A CAPACITY PLANNING MODEL

FOR

CANADIAN MILITARY AIRLIFT REQUESTS

BY

LIEUTENANT-COLONEL BARRY A. STANNARD

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of

MASTER OF SCIENCE

Department of Actuarial and Management Science
University of Manitoba
Winnipeg, Manitoba

ISBN 0-315-86079-0

Name BARRY ANTHONY STANWARD

Dissertation Abstracts International is arranged by broad, general subject categories. Please select the one subject which most nearly describes the content of your dissertation. Enter the corresponding four-digit code in the spaces provided.

OPERATIONS RESEARCH — SUBJECT TERM

0 7 9 6 — SUBJECT CODE

U·M·I

Subject Categories

THE HUMANITIES AND SOCIAL SCIENCES

COMMUNICATIONS AND THE ARTS
- Architecture 0729
- Art History 0377
- Cinema 0900
- Dance 0378
- Fine Arts 0357
- Information Science 0723
- Journalism 0391
- Library Science 0399
- Mass Communications 0708
- Music 0413
- Speech Communication 0459
- Theater 0465

EDUCATION
- General 0515
- Administration 0514
- Adult and Continuing 0516
- Agricultural 0517
- Art 0273
- Bilingual and Multicultural 0282
- Business 0688
- Community College 0275
- Curriculum and Instruction 0727
- Early Childhood 0518
- Elementary 0524
- Finance 0277
- Guidance and Counseling 0519
- Health 0680
- Higher 0745
- History of 0520
- Home Economics 0278
- Industrial 0521
- Language and Literature 0279
- Mathematics 0280
- Music 0522
- Philosophy of 0998
- Physical 0523
- Psychology 0525
- Reading 0535
- Religious 0527
- Sciences 0714
- Secondary 0533
- Social Sciences 0534
- Sociology of 0340
- Special 0529
- Teacher Training 0530
- Technology 0710
- Tests and Measurements 0288
- Vocational 0747

LANGUAGE, LITERATURE AND LINGUISTICS
- Language
  - General 0679
  - Ancient 0289
  - Linguistics 0290
  - Modern 0291
- Literature
  - General 0401
  - Classical 0294
  - Comparative 0295
  - Medieval 0297
  - Modern 0298
  - African 0316
  - American 0591
  - Asian 0305
  - Canadian (English) 0352
  - Canadian (French) 0355
  - English 0593
  - Germanic 0311
  - Latin American 0312
  - Middle Eastern 0315
  - Romance 0313
  - Slavic and East European 0314

PHILOSOPHY, RELIGION AND THEOLOGY
- Philosophy 0422
- Religion
  - General 0318
  - Biblical Studies 0321
  - Clergy 0319
  - History of 0320
  - Philosophy of 0322
- Theology 0469

SOCIAL SCIENCES
- American Studies 0323
- Anthropology
  - Archaeology 0324
  - Cultural 0326
  - Physical 0327
- Business Administration
  - General 0310
  - Accounting 0272
  - Banking 0770
  - Management 0454
  - Marketing 0338
- Canadian Studies 0385
- Economics
  - General 0501
  - Agricultural 0503
  - Commerce-Business 0505
  - Finance 0508
  - History 0509
  - Labor 0510
  - Theory 0511
- Folklore 0358
- Geography 0366
- Gerontology 0351
- History
  - General 0578
  - Ancient 0579
  - Medieval 0581
  - Modern 0582
  - Black 0328
  - African 0331
  - Asia, Australia and Oceania 0332
  - Canadian 0334
  - European 0335
  - Latin American 0336
  - Middle Eastern 0333
  - United States 0337
- History of Science 0585
- Law 0398
- Political Science
  - General 0615
  - International Law and Relations 0616
  - Public Administration 0617
- Recreation 0814
- Social Work 0452
- Sociology
  - General 0626
  - Criminology and Penology 0627
  - Demography 0938
  - Ethnic and Racial Studies 0631
  - Individual and Family Studies 0628
  - Industrial and Labor Relations 0629
  - Public and Social Welfare 0630
  - Social Structure and Development 0700
  - Theory and Methods 0344
- Transportation 0709
- Urban and Regional Planning 0999
- Women's Studies 0453

THE SCIENCES AND ENGINEERING

BIOLOGICAL SCIENCES
- Agriculture
  - General 0473
  - Agronomy 0285
  - Animal Culture and Nutrition 0475
  - Animal Pathology 0476
  - Food Science and Technology 0359
  - Forestry and Wildlife 0478
  - Plant Culture 0479
  - Plant Pathology 0480
  - Plant Physiology 0817
  - Range Management 0777
  - Wood Technology 0746
- Biology
  - General 0306
  - Anatomy 0287
  - Biostatistics 0308
  - Botany 0309
  - Cell 0379
  - Ecology 0329
  - Entomology 0353
  - Genetics 0369
  - Limnology 0793
  - Microbiology 0410
  - Molecular 0307
  - Neuroscience 0317
  - Oceanography 0416
  - Physiology 0433
  - Radiation 0821
  - Veterinary Science 0778
  - Zoology 0472
- Biophysics
  - General 0786
  - Medical 0760

EARTH SCIENCES
- Biogeochemistry 0425
- Geochemistry 0996
- Geodesy 0370
- Geology 0372
- Geophysics 0373
- Hydrology 0388
- Mineralogy 0411
- Paleobotany 0345
- Paleoecology 0426
- Paleontology 0418
- Paleozoology 0985
- Palynology 0427
- Physical Geography 0368
- Physical Oceanography 0415

HEALTH AND ENVIRONMENTAL SCIENCES
- Environmental Sciences 0768
- Health Sciences
  - General 0566
  - Audiology 0300
  - Chemotherapy 0992
  - Dentistry 0567
  - Education 0350
  - Hospital Management 0769
  - Human Development 0758
  - Immunology 0982
  - Medicine and Surgery 0564
  - Mental Health 0347
  - Nursing 0569
  - Nutrition 0570
  - Obstetrics and Gynecology 0380
  - Occupational Health and Therapy 0354
  - Ophthalmology 0381
  - Pathology 0571
  - Pharmacology 0419
  - Pharmacy 0572
  - Physical Therapy 0382
  - Public Health 0573
  - Radiology 0574
  - Recreation 0575
  - Speech Pathology 0460
  - Toxicology 0383
- Home Economics 0386

PHYSICAL SCIENCES

Pure Sciences
- Chemistry
  - General 0485
  - Agricultural 0749
  - Analytical 0486
  - Biochemistry 0487
  - Inorganic 0488
  - Nuclear 0738
  - Organic 0490
  - Pharmaceutical 0491
  - Physical 0494
  - Polymer 0495
  - Radiation 0754
- Mathematics 0405
- Physics
  - General 0605
  - Acoustics 0986
  - Astronomy and Astrophysics 0606
  - Atmospheric Science 0608
  - Atomic 0748
  - Electronics and Electricity 0607
  - Elementary Particles and High Energy 0798
  - Fluid and Plasma 0759
  - Molecular 0609
  - Nuclear 0610
  - Optics 0752
  - Radiation 0756
  - Solid State 0611
- Statistics 0463

Applied Sciences
- Applied Mechanics 0346
- Computer Science 0984
- Engineering
  - General 0537
  - Aerospace 0538
  - Agricultural 0539
  - Automotive 0540
  - Biomedical 0541
  - Chemical 0542
  - Civil 0543
  - Electronics and Electrical 0544
  - Heat and Thermodynamics 0348
  - Hydraulic 0545
  - Industrial 0546
  - Marine 0547
  - Materials Science 0794
  - Mechanical 0548
  - Metallurgy 0743
  - Mining 0551
  - Nuclear 0552
  - Packaging 0549
  - Petroleum 0765
  - Sanitary and Municipal 0554
  - System Science 0790
- Geotechnology 0428
- Operations Research 0796
- Plastics Technology 0795
- Textile Technology 0994

PSYCHOLOGY
- General 0621
- Behavioral 0384
- Clinical 0622
- Developmental 0620
- Experimental 0623
- Industrial 0624
- Personality 0625
- Physiological 0989
- Psychobiology 0349
- Psychometrics 0632
- Social 0451

Nom ____________________

Dissertation Abstracts International est organisé en catégories de sujets. Veuillez s.v.p. choisir le sujet qui décrit le mieux votre thèse et inscrivez le code numérique approprié dans l'espace réservé ci-dessous.

____________________ SUJET

☐☐☐☐ CODE DE SUJET

U·M·I

Catégories par sujets

HUMANITÉS ET SCIENCES SOCIALES

COMMUNICATIONS ET LES ARTS
- Architecture 0729
- Beaux-arts 0357
- Bibliothéconomie 0399
- Cinéma 0900
- Communication verbale 0459
- Communications 0708
- Danse 0378
- Histoire de l'art 0377
- Journalisme 0391
- Musique 0413
- Sciences de l'information 0723
- Théâtre 0465

ÉDUCATION
- Généralités 515
- Administration 0514
- Art 0273
- Collèges communautaires 0275
- Commerce 0688
- Économie domestique 0278
- Éducation permanente 0516
- Éducation préscolaire 0518
- Éducation sanitaire 0680
- Enseignement agricole 0517
- Enseignement bilingue et multiculturel 0282
- Enseignement industriel 0521
- Enseignement primaire. 0524
- Enseignement professionnel 0747
- Enseignement religieux 0527
- Enseignement secondaire 0533
- Enseignement spécial 0529
- Enseignement supérieur 0745
- Évaluation 0288
- Finances 0277
- Formation des enseignants 0530
- Histoire de l'éducation 0520
- Langues et littérature 0279
- Lecture 0535
- Mathématiques 0280
- Musique 0522
- Orientation et consultation 0519
- Philosophie de l'éducation 0998
- Physique 0523
- Programmes d'études et enseignement 0727
- Psychologie 0525
- Sciences 0714
- Sciences sociales 0534
- Sociologie de l'éducation 0340
- Technologie 0710

LANGUE, LITTÉRATURE ET LINGUISTIQUE
- Langues
  - Généralités 0679
  - Anciennes 0289
  - Linguistique 0290
  - Modernes 0291
- Littérature
  - Généralités 0401
  - Anciennes 0294
  - Comparée 0295
  - Mediévale 0297
  - Moderne 0298
  - Africaine 0316
  - Américaine 0591
  - Anglaise 0593
  - Asiatique 0305
  - Canadienne (Anglaise) 0352
  - Canadienne (Française) 0355
  - Germanique 0311
  - Latino-américaine 0312
  - Moyen-orientale 0315
  - Romane 0313
  - Slave et est-européenne 0314

PHILOSOPHIE, RELIGION ET THEOLOGIE
- Philosophie 0422
- Religion
  - Généralités 0318
  - Clergé 0319
  - Études bibliques 0321
  - Histoire des religions 0320
  - Philosophie de la religion 0322
- Théologie 0469

SCIENCES SOCIALES
- Anthropologie
  - Archéologie 0324
  - Culturelle 0326
  - Physique 0327
- Droit 0398
- Économie
  - Généralités 0501
  - Commerce-Affaires 0505
  - Économie agricole 0503
  - Économie du travail 0510
  - Finances 0508
  - Histoire 0509
  - Théorie 0511
- Études américaines 0323
- Études canadiennes 0385
- Études féministes 0453
- Folklore 0358
- Géographie 0366
- Gérontologie 0351
- Gestion des affaires
  - Généralités 0310
  - Administration 0454
  - Banques 0770
  - Comptabilité 0272
  - Marketing 0338
- Histoire
  - Histoire générale 0578
  - Ancienne 0579
  - Médiévale 0581
  - Moderne 0582
  - Histoire des noirs 0328
  - Africaine 0331
  - Canadienne 0334
  - États-Unis 0337
  - Européenne 0335
  - Moyen-orientale 0333
  - Latino-américaine 0336
  - Asie, Australie et Océanie 0332
- Histoire des sciences 0585
- Loisirs 0814
- Planification urbaine et régionale 0999
- Science politique
  - Généralités 0615
  - Administration publique 0617
  - Droit et relations internationales 0616
- Sociologie
  - Généralités 0626
  - Aide et bien-àtre social 0630
  - Criminologie et établissements pénitentiaires 0627
  - Démographie 0938
  - Études de l' individu et de la famille 0628
  - Études des relations interethniques et des relations raciales 0631
  - Structure et développement social 0700
  - Théorie et méthodes. 0344
  - Travail et relations industrielles 0629
- Transports 0709
- Travail social 0452

SCIENCES ET INGÉNIERIE

SCIENCES BIOLOGIQUES
- Agriculture
  - Généralités 0473
  - Agronomie. 0285
  - Alimentation et technologie alimentaire 0359
  - Culture 0479
  - Élevage et alimentation 0475
  - Exploitation des péturages 0777
  - Pathologie animale 0476
  - Pathologie végétale 0480
  - Physiologie végétale 0817
  - Sylviculture et faune 0478
  - Technologie du bois 0746
- Biologie
  - Généralités 0306
  - Anatomie 0287
  - Biologie (Statistiques) 0308
  - Biologie moléculaire 0307
  - Botanique 0309
  - Cellule 0379
  - Écologie 0329
  - Entomologie 0353
  - Génétique 0369
  - Limnologie 0793
  - Microbiologie 0410
  - Neurologie 0317
  - Océanographie 0416
  - Physiologie 0433
  - Radiation 0821
  - Science vétérinaire 0778
  - Zoologie 0472
- Biophysique
  - Généralités 0786
  - Medicale 0760

SCIENCES DE LA TERRE
- Biogéochimie 0425
- Géochimie 0996
- Géodésie 0370
- Géographie physique 0368
- Géologie 0372
- Géophysique 0373
- Hydrologie 0388
- Minéralogie 0411
- Océanographie physique 0415
- Paléobotanique 0345
- Paléoécologie 0426
- Paléontologie 0418
- Paléozoologie 0985
- Palynologie 0427

SCIENCES DE LA SANTÉ ET DE L'ENVIRONNEMENT
- Économie domestique 0386
- Sciences de l'environnement 0768
- Sciences de la santé
  - Généralités 0566
  - Administration des hipitaux 0769
  - Alimentation et nutrition 0570
  - Audiologie 0300
  - Chimiothérapie 0992
  - Dentisterie 0567
  - Développement humain 0758
  - Enseignement 0350
  - Immunologie 0982
  - Loisirs 0575
  - Médecine du travail et thérapie 0354
  - Médecine et chirurgie 0564
  - Obstétrique et gynécologie 0380
  - Ophtalmologie 0381
  - Orthophonie 0460
  - Pathologie 0571
  - Pharmacie 0572
  - Pharmacologie 0419
  - Physiothérapie 0382
  - Radiologie 0574
  - Santé mentale 0347
  - Santé publique 0573
  - Soins infirmiers 0569
  - Toxicologie 0383

SCIENCES PHYSIQUES

Sciences Pures
- Chimie
  - Genéralités 0485
  - Biochimie 487
  - Chimie agricole 0749
  - Chimie analytique 0486
  - Chimie minérale 0488
  - Chimie nucléaire 0738
  - Chimie organique 0490
  - Chimie pharmaceutique 0491
  - Physique 0494
  - PolymÇres 0495
  - Radiation 0754
- Mathématiques 0405
- Physique
  - Généralités 0605
  - Acoustique 0986
  - Astronomie et astrophysique 0606
  - Electronique et électricité 0607
  - Fluides et plasma 0759
  - Météorologie 0608
  - Optique 0752
  - Particules (Physique nucléaire) 0798
  - Physique atomique 0748
  - Physique de l'état solide 0611
  - Physique moléculaire 0609
  - Physique nucléaire 0610
  - Radiation 0756
- Statistiques 0463

Sciences Appliqués Et Technologie
- Informatique 0984
- Ingénierie
  - Généralités 0537
  - Agricole 0539
  - Automobile 0540
  - Biomédicale 0541
  - Chaleur et ther modynamique 0348
  - Conditionnement (Emballage) 0549
  - Génie aérospatial 0538
  - Génie chimique 0542
  - Génie civil 0543
  - Génie électronique et électrique 0544
  - Génie industriel 0546
  - Génie mécanique 0548
  - Génie nucléaire 0552
  - Ingénierie des systämes 0790
  - Mécanique navale 0547
  - Métallurgie 0743
  - Science des matériaux 0794
  - Technique du pétrole 0765
  - Technique minière 0551
  - Techniques sanitaires et municipales 0554
  - Technologie hydraulique 0545
- Mécanique appliquée 0346
- Géotechnologie 0428
- Matières plastiques (Technologie) 0795
- Recherche opérationnelle 0796
- Textiles et tissus (Technologie) 0794

PSYCHOLOGIE
- Généralités 0621
- Personnalité 0625
- Psychobiologie 0349
- Psychologie clinique 0622
- Psychologie du comportement 0384
- Psychologie du développement 0620
- Psychologie expérimentale 0623
- Psychologie industrielle 0624
- Psychologie physiologique 0989
- Psychologie sociale 0451
- Psychométrie 0632

A CAPACITY PLANNING MODEL FOR

CANADIAN MILITARY AIFRLIFT REQUESTS

BY

LIEUTENANT-COLONEL BARRY A. STANNARD

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

TABLE OF CONTENTS

ABSTRACT

This thesis research was motivated by capacity planning problems encountered by the Canadian Forces Air Transport Group airlift planners who usually have more airlift mission requests than can be satisfied with the resources available. The problem is the constrained assignment of n variable length tasks (missions), integrating many airlift requests from 13 users with eight priorities, to m parallel machines (CC130 "Hercules" airframes). A general mathematical model was developed which is suitable for assisting airlift planners in deciding which airlift mission requests to accept. The model, which can be implemented on a micro computer, is essentially a computational subroutine for a larger Decision Support System.

A high quality airlift capacity plan resulted from the application of a group of management science techniques. Analytic Hierarchy Process was used to quantify each mission request. A sequential linear programming model proved to be a computationally efficient approach for producing an automated planning aid to assist the airlift capacity planners. The model is flexible, computationally quick and accurate. It handles linked missions, either as a pair or as a minimum out of an optimal number. User hour and fleet flying hour constraints are modelled and missions can be added, deleted or modified. While the model has been developed for the Canadian Forces it can be adapted for other similar military and civilian situations.

ACKNOWLEDGEMENTS

The author is sincerely grateful to the following people:

a. Doctor E.S. Rosenbloom, who unfailingly and willingly made considerable time available, for his wisdom, patience, friendship and motivation as my thesis supervisor;

b. Doctor N.S. Bruning for providing editorial advice as a committee member and for her significant help with the statistical content;

c. Professor H.J. Boom for his exhaustive, extremely helpful editorial advice as a committee member;

d. Doctor S.K. Bhatt for his service as a committee member and for his assistance in developing an early version of the mathematical model;

e. Doctor C.R. Bector for his encouragement and for exposing a number of mathematical models from which the crucial modelling approach sprang;

f. Mr. Ivan Taylor, Operational Research Advisor to the Commander of Air Transport Group, for his encouragement and for having provided virtually all the required data;

g. Mr. Peter Hypher, former OR Advisor to Commander Air Transport Group, who was instrumental in encouraging pursuit of this goal; and

h. Mavis, my wife, Katherine and Anthony, our children, for their patience and support without which this would not have been possible.

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE

DESCRIPTION OF THE RESEARCH PROBLEM

INTRODUCTION

Military airlift planners usually have more airlift mission requests than can be satisfied with the resources available. This thesis proposes an Operational Research model which can assist military airlift planners in deciding which airlift mission requests to accept. This model can be implemented on a micro computer. The research was motivated by scheduling problems encountered by the Canadian Forces Air Transport Group airlift planners. The model in this thesis is developed for the Canadian Forces but can be generalised to other military and civilian situations. The purpose of this chapter is to describe the Canadian Forces airlift capacity planning problem. The background section provides an overview of the planning environment followed by a statement of the thesis objective and an outline of its presentation.

BACKGROUND

Air Transport Group (ATG) is the primary national military air transport formation that is tasked to provide operationally ready air transport including strategic airlift, air-to-air refuelling, tactical airlift, as well as very important person transport and utility airlift. In addition, ATG provides air Search and Rescue (SAR) forces for the Canadian SAR Regions and conducts SAR operations in the Trenton and Edmonton SAR regions. Air Transport Group, with headquarters in Trenton, Ontario, is a subordinate formation of Air Command which, in turn, is a subordinate formation of the Department of National Defence. Air Command Headquarters is located in Winnipeg and National Defence Headquarters in Ottawa.

Air Transport Group consists of four bases, ten squadrons and 14 units. It has 5000 military and 2000 civilian personnel in locations from Comox, B.C. to Lahr, Germany. ATG's aircraft and personnel regularly operate worldwide. The main airlift operating bases are Trenton, Ottawa and Edmonton. To illustrate a typical yearly workload, in fiscal year 1991/92[1], ATG squadrons flew 9,805 flights that carried 229,969 passengers and airlifted 31,742,653 pounds of freight, baggage and mail. ATG's 84 aircraft of seven different types flew 65,761 hours. The 31 CC130 Hercules aircraft comprise the largest component of the ATG fleet logging 33,831 hours of that total.

The 31 CC130 aircraft are based at Trenton (19), Edmonton (10) and Greenwood (2). As the two based at Greenwood are essentially SAR resources and are currently rarely tasked for transport missions, their planning and scheduling will not be included in this discussion. The remaining 29 airframes vary somewhat in their configurations due to acquisition of different CC130 models over a span of more than 25 years and to subsequent modifications. For example, only the five acquired in 1991 will be capable of air-to-air refuelling. Others embody differences in electronics while others have certain airframe differences. About two thirds of the fleet are older "E" models and have engines with different operating characteristics that, under certain operating conditions, can affect the efficiency of airlift. However, as far as strategic (generally longer range hauling) and certain tactical missions are concerned, all CC130 airframes are equally taskable. Tactical missions in certain hostile operations could be airframe dependent.

Air Transport Group identifies 13 airlift *users*[2]. As the Government of Canada or the Department of National Defence evolves, the following list could change:

1 - NDHQ/Director General Transport (DGT) scheduled flights

2 - NDHQ/Director General Transport forecast special flights

3 - NDHQ contingency reserve

4 - Air Command (except ATG)

5 - Air Transport Group

6 - Force Mobile Command (FMC)

7 - Maritime Command (MARCOM)

8 - Rescue Coordination Centre (RCC) for SAR

9 - Northern Region Headquarters (NRHQ)

10 - Canadian Forces Support Unit (CFSU) Colorado Springs

11 - Canadian Forces Communications Command (CFCC)

12 - Canadian Forces Europe (CFE)

13 - United Nations.

As requests for airlift resources normally exceed capabilities, the following order of *priorities* has been established for tasking the airlift resources of Air Transport Group:

1 - emergency and code 1 VIP flights

2 - ATG conversion and continuation training including the route training

programme (for aircrew) and one 12 plane CC130 formation exercise per year

3 - scheduled northern and deployed peacekeeping forces resupply flights

4 - CF exercises approved by NDHQ which evaluate portions of the national contingency plan

5 - joint exercises (includes Mobile Command/Maritime Command/Communications Command/Air Command)

6 - scheduled passenger flights

7 - scheduled freight flights

8 - special flights, including command exercises not covered under priority 4 or 5.

Annual demand for airlift support has always been greater than the military airlift system has been able to provide. Therefore, equitable allocation of scarce airlift resources has been the focus of multi-level iterative planning striving to maximize use of resources to meet as much of the demand as possible within the constrained availability of airframe types, personnel, funding and infrastructure. For many years airlift planning has used a yearly forecast, updated and republished quarterly to all involved. Planning has been in the domain of Air Transport Group Headquarters and higher headquarters, while the actual scheduling of the airframes has been done at the Base level.

Following orders and set procedures, users identify airlift requirements in terms of the amount of freight or passengers to be moved from a departure point to a destination on a specific date or within a specific time window. Air Transport Group staff officers distil the request into an itinerary for one or more aircraft to accomplish the task. These itineraries, as well as the aircrew training missions, are an input to the production of the airlift capacity plan. Some of the elements affecting airlift capacity are: priority of user request, enroute time estimates, amount of required airlift in hours by aircraft type, amount of hours available to user, planned operating hours and maximum traffic handling capabilities of airports to be used, maximum number of aircraft available for tasking by Base and type, specific requirements of foreign countries affecting overflight clearances, and contract maintenance of specific aircraft requiring significant downtime.

In spite of a multitude of factors such as the changeability of user demand including: additional airlift requests, the lack of suitable computer support to the personnel involved, and the rotation of staff every three to four years, the current system has worked reasonably well. It is, however, very labour intensive. Further, the time and personnel constraints of the current system have not permitted comparisons to alternative possible plans. Thus, the quality of the manually developed plan has not been quantitatively assessed.

The airlift plan is designed to do valuable work and keep the entire airlift system operationally current while awaiting the development of emergency situations. These are usually humanitarian relief flights, UN peacekeeping tasks and missions associated with military necessity like the Gulf Crisis. When these events occur, much of the preplanned airlift is cancelled and it is necessary to rework the airlift plan to do as much as can be done with the resources left and then be ready to pick up immediately after the emergency is over.

Flexibility and timeliness depend totally upon the knowledge and dedication of the staff officers involved. They do this process by hand using pencil, paper and eraser. The results are displayed on magnetic boards. The two officers who actually do the CC130 planning are Staff Officer Operations Planning (SOOPSP-5) and Staff Officer Airlift Programs (SOAP-3) located at Air Transport Group Headquarters. Both exercise their judgement and operate within the framework of a set of decision rules concerning their work.

The output of the current Air Transport Group Headquarters planning method is a Gantt chart-like-matrix with rows being CC130s (only 10 "lines" per day to Trenton, 7 to Edmonton and 2 to Greenwood) and columns being the days of the year. Airlift tasks are manually scheduled into the grid integrating airlift mission requests from the 13 users ranging over the 8 priority levels. The sheer size and complexity of the resultant matrix for the CC130 airlift plan makes changes difficult and very labour intensive. The emphasis is on finding a feasible schedule.

Each Base has a given allocation of aircraft and Air Transport Group Headquarters does not concern itself with the specific aircraft which actually carries out a task assigned to a Base. Since Air Transport Group Headquarters usually only tasks 70% or less of a Base's aircraft inventory per day, many of the considerations that would otherwise complicate the planning issue are avoided. Some of these considerations are: number of hours left on each airframe before the next required inspection, time for required aircraft maintenance, maintenance capability, aircraft configuration and reconfiguration times, maximum number of crew available for tasking by squadron, time on ramp for loading, time on ramp for refuelling and time on ramp for crew preflight checks.

Assignment of a specific airframe to a mission is a function of Base Operations (an organisation responsible for coordinating all aspects of a Base's daily flying activities). One of their objectives is to maximise the number of fleet flying hours available to provide maximum airlift capability at any point in time specifically keeping in mind the number of flying hours remaining to required major airframe maintenance. When a Base receives a tasking from Air Transport Group Headquarters a scheduling officer (an experienced aircrew officer) assigns a specific airframe (or airframes) by tail number. Developing the best scheduling plan incorporating the task again depends entirely upon the knowledge of the scheduling officer and the hours available to do the job. Response to last minute changes in previously scheduled missions, aircraft unserviceabilities or emergency tasking can precipitate a scheduling situation that does not result in a better solution due to the finite number of hours available for manually integrating all the factors required to produce an amended schedule. The full benefit of the scheduling officer's knowledge may not be realised, neither may the fleet hour allocation be the best it could be. At present the schedulers display the results on a large magnetic matrix board with airframe tail numbers as rows and days of the year as columns. The missions are represented on magnetic strips placed on the grid resulting in a picture of the day-to-day disposition of the aircraft in each Base's fleet. It would be a complex and a labour intensive procedure to test the different available patterns of possible airframe assignments for the best possible fit. While the first schedule produced may be reasonable, subsequent rapid or major disruptions often result in less than efficient but still workable solutions.

The current system does have some negative human factors. For example, all scheduling officers are highly paid experienced pilot or navigator aircrew officers. As such, they virtually all see "ground tours" such as these planning and scheduling jobs, although recognised and responded to as very important, in a lesser light than "flying tours". This is compounded by a lack of modern technology with which to do the job. At a handover briefing between the outgoing and incoming planning officers very specific direction was given as to precisely which type of eraser was best for the job. At one of the Base airframe scheduling desks I was once told that some days all that could be expected was production of a workable solution rather than the best possible one, strictly due to the pressure of time. This same situation occurs for HQ planners as well.

In summary, the problem is the constrained assignment of n variable length (possibly airframe type dependent) tasks (missions), integrating hundreds of airlift requests from 13 users with eight priorities, to m CC130 airframes of different variants located at two geographically widely separated sites (Bases). Given the realities faced by Air Transport Group, it must be possible to implement this solution using microprocessors at modest cost. The solution must be sufficiently timesensitive. Staff Officer Operations Planning-5, Staff Officer Airlift Programs-3, and Base schedulers need solutions in near real time.

THESIS OBJECTIVE AND OUTLINE

Operations Research Advisor to the Commander of Air Transport Group, Ivan Taylor, proposed this area of research to meet a long standing need. The Senior Staff Officer Operations (SSOOPS), who is responsible to the Commander for airlift operations including airlift programs, is the officer of primary interest (OPI) for this command and control project. It is his staff who would use a system fully developed from a prototype. It should be noted that some researchers at the Royal Military College are using an Expert Systems approach to the same problem.

For the purposes of this thesis, only CC130 "Hercules" fleet planning is considered. However, it is fully expected that the results will be applicable to the other smaller fleets as well. Further, the thesis is limited to ATG Headquarters (HQ) planning only. Due to time constraints, the additional complexity of Base level scheduling is beyond the scope of this research. Information used in this thesis is dated 1992 or earlier.

The prototype planning model for planning officers to be developed in this thesis should:

a. develop an airlift plan including linked mission requests,

b. respond to user requested changes to the original airlift plan including mission request additions, deletions and modifications,

d. respond to changes in system constraints such as the number of airframes that can be tasked on a given day, the number of flying hours available to each user, and the total fleet yearly flying rate hours,

e. improve the quality of work life for those involved in the planning process and

f. improve the quality and timeliness of the information available to those in command and control positions.

This prototype model is to be imbedded in a larger command and control system which, at a minimum, incorporates:

a. a user request database for airlift missions in terms of user identity, dates of a specific request, hours of airframe usage per mission, priority and category of a request, linkage to any other missions;

b. a system constraint database including fleet flying hour limitations, maximum number of taskable aircraft by type and Base; and

c. an appropriate user input/output interface.

The thesis is developed in the following outline by chapter. Chapter two develops the relationship between operational research (OR) and the military. The Analytic Hierarchy Process, a method for discriminating between competing alternatives, is presented in chapter three. Chapter four investigates methods of forecasting the number of aircraft available to task each day. Chapter five presents the use of the mathematical planning model. Chapter six contains the conclusions, recommendations and limitations.

CHAPTER TWO

OPERATIONAL RESEARCH AND THE MILITARY

THE BEGINNING OF OPERATIONAL RESEARCH

Operational Research, like much else, was born out of necessity. The purpose of this chapter is to describe its historical beginnings as well as its current connection to the military.

The application of the methods and techniques of science to decision making is known as Operational Research (a.k.a. as Operations Research), in military circles and as Management Science in civilian organizations. Several definitions of Operational Research or Management Science (OR/MS) exist. From one point of view, Cook and Russell[3] note that it is an interdisciplinary field, comprising elements of mathematics, economics, computer science and engineering, devoted to studying and developing procedures to help in the process of decision making. From another, Woolsey[4] states "operations research is the application of logic and mathematics to real world problems in such a way that the method doesn't get in the way of common sense". Further, he emphasises that applicational success is the only proper measure of the profession. Whatever the view point, the hallmark of operational research is the application of the scientific method to management problems so as to enable better decisions for successful implementation.

During the latter parts of the previous century and throughout this century there has been an ever increasing effort to apply scientific techniques to management. Cook and Russell[3] give a brief review of the early days. They note that Charles

Babbage, a brilliant English mathematician and mechanical inventor, wrote a "seminal treatise titled *On Economy of Machines and Manufactures* (1832)"[3]. In it he discussed relevant management science issues such as skill-related differentials in wages and concepts of industrial engineering. Later, the American engineer Frederick Taylor[3] postulated that there was one best or most efficient way to accomplish a given task. He used time studies to rate worker performance and examine work methods. At the same time, Henry L. Gantt[3] brought the consideration of the human factor into management's attitude towards labour, championing the importance of a personnel department to the scientific approach to management. Most important is that his development of a method for scheduling jobs on machines endures today. His Gantt chart method, essentially a manual recording system, facilitated minimising job completion delays permitting machine loadings to be planned months in advance. These developments were concentrated on the working levels of organizations and were significant advances at the time.

Mathematical modelling of decision problems was apparent by 1914. Frederick W. Lanchester[3] attempted to predict the outcome of military battles based on numerical personnel strength and weaponry. Development of a simple lot-sized formula by Ford W. Harris[3] followed and it remains in use today. Amongst other work, A. K. Erlang[3], a Danish mathematician, founded queueing theory which includes mathematical formulas to predict waiting times for callers using automatic telephone systems. World War II saw the emergence of operational research as a recognised discipline.

OPERATIONAL RESEARCH COMES OF AGE IN WORLD WAR II

As one of the people involved, Harold Larnder, past president of the Canadian Operational Research Society, provides a superb look at *The Origin of Operational Research*[5]. The following is a summary with quoted excerpts. From 1933 to 1939, Hitler's goal was to create a Luftwaffe equal in power to the combined air forces of Britain and France. Britain was determined to create an air defence that could resist an attack on the British Isles. Through 1933 and 1934, no solution to this problem could be seen. The Committee for the Scientific Survey of Air Defence was established in Britain to consider "how far recent advances in scientific and technical knowledge can be used to strengthen the present methods against hostile aircraft". In 1935 the Committee asked Robert Watson-Watt to see if a "death ray" might be developed to kill or incapacitate the pilot or disable the aircraft. Watson-Watt and his team found that the essential problem was locating the incoming aircraft. Further, although a "death ray" was beyond the technology of the time, Watson-Watt was able to demonstrate that he could locate an aircraft by radio. In 1937, the first major air defence exercise was held. Radar results were encouraging but obvious command and control problems arose. After these finding were confirmed by another exercise in 1938, A.P. Rowe proposed that research be carried out into the operational aspects of the system. Larnder notes that the term "Operational Research" was coined to name this new branch of applied science. The results were so effective that Air Chief Marshall Sir Hugh Dowding, then Air Officer Commanding-in-Chief Royal Air Force Fighter Command, ensured that the research teams be attached to his headquarters.

Under the direction of Harold Larnder, the Operational Research Section was formalised in 1939.

From 1939 onwards, every failure in intercepting a daylight raid was analysed. This resulted in high air defence system efficiency. In addition, research was extended beyond warning and control systems to the deployment and handling of air defence fighters. Further, noting that enemy mine-laying aircraft left fragmentary radar tracks due to low altitude flying, the section postulated that when the targets disappeared they were often laying mines. Given the subsequent positions, the navy was able to take appropriate action. So, the early operational research work was intertwined with radar. However, this was only the beginning.

In 1940, the RAF was fighting on the continent and suffering significant losses. Dowding, faced with this high loss rate and Churchill's impending deployment of yet another ten squadrons to support the French, was determined to keep the aircraft in Britain. Here starts one of the best known of all operational research war stories. On the morning of 15 May 1940, Dowding asked Larnder "is there anything you scientists can suggest bearing on this matter?".

Only two hours were available before the War Cabinet meeting. Larnder recounts "at the suggestion of E.C. Williams, a rapid study was carried out based on current daily losses and replacement rates to show how rapidly the Command's strength was being sapped and how much more rapid this would become if its losses were to be doubled while the replacement rate remained constant. For ease of display and understanding the findings were presented in graphical form". The meeting ensues. According to Larnder, quoting Collier (1957), Dowding sensed the need for more persuasion and walked around to Churchill saying "if the present rate of wastage continues for another fortnight, we shall not have a single Hurricane left in France or in this country". Laying down the graphs won the day. Not only was the deployment cancelled but the aircraft on the continent were recalled. Larnder notes that the important lesson here was in providing the Commander-in-Chief with information in a form (graphs) that would give Dowding the means to oppose what he knew would have been a fatal decision.

The winning of the Battle of Britain was crucial to the outcome of World War II. Larnder notes "there seems little doubt that, had Dowding not won his battle with Churchill in May, he would almost have lost the Battle of Britain in September". Historian William L. Shirer[6] quotes Adolph Galland, the famous German fighter ace: "We realised that the RAF fighter squadrons must be controlled from the ground by some new procedure because we heard commands skilfully and accurately directing Spitfires and Hurricanes on to German formations...For us this radar and fighter control was a surprise and a very bitter one." Larnder notes that operational research contributions were significant. When Sir Hugh Dowding turned over his Command he responded to Larnder: "Thanks. This war will be won by science thoughtfully applied to operational needs."

Cook and Russell[3] note that other major problem areas studied in World War II were: guidance systems for long range bombing, antisubmarine warfare weapon systems and methods as well as civilian defence and the optimal deployment of convoy escort vessels. Further, the multidisciplinary teams formed have become "characteristic of operational research/management science".

Cook and Russell[3] observe that the successes of the British operational research teams convinced the United States military to include "operations analysis" groups. These were comprised of mathematicians, statisticians, probability theorists and computer experts. John Von Neumann[3] made huge contributions in the area of game and utility theory. George Dantzig[3] worked on the simplex method of linear programming, a technique that uses linear algebra to determine the optimal allocation of scarce resources. At the end of the 1950's, the major tools of operational research were fairly well developed. These included linear programming, dynamic programming, inventory control theory and queueing theory. In the 1960s, decision analysis was initiated for dealing with decisions under uncertainty. Goal programming and multiobjective linear programming were introduced to solve decision problems with multiple or conflicting goals.

One of the most crucial developments in support of operational research activities has been the maturing of computer technology, methods and software. Much of what operational research professionals do requires powerful computational ability. Development of mathematical models such as those used in this thesis would have been much more difficult without digital computers and associated software.

OPERATIONAL RESEARCH AND CANADIAN MILITARY

Operational research was formally established in Canada as the Defence Research Board in the late 1950s. Currently there are operational research sections in

National Defence Headquarters, Air Command Headquarters, Fighter Group, Maritime Air Group and Transport Group Headquarters. Within Air Transport, the position of operational research advisor to the Commander was set up in the mid 1960s. It was first staffed by the multitalented Peter Hypher who inspired the author, while seconded to his staff in 1982, to seek professional development in the operational research community.

Throughout the years in Air Transport Group, the Operational Research section has developed automated tools to assist in airlift load planning, airlift itinerary generation for multiple aircraft and crews, and airlift simulation. They continually carry out detailed post-operation analysis of major airlifts. As a result of their research, they have published general planning guidelines for airlift planners. Moreover, significant studies have been done concerning the Search and Rescue system, the transport aircrew training system, replacement aircraft selection, operational characteristics of the various aircraft fleets, and aircrew experience levels, to name but a few.

Air Transport Group was heavily involved in the Gulf crisis. One of the consequences was the observation "the [Commander's] Command and Control system needs 'user-friendly', fast, reliable and deployable automated airlift planning tools"[7]. The prototype models developed in this thesis are expected to be of some use in this area.

In closing this chapter it is worthwhile to note that, in the military context, work as an Operational Research professional is a staff, as opposed to line, function.

Simply put, this means Operational Research personnel do not command, they advise those military officers who do command. Thus, the objective must be to provide the military commander with the best possible advice to enable that officer to make the best possible decision. In doing this, one should remember Woolsey's primary law, "People would rather live with a problem they cannot solve than accept a solution they cannot understand"[8].

Penultimately, to practise military operational research one must be mindful of the simple but sometimes forgotten fact that the military environment functions under particular laws and ethics germane only to the military. Thus, what may be a most suitable model in business or other civilian disciplines must be carefully scrutinised for acceptability.

Finally, given the ever continuing spiral of decreasing resources made available to the military and the high expectations of government and miliary leadership, it seems reasonable that operational research professionals will not be short of work. Analysis and modelling of systems with scarce resources to provide acceptable options will become ever more necessary.

CHAPTER THREE

MISSION WORTH ASSESSMENT USING ANALYTIC HIERARCHY PROCESS

INTRODUCTION

A means of describing an airlift mission request numerically is required in order to quantitatively and selectively discriminate amongst competing alternatives. This becomes challenging when a number of categorical mission criteria, such as importance to a user, training value, and effective aircraft use, are used to identify a mission. In chapter five, an algorithm involving linear programming is developed for fleet capacity planning. The algorithm requires the numerical values calculated here as the objective function coefficients. Parts of this chapter were inspired by a joint course project[9].

Thomas Saaty's Analytic Hierarchy Process (AHP)[10], first proposed in the late 1970s, is a powerful tool suited to this type of multiple criteria decision problem. As the criteria measurement is not probabilistic, an alternative such as Multi-Attribute Utility Theory (MAUT) is not appropriate. Saaty notes that AHP is a systematic procedure for representing the elements of any hierarchic structure. It organises one's basic reasoning disposition by breaking down the structure into its smaller constituent parts and then calls for simple pairwise comparison judgements to develop the priorities in each hierarchy. Schoner[11] observes that AHP involves three stages: first, decomposition of the problem into a hierarchy; second, paired comparisons of items on any hierarchical level relative to their contribution towards the immediately higher level; and third, composition of the resulting local priorities, known as importance weights, into ratio-scaled composite values that reflect the overall importance of each objective. Application of AHP to the numerical description of airlift mission requests is the aim of this chapter; a more detailed explanation of AHP is incorporated.

Currently, the only means of differentiating between airlift support requests in a given priority level, as described in chapter one, is assigning one of three following *categories*[12].

Category A - missions in direct support of planned operations, such as personnel rotations and exercise reconnaissance.

Category B - missions in support of the day to day functioning of the Department of National Defence, such as staff liaison visits and the movement of personnel as part of a formalised Canadian Forces course.

Category C - missions in support of other activities such as parades, ceremonies and official sports competitions.

Thus, the worth of a mission in a given priority could be represented mathematically by simple weighting factors such as 0.6 for category A, 0.3 for B and 0.1 for C. The priority and category capture the user's measure of importance of a mission. However, there are other significant aspects of an airlift mission that could aid in the discrimination if incorporated with category, such as system training value and effective use of the aircraft.

With an input assessment of the training, category and effective use decision attributes, the Analytic Hierarchy Process (AHP) can be used to produce a ranking of airlift requests. AHP is well suited to "converting subjective assessments of relative importance into a linear set of weights which can be used to rank alternatives or to serve as an objective function in other techniques"[13].

SCOPE

As a matter of Departmental policy, the order of priority is virtually absolute. A priority 3 mission should not be planned at the expense of a priority 2 mission. Operational exigencies can cause senior departmental officials to override this limitation but this is rarely done. For the purposes of this paper, priority is overriding and therefore has not been included in the AHP. Rather, AHP will be used to rank requests within a given priority.

AHP MODEL STRUCTURE

Ranking of airlift support requests implies that some value must be calculated for each request. The only currently documented factor in addition to priority is user category. Many years of experience and discussions with several decision makers within ATG made it clear that more than just priority and user category is involved in a mission's value. Particularly in peacetime, a major component of the value of a mission to ATG is the amount of training it provides. Further, the effective use of the aircraft related to other requests has a value. While these attributes are only representative and others may be deemed significant by other decision makers at other points in time, they are assessed to find the relative worth of a mission for the prototype model. The resultant AHP hierarchical structure is presented in Figure 1 and represents the first step in the AHP.

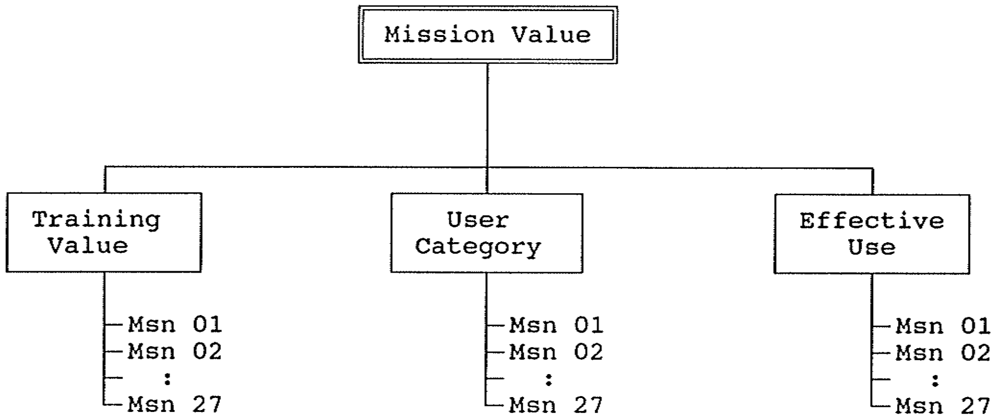


Figure 1. Decision Hierarchy

The next step is to assess the relative importance of the decision attributes using pairwise comparisons of relative importance as shown in Table 1.

Mission Attribute	Training Value	User Category	Effective Use
Training Value	1	3	7
User Category	1/3	1	4
Effective Use	1/7	1/4	1

Table 1. Pairwise Comparison - Decision Attributes.

Table 1 is representative of the type of management assessment that puts a higher importance on training value more typical of a peacetime scenario. Like all other such comparisons in this project, the accuracy of the assessments depends on a particular point of view. Therefore, sensitivity analysis of this matrix is important and various management "importance" assessments are developed later in this section.

Standard AHP "importance" comparisons have been used where the decision attributes are compared pairwise to determine the relative importance based on a scale of 1-9 as per Table 2.

Value of a_{ij}	Interpretation
1	Attribute i and j are of equal importance.
3	Attribute i is weakly more important than j.
5	Experience and judgement indicate that attribute i is strongly more important than attribute j.
7	Attribute i is very strongly or demonstrably more important than attribute j.
9	Attribute i is absolutely more important than j.
2,4,6,8	Intermediate values - ie. a value of 2 indicates that attribute i is midway between equal and weakly more important than j.

Table 2. Interpretation of Entries in a Pairwise Comparison Matrix.

The preferred method of computing AHP values is to use an eigenvector based method such as Expert Choice, a licensed software product not available to the author. As an alternative, the spreadsheet technique described by Winston[14] was used with recent verification using MathCad[15], see Figure 2. As shown in Table 3, the pairwise comparison matrix has been normalised and from this normalised matrix, the weights for each attribute have been determined. Part of the AHP is a consistency check to ensure that the decision makers' comparisons of importance between the decision attributes are consistent. Referring to Table 3, the measure of consistency (CI/RI) is 0.028 which is well within the maximum limit of 0.10.

$$A := \begin{bmatrix} 1.0 & 3.0 & 7.0 \\ \frac{1}{3} & 1.0 & 4.0 \\ \frac{1}{7} & \frac{1}{4} & 1.0 \end{bmatrix} \qquad c := \mathrm{eigenvals}(A) \qquad c = \begin{pmatrix} 3.0324 \\ -0.0162 - 0.3129i \\ -0.0162 + 0.3129i \end{pmatrix}$$

$$v := \mathrm{eigenvec}(A, c_0) \qquad v = \begin{pmatrix} 0.9232 \\ 0.3683 \\ 0.1102 \end{pmatrix} \qquad |v| = 1 \qquad s := \sum v \qquad s = 1.4016$$

Check

$$A \cdot v = \begin{pmatrix} 2.7994 \\ 1.1168 \\ 0.3341 \end{pmatrix} \qquad c_0 \cdot v = \begin{pmatrix} 2.7994 \\ 1.1168 \\ 0.3341 \end{pmatrix}$$

$$T := \frac{v_0}{s} \qquad T = 0.6586$$

$$U := \frac{v_1}{s} \qquad U = 0.2628$$

$$E := \frac{v_2}{s} \qquad E = 0.0786$$

$$N := T + U + E \qquad N = 1$$

Figure 2. Eigenvector Calculation of Attribute Weights.

Comparing the W_MAX values for training, user and effectiveness from Table 3 and the T, U and E values from Figure 2 show that the maximum difference is .003 (for the training value attribute). As shown in Table 9, further analysis of the effect of the eigenvector values on the overall ranking of mission requests has revealed that the ranking order did not change nor was any difference greater than .0002. Thus, the approximation is sufficiently accurate for this prototype model. The eigenvector method, however, is recommended for an operational implementation.

Like most military personnel, those in ATG experience three to four year posting cycles. The effect of this is that 1/4 to 1/3 of a military unit's personnel posting cycles. The effect of this is that 1/4 to 1/3 of a military unit's personnel

PAIRWISE COMPARISON MATRIX					
COMPARE	TRG	USER	EFFECT		
TRG VAL	1.0000	3.0000	7.0000		
USER CAT	0.3333	1.0000	4.0000		
EFFECT	0.1429	0.2500	1.0000		
COL SUM	1.4762	4.2500	12.0000		
NORMALISE	TRG	USER	EFFECT		**W_MAX**
TRG VAL	0.6774	0.7059	0.5833	2.0075	0.6555
USER CAT	0.2258	0.2353	0.3333	0.8019	0.2648
EFFECT	0.0968	0.0588	0.0833	0.2395	0.0796
CONSISTENCY INDEX CALCULATIONS					
3.0623			CI=		0.0163
3.0282	SUM/3=	3.0325	RI=		0.58
3.0071			**CI/RI=**		0.0280

Table 3. Spreadsheet Decision Attribute Matrix Calculations.

require training to some degree every year to accomplish the tasks associated with their new positions. Further, due to the necessarily very high performance standards, personnel undergo training and evaluation to varying degrees every year. In order for ATG to meet its mandate of being operationally ready, the entire system must provide those necessary training opportunities. Therefore it is reasonable to include Training Value as an AHP attribute. In relation to the other two attributes, ATG decision makers feel that Training Value is more important. When comparing the Training Value offered by various missions, it was felt that a subjective rating system of High (H), Average (V) and Low (L) could be implemented. For example, a transoceanic flight from Canada to Europe with a freight load would be considered to have a high ATG Training Value. A fly past for an air display would have a relatively low system training value. Routine passenger or freight flights in southern Canada would be rated as average. Although some point-scoring method for given missions would provide better discrimination, the resources to do this are not currently available. Table 4 shows a typical baseline pairwise comparison matrix to assist in assessing airlift support requests, given that the decision maker can accomplish a three point assessment.

Training Value	High	Average	Low
High	1	3	5
Average	1/3	1	3
Low	1/5	1/3	1

Table 4. Pairwise Comparison - Training Value Attribute.

The Canadian Forces Administration Orders(CFAO)[12] directs that airlift users thoroughly screen and categorise airlift requests in accordance with the A,B, or C category system, so that the users can indicate relative importance of their mission requests. Because there are 13 users, consistency is an important issue. Should the model be adopted it may be necessary to amplify this CFAO to provide better guidance to avoid over-rating in the user category. Table 5 shows a probable baseline pairwise comparison matrix for the user category attribute to assist in assessing airlift support requests.

Category	A	B	C
A	1	5	9
B	1/5	1	5
C	1/9	1/5	1

Table 5. Pairwise Comparison - User Category Attribute.

In the past, decision makers involved in the airlift system have expressed a desire to assess the effective use of airlift. This, however, is a complex issue due to the influence of many absolute factors such as aircraft maximum load bearing capacity, maximum volume, maximum seating, and maximum all-up-weight. These constraints are affected by the range-payload dichotomy. The weight of the aircraft when empty combined with the weight of fuel required and the weight of the freight or passengers cannot exceed the maximum all-up-weight for takeoff. Therefore, the actual usable "maxima" for a given flight over a given range are often less than the absolute maxima. An additional important factor for effective use assessment is the average number of flying hours used per day during the mission. For example, Service Flight 85/86 between Trenton and Alert typically uses 19 hours in 2 days for an average of 8.5 hours per day, while a passenger airlift mission for an essential training course has used 35 hours in 14 days for 2.5 hours per day. The former is a much more effective use of the aircraft than the latter. Both would likely be full to capacity and the decision maker would have to weight both aspects in judging the relative worth of the two missions. Again, a three point High (H), Average (V) and Low (L) assessment of relative effective use can be instituted. For example, if the airlift support request will clearly fill the capacity of the cargo compartment in any of weight, volume or seating factors, the mission would be rated as High. Likewise, if it is a long range flight and again the cargo compartment is filled to the maximum for that given mission, a rating of High would be appropriate. Arbitrarily, greater than 75% could be considered High, less than 25% could be considered low with Average lying in between. Table 6 shows a baseline pairwise comparison matrix for the Effective Use attribute to assist in assessing airlift support requests.

Effective Use	High	Average	Low
High	1	3	5
Average	1/3	1	3
Low	1/5	1/3	1

Table 6. Pairwise Comparison - Effective Use Attribute.

Once the relative weights of the attributes have been decided upon, the mission requests are similarly compared pairwise to determine their relative importance with respect to each attribute. Finally, the weights for each decision attribute are combined with the weights for each alternative with respect to that attribute and a final weight is produced for each mission request. The end result is a comparative rank for each triplet of mission attributes amongst the possible attribute combinations. Given the three decision attributes of training value, category and effective use, only 27 possible combinations exist (see Table 7)

Attribute Combinations			
Mission #	Training Value	User Category	Effective Use
1	H	A	H
2	H	A	V
3	H	A	L
4	H	B	H
5	H	B	V
6	H	B	L
7	H	C	H
8	H	C	V
9	H	C	L
10	V	A	H
11	V	A	V
12	V	A	L
13	V	B	H
14	V	B	V
15	V	B	L
16	V	C	H
17	V	C	V
18	V	C	L
19	L	A	H
20	L	A	V
21	L	A	L
22	L	B	H
23	L	B	V
24	L	B	L
25	L	C	H
26	L	C	V
27	L	C	L

Table 7. All Possible Mission Attribute Combinations.

One of the most important steps for gaining management acceptance of the proposed model is demonstrating the robustness of this approach. In addition to the 1-3-7 baseline calculations, three other sets have been produced (see Table 8). To differentiate between the four sets of calculations, the second row of Table 8 refers to management's importance ratings on the associated upper row of the pairwise decision attribute comparison matrix. Comparing the mission number (MSN #) sequence (1-27) of all possible options from Table 7, one can see in Table 8 that for each decision attribute comparison matrix, a new sequence of mission numbers results. This occurs as the AHP produces a new set of values for ranking the 27 combinations for each additional set. Set four, for example, shows the results of management postulating that the training value and user category are of equal importance, thus allowing effective use of the aircraft to be the discriminating attribute for ranking the mission requests. The AHP-computed values for each set of calculations are shown in Figure 3.

SET 1	ATTRIB	SET 2	ATTRIB	SET 3	ATTRIB	SET 4	ATTRIB
MSN #	1-3-7	MSN #	1-3-9	MSN #	1-2-5	MSN #	1-1-5
1	HAH	1	HAH	1	HAH	1	HAH
2	HAV	2	HAV	2	HAV	2	HAV
3	HAL	3	HAL	3	HAL	3	HAL
4	HBH	4	HBH	4	HBH	10	VAH
5	HBV	5	HBV	5	HBV	11	VAV
7	HCH	6	HBL	7	HCH	4	HBH
6	HBL	7	HCH	6	HBL	12	VAL
8	HCV	8	HCV	10	VAH	19	LAH
9	HCL	9	HCL	8	HCV	5	HBV
10	VAH	10	VAH	11	VAV	6	HBL
11	VAV	11	VAV	9	HCL	20	LAV
12	VAL	12	VAL	12	VAL	7	HCH
19	LAH	19	LAH	19	LAH	21	LAL
20	LAV	20	LAV	20	LAV	8	HCV
13	VBH	21	LAL	21	LAL	9	HCL
21	LAL	13	VBH	13	VBH	13	VBH
14	VBV	14	VBV	14	VBV	14	VBV
16	VCH	15	VBL	16	VCH	15	VBL
15	VBL	16	VCH	15	VBL	16	VCH
17	VCV	17	VCV	17	VCV	22	LBH
18	VCL	18	VCL	22	LBH	17	VCV
22	LBH	22	LBH	18	VCL	23	LBV
23	LBV	23	LBV	23	LBV	18	VCL
25	LCH	24	LBL	25	LCH	24	LBL
24	LBL	25	LCH	24	LBL	25	LCH
26	LCV	26	LCV	26	LCV	26	LCV
27	LCL	27	LCL	27	LCL	27	LCL

Table 8. Sensitivity to Decision Attribute Importance Values.

The x-axis mission numbers of the graph refer to the original mission numbers from Table 7. As expected, the airlift mission requests with the absolute highest and lowest decision attribute values (HAH and LCL) always appear highest and lowest respectively with the rearrangement due to AHP value calculations occurring in between.

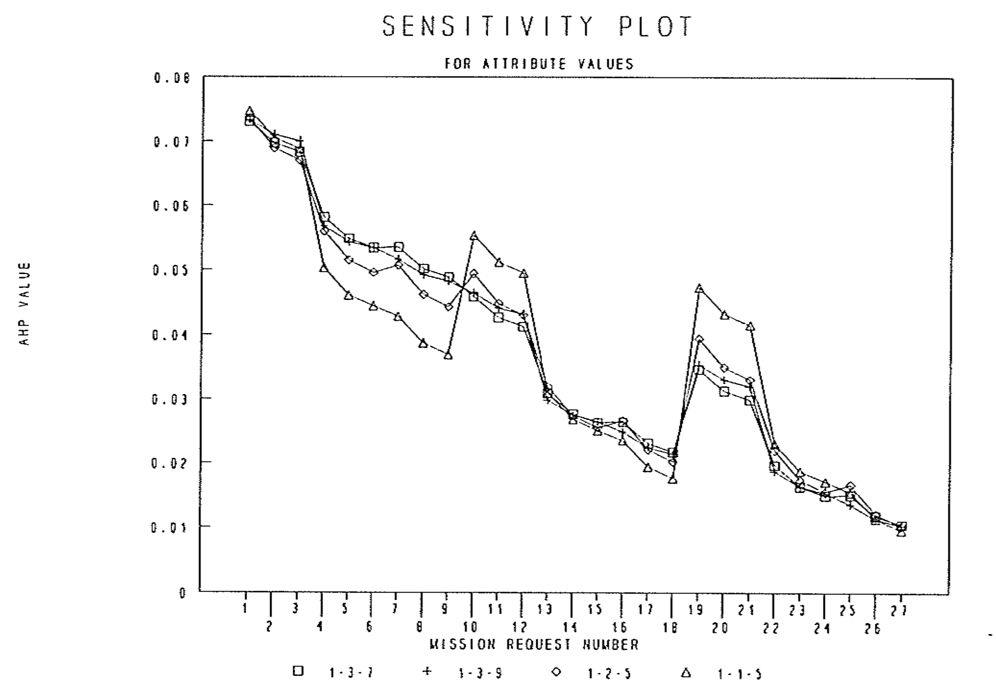


Figure 3. Sensitivity Plot for Decision Attribute Values.

The senior officers in the ATG hierarchy would have positive control over the particular importance values for the decision attribute comparison matrix. They could direct the use of certain set matrices for given military circumstances. The results for the planners are shown in Table 9 which is representative of peacetime operations.

SELECTION SEQUENCE	MISSION NUMBER	MISSION REQUEST ATTRIBUTES	AHP VALUE Lotus	AHP VALUE MathCad
1	1	HAH	0.0730	0.0730
2	2	HAV	0.0697	0.0697
3	3	HAL	0.0683	0.0684
4	4	HBH	0.0581	0.0582
5	5	HBV	0.0548	0.0549
6	7	HCH	0.0535	0.0537
7	6	HBL	0.0534	0.0536
8	8	HCV	0.0502	0.0504
9	9	HCL	0.0489	0.0491
10	10	VAH	0.0459	0.0457
11	11	VAV	0.0426	0.0425
12	12	VAL	0.0412	0.0411
13	19	LAH	0.0346	0.0344
14	20	LAV	0.0313	0.0312
15	13	VBH	0.0309	0.0309
16	21	LAL	0.0299	0.0298
17	14	VBV	0.0276	0.0276
18	16	VCH	0.0264	0.0264
19	15	VBL	0.0263	0.0263
20	17	VCV	0.0231	0.0231
21	18	VCL	0.0217	0.0218
22	22	LBH	0.0197	0.0196
23	23	LBV	0.0164	0.0163
24	25	LCH	0.0151	0.0151
25	24	LBL	0.0150	0.0150
26	26	LCV	0.0118	0.0118
27	27	LCL	0.0105	0.0105

Table 9. Ranking of Missions Representative of Peacetime Conditions.

RANK REVERSAL

The AHP produces an ordered set from a set of choices. Sometimes, when an alternative is added or deleted from the choice set, the order for the choices in the new set may change. If it does, this is known as rank reversal. For example, from a choice set of four items, suppose the initial ordered set from the AHP was items 1,2,4 and 3. If item 2 is removed from the choice set, one expects the order to be 1,4 and 3. If the AHP produces an ordered set of 1,3 and 4, rank reversal has occurred.

Invariably questions are asked about rank reversal when the AHP is used. In this case, is rank reversal a threat to the model? First, we review the academic argument between proponents and opponents of the Analytic Hierarchy Process (AHP) in its conventional form. Then, a discussion of the AHP and the proposed use of the model specifically.

Dyer[16] states "the Analytic Hierarchy Process (AHP) is flawed as a procedure for ranking alternatives in that the rankings produced by this procedure are arbitrary". Howard[17], using a religious metaphor, claims that those who embrace AHP (and fuzzy set theory) are "heathens". Schoner[11] claims "the case against conventional AHP is ironclad".

The phenomenon of rank reversal is identified by Dyer as the most controversial aspect of AHP. Under certain circumstances, rank reversal can occur when another alternative is added to a group of alternatives previously ranked by AHP. He concludes that "rank reversal is a symptom of a much more profound problem with AHP: the rankings provided by the methodology are arbitrary". Dyer attributes the problem to the AHP principle of "hierarchic composition" (the weights assigned to the criteria (decision attributes) do not depend on the alternatives under consideration). He further argues that this principle is always violated when evaluating alternatives on multiple criteria. Dyer concludes that the solution lies in a synthesis of AHP and Multi-Attribute Utility Theory (MAUT).

Howard is biased toward the Utility Theory view held by what, to extend Howard's religious metaphor, might be called the "true believers". He defines "heathens" as those who are external challengers to the "usual axiomatic structure" of decision analysis and includes proponents of AHP in this group. His main reason for rejecting AHP is that it does not measure up to his self-defined "warranties"[criteria]. He also, but without the rigour of Dyer, identifies the possibility of rank reversal as "particularly bothersome". Howard does not propose any remedial fixes. It appears celibacy may reflect his approach to AHP.

Schoner has been actively involved in the discussions on AHP. The following is from his article *Correcting the Analytic Hierarchy Process*[11]. He notes that Watson and Freeling (1982) identified the manner in which criteria weights are assigned as the cause of rank reversal. He further notes that an example of protection against rank reversal by Saaty, Vargas and Wendell (1983), required that the criteria decision attribute weights be constrained so that "the ratio of the weights of two criteria equalled the ratio of the sum (or average) of the measurements of the alternatives on each of the criteria". In 1988, Schoner and Wedley coined the term "Mean Referenced Condition" for this concept and showed that the Mean Referenced

Condition is essential. If it is violated, "the estimated composite priorities of all alternatives are incorrect". While conventional AHP axioms state that the higher levels are not dependent on the lower levels in the hierarchy, Schoner notes the Mean Referenced Condition "makes criteria weights completely dependent on the alternatives in the choice set". Schoner concludes by stating that the AHP should be modified to overcome the identified deficiencies and retain its positive features. He suggests a *Vertical Linking Pins* model, discussed below.

Saaty[18], responding to Dyer, notes "there is good reason, even a need, for rank reversal in the relative measurement mode of AHP for which there is no parallel in utility theory. This is an advantage of relative measurement, rather than being flawed...". Harker and Vargas[19] state that Dyer's "criticism arises out of a lack of understanding of the theory of AHP". Thus, between the two camps, we have an ongoing strenuous argument.

Saaty[20] goes to great lengths to respond to Dyer's criticism. He points out that AHP is a "different and independent theory of decision making from utility theory". Utility theory is a normative process while AHP is a descriptive process capable of dealing with "outcomes not accounted for by the demanding assumptions of a normative theory". Further, and apparently to remind the reader, Saaty notes that the utility theory rival also makes some "unrealistic assumptions about transitivity, consistency of preferences and the difficult use of lotteries leaving a long trail of paradoxes behind that diminish its validity and relevance". It appears that Saaty's point is that utility theory is also "flawed" so direct comparisons to it are not necessarily valid.

The main point made by Saaty is that addition or deletion of an alternative changes the fundamental nature of the decision to be made. The change is one of information concerning the dominance of one alternative over another. He uses the analogy of adding or deleting variables to a linear program from which a new optimal solution does not usually coincide with the previous one for some of the variables. Saaty notes "this is not like anything encountered in utility theory. It is new and logical, but certainly not arbitrary" as suggested by Dyer. Further, Saaty indicates that relative measurements based on ratios, as used in AHP, involve a kind of dependence among alternatives that is not encountered in absolute measurement nor in utility theory. Saaty also agrees that the addition of an exact copy of one of the alternatives in relative AHP measurement can change the rank of alternatives, but argues that this is because what appears to be a copy using absolute measurement may not be so under the AHP relative measurement paradigm. Saaty dismisses Dyer's MAUT fix by again marshalling the inadequacies of utility theory, with examples to conclude that Dyer's fix produces no better decision than the conventional AHP.

Dyer had reasoned that AHP does not have an independence axiom and concluded AHP yields arbitrary rankings. Harker and Vargas[21] point out that AHP Axiom 3 "states very clearly what independence means in the context of AHP". They also point out that the example used by Dyer does not comply with Axiom 3 and is therefore invalid. Further, they go on to show by example that Dyer's proposed MAUT fix doesn't work. Harker and Vargas support Saaty by concluding "the reason why rank can reverse in the AHP with relative measurement is clear. It is because the alternatives depend on what alternatives are considered, hence, adding or deleting alternatives can lead to change in the final rank". They sign off by firing a broadside: "utility theorists should direct their energy to preserving rank in their theory in a mathematically justifiable way rather than banning rank reversals from the domain of what constitutes rational behaviour."

We left Schoner above with a promise to discuss Vertical Linking Pins. Schoner[11] shows why the Mean Referenced Condition is necessary with respect to conventional AHP and notes that, in his experience, it is "extremely difficult to implement". This led Schoner, Wedley and Choo[22] to develop a class of AHP methods involving Vertical Linking Pins to overcome the view that "AHP is not consistent with the principle of the independence of irrelevant alternatives". They discuss three approaches that are consistent: referenced AHP, normalisation to the maximum entry, and normalisation to the minimum entry. They then present an approach that unifies all three and continue to compare their approach to Saaty's supermatrix approach. All give the same answer to a test case. Furthermore, their approach does not require implementation of the Mean Referenced Condition. The reader is directed to Schoner et al. for a complete description but, briefly, "local priorities of attributes are normalised so that one entry in each vector of local priorities is assigned a value of unity, and comparing the importance weights of criteria consists of comparing the corresponding values of the alternatives assigned unity. For example, if the styling of car 1 in the vector of local priorities under styling, and the engineering of car 2 in the vector of local priorities under engineering were each assigned values of one, the appropriate question to assess criteria importance would be 'Which is more important, the styling of car 1 or the engineering of car 2, and by how many times?"[11]. Schoner et al. also note that their method is essentially a simple but effective subset of Saaty's supermatrix approach requiring many fewer estimates by the decision maker.

It is clear that care needs to be taken with the AHP concerning possible rank reversals. Use of the Mean Referenced Condition and Vertical Linking Pins offers a more defensively robust option to conventional AHP when needed.

As has been shown above, the purpose of the AHP model constructed in this thesis is to quantitatively describe the entire set of possible qualitative descriptions of airlift missions within the constraints of the three decision attributes presented. The AHP quantitative value associated with each mission description becomes the weighting factor for a unique mission variable indicating the worth of a specific mission when compared against the worth of another mission at the same priority level. With three decision attributes, each with three possible values, only 27 alternatives are possible. Rank reversal occurs when a change to the list of alternatives is introduced. This will not occur within the context of this paper. Should the number or type of factors within the decision attributes be changed, then a new set of quantitative values to describe the worth of a mission could be generated for use within the linear program.

SUMMARY

In summary, we have shown that the AHP process can be used to produce a ranking of airlift missions within a given priority. Further, management can change the "importance" values of the decision attribute matrix to reflect the military situation be it peacetime or otherwise. Moreover, rank reversal is not a factor within the model constraints.

CHAPTER FOUR

AIRCRAFT AVAILABILITY MODELS

INTRODUCTION

The linear programming model developed in chapter five contains one aircraft availability constraint for each day of the period under consideration. Air Transport Group currently plans day-to-day tasking of the CC130 Hercules fleet at a rate of 70%. This means that a Base with 10 aircraft is expected to have 7 available for tasking of various sorts. The purpose of this chapter is to assess the validity of the standard 70% forecasting model and to investigate whether there are other, possibly better, forecast models The results of this research define the form of the availability constraints. Portions of this chapter result from joint course project work[23][24].

The future state of a pool of resources is often unknown to those who plan the optimal allocation of those resources. Accurate planning of the pool for future use is time consuming and difficult if the projected availability of the resources is not known with appropriate precision. The CC130 aircraft fleet represents such a pool of resources which must be allocated to specific tasks ahead of time.

The decision makers (DM) are the planning staff officers at Air Transport Group Headquarters. The planners currently assume that 70% of the aircraft will be serviceable on any given day in the future and task Bases to fly missions based on this assumption. The remaining 30% of aircraft are expected to be in an unavailable state. Upon occasion, Air Transport Group planners, by consensus, do task a Base to provide more than the normal 70%. Optimal planning requires this flexibility.

The operational research office of Air Transport Group has studied the operational characteristics of the CC130 fleet. One area of interest has been the utilisation rate of the fleet both in day-to-day operations and during emergency airlift. The 70% standard for day-to-day tasking appears to have existed for many years. Funding personnel and infrastructure have been provided to accomplish this objective. Taylor[25] believes that queueing theory applies to the provision of air transport services. He notes that, for large fleets, a 70% standard provides a small cancellation rate.

Few detailed studies have been undertaken to determine to what degree and how well this level of utilisation is being achieved. Further, it appears that no dynamic mathematical model has been developed to predict the operational state of the CC130 fleet.

Individual aircraft can be found in various states of serviceability and unserviceability. The transition from one state to another does not appear to depend on history. Therefore, a Markov Transition Matrix is a natural tool for modelling the availability of CC130 aircraft. Knowledge of the mean and standard deviation of flyable (serviceable) aircraft availability and the Markov transition matrices for fleet status are essential for further development of a microcomputer based capacity planning model for the CC130 fleet. The major benefit of a more accurate availability forecast would be an airlift capacity plan with improved user (customer) satisfaction resulting from making constraints on the daily number of taskable aircraft more accurate and responsive to the planner's needs. Thus, the purpose of this chapter is to develop and examine, in comparison to the current 70% standard, the application of the following forecast models to the CC130 aircraft fleet:

a. researched statistical means,

b. Markov steady state probabilities,

c. Markov chain prediction.

Specifically, the aim is to ascertain the best model for predicting the number of aircraft available to task.

SCOPE

During the period of this study, the main fleet of CC130 aircraft was situated at Canadian Forces Base (CFB) Edmonton, Alberta and CFB Trenton, Ontario (CC130 aircraft at Winnipeg have been excluded from the study). The study examined each location as a separate entity since the unique characteristics of each Base make aggregation into a single fleet unreasonable.

AVAILABLE DATA

Military maintenance personnel track the status of individual aircraft hour by hour using a "Rainbow" sheet. Four and a half years of these raw data were obtained from Air Transport Group Headquarters in Trenton, Ontario. It is colour-coded according to aircraft state by hour for each day of the year for each airplane based at Edmonton and Trenton. Although the sheets code only five states, the yellow state was broken into two states as defined below. This decision was made to avoid a possible confounding variable arising from interaction between routine inspections and long-term contractor inspections. Each of the approximately 25 airplanes was in one of six states. These were:

(1) RED - unserviceable, needs repair,
(2) BROWN - unserviceable, awaiting parts,
(3) YELLOW 1 - undergoing routine inspection,
(4) YELLOW 2 - undergoing long term contractor inspections
(5) GREEN - serviceable, not flying,
(6) BLUE - serviceable, flying.

RED, BROWN, and YELLOW collectively mean that the airplane was not available for flying operations, while GREEN and BLUE collectively mean that the airplane was available for flying operations. The probability of a plane moving between the states was the transition probability to be determined for Markov modelling.

SAMPLING

The two main bases for the CC130s are Trenton and Edmonton; they account for most of the CC130 airlift missions. Several years of daily airplane activity data have been recorded by the two Air Command bases but it has not been analysed to verify the level of aircraft availability. The complete years, 1987 to 1990 inclusive, were made available. The Persian Gulf crisis in 1990 resulted in a decision to discard that year's data because this thesis was intended to focus on the level of serviceability in peacetime service only and the crisis altered the tasking of the planes. Given the objectives and the type of data being used, the study dealt with inference for count data. A simple random sample size was selected for a desired confidence level and set confidence interval width according to the formula[26]:

$$n = [(2z^*\sigma)/w]^2(p^*)(1-p^*)$$

where: z^* = the upper $\alpha/2$ normal critical value
σ = standard deviation
w = confidence interval width
p^* = the guessed value of the true proportion.

For a confidence level of 95%, a desired confidence interval of 0.1 and a highly probable estimate of p = 0.7 for the true proportion, the sample size would be 332. This study dealt with data where the independent variable was time and the possibility of trends and seasonality were explored concerning the aircraft state as the dependent variable. For the Markov transition matrix, however, sequential pairs of days were needed for the calculation of the Markov transition probabilities. After pairing the random sample days, difficulty was encountered in processing the data. As the sample days numbered approximately two-thirds of the year, putting in the other third of the sample days simplified the processing. Thus, a census of the 1989 data, being the most recent peacetime data, was used.

DESIGN

This section provides the rationale for using a formal descriptive design. Air Transport Group Headquarters has set a standard of 70% aircraft utilisation at each base as appropriate to meet peacetime needs. It is possible to ascertain if the two bases have actually met this level. Given the standard desired by Air Transport Group, further exploratory research to determine the appropriate desired level was not needed. Therefore, the study was a formal one, whereby the hypothesis of actual versus standard utilisation was tested:

$$\left.\begin{array}{l}Ho: p = 0.7\\ Ha: p \neq 0.7\\ Ha: p > 0.7\\ Ha: p < 0.7\end{array}\right\} \text{ as applicable.}$$

By using the previously collected data, the study has been observational, rather than interrogative. The objective of confirmation of the 70% standard with hypothesis testing and the development of the Markov model for state transitions makes this study descriptive, rather than causal. The objective was not to determine why the actual utilization may or may not differ from the standard utilization.

Other than ensuring the accuracy of data entry, no control was exercised over the data variables collected by each base. The study design was ex post facto, rather than experimental. The aircrews and maintenance crews were aware of the data collection but not of its use in this study. The data collection was unlikely to have affected their actions as it is used primarily as a record of each aircraft's activities.

The determination of whether seasonality affects aircraft availability required a longitudinal study and three years of data were selected. Since a plane may have its availability status changed during the day, analysing every status change would have been extraordinarily time consuming. Discussion with airlift planners and review of an Air Transport Group draft study of mission departure times has indicated that the highest frequency of departures occurs at 0900 hours local for both bases. Each airplane is generally prepared for a day's flying three hours before take-off. Therefore, a sample taken daily at 0600 hours would be appropriate.

Given the amount of data, a statistical study to capture the breadth of aircraft availability was appropriate. By using the sample, the statistics generated from it should exhibit the characteristics of each base's activities over the three year period.

DATA COLLECTION

Each day of the three year period (1987-89) was assigned a sequential number, with January 1, 1987 and December 31, 1989 being assigned number 1 and number 1095 respectively. A random sample of 332 from the 1095 days was generated in Lotus using the random number generator. With the sample set, the corresponding data ($\approx$ 8600 items) were tabulated for those days to sample actual availability. To establish the Markov transition matrices, the Rainbow sheets for the entire 1989 calendar year were used. A census of the fleet's availability was taken from the data for both Edmonton and Trenton with the 9 aircraft at Edmonton and 15 aircraft at Trenton accounting for more than 9,000 transitions. As the state of an aircraft was recorded at 0600 hours local time every day, the 24 hour period was used as the Markov period t for extracting the state of both bases' fleets. The condition of the plane at this time was recorded as indicative of the state for the period. The states of 1 to 6, as discussed earlier were used as the primary classification. Other classification states used were:

State 8	-	Airplane crashed
State 9	-	Airplane data missing
States 11 to 16	-	The first digit, the "1", designates a Trenton airplane at the Edmonton base and the second number denotes its state ("16" means a Trenton airplane at Edmonton in state 6).

States 21 to 26 - The first digit, the "2", designates an Edmonton airplane at the Trenton base and the second number denotes its state.

DATA MANIPULATION

Data entry and initial processing was accomplished by constructing a detailed spreadsheet for both Edmonton and Trenton. Figure 4 shows the layout.

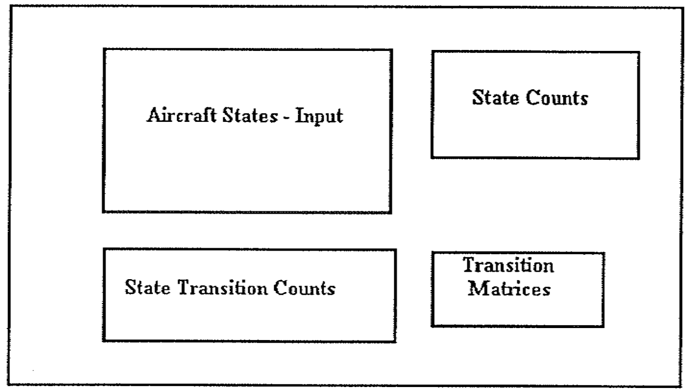


Figure 4. Spreadsheet Representation.

Lotus was used to record the visually extracted input data and two macros were developed to first calculate the number of airplanes in each state daily and then calculate the probability of an airplane moving from one state to another. The first macro counted the number of airplanes in each state and tabulated the total for each state every sample day. The purpose of the second macro was to count the number of transitions from one state at time *t* to another, including the original state, at time $t+1$. The combined count for each state i to j for j = 1 to 6 was divided into the count for each state ij. This was repeated for all i = 1 to 6. Finally, the Markov transition matrices for Edmonton and Trenton were calculated from the state transition counts for the 1989 census data only. Table 10 illustrates the transition matrices for Edmonton and Trenton.

	EDMONTON TRANSITION MATRIX						
STATE	1	2	3	4	5	6	SUM
1	0.294	0.057	0.011	0.003	0.529	0.106	1.0
2	0.153	0.389	0.014	0.014	0.431	0.000	1.0
3	0.039	0.000	0.872	0.010	0.074	0.005	1.0
4	0.011	0.000	0.000	0.973	0.011	0.005	1.0
5	0.087	0.014	0.010	0.002	0.750	0.137	1.0
6	0.098	0.004	0.005	0.002	0.127	0.764	1.0
	TRENTON TRANSITION MATRIX						
STATE	1	2	3	4	5	6	
1	0.406	0.034	0.026	0.000	0.409	0.124	1.0
2	0.191	0.649	0.053	0.000	0.106	0.000	1.0
3	0.046	0.007	0.889	0.000	0.054	0.004	1.0
4	0.008	0.000	0.002	0.984	0.006	0.000	1.0
5	0.119	0.004	0.006	0.005	0.580	0.285	1.0
6	0.136	0.003	0.001	0.001	0.124	0.735	1.0

Table 10. Edmonton and Trenton Transition Matrices.

Table 11 displays the minimal effect of removing state 4 from the matrices. This was done to see if the other states would be affected by removing a state which had such a comparatively long duration and could be argued as being somewhat deterministic. Since the effect appeared minimal it was decided to proceed with the original six states.

	EDMONTON WITH STATE 4 REMOVED					
STATE	1	2	3	5	6	Sum
1	0.295	0.057	0.011	0.530	0.106	1.0
2	0.155	0.394	0.014	0.437	0.000	1.0
3	0.040	0.000	0.881	0.075	0.005	1.0
5	0.087	0.014	0.010	0.752	0.137	1.0
6	0.098	0.004	0.005	0.128	0.766	1.0
	TRENTON WITH STATE 4 REMOVED					
STATE	1	2	3	5	6	Sum
1	0.406	0.034	0.026	0.409	0.124	1.0
2	0.191	0.649	0.053	0.106	0.000	1.0
3	0.046	0.007	0.889	0.054	0.004	1.0
5	0.120	0.004	0.006	0.583	0.287	1.0
6	0.136	0.003	0.001	0.124	0.736	1.0

Table 11. Edmonton and Trenton Transition Matrices Without State 4.

INITIAL DATA ANALYSIS

An initial overview of the data was obtained using Lotus pie charts that showed the percentage of aircraft in each of the states for each base. For example, using the census 1989 data and observing the two sectors Flyable at Base and Flyable Away, the Edmonton fleet pie chart in Figure 5 indicates that the 70% standard was slightly exceeded with the serviceable total being 71.0%. The Trenton fleet pie chart in Figure 6 indicates that the 70% standard was not met, with the total being 63.0%. Pie charts for Edmonton and Trenton covering the years 1987-1989 are similar.

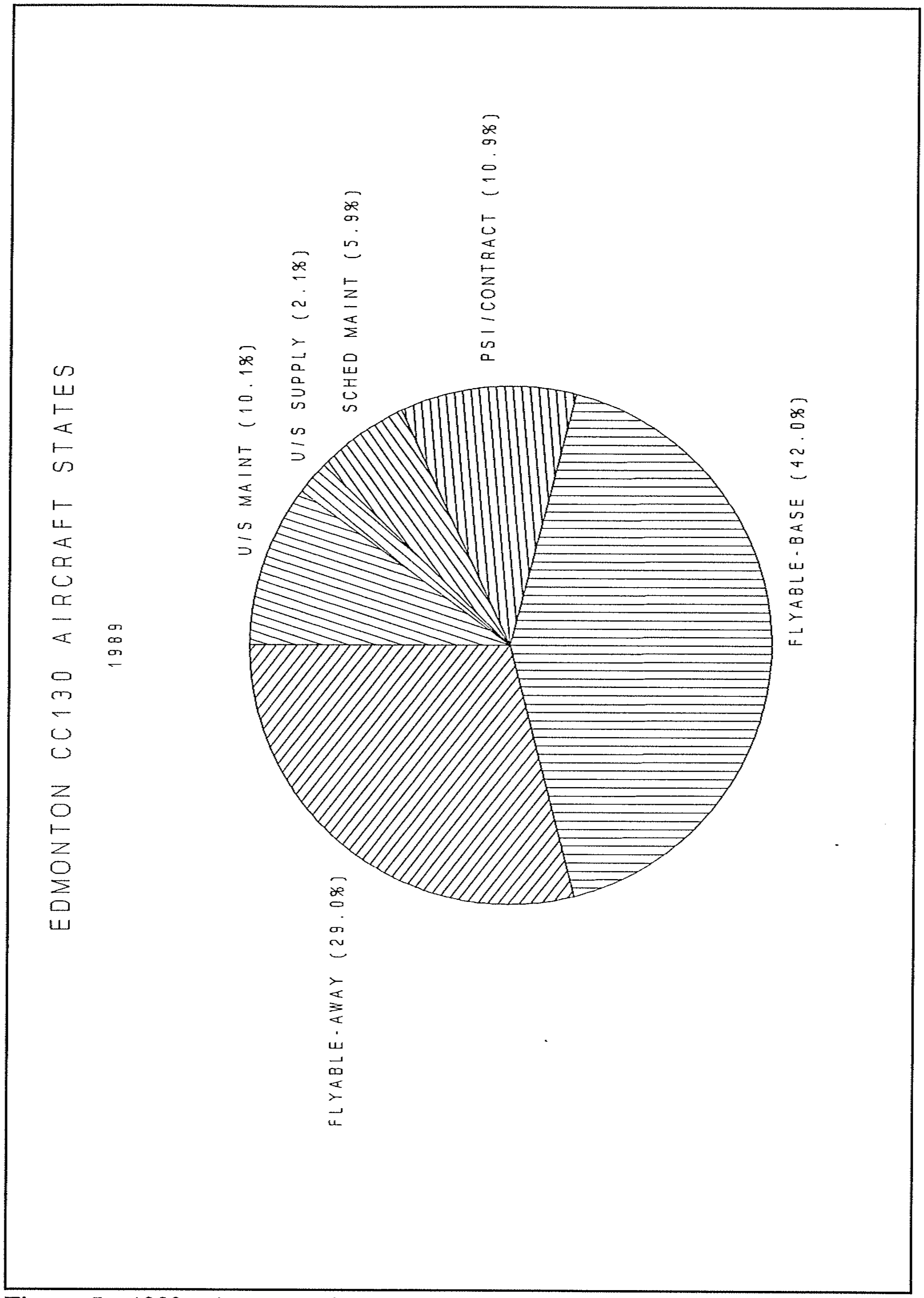


Figure 5. 1989 Edmonton Aircraft States.

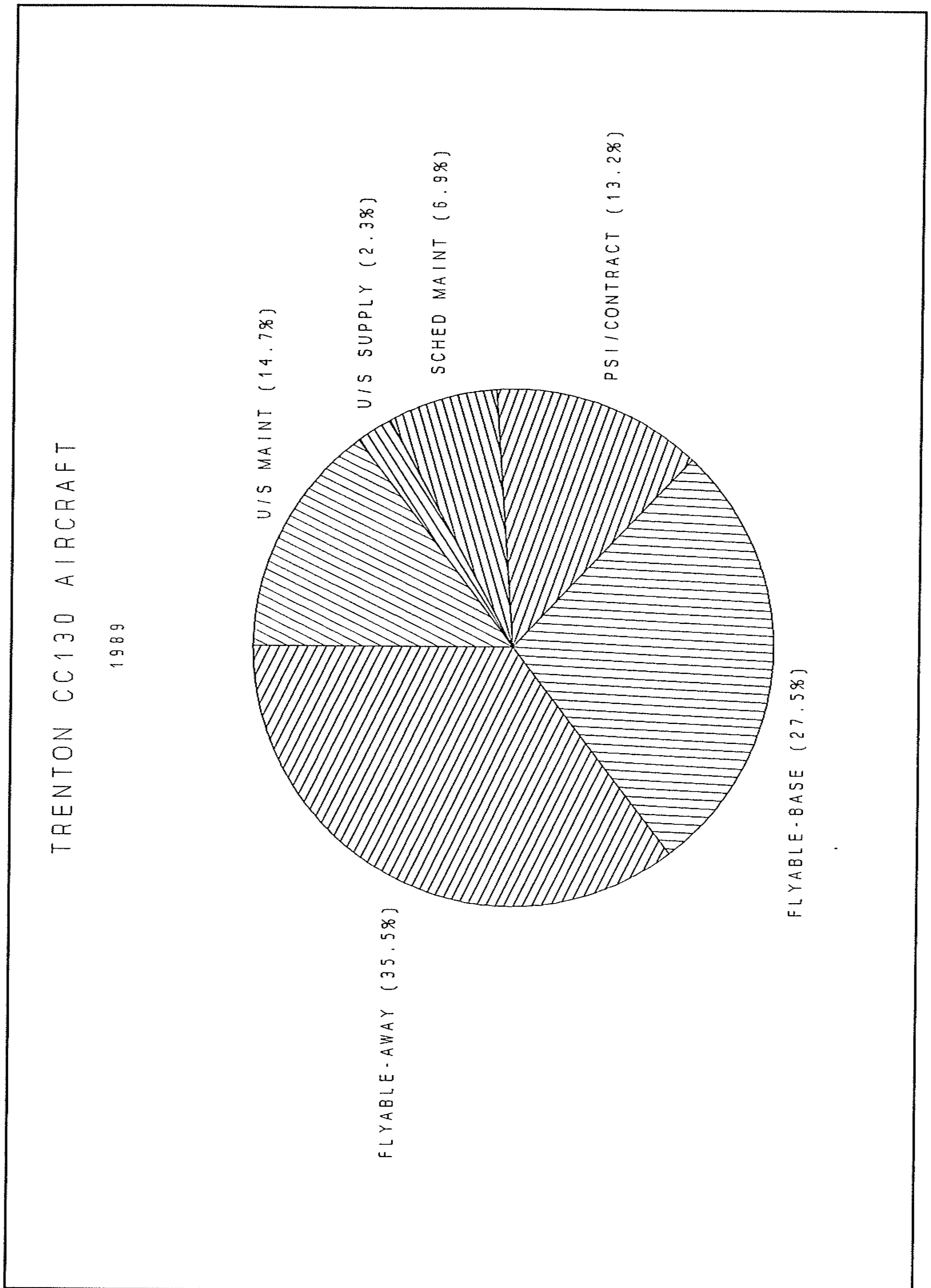


Figure 6. 1989 Trenton Aircraft States.

The data used in this study were classified into states by counts and proportion. This is referred to as classification by attribute in control chart theory. At each Base, the sample size is common (well within the $\pm$ 25% variation allowed) to all daily samples and thus a P chart[27] was appropriate for an initial longitudinal overview of the data. The essential chart structure consists of a centreline (CL) and upper and lower control limits (UCL and LCL). In the P chart, the CL usually represents the fraction defective p but in this case it represents the fraction of aircraft serviceable (combined flying and ready-to-fly). The sampling distribution of the fraction serviceable in an infinite frame is defined[28] in terms of p and sample size n as

$$\mu = p \qquad \sigma_p = \sqrt{\frac{p(1-p)}{n}} .$$

A certain proportion of the data will tend to fall within one, two or three standard errors from the mean μ of the process, also the CL. The UCL and LCL represent $\pm$ 3 standard errors. Although application of interpretation rules can define whether or not a process is stable, it was not the objective of this study to do so. Rather, the P chart has been used to show the variability of the fraction serviceable and give the reader a clear picture of the sample being studied.

The P charts for Edmonton and Trenton, covering the years 1987-1989, summarise the proportion serviceable against mean proportion and the upper and lower control limits for the data. The key finding from the P charts is that the actual serviceable rate is highly variable. The P chart for the Edmonton fleet, as shown in Figure 7, shows variability between approximately .33 and 1.00, with several sample

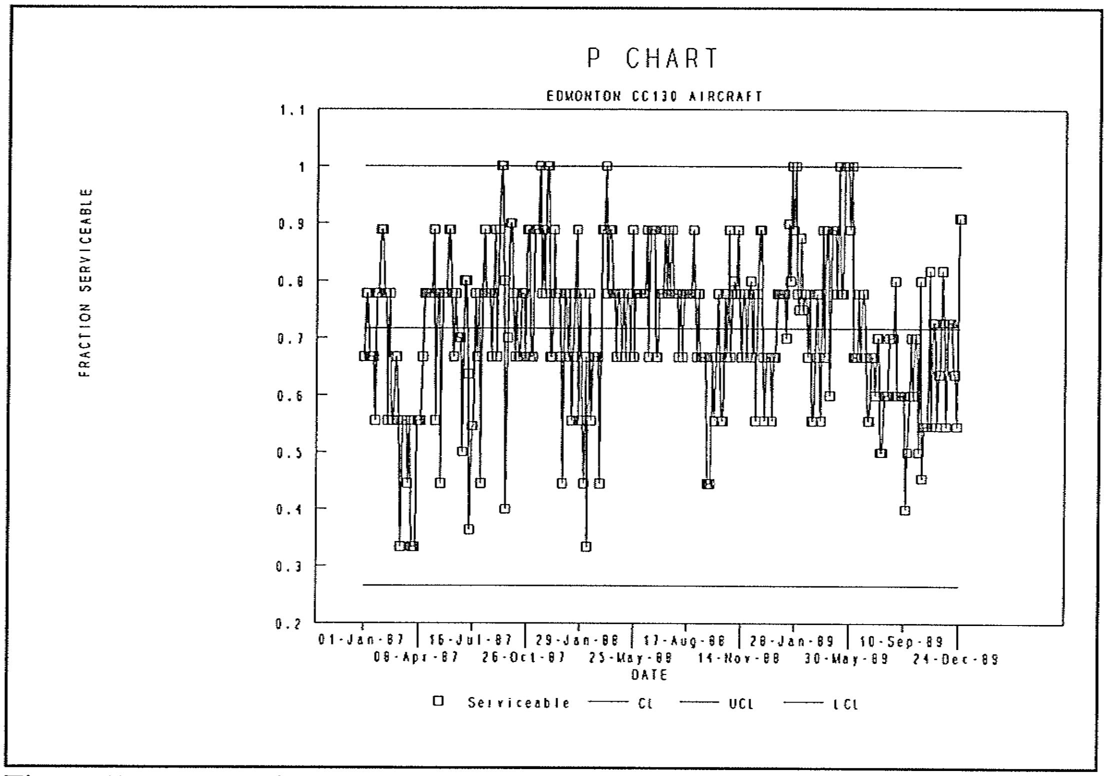


Figure 7. P Chart for Edmonton Serviceability 1987-1989.

days at the upper control limit. No quarterly trends or seasonality seem to exist for the Edmonton data.

The P chart for Trenton, as shown in Figure 8, exhibits the same high variability as Edmonton, between approximately .30 and .95. No quarterly trends or seasonality seems to exist for the Trenton data, although here a large number of the sample days are clustered closer to the centre line than in Edmonton. As the process appears to be highly variable, one might expect that forecasting the availability of aircraft using the same proportion for each day may not be the best approach.

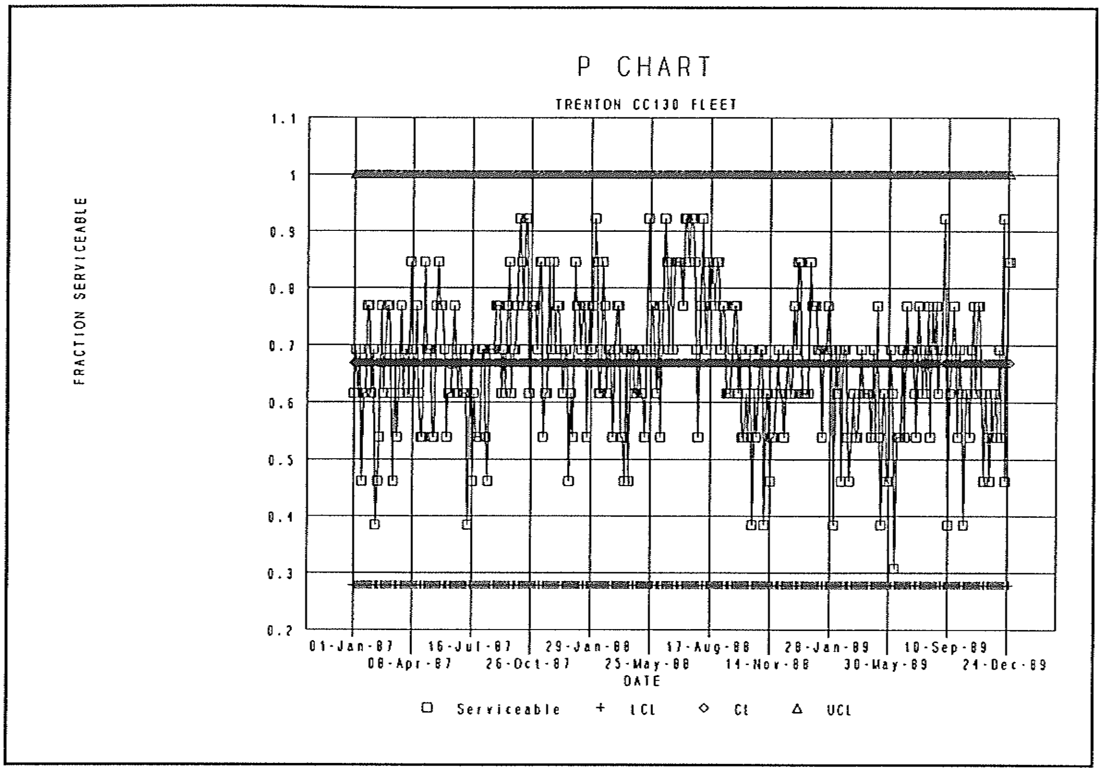


Figure 8. P Chart for Trenton Serviceability 1987-1989.

RESEARCHED MEANS MODEL

Statistical Processing for the Social Sciences (SPSS) was used to obtain the descriptive statistics including the mean number of aircraft serviceable for each base, standard deviation, maximum and minimum of aircraft serviceable and unserviceable. Histogram plots were generated to give a visual representation of the data. Recoding was done where needed. The mean number of aircraft serviceable and unserviceable and the standard deviation for the two bases are shown in Table 12. These means, converted to %, are referred to as the Researched Means for comparison with the 70% standard.

BASE	Edmonton		Trenton	
STATE	Mean	Std Dev	Mean	Std Dev
Serviceable	6.65	1.26	8.70	1.58
Unserviceable	2.63	1.34	4.30	1.58

Table 12. Summary of Means and Standard Deviations.

Hypothesis tests were performed on each base's fleets to determine if there was a statistically significant difference between the actual utilisation and the 70% standard (Ho: $p=0.7$). Also, testing was done to see if there was a statistical difference between the Bases (Ho: $p_1=p_2$). Testing was conducted using both 95% and 99% confidence intervals. The results are shown in Table 13.

STATS \ BASE	Ho	Ha	Zcrit $\alpha=.05$ $\alpha=.01$	Zstat $\alpha=.05$ $\alpha=.01$	Reject Ho $\alpha=.05$?	Reject Ho $\alpha=.01$?
EDMONTON	$p=0.7$	$p>0.7$	1.645	2.028	Yes	
			2.330	2.028		No
TRENTON	$p=0.7$	$p<0.7$	-1.645	-4.435	Yes	
			-2.330	-4.435		Yes
TRENTON SAME AS EDMONTON	$p_1=p_2$	$p_1<p_2$	-1.645	-4.396	Yes	
			-2.330	-4.396		Yes

Table 13. Summary of Statistical Tests.

Statistically, given an $\alpha = 0.05$, it appears that Edmonton exceeds the goal while Trenton does not. Trenton and Edmonton do not appear equal in terms of fraction of CC130 fleet serviceable.

MARKOV MODELS

It is possible to create a CC130 aircraft state forecast model using Markov theory. Referring to the Markov transition matrices shown in Table 10, the matrix elements identify the probability of an aircraft moving from one state to another in one time period. For example, using the Trenton state matrix, the probability of going from state 1 (unserviceable - maintenance) to state 5 (serviceable at Base) in one day is 0.409.

Several conditions must be satisfied to allow application of a Markov chain model to the airlift system. The first property that the model must display is that there is a finite number of states. The state space consists of six unique states which an individual aircraft can occupy. The states are as follows:

1) Unserviceable - needs repair. The aircraft cannot be flown until repairs are made. The aircraft is usually in this state for a short periods only.

2) Unserviceable - awaiting parts. The aircraft cannot be flown until replacement parts are received and installed. Typically the plane is in this state for reasonably short periods.

3) Unserviceable - routine inspection. An inspection by Canadian Forces personnel is in progress and the aircraft cannot be flown. The aircraft is typically unserviceable for a few days to weeks.

4) Unserviceable - long-term contractor maintenance. The aircraft is being refurbished by a private contractor and is unserviceable in this sate for a few months.

5) Serviceable - not flying. The aircraft is in flying condition but is on the ground.

6) Serviceable - flying. The aircraft is in on a mission and expected to be serviceable.

(The current planning model uses a 70% standard to represent those aircraft in states 5 and 6.)

The model must also display the Markov property. If we consider the state of each aircraft at a specific point in time to be a random variable then the availability of a CC130 aircraft is a discrete time stochastic process. It is reasonable to assume that the probability distribution of the state j at time t+1 depends on state i at time t and does not depend on the states the aircraft passed through on the way to state i at time t, so $P(x_{t+1} = j | X_t = i) = P_{ij}$.

The process must also be stationary if a Markov chain model is to be applied. The model is stationary if the probability of going from state i to state j is independent of the time at which the transition is made. Based on a historical perspective, it is reasonable to assume that the probability of an aircraft changing from one state to another is independent of time. The serviceability of the fleet does not appear to display seasonal fluctuations. The occurrence of states 1, 2, 3, 5 and 6 is random. State 4 is somewhat less random than the other states as aircraft are scheduled for long-term maintenance far in advance.

The aircraft in the fleet must also be homogeneous if the transition matrices for each separate aircraft are to be combined to form the fleet transition matrix. The assumption that all aircraft are homogeneous is valid. First, all aircraft are the same type, CC130s. Second, although the aircraft are of different variants and ages, each aircraft in the fleet is maintained and operated under the same rules.

In summary, the modelling of the aircraft states as a Markov chain is valid since the underlying conditions for such a model are satisfied.

The Markov chain modelled is ergodic as all states are recurrent, aperiodic, and communicate with each other. A state is said to be recurrent if it is not transient. Transient implies that once a state is exited it can never be entered again. All states in the model may be re-entered at some time in the future so they are recurrent. All states are aperiodic because there is no cyclical period k which leads from state i back to state i. Finally, all states can be reached from all others, so they are said to communicate with each other. The ergodic nature of the aircraft states will allow mean first passage times to be calculated from each state i to each state j.

A PL/1 program was used to solve a system of 36 linear equations with 36 unknowns to produce the mean first passage times. Tables 14 and 15 show the results for Trenton and Edmonton. Although further detailed analysis was not required in this thesis, these results have been recorded to provide a more complete picture and to enable future assessment.

STATES	1	2	3	4	5	6
1	6.5	120.9	123.5	508.6	3.8	6.7
2	7.8	44.2	109.1	511.6	6.6	10.4
3	14.3	125.3	14.9	516.2	11.2	15.1
4	67.8	186.1	172.4	9.1	65.8	69.8
5	9.3	126.6	128.3	505.3	3.6	5.4
6	8.5	126.3	129.1	509.0	6.1	2.7

(**Note:** Values represent the number of transitions (days) to get from state i to state j on average.)

Table 14. Mean First Passage Times for Trenton.

STATES	1	2	3	4	5	6
1	9.7	71.1	128.4	345.0	3.0	9.8
2	11.7	46.9	128.1	338.9	3.6	12.1
3	18.8	84.6	17.8	326.4	12.0	19.6
4	44.2	110.9	165.9	10.3	39.4	44.8
5	12.2	75.1	128.6	345.9	2.3	9.2
6	11.8	76.9	130.4	346.3	6.1	3.4

Table 15. Mean First Passage Times for Edmonton.

The model does not include any absorbing states (the ergodic property would be lost). A crashed aircraft would be representative of an absorbing state. Since the frequency of crashes is minimal they will not be included in the model.

MARKOV STEADY STATE MODEL

An Ergodic Markov chain will converge to a steady-state or equilibrium, that is, as the number of periods grows larger the state values tend to stabilise at a steady-state independent of the initial state. Quantitative Systems for Business Plus was used to calculate the Markov Steady State probabilities from the transition matrices. (see Table 16). States five and six were combined to get an estimate of serviceability for each Base and used as one of the forecast methods.

STATE / BASE	EDMONTON	TRENTON
STATE 1	0.1029	0.1541
STATE 2	0.0213	0.0226
STATE 3	0.0561	0.0671
STATE 4	0.0968	0.1095
STATE 5	0.4261	0.2763
STATE 6	0.2968	0.3703
TOTAL ITERATIONS	258	380

Table 16. Markov Steady State Probabilities.

MARKOV TRANSITION PREDICTION MODEL

A useful application of the Markov transition probabilities is that it enables the prediction of future states. To predict a future state, one needs to know the initial state of the system and the transition probabilities. The successive future states of a Markov process are called chains. Exhibit 1 shows the n-step transition calculations used to predict the future state of the fleet. The initial starting days were selected to ensure that the effect of high and low initial serviceability states could be observed. These represent the extremes of the system.

FINDINGS

The P charts in Figures 7 and 8 show considerable variability of the fraction of aircraft serviceable at both Edmonton and Trenton. This leads one to suspect that a linear forecast model may not be the most appropriate; any model of the system must take this variability into account.

A way of comparing the four forecast methods was needed. To observe the relative effectiveness of the current 70% linear standard against the values produced by this research, Mean Forecast Error (MFE) and Mean Absolute Deviation techniques were employed; they are complementary means of comparison. MFE was selected as it produces a measure of comparability with a directional component which reveals the under- or over-forecasting tendency of a forecast model. MAD gives a better sense of the accuracy of the forecast model as the positive and negative deviations do not cancel out and produce a more optimistic measure of accuracy as is the case with MFE . Statistical Processing for the Social Sciences was used for detailed analysis of the forecast methods.

Tables 17 and 18 present a summary of Exhibit 2, MFE and MAD calculations. For Edmonton samples it was found that, following an initial high serviceability state, the Markov Prediction appeared best, using either MFE or MAD. However, for similar Trenton samples, the current 70% standard appeared best. Thus, for these conditions, there is no dominant method. It was found that, for both Edmonton and Trenton samples following an initial low serviceability state, the Markov Prediction appeared best using either MFE or MAD. As the initial condition of low serviceability is operationally more critical, these results are considered more important. However, it is not clear whether or not these are statistically different.

	EDMONTON		TRENTON	
METHOD	MFE	MAD	MFE	MAD
70% STANDARD	1.400	1.563	-0.038	0.713
RESEARCH	1.253	1.435	0.366	0.839
STEADY STATE	1.202	1.391	0.657	0.985
PREDICTION	0.213	0.855	0.751	1.040
Note. Values closer to zero are better.				

Table 17. MFE and MAD Comparisons - High Initial Serviceability.

	EDMONTON		TRENTON	
METHOD	MFE	MAD	MFE	MAD
70% STANDARD	-0.981	1.094	-1.538	1.537
RESEARCH	-1.163	1.205	-1.135	1.324
STEADY STATE	-1.226	1.244	-0.843	1.215
PREDICTION	-0.144	0.824	-0.397	1.190
Note. Values closer to zero are better.				

Table 18. MFE and MAD Comparisons - Low Initial Serviceability.

Exhibit 3 comprises the Statistical Processing for the Social Sciences (SPSS) statistical output. The sample forecasts were grouped by Method and Base, differentiating the high and low initial serviceability conditions. Taking starting states from extreme values, limited samples of days with high and low actual serviceability levels were selected to capture the extremes of the systems. This was possible by taking the sample forecasts and grouping them by Method (1 = 70% standard and 2 = means researched) and Base (1 = high serviceability Edmonton, 2 = high serviceability Trenton, 3 = low serviceability Edmonton, 4 = low serviceability Trenton). First, the Differences (DIFF) were calculated by subtracting Forecast (FORESCT) from Actual (ACTUAL). Starting at page 5 of Exhibit 3, the Differences were examined for normality to ascertain the need for parametric or non-parametric tests. The results, including the Lilliefors significance > 0.2, indicated that parametric tests should be acceptable. However, where possible, equivalent non-parametric tests were run to ensure accuracy. The boxplot on page 9 confirms the similarity of all methods as seen previously. The null hypothesis is that the population means for the four methods are equal, the alternative being that at least one is not equal. A ONEWAY ANOVA was run with the results (page 11) showing that "No two groups are significantly different at the 0.05 α level". This was confirmed by the NPAR /KRUSKAL-WALLIS test on page 11. This did not change at the 0.10 α level as shown on page 12. The ANOVA (page 20) shows a significant interaction effect between Method and Base. To find out the source of the interaction a new variable INTER was defined to facilitate a ONEWAY ANOVA (page 20). This enabled assessment of INTER pairs. The results "*" (page 22) show that INTER Grp values 1 to 8 (representing the samples from a high initial serviceability state) are significantly different from values 9 to 16 (representing the samples from an initial low serviceability state). There is no statistical support to reject the null hypothesis that the means of the four methods are the same. However, the conclusion is that all testing methods have proved the Markov Steady State and Markov Prediction at least as effective as the current 70% standard or the newly researched values.

LIMITATIONS

The decision to use 0600 hours local time for the sampling time for each day may not represent the actual daily condition of the fleet with the best accuracy. The distribution of departure times led to picking 0600 hours but the departure times are spread over approximately six hours. Consideration was given to using a window of time but Markov chain models require equal time intervals for calculation of the transition probabilities. Another approach, considered but not used, would be to divide the day into equal time periods and select the period that covers the majority of a Base's departures to better ascertain the status of the fleet.

The data were collected by maintenance personnel. There is no way to ensure that bias has not been injected into the record keeping. However, given the professionalism of the personnel involved, bias is not expected. Only one day in 1989 was missed for one base. This day had no effect on the three year sample. It was coded as missing data for the Markov transition calculation.

Historical trends are not reliably indicative of the future. The results from the data used are applicable to the system as it existed during the period of the study. Since the current fleet size, the distribution of aircraft, and the distribution of types of missions have changed, these results are less applicable to the current system.

However, the methods used in the study can be applied to the present system to obtain current results. Time did not permit the analysis of the system using the transition matrices with state four removed. This could be pursued to see if a better solution will result.

The fleet sizes at both Edmonton and Trenton are relatively small. To change the expected number of taskable aircraft per day using a % standard would require that the current 70% be found inaccurate by about 5% for Edmonton and about 8% for Trenton for the 1989 year. For example, suppose a fleet size of 10 at Edmonton which results in 7 aircraft being tasked. To change this to 6 or 8 aircraft would require a goal of less than 65% or greater than 75% respectively. This concept must be considered in operationally assessing the significance of any similar statistical analysis.

CONCLUDING REMARKS

Statistically Trenton did not achieve the standard serviceability of 70% while Edmonton exceeded the standard. However, the 70% standard for tasking missions for both Bases remains usable as these findings are not operationally significant. Second, the serviceability fraction is highly variable and it is therefore recommended that further research be pursued on a non-linear prediction model for availability forecasting. The Markov state transition matrices provided are the first step towards building Markov steady state and prediction models for a micro-computer based optimised decision support system.

Given the limitations of this study, it is concluded that all four forecast methods can be considered statistically equivalent across both bases and initial system states.

It is recommended that the current 70% linear standard be maintained for capacity planning because it is simple and easily understood. Further, because of the inherent variability in the system, another study should be undertaken with current fleet data, to model and more thoroughly test the Markov Prediction Model. As shown by this study, in the MAD and MFE detailed calculations, there is potential for use in near term capacity planning. The accuracy period for the resultant Markov model should be established by statistical testing. Finally, any mathematical planning model, developed for capacity planning, must take into account the need to accommodate the highly variable availability of aircraft at both Bases.

CHAPTER FIVE

A SEQUENTIAL LINEAR PROGRAMMING MODEL FOR AIRLIFT MISSION REQUEST ALLOCATION

WHAT MUST THE MODEL DO?

The model must support two planning officers with different needs. The Staff Officer Operations Planning (SOOPSP-5) requires a model that will identify the airlift missions to select in accordance with the priority of the mission, the category or other mission discrimination features, the limit of taskable aircraft by Base, the user flying hours budget, missions that are linked together, and the fleet flying hour limit. The Staff Officer Airlift Programs (SOAP 3) requirements are the same, with the addition of the ability to enable overtasking of a Base for a specific number of aircraft for specific days. Both require an ability to add, delete and modify airlift requests. Both require near real time response. Most important is that the priority criteria must not be violated (a lower priority mission must not unthinkingly be selected at the expense of a higher priority mission)

GENERAL MATHEMATICAL MODEL

The fact that the airlift missions are partitioned into a number of priority levels suggests a pre-emptive goal programming like approach. Since the system priority levels are stated in terms of ordinal measurement, pre-emptive versus archimedean goal programming is pertinent. Goal programming operates in such a way that lower priority goals are addressed only after higher priority goals have been satisfied as well as possible. While linear programming yields the solution that optimises a single objective, goal programming identifies the overall solution that best satisfies all problem goals at the cost of sacrificing some individual ones. This is called *satisficing*.

Unfortunately there are no *a priori* target levels for each priority. Therefore, formulating the problem as a goal programme where goal j is the total value of priority j missions selected is inconsistent with the *satisficing* philosophy of goal programming, since solving such a goal program would almost certainly result in ignoring the lower priority goals. While the priority criteria for airlift mission selection must not be automatically compromised, a selection of lower priority missions should be made given that the resources are available.

A sequential linear program approach is suggested for multi-objective problems without *a priori* target levels. Such an approach can be applied to the problem described below. The decision variables are 0-1 integers $X_1, X_2, \ldots, X_n$. There are k goals ($j=1,\ldots,k$), one for each priority level, numbered so that goal j has a higher priority than goal j+1 ($j=1,\ldots,k-1$) The objective function of goal j and the linear constraints are represented by

$$Maximize \sum_i C_{ij} X_i$$

$$AX \leq b \ .$$

Although goal j represents a more important priority than goal j+1, the decision maker would prefer a solution with the highest possible value for priority j+1 if the corresponding value for priority j is at least a fraction γ of the highest possible value for priority j. A solution $X = \{X_1, X_2, \ldots, X_n\}$ is defined to be γ_t preferable if it maximises priority t subject to the constraints that it is at least a fraction γ of the highest possible values for priorities 1,...,t-1. This relaxation may make it possible to select more missions.

A sequential linear programming approach involves solving a sequence of linear programs LP_1, $LP_2, \ldots, LP_k$, formulated as follows:

for j = 1

$$LP_1 \ : \ Max \ Z_1 = \sum_i^n C_{i1} X_i$$

$$S.T. \quad AX \le b, \quad C_{i1} \ge 0, \quad X_i = 0, 1$$

and for j = 2,...,k

$$LP_j: \ Max \ Z_j = \sum_i^n C_{ij} X_i$$

$$S.T. \quad AX \le b$$

$$\sum_i C_{it} X_i \ge \gamma Z_t \qquad t = 1, \ldots, j-1$$

$$X_i = 0, 1$$

$$C_{ij} \ge 0$$

The optimal solution to LP_k would be γ_k *preferable.* If γ equals 1, the problem is reduced to pre-emptive goal programming. Choosing γ less than 1, say 0.95, gives a γ_k solution which scores close to the highest possible value for the more important goals but enables more missions to possibly be selected at the lower priorities.

MODEL APPLIED TO REPRESENTATIVE SCENARIO

The following application of the general model to the mission request scenario is amplified by reference to Exhibit 4 which displays the last of five passes for the initial run of a prototype model demonstration. The decision variables are 0-1 integers $X_1, X_2, \ldots X_n$. If $X_i = 1$, mission i is accepted. If $X_i = 0$, then mission i is not accepted. C_{ij} is the AHP value of mission i on priority j. The linear program LP_j is run once for each priority level j, with the objective function Z_j maximising the worth C_{ij} of the missions X_i at that specific priority level. The γ fraction chosen for the test runs was 1.0, thus the model was one of preemptive goal programming.

Taskable Aircraft Constraints

The major system constraint is TAC, the total number of aircraft available to task each day. For each Base there is a target represented by the parameter TAC. For every day d (1-30 for the prototype), the sum of all missions X_i selected for that day, plus the number of undertaskings (A_dM) minus the number of overtaskings (A_dP) must equal TAC. Let $a_{id} = 1$, if mission i uses day d and 0 otherwise, then we must have

$$\sum_{d} a_{id} X_i + A_d M - A_d P = TAC_d \ .$$

These constraints are shown in lines 2 to 31 of Exhibit 4. Note that TAC has been set to 6 for this model. This represents a Base with ten aircraft whose 70% standard expected availability is seven. As one tasking line is permanently assigned to Search and Rescue missions only six tasking lines are left.

Overtasking Constraints

The use of the deviational variables A_dM and A_dP provides flexibility to the model. As previously seen, the actual level of serviceability is highly variable. In the short term this can affect the number of tasking lines available at a Base. Further, the SOAP 3 planner sometimes coordinates the deliberate overtasking of a Base. Both planners need to know the number of aircraft tasking lines available on any given day. The use of a deviational variable handles these variations. For example, the A_dM value for day d shows the number of aircraft lines still available to task under normal conditions (that is, less than the value of TAC). Observing the A_dP values indicates on which days overtasking is required and by how many aircraft lines. Overtasking may be constrained in terms of the number of days a Base can be overtasked (OT) during the period. Line 32 of Exhibit 4 shows that overtasking is disabled by setting the constraint equal to zero. More likely, the planner would choose which days to overtask and by how many aircraft by setting the right hand side values (HM_d) of the specific A_dPs. Such is the case as shown in lines 33 to 62 of Exhibit 7. The general representation is

$$\sum A_d P \leq OT$$
$$A_d P \leq HM_d \ .$$

User Hour Constraints

Each user r of the system has a budget of allocated flying hours F_r. Some small flexibility is normal. The model again uses deviational variables, U_rM for underflying and U_rP for overflying the user budget goal. The corresponding constraints are shown in lines 33 to 42 of Exhibit 4. For the purposes of this model, mission requests are sorted according to user. The lower limit for user r is L_r while the upper limit is M_r. H_i denotes the flying hours associated with mission request X_i. The general representation of the user hour budget constraint is

$$\{\sum_{i=L_r}^{M_r} H_i X_i\} + U_r M - U_r P = F_r \ .$$

Total Fleet Hours Constraints

The model must also accommodate flexibility in total fleet flying hour allocation. The total yearly flying rate for the CC130 fleet is represented by the variable YFR. The sum of hours for all mission requests accepted cannot exceed YFR as shown in the constraint lines 43 and 44 of Exhibit 4. This constraint can be changed by a decision higher in the chain of command. Underflying the fleet yearly flying rate goal is denoted by FM and overflying by FP, as shown in line 43 of

Exhibit 4. The generalised constraint is

$$\{\sum_{i=1}^{n} H_i X_i\} + FM - FP = YFR \ .$$

Linked Mission Constraints

Certain missions are operationally linked to other missions. If one is selected then all must be selected; if one is not selected then none are selected. The constraint for a pair of missions is quite simple; just equate one X_i to the other X_i as shown in line 45 of Exhibit 4 which links missions 17 and 19. The constraint for more than two can be modelled using the standard "either or" pattern of 0-1 integer programming. This involves the introduction of a 0-1 variable, say Y, and M, an arbitrarily large value. As an example, suppose out of five missions (X_{21},X_{22},X_{23},X_{24},X_{25}) at least four must be selected as demonstrated in lines 46 and 47. Then we wish one of the two following constraints to become relevant.

$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25} \geq 4$$

$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25} = 0.$$

Re-writing,

$$4 - X_{21}-X_{22}-X_{23}-X_{24}-X_{25} \leq 0$$

$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25} \leq 0.$$

Then, introducing M and Y,

$$4 - X_{21}-X_{22}-X_{23}-X_{24}-X_{25} \leq 0 + MY$$

$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25} \leq 0 + M(1-Y).$$

Then, using M = 1000

$$4 - X_{21}-X_{22}-X_{23}-X_{24}-X_{25} \leq 0 + 1000Y$$

$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25} \leq 0 + 1000(1-Y).$$

Finally, rewriting (A) $1000Y+X_{21}+X_{22}+X_{23}+X_{24}+X_{25} \geq 4$

(B) $1000Y+X_{21}+X_{22}+X_{23}+X_{24}+X_{25} \leq 1000.$

These constraints are interpreted as follows. If mission requests $X_{21},X_{22},X_{23},X_{24}$ and X_{25} take on a total value of 4 or 5, the 0-1 variable Y in constraint A becomes 0. This causes constraint A to be relevant and B to become redundant. If these missions take on a total value of three or less, Y is forced to be 1 due to constraint A. In turn, this makes constraint B relevant and forces the values of $X_{21},X_{22},X_{23},X_{24}$ and X_{25} to become 0.

Previous Goal Constraints

In all passes of the linear program, except the first, it is necessary to introduce an additional constraint to reflect the solution attained at the previous priority level. For example, if the first pass for priority 1 is solved with missions X_1, X_5, and X_6 being selected with an objective function value (Z_j) of 0.1408, then this must be introduced into the next pass for priority 2 (see line 48). Lines 49 to 51 similarly represent passes for priority 2 to 4. The general formulation is as follows.

$$\sum C_i X_i \mid i \in Priority\ t \geq \gamma Z_t$$
$$t=1,\ldots,j-1$$

This constraint ensures that each subsequent pass maintains at least as good a solution for the previous passes. In the event that there are alternative optima (different sets of missions that have the same Z_j) at a specific priority level, this formulation allows the selection of the missions forming that particular alternative.

PROTOTYPE MODEL USAGE

To demonstrate the use of the model, a representative set of 33 mission requests from users 2,4,5,6 and 7 was developed as test data (see Table 19) and a hypothetical, abridged airlift planning process was used to develop the sequence of model applications. Note that mission requests 1 to 31 represent what the SOOPSP-5 planner might initially be faced with for one month for one Base. Mission 32 represents additional missions for SOOPSP-5. Likewise, mission 33 is a new request to SOAP-3. The runs of the model presented here approximate the type of processing done by both planners; SOOPSP in the long term and SOAP in the nearer term (90 days). All model calculations were made using Hyper LINDO[29].

Consider the SOOPSP-5 task of initially forming an airlift mission capacity plan for missions 1 to 31. Exhibit 4 is the edited output of the fifth pass of initial SOOPSP-5 planning, representing priority 5. Recall that the variable values for X1 to X31 equal 1 if the mission was selected and 0 if not. Like further runs of the model, the results are summarised in Table 20. The algorithm correctly did not select mission 4 because user 2 had insufficient hours. Missions 14, 16, 25,29 and 31 were not selected due to lack of aircraft tasking lines.

Although the weighting system used in these particular model runs is taken from the output of the AHP analysis in chapter 3, any weighting could have been used. For example, currently the attribute category is used to discriminate amongst missions at a given priority level and categories A,B and C were allocated weights

MISSION REQUEST	DATES (INCL)	PRIORITY	USER	HOURS	ATTRIBUTES	AHP WORTH
1	1-2	1	2	12	LAV	0.0313
2	3-10	4	2	41	VAH	0.0459
3	5-8	4	2	29	HBH	0.0581
4	12-17	5	2	18	VBH	0.0309
5	19-22	1	2	10	VAL	0.0412
6	22-26	1	2	14	HAL	0.0683
7	1-4	3	4	24	LAH	0.0346
8	2-4	5	4	19	LAV	0.0313
9	3	5	4	7	VCH	0.0264
10	5-8	5	4	20	HBL	0.0534
11	9-11	5	4	22	VAV	0.0426
12	4-10	4	4	35	VAH	0.0459
13	27-30	3	4	8	LBL	0.0150
14	4-10	5	4	45	HBH	0.0581
15	1-2	2	5	14	LAH	0.0346
16	1-3	2	5	20	VBH	0.0309
17	3-5	2	5	11	VAV	0.0426
18	22-25	2	5	28	HCV	0.0502
19	21-23	2	5	16	VBV	0.0276
20	1-13	2	5	90	HAL	0.0683
21	11-21	4	6	60	HAV	0.0697
22	11-21	4	6	60	HAV	0.0697
23	11-21	4	6	60	HAV	0.0697
24	11-21	4	6	60	HAV	0.0697
25	11-21	4	6	60	HAV	0.0697
26	23-30	3	6	50	HAL	0.0683
27	23-28	3	6	41	HBV	0.0548
28	23-27	3	6	35	HCV	0.0502
29	23-27	3	7	36	VAV	0.0426
30	24-28	3	7	34	HBL	0.0534
31	25-28	3	7	22	VAV	0.0426
32	6-9	3	4	24	VBV	0.0276
33	24-26	3	7	12	HAV	0.0697

Table 19. Inputs to Prototype Sequential Linear Model.

0.6, 0.3, and 0.1 respectively. The output of this model run is summarised in Table 20 under the heading SOOPSP CATEGORY. The results have been manually confirmed.

Note the values of the A_dM and A_dP variables, particularly A_6M to A_9M. They show that 1,1,1 and 2 aircraft tasking lines are available on days 6 to 9. Note also that only four of the five missions $X_{21},X_{22},X_{23},X_{24},X_{25}$ were selected on days 11 to 21. Inspection of the associated A_dMs reveals that only on days 11 and 21 no aircraft is available. Suppose SOOPSP-5, in consultation, decides to task the fifth aircraft now from day 12 to 20. Suppose also that user 4 calls with a request to add mission 32 which requires an aircraft for days 6 to 9 and 24 flying hours. SOOPSP-5 already knows that an aircraft is available and only needs to confirm that user 4 has sufficient hours available. Variable U4M shows that user 4 has sufficient hours. Thus the planner is able to confirm immediately that user 4 can have the requested mission. Exhibit 5 is the output of the SOOPSP run which represents the state of the airlift plan when SOOPSP-5 hands it over to SOAP-3.

User 7 has not given up on the request for mission 31 which did not get selected in the airlift plan (see variable value for X31 in Exhibit 5). SOAP notes that, if this mission could be shifted to start on the 27th instead of the 25th, resources are available and user 7 agrees to this fix. User 7 also introduces mission 33 as another request. SOAP knows that aircraft are not available without cancelling another already programmed mission. Exhibit 6 shows the edited output of the subsequent SOAP model run as the planning officer establishes the effect of introducing this new

MISSION REQUEST	DATES (INCL)	PRIORITY	SOOPSP CATEGORY	SOOPSP INITIAL	SOOPSP MODIFIED	SOAP	SOAP OVERTASK
1	1-2	1	1	1	1	1	1
2	3-10	4	1	1	1	1	1
3	5-8	4	1	1	1	1	1
4	12-17	5	0	0	0	0	0
5	19-22	1	1	1	1	1	1
6	22-26	1	1	1	1	1	1
7	1-4	3	1	1	1	1	1
8	2-4	5	1	1	1	1	1
9	3	5	0	1	1	1	1
10	5-8	5	1	1	1	1	1
11	9-11	5	1	1	1	1	1
12	4-10	4	1	1	1	1	1
13	27-30	3	1	1	1	1	1
14	4-10	5	0	0	0	0	0
15	1-2	2	1	1	1	1	1
16	1-3	2	1	0	0	0	0
17	3-5	2	1	1	1	1	1
18	22-25	2	0	1	1	1	1
19	21-23	2	1	1	1	1	1
20	1-13	2	1	1	1	1	1
21	11-21	4	1	1	1	1	1
22	11-21	4	1	1	1	1	1
23	11-21	4	0	1	1	1	1
24	11-21	4	1	1	1	1	1
25	11-21	4	1	0	1	1	1
26	23-30	3	1	1	1	1	1
27	23-28	3	1	1	1	1	1
28	23-27	3	1	1	1	1	1
29	23-27	3	1	0	0	0	0
30	24-28	3	0	1	1	1	1
31	25-28	3	1	0	1	1	1
32	6-9	3	N/A	N/A	1	1	1
33	24-26	3	N/A	N/A	N/A	1	1

Note. 1 means mission is selected, 0 not selected.

Table 20. Outputs of Various Runs of the Prototype Sequential Linear Model.

mission. Note that mission 31 is selected on its new dates and that mission 33 is also selected but at the expense of mission 28. Mission 28 is another priority 3 mission belonging to user 6 who would not be pleased if it were cancelled for a priority three mission for user 7 even if it is judged to be worth more to the system. This decision could be taken but an unhappy customer would be a negative result. Thus SOAP investigates the possibility of overtasking and notes that if the Base could provide an extra aircraft for days 24 and 25, both could be satisfied. Exhibit 7 shows the resultant airlift model with overtasking enabled for the two days. Line 32 controls the total amount of overtasking allowed while lines 56 and 57 control the specific days for this scenario.

COMPUTATIONAL SPEED

All models were constructed and run on a 386DX33 IBM compatible PC with a math coprocessor. Each of the model runs consists of five sequential submissions to LINDO which will accept ASCII files. Although the software does not identify the amount of time used for each run, personal observation revealed that no single pass took more than 10 seconds with most being around 5. Thus it is reasonable to assume that the total processing time for a model run is on the order of 25 to 50 seconds. It is also reasonable to expect that dedicated software on a higher speed processor would reduce this interval. The total time needed for an operational system will need to be determined but the speed results of this prototype are promising.

PROTOTYPE MODEL SUMMARY

The prototype model was developed to provide direct computational support to the two planning officers as they currently do their work. The individual mission weighting values used can be taken from any system that will provide discrimination amongst missions at a given priority level. The prototype model has demonstrated flexibility and accuracy.

CHAPTER SIX

CONCLUSIONS, LIMITATIONS AND RECOMMENDATIONS

CONCLUSIONS

Operational Research and the military have a historical connection. Indeed, for many years OR professionals have carried out many effective studies centred on military airlift in Canada and other countries. The problem, addressed in this thesis, of ranking many airlift mission requests from several users with a constrained number of taskable aircraft in order of priority to produce of a high quality airlift capacity plan was made tractable by the application of a group of management science techniques.

To create a mathematical model of the problem, a means was required to numerically describe an airlift mission request. The category method of discriminating amongst missions at a given priority level provides a workable, if qualitatively and quantitatively rudimentary approach. We have seen that the Analytic Hierarchy Process can numerically integrate the assessment of a mission's category with other important mission attributes such as system training value and effective use of the aircraft to provide better discrimination. The 27 numerical values produced by the Analytical Hierarchy Process, compared to the three values resulting from using the category attribute alone, result in a more differentiated airlift plan. Further, the Analytic Hierarchy Process approach provides the decision makers with a flexible, robust way of reflecting the importance of certain type of missions dependent upon the prevailing military situation.

The detailed study of aircraft availability assessed four forecast models. The aim was to validate the current 70% planning standard. All four models were found to be statistically equivalent and we conclude that the 70% standard should be maintained for capacity planning because it is simple and easily understood. The analysis also showed a high variability of flyable aircraft on any given day. Potential exists for the development of a more refined Markov prediction model and, if a suitable current database is available, this should be pursued. The most important conclusion from the research was that any mathematical model of the planning system must allow for the highly variable availability. While this is less applicable for that portion of the model used by SOOPSP-5 in generating the long range airlift capacity plan, it is important to SOAP-3 modelling.

The prototype sequential linear programming model meets the objective of the thesis. The model is flexible and accurate. The model appears to be computationally quick, it does not violate the priority criteria and it handles the selection of multiple priorised mission requests from multiple users. It also handles linked missions, either as a pair or as a minimum out of an optimal number. User hour and fleet flying hour constraints are modelled and missions can be added, deleted or modified. In short, a computationally efficient approach has been found to produce an automated planning aid to assist the airlift capacity planners.

LIMITATIONS

Currently, the main limitation of the model is that it does not support a sliding time window for departure dates. Still, this can be done manually by generating another input file based on a planner's decision as to which date to try. To automate this feature, further development of the model is necessary.

The model is essentially a computational subroutine for a larger Air Transport Group Decision Support System. Although it is possible to construct the necessary file to pass to LINDO using commonly available word processing packages, this would require extensive training for the planners to use the model operationally. Appropriate database, processing and interface software needs to be developed.

The model does not currently support more than one Base per model run. It requires a separate run for each of two or more Bases. Further development needs to be done if the model is to identify surplus capacity at one Base that can be used to make up for a shortage at another. Given the current development of the model, this must be done manually and a new file submitted for processing.

The model does not support the needs of the Base level scheduler who must match missions to aircraft tail numbers. Further development is required to enable a combined planning and scheduling package.

RECOMMENDATIONS

The prototype model only shows that the approach merits serious consideration for full operational development. It is strongly recommended that this approach be incorporated in the current development of the Decision Support System for Air Transport Group.

REFERENCES

[1]. 1326-1 (Comd) 26 Mar 92 - Air Transport Group Annual Historical Report

[2]. Canadian Forces Administration Order (CFAO) 20-22.

[3]. Thomas M. Cook and Robert A. Russell, Introduction to Management Science, (Prentice Hall: Englewood Cliffs, New Jersey, 1989) 3-5.

[4]. Robert E.D. Woolsey, Operations Research for Immediate Application: A Quick and Dirty Manual, (Harper and Row Publishers: New York, N.Y., 1969) xiv.

[5]. Harold Larnder, "The Origin of Operational Research", Proceedings of the Eighth IFORS International Conference on Operational Research, Toronto, 1978, ed. K.B. Haley, (North Holland Publishing Company: New York, 1979) 3-12.

[6]. William L. Shirer, The Rise and Fall of the Third Reich, (Simon and Schuster: New York, 1960) 776.

[7]. Taylor, Ivan; Operational Support to Air Transport Group During Operation Friction, ATGOR Internal Working Paper 5/91, August 1991.

[8]. Woolsey, 169.

[9]. Danielson and Stannard, "A Developmental Model Using Analytic Hierarchy Process for Ranking Military Airlift Support Requests", Unpublished Project Report, 14 December 1992.

[10]. Thomas L Saaty, "A Natural Way to Make Momentous Decisions", Journal of Scientific and Industrial Research, Vol.51, August-September (1992): 561-571.

[11]. Bertram Schoner, "Correcting the Analytic Hierarchy Process", Proceedings of Administration Sciences Association of Canada (1992) Quebec, 62-68.

[12]. Canadian Forces Administration Order (CFAO) 20-22, pg 9, para 14 Note.

[13]. D.L. Olson and J.F. Courtney Jr., Decision Support Models and Expert Systems, (MacMillan Publishing Company, New York, N.Y., 1992) 35.

[14]. Wayne L. Winston, Operations Research: Applications and Algorithms (PWS-Kent Publishing Company, Boston, 1991) 753-760.

[15]. MathCad, computer software, MathSoft Inc., Cambridge, MA, USA, 1992.

[16]. James S. Dyer, "Remarks on the Analytic Hierarchy Process", Management Science Vol.36, No.3, March (1990): 249-258.

[17]. Ronald A. Howard, "Heathens, Heretics, and Cults: The Religious Spectrum of Decision Aiding", Interfaces 22:6, November-December (1992): 15-27.

[18]. Thomas L Saaty, "An Exposition of the AHP in Reply to the Paper 'Remarks on the Analytic Hierarchy Process'", Management Science, Vol.36, No.3, March(1990): 261.

[19]. Patrick T. Harker and Luis G. Vargas, "Reply to 'Remarks on the Analytic Hierarchy Process'", Management Science, Vol.36, N0.3, March (1990): 269.

[20]. Saaty, 259-268.

[21]. Harker and Vargas, 271.

[22]. Bertram Schoner, William C. Wedley, and Eng Ung Choo, "A Unified Approach to AHP with Linking Pins", European Journal of Operational Research, 64 (1993):384-392.

[23]. Stannard and Wong, "The Accuracy of the 70% Heuristic for Airlift Availability", Unpublished Project Report, 15 April 1993.

[24]. Kummen and Stannard, "Markov Model of CC130 Aircraft Fleet States", Unpublished Project Report, 15 April 1993.

[25]. Ivan Taylor and Kevin Naismith, "The Theory of Tasking: A Discussion Paper", Air Transport Group Headquarters, October, 1992.

[26]. D.S. Moore and G.P. McCabe, Introduction to the Practise of Statistics (Freeman: New York,1989) 587-588.

[27]. Gitlow, Gitlow, Oppenheim and Oppenheim, Tools and Methods for the Improvement of Quality, (Irwin: Homewood, 1989) 223-233.

[28]. Gitlow et al., 145.

[29]. Hyper LINDO, computer software, LINDO Systems Inc., Chicago, IL, USA.

EXHIBIT 1 - N-STEP TRANSITION CALCULATIONS

EDMONTON - HIGH SERVICEABILITY

On 22 January 1989 the number of aircraft in states 1 to 6 was 2 ,0 ,0 ,0 ,3 and 4 respectively. Given the transition matrix (TM) for Edmonton and using MathCad, the transition states (T_n) for subsequent days 1 to 14 , 21 and 28 were calculated using the formula: $T_n = v \bullet TM^n$

Table 1 shows the results with the cell values representing the predicted aircraft in each state for each T_n

22Jan89	STATES						SUM
T_n	1	2	3	4	5	6	S5+S6
T_1	1.241	0.172	0.072	0.020	3.816	3.679	7.495
T_2	1.087	0.206	0.135	0.041	4.065	3.466	7.531
T_3	1.050	0.213	0.191	0.063	4.163	3.321	7.484
T_4	1.037	0.214	0.239	0.084	4.206	3.220	7.426
T_5	1.029	0.214	0.281	0.105	4.223	3.148	7.371
T_6	1.023	0.214	0.317	0.126	4.226	3.095	7.321
T_7	1.018	0.213	0.349	0.146	4.221	3.054	7.275
T_8	1.014	0.212	0.376	0.167	4.211	3.022	7.233
T_9	1.010	0.211	0.399	0.186	4.200	2.996	7.196
T_{10}	1.006	0.211	0.419	0.206	4.187	2.974	7.161
T_{11}	1.002	0.210	0.436	0.225	4.174	2.956	7.130
T_{12}	0.999	0.209	0.451	0.243	4.161	2.939	7.100
T_{13}	0.996	0.208	0.463	0.261	4.149	2.925	7.074
T_{14}	0.993	0.208	0.474	0.279	4.137	2.912	7.049
T_{21}	0.978	0.204	0.515	0.390	4.069	2.849	6.918
T_{28}	0.967	0.201	0.525	0.482	4.021	2.809	6.830

Table 1. Edmonton State Predictions - High Initial Serviceability.

EDMONTON - LOW SERVICEABILITY

Similarly, on 3 October 1989, the state vector was 4, 1, 0, 2, 0, 3. The state transitions are shown in Table 2.

3Oct89	STATES						SUM
T_n	1	2	3	4	5	6	S5+S6
T_1	1.645	0.629	0.073	1.978	2.950	2.726	5.676
T_2	1.128	0.391	0.134	1.950	3.727	2.671	6.398
T_3	1.004	0.279	0.185	1.921	3.931	2.682	6.613
T_4	0.971	0.232	0.229	1.891	3.975	2.704	6.679
T_5	0.962	0.212	0.267	1.862	3.976	2.724	6.700
T_6	0.959	0.204	0.300	1.833	3.968	2.739	6.707
T_7	0.959	0.200	0.328	1.806	3.962	2.748	6.710
T_8	0.959	0.199	0.353	1.780	3.958	2.755	6.713
T_9	0.960	0.199	0.374	1.754	3.957	2.759	6.716
T_{10}	0.961	0.198	0.393	1.730	3.959	2.763	6.722
T_{11}	0.962	0.198	0.410	1.706	3.962	2.765	6.727
T_{12}	0.964	0.199	0.424	1.683	3.966	2.768	6.734
T_{13}	0.965	0.199	0.437	1.661	3.972	2.771	6.743
T_{14}	0.967	0.199	0.448	1.640	3.977	2.774	6.751
T_{21}	0.977	0.201	0.496	1.509	4.023	2.799	6.822
T_{28}	0.986	0.230	0.519	1.404	4.067	2.828	6.895

Table 2. Edmonton State Predictions - Low Initial Serviceability.

TRENTON HIGH SERVICEABILITY

Using the Trenton transition matrix, the calculations were repeated for a low initial serviceability state. Table 3 shows the results for a starting vector of 2, 0, 2, 1, 4, 4 on 24 July 1989.

24Jul89	STATES						SUM
T_n	1	2	3	4	5	6	S5+S6
T_1	1.932	0.110	1.860	1.008	3.748	4.336	8.084
T_2	1.935	0.178	1.738	1.015	3.620	4.502	8.122
T_3	1.951	0.222	1.633	1.021	3.568	4.588	8.156
T_4	1.966	0.250	1.543	1.027	3.554	4.637	8.191
T_5	1.979	0.268	1.464	1.033	3.556	4.671	8.227
T_6	1.989	0.280	1.395	1.039	3.565	4.698	8.263
T_7	1.996	0.287	1.335	1.045	3.575	4.721	8.296
T_8	2.003	0.292	1.282	1.051	3.584	4.742	8.326
T_9	2.008	0.295	1.236	1.057	3.592	4.760	8.352
T_{10}	2.012	0.297	1.195	1.063	3.599	4.776	8.375
T_{11}	2.015	0.298	1.159	1.068	3.605	4.791	8.396
T_{12}	2.017	0.299	1.127	1.074	3.610	4.803	8.413
T_{13}	2.019	0.299	1.099	1.080	3.613	4.814	8.427
T_{14}	2.021	0.300	1.074	1.085	3.616	4.823	8.439
T_{21}	2.022	0.298	0.963	1.123	3.619	4.851	8.470
T_{28}	2.016	0.297	0.913	1.156	3.608	4.845	8.453

Table 3. Trenton State Predictions - High Initial Serviceability.

TRENTON LOW SERVICEABILITY

The initial state vector for Trenton in a high serviceability state was 6, 1, 0, 2, 2, 2 on 5 Feb 1989. The subsequent transition states are shown in Table 4.

5Feb89	STATES						SUM
T_n	1	2	3	4	5	6	S5+S6
T_1	3.153	0.867	0.227	1.980	3.980	2.784	6.764
T_2	2.324	0.696	0.36	1.971	4.059	3.572	7.631
T_3	2.078	0.560	0.450	1.963	3.853	4.072	7.925
T_4	1.999	0.465	0.514	1.955	3.685	4.351	8.036
T_5	1.970	0.401	0.564	1.947	3.583	4.498	8.081
T_6	1.956	0.359	0.604	1.938	3.526	4.574	8.100
T_7	1.948	0.332	0.637	1.929	3.495	4.612	8.107
T_8	1.942	0.314	0.664	1.920	3.477	4.630	8.107
T_9	1.938	0.302	0.686	1.912	3.465	4.637	8.102
T_{10}	1.934	0.294	0.706	1.903	3.458	4.639	8.097
T_{11}	1.932	0.290	0.723	1.895	3.453	4.638	8.091
T_{12}	1.930	0.286	0.737	1.886	3.449	4.635	8.084
T_{13}	1.928	0.284	0.750	1.878	3.446	4.632	8.078
T_{14}	1.926	0.283	0.761	1.870	3.443	4.629	8.072
T_{21}	1.922	0.281	0.808	1.816	3.435	4.611	8.046
T_{28}	1.920	0.281	0.828	1.767	3.432	4.604	8.036

Table 4. Trenton State Predictions - Low Initial Serviceability.

EXHIBIT 2
MEAN FORECAST ERROR - LOW SERVICEABILITY

EDMONTON T+0 is 3 October 1989

AIRCRAFT ON BASE	T+0	ACTUAL	CURRENT HEURISTIC FORECAST	CURRENT HEURISTIC DEVIATION	RESEARCHED MEAN ACTUAL	RESEARCHED MEAN FORECAST	RESEARCHED MEAN DEVIATION	MARKOV STEADY STATE ACTUAL	MARKOV STEADY STATE FORECAST	MARKOV STEADY STATE DEVIATION	MARKOV PREDICTION ACTUAL	MARKOV PREDICTION FORECAST	MARKOV PREDICTION DEVIATION
		3											
10	1	6	7	-1	6	7.17	-1.17	6	7.229	-1.229	6	5.676	0.324
10	2	7	7	0	7	7.17	-0.17	7	7.229	-0.229	7	6.398	0.602
10	3	7	7	0	7	7.17	-0.17	7	7.229	-0.229	7	6.613	0.387
10	4	6	7	-1	6	7.17	-1.17	6	7.229	-1.229	6	6.679	-0.679
10	5	6	7	-1	6	7.17	-1.17	6	7.229	-1.229	6	6.7	-0.7
11	6	7	7.7	-0.7	7	7.887	-0.887	7	7.9519	-0.9519	7	6.707	0.293
11	7	8	7.7	0.3	8	7.887	0.113	8	7.9519	0.0481	8	6.71	1.29
11	8	5	7.7	-2.7	5	7.887	-2.887	5	7.9519	-2.9519	5	6.713	-1.713
11	9	6	7.7	-1.7	6	7.887	-1.887	6	7.9519	-1.9519	6	6.716	-0.716
11	10	6	7.7	-1.7	6	7.887	-1.887	6	7.9519	-1.9519	6	6.722	-0.722
11	11	8	7.7	0.3	8	7.887	0.113	8	7.9519	0.0481	8	6.727	1.273
11	12	8	7.7	0.3	8	7.887	0.113	8	7.9519	0.0481	8	6.734	1.266
11	13	6	7.7	-1.7	6	7.887	-1.887	6	7.9519	-1.9519	6	6.743	-0.743
11	14	6	7.7	-1.7	6	7.887	-1.887	6	7.9519	-1.9519	6	6.751	-0.751
11	21	6	7.7	-1.7	6	7.887	-1.887	6	7.9519	-1.9519	6	6.822	-0.822
11	28	6	7.7	-1.7	6	7.887	-1.887	6	7.9519	-1.9519	6	6.895	-0.895
			TOTAL	-15.7		TOTAL	-18.607		TOTAL	-19.6159		TOTAL	-2.306
			MFE =	-0.98125		MFE =	-1.16294		MFE =	-1.22599		MFE =	-0.144125

TRENTON T+0 is 5 February 1989

AIRCRAFT ON BASE	T+0	ACTUAL	CURRENT HEURISTIC FORECAST	CURRENT HEURISTIC DEVIATION	RESEARCHED MEAN ACTUAL	RESEARCHED MEAN FORECAST	RESEARCHED MEAN DEVIATION	MARKOV STEADY STATE ACTUAL	MARKOV STEADY STATE FORECAST	MARKOV STEADY STATE DEVIATION	MARKOV PREDICTION ACTUAL	MARKOV PREDICTION FORECAST	MARKOV PREDICTION DEVIATION
13		4											
13	1	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	6.764	1.236
13	2	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	7.631	1.369
13	3	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	7.925	0.075
13	4	6	9.1	-3.1	6	8.697	-2.697	6	8.4058	-2.4058	6	8.036	-2.036
13	5	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	8.081	-0.081
13	6	5	9.1	-4.1	5	8.697	-3.697	5	8.4058	-3.4058	5	8.1	-3.1
13	7	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.107	0.893
13	8	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	8.107	-0.107
13	9	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	8.102	-0.102
13	10	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.097	0.903
13	11	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.091	0.909
13	12	6	9.1	-3.1	6	8.697	-2.697	6	8.4058	-2.4058	6	8.084	-2.084
13	13	5	9.1	-4.1	5	8.697	-3.697	5	8.4058	-3.4058	5	8.078	-3.078
13	14	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	8.072	-0.072
13	21	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.046	0.954
13	28	6	9.1	-3.1	6	8.697	-2.697	6	8.4058	-2.4058	6	8.036	-2.036
			TOTAL	-24.6		TOTAL	-18.152		TOTAL	-13.4928		TOTAL	-6.357
			MFE	-1.5375		MFE =	-1.1345		MFE =	-0.8433		MFE =	-0.3973125

FORECAST ACCURACY COMPARISON TABLE

METHOD	EDMONTON	TRENTON
HEURISTIC	-0.981	-1.537
RESEARCH	-1.163	-1.134
STEADY STATE	-1.226	-0.843
PREDICTION	-0.144	-0.397

EXHIBIT 2
MEAN FORECAST ERROR - HIGH SERVICEABILITY

EDMONTON T+0 is 22 January 1989

AIRCRAFT ON BASE	T+0	ACTUAL	CURRENT HEURISTIC FORECAST	CURRENT HEURISTIC DEVIATION	RESEARCHED MEAN ACTUAL	RESEARCHED MEAN FORECAST	RESEARCHED MEAN DEVIATION	MARKOV STEADY STATE ACTUAL	MARKOV STEADY STATE FORECAST	MARKOV STEADY STATE DEVIATION	MARKOV PREDICTION ACTUAL	MARKOV PREDICTION FORECAST	MARKOV PREDICTION DEVIATION
		7											
9	1	9	6.3	2.7	9	6.453	2.547	9	6.5061	2.4939	9	7.495	1.505
10	2	10	7	3	10	7.17	2.83	10	7.229	2.771	10	7.531	2.469
9	3	9	6.3	2.7	9	6.453	2.547	9	6.5061	2.4939	9	7.484	1.516
9	4	8	6.3	1.7	8	6.453	1.547	8	6.5061	1.4939	8	7.426	0.574
9	5	8	6.3	1.7	8	6.453	1.547	8	6.5061	1.4939	8	7.371	0.629
9	6	9	6.3	2.7	9	6.453	2.547	9	6.5061	2.4939	9	7.321	1.679
9	7	7	6.3	0.7	7	6.453	0.547	7	6.5061	0.4939	7	7.275	-0.275
8	8	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.233	-0.233
8	9	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.196	-0.196
8	10	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.161	-0.161
8	11	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.13	-0.13
8	12	6	5.6	0.4	6	5.736	0.264	6	5.7832	0.2168	6	7.1	-1.1
8	13	6	5.6	0.4	6	5.736	0.264	6	5.7832	0.2168	6	7.074	-1.074
8	14	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.049	-0.049
9	21	5	6.3	-1.3	5	6.453	-1.453	5	6.5061	-1.5061	5	6.918	-1.918
9	28	7	6.3	0.7	7	6.453	0.547	7	6.5061	0.4939	7	6.83	0.17
			TOTAL	22.4		TOTAL	20.054		TOTAL	19.2398		TOTAL	3.406
			MFE =	1.4		MFE =	1.253375		MFE =	1.202488		MFE =	0.212875

TRENTON T0 is 24 July 1989

AIRCRAFT ON BASE	T+0	ACTUAL	CURRENT HEURISTIC FORECAST	CURRENT HEURISTIC DEVIATION	RESEARCHED MEAN ACTUAL	RESEARCHED MEAN FORECAST	RESEARCHED MEAN DEVIATION	MARKOV STEADY STATE ACTUAL	MARKOV STEADY STATE FORECAST	MARKOV STEADY STATE DEVIATION	MARKOV PREDICTION ACTUAL	MARKOV PREDICTION FORECAST	MARKOV PREDICTION DEVIATION
13		8											
13	1	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.084	1.916
13	2	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.122	1.878
13	3	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.156	1.844
13	4	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.191	1.809
13	5	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	8.227	-0.227
13	6	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.263	0.737
13	7	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.296	0.704
13	8	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	8.326	-0.326
13	9	7	9.1	-2.1	7	8.697	-1.697	7	8.4058	-1.4058	7	8.352	-1.352
13	10	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.375	0.625
13	11	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.396	0.604
13	12	8	9.1	-1.1	8	8.697	-0.697	8	8.4058	-0.4058	8	8.413	-0.413
13	13	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.427	0.573
13	14	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.439	1.561
13	21	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.47	1.53
13	28	9	9.1	-0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.453	0.547
		14	TOTAL	-0.6		TOTAL	5.848		TOTAL	10.5072		TOTAL	12.01
			MFE =	-0.0375		MFE =	0.3655		MFE =	0.6567		MFE =	0.750625

FORECAST ACCURACY COMPARISON TABLE

METHOD	EDMONTON	TRENTON
HEURISTIC	1.400	-0.037
RESEARCH	1.253	0.366
STEADY STATE	1.202	0.657
PREDICTION	0.213	0.751

EXHIBIT 2
MEAN ABSOLUTE DEVIATION - LOW SERVICEABILITY

EDMONTON T+0 is 3 October 1989

AIRCRAFT ON BASE	T+0	ACTUAL	CURRENT HEURISTIC FORECAST	CURRENT HEURISTIC DEVIATION	RESEARCHED MEAN ACTUAL	RESEARCHED MEAN FORECAST	RESEARCHED MEAN DEVIATION	MARKOV STEADY STATE ACTUAL	MARKOV STEADY STATE FORECAST	MARKOV STEADY STATE DEVIATION	MARKOV PREDICTION ACTUAL	MARKOV PREDICTION FORECAST	MARKOV PREDICTION DEVIATION
		3											
10	1	6	7	1	6	7.17	1.17	6	7.229	1.229	6	5.676	0.324
10	2	7	7	0	7	7.17	0.17	7	7.229	0.229	7	6.398	0.602
10	3	7	7	0	7	7.17	0.17	7	7.229	0.229	7	6.613	0.387
10	4	6	7	1	6	7.17	1.17	6	7.229	1.229	6	6.679	0.679
10	5	6	7	1	6	7.17	1.17	6	7.229	1.229	6	6.7	0.7
11	6	7	7.7	0.7	7	7.887	0.887	7	7.9519	0.9519	7	6.707	0.293
11	7	8	7.7	0.3	8	7.887	0.113	8	7.9519	0.0481	8	6.71	1.29
11	8	5	7.7	2.7	5	7.887	2.887	5	7.9519	2.9519	5	6.713	1.713
11	9	6	7.7	1.7	6	7.887	1.887	6	7.9519	1.9519	6	6.716	0.716
11	10	6	7.7	1.7	6	7.887	1.887	6	7.9519	1.9519	6	6.722	0.722
11	11	8	7.7	0.3	8	7.887	0.113	8	7.9519	0.0481	8	6.727	1.273
11	12	8	7.7	0.3	8	7.887	0.113	8	7.9519	0.0481	8	6.734	1.266
11	13	6	7.7	1.7	6	7.887	1.887	6	7.9519	1.9519	6	6.743	0.743
11	14	6	7.7	1.7	6	7.887	1.887	6	7.9519	1.9519	6	6.751	0.751
11	21	6	7.7	1.7	6	7.887	1.887	6	7.9519	1.9519	6	6.822	0.822
11	28	6'	7.7	1.7	6	7.887	1.887	6	7.9519	1.9519	6	6.895	0.895
			TOTAL	17.5		TOTAL	19.286		TOTAL	19.9045		TOTAL	13.176
			MAD =	1.09375		MAD =	1.205313		MAD =	1.244031		MAD =	0.8235

TRENTON T+0 is 5 February 1989

AIRCRAFT ON BASE	T+0	ACTUAL	CURRENT HEURISTIC FORECAST	CURRENT HEURISTIC DEVIATION	RESEARCHED MEAN ACTUAL	RESEARCHED MEAN FORECAST	RESEARCHED MEAN DEVIATION	MARKOV STEADY STATE ACTUAL	MARKOV STEADY STATE FORECAST	MARKOV STEADY STATE DEVIATION	MARKOV PREDICTION ACTUAL	MARKOV PREDICTION FORECAST	MARKOV PREDICTION DEVIATION
13		4											
13	1	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	6.764	1.236
13	2	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	7.631	1.369
13	3	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	7.925	0.075
13	4	6	9.1	3.1	6	8.697	2.697	6	8.4058	2.4058	6	8.036	2.036
13	5	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	8.081	0.081
13	6	5	9.1	4.1	5	8.697	3.697	5	8.4058	3.4058	5	8.1	3.1
13	7	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.107	0.893
13	8	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	8.107	0.107
13	9	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	8.102	0.102
13	10	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.097	0.903
13	11	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.091	0.909
13	12	6	9.1	3.1	6	8.697	2.697	6	8.4058	2.4058	6	8.084	2.084
13	13	5	9.1	4.1	5	8.697	3.697	5	8.4058	3.4058	5	8.078	3.078
13	14	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	8.072	0.072
13	21	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.046	0.954
13	28	6	9.1	3.1	6	8.697	2.697	6	8.4058	2.4058	6	8.036	2.036
			TOTAL	24.6		TOTAL	21.182		TOTAL	19.4348		TOTAL	19.035
			MAD =	1.5375		MAD =	1.323875		MAD =	1.214675		MAD =	1.1896875

FORECAST ACCURACY COMPARISON TABLE

METHOD	EDMONTON	TRENTON
HEURISTIC	1.094	1.537
RESEARCH	1.205	1.324
STEADY STATE	1.244	1.215
PREDICTION	0.824	1.190

EXHIBIT 2
MEAN ABSOLUTE DEVIATION - HIGH SERVICEABILITY

EDMONTON T+0 is 22 January 1989

AIRCRAFT ON BASE	T+0	CURRENT HEURISTIC ACTUAL	FORECAST	DEVIATION	RESEARCHED MEAN ACTUAL	FORECAST	DEVIATION	MARKOV STEADY STATE ACTUAL	FORECAST	DEVIATION	MARKOV PREDICTION ACTUAL	FORECAST	DEVIATION
		7											
9	1	9	6.3	2.7	9	6.453	2.547	9	6.5061	2.4939	9	7.495	1.505
10	2	10	7	3	10	7.17	2.83	10	7.229	2.771	10	7.531	2.469
9	3	9	6.3	2.7	9	6.453	2.547	9	6.5061	2.4939	9	7.484	1.516
9	4	8	6.3	1.7	8	6.453	1.547	8	6.5061	1.4939	8	7.426	0.574
9	5	8	6.3	1.7	8	6.453	1.547	8	6.5061	1.4939	8	7.371	0.629
9	6	9	6.3	2.7	9	6.453	2.547	9	6.5061	2.4939	9	7.321	1.679
9	7	7	6.3	0.7	7	6.453	0.547	7	6.5061	0.4939	7	7.275	0.275
8	8	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.233	0.233
8	9	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.196	0.196
8	10	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.161	0.161
8	11	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.13	0.13
8	12	6	5.6	0.4	6	5.736	0.264	6	5.7832	0.2168	6	7.1	1.1
8	13	6	5.6	0.4	6	5.736	0.264	6	5.7832	0.2168	6	7.074	1.074
8	14	7	5.6	1.4	7	5.736	1.264	7	5.7832	1.2168	7	7.049	0.049
9	21	5	6.3	1.3	5	6.453	1.453	5	6.5061	1.5061	5	6.918	1.918
9	28	7	6.3	0.7	7	6.453	0.547	7	6.5061	0.4939	7	6.83	0.17
			TOTAL	25		TOTAL	22.96		TOTAL	22.252		TOTAL	13.678
			MAD =	1.5625		MAD =	1.435		MAD =	1.39075		MFE =	0.854875

TRENTON T0 is 24 July 1989

AIRCRAFT ON BASE	T+0	CURRENT HEURISTIC ACTUAL	FORECAST	DEVIATION	RESEARCHED MEAN ACTUAL	FORECAST	DEVIATION	MARKOV STEADY STATE ACTUAL	FORECAST	DEVIATION	MARKOV PREDICTION ACTUAL	FORECAST	DEVIATION
13		8											
13	1	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.084	1.916
13	2	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.122	1.878
13	3	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.156	1.844
13	4	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.191	1.809
13	5	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	8.227	0.227
13	6	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.263	0.737
13	7	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.296	0.704
13	8	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	8.326	0.326
13	9	7	9.1	2.1	7	8.697	1.697	7	8.4058	1.4058	7	8.352	1.352
13	10	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.375	0.625
13	11	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.396	0.604
13	12	8	9.1	1.1	8	8.697	0.697	8	8.4058	0.4058	8	8.413	0.413
13	13	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.427	0.573
13	14	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.439	1.561
13	21	10	9.1	0.9	10	8.697	1.303	10	8.4058	1.5942	10	8.47	1.53
13	28	9	9.1	0.1	9	8.697	0.303	9	8.4058	0.5942	9	8.453	0.547
			TOTAL	11.4		TOTAL	13.424		TOTAL	15.7536		TOTAL	16.646
			MAD =	0.7125		MAD =	0.839		MAD =	0.9846		MAD =	1.040375

FORECAST ACCURACY COMPARISON TABLE

METHOD	EDMONTON	TRENTON
HEURISTIC	1.563	0.712
RESEARCH	1.435	0.839
STEADY STATE	1.391	0.985
PREDICTION	0.855	1.040

EXHIBIT 3 - SPSS EDITED OUTPUT

SPSS/PC+ Studentware+ for IBM PC 4/10/93

```
The SPSS/SW+ system file is read from
    file proj750.sys
The file was created on  4/10/93 at  0:12:01
and is titled                 SPSS/PC+ Studentware+
The SPSS/SW+ system file contains
    256 cases, each consisting of
      7 variables (including system variables).
      7 variables will be used in this session.

This procedure was completed at 10:06:52
COMPUTE DIFF = ACTUAL-FORECST.
LIST.
The raw data or transformation pass is proceeding
    256 cases are written to the compressed active file.
```

BASE	METHOD	ACTUAL	FORECST	DIFF
1.0	1.0	9.0	6.3	2.70
1.0	1.0	10.0	7.0	3.00
1.0	1.0	9.0	6.3	2.70
1.0	1.0	8.0	6.3	1.70
1.0	1.0	8.0	6.3	1.70
1.0	1.0	9.0	6.3	2.70
1.0	1.0	7.0	6.3	.70
1.0	1.0	7.0	5.6	1.40
1.0	1.0	7.0	5.6	1.40
1.0	1.0	7.0	5.6	1.40
1.0	1.0	7.0	5.6	1.40
1.0	1.0	6.0	5.6	.40
1.0	1.0	6.0	5.6	.40
1.0	1.0	7.0	5.6	1.40
1.0	1.0	5.0	6.3	-1.30
1.0	1.0	7.0	6.3	.70
1.0	2.0	9.0	6.5	2.55
1.0	2.0	10.0	7.2	2.83
1.0	2.0	9.0	6.5	2.55
1.0	2.0	8.0	6.5	1.55
1.0	2.0	8.0	6.5	1.55
1.0	2.0	9.0	6.5	2.55
1.0	2.0	7.0	6.5	.55
1.0	2.0	7.0	5.7	1.26
1.0	2.0	7.0	5.7	1.26
1.0	2.0	7.0	5.7	1.26
1.0	2.0	7.0	5.7	1.26
1.0	2.0	6.0	5.7	.26
1.0	2.0	6.0	5.7	.26
1.0	2.0	7.0	5.7	1.26
1.0	2.0	5.0	6.5	-1.45
1.0	2.0	7.0	6.5	.55
1.0	3.0	9.0	6.5	2.49
1.0	3.0	10.0	7.2	2.77
1.0	3.0	9.0	6.5	2.49
1.0	3.0	8.0	6.5	1.49
1.0	3.0	8.0	6.5	1.49
1.0	3.0	9.0	6.5	2.49
1.0	3.0	7.0	6.5	.49
1.0	3.0	7.0	5.8	1.22
1.0	3.0	7.0	5.8	1.22
1.0	3.0	7.0	5.8	1.22
1.0	3.0	7.0	5.8	1.22
1.0	3.0	6.0	5.8	.22
1.0	3.0	6.0	5.8	.22
1.0	3.0	7.0	5.8	1.22
1.0	3.0	5.0	6.5	-1.51

1.0	3.0	7.0	6.5	.49
1.0	4.0	9.0	7.5	1.51
1.0	4.0	10.0	7.5	2.47
1.0	4.0	9.0	7.5	1.52
1.0	4.0	8.0	7.4	.57
1.0	4.0	8.0	7.4	.63
1.0	4.0	9.0	7.3	1.68
1.0	4.0	7.0	7.3	-.28
1.0	4.0	7.0	7.2	-.23
1.0	4.0	7.0	7.2	-.20
1.0	4.0	7.0	7.2	-.16
1.0	4.0	7.0	7.1	-.13
1.0	4.0	6.0	7.1	-1.10
1.0	4.0	6.0	7.1	-1.07
1.0	4.0	7.0	7.0	-.05
1.0	4.0	5.0	6.9	-1.92
1.0	4.0	7.0	6.8	.17
2.0	1.0	10.0	9.1	.90
2.0	1.0	10.0	9.1	.90
2.0	1.0	10.0	9.1	.90
2.0	1.0	10.0	9.1	.90
2.0	1.0	8.0	9.1	-1.10
2.0	1.0	9.0	9.1	-.10
2.0	1.0	9.0	9.1	-.10
2.0	1.0	8.0	9.1	-1.10
2.0	1.0	7.0	9.1	-2.10
2.0	1.0	9.0	9.1	-.10
2.0	1.0	9.0	9.1	-.10
2.0	1.0	8.0	9.1	-1.10
2.0	1.0	9.0	9.1	-.10
2.0	1.0	10.0	9.1	.90
2.0	1.0	10.0	9.1	.90
2.0	1.0	9.0	9.1	-.10
2.0	2.0	10.0	8.7	1.30
2.0	2.0	10.0	8.7	1.30
2.0	2.0	10.0	8.7	1.30
2.0	2.0	10.0	8.7	1.30
2.0	2.0	8.0	8.7	-.70
2.0	2.0	9.0	8.7	.30
2.0	2.0	9.0	8.7	.30
2.0	2.0	8.0	8.7	-.70
2.0	2.0	7.0	8.7	-1.70
2.0	2.0	9.0	8.7	.30
2.0	2.0	9.0	8.7	.30
2.0	2.0	8.0	8.7	-.70
2.0	2.0	9.0	8.7	.30
2.0	2.0	10.0	8.7	1.30
2.0	2.0	10.0	8.7	1.30
2.0	2.0	9.0	8.7	.30
2.0	3.0	10.0	8.4	1.59
2.0	3.0	10.0	8.4	1.59
2.0	3.0	10.0	8.4	1.59
2.0	3.0	10.0	8.4	1.59
2.0	3.0	8.0	8.4	-.41
2.0	3.0	9.0	8.4	.59
2.0	3.0	9.0	8.4	.59
2.0	3.0	8.0	8.4	-.41
2.0	3.0	7.0	8.4	-1.41
2.0	3.0	9.0	8.4	.59
2.0	3.0	9.0	8.4	.59
2.0	3.0	8.0	8.4	-.41
2.0	3.0	9.0	8.4	.59
2.0	3.0	10.0	8.4	1.59
2.0	3.0	10.0	8.4	1.59
2.0	3.0	9.0	8.4	.59
2.0	4.0	10.0	8.1	1.92
2.0	4.0	10.0	8.1	1.88
2.0	4.0	10.0	8.2	1.84
2.0	4.0	10.0	8.2	1.81

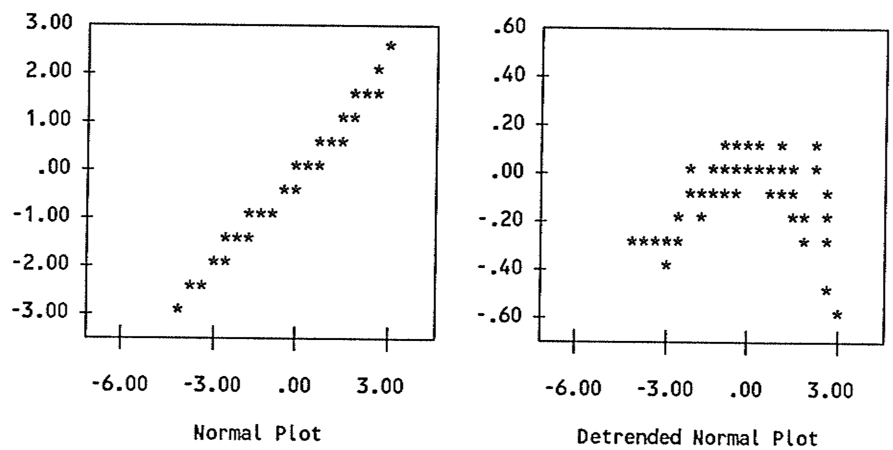
2.0	4.0	8.0	8.2	-.23
2.0	4.0	9.0	8.3	.74
2.0	4.0	9.0	8.3	.70
2.0	4.0	8.0	8.3	-.33
2.0	4.0	7.0	8.4	-1.35
2.0	4.0	9.0	8.4	.63
2.0	4.0	9.0	8.4	.60
2.0	4.0	8.0	8.4	-.41
2.0	4.0	9.0	8.4	.57
2.0	4.0	10.0	8.4	1.56
2.0	4.0	10.0	8.5	1.53
2.0	4.0	9.0	8.5	.55
3.0	1.0	6.0	7.0	-1.00
3.0	1.0	7.0	7.0	.00
3.0	1.0	7.0	7.0	.00
3.0	1.0	6.0	7.0	-1.00
3.0	1.0	6.0	7.0	-1.00
3.0	1.0	7.0	7.7	-.70
3.0	1.0	8.0	7.7	.30
3.0	1.0	5.0	7.7	-2.70
3.0	1.0	6.0	7.7	-1.70
3.0	1.0	6.0	7.7	-1.70
3.0	1.0	8.0	7.7	.30
3.0	1.0	8.0	7.7	.30
3.0	1.0	6.0	7.7	-1.70
3.0	1.0	6.0	7.7	-1.70
3.0	1.0	6.0	7.7	-1.70
3.0	1.0	6.0	7.7	-1.70
3.0	2.0	6.0	7.2	-1.17
3.0	2.0	7.0	7.2	-.17
3.0	2.0	7.0	7.2	-.17
3.0	2.0	6.0	7.2	-1.17
3.0	2.0	6.0	7.2	-1.17
3.0	2.0	7.0	7.9	-.89
3.0	2.0	8.0	7.9	.11
3.0	2.0	5.0	7.9	-2.89
3.0	2.0	6.0	7.9	-1.89
3.0	2.0	6.0	7.9	-1.89
3.0	2.0	8.0	7.9	.11
3.0	2.0	8.0	7.9	.11
3.0	2.0	6.0	7.9	-1.89
3.0	2.0	6.0	7.9	-1.89
3.0	2.0	6.0	7.9	-1.89
3.0	2.0	6.0	7.9	-1.89
3.0	3.0	6.0	7.2	-1.23
3.0	3.0	7.0	7.2	-.23
3.0	3.0	7.0	7.2	-.23
3.0	3.0	6.0	7.2	-1.23
3.0	3.0	6.0	7.2	-1.23
3.0	3.0	7.0	8.0	-.95
3.0	3.0	8.0	8.0	.05
3.0	3.0	5.0	8.0	-2.95
3.0	3.0	6.0	8.0	-1.95
3.0	3.0	6.0	8.0	-1.95
3.0	3.0	8.0	8.0	.05
3.0	3.0	8.0	8.0	.05
3.0	3.0	6.0	8.0	-1.95
3.0	3.0	6.0	8.0	-1.95
3.0	3.0	6.0	8.0	-1.95
3.0	3.0	6.0	8.0	-1.95
3.0	4.0	6.0	5.7	.32
3.0	4.0	7.0	6.4	.60
3.0	4.0	7.0	6.6	.39
3.0	4.0	6.0	6.7	-.68
3.0	4.0	6.0	6.7	-.70
3.0	4.0	7.0	6.7	.29
3.0	4.0	8.0	6.7	1.29
3.0	4.0	5.0	6.7	-1.71
3.0	4.0	6.0	6.7	-.72

3.0	4.0	6.0	6.7	-.72
3.0	4.0	8.0	6.7	1.27
3.0	4.0	8.0	6.7	1.27
3.0	4.0	6.0	6.7	-.74
3.0	4.0	6.0	6.8	-.75
3.0	4.0	6.0	6.8	-.82
3.0	4.0	6.0	6.9	-.90
4.0	1.0	8.0	9.1	-1.10
4.0	1.0	9.0	9.1	-.10
4.0	1.0	8.0	9.1	-1.10
4.0	1.0	6.0	9.1	-3.10
4.0	1.0	8.0	9.1	-1.10
4.0	1.0	5.0	9.1	-4.10
4.0	1.0	9.0	9.1	-.10
4.0	1.0	8.0	9.1	-1.10
4.0	1.0	8.0	9.1	-1.10
4.0	1.0	9.0	9.1	-.10
4.0	1.0	9.0	9.1	-.10
4.0	1.0	6.0	9.1	-3.10
4.0	1.0	5.0	9.1	-4.10
4.0	1.0	8.0	9.1	-1.10
4.0	1.0	9.0	9.1	-.10
4.0	1.0	6.0	9.1	-3.10
4.0	2.0	8.0	8.7	-.70
4.0	2.0	9.0	8.7	.30
4.0	2.0	8.0	8.7	-.70
4.0	2.0	6.0	8.7	-2.70
4.0	2.0	8.0	8.7	-.70
4.0	2.0	5.0	8.7	-3.70
4.0	2.0	9.0	8.7	.30
4.0	2.0	8.0	8.7	-.70
4.0	2.0	8.0	8.7	-.70
4.0	2.0	9.0	8.7	.30
4.0	2.0	9.0	8.7	.30
4.0	2.0	6.0	8.7	-2.70
4.0	2.0	5.0	8.7	-3.70
4.0	2.0	8.0	8.7	-.70
4.0	2.0	9.0	8.7	.30
4.0	2.0	6.0	8.7	-2.70
4.0	3.0	8.0	8.4	-.41
4.0	3.0	9.0	8.4	.59
4.0	3.0	8.0	8.4	-.41
4.0	3.0	6.0	8.4	-2.41
4.0	3.0	8.0	8.4	-.41
4.0	3.0	5.0	8.4	-3.41
4.0	3.0	9.0	8.4	.59
4.0	3.0	8.0	8.4	-.41
4.0	3.0	8.0	8.4	-.41
4.0	3.0	9.0	8.4	.59
4.0	3.0	9.0	8.4	.59
4.0	3.0	6.0	8.4	-2.41
4.0	3.0	5.0	8.4	-3.41
4.0	3.0	8.0	8.4	-.41
4.0	3.0	9.0	8.4	.59
4.0	3.0	6.0	8.4	-2.41
4.0	4.0	8.0	6.8	1.24
4.0	4.0	9.0	7.6	1.37
4.0	4.0	8.0	7.9	.07
4.0	4.0	6.0	8.0	-2.04
4.0	4.0	8.0	8.1	-.08
4.0	4.0	5.0	8.1	-3.10
4.0	4.0	9.0	8.1	.89
4.0	4.0	8.0	8.1	-.11
4.0	4.0	8.0	8.1	-.10
4.0	4.0	9.0	8.1	.90
4.0	4.0	9.0	8.1	.91
4.0	4.0	6.0	8.1	-2.08
4.0	4.0	5.0	8.1	-3.08
4.0	4.0	8.0	8.1	-.07

```
4.0    4.0      9.0      8.0      .95
4.0    4.0      6.0      8.0    -2.04

Number of cases read =     256    Number of cases listed =     256
```

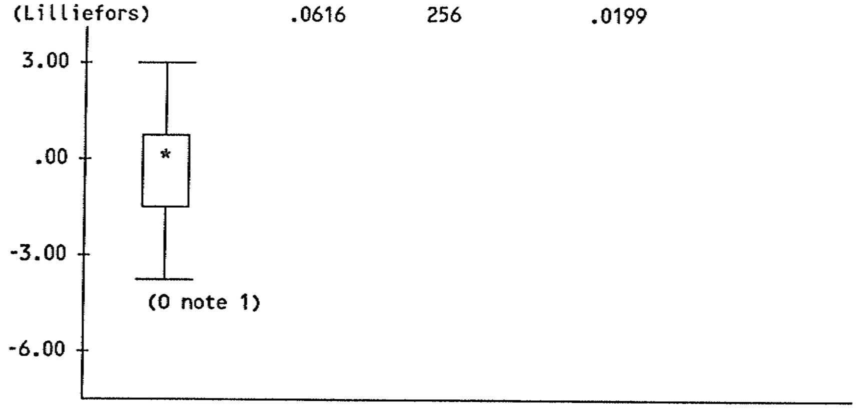
EXAMINE /DIFF BY METHOD /PLOT BOXPLOT NPPLOT HISTOGRAM SPREADLEVEL.

```
    DIFF

 Valid cases:        256.0  Missing cases:        .0  Percent missing:       .0

 Mean          -.1014  Std Err      .0909  Min        -4.1000  Skewness     -.3650
 Median         .0000  Variance    2.1175  Max         3.0000  S E Skew      .1522
 5% Trim       -.0721  Std Dev     1.4552  Range       7.1000  Kurtosis     -.1409
                                           IQR         2.0000  S E Kurt      .3033

    DIFF
 Frequency    Bin Center

     2.00  Extremes    *
     9.00    -3.500    ****
    13.00    -2.500    ******
    46.00    -1.500    ***********************
    57.00     -.500    ****************************
    69.00      .500    **********************************
    47.00     1.500    ***********************
    12.00     2.500    ******
     1.00  Extremes

 Bin width :      1.000
 Each star:        2 case(s)
     DIFF
```



DIFF

	Statistic	df	Significance
K-S (Lilliefors)	.0616	256	.0199



Variable DIFF

N of Cases 256.00

Symbol Key: * - Median (O) - Outlier (E) - Extreme

Boxplot footnotes denote the following:

1) CASE198, CASE205

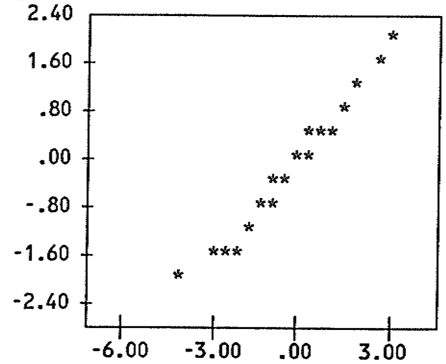
DIFF
By METHOD 1.0

Valid cases:	64.0	Missing cases:			.0	Percent missing:	.0
Mean	-.2891	Std Err	.1962	Min	-4.1000	Skewness	-.2211
Median	-.1000	Variance	2.4635	Max	3.0000	S E Skew	.2993
5% Trim	-.2694	Std Dev	1.5696	Range	7.1000	Kurtosis	.0923
				IQR	2.0000	S E Kurt	.5905

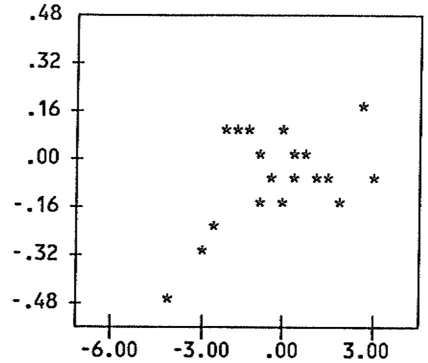
DIFF
By METHOD 1.0

```
Frequency   Bin Center
    2.00  Extremes   **
    3.00    -3.250   ***
    1.00    -2.750   *
    1.00    -2.250   *
    6.00    -1.750   ******
   13.00    -1.250   *************
    1.00     -.750   *
   11.00     -.250   ***********
    7.00      .250   *******
    8.00      .750   ********
    5.00     1.250   *****
    2.00     1.750   **
     .00     2.250
    3.00     2.750   ***
    1.00  Extremes   *

Bin width :     .500
Each star:      1 case(s)
```

DIFF
By METHOD 1.0



Normal Plot



Detrended Normal Plot

	Statistic	df	Significance
K-S (Lilliefors)	.0810	64	> .2000

DIFF
By METHOD 2.0

Valid cases: 64.0 Missing cases: .0 Percent missing: .0

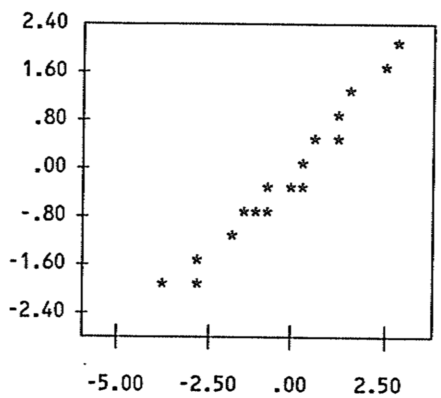
Mean	-.1696	Std Err	.1887	Min	-3.6970	Skewness	-.3037
Median	.1885	Variance	2.2785	Max	2.8300	S E Skew	.2993
5% Trim	-.1470	Std Dev	1.5095	Range	6.5270	Kurtosis	-.2458
				IQR	2.4340	S E Kurt	.5905

DIFF
By METHOD 2.0

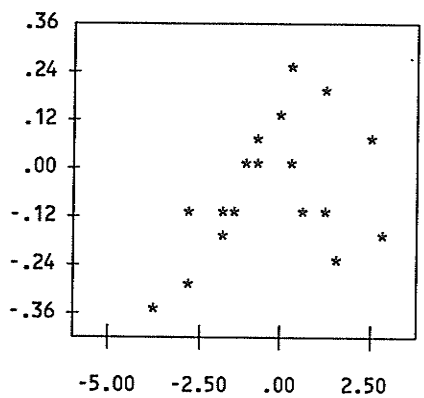
Frequency	Bin Center	
2.00	-3.500	**
4.00	-2.500	****
11.00	-1.500	***********
12.00	-.500	************
18.00	.500	******************
13.00	1.500	*************
4.00	2.500	****

Bin width : 1.000
Each star: 1 case(s)

DIFF
By METHOD 2.0



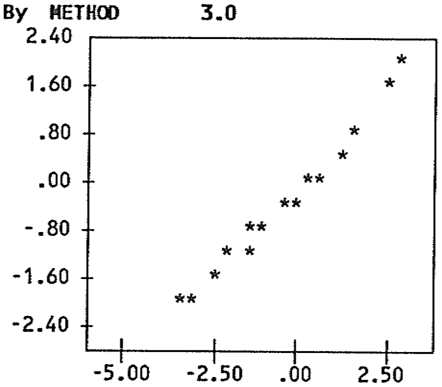
Normal Plot



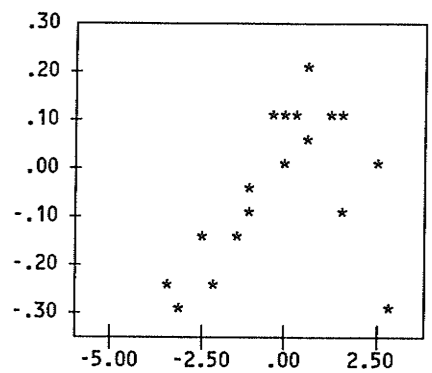
Detrended Normal Plot

Statistic df Significance

```
K-S (Lilliefors)            .0818        64          > .2000

    DIFF
By  METHOD        3.0

Valid cases:          64.0   Missing cases:         .0   Percent missing:      .0
Mean          -.0525  Std Err       .1871  Min        -3.4058  Skewness      -.3506
Median         .1324  Variance     2.2405  Max         2.7710  S E Skew       .2993
5% Trim       -.0239  Std Dev      1.4968  Range       6.1768  Kurtosis      -.4760
                                           IQR         2.4458  S E Kurt       .5905
    DIFF
By  METHOD        3.0

Frequency    Bin Center
    2.00     -3.250    **
    1.00     -2.750    *
    3.00     -2.250    ***
    7.00     -1.750    *******
    4.00     -1.250    ****
    1.00      -.750    *
   11.00      -.250    ***********
    7.00       .250    *******
   11.00       .750    ***********
    7.00      1.250    *******
    6.00      1.750    ******
    3.00      2.250    ***
    1.00      2.750    *

Bin width :       .500
Each star:        1 case(s)

    DIFF
By  METHOD        3.0
```



Normal Plot

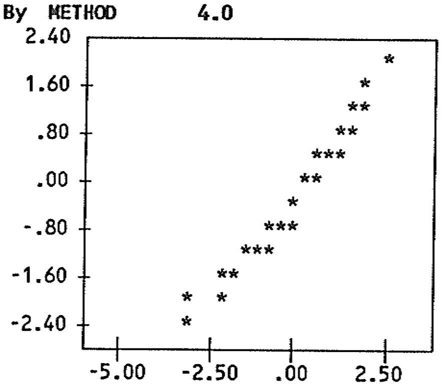


Detrended Normal Plot

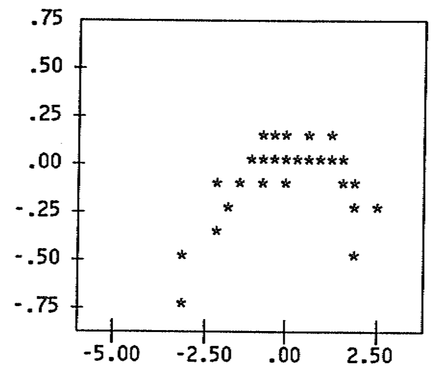
```
                             Statistic        df         Significance
K-S (Lilliefors)               .0853          64          > .2000

    DIFF
By  METHOD        4.0

Valid cases:          64.0   Missing cases:         .0   Percent missing:      .0

Mean           .1055  Std Err       .1532  Min        -3.1000  Skewness      -.5314
Median         .1225  Variance     1.5018  Max         2.4690  S E Skew       .2993
5% Trim        .1526  Std Dev      1.2255  Range       5.5690  Kurtosis       .0911
                                           IQR         1.6548  S E Kurt       .5905
```

```
    DIFF
By  METHOD       4.0

Frequency    Bin Center
    2.00     -3.250   **
     .00     -2.750
    3.00     -2.250   ***
    2.00     -1.750   **
    3.00     -1.250   ***
    8.00      -.750   ********
   13.00      -.250   *************
    5.00       .250   *****
   13.00       .750   *************
    5.00      1.250   *****
    9.00      1.750   *********
    1.00      2.250   *

Bin width :      .500
Each star:       1 case(s)
```

DIFF
By METHOD 4.0

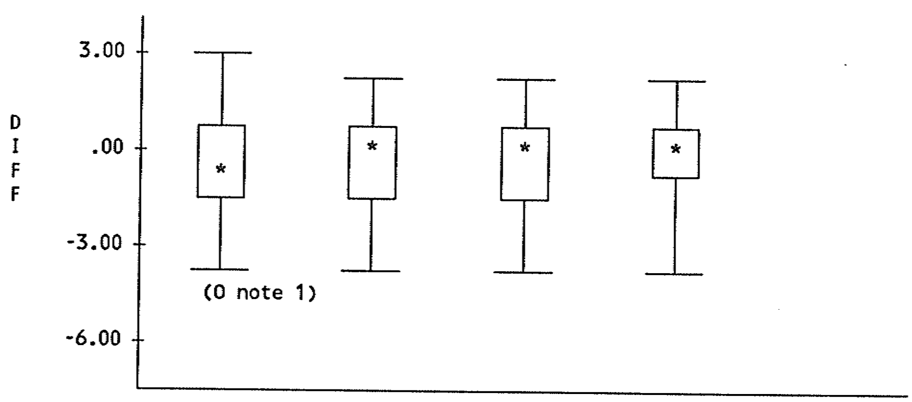


Normal Plot



Detrended Normal Plot

	Statistic	df	Significance
K-S (Lilliefors)	.0625	64	> .2000



METHOD	1.0	2.0	3.0	4.0
N of Cases	64.00	64.00	64.00	64.00

Symbol Key: * - Median (O) - Outlier (E) - Extreme
Boxplot footnotes denote the following:
1) CASE198, CASE205

Test of homogeneity of variance		df1	df2	Significance
Levene Statistic	1.3348	3	252	.2636

T-TEST /GROUPS METHOD (1,2) /VARIABLES DIFF.

t-tests for independent samples of METHOD

Variable	Number of Cases	Mean	SD	SE of Mean
DIFF				
METHOD 1.0	64	-.2891	1.570	.196
METHOD 2.0	64	-.1696	1.509	.189

Mean Difference = -.1194

Levene's Test for Equality of Variances: F= .016 P= .899

Variances	t-value	df	2-Tail Sig	SE of Diff	95% CI for Diff
Equal	-.44	126	.662	.272	(-.658, .419)
Unequal	-.44	125.81	.662	.272	(-.658, .419)

(t-test for Equality of Means)

NPAR TESTS /MANN-WHITNEY DIFF BY METHOD (1,2).

- - - - - Mann-Whitney U - Wilcoxon Rank Sum W Test
DIFF
by METHOD

Mean Rank	Cases	
62.67	64	METHOD = 1.0
66.33	64	METHOD = 2.0

	128	Total

U	W	Z (Corrected for Ties)	2-tailed P (Corrected for Ties)
1931.0	4011.0	-.5583	.5766

ONEWAY /VARIABLES DIFF BY METHOD (1,4) /RANGES BTUKEY /STATISTICS ALL.

- - - - - - - - - - - O N E W A Y - - - - - - - - - - -

Variable DIFF By Variable
METHOD
Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	3	5.4449	1.8150	.8557	.4647
Within Groups	252	534.5155	2.1211		
Total	255	539.9604			

- - - - - - - - - - - O N E W A Y - - - - - - - - - - -

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean		
Grp 1	64	-.2891	1.5696	.1962	-.6811	To	.1030
Grp 2	64	-.1696	1.5095	.1887	-.5467	To	.2074
Grp 3	64	-.0525	1.4968	.1871	-.4264	To	.3214
Grp 4	64	.1055	1.2255	.1532	-.2006	To	.4116
Total	256	-.1014	1.4552	.0909	-.2805	To	.0777
Fixed Effects Model			1.4564	.0910	-.2807	To	.0778
Random Effects Model				.0910	-.3911	To	.1882

```
WARNING - Between component variance is negative
          it was replaced by 0.0 in computing above random effects measures

Random Effects Model - Estimate of Between Component Variance        -.0048
               - - - - - - - - - - O N E W A Y - - - - - - - - - -

Group        Minimum     Maximum

Grp 1        -4.1000      3.0000
Grp 2        -3.6970      2.8300
Grp 3        -3.4058      2.7710
Grp 4        -3.1000      2.4690

Total        -4.1000      3.0000


Tests for Homogeneity of Variances

     Cochrans C = Max. Variance/Sum(Variances) =  .2904, P =  .592 (Approx.)
     Bartlett-Box F =                            1.444 , P =  .228
     Maximum Variance / Minimum Variance         1.640

               - - - - - - - - - - O N E W A Y - - - - - - - - - -

     Variable  DIFF
  By Variable  METHOD

Multiple Range Test

Tukey-B Procedure
Ranges for the  .050 level -

         3.24   3.50   3.66

The ranges above are table ranges.
The value actually compared with Mean(J)-Mean(I) is..
       1.0298 * Range * Sqrt(1/N(I) + 1/N(J))


No two groups are significantly different at the  .050 level

  Homogeneous Subsets   (Subsets of groups, whose highest and lowest means
                         do not differ by more than the shortest
                         significant range for a subset of that size)

SUBSET  1

Group       Grp 1         Grp 2         Grp 3         Grp 4
Mean        -.2891        -.1696        -.0525         .1055
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

NPAR TESTS /KRUSKAL-WALLIS DIFF BY METHOD (1,4) /STATISTICS 2.

                                  (Median)
                       25th         50th        75th
               N   Percentile  Percentile  Percentile

DIFF          256    -1.1000       .0000       .9000
METHOD        256     1.2500      2.5000      3.7500

- - - - - Kruskal-Wallis 1-way ANOVA

    DIFF
 by METHOD

     Mean Rank    Cases

        119.04       64   METHOD =   1
        125.42       64   METHOD =   2
```

130.78	64	METHOD =	3
138.76	64	METHOD =	4

	256	Total	

			Corrected for Ties	
CASES	Chi-Square	Significance	Chi-Square	Significance
256	2.4445	.4854	2.4457	.4852

T-TEST /GROUPS METHOD (1,2) /VARIABLES DIFF /CRITERIA=CI (.9).

t-tests for independent samples of METHOD

Variable	Number of Cases	Mean	SD	SE of Mean
DIFF				
METHOD 1.0	64	-.2891	1.570	.196
METHOD 2.0	64	-.1696	1.509	.189

Mean Difference = -.1194

Levene's Test for Equality of Variances: F= .016 P= .899

t-test for Equality of Means					90%
Variances	t-value	df	2-Tail Sig	SE of Diff	CI for Diff
Equal	-.44	126	.662	.272	(-.571, .332)
Unequal	-.44	125.81	.662	.272	(-.571, .332)

ONEWAY /VARIABLES DIFF BY METHOD (1,4) /RANGES LSD (.1).

- - - - - - - - - - O N E W A Y - - - - - - - - - -

Variable DIFF

By Variable METHOD

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	3	5.4449	1.8150	.8557	.4647
Within Groups	252	534.5155	2.1211		
Total	255	539.9604			

- - - - - - - - - - **O N E W A Y** - - - - - - - - - -

Variable DIFF
By Variable METHOD

Multiple Range Test

LSD Procedure
Ranges for the .100 level -

2.33 2.33 2.33

The ranges above are table ranges.
The value actually compared with Mean(J)-Mean(I) is..
1.0298 * Range * Sqrt(1/N(I) + 1/N(J))

No two groups are significantly different at the .100 level

```
Homogeneous Subsets    (Subsets of groups, whose highest and lowest means
                        do not differ by more than the shortest
                        significant range for a subset of that size)

SUBSET  1

Group        Grp 1        Grp 2        Grp 3        Grp 4
Mean         -.2891       -.1696       -.0525        .1055
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

ANOVA /VARIABLES DIFF BY METHOD (1,4) BASE (3,4) /STATISTICS ALL.

```
                        * * *  C E L L   M E A N S  * * *
             DIFF
          BY METHOD
             BASE

TOTAL POPULATION

     -.93
 (   128)

METHOD
        1          2          3          4
     -1.26      -1.15      -1.03       -.27
 (     32) (     32) (     32) (     32)

BASE
        3          4
      -.88       -.98
 (     64) (     64)

          BASE
                 3          4
METHOD
        1       -.98      -1.54
           (     16) (     16)
        2      -1.16      -1.13
           (     16) (     16)
        3      -1.23       -.84
           (     16) (     16)
        4       -.14       -.40
           (     16) (     16)

          * * *  A N A L Y S I S   O F   V A R I A N C E  * * *

             DIFF
          BY METHOD
             BASE
```

Source of Variation	Sum of Squares	DF	Mean Square	F	Signif of F
Main Effects	19.579	4	4.895	3.198	.016
METHOD	19.261	3	6.420	4.195	.007
BASE	.317	1	.317	.207	.650
2-way Interactions	3.849	3	1.283	.838	.475
METHOD BASE	3.849	3	1.283	.838	.475
Explained	23.428	7	3.347	2.187	.040
Residual	183.673	120	1.531		
Total	207.101	127	1.631		

256 Cases were processed.
128 Cases (50.0 PCT) were missing.

*** MULTIPLE CLASSIFICATION ANALYSIS ***

DIFF
By METHOD
BASE

Grand Mean = -.928

Variable + Category	N	Unadjusted Dev'n	Eta	Adjusted for Independents Dev'n	Beta	Adjusted for Independents + Covariates Dev'n	Beta
METHOD							
1	32	-.33		-.33			
2	32	-.22		-.22			
3	32	-.11		-.11			
4	32	.66		.66			
			.30		.30		
BASE							

*** MULTIPLE CLASSIFICATION ANALYSIS ***

DIFF
By METHOD
BASE

Grand Mean = -.928

Variable + Category	N	Unadjusted Dev'n	Eta	Adjusted for Independents Dev'n	Beta	Adjusted for Independents + Covariates Dev'n	Beta
3	64	.05		.05			
4	64	-.05		-.05			
			.04		.04		

Multiple R Squared .095
Multiple R .307

ANOVA /VARIABLES DIFF BY METHOD (1,4) BASE (1,2) /STATISTICS ALL.

*** CELL MEANS ***

DIFF
BY METHOD
BASE

TOTAL POPULATION

.73
(128)

METHOD

1	2	3	4
.68	.81	.93	.48
(32)	(32)	(32)	(32)

BASE

1	2
1.02	.43
(64)	(64)

	BASE 1	2
METHOD		
1	1.40	-.04
	(16)	(16)
2	1.25	.37
	(16)	(16)
3	1.20	.66
	(16)	(16)
4	.21	.75
	(16)	(16)

*** ANALYSIS OF VARIANCE ***

DIFF
BY METHOD
BASE

Source of Variation	Sum of Squares	DF	Mean Square	F	Signif of F
Main Effects	14.412	4	3.603	3.411	.011
METHOD	3.522	3	1.174	1.112	.347
BASE	10.890	1	10.890	10.310	.002
2-way Interactions	16.645	3	5.548	5.253	.002
METHOD BASE	16.645	3	5.548	5.253	.002
Explained	31.057	7	4.437	4.201	.000
Residual	126.744	120	1.056		
Total	157.801	127	1.243		

256 Cases were processed.
128 Cases (50.0 PCT) were missing.

*** MULTIPLE CLASSIFICATION ANALYSIS ***

DIFF
By METHOD
BASE

Grand Mean = .726

Variable + Category	N	Unadjusted Dev'n	Unadjusted Eta	Adjusted for Independents Dev'n	Adjusted for Independents Beta	Adjusted for Independents + Covariates Dev'n	Adjusted for Independents + Covariates Beta
METHOD							
1	32	-.04		-.04			
2	32	.08		.08			
3	32	.20		.20			
4	32	-.24		-.24			
			.15		.15		
BASE							

*** MULTIPLE CLASSIFICATION ANALYSIS ***

DIFF
By METHOD
BASE

Grand Mean = .726

Variable + Category	N	Unadjusted Dev'n	Unadjusted Eta	Adjusted for Independents Dev'n	Adjusted for Independents Beta	Adjusted for Independents + Covariates Dev'n	Adjusted for Independents + Covariates Beta
1	64	.29		.29			
2	64	-.29		-.29			
			.26		.26		
Multiple R Squared					.091		
Multiple R					.302		

ANOVA /VARIABLES DIFF BY METHOD (1,4) BASE (1,4) /STATISTICS ALL.

*** CELL MEANS ***

DIFF
BY METHOD
BASE

TOTAL POPULATION

-.10
(256)

METHOD

1	2	3	4
-.29	-.17	-.05	.11
(64)	(64)	(64)	(64)

BASE

1	2	3	4
1.02	.43	-.88	-.98
(64)	(64)	(64)	(64)

METHOD \ BASE	1	2	3	4
1	1.40	-.04	-.98	-1.54
	(16)	(16)	(16)	(16)
2	1.25	.37	-1.16	-1.13
	(16)	(16)	(16)	(16)
3	1.20	.66	-1.23	-.84
	(16)	(16)	(16)	(16)
4	.21	.75	-.14	-.40
	(16)	(16)	(16)	(16)

*** ANALYSIS OF VARIANCE ***

DIFF
BY METHOD
BASE

Source of Variation	Sum of Squares	DF	Mean Square	F	Signif of F
Main Effects	191.711	6	31.952	24.704	.000
METHOD	5.445	3	1.815	1.403	.242
BASE	186.266	3	62.089	48.004	.000
2-way Interactions	37.832	9	4.204	3.250	.001
METHOD BASE	37.832	9	4.204	3.250	.001
Explained	229.543	15	15.303	11.831	.000
Residual	310.417	240	1.293		
Total	539.960	255	2.117		

256 Cases were processed.
0 Cases (.0 PCT) were missing.

*** MULTIPLE CLASSIFICATION ANALYSIS ***

DIFF
By METHOD
BASE

Grand Mean = -.101

Variable + Category	N	Unadjusted Dev'n	Unadjusted Eta	Adjusted for Independents Dev'n	Adjusted for Independents Beta	Adjusted for Independents + Covariates Dev'n	Adjusted for Independents + Covariates Beta
METHOD							
1	64	-.19		-.19			
2	64	-.07		-.07			
3	64	.05		.05			
4	64	.21		.21			
			.10		.10		

*** MULTIPLE CLASSIFICATION ANALYSIS ***

DIFF
By METHOD
BASE

Grand Mean = -.101

Variable + Category	N	Unadjusted Dev'n	Unadjusted Eta	Adjusted for Independents Dev'n	Adjusted for Independents Beta	Adjusted for Independents + Covariates Dev'n	Adjusted for Independents + Covariates Beta
BASE							
1	64	1.12		1.12			
2	64	.54		.54			
3	64	-.78		-.78			
4	64	-.88		-.88			
			.59		.59		
Multiple R Squared					.355		
Multiple R					.596		

```
IF (BASE=1 AND METHOD=1) INTER=1.
IF (BASE=1 AND METHOD=2) INTER=2.
IF (BASE=1 AND METHOD=3) INTER=3.
IF (BASE=1 AND METHOD=4) INTER=4.
IF (BASE=2 AND METHOD=1) INTER=5.
IF (BASE=2 AND METHOD=2) INTER=6.
IF (BASE=2 AND METHOD=3) INTER=7.
IF (BASE=2 AND METHOD=4) INTER=8.
IF (BASE=3 AND METHOD=1) INTER=9.
IF (BASE=3 AND METHOD=2) INTER=10.
IF (BASE=3 AND METHOD=3) INTER=11.
IF (BASE=3 AND METHOD=4) INTER=12.
IF (BASE=4 AND METHOD=1) INTER=13.
IF (BASE=4 AND METHOD=2) INTER=14.
IF (BASE=4 AND METHOD=3) INTER=15.
IF (BASE=4 AND METHOD=4) INTER=16.
```

ONEWAY /VARIABLES DIFF BY BASE (1,4) /RANGES BTUKEY /STATISTICS ALL.

- - - - - - - - - - - O N E W A Y - - - - - - - - - -

Variable DIFF **By Variable BASE**

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	3	186.2658	62.0886	44.2368	.0000
Within Groups	252	353.6946	1.4036		
Total	255	539.9604			

- - - - - - - - - - - O N E W A Y - - - - - - - - - -

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean		
Grp 1	64	1.0172	1.1809	.1476	.7222	To	1.3122
Grp 2	64	.4338	.9682	.1210	.1920	To	.6757
Grp 3	64	-.8786	1.0071	.1259	-1.1302	To	-.6270
Grp 4	64	-.9782	1.5060	.1882	-1.3543	To	-.6020
Total	256	-.1014	1.4552	.0909	-.2805	To	.0777
	Fixed Effects Model		1.1847	.0740	-.2473	To	.0444
	Random Effects Model			.4925	-1.6687	To	1.4658

Random Effects Model - Estimate of Between Component Variance .9482

- - - - - - - - - - - O N E W A Y - - - - - - - - - -

Group	Minimum	Maximum
Grp 1	-1.9180	3.0000
Grp 2	-2.1000	1.9160
Grp 3	-2.9519	1.2900
Grp 4	-4.1000	1.3690
Total	-4.1000	3.0000

```
Tests for Homogeneity of Variances

      Cochrans C = Max. Variance/Sum(Variances) =  .4040, P =  .001 (Approx.)
      Bartlett-Box F =                            5.321 , P =  .001
      Maximum Variance / Minimum Variance         2.420

              - - - - - - - - - - O N E W A Y - - - - - - - - - -

      Variable  DIFF
   By Variable  BASE

Multiple Range Test

Tukey-B Procedure
Ranges for the  .050 level -

         3.24   3.50   3.66

The ranges above are table ranges.
The value actually compared with Mean(J)-Mean(I) is..
        .8377 * Range * Sqrt(1/N(I) + 1/N(J))

(*) Denotes pairs of groups significantly different at the  .050 level

              - - - - - - - - - - O N E W A Y - - - - - - - - - -

      Variable  DIFF
      (Continued)

                             G G G G
                             r r r r
                             p p p p

     Mean        Group       4 3 2 1

     -.9782      Grp 4
     -.8786      Grp 3
      .4338      Grp 2       * *
     1.0172      Grp 1       * * *

  Homogeneous Subsets    (Subsets of groups, whose highest and lowest means
                          do not differ by more than the shortest
                          significant range for a subset of that size)

SUBSET  1

Group        Grp 4         Grp 3
Mean         -.9782        -.8786
- - - - - - - - - - - - - - - - -

SUBSET  2

Group        Grp 2
Mean          .4338
- - - - - - - - - -

SUBSET  3

Group        Grp 1
Mean         1.0172
```

ONEWAY /VARIABLES DIFF BY INTER (1,16)/RANGES BTUKEY /STATISTICS ALL.

- - - - - - - - - - - O N E W A Y - - - - - - - - - - -

Variable DIFF By Variable INTER

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	15	229.5431	15.3029	11.8315	.0000
Within Groups	240	310.4173	1.2934		
Total	255	539.9604			

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean		
Grp 1	16	1.4000	1.0979	.2745	.8150	To	1.9850
Grp 2	16	1.2534	1.0942	.2736	.6703	To	1.8364
Grp 3	16	1.2025	1.0930	.2732	.6201	To	1.7849

- - - - - - - - - - - O N E W A Y - - - - - - - - - - -

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean		
Grp 4	16	.2129	1.1478	.2870	-.3988	To	.8245
Grp 5	16	-.0375	.9287	.2322	-.5324	To	.4574
Grp 6	16	.3655	.9287	.2322	-.1294	To	.8604
Grp 7	16	.6567	.9287	.2322	.1618	To	1.1516
Grp 8	16	.7506	.9734	.2433	.2320	To	1.2693
Grp 9	16	-.9812	.9304	.2326	-1.4770	To	-.4855
Grp10	16	-1.1629	.9327	.2332	-1.6600	To	-.6659
Grp11	16	-1.2260	.9336	.2334	-1.7235	To	-.7285
Grp12	16	-.1441	.9227	.2307	-.6358	To	.3475
Grp13	16	-1.5375	1.4592	.3648	-2.3150	To	-.7600
Grp14	16	-1.1345	1.4592	.3648	-1.9120	To	-.3570
Grp15	16	-.8433	1.4592	.3648	-1.6208	To	-.0658
Grp16	16	-.3973	1.5488	.3872	-1.2226	To	.4280
Total	256	-.1014	1.4552	.0909	-.2805	To	.0777
	Fixed Effects Model		1.1373	.0711	-.2414	To	.0386

- - - - - - - - - - - O N E W A Y - - - - - - - - - - -

	Random Effects Model			.2445	-.6226	To	.4197

Random Effects Model - Estimate of Between Component Variance .8756

Group	Minimum	Maximum
Grp 1	-1.3000	3.0000
Grp 2	-1.4530	2.8300
Grp 3	-1.5061	2.7710
Grp 4	-1.9180	2.4690
Grp 5	-2.1000	.9000
Grp 6	-1.6970	1.3030
Grp 7	-1.4058	1.5942
Grp 8	-1.3520	1.9160
Grp 9	-2.7000	.3000
Grp10	-2.8870	.1130
Grp11	-2.9519	.0481
Grp12	-1.7130	1.2900
Grp13	-4.1000	-.1000

- - - - - - - - - - O N E W A Y - - - - - - - - - -

Group	Minimum	Maximum
Grp14	-3.6970	.3030
Grp15	-3.4058	.5942
Grp16	-3.1000	1.3690
Total	-4.1000	3.0000

Tests for Homogeneity of Variances

Cochrans C = Max. Variance/Sum(Variances) = .1159, P = .298 (Approx.)
Bartlett-Box F = 1.205 , P = .259
Maximum Variance / Minimum Variance 2.818

- - - - - - - - - - O N E W A Y - - - - - - - - - -

Variable DIFF
By Variable INTER

Multiple Range Test

Tukey-B Procedure
Ranges for the .050 level -

3.85 4.12 4.28 4.39 4.48 4.55 4.61 4.66 4.70 4.74
4.78 4.81 4.84 4.87 4.89

The ranges above are table ranges.
The value actually compared with Mean(J)-Mean(I) is..
.8042 * Range * Sqrt(1/N(I) + 1/N(J))

(*) Denotes pairs of groups significantly different at the .050 level

```
                          G G G G G G G G G G G G G G G G
                          r r r r r r r r r r r r r r r r
                          p p p p p p p p p p p p p p p p
                          1 1 1 1   1 1 1
     Mean      Group      3 1 0 4 9 5 6 2 5 4 6 7 8 3 2 1

   -1.5375     Grp13
   -1.2260     Grp11
   -1.1629     Grp10
   -1.1345     Grp14
    -.9812     Grp 9
    -.8433     Grp15
    -.3973     Grp16
    -.1441     Grp12      *
    -.0375     Grp 5      *
     .2129     Grp 4      * * * *
     .3655     Grp 6      * * * * *
     .6567     Grp 7      * * * * * *
     .7506     Grp 8      * * * * * *
    1.2025     Grp 3      * * * * * * * *
    1.2534     Grp 2      * * * * * * * *
    1.4000     Grp 1      * * * * * * * * *
```

Homogeneous Subsets (Subsets of groups, whose highest and lowest means do not differ by more than the shortest significant range for a subset of that size)

SUBSET 1

Group	Grp13	Grp11	Grp10	Grp14	Grp 9
Mean	-1.5375	-1.2260	-1.1629	-1.1345	-.9812

Group	Grp15	Grp16
Mean	-.8433	-.3973

- -

SUBSET 2

Group	Grp11	Grp10	Grp14	Grp 9	Grp15
Mean	-1.2260	-1.1629	-1.1345	-.9812	-.8433

Group	Grp16	Grp12	Grp 5
Mean	-.3973	-.1441	-.0375

- -

SUBSET 3

Group	Grp 9	Grp15	Grp16	Grp12	Grp 5
Mean	-.9812	-.8433	-.3973	-.1441	-.0375

Group	Grp 4
Mean	.2129

- -

SUBSET 4

Group	Grp15	Grp16	Grp12	Grp 5	Grp 4
Mean	-.8433	-.3973	-.1441	-.0375	.2129

Group	Grp 6
Mean	.3655

SUBSET 5

Group	Grp16	Grp12	Grp 5	Grp 4	Grp 6
Mean	-.3973	-.1441	-.0375	.2129	.3655
Group	Grp 7	Grp 8			
Mean	.6567	.7506			

SUBSET 6

Group	Grp 5	Grp 4	Grp 6	Grp 7	Grp 8
Mean	-.0375	.2129	.3655	.6567	.7506
Group	Grp 3	Grp 2			
Mean	1.2025	1.2534			

SUBSET 7

Group	Grp 4	Grp 6	Grp 7	Grp 8	Grp 3
Mean	.2129	.3655	.6567	.7506	1.2025
Group	Grp 2	Grp 1			
Mean	1.2534	1.4000			

EXHIBIT 4 - EDITED OUTPUT OF SOOPSP INITIAL RUN

```
MAX      0.0309 X4 + 0.0313 X8 + 0.0264 X9 + 0.0534 X10 + 0.0426 X11
     + 0.0581 X14
SUBJECT TO
       2)   X1 + X7 + X15 + X16 + X20 + A1M - A1P =    6
       3)   X1 + X7 + X8 + X15 + X16 + X20 + A2M - A2P =    6
       4)   X2 + X7 + X8 + X9 + X16 + X17 + X20 + A3M - A3P =    6
       5)   X2 + X7 + X8 + X12 + X14 + X17 + X20 + A4M - A4P =    6
       6)   X2 + X3 + X10 + X12 + X14 + X17 + X20 + A5M - A5P =    6
       7)   X2 + X3 + X10 + X12 + X14 + X20 + A6M - A6P =    6
       8)   X2 + X3 + X10 + X12 + X14 + X20 + A7M - A7P =    6
       9)   X2 + X3 + X10 + X12 + X14 + X20 + A8M - A8P =    6
      10)   X2 + X11 + X12 + X14 + X20 + A9M - A9P =    6
      11)   X2 + X11 + X12 + X14 + X20 + A10M - A10P =    6
      12)   X11 + X20 + X21 + X22 + X23 + X24 + X25 + A11M - A11P =    6
      13)   X4 + X20 + X21 + X22 + X23 + X24 + X25 + A12M - A12P =    6
      14)   X4 + X20 + X21 + X22 + X23 + X24 + X25 + A13M - A13P =    6
      15)   X4 + X21 + X22 + X23 + X24 + X25 + A14M - A14P =    6
      16)   X4 + X21 + X22 + X23 + X24 + X25 + A15M - A15P =    6
      17)   X4 + X21 + X22 + X23 + X24 + X25 + A16M - A16P =    6
      18)   X4 + X21 + X22 + X23 + X24 + X25 + A17M - A17P =    6
      19)   X21 + X22 + X23 + X24 + X25 + A18M - A18P =    6
      20)   X5 + X21 + X22 + X23 + X24 + X25 + A19M - A19P =    6
      21)   X5 + X21 + X22 + X23 + X24 + X25 + A20M - A20P =    6
      22)   X5 + X19 + X21 + X22 + X23 + X24 + X25 + A21M - A21P =    6
      23)   X5 + X6 + X18 + X19 + A22M - A22P =    6
      24)   X6 + X18 + X19 + X26 + X27 + X28 + X29 + A23M - A23P =    6
      25)   X6 + X18 + X26 + X27 + X28 + X29 + X30 + A24M - A24P =    6
      26)   X6 + X18 + X26 + X27 + X28 + X29 + X30 + X31 + A25M - A25P
     =    6
      27)   X6 + X26 + X27 + X28 + X29 + X30 + X31 + A26M - A26P =    6
      28)   X13 + X26 + X27 + X28 + X29 + X30 + X31 + A27M - A27P =    6
      29)   X13 + X26 + X27 + X30 + X31 + A28M - A28P =    6
      30)   X13 + X26 + A29M - A29P =    6
      31)   X13 + X26 + A30M - A30P =    6
      32)   A1P + A2P + A3P + A4P + A5P + A6P + A7P + A8P + A9P + A10P
     + A11P + A12P + A13P + A14P + A15P + A16P + A17P + A18P + A19P +
A20P
     + A21P + A22P + A23P + A24P + A25P + A26P + A27P + A28P + A29P +
A30P
     =    0
      33)   12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + U2M - U2P
     =    112
      34)   24 X7 + 19 X8 + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45
X14
     + U4M - U4P =    162
      35)   14 X15 + 20 X16 + 11 X17 + 28 X18 + 16 X19 + 90 X20 + U5M -
U5P
     =    161
      36)   60 X21 + 60 X22 + 60 X23 + 60 X24 + 60 X25 + 50 X26 + 41 X27
     + 35 X28 + U6M - U6P =    383
      37)   36 X29 + 34 X30 + 22 X31 + U7M - U7P =    83
      38)   U2P <=   11
      39)   U4P <=   16
      40)   U5P <=   16
      41)   U6P <=   38
      42)   U7P <=   8
      43)   12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + 24 X7 + 19 X8
     + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45 X14 + 14 X15 + 20 X16
     + 11 X17 + 28 X18 + 16 X19 + 90 X20 + 60 X21 + 60 X22 + 60 X23
```

```
      + 60 X24 + 60 X25 + 50 X26 + 41 X27 + 35 X28 + 36 X29 + 34 X30
      + 22 X31 + FM - FP =     900
       44)    FP <=    100
       45)    X17 - X19 =     0
       46)    X21 + X22 + X23 + X24 + X25 + 1000 Y >=    4
       47)    X21 + X22 + X23 + X24 + X25 + 1000 Y <=    1000
       48)    0.0313 X1 + 0.0412 X5 + 0.0683 X6 >=    0.1408
       49)    0.0346 X15 + 0.0309 X16 + 0.0426 X17 + 0.0502 X18 + 0.0276
 X19
      + 0.0683 X20 >=    0.2233
       50)    0.0346 X7 + 0.015 X13 + 0.0683 X26 + 0.0548 X27 + 0.0502 X28
      + 0.0426 X29 + 0.0534 X30 + 0.0426 X31 >=    0.2763
       51)    0.0459 X2 + 0.0581 X3 + 0.0459 X12 + 0.0697 X21 + 0.0697 X22
      + 0.0697 X23 + 0.0697 X24 + 0.0697 X25 >=    0.4287
 END
 INTE        X1
 INTE        X2
 INTE        X3
 INTE        X4
 INTE        X5
 INTE        X6
 INTE        X7
 INTE        X8
 INTE        X9
 INTE       X10
 INTE       X11
 INTE       X12
 INTE       X13
 INTE       X14
 INTE       X15
 INTE       X16
 INTE       X17
 INTE       X18
 INTE       X19
 INTE       X20
 INTE       X21
 INTE       X22
 INTE       X23
 INTE       X24
 INTE       X25
 INTE       X26
 INTE       X27
 INTE       X28
 INTE       X29
 INTE       X30
 INTE       X31
 INTE         Y
 ENUMERATION COMPLETE. BRANCHES=     6 PIVOTS=    195

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)     .153700000

  VARIABLE         VALUE          REDUCED COST
        X1          1.000000           .000000
        X2          1.000000           .000000
        X3          1.000000           .000000
        X4           .000000         -.030900
        X5          1.000000           .000000
```

X6	1.000000	.000000
X7	1.000000	.000000
X8	1.000000	-.031300
X9	1.000000	-.026400
X10	1.000000	-.053400
X11	1.000000	-.042600
X12	1.000000	.000000
X13	1.000000	.000000
X14	.000000	-.058100
X15	1.000000	.000000
X16	.000000	.000000
X17	1.000000	.000000
X18	1.000000	.000000
X19	1.000000	.000000
X20	1.000000	.000000
X21	1.000000	.000000
X22	1.000000	.000000
X23	1.000000	.000000
X24	1.000000	.000000
X25	.000000	.000000
X26	1.000000	.000000
X27	1.000000	.000000
X28	1.000000	.000000
X29	.000000	.000000
X30	1.000000	.000000
X31	.000000	.000000
Y	.000000	.000000
A1M	2.000000	.000000
A1P	.000000	.000000
A2M	1.000000	.000000
A2P	.000000	.000000
A3M	.000000	.000000
A3P	.000000	.000000
A4M	.000000	.000000
A4P	.000000	.000000
A5M	.000000	.000000
A5P	.000000	.000000
A6M	1.000000	.000000
A6P	.000000	.000000
A7M	1.000000	.000000
A7P	.000000	.000000
A8M	1.000000	.000000
A8P	.000000	.000000
A9M	2.000000	.000000
A9P	.000000	.000000
A10M	2.000000	.000000
A10P	.000000	.000000
A11M	.000000	.000000
A11P	.000000	.000000
A12M	1.000000	.000000
A12P	.000000	.000000
A13M	1.000000	.000000
A13P	.000000	.000000
A14M	2.000000	.000000
A14P	.000000	.000000
A15M	2.000000	.000000
A15P	.000000	.000000
A16M	2.000000	.000000
A16P	.000000	.000000
A17M	2.000000	.000000
A17P	.000000	.000000
A18M	2.000000	.000000

A18P	.000000	.000000
A19M	1.000000	.000000
A19P	.000000	.000000
A20M	1.000000	.000000
A20P	.000000	.000000
A21M	.000000	.000000
A21P	.000000	.000000
A22M	2.000000	.000000
A22P	.000000	.000000
A23M	.000000	.000000
A23P	.000000	.000000
A24M	.000000	.000000
A24P	.000000	.000000
A25M	.000000	.000000
A25P	.000000	.000000
A26M	1.000000	.000000
A26P	.000000	.000000
A27M	1.000000	.000000
A27P	.000000	.000000
A28M	2.000000	.000000
A28P	.000000	.000000
A29M	4.000000	.000000
A29P	.000000	.000000
A30M	4.000000	.000000
A30P	.000000	.000000
U2M	6.000000	.000000
U2P	.000000	.000000
U4M	43.000000	.000000
U4P	16.000000	.000000
U5M	2.000000	.000000
U5P	.000000	.000000
U6M	17.000000	.000000
U6P	.000000	.000000
U7M	49.000000	.000000
U7P	.000000	.000000
FM	100.000000	.000000
FP	.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.000000	.000000
23)	.000000	.000000

24)	.000000	.000000
25)	.000000	.000000
26)	.000000	.000000
27)	.000000	.000000
28)	.000000	.000000
29)	.000000	.000000
30)	.000000	.000000
31)	.000000	.000000
32)	.000000	.000000
33)	.000000	.000000
34)	.000000	.000000
35)	.000000	.000000
36)	.000000	.000000
37)	.000000	.000000
38)	11.000000	.000000
39)	.000000	.000000
40)	16.000000	.000000
41)	38.000000	.000000
42)	8.000000	.000000
43)	.000000	.000000
44)	100.000000	.000000
45)	.000000	.000000
46)	.000000	.000000
47)	996.000000	.000000
48)	.000000	.000000
49)	.000000	.000000
50)	.000000	.000000
51)	.000000	.000000

NO. ITERATIONS= 198
BRANCHES= 6 DETERM.= -1.000E 0

EXHIBIT 5 - EDITED OUTPUT SOOPSP MODEL RUN

```
MAX      0.0309 X4 + 0.0313 X8 + 0.0264 X9 + 0.0534 X10 + 0.0426 X11
     + 0.0581 X14
SUBJECT TO
        2)    X1 + X7 + X15 + X16 + X20 + A1M - A1P =    6
        3)    X1 + X7 + X8 + X15 + X16 + X20 + A2M - A2P =    6
        4)    X2 + X7 + X8 + X9 + X16 + X17 + X20 + A3M - A3P =    6
        5)    X2 + X7 + X8 + X12 + X14 + X17 + X20 + A4M - A4P =    6
        6)    X2 + X3 + X10 + X12 + X14 + X17 + X20 + A5M - A5P =    6
        7)    X2 + X3 + X10 + X12 + X14 + X20 + X32 + A6M - A6P =    6
        8)    X2 + X3 + X10 + X12 + X14 + X20 + X32 + A7M - A7P =    6
        9)    X2 + X3 + X10 + X12 + X14 + X20 + X32 + A7M - A8P =    6
       10)    X2 + X11 + X12 + X14 + X20 + X32 + A9M - A9P =    6
       11)    X2 + X11 + X12 + X14 + X20 + A10M - A10P =    6
       12)    X11 + X20 + X22 + X23 + X24 + X25 + A11M - A11P =    6
       13)    X4 + X20 + X21 + X22 + X23 + X24 + X25 + A12M - A12P =    6
       14)    X4 + X20 + X21 + X22 + X23 + X24 + X25 + A13M - A13P =    6
       15)    X4 + X21 + X22 + X23 + X24 + X25 + A14M - A14P =    6
       16)    X4 + X21 + X22 + X23 + X24 + X25 + A15M - A15P =    6
       17)    X4 + X21 + X22 + X23 + X24 + X25 + A16M - A16P =    6
       18)    X4 + X21 + X22 + X23 + X24 + X25 + A17M - A17P =    6
       19)    X21 + X22 + X23 + X24 + X25 + A18M - A18P =    6
       20)    X5 + X21 + X22 + X23 + X24 + X25 + A19M - A19P =    6
       21)    X5 + X21 + X22 + X23 + X24 + X25 + A20M - A20P =    6
       22)    X5 + X19 + X22 + X23 + X24 + X25 + A21M - A21P =    6
       23)    X5 + X6 + X18 + X19 + A22M - A22P =    6
       24)    X6 + X18 + X19 + X26 + X27 + X28 + X29 + A23M - A23P =    6
       25)    X6 + X18 + X26 + X27 + X28 + X29 + X30 + A24M - A24P =    6
       26)    X6 + X18 + X26 + X27 + X28 + X29 + X30 + A25M - A25P =    6
       27)    X6 + X26 + X27 + X28 + X29 + X30 + A26M - A26P =    6
       28)    X13 + X26 + X27 + X28 + X29 + X30 + X31 + A27M - A27P =    6
       29)    X13 + X26 + X27 + X30 + X31 + A28M - A28P =    6
       30)    X13 + X26 + X31 + A29M - A29P =    6
       31)    X13 + X26 + X31 + A30M - A30P =    6
       32)    A1P + A2P + A3P + A4P + A5P + A6P + A7P + A8P + A9P + A10P
      + A11P + A12P + A13P + A14P + A15P + A16P + A17P + A18P + A19P +
A20P
      + A21P + A22P + A23P + A24P + A25P + A26P + A27P + A28P + A29P +
A30P
      =    0
       33)    12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + U2M - U2P
      =    112
       34)    24 X7 + 19 X8 + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45
X14
      + 24 X32 + U4M - U4P =    162
       35)    14 X15 + 20 X16 + 11 X17 + 28 X18 + 16 X19 + 90 X20 + U5M -
U5P
      =    161
       36)    60 X21 + 60 X22 + 60 X23 + 60 X24 + 60 X25 + 50 X26 + 41 X27
      + 35 X28 + U6M - U6P =    383
       37)    36 X29 + 34 X30 + 22 X31 + U7M - U7P =    83
       38)    U2P <=   11
       39)    U4P <=   16
       40)    U5P <=   16
       41)    U6P <=   43
       42)    U7P <=   8
       43)    12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + 24 X7 + 19 X8
      + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45 X14 + 14 X15 + 20 X16
      + 11 X17 + 28 X18 + 16 X19 + 90 X20 + 60 X21 + 60 X22 + 60 X23
      + 60 X24 + 60 X25 + 50 X26 + 41 X27 + 35 X28 + 36 X29 + 34 X30
```

```
  + 22 X31 + 24 X32 + FM - FP =     900
   44)    FP <=    100
   45)    X17 - X19 =      0
   46)    X21 + X22 + X23 + X24 + X25 + 1000 Y >=    4
   47)    X21 + X22 + X23 + X24 + X25 + 1000 Y <=    1000
   48)    0.0313 X1 + 0.0412 X5 + 0.0683 X6 >=    0.1408
   49)    0.0346 X15 + 0.0309 X16 + 0.0426 X17 + 0.0502 X18 + 0.0276
X19
  + 0.0683 X20 >=    0.2233
   50)    0.0346 X7 + 0.015 X13 + 0.0683 X26 + 0.0548 X27 + 0.0502 X28
  + 0.0426 X29 + 0.0534 X30 + 0.0426 X31 + 0.0276 X32 >=    0.3465
   51)    0.0459 X2 + 0.0581 X3 + 0.0459 X12 + 0.0697 X21 + 0.0697 X22
  + 0.0697 X23 + 0.0697 X24 + 0.0697 X25 >=    0.4984
   52)  - X1 - X5 - X6 + N1 =     0
   53)  - X15 - X16 - X17 - X18 - X19 - X20 + N2 =     0
   54)  - X7 - X13 - X26 - X27 - X28 - X29 - X30 - X31 - X32 + N3 =
0
   55)  - X2 - X3 - X12 - X21 - X22 - X23 - X24 - X25 + N4 =     0
   56)  - X4 - X8 - X9 - X10 - X11 - X14 + N5 =     0
   57)  - X1 - X2 - X3 - X4 - X5 - X6 - X7 - X8 - X9 - X10 - X11 - X12
  - X13 - X14 - X15 - X16 - X17 - X18 - X19 - X20 - X21 - X22 - X23
  - X24 - X25 - X26 - X27 - X28 - X29 - X30 - X31 - X32 + T =     0
 END
 INTE        X1
 INTE        X2
 INTE        X3
 INTE        X4
 INTE        X5
 INTE        X6
 INTE        X7
 INTE        X8
 INTE        X9
 INTE       X10
 INTE       X11
 INTE       X12
 INTE       X13
 INTE       X14
 INTE       X15
 INTE       X16
 INTE       X17
 INTE       X18
 INTE       X19
 INTE       X20
 INTE       X21
 INTE       X22
 INTE       X23
 INTE       X24
 INTE       X25
 INTE       X26
 INTE       X27
 INTE       X28
 INTE       X29
 INTE       X30
 INTE       X31
 INTE       X32
 INTE         Y

ENUMERATION COMPLETE. BRANCHES=      1 PIVOTS=      74

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...
```

OBJECTIVE FUNCTION VALUE

1) .153700000

VARIABLE	VALUE	REDUCED COST
X1	1.000000	.000000
X2	1.000000	.000000
X3	1.000000	.000000
X4	.000000	-.030900
X5	1.000000	.000000
X6	1.000000	.000000
X7	1.000000	.000000
X8	1.000000	-.031300
X9	1.000000	-.026400
X10	1.000000	-.053400
X11	1.000000	-.042600
X12	1.000000	.000000
X13	1.000000	.000000
X14	.000000	-.058100
X15	1.000000	.000000
X16	.000000	.000000
X17	1.000000	.000000
X18	1.000000	.000000
X19	1.000000	.000000
X20	1.000000	.000000
X21	1.000000	.000000
X22	1.000000	.000000
X23	1.000000	.000000
X24	1.000000	.000000
X25	1.000000	.000000
X26	1.000000	.000000
X27	1.000000	.000000
X28	1.000000	.000000
X29	.000000	.000000
X30	1.000000	.000000
X31	1.000000	.000000
X32	1.000000	.000000
Y	.000000	.000000
A1M	2.000000	.000000
A1P	.000000	.000000
A2M	1.000000	.000000
A2P	.000000	.000000
A3M	.000000	.000000
A3P	.000000	.000000
A4M	.000000	.000000
A4P	.000000	.000000
A5M	.000000	.000000
A5P	.000000	.000000
A6M	.000000	.000000
A6P	.000000	.000000
A7M	.000000	.000000
A7P	.000000	.000000
A8P	.000000	.000000
A9M	1.000000	.000000
A9P	.000000	.000000
A10M	2.000000	.000000
A10P	.000000	.000000
A11M	.000000	.000000
A11P	.000000	.000000
A12M	.000000	.000000
A12P	.000000	.000000
A13M	.000000	.000000

A13P	.000000	.000000
A14M	1.000000	.000000
A14P	.000000	.000000
A15M	1.000000	.000000
A15P	.000000	.000000
A16M	1.000000	.000000
A16P	.000000	.000000
A17M	1.000000	.000000
A17P	.000000	.000000
A18M	1.000000	.000000
A18P	.000000	.000000
A19M	.000000	.000000
A19P	.000000	.000000
A20M	.000000	.000000
A20P	.000000	.000000
A21M	.000000	.000000
A21P	.000000	.000000
A22M	2.000000	.000000
A22P	.000000	.000000
A23M	.000000	.000000
A23P	.000000	.000000
A24M	.000000	.000000
A24P	.000000	.000000
A25M	.000000	.000000
A25P	.000000	.000000
A26M	1.000000	.000000
A26P	.000000	.000000
A27M	.000000	.000000
A27P	.000000	.000000
A28M	1.000000	.000000
A28P	.000000	.000000
A29M	3.000000	.000000
A29P	.000000	.000000
A30M	3.000000	.000000
A30P	.000000	.000000
U2M	17.000000	.000000
U2P	11.000000	.000000
U4M	19.000000	.000000
U4P	16.000000	.000000
U5M	2.000000	.000000
U5P	.000000	.000000
U6M	.000000	.000000
U6P	43.000000	.000000
U7M	27.000000	.000000
U7P	.000000	.000000
FM	.000000	.000000
FP	6.000000	.000000
N1	3.000000	.000000
N2	5.000000	.000000
N3	8.000000	.000000
N4	8.000000	.000000
N5	4.000000	.000000
T	28.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000

8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.000000	.000000
23)	.000000	.000000
24)	.000000	.000000
25)	.000000	.000000
26)	.000000	.000000
27)	.000000	.000000
28)	.000000	.000000
29)	.000000	.000000
30)	.000000	.000000
31)	.000000	.000000
32)	.000000	.000000
33)	.000000	.000000
34)	.000000	.000000
35)	.000000	.000000
36)	.000000	.000000
37)	.000000	.000000
38)	.000000	.000000
39)	.000000	.000000
40)	16.000000	.000000
41)	.000000	.000000
42)	8.000000	.000000
43)	.000000	.000000
44)	94.000000	.000000
45)	.000000	.000000
46)	1.000000	.000000
47)	995.000000	.000000
48)	.000000	.000000
49)	.000000	.000000
50)	.000000	.000000
51)	.000000	.000000
52)	.000000	.000000
53)	.000000	.000000
54)	.000000	.000000
55)	.000000	.000000
56)	.000000	.000000
57)	.000000	.000000

NO. ITERATIONS= 94
BRANCHES= 1 DETERM.= 1.000E 0

EXHIBIT 6 - EDITED OUTPUT SOAP MODEL RUN

```
MAX      0.0309 X4 + 0.0313 X8 + 0.0264 X9 + 0.0534 X10 + 0.0426 X11
   + 0.0581 X14
SUBJECT TO
        2)   X1 + X7 + X15 + X16 + X20 + A1M - A1P =    6
        3)   X1 + X7 + X8 + X15 + X16 + X20 + A2M - A2P =    6
        4)   X2 + X7 + X8 + X9 + X16 + X17 + X20 + A3M - A3P =    6
        5)   X2 + X7 + X8 + X12 + X14 + X17 + X20 + A4M - A4P =    6
        6)   X2 + X3 + X10 + X12 + X14 + X17 + X20 + A5M - A5P =    6
        7)   X2 + X3 + X10 + X12 + X14 + X20 + X32 + A6M - A6P =    6
        8)   X2 + X3 + X10 + X12 + X14 + X20 + X32 + A7M - A7P =    6
        9)   X2 + X3 + X10 + X12 + X14 + X20 + X32 + A7M - A8P =    6
       10)   X2 + X11 + X12 + X14 + X20 + X32 + A9M - A9P =    6
       11)   X2 + X11 + X12 + X14 + X20 + A10M - A10P =    6
       12)   X11 + X20 + X22 + X23 + X24 + X25 + A11M - A11P =    6
       13)   X4 + X20 + X21 + X22 + X23 + X24 + X25 + A12M - A12P =    6
       14)   X4 + X20 + X21 + X22 + X23 + X24 + X25 + A13M - A13P =    6
       15)   X4 + X21 + X22 + X23 + X24 + X25 + A14M - A14P =    6
       16)   X4 + X21 + X22 + X23 + X24 + X25 + A15M - A15P =    6
       17)   X4 + X21 + X22 + X23 + X24 + X25 + A16M - A16P =    6
       18)   X4 + X21 + X22 + X23 + X24 + X25 + A17M - A17P =    6
       19)   X21 + X22 + X23 + X24 + X25 + A18M - A18P =    6
       20)   X5 + X21 + X22 + X23 + X24 + X25 + A19M - A19P =    6
       21)   X5 + X21 + X22 + X23 + X24 + X25 + A20M - A20P =    6
       22)   X5 + X19 + X22 + X23 + X24 + X25 + A21M - A21P =    6
       23)   X5 + X6 + X18 + X19 + A22M - A22P =    6
       24)   X6 + X18 + X19 + X26 + X27 + X28 + X29 + A23M - A23P =    6
       25)   X6 + X18 + X26 + X27 + X28 + X29 + X30 + X33 + A24M - A24P
     =    6
       26)   X6 + X18 + X26 + X27 + X28 + X29 + X30 + X33 + A25M - A25P
     =    6
       27)   X6 + X26 + X27 + X28 + X29 + X30 + X33 + A26M - A26P =    6
       28)   X13 + X26 + X27 + X28 + X29 + X30 + X31 + A27M - A27P =    6
       29)   X13 + X26 + X27 + X30 + X31 + A28M - A28P =    6
       30)   X13 + X26 + X31 + A29M - A29P =    6
       31)   X13 + X26 + X31 + A30M - A30P =    6
       32)   A1P + A2P + A3P + A4P + A5P + A6P + A7P + A8P + A9P + A10P
     + A11P + A12P + A13P + A14P + A15P + A16P + A17P + A18P + A19P +
A20P
     + A21P + A22P + A23P + A24P + A25P + A26P + A27P + A28P + A29P +
A30P
     =    0
       33)   12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + U2M - U2P
     =    112
       34)   24 X7 + 19 X8 + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45
X14
     + 24 X32 + U4M - U4P =    162
       35)   14 X15 + 20 X16 + 11 X17 + 28 X18 + 16 X19 + 90 X20 + U5M -
U5P
     =    161
       36)   60 X21 + 60 X22 + 60 X23 + 60 X24 + 60 X25 + 50 X26 + 41 X27
     + 35 X28 + U6M - U6P =    383
       37)   36 X29 + 34 X30 + 22 X31 + 12 X33 + U7M - U7P =    83
       38)   U2P <=   11
       39)   U4P <=   16
       40)   U5P <=   16
       41)   U6P <=   43
       42)   U7P <=   8
       43)   12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + 24 X7 + 19 X8
     + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45 X14 + 14 X15 + 20 X16
```

```
+ 11 X17 + 28 X18 + 16 X19 + 90 X20 + 60 X21 + 60 X22 + 60 X23
+ 60 X24 + 60 X25 + 50 X26 + 41 X27 + 35 X28 + 36 X29 + 34 X30
+ 22 X31 + 24 X32 + 12 X33 + FM - FP =     900
 44)    FP <=    100
 45)    X17 - X19 =     0
 46)    X21 + X22 + X23 + X24 + X25 + 1000 Y >=    4
 47)    X21 + X22 + X23 + X24 + X25 + 1000 Y <=    1000
 48)    0.0313 X1 + 0.0412 X5 + 0.0683 X6 >=    0.1408
 49)    0.0346 X15 + 0.0309 X16 + 0.0426 X17 + 0.0502 X18 + 0.0276
X19
+ 0.0683 X20 >=    0.2233
 50)    0.0346 X7 + 0.015 X13 + 0.0683 X26 + 0.0548 X27 + 0.0502 X28
+ 0.0426 X29 + 0.0534 X30 + 0.0426 X31 + 0.0276 X32 + 0.0697 X33
>=    0.366
 51)    0.0459 X2 + 0.0581 X3 + 0.0459 X12 + 0.0697 X21 + 0.0697 X22
+ 0.0697 X23 + 0.0697 X24 + 0.0697 X25 >=    0.4984
END
ENUMERATION COMPLETE. BRANCHES=     3 PIVOTS=    111

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     .153700000
```

VARIABLE	VALUE	REDUCED COST
X1	1.000000	.000000
X2	1.000000	.000000
X3	1.000000	.000000
X4	.000000	-.030900
X5	1.000000	.000000
X6	1.000000	.000000
X7	1.000000	.000000
X8	1.000000	-.031300
X9	1.000000	-.026400
X10	1.000000	-.053400
X11	1.000000	-.042600
X12	1.000000	.000000
X13	1.000000	.000000
X14	.000000	-.058100
X15	1.000000	.000000
X16	.000000	.000000
X17	1.000000	.000000
X18	1.000000	.000000
X19	1.000000	.000000
X20	1.000000	.000000
X21	1.000000	.000000
X22	1.000000	.000000
X23	1.000000	.000000
X24	1.000000	.000000
X25	1.000000	.000000
X26	1.000000	.000000
X27	1.000000	.000000
X28	.000000	.000000
X29	.000000	.000000
X30	1.000000	.000000
X31	1.000000	.000000
X32	1.000000	.000000
X33	1.000000	.000000
Y	.000000	.000000
A1M	2.000000	.000000

A1P	.000000	.000000
A2M	1.000000	.000000
A2P	.000000	.000000
A3M	.000000	.000000
A3P	.000000	.000000
A4M	.000000	.000000
A4P	.000000	.000000
A5M	.000000	.000000
A5P	.000000	.000000
A6M	.000000	.000000
A6P	.000000	.000000
A7M	.000000	.000000
A7P	.000000	.000000
A8P	.000000	.000000
A9M	1.000000	.000000
A9P	.000000	.000000
A10M	2.000000	.000000
A10P	.000000	.000000
A11M	.000000	.000000
A11P	.000000	.000000
A12M	.000000	.000000
A12P	.000000	.000000
A13M	.000000	.000000
A13P	.000000	.000000
A14M	1.000000	.000000
A14P	.000000	.000000
A15M	1.000000	.000000
A15P	.000000	.000000
A16M	1.000000	.000000
A16P	.000000	.000000
A17M	1.000000	.000000
A17P	.000000	.000000
A18M	1.000000	.000000
A18P	.000000	.000000
A19M	.000000	.000000
A19P	.000000	.000000
A20M	.000000	.000000
A20P	.000000	.000000
A21M	.000000	.000000
A21P	.000000	.000000
A22M	2.000000	.000000
A22P	.000000	.000000
A23M	1.000000	.000000
A23P	.000000	.000000
A24M	.000000	.000000
A24P	.000000	.000000
A25M	.000000	.000000
A25P	.000000	.000000
A26M	1.000000	.000000
A26P	.000000	.000000
A27M	1.000000	.000000
A27P	.000000	.000000
A28M	1.000000	.000000
A28P	.000000	.000000
A29M	3.000000	.000000
A29P	.000000	.000000
A30M	3.000000	.000000
A30P	.000000	.000000
U2M	6.000000	.000000
U2P	.000000	.000000
U4M	19.000000	.000000
U4P	16.000000	.000000

U5M	2.000000	.000000
U5P	.000000	.000000
U6M	.000000	.000000
U6P	8.000000	.000000
U7M	15.000000	.000000
U7P	.000000	.000000
FM	17.000000	.000000
FP	.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.000000	.000000
23)	.000000	.000000
24)	.000000	.000000
25)	.000000	.000000
26)	.000000	.000000
27)	.000000	.000000
28)	.000000	.000000
29)	.000000	.000000
30)	.000000	.000000
31)	.000000	.000000
32)	.000000	.000000
33)	.000000	.000000
34)	.000000	.000000
35)	.000000	.000000
36)	.000000	.000000
37)	.000000	.000000
38)	11.000000	.000000
39)	.000000	.000000
40)	16.000000	.000000
41)	35.000000	.000000
42)	8.000000	.000000
43)	.000000	.000000
44)	100.000000	.000000
45)	.000000	.000000
46)	1.000000	.000000
47)	995.000000	.000000
48)	.000000	.000000
49)	.000000	.000000
50)	.000000	.000000
51)	.000000	.000000

```
NO. ITERATIONS=      120
BRANCHES=    3 DETERM.= -1.000E    0
```

EXHIBIT 7 - EDITED OUTPUT OVERTASKING MODEL RUN

```
MAX      0.0309 X4 + 0.0313 X8 + 0.0264 X9 + 0.0534 X10 + 0.0426 X11
     + 0.0581 X14
SUBJECT TO
        2)   X1 + X7 + X15 + X16 + X20 + A1M - A1P =    6
        3)   X1 + X7 + X8 + X15 + X16 + X20 + A2M - A2P =    6
        4)   X2 + X7 + X8 + X9 + X16 + X17 + X20 + A3M - A3P =    6
        5)   X2 + X7 + X8 + X12 + X14 + X17 + X20 + A4M - A4P =    6
        6)   X2 + X3 + X10 + X12 + X14 + X17 + X20 + A5M - A5P =    6
        7)   X2 + X3 + X10 + X12 + X14 + X20 + X32 + A6M - A6P =    6
        8)   X2 + X3 + X10 + X12 + X14 + X20 + X32 + A7M - A7P =    6
        9)   X2 + X3 + X10 + X12 + X14 + X20 + X32 + A7M - A8P =    6
       10)   X2 + X11 + X12 + X14 + X20 + X32 + A9M - A9P =    6
       11)   X2 + X11 + X12 + X14 + X20 + A10M - A10P =    6
       12)   X11 + X20 + X22 + X23 + X24 + X25 + A11M - A11P =    6
       13)   X4 + X20 + X21 + X22 + X23 + X24 + X25 + A12M - A12P =    6
       14)   X4 + X20 + X21 + X22 + X23 + X24 + X25 + A13M - A13P =    6
       15)   X4 + X21 + X22 + X23 + X24 + X25 + A14M - A14P =    6
       16)   X4 + X21 + X22 + X23 + X24 + X25 + A15M - A15P =    6
       17)   X4 + X21 + X22 + X23 + X24 + X25 + A16M - A16P =    6
       18)   X4 + X21 + X22 + X23 + X24 + X25 + A17M - A17P =    6
       19)   X21 + X22 + X23 + X24 + X25 + A18M - A18P =    6
       20)   X5 + X21 + X22 + X23 + X24 + X25 + A19M - A19P =    6
       21)   X5 + X21 + X22 + X23 + X24 + X25 + A20M - A20P =    6
       22)   X5 + X19 + X22 + X23 + X24 + X25 + A21M - A21P =    6
       23)   X5 + X6 + X18 + X19 + A22M - A22P =    6
       24)   X6 + X18 + X19 + X26 + X27 + X28 + X29 + A23M - A23P =    6
       25)   X6 + X18 + X26 + X27 + X28 + X29 + X30 + X33 + A24M - A24P
     =    6
       26)   X6 + X18 + X26 + X27 + X28 + X29 + X30 + X33 + A25M - A25P
     =    6
       27)   X6 + X26 + X27 + X28 + X29 + X30 + X33 + A26M - A26P =    6
       28)   X13 + X26 + X27 + X28 + X29 + X30 + X31 + A27M - A27P =    6
       29)   X13 + X26 + X27 + X30 + X31 + A28M - A28P =    6
       30)   X13 + X26 + X31 + A29M - A29P =    6
       31)   X13 + X26 + X31 + A30M - A30P =    6
       32)   A1P + A2P + A3P + A4P + A5P + A6P + A7P + A8P + A9P + A10P
     + A11P + A12P + A13P + A14P + A15P + A16P + A17P + A18P + A19P +
A20P
     + A21P + A22P + A23P + A24P + A25P + A26P + A27P + A28P + A29P +
A30P
     =    2
       33)   A1P <=   0
       34)   A2P <=   0
       35)   A3P <=   0
       36)   A4P <=   0
       37)   A5P <=   0
       38)   A6P <=   0
       39)   A7P <=   0
       40)   A8P <=   0
       41)   A9P <=   0
       42)   A10 <=   0
       43)   A11P <=   0
       44)   A12P <=   0
       45)   A13P <=   0
       46)   A14P <=   0
       47)   A15P <=   0
       48)   A16P <=   0
       49)   A17P <=   0
       50)   A18P <=   0
```

```
 51)    A19P <=    0
 52)    A20P <=    0
 53)    A21P <=    0
 54)    A22P <=    0
 55)    A23P <=    0
 56)    A24P <=    1
 57)    A25P <=    1
 58)    A26P <=    0
 59)    A27P <=    0
 60)    A28P <=    0
 61)    A29P <=    0
 62)    A30P <=    0
 63)    12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + U2M - U2P
 =    112
 64)    24 X7 + 19 X8 + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45
X14
 + 24 X32 + U4M - U4P =    162
 65)    14 X15 + 20 X16 + 11 X17 + 28 X18 + 16 X19 + 90 X20 + U5M -
U5P
 =    161
 66)    60 X21 + 60 X22 + 60 X23 + 60 X24 + 60 X25 + 50 X26 + 41 X27
 + 35 X28 + U6M - U6P =    383
 67)    36 X29 + 34 X30 + 22 X31 + 12 X33 + U7M - U7P =    83
 68)    U2P <=    11
 69)    U4P <=    16
 70)    U5P <=    16
 71)    U6P <=    43
 72)    U7P <=    8
 73)    12 X1 + 41 X2 + 29 X3 + 18 X4 + 10 X5 + 14 X6 + 24 X7 + 19 X8
 + 7 X9 + 20 X10 + 22 X11 + 35 X12 + 8 X13 + 45 X14 + 14 X15 + 20 X16
 + 11 X17 + 28 X18 + 16 X19 + 90 X20 + 60 X21 + 60 X22 + 60 X23
 + 60 X24 + 60 X25 + 50 X26 + 41 X27 + 35 X28 + 36 X29 + 34 X30
 + 22 X31 + 24 X32 + 12 X33 + FM - FP =    900
 74)    FP <=    100
 75)    X17 - X19 =    0
 76)    X21 + X22 + X23 + X24 + X25 + 1000 Y >=    4
 77)    X21 + X22 + X23 + X24 + X25 + 1000 Y <=    1000
 78)    0.0313 X1 + 0.0412 X5 + 0.0683 X6 >=    0.1408
 79)    0.0346 X15 + 0.0309 X16 + 0.0426 X17 + 0.0502 X18 + 0.0276
X19
 + 0.0683 X20 >=    0.2233
 80)    0.0346 X7 + 0.015 X13 + 0.0683 X26 + 0.0548 X27 + 0.0502 X28
 + 0.0426 X29 + 0.0534 X30 + 0.0426 X31 + 0.0276 X32 + 0.0697 X33
 >=    0.4162
 81)    0.0459 X2 + 0.0581 X3 + 0.0459 X12 + 0.0697 X21 + 0.0697 X22
 + 0.0697 X23 + 0.0697 X24 + 0.0697 X25 >=    0.4984
END
INTE       X1
INTE       X2
INTE       X3
INTE       X4
INTE       X5
INTE       X6
INTE       X7
INTE       X8
INTE       X9
INTE      X10
INTE      X11
INTE      X12
INTE      X13
INTE      X14
INTE      X15
```

```
INTE      X16
INTE      X17
INTE      X18
INTE      X19
INTE      X20
INTE      X21
INTE      X22
INTE      X23
INTE      X24
INTE      X25
INTE      X26
INTE      X27
INTE      X28
INTE      X29
INTE      X30
INTE      X31
INTE      X32
INTE      X33
INTE        Y

ENUMERATION COMPLETE. BRANCHES=     1 PIVOTS=    124

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     .153700000

 VARIABLE        VALUE          REDUCED COST
       X1        1.000000           .000000
       X2        1.000000           .000000
       X3        1.000000           .000000
       X4         .000000          -.030900
       X5        1.000000           .000000
       X6        1.000000           .000000
       X7        1.000000           .000000
       X8        1.000000          -.031300
       X9        1.000000          -.026400
      X10        1.000000          -.053400
      X11        1.000000          -.042600
      X12        1.000000           .000000
      X13        1.000000           .000000
      X14         .000000          -.058100
      X15        1.000000           .000000
      X16         .000000           .000000
      X17        1.000000           .000000
      X18        1.000000           .000000
      X19        1.000000           .000000
      X20        1.000000           .000000
      X21        1.000000           .000000
      X22        1.000000           .000000
      X23        1.000000           .000000
      X24        1.000000           .000000
      X25        1.000000           .000000
      X26        1.000000           .000000
      X27        1.000000           .000000
      X28        1.000000           .000000
      X29         .000000           .000000
      X30        1.000000           .000000
      X31        1.000000           .000000
      X32        1.000000           .000000
```

X33	1.000000	.000000
Y	.000000	.000000
A1M	2.000000	.000000
A1P	.000000	.000000
A2M	1.000000	.000000
A2P	.000000	.000000
A3M	.000000	.000000
A3P	.000000	.000000
A4M	.000000	.000000
A4P	.000000	.000000
A5M	.000000	.000000
A5P	.000000	.000000
A6M	.000000	.000000
A6P	.000000	.000000
A7M	.000000	.000000
A7P	.000000	.000000
A8P	.000000	.000000
A9M	1.000000	.000000
A9P	.000000	.000000
A10M	2.000000	.000000
A10P	.000000	.000000
A11M	.000000	.000000
A11P	.000000	.000000
A12M	.000000	.000000
A12P	.000000	.000000
A13M	.000000	.000000
A13P	.000000	.000000
A14M	1.000000	.000000
A14P	.000000	.000000
A15M	1.000000	.000000
A15P	.000000	.000000
A16M	1.000000	.000000
A16P	.000000	.000000
A17M	1.000000	.000000
A17P	.000000	.000000
A18M	1.000000	.000000
A18P	.000000	.000000
A19M	.000000	.000000
A19P	.000000	.000000
A20M	.000000	.000000
A20P	.000000	.000000
A21M	.000000	.000000
A21P	.000000	.000000
A22M	2.000000	.000000
A22P	.000000	.000000
A23M	.000000	.000000
A23P	.000000	.000000
A24M	.000000	.000000
A24P	1.000000	.000000
A25M	.000000	.000000
A25P	1.000000	.000000
A26M	.000000	.000000
A26P	.000000	.000000
A27M	.000000	.000000
A27P	.000000	.000000
A28M	1.000000	.000000
A28P	.000000	.000000
A29M	3.000000	.000000
A29P	.000000	.000000
A30M	3.000000	.000000
A30P	.000000	.000000
A10	.000000	.000000

U2M	17.000000	.000000
U2P	11.000000	.000000
U4M	3.000000	.000000
U4P	.000000	.000000
U5M	2.000000	.000000
U5P	.000000	.000000
U6M	.000000	.000000
U6P	43.000000	.000000
U7M	23.000000	.000000
U7P	8.000000	.000000
FM	82.000000	.000000
FP	100.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.000000	.000000
23)	.000000	.000000
24)	.000000	.000000
25)	.000000	.000000
26)	.000000	.000000
27)	.000000	.000000
28)	.000000	.000000
29)	.000000	.000000
30)	.000000	.000000
31)	.000000	.000000
32)	.000000	.000000
33)	.000000	.000000
34)	.000000	.000000
35)	.000000	.000000
36)	.000000	.000000
37)	.000000	.000000
38)	.000000	.000000
39)	.000000	.000000
40)	.000000	.000000
41)	.000000	.000000
42)	.000000	.000000
43)	.000000	.000000
44)	.000000	.000000
45)	.000000	.000000
46)	.000000	.000000
47)	.000000	.000000
48)	.000000	.000000

49)	.000000	.000000
50)	.000000	.000000
51)	.000000	.000000
52)	.000000	.000000
53)	.000000	.000000
54)	.000000	.000000
55)	.000000	.000000
56)	.000000	.000000
57)	.000000	.000000
58)	.000000	.000000
59)	.000000	.000000
60)	.000000	.000000
61)	.000000	.000000
62)	.000000	.000000
63)	.000000	.000000
64)	.000000	.000000
65)	.000000	.000000
66)	.000000	.000000
67)	.000000	.000000
68)	.000000	.000000
69)	16.000000	.000000
70)	16.000000	.000000
71)	.000000	.000000
72)	.000000	.000000
73)	.000000	.000000
74)	.000000	.000000
75)	.000000	.000000
76)	1.000000	.000000
77)	995.000000	.000000
78)	.000000	.000000
79)	.000000	.000000
80)	.000000	.000000
81)	.000000	.000000

NO. ITERATIONS= 140
BRANCHES= 1 DETERM.= -1.000E 0