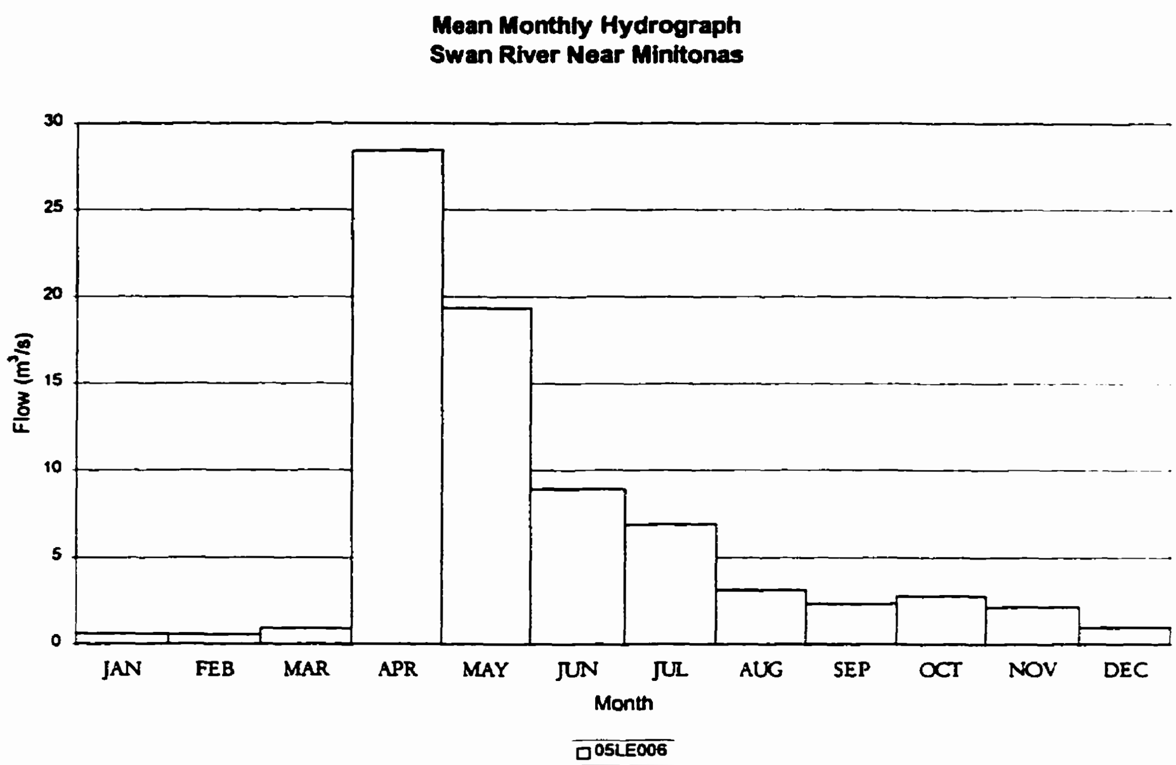
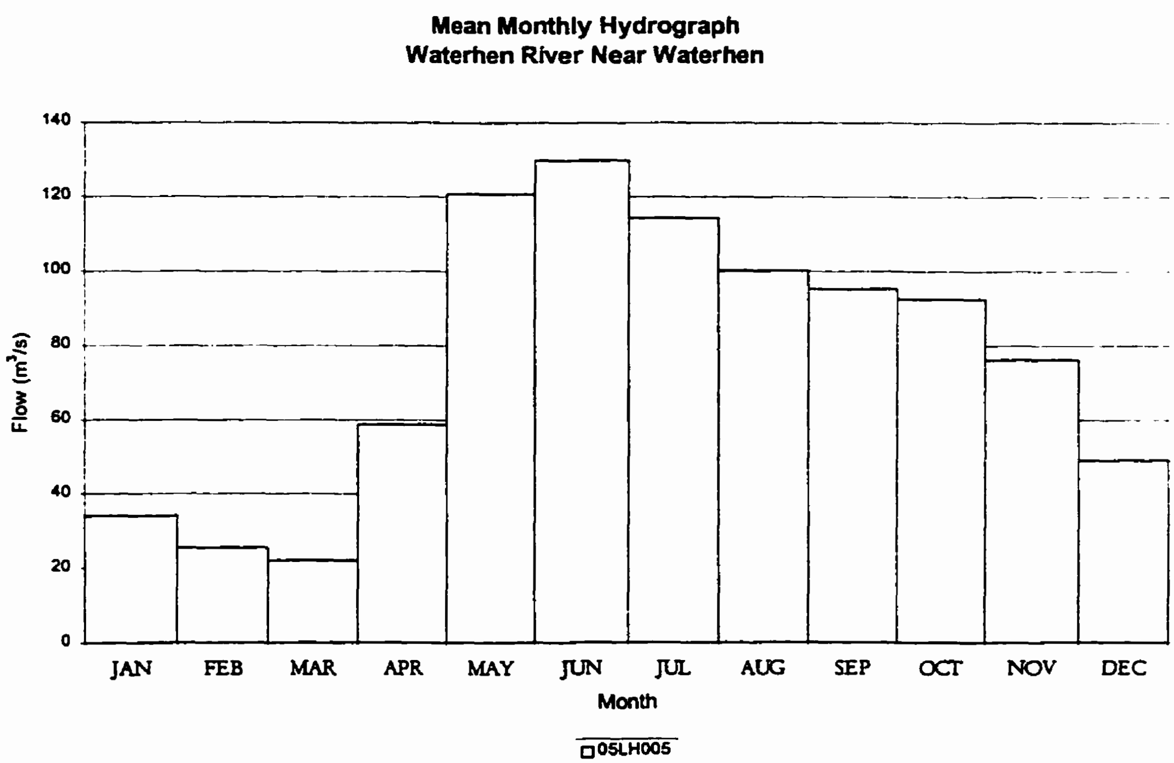
APPENDIX A

HYDROGRAPHS

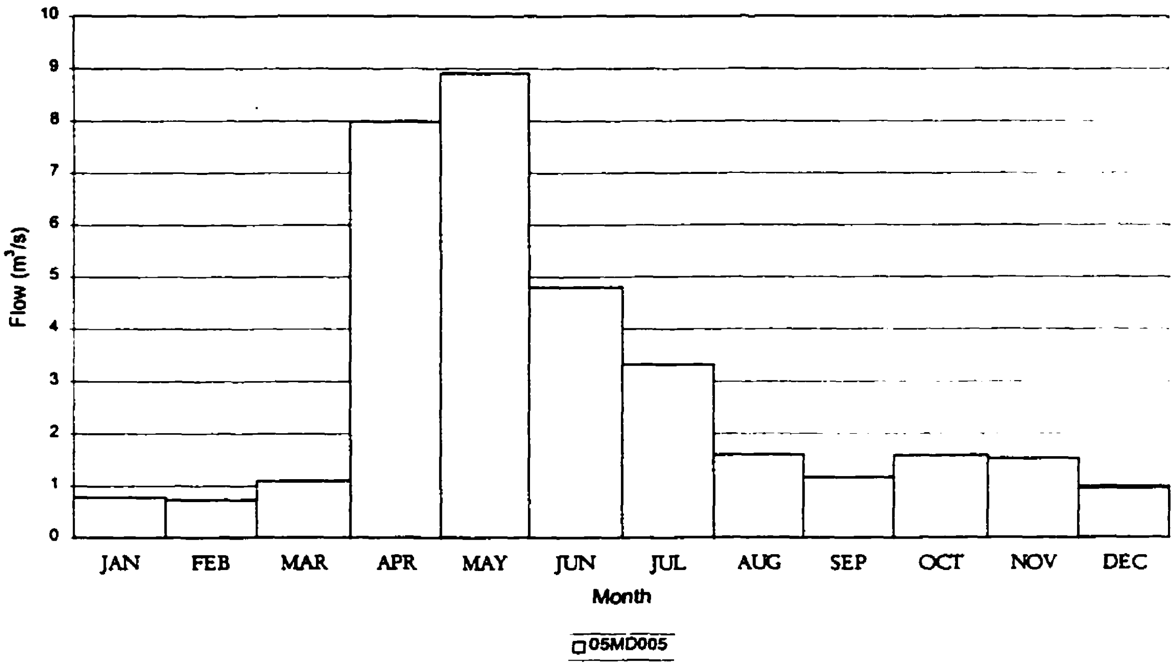

Mean Monthly Hydrograph
Gods River Below Allen Rapids
250
200
150
100
50
0
Flow (m³/s)
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
04AC005


Mean Monthly Hydrograph
Island Lake River Near Island Lake
160
140
120
100
80
60
40
20
0
Flow (m³/s)
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
04AC007

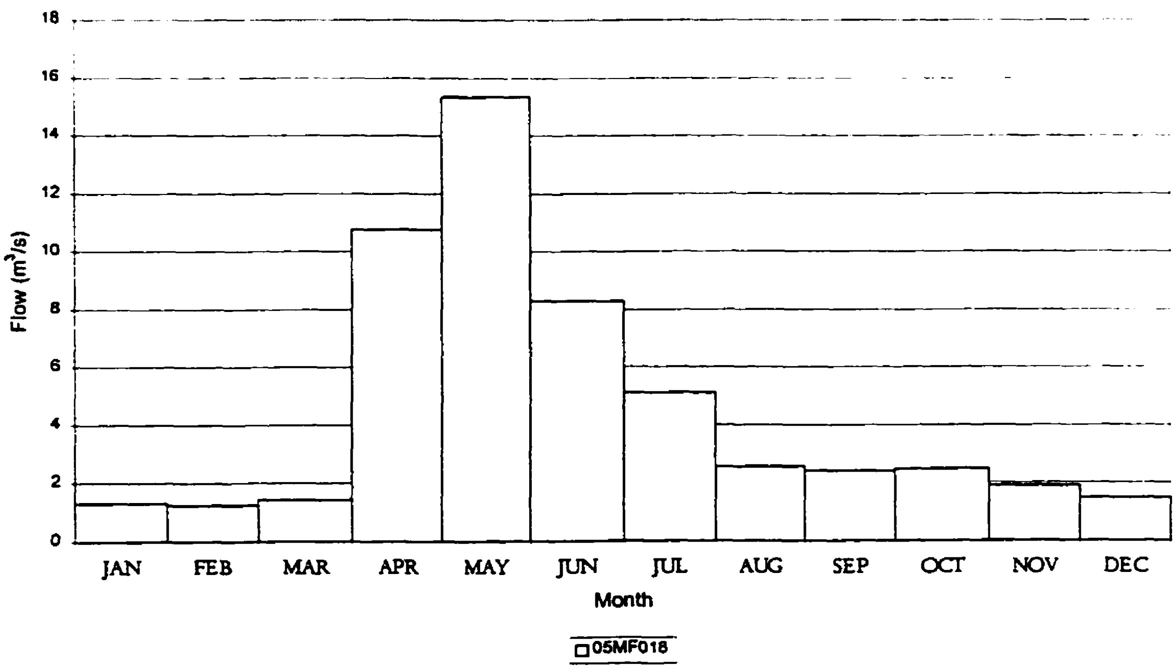

Mean Monthly Hydrograph
Swan River Near Minitonas
Flow (m³/s)
30
25
20
15
10
5
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
05LE006


Mean Monthly Hydrograph
Waterhen River Near Waterhen
Flow (m³/s)
140
120
100
80
60
40
20
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
05LH005

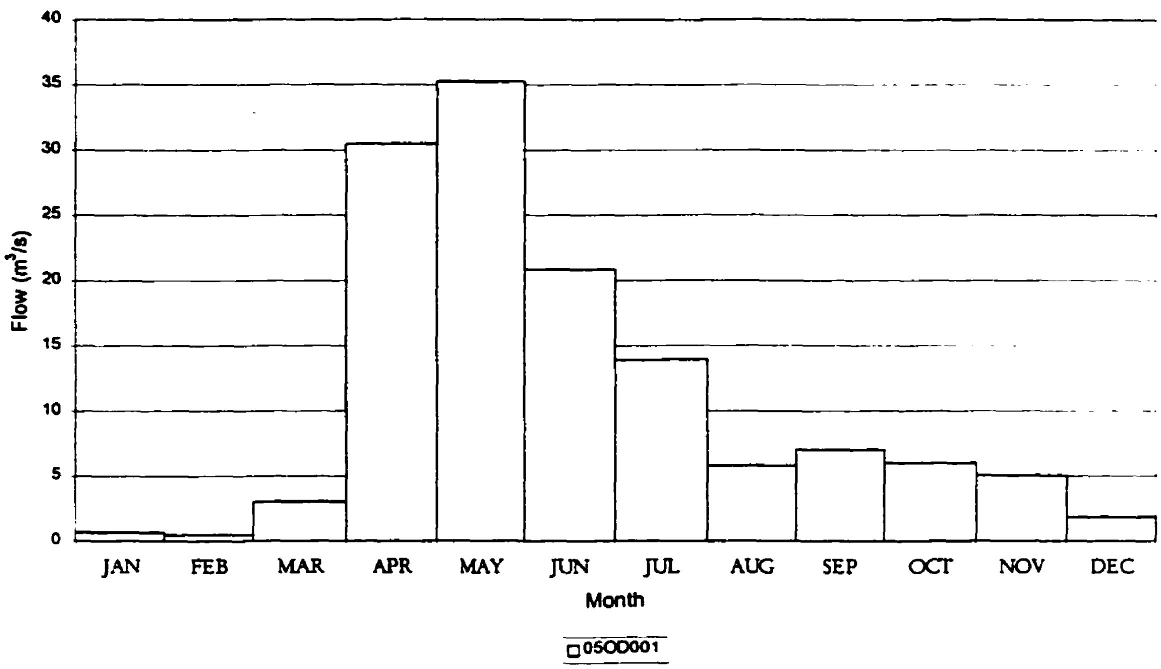
Mean Monthly Hydrograph
Shell River Near Inglis



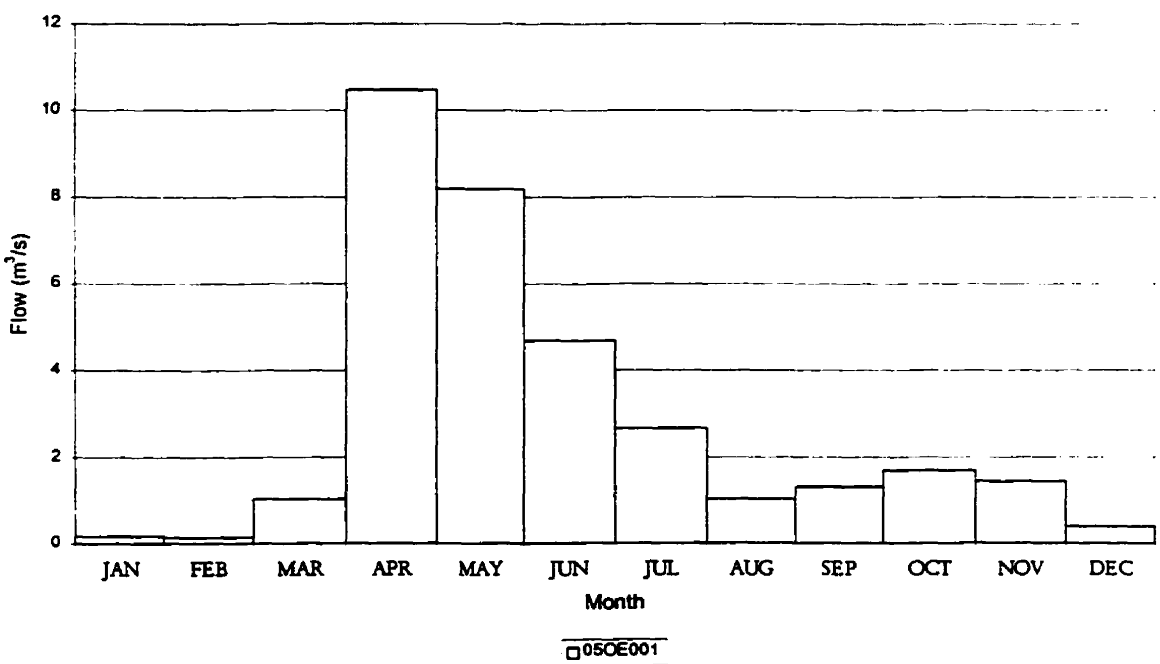
Mean Monthly Hydrograph
Little Saskatchewan River Near Rivers



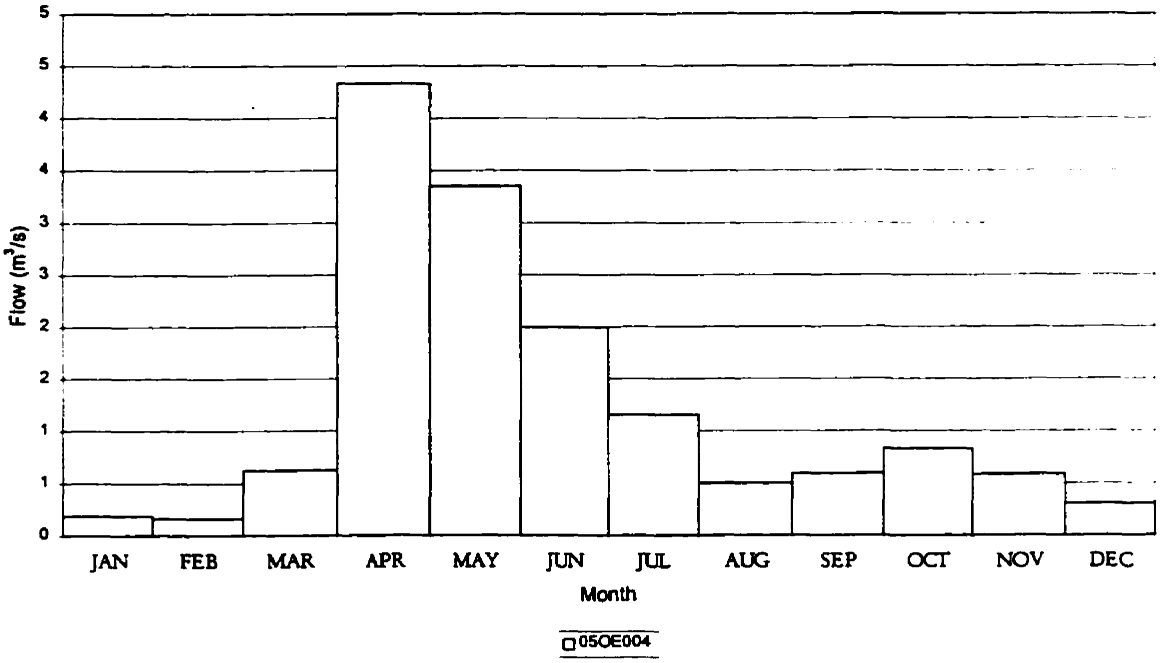
Mean Monthly Hydrograph
Roseau River Near Dominion City



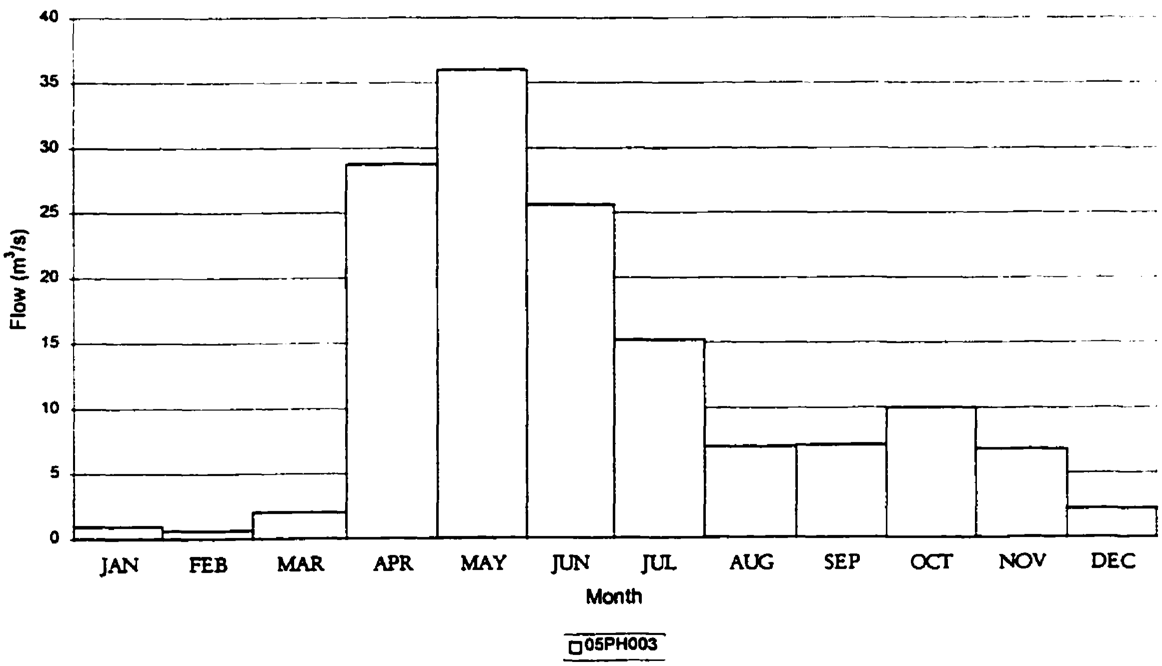
Mean Monthly Hydrograph
Rat River Near Otterburne



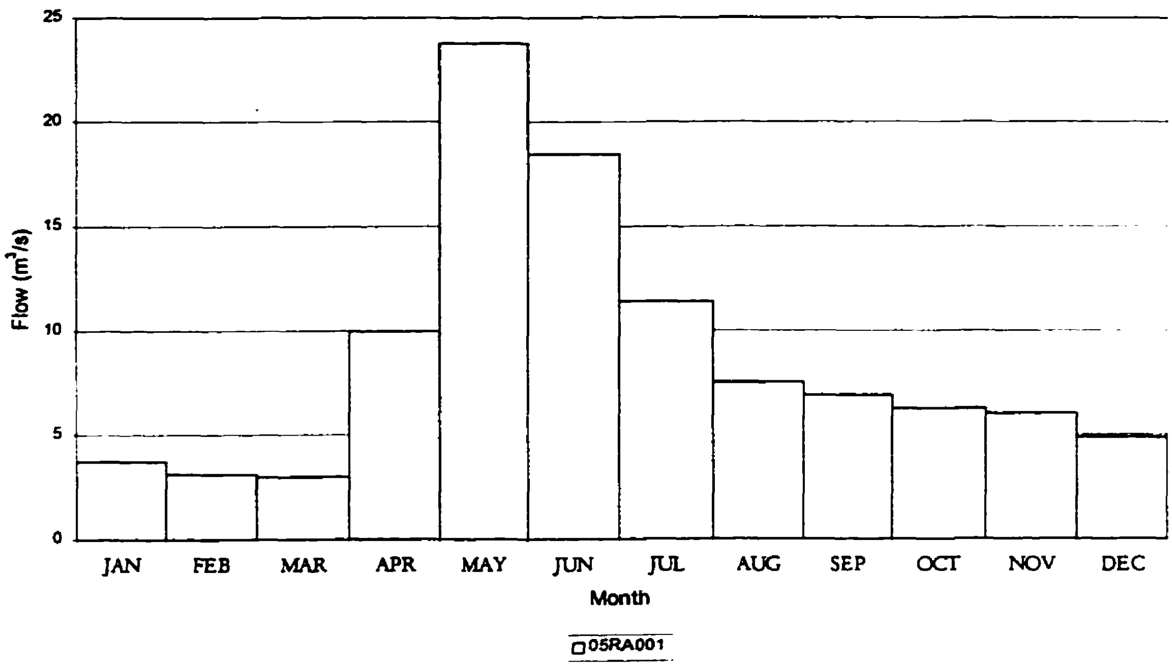
Mean Monthly Hydrograph
Rat River Near Sundown



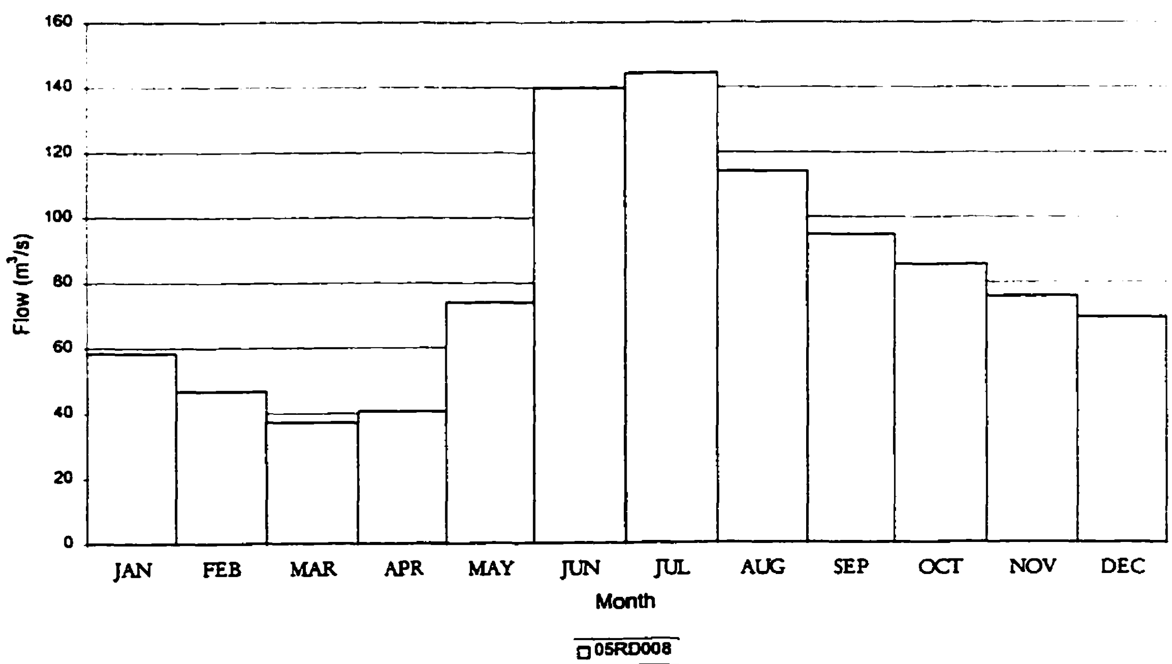
Mean Monthly Hydrograph
Whitemouth River Near Whitemouth



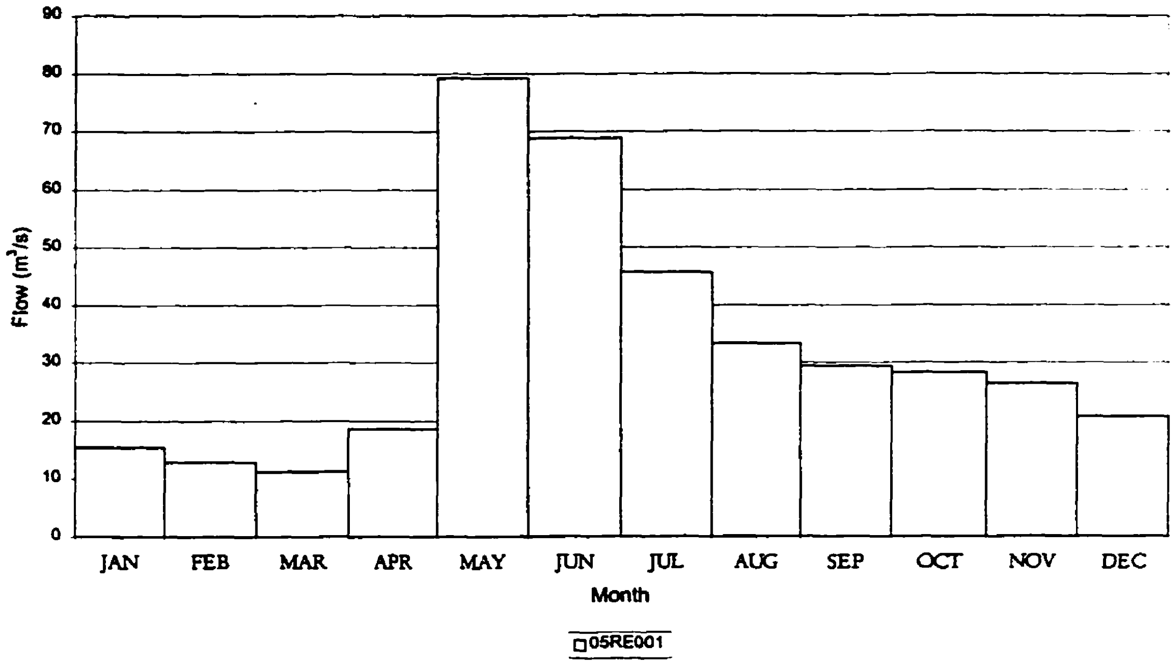
Mean Monthly Hydrograph
Manigotagan River Near Manigotagan



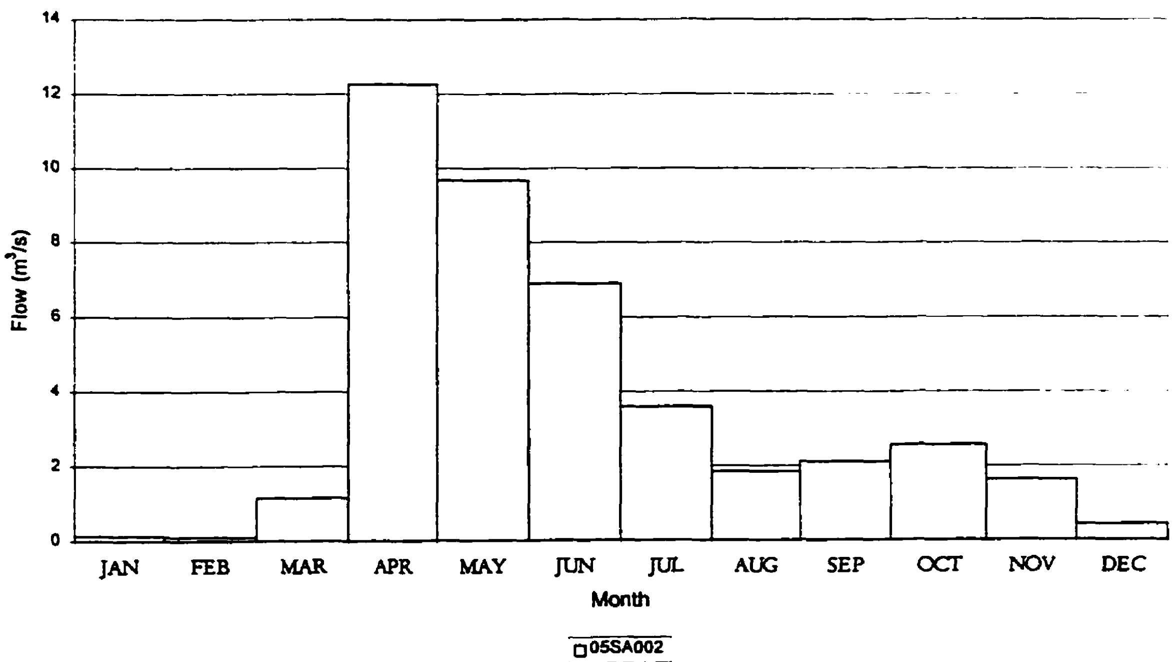
Mean Monthly Hydrograph
Pigeon River at Outlet of Round Lake



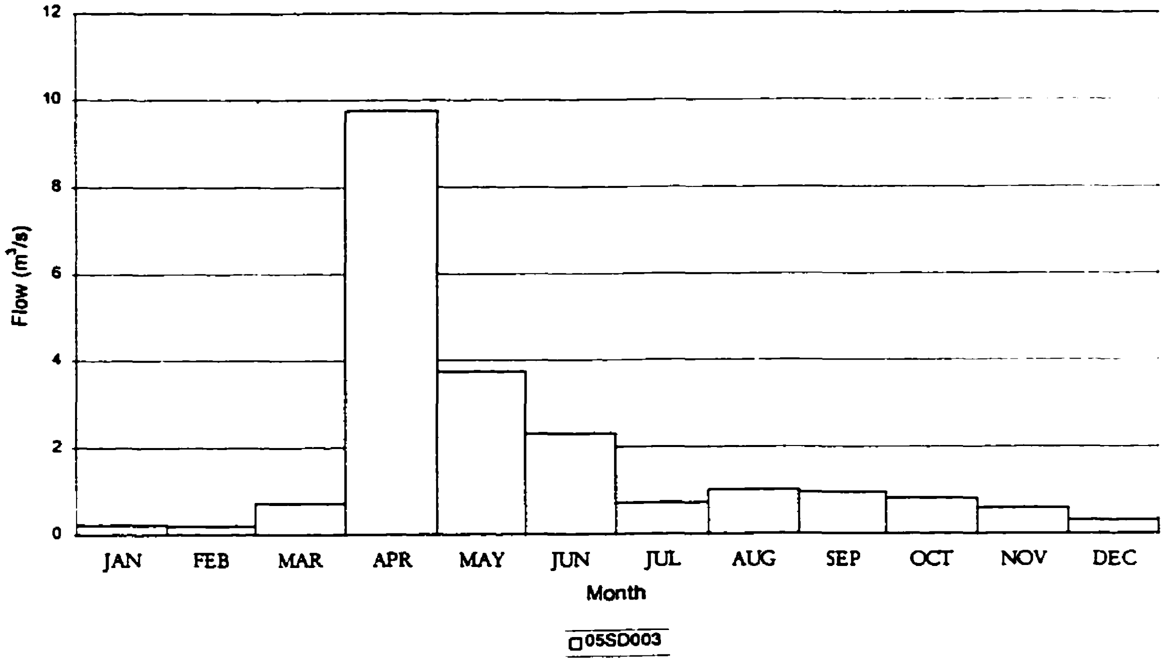
Mean Monthly Hydrograph
Poplar River at Outlet of Weaver Lake



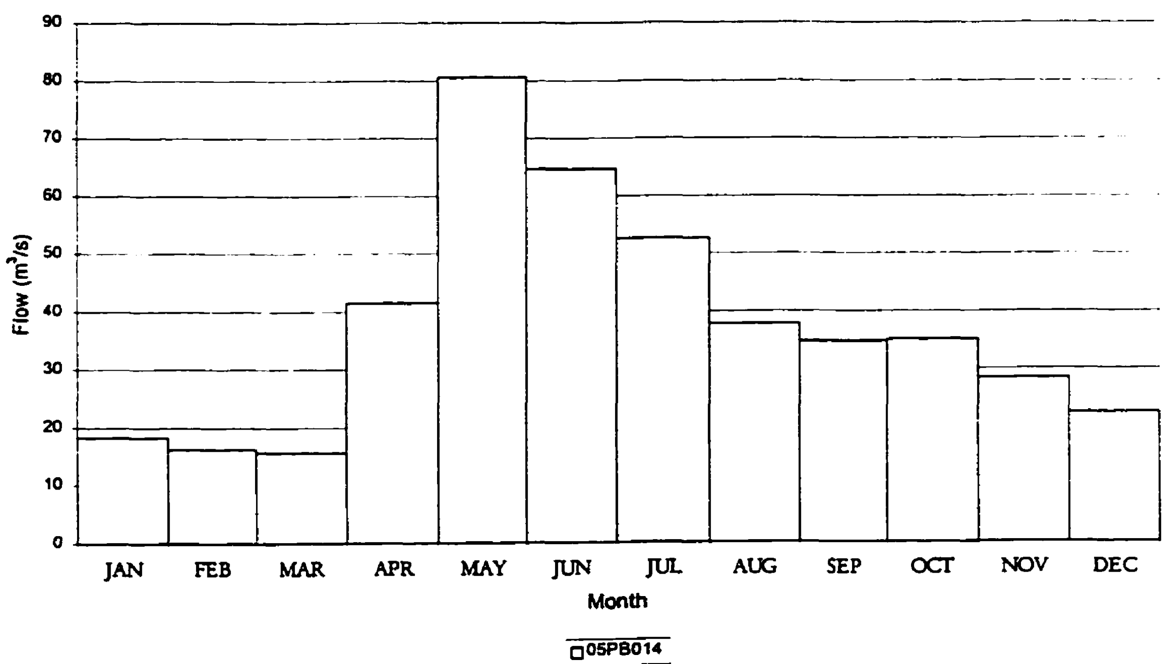
Mean Monthly Hydrograph
Brokenhead River Near Beausejour

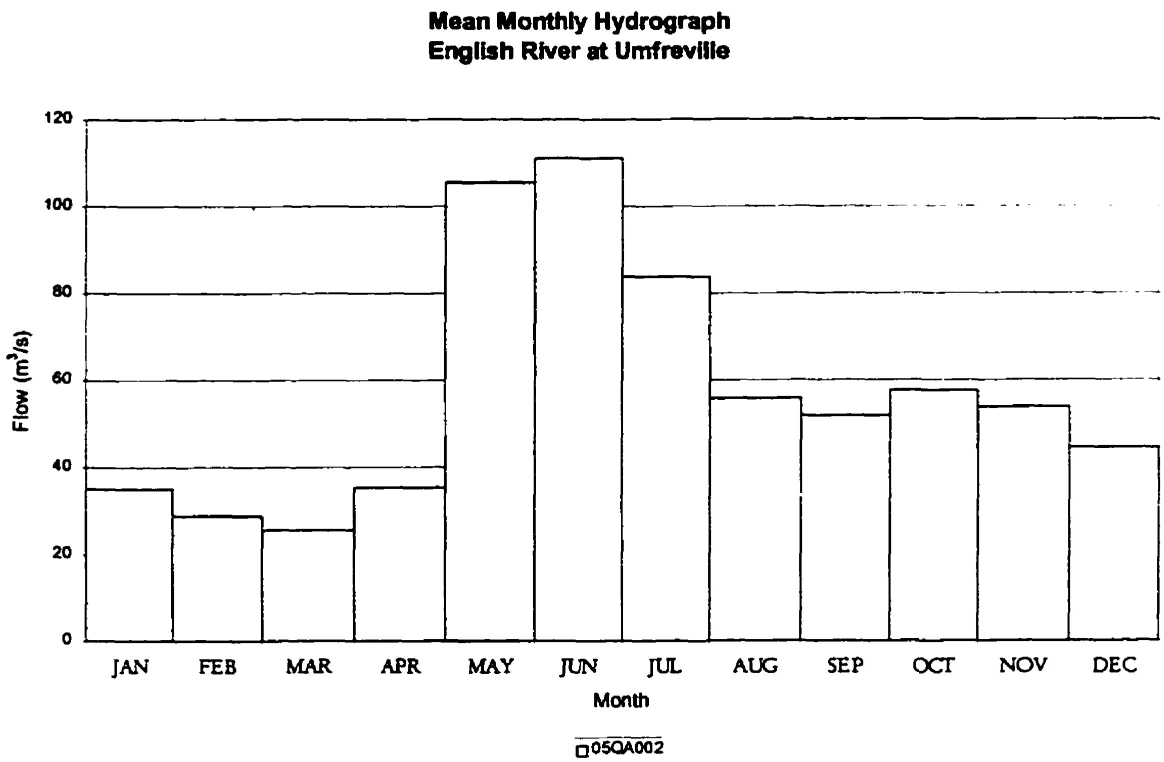


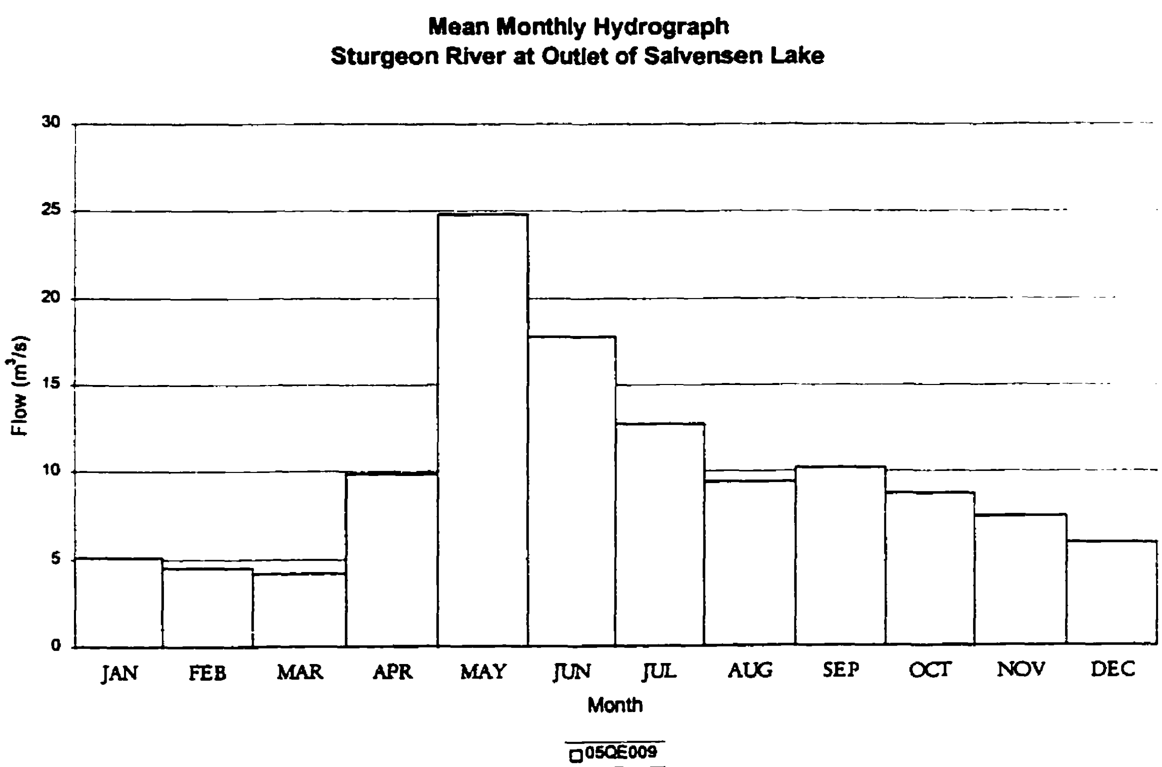
Mean Monthly Hydrograph
Fisher River Near Dallas



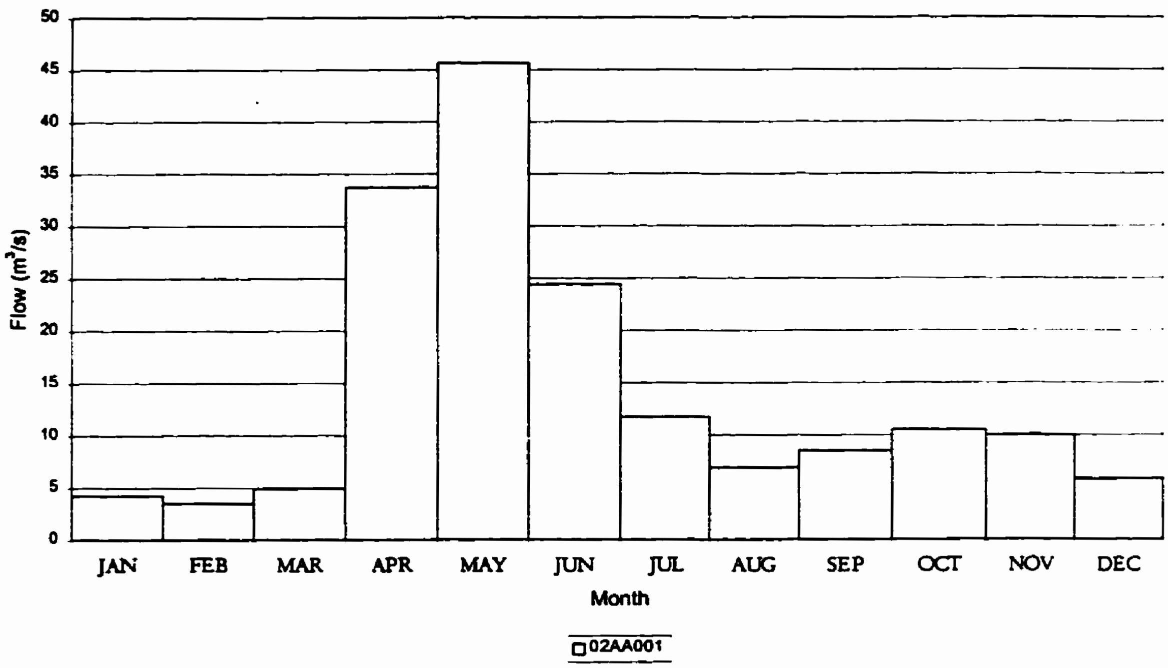
Mean Monthly Hydrograph
Turle River Near Mine Centre



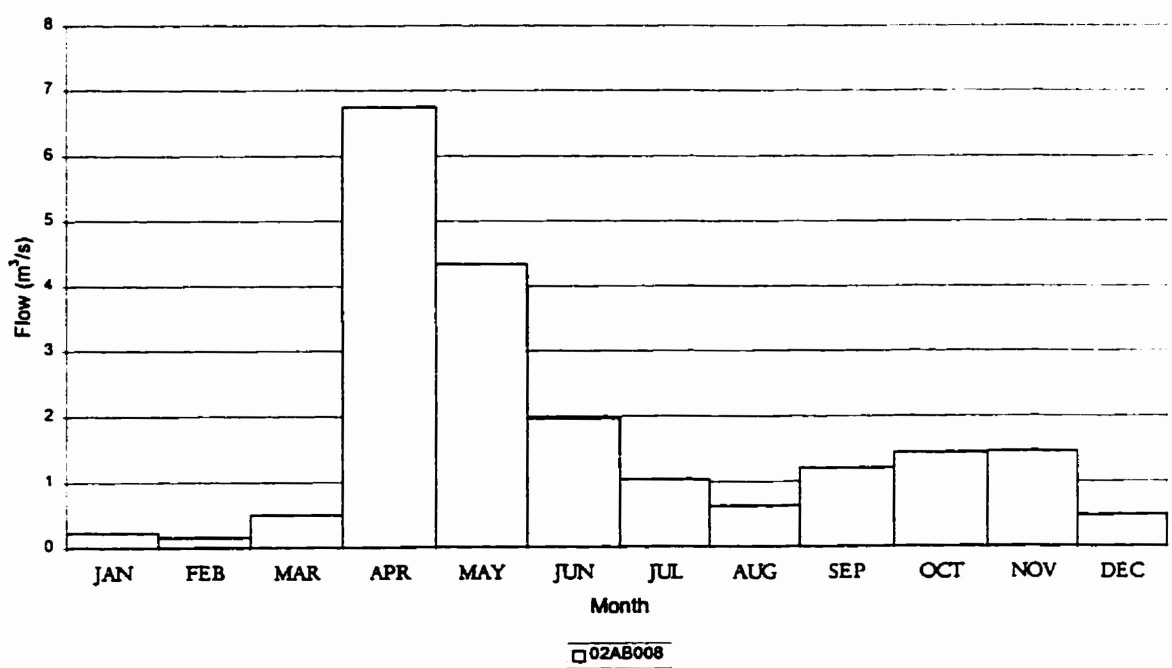

Mean Monthly Hydrograph
English River at Umfreville
120
100
80
60
40
20
0
Flow (m³/s)
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
05QA002

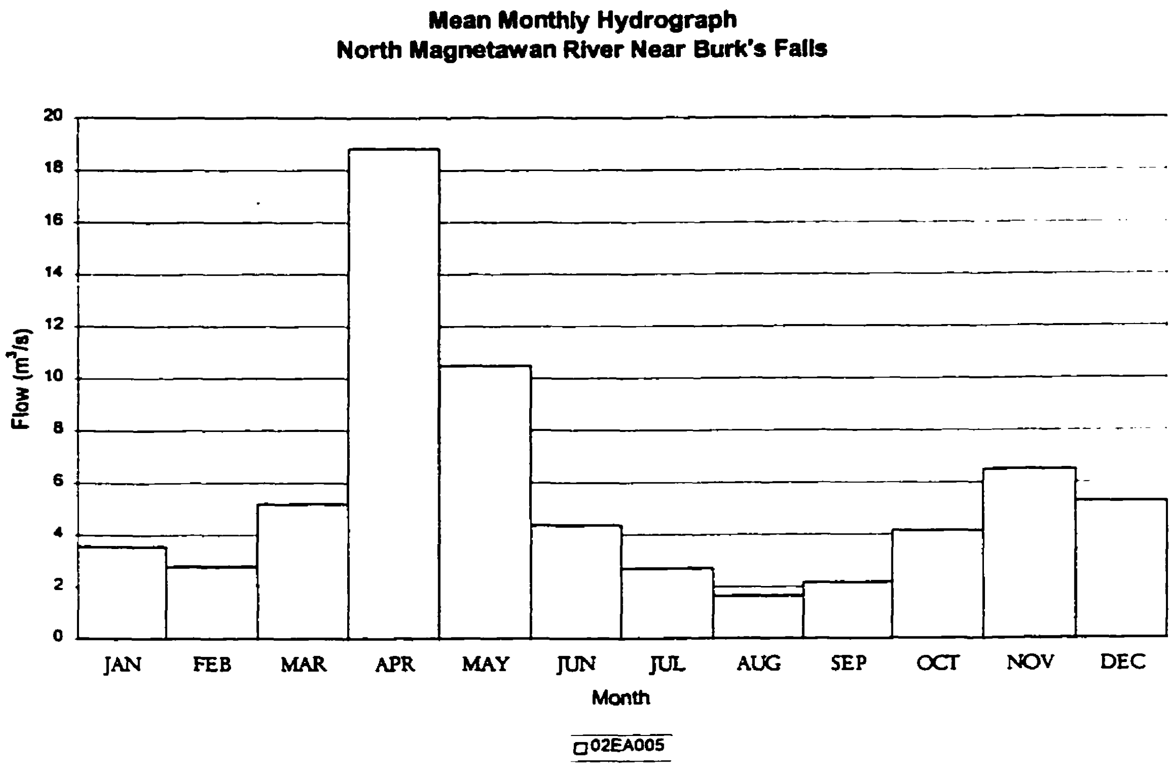

Mean Monthly Hydrograph
Sturgeon River at Outlet of Salvensen Lake
30
25
20
15
10
5
0
Flow (m³/s)
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
05QE009

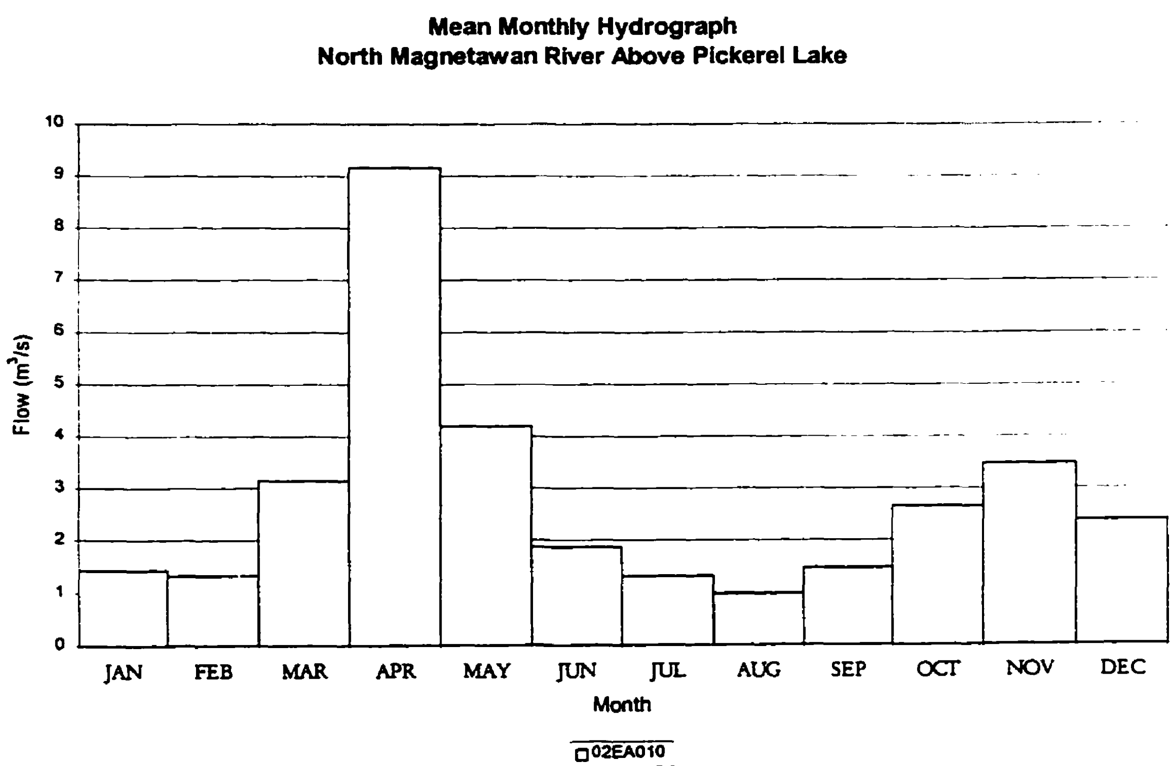
Mean Monthly Hydrograph
Pigeon River at Middle Falls



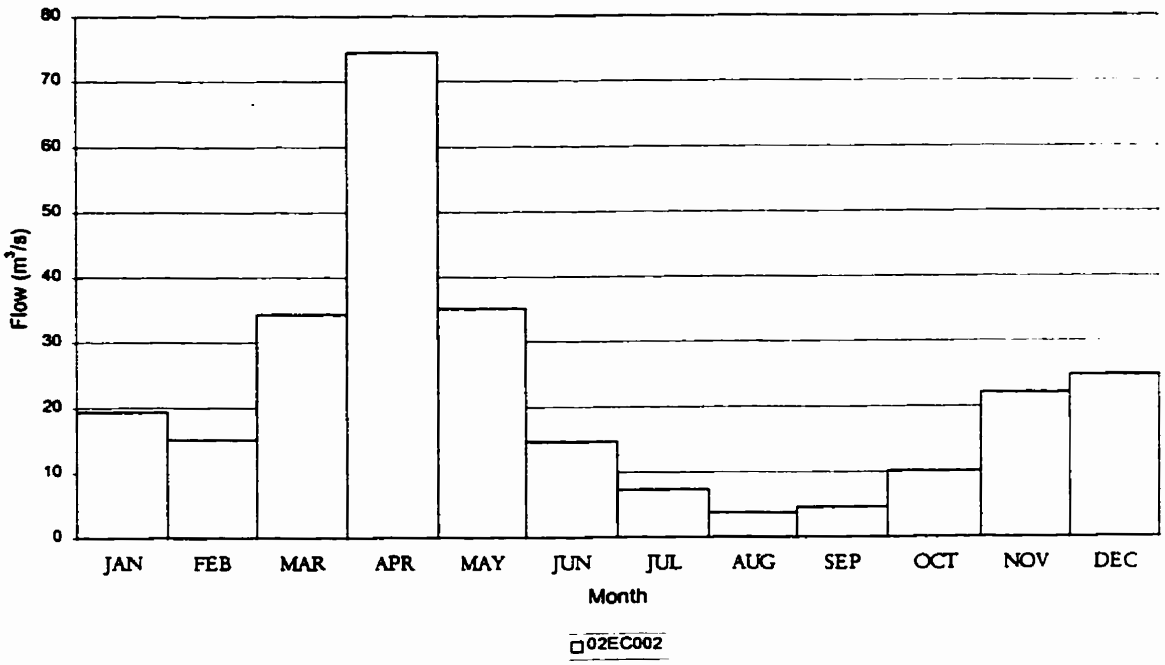
Mean Monthly Hydrograph
Neebing River Near Thunder Bay



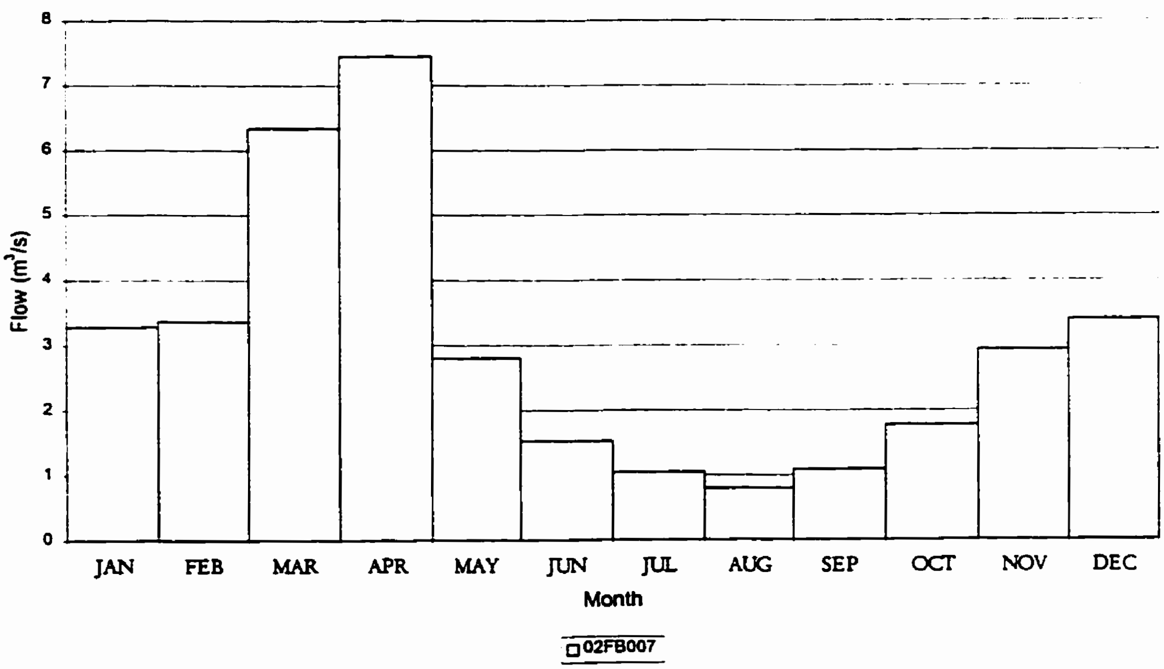

Mean Monthly Hydrograph
North Magnetawan River Near Burk's Falls
Flow (m³/s)
20
18
16
14
12
10
8
6
4
2
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
02EA005


Mean Monthly Hydrograph
North Magnetawan River Above Pickerel Lake
Flow (m³/s)
10
9
8
7
6
5
4
3
2
1
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
02EA010

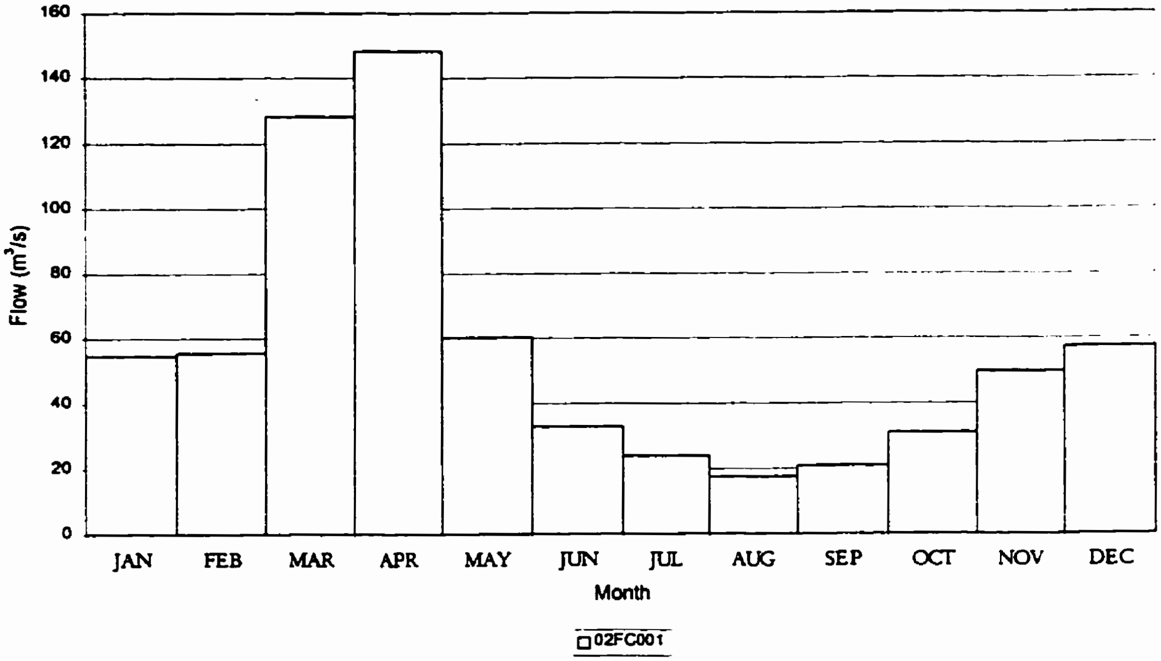
Mean Monthly Hydrograph
Black River Near Washago



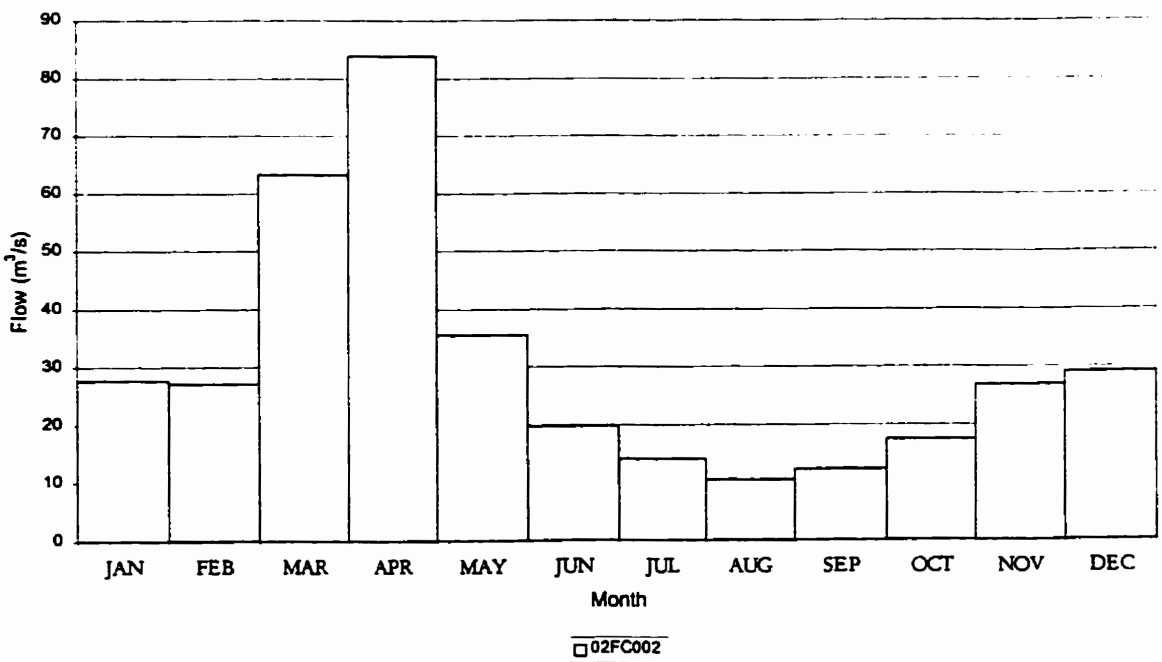
Mean Monthly Hydrograph
Sydenham River Near Owen Sound



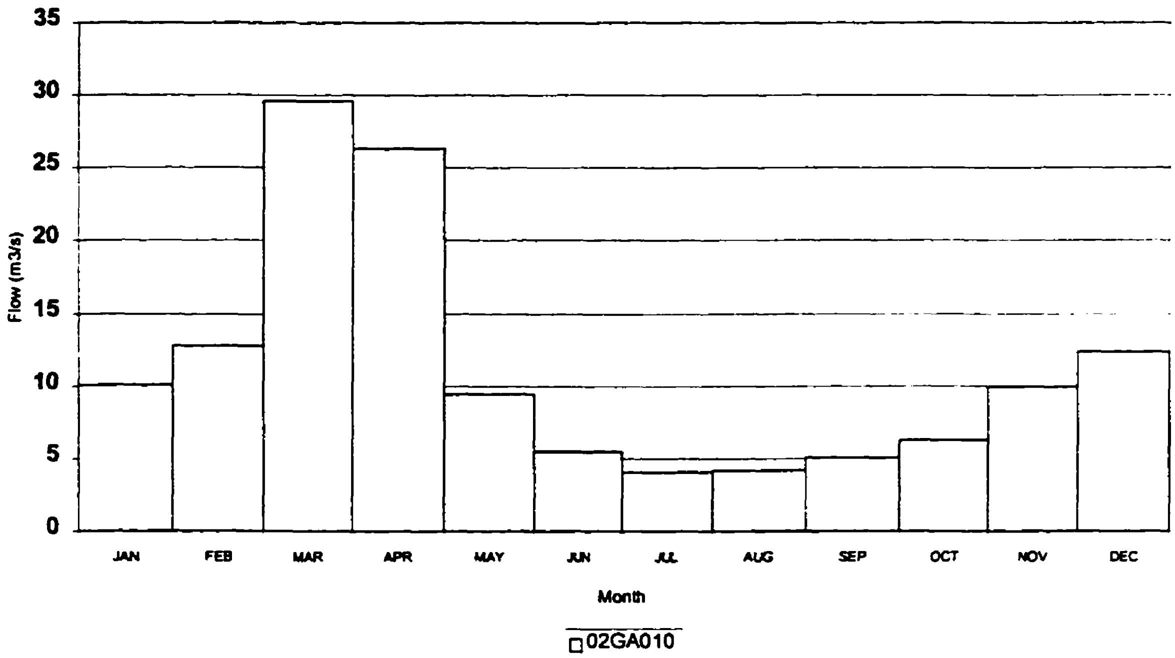
Mean Monthly Hydrograph
Saugeen River Near Port Elgin



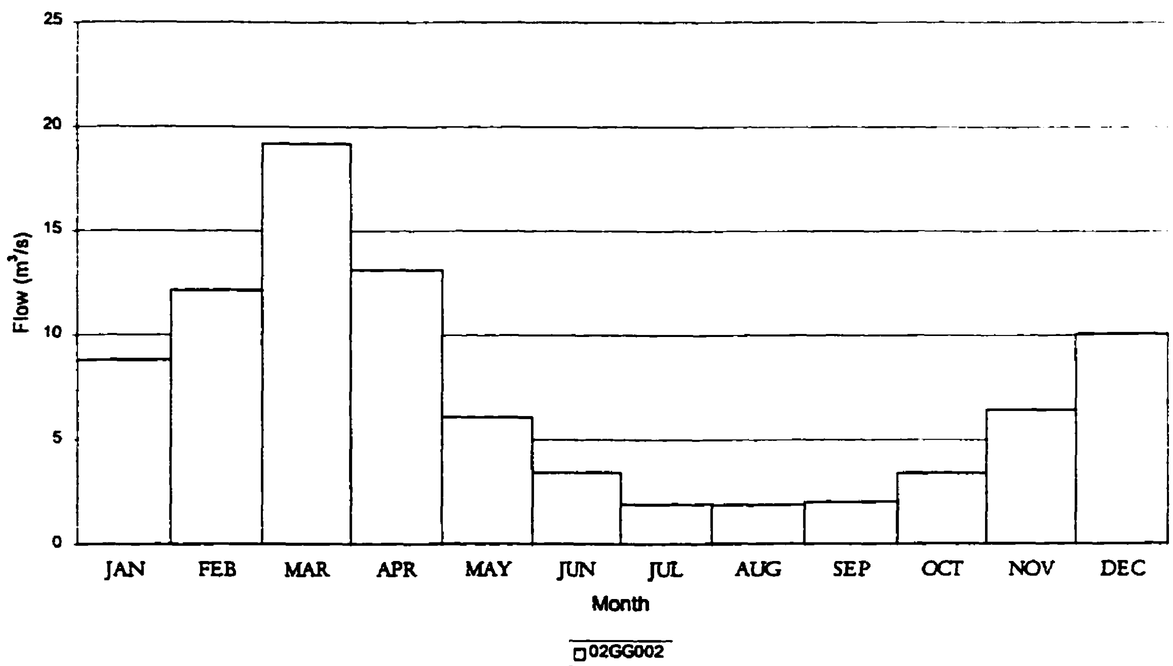
Mean Monthly Hydrograph
Saugeen River Near Walkerton

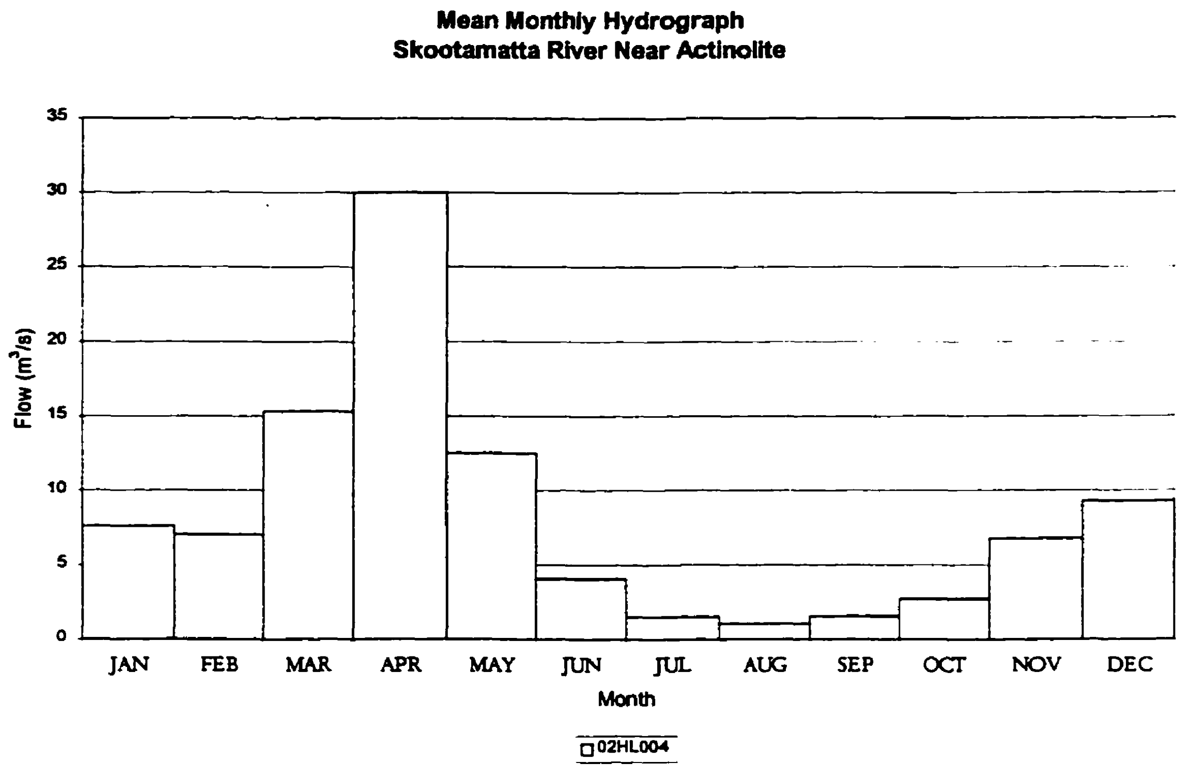


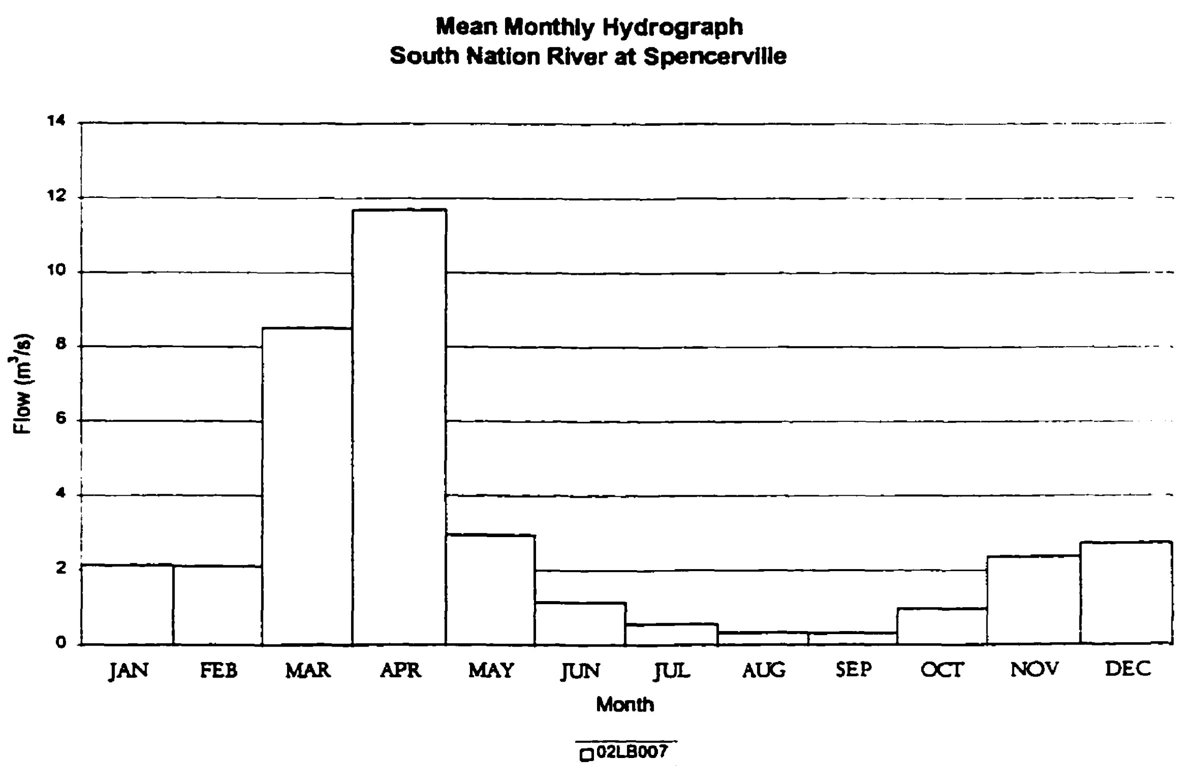
Mean Monthly Hydrograph
Nith River Near Canning



Mean Monthly Hydrograph
Sydenham River Near Alvinston




Mean Monthly Hydrograph
Skootamatta River Near Actinolite
35
30
25
20
15
10
5
0
Flow (m³/s)
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
02HL004


Mean Monthly Hydrograph
South Nation River at Spencerville
14
12
10
8
6
4
2
0
Flow (m³/s)
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
Month
02LB007

APPENDIX B

NUMBER OF NEAREST NEIGHBORS USED BY NONPARAMETRIC TECHNIQUES

Table B-1 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, $N_1 = 5$ Years, $N_2 = 5$ to 75 Years, $r_{xy} > 0.666$

N1 =	5														
N2 =	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
NP1A	3	3	4	5	6	6	7	9	13	11	13	14	16	10	12
STDV:	1.18	1.73	2.01	2.38	3.82	3.86	5.98	6.36	10.36	8.86	9.91	11.19	13.75	13.45	14.82
NP1B	3	4	4	4	6	7	7	8	13	15	15	19	20	20	14
STDV:	1.07	1.77	2.35	2.62	4.02	4.91	5.84	7.07	11.58	12.79	13.65	14.93	16.53	16.69	19.33
NP1C	3	4	5	6	4	5	6	6	7	6	7	8	6	9	8
STDV:	1.67	3.41	4.35	5.86	2.58	5.96	8.19	4.06	3.78	1.34	2.19	1.1	2.19	8.47	2.55
NP2A	3	4	4	5	6	6	7	9	15	15	16	15	19	12	13
STDV:	1.32	1.86	1.96	2.96	3.89	4.31	6.29	8.74	11.34	12.76	14.46	14.13	17.57	16.48	17.34
NP2B	3	3	4	5	5	5	7	9	12	13	14	12	14	12	11
STDV:	0.96	1.49	1.78	2.42	3.26	4.33	6	7.66	7.35	7.89	9.14	15.97	16.81	13.75	16.47
NP2C	3	3	4	4	5	7	7	9	14	14	16	17	20	20	22
STDV:	1.28	2.03	2.63	3.22	3.83	4.7	5.06	7.7	11.1	13.2	15.29	15.55	19.49	17.64	18.27
NP3A	3	4	6	7	7	6	8	8	15	18	10	12	13	15	11
STDV:	1.34	2.46	4.15	5.96	6.63	6.06	6.54	9.58	15.04	16.6	9.96	12.03	12.08	12.95	4.87
NP3B	3	5	6	6	7	8	10	10	18	20	15	17	18	20	10
STDV:	1.41	2.46	4.37	4.52	6.63	5.75	8.11	10.52	14.01	16.74	11.7	13.32	14.52	13.93	4.66
NP3C	3	5	6	5	6	7	6	8	14	5	11	14	13	9	10
STDV:	1.44	2.98	3.88	4.69	5.62	6.74	4.58	5.82	6.69	2.05	11.08	7.83	9.18	4.06	4.47
NPM	2	4	5	6	6	6	7	6	8	7	9	8	11	15	14
STDV:	1.36	1.5	2.11	3.17	2.62	3.6	3.01	3.57	5.4	5.5	4.93	3.96	5.54	4.55	4.51

Table B-2 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, N_1 = 10 Years, N_2 = 10 to 70 Years, $r_{xy} > 0.666$

N1 =	10												
N2 =	10	15	20	25	30	35	40	45	50	55	60	65	70
NP1A	3	5	5	7	8	7	10	11	9	16	17	17	14
STDV:	1.4	2.19	2.59	4.37	4.97	4.65	6.06	7.14	7.54	10.37	10.8	10.49	12
NP1B	3	5	4	6	7	7	10	13	12	12	14	15	13
STDV:	1.45	2.25	2.16	4.61	3.42	4.69	6.07	7.64	7.35	8.41	8.79	9.45	12.03
NP1C	5	5	4	6	6	6	6	6	6	9	12	11	8
STDV:	3.15	4.39	2.31	4.36	4.59	3.51	2.05	1.82	2.17	5.13	5.32	7.54	5.76
NP2A	4	4	4	6	7	9	11	13	12	16	15	18	12
STDV:	1.63	1.89	2.04	3.7	4.5	5.72	7.19	8.93	10.63	9.2	12.12	11.55	4.53
NP2B	4	5	4	6	8	8	11	12	12	13	16	14	15
STDV:	2.08	2.31	1.83	4.33	4.81	4.6	6.69	7.27	7.23	9.47	9.26	11.22	13.25
NP2C	4	5	5	7	8	7	12	10	11	14	13	12	14
STDV:	1.97	2.96	2.86	5.76	5.77	2.98	7.4	5.1	7.98	8.53	4.93	5.59	3.83
NP3A	4	6	5	6	8	6	8	9	6	11	11	10	10
STDV:	2.78	4.13	4.11	3.38	4.78	3.03	4.85	6.07	2.3	4.09	5.18	3.74	3.96
NP3B	5	5	6	6	8	7	12	11	10	12	14	12	11
STDV:	2.96	3.81	4.92	3.5	4.11	4.43	3.29	4.87	6.57	4.16	6.91	5.67	3.11
NP3C	4	6	6	6	6	5	9	8	7	7	6	8	10
STDV:	2.1	3.87	4.88	5.31	5.3	3.37	5.22	6.46	3.05	3.78	3.65	3.94	3.05
NPM	3	4	5	6	8	8	8	10	10	12	16	17	17
STDV:	1.62	2.54	2.11	3.36	3.73	4.7	1.92	3.83	2.17	2.7	3.83	4.72	5.07

Table B-3 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, N_1 = 15 Years, N_2 = 15 to 65 Years, $r_{xy} > 0.666$

N1 =	15									
N2 =	15	20	25	30	35	40	45	50	55	60	65
NP1A	4	5	6	6	7	7	8	10	11	9	9
STDV:	1.35	3.28	2.69	2.25	3.94	4.39	3.56	6.02	7.04	2.49	4.34
NP1B	4	5	6	6	7	8	10	11	12	9	9
STDV:	1.62	2.69	2.41	3.43	5.57	7.13	7.38	7.7	9.11	3.35	3.63
NP1C	4	4	5	6	6	8	8	10	11	8	8
STDV:	3.74	2.15	1.55	2.49	3.58	2.7	1.34	4.58	4.87	2.77	2.7
NP2A	4	5	7	6	6	7	6	8	8	8	8
STDV:	1.47	2.76	2.83	3.33	3.71	4.06	1.73	2.77	3.77	3.96	4.34
NP2B	4	6	7	6	6	8	7	8	11	6	9
STDV:	2.19	3.12	3.09	3.06	5.5	7.6	2.59	6.46	11.03	3.91	3.03
NP2C	4	5	6	6	11	9	7	10	9	9	10
STDV:	1.63	1.86	1.91	2.26	2.05	4.38	3.32	7.09	3.67	3.9	5.07
NP3A	5	5	6	6	8	8	8	8	9	10	9
STDV:	2.22	1.98	2.61	3.5	4.18	3.77	4.15	5.1	6.31	6.48	4.21
NP3B	5	5	7	9	8	10	11	8	11	11	8
STDV:	1.91	2.76	2.51	4.62	3.85	3.36	4.6	4.92	5.4	5.93	3.7
NP3C	5	6	6	6	8	8	8	8	9	9	8
STDV:	3.19	4.16	3.42	3.41	3.29	3.58	4.34	5.18	5.31	6.2	4.1
NPM	4	6	6	5	8	7	8	10	11	12	12
STDV:	1.94	2.04	1.98	2.91	2.51	2.79	3.65	4.97	3.74	3.87	3.91

Table B-4 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, $N_1 = 20$ Years, $N_2 = 20$ to 60 Years, $r_{xy} > 0.666$

N1 =	20								
N2 =	20	25	30	35	40	45	50	55	60
NP1A	6	6	7	7	9	9	8	8	9
STDV:	3.74	1.64	4.1	4.04	5.26	4	2.68	2.07	2.7
NP1B	6	6	8	9	9	9	7	7	7
STDV:	2.47	2.4	3.78	5.1	6.02	6.8	2.05	2.39	2.45
NP1C	5	5	6	6	7	8	7	6	6
STDV:	4.28	1.63	2.51	1.67	1.3	2.92	2.17	2.35	0.84
NP2A	5	5	7	8	7	7	8	7	11
STDV:	1.9	2.41	3.27	3.21	2.28	2.83	3.49	3.13	4.44
NP2B	6	5	7	8	7	6	6	7	9
STDV:	2.76	2.76	3.94	6.38	1.82	3.05	2.88	2.41	3.29
NP2C	6	6	7	6	7	8	8	8	8
STDV:	4.15	2.5	2.95	2.39	2.88	3.03	3.7	3.42	2.83
NP3A	5	6	8	6	6	8	9	7	9
STDV:	2.09	2.53	4.62	3.91	2.95	4.83	5.22	3.49	3.36
NP3B	6	6	7	7	8	7	7	7	8
STDV:	1.4	2.2	4.1	5.41	5.36	5.13	6.16	4.69	3.85
NP3C	5	6	7	9	7	7	8	7	7
STDV:	1.85	2.79	3.03	3.58	2.61	1.87	5.63	3.24	3.36
NPM	5	6	8	7	7	10	10	12	11
STDV:	1.99	3.27	3.51	2.83	3.13	3.78	3.9	4.64	6.3

Table B-5 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, N_1 = 5 Years, N_2 = 5 to 75 Years, $r_{xy} < 0.666$

N1 =	5														
N2 =	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
NP1A	4	6	8·	11	14	16	19	22	40	41	45	40	43	39	36
STDV:	1.55	3.23	5.01	7.14	8.05	10.38	12.98	15.51	8.39	15.31	6.43	9.87	10.44		
NP1B	4	6	8	10	14	15	16	24	36	44	45	46	48	45	48
STDV:	1.45	3.22	4.73	6.89	8.18	10.33	11.05	14.21	14.15	9.54	5.29	4.16	2		
NP1C	4	6	10	11	14	16	23	18	30	32	26	23	30	24	10
STDV:	1.56	3.81	5.69	7.77	9.33	10.11	12.16	15.16	21.22	21.39	15.53	15.1	18.15		
NP2A	3	6	9	11	13	16	18	23	40	45	44	45	43	42	45
STDV:	1.64	3.27	4.81	6.55	8.15	10.8	13.74	16.59	8.39	8.39	5.57	7.23	6.24		
NP2B	3	7	9	12	16	16	19	23	36	40	41	47	41	36	40
STDV:	1.44	2.97	4.63	6.56	7.07	10.12	12.47	13.53	14.15	15.89	12.29	5.77	8.14		
NP2C	3	6	10	13	14	16	20	20	39	48	35	39	44	45	41
STDV:	1.58	3.29	4.61	6.4	9.14	11.06	12.58	13.45	10.97	4.04	3.46	4.62	3.79		
NP3A	4	6	9	12	11	14	16	20	19	42	29	38	41	37	13
STDV:	1.47	3.34	5.43	6.95	8.71	9.88	6.85	12.09	9.85	13.58	1.53	5.69	6.43		
NP3B	4	6	9	11	12	17	20	22	29	45	30	40	36	36	12
STDV:	1.41	3.48	5.01	7.01	8.88	11.62	10	12.09	15.52	8.39	11.5	4.73	21.66		
NP3C	4	7	11	14	17	19	20	17	27	22	29	33	35	9	9
STDV:	1.37	3.07	4.76	6.47	8.83	11.15	10.35	11.07	7.64	17.16	18.93	19.47	22.37		
NPM	3	5	8	9	12	16	16	15	25	37	18	18	22	19	20
STDV:	1.2	2.96	3.59	5.88	5.82	7.89	9.02	12.08	19.08	23.09	16.74	19.73	20.4		

Table B-6 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, N_1 = 10 Years, N_2 = 10 to 70 Years, r_{xy} < 0.666

N1 =	10												
N2 =	10	15	20	25	30	35	40	45	50	55	60	65	70
NP1A	6	9	10	14	16	18	30	35	42	43	41	33	31
STDV:	2.36	4.24	6.38	7.6	9.98	12.45	13.43	15.89	13.58	12.7	9.81		
NP1B	7	9	11	13	14	17	33	37	40	42	41	32	33
STDV:	2.82	4.88	5.87	8.09	9.7	12.37	11.85	13	15.89	13.28	10.39		
NP1C	9	10	13	15	17	18	34	29	23	30	24	23	18
STDV:	1.76	5.31	6.63	8.37	9.38	12.92	3	19.16	20.98	16.17	14		
NP2A	6	9	10	13	16	19	34	38	43	42	43	36	39
STDV:	2.53	3.98	6.24	7.17	9.63	12.79	9.29	9.87	10.44	9.71	9.87		
NP2B	7	9	11	12	16	19	32	37	41	41	43	33	38
STDV:	2.89	4.38	6.22	7.19	9.1	11.85	13	12.42	14.73	12.1	9.81		
NP2C	7	10	12	14	16	16	26	30	42	37	37	26	22
STDV:	2.44	4.49	7.33	8.69	9.98	11.43	12.66	13.65	14.43	12.01	11.37		
NP3A	6	9	11	12	15	17	22	23	20	22	21	11	11
STDV:	3.47	4.3	6.03	6.74	8.36	12.47	16.26	19.86	26	14.18	18.82		
NP3B	6	9	11	13	11	14	22	35	40	33	31	27	12
STDV:	3.38	4.42	7.28	6.69	8.42	7.7	15.95	15.89	16.46	10.41	12.17		
NP3C	8	10	14	16	19	16	17	22	25	21	32	9	9
STDV:	2.4	5.28	7.1	8.15	6.56	10.11	12.22	20.6	22.74	18.25	20.07		
NPM	7	9	11	14	14	16	25	29	36	34	36	19	20
STDV:	2.23	4.4	5.21	6.53	8.62	9.75	16.09	17.16	23.67	17.79	16.82		

Table B-7 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, N_1 = 15 Years, N_2 = 15 to 65 Years, $r_{xy} < 0.666$

N1 =	15										
N2 =	15	20	25	30	35	40	45	50	55	60	65
NP1A	8	10	11	12	24	28	32	36	35	24	15
STDV:	3.5	4.48	6.75	6.58	11	13.05	14.19	15.63	11.85		
NP1B	8	9	11	13	22	27	30	33	36	12	13
STDV:	3.8	4.81	5.23	6.32	5.51	6.08	9.54	9.54	11.59		
NP1C	10	10	12	11	16	19	21	15	18	6	7
STDV:	4.67	6.24	8.98	11.3	14.18	16.65	17.78	8.02	9.54		
NP2A	7	8	12	15	23	26	26	33	31	13	15
STDV:	3.49	4.62	5.45	8.61	11.14	13.05	18.04	16.52	16.82		
NP2B	8	10	12	12	22	29	27	32	33	10	11
STDV:	3.32	5.27	5.53	8.2	7.09	12.12	10.02	11.37	11.59		
NP2C	8	11	9	11	18	27	26	27	28	18	21
STDV:	4.38	6.93	3.52	3.76	4.36	12.22	4.73	5.57	9.29		
NP3A	7	8	11	13	12	20	16	18	21	8	11
STDV:	4.11	3.31	3.78	8.85	6.51	6.24	9.07	10.69	11.68		
NP3B	7	8	9	11	17	19	20	22	24	10	14
STDV:	4.21	4.72	4.04	5.12	4.73	5.29	6.11	9.07	7		
NP3C	11	12	14	12	16	16	17	22	24	11	12
STDV:	4.62	5.89	5.66	6.18	9.07	13.45	15.13	14	15.37		
NPM	7	11	11	13	22	25	27	33	34	15	17
STDV:	3.16	4.13	6.42	8.58	13.58	16.09	19.67	18.08	17.79		

Table B-8 Avg. No. of Nearest Neighbors Used by Nonparametric Techniques Averaged Over All Trials, and Rounded to Nearest Integer. Standard Deviation of Nearest Neighbors Used Shown Below Average Value, N_1 = 20 Years, N_2 = 20 to 60 Years, r_{xy} < 0.666

N1 =	20								
N2 =	20	25	30	35	40	45	50	55	60
NP1A	10	10	22	26	19	22	27	9	12
STDV:	4.21	4.86	9.85	11.53	10.69	11.59	11.59		
NP1B	9	11	20	24	20	21	21	9	9
STDV:	4.14	5.38	5.51	7.02	9.61	12.29	15.87		
NP1C	10	11	15	13	13	18	16	10	7
STDV:	7.12	7.55	9.07	10.44	7.55	9.64	8.62		
NP2A	9	10	19	23	19	15	24	12	10
STDV:	2.71	5.15	9.64	11.59	18.01	9.29	20.88		
NP2B	10	12	21	25	27	23	23	9	10
STDV:	4.26	5.68	6.11	8.74	11.93	17.24	20.79		
NP2C	8	10	13	23	25	19	27	15	14
STDV:	3.07	4.81	3.51	11.59	14.53	9.07	19.86		
NP3A	11	12	16	18	15	16	18	10	10
STDV:	3.64	7.07	3.06	14.93	9.64	13.89	15.59		
NP3B	9	11	14	18	17	19	19	10	11
STDV:	4.05	6	3.06	6.43	5.51	7.37	10.07		
NP3C	11	11	15	15	15	18	23	10	11
STDV:	4.96	5.19	6.08	10.82	13.08	14.15	13.32		
NPM	9	11	19	23	24	28	30	16	14
STDV:	2.33	6.97	11	13.05	17.04	15.01	17.16		

APPENDIX C

EVALUATION RESULTS

Table C-1a Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, N_1 = 5 years, r_{xy} > 0.666

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	MV2	-0.16	0.10
	MV3	-0.16	0.10
10	REGM	-0.11	0.07
15	REGM	-0.10	0.07
20	REGM	-0.10	0.07
	MV2M	-0.10	0.06
25	REGM	-0.10	0.06
	MV1M	-0.10	0.05
	MV2M	-0.10	0.05
30	REGM	-0.10	0.06
	MV2M	-0.10	0.05
35	REGM	-0.09	0.05
	MV2M	-0.09	0.05
40	REGM	-0.10	0.05
45	NP1B	-0.08	0.03
	NP2B	-0.08	0.03
	REGM	-0.08	0.05
	NPM	-0.08	0.04
50	REGM	-0.08	0.05
	NPM	-0.08	0.05
55	REGM	-0.08	0.05
	NPM	-0.08	0.05
60	REGM	-0.08	0.05
	NPM	-0.08	0.04
65	NPM	-0.08	0.04
70	NP1B	-0.08	0.03
	REGM	-0.08	0.05
	NPM	-0.08	0.04
75	NP1B	-0.08	0.03
	REGM	-0.08	0.05
	NPM	-0.08	0.04

Table C-1b Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, N_1 = 10 years, r_{xy} > 0.666

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	MV2M	-0.14	0.07
	NPM	-0.14	0.07
15	REGM	-0.13	0.06
	NPM	-0.13	0.07
20	MV2M	-0.13	0.07
	NPM	-0.13	0.06
25	NPM	-0.12	0.07
30	REGM	-0.10	0.05
35	REGM	-0.11	0.05
40	REGM	-0.08	0.04
	NPM	-0.08	0.04
45	REGM	-0.08	0.04
	NPM	-0.08	0.04
50	REGM	-0.08	0.04
	NPM	-0.08	0.04
55	REGM	-0.08	0.04
	NPM	-0.08	0.04
60	REGM	-0.08	0.04
	NPM	-0.08	0.04
65	NP1A	-0.08	0.03
	NP2A	-0.08	0.03
	REGM	-0.08	0.04
	NPM	-0.08	0.04
70	NP1A	-0.08	0.03
	NP1B	-0.08	0.03
	NP2B	-0.08	0.03
	REGM	-0.08	0.04
	NPM	-0.08	0.04

Table C-1c Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, N_1 = 15 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	MV2M	-0.14	0.07
	MV3M	-0.14	0.07
	NPM	-0.14	0.05
20	REGM	-0.13	0.06
	NPM	-0.13	0.06
25	REGM	-0.11	0.04
30	REGM	-0.11	0.05
35	NPM	-0.08	0.04
40	NPM	-0.08	0.04
45	NPM	-0.08	0.04
50	NPM	-0.08	0.04
55	NPM	-0.08	0.04
60	NPM	-0.08	0.04
65	NPM	-0.08	0.04

Table C-1d Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, N_1 = 20 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	REGM	-0.11	0.04
25	REGM	-0.11	0.05
30	REGM	-0.08	0.05
	NPM	-0.08	0.04
35	REGM	-0.08	0.05
	NPM	-0.08	0.04
40	REGM	-0.08	0.05
	NPM	-0.08	0.04
45	REGM	-0.08	0.05
	NPM	-0.08	0.04
50	REGM	-0.08	0.05
	NPM	-0.08	0.04
55	REGM	-0.08	0.05
	NPM	-0.08	0.04
60	REGM	-0.08	0.05
	NPM	-0.08	0.04

Table C-2a Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, N_1 = 5 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	NP1A	-0.26	0.11
	NP1B	-0.26	0.11
	NP2B	-0.26	0.11
10	NP1A	-0.26	0.11
	NP1B	-0.26	0.12
	NP1C	-0.26	0.12
	NP2A	-0.26	0.11
	NP2B	-0.26	0.11
	NP2C	-0.26	0.12
	NP3B	-0.26	0.10
	NPM	-0.26	0.11
15	NP3A	-0.24	0.12
	NP3B	-0.24	0.12
20	NP3B	-0.23	0.09
25	NP1A	-0.22	0.08
	NP2A	-0.22	0.08
30	NP2A	-0.22	0.08
35	NP1A	-0.20	0.05
	NP2A	-0.20	0.05
	NPM	-0.20	0.06
40	NP2A	-0.20	0.05
	NPM	-0.20	0.06
45	NP3A	-0.17	0.03
50	NP3C	-0.17	0.03
55	NP3C	-0.17	0.03
60	NP3C	-0.17	0.03
65	NP3C	-0.17	0.03
	NPM	-0.17	0.05
70	REGM	-0.11	(n/a, one trial)
75	REGM	-0.11	(n/a, one trial)
	NPM	-0.11	(n/a, one trial)

Table C-2b Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, $N_1 = 10$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	NP2A	-0.25	0.08
	NPM	-0.25	0.08
15	NP1B	-0.24	0.07
	NP3A	-0.24	0.08
20	NP1A	-0.24	0.06
	NP1B	-0.24	0.06
	NP2A	-0.24	0.06
	NP2B	-0.24	0.06
	NP3A	-0.24	0.07
	NP3B	-0.24	0.07
	NPM	-0.24	0.07
25	NP1B	-0.23	0.07
30	NPM	-0.21	0.05
35	NPM	-0.21	0.05
40	NPM	-0.18	0.05
45	NP3C	-0.18	0.04
	NPM	-0.18	0.05
50	NP3C	-0.18	0.04
	NPM	-0.18	0.05
55	NP1A	-0.18	0.05
	NP1B	-0.18	0.05
	NP2B	-0.18	0.05
	NP3B	-0.18	0.05
	NP3C	-0.18	0.04
	NPM	-0.18	0.05
60	NP1A	-0.18	0.05
	NP1B	-0.18	0.05
	NP3B	-0.18	0.05
	NP3C	-0.18	0.04
	NPM	-0.18	0.05
65	NPM	-0.11	(n/a, one trial)
70	NPM	-0.11	(n/a, one trial)

Table C-2c Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, $N_1 = 15$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	NP2B	-0.23	0.06
	NP3A	-0.23	0.06
	NP3B	-0.23	0.07
20	NP3B	-0.22	0.06
25	NPM	-0.20	0.05
30	NPM	-0.20	0.05
35	NPM	-0.16	0.03
40	NPM	-0.16	0.03
45	NP3A	-0.16	0.01
	NP3B	-0.16	0.01
	NP3C	-0.16	0.02
	NPM	-0.16	0.03
50	NP1A	-0.16	0.02
	NP1B	-0.16	0.02
	NP2C	-0.16	0.01
	NP3A	-0.16	0.01
	NP3B	-0.16	0.01
	NP3C	-0.16	0.02
	NPM	-0.16	0.03
55	NP1A	-0.16	0.02
	NP3A	-0.16	0.02
	NP3B	-0.16	0.01
	NP3C	-0.16	0.02
	NPM	-0.16	0.03
60	NPM	-0.13	(n/a, one trial)
65	NPM	-0.13	(n/a, one trial)

Table C-2d Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, N_1 = 20 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	NPM	-0.20	0.06
25	NPM	-0.20	0.05
30	NPM	-0.15	0.01
35	NPM	-0.15	0.02
40	NP1A	-0.15	0.01
	NP1B	-0.15	0.01
	NP3C	-0.15	0.03
	NPM	-0.15	0.01
45	NP1A	-0.15	0.01
	NP1B	-0.15	0.01
	NP3A	-0.15	0.01
	NP3B	-0.15	0.01
	NP3C	-0.15	0.03
	NPM	-0.15	0.01
50	NP1A	-0.15	0.01
	NP1B	-0.15	0.01
	NP2C	-0.15	0.01
	NP3A	-0.15	0.01
	NP3C	-0.15	0.02
	NPM	-0.15	0.01
55	NPM	-0.13	(n/a, one trial)
60	NPM	-0.13	(n/a, one trial)

Table C-3a Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, $N_1 = 5$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	MV2	0.00	0.36
	MV3	0.00	0.37
	REGM	0.00	0.45
10	MV1	0.00	0.33
15	MV1	0.01	0.33
	MV2	0.01	0.34
	MV3	0.01	0.35
20	MV3M	0.00	0.25
25	MV3M	0.01	0.25
30	MV3M	0.01	0.27
35	MV1M	-0.01	0.25
	MV2M	0.01	0.23
40	MV1	-0.03	0.26
	MV2	-0.03	0.26
	MV2M	-0.03	0.24
45	MV3M	0.00	0.11
50	MV3M	0.00	0.11
55	MV3M	0.01	0.11
60	MV3M	0.00	0.12
65	MV3M	0.00	0.12
70	MV3M	0.00	0.11
75	MV3M	0.01	0.11

Table C-3b Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, $N_1 = 10$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	MV2	-0.05	0.27
	MV3	-0.05	0.27
15	REGM	0.00	0.19
20	REGM	0.00	0.29
25	MV1	-0.02	0.27
	MV2	-0.02	0.27
	REGM	0.02	0.22
30	REG	0.02	0.25
35	REG	0.01	0.20
40	MV1	0.01	0.18
	MV2	0.01	0.18
	MV3	0.01	0.18
	RPNM	-0.01	0.08
45	MV1	0.01	0.19
	MV2	0.01	0.19
	MV3	0.01	0.19
50	MV1	0.01	0.20
	MV2	0.01	0.20
	MV3	0.01	0.19
55	MV2	0.01	0.20
60	MV1	0.02	0.20
	MV2	0.02	0.20
	MV3	0.02	0.20
65	MV1	0.02	0.20
	MV2	0.02	0.20
	MV3	0.02	0.20
70	MV1	0.02	0.21
	MV2	0.02	0.20
	MV3	0.02	0.20

Table C-3c Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, $N_1 = 15$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	REGM	-0.02	0.25
20	REGM	-0.01	0.17
25	REGM	-0.01	0.13
30	REG	0.00	0.21
35	NP3C	-0.01	0.12
40	RPNM	0.00	0.05
45	RPNM	0.00	0.10
50	NP3B	0.00	0.14
55	NP3A	0.00	0.11
60	NP3B	0.00	0.12
65	MV2	-0.01	0.20
	MV3	-0.01	0.20

Table C-3d Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, $N_1 = 20$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	NP3C	-0.01	0.13
25	REG	-0.01	0.23
30	NP3B	0.01	0.22
	RPNM	-0.01	0.03
35	NP3A	0.00	0.15
	NP3B	0.00	0.19
40	NP1C	0.01	0.16
	NP3B	0.01	0.17
45	NP1C	0.02	0.11
50	NP1C	0.00	0.14
55	NP1C	0.00	0.15
60	MV2	-0.02	0.21
	MV3	-0.02	0.20
	NP1C	-0.02	0.16

Table C-4a Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, N1 = 5 years, rxy < 0.666

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	NP2A	0.08	0.42
10	NP1A	0.03	0.44
15	MV2M	0.00	0.37
20	MV2M	0.00	0.36
25	MV2M	0.01	0.33
	MV3M	-0.01	0.32
30	MV3M	-0.01	0.33
35	MV3M	-0.03	0.33
40	RPNM	-0.03	0.16
45	REG	-0.03	0.10
50	RPNM	0.00	0.12
55	RPNM	-0.01	0.15
60	RPNM	-0.02	0.15
65	REG	-0.02	0.10
	RPNM	-0.02	0.15
70	MV1M	0.03	0.00
75	REG	0.05	0.00
	MV1	-0.05	0.00
	MV2	-0.05	0.00
	MV1M	0.05	0.00

Table C-4b Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, N_1 = 10 years, r_{xy} < 0.666

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	MV2M	-0.03	0.21
15	MV2M	-0.02	0.25
20	REGM	0.05	0.28
	MV2M	-0.05	0.24
25	MV2M	-0.02	0.24
30	RPNM	0.04	0.20
35	RPNM	-0.03	0.09
40	REG	0.07	0.18
45	RPNM	0.03	0.16
50	REGM	0.04	0.12
55	REGM	0.02	0.12
60	REGM	0.04	0.11
65	REG	0.00	0.00
70	REG	0.00	0.00

Table C-4c Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, $N_1 = 15$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	MV3M	0.00	0.26
20	MV1M	0.00	0.32
	MV2M	0.00	0.26
25	REG	-0.03	0.26
	RPNM	0.03	0.13
30	RPNM	0.02	0.09
35	RPNM	0.02	0.07
40	RPNM	0.04	0.06
45	REGM	0.09	0.09
50	RPNM	-0.02	0.08
55	RPNM	-0.01	0.08
60	REG	0.00	0.00
65	REG	0.00	0.00

Table C-4d Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, $N_1 = 20$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	REG	-0.06	0.21
25	RPNM	-0.03	0.14
30	REG	0.05	0.17
35	REGM	0.02	0.16
40	REGM	0.00	0.17
45	REGM	-0.02	0.17
50	REGM	0.00	0.17
55	REG	-0.02	0.00
60	REG	-0.02	0.00

Table C-5a Minimum Deviation from Target Variance, as a Fraction of Target Value, N_1 = 5 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	NP3C	-0.22	0.44
10	REGM	0.02	0.63
	RPNM	-0.02	0.53
15	RPNM	0.02	0.38
20	NP3C	-0.05	0.55
25	NP1C	0.01	0.56
30	NP1C	0.00	0.55
35	REGM	-0.02	0.67
40	MV1	-0.01	0.49
	MV3	-0.01	0.32
45	RPNM	0.00	0.08
50	NP1C	0.01	0.14
55	NP1C	0.01	0.15
60	NP1C	-0.01	0.10
65	NP1C	0.03	0.16
	MV1M	0.03	0.05
	MV2M	0.03	0.05
70	MV1M	0.00	0.06
	MV2M	0.00	0.06
75	MV1M	0.00	0.05
	MV2M	0.00	0.06

Table C-5b Minimum Deviation from Target Variance, as a Fraction of Target Value, N_1 = 10 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	NP3C	-0.03	0.42
15	REGM	0.03	0.45
	RPNM	0.03	0.40
20	NP1C	-0.03	0.25
25	REGM	0.03	0.56
30	REGM	0.03	0.53
35	RPN	-0.05	0.42
40	REGM	0.02	0.10
45	REGM	0.03	0.09
50	REGM	0.02	0.08
55	NP1C	0.00	0.19
60	REGM	0.00	0.08
65	NP1C	-0.02	0.19
	NP3C	-0.02	0.18
70	NP1C	0.01	0.19

Table C-5c Minimum Deviation from Target Variance, as a Fraction of Target Value, N_1 = 15 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	REGM	0.03	0.58
20	REGM	-0.02	0.70
25	REGM	0.01	0.60
30	REGM	0.09	0.73
35	RPNM	-0.02	0.09
40	REGM	-0.05	0.10
	RPNM	0.05	0.06
45	REGM	-0.06	0.09
	RPNM	0.06	0.06
50	MV3	0.06	0.48
55	MV3	0.04	0.48
60	RPNM	0.01	0.07
65	RPNM	0.00	0.08

Table C-5d Minimum Deviation from Target Variance, as a Fraction of Target Value, $N_1 = 20$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	REGM	0.02	0.67
25	MV3	0.09	0.37
30	NP1C	-0.05	0.12
35	NP1C	-0.06	0.09
40	RPNM	0.05	0.04
45	RPNM	0.00	0.02
50	RPNM	0.00	0.02
55	MV3	0.04	0.51
	RPNM	-0.04	0.04
60	MV3	0.03	0.52

Table C-6a Minimum Deviation from Target Variance, as a Fraction of Target Value, $N_1 = 5$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	MV2	0.01	0.75
10	NP1C	-0.03	0.95
15	MV2	0.01	0.72
	NP1C	-0.01	0.83
20	RPNM	-0.01	0.76
25	MV3	0.05	0.67
	NP3C	-0.05	0.80
30	MV1	-0.03	0.63
	MV3	-0.03	0.62
35	MV3	0.12	0.37
40	RPNM	-0.05	0.57
45	REGM	-0.03	0.39
50	REGM	0.09	0.50
55	REGM	0.07	0.46
60	REGM	0.08	0.47
65	REGM	0.06	0.44
70	MV3M	0.04	(n/a, one trial)
75	MV3M	0.02	(n/a, one trial)

Table C-6b Minimum Deviation from Target Variance, as a Fraction of Target Value, N_1 = 10 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	NP2A	0.00	0.60
	NP2B	0.00	0.60
15	MV3	-0.01	0.61
	NP1B	-0.01	0.54
20	NP1B	0.00	0.49
25	NP1B	0.00	0.48
	NP1C	0.00	0.43
	NP3C	0.00	0.42
30	RPNM	-0.10	0.33
35	MV3	0.13	0.34
40	NP3A	0.05	0.39
45	NP2A	0.01	0.36
	NP2B	0.01	0.36
	NP3B	0.01	0.35
50	NP2A	0.00	0.34
	NP3A	0.00	0.24
55	NP2B	-0.01	0.27
	NP3C	0.01	0.22
60	NP3C	-0.01	0.20
65	MV1	-0.01	(n/a, one trial)
70	MV1	-0.01	(n/a, one trial)

Table C-6c Minimum Deviation from Target Variance, as a Fraction of Target Value, N_1 = 15 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	MV1	-0.01	0.64
20	RPNM	0.05	0.56
25	MV2	0.07	0.30
30	MV1	0.04	0.33
	MV2	0.04	0.31
35	MV3	0.00	0.23
40	REGM	-0.10	0.19
45	MV3	0.11	0.30
50	MV3	0.05	0.26
55	MV3	0.02	0.24
60	MV1	0.00	(n/a, one trial)
65	MV1	0.00	(n/a, one trial)

Table C-6d Minimum Deviation from Target Variance, as a Fraction of Target Value, $N_1 = 20$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	MV2	0.00	0.26
25	MV3	-0.01	0.28
30	MV3	0.01	0.21
35	REGM	0.02	0.26
40	REGM	-0.01	0.22
45	REGM	-0.02	0.20
50	MV3	0.04	0.21
	REGM	-0.04	0.20
55	MV3	-0.02	(n/a, one trial)
60	MV2	0.03	(n/a, one trial)
	MV3	-0.03	(n/a, one trial)

Table C-7a Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, $N_1 = 5$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	NPM	0.00	0.22
10	NPM	0.02	0.21
15	REGM	0.01	0.10
20	RPN	0.01	0.18
	REGM	-0.01	0.11
	NPM	0.01	0.19
25	REGM	0.00	0.11
30	REGM	-0.01	0.11
35	REGM	0.02	0.07
	NPM	0.02	0.05
40	NPM	0.00	0.06
45	NP2A	0.00	0.06
	NP2B	0.00	0.05
	REGM	0.00	0.04
50	RPNM	0.00	0.04
55	REGM	0.00	0.04
60	RPNM	0.00	0.04
65	RPNM	0.00	0.04
70	RPNM	0.00	0.05
75	NP1B	0.01	0.06
	NP3A	-0.01	0.04
	REGM	0.01	0.04
	RPNM	-0.01	0.05

Table C-7b Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, N_1 = 10 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	NPM	0.01	0.19
15	NP1B	-0.01	0.26
	NP3A	0.01	0.29
	NP3B	-0.01	0.29
20	NP3B	0.00	0.28
25	MV2	0.00	0.17
	MV3	0.00	0.18
	NPM	0.00	0.12
30	REGM	-0.01	0.08
35	MV1	0.00	0.11
40	NP2A	0.00	0.05
	NP2B	0.00	0.05
45	NP1B	0.00	0.05
50	RPN	0.00	0.03
55	RPN	0.01	0.03
	NP3A	-0.01	0.06
60	RPN	0.01	0.03
	NP3A	0.01	0.06
65	NP3A	0.00	0.05
70	RPN	0.01	0.03
	NP3A	0.01	0.05
	NP3B	-0.01	0.07

Table C-7c Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, N_1 = 15 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	MV3	0.00	0.17
20	RPNM	0.00	0.17
25	RPNM	0.00	0.12
	NPM	0.00	0.07
30	MV1	0.01	0.08
	NPM	-0.01	0.08
35	NPM	-0.01	0.04
40	NPM	0.00	0.04
45	NPM	0.00	0.04
50	NP3C	0.00	0.07
	NPM	0.00	0.03
55	NP1C	0.00	0.03
	NP3C	0.00	0.07
	NPM	0.00	0.04
60	NP3C	0.00	0.07
	NPM	0.00	0.04
65	NPM	0.00	0.03

Table C-7d Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, $N_1 = 20$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	RPNM	0.00	0.10
25	MV1	0.00	0.11
	NP1A	0.00	0.07
	REGM	0.00	0.12
30	NPM	0.00	0.03
35	NP1A	0.00	0.05
	REGM	0.00	0.04
40	NP1C	0.00	0.04
	NP2A	0.00	0.03
45	NP1B	0.00	0.04
	NP2A	0.00	0.05
	NP2B	0.00	0.04
	REGM	0.00	0.04
50	NP1B	0.00	0.04
	NP2A	0.00	0.05
	NP2B	0.00	0.04
	REGM	0.00	0.04
55	NP2A	0.00	0.04
	NP2B	0.00	0.04
60	NP2A	0.00	0.04
	NP2B	0.00	0.04

Table C-8a Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, $N_t = 5$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	NPM	0.00	0.32
10	MV1	0.05	0.39
	MV3	0.05	0.36
15	REGM	0.02	0.28
20	REGM	-0.01	0.23
25	REGM	0.02	0.23
30	REGM	0.00	0.21
35	NPM	-0.02	0.07
40	NP2B	0.00	0.05
	NP3A	0.00	0.07
45	NPM	0.01	0.03
50	NPM	0.00	0.01
55	NPM	0.01	0.04
60	NPM	0.02	0.03
65	NPM	-0.01	0.01
70	NP1A	0.00	0.00
	NP2B	0.00	0.00
75	NP1A	0.00	0.00
	NP1B	0.00	0.00

Table C-8b Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, N_1 = 10 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	MV3	0.07	0.41
15	MV3	0.04	0.32
20	MV3	0.06	0.31
25	MV3	0.03	0.27
	RPNM	0.03	0.22
30	RPNM	-0.01	0.11
35	NP3A	0.00	0.10
	NP3B	0.00	0.11
40	REGM	0.02	0.05
	RPNM	0.02	0.03
45	REGM	0.03	0.04
50	REGM	0.02	0.03
55	REGM	0.03	0.03
60	REGM	0.03	0.03
65	NP3A	-0.01	(n/a, one trial)
	NP3B	-0.01	(n/a, one trial)
70	NP3A	0.00	(n/a, one trial)

Table C-8c Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, N_1 = 15 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	MV3	0.03	0.25
	REGM	0.03	0.20
20	MV3	0.00	0.24
25	NPM	0.02	0.08
30	MV1	0.00	0.08
	NPM	0.00	0.06
35	NP3A	0.00	0.13
40	NP1C	0.03	0.08
45	RPNM	-0.01	0.02
50	NP3A	0.02	0.10
	RPNM	0.02	0.03
55	RPNM	0.02	0.03
60	MV3	0.00	(n/a, one trial)
	NP1C	0.00	(n/a, one trial)
65	MV3	0.00	(n/a, one trial)

Table C-8d Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, $N_1 = 20$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	MV1	0.00	0.08
25	RPNM	0.00	0.08
30	REGM	-0.03	0.03
35	REGM	-0.02	0.02
40	REGM	-0.03	0.01
45	REGM	-0.02	0.01
50	NP1C	0.03	0.04
	REGM	-0.03	0.01
	RPNM	0.03	0.03
55	NP1B	0.00	(n/a, one trial)
	NP2B	0.00	(n/a, one trial)
	NPM	0.00	(n/a, one trial)
60	NP2B	0.00	(n/a, one trial)

Table C-9a Minimum Deviation from Target Low Flow, as a Fraction of Target Value, N_t = 5 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	MV3M	0.18	0.55
10	MV1M	0.30	0.81
15	RPNM	0.28	0.63
20	MV1M	0.29	0.61
25	MV3M	0.32	0.57
30	MV3M	0.31	0.52
35	RPNM	0.11	0.15
40	MV1M	0.07	0.14
	MV2M	0.07	0.16
45	MV3M	0.02	0.06
50	RPNM	0.02	0.07
55	RPNM	0.01	0.07
60	RPNM	0.04	0.09
	MV1M	0.04	0.06
65	RPNM	0.04	0.10
	MV1M	0.04	0.07
	MV2M	0.04	0.07
70	RPNM	0.05	0.11
75	RPNM	0.05	0.10

Table C-9b Minimum Deviation from Target Low Flow, as a Fraction of Target Value, N_1 = 10 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	MV1M	0.17	0.39
15	MV1M	0.14	0.31
20	MV1M	0.13	0.30
25	MV1M	0.11	0.34
30	MV1M	0.10	0.32
35	MV1M	0.01	0.23
	MV3M	0.01	0.17
40	MV1	0.00	0.13
	MV2	0.00	0.13
45	NP3A	0.00	0.07
	RPNM	0.00	0.16
50	MV3M	0.00	0.18
55	MV1M	0.00	0.23
60	MV1M	0.00	0.23
	MV2M	0.00	0.23
65	MV1M	0.01	0.24
	MV2M	0.01	0.24
	MV3M	0.01	0.15
70	MV1M	0.00	0.23
	MV2M	0.00	0.23

Table C-9c Minimum Deviation from Target Low Flow, as a Fraction of Target Value, N_l = 15 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	MV1M	0.08	0.25
20	MV1M	0.06	0.27
25	MV1M	0.06	0.29
30	MV3M	0.04	0.29
35	MV2	0.00	0.15
40	MV3	0.01	0.16
	NP3A	-0.01	0.11
	NP3B	0.01	0.08
	MV1M	-0.01	0.24
45	MV3	0.01	0.17
	NP3B	0.01	0.07
50	REGM	0.00	0.21
	MV2M	0.00	0.26
55	MV1M	0.00	0.28
60	MV3M	0.00	0.20
65	MV3M	0.00	0.20

Table C-9d Minimum Deviation from Target Low Flow, as a Fraction of Target Value, N_l = 20 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	MV1M	0.03	0.35
25	MV3M	0.08	0.29
30	REGM	0.01	0.14
	MV1M	-0.01	0.21
	MV2M	-0.01	0.19
35	MV2M	0.00	0.22
40	MV1M	0.00	0.24
	MV2M	0.00	0.23
45	MV3M	0.01	0.20
50	MV3M	0.01	0.20
55	MV1M	0.04	0.27
	MV3M	0.04	0.19
60	MV1M	0.04	0.27
	MV2M	0.04	0.25
	MV3M	0.04	0.18

Table C-10a Minimum Deviation from Target Low Flow, as a Fraction of Target Value, N_1 = 5 years, r_{xy} < 0.666

Common Period (Years)	Models	Average Deviation	Standard Deviation
5	MV1M	0.00	0.51
10	MV3	-0.02	0.40
15	MV3	0.08	0.47
20	MV1M	0.05	0.33
25	MV1M	0.11	0.42
30	MV1M	0.11	0.39
35	MV1M	0.17	0.16
	MV2M	0.17	0.15
	MV3M	0.17	0.17
40	MV2M	0.13	0.14
45	MV3	0.01	0.13
50	MV3M	0.00	0.05
55	MV3M	0.01	0.03
60	MV3	0.00	0.16
65	MV3	0.00	0.16
70	RPNM	0.15	(n/a, one trial)
	NPM	0.15	(n/a, one trial)
75	RPNM	0.13	(n/a, one trial)
	NPM	0.13	(n/a, one trial)

Table C-10b Minimum Deviation from Target Low Flow, as a Fraction of Target Value, $N_1 = 10$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
10	MV1M	0.06	0.57
15	MV1M	0.01	0.48
20	MV1M	0.11	0.53
25	MV1M	0.09	0.48
30	MV2M	0.18	0.31
35	MV3M	0.09	0.12
40	MV1M	0.00	0.15
	MV2M	0.00	0.13
45	RPNM	0.03	0.02
	MV3M	-0.03	0.07
50	MV3M	-0.02	0.07
55	MV3M	0.01	0.06
60	MV3M	0.01	0.06
65	MV1M	0.06	(n/a, one trial)
70	MV1M	0.04	(n/a, one trial)

Table C-10c Minimum Deviation from Target Low Flow, as a Fraction of Target Value, $N_1 = 15$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
15	MV1M	0.07	0.37
20	MV1M	0.06	0.31
25	MV1M	0.12	0.17
30	MV1M	0.10	0.15
35	MV3M	0.00	0.08
40	MV1	-0.03	0.16
	MV2M	-0.03	0.10
45	MV3M	-0.02	0.04
50	MV3M	0.00	0.03
55	MV3M	0.00	0.03
60	MV2M	-0.01	(n/a, one trial)
	MV3M	0.01	(n/a, one trial)
65	MV3M	-0.01	(n/a, one trial)

Table C-10d Minimum Deviation from Target Low Flow, as a Fraction of Target Value, $N_1 = 20$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Average Deviation	Standard Deviation
20	MV1M	0.11	0.20
25	MV1M	0.14	0.20
30	MV3M	0.05	0.11
35	MV3	0.00	0.08
	MV1M	0.00	0.16
	MV3M	0.00	0.06
40	MV3	0.00	0.07
	MV2M	0.00	0.13
45	MV1M	0.00	0.13
50	MV2M	0.01	0.10
55	MV3M	0.02	(n/a, one trial)
60	MV3M	0.02	(n/a, one trial)

Table C-11a Average Mean Percentage Error, $N_1 = 5$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
5	MV3	100.5	149.2
10	MV3	110.1	192.3
15	RPNM	95.4	162.9
20	NP1B	75.5	117.3
25	NP3A	91.9	153.0
30	NP3A	73.5	119.7
35	NP3B	33.8	28.2
40	NP3B	20.0	12.6
45	NP3B	10.5	7.1
50	NP3B	12.0	8.2
55	NP3A	10.4	4.1
60	NP3B	12.6	7.2
65	RPNM	13.5	4.9
70	RPNM	13.7	5.2
75	RPNM	13.6	5.1

Table C-11b Average Mean Percentage Error,
N_1 = 10 years, $r_{xy} > 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
10	MV1M	65.3	102.4
15	MV1M	62.2	95.0
20	MV1M	62.2	90.0
25	MV1M	60.2	85.2
30	MV1M	29.2	23.2
35	REGM	15.2	15.5
40	REGM	10.0	5.2
45	REGM	11.2	6.0
50	REGM	10.9	5.8
55	REGM	12.0	6.7
60	REGM	12.4	6.9
65	REGM	13.2	7.1
70	REGM	13.1	7.1

Table C-11c Average Mean Percentage Error,
N_1 = 15 years, $r_{xy} > 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
15	MV3M	53.2	71.7
20	MV3M	51.2	68.4
25	MV1M	29.2	28.0
30	REGM	17.4	29.0
35	NP3B	0.9	3.6
40	NP3B	4.0	4.1
45	NP3B	4.8	5.1
50	NP3B	4.9	3.4
55	NP3B	7.7	4.5
60	REGM	10.1	7.1
65	NP3B	8.4	3.5

Table C-11d Average Mean Percentage Error, $N_1 = 20$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
20	REGM	28.9	25.9
25	REGM	21.4	33.7
30	NP3B	8.3	7.5
35	NP3B	10.7	7.6
40	REGM	11.0	7.5
45	REGM	12.3	8.6
50	REGM	12.7	8.9
55	REGM	13.7	9.1
60	REGM	13.6	8.9

Table C-12a Average Mean Percentage Error, $N_1 = 5$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
5	REG	83.37	123.39
10	REG	86.68	132.47
15	NP3B	95.08	126.53
20	NP3B	87.6	121.4
25	MV1M	108.15	175.01
30	NP2A	102.1	123.14
35	RPNM	31.85	25.97
40	REGM	27.49	16.35
45	REGM	15.03	2.47
50	REGM	15.83	2.28
55	REGM	15.37	1.99
60	REGM	17.29	2.48
65	REGM	17.58	1.9
70	REGM	18.27	(n/a, one trial)
75	REGM	18.06	(n/a, one trial)

Table C-12b Average Mean Percentage Error,
N_1 = 10 years, $r_{xy} < 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
10	MV1M	108.79	155.13
15	MV3M	100.15	136.51
20	MV3M	113.44	132.29
25	RPNM	110.39	142.88
30	REGM	36.07	34.11
35	REGM	24.6	13.38
40	RPNM	17.02	3.77
45	REGM	18.38	3.9
50	REGM	18	3.75
55	REGM	19.89	3.98
60	REGM	19.98	3.96
65	REGM	16.43	(n/a, one trial)
70	REGM	16.28	(n/a, one trial)

Table C-12c Average Mean Percentage Error,
N_1 = 15 years, $r_{xy} < 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
15	MV3M	117.82	132.25
20	MV3M	125.06	142.36
25	REGM	34.12	24.97
30	REGM	26.1	14.81
35	REGM	15.97	7.76
40	REGM	15.89	5.8
45	REGM	15.93	6.13
50	REGM	18.07	6.53
55	REGM	18	6.54
60	REGM	11.34	(n/a, one trial)
65	REGM	11.17	(n/a, one trial)

Table C-12d Average Mean Percentage Error,
N_1 = 20 years, $r_{xy} < 0.666$

Common Period (Years)	Models	MPE	Standard Deviation
20	REGM	34.9	21.63
25	REGM	28.3	17.38
30	REGM	20.37	8.67
35	REGM	19.46	6.02
40	REGM	19.57	6.54
45	REGM	21.85	6.99
50	REGM	21.75	7.02
55	REGM	14.65	(n/a, one trial)
60	REGM	14.39	(n/a, one trial)

Table C-13a Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, N_1 = 5 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
5	MV3M	0.19
10	REGM	0.19
15	REGM	0.33
20	REGM	0.19
25	MV1M	0.19
30	RPNM	0.24
35	REGM	0.40
40	REGM	0.40
45	NP1C	0.20
	NP3A	0.20
	REGM	0.20
	MV3M	0.20
	NPM	0.20
50	NP1C	0.20
	NP3A	0.20
	REGM	0.20
	RPNM	0.20
	NPM	0.20
55	NP1C	0.40
60	NP1C	0.20
	NP3B	0.20
	RPNM	0.20
	MV3M	0.20
	NPM	0.20
65	NP1C	0.20
	NP3B	0.20
	REGM	0.20
	RPNM	0.20
	NPM	0.20
70	REGM	0.40
75	NP1B	0.20
	REGM	0.20
	RPNM	0.20
	MV1M	0.20
	MV3M	0.20

Table C-13b Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, N_1 = 10 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
10	REGM	0.24
15	REGM	0.24
	MV2M	0.24
	NPM	0.24
20	REGM	0.29
	MV2M	0.29
25	REGM	0.29
30	REGM	0.33
35	REGM	0.30
	NPM	0.30
40	REGM	0.40
45	NPM	0.40
50	NPM	0.40
55	NPM	0.40
60	NP2A	0.40
	NPM	0.40
65	REGM	0.40
70	NP1C	0.40
	REGM	0.40

Table C-13c Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, $N_1 = 15$ years, $r_{xy} > 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
15	REGM	0.24
20	REGM	0.29
25	REGM	0.47
30	REGM	0.30
	MV3M	0.30
35	NP2A	0.40
	REGM	0.40
40	NP2A	0.40
	RPNM	0.40
45	REGM	0.40
	RPNM	0.40
50	NP1A	0.20
	NP2B	0.20
	REGM	0.20
	RPNM	0.20
	NPM	0.20
55	REGM	0.40
	NPM	0.40
60	RPNM	0.40
	NPM	0.40
65	RPNM	0.40
	NPM	0.40

Table C-13d Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, N_1 = 20 years, $r_{xy} > 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
20	REGM	0.53
25	REGM	0.50
30	REGM	0.40
	NPM	0.40
35	REGM	0.60
40	REGM	0.60
45	REGM	1.00
50	REGM	0.80
55	REGM	0.40
60	REGM	0.40

Table C-14a Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, $N_1 = 5$ years, $r_{xy} < 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
5	NP2B	0.13
	NP3A	0.13
	REGM	0.13
	MV2M	0.13
	MV3M	0.13
10	REGM	0.33
15	REGM	0.40
20	REGM	0.40
25	REGM	0.54
30	REGM	0.31
35	REGM	0.25
	MV3M	0.25
40	MV2M	0.29
45	REGM	0.67
50	REGM	0.67
55	NPM	0.67
60	NPM	0.67
65	NPM	0.67
70	MV3M	1.00
75	MV3M	1.00

Table C-14b Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, N_1 = 10 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
10	REGM	0.40
15	REGM	0.40
20	REGM	0.31
25	REGM	0.46
30	REGM	0.38
	MV3M	0.38
35	REGM	0.57
40	RPNM	0.67
45	REGM	1.00
50	REGM	1.00
55	REGM	1.00
60	REGM	1.00
65	REGM	1.00
70	REGM	1.00

Table C-14c Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, N_1 = 15 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
15	REGM	0.46
20	REGM	0.46
25	REGM	0.50
30	RPNM	0.43
35	REGM	0.67
40	REGM	0.67
45	REGM	0.67
50	RPNM	0.67
55	REGM	0.67
60	MV3M	1.00
65	MV3M	1.00

Table C-14d Occurrence of Minimum Objective Function Value, as a Fraction of Total Trials, N_1 = 20 years, $r_{xy} < 0.666$

Common Period (Years)	Models	Occurrence of Minimum Objective Function Value
20	MV2M	0.63
25	MV2M	0.43
30	REGM	1.00
35	REGM	0.67
40	REGM	0.67
45	REGM	1.00
50	REGM	0.67
55	MV3M	1.00
60	MV3M	1.00

APPENDIX D

VERIFICATION RESULTS

Table D-1a Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 1, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV1M	-0.14		
	MV2M	-0.14		
	MV3M	-0.14		
15	MV1M	-0.11	NP2C	-0.13
	MV2M	-0.11		
	MV3M	-0.11		
20	NP3A	-0.09		
	NP3B	-0.09		
	MV2M	-0.09		
	MV3M	-0.09		
25	NP3A	-0.08		
	MV1M	-0.08		
	MV2M	-0.08		
	MV3M	-0.08		
30	NP3A	-0.08		
35	MV1M	-0.08	NP1A	-0.09
	MV2M	-0.08		
	MV3M	-0.08		
40	REGM	-0.08	NP1A	-0.09
	MV1M	-0.08	NP1B	-0.09
	MV2M	-0.08	NP2A	-0.09
			NP3A	-0.09
45	NP3A	-0.08		
	REGM	-0.08		
	MV1M	-0.08		
	MV2M	-0.08		
	MV3M	-0.08		
50	REGM	-0.08	NP1A	-0.09
	MV1M	-0.08	NP1B	-0.09
	MV2M	-0.08	NP1C	-0.09
	MV3M	-0.08	NP2A	-0.09
			NP2B	-0.09
			NP3A	-0.09

Table D-1b Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 1, N1 = 15 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV1M	-0.09	NP3A	-0.11
	MV2M	-0.09	NP3B	-0.11
	MV3M	-0.09		
20	MV1M	-0.08	NP3A	-0.10
	MV2M	-0.08		
25	REGM	-0.09	NP1A	-0.11
	MV1M	-0.09	NP1B	-0.11
	MV2M	-0.09	NP2B	-0.11
			NP3A	-0.11
30	REGM	-0.09	NP1A	-0.11
	MV1M	-0.09	NP1B	-0.11
	MV2M	-0.09	NP1C	-0.11
	MV3M	-0.09	NP2B	-0.11
			NP2C	-0.11
			NP3A	-0.11
35	REGM	-0.08	NP3A	-0.09
40	REGM	-0.08	NP3A	-0.09
45	REGM	-0.08	NP1A	-0.10
			NP1B	-0.10
			NP3A	-0.10

Table D-1c Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 1, N_1 = 20 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
20	MV2M	-0.10	NP1A	-0.16
	MV3M	-0.10	NP1B	-0.16
			NP2A	-0.16
			NPM	-0.16
25	MV1M	-0.10	NP1A	-0.15
	MV2M	-0.10	NP1B	-0.15
	MV3M	-0.10	NP2A	-0.15
30	MV1M	-0.10	NP1A	-0.14
	MV2M	-0.10	NP2A	-0.14
	MV3M	-0.10		
35	MV1M	-0.10	NP1A	-0.14
	MV2M	-0.10		
	MV3M	-0.10		
40	MV1M	-0.10	NP1B	-0.14
	MV2M	-0.10		
	MV3M	-0.10		

Table D-2a Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 1, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	RPNM	0.01		
15	RPNM	0.02	NPM	0.02
	NPM	0.02		
20	RPNM	-0.02	NPM	0.03
25	NP3C	0.01		
30	RPNM	0.00	NP3B	0.03
			NPM	0.03
35	NP3B	0.03		
	NP3C	0.03		
	RPNM	-0.03		
40	NP3B	0.03		
	NP3C	0.03		
45	NP3B	0.02		
50	NP3B	0.01		
	NP3C	0.01		

Table D-2b Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 1, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}
15	NP3C	-0.01
	REGM	0.01
	NPM	-0.01
20	NP1A	0.00
	NP2A	0.00
	REGM	0.00
25	NP1B	0.00
	NP3B	0.00
30	NP1C	0.01
	NP3A	0.01
	NP3B	-0.01
	NP3C	-0.01
	REGM	0.01
	NPM	0.01
35	NP1C	0.00
40	NP3A	0.00
45	NP2C	0.00

Table D-2c Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 1, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
20	MV1M	0.00	NP1A	-0.05
	MV2M	0.00	NP2A	-0.05
	MV3M	0.00		
25	MV1M	-0.01	NP1A	-0.05
	MV2M	-0.01	NP2A	-0.05
	MV3M	-0.01		
30	MV1M	-0.01	NP1A	-0.05
	MV2M	-0.01	NP2A	-0.05
	MV3M	-0.01	NP2B	-0.05
35	MV1M	0.00	NP1A	-0.05
	MV2M	0.00		
	MV3M	0.00		
40	MV1M	0.00	NPM	-0.05
	MV2M	0.00		
	MV3M	0.00		

Table D-3a Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 1, $N_1 = 10$ years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	NP2C	-0.01		
15	NP3C	0.08		
20	REGM	0.02	NP2C	-0.11
25	REGM	-0.01	NP2C	0.05
			NP3A	0.05
30	REGM	0.00	NP2C	0.02
35	NP2C	0.00		
40	MV2	0.00	NP3A	-0.04
45	MV2	-0.02	NP3A	-0.07
50	RPNM	0.03	NP3A	-0.06

Table D-3b Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 1, $N_1 = 15$ years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	RPNM	-0.02	NP1C	0.06
20	NP1C	-0.01		
25	NP1C	0.00		
30	MV1M	0.00	NP3C	-0.03
35	MV2M	0.00	NP3C	-0.04
40	MV3	0.01	NP3C	-0.06
45	MV3	0.00	NP1C	-0.04

Table D-3c Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 1, $N_1 = 20$ years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
20	MV3	0.03	NP1C	-0.15
25	MV3	0.01	NP1C	-0.15
30	MV3	-0.02	NP1C	-0.20
35	MV2	0.02	NP1C	-0.19
40	MV2	0.01	NP1C	-0.17

Table D-4a Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 1, N_1 = 10 years, r_{xy} > 0.666

N_2	Models	f_{ij}
10	REG	0.04
15	REG	0.04
20	REG	0.04
25	REG	0.05
30	REG	0.06
35	REG	0.08
40	REG	0.09
45	REG	0.09
50	REG	0.09

Table D-4b Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 1, N_1 = 15 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV1	0.00	RPN	-0.01
20	RPN	0.00		
25	REG	0.01		
30	REG	0.02		
35	REG	0.04		
40	REG	0.03		
45	REG	0.03		
	RPN	0.03		

Table D-4c Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 1, N_1 = 20 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
20	RPN	0.00		
25	RPN	0.01		
30	REG	0.03	RPN	0.03
	RPN	0.03		
35	RPN	0.02		
40	RPN	0.01		

Table D-5a Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 1, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV2M	-0.01		
	MV3M	0.01		
15	MV2M	0.07	NP3A	0.21
			NP3B	0.21
20	NP1C	0.07		
25	NP1C	0.09		
30	NP1C	0.13		
35	NP1C	0.12		
40	RPNM	0.17	NP1A	0.22
			NP3A	0.22
			NP3B	0.22
45	NP1C	0.19		
50	MV3M	0.19	NP1C	0.21

Table D-5b Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 1, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	NP1C	0.05		
20	NP1C	0.06		
25	NP1C	0.08		
30	MV3M	0.10	NP3B	0.13
35	MV3M	0.13	NP3B	0.14
40	NP3B	0.14		
	MV3M	0.14		
45	NP1C	0.13		
	MV3M	0.13		

Table D-5c Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 1, $N_1 = 20$ years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
20	MV1	0.10	NPM	0.19
25	MV1	0.15	NP3A	0.17
	MV2	0.15		
	MV3	0.15		
30	MV3	0.18	NP3A	0.20
	MV1M	0.18		
	MV2M	0.18		
	MV3M	0.18		
35	MV1	0.19	NP3A	0.19
	MV2	0.19		
	MV3	0.19		
	NP3A	0.19		
	MV2M	0.19		
40	MV3	0.17	NP3A	0.18
	MV3M	0.17		

Table D-6a Minimum Mean Percentage Error, Set 1, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	MPE	Alternate	MPE
10	NP3A	9.73		
15	MV1	14.69	NP3A	15.85
20	NP3B	11.71		
25	NP1A	15.02		
30	NP1A	17.03		
35	NP3B	15.88		
40	NP1A	17.54		
45	NP1A	17.85		
50	NP3B	16.21		

Table D-6b Minimum Mean Percentage Error, Set 1, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	MPE	Alternate	MPE
15	NP3A	12.38		
20	MV2M	15.87	NP3A	17.44
25	MV2M	17.63	NP3B	18.89
30	NP3B	17.50		
35	NP3B	17.55		
40	NP3B	18.57		
45	NP3A	17.04		

Table D-6c Minimum Mean Percentage Error, Set 1, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	MPE	Alternate	MPE
20	MV1M	15.30	NP3A	17.21
25	NP3B	14.50		
30	NP3B	14.43		
35	NP3A	13.91		
40	NP3A	11.40		

Table D-7a Minimum SSE,
Set 1, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	SSE
10	NP3A	76677.10
15	NP1A	58816.00
20	NP3A	39194.90
25	NP3A	42069.20
30	NP1A	43339.40
35	NP1A	44486.30
40	NP1A	42767.10
45	NP1A	42401.00
50	NP1A	40991.00

Table D-7b Minimum SSE,
Set 1, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	SSE	Alternate	SSE
15	NP3A	50849.00		
20	REGM	52116.70	NP3A	54544.10
25	REGM	57308.30	NP1A	62303.50
30	NP3B	58328.30		
35	NP3B	52867.80		
40	NP3A	52716.80		
45	NP3A	50498.40		

Table D-7c Minimum SSE,
Set 1, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	SSE	Alternate	SSE
20	REGM	104230.70	NP1A	142921.30
25	REGM	110241.90	NP1A	137105.00
30	REGM	109644.90	NP1A	130071.90
35	REGM	108094.70	NP1A	128832.70
40	REGM	103617.90	NP1B	129985.60

Table D-8a Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 2, N_1 = 5 years, r_{xy} > 0.666

N_2	Models	f_{ij}
5	REGM	-0.12
	MV1M	-0.12
	MV2M	-0.12
	MV3M	-0.12
10	MV1M	-0.11
	MV2M	-0.11
15	REGM	-0.11
20	REGM	-0.12
25	REGM	-0.12
30	REGM	-0.12
35	REGM	-0.12
40	REGM	-0.12
	MV3M	-0.12
45	REGM	-0.11
50	REGM	-0.11
55	REGM	-0.12
	MV1M	-0.12
	MV2M	-0.12
	MV3M	-0.12
	NPM	-0.12
60	REGM	-0.12
	MV1M	-0.12
	MV2M	-0.12
	MV3M	-0.12
	NPM	-0.12
65	REGM	-0.11

Table D-8b Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 2, N_1 = 10 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV1M	-0.06		
15	REGM	-0.07		
	RPNM	-0.07		
	MV1M	-0.07		
	MV2M	-0.07		
	MV3M	-0.07		
20	REGM	-0.07		
	MV1M	-0.07		
	MV2M	-0.07		
	MV3M	-0.07		
25	REGM	-0.07		
	MV2M	-0.07		
	MV3M	-0.07		
30	REGM	-0.07		
	MV1M	-0.07		
	MV2M	-0.07		
	MV3M	-0.07		
35	MV3M	-0.06	REGM	-0.07
40	MV2M	-0.06	REGM	-0.07
	MV3M	-0.06		
45	MV3M	-0.06	REGM	-0.07
50	MV3M	-0.06	REGM	-0.07
55	MV3M	-0.06	REGM	-0.07
60	REGM	-0.06		
	MV1M	-0.06		
	MV2M	-0.06		
	MV3M	-0.06		

Table D-8c Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 2, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	REGM	-0.11	NPM	-0.11
	RPNM	-0.11		
	NPM	-0.11		
20	REGM	-0.10	NPM	-0.11
25	REGM	-0.10		
30	NPM	-0.09		
35	NPM	-0.09		
40	NPM	-0.08		
45	NPM	-0.08		
50	NPM	-0.08		
55	REGM	-0.08		
	NPM	-0.08		

Table D-8d Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 2, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
20	MV1M	-0.08	RPN	-0.15
25	MV1M	-0.08	NP2A	-0.14
	MV2M	-0.08		
	MV3M	-0.08		
30	MV2M	-0.07	RPN	-0.14
	MV3M	-0.07	NP2A	-0.14
			NP2B	-0.14
			NP2C	-0.14
35	MV2M	-0.07	RPN	-0.14
	MV3M	-0.07	NP2A	-0.14
			NP2B	-0.14
			NP2C	-0.14
40	MV3M	-0.07	MV1M	-0.08
45	REGM	-0.07	NP2B	-0.12
	MV2M	-0.07		
	MV3M	-0.07		
50	MV3M	-0.06	RPNM	-0.08

Table D-9a Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 2, N_1 = 5 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}
5	NP1A	0.02
	NP1B	0.02
10	NP3B	-0.05
15	NP3B	-0.01
20	NP1C	-0.04
25	NP1C	-0.06
30	NP1C	-0.03
35	NP1C	-0.04
40	NP1C	-0.04
45	NP1C	-0.05
	NP3B	-0.05
50	NP3C	-0.06
55	NP3B	-0.05
60	NP1C	-0.05
	NP3A	-0.05
	NP3B	-0.05
65	NP1C	-0.03

Table D-9b Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 2, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}
10	REG	-0.02
15	REG	-0.01
20	REG	0.00
25	REG	0.00
30	REG	0.00
35	REG	0.00
40	REG	0.00
	NP1C	0.00
45	REG	0.00
50	REG	0.00
55	REG	0.00
60	REG	0.00

Table D-9c Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 2, N_1 = 15 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
15	NP3C	-0.02		
	REGM	0.02		
20	NP3C	-0.01		
25	NP1C	0.00		
30	NP3C	-0.02		
	REGM	0.02		
	NPM	-0.02		
35	NP2B	-0.01		
	NP2C	0.01		
	NP3C	0.01		
	RPNM	0.01		
	NPM	-0.01		
40	NP2C	0.00		
	NP3C	0.00		
	NPM	0.00		
45	MV2M	0.00	NPM	0.00
	NPM	0.00		
50	NP2B	0.00		
	NP3C	0.00		
55	NP2A	0.00		
	RPNM	0.00		

Table D-9d Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 2, N_1 = 20 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
20	REGM	0.00	NP3C	-0.01
	MV1M	0.00		
25	NP3C	0.00		
30	NP2A	-0.01		
	RPNM	0.01		
	MV1M	0.01		
35	RPNM	-0.01	NP2B	-0.02
			NP2C	0.02
			NPM	-0.02
40	NP1B	-0.01		
	NP2B	-0.01		
	RPNM	-0.01		
	MV2M	-0.01		
45	NP1B	0.00		
	NP2A	0.00		
	MV2M	0.00		
50	MV1M	0.00	NP1A	0.01
			NP1B	0.01
			NP3B	-0.01

Table D-10a Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 2, N_1 = 5 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
5	MV3	-0.02	MV3M	-0.02
	MV3M	-0.02		
10	RPN	-0.05		
	NP1C	-0.05		
15	NP3B	0.01		
20	NP1A	0.00		
	NP1B	0.00		
	NP2B	0.00		
25	NP3B	0.01		
30	NP2A	-0.03		
35	NP2A	0.00		
40	NP2A	0.00		
	NP3B	0.00		
45	NP3A	0.00		
50	NP1B	-0.02		
	NP2A	-0.02		
	NP3A	-0.02		
55	NP2A	-0.01		
	NPM	-0.01		
60	NP1A	0.01		
	NP3A	-0.01		
	RPNM	0.01		
	NPM	0.01		
65	NPM	0.01		

Table D-10b Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 2, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}
10	NP3C	-0.12
15	REG	-0.08
20	RPN	0.01
	NP3C	-0.01
25	RPN	0.02
	NP3C	-0.02
30	RPN	0.01
35	REGM	0.00
40	RPN	0.01
45	RPN	0.01
	RPNM	0.01
50	RPN	0.00
55	RPN	-0.02
	NP1C	-0.02
60	RPN	-0.03
	REGM	-0.03

Table D-10c Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 2, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV2	0.00	NP3C	-0.06
20	MV1	0.02	NP3C	0.04
25	NP3C	-0.04		
30	MV2	-0.02	NP3C	0.07
35	MV1	0.01	NP3C	0.11
40	NP3C	-0.03		
45	NP3C	-0.01		
50	MV1	0.01	RPNM	-0.03
55	MV1	0.02	NP3C	-0.08
	MV2M	0.02	MV3M	0.08

Table D-10d Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 2, $N_1 = 20$ years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
20	MV2M	0.00	NP3C	-0.05
25	MV2M	-0.03	NP3C	-0.07
30	NP3C	-0.01		
35	RPNM	-0.03	NP3C	-0.11
40	MV1	-0.02	NP3C	-0.09
45	RPNM	0.00	NP3C	-0.17
50	MV1	-0.05	RPNM	-0.18

Table D-11a Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 2, N_1 = 5 years, r_{xy} > 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
5	NP2C	0.00		
10	MV3	-0.03	MV3M	-0.03
	MV3M	-0.03	NPM	-0.03
	NPM	-0.03		
15	NP3C	0.01		
20	NP3C	0.02		
25	MV3M	-0.02		
30	MV3	0.01	NPM	-0.01
	NPM	-0.01		
35	NPM	0.00		
40	MV3	0.01	RPN	-0.02
			MV3M	0.02
			NPM	0.02
45	RPN	-0.01		
	NP1B	-0.01		
	NP2B	-0.01		
	REGM	-0.01		
50	RPN	-0.02		
	MV3	0.02		
	NP1A	-0.02		
	NP1B	-0.02		
	NP2B	-0.02		
	REGM	-0.02		
	NPM	0.02		
55	MV3	0.00	NPM	0.00
	NPM	0.00		
60	MV3	0.00	MV3M	0.01
			NPM	-0.01
65	NPM	0.00		

Table D-11b Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 2, $N_1 = 10$ years, $r_{xy} > 0.666$

N_2	Models	f_{ij}
10	REG	0.02
	NP3C	0.02
15	REG	-0.01
	RPN	0.01
20	REG	0.00
	NPM	0.00
25	NP2B	0.00
30	NP3A	0.00
	NP3B	0.00
35	REG	0.02
40	REG	0.05
	RPNM	0.05
45	REG	0.03
50	REG	0.02
55	REG	0.01
60	REG	0.02

Table D-11c Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 2, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV3	0.00	MV3M	-0.01
20	RPN	0.00		
	RPNM	0.00		
25	REGM	0.00		
30	RPN	0.00		
	MV1	0.00		
	MV1M	0.00		
	MV2M	0.00		
35	REGM	0.00		
40	RPN	0.00		
	MV1M	0.00		
45	MV2	0.00	NP3A	0.00
	NP3A	0.00	NP3B	0.00
	NP3B	0.00		
50	NP2A	0.00		
	RPNM	0.00		
55	MV1	0.00	NPM	0.00
	MV2	0.00		
	MV2M	0.00		
	NPM	0.00		

Table D-11d Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 2, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
20	REG	0.01	RPN	0.03
25	REG	-0.01	RPN	0.01
	RPN	0.01		
30	REG	0.02	RPN	0.04
35	REG	0.00	RPN	0.02
40	RPN	0.00		
45	MV1	0.00	RPN	-0.01
50	RPN	0.00		

Table D-12a Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 2, $N_1 = 5$ years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
5	RPN	-0.02		
10	REG	-0.01	NP2B	-0.01
	NP2B	-0.01	NP3A	-0.01
	NP3A	-0.01	REGM	-0.01
	REGM	-0.01		
15	RPN	0.00		
20	REG	-0.01	NP3C	-0.01
	NP3C	-0.01	NPM	0.01
	NPM	0.01		
25	REGM	0.01		
	MV1M	-0.01		
30	MV1	-0.01	MV3M	-0.01
	MV2	-0.01		
	MV3M	-0.01		
35	MV1	0.00	MV3M	0.00
	MV2	0.00		
	MV3M	0.00		
40	MV1	0.00	MV3M	0.01
	MV2	0.00		
45	MV2	0.00	MV3M	0.03
50	MV1	-0.02	MV3M	0.02
	MV2	-0.02		
	MV3	-0.02		
	MV3M	0.02		
55	MV2M	0.00		
60	MV2M	0.00		
65	MV1	-0.01	MV3M	0.01
	MV3M	0.01		

Table D-12b Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 2, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	NP2B	0.02		
	REGM	-0.02		
	MV1M	-0.02		
	MV2M	-0.02		
15	NP3A	0.00		
	REGM	0.00		
20	MV3M	0.00		
25	MV1	0.00		
30	MV1	0.00		
	MV2	0.00		
35	MV1	0.00		
	MV2	0.00		
40	MV1	0.01		
	MV2	0.01		
45	MV3	-0.01		
50	MV3	-0.03		
	MV2M	0.03		
55	MV1M	0.02	MV3	-0.03
	MV2M	0.02		
60	MV3	0.00		

Table D-12c Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 2, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV1M	0.00	NP1C	-0.01
	MV2M	0.00		
20	NP1C	-0.01		
25	RPNM	0.06		
30	MV3M	0.08	NP1C	0.11
			RPNM	0.11
35	MV1M	0.10	NP1C	0.11
40	MV2M	0.05	NP1C	0.11
			RPNM	0.11
45	MV1M	0.02	NP1C	0.04
	MV2M	0.02		
50	MV1M	0.03	NP1C	0.07
55	MV1M	0.06	MV3M	0.08

Table D-12d Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 2, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
20	MV1	0.11	RPN	0.34
25	MV1	0.09	RPN	0.29
30	MV1	0.08	RPN	0.28
35	MV1	0.02	RPN	0.22
40	MV2	0.02	MV1M	0.06
45	MV2	0.00	RPN	0.15
50	MV1	-0.01	RPN	0.18

Table D-13a Minimum Mean Percentage Error,
Set 2, N_1 = 5 years, $r_{xy} > 0.666$

N_2	Models	MPE
5	NP3A	-2.1
10	NP3A	-1.1
15	RPN	3.0
20	NPM	0.1
25	NP2A	0.0
30	REGM	0.7
35	MV2M	-0.3
40	NP3B	0.8
45	MV1M	-0.3
50	RPNM	-0.4
55	REGM	0.1
60	REGM	-0.2
65	RPNM	-0.5

Table D-13b Minimum Mean Percentage Error,
Set 2, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	MPE	Alternate	MPE
10	NP2A	-1.4		
15	NP1A	2.4		
20	NP1C	-1.6		
25	NP1C	0.7		
30	NP1C	7.0		
35	MV1M	10.7	MV1	12.3
40	MV1M	11.5	RPNM	13.2
45	MV1M	9.9	MV1	11.3
50	MV1M	8.2	MV1	9.7
55	MV1M	7.3	MV1	8.7
60	MV1M	9.2	MV1	10.8

Table D-13c Minimum Mean Percentage Error, Set 2, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	MPE	Alternate	MPE
15	MV1M	0.9	NP2B	2.1
20	NP1C	-0.7		
25	NP1C	3.4		
30	MV1M	5.8	NP1C	6.7
35	MV1M	6.8	REGM	10.2
40	MV1M	4.8	REGM	8.4
45	MV1M	3.1	REGM	6.3
50	MV1M	2.4	REGM	5.9
55	MV1M	4.7	NP1C	7.9

Table D-13d Minimum Mean Percentage Error, Set 2, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	MPE	Alternate	MPE
20	MV1M	20.3	RPN	28.4
25	MV1M	16.6	RPN	24.9
30	MV1M	17.4	RPN	26.1
35	MV1M	12.9	RPN	22.2
40	MV1M	9.0		
45	MV1M	7.6	RPN	17.1
50	MV1M	10.5	RPNM	17.6

Table D-14a Minimum SSE, Set 2, N_1 = 5 years, $r_{xy} > 0.666$

N_2	Models	SSE
5	MV1M 1	1983.4
10	NP1C	8365.0
15	REGM 1	201.5
20	REGM 1	724.9
25	REGM 1	689.6
30	REGM 1	249.0
35	REGM 1	229.2
40	REGM	9694.1
45	REGM	9320.8
50	REGM	9533.3
55	NPM	9780.5
60	REGM	9703.3
65	REGM	9363.6

Table D-14b Minimum SSE, Set 2, N_1 = 10 years, $r_{xy} > 0.666$

N_2	Models	SSE
10	REG	29141.0
15	REG	26063.8
20	REG	26096.4
25	REG	26600.3
30	REG	26791.4
35	REGM	25037.1
40	REGM	26457.2
45	REGM	25985.0
50	REG	26253.2
55	REGM	23763.9
60	REGM	21540.7

Table D-14c Minimum SSE,
Set 2, N_1 = 15 years, $r_{xy} > 0.666$

N_2	Models	SSE	Alternate	SSE
15	REGM	58581.0	REG	68575.3
20	REGM	56487.6	RPNM	62411.1
25	REGM	53534.4		
30	REGM	51825.7	NPM	61052.1
35	REGM	52154.1		
40	NPM	51980.0		
45	NPM	53137.4		
50	REGM	51572.9		
55	REGM	46956.8		

Table D-14d Minimum SSE,
Set 2, N_1 = 20 years, $r_{xy} > 0.666$

N_2	Models	SSE	Alternate	SSE
20	MV1	88330.2	NP2B	109731.3
25	MV2M	74170.7	NP2A	103933.0
30	REGM	75377.0	NP2B	105861.7
35	REGM	74748.8	NP2B	103218.7
40	REGM	74503.8	NP2B	94353.5
45	REGM	70478.7	NP2B	86072.0
50	MV3M	66157.4	RPNM	73784.6

Table D-15a Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 3, $N_1 = 5$ years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	REGM	-0.11	NP1B	-0.18
			NPM	-0.18
15	REGM	-0.15	NP2B	-0.17
	MV3M	-0.15	NPM	-0.17
20	MV2M	-0.13	NP2A	-0.17
			NP2C	-0.17
25	MV2M	-0.13	NP2C	-0.15
30	MV2M	-0.13	NP2A	-0.15
			NP2C	-0.15

Table D-15b Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 3, $N_1 = 10$ years, $r_{xy} < 0.666$

N_2	Models	f_{ij}
10	NP2C	-0.18
	NP3C	-0.18
15	NP2C	-0.16
20	NP2C	-0.16
25	NP2A	-0.15
	NP2C	-0.15

Table D-15c Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 3, $N_1 = 15$ years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	REGM	-0.15	NP2A	-0.17
	MV2M	-0.15		
20	REGM	-0.15	NP2A	-0.17
	MV2M	-0.15		

Table D-16a Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 3, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	NP3C	0.01		
15	MV3M	0.04	MV1M	0.05
20	MV2M	0.05	NP2C	0.11
25	MV2M	0.07	NP2C	0.11
			NP3C	0.11
			NPM	0.11
30	MV2M	0.09	NP2C	0.11

Table D-16b Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 3, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV1M	0.00	NP3C	0.03
			NPM	0.03
15	MV1M	-0.01	NPM	0.07
20	REGM	0.00	NPM	0.07
25	REGM	-0.02	NP2C	0.05

Table D-16c Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 3, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	REGM	-0.02	NPM	0.13
20	MV1M	-0.01	NPM	0.10

Table D-17a Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 3, N_1 = 5 years, r_{xy} < 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
10	NP2C	-0.42		
15	MV1M	-0.38		
20	MV2M	-0.42	NP2C	-0.52
25	MV2M	-0.38	NP2C	-0.52
30	MV2M	-0.38	NP2C	-0.61

Table D-17b Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 3, N_1 = 10 years, r_{xy} < 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV1M	0.02	NP2C	-0.18
	MV3M	0.02		
15	MV3M	-0.08	NP2C	-0.39
20	MV3M	-0.02	NP2C	-0.50
25	MV3M	-0.02	NP2C	-0.50

Table D-17c Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 3, N_1 = 15 years, r_{xy} < 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV3M	0.00	NP2C	-0.73
20	MV3M	0.00	NPM	-0.74

Table D-18a Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 3, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}
10	NP3C	-0.01
15	NP2C	0.02
20	NP2C	-0.05
25	NP2C	-0.05
30	NP2C	-0.05

Table D-18b Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 3, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	NP3C	0.03		
	MV1M	0.03		
15	NP2C	0.01		
20	NP2C	-0.03		
25	MV3M	-0.03	NP2C	0.04

Table D-18c Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 3, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}		
15	MV3M	-0.03	NP2C	-0.14
20	MV3M	0.00	NP2C	-0.09

Table D-19a Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 3, N_1 = 5 years, r_{xy} < 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV1M	0.16	NPM	1.74
15	MV1	0.49	MV1M	0.56
20	MV1	0.64	NP1B	1.21
25	MV1	0.51	NP3B	1.26
30	MV1	0.51	NP3B	1.19

Table D-19b Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 3, N_1 = 10 years, r_{xy} < 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV3M	0.50	NP3A	0.74
			NP3B	0.74
15	MV3M	0.65	NP3B	0.72
20	MV3M	0.52	NP3B	0.68
25	MV3M	0.54	NP3B	0.78

Table D-19c Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 3, N_1 = 15 years, r_{xy} < 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV3	0.07	NP3B	0.21
20	MV3	0.10	NP3B	0.24

Table D-20a Minimum Mean Percentage Error, Set 3, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	MPE	Alternate	MPE
10	MV1M	95.9	NPM	362.4
15	MV1M	139.6		
20	MV1M	159.4	NP1B	314.7
25	MV1M	139.7	NPM	290.7
30	MV1M	142.0	NPM	291.9

Table D-20b Minimum Mean Percentage Error, Set 3, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	MPE	Alternate	MPE
10	MV3M	82.5	NPM	142.8
15	MV3M	89.4	NP1B	144.3
20	MV3M	77.5	NP3B	137.4
25	MV3	82.9	NP3B	147.9

Table D-20c Minimum Mean Percentage Error, Set 3, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	MPE	Alternate	MPE
15	MV3M	59.5	NP3B	87.1
20	MV3	64.5	NP3B	96.6

Table D-21a Minimum SSE,
Set 3, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	SSE	Alternate	SSE
10	MV3M	2612.6	NP3C	3439.9
15	MV3M	3042.0	NP2C	3104.8
20	MV2M	2634.8	NP2C	3264.4
25	MV2M	2500.9	NP2C	2988.5
30	MV2M	2557.2	NP2C	3256.0

Table D-21b Minimum SSE,
Set 3, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	SSE
10	NP2C	4387.6
15	NP2C	4219.2
20	NP2C	4508.9
25	NP2C	4300.4

Table D-21c Minimum SSE,
Set 3, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	SSE	Alternate	SSE
15	REGM	6945.8	NP2C	10930.8
20	REGM	6881.0	NP2A	11203.7

Table D-22a Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 4, $N_1 = 5$ years, $r_{xy} < 0.666$

N_2	Models	f_{ij}
5	NP1C	-0.22
	NP3C	-0.22
10	NP1C	-0.20
	NP3C	-0.20
15	NP1C	-0.18
	NP3B	-0.18
	NP3C	-0.18
20	NP3B	-0.15
25	NP3B	-0.17
	NP3C	-0.17

Table D-22b Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 4, $N_1 = 10$ years, $r_{xy} < 0.666$

N_2	Models	f_{ij}
10	NP1A	-0.25
	NP2A	-0.25
	NP3B	-0.25
15	NP3B	-0.23
20	NP3B	-0.24

Table D-22c Minimum Deviation from Target Cross Correlation, as a Fraction of Target Value, Set 4, $N_1 = 15$ years, $r_{xy} < 0.666$

N_2	Models	f_{ij}
15	NP2C	-0.24

Table D-23a Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 4, N_1 = 5 years, r_{xy} < 0.666

N_2	Models	f_{ij}	Alternate	f_{ij}
5	MV2M	-0.03	MV3M	0.05
10	RPNM	0.00		
15	NP2B	0.00		
	MV1M	0.00		
20	NP1B	0.01		
	NP2B	0.01		
25	NP2C	-0.01		

Table D-23b Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 4, N_1 = 10 years, r_{xy} < 0.666

N_2	Models	f_{ij}
10	NP1B	0.02
	MV3M	-0.02
15	NP2C	0.00
20	NP2C	0.00

Table D-23c Minimum Deviation from Target Serial Correlation, as a Fraction of Target Value, Set 4, N_1 = 15 years, r_{xy} < 0.666

N_2	Models	f_{ij}
15	RPNM	-0.02

Table D-24a Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 4, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}
5	NP1A	-0.06
	NP1B	-0.06
10	NP2B	-0.04
15	NP2C	-0.11
20	RPNM	0.06
25	NP2C	-0.18

Table D-24b Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 4, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	RPNM	0.02		
15	MV3M	0.03		
20	MV2	0.01	MV3M	-0.14

Table D-24c Minimum Deviation from Target Variance, as a Fraction of Target Value, Set 4, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV1M	-0.17	RPNM	-0.60

Table D-25a Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 4, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
5	REG	-0.04	RPN	0.05
10	REG	0.04		
15	NP1B	0.00		
	NP2A	0.00		
20	NP2A	0.00		
25	MV3	0.00		
	NP2C	0.00		

Table D-25b Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 4, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV3M	0.00		
15	MV1	0.05	MV3M	-0.15
	MV2	-0.05		
	MV2M	-0.05		
20	MV1	-0.01	MV3M	-0.15

Table D-25c Minimum Deviation from Target Mean Flow, as a Fraction of Target Value, Set 4, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
15	MV1M	-0.13	NP2C	-0.24

Table D-26a Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 4, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
5	MV2	0.01		
10	RPNM	0.04		
15	NP1B	0.00		
20	NP1A	-0.01		
	NP2A	0.01		
	RPNM	-0.01		
25	REG	0.00	NP2A	0.00
	NP2A	0.00		

Table D-26b Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 4, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}	Alternate	f_{ij}
10	MV2	0.00	RPNM	0.01
15	REG	0.00	RPN	-0.01
	MV2M	0.00		
20	REGM	0.01	RPN	-0.03

Table D-26c Minimum Deviation from Target Low Flow, as a Fraction of Target Value, Set 4, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	f_{ij}
15	NP3B	0.00

Table D-27a Minimum Mean Percentage Error, Set 4, N_1 = 5 years, $r_{xy} < 0.666$

N_2	Models	MPE
5	MV1	21.9
10	REG	14.6
15	NP1C	0.7
20	NPM	0.7
25	MV3	-0.1

Table D-27b Minimum Mean Percentage Error, Set 4, N_1 = 10 years, $r_{xy} < 0.666$

N_2	Models	MPE	Alternate	MPE
10	REGM	-1.2	RPNM	5.9
15	RPN	-1.4		
20	MV3	1.1	RPN	-2.2

Table D-27c Minimum Mean Percentage Error, Set 4, N_1 = 15 years, $r_{xy} < 0.666$

N_2	Models	MPE	Alternate	MPE
15	MV1	0.1	NP1C	0.5

Table D-28a Minimum SSE, Set 4, $N_1 = 5$ years, $r_{xy} < 0.666$

N_2	Models	SSE
5	NP3C	2636.5
10	NP1C	1969.2
15	NP3C	1630.4
20	NP3C	1472.0
25	NP3C	1615.8

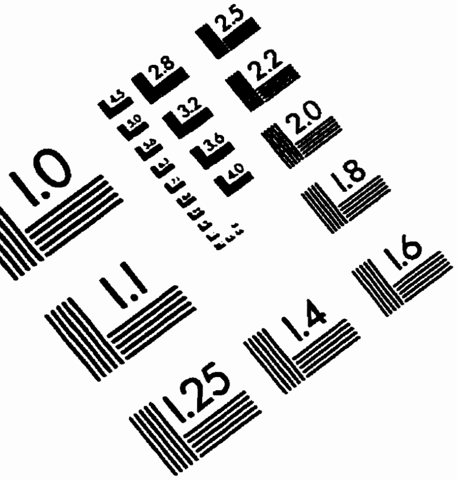
Table D-28b Minimum SSE, Set 4, $N_1 = 10$ years, $r_{xy} < 0.666$

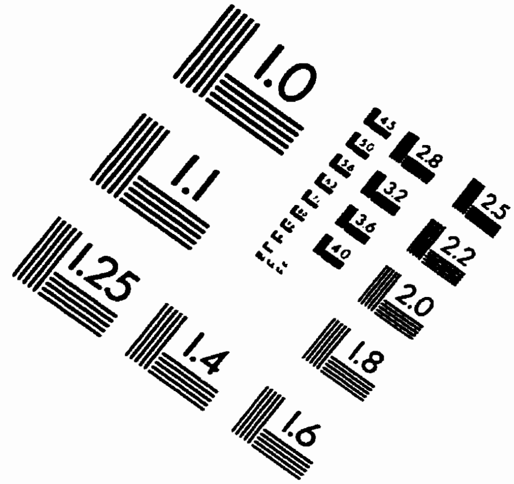
N_2	Models	SSE
10	NP1B	8739.5
15	NP1B	8207.5
20	NP2C	8458.0

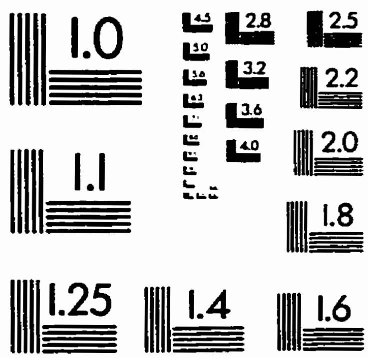
Table D-28c Minimum SSE, Set 4, $N_1 = 15$ years, $r_{xy} < 0.666$

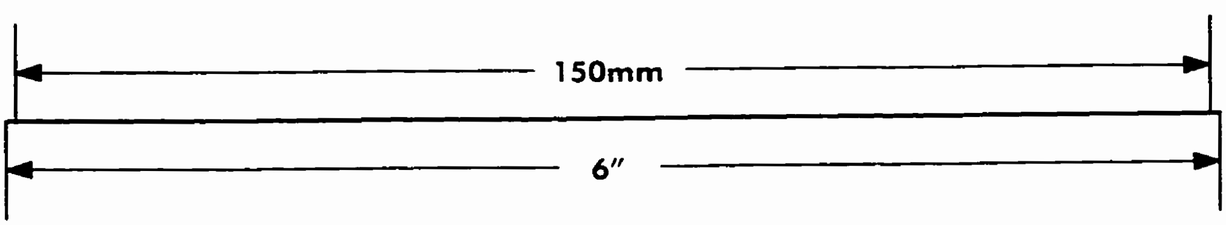
N_2	Models	SSE
15	NP2C	11829.0

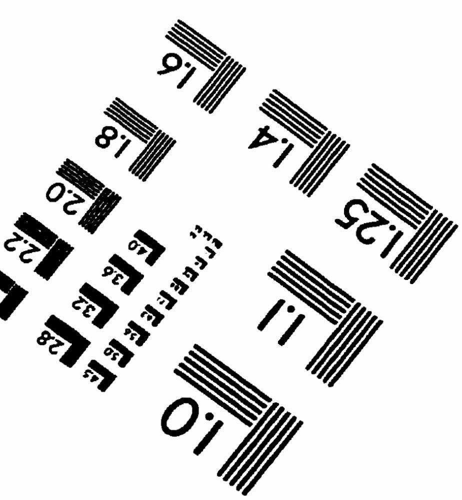
IMAGE EVALUATION TEST TARGET (QA-3)

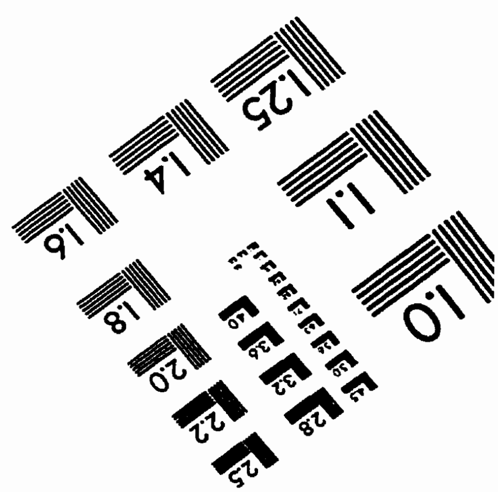












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