

Numerical Modeling in Geotechnical Engineering with Applications in Cold Regions

By

Hongwei Liu

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Abstract

The objective of this thesis is to apply thermal analysis, coupled thermo-hydraulic analysis, and fully coupled thermo-hydro-mechanical analysis developed in this research to different engineering designs in cold regions. The applications vary from *Geothermal Snow-Melting System Design* to *Structural and Thermal Integrity of Buried Infrastructure* in cold regions. A feasibility study of geothermal energy pile based snow melting system was performed for six major cities in Canada. The coefficient of performance (COP) of heat pump is derived and the number of geothermal piles are determined for each city based on its specific local geological condition and heating demand.

Also, the structural and thermal integrity of buried infrastructure were studied by performing an optimum design of rigid plastic foam insulations to protect buried utilities against frost damage and reduce the excavation cost. Furthermore, the effect of frost heave on the structural integrity of pavement structure and culverts are assessed.

Résumé

Cette thèse a pour but d'appliquer les modèles de transfert de chaleur, de transfert couplé de masse et de chaleur, ainsi qu'un modèle thermo-hydro-mécanique complètement couplé, développé dans cette recherche, à différentes conceptions d'ingénierie dans les régions froides. Les applications abordées dans ce travail varient de la *Conception Géothermique du Système de Dénéigement* à *L'Intégrité Structurelle et Thermique des Infrastructures Enterrées* dans les régions froides.

Une étude de faisabilité du système de déneigement basé sur des pieux géothermiques a été réalisée pour six grandes villes du Canada. Le coefficient de performance (COP) de la pompe à chaleur en est dérivé et le nombre des pieux géothermiques est déterminé pour chaque ville en fonction de ses conditions géologiques locales et de la demande de chauffage. De plus, une étude de faisabilité économique a été réalisée pour comparer les coûts entre le système de déneigement des tabliers de pont utilisant des pieux géothermique et un système de chauffage à base d'électricité.

Ainsi, l'intégrité structurelle et thermique des infrastructures enterrées a été étudiée en réalisant une conception optimale des isolants rigides pour protéger les services publics enterrés contre les dommages causés par le gel et réduire les coûts d'excavation. De plus, l'effet du soulèvement dû au gel sur l'intégrité structurelle de la structure de la chaussée et des ponceaux est évalué.

Synopsis

Nowadays, numerical modeling provides engineers powerful tools to predict and assess the performance of infrastructure under different conditions. Geotechnical engineering design and practice in cold regions are challenging due to complex physical processes associated with the freeze/thaw cycles in soils and extreme cold weather conditions applied at the ground surface. To improve the accuracy of numerical tools in predicting the geotechnical infrastructure's response in cold regions, such physical processes must be appropriately addressed in numerical models.

This thesis deals with thermal modeling, coupled thermo-hydraulic modeling, and coupled thermo-hydro-mechanical modeling in freezing soils. Since the system of governing equations may be highly non-linear, the numerical linearization technique was addressed. Then, the implemented models were applied to three sets of problems encountered in cold regions.

First, the feasibility of innovative snow melting system of bridge decks and pavement slabs using geothermal energy piles was studied in six major cities (Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg) in Canada. Snow melting system using geothermal energy piles is a promising clean technology to overcome the problems of traditional chemical-based snow melting methods. It was concluded that the proposed system is efficient and cost effective. However, the extent of efficiency and saving varies with implementation areas.

Second, the effectiveness of various insulation systems for the buried utilities frost protection was investigated by considering two different conductive heat transfer models,

with and without the effect of phase change, as well as a heat and mass transfer model for freezing soils. It was concluded that the latent heat released during the phase change can significantly affect the heat transfer process and neglecting the effects of phase change will eventually lead to an erroneous design of the insulation. By comparing numerical results with field measurement data, it was found that the mass transfer has an insignificant effect on the temperature and unfrozen water content distribution in frost-susceptible foundation soils. A parametric study was carried out to assess the effects of different factors such as backfill materials, insulation geometry and thickness, burial depth of pipe on the thermal performance of insulation. Furthermore, the Nelder-Mead algorithm was implemented to determine the optimum insulation design so that the overall cost is minimized while the pipe is protected from freezing.

Finally, the distribution of temperature and pore-water pressure during the freezing process as well as the deformation in the soil due to frost heave were studied in two double-barrel culvert sites in Manitoba, Canada. Also, the performance of insulation was studied to mitigate the effect of frost heave and differential deformation at the road surface. It was concluded that for double-barrel culverts with a deeper burial depth, using an insulation at the top of the frost-susceptible subgrade below the culverts and at the sides of the culvert backfill structure can effectively decrease the frost heave and the corresponding differential displacement at the pavement surface. However, for double-barrel culverts with a shallower burial depth, the same insulating method must be combined with an adequate overburden pressure to minimize the overall frost heave and differential displacement at the pavement surface. This can be done by increasing the thickness of the granular base or replacing frost-susceptible subgrade by non-frost-susceptible backfill. Furthermore, the applicability of the insulating methods depends mainly on the importance of the road, the level of the driving comfort, and how much differential deformation at the pavement surface is tolerated.

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Introduction

” *The journey with a 1000 miles begins with one step.*

— Confucius

In Northern countries, cold climates may lead to the freezing of soil pore-water near the ground surface. Freezing process under the temperature gradient may generate moisture migration in the soil due to the development of cryogenic suction and change thermal, hydraulic, and physical properties of soils. These processes, with or without frost heaving, have important consequences in engineering design in both seasonal frost and permafrost areas.

Nowadays, numerical modeling is an integral part of any engineering design and rehabilitation program. This can provide civil engineers and designers powerful tools to design new infrastructure and/or assess the performance of natural and built environment under different conditions. Complex physical processes associated with the freeze/thaw cycles in soils must be appropriately addressed in such models in cold regions engineering.

The objective of this thesis is to apply thermal analysis, coupled thermo-hydraulic analysis, and fully coupled thermo-hydro-mechanical analysis (THM) developed in this research to different engineering designs in cold regions. The applications addressed in this work vary from *Geothermal Snow-Melting System Design* to *Structural and Thermal Integrity of Buried Infrastructure* in cold regions. More details about these sets of applications are presented hereafter.

1.1 Problem Statement

1.1.1 Geothermal Snow-Melting System Design

Snow accumulation on pavement slabs and bridge decks during winter causes inconvenience for drivers and traffic accidents. For example, almost 30% of car accidents in Canada

occur due to snowy or icy roads. A traditional approach for snow melting and de-icing of bridge decks is to use salt and other kinds of chemical materials, which can lower the freezing point of water to prevent the formation of ice. However, this method can induce some problems. Salt will not only be ineffective for snow melting or de-icing if temperature falls below -3.9°C , but also penetrate down to the slab and accelerate the corrosion of concrete and steel used in a bridge deck [BE10].

The heat energy stored in the ground (geothermal energy) can be extracted and used for the snow melting and de-icing of bridge decks. Geothermal energy, as a heat source for hydronic system, coupled with a heat pump is very appealing for the snow melting and de-icing application, because it is economic and environmentally friendly. Traditionally, the ground source heat pump system requires a certain number of heat exchanger boreholes, which are independent from the foundation structure. The pile foundations designed to support the loads of bridge structures can also be used as ground heat exchangers. Therefore, these bi-functional foundations called geothermal energy piles, have less initial installation costs.

The appropriate design of such system needs to consider the local geology, groundwater, and climatic conditions. However, the design of such systems in Canada is not documented in the literature.

Although the Canadian Government invested in clean and renewable energy, the application of geothermal energy is still limited due to the lack of data and appropriate analysis of performance of such technology in Canadian climate. Consequently, the outcomes of this thesis will provide information required by practitioners for design information parameters.

1.1.2 Buried Infrastructure subject to Frost Actions

Buried utilities such as conduits and pipelines are a main part of infrastructure in urban areas. In cold regions, the effect of frost actions on the structural integrity must be appropriately addressed in the design process of such utilities.

Typically, buried pipelines must be located below the frost depth to avoid frost-induced damage due to differential heave or freezing of water inside the pipe. A deeper depth of the frost penetration increases the construction cost of buried utilities dramatically. Despite all the efforts in design and operation to prevent buried pipes from frost damage, pipe failure due to the harsh climate and soil deformations currently costs the City of Winnipeg over \$8 million annually. Most parts of Manitoba, with freezing indices ranging from 1550 to 3800 degree-days and frost depth as deep as 2.5 m, are susceptible to frost actions, surface heave and eventually distress of the structures.

A practical way to control the frost action in a frost susceptible soil is to prevent the penetration of the freezing front through the use of insulation or apply a desired amount of overburden pressure by increasing the backfill thickness or other mitigation actions. The low thermal conductivity of insulating materials controls the heat transfer so that the heat is conserved in the soil during winter and the formation of ice lenses is halted. The current practice to design the thickness and width of insulation materials used to protect buried pipes from freezing is based on recommendations provided by manufacturers. Also, to the best of the author's knowledge, the desired amount of overburden to prevent the formation of ice lenses in frost-susceptible soils depends on many factors such as the type of structure and backfill materials, the presence of insulating materials in the ground, the vicinity of cooling sources and so on. In addition, no recommendation is provided in the construction specifications to address such problems. The outcomes of this thesis can provide practitioners and engineers a better insight into the design of buried infrastructure considering the effects of frost actions.

1.2 Hypothesis and Objectives

The hypothesis and objectives for each application are presented hereafter.

1.2.1 Geothermal Snow-Melting System Design

Hypothesis:

The modeling of the hydronic heating system is performed considering the heat transfer in pipe and concrete materials used for bridge decks. The important assumptions applied in this model are: a) the velocity profile is fully developed with entire pipe section; b) the average bulk mean velocity remains constant along the pipe; c) all velocity components normal to pipe axis is assumed to be 0. The snow melting system is designed to melt the snow and drain the water instantaneously.

Since the underground temperature at a deeper depth remains constant during the year, the induced mass transfer due to the freezing effect is not applicable. The analysis will be based on a transient energy balance at the slab surface by considering inputs such as air temperature, snowfall rate, wind speed, and solar radiation as provided by Environment Canada for typical climate conditions for each city.

Objective of Research:

The objective of this research is to study the feasibility of snow melting system using geothermal energy piles in six major cities (Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg) in Canada. This study will focus on the evaluation of the amount of energy and inlet temperature of the hydronic system required to warm up a hypothetical slab and keep its temperature above 0°C during a typical snowfall. The coefficient of performance (COP) of heat pump and the number of piles will be derived for each city based on its specific local geological condition and heating demand. The heat transfer in geothermal heat exchanger pile and surrounding soil will be analyzed to evaluate the amount of energy extracted from the ground. Also, a comparative economic feasibility study will be performed between snow melting system for bridge decks using geothermal energy piles and electricity-based systems.

1.2.2 Buried Infrastructure subject to Frost Actions

This application is targeted by performing two different, but related, studies: I) optimum design of rigid plastic foam insulation for buried utilities in frost susceptible soils (Study 1), and II) Thermo-Hydro-Mechanical modeling of freezing soil to assess the effect of frost heave on the structural integrity of pavement structure and culverts (Study 2).

Hypothesis:

In both studies, the temperature of soil varies with seasonal weather conditions. A transient energy balance at the ground surface is used by considering inputs such as air temperature, snowfall rate, wind speed, and solar radiation as provided by Environment Canada for typical climate conditions of Winnipeg, Manitoba. The coupled transfer of heat and mass is considered. Thermal and hydraulic properties of soils will change by the phase change and will be affected by the heat and mass transfer processes.

In the second study, the THM model takes into considerations the effect of both frost heave and soil consolidation by adapting the effective stress analysis approach for the transient problems. This can be considered as a considerable advantage for modeling frost heave and studying the effect of overburden pressure on frost-susceptible soils. The model is developed in the framework of the poroelasticity theory using a porosity rate function and thermodynamics principles and can be easily implemented to commercially available software.

Objective of Research:

In the Study 1, the performance of the insulating foam with various backfill materials, insulation length, insulation thickness, distance between pipe and insulation as well as pipe burial depth will be studied. The advanced Nelder-Mead algorithm will be implemented to determine the optimum insulation design based on local construction and insulation costs. In the Study 2, the distribution of temperature and pore-water pressure in the ground as well as the amount of frost heave and differential displacement at the pavement surface

will be addressed in two double-barrel culvert sites in Manitoba, Canada. Also, the effect of overburden pressure and thermal variation will be studied. Also, to mitigate the frost heave and differential displacement at the pavement surface, different insulating methods will be studied to evaluate the effect of the position of insulating materials near the culverts on the frost heave at the pavement surface.

The outcomes of this research can be an asset for the assessment of the structural integrity of infrastructure subject to freezing and thawing cycles.

1.3 Thesis Structure

Chapter 2

Chapter 2 aims to study the snow melting system using geothermal energy pile for six major Canadian cities : Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg. A transient snow melting model is used for the calculation of required heat flux to achieve the desired snow free area by considering a typical winter condition during snowfall for each city. Transient conduction and convection mechanisms are considered in the analysis of heat transfer through a hypothetical bridge deck ($4\text{ m} \times 7\text{ m}$) and a geothermal energy pile with a depth of 30 m . The required inlet temperature of geothermal heat exchanger is derived based on the heating load. The coefficient of performance (COP) of heat pump for each city is determined. Then, an economic feasibility study is performed to determine the relative reduction in the electricity consumption and corresponding cost savings between the snow melting system for bridge decks using geothermal energy piles and electricity-based heating system.

Chapter 3

Chapter 3 aims to study the optimum insulation design for buried utilities subject to frost action in cold regions. Various heat transfer models including conductive heat transfer (with and without phase change) and coupled heat and mass transfer are studied. A para-

metric study is carried out to assess the effects of different factors such as backfill materials, insulation geometry and thickness, burial depth of pipe on the thermal performance of insulation. Finally, the Nelder-Mead algorithm is implemented to determine the optimum insulation design so that the overall cost is minimized while the pipe is protected from freezing.

Chapter 4

Chapter 4 presents a fully coupled therm-hydro-mechanical model that can be used to study the distribution of temperature and pore-water pressure during the freezing process as well as the deformation in the soil due to frost heave under various thermal, hydraulic and mechanical boundary conditions. The numerical results are verified by comparing with different laboratory test results found in the literature. Afterwards, the model is applied to study the distribution of temperature and pore-water pressure in the ground as well as the amount of frost heave and differential displacement at the pavement surface in two double-barrel culvert sites in Manitoba, Canada. The culvert sites have been instrumented with thermistors as part of the monitoring program of the ground thermal profile near the culvert openings since 2015. The frost heave phenomena for culvert structures is comprehensively investigated. Also, to mitigate the frost heave and differential displacement at the pavement surface, different insulating methods is considered to evaluate the effect of the position of insulating materials near the culverts on the frost heave at the pavement surface. Two different approaches are presented to minimize frost heave for the culverts with various burial depth.

Chapter 5

Chapter 5 summarizes the overall work and indicates future work.

1.4 Dissemination

The outcomes of this research have been disseminated as follows:

Journal Papers:

- Liu H., Maghoul P., Bahari A., Kavagic M., 2017. Feasibility Study of Snow Melting System for Bridge Decks Using Geothermal Energy Piles Integrated with Heat Pump in Canada, *International Journal of Renewable Energy* (Elsevier), Under Review.
- Liu H., Maghoul P., Shalaby A., 2018. Optimum Insulation Design for Buried Utilities Subject to Frost Action in Cold Regions Using Nelder-Mead Algorithm, *International Journal of Heat and Mass Transfer* (Elsevier), Under Review.
- Liu H., Maghoul P., Shalaby A., Bahari A., 2018. Thermo-Hydro-Mechanical Modeling of Frost Heave Using the Theory of Poroelasticity in Frost-Susceptible Soils in Double-Barrel Culvert Sites, *Transportation Geotechnics* (Elsevier), Under Review.
- Liu H., Maghoul P., Hollaender H., 2018. Optimum Design of a Ground-Source Hydronic Heating System of Bridge Decks in Cold Regions, *Physics and Chemistry of the Earth* (Elsevier), Under Review.

Conference Papers:

- Liu H., Maghoul P., Bahari A., Shahmohammadi S., 2018. Sensitivity Analysis and Optimum Design of a Ground-Source Hydronic Heating System of Bridge Decks during Snowfall, 26th Annual Conference of the Computational Fluid Dynamics Society of Canada, Winnipeg, Canada.
- Liu H., Maghoul P., Bahari A., 2018. Feasibility Study of Snow Melting System using Geothermal Energy Piles in Canadian Prairies, 71th Canadian Geotechnical Conference (GeoEdmonton 2018), Edmonton, Canada.
- Katebi M., Liu H., Maghoul P., Blatz J., 2018. Numerical Analysis of Buried Pipelines subject to Slope Failure and Seasonal Temperature Variation, International Pipeline Conference, Calgary, Canada.

Feasibility Study of Snow Melting System for Bridge Decks Using Geothermal Energy Piles Integrated with Heat Pump in Canada

Abstract

Snow melting system using geothermal energy piles is a clean technology to overcome the problems of traditional chemical-based snow melting methods. This chapter aims to study the feasibility of snow melting system using geothermal energy piles in six major cities (Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg) in Canada. The energy and inlet temperature of the hydronic system required to warm up a hypothetical slab and keep its temperature above 0°C during a typical snowfall are derived based on a transient energy balance at the slab surface and weather conditions for each city. The coefficient of performance (COP) of heat pump is derived and the number of piles are determined for each city based on its specific local geological condition and heating demand. Also, the economic feasibility study was performed to compare costs between the snow melting system for bridge decks using geothermal energy piles and electricity-based heating system. It was concluded that

- The snow melting system using geothermal energy pile is efficient and cost effective. However, the extent of efficiency and saving varies with implementation areas.

- The COP of the heat pump, as the indicator of the energy efficiency, depends on the different weather conditions, which determine the heating load. In terms of COP, Winnipeg and Toronto are the most suitable places for the geothermal energy pile based snow melting system among the six cities studied in this chapter.
- Despite of different values of COP in six cities, the number of piles required for snow melting is close because of the similar ground temperature and thermal conductivity in six cities. Montreal and Ottawa are expected to have longer length of geothermal heat exchanger pipe because of their relatively higher heating load.
- Snow melting using the geothermal energy pile system is much more cost effective than electrical-based system. However, the relative savings vary with local price of electricity, the value of COP and the local heating load. Toronto is predicted to have the largest saving due to higher price of electricity and higher COP value.

2.1 Introduction

Snow and ice accumulation on bridge decks during winter causes inconvenience for drivers and traffic accidents. For example, almost 30% of car accidents in Canada occur due to snowy or icy roads. A traditional approach for snow melting and de-icing of pavement slabs and bridge decks is to use salt and other kinds of chemical materials, which can lower the freezing point of water to prevent the formation of ice. However, this method can induce some problems. Salt will not only be ineffective for snow melting or de-icing if temperature falls below -3.9°C , but also penetrate down to the slab and accelerate the corrosion of concrete and steel used in a bridge deck [BE10].

In recent decades, researchers and practitioners have been looking for effective and economical alternatives for the snow melting and de-icing system. Some of the most efficient techniques are hydronic and electrical melting system [BE10]. In an electrical system, electrical current flows through the metallic cable in a serpentine pattern in order to heat bridge

deck uniformly. As cables generate heat through the electrical current, induced heat is conveyed to concrete and snow can be melted. In a hydronic system, heat is transferred in the form of convection as the heat carrier fluid circulates through a closed-circuit by a hydraulic pump and releases heat to the surrounding medium [Hop00].

Hydronic systems are becoming more popular as a proved technology by many applications, for instance, radiant heating floor systems for buildings. It has also received more attention in recent years for the application of snow melting on bridge decks. An experimental bridge deck with embedded hydronic tubes built at Oklahoma State University by [LRS03; LRS07] showed that this system could successfully maintain the surface snow-free even during severe weather conditions. There are also several numerical studies of the bridge deck de-icing system. A two-dimensional numerical model implemented by [RSX02] stated that the system idling will probably be required to achieve the surface with a snow-free area ratio of 1 ($A_r = 1$). [LRS03] enhanced the numerical model developed by [RSX02] in order to simulate the hydronic heating system for bridge surface. This model contains four sub-models: hydronic heating system for bridge deck, underground heat exchanger, water-to-water heat pump and system controlling. This model was able to predict the average surface temperature and outlet fluid temperature; however the results obtained in this study were slightly higher than experimental observation [LRS03]. [LRS07] found that the system's performance can be enhanced by running the system at full capacity just before a snowfall. Geothermal energy, a renewable energy heat source of hydronic system, is appealing for the snow melting and de-icing application, because it is economic and environmentally friendly. In the 1990s, several pilot plants for geothermal heat pump system were realized in Japan and central Switzerland [Eug07]. The practicability of a ground source heat pump system composed of vertical single U-shaped heat exchangers investigated by [BE10] showed that bridge slab can be heated effectively. Traditionally, the ground source heat pump system requires a certain number of heat exchanger boreholes which are independent from

the foundation structure. The pile foundations designed to support the loads of bridge structures can also be used as the ground heat exchangers. Therefore, these bi-functional foundations called geothermal energy piles, have less initial installation costs. The thermal performance of the geothermal energy pile for different cities in the USA has been studied by [HY17]. However, the thermal load estimation due to snowfall was calculated based on the steady state ASHREA recommendations. In addition, the energy absorbed by the geothermal energy pile is based on the assumption that the inlet temperature of the geothermal heat exchanger is $-4\text{ }^{\circ}\text{C}$ for each city, which is not realistic.

In this chapter, the snow melting system using geothermal energy pile is studied for six major Canadian cities : Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg. A transient snow melting model is used for the calculation of required heat flux (q_H) to achieve the desired snow free area by considering a typical winter condition during snowfall for each city. Transient conduction and convection mechanisms are considered in the analysis of heat transfer through a hypothetical bridge slab ($4\text{ m} \times 7\text{ m}$) and a geothermal energy pile with a depth of 30 m . The required inlet temperature of geothermal heat exchanger is derived based on the heating load. The coefficient of performance (COP) of heat pump for each city is determined. Then an economic feasibility study is performed to determine the relative reduction in the electricity consumption and corresponding cost savings between the snow melting system for bridge decks using geothermal energy piles and electricity-based heating system.

2.2 Design of a Snow Melting System

In this study, the design of a snow melting system using geothermal energy pile includes (Fig. 2.1): i) evaluation of required heat flux to keep the average temperature at the pavement surface above $0\text{ }^{\circ}\text{C}$ based on local weather conditions; ii) numerical simulation of a hydronic heating system for the slab to determine the high working temperature of the heat pump in order to achieve a desired snow free area; iii) heat transfer analysis in geothermal energy

pile to evaluate the required length of the heat exchanger; iv) determination of the number of geothermal energy piles for the snow-melting system and cost analysis to determine the economic feasibility of the system.

The bridge slab dimension considered in this analysis is assumed to be 4 m by 7 m.

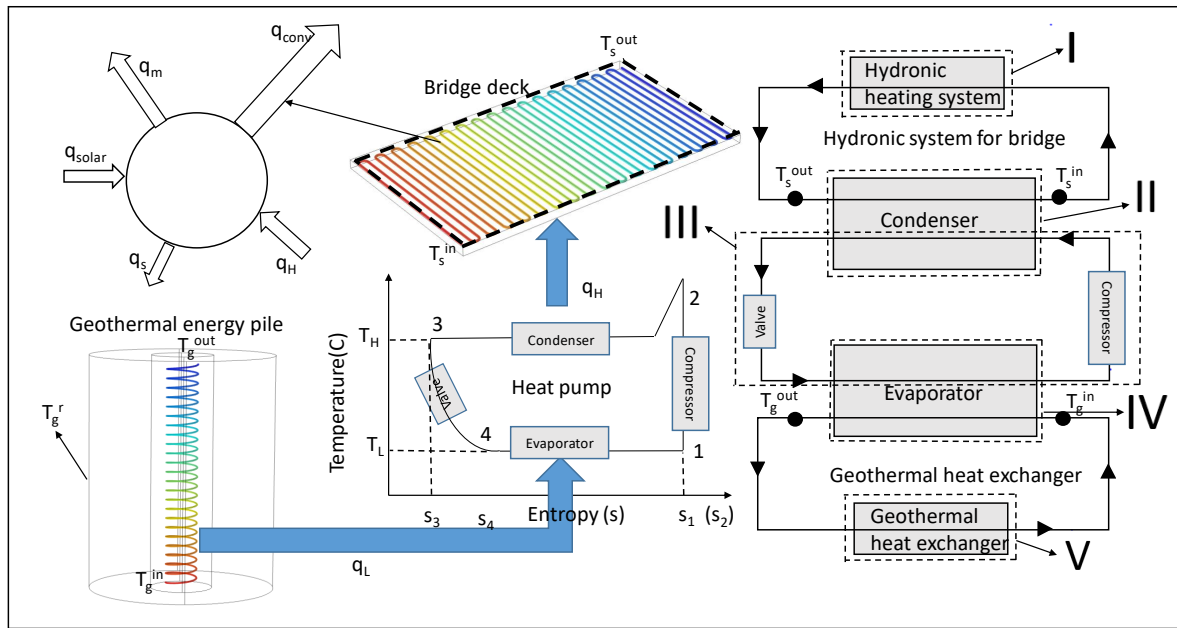


Fig. 2.1: Snow-melting system using geothermal energy pile

2.3 Energy Balance at the Surface of Bridge Slab for Snow Melting/De-Icing

The energy balance equation described in 2007 ASHREA handbook states that the heat flux depends on surface convection, radiation and evaporation whose values are based on five atmospheric factors: rate of snowfall, air temperature, humidity, wind speed near the heated surface and sky temperature. For the control volume I in Fig. 2.1, the complete energy balance equation is expressed as:

$$q = q_s + q_m + A_r (q_{c,r} + q_e) \quad (2.1)$$

where q (W/m^2) represents the required heat flux for snow-melting at the surface; q_s (W/m^2) denotes the sensible heat flux; q_m (W/m^2) is the heat flux due to latent heat during phase change; A_r stands for the snow free area ratio, defined as the ratio of snow free area to snow cover area; $q_{c,r}$ (W/m^2) is the convective and radiative heat flux; And q_e (W/m^2) is the heat flux required for evaporation.

The assumption made in this energy balance equation is that the area covered with snow is free of convection, radiation and heat flux due to evaporation. Also the melted water needs to be heated up to 0.56 °C before being drained. This model provides a simple way of the evaluation of the required heat flux to achieve the desired snow free area. In this paper, the snow melting system is designed to melt the snow and drain the water instantaneously. Then, the energy balance equation for the control volume I in Fig. 2.1) becomes:

$$q = q_{conv} + q_s + q_m + q_{solar} \quad (2.2)$$

where q_{conv} (W/m^2) represents the heat flux due to convection at the bridge deck surface; q_{solar} (W/m^2) is the short-wave solar radiation. Sensible heat flux q_s is used to increase the temperature of fresh snow to its melting point and then increase the temperature of water to liquid film temperature (0.56 °C). This sensible heat flux can be calculated using the following equation:

$$q_s(t) = \rho_w s(t) \left(c_p^i (T_m - T_a(t)) + c_p^w (T_f - T_m) \right) \quad (2.3)$$

where ρ_w ($1000 \text{ kg}/m^3$) is the density of the liquid water; s (m/s) is the snowfall rate expressed in water equivalent; c_p^i ($2100 \text{ J}/\text{kg}\cdot^\circ\text{C}$) is the specific heat of ice; c_p^w ($4200 \text{ J}/\text{kg}\cdot^\circ\text{C}$) is the specific heat of water; T_f ($^\circ\text{C}$) is the liquid film temperature which is usually taken as 0.56 °C; T_a ($^\circ\text{C}$) is the ambient temperature that varies with time; T_m is the melting temperature taken as 0 °C; t (s) denotes time.

Latent heat of fusion of snow q_m is the heat that snow needs to absorb during the phase change:

$$q_m(t) = \rho_w s(t) h_{if} \quad (2.4)$$

where h_{if} is the heat of fusion, usually taken as $3.3 \times 10^5 \text{ J/kg}$.

Short-wave radiation q_{solar} is evaluated by the combination of beam radiation and diffuse radiation on the horizontal plane. Beam radiation, also termed as direct radiation, refers to the component of total solar radiation that travels from the sun to the earth surface without changing its direction. Diffuse radiation, on the other hand, is the sunlight scattered by air particles that arrive at the earth surface. The data used in this analysis is obtained from Natural Resource Canada.

The convection term q_{conv} is temperature- and time-dependent. Because of the huge thermal mass of concrete, temperature of slab takes time to change. In this analysis, the surface convection heat flux can be calculated using the following equation:

$$q_{conv}(t) = h_{conv} (T_a(t) - T_s(t)) \quad (2.5)$$

where T_a ($^{\circ}\text{C}$) is the ambient air temperature and T_s ($^{\circ}\text{C}$) is the temperature on the slab surface. The convection heat transfer mode is comprised of two mechanism: diffusion and advection [I+]. The convection heat transfer coefficient, h_{conv} , mainly depends on conditions on the boundary layer, for example, wind speed and ambient temperature

[I+]. The average convective heat transfer coefficient for external forced convection on a horizontal surface can be calculated by the following formula [WRF14]:

$$h_{conv} = 2 \frac{k_a}{L} \begin{cases} \frac{0.3387 P_r^{1/3} Re_L^{1/2}}{\left(1 + \left(\frac{0.0468}{P_r}\right)^{2/3}\right)^{1/4}} & \text{if } Re_L \leq 5 \cdot 10^5 \\ P_r^{1/3} (0.037 Re_L^{4/5} - 871) & \text{if } Re_L > 5 \cdot 10^5 \end{cases} \quad (2.6)$$

in which k ($W/(m \cdot K)$) is the thermal conductivity of air; L (m) is the characteristic length of bridge slab in the direction of wind. Pr stands for Prandtl number of air, the ratio of thermal diffusivity to viscosity, taken as 0.7. And Re_L is Reynolds number based on characteristic length L , calculated as follows

$$Re_L = \frac{V L}{\nu_{air}} \quad (2.7)$$

where V (m/s) is the wind speed at the slab surface and ν_{air} (m^2/s) is the kinematic viscosity of air. The design weather is a very important parameter for sizing heat pump so that the system can handle most of snowy situations. In this analysis, the design weather is not the worst weather condition since it will overestimate the heating load of the snow melting system and hence dramatically increase the cost. In this study, it is assumed that the design weather covers at least 90% of snowfalls.

Environment and Climate Change Canada provides the statistic information based on weather data from 1981 to 2010 as well as the days with snow for different cities in Canada (Fig. 2.2). The daily snowfall that are smaller than 10 cm for Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg is 97%, 97%, 91%, 92%, 93% and 96%, respectively. Therefore,

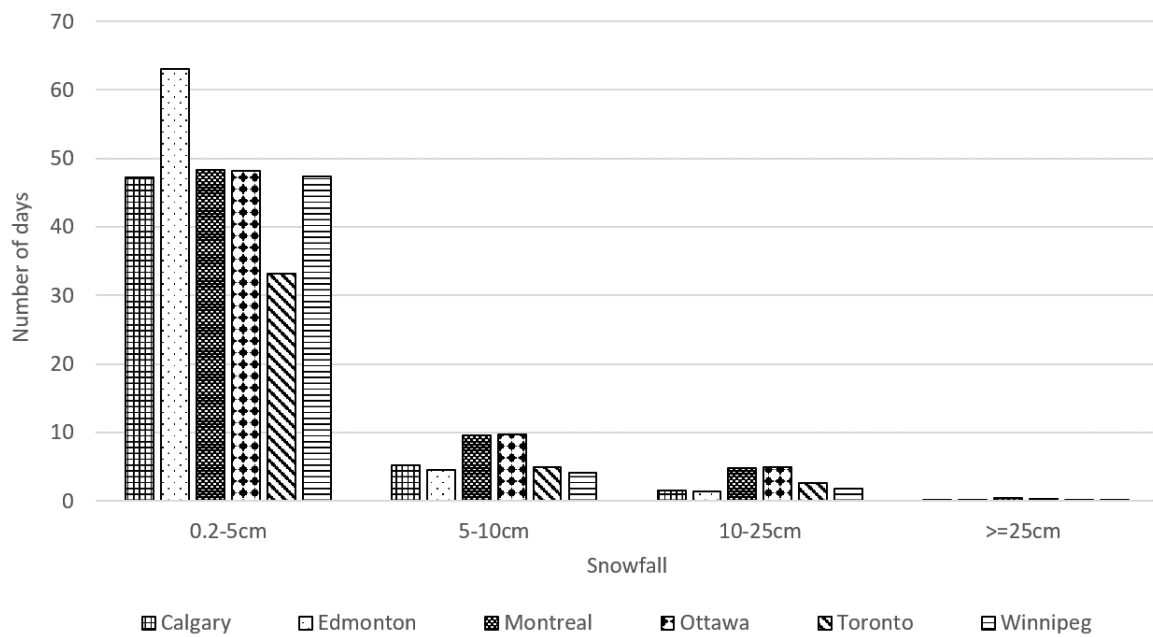


Fig. 2.2: Average number of days per year with snowfall from 1981 to 2010

the design weather is a day with an approximate daily snow of 5-10 *cm*. The selected weather condition (average value) for the analysis can be found in Table 2.1.

Tab. 2.1: Summary of selected weather conditions for six Canadian cities

City	Evaluated data	Wind speed (m/s)	Air temperature (°C)	Snowfall (cm)	Snowfall rate(cm/h)	Solar radiation (W/m2)
Calgary	2014-03-30	4.4	-9.4	7.7	0.32	168
Edmonton	2010-11-17	4.4	-12	4.8	0.21	46
Montreal	2008-01-22	1.6	-12.8	10.6	0.48	92
Ottawa	2017-02-14	2.8	-7.8	9.1	0.48	115
Toronto	2012-12-17	2.7	-2.6	6	0.43	68
Winnipeg	2007-02-15	3.9	-4.8	6.6	0.3	136

*: Snowfall rate is calculated by the ratio of snowfall to the corresponding snowfall duration

In this analysis, the net heat flux q_{net} (W/m^2) which represents the heat flux that should be provided by the hydronic heating system on the pavement surface excluding the heat transfer due to q_{conv} is defined as:

$$q_{net} = q_{solar} + q_m + q_s \quad (2.8)$$

Fig. 2.3a to Fig. 2.3d present the net heat flux for different cities.

2.4 Design of Hydronic Heating System for Bridge Decks

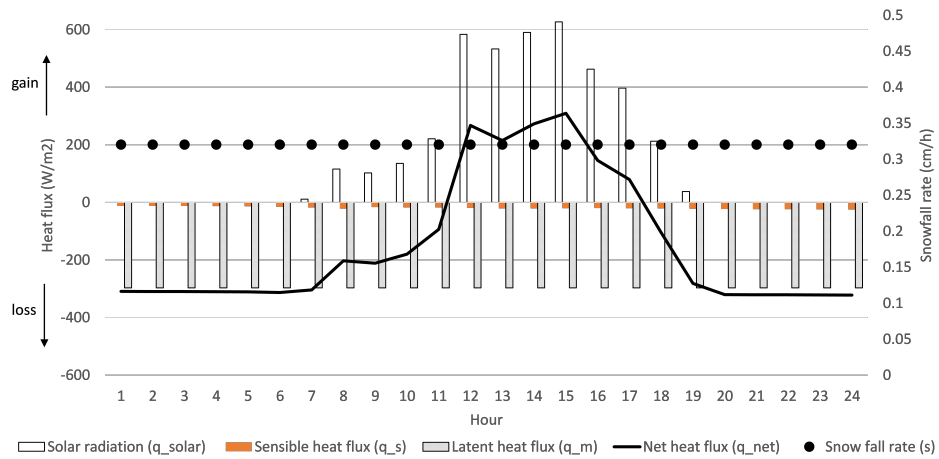
The modeling of the hydronic heating system is performed considering the heat transfer in pipe and concrete materials used for bridge decks. The important assumptions applied in this model are: a) the velocity profile is fully developed with entire pipe section; b) the average bulk mean velocity remains constant along the pipe; c) all velocity components normal to pipe axis is assumed to be 0 [Bar+66]. The governing equation of the heat transfer in the pipe is used to determine the temperature distribution profile in the fluid flow (control volume I) [LS08]:

$$\rho_w A C_p \frac{\partial T}{\partial t} + \rho_w A C_p v_s \cdot \nabla T = \nabla \cdot (A k \nabla T) + Q_{w1} \quad (2.9)$$

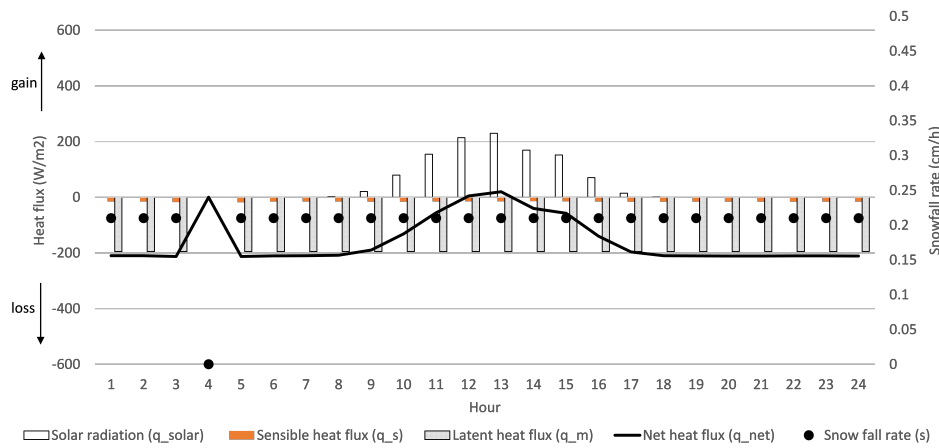
where k ($W/(m \cdot K)$) is thermal conductivity; C_p ($J/(kg \cdot K)$) is the heat capacity of water at constant pressure; v_s (m/s) is the fluid velocity. Q_{w1} (W/m) is the external heat source through pipe wall. For unit area, Q_{w1} can be written as follows:

$$Q_{w1} = (hZ)_{eff} (T_{ext} - T) \quad (2.10)$$

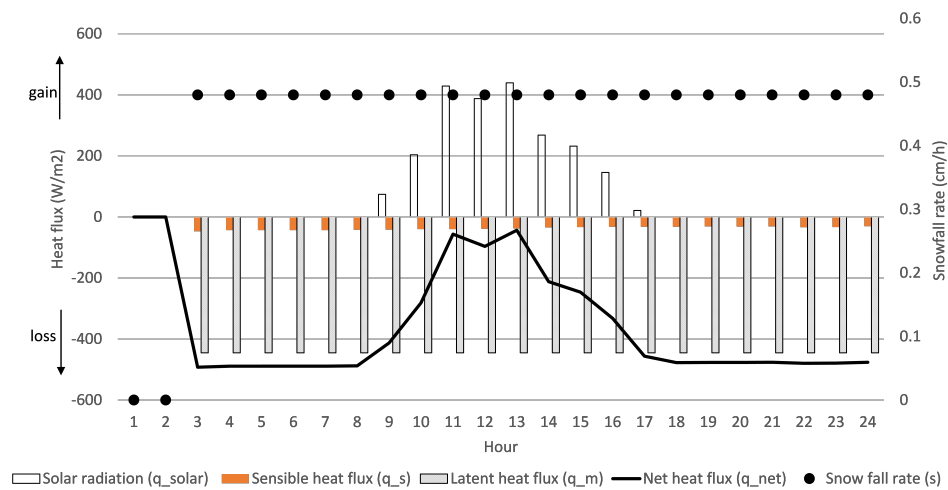
where $(hZ)_{eff}$ ($W/m \cdot K$) is the effective heat transfer coefficient; T_{ext} is the temperature of the outer surface of the pipe. It should be mentioned that the product of Q_{w1} and the length of fluid flow is the energy output rate (q_H) from the condenser (Fig. 2.1).



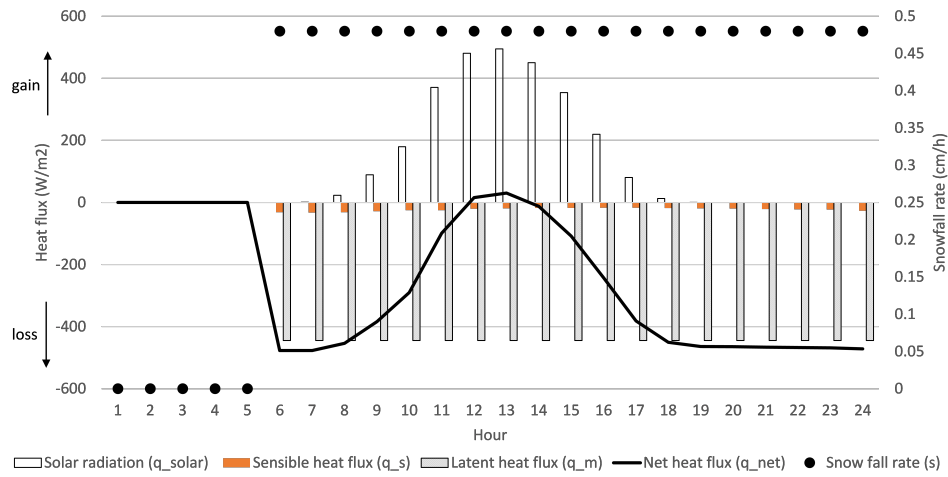
(a) Heat flux calculation results for Calgary



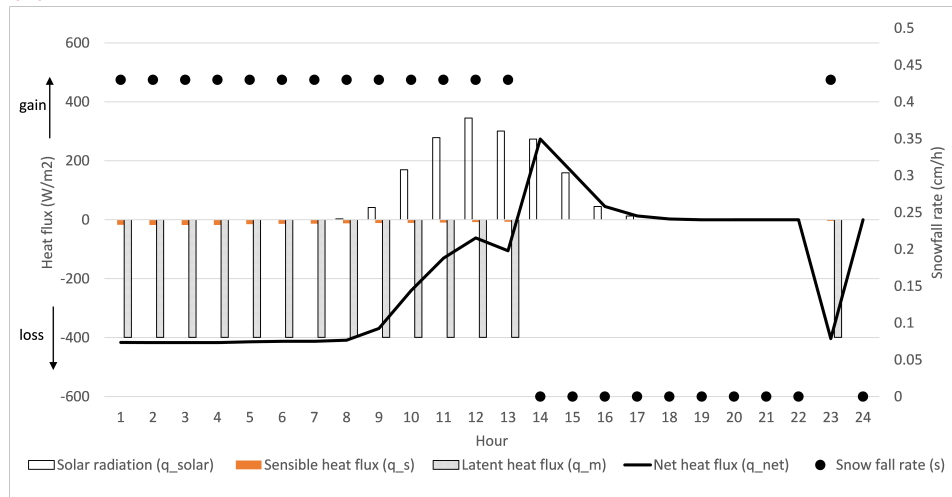
(b) Heat flux calculation results for Edmonton



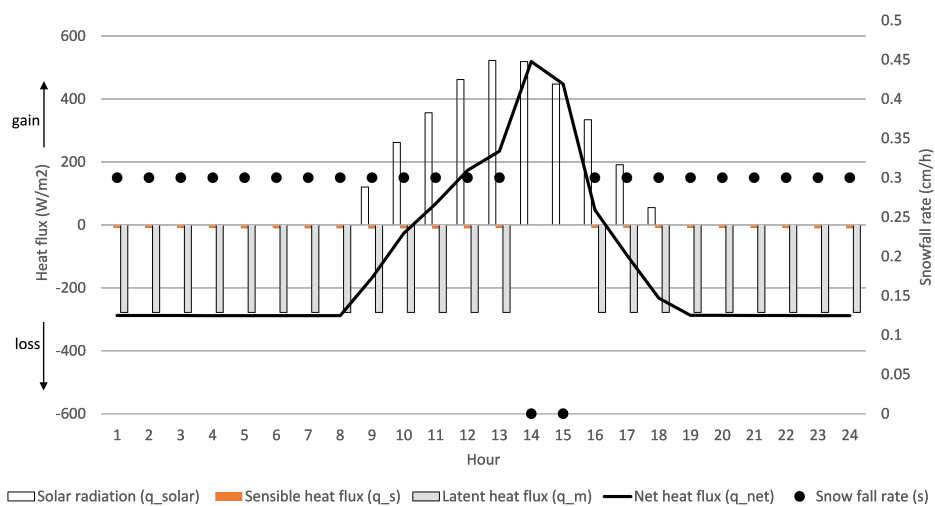
(c) Heat flux calculation results for Montreal



(d) Heat flux calculation results for Ottawa



(e) Heat flux calculation results for Toronto



(f) Heat flux calculation results for Winnipeg

Fig. 2.3: Heat flux calculation summary for a) Calgary, b) Edmonton, c) Montreal, d) Ottawa, e) Toronto and f) Winnipeg

The heat transfer through the surrounding concrete slab is described as follows:

$$\rho_s A C_p^s \frac{\partial T}{\partial t} = \nabla \cdot (A k_s \nabla T) - Q_{w1} \quad (2.11)$$

where ρ_s (2300 kg/m^3) is the density of concrete; C_p^s ($880 \text{ J/kg} \cdot \text{K}$) is the heat capacity of concrete; T (K) is the temperature of concrete slab; k_s ($1.8 \text{ W/m} \cdot \text{K}$) is the thermal conductivity of concrete; Q_{w1} (W/m) is a source term because of the heat transfer within the hydronic heating system.

The boundary conditions at the top surface of the bridge slab include the forced convection (q_{conv}), short-wave radiation (q_{solar}), sensible heat (q_s) and latent heat (q_m) during the phase change. The bottom and side surfaces are assumed to be insulated to make sure no heat loss occurs.

According to ASHREA (2011) handbook, the satisfactory standard installation place of pipe is at least 5 cm from the top and bottom of the slab [Ash11]. The typical spacing between pipes ranges from 150 mm to 300 mm [SR00]. Nominal pipe diameters are 18 mm to 25 mm [SR00].

The parameters used in the model is shown in the table 2.2:

2.5 Design of Heat Pump for Snow-Melting System

A heat pump is defined as a device that pumps thermal energy opposite to the direction of spontaneous heat flow by absorbing heat from a cold space and releasing it to a warmer one [Chi16]. A heat pump has four sub-components which are condenser, evaporator, expansion valve and compressor. During winter, because of the existing temperature gradient

Tab. 2.2: Snow melting parameters used for the bridge slab

	Parameter	Description
Hydronic heating system	Diameter of pipe	2 cm
	Space	0.2 m
	Depth of pipe	5 cm
Bridge slab	length	7 m
	Width	4 m
	Height	0.2 m
Constant parameter value	Heat capacity of air	1004 J/kg · K
	Viscosity of air	0.000011 Pa · s
	Air conductivity	0.02 W/m · K
	Melting point	0 °C
	Film temperature	0.56 °C
	Pr number of air	0.7
	Sc number of air	0.6

between the refrigerant in the evaporator and the underground temperature, geothermal heat is absorbed by the refrigerant and transferred to the compressor. By compressing the refrigerant and increasing its pressure, the temperature will be increased. Finally, the refrigerant is circulated to the condenser to release the heat for melting snow and ice on the bridge deck. Then expansion valve acts to reduce the pressure and temperature of the refrigerant liquid and this system keeps going on.

In this study, a conceptual heat pump is designed using the refrigerant R404a. The inlet temperature T_s^{in} and outlet temperature T_s^{out} can be designed to meet the requirement for the snow melting based on various weather conditions. As shown in Fig. 2.1, the average value of T_s^{in} is taken as the high working temperature of the heat pump (T_H).

In this application, the energy output rate of condenser (q_H) is the heat source for the hydronic snow melting system. In order to save computational time, the low working temperature (T_L) of the heat pump is 1 °C lower than constant underground temperature (T_g^r) in each city since the thermal equilibrium is hard to achieve. After determining the high and low working temperatures of the heat pump, the enthalpy for each state (h_1, h_2, h_3

and h_4) can be obtained using the thermodynamic tables [Han+01]. The heat exchange for the condenser of heat pump (control volume II) can be expressed as :

$$q_H \equiv \dot{m}_R (h_2 - h_3) = \dot{m}_s C_p (T_s^{in} - T_s^{out}) \quad (2.12)$$

Then $\dot{m}_R$ can be calculated using equation 2.12. The heat exchange for the evaporator of the heat pump (control volume IV) can be expressed as :

$$q_L \equiv \dot{m}_R (h_1 - h_4) = \dot{m}_g C_p (T_g^{in} - T_g^{out}) \quad (2.13)$$

where $\dot{m}_R$ (kg/s) is the mass flow rate of fluid in the heat pump; $\dot{m}_s$ (kg/s) and $\dot{m}_g$ (kg/s) denote the mass flow rate for hydronic heating system of bridge deck and the geothermal heat exchanger, respectively; T_g^{in} ($^{\circ}C$) is the inlet fluid temperature of the geothermal energy pile in contact with the evaporator at the beginning of the cycle; T_g^{out} ($^{\circ}C$) is the outlet fluid temperature in contact with the evaporator at the end of the cycle. T_s^{in} ($^{\circ}C$) is the inlet fluid temperature of the hydronic heating system in contact with the condenser at the beginning of the cycle; T_s^{out} ($^{\circ}C$) is the outlet fluid temperature of the hydronic heating system in contact with the condenser at the end of the cycle. h (J/kg) is the enthalpy.

Based on the energy conservation, the energy absorbed by the evaporator of the heat pump is equal to the energy absorbed by the underground heat exchanger through soil. The energy released by the condenser of the heat pump is equal to the energy consumption of the concrete slab. It should be noted that the heat pump always requires the electricity input in order to utilize energy with low temperature. Also, T_g^{in} is designed to reach the low working temperature of the heat pump (T_L). Therefore, the T_g^{out} for the geothermal heat exchanger can be easily calculated using Eq. 2.13.

The COP of the heat pump (control volume III) can be calculated for the six major cities in Canada by using the following equation:

$$COP \equiv \frac{q_H}{q_H - q_L} = \frac{h_2 - h_3}{h_2 - h_1} \quad (2.14)$$

2.6 Design of Geothermal Energy Pile

The first few meters of geothermal piles are usually insulated because of the freezing and thawing effect of the ground. The governing equation for the heat transfer in geothermal heat exchanger fluid (control volume V) can be expressed as:

$$\rho_w A C_p \frac{\partial T}{\partial t} + \rho_w A C_p v_g \cdot \nabla T = \nabla \cdot (A k \nabla T) + Q_{w2} \quad (2.15)$$

where Q_{w2} (W/m) is the heat sink term due to the phase change in the evaporator of the heat pump; v_g (m/s) is the velocity of the geothermal heat exchanger pipe flow.

The heat transfer in the soil and concrete pile are described as:

$$\rho A C_p \frac{\partial T}{\partial t} = \nabla \cdot (k A \nabla T) - Q_{w2} \quad (2.16)$$

where C_p ($J/kg \cdot K$) is the heat capacity for concrete or soil; k ($W/m \cdot K$) is the thermal conductivity for concrete or soil; ρ (kg/m^3) stands for the density of soil or concrete. This equation gives the temperature distribution in soil and concrete due to the heat conduction mechanism.

Constant underground temperature is used as the boundary condition for the geothermal energy pile. It should be noted that the soil medium should be large enough so that the constant soil temperature can be considered as a valid boundary condition. The heat exchanger pipe is assumed to be polyvinyl chloride(PVC) with an inner diameter of 2 cm

and wall thickness of 4 *mm*. The computational domain of the surrounding soil medium is set to be a cylinder with a diameter of 12 *m* and 1.2 *m* for the concrete pile.

The performance of the geothermal energy pile requires a good understanding of the ground temperature distribution at various depth. Also, the length of the pile as well as the length of the heat exchanger pipe could have impact on its working efficiency. One of the most important properties of the soil is its very high thermal mass, which causes a slow response to temperature variations at the ground surface.

The constant underground temperature for different cities in Canada is provided by Natural Resource Canada, as can be seen in Fig. 2.4. In this study, the constant underground temperature considered for Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg, is 6°C, 5°C, 7°C, 7°C, 9°C and 7°C, respectively.

Natural Resource Canada provided the thermal conductivity of the soil across Canada, as can be seen in Fig. 2.5. According to Fig. 2.5, the thermal conductivity of soil for Calgary, Edmonton, Montreal, Ottawa, Toronto and Winnipeg is 2.3 W/m·K, 1.8 W/m·K, 3 W/m·K, 2.8 W/m·K, 2.8 W/m·K, and 3 W/m·K, respectively.

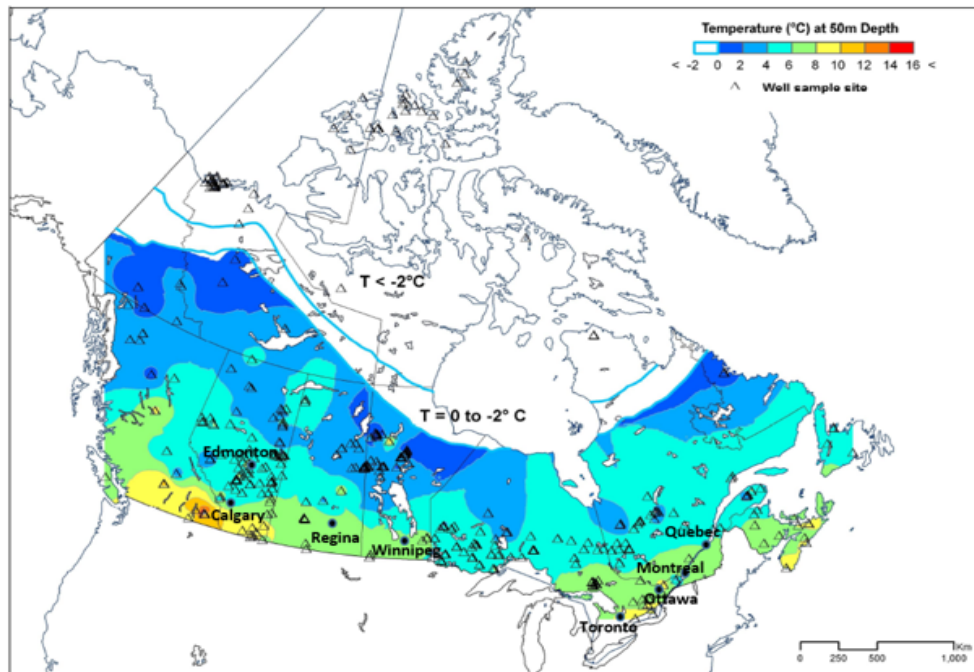


Fig. 2.4: Underground temperature distribution across Canada at a depth of 50m [Gra12]

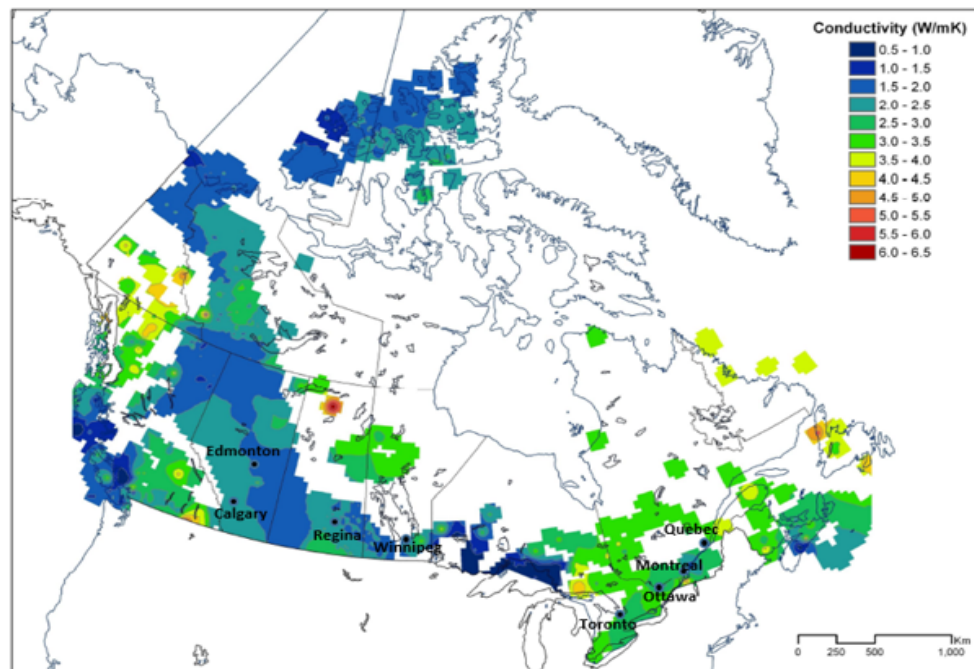


Fig. 2.5: Thermal conductivity of soil across Canada at a depth of 50m [Gra12]

Tab. 2.3: Geothermal energy pile heat exchanger design parameters

Parameter	Description
Arrange type of heat exchanger	Spiral shape/ W shape/ U shape
Pile depth	30m
Pile diameter	1.2m
Pipe material	PVC
Pipe inner diameter	2cm
Pipe wall thickness	4mm
Thermal conductivity of pipe wall	0.46W/(m · K)
Thermal conductivity of concrete	1.8 W/(m · K)
Specific heat capacity of concrete	880 J/(kg · K)
Density of concrete	2300 kg/m ³
Density of soil	1600 kg/m ³

The influence of the initial inlet temperature of the heat exchanger fluid has been analyzed in this study. As it is showed in Section 2.7.2, the performance of geothermal piles is not affected by this value after one hour. In this analysis, the outlet temperature from geothermal energy pile (T_g^{out}) is determined by using Eq. 2.13.

In general, a heat exchanger pipe can have three different arrangement types: U shape, W shape and spiral shape. Considering the dimension of the geothermal pile structure presented in Table 2.3, it is assumed that the maximum length of 60 m, 120 m and 180 m is used for the U shape, W shape and spiral shape heat exchanger pipe in the geothermal pile, respectively.

The geometrical parameters and material properties used in the model are listed in Table 2.3.

2.7 Results and Discussions

2.7.1 Determination of COP of Heat Pump

The COP of the heat pump can be determined by using Eq. 2.14. In this equation, the q_H and q_L can be calculated by using the thermodynamic tables and corresponding low working temperature (T_L) and high working temperature (T_H). As shown in part (a) in Fig.

Tab. 2.4: The summary for working temperature and enthalpy for each city

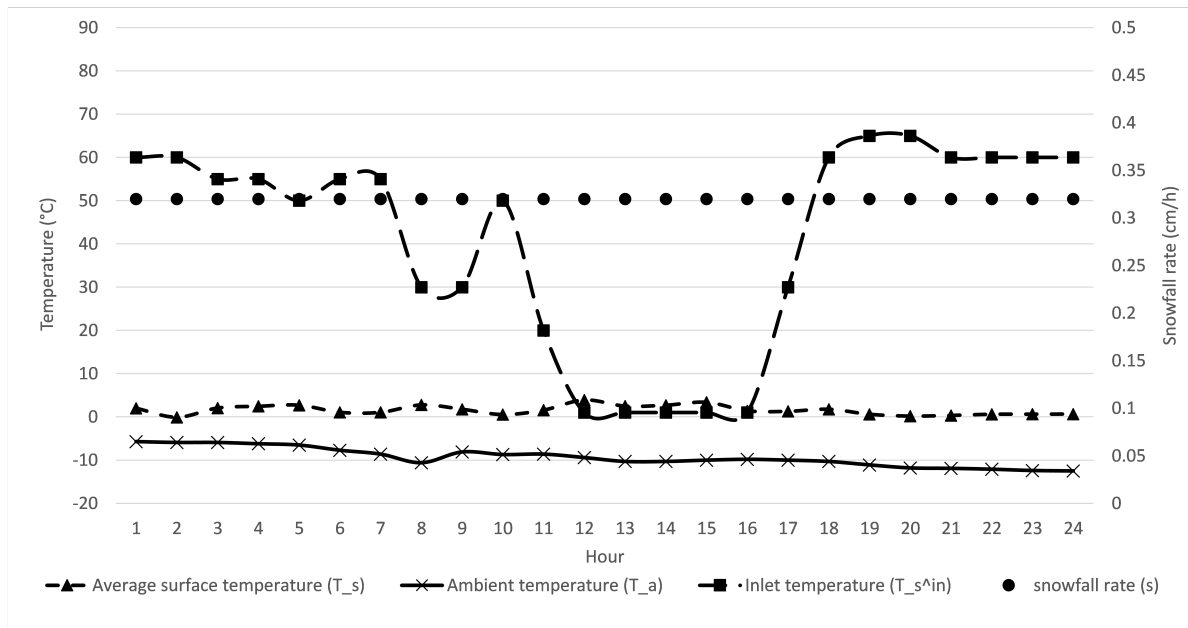
City	T_H or T_s^{in} (°C)	T_L or T_g^{in} (°C)	T_g^{out} (°C)	h_1 (kJ/kg)	h_2 (kJ/kg)	h_3 (kJ/kg)	h_4 (kJ/kg)
Winnipeg	40	6	2	371.5	389.825	263.8	263.8
Edmonton	58	4	-1	370.5	396.475	300.9	300.9
Montreal	50	6	0	371.5	394.675	283.2	283.2
Ottawa	60	6	0	371.5	400.55	305.8	305.8
Calgary	50	5	0	371	394.525	283.2	283.2
Toronto	50	8	3	372.6	394.525	283.2	283.2

2.6 to Fig. 2.11, the inlet temperature (T_s^{in}) is optimized according to the weather conditions (air temperature, wind speed, solar radiation and snowfall rate) to maintain the average surface temperature of the slab above 0 °C. The part (b) in Fig. 2.6 to Fig. 2.11 shows the distribution of the slab's surface temperature (T_s) at a certain time during the snowfall for each city. The average inlet temperature (T_s^{in}) during the snowfall is used as the high working temperature (T_H) of the heat pump. Therefore, the average inlet temperature (T_s^{in}) of the hydronic heating system for the bridge slab during the snowfall can be obtained. The flow rate (v_g) in this analysis is 1.25 m/s. The temperature and enthalpy are shown in Table 2.4. Consequently, the values of q_H , q_L and COP for each city can be derived.

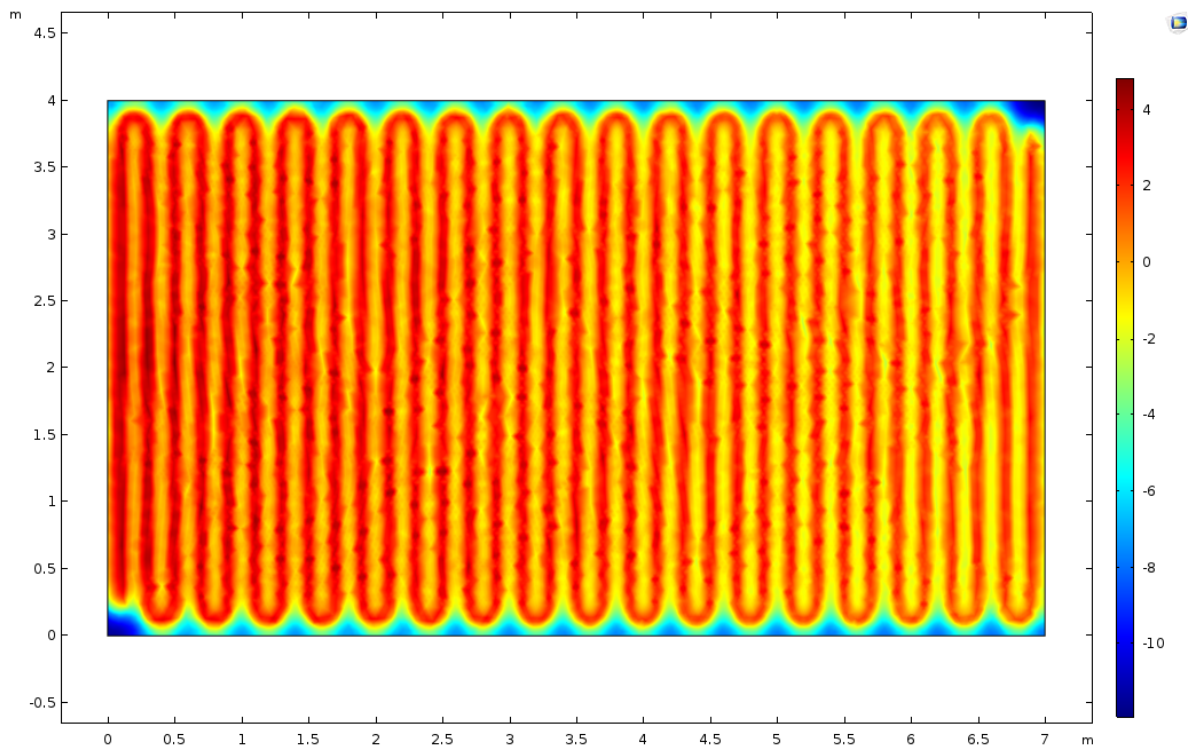
Tab. 2.5: The COP of heat pump for each city

City	q_L (kW)	q_H (kW)	q_w (kW)	m_R (kg/s)	COP
Winnipeg	8.5	9.9	1.4	0.1	6.9
Edmonton	10.7	14.7	4.0	0.2	3.7
Montreal	12.4	15.6	3.2	0.1	4.8
Ottawa	11.8	17	5.2	0.2	3.3
Calgary	9.9	12.5	2.6	0.1	4.7
Toronto	10.8	13.4	2.6	0.1	5.1

It should be mentioned that the value of q_H mainly depends on the weather conditions (wind speed, air temperature, snowfall rate and solar radiation). The higher wind speed and snowfall rate will lead to a higher energy consumption rate (q_H), whereas the higher air

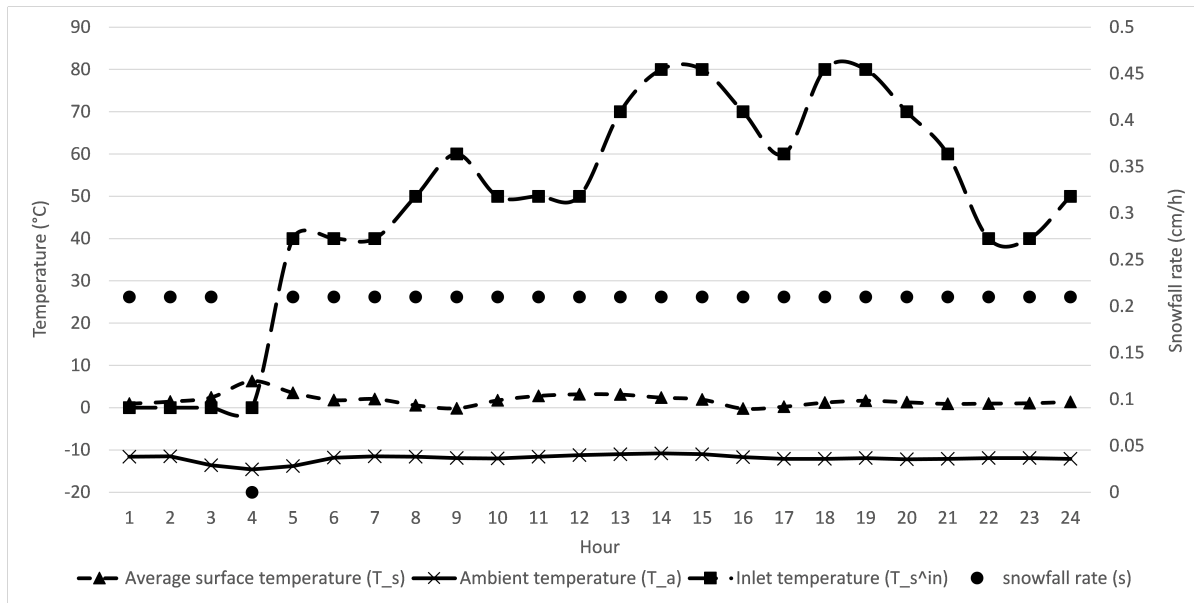


(a) Hourly inlet temperature of hydronic heating system for Calgary

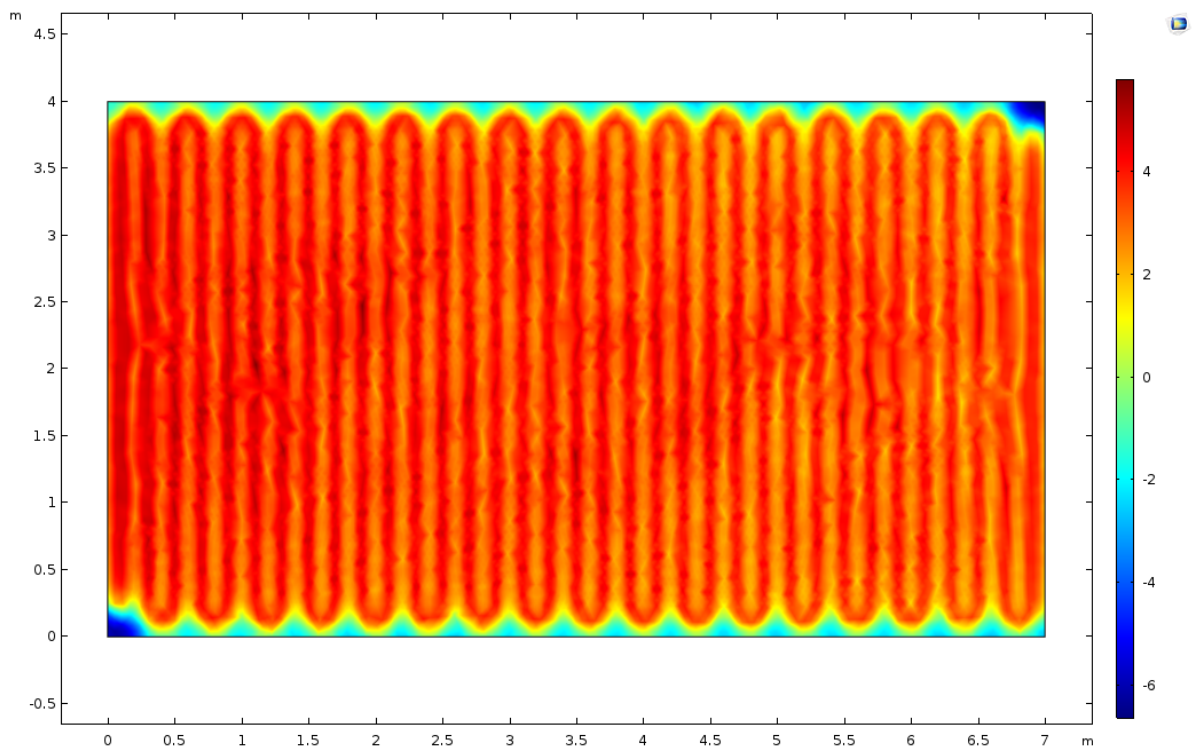


(b) Distribution of the slab's surface temperature for Calgary (7h)

Fig. 2.6: (a) Hourly inlet temperature of hydronic heating system for Calgary; (b) Distribution of the slab's surface temperature for Calgary (7h)

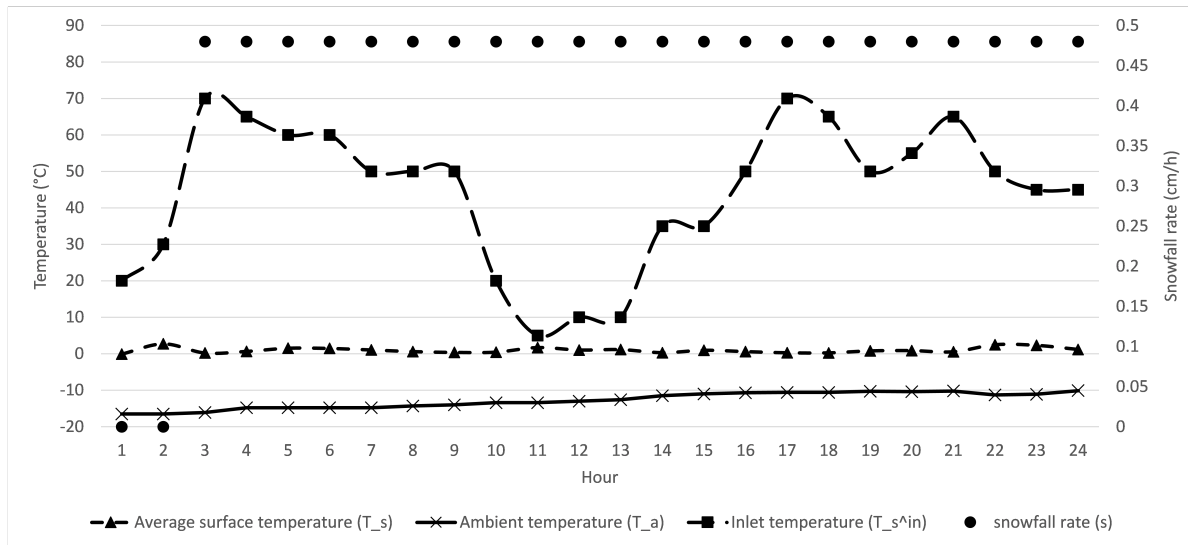


(a) Hourly inlet temperature of hydronic heating system for Edmonton

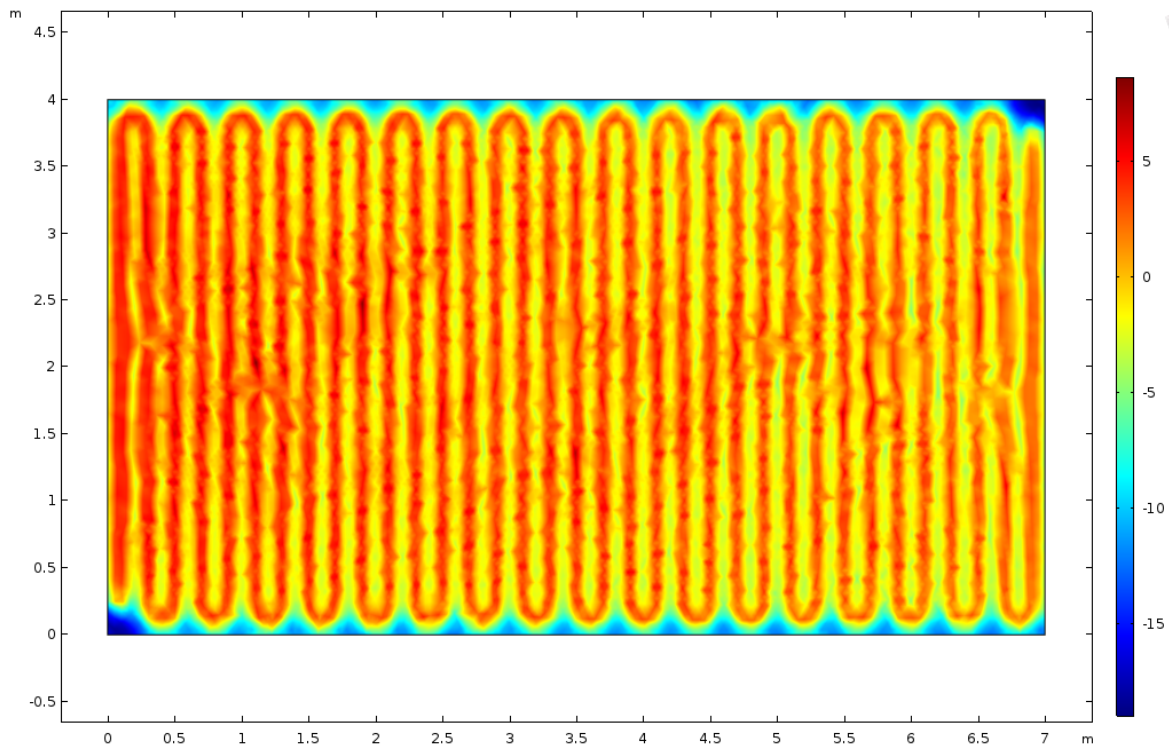


(b) Distribution of the slab's surface temperature for Edmonton (12h)

Fig. 2.7: (a) Hourly inlet temperature of hydronic heating system for Edmonton; (b) Distribution of the slab's surface temperature for Edmonton (12h)

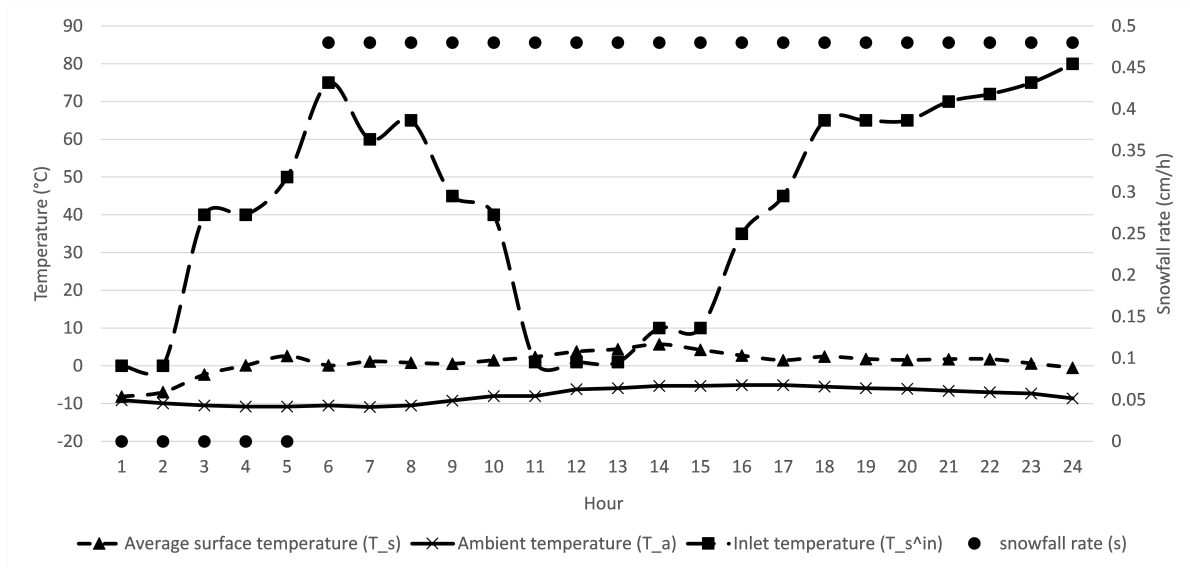


(a) Hourly inlet temperature of hydronic heating system for Montreal

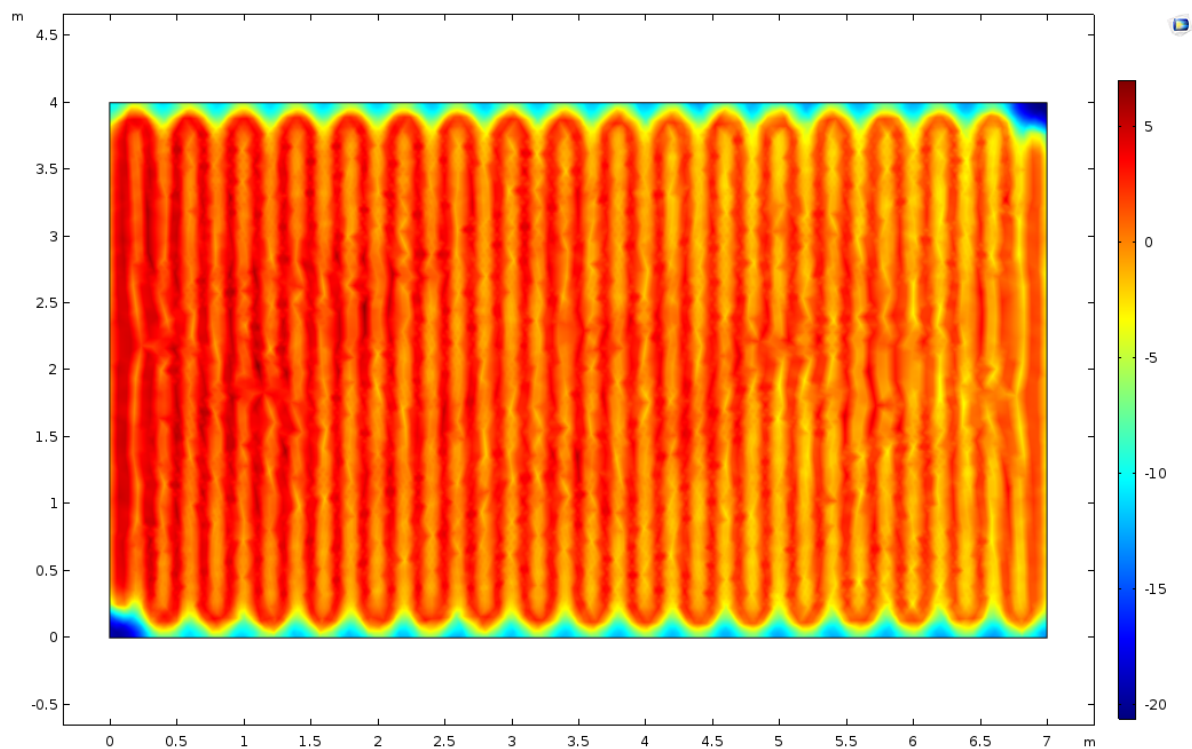


(b) Distribution of the slab's surface temperature for Montreal (17h)

Fig. 2.8: (a) Hourly inlet temperature of hydronic heating system for Montreal; (b) Distribution of the slab's surface temperature for Montreal (17h)

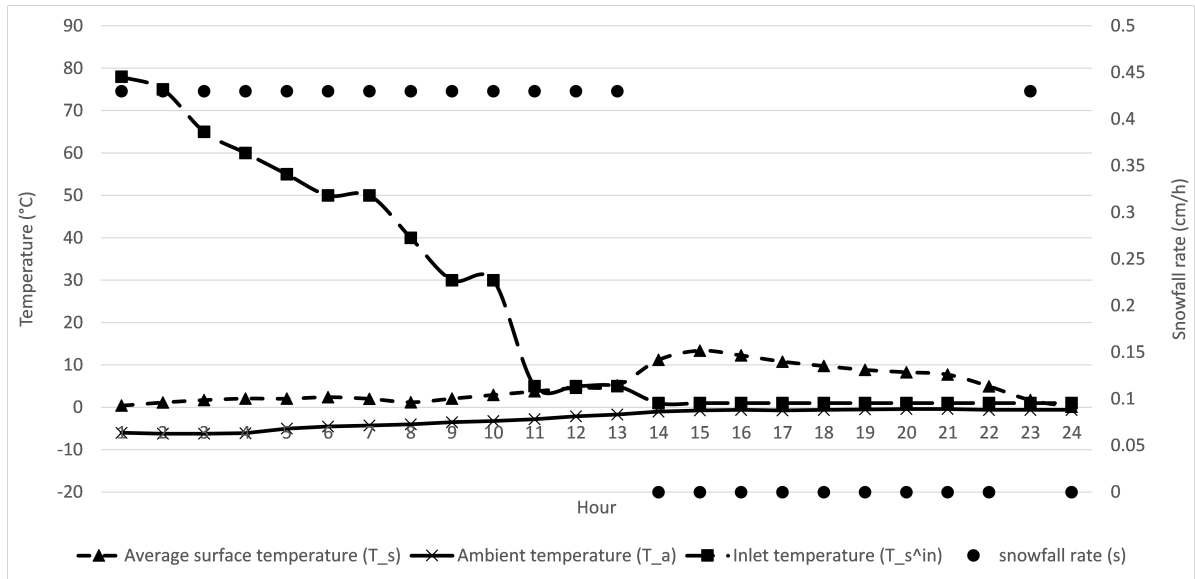


(a) Hourly inlet temperature of hydronic heating system for Ottawa

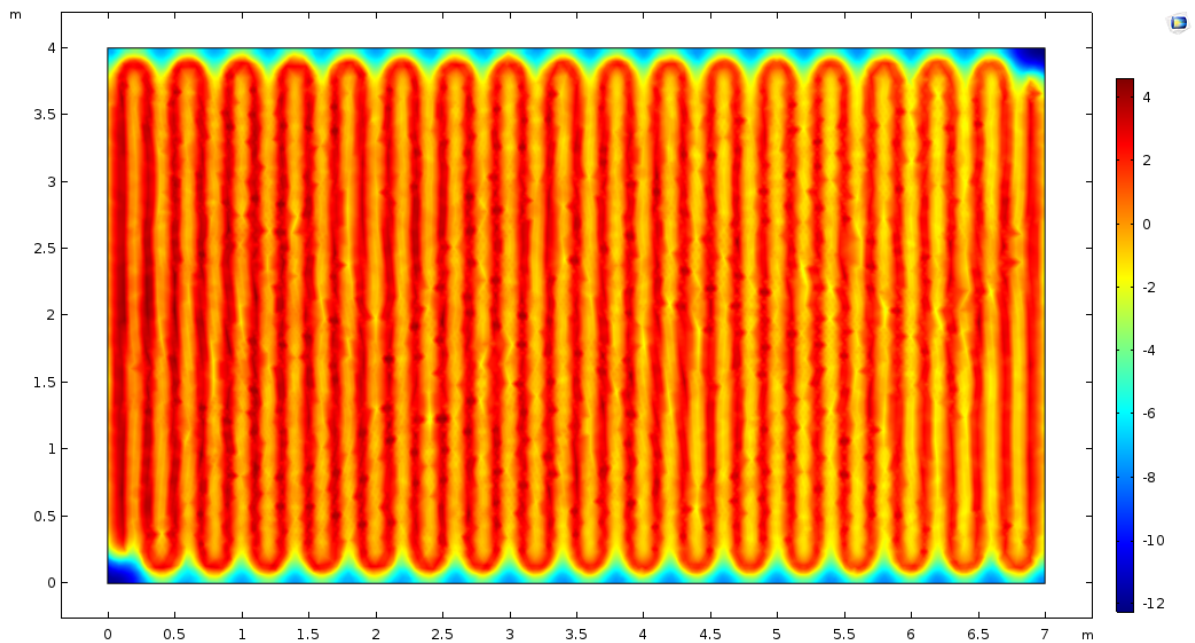


(b) Distribution of the slab's surface temperature for Ottawa (9h)

Fig. 2.9: (a) Hourly inlet temperature of hydronic heating system for Ottawa; (b) Distribution of the slab's surface temperature for Ottawa (9h)

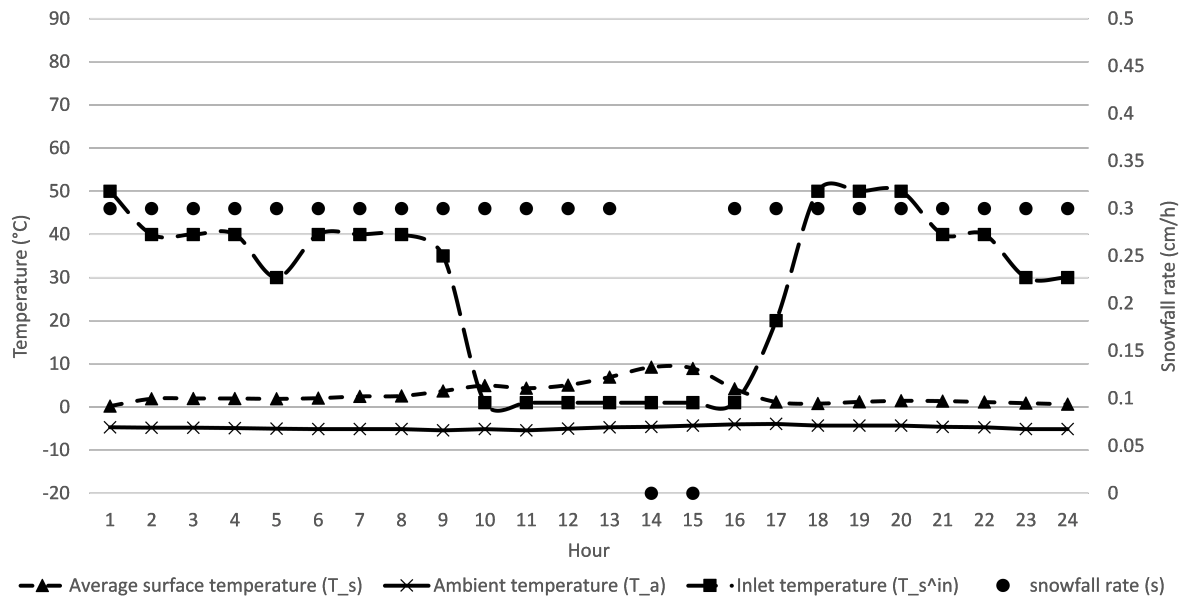


(a) Hourly inlet temperature of hydronic heating system for Toronto

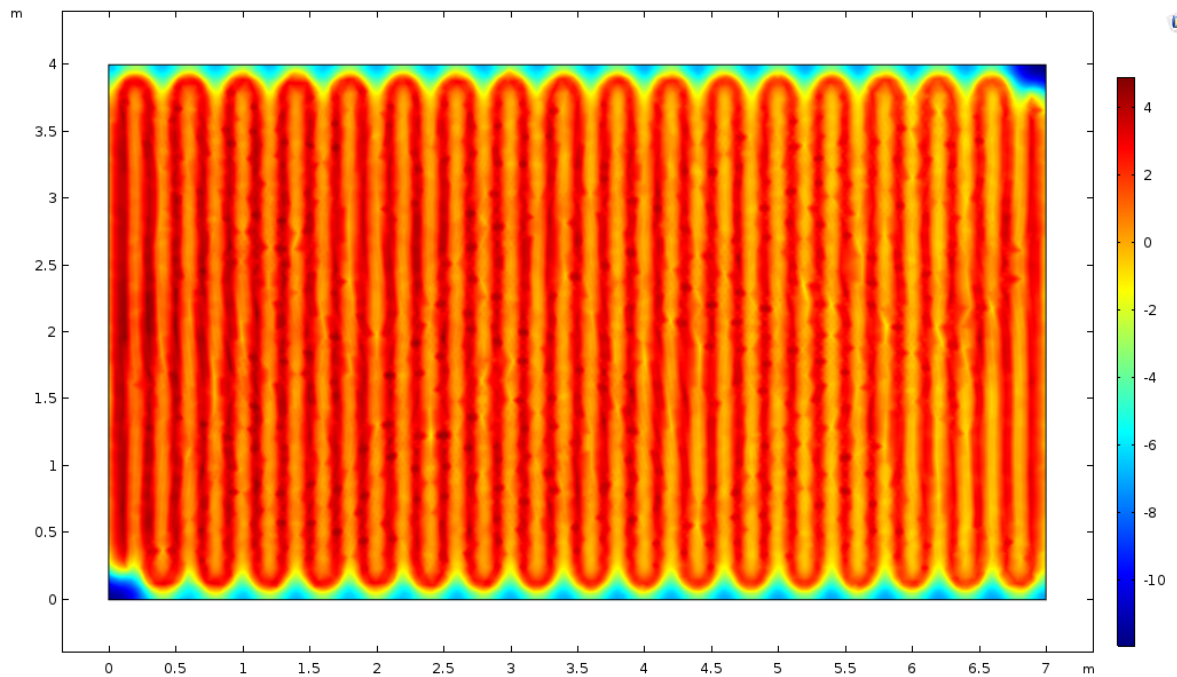


(b) Distribution of the slab's surface temperature for Toronto (7h)

Fig. 2.10: (a) Hourly inlet temperature of hydronic heating system for Toronto; (b) Distribution of the slab's surface temperature for Toronto (7h)



(a) Hourly inlet temperature of hydronic heating system for Winnipeg



(b) Distribution of the slab's surface temperature for Winnipeg (7h)

Fig. 2.11: (a) Hourly inlet temperature of hydronic heating system for Winnipeg; (b) Distribution of the slab's surface temperature for Winnipeg (7h)

temperature and solar radiation will cause a decrease in the energy consumption rate (q_H). The value of q_L is dependent on the ground temperature (T_g^r). Due to the close ground temperature (T_g^r), as shown in Fig. 2.4, the energy consumption rates (q_H) are also close in these six cities.

The COP of the heat pump, as the indicator of the energy efficiency, varies with cities largely due to the different weather conditions. As shown in Table 2.5, Winnipeg and Toronto are the most suitable places for the snow melting system using the geothermal energy pile among the six cities studied in this research.

2.7.2 Determination of the Number of Geothermal Piles

The flow rate of fluid (ν_g) inside the geothermal heat exchanger pipe is an important parameter in terms of the heat absorption rate. A higher flow rate intends to achieve a faster heat exchange rate. However the larger flow rate is also accompanied with a greater cost of electricity consumption from water pump. Therefore, it is important to determine the optimum flow rate. In this section, the impact of the flow rate for different loop arrangements is studied.

From Fig. 2.12 and Fig. 2.13, one can see that the average heat exchange rate between the soil and heat exchanger inside the energy pile increases with flow rate and the rate of average heat flux decreases dramatically after 0.5 m/s and almost reaches a plateau after 1.25 m/s . A higher mass flow rate (ν_g) will lead to a decrease of the T_g^{in} since the traveling time of fluid is reduced in one cycle.

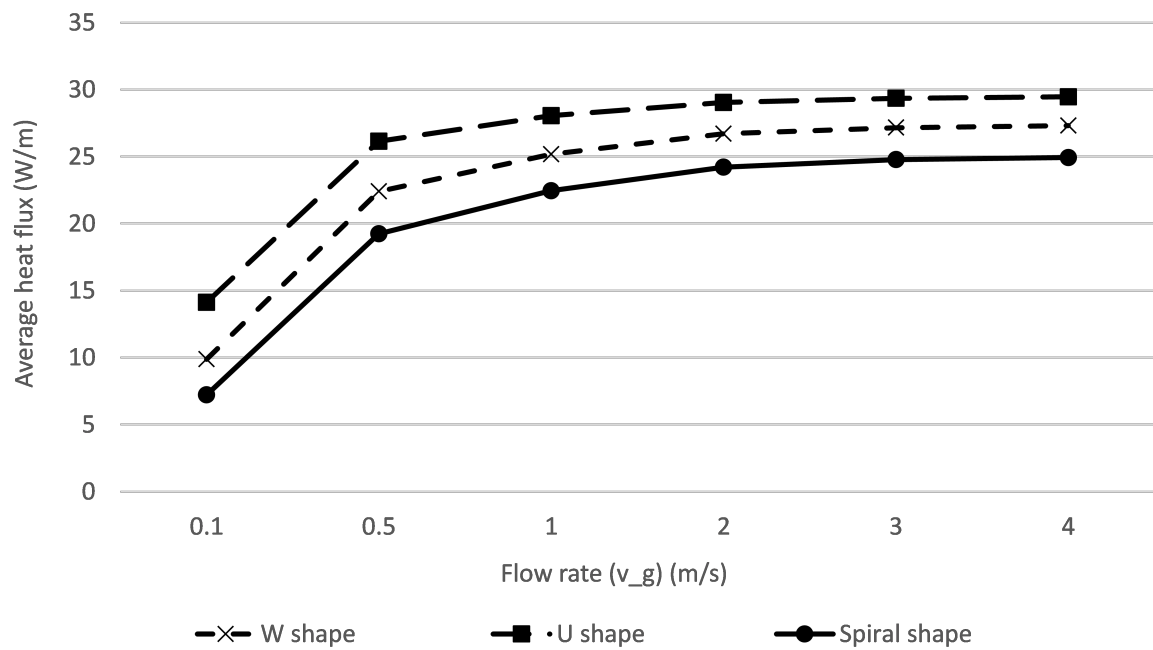


Fig. 2.12: Average heat exchange rate for various flow rate

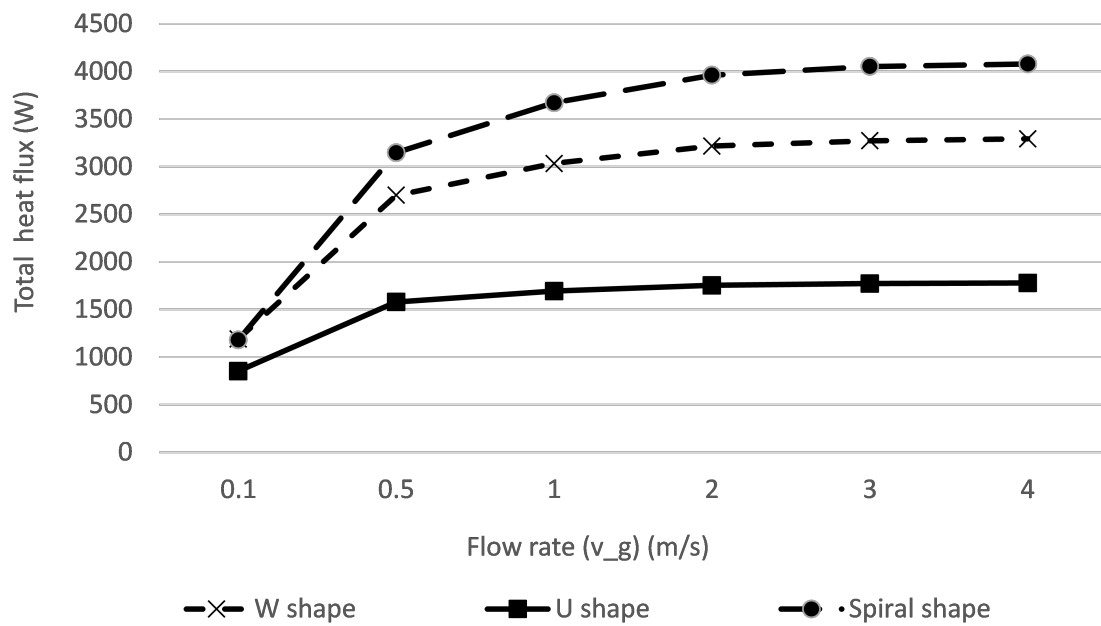
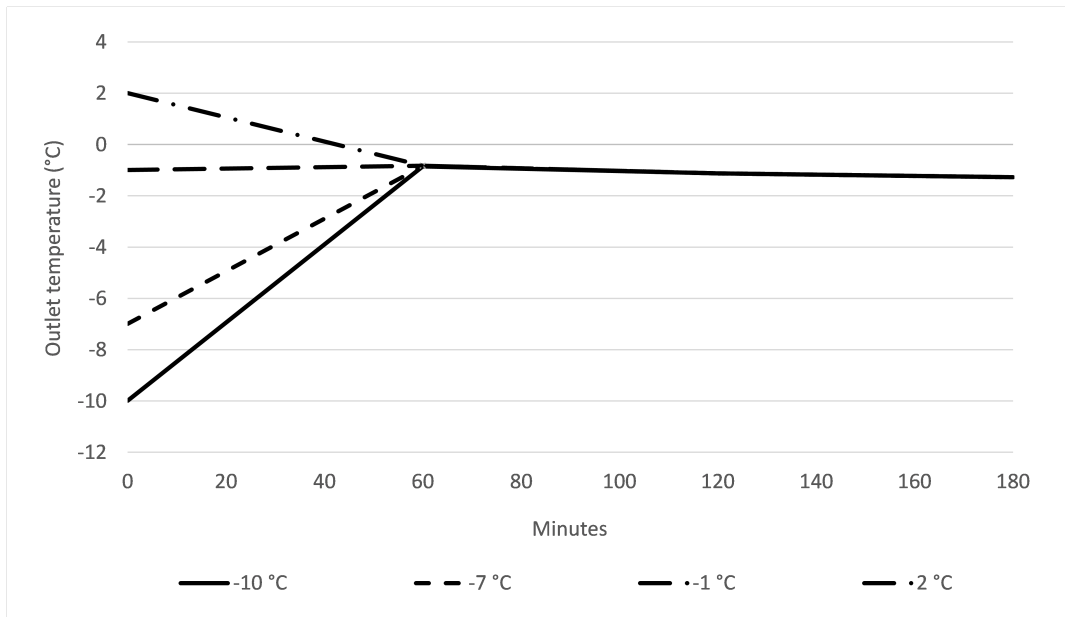
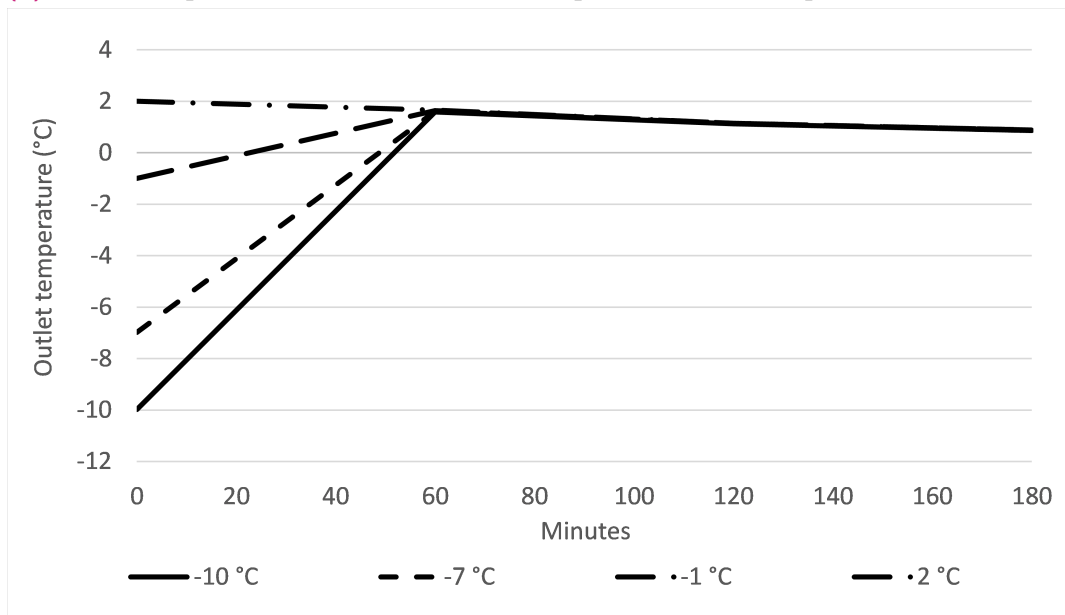


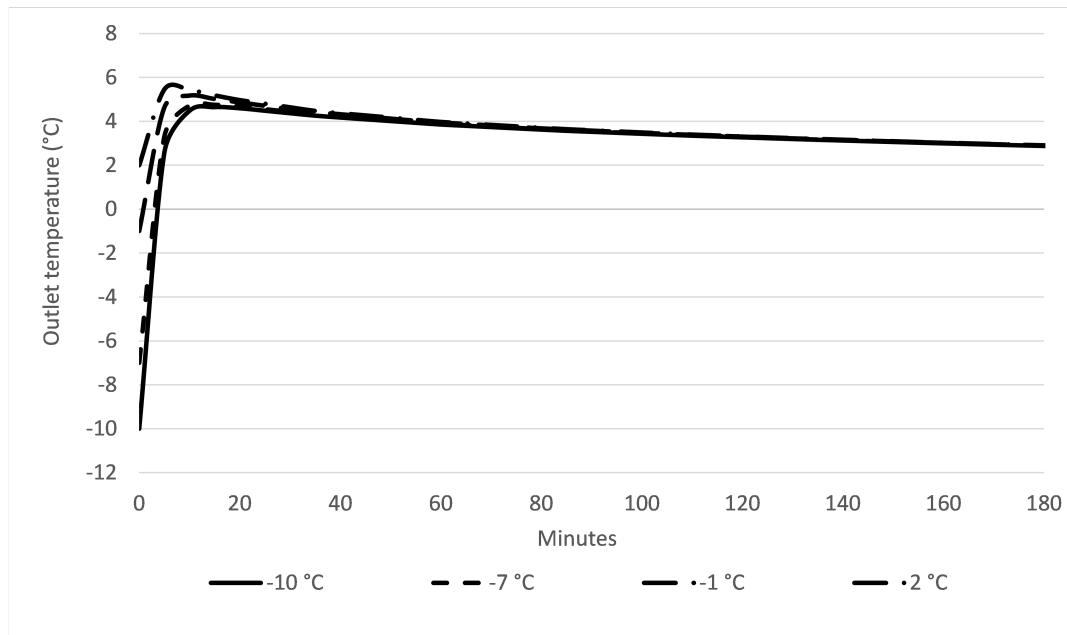
Fig. 2.13: Total heat exchange rate for various flow rate



(a) Outlet temperature with various initial temperatures for U shape



(b) Outlet temperature with various initial temperatures for W shape



(c) Outlet temperature with various initial temperatures for spiral shape

Fig. 2.14: Outlet temperature with various initial temperatures for a) U shape, b) W shape and c) spiral shape

The numerical results show that not only the length of the heat exchanger pipe affects the heat exchange rate, but also the available heat source of the surrounding soil. For example, having 200 *m* of heat exchanger pipe instead of 180 *m* does not improve the efficiency but increases the cost. It can be concluded from Fig. 2.13 that the flow rate of fluid have the most significant impact on spiral loops. The total heat exchange rate by using U shape, W shape and spiral shape is increased by 2 times, 2.7 times and 3.4 times respectively from a flow rate of 0.1 *m/s* to 4 *m/s*. Therefore, the flow rate tends to play a more important role when loop becomes longer.

The influence of the initial temperature of heat exchanger fluid is analyzed by performing a parametric study. Fig. 2.14a to Fig. 2.14c show the outlet temperature distribution with inlet temperature of -4 °C and initial temperatures of -10 °C, -7 °C, -1 °C, and 2 °C.

The results show that the initial temperature of fluid is more important for loops with shorter length and it does not affect the performance of the heat exchanger after one hour.

It should be mentioned that the heat exchange rate decays with the time because of the smaller temperature difference between the soil and fluid as fluid gets warmer with time. When the length of the pipe reaches a certain value, heat transfer will almost stop because of the thermal equilibrium. Consequently, increasing the length of the heat exchanger pipe is not effective anymore. Since the number of pile is predetermined by the structural design, spiral shape is likely to be used for large demands of heat supply.

In order to achieve the desired inlet temperature (T_g^{in}), the length of the heat exchanger pipe inside the pile can be determined. Therefore, the required number of piles integrating the spiral heat exchanger pipe can be simply calculated by:

$$N = \frac{L_{tot}}{180m} \quad (2.17)$$

where N is the required number of piles; L_{tot} (m) is the total length of the heat exchanger pipe inside the piles.

The required number of piles depends on the local ground temperature and the heating load. By using a flow rate (v_g) of 1.25 m/s , the total length of the heat exchanger pipe and the required number of piles for each city can be seen in Table 2.6:

Tab. 2.6: The summary for heat pump in six Canadian cities

City	T_g^r (°C)	T_g^{in} (°C)	T_g^{out} (°C)	L_{tot} (m)	N
Winnipeg	7	6	2	406.5	2.3
Edmonton	5	4	-1	454	2.5
Montreal	7	6	0	493	2.7
Ottawa	7	6	0	493	2.7
Calgary	6	5	0	452	2.5
Toronto	9	8	3	462	2.6

The length of the heat exchanger pipe or the number of pile is largely influenced by the difference between T_g^{in} and T_g^{out} . The higher temperature gradient normally requires longer heat exchanger pipe. Montreal and Ottawa require the longest geothermal heat exchanger pipes due to the temperature difference of 6°C, which in essence is because of the higher heating load in these two areas.

2.7.3 Cost Analysis of the Snow Melting System

In this section, an economic feasibility study is performed to determine the savings of snow melting system using pile-based geothermal energy versus electrical based system.

The initial cost mainly includes the the cost of the heat pump, engineering designs and costs related to the constructions. The annual cost consists of the cost of the electrical consumption of heat pumps, repair and replacement and the cost of system monitoring and verification. The cost related to carbon emission analysis is also performed in this study. Because of its environment-friendly nature, the Canadian government grants and incentives are also provided and they are considered in this analysis.

The cost analysis is performed in Winnipeg area where the average snow melting power is taken as $0.4 \text{ kW} / \text{m}^2$. It is important to realize that the analysis is based on the running time of the system instead of the life service time of bridge. We also assume the installation cost for the hydronic heating system and electrical system is the same. The conceptual COP of the heat pump derived previously (Table 2.5) is 6.9. The bridge dimension is assumed to be $100 \text{ m} \times 8 \text{ m}$; The detailed parameters can be seen in Table 2.7.

Tab. 2.7: Cost analysis based on snow melting system in Winnipeg

Baes case			Proposed case		
Heating system	Electriccal heating		Heating system	Geothermal heat pump	
Heated surface	800	m^2	Coefficient of performance	6.9	
Fuel type	Electricity		Fuel type	Electricity	
Fuel rate (Including taxes)* ¹	0.10	C\$/kWh	Fuel rate (including tax)* ¹	0.10	C\$/kWh
Heating load for bridge deck	0.4	kW/m ²	Heating load using electricity	0.05	kW/m ²
Total heating load	407,314	kWh/year	Total heating load using electricity	59,031	kWh/year
Annual heating cost	40,731	C\$	Annual heating cost	3,025	C\$
Cost analysis for heating with geothermal energy piles					
Initial cost			Annual cost		
Heat pump* ²	1,043	C\$/kW	Fuel cost	5,903	C\$
Total cost for heat pump	42,757	C\$	GHG monitoring/verification* ²	500	C\$
Mechanical design* ²	30,000	C\$	Repair and maintainess	1,000	C\$
Electrical design* ²	30,000	C\$	Miscellaneous	500	C\$
Civil design* ²	30,000	C\$	Total annual cost	7,903	C\$
Construction cost* ²	100,000	C\$			
Miscellaneous	5,000	C\$			
Total initial cost	237,757	C\$			
Emission analysis			Financial parameter		
GHG emission factor* ²	0.04	kgCO ₂ /kWh	Inflation rate * ²	0.02	
GHG reduction	12,190	kgCO ₂ /year	project operation life	30	years
Price of GHG* ²	216	C\$/t	Total initial cost	237,757	C\$
GHG reduction revenue	2,633	C\$/year	Incentives and grants	10,000	C\$
GHG reduction over life time	365,697	kgCO ₂	Simple payback	6.5	years
GHG reduction revenue for life time	78,991	C\$	Total saving	747,093	C\$
* ¹ : data is from hydro-Québec who performed the comparative analysis of utility price on April 1, 2017					
* ² : The recommended data is taken from Retcreen					

In this calculation, it is assumed that both snow-melting systems operate for 60 days per year (snowfall time). It is concluded that the simple pay back time for the geothermal pile-based snow melting system in comparison with the electrical heating system is approximately 6.5 years in Winnipeg. This design is expected to save around 747,000 C\$ (935 C\$/m²) by the end of 30 years of operation, as can be seen in Table 2.7.

However the relative saving varies with the utility price, the value of COP and the heating load in each city. Table 2.8 summarizes the relative savings by using the geothermal energy in comparison to using the electrical heating system for the six cities in Canada.

Tab. 2.8: Relative savings for six major cities in Canada

City	Price of electricity (C\$/kW · h)	COP	Heating load (kW/m ²)	Relative saving (C\$)
Winnipeg	0.1	6.9	0.4	747,093
Edmonton	0.11	3.7	0.5	1,083,029
Montreal	0.08	4.8	0.6	867,624
Ottawa	0.17	3.3	0.6	2,077,635
Calgary	0.11	4.7	0.4	1,001,793
Toronto	0.19	5.1	0.5	2,193,019

The relative saving is a function of the local price of the electricity, the value of COP and the local heating load. Renewable energy such as geothermal energy is economically more efficient in cities that have higher price of electricity and higher heating load. Also, a higher value of COP will reduce the cost of the system by reducing the electricity consumption. Toronto has the largest relative saving due to its higher price of electricity and higher COP value. On the other hand, Winnipeg is estimated to have the smallest relative saving because of the lower electricity price and lower heating load.

2.8 Conclusions

In this study, we developed a transient snow melting model which evaluates the heat flux required for instantaneous snow melting based on typical Canadian weather. By performing a 3D transient heat transfer analysis, we evaluated the required energy consumption (q_H) to achieve a desired snow melting performance for the bridge slab and the energy input (q_L) provided by geothermal energy piles. Some important conclusions can be drawn as follows:

- The COP of the heat pump, as the indicator of the energy efficiency, depends on the different weather conditions which determine the heating load. In terms of COP, Winnipeg and Toronto are the most suitable places for the geothermal energy pile based snow melting system among the six cities studied in this paper.
- Despite of different values of COP in these six cities, the number of piles required for snow melting is close because of the similar ground temperature and thermal conductivity in these cities. Montreal and Ottawa are expected to have a longer length of geothermal heat exchanger pipe because of their relatively higher heating load.
- Snow melting using the geothermal energy pile system is much more cost effective than electrical-based systems. However, the relative savings vary with local price of electricity, the value of COP and the local heating load. Toronto is predicted to have the largest saving due to the higher price of electricity and higher COP value.

Optimum Insulation Design for Buried Utilities subject to Frost Actions in Cold Resins Using Nelder-Mead Algorithm

Abstract

Damage due to frost actions in soils can be detrimental to buried utilities such as water and gas pipelines. One of the promising approaches to protect buried utilities against frost damage and reduce the excavation cost is to install thermal insulation over and around the pipe. This study considers two different conductive heat transfer models, with and without the effect of phase change, as well as a heat and mass transfer model for freezing soils to investigate the effectiveness of various insulation systems for the pipe frost protection. A parametric study was carried out to assess the effects of different factors such as backfill materials, insulation geometry and thickness, burial depth of pipe on the thermal performance of insulation. Finally, the Nelder-Mead algorithm is implemented to determine the optimum insulation design so that the overall cost is minimized while the pipe is protected from freezing. It was concluded that

- The latent heat released during the phase change can significantly affect the heat transfer process and decrease the freezing rate of the soil. Neglecting the phase change effects will eventually lead to an erroneous design of the insulation.
- The mass transfer mechanism is negligible in frost-susceptible foundation soil for predicting the distribution of temperature and unfrozen water content.

- Using granular soil (sand) as backfill is more critical than clay due to the higher thermal conductivity of sand.
- The cylindrical shape insulation is not recommended. The heat loss mainly occurs at the top pipe. Therefore, the insulation located at the bottom water pipe does not effectively reduce the heat loss.
- The optimum design should consider the effect of insulation length, distance between water pipe and insulation, pipe burial depth and insulation thickness.
- The Nelder-Mead algorithm is efficient and effective to determine the optimum burial depth and the designing insulation parameters.

3.1 Introduction

Buried utilities such as conduits and pipelines are a main part of infrastructure in urban areas. In cold regions, the effect of frost actions on the structural integrity should be addressed appropriately in the design process of such utilities. Typically, buried pipelines must be located below the frost depth to avoid frost-induced damage due to differential heave, or freezing of water inside the pipe.

A deeper depth of the frost penetration in Canada is accompanied with increasing the construction cost of buried utilities. Despite all the efforts in design and operation to prevent buried pipes from the frost damage, pipe failure due to the harsh climate and soil deformations currently costs the City of Winnipeg (MB), Canada, over \$8 million annually. The three essential factors for detrimental frost actions are : a) subfreezing temperature, b) a continual supply of water (a high water table), and c) presence of frost-susceptible soils. Frost-susceptible soils, e.g. silty soils, have moderate hydraulic conductivity due to their pore size distribution, which promotes capillary flow. In soils with high hydraulic conductivity, e.g., coarse-grained soils, water cannot rise by capillarity to the freezing front

while in clayey soils with low hydraulic conductivity, not enough water will get to the freezing front to produce significant heaving.

If the soil temperature drops below the freezing point of the pore-water, some of the water freezes and releases the latent heat. In the beginning, freezing of pore-water only produces 9% of the volume of the amount of water already in the soil. As the pore-water continues freezing, the amount of water remaining in the liquid phase reduces and negative pore-water pressures (or suctions) develop in frost-susceptible soils. It has been assumed that the freezing process in a frozen soil is similar to the drying process in an unsaturated soil. In response to the pressure and temperature gradient, water migrates in the unfrozen soil towards the freezing front where it freezes and releases the latent heat. The advance of the frost front continues until the heat sink (i.e., cold surface air) and frozen ground can no longer remove the latent energy released by the water freezing at the frost front. Ice lenses are formed when enough water migrates to the freezing front and it causes excessive deformation (heave) in frost-susceptible soils.

Most parts of Manitoba, with freezing indices ranging from 1550 to 3800 ($^{\circ}\text{C}$ -days) and frost depth as deep as 3 m, are susceptible to frost actions, surface heave and eventually distress of the structures.

A practical way to control the frost action in a soil is to prevent the penetration of the freezing front through using thermal insulation. The low thermal conductivity of insulating materials controls the heat transfer so that the heat is conserved in the soil during winter and the formation of ice lenses is halted.

The current practice to design the thickness and width of insulation materials used to protect buried pipes from freezing is based on recommendations provided by manufacturers. In these recommendations, the thickness of insulation is determined by knowing the amount of backfill to be used on top of the insulation and design freezing index ($^{\circ}\text{C}$ -days), which depends on location of the project. Also, the total insulation width is calculated by knowing

the pipe diameter, design frost depth as well as the insulation depth [Tab60]. In these recommendations, the effects of pore-water phase change and water movement toward the frozen fringe on the geometry of insulation materials are not considered explicitly.

The optimum design of the rigid plastic foam insulation for the frost protection of buried pipes is the main focus of this study. For this purpose, a transient heat and mass transfer model for freezing soil as well as two transient conductive heat transfer models, with and without considering the pore-water phase change, are implemented to study the thermal regime within the soil during freezing and thawing periods. Results of these analyses are compared in order to highlight the importance of the water movement as well as the pore-water phase change in the heat transfer process in the soil. A transient energy balance at the ground surface is used by considering inputs such as ambient temperature and wind speed as provided by Environment Canada for typical climate conditions of Winnipeg, Manitoba. Then, the effect of mass transfer on the thermo-hydraulic regime of frost-susceptible foundation is evaluated by comparing the numerical models with field measurement data of two sections on the provincial trunk highway 11 (PTH11) and provincial road 304 (PR304) near Pine Falls in Manitoba, Canada. Also, the performance of rigid plastic foam insulation with various backfill materials, insulation geometry, total length, and thickness, distance between pipe and insulation as well as the pipe burial depth is studied. Finally, the advanced Nelder-Mead algorithm is implemented to determine the optimum insulation design based on insulation and local construction costs.

3.2 Model Description

3.2.1 Conductive Heat Transfer Without Considering Phase Change (H)

The governing equation for the transient conductive heat transfer, i.e. by assuming no water movement associated with the heat transfer process, without considering the pore-water phase change in a soil can be written as follows:

$$\rho^n C_p^n \frac{\partial T}{\partial t} = \nabla \cdot (k^n \nabla T) + Q \quad (3.1)$$

where ρ^n (kg/m^3) is the bulk density of soil; C_p^n ($J/(kg \cdot K)$) is the specific heat capacity of soil at constant pressure; T (K) is the temperature of soil; k^n ($W/(m \cdot K)$) is the thermal conductivity of soil; and Q (W/m^3) is a heat source.

The density of a soil (ρ^n) can be obtained as follows:

$$\rho^n = \rho_w \theta_w + \rho_s (1 - \theta_s) \quad (3.2)$$

where ρ_w (kg/m^3) and ρ_s (kg/m^3) are the density of water and soil skeleton, respectively; θ_w and θ_s are the volumetric water content and the porosity (saturated volumetric water content) of the soil.

Similarly, the specific heat capacity of a soil (C_p^n) is defined by:

$$C_p^n = \theta_w C_{pw} + (1 - \theta_s) C_{ps} \quad (3.3)$$

where C_{pw} ($J/(kg \cdot K)$) and C_{ps} ($J/(kg \cdot K)$) are the specific heat capacity of water and solid particles, respectively.

Also, the thermal conductivity (k) can be calculated as follows:

$$k^n = \theta_w k_w + (1 - \theta_s) k_s \quad (3.4)$$

where k_w ($W/(m \cdot K)$) and k_s ($W/(m \cdot K)$) are the thermal conductivity of water and the solid particles, respectively.

3.2.2 Conductive Heat Transfer Considering Phase Change (H-PhC)

The governing equation for the transient conductive heat transfer by considering the effects of pore-water phase change in a soil can be described as follows:

$$C_{app}^{ph} \frac{\partial T}{\partial t} = \nabla \cdot (q_T) + Q \quad (3.5)$$

where $C_{app}^{ph} = \rho^{ph} C_{ph} + L_f \rho_w \frac{\partial \theta_w}{\partial T}$ is the apparent volumetric heat capacity; L_f (333 kJ/kg) is the latent heat; $\theta_i = \theta_s - \theta_w$ is the volumetric ice content; θ_w is the unfrozen volumetric water content; ρ_i (kg/m^3) is the density of ice; the superscript ph stands for the properties affected by the pore-water phase change; and q_T ($W \cdot m^{-2}$) is the conduction flux through the volume boundaries and, following the Fourier's law, it can be written as:

$$q_T = k_T^{ph} \nabla T \quad (3.6)$$

in which k_T^{ph} ($W/(m \cdot K)$) is the total thermal conductivity of the soil:

$$k_T^{ph} = \theta_w k_w + \theta_i k_i + (1 - \theta_s) k_s \quad (3.7)$$

where k_i ($W/(m \cdot K)$) is the thermal conductivity of ice.

An empirical relationship is used to relate the unfrozen water content (θ_w) to the temperature [MZ06]:

$$\theta_w = \theta_r + (\theta_{wo} - \theta_r) e^{a(T-T_0)} \quad (3.8)$$

where θ_r is the residual volumetric water content; a is the curve fitting parameter; T_0 is the freezing temperature, considered as 0°C .

By considering the effect of phase change, the pore-water can exist in both liquid and solid states, simultaneously. Therefore, the density of a soil (ρ^{ph}) can be calculated as follows:

$$\rho^{ph} = \rho_w \theta_w + \rho_i \theta_i + \rho_s (1 - \theta_s) \quad (3.9)$$

Similarly, the specific heat capacity of a soil (C_p^{ph}) is defined by:

$$C_p^{ph} = \theta_w C_{pw} + \theta_i C_{pi} + \rho_s (1 - \theta_s) C_{ps} \quad (3.10)$$

where C_{pi} ($\text{J}/(\text{kg} \cdot \text{K})$) is the specific heat capacity of ice.

3.2.3 Heat and Mass Transfer in Unsaturated Freezing Soils (H&M-PhC)

To derive the governing equations for heat and mass transfer in unsaturated freezing soils, it is assumed that:

- No deformation of the solid matrix occurs during freezing. Therefore, the density of water ρ_w and ice ρ_i must be considered equal.
- The water is chemically pure, i.e. no simultaneous solute transport occurs.
- In frozen soil, the water flow is assumed analogous to that in unsaturated soil. Therefore, the relationships for matric potential and hydraulic conductivity of unsaturated soils are assumed valid for freezing soil.

- The *freezing = drying* assumption is considered [Mil65]. This implies that:
 - Both air and ice phases are assumed to be at atmospheric pressure.
 - The water content θ_w and ice content θ_i in the soil may be related to the soil matric potential according to the soil water characteristic curve (SWCC) used in the mechanics of unsaturated soils. The latter will be obtained by combining a generalized form of the Clapeyron equation with the SWCC and the thermodynamic equilibrium.

The soil water characteristic curve (SWCC) can be defined based on the properties of soil and some empirical values. In this study, the Van Genuchten SWCC model is used [Van80]. Therefore, the total water content (liquid and ice), Θ , as well as the liquid water content, θ_w , can be written as follows:

$$\Theta = F_{VG}(\Psi_{w0}) = \theta_r + (\theta_s - \theta_r) \left(1 + |\alpha \Psi_{w0}|^n \right)^{-m} \quad (3.11)$$

$$\theta_w = F_{VG}(\Psi_{w1}) = \theta_r + (\theta_s - \theta_r) \left(1 + |\alpha \Psi_{w1}|^n \right)^{-m} \quad (3.12)$$

where $\Psi_{w0}(m)$ is the matric potential corresponding to the total water content (liquid and ice); $\Psi_{w1}(m)$ is the matric potential corresponding to the liquid water content; α ($1/m$), n ($-$) and m ($-$) are the curve fitting parameters.

The Clausius-Clapeyron relation can be used to characterize the relation between the matric potential and temperature during phase change. Amico et al. derived a generalized Clausius-Clapeyron relation by assuming that both air and ice phases are at atmospheric pressure (*freezing = drying* assumption) as follows [Dal+11]:

$$\Psi_{w1}(T) = \Psi_{w0} + \frac{L_f}{g T^*} H(T^* - T) \quad (3.13)$$

where g (9.81 m/s^2) is the gravitational acceleration constant; $H()$ is the Heaviside function; T^* ($^{\circ}\text{K}$) is the water melting temperature, which is a function of the melting temperature at atmospheric pressure $T_m = 273.15\text{K}$ and the matric potential:

$$T^* = T_m + \frac{g T_m}{L_f} \Psi_{w0} \quad (3.14)$$

The change of total volumetric water content (unfrozen water content and ice content) can be triggered by water movement when the metric suction of freezing soil varies with temperature. The mass conservation is derived by considering the phase change and the water flux, as follows:

$$\frac{\partial \Theta}{\partial t} + \nabla \cdot (q_w) + S = 0 \quad (3.15)$$

where $\Theta = \theta_w + (\rho_i / \rho_w) \theta_i$ (-) is the total volumetric water content; S ($1/s$) is the term due to evaporation; and q_w is the water flux within the soil following the Darcy-Buckingham formulation which can be written as

$$q_w = -K_H \nabla (\Psi_{w1} + z) \quad (3.16)$$

K_H (m/s) is the hydraulic conductivity and z (m) is the elevation head. The hydraulic conductivity (K_H) can be described as:

$$K = K_{sat} f(T) S_r^{\frac{1}{2}} \left(1 - \left(1 - S_r^{\frac{1}{m}} \right)^m \right)^2 \quad (3.17)$$

where K_{sat} (m/s) is the saturated hydraulic conductivity and $S_r = (\theta_w - \theta_r) / (\theta_s - \theta_r)$ is the degree of saturation.

Energy conservation is derived by considering phase change, conduction and convection of heat as follows:

$$C_{app}^{ph} \frac{\partial T}{\partial t} = \nabla \cdot (q_T + q_{conv}) \quad (3.18)$$

where q_{conv} ($W.m^{-2}$) is the convection heat flux given by the heat transported by water flow:

$$q_{conv} = \rho_w \left(L_f + C_{pw} (T - T_{ref}) \right) q_w \quad (3.19)$$

where $T_{ref} = 273.15K$ is the reference temperature.

The system of equations to be solved then consists of the mass conservation (Eq. 3.15), energy conservation (Eq. 3.18), and a set of boundary conditions assigned to the boundaries of the domain. This results in a system of coupled and highly non-linear differential equations based on Ψ_{w1} and temperature T as state variables. The secondary variables, e.g., Ψ_{w0} , T^* , and $\Theta = f(\Psi_{w0})$ will be obtained and updated at each time step.

The systems of governing partial differential equations (PDEs) for three different models explained above (H, H-PhC, H&M-PhC) are implemented and solved in the finite element model builder COMSOL Multiphysics. A Galerkin weighted-residual approach is employed to minimize the resulting residuals.

3.2.4 Numerical Linearization Technique

As the system of equations in H&M-PhC are highly non-linear due to the discontinuity of the physical and thermal properties near the freezing fringe, a smoothed Heaviside step function, $f(T) = H(T - T^*, \Delta T)$, is used to facilitate the transition of the phase change around the melting point. So, Ψ_{w0} in Eq. 3.13 can be smoothed as follows:

$$\Psi_{w0}(T) = \Psi_{w1} - \frac{L_f}{g T^*} H(T^* - T, \Delta T) \quad (3.20)$$

This strategy will help a smooth transition of the phase change around the melting point in physical and thermal properties, which are functions of Ψ_{w0} .

Another source of non-linearity in the numerical simulation, $\partial\theta_w/\partial T$ in C_{app}^{ph} , is treated as follows:

$$L_f \rho_w \frac{\partial\theta_w}{\partial T} = L_f \rho_w \frac{df(T)}{dT} a \quad (3.21)$$

The function $df(T)/dT$ is used to approximate the unfrozen water content variation around the freezing point. The size of the transition zone, ΔT , should be chosen in a way to avoid the convergence difficulties. It is realized that ΔT determines the slope of $f(T)/dT$ in Eq. 3.21. In order to represent the water content variation with respect to the temperature and avoid convergence problems, a correction factor, a , is required.

In this study, the best fitted values for the phase change transition size, ΔT , and the correction factor, a , are determined by try and error, as $0.04K$ and 0.5 , respectively (Fig. 3.1).

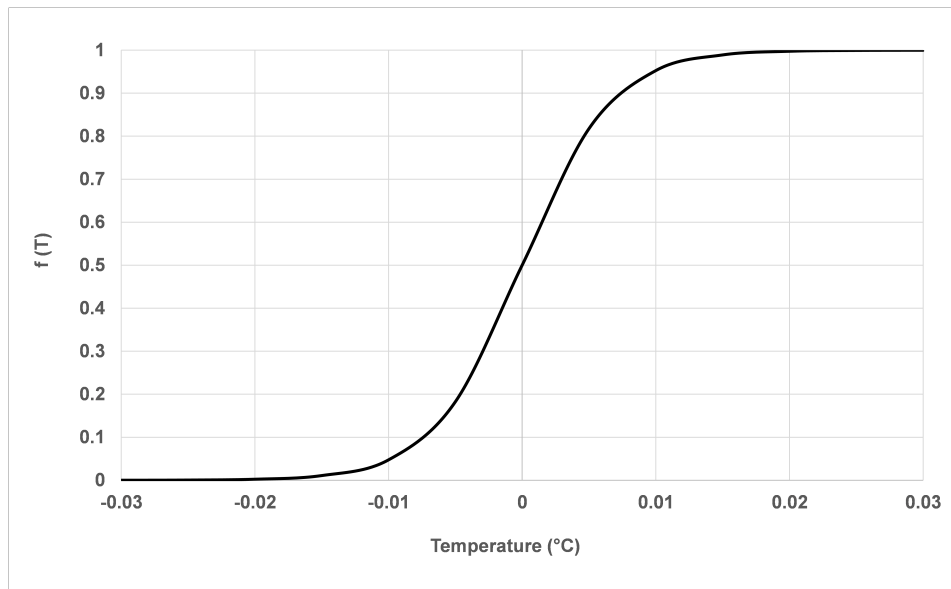


Fig. 3.1: Transitional function $f(T)$

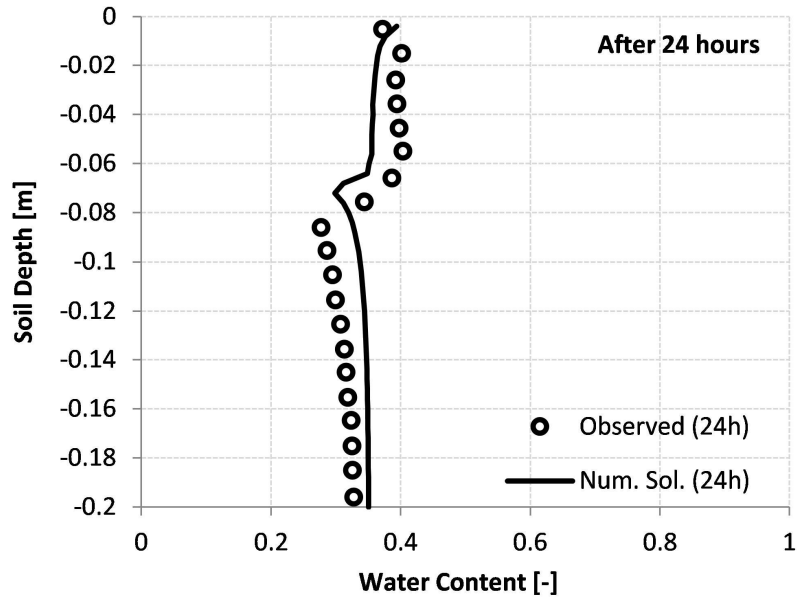
3.2.5 Model Validation and Verification

The above-mentioned models are validated by comparison with analytic solutions for H and H-PhC. Also, the H&M-PhC model is verified by comparison with experimental data for a soil column which was forced to freeze from the top [Han+04]. For the sake of brevity, only the latter is presented here.

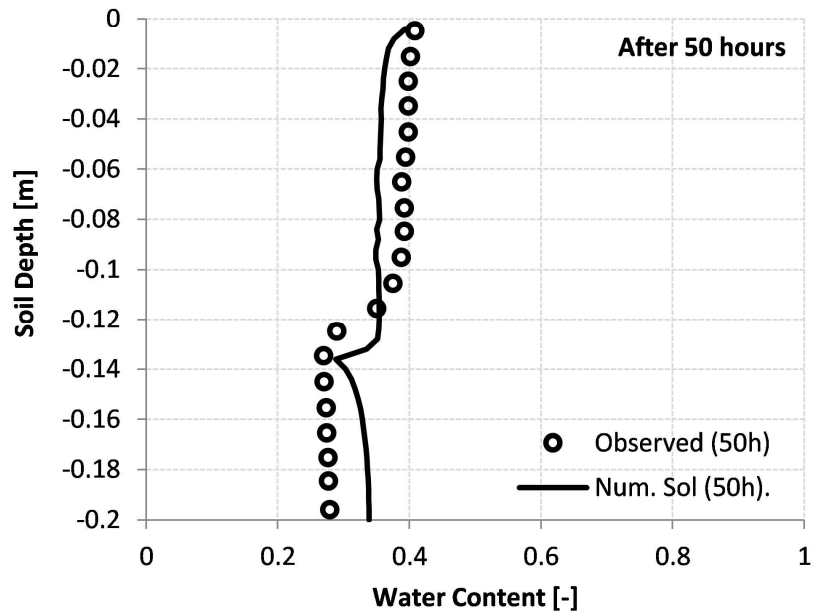
The soil considered in this analysis represents a Kanagawa sandy loam, with the following parameters: $\theta_s = 0.535$, $\theta_r = 0.05$, $\alpha = 1.11(m^{-1})$, $n = 1.48$, $\lambda_{gs} = 2.5(W/(m \cdot K))$, $C_{gs} =$

$2.3 \times 10^6 (J/(m^3 \cdot K))$, and $K_{sat} = 3.2 \times 10^{-6} (m/s)$. The column was initially unfrozen, with a uniform total water content $\theta_w = 0.33$ and a uniform initial temperature distribution at $T = 6.7^\circ C$. The boundary condition considered at the top surface is of Neumann type, which is described as: $q_T = -28(T_{top} + 6)$, where T_{top} is the temperature at the top surface. The boundary condition at the bottom surface is considered adiabatic, i.e. no heat flux. For the mass balance equation, a zero flux was applied at both top and bottom boundaries.

As can be seen in Fig. 3.2, the numerical results have a good agreement with the experimental test results. It should be noted that the jump in the volumetric water content profile happens where the phase change occurs. During the freezing process, the unfrozen water content dramatically decreases and ice content significantly increases. The freezing, similar to drying, induces the migration of free pore-water to the freezing fringe. The volumetric water content below the freezing fringe is still remain unfrozen.



(a) Total water (liquid plus ice) after 24 hours



(b) Total water (liquid plus ice) after 50 hours

Fig. 3.2: Comparison between the numerical (solid line) and the experimental results (points) obtained by Hansson et al. (2004) for the total water (liquid plus ice) after 24 hours (a) and 50 hours (b)

3.3 Effect of Mass Transfer on Thermo-Hydraulic Profile of Frost-Susceptible Foundation Soils

The moisture regime in pavement has a great impact on the performance and serviceability of pavement structures. The excess moisture content in the subgrade materials weakens the internal resistance to the shear, which largely reduce the soil bearing capacity [Zap+07]. The moisture condition of pavement mainly depends on the pavement materials, precipitation and phase change of the pore-water of the soil [Oli78]. There are several factors contributing to the moisture change in pavement system. As the capillary force existing in unsaturated soil suck water up from the groundwater table zone, the moisture content could be redistributed [Haz30]. Infiltration of water due to precipitation is one of the main sources of water in the pavement system. The rainfall or melted water from snowfall are likely to penetrate into pavement through cracks and unprotected gravel surface [JC84]. In addition, the pavement drainage system could strongly affect the moisture condition of pavement system. Another important source of water in the pavement system is the frost action. As the air temperature drops, the temperature of the ground surface decreases while the temperature at depth remains relatively warm and constant during the whole seasons. Consequently, a thermal gradient develops in the ground. As the soil temperature drops below the freezing point, some of the pore water freezes in soil. This freezing effect could induce negative pore water pressure, which generates the pressure gradient at various depth of soil. The free water from surrounding voids migrates to freezing fringe due to the generated pressure gradient. The ice lenses then grow until the water ceases to move into the frozen fringe [Zha14].

This section aims to study the effect of mass transfer on the distribution of temperature and moisture in the frost-susceptible foundation (subgrade) of pavement structure in cold regions. For this purpose, two different analyses including H-PhC and H&M-PhC are carried out to predict the thermo-hydraulic profile of the pavement structure and compared with

field measurement data. Then, the most suitable numerical model is selected to investigate the effectiveness of various insulation systems for the frost protection of buried pipes resting on frost-susceptible soils.

3.3.1 Field Monitoring Program

Project Location

The distributions of temperature and moisture content of pavement structure at different depths were monitored during 2008 and 2009 for two test sections on the provincial trunk highway 11 (PTH11) and provincial road 304 (PR304) near Pine Falls in Manitoba, Canada [Qin11]. The test section 1 at PTH11 was located 2.70 km South-East of the intersection of PR304 and PTH11 and the test section 2 was located 2.40 km North-East of the intersection of PR304 and PTH11. Figure 3.3 shows the location of these sites.

Road Structure

The pavement structure at PTH11 is comprised of an asphalt concrete (130 mm), a granular base (250 mm) underlain by a highly plastic clay subgrade. The test section at PR304 has an asphalt surface treatment (30 mm) over an aggregate base (200 mm) followed by a 500 mm thick rockfill layer underlain by a sandy clay subgrade.



Fig. 3.3: Locations of test sites at PTH11 and PR304.

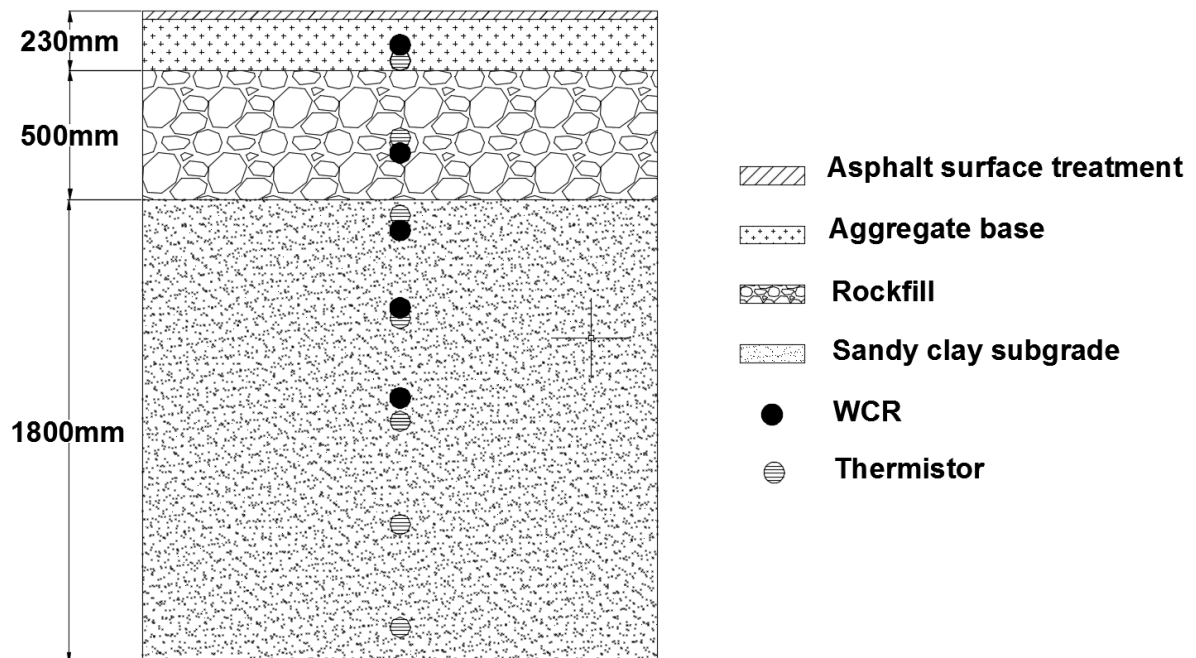


Fig. 3.4: Cross-sectional profile of the pavement structure and the locations of thermistors and WCR at PR304

Moisture Content and Temperature Instrumentation

The overall behavior of flexible pavements are highly influenced by the variation of moisture content within different layers of pavement. For this project, the Campbell Scientific Water Content Reflectometer (WCR) probe was selected to monitor moisture content of the various pavement layers. Five WCRs were installed at each site. At test section PR304, the sensors were located at depths 130 *mm*, 550 *mm*, 850 *mm*, 1150 *mm*, 1500 *mm* from the pavement surface while at PTH11, the sensors were located at 200 *mm*, 510 *mm*, 830 *mm*, 1170 *mm*, 1520 *mm* depth from the surface.

The daily temperature, which is the average of the 12 readings for each day, was monitored by using Campbell Scientific thermistors installed at depths 191 *mm*, 491 *mm*, 791 *mm*, 1191 *mm*, 1591 *mm*, 1991 *mm*, and 2391 *mm* at PR304, and 200 *mm*, 500 *mm*, 800 *mm*, 1200 *mm*, 1600 *mm*, 2000 *mm*, 2400 *mm* at PTH11.

The cross-sectional layout of the pavement structure and the location of thermistors and WCR at test section PR304 are illustrated in Fig. 3.4.

3.3.2 Numerical Modeling

As mentioned earlier, two models (H-PhC and H&M-PhC) are considered to evaluate the distribution of temperature and water content in the pavement structure. The goal is to evaluate the importance of mass (water) transfer in the thermo-hydraulic profile of the pavement structure.

The first model, i.e. H-PhC, assumes no convective heat transfer. Then, no water movement occurs during the phase change process to the freezing fringe. The soil is assumed fully saturated ($\theta_w = \theta_s$). The second model, i.e. H&M-PhC, assumes the soil subgrade in the pavement structure to be unsaturated initially with a volumetric water content changing from 0.2 to 0.38 at different depths and a constant porosity of 0.5. The saturated hydraulic conductivity is $3.2 \times 10^{-6} \text{ m/s}$. The typical values of clay are used in Van Genuchten model

to describe the relationship between unfrozen volumetric water content and matric suction ($\alpha = 1.11(m^{-1})$, $n = 1.48$, $m = 1 - 1/n$). The other material properties are presented in Table 3.1.

Tab. 3.1: Pavement material properties

Material	Density (kg/m^3)	Thermal conductivity ($W/(m \cdot K)$)	Heat capacity ($J/(kg \cdot K)$)
Clay	2,650	2.5	868
Water	1,000	0.6	4,187
Ice	1,000	2.2	2,093
Rockfill	2,000	1.3	1,000

Initial and Boundary Conditions

Initial conditions are based on the temperature and moisture measurements on November 31st, 2008. The initial temperature of pavement varies from 0°C to 10 °C. The initial volumetric water content varies from 20% to 38% at different depths.

A forced convection is considered as boundary conditions at the pavement surface. The convection term, q_{conv} , is temperature- and time-dependent and can be calculated using the following equation:

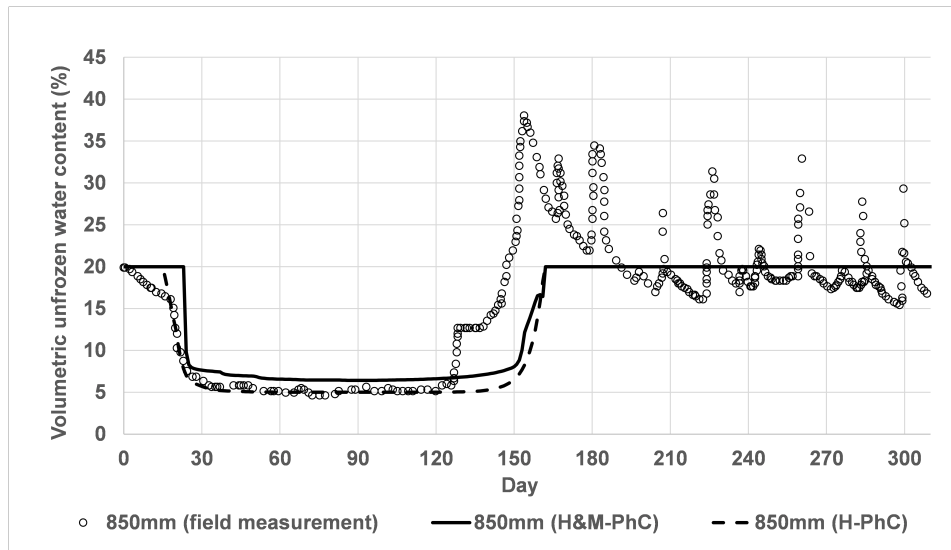
$$q_{conv}(t) = h_{conv}(T_a(t) - T_s(t)) \quad (3.22)$$

where $T_a(^{\circ}C)$ is the ambient air temperature and $T_s(^{\circ}C)$ is the temperature at the pavement surface.

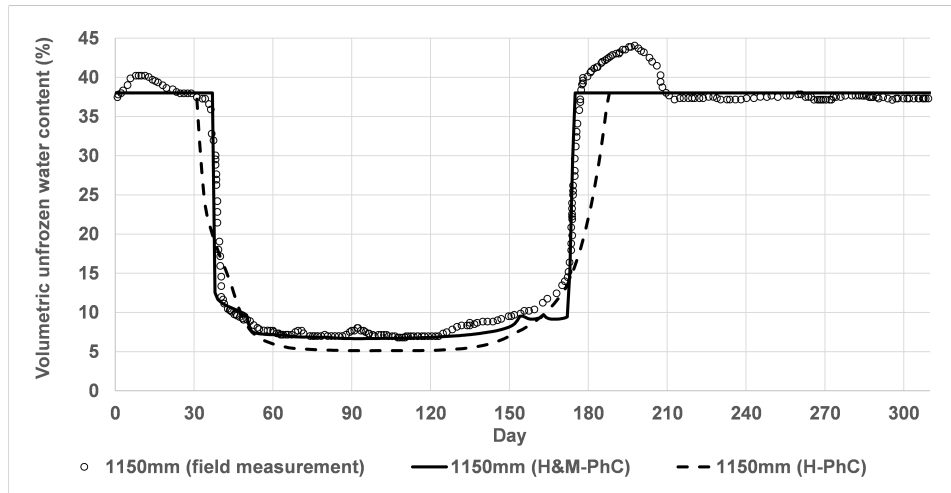
The convection heat transfer mode is comprised of two mechanism: diffusion and advection [I+]. The calculation of convection heat transfer coefficient, h_{conv} , can be referred to Chapter 2.

3.3.3 Comparison between Field Data and Numerical Results

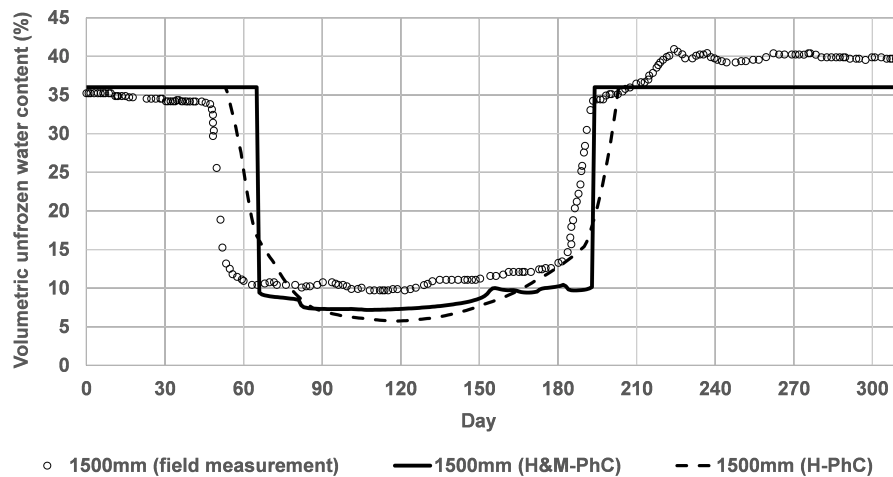
For the sake of brevity, only the comparison between the numerical results and field data measured at PR304 are presented hereafter.



(a) At 850 mm below the pavement surface



(b) At 1150 mm below the pavement surface



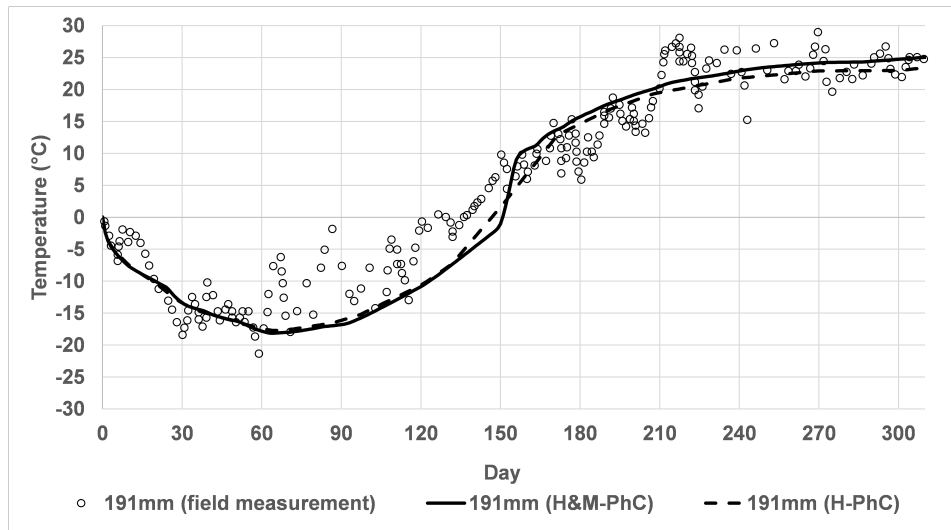
(c) At 1500 mm below the pavement surface

Fig. 3.5: Moisture distribution obtained by numerical simulations, H-PhC (dashed line) and H&M-PhC (solid line), and measured in the field (points) at 850 mm (a), 1150 mm (b), and 1500 mm (c) below the pavement surface

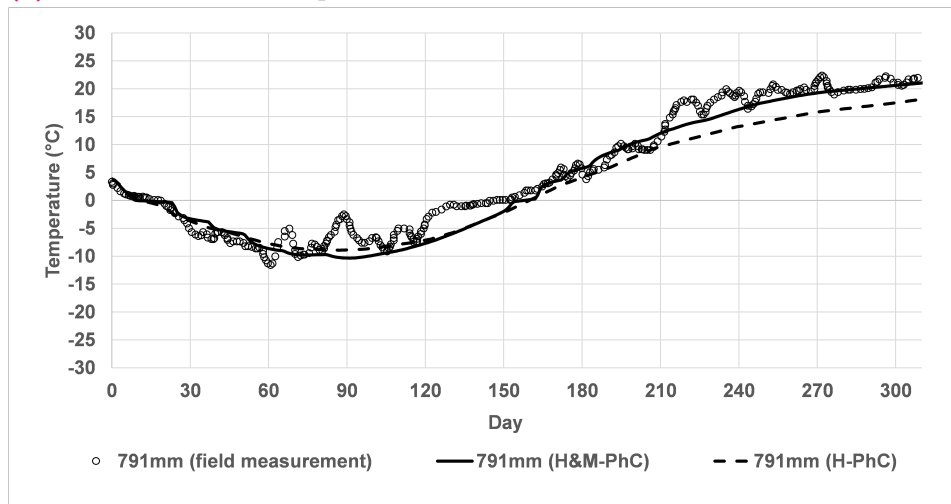
Moisture Content

The distribution of unfrozen volumetric water content in the pavement structure measured in the field and obtained by numerical simulations (H-PhC and H&M-PhC) is illustrated in Fig. 3.5.

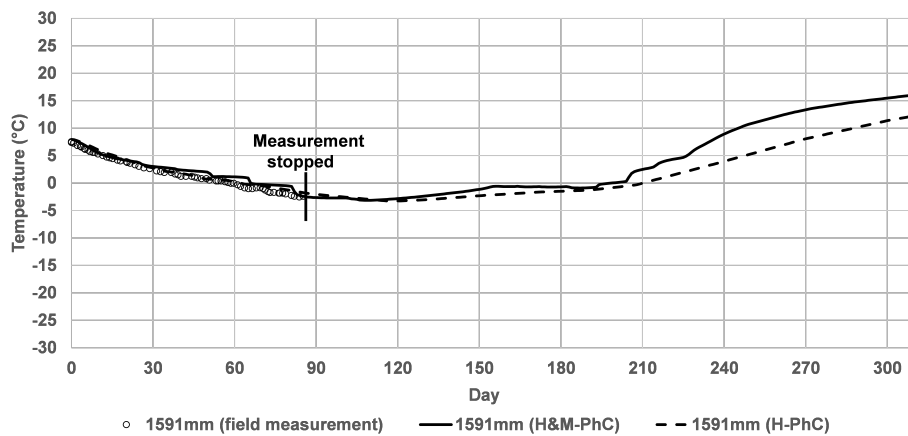
As can be seen in Fig. 3.5, the unfrozen volumetric water content of subgrade soils decreases dramatically to the residual water content during the the winter time. The fluctuation in the distribution of unfrozen water content, especially in a shallower depth, can be caused by various factors such as a poor drainage system, penetration of rainfall into the pavement structure, or even the sensitivity of measurement instrumentation. In this analysis, it is assumed that the drainage system works ideally and no water flows into the subgrade soil. Both H-PhC and H&M-PhC models accurately predict the unfrozen volumetric water content in the pavement structure and represent the general trend of the moisture distribution.



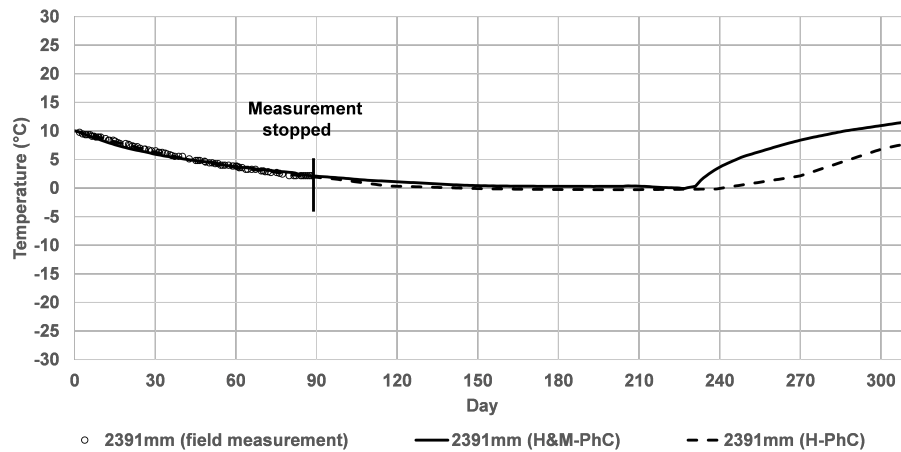
(a) At 191 mm below the pavement surface



(b) At 791 mm below the pavement surface



(c) At 1591 mm below the pavement surface



(d) At 2391 mm below the pavement surface

Fig. 3.6: Temperature distribution obtained by numerical simulations, H-PhC (dashed line) and H&M-PhC (solid line), and measured in the field (points) at 191 mm (a), 791 mm (b), 1591 mm (c), and 2391 mm (d) below the pavement surface

Temperature

Similarly, the temperature distribution at various depth in the pavement structure measured in the field and obtained by numerical simulations (H-PhC and H&M-PhC) is illustrated in Fig. 3.6.

Figs. 3.6a to 3.6b show the temperature variation in the granular backfill (granular base and rockfill) while Figs. 3.6c to 3.6d represent the temperature variation in the subgrade soils. It can be seen that the pavement temperature at 191mm varies dramatically, whereas the temperature at 2391 mm remains relatively stable during the testing period. The amplitude of the fluctuations for temperature distribution is dampened with depth. This can be attributed to the large thermal mass of pavement materials (granular base, rockfill and subgrades soil), which causes a slow response to changes in temperature variations at the surface.

It can be concluded from Fig. 3.5 and Fig. 3.6 that both numerical models (H-PhC and H&M-PhC) predict reasonably the temperature and water content distributions in the pavement structure. Also, by comparing the results of H-PhC and H&M-PhC, it can be seen that the effect of mass transfer on the distribution of unfrozen water content and temperature in the pavement structure is almost insignificant. Therefore, to reduce the computational time, the mass transfer due to the freezing effect is not considered in the future analysis.

3.4 Factors Affecting the Buried Insulation Performance

3.4.1 The Effect of Pore-Water Phase Change

This section aims to investigate the importance of considering the phase change of pore-water in the design of insulation for protecting buried water pipes from frost actions.

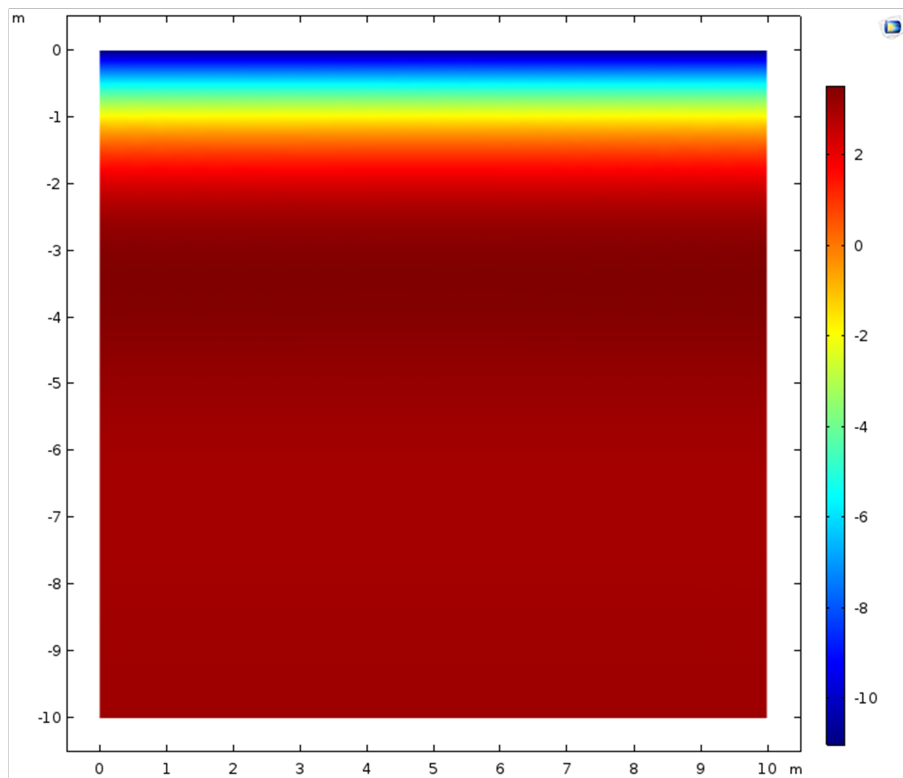
A 400 mm diameter water pipe buried at a depth of 2,250 mm from the ground surface is considered. A trench with a width of 900 mm is backfilled with non-shrinkable materials (1500 mm) and sand (1000 mm). This geometry is representative of the typical design practice in Western Canada [SG94].

The sand and non-shrinkable fill are assumed to be dry (drained condition) so that the phase change only occurs in the clay subgrade (natural soil). The soil is assumed to be initially saturated with a volumetric water content of 0.5. The heat transfer through the water pipe wall is assumed to be negligible. The thermal properties of material used in the analysis are presented in Table 3.2.

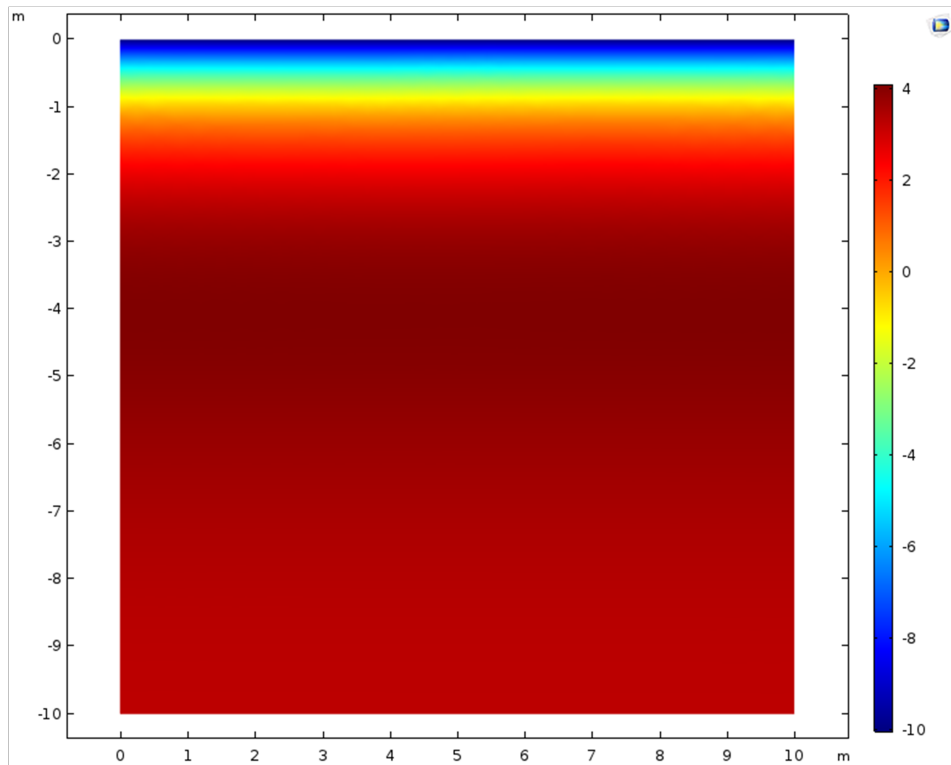
Tab. 3.2: Material properties used in the analyses

Material	Density (kg/m^3)	Thermal conductivity ($W/(m \cdot K)$)	Heat capacity ($J/(kg \cdot K)$)
Sand	1,710	2.7	1,255
Non-shrinkable fill	1,800	2.75	1,465
water	1,000	0.6	4,187
Ice	1,000	2.2	2,093
Soil skeleton	2,650	2.5	868
Asphalt	1,850	1.5	1,674
Extruded polystyrene insulation	60	0.035	1,225

In the numerical simulation, the initial ground temperature is determined by a sensitivity analysis. Heat transfer analyses, with and without considering the phase change (H vs H-PhC), by considering different initial ground temperatures are carried out for 20 years (1995-2015) based on Winnipeg weather data. It was seen that regardless of the initial value, the ground temperature profile reaches the equilibrium and becomes stationary after 20 years. These temperature profiles are then used as the initial condition for the analysis, as shown in Fig. 3.7. For example, the initial ground temperature profiles along the centerline of the trench are presented in Fig. 3.8.



(a) By considering heat transfer without phase change



(b) By considering heat transfer with phase change

Fig. 3.7: Stationary temperature distribution obtained by heat transfer analysis without considering phase change (a) and with considering phase change (b)

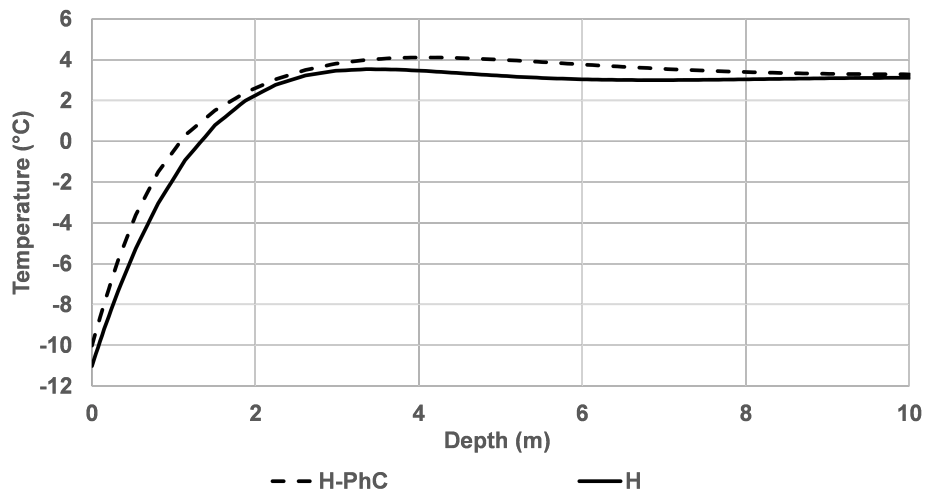


Fig. 3.8: Stationary temperature distribution along the center-line of the trench.

Using the stationary ground temperature profiles (Fig. 3.7) as the initial conditions, different geometries of the extruded polystyrene insulation including horizontal, inverted U-shaped and cylindrical are studied based on Winnipeg weather data of 2015.

The thickness of the insulation is 100 mm for all cases. The horizontal insulation covers all width of the trench (900 mm) and is located at 100 mm above the pipe wall. For the inverted U-shaped, the top and side insulation are 900 mm wide. The distance of the insulation from the pipe wall on the sides is 350 mm and on the top is 100 mm. For the cylindrical insulation, there is no space between the insulation and the pipe. The detailed dimensions can be seen in Fig. 3.9.

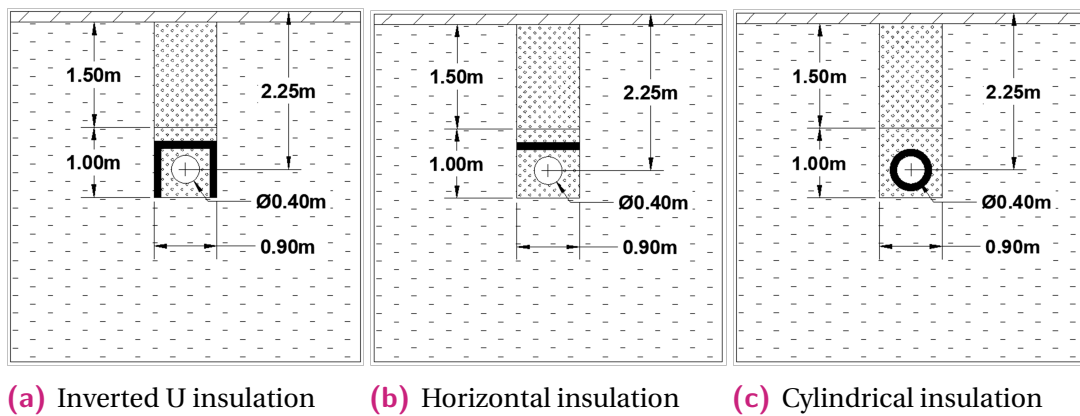
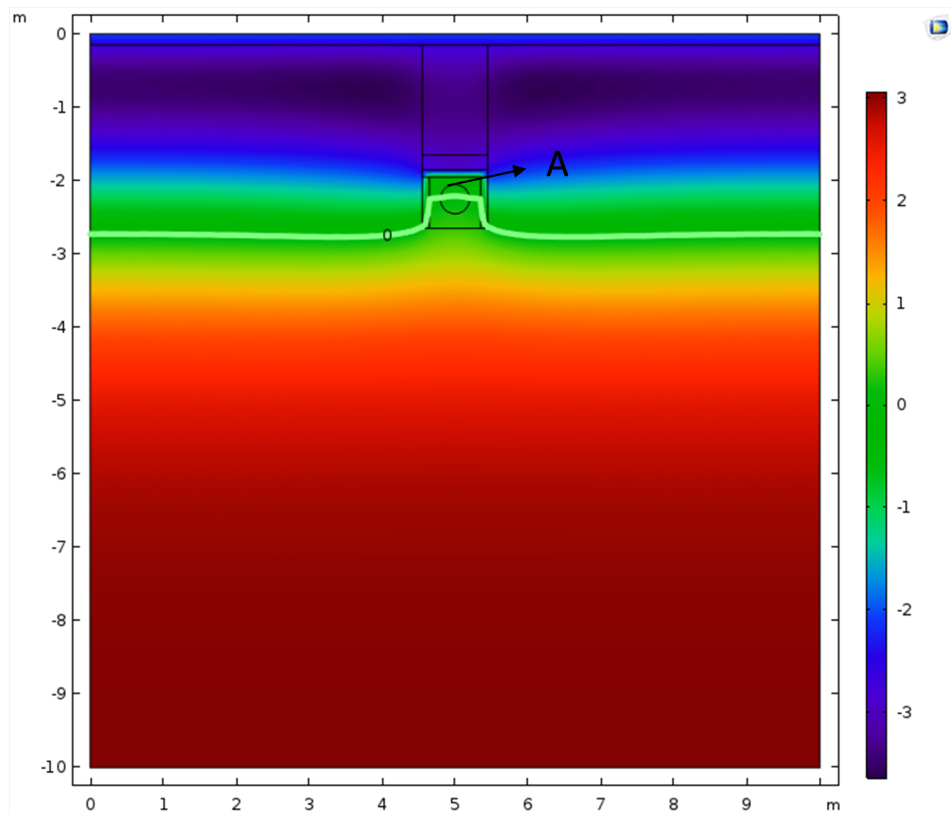
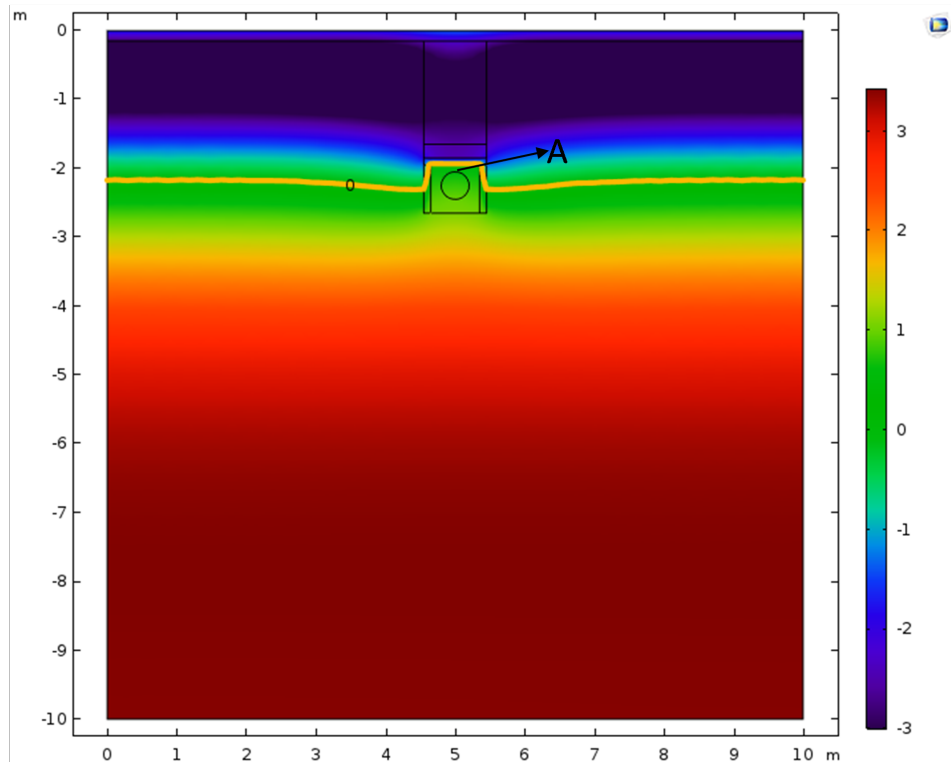


Fig. 3.9: The dimension of backfill, water pipe and (a) inverted U-shaped insulation, (b) horizontal insulation and (c) cylindrical insulation

Fig. 3.10 to Fig. 3.12 show the temperature distribution and the maximum frost penetration line using inverted U-shaped, horizontal and cylindrical insulation around the buried pipe, respectively. The simulations are carried out for H and H-PhC heat transfer models for one year starting on Jan. 1st, 2015 based on Winnipeg weather data. The results are for a time corresponding to the maximum frost penetration line so that the worst condition can be considered for the design.

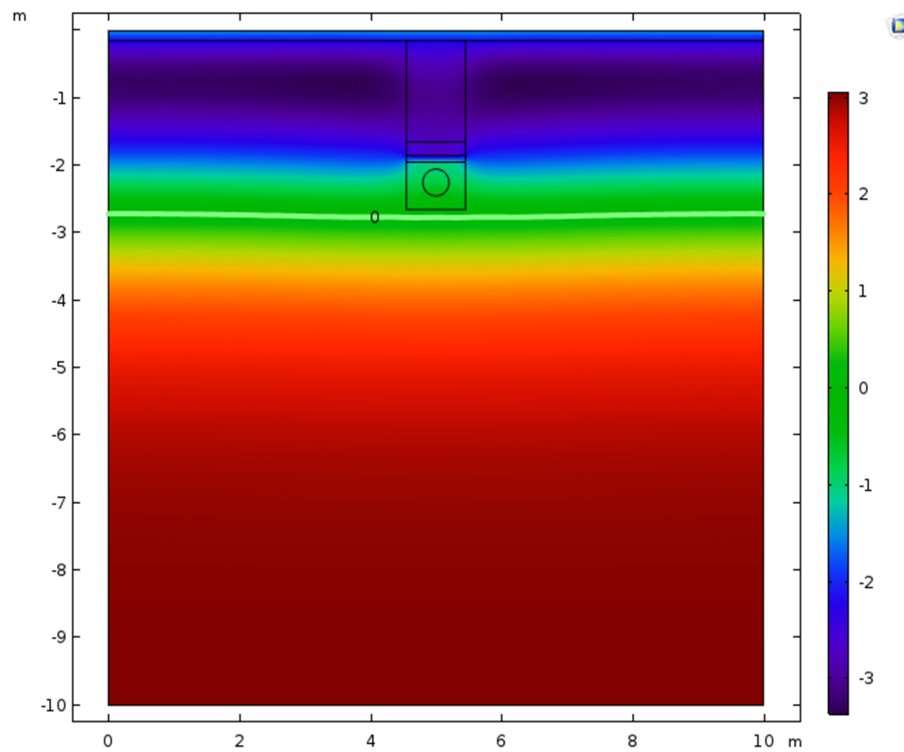


(a) Without phase change

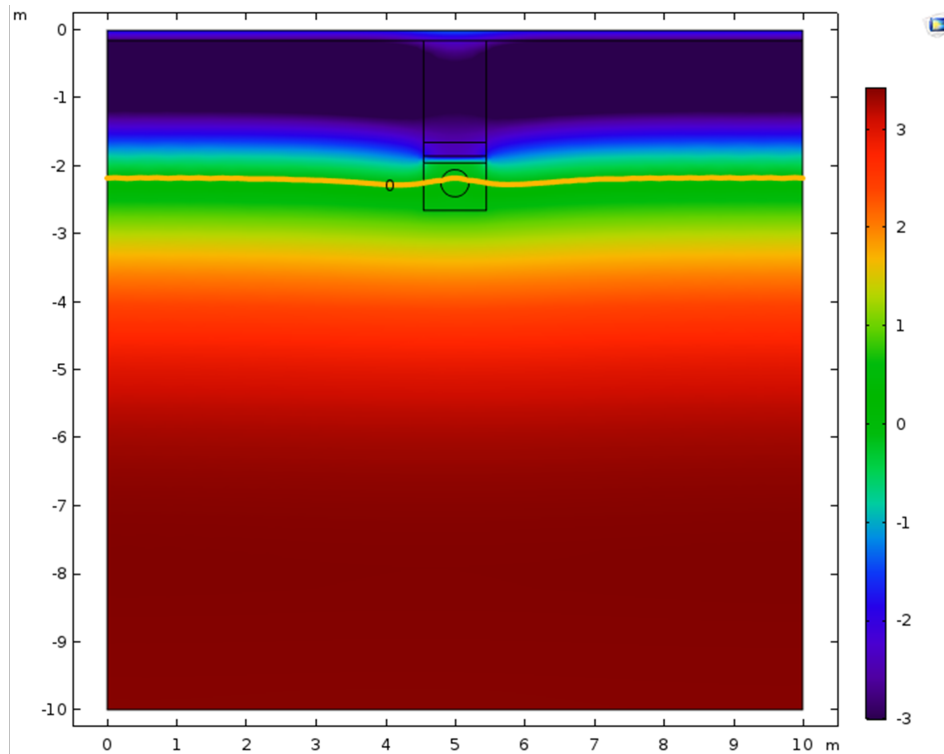


(b) With phase change

Fig. 3.10: Temperature distribution and the maximum frost penetration line using inverted U-shaped insulation without phase change (a) and with phase change (b)

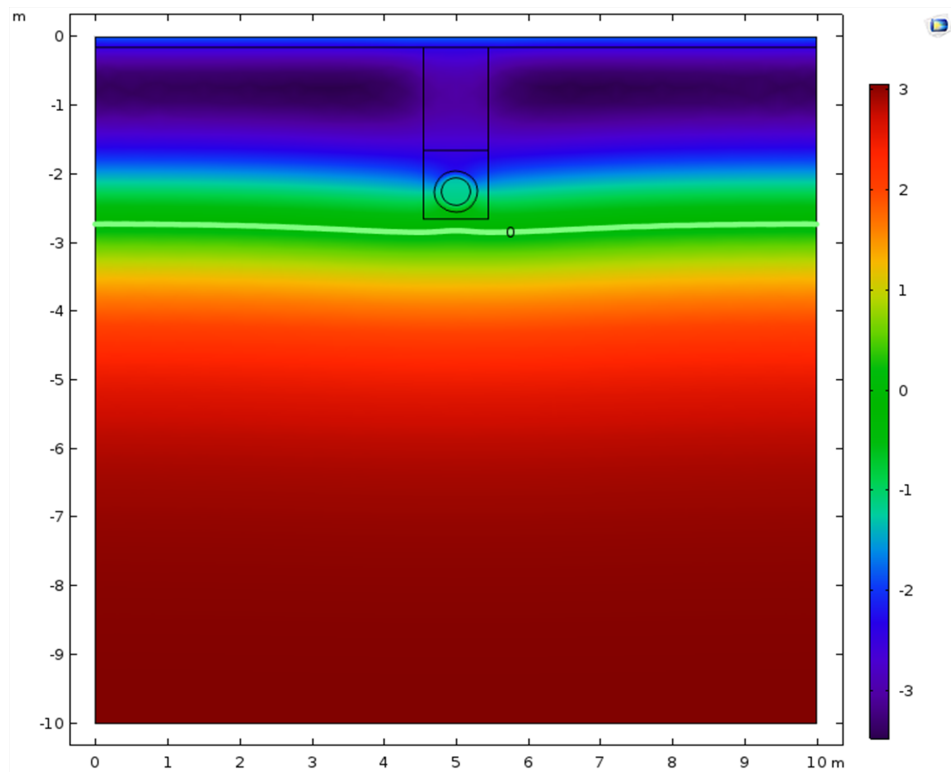


(a) Without phase change

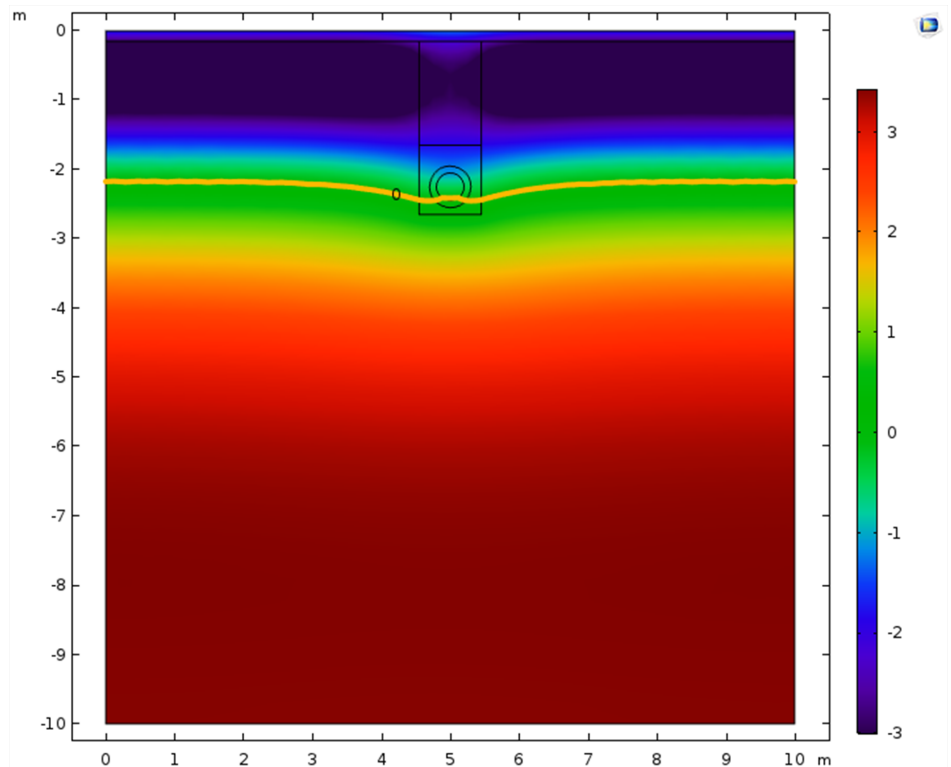


(b) With phase change

Fig. 3.11: Temperature distribution and the maximum frost penetration line using horizontal insulation without phase change (a) and with phase change (b)



(a) Without phase change



(b) With phase change

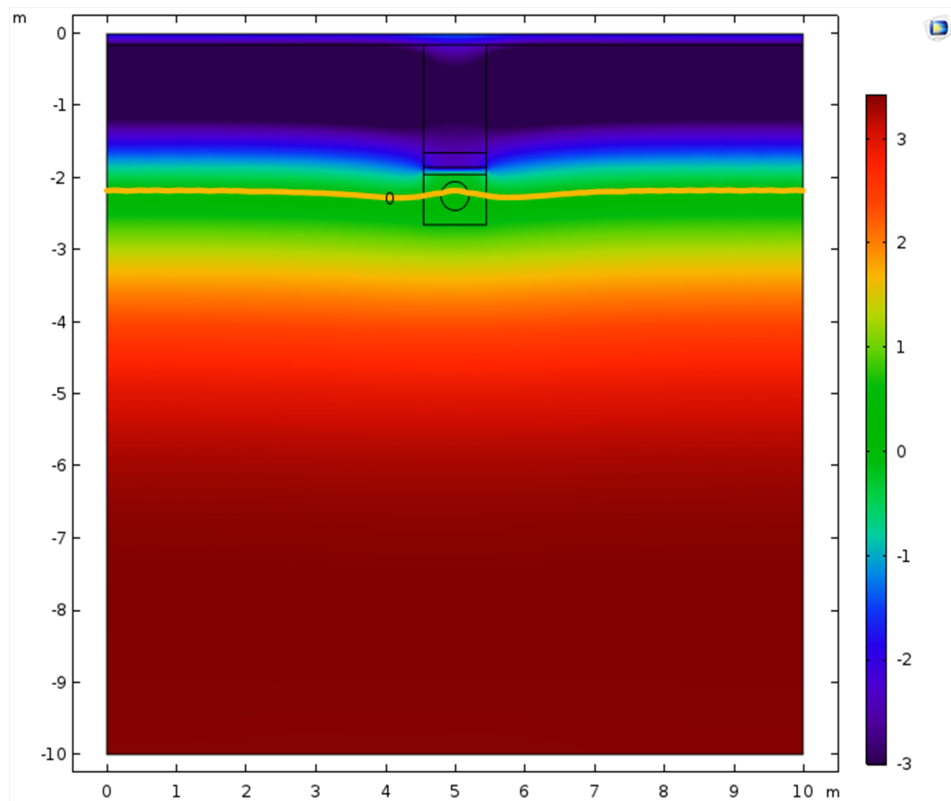
Fig. 3.12: Temperature distribution and the maximum frost penetration line using cylindrical insulation without phase change (a) and with phase change (b)

As can be seen in Figs. 3.10 to 3.12, considering the phase change mechanism plays an important role in evaluating the performance of insulation. The latent heat released during the phase change significantly impacts the heat transfer process. During the phase change, the released latent heat decreases the freezing rate. In addition, the changes in thermal properties of geomaterials by considering the phase change affect the heat transfer in the ground. As a result, designing the insulation without considering the effect of phase change leads to an erroneous design.

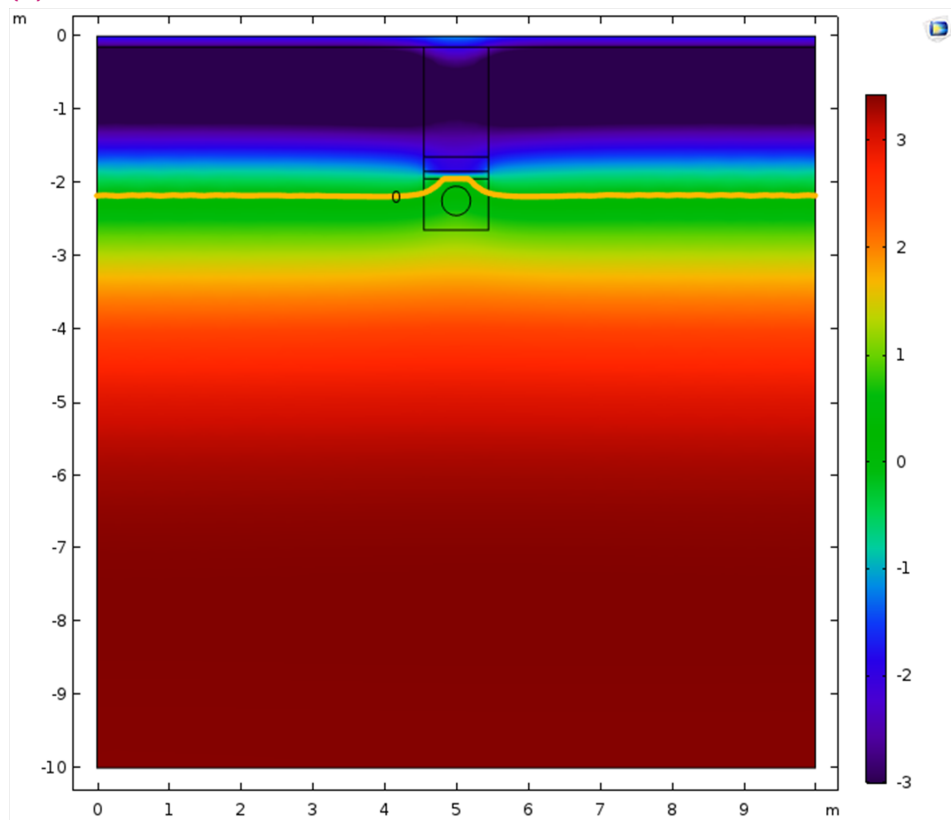
3.4.2 The Effect of Backfill

In this section, the trench is assumed to have a well-designed drainage system so that water in backfill materials is drained instantaneously. Therefore, backfill covering the buried pipe remains dry all the time and the phase change does not occur. In this analysis, two types of backfill materials are considered to study their impact on the thermal performance of insulation: fine-grained soil (clay) and granular soil (sand and non-shrinkable soil).

Fig. 3.13 shows the temperature and maximum frost penetration line using granular soils (sand and non-shrinkable soil) and fine-grained soils (clay) as backfill. The results show that using granular soil as backfill is more critical. The granular soil normally has a higher thermal conductivity and lower specific heat capacity than clay [DZ99]. Therefore, a faster heat loss is induced and the depth of maximum frost penetration line increases, as shown in Fig. 3.13. To be conservative, the granular soil will be used as backfill in the future analysis.

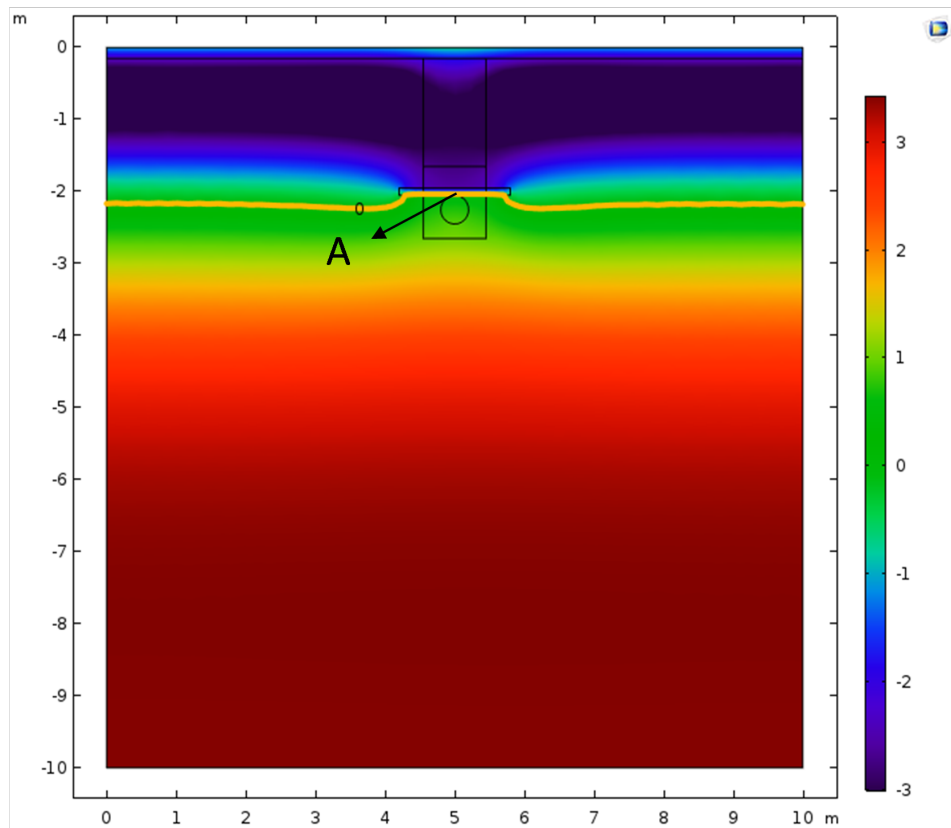


(a) Sand and non-shrinkable soil as backfill

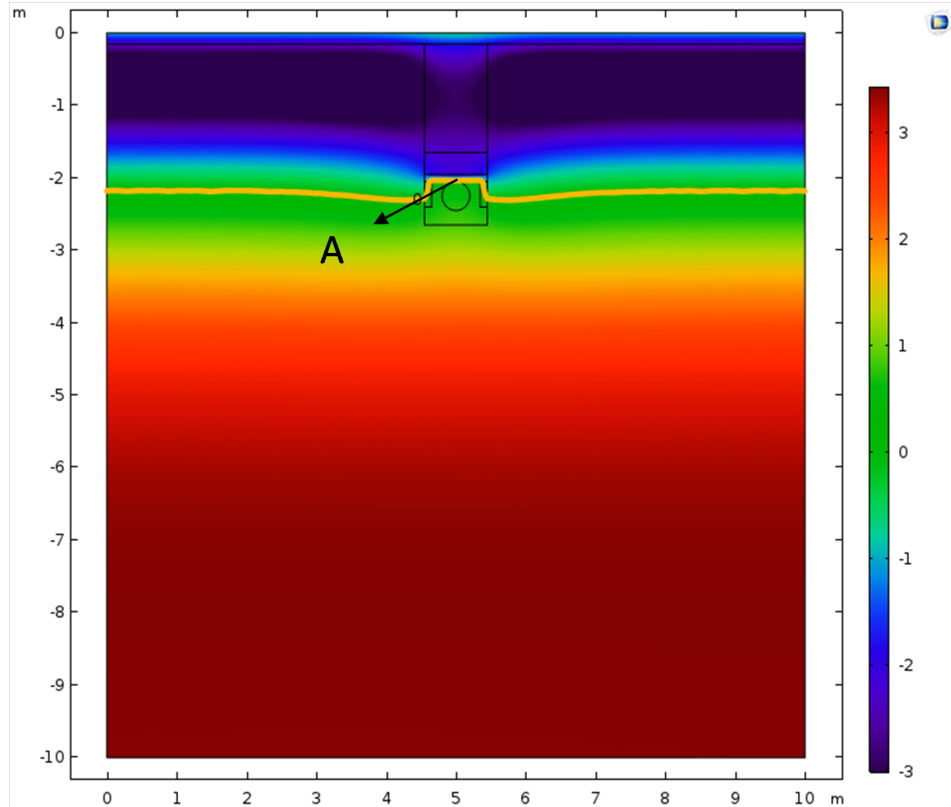


(b) Clay soil as backfill

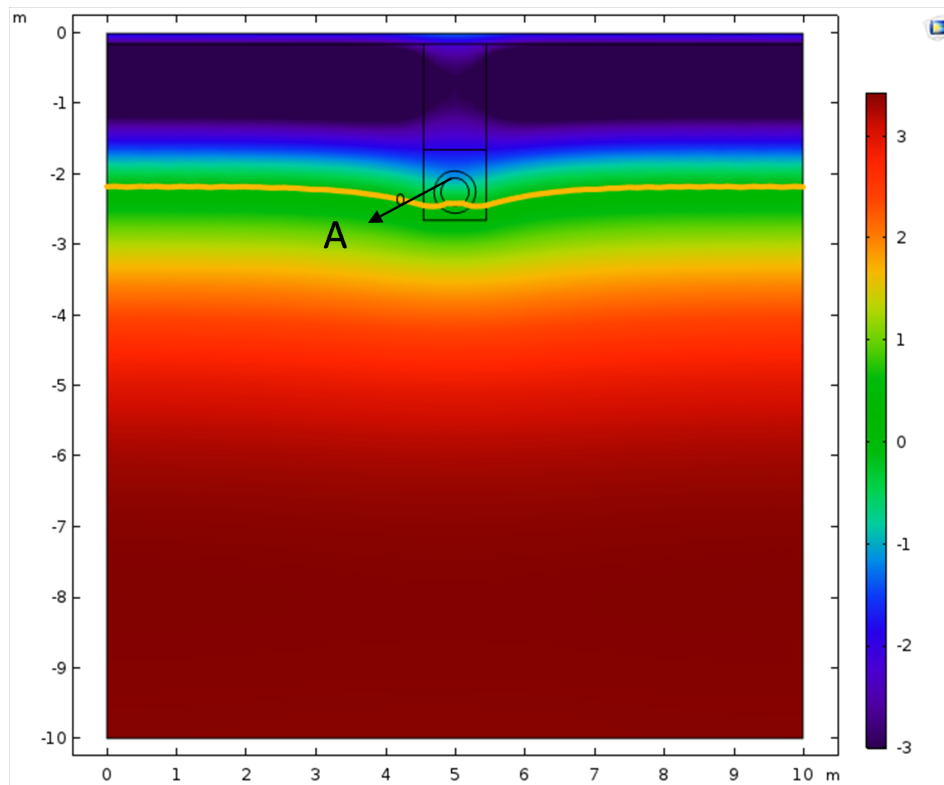
Fig. 3.13: Temperature distribution and the maximum frost penetration line using sand and non-shrinkable soil as backfill (a) and clay soil as backfill (b)



(a) Horizontal shape insulation



(b) Inverted U-shaped insulation



(c) Cylindrical shape insulation

Fig. 3.14: Temperature distribution and the maximum frost penetration line with horizontal shape (a), inverted U-shaped (b) and cylindrical shape insulation (c)

3.4.3 The Effect of Geometry

In order to study the effect of geometry of insulation on the thermal profile around the buried pipe, different configurations including inverted U-shaped, horizontal and cylindrical insulation are studied.

The same area of insulation (0.16 m^2) is used for the three different geometries. The thickness is assumed to be 100 mm. The insulation is assumed to be attached to the water pipe directly. The temperature distribution and maximum frost penetration line using different geometries can be seen in Fig. 3.14.

As can be seen in Fig. 3.14, the inverted U-shaped and horizontal insulation are able to protect the pipe from freezing, whereas the frost penetration line has penetrated into the pipe using the cylindrical shape insulation. Therefore, using cylindrical shape insulation

is not as effective as horizontal shape or inverted U-shaped insulation. However, it has been concluded [Mou+18] and [Liu+18] that the cylindrical shape insulation is the most efficient configuration to protect double-barrel culverts against frost actions. The contrary conclusion drawn in this study is because of the different boundary conditions assumed on the pipe wall. The heat loss mainly occurs at the top pipe. Therefore, the insulation located at the bottom water pipe does not effectively reduce the heat loss.

Fig. 3.15 shows the temperature distribution at the crown of the pipe (location A) using these three different insulation geometries for entire 2015.

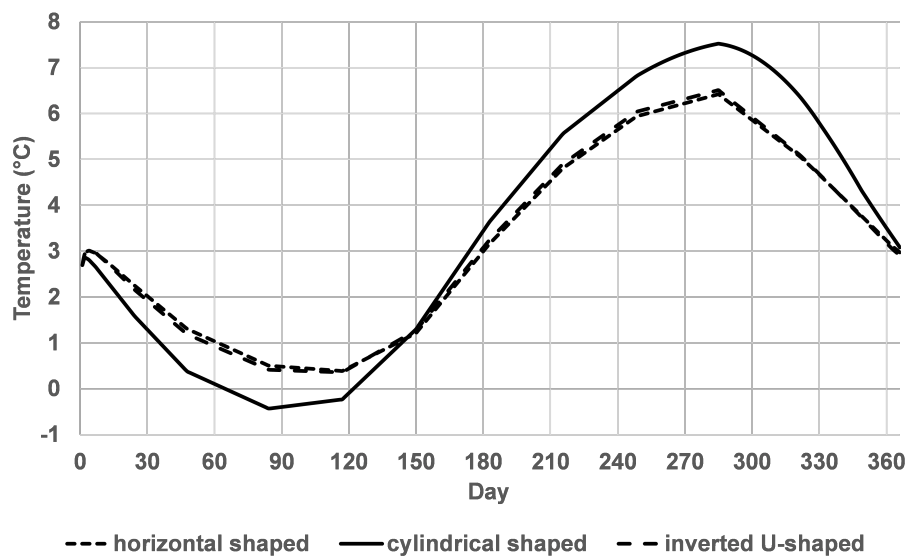


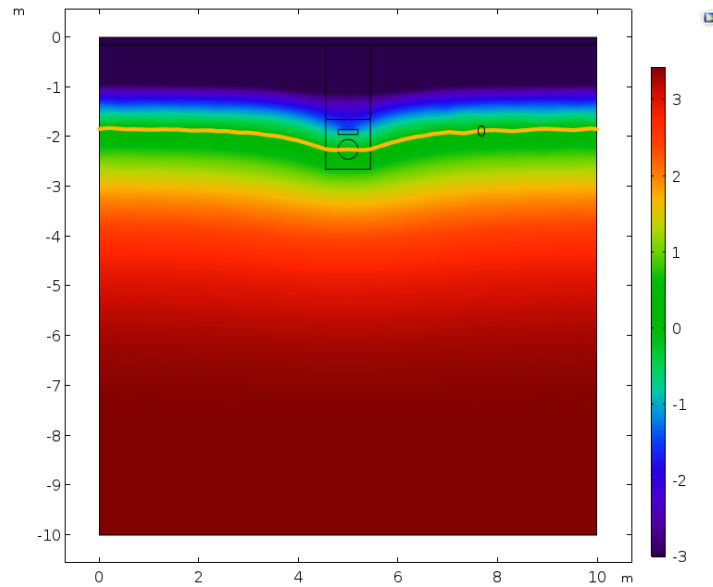
Fig. 3.15: Temperature distribution at the crown of the pipe (location A) with horizontal, cylindrical and inverted U-shaped insulation

3.4.4 The Effect of Total Insulation Length

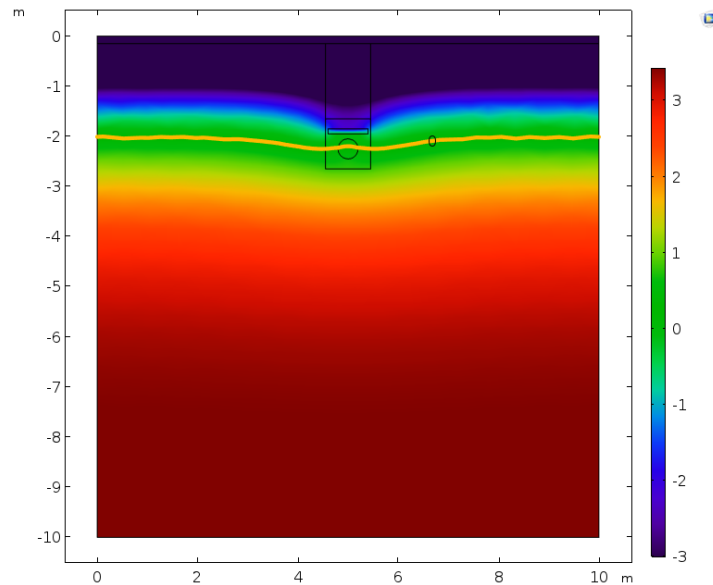
In this section, the horizontal insulation with various length are studied. The distance between the insulation and water pipe is assumed to be 100 mm.

Fig. 3.16 shows the maximum frost penetration location and temperature distribution using a horizontal insulation with a length which varies from $L = D = 400$ mm to $L = 3D = 1200$ mm. The results show that the increase of the insulation length could improve the insulation performance. The heat exchange can occur at any direction. Therefore, the

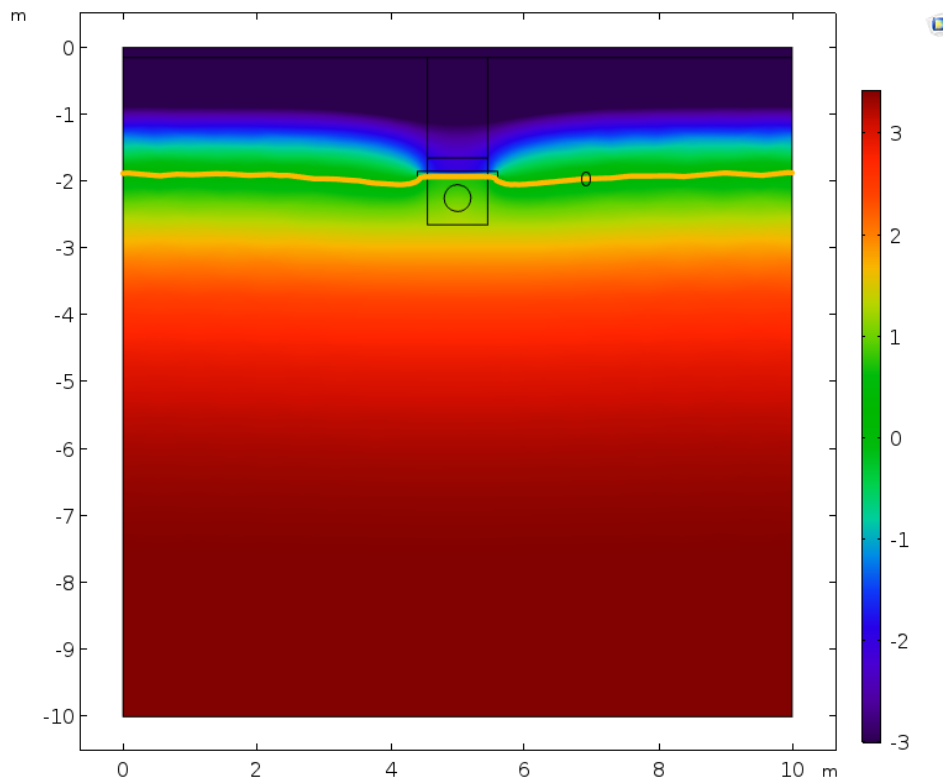
insulation should be extended to protect the pipe from freezing in all directions. In this case, the length should be at least 3 times the diameter of the pipe (1200 mm) to prevent the water pipe from freezing, as shown in Fig. 3.16. However, this approach may not be always practical due to the trench width limitation.



(a) Horizontal insulation with a length of 400 mm ($L = D$)



(b) Horizontal insulation with a length of 800 mm ($L = 2D$)

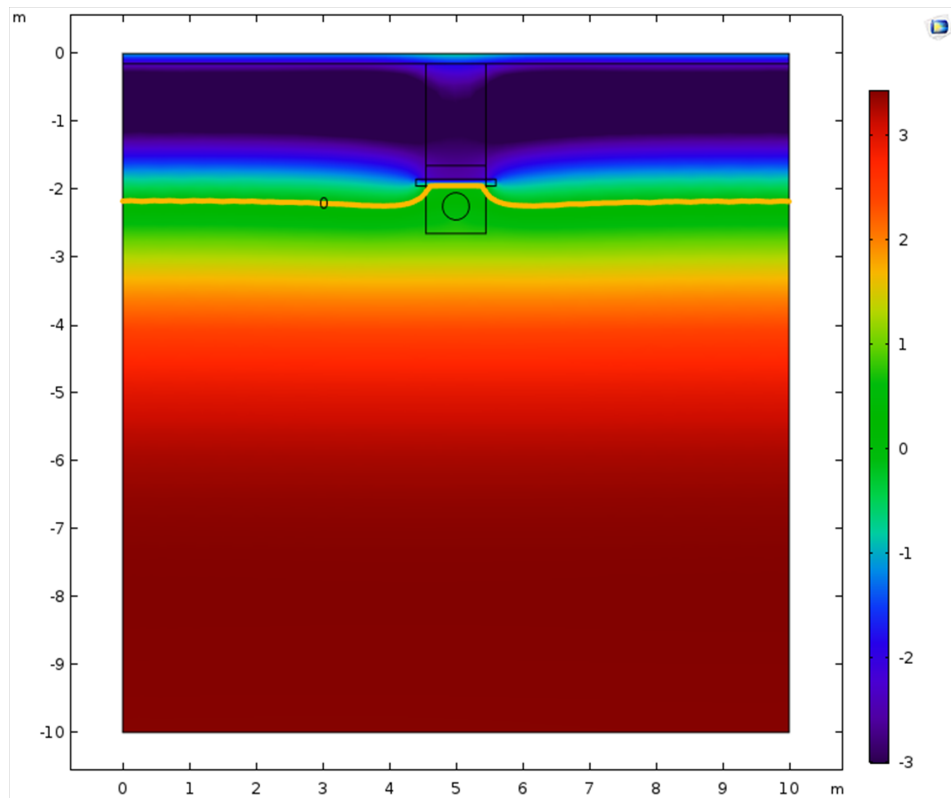


(c) Horizontal insulation with a length of 1200 mm ($L = 3D$)

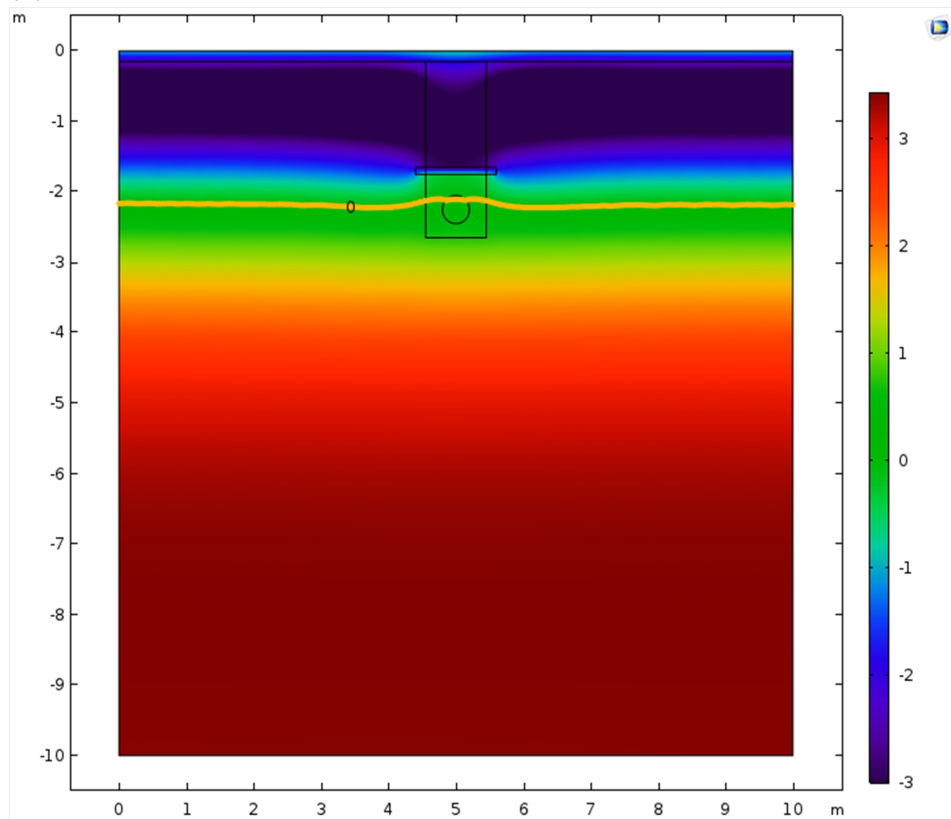
Fig. 3.16: Temperature distribution and the maximum frost penetration line with an insulation length of 400 mm (a), 800 mm (b) and 1200 mm (c)

3.4.5 The Effect of the Distance Between Insulation and Buried Utilities

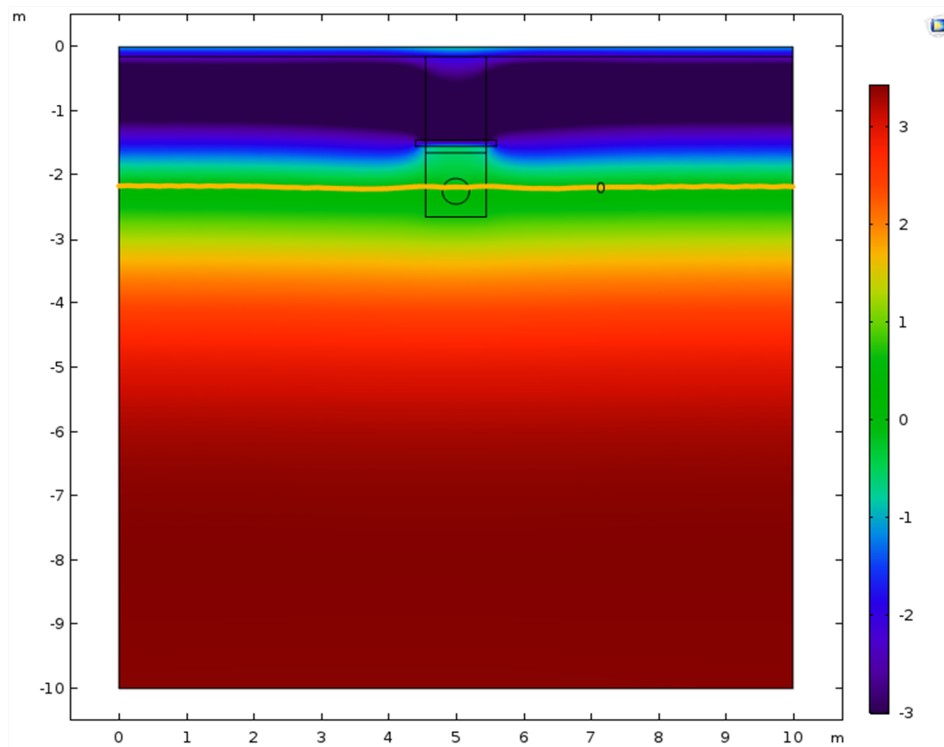
The distance between insulation and water pipe could be another factor influencing the insulation performance. Fig. 3.17 shows the temperature and maximum frost penetration line by having different distances of 100 mm, 300 mm, and 500 mm between a horizontal insulation and water pipe.



(a) Distance of 100mm



(b) Distance of 300mm



(c) Distance of 500mm

Fig. 3.17: Temperature distribution and the maximum frost penetration having a distance between a horizontal insulation and water pipe of 100 mm (a), 300 mm (b) and 500 mm (c)

As can be seen in Fig. 3.17, the water pipe can be protected from freezing by having a 100 mm distance between insulation and water pipe. However, a larger distance (300 mm or 500 mm) will lead to a failure in design. Because the frost action could penetrate into the soil in all directions, a larger distance between insulation and water pipe increases the chance for the frost penetration. It is worth noting that due to structural considerations of the backfill (compaction, dimensions of the backfill envelope and so on.), a distance between water pipe and insulation might be required. However, from the thermal performance point of view, the closer distance between the insulation and pipe will lead to a better insulation performance.

3.4.6 The Effect of the Burial Depth of Buried Utilities

A deeper burial depth of buried utilities normally increases the excavation and maintenance cost. Therefore, it is important to study the insulation performance with various pipe burial depth.

It can be seen in Fig. 3.19 that the design with a shallower burial depth (1500 mm) is more critical than that with a burial depth of 2250 mm. Because the soil temperature tends to become lower when the burial depth is shallower in the winter, the same design that works at a burial depth of 2250 mm becomes insufficient at a depth of 1500 mm.

Furthermore, Fig. 3.18 shows that the frost penetrates into the water pipe having a shallower burial depth (1500 mm). Therefore, the length of insulation, the distance between insulation and water pipe and other design parameters should be changed to accommodate the severe conditions induced by the shallower burial depth.

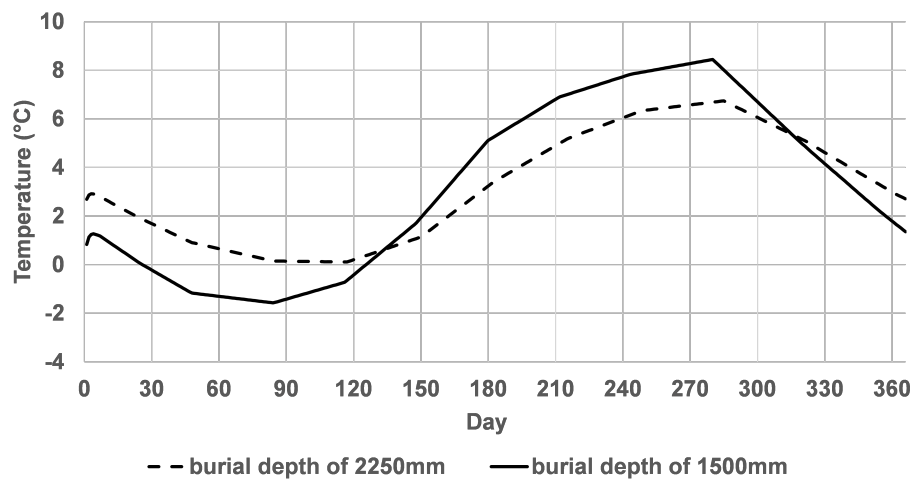
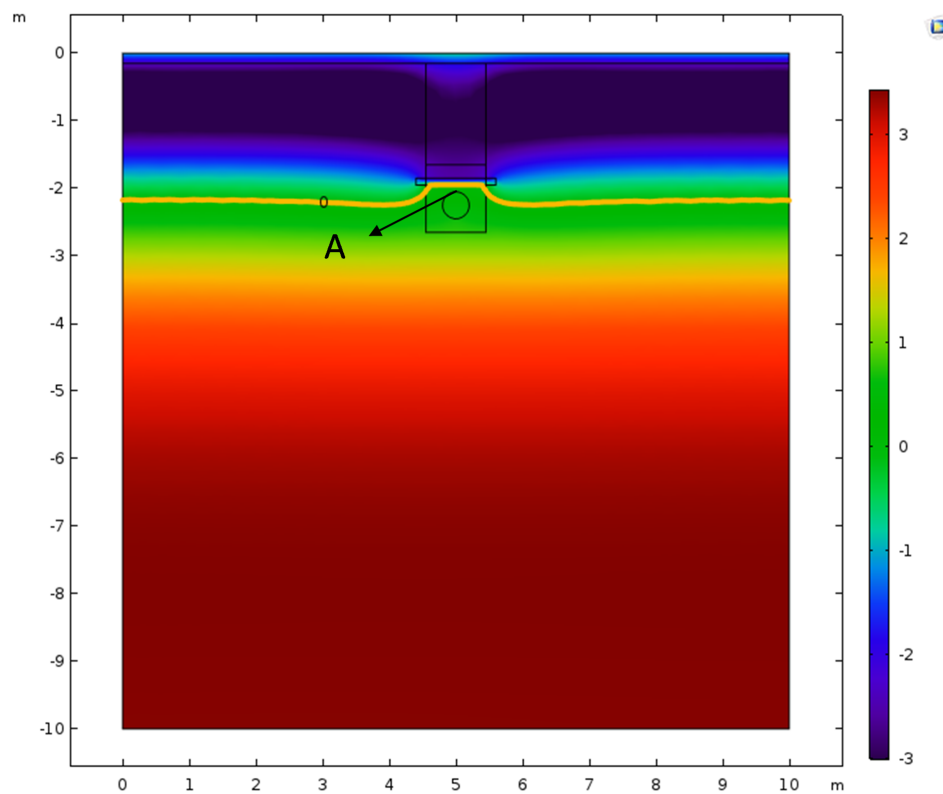
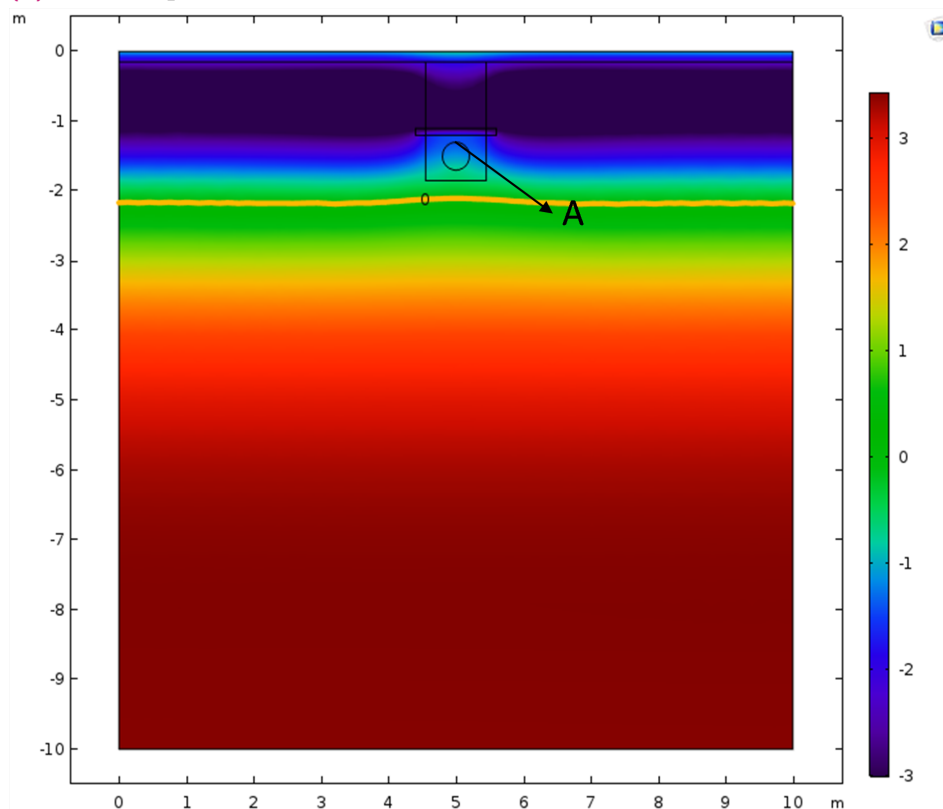


Fig. 3.18: Temperature distribution at the crown of the pipe (location A) with pipe burial depth of 2250 mm and 1500 mm.

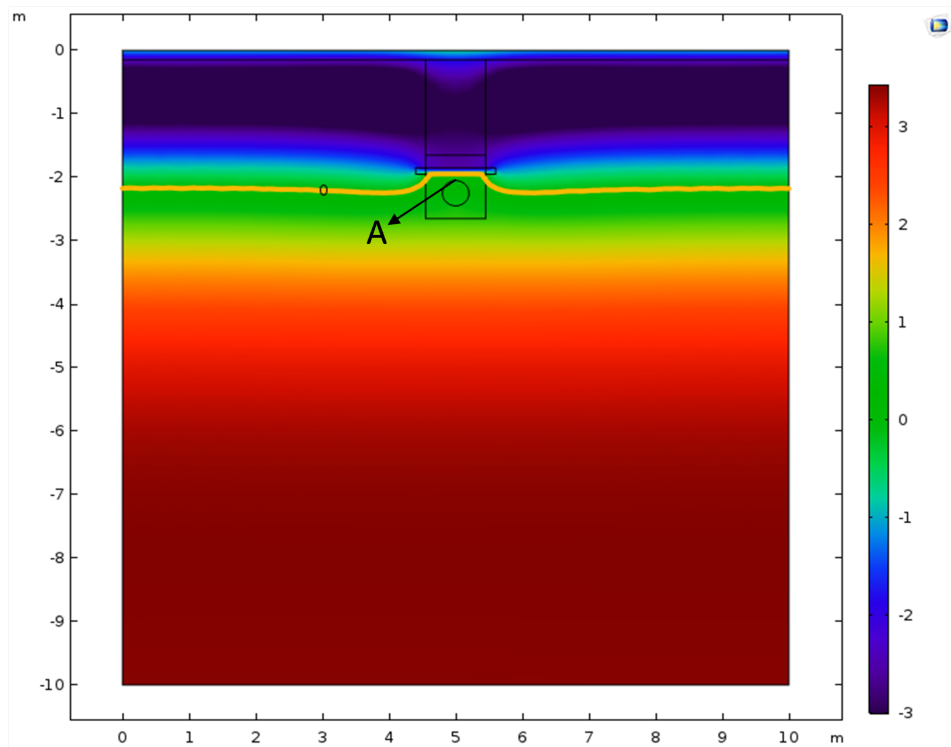


(a) Burial depth of 2250 mm

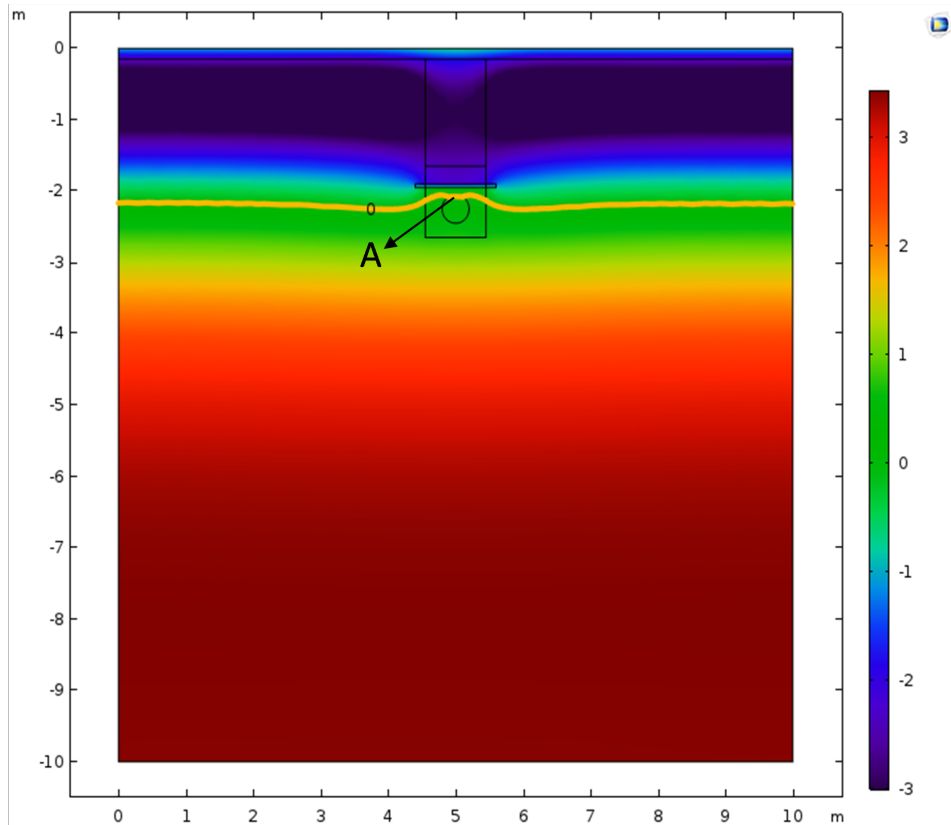


(b) Burial depth of 1500 mm

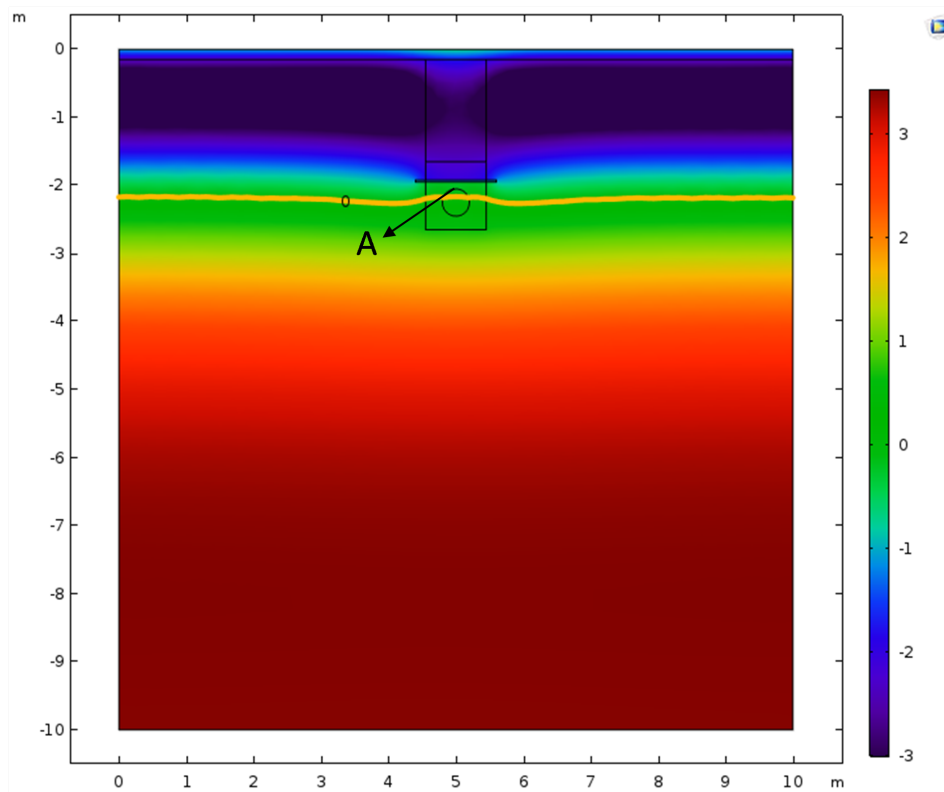
Fig. 3.19: Temperature distribution and the maximum frost penetration line with a pipe burial depth of 2250 mm (a) and 1500 mm (b)



(a) Insulation thickness of 100 mm



(b) Insulation thickness of 50 mm



(c) Insulation thickness of 30 mm

Fig. 3.20: Temperature distribution and the maximum frost penetration line by having an insulation thickness of 100 mm (a), 50 mm (b) and 30 mm (c)

3.4.7 The Effect of Insulation Thickness

The thickness of the insulation is one of the important factors influencing the insulation design. A thicker insulation will have a better insulation performance. But it is also accompanied with a higher construction costs. Therefore, it is important to determine the optimum thickness of the insulation. Fig. 3.20 shows the temperature distribution and the maximum frost penetration depth by having various thicknesses for horizontal insulation. Also, Fig. 3.21 shows the corresponding temperature distribution at the crown of the water pipe.

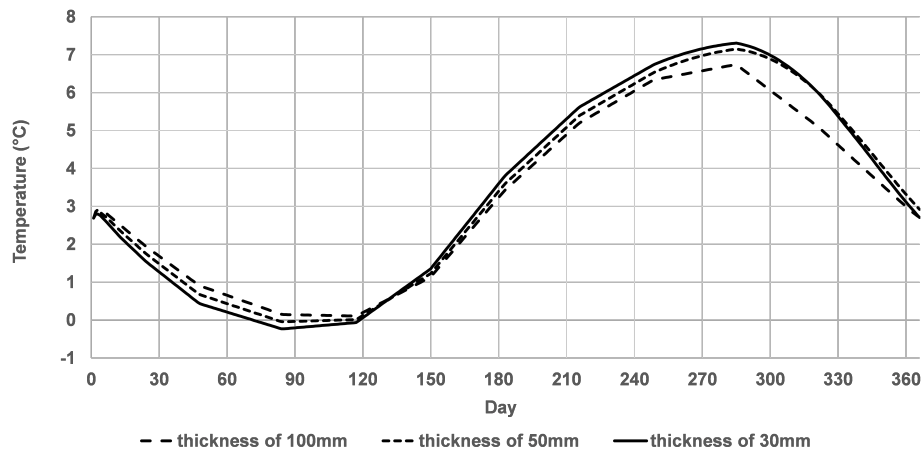


Fig. 3.21: Temperature distribution at the top pipe by having an insulation thickness of 0.1m, 0.05 m and 0.03 m

It can be seen from Fig. 3.20 and Fig. 3.21 that the critical insulation thickness in this case is 50 mm. The frost will penetrate into the water pipe when the insulation thickness smaller than 50 mm.

3.5 Nelder-Mead Optimum Algorithm

The optimum design should not only protect the water pipe from freezing, but also have the minimum cost. The major costs include the cost of excavation and insulation materials. The insulation material cost will be reduced for a pipe buried at a deeper depth where soil temperature remains relatively warmer. However, a deeper pipe burial depth also leads to a higher excavation cost. In order to find the optimum burial depth at which pipe is free from freezing and the overall cost is minimum, the Nelder-Mead optimum algorithm is used.

The Nelder-Mead is one of the effective and computationally compact optimum methods. In contrary to the gradient-based method, Nelder-Mead method is a direct search method which use only the objective function values [RRR06]. The Nelder-Mead method generates a series of simplexes to minimize the objective function. For three variables, a simplex is a tetrahedron which has four vertices. The objective function value is evaluated at every vertex. The worst vertex, where the functional value is maximum among the four vertices, will be

replaced by a new vertex. During this process, a series of tetrahedron can be generated. The size of tetrahedron gets smaller and objective function value decreases. To do so, four different operations are required including reflection, expansion, contraction and shrinkage.

3.5.1 Design Variables

In this paper, the variables influencing the insulation performance and overall cost includes the backfill materials, insulation length, insulation thickness, distance between pipe and insulation as well as pipe burial depth. To be conservative, the backfill is assumed to be granular soil (sand). The pipe diameter is assumed to be 400 mm. The distance between insulation and water pipe is 100 mm. The distance between pipe bottom and trench bottom (foundation) is set to be 100 mm. The minimum trench width needs to be 300 mm greater than the external pipe diameter. The maximum trench width is 450 mm greater than the external pipe diameter. Therefore, the trench width ranges from 700 mm to 850 mm. The insulation length and thickness should depend on the burial depth of the water pipe. A pipe located at a shallower burial depth normally requires more insulation materials because the heat loss is more severe in the area close to ground surface.

Therefore, three optimization variables, pipe burial depth, insulation length, and insulation thickness, are selected in the optimum design of insulation for the buried utilities. As can be seen in Fig. 3.22, these three parameters construct a three dimensional space in which a tetrahedron is formed.

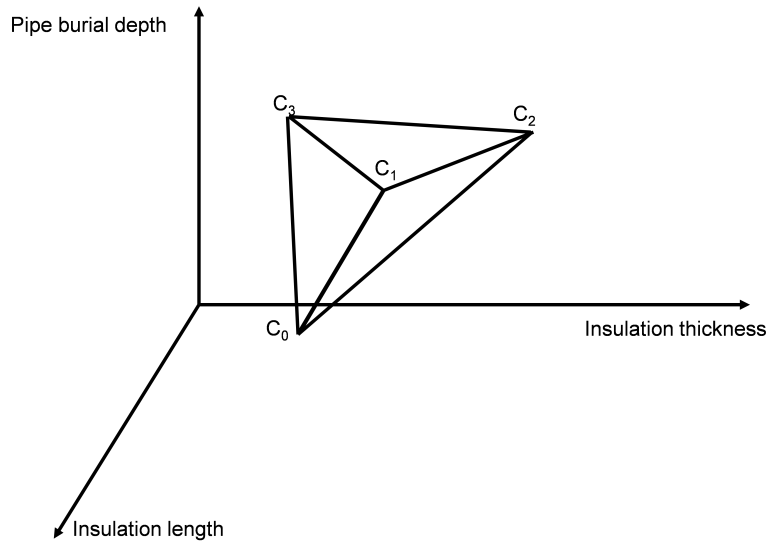


Fig. 3.22: Nelder-Mead simplex for three optimization parameters

3.5.2 Objective Function and Constraints

The objective function in this analysis is defined as the overall cost (C). The major costs include the cost of excavation and insulation materials. The polystyrene foam (Dow Chemical) is used as the insulation materials whose cost is taken as $560 \text{ \$/m}^3$. The average trenching cost including labour, backfill materials and excavation equipment is taken as $220 \text{ \$/m}^3$ (Baca et al. 2010) [Bac10].

The overall cost (C) is expressed as:

$$C = 220(L \times (D + 0.6)) + 560 (L \times T) \quad (3.23)$$

where D (m) is the burial depth of the insulation; L (m) is the length (width) of the insulation; T (m) is the thickness of the insulation; 0.6 m in the first term corresponds to the distance between insulation the trench bottom (foundation).

The constraint of the objective function is that the minimum temperature at the top pipe should be above freezing point all the time. It should be noted that the initial value of the insulation length, thickness and depth should be sufficient to protect pipe from freezing.

This is very important for the optimum algorithm because the initial value should be located in a feasible zone.

3.5.3 Iteration of Nelder-Mead Optimum Algorithm

In this section, the standard coefficients are used in the iteration of the Nelder-Mead algorithm as follows [CSV09].

$$\gamma^s = 0.5, \delta^{ic} = -0.5, \delta^{oc} = 0.5, \delta^r = 1, \delta^e = 2 \quad (3.24)$$

where γ^s is the shrinkage coefficient; δ^{ic} and δ^{oc} is the inside and outside contraction coefficient respectively; δ^r is the reflection coefficient; δ^e is the expansion coefficient.

1. Order: Order the 4 vertices as shown in Fig. 3.22 so that

$$C_0 \leq C_1 \leq C_2 \leq C_3$$

2. Reflect: Reflect C_3 over the centroid (C_c) of C_0, C_1 and C_2 so that

$$C_r = C_c + \delta^r (C_c - C_3)$$

If $C_0 \leq C_r < C_3$, then replace C_3 by C_r .

3. Expand: If $C_r < C_0$, then determine the expansion point:

$$C_e = C_c + \delta^e (C_c - C_3)$$

If $C_e \leq C_r$, replace C_3 by C_e

4. Contract: If $C_r \geq C_2$, then a contraction is performed between the best of C_r and C_3

Outside contraction: If $C_r < C_n$, perform an outside contraction:

$$C_{oc} = C_c + \delta_{oc} (C_c - C_3)$$

If $C_{oc} \leq C_r$, replace C_3 by the C_{oc} . Otherwise, perform s a shrinkage.

Inside contraction: If $C_r \geq C_4$, perform an inside contraction

$$C_{ic} = C_c + \delta_{ic} (C_c - C_3)$$

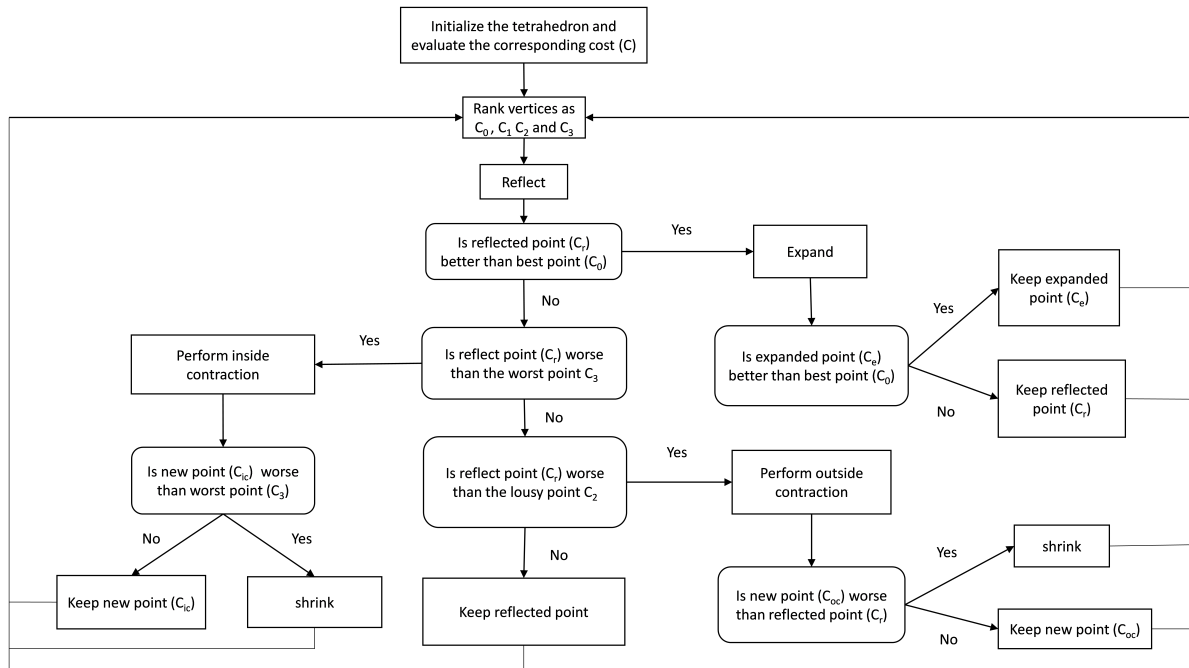


Fig. 3.23: Iteration of Nelder-Mead optimum algorithm flowchart

If $C_{ic} < C_3$, replace C_3 by the inside contraction point C_{ic} . Otherwise, perform shrinkage.

5. Shrinkage: Evaluate $C_0 + \gamma_s(C_i - C_0)$, $i = 1, 2$ and 3 and replace C_1 , C_2 and C_3 by these points.

The iteration flow chart can be seen in Fig. 3.23.

3.5.4 Optimum Design for Horizontal Shape Insulation

In this section, the horizontal shape insulation is evaluated. The maximum insulation width is assumed to be the trench width. By using the following initial design parameters (800 mm length, 2000 mm burial depth, and 100 mm thickness), the feasible design options can be seen in Table 3.3.

Tab. 3.3: The design with various insulation length, thickness, burial depth and corresponding cost

Length (m)	Thickness (m)	Depth (m)	Cost (\$/m)
0.80	0.10	2.00	503.20
0.80	0.40	2.00	640.00
0.85	0.10	2.00	534.65
0.80	0.10	2.30	556.00
0.81	0.25	2.05	586.45
0.81	0.18	2.08	559.20
0.82	0.05	2.12	513.11
0.83	0.08	1.91	496.31
0.79	0.05	2.02	474.94
0.79	0.10	1.83	470.58
0.81	0.06	1.88	469.27
0.76	0.07	1.91	447.21
0.79	0.09	1.80	456.14
0.78	0.05	1.89	446.81
0.74	0.08	1.86	430.86
0.74	0.05	1.93	434.35
0.75	0.06	1.90	442.29
0.72	0.08	1.90	424.34
0.72	0.07	1.89	423.68

Accordingly, the optimum design for a horizontal insulation consists of 720 mm insulation length, 70 mm insulation thickness, and 1890 mm burial depth, as illustrated in Fig.3.24:

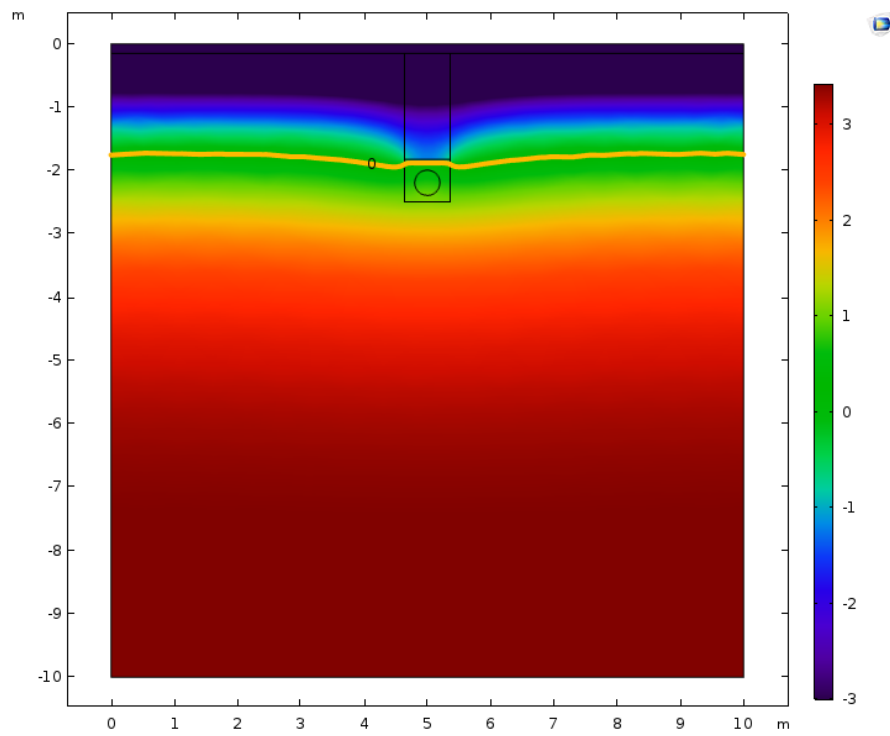


Fig. 3.24: Optimum design for horizontal insulation

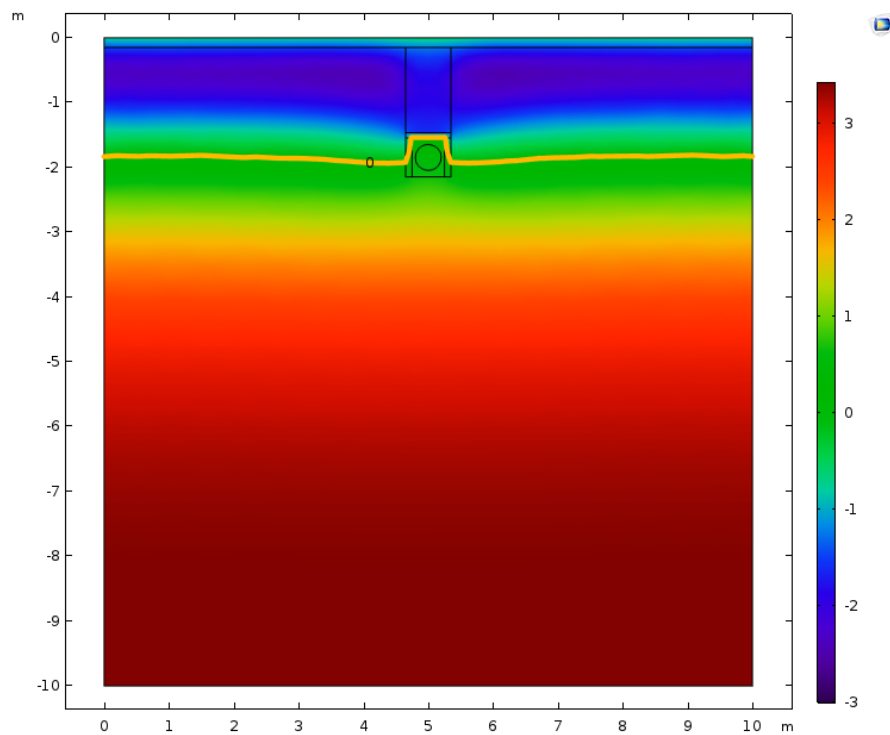


Fig. 3.25: Optimum design of inverted U-shaped insulation

Because the insulation width is limited by the trench width, the functional design option is limited. The corresponding minimum cost is 424 \$/m.

3.5.5 Optimum Design for Inverted U-shaped Insulation

Because the length (width) of horizontal shape insulation is limited by the width of the trench, it is interesting to study the performance of insulation with a longer length so that the burial depth (D) can be decreased. For this purpose, the inverted U-shaped insulation is studied. However, the cost of insulation will be increased. Therefore, the Nelder-Mead algorithm needs to be applied to determine the optimum burial depth and total insulation length. The initial design parameters used in this analysis are 800 mm horizontal length, 100 mm thickness, and 1800 mm insulation burial depth. The insulation on the sides of the water pipe has 600 mm height and 100 mm thickness. The feasible design option can be seen in Table 3.4. The length in Table 3.4 only refers to the length of the horizontal insulation on top of the pipe.

The inverted U-shaped insulation can be used when a shallower burial depth is required to accommodate special conditions. It is concluded that optimum insulation design consists of: top insulation with 700 mm width and 80 mm thickness; 1550 mm insulation burial depth; side insulation with 600 mm height and 100 mm thickness. The corresponding minimum cost is 430 \$/m. The temperature distribution using the obtained optimum design parameters can be seen in Fig. 3.25

Tab. 3.4: Inverted U-shaped Insulation

Length (m)	Thickness (m)	Depth (m)	Cost (\$/m)
0.80	0.10	1.80	534.40
0.70	0.10	1.80	476.00
0.80	0.40	1.80	668.80
0.80	0.10	1.50	481.60
0.78	0.25	1.75	581.85
0.78	0.18	1.73	539.56
0.76	0.05	1.68	470.84
0.73	0.08	1.59	450.13
0.77	0.09	1.60	474.39
0.73	0.09	1.71	471.26
0.73	0.06	1.70	458.97
0.75	0.05	1.63	454.13
0.71	0.07	1.59	438.26
0.73	0.07	1.56	441.81
0.70	0.09	1.54	433.27
0.70	0.08	1.55	429.89
0.70	0.09	1.54	431.58
0.71	0.08	1.55	435.88
0.70	0.08	1.55	429.89

3.6 CONCLUSION

The insulation thermal performance has been studied using the heat transfer with and without considering phase change as well as the heat and mass transfer by considering phase change in unsaturated soils. The Nelder-Mead algorithm is introduced to find the

optimum pipe burial depth and the corresponding insulation design so that the overall cost is minimized. The major conclusions include:

- The latent heat released during the phase change can significantly affect the heat transfer process and decrease the freezing rate of the soil. Neglecting the phase change effects will eventually lead to an erroneous design of the insulation.
- The mass transfer mechanism is negligible in frost-susceptible foundation soil for predicting temperature and unfrozen water content distribution.
- Using granular soil (sand) as backfill is more critical than clay due to the higher thermal conductivity of sand.
- The cylindrical shape insulation is less effective in thermally closed-system buried pipes. The heat loss mainly occurs at the top pipe. Therefore, the insulation located at the bottom water pipe does not effectively reduce the heat loss.
- The optimum design should consider the effect of insulation length, distance between water pipe and insulation, pipe burial depth and insulation thickness.
- The Nelder-Mead algorithm is efficient and effective to determine the optimum burial depth and the designing insulation parameters.

Thermo-Hydro-Mechanical Modeling of Frost Heave Using the Theory of Poroelasticity in Frost-Susceptible Soils in Double-Barrel Culvert Sites

Abstract

The upward movement of soil due to the formation of ice lenses, known as the frost heave, is detrimental to buried utilities, pavement structure and building foundations. In this paper, a fully coupled thermo-hydro-mechanical model is developed by using the theory of poroelasticity and a porosity rate function aiming to predict the frost heave, temperature distribution, as well as pore water pressure in the ground under various thermal, hydraulic and mechanical boundary conditions. The proposed model is validated by comparing the numerical results with experimental tests data. Then, the model is applied to study the frost heave in two double-barrel concrete culvert sites in Manitoba, Canada. The results are verified by temperature field data and consequently frost heave and pore-water pressure distribution are predicted numerically. Furthermore, the performance of insulation is studied to mitigate the effect of frost heave and differential deformation at the road surface. It is concluded that

- The THM model proposed in this study is able to accurately predict the frost heave, temperature and pore-water pressure distribution in the ground. The numerical

results showed a good agreement with the experimental test data and field measurement.

- The negative pore-water pressure (suction) can be generated when frost heave occurs. The value of suction is related to the rate of the frost heave. The maximum suction occurs when the rate of frost heave is at the highest level.
- For double-barrel culverts with a deeper burial depth, using an insulation at the top of the frost-susceptible subgrades below the culverts and at the sides of the culvert structure (insulating method C) can be applied to effectively decrease the frost heave and the corresponding differential displacement at the pavement surface.
- For double-barrel culverts with a shallower burial depth, the insulating method C combined with an adequate overburden pressure are required to minimize the overall frost heave and differential displacement at the pavement surface. This can be done by increasing the thickness of the granular base or replacing frost-susceptible subgrades by non-frost-susceptible backfill.
- The applicability of the insulating methods studied in the paper depends mainly on the important of the road, the level of the driving comfort, and how much differential deformation at the pavement surface is tolerated.

4.1 Introduction

The serviceability of infrastructure such as buried utilities, pavement structures, and building foundations in cold regions can be adversely affected by seasonal frost and frost action. Frost actions on road structures including culverts can be more difficult to assess and mitigate due to the existing double-cooling sources, i.e. pavement surface and culverts openings. Culverts are used as a waterway under a road and their structural integrity has an impact on the road roughness.

The three essential factors for detrimental frost actions are (a) subfreezing temperature, (b) a continual supply of water (a high water table), and (c) presence of frost-susceptible soils. If one of these conditions is not met, frost actions will not occur. For example, in a pavement structure, as far as the temperature at the top surface of the frost-susceptible subgrades, by increasing the base thickness or using insulation, is kept above the freezing point, frost heave will be preventable. Furthermore, it is worth noting that a good infiltration drainage system in the pavement structure, per se, cannot prevent frost actions as long as these conditions exist: subfreezing temperature and a high water table that provides the frost-susceptible subgrades with continual upward water supply.

Frost-susceptible soils, e.g. silty soils, have moderate hydraulic conductivity due to their pore size distribution, which promotes capillary flow. In soils with high hydraulic conductivity, e.g., coarse-grained soils, water cannot rise by capillarity to the freezing front while in clayey soils with low hydraulic conductivity, not enough water will get to the freezing front to produce significant heaving. Ice lenses are formed when enough water migrates to the freezing front and it causes excessive deformation in frost-susceptible soils. The upward displacement of soil surface due to the formation of ice lenses is called *frost heave*. The formation of ice lenses is also very sensitive to both the duration and intensity of the freezing temperature. A fast propagation of freezing front due to the quick drop of temperature will not lead to the formation of ice lenses and all water will freeze in the pores (in-situ freezing); A slow rate of cooling, however, provides sufficient time for water migration and ice lenses likely to grow.

Both the heaving and thawing phases of frost action often lead to cracking of the pavement surface, which results in annual losses in riding quality and a reduction in the life of a pavement. The damage cost due to the adverse effects of frost heave on infrastructure is estimated to be up to billions of dollars every year.

Multi-physics analysis including heat transfer, mass transfer and mechanical analysis is required to explain thoroughly the formation of ice lenses in frost-susceptible soils. The heat transfer determines the temperature profile and thus determines the primary condition for the ice lens formation (porosity growth) or volumetric expansion in freezing soil [Zha14]. Heat transfer also affects the hydraulic conductivity and pore water pressure in soil during the freezing and thawing process [Zha14]. If the soil temperature drops below the freezing point of the pore-water, some of the water freezes and releases latent heat. In the beginning, freezing of contained water only produces 9% of the volume of the amount of water already in the soil. As the pore-water continues freezing, the amount of water remaining in the liquid phase reduces and negative pore-water pressures (or suctions) develop in frost-susceptible soils. In response to the pressure and temperature gradient, water migrates in the unfrozen soil towards the freezing front where it freezes and releases latent heat. The advance of the frost front continues until the heat sink (i.e., cold surface air) and frozen ground can no longer remove the latent energy released by the water freezing at the frost front. Mechanical forces affect the volumetric strain of soil, which influences the pore-water pressure and later the thermal properties.

The mechanism of frost heave has been studied by many researchers. A semi-empirical model has been proposed by [KM81]. The two major issues for frost heave, the amount and spatial distribution of frost heave and water content distribution, were addressed using the concept of segregation potential and consolidation theory, respectively. The incremental frost heave δh is computed based on segregation potential, normal stress and temperature gradient. The total suction potential P_w was determined by assuming that the ice pressure is the same as overburden pressure [Kon87]. Many other researchers investigate the frost heave from the perspective of fundamental thermodynamic principles. The rigid ice model proposed by [OM85] successfully predicted the distribution of ice lenses and the amount of frost heave under various freezing rates and overburden pressures. In their model, the

pre-melted liquid flowing from the warm side to cold side is considered as frost heave. The coupled mass and heat transfer equations and Clapeyron equation were used for the rigid ice model. A simplified numerical model of rigid ice model was developed by [FN93] by only using two linear ordinary differential equations. The model neglected the heat transfer due to convection and gravity effect. Another model was proposed by [Gil80] to predict the frost heave rate based on applied thermal and overburden load as well as basic soil properties including particle size and thermal properties. The pre-melting mechanism was further studied by [RWW04] to predict the frost heave rate, ice lens spacing and thickness. [MZ06] proposed a material function, known as the porosity rate function, that predicts the overall frost heave of various soils. The proposed porosity rate function was the first approach, from the continuum mechanics point of view, to model frost heave. [MZ06] investigated the frost heave under various thermal and loading conditions using elastic constitutive model based on total stress analysis method. [ZM15] proposed a new frost heave model by coupling the porosity rate function with the modified Cam Clay model. The model was able to capture the frost heave and thaw settlement under various boundary conditions. However, the frost heave models described here-above cannot consider the effect of soil consolidation under surcharge on frost heave.

This study aims to present a coupled Thermo-Hydro-Mechanical (THM) model to numerically study the distribution of temperature and pore-water pressure during the freezing process as well as the deformation in the soil due to frost heave. The model takes into considerations the effect of both frost heave and soil consolidation by adapting the effective stress analysis approach for the transient problems. This can be considered as a considerable advantage for modeling frost heave and studying the effect of overburden pressure on frost-susceptible soils. The model is developed in the framework of the poroelasticity theory using a porosity rate function and thermodynamics principles and can be easily implemented to commercially available software. The numerical results are verified by

comparing with different laboratory test results found in the literature. Afterwards, the model is applied to study the distribution of temperature and pore-water pressure in the ground as well as the amount of frost heave and differential displacement at the pavement surface in two double-barrel culvert sites in Manitoba, Canada. The culvert sites have been instrumented with thermistors as part of the monitoring program of the ground thermal profile near the culvert openings since 2015 [Mou+18]. The frost heave phenomena for culvert structures is comprehensively investigated. The real weather data are used for the numerical modelling. The effect of overburden pressure and thermal variation are studied. Also, to mitigate the frost heave and differential displacement at the pavement surface, different insulating methods is considered to evaluate the effect of the position of insulating materials near the culverts on the frost heave at the pavement surface. The outcomes of this research can be used for the design of culvert structures in cold regions.

4.2 A Fully Coupled Thermo-Hydro-Mechanical Model

Conventions and common assumptions are given after which a general configuration of the problem is defined. Kinematic assumptions are given next. The constitutive models are applied to include the effects of temperature change and pore-water migration into freezing fringes on the deformation of the soil skeleton in frost-susceptible soils. Conservation principles of mass, linear momentum and energy are applied. General boundary conditions are explained finally.

4.2.1 Problem Definition

A general schematic of the problem is illustrated in Figure 4.1. The domain is composed a multilayered porous formations which can sustain frost heave. Buried utilities are assumed, here, to be rigid. Different types of the material properties can be assigned to each layers. A typical practical example can be a double-barrel culverts site having a pavement structure at the top and a culvert back-fill resting on a natural clay soil.

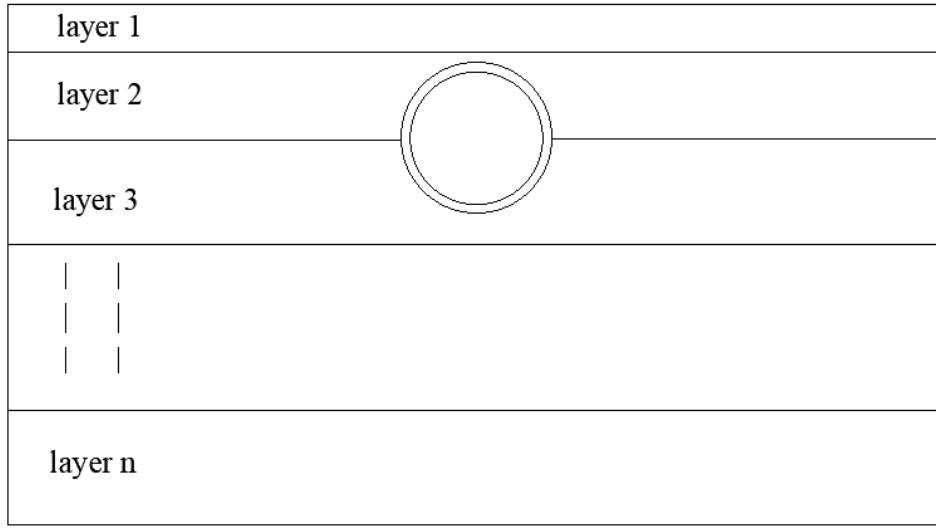


Fig. 4.1: A general configuration of the problem.

4.2.2 Conventions and Assumptions

The current state variables are the displacements, u_i , pore-water pressure, p , and temperature, T . Number of assumptions and conventions used through out such that

- The soil consists of the superposition a continuum body consisting of a deformable skeleton, s , and porous spaces filled with water, w ;
- The volumetric fractions of soil skeleton, θ_s , unfrozen water, θ_w , and ice content, θ_i , during frost heave are

$$\theta_s = 1 - n, \quad \theta_w = nS_w, \quad \theta_i = nS_i, \quad \text{and} \quad n = \theta_w + \theta_i \quad (4.1)$$

in which $n = V_v/V_{tot}$, V_v , V_{tot} , $S_w = V_w/V_v$, $S_i = V_i/V_v$, V_i and V_w are the porosity, volume of voids in a soil, the total volume, the degrees of saturation relative to water and air, the volume of water in voids and is the volume of ice in voids, respectively. An

empirical relationship is used to relate the unfrozen water content, θ_w , to the negative temperature as given by [MZ06]. Thus

$$\theta_w = \theta_r + (\theta_{w0} - \theta_r)e^{a(T-T_0)} \quad (4.2)$$

where θ_r , θ_{w0} , a and T_0 are the residual volumetric water content, the initial volumetric unfrozen water, a curve fitting parameter and the freezing temperature which is assumed to be 0 °C;

- The poroelastic medium of the skeleton is homogeneous, isotropic and linear;
- Quasi-static conditions and small transformation are considered;
- Generalized Darcy's law is valid for motion of water in freezing soils;
- Fourier's law stays valid in the range of application;
- Porosity, n , increases with the freezing of in-situ pore-water and the formation of ice lens.

The kinematic assumption are addressed next.

4.2.3 Kinematics Assumptions

The position vector of a material point on the solid skeleton at a given time t is $x_i = x_i(X_i, t)$ in which X_i is its initial position vector. Having set the displacement vector $u_i = x_i - X_i$, the deformation gradient F_{ij} becomes

$$F_{ij} = \frac{\partial x_i}{\partial X_j} = \delta_{ij} + \frac{\partial u_i}{\partial X_j} \quad (4.3)$$

where δ_{ij} is the identity tensor. The linearized form of Green-Lagrange strain tensor, ε_{ij} , for infinitesimal deformations is

$$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (4.4)$$

The skeleton's volume dilatation ε_{ii} is required to match variations of connected pore spaces (or porosity) n because of the incompressibility of solid grains. Assumed constitutive rules are described next.

4.2.4 Constitutive Models

The constitutive law of the solid skeleton describing the stress-strain relations, in a rate form, is defined in terms of effective stresses, $\sigma'_{ij} \equiv \sigma_{ij} - \alpha p \delta_{ij}$, as

$$d\sigma'_{ij} = D_{ijkl} d\varepsilon_{kl}^m \quad (4.5)$$

where σ_{ij} , α , p , D_{ijkl} and ε_{kl}^m are the total stress, the Biot coefficient, the pore-water pressure, the fourth order stiffness tensor and the strain tensor due to effective stress in a freezing soil, respectively.

Total strain increment consists of two parts due to: (i) the expel of the pore water, $d\varepsilon_{ij}^m$, and (ii) the porosity growth because of the ice lenses' formation, $d\varepsilon_{ij}^{ice}$. Subsequently

$$d\varepsilon_{ij}^m = d\varepsilon_{ij} - d\varepsilon_{ij}^{ice} \quad (4.6)$$

where $d\varepsilon_{ij}^{ice}$ is defined in terms of the porosity rate function and porosity growth tensor in section 4.2.4.

For the fluid phase, on the other hand, the adopted constitutive model in the water flux within the soil skeleton, q_i^w , is the one's of Darcy-Buckingham as

$$q_i^w = -K_{ij}^H \left(\frac{p_{,j}}{\gamma_w} + g_j z \right) \quad (4.7)$$

where $-K_{ij}^H$ and z are respectively the hydraulic conductivity tensor and a elevation head. Hence $-K_{ij}^H = -K^H \delta_{ij}$ in an isotropic situation.

The well-known Furrier's law is chosen as the constitutive equation relating temperature, T , and heat flux, q_i^T . Thus

$$q_i^T = -\lambda_T T_{,i} \quad (4.8)$$

in which λ_T is the total thermal conductivity of the soil is defined as

$$\lambda_T = \theta_w \lambda_w + \theta_i \lambda_i + \theta_s \lambda_s \quad (4.9)$$

where λ_w , λ_i , and λ_s are the thermal conductivity of water, ice, and solid skeleton, respectively.

Porosity Rate Function and Porosity Growth Tensor

The porosity rate function was introduced originally by [MZ06] to determine the soil volume increase due to the ice growth. This describes an increase in the soil porosity that is induced by the migration of pore-water into the freezing fringe where the pore-water freezes. The porosity rate function allows to take into account the effect of the ice lenses' formation on the solid skeleton deformation without introducing the ice pressure in the constitutive rule. The volume increase rate $\dot{n}$ given rise by in-situ freezing and ice lens formation, such that

$$\dot{n} = \dot{n}_m \left(\frac{T - T_0}{T_m} \right)^2 e^{1 - \left(\frac{T - T_0}{T_m} \right)^2} \left| \frac{\partial T}{\partial l} \right| \frac{1}{g_T} e^{-|\sigma|/\zeta} \quad (4.10)$$

where $\dot{n}_m$ is the maximum porosity rate. Furthermore, T_m , T_0 , T , $\left| \frac{\partial T}{\partial l} \right|$ and σ denote temperature occurring at the maximum porosity rate, freezing point of water, the soil's temperature, the temperature gradient and overburden pressure, respectively. The g_T and ζ are two constant values. The porosity growth tensor is defined to specify the growth direction of ice lenses, such that

$$\dot{n}_{ij} = \dot{n} \alpha_{ij}, \quad (4.11)$$

where

$$\alpha_{ij} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} = \begin{pmatrix} \xi & 0 & 0 \\ 0 & (1-\xi)/2 & 0 \\ 0 & 0 & (1-\xi)/2 \end{pmatrix}. \quad (4.12)$$

The value of ξ ranges from 0.33 to 1. An isotropic and one dimensional ice lenses growth correspond to $\xi = 0.33$ and $\xi = 1$, respectively. The volumetric strain due to the frost effect, $d\epsilon^{ice}$, is

$$d\epsilon_{ij}^{ice} = \dot{n}_{ij} dt. \quad (4.13)$$

The stress-strain, constitutive equation given in Equation (4.5) can be rewritten as

$$d\epsilon_{ij}^m = \frac{1+\nu}{E} d\sigma'_{ij} - \frac{\nu}{E} d\sigma'_{kk} \delta_{ij} \quad (4.14)$$

for an isotropic material where E and μ are the Young's modulus and Poisson's ratio, respectively. This equation in terms of total stress and pore water pressure becomes

$$d\epsilon_{ij}^m = \frac{1}{2G} \left(d\sigma_{ij} - \frac{\nu}{1+\nu} d\sigma_{kk} \delta_{ij} \right) + \frac{\alpha}{3K} dp \delta_{ij} \quad (4.15)$$

The constitutive equation in terms of total strains obtained by inserting equation (4.6) into the latter equation as

$$d\epsilon_{ij} = \frac{1}{2G} \left(d\sigma_{ij} - \frac{\nu}{1+\nu} d\sigma_{kk} \delta_{ij} \right) + \frac{\alpha}{3K} dp \delta_{ij} + d\epsilon_{ij}^{ice}. \quad (4.16)$$

Conservation laws of linear momentum, mass and energy are implemented in the following section.

4.2.5 Conservation Laws

The conservation of linear momentum in absence of inertial terms becomes

$$\sigma_{ji,j} + f_i = 0 \quad (4.17)$$

where $\rho = \rho_w \theta_w + \rho_i \theta_i + \rho_s \theta_s$ is the bulk density of soil. The ρ_w , ρ_i , and ρ_s are the density of water, ice and solid skeleton, respectively. The $f_i = -\rho g_i$ where g_i is the gravitational acceleration vector. The mass of the solid skeleton in a representative elementary volume is

$$M_s = \int_V (1 - n) \rho_s dV \quad (4.18)$$

where $(1 - n) \rho_s$ represents the volume density of solid skeleton mass in the current configuration. The conservation of mass requires that $dM_s/dt = 0$, so

$$\frac{\partial n}{\partial t} = (1 - n) \dot{u}_{i,i}. \quad (4.19)$$

The mass conservation equation for pore-water in a representative elementary volume of a freezing soil can be written as

$$\frac{\partial n}{\partial t} + q_{i,i}^w + S = 0. \quad (4.20)$$

In a fully saturated condition $n = \theta_w + \theta_i$ represents the total volumetric water (unfrozen and frozen) content and S is a source term. Conservation of energy by considering the phase change becomes

$$C_{app} \frac{\partial T}{\partial t} + q_{i,i}^T = 0 \quad (4.21)$$

where a conduction flux through the volume boundaries, q_i^T , exists. Moreover

$$C_{app} = C_{ph} - \rho_i L_f \frac{\partial \theta_i}{\partial T}$$

is the apparent volumetric heat capacity in which

$$C_{ph} = \rho_w \theta_w C_{pw} + \rho_i \theta_i C_{pi} + \rho_s \theta_s C_{ps}$$

is the volumetric heat capacity of a soil. The C_{pw} , C_{pi} and C_{ps} are the specific heat capacity of water, ice and soil grains, respectively. The value of latent heat L_f is 333 kJ/kg .

4.2.6 Boundary Conditions

Three sets of boundary conditions are enforced relating to the displacement and Cauchy stress vectors; heat flux and temperatures; and water flux and pore pressures. Two sets of vectors must stay continuous at any given traction surface, $\partial\Gamma$. They are the Cauchy stress vector, $t_i \equiv \sigma_{ij} n_j$ where n_j is a normal unit vector to the surface and the particle displacement vector, u_i . The heat communication boundary conditions, on the other hand, are generally of three types: prescribed temperatures, T , heat flux, $T_{,ni} \equiv -q_0/k_T$, and convective heat exchange, $\alpha_k(T - T_f)$. Finally, the prescribed pore water pressure, p and water flux $p_{,n} = -q_0^w/k_w$ are applied as the hydraulic boundary conditions.

The coupled system of the constitutive (4.2.4), kinematic (4.2.3) and conservation (4.2.5) equations along with the pertinent boundary conditions (4.2.6) are implemented simultaneously in the commercial finite element package COMSOL Multiphysics.

4.3 Model Verification

The THM model developed in this study are verified by comparing the numerical results with two sets of experimental data found in the literature as well as the temperature field data as explained in section 4.4.3.

The temperature distribution in the soil sample as well as the amount of frost heave at the surface of the sample are compared with experimental test performed by [Kon05]. He performed a freezing experimental test using a limestone with an initial water content of 21%. The soil column was subject to a surcharge of 20 kPa . The initial soil column was

97mm. The temperature of the soil column at the top and bottom was kept at $-4.1\text{ }^{\circ}\text{C}$ and $1.5\text{ }^{\circ}\text{C}$, respectively. The initial temperature was $3.6\text{ }^{\circ}\text{C}$. The saturated samples were frozen in the axial direction from the top downwards. Water was supplied freely at the warm end. Freezing condition was applied until the steady state condition was reached. During this test, only the temperature profile along the soil sample and the total frost heave at the top of the sample were monitored.

Furthermore, the pore-water pressure at the base of the soil column and frost heave at the top of the soil column are verified with experimental test results provided by [Eri15]. The sandy-silt soil was used to conduct the freezing test to examine the frost heave. The 0.2m -height soil column had an initial void ratio of 0.53 (initial porosity of 0.35). The soil sample was initially saturated. The initial soil temperature was $4.5\text{ }^{\circ}\text{C}$. A constant temperature of $-3.0\text{ }^{\circ}\text{C}$ and $3.2\text{ }^{\circ}\text{C}$ was applied to the top and bottom surface, respectively. The applied overburden pressure was 20 kPa . The entire test lasted for 212 hours. The test was conducted without external water supply.

In the numerical simulation performed by the THM model developed in this study, the input values are as follows:

- In the unfrozen water content curve (Eq.4.2): $\theta_r = 0.058$, $T_0 = 0\text{ }^{\circ}\text{C}$ and $a = 0.16\text{ }^{\circ}\text{C}^{-1}$
- In the porosity rate function (Eq. 4.10): $\dot{n}_m = 6.02 \times 10^{-5}\text{ s}^{-1}$, $T_m = -0.87\text{ }^{\circ}\text{C}^{-1}$, $g_T = 100\text{ }^{\circ}\text{C}/\text{m}$ and $\zeta = 0.73\text{ MPa}$
- In the poroelastic constitutive model: $\alpha = 0.99975$, the frozen and unfrozen Young's module of the limestone is assumed to be $4,000\text{ kPa}$ and $10,000\text{ kPa}$; for sandy-silt soil, Young's module is assumed to be $1,000\text{ kPa}$ and $5,000\text{ kPa}$
- The hydraulic conductivity is $K_H = 10^{-10}\text{ m/s}$ is used for of the limestone; For sandy-silt soil, the hydraulic conductivity is defined as a smoothed step function in which the

unfrozen and frozen hydraulic conductivity is $10^{-7} m/s$ and $10^{-10} m/s$, respectively. The transitional zone is defined as $2.5K$

- For the thermal analysis, the typical values for the thermal properties of limestone can be seen in Table 4.1

Tab. 4.1: Soil thermal properties

Material	Density (kg/m^3)	Thermal conductivity ($W/(m \cdot K)$)	Heat capacity ($J/(kg \cdot K)$)
soil skeleton	2,620	1.95	900
Water	1,000	0.56	4,180
Ice	917	2.24	2,000

The experimental and numerical results for the the frost heave at the top of the soil sample over the testing period are compared in Fig. 4.2. The frost heave increases as more free water migrates to the freezing front to form ice lenses. As can be seen, the THM numerical results show a good agreement with the experimental data.

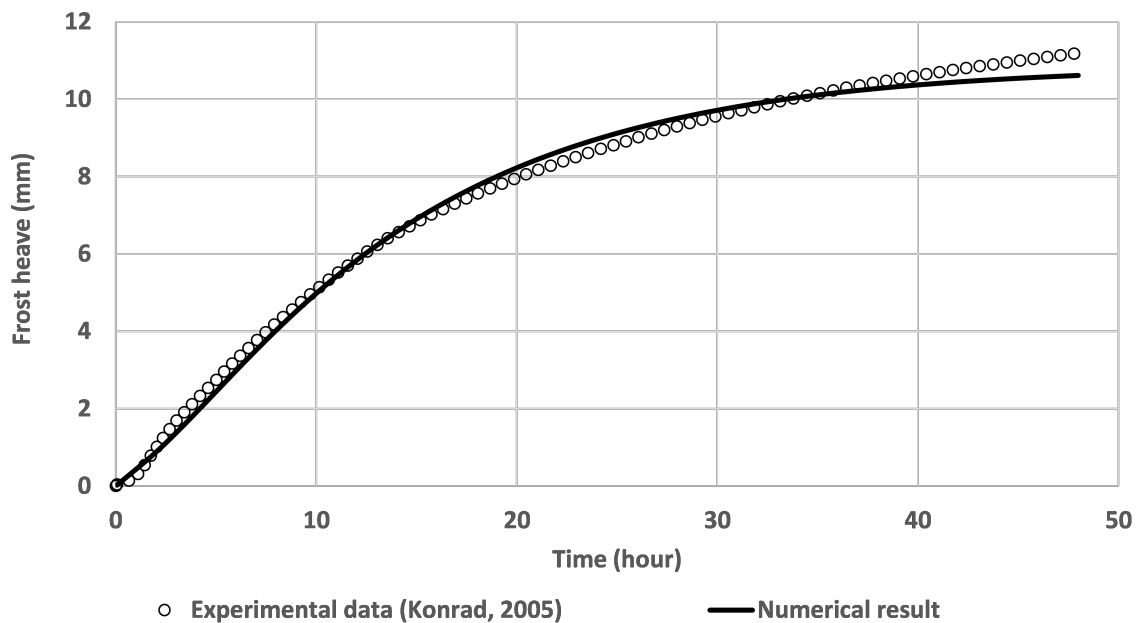


Fig. 4.2: Frost heave during testing period from experimental and numerical results for Konrad's test [Kon05]

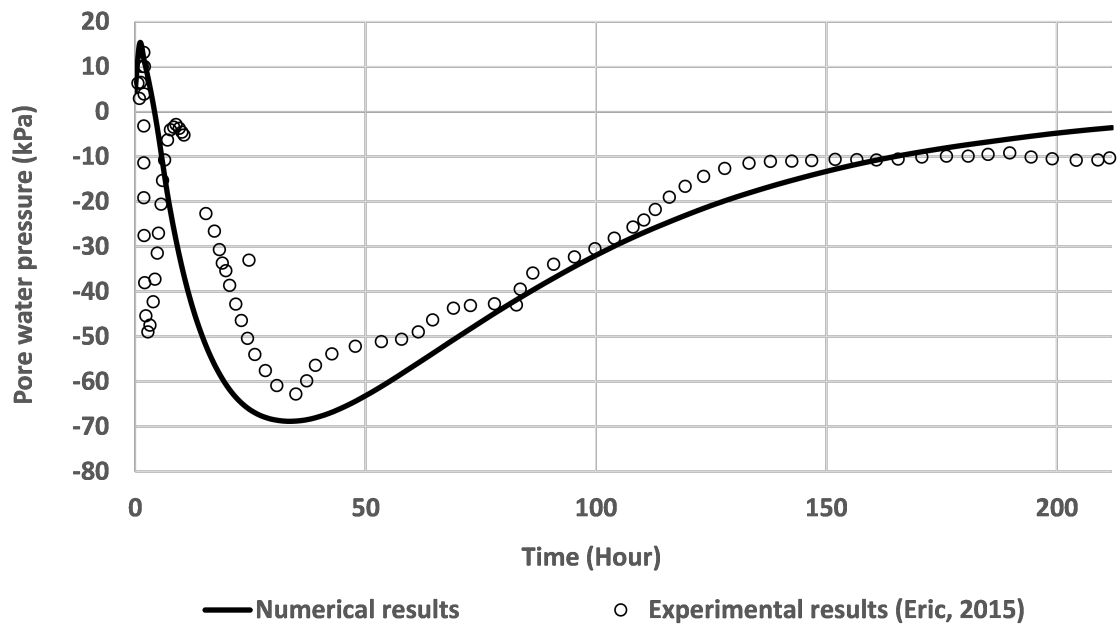
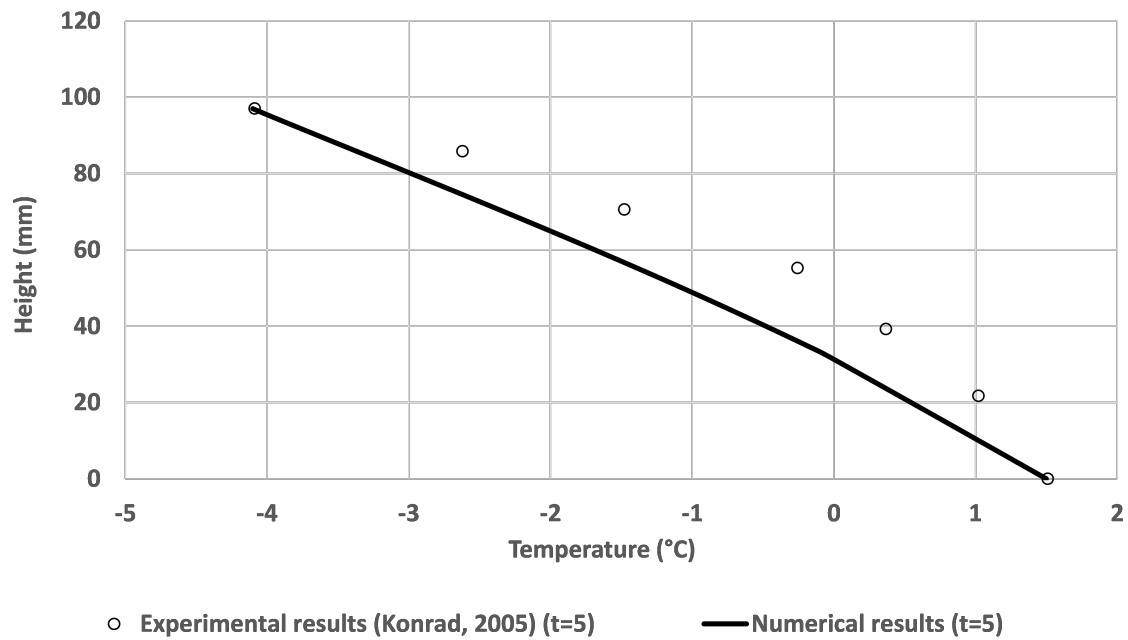
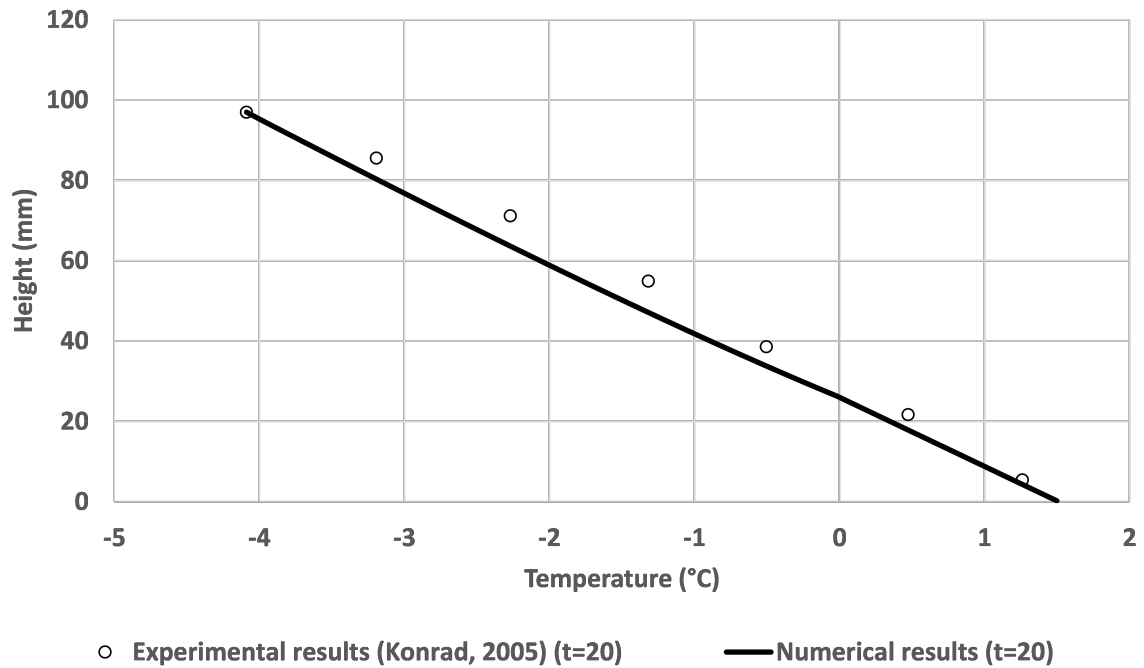


Fig. 4.3: Pore-water pressure distribution at the base during testing period from experimental and numerical results [Eri15]



(a) Temperature distribution after 5 hours



(b) Temperature distribution at 20 hours

Fig. 4.4: Comparison between the numerical (solid line) and the experimental results (points) obtained by Konrad (2005) for the temperature distribution after 5 hours (b) and 20 hours (c) [Kon05]

Similarly, Fig. 4.4 illustrates the corresponding temperature distribution along the soil sample at different time. As can be seen, the numerical model predicts reasonably the temperature distribution in the soil column.

The distribution of pore-water pressure at the base of the soil column during the test can be seen in Fig. 4.3. Initially, the pore-water pressure at the base is at the hydro-static pressure. With an applied overburden pressure of 20 *kPa* combined with frost heave at the top, the pore-water pressure at the base started to increase to 15 *kPa*. As the frost heave became dominant, the negative pore-water pressure (suction) occurred. The maximum suction occurred when the the frost heave rate was maximum. The suction decreased as the frost heave rate declined.

4.4 THM Modeling of Frost-Susceptible Soils in Double-Barrel Culvert Sites: A Case Study

The THM model developed in this study has been used to predict the distribution of temperature and pore-water pressure in frost-susceptible soils as well as the amount of frost heave and differential displacement at the pavement surface in two double-barrel precast culvert sites in Manitoba, Canada.

4.4.1 Field Monitoring Program

The double-barrel culvert sites studied in this paper is part of a comprehensive project consists of five instrumented culvert sites located along the provincial trunk highway 68 (PTH-68), between the Town of Arborg and Lake Winnipeg in Manitoba, Canada. The project aims to study different innovative culvert construction methods including the effect of the transition slope between the natural soil and backfill of the culverts, backfill materials, and the application of geosynthetics to reduce the road roughness and deformation over culverts. For more information about the features of each site, see [Kav+17] and [Mou+18]. Two project sites studied in this paper (site 4 and site 5) are illustrated in Fig. 4.5.

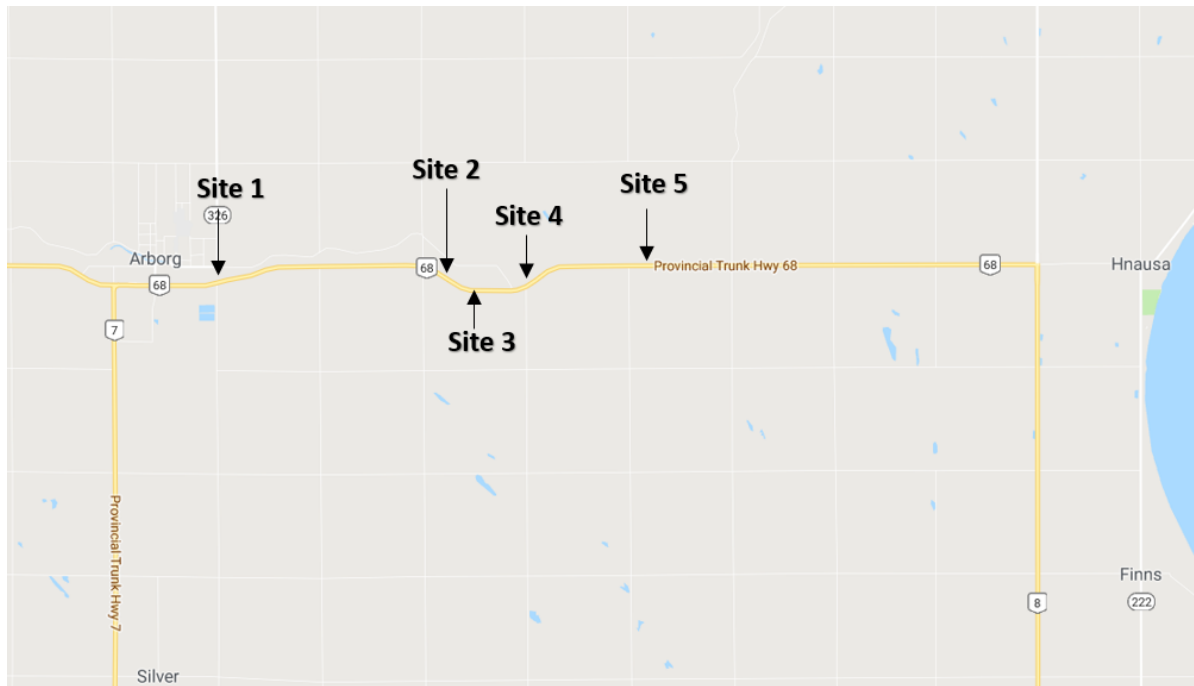
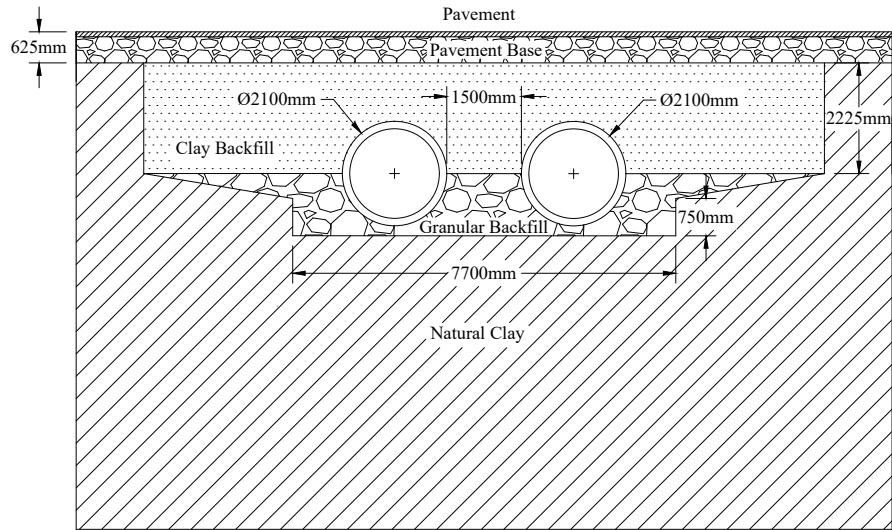


Fig. 4.5: Location of project site 4 and site 5

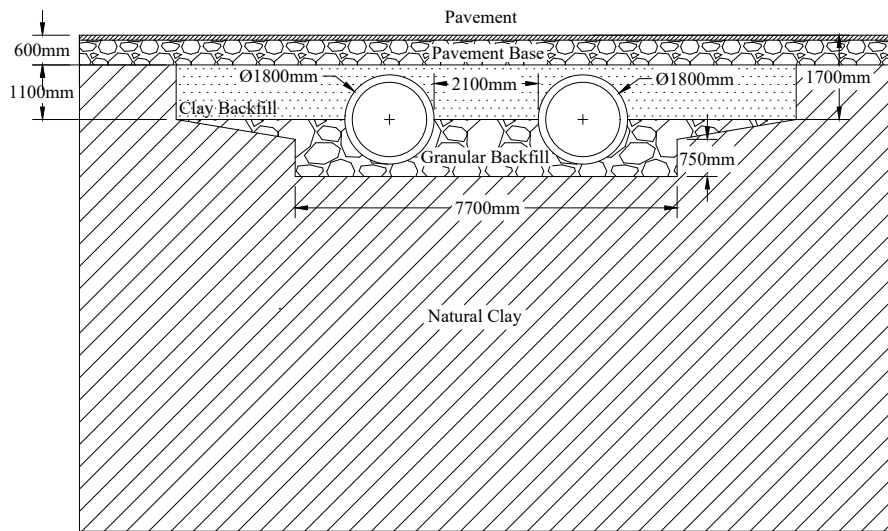
Both sites have a pavement structure comprising of 100 *mm* asphalt layer and 525 *mm* and 500 *mm* granular base for sites 4 and 5, respectively. The granular base in both sites is underlain by the culvert backfill rest on high plastic clay subgrade classified as (A-7-6) according to the AASHTO soil classification system. The culvert backfill in the project site 4 is composed of a 2,225 *mm* clay backfill followed by a 1,250 *mm* granular backfill while in the project site 5, the culvert backfill is composed of a 1,100 *mm* clay backfill underlain by a 1,150 *mm* granular backfill. In sites 4 and 5, the double-barrel culverts are located at 1.8 *m* and 0.8 *m* below the road surface, respectively. The double-barrel culverts have an outer diameter of 2.1 *m* and 1.8 *m* in site 4 and site 5, respectively. The thickness of the culverts in both sites is 15 *cm*.

The instrumentation program consists of monitoring the ground thermal profile near the culverts openings by the equally-spaced thermistors, which are placed around the culvert barrels in U-shaped and L-shaped strings [Mou+18].

The detailed cross-sectional layout of the pavement/culvert structure for the project sites 4 and 5 are shown in Fig. 4.6.



(a) Project site 4



(b) Project site 5

Fig. 4.6: Cross-sectional profile of the road structure for the (a) project site 4 and (b) project site 5

4.4.2 Numerical Modeling

The THM model is applied to evaluate the distribution of temperature in the road structure, the pore-water pressure at the top of the clay subgrades, and frost heave and differential deformation at the pavement surface in the project sites 4 and 5. Since only the thermal profile is monitored in these sites, the numerical results are partly verified by field measurement data. Furthermore, the model is applied to assess the effect of insulation located close to the culvert openings on the frost heave and differential displacement on the pavement surface. The conclusions can be used for the evaluation of the road roughness.

It is assumed that the clay backfill is free from frost heave because the well-compacted clay has a very low hydraulic conductivity so that it is almost impossible for free water to reach the freezing front. Meanwhile, the granular backfill at the bottom of the clay backfill is able to drain water instantaneously. Consequently, without the continuous water supply, the frost heave is unlikely to occur in the the clay backfill.

Initial and Boundary conditions

- Thermal Initial and Boundary Conditions

An external forced convection is considered as boundary conditions at the pavement and culvert surface. The convection term, q_{conv} , is temperature- and time-dependent and can be calculated using the following equation:

$$q_{conv}(t) = h_{conv} (T_a(t) - T_s(t)) \quad (4.22)$$

where $T_a(^{\circ}C)$ is the ambient air temperature and $T_s(^{\circ}C)$ is the temperature at the pavement and culverts surface. The convection heat transfer coefficient, h_{conv} , depends mainly on conditions on the boundary layer such as wind speed and ambient temperature [I+].

An adiabatic condition is considered at both vertical sides and the bottom of the domain.

Tab. 4.2: Material thermal properties

Material	Thermal conductivity ($W/(m \cdot K)$)	Heat capacity ($J/(kg \cdot K)$)
Asphalt	1.8	674
Granular base	1.5	1200
Clay backfill	1.1	2,000
Granular backfill	1.5	1,200
Precast culvert	0.25	1,174

The thermal properties used in the numerical analysis for different soil layers and the precast culvert are presented in table 4.2.

In this analysis, the initial ground temperature is obtained by a sensitivity analysis. The simulation, by considering different initial ground temperatures, is carried out for 10 years (2005-2015) based on Winnipeg weather data. It was seen that regardless of the initial value, the ground temperature profile reaches the equilibrium and becomes stationary after 10 years. Therefore, the temperature distribution at equilibrium is considered as the initial temperature condition in the simulations (Fig. 4.7).

- Hydraulic Initial and Boundary Conditions

In this analysis, the pore-water pressure is assumed to be initially hydrostatic. The pavement structure is assumed to be dry (drained condition). Also, since the clay backfill is underlain by a granular backfill, it is supposed that a drained condition is applied to this layer. On the other hand, it is assumed that the drainage system works ideally and no water can infiltrate to the clay subgrade. Consequently, it is assumed that the ground water table is situated at the bottom of the granular backfill layer. The permeable layer is defined at the soil bottom because of the high water table in the field. This permeable boundary condition allows the continues water supply to the freezing fringe. Both vertical sides of the soil domain are considered impervious (i.e. no water flux).

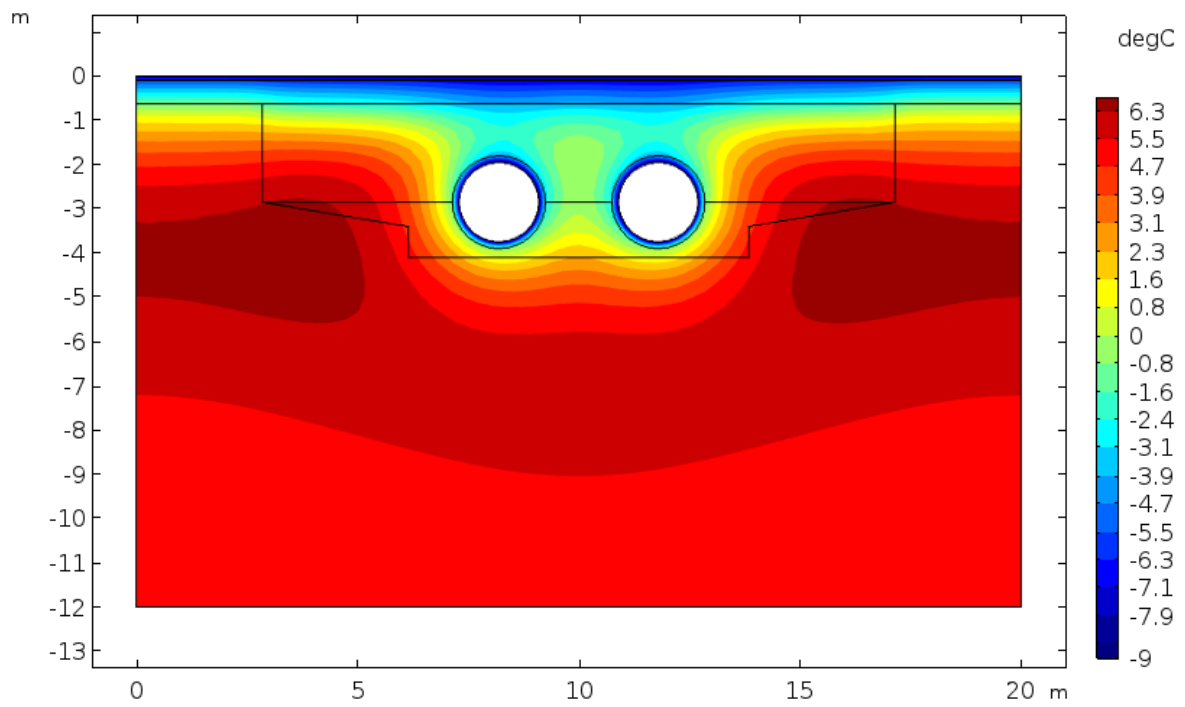


Fig. 4.7: Initial temperature distribution

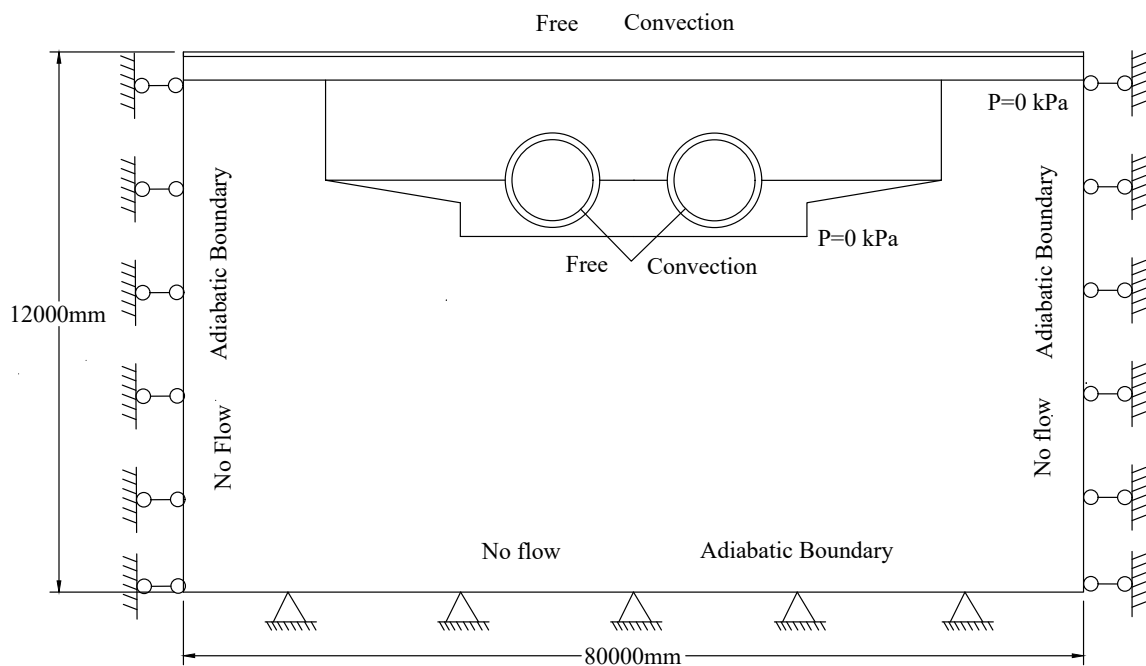


Fig. 4.8: Summary of boundary conditions

- Mechanical Initial and Boundary Conditions

It is considered that the culverts and the pavement surface are free to move. The lateral boundaries of the domain are only allowed to move vertically. A fixed constraint is applied to the bottom of the domain.

The boundary conditions considered in the analysis are summarized in Fig. 4.8. The mechanical properties used in the numerical analysis for different soil layers and the precast culvert can be seen in table 4.3. It should be noted that the contour plots presented hereafter only illustrate the field close to the culverts openings (a total width of 20 m) to highlight their effect.

Tab. 4.3: Material mechanical properties

Material	Modulus of Elasticity (MPa)	Poisson's ratio (-)	density (kg/m ³)
Asphalt concrete	2.7×10^4	0.2	2,371
Granular base	200	0.45	1,500
Clay backfill	100	0.34	1,600
Granular backfill	200	0.45	1,500
Culvert	2×10^4	0.15	3,200

The input values in the THM model for the clay subgrade are: 1) in the unfrozen water content curve (Eq. 4.2): $\theta_r = 0.058$, $\theta_{w0} = 0.29$, $T_0 = 0^\circ\text{C}$ and $a = 0.16^\circ\text{C}^{-1}$; 2) in the porosity rate function (Eq. 4.10): $\dot{n}_m = 5.78 \times 10^{-7} \text{ s}^{-1}$, $T_m = -0.87^\circ\text{C}^{-1}$, $g_T = 100^\circ\text{C}/\text{m}$ and $\xi = 0.73\text{MPa}$; 3) in the poroelastic constitutive model: $\alpha = 0.99975$, the unfrozen and frozen Young's module is assumed to be 5,000 kPa and 10,000 kPa; The hydraulic conductivity is defined as smoothed step function in which the unfrozen and frozen hydraulic conductivity is 10^{-8} m/s and 10^{-12} m/s , respectively. The transitional zone is defined as 2K.

4.4.3 Results and Discussions

Project Site 4: Culverts with a Deeper Burial Depth

This section aims to study the frost heave of culverts with a deeper burial depth. The effect of burial depth on temperature distribution, frost heave and corresponding pore-water pressure distribution will be investigated.

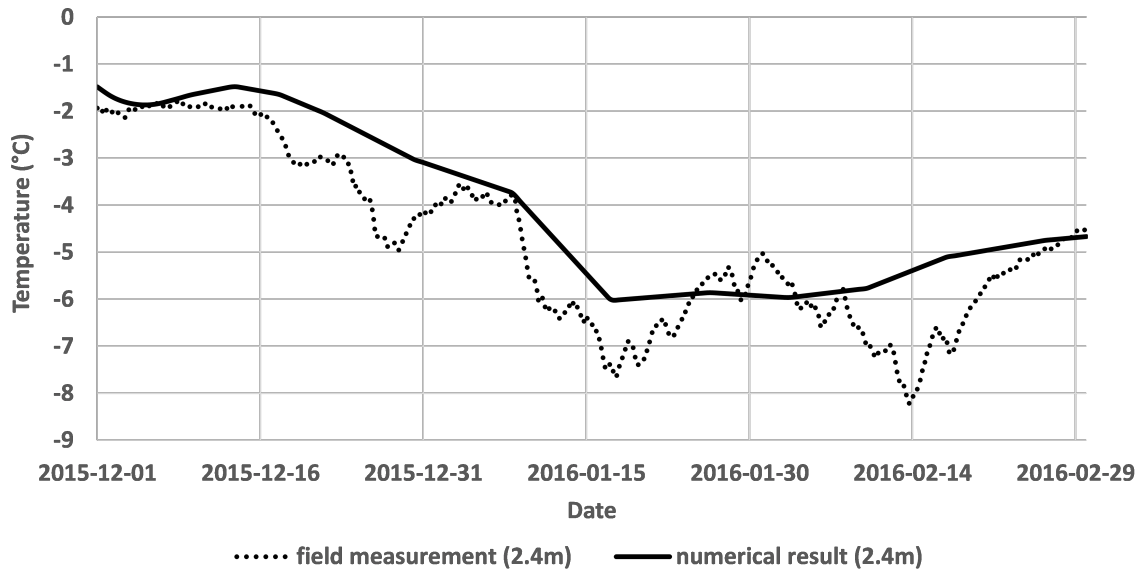
- Temperature Distribution: Verification:

The temperature distribution obtained by the numerical simulation (THM model) is compared with field measurement data. For the sake of brevity, only the results for the project site 4 at depths of 2.4 m and 4.0 m below the pavement surface are illustrated in Fig. 4.9. According to this figure, the numerical results show a good agreement with the field measurement data. The temperature profile at a depth of 1.8 m has the lowest values in comparison to deeper profiles due to its closeness to the cooling sources (pavement surface and culverts openings).

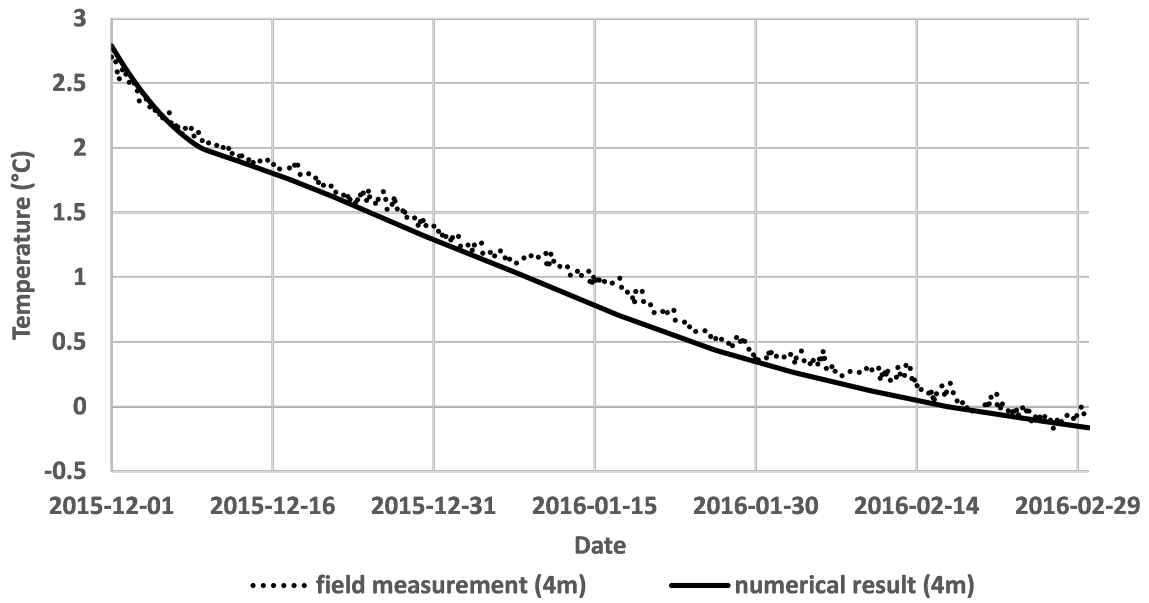
- Frost Heave and Differential Displacement:

The frost heave in clay subgrade and the corresponding differential displacement at the pavement surface is studied. As temperature drops below the pore-water freezing point in the soil, the pore-water starts to freeze. This leads to a pore-water pressure gradient and consequently water movement in the soil. As more free pore-water migrates to the freezing front, the ice lenses form, which is visualized as frost heaving. The frost heave in clay subgrade induces the differential displacement at the pavement surface.

Fig. 4.10 illustrates the deformation (magnified 10 times) and the temperature distribution in the domain after 60 days and 90 days from the start of the freezing period (December 1st, 2015) in the project site 4.

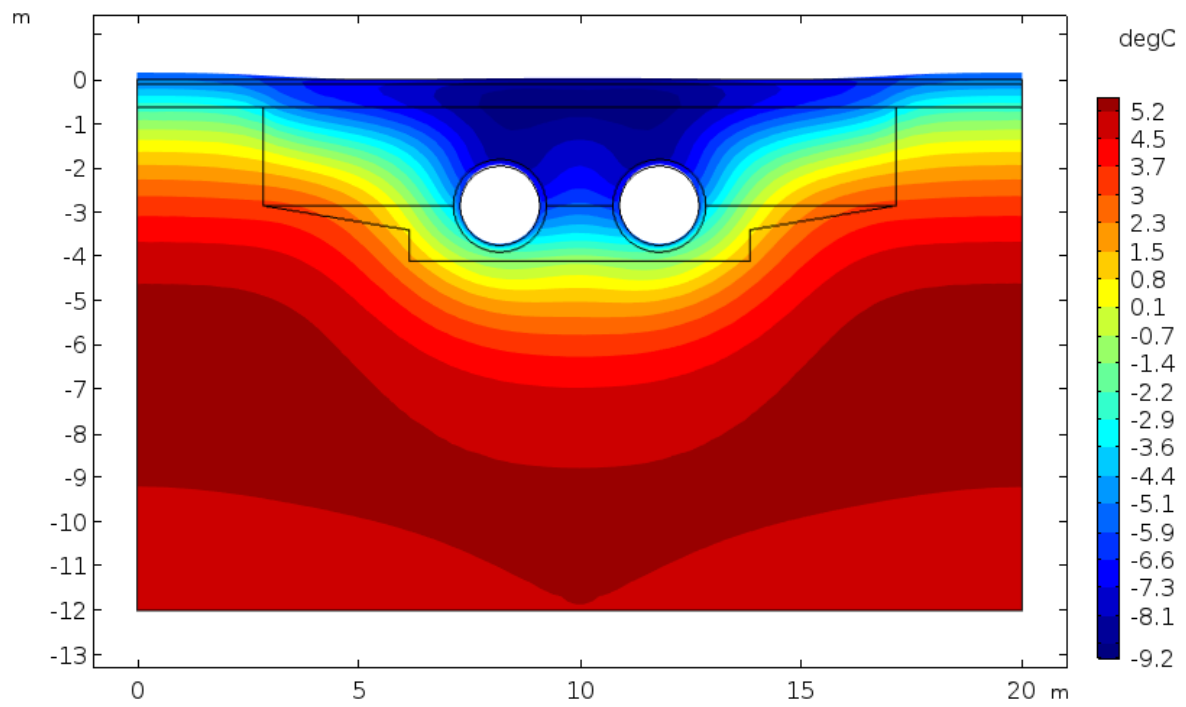


(a) Temperature distribution at 2.4 m depth

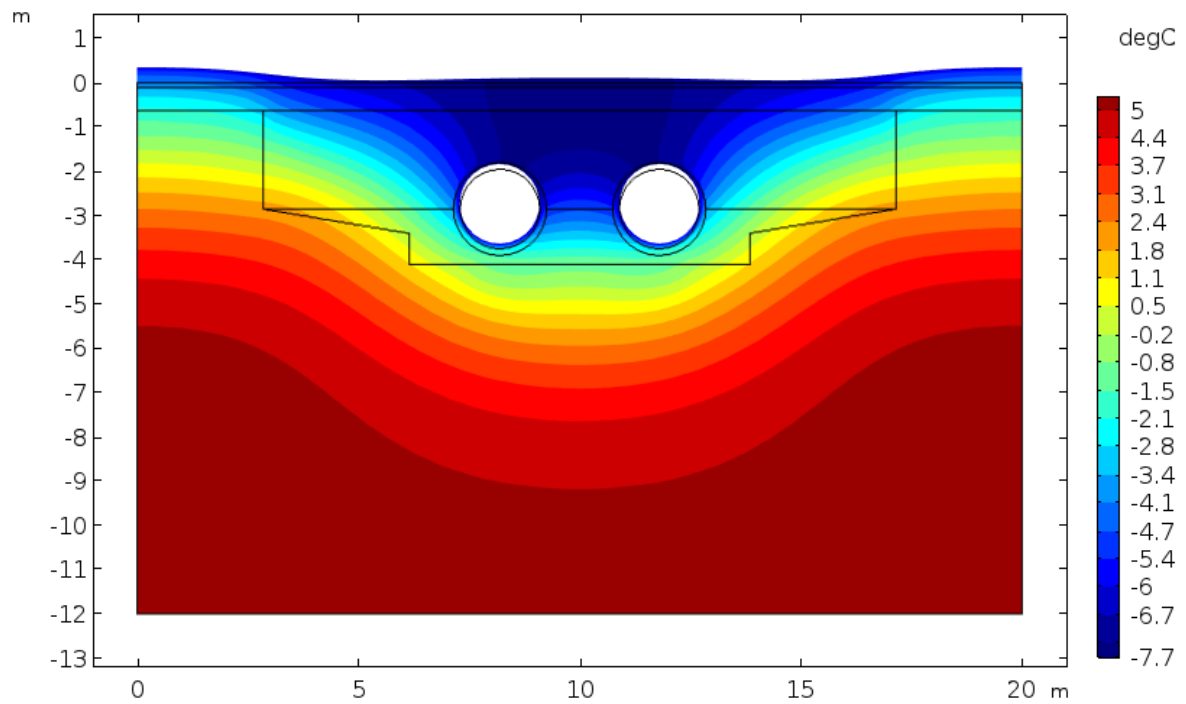


(b) Temperature distribution at 4 m depth

Fig. 4.9: Comparison between the numerical (solid line) and the field measurement results (dot line) for the temperature distribution at a depth of 2.4 m (b) and 4 m (c)



(a) Deformation and temperature distribution after 60 days



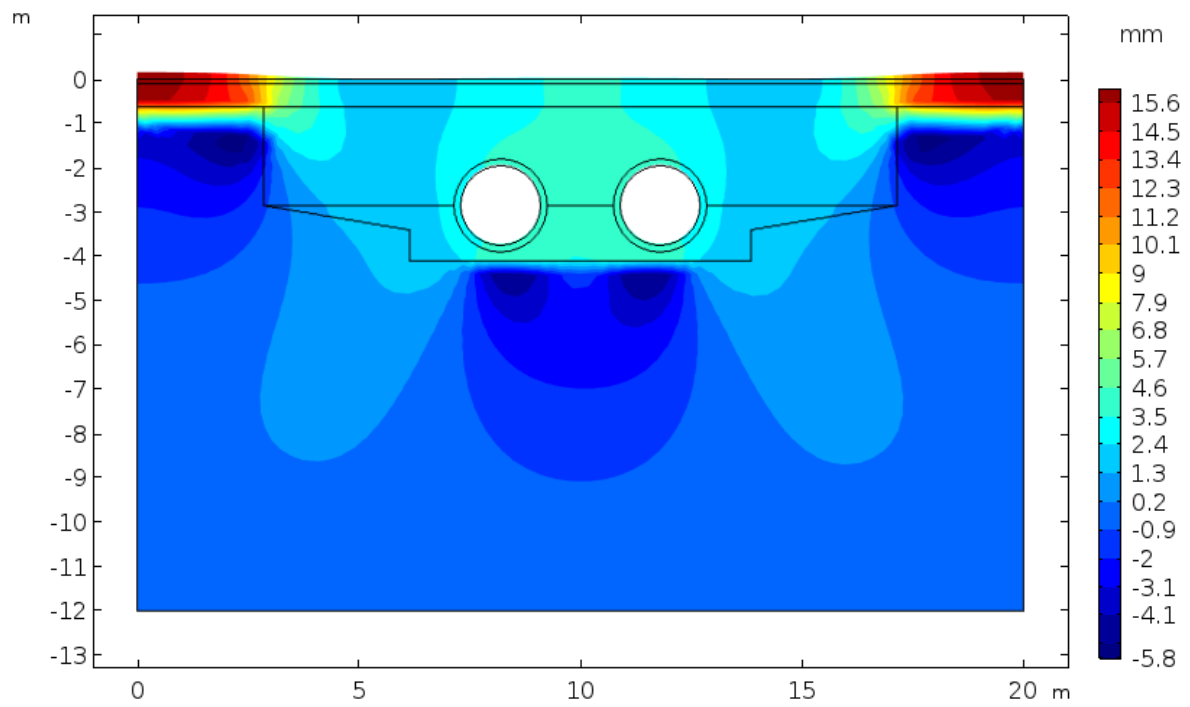
(b) Deformation and temperature distribution after 90 days

Fig. 4.10: Deformation and temperature distribution (a) after 60 days (b) after 90 days in the project site 4

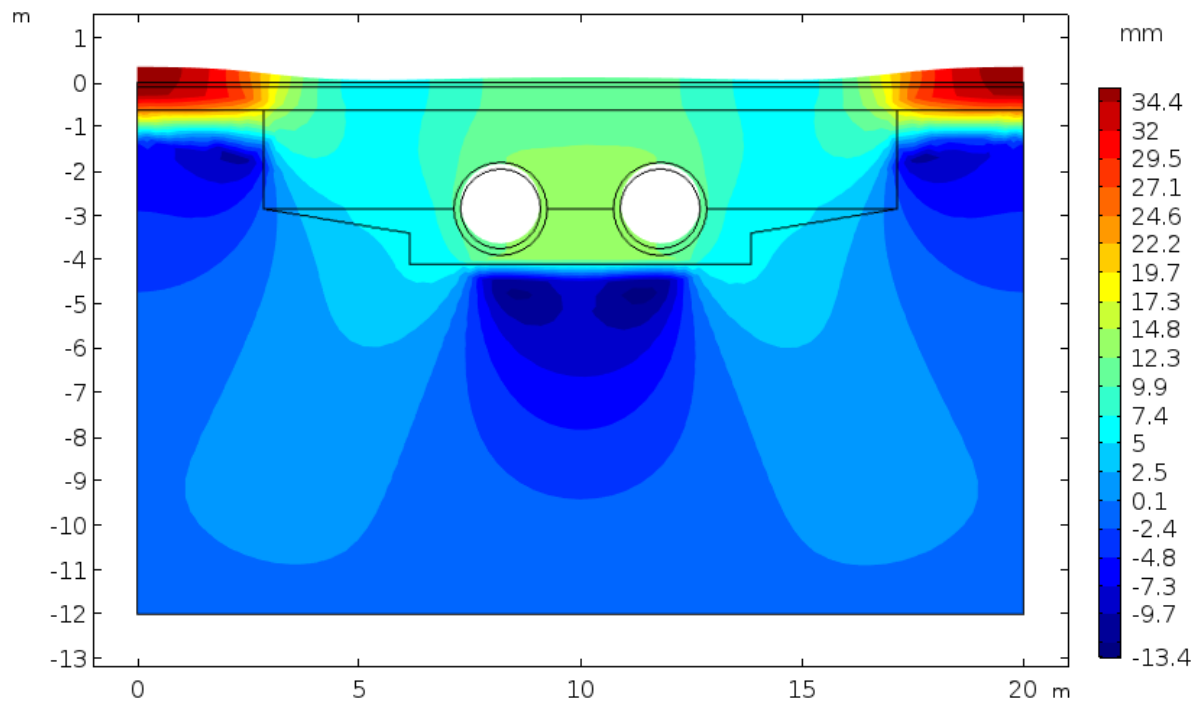
Initially, the temperature in the clay subgrade is above the freezing point everywhere in the domain. Due to the heat convection at the surface of pavement and culverts openings, the temperature of the soil located just below the culverts starts to decrease below the freezing point. As can be seen in Fig. 4.10, this freezing effect is accompanied with the frost heave lifting the culverts. To give a better insight into the deformation caused by frost heave, the corresponding vertical displacement contours are depicted in Fig. 4.11.

As can be seen in Figs. 4.10 and 4.11, the displacement (frost heave) is mainly affected by a combination of temperature distribution and overburden pressure. The culverts openings caused a deeper frost depth in the middle of the domain in comparison to both sides of the domain. It is worth noting that, to help the structural integrity of the culverts, the frost-susceptible soils are replaced by granular materials above the culverts. This results in a higher overburden pressure on the clay subgrade below the double-barrel culverts in comparison to side profiles away from the culvert structure due to the heavier weight of such materials as well as the weight of culvert structures. The frost-susceptible soils at both sides of the domain are in the direct contact with the outdoor subfreezing temperatures through the pavement surface. The presence of subfreezing temperature as well as the continual water supply provide the frost-susceptible soils at both sides of the domain all the requirements for frost heave. Furthermore, it is important to mention that due to the higher thermal gradient and smaller amount of overburden pressure, the frost heave develops faster and is more significant at both sides of the domain in comparison to the natural soil beneath the granular backfill.

The cumulative displacement along the road surface, after 90 days from the start of the freezing period, is illustrated in Fig. 4.12. It can be seen that frost heave, at the pavement surface, at a distance of 10 *m* away from the centerline and at the mid-distance between the culvert structures are 31 *mm* and 12 *mm*, respectively after 90 days.



(a) Deformation and pore-water distribution after 60 days



(b) Deformation and vertical displacement after 90 days

Fig. 4.11: Deformation and vertical displacement distribution (a) after 60 days (b) after 90 days in the project site 4

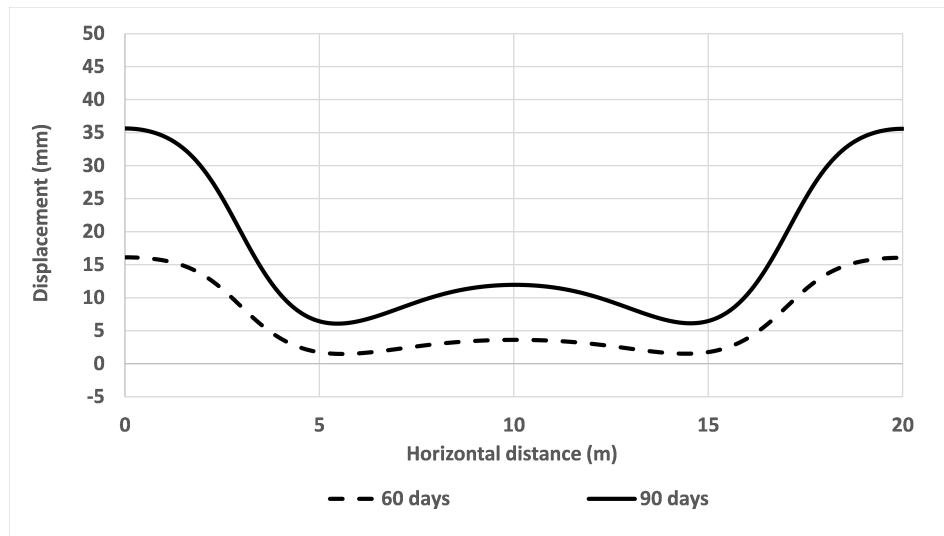


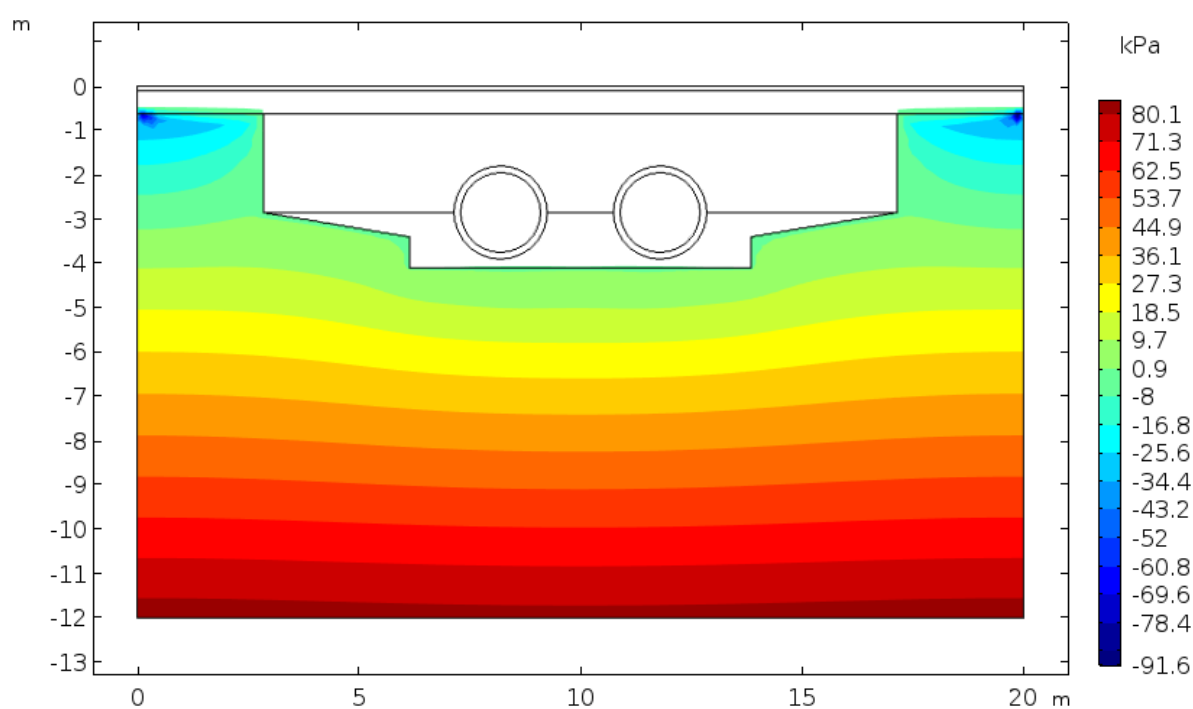
Fig. 4.12: Frost heave along the pavement surface

This conclusion is in accordance with field observations that culverts seem to settle during the freezing period. This can be explained from Figs. 4.11 and 4.12 that the more significant upward displacement due to frost heave at the sides of the culverts in comparison to regions limited by the culvert structure can mislead to the settlement of the culvert. Furthermore, it can be seen that the double-cooling effect induced by the culverts openings is able to affect a distance of 10 m at both sides of the mid-distance between the culverts. On the other hand, the uneven differential deformation at the road surface is more significant in an interval of 20 m around the culvert structures.

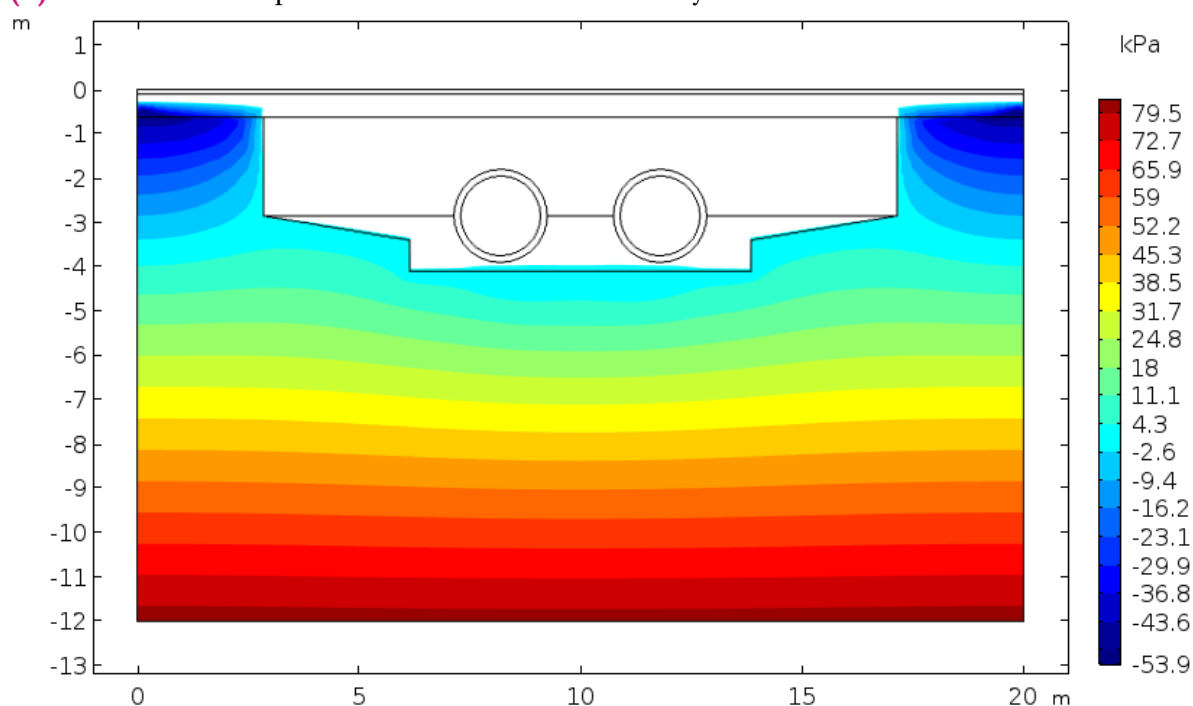
- Pore-Water Pressure Distribution

The distribution of pore-water pressure in soil is important in terms of the soil strength and the bearing capacity of the pavement foundation. The freezing of pore-water generates negative pore-water pressure or suction.

It can be seen from Fig. 4.14 that suction is quickly built up in the side soils due to the faster heat loss. The maximum suction is estimated to be -35 kPa and -50 kPa after 60 days and 90 days, respectively.



(a) Deformation and pore-water distribution after 60 days



(b) Deformation and pore-water distribution after 90 days

Fig. 4.13: Deformation and pore-water distribution in soil subgrades (a) after 60 days (b) after 90 days in the project site 4

Project Site 5: Culverts with a Shallower Burial Depth

This section aims to study the frost heave of culverts with a shallower burial depth. The effect of burial depth on temperature distribution, frost heave and corresponding pore-water pressure distribution will be investigated.

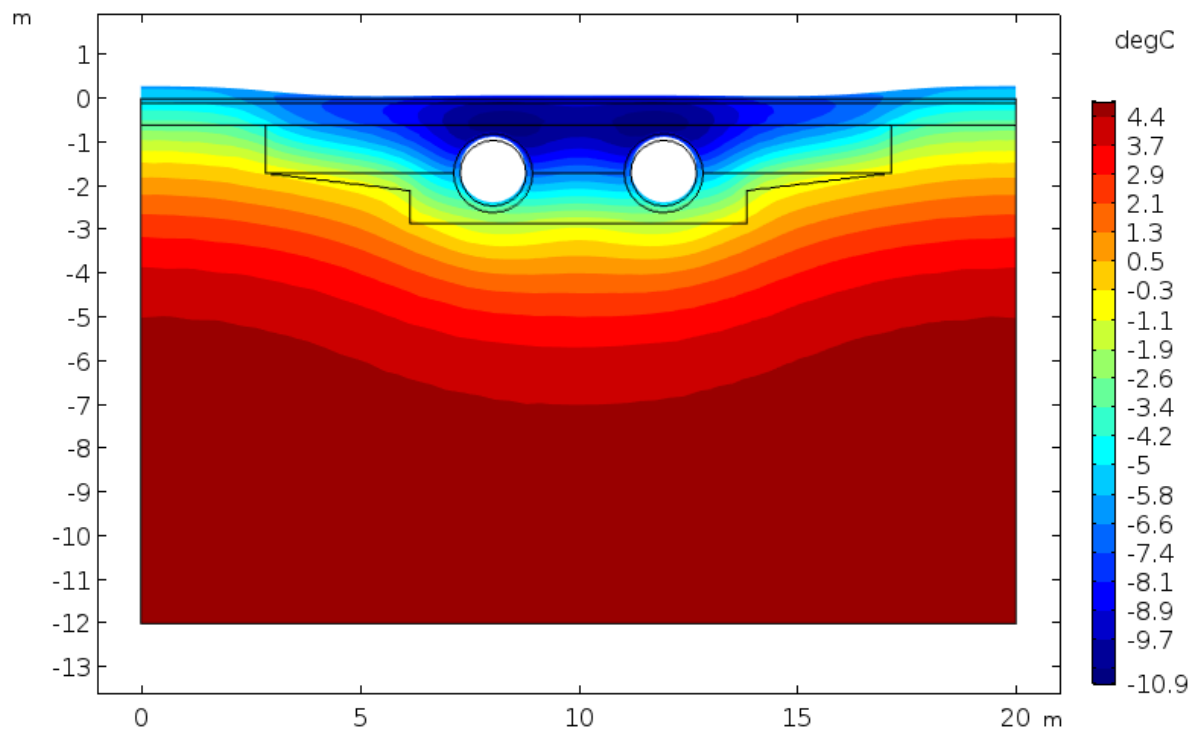
- Temperature and Deformation of Road Structure during Freezing

The temperature and deformation contours for the project site 5 after 60 and 90 days from the start of the freezing period are shown in Fig. 4.14.

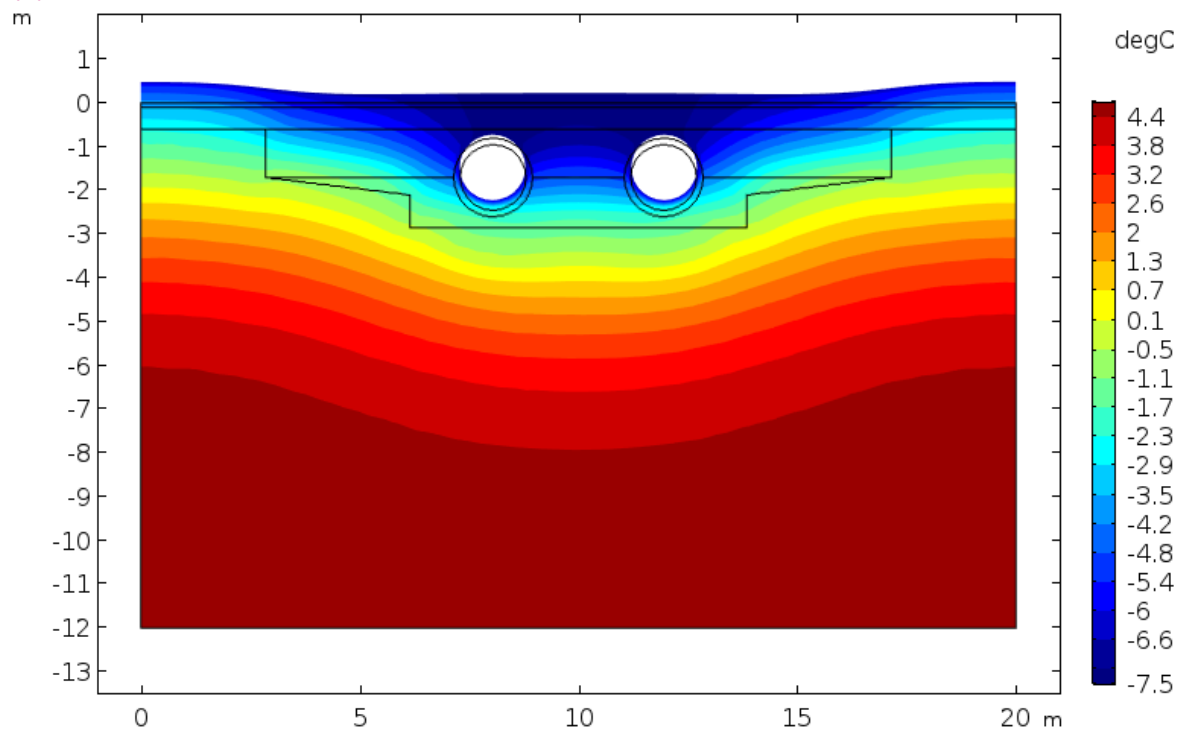
Since the culverts openings in the project site 5 are closer to the road surface, the effect of the heat exchange between the outside air and soil through the culverts' wall attenuates faster and is less significant in depth. That is the reason why the frost depth is almost 1 *m* shallower in the project site 5 in comparison with the site 4 with a deeper burial depth. Also, in the project site 5 the thickness of the road structure including granular backfill materials is smaller compared with the project site 4. This leads to a decrease in overburden pressure on the clay subgrade in the project site 5 compared with the site 4.

The combination of the subfreezing thermal profile and smaller overburden pressure induces higher values of displacement at the pavement surface above the culverts in the project site 5. The corresponding vertical displacement contour and vertical displacements at the pavement surface are shown in Fig. 4.15 and Fig. 4.16, respectively.

In comparison with the project site 4, the frost heave at the pavement surface at 5 *m* distance from the mid-distance between the culverts is increased from 6 *mm* to 13 *mm* after 90 days from the start of freezing. Similarly, the frost heave increased from 12 *mm* to 17 *mm* at the mid-distance between the culverts. The shallower granular backfill applies less overburden pressure to the frost-susceptible soil while, at the same time, the negative temperature is damped more quickly in the depth in comparison to the project site 5. This combined effect, the double-cooling source and the culverts' burial depth, is the reason of almost unchanged frost heave at 10 *m* away from the centerline in comparison with the project site 5.

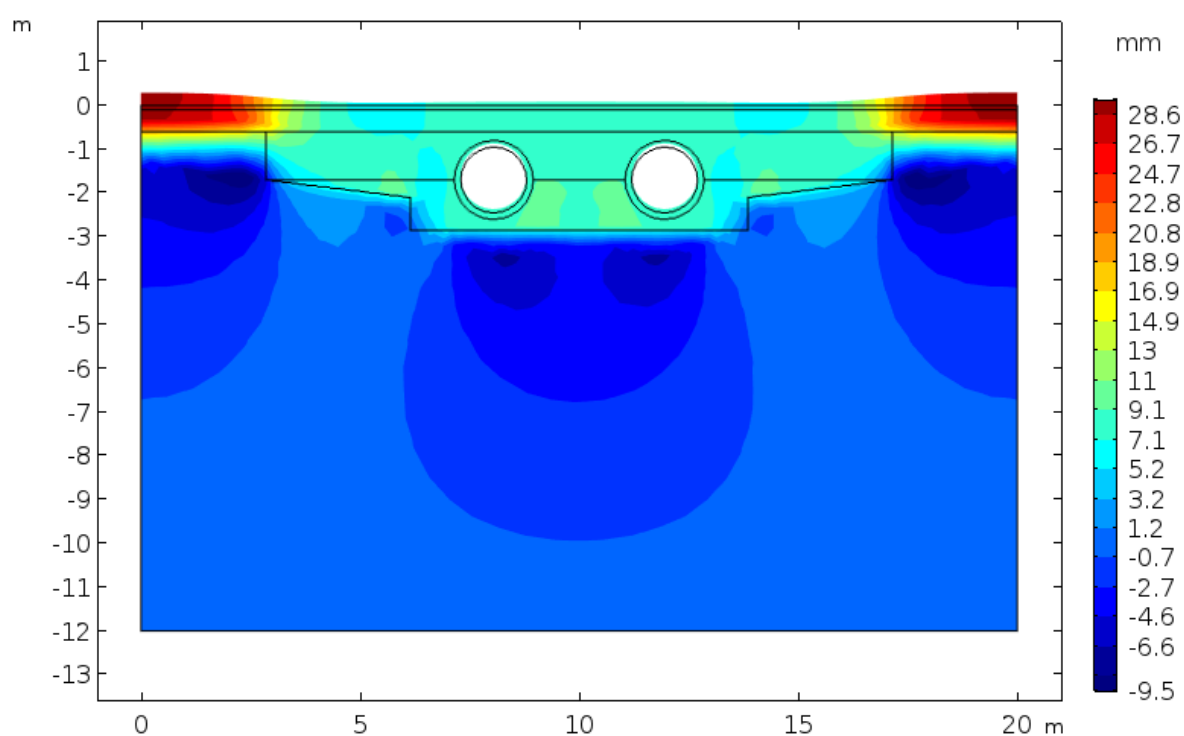


(a) Deformation and temperature distribution after 60 days

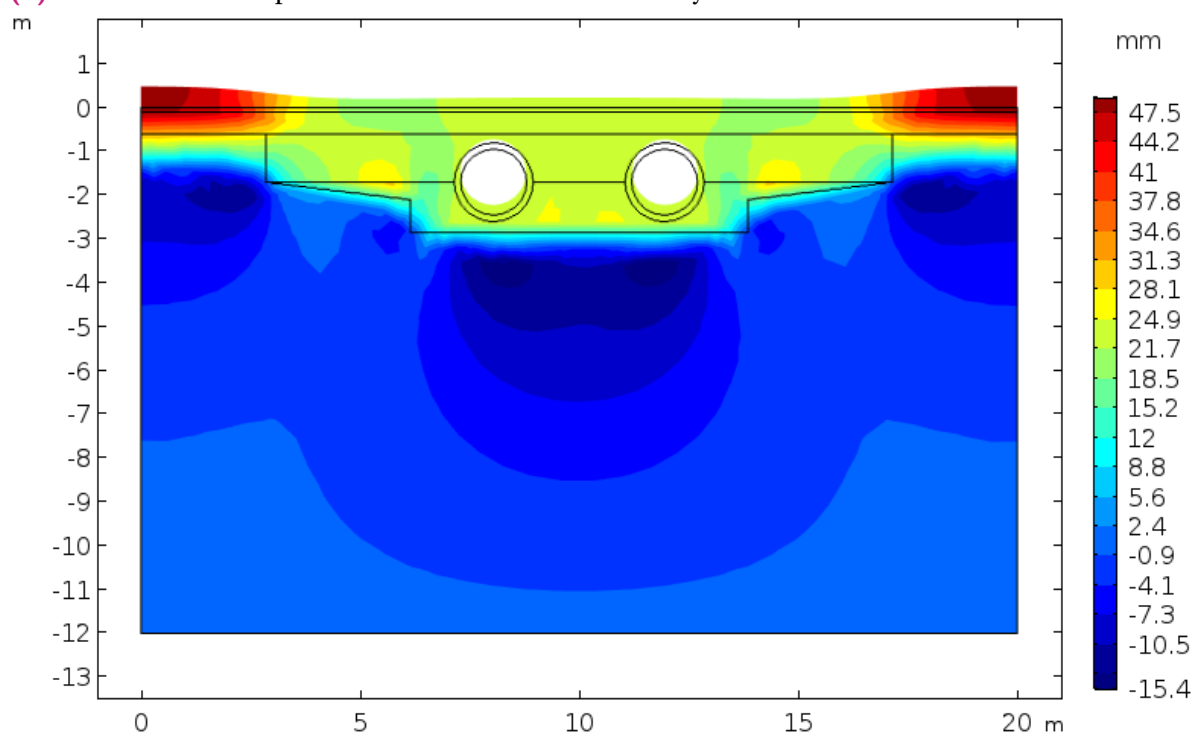


(b) Deformation and temperature distribution after 90 days

Fig. 4.14: Deformation and temperature distribution (a) after 60 days (b) after 90 days in the project site 5



(a) Deformation and pore-water distribution after 60 days



(b) Deformation and vertical displacement after 90 days

Fig. 4.15: Deformation and vertical displacement (a) after 60 days (b) after 90 days in the project site 5

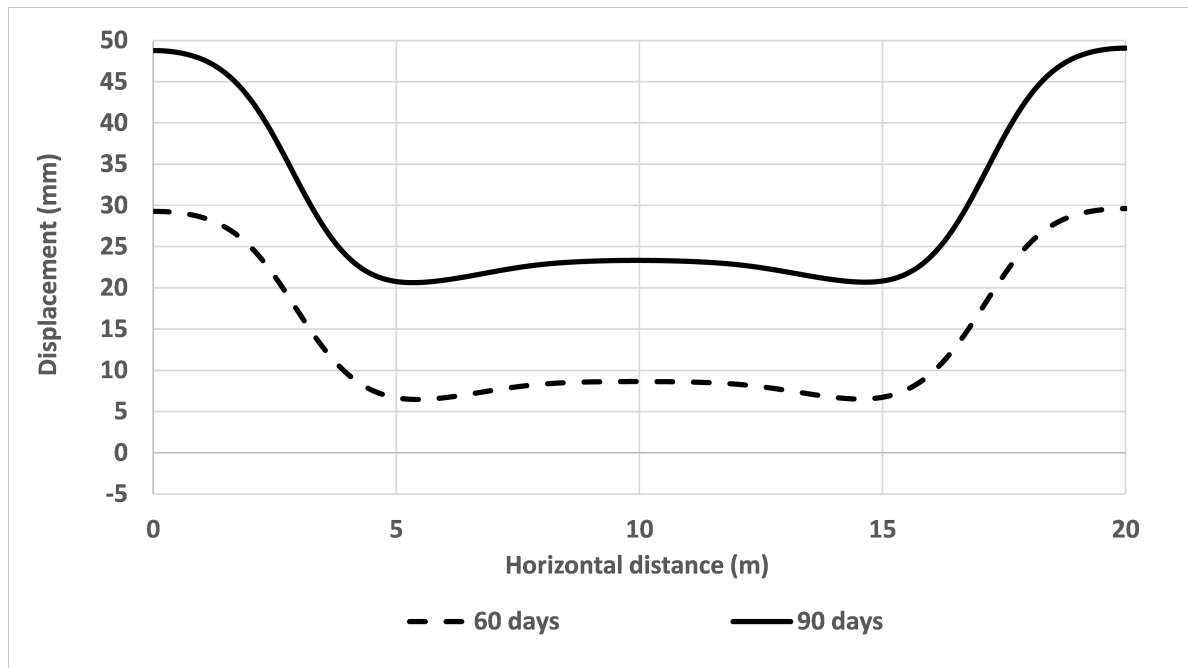
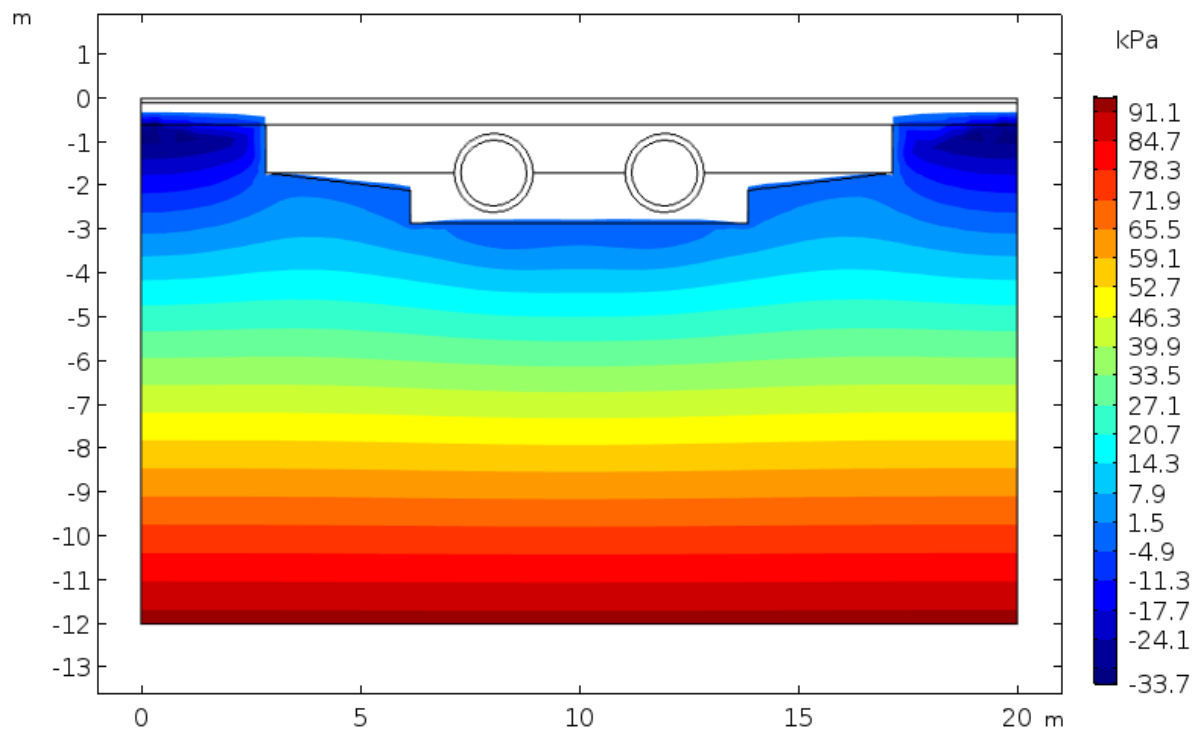


Fig. 4.16: Frost heave along the pavement surface

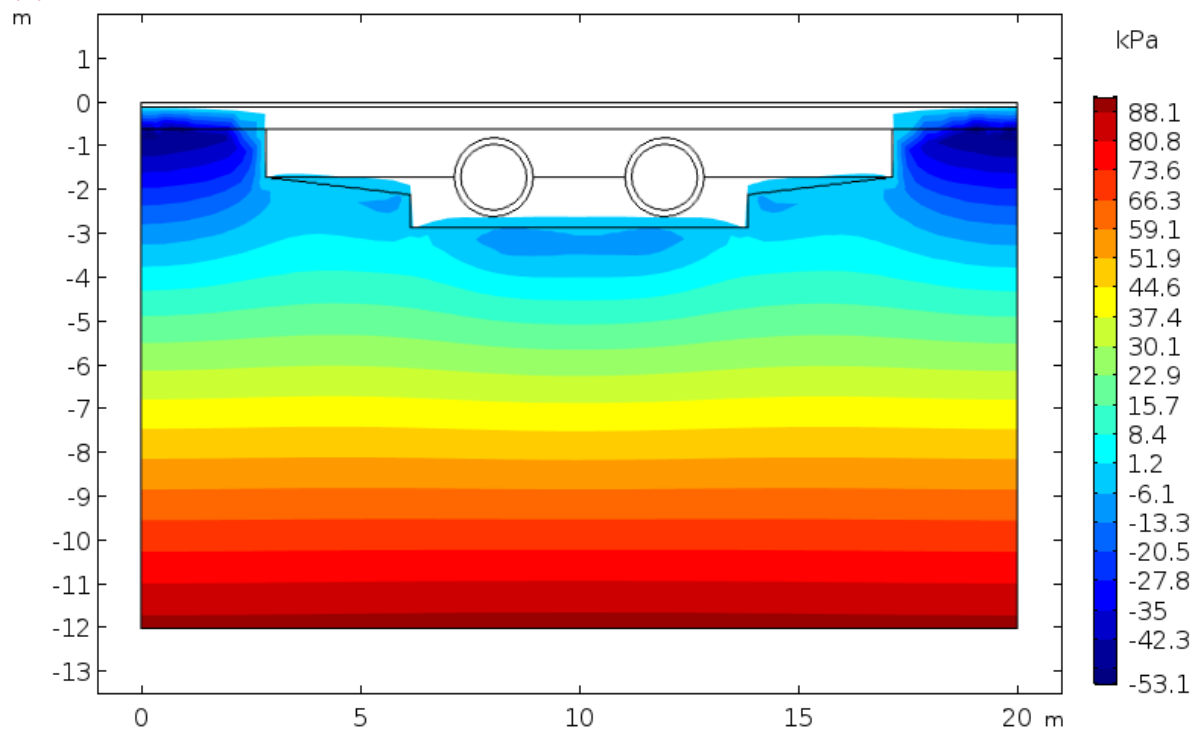
- Pore-Water Pressure Distribution

The pore-water pressure distribution contours after 60 days and 90 days at the project site 5 are depicted in Fig.4.17.

A similar trend in the distribution of the pore-water pressure is obtained for the project site 5. However, the suction in the clay subgrade is estimated to increase overall with a shallower burial depth of culverts. In fact, the smaller overburden pressure makes easier the formation of ice lenses, which consequently promotes more free water to migrate to the freezing front.



(a) Deformation and pore-water distribution after 60 days



(b) Deformation and pore-water distribution after 90 days

Fig. 4.17: Deformation and pore-water distribution in soil subgrades (a) after 60 days (b) after 90 days in the project site 5

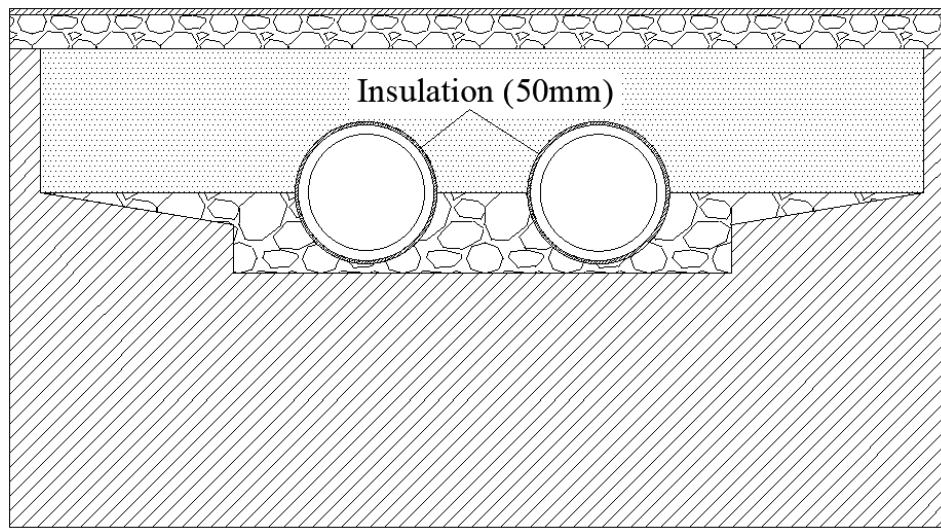
4.5 Effect of Insulation on Frost Heave in Double-Barrel Culvert Sites

The differential displacement is detrimental to the structural integrity of culverts and pavement. The differential displacement occurs mainly due to the different frost heave rate for natural soils at the sides of culverts and soils beneath the culverts. As explained in numerical modeling section, both soil temperature distribution and overburden pressure have impact on the frost heave. Since overburden pressure is predetermined by the structure design, controlling the temperature distribution becomes the key to minimize the frost heave and differential displacement.

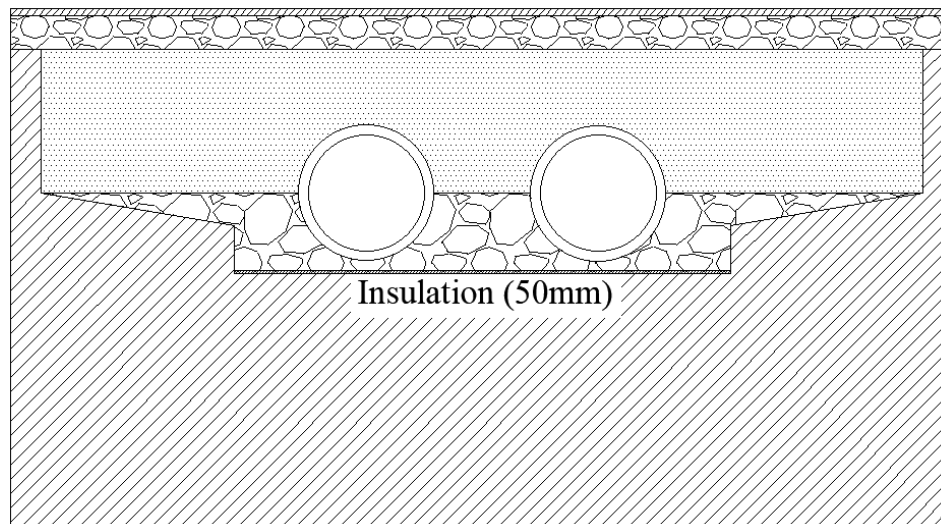
Insulating materials can be used to minimize the heat exchange between the cooling sources (such as pavement surface and culvert openings) and surrounding soils so that the soil temperature can be maintained above the freezing point. In these conditions, the frost heave can be largely decreased or even avoided.

To the best of the authors' knowledge, the effect of the distribution of thermal insulation and its geometry on differential frost actions and frost heave of double-barrel culvert is poorly documented. In a recent study [Mou+18], a parametric study is carried out to determine the most effective position of insulation near culvert to minimize the frost depth in double-barrel culvert sites.

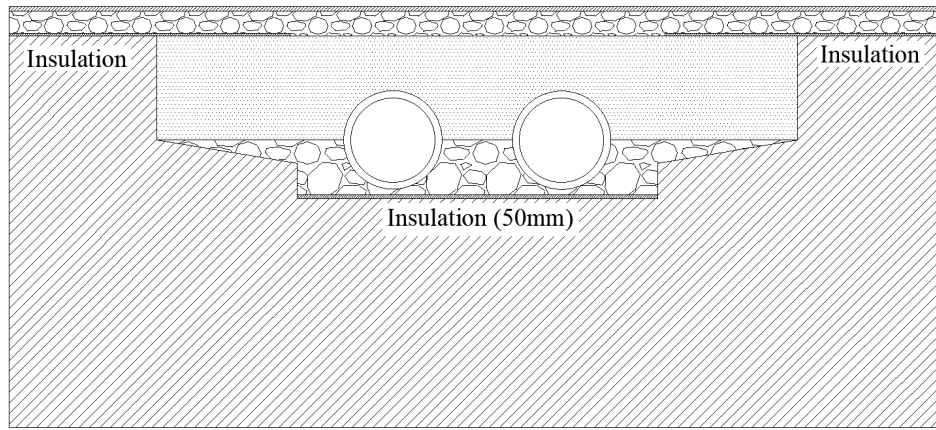
This section aims to study the effect of insulation's location with respect to culverts openings on frost heave and differential displacement at the pavement surface. Three different insulation configurations are studied for the project sites 4 and 5 in this paper: Method A) a circular insulation on the outer surface of the culverts; Method B) a horizontal insulation at the top of the clay subgrade below the culverts; and Method C) a horizontal insulation at the top of the frost-susceptible subgrade below the culverts and at the sides of the culvert structure. The culverts layout with different insulating methods can be seen in Fig.4.19. The first two configurations have been found to be the most effective in terms of thermal performance according to [Mou+18].



(a) Insulating method A



(b) Insulating method B



(a) Insulating method C

Fig. 4.19: Summary of insulating methods studied in this paper (a) method A (b) method B (c) method C

The following insulation properties are considered in this analysis: $\rho = 50 \text{ kg/m}^3$, $C_p = 2000 \text{ J/(kg} \cdot \text{K)}$, $\lambda_T = 0.02 \text{ W/(m} \cdot \text{K)}$. The insulation thickness considered in the simulation is 50 mm.

4.5.1 Insulation Design for the Project Site 4 with a Deeper Burial Depth of Culverts (3.9m)

The culverts in the project site 4 is located at a relatively deeper depth. This section aims to study the effective insulation design for culverts with a deeper burial depth in the project site 4.

Method A: Circular Insulation on the Outer Surface of the Culverts

The deformation and temperature distribution in the domain by using a circular insulation on the outer surface of the culverts, after 90 days from the start of the freezing period, are illustrated in Fig. 4.20 for the project site 4.

To give a better insight into the structural integrity and roughness of the road, the differential deformation due to frost heave at the pavement surface is shown in Fig. 4.21. It can be seen that the circular insulation on the outer surface of the culverts reduces effectively

the heat exchange between the culverts openings and the surrounding soils. In this case, the frost heave can be avoided in frost-susceptible soils below the culverts. However, with the advance of frost penetration, the frost heave at the side soils develops gradually. On the other hand, the circular insulation on the outer surface of the culverts does not have any impact on the frost heave of the side soils. Consequently, by preventing frost heave in frost-susceptible soils below the culverts by using an insulation on the outer surface of the culverts, the differential displacement at the pavement surface becomes more significant.

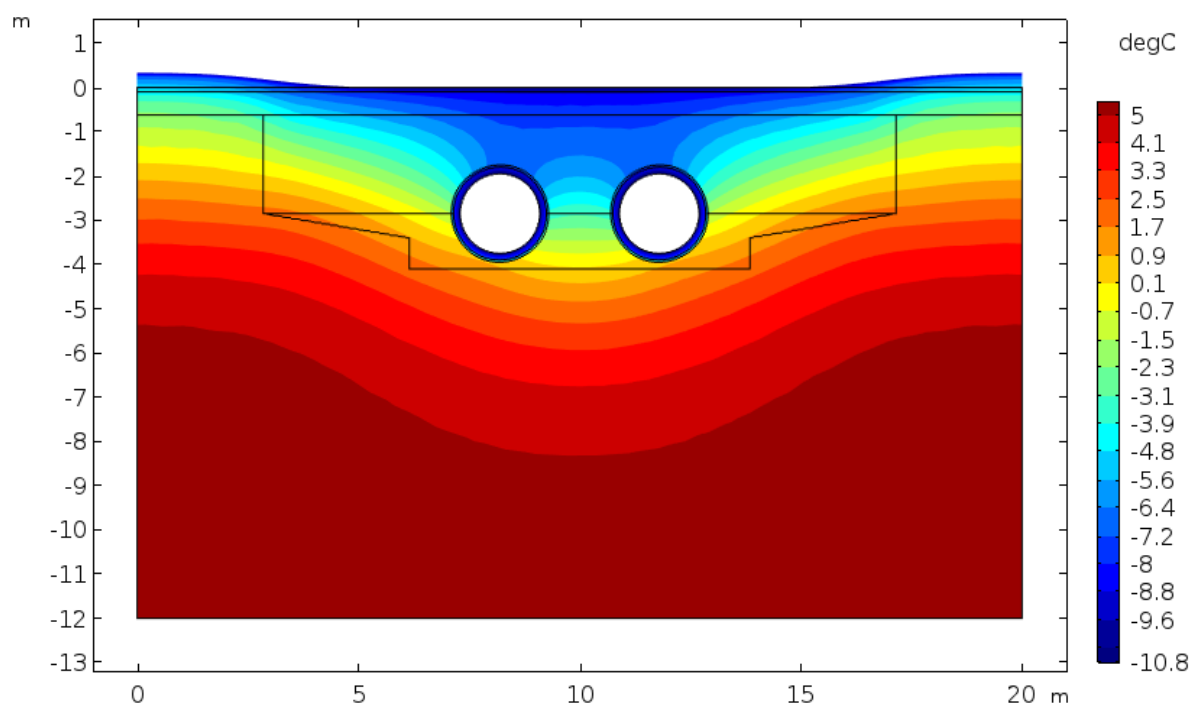


Fig. 4.20: Deformation and temperature distribution after 90 days by using insulating method A for site 4

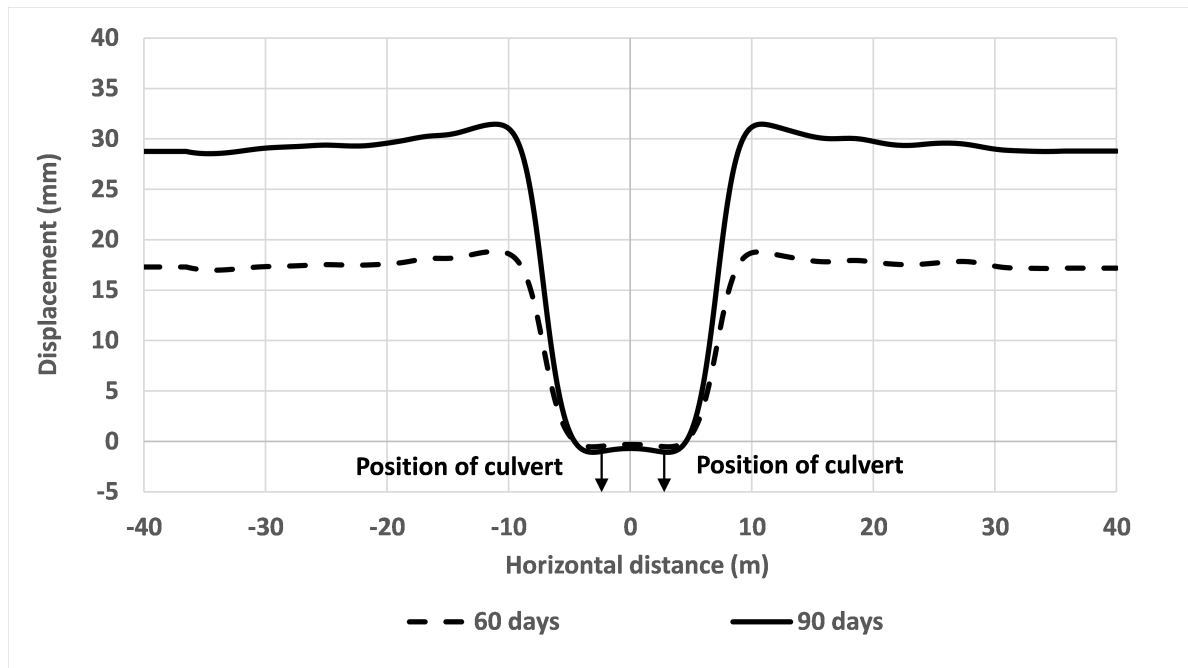


Fig. 4.21: Deformation along the pavement surface after 60 days and 90 days by using insulating method A for site 4

Method B: Horizontal Insulation at the Top of the Clay Subgrades below the Culverts

The deformation and temperature distribution in the domain by using a horizontal insulation at the top of the clay subgrades below the culverts, after 90 days from the start of the freezing period, are illustrated in Fig. 4.22 for the project site 4. Furthermore, the deformation along the pavement surface is shown in Fig. 4.23.

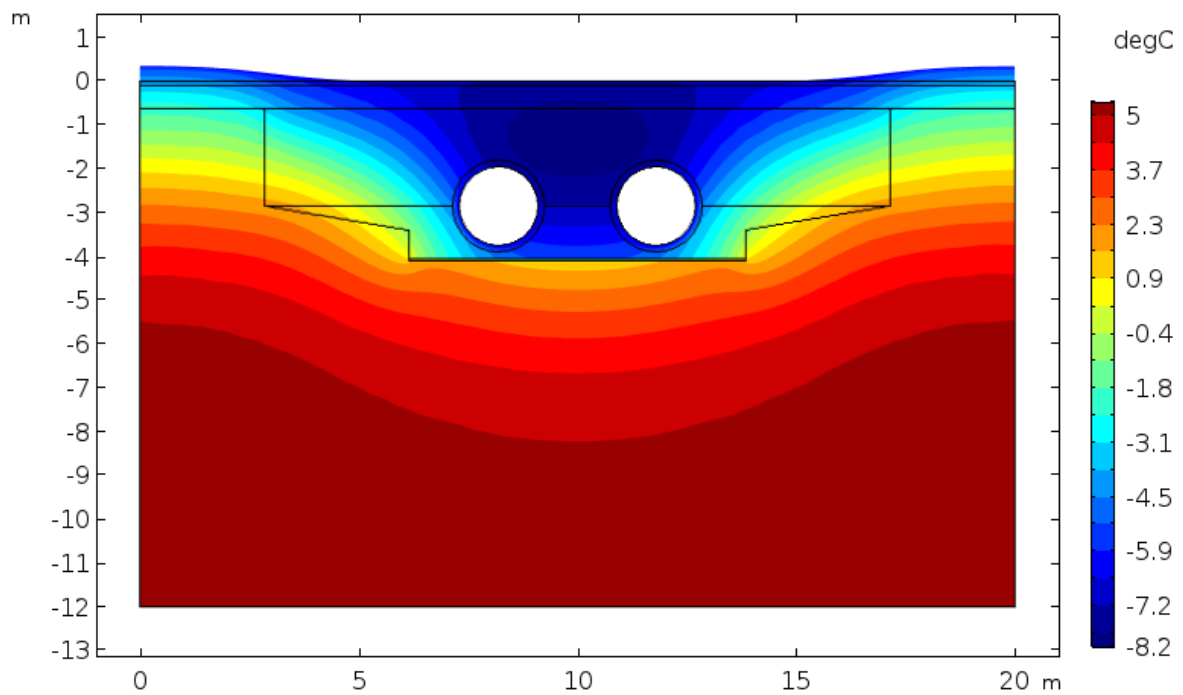


Fig. 4.22: Deformation and temperature distribution after 90 days by using insulating method B for the site 4

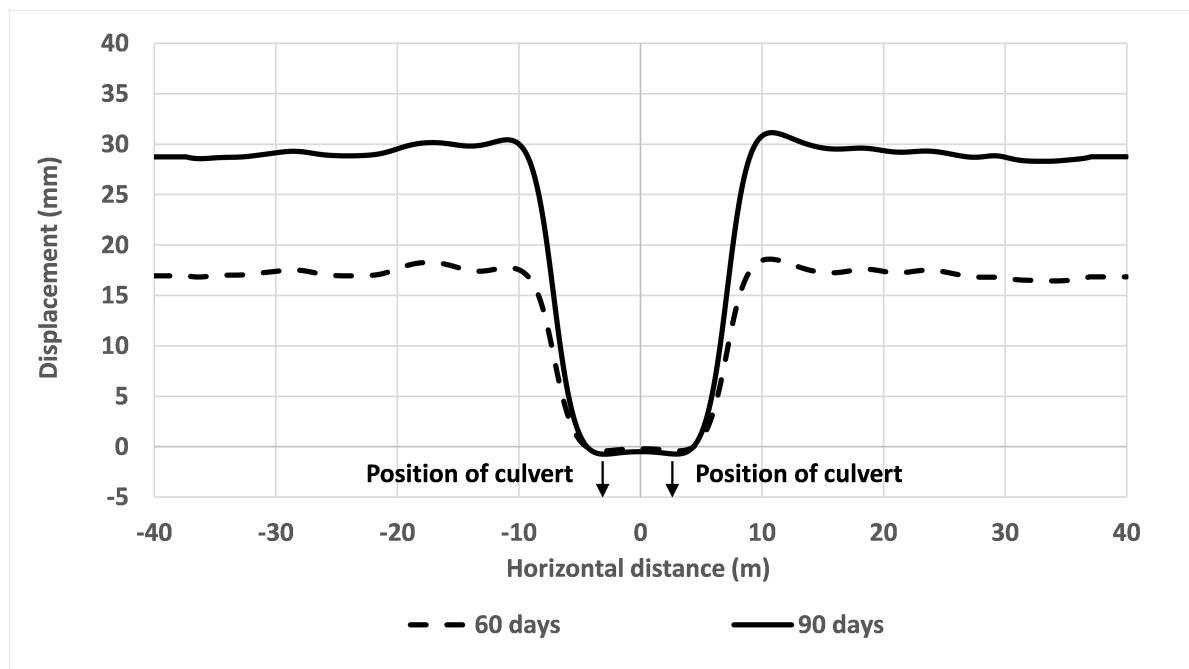


Fig. 4.23: Deformation along the pavement surface after 60 days and 90 days by using insulating method B for site 4

It is interesting to see that although the temperature distribution in the ground using insulating methods A and B are different, the overall frost heave and differential displacement are similar. Both methods A and B can largely reduce the frost heave in the clay subgrades below the culverts. However, the differential displacement using both methods is increased. On the other hand, the insulating methods A and B cannot efficiently remove differential deformations at the pavement surface.

Method C: Horizontal Insulation at Both Sides of the Culvert Structure Combined with Insulating Method B

To effectively remove the differential deformation at the pavement surface, a horizontal insulation is placed at the top of the natural frost-susceptible subgrade below the culverts and at both sides of the double-barrel culvert structure. An insulation thickness of 50 *mm* and 100 *mm* for the sides are studied in this section. The deformation and temperature distribution using insulating method C (100 *mm*) are illustrated in Fig. 4.24. The displacements after 90 days along the pavement surface by using the insulating methods B and C are compared in Fig. 4.25.

It can be seen that differential displacement can be more than 25 *mm* by using 50 *mm* thickness insulation at the sides. It is concluded that placing insulation at the top of the frost-susceptible soil is necessary to mitigate the differential deformation along the pavement surface. By applying the insulating method C with 100 *mm* thickness insulation at sides, the differential displacement and the maximum frost heave are reduced to 2.4 *mm*, as shown in Fig. 4.25. Therefore, for the project site 4 with a deeper burial depth of culverts, the insulating method C is recommended.

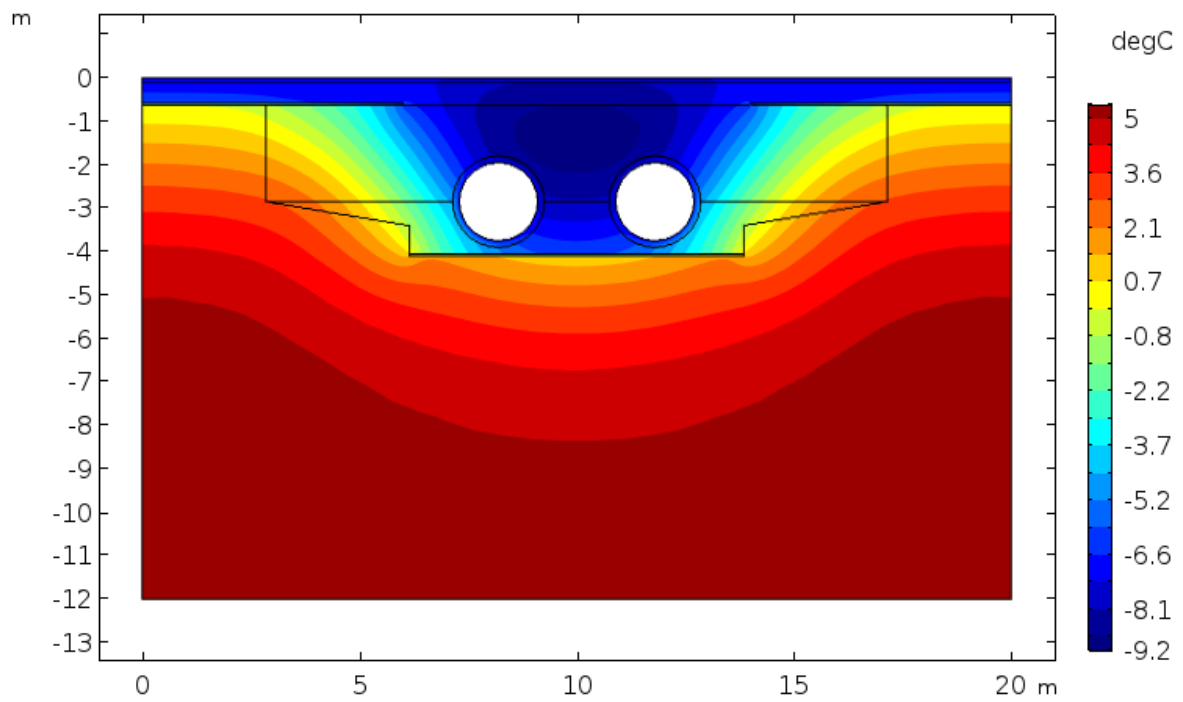


Fig. 4.24: Deformation and temperature distribution in the domain by using the insulating method C after 90 days for the project site 4

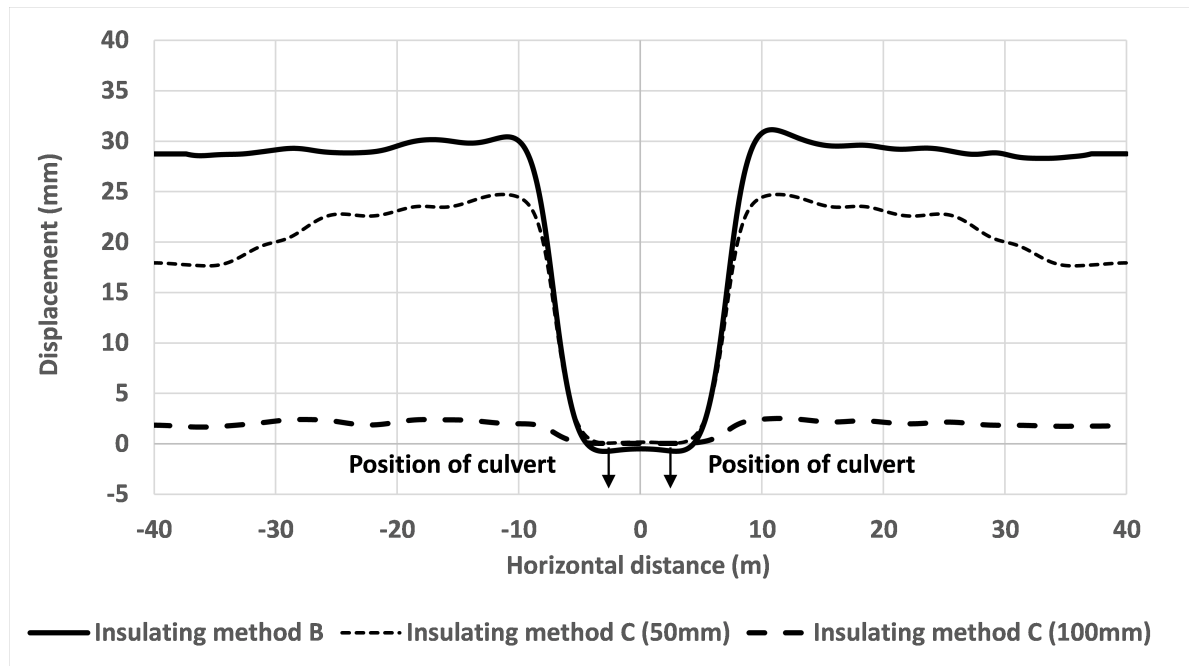


Fig. 4.25: Deformation along the pavement surface by using the insulating methods B and C after 90 days for the project site 4

4.5.2 Insulation Design for the Project Site 5 with a Shallower Burial Depth of Culverts (2.75m)

This section aims to study the effective insulation design for the culverts with a shallower burial depth in the project site 5.

Method A: Circular Insulation on the Outer Surface of the Culverts

The performance of a circular insulation placed on the outer surface of the culverts are investigated for the project site 5. Fig. 4.26 illustrates the deformation and temperature distribution using the insulating method A after 90 days from the start of the freezing period. Furthermore, the deformation along the pavement surface by using the method A is shown in Fig. 4.27.

It can be seen that by using the insulating method A, the frost heave can be significantly reduced in frost-susceptible soils below the culverts. However, the frost heave at the sides of the double-barrel culvert structure is not affected by the insulation. Consequently, the differential deformation at the pavement surface by using a circular insulation placed on the outer surface of the culverts increased. In general, the insulating method A cannot efficiently reduce the differential deformation at the pavement surface and is not recommended.

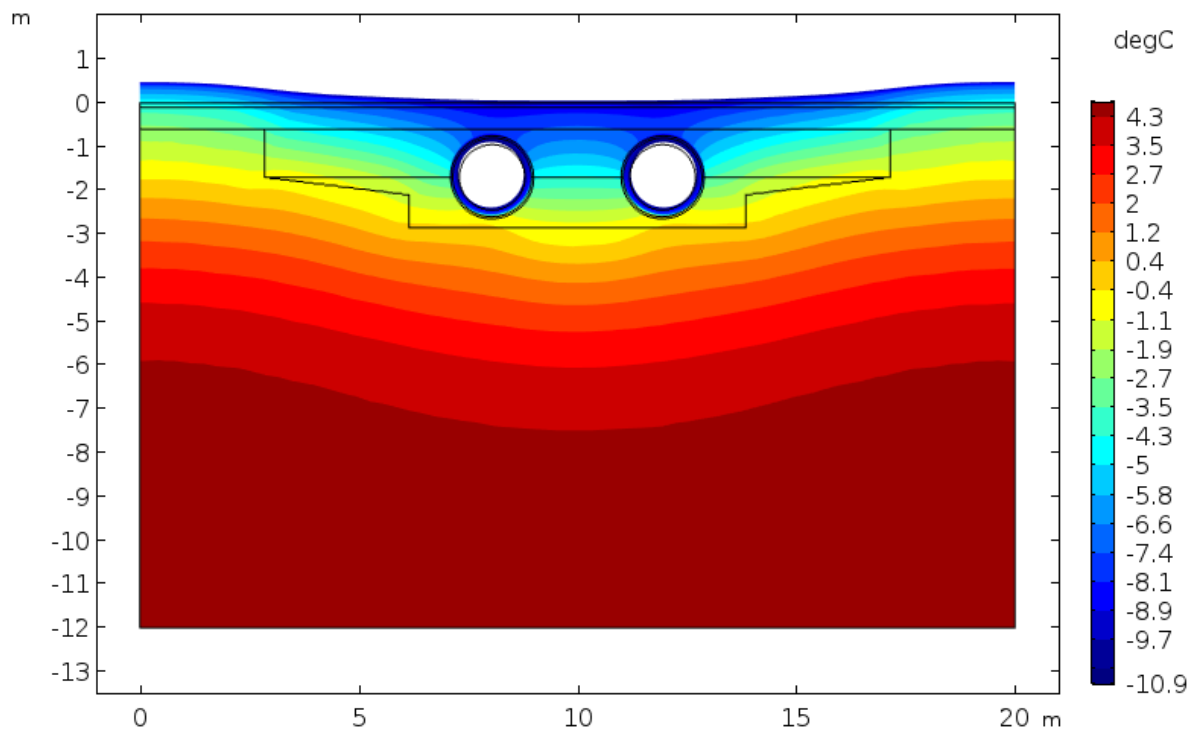


Fig. 4.26: Deformation and temperature distribution in the domain by using the insulating method A after 90 days for the project site 5

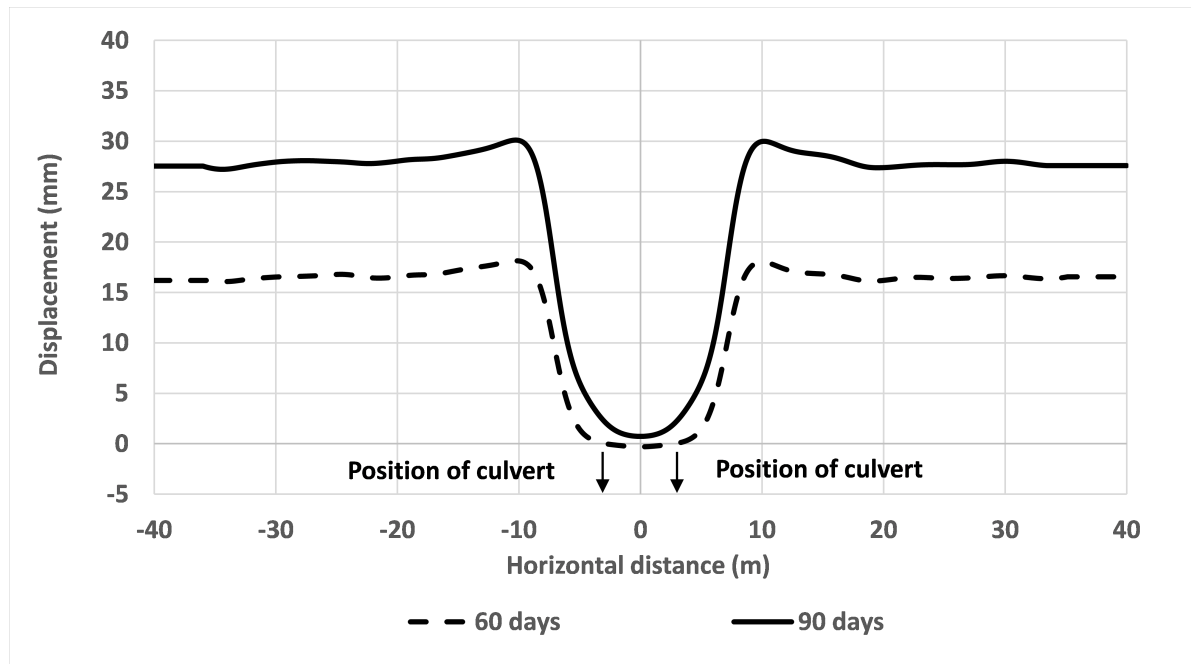


Fig. 4.27: Deformation along the pavement surface by using the insulating method A after 60 days and 90 days for the project site 5

Method B: Horizontal Insulation at the Top of the Clay Subgrade Below the Culverts

This section aims to compare the overall performance of insulating methods A and B. Fig. 4.28 illustrates the deformation and temperature distribution using the insulating method B after 90 days from the start of the freezing period. Furthermore, the deformation along the pavement surface by using the method B is shown in Fig. 4.28.

It can be seen that the frost heave in the frost-susceptible subgrade below the culverts by using the insulating method B is almost prevented while the frost heave of soil sides is not affected by the insulation. It is concluded that even though the insulating method A and B prevent the frost heave in frost-susceptible soils below the culvert structure, the differential deformation are increased at the pavement surface.

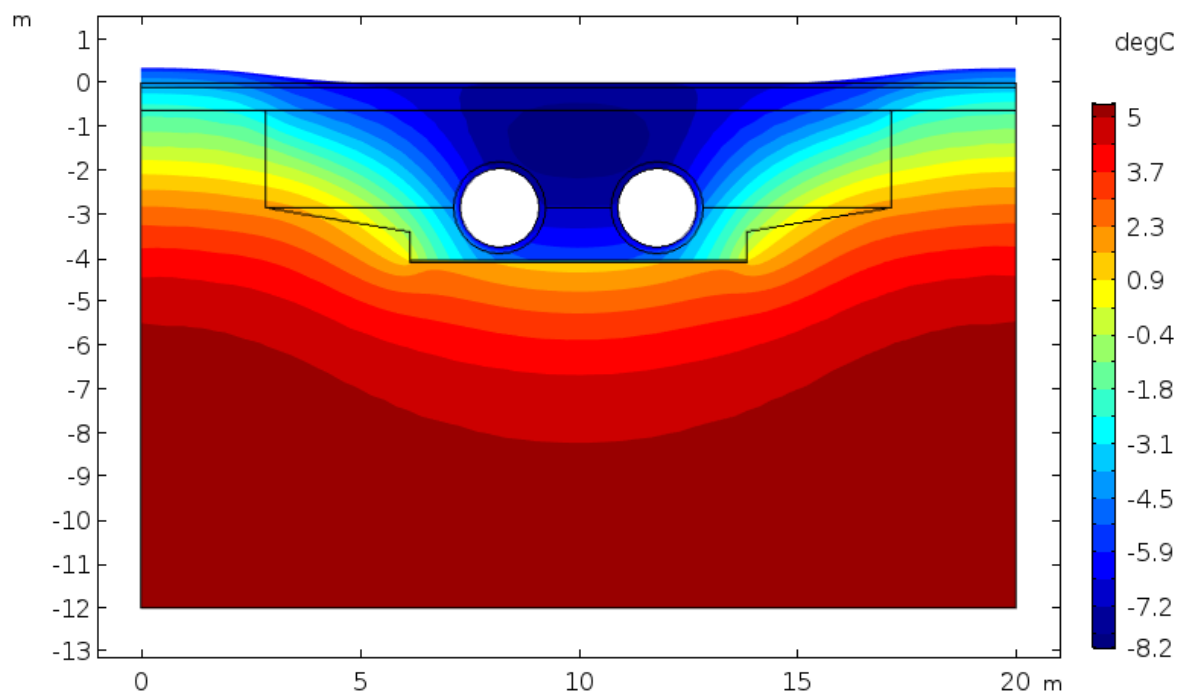


Fig. 4.28: Deformation and temperature distribution in the domain by using the insulating method B after 90 days for the project site 5

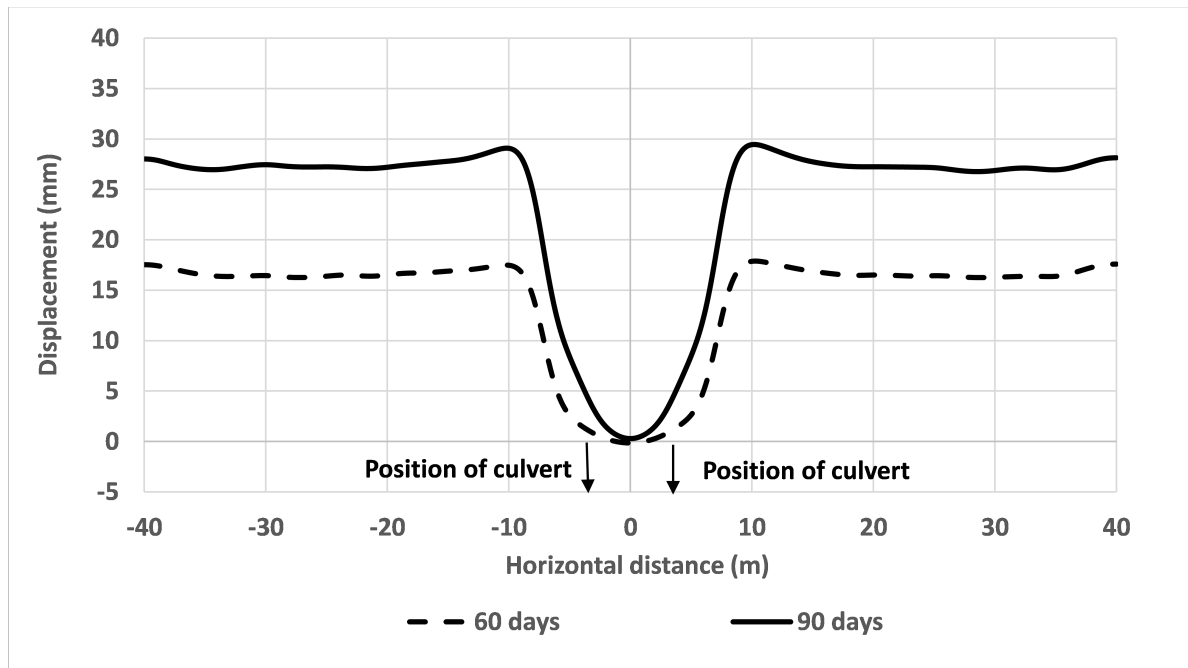


Fig. 4.28: Deformation along the pavement surface by using the insulating method B after 60 days and 90 days for the project site 5

Method C: Horizontal Insulation at Both Sides of the Culvert Structures Combined with the Method B

The insulating method C is investigated in this section. Various thickness of insulation at sites are investigated. The deformation and temperature distribution by using the insulating method C (with 50 mm thickness insulation at sides) can be seen in Fig. 4.29.

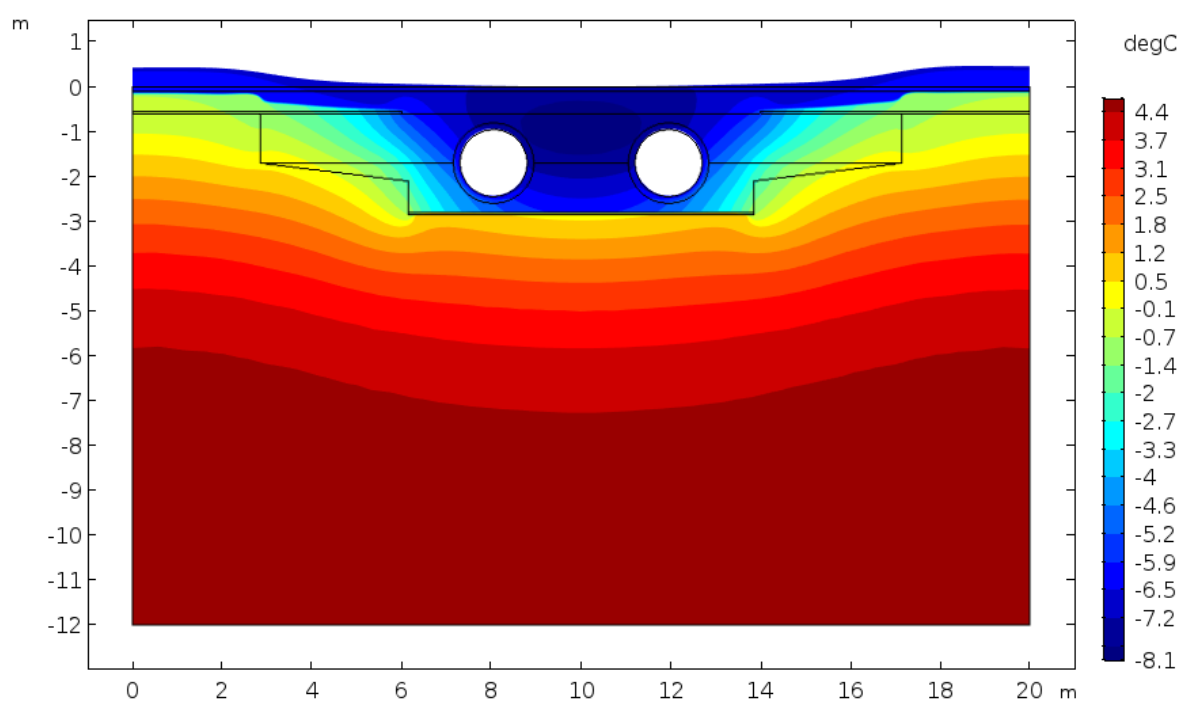


Fig. 4.29: Deformation and temperature distribution in the domain by using the insulating method C after 90 days for project site 5

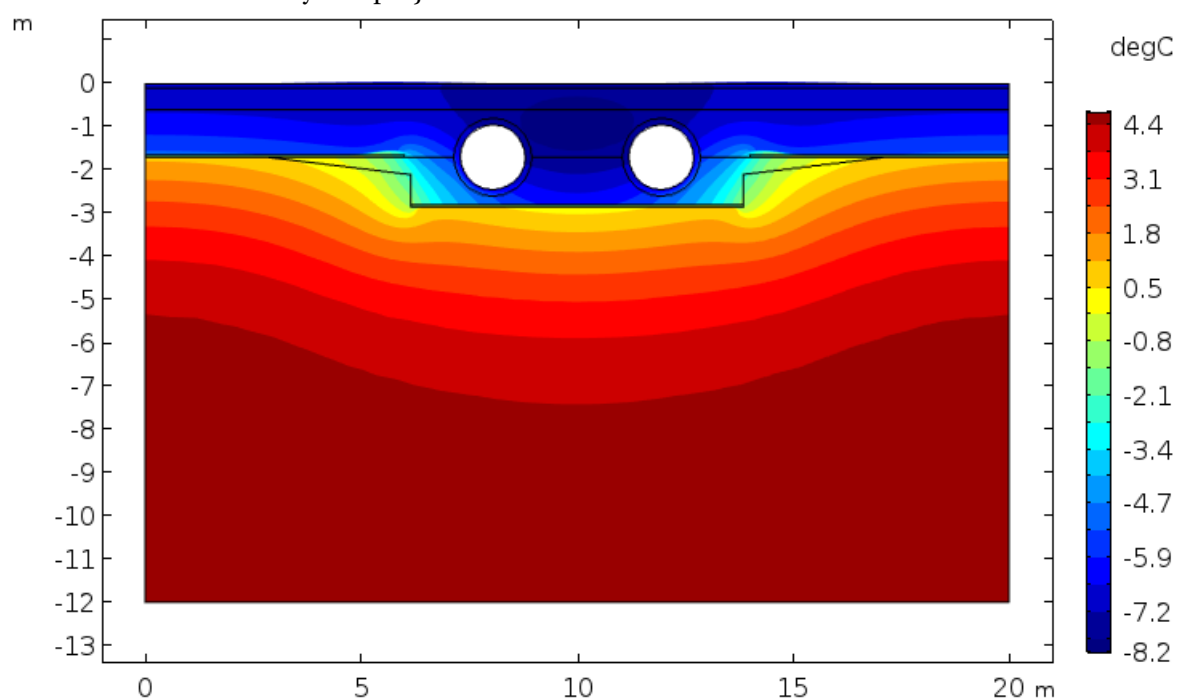


Fig. 4.30: Deformation and temperature distribution using insulating method C and non-frost-susceptible backfill after 90 days for project site 5

It can be concluded that even by using an insulation at the top of the clay subgrade in the sides of the culvert structure, the frost still penetrates into the natural frost-susceptible soils and causes frost heave and differential deformation at the pavement surface. Then, two alternatives can be applied. The first alternative is to increase the insulation thickness so that soil remains unfrozen. The second alternative is to increase the overburden pressure on the frost-susceptible subgrade by replacing the natural subgrade by well-compacted clay backfill with very low permeability or well-drained granular backfill. In this case, a depth of 1.1 *m* of natural soil is replaced by the compacted clay backfill. The deformation and temperature contour using the insulating method C combined with non-frost-susceptible backfill are shown in Fig. 4.30.

The differential displacements along the pavement surface using the insulating methods B and C (with 100 *mm* thickness insulation at sites) and the insulating method C (with 50 *mm* thickness insulation at sites) combined with non-frost-susceptible backfill are summarized in Fig. 4.31.

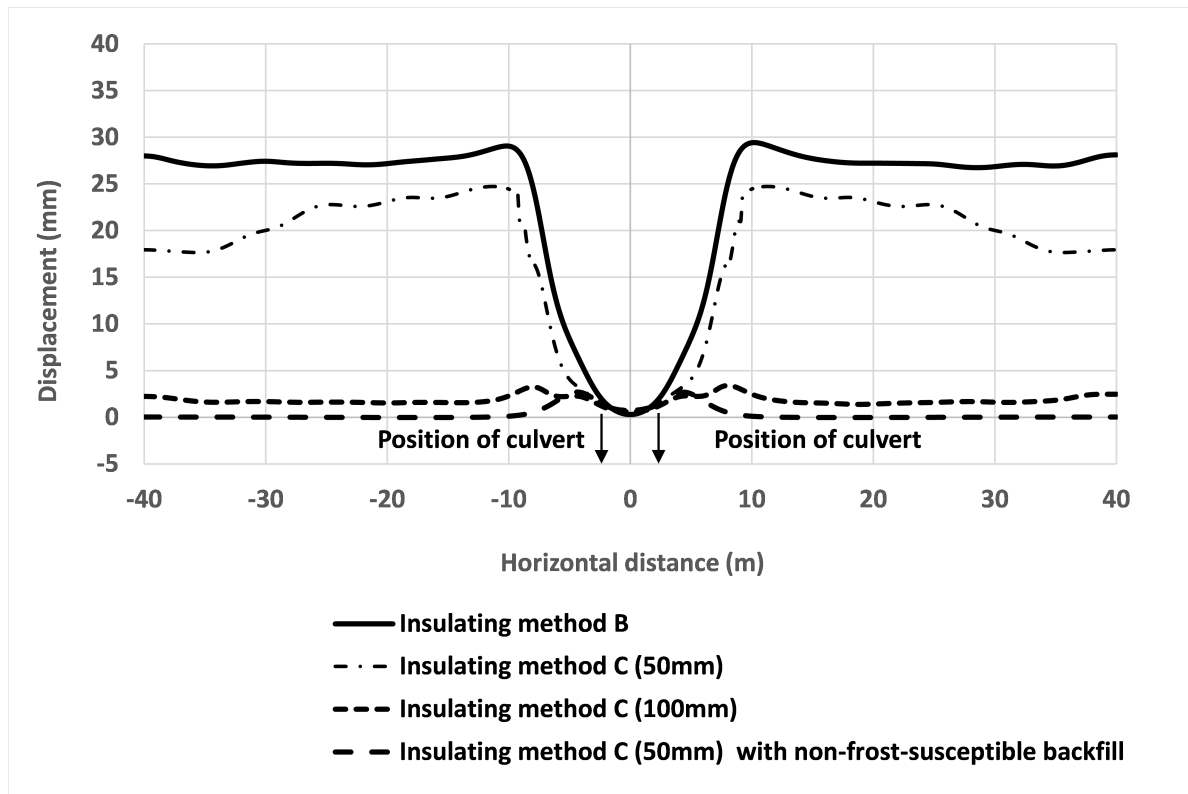


Fig. 4.31: Differential deformation at the pavement surface by using insulating method B, method C and method C with non-frost-susceptible backfill after 90 days for the project site 5

It can be seen that the insulating method C (50 *mm* insulation at sides) alone in the project site 5 is not effective. However, by increasing insulation thickness to 100 *mm* (first alternative), the differential displacement can be largely reduced. Using the insulating method C (50 *mm*) combined with non-frost-susceptible backfill (second alternative) is a very promising approach to decrease the overall frost heave and differential displacement at the pavement surface. In this case, the differential displacement is reduced to only 5 *mm*. The second alternative can be seen is slightly more effective than the latter alternative. But cost analysis might be required to determine the most appropriate alternative to reduce differential displacement at the road surface.

4.6 Conclusion

In this study, a fully coupled thermal-hydro-mechanical model is developed by using the theory of poroelasticity and a porosity rate function to numerically study the phenomena of frost heave in double-barrel culvert sites. The major conclusions in this paper can be drawn as follows:

- The THM model proposed in this study is able to accurately predict frost heave, temperature and pore-water pressure distribution in the ground. The numerical results showed a good agreement with the experimental test data and field measurement.
- The negative pore-water pressure (suction) can be generated when frost heave occurs. The value of suction is related to the rate of frost heave. The maximum suction occurs when the rate of frost heave is at the highest level.
- The double cooling effect induced by culverts impacts the surround soils in a range of 20 *m*. The temperature and displacement distribution flattened 10 *m* away from the culverts center.
- Two alternatives can be applied to reduce the differential displacement at the pavement surface. The thickness of insulation can be increased to protect soil from freezing. The insulating method C (50 *mm*) combined with an adequate overburden pressure can be used to minimize the overall frost heave and differential displacement at the pavement surface. This can be done by increasing the thickness of the granular base or replacing frost-susceptible subgrade by non-frost-susceptible backfill.
- For double-barrel culverts in both sides 4 and 5, using an insulation at the top of the frost-susceptible subgrade below the culverts and at the sides of the culvert structure (insulating method C with 100 *mm* insulation thickness at sides) can be applied to

effectively decrease the frost heave and the corresponding differential displacement at the pavement surface.

- The applicability of the insulating methods studied in the paper depends mainly on the important of the road, the level of the driving comfort, and how much differential deformation at the pavement surface is tolerated.

Conclusions and Recommendations

5.1 Conclusions

In this thesis, a comprehensive numerical modeling including heat transfer analysis, coupled heat and mass transfer analysis, and fully coupled thermo-hydro-mechanical analysis in freezing soils have been performed. The systems of governing equations were implemented in COMSOL Multiphysics. The implemented models have been verified with analytical solutions, experimental data, as well as field measurement data. Then, three sets of problems encountered in cold regions have been targeted as follows:

- A feasibility study of geothermal energy pile based snow melting system was performed for six major cities in Canada. The energy efficiency in the form of COP for each city were derived based on transient heat transfer analysis and fundamental thermodynamics. A transient snow melting model is developed to evaluate the heat flux required for instantaneous snow melting based on typical Canadian weather.
- The insulation thermal performance was studied using a heat transfer analysis with and without considering phase change as well as a coupled heat and mass transfer analysis by considering phase change in unsaturated soils. The Nelder-Mead algorithm is introduced to find the optimum pipe burial depth and the corresponding insulation design so that the overall cost is minimized.
- The fully coupled thermal-hydro-mechanical model in this study was used to numerically study the phenomena of frost heave in double-barrel culvert sites.

The important conclusions can be draws as follows.

5.1.1 Geothermal Snow-Melting System Design

- The snow melting system using geothermal energy pile is efficient and cost effective. However, the extent of efficiency and saving varies with implementation areas.
- The COP of the heat pump, as the indicator of the energy efficiency, depends on the different weather conditions which determine the heating load. In terms of COP, Winnipeg and Toronto are the most suitable places for the geothermal energy pile based snow melting system among the six cities studied.
- Despite of different values of COP in six cities, the number of piles required for snow melting is close because of the similar ground temperature and thermal conductivity in six cities. Montreal and Ottawa are expected to have longer length of geothermal heat exchanger pipe because of their relatively higher heating load.
- Snow melting using the geothermal energy pile system is much more cost effective than electrical-based system. However, the relative savings vary with local price of electricity, the value of COP and the local heating load. Toronto is predicted to have the largest saving due to higher price of electricity and higher COP value.

5.1.2 Structural and Thermal Integrity of Buried Infrastructure subject to Frost Actions

Optimum design of rigid plastic foam insulations for buried utilities

- The latent heat released during the phase change can significantly affect the heat transfer process and decrease the freezing rate of the soil. Neglecting the phase change effects will eventually lead to an erroneous design of the insulation.
- The mass transfer mechanism is negligible in clay subgrade for predicting temperature and unfrozen water content distribution.

- Using granular soil (sand) as backfill is more critical than clay due to the higher thermal conductivity of sand. The cylindrical shape insulation is not recommended. The heat loss mainly occurs at the top pipe. Therefore, the insulation located at the bottom water pipe does not effectively reduce the heat loss.
- The optimum design should consider the effect of insulation length, distance between water pipe and insulation, pipe burial depth and insulation thickness. The Nelder-Mead algorithm is efficient and effective to determine the optimum burial depth and the designing insulation parameters.

Thermo-Hydro-Mechanical modeling of freezing soil

- The THM model proposed in this study is able to accurately predict frost heave, temperature and pore-water pressure distribution in the ground. The numerical results showed a good agreement with the experimental test data and field measurement.
- The negative pore-water pressure (suction) can be generated when frost heave occurs. The value of suction is related to the rate of frost heave. The maximum suction occurs when the rate of frost heave is at the highest level.
- The double cooling effect induced by culverts impacts the surround soils in a range of 20 *m*. The temperature and displacement distribution flattened 10 *m* away from the culverts center.
- Two alternatives can be applied to reduce the differential displacement at the pavement surface. The thickness of insulation can be increased to protect soil from freezing. The insulating method C (50 *mm*) combined with an adequate overburden pressure can be used to minimize the overall frost heave and differential displacement at the pavement surface. This can be done by increasing the thickness of the granular base or replacing frost-susceptible subgrade by non-frost-susceptible backfill.

5.2 Recommendations for Further Work

The recommendations for future work can be concluded as follows:

5.2.1 Numerical Modeling

- The coupled heat and mass transfer induced numerical convergence problems due to the jump of enthalpy during the phase change. A smoothed transition function was defined to accommodate this numerical problem. However, with complex geometry and boundary conditions, the model experienced the convergence problem. More numerical linearization techniques must be applied to mitigate the convergence difficulties.
- The Thermo-Hydro-Mechanical model developed in this study is limited to elastic deformation. This assumption can reasonably predict deformations due to frost heave. However, to model thaw settlements, an elasto-plastic constitutive model should be implemented.
- In the THM model, the parameters used in the porosity rate function should be experimentally tested for different types of frost-susceptible soils.

5.2.2 Geothermal Snow-Melting System Design

- The performance of snow melting system using geothermal energy is highly dependent on the local weather temperature during snowfall. The more analysis should be conducted to evaluate the feasibility of snow melting system in each city.
- In terms of cost analysis, the more reliable data should be provided in the future to compare the relative saving of using geothermal energy to traditional snow melting method and other alternatives.

- For the heat flux required for snow melting, only a snow free area of 100% is considered in this thesis. For other desired snow free area design, a more complete energy balance at the pavement slab must be considered.

5.2.3 Structural Integrity of Buried Infrastructure

- The displacement and pore-water pressure distribution in frost-susceptible soils beneath double-barrel culvert predicted by the proposed THM model should be validated by field measurements in the future.
- The effect of overburden pressure on frost heave should be verified by some pilot studies.

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