THE UNIVERSITY OF MANITOBA

BEHAVIOR OF THREE DIMENSIONAL FLEXIBLY CONNECTED STEEL FRAMES

by

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ABSTRACT

A procedure is presented for analyzing three dimensional rectangular steel frames which incorporate any of five commonly used beam-column connection types. A method is described for expressing the moment-rotation behavior of all connections of a given type (for example all end plate connections), in terms of a single standardized Ramberg-Osgood function.

The method involves an examination of experimental information on the moment-rotation behavior of a given connection type to determine the influence of various size parameters, such as end plate thickness. The method has been used to generate standardized moment-rotation functions for five common connection types and the functions are presented.

An iterative, successive approximation structural analysis procedure is described, in which repeated approximations are made to assumed stiffness characteristics of all connections in the structure. When the appropriate connection stiffnesses have been determined, a single linear analysis is performed to determine the correct structural displacements and internal forces. Thus, the non-linear behavior of the connections is properly accounted for.

Examples are presented to demonstrate the influence of connection deformation on structural displacements and internal forces.

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NOMENCLATURE

A	cross-sectional area of member
a_j	dimensionless exponent
D_a D_b	beam end displacement vectors
D_u D_ℓ	column end displacement vectors
D_{uo} $D_{\ell o}$	out-of-plane displacement vectors
D_{ui} $D_{\ell i}$	in-plane displacement vectors
D'_{vi} D'_{mi}	in-plane displacement vectors in global system at master joints v and m
D'_{uo} $D'_{\ell o}$	out-of-plane displacement vectors in global system at slave joint u and ℓ
E	modulus of elasticity
G	shearing modulus
H'_{vu}	translation matrix in global system from joint u to joint v
$H'_{m\ell}$	translation matrix in global system from joint ℓ to joint m
I_1 or J	torsional constant for columns and beams
I_2 I_3	second moment of area about x_2 and x_3 axes
K	standardization factor for connections
K_{bb}	stiffness matrix at b for beam member ab
K_{uu}	stiffness matrix at u for column member uℓ
k	constant in the Ramberg Osgood function
M	applied moment at connections
m	number of size parameters that affect the M-ϕ rotation curve or master joint

n	exponent in the Ramberg Osgood function
P_a P_b	beam end force vectors
P_u P_ℓ	column end force vectors
P_{uo} $P_{\ell o}$	out-of-plane force vectors
P_{ui} $P_{\ell i}$	in-plane force vectors
P'_{vc} P'_{mc}	in-plane force vectors in global system at master joints v and m
$P'_{\ell o}$ P'_{uo}	out-of-plane force vectors in global system at joints ℓ and u
q_j	connection i^{th} size parameter
R_b	rotation matrix for beam
R_c	rotation matrix for column
S	slope of the initial tangent to the M-ϕ curve
ε	strain
σ	stress
ϕ	connection rotational deformation

CHAPTER I

INTRODUCTION

1.1 Introduction

The deformations within the connections in a structural steel frame sometimes contribute substantially to the frame displacements. Often too, they affect significantly the internal force distribution. Research over the past fifty years has shown the effect of connection deformation to be more significant than that due to axial and direct shearing deformations in beams and columns. Nonetheless, while the latter have been incorporated into structural analysis computer programs, connection deformations have either not been modelled or have been accounted for by crude, simplistic means.

For example, Canadian Standards Association specification Can 3-S16.1-M78 (1978) recognizes two types of construction for steel frames. Continuous construction may be assumed when the beam-column connections are perfectly rigid. If the beam-column connections are relatively flexible, simple construction may be assumed, in which the beam-column connections are considered to have no moment resistance. The procedures permitted by the specification may lead to an unconservative design in which either structural deflections or internal forces at certain locations are underestimated.

In this study, a computer program has been developed which incorporates the rotational deformations of commonly used steel beam-column connection types in a static structural analysis of three-dimensional steel frames. The use of the program permits a new factor to be considered in structural steel design. The inclusion of connection behavior leads to more accurate structural analysis. Furthermore, by selecting connections with appropriate moment-rotation characteristics, the designer can dictate more reliably than in the past, the stiffness of and internal force distribution in the structure.

1.2 Scope of Study

To account for the effect of connection deformation in a structural analysis program, it is first necessary to have available force-deformation information for the different types of connections in use. Secondly this information must be put into a form which requires a minimum of computer storage. Finally, the connection characteristics must be incorporated into the member force-deformation relationships.

Based on these requirements and considering only the rotational deformation of connections (thus ignoring axial, torsional and direct shearing deformation), this study includes three distinct phases:

a) the assembly of all available experimentally obtained force-deformation information on the most commonly used connection types,

b) the use of the Ramberg-Osgood function (Ramberg and Osgood, 1943) to generate a standardized moment-curvature relationship for each connection type, in order to minimize the amount of connection information that must be stored and

c) the generation of a three-dimensional structural analysis program which incorporates the effect of connection deformation. Because of the non-linearity of the connection behavior, the program must employ an iterative technique. Also incorporated in the program is the P-Δ effect described in Appendix J of CSA Specification CAN 3-S16.1-M78 (1978).

1.3 Relationship to Previous Studies

The end restraint effect of beam-to-column connections under static loading conditions has long been recognized and various investigations of connection behavior have been carried out. Although information is available on the actual load-deformation behavior of connections, due to the limited possibilities of long-hand design and analysis methods this information seldom found practical application prior to the introduction of computer analysis. Nevertheless, efforts had been made to take into consideration semi-rigid connection effects.

Batho and Rowan (1930) introduced a beam line method for the analysis of frames with semi-rigid connections. Slope-deflection and modified moment distribution methods were applied to the problem by J.F. Baker (1931) and J.C. Rathburn (1935). Lothers (1950) published a paper on elastic restraint equations for semi-rigid connections. Lewitt, Chesson, and Munse (1966) presented a method for moment-rotation prediction based upon load-deformation characteristics. Somner (1969) also predicted connection behavior by the formulation of a series function which had different coefficients for various connection types and standardized parameters.

With the advent of the high speed digital computer, the application of matrix methods to structural analysis has permitted exact, systematic methods to be implemented and several papers and books have been published in the field of analysis of structures with semi-rigid connections. Gere and Weaver (1965), Livesley (1964), Monforton and Wu (1963), Lightfoot and Le Messurier (1974), and Goble (1963) presented analysis methods assuming linearly elastic connections with varying rigidities. Romstad and Subramanian (1970) studied the buckling of a frame with flexible connections using a bi-linear approximation to the moment-rotation relationship. Jones, Kirby and Nethercot (1979) investigated the effect of semi-rigid end restraint on the strength and behavior of steel columns.

Frye (1971), using Somner's method, modelled the moment-rotation behavior of connections and incorporated the effects of connection deformation into a planar frame analysis program. He employed an iterative technique to account for the non-linear behavior of connections. The study described herein is a continuation of Frye's work. The non-linear behavior of the connection is modelled using the Ramberg-Osgood function and the iterative technique employed by Frye is used for the analysis of three-dimensional rectangular frames.

1.4 Assumptions and Limitations

The assumptions employed and the limitations of the analysis program developed in this study are the following:

a) Axial and direct shearing deformations in connections are ignored and connections are assumed to have infinite torsional stiffness. Thus, only the moment-rotation behavior of the connection is modelled as a non-linear function.

b) All members are assumed to be prismatic and straight.

c) The material in the members is linearly elastic.

d) Possible buckling of individual members or portions of the structure is ignored.

e) Small deflection theory is employed. Hence, it is assumed that the only cause of non-linear structural behavior is the non-linear force-deformation characteristics of the connections.

f) All columns are assumed to be rigidly fixed at their bases.

g) Floors are assumed to act as rigid diaphragms for resisting in-plane forces.

h) Connection dimensions are assumed to be negligible compared to the lengths of the beams and columns. Thus, the rotational deformation of each connection is assumed to be concentrated at a point.

i) All joints (points of intersection between framing elements) are assumed to lie at floor level.

j) Only static loading, in the form of concentrated loads and uniformly distributed loads, can be accommodated.

k) Lateral loads can be applied at floor levels only. The lateral load at each floor level must be treated as a single three-component concentrated load vector.

l) The program uses an "in-core" equation solver. Hence the size of the structure that can be analyzed may be limited by computer primary storage capacity.

CHAPTER II

CONNECTION BEHAVIOR

2.1 Review of Research on Beam-to-Column Connections

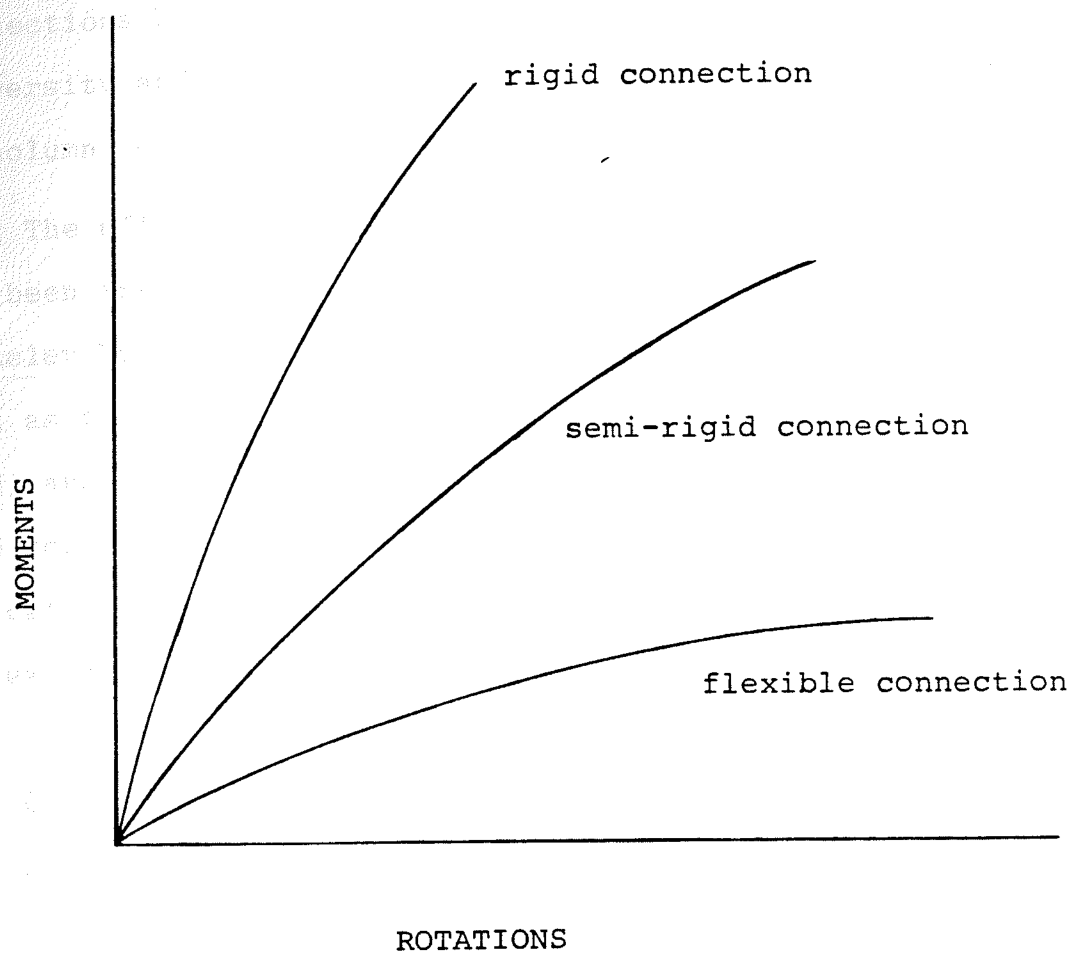
Beam-to-column connections have been a topic of research for the past fifty years (S.W. Jones et al. 1979). Over the years many types of connections have been developed as a consequence of the research effort expended in this area.

The first comprehensive research into the behavior of beam-to-column connections was conducted by C. Batho et al. (1934) and J.C. Rathbun (1935). Their work was restricted to riveted beam and stanchion connections suitable only for elastic design. Tests were performed mainly to determine the rigidity of the connections and the load-deformation behavior of the connection was given little attention.

With the introduction of plastic theory in analysis and design, it was recognized that connections are required to sustain large inelastic deformations. Hence tests on welded connections providing full continuity were conducted to investigate the connection effectiveness in resisting load in the plastic region. In the 1950's considerable experimental work that was most informative on the practical detail of assembly, was carried out by L.G. Johnson (1959). Horne (1958), among other investigators, discussed the design of one-way and two-way welded connections and proposed simple rules based on the observed failure of the component parts of connections. Although empirical in nature, these rules provided conservative estimates of joint behavior.

While investigations on welded connections were underway, bolted connections, having the advantage of lower fabrication and erection cost, also received attention. Work by Munse (1959) and Schultz (1959) represented an attempt to understand the fundamental behavior of the high strength bolt under static and dynamic loading, and an attempt to outline design methods for bolted connections. Johnson et al. (1960), Sherbourne (1961), and Charlton (1960) investigated the problem of bolted connections in the plastic range. As with welded connections, design formulae were derived, based purely on experimental observations.

Very little theoretical analysis was done in the early investigations. However work by Graham et al. (1959), which formed the basis of current design practice, was the first systematic, analytical and experimental treatment of connection problems. Since the early 1960's, there have been analytical testing programs to determine overall connection behavior and the interaction between the connections and the adjoining members. Bose (1972) and many other investigators, using the finite element formulation, investigated the effect



CLASSIFICATION OF CONNECTIONS BY THE MOMENT-ROTATION CURVES DEPICTED

FIG. 2.1

of geometric parameters on connection behavior and arrived at simple design equations affecting the strength and stability of column webs. The extensive testing program on beam-column connections by W.F. Chen and K.V. Patel (1980) at Lehigh University added valuable information on the behavior of beam-to-column connections.

The effect of cyclic loading on beam-to-column connections has been investigated at the University of California, Berkeley by H. Krawinkler et al. (1981). Other problems, such as the effect of eccentric loading (John L. Dawe et al. 1974) and prying action (N. Krishnamurthy 1978) on connections, have received attention in recent years. All of this work indicates that current design practice is constantly under review, and that better and safer designs can be expected.

2.2 General Discussion of Connection Behavior

The tests conducted during the past fifty years indicate that the primary distortion of connections, which is of interest in both analysis and design, is the rotational deformation caused by bending moment. Indeed, as illustrated in Fig. 2.1, connections have been classified as "flexible", "semi-rigid" or "rigid" (American Institute Steel Construction, 1971), depending upon their moment rotation curves. Tests have shown that almost all connections have nonlinear moment-rotation curves. The less stiff connections have rounded curves, as yielding begins upon application of a small amount of moment. The stiffer connections initially behave linearly, then local yielding causes a nonlinear response.

The response to applied moment is different for each connection type and it varies even among connections of a given type. Methods have been proposed for calculating the moment-rotation (M-ϕ) relationship for semi-rigid connections, but most M-ϕ curves must be determined experimentally. However, experimentation can be expensive and time consuming. Furthermore, the results obtained for a connection type show wide variation depending upon beam depth, number of bolts, angle size, column width, thickness of beam flange and web, and thickness of column flange. Frye (1971) considered these geometric parameters in studying the connection response to applied moment. He developed "standardized", dimensionless moment-rotation functions for different connection types. Each standardized function was expressed in terms of the geometric parameters for the connection. In this study, connection behavior is examined in the same context. The experimental moment-rotation functions for the connections considered in this study are presented in Appendix A. Also presented are the geometric parameters that affect the moment-rotation behavior for each connection type.

CHAPTER III

MODELLING OF MOMENT-ROTATION BEHAVIOR OF CONNECTIONS

3.1 Moment-Rotation Modelling by Ramberg-Osgood Function

W. Ramberg and W.R. Osgood (1943) suggested an analytical expression for describing stress-strain relationships in terms of three parameters; namely the Young's Modulus and two secant yield strengths. The original expression had the form

$$\varepsilon = \frac{\sigma}{E} + k \left(\frac{\sigma}{E}\right)^{n} \tag{3.1}$$

where

ε = strain,

σ = stress,

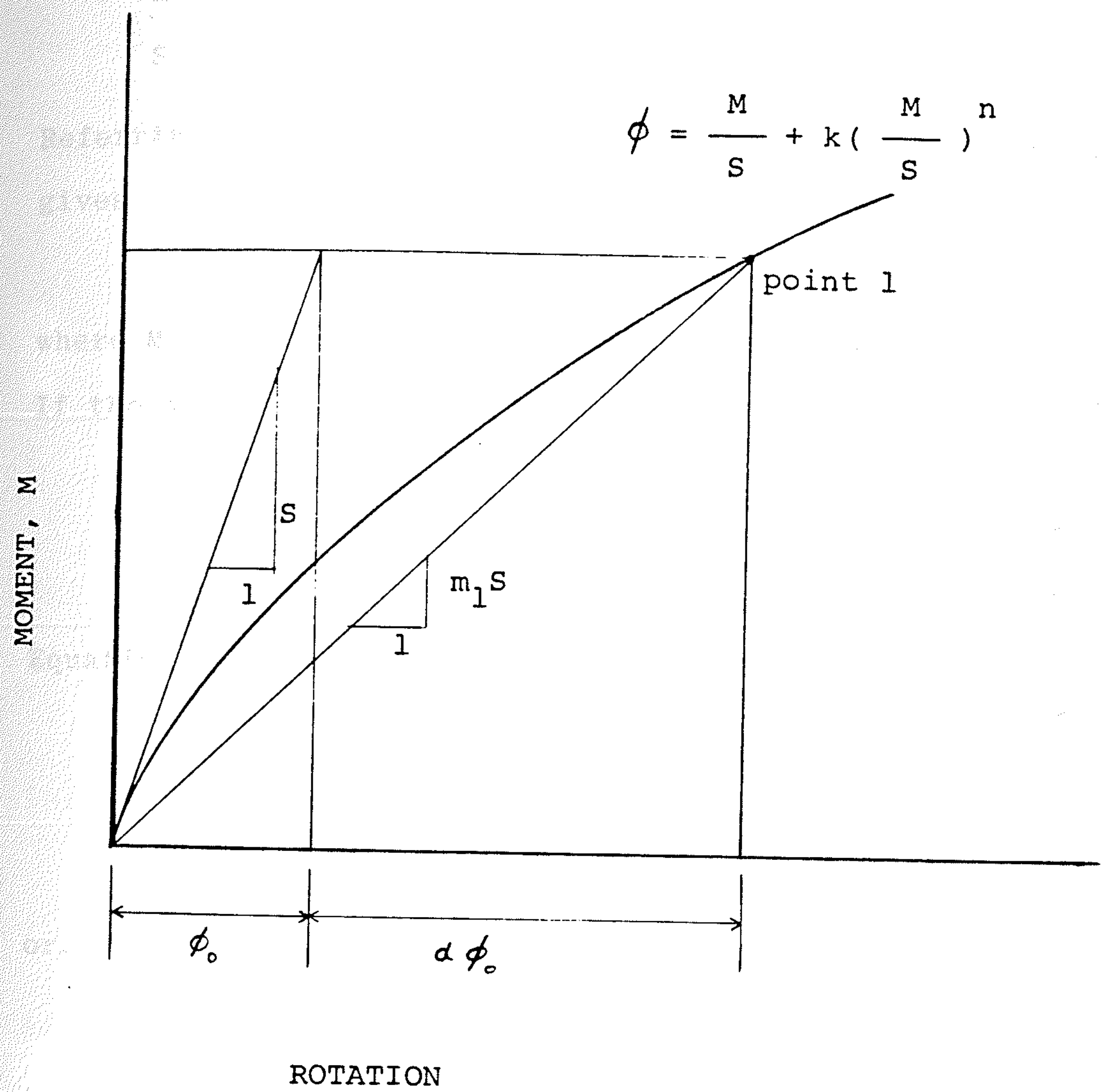
E = Young's Modulus, and

k and n are constants.

This expression can also be used to describe the moment-rotation relationship for a connection, as illustrated in Fig. 3.1.

For a connection, the moment-curvature function can be approximated by

$$\phi = \frac{M}{S} + k \left(\frac{M}{S}\right)^{n} \tag{3.2}$$



RAMBERG OSGOOD FUNCTION FOR MOMENT-ROTATION CURVE

FIG 3.1

where

ϕ = rotational deformation of the connection,

M = applied moment,

S = slope of the initial tangent to the M-ϕ curve

Referring to Fig. 3.1, equation (3.2) written at point 1 gives

$$(1 + \alpha)\ \phi_o = \frac{M_o}{S} + k\ \left(\frac{M_o}{S}\right)^n \tag{3.3}$$

where M_o, ϕ_o, S and α are as illustrated in the figure.

If the slope of secant 0-1 is m_1S then

$$(1 + \alpha)\ \phi_o = \frac{M_o}{m_1 S} \tag{3.4}$$

Equating (3.3) and (3.4),

$$\frac{M_o}{m_1 S} = \frac{M_o}{S} + k\ \left(\frac{M_o}{S}\right)^n \tag{3.5}$$

or,

$$\frac{1}{m_1} = 1 + k\ \left(\frac{M_o}{S}\right)^{n-1}$$

thus,

$$k\ \left(\frac{M_o}{S}\right)^{n-1} = \frac{1 - m_1}{m_1} \tag{3.6}$$

But from similar triangles in Fig. 3.1, it can be seen that

$$\frac{M_o}{\phi_o} = S$$

and

$$\frac{M_o}{(1 + \alpha)\ \phi_o} = m_1\ S \quad .$$

Thus,

$$\frac{M_o}{(1 + \alpha)\ \phi_o} = m_1 \frac{M_o}{\phi_o}$$

or,

$$m_1 = \frac{1}{1 + \alpha} \quad . \tag{3.7}$$

Solving (3.6) for α

$$(1 + \alpha) = \frac{1}{m_1}$$

and

$$\alpha = \frac{1}{m_1} - 1 = \frac{1 - m_1}{m_1} \quad . \tag{3.8}$$

Substituting (3.8) into (3.6),

$$k\ (\frac{M_o}{S})^{n-1} = \alpha \tag{3.9}$$

Multiplying (3.2) by $\frac{S}{M_o}$ gives

$$\phi \frac{S}{M_o} = \frac{M}{M_o} + k \left(\frac{M}{S}\right)^n \left(\frac{S}{M_o}\right) .$$

Then, since $\frac{M_o}{S} = \phi_o$,

$$\frac{\phi}{\phi_o} = \frac{M}{M_o} + k \left(\frac{M_o}{S}\right)^{n-1} \left(\frac{M}{M_o}\right)^n . \qquad (3.10)$$

Substituting (3.9) into (3.10), the following equation is obtained:

$$\frac{\phi}{\phi_o} = \frac{M}{M_o} + \alpha \left(\frac{M}{M_o}\right)^n . \qquad (3.11a)$$

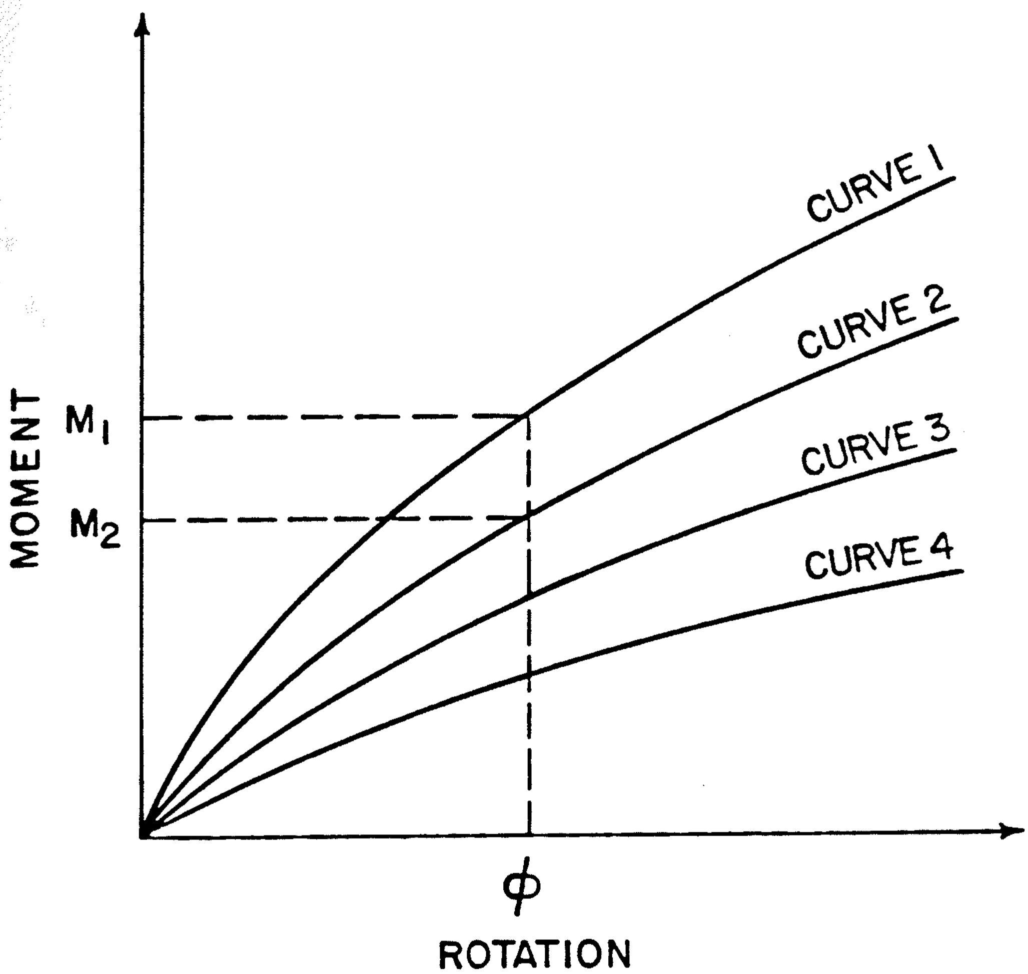
Equation (3.11a) expresses the Ramberg-Osgood function for moment-rotation relationships in terms of the four parameters: ϕ_o, M_o, α, and n. In the equation, ϕ_o, M_o, and α define the location of point 1 and n defines the sharpness of the 'knee' for any of a family of curves passing through point 1. For the special case where $\alpha = 1$, eq. (3.11a) becomes

$$\frac{\phi}{\phi_o} = \frac{M}{M_o} + \left(\frac{M}{M_o}\right)^n . \qquad (3.11b)$$

In this study, Equation (3.11b) has been chosen to describe the moment-rotation relationship for connections.

3.2 Standardization of Moment-Rotation Functions

In order for a structural analysis computer program to incorporate the effects of connection deformation, the constitutive relationship between moment and rotation for the connections used must be available. As illustrated in Fig. 3.2, except for the difference in sharpness of curvature, all connections of a given type have similar moment-rotation curves. Obviously, to store all of the moment-rotation information for every connection of every type in the computer would be very expensive. Therefore it is necessary to derive a 'standardized' moment-rotation function for each connection type. The standardized relationship is a function of several geometric parameters; depth, end plate thickness, etc., for that connection type. The moment-rotation characteristics for a particular connection can then be generated by substituting the values of its geometric parameters into the standardized relationship. The procedure has the obvious advantage that it reduces drastically the amount of connection information that must be stored.



MOMENT - ROTATION CURVE FOR CONNECTIONS WITH DIFFERENT VALUES OF q_j

FIG. 3.2

Using Equation (3.11b), the standardized moment-rotation relationship can be expressed in the form

$$\frac{\phi}{\phi_o} = \frac{K \times M}{[KM]_o} \left[1 + \left[\frac{K \times M}{[KM]_o}\right]^{n-1}\right] \tag{3.12}$$

where

ϕ_o, $[KM]_o$, and n are constants to be evaluated from a Ramberg-Osgood curve fitting program,

ϕ is the rotation at one side of the connection relative to that at the other side,

M is the bending moment at the connection, and

K is a dimensionless factor which standardizes the moment-rotation function by scaling the ordinates, thus accounting for their dependence on the magnitude of the various joint size parameters. The factor K has the form

$$K = \prod_{j=1}^{m} q_j^{a_j} \tag{3.13}$$

where

q_j = numerical value of jth size parameter, such as connection depth, d, plate thickness, t, etc.

a_j = a dimensionless exponent which indicates the effect of the jth size parameter on the moment-rotation relationship.

m = total number of size parameters which are assumed to influence the moment-rotation relationships for the connection type considered.

The evaluation of the exponents a_j in Eq. (3.13) can be illustrated by considering a family of experimentally obtained moment-rotation curves for connections which are identical except for parameter q_j, (see Fig. 3.2). For a given value of ϕ, moments M_1, M_2, M_3 and M_4 are measured from experimental M-ϕ curves, or obtained from tabulated experimental data.

A pair of curves, say curves 1 and 2, is considered and the relationship between moments M_1 and M_2 at a particular rotation, ϕ, is assumed to have the form

$$\frac{M_1}{M_2} = \left(\frac{q_{j2}}{q_{j1}}\right)^{a_j} \tag{3.14}$$

where q_{j1} and q_{j2} are the numerical values of jth parameter q for connections 1 and 2 (corresponding to curves 1 and 2) respectively. M_1 and M_2 are the moment values at rotation ϕ, for curves 1 and 2 respectively.

Equation (3.14) is rewritten and solved for a_j as follows:

$$a_j = \frac{\log \frac{M_1}{M_2}}{\log \frac{q_{j2}}{q_{j1}}} \qquad (3.15)$$

Equation (3.15) is used to calculate a_j values corresponding to several different rotations for each combination of experimental curves, such as 1 and 2, 1 and 3, 1 and 4, 2 and 3, 2 and 4, etc. The mean of the a_j values thus obtained is used in Equation (3.13).

When average values have been calculated for all m exponents a_j in Equation (3.13), they are used to evaluate the factor K. Values of K x M for the various combinations of parameters are then calculated for a number of ϕ values and the average value of K x M is computed. A number of pairs of coordinates (K x M and ϕ) are then fed into a Ramberg-Osgood curve-fitting program, which fits the curve represented by Equation (3.12). The values of $[KM]_o$, ϕ_o, and η are thus obtained. The standardized equations for the five types of connections considered in this study are presented in Table 3-1. The comparisons of experimentally obtained moment-

Connection Type	Dimensionless Factor	Standardized Moment-Rotation Equations	Max. Deviation Standardized Curves From Exp. Curves	References for Experimental Curves	Number of Experimental Curves Re-Produced
Single Web Angle Connection	$K = d^{-2.09} t^{-1.64} g^{2.06}$	$\frac{\phi}{1.03\times10^{-2}} = \frac{KxM}{32.75}\left[1 + \left(\frac{K\times M}{32.75}\right)^{2.93}\right]$	-11%	Lipson (1968)	6
Double Web Angle Connection	$K = d^{-2.2} t^{0.08} g^{-0.28}$	$\frac{\phi}{3.98\times10^{-3}} = \frac{KxM}{0.63}\left[1 + \left(\frac{KxM}{0.63^{-1}}\right)^{3.94}\right]$	-18%	Batho and Rowan (1934) Lewitt, Chesson and Munse (1966) Somner (1969)	7
Header Plate Connection	$K = d^{-2.41} t^{-1.54} g^{2.12} w^{-0.45}$	$\frac{\phi}{7.04\times10^{-3}} = \frac{K\times M}{186.77}\left[1 + \left(\frac{K\times M}{186.77}\right)^{3.32}\right]$	-12%	Somner (1969)	7
Top and Seat Angle Connection	$K = d^{-1.06} t^{-0.54} \ell^{0.85} f^{-1.28}$	$\frac{\phi}{5.17\times10^{-3}} = \frac{K\times M}{745.94}\left[1 + \left(\frac{K\times M}{745.94}\right)^{4.61}\right]$	-4%	Hetchman and Johnston (1947)	7
Strap Angle Connection	$K = h^{-.059} t^{-0.85} \left(\frac{H}{p}\right)^{-1.06}$	$\frac{\phi}{4.58\times10^{-5}} = \frac{K\times M}{753.26}\left[1 + \left(\frac{K\times M}{753.26}\right)^{4.98}\right]$	-5%	Brun and Picard (1976) Beaulieu, Giroux, Picard (1974)	5

Table 3.1 Standardized connection moment-rotation functions.

rotation curves with those obtained from the standardized equation are presented in Appendix B.

3.3 Accuracy of Standardized Moment-Rotation Functions

Perfect agreement between the original experimental moment-rotation data and the standardized functions is impossible to achieve. Usually the standardized function derived for a given connection type is based on data from several different experimental investigations. Furthermore, the investigations often have been conducted to determine only the maximum joint strength, rather than its moment rotation function. Hence the available information sometimes does not lend itself to an accurate determination of the standardization factor K. In some instances, the small amount of experimental moment-rotation data available for a particular type of connection limits the accuracy that can be achieved in the standardized function.

Nevertheless, since the standardization process involves averaging of the available experimental moment-rotation information, reasonably good accuracy is obtained. In Table 3-1, the maximum difference between the connection moment, calculated using the standardized function, and the experimentally measured moment, is shown for each connection type. Also shown is the number of experimental curves used to generate the standardized function. While maximum deviations of 11, 18 and 12 percent are observed for three of the connection types, they occurred for one connection only, of that type. In all other cases, the deviations were within five percent. What this means is that when experimental data have been used to develop a standardized M-ϕ function and when that function has been used to "reconstruct" the function for a specific connection, the agreement between the original data and the reconstructed function was nearly always within 5 percent.

CHAPTER IV

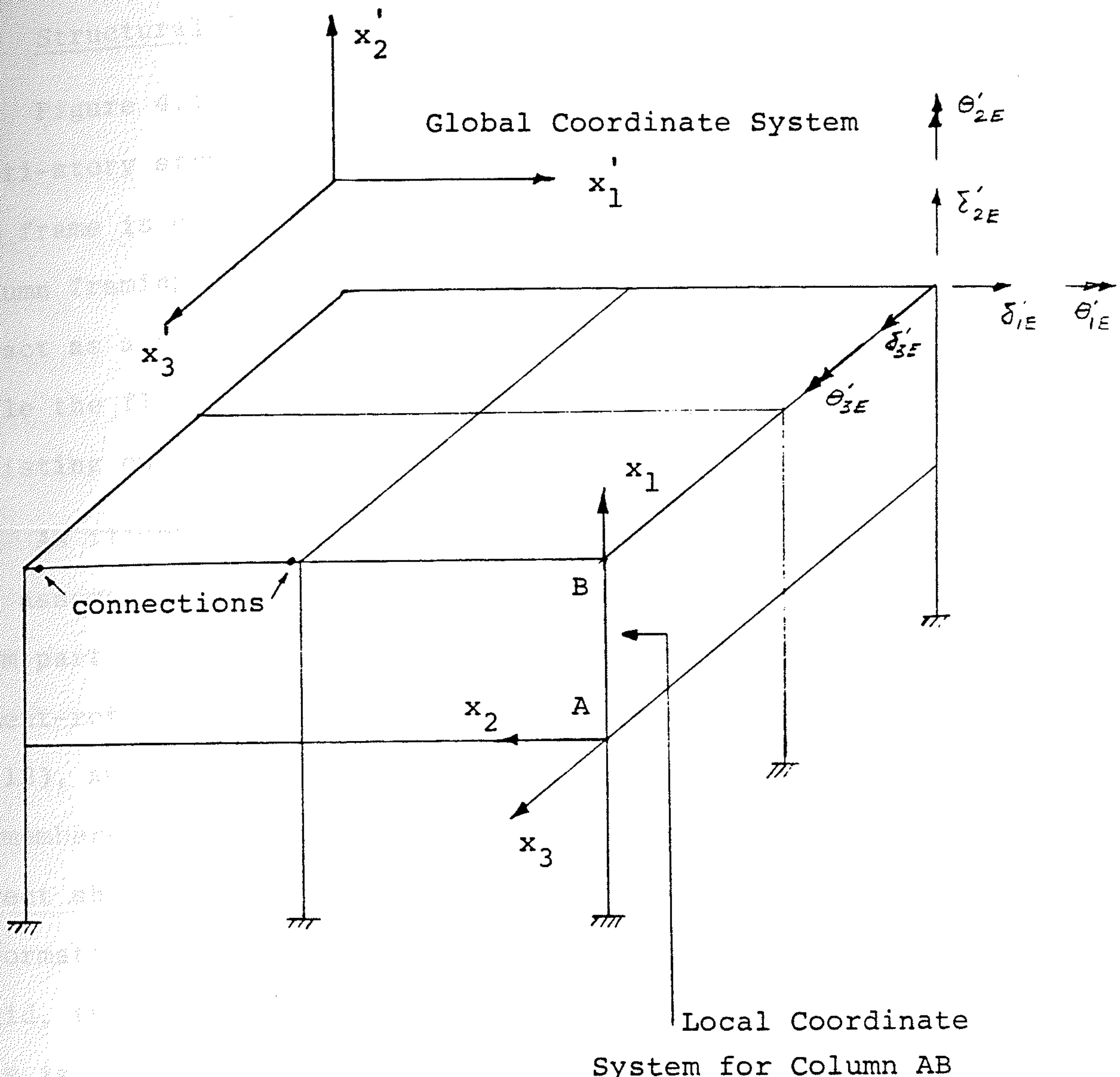
ANALYSIS OF FRAMES WITH FLEXIBLE CONNECTIONS

4.1 Coordinate Systems Used

As illustrated in Fig. 4.1, two types of coordinate systems, the global system and the local system, are employed in structural analysis. Right-handed systems are used for both.

The global system is a single coordinate system for the whole structure, used to express the external loading and all joint displacements. When a load or displacement component or vector is expressed in the global system it is represented by a primed symbol. For example in Fig. 4.1, displacement component δ'_{1E} is in the direction of the global x'_1 axis.

As also illustrated in Fig. 4.1, there is a local coordinate system associated with each member in the structure. The local 1 axis is always coincident with the member axis and the member is assigned the direction of the positive 1 axis. The local 3 axis always lies in a horizontal plane perpendicular to x'_2 axis. Internal member forces and deformation are expressed in the local systems and are designated by unprime symbols. For example P_2 is a force component in the direction of the local 2 axis.



COORDINATE SYSTEM

FIG. 4.1

4.2 Structural Idealization

Figure 4.1 shows the structural framing system for a multi-story structure of the type considered in this study. The frame is composed of a rectangular layout of beam and column framing elements. The floor at each level is assumed to act as a rigid diaphragm in resisting in-plane forces while the floor beams provide their normal stiffness in resisting out-of-plan forces.

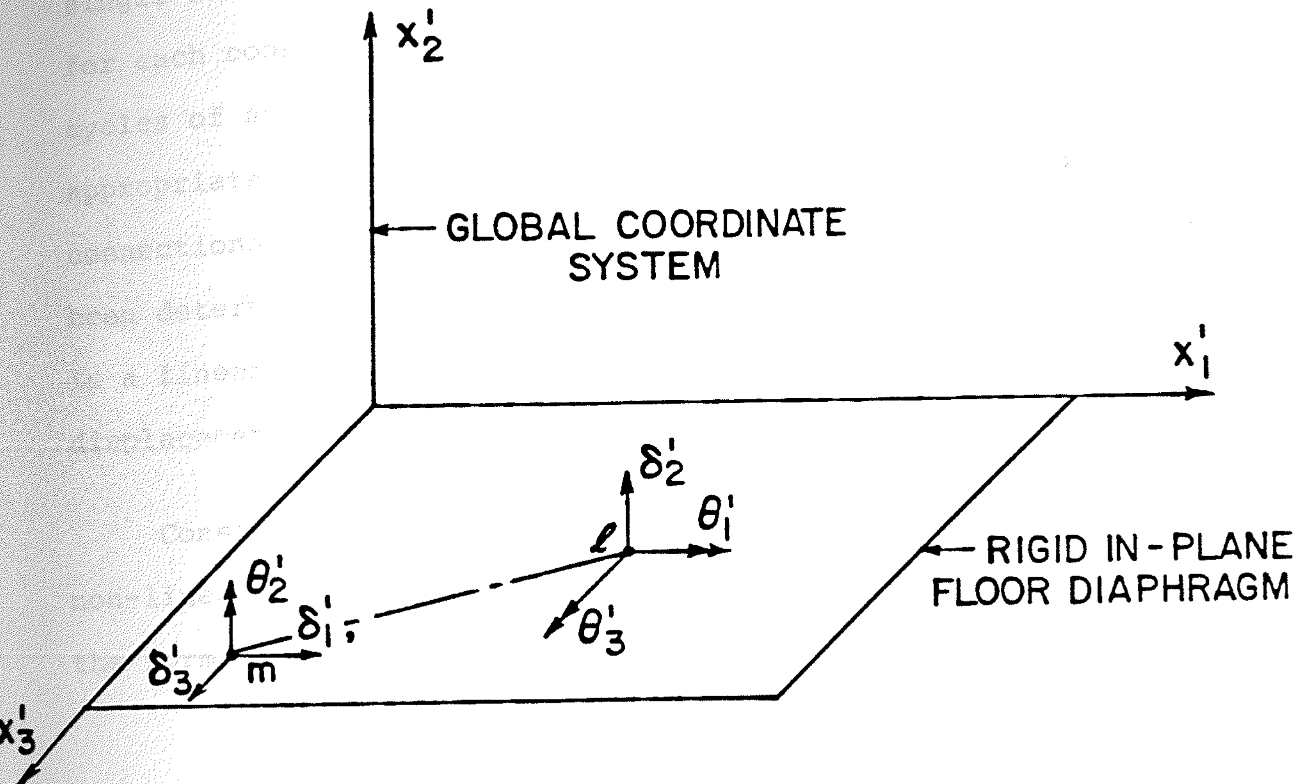
As illustrated for beam CD, the beam-to-column connections are assumed to be attached to the ends of the beams and to form part of them. The connections are assumed to have moment-rotation functions of the form indicated in Equation (3.12), and to have negligible size as compared to the lengths of members. Furthermore, they are assumed to permit no direct shearing deformation, axial deformation or torsional deformation. Since the floor diaphragms are assumed to be rigid, the bending stiffness of the connection about a vertical axis is assumed to be infinite. All joints are assumed to lie at floor level.

With this structural idealization, three independent degrees of freedom need to be considered for each joint. They are the two rotations, θ_1' and θ_3' about the x_1' and x_3' axes, and a translation δ_2' in the x_2' direction, as illustrated in Fig. 4.2. They will be referred to as out-of-plane displacement components. The other three; δ_1' and δ_3', the two translations in direction x_1' and x_3', and θ_2', a rotation about an axis in the x_2' direction, are associated with the in-plane rigid body displacements of the floor diaphragm. The latter, refered to as in-plane components, impose certain displacements on all of the joints at the given floor level. Therefore, if there are n joints at a given floor level, the number of independent displacement components to be considered at that level is 3n+3. Without the assumption of rigid in-plane floor diaphragm, 6n independent components would have to be considered at each floor level.

In the structure considered, the beams and columns are assumed to be linearly elastic while the connections behave non-linearly even at working load levels. Consequently an iterative procedure, in which the non-linear connection stiffness is linearized in successive cycles of linear analysis, is implemented in order to predict accurately the forces and displacements induced in the structure by external loads.

4.3 Non-Linear Analysis Procedure

The basic premise of the iterative analysis procedure is that the correct deflections and internal forces for a



IN - PLANE AND OUT - OF - PLANE DEGREES OF FREEDOM

FIG. 4.2

structure with nonlinear connections can be obtained from a single linear analysis, provided the correct stiffness is assumed for each connection. The procedure thus involves repeated cycles of an iterative procedure whose purpose is to determine appropriate flexibility characteristics for the various connections in a structure. When these characteristics have been determined with sufficient accuracy, they are employed in a linear analysis to calculate the correct structural displacements and forces.

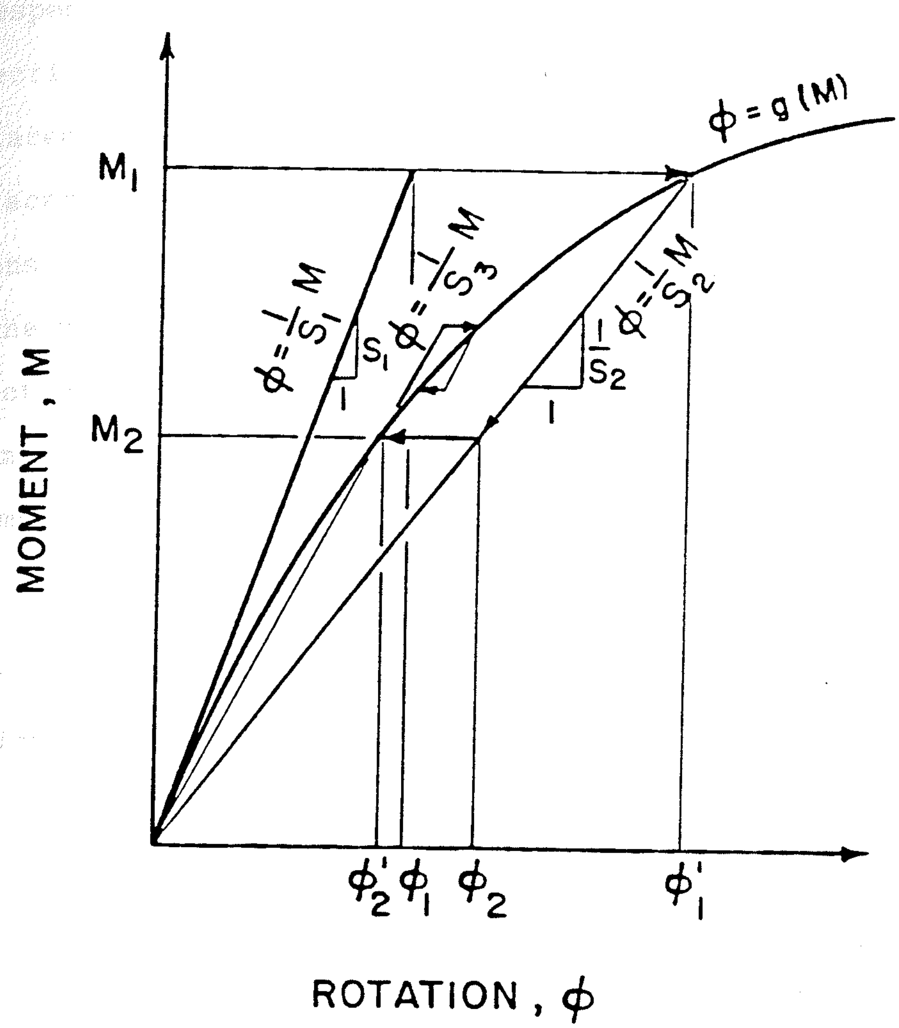
Consider a structure whose member end connections have non-linear moment-rotation functions, as in Fig. 4.3, of the form

$$\phi = g(m) \tag{4.1}$$

where g(m) is a non-linear function of the moment acting on the connection. The function is replaced by a linear relationship of the form

$$\phi = M/S_1 \tag{4.2}$$

where S_1 is the slope of the initial tangent to the M-ϕ curve.



MODIFICATION OF CONNECTION FLEXIBILITY

FIG 4.3

The moment-rotation relationships for all other connections in the structure are similarly linearized Corresponding to the linearized $M-\phi$ relationships for the connections at the ends of a given beam AB, the member force-displacement relationship can be determined. Similar force-displacement relationships for all beams and those for the columns in the structure are used in a linear analysis, and the member end forces, and hence the moments at all connections, are calculated. If the moment at the connection originally considered is M_1, the corresponding rotational deformation is

$$\phi_1 = M_1/S_1 \quad . \tag{4.3}$$

However, the rotation calculated from the correct nonlinear relationship of Equation (4.1) is

$$\phi_1' = g(M_1) \quad . \tag{4.4}$$

A better approximation to the connection moment-rotation function is thus

$$\phi = M/S_2 \tag{4.5}$$

where

$$S_2 = M_1/\phi_1' \tag{4.6}$$

as illustrates in Fig. 4.3. Equation (4.5) and similar relationships for all other connections are then used to calculate the new member force-displacement relationships and a second linear analysis is performed. The procedure is repeated until the rotation of each connection, calculated from the linear relationship for the current cycle, is sufficiently close to that given by the appropriate nonlinear relationship of the form of Equation (4.1).

The convergence of the above procedure can be hastened by using only some fraction of the difference between ϕ' and ϕ, rather than the total difference, when modifying the connection flexibility. A factor of one-half was arbitrarily employed in this study.

4.4 Member Stiffness Matrices

The development of the stiffness matrix for the typical column and for the typical beam with end connections is presented in this section.

4.4.1 Column Stiffness Matrix

Consider the column shown in Fig. 4.4. The stiffness matrix expressed in the local system is defined by the

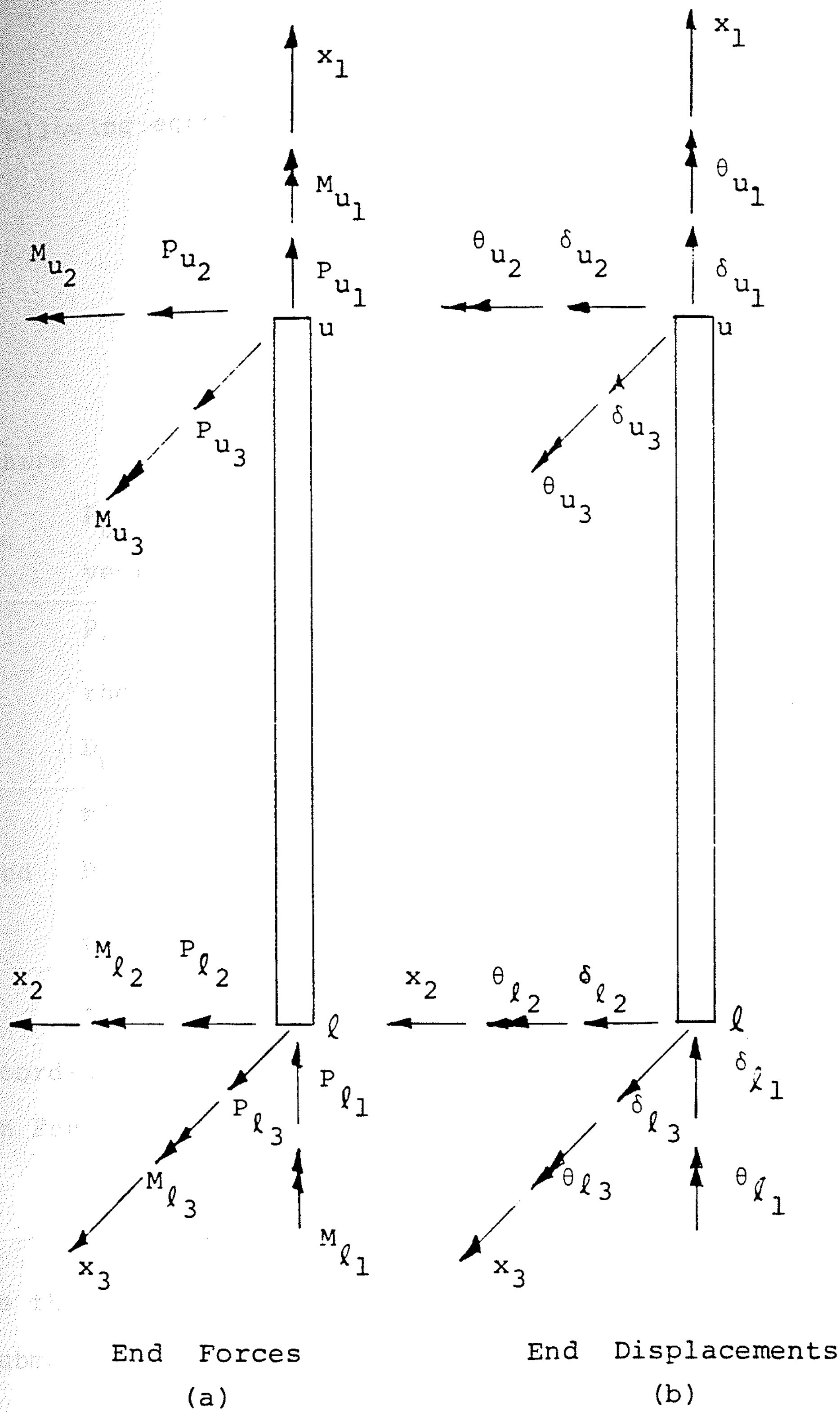


FIG 4.4 TYPICAL COLUMN ℓ-u

following equation

$$\begin{Bmatrix} P_u \\ P_\ell \end{Bmatrix} = \begin{bmatrix} K_{uu} & K_{u\ell} \\ K_{\ell u} & K_{\ell\ell} \end{bmatrix} \begin{Bmatrix} D_u \\ D_\ell \end{Bmatrix} \qquad (4.7)$$

where

$P_u = < P_{u_1} \quad P_{u_2} \quad P_{u_3} \quad M_{u_1} \quad M_{u_2} \quad M_{u_3} >^T$ is the force vector at the upper end of the column,

$P_\ell = < P_{\ell_1} \quad P_{\ell_2} \quad P_{\ell_3} \quad M_{\ell_1} \quad M_{\ell_2} \quad M_{\ell_3} >^T$ is that at the lower end,

$D_u = < \delta_{u_1} \quad \delta_{u_2} \quad \delta_{u_3} \quad \theta_{u_1} \quad \theta_{u_2} \quad \theta_{u_3} >^T$ is the displacement vector at the upper end,

and $D_\ell = < \delta_\ell \quad \delta_\ell \quad \delta_\ell \quad \theta_\ell \quad \theta_\ell \quad \theta_\ell >^T$ is that at the lower end.

All of the above vectors are expressed in the local coordinate system for the column.

In Equation (4.7),

$$\begin{bmatrix} K_{uu} & K_{u\ell} \\ K_{\ell u} & K_{\ell\ell} \end{bmatrix}$$

is the stiffnesses matrix for the column, partitioned into submatrices for the upper and lower ends.

In the stiffness matrix,

$$K_{uu} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & 0 & 0 & 0 & \frac{-6EI_3}{L^2} \\ 0 & 0 & \frac{12EI_2}{L^3} & 0 & \frac{6EI_2}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI_1}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_2}{L^2} & 0 & \frac{4EI_2}{L} & 0 \\ 0 & \frac{-6EI_3}{L^2} & 0 & 0 & 0 & \frac{4EI_3}{L} \end{bmatrix}, \quad (4.8)$$

$$K_{u\ell} = \begin{bmatrix} -\frac{AE}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & 0 & 0 & 0 & \frac{-6EI_3}{L^2} \\ 0 & 0 & \frac{-12EI_3}{L^3} & 0 & \frac{6EI_2}{L^2} & 0 \\ 0 & 0 & 0 & \frac{-GI_1}{L} & 0 & 0 \\ 0 & 0 & \frac{-6EI_2}{L^2} & 0 & \frac{2EI_2}{L} & 0 \\ 0 & \frac{6EI_3}{L^2} & 0 & 0 & 0 & \frac{2EI_3}{L} \end{bmatrix}, \quad (4.9)$$

$$K_{\ell u} = (K_{u\ell})^T , \qquad (4.10)$$

and

$$K_{\ell\ell} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & 0 & 0 & 0 & \frac{6EI_3}{L^2} \\ 0 & 0 & \frac{12EI_2}{L^3} & 0 & \frac{-6EI_2}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI_1}{L} & 0 & 0 \\ 0 & 0 & \frac{-6EI_2}{L^2} & 0 & \frac{4EI_2}{L} & 0 \\ 0 & \frac{6EI_3}{L^2} & 0 & 0 & 0 & \frac{4EI_3}{L} \end{bmatrix} \qquad (4.11)$$

In the submatrices,

I_1 = torsion constant for the column,

I_2 and I_3 = second moments of area about cross-sectional axes X_2 and X_3 respectively,

G = shearing modulus,

E = Young's modulus,

L = member length, and

A = member cross-sectional area.

4.4.2 Beam Stiffness Matrix

Consider a typical beam with flexible connections at ends A and B as shown in Fig. 4.5. The stiffness matrix for the beam plus connections, expressed in the local system, is defined by the following equation.

$$\begin{Bmatrix} P_b \\ P_a \end{Bmatrix} = \begin{bmatrix} K_{bb} & K_{ba} \\ K_{ab} & K_{aa} \end{bmatrix} \begin{Bmatrix} D_b \\ D_a \end{Bmatrix} \qquad (4.12)$$

where

$P_b = < P_{b_2} \quad M_{b_1} \quad M_{b_3} >^T$ is the force vector at end B,

$P_a = < P_{a_2} \quad M_{a_1} \quad M_{a_3} >^T$ is that at end A,

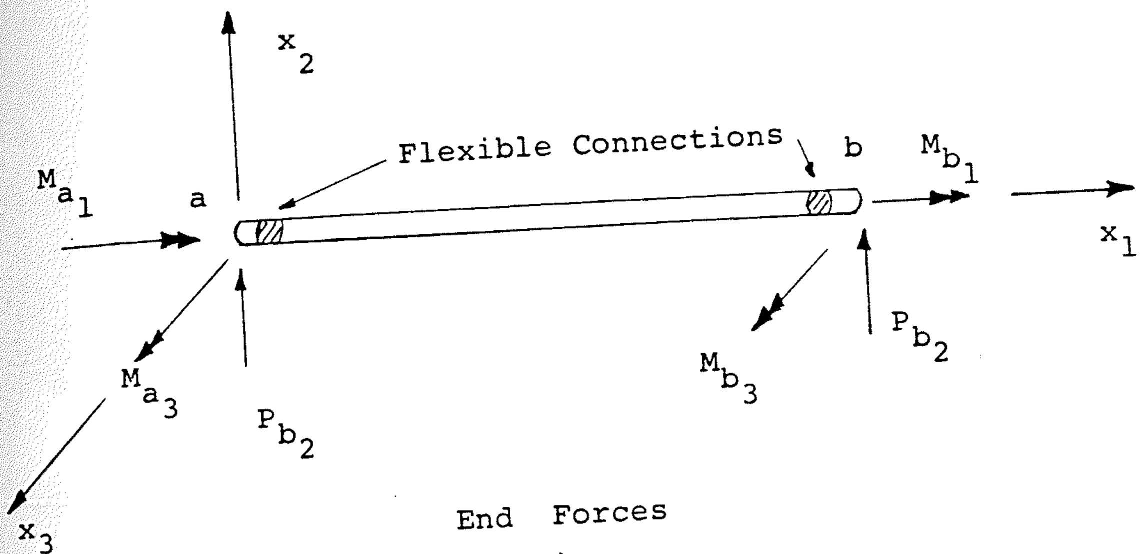
$D_b = < \delta_{b_2} \quad \theta_{b_1} \quad \theta_{b_3} >^T$ is the displacement vector at end B,

and $D_a = < \delta_{a_2} \quad \theta_{a_1} \quad \theta_{a_3} >^T$ is that at end A.

In Equation (4.12),

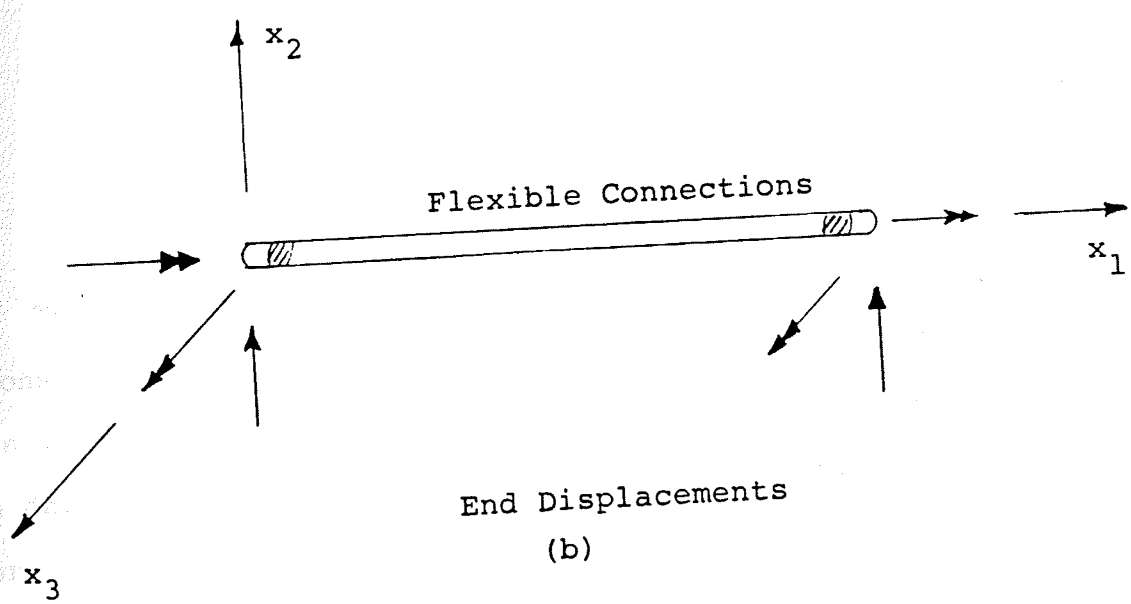
$$\begin{bmatrix} K_{bb} & K_{ba} \\ K_{ab} & K_{aa} \end{bmatrix}$$

is the stiffness matrix which relates forces and displacements at A and B. The submatrices in the above matrix are:



End Forces

(a)



End Displacements

(b)

TYPICAL BEAM a-b

FIG. 4.5

$$K_{bb} = \begin{bmatrix} S1 & 0 & -S2 \\ 0 & S3 & 0 \\ -S2 & 0 & S4 \end{bmatrix} \tag{4.13}$$

$$K_{ba} = \begin{bmatrix} -S1 & 0 & -S5 \\ 0 & -S3 & 0 \\ S2 & 0 & S7 \end{bmatrix} \tag{4.14}$$

$$K_{ab} = (K_{ba})^T \tag{4.15}$$

and

$$K_{aa} = \begin{bmatrix} S1 & 0 & S5 \\ 0 & S3 & 0 \\ S5 & 0 & S6 \end{bmatrix} . \tag{4.16}$$

The elements in the submatrices can be derived from a consideration of the equilibrium, compatibility and constitutive relationships for beam ab and its connections. The derivations are presented in Appendix C and they can be expressed as follows:

Let

$$\beta = \frac{3}{4\left(1 + \frac{3EI}{S_a L}\right)\left(1 + \frac{3EI}{S_b L}\right) - 1}$$

$$\beta_1 = \beta\left(1 + \frac{EI(S_a + S_b)}{S_a S_b L}\right)$$

$$\beta_2 = \beta\left(1 + \frac{2EI}{S_b L}\right)$$

$$\beta_3 = \beta\left(1 + \frac{2EI}{S_a L}\right)$$

$$\beta_4 = \beta\left(1 + \frac{3EI}{S_b L}\right)$$

$$\beta_5 = \beta\left(1 + \frac{3EI}{S_a L}\right)$$

Then,

$$S1 = \frac{12EI}{L^3}\beta_1$$

$$S2 = \frac{6EI}{L^2}\beta_2$$

$$S3 = \frac{GJ}{L}$$

$$S4 = \frac{4EI}{L} \beta_5$$

$$S5 = \frac{6EI}{L^2} \beta_2$$

$$S6 = \frac{4EI}{L} \beta_4$$

$$S7 = \frac{2EI}{L} \beta$$

E = modulus of elasticity

G = shearing modulus

L = beam length

I = second moment of area about bending axis

J = torsion constant

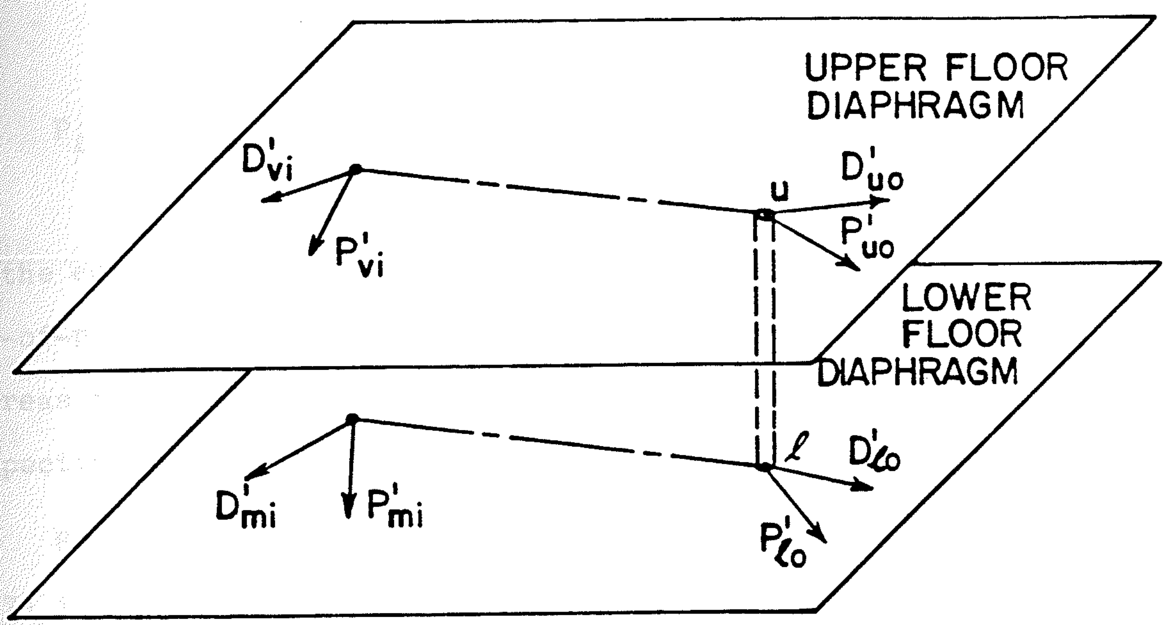
In the above expressions, S_a, S_b are the stiffness coefficients for the connections at a and b respectively. They vary with changes in load, but with the iterative procedure used in this study, repeated linear analyses are performed and for each analysis, the coefficients are constant.

4.5 Formation of Structure Stiffness Matrix

In formulating the structure stiffness matrix, the columns and beams are considered in turn and their contributions are computed and incorporated. The member stiffness coefficients are evaluated, appropriately transformed to the global coordinate system and then added, in the appropriate locations, to the structure stiffness matrix. Because of the assumption that the floors act as rigid in-plane diaphragms, the treatment of columns is different from that of beams, as described below.

4.5.1 Contribution of Column Stiffness

Consider a typical column $u\ell$, which lies between two floor diaphragms, as illustrated in Fig. 4.6. The end displacement and end force vectors in Equation (4.7) can be partitioned into out-of-plane sub-vectors and in-plane sub-vectors, as described in Section 4.2. The vectors are therefore the following:



END FORCES AND DISPLACEMENTS, COLUMN u – ℓ

FIG. 4.6

$$D_u = < D_{uo} \mid D_{ui} >^T = < \delta_{u_1} \quad \theta_{u_2} \quad \theta_{u_3} \mid \delta_{u_2} \; \delta_{u_3} \; \theta_{u_1} >^T$$

$$D_\ell = < D_{\ell o} \mid D_{\ell i} >^T = < \delta_{\ell_1} \quad \theta_{\ell_2} \quad \theta_{\ell_3} \mid \delta_{\ell_2} \; \delta_{\ell_3} \; \theta_{\ell_1} >^T$$

$$P_u = < P_{uo} \mid P_{ui} >^T = < P_{u_1} \quad M_{u_2} \quad M_{u_3} \mid P_{u_2} \; P_{u_3} \; M_{u_1} >^T ,$$

and

$$P_\ell = < P_{\ell o} \mid P_{\ell i} >^T = < P_{\ell_1} \quad M_{\ell_2} \quad M_{\ell_3} \mid P_{\ell_2} \; P_{\ell_3} \; M_{\ell_1} >^T$$

In the equations, the o and i in the subscript refer to the out-of-plane components and the in-plane components respectively whereas the u and ℓ indicate the upper and lower ends respectively of column $u\ell$.

By also partitioning the stiffness matrix of Equation (4.7) in accordance with the partitioning of the end force and displacement vectors, the equation which relates the forces and displacements at the upper and lower ends of the column can be rewritten as follows.

$$\begin{Bmatrix} P_{uo} \\ P_{ui} \\ P_{\ell o} \\ P_{\ell i} \end{Bmatrix} = \begin{bmatrix} K_{uuoo} & K_{uuoi} & K_{uioo} & K_{uioi} \\ K_{uuio} & K_{uuii} & K_{u\ell io} & K_{u\ell ii} \\ K_{\ell uoo} & K_{\ell uoi} & K_{\ell\ell oo} & K_{\ell\ell oi} \\ K_{\ell uio} & K_{\ell uii} & K_{\ell\ell io} & K_{\ell\ell ii} \end{bmatrix} \begin{Bmatrix} D_{uo} \\ D_{ui} \\ D_{\ell o} \\ D_{\ell i} \end{Bmatrix} \qquad (4.17)$$

The submatrices in the above stiffness matrix are defined as follows:

$$K_{uuoo} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{4EI_2}{L} & 0 \\ 0 & 0 & \frac{4EI_3}{L} \end{bmatrix} \quad (4.18) \qquad K_{uuio} = \begin{bmatrix} 0 & 0 & \frac{-6EI_3}{L^2} \\ 0 & \frac{6EI_2}{L^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.19)$$

$$K_{uuii} = \begin{bmatrix} \frac{12EI_3}{L^3} & 0 & 0 \\ 0 & \frac{12EI_2}{L^3} & 0 \\ 0 & 0 & \frac{GI_1}{L} \end{bmatrix} \quad (4.20) \qquad K_{\ell uoo} = \begin{bmatrix} \frac{-AE}{L} & 0 & 0 \\ 0 & \frac{2EI_2}{L} & 0 \\ 0 & 0 & \frac{2EI_3}{L} \end{bmatrix} \quad (4.21)$$

$$K_{uuoi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{6EI_2}{L^2} & 0 \\ \frac{-6EI_3}{L^2} & 0 & 0 \end{bmatrix} \quad (4.22) \qquad K_{\ell\ell oo} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{4EI_2}{L} & 0 \\ 0 & 0 & \frac{4EI_3}{L} \end{bmatrix} \quad (4.23)$$

$$K_{\ell uio} = \begin{bmatrix} 0 & 0 & \frac{6EI_3}{L^2} \\ 0 & \frac{-6EI_2}{L^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.24) \qquad K_{\ell uii} = \begin{bmatrix} \frac{-12EI_3}{L^3} & 0 & 0 \\ 0 & \frac{-12EI_2}{L^2} & 0 \\ 0 & 0 & \frac{-GI_1}{L} \end{bmatrix} \quad (4.25)$$

$$K_{\ell\ell io} = \begin{bmatrix} 0 & 0 & \frac{6EI_3}{L^2} \\ 0 & \frac{-6EI_2}{L^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.26) \qquad K_{\ell\ell ii} = \begin{bmatrix} \frac{12EI_3}{L^3} & 0 & 0 \\ 0 & \frac{12EI_2}{L^3} & 0 \\ 0 & 0 & \frac{GI_1}{L} \end{bmatrix} \quad (4.27)$$

The partitioned end displacement and end force vectors can be transformed from the local system to the global system using the following rotation transformations.

$$\begin{aligned} P'_u &= R_c\ P_u \\ P'_\ell &= R_c\ P_\ell \\ D_u &= R_c^T\ D'_u \\ D_\ell &= R_c^T\ D'_\ell \end{aligned} \qquad (4.28)$$

where

R_c is the rotation transformation matrix from the local system for column ℓu to the global system.

$$R_c = \begin{bmatrix} R11 & 0 \\ 0 & R22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Then substituting Equation (4.28) into Equation (4.17), the stiffness matrix, expressed in the global system, can be written as:

$$\begin{Bmatrix} P'_{uo} \\ P'_{ui} \\ P'_{\ell o} \\ P'_{\ell i} \end{Bmatrix} = \begin{bmatrix} K'_{uuoo} & K'_{uuoi} & K'_{u\ell oo} & K'_{u\ell oi} \\ K'_{uuio} & K'_{uuii} & K'_{u\ell io} & K'_{u\ell ii} \\ K'_{\ell uoo} & K'_{\ell uoi} & K'_{\ell\ell oo} & K'_{\ell\ell oi} \\ K'_{\ell uio} & K'_{\ell uii} & K'_{\ell\ell io} & K'_{\ell\ell ii} \end{bmatrix} \begin{Bmatrix} D'_{uo} \\ D'_{ui} \\ D'_{\ell o} \\ D'_{\ell i} \end{Bmatrix} \qquad (4.30)$$

Each submatrix in the above stiffness matrix is the triple product of three 3 x 3 submatrices and is as defined below:

$$K'_{uuoo} = R11\ K_{uuoo}\ R11^T,$$

$$K'_{uuoi} = R11\ K_{uuoi}\ R22^T,$$

$$K'_{u\ell oo} = R11\ K_{u\ell oo}\ R11^T,$$

$$K'_{u\ell oi} = R11\ K_{u\ell oi}\ R22^T,$$

$$K'_{uuio} = R22\ K_{uuio}\ R11^T,$$

$$K'_{uuii} = R22\ K_{uuii}\ R22^T,$$

$$K'_{u\ell io} = R22\ K_{u\ell io}\ R11^T,$$

$$K'_{u\ell ii} = R22\ K_{u\ell ii}\ R22^T,$$

$$K'_{\ell uoo} = R11\ K_{\ell uoo}\ R11^T,$$

$$K'_{\ell uoi} = R11\ K_{\ell uoi}\ R22^T,$$

$$K'_{\ell\ell oo} = R11\ K_{\ell\ell oo}\ R11^T,$$

$$K'_{\ell\ell oi} = R11\ K_{\ell\ell oi}\ R22^T,$$

$$K'_{\ell uio} = R22\ K_{\ell uio}\ R11^T,$$

$$K'_{\ell uii} = R22\ K_{\ell uii}\ R22^T,$$

$$K'_{\ell\ell io} = R22\ K_{\ell\ell io}\ R11^T,$$

and

$$K'_{\ell\ell ii} = R22\ K_{\ell\ell ii}\ R22^T.$$

Since the floor at each level is assumed to be a rigid diaphragm in resisting the in-plane forces, the in-plane force and displacement components for each column can be translated from the column ends to a common "master node" in the floor diaphragm. The node from which the in-plane components are translated is referred to as the "slave node". The master node in the floor at the upper end of the column is designated v and that in the floor at the lower end is designated m. The translation transformations for the in-plane components from the slave to the master node are given by the following equations

$$\begin{aligned} P'_{vi} &= H'_{vu}\ P'_{ui} \\ P'_{mi} &= H'_{m\ell}\ P'_{\ell i} \\ D'_{ui} &= H'_{uv}\ D'_{vi} \\ D'_{\ell i} &= H'_{\ell m}\ D'_{\ell mi} \end{aligned} \qquad (4.31)$$

The vectors P'_{vi} P'_{mi} D'_{ui} and $D'_{\ell i}$ are the in-plane force and displacement components at nodes v and m as illustrated in Fig. 4.6. The translation transformation matrices are

$$H'_{vu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (X'_{3_u} - X'_{3_v}) & (X'_{1_u} - X'_{1_v}) & 1 \end{bmatrix} \qquad (4.32)$$

$$H'_{m\ell} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (X'_{3_\ell} - X'_{3_m}) & (X'_{1_\ell} - X'_{1_m}) & 1 \end{bmatrix} \qquad (4.33)$$

and displacement components for each column can be translated from the column ends to a common "master node" in the floor diaphragm. The node from which the in-plane components are translated is referred to as the "slave node". The master node in the floor at the upper end of the column is designated v and that in the floor at the lower end is designated m. The translation transformations for the in-plane components from the slave to the master node are given by the following equations

$$\begin{aligned} P'_{vi} &= H'_{vu}\ P'_{ui} \\ P'_{mi} &= H'_{m\ell}\ P'_{\ell i} \\ D'_{ui} &= H'_{uv}\ D'_{vi} \\ D'_{\ell i} &= H'_{\ell m}\ D'_{\ell mi} \end{aligned} \qquad (4.31)$$

The vectors P'_{vi} P'_{mi} D'_{ui} and $D'_{\ell i}$ are the in-plane force and displacement components at nodes v and m as illustrated in Fig. 4.6. The translation transformation matrices are

$$H'_{vu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (X'_{3_u} - X'_{3_v}) & (X'_{1_u} - X'_{1_v}) & 1 \end{bmatrix} \qquad (4.32)$$

$$H'_{m\ell} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (X'_{3_\ell} - X'_{3_m}) & (X'_{1_\ell} - X'_{1_m}) & 1 \end{bmatrix} \qquad (4.33)$$

$$H'_{uv} = \begin{bmatrix} 1 & 0 & -(X'_{3_u} - X'_{3_v}) \\ 0 & 1 & (X'_{1_u} - X'_{1_v}) \\ 0 & 0 & 1 \end{bmatrix} \qquad (4.34)$$

$$H'_{\ell m} = \begin{bmatrix} 1 & 0 & -(X'_{3_\ell} - X'_{3_m}) \\ 0 & 1 & (X'_{1_\ell} - X'_{1_m}) \\ 0 & 0 & 1 \end{bmatrix} \qquad (4.35)$$

X'_{1_u} and X'_{1_ℓ}, X'_{3_u} and X'_{3_ℓ} are coordinates in X'_1 and X'_3 directions respectively of slave nodes u and ℓ.

X'_{1_v} and X'_{1_m}, X'_{3_v} and X'_{3_m} are coordinates in X_1 and X'_3 directions respectively of master nodes v and m.

Hence, by substituting Equation (4.31) into (4.30), the stiffness matrix for a typical column, expressed in terms of the master and slave node, are

$$\begin{Bmatrix} P'_{uo} \\ P'_{vi} \\ P'_{\ell o} \\ P'_{mi} \end{Bmatrix} = \begin{bmatrix} K'_{uu} & K'_{uv} & K'_{u\ell} & K_{um} \\ K'_{vu} & K'_{vv} & K'_{v\ell} & K'_{vm} \\ K'_{\ell u} & K'_{\ell v} & K'_{\ell\ell} & K'_{\ell m} \\ K'_{mu} & K'_{mv} & K'_{m\ell} & K'_{mm} \end{bmatrix} \begin{Bmatrix} D'_{uo} \\ D'_{vi} \\ D'_{\ell o} \\ D'_{mi} \end{Bmatrix} \qquad (4.36)$$

where

$$K'_{uu} = K'_{uuoo}$$

$$K'_{uv} = K'_{uuoi} \; H'_{uv}$$

$$K'_{u\ell} = K'_{u\ell oo}$$

$$K'_{um} = K'_{u\ell oi} \; H'_{\ell m}$$

$$K'_{vu} = H'_{vu} \; K'_{uuio}$$

$$K'_{vv} = H'_{vu} \; K'_{uuii} \; H'_{uv}$$

$$K'_{v\ell} = H'_{vu} \; K'_{u\ell io}$$

$$K'_{vm} = H'_{vu} \; K'_{u\ell ii} \; H'_{\ell m}$$

$$K'_{\ell u} = K'_{\ell uoo}$$

$$K'_{\ell v} = K'_{\ell\ell uoi} \; H'_{uv}$$

$$K'_{\ell\ell} = K'_{\ell\ell oo}$$

$$K'_{\ell m} = K'_{\ell\ell oi} \; H'_{\ell m}$$

$$K'_{mu} = H'_{m} \; K'_{\ell uio}$$

$$K'_{mv} = H'_{m\ell} \; K'_{\ell uii} \; H'_{uv}$$

$$K'_{m\ell} = H'_{m\ell} \; K'_{\ell uio}$$

$$K'_{mm} = H'_{m\ell} \; K'_{\ell\ell ii} \; H'_{\ell m}$$

The contributions to the structure stiffness matrix of the typical column $u\ell$ are as illustrated in Fig. 4.7.

4.5.2 Contributions of Beam Stiffness

For the typical beam with end connections as shown in Fig. 4.5 , the end displacement and force vectors comprise

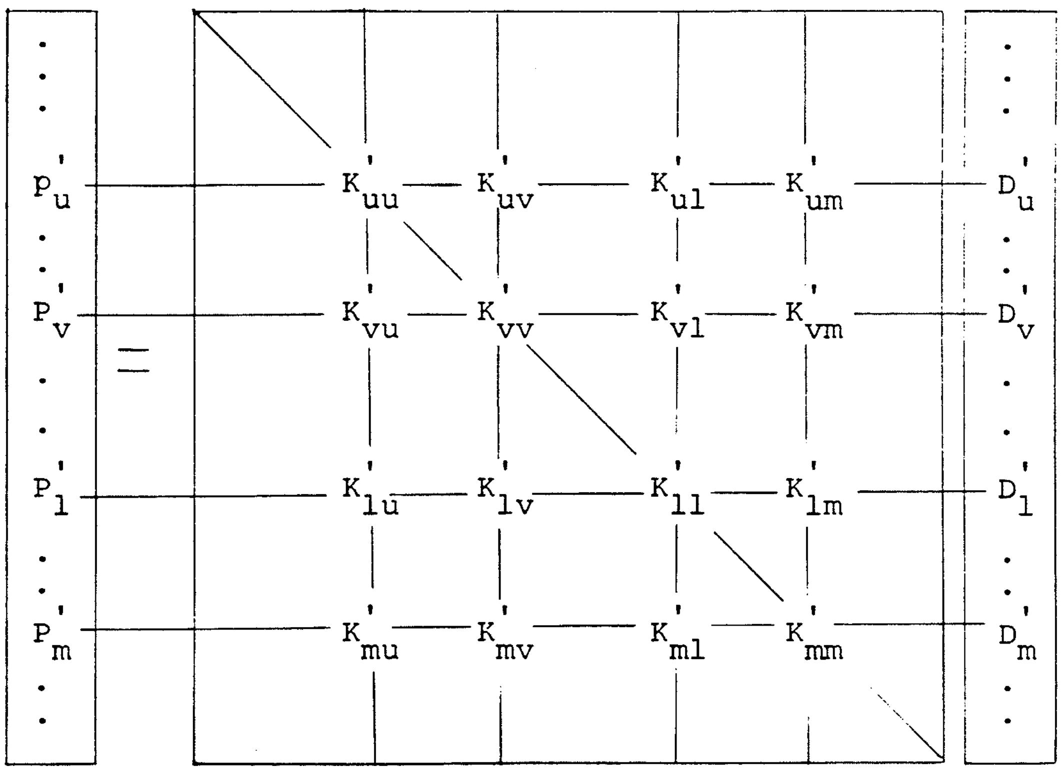


FIG. 4.7 CONTRIBUTION OF A TYPICAL COLUMN STIFFNESSES TO THE STRUCTURE STIFFNESS MATRIX

only the out-of-plane components. Hence no partitioning or translation transformations, as described for the columns, are required.

However, as illustrated in Fig. 4.8, two different beam orientations are possible. The transformations for rotating force vectors from the local to the global system and for rotating displacement vectors from the global to the local system are the following

$$\begin{aligned} P'_b &= R_b\ P_b \\ P'_a &= R_a\ P_b \\ D_b &= R_b^T\ D'_b \\ D_a &= R_b^T\ D_a \end{aligned} \tag{4.37}$$

In Equation (4.37) the rotation transformation R_b has the form

$$R_b = \begin{bmatrix} r11 & 0 & 0 \\ 0 & r22 & r23 \\ 0 & r22 & r33 \end{bmatrix} \tag{4.38}$$

where, for a beam whose axis is parallel to the global axis X_i,

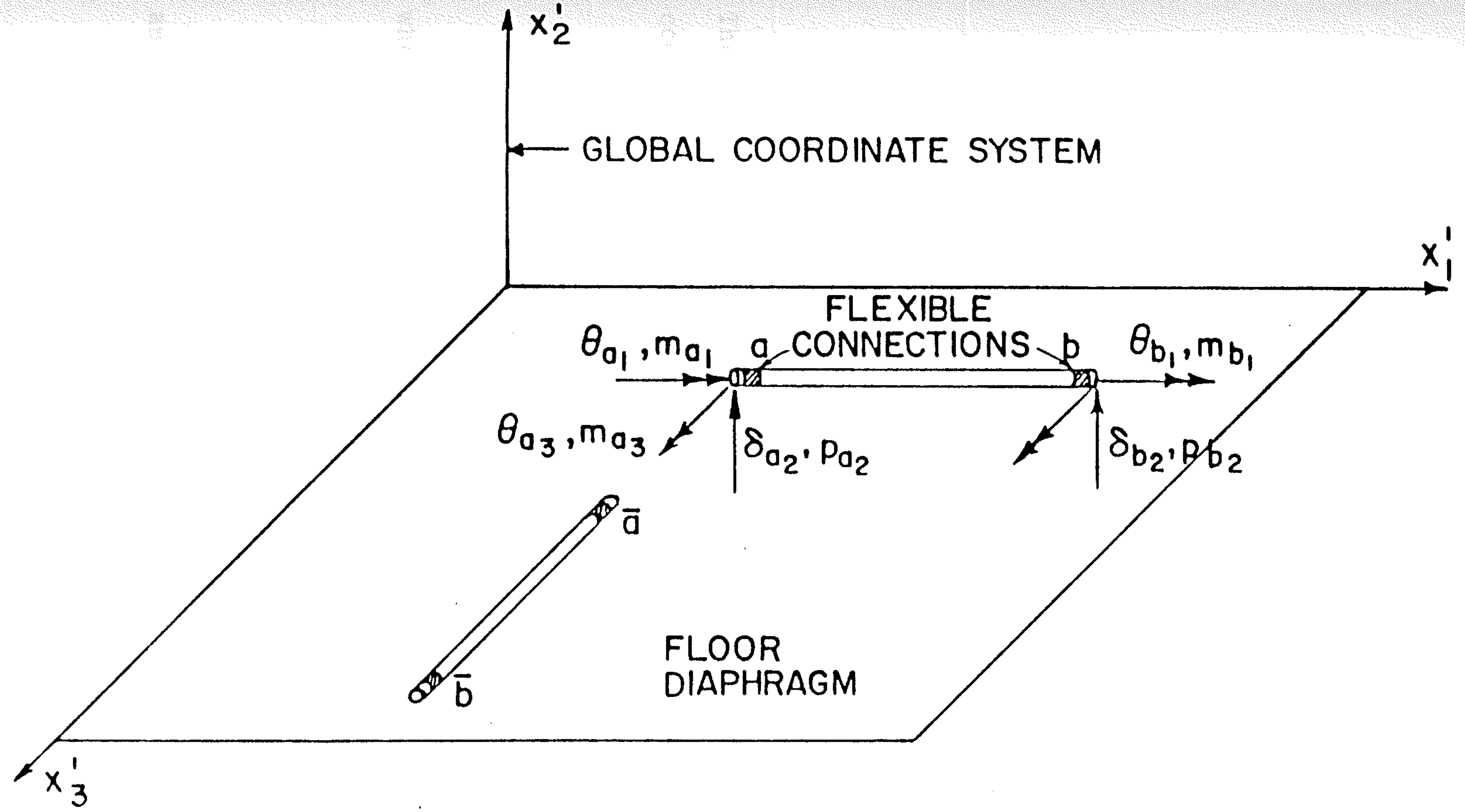


Fig. 4.8 TYPICAL BEAM ab WITH FLEXIBLE CONNECTIONS

$r11 = 1$

$r22 = 1$

$r23 = 0$

$r32 = 0$

$r33 = 1$

For a beam whose axis is parallel to global axis X_3'

$r11 = 1$

$r22 = 0$

$r23 = -1$

$r32 = 1$

$r33 = 0$

Therefore the beam stiffness matrix expressed in the global coordinate system can be defined by the following equation

$$\begin{Bmatrix} P_b' \\ P_a' \end{Bmatrix} = \begin{bmatrix} K_{bb}' & K_{ba}' \\ K_{ab}' & K_{aa}' \end{bmatrix} \begin{Bmatrix} D_b' \\ D_a' \end{Bmatrix} \tag{4.39}$$

where

$$K_{bb}' = R_b \; K_{bb} \; R_b^T \; ,$$

$$K_{ba}' = R_b \; K_{ba} \; R_b^T \; ,$$

$$K_{ab}' = R_b \; K_{ab} \; R_b^T \; ,$$

and

$$K_{aa}' = R_b \; K_{aa} \; R_b^T \; .$$

The contributions of a typical beam to the structure stiffness matrix are illustrated by Fig. 4.9.

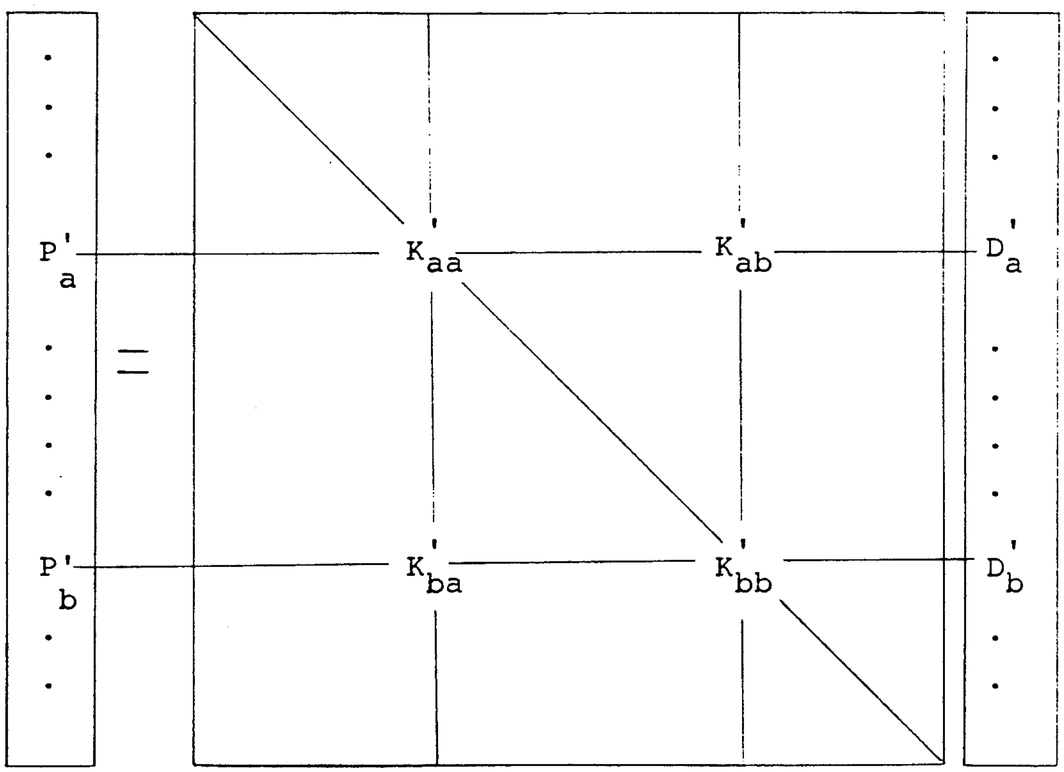


FIG. 4.9 CONTRIBUTION OF A TYPICAL BEAM STIFFNESSES TO THE STRUCTURE STIFFNESS MATRIX

CHAPTER V

STRUCTURAL ANALYSIS COMPUTER PROGRAM

5.1 Introduction

A structural analysis computer program, named TFNCSAP and written in FORTRAN, has been developed in this study, to perform the structural analysis of three dimensional rectangular steel frames under static loading conditions. The structure is assumed to have a rigid-in-plane diaphragm at each floor level, and it can comprise any rectangular network of beams and columns connected together by any of the following types of beam-to-column connections.

(a) Single-Web Angle,

(b) Double-Web Angle,

(c) Header-Plate,

(d) Top-and-Seat Angle

(e) Strap-Angle.

For each connection type used, the associated size parameters must be specified. These size parameters allow the program to generate the moment-rotation relationship for the connection. Alternatively, any of the beam-to-column connections may be assumed to be perfectly rigid or to have bending moment releases. The column bases are assumed to be fixed.

The structure loading may consist of any number of nodal loads and concentrated or uniformly distributed loads applied to the beams. Separate gravity and wind loading systems can be accommodated. Gravity loading may be applied at the slave nodes or on the beam members, whereas wind loading must be applied as a series of three-component vectors at the master nodes only. Because of the non-linearity of the moment-rotation characteristics of the connections, the principle of superposition cannot be applied to combine the results for the gravity load analysis with those for wind load analysis.

The program can accommodate either SI or Imperial units. The analysis procedure, in general, consists of initialization, followed by repeated cycles of the following steps:

(a) linear analysis,

(b) test for termination,

(c) modification of the connection stiffness characteristics.

The program also incorporates the iterative procedure described in Appendix J of CSA Specification CAN 3-S16.1-M78 (1978) to account for the displacements and internal forces produced by the PΔ effect. The procedure is summarized in Appendix E

A flow diagram for the program is presented in Fig. 5.1 and the various program phases are described in the following sections.

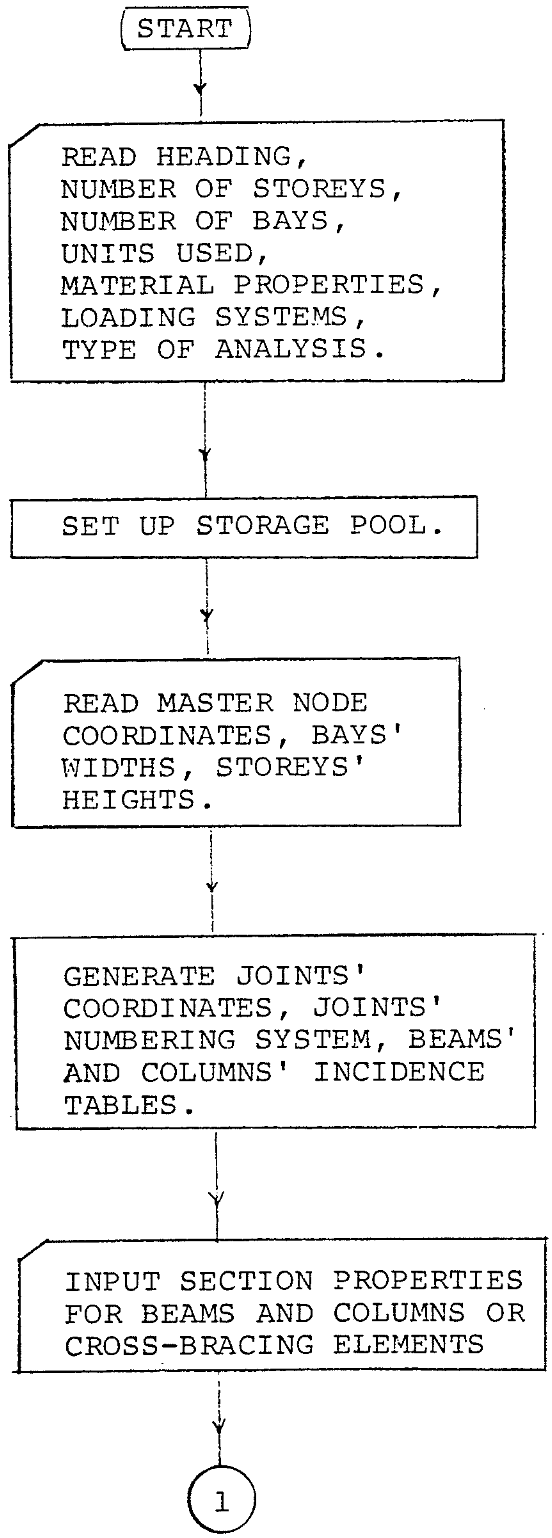


FIG 5.1 FLOW CHART FOR THE STRUCTURAL ANALYSIS COMPUTER PROGRAM

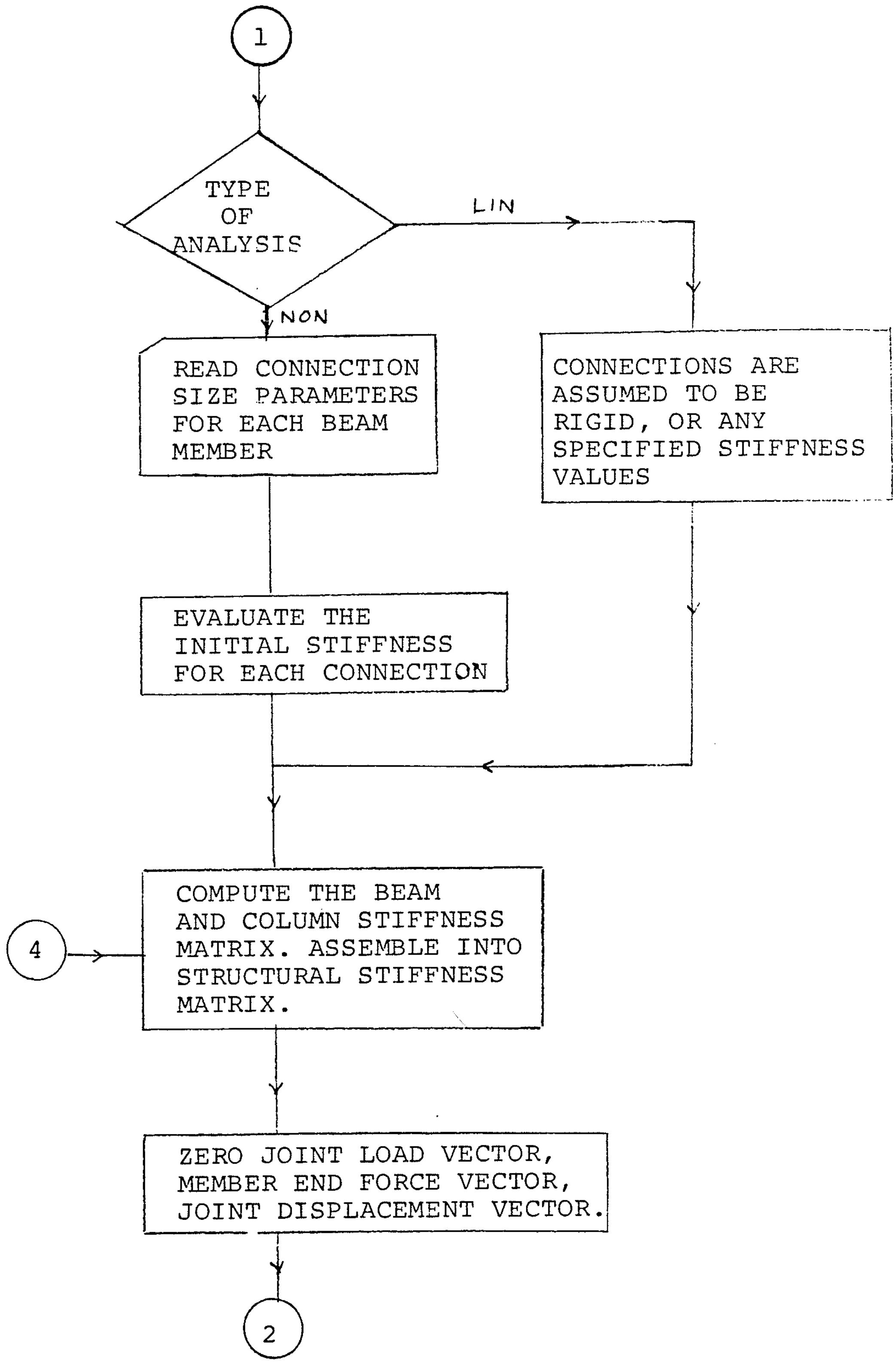


FIG 5.1 (CONTINUE)

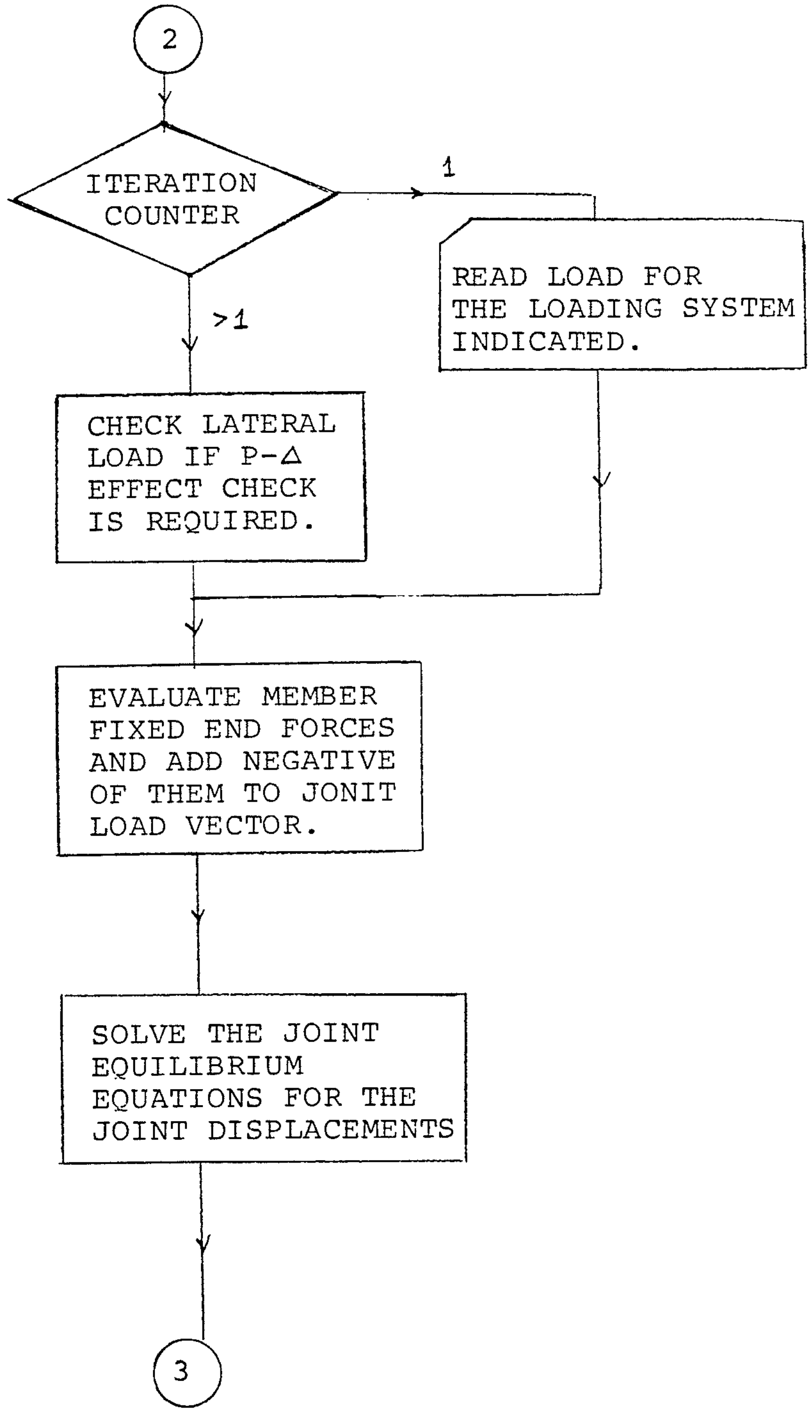


FIG. 5.1 (CONTINUE)

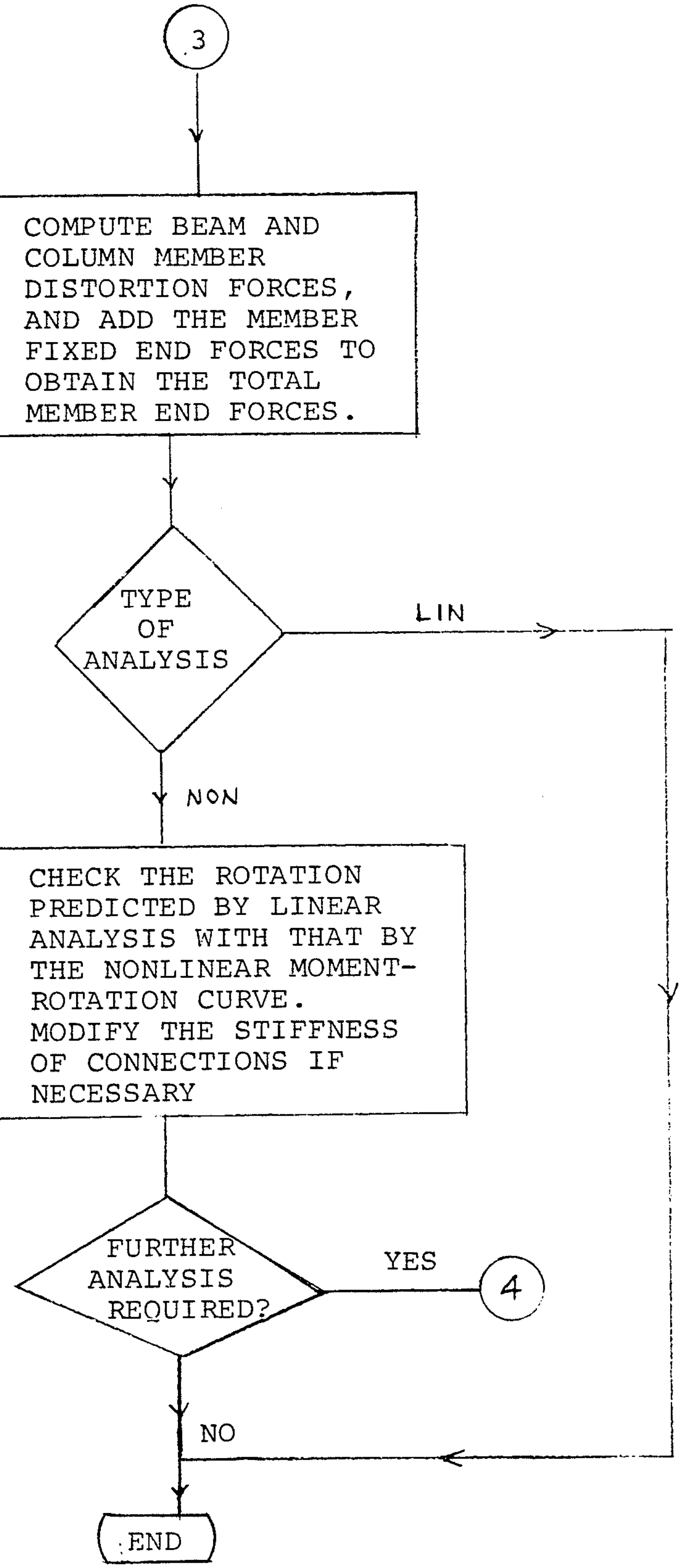


FIG. 5.1 (CONTINUE)

5.2 Initialization

The initialization involves the specification of the units to be employed (S.I. or Imperial), the type of analysis (linear or nonlinear, with or without the PΔ effect), the characteristics of the structure, the material properties and the description of the loading system. The default system of units is the S.I. system.

The type of analysis is specified by indicating LIN or NON, and PY or PN. If LIN is specified, values of the flexural stiffnesses of all beam column connections are input. The values may be very small (e.g. 10^{-10}), indicating a moment release, very large (e.g. 10^{20}), indicating complete continuity between the beam and the column, or any value in between.

If NON is specified, all beam-column connection types must be input along with the connection size parameters. PY indicates PΔ effect is to be included in the analysis; PN indicates that it is to be ignored.

The following information is input to describe the structure:

(a) the number of bays in the X_1' and X_3' directions,

(b) the number of storeys,

(c) the width of each bay in each direction,

(d) the height of each storey

(e) the global X_1', X_2', and X_3' coordinates of the master nodes at all floor levels,

(f) the member cross-section properties.

With this information, the program establishes the global coordinates of all joints, the interconnectivity of all members and joints, numbering systems for all joints, columns and beams, and a table of member cross-sectional properties.

Unless otherwise specified, the modulus of elasticity is taken as 30,000 Ksi or 200. GPa, and the shearing modulus as 12,000 Ksi or 80. GPA.

5.3 Linear Analysis

The stiffness method is used to perform the linear analysis. The program employs an in-core, constant bandwidth equation solver. Because of symmetry of the structure stiffness matrix, only the elements below the main diagonal are stored as a one-dimensional array. The beam stiffness coefficients, which incorporate the effects of flexible connections at the ends of the beams, are recalculated each time they are needed. The member fixed-end forces are also dependent on the connection characteristics and thus they are recalculated for each linear analysis.

5.4 Termination Criteria

Two different criteria are used to determine when the analysis should be terminated. Firstly, a test is performed to determine when the connection rotations have converged to the values indicated by the nonlinear functions. Secondly, a test is performed to determine that the secondary forces contributed by the P-Δ effect have become negligibly small. The former is indicated when the rotation for each connection, computed on the basis of the current linearized stiffness, is equal to that calculated from the nonlinear moment-rotation function. The latter is satisfied when the secondary forces computed in the current cycle of analysis are negligible as compared to those calculated in the previous cycle.

For frames with very flexible connections and large loads, it is possible that the iterative procedure will not converge on a value of connection stiffness. In this event, the connection end moments obtained from the first linear analysis exceed the maximum capacity of the connection, as illustrated for a typical connection, in Fig. 5.2. A decrease in the stiffness of a connection causes a reduction in the moment carried by that connection and a redistribution of the moment to other connections and other points in the structure.

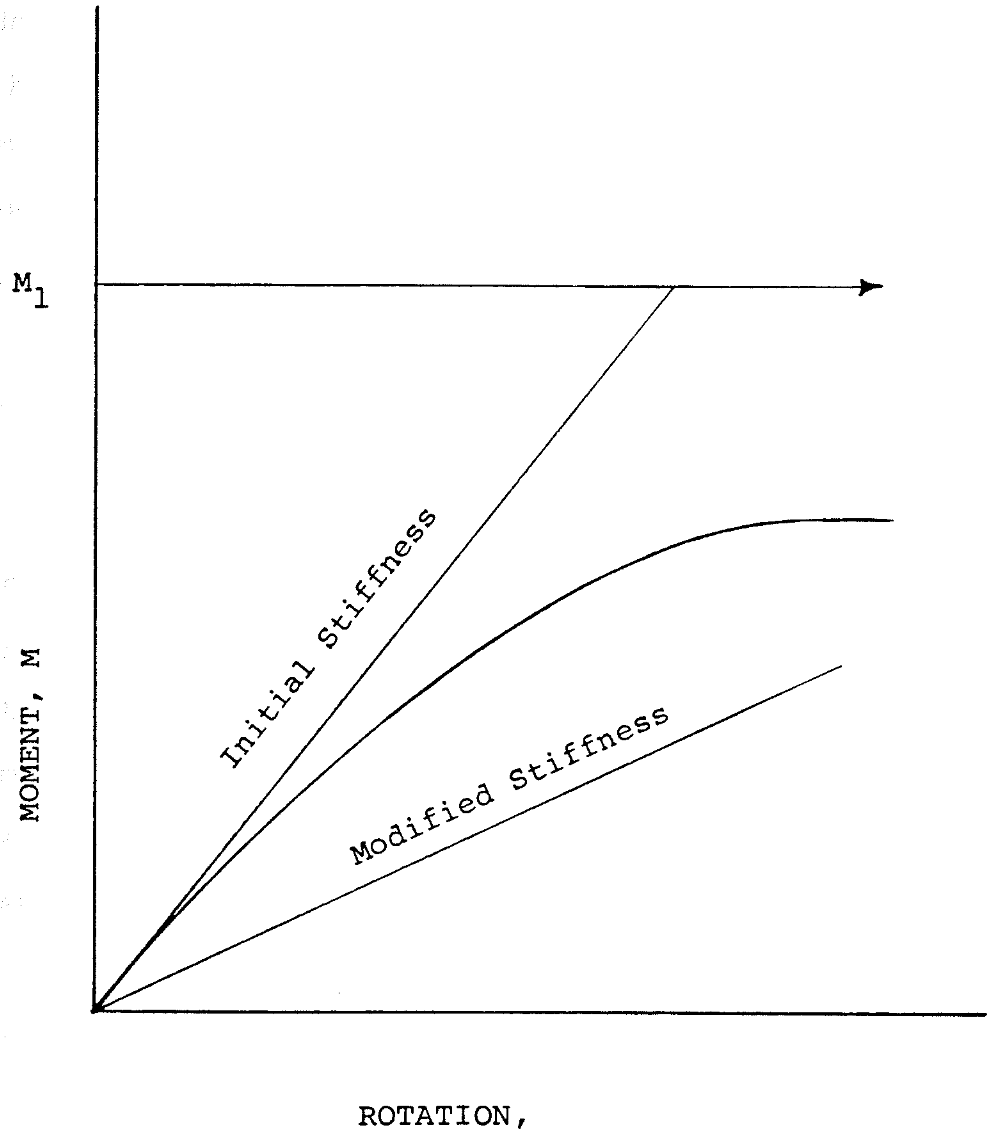


FIG. 5.2 MOMENT ROTATION CURVE FOR NON CONVERGING ITERATIVE PROCEDURE

However, if the other connections have already exceeded their maximum moment capacity, the analysis fails to converge. Therefore, a counter has been incorporated in the program and the analysis is automatically terminated after a specified number of cycles of iteration.

5.5 Modification of the Connection Stiffness and the Loading System

After each analysis, the assumed stiffness of each connection is modified if the connection rotation predicted by the linear analysis is different from that predicted by the nonlinear moment-rotation curve by more than an acceptable amount. Similarly, the loading system is modified if the secondary forces contributed by the P-Δ effect are too large (approximately more than 10%) to be ignored.

5.6 Program for Generating Standardized Moment-Rotation Functions

To facilitate the addition of new connection types for which experimental data are available into the structural analysis program (TFNCSAP), a separate program, named CONNECTION, has been written. This program is used to compute the required exponent values a_j in the expression (Equation 3.13) for the dimensionless factor K, and then to compute the constant $[KM]_o$, ϕ_o and η for the standardized moment-rotation function. The expression for the dimensionless factor K and the standardized function can then be coded in FORTRAN and can be easily added into the structural analysis program (TFNCSAP).

The program requires as input the number of experimental moment-rotation curves available, the size parameters q_j and the experimental moment-rotation data associated with each connection. The moment-rotation data enable the program to curve-fit a Ramberg-Osgood function to the data.

Also required for input are the number of rotation values, and the values, to be used in computing the exponents a_j. The exponents a_j, and the constants $[KM]_o$, ϕ_o, and η are computed according to the procedure specified in Section 3.2.

CHAPTER VI

ILLUSTRATIVE EXAMPLES

6.1 Introduction

In this chapter, several examples are presented to illustrate the application of, and to check the validity and accuracy of, the analysis procedure. Examples have been selected which involve simple structures and loadings, but which illustrate the significance of connection deformation, and the correctness and the capabilities of the computer program. The procedure for generating the standardized moment-rotation curves has also been illustrated by a numerical example. For the sake of simplicity, only selected data and results are presented and discussed.

6.2 Standardized Moment-Rotation Curve for Single-Web Angle Connections

The purpose of this example is to illustrate numerically the procedure, described in Section 3.2, for generating the standardized moment-rotation curve for a particular connection type, given experimental moment-rotation data. The single web angle connections are used in the example.

Example 1:

For single-web angle connections, the geometric parameters which most strongly affect the moment-rotation characteristics are:

depth of connection - d

gage of column flange bolts - g, and

thickness of angles - t,

as illustrated in Figure A.1 in Appendix A.

For illustrative purposes, the experimental moment-rotation data and the geometric properties of the connections are presented in Tables 6-1 and 6-2. The calculation of the values of the exponents a_j in Equation (3.13) is illustrated below.

Parameter 1 - depth of connections

Consider test specimens AA3/1 and AA4/1 in Tables 6-1 and 6-2. For a rotation value of $\phi = 0.015$ radians, the moments values are

$$M_1 \text{ (AA3/1)} = 28 \text{ in-Kip}$$

$$M_2 \text{ (AA4/1)} = 57 \text{ in-Kip.}$$

The corresponding depth parameters are

$$d_1 \text{ (AA3/1)} = 7.5 \text{ in and}$$

$$d_2 \text{ (AA4/1)} = 10.5 \text{ in.}$$

Hence, by Equation (3.15)

$$a_1 = \frac{\log\left(\frac{28}{57}\right)}{\log\left(\frac{10.5}{7.5}\right)} = \frac{-0.3087}{0.1461} = -2.11 \ .$$

Rotation ϕ (radian)	MOMENT (Kip-in) TEST SPECIMEN					
	BB4/1	C4	AA3/1	AA4/1	AA5/1	AA6/1
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01	68	138	22	44	79	126
0.015	84	154	28	57	97	151
0.020	95	165	33	67	109	169
0.025	104	174	37	74	119	184
0.030	111	182	40	81	127	197

Table 6-1 Experimental Moment Rotation Values for Single-Web Angles Connections (Lipson, 1968).

Geometric Parameters P_j	TEST SPECIMEN					
	BB4/1	C4	AA3/1	AA4/1	AA5/1	AA6/1
d (in)	10.5	10.5	7.5	10.5	13.5	16.5
t (in)	0.3125	0.3125	0.25	0.25	0.25	0.25
g (in)	2.5625	1.9375	2.5625	2.5625	2.5625	2.5625

Table 6-2 Values of the geometric parameters that affects the M-ϕ curves for Example 1.

Test Specimen	BB4/1	C4	AA3/1	AA4/1	AA5/1	AA6/1
Standardization Factor K	0.3404	0.1913	0.0024	0.4907	0.2899	0.1904

Table 6-3 Standardization factors, K.

The above procedure is repeated for different values of the depth parameter and for other rotation values. Averaging the resulting exponents, a mean value of $a_1 = -2.09$ is obtained.

Similarly, following the same procedure, the values of the exponents for the gage parameter and the thickness parameter are found to be 2.06 and -1.64 respectively.

Hence the standardization factor K for single-web angle connections, obtained by substituting exponents a_1, a_2 and a_3 into Equation (3.13) is

$$K = d^{-2.09} \, g^{2.06} \quad t^{-1.64}$$

The standardization factor K for each of the single-web angle connections in Table 6-2 can be calculated. For each rotation value, the test moments for the various connections, after multiplying by the corresponding factor K, are averaged and the averaged moments are as shown in Table 6-4. Pairs of averaged moments and their corresponding rotations are input to a least-squares Ramberg-Osgood curve-fitting computer program which produces the following equation for the standardized moment-rotation curve for the single-web angle connection:

$$\frac{\phi}{0.0103} = \frac{K \times M}{32.7} \left[1 + \left(\frac{K \times M}{32.7}\right)^{2.93}\right]$$

Rotation ϕ (radian)	TEST SPECIMEN Modified Moment KxM (Kip-in)						Average KxM
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.010	23.0	26.4	21.5	21.5	22.8	24.0	23.2
0.015	28.5	29.5	28.2	28.0	28.2	18.8	28.5
0.020	32.3	21.6	33.1	32.7	31.8	32.3	32.3
0.25	35.3	33.3	36.9	36.4	34.6	35.2	35.3
0.030	37.8	34.7	40.0	39.5	36.9	37.6	37.8

Table 6-4 Modified moment KM and average KM vs ϕ.

This equation can be used to reproduce the moment-rotation curves for single-web angle connections within the range of the test results

6.3 Verification of Structural Analysis Program (TFNCSAP)

In the second example, a simple frame is analyzed and the results are compared with those obtained from a space frame analysis program SFRAME (Pinkney, 1982). The results verify as far as possible, the correctness of the program TFNCSAP developed in this study.

Example 2

Consider the structure and loading shown in Figure 6-1, for which all beam-column connections are assumed to be rigid. The modulus of elasticity and shearing modulus are assumed to be 30000 Ksi and 12000 Ksi respectively. Furthermore the torsion constant J, and moments of area I_2 and I_3 are assumed to be 100 in^4, 200 in^4, and 200 in^4 respectively, for all members. The structure was analyzed using TFNCSAP and then using SFRAME.

The joint displacements and member end moments, as computed by the two programs are summarized in Tables 6-5 and 6-6. It is noted that there are practically no differences between the two sets of results.

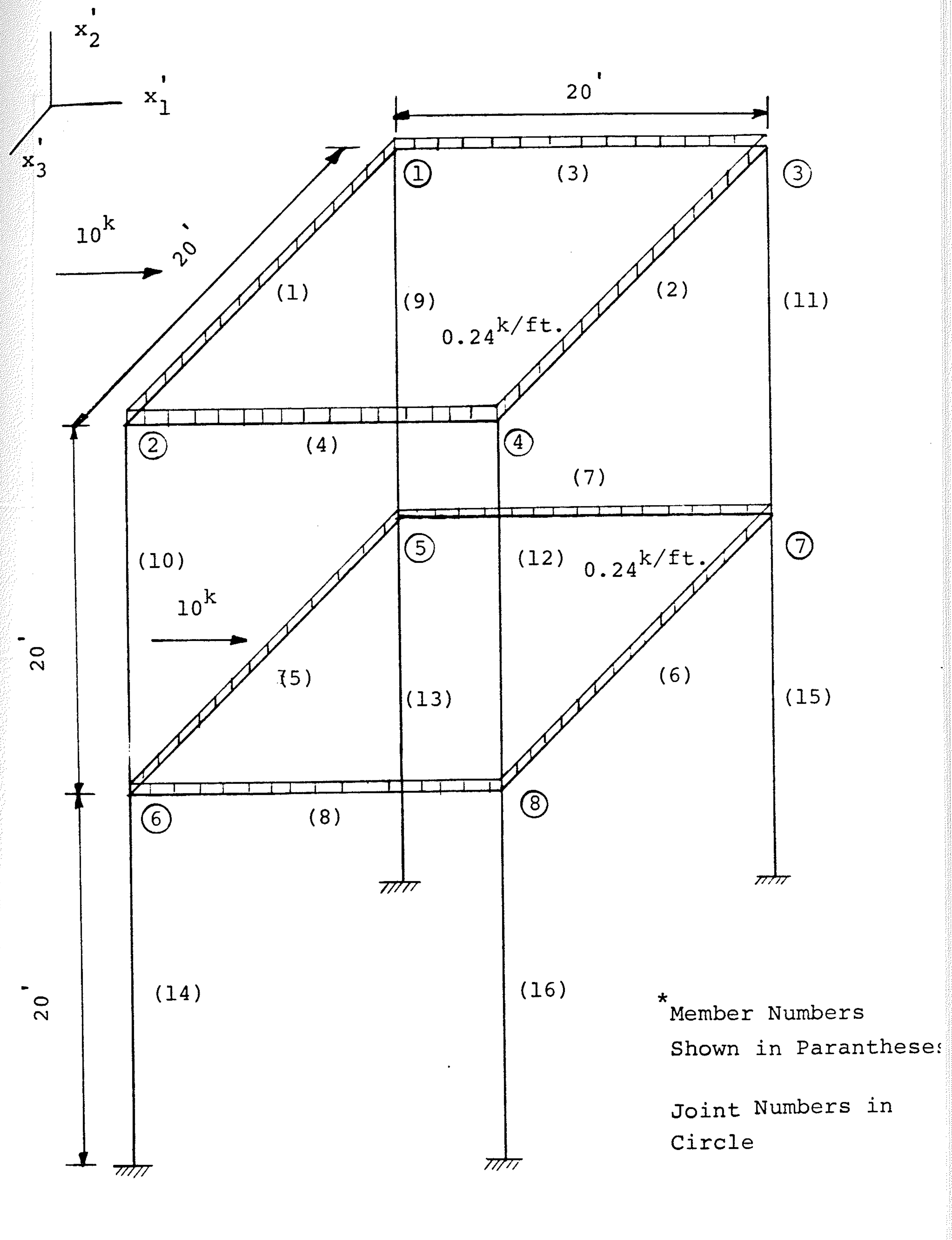


FIG. 6.1 TWO STOREY SPACE FRAME FOR EXAMPLE 2

Joint No.	X_1'-Translation (in)		X_3'-Rotation x 10^3 (rad)	
	TENCSAP	SFRAME	TENCSAP	SFRAME
1	0.50	0.50	-1.27	-1.27
2	0.50	0.50	-1.27	-1.27
3	0.50	0.50	-1.27	-1.27
4	0.50	0.50	-1.27	-1.27
5	0.24	0.24	-2.06	-2.06
6	0.24	0.24	-2.06	-2.06
7	0.24	0.24	-2.06	-2.06
8	0.24	0.24	-2.06	-2.06

EXAMPLE 2

Table 6-5 Joint displacements due to lateral load.

Member	End Moments Kip-ft (left end or bottom) about X_3		End Moments Kip-ft (left end or bottom) about X_3	
	SFRAME	TFNCSAP	SFRAME	TFNCSAP
1	6.61	6.63	-6.61	-6.63
2	6.61	6.63	-6.61	-6.63
3	6.61	6.63	-6.61	-6.63
4	6.61	6.63	-6.61	-6.63
5	7.41	7.41	-7.41	-7.41
6	7.41	7.41	-7.41	-7.41
7	7.41	7.41	-7.41	-7.41
8	7.41	7.41	-7.41	-7.41
9	-5.02	-5.07	-6.61	-6.63
10	-5.02	-5.07	-6.61	-6.63
11	5.02	5.07	6.61	6.63
12	5.02	5.07	6.61	6.63
13	-1.22	-1.17	-2.40	-2.34
14	-1.22	-1.17	-2.40	-2.34
15	1.22	1.17	2.40	2.34
16	1.22	1.17	2.40	2.34

EXAMPLE 2

Table 6-6a Member end moments about member axis X_3 direction due to gravity load.

Member	End Moments Kip-ft (left end or bottom) about X_3		End Moments Kip-ft (left end or bottom) about X_3	
	SFRAME	TFNCSAP	SFRAME	TFNCSAP
1	6.61	6.63	-6.61	-6.63
2	6.61	6.63	-6.61	-6.63
3	-9.16	-9.14	-22.39	-22.41
4	-9.16	-9.14	-22.39	-22.41
5	7.41	7.41	-7.41	-7.41
6	7.41	7.41	-7.41	-7.41
7	-18.23	-18.22	-33.05	-33.05
8	-18.23	-18.23	-33.05	-33.06
9	4.21	4.15	9.16	9.14
10	4.21	4.15	9.16	9.14
11	14.24	14.29	22.39	22. 40
12	14.24	14.29	22.39	22. 40
13	32.36	32.41	14.02	14.07
14	32.36	32.41	14.02	14.07
15	34.81	34.75	18.81	18.76
16	34.81	34.75	18.81	18.76

Table 6-6b Member end moments due to gravity and lateral load for Example 2.

6.4 Effect of Connection Deformation on Member End Forces

While the connections in a typical steel structure constitute only a small proportion of the weight of the structure, because of their high labor content, they sometimes contribute substantially to the total framing cost. They may affect significantly the internal force distribution, and it may be unconservative to ignore the deformations of the connections in a structural analysis. The purpose of the third example is to demonstrate that connections may, in fact, significantly affect the internal force distribution in the structure.

Example 3

The braced steel frame for a two storey office building is shown in Fig. 6.2. The structure is designed for the following loads:

Roof

Snow Load = 1.72 KN/m^2

Dead Load = 0.91 KN/m^2

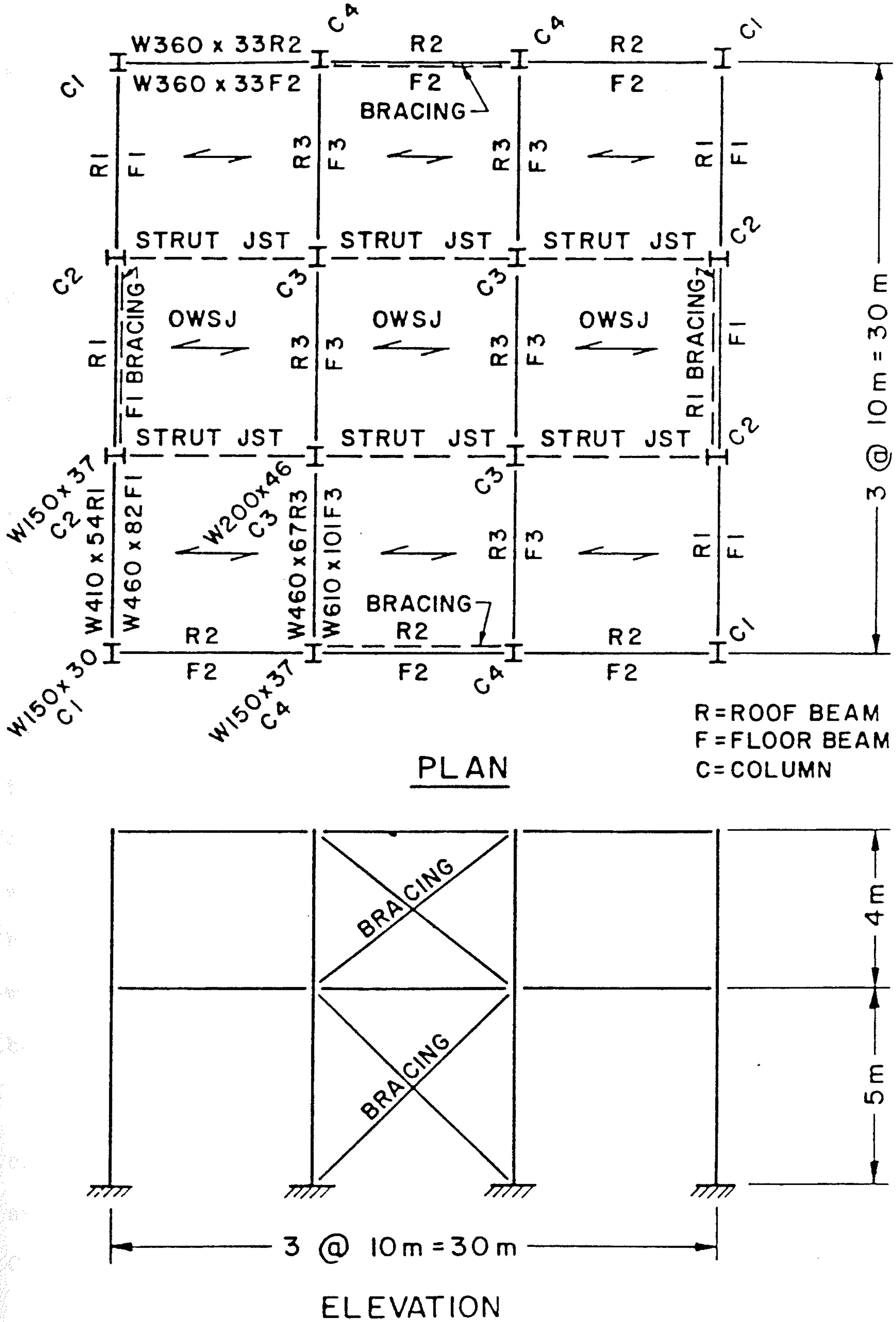
Floor

Dead Load = 2.39 KN/m^2

Live Load = 2.39 KN/m^2

Wall

Dead Load = 1.20 KN/m^2



EXAMPLE III-2 STOREY BRACED FRAME.

Fig. 6.2

Normally, the beam-to-columns connections would be assumed to act as pins and the columns would be designed in the absence of moments applied by the beams.

The frame was analyzed firstly assuming all beam-to-column connections to be of the double-web angle type, then assuming them to be of the header plate type and finally, assuming them to be rigid.

The column end moments for the three analyses are shown in Tables 6-7 and the axial forces are shown in Table 6-8. Very substantial moment increases can be observed in both the "strong direction" and the "weak direction" moments in changing from double web angle to header plate to rigid connections.

What is very significant is the influence of connection type on the maximum normal stresses on the various column cross-sections. Normally a braced frame of the type considered would be assumed, for analysis purposes, to have "pinned" beam-column connections. Consequently, the column stresses would be assumed to be those due to axial force only, there being no column bending moments.

However, when the connection behavior is modelled correctly, the columns are found to be subject to axial force and biaxial bending. The resultant normal stresses in columns C1 and C4 are tabulated in Table 6-9 for the three analyses.

Column No.		Upper End Moments (KN - m)			Lower End Moments (Kn - m)		
		Double Web Angle	Header Plate	Rigid Connection	Double Web Angle	Header Plate	Rigid Connection
Upper Floor	C1	-23	-39	-53	-17	-44	-52
	C2	-23	-34	-50	-18	-27	-44
	C3	0.07	-0.06	29	-0.01	0.04	16
	C4	-24	-41	-58	-13	-24	-35
Lower Floor	C1	-6	-26	-28	-3	-13	-14
	C2	-7	-11	-20	-3	-5	-10
	C3	-0.05	0.07	2	-0.03	0.03	1
	C4	-0.6	-4	-6	-0.3	-2	-3

Table 6-7a Column end moments about x-x strong axis for Example 3.

	Column No.	Upper End Moments (KN - m)			Lower End Moments (KN - m)		
		Double Web Angle	Header Plate	Rigid Connection	Double Web Angle	Header Plate	Rigid Connection
Upper Floor	C1	-19	-23	-22	-17	-22	-22
	C2	-0.03	0.09	6	-0.03	0.21	6
	C3	0.05	0.2	8	0.1	0.3	7
	C4	0.07	0.5	7	0.06	0.3	7
Lower Floor	C1	-7	-11	-12	-4	-5	-6
	C2	-0.02	0.17	3	-0.01	0.08	2
	C3	0.1	0.2	3	0.05	0.08	1
	C4	0.02	0.06	4	0.006	0.02	2

Table 6-7b Column end moments about weak axis y-y for Example 3.

	Column No.	Axial Forces (KN)		
		Double Web Angle	Header Plate	Rigid Connection
Upper Floor	C1	165	163	140
	C2	291	291	297
	C3	523	524	564
	C4	301	302	278
Lower Floor	C1	426	421	363
	C2	738	740	770
	C3	788	800	846
	C4	510	511	486

Table 6-8 Column axial forces for Example 3.

	Axial	Bending X-axis	Bending Y-axis	Bending axial
Corner Column, C1				
(a) Double Angle	44	105	262	8.34
(b) Header Plate	43	178	317	11.51
(c) Rigid	37	237	303	14.59
Side Column, C4				
(a) Double Angle	79	110	1	1.41
(b) Header Plate	80	187	7	2.43
(c) Rigid	73	265	96	4.95

Table 6-9 Example 3 - Column stresses (MPa)

It can be seen that for the corner column C1, even for the double angle connections, which are normally treated as "pin connections", the ratio of bending stress to axial force stress is 8.34.

Obviously, it can be highly unconservative to ignore the moment resistant capacity of connections

6.5 Effect of Connection Deformation on the Lateral Deflections of Structures

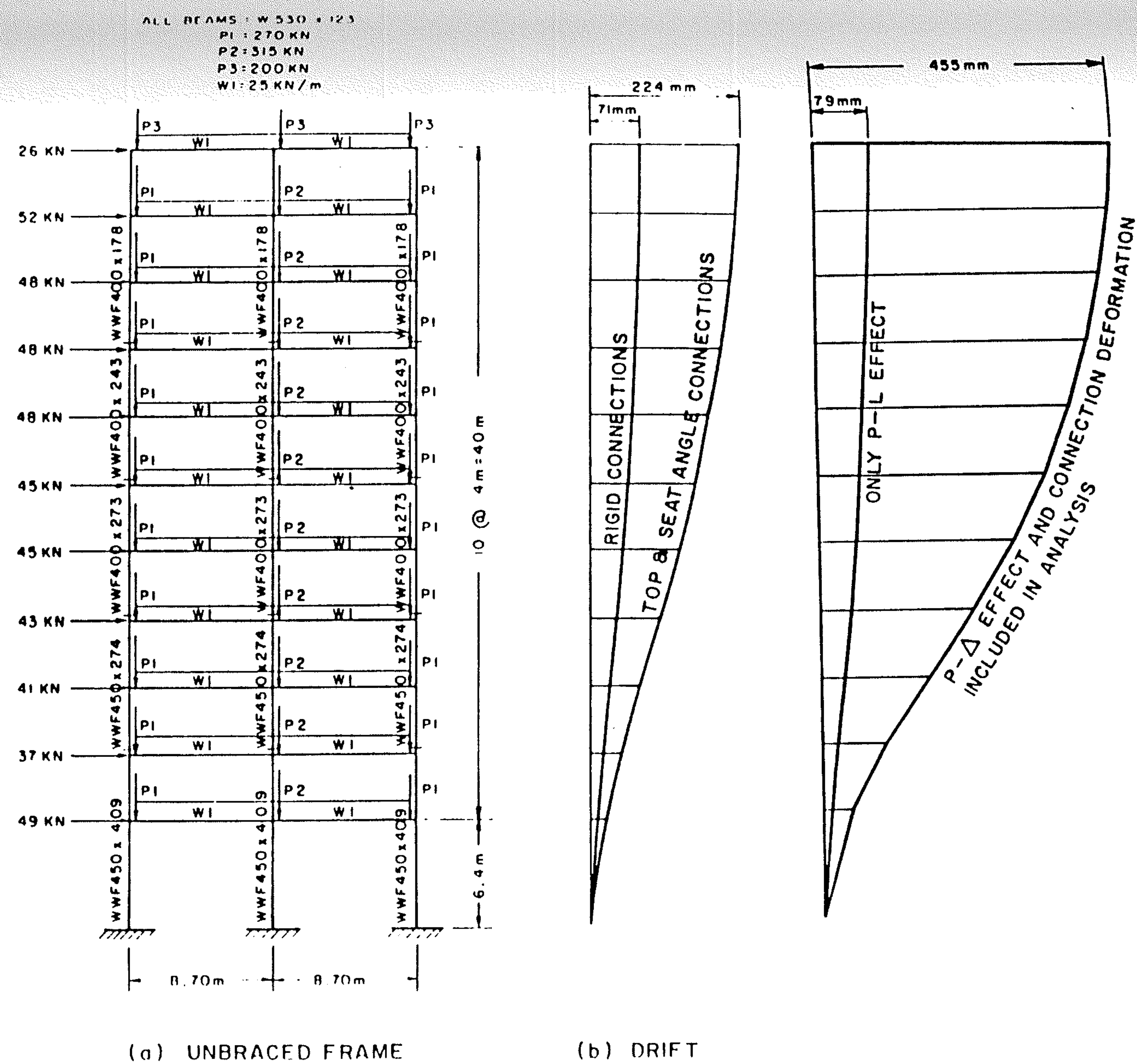
Besides having to satisfy strength requirements, a structure may have to satisfy specified deflection limitations. The deflections of structures, as intuition might predict, may be affected significantly by connection deformation. Hence it is instructive to know the effect of connection deformation on the displacements of structures. The following example demonstrates the effect of connection deformation on the lateral deflection of a multi-story steel frame. Also investigated in the example is the P-Δ effect due to the axial load acting on columns.

Example 4

This example involves the analysis of a typical transverse frame from an 11-storey unbraced steel frame, loaded by gravity and wind loads, as illustrated in Figure 6.3. The frame was first analyzed with all connections assumed to be completely rigid. The same frame was subsequently analyzed assuming top-and-seat-angle beam-column connections. Although the top-and-seat-angle connections are perhaps more flexible than desirable for this application, they demonstrate the sometimes dramatic contribution of connection deformation to structural deflection.

As illustrated in Figure 6.3, and Table 6-10, the transverse deflection of the frame with top-and-seat-angle connections is more than three times that for the frame with assumed rigid beam-column connections. On the other hand, the P-Δ effect increases the deflection of the frame with rigid connections by only 11 percent.

It is interesting to note that when connection deformations are accounted for in the analysis, they amplify the P-Δ effect and the result is excessively large deflections. Thus, for the rigid connections, the P-Δ effect increases the 11th storey drift only from 70 mm to 79mm, while for the top and seat angle connections, the increase is from 224mm to 455mm.



EXAMPLE IV

Fig. 6.4

Storey Level	Rigid Connection	Rigid Connection and P-Δ Effect	Top-and-Seat-Angle	Top-and-Seat-Angle plus P-Δ Effect
3	30	34	78	167
6	53	60	160	350
9	67	75	209	435
11	70	79	224	455

Table 6-10 Lateral deflection in mm for the structure in Example 4.

CHAPTER VII

CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

7.1 Conclusions

In this study, a procedure has been developed for generating a standardized moment-rotation function for steel beam-column connections of a given type, in terms of several geometric parameters, such as depth, plate thickness.
Using experimentally obtained moment-rotation data, standardized moment-rotation functions, expressed in terms of a Ramberg-Osgood function, have been derived for five commonly used beam-column connection types.

The standardized functions have been incorporated into a computer program which performs a linear structural analysis of three-dimensional rectangular steel frames with rigid in-plane floor diaphragms and deformable beam-column connections. The standardized functions permit the moment-rotation behavior for a given connection to be reproduced accurately, when the connection geometric parameters have been specified. The program will also analyze structures with rigid or "pinned" beam-column connections. It also incorporates the P-Δ effect prescribed in Specification CAN3-S16.1-M78 for accounting for the secondary bending moments caused by axial column loads.

Since the moment-rotation behavior of connections is markedly non-linear, an iterative, successive approximation procedure has been incorporated into the computer program. The procedure involves repeated modifications to the assumed stiffness characteristics of all connections in the structure, the stiffness matrix and the fixed-end-force vectors for any member with flexible connections at its ends.

Examples have been included to demonstrate that connection deformation has a very significant effect on the internal force distribution in, and the deflection of, a structure.

7.2 Suggestions for Further Study

It is suggested that, in the future study of the behavior of flexibly connected frames, a comprehensive, experimental study of the force-deformation behavior of all common connection types be conducted to supplement the presently available information, and perhaps to arrive at more rational constitutive relationships. Since experimentation is relatively expensive, analytical methods should be considered for developing the complete force-deformation curves for connections.

In the present study, the axial, shearing, and torsional deformations of connections were ignored and possible buckling of members and/or portions of the structure was ignored. However, future analyses could incorporate all of these effects.

To facilitate the design of frames with flexible connections in a design office environment, design tools, perhaps in the form of design charts or tables, could be developed.

The structural analysis computer program developed in this study could be extended to be capable of treating both statically and dynamically loaded structures. The dynamic behavior of connections has yet to be investigated.

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APPENDIX A

EXPERIMENTAL MOMENT-ROTATION CURVES

This Appendix contains the figures and experimental moment-rotation curves for the following connection types:

(a) Single web-angle connections

(b) Double web-angle connections

(c) Header-plate connections

(d) Top-and seat-angle connections

(e) Strap-angle connection.

The pertinent parameters for the above connection types are also summarized in Tables A-1 to A-5. The test numbers refer to the actual experimental test number.

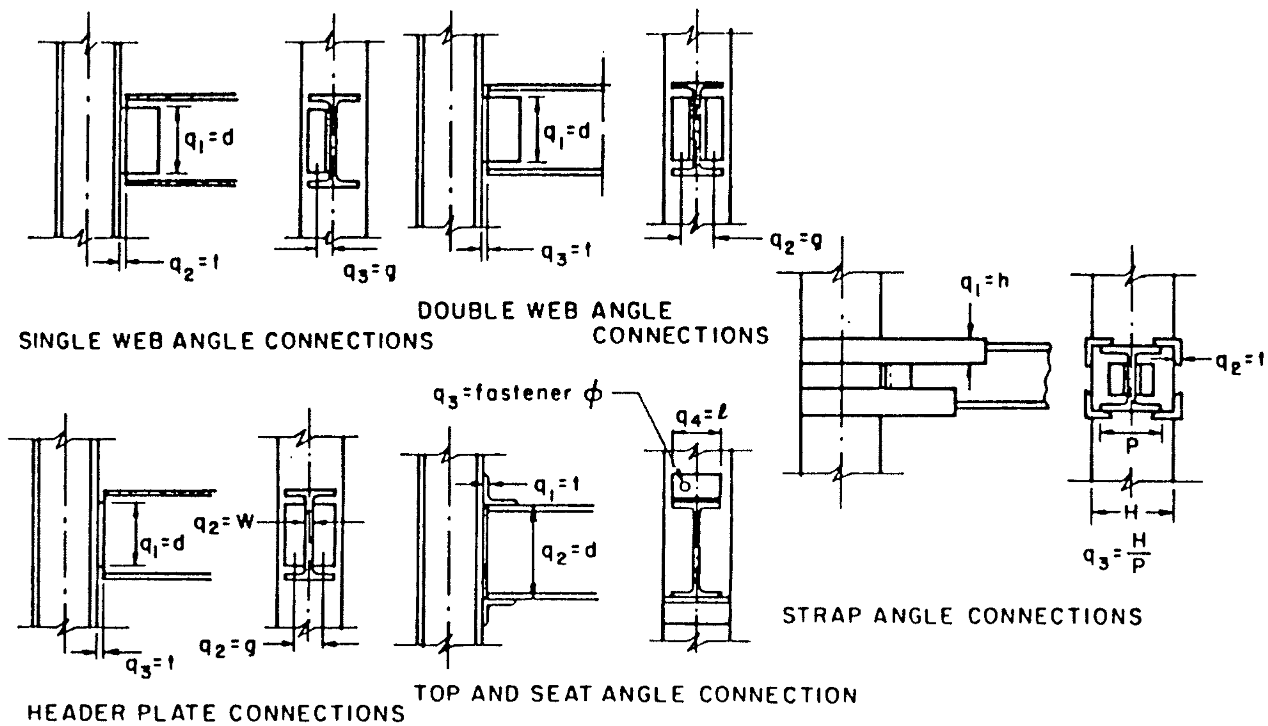


FIG. A.1 CONNECTION TYPES AND STANDARDIZATION PARAMETERS

Investigator	Test No.	Beam Size	Column Size	Web Angle	Gage
S.L. Lipson	BB4-1	21WF62	$\frac{3}{4}$ - Plate	$4 \times 3\frac{1}{2} \times \frac{5}{16} \times 13\frac{1}{2}$	$2\frac{9}{16}$
	C4	21WF62	$\frac{3}{4}$ - Plate	$3\frac{1}{2} \times 5 \times \frac{5}{16} \times 13\frac{1}{2}$	$1\frac{15}{16}$
	AA3-1	21WF62	$\frac{3}{4}$ - Plate	$4 \times 3\frac{1}{2} \times \frac{1}{4} \times 10\frac{1}{2}$	$2\frac{9}{16}$
	AA4-1	21WF62	$\frac{3}{4}$ - Plate	$4 \times 3\frac{1}{2} \times \frac{1}{4} \times 13\frac{1}{2}$	$2\frac{9}{16}$
	AA5-1	21WF62	$\frac{3}{4}$ - Plate	$4 \times 3\frac{1}{2} \times \frac{1}{4} \times 16\frac{1}{2}$	$2\frac{9}{16}$
	AA6-1	21WF62	$\frac{3}{4}$ - Plate	$4 \times 3\frac{1}{2} \times \frac{1}{4} \times 19\frac{1}{2}$	$2\frac{9}{16}$

Table A-1 Single web-angle connections

Investigator	Test No.	Beam Size	Column Size	Web Angle	Gage
Lewitt, Chesson and Munse	FK-3	12WF27	10WF49	$6 \times 4 \times \frac{3}{8} \times 8\frac{1}{2}$	$5\frac{1}{2}$
	FK-4P	18WF50	12WF65	$6 \times 4 \times \frac{3}{8} \times 11\frac{1}{2}$	$5\frac{1}{2}$
Somner	23	24WF76	12WF38	$4 \times 3 \times \frac{3}{8} \times 15$	$5\frac{1}{2}$
	24	24WF76	14WF38	$4 \times 3 \times \frac{3}{8} \times 20$	$5\frac{1}{2}$
Batho and Rowan	8	12 x 5 @ 30	12 x 8 @ 65	$6 \times 3\frac{1}{2} \times \frac{3}{8} \times 9$	4.0
	9	12 x 5 @ 30	12 x 8 @ 65	$6 \times 3\frac{1}{2} \times \frac{1}{2} \times 9$	4.0
	10	12 x 5 @ 30	12 x 8 @ 65	$6 \times 3\frac{1}{2} \times \frac{5}{8} \times 9$	4.0
Rathbun	4	121 @ 31.8	$9 \times \frac{1}{2} \times 16$-Pl	$4 \times 3\frac{1}{2} \times \frac{3}{8} \times 9$	$5\frac{1}{2}$

Table A-2 Double web-angle connections.

Investigator	Test No.	Beam Size	Column Size	Plate Size	Gage
Somner	15	24WF76	14WF38	$15 \times 7\frac{1}{2} \times \frac{3}{8}$	$5\frac{1}{2}$
	18	24WF76	14WF38	$15 \times 7\frac{1}{2} \times \frac{1}{4}$	$5\frac{1}{2}$
	20	24WF76	14WF38	$15 \times 7\frac{1}{2} \times \frac{1}{4}$	$5\frac{1}{2}$
	8	24WF76	14WF38	$15 \times 6 \times \frac{1}{4}$	4.0
	7	24WF76	14WF38	$12 \times 6 \times \frac{1}{4}$	4.0
	9	24WF76	14WF38	$18 \times 6 \times \frac{1}{4}$	4.0
	25	18WF45	14WF38	$12 \times 6 \times \frac{1}{4}$	4.0

Table A-3 Header-plate angle connections

Investigator	Test No.	Beam Size	Column Size	Top Angle	Seat Angle	Fastener Diameter
Hetchman and Johnston	9	18WF47	12WF65	$6 \times 4 \times \frac{5}{8} \times 12$	$6 \times 6 \times \frac{7}{8} \times 7\frac{1}{2}$	$\frac{3}{4}$
	20	14WF34	12WF65	$6 \times 4 \times \frac{5}{8} \times 12$	$6 \times 6 \times \frac{5}{8} \times 7\frac{1}{4}$	$\frac{3}{4}$
	22	16WF40	12WF65	$6 \times 4 \times \frac{5}{8} \times 12$	$6 \times 6 \times \frac{3}{4} \times 7\frac{1}{4}$	$\frac{3}{4}$
	5	18WF47	12WF65	$6 \times 4 \times \frac{1}{2} \times 12$	$6 \times 6 \times \frac{7}{8} \times 7\frac{1}{2}$	$\frac{3}{4}$
	10	18WF47	12WF65	$6 \times 4 \times \frac{3}{4} \times 12$	$6 \times 6 \times \frac{7}{8} \times 7\frac{1}{2}$	$\frac{3}{4}$
	24	18WF47	12WF65	$6 \times 4 \times \frac{5}{8} \times 12\frac{1}{2}$	$6 \times 6 \times \frac{7}{8} \times 7\frac{1}{2}$	$\frac{7}{8}$
	37	18WF47	14WF58	$6 \times 4 \times \frac{5}{8} \times 11\frac{1}{4}$	$6 \times 6 \times \frac{7}{8} \times 11\frac{1}{4}$	$\frac{7}{8}$

Table A-4 Top-and seat-angle connections

Investigators	Test No.	h	t	$\frac{H}{P}$
Brun and Picard	WT-6-04-03 WT-6-05-01	5 4	.75 .75	1.5 1.5
Beaulieu and Giroux	WT-5-01-03 WT-5-02-02 WT-5-03-02	4 4 4	.75 .6250 .5625	1 1 1

Table A-5 Strap-angle connections

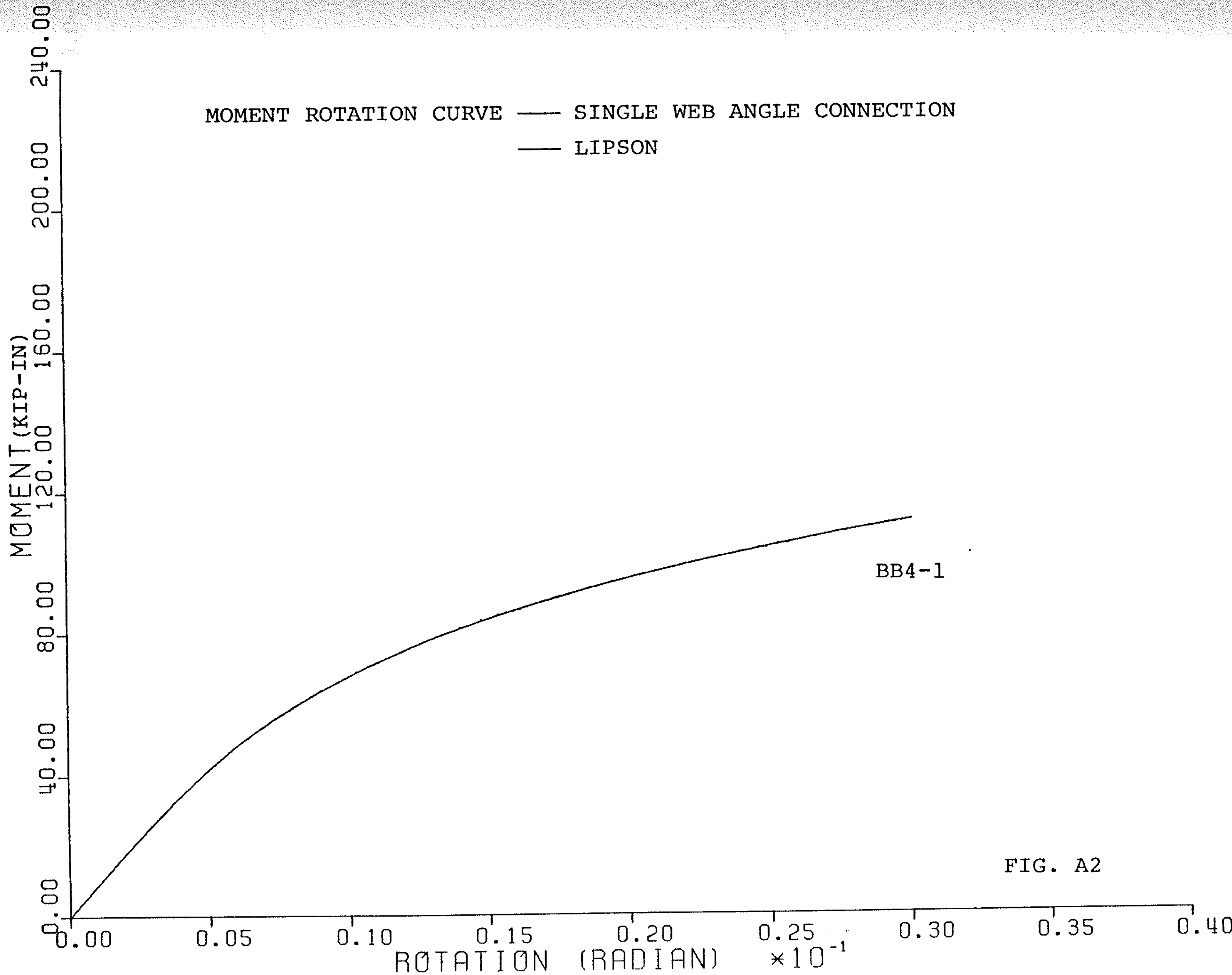

MOMENT ROTATION CURVE — SINGLE WEB ANGLE CONNECTION
— LIPSON
BB4-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A2

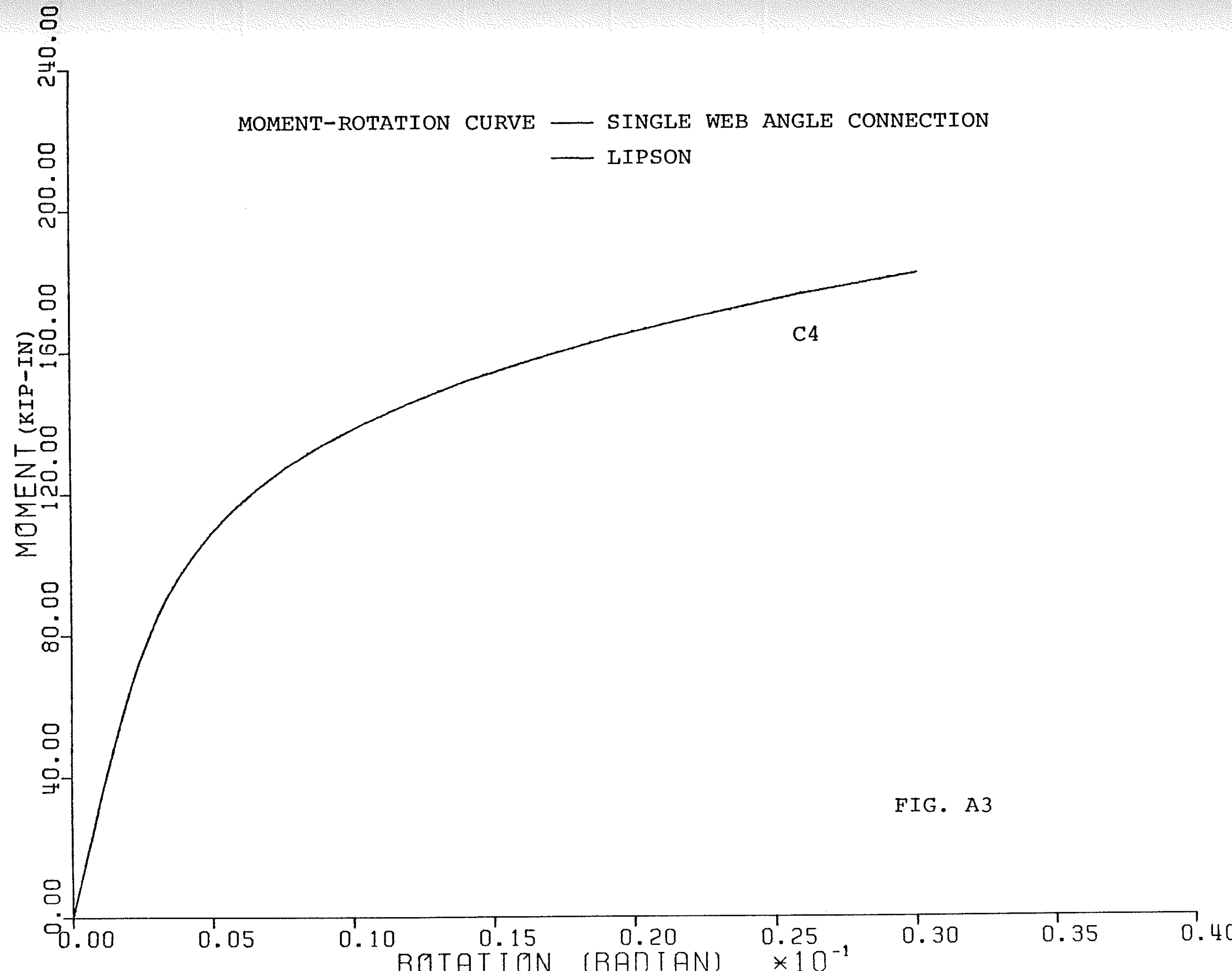

MOMENT-ROTATION CURVE —— SINGLE WEB ANGLE CONNECTION
—— LIPSON
C4
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. A3

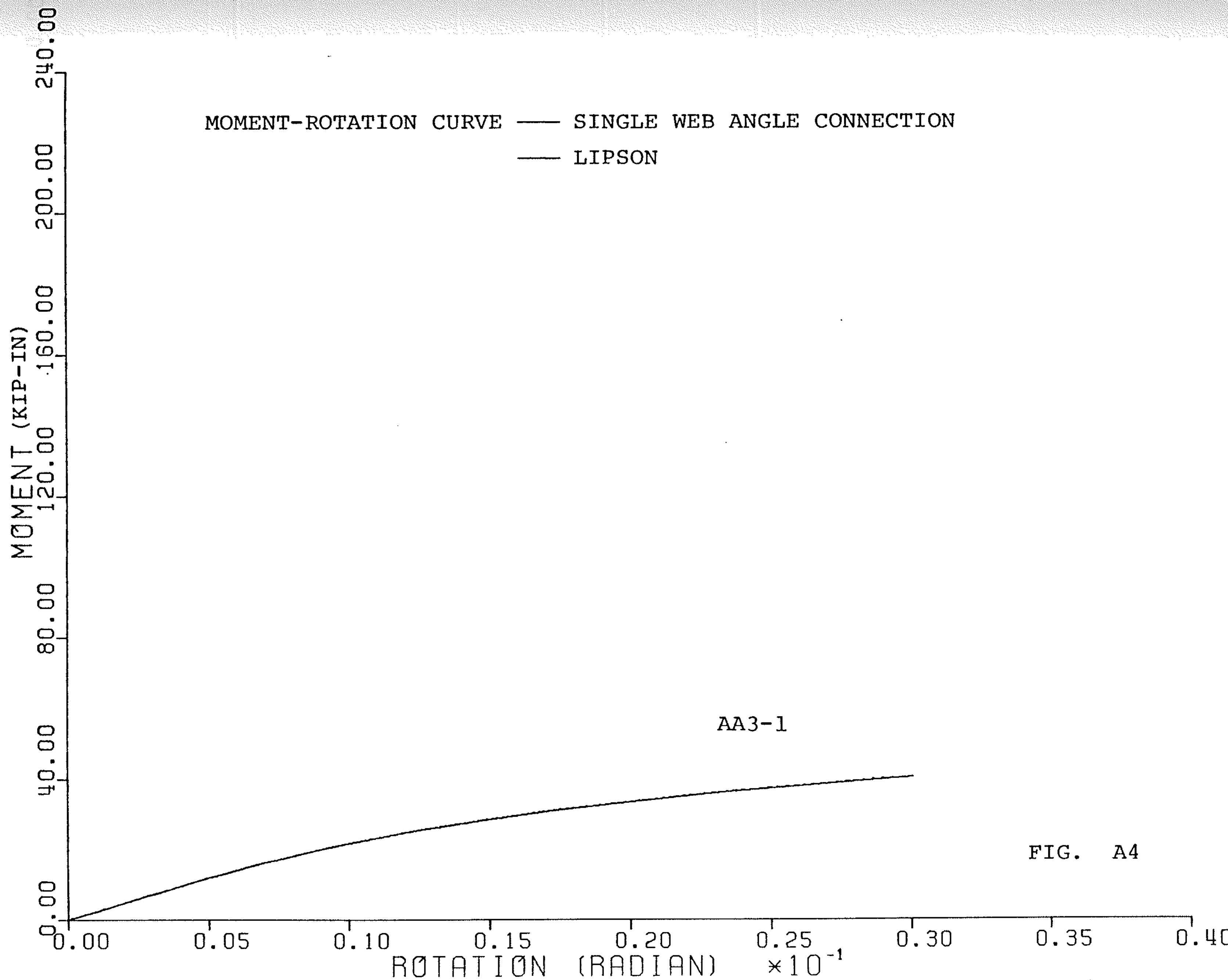

MOMENT-ROTATION CURVE —— SINGLE WEB ANGLE CONNECTION
—— LIPSON
AA3-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A4

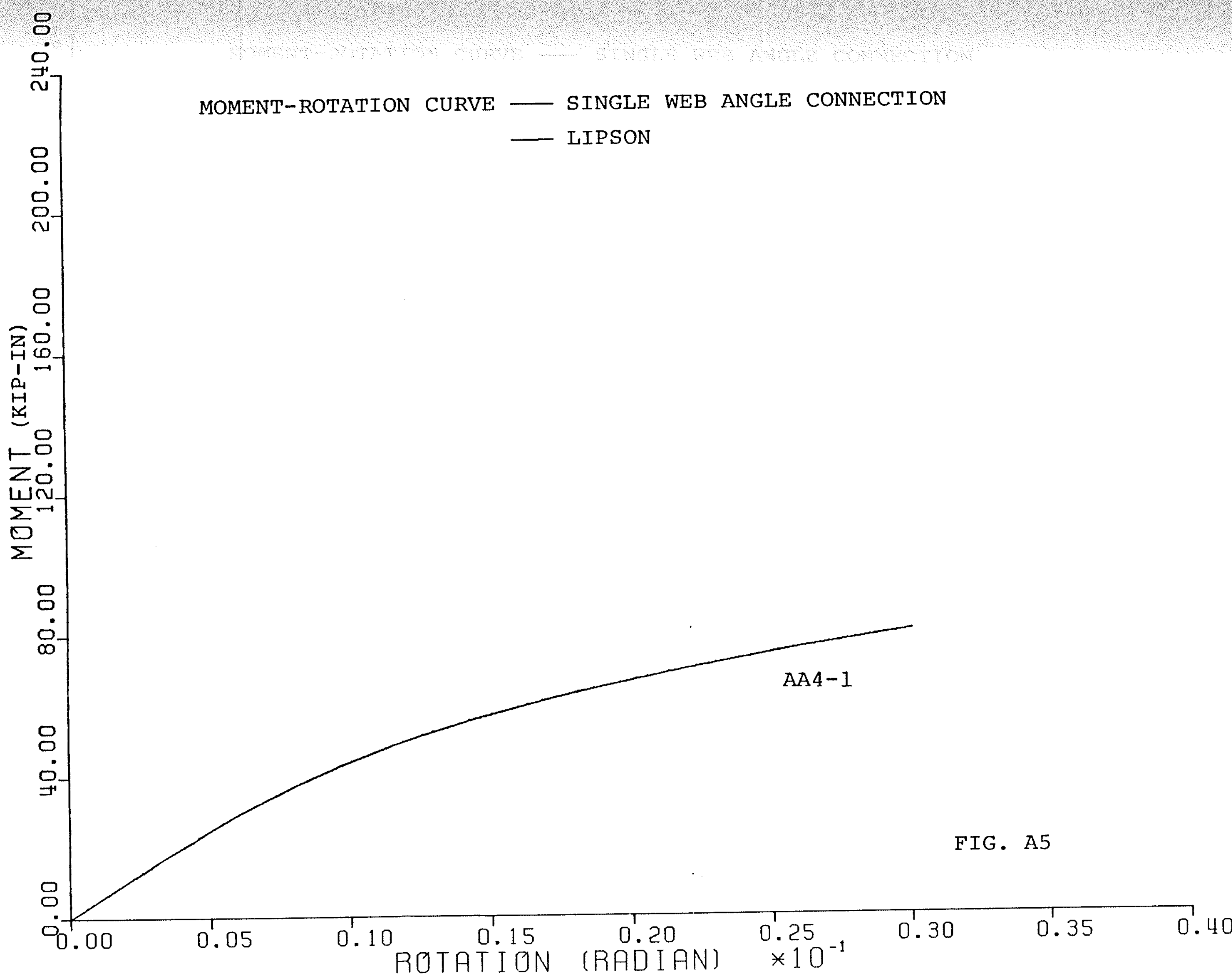
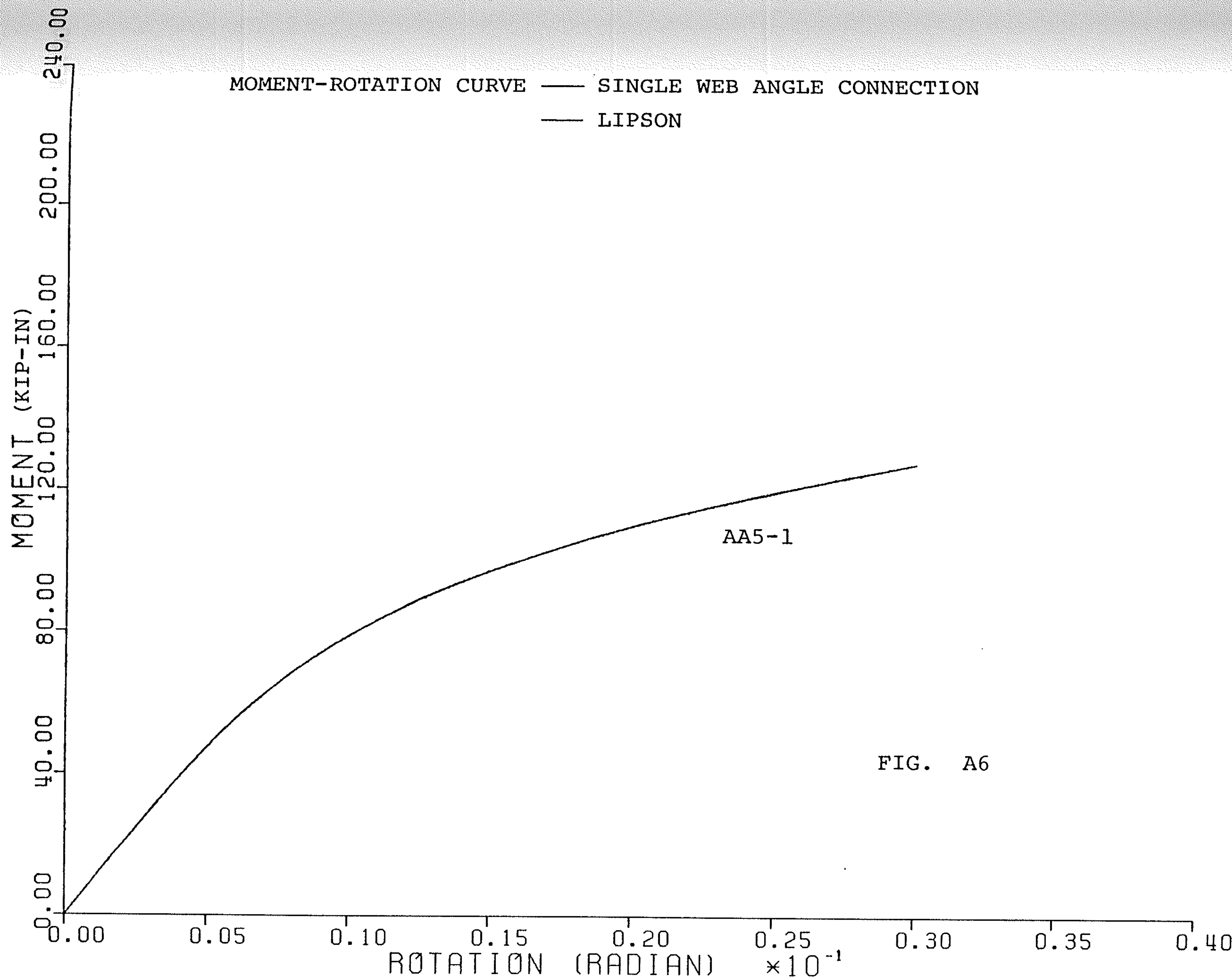

MOMENT-ROTATION CURVE —— SINGLE WEB ANGLE CONNECTION
—— LIPSON
AA4-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. A5



MOMENT-ROTATION CURVE —— SINGLE WEB ANGLE CONNECTION
—— LIPSON
AA5-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) $\times 10^{-1}$

FIG. A6

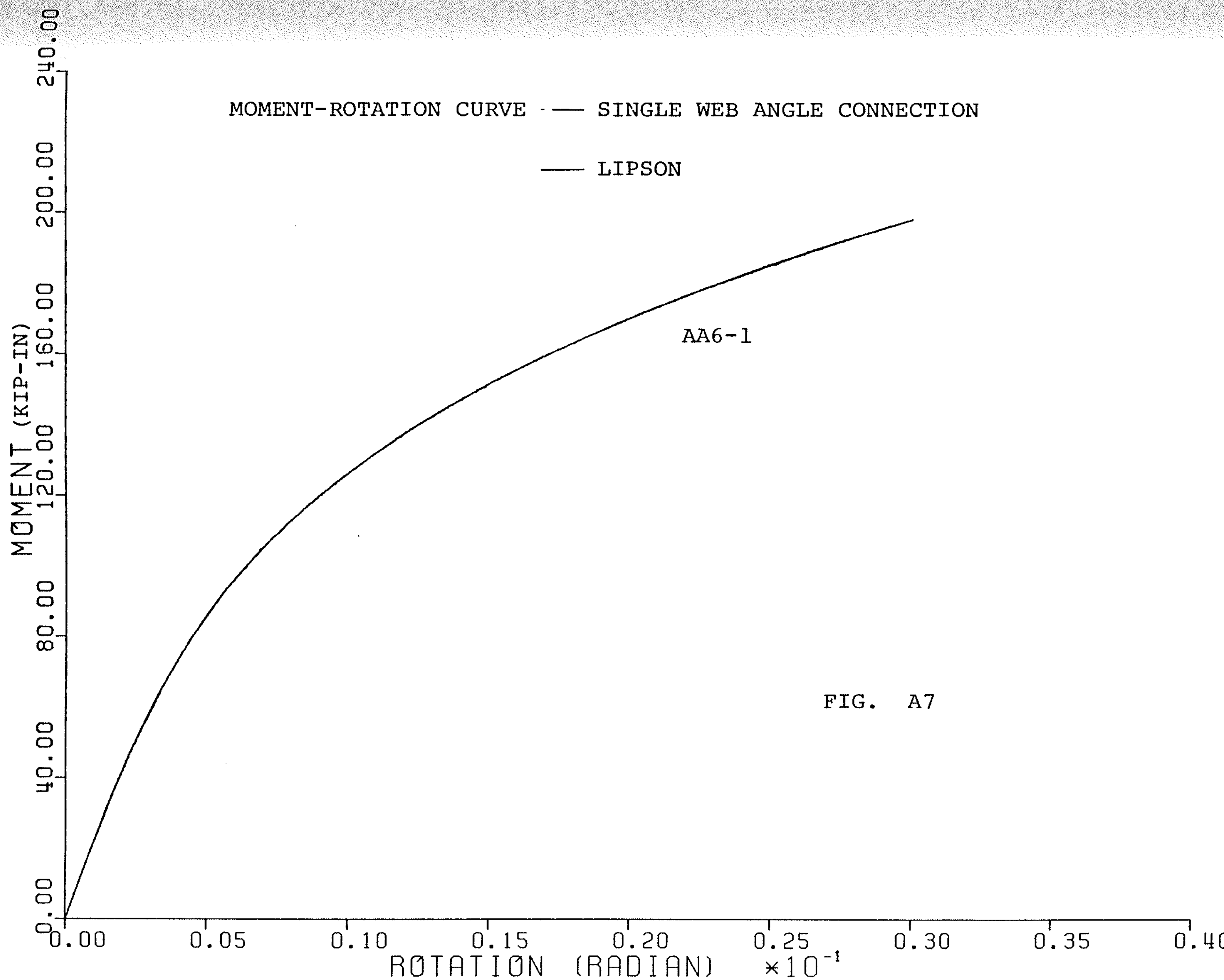

MOMENT-ROTATION CURVE — SINGLE WEB ANGLE CONNECTION
— LIPSON
AA6-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) $\times 10^{-1}$

FIG. A7

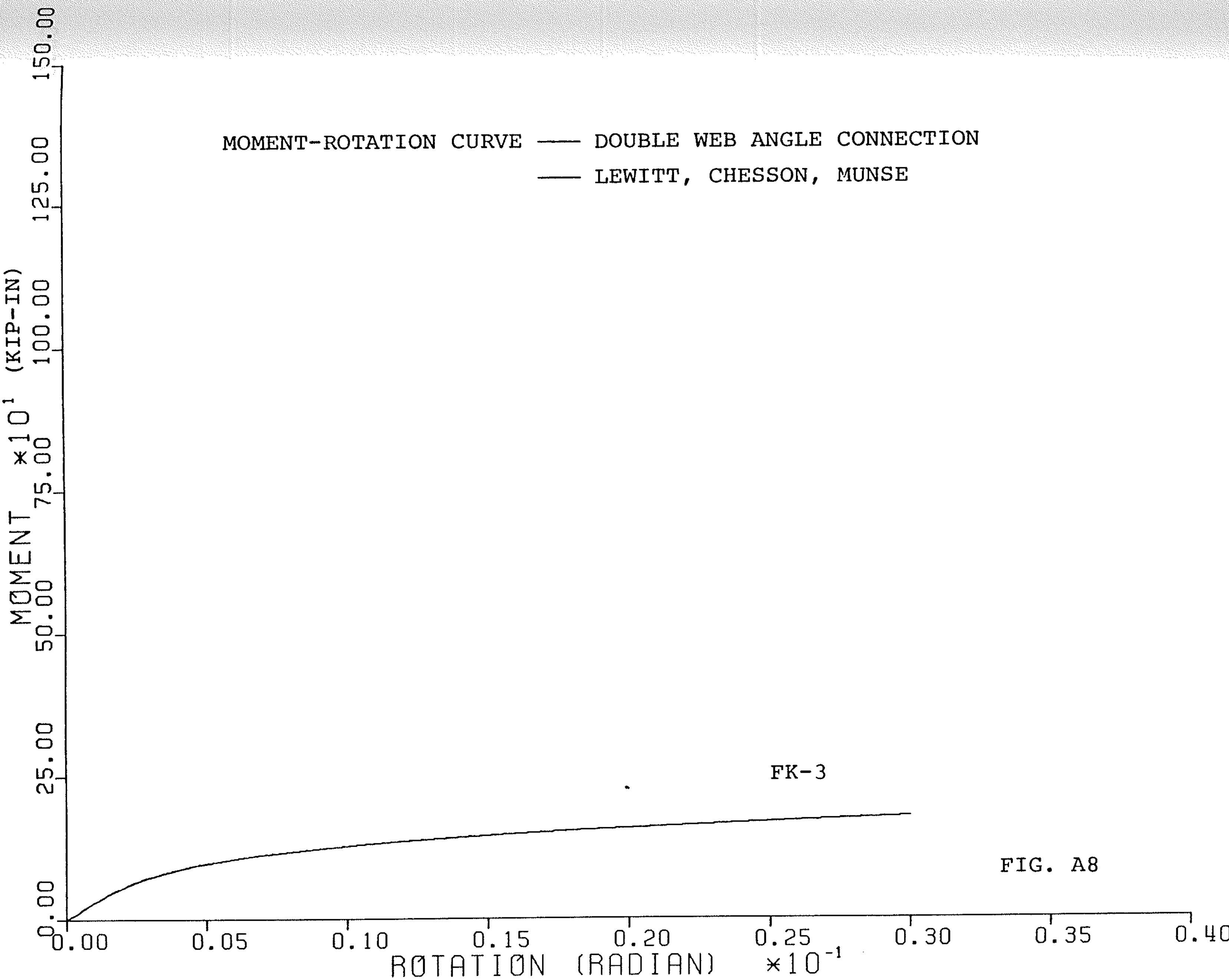

MOMENT-ROTATION CURVE — DOUBLE WEB ANGLE CONNECTION
— LEWITT, CHESSON, MUNSE
FK-3
MOMENT ×10^1 (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10^-1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A8

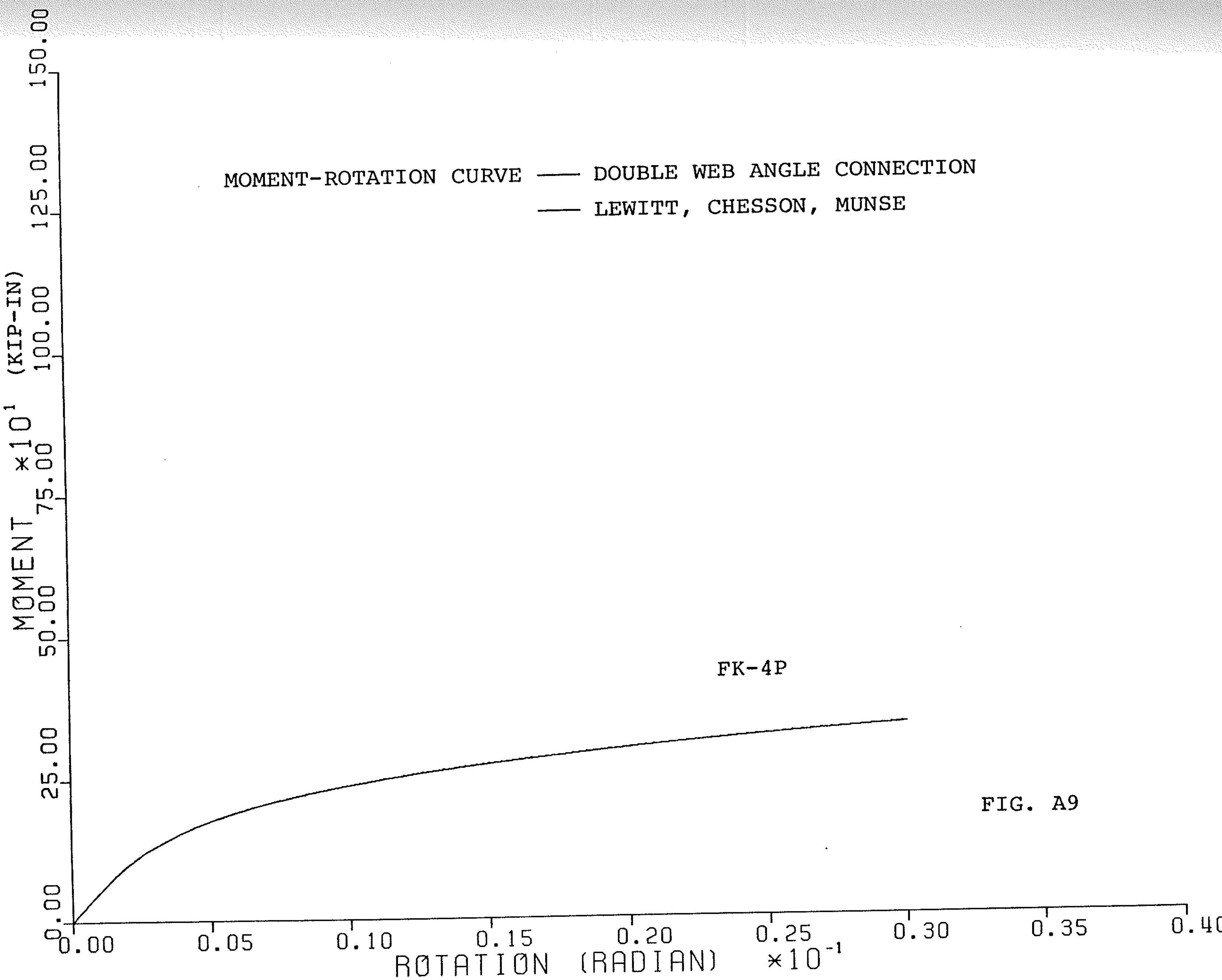

MOMENT-ROTATION CURVE — DOUBLE WEB ANGLE CONNECTION
— LEWITT, CHESSON, MUNSE
FK-4P
MOMENT *10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) *10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A9

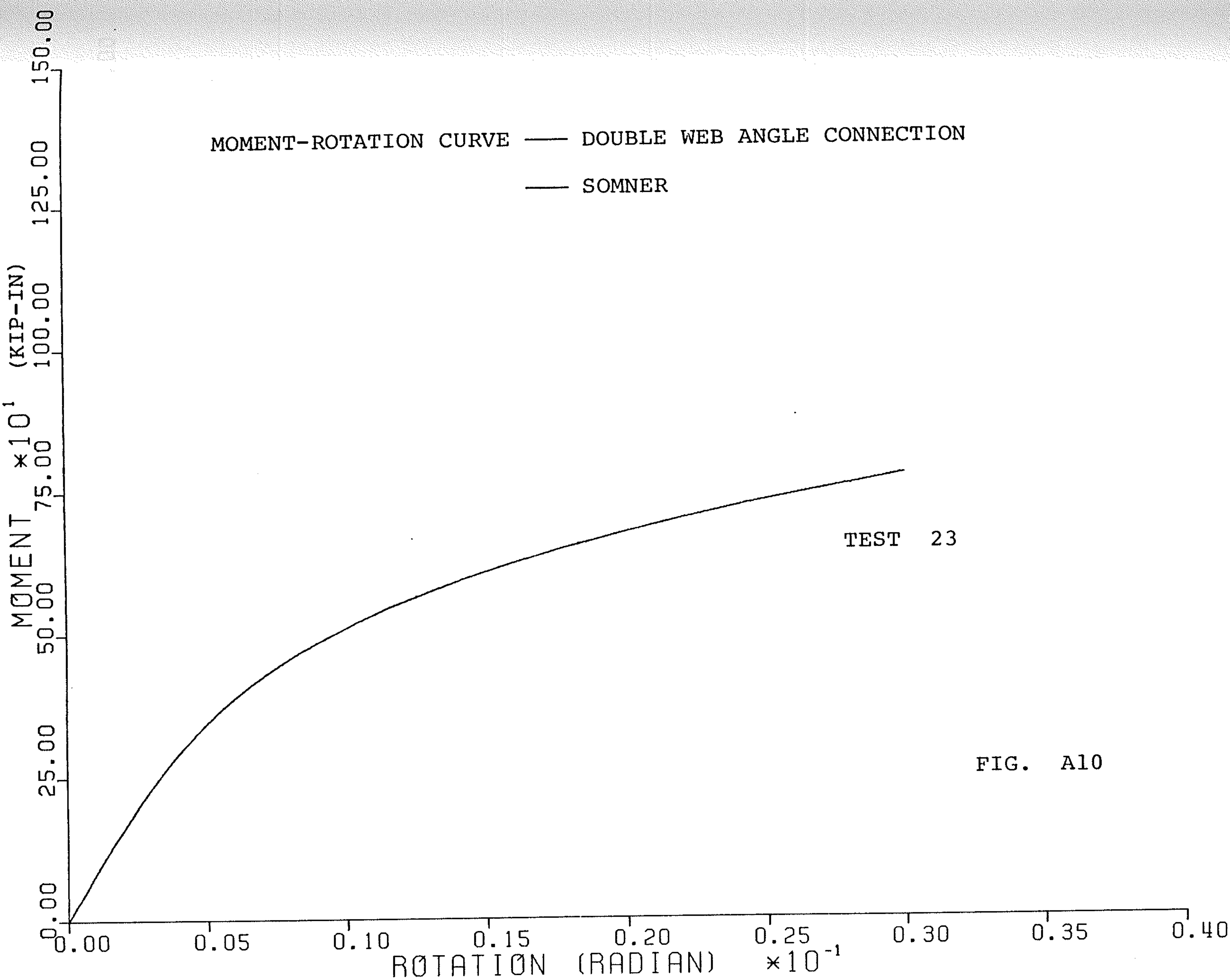

MOMENT-ROTATION CURVE — DOUBLE WEB ANGLE CONNECTION
— SOMNER
TEST 23
MØMENT *10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
RØTATIØN (RADIAN) *10⁻¹

FIG. A10

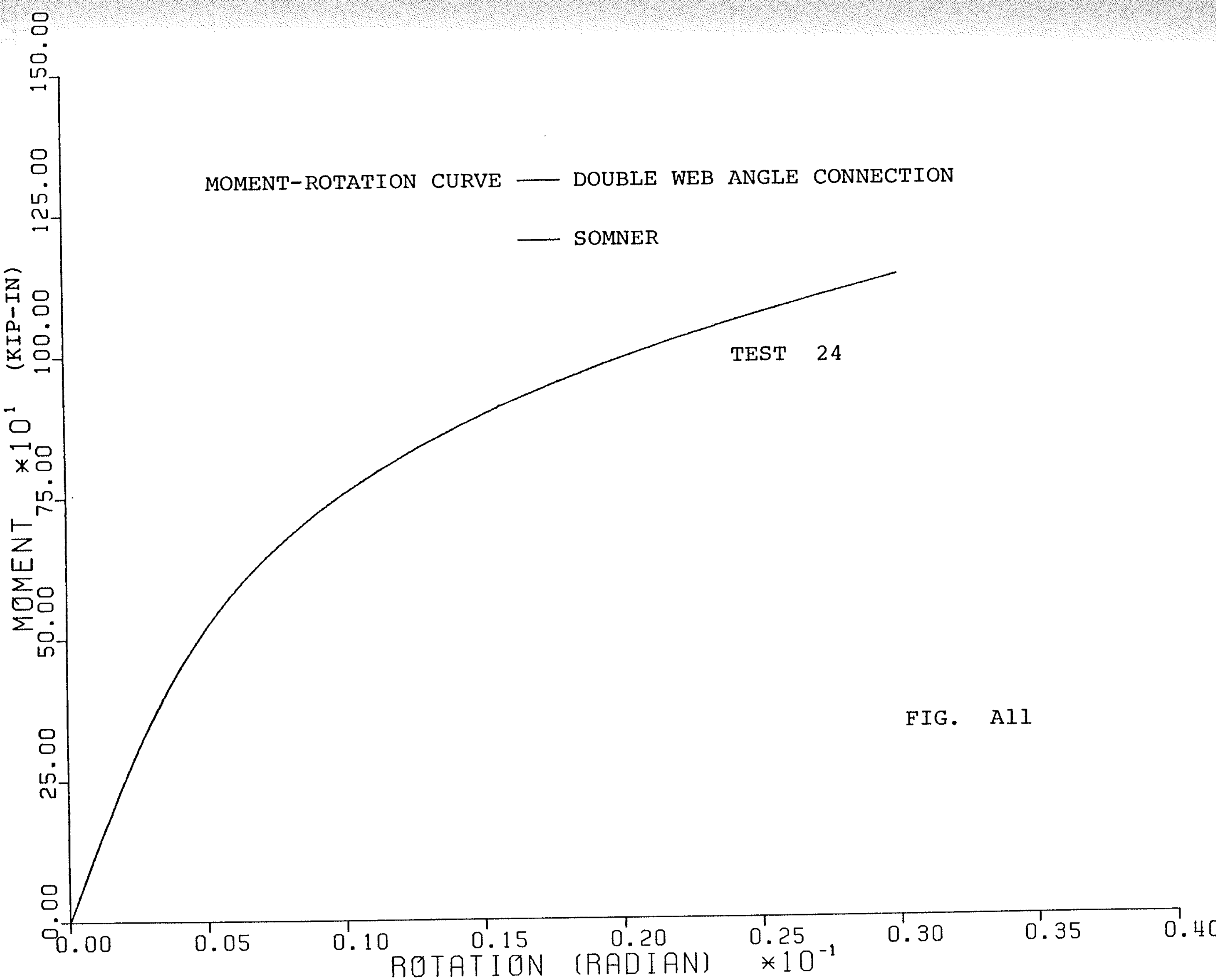

MOMENT-ROTATION CURVE —— DOUBLE WEB ANGLE CONNECTION
—— SOMNER
TEST 24
MØMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
RØTATIØN (RADIAN) ×10⁻¹

FIG. A11

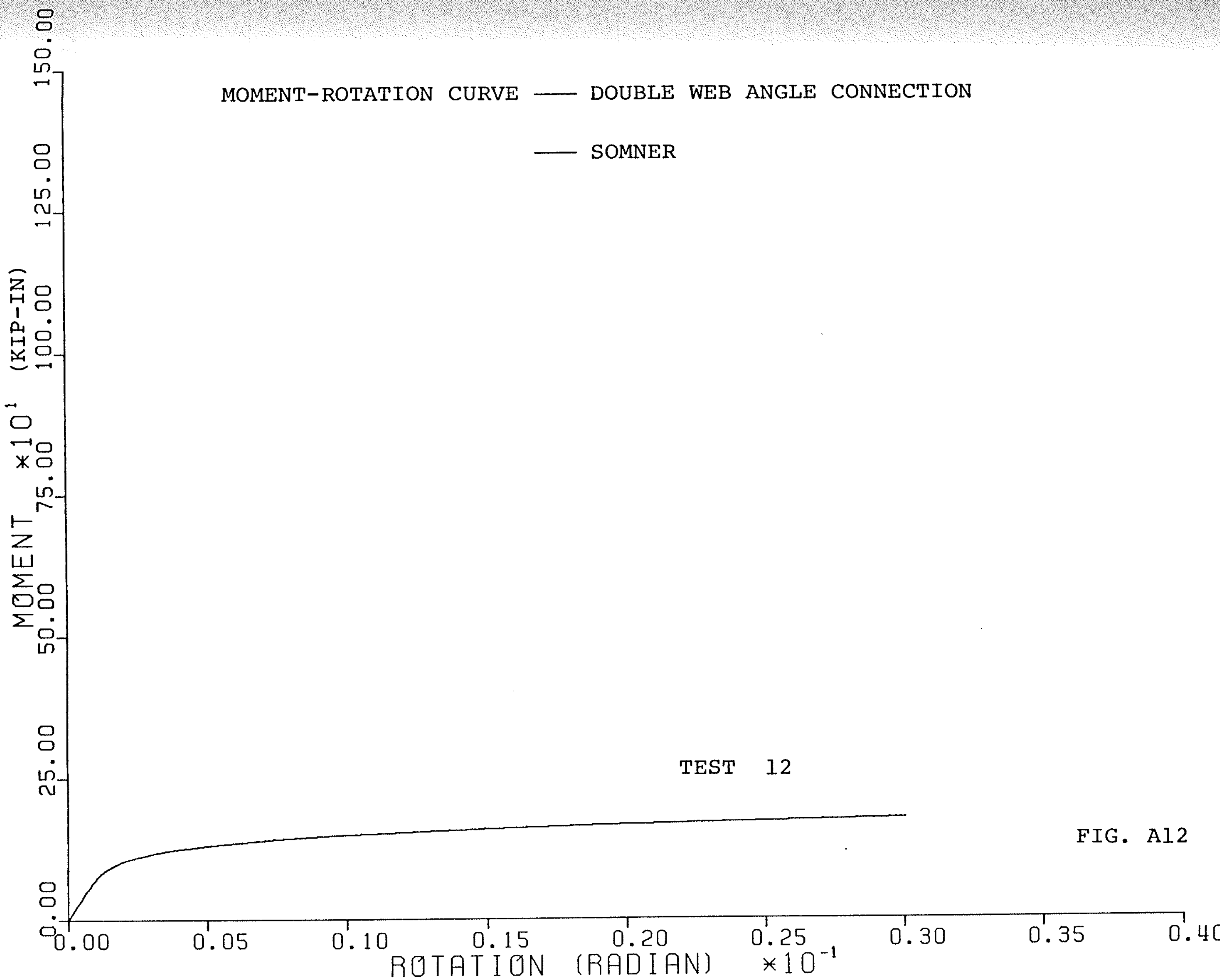

MOMENT-ROTATION CURVE —— DOUBLE WEB ANGLE CONNECTION
—— SOMNER
TEST 12
MOMENT ×10^1 (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10^-1

FIG. A12

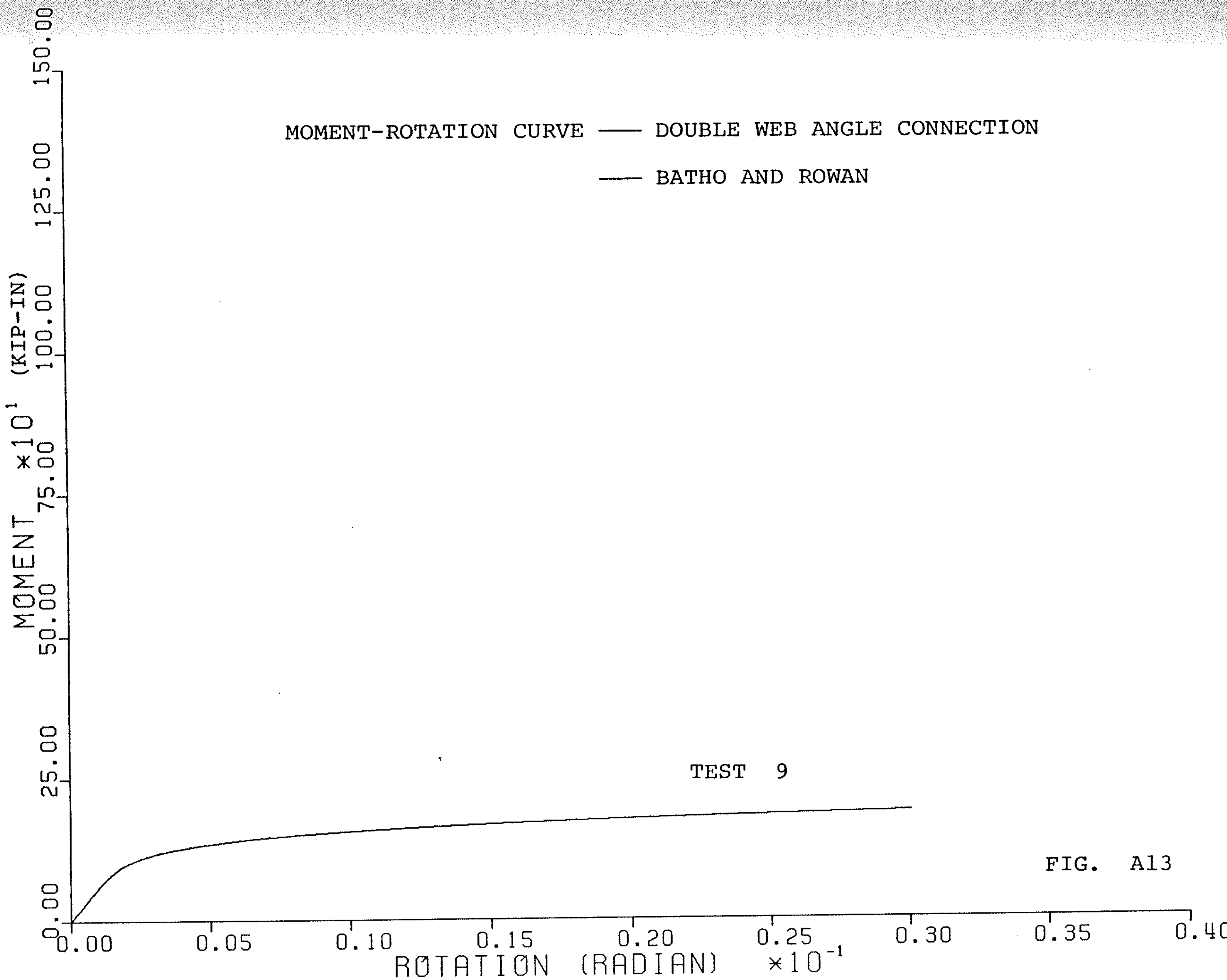

MOMENT-ROTATION CURVE —— DOUBLE WEB ANGLE CONNECTION
—— BATHO AND ROWAN
TEST 9
MØMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
RØTATIØN (RADIAN) ×10⁻¹

FIG. A13

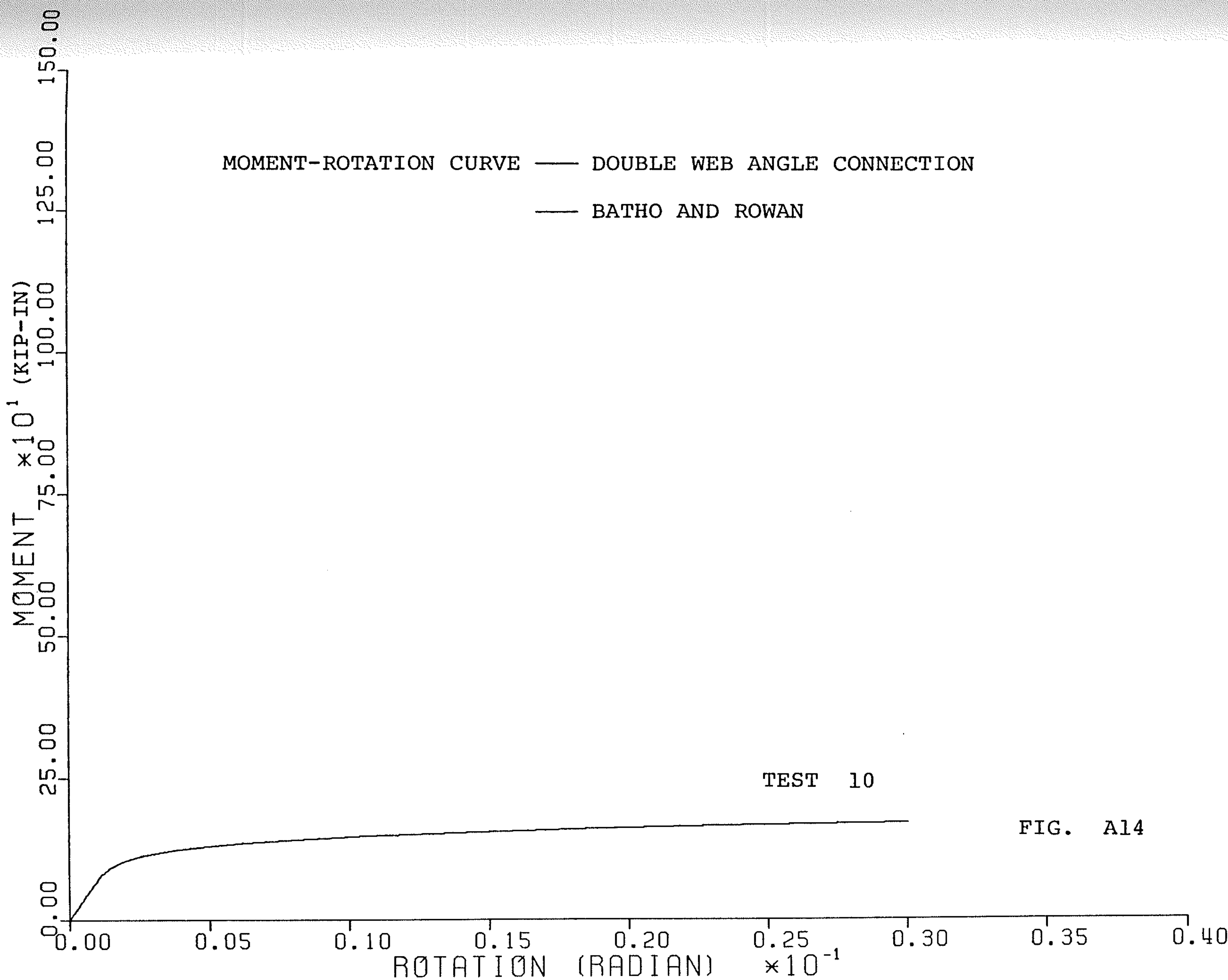

MOMENT-ROTATION CURVE —— DOUBLE WEB ANGLE CONNECTION
—— BATHO AND ROWAN
TEST 10
MOMENT ×10^1 (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10^-1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A14

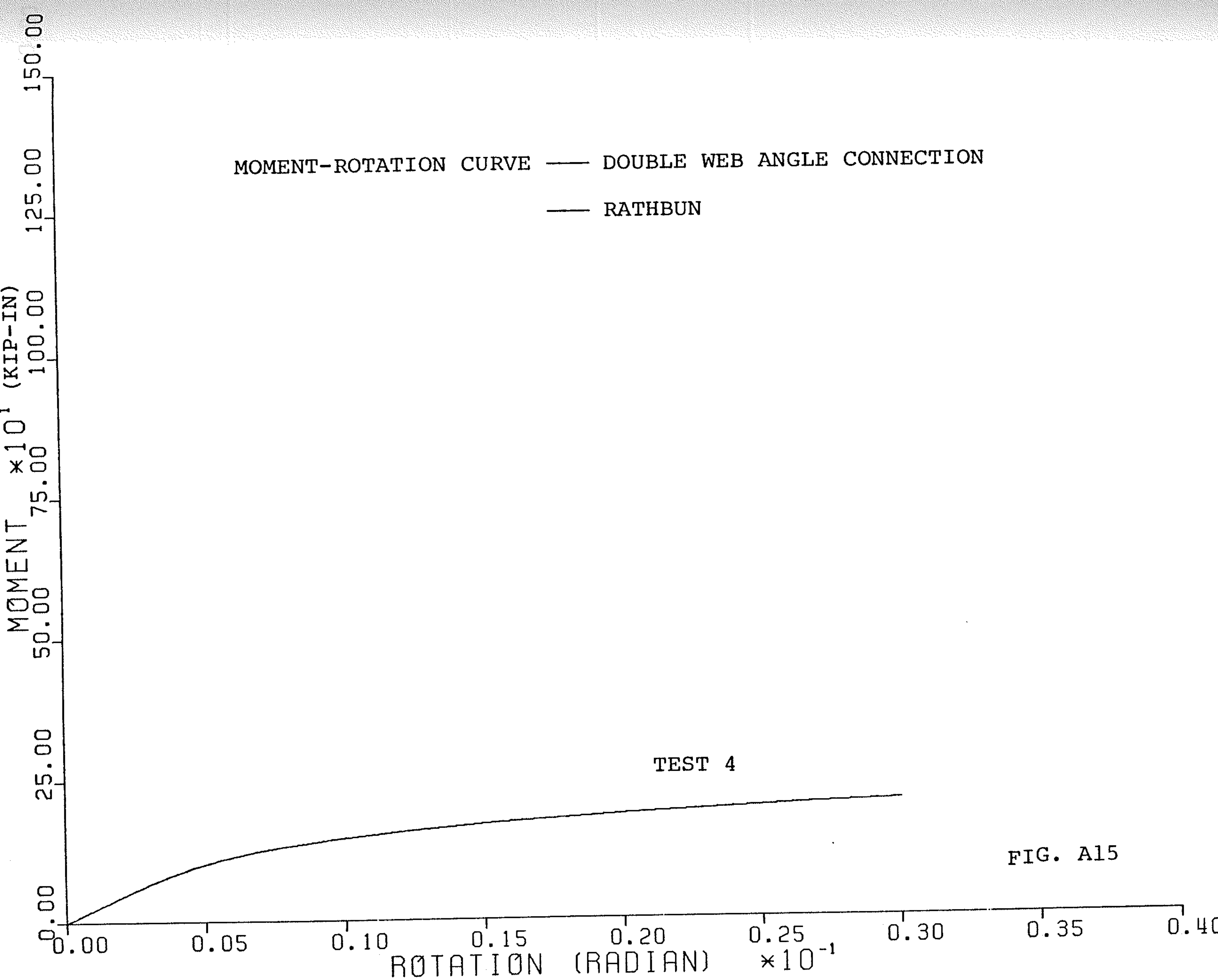
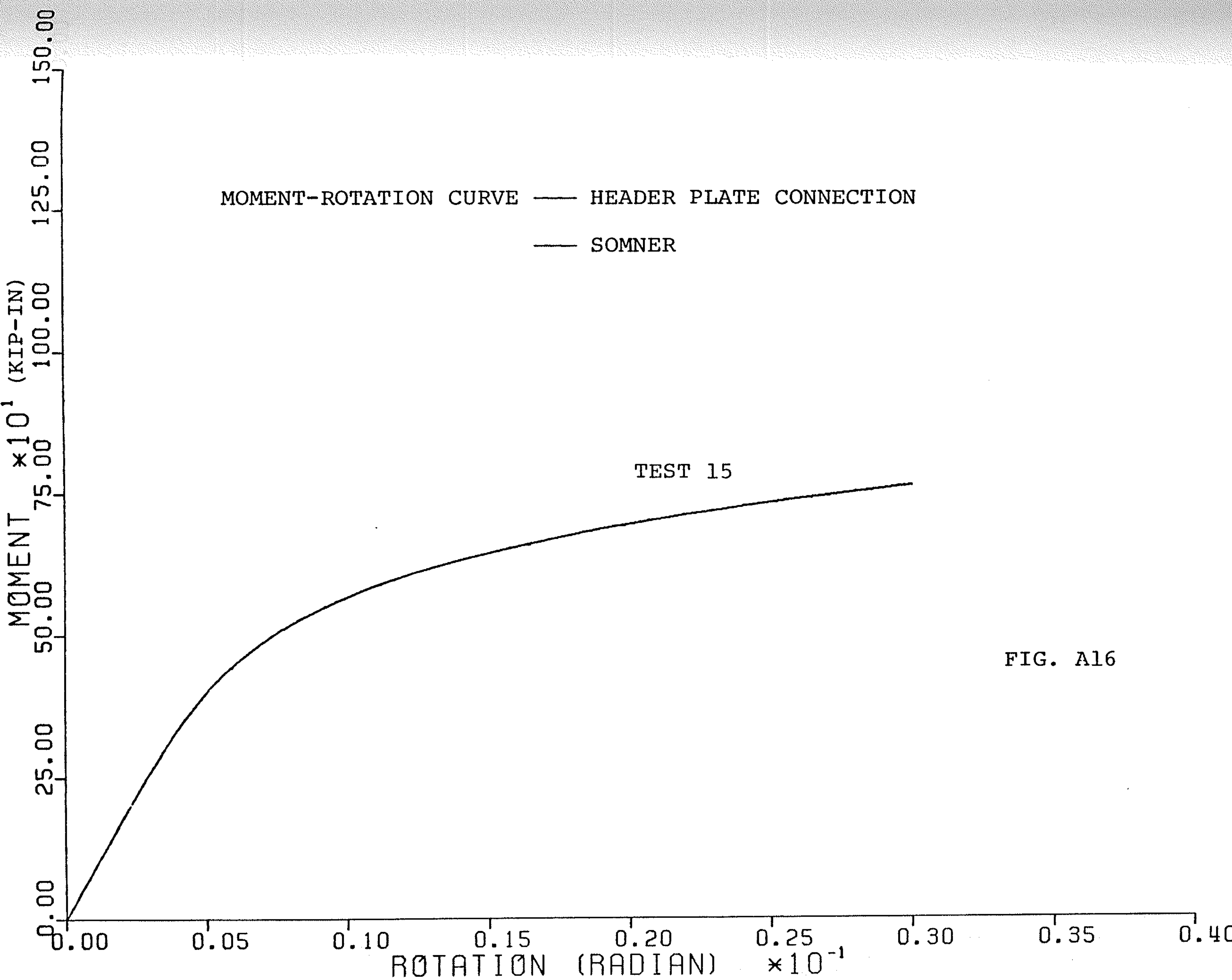

MOMENT-ROTATION CURVE —— DOUBLE WEB ANGLE CONNECTION
—— RATHBUN
TEST 4
MØMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
RØTATIØN (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A15



MOMENT-ROTATION CURVE —— HEADER PLATE CONNECTION
—— SOMNER
TEST 15
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A16

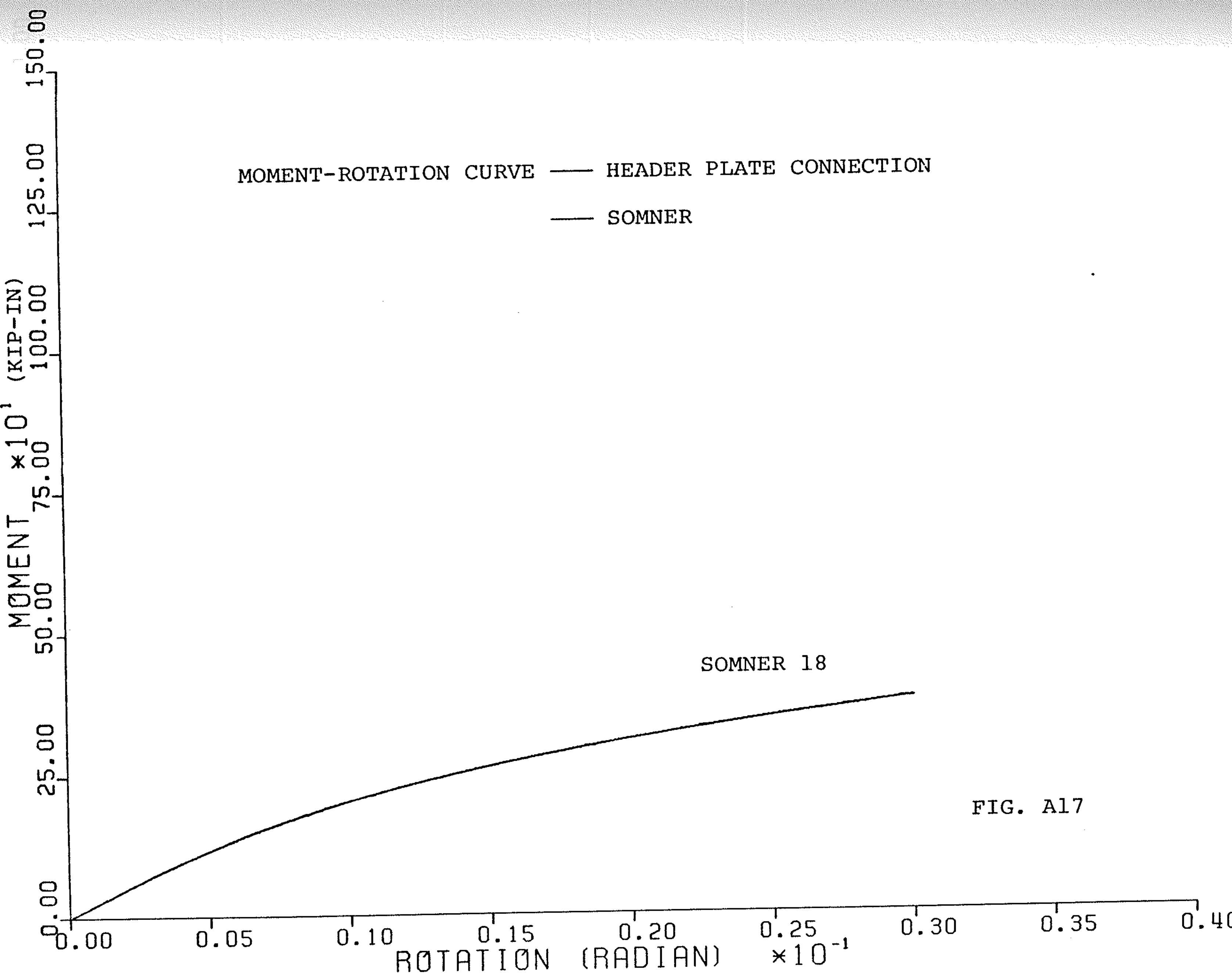

MOMENT-ROTATION CURVE — HEADER PLATE CONNECTION
— SOMNER
SOMNER 18
MØMENT *10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
RØTATIØN (RADIAN) *10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A17

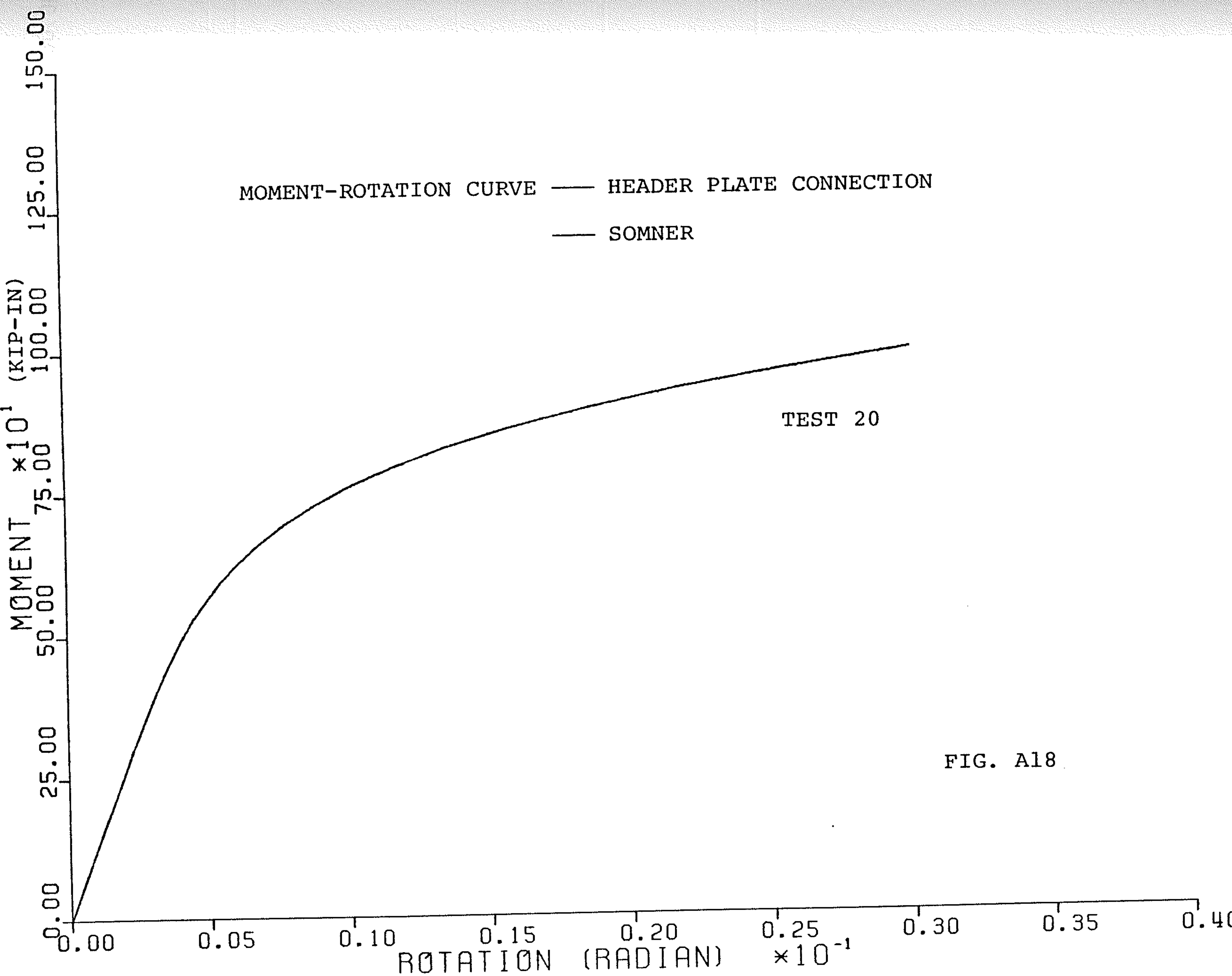

MOMENT-ROTATION CURVE — HEADER PLATE CONNECTION
— SOMNER
TEST 20
MOMENT ×10[1] (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10[-1]
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A18

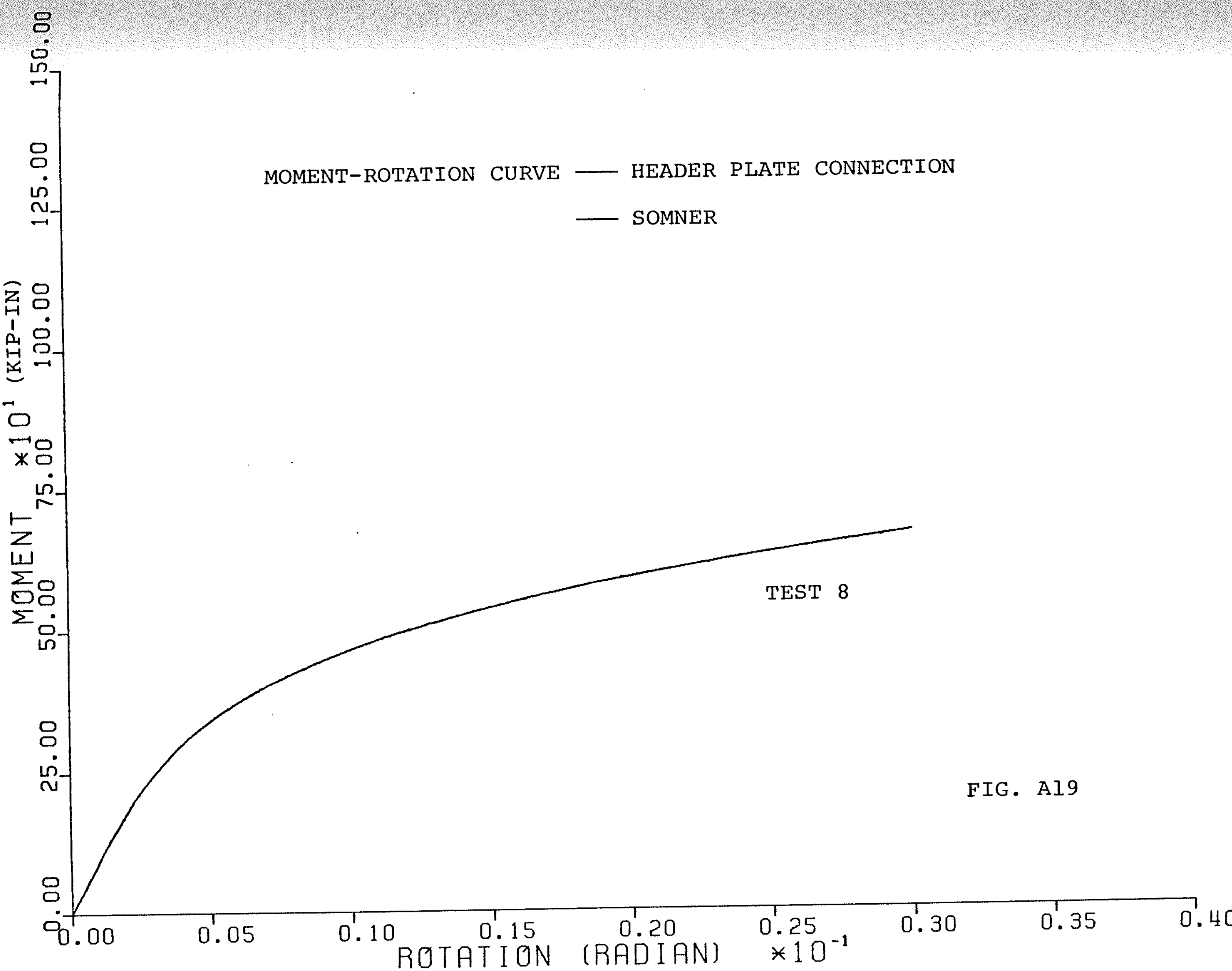

MOMENT-ROTATION CURVE —— HEADER PLATE CONNECTION
—— SOMNER
TEST 8
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A19

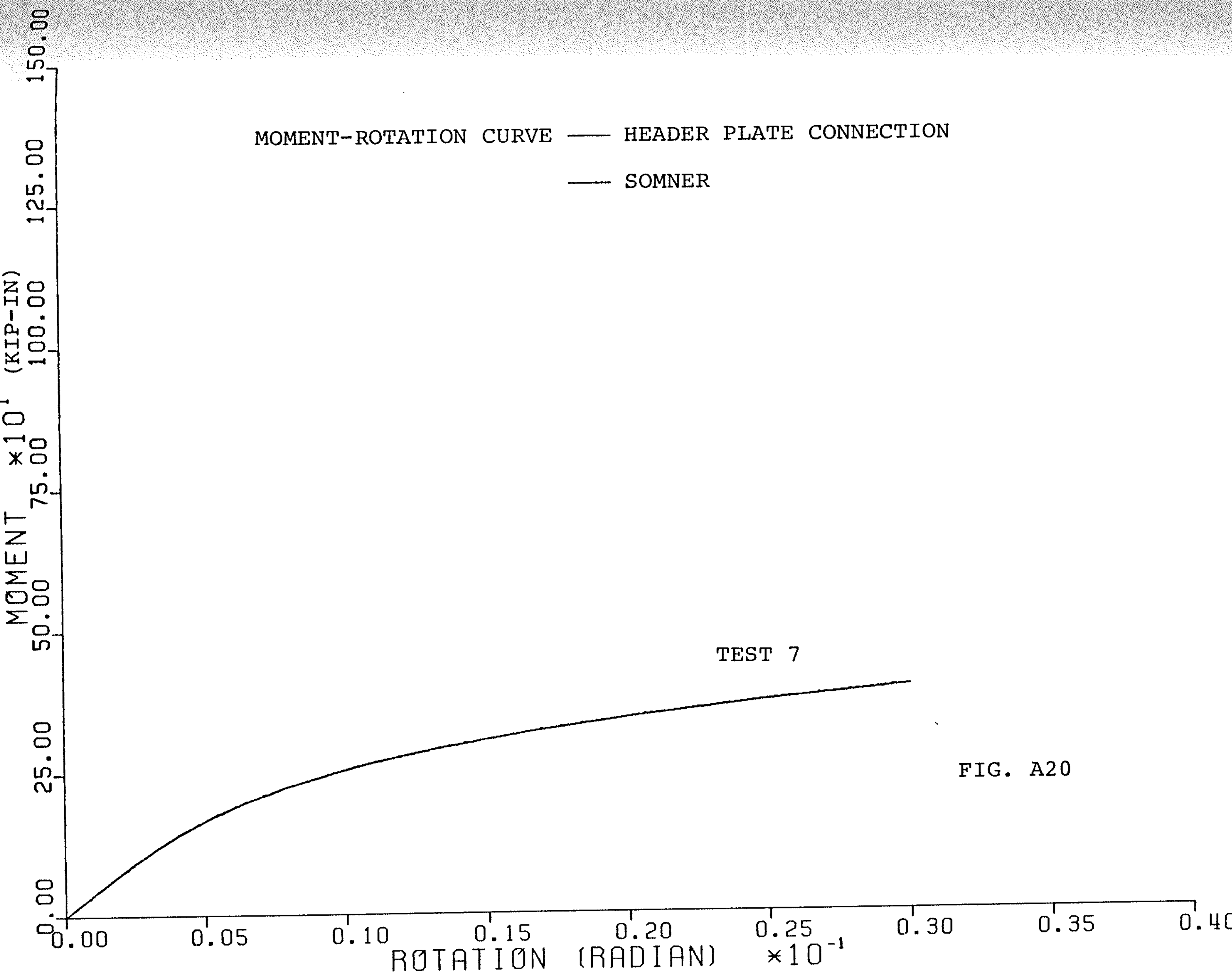
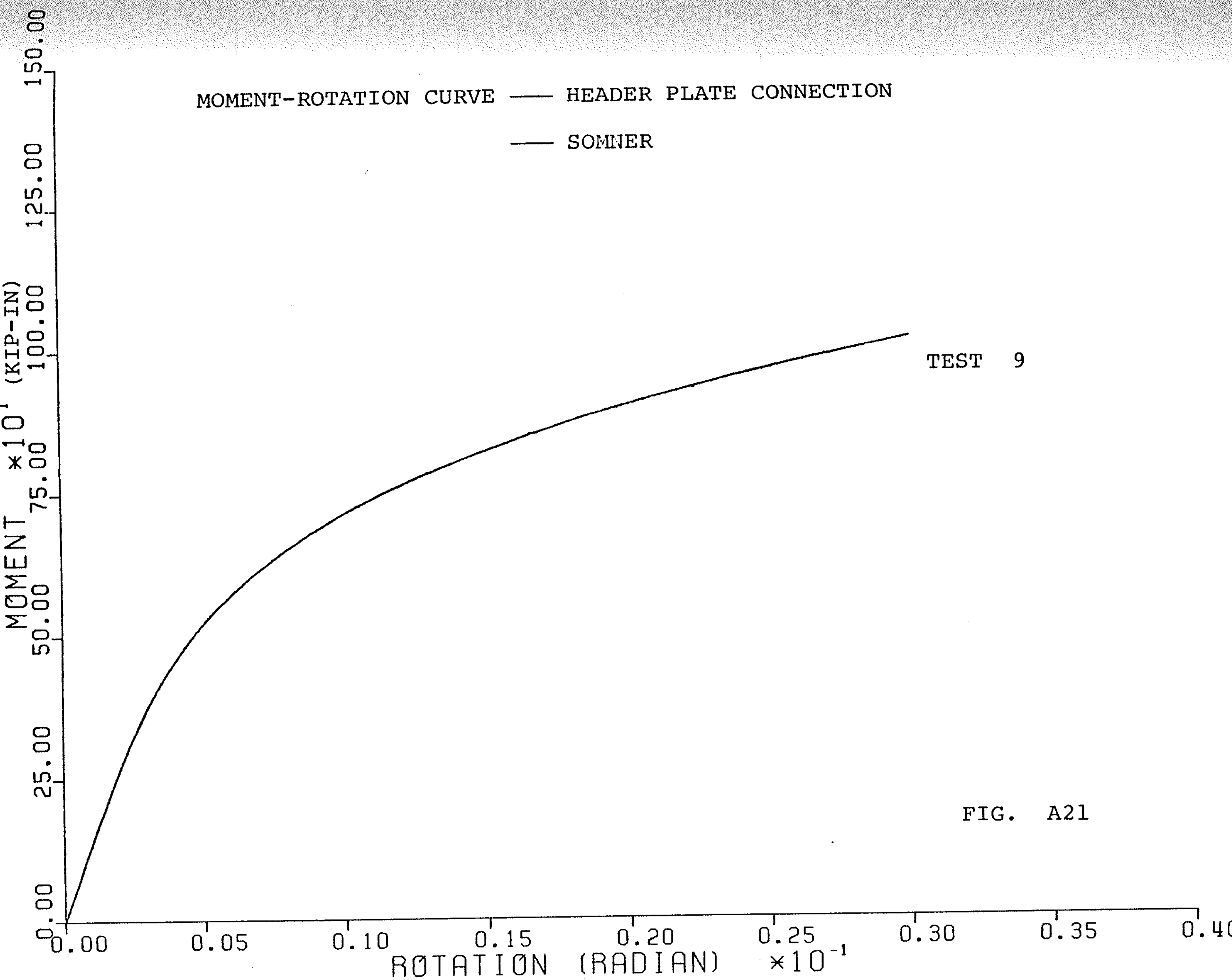

MOMENT-ROTATION CURVE — HEADER PLATE CONNECTION
— SOMNER
TEST 7
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A20



MOMENT-ROTATION CURVE — HEADER PLATE CONNECTION
— SOMNER
TEST 9
MOMENT *10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. A21

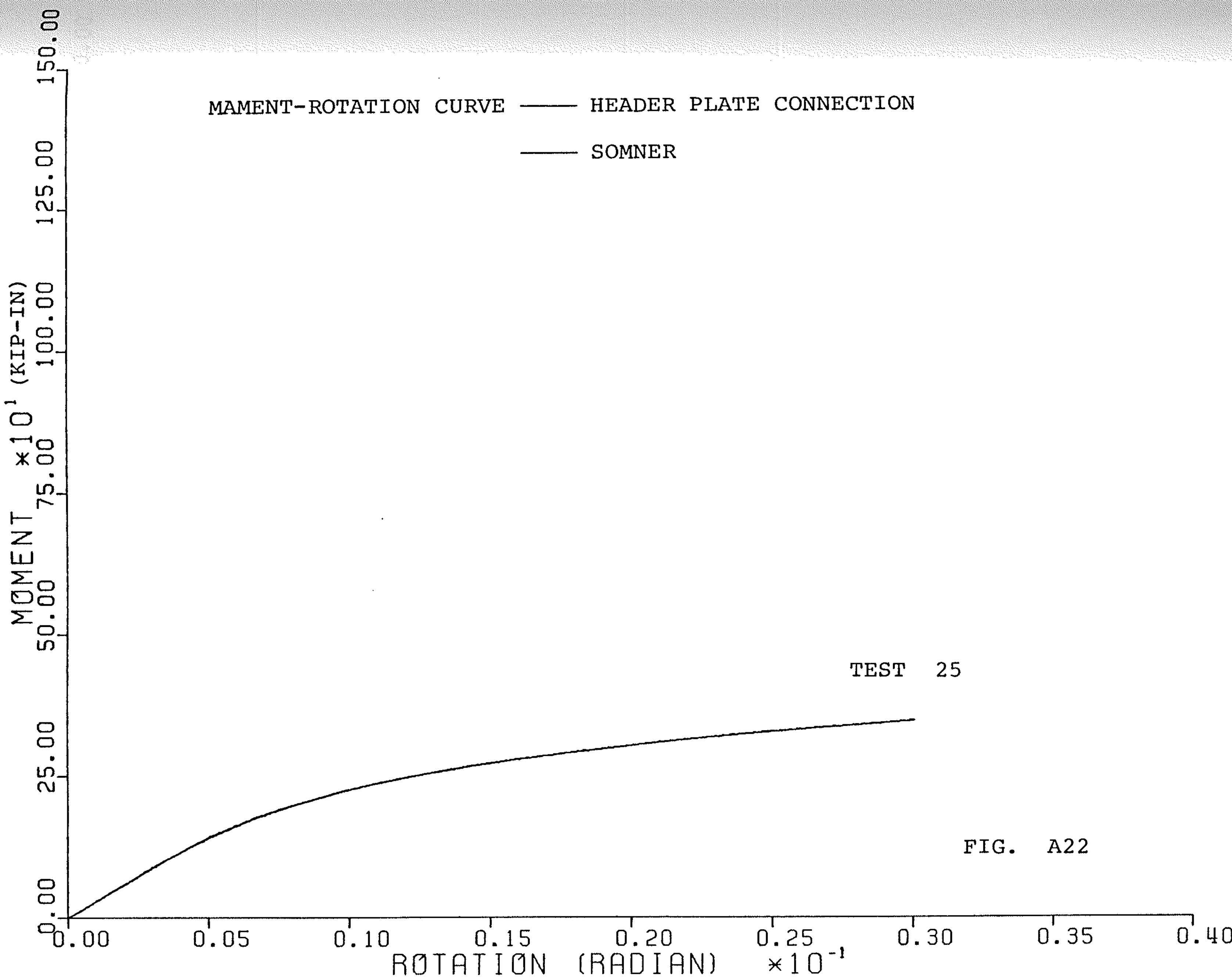

MAMENT-ROTATION CURVE —— HEADER PLATE CONNECTION
—— SOMNER
TEST 25
MØMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
RØTATIØN (RADIAN) ×10⁻¹

FIG. A22

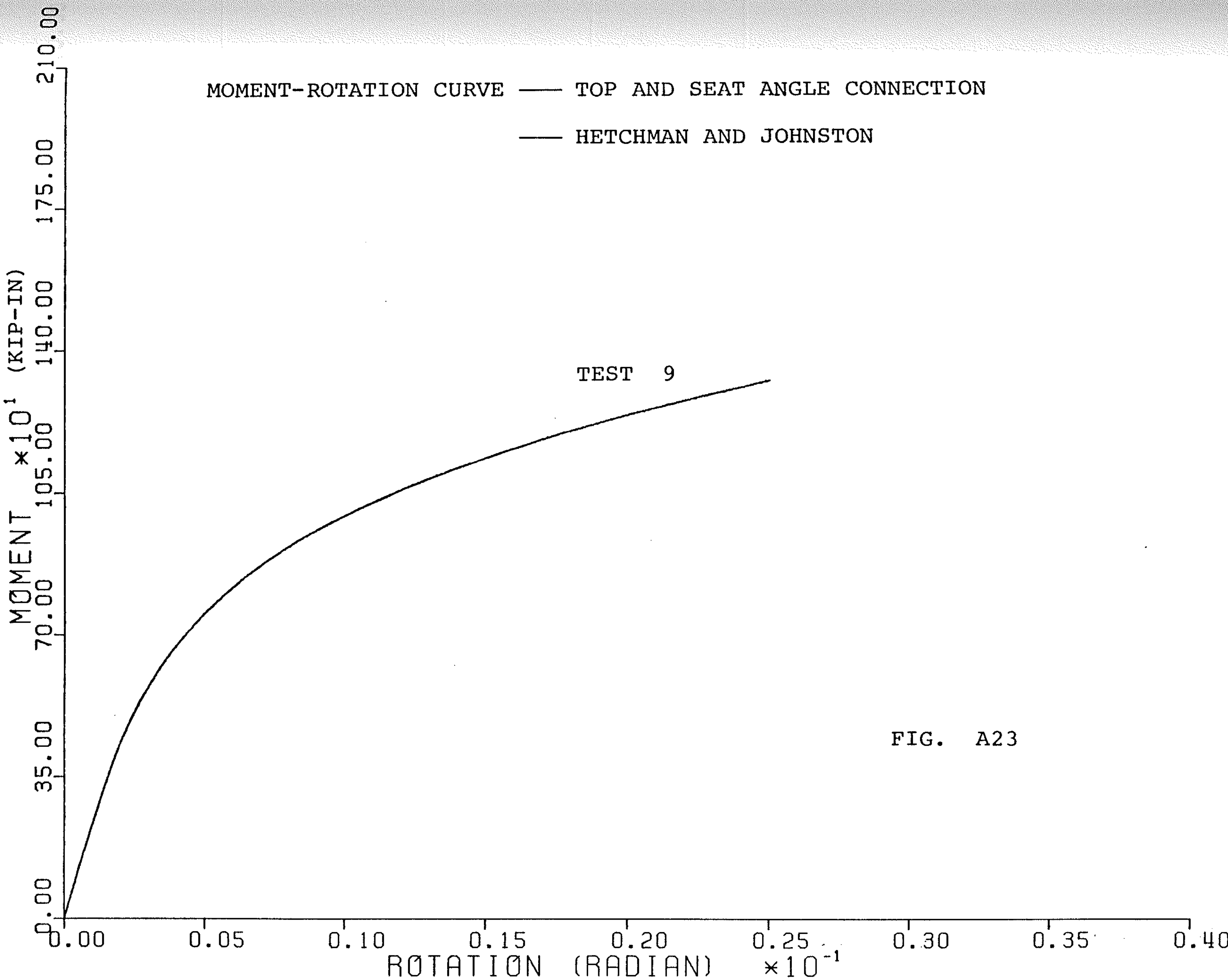

MOMENT-ROTATION CURVE — TOP AND SEAT ANGLE CONNECTION
— HETCHMAN AND JOHNSTON
TEST 9
MOMENT ×10^1 (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) ×10^-1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A23

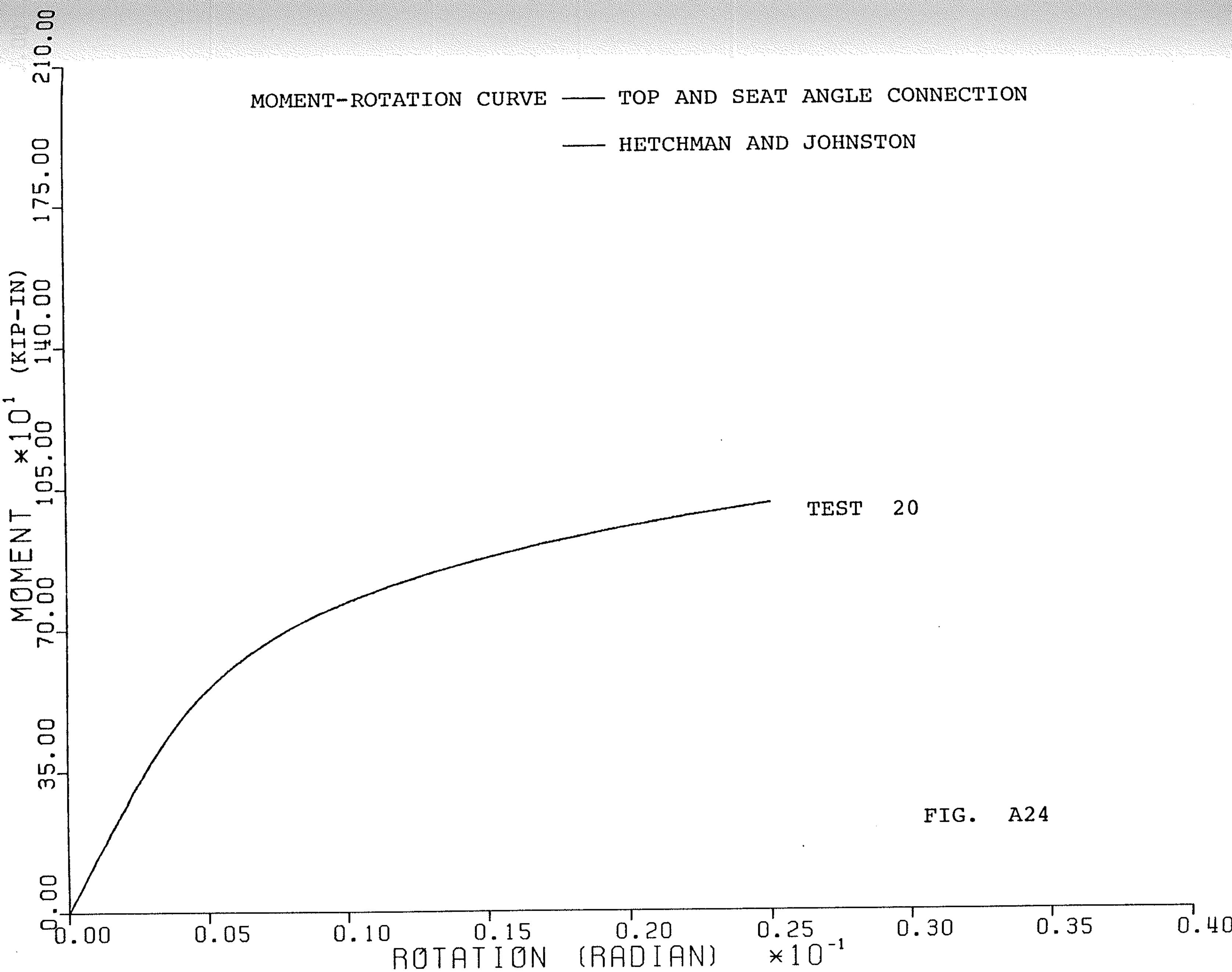

MOMENT-ROTATION CURVE —— TOP AND SEAT ANGLE CONNECTION
—— HETCHMAN AND JOHNSTON
TEST 20
MOMENT *10^1 (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) *10^-1

FIG. A24

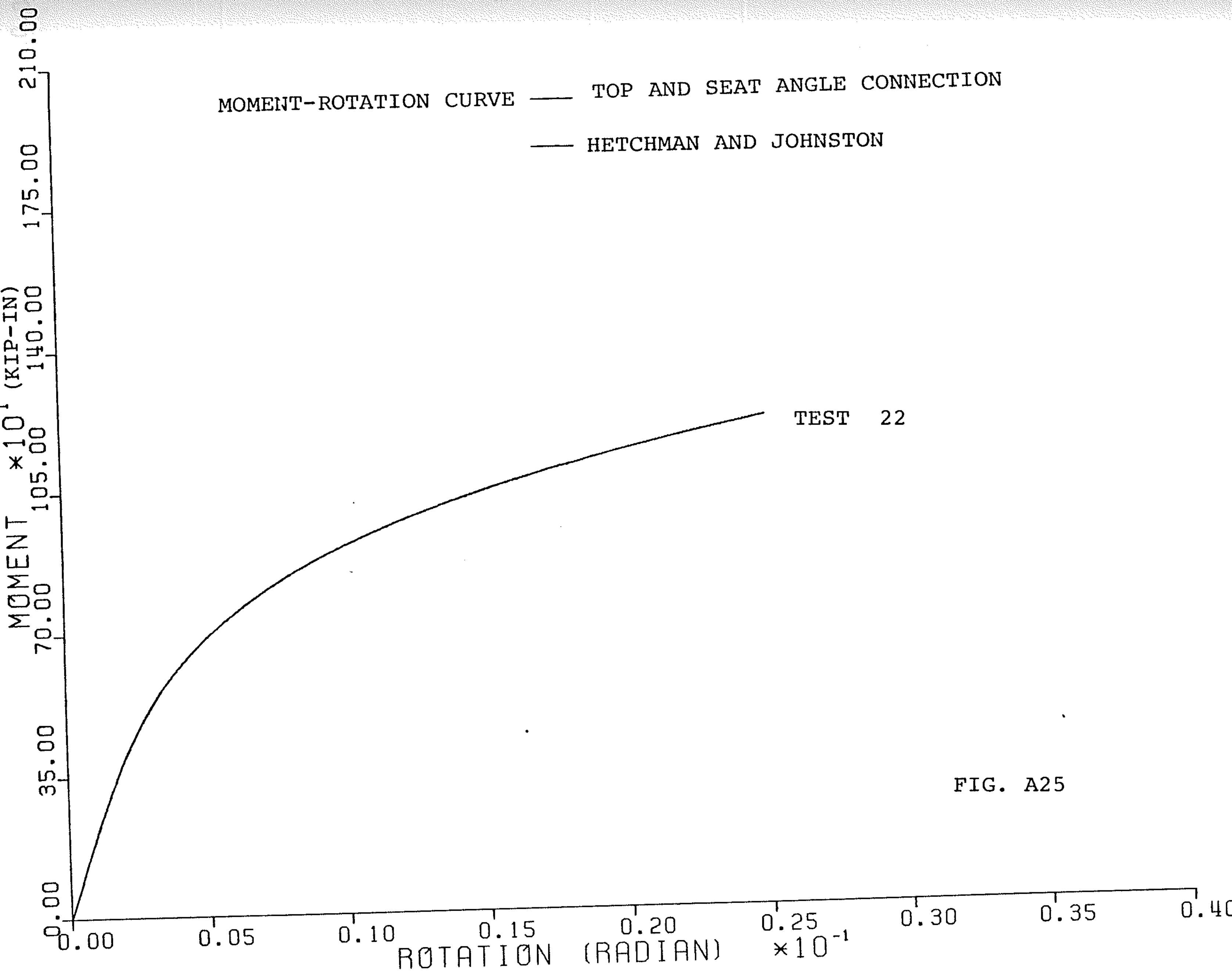

MOMENT-ROTATION CURVE — TOP AND SEAT ANGLE CONNECTION
— HETCHMAN AND JOHNSTON
TEST 22
MOMENT *10¹ (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) *10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A25

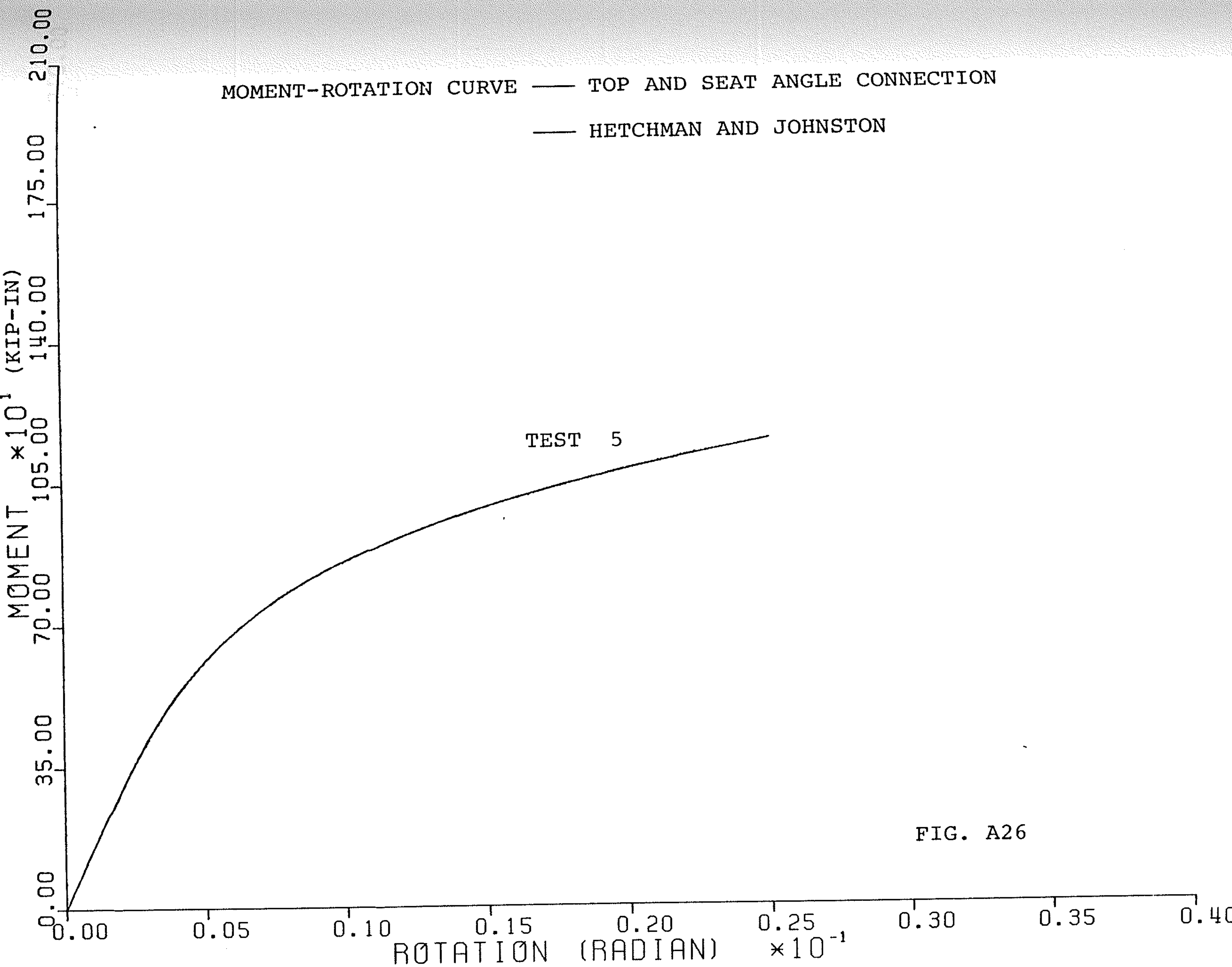

MOMENT-ROTATION CURVE — TOP AND SEAT ANGLE CONNECTION
— HETCHMAN AND JOHNSTON
TEST 5
MOMENT *10¹ (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. A26

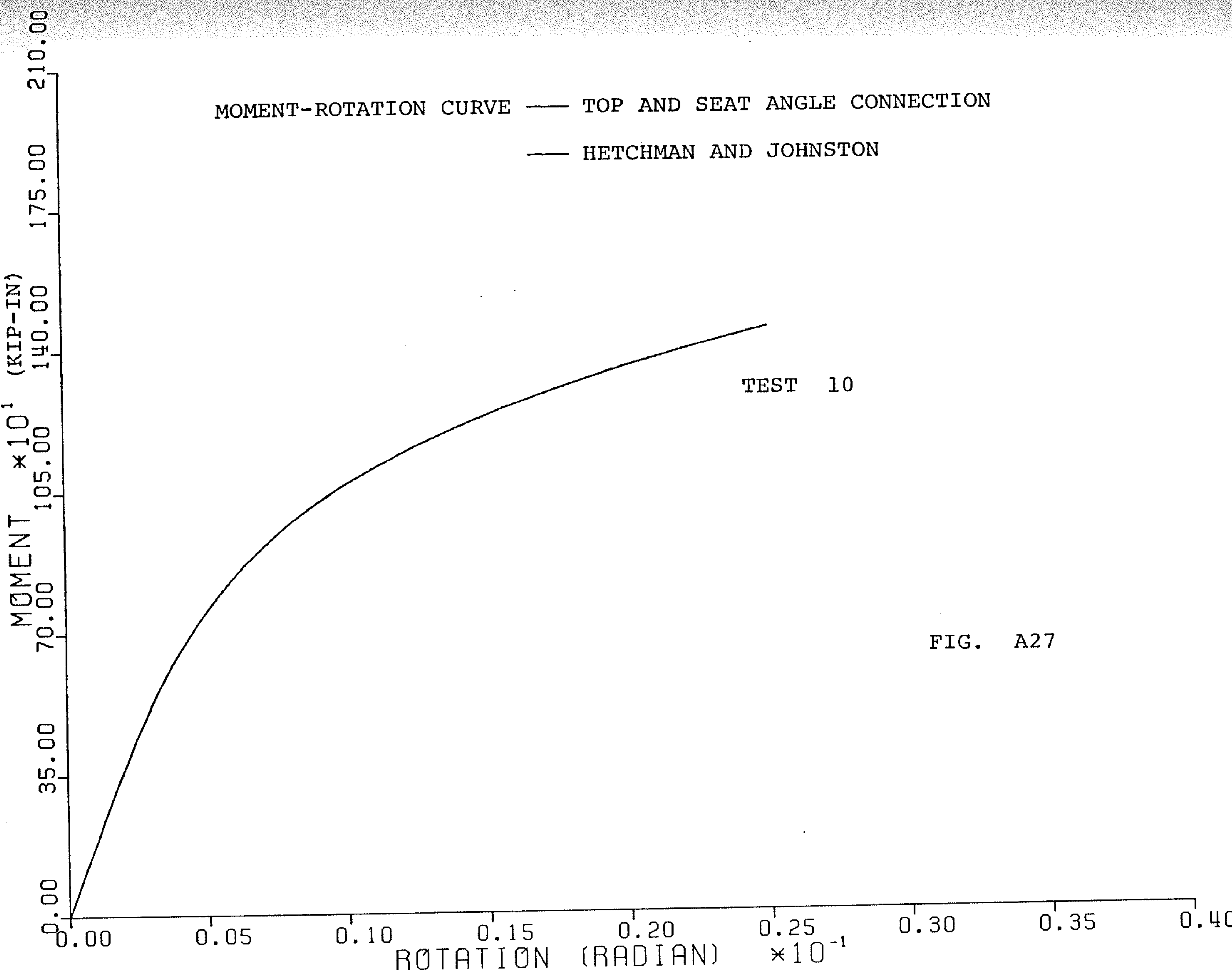

MOMENT-ROTATION CURVE — TOP AND SEAT ANGLE CONNECTION
— HETCHMAN AND JOHNSTON
TEST 10
MØMENT *10¹ (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
RØTATIØN (RADIAN) *10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A27

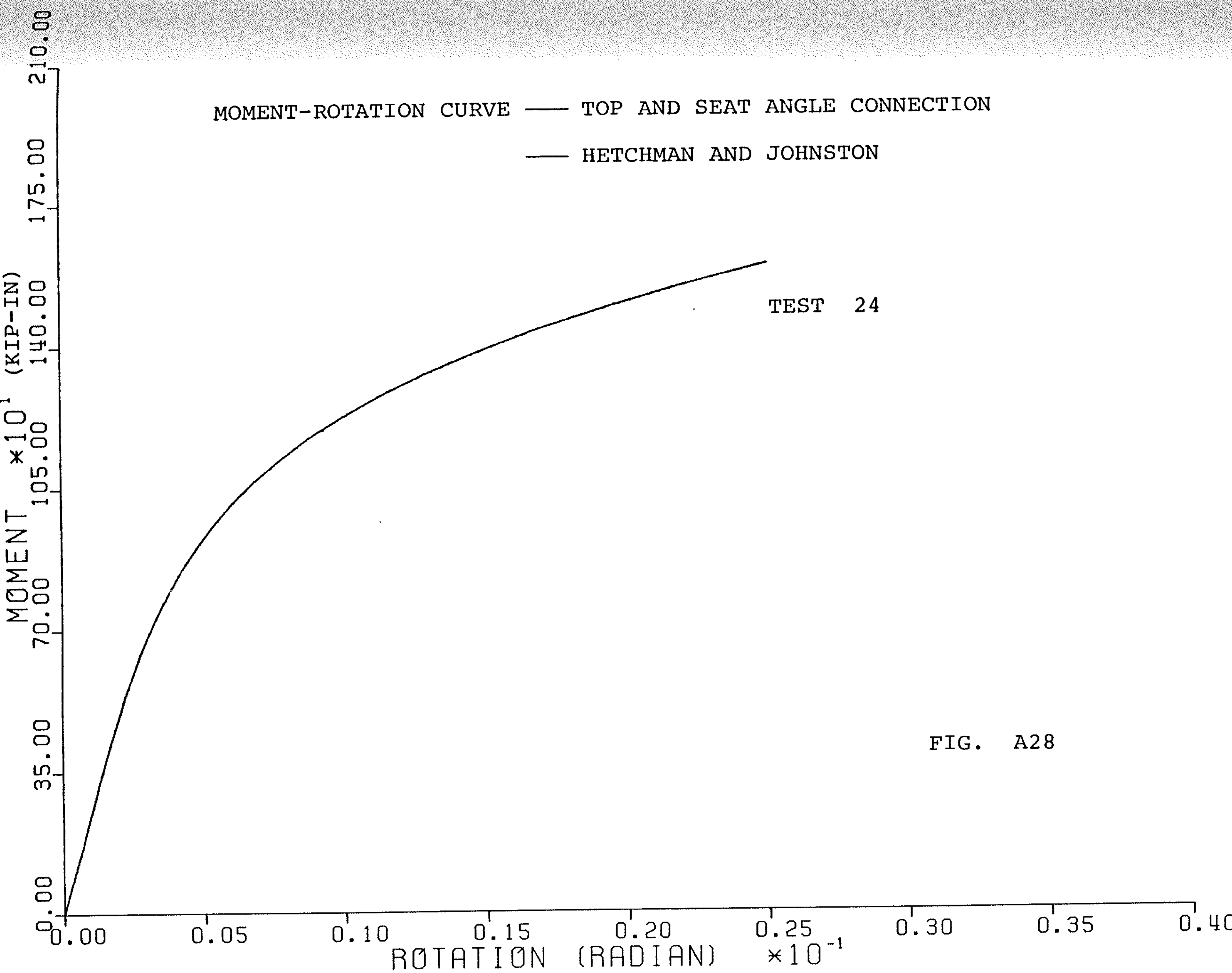

MOMENT-ROTATION CURVE — TOP AND SEAT ANGLE CONNECTION
— HETCHMAN AND JOHNSTON
TEST 24
MOMENT ×10¹ (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A28

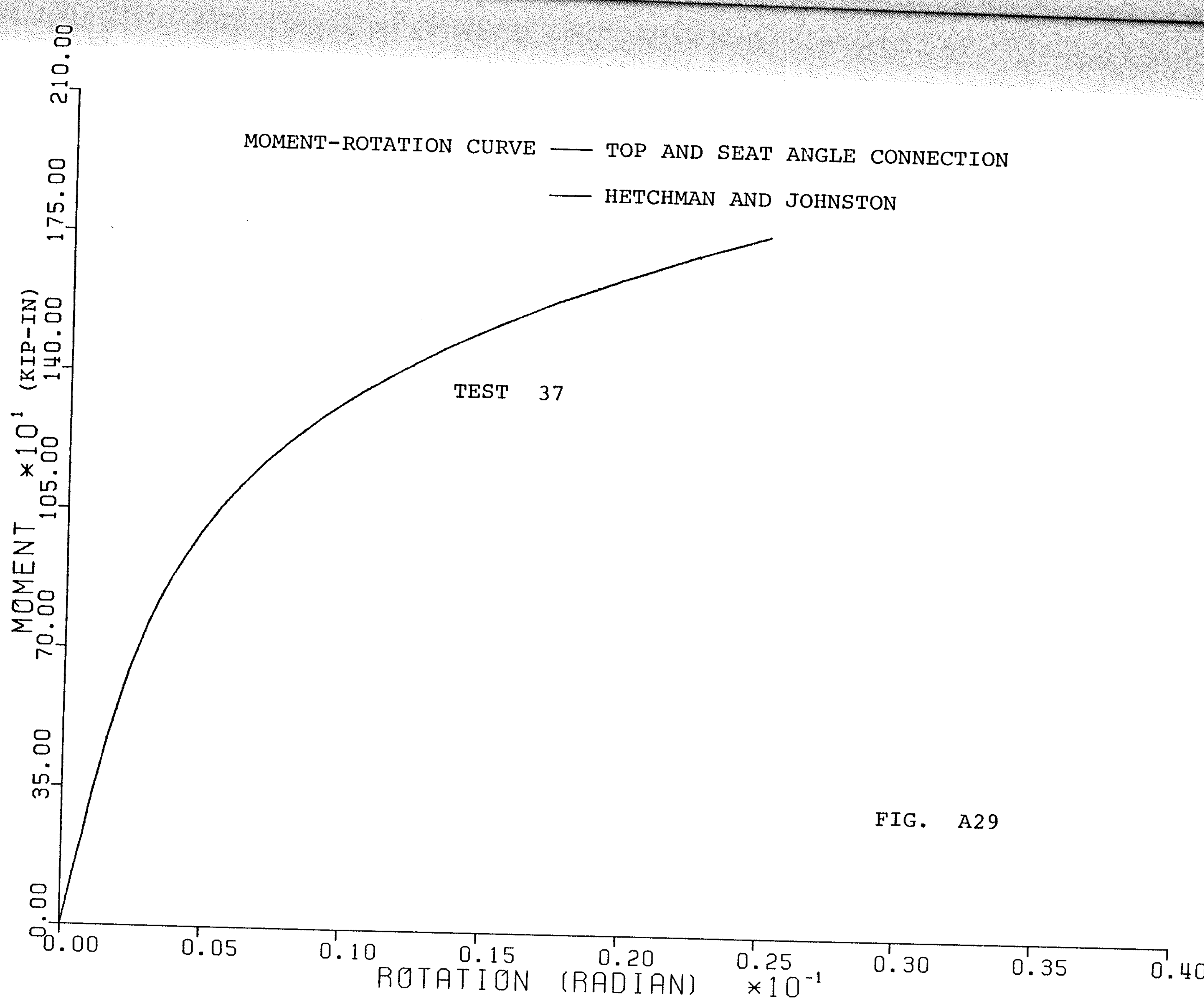

MOMENT-ROTATION CURVE — TOP AND SEAT ANGLE CONNECTION
— HETCHMAN AND JOHNSTON
TEST 37
MOMENT *10¹ (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) *10⁻¹

FIG. A29

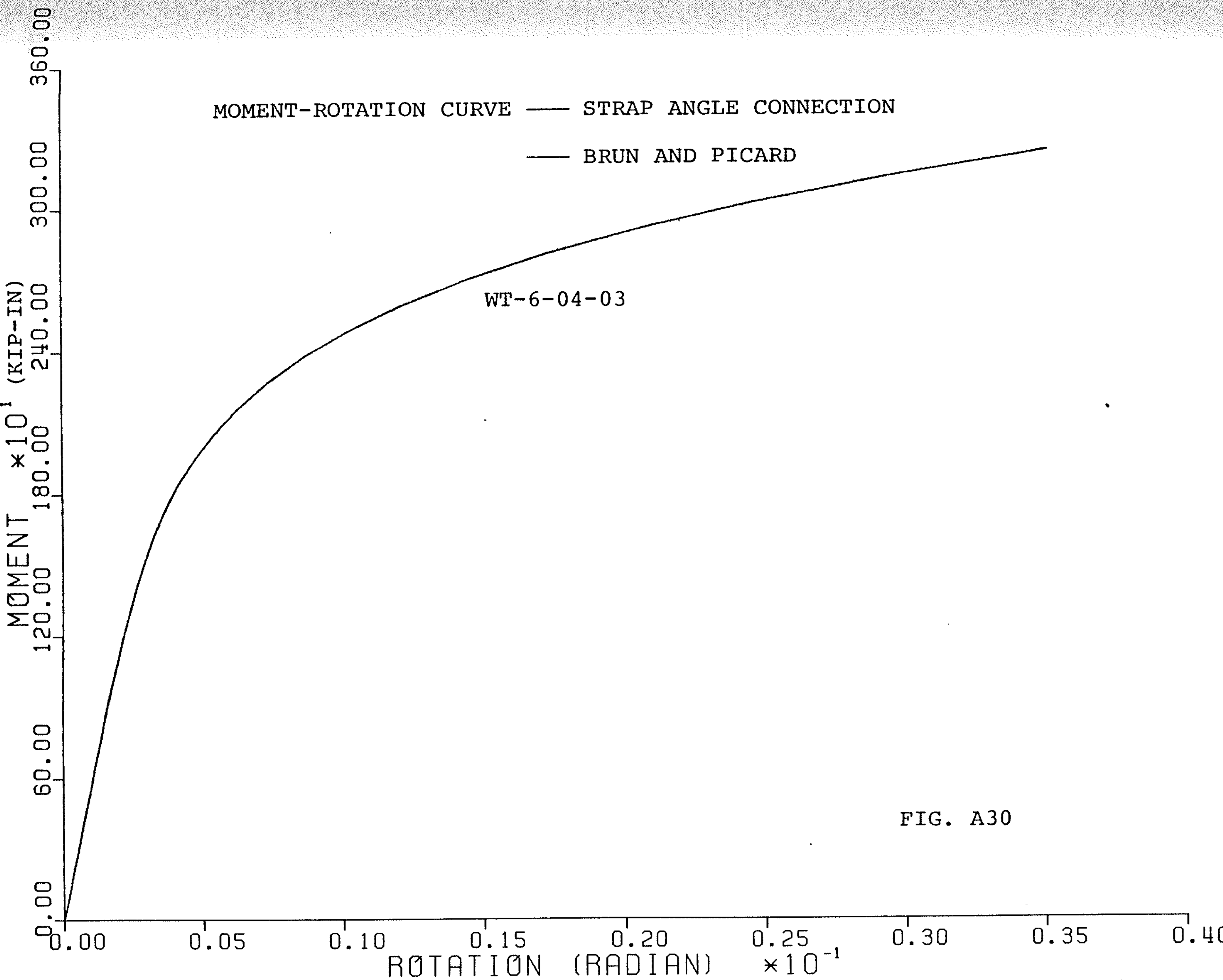

MOMENT-ROTATION CURVE —— STRAP ANGLE CONNECTION
—— BRUN AND PICARD
WT-6-04-03
MOMENT *10¹ (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
ROTATION (RADIAN) *10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. A30

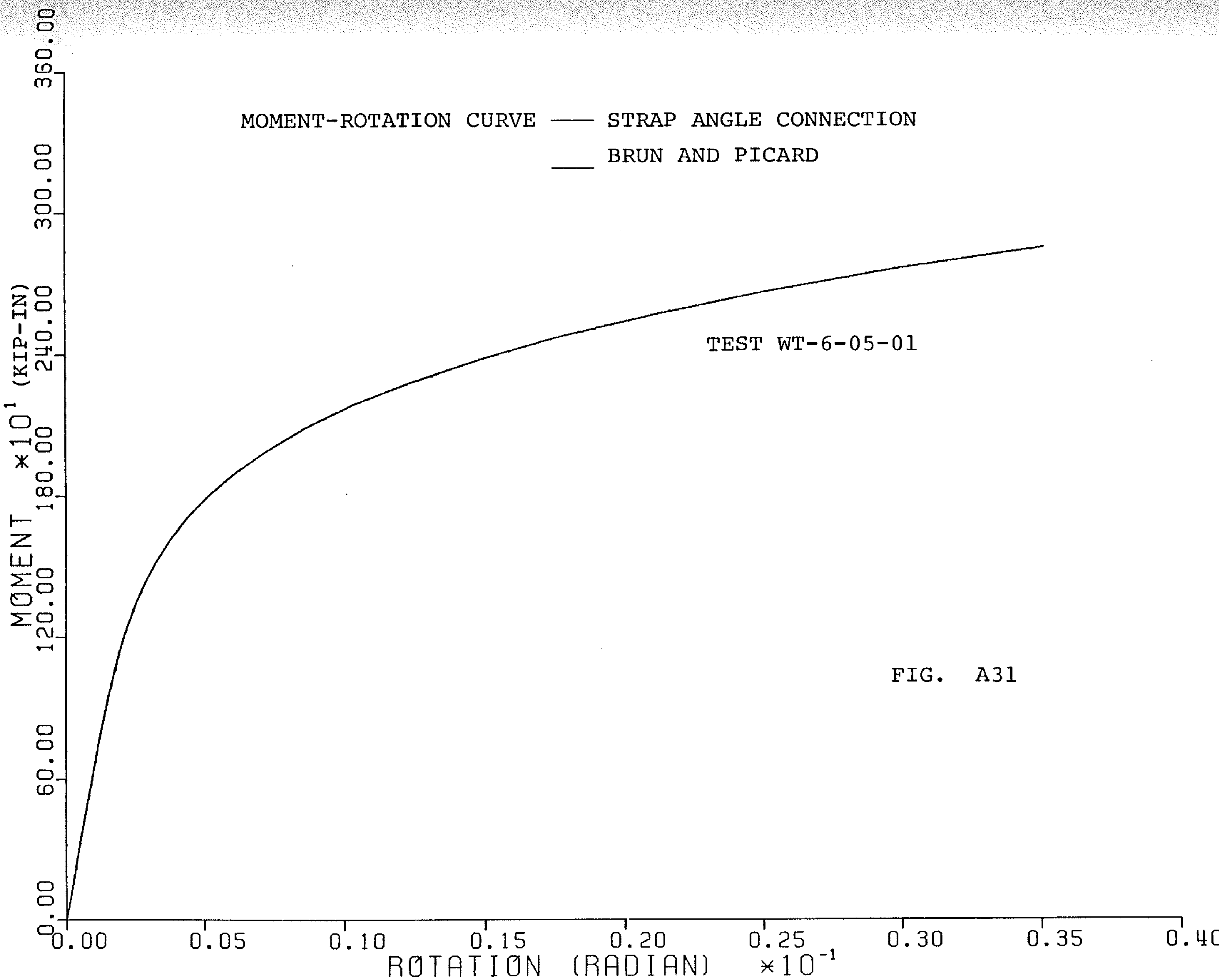

MOMENT-ROTATION CURVE —— STRAP ANGLE CONNECTION
—— BRUN AND PICARD
TEST WT-6-05-01
MOMENT *10¹ (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) *10⁻¹

FIG. A31

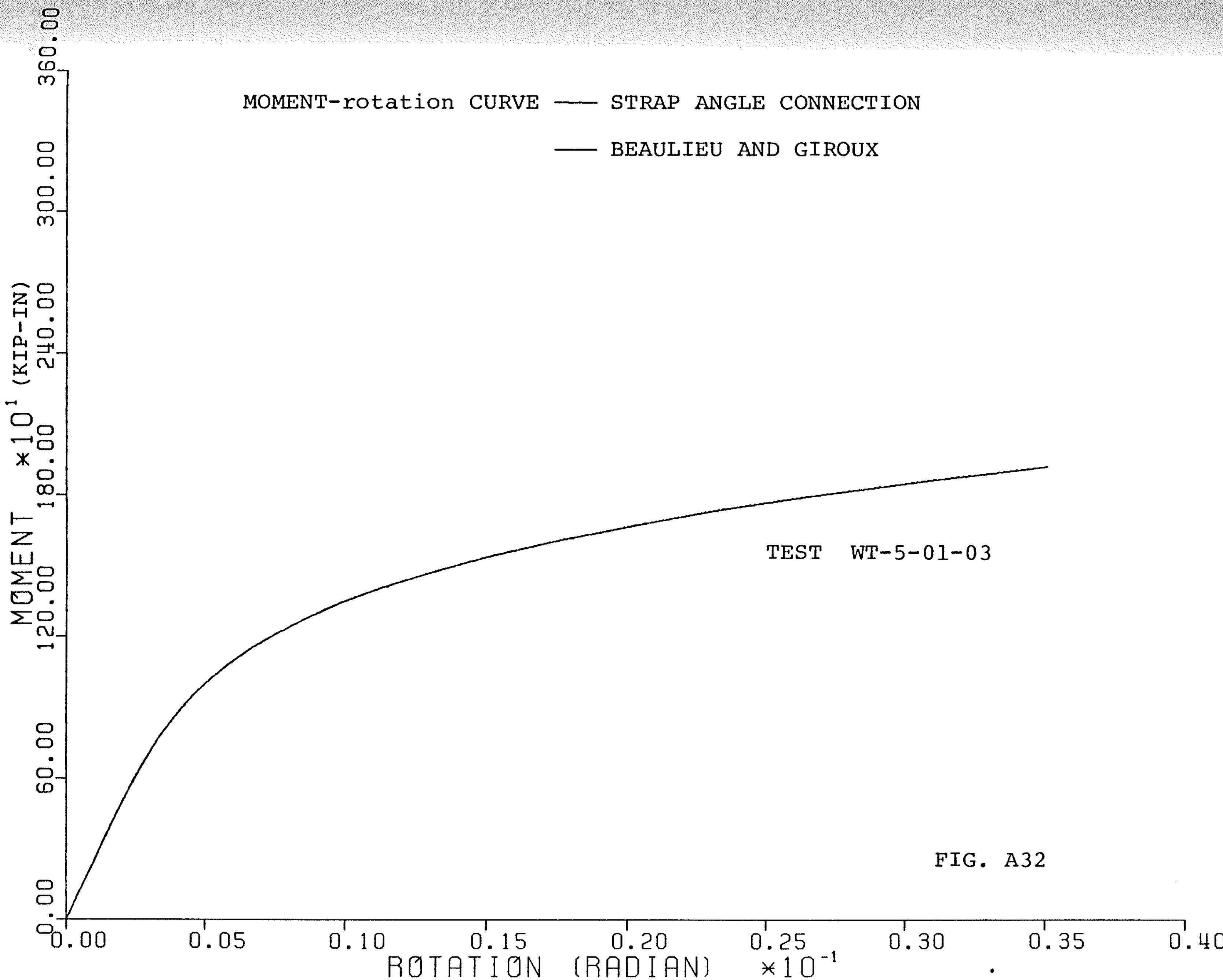

MOMENT-rotation CURVE — STRAP ANGLE CONNECTION
— BEAULIEU AND GIROUX
TEST WT-5-01-03
MOMENT *10^1 (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) *10^-1

FIG. A32

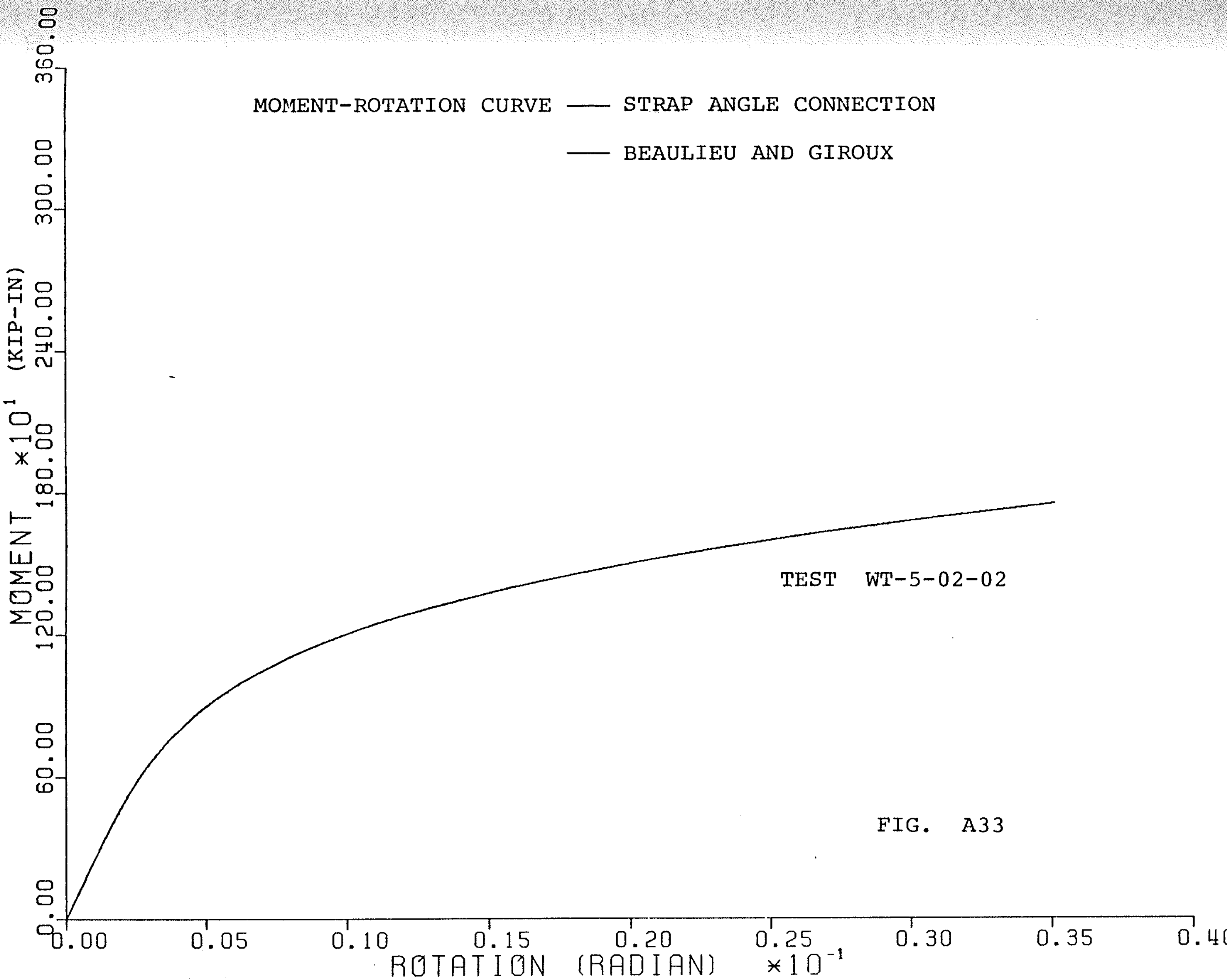
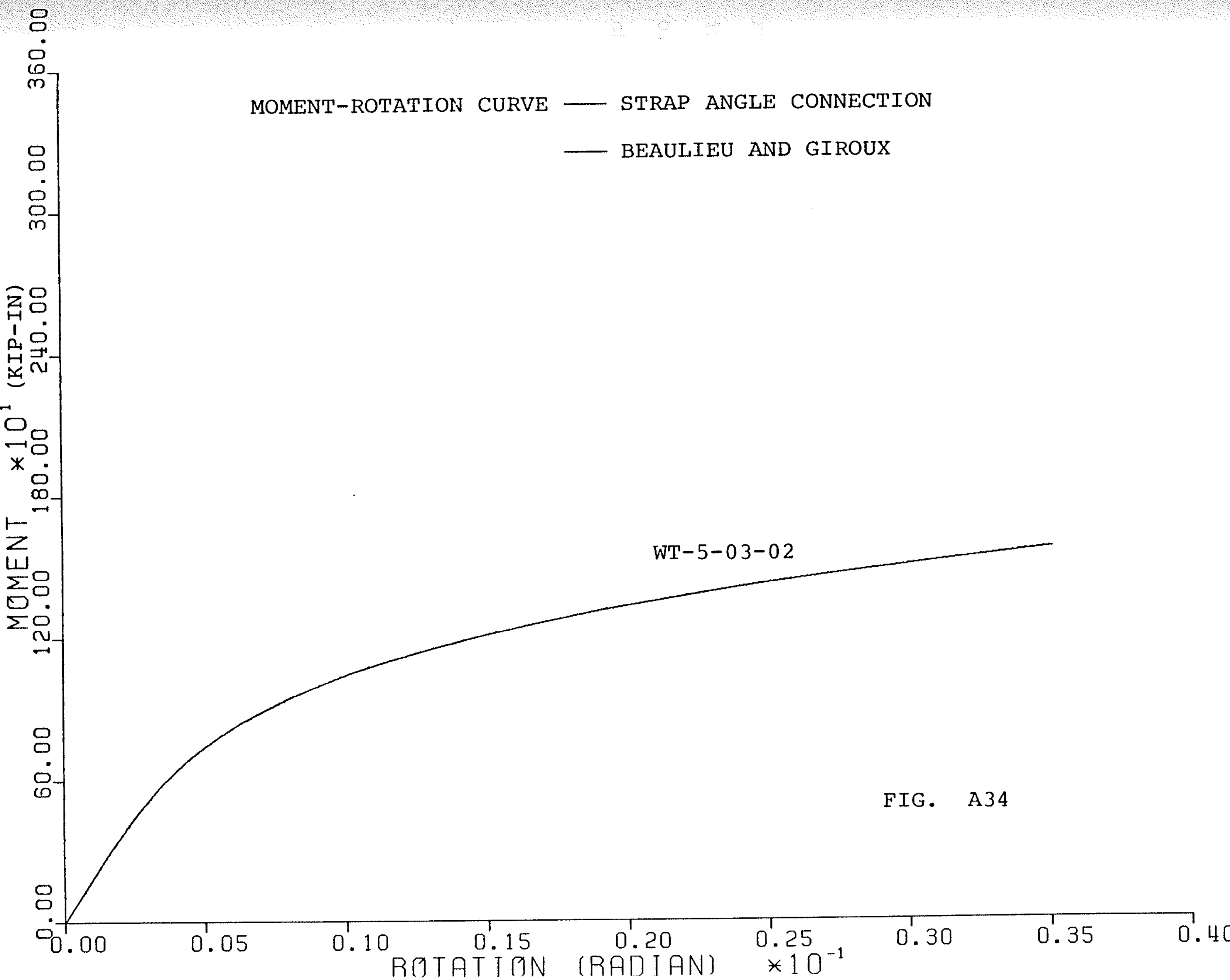

MOMENT-ROTATION CURVE —— STRAP ANGLE CONNECTION
—— BEAULIEU AND GIROUX
TEST WT-5-02-02
MOMENT *10^1 (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) *10^-1

FIG. A33



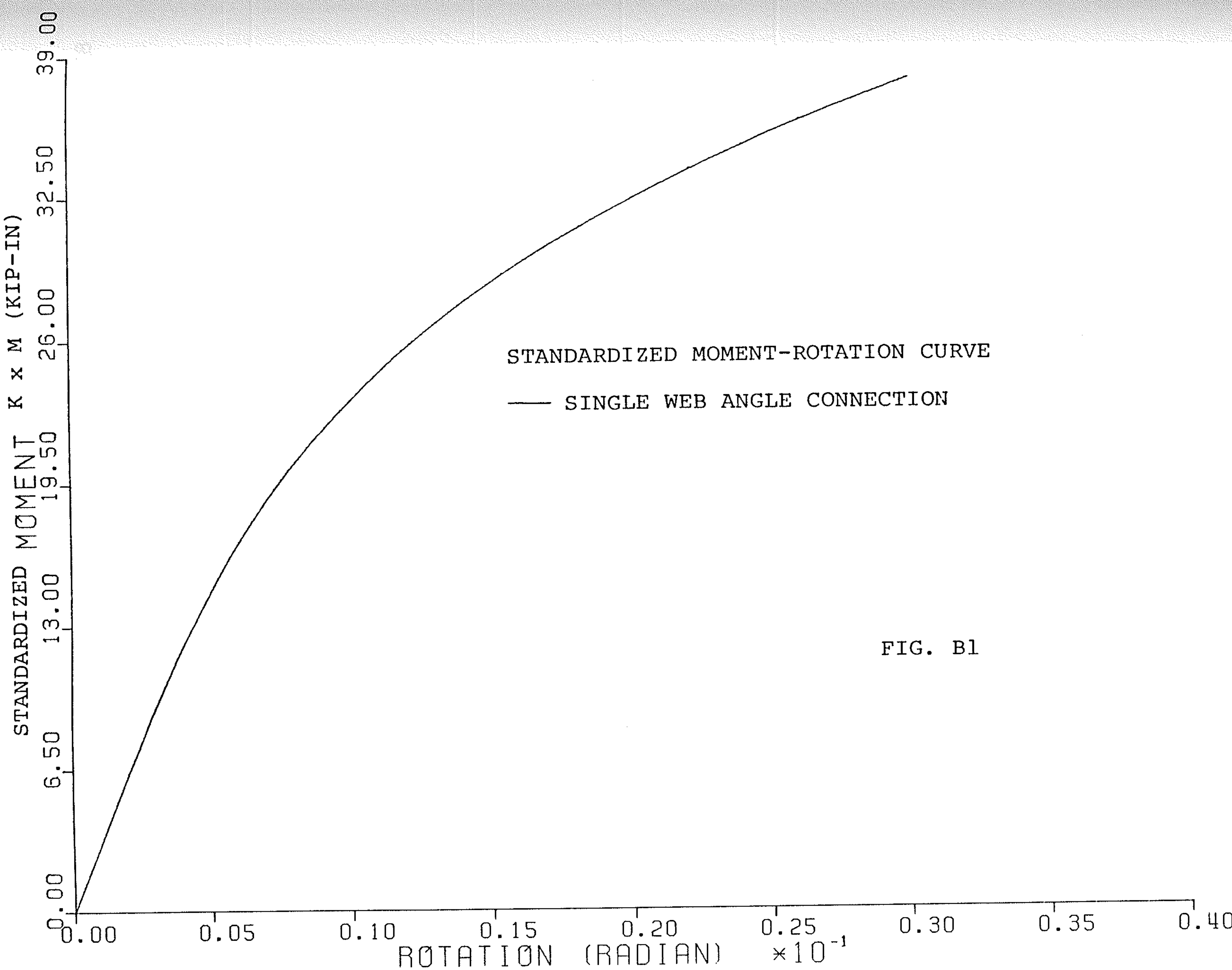
MOMENT-ROTATION CURVE — STRAP ANGLE CONNECTION
— BEAULIEU AND GIROUX
WT-5-03-02
MOMENT ×10¹ (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. A34

APPENDIX B

STANDARDIZED MOMENT-ROTATION CURVES

This Appendix contains the standardized moment-rotation curves for the various connection types considered in this study. Also presented are figures for the comparison of experimentally obtained moment-rotation curves with those obtained from the standardized equations.



STANDARDIZED MOMENT-ROTATION CURVE
—— SINGLE WEB ANGLE CONNECTION
STANDARDIZED MOMENT K x M (KIP-IN)
0.00
6.50
13.00
19.50
26.00
32.50
39.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. B1

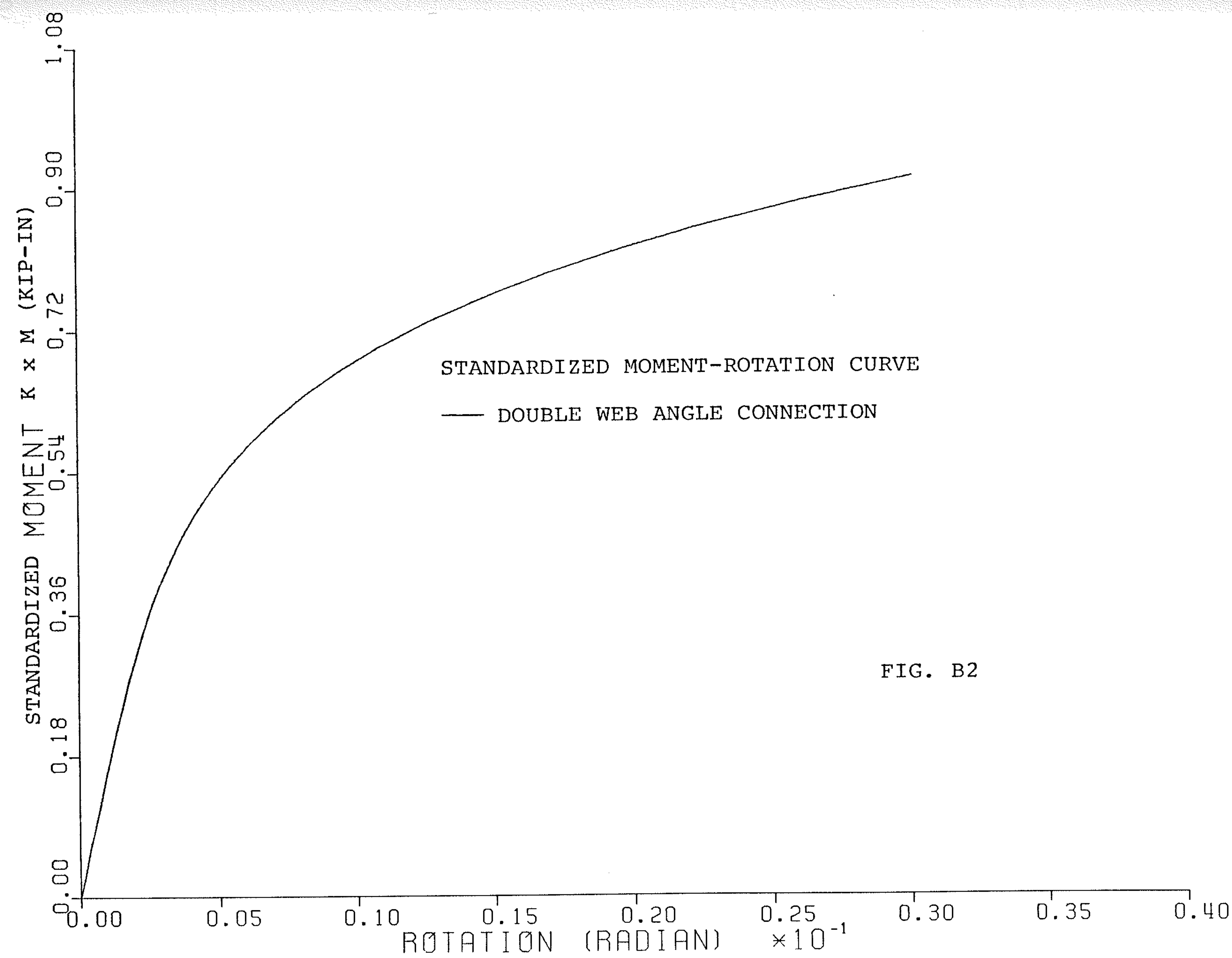

STANDARDIZED MOMENT-ROTATION CURVE
DOUBLE WEB ANGLE CONNECTION
STANDARDIZED MØMENT K x M (KIP-IN)
0.00
0.18
0.36
0.54
0.72
0.90
1.08
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
RØTATIØN (RADIAN) ×10⁻¹

FIG. B2

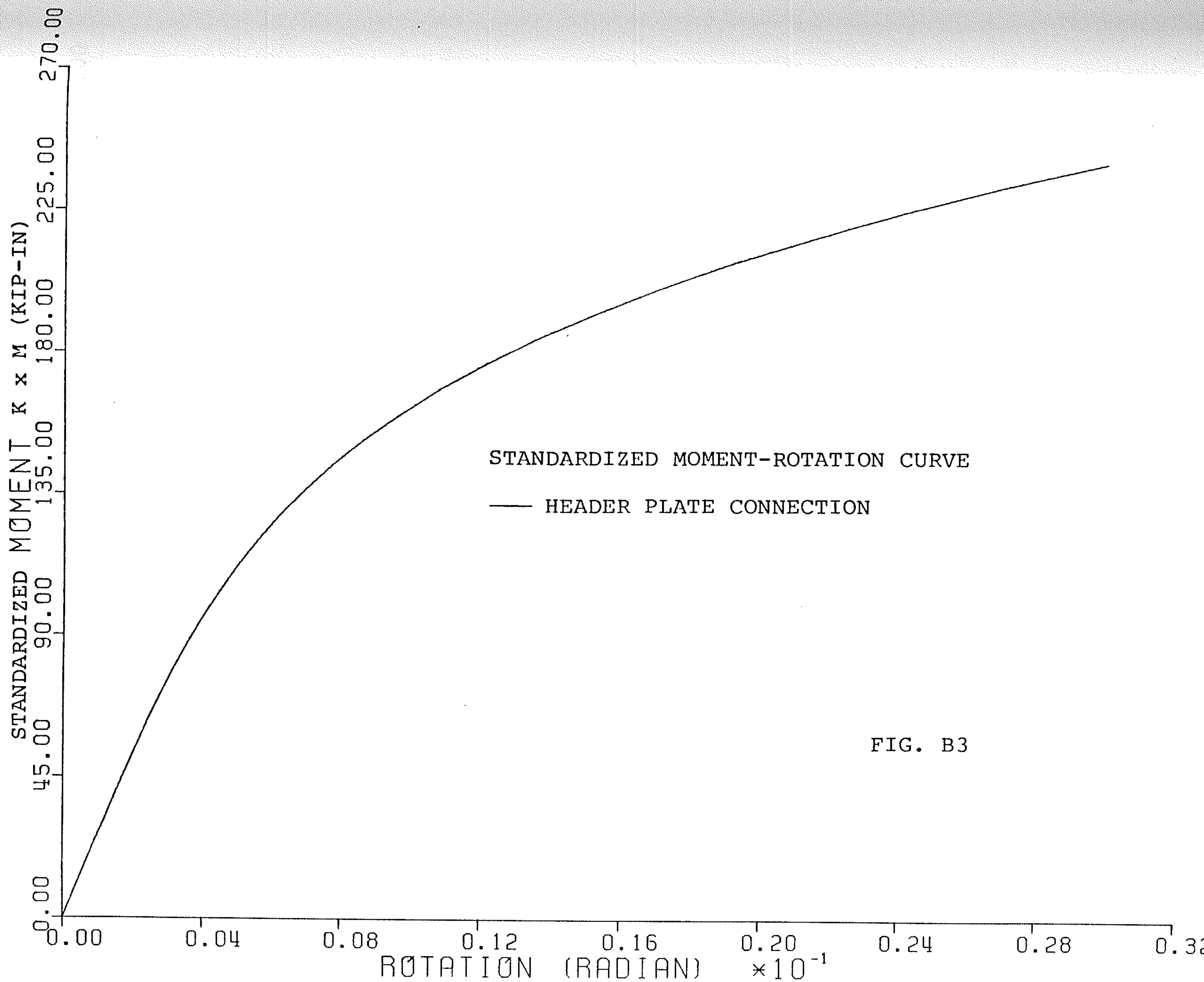
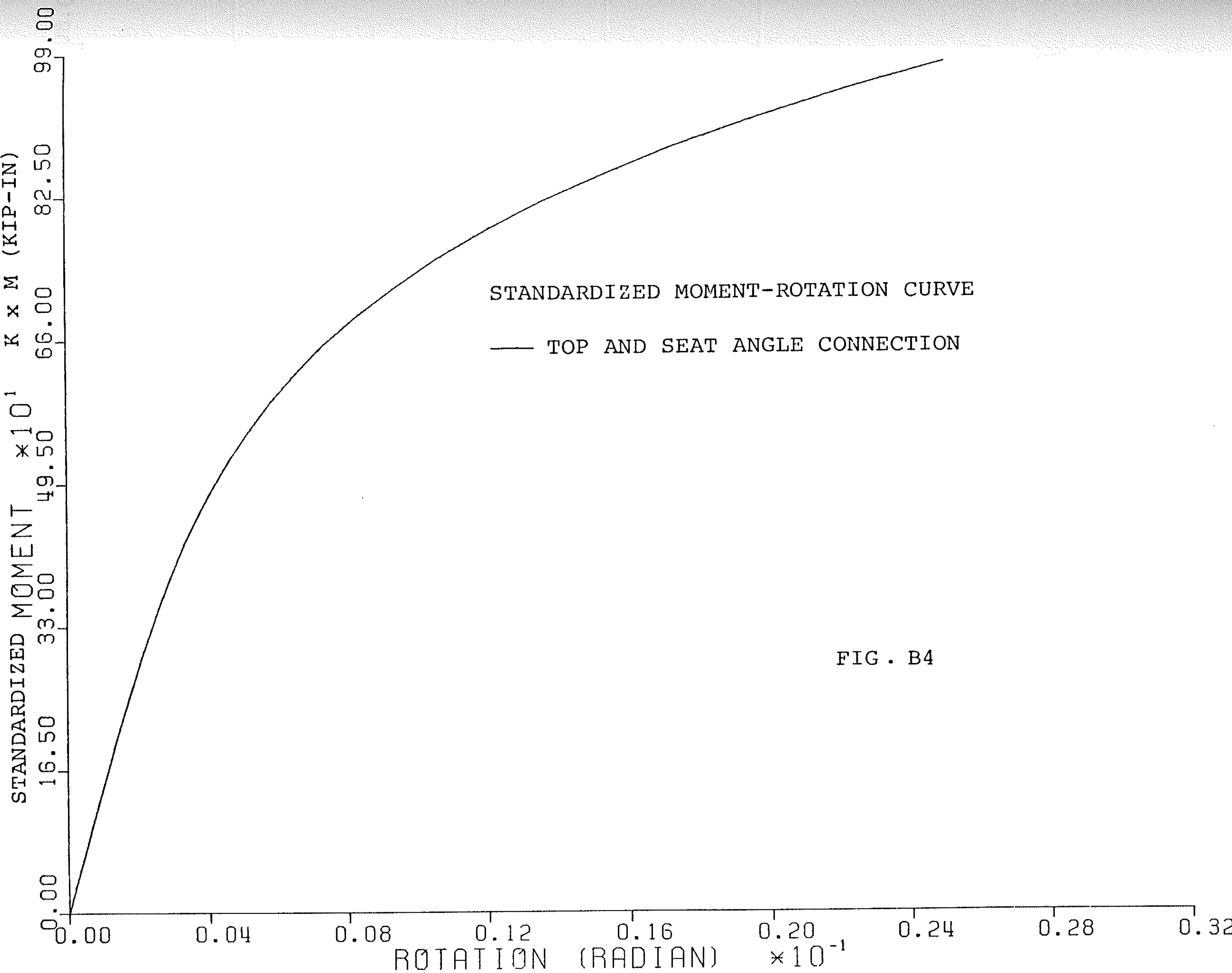

STANDARDIZED MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
STANDARDIZED MOMENT K x M (KIP-IN)
ROTATION (RADIAN) *10^-1
0.00
45.00
90.00
135.00
180.00
225.00
270.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B3



STANDARDIZED MOMENT-ROTATION CURVE
—— TOP AND SEAT ANGLE CONNECTION
STANDARDIZED MOMENT ×10¹ K x M (KIP-IN)
0.00
16.50
33.00
49.50
66.00
82.50
99.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) ×10⁻¹

FIG. B4

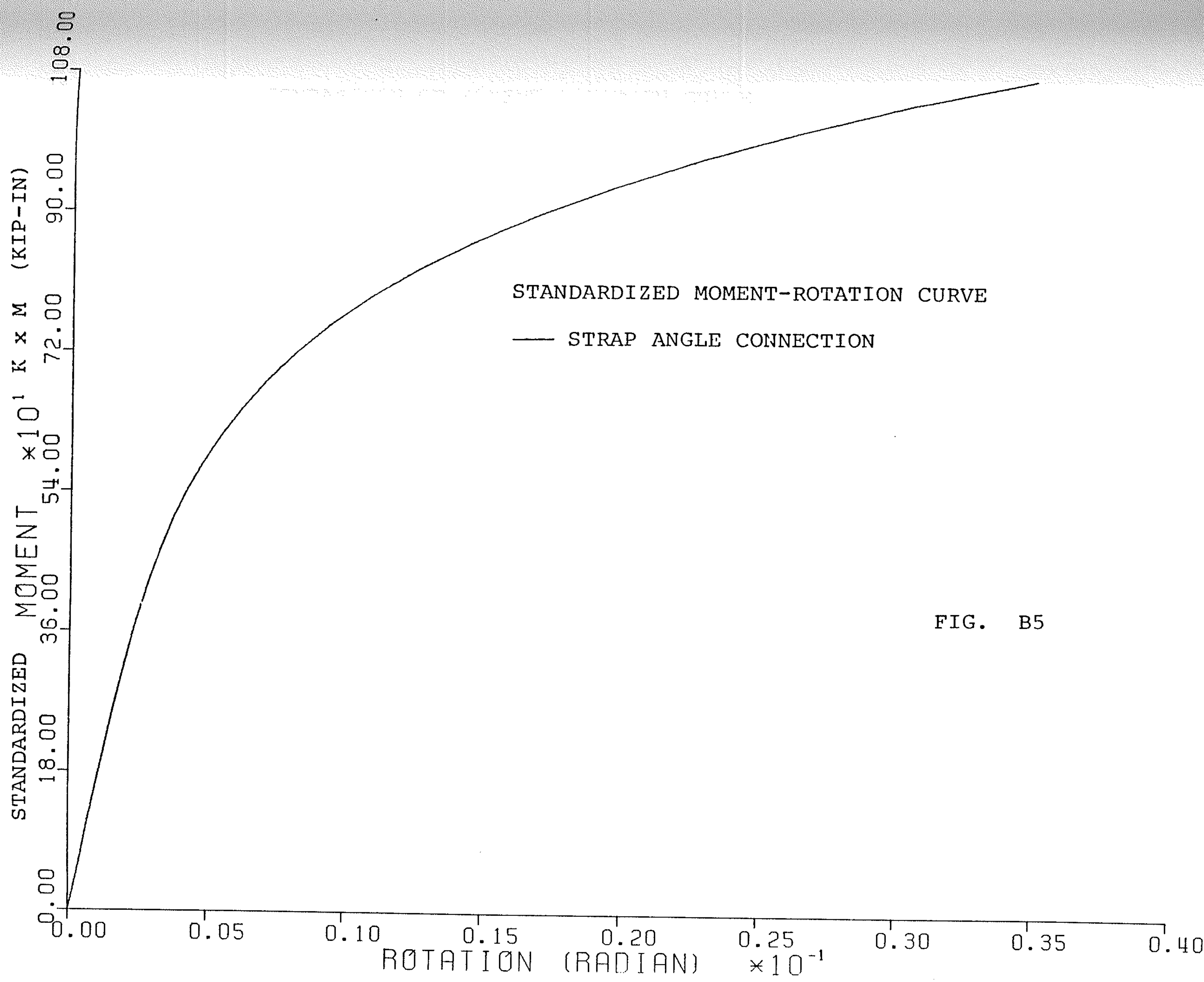

STANDARDIZED MOMENT-ROTATION CURVE
—— STRAP ANGLE CONNECTION
STANDARDIZED MOMENT *10¹ K x M (KIP-IN)
0.00
18.00
36.00
54.00
72.00
90.00
108.00
ROTATION (RADIAN) *10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B5

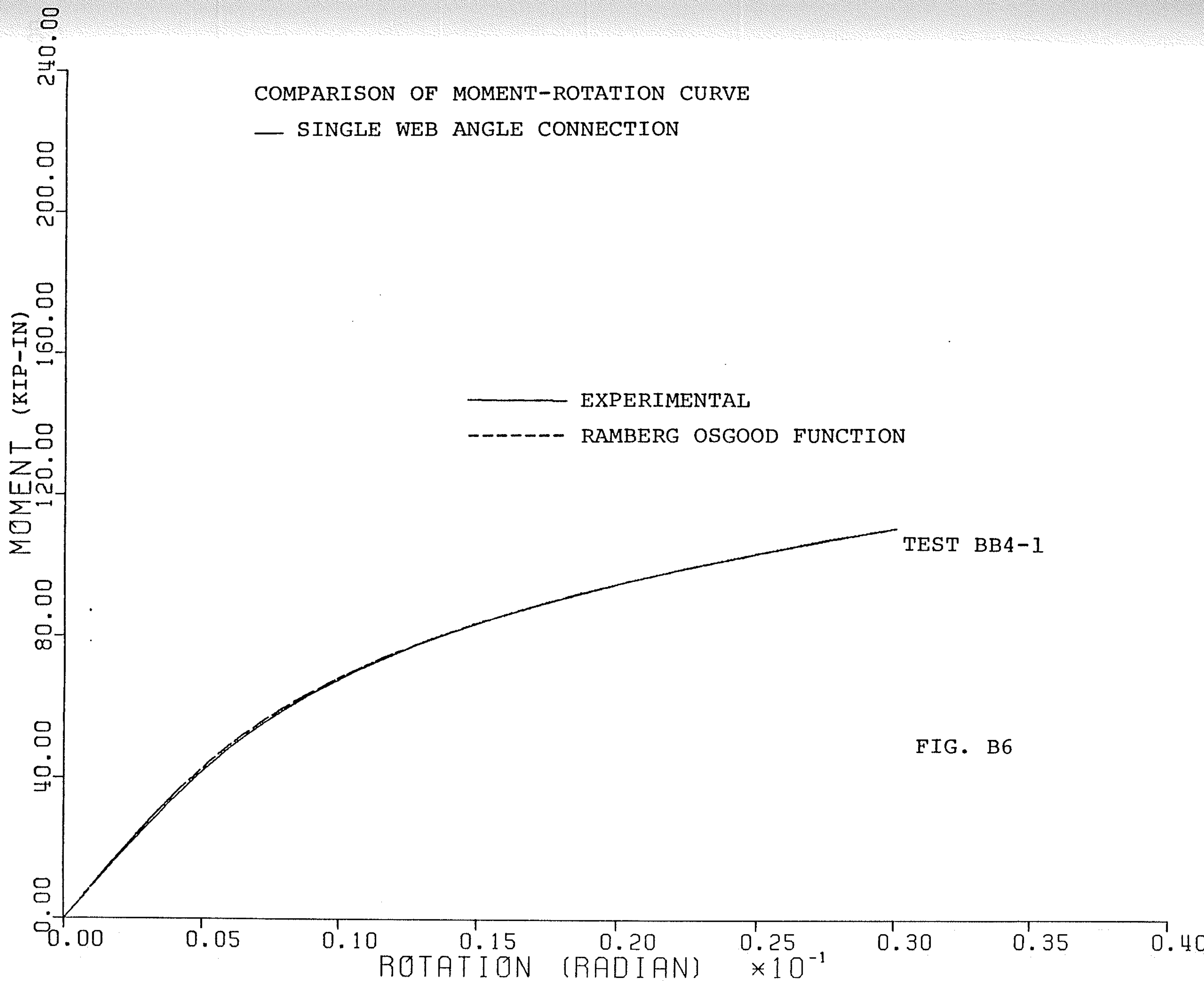

COMPARISON OF MOMENT-ROTATION CURVE
— SINGLE WEB ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
TEST BB4-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B6

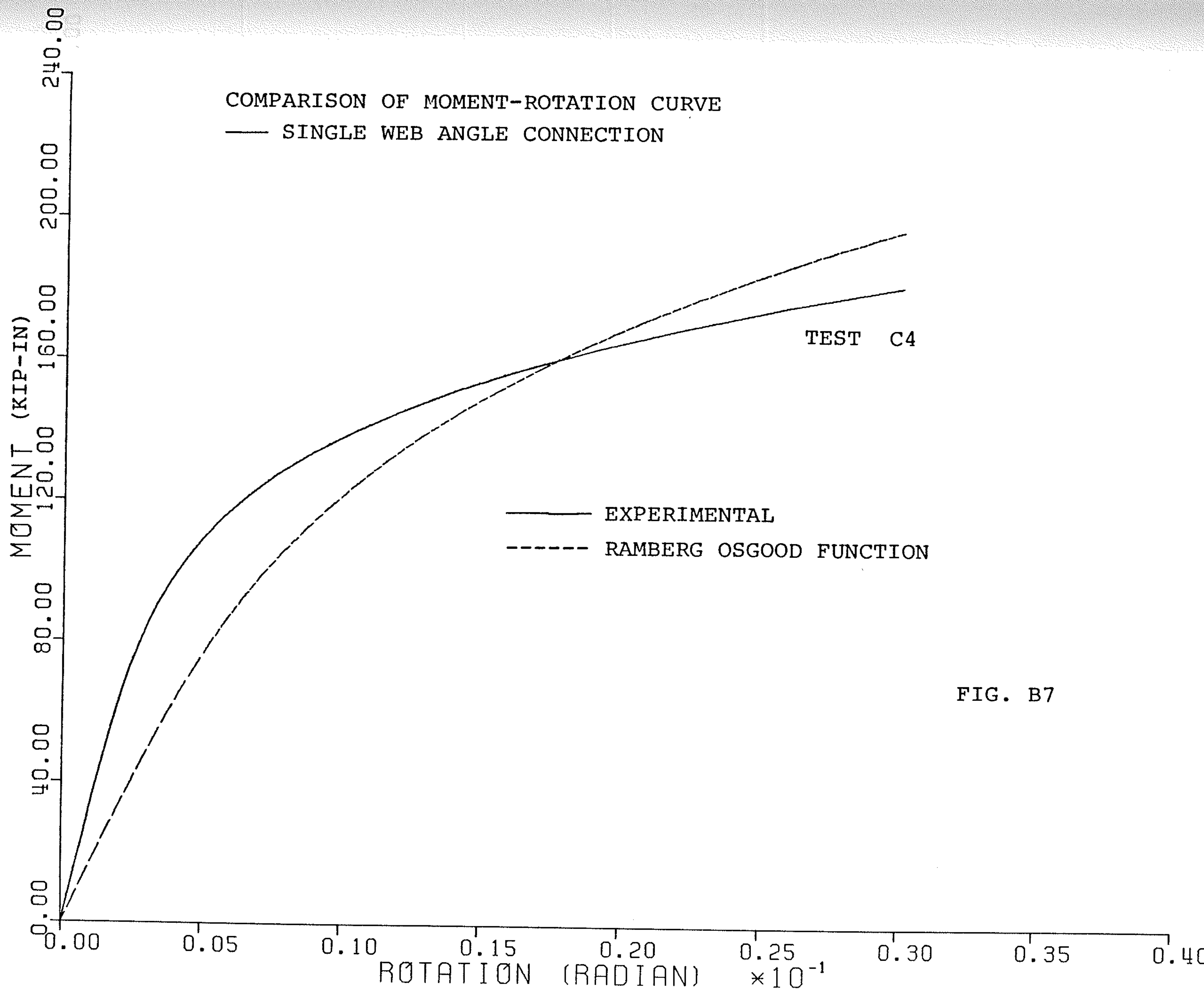

COMPARISON OF MOMENT-ROTATION CURVE
—— SINGLE WEB ANGLE CONNECTION
TEST C4
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MØMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
RØTATIØN (RADIAN) ×10^-1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B7

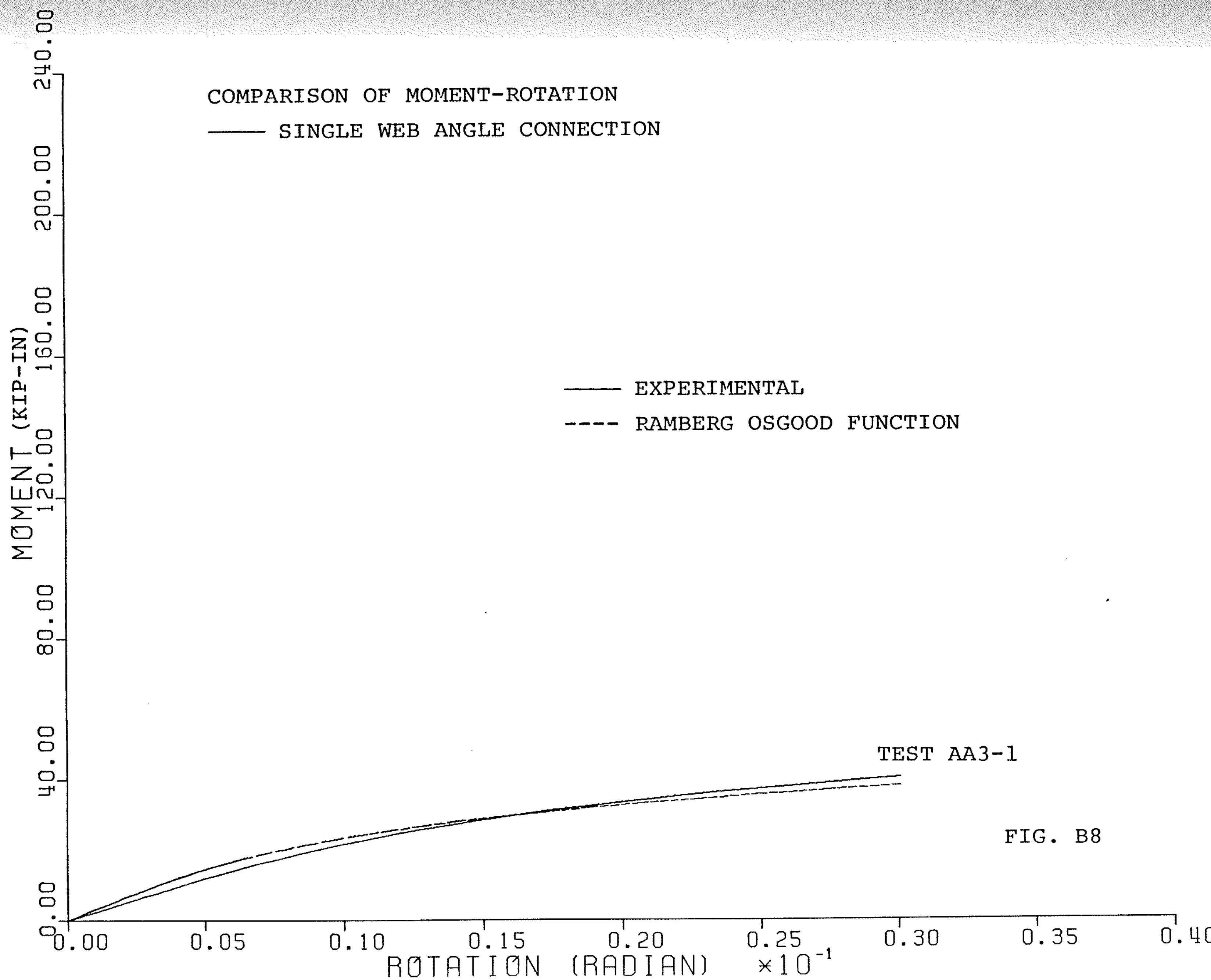
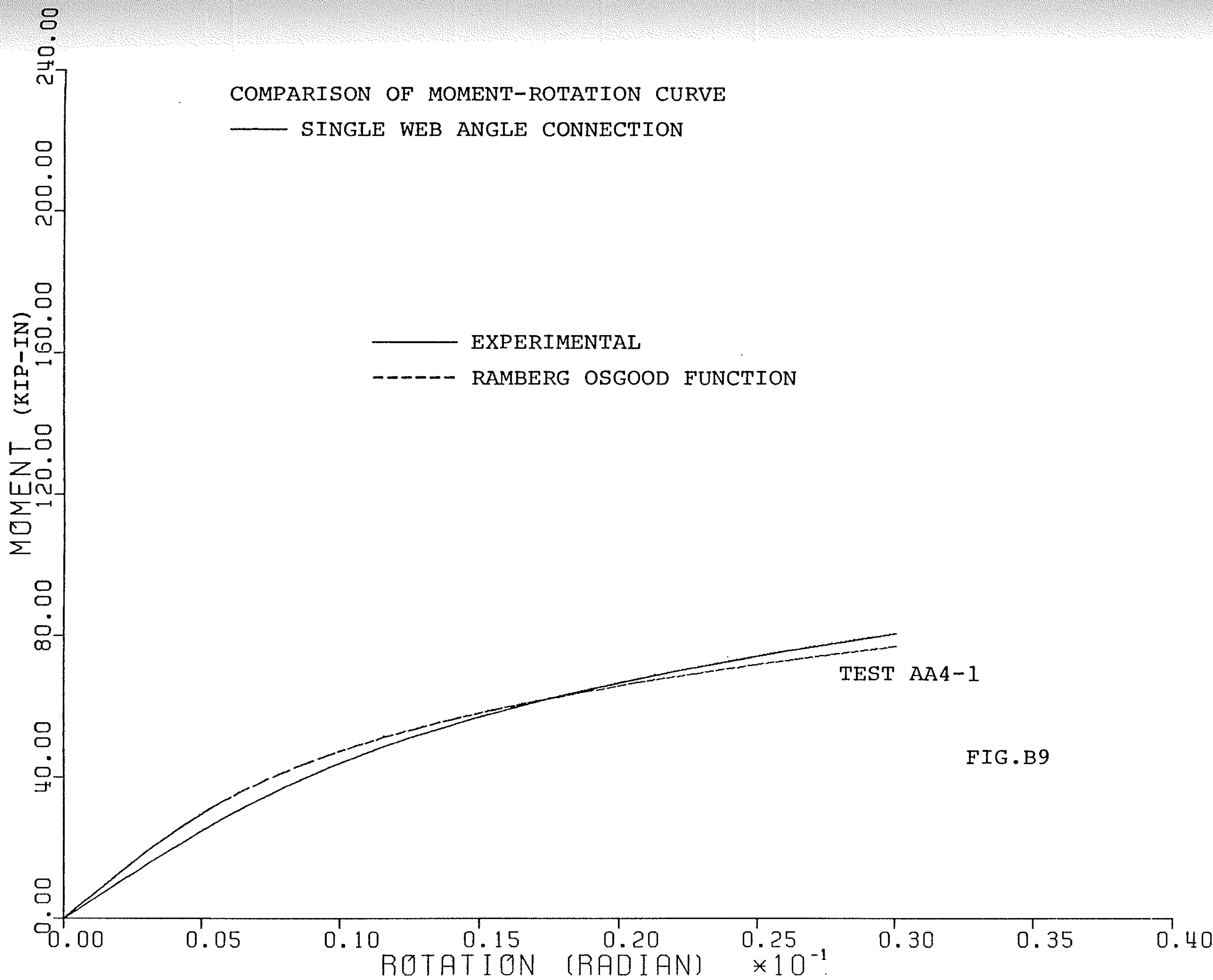

COMPARISON OF MOMENT-ROTATION
SINGLE WEB ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
TEST AA3-1
MØMENT (KIP-IN)
RØTATIØN (RADIAN) ×10⁻¹
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B8


COMPARISON OF MOMENT-ROTATION CURVE
—— SINGLE WEB ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
TEST AA4-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹
FIG.B9

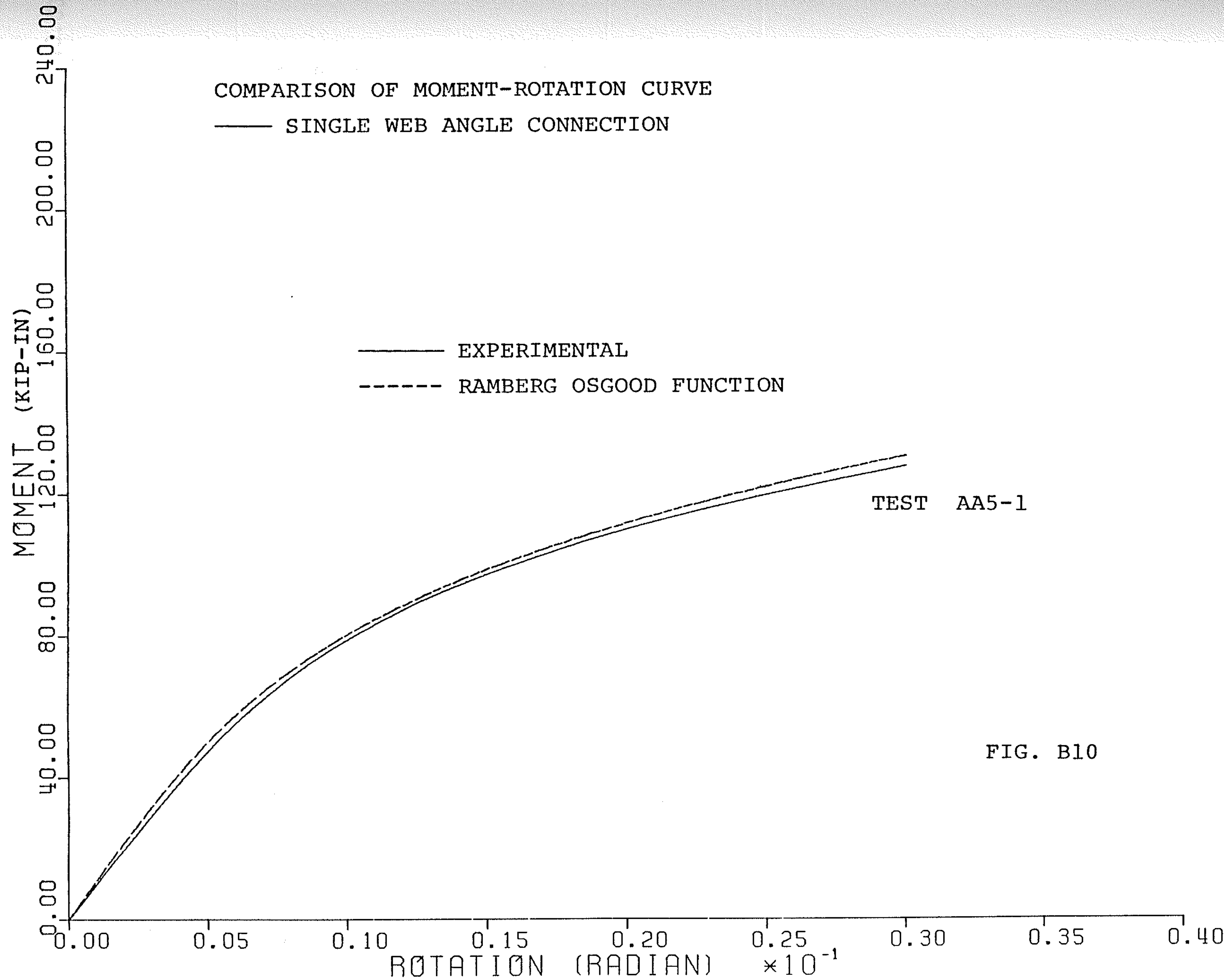

COMPARISON OF MOMENT-ROTATION CURVE
—— SINGLE WEB ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
TEST AA5-1
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. B10

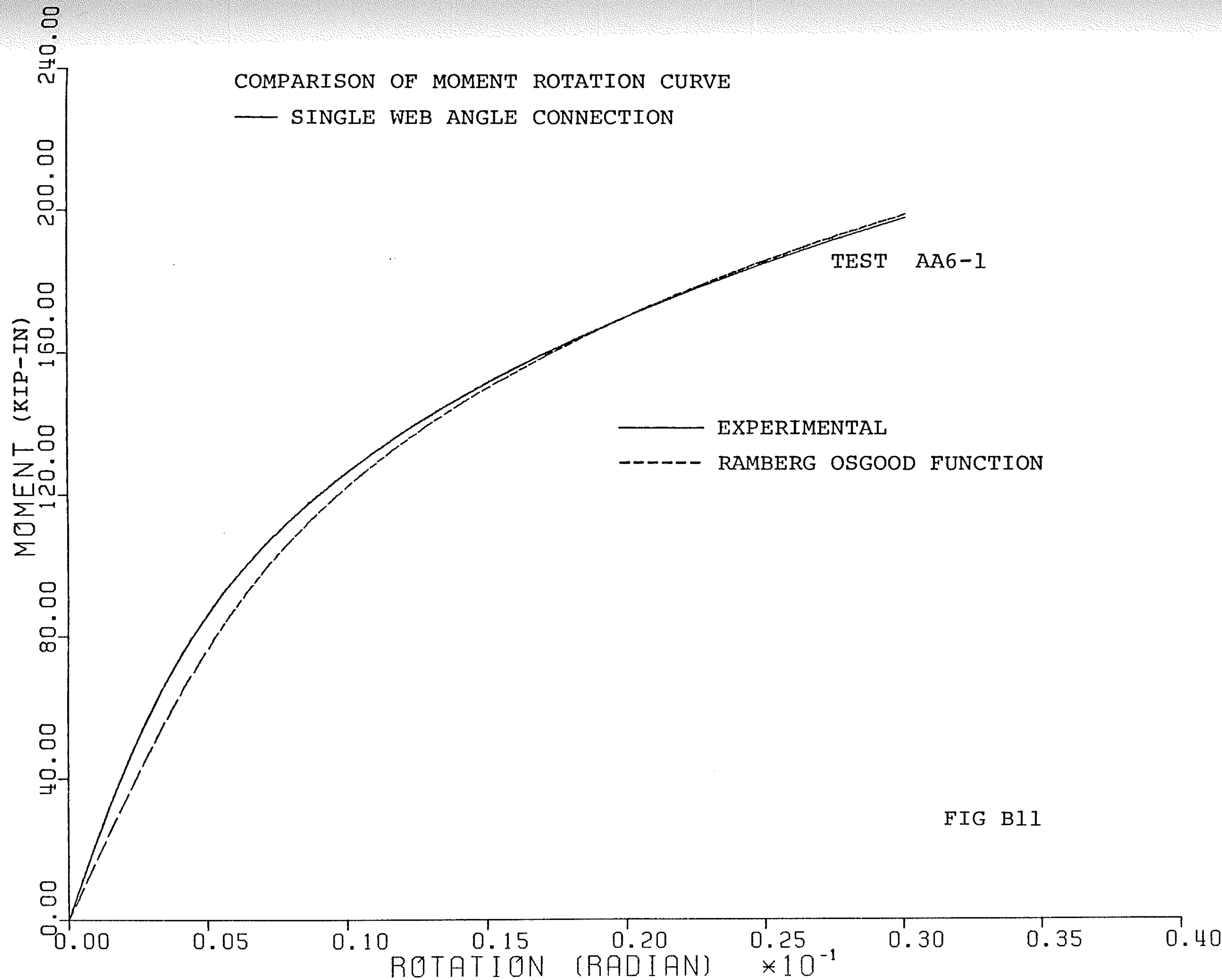
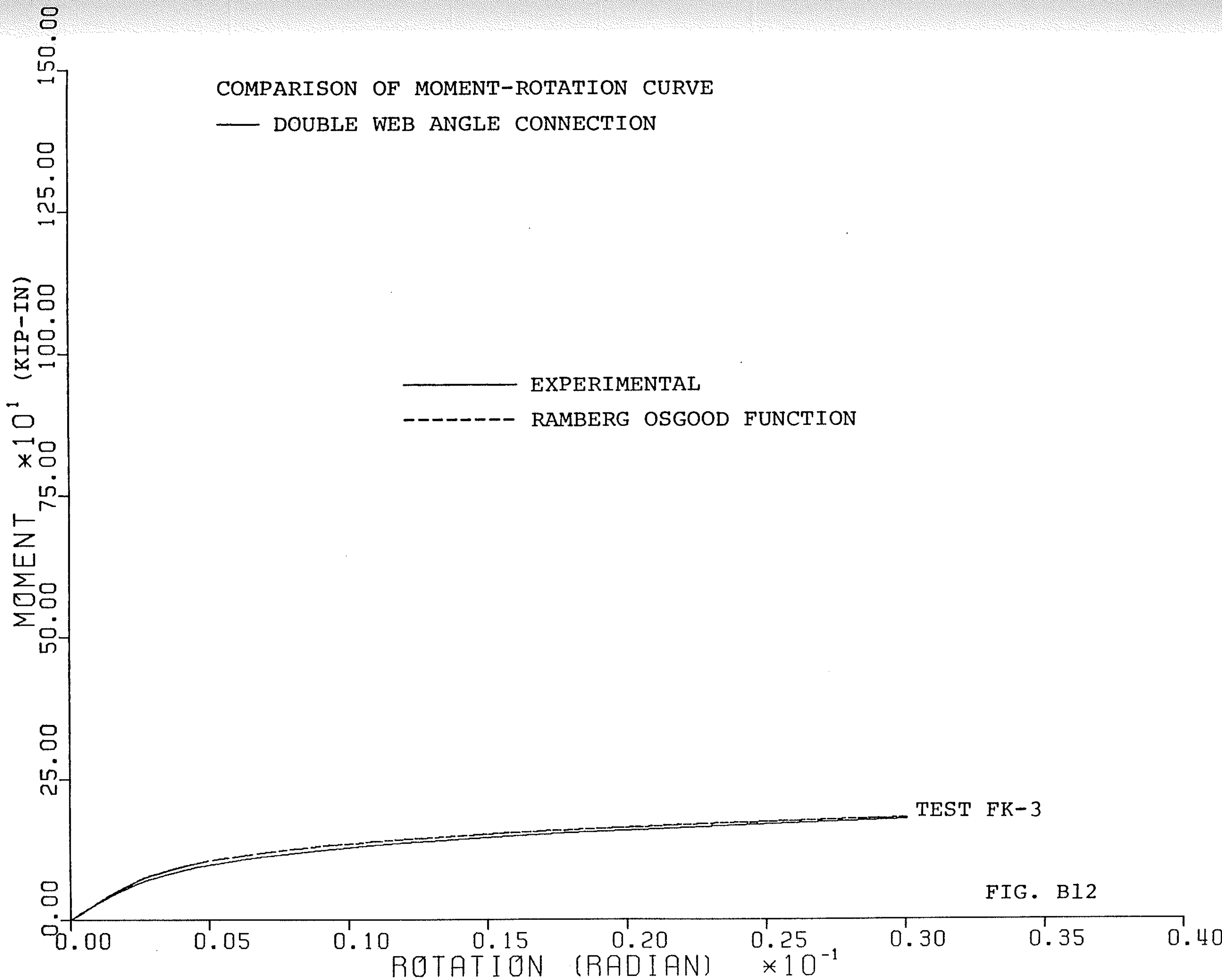

COMPARISON OF MOMENT ROTATION CURVE
—— SINGLE WEB ANGLE CONNECTION
TEST AA6-1
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MOMENT (KIP-IN)
0.00
40.00
80.00
120.00
160.00
200.00
240.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG B11



COMPARISON OF MOMENT-ROTATION CURVE
—— DOUBLE WEB ANGLE CONNECTION
———— EXPERIMENTAL
-------- RAMBERG OSGOOD FUNCTION
TEST FK-3
MOMENT ×10[1] (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10[-1]

FIG. B12

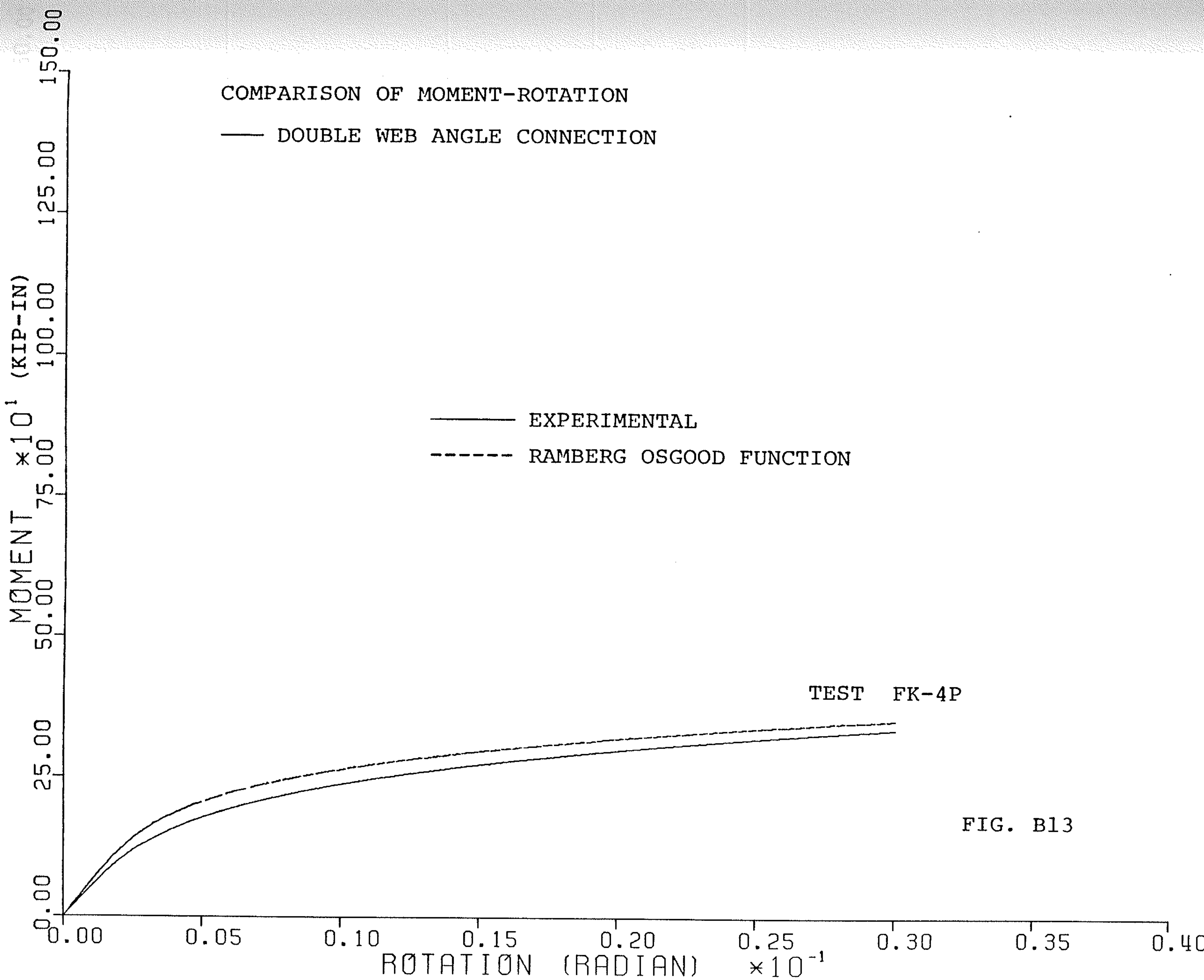

COMPARISON OF MOMENT-ROTATION
—— DOUBLE WEB ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
TEST FK-4P
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B13

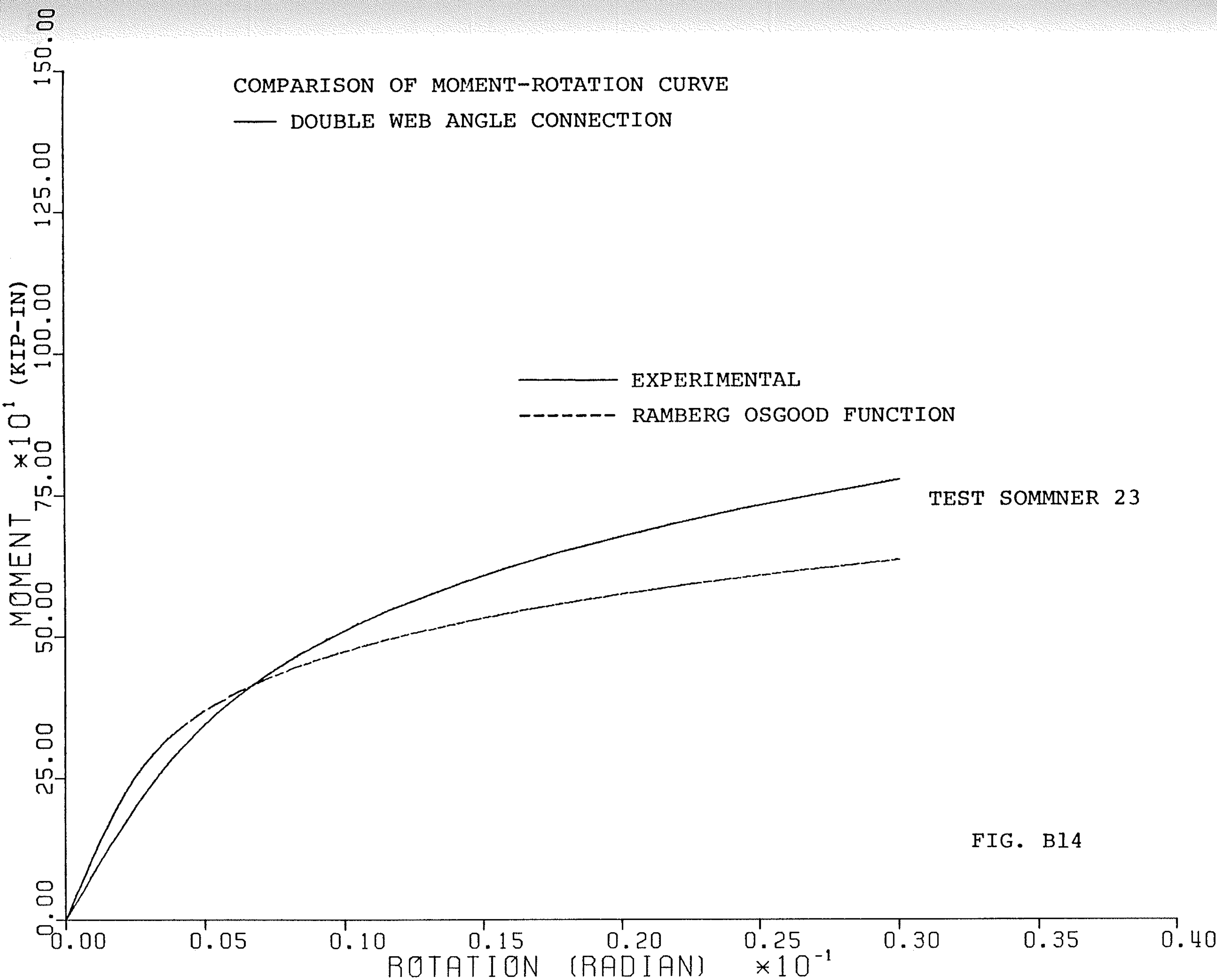

COMPARISON OF MOMENT-ROTATION CURVE
— DOUBLE WEB ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
TEST SOMMNER 23
MOMENT ×10¹ (KIP-IN)
ROTATION (RADIAN) ×10⁻¹
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B14

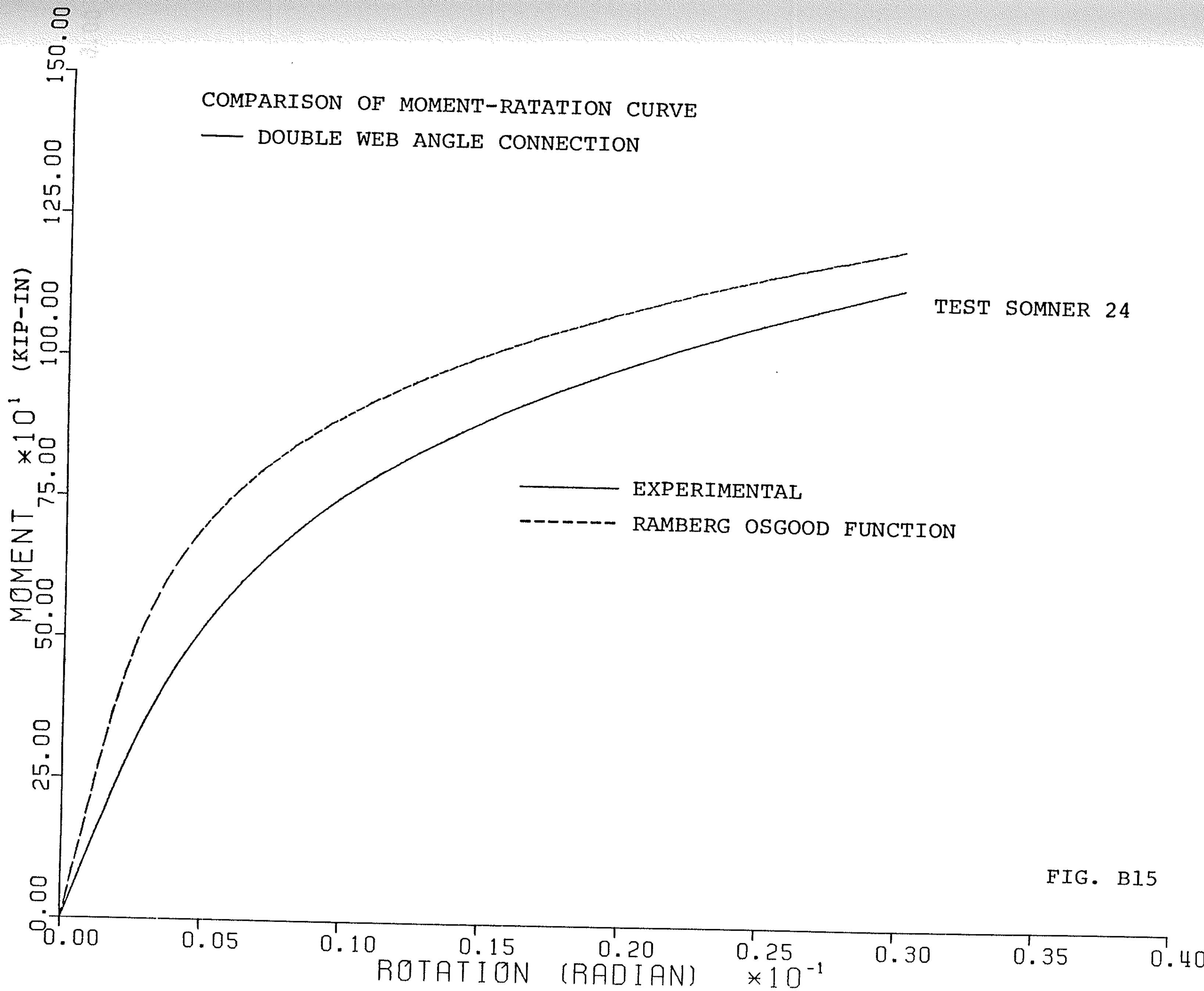

COMPARISON OF MOMENT-RATATION CURVE
— DOUBLE WEB ANGLE CONNECTION
TEST SOMNER 24
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MOMENT *10^1 (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) *10^-1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B15

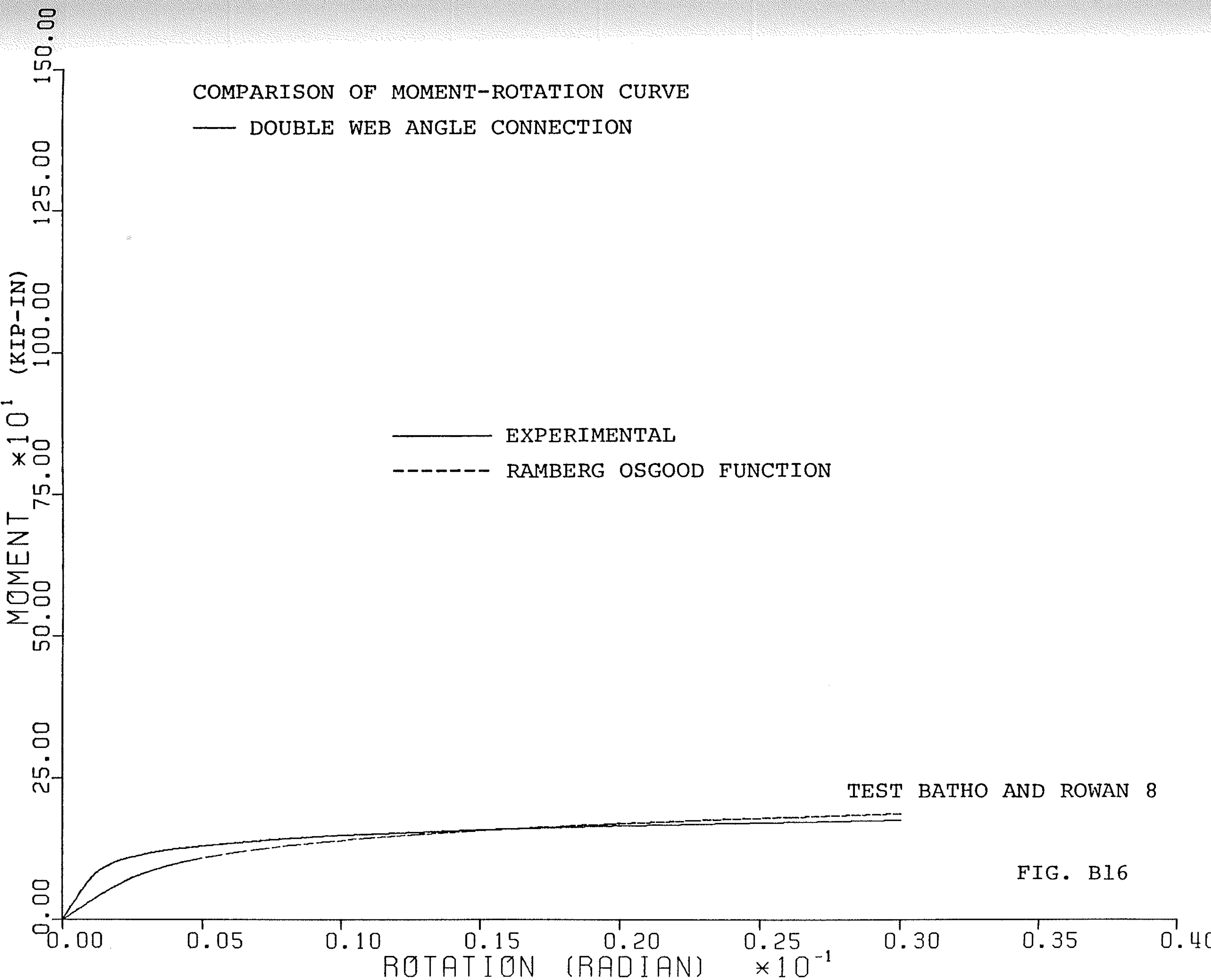

COMPARISON OF MOMENT-ROTATION CURVE
—— DOUBLE WEB ANGLE CONNECTION
——— EXPERIMENTAL
------- RAMBERG OSGOOD FUNCTION
TEST BATHO AND ROWAN 8
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B16

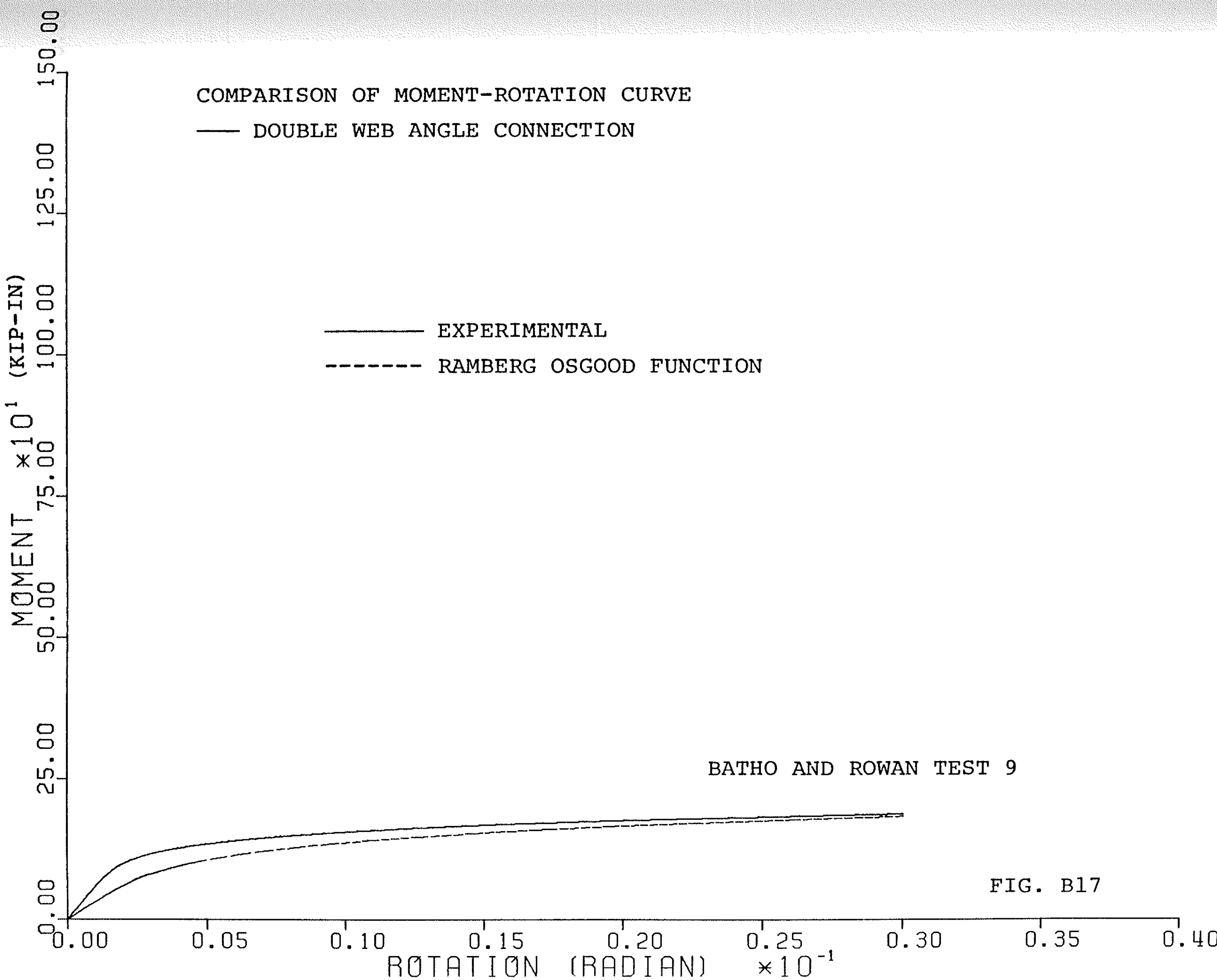
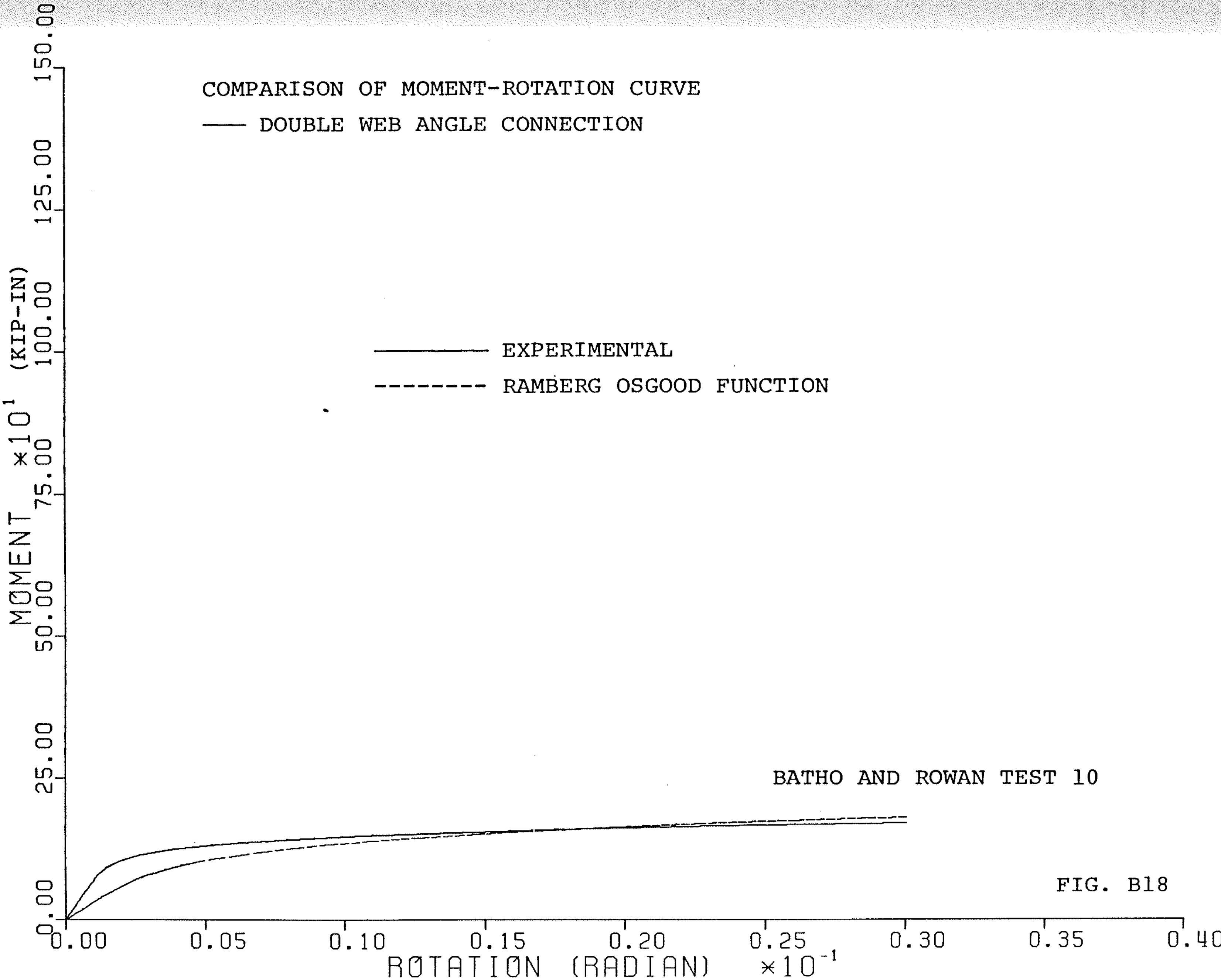
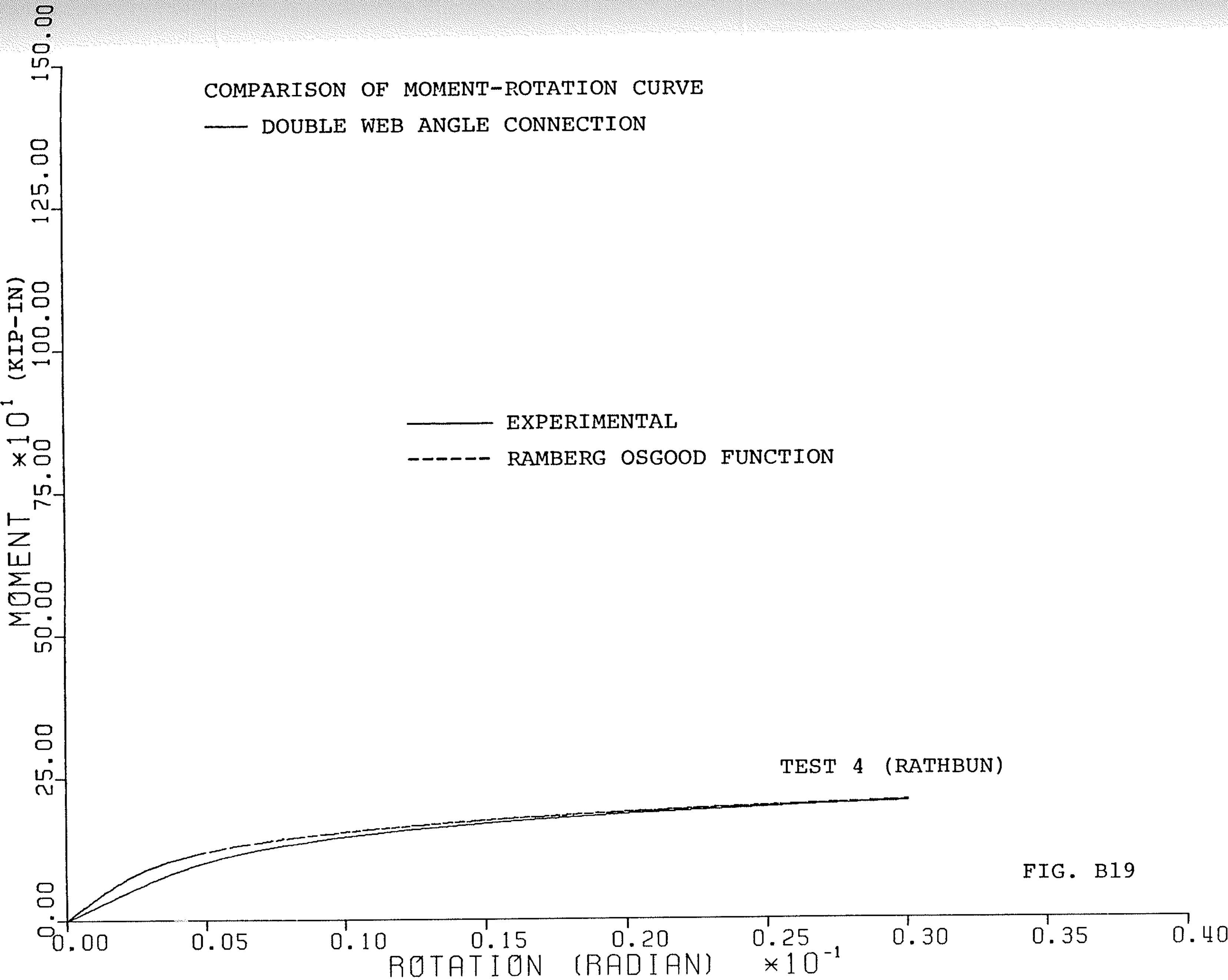

COMPARISON OF MOMENT-ROTATION CURVE
—— DOUBLE WEB ANGLE CONNECTION
———— EXPERIMENTAL
------- RAMBERG OSGOOD FUNCTION
BATHO AND ROWAN TEST 9
MØMENT ×10¹ (KIP-IN)
150.00
125.00
100.00
75.00
50.00
25.00
0.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
RØTATIØN (RADIAN) ×10⁻¹

FIG. B17



COMPARISON OF MOMENT-ROTATION CURVE
— DOUBLE WEB ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
BATHO AND ROWAN TEST 10
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
ROTATION (RADIAN) ×10⁻¹

FIG. B18



COMPARISON OF MOMENT-ROTATION CURVE
—— DOUBLE WEB ANGLE CONNECTION
———— EXPERIMENTAL
------ RAMBERG OSGOOD FUNCTION
TEST 4 (RATHBUN)
MOMENT ×10^1 (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10^-1
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

FIG. B19

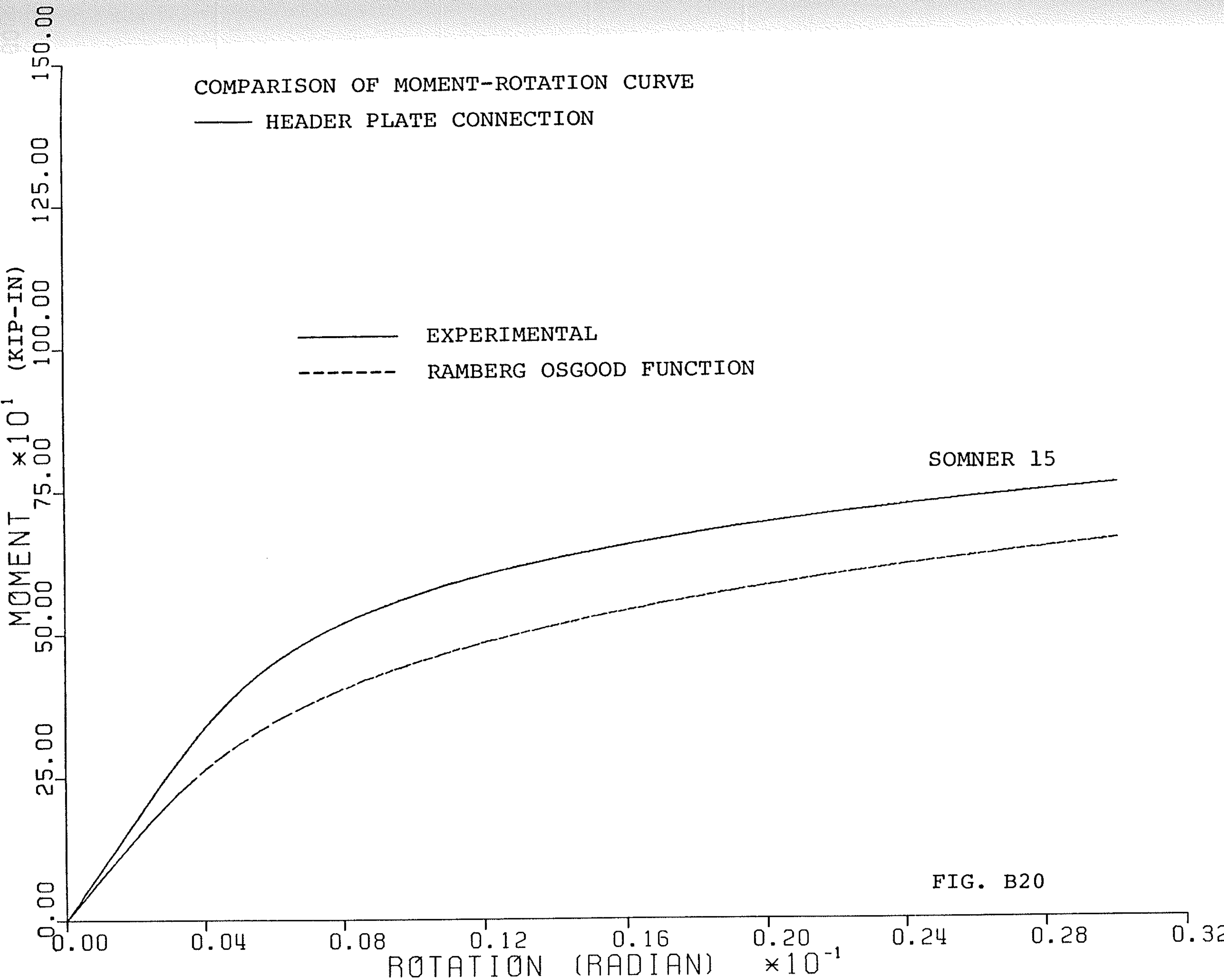

COMPARISON OF MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
SOMNER 15
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) ×10⁻¹

FIG. B20

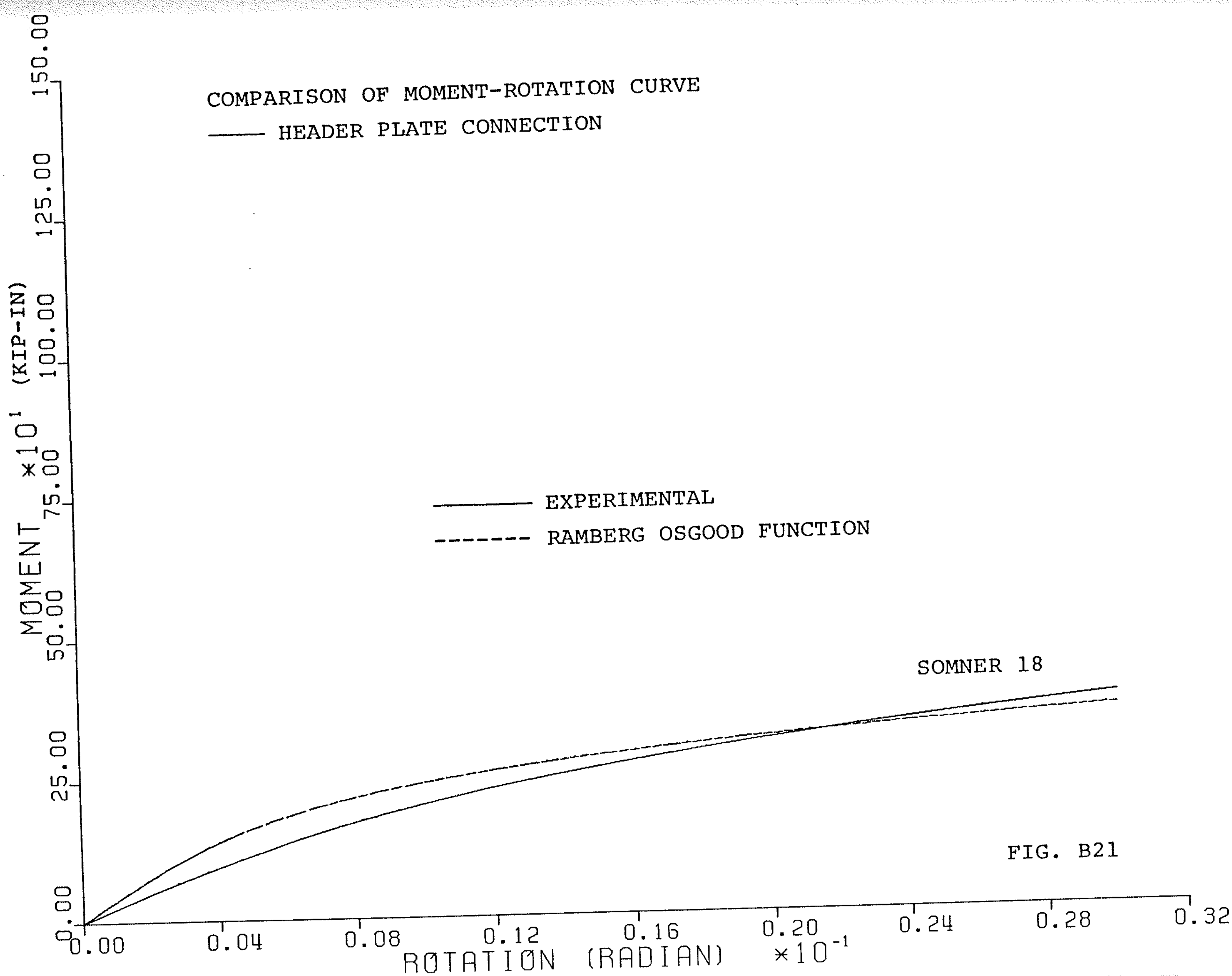

COMPARISON OF MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
—— EXPERIMENTAL
------- RAMBERG OSGOOD FUNCTION
SOMNER 18
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B21

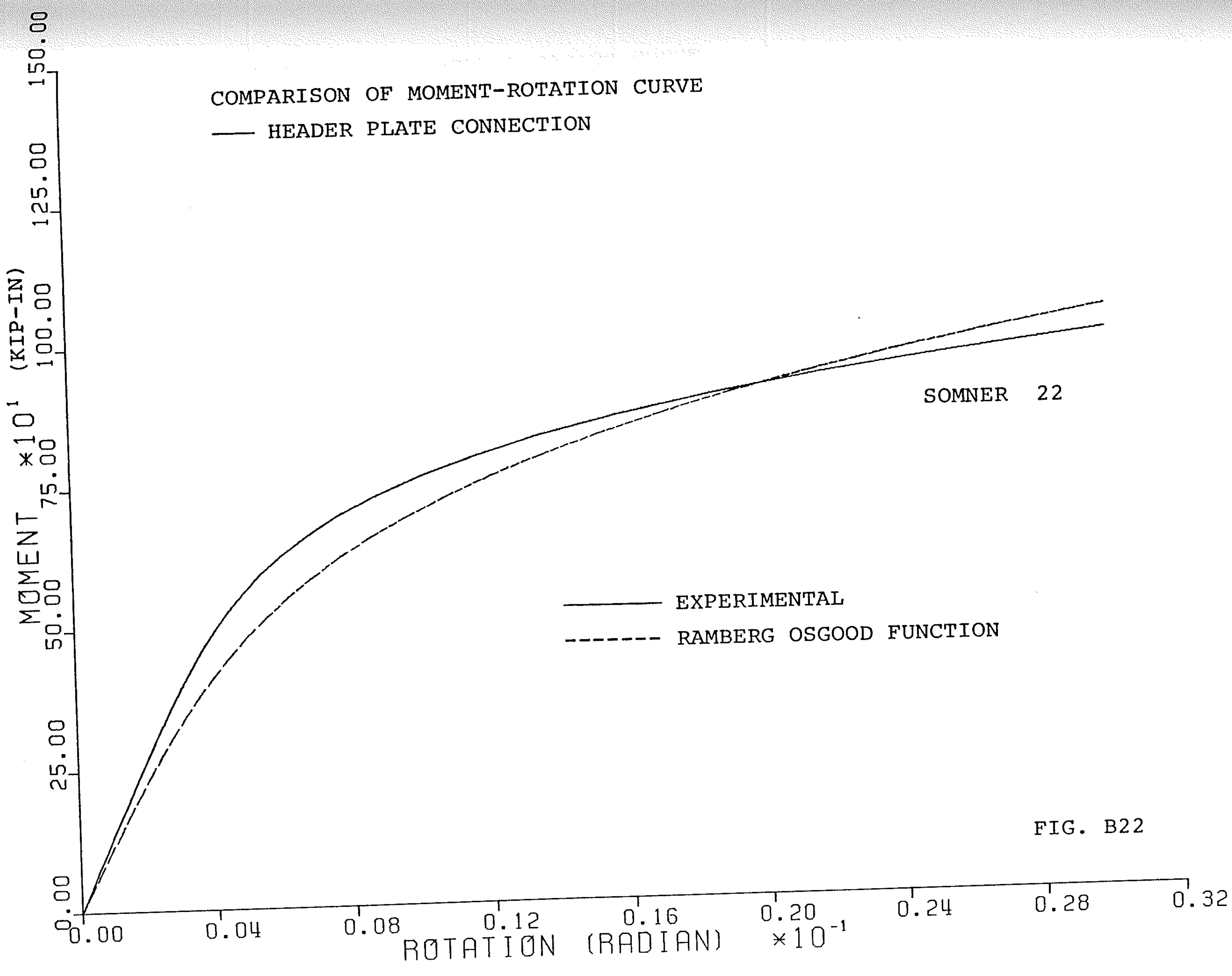

COMPARISON OF MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
SOMNER 22
—— EXPERIMENTAL
------- RAMBERG OSGOOD FUNCTION
MOMENT *10^1 (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) *10^-1
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B22

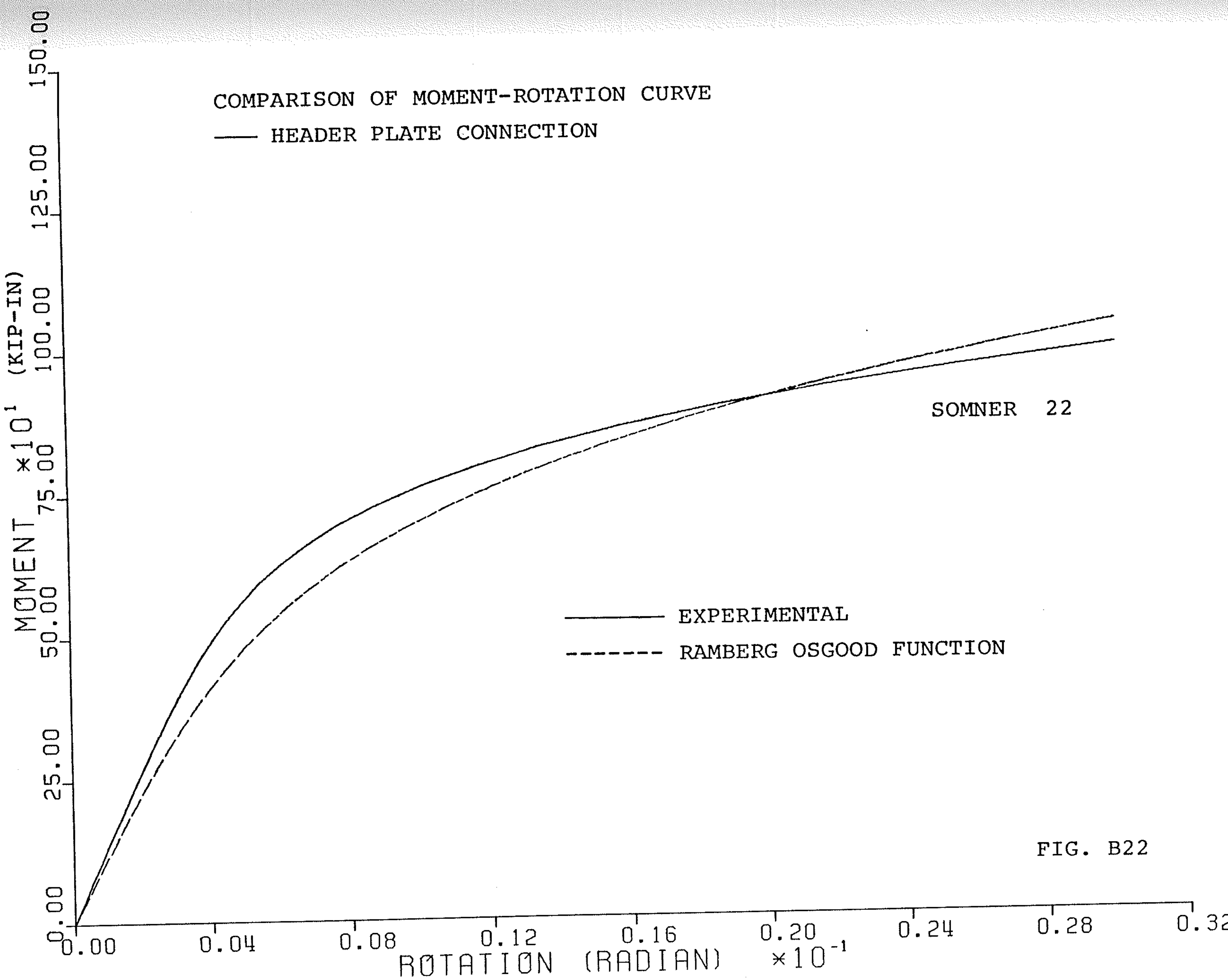

COMPARISON OF MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
SOMNER 22
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) ×10⁻¹
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B22

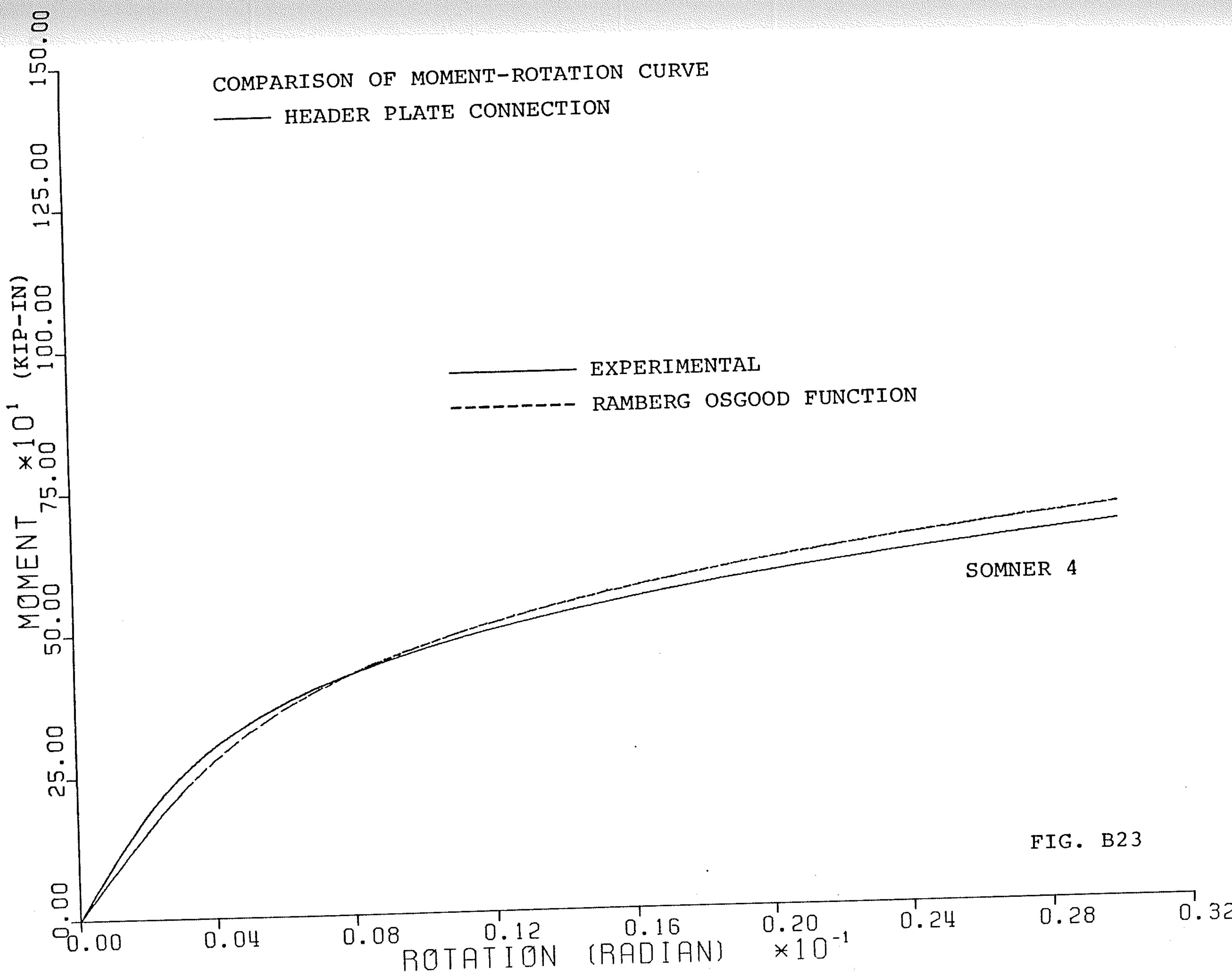
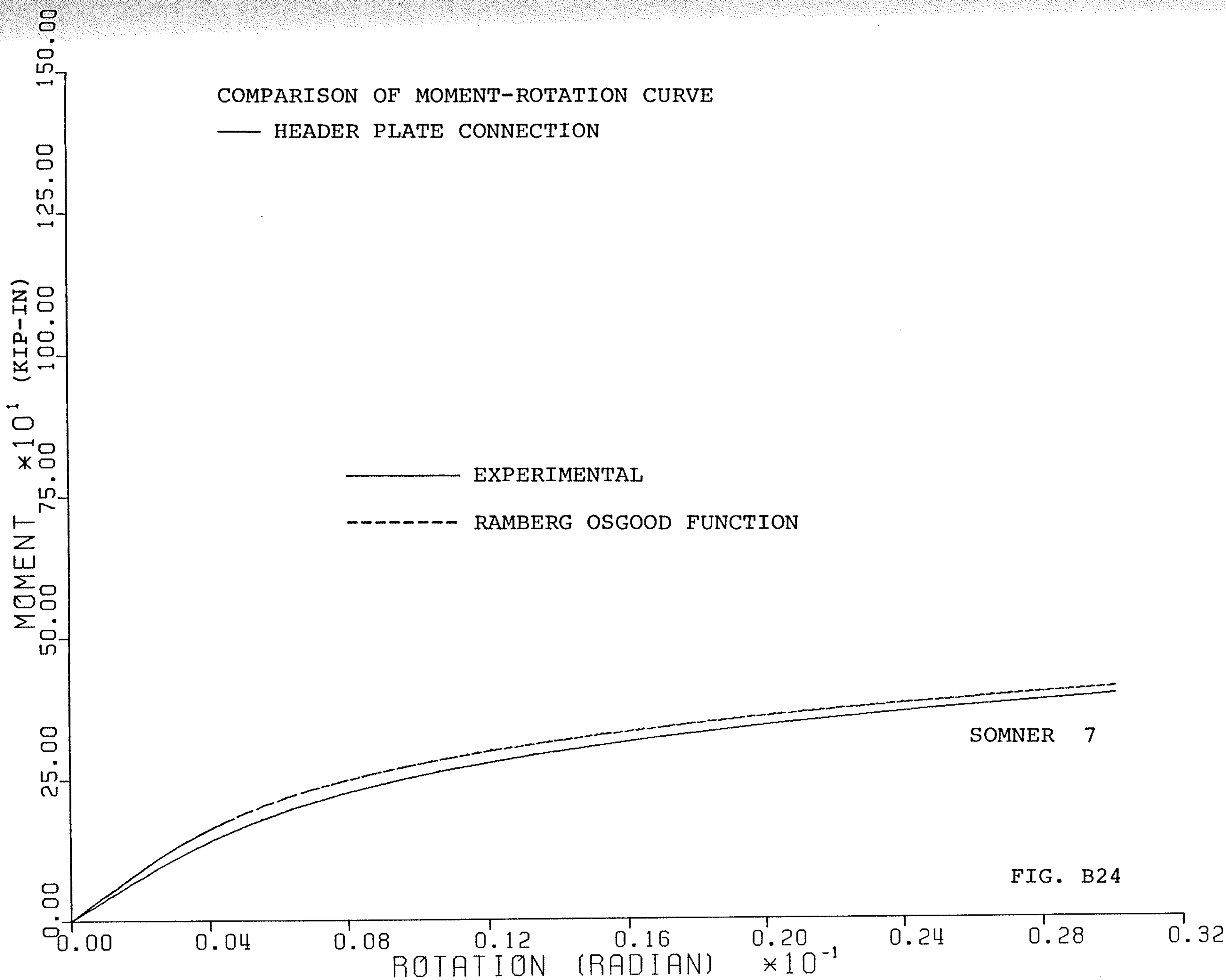

COMPARISON OF MOMENT-ROTATION CURVE
HEADER PLATE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
SOMNER 4
MOMENT ×10¹ (KIP-IN)
ROTATION (RADIAN) ×10⁻¹
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B23


COMPARISON OF MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
———— EXPERIMENTAL
-------- RAMBERG OSGOOD FUNCTION
SOMNER 7
FIG. B24
MOMENT *10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
ROTATION (RADIAN) *10⁻¹
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

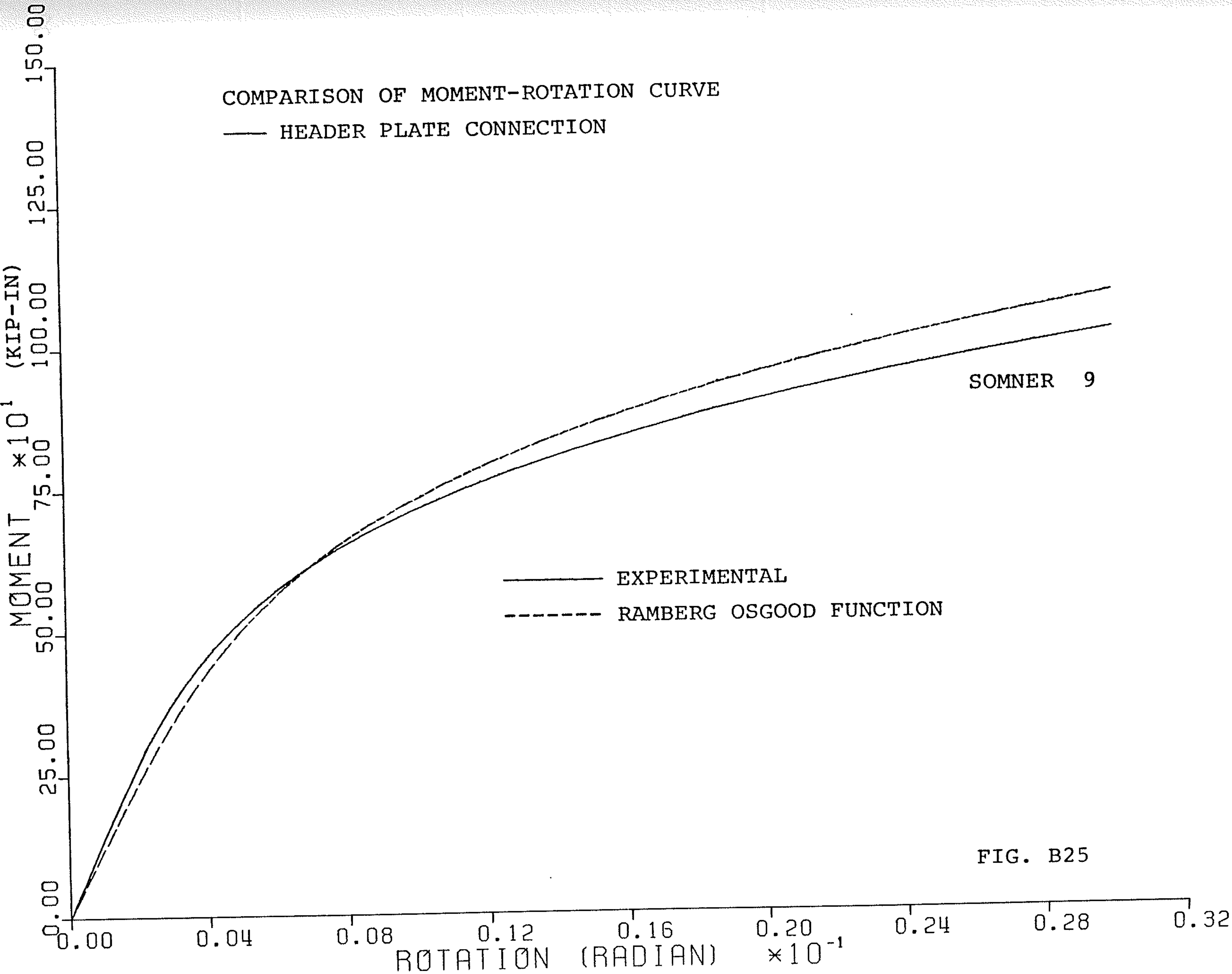

COMPARISON OF MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
SOMNER 9
———— EXPERIMENTAL
------- RAMBERG OSGOOD FUNCTION
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) ×10⁻¹

FIG. B25

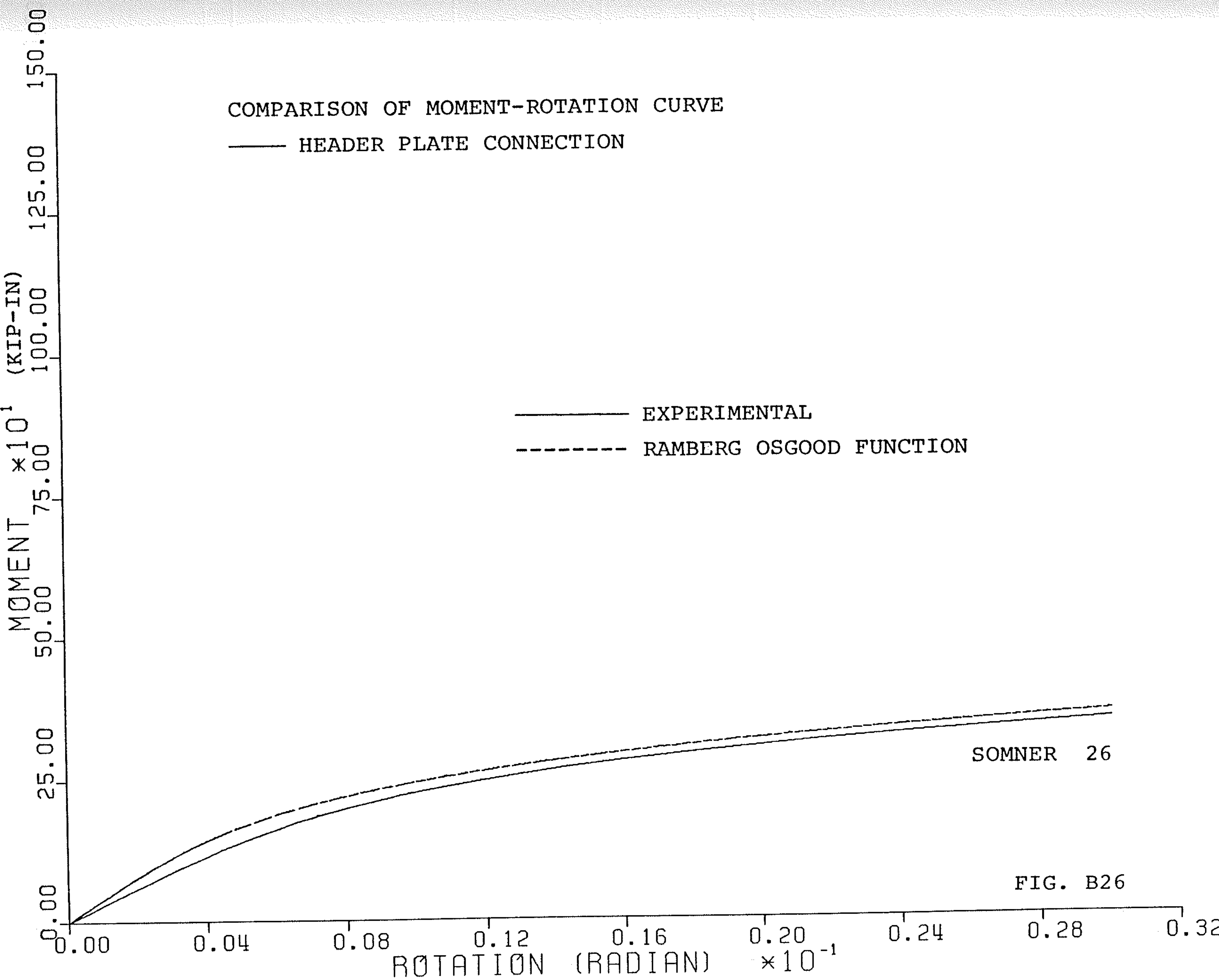

COMPARISON OF MOMENT-ROTATION CURVE
—— HEADER PLATE CONNECTION
———— EXPERIMENTAL
-------- RAMBERG OSGOOD FUNCTION
SOMNER 26
MOMENT ×10¹ (KIP-IN)
0.00
25.00
50.00
75.00
100.00
125.00
150.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) ×10⁻¹

FIG. B26

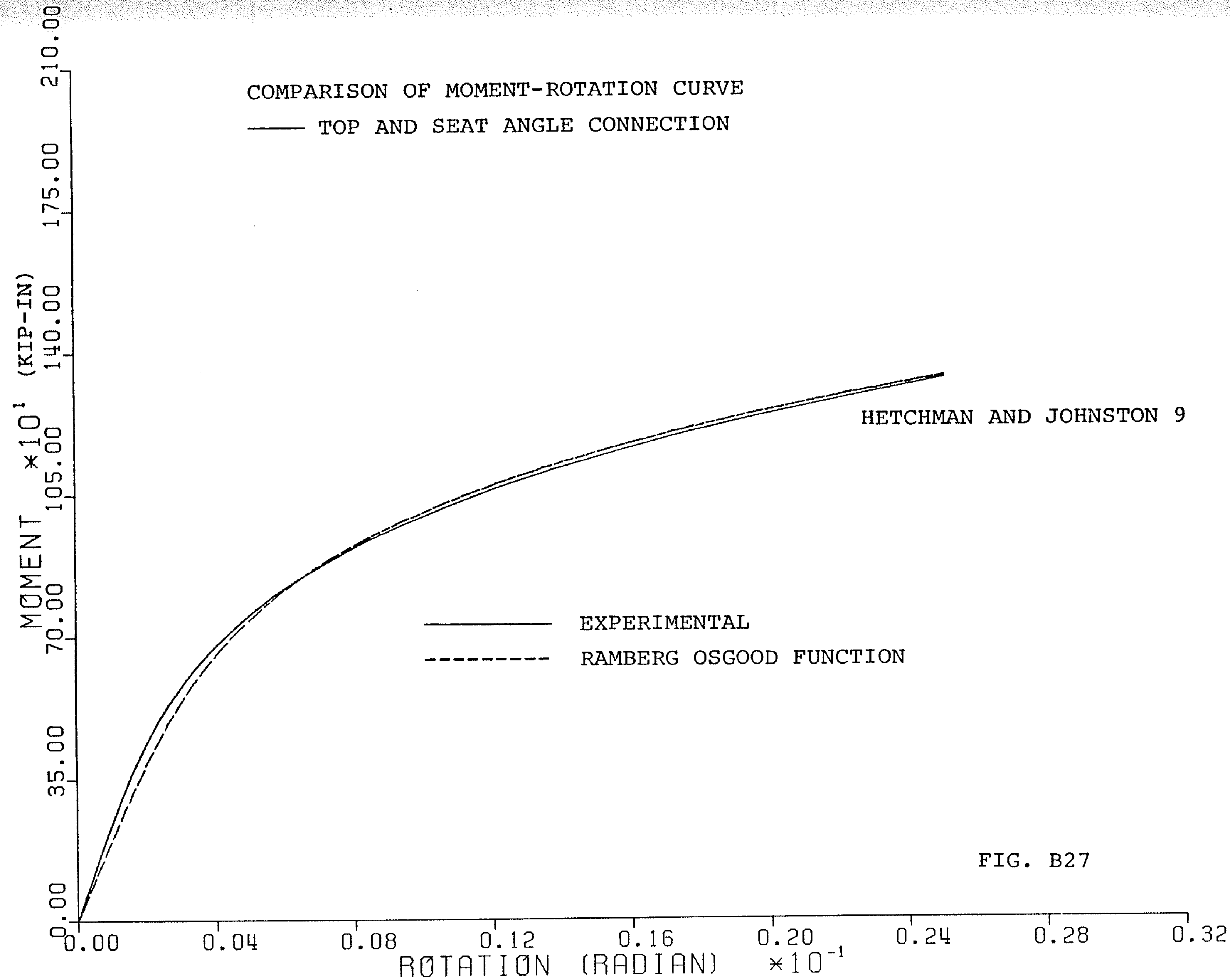

COMPARISON OF MOMENT-ROTATION CURVE
—— TOP AND SEAT ANGLE CONNECTION
HETCHMAN AND JOHNSTON 9
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MOMENT *10^1 (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) *10^-1
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B27

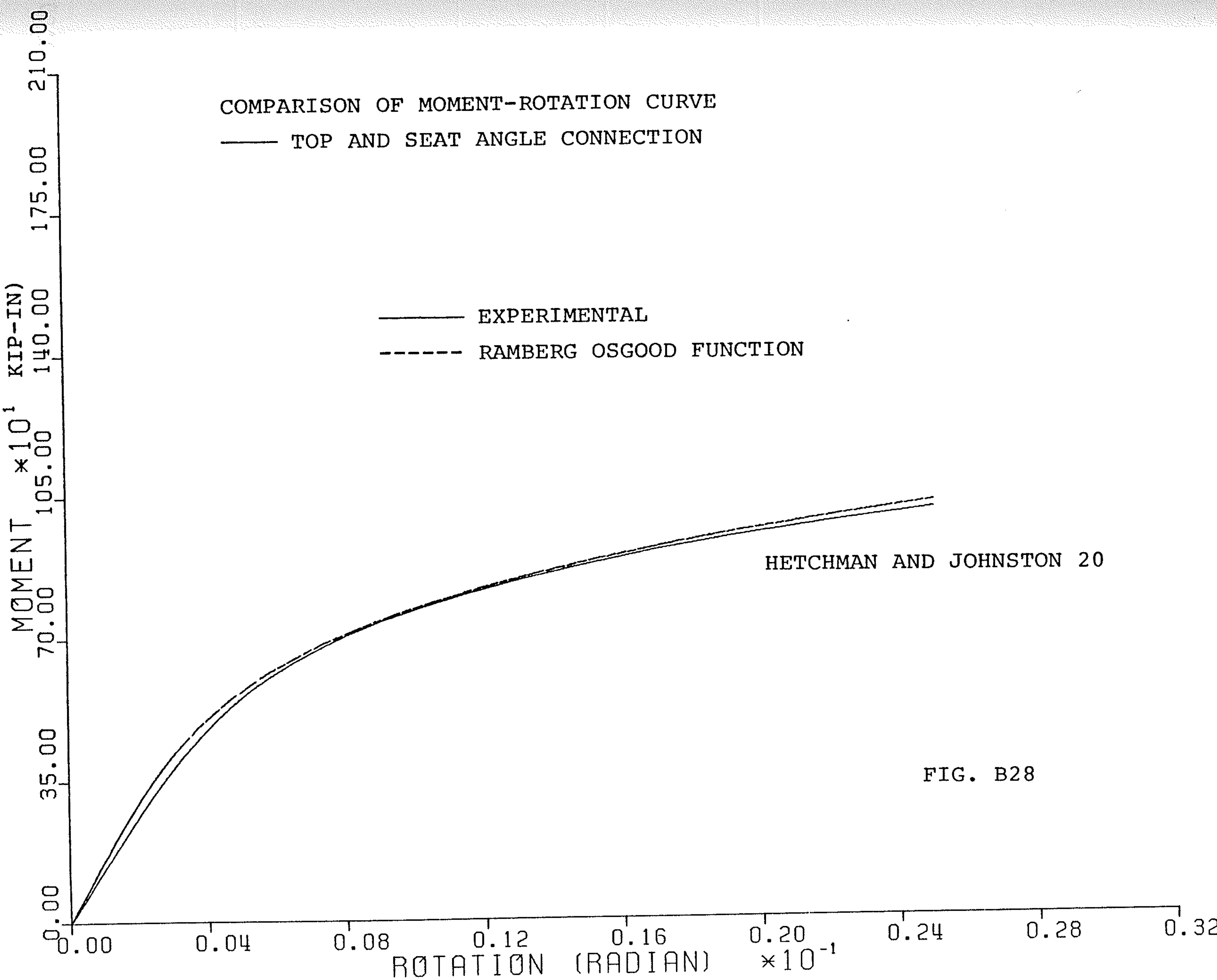

COMPARISON OF MOMENT-ROTATION CURVE
—— TOP AND SEAT ANGLE CONNECTION
EXPERIMENTAL
------ RAMBERG OSGOOD FUNCTION
HETCHMAN AND JOHNSTON 20
MOMENT *10^1 KIP-IN)
ROTATION (RADIAN) ×10^-1
0.00
35.00
70.00
105.00
140.00
175.00
210.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B28

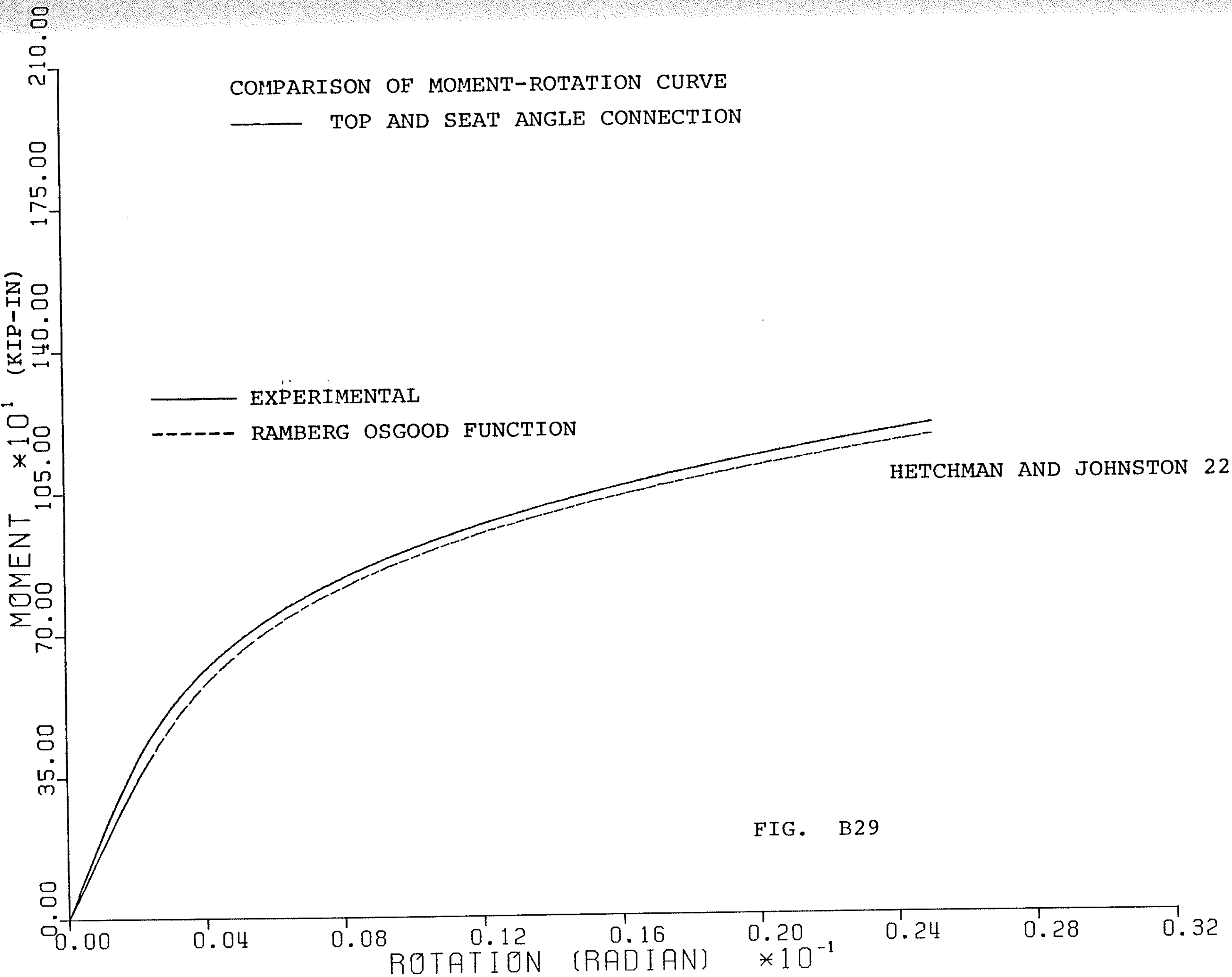

COMPARISON OF MOMENT-ROTATION CURVE
——— TOP AND SEAT ANGLE CONNECTION
——— EXPERIMENTAL
------ RAMBERG OSGOOD FUNCTION
HETCHMAN AND JOHNSTON 22
MOMENT *10¹ (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) *10⁻¹

FIG. B29

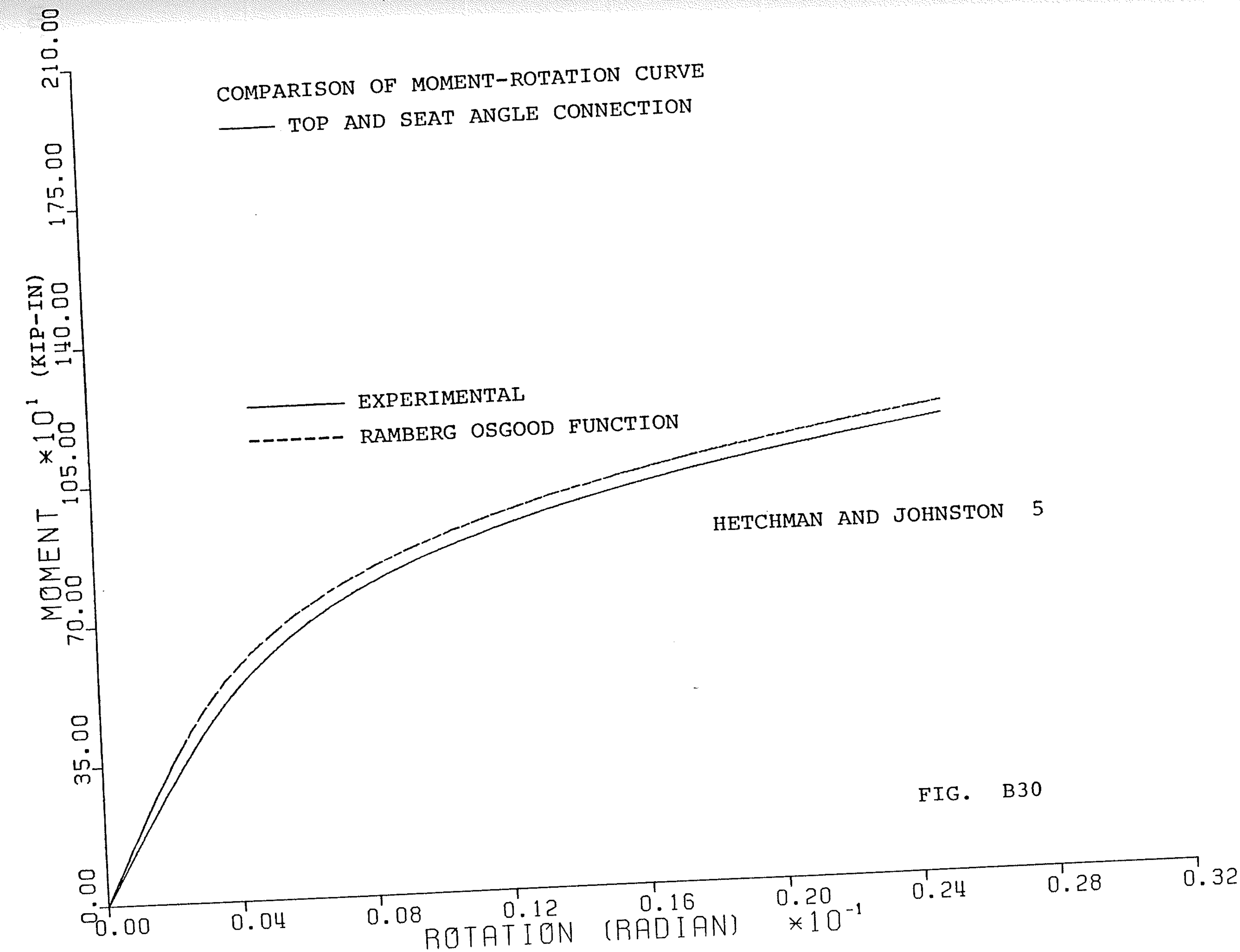

COMPARISON OF MOMENT-ROTATION CURVE
—— TOP AND SEAT ANGLE CONNECTION
—— EXPERIMENTAL
------- RAMBERG OSGOOD FUNCTION
HETCHMAN AND JOHNSTON 5
MOMENT *10^1 (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) *10^-1
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B30

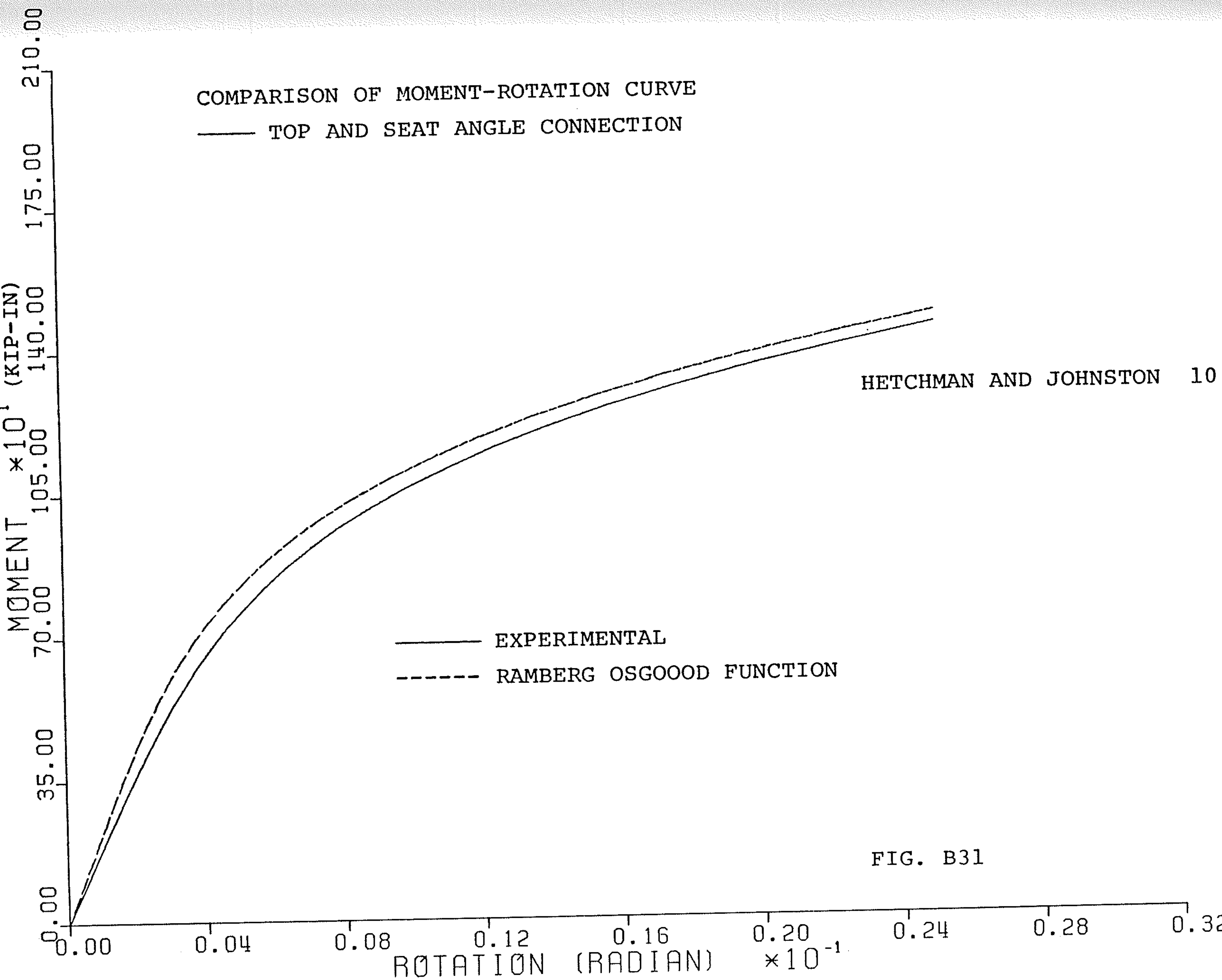

COMPARISON OF MOMENT-ROTATION CURVE
—— TOP AND SEAT ANGLE CONNECTION
HETCHMAN AND JOHNSTON 10
—— EXPERIMENTAL
------ RAMBERG OSGOOOD FUNCTION
MOMENT *10^1 (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) *10^-1
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B31

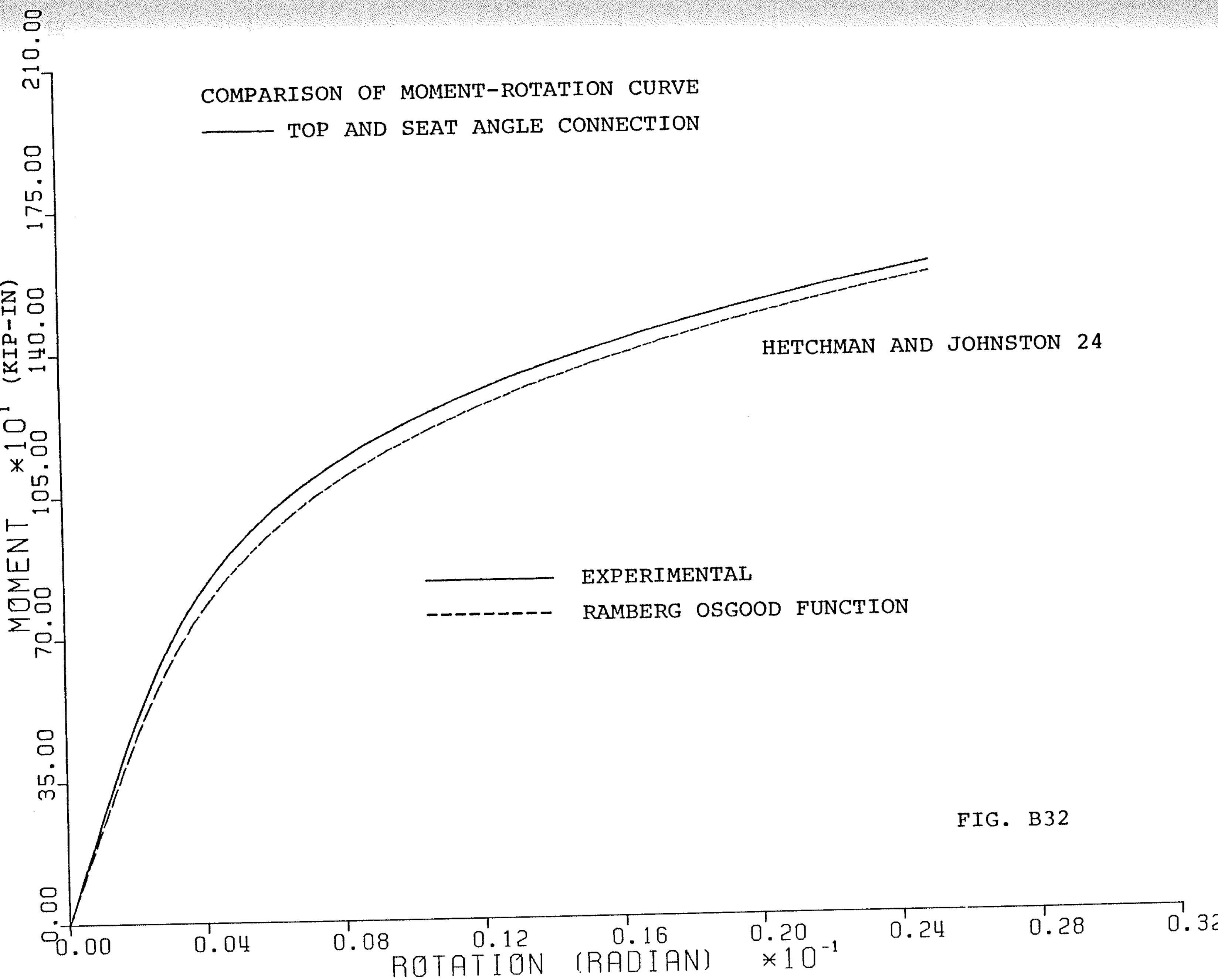
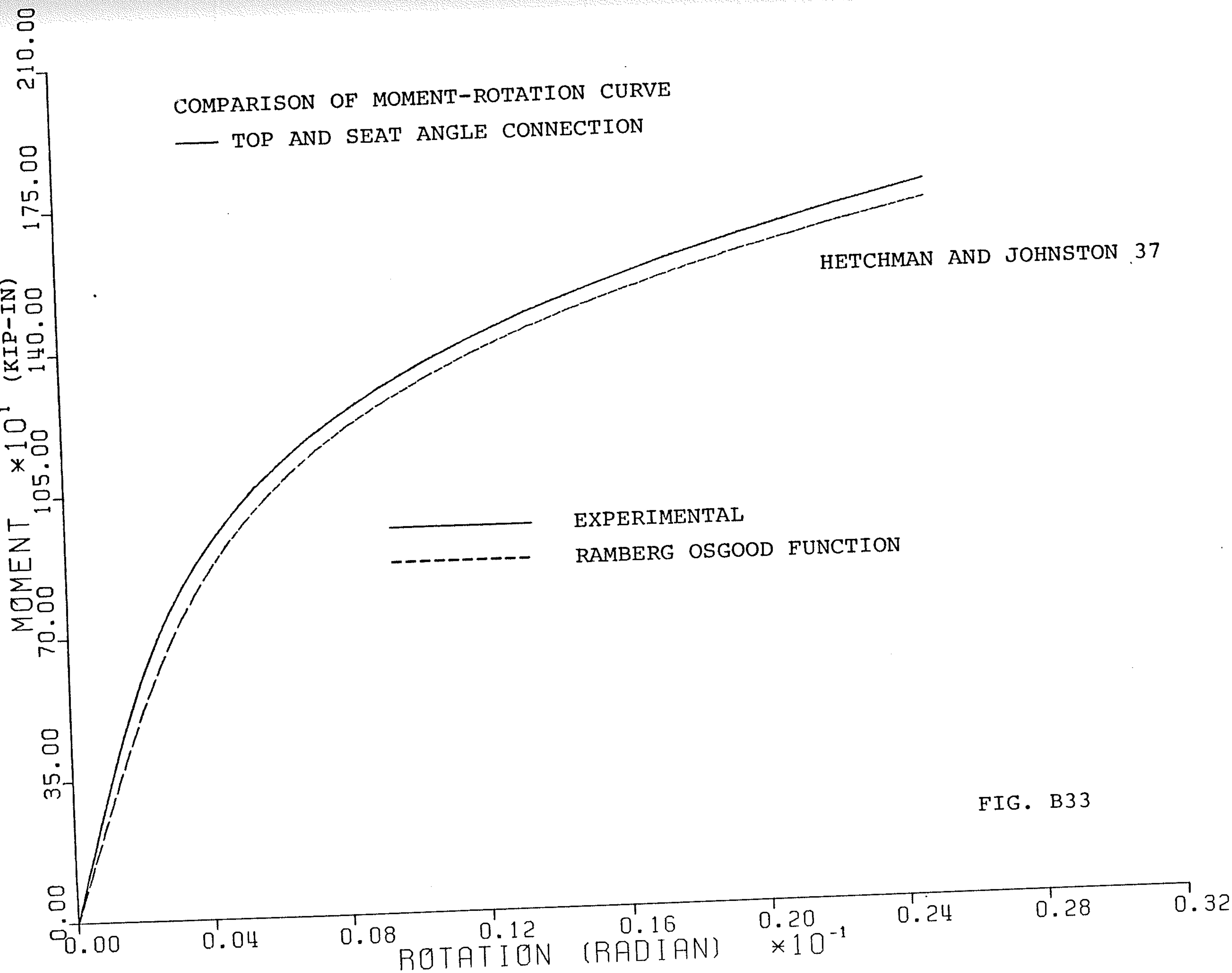

COMPARISON OF MOMENT-ROTATION CURVE
——— TOP AND SEAT ANGLE CONNECTION
HETCHMAN AND JOHNSTON 24
——— EXPERIMENTAL
--------- RAMBERG OSGOOD FUNCTION
MOMENT *10^1 (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) *10^-1
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B32



COMPARISON OF MOMENT-ROTATION CURVE
—— TOP AND SEAT ANGLE CONNECTION
HETCHMAN AND JOHNSTON 37
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MOMENT *10^1 (KIP-IN)
0.00
35.00
70.00
105.00
140.00
175.00
210.00
ROTATION (RADIAN) *10^-1
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B33

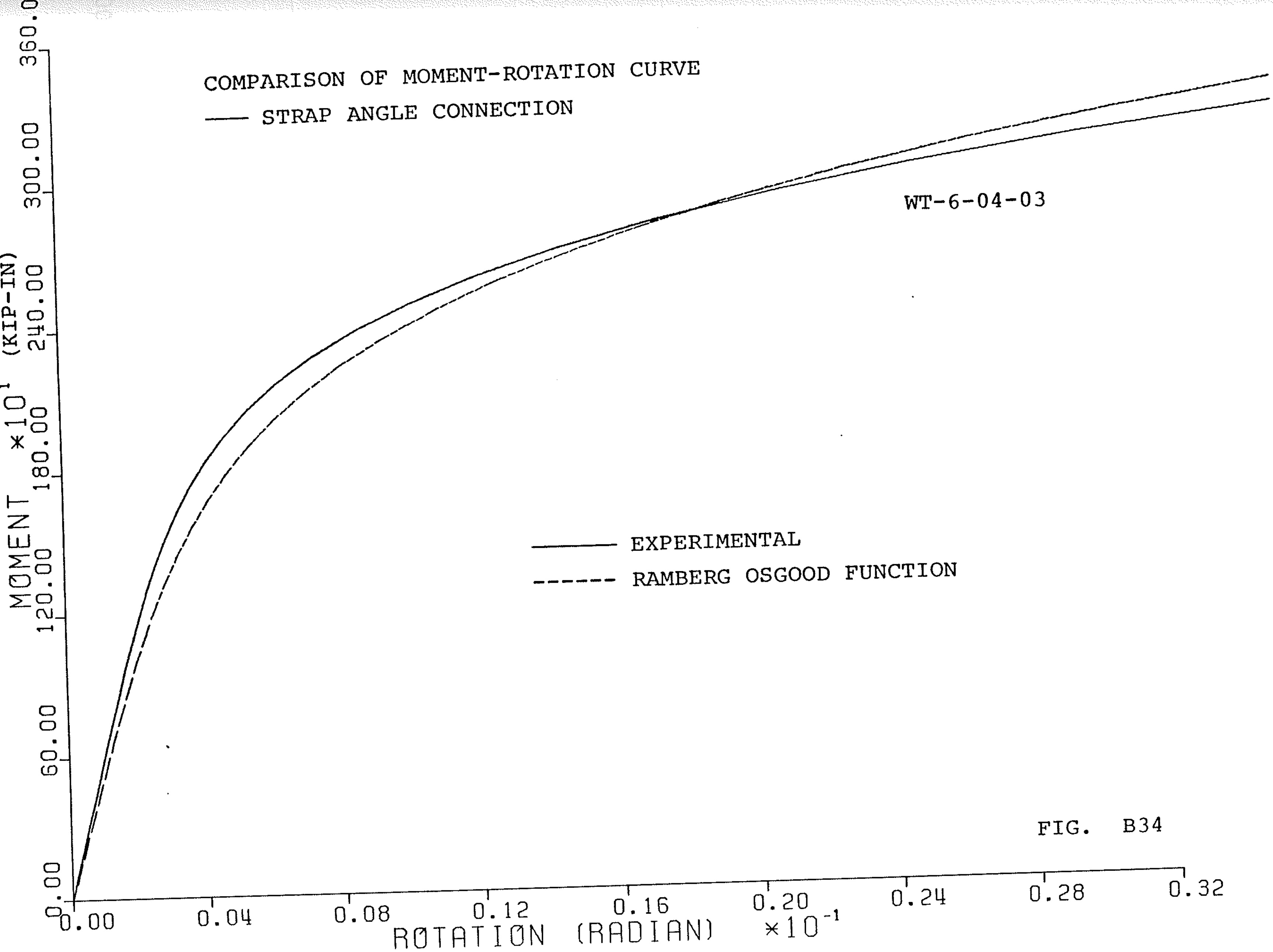

COMPARISON OF MOMENT-ROTATION CURVE
— STRAP ANGLE CONNECTION
WT-6-04-03
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MOMENT *10[1] (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
ROTATION (RADIAN) *10[-1]
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32

FIG. B34

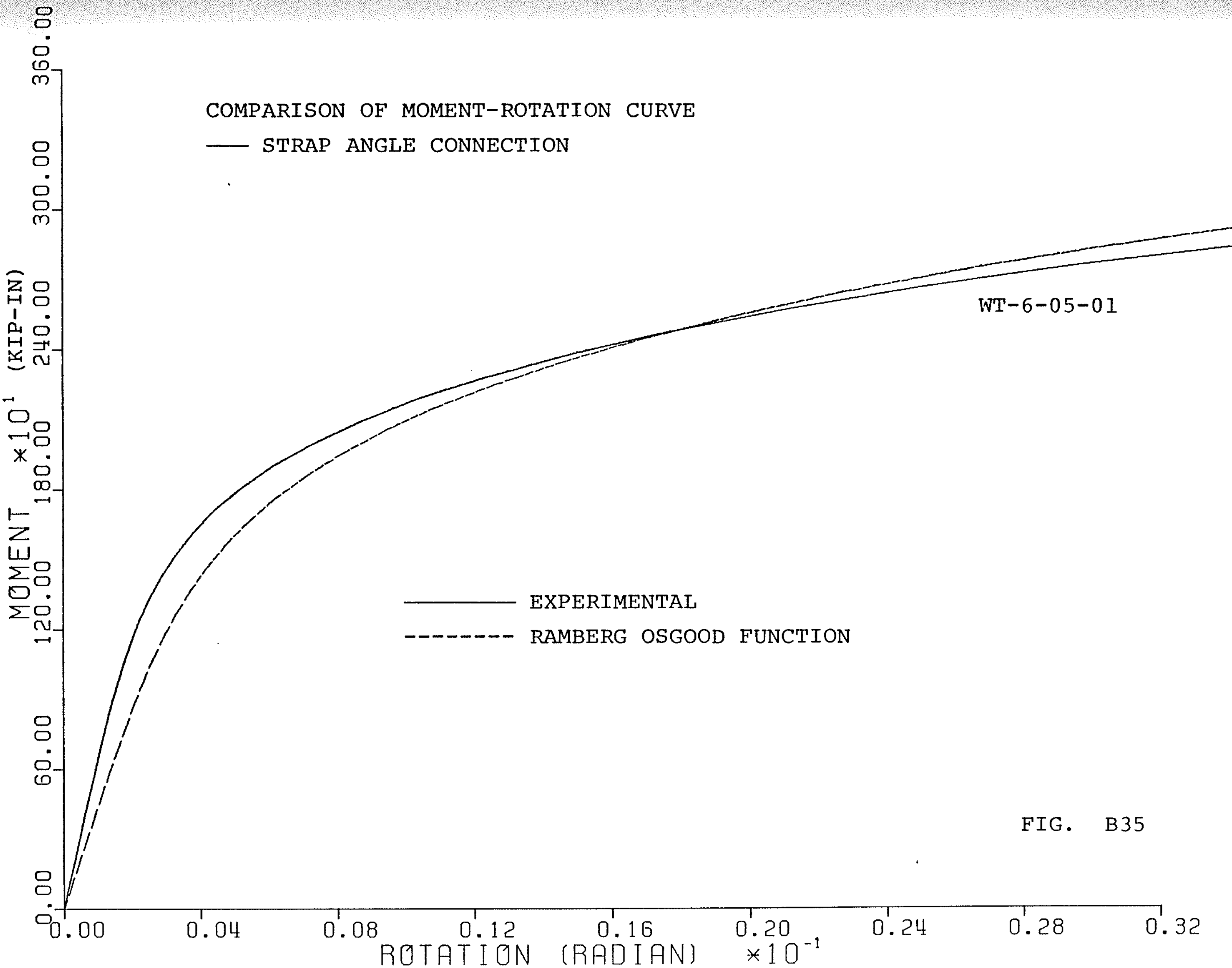

COMPARISON OF MOMENT-ROTATION CURVE
—— STRAP ANGLE CONNECTION
WT-6-05-01
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
MØMENT ×10¹ (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
RØTATIØN (RADIAN) ×10⁻¹

FIG. B35

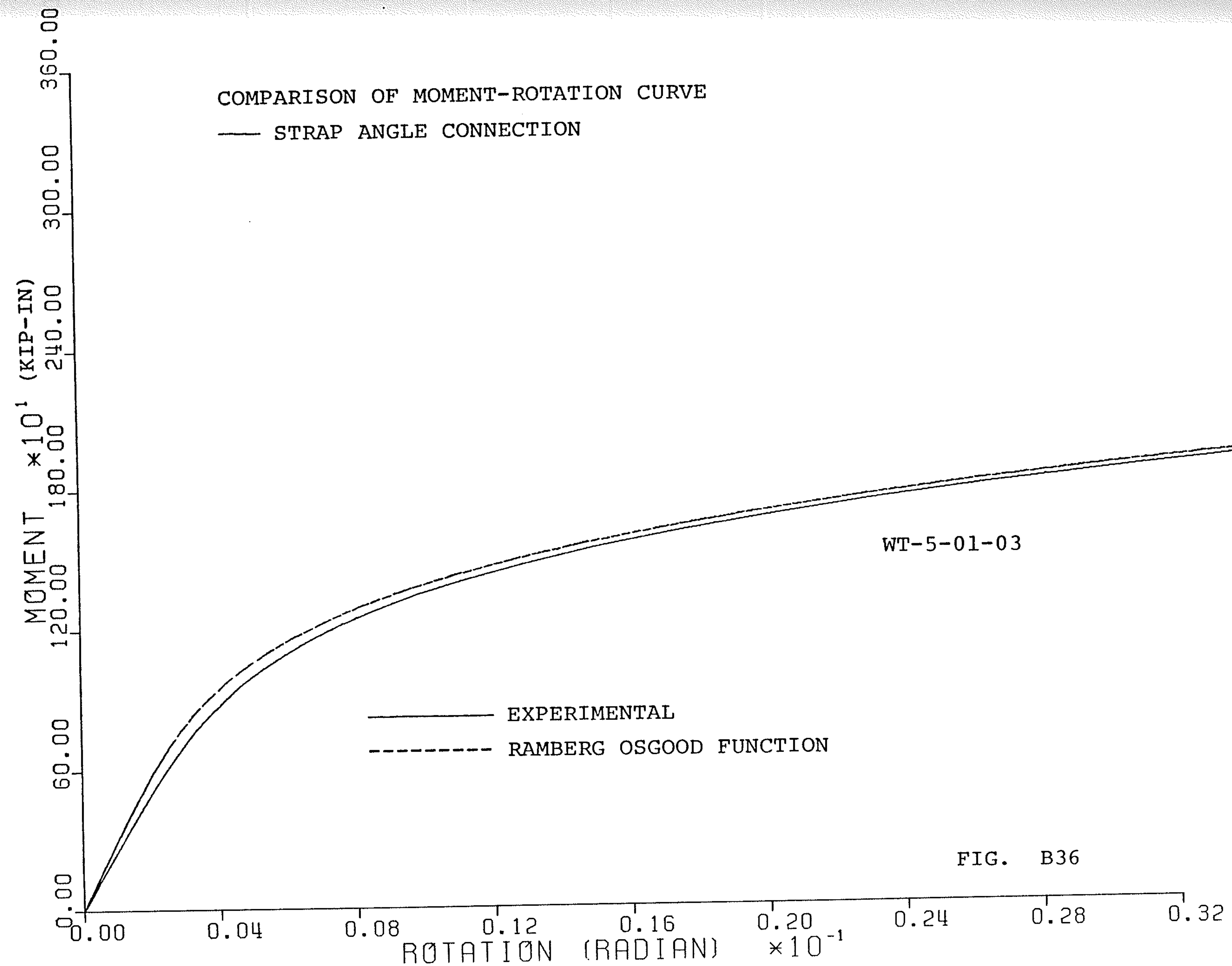

COMPARISON OF MOMENT-ROTATION CURVE
—— STRAP ANGLE CONNECTION
WT-5-01-03
———— EXPERIMENTAL
-------- RAMBERG OSGOOD FUNCTION
MOMENT *10^1 (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) ×10^-1

FIG. B36

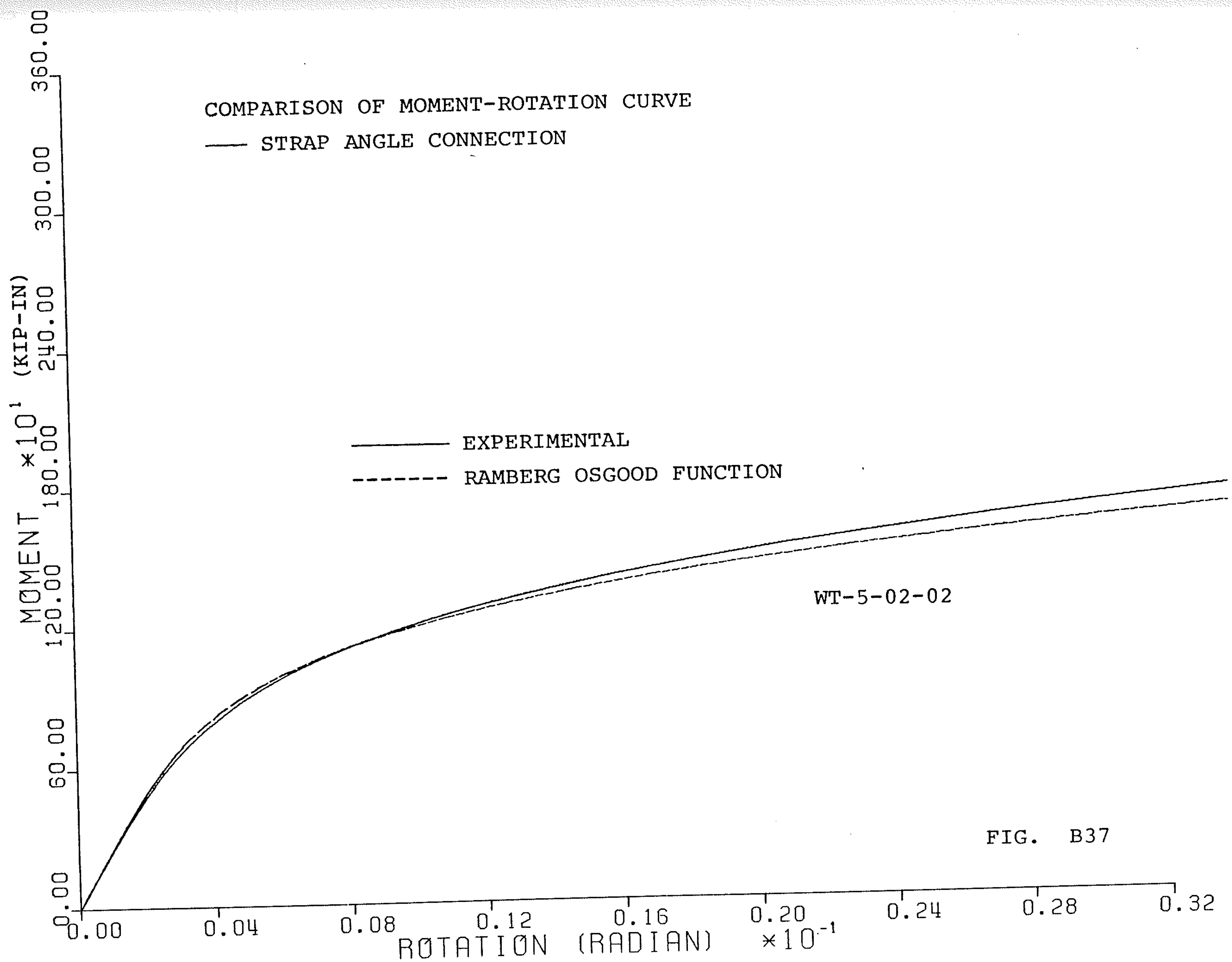

COMPARISON OF MOMENT-ROTATION CURVE
—— STRAP ANGLE CONNECTION
———— EXPERIMENTAL
------- RAMBERG OSGOOD FUNCTION
WT-5-02-02
MOMENT *10^1 (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) *10^-1

FIG. B37

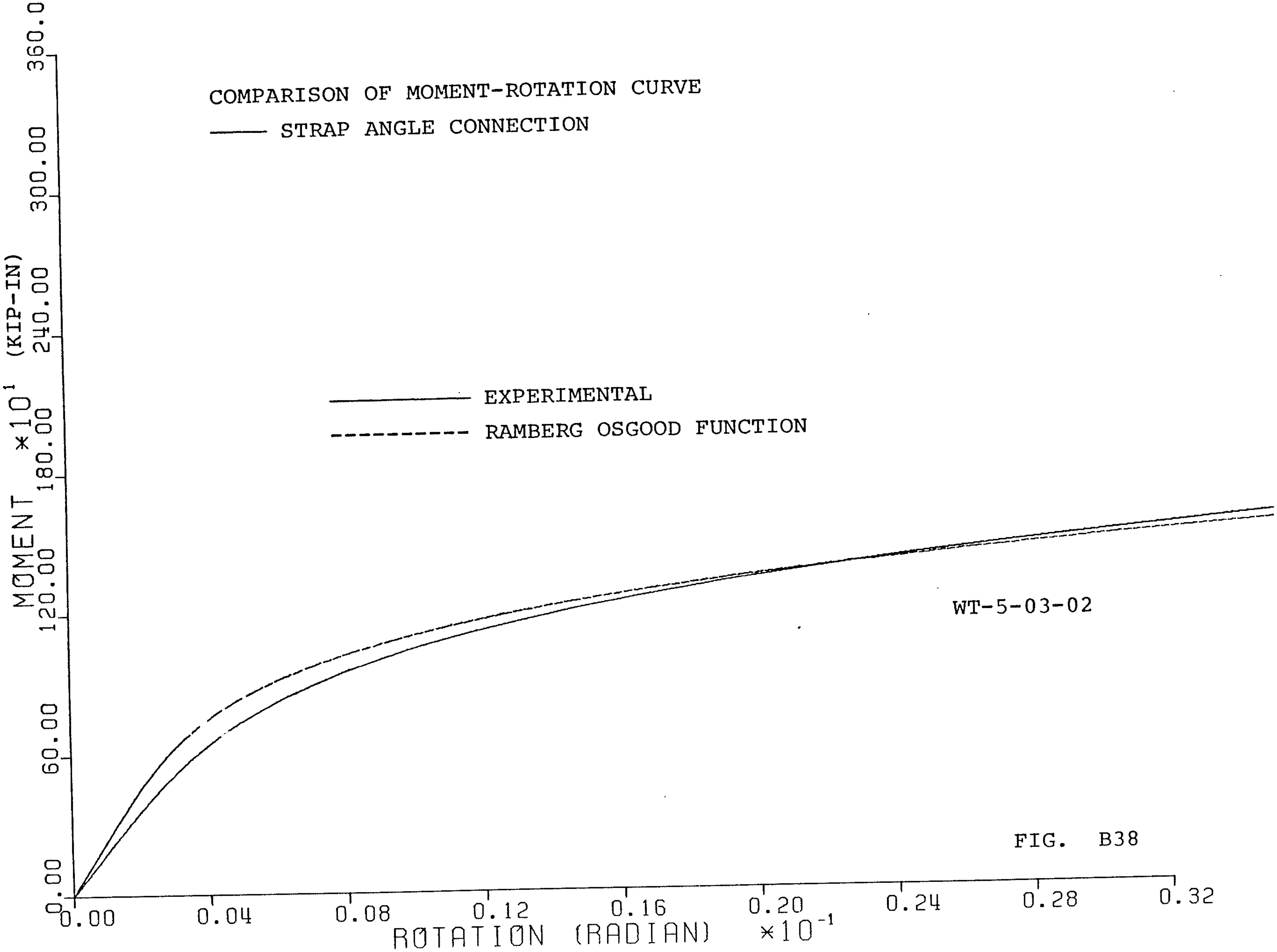

COMPARISON OF MOMENT-ROTATION CURVE
—— STRAP ANGLE CONNECTION
EXPERIMENTAL
RAMBERG OSGOOD FUNCTION
WT-5-03-02
MOMENT *10¹ (KIP-IN)
0.00
60.00
120.00
180.00
240.00
300.00
360.00
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
ROTATION (RADIAN) *10⁻¹

FIG. B38

APPENDIX C

DERIVATION OF STIFFNESS MATRIX FOR MEMBERS WITH ELASTIC END CONNECTIONS

Consider the member AB, as shown in Fig. C.1, with elastic connections at end A and B. Assume end A to be fixed and apply force P_B at B. The resulting displacement at B is:

$$D_B = L_{BB} P_B + H_{AE}^T L_A^C H_{AB} P_B + L_B^C P_B \qquad (a)$$

L_{BB} is the flexibility matrix at B for elastic member AB. L_A^C and L_B^C are the flexibility matrices for elastic connections at A and B.

For example:

$$L_A^C = \begin{bmatrix} 1/S_{1A} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/S_{2A} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/S_{3A} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/S_{4A} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/S_{5A} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/S_{6A} \end{bmatrix}$$

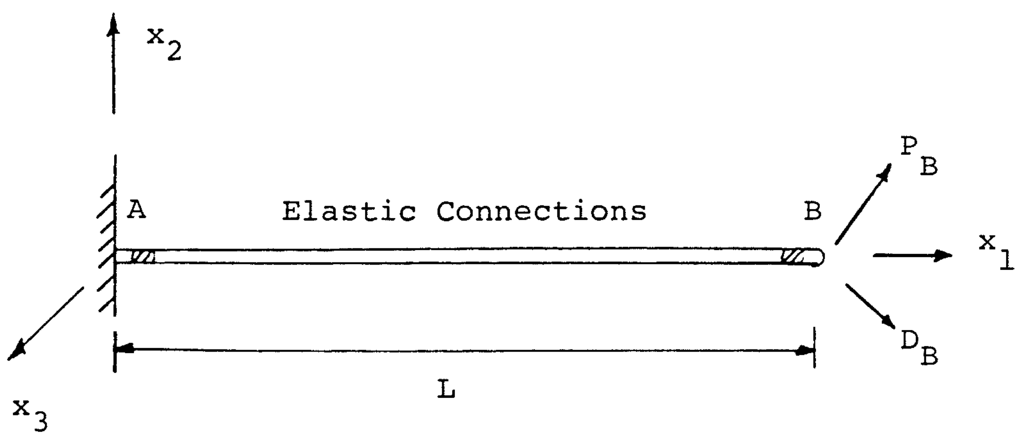


FIG. C.1 MEMBER WITH ELASTIC END CONNECTIONS

where

S_{1A}, S_{2A}, S_{4A}, S_{5A} = stiffness of connection at A in direction of force components 1, 2, 4 and 5. That is, axial, shearing, torsional and bending stiffness.

H_{AB} is the force vector translation matrix.

For most member geometries and connection types, the axial, shearing and torsional deformations are negligibly small, while the bending deformations of the connections are often significant. Consequently $1/S_1 = 1/S_2 = 1/S_4 = 0$, and the flexibility matrix at B, from Equation (a), for the member with connections is:

$$L_{BB}^{*} = L_{BB} + H_{AB}^{T} L_{A}^{C} H_{AB} + L_{B}^{C} \quad . \tag{b}$$

Substituting the appropriate material and geometric properties for the member, the flexibility matrix L_{BB}^{*} is:

$$L^*_{BB} = \begin{bmatrix} L_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{22} & 0 & 0 & 0 & L_{26} \\ 0 & 0 & L_{33} & 0 & L_{35} & 0 \\ 0 & 0 & 0 & L_{44} & 0 & 0 \\ 0 & 0 & L_{53} & 0 & L_{55} & 0 \\ 0 & L_{62} & 0 & 0 & 0 & L_{66} \end{bmatrix}$$

where

$$L_{11} = \frac{L}{AE}$$

$$L_{22} = \left(\frac{L^3}{3EI_3} + \frac{L^2}{S6_A}\right)$$

$$L_{26} = \left(\frac{L^2}{2EI_3} + \frac{L}{S6_A}\right) = L_{62}$$

$$L_{33} = \left(\frac{L^3}{3EI_2} + \frac{L^2}{S_{5A}}\right)$$

$$L_{35} = \left(\frac{-L^2}{2EI_2} - \frac{L}{S_{5A}}\right) = L_{53}$$

$$L_{44} = \frac{L}{GI_1}$$

$$L_{45} = \left(\frac{L}{EI_2} + \frac{1}{S_{5A}} + \frac{1}{S_{5B}}\right)$$

$$L_{66} = \left(\frac{L}{EI_3} + \frac{1}{S_{6A}} + \frac{1}{S_{6B}}\right)$$

I_1, I_2, I_3, E and G are as defined in Chapter IV.

The stiffness matrix for the member plus connections can thus be computed as

$$K^*_{BB} = L^{*\ -1}_{BB} \quad .$$

Finally, the stiffness matrix relating forces and displacements at both ends of the bar is:

$$K^* \quad \begin{bmatrix} K^*_{BB} & -K^*_{BB}\ H^T_{AB} \\ -H_{AB}\ K^*_{BB} & H_{AB}\ K^*_{BB}\ H^T_{AB} \end{bmatrix}$$

APPENDIX D

DERIVATION FOR FIXED END FORCE VECTORS FOR MEMBERS WITH ELASTIC END CONNECTIONS

D.1 Uniformly Distributed Load

Consider member AB, as shown in Fig. D.1, with end elastic connections and loaded with uniformly distributed load. Assume end A to be fixed. The deflection at B due to the applied load W, considering only the connection bending stiffness in X_3 direction, is:

$$D_B = \int_o^L H_{CB}^T L_C \left(\int_o^Z H_{CD} W ds\right) dZ$$

$$+ H_{AB}^T L_A^C \int_o^L H_{AD} W dx_1 \qquad (D.1)$$

where

$$H_{CD} = \begin{bmatrix} 1 & 0 \\ S & 1 \end{bmatrix}$$

$$W = \begin{pmatrix} W \\ 0 \end{pmatrix}$$

$$H_{CB}^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

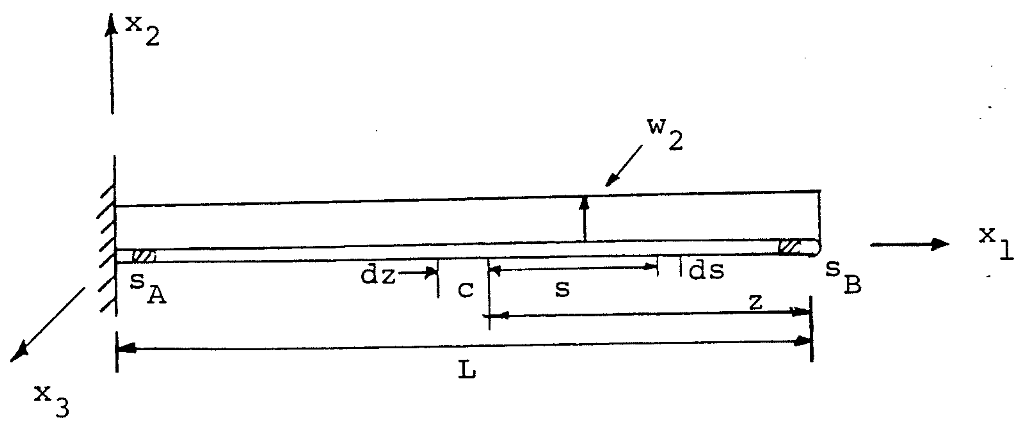


FIG. D.1 UNIFORMLY DISTRIBUTED LOAD OVER MEMBER LENGTH

$$L_C = \begin{bmatrix} \frac{1}{A_2 G} & 0 \\ 0 & \frac{1}{EI_3} \end{bmatrix}$$ = flexibility matrix, for a unit length of the member at C

$$L_A^C = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{S_A} \end{bmatrix}$$ S_A is the connection bending stiffness in X_3 direction.

Integrating D-1, the deflection at B is:

$$D_B = \begin{Bmatrix} \frac{WL^4}{8EI_3} \\ \frac{WL^3}{6EI_3} + \frac{WL^2}{2S_A} \end{Bmatrix} . \tag{D.2}$$

Having calculated the cantilever deflection at B, the fixed-end-force vector at B, P_B^F, is the force required at B to reduce the deflection there to zero.

Thus,

$$D_B^C + L_{BB}^* P_B^F = 0 \tag{D.3}$$

L_{BB}^* is as defined in Appendix C.

The force vector P_B^F at B is

$$P_B^F = 1\ L_{BB}^{*-1}\ D_B = -K_{BB}^{*}\ D_B \quad . \tag{D.4}$$

The corresponding fixed-end-force vector at A can then be calculated from statics:

$$P_A^F + \int_o^L H_{AD}\ \omega\ dx_1 + H_{AB}\ P_B^F = 0 \tag{D.5}$$

or,

$$P_A^F = - \int_o^L H_{AD}\ \omega\ dx_1 + H_{AB}\ K_{BB}^{*}\ D_B \tag{D.6}$$

D.2 A Single Concentrated Load

Consider member AB, as shown in Fig. D.2, loaded with a concentrated load vector P_D. The end deflection B due to P_D is

$$D_B = H_{DB}^T\ P_D + H_{AB}^T\ L_A^C\ H_{AD}\ P_D \tag{D.7}$$

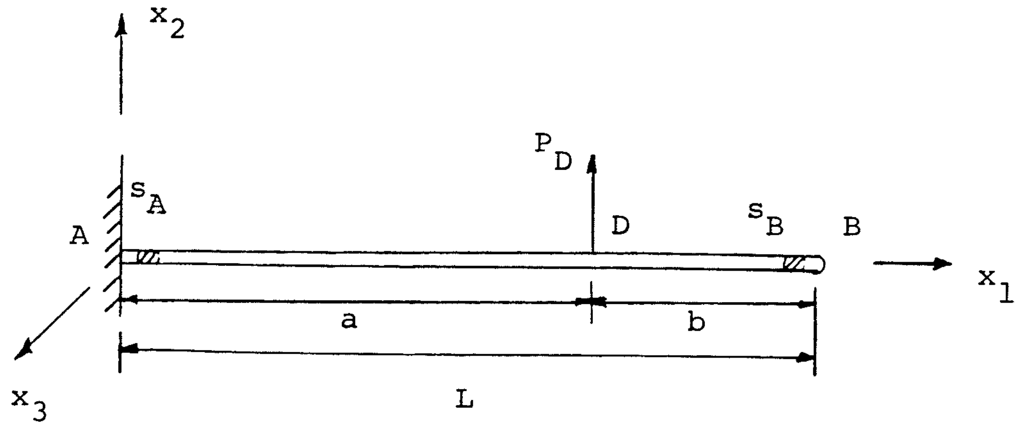


FIG. D.2 A SINGLE CONCENTRATED LOAD ON MEMBER AB where,

$$P_D = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

$$L_{DD} = \begin{bmatrix} \frac{a^3}{3EI_3} & \frac{a^2}{2EI_3} \\ \frac{a^2}{2EI_3} & \frac{a}{EI_3} \end{bmatrix}$$

$$H_{DB}^T = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$

$$H_{AD} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$$

L_A^C and H_{AB}^T are as defined in D.1. Hence,

$$D_B = \begin{Bmatrix} \frac{Pa^3}{3EI_3} + \frac{Pa^2b}{2EI_3} + \frac{PLa}{S_A} \\ \frac{Pa^2}{2EI_3} + \frac{Pa}{S_A} \end{Bmatrix}$$

Using Equation (D.4), the fixed and force P_B^F at B due to the load P_D is:

$$P_B^F = K_{BB}^* D_B \quad .$$

Similarly, the fixed end force at A is:

$$P_A^F = -H_{AD} P_D + H_{AB} K_{BB}^* P_B^F \quad .$$

APPENDIX E: P-Δ EFFECT CALCULATION

Guide to Calculation of Stability Effects*

J1. General

J1.1 This Appendix gives one approach to the calculation of the additional bending moments and forces generated by the vertical loads acting through the deflected shape of the structure. By this approach the above moments and forces are incorporated into the results of the analysis of the structure; alternatively a second order analysis, which formulates equilibrium on the deformed structure, may be used to include the stability effects.

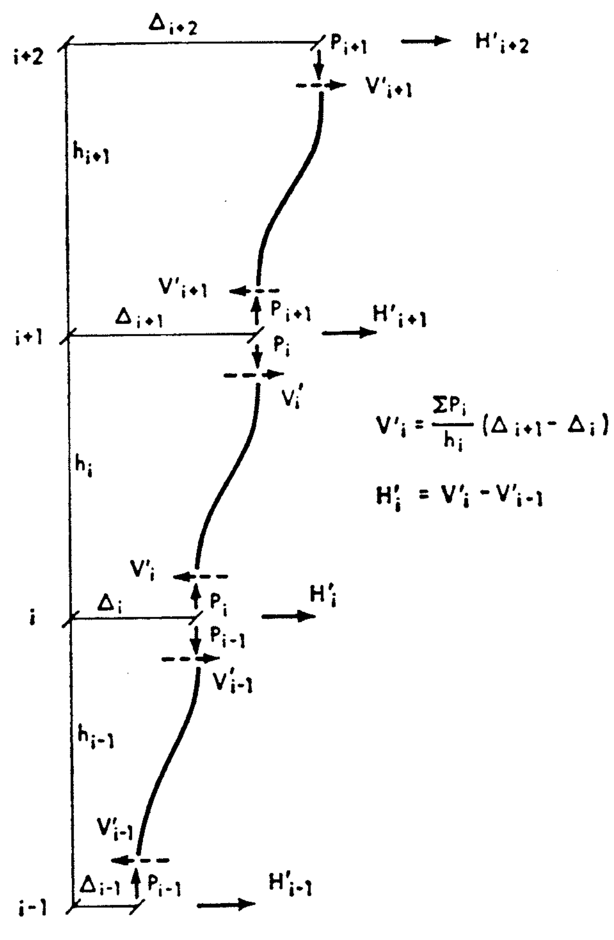


Figure J1
Sway Forces Due to Vertical Loads

*Appendix J of CAN3-S16.1-M78

J2. Combined Loading Case

J2.1 Step 1 — Apply the factored load combination to the structure (see Clause 7.2.2).

Step 2 — Compute the lateral deflections at each floor level (Δ_i) by first order elastic analysis.

Step 3 — Compute the artificial storey shears V'_i due to the sway forces.

where

$$V'_i = \frac{\Sigma P_i}{h_i}\left(\Delta_{i+1} - \Delta_i\right)$$

= artificial shear in storey i due to the sway forces

ΣP_i = sum of the column axial loads in storey i

h_i = height of storey i

Δ_{i+1}, Δ_i = displacements of level i + 1 and i, respectively

Step 4 — Compute the artificial lateral loads H'

$H'_i = V'_{i+1} - V'_i$

Step 5 — Repeat Step 1 applying the artificial lateral loads H'_i in addition to the factored load combination.

Step 6 — Repeat Steps 2 through 5 until satisfactory convergence is achieved. Lack of convergence within 5 cycles may indicate an excessively flexible structure.

J3. Vertical Loads Only

J3.1 Since vertical loads do not normally produce significant sway deflections of the structure the initial sway forces are computed on the basis of the sway displacements in each storey equal to the erection tolerance permitted by Clause 28.7.1. Using these deflections the calculations are commenced at Step 3 of the procedure described in Clause J2.1.

APPENDIX F

COMPUTER PROGRAM: CONNECTION

This Appendix contains the input instruction to, and the listing of, the computer program, named CONNECTION, described in Chapter V. A sample example for the input and a list of the output items are also included.

F.1 Input Data

The variables in each card type represent the data items to be entered and more than one card may be needed to accommodate all the data items in each type of card. Input is in free format except as indicated.

1. Job Title Card 18A(4)

   Title to be printed with the output.

2. Control Information Cards

   Card Type 1

Variables	Descriptions
NDS	Number of tested connections to be used in generating the standardized function for a given type of connections.
NCOM	Number of geometric parameters that will affect the moment-rotation curves.
NPC	Number of rotation values to be used to compute the exponent a_j in Equation (3.15).

2. <u>Control Information Cards</u> (Continued)

<u>Card Type 2</u>

X(I)	The rotation values referred to in NPC. The first value must be 0. NPC number of values must be input.

<u>Card Type 3</u>

GP(I,J)	Values of the geometric parameters for each connection tested. One card is needed for each connection tested.

3. <u>Connection Information Cards</u>

The following types of cards are prepared for each connection tested.

<u>Card Type 1</u>

Variables	Descriptions
LABEL (2A5)	Test specimen identification.

<u>Card Type 2</u>

NP	Number of test data in each connection tested.
F1	Multiplication factor for the rotation data. Enter 1.0 if it is not required.
F2	Moment multiplication factor. Enter 1.0 if it is not needed.

<u>Card Type 3</u>

Prepare NP cards for this type of card.

PGG	Experimental moment values
D1	First rotation values
D2	Second rotation values. Enter 0 if not applicable

```
TEST 3
4          1.0        1.0
0.0        0.0        0.0
10.0       0.004      0.0
15.0       0.006      0.0
20.0       0.008      0.0
TEST 4
3          1.0        1.0
0.0        0.0        0.0
45.0       0.008      0.0
60.0       0.012      0.0
TEST 5
5          1.0        1.0
20.0       0.006      0.0
40.0       0.012      0.0
60.0       0.015      0.0
80.0       0.018      0.0
100.0      0.020      0.0
3
12
13
23
2
34
2
25
```

F.3 Output

The output consists of

(1) Title for the job

(2) Control information

(3) Connection information

(4) The parameters ϕ_o, $[KM]_o$ and η for the standardized function for the type of connection considered.

(5) Table showing the deviation of the standardized function for the experimental curve.

F. 4 Listing of the Program: CONNECTION

```
C*************************************************
C                                                *
C   CONNECTION: THIS PROGRAM IS USED TO          *
C               GENERATE THE STANDARDIZED        *
C               MOMENT-ROTATION FUNCTIONS        *
C               OF ANY CONNECTION TYPES FOR      *
C               WHICH THE EXPERIMENTAL MOMENT-   *
C               ROTATION DATA ARE AVAILABLE.     *
C                                                *
C*************************************************
C                                                *
C   10 SETS OF TESTED CONNECTION MOMENT-         *
C   ROTATION DATA CAN BE ACCEPTED  .IF           *
C   MORE THAN TEN SETS ARE AVAILABLE, CHANGE     *
C   THE DIMEMSION SIZE OF ARRAY E0, Q0, AN,      *
C   GP, AVG, FK, NDP FROM 10 TO THE APPROPRIATE  *
C   VALUES.                                      *
C                                                *
C*************************************************
C
C
C      E0 , Q0, AN -- CONSTANTS FROM THE RAMBERG
C                     OSGOOD CURVE FITTING PROGRAM
C         X     Q        Q   AN
C        ___ = ___  +  (___)
C         E0    Q0       Q0
C              GP -- CONNECTION SIZE PARAMETERS
C              FK -- STANDARDIZATION FACTOR EXPRESSION
C             AVG -- DIMENSIONLESS EXPONENT
C               X -- ROTATION VALUES OF CONNECTION
C               Y -- MOMENT VALUES OF CONNECTIONS
C               Z -- STANDARDIZED MOMENT VALUES
C
C
      DIMENSION E0(10),Q0(10),AN(10),NDP(10),X(20),Y(20,20),TITLE(18),
     & GP(5,10), AVG(10), A(200), B(200), FK(10), C(600),YY(20),Z(20)
C
C
      READ(5,5) TITLE
5     FORMAT( 18A4)
      READ(5,*) NDS, NCOM, NPC
      READ(5,*) ( X(I), I=1, NPC)
C
C
      WRITE(6,7) TITLE, NDS, NCOM, NPC, ( X(I), I=1,NPC)
7     FORMAT('1'//1H ,18A4//' NUMBER OF SETS OF DATA',I3//
     & ' NUMBER OF SIZE PARAMETERS',I3//
     & ' NUMBER OF ROTATION VALUES CONSIDERED IN COMPUTING THE DIMENSIO
     &NLESS EXPONENT',I3//
     & ' ROTATION VALUES (RADIAN)',20F10.5//)
       WRITE(6,6)
6     FORMAT(' CONNECTION SIZE PARAMETES'//
     & ' CONNECTION',2X,' SIZE PARAMETERS')
C
```

```
C
      DO 10 J=1, NDS
      READ(5,*)  (GP(I,J),I=1,NCOM)
      WRITE(6,8) J, (GP(I,J), I=1,NCOM)
 8    FORMAT(I9,4X, 5F10.5)
10    CONTINUE
C
C
      CALL READA(NDS,E0,Q0,AN,X1)
      CALL  GEN(X,Y,E0,Q0,AN,NDS,NPC)
      CALL  EXPNT(GP,Y,AVG,NDS,NCOM,NPC)
C
C
C     COMPUTE STANDARDIZATION FACTOR K FOR EACH CONNECTION
C
      DO 20  J=1, NDS
      FK(J)=1.
      DO 21 K=1, NCOM
21    FK(J)=GP(K,J)**AVG(K)*FK(J)
20    CONTINUE
C
C
C     START TO DETERMINE THE THREE CONSTANTS IN THE RAMBERG
C     OSGOOD STANDARDIZED FUNCTION
C
      A(1)=0.0
      B(1)=0.0
      DX=.05
      DO 30  J=2,NPC
      SUM=0.0
      DO  26  I=1, NDS
      SUM=SUM+FK(I)*Y(J,I)
26    CONTINUE
      A(J)=X(J)
      B(J)=SUM/NDS
30    CONTINUE
      NP=NPC
      NNP=NP-1
      M1=1
      M2=M1+3*NNP
      M3=M2+NNP
      CALL  CFIT(A,B,C(M1),C(M2),C(M3),P1,P2,P3,NNP,NP,DX)
      WRITE(6,9) P1,P2,P3
9     FORMAT('1',' THE 3 CONSTANTS OF THE RAMBERG OSGOOD FUNCTION FOR  T
     &HE STANDARDIZED FUNCTION E0 Q0 AN ARE:'/3F15.5)
      WRITE(6,11)
11    FORMAT('1',//'COMPARISION OF EXPERIMENTAL STANDARDIZED FUNCTION
     & MOMENT VALUES'/
     & ' NOTE EXPERIMENTAL VALUES MULTIPLIED BY STANDARDIZED FACTOR'/
     & ' EXPERIMENTAL',5X,'STANDARDIZED FUNCTION',5X,'PERCENT DIFFER.'/)
C
C
C     COMPUTE STANDARDIZED MOMENTS
C
```

```
      RM1=0.0
      DO  60  JK=2, NPC
      RM=RM1
      C1=X(JK)/P1
      C2=1./P2
      C3=1./(P2**P3)
55    FM=C1-C2*RM-C3*RM**P3
      FMP=-C2-P3*C3*RM**(P3-1.)
      RM1=RM-FM/FMP
      IF (ABS(RM1-RM)/RM1 .LT. .001) GOTO  65
      RM=RM1
      GOTO 55
65    Z(JK)=RM1
      RM1=0.0
60    CONTINUE
      DO 50 JJ=1, NDS
      DO 52 JK=2, NPC
      YY(JK)=Y(JK,JJ)*FK(JJ)
      PER= ((Z(JK)-YY(JK) )/YY(JK) )*100.
      WRITE(6,12) YY(JK), Z(JK), PER
12    FORMAT(F13.4,5X,F21.4,5X,F15.2)
 52   CONTINUE
50    CONTINUE
      STOP
      END
C
C
C
C     THIS SUBROUTINE READS IN THE EXPERIMENTAL
C     MOMENT-ROTATION VALUES FOR EACH TESTED CONNECTION
C     AND FITTED THE DATA WITH RAMBERG OSGOOD FUNCTION
C
C
      SUBROUTINE READA(NDS,E0,Q0,AN,X1 )
      DIMENSION E0(NDS),Q0(NDS),AN(NDS),X(50),Y(50),B(500)
      INTEGER*2 LABEL(5)
C
C
      DX=.1
      DO 10 N=1, NDS
      READ(5,1) LABEL
1     FORMAT(5A2)
      READ(5,*)  NP, F1, F2
      WRITE(6,2) LABEL, NP, F1, F2
2     FORMAT('1'//' TEST LABEL:',5A2/' NO. OF DATA',I3,/
     &' ROTATION AND NONENT CORRECTION FACTOR RESPECTIVELY',2F15.5/
     &' EXPERIMENTAL VALUES AS INPUT'/' MOMENTS',5X,'ROTATIONS')
      DO 20 K=1,NP
      READ(5,*) PGG,D1,D2
      WRITE(6,3) PGG,D1,D2
3     FORMAT(F10.5,6X,F10.5)
      X(K)=(D1+D2)/F1
      Y(K)=PGG*F2
20    CONTINUE
```

```
      NNP=NP-1
      M1=1
      M2=M1+3*NNP
      M3=M2+NNP
      CALL CFIT(X,Y,B(M1),B(M2),B(M3),P1,P2,P3,NNP,NP,DX)
      E0(N)=P1
      Q0(N)=P2
      AN(N)=P3
10    CONTINUE
      WRITE(6,5)
5     FORMAT(/////' THE VALUES OF E0, Q0, AN FOR INDIVIDUAL CONNECTION
     &CURVE-FITTED WITH THE RAMBERG OSGOOD FUNCTION ARE-'/
     &' CONNECTION',13X,'E0',13X,'Q0',13X,'AN')
      DO 30  J=1, NDS
      WRITE(6,4) J,E0(J),Q0(J),AN(J)
4     FORMAT(I11,3F15.5)
30    CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE GENERATES THE MOMENT VALUES CORRESPONDING TO
C     THE INPUT ROTATION VALUES FOR COMPUTING THE DIMENSIONLESS
C     EXPONENT FROM EACH CONNECTION'S RAMBERG OSGOOD FUNCTION
C
C
      SUBROUTINE GEN(X,Y,E0,Q0,AN,NDS,NPC)
      DIMENSION X(NPC),Y(20,NDS),E0(NDS),Q0(NDS),AN(NDS)
C
C
      DO 10 I=1, NDS
      A=E0(I)
      B=Q0(I)
      C=AN(I)
      RM1=0.0
      Y(1,I)=RM1
      DO  20  J=2,NPC
      RM=RM1
      C1=X(J)/A
      C2=1./B
      C3=1./(B**C)
40    FM=C1-C2*RM-C3*RM**C
      FMP=-C2-C*C3*RM**(C-1.)
      RM1=RM-FM/FMP
      IF( ABS(RM1-RM)/RM1.LT. .001) GOTO 30
      RM=RM1
      GOTO 40
30    Y(J,I)=RM1
20    CONTINUE
10    CONTINUE
      WRITE(6,1)
1     FORMAT('1'//' ROTATION',5X,'MOMENT FOR EACH CONNECTION')
      DO  50 I=1, NPC
```

```
      WRITE(6,2) X(I), ( Y(I,J), J=1,NDS)
2     FORMAT(F9.5,5X,10F10.2)
50    CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE COMPUTES THE DIMENSIONLESS
C     EXPONENT CORRESPONDING TO THE SIZE PARAMETER CONSIDERED
C
C
      SUBROUTINE  EXPNT(GP,Y,AVG,NDS,NCOM,NPC)
      DIMENSION GP(5,NDS), Y(20,NDS), NCN(10), AVG(NCOM)
C
C
      DO  5 I=1, NCOM
      SUM=0.0
      NCOUNT=0
      READ(5,*) NC
      READ(5,*) ( NCN(J), J=1, NC )
      NNC=NC-1
      DO  10  J=2, NPC
      DO  15  K=1, NNC
      N1=NCN(K)
      KK=K+1
      DO  20  KM=KK, NC
      N2=NCN(KM)
      AT=ALOG( Y(J,N1)/Y(J,N2) )/ALOG( GP(I,N2)/GP(I,N1) )
      SUM=SUM+AT
      NCOUNT=NCOUNT+1
20    CONTINUE
15    CONTINUE
10    CONTINUE
      AVG(I)=SUM/NCOUNT
      WRITE(6,3) I, AVG(I)
3     FORMAT(//,1H ,'AVERAGE VALUE OF EXPONENT FOR PARAMETER',I3,
     & 'IS ',F15.5)
5     CONTINUE
      RETURN
      END
C
C
C
C     THIS IS THE RAMBERG OSGOOD CURVE-FITTING SUBROUTINE
C
C
      SUBROUTINE  CFIT(X,Q, F, F0, L ,E0,Q0,AN,NNP,NP,DX)
      DIMENSION  X(NP),Q(NP),F(NNP,3),F0(NNP),L(NNP),
     1 A(3,3), B(3)
      REAL L
C
C
      Q0 = ABS(( Q(NP) - Q(1))/2)
```

```
Q1 = Q0
DO  10  J=2, NP
Q2 = ABS( Q(J) - Q(1) )
IF ( ABS(Q0 - Q2) .GE. Q1)  GOTO 10
Q1 = ABS(Q0 - Q2)
K = J
CONTINUE
IF ( ABS( Q(K) - Q(1) ) .LE. Q0 ) GOTO 200
K = K - 1
K = MAX0(K,2)
E0 = ABS( X(K) - X(1) )
Q0 = ABS( Q(K) - Q(1) )
AN = 10.
S1 = Q0*1.0E10
ITERATE  PARAMETERS
WRITE(6,25)
FORMAT(//' VALUES OF E0 Q0 AN S1 IN EACH CYCLE OF ITERATION')
S0 = S1
FORM F0, FP, L INVERSE
DO 20 J = 2,  NP
I = J - 1
Q9 = ( Q(J) - Q(1) )/ Q0
Q8 = ABS(Q9)
Q7 = Q8**(AN-1)
F(I,1) = - ( X(J) - X(1) )/E0**2
F(I,2) = Q9/Q0*(1. + AN*Q7)
F(I,3) = -Q9*Q7*ALOG(Q8)
F0(I) = ( X(J) - X(1) )/E0 - Q9*(1. + Q8**(AN-1) )
L(I) = 1./( 1. + ( 1. + AN*ABS((Q(J) - Q(1))/Q0)
1**(AN-1))**2)
CONTINUE
DO 30 I = 1,  3
DO 40 J = 1,  3
C = 0.0
DO  50  K = 1,NNP
C = C + F(K,I)*L(K)*F(K,J)
CONTINUE
A(I,J) = C
CONTINUE
C = 0.0
DO  60  K = 1, NNP
C = C + F(K,I)*L(K)*F0(K)
CONTINUE
B(I) = C
CONTINUE
N0 = 3.0
N1 = N0  -1
DO  70  J = 1,  N1
JJ = J + 1
DO  80  I = JJ, N0
A(I,J) = A(J,I)/A(J,J)
DO  90  K = I,  N0
A(I,K) = A(I,K) - A(I,J)*A(J,K)
CONTINUE
```

```
B(I) = B(I) - A(I,J)*B(J)
CONTINUE
CONTINUE
DO  101  I = 1, NO
II = NO + 1 - I
IF (II.EQ.NO ) GOTO 400
JJ = II + 1
DO  111  JK = JJ, NO
B( II) = B(II) - A(II,JK)*B(JK)
CONTINUE
B(II) = B(II)/A(II,II)
CONTINUE
E0 = E0 - B(1)
Q0 = Q0 - B(2)
AN = AN - B(3)
S1 = 0.0
DO  121  I = 1,  NNP
C = 0.0
DO  131  J = 1, 3
C = C - F(I,J)*B(J)
CONTINUE
S1 = S1 + ( F0(I) - C )*L(I)*F0(I)
CONTINUE
WRITE(6,*) E0, Q0, AN, S1
IF ( ABS(S0-S1)/ABS(S1).GT. DX    ) GOTO 300
RETURN
END
```

APPENDIX G

COMPUTER PROGRAM: TFNCSAP

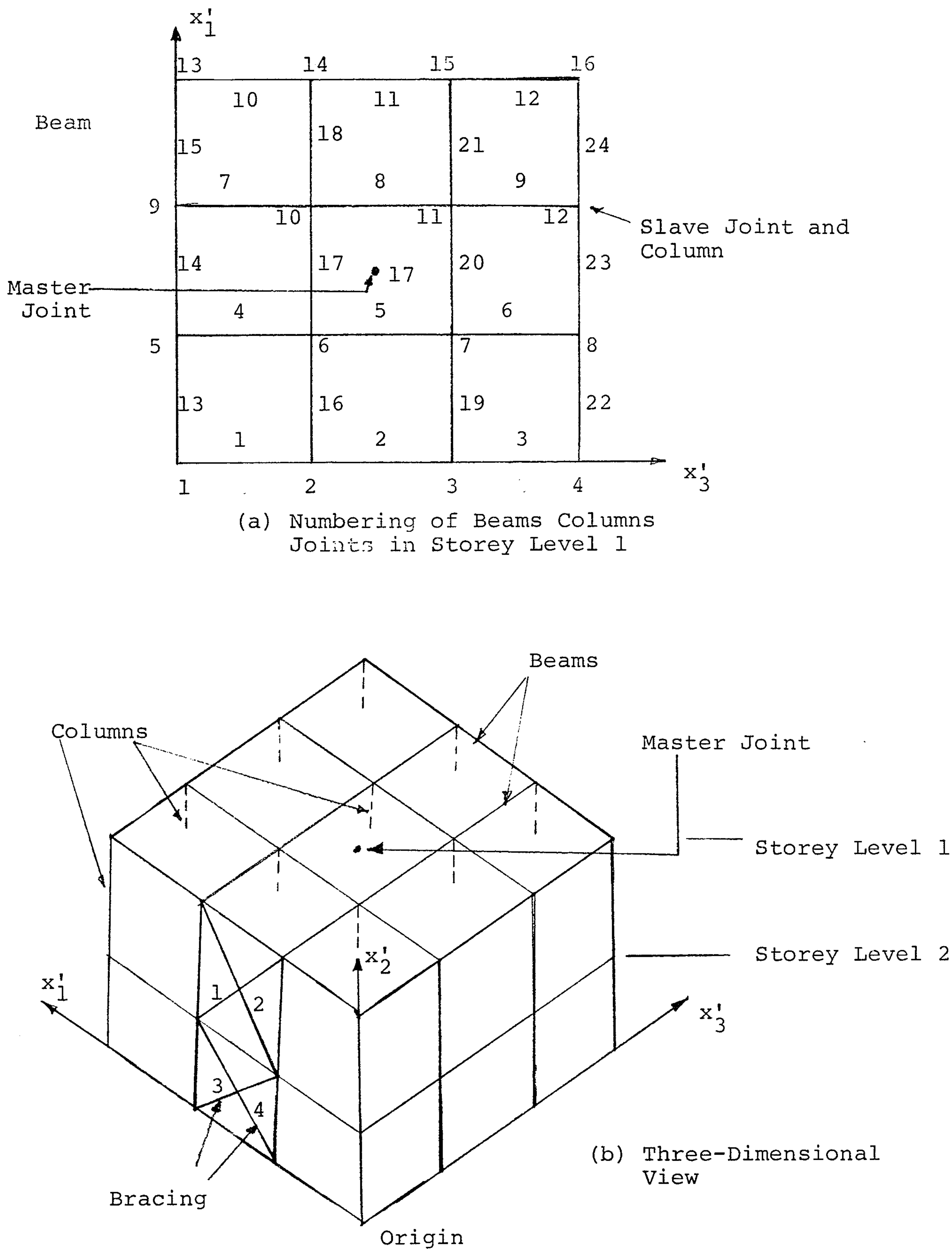
In this Appendix, the user manual for the computer program TFNCSAP is presented. An example and the listing of the program are also included.

G.1 Numerical Definition of the Structure

In this analysis, the overall structure is considered as a single three-dimensional frame, as illustrated in Fig. G.1. A global coordinate system is chosen at one corner of the structure such that the joint coordinates generated by the program are always positive. The numbering of the joints, beam members and column members, and storey levels are to be exactly the same as that shown in Fig. G.1. The numbering is automatically generated by the program. Cross-bracing elements are used, and are numbered from the top floor to the bottom floor, from one braced bay to another, as shown in Fig. G.1. Each braced bay is defined as the bay having a cross-bracing element from the top floor to the bottom floor in that bay.

G.2 Input Data

For convenience, input is in free format except as indicated.



(a) Numbering of Beams Columns Joints in Storey Level 1

(b) Three-Dimensional View

FIG G.1 TYPICAL STRUCTURE

1. Job Title Card (9A8)

Job title to be printed with the output.

2. Control Information Cards

Card Type 1

Variables	Descriptions
NST(I5)	Number of storeys
NBAY1(I5)	Number of bays in X_1' direction
NBAY3(I5)	Number of bays in X_3' direction
I2(A5)	System of units used (SI or BR). Default is SI
PDEL(A5)	Enter PY = P-Δ effect included or PN = P-Δ effect not included
EE(10.0)	Modulus of elasticity. Default is 30000 ksi or 200 GPa
GG(10.0)	Shearing Modulus. Default is 12000 ksi or 80 GPa

Card Type 2

NLJ	Number of loaded joint. If none enter 0
NIMC	Number of concentrated load on all beam members. If none enter 0
NLMU	Number of uniformly loaded beam. If none, enter 0
NTA	Enter either 0 = pinned connection or 1 = rigid connection or 2 = connection stiffness is any constant value or 3 = connection stiffness is non-linear

Card Type 2 (Continued)

NFT	Number of bracing elements. If none enter 0
NBF	Number of braced bay. If NFT is zero, NBF should also be zero

3. Structural Geometry Cards

Card Type 1

Variables	Descriptions
ANODE1	Master joint coordinate in X_1' direction
ANODE3	Master joint coordinate in X_3' direction
HEI	Height of the structure
SPAN1(N)	Length of each bay in X_1' direction. NBAY1 values will be entered
SPAN3(N)	Length of each bay in X_3' direction. NBAY3 values will be entered
DHE(N)	Height between floor level. NST values will be entered

Card Type 2

NBP	Number of beam types. Each type has its own distinct set of I_1, I_2 I_3 and A values as defined in Fig. G.2

Card Type 3

Prepare a card for each beam type, and each contains four data items

BP(I, J)	Beam section properties card. The four data items are entered in the order I_1, I_2, I_3, and A.

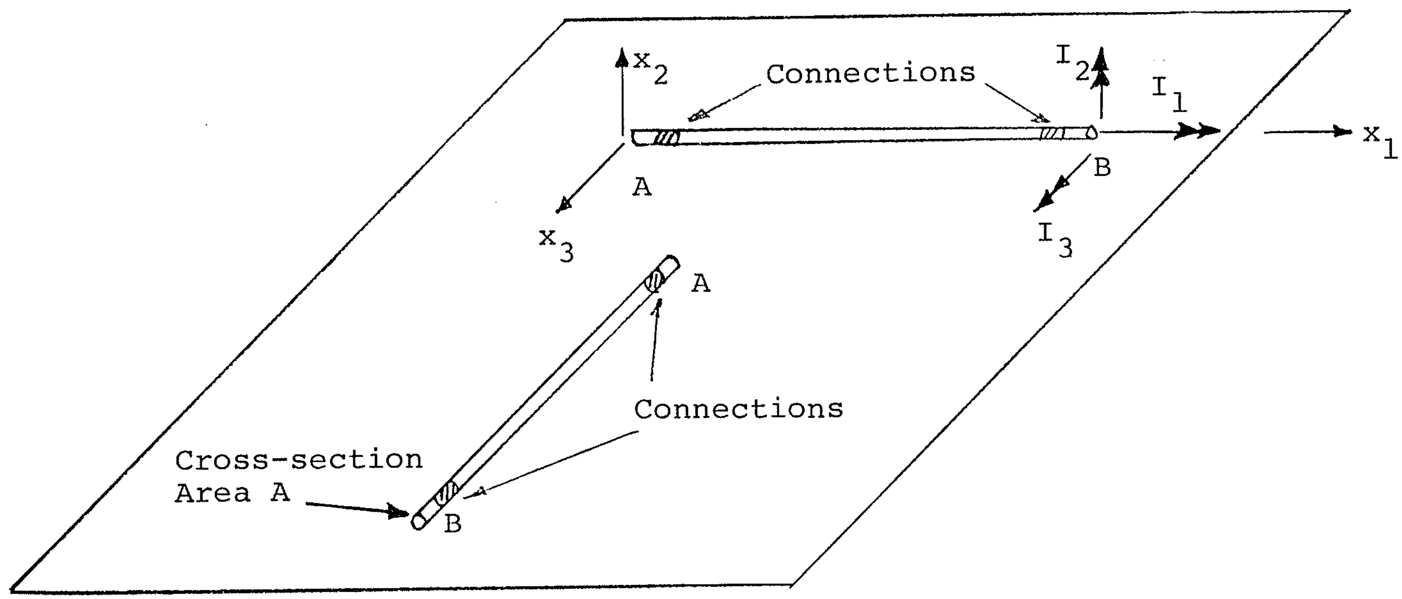


FIG G.2 BEAM SECTION PROPERTIES IN A TYPICAL FLOOR

Card Type 4

NCP — Number of column types. Each type has its own distinct set of I_1, I_2, I_3, and A values as defined in Fig. G.3

Card Type 5

CP(I, J) — Column section properties card. The four section properties are entered in the order I_1, I_2, I_3, and A

Card Type 6

Prepare one or more cards for the identification of the different types of beam. Each type is identified by a number corresponding to the sequence number in which it appears in Card Type 3.

Each beam should have a section property identification number associated with it.

IBP(N) — Identification number for each beam

Card Type 7

Prepare one or more cards for the identification of the different types of columns. Each type is identified by a number corresponding to the sequence number in which it appears in Card Type 5. Each column should have a section property identification number associated with it.

ICP(N) — Identification number for each column

4. Connection Information Card

This set of cards is required when NTA is either 2 or 3. The total number of cards required is NFB. The cards should be placed corresponding to the order of the numbering of the beam members.

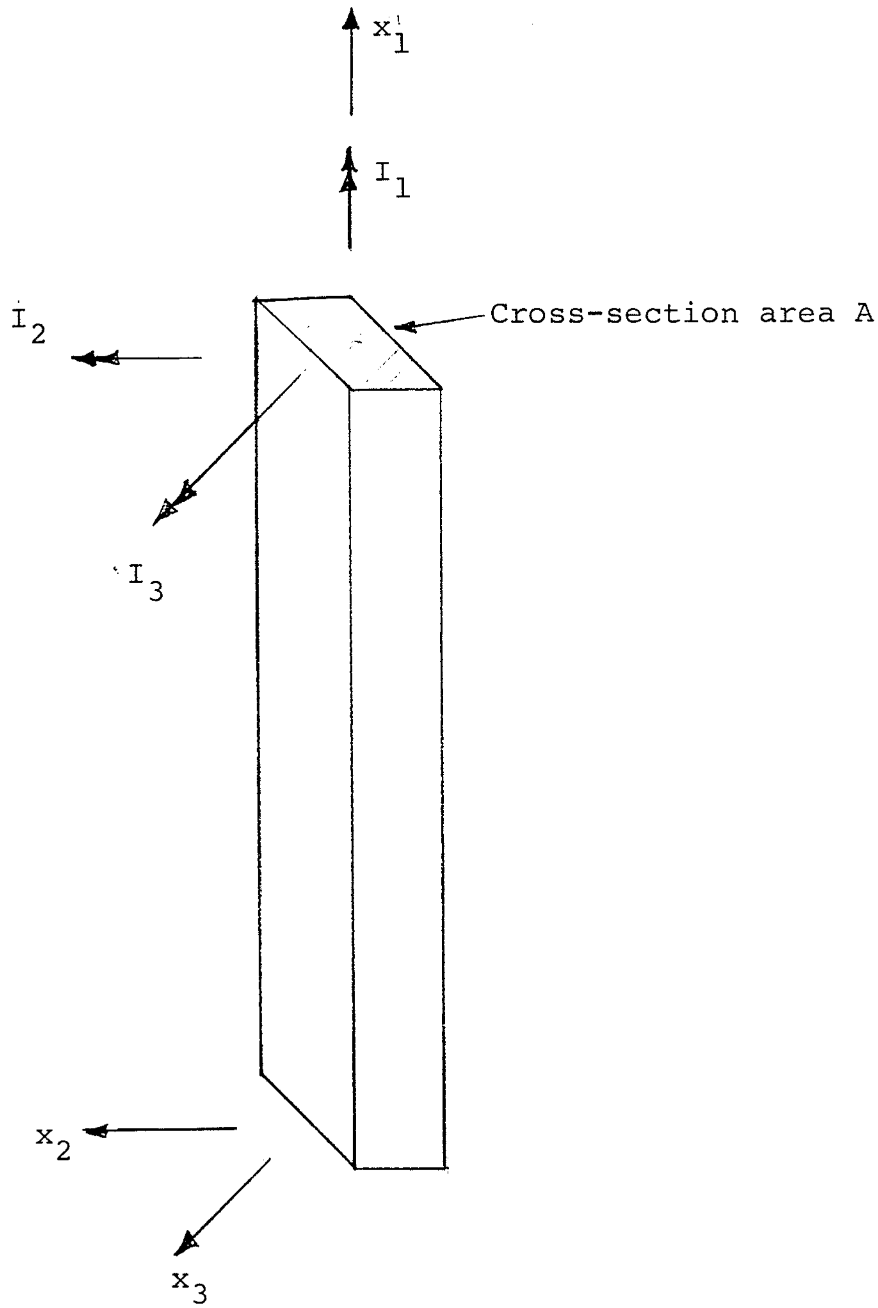


FIG G.3 COLUMN SECTION PROPERTIES

4. <u>Connection Information Card</u> (Continued)

Enter the following data items in each card when NTA = 2:

Variables	Descriptions
NM	Beam member number
SA	Connection stiffness at end A as shown in Fig. G.2
SB	Connection stiffness at end B

Enter the following data items in each card when NTA = 3:

ICT(I,M)	Type of connections: 1 = single web 2 = double web 3 = header plate 4 = top and seat angle 5 = strap angle Selection the one that is used
P1, P2, P3, P4	The geometric parameters as shown in Fig. A.1. If a particular parameter is not applicable, enter 0.

5. <u>Bracing Element Cards</u>

<u>Card Type 1</u>

Prepare one card for each bracing element

Variables	Description
N	Bracing element number. Number as explained in G.1
INT(1, N)	Joint number at the lower end of the bracing element. This number is the number generated by the program as explained in G.1
INT(2, N)	Joint number at the upper end

Card Type 2

NTP	Number of types of different cross-section area of bracing elements

Card Type 3

Prepare a card for each bracing element type. Each has one data item.

TP(N)	Bracing element cross-sectional area

Card Type 4

Prepare one or more cards for the identification of the different bracing element type. Each type of bracing element is identified by a number corresponding to the sequence number in which it appears in Card Type 3.

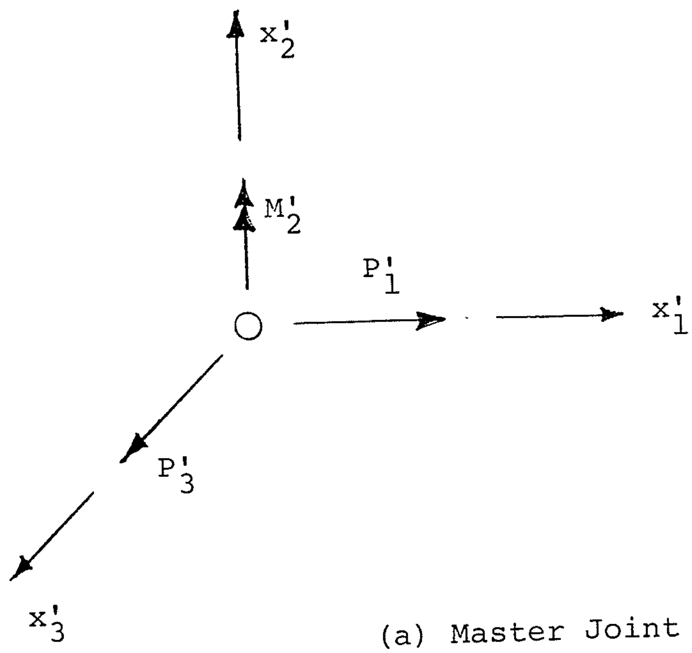
Each bracing element should have a section property identification number associated with it.

6. Loading Information Cards

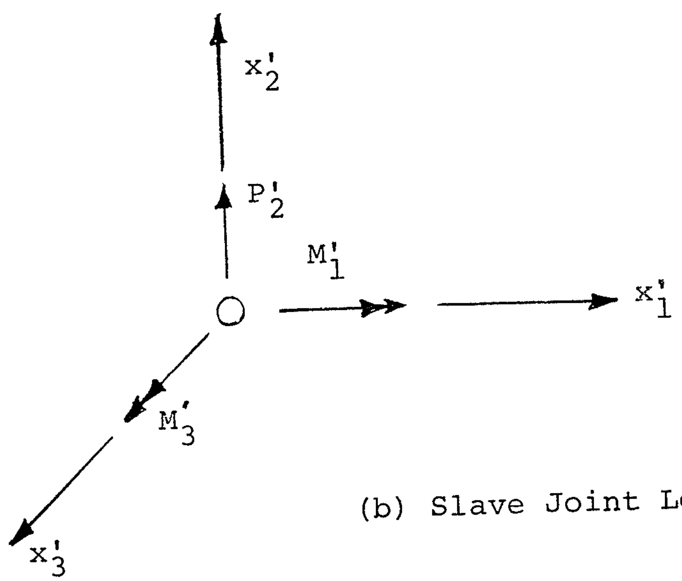
(a) Joint Load

Prepare one card for each loaded joint. Ignore if NLJ is zero. Positive direction of joint load is as shown in G.4.

Variable	Descriptions
K	Loaded joint number
P	Force in X_1' or X_2' direction
T	Force in X_3' or Moment about X_1' directio
B	Moment about X_2' or X_3' direction



(a) Master Joint Loads



(b) Slave Joint Loads

FIG. G.4 POSITIVE DIRECTION OF JOINT LOADS

6. Loading Information Cards (Continued)

(b) Beam Concentrated Load

Prepare one card for each beam with different concentrated load. Ignore if NLMC is zero. Positive direction of the beam concentrated load is as shown in Fig. G.5.

Variable	Descriptions
K	Beam member No.
P	Force on beam in X_2' direction
AL	Distance of the force P from the A end as shown in G.5

(c) Beam with Uniformly Distributed Load

Prepare a card for each beam with uniformly distributed load over its full length. Ignore if NLMU is zero. Positive direction of the load is as shown in Fig. G.6.

Variable	Descriptions
K	Beam member No.
P	Uniformly distributed load in X_2' direction

G.3 Sample Example for Input

```
//G0.SYSN DD *
INPUT FOR EXAMPLE 2 IN CHAPTER 6

      2       1 ·     1       SI       PN
2 0 8 1 0 0
10.0 10.0 40.0 20.0 20.0 20.0 20.0 20.0 20.0

1
100.0 200.0 200.0 15.0
1
100.0 200.0 200.0 15.0
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
```

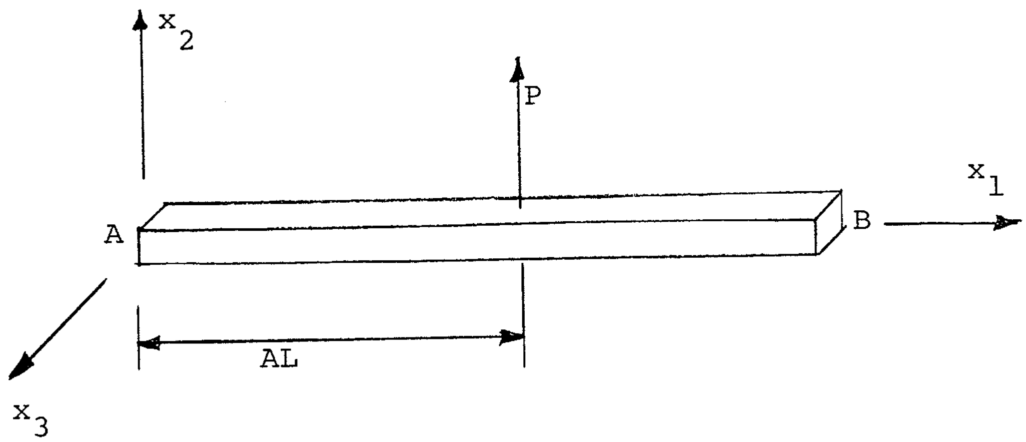


FIG G.5 POSITIVE DIRECTION OF BEAM CONCENTRATED LOAD

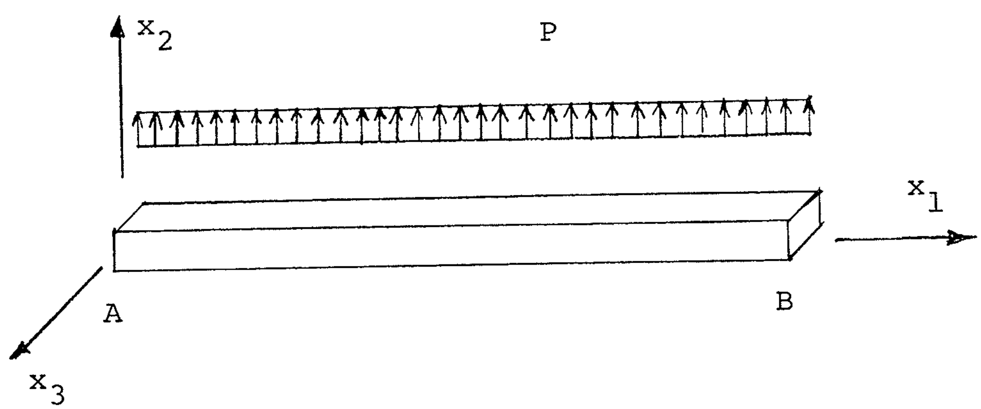


FIG. G.6 POSITIVE DIRECTION OF BEAM UNIFORMLY DISTRIBUTED LOAD

G.3 Sample Example for Input (Continued)

5	10.0	0.0	0.0
10	10.0	0.0	0.0
1	- 0.24		
2	- 0.24		
3	- 0.24		
4	- 0.24		
5	- 0.24		
6	- 0.24		
7	- 0.24		
8	- 0.24		

LISTING OF COMPUTER PROGRAM:

TFNCSAP

```
C  ******************************************************
C  *                                                    *
C  *    TFNCSAP: THREE DIMENSIONAL STEEL FRAMES         *
C  *             ANALYSIS PROGRAM WITH OR WITHOUT       *
C  *             CONSIDERING CONNECTION DEFORMATION     *
C  *                                                    *
C  *           PROGRAMMED BY:                           *
C  *              K. M. ANG                             *
C  *              DEPARTMENT OF CIVIL ENGINEERING       *
C  *              THE UNIVERSITY OF MANITOBA            *
C  *              WINNIPEG  MANITOBA                    *
C  *                                                    *
C  *                                                    *
C  *           DATE: APRIL 1983                         *
C  *                                                    *
C  ******************************************************
C  *                                                    *
C  *                                                    *
C  *      PROGRAM LIMITATIONS:                          *
C  *          (1) ANALYZE RECTANGULAR STRUCTURE         *
C  *          (2) TYPES OF BEAM AND COLUMN SECTION      *
C  *              RESTRICTED TO 20                      *
C  *          (3) DIMENSION SIZE OF ARRAY A CONTROL     *
C  *              THE STORAGE REQUIREMENTS OF           *
C  *              STRUCTURES ANALYZED                   *
C  *                                                    *
C  ******************************************************
C
C     EXTERNAL  SFRAME
C     REAL *8 IHO208
C     CALL  ERRSAV ( 208, IHO208 )
C     CALL  ERRSET ( 208, 999, -1, 1 )
C     CALL  GETCOR ( SFRAME )
C     CALL  ERRSTR ( 208, IHO208 )
C     STOP
C     END
C     SUBROUTINE  SFRAME( A, NZZ )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(120000),BHED(9)
      COMMON /IPT/ NST, NBAY1, NBAY3
      COMMON /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /TBRACE/ NFT, NBF
      COMMON /MAT/ EM, GM
      COMMON /ALOAD/ NLJ, NLMC, NLMU, NLT1, NLT2, NLT3
      COMMON /SECT/ BP(4,20), CP(4,20)
      COMMON /SECT2/ TP(20)
      COMMON /CON/ FACTOR,FAC2,FAC3,FAC4
      COMMON /UNITS/ UNIT1,UNIT2,UNIT3,UNIT4,UNIT5,UNIT6,UNIT7
      REAL*8 UNIT1,UNIT2,UNIT3,UNIT4,UNIT5,UNIT6,UNIT7
      REAL*8 INCHES/'(IN)'/,FEET/'(FT)'/,SQIN/'(IN**2)'/,IN4/'(IN**4)'/,
     & KIPS/'(KIPS)'/,KIPFT/'(KIP-FT)'/,KIPPFT/'(K/FT)'/
      REAL*8 MM/'(MM)'/,METRES/'(METRES)'/,SQMM/'(MM**2)'/,
     & MM4/'(MM**4)'/,KN/'(KN)'/,KNM/'(KN-M)'/,KNPM/'(KN/M)'/
```

```
      REAL *8  GPA/'(GPA)'/, KSI/'(KSI)'/
      INTEGER*2 I2  , BRUNIT/'BR'/, BLANK/'  '/, I3/'SI'/,PDEL,PY/'PY'/
      LOGICAL SSI, PPY
C
C
C        READ  IN  TITLE
C
      READ(5,100) BHED
100   FORMAT( 9A8 )
      WRITE(6,200) BHED
200   FORMAT('1',10X,9A8//' **** BUILDING GENERAL INFORMATION ****'/)
C
C        READ IN BUILDING INFORMATION
C
      READ(5,250) NST, NBAY1, NBAY3, I2, PDEL, EE, GG
250   FORMAT( 3I5, 2A5, 2F10.0 )
      READ(5,*) NLJ, NLMC, NLMU, NTA, NFT, NBF
C
C          TO AVOID ZERO SUBSCRIPT IN LOAD VECTOR
C
      NLT1=NLJ
      NLT2=NLMC
      NLT3=NLMU
      IF(NLT1 .EQ. 0) NLT1=1
      IF(NLT2 .EQ. 0) NLT2=1
      IF(NLT3.EQ.0)NLT3=1
C
C          CHOOSE THE APPROPRIATE INPUT SYSTEM OF UNIT
C
      SSI = .TRUE.
      IF ( I2 .EQ. BRUNIT ) SSI = .FALSE.
      IF ( I2 .EQ. BLANK )  I2 = I3
C
C     PPY=.FALSE. --- NO PDELTA EFFECT CHECK WANTED
C     PDEL=PY OR PN
C        PY---PDELTA EFFECT INCLUDED IN ANALYSIS
C        PN---PDELTA EFFECT NOT INCLUDED IN ANALYSIS
C
      PPY=.FALSE.
      IF ( PDEL .EQ. PY ) PPY=.TRUE.
C
C
      IF ( SSI ) GOTO 300
      DFLTE = 30000.
      DFLTG = 12000.
      FACTOR = 12.
      FAC2=1.0
      FAC3=1.0
      FAC4=1.0
      UNIT = KSI
      UNIT1=INCHES
      UNIT2=FEET
      UNIT3=SQIN
      UNIT4=IN4
```

```
      UNIT5=KIPS
      UNIT6=KIPFT
      UNIT7=KIPPFT
      GOTO  400
300   DFLTE = 200.
      DFLTG = 80.
      FACTOR = 1000.
      FAC2=0.0394
      FAC3=0.7375
      FAC4=8.8503
      UNIT = GPA
      UNIT1=MM
      UNIT2=METRES
      UNIT3=SQMM
      UNIT4=MM4
      UNIT5=KN
      UNIT6=KNM
      UNIT7=KNPM
400   EM = DFLTE
      GM  = DFLTG
      IF ( EE .NE. 0.0 ) EM  = EE
      IF ( GG .NE. 0.0 ) GM  = GG
C
      WRITE ( 6, 430 ) NST, NBAY1, NBAY3,NTA,NFT,NBF
430   FORMAT ('  NUMBER OF STOREYS . . . . . . . . . . . . . ',I10/
     &         '  NUMBER OF BAY IN X1 . . . . . . . . . . . . ',I10/
     &         '  NUMBER OF BAY IN X3 . . . . . . . . . . . . ',I10//
     &         '  TYPE OF ANALYSIS: . . . . . . . . . . . . . ',I10/
     &         '     =0 BEAMS PINNED TO COLUMNS'/
     &         '     =1 BEAMS FIXED TO COLUMNS'/
     &         '     =2 BEAM_COLUMN CONNECTIONS CAN TAKE '/
     &         '        ALL POSSIBLE VALUES OF STIFFNESS '/
     &         '     =3 CONNECTIONS ARE NON-LINEAR '//
     &         '  NUMBER OF BRACING . . . . . . . . . . . . . ',I10/
     &         '  NUMBER OF BRACED FRAME  . . . . . . . . . . ',I10/)
      WRITE(6, 440 ) PDEL, I2, UNIT, EM, UNIT, GM, NLJ, NLMU, NLMC
440   FORMAT ('  P-DELTA EFFECT CHECK  . . . . . . . . . . . ',A10/
     &         '    = PY--PDELTA INCLUDED IN ANALYSIS'/
     &         '    = PN--PDELTA NOT INCLUDED IN ANALYSIS'//
     &         '  SYSTEM OF UNIT USED . . . . . . . . . . . . ',A10/
     &         '  MODULUS OF ELASTICITY',A5,' . . . . . . . . .',F10.1/
     &         '  SHEAR MODULUS',A5,' . . . . . . . . . . . . .',F10.1/
     &         '  NUMBER OF LOADED JOINTS . . . . . . . . . . ',I10/
     &         '  NUMBER OF UNIFORMLY LOADED BEAM . . . . . . ',I10/
     &         '  NUMBER OF LOADED BEAM-CONCENTRATED LOAD . ',I10/)
C
C      NFRJ = # OF FREE NODE INCLUDE MASTER NODE
C      NSJ = # OF SUPPORT JOINTS
C      NTJ = TOTAL NO. OF JOINTS
C      NFB = TOTAL NO. OF BEAMS
C      NFC = TOTAL NO. OF COLUMNS
C      NBAND = BAND WIDTH
C      NEQ = NO OF EQUATIONS
C
```

```
      NFRJ = ((NBAY1 + 1)*(NBAY3 + 1) + 1)*NST
      NSJ = (NBAY1 + 1)*(NBAY3 + 1) + 1
      NFB = (NBAY3*(NBAY1 + 1) + NBAY1*(NBAY3 + 1))*NST
      NFC = (NBAY3 + 1)*(NBAY1 + 1)*NST
      NTJ = NFRJ + NSJ
      NBAND = (NTJ/(NST + 1))*2*3
      NEQ = NTJ*3
      NXM = NEQ*NBAND
C
C *** SET UP STORAGE MAP
C   N1  CN(3,NTJ)         R*8  JOINT COORIDINATES
C   N2  INB(2,NFB)        I*4  BEAM INCIDENCE TABLE
C   N3  SPAN1(NBAY1)      R*8  LENGTH/BAY IN  X1 DIRECTION
C   N4  SPAN3(NBAY3)      R*8  LENGTH/BAY IN  X3 DIRECTION
C   N5  DHE(NST)          R*8  HEIGHT PER STORY
C   N6  INC(2,NFC)        I*4  COLUMN INCIDENCE TABLE
C   N7  IBP(NFB)          I*4  BEAM SECTION  IDENTIFICATION NO.
C   N8  ICP(NFC)          I*4  COLUMN SECTION  IDENTIFICATION NO.
C   N9  ITC(2,NFB)        I*4  CONNECTION TYPE IDENTIFICATION NO.
C   N10 FK(2,NFB)         R*8  CONNECTION MOMENT MULTIPLICATION  FACTOR
C   N11 SC(2,NFB)         R*8  CONNECTION STIFFNESS
C   N12 SS(NBAND,NEQ)     R*8  GLOBAL STIFFNESS MATRIX
C   N13 JD(3,NTJ)         R*8  CALCULATED DISPLACEMENT VECTOR
C   N14 AJL(3,NLTI)       R*8  JOINT LOAD VECTOR
C   N15 BFA(3,NFB)        R*8  BEAM A  END FORCES
C   N16 BFB(3,NFB)        R*8  BEAM B END FORCES
C   N17 CFA(3,NFC)        R*8  COLUMN A END FORCES
C   N18 CFB(3,NFC)        R*8  COLUMN B END FORCES
C   N19 CIA(3,NFC)        R*8  COLUMN A END IN-PLANE FORCES
C   N20 CIB(3,NFC)        R*8  COLUMN B END IN-PLANE FORCES
C   N21 ALMC(3,NLT2)      R*8  BEAM CONCENTRATED LOAD
C   N22 ALMU(3,NLT3)      R*8  BEAM UNIFORMLY DISTRIBUTED LOAD
C   N23 MU(NLMU)          I*4  UNIFORMILY LOADED BEAM MEMBER NO.
C   N24 MJ(NLJ)           I*4  LOADED JOINT NUMBER
C   N25 MC(NLMC)          I*4  CONCENTRATED LOADED BEAM MEMBER NO.
C   N26 INT(2,NFT)        I*4  BRACING INCIDENCE TABLE
C   N27 ITP(NFT)          I*4  BRACING PROPERTY IDENTIFICATION NUMBER
C   N28 TFA(NFT)          R*8  BRACING A END FORCE
C   N29 TFB(NFT)          R*8  BRACING B END FORCE
C   N30 NEXT STORAGE LOCATION
C
      N1=1
      N2=N1+3*NTJ
      N3=N2+2*NFB
      N4=N3+NBAY1
      N5=N4+NBAY3
      N6=N5+NST
      N7=N6+2*NFC
      N8=N7+NFB
      N9=N8+NFC
      N10=N9+2*NFB
      N11=N10+2*NFB
      N12=N11+2*NFB
      N13=N12+NXM
```

```
      N14=N13+NEQ
      N15=N14+3*NLT1
      N16=N15+3*NFB
      N17=N16+3*NFB
      N18=N17+3*NFC
      N19=N18+3*NFC
      N20=N19+3*NFC
      N21=N20+3*NFC
      N22=N21+2*NLT2
      N23=N22+NLT3
      N24=N23+NLT3
      N25=N24+NLT1
      N26=N25+NLT2
      NITER=1
      NPDT=1
      NPD=1
      NCOUNT=1
C
      CALL  GENINP( A(N1), A(N2), A(N3), A(N4), A(N5), A(N6),
     & A(N7), A(N8) )
      CALL  CINFO( A(N9), A(N10), A(N11), A(N15), A(N16),NTA, SSI )
      IF (NFT .EQ. 0 ) GOTO 450
      N27=N26+2*NFT
      N28=N27+NFT
      N29=N28+NFT
      N30=N29+NFT
      CALL  INTRS( A(N26), A(N27) )
450   CALL  COL( A(N1), A(N6), A(N8), A(N12) )
      CALL  BEAM( A(N1), A(N2), A(N7), A(N11), A(N12) )
      IF(NBF .GT. 0) CALL BRACE(A(N1), A(N26), A(N12), A(N27) )
      IF (NITER .GT. 1 .OR. NPDT .GT. 1 ) GOTO 401
      CALL  LOAD( A(N1), A(N2), A(N7), A(N11), A(N13),
     & A(N14), A(N15), A(N16), A(N21), A(N22), A(N23), A(N24), A(N25) )
      GOTO 402
401   CALL RELOAD(A(N11), A(N13), A(N15), A(N16) )
402   CALL  SPT( A(N12), A(N13) )
      CALL  EQN( A(N1), A(N12), A(N13) )
      CALL  BMF( A(N1), A(N2), A(N7), A(N11), A(N13) , A(N15), A(N16) )
      CALL  CMF( A(N1), A(N6), A(N8), A(N13), A(N17), A(N18),
     & A(N19), A(N20) )
      IF( NBF .GT. 0 ) CALL FBRACE( A(N1), A(N26), A(N27), A(N13),
     & A(N28), A(N29) )
C
      NPCODE=0
      NCCODE=0
C
C
C     PDELTA AND CONNECTION DEFORMATION NOT INCLUDED IN ANALYSIS
C
      IF ( .NOT.PPY .AND. NTA .LT. 3) STOP
C
C
C     CONNECTION DEFORMATION AND PDELTA INCLUDED IN ANALYSIS
C
```

```
      IF (PPY .AND. NTA .EQ. 3) GOTO 452
C
C
C     ONLY PDELTA INCLUDED IN ANALYSIS
C
      IF (PPY .AND. NTA .LT. 3) GOTO 452
C
C
C
C     ONLY CONNECTION DEFORMATION INCLUDED IN ANALYSIS
C
      IF (.NOT.PPY .AND. NTA .EQ. 3) GOTO 466
C
C
C
452   IF ( NPDT .LT. 20) GOTO 456
      WRITE(6,455)
455   FORMAT('  P-DELTA EFFECT DOES NOT STABILIZE')
      NPCODE=2
      IF (NTA .LT.3) STOP
      GOTO  466
C
456   IF (NPD .GT. 0) GOTO 457
      NPCODE=1
      IF(NTA .LT.3) STOP
      GOTO  466
C
457   CALL PDELTA( A(N5), A(N13), A(N14), A(N17), A(N24),NPD,NPDT)
      NPDT=NPDT+1
      IF ( NTA .LT. 3) GOTO 450
C
466   IF ( NITER .LT. 20 ) GOTO 468
C
      WRITE(6,467)
467   FORMAT( '  CONNECTIONS FELIXIBLE, STIFFER CONNECTION NEEDED')
      GOTO 471
C
468   IF (NPCODE .EQ. 2) STOP
      IF (NCOUNT .GT.0) GOTO 469
      NCCODE=1
      IF (NPCODE .EQ. 1 .AND. NCCODE .EQ. 1) STOP
      IF (.NOT. PPY) STOP
      GOTO 450
C
469   CALL ITER( A(N11), A(N15), A(N16), NCOUNT)
      NITER=NITER+1
      IF ( .NOT. PPY) GOTO 450
      IF (  NPCODE .EQ. 1) NPD=1
      GOTO 450
471   STOP
      END
C
C
C
```

```
C      THIS SUBROUTINE GENERATES THE JOINT NUMBERS, JOINT COORIDINATES
C      BEAM MEMBER NUMBERS AND COLUMN MEMBER NUMBERS, BEAM AND COLUMN
C      INCIDENCE TABLES FOR THE RECTANGULAR STRUCTURE ANALYZED.
C
C
      SUBROUTINE GENINP ( CN, INB, SPAN1, SPAN3, DHE, INC, IBP, ICP )
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /IPT/ NST, NBAY1, NBAY3
      COMMON /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /SECT/ BP(4,20), CP(4,20)
      COMMON /CON/ FACTOR, FAC2, FAC3, FAC4
      COMMON /UNITS/ UNIT1,UNIT2,UNIT3,UNIT4,UNIT5,UNIT6,UNIT7
      DIMENSION CN(3,NTJ), SPAN1(NBAY1), SPAN3(NBAY3), DHE(NST),
     1INB(2,NFB), INC(2,NFC), IBP(NFB), ICP(NFC)
      REAL *8 MNODE1, MNODE3
C
C
C     INPUT: MASTER NODE COORDINATES- X1' AND X3'  IN FT.
C            OVERALL HEIGHT OF STRUCTURE IN FT.
C            LENGTH OF EACH BAY IN X1' DIRECTION IN FT.
C            LENGTH OF EACH BAY IN X3' DIRECTION IN FT.
C            HEIGHT OF EACH FLOOR IN FT.
C
      READ (5,*) MNODE1, MNODE3,  HEI, (SPAN1(N),N=1,NBAY1)
     1,(SPAN3(N),N=1,NBAY3), (DHE(N),N=1,NST)
      WRITE (6,1001) UNIT2, MNODE1, UNIT2,MNODE3,UNIT2,HEI
1001  FORMAT( ' MASTER NODE IN X1',1H',' DIRECTION',A10,F10.2/
     1        ' MASTER NODE IN X3',1H',' DIRECTION',A10,F10.2/
     1        ' HEIGHT OF BUILDING          ',A10,F10.2/)
      WRITE(6,1011) UNIT2
 1011 FORMAT(//' BAY NO. IN X1',1H',' DIRECTION',10X,'LENGTH',A8)
      DO  10  NB=1, NBAY1
      WRITE(6,1012) NB, SPAN1(NB)
 1012 FORMAT(10X,I5,18X,F10.2)
10    CONTINUE
      WRITE(6,1013) UNIT2
1013  FORMAT(//' BAY NO. IN X3',1H',' DIRECTION',10X,'LENGTH',A8)
      DO  11  NB=1,  NBAY3
      WRITE(6,1014) NB, SPAN3(NB)
1014  FORMAT(10X,I5,18X,F10.2)
11    CONTINUE
      WRITE(6,1015) UNIT2
1015  FORMAT(//' FLOOR NO.',5X,'STOREY HEIGHT',A8)
      DO  12  NB=1, NST
      WRITE(6,1016) NB, DHE(NB)
1016  FORMAT(3X,I5,10X,F10.2)
12    CONTINUE
C
C
C   GENERATE JOINTS NO. AND COORIDINATES
C
      NJ = 0
      TEMP1 = 0.D0
      TEMP3 = 0.D0
```

```
      NSH = NST + 1
      NBAY1P = NBAY1 + 1
      NBAY3P = NBAY3 + 1
      DO 400 N = 1, NSH
      NJ = NJ + 1
      CN(1,NJ) = TEMP1
      CN(2,NJ) = HEI
      CN(3,NJ) = TEMP3
      DO 300 NN=1, NBAY1P
      DO 200 MN=1, NBAY3
      NJ = NJ + 1
      CN(1,NJ) = TEMP1
      CN(2,NJ) = HEI
      CN(3,NJ) = CN(3,NJ-1) + SPAN3(MN)
200   CONTINUE
      IF ( NN.EQ.NBAY1P )  GOTO 100
      TEMP1  = TEMP1 + SPAN1(NN)
      NJ = NJ + 1
      CN(1,NJ) = TEMP1
      CN(3,NJ) = TEMP3
      CN(2,NJ) = HEI
300   CONTINUE
100   NJ= NJ + 1
      CN(1,NJ) = MNODE1
      CN(2,NJ) = HEI
      CN(3,NJ) = MNODE3
      TEMP1 = 0.D0
      IF  (N.EQ.NSH)GOTO 400
      HEI = HEI - DHE(N)
400   CONTINUE
C
C  CONVERT  TO  IN. OR MM.
C
      DO   111  KK=1, NTJ
      CN(1,KK) = CN(1,KK)*FACTOR
      CN(2,KK) = CN(2,KK)*FACTOR
      CN(3,KK) = CN(3,KK)*FACTOR
111   CONTINUE
C
C   GENERATE BEAM INCIDENCE TABLE
C   NB = FIRST JOINT
C   NE = LAST JOINT
C
      NB = 1
      NE = (NBAY1 + 1)*(NBAY3 + 1)
      NEB = NE
      JS = NB - 1
      JE = JS + 1
      MM = 0
      DO 950 NN = 1, NST
      DO 700 N = 1, NBAY1P
      DO 600 M = 1, NBAY3
      MM = MM + 1
      JS = JS + 1
```

```
      JE = JE + 1
      INB(1,MM) = JS
      INB(2,MM) = JE
600   CONTINUE
      JS = JE
      JE = JS + 1
700   CONTINUE
      JS = NB
      JE = JS + NBAY3 + 1
      DO 900 N = 1, NBAY3P
      DO 800 M = 1, NBAY1
      MM = MM + 1
      INB(1,MM) = JS
      INB(2,MM) = JE
      JS = JE
      JE = JE + (NBAY3 + 1)
800   CONTINUE
      JS = NB + N
      JE = JS + NBAY3 + 1
900   CONTINUE
      NB = NE + 1
      NE = NB + NEB
      JS = NB
      JE = JS + 1
      NB = NB + 1
950   CONTINUE
C
C    GENERATE COLUMN INCIDENCE TABLE
C
      MM = 0
      NEC = NFC/NST
      NE = 0
      NB = NFC/NST  + 1
      JS = NB
      JE = NE
      DO 650 NN = 1,  NST
      DO 750 N = 1,  NBAY1P
      DO 850 M = 1,  NBAY3P
      MM = MM + 1
      JS = JS + 1
      JE = JE + 1
      INC(1,MM) = JS
      INC(2,MM) = JE
850   CONTINUE
750   CONTINUE
      NE = NB
      NB = NB + NEC + 1
      JS = NB
      JE = NE
650   CONTINUE
C
C
      WRITE (6,1002) UNIT1
1002  FORMAT(///' GENERATED JOINT NO. AND JOINT COORDINATES IN ',A8/
```

```
     &           ' JOINT NO',5X,'X1',1H',' COORDINATE',5X,'X2',1H','COORD
     &INATE',5X,'X3',1H',' COORDINATE')
      DO 105 N = 1, NTJ
      WRITE (6,1003) N, CN(1,N), CN(2,N), CN(3,N)
1003  FORMAT ( 3X,I5,6X,F10.2,8X,F10.2,10X,F10.2)
105   CONTINUE
C
C
      WRITE (6,1005)
1005  FORMAT ( ///' GENERATED BEAM INCIDENCE TABLE'/
     &           21X,'BEAM NO',10X,'JOINT AT A-END',10X,'JOINT AT B-END')
      NBE = NFB/NST
      MM = 1
      MN = NBE
      DO 107 N = 1, NST
      WRITE (6, 1004) N
1004  FORMAT (15H0STORY LEVEL---,I5)
      DO 108 M = MM,  MN
      WRITE (6,1006) M, INB(1,M), INB(2,M)
1006  FORMAT (' ', 20X, I5, 12X, I10, 20X, I5)
108   CONTINUE
      MM = MN + 1
      MN = (N+1)*NBE
107   CONTINUE
C
C
      WRITE (6,1105)
1105  FORMAT ( ///' GENERATED COLUMN INCIDENCE TABLE'/
     &       21X,'COLUMN NO',10X,'JOINT AT A-END',10X,'JOINT AT B-END')
      MM = 1
      MN = NEC
      DO 110 N = 1, NST
      WRITE (6,1004) N
      DHE(N)=DHE(N)*FACTOR
      DO 109 M = MM, MN
      WRITE (6, 1006) M, INC(1,M), INC(2,M)
109   CONTINUE
      MM = MN + 1
      MN = (N + 1)*NEC
110   CONTINUE
C
C
C      INPUT BEAM   PROPERTIES
C
      READ(5,*) NBP
      WRITE(6,1007) NBP, UNIT4,UNIT4,UNIT4,UNIT3
1007  FORMAT(///' NUMBER OF BEAM TYPE - - -',I5/
     &           ' BEAM TYPE',5X,'TORSION CONSTANT J',A8,5X,
     &           '2ND MOMENT AREA I2',A8,5X,'3RD MOMENT AREA I3',A8,
     &           5X,'X-SECT AREA',A8)
      DO 5 I=1, NBP
      READ(5,*)    ( BP(J,I), J=1,4 )
      WRITE(6,1008)  I, (BP(J,I), J=1, 4)
1008  FORMAT(4X, I5, 12X, F15.1,20X,F15.1,16X,F15.1,10X,F10.1)
```

```
5       CONTINUE
C
C
C        INPUT COLUMN PROPERTIES
C
        READ(5,*) NCP
        WRITE(6,1107) NCP, UNIT4, UNIT4, UNIT4,UNIT3
1107    FORMAT(//' NUMBER OF COLUMN TYPE - - -',I5/
     &           ' COLUMN TYPE',5X,'TORSION CONSTANT J',A8,5X,
     &           '2ND MOMENT AREA I2',A8,5X,'3RD MOMENT AREA I3',A8,
     &           5X,'X-SECT AREA',A8)
        DO 6 I=1, NCP
        READ(5,*) ( CP(J,I), J=1,4)
        WRITE(6,1008)  I,( CP(J,I), J=1, 4)
6       CONTINUE
C
C       INPUT BEAM ID. NO FOR EACH BEAM
C
        READ(5,*)  (IBP(I), I=1,NFB)
        WRITE(6,1111)
1111    FORMAT(//'  BEAM NO.',10X, 'BEAM TYPE ID. NO.')
        DO  17 M=1, NFB
        WRITE(6,1112) M , IBP(M)
1112    FORMAT(7X,I3,11X,I12)
17      CONTINUE
C
C       INPUT COLUMN ID. NO. FOR EACH COLUMN
C
        READ(5,*) (ICP(I), I=1,NFC)
        WRITE(6,1113)
1113    FORMAT(//'  COLUMN NO.',10X, 'COLUMN TYPE ID. NO.')
        DO  18 M=1, NFC
        WRITE(6,1112)  M ,ICP(M)
18      CONTINUE
        RETURN
        END
C
C
C
C      THIS SUBROUTINE PERFORMS AN ASSEMBLY OF THE COLUMN
C      STIFFNESS COEFFICIENTS FOR EACH INDIVIDUAL COLUMN.
C
C
        SUBROUTINE  COL( CN, INC, ICP, SS )
        IMPLICIT REAL*8 (A-H,O-Z)
        COMMON /MAT/ EM, GM
        COMMON /SECT/ BP(4,20), CP(4,20)
        COMMON  /GEN/ NTJ, NFB, NFC, NBAND, NEQ,NXM
        COMMON /IPT/ NST, NBAY1, NBAY3
        INTEGER V, M, U, L
        DIMENSION  CN(3,NTJ), INC(2,NFC), SS(NXM), ICP(NFC)
C
C
        KK(L1,M1,L3,M3) = (3*L1 - 3*M1 + L3 - M3)*NEQ + 3*M1 + M3 - 3
```

```
      DO  100 I = 1,  NXM
      SS(I) = 0.D0
100   CONTINUE
      MM = 1
      MN = NFC/NST
      DO  300 N = 1, NST
      M = (N + 1)*NTJ/(NST + 1)
      V = N*NTJ/(NST + 1)
      DO  310  J = MM, MN
      CALL  CELM( J, CN,ICP,INC, A, B, C, D, E, F, G, H, P, Q,
     1 U, L, V, M, ZU, XU, ZL, XL )
      K = KK(U,U,1,1)
      SS(K) = SS(K) + A
      K = KK(U,U,2,2)
      SS(K) = SS(K) + B
      K = KK(U,U,3,3)
      SS(K) = SS(K) + C
      K = KK(V,U,1,3)
      SS(K) = SS(K) + D
      K = KK(V,U,2,2)
      SS(K) = SS(K) - E
      K = KK(V,U,3,2)
      SS(K) = SS(K) + XU*E
      K = KK(V,U,3,3)
      SS(K) = SS(K) + ZU*D
      K = KK(V,V,1,1)
      SS(K) = SS(K) + F
      K = KK(V,V,2,2)
      SS(K) = SS(K) + G
      K = KK(V,V,3,1)
      SS(K) = SS(K) + ZU*F
      K = KK(V,V,3,2)
      SS(K) = SS(K) - XU*G
      K = KK(V,V,3,3)
      SS(K) = SS(K) + (ZU**2)*F + (XU**2)*G + H
      K = KK(L,U,1,1)
      SS(K) = SS(K) - A
      K = KK(L,U,2,2)
      SS(K) = SS(K) + P
      K = KK(L,U,3,3)
      SS(K) = SS(K) + Q
      K = KK(L,V,2,2)
      SS(K) = SS(K) - E
      K = KK(L,V,2,3)
      SS(K) = SS(K) + E*XL
      K = KK(L,V,3,1)
      SS(K) = SS(K) + D
      K = KK(L,V,3,3)
      SS(K) = SS(K) + D*ZL
      K = KK(L,L,1,1)
      SS(K) = SS(K) + A
      K= KK(L,L,2,2)
      SS(K) = SS(K) + B
      K = KK(L,L,3,3)
```

```
      SS(K) = SS(K) + C
      K = KK(M,U,1,3)
      SS(K) = SS(K) - D
      K = KK(M,U,2,2)
      SS(K) = SS(K) + E
      K = KK(M,U,3,2)
      SS(K) = SS(K) - XL*E
      K = KK(M,U,3,3)
      SS(K) = SS(K) - ZL*D
      K = KK(M,V,1,1)
      SS(K) = SS(K) - F
      K = KK(M,V,1,3)
      SS(K) = SS(K) - ZL*F
      K = KK(M,V,2,2)
      SS(K) = SS(K) - G
      K = KK(M,V,2,3)
      SS(K) = SS(K) + XL*G
      K = KK(M,V,3,1)
      SS(K) = SS(K) - ZL*F
      K = KK(M,V,3,2)
      SS(K) = SS(K) + XL*G
      K= KK(M,V,3,3)
      SS(K) = SS(K) - ZL**2*F -XL**2*G - H
      K= KK(M,L,1,3)
      SS(K) = SS(K) - D
      K = KK(M,L,2,2)
      SS(K) = SS(K) + E
      K = KK(M,L,3,2)
      SS(K) = SS(K) - XL*E
      K = KK(M,L,3,3)
      SS(K) = SS(K) - ZL*D
      K = KK(M,M,1,1)
      SS(K) = SS(K) + F
      K = KK(M,M,2,2)
      SS(K) = SS(K) + G
      K = KK(M,M,3,1)
      SS(K) = SS(K) + ZL*F
      K = KK(M,M,3,2)
      SS(K) = SS(K) - XL*G
      K = KK (M,M,3,3)
      SS(K) = SS(K) + (ZL**2)*F + (XL**2)*G + H
310   CONTINUE
      MM = MN + 1
      MN = (N+1)*NFC/NST
300   CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE COMPUTES THE STIFFNESS COEFFICIENTS
C     FOR EACH COLUMN MEMBER
C
C
```

```
      SUBROUTINE   CELM(J,CN,ICP,INC,A,B,C,D,E,F,G,H,
     1P,Q,U,L,V,M,ZU,XU,ZL,XL)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /MAT/ EM, GM
      COMMON /SECT/ BP(4,20), CP(4,20)
      DIMENSION  CN(3,NTJ), INC(2,NFC), ICP(NFC)
      INTEGER  V, M, U, L
      REAL *8 I1, I2, I3
C
C
      MP = ICP(J)
      I1 = CP(1,MP)
      I2 = CP(2,MP)
      I3 = CP(3,MP)
      AA = CP(4,MP)
      L = INC(1,J)
      U = INC(2,J)
      X1 = CN(1,L)
      Y1 = CN(2,L)
      Z1 = CN(3,L)
      X2 = CN(1,U)
      Y2 = CN(2,U)
      Z2 = CN(3,U)
      X3 = CN(1,M)
      Y3 = CN(2,M)
      Z3 = CN(3,M)
      X4 = CN(1,V)
      Y4 = CN(2,V)
      Z4 = CN(3,V)
      DL = DABS(Y2 - Y1)
      IF  ( DL.GT. 10**(-6) ) GOTO 20
      WRITE ( 6,100) M,L,U,DL
100   FORMAT (//////'  ELEMENT',I4,'  CONNECTING
     1NODES',I4,' AND',I4,/'HAS LENGTH OF ',F20.7,
     1' CHECK COORIDINATES'///)
      CALL EXIT
20    ZU = Z2 - Z4
      XU = X2 - X4
      ZL = Z1 - Z3
      XL = X1 - X3
      P = 2.0*EM*I2/DL
      B = 2.0*P
      E = 1.5*B/DL
      G = 2.0*E/DL
      Q = 2.0*EM*I3/DL
      C = 2.0*Q
      D = 1.5*C/DL
      F = 2.0*D/DL
      H = GM*I1/DL
      A = AA*EM/DL
      RETURN
      END
C
```

```
C
C
C     THIS SUBROUTINE PERFORMS AN ASSEMBLY OF THE
C     STIFFNESS COEFFICIENTS FOR EACH INDIVIDUAL BEAM
C     MEMBER INTO THE STRUCTURE STIFFNESS MATRIX.
C
C
      SUBROUTINE  BEAM( CN, INB, IBP, SC, SS )
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON  /GEN/ NTJ, NFB, NFC, NBAND, NEQ,NXM
      COMMON  /IPT/NST, NBAY1, NBAY3
      COMMON /MAT/ EM, GM
      COMMON /SECT/ BP(4,20), CP(4,20)
      DIMENSION  CN(3,NTJ), INB(2,NFB), SS(NXM), IBP(NFB), SC(2,NFB)
C
C
      KK(L1,M1,L3,M3) = (3*L1 - 3*M1 + L3 - M3)*NEQ + 3*M1+M3-3
      MM = 1
      MN = NFB/NST
      DO   100  N = 1, NST
      DO   200  M = MM, MN
C
C
C    SA, SB ARE THE CONNECTION STIFFNESS AT END A AND B
C    OF BEAM. IF THE CONNECTION IS ASSUMED TO BE RIGID,
C    SA AND SB IS SET TO A VERY LARGE NUMBER IN SUBROUTINE CINFO.
C
      SA=SC(1,M)
      SB=SC(2,M)
      CALL   BELM( M, CN,INB, IBP, SA, SB, S1, S2, S3, S4, S5, S22,
     1 S44, R11, R22, R23,R32, R33, I, J )
      K = KK(J,J,1,1)
      SS(K) = SS(K) + R11*S1*R11
      K = KK(J,J,2,1)
      SS(K) = SS(K) + R11*(-S2)*R23
      K = KK(J,J,2,2)
      SS(K) = SS(K) + R22*S3*R22 + R23*S4*R23
      K = KK(J,J,3,1)
      SS(K) = SS(K) + R33*(-S2)*R11
      K = KK(J,J,3,2)
      SS(K) = SS(K) + R32*S3*R22 + R33*S4*R23
      K = KK(J,J,3,3)
      SS(K) = SS(K) + R32*S3*R32 + R33*S4*R33
      K = KK(J,I,1,1)
      SS(K) = SS(K) + R11*(-S1)*R11
      K = KK(J,I,1,2)
      SS(K) = SS(K) + R11*(-S22)*R23
      K= KK(J,I,1,3)
      SS(K) = SS(K) + R11*(-S22)*R33
      K = KK(J,I,2,1)
      SS(K) = SS(K) +   R23 *S2*R11
      K = KK(J,I,2,2)
      SS(K) = SS(K) + R22*(-S3)*R22 + R23*S5*R23
      K = KK(J,I,2,3)
```

```
      SS(K) = SS(K) + R22*(-S3)*R32 + R23*S5*R33
      K = KK(J,I,3,1)
      SS(K) = SS(K) +   R33 *S2*R11
      K = KK(J,I,3,2)
      SS(K) = SS(K) + R32*(-S3)*R22 + R33*S5*R23
      K = KK(J,I,3,3)
      SS(K) = SS(K) + R32*(-S3)*R32 + R33*S5*R33
      K = KK(I,I,1,1)
      SS(K) = SS(K) + R11*S1*R11
      K = KK(I,I,2,1)
      SS(K) = SS(K) + R23*S22*R11
      K = KK(I,I,2,2)
      SS(K) = SS(K) + R22*S3*R22 + R23*S44*R23
      K = KK(I,I,3,1)
      SS(K) = SS(K) + R33*S22*R11
      K = KK(I,I,3,2)
      SS(K) = SS(K) + R32*S3*R22 + R33*S44*R23
      K = KK(I,I,3,3)
      SS(K) = SS(K) + R32*S3*R32 + R33*S44*R33
200    CONTINUE
      MM = MN + 1
      MN = (N + 1)*NFB/NST
100   CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE COMPUTES THE STIFFNESS COEFFICIENTS
C     FOR EACH BEAM MEMBER.
C
C
      SUBROUTINE   BELM( M, CN,INB, IBP, SA, SB, S1, S2, S3, S4, S5,
     1S22, S44, R11, R22, R23, R32, R33, I, J)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON  /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /MAT/EM, GM
      COMMON /SECT/ BP(4,20), CP(4,20)
      DIMENSION CN(3,NTJ), INB(2,NFB), IBP(NFB)
      REAL*8 I1, I2, I3
C
C
      MP = IBP(M)
      I1 = BP(1,MP)
      I2 = BP(2,MP)
      I3 = BP(3,MP)
      AA=  BP(4,MP)
      I = INB(1,M)
      J = INB(2,M)
      X1 = CN(1,I)
      Y1 = CN(2,I)
      Z1 = CN(3,I)
      X2 = CN(1,J)
      Y2 = CN(2,J)
```

```
      Z2 = CN(3,J)
      R11 =1.
      R22 = 1.0
      R23 = 0.D0
      R32 = 0.D0
      R33 = 1.
      IF  ( Z1 .EQ. Z2 )  GOTO 10
      DL = DABS(Z1 - Z2)
      R22 = 0.
      R23 = -1.
      R32 = 1.
      R33 = 0.D0
      GOTO  20
10    DL = DABS(X1 - X2)
20    CONTINUE
      COM=DL**4/(12.*EM**2*I3**2)+DL**3/(3.*EM*I3*SA)
     & + DL**3/(3.*EM*I3*SB) + DL**2/(SA*SB)
      S5=(DL**3/(6.*EM*I3))/COM
      S2=(DL**2/(2.*EM*I3)+DL/SA)/COM
      S4=(DL**3/(3.*EM*I3)+DL**2/SA)/COM
      S22=(DL**2/(2.*EM*I3)+DL/SB)/COM
      S3=GM*I1/DL
      S44=(DL**3/(3.*EM*I3)+DL**2/SB)/COM
      S1=(DL/(EM*I3)+1./SA+1./SB)/COM
      RETURN
      END
C
C
C
C     THIS SUBROUTINE SETS THE SUPPORT CONDITION BY ASSUMING
C     THAT THE BUILDING IS RIGIDLY CONNECTED TO THE FOUNDATION
C
C
      SUBROUTINE   SPT( SS, JL )
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON  /GEN/NTJ,NFB,NFC,NBAND,NEQ,NXM
      COMMON  /IPT/NST, NBAY1, NBAY3
      DIMENSION  SS(NXM),  JL(3,NTJ)
      REAL*8JL
C
C
      KK(L1,M1,L3,M3) = (3*L1 - 3*M1 + L3 - M3)*NEQ + 3*M1 + M3 -3
      NS = NTJ/(NST + 1) *NST + 1
      DO  5  N = NS, NTJ
      DO  6  L = 1,3
      K = KK(N,N,L,L)
      SS(K) = 1.0D20
      JL(L,N) = JL(L,N)*1.0D20
 6    CONTINUE
 5    CONTINUE
      RETURN
      END
C
C
```

```
C
C     THIS SUBROUTINE SOLVES THE LINEAR FORCE-DISPLACEMENT
C     EQUATIONS BY CHOLESKY-DECOMPOSITION METHOD.
C
C
      SUBROUTINE  EQN (CN, A, JD )
      IMPLICIT REAL *8 (A-H,O-Z)
      COMMON  /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /IPT/ NST, NBAY1, NBAY3
      COMMON /UNITS/ UNIT1,UNIT2,UNIT3,UNIT4,UNIT5,UNIT6,UNIT7
      DIMENSION  A(NXM), CN(3,NTJ)
      REAL *8 JD(NEQ)
C
C
      NM=NEQ-1
      MS = NBAND
      JWT = 6
      DO 200  K=1,NM
      PIVOT = A(K)
      MR = MIN0(MS,NEQ-K+1)
      DO 180 L=2,MR
      LK = (L-1)*NEQ+K
      C = A(LK)/PIVOT
      IF(C)   150,180,150
 150  I=0
      J=K+L-1
      DO 160 JL=L,MR
      I=I+1
      IJ=(I-1)*NEQ+J
      JLK=(JL-1)*NEQ+K
  160 A(IJ) = A(IJ)-A(JLK)*C
  180 A(LK) = C
  200 CONTINUE
C
C     ' FORWARD SUBSTITUTION '  (LD)*Y=B TO STORE Y IN JD O
C
      M1 = MS-1
      JD(1) = JD(1)/A(1)
      DO 20 I=2,NEQ
      L=MAX0(1,I-M1)
      SUM =0.
      II=I+1
      I1=I-1
      DO 10 K=L,I1
      J=II-K
      JK=(J-1)*NEQ+K
      SUM=SUM+A(JK)*JD(K)*A(K)
 10   CONTINUE
   20 JD(I)=(JD(I)-SUM)/A(I)
C
C     ' BACK SUBSTITUTION '  LTR*X=Y  TO STORE X IN JD O
C
      DO 40 KK=1,NM
      I=NEQ-KK
```

```
      I1=MIN0(NEQ,I+M1)
      SUM=0.
      L =I+1
      DO 30 K=L,I1
      J=K-I+1
      JI=(J-1)*NEQ+I
      SUM = SUM+A(JI)*JD(K)
   30 CONTINUE
      JD(I) = JD(I)-SUM
   40 CONTINUE
      WRITE(JWT,300) UNIT1
  300 FORMAT ('1',//' ******* JOINT DISPLACEMENTS ',A8,'AND (RADIAN)'/
     &' JOINT',7X,'TRANS X1',1H',7X,'TRANS X2',1H',7X,'TRANS X3',1H',9X,
     &'ROT-X1',1H',9X,'ROT-X2',1H',9X,'ROT-X3',1H')
C
C     TRANSLATE DISPLACEMENT AT MASTER NODE TO
C     FOR PRINTING DISPLACEMENT VALUES
C
C
       NSTT=NST+1
       NODE=1
C
       DO  55  N=1,  NSTT
       MSN=N*NTJ/NSTT
       NMSN=MSN-1
C
       K1=3*MSN-2
       K2=K1+1
       K3=K2+1
       TX=JD(K1)
       TZ=JD(K2)
       RY=JD(K3)
C
       DO  60  J=NODE,  NMSN
       ZU=CN(3,J)-CN(3,MSN)
       XU=CN(1,J)-CN(1,MSN)
       TRANX=TX+ZU*RY
       TRANZ=TZ-XU*RY
       I1=3*J-2
       I2=I1+1
       I3=I2+1
       WRITE(6,310) J, TRANX,JD(I1),TRANZ,JD(I2),RY,JD(I3)
 60    CONTINUE
       NODE=MSN+1
 55    CONTINUE
 310   FORMAT(I5,1X,3F15.2,3F15.5)
       RETURN
       END
C
C
C
C      THIS SUBROUTINE COMPUTE THE BEAM END FORCES.
C
C
```

```
      SUBROUTINE  BMF ( CN, INB, IBP, SC, JL, BFA, BFB )
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON  /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /SECT/ BP(4,20), CP(4,20)
      COMMON /CON/ FACTOR, FAC2, FAC3, FAC4
      COMMON /MAT/ EM,GM
      COMMON /UNITS/ UNIT1,UNIT2,UNIT3,UNIT4,UNIT5,UNIT6,UNIT7
      DIMENSION  CN(3,NTJ), INB(2,NFB), JL(3,NTJ), BFA(3,NFB),
     1 BFB(3,NFB), IBP(NFB), SC(2,NFB)
      REAL*8 JL
C
C
      DO  30  M=1, NFB
      SA=SC(1,M)
      SB=SC(2,M)
      CALL  BELM(M, CN,INB, IBP, SA, SB, S1, S2, S3, S4, S5,
     & S22, S44, R11, R22, R23, R32, R33, I, J)
      F1 = S1*R11*(JL(1,J) - JL(1,I)) - R23*(S2*JL(2,J)+S22*JL(2,I))
     1 - R33*(S2*JL(3,J)+S22*JL(3,I))
      BFA(1,M) = BFA(1,M) - F1
      BFB(1,M) = BFB(1,M) + F1
      F1 = S3*R22*(JL(2,J) - JL(2,I)) + S3*R32*(JL(3,J) - JL(3,I))
      BFA(2,M) = BFA(2,M) - F1
      BFB(2,M) = BFB(2,M) + F1
      BFB(3,M) = BFB(3,M) - R11*S2*(JL(1,J) - JL(1,I))
     1 + R23*(S4*JL(2,J) + S5*JL(2,I)) + R33*( S4*JL(3,J) +
     1 S5*JL(3,I))
      BFA(3,M) = BFA(3,M) - R11*S22*(JL(1,J) -JL(1,I))
     1 + R23*(S5*JL(2,J) + S44*JL(2,I)) + R33*(S5*JL(3,J)
     1 + S44*JL(3,I))
30    CONTINUE
      WRITE(6,35) UNIT5, UNIT6, UNIT6, UNIT5, UNIT6, UNIT6
35    FORMAT(////' *****   BEAM  END  FORCES  AND  MOMENTS   *****'//
     &12X,'****  A-END ****',50X,'****  B-END  ****'/
     &' BEAM NO.',3X,'JOINT',5X,'P2',A8,5X,'M1',A8,5X,'M3',A8,5X,'JOINT'
     &5X,'P2',A8,5X,'M1',A8,5X,'M3',A8)
      DO  40  K = 1, NFB
      BFA(2,K)=BFA(2,K)/FACTOR
      BFA(3,K)=BFA(3,K)/FACTOR
      BFB(2,K)=BFB(2,K)/FACTOR
      BFB(3,K)=BFB(3,K)/FACTOR
      WRITE(6,45) K,INB(1,K),( BFA(J,K), J=1,3),INB(2,K),(BFB(J,K), J
     & =1, 3)
45    FORMAT(I5,6X,I4,1X,3F15.2,5X,I4,1X,3F15.2)
40    CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE COMPUTES THE COLUMN END FORCES
C     AND MOMENTS FOR EACH COLUMN.  THE END FORCES AND MOMENTS
C     ARE PRINTED AS OUT-OF-PLANE AND IN-PLANE VECTORS SEPERATELY.
C
```

```
C
      SUBROUTINE CMF (CN, INC, ICP, JL, CFA, CFB, CIA, CIB)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON  /IPT/ NST, NBAY1,NBAY3
      COMMON  /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /SECT/ BP(4,20), CP(4,20)
      COMMON /CON/ FACTOR, FAC2, FAC3, FAC4
      COMMON /UNITS/ UNIT1, UNIT2, UNIT3, UNIT4, UNIT5, UNIT6, UNIT7
      DIMENSION  CN(3,NTJ), INC(2,NFC), JL(3,NTJ),
     1 CFA(3,NFC), CFB(3,NFC),CIA(3,NFC), CIB(3,NFC), ICP(NFC)
      REAL*8   JL
      INTEGER V,M,U,L
C
C
      DO  10  K = 1, NFC
      DO  20  KK = 1, 3
      CFA(KK,K) = 0.D0
      CFB(KK,K) = 0.D0
      CIA(KK,K) = 0.D0
      CIB(KK,K) = 0.D0
 20   CONTINUE
 10   CONTINUE
      MM = 1
      MN = NFC/NST
      DO  30  N = 1, NST
      M = ( N+1 )*NTJ/ (NST + 1)
      V = N*NTJ/(NST + 1)
      DO  40  J = MM, MN
      CALL  CELM ( J, CN,ICP, INC, A, B, C, D, E, F, G, H,
     1  P, Q, U, L, V, M, ZU, XU, ZL, XL )
      F1 = A*(JL(1,U) - JL(1,L))
      CFB(1,J) = F1 + CFB(1,J)
      CFA(1,J) = - F1 + CFA(1,J)
      CFB(2,J) = CFB(2, J) + E*((JL(2,V) - JL(3,V)*XU) -
     1  (JL(2,M) - JL(3,M)*XL)) - B*JL(2,U) - P*JL(2,L)
      CFA(2,J) = CFA(2,J) + E*((JL(2,V)  - JL(3,V)*XU) -
     1 (JL(2,M) - JL(3,M)*XL)) - P*JL(2,U) - B*JL(2,L)
      CFB(3,J) = CFB(3,J) + D*((-JL(1,M) - JL(3,M)*ZL) -
     1 (-JL(1,V) - JL(3,V)*ZU)) + C*JL(3,U) + Q*JL(3,L)
      CFA(3,J) = CFA(3,J) + D*((-JL(1,M) - JL(3,M)*ZL) -
     1  (-JL(1,V) - JL(3,V)*ZU)) + Q*JL(3,U) + C*JL(3,L)
      F1 = F*( -JL(1,V) - JL(3,V)*ZU) - D*(JL(3,U) + JL(3,L))
     1  - F*(-JL(1,M) - JL(3,M)*ZL)
       CIB(1,J) = CIB(1,J) + F1
       CIA(1,J) = CIA(1,J) - F1
      F1 = G*( JL(2,V) - JL(3,V)*XU) + E*(-JL(2,U))
     1- G*(JL(2,M) - JL(3,M)*XL) + E*(-JL(2,L))
       CIB(2,J) = CIB(2,J) + F1
       CIA(2,J) = CIA(2,J) - F1
      F1 = H*(JL(3,V) - JL(3,M))
      CIB(3,J) = CIB(3,J) + F1
      CIA(3,J) = CIA(3,J) - F1
 40   CONTINUE
      MM = MN + 1
```

```
      MN = (N+1)*NFC/NST
30    CONTINUE
      WRITE(6,35) UNIT5, UNIT6, UNIT6, UNIT5, UNIT6, UNIT6
35    FORMAT(////' ****  COLUMN OUT-OF-PLANE END FORCES AND MOMENTS '//
     &12X,'****  L-END  ****',50X,'****  U-END  ****'/
     &' COLUMN',4X,'JOINT',5X,'P1',A8,5X,'M2',A8,5X,'M3',A8,5X,'JOINT',5
     &X,'P1',A8,5X,'M2',A8,5X,'M3',A8)
      DO  50  NN = 1, NFC
      CFA(2,NN)=CFA(2,NN)/FACTOR
      CFA(3,NN)=CFA(3,NN)/FACTOR
      CFB(2,NN)=CFB(2,NN)/FACTOR
      CFB(3,NN)=CFB(3,NN)/FACTOR
      WRITE(6,36)NN,INC(1,NN),(CFA(K,NN),K=1,3),INC(2,NN),
     & (CFB(K,NN),K=1,3)
 36   FORMAT(I5,6X,I4,1X,3F15.2,5X,I4,1X,3F15.2)
50    CONTINUE
      WRITE(6,45) UNIT5, UNIT5, UNIT6, UNIT5, UNIT5, UNIT6
45    FORMAT(////' ****  COLUMN  IN-PLANE  END  FORCES  AND MOMENTS '//
     &12X,'****  L-END  ****',50X,'****  U-END  ****'/
     &' COLUMN',4X,'JOINT',5X,'P1',A8,5X,'M2',A8,5X,'M3',A8,5X,'JOINT',5
     &X,'P1',A8,5X,'M2',A8,5X,'M3',A8)
      DO 60  L=1, NFC
      CIA(3,L)=CIA(3,L)/FACTOR
      CIB(3,L)=CIB(3,L)/FACTOR
      WRITE(6,46)L,INC(1,L),(CIA(K,L),K=1,3),INC(2,L),
     & (CIB(K,L),K=1,3)
 46   FORMAT(I5,6X,I4,1X,3F15.2,5X,I4,1X,3F15.2)
60    CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE PROCESSES THE CONNECTION
C     INFORMATION OR THE ASSUMED CONNECTION STIFFNESS
C     INPUT TO ANALYZE THE STRUCTURE CONSIDERED.
C
C
C     NOTE: THE RAMBERG OSGOOD STANDARDIZED FUNCTION
C     FOR THE VARIOUS CONNECTION TYPES IS DERIVED
C     USING EXPERIMENTAL DATA RECORED IN IMPERIAL UNIT.
C     HOWEVER NECESSARY CONVERSION IS MADE WHEN THE STRUCTURE
C     IS IN SI UNIT.
C
C
      SUBROUTINE CINFO( ICT, FK, SC, BFA, BFB,NTA, SSI )
C
      IMPLICIT REAL*8  (A-H,O-Z)
      COMMON /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /MAT/ EM, GM
      COMMON /CON/ FACTOR, FAC2, FAC3, FAC4
      DIMENSION ICT(2,NFB), FK(2,NFB), SC(2,NFB), BFA(3,NFB),
     & BFB(3,NFB)
      LOGICAL SSI
```

```
C
      FAC1=1.
      IF (SSI) FAC1=FACTOR
      IF (NTA .EQ. 0) GOTO 200
      IF (NTA.EQ.1) GOTO 300
      IF (NTA.EQ.2) GOTO 400
      WRITE(6,100)
100   FORMAT(////,' CONNECTION INFORMATION AS INPUT'/
     & ' *****    GEOMETRIC  PARAMETER  AS  INPUT  *****'/
     &' CONNECTION TYPE: 1=SINGLE WEB ANGLE CONNECTION'/
     &'                 ','2=DOUBLE WEB ANGLE CONNECTION'/
     &'                 ','3=HEADER PLATE ANGLE CONNECTION'/
     &'                 ','4=TOP AND SEAT ANGLE CONNECTION'/
     &'                 ','5=STRAP ANGLE CONNECTION'/
     &' BEAM',5X,'CONNECTION TYPE',5X,'PARAMETER 1',5X,'PARAMETER 2',5X
     &,'PARAMETER 3',5X,'PARAMETER 4')
C
C
C     THE PARAMETERS P1, P2, P3, P4 FOR THE VARIOUS
C     TYPES OF CONNECTION ARE THE FOLLOWING:
C
C     SINGLE WEB ANGLE:
C        P1 = DEPTH OF WEB ANGLE
C        P2 = GAGE OF COLUMN
C        P3 = THICKNESS OF ANGLE
C
C     DOUBLE WEB ANGLE:
C        P1 = DEPTH OF WEB ANGLE
C        P2 = THICKNESS OF ANGLE
C        P3 = GAGE OF COLUMN
C
C     HEADER PLATE:
C        P1 = THICKNESS OF PLATE
C        P2 = GAGE OF COLUMN
C        P3 = LENGTH OF PLATE
C        P4 = THICKNESS OF BEAM WEB
C
C     TOP AND SEAT ANGLE:
C        P1 = DEPTH OF BEAM
C        P2 = THICKNESS OF PLATE
C        P3 = LENGTH OF TOP ANGLE
C        P4 = DIAMETER OF BOLT
C
C     STRAP ANGLE CONNECTION:
C        P1 = WIDTH OF STRAP ANGLE
C        P2 = THICKNESS OF STRAP ANGLE
C        P3 = H/P
C
C
      DO 10  M=1, NFB
      DO 20  I=1, 2
      READ(5,*) ICT(I,M), P1, P2, P3, P4
      WRITE(6,101)   M,  ICT(I,M), P1, P2, P3, P4
101   FORMAT(I5,5X,11X,I3,6X,F9.2,7X,F9.2,7X,F9.2,7X,F9.2)
```

```
      IF (SSI) GOTO 2
      GOTO 4
2     P1=P1*FAC2
      P2=P2*FAC2
      P3=P3*FAC2
      P4=P4*FAC2
C
4     IC=ICT(I,M)
      GOTO( 5, 15, 25, 35, 45 ),IC
C
C - - -  SINGLE WEB ANGLE CONNECTION
C
5     E01=0.01033
      Q01=32.7476
      AN1=3.93419
      A1=-2.09354
      A2=2.06227
      A3=-1.63744
      FK(I,M)=P1**A1*P2**A2*P3**A3
      SC(I,M)=(Q01/(FK(I,M)*E01)/FAC4)*FAC1
      GOTO 20
C
C - - -  DOUBLE WEB ANGLE CONNECTION
C
15    E02=0.00398
      Q02=0.63324
      AN2=4.94052
      A1=-2.21594
      A2=0.07617
      A3=-0.2750
      FK(I,M)=P1**A1*P2**A2*P3**A3
      SC(I,M)=(Q02/(FK(I,M)*E02)/FAC4)*FAC1
      GOTO  20
C
C - - -  HEADER PLATE CONNECTION
C
25    E03=0.00704
      Q03=186.77160
      AN3=4.32072
      A1=-1.54304
      A2=2.12088
      A3=-2.40526
      A4=-0.45137
      FK(I,M)=P1**A1*P2**A2*P3**A3*P4**A4
      SC(I,M)=(Q03/(FK(I,M)*E03)/FAC4)*FAC1
      GOTO  20
C
C - - -  TOP AND SEAT ANGLE CONNECTION
C
35    E04=0.00517
      Q04=745.93994
      AN4=4.61637
      A1=-1.06054
      A2=-0.53970
```

```
      A3=0.85885
      A4=-1.27911
      FK(I,M)=P1**A1*P2**A2*P3**A3*P4**A4
      SC(I,M)=(Q04/(FK(I,M)*E04)/FAC4)*FAC1
      GOTO  20
C
C - - -  STRAP ANGLE CONNECTION
C
45    E05=0.00458
      Q05=753.25757
      AN5=4.98240
      A1=-0.59211
      A2=-0.85055
      A3=-1.06018
      FK(I,M)=P1**A1*P2**A2*P3**A3
      SC(I,M)=(Q05/(FK(I,M)*E05)/FAC4)*FAC1
C
20    CONTINUE
10    CONTINUE
      RETURN
C
C
C
      ENTRY  ITER(   SC, BFA, BFB, NCOUNT )
      NCOUNT=0
      DO  30 M=1,  NFB
      DO  40 I=1,  2
      IC=ICT(I,M)
      AM=DABS(BFA(3,M))*FAC3*12.
      IF ( I.EQ.2 ) AM=DABS(BFB(3,M))*FAC3*12.
      GOTO( 55, 65, 75, 85, 95 ),IC
C
C - - -  SINGLE WEB ANGLE CONNECTION
C
55    R=(E01*AM*FK(I,M)/Q01)*(1. + (AM*FK(I,M)/Q01)**(AN1-1.))
      RP=(AM/(SC(I,M)*FAC4))*FAC1
      GOTO  50
C
C - - -  DOUBLE WEB ANGLE CONNECTION
C
65    R=(E02*AM*FK(I,M)/Q02)*(1. + (AM*FK(I,M)/Q02)**(AN2-1.))
      RP=(AM/(SC(I,M)*FAC4))*FAC1
      WRITE(6,16) R, RP, AM, FK(I,M), SC(I,M)
16    FORMAT('  CONNECTION ROTATION R AND RP', 5F20.5)
      GOTO  50
C
C - - -  HEADER PLATE CONNECTION
C
75    R=(E03*AM*FK(I,M)/Q03)*(1. + (AM*FK(I,M)/Q03)**(AN3-1.))
      RP=(AM/(SC(I,M)*FAC4))*FAC1
      GOTO  50
C
C - - -  TOP AND SEAT ANGLE
C
```

```
85     R=(E04*AM*FK(I,M)/Q04)*(1. + (AM*FK(I,M)/Q04)**(AN4-1.))
       RP=(AM/(SC(I,M)*FAC4))*FAC1
       GOTO  50
C
C - - -  STRAP ANGLE CONNECTION
C
95     R=(E05*AM*FK(I,M)/Q05)*(1. + (AM*FK(I,M)/Q05)**(AN-1.))
        RP=(AM/(SC(I,M)*FAC4))*FAC1
C
50     DR=DABS(R-RP)/R
       IF ( DR.LT.0.07 ) GOTO 40
       SC(I,M)=(2.*AM/(R+RP)/FAC4)*FAC1
       NCOUNT=NCOUNT+1
40     CONTINUE
30     CONTINUE
       RETURN
C
200    CONTINUE
C
C      SET CONNECTION STIFFNESS TO BE VERY FELIXIBLE
C
       SA=1.E-3
       SB=1.E-3
       GOTO 350
C
C      SET CONNECTION STIFFNESS TO BE VERY RIGID
C
300    SA=1.0E25
       SB=1.0E25
350    DO 110 M=1, NFB
       SC(1,M)=SA
       SC(2,M)=SB
 110   CONTINUE
       RETURN
400    CONTINUE
C
C      INPUT ANY SPECIFIED CONNECTION STIFFNESS
C      VALUES FOR BOTH ENDS  OF A BEAM MEMBER.
C
       WRITE(6,104)
104    FORMAT(////' ***** INPUT CONNECTION STIFFNESS AT BOTH ENDS OF A BE
      &AM'/
      &' BEAM',5X,'STIFFNESS @ A-END',5X,'STIFFNESS @ B-END')
       DO  450 M=1, NFB
       READ(5,*) MM ,SA, SB
       WRITE(6,105) MM, SA, SB
105    FORMAT(I5,5X,F12.2,10X,F12.2)
       SC(1,M)=SA
       SC(2,M)=SB
450    CONTINUE
       RETURN
       END
C
C
```

```
C
C     SUBROUTINE LOAD PROCESSES LOADING INFORMATION.
C     FIXED END FORCE VECTOR IS COMPUTED.
C     ENTRY STATEMENT IS USED FOR RELOAD THE STRUCTURE
C     FOR NONLINEAR  ANALYSIS.
C
C
      SUBROUTINE LOAD(CN, INB, IBP, SC, JD, AJL, BFA, BFB, AMLC,
     & AMLU, MU, MJ, MC )
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /GEN/NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /MAT/EM, GM
      COMMON /SECT/BP(4,20), CP(4,20)
      COMMON /CON/ FACTOR, FAC2, FAC3, FAC4
      COMMON /ALOAD/NLJ, NLMC, NLMU, NLT1, NLT2, NLT3
      COMMON /UNITS/UNIT1, UNIT2, UNIT3, UNIT4, UNIT5, UNIT6, UNIT7
      DIMENSION CN(3,NTJ), INB(3,NFB), IBP(NFB), SC(2,NFB), JD(3,NTJ),
     & AJL(3,NLT1), BFA(3,NFB), BFB(3,NFB), AMLC(2,NLT2),
     & AMLU(NLT3), MU(NLT3), MC(NLT2), MJ(NLT1)
      REAL*8 JD, I3
C
C
      WRITE(6,1000)
1000  FORMAT(/////' **** INPUT LOADING INFORMATION ****')
C
      IF(NLJ.EQ.0)GOTO 100
C
C
C     INPUT JOINT LOAD AND STORE IN A TEMPORARY
C     JOINT LOAD VECTOR.
C
      WRITE(6,1001) UNIT5, UNIT5,UNIT6,UNIT5,UNIT6
1001  FORMAT(/' LOADED JOINT NO.',5X,'FORCE',A8,5X,'FORCE',A8,' OR MOMEN
     C MOMENT',A8)
C
      DO 10 J=1,NLJ
      READ(5,*) K, P, T, B
      WRITE(6,1002)K,P,T,B
1002  FORMAT(12X,I3,6X,F8.2,18X,F10.2,20X,F10.2)
      MJ(J)=K
      AJL(1,J)=P
      AJL(2,J)=T
      AJL(3,J)=B
10    CONTINUE
C
100   IF(NLMC.EQ.0) GOTO 200
C
C
C     INPUT  CONCENTRATED GRAVITY LOAD ON BEAM
C     AND STORE IN A TEMPORARY BEAM LOAD VECTOR
C
      WRITE(1003) UNIT5, UNIT2
1003  FORMAT(//' CONCENTRATED GRAVITY LOAD ON BEAM'/
     &     ' BEAM NO.',5X,'LOAD',A8,5X,'DISTANCE FROM A-END',A8)
```

```
C
      DO 20 M=1, NLMC
      READ(5,*) K, P, AL
      WRITE(6,1004)K,P,AL
1004  FORMAT(4X,I5,5X,F10.2,15X,F10.2)
      MC(M)=K
      AMLC(1,M)=P
      AMLC(2,M)=AL*FACTOR
20    CONTINUE
C
200   IF(NLMU.EQ.0) GOTO 300
C
C
C     INPUT UNIFORMLY DISTRIBUTED LOAD ON BEAM
C     AND STORE IN TEMPORARY BEAM LOAD VECTOR
C
      WRITE(6,1005) UNIT7
1005  FORMAT(//' UNIFORMLY DISTRIBUTED LOAD ON BEAM'/
     &      ' BEAM NO.',5X,'LOAD',A8)
C
      DO 30 M=1, NLMU
      READ(5,*) K, P
      WRITE(6,1006)K, P
1006  FORMAT(4X,I5,5X,F10.2)
      MU(M)=K
      AMLU(M)=P/FACTOR
30    CONTINUE
C
300   CONTINUE
      ENTRY RELOAD(SC,JD,BFA,BFB)
C
C     ZERO BEAM END FORCE VECTOR
C
      DO 40 M=1, NFB
      DO 40 MM=1, 3
      BFA(MM,M)=0.0
      BFB(MM,M)=0.0
40    CONTINUE
C
C     ZERO JOINT LOAD VECTOR
C
      DO 50 J=1,NTJ
      DO 50 JJ=1,3
      JD(JJ,J)=0.0
50    CONTINUE
C
      IF(NLJ.EQ.0) GOTO 400
C
C     APPLY INPUT JOINT LOAD TO JOINT LOAD VECTOR
C
      DO 60 J=1, NLJ
      K=MJ(J)
      JD(1,K)=AJL(1,J)
      JD(2,K)=AJL(2,J)
```

```
      JD(3,K)=AJL(3,J)
60    CONTINUE
C
400   IF (NLMC.EQ.0) GOTO 500
C
C     COMPUTE FIXED END FORCE DUE TO CONCENTRATED GRAVITY LOAD,
C     AND STORE IN FIXED END FORCE VECTOR AND ADD THE NEGATIVE
C     TO THE JOINT LOAD VECTOR.
C
      DO 70 M=1,NLMC
      K=MC(M)
      P=AMLC(1,M)
      AL=AMLC(2,M)
      SA=SC(1,K)
      SB=SC(2,K)
      CALL BELM(K,CN,INB,IBP,SA,SB,S1,S2,S3,S4,S5,S22,S44,R11,R22,
     & R23,R32,R33,I,J)
      IR2=R22
      IR3=R23
      IF(IR2.NE.0) DL=DABS(CN(1,I)-CN(1,J))
      IF(IR3.NE.0) DL=DABS(CN(3,I)-CN(3,J))
      BL=DL-AL
      MP=IBP(K)
      I3=BP(3,MP)
      DB1=(P*AL**3)/(3.*EM*I3)+(P*AL**2*BL)/(2.*EM*I3)+(P*DL*AL)/SA
      DB3=P*AL**2/(2.*EM*I3)+(P*AL)/SA
      PBF1=S2*DB3-S1*DB1
      PBF3=S2*DB1-S4*DB3
      PAF1=-(P+PBF1)
      PAF3=-(P*AL+DL*PBF1+PBF3)
      BFA(1,K)=BFA(1,K)+PAF1
      BFB(1,K)=BFB(1,K)+PBF1
      BFA(3,K)=BFA(3,K)+PAF3
      BFB(3,K)=BFB(3,K)+PBF3
      JD(1,I)=JD(1,I)-PAF1
      JD(1,J)=JD(1,J)-PBF1
      IF(IR3.NE.0)GOTO 65
      JD(3,I)=JD(3,I)-(PAF3*R33)
      JD(3,J)=JD(3,J)-(PBF3*R33)
      GOTO 70
65    CONTINUE
      JD(2,I)=JD(2,I)-(PAF3*R23)
      JD(2,J)=JD(2,J)-(PBF3*R23)
70    CONTINUE
C
500   IF(NLMU.EQ.0)RETURN
C
C     COMPUTE FIXED END FORCE DUE TO UNIFORMLY DISTRIBUTED
C     LOAD VECTOR, AND STORE IN THE BEAM END FORCE VECTOR.THE
C     NEGATIVE OF THE FIXED END FORCE VECTOR IS ADDED TO THE
C     JOINT LOAD VECTOR
C
      DO 80 M=1, NLMU
      K=MU(M)
```

```
      SA=SC(1,K)
      SB=SC(2,K)
      W=AMLU(M)
      MP=IBP(K)
      I3=BP(3,MP)
      CALL BELM(K,CN,INB,IBP,SA,SB,S1,S2,S3,S4,S5,S22,S44,R11,R22,
     & R23,R32,R33,I,J)
      IR2=R22
      IR3=R32
      IF(IR2.NE.0)DL=DABS(CN(1,I)-CN(1,J))
      IF(IR3.NE.0)DL=DABS(CN(3,I)-CN(3,J))
      DB1=(W*DL**4)/(8.*EM*I3)+(W*DL**3)/(2.*SA)
      DB3=(W*DL**3)/(6.*EM*I3)+(W*DL**2)/(2.*SA)
      WFB1=S2*DB3-S1*DB1
      WFB3=S2*DB1-S4*DB3
      WFA1=-(W*DL+WFB1)
      WFA3=-(W*DL**2/2.+DL*WFB1+WFB3)
      BFA(1,K)=BFA(1,K)+WFA1
      BFB(1,K)=BFB(1,K)+WFB1
      BFA(3,K)=BFA(3,K)+WFA3
      BFB(3,K)=BFB(3,K)+WFB3
      JD(1,I)=JD(1,I)-WFA1
      JD(1,J)=JD(1,J)-WFB1
      IF(IR3.NE.0) GOTO 75
      JD(3,I)=JD(3,I)-(WFA3*R33)
      JD(3,J)=JD(3,J)-(WFB3*R33)
      GOTO 80
75    CONTINUE
      JD(2,I)=JD(2,I)-(WFA3*R23)
      JD(2,J)=JD(2,J)-(WFB3*R23)
80    CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE TAKE IN THE BRACING ELEMENTS
C     END INCIDENCE TO THE JOINTS.
C
C
      SUBROUTINE  INTRS( INT,ITP )
      IMPLICIT  REAL*8 (A-H,O-Z)
      COMMON /TBRACE/ NFT, NBF
      COMMON /SECT2/ TP(20)
      COMMON /UNITS/ UNIT1, UNIT2, UNIT3, UNIT4, UNIT5, UNIT6, UNIT7
      DIMENSION  INT(2,NFT), ITP(NFT)
C
C
      WRITE(6,99)
 99    FORMAT(////' ***** BRACING ELEMENTS INCIDENCE AS INPUT *****'//
     &  ' BRACING MEMBER',5X,'LOWER END CONNECTED TO JOINT',5X,'UPPER EN
     &D CONNECTED TO JOINT')
C
C     INPUT BRACING INCIDENCE
```

```
C
      DO  100  M=1, NFT
      READ (5,*) N, INT(1,N), INT(2,N)
      WRITE(6,101) N , INT(1,N),INT(2,N)
101   FORMAT(7X,I5,29X,I4,27X,I4)
100   CONTINUE
C
C     INPUT BRACING PROPERTIES
C
      READ (5,*) NTP
      WRITE(6,102) NTP, UNIT3
102   FORMAT(/'  NUMBER OF BRACING TYPE-',I3//
     &' BRACING TYPE',5X,'X-SECTION AREA',A8)
      DO  104 N=1, NTP
      READ (5,*)  TP(N)
      WRITE(6,103) N, TP(N)
103   FORMAT(9X,I3,5X,F11.2)
 104  CONTINUE
C
C     INPUT BRACING PROPERTIES ID NO..
C
      READ (5,*) (ITP(N), N=1,NFT)
      WRITE(6,107)
107   FORMAT(' BRACING',5X,'BRACING TYPE')
      DO  108 N=1, NFT
      WRITE(6,109) N,  ITP(N)
109   FORMAT(4X,I3,13X,I3)
108   CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE PERFORMS AN ASSEMBLY OF THE STIFFNESS
C     COEFFICIENTS OF EACH INDIVIDUAL BRACING ELEMENT
C     INTO THE STRUCTURE STIFFNESS MATRIX.
C
C
      SUBROUTINE  BRACE( CN, INT, SS, ITP )
      IMPLICIT  REAL*8 (A-H,O-Z)
      COMMON /MAT/ EM, GM
      COMMON /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /TBRACE/ NFT, NBF
      COMMON /SECT2/ TP(20)
      COMMON /IPT/ NST, NBAY1, NBAY3
      INTEGER V,M,U,L
      DIMENSION  CN(3,NTJ), INT(2,NFT), SS(NXM), ITP(NFT)
C
C
      KK(L1,M1,L3,M3)=(3*L1-3*M1+L3-M3)*NEQ + 3*M1 + M3 - 3
      J=0
      DO 50 JJ=1, NBF
      DO 100 N=1,NST
      M=(N+1)*NTJ/(NST+1)
```

```
      V=N*NTJ/(NST+1)
      DO 200 NN=1,2
      J=J+1
C
C     KODE IS USED TO KEEP TRACK OF THE DEGREE OF FREEDOM
C     OF BRACING ELEMENTS IN TWO POSSIBLE ORIENTATION OF
C     BRACING IN A RECTANGULAR BUILDING
C
      CALL TELM( J, CN, ITP, INT, S1, S2, S3, U, L, KODE )
      K=KK(U,U,1,1)
      SS(K)=SS(K) + S2
      K=KK(V,U,KODE,1)
      SS(K)=SS(K) + S3
      K=KK(V,V,KODE,KODE)
      SS(K)=SS(K) + S1
      K=KK(L,U,1,1)
      SS(K)=SS(K) - S2
      K=KK(L,V,1,KODE)
      SS(K)=SS(K) - S3
      K=KK(L,L,1,1)
      SS(K)=SS(K) + S2
      K=KK(M,U,KODE,1)
      SS(K)=SS(K) - S3
      K=KK(M,V,KODE,KODE)
      SS(K)=SS(K) - S1
      K=KK(M,L,KODE,1)
      SS(K)=SS(K) + S3
      K=KK(M,M,KODE,KODE)
      SS(K)=SS(K) + S1
200   CONTINUE
100   CONTINUE
50    CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE COMPUTES THE STIFFNESS COEFICIENTS
C     OF EACH BRACING ELEMENT.
C
C
      SUBROUTINE  TELM(M, CN, ITP, INT, S1, S2, S3, U, L, KODE)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /MAT/ EM, GM
      COMMON /TBRACE/ NFT, NBF
      COMMON /SECT2/ TP(20)
      INTEGER U, L
      DIMENSION CN(3,NTJ), ITP(NFT), INT(2,NFT)
C
C
      NCASE=1
      GOTO 100
      ENTRY BFORCE( M, COSA, SINA, AEL, KODE , U, L )
```

```
      NCASE=2
100   MP=ITP(M)
      A=TP(MP)
      L=INT(1,M)
      U=INT(2,M)
      X1=CN(1,L)
      Y1=CN(2,L)
      Z1=CN(3,L)
      X2=CN(1,U)
      Y2=CN(2,U)
      Z2=CN(3,U)
      KODE=1
      IF( DABS(X1-X2) .LT. 0.0001 ) KODE=2
      GOTO(150,160), KODE
150   DL=DSQRT((X2-X1)**2+(Y2-Y1)**2)
      AL=X2-X1
      GOTO 170
160   DL=DSQRT((Z2-Z1)**2+(Y2-Y1)**2)
      AL=Z2-Z1
170   COSA=AL/DL
      SINA=(Y2-Y1)/DL
      AEL=A*EM/DL
      GOTO( 200, 300) , NCASE
200   S1=COSA*AEL*COSA
      S2=SINA*AEL*SINA
      S3=SINA*AEL*COSA
300   RETURN
      END
C
C
C
C     THIS SUBROUTINE COMPUTES THE BRACING END FORCES.
C
C
      SUBROUTINE  FBRACE( CN, INT, ITP, JD, TFA, TFB )
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON  /IPT/ NST, NBAY1, NBAY3
      COMMON  /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON  /TBRACE/ NFT, NBF
      COMMON  /SECT2/ TP(20)
      COMMON /UNITS/ UNIT1, UNIT2, UNIT3, UNIT4, UNIT5, UNIT6, UNIT7
      DIMENSION  CN(3,NTJ), INT(2,NFT), JD(3,NTJ), TFA(NFT),
     & TFB(NFT), ITP(NFT)
      REAL*8 JD
      INTEGER V,M,U,L
C
C
      WRITE(6,5) UNIT5, UNIT5
5     FORMAT(///' ****  BRACING ELEMENT END FORCES'//
     & 21X,'LOWER END',30X,'UPPER END'/
     & ' BRACING',13X,'JOINT',5X,'AXIAL FORCE',A8,5X,'JOINT',5X,'AXIAL F
     &ORCE',A8)
      DO  10  K=1, NFT
      TFA(K)=0.D0
```

```
      TFB(K)=0.D0
10    CONTINUE
      MT=0
      DO 20 J=1, NBF
      DO 30 N=1, NST
      M=(N+1)*NTJ/(NST+1)
      V=N*NTJ/(NST+1)
      DO 40 NT=1, 2
      MT=MT+1
      CALL BFORCE( MT, COSA, SINA, AEL, KODE , U, L )
      ZU=CN(3,U)-CN(3,V)
      XU=CN(1,U)-CN(1,V)
      ZL=CN(3,L)-CN(3,M)
      XL=CN(1,L)-CN(1,M)
      GOTO (100, 200) , KODE
100   DSPV=JD(1,V)+ZU*JD(3,V)
      DSPM=JD(1,M)+ZL*JD(3,M)
      GOTO 300
200   DSPV=JD(2,V)-XU*JD(3,V)
      DSPM=JD(2,M)-XL*JD(3,M)
300   DSPU=JD(1,U)
      DSPL=JD(1,L)
      F1=AEL*(COSA*(DSPV-DSPM)+SINA*(DSPU-DSPL))
      TFB(MT)=F1
      TFA(MT)=-F1
      WRITE(6,400) MT, INT(1,MT),TFA(MT),INT(2,MT),TFB(MT)
400   FORMAT( 6X,I2,13X,I4,6X,F12.2,12X,I4,6X,F12.2)
40    CONTINUE
30    CONTINUE
20    CONTINUE
      RETURN
      END
C
C
C
C     THIS SUBROUTINE PERFORMS THE P-DELTA EFFECT CHECK
C     ACCORDING TO THE CSA CAN 3 S16.1 M78 IN APPENDIX J.
C
C
      SUBROUTINE  PDELTA( DHE, JD, AJL, CFA, MJ, NPD,NPDT)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /IPT/ NST, NBAY1, NBAY3
      COMMON /GEN/ NTJ, NFB, NFC, NBAND, NEQ, NXM
      COMMON /ALOAD/ NLJ, NLMC, NLMU, NLT1, NLT2, NLT3
      COMMON /UNITS/ UNIT1,UNIT2,UNIT3,UNIT4,UNIT5,UNIT6,UNIT7
      DIMENSION DHE(NST), CFA(3,NFC), AJL(3,NLT1), MJ(NLT1), JD(3,NTJ)
      REAL *8 JD, HLOADX(20),  HLOADZ(20)
      INTEGER V, M
C
C
      WRITE(6,5) UNIT5, UNIT5
5     FORMAT(/// '   P-DELTA EFFECT CHECK '//
     &' ADDITIONAL LATERAL LOAD DUE TO P-DELTA EFFECT'//
     &' STOREY',3X,'ADD. LATERAL LOAD IN X1',1H',A8,10X,'LATERAL LOAD IN
```

```
      &X3',1H',A8)
C
       NSTT=NST+1
       NODE=1
       NCPF=NFC/NST
       TEMPX=0.D0
       TEMPZ=0.D0
       NPD=0
       IF ( NPDT .GT. 1) GOTO 6
       DO  7  N=1,NST
       HLOADX(N)=0.D0
       HLOADZ(N)=0.D0
7      CONTINUE
C
6      DO 10 N=1, NST
       V=N*NTJ/NSTT
       M=(N+1)*NTJ/NSTT
       NCODEX=0
       NCODEZ=0
       SUM=0.D0
       DVPX=JD(1,V)
       DVPZ=JD(2,V)
       DMPX=JD(1,M)
       DMPZ=JD(2,M)
C
       DO  20  J=NODE, NCPF
       SUM=SUM+CFA(1,J)
20     CONTINUE
C
       HEIGHT=DHE(N)
       DELX=DVPX-DMPX
       DELZ=DVPZ-DMPZ
       SHEARX=SUM*DELX/HEIGHT
       SHEARZ=SUM*DELZ/HEIGHT
       HLX=SHEARX+TEMPX
       HLZ=SHEARZ+TEMPZ
       WRITE(6,27) N,HLX,HLZ
27     FORMAT(I5,16X,F15.2,16X,F15.2)
       TEMPX=-SHEARX
       TEMPZ=-SHEARZ
       DHX=DABS(HLX-HLOADX(N))
       DHZ=DABS(HLZ-HLOADZ(N))
       IF (DHX .GE. 0.1) DHX=DHX/HLX
       IF (DHZ .GE. 0.1) DHZ=DHZ/HLZ
       IF (DHX .GT. 0.1) NCODEX=1
       IF (DHZ .GT. 0.1)  NCODEZ=3
C
       DO 25  K=1, NLJ
       KJ=MJ(K)
       IF ( KJ .EQ. V) GOTO 26
       GOTO  25
26     IF (NCODEX .EQ. 1) AJL(1,K)=AJL(1,K)+HLX-HLOADX(N)
       IF (NCODEZ .EQ. 3) AJL(2,K)=AJL(2,K)+HLZ-HLOADZ(N)
       IF (NCODEX.EQ.1 .OR. NCODEZ.EQ.3) NPD=NPD+1
```

```
25    CONTINUE
C
      HLOADX(N)=HLX
      HLOADZ(N)=HLZ
      NODE=NCPF+1
      NCPF=(N+1)*NFC/NST
10    CONTINUE
      RETURN
      END
```