MATHEMATICS PROGNOSIS:
PREDICTING SENIOR HIGH SCHOOL MATHEMATICS ACHIEVEMENT BASED ON NINTH-GRADE MATHEMATICS MARKS IN MORDEN COLLEGIATE

A Thesis

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Master of Education

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ABSTRACT

Problem

The purpose of this study was to evaluate how well grade nine students were achieving in the different mathematics courses in senior high grades and to establish linear and multiple regression equations for predicting success in those courses. The independent variables selected as predictors were the final mathematics marks in the previous grades beginning with grade nine.

Procedure

Final examination scores of 360 students in grade nine, ten, eleven, and twelve were analyzed by the computation of means, standard deviations, correlation coefficients and regression equations. The relationships were also illustrated by the use of scattergrams. The results recorded below emerged from the analysis of these data in the three courses--University Entrance, General, and Business Education.

Results

Positive and significant relationships existed between preceding final marks and the criterion scores.

There was no difference in the probability of success between the three courses at the grade ten level and students who scored below fifty percent in Mathematics IX should be required to repeat the course.

For predicting grade eleven and twelve scores the multiple regression equations were preferable since the last score (the most recent preceding score) had the most weight.

Recommendations

1. The choice of mathematics course in grade ten should be left entirely with the student--depending strictly on motivation and desires but that students of the ninth grade be given more guidance with respect to the nature and demands of high school mathematics.

2. Mathematics teachers should develop cooperatively a series of prognostic tests which could be used to predict student success in the next grade.

3. School counsellors and classroom teachers should develop a system by which prediction techniques may be used effectively as an aid to forecasting academic success in high school.

4. Teachers should utilize this type of statistical analysis in the classrooms in an effort to motivate their students.

5. Additional research should be undertaken utilizing the grade-point average in conjunction with the subject mark as a multi-predictor for achievement in senior high mathematics.

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to the following people for their initial motivation for this thesis and their assistance throughout its progress:

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CHAPTER I

THE PROBLEM AND ITS IMPORTANCE

INTRODUCTION

At the completion of the ninth grade and prior to their embarking upon a senior high program, many students in Manitoba must elect to enrol in a specific course pattern: 100, 101, 102, 103, 104. In the majority of schools, the choice of a certain course precludes the choice on a subject basis since courses and subjects are time-tabled. In the "local" school, students may choose on an individual subject basis but the choice is often crucial for the student's future goals, in view of the fact that many subjects act as prerequisites for the attainment of particular academic or vocational goals. Consequently, in many situations sound information based on statistical evidence involving previous marks should be made available to the student, the parent, the teacher, the guidance counsellor and the principal.

Students are promoted from grade nine to grade ten in one of two ways, either they achieve a "complete" standing or a "conditional" standing which may include a pass or failure in Mathematics. As the content in mathematics is cumulative in nature, failure at one grade level makes success in the succeeding grade doubtful. For this reason mathematics requires very careful consideration. On the basis of success or failure in grade nine, what are the chances of success in grade ten or in the senior years?

STATEMENT OF THE PROBLEM

The basic purpose of the study was to appraise the extent to which achievement in grade nine mathematics serves as a predictor of achievement in the senior grades. The study also proposed to ascertain the means by which course selection is effected and the extent to which grade nine achievement is considered in that selection. More specifically, the study sought to establish relationship that exists between:

1. the final grade nine mathematics mark and the final mark in grades ten, eleven, and twelve

2. the grade nine and ten mathematics marks and the final mark in grade eleven mathematics

3. the grade nine, ten, and eleven mathematics marks in grade twelve mathematics.

It also sought to determine simple linear and multiple regression equations for predicting success in senior high mathematics courses based on grade nine only or a combination of nine and ten, or nine, ten, and eleven.

IMPORTANCE OF THE STUDY

The answers to the stated questions could be of invaluable benefit to the students, parents, teachers, guidance counsellors and the principal of Morden Collegiate. The format used in this study could be applied other subjects which are prerequisites for certain courses throughout high school. This type of study is imperative for the principals who follow the guidelines for organizational and educational purposes of the secondary program:

1. Grade nine is the final year of broad general education for all students. During this year the student is exposed to learning experiences which should assist in helping the student, the teacher, and the parents ascertain the aptitudes, interests, and inclinations of the individual and which will be of value in guiding the student in his choice of the most suitable pattern to pursue in grades 10, 11, and 12.

2. The high school program in Grades 10, 11, and 12 provides differentiation of studies to meet the varying needs, interests, and abilities of students.[1]

PROCEDURES

Population

It was proposed that the study be conducted in the Morden Collegiate since it is the only collegiate in the Western School Division No. 47. The population was defined as all grade nine students who attended Morden Collegiate from September, 1966 to June 1971 inclusive. Because the population was relatively small (360 students) the total population constituted the sample.

Predictors and Criterion

(a) The predictor was the final grade nine mark in mathematics.

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[1]Province of Manitoba, High School Program of Studies, Administrative Handbook, grades 9-12, 1970-1, Authorized by the Minister of Youth and Education, page 3.

The criterion was the final grade ten, eleven, and twelve marks in Mathematics in either _00, Mathematics _01, or Mathematics _02.

(b) For determining the multiple regression the predictors was the final mathematic marks in grade nine and ten and grade nine, ten and eleven. The criteria was the final mathematic mark in grade eleven and in grade twelve.

Analysis of Data

Once the sample was determined the scores for each of the required grades were recorded from the student histories. These scores were then analyzed using statistical procedures outlined by Glass and Stanley.[2] The required relationships were determined by computing the means, standard deviations, coefficients of correlation and regression equations. Scattergrams were also employed to illustrate the relationships.

LIMITATIONS

The study was limited by considering a population which consisted only of those grade nine students who subsequently completed grade ten mathematics in Morden Collegiate. The findings of the study would therefore apply only to this school division.

The study was restricted to a consideration of only one

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[2]Gene Glass and Julian Stanley, Statistical Methods in Education and Psychology. (Englewood Cliffs, N.J.: Prentice-Hall Inc., 1960) page 590.

independent variable as a predictor of academic achievement. This independent variable consisted of data which was annually accumulated in the local school setting where this study was performed. Consequently, the study was further restricted to a consideration of the standards set by three teachers who were on staff for a least ten years and who might probably remain so for another ten years. All grade nine students received their mathematics instruction and summative evaluation from one teacher. All grade ten Mathematics 100 students were taught and evaluated by a second teacher. All grade ten Mathematics 101 students were taught and evaluated by a third teacher. All grade eleven and twelve students were evaluated by the same teachers as the grade tens except those taking 102 and 202.

In calculating the multiple regression equations certain other restrictions were considered. The name of any student who changed from 101 to 200 or from 100 to 201 was deleted i.e. the source of the independent variables or predictors had to be consistent with the source of the criterion measure. By the same token no data were considered for grade twelve regression equation unless the student remained in the same course throughout high school.

ASSUMPTIONS

It was assumed that the data used in this study were based on valid, academic achievement in grade nine Mathematics in accordance with a school standard set for that grade level. It was further assumed that each student had an equal chance of establishing his maximum potential in view of the fact that the final mark was the result of a series of short objective tests and formal examinations. It was also assumed that the final mark in grades ten, eleven, and twelve consisted of a series of tests and formal examinations. Finally it was assumed that only lack of knowledge and computational skills and not some other factor was instrumental in any student achieving poorly to the extent that he would fail.

DEFINITIONS

Success. For the purpose of the study success referred to a final mark of fifty per cent or over in academic achievment. In the classroom setting success frequently implied a change of attitude or of ability to cooperate with a peer group; however only the cognitive aspect was considered in the study.

Failure. For the purpose of this study failure referred to a final mark below fifty per cent in academic achievement. No consideration was given to the affective attributes which were evidenced in the classroom.

Local School. For the purpose of this study local school referred to Morden Collegiate.

Mathematics _00. For the purpose of this study Mathematics 100, 200, or 200 referred to the University Entrance mathematics (a nomenclature which has been discarded).

Mathematics _01. For the purpose of this study Mathematics 101, 201, or 301 referred to the General Course mathematics (a nomenclature which has been discontinued).

Mathematics _02. For the purpose of this study Mathematics 102 or 202 referred to Business Education mathematics.

CHAPTER II

THE REVIEW OF THE LITERATURE

This chapter will present a review of the related literature with emphasis upon methods of prediction and some of the more pertinent predictive studies.

BACKGROUND MATERIAL

Recently tests have been developed in both Canada and the United States to measure skills on performance objectives in accordance with the rationale that skills should be operationally defined. Guidance counsellors, teachers, and principals are aware of the pronounced need to help the students ascertain these skills and to help the students increase their ability for self-appraisal and decision-making concerning their choice of a mathematics program.

According to Scandura[1] the goals of a good mathematical education in affluent suburbs may be very different from the objectives in the inner city of our metropolitan centres or in farm communities across the country.

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[1]Joseph M. Scandura, "A Research Basis for Mathematics Education," The High School Journal, Vol. 53, (Feb. 1970), p. 269.

Kolb,[2] in his discussion of the goals of mathematics instruction stated that the latter part of the nineteenth century was a period in which mathematics was thought to be an exact field of study because it disciplined the learner to think logically and rationally.

But the goals of mathematics instruction undergo a revision almost every decade. They vascillate from the utilitarian and practical to the purely philosophical and theoretical. Consequently, it may be advisable to examine the objectives as they are listed for grade seven:

1. To develop an understanding of the basic concepts and processes of mathematics through study of the structure of the number system.

2. To develop mastery of the skills of computation.

3. To provide a learning situation in which pupils can "discover" the basic principles and relationships of mathematics.

4. To develop competence in the application of mathematical concepts, laws, and skills to problem solving in situations within the experience of the modern child.

5. To develop desirable attitudes and appreciations with respect to mathematics.[3]

The study of mathematics cannot be a conglomeration of disjointed facts. Rather it must be a transitional,

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[2]John Kolb, "Goals of Mathematics Instruction," The High School Journal, Vol. 53, (Feb. 1970), p. 253.

[3]Province of Manitoba, Department of Education, Grade VII Mathematics, Authorized by The Minister of Education, 1967, Mineographed, p. 2.

spiralling continuum. For that reason it might be advisable to survey the major aims set out for the grade nine student:

1. To further develop sound foundations in the study of algebra.

2. To extend the study of algebra of the real number system.

3. To extend the study of such set language as is immediately required and useful to the students.[4]

However, it is at this juncture that the student must choose which course he will follow in senior high school. He, then, should be informed of the dichotomy in the mathematics course and informed of the objectives in each one.

The objectives of the General Course (Mathematics 101) are:

1. Provision of the basic core of mathematical knowledge necessary for every citizen in an atomic age.

2. Ability and willingness to apply problem solving techniques, both in familiar situations and in genuinely new situations.

3. Awareness that mathematics is a vast, complex and rapidly changing field of knowledge, and of the appeal pure mathematics has for many people.

4. Awareness and appreciation of the importance of mathematics in the natural and social sciences, and in business and government.

5. Acquaintance with the basic mathematical operations used in business and government,

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[4]Province of Manitoba, Department of Education, Grade X Mathematics, Authorized by The Minister of Education, September, 1968, Mimeographed, p. 1.

and development of the ability and desire to learn such operations on the job.

6. Understanding of the mathematics involved in personal and family finance, and willingness to use this understanding.

7. Development of a foundation in mathematics sufficient to permit further study if so desired.

8. Development of sound judgment and a critical attitude in assessing mathematical reasoning, processes and results.

9. Development of an appreciation of the value of mathematics in relation to the school subjects.[5]

The general objectives of the Mathematics 100 course are listed in separate outlines. For the Algebra, the student should:

1. Learn to appreciate mathematics,

2. Acquire skill in using algebraic technique with understanding and accuracy,

3. Become acquainted with mathematical facts, processes and concepts,

4. Learn to use his creative and analytical powers (in solving problems)[6]

while for the geometry section, the student should:

1. Learn to appreciate Geometry,

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[5]Province of Manitoba, Department of Education, General Course, Mathematics 101, 201, 301, Authorized by the Minister of Education, 1967, mimeographed - p. 2-3 left page.

[6]Province of Manitoba, Department of Education, University Entrance Course, Mathematics, Algebra 100, Algebra 200, Authorized by the Minister of Education, 1968, Mimeographed, p. 2.

2. Become aware of the structure of Geometry, (i.e. acquire a knowledge of geometrical ideas including undefined terms, definitions, postulates and theorems.)

3. Learn the elementary relationships of points, lines and planes in space and the basic properties of angles, triangles and quadrilaterals.

4. Learn to understand and to develop deductive proofs.[7]

PREVIOUS STUDIES IN PREDICTION

The writer found many studies dealing with prediction of academic achievement but the majority of them dealt with multiple predictors and/or multiple criteria and none dealt with a single subject predictor and a single criterion.

Klugh and Bierley[8] carried out a study using the School and College Ability Test to determine whether a significant improvement in prediction could be obtained by using the student's high school grade point average in addition to the SCAT test. The criterion measure was the students' grade point average computed at the end of the first semester of college work. Their findings prove that the SCAT is as valid as high school average in predicting the first semester grades.

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[7]Province of Manitoba, Department of Youth and Education University Entrance Course, Mathematics 100, Geometry, Authorized by the Minister of Youth and Education, 1969, Mimeographed, p. 1.

[8]H. Klugh and R. Bierley, "The School and College Ability Test and High School Grades as Predictors of College Achievement," Educational and Psychological Measurement, Vol. 19, (1959) p. 625-6.

Caldwell and Schrader[9] performed a study comparing the validity and administering times for composites of structure-of-intellect tests and algebra grades and composites of commercial tests and algebra grades for predicting success in tenth-grade geometry. The results indicate that correlations using the SI-Alg. Composites exhibited higher validity coefficients (.59 - .83) than the correlations using Com-Alg. Composites (.47 - .78) in twenty-one out of twenty-two instances. However, they included teacher-made final examination scores in the Commercial tests and did not indicate separately what relationships existed by using the analysis of variance.

Scannell[10] made an interesting observation as a result of his study using various measures of achievement from grade four through grade twelve. The combinations of achievement test data obtained at several points in the students' careers were only slightly more predictive than the most recent results. High school GPA was the best single predictor of college success yielding correlations of .67 and .59 with the criterion freshman GPA. He also suggests that

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[9]James Caldwell et al, "Comparative Validities and Working Times for Composites of Structure-of-intellect Tests and Algebra grades and Composites of Traditional Measures and Algebra grades in the Prediction of Success in Tenth-grade Geometry," Educational and Psychological Measurement, Vol. 30, (1970), p. 955-959.

[10]Dale P. Scannell, "Prediction of College Success from Elementary and Secondary Schools Performance," Journal of Educational Phychology, (June, 1960), Vol. 51, p. 130-134.

college success based on elementary school test scores can be as useful as predictions from high school data.

Seigle[11] at Washington Municipal University, Kansas, stated that students have often been placed in courses or they have chosen them without any knowledge of their fitness for these courses-resulting in a failure or withdrawal from the course. For predictive purposes the first consideration might be the previous records in that subject. The high school grade average was a better predictive factor than the Washburn Entrance Mathematics Test with an r of .6623 in the case of the prediction of success in analytic geometry.

Young, Knapp and Michael[12] utilized a Test of Achievement in Basic Skills (TABS-Math) to determine the validity in predicting achievement in general mathematics and algebra. The rationale for the construction of the test was that skills should be operationally defined and measured by means of test items based on sixty-four performance objectives. Sample students were chosen from several California junior high schools. In grade nine the correlations between the three sections of the predictor were .67, .51, .64 and the

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[11]William F. Seigle, "Prediction of Success in College Mathematics at Washburn University," Journal of Educational Research, Vol. 47, (1953-4), p. 577-588.

[12]James C. Young, Robert R. Knapp, and William B. Michael, "The Validity of the Tests of Achievement in Basic Skills for Prediciting Achievement in General Mathematics and Algebra," Educational and Psychological Measurement, Vol. 30 (1970) ;. 951-954.

correlation between teacher marks and the individual section were .69, .60, .49. The correlation between the total TABS predictor and the criterion teacher marks was .71. The present results do not reflect on the usefulness of TABS scores for the purpòse of assessing the level of achievement in specific performance objectives.

Several other studies were considered but a report on each was not submitted because of the multiplicity of predictors. However, the findings would be extremely fruitful to anyone comtemplating using more than one predictor. Shevel and Whitney[13] found that Mathematics Placement Examination is no more effective for predicting first semester mathematics grades for lower ability classes than are high school grades. Hanna et al,[14] Kansas State University, supported the theory that the use of teacher predictions were significant at the five per cent level. Dinkel,[15] Culver City, California, performed a two-year test on grades seven and eight. He had derived a prognosis battery with a multiple coefficient of correlation of .78 for the first year and .86 for the second year.

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[13]Linda R. Shevel and Douglas R. Whitney, "Predictive Validity of the Mathematics Placement Examination," Educational and Psychological Measurement, (1969), p. 895-901

[14]G. Hanna et al, "Predicting Algebra Achievement with an Algebra Prognosis Test, IQS, Teacher Predictions, and Mathematics Grades," Educational and Psychological Measurement, (1969) p. 903-907.

[15]Robert E. Dinkel, "Prognosis for Studying Algebra," The Arithmetic Teacher, Vol. 6, (1959), p. 317-319.

Abers and Feldt[16] developed two forty-minute achievement tests and slso asked teachers to assign a mark to each student representing his level of achievement. The above criteria were used to validate the predictive ability of the Iowa Algebra Aptitude Test. The correlations among the predictor variables ranged from .75 - .82. Teacher's marks correlated from .64 - .72 with the predictors and .70 with the criterion achievement in modern mathematics. The fact that teachers' marks correlated high with the achievement test indicates that special achievement tests are a luxury item.

Barnes and Asher[17] used a method of predicting success in grade nine mathematics based on the student's eighth-grade mathematics marks and an algebra progsosis test given during the eigth grade. To determine the regression equation they used an IBM 7070 into which he fed information which was available from school records. The maximum computed multiple correlation was .6610 between the predictors and the criterion. The correlations among the multiple predictors ranged from .391 - .627. They concluded that the best single predictor of success in algebra

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[16] Darrell L. Sabers and Leonard D. Feldt, "The Predictive Validity of the Iowa Algebra Aptitude Test for Achievement in Modern Mathematics and Algebra," Educational and Phychological Measurement, (1968), p. 901-907.

[17] Ward E. Barnes and John W. Asher, "Predicting Students' Success in First-year Algebra," The Mathematics Teacher, Vol. 55, (1962). p. 651-654.

in their school system was the eight-grade mathematics mark with a correlation of .5881. The only other factor which greatly raised the multiple correlation was the grade equivalent on the arithmetic part of the achievement test given near the end of the seventh grade. This raised the multiple correlation to .6245. The guidance people in their school were correct in theorizing that the eigth-grade mathematics mark was useful as a predictor and were correct in not using any results of reading tests. The Orleans Algebra Prognosis Test did not detract from their prediction, but neither did it help. It can be seen that it adds so little to the multiple correlations that it probably is not worth the expense nor the time involved in giving and correcting the test.

Ivanoff and De Wane[18] observed from their study that because of the error or overlapping component of multiple predictors the regression equation would have to be used cautiously with the marginal student. But, at least, it should provide the student with an appraisal of his marginal situation and thus bring out the need for real effort if he is to succeed in an algebra program. They also suggested that although the four predictor variables were significant, for all practical purposes the school system should probably use a simpler equation with fewer variables.

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[18]John M. Ivanoff and Evermode T. De Wane, "Use of Discriminant Analysis for Selecting Students for Ninth-grade Algebra or General Mathematics," The Mathematics Teacher, (May, 1965), p. 412-416.

The writer was also advised by his chief advisor to peruse some Manitoba studies dome on prediction. Cross[19] in his study on schievement and attitude in a modern and a traditional grade ten geometry program found a significant correlation between grade nine criteria variables. Froese[20] performed a study on the prediction on success in grade twelve based on ninth-grade level. However, he used multiple predictors and concluded that the nine independent variables did not account for a sufficient amount of the variance of individual grade twelve subjects to make accurate prediction by multiple regression equations possible.

SUMMARY

The review of the literature consisted of two parts.

The background material supplied certain current objectives and philosophy to which schools must adhere in order to provide for the students the continuum which is imperative for a sound mathematics education.

The consensus of researchers who studied predictions

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[19]Robert W. Cross, "Student Achievement and Attitude in a Modern and a Traditional Grade Ten Geometry Program," Unpublished Master of Education Thesis, University of Manitoba, Winnipeg, 1968.

[20]Frank J. Froese, "Predictive Indices of Junior High School Test Scores with Respect to Academic Performance in Twelfth-grade Subjects of the University Entrance Course," Unpublished Master of Education Thesis, University of Manitoba, 1969.

is that studies using multiple predictors are not definitive. In the main, the relationship showed no significant correlation or proved that a similar correlation could have been achieved by using fewer predictors. Frequently the final subject mark from the previous grade level was used. The most significant results were obtained when the previous mark was used in conjunction with the mark in that subject from the year before or with a special achievement test devised to evaluate specific skills.

CHAPTER III

COLLECTION AND ORGANIZATION OF DATA

The purpose of this chapter was to assemble and organize the data through the utilization of statistical procedures in preparation for the analysis which will be presented in chapter four. An outline of the devices used at specific points was presented in this chapter.

COLLECTION OF DATA

The data were based on a population of 462 grade nine students who attended the local school i.e. Morden Collegiate during a period from September, 1966 to June, 1970. Because the population was not too unwieldy all students were considered in the sample and consequently no random sampling was required. From the cumulative records the final grade nine, ten, eleven, and twelve marks in mathematics were obtained and recorded beside the corresponding names.

The names of any students who did not complete grade nine or who did not return to complete the grade ten course were dropped from the list, that is, 102 names were deleted from the sample. New students who transferred into this school for any grade above grade nine were not added to the list. The final population which was used in this study was 360 students, of which 173 were males.

The list of 360 names was then divided to form three separate groups, that is, a group made up of all students enrolled in the _00 courses, one for the _01 courses and a third for the _02 courses. To facilitate discussion of these groupings of students the writer arbitrarily labelled eight. subgroups as follows:

TABLE I

A LIST OF COURSES AND CORRESPONDING NUMBERS
SHOWING THE GROUP ASSIGNMENTS

Course and Number		Assigned Letter
Mathematics	100	Group A
Mathematics	101	Group B
Mathematics	102	Group C
Mathematics	200	Group D
Mathematics	201	Group E
Mathematics	202	Group F
Mathematics	300	Group G
Mathematics	301	Group H

Note: Group I includes subgroups A, D, G.
Group II includes subgroups B, E, H.
Group III includes subgoups C, F.

ORGANIZATION OF DATA

The data in this study were analyzed using two different systems:

1. Through the use of standard statistical procedures, simple linear regression equations were derived.
2. Through the use of computer programming, multiple regression equations were obtained.

SIMPLE REGRESSION EQUATIONS

In order to complete the data in Table II page 23 the following formulae were used:

(a) Calculation of the means ($\overline{X}$, $\overline{Y}$)

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{N}$$

(b) After the above measures of central tendency were completed, the measures of variability were calculated by using the formula for the standard deviations (Sx, Sy).

$$Sx = \frac{\sum_{i=1}^{n} (x_i - \overline{X})^2}{n - 1}$$

The standard deviation was a measure of the scatter of the cases about their mean, that is, the standard deviation was associated with the arithmetic mean. One standard deviation distance, lying on each side of the mean, included approximately sixty-eight percent of the individual measures. A standard deviation of small numerical value indicated that the scores were relatively homogeneous while a large standard deviation meant that the distribution was heterogeneous. In other words the standard deviation measured the dispersion or the spread of the cases.

(c) For measures of relationship, the Pearson Product-moment

Correlation Coefficients, the formula used was

$$r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{X})(y_i - \bar{Y})}{\sqrt{\left[\sum_{i=1}^{n} (x_i - \bar{X})^2\right] \left[\sum_{i=1}^{n} (y_i - \bar{Y})^2\right]}}$$

The coefficient of correlation indicated the degree of relationship which existed between the independent variable, x, and the criterion variable, y, that is, the extent to which high, medium and low values of variable-x corresponded respectively to high, medium, and low values of variable-y. A high positive value of the coefficient of correlation indicated that the paired scores varied together, with the higher values of x associated with the higher values of y, that is, both scores tended to lie on the same side of the mean.

(e) For the calculation of the linear regression equations (or prediction lines) $\hat{y}_i = b_1 x_i + b_o$ the formulae used were:

$$b_1 = \frac{N \sum_{i=1}^{n} x_i y_i - \left[\sum_{i=1}^{n} x_i\right] \left[\sum_{i=1}^{n} y_i\right]}{N \sum_{i=1}^{n} x_i^2 - \left[\sum_{i=1}^{n} x_i\right]^2}$$

$$b_o = \bar{Y} - b_1 \bar{X}$$

TABLE II

A SUMMARY OF THE MEANS STANDARD DEVIATION, CORRELATION COEFFICIENTS, AND REGRESSION EQUATIONS

Group Course No. & Sample Size	Mean		S. Deviation		Correlation and Regression Equations
	IX	X	IX	X	
A 100 N=177	77	72.3	11.0	13.2	$r = .712$ $\hat{y}_i = .75x + 15$
B 101 N=144	62	60.5	12.0	14.5	$r = .658$ $\hat{y}_i = .78x + 12$
C 102 N=39	65	62	13.3	10.1	$r = .760$ $\hat{y}_i = .57x + 25$
	IX	XI	IX	XI	
D 200 N=144	80.5	70.5	10.3	13.2	$r = .666$ $\hat{y}_i = .87x + 0.5$
E 201 N=89	67.5	60.7	10.7	13.5	$r = .508$ $\hat{y}_i = .65x + 17$
F 202 N=49	63	63.7	11.1	13.3	$r = .419$ $\hat{y}_i = .5x + 31$
	IX	XII	IX	XII	
G 300 N=63	85.5	70	7.49	13.2	$r = .677$ $\hat{y}_i = 1.106x - 25$
H 301 N=67	73	67.5	10.1	10.9	$r = .390$ $\hat{y}_i = .43x + 36$

The equation which predicted the y-value from the x-value had two constants, b_1 and b_o. When the x-value was multiplied by b_1 and the constant b_o was added the predicted value was closer to the actual y score than could be obtained by any other method.

The Pearson Product-moment correlation coefficient was a measure of relationship or the total degree of association between paired scores. In order to illustrate this relationship the scores were plotted on the graphs on pages 25 to 32 inclusive.

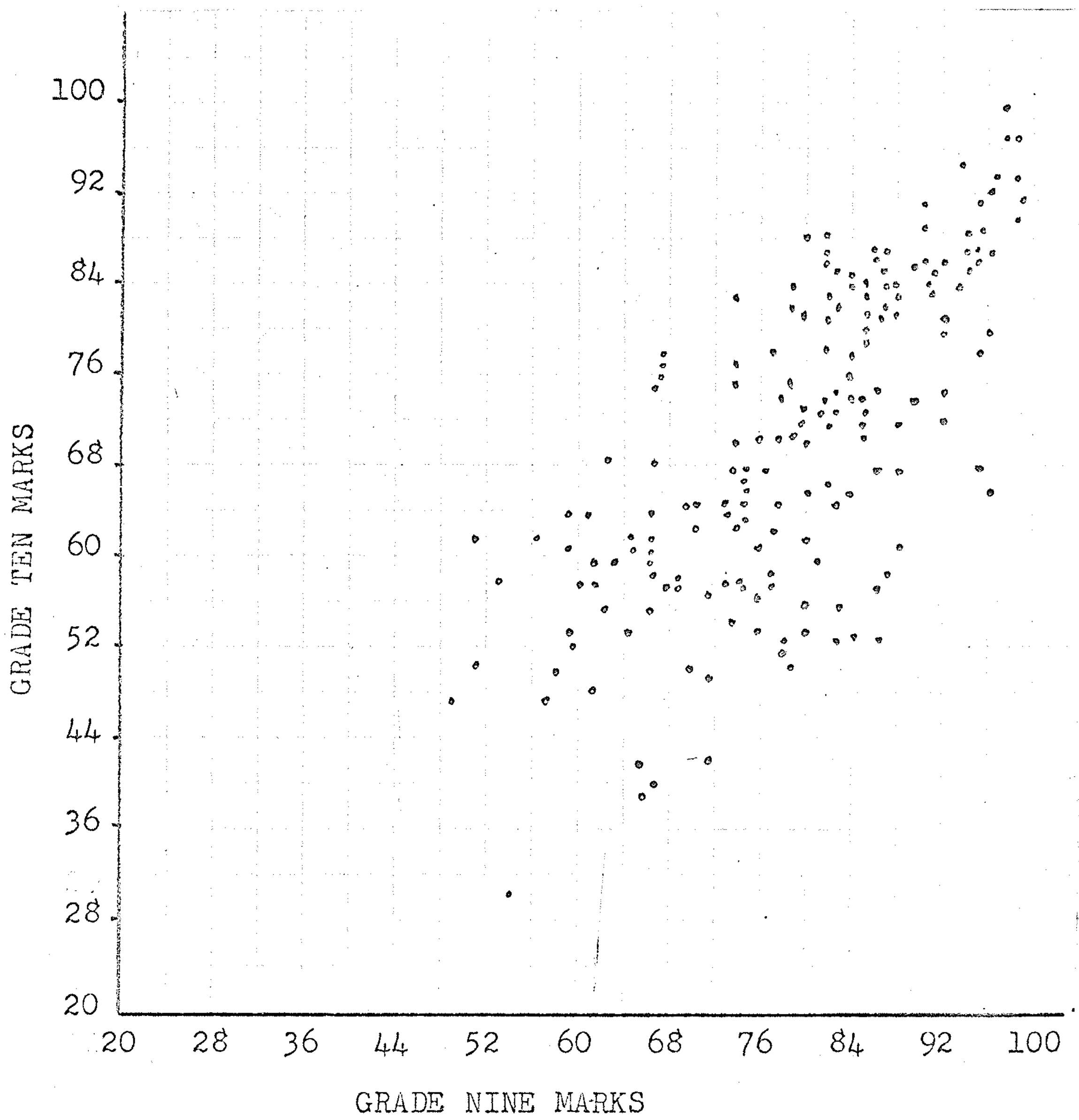
Each pair of scores was represented by a point placed on a cartesian graph. A dot was placed at the intersection of the abscissa (distance along the x-axis) and the ordinate (distance along the y-axis).

If a student did well in grade nine and also in grade ten his scores were represented by a dot in the upper right hand part of the graph. A student who did poorly in both grades had his score recorded in the lower left hand section of the graph. Where a good score in one grade went with a poor score in the other grade the points fell in the upper left and lower right sections.

An inspection of the graphs indicated that there was some tendency for the scores to run in the lower-left to upper-right direction.

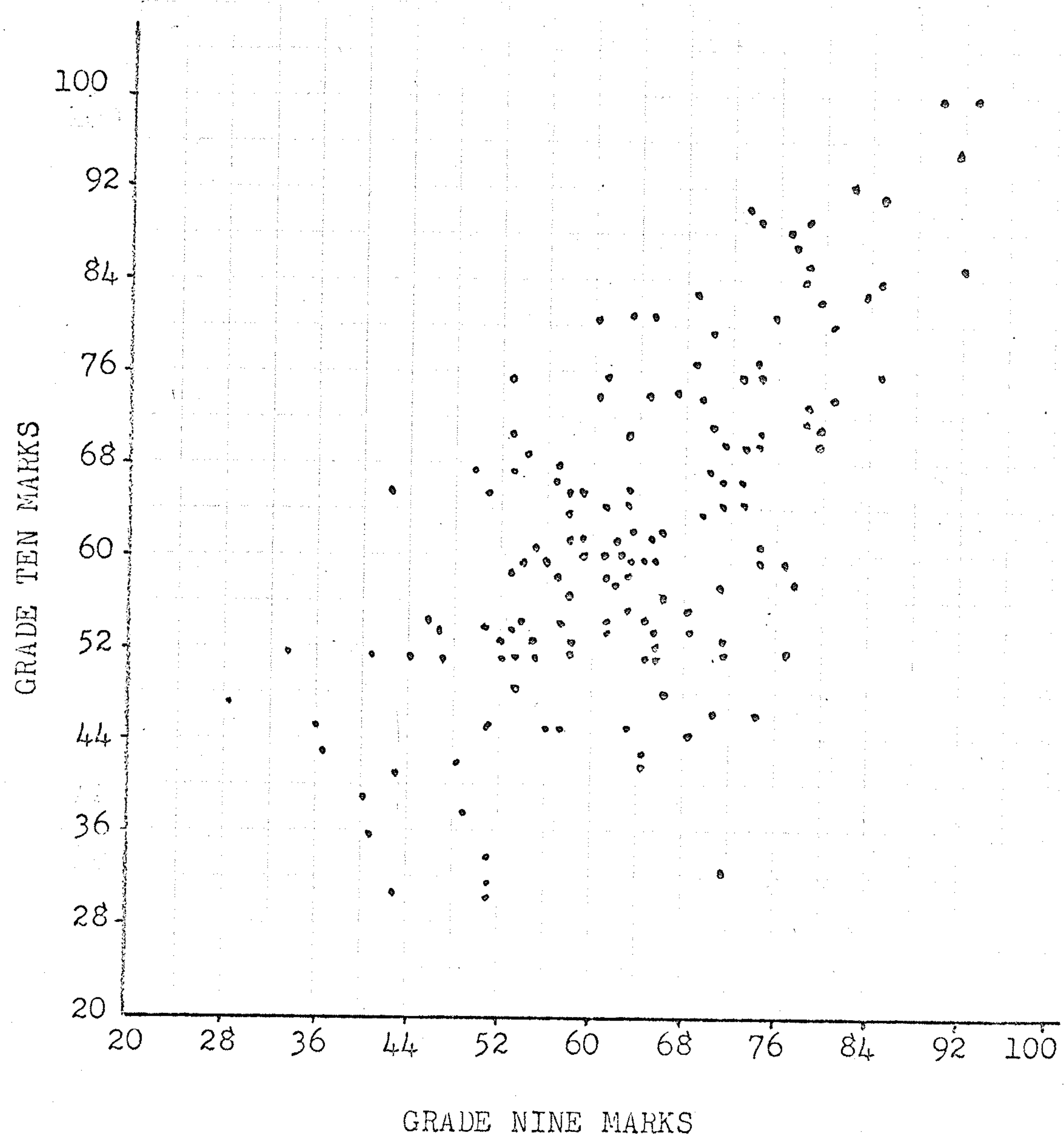
SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF ONE HUNDRED AND SEVENTY SEVEN GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE TEN MATHEMATICS 100 MARKS

(GROUP A)



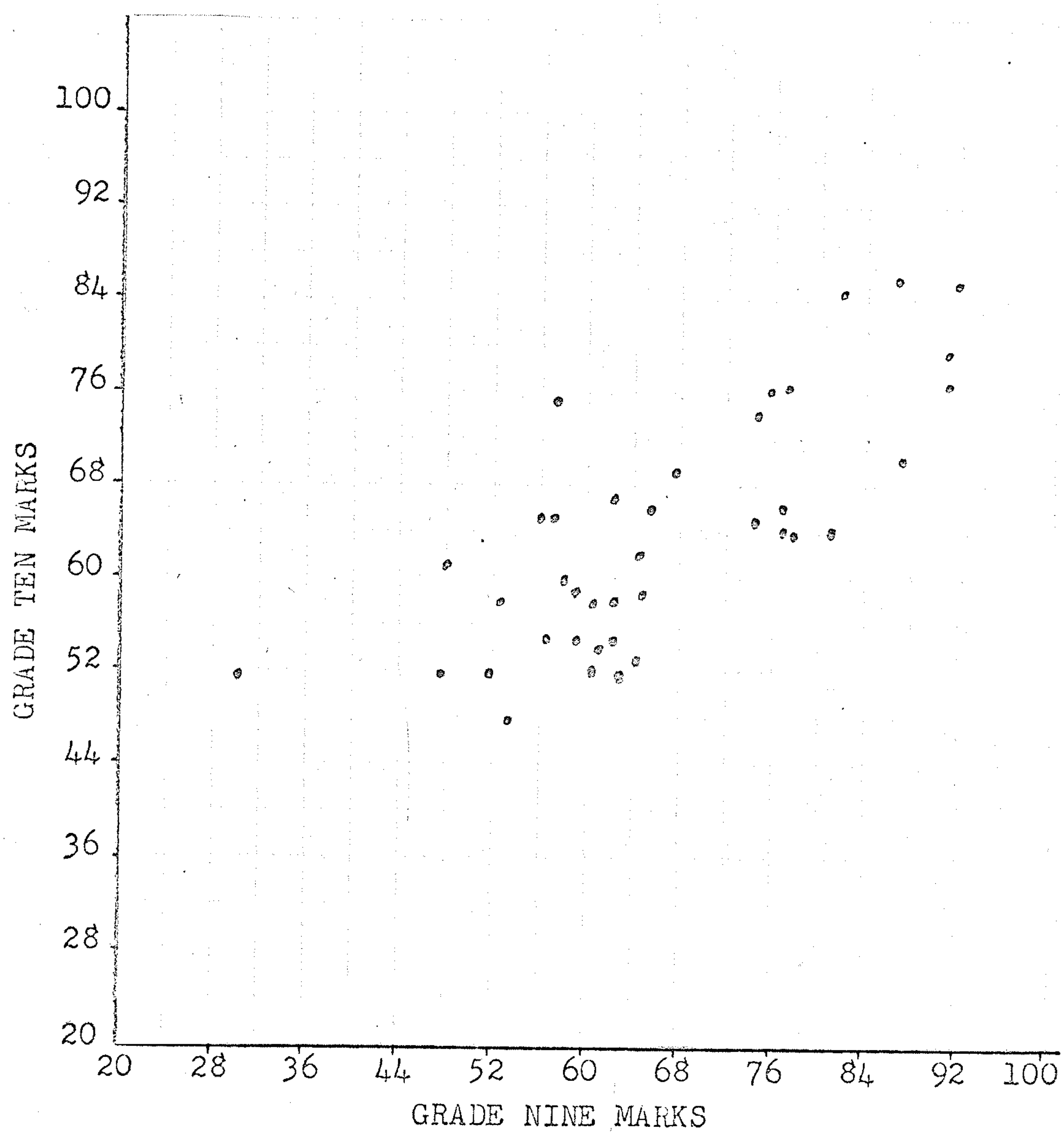
SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF ONE HUNDRED AND FORTY FOUR GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE TEN MATHEMATICS 101 MARKS

(GROUP B)



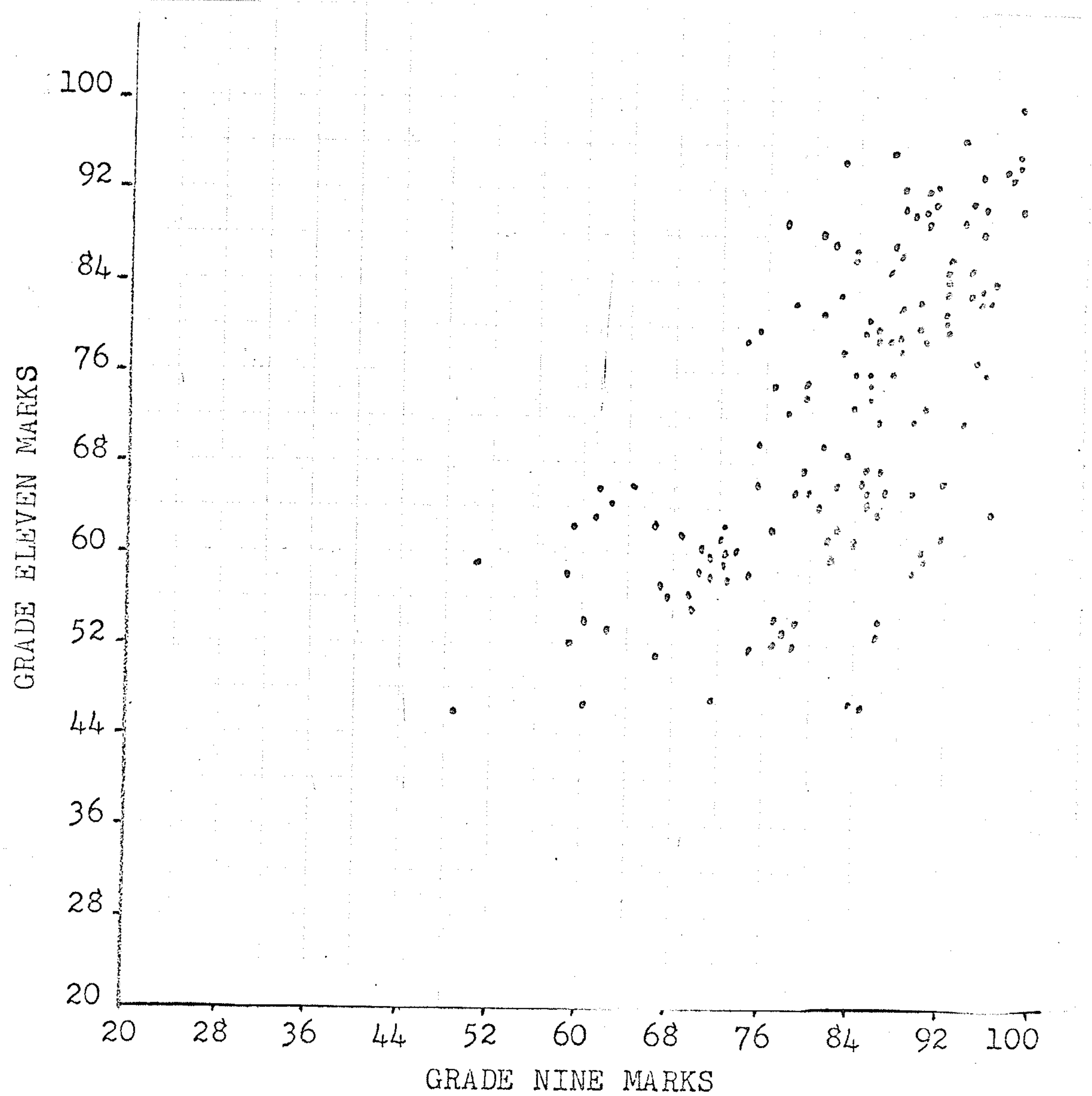
SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF THIRTY NINE GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE TEN MATHEMATICS 102 MARKS

(GROUP C)



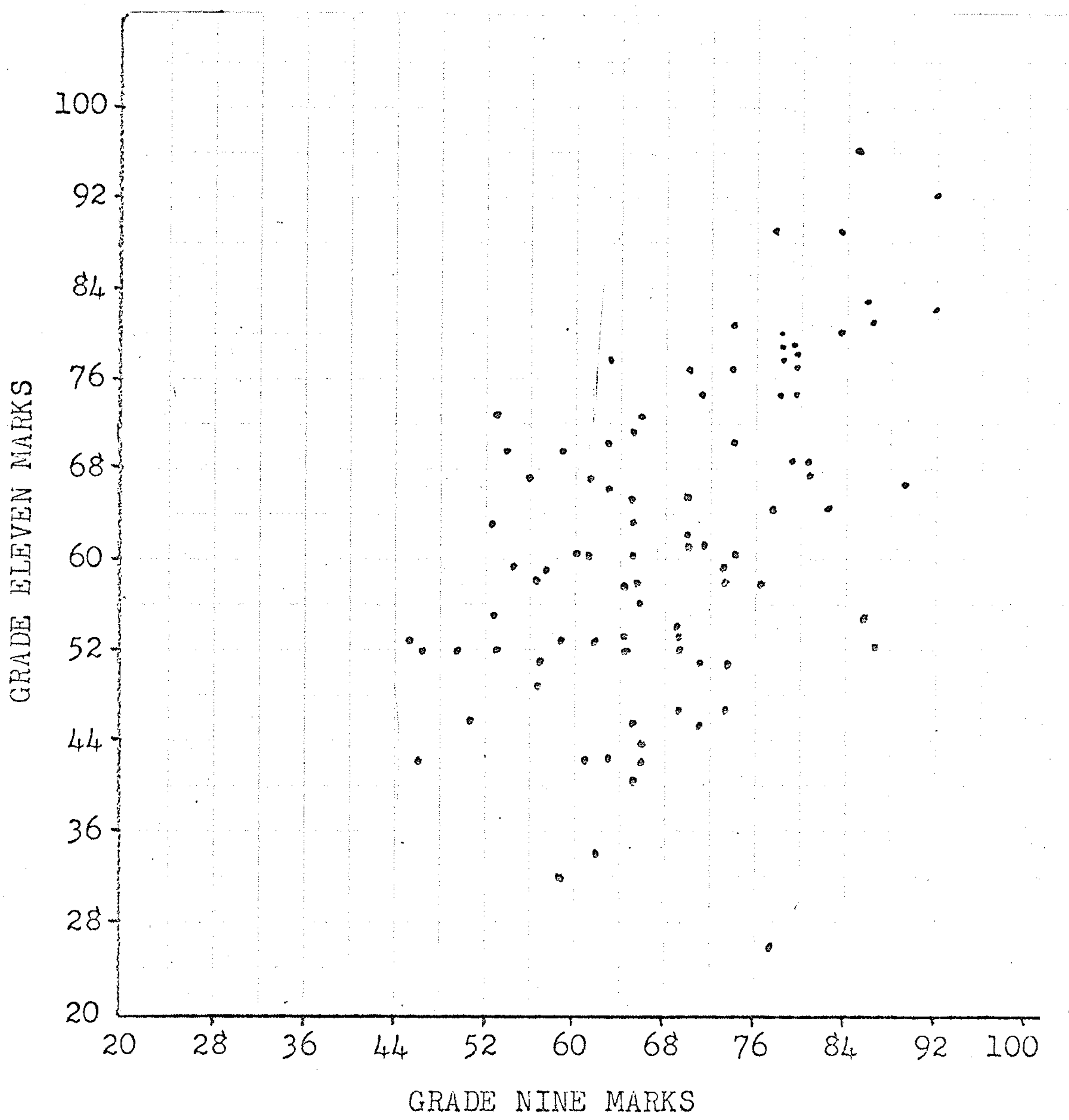
SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF ONE HUNDRED AND FORTY FOUR GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE ELEVEN MATHEMATICS 200 MARKS

(GROUP D)



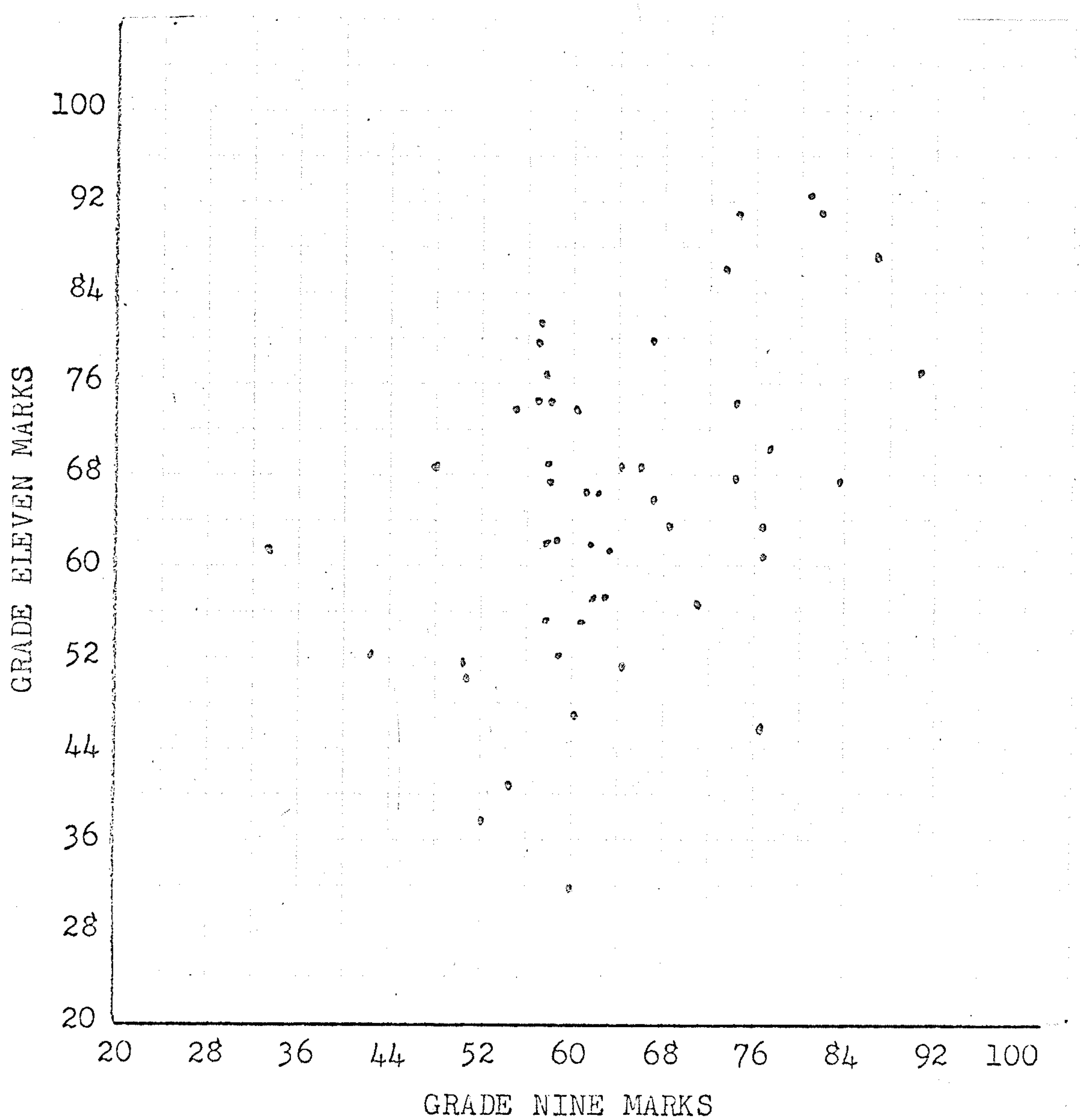
SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF EIGHTY NINE GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE ELEVEN MATHEMATICS 201 MARKS

(GROUP E)



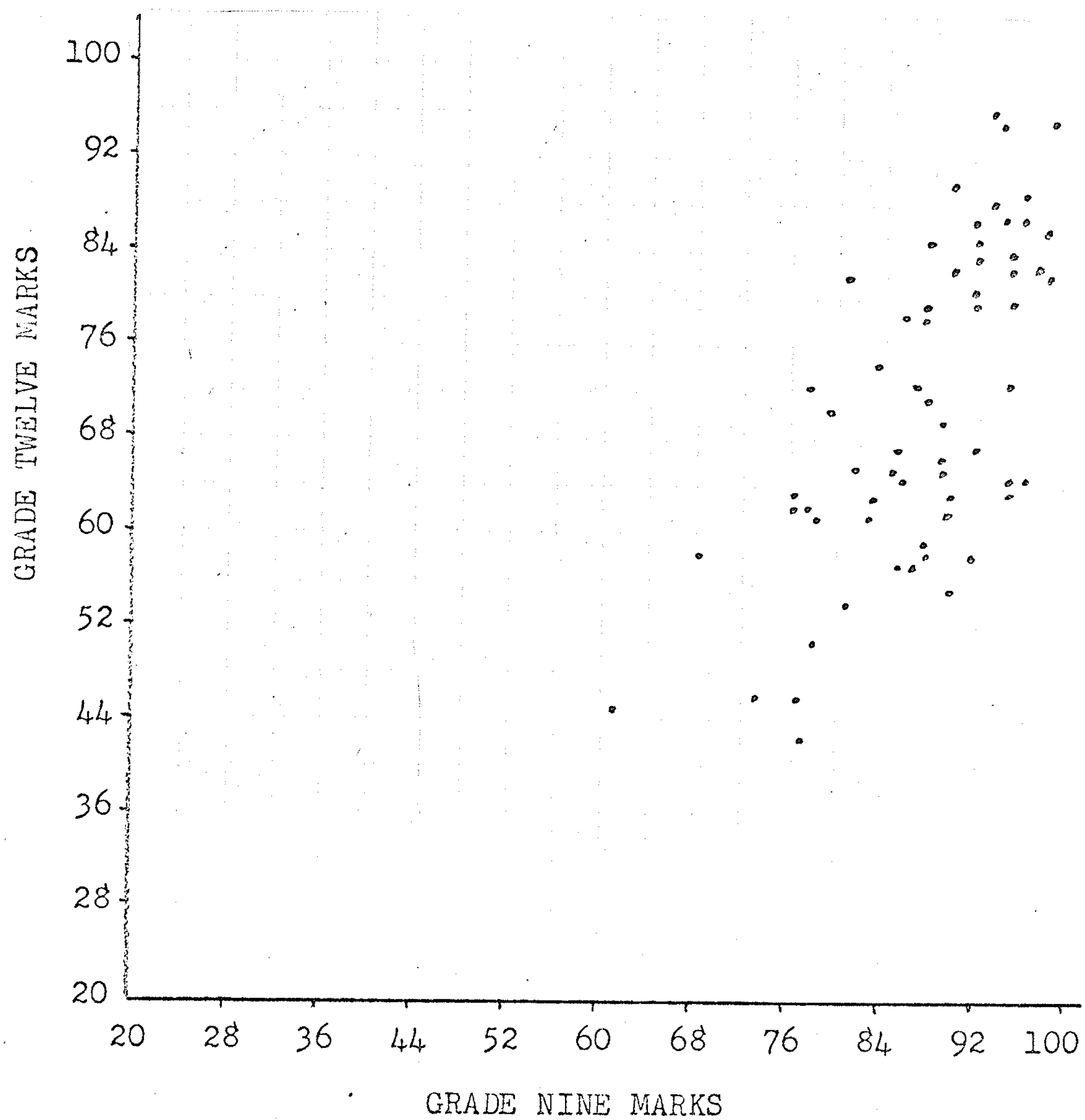
SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF FORTY NINE GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE ELEVEN MATHEMATICS 202 MARKS

(GROUP F)



SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF SIXTY THREE GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE TWELVE MATHEMATICS 300 MARKS

(GROUP G)



SCATTER DIAGRAM OF THE CORRELATION OF SCORES OF SIXTY SEVEN GRADE NINE MATHEMATICS MARKS AND THE CORRESPONDING GRADE TWELVE MATHEMATICS 301 MARKS

(GROUP H)

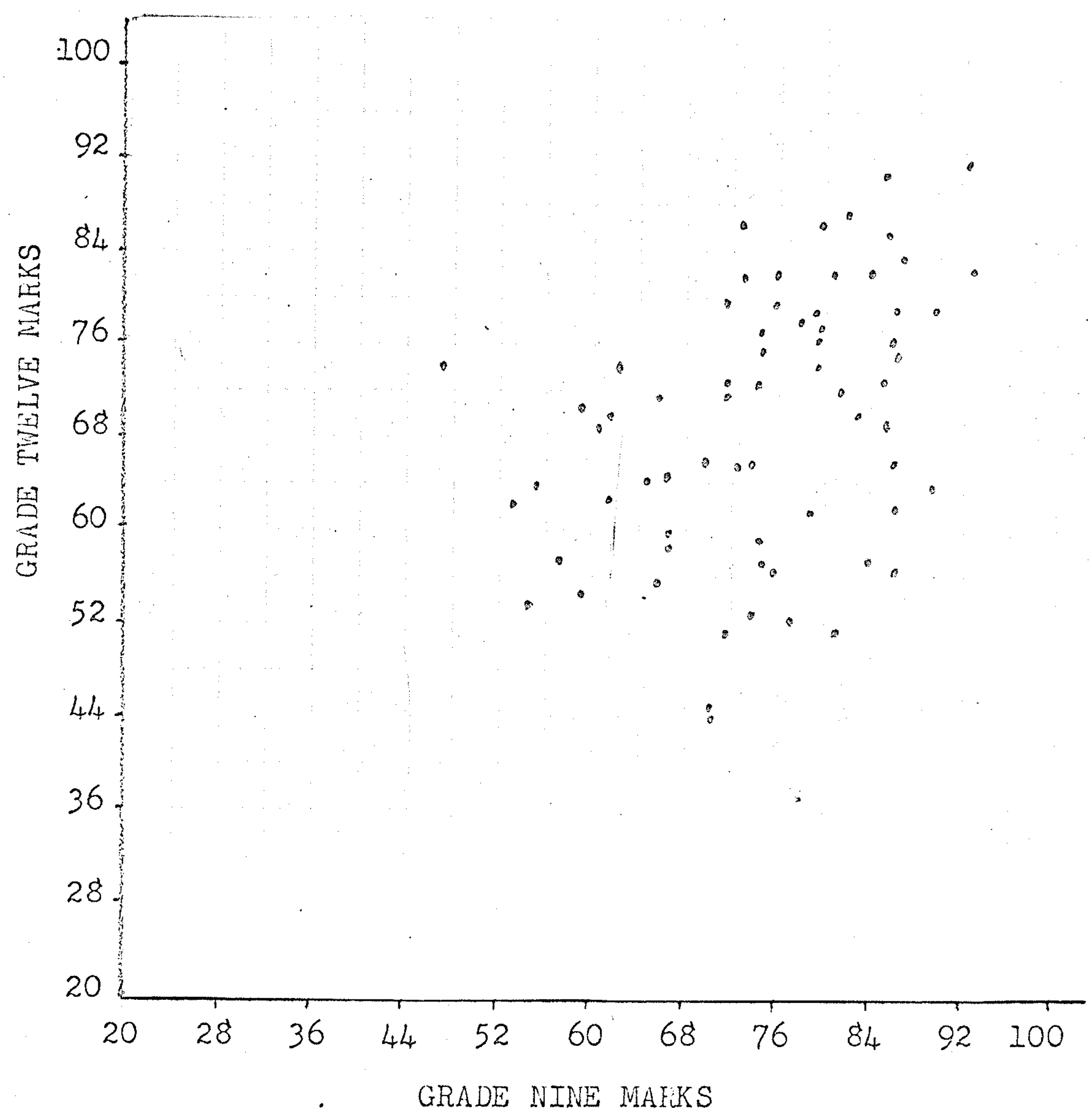
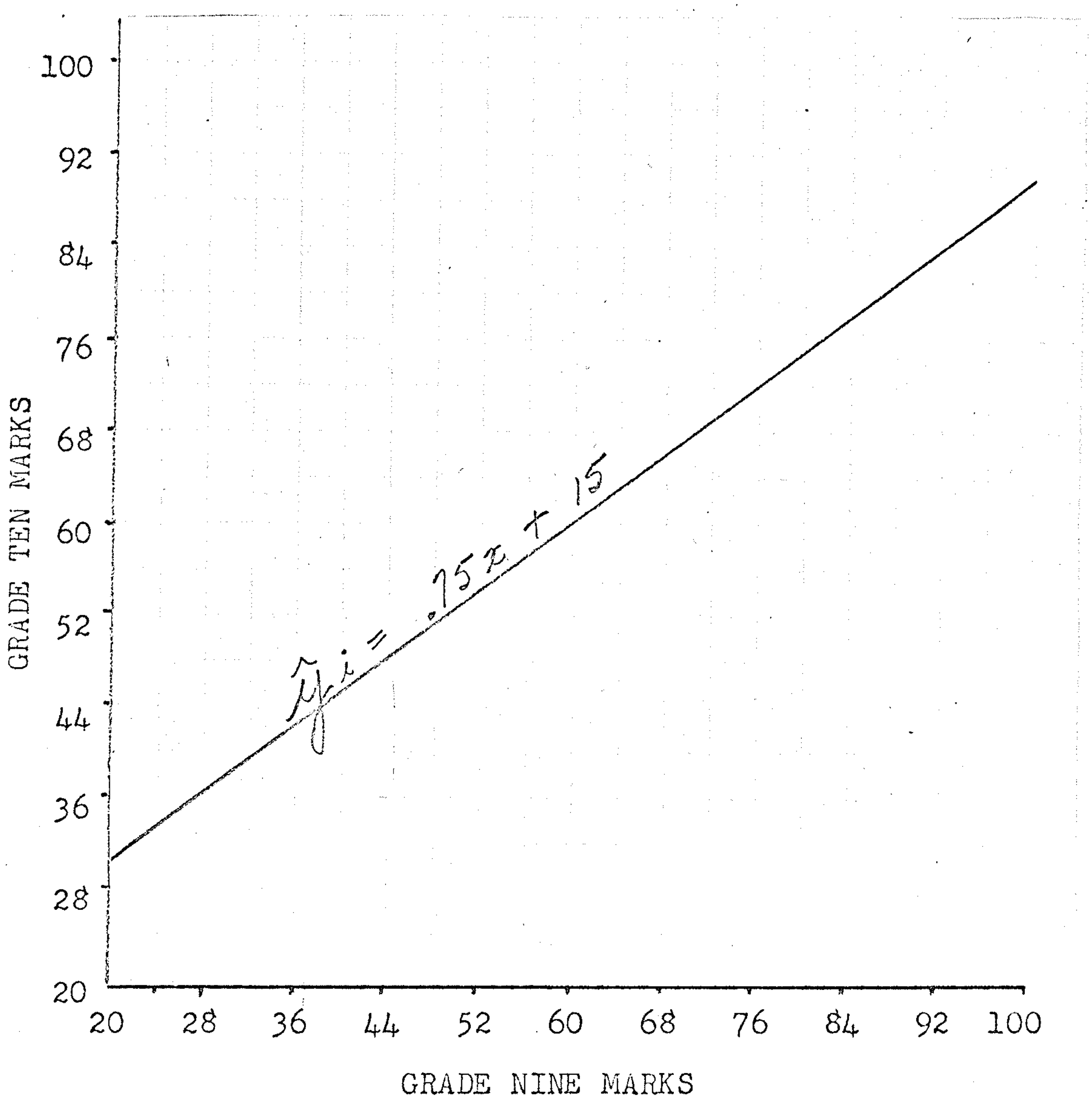


Table II page 23 contained a summary of regression equations for the groups. The equations were of the general form $y = b_1x + b_o$ where b_1 and b_o were constants, consequently where b_1 was the slope of a straight line and b_o was the y-intercept (since $y = mx + b$ is the general equation of a straight line with slope, m, and y-intercept, b).

From the linear graphs (pages 34 to 41 of the simple regression equations it was possible to predict marks in the senior high grades by moving horizontally until the abscissa corresponded to the student's grade nine mark and then proceeding vertically upward from that point to the graph. Where the graph and the vertical line intersected, the ordinate indicated the predicted mark.

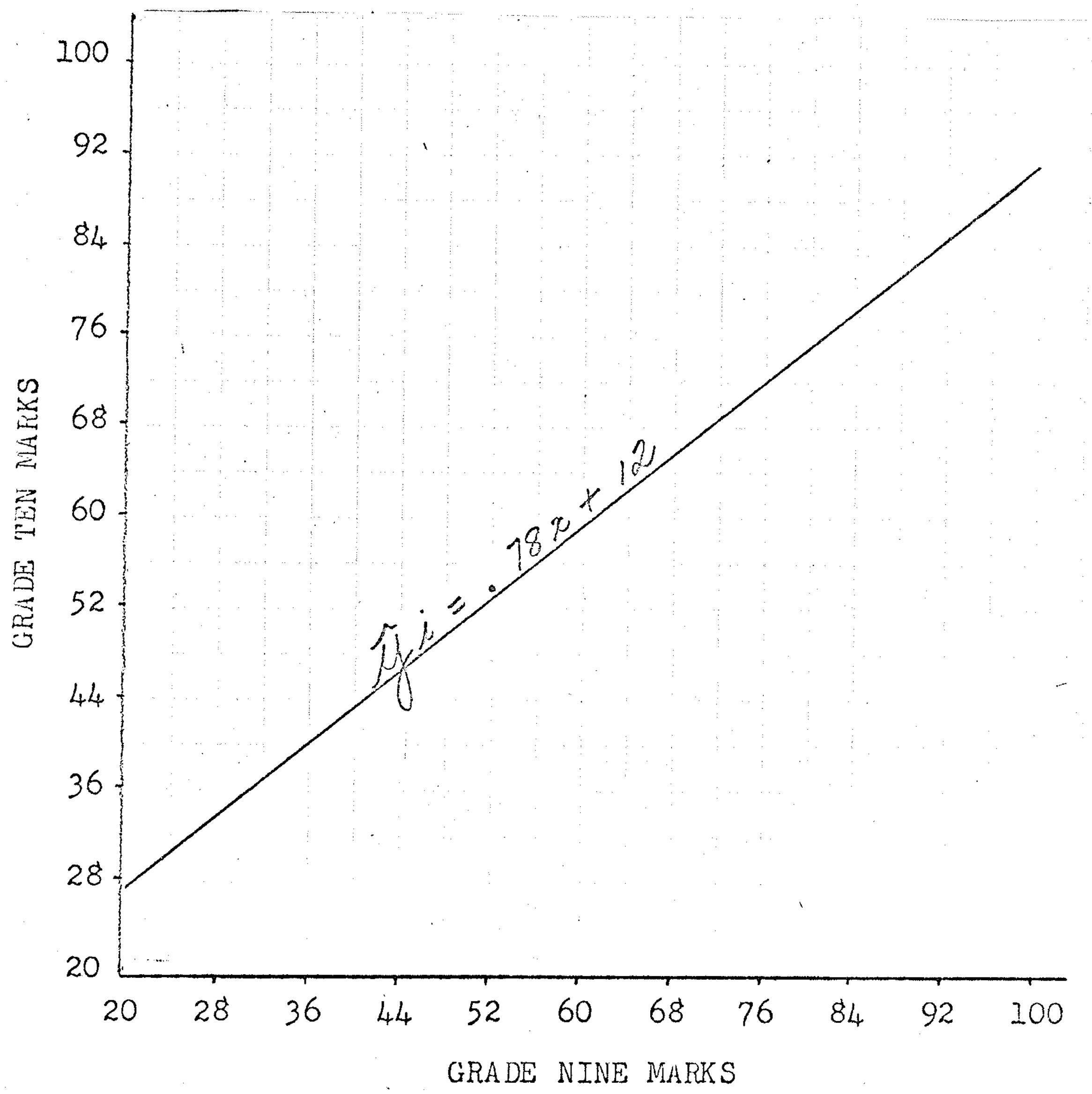
GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP A)



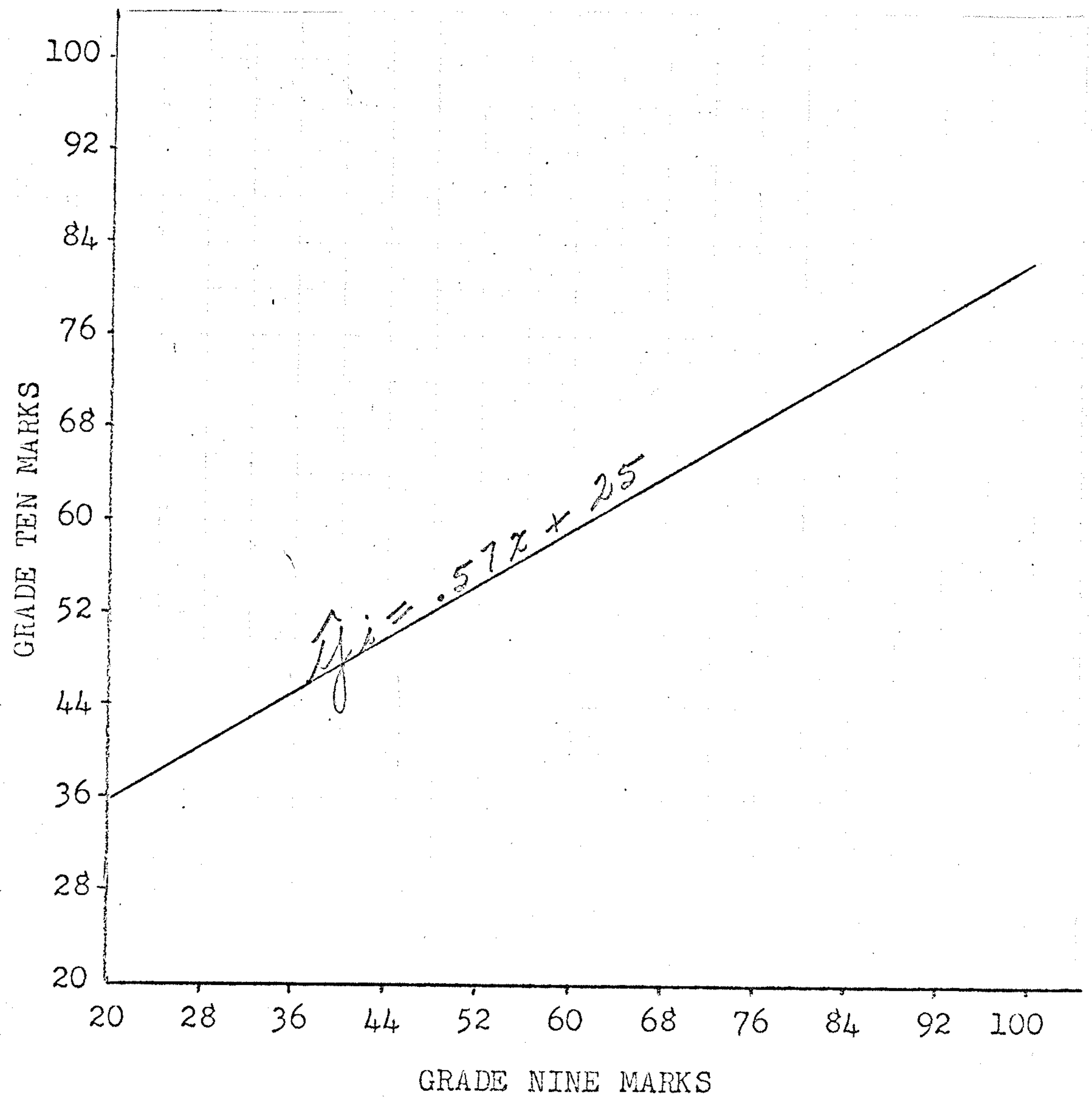
GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP B)



GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

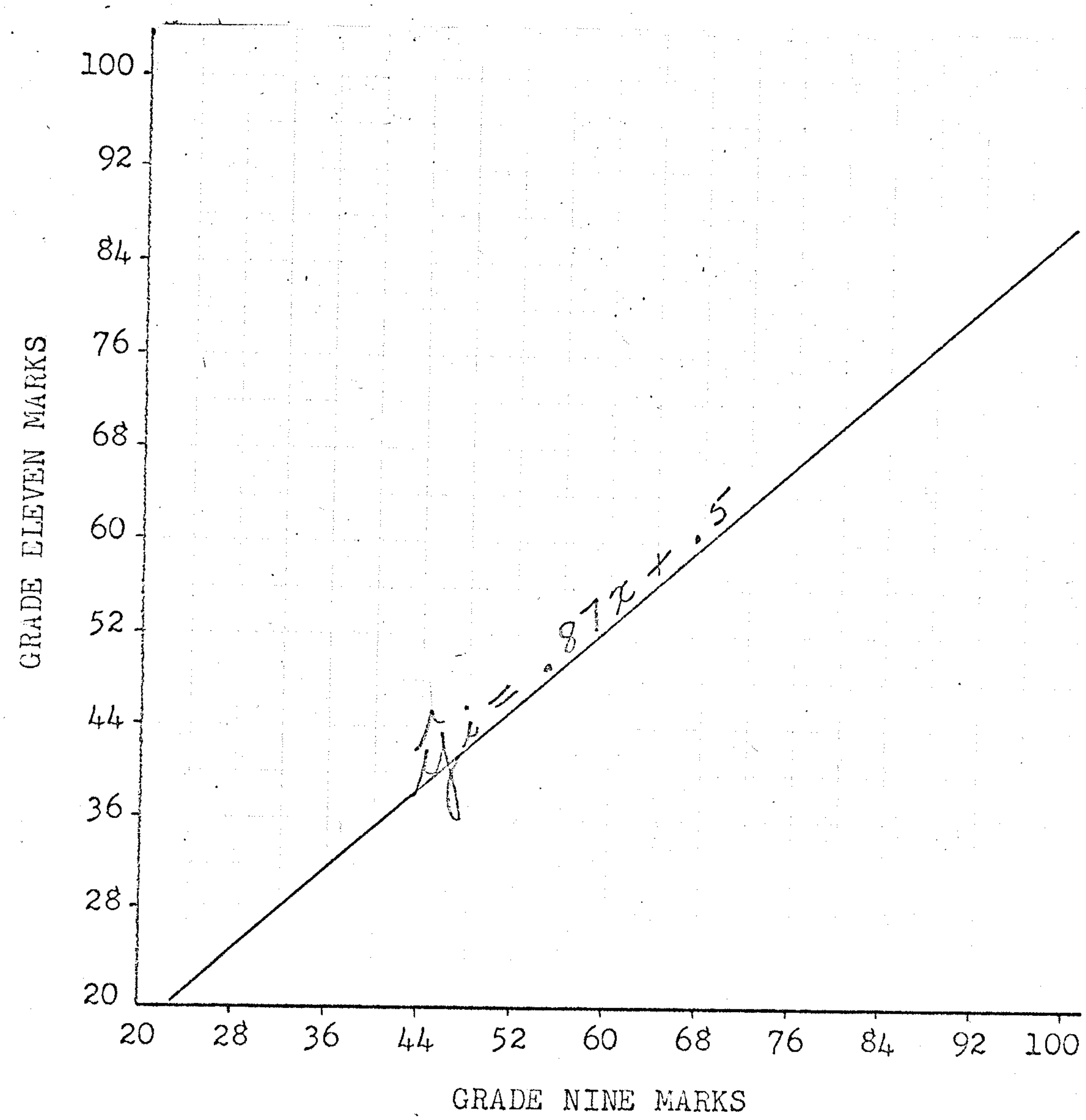
(GROUP C)



GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING

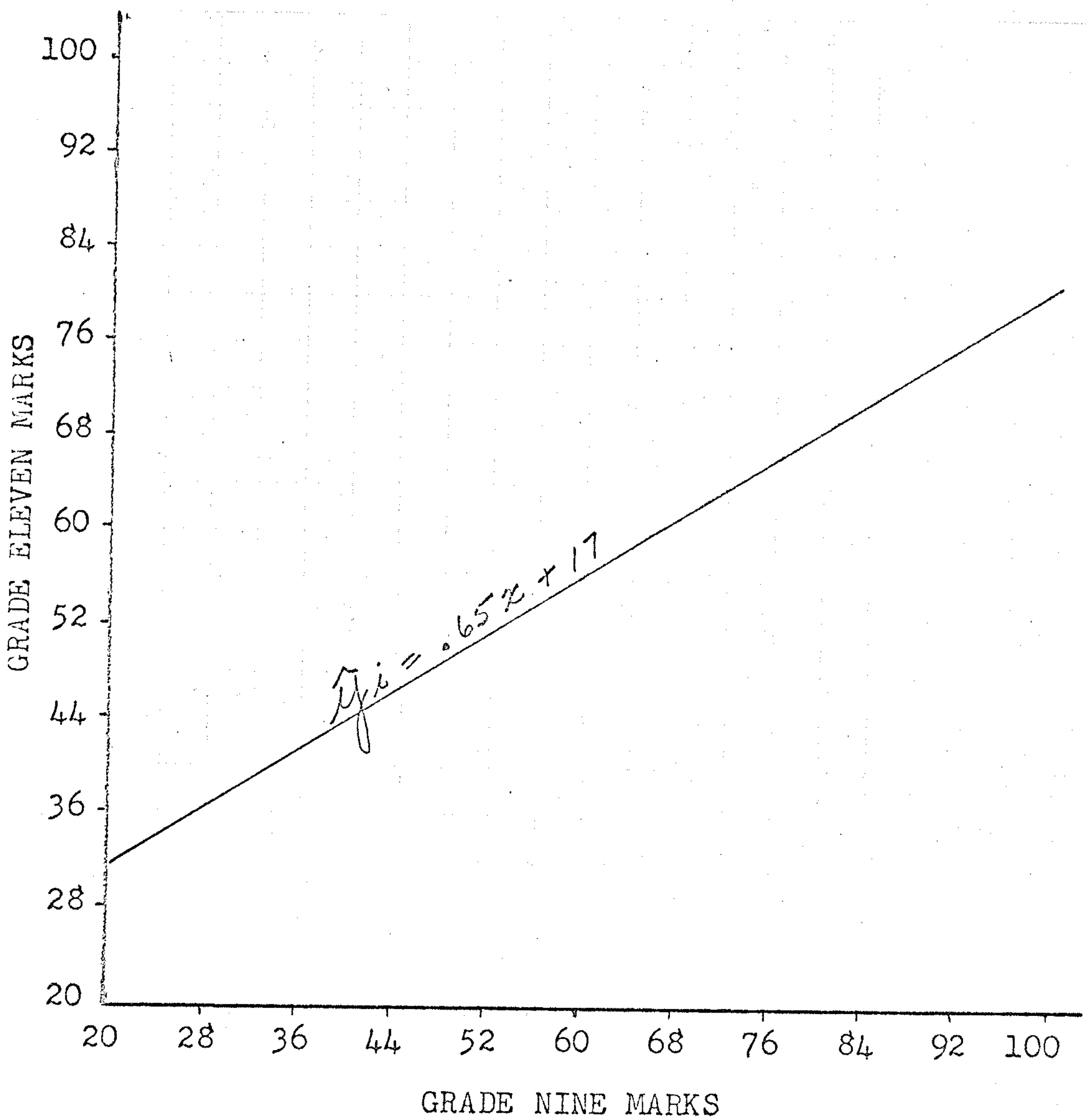
THE LINEAR REGRESSION EQUATION

(GROUP D)



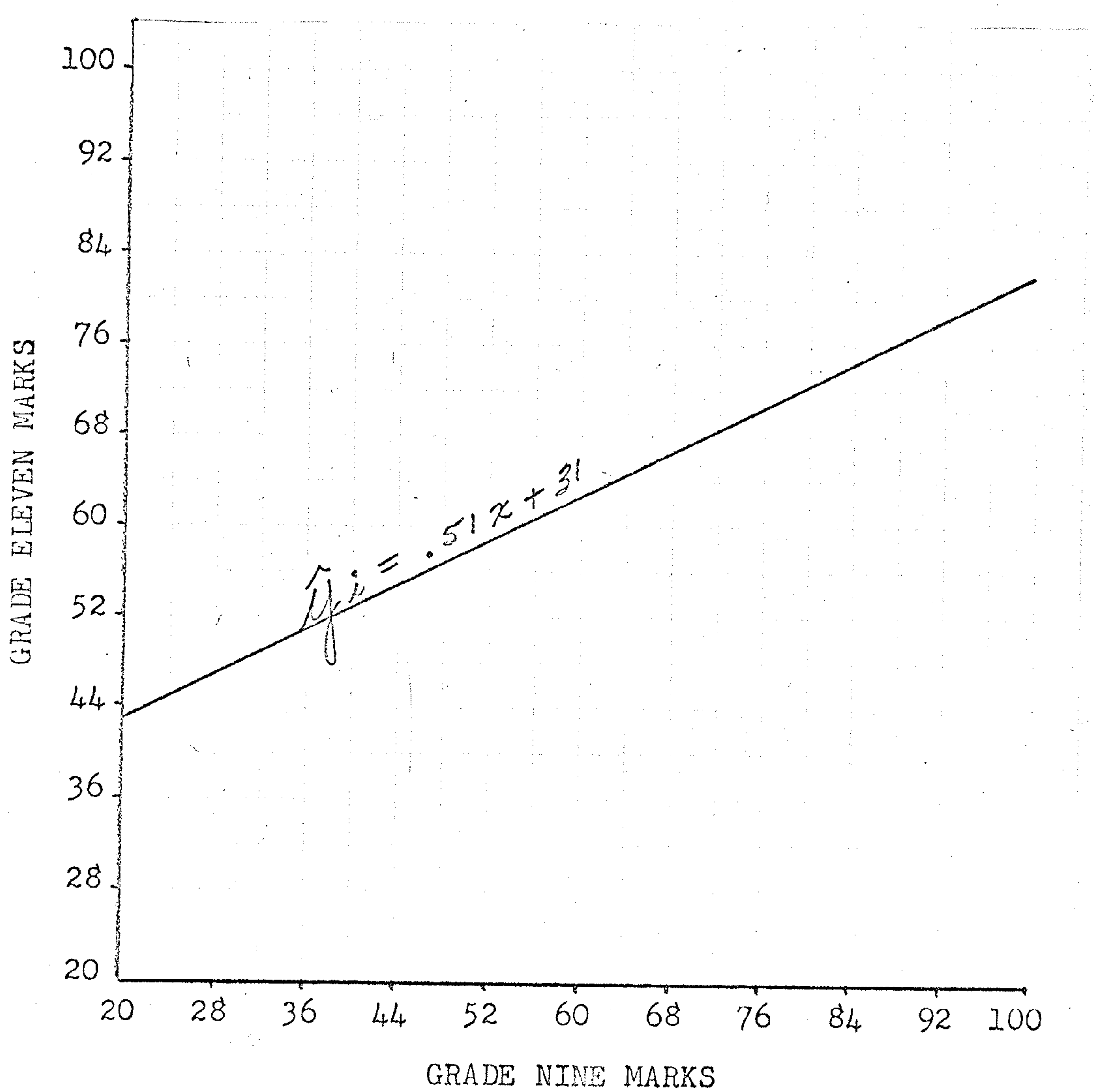
GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP E)



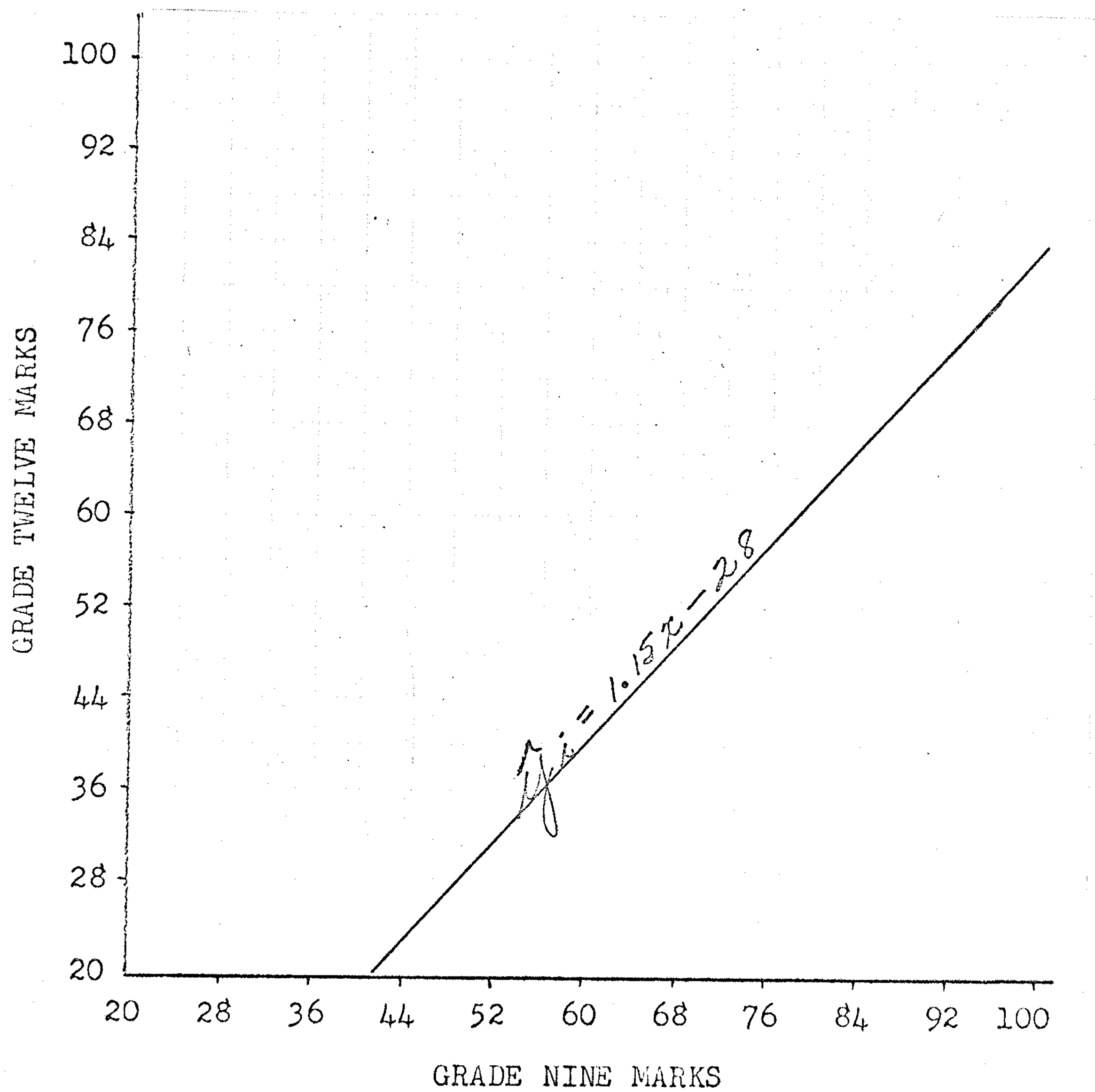
GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP F)



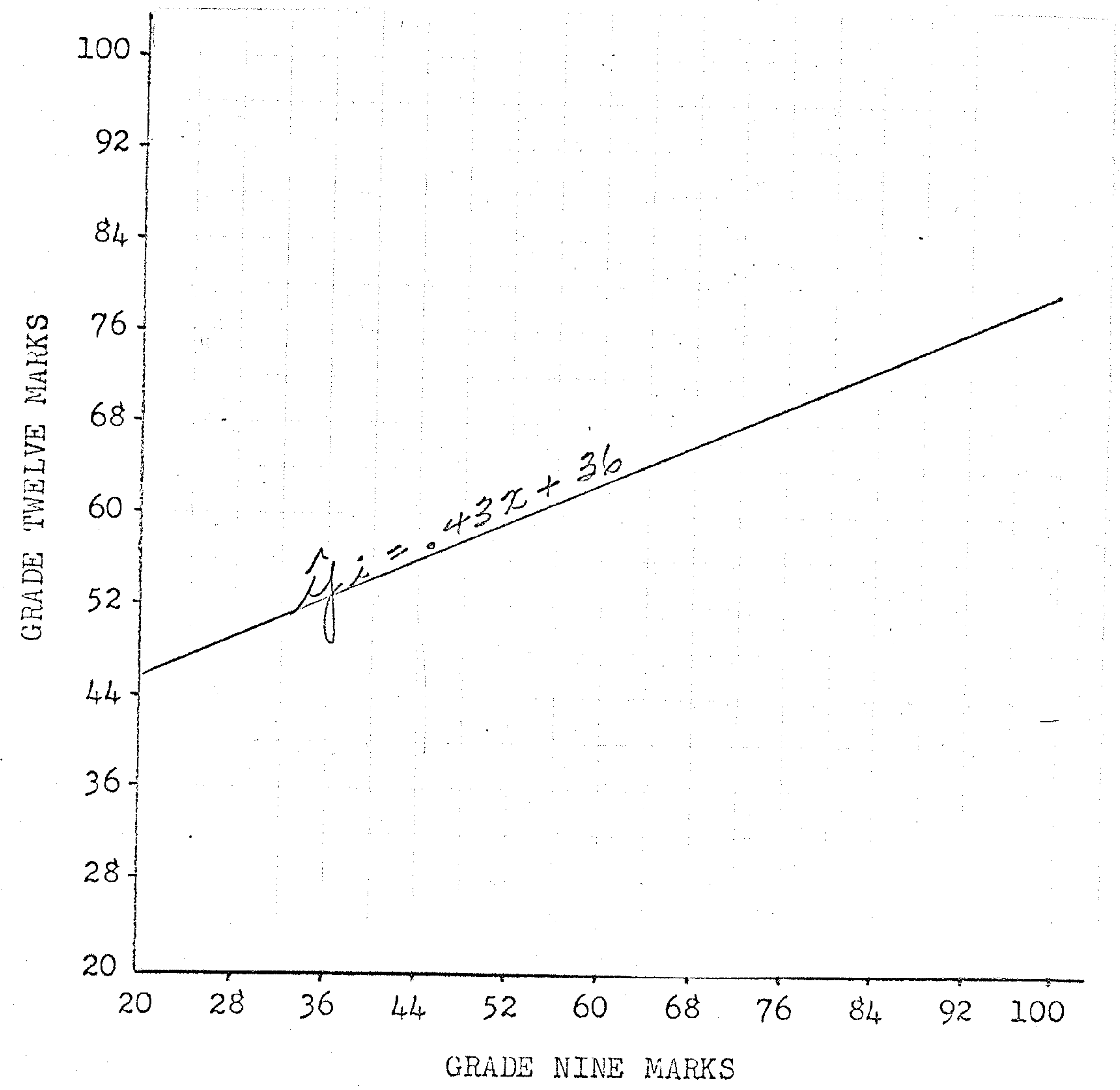
GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP G)



GRAPHICAL ILLUSTRATION OF THE PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP H)



MULTIPLE REGRESSION EQUATION

Because multiple correlation deals with the calculation of weights which produce the maximum possible correlation between the criterion variable and the weighted sum of two or more predictor variables and because the steps were rather tedious the raw scores were key-punched on computer cards and fed into the computer to obtain information for the general equation Criterion $\hat{y}_i$ = A (Predictor 1) + B (Predictor 2) + C)Predictor 3) + Constant. This multiple regression equation consisted of an algebraically determined constant value and the regression weight of each of the independent variables which functioned as predictors. Each regression weight indicated how many units the criterion variable increased (or decreased) for each variation of one unit in the independent variable.

This general equation for predicting the score in grade twelve, for example, was derived by computing the numerical coefficients for each of the letters A, B, and C. The technique used to determine the relationships between two or more independent variables and the dependent variable was done in three steps or stages.

The table below was an extract from a print-out summary for the G group showing the effect of stepwise regression.

Step no. 1 determined the numerical coefficient C, to be .839 and a constant of 7.331 which yielded and equation of $\hat{y}$ = .839 (XI) + 7.331. But since it was necessary to add the effect of grade ten scores step no. 2 was computed.

TABLE III

EXTRACT FROM PRINT-OUT SHOWING REGRESSION WEIGHTS, REGRESSION CONSTANTS, PER CENT VARIANCE ACCOUNTED FOR, AND PROBABILITY LEVEL FOR GROUP G

STEP NO	VARIABLE (GRADE)	WEIGHT	CONSTANT	VARIANCE	PROBABILITY
ONE	Eleven	.839	7.331	65.03	.000
TWO	Ten Eleven	.478 .504	-4.181	70.81	.001
THREE	Nine Ten Eleven	.262 .408 .455	-17.535	72.05	.121

* Complete print-outs included in appendix B.

The inclusion of grade ten score raised the variance of the predicted score accounted for to 70% from 65%. The coefficient C was altered to .504243 and the numerical coefficient B was introduced as .477581 with a new constant of -4.181. At that point the new equation was $\hat{y}_i$ = .478 (Grade X) + .504 (Grade XI) - 4.18.

In order to consider grade nine scores Step no. 3 redistributed the weights and brought out a new constant. The inclusion of those scores raised the percent variance accounted for to 72. Consequently, the final equation became $\hat{y}_i$ = .262 (Grade IX) + .408 (Grade X) + .455 (Grade XI) - 18.54.

The coefficient of multiple correlation indicated the extent of the relationship that existed between the dependent variable and the independent variable. The square of the multiple correlation coefficient (coefficient of determination) expressed the amount of variance of the independent variable that was accounted for by the independent variable, that is, the greater the number of independent variables the greater the portion of variance that was accounted for in the criterion.

For the multiple regression study the population (or sample) was decreased to 222 since many students switched from the 100 course to 201 or 202 or from 101 to 200 or 202 and were deleted from the original sample.

There was no way of graphically illustrating the above results. Table IV page 45 contains the summary of the multiple regression equations taken from the print-outs.

TABLE IV

A SUMMARY OF THE MEANS, STANDARD DEVIATIONS, CORRELATION COEFFICIENTS, AND MULTIPLE REGRESSION EQUATIONS

Group & Sample Size	(a) Mean (b) Standard Deviation				Correlation and Regression Equations
	IX	X	XI	XII	
A	(a) 80.9	74.9			$r_{12} = .70$
N = 137	(b) 10.4	11.9			$\hat{y} = .795(\text{Gr. IX}) + 10.64$
B	(a) 66.3	59.9			$r_{12} = .53$
N = 62	(b) 11.1	15.9			$\hat{y} = .758(\text{IX}) + 9.69$
C	(a) 64.1	62.2			$r_{12} = .77$
N = 23	(b) 11.8	10.9			$\hat{y} = .710(\text{IX}) + 16.66$
D	(a) 80.9	74.9	70.8		$r_{13} = .67,\ r_{23} = .82$
N = 137	(b) 10.4	11.9	13.3		$\hat{y} = .244(\text{IX}) + .765(\text{X}) - 6.20$
E	(a) 66.3	60.0	60.9		$r_{13} = .57,\ r_{23} = .73$
N = 62	(b) 11.1	15.9	13.8		$\hat{y} = .320(\text{IX}) + .517(\text{X}) + 8.75$
F	(a) 64.1	62.2	62.1		$r_{13} = .59,\ r_{23} = .81$
N = 23	(b) 11.8	10.9	14.4		$\hat{y} = -.100(\text{IX}) + 1.149(\text{X}) - 2.97$
G	(a) 86.0	78.5	74.5	69.9	$r_{14} = .65,\ r_{24} = .79,\ r_{34} = .81$
N = 60	(b) 7.3	10.7	12.3	12.8	$\hat{y} = .260(\text{IX}) + .408(\text{X}) + .455(\text{XI}) - 18.54$
H	(a) 68.8	60.6	64.2	65.5	$r_{14} = .49,\ r_{24} = .52,\ r_{34} = .54$
N = 25	(b) 11.2	14.9	12.0	12.1	$\hat{y} = .192(\text{IX}) + .184(\text{X}) + .271(\text{XI}) + 23.77$

The remainder of the results in the form of print-outs will appear in appendix B.

The hypothesis that there was no significant difference between the means of the predictor variables and the criterion variable was rejected at the five percent level of significance. In other words, if the probability level was above .05 the independent variable was rejected. If the probability level was below .05 a large F factor could not happen by chance and consequently the null hypothesis was accepted.

Because some of the independent variables were rejected due to significant variations or differences it was necessary to calculate new equations for some of the groups , in particular for F, G, and H.

TABLE V

LIST OF REVISED MULTIPLE REGRESSION EQUATIONS

GROUP	MULTIPLE REGRESSION EQUATION
F	1.066 (Grade X) - 4.165
G	.478 (Grade X) + .504 (Grade XI) - 5.18
H	.542 (Grade XI) + 30.74

SUMMARY

This chapter dealt with the organization of the data into two categories. The first part dealt with a description of the procedures utilized and some analysis regarding linear regression equations, while the second section was devoted entirely to a discussion and analysis related to multiple regression techniques.

In the discussion of the first section a sample of 360 scores was employed. In order to obtain the linear regression equations it was imperative to work through a sequence of steps, that is, calculate the means, standard deviations, and coefficients of correlation. To make the equations more comprehensible graphical illustrations were included. The scattergrams illustrated the standard deviations and the correlation of scores.

The multiple regression technique was discussed in some detail in spite of the fact that all the information was derived from the summary print-outs. The general multiple regression equation $\hat{y}_i$ = A (Predictor I) + B (Predictor II) + C (Predictor III) + Constant was defined. The derivation of the numerical coefficients was explained. It was also pointed out that the size of the sample was reduced to 222 due to the fact that some students changed courses during their senior years. Finally a summary of the equations was included.

The findings of the investigation and a comparison of these findings were reported in the succeeding chapters.

CHAPTER IV

ANALYSIS OF DATA

In this chapter an analysis of data is reported and the regression equations applied to the grade nine scores in order to predict student achievement in the final year of high school. Comparisons of the predicted scores and the actual results are also presented.

A discussion of the inferential properties of the statistical investigation necessitates consideration of a number of factors. These factors are correlation, regression, simple and multiple regression equations, and standard errors of estimate. Through a study of these factors the writer sought to establish practical means by which prediction of success in high school may be accomplished at a reasonable level of accuracy.

Correlation

The correlation coefficient, r, is a term used to describe the relationship between the independent variable x and the dependent variable y in this case the grade nine mark and the mark in a later grade. The presence of a correlation between the two variables does not necessarily mean that there exists a causal link between them but that a statistical relationship exists.

For group A, an "r" of .71 indicated that there was a high correlation between the marks achieved by the students in grade nine mathematics and the marks achieved by the same students a year later in Mathematics 100. The scatter diagram presented on page 25 indicated the relationship in a non-numerical way, that is, there was a high density of location points along a diagonal line running upward and to the right. Neither the r of .71 nor the diagram gave any evidence of "why" or "cause" for this relationship but only that such a relationship existed.

For group H, the "r" of .39 indicated that there was a low correlation between the marks obtained by the students in grade nine mathematics and the marks achieved by the same student three years later in Mathematics 301. The scatter diagram on page 32 indicated this poor relationship in a non-numerical way since the location points did not present a definite direction.

In the table below, the reader can find the numerical correlation coefficient for each group and the corresponding magnitude of relationship. There is no definitive explanation for the magnitude of relationship since it is strictly a subjective concept. The writer has arbitrarily labelled a correlation of .70 plus as high, .60 - .69 as moderate, and .59 minus as a low magnitude of relationship.

To examine the results presented above in another way, the coefficient of determination (r^2) was calculated. This coefficient expresses the amount of variance in y (the dependent variable) which may be accounted for by x (the independent variable). For Group A, $r^2 = (.71)^2 = .50$ which indicated that approximately one half of the variance of the grade ten marks may be accounted for by the grade nine marks.

Similarly for group H, $r^2 = (.39)^2 = .15$ which indicated that approximately fifteen percent of the variance in y was accounted for by the grade nine marks. The variance for all the groups is summarized in Table VI.

TABLE VI

A SUMMARY OF THE CORRELATION COEFFICIENTS, VARIANCE, AND MAGNITUDE OF RELATIONSHIP

GROUP	CORRELATION	VARIANCE	MAGNITUDE
A	.71	50	High
B	.66	44	Moderate
C	.76	58	High
D	.67	45	Moderate
E	.51	26	Low
F	.42	18	Low
G	.68	47	Moderate
H	.39	15	Low

The correlation between the independent variables and the criterion variable was also required for the multiple regression analysis. The correlation matrix in Table VII below is an excerpt from the computer print-out. This table illustrates the moderate association between grade nine and ten marks, the moderate association between grade nine and eleven marks, and the low association between grade nine and twelve marks.

TABLE VII

CORRELATION MATRIX FOR MATHEMATICS 301

	9	10	11	12
9	1.000000	0.610988	0.634796	0.487590
10	0.610988	1.000000	0.690395	0.521760
11	0.634796	0.690395	1.000000	0.539009
12	0.487590	0.521760	0.539009	1.000000

A computer program designed to provide step-wise regression was used to analyze the results obtained by following the students through from grade nine to twelve. (This analysis is presented in Appendix B) It was noted that in Step 2 the coefficient of determination, r^2, between grade eleven and grade twelve was 29.05%. Step 2 included grade ten and together with the grade eleven accounted for 33.33% of the variance. Step 3 (or the inclusion of grade nine) indicated that 35.04% of the variance in grade twelve was accounted for by grades nine, ten, and eleven combined.

Regression

The regression line on the graph provides a means, whether the equation is used alone or in conjunction with the graph, by which the student could predict his score in grade ten on the basis of his grade nine mark. The regression equation is as dependable as its components, especially the y -intercept, b_o, which is dependent on the differential between the means.

For group A, a grade nine mark of 70 would yield a grade ten predition score of 67.5 from the equation $\hat{y}_i = .75x + 15$. In the table below, the reader can find corresponding marks in each group based on certain grade nine marks.

TABLE VIII

CORRESPONDING MARKS IN EACH GROUP BASED ON CERTAIN GRADE NINE MARKS

GROUP	PREDICTED SCORES BASED ON		
	52	68	78
A	54	66	78
B	53	65	78
C	55	64	73
D	47	60	74
E	51	61	71
F	58	66	74
G	33	50	68
H	58	65	72

These predicted scores would be exact if the correlation were equal to one and/or the standard deviation of y were zero. A significant result of the regression investigation revealed a considerable error of estimate when the prediction was based on the grade nine marks.

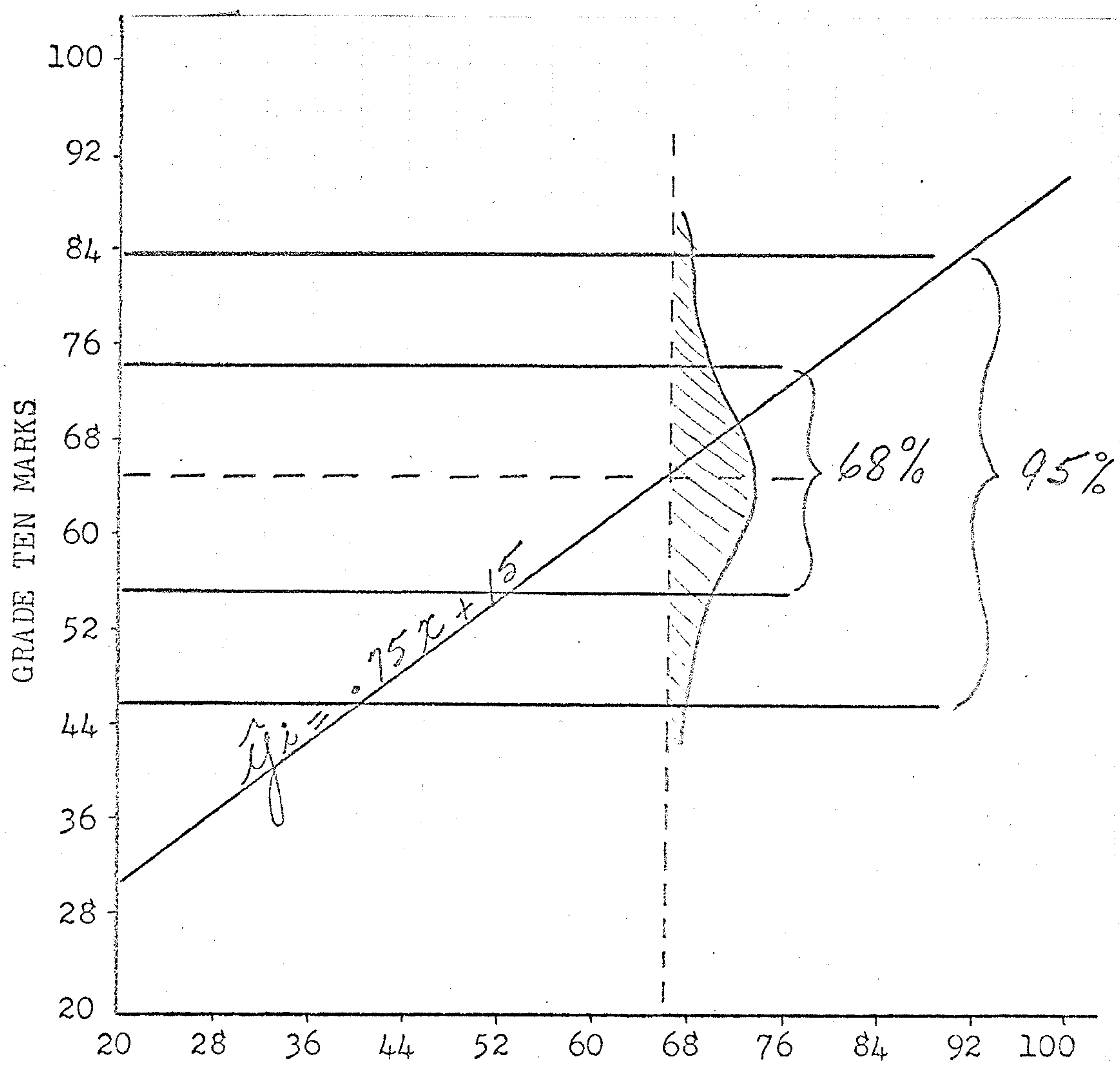
In using the prediction equation or prediction line graphs errors of estimate were inevitable, that is, there were differences between the estimate score and the actual score. Those differences were labelled "e_i". The variance, Se^2, would be the result of summing all the e_i^2 and dividing it by n - 1. The square root of the variance yielded the standard error of estimate, Se. A shorter way of acquiring the standard error of estimate was by using the formula $Se = Sy\sqrt{1 - r^2xy}$. That standard error of estimate could then be used to set limits around the predicted score y, within which a person's actual score was likely to lie.

The standard error of estimate, calculated by using the formula $S_e = sy\sqrt{1 - r_{xy}^2}$, implied that 68 per cent of the predicted scores would be within the limits plus or minus that value of the actual score. The standard error or estimate for group A was 9.2 which meant that the original mark of 70 in grade nine and a predicted mark of 67.5 in grade ten would in essence range from 58.3 to 76.7 for sixty-eight per cent of the population to which the regression equation was applied, and from 49.1 to 85.9 for two standard errors or ninety-five per cent of the population. Graphical illustrations can be found on pages 54 to 61 inclusive.

The prediction lines selected were those which minimized the sum of the squared deviations i.e. the sum of the distances of the points from the line. The graphs in appendix A, pages 78 to 85 illustrated how the prediction lines related to the distribution of scores. The lines were drawn in such a direction as to create the least-squares regression line.

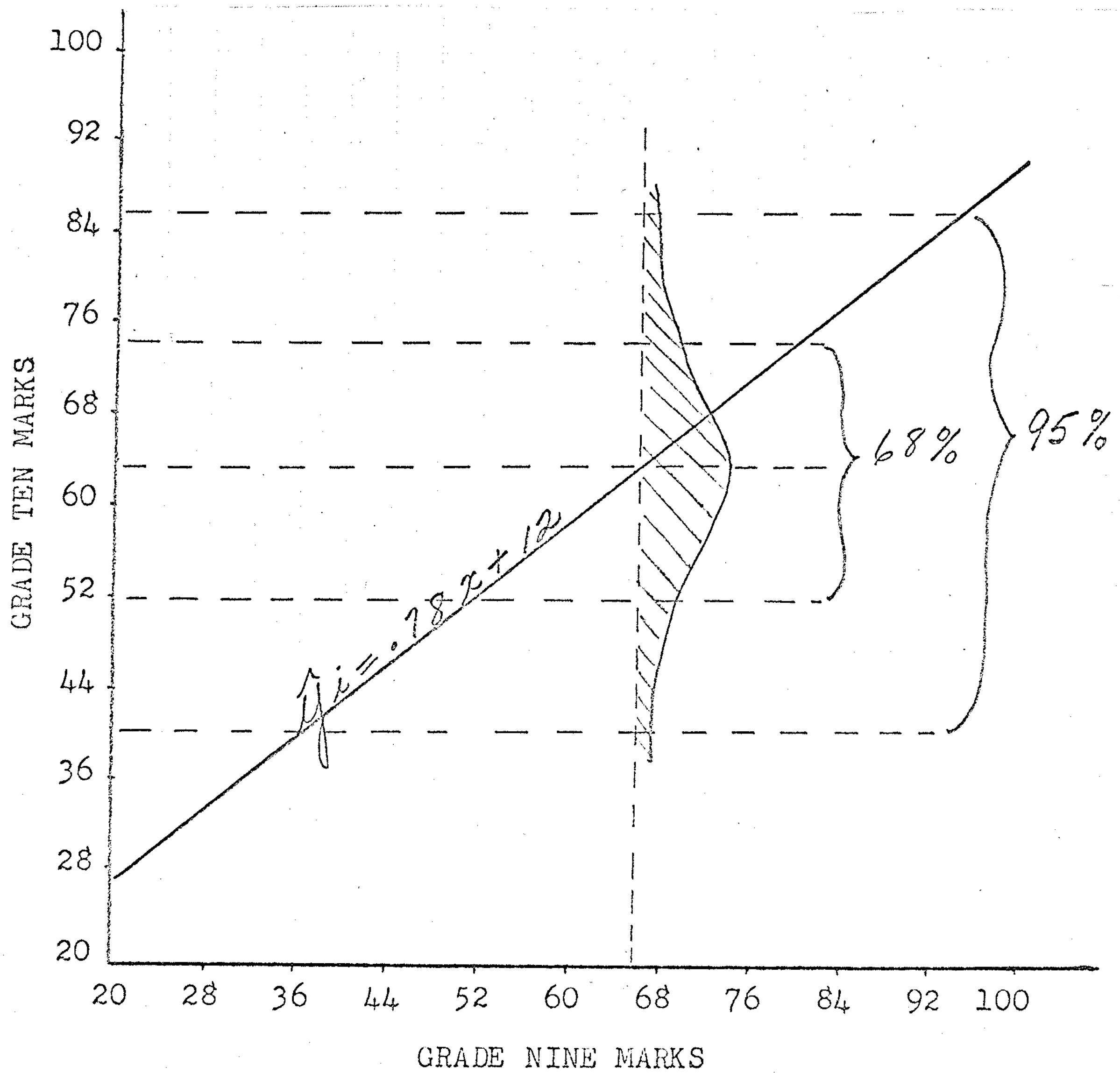
GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

S_e = 14.6 FOR GROUP A



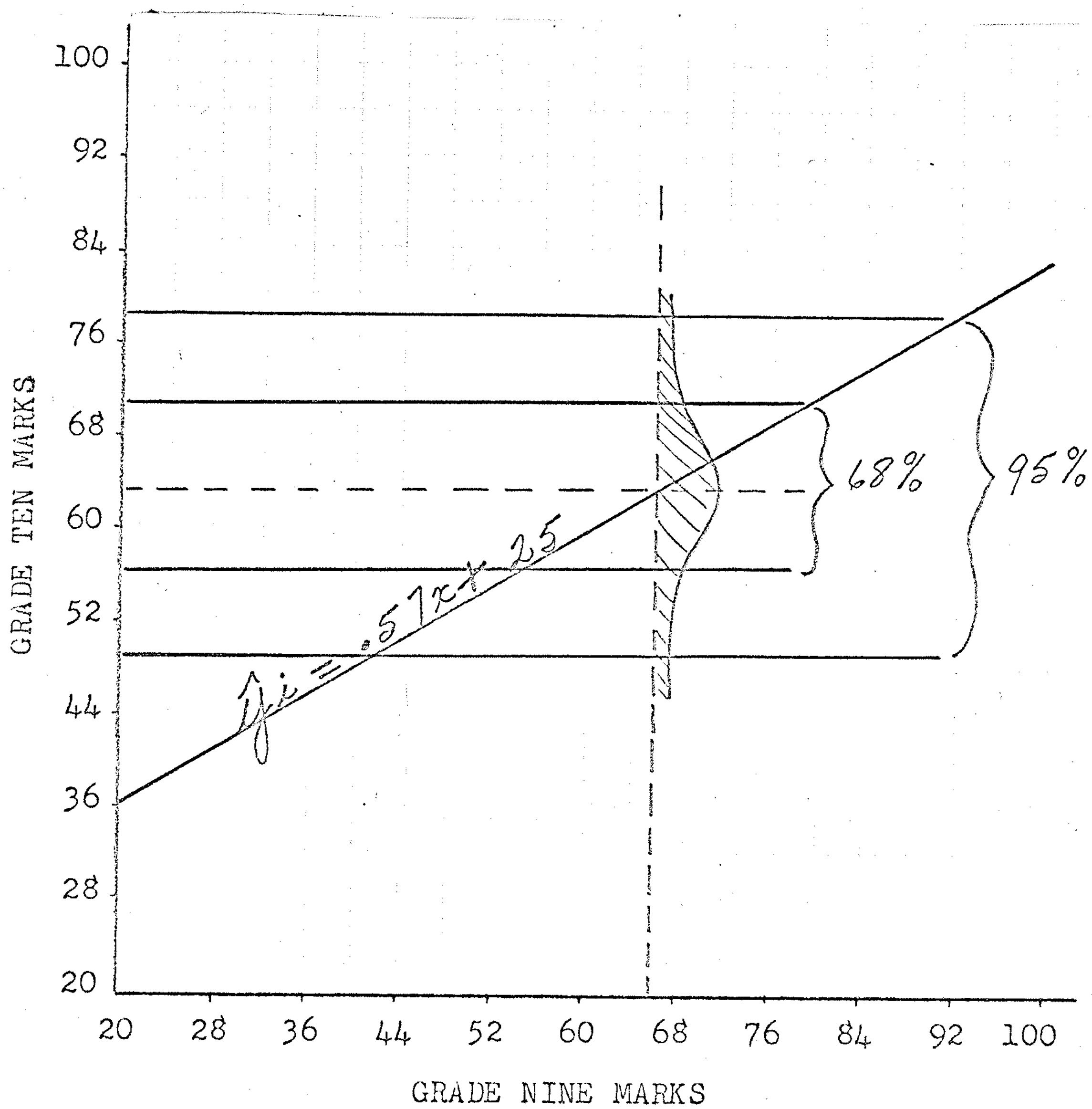
GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

S_e = 12.1 FOR GROUP B



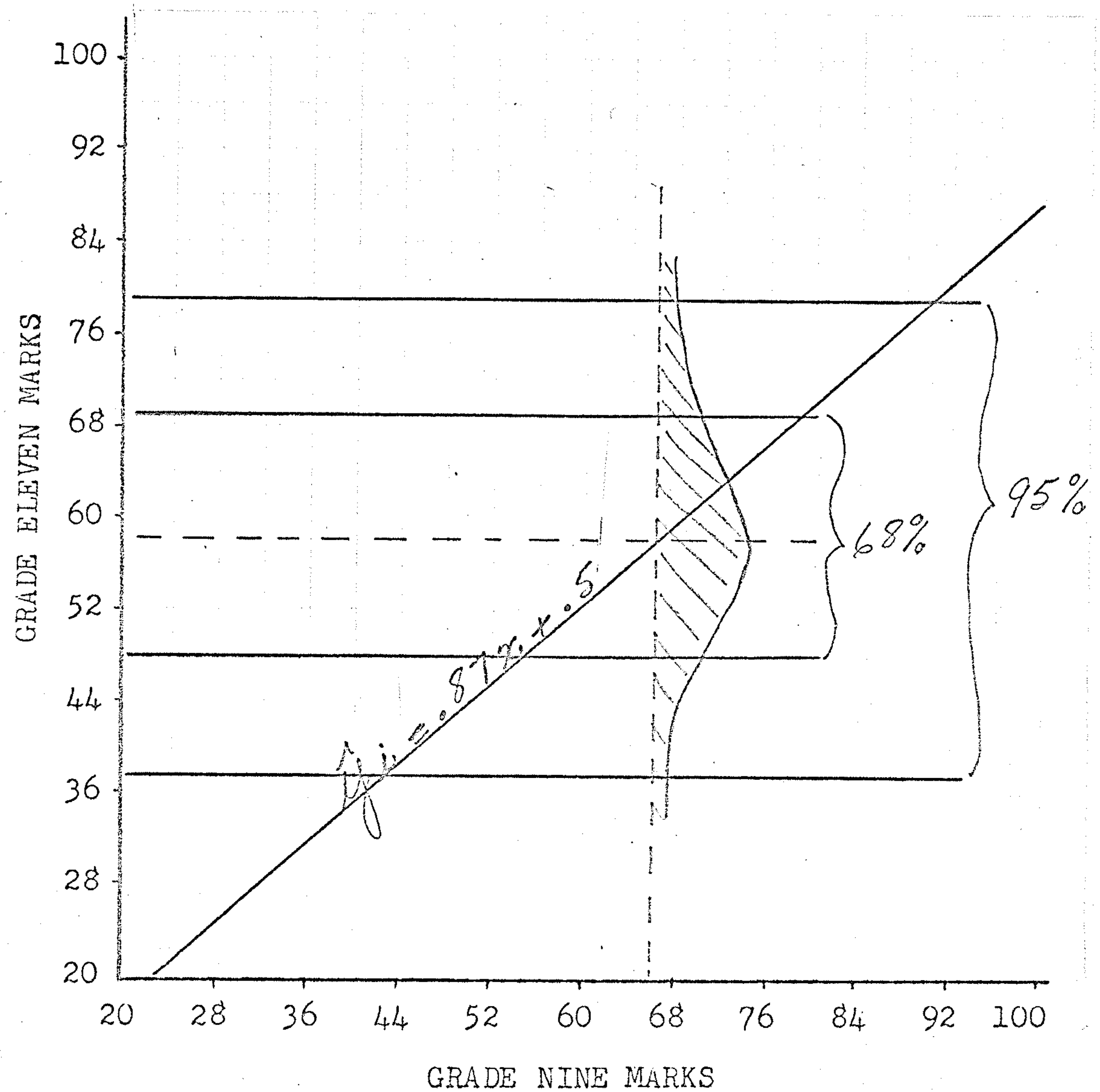
GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

$S_e = 6.6$ FOR GROUP C



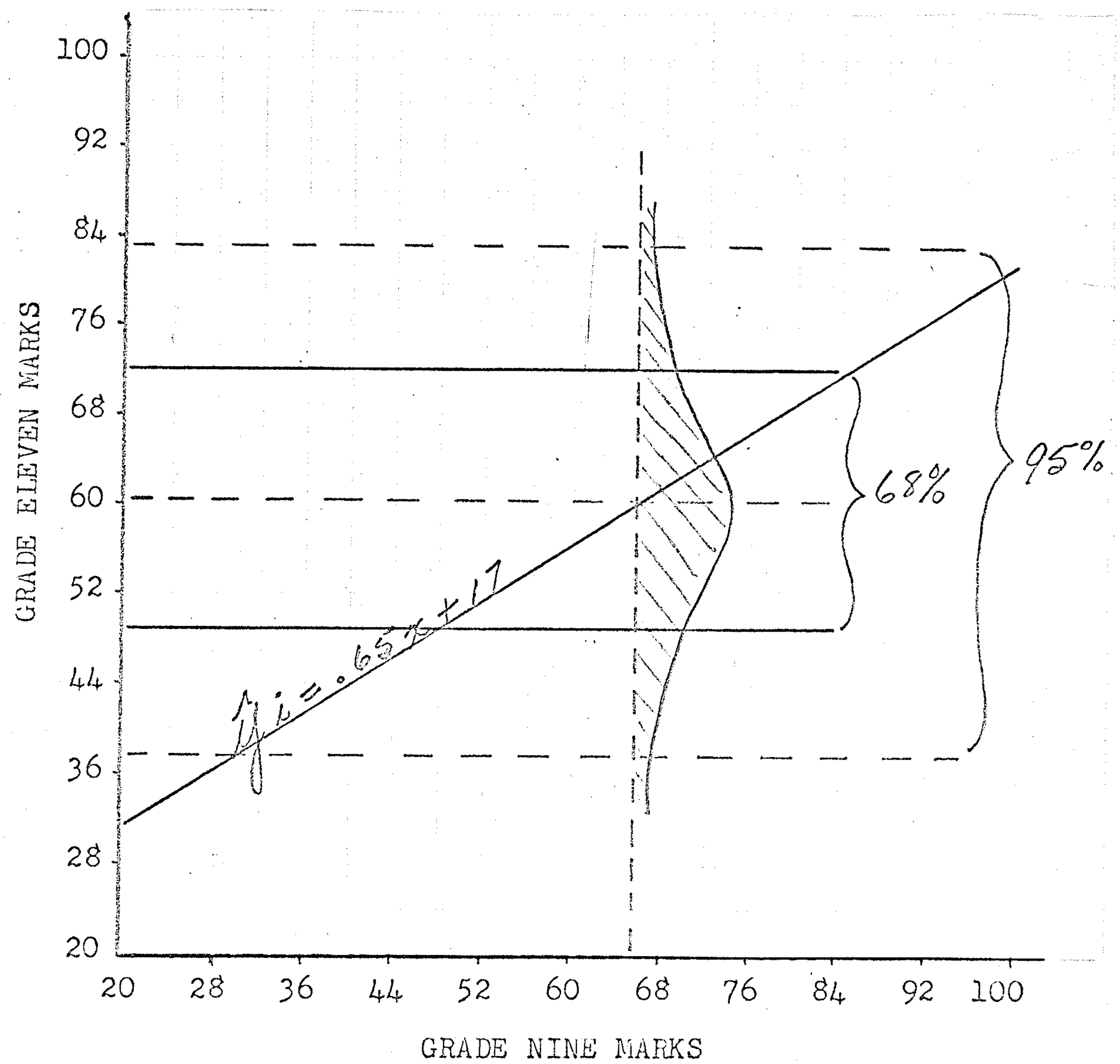
GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

$S_e = 9.9$ FOR GROUP D



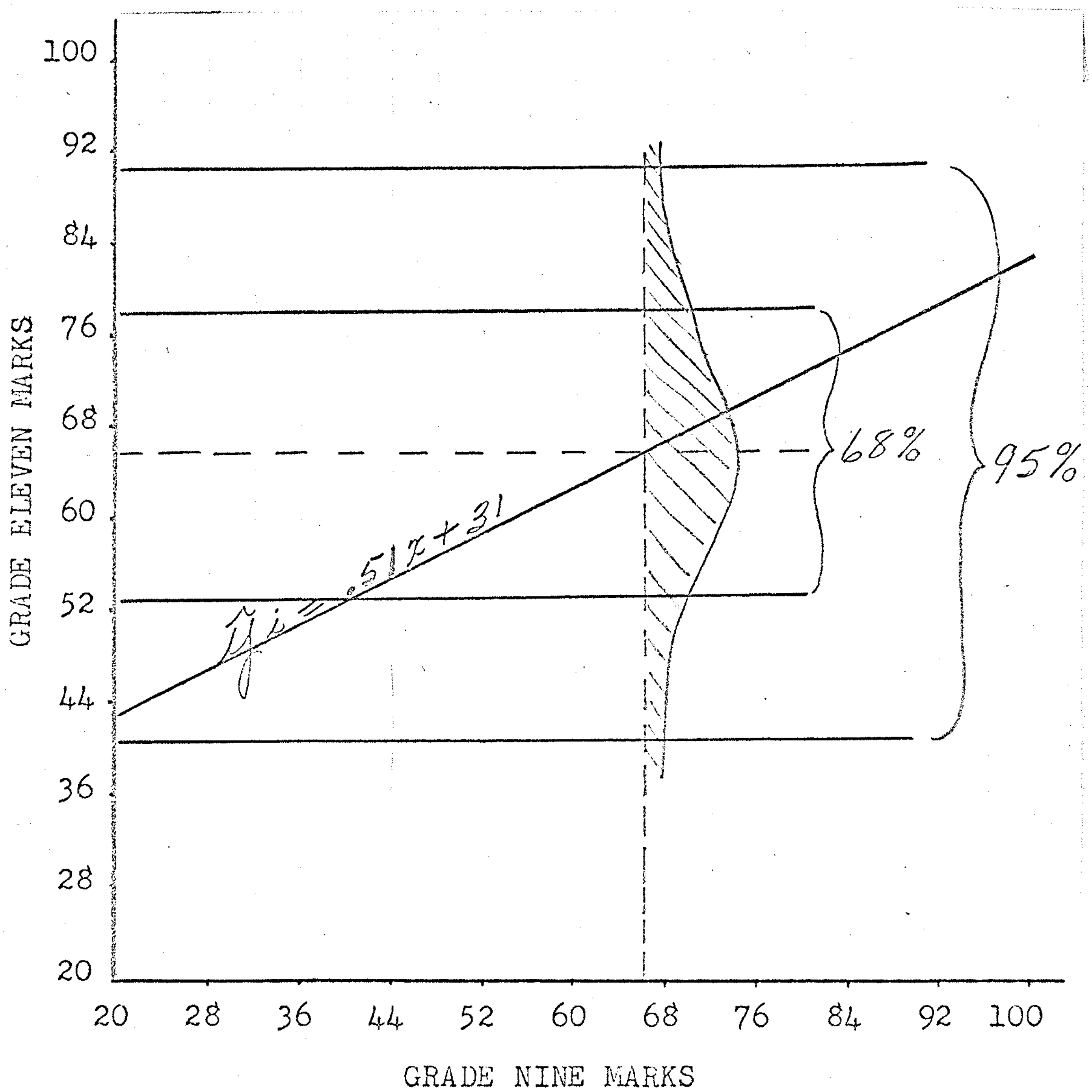
GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

S_e = 11.5 FOR GROUP E



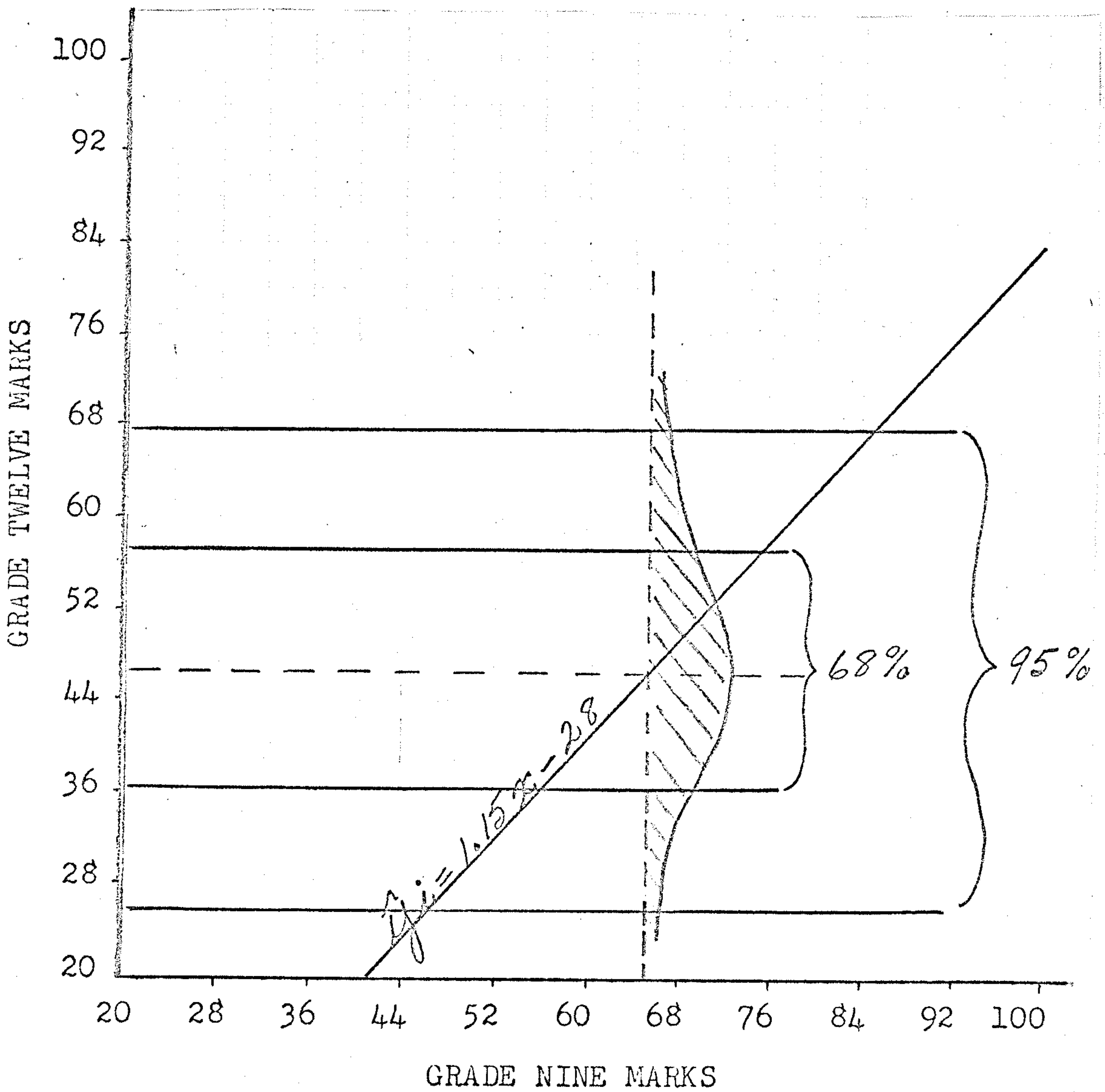
GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

S_e = 12 FOR GROUP F



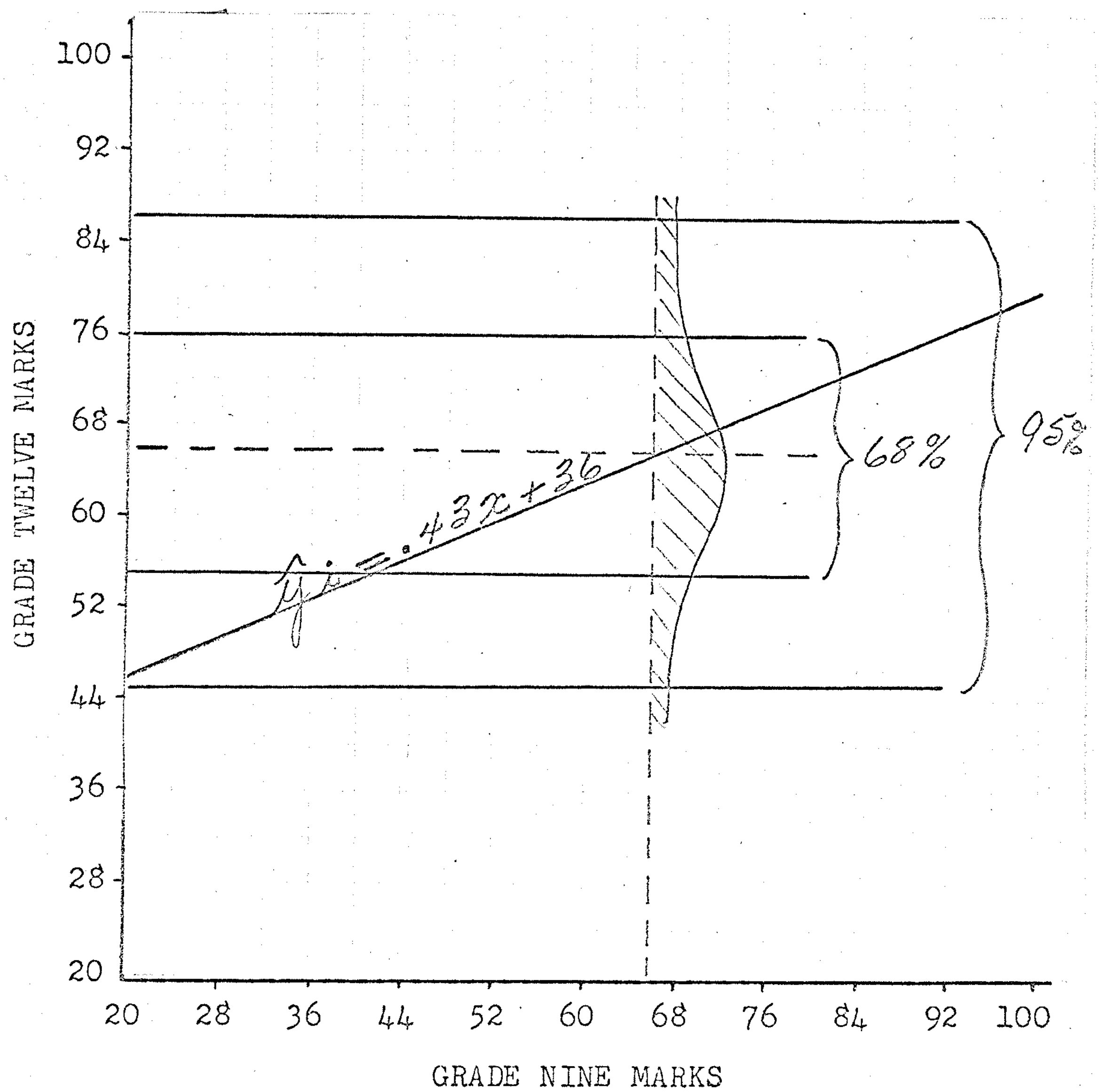
GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

$S_e = 9.6$ FOR GROUP G



GRAPHICAL ILLUSTRATION OF THE STANDARD ERROR OF ESTIMATE,

S_e = 10 FOR GROUP H



To facilitate the collecting of information which was illustrated by the graphs a summary of the equations with corresponding errors was prepared in the table below.

TABLE IX

A SUMMARY OF LINEAR REGRESSION EQUATION WITH CORRESPONDING STANDARD ERRORS OF ESTIMATE

GROUP	EQUATION	S_e
A	$y_i = .75x + 15$	9.2
B	$y_i = .78x + 12$	10.7
C	$y_i = .57x + 25$	6.6
D	$y_i = .87x + .5$	9.9
E	$y_i = .65x + 17$	11.5
F	$y_i = .51x + 31$	12.0
G	$y_i = 1.15x - 28$	9.6
H	$y_i = .43x + 36$	10.0

Due to their complex nature, it is impossible to graph the multiple regression equations. Consequently, these equations and the corresponding standard errors of predicted y are tabulated in Table X on the following page.

TABLE X

A SUMMARY OF MULTIPLE REGRESSION EQUATIONS WITH CORRESPONDING STANDARD ERRORS OF ESTIMATE

Group	Equation	S_e
A	$\hat{y} = .795(IX) + 10.64$	8.5
B	$\hat{y} = .758(IX) + 9.69$	13.5
C	$\hat{y} = .710(IX) + 16.66$	7.1
D	$\hat{y} = .244(IX) + .765(X) - 6.20$	7.5
E	$\hat{y} = .320(IX) + .517(X) + 8.75$	9.1
F	$\hat{y} = -.100(IX) + 1.149(X) - 2.97$	8.9
G	$\hat{y} = .260(IX) + .408(X) + .455(XI) - 18.54$	6.9
H	$\hat{y} = .192(IX) + .184(X) + .271(XI) + 23.77$	10.4

From Table X it is possible to calculate the predicted mark for a student, for example, in Group H, if the student who obtained marks of 53, 58, and 68 in grades nine, ten, and eleven respectively inquired about his probable success in grade twelve 301 course his equation $\hat{y} = .192(IX) + .184(X) + .271(XI) + 23.77$ would yield a prediction score of 63. But a standard error of 10.4 implies that 68 per cent of the predicted scores would be within the limits of plus or minus 10.4 of 63, ranging from 53 to 73. To predict his score with a greater degree of accuracy, say 95% or two standard errors, his score would lie in the range 43 - 83.

Summary

This chapter deals with the factors which were employed in the analysis of data.

The coefficient of correlation was a true indicator of the relationship between the predictors and the dependent or criterion variables. The square of the coefficient (known as the coefficient of determination) expressed the amount of variance in the dependent variable that was due to the independent variable.

Linear regression equations were discussed together with some predicted scores (see Table VIII, page 52). Because standard errors of estimate were inevitable, graphical illustrations of these were also included. A summary of the linear equations was presented on page 62 and of the multiple equations on page 63.

The inferences from the foregoing analysis will be presented in detail in the following chapter.

CHAPTER V

SUMMARY, CONCLUSION AND RECOMMENDATIONS

This chapter presented a brief review of the study, the findings and cross-validation of the equations, and some concluding remarks including recommendations.

SUMMARY

In this study 360 students who were enrolled in a grade nine mathematics course were given the freedom to choose any Mathematics course in grades ten, eleven, and twelve. All students attended Morden Collegiate and almost all of them had attended the Morden Elementary School for their grade eight.

The general purpose of this study was to examine pupil achievement in the senior high grades and in terms of their achievement to determine whether or not there should be more directed guidance offered to the students at the completion of grade nine regarding course choice and prediction of success.

Examination of the correlation indices revealed positive and significant values (ranging from between .488 - .817) between the predictor variables (grade IX marks) and the criterion variables (see Tables XIII - XIX in appendix A). The regression equations were based on the correlations and the means between the independent variables and the dependent variables. The positive and significant correlation values produced valid regression equations.[1]

These prediction or regression equations were cross-validated at the grade twelve level by selecting five students at random from each group G (300) and H (301). Each student's predicted score was recorded and compared with his actual score. This investigation was designed to determine the efficiency of predicting grade twelve marks by means of simple linear regression equations and also by means of multiple regression equations and then comparing the results with the actual scores.

TABLE XI

CROSS-VALIDATION OF TEN GRADE TWELVE RESULTS BY COMPARING PREDICTION SCORES WITH ACTUAL SCORES

SUBJECT NUMBER	SCORES ACHIEVED IN			MULTIPLE REGRESSION SCORE	LINEAR REGRESSION SCORE	ACTUAL SCORE
	IX	X	XI			
059	96	95	88	85	82	84
053	75	65	53	52	58	61
037	84	87	77	74	69	63
029	75	57	51	47	58	45
138	75	80	73	67	58	62
154	53	58	68	63	59	52
009	63	33	51	56	63	63
102	69	78	60	68	66	43
052	64	52	59	62	64	54
076	78	69	67	70	70	84

[1]James N. Jacobs, "Aptitude and Achievement Measures in Predicting High School Academic Success," Personnel and Guidance Journal, Vol. 37 1959, p. 335.

The results in the above table indicated that reasonably accurate predictions of Grade twelve results could be made from grade nine mathematics results, using either the linear or the multiple regression equations.

CONCLUSIONS

It has been shown that students achieving fifty per cent or better in grade nine have a very realistic chance of being successful in any one of the grade ten mathematics courses. In fact it would appear that essentially there is no difference in the probability of success or the predicted scores at this level. The grade nine marks are valid prediction of success in grade ten. Consequently, the decision as to which course students should choose should rest entirely upon the individual student's wishes, needs, and abilities. This means that students who wish to take a course of the practical nature would be satisfied by Mathematics 102. Those students who desire a formal mathematics course dealing with algebra and its applications would be encouraged to take Mathematics 101. The students who wish to explore all the facets of Mathematics and who enjoy the theoretical approach would enrol in Mathematics 100.

It would also appear that the students who scored below fifty percent in Mathematics IX should be required to repeat the course in grade nine unless there were some extenuating circumstances, such as a prolonged illness during grade nine which is not likely to recur or where a student is much older than his classmates.

As the reader can see from Table XII below, if a student received less than 50 in grade IX, the probability of that student passing grade ten is indeed very low.

TABLE XII

GRADE TEN PREDICTED SCORES USING LINEAR AND MULTIPLE REGRESSION EQUATIONS

GROUP	SIMPLE LINEAR			MULTIPLE		
	40	44	48	40	44	48
A	45	48	51	42	46	49
B	43	45	49	40	43	46
C	48	50	52	45	48	51

In predicting grade eleven and twelve scores it is preferable to insert the most recent scores, that is, those achieved in the highest grade, if prediction is made on the basis of a single score. However, when more than one score is to be inserted the best prediction is achieved when the scores of all preceding grades are used.

Since the ultimate objective of this study was to provide statistical information as evidence that would enable teachers, counsellors, and principals to council students with respect to the selection of subjects in grade ten, particularly in the selection of mathematics courses, this objective was achieved but the accuracy of the prediction is not as high as anticipated. Statistically, the student has an equal chance of success in any one of the three courses.

This is also true for a student going into grade eleven but with certain limitations since a student could not function in a 200 course if he had not previously taken 100.

However, the fact that a study has been performed and that regression equations and corresponding graphs have been developed and could be explained to the student or the parents would conceivably inspire the students to maximize his potential. Hopefully the findings of this study, i.e. the regression equations and graphs, will be of value to the student in determining the probable score in the next grade or at the end of his high school career. In spite of the relatively large error of estimate it is superior to the so-called "educated guess".

RECOMMENDATIONS

Based upon the analysis of data collected in this investigation and the literature reviewed in the study, the following recommendations are submitted.

It is recommended that school counsellors and classroom teacher develop a system by which prediction techniques such as described in this paper be used effectively as an aid to forecasting academic success in high school. This system enables counsellors to employ existing data available in the student records to predict future achievement. As noted in the conclusions a single mark bears only a moderate predictor value but the results achieved over several grades are valuable as predictors of future achievement.

It is also recommended that students of the ninth grade be given more guidance with respect to the nature and demands of high school mathematics. As most of the information required in this respect is related to the subject itself, it is provided by the subject area instructor.

Since it is noted by several authorities that prognostic tests which are developed by classroom teachers are frequently more valid than commercially prepared tests, it is recommended that the mathematics teachers of the Western School Division develop cooperatively a series of prognostic tests. If these instruments were analyzed item by item each semester, in a short period of time a valuable prognostic test would be available.

It is further recommended that teachers utilize this type of research or statistical analysis in an effort to motivate their students. If students are permitted to calculate their predicted scores, based on past performance, from the regression equations or the graphs they will attempt to better their predicted mark due to the "Hawthorn" effect, a consequence of any group being considered for an experiment. Also the teacher maintains a keener interest in his methodology and the student. Every year the sample size would be increased and consequently the predictions would become more valid.

It is also recommended that another study might be undertaken utilizing the grade-point average in conjunction with the subject as a multi-predictor for achievement in senior high mathematics.

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BIBLIOGRAPHY

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APPENDIX A

APPENDIX A

TABLE XIII

CORRELATION MATRIX FOR MATHEMATICS 100 (GROUP A)

Grades	9	10
9	1.000006	0.698569
10	0.698569	1.000005

TABLE XIV

CORRELATION MATRIX FOR MATHEMATICS 101 (GROUP B)

Grades	9	10
9	1.000004	0.531724
10	0.531724	1.000000

TABLE XV

CORRELATION MATRIX FOR MATHEMATICS 102 (GROUP C)

Grades	9	10
9	1.000000	0.771440
10	0.771440	1.000000

TABLE XVI

CORRELATION MATRIX FOR MATHEMATICS 200 (GROUP D)

Grade	9	10	11
9	1.000006	0.698569	0.669175
10	0.698569	1.000005	0.817409
11	0.669175	0.817409	1.000004

TABLE XVII

CORRELATION MATRIX FOR MATHEMATICS 201 (GROUP E)

Grade	9	10	11
9	1.000004	0.531724	0.573096
10	0.531624	1.000000	0.729970
11	0.573096	0.739970	1.000000

TABLE XVIII

CORRELATION MATRIX FOR MATHEMATICS 202 (GROUP F)

Grade	9	10	11
9	1.000000	0.661440	0.588889
10	0.771440	1.000000	0.806302
11	0.588889	0.816302	1.000000

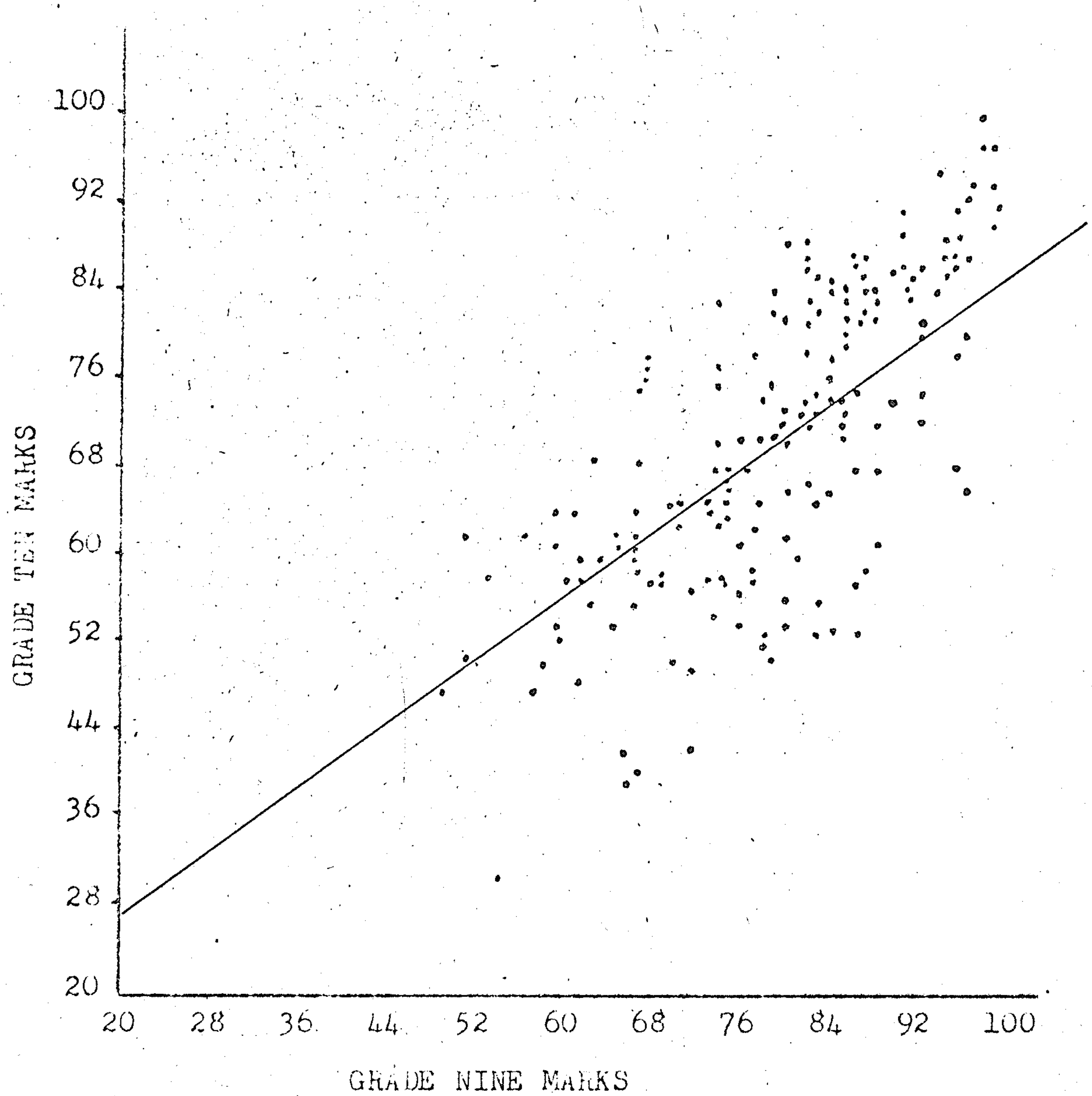
TABLE XIX

CORRELATION MATRIX FOR MATHEMATICS 300 (GROUP G)

Grade	9	10	11	12
9	1.000000	0.643896	0.628735	0.645837
10	0.643896	1.000020	0.801155	0.789990
11	0.628735	0.801155	1.000008	0.806422
12	0.645837	0.789990	0.806422	1.000000

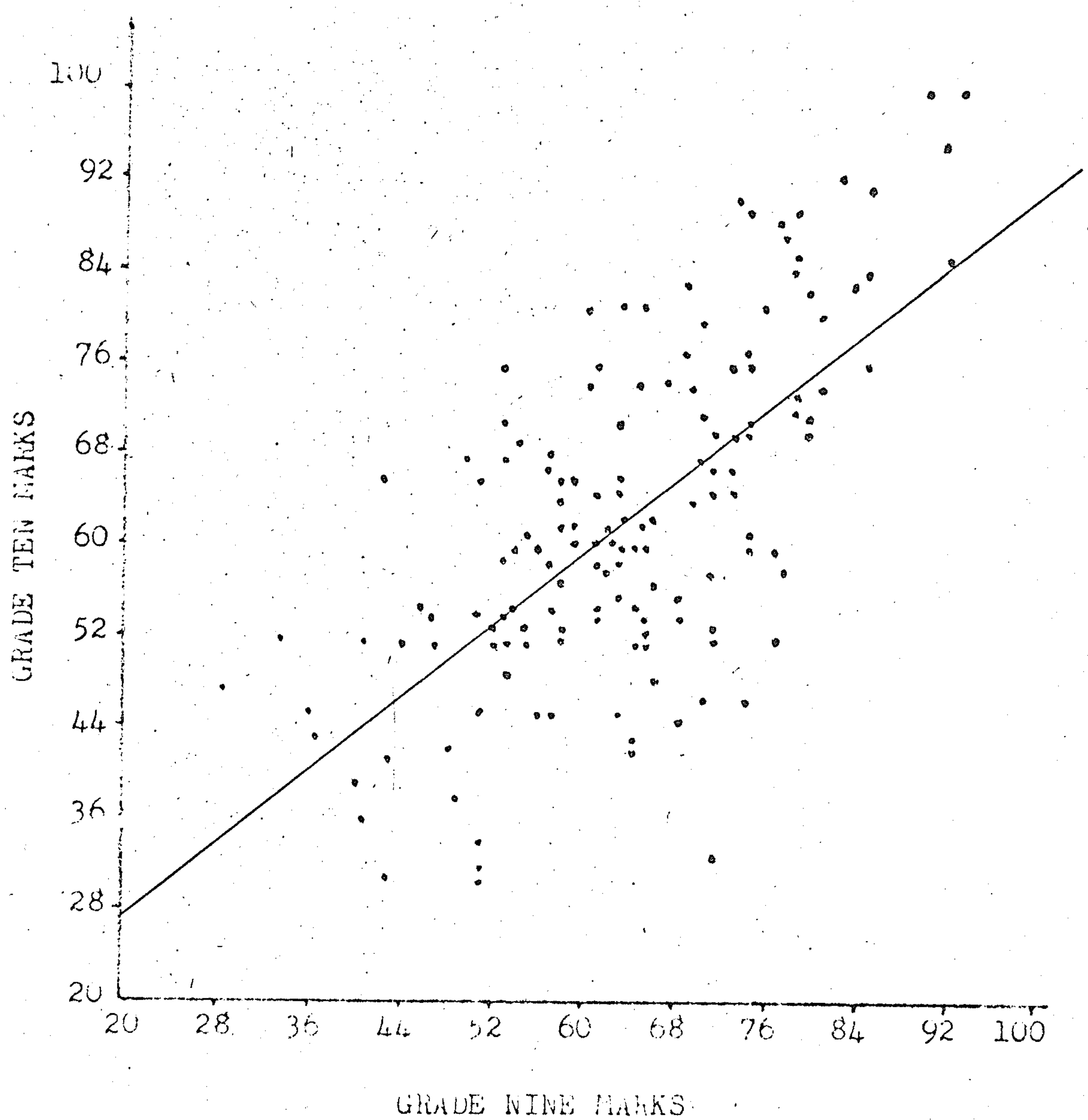
SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRAPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP A)



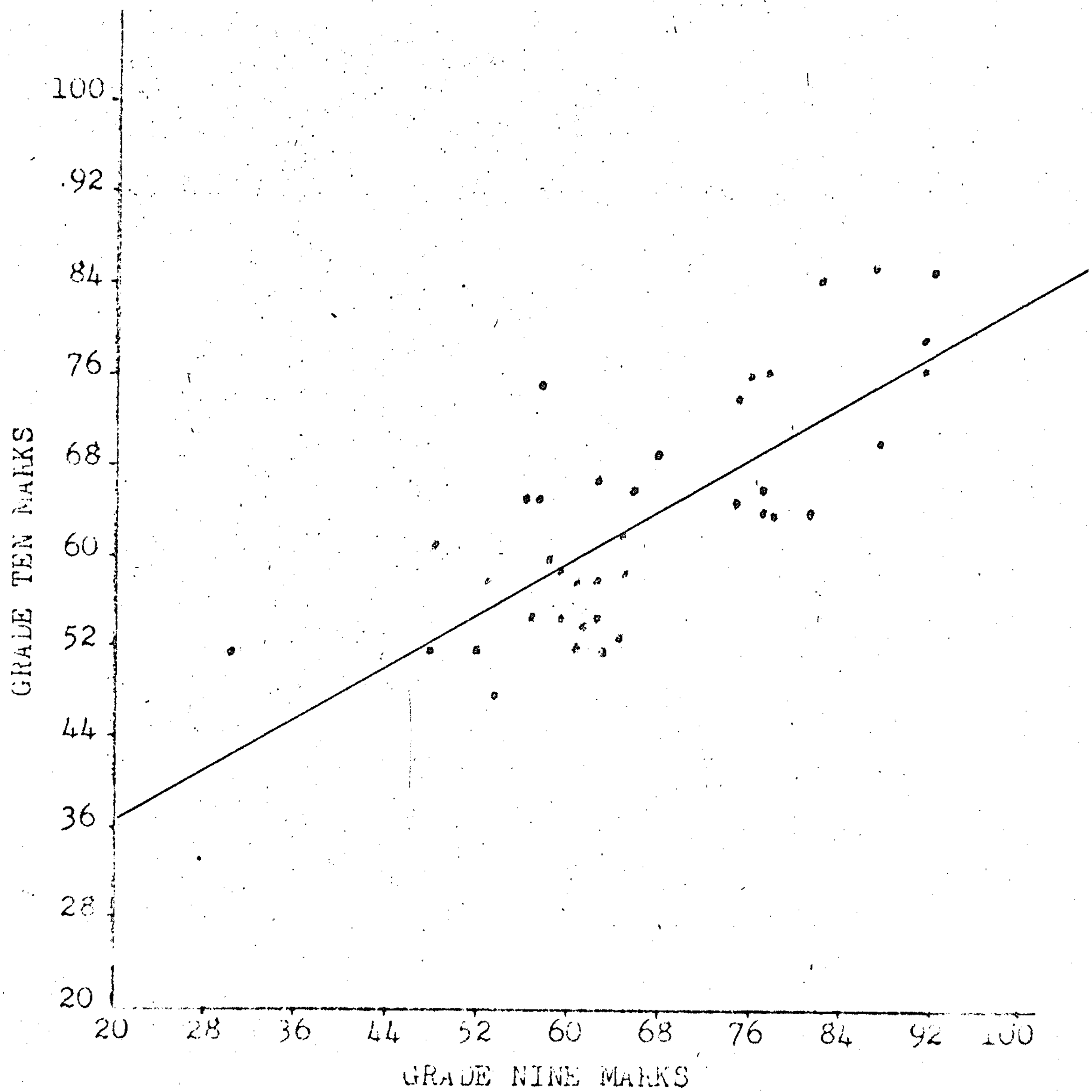
SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRAPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP B)



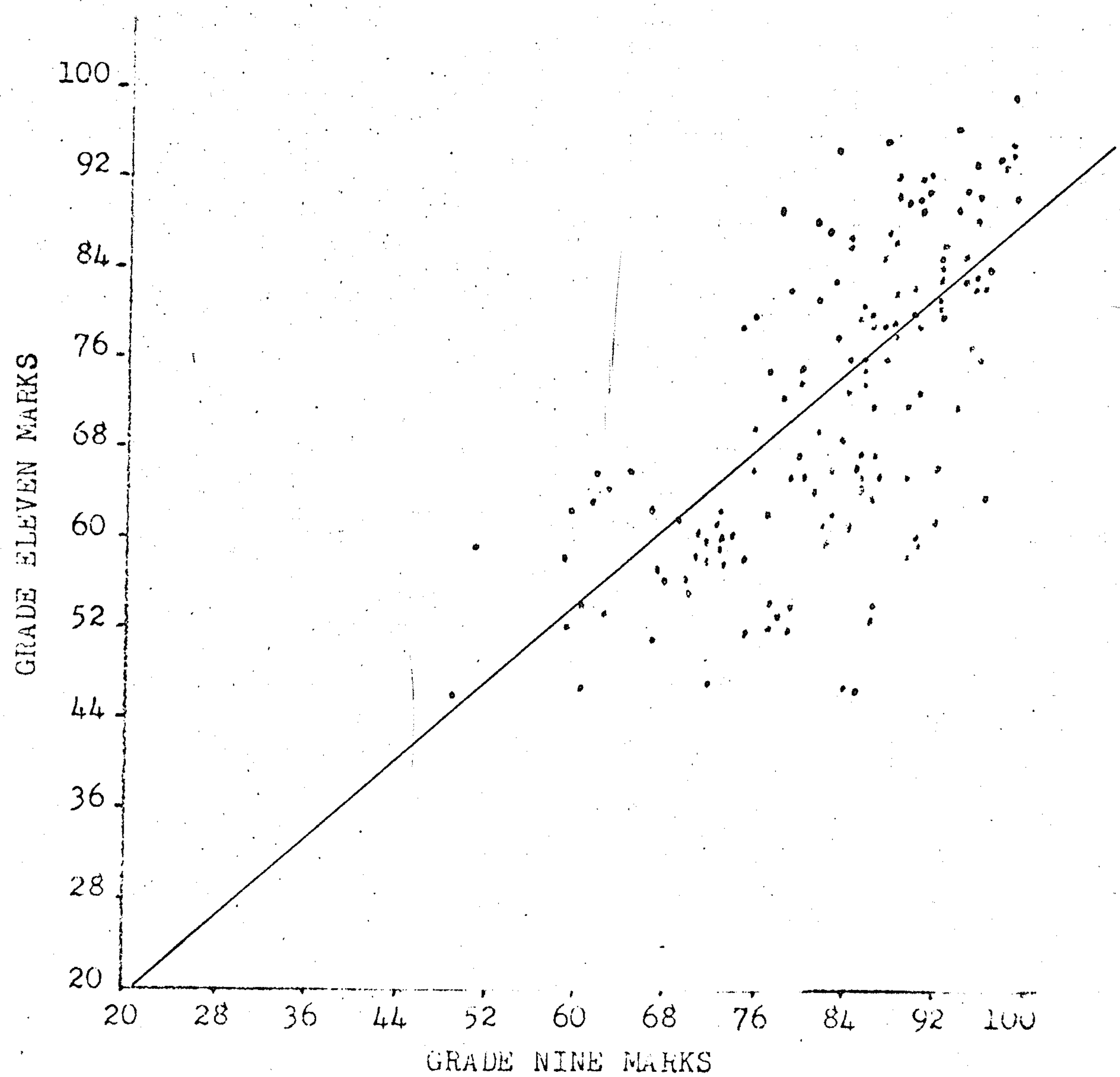
SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRAPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP C)



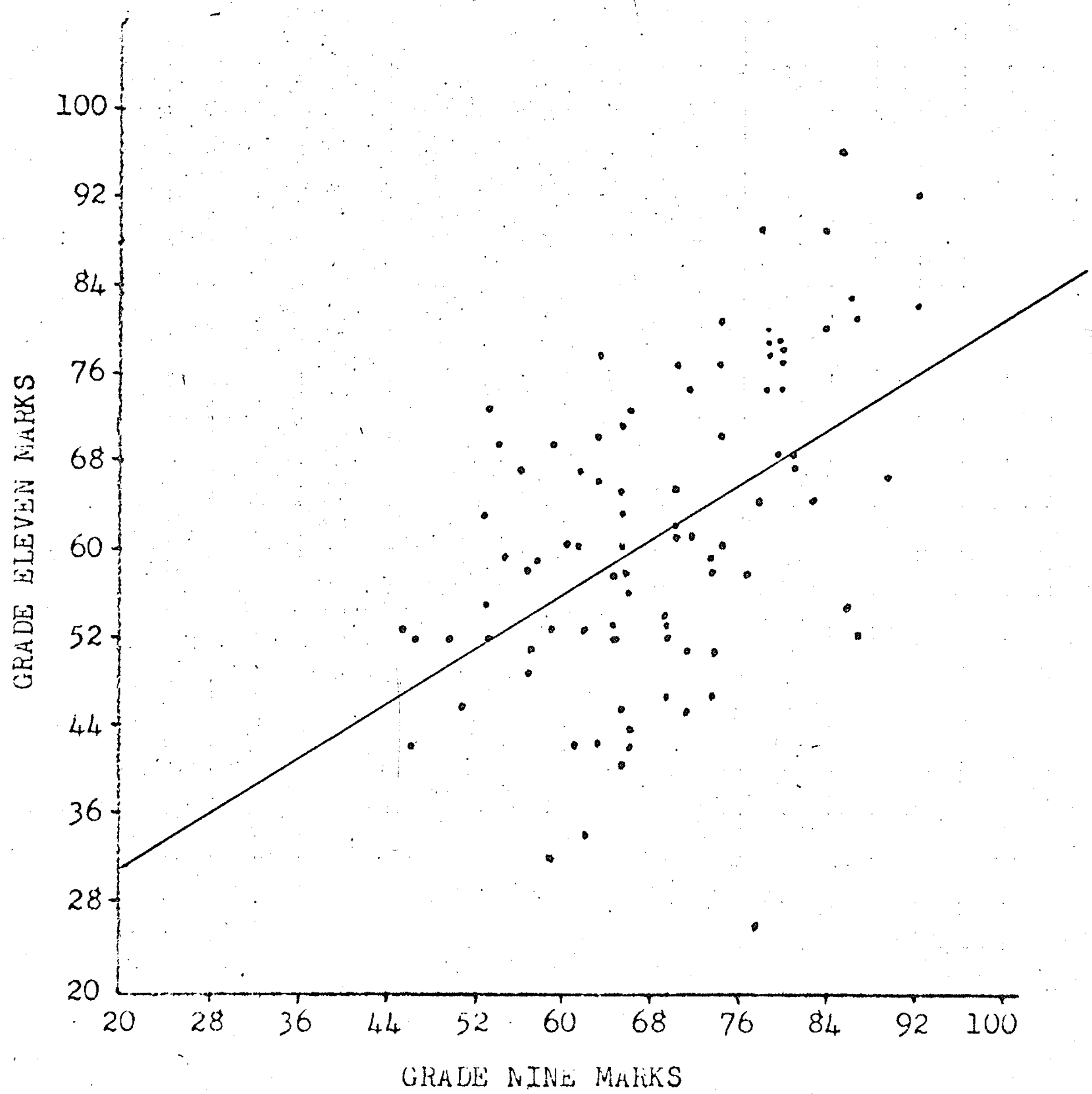
SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRAPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP D)



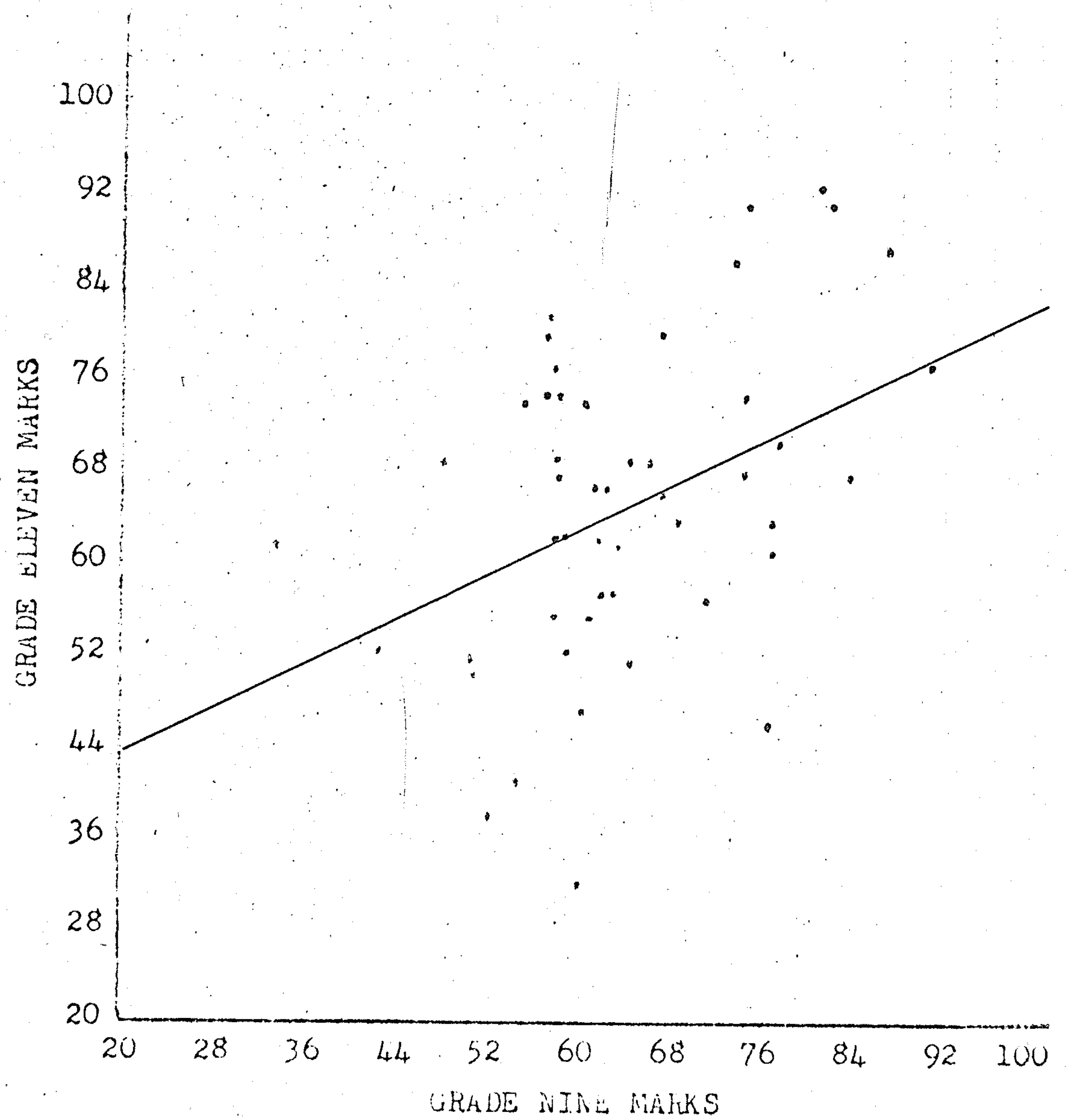
SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRAPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP E)



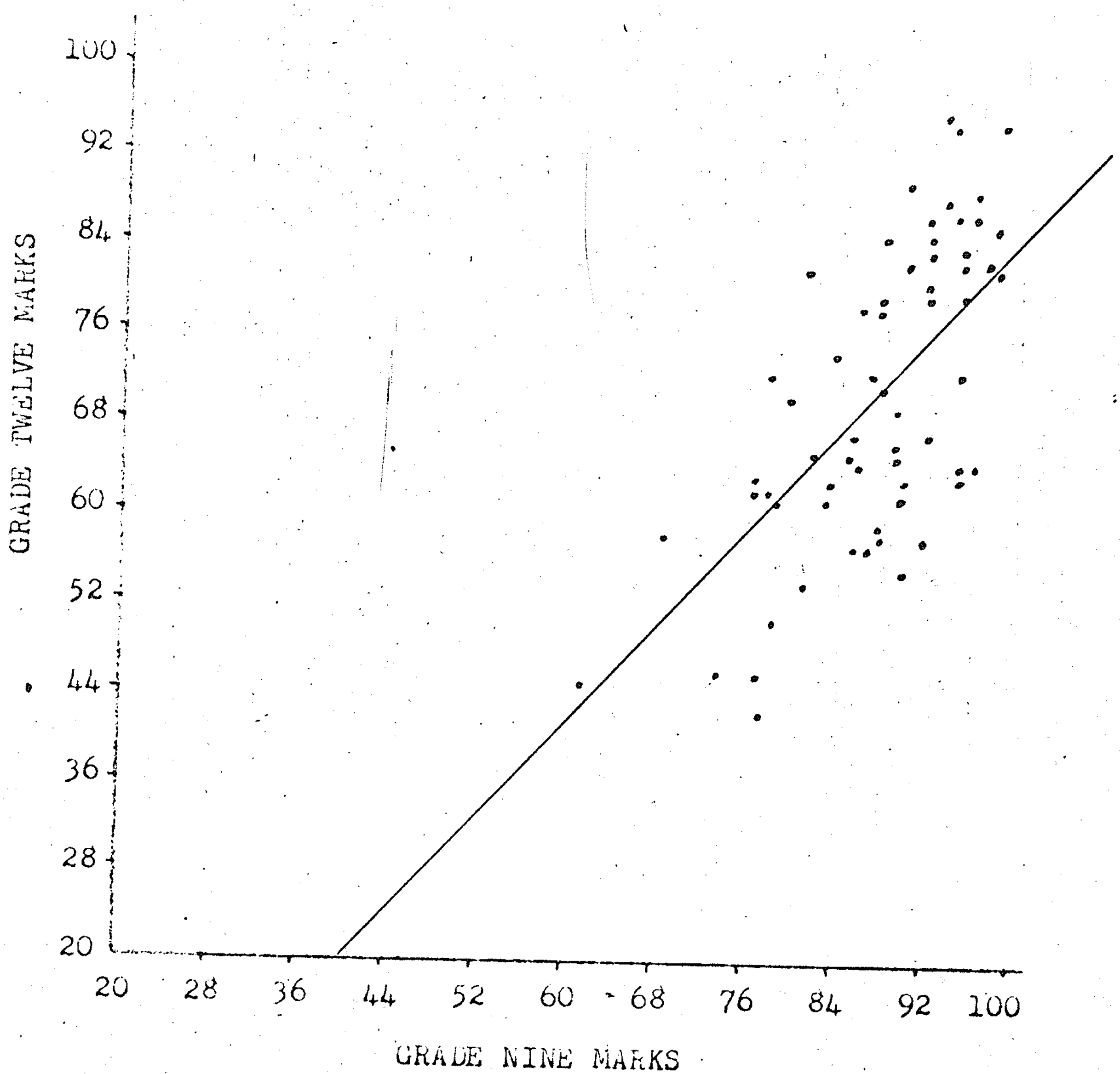
SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRAPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP F)



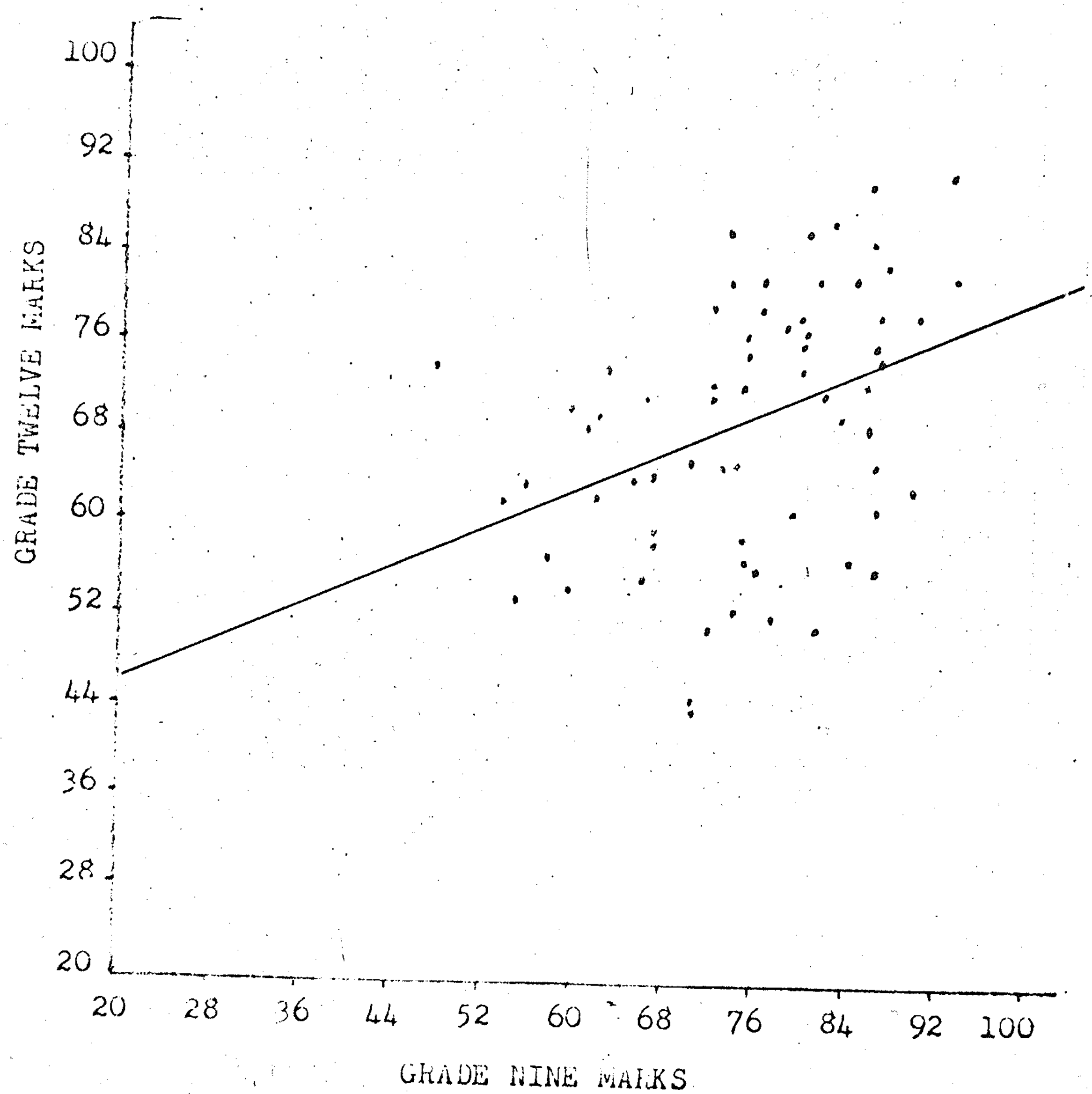
SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRAPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP G)



SCATTER DIAGRAM OF THE CORRELATION OF SCORES SUPERIMPOSED ON THE GRPHICAL ILLUSTRATION OF THE CORRESPONDING PREDICTION LINE REPRESENTING THE LINEAR REGRESSION EQUATION

(GROUP H)



APPENDIX B

TT REGRESSION ANALYSIS PREDICTOR GR IX MATH   CRITERION 100 MATH

NO. OF FORMAT CARDS =   1
NO. OF OBSERVATIONS =   137
NO. OF VARIABLES =   2
P LEVEL TO ADD VARIABLE =   1.000
P LEVEL TO DELETE VARIABLE =   1.000
FORMAT(4X,F3.0,1X,F3.0)

MEANS

	1	2
1	80.854004	74.934296

STANDARD DEVIATIONS-UNBIASED

	1	2
1	10.437893	11.881906

CORRELATIONS

	1	2
1	1.000006	0.698569
2	0.698569	1.000005

GR 9 MATH + MATH 100

NO. OF PREDICTORS = 1
CRITERION = VARIABLE 2
NEW DESIGNATION OF CRITERION VARIABLE = 2
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1
NEW DESIGNATION OF PREDICTOR VARIABLES = 1

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2
1	1.000006	0.698569
2	0.698569	1.000005

****STEP NO. 1

VARIABLE ENTERING	1
F VALUE FOR VARIABLE ENTERING	128.668549
PROBABILITY LEVEL	0.000000
PERCENT VARIANCE ACCOUNTED FOR	48.799118
STANDARD ERROR OF PREDICTED Y	8.533515

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	1.	9369.680	9369.680	128.667	0.000000
RESID	135.	9830.820	72.821		
TOTAL	136.	19200.500			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
1	0.698565	0.795207	0.070104

CONSTANT = 10.639

****NO FURTHER STEPS REQUIRED
END OF THIS JOB

TT REGRESSION ANALYSIS PREDICTOR GR IX MATH   CRITERION 101 MATH

NO. OF FORMAT CARDS = 1
NO. OF OBSERVATIONS = 62
NO. OF VARIABLES = 2
P LEVEL TO ADD VARIABLE = 1.000
P LEVEL TO DELETE VARIABLE = 1.000
FORMAT(4X,F3.0,5X,F3.0)

MEANS

	1	2
1	66.306442	59.935471

STANDARD DEVIATIONS-UNBIASED

	1	2
1	11.130517	15.864044

CORRELATIONS

	1	2
1	1.000004	0.531724
2	0.531724	1.000000

GR 9 MATH + MATH 101

NO. OF PREDICTORS = 1
CRITERION = VARIABLE 2
NEW DESIGNATION OF CRITERION VARIABLE = 2
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1
NEW DESIGNATION OF PREDICTOR VARIABLES = 1

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2
1	1.000004	0.531724
2	0.531724	1.000000

```
****STEP NO.  1
    VARIABLE ENTERING                                    1
    F VALUE FOR VARIABLE ENTERING                23.650375
    PROBABILITY LEVEL                             0.000009
    PERCENT VARIANCE ACCOUNTED FOR               28.272888
    STANDARD ERROR OF PREDICTED Y                13.547056

                ANALYSIS OF VARIANCE TABLE
   SOURCE   DF          SS              MS              F              P
   REGRES    1.      4340.387        4340.387        23.650        0.000009
   RESID    60.     11011.363         183.523
   TOTAL    61.     15351.750

 REGRESSION WEIGHTS

   VARIABLES      STANDARD WEIGHT      WEIGHT      STANDARD ERROR
       1             0.531722        0.757847        0.155834

              CONSTANT =       -9.685


****NO FURTHER STEPS REQUIRED
    END OF THIS JOB
```

TT REGRESSION ANALYSIS PREDICTOR GR IX MATH CRITERION 102 MATH

NO. OF FORMAT CARDS = 1
NO. OF OBSERVATIONS = 23
NO. OF VARIABLES = 2
P LEVEL TO ADD VARIABLE = 1.000
P LEVEL TO DELETE VARIABLE = 1.000
FORMAT(4X,F3.0,9X,F3.0)

MEANS

	1	2
1	64.086945	62.217377

STANDARD DEVIATIONS-UNBIASED

	1	2
1	11.797199	10.870964

CORRELATIONS

	1	2
1	1.000000	0.771440
2	0.771440	1.000000

GR 9 MATH + MATH 102

NO. OF PREDICTORS = 1
CRITERION = VARIABLE 2
NEW DESIGNATION OF CRITERION VARIABLE = 2
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1
NEW DESIGNATION OF PREDICTOR VARIABLES = 1

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2
1	1.000000	0.771440
2	0.771440	1.000000

JULY

****STEP NO. 1

VARIABLE ENTERING	1
F VALUE FOR VARIABLE ENTERING	30.867111
PROBABILITY LEVEL	0.000016
PERCENT VARIANCE ACCOUNTED FOR	59.511917
STANDARD ERROR OF PREDICTED Y	7.080032

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	1.	1547.273	1547.273	30.867	0.000016
RESID	21.	1052.664	50.127		
TOTAL	22.	2599.938			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
1	0.771440	0.710869	0.127950

CONSTANT = 16.660

****NO FURTHER STEPS REQUIRED
END OF THIS JOB

TT REGRESSION ANALYSIS PRED MATH 9 100 CRITERION MATH 200

NO. OF FORMAT CARDS = 1
NO. OF OBSERVATIONS = 137
NO. OF VARIABLES = 3
P LEVEL TO ADD VARIABLE = 1.000
P LEVEL TO DELETE VARIABLE = 1.000
FORMAT(4X,F3.0,1X,F3.0,9X,F3.0)

MEANS

	1	2	3
1	80.854004	74.934296	70.846710

STANDARD DEVIATIONS-UNBIASED

	1	2	3
1	10.437893	11.881906	13.294683

CORRELATIONS

	1	2	3
1	1.000006	0.698569	0.669175
2	0.698569	1.000005	0.817409
3	0.669175	0.817409	1.000004

PRED 9 100 CRIT 200

NO. OF PREDICTORS = 2
CRITERION = VARIABLE 3
NEW DESIGNATION OF CRITERION VARIABLE = 3
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1 2
NEW DESIGNATION OF PREDICTOR VARIABLES = 1 2

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2	3
1	1.000006	0.698569	0.669175
2	0.698569	1.000005	0.817409
3	0.669175	0.817409	1.000004

TOLU

****STEP NO. 1

VARIABLE ENTERING	2
F VALUE FOR VARIABLE ENTERING	271.812256
PROBABILITY LEVEL	0.0
PERCENT VARIANCE ACCOUNTED FOR	66.815048
STANDARD ERROR OF PREDICTED Y	7.686897

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	1.	16060.879	16060.879	271.811	0.0
RESID	135.	7976.934	59.088		
TOTAL	136.	24037.813			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
2	0.817405	0.914594	0.055475

CONSTANT = 2.312

```
****STEP NO.  2
    VARIABLE ENTERING                                   1
    F VALUE FOR VARIABLE ENTERING                       8.055966
    PROBABILITY LEVEL                                   0.005241
    PERCENT VARIANCE ACCOUNTED FOR                     68.696945
    STANDARD ERROR OF PREDICTED Y                       7.493562
```

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	2.	16513.246	8256.621	147.037	0.0
RESID	134.	7524.566	56.153		
TOTAL	136.	24037.813			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
1	0.191717	0.244188	0.086033
2	0.683478	0.764743	0.075578

CONSTANT = -6.202

****NO FURTHER STEPS REQUIRED
END OF THIS JOB

1000

TT REGRESSION ANALYSIS PRED MATH 9 101 CRITERION MATH 201

NO. OF FORMAT CARDS = 1
NO. OF OBSERVATIONS = 62
NO. OF VARIABLES = 3
P LEVEL TO ADD VARIABLE = 1.000
P LEVEL TO DELETE VARIABLE = 1.000
FORMAT(4X,F3.0,5X,F3.0,9X,F3.0)

MEANS

	1	2	3
1	66.306442	59.935471	60.935471

STANDARD DEVIATIONS-UNBIASED

	1	2	3
1	11.130517	15.864044	13.821901

CORRELATIONS

	1	2	3
1	1.000004	0.531724	0.573096
2	0.531724	1.000000	0.729970
3	0.573096	0.729970	1.000000

PRED 9 101 CRIT 201

NO. OF PREDICTORS = 2
CRITERION = VARIABLE 3
NEW DESIGNATION OF CRITERION VARIABLE = 3
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1 2
NEW DESIGNATION OF PREDICTOR VARIABLES = 1 2

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2	3
1	1.000004	0.531724	0.573096
2	0.531724	1.000000	0.729970
3	0.573096	0.729970	1.000000

1005

****STEP NO. 1

VARIABLE ENTERING	2
F VALUE FOR VARIABLE ENTERING	68.439911
PROBABILITY LEVEL	0.000000
PERCENT VARIANCE ACCOUNTED FOR	53.285553
STANDARD ERROR OF PREDICTED Y	9.525387

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	1.	6209.770	6209.770	68.440	0.000000
RESID	60.	5443.980	90.733		
TOTAL	61.	11653.750			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
2	0.729970	0.636002	0.076878

CONSTANT = - 22.816

****STEP NO. 2

VARIABLE ENTERING	1
F VALUE FOR VARIABLE ENTERING	6.708223
PROBABILITY LEVEL	0.012070
PERCENT VARIANCE ACCOUNTED FOR	58.054688
STANDARD ERROR OF PREDICTED Y	9.102243

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	2.	6765.551	3382.775	40.830	0.000000
RESID	59.	4888.199	82.851		
TOTAL	61.	11653.750			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
1	0.257856	0.320205	0.123630
2	0.592861	0.516544	0.086742

CONSTANT = 8.745

****NO FURTHER STEPS REQUIRED
END OF THIS JOB

TT REGRESSION ANALYSIS PRED MATH 9 102 CRITERION MATH 202

NO. OF FORMAT CARDS = 1
NO. OF OBSERVATIONS = 23
NO. OF VARIABLES = 3
P LEVEL TO ADD VARIABLE = 1.000
P LEVEL TO DELETE VARIABLE = 1.000
FORMAT(4X,F3.0,9X,F3.0,9X,F3.0)

MEANS

	1	2	3
1	64.086945	62.217377	62.130432

STANDARD DEVIATIONS-UNBIASED

	1	2	3
1	11.797199	10.870964	14.366325

CORRELATIONS

	1	2	3
1	1.000000	0.771440	0.588889
2	0.771440	1.000000	0.806302
3	0.588889	0.806302	1.000000

PRED 9 102 CRIT 202

NO. OF PREDICTORS = 2
CRITERION = VARIABLE 3
NEW DESIGNATION OF CRITERION VARIABLE = 3
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1 2
NEW DESIGNATION OF PREDICTOR VARIABLES = 1 2

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2	3
1	1.000000	0.771440	0.588889
2	0.771440	1.000000	0.806302
3	0.588889	0.806302	1.000000

****STEP NO. 1

VARIABLE ENTERING	2
F VALUE FOR VARIABLE ENTERING	39.021194
PROBABILITY LEVEL	0.000003
PERCENT VARIANCE ACCOUNTED FOR	65.012360
STANDARD ERROR OF PREDICTED Y	8.697720

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	1.	2951.968	2951.968	39.021	0.000003
RESID	21.	1588.657	75.650		
TOTAL	22.	4540.625			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
2	0.806302	1.065551	0.170578

CONSTANT = -4.165

```
****STEP NO.  2
    VARIABLE ENTERING                              1
    F VALUE FOR VARIABLE ENTERING                  0.156122
    PROBABILITY LEVEL                              0.696883
    PERCENT VARIANCE ACCOUNTED FOR                65.283356
    STANDARD ERROR OF PREDICTED Y                  8.877927

              ANALYSIS OF VARIANCE TABLE
   SOURCE  DF          SS             MS               F            P
   REGRES    2.     2964.273      1482.136          18.805       0.000025
   RESID    20.     1576.352        78.818
   TOTAL    22.     4540.625

 REGRESSION WEIGHTS

   VARIABLES      STANDARD WEIGHT      WEIGHT      STANDARD ERROR
       1            -0.081813        -0.099629        0.252147
       2             0.869416         1.148957        0.273632

              CONSTANT =      -2.970


****NO FURTHER STEPS REQUIRED
   END OF THIS JOB
```

TT REGRESSION ANALYSIS PRED MATH 9 100 200 CRIT 300 MATH

NO. OF FORMAT CARDS = 1
NO. OF OBSERVATIONS = 60
NO. OF VARIABLES = 4
P LEVEL TO ADD VARIABLE = 1.000
P LEVEL TO DELETE VARIABLE = 1.000
FORMAT(4X,F3.0,1X,F3.0,9X,F3.0,9X,F3.0)

MEANS

	1	2	3	4
1	86.033325	78.500000	74.549988	69.899994

STANDARD DEVIATIONS-UNBIASED

	1	2	3	4
1	7.323054	10.741230	12.266025	12.766127

CORRELATIONS

	1	2	3	4
1	1.000000	0.643896	0.628735	0.645837
2	0.643896	1.000020	0.801155	0.789990
3	0.628735	0.801155	1.000008	0.806422
4	0.645837	0.789990	0.806422	1.000000

PRE 9 100 200 C 300

NO. OF PREDICTORS = 3
CRITERION = VARIABLE 4
NEW DESIGNATION OF CRITERION VARIABLE = 4
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1 2 3
NEW DESIGNATION OF PREDICTOR VARIABLES = 1 2 3

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2	3	4
1	1.000000	0.643896	0.628735	0.645837
2	0.643896	1.000020	0.801155	0.789990
3	0.628735	0.801155	1.000008	0.806422
4	0.645837	0.789990	0.806422	1.000000

****STEP NO. 1

VARIABLE ENTERING	3
F VALUE FOR VARIABLE ENTERING	107.861847
PROBABILITY LEVEL	0.000000
PERCENT VARIANCE ACCOUNTED FOR	65.031128
STANDARD ERROR OF PREDICTED Y	7.613996

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	1.	6253.066	6253.066	107.862	0.000000
RESID	58.	3362.430	57.973		
TOTAL	59.	9615.500			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
3	0.806416	0.839296	0.030813

CONSTANT = - 7.331

****STEP NO. 2

VARIABLE ENTERING	2
F VALUE FOR VARIABLE ENTERING	11.295044
PROBABILITY LEVEL	0.001392
PERCENT VARIANCE ACCOUNTED FOR	70.814499
STANDARD ERROR OF PREDICTED Y	7.016685

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	2.	6809.168	3404.584	69.151	0.000000
RESID	57.	2806.331	49.234		
TOTAL	59.	9615.500			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
2	0.401830	0.477581	0.142103
3	0.484490	0.504243	0.124439

CONSTANT = -5.181

****STEP NO. 3

VARIABLE ENTERING	1
F VALUE FOR VARIABLE ENTERING	2.479457
PROBABILITY LEVEL	0.120974
PERCENT VARIANCE ACCOUNTED FOR	72.051926
STANDARD ERROR OF PREDICTED Y	6.927361

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	3.	6928.152	2309.384	48.124	0.000000
RESID	56.	2687.346	47.988		
TOTAL	59.	9615.500			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
1	0.150019	0.261526	0.166087
2	0.343115	0.407796	0.147127
3	0.437208	0.455034	0.126768

CONSTANT = -18.535

****NO FURTHER STEPS REQUIRED
END OF THIS JOB

TT REGRESSION ANALYSIS PRED MATH 9 10[illegible] 201 CRIT 301 MATH

NO. OF FORMAT CARDS = 1
NO. OF OBSERVATIONS = 25
NO. OF VARIABLES = 4
P LEVEL TO ADD VARIABLE = 1.000
P LEVEL TO DELETE VARIABLE = 1.000
FORMAT(4X,F3.0,5X,F3.0,9X,F3.0,9X,F3.0)

MEANS

	1	2	3	4
1	68.839996	60.599991	64.159988	65.519989

STANDARD DEVIATIONS-UNBIASED

	1	2	3	4
1	11.190323	14.866069	11.998889	12.066206

CORRELATIONS

	1	2	3	4
1	1.000000	0.610988	0.634796	0.487590
2	0.610988	1.000000	0.690395	0.521760
3	0.634796	0.690395	1.000000	0.539009
4	0.487590	0.521760	0.539009	1.000000

PRE 9 10[illegible] 201 C 301

NO. OF PREDICTORS = 3
CRITERION = VARIABLE 4
NEW DESIGNATION OF CRITERION VARIABLE = 4
ORIGINAL DESIGNATION OF PREDICTOR VARIABLES = 1 2 3
NEW DESIGNATION OF PREDICTOR VARIABLES = 1 2 3

CORRELATION MATRIX FOR THIS REGRESSION PROBLEM

	1	2	3	4
1	1.000000	0.610988	0.634796	0.487590
2	0.610988	1.000000	0.690395	0.521760
3	0.634796	0.690395	1.000000	0.539009
4	0.487590	0.521760	0.539009	1.000000

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****STEP NO. 1

VARIABLE ENTERING	3
F VALUE FOR VARIABLE ENTERING	9.418612
PROBABILITY LEVEL	0.005432
PERCENT VARIANCE ACCOUNTED FOR	29.053101
STANDARD ERROR OF PREDICTED Y	10.381969

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	1.	1015.188	1015.188	9.419	0.005432
RESID	23.	2479.062	107.785		
TOTAL	24.	3494.250			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
3	0.539009	0.542033	0.176617

CONSTANT = 30.743

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****STEP NO. 2

VARIABLE ENTERING	2
F VALUE FOR VARIABLE ENTERING	1.411702
PROBABILITY LEVEL	0.247420
PERCENT VARIANCE ACCOUNTED FOR	33.331131
STANDARD ERROR OF PREDICTED Y	10.290279

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	2.	1164.673	582.337	5.499	0.011565
RESID	22.	2329.577	105.890		
TOTAL	24.	3494.250			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
2	0.285906	0.232059	0.195311
3	0.341621	0.343538	0.241981

CONSTANT = 29.416

****STEP NO. 3

VARIABLE ENTERING	1
F VALUE FOR VARIABLE ENTERING	0.552204
PROBABILITY LEVEL	0.465636
PERCENT VARIANCE ACCOUNTED FOR	35.C39307
STANDARD ERROR OF PREDICTED Y	10.396631

ANALYSIS OF VARIANCE TABLE

SOURCE	DF	SS	MS	F	P
REGRES	3.	1224.361	408.120	3.776	0.025991
RESID	21.	2269.889	108.090		
TOTAL	24.	3494.250			

REGRESSION WEIGHTS

VARIABLES	STANDARD WEIGHT	WEIGHT	STANDARD ERROR
1	0.177852	0.191772	0.258069
2	0.227208	0.184416	0.207484
3	0.269247	0.270757	0.263370

CONSTANT = 23.771

****NO FURTHER STEPS REQUIRED
END OF THIS JOB