A MONTE CARLO INVESTIGATION OF SEVERAL PROCEDURES FOR TESTING THE SIGNIFICANCE OF INDIVIDUAL DISCRIMINANT FUNCTIONS FOLLOWING MULTIVARIATE ANALYSIS OF VARIANCE

A Thesis submitted in conformity with the requirements
for the Degree of Doctor of Philosophy
at the University of Manitoba

Colin Douglas Meredith

Department of Psychology
University of Manitoba
July 1978

A MONTE CARLO INVESTIGATION OF SEVERAL PROCEDURES FOR TESTING THE SIGNIFICANCE OF INDIVIDUAL DISCRIMINANT FUNCTIONS FOLLOWING MULTIVARIATE ANALYSIS OF VARIANCE

OF

COLIN DOUGLAS MEREDITH

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

©1978

Permission has been granted to the LIBRARY OF THE UNIVERSITY OF MANITOBA to lend or sell copies of this dissertation, to the NATIONAL LIBRARY OF CANADA to microfilm this dissertation and to lend or sell copies of the film, and UNIVERSITY MICROFILMS to publish an abstract of this dissertation.

The author reserves other publication rights, and neither the dissertation nor extensive extracts from it may be printed or otherwise reproduced without the author's written permission.

Abstract

The performance of five tests purported to evaluate the significance of individual discriminant functions following a multivariate analysis of variance was investigated using Monte Carlo simulation methods. The five procedures examined were Bartlett's partitioned-U, Marriott's chi-square procedure, and three procedures which utilized Roy's largest root criterion in some form. The simulation procedure did not involve any violation of distributional assumptions. In terms of type I error rates, the partitioned-U procedure was found to falsely reject the null hypothesis at levels which most closely corresponded to the theoretical rate, provided the sample size was large relative to the number of dependent variables. This procedure was also found to have, in general, greater power against a variety of alternative hypotheses than the other procedures considered. In addition, certain published criticisms of the partitioned-U procedure were examined and not supported by the present results. In view of these findings, and due to its relative ease of application, the partitioned-U was recommended as the method of choice.

ACKNOWLEDGEMENTS

Thanks are due to Dr. Kathleen Subrahmaniam, Dr. Larry Breen, Dr. Warren Eaton and Dr. Chester Olson for serving on the committee which supervised this research, and for the assistance which they generously provided. A special expression of gratitude and respect is tendered to my advisor, Dr. Roy Gabriel, who has provided continual support and encouragement throughout my doctoral program.

Finally, I would like to thank Mrs. W. Neil, whose care and diligence in typing this paper, as well as my candidacy examination are much appreciated.

TABLE OF CONTENTS

ABSTRACT	ii
ACKNOWLEDGEMENTS	iii
LIST OF TABLES	v
LIST OF FIGURES	vi
TEXT	1
INTRODUCTION	1
METHOD	11
RESULTS	16
STUDY 1	19
STUDY 2	29
DISCUSSION	39
LITERATURE REVIEW	43
THE MULTIVARIATE LINEAR MODEL	45
MANOVA TEST STATISTICS	47
LINEAR DISCRIMINANT ANALYSIS	48
THE TEST CRITERIA	52
REFERENCES	65
APPENDIX 1. THEORETICAL CRITICAL VALUES	70
APPENDIX 2. NONCENTRALITY ROOTS	77
APPENDIX 3. OVERALL MANOVA TEST RESULTS	79
APPENDIX 4. COMPUTATIONAL DETAILS	81
APPENDIX 5. AN ILLUSTRATIVE EXAMPLE	82
APPENDIX 6. RAW DATA	90
APPENDIX 7. ANALYSIS OF VARIANCE SUMMARY TABLE	118

LIST OF TABLES

Table		Page
1	Levels of the independent variables studied by Olson(1973), Schatzoff(1966b) and Meredith (1978).	14
2	Four test criteria for MANOVA.	48
3	Degree of freedom parameters for referencing the greatest characteristic root distribution.	54
4	Empirical estimates of type I error rate for Roy's largest root test, the partitioned-U test and Wilks' lambda overall MANOVA test (Harris,1976).	58

LIST OF FIGURES

Figure		Page
1	Results for tests of the first null root for combinations of p,q and n.	20-2
2	Results for tests of the first null root as a function of s.	24
3	Results for tests of the second null root for combinations of p,q and n.	26-8
4	Results for the concentrated noncentrality situation as a function of the degree of noncentrality.	30
5	Results for various situations in which more than one population root was nonzero as a function of degree and pattern of noncentrality.	33-7
6	A flow chart showing the steps in Harris'(1976) mixed procedure for testing roots 1 through s.	62

This paper is concerned with the problem of determining, for a one-way, fixed effects multivariate analysis of variance (MANOVA) design, how many significant discriminant functions are present in a data set. The most frequently recommended procedure (e.g., Tatsuoka, 1971; Overall and Klett, 1972; Timm, 1975) for determining how many of a possible s discriminant functions are significant (where s is equal to the lesser of p, the number of dependent variables, and df_h, the hypothesis degrees of freedom) is the partitioned-U procedure, first proposed by Bartlett (1941). Despite its seeming popularity and acceptance, the partitioned-U procedure is not without its critics. Notably, Harris (1975, 1976) contends that the procedure is both logically and statistically invalid. Specifically, Harris (1976) claims that the procedure neither tests the first root, nor takes into account the possibility that the size order of the sample roots may not correspond with the order of the population roots. The purpose of the research reported here is to compare a number of procedures purported to test the hypotheses alluded to above, in terms of their type I error rates and power to detect true population differences.

The past decade has seen a substantial growth in the application of multivariate analysis procedures by social scientists. While the work of researchers such as Olson (1974, 1976) has provided guidelines for choosing a 'best test' from among a number of alternative overall MANOVA test statistics, such guidelines are frequently not available to the analyst uncertain about which procedure to apply after the overall MANOVA null hypothesis has been rejected. A number of options are available

including univariate analysis of variance on each of the p dependent variables (Cramer and Bock, 1966), calculation of step-down or residual F-statistics (Roy and Bargmann, 1958), and discriminant function analysis (for a review of this procedure, see Huberty, 1975). It is the application of this third procedure, discriminant function analysis, as a follow-up procedure to MANOVA, which is investigated by the research reported here. A recent general review of these procedures is provided by Miller (1977).

The model under consideration is the one-way fixed effects design in which p (>1) normally distributed response measures are available for n experimental units in each of q treatment groups. The overall MANOVA null hypothesis for such a design can be stated as:

$$H_{ov} : \mu_1 = \mu_2 = \cdots = \mu_q ,$$

where the μ_q are the p-variate mean parameter vectors associated with each of the q levels of the treatment. Importantly, all of the frequently-applied MANOVA test criteria are functions of the latent roots, or eigenvalues of the matrix product $E^{-1}H$ where H and E are pxp sums of squares and cross products (SSCP) matrices defined in the one-way case as

$$H = \sum_{i=1}^{q} n_i \, (\overline{Y}_i - \overline{Y}) \, (\overline{Y}_i - \overline{Y})' \tag{1.1}$$

$$E = \sum_{i=1}^{q} \sum_{j=1}^{n} (Y_{ij} - \overline{Y}_i)(Y_{ij} - \overline{Y})' \tag{1.2}$$

where Y_{ij} is the jth of n_i observation vectors in group i, $\overline{Y}_i$ is the mean vector for the ith group and $\overline{Y}$ is the grand mean vector. Examples of

overall MANOVA test criteria which are functions of the eigenvalues of $E^{-1}H$ are Roy's largest root criterion (Roy, 1945), Wilks' likelihood ratio (Wilks, 1932), the Lawley-Hotelling trace criterion (Lawley, 1938; Hotelling, 1951) and Pillai's trace criterion (Pillai, 1955). The distribution theory of the eigenvalues used in these tests is reviewed and discussed in some detail by Pillai (1976, 1977). For a comparison of these procedures in terms of their robustness to violation of certain of their distributional assumptions, the reader is referred to Olson (1974).

As is clear from the statement of the overall MANOVA null hypothesis above, a significant overall statistic implies only that, in terms of the parameter mean vectors, some group (or groups) differ from others on some variate (or variates). No indication is given as to where exactly the population differences are to be found. Discriminant analysis addresses this question by determining the number of distinct ways in which the q groups differ significantly on the p variates, sometimes referred to as dimensions of difference (Huberty, 1975). This is accomplished by forming linear composites of the original variables, called discriminant functions, which weight the variables in a manner which maximizes the ratio of between-group to within-group variation while taking into account the intercorrelations among the dependent variables. As many as s functions are possible, each of which is associated with one of the latent roots of $E^{-1}H$. The maximization procedure employed results in the extraction of the sample roots (denoted ℓ_1) in decreasing order of magnitude, with successively derived functions uncorrelated (in the total sample) with those already derived. The problem then becomes one of determining how many of these sample roots are statistically significant, meaning that they represent reliable dimensions of difference in the population. Huberty (1975), in his comprehensive review of discriminant analysis, comments that "in the literature, the

process of testing the significance of a function has been lacking in clarity" (p.548). To date, five procedures have been proposed in response to this situation. They are:

1) The partitioned-U procedure due to Bartlett (1941);

2) A variation on Roy's largest root test proposed by Finn (1974);

3) Another variation on Roy's test proposed by Harris (1976);

4) A variation of Bartlett's procedure proposed by Marriott (1952); and

5) A combination of Bartlett's and Roy's tests proposed by Harris (1976).

Before discussing the details of these procedures, it will be convenient to introduce the following notation. Let the pxp matrix M, the parametric counterpart of H, be defined as

$$M = \sum_{i=1}^{q} n_i (\mu_i - \mu)(\mu_i - \mu)' \tag{1.3}$$

where μ_i is the population mean vector for the ith of q treatments, μ is the grand mean vector in the population, and n_i is the number of observations in the ith group. Further, let Σ denote the pxp population SSCP matrix common to each of the q levels of the treatment populations, estimated by E, and let Ξ denote the pxp population variance-covariance matrix. It is the nonzero latent roots of $\Sigma^{-1}M$, denoted $\lambda_1, \lambda_2, \ldots, \lambda_r$, (where r is the rank of $\Sigma^{-1}M$) which are estimated by the roots of $E^{-1}H$ ($\ell_1, \ell_2, \ldots, \ell_r$), and which correspond to the true population discriminant functions. The following sections describe in some detail the five procedures mentioned above, and present statements of the null hypotheses tested by each in terms of the population roots.

The Partitioned-U Procedure. One approach to testing the significance of a given value of U (the Wilks' lambda (Wilks, 1932) overall MANOVA statistic), where U is defined as

$$U = \prod_{i=1}^{s} \frac{1}{1 + \ell_i} \tag{1.4}$$

involves calculating Bartlett's V statistic as

$$V = [N - 1-(p+q) / 2] \sum_{i=1}^{s} \ln (1+\ell_i) \tag{1.5}$$

where N is the total sample size, and p and q are as defined previously. V is distributed approximately as a chi-square with p(q-1) degrees of freedom when used as a test of the overall MANOVA null hypothesis. Since the successive terms $\ln(1+\ell_i)$ in expression (1.5) are statistically independent (assuming multivariate normality of the original p variables) the additive components of V are each approximately distributed as chi-square variates with p+q-2i degrees of freedom where i is the number of components removed from the expression for V.

The partitioned-U procedure per se involves the following steps (as outlined by Harris, 1976):

(1) Test the overall null hypothesis by comparing V to χ^2 with p(q-1) degrees of freedom. If H_o overall is not rejected, it is concluded that none of the discriminant functions are statistically significant, and testing stops. If, however, this null hypothesis is rejected, then at least the first discriminant function is considered significant, and the researcher proceeds to step 2.

(2) Test the significance of the residual after successively removing the chi-square components (i.e., $\ln(1+\ell_i)$) corresponding to the first

i discriminant functions. As soon as the residual becomes smaller than the critical value of the appropriate chi-square distribution, it is concluded that only these first i functions are significant (at that α level). The successive test statistics and their associated degrees of freedom (as shown in Tatsuoka, 1971, p. 165) may be summarized as follows:

Residual after Removing	Approximate χ^2 - Statistic	Degrees of Freedom
First discriminant function	$V - V_1$	$p(q-1) - (p+q-2) = (p-1)(q-2)$
First 2 discriminant function	$V - V_1 - V_2$	$(p-1)(q-2) - (p+q-4) = (p-2)(q-3)$
First 3 discriminant function	$V - V_1 - V_2 - V_3$	$(p-2)(q-3) - (p+q-6) = (p-3)(q-4)$
⋮	⋮	⋮

The null hypothesis tested by this procedure is stated succinctly by Kshirsagar (1972) as follows: for any value of i greater than zero (meaning <u>not</u> the overall test) the first i population roots are nonzero, while the remaining s-i roots are equal to zero. Hence, this procedure implicitly rejects the hypothesis that $\lambda_1 = 0$ when the overall statistic is found to be significant.

Finn's (1974) variation of Roy's criterion. As mentioned above, Roy's largest root criterion defined in terms of the roots of $E^{-1}H$ as

$$R = \frac{\ell_1}{\ell_1 + 1} \tag{1.6}$$

was developed as a test of the overall MANOVA null hypothesis. This fact notwithstanding, Finn (1974) and Harris (see below) have recommended

using this statistic to test null hypotheses of the form $H_o : \lambda_i = 0$, when the previous (larger) i-1 sample roots have been tested. Finn's (1974) procedure involves reducing two of the components of the degrees of freedom by one for each prior root tested. As a test of the overall hypothesis, R is compared to the critical value of the greatest characteristic root distribution (Pillai, 1965) with the following parameters:

$$s = \min (p, q-1) \tag{1.7}$$

$$m = (\left| p-df_h \right| - 1) / 2 \tag{1.8}$$

$$n = (df_e - p - 1) / 2 \tag{1.9}$$

where df_h and df_e are the degrees of freedom for hypothesis and error, respectively. As a test of the significance of discriminant functions other than the first (presumably proven significant by the overall test) Finn would calculate R as

$$R = \frac{\ell_i}{\ell_i + 1} \tag{1.10}$$

and evaluate this statistic against the greatest root distribution with df_h and df_e in the m and n arguments, respectively, reduced by one for each prior root tested. Testing would terminate with the finding of a nonsignificant statistic. The null hypothesis tested at any given stage is $H_o : \lambda_i = 0$, given that the previous i-1 roots have been found to be nonzero.

Harris' (1976) variation of Roy's criterion. This procedure is similar in all respects to Finn's (1974) procedure except that, rather than reducing df_e and df_h, Harris recommends reducing s by one for each prior

root tested, while leaving the m and n arguments unchanged.

Marriott's (1952) Chi-square Procedure. Marriott (1952) proposed a variation on the partitioned-U procedure which both takes into account the possibility that the ordering of the sample roots does not correspond to the ordering of the population roots, and provides a direct test of λ_1 . The procedure involves the following steps:

1) Test the hypothesis $H_o : \lambda_1 = 0$ by evaluating the significance of

$$V_1 = - [N - 1 - (p+q) / 2] \ \ln \ell_1 \tag{1.11}$$

as a χ^2 variate with degrees of freedom equal to

$$p + q - 2 + 1/2 \ [(p-1) \ (q-2)]^{2/3} \tag{1.12}$$

2) Test the hypothesis that $\lambda_{i+1} = 0$ when the previous i roots have been shown to be significant by evaluating

$$V_i = - [N - 1 - (p-i + q-i) / 2] \ \ln \ \ell_i \tag{1.13}$$

as a χ^2 variate with degrees of freedom

$$p - i + q - i - 2 + 1/2 \ [(p-i-1) \ (q-i-2)]^{2/3} \tag{1.14}$$

Step 2 is repeated with i increasing until either nonsignificance is obtained or i=s. Marriott (1952) reports some data indicating that this procedure performs well when p is small (4 or less) but may become liberal as p increases. The validity of these assertions has not, before the present study, been put to an empirical test.

Harris' (1976) Mixed Procedure. As an alternative to the use of Roy's criterion for testing roots other than the first (see above) Harris (1976) has proposed a procedure which combines the partitioned-U test and a variation on the largest root test. The steps in this 'mixed' procedure are as follows:

1) Test the overall null hypothesis using Bartlett's V with p(q-1) degrees of freedom. If this null hypothesis is rejected, conclude that there is at least one significant discriminant function, and proceed to step (2). If the observed V does not exceed the critical value, testing stops.

2) Test the significance of the largest population root by comparing R_1 to the tabled critical value of the greatest characteristic root distribution with values for s, m and n calculated in the manner described above. If this test achieves significance, conclude that the population value of λ_1 does not equal zero, and proceed to step (3). Otherwise conclude that the nature of the relationship between group membership and the dependent variables cannot be specified, and stop testing.

3) Test the hypothesis that the remaining s-i population roots are all equal to zero by comparing

$$V = [N - 1 - (p+q) / 2] \sum_{K=i+1}^{s} \ln (1+\ell_K) \qquad (1.15)$$

to χ^2_α with (p-i)(q-i-1) degrees of freedom. If this test fails to achieve significance, conclude that only the first i population roots are nonzero. If it achieves significance, conclude that not all of the last s-i population roots are zero, and proceed to step (4).

4) Test the null hypothesis that the i + 1th population root is equal to zero by comparing R_{i+1} to the greatest characteristic root distribution with s' = s-i. If nonsignificant conclude that the nature of the relationship associated with the last s-i roots cannot be specified. If significant, conclude that the i + 1th discriminant function is nonzero (in the population), and return to step (3). Steps (3) and (4) are repeated until a nonsignificant test terminates the procedure, or until i=s.

To date, little, if any, empirical investigation of these procedures has been reported in the literature. Bartlett's (1941) discussion of the partitioned-U procedure is largely analytical in content. Harris (1976) comments that his proposed variation of Roy's test may be conservative in its evaluation of roots other than the first, while the mixed procedure may prove frustrating to use due to the fact that different sampling distributions are referenced at different stages of the procedure, resulting in potentially contradictory conclusions. Marriott (1952) investigated the behaviour of his proposed procedure for selected levels of p and q, and noted some overestimation of significance levels as p increased. These comments pertain solely to the type I error rate levels of these tests. No individual or comparative data are at present available regarding the power of any of these tests to detect the existence of nonzero population roots given a set of sample data. It will be convenient at this point to introduce the MANOVA noncentrality parameter, a standardized measure of the distance between group centroids in the population. Given a

pxp parameter hypothesis SSCP matrix, M as defined above, and the population covariance matrix, Ξ , let

$$H^* = M \Xi^{-1} . \tag{1.16}$$

The noncentrality parameter is the sum of the eigenvalues, h_j (for j = 1,2,...,p) of H*, or simply tr(H*), the trace of H* (Schatzoff, 1966b; Olson, 1973).

In response to the need for both of the types of information mentioned above, the research reported here comprised two distinct phases. In study I, the procedures were compared in terms of their type I error rates for various combinations of p, q and n. In study II, varying degrees and patterns of multivariate noncentrality were incorporated into the simulation procedure, and the procedures were compared in terms of their power to detect these population discriminant functions. The simulation procedure did not incorporate violations of the distributional assumptions of the overall tests.

Method

Even in the absence of assumption violation, the researcher who wishes to investigate almost any aspect of multivariate analysis is faced with the problem of choosing from among a seemingly infinite number of combinations of independent variables. Central to this problem in the present context of a one-way MANOVA is the fact that, subject to the constraint of s always being limited to the lesser of p and df_h, any given set of population roots can result from an infinite number of combinations of p, q, n, that form the q p-variate population mean vectors, and the

common population covariance matrix. Even holding all but the mean vectors constant, there exists an infinite number of sets of mean vectors which, in combination with the other factors, will yield a given set of roots. This problem makes simulation of a particular set of roots by starting with the raw data a somewhat confusing and arduous task (although not an impossible task - see Olson, 1973). In order to circumvent this problem, the data generation procedure utilized by Schatzoff (1966b) was employed in the present study. Details of this procedure are presented below, following an enumeration and description of the independent variables investigated.

1. Number of dependent variables: p. The levels investigated were 2, 3, 6 and 10 in the null situations (study I) and 2, 3 and 6 in the noncentral cases (study II).

2. Number of groups: q. All of the simulated experiments had a one-way design with 3, 6 or 10 levels of the treatment factor.

3. Number of observations per group: n. The levels of this variable examined were 5, 10 and 50 with all groups in any simulated experiment being equal in size.

The following factors relate to the manipulation of multivariate noncentrality and were, accordingly, investigated only in study II.

4. Number of nonzero population roots: r. Subject to the limitation that r not exceed s for any combination of p and q, experiments were simulated in which r, the population dimensionality, was equal to 1, 2, 3 and 5.

5. Degree of multivariate noncentrality: tr(H*). The values of this parameter investigated were 0 (indicating the null case, as in study

I), 10, 40 and 90.

6. Pattern of noncentrality. The following patterns of the roots of H* were investigated:

a) concentrated: in which only the first root of H* is nonzero and is therefore itself equal to tr(H*).

b) equal: in which the r nonzero roots of H* are equal in size and sum to tr(H*).

c) linear: in which the size of the roots from 1 to r decreases by a constant amount in a linear fashion.

d) geometric: in which the size of the roots from 1 to r decreases by half in a geometric fashion.

The values of the factor levels were chosen to facilitate comparison of the results of the present study with those of Olson (1973, 1976) and Schatzoff (1966b). Table 1 displays the levels of these factors investigated in each of the three studies in question.

The various combinations of these factors examined in the present investigation allowed for 36 (pxqxn) distant cases to be simulated in study I, while 405 distinct factor combinations were possible in the noncentral situation. In each of the null cases, except those in which n=50, for p at least 6, or with p=q=n=10, 5000 replications were generated. In all remaining cases 1000 replications were run.

Table 1

The values of the independent variables investigated by Olson (1973), Schatzoff (1966b) and Meredith (1978)

Factor	Study		
	Olson (1973)	Schatzoff (1966b)	Meredith (1978)
p	2, 3, 6, 10	2, 4, 6	2, 3, 6, 10
q	2, 3, 6, 10	3, 5, 7	3, 6, 10
n	5, 10, 50	25, 100	5, 10, 50
tr(H*)	0, 10, 40, 90	12, 24	0, 10, 40, 90
Pattern	concentrated	equal	concentrated
	diffuse*	linear	equal
		geometric	linear
			geometric

* differences on all canonical variates

The data generation procedure employed was that outlined by Schatzoff (1966b). Specifically, a p x df_e matrix, T_1, is generated the elements of which are independently and identically distributed N (0,1) random variables. The sample error SSCP matrix E is then computed as

$$E = T_1 T_1'$$

In similar fashion, a p x df_h matrix, T_2, is generated, also composed of N (0,1) variates, and the hypothesis SSCP matrix, H, is computed as

$$H = T_2 T_2' .$$

This definition of H applies to the null case only. In the noncentral case values equal to the square roots of h_j, the eigenvalues of H*, are added to the elements of the principal diagonal of T_2 prior to computing H, in order to achieve the desired degree and form of multivariate noncentrality. The theory underlying this procedure is discussed by Dempster (1969). Briefly, it relies upon the fact that if H and E are independent random pxp matrices distributed as Wishart matrices with q-1 and N-q degrees of freedom respectively, then the quantity

$$\frac{|E|}{|H + E|}$$

has the desired distribution of Wilks' Λ statistic (Dempster, 1969, p. 311), and the roots of $E^{-1}H$ are the desired estimates of $\lambda_1, \lambda_2, \ldots \lambda_s$ as defined above.

Once $E^{-1}H$ was computed for each replication, the sample eigenvalues were obtained as the solutions to the determinantal equation

$$\left| E^{-1}H - \ell I \right| = 0$$

Values of the five statistics under consideration as well as Bartlett's V and Roy's overall MANOVA tests were calculated as described above. Counts were kept of the number of experiments in which the statistics exceeded the tabled critical values. Successively smaller ℓ_i were tested either until a nonsignificant result was obtained or until s roots had been evaluated. These procedures were carried out for nominal significance (α) levels of .05 and .01. The raw data were converted to percentage

exceedance data in which the rejection count for the tests of any given root are expressed as a percentage of the number of samples in which the test of next largest root was found significant. For example, if a given procedure rejected $H_o : \lambda_3 = 0$ in 500 samples, and then rejected $H_o : \lambda_4 = 0$ in 200 of these samples, then the percentage exceedance for this procedure as a test of the fourth population root would be (200/500) x 10 = 40%.

In all, 575,000 simulated experiments were generated and analyzed in the present study.

Results

Certain data relevant to the practicability of three of the procedures become evident in advance of any simulation. With respect to Harris' (1976) mixed procedure, and Harris' (1976) variation of Roy's test (hereafter these procedures will be referred to as MX and H, respectively), the procedures could not be applied exactly as originally described since s will necessarily equal 1 for the test of the last (sth) root, and no tables of the greatest characteristic root are available for s=1. This is not surprising, in that any situation where s=1 is analyzable as a multiple regression problem. In the present study, MX and H tested the significance of the sth root as an F-statistic with k and N-k-1 degrees of freedom, where N is the total sample size, and k is the absolute value of the difference between p and $df_h + 1$. This procedure is outlined by Marriott (1974) and falls within the general format for testing the significance of multiple correlation coefficients.

A second pre-simulation finding concerned Finn's (1974) variation

of Roy's test (hereafter referred to as F). It was discovered in the process of obtaining the tabled critical values, that reducing df_h in the m argument could result in the critical value increasing in some cases across roots, decreasing in other cases, and even decreasing, and then increasing again across certain sets of roots. The size pattern of m across roots (and thence, of the critical values) depended upon the relative sizes of p and q. For example, if p=3 and q=6, the value of the m parameter for the test of the first root is 1/2, for the second root m=0, and for the third root m = - 1/2. Here, as would be expected, the size of the critical value decreases as successively smaller roots are tested. If, however, p=6 and q=3, then m_1 = 1 1/2 while m_2 = 2. Finally, if p=6 and q=10, then the values of m associated with the critical values used to test the successively smaller sample roots are 1, 1/2, 0, -1/2, 0, 1/2, a nonmonotonic sequence. This serious logical inconsistency in F resulted in this procedure being eliminated from further consideration in the present study.

The principal mode of presentation of the results of studies I and II is via the use of graphs depicting the behaviour of the four procedures in various situations. As an aid to interpreting the figures, estimates of the variability inherent in the data were calculated in two different ways. In the first, a factorial analysis of variance was performed on the percentage exceedance data for tests of the first root in the null case at a nominal significance level of .05. The factors in the analysis were: p, q, n and the test. The four-way interaction, used as a residual error mean square, was found to be 0.128 based on 36

degrees of freedom. (A summary table of this ANOVA as well as tables of raw data are available in a longer version of this report (Meredith, 1978).) The approximate standard error of a point in the graphs is then $\sqrt{.128\%} = .37\%$. Caution is therefore recommended when drawing conclusions based on differences of less than .75 percentage points. Olson (1973) points out that while such estimates are useful for assessing the effects of the various independent variables within a study, they do not address the issue of how accurately the data are estimating theoretical values. For such comparisons it is useful to calculate the standard errors of the proportional exceedance rates as $((\alpha)(1-\alpha)/NR)^{1/2}$ where NR is the number of replications, and α reflects the proportion of the replicates exceeding the critical value. Hence, when $\alpha = .05$ and 5000 replications were generated, the standard error of the percentage data is .31. In cases where 1000 replications were generated, the standard error is .69. When inspecting figures for tests of the second root in the null case, it should be remembered that the number of replications is really NR times α, since tests of the second root only follow rejections of the test of λ_1, of which approximately $\alpha \times NR$ are expected. For example, if NR=5000 and $\alpha = .05$, then the approximate number of expected replications for the test of λ_2 is $5000 \times .05 = 250$, resulting in an approximate standard error for these data of $(250(.05)(.95))^{1/2} = 1.38$. Accordingly, differences between the tests in the figures for λ_2 at a nominal significance level of .05 should be interpreted cautiously if they are less than approximately three percentage points.

Study I. In this section are reported the results for the null conditions, i.e., the cases of no nonzero population eigenvalues, where the value of tr(H*), the noncentrality parameter, is equal to zero. Figures 1 through 3 present percentage exceedances of tabled critical values for the tests of the first two sample roots at a nominal significance level of .05. Dealing first with the tests of λ_1 , inspection of figures 1(a), (b) and (c) reveals that, in most cases, the partitioned-U (P-U) and H were found to perform at around the 5% level, meaning that the empirical rejection rate corresponded well with the theoretical level. In contrast, Marriott's (1952) procedure (MR) and MX were found in general to be conservative, meaning that empirical α was below the theoretical level. The satisfactory performance of P-U and H was not surprising since these procedures are in fact the overall V and Roy's overall test, procedures whose operating characteristics have been well documented. The consistency of the difference between P-U and H, and MR and MX is apparent in figures 1(a), (b) and (c). To the extent that it is possible to make general statements about the individual independent variables under investigation here, the following conclusions seem warranted. The effect of increasing either of p or q is, in general, to lower the rejection rates, especially for MR and MX. The most marked effect of increasing n, the cell size, is to improve the performance of P-U especially when p is large relative to q, as in the cases of p=6, q=3 and p=10, q=3. This finding is not unexpected in that P-U was developed as a large sample procedure. The effect of significance level was entirely predictable in that rejection rates were lower for the

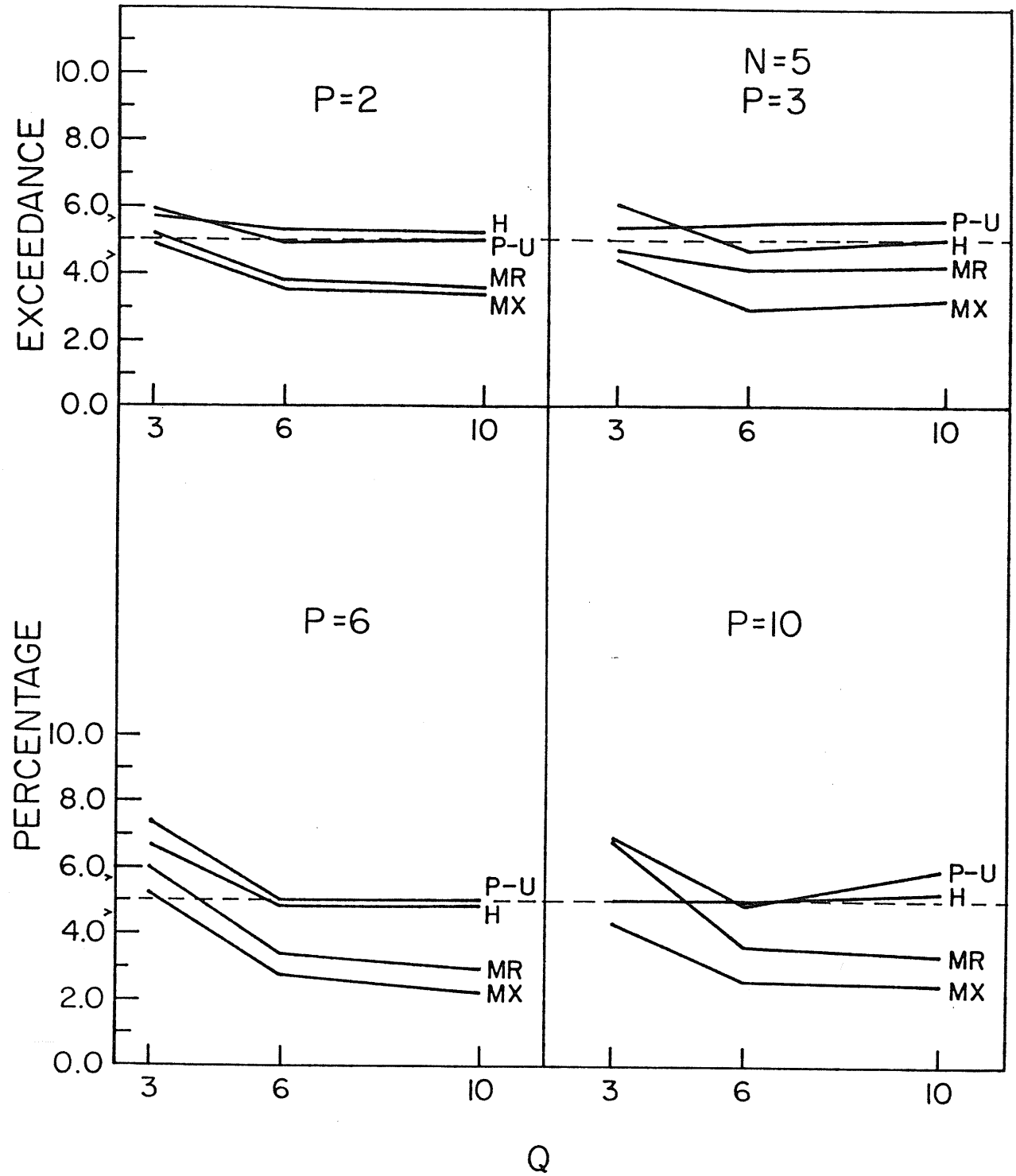

Figure 1(a). Percentage exceedance of theoretical critical values of P-U, H MX, and MR as tests of the first root in the null case for combinations of p and q given n=5. The nominal significance level of 5% is indicated by the dotted line, while the arrows beside the ordinate denote the boundaries of an approximately two standard error of proportion region around 5%.

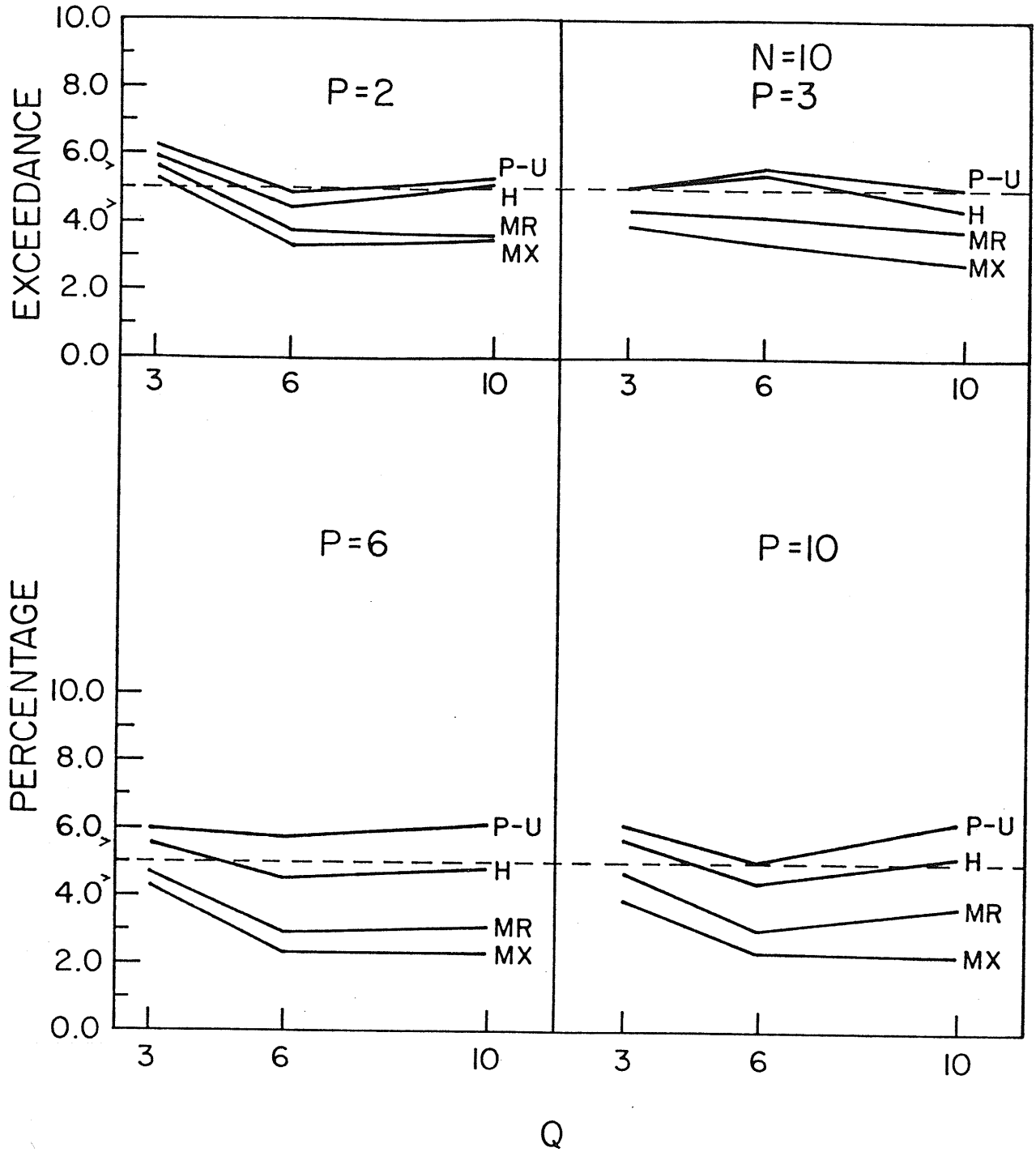

Figure 1(b). Percentage exceedance of theoretical critical values of P-U, H, MX, and MR as tests of the first root in the null case for combinations of p and q given n=10. The nominal significance level of 5% is indicated by the dotted line, while the arrows beside the ordinate denote the boundaries of an approximately two standard error of proportion region around 5%.

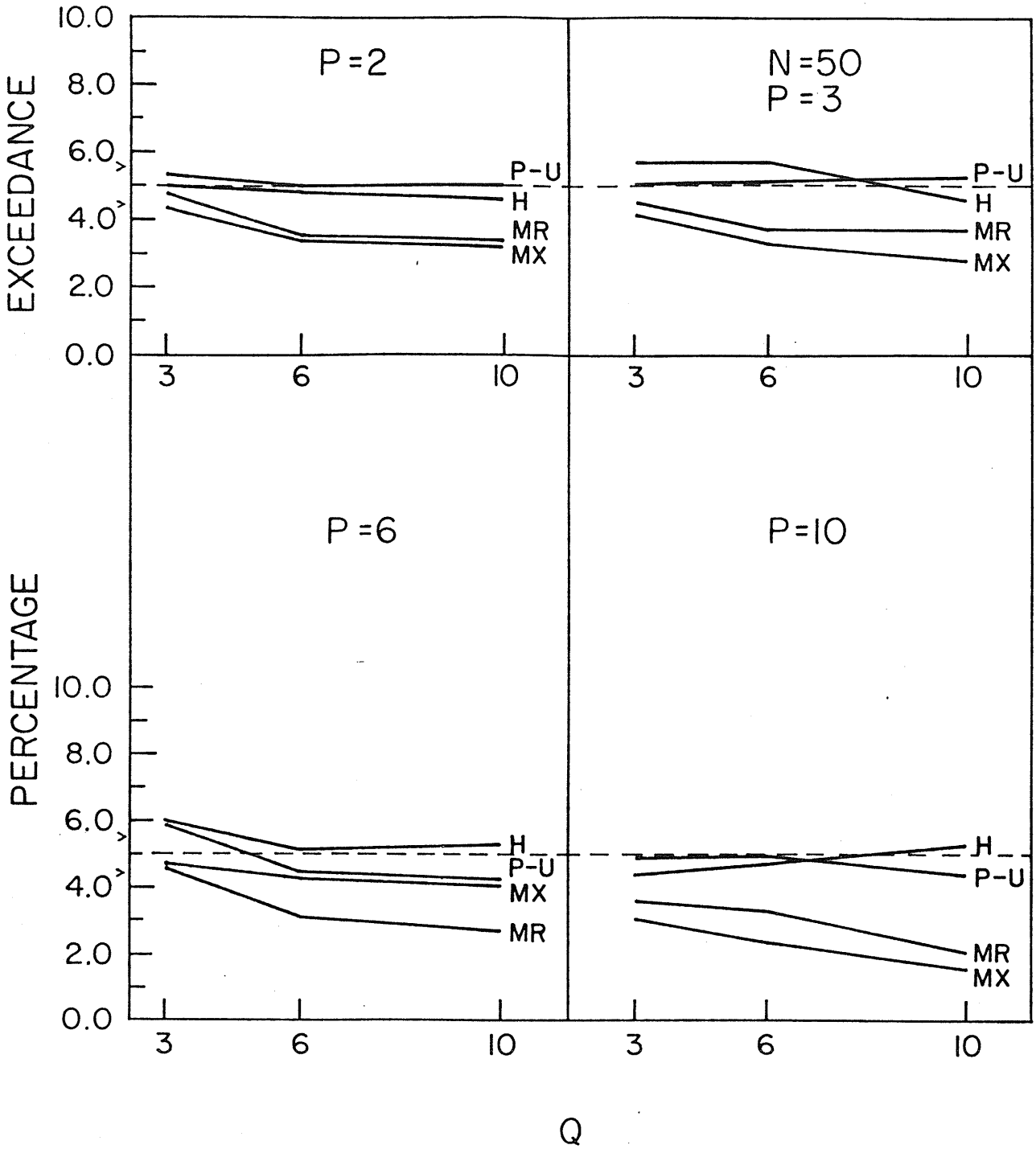

Figure 1(c). Percentage exceedance of theoretical critical values of P-U, H, MX, and MR as tests of the first root in the null case for combinations of p and q given n=50. The nominal significance level of 5% is indicated by the dotted line, while the arrows beside the ordinate denote the boundaries of an approximately two standard error of proportion region around 5%.

smaller value of α. Since the findings for $\alpha = .01$ were similar in form to those for .05, only the .05 results are presented since the rejection rates at .05 are less susceptible to floor effects.

It was thought possible that the effects of changing p and q on the tests might be due to the joint influence of these factors on s, the maximum possible number of roots. Accordingly, figure 2 was drawn to show for levels of n, the effect of s on the percentage exceedance rates of the four procedures. Inspection of figure 2 serves to confirm the general finding noted above that increasing the maximum possible dimensionality of a problem lowers the rejection rates of the procedures, especially MR and MX. Further, it was found that within values of s, the rejection rate was a decreasing function of q, the number of groups, and did not vary consistently with changes in p. This may well have been due to the influence of q on df_e.

The performance of the four procedures as tests of the second discriminant function revealed a considerable change in their relative alignment. Figures 3(a), (b), and (c) present the percentage exceedance data at a nominal significance level of .05 for the four procedures for all 36 combinations of p, q and n investigated. Whereas for λ_1, the performance of P-U and H was in sharp contrast with that of MR and MX, inspection of figure 3 reveals that, for λ_2, it is P-U and MR which differ consistently from the other two tests. Another general difference between the results for λ_2 and those for λ_1 is the lack of smoothness in the curves for λ_2 compared to those for λ_1. This is surely due at least in part to the fact that the λ_1 results were taken as proportions of 5000 (or 1000 for selected conditions) while the λ_2

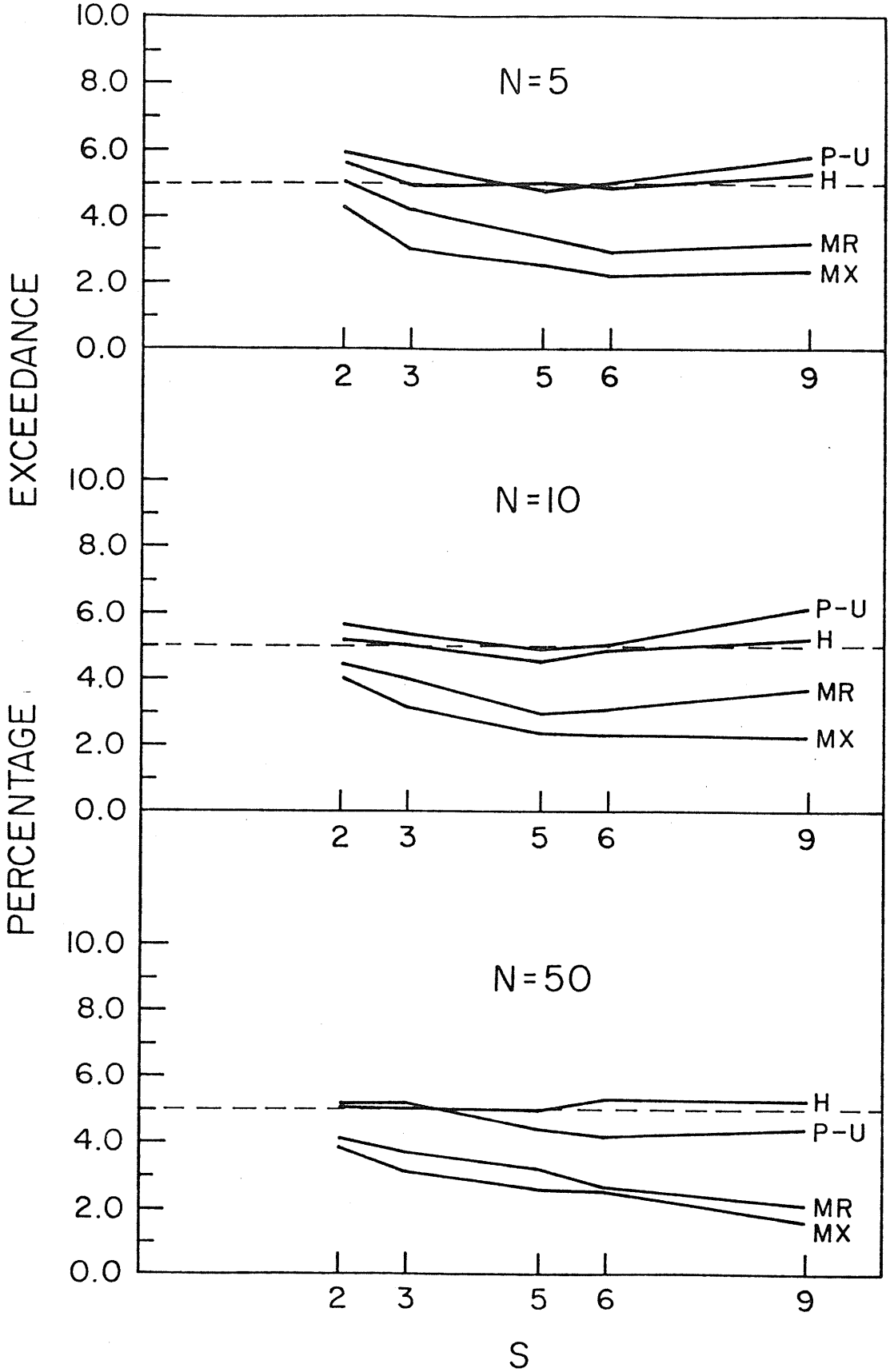

Figure 2. Percentage exceedance of theoretical critical values of P-U, H, MX, and MR as tests of the first root in the null case as a function of s for levels of n. The dotted line denotes the 5% level.

results were taken as proportions of 250 or 50 (5% of 5000 and 1000, respectively). Hence the greater variability of the estimates and concomitant lack of smoothness in the curves.

The caveat notwithstanding, the following conclusions regarding the performance of these procedures as tests of the second discriminant function seem warranted. The reader is referred to figures 3(a), (b), and (c) for visual corroboration of these conclusions. As mentioned above, P-U and MR are performing similarly as are MX and H. In general, the rejection rates of P-U and MR are closer to the nominal level of 5% than are the rates of MX and H. The rates for MX and H are generally conservative, particularly for larger values of q. In contrast to this finding is the highly anticonservative performance of P-U and MR in the condition where p=10, q=3 and n=5 (percentage exceedances of 12% and 27%, respectively), even allowing for a standard error of these points of 1.38. The liberality of P-U may be due to the fact that it is a large sample test, making it susceptible to sampling error when n is small, or when p approaches df_e , as in this case. In terms of the general effects of the independent variables individually, percentage exceedance was found to be a decreasing function of p, q, s and n.

The tests of λ_2 at $\alpha = .01$ showed very few rejections by any of the tests since, even with 5000 replications, if the nominal rate was operating one would expect only .01 x .01 x 5000 = .5 rejections of the null hypothesis for λ_2. The results for tests of roots 3 through 5 revealed only a few rejections by P-U and MR for λ_3 at $\alpha = .05$. No other rejections were detected.

Although these data address the issue of the relative propensity

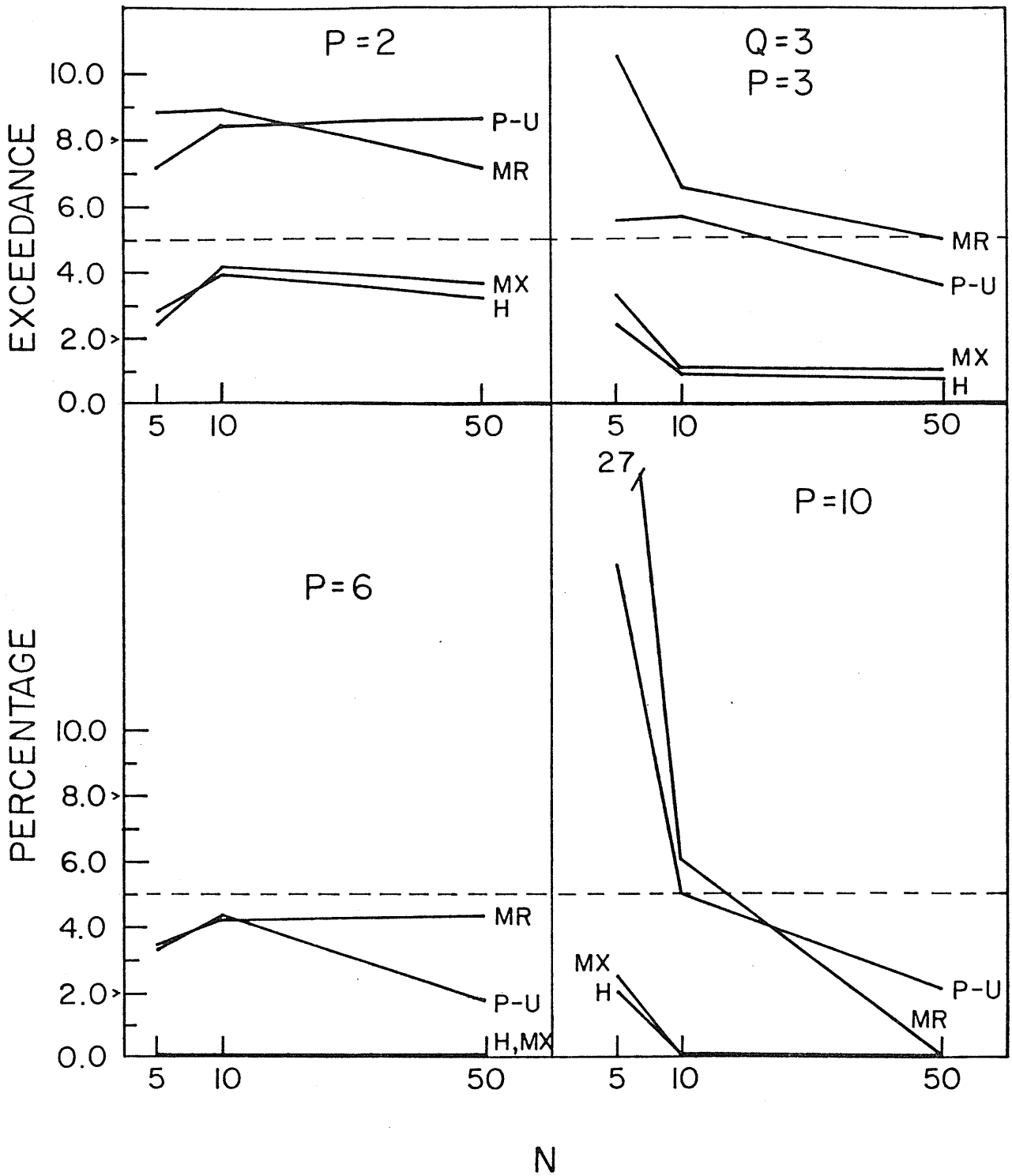

Figure 3(a). Percentage exceedance of theoretical critical values of P-U, H, MX, and MR as tests of the second root in the null case for combinations of p and n given q=3. The nominal significance level of 5% is indicated by the dotted line, while the arrows beside the ordinate denote the boundaries of an approximately two standard error of proportion region around 5%.

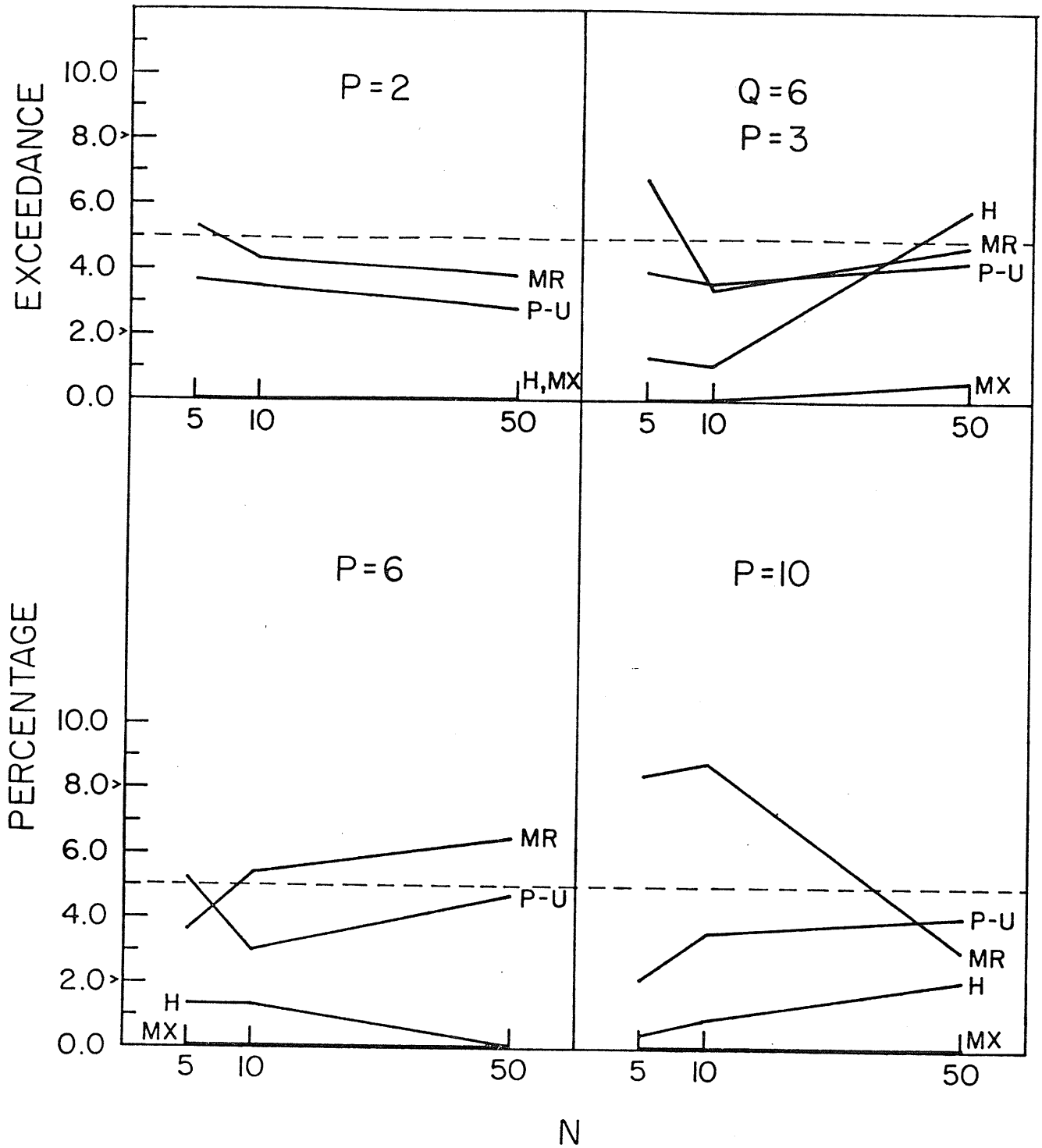

Figure 3(b). Percentage exceedance of theoretical critical values of P-U, H, MX, and MR as tests of the second root in the null case for combinations of p and n given q=6. The nominal significance level of 5% is indicated by the dotted line, while the arrows beside the ordinate denote the boundaries of an approximately two standard error of proportion region around 5%.

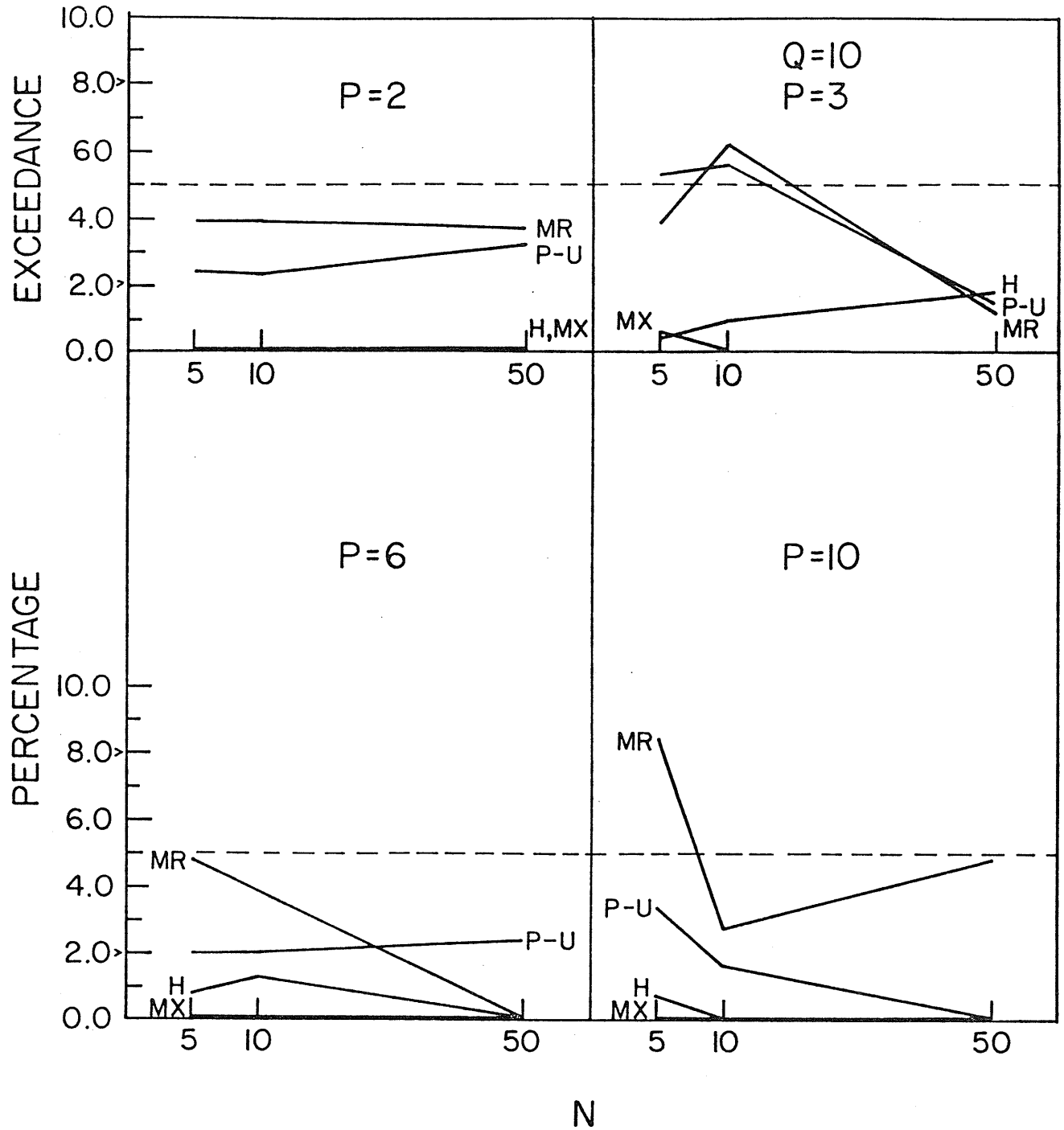

Figure 3(c). Percentage exceedance of theoretical critical values of P-U, H, MX, and MR as tests of the second root in the null case for combinations of p and n given q=10. The nominal significance level of 5% is indicated by the dotted line, while the arrows beside the ordinate denote the boundaries of an approximately two standard error of proportion region around 5%.

of the procedures examined to falsely reject null hypotheses pertaining to the second discriminant function, it was of interest to determine more precisely the behaviour of P-U as a test for roots beyond the first. This interest was generated by Harris' (1976) claim that the procedure would be conservative in its evaluation of roots beyond the first. Accordingly, 200 sets of 1000 replications were generated for condition p=2, q=3, n=10; and 100 sets of 1000 replications were generated for the case of p=3, q=3, n=10. The mean percentage exceedance for the test of the first residual (hence, of λ_2) was found to be 5.7% for the two variate case, and 5.1% for the three variate case. These findings strongly suggest that, for the cases examined at least, P-U is rejecting at or near the nominal rate for both the overall test and the test of the first residual.

Study II. The results reported in this section pertain to the comparative powers of the four procedures to detect nonzero population discriminant functions given various degrees and patterns of multivariate noncentrality. The first cases to be considered are those in which the population noncentrality was concentrated in a single noncentrality root. Figure 4 presents percentage exceedance data for p=6 and combinations of q (3, 6 and 10), n (5 and 10) and the multivariate noncentrality parameter (10, 40 and 90) at a nominal significance level of .05. Inspection of this figure reveals that in general, the tests are performing at approximately the same level. Exceptions to this are the cases in which p=6, where H shows consistently greater sensitivity to an intermediate level of noncentrality (tr(H*) = 40). All of the

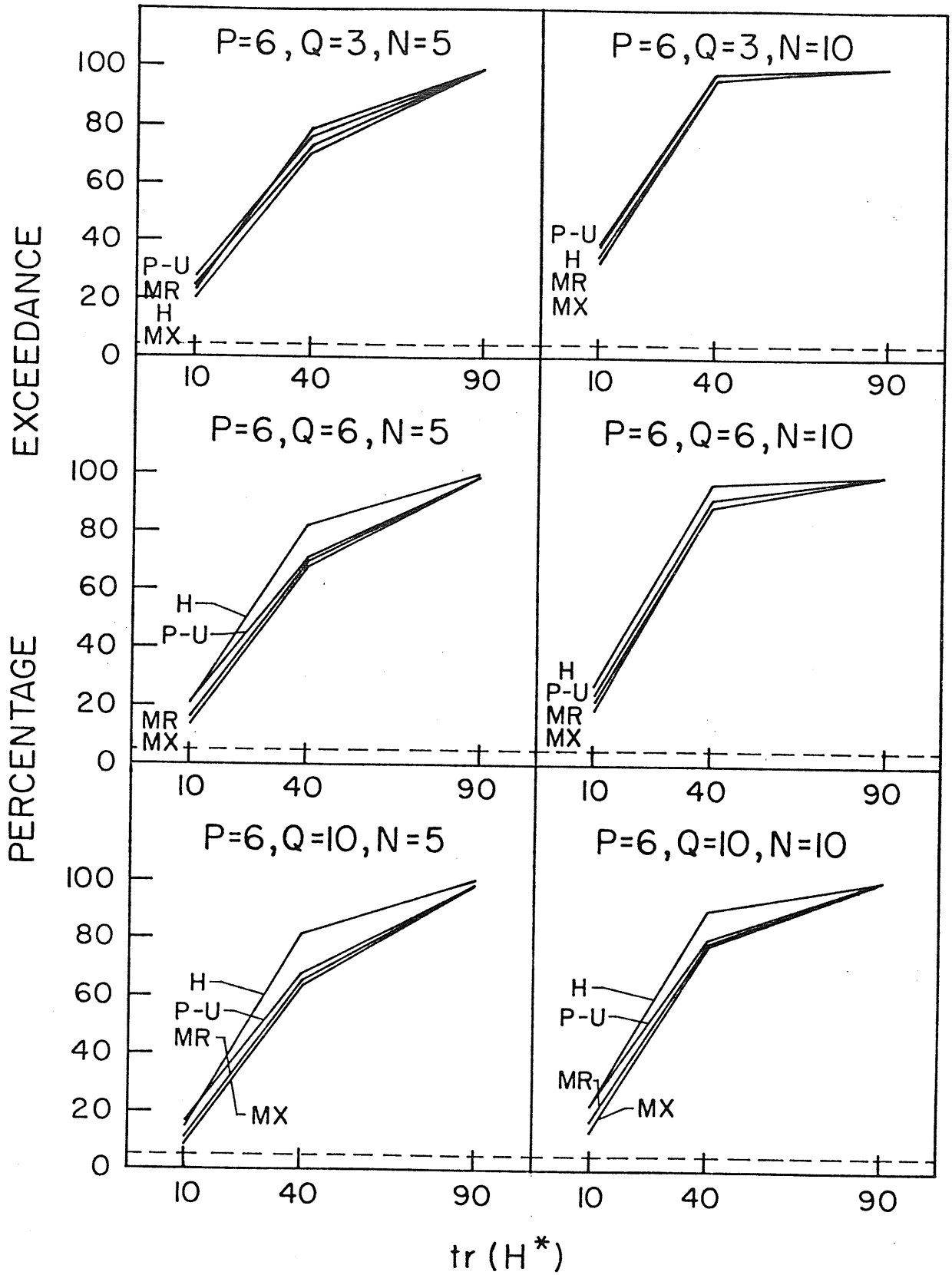

Figure 4. Percentage exceedance of theoretical critical values of P-U, H, MX, and MR as tests of the first root in the concentrated noncentrality case for combinations of p, q, and n as a function of tr(H*). The dotted line denotes the nominal significance level of 5%.

tests rejected H_o in approximately 100% of the samples when the value of tr(H*) simulated was 90, at both nominal significance levels of .05 and .01. The general effects of independent variables may be summarized as follows: percentage exceedance decreased with increases in p or q (hence s), and increased with increases in n, tr(H*) and α . The superiority of H is consistent with earlier findings comparing V and Roy's overall tests among others (e.g., Olson, 1974; Schatzoff, 1966b) which indicated greater power for Roy's test than for V to detect significant discriminant dimensions when all but one of the noncentrality roots are zero.

The results for those cases in which more than one population root was nonzero showed a slight realignment of the four procedures from the concentrated noncentrality pattern. Figures 5(a) to 5(e) present percentage exceedances for selected combinations of p, q and n at a nominal significance level of .05. Note that in cases where two nonzero parameter roots were simulated, only two patterns are presented, equal and linear, since the linear and geometric patterns are identical for two roots. The consistency of the results presented is representative of all of the conditions simulated at both nominal significance levels. The data for each successive root are expressed as a percentage of the number of samples in which the test of the preceding root was significant. For example, if, for a given procedure, the test of λ_1 is significant in 500 of the 1000 replications, then the percentage for the test of λ_2 is taken as a proportion of 500, and not of 1000. Inspection of figure 5 will reveal the following consistent ordering of the four procedures: for all but the last nonzero population root in any set, P-U is the most

powerful procedure followed by MR, H and MX, with the latter three tests generally closer to each other than they are to P-U. For the test of the last nonzero population root, the alignment changes somewhat, with MR slightly superior to P-U, and both of these tests rejecting more frequently than either H or MX. MX is uniformly the least powerful procedure, particularly for tests of roots beyond the first. In terms of the effects of the independent variables individually, the following general conclusions seem warranted: power is a decreasing function of p and q, and increases with larger values of n, tr(H*) and the nominal significance level. The size pattern of the roots was found to be a less potent factor than anticipated, with little difference evident between the linear and geometric patterns, and slightly higher rejection rates for roots tested later in the equal pattern, than in either of the other two. The relatively small size of this last difference underlines the insensitivity of all of the procedures when faced with equal sized population roots. In all cases, the tests were more sensitive to the first root than to the second, more sensitive to the second root than to the third, and so on, despite the equal size of the parameter roots.

In evaluating the performance of the procedures under investigation in the noncentral case, one might be concerned with the comparative frequency with which the procedures overestimate the true dimensionality of a set of sample data. Stated another way, one might ask: what is the comparative likelihood of falsely rejecting $H_o : \lambda_k = 0$ after correctly rejecting $H_o : \lambda_{k-1} = 0$, for the four procedures? Data bearing on this question were examined in the present study for cases in which

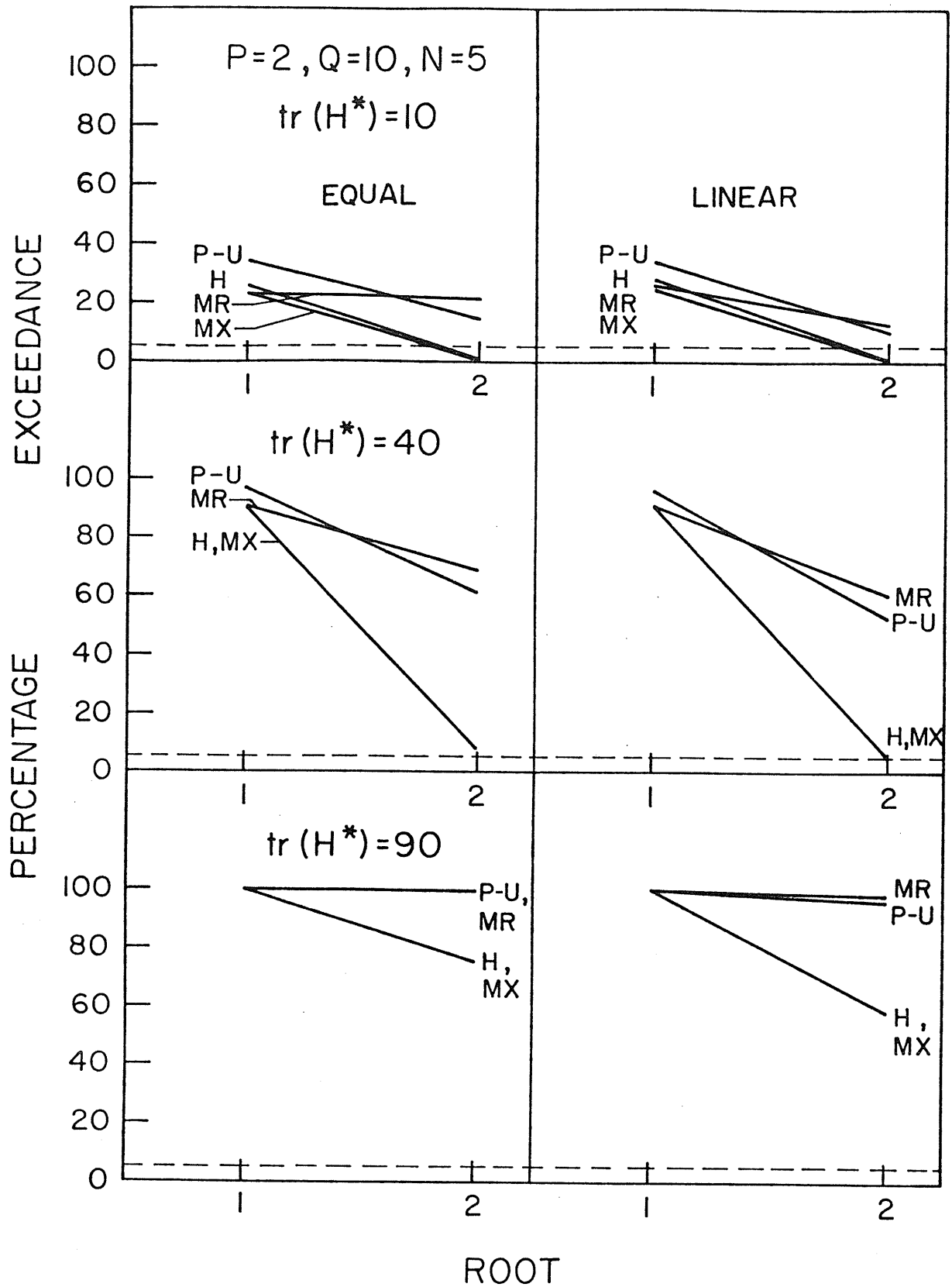

Figure 5(a). Percentage exceedance of the procedures as tests of two non-zero population roots for combinations of tr(H*) and pattern in the case of p=2, q=10, n=5. The dotted line denotes the nominal significance level of 5%.

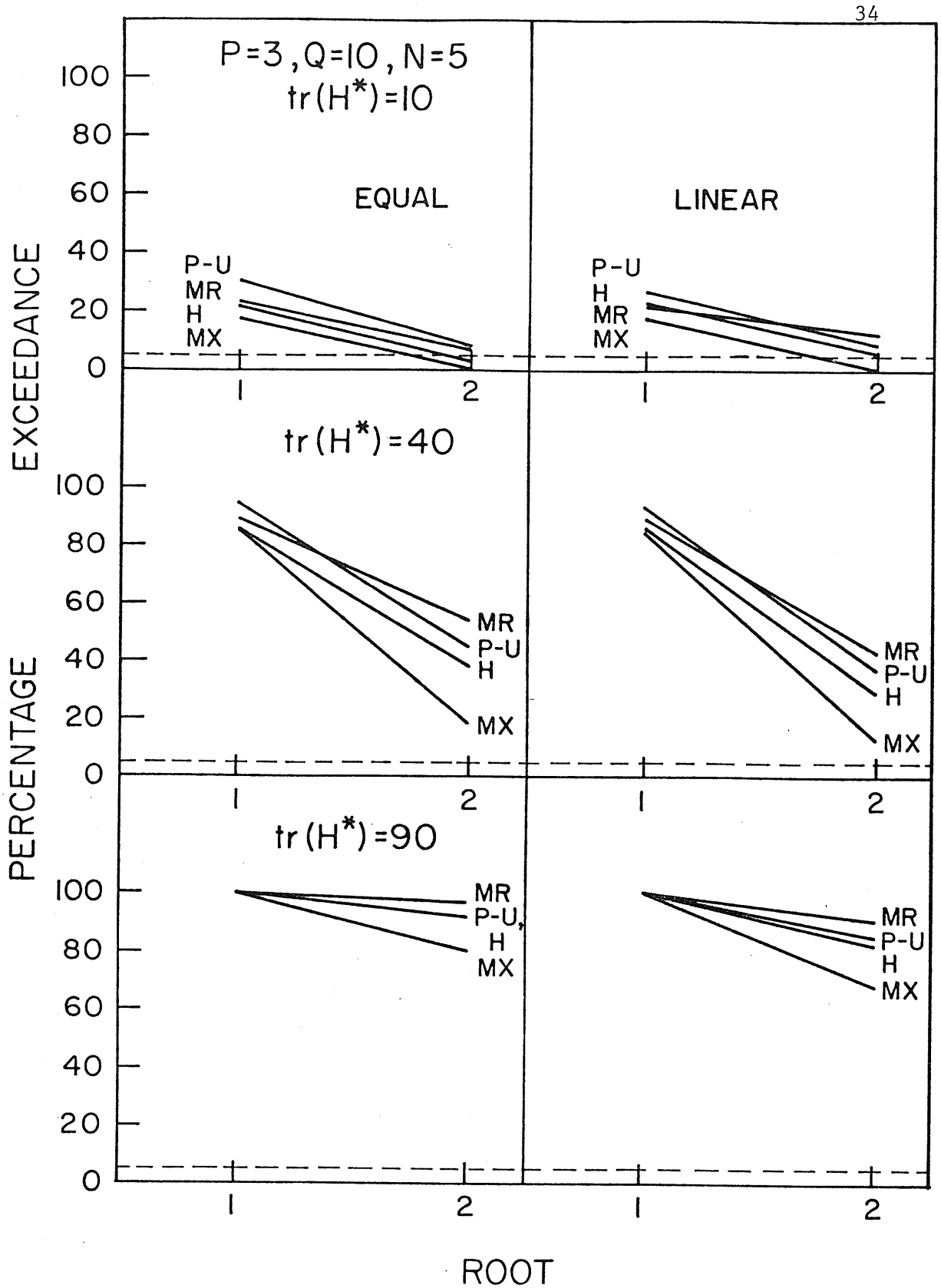

Figure 5(b). Percentage exceedance of the procedures as tests of two non-zero population roots for combinations of tr(H*) and pattern in the case of p=3, q=10, n=5. The dotted line denotes the nominal significance level of 5%.

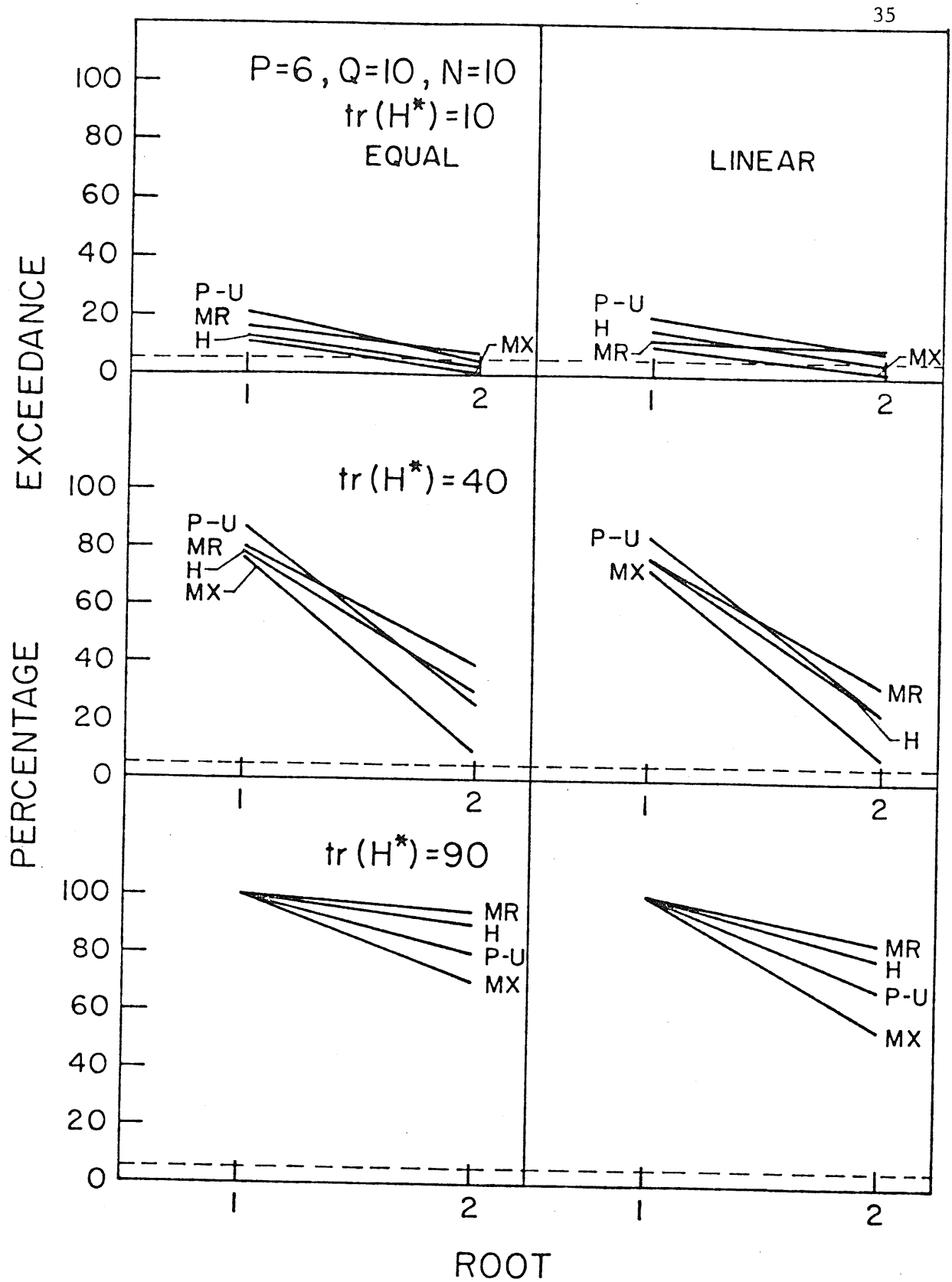

Figure 5(c). Percentage exceedance of the procedures as tests of two non-zero population roots for combinations of tr(H*) and pattern in the case of p=6, q=10, n=10. The dotted line denotes the nominal significance level of 5%.

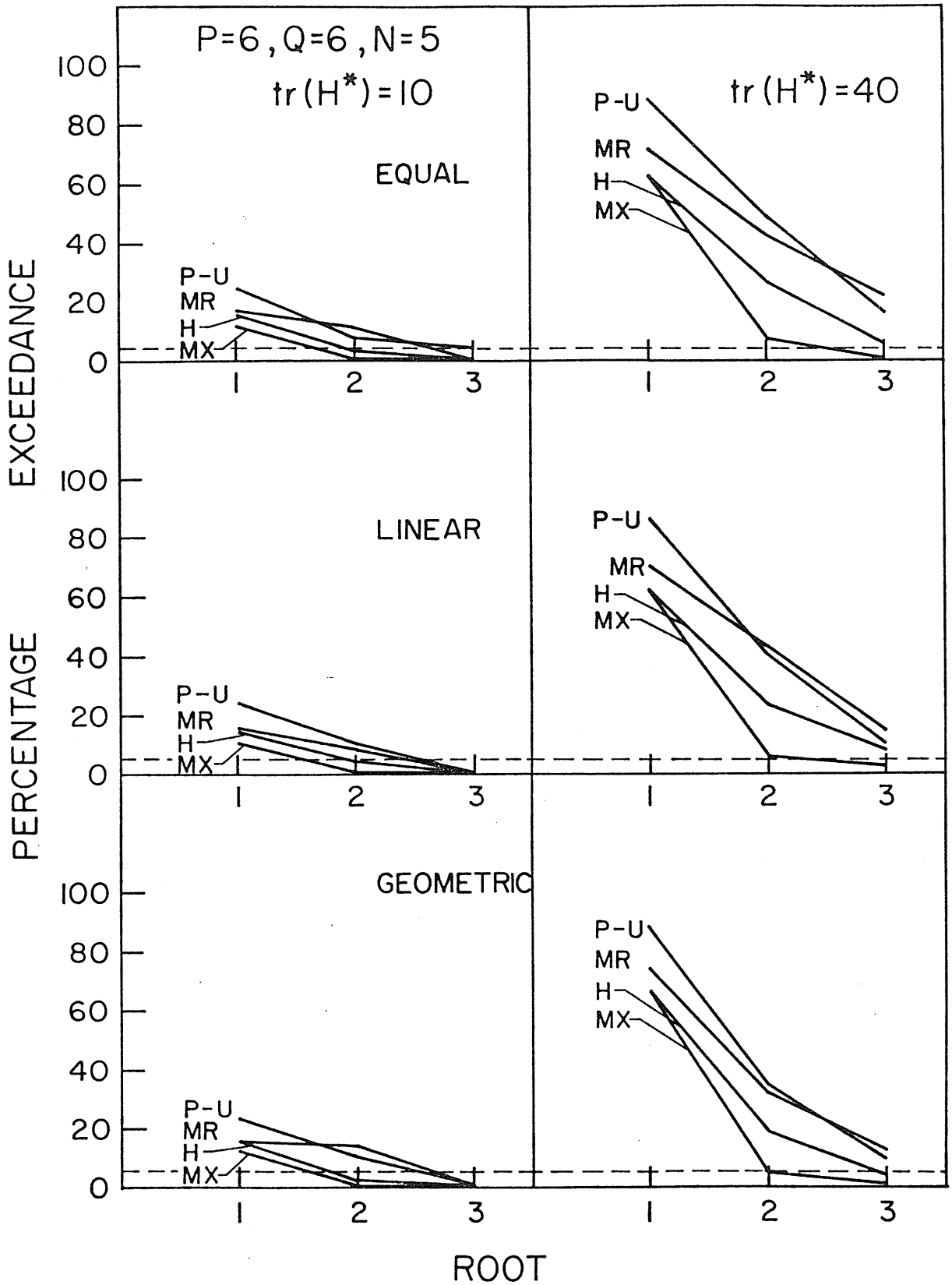

Figure 5(d). Percentage exceedances of the procedures as tests of three nonzero population roots for combinations of tr(H*) and pattern in the case of p=6,q=6,n=5. The dotted line denotes the nominal significance level of 5%.

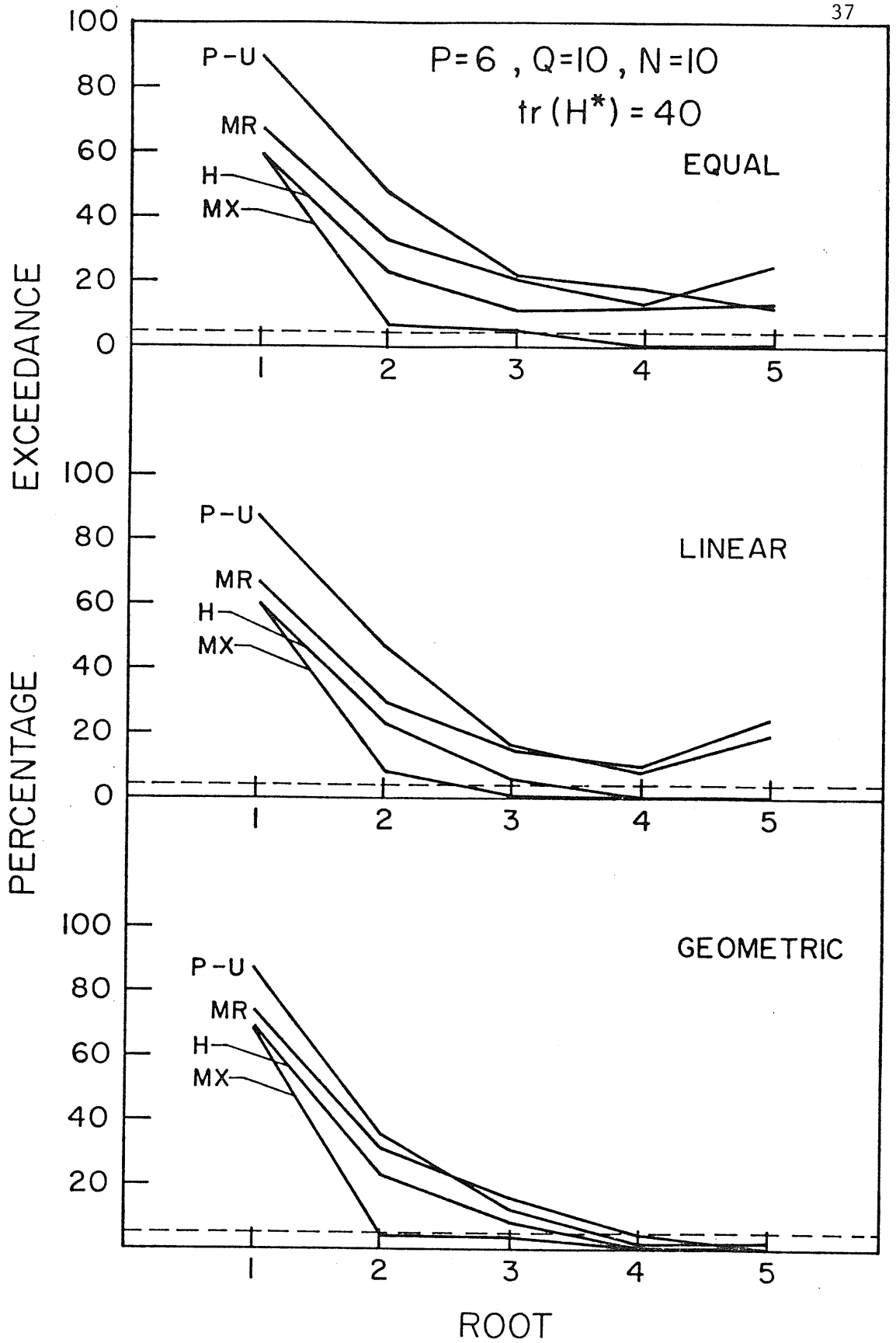

Figure 5(e). Percentage exceedance of the procedures as tests of five nonzero population roots for various patterns in the case p=6,q=10,n=10 and tr(H*) 40. The dotted line denotes the nominal significance level.

dimensionalities of two and three were simulated. The results were similar to those reported for the tests of λ_2 in study I, with P-U and MR rejecting at or near the nominal rate, and, MX and H rejecting at a level well below the nominal rate. In general, the likelihood of such false rejections was a decreasing function of p and q.

Computational Checks. In any simulation study it is imperative to ensure that the data generation procedure is working properly, and yielding reasonable results. In the present study this was accomplished by comparing the rejection rates of Bartlett's V and Roy's overall MANOVA tests to the anticipated theoretical rates in the null case. The direct comparison is feasible in the present study since no violation of distributional assumptions was incorporated into the simulation procedure. With few exceptions, the empirical rejection rates were found to correspond closely to the theoretical rates. The exceptions to this general finding frequently involved Bartlett's V in cases where n was small. Over all combinations of p, q and n, the mean rejection rate at $\alpha = .05$ was 5.34% for V, and 5.1% for Roy's test. Certain comparisons were also possible in the noncentral case. Specifically, Roy (1966) tabulated values of the power function for V for values of tr(H*) - 10, 40 in the concentrated noncentrality situation. Comparison with the results of the present study showed consistent agreement to within approximately two percentage points.

Discussion

The overall results of this study may be summarized as follows: in the null case P-U and H closely approximate the nominal type I error rate, while MR and MX are conservative as tests of the first discriminant function. As tests of the second root (again in the null case) P-U and MR perform at a level more consistent with the nominal rate than do H and MX, which are again conservative. In the noncentral case, P-U generally detects true population discriminant functions as frequently, or more frequently than the other procedures. The only major exception to this is the concentrated noncentrality situation in which H is slightly superior to P-U. The satisfactory performance of P-U and H as tests of the first root in the null case is not surprising, since, as noted above, as tests of λ_1, these procedures are, in fact, Bartlett's V and Roy's criterion, respectively. It is important to note that neither of these procedures directly tests a null hypothesis of the form $\lambda_1 = 0$, but are instead tests of the overall MANOVA null hypothesis, the rejection of which only indirectly implies that at least one population discriminant function is nonzero.

The results of the present study bear directly on Harris' (1976) criticisms of P-U as liberal in its evaluation of λ_1 and conservative in its evaluation of smaller roots. Unlike Harris' (1976) simulation, the procedure employed in the present study is consistent with the null hypothesis tested by P-U, namely that the first k roots are nonzero, while the remaining s-k are zero. Harris' (1976) simulation tested a null hypothesis of the form $\lambda_1 = 0$, which is not the null hypothesis

tested by P-U (Kshirsagar, 1972). Across almost all combinations of p, q and n examined in the present study, P-U rejected correct null hypotheses (the null case) at a level close to the nominal significance level. The most noteworthy exception to this was the case of p=10, q=3, n=5 in which P-U was liberal in its evaluations of both λ_1 and (especially) λ_2. The finding of a 12% rejection rate by P-U (nominal $\alpha = .05$) for the test of the second root in this case (based on 15,000 replications) reflects the instability of this procedure as p approaches df_e (here, 10 and 12, respectively). Only the 27% rejection rate of MR under the same conditions was farther from the nominal rate of 5%. In any event, it seems clear, on the basis of these results, that P-U, correctly applied, is neither liberal in its evaluation of λ_1, nor conservative in its evaluation of λ_2.

The unsatisfactory performance of the procedure proposed by Finn (1974) documents the danger inherent in ad hoc solutions to problems in inferential statistics. Not only did Finn's (1974) procedure lack distributional theory support, but its application in certain situations is clearly illogical due to the degree of freedom problem discussed above. Users of the widely-distributed multivariate analysis program MULTIVARIANCE (Finn, 1972) should be aware that this procedure is included as part of the discriminant analysis output (values of s, m and n are printed for each discriminant function), and should ignore this information when interpreting their results.

A principal aim of this research project was to attempt to provide guidelines for researchers who wished to determine the number of reliable discriminant functions present in their data. (An example

application of the four procedures considered here is provided in Meredith (1978).) The overwhelming recommendation is to use P-U provided that df_e is large relative to p. Across all combinations of p, q and n investigated (except p=10, df_e = 12) P-U was found to be neither unacceptably liberal nor conservative in its rate of false rejections (type I error rate), and to have adequate power against a variety of alternative hypotheses. In addition to these factors P-U has distributional theory support, is widely available in statistical program packages, and references the chi-square distribution (available in most statistical texts) in a straightforward and simple manner. This last consideration casts H and MR in an unfavourable light since use of the published tables of the greatest characteristic root distribution (for H) often requires substantial interpolation, and occasionally extrapolation, while MR requires calculation of non-integral degrees of freedom for referencing the chi-square critical values, again with frequent interpolation. It seems reasonable to recommend against use of MX since it lacks distributional theory support, would be tedious to apply in that two sampling distributions would have to be referenced, is conservative in terms of type I error rate, and lacks power.

As noted above, the performance of P-U deteriorated as p ap-approached df_e . Several possible tacks to follow in such situations might be a) use P-U but with a more stringent significance level (say .01 instead of .05) in order to reduce the probability of over-interpreting the number of discriminant functions present in one's data; b) attempt to reduce p by, for example, performing a principal

components analysis on the dependent variables prior to the MANOVA; c) increase n; or d) eliminate dependent variables on the basis of theoretical importance or interest.

The recommendation of the present study to employ P-U must be tempered with the following considerations. Importantly, the present study simulated data in which all distributional assumptions were satisfied. Further research will be required to determine whether the procedures considered here are differentially sensitive to assumption violation. Certainly the results reported by Olson (1973, 1974) indicate that both V and Roy's overall tests are sensitive to violations of the assumption of population variance-covariance matrix homogeneity. It is not clear whether these findings can be extended to P-U and H, but in view of the probable high frequency of violation of this assumption, this question would seem to warrant examination. Another characteristic of the data simulated in this study which is likely atypical of 'real' data is that all groups were equal in size, and the data were complete (i.e., no data were missing). A complete evaluation of the performance of P-U and the other procedures would require simulation of mixed cell sizes and the application of several procedures to compensate for missing data.

THE UNIVERSITY OF MANITOBA LIBRARIES

LITERATURE REVIEW

In the past, research in experimental psychology has typically involved investigating the effect of one or more manipulations or treatments on only one outcome measure or dependent variable. This concentration on a single dependent variable defines the univariate approach. In contrast, the multivariate approach simultaneously investigates the effects of the manipulation(s) on two or more dependent variables. The univariate approach has predominated despite general recognition among researchers that any given treatment is liable to affect more than one aspect of the subjects' behaviour.

Although the basic principles of many multivariate analysis procedures were worked out by mathematical statisticians several decades ago (e.g., Fisher, 1936; Hotelling, 1936; Rao, 1948), the application of these procedures to social science research has only become widespread during the last decade or so. A major factor in the increasing popularity of multivariate techniques is, without doubt, the proliferation of digital computing facilities, and the concurrent availability of statistical program packages such as SPSS (Nie, Hull, Jenkins, Steinbrenner and Bent, 1975) and BMDP (Dixon, 1975).

As more textbooks on multivariate analysis are written for social scientists (e.g., Overall and Klett, 1972; Tatsuoka, 1971; Finn, 1974; Harris, 1975), the rationale and procedures for applying multivariate analysis of variance (MANOVA) are becoming increasingly familiar

to psychological researchers. The greater complexity of multivariate, as opposed to univariate procedures, which results from the multidimensional nature of the analysis,[1] demands that researchers be sensitive to a number of procedural choice points which they face. For example, several statistics are available for use in testing the overall hypothesis in MANOVA. Fortunately, the results of simulation research such as those reported by Olson (1974) enable practitioners to make reasoned choices when no single procedure is uniformly preferable.

The overall null hypothesis in MANOVA alluded to above can be stated for a one-way (single factor) design as: the p-element population group mean vectors are identical for each of q groups, where p is the number of dependent variables. Rejection of this overall null hypothesis, given a significant multivariate test statistic, implies to the researcher that the mean vector associated with at least one of the q factor levels differs from the others (in terms of the population parameter vectors). Often however, this general finding will not answer all of the researcher's questions about his data. He may want to know which of the p dependent variables are contributing significantly to the multivariate differences among the levels of the factor. A number of procedures are currently available for addressing this question. The simplest procedure involves evaluating the size of the univariate ANOVA

---

[1]For example, given a four group design with two dependent measures, groups 1 and 2 might differ from groups 3 and 4 on the first dependent variable, while on the second variable, groups 1 and 3 could differ from 2 and 4. In this case, there might be said to be two dimensions of difference in these data: dimension 1 differentiates groups 1 and 2 from 3 and 4 while dimension 2 differentiates groups 1 and 3 from 2 and 4.

F statistics for each dependent variable (Cramer and Bock, 1966). Another approach involves the calculation of step-down or residual F-statistics (Roy and Bargmann, 1958). It is the applicability of a third procedure, linear discriminant analysis, which is addressed by the research described here. (For a review of this procedure, the reader is referred to Huberty, 1975.) Specifically, several procedures for determining the number of significant dimensions of difference present in a set of multivariate mean vectors are compared. The broad aims of this research are to comment on the type I error rate and power characteristics of the procedures compared, and, thereby, provide guidelines for the practitioner which will enable him to analyze his multivariate data in the manner which best addresses his research interests.

The Multivariate Linear Model

It is necessary at this point to introduce the model underlying MANOVA, and the notation to be employed through this paper. The multivariate linear model for a one-way fixed effects design is defined as follows:

$$Y = X\beta + \varepsilon \qquad (2.1)$$

where $\underset{N \times p}{Y}$ is the observation matrix consisting of a p-element response vector for each of N subjects.

X is an N x q design matrix specifying the multivariate linear model components associated with each of the q treatment levels,

β is a q x p matrix of parameters representing each variate's overall mean in the population, and the q - 1 fixed treatment effects, and

ε is an N x p error matrix whose rows are assumed to be randomly sampled from $N_{1xp}(0', \Xi_{pxp})$, that is, from a p-variate normal distribution with covariance matrix Ξ and the null vector as mean.

The primary assumptions of the model can be reworded succinctly in terms of the observation matrix Y: the N row vectors Y'_i are assumed to be <u>independent</u> samples from a multivariate normal distribution with a <u>common variance matrix</u> Ξ (Olson, 1973).

In order to help clarify the meaning of the components of equation 2.1 the following symbolic expansion is provided. Given N = 6 subjects randomly assigned to one of three levels of treatment A (q=3) such that $n_1 = 2$, $n_2 = 2$ and $n_3 = 2$, and p = 4 dependent variables measured on each subject, the fixed effects linear model would take the form:

$$\underset{6 \times 4}{Y} = \underset{6 \times 3}{X} \; \underset{3 \times 4}{\beta^*} + \underset{6 \times 4}{\varepsilon}$$

$$\begin{array}{c} A1 \\ A1 \\ \\ A2 \\ A2 \\ \\ A3 \\ A3 \end{array}
\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \\ \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} \\ Y_{61} & Y_{62} & Y_{63} & Y_{64} \end{bmatrix}
=
\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}
\begin{bmatrix} \mu(1) & \mu(2) & \mu(3) & \mu(4) \\ \alpha_1(1) & \alpha_1(2) & \alpha_1(3) & \alpha_1(4) \\ \\ \alpha_2(1) & \alpha_2(2) & \alpha_2(3) & \alpha_2(4) \end{bmatrix}
+
\begin{bmatrix} \varepsilon_{11} & \cdots & \varepsilon_{14} \\ \vdots & & \vdots \\ \varepsilon_{61} & & \varepsilon_{64} \end{bmatrix}$$

* p subscript in parentheses

Hence, taking Y_{33}, for example, and performing the $X\beta$ multiplication, the equation for this particular score would be:

$$Y_{33} = (1)\mu_{(3)} + (0)\alpha_{1(3)} + (1)\alpha_{2(3)} + \varepsilon_{33}$$

$$= \mu_{(3)} + \alpha_{2(3)} + \varepsilon_{33}$$

To take a second example

$$Y_{64} = (1)\mu_{(4)} + (-1)\alpha_{1(4)} + (-1)\alpha_{2(4)} + \varepsilon_{64}$$

$$= \mu_{(4)} + \alpha_{1(4)} + \alpha_{2(4)} + \varepsilon_{64}$$

Equation (2.2) demonstrates an important characteristic of fixed effect linear models, namely that the effect associated with the qth level is equal to:

$$-n_q \sum_{i=1}^{q-1} \alpha_i \qquad (2.2)$$

indicating that the q treatment effects sum to zero.

MANOVA Test Statistics

As noted above, the overall null hypothesis to be tested via the MANOVA is that of no differences among the population group mean vectors. As pointed out in many sources (e.g., Tatsuoka, 1971; Olson, 1976) any of several test criteria may be used to perform this test. The most frequently mentioned of these are: Wilks' likelihood ratio (Wilks, 1932); Roy's largest root test (Roy, 1945); the Lawley-Hotelling trace criterion (Lawley, 1938; Hotelling, 1951); and Pillai's trace criterion (Pillai,

1955). Importantly, all of these statistics are functions of the eigenvalues or latent roots of the matrix product $E^{-1}H$ where E is a sample estimate of the pxp within groups or error sum of squares and cross products (SSCP) matrix, Σ, and H is a sample estimate of the pxp between groups or hypothesis SSCP matrix, M. The maximum possible number of nonzero eigenvalues for any given $\Sigma^{-1}M$, denoted s, is equal to the lesser of p, the number of dependent variables and df_h , the hypothesis degrees of freedom (here, q-1). Hence

$$s = \min (p, df_h). \qquad (2.3)$$

Table 2 presents formulae for the MANOVA statistics listed above in

Table 2 . Four Test Criteria for MANOVA

Descriptive Label of Criterion	Symbol and Range	Criterion in terms of $E^{-1}H$
Roy's largest root	$0 < R < 1$	$\frac{\ell_1}{1 + \ell_1}$
Hotelling's trace	$0 < T < \infty$	$\sum_1^s \ell_i$
Wilks' likelihood ratio	$0 < U < 1$	$\prod_1^s \frac{1}{1 + \ell_i}$
Pillai's trace	$0 < P < s$	$\sum_1^s \frac{\ell_i}{1 + \ell_i}$

terms of the latent roots of $E^{-1}H$ (denoted ℓ_i). The null case is represented by the situation in which all of a possible s roots are equal to 0. It will be convenient here to introduce the s x s diagonal matrix Λ whose elements are the roots of $\Sigma^{-1}M$ (some or all of which may be zero):

$$\underset{s \times s}{\Lambda} = \begin{bmatrix} \lambda_i & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \lambda_2 & & & & \cdot \\ \cdot & & \cdot & & & \cdot \\ \cdot & & & \cdot & & \cdot \\ \cdot & & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \lambda_s \end{bmatrix}$$

Inspection of Table 2 will reveal that all of the statistics employed to test the overall null hypothesis utilize all s sample roots, with the exception of Roy's test which utilizes only the first (largest) sample root.

Linear Discriminant Analysis

While the sizes of the overall MANOVA test statistics described above are influenced by either all r ($\leq$ s) nonzero sample roots, or by only the first (largest) sample root (in the case of Roy's test), discriminant analysis involves testing the significance of each nonzero root individually. The r nonzero eigenvalues of $E^{-1}H$, denoted $\ell_1, \ell_2, \ldots, \ell_r$, are important for discriminant analysis in two major ways. Specifically, there is associated with each nonzero ℓ_i, a p-element vector of discriminant function coefficients denoted v_i, which

are defined such that when each of the p original variables is multiplied by its corresponding discriminant function coefficient to yield a composite variable Y'_i , as

$$Y'_i = v_i X \qquad (2.4)$$

where X is a p-element vector of raw scores,
the ratio of the hypothesis sum of squares to the error sum of squares is maximized. This ratio of hypothesis to error components, called the determinantal criterion and denoted λ_i , can be expressed in terms of the raw variables as

$$v_i H v'_i \;/\; v_i E v'_i \;(= \lambda_i) \;. \qquad (2.5)$$

Successively derived discriminant functions satisfy the same criterion subject to the constraint that each 'new' discriminant function be uncorrelated (in the total sample) with those already derived. The maximization procedure results in the extraction of the sample roots in decreasing order of magnitude. Hence, the first sample root is always the largest of the r nonzero roots, ℓ_2 is the second largest, and so on.

Interpretively, the r discriminant functions associated with each nonzero ℓ_i (where r is the rank of $E^{-1}H$) can be thought of as representing 'dimensions of difference' among the q groups in the p-variate test space. The potential existence of more than one dimension of difference (provided $p > 1$ and $q > 2$) reflects the fact that the

mean of any given group can differ from that of any of the other q - 1 groups on any of the p dependent variables. In general, increasing the differences among the groups on the dependent variables, and decreasing the degree of intercorrelation (within group) among the variables, will tend to increase r (to a maximum of s), thereby increasing the dimensionality of the test space required to represent all of the dimensions along which the groups differ. As mentioned previously, the maximum value which s can attain is the lesser of p and q - 1. Hence, in situations where only two groups are being compared, only one discriminant function is possible, the significance of which can be tested by any of several statistics including Hotelling's T^2 (Hotelling, 1931), Wilks' likelihood ratio (Wilks, 1932), Roy's test (Roy, 1945) and the Lawley-Hotelling trace criterion (Lawley, 1938; Hotelling, 1951). As Timm (1975) points out, in the two sample cases (where s=1), these statistics are equivalent.

In cases where the reduced dimensionality of the test space may exceed one, the researcher is faced with the problem of deciding how many r of a possible s sample discriminant functions are important in the sense that they represent reliable dimensions of difference in the population. Huberty (1975) comments that "in the literature, the process of testing the significance of a function has been lacking in clarity" (p. 548). The method which is recommended by most authors of multivariate texts written for behavioural scientists (e.g., Tatsuoka, 1971; Timm, 1975; Overall and Klett, 1972), is a procedure first suggested by Bartlett (1941) called the "partitioned-U" sequential procedure. The

"U" referred to here is Wilks' lambda criterion, the overall MANOVA statistic discussed earlier. As shown in Table 2

$$U = \prod_{i=1}^{s} \frac{1}{1 + \ell_i} \qquad (2.6)$$

where the ℓ_i are the sample eigenvalues associated with the successively derived discriminant functions. In essence, the partitioned-U procedure tests the statistical significance of each function by evaluating the size of the residual U's after successive ℓ_i have been subtracted. This procedure and others will be described in detail below. Suffice it to say at this point that despite the apparent popularity of the partitioned-U procedure among writers of both texts and packaged statistical programs such as SPSS (Klecka, 1975), MANOVA (Clyde, 1969) and MULTIVARIANCE (Finn, 1972), there is less than complete agreement as to its logical and statistical validity (Harris, 1975; Harris, 1976). The next section of this paper will describe the several approaches currently available for testing the significance of individual discriminant functions, and elaborate upon any criticisms of these procedures which have been published to date.

The Test Criteria

Roy's Largest Root Test. This statistic, which was introduced earlier (see Table 2) is generally considered to be a test of the overall null hypothesis of equality of population mean vectors across q groups ($q \geq 2$). As is evident from the formula for this statistic (reproduced

for convenience below), in its "classical" form, only the largest sample eigenroot of $E^{-1}H$, ℓ_1 , is involved in its computation. Hence, as a test of the first discriminant function (that associated with ℓ_1) ,

$$\text{Roy's criterion, } R = \frac{\ell_1}{\ell_1 + 1} \qquad (2.7)$$

which is compared to the critical value of the greatest characteristic root distribution (Pillai, 1965) with the following parameters:

$$s = \min (p, q-1)$$
$$m = (\left| p-df_h \right| -1) / 2$$
$$n = (df_e - p-1) / 2$$

where df_h and df_e are the degrees of freedom for hypothesis and error, respectively.

As a test of the significance of discriminant functions other than the first, Harris (1976) and Finn (1974) recommend using Roy's criterion on roots 2 through s, but with reduced values for the degree of freedom parameters. Specifically, Harris recommends that the value of s be set equal to s-i, where i is the number of functions already tested, while m and n remain unchanged from the values employed in the overall test. Finn, on the other hand, would leave s unchanged but would reduce df_h and df_e (in the m and n arguments, respectively) by one for each prior function tested. Hence, a test of the second sample discriminant function, ℓ_2 , using these procedures would reference the tables of the greatest characteristic root distribution (Pillai, 1965) with the parameter values shown in Table 3.

Table 3. Degree of freedom parameters for referencing the greatest characteristic root distribution to test the significance of λ_2 (hence i+1)

	Harris (1976)	Finn (1974)
s	$s - 1$	s
m	$(\lvert p-df_h \rvert -1) / 2$	$(\lvert p- (df_h-1) \rvert -1) /2$
n	$(df_e - p-1) / 2$	$(df_e - p-2) / 2$

Finn presents his procedure without comment. Harris, however, notes that the tests of roots other than the first will be conservative since "this procedure does not take into account the capitalization on chance which leads to the first sample root's 'absorbing' a larger share of the total (degree of relationship) than does the corresponding population root" (1976, p. 363). This conclusion is based on a logical argument, and has not, to date, been empirically supported. In fact, simulation data on the type I error and power characteristics of these tests are unavailable for either Harris' or Finn's procedures.

The Partitioned-U Procedure. As mentioned above, this procedure was first suggested by Bartlett (1941) and has been adopted as the method of choice by the majority of current multivariate texts (e.g., Tatsuoka, 1971; Overall and Klett, 1972; Finn, 1974; Timm, 1975). One approach to testing the significance of a given value of U as a test of the overall null hypothesis involves calculating Bartlett's V statistic as

$$V = [N - 1-(p + q) / 2] \quad \ln U \qquad (2.8)$$

which is distributed approximately as a chi-square with p(q-1) degrees of freedom. The fact that successive discriminant functions are uncorrelated allows for the following alternate formula for V expressed in terms of the nonzero eigenvalues associated with the first r discriminant functions:

$$V = [N - 1-(p + q)/ 2] \sum_{i=1}^{r} \ln (1 + \ell_i) . \qquad (2.9)$$

Because the successive terms $\ln (1+\ell_i)$ in this expression are statistically independent (assuming multivariate normality of the original p variables) the additive components of V are each approximately distributed as chi-square variates with $p + q - 2i$ degrees of freedom.

The partitioned-U procedure per se, then, involves the following steps (as outlined by Harris, 1976):

(1) Test the overall null by comparing V to χ^2_α with p(q-1) degrees of freedom. If H_o overall is not rejected, it is concluded that none of the discriminant functions are statistically significant, and testing stops. If, however, this null hypothesis is rejected, then at least the first discriminant function is considered significant, and the researcher proceeds to step 2.

(2) Test the significance of the residual after successively removing the first k discriminant functions. As soon as the residual becomes smaller than the critical value of the appropriate chi-square distribution, it is concluded that only these first k functions are significant (at that α level). The successive test statistics and their associated degrees of freedom (as shown in Tatsuoka, 1971, p. 165) may be

summarized as follows:

Residual after Removing	Approximate χ^2 Statistic	Degrees of Freedom
First discriminant function	$V - V_1$	$p(q-1) - (p+q-2) = (p-1)(q-2)$
First 2 discriminant function	$V - V_1 - V_2$	$(p-1)(q-2) - (p+q-4) = (p-2)(q-3)$
First 3 discriminant function	$V - V_1 - V_2 - V_3$	$(p-2)(q-3) - (p+q-6) = (p-3)(q-4)$
⋮	⋮	⋮

The largest number of significant discriminant functions which is possible in a given set of data is $s = \min(p,q-1)$. Typically, the number of functions which prove significant (k) will be less than s. The remaining $s-k$ dimensions can be regarded as immaterial for population differentiation since any sample differences along these dimensions can be attributed to sampling error. The practical experience of two researchers leads them to report that k is very seldom greater than two (Huberty, 1975) or three (Olson, 1976) in educational/psychological research.

Despite its seeming popularity, the partitioned-U procedure is not without its critics. In particular, Harris (1976) has argued that this testing sequence is both logically and statistically invalid. The dissatisfaction with the logic of the procedure centres on the fact that the significance of the first discriminant function is never directly tested. Instead, it is inferred on the basis of the overall test that

if at least one of the population roots is nonzero, then that will necessarily be the first (the largest). Harris argues that to test the significance of ℓ_1 , it is necessary to refer to the sampling distribution of largest roots, namely, the greatest characteristic root distribution (Pillai, 1965). To support this position, Harris cites Bartlett (1941): "for any test on [the population value of the largest root] we have little choice but to consider [ℓ_1 , the largest sample root]" (p. 30).

The statistically-based criticisms of the partitioned-U test focus on two distinct issues. The first concerns the partitioning procedure itself. Harris (1976) provides simulation data which suggest that the use of the partitioned-U procedure to evaluate the significance of the largest root (in the null case, i.e., where the population roots were all zero) led to type I error rates (false rejection of H_o : $\lambda_1 = 0$) as high as .61 when the nominal rate was set at .05. Harris' (1976) Table 2 is reproduced below as Table 4 to provide a complete picture of his findings. While these data were obtained from simulated canonical correlation results, their extension to MANOVA is justified on the grounds that both analyses fall within a general linear model framework.

Table 4. Empirical estimates of type I error rate α for Roy's largest root test, the partitioned-U (P.U.) test of λ_1, and Wilks' lambda overall MANOVA test (Harris, 1976).

Test	α = .05 Critical Value	α = .05 % Significant	α = .01 Critical Value	α = .01 % Significant
	s = 2			
Roy's	.05151	6.4	.06934	1.2
P.-U.	.04566	9.0***	.06331	2.2**
Wilks'	.93986	5.8	.92052	1.2
	s = 10			
Roy's	.19263	2.0	.21533	0.5
P.-U.	.13831	61.0	.16309	24.0***
Wilks'	.52619	1.0	.49725	0.0

*Significantly different from the nominal α rate at the .05 level
**Significantly different from the nominal α rate at the .01 level
***Significantly different from the nominal α rate at the .001 level

On the basis of these findings, Harris argues that since the partitioned-U procedure consistently overestimates the significance of λ_1, "it would seem absurd to claim that subtracting this grossly inflated estimate of the contribution of [λ_1] to [V] . . . would leave us with a statistic which, a) truly represents that portion of the relationship (between group membership and the dependent variables) summarized by [V] which is not accounted for by the first [discriminant function]; and, b) is distributed as a chi-square with [p(q-1) - (p+q-2)] degrees of freedom" (Harris, 1976, p. 361).

Unfortunately, it seems clear that Harris has not used the proper simulation procedure to examine the partitioned-U procedure's test of λ_1 What Harris has done, is to assume that since the components of V are independently chi-square distributed, then the statistic associated with each of the ℓ_i could be used to test the null hypothesis that each $\lambda_i = 0$, by comparing each statistic to the chi-square distribtuion with p+q-2i degrees of freedom. One possible reason for the lack of agreement regarding the appropriateness of the partitioned procedure may be the existence of some confusion about the exact hypotheses being tested. Kshirsagar (1972) makes a particular point of stating that the hypothesis being tested at any given point following rejection of the overall null hypothesis, i.e., for any value of i, is the first i population roots are nonzero, while the remaining s-i are zero. He points out that this hypothesis is correctly tested by evaluating the size of the statistic associated with the s-i remaining roots as a χ^2 with (p-i) (q-1-i) degrees of freedom, and not by evaluating the statistics associated with just the i roots individually. Accordingly, Harris' assertion that the partitioned-U procedure is liberal in its evaluation of the significance of λ_1 is neither supported nor refuted by the simulation results he reports since the null hypothesis that $\lambda_1=0$ is evaluated indirectly by the overall test and not by evaluating the size of a V statistic based on ℓ_1 alone (as Harris did).

Harris' second statistical basis for criticizing the use of the partitioned-U procedure stems from the fact that the ordering of the sample roots of $E^{-1}H$ (in terms of size) may not correspond to the ordering of the population roots with the result that sample roots which are "out of order" are not evaluated against the correct chi-square distribution (as determined by the degree of freedom parameter). While Harris' arguments concerning both this order problem, and the lack of any direct test of λ_1 in the partitioned-U procedure are not logically unappealing, an empirical determination of a) the relative frequency of noncorrespondence (in terms of size ordering) of the sample and population roots, and b) the relative frequency of a significant overall test being followed by a nonsignificant test of λ_1, would shed some light on the practical importance of these problems.

Harris' (1976) Mixed Procedure. As an alternative to the use of Roy's criterion for testing roots other than the first (see above) Harris (1976) has proposed a procedure which combines the partitioned-U test and a variation on the largest root test. The steps in this 'mixed' procedure are as follows:

(1) Test the overall null hypothesis using Bartlett's V with $p(q-1)$ degrees of freedom. If this null hypothesis is rejected, conclude that there is at least one significant discriminant function, and proceed to step (2). If the observed V does not exceed the critical value, testing stops.

(2) Test the significance of the largest population root by

comparing ℓ_1 to the tabled critical value of the greatest characteristic root distribution with values for s, m and n calculated in the manner described above. If this test achieves significance, conclude that the population value of λ_1 does not equal zero, and proceed to step (3). Otherwise conclude that the nature of the relationship between group membership and the dependent variables cannot be specified, and stop testing.

(3) Test the hypothesis that the remaining s-i population roots are all equal to zero by comparing

$$V = [N - 1 - (p+q) / 2] \sum_{K=i+1}^{s} \ln(1+\ell_K) \qquad (2.10)$$

to χ^2_α with (p-i)(q-i-1) degrees of freedom. If this test fails to achieve significance, conclude that only the first i population roots are nonzero. If it achieves significance, conclude that not all of the last s-i population roots are zero, and proceed to step (4).

(4) Test the null hypothesis that the i + 1th population root is equal to zero by comparing R_{i+1} to the greatest characteristic root distribution with s' = s-i. If nonsignificant conclude that the nature of the relationship associated with the last s-i roots cannot be specified. If significant, conclude that the i+1th discriminant function is nonzero (in the population), and return to step (3). Steps (3) and (4) are repeated until a nonsignificant test terminates the procedure, or until i=s. Figure 6 presents the steps in this procedure in flow chart form.

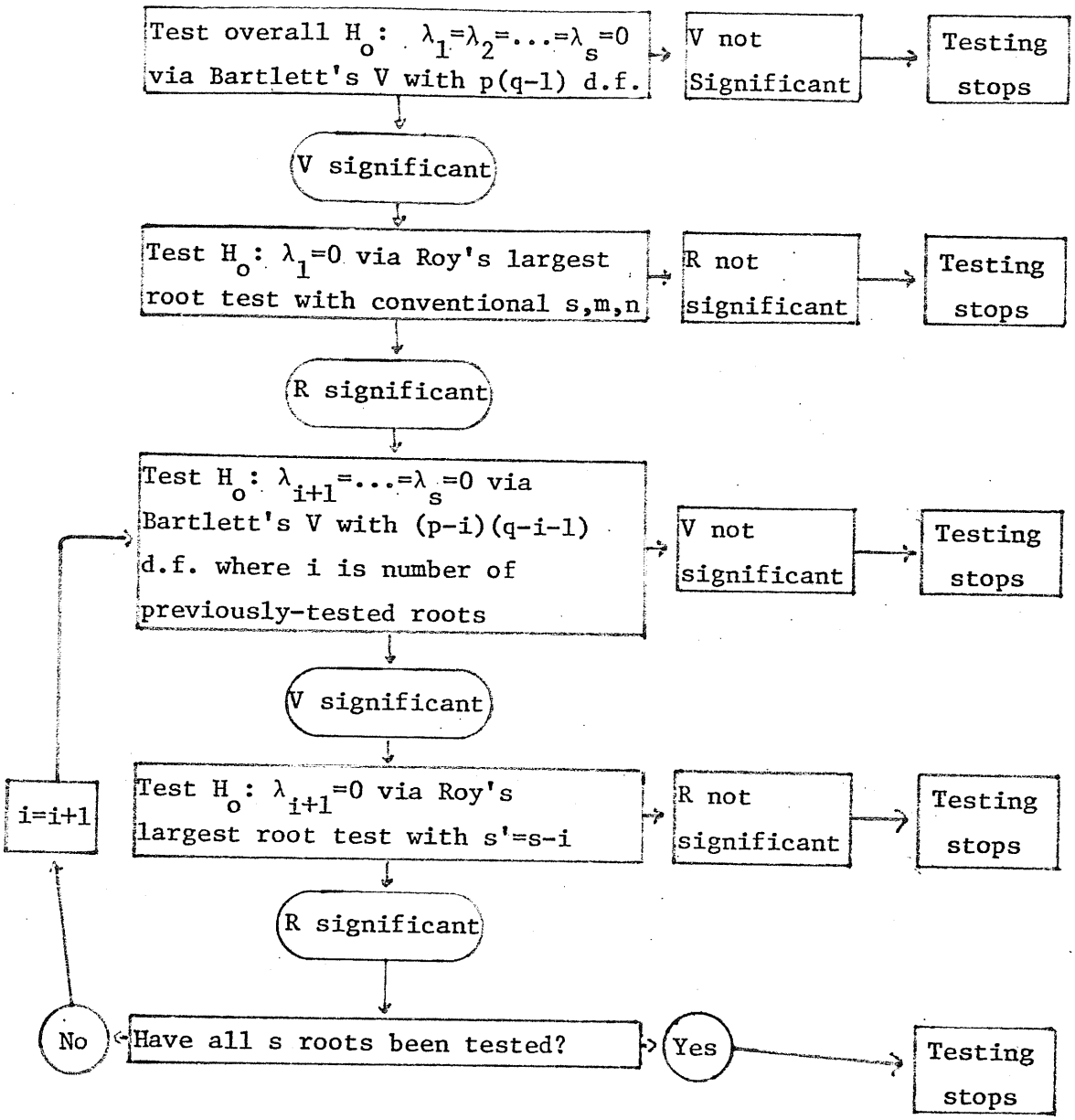

Figure 6. A flow chart showing the steps in Harris' (1976) mixed procedure for testing roots 1 through s.

Harris comments that this procedure might prove frustrating since conflicting conclusions might be suggested by the results of different steps in the procedure. For example, a significant V, which in the conventional partitioned-U procedure would be considered sufficient to reject the null hypothesis that all s roots (including the largest) are zero, might in step (2) be contradicted by a nonsignificant largest root test (of λ_1). A similar type of situation could occur between steps (3) and (4). The reason for these seeming discrepancies is that different sampling distributions are being referred to by the different tests, making this procedure nonconsonant (in Gabriel's (1969) terms).

Marriott's (1952) Procedure. Marriott (1952) proposed a variation on the partitioned-U procedure which both takes into account the possibility that the ordering of the sample roots does not correspond to the ordering of the population roots, and provides a direct test of λ_1. The procedure involves the following steps:

1) Test the hypothesis H_o : $\lambda_1 = 0$ by evaluating the significance of

$$V_1 = - [N - 1 - (p+q) / 2] \quad \ln \ell_1 \qquad (2.11)$$

as a χ^2 variate with degrees of freedom equal to

$$p + q - 2 + 1/2 \, [(p-1)(q-2)]^{2/3}$$

2) Test the hypothesis that $\lambda_{i+1} = 0$ when the previous i roots have been shown to be significant by evaluating

$$V_i = -[N - 1 - (p-i + q-i) / 2] \ \ln \ell_i \qquad (2.12)$$

as a χ^2 variate with degrees of freedom

$$p - i + q - i - 2 + 1/2[(p-i-1)(q-i-2)]^{2/3}$$

Step 2 is repeated with i increasing until either nonsignificance is obtained or i=s. Marriott (1952) reports some data indicating that this procedure performs well when p is small (4 or less) but may become liberal as p increases. The validity of these assertions has not, before the present study, been put to an empirical test.

References

Barnard, M.M. The secular variation of skull characters in four series of Egyptian skulls. Annals of Eugenics, 1935, 6, 189-195.

Bartlett, M.S. The statistical significance of canonical correlations. Biometrika, 1941, 32, 29-37.

Box, G.E.P. and Muller, M.E. A note on the generation of random normal deviates. Annals of Mathematical Statistics, 1958, 29, 610-611.

Clyde, D.J. Multivariate Analysis of Variance on Large Computers. Miami: Clyde Computing Service, 1969.

Cramer, E.M. and Bock, R.D. Multivariate Analysis. Review of Educational Research, 1966, 36, 604-617.

Dempster, A.P. Elements of Continuous Multivariate Analysis. Reading: Addison-Wesley, 1969.

Dixon, W.J. (ed.). BMDP: Biomedical Computer Programs. Berkeley: University of California Press, 1975.

Finn, J.D. MULTIVARIANCE: Univariate and multivariate analysis of variance, covariance and regression. Ann Harbor: National Educational Resources, 1972.

Finn, J.D. A General Model for Multivariate Analysis. New York: Holt, Rinehart and Winston, 1974.

Fisher, R.A. The use of multiple measurements in taxonomic problems. Annals of Eugenics, 1936, 7, 179-188.

Gabriel, K.R. A comparison of some methods of simultaneous inference in MANOVA. In P.R. Krishnaiah (Ed.), Multivariate Analysis - II. New York: Academic Press, 1969.

Harris, R.J. A Primer of Multivariate Statistics. New York: Academic Press, 1975.

Harris, R.J. The invalidity of partitioned-U tests in canonical correlation and multivariate analysis of variance. Multivariate Behavioral Research, 1976, 11, 353-365.

Hotelling, H. The generalization of Student's ratio. Annals of Mathematical Statistics, 1931, 2, 360-378.

Hotelling, H. Relations between two sets of variates. Biometrika, 1936, 28, 321-377.

Hotelling, H. A generalized T test and measure of multivariate dispersion. Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, 1951, 23-41.

Huberty, C.J. Discriminant analysis. Review of Educational Research, 1975, 4, 543-598.

Klecka, W.R. Discriminant analysis. In N.H. Nie, C.H. Hull, J.G. Jenkins, K. Steinbrenner and D.H. Bent (Eds.), SPSS: Statistical Package for the Social Sciences (2nd Edition). New York: McGraw-Hill, 1975.

Kshirsagar, A.M. Multivariate Analysis. New York: Dekker, 1972.

Lawley, D.N. A generalization of Fisher's z test. Biometrika, 1938, 30, 180-187 (corrections 467-469).

Marriott, F.H.C. Tests of significance in canonical analysis. Biometrika, 1952, 39, 58-64.

Marriott, F.H.C. The Interpretation of Multiple Observations. London: Academic Press, 1974.

Marsaglia, G., MacLaren, M.D. and Bray, T.A. A fast procedure for generating normal random variables. Communications of the ACM, 1964, 4-10.

Meredith, C.D. A Monte Carlo investigation of several procedures for testing the significance of individual discriminant functions following multivariate analysis of variance. Doctoral Dissertation, University of Manitoba, 1978.

Miller, R.G. Developments in Multiple Comparisons 1966-1976. Journal of the American Statistical Association, 1977, 72, 779-788.

Nie, N.H., Hull, C.H., Jenkins, J.G., Steinbrenner, K. and Bent, D.H. (Eds.), SPSS: Statistical Package for the Social Sciences (2nd Edition). New York: McGraw-Hill, 1975.

Olson, C.L. A Monte Carlo investigation of the robustness of multivariate analysis of variance. Doctoral Dissertation, University of Toronto, 1973.

Olson, C.L. Comparative robustness of six tests in multivariate analysis of variance. Journal of the American Statistical Association, 1974, 69, 894-908.

Olson, C.L. On choosing a test statistic in multivariate analysis of variance. Psychological Bulletin, 1976, 83, 579-586.

Overall, J.E. and Klett, C.J. Applied Multivariate Analysis. New York: McGraw-Hill, 1972.

Pearson, E.S. and Hartley, H.O. (Eds.). Biometrika Tables for Statisticians. Vol. 2. Cambridge: Cambridge University Press, 1972.

Pillai, K.C.S. Some new test criteria in multivariate analysis. Annals of Mathematical Statistics, 1955, 26, 117-121.

Pillai, K.C.S. Statistical Tables for Tests of Multivariate Hypotheses. Manila: Statistical Center, University of the Philippines, 1960.

Pillai, K.C.S. On the distribution of the largest characteristic root of a matrix in multivariate analysis. Biometrika, 1965, 52, 405-414.

Pillai, K.C.S. Distributions of characteristic roots in multivariate analysis. Part I. Null distributions. The Canadian Journal of Statistics, 1976, 4, 157-184.

Pillai, K.C.S. Distribution of characteristic work in multivariate analysis, Part II. Non-null distributions. The Canadian Journal of Statistics, 1977, 5, 1-62.

Rao, C.R. The ultilization of multiple measurements in problems of biological classification. Journal of the Royal Statistical Society, Series B, 1948, 10, 159-193.

Rao, C.R. Linear Statistical Inference and its Applications. New York: Wiley, 1965.

Roy, J. Power of the likelihood-ratio test in analysis of dispersion. Multivariate Analysis, 1966, 1, 105-127.

Roy, S.N. The individual sampling distribution of the maximum, the minimum and any intermediate one of the p-statistics on the null-hypothesis. Sankhyà, 1945, 7, 133-158.

Roy, S.N. and Bargmann, R.E. Tests of multiple independence and the associated confidence bounds. Annals of Mathematical Statistics, 1958, 29, 491-503.

Schatzoff, M. Exact distributions of Wilks's likelihood ratio criterion. Biometrika, 1966, 53, 347-358. (a) (Corrections in Biometrika, 1967, 54, 688.)

Schatzoff, M. Sensitivity comparisons among tests of the general linear hypothesis. Journal of the American Statistical Association, 1966, 61, 415-435. (b)

Tatsuoka, M.M. Multivariate Analysis: Techniques for Educational and Psychological Research. New York: Wiley, 1971.

Wilkinson, J.H. The Algebraic Eigenvalue Problem. Oxford: Clarendon Press, 1965.

Wilks, S.S. Certain generalizations in the analysis of variance. Biometrika, 1932, 24, 471-494.

Wishart, J. The generalized product moment distribution in samples from a normal multivariate population. Biometrika, 20A, 32-52.

APPENDIX 1

THEORETICAL CRITICAL VALUES

Tables A to E present the theoretical critical values employed in both the overall MANOVA tests, and the tests of individual discriminant functions. No critical values are presented for Finn's (1974) procedure for reasons discussed in the text. Four- and five-point Lagrangian interpolation was used where necessary in obtaining the tabled critical values.

Table A

Theoretical critical values for Bartlett's V

		α=.05			α=.01		
		n=5	n=10	n=50	n=5	n=10	n=50
P	Q						
2	3	9.52	9.49	9.49	13.30	13.30	13.30
	6	18.40	18.30	18.30	23.30	23.20	23.20
	10	28.90	28.90	28.87	34.80	34.80	34.80
3	3	12.68	12.61	12.59	16.96	16.84	16.81
	6	25.18	25.12	25.00	30.89	30.61	30.58
	10	40.30	40.11	40.11	47.20	47.00	46.96
6	3	21.93	21.15	21.07	27.53	26.40	26.30
	6	44.30	43.90	43.80	51.50	51.00	50.90
	10	72.60	72.20	72.15	81.60	81.20	81.07
10	3	39.48	32.00	31.40	48.69	38.50	37.60
	6	69.40	67.80	67.50	78.40	76.50	76.20
	10	114.00	113.20	113.15	125.00	124.20	124.12

The values in this table were obtained from Schatzoff's (1966a) tables, as reproduced in Pearson and Hartley (1972).

Table B

Theoretical critical values for Roy's criterion

		α=.05			α=.01		
		n=5	n=10	n=50	n=5	n=10	n=50
p	Q						
2	3	.520	.277	.057	.650	.368	.080
	6	.440	.235	.048	.535	.295	.062
	10	.390	.205	.042	.460	.245	.052
3	3	.619	.340	.070	.734	.435	.093
	6	.507	.271	.055	.590	.328	.070
	10	.438	.235	.049	.501	.275	.059
6	3	.824	.485	.106	.904	.575	.133
	6	.655	.365	.078	.725	.420	.093
	10	.549	.300	.062	.606	.342	.072
10	3	.981 *	.640	.150	.994 *	.725	.180
	6	.795	.465	.105	.845	.515	.120
	10	.657	.370	.080	.707	.410	.091

The values in this table were obtained from Pillai's (1960, 1965) results for the distribution of the greatest characteristic root, as reproduced in Pearson and Hartley (1972). The asterisk denotes values obtained from Olson's (1973) table of Monte Carlo critical values.

Table C

Theoretical critical values for the partitioned-U procedure

P	Q	Residual 1	2	3	4	5	6	7	8
					α = .05				
2	3	3.84							
	6	9.48							
	10	15.51							
3	3	5.99							
	6	15.51	7.81						
	10	26.30	14.07						
6	3	11.07							
	6	31.41	21.03	12.59	5.99				
	10	55.76	41.34	28.87	18.31	9.85			
10	3	16.92							
	6	51.00	36.42	26.68	12.59				
	10	92.81	74.47	58.12	43.77	31.41	21.03	12.59	5.99
					α = .01				
2	3	6.63							
	6	13.28							
	10	20.09							
3	3	9.21							
	6	20.09	11.34						
	10	32.00	18.48						
6	3	15.09							
	6	37.57	26.22	16.81	9.21				
	10	63.69	48.28	34.81	23.21	13.28			
10	3	21.67							
	6	58.62	42.98	29.14	16.81				
	10	102.82	83.51	66.21	50.89	37.57	26.22	16.81	9.21

The values in this table were obtained from tables of the χ^2 distribution as reproduced in Pearson and Hartley (1972).

Table D

Theoretical critical values for Harris' (1976) variation on Roy's test

P	Q	n	Root 1	2	3	4	5	6	7	8	9
						$\alpha = .01$					
2	3	5	.650	9.07							
		10	.368	7.64							
		50	.080	6.81							
	6	5	.535	4.18							
		10	.295	3.64							
		50	.062	3.42							
	10	5	.460	2.99							
		10	.245	2.68							
		50	.052	2.53							
3	3	5	.734	6.93							
		10	.435	5.49							
		50	.093	4.76							
	6	5	.590	.511	4.64						
		10	.348	.268	4.14						
		50	.070	.048	3.91						
	10	5	.501	.455	3.12						
		10	.275	.242	2.83						
		50	.059	.049	2.66						
6	3	5	.904	6.06							
		10	.575	3.90							
		50	.133	3.16							
	6	5	.725	.673	.608	.519	5.49				
		10	.420	.373	.318	.252	5.01				
		50	.093	.085	.069	.052	4.72				
	10	5	.606	.566	.520	.465	.398	3.79			
		10	.342	.308	.274	.236	.194	3.58			
		50	.072	.065	.056	.047	.038	3.41			
10	3	5	.994	10.16							
		10	.725	3.46							
		50	.180	2.53							
	6	5	.845	.813	.775	.721	3.71				
		10	.515	.478	.431	.375	3.17				
		50	.120	.105	.090	.075	2.88				
	10	5	.707	.679	.646	.609	.565	.512	.447	.366	5.14
		10	.410	.383	.354	.322	.288	.251	.210	.163	4.83
		50	.091	.083	.075	.066	.058	.049	.040	.030	4.62

$\alpha = .05$

2	3	5	.520	4.67							
		10	.277	4.20							
		50	.057	3.91							
	6	5	.440	2.76							
		10	.235	2.50							
		50	.048	2.41							
	10	5	.390	2.17							
		10	.205	2.04							
		50	.042	1.98							
3	3	5	.619	3.89							
		10	.340	3.35							
		50	.070	3.05							
	6	5	.507	.418	2.98						
		10	.271	.212	2.77						
		50	.055	.035	2.66						
	10	5	.438	.389	2.24						
		10	.235	.203	2.11						
		50	.049	.039	2.03						
6	3	5	.824	3.48							
		10	.485	2.62							
		50	.106	2.27							
	6	5	.655	.595	.519	.417	3.35				
		10	.365	.315	.258	.192	3.16				
		50	.078	.069	.055	.039	3.04				
	10	5	.549	.505	.455	.396	.325	2.58			
		10	.300	.267	.233	.195	.154	2.47			
		50	.062	.055	.047	.038	.029	2.39			
10	3	5	.981	4.77							
		10	.640	2.39							
		50	.150	1.94							
	6	5	.795	.754	.706	.637	2.53				
		10	.465	.421	.371	.311	2.27				
		50	.105	.090	.075	.060	2.15				
	10	5	.657	.625	.589	.547	.498	.440	.371	.285	3.21
		10	.370	.342	.312	.280	.246	.208	.168	.123	3.11
		50	.080	.072	.064	.056	.048	.040	.032	.022	3.03

The values in this table, other than the right-most in each row, were obtained from Pillai's (1960, 1965) results for the greatest characteristic root distribution as reproduced in Pearson and Hartley (1972). The right-most entry in each row is a critical value obtained from the F-distribution also as reproduced in Pearson and Hartley (1972). The reason for this change of statistics is discussed in the text.

Table E

Theoretical Critical values for Marriott's (1952) chi-square procedure

P	Q	Root 1	2	3	4	5	6	7	8	9
					$\alpha = .05$					
2	3	8.67	3.84							
	6	14.43	9.49							
	10	21.03	15.51							
3	3	10.75	5.99							
	6	16.92	12.65	7.81						
	10	23.91	19.44	14.07						
6	3	16.16	11.07							
	6	23.27	19.16	15.01	10.76	5.99				
	10	31.22	27.08	22.94	18.75	14.44	9.49			
10	3	22.57	16.92							
	6	30.71	26.50	22.23	17.82	12.59				
	10	39.68	35.57	31.47	27.37	23.26	19.16	15.01	10.76	5.99
					$\alpha = .01$					
2	3	12.33	6.63							
	6	18.89	13.28							
	10	26.22	20.09							
3	3	14.72	9.21							
	6	21.67	16.88	11.34						
	10	29.39	24.47	18.48						
6	3	20.82	15.09							
	6	28.68	24.15	19.53	14.73	9.21				
	10	37.36	32.86	28.32	23.70	18.90	13.28			
10	3	27.92	21.67							
	6	36.81	32.23	27.54	22.67	16.81				
	10	46.50	42.07	37.62	33.17	28.68	24.15	19.52	14.73	9.21

The values in this table were obtained from tables of the χ^2 distribution as reproduced in Pearson and Hartley (1972).

APPENDIX 2

Table F presents the noncentrality roots (the eigenvalues of H*) used to achieve the desired degrees and patterns of multivariate noncentrality.

Table F

Values of the roots of H* (h_j, j=1,r) for combinations of r, tr (H*) and pattern of noncentrality

Tr(H*)	Pattern	Roots of H*				
		h_1	h_2	h_3	h_4	h_5
	r = 1					
10	concentrated	10.00				
40	concentrated	40.00				
90	concentrated	90.00				
	r = 2					
10	equal	5.00	5.00			
	linear/geometric	6.67	3.33			
40	equal	20.00	20.00			
	linear geometric	26.67	13.33			
90	equal	45.00	45.00			
	linear/geometric	60.00	30.00			
	r = 3					
10	equal	3.33	3.33	3.33		
	linear	5.00	3.33	1.67		
	geometric	5.71	2.86	1.43		
40	equal	13.33	13.33	13.33		
	linear	20.00	13.33	6.67		
	geometric	22.86	11.43	5.71		
90	equal	30.00	30.00	30.00		
	linear	45.00	30.00	15.00		
	geometric	51.43	25.71	12.86		
	r = 5					
10	equal	2.00	2.00	2.00	2.00	2.00
	linear	3.33	2.67	2.00	1.33	.67
	geometric	5.16	2.58	1.29	.65	.32
40	equal	8.00	8.00	8.00	8.00	8.00
	linear	13.33	10.67	8.00	5.33	2.67
	geometric	20.65	10.32	5.16	2.58	1.29
90	equal	18.00	18.00	18.00	18.00	18.00
	linear	30.00	24.00	18.00	12.00	6.00
	geometric	46.45	23.23	11.61	5.81	2.90

APPENDIX 3

OVERALL MANOVA TEST RESULTS

Table G presents rates of rejection of the overall MANOVA null hypothesis by Bartlett's V and Roy's largest root criterion for combinations of p, q and n at α levels of .05 and .01. All data in this table were obtained in the course of conducting study I and therefore represent estimates of type I error rate. These data were compiled for, and are presented as checks on the data generation procedure.

Table G

Rejection rates for Bartlett's V and Roy's largest root (R) overall MANOVA tests for combinations of p, q and n

			$\alpha = .05$		$\alpha = .01$	
p	q	n	V	R	V	R
2	3	5	.059	.057	.013	.010
		10	.062	.059	.013	.011
		50	.054	.051	.009	.009
2	6	5	.049	.052	.010	.007
		10	.048	.043	.008	.007
		50	.050	.049	.009	.009
2	10	5	.050	.052	.012	.018
		10	.053	.051	.011	.012
		50	.050	.046	.010	.006
3	3	5	.054	.061	.011	.011
		10	.050	.050	.010	.009
		50	.052	.058	.011	.012
3	6	5	.055	.047	.010	.008
		10	.056	.055	.012	.012
		50	.051	.057	.010	.011
3	10	5	.056	.050	.013	.009
		10	.050	.044	.014	.010
		50	.054	.045	.009	.009
6	3	5	.074	.067	.018	.012
		10	.061	.056	.012	.012
		50 *	.059	.060	.012	.013
6	6	5	.050	.048	.011	.012
		10	.047	.044	.009	.008
		50 *	.043	.051	.009	.006
6	10	5	.051	.049	.012	.009
		10	.051	.048	.012	.008
		50 *	.041	.054	.007	.013
10	3	5	.069	.050	.019	.009
		10	.062	.057	.013	.008
		50 *	.049	.044	.008	.008
10	6	5	.048	.049	.011	.010
		10	.050	.044	.010	.009
		50 *	.049	.047	.007	.006
10	10	5	.059	.053	.014	.010
		10 *	.062	.052	.008	.010
		50 *	.044	.053	.006	.011

* Estimate based on 1000 rather than 5000 replications.

APPENDIX 4

COMPUTATIONAL DETAILS

The pseudorandom unit normal deviates which comprised the elements of T_1 and T_2 were generated by the procedure due to Marsaglia, MacLaren and Bray (1964). In this method, pairs of independent pseudorandom numbers (U_1 , U_2) in the range of zero to one are generated from a uniform distribution and are then transformed into pairs of pseudorandom unit normal variates (z_1 , z_2) by the following relations due to Box and Muller (1958):

$$z_1 = (-2 \ln U_1)^{1/2} \cos (2 \Pi U_2)$$

$$z_2 = (-2 \ln U_1)^{1/2} \sin (2 \Pi U_2) \quad .$$

Seed values for the generator for each condition run were obtained from preceding runs in order to ensure independence of the samples. Calculation of E and H was accomplished in double precision using subroutine VMULFP in the International Mathematical and Statistical Library (IMSL) package. Subsequently, E was inverted using IMSL subroutine LINV2F, which utilizes the Gaussian elimination method. E^{-1} and H were then multiplied together and the eigenvalues of $E^{-1}H$ were obtained using IMSL subroutine EIGRF in which $E^{-1}H$ is first reduced to upper Hessenberg form, and the roots are solved for using the QR method (Wilkinson, 1965). The simulation program was written in FORTRAN for execution on the IBM 370/168 computer at the University of Manitoba.

APPENDIX 5

AN ILLUSTRATIVE EXAMPLE

In an effort to completely describe the procedures investigated in the present study, they were applied to a real data set, namely the much-analyzed "Egyptian skulls" data reported by Barnard (1935). Briefly, the data set comprises 398 observations divided into four groups dating from different historical periods as follows:

1) predynastic ($n_1 = 91$)
2) dynasties 6 to 12 ($n_2 = 162$)
3) dynasties 12 and 13 ($n_3 = 70$)
4) ptolemaic dynasties ($n_4 = 75$)

The dependent variables were measurements of four characteristics of skull size, namely:

dv_1 : maximum breadth (the width of the skull across the temples)

dv_2 : basi-alveolar height (the distance from the base of the skull to between the upper front teeth)

dv_3 : nasal height (the vertical distance between the top and base of the nasal aperture)

dv_4 : basi-bregmatic height (the distance between the base and the top of the skull)

Marriott (1974) reports that these data are believed to conform reasonably well to the assumptions of classical multivariate analysis procedures, and are complete. Table H presents group means on the four dependent variables. As shown in this table, the means across time

increase on dv_1 , decrease on dv_2 and vary nonmonotonically on dv_3 and dv_4 .

Table H

Group means on the four dependent variables
(from Barnard, 1935)

		group			
		1	2	3	4
dependent variable	1	133.58	134.27	134.38	135.31
	2	98.31	96.46	95.86	95.04
	3	50.84	51.15	50.10	52.09
	4	133.00	134.88	133.64	131.47

Table I presents the variances and intercorrelations (within groups) of the four dependent variables. Inspection of this table will reveal the variables to be for the most part moderately positively correlated with the exception of variables one and two (r_{12} = .048).

Table I

Matrix of within-group sample correlations (off diagonal)
and sample variances (on the principal diagonal)
for the four dependent variables

		dependent variable			
		1	2	3	4
dependent variable	1	24.520			
	2	.048	23.030		
	3	.183	.208	10.000	
	4	.234	.253	.217	22.190

The Test Procedures

Calculation of the sample eigenvalues for these data revealed the following values: $\ell_1 = .123$, $\ell_2 = .063$, and $\ell_3 = .019$. Values of the statistics for the partitioned-U procedure, Harris' (1976) variation of Roy's test, Harris' (1976) mixed procedure, and Marriott's (1952) chi-square procedure, as well as the appropriate overall MANOVA tests were calculated as shown below, with the results indicated. To recap the numerical values used in these computations: p=4, q=4, N=398, $\ell_1 = .123$, $\ell_2 = .063$, and $\ell_3 = .019$. All tests were performed at the .01 level of significance.

1) The partitioned-U procedure. The first step would be to test the overall MANOVA null hypothesis via Bartlett's V as

$$V = [N-1-(p+q) / 2] \sum_{i=1}^{r} \ln(1+\ell_i)$$

as a chi-square with p(q-1) degrees of freedom. In the present case

$$\begin{aligned} V &= [398-1-(4+4)/2] (\ln(1.123) + \ln(1.063) + \ln(1.019) \\ &= 393 \times .196 \\ &= 77.03 \end{aligned}$$

which is compared to a critical value of $\chi^2_{(12)} = 26.22$ resulting in a rejection of the overall MANOVA null hypothesis. The next step is then to test the null hypothesis $H_o : \lambda_1 \neq 0 , \lambda_2 \ldots \lambda_s = 0$ by evaluating the first residual V as a chi-square with (p-1)(q-2) degrees of freedom. Hence,

$$V_1 = 393 \times (\ln(1.063) + \ln(1.019))$$

$$= 393 \times .08$$

$$= 31.44$$

which, when compared to critical value $\chi^2_{(6)} = 16.81$ leads to rejection of the null hypothesis. The second residual is then tested as

$$V_2 = 393 \times \ln(1.019)$$

$$= 393 \times .019$$

$$= 7.47$$

which is evaluated against a critical $\chi^2_{(2)} = 9.21$ and found not to be significant. To summarize the results of the partitioned-U procedure, λ_1 and λ_2 are concluded to be nonzero while $\lambda_3 = 0$.

2) Harris' variation of Roy's test. The test of the first root (and simultaneously of the overall null hypothesis) is accomplished by calculating R_1 as

$$R_1 = \frac{\ell_1}{1 + \ell_1} = \frac{.123}{1.123} = .110$$

This value is compared to the greatest characteristic root distribution critical value (with degrees of freedom s=3, m=0, n=195) of .050, with the result that the null hypothesis $H_o : \lambda_1 = 0$ is rejected.

Next, the significance of the second root is tested as

$$R_2 = \frac{\ell_2}{1 + \ell_2} = \frac{.063}{1.063} = .059$$

by comparing it to a critical value of .037 (degrees of freedom: s'=2, m=0, n=195, note the reduction in s). This finding leads to a rejection of $H_o : \lambda_2 = 0$ and testing continues.

Since s=3 for this data set, the third root must be tested as an F-statistic as no tables of the greatest characteristic root exist for s' = 3-2 = 1 (two prior roots tested). This is accomplished by converting ℓ_3 to its squared canonical correlation form as

$$r_3^2 = \frac{\ell_3}{1 + \ell_3} = \frac{.019}{1.019} = .0187$$

and converting this squared multiple correlation (since s'=1) to an F-statistic as

$$F = \frac{r_3^2 / k}{1 - r_3^2 / N-k-1} = \frac{.0187/2}{1 - .0187 / 398-2-1} = 3.76$$

where k is the absolute value of the difference between p and df_h plus one. This value is referred to the F distribution with k and N-k-1 (here 2, 395) degrees of freedom. Since the critical F is approximately 4.65, the third discriminant function is deemed nonsignificant. In summary, use of this procedure would lead one to conclude that the first two population roots were nonzero while $\lambda_3 = 0$.

3) Harris' mixed procedure. This procedure involves combining the results of the preceding two tests with the following conslusions:

a) V overall was found significant as was R_1 . Hence λ_1 is concluded to be nonzero.

b) V_1 was found to be significantly nonzero with the result that testing continued.

c) R_2 was evaluated and found to be significant with the result that λ_2 was concluded to be nonzero.

d) V_2 was tested and found to be nonsignificant. Testing stops.

Hence use of this procedure would also lead to the conclusion that only the first two functions are significant.

4) Marriott's chi-square procedure. Use of this test would follow a rejection of the overall null hypothesis by V as described in (1). Assuming a significant V (as in the present data) the significance of the largest sample root would proceed as

$$M_1 = - [N-1-(p+q) / 2] \ln \ell_1$$

$$= - [398-1-4] \ln .123$$

$$= 45.59$$

which is evaluated as a chi-square with

$$p + q - 2 + 1/2 [(p-1)(q-2)]^{2/3} = 7.65$$

degrees of freedom. Since M_1 exceeds the critical value of $\chi^2_{(7.65)}$ = 19.52, λ_1 is concluded to be nonzero.

Testing proceeds with an evaluation of ℓ_2 as

$$M_2 = - [N-1-(p-1+q-1) / 2] \ln \ell_2$$

$$= - [398-1-3] \ln .06$$

$$= 24.03$$

which is evaluated as a chi-square with

$$p - 1 + q - 3 + 1/2\ [(p-2)(q-3)]^{2/3} = 4.8$$

degrees of freedom. Since M_2 exceeds $\chi^2_{(48)} = 14.73$, λ_2 is concluded to be nonzero.

In the same manner λ_3 is tested with p and q again reduced by one yielding the following values: $M_3 = 7.51$, $\chi^2_{(2)} = 9.21$. Hence λ_3 is concluded to equal zero. Use of this procedure, as with the others, leads to the conclusion that the first two discriminant functions are significant, while the third reflects only sampling error.

Interpretation of the results

Having determined that there exist two reliable discriminant functions, or 'dimensions of difference' present in these data, the next task becomes that of interpreting the meaning of the dimensions in terms of the original variables. This is accomplished by examining the discriminant function coefficients (presented in both raw and standardized form in table J) and using these weights to calculate group means on the composites (presented in table K). As a further aid to interpretation univariate ANOVAs were performed on the original dependent variables

Table J

Discriminant function coefficients

	function 1		function 2	
variable	raw	standardized	raw	standardized
1) skull breadth	-.0713	-.3531	.0241	.1192
2) skull depth	.0702	.3369	-.1870	-.8963
3) nasal height	-.1900	-.6001	.0415	.1312
4) skull height	.1671	.7871	.1419	.6685

Table K

Group means on the composites (raw scores)

group	function 1	function 2
1	9.909	5.847
2	9.985	6.485
3	9.929	6.380
4	9.064	6.334

with the following results: the group effect was significant at the .005 level for variables 2, 3 and 4 (skull depth, nasal height and skull height, respectively) and not significant for skull breadth ($p > .16$).

As shown in table J, function 1 weights variables 3 and 4 heavily relative to variables 1 and 2, with skull height positively weighted while nasal height is negatively weighted. Inspection of table K reveals that function 1 is differentiating group 4 from the other three groups. Taken along with the pattern of group means presented in table H, these values suggest that dimension 1 is one of 'noseyness', that is of nasal height relative to skull height with group 4 having the greatest nasal height and the least skull height. In contrast, function 2 differentiates group 1 from the other groups on variables 2 and 4 (skull depth and skull height, respectively) reflecting the greater overall size (except for breadth) of the group 1 skulls relative to the other groups.

APPENDIX 6

RAW DATA

The raw data from this investigation are presented in table L as frequencies with which the critical values were exceeded in each condition. As noted in the text, these frequencies were obtained from 5000 replications in all null cases except those in which n=50, for p at least 6, or with p= q= n= 10. In these cases, as well as all of the noncentral cases, 1000 replications were simulated.

The first six columns of table L specify the levels of the factors in the study as follows:

column	factor	levels
1	p, the number of variates	2, 3, 6, 10
2	q, the number of groups	3, 6, 10
3	n, the group size	5, 10, 50
4	r, the number of nonzero population roots	1, 2, 3, 5
5	tr(H*), the noncentrality parameter	0, 10, 40, 90 (0 for the null cases)
6	pattern of roots	1 = concentrated 2 = equal 3 = linear 4 = geometric

Hence, the null cases are those in which r=0, tr(H*)=0 and pattern=0.

Column seven in table L specifies the test criterion referred to in the remaining columns for that line of the table, with 1 indicating the partitioned-U procedure; 2, Harris' mixed procedure; 3, Harris' variation of Roy's test; and, 4, Marriott's procedure. The next seven columns contain the frequency of exceedance of the critical values for tests of roots one through seven (limited by s) for a nominal significance level of .05. The final seven columns present the same information for a nominal significance level of .01.

Table L

							alpha=.05			alpha=.01		
p	q	n	r	trh	pat	t	1	2	3	1	2	3
3	10	10	0	0	0	1	250	14	0	69	0	0
3	10	10	0	0	0	2	142	0	0	29	0	0
3	10	10	0	0	0	3	229	2	0	51	0	0
3	10	10	0	0	0	4	192	12	0	32	0	0
6	6	10	0	0	0	1	233	7	0	46	0	0
6	6	10	0	0	0	2	113	0	0	19	0	0
6	6	10	0	0	0	3	222	3	0	40	0	0
6	6	10	0	0	0	4	143	8	0	21	0	0
6	3	5	0	0	0	1	369	12	0	91	0	0
6	3	5	0	0	0	2	263	0	0	49	0	0
6	3	5	0	0	0	3	335	0	0	58	0	0
6	3	5	0	0	0	4	301	10	0	77	0	0
6	6	5	0	0	0	1	249	13	2	54	1	0
6	6	5	0	0	0	2	136	0	0	23	0	0
6	6	5	0	0	0	3	241	3	0	59	0	0
6	6	5	0	0	0	4	168	6	0	31	0	0
6	3	10	0	0	0	1	303	13	0	62	1	0
6	3	10	0	0	0	2	217	0	0	39	0	0
6	3	10	0	0	0	3	282	0	0	59	0	0
6	3	10	0	0	0	4	234	10	0	50	0	0
3	10	50	0	0	0	1	268	4	0	44	0	0
3	10	50	0	0	0	2	143	0	0	19	0	0
3	10	50	0	0	0	3	227	4	0	43	0	0
3	10	50	0	0	0	4	186	4	0	22	0	0
3	6	10	0	0	0	1	279	10	1	61	0	0
3	6	10	0	0	0	2	168	0	0	34	0	0
3	6	10	0	0	0	3	277	3	0	29	0	0
3	6	10	0	0	0	4	208	7	0	42	0	0
6	10	5	0	0	0	1	254	5	0	60	2	0
6	10	5	0	0	0	2	111	0	0	21	0	0
6	10	5	0	0	0	3	245	2	0	47	0	0
6	10	5	0	0	0	4	147	7	0	29	2	0
3	6	5	0	0	0	1	275	11	2	51	1	0
3	6	5	0	0	0	2	145	0	0	17	0	0
3	6	5	0	0	0	3	237	3	0	40	0	0
3	6	5	0	0	0	4	206	14	1	23	0	0
10	3	10	0	0	0	1	309	15	0	66	1	0
10	3	10	0	0	0	2	193	0	0	27	0	0
10	3	10	0	0	0	3	284	0	0	40	0	0
10	3	10	0	0	0	4	233	15	0	43	2	0
3	3	10	0	0	0	1	245	14	0	49	1	0
3	3	10	0	0	0	2	195	2	0	32	0	0
3	3	10	0	0	0	3	252	2	0	45	0	0
3	3	10	0	0	0	4	214	14	0	36	0	0
3	6	50	0	0	0	1	253	11	1	52	1	0
3	6	50	0	0	0	2	169	1	0	32	0	0
3	6	50	0	0	0	3	287	17	0	53	3	0
3	6	50	0	0	0	4	186	9	1	33	2	0
3	3	50	0	0	0	1	253	9	0	54	0	0
3	3	50	0	0	0	2	212	2	0	39	0	0
3	3	50	0	0	0	3	289	2	0	61	0	0
3	3	50	0	0	0	4	224	11	0	40	0	0
3	10	5	0	0	0	1	280	15	1	67	5	0
3	10	5	0	0	0	2	160	1	0	24	0	0
3	10	5	0	0	0	3	250	1	0	45	0	0
3	10	5	0	0	0	4	209	8	1	28	0	0
6	10	10	0	0	0	1	254	5	0	58	0	0
6	10	10	0	0	0	2	117	0	0	16	0	0
6	10	10	0	0	0	3	238	3	0	42	0	0
6	10	10	0	0	0	4	153	6	0	21	0	0
10	6	10	0	0	0	1	250	9	0	48	1	0
10	6	10	0	0	0	2	117	0	0	11	0	0
10	6	10	0	0	0	3	220	2	0	45	0	0
10	6	10	0	0	0	4	147	13	0	17	0	0
10	10	5	0	0	0	1	294	10	0	71	0	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

							alpha=.05			alpha=.01		
p	q	n	r	trh	pat	t	1	2	3	1	2	3
10	10	5	0	0	0	3	264	2	0	50	0	0
10	10	5	0	0	0	4	167	14	0	24	0	0
10	6	5	0	0	0	1	242	5	0	56	0	0
10	6	5	0	0	0	2	126	0	0	25	0	0
10	6	5	0	0	0	3	247	1	0	51	0	0
10	6	5	0	0	0	4	173	15	0	39	1	0
10	3	5	0	0	0	1	347	52	0	96	5	0
10	3	5	0	0	0	2	213	5	0	38	0	0
10	3	5	0	0	0	3	251	5	0	45	0	0
10	3	5	0	0	0	4	346	93	0	94	10	0
3	3	5	0	0	0	1	271	15	0	57	0	0
3	3	5	0	0	0	2	219	7	0	40	0	0
3	3	5	0	0	0	3	307	7	0	55	0	0
3	3	5	0	0	0	4	236	25	0	40	0	0
2	3	50	0	0	0	1	268	23	0	46	1	0
2	3	50	0	0	0	2	220	8	0	36	1	0
2	3	50	0	0	0	3	253	8	0	43	1	0
2	3	50	0	0	0	4	243	17	0	37	1	0
2	10	50	0	0	0	1	252	8	0	50	1	0
2	10	50	0	0	0	2	160	0	0	18	0	0
2	10	50	0	0	0	3	231	0	0	28	0	0
2	10	50	0	0	0	4	164	6	0	22	0	0
2	3	10	0	0	0	1	309	26	0	69	0	0
2	3	10	0	0	0	2	266	11	0	49	0	0
2	3	10	0	0	0	3	297	12	0	56	0	0
2	3	10	0	0	0	4	282	25	0	52	1	0
2	3	5	0	0	0	1	296	21	0	66	1	0
2	3	5	0	0	0	2	247	6	0	49	1	0
2	3	5	0	0	0	3	285	8	0	52	1	0
2	3	5	0	0	0	4	261	23	0	53	1	0
2	10	10	0	0	0	1	263	6	0	57	1	0
2	10	10	0	0	0	2	173	0	0	33	1	0
2	10	10	0	0	0	3	255	0	0	56	1	0
2	10	10	0	0	0	4	180	7	0	33	1	0
2	6	10	0	0	0	1	240	8	0	41	1	0
2	6	10	0	0	0	2	165	0	0	21	1	0
2	6	10	0	0	0	3	216	0	0	34	1	0
2	6	10	0	0	0	4	187	8	0	31	1	0
2	6	50	0	0	0	1	250	7	0	43	1	0
2	6	50	0	0	0	2	171	0	0	23	1	0
2	6	50	0	0	0	3	244	0	0	45	1	0
2	6	50	0	0	0	4	184	7	0	28	1	0
2	10	5	0	0	0	1	251	6	0	59	0	0
2	10	5	0	0	0	2	171	0	0	36	0	0
2	10	5	0	0	0	3	259	0	0	59	0	0
2	10	5	0	0	0	4	179	7	0	37	0	0
2	6	5	0	0	0	1	246	9	0	48	0	0
2	6	5	0	0	0	2	178	0	0	19	0	0
2	6	5	0	0	0	3	262	0	0	36	0	0
2	6	5	0	0	0	4	190	10	0	30	0	0
10	10	50	0	0	0	1	44	0	0	6	0	0
10	10	50	0	0	0	2	16	0	0	2	0	0
10	10	50	0	0	0	3	53	0	0	11	0	0
10	10	50	0	0	0	4	21	1	0	2	0	0
6	6	50	0	0	0	1	43	2	0	9	0	0
6	6	50	0	0	0	2	25	0	0	3	0	0
6	6	50	0	0	0	3	51	0	0	6	0	0
6	6	50	0	0	0	4	31	2	0	3	0	0
6	3	50	0	0	0	1	59	1	0	12	0	0
6	3	50	0	0	0	2	46	0	0	8	0	0
6	3	50	0	0	0	3	60	0	0	13	0	0
6	3	50	0	0	0	4	47	2	0	10	0	0
6	10	50	0	0	0	1	41	1	0	7	0	0
6	10	50	0	0	0	2	26	0	0	5	0	0
6	10	50	0	0	0	3	54	0	0	13	0	0
6	10	50	0	0	0	4	27	0	0	5	0	0

p	q	n	r	trh	pat	t	alpha=.05 1	2	3	alpha=.01 1	2	3
10	3	50	0	0	0	1	49	1	0	8	0	0
10	3	50	0	0	0	2	31	0	0	2	0	0
10	3	50	0	0	0	3	44	0	0	8	0	0
10	3	50	0	0	0	4	36	0	0	5	0	0
10	6	50	0	0	0	1	49	2	0	7	0	0
10	6	50	0	0	0	2	23	0	0	1	0	0
10	6	50	0	0	0	3	47	1	0	6	0	0
10	6	50	0	0	0	4	33	1	0	2	0	0
10	10	10	0	0	0	1	62	1	0	8	0	0
10	10	10	0	0	0	2	23	0	0	1	0	0
10	10	10	0	0	0	3	52	0	0	10	0	0
10	10	10	0	0	0	4	37	1	0	2	0	0

The data below are from the noncentral cases.

p	q	n	r	trh	pat	t	alpha=.05 1	2	3	4	5	6	7	alpha=.01 1	2	3	4	5	6	7
3	6	50	3	10	2	1	439	80	3	0	0	0	0	221	14	0	0	0	0	0
3	6	50	3	10	2	2	335	7	0	0	0	0	0	129	1	0	0	0	0	0
3	6	50	3	10	2	3	378	90	0	0	0	0	0	147	18	0	0	0	0	0
3	6	50	3	10	2	4	363	67	4	0	0	0	0	137	10	0	0	0	0	0
3	6	50	3	10	3	1	491	80	11	0	0	0	0	254	18	1	0	0	0	0
3	6	50	3	10	3	2	372	10	0	0	0	0	0	153	1	0	0	0	0	0
3	6	50	3	10	3	3	423	100	0	0	0	0	0	170	19	0	0	0	0	0
3	6	50	3	10	3	4	408	74	8	0	0	0	0	158	11	0	0	0	0	0
3	6	50	3	10	4	1	454	62	3	0	0	0	0	220	12	0	0	0	0	0
3	6	50	3	10	4	2	344	7	0	0	0	0	0	128	2	0	0	0	0	0
3	6	50	3	10	4	3	399	91	0	0	0	0	0	149	8	0	0	0	0	0
3	6	50	3	10	4	4	368	53	5	0	0	0	0	141	5	0	0	0	0	0
3	6	50	3	40	2	1	991	878	430	0	0	0	0	975	666	147	0	0	0	0
3	6	50	3	40	2	2	964	430	146	0	0	0	0	847	155	2	0	0	0	0
3	6	50	3	40	2	3	964	860	146	0	0	0	0	848	571	2	0	0	0	0
3	6	50	3	40	2	4	974	775	483	0	0	0	0	860	480	138	0	0	0	0
3	6	50	3	40	3	1	994	810	283	0	0	0	0	967	583	90	0	0	0	0
3	6	50	3	40	3	2	977	432	90	0	0	0	0	870	166	0	0	0	0	0
3	6	50	3	40	3	3	977	818	90	0	0	0	0	870	551	0	0	0	0	0
3	6	50	3	40	3	4	983	750	328	0	0	0	0	881	467	77	0	0	0	0
3	6	50	3	40	4	1	995	754	238	0	0	0	0	972	502	69	0	0	0	0
3	6	50	3	40	4	2	975	367	68	0	0	0	0	893	143	0	0	0	0	0
3	6	50	3	40	4	3	975	778	68	0	0	0	0	893	519	0	0	0	0	0
3	6	50	3	40	4	4	982	698	269	0	0	0	0	903	417	67	0	0	0	0
3	6	50	3	90	2	1	1000	1000	964	0	0	0	0	1000	999	877	0	0	0	0
3	6	50	3	90	2	2	1000	990	877	0	0	0	0	1000	936	414	0	0	0	0
3	6	50	3	90	2	3	1000	1000	877	0	0	0	0	1000	998	414	0	0	0	0
3	6	50	3	90	2	4	1000	1000	981	0	0	0	0	1000	993	887	0	0	0	0
3	6	50	3	90	3	1	1000	1000	858	0	0	0	0	1000	996	659	0	0	0	0
3	6	50	3	90	3	2	1000	974	659	0	0	0	0	1000	885	163	0	0	0	0
3	6	50	3	90	3	3	1000	1000	659	0	0	0	0	1000	994	163	0	0	0	0
3	6	50	3	90	3	4	1000	999	887	0	0	0	0	1000	980	674	0	0	0	0
3	6	50	3	90	4	1	1000	999	786	0	0	0	0	1000	991	530	0	0	0	0
3	6	50	3	90	4	2	1000	959	526	0	0	0	0	1000	826	86	0	0	0	0
3	6	50	3	90	4	3	1000	999	526	0	0	0	0	1000	986	86	0	0	0	0
3	6	50	3	90	4	4	1000	994	829	0	0	0	0	1000	969	557	0	0	0	0

p	q	n	r	trh	pat	t	alpha=.05 1	2	3	4	5	6	7	alpha=.01 1	2	3	4	5	6	7
2	3	5	1	10	1	1	549	40	0	0	0	0	0	271	2	0	0	0	0	0
2	3	5	1	10	1	2	513	32	0	0	0	0	0	223	2	0	0	0	0	0
2	3	5	1	10	1	3	537	34	0	0	0	0	0	238	3	0	0	0	0	0
2	3	5	1	10	1	4	529	60	0	0	0	0	0	244	4	0	0	0	0	0
2	3	5	2	10	2	1	631	231	0	0	0	0	0	360	73	0	0	0	0	0
2	3	5	2	10	2	2	485	172	0	0	0	0	0	202	33	0	0	0	0	0
2	3	5	2	10	2	3	488	180	0	0	0	0	0	203	37	0	0	0	0	0
2	3	5	2	10	2	4	555	299	0	0	0	0	0	232	54	0	0	0	0	0
2	3	5	2	10	3	1	597	199	0	0	0	0	0	331	47	0	0	0	0	0
2	3	5	2	10	3	2	471	152	0	0	0	0	0	190	25	0	0	0	0	0
2	3	5	2	10	3	3	478	160	0	0	0	0	0	197	28	0	0	0	0	0
2	3	5	2	10	3	4	525	228	0	0	0	0	0	216	46	0	0	0	0	0
2	3	5	1	40	1	1	995	58	0	0	0	0	0	938	19	0	0	0	0	0
2	3	5	1	40	1	2	996	38	0	0	0	0	0	922	16	0	0	0	0	0
2	3	5	1	40	1	3	996	63	0	0	0	0	0	933	17	0	0	0	0	0
2	3	5	1	40	1	4	996	102	0	0	0	0	0	929	24	0	0	0	0	0
2	3	5	2	40	2	1	997	940	0	0	0	0	0	983	727	0	0	0	0	0
2	3	5	2	40	2	2	977	924	0	0	0	0	0	845	652	0	0	0	0	0
2	3	5	2	40	2	3	977	933	0	0	0	0	0	845	677	0	0	0	0	0
2	3	5	2	40	2	4	992	958	0	0	0	0	0	878	752	0	0	0	0	0
2	3	5	2	40	3	1	993	892	0	0	0	0	0	970	645	0	0	0	0	0
2	3	5	2	40	3	2	980	879	0	0	0	0	0	858	596	0	0	0	0	0
2	3	5	2	40	3	3	981	891	0	0	0	0	0	858	610	0	0	0	0	0
2	3	5	2	40	3	4	987	935	0	0	0	0	0	880	670	0	0	0	0	0
2	3	5	1	90	1	1	1000	42	0	0	0	0	0	999	4	0	0	0	0	0
2	3	5	1	90	1	2	1000	42	0	0	0	0	0	999	4	0	0	0	0	0
2	3	5	1	90	1	3	1000	44	0	0	0	0	0	999	4	0	0	0	0	0
2	3	5	1	90	1	4	1000	84	0	0	0	0	0	999	11	0	0	0	0	0
2	3	5	2	90	2	1	1000	1000	0	0	0	0	0	1000	996	0	0	0	0	0
2	3	5	2	90	2	2	1000	1000	0	0	0	0	0	999	995	0	0	0	0	0
2	3	5	2	90	2	3	1000	1000	0	0	0	0	0	999	996	0	0	0	0	0
2	3	5	2	90	2	4	1000	1000	0	0	0	0	0	999	996	0	0	0	0	0
2	3	5	2	90	3	1	1000	998	0	0	0	0	0	1000	976	0	0	0	0	0
2	3	5	2	90	3	2	1000	998	0	0	0	0	0	1000	976	0	0	0	0	0
2	3	5	2	90	3	3	1000	998	0	0	0	0	0	1000	980	0	0	0	0	0
2	3	5	2	90	3	4	1000	999	0	0	0	0	0	1000	989	0	0	0	0	0
2	3	50	1	40	1	2	1000	46	0	0	0	0	0	1000	5	0	0	0	0	0
2	3	50	1	40	1	3	1000	46	0	0	0	0	0	1000	5	0	0	0	0	0
2	3	50	1	40	1	4	1000	82	0	0	0	0	0	1000	12	0	0	0	0	0
2	3	50	2	40	2	1	999	974	0	0	0	0	0	995	890	0	0	0	0	0
2	3	50	2	40	2	2	995	971	0	0	0	0	0	986	887	0	0	0	0	0
2	3	50	2	40	2	3	995	971	0	0	0	0	0	986	887	0	0	0	0	0
2	3	50	2	40	2	4	998	983	0	0	0	0	0	986	908	0	0	0	0	0
2	3	50	2	40	3	1	1000	916	0	0	0	0	0	998	787	0	0	0	0	0
2	3	50	2	40	3	2	997	913	0	0	0	0	0	985	783	0	0	0	0	0
2	3	50	2	40	3	3	997	914	0	0	0	0	0	985	783	0	0	0	0	0
2	3	50	2	40	3	4	997	941	0	0	0	0	0	985	820	0	0	0	0	0
2	3	50	1	90	1	1	1000	56	0	0	0	0	0	1000	10	0	0	0	0	0
2	3	50	1	90	1	2	1000	56	0	0	0	0	0	1000	10	0	0	0	0	0
2	3	50	1	90	1	3	1000	56	0	0	0	0	0	1000	10	0	0	0	0	0
2	3	50	1	90	1	4	1000	95	0	0	0	0	0	1000	15	0	0	0	0	0
2	3	50	2	90	2	1	1000	1000	0	0	0	0	0	1000	999	0	0	0	0	0
2	3	50	2	90	2	2	1000	1000	0	0	0	0	0	1000	999	0	0	0	0	0
2	3	50	2	90	2	3	1000	1000	0	0	0	0	0	1000	999	0	0	0	0	0
2	3	50	2	90	2	4	1000	1000	0	0	0	0	0	1000	1000	0	0	0	0	0
2	3	50	2	90	3	1	1000	1000	0	0	0	0	0	1000	998	0	0	0	0	0
2	3	50	2	90	3	2	1000	1000	0	0	0	0	0	1000	998	0	0	0	0	0
2	3	50	2	90	3	3	1000	1000	0	0	0	0	0	1000	998	0	0	0	0	0
2	3	50	2	90	3	4	1000	1000	0	0	0	0	0	1000	998	0	0	0	0	0
2	3	10	1	10	1	1	649	24	0	0	0	0	0	381	1	0	0	0	0	0
2	3	10	1	10	1	2	622	17	0	0	0	0	0	351	1	0	0	0	0	0
2	3	10	1	10	1	3	648	18	0	0	0	0	0	377	1	0	0	0	0	0
2	3	10	1	10	1	4	633	47	0	0	0	0	0	359	2	0	0	0	0	0
2	3	10	2	10	2	1	692	247	0	0	0	0	0	409	64	0	0	0	0	0
2	3	10	2	10	2	2	563	206	0	0	0	0	0	264	41	0	0	0	0	0
2	3	10	2	10	2	3	571	212	0	0	0	0	0	274	43	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
2	3	10	2	10	2	4	629	311	0	0	0	0	0	281	63	0	0	0	0	0
2	3	10	2	10	3	1	617	209	0	0	0	0	0	389	59	0	0	0	0	0
2	3	10	2	10	3	2	515	167	0	0	0	0	0	267	37	0	0	0	0	0
2	3	10	2	10	3	3	529	171	0	0	0	0	0	276	39	0	0	0	0	0
2	3	10	2	10	3	4	553	234	0	0	0	0	0	279	49	0	0	0	0	0
2	3	10	1	40	1	1	999	40	0	0	0	0	0	992	8	0	0	0	0	0
2	3	10	1	40	1	2	999	40	0	0	0	0	0	991	8	0	0	0	0	0
2	3	10	1	40	1	3	1000	41	0	0	0	0	0	991	11	0	0	0	0	0
2	3	10	1	40	1	4	999	74	0	0	0	0	0	992	16	0	0	0	0	0
2	3	10	2	40	2	1	1000	972	0	0	0	0	0	993	853	0	0	0	0	0
2	3	10	2	40	2	2	996	970	0	0	0	0	0	960	832	0	0	0	0	0
2	3	10	2	40	2	3	996	970	0	0	0	0	0	960	836	0	0	0	0	0
2	3	10	2	40	2	4	998	985	0	0	0	0	0	966	872	0	0	0	0	0
2	3	10	2	40	3	1	1000	926	0	0	0	0	0	995	732	0	0	0	0	0
2	3	10	2	40	3	2	998	924	0	0	0	0	0	966	719	0	0	0	0	0
2	3	10	2	40	3	3	998	927	0	0	0	0	0	966	728	0	0	0	0	0
2	3	10	2	40	3	4	999	958	0	0	0	0	0	972	776	0	0	0	0	0
2	3	10	1	90	1	1	1000	48	0	0	0	0	0	1000	11	0	0	0	0	0
2	3	10	1	90	1	2	1000	48	0	0	0	0	0	1000	11	0	0	0	0	0
2	3	10	1	90	1	3	1000	48	0	0	0	0	0	1000	11	0	0	0	0	0
2	3	10	1	90	1	4	1000	91	0	0	0	0	0	1000	16	0	0	0	0	0
2	3	10	2	90	2	1	1000	1000	0	0	0	0	0	1000	1000	0	0	0	0	0
2	3	10	2	90	2	2	1000	1000	0	0	0	0	0	1000	1000	0	0	0	0	0
2	3	10	2	90	2	3	1000	1000	0	0	0	0	0	1000	1000	0	0	0	0	0
2	3	10	2	90	2	4	1000	1000	0	0	0	0	0	1000	1000	0	0	0	0	0
2	3	10	2	90	3	1	1000	1000	0	0	0	0	0	1000	998	0	0	0	0	0
2	3	10	2	90	3	2	1000	1000	0	0	0	0	0	1000	998	0	0	0	0	0
2	3	10	2	90	3	3	1000	1000	0	0	0	0	0	1000	998	0	0	0	0	0
2	3	10	2	90	3	4	1000	1000	0	0	0	0	0	1000	999	0	0	0	0	0
2	6	50	1	10	1	1	545	32	0	0	0	0	0	313	1	0	0	0	0	0
2	6	50	1	10	1	2	506	0	0	0	0	0	0	279	0	0	0	0	0	0
2	6	50	1	10	1	3	566	0	0	0	0	0	0	340	0	0	0	0	0	0
2	6	50	1	10	1	4	516	35	0	0	0	0	0	289	1	0	0	0	0	0
2	6	50	2	10	2	1	548	101	0	0	0	0	0	305	24	0	0	0	0	0
2	6	50	2	10	2	2	433	8	0	0	0	0	0	202	0	0	0	0	0	0
2	6	50	2	10	2	3	455	8	0	0	0	0	0	217	0	0	0	0	0	0
2	6	50	2	10	2	4	449	108	0	0	0	0	0	224	19	0	0	0	0	0
2	6	50	2	10	3	1	532	97	0	0	0	0	0	312	18	0	0	0	0	0
2	6	50	2	10	3	2	423	8	0	0	0	0	0	214	0	0	0	0	0	0
2	6	50	2	10	3	3	454	8	0	0	0	0	0	230	0	0	0	0	0	0
2	6	50	2	10	3	4	447	97	0	0	0	0	0	233	17	0	0	0	0	0
2	6	50	1	40	1	1	1000	43	0	0	0	0	0	991	9	0	0	0	0	0
2	6	50	1	40	1	2	1000	5	0	0	0	0	0	988	0	0	0	0	0	0
2	6	50	1	40	1	3	1000	5	0	0	0	0	0	995	0	0	0	0	0	0
2	6	50	1	40	1	4	1000	56	0	0	0	0	0	989	12	0	0	0	0	0
2	6	50	2	40	2	1	998	853	0	0	0	0	0	986	649	0	0	0	0	0
2	6	50	2	40	2	2	992	517	0	0	0	0	0	946	85	0	0	0	0	0
2	6	50	2	40	2	3	993	517	0	0	0	0	0	946	85	0	0	0	0	0
2	6	50	2	40	2	4	993	875	0	0	0	0	0	966	659	0	0	0	0	0
2	6	50	2	40	3	1	997	783	0	0	0	0	0	985	543	0	0	0	0	0
2	6	50	2	40	3	2	991	410	0	0	0	0	0	960	37	0	0	0	0	0
2	6	50	2	40	3	3	991	410	0	0	0	0	0	960	37	0	0	0	0	0
2	6	50	2	40	3	4	992	910	0	0	0	0	0	968	560	0	0	0	0	0
2	6	50	1	90	1	1	1000	56	0	0	0	0	0	1000	13	0	0	0	0	0
2	6	50	1	90	1	2	1000	6	0	0	0	0	0	1000	0	0	0	0	0	0
2	6	50	1	90	1	3	1000	6	0	0	0	0	0	1000	0	0	0	0	0	0
2	6	50	1	90	1	4	1000	71	0	0	0	0	0	1000	13	0	0	0	0	0
2	6	50	2	90	2	1	1000	999	0	0	0	0	0	1000	995	0	0	0	0	0
2	6	50	2	90	2	2	1000	993	0	0	0	0	0	1000	860	0	0	0	0	0
2	6	5	1	10	1	1	409	21	0	0	0	0	0	176	1	0	0	0	0	0
2	6	5	1	10	1	2	352	1	0	0	0	0	0	128	0	0	0	0	0	0
2	6	5	1	10	1	3	413	1	0	0	0	0	0	161	0	0	0	0	0	0
2	6	5	1	10	1	4	363	21	0	0	0	0	0	145	1	0	0	0	0	0
2	6	5	2	10	2	1	405	46	0	0	0	0	0	174	7	0	0	0	0	0
2	6	5	2	10	2	2	293	9	0	0	0	0	0	95	0	0	0	0	0	0
2	6	5	2	10	2	3	324	9	0	0	0	0	0	103	0	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
2	6	5	2	10	2	4	311	56	0	0	0	0	0	116	9	0	0	0	0	0
2	6	5	2	10	3	1	472	72	0	0	0	0	0	211	14	0	0	0	0	0
2	6	5	2	10	3	2	355	15	0	0	0	0	0	119	0	0	0	0	0	0
2	6	5	2	10	3	3	384	15	0	0	0	0	0	133	0	0	0	0	0	0
2	6	5	2	10	3	4	379	79	0	0	0	0	0	145	10	0	0	0	0	0
2	6	5	1	40	1	1	984	36	0	0	0	0	0	887	10	0	0	0	0	0
2	6	5	1	40	1	2	984	11	0	0	0	0	0	870	1	0	0	0	0	0
2	6	5	1	40	1	3	993	11	0	0	0	0	0	917	1	0	0	0	0	0
2	6	5	1	40	1	4	984	54	0	0	0	0	0	876	14	0	0	0	0	0
2	6	5	2	40	2	1	992	764	0	0	0	0	0	937	442	0	0	0	0	0
2	6	5	2	40	2	2	952	464	0	0	0	0	0	766	61	0	0	0	0	0
2	6	5	2	40	2	3	952	464	0	0	0	0	0	767	61	0	0	0	0	0
2	6	5	2	40	2	4	961	805	0	0	0	0	0	819	479	0	0	0	0	0
2	6	5	2	40	3	1	986	666	0	0	0	0	0	940	341	0	0	0	0	0
2	6	5	2	40	3	2	964	377	0	0	0	0	0	797	42	0	0	0	0	0
2	6	5	2	40	3	3	966	377	0	0	0	0	0	800	42	0	0	0	0	0
2	6	5	2	40	3	4	971	728	0	0	0	0	0	850	383	0	0	0	0	0
2	6	5	1	90	1	1	1000	47	0	0	0	0	0	999	6	0	0	0	0	0
2	6	5	1	90	1	2	1000	7	0	0	0	0	0	999	1	0	0	0	0	0
2	6	5	1	90	1	3	1000	7	0	0	0	0	0	999	1	0	0	0	0	0
2	6	5	1	90	1	4	1000	66	0	0	0	0	0	999	8	0	0	0	0	0
2	6	5	2	90	2	1	1000	997	0	0	0	0	0	1000	968	0	0	0	0	0
2	6	5	2	90	2	2	1000	972	0	0	0	0	0	997	662	0	0	0	0	0
2	6	5	2	90	2	3	1000	972	0	0	0	0	0	997	662	0	0	0	0	0
2	6	5	2	90	2	4	1000	998	0	0	0	0	0	1000	975	0	0	0	0	0
2	6	5	2	90	3	1	1000	984	0	0	0	0	0	1000	916	0	0	0	0	0
2	6	5	2	90	3	2	1000	926	0	0	0	0	0	1000	514	0	0	0	0	0
2	6	5	2	90	3	3	1000	926	0	0	0	0	0	1000	514	0	0	0	0	0
2	6	5	2	90	3	4	1000	989	0	0	0	0	0	1000	938	0	0	0	0	0
2	6	10	1	10	1	1	446	23	0	0	0	0	0	223	4	0	0	0	0	0
2	6	10	1	10	1	2	399	2	0	0	0	0	0	178	0	0	0	0	0	0
2	6	10	1	10	1	3	456	2	0	0	0	0	0	216	0	0	0	0	0	0
2	6	10	1	10	1	4	412	21	0	0	0	0	0	201	5	0	0	0	0	0
2	6	10	2	10	2	1	517	99	0	0	0	0	0	260	17	0	0	0	0	0
2	6	10	2	10	2	2	389	10	0	0	0	0	0	154	0	0	0	0	0	0
2	6	10	2	10	2	3	417	10	0	0	0	0	0	167	0	0	0	0	0	0
2	6	10	2	10	2	4	414	32	0	0	0	0	0	184	18	0	0	0	0	0
2	6	10	2	10	3	1	486	78	0	0	0	0	0	255	7	0	0	0	0	0
2	6	10	2	10	3	2	361	6	0	0	0	0	0	150	0	0	0	0	0	0
2	6	10	2	10	3	3	386	6	0	0	0	0	0	160	0	0	0	0	0	0
2	6	10	2	10	3	4	394	75	0	0	0	0	0	185	6	0	0	0	0	0
2	6	10	1	40	1	1	993	41	0	0	0	0	0	958	7	0	0	0	0	0
2	6	10	1	40	1	2	991	6	0	0	0	0	0	953	0	0	0	0	0	0
2	6	10	1	40	1	3	995	6	0	0	0	0	0	970	0	0	0	0	0	0
2	6	10	1	40	1	4	992	52	0	0	0	0	0	957	10	0	0	0	0	0
2	6	10	2	40	2	1	1000	845	0	0	0	0	0	984	592	0	0	0	0	0
2	6	10	2	40	2	2	987	529	0	0	0	0	0	894	66	0	0	0	0	0
2	6	10	2	40	2	3	987	529	0	0	0	0	0	894	66	0	0	0	0	0
2	6	10	2	40	2	4	990	872	0	0	0	0	0	933	621	0	0	0	0	0
2	6	10	2	40	3	1	994	731	0	0	0	0	0	972	448	0	0	0	0	0
2	6	10	2	40	3	2	982	389	0	0	0	0	0	900	47	0	0	0	0	0
2	6	10	2	40	3	3	982	389	0	0	0	0	0	903	47	0	0	0	0	0
2	6	10	2	40	3	4	986	766	0	0	0	0	0	926	482	0	0	0	0	0
2	6	10	1	90	1	1	1000	59	0	0	0	0	0	1000	14	0	0	0	0	0
2	6	10	1	90	1	2	1000	11	0	0	0	0	0	1000	0	0	0	0	0	0
2	6	10	1	90	1	3	1000	11	0	0	0	0	0	1000	0	0	0	0	0	0
2	6	10	1	90	1	4	1000	74	0	0	0	0	0	1000	14	0	0	0	0	0
2	6	10	2	90	2	1	1000	999	0	0	0	0	0	1000	994	0	0	0	0	0
2	6	10	2	90	2	2	1000	993	0	0	0	0	0	1000	780	0	0	0	0	0
2	6	10	2	90	2	3	1000	993	0	0	0	0	0	1000	780	0	0	0	0	0
2	6	10	2	90	2	4	1000	999	0	0	0	0	0	1000	996	0	0	0	0	0
2	6	10	2	90	3	1	1000	996	0	0	0	0	0	1000	957	0	0	0	0	0
2	6	10	2	90	3	2	1000	942	0	0	0	0	0	1000	589	0	0	0	0	0
2	6	10	2	90	3	3	1000	942	0	0	0	0	0	1000	589	0	0	0	0	0
2	6	10	2	90	3	4	1000	998	0	0	0	0	0	1000	965	0	0	0	0	0
2	10	5	1	10	1	1	335	15	0	0	0	0	0	137	2	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
2	10	5	1	10	1	2	268	0	0	0	0	0	0	96	0	0	0	0	0	0
2	10	5	1	10	1	3	342	0	0	0	0	0	0	134	0	0	0	0	0	0
2	10	5	1	10	1	4	278	15	0	0	0	0	0	101	1	0	0	0	0	0
2	10	5	2	10	2	1	337	47	0	0	0	0	0	157	5	0	0	0	0	0
2	10	5	2	10	2	2	231	0	0	0	0	0	0	77	0	0	0	0	0	0
2	10	5	2	10	2	3	259	0	0	0	0	0	0	88	0	0	0	0	0	0
2	10	5	2	10	2	4	236	49	0	0	0	0	0	84	3	0	0	0	0	0
2	10	5	2	10	3	1	333	32	0	0	0	0	0	152	3	0	0	0	0	0
2	10	5	2	10	3	2	246	0	0	0	0	0	0	84	0	0	0	0	0	0
2	10	5	2	10	3	3	277	0	0	0	0	0	0	99	0	0	0	0	0	0
2	10	5	2	10	3	4	262	33	0	0	0	0	0	91	2	0	0	0	0	0
2	10	5	1	40	1	1	961	34	0	0	0	0	0	841	6	0	0	0	0	0
2	10	5	1	40	1	2	956	1	0	0	0	0	0	818	0	0	0	0	0	0
2	10	5	1	40	1	3	974	1	0	0	0	0	0	874	0	0	0	0	0	0
2	10	5	1	40	1	4	957	47	0	0	0	0	0	822	8	0	0	0	0	0
2	10	5	2	40	2	1	957	585	0	0	0	0	0	891	285	0	0	0	0	0
2	10	5	2	40	2	2	900	69	0	0	0	0	0	662	0	0	0	0	0	0
2	10	5	2	40	2	3	903	69	0	0	0	0	0	665	0	0	0	0	0	0
2	10	5	2	40	2	4	905	622	0	0	0	0	0	692	278	0	0	0	0	0
2	10	5	2	40	3	1	959	511	0	0	0	0	0	880	221	0	0	0	0	0
2	10	5	2	40	3	2	910	54	0	0	0	0	0	721	0	0	0	0	0	0
2	10	5	2	40	3	3	912	54	0	0	0	0	0	728	0	0	0	0	0	0
2	10	5	2	40	3	4	915	558	0	0	0	0	0	738	235	0	0	0	0	0
2	10	5	1	90	1	1	1000	48	0	0	0	0	0	1000	8	0	0	0	0	0
2	10	5	1	90	1	2	1000	1	0	0	0	0	0	1000	0	0	0	0	0	0
2	10	5	1	90	1	3	1000	1	0	0	0	0	0	1000	0	0	0	0	0	0
2	10	5	1	90	1	4	1000	69	0	0	0	0	0	1000	11	0	0	0	0	0
2	10	5	2	90	2	1	1000	991	0	0	0	0	0	1000	940	0	0	0	0	0
2	10	5	2	90	2	2	1000	749	0	0	0	0	0	998	123	0	0	0	0	0
2	10	5	2	90	2	3	1000	749	0	0	0	0	0	998	123	0	0	0	0	0
2	10	5	2	90	2	4	1000	994	0	0	0	0	0	998	951	0	0	0	0	0
2	10	5	2	90	3	1	1000	960	0	0	0	0	0	1000	838	0	0	0	0	0
2	10	5	2	90	3	2	1000	576	0	0	0	0	0	1000	52	0	0	0	0	0
2	10	5	2	90	3	3	1000	576	0	0	0	0	0	1000	52	0	0	0	0	0
2	10	5	2	90	3	4	1000	975	0	0	0	0	0	1000	860	0	0	0	0	0
2	10	10	1	10	1	1	361	18	0	0	0	0	0	172	2	0	0	0	0	0
2	10	10	1	10	1	2	305	0	0	0	0	0	0	132	0	0	0	0	0	0
2	10	10	1	10	1	3	370	0	0	0	0	0	0	177	0	0	0	0	0	0
2	10	10	1	10	1	4	316	22	0	0	0	0	0	133	1	0	0	0	0	0
2	10	10	2	10	2	1	383	43	0	0	0	0	0	178	0	0	0	0	0	0
2	10	10	2	10	2	2	275	0	0	0	0	0	0	89	0	0	0	0	0	0
2	10	10	2	10	2	3	299	0	0	0	0	0	0	110	0	0	0	0	0	0
2	10	10	2	10	2	4	285	43	0	0	0	0	0	94	1	0	0	0	0	0
2	10	10	2	10	3	1	371	37	0	0	0	0	0	174	3	0	0	0	0	0
2	10	10	2	10	3	2	266	0	0	0	0	0	0	111	0	0	0	0	0	0
2	10	10	2	10	3	3	299	0	0	0	0	0	0	128	0	0	0	0	0	0
2	10	10	2	10	3	4	278	37	0	0	0	0	0	111	2	0	0	0	0	0
2	10	10	1	40	1	1	979	41	0	0	0	0	0	926	8	0	0	0	0	0
2	10	10	1	40	1	2	979	0	0	0	0	0	0	921	0	0	0	0	0	0
2	10	10	1	40	1	3	988	0	0	0	0	0	0	959	0	0	0	0	0	0
2	10	10	1	40	1	4	979	47	0	0	0	0	0	921	9	0	0	0	0	0
2	10	10	2	40	2	1	982	661	0	0	0	0	0	937	354	0	0	0	0	0
2	10	10	2	40	2	2	933	66	0	0	0	0	0	792	0	0	0	0	0	0
2	10	10	2	40	2	3	934	66	0	0	0	0	0	795	0	0	0	0	0	0
2	10	10	2	40	2	4	941	688	0	0	0	0	0	797	354	0	0	0	0	0
2	10	10	2	40	3	1	988	611	0	0	0	0	0	950	320	0	0	0	0	0
2	10	10	2	40	3	2	959	61	0	0	0	0	0	857	0	0	0	0	0	0
2	10	10	2	40	3	3	959	61	0	0	0	0	0	858	0	0	0	0	0	0
2	10	10	2	40	3	4	961	647	0	0	0	0	0	859	325	0	0	0	0	0
2	10	10	1	90	1	1	1000	39	0	0	0	0	0	1000	7	0	0	0	0	0
2	10	10	1	90	1	2	1000	1	0	0	0	0	0	1000	0	0	0	0	0	0
2	10	10	1	90	1	3	1000	1	0	0	0	0	0	1000	0	0	0	0	0	0
2	10	10	1	90	1	4	1000	50	0	0	0	0	0	1000	8	0	0	0	0	0
2	10	10	2	90	2	1	1000	995	0	0	0	0	0	1000	970	0	0	0	0	0
2	10	10	2	90	2	2	1000	804	0	0	0	0	0	1000	118	0	0	0	0	0
2	10	10	2	90	2	3	1000	804	0	0	0	0	0	1000	118	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
2	10	10	2	90	2	4	1000	998	0	0	0	0	0	1000	976	0	0	0	0	0
2	10	10	2	90	3	1	1000	987	0	0	0	0	0	1000	916	0	0	0	0	0
2	10	10	2	90	3	2	1000	643	0	0	0	0	0	1000	56	0	0	0	0	0
2	10	10	2	90	3	3	1000	643	0	0	0	0	0	1000	56	0	0	0	0	0
2	10	10	2	90	3	4	1000	987	0	0	0	0	0	1000	918	0	0	0	0	0
2	3	50	1	40	1	1	1000	46	0	0	0	0	0	1000	5	0	0	0	0	0
2	3	50	1	10	1	1	697	40	0	0	0	0	0	473	3	0	0	0	0	0
2	3	50	1	10	1	2	678	33	0	0	0	0	0	438	1	0	0	0	0	0
2	3	50	1	10	1	3	714	34	0	0	0	0	0	461	1	0	0	0	0	0
2	3	50	1	10	1	4	689	55	0	0	0	0	0	442	5	0	0	0	0	0
2	3	50	2	10	2	1	685	240	0	0	0	0	0	438	76	0	0	0	0	0
2	3	50	2	10	2	2	581	201	0	0	0	0	0	326	53	0	0	0	0	0
2	3	50	2	10	2	3	591	202	0	0	0	0	0	334	53	0	0	0	0	0
2	3	50	2	10	2	4	617	285	0	0	0	0	0	340	79	0	0	0	0	0
2	3	50	2	10	3	1	731	235	0	0	0	0	0	479	68	0	0	0	0	0
2	3	50	2	10	3	2	639	203	0	0	0	0	0	371	48	0	0	0	0	0
2	3	50	2	10	3	3	649	203	0	0	0	0	0	376	49	0	0	0	0	0
2	3	50	2	10	3	4	664	280	0	0	0	0	0	386	70	0	0	0	0	0
6	10	10	2	90	2	1	999	800	45	0	0	0	0	996	541	4	0	0	0	0
6	10	10	2	90	2	2	997	697	8	0	0	0	0	983	424	0	0	0	0	0
6	10	10	2	90	2	3	997	901	22	0	0	0	0	984	711	5	0	0	0	0
6	10	10	2	90	2	4	998	935	64	1	0	0	0	984	795	10	0	0	0	0
6	10	10	2	90	3	1	1000	680	19	0	0	0	0	996	370	1	0	0	0	0
6	10	10	2	90	3	2	1000	536	3	0	0	0	0	989	244	0	0	0	0	0
6	10	10	2	90	3	3	1000	787	21	0	0	0	0	993	562	2	0	0	0	0
6	10	10	2	90	3	4	1000	338	45	0	0	0	0	992	645	5	0	0	0	0
6	10	10	3	10	2	1	214	10	0	0	0	0	0	63	0	0	0	0	0	0
6	10	10	3	10	2	2	106	2	0	0	0	0	0	21	0	0	0	0	0	0
6	10	10	3	10	2	3	146	3	0	0	0	0	0	38	0	0	0	0	0	0
6	10	10	3	10	2	4	130	6	0	0	0	0	0	24	1	0	0	0	0	0
6	10	10	3	10	3	1	252	17	1	0	0	0	0	90	5	0	0	0	0	0
6	10	10	3	10	3	2	134	0	0	0	0	0	0	30	0	0	0	0	0	0
6	10	10	3	10	3	3	183	8	0	0	0	0	0	43	0	0	0	0	0	0
6	10	10	3	10	3	4	170	16	0	0	0	0	0	41	0	0	0	0	0	0
6	10	10	3	10	4	1	189	16	0	0	0	0	0	78	4	0	0	0	0	0
6	10	10	3	10	4	2	99	1	0	0	0	0	0	28	0	0	0	0	0	0
6	10	10	3	10	4	3	154	5	0	0	0	0	0	42	1	0	0	0	0	0
6	10	10	3	10	4	4	126	14	0	0	0	0	0	36	1	0	0	0	0	0
6	10	10	3	40	2	1	867	310	28	0	0	0	0	664	107	4	0	0	0	0
6	10	10	3	40	2	2	655	43	4	0	0	0	0	344	7	0	0	0	0	0
6	10	10	3	40	2	3	672	170	8	0	0	0	0	367	30	2	0	0	0	0
6	10	10	3	40	2	4	724	253	30	0	0	0	0	388	55	3	0	0	0	0
6	10	10	3	40	3	1	856	290	21	2	0	0	0	652	92	2	0	0	0	0
6	10	10	3	40	3	2	687	55	0	0	0	0	0	397	3	0	0	0	0	0
6	10	10	3	40	3	3	710	168	4	0	0	0	0	432	36	0	0	0	0	0
6	10	10	3	40	3	4	750	239	25	0	0	0	0	428	62	1	0	0	0	0
6	10	10	3	40	4	1	861	252	13	0	0	0	0	661	70	2	0	0	0	0
6	10	10	3	40	4	2	706	54	0	0	0	0	0	418	4	0	0	0	0	0
6	10	10	3	40	4	3	734	162	5	0	0	0	0	442	42	0	0	0	0	0
6	10	10	3	40	4	4	755	232	13	1	0	0	0	460	65	0	0	0	0	0
6	10	10	3	90	2	1	1000	950	398	17	0	0	0	1000	812	143	2	0	0	0
6	10	10	3	90	2	2	993	693	182	1	0	0	0	942	352	37	0	0	0	0
6	10	10	3	90	2	3	993	858	415	8	0	0	0	942	616	155	0	0	0	0
6	10	10	3	90	2	4	996	910	601	23	0	0	0	969	721	232	1	0	0	0
6	10	5	3	10	2	1	178	13	0	0	0	0	0	56	0	0	0	0	0	0
6	10	5	3	10	2	2	80	0	0	0	0	0	0	14	0	0	0	0	0	0
6	10	5	3	10	2	3	115	4	0	0	0	0	0	24	0	0	0	0	0	0
6	10	5	3	10	2	4	106	10	0	0	0	0	0	22	0	0	0	0	0	0
6	10	5	3	10	3	1	226	19	0	0	0	0	0	78	0	0	0	0	0	0
6	10	5	3	10	3	2	113	0	0	0	0	0	0	24	0	0	0	0	0	0
6	10	5	3	10	3	3	147	4	0	0	0	0	0	40	0	0	0	0	0	0
6	10	5	3	10	3	4	142	10	0	0	0	0	0	32	0	0	0	0	0	0
6	10	5	3	10	4	1	187	18	0	0	0	0	0	60	1	0	0	0	0	0
6	10	5	3	10	4	2	80	0	0	0	0	0	0	18	0	0	0	0	0	0
6	10	5	3	10	4	3	114	4	0	0	0	0	0	35	0	0	0	0	0	0
6	10	5	3	10	4	4	107	12	0	0	0	0	0	36	0	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	10	5	3	40	2	1	791	260	24	0	0	0	0	560	90	5	0	0	0	0
6	10	5	3	40	2	2	512	30	1	0	0	0	0	242	4	0	0	0	0	0
6	10	5	3	40	2	3	530	99	7	0	0	0	0	260	16	1	0	0	0	0
6	10	5	3	40	2	4	584	176	32	2	0	0	0	276	38	2	0	0	0	0
6	10	5	3	40	3	1	786	225	20	0	0	0	0	536	77	0	0	0	0	0
6	10	5	3	40	3	2	542	25	1	0	0	0	0	247	0	0	0	0	0	0
6	10	5	3	40	3	3	561	87	1	0	0	0	0	272	12	0	0	0	0	0
6	10	5	3	40	3	4	620	173	21	0	0	0	0	289	37	1	0	0	0	0
6	10	5	3	40	4	1	799	202	18	1	0	0	0	562	67	0	0	0	0	0
6	10	5	3	40	4	2	584	16	0	0	0	0	0	283	1	0	0	0	0	0
6	10	5	3	40	4	3	612	80	2	0	0	0	0	315	10	0	0	0	0	0
6	10	5	3	40	4	4	658	182	18	0	0	0	0	322	29	0	0	0	0	0
6	10	5	3	90	2	1	998	866	281	12	0	0	0	984	668	63	2	0	0	0
6	10	5	3	90	2	2	968	437	71	2	0	0	0	827	152	3	0	0	0	0
6	10	5	3	90	2	3	969	665	227	2	0	0	0	828	325	37	0	0	0	0
6	10	5	3	90	2	4	980	815	482	23	0	0	0	876	519	145	2	0	0	0
6	10	5	3	90	3	1	998	836	199	8	0	0	0	983	579	46	0	0	0	0
6	10	5	3	90	3	2	967	420	27	0	0	0	0	842	131	1	0	0	0	0
6	10	5	3	90	3	3	968	656	129	3	0	0	0	843	307	13	0	0	0	0
6	10	5	3	90	3	4	980	793	330	15	1	0	0	880	490	58	0	0	0	0
6	10	5	3	90	4	1	998	770	144	8	0	0	0	980	496	23	1	0	0	0
6	10	5	3	90	4	2	977	355	16	0	0	0	0	870	107	1	0	0	0	0
6	10	5	3	90	4	3	977	591	82	1	0	0	0	874	267	11	0	0	0	0
6	10	5	3	90	4	4	989	745	236	11	0	0	0	892	433	38	0	0	0	0
6	10	5	5	10	2	1	188	20	1	0	0	0	0	57	3	0	0	0	0	0
6	10	5	5	10	2	2	82	0	0	0	0	0	0	17	0	0	0	0	0	0
6	10	5	5	10	2	3	109	2	0	0	0	0	0	24	0	0	0	0	0	0
6	10	5	5	10	2	4	112	7	0	0	0	0	0	19	0	0	0	0	0	0
6	10	5	5	10	3	1	192	21	1	0	0	0	0	67	5	1	0	0	0	0
6	10	5	5	10	3	2	75	1	0	0	0	0	0	17	0	0	0	0	0	0
6	10	5	5	10	3	3	107	3	1	0	0	0	0	28	1	0	0	0	0	0
6	10	5	5	10	3	4	97	10	2	0	0	0	0	23	1	0	0	0	0	0
6	10	5	5	10	4	1	202	13	1	0	0	0	0	77	1	0	0	0	0	0
6	10	5	5	10	4	2	100	1	0	0	0	0	0	27	0	0	0	0	0	0
6	10	5	5	10	4	3	131	3	0	0	0	0	0	41	0	0	0	0	0	0
6	10	5	5	10	4	4	116	10	1	0	0	0	0	31	1	0	0	0	0	0
6	10	5	5	40	2	1	829	338	76	5	0	0	0	595	148	14	0	0	0	0
6	10	5	5	40	2	2	476	20	0	0	0	0	0	202	1	0	0	0	0	0
6	10	5	5	40	2	3	481	87	9	0	0	0	0	209	8	0	0	0	0	0
6	10	5	5	40	2	4	571	172	47	5	0	0	0	258	41	0	0	0	0	0
6	10	5	5	40	3	1	824	317	55	2	0	0	0	604	117	7	0	0	0	0
6	10	5	5	40	3	2	502	21	0	0	0	0	0	243	2	0	0	0	0	0
6	10	5	5	40	3	3	514	91	1	0	0	0	0	254	11	0	0	0	0	0
6	10	5	5	40	3	4	597	178	34	3	0	0	0	279	34	0	0	0	0	0
6	10	5	5	40	4	1	804	271	32	1	0	0	0	550	75	3	0	0	0	0
6	10	5	5	40	4	2	548	22	0	0	0	0	0	252	4	0	0	0	0	0
6	10	5	5	40	4	3	567	70	2	0	0	0	0	273	14	0	0	0	0	0
6	10	5	5	40	4	4	627	164	23	0	0	0	0	292	31	0	0	0	0	0
6	10	5	5	90	2	1	999	965	769	292	41	0	0	993	867	464	83	0	0	0
6	10	5	5	90	2	2	920	317	52	2	0	0	0	696	80	8	0	0	0	0
6	10	5	5	90	2	3	920	575	187	27	3	0	0	696	218	31	1	0	0	0
6	10	5	5	90	2	4	951	752	513	216	58	0	0	759	375	90	11	1	0	0
6	10	5	5	90	3	1	999	936	601	175	13	0	0	990	829	303	31	1	0	0
6	10	5	5	90	3	2	939	328	47	2	0	0	0	743	81	3	0	0	0	0
6	10	5	5	90	3	3	939	603	179	24	2	0	0	743	233	28	1	0	0	0
6	10	5	5	90	3	4	963	750	447	151	15	1	0	796	400	96	14	1	0	0
6	10	5	5	90	4	1	997	869	343	47	1	0	0	986	664	117	5	0	0	0
6	10	5	5	90	4	2	965	357	20	1	0	0	0	820	96	1	0	0	0	0
6	10	5	5	90	4	3	965	598	106	7	1	0	0	824	246	9	0	0	0	0
6	10	5	5	90	4	4	985	738	307	41	4	0	0	875	435	55	4	1	0	0
6	10	10	3	90	3	1	1000	904	222	6	0	0	0	998	698	65	3	0	0	0
6	10	10	3	90	3	2	988	612	59	1	0	0	0	964	299	11	0	0	0	0
6	10	10	3	90	3	3	989	819	226	5	0	0	0	964	564	54	1	0	0	0
6	10	10	3	90	3	4	996	882	385	17	0	0	0	973	664	101	2	0	0	0
6	10	10	3	90	4	1	998	861	215	5	0	0	0	998	660	41	0	0	0	0
6	10	10	3	90	4	2	995	564	50	0	0	0	0	974	269	9	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	10	10	3	90	4	3	995	792	181	1	0	0	0	974	517	44	0	0	0	0
6	10	10	3	90	4	4	997	854	319	3	0	0	0	981	620	67	0	0	0	0
6	10	10	5	10	2	1	205	16	2	0	0	0	0	65	3	0	0	0	0	0
6	10	10	5	10	2	2	91	0	0	0	0	0	0	20	0	0	0	0	0	0
6	10	10	5	10	2	3	132	5	0	0	0	0	0	31	0	0	0	0	0	0
6	10	10	5	10	2	4	113	9	0	0	0	0	0	25	1	0	0	0	0	0
6	10	10	5	10	3	1	222	17	2	0	0	0	0	76	2	0	0	0	0	0
6	10	10	5	10	3	2	107	0	0	0	0	0	0	22	0	0	0	0	0	0
6	10	10	5	10	3	3	145	8	0	0	0	0	0	40	0	0	0	0	0	0
6	10	10	5	10	3	4	145	13	1	0	0	0	0	32	0	0	0	0	0	0
6	10	10	5	10	4	1	219	18	0	0	0	0	0	72	1	0	0	0	0	0
6	10	10	5	10	4	2	99	2	0	0	0	0	0	20	0	0	0	0	0	0
6	10	10	5	10	4	3	135	3	0	0	0	0	0	29	0	0	0	0	0	0
6	10	10	5	10	4	4	131	7	0	0	0	0	0	22	2	0	0	0	0	0
6	10	10	5	40	2	1	886	425	91	17	2	0	0	716	199	24	3	1	0	0
6	10	10	5	40	2	2	597	41	2	0	0	0	0	290	2	0	0	0	0	0
6	10	10	5	40	2	3	605	141	16	2	0	0	0	304	24	2	0	0	0	0
6	10	10	5	40	2	4	667	217	45	6	2	0	0	347	41	3	0	0	0	0
6	10	10	5	40	3	1	878	413	66	5	1	0	0	695	176	6	1	0	0	0
6	10	10	5	40	3	2	604	46	0	0	0	0	0	327	4	0	0	0	0	0
6	10	10	5	40	3	3	612	140	8	0	0	0	0	335	31	0	0	0	0	0
6	10	10	5	40	3	4	682	211	34	4	1	0	0	375	58	0	0	0	0	0
6	10	10	5	40	4	1	866	310	38	1	1	0	0	669	113	6	0	0	0	0
6	10	10	5	40	4	2	680	35	2	0	0	0	0	374	3	0	0	0	0	0
6	10	10	5	40	4	3	699	159	14	0	0	0	0	393	20	0	0	0	0	0
6	10	10	5	40	4	4	738	230	37	2	0	0	0	415	51	0	0	0	0	0
6	10	10	5	90	2	1	1000	938	803	366	48	1	0	1000	928	557	119	4	0	0
6	10	10	5	90	2	2	968	513	142	19	0	0	0	856	183	23	0	0	0	0
6	10	10	5	90	2	3	968	778	379	113	15	0	0	856	417	94	10	0	0	0
6	10	10	5	90	2	4	987	846	584	256	63	2	0	891	549	165	36	2	0	0
6	10	10	5	90	3	1	1000	970	679	206	23	0	0	995	880	405	44	0	0	0
6	10	10	5	90	3	2	979	542	130	6	1	0	0	891	209	19	0	0	0	0
6	10	10	5	90	3	3	979	787	359	65	5	0	0	891	470	94	6	1	0	0
6	10	10	5	90	3	4	985	848	539	169	20	0	0	919	584	152	17	1	0	0
6	10	10	5	90	4	1	999	899	384	58	0	0	0	999	717	149	7	0	0	0
6	10	10	5	90	4	2	996	493	61	1	0	0	0	965	231	7	0	0	0	0
6	10	10	5	90	4	3	996	731	203	22	0	0	0	965	431	40	0	0	0	0
6	10	10	5	90	4	4	998	799	359	60	0	0	0	970	529	76	5	0	0	0
6	6	50	1	10	1	1	321	15	1	0	0	0	0	133	4	0	0	0	0	0
6	6	50	1	10	1	2	254	0	0	0	0	0	0	90	0	0	0	0	0	0
6	6	50	1	10	1	3	349	5	0	0	0	0	0	155	0	0	0	0	0	0
6	6	50	1	10	1	4	268	10	0	0	0	0	0	98	0	0	0	0	0	0
6	6	50	2	10	2	1	306	25	1	0	0	0	0	139	1	0	0	0	0	0
6	6	50	2	10	2	2	204	0	0	0	0	0	0	76	0	0	0	0	0	0
6	6	50	2	10	2	3	255	12	0	0	0	0	0	109	0	0	0	0	0	0
6	6	50	2	10	2	4	225	24	1	0	0	0	0	82	0	0	0	0	0	0
6	6	50	2	10	3	1	329	27	0	0	0	0	0	131	4	0	0	0	0	0
6	6	50	2	10	3	2	235	3	0	0	0	0	0	75	1	0	0	0	0	0
6	6	50	2	10	3	3	299	8	0	0	0	0	0	121	3	0	0	0	0	0
6	6	50	2	10	3	4	256	27	0	0	0	0	0	80	4	0	0	0	0	0
6	6	50	1	40	1	1	960	49	3	0	0	0	0	873	10	0	0	0	0	0
6	6	50	1	40	1	2	957	8	0	0	0	0	0	865	1	0	0	0	0	0
6	6	50	1	40	1	3	985	24	0	0	0	0	0	950	2	0	0	0	0	0
6	6	50	1	40	1	4	959	50	2	0	0	0	0	867	9	0	0	0	0	0
6	6	50	2	40	2	1	960	470	14	0	0	0	0	886	209	1	0	0	0	0
6	6	50	2	40	2	2	932	270	0	0	0	0	0	791	89	0	0	0	0	0
6	6	50	2	40	2	3	941	433	8	0	0	0	0	811	189	0	0	0	0	0
6	6	50	2	40	2	4	943	573	15	0	0	0	0	817	278	2	0	0	0	0
6	6	50	2	40	3	1	956	352	17	0	0	0	0	878	148	2	0	0	0	0
6	6	50	2	40	3	2	927	178	1	0	0	0	0	794	49	0	0	0	0	0
6	6	50	2	40	3	3	944	320	6	0	0	0	0	820	122	1	0	0	0	0
6	6	50	2	40	3	4	938	446	20	0	0	0	0	814	184	2	0	0	0	0
6	6	50	1	90	1	1	1000	53	4	0	0	0	0	1000	10	0	0	0	0	0
6	6	50	1	90	1	2	1000	14	0	0	0	0	0	1000	2	0	0	0	0	0
6	6	50	1	90	1	3	1000	33	0	0	0	0	0	1000	6	0	0	0	0	0
6	6	50	1	90	1	4	1000	58	1	0	0	0	0	1000	14	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	6	50	2	90	2	1	1000	976	42	1	0	0	0	1000	917	9	0	0	0	0
6	6	50	2	90	2	2	1000	957	7	0	0	0	0	1000	863	0	0	0	0	0
6	6	50	2	90	2	3	1000	987	28	0	0	0	0	1000	937	6	0	0	0	0
6	6	50	2	90	2	4	1000	992	48	0	0	0	0	1000	961	11	0	0	0	0
6	6	50	2	90	3	1	1000	920	28	1	0	0	0	1000	758	4	0	0	0	0
6	6	50	2	90	3	2	1000	848	2	0	0	0	0	1000	661	1	0	0	0	0
6	6	50	2	90	3	3	1000	936	12	0	0	0	0	1000	795	2	0	0	0	0
6	6	50	2	90	3	4	1000	972	27	1	0	0	0	1000	856	4	0	0	0	0
6	6	50	3	10	2	1	339	32	2	0	0	0	0	142	6	0	0	0	0	0
6	6	50	3	10	2	2	203	3	0	0	0	0	0	71	1	0	0	0	0	0
6	6	50	3	10	2	3	253	11	0	0	0	0	0	101	2	0	0	0	0	0
6	6	50	3	10	2	4	230	21	0	0	0	0	0	82	2	0	0	0	0	0
6	6	50	3	10	3	1	358	36	3	2	0	0	0	143	4	1	0	0	0	0
6	6	50	3	10	3	2	224	0	0	0	0	0	0	78	0	0	0	0	0	0
6	6	50	3	10	3	3	270	9	0	0	0	0	0	99	0	0	0	0	0	0
6	6	50	3	10	3	4	255	17	0	0	0	0	0	83	0	0	0	0	0	0
6	6	50	3	10	4	1	324	41	3	0	0	0	0	131	5	0	0	0	0	0
6	6	50	3	10	4	2	196	1	0	0	0	0	0	62	0	0	0	0	0	0
6	6	50	3	10	4	3	261	9	0	0	0	0	0	96	0	0	0	0	0	0
6	6	50	3	10	4	4	216	19	0	0	0	0	0	69	1	0	0	0	0	0
6	6	50	3	40	2	1	973	651	124	5	0	0	0	905	370	20	0	0	0	0
6	6	50	3	40	2	2	903	215	15	0	0	0	0	716	70	1	0	0	0	0
6	6	50	3	40	2	3	907	409	65	0	0	0	0	722	123	6	0	0	0	0
6	6	50	3	40	2	4	917	557	118	2	1	0	0	745	204	18	0	0	0	0
6	6	50	3	40	3	1	970	546	77	3	0	0	0	879	296	14	1	0	0	0
6	6	50	3	40	3	2	894	230	5	0	0	0	0	693	68	0	0	0	0	0
6	6	50	3	40	3	3	899	370	27	0	0	0	0	707	127	3	0	0	0	0
6	6	50	3	40	3	4	911	487	68	4	1	0	0	733	224	10	0	0	0	0
6	6	50	3	40	4	1	975	499	85	3	0	0	0	892	255	14	0	0	0	0
6	6	50	3	40	4	2	916	171	4	0	0	0	0	761	45	0	0	0	0	0
6	6	50	3	40	4	3	920	322	39	1	0	0	0	777	109	4	0	0	0	0
6	6	50	3	40	4	4	939	443	79	4	0	0	0	780	169	6	0	0	0	0
6	10	50	3	40	3	1	917	347	27	0	0	0	0	749	139	2	0	0	0	0
6	10	50	3	40	3	2	805	111	1	0	0	0	0	567	23	0	0	0	0	0
6	10	50	3	40	3	3	824	277	22	0	0	0	0	601	76	0	0	0	0	0
6	10	50	3	40	3	4	818	307	39	0	0	0	0	560	96	1	0	0	0	0
6	10	50	5	10	2	1	225	18	2	0	0	0	0	79	3	0	0	0	0	0
6	10	50	5	10	2	2	135	1	0	0	0	0	0	35	0	0	0	0	0	0
6	10	50	5	10	2	3	204	6	0	0	0	0	0	72	0	0	0	0	0	0
6	10	50	5	10	2	4	140	7	0	0	0	0	0	34	1	0	0	0	0	0
6	10	50	5	10	3	1	263	26	2	1	0	0	0	98	5	0	0	0	0	0
6	10	50	5	10	3	2	158	0	0	0	0	0	0	40	0	0	0	0	0	0
6	10	50	5	10	3	3	225	14	0	0	0	0	0	74	0	0	0	0	0	0
6	10	50	5	10	3	4	163	19	1	0	0	0	0	39	0	0	0	0	0	0
6	10	50	3	90	3	1	1000	944	318	8	0	0	0	1000	816	118	0	0	0	0
6	10	50	3	90	3	2	998	806	141	1	0	0	0	991	555	42	0	0	0	0
6	10	50	3	90	3	3	998	899	352	7	0	0	0	991	757	139	1	0	0	0
6	10	50	3	90	3	4	998	919	456	14	0	0	0	990	796	154	1	0	0	0
6	10	50	3	90	4	1	1000	899	258	9	0	0	0	999	744	86	2	0	0	0
6	10	50	3	90	4	2	1000	720	91	1	0	0	0	994	502	26	0	0	0	0
6	10	50	3	90	4	3	1000	861	253	6	0	0	0	994	671	86	2	0	0	0
6	10	50	3	90	4	4	1000	880	352	12	0	0	0	993	711	98	2	0	0	0
6	10	50	3	40	4	1	907	329	35	2	0	0	0	742	126	6	0	0	0	0
6	10	50	3	40	4	2	824	91	3	0	0	0	0	601	24	0	0	0	0	0
6	10	50	3	40	4	3	849	223	13	1	0	0	0	650	75	2	0	0	0	0
6	10	50	3	40	4	4	831	273	33	4	0	0	0	590	89	3	0	0	0	0
6	10	50	3	90	2	1	1000	975	496	10	0	0	0	1000	913	218	1	0	0	0
6	10	50	3	90	2	2	999	958	300	1	0	0	0	988	644	101	1	0	0	0
6	10	50	3	90	2	3	999	950	583	18	0	0	0	988	804	304	1	0	0	0
6	10	50	3	90	2	4	1000	966	696	31	0	0	0	987	837	328	2	0	0	0
6	10	50	5	90	3	1	1000	992	786	274	24	1	0	999	954	535	92	0	0	0
6	10	50	5	90	3	2	996	756	262	35	0	0	0	970	476	61	2	0	0	0
6	10	50	5	90	3	3	996	910	548	159	16	0	0	970	676	223	28	0	0	0
6	10	50	5	90	3	4	997	927	666	234	23	0	0	967	728	250	37	0	0	0
6	10	50	5	90	4	1	1000	948	440	64	7	0	0	999	811	206	15	0	0	0
6	10	50	5	90	4	2	998	679	125	3	0	0	0	985	421	23	1	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	10	50	5	90	4	3	998	826	303	40	2	0	0	985	613	107	1	0	0	0
6	10	50	5	90	4	4	999	857	401	62	4	0	0	985	658	122	4	0	0	0
6	10	50	5	10	4	1	223	25	1	0	0	0	0	90	2	0	0	0	0	0
6	10	50	5	10	4	2	135	0	0	0	0	0	0	37	0	0	0	0	0	0
6	10	50	5	10	4	3	207	9	1	0	0	0	0	65	0	0	0	0	0	0
6	10	50	5	10	4	4	143	15	1	0	0	0	0	35	0	0	0	0	0	0
6	10	50	5	40	2	1	909	518	132	10	0	0	0	766	267	22	0	0	0	0
6	10	50	5	40	2	2	706	91	3	0	0	0	0	442	12	0	0	0	0	0
6	10	50	5	40	2	3	716	219	23	1	0	0	0	455	53	0	0	0	0	0
6	10	50	5	40	2	4	733	267	53	4	0	0	0	428	75	2	0	0	0	0
6	10	50	5	40	3	1	905	479	91	5	0	0	0	768	231	17	0	0	0	0
6	10	50	5	40	3	2	757	115	8	0	0	0	0	489	13	1	0	0	0	0
6	10	50	5	40	3	3	772	259	36	5	0	0	0	512	69	5	0	0	0	0
6	10	50	5	40	3	4	772	296	64	7	0	0	0	475	92	6	0	0	0	0
6	10	50	5	40	4	1	906	393	48	2	0	0	0	754	154	9	0	0	0	0
6	10	50	5	40	4	2	798	99	5	0	0	0	0	562	15	0	0	0	0	0
6	10	50	5	40	4	3	816	239	25	3	0	0	0	594	71	3	0	0	0	0
6	10	50	5	40	4	4	810	279	45	5	0	0	0	554	94	5	0	0	0	0
6	10	50	5	90	2	1	1000	993	853	438	70	1	0	1000	965	656	182	8	0	0
6	10	50	5	90	2	2	992	727	300	64	9	0	0	938	436	68	8	1	0	0
6	10	50	5	90	2	3	992	889	548	240	44	0	0	938	640	254	59	6	0	0
6	10	50	5	90	2	4	993	917	686	326	70	2	0	936	691	277	72	12	0	0
6	10	50	1	90	1	1	1000	41	1	1	0	0	0	999	8	1	0	0	0	0
6	10	50	1	90	1	2	1000	3	0	0	0	0	0	999	0	0	0	0	0	0
6	10	50	1	90	1	3	1000	46	0	0	0	0	0	1000	5	0	0	0	0	0
6	10	50	1	90	1	4	1000	53	0	0	0	0	0	999	7	0	0	0	0	0
6	10	50	2	90	2	1	1000	882	32	0	0	0	0	999	706	5	0	0	0	0
6	10	50	2	90	2	2	1000	846	6	0	0	0	0	993	645	0	0	0	0	0
6	10	50	2	90	2	3	1000	961	30	0	0	0	0	994	861	8	0	0	0	0
6	10	50	2	90	2	4	1000	973	58	0	0	0	0	993	891	8	0	0	0	0
6	10	50	2	40	2	1	896	294	13	0	0	0	0	745	102	2	0	0	0	0
6	10	50	2	40	2	2	842	152	1	0	0	0	0	632	43	0	0	0	0	0
6	10	50	2	40	2	3	877	317	6	0	0	0	0	694	128	0	0	0	0	0
6	10	50	2	40	2	4	847	361	14	0	0	0	0	623	151	0	0	0	0	0
6	10	50	2	40	3	1	908	227	11	0	0	0	0	755	70	1	0	0	0	0
6	10	50	2	40	3	2	866	90	0	0	0	0	0	666	18	0	0	0	0	0
6	10	50	2	40	3	3	902	247	9	0	0	0	0	744	73	0	0	0	0	0
6	10	50	2	40	3	4	870	286	15	0	0	0	0	660	92	0	0	0	0	0
6	10	5	1	10	1	1	164	11	1	0	0	0	0	54	2	0	0	0	0	0
6	10	5	1	10	1	2	87	0	0	0	0	0	0	19	0	0	0	0	0	0
6	10	5	1	10	1	3	133	2	0	0	0	0	0	37	0	0	0	0	0	0
6	10	5	1	10	1	4	114	9	0	0	0	0	0	26	0	0	0	0	0	0
6	10	5	2	10	2	1	160	16	1	0	0	0	0	46	2	0	0	0	0	0
6	10	5	2	10	2	2	74	0	0	0	0	0	0	17	0	0	0	0	0	0
6	10	5	2	10	2	3	118	6	0	0	0	0	0	30	0	0	0	0	0	0
6	10	5	2	10	2	4	96	11	0	0	0	0	0	20	1	0	0	0	0	0
6	10	5	2	10	3	1	160	9	0	0	0	0	0	47	0	0	0	0	0	0
6	10	5	2	10	3	2	82	1	0	0	0	0	0	17	0	0	0	0	0	0
6	10	5	2	10	3	3	131	3	0	0	0	0	0	28	0	0	0	0	0	0
6	10	5	2	10	3	4	105	6	0	0	0	0	0	19	0	0	0	0	0	0
6	10	5	1	40	1	1	672	37	1	0	0	0	0	408	4	0	0	0	0	0
6	10	5	1	40	1	2	630	5	0	0	0	0	0	342	2	0	0	0	0	0
6	10	5	1	40	1	3	808	30	0	0	0	0	0	557	4	0	0	0	0	0
6	10	5	1	40	1	4	648	68	1	0	0	0	0	356	7	0	0	0	0	0
6	10	5	2	40	2	1	765	165	10	1	0	0	0	506	36	1	0	0	0	0
6	10	5	2	40	2	2	588	32	1	0	0	0	0	289	2	0	0	0	0	0
6	10	5	2	40	2	3	634	122	3	0	0	0	0	337	17	0	0	0	0	0
6	10	5	2	40	2	4	645	214	16	1	0	0	0	331	58	1	0	0	0	0
6	10	5	2	40	3	1	772	168	6	0	0	0	0	520	37	0	0	0	0	0
6	10	5	2	40	3	2	606	27	0	0	0	0	0	301	3	0	0	0	0	0
6	10	5	2	40	3	3	648	94	1	0	0	0	0	359	19	0	0	0	0	0
6	10	5	2	40	3	4	668	203	11	0	0	0	0	347	49	0	0	0	0	0
6	10	5	1	90	1	1	975	53	4	0	0	0	0	904	7	0	0	0	0	0
6	10	5	1	90	1	2	975	5	0	0	0	0	0	899	0	0	0	0	0	0
6	10	5	1	90	1	3	998	37	0	0	0	0	0	980	6	0	0	0	0	0
6	10	5	1	90	1	4	975	77	2	0	0	0	0	903	19	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	10	5	2	90	2	1	999	647	26	1	0	0	0	975	330	4	0	0	0	0
6	10	5	2	90	2	2	984	451	3	0	0	0	0	912	154	0	0	0	0	0
6	10	5	2	90	2	3	984	734	15	0	0	0	0	919	405	2	0	0	0	0
6	10	5	2	90	2	4	993	870	59	1	0	0	0	941	595	7	0	0	0	0
6	10	5	2	90	3	1	991	522	26	0	0	0	0	956	242	4	0	0	0	0
6	10	5	2	90	3	2	977	303	2	0	0	0	0	897	109	1	0	0	0	0
6	10	5	2	90	3	3	979	575	12	0	0	0	0	911	276	2	0	0	0	0
6	10	5	2	90	3	4	986	715	56	0	0	0	0	921	421	5	0	0	0	0
6	10	10	1	10	1	1	223	14	1	0	0	0	0	71	4	0	0	0	0	0
6	10	10	1	10	1	2	139	0	0	0	0	0	0	33	0	0	0	0	0	0
6	10	10	1	10	1	3	220	3	0	0	0	0	0	75	0	0	0	0	0	0
6	10	10	1	10	1	4	160	10	1	0	0	0	0	35	0	0	0	0	0	0
6	10	10	2	10	2	1	212	10	0	0	0	0	0	58	0	0	0	0	0	0
6	10	10	2	10	2	2	110	0	0	0	0	0	0	23	0	0	0	0	0	0
6	10	10	2	10	2	3	162	5	0	0	0	0	0	39	1	0	0	0	0	0
6	10	10	2	10	2	4	130	9	0	0	0	0	0	26	1	0	0	0	0	0
6	10	10	2	10	3	1	204	16	0	0	0	0	0	63	0	0	0	0	0	0
6	10	10	2	10	3	2	101	1	0	0	0	0	0	29	0	0	0	0	0	0
6	10	10	2	10	3	3	147	7	0	0	0	0	0	46	1	0	0	0	0	0
6	10	10	2	10	3	4	123	11	1	0	0	0	0	35	2	0	0	0	0	0
6	10	10	1	40	1	1	802	33	0	0	0	0	0	585	6	0	0	0	0	0
6	10	10	1	40	1	2	776	2	0	0	0	0	0	544	0	0	0	0	0	0
6	10	10	1	40	1	3	905	35	0	0	0	0	0	753	1	0	0	0	0	0
6	10	10	1	40	1	4	788	55	0	0	0	0	0	556	5	0	0	0	0	0
6	10	10	2	40	2	1	868	225	6	0	0	0	0	678	56	0	0	0	0	0
6	10	10	2	40	2	2	755	65	1	0	0	0	0	466	8	0	0	0	0	0
6	10	10	2	40	2	3	784	234	5	0	0	0	0	531	65	0	0	0	0	0
6	10	10	2	40	2	4	807	314	13	0	0	0	0	520	94	1	0	0	0	0
6	10	10	2	40	3	1	841	190	7	0	0	0	0	643	61	0	0	0	0	0
6	10	10	2	40	3	2	728	57	0	0	0	0	0	460	7	0	0	0	0	0
6	10	10	2	40	3	3	773	174	1	0	0	0	0	515	54	0	0	0	0	0
6	10	10	2	40	3	4	766	244	7	0	0	0	0	504	83	0	0	0	0	0
6	10	10	1	90	1	1	998	44	3	0	0	0	0	986	8	0	0	0	0	0
6	10	10	1	90	1	2	998	8	0	0	0	0	0	986	0	0	0	0	0	0
6	10	10	1	90	1	3	1000	35	1	0	0	0	0	999	7	0	0	0	0	0
6	10	10	1	90	1	4	998	63	3	0	0	0	0	986	13	0	0	0	0	0
6	10	50	2	90	3	1	1000	772	23	0	0	0	0	1000	552	5	0	0	0	0
6	10	50	2	90	3	2	1000	698	6	0	0	0	0	998	632	0	0	0	0	0
6	10	50	2	90	3	3	1000	885	30	0	0	0	0	998	714	4	0	0	0	0
6	10	50	2	90	3	4	1000	905	63	0	0	0	0	998	754	7	0	0	0	0
6	10	50	3	10	2	1	254	22	4	0	0	0	0	100	6	0	0	0	0	0
6	10	50	3	10	2	2	150	2	0	0	0	0	0	52	0	0	0	0	0	0
6	10	50	3	10	2	3	222	10	1	0	0	0	0	82	1	0	0	0	0	0
6	10	50	3	10	2	4	157	15	1	0	0	0	0	46	2	0	0	0	0	0
6	6	5	5	90	2	2	943	354	93	19	2	0	0	708	77	5	0	0	0	0
6	6	5	5	90	2	3	943	687	354	157	99	0	0	708	246	47	9	0	0	0
6	6	5	5	90	2	4	965	865	686	536	393	0	0	794	458	233	115	49	0	0
6	6	5	5	90	3	1	1000	985	829	448	127	0	0	998	923	573	137	14	0	0
6	6	5	5	90	3	2	968	369	91	14	1	0	0	761	87	11	0	0	0	0
6	6	5	5	90	3	3	968	689	333	78	33	0	0	761	264	55	8	2	0	0
6	6	5	5	90	3	4	981	861	614	371	168	0	0	839	477	228	61	18	0	0
6	6	5	5	90	4	1	1000	947	552	124	22	0	0	997	789	237	27	1	0	0
6	6	5	5	90	4	2	985	368	43	1	0	0	0	838	90	1	0	0	0	0
6	6	5	5	90	4	3	985	651	162	16	3	0	0	838	270	25	0	0	0	0
6	6	5	5	90	4	4	994	844	418	135	33	0	0	899	463	111	41	2	0	0
6	10	50	3	10	3	1	251	23	1	0	0	0	0	108	3	0	0	0	0	0
6	10	50	3	10	3	2	150	1	0	0	0	0	0	48	0	0	0	0	0	0
6	10	50	3	10	3	3	226	8	0	0	0	0	0	76	0	0	0	0	0	0
6	10	50	3	10	3	4	162	10	0	0	0	0	0	46	0	0	0	0	0	0
6	10	50	3	10	4	1	216	18	0	0	0	0	0	79	0	0	0	0	0	0
6	10	50	3	10	4	2	123	0	0	0	0	0	0	37	0	0	0	0	0	0
6	10	50	3	10	4	3	183	3	0	0	0	0	0	71	0	0	0	0	0	0
6	10	50	3	10	4	4	128	8	0	0	0	0	0	34	0	0	0	0	0	0
6	10	50	3	40	2	1	904	398	51	1	0	0	0	750	180	14	0	0	0	0
6	10	50	3	40	2	2	783	129	7	0	0	0	0	521	37	2	0	0	0	0
6	10	50	3	40	2	3	801	296	32	2	0	0	0	554	94	6	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	10	50	3	40	2	4	804	330	54	3	0	0	0	515	114	6	0	0	0	0
2	6	50	2	90	2	3	1000	993	0	0	0	0	0	1000	860	0	0	0	0	0
2	6	50	2	90	2	4	1000	1000	0	0	0	0	0	1000	995	0	0	0	0	0
2	6	50	2	90	3	1	1000	998	0	0	0	0	0	1000	986	0	0	0	0	0
2	6	50	2	90	3	2	1000	957	0	0	0	0	0	1000	644	0	0	0	0	0
2	6	50	2	90	3	3	1000	957	0	0	0	0	0	1000	644	0	0	0	0	0
2	6	50	2	90	3	4	1000	999	0	0	0	0	0	1000	987	0	0	0	0	0
2	10	50	1	10	1	1	388	17	0	0	0	0	0	193	2	0	0	0	0	0
2	10	50	1	10	1	2	330	0	0	0	0	0	0	155	0	0	0	0	0	0
2	10	50	1	10	1	3	406	0	0	0	0	0	0	204	0	0	0	0	0	0
2	10	50	1	10	1	4	333	12	0	0	0	0	0	156	1	0	0	0	0	0
2	10	50	2	10	2	1	453	38	0	0	0	0	0	204	7	0	0	0	0	0
2	10	50	2	10	2	2	325	0	0	0	0	0	0	116	0	0	0	0	0	0
2	10	50	2	10	2	3	361	0	0	0	0	0	0	131	0	0	0	0	0	0
2	10	50	2	10	2	4	327	35	0	0	0	0	0	125	6	0	0	0	0	0
2	10	50	2	10	3	1	387	45	0	0	0	0	0	186	5	0	0	0	0	0
2	10	50	2	10	3	2	299	0	0	0	0	0	0	114	0	0	0	0	0	0
2	10	50	2	10	3	3	330	0	0	0	0	0	0	132	0	0	0	0	0	0
2	10	50	2	10	3	4	306	45	0	0	0	0	0	118	4	0	0	0	0	0
2	10	50	1	40	1	1	993	34	0	0	0	0	0	954	6	0	0	0	0	0
2	10	50	1	40	1	2	991	0	0	0	0	0	0	949	0	0	0	0	0	0
2	10	50	1	40	1	3	994	0	0	0	0	0	0	973	0	0	0	0	0	0
2	10	50	1	40	1	4	991	44	0	0	0	0	0	949	7	0	0	0	0	0
2	10	50	2	40	2	1	991	723	0	0	0	0	0	966	461	0	0	0	0	0
2	10	50	2	40	2	2	970	71	0	0	0	0	0	873	0	0	0	0	0	0
2	10	50	2	40	2	3	970	71	0	0	0	0	0	874	0	0	0	0	0	0
2	10	50	2	40	2	4	972	749	0	0	0	0	0	879	452	0	0	0	0	0
2	10	50	2	40	3	1	991	655	0	0	0	0	0	969	386	0	0	0	0	0
2	10	50	2	40	3	2	978	65	0	0	0	0	0	890	0	0	0	0	0	0
2	10	50	2	40	3	3	978	65	0	0	0	0	0	892	0	0	0	0	0	0
2	10	50	2	40	3	4	979	689	0	0	0	0	0	898	387	0	0	0	0	0
2	10	50	1	90	1	1	1000	32	0	0	0	0	0	1000	7	0	0	0	0	0
2	10	50	1	90	1	2	1000	1	0	0	0	0	0	1000	0	0	0	0	0	0
2	10	50	1	90	1	3	1000	1	0	0	0	0	0	1000	0	0	0	0	0	0
2	10	50	1	90	1	4	1000	44	0	0	0	0	0	1000	7	0	0	0	0	0
2	10	50	2	90	2	1	1000	998	0	0	0	0	0	1000	988	0	0	0	0	0
2	10	50	2	90	2	2	1000	842	0	0	0	0	0	1000	146	0	0	0	0	0
2	10	50	2	90	2	3	1000	842	0	0	0	0	0	1000	146	0	0	0	0	0
2	10	50	2	90	2	4	1000	998	0	0	0	0	0	1000	988	0	0	0	0	0
2	10	50	2	90	3	1	1000	986	0	0	0	0	0	1000	925	0	0	0	0	0
2	10	50	2	90	3	2	1000	644	0	0	0	0	0	1000	60	0	0	0	0	0
2	10	50	2	90	3	3	1000	644	0	0	0	0	0	1000	60	0	0	0	0	0
2	10	50	2	90	3	4	1000	989	0	0	0	0	0	1000	926	0	0	0	0	0
6	6	50	3	90	2	1	1000	997	758	43	0	0	0	1000	988	524	7	0	0	0
6	6	50	3	90	2	2	1000	949	513	4	0	0	0	999	793	219	1	0	0	0
6	6	50	3	90	2	3	1000	987	757	30	0	0	0	999	890	499	2	0	0	0
6	6	50	3	90	2	4	1000	996	844	50	1	0	0	999	949	626	9	0	0	0
6	6	50	3	90	3	1	1000	989	551	10	1	0	0	1000	953	273	0	0	0	0
6	6	50	3	90	3	2	1000	897	249	0	0	0	0	999	727	69	0	0	0	0
6	6	50	3	90	3	3	1000	955	498	4	0	0	0	999	835	237	0	0	0	0
6	6	50	3	90	3	4	1000	977	619	15	0	0	0	1000	897	345	2	0	0	0
6	6	50	3	90	4	1	1000	975	479	17	0	0	0	1000	916	216	3	0	0	0
6	6	50	3	90	4	2	1000	833	193	1	0	0	0	999	628	50	0	0	0	0
6	6	50	3	90	4	3	1000	918	411	8	0	0	0	999	751	184	1	0	0	0
6	6	50	3	90	4	4	1000	953	533	18	1	0	0	999	833	283	4	0	0	0
6	6	50	5	10	2	1	308	50	10	1	0	0	0	132	15	1	0	0	0	0
6	6	50	5	10	2	2	193	2	0	0	0	0	0	49	0	0	0	0	0	0
6	6	50	5	10	2	3	225	10	2	1	0	0	0	67	0	0	0	0	0	0
6	6	50	5	10	2	4	214	29	2	1	0	0	0	60	2	1	1	0	0	0
6	6	50	5	10	3	1	334	33	3	0	0	0	0	149	5	1	0	0	0	0
6	6	50	5	10	3	2	207	2	0	0	0	0	0	66	0	0	0	0	0	0
6	6	50	5	10	3	3	239	8	0	0	0	0	0	87	2	0	0	0	0	0
6	6	50	5	10	3	4	236	19	1	0	0	0	0	80	2	0	0	0	0	0
6	6	50	5	10	4	1	296	30	2	0	0	0	0	128	6	1	0	0	0	0
6	6	50	5	10	4	2	184	2	0	0	0	0	0	63	0	0	0	0	0	0
6	6	50	5	10	4	3	235	9	1	0	0	0	0	91	0	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	6	50	5	10	4	4	213	24	1	0	0	0	0	72	2	0	0	0	0	0
3	10	5	1	10	1	1	271	18	1	0	0	0	0	105	2	0	0	0	0	0
3	10	5	1	10	1	2	196	3	0	0	0	0	0	61	0	0	0	0	0	0
3	10	5	1	10	1	3	285	8	0	0	0	0	0	96	0	0	0	0	0	0
3	10	5	1	10	1	4	231	16	0	0	0	0	0	69	3	0	0	0	0	0
3	10	5	2	10	2	1	296	25	2	0	0	0	0	109	5	0	0	0	0	0
3	10	5	2	10	2	2	177	1	0	0	0	0	0	51	0	0	0	0	0	0
3	10	5	2	10	2	3	217	6	0	0	0	0	0	74	1	0	0	0	0	0
3	10	5	2	10	2	4	225	17	1	0	0	0	0	63	1	0	0	0	0	0
3	10	5	2	10	3	1	272	24	2	0	0	0	0	94	7	0	0	0	0	0
3	10	5	2	10	3	2	181	1	0	0	0	0	0	44	0	0	0	0	0	0
3	10	5	2	10	3	3	227	13	0	0	0	0	0	68	0	0	0	0	0	0
3	10	5	2	10	3	4	218	29	2	0	0	0	0	50	2	0	0	0	0	0
3	10	5	1	40	1	1	898	56	2	0	0	0	0	713	11	0	0	0	0	0
3	10	5	1	40	1	2	884	6	0	0	0	0	0	675	1	0	0	0	0	0
3	10	5	1	40	1	3	943	31	0	0	0	0	0	801	4	0	0	0	0	0
3	10	5	1	40	1	4	889	68	4	0	0	0	0	685	12	0	0	0	0	0
3	10	5	2	40	2	1	935	414	19	0	0	0	0	810	152	0	0	0	0	0
3	10	5	2	40	2	2	845	148	0	0	0	0	0	598	35	0	0	0	0	0
3	10	5	2	40	2	3	851	322	0	0	0	0	0	611	106	0	0	0	0	0
3	10	5	2	40	2	4	892	482	16	0	0	0	0	643	161	0	0	0	0	0
3	10	5	2	40	3	1	930	343	14	0	0	0	0	793	114	1	0	0	0	0
3	10	5	2	40	3	2	845	107	0	0	0	0	0	599	17	0	0	0	0	0
3	10	5	2	40	3	3	859	247	0	0	0	0	0	627	69	0	0	0	0	0
3	10	5	2	40	3	4	889	384	14	0	0	0	0	640	135	1	0	0	0	0
3	10	5	1	90	1	1	1000	43	1	0	0	0	0	996	9	0	0	0	0	0
3	10	5	1	90	1	2	1000	11	0	0	0	0	0	996	3	0	0	0	0	0
3	10	5	1	90	1	3	1000	29	0	0	0	0	0	1000	7	0	0	0	0	0
3	10	5	1	90	1	4	1000	69	0	0	0	0	0	996	14	0	0	0	0	0
3	10	5	2	90	2	1	1000	924	32	0	0	0	0	999	722	5	0	0	0	0
3	10	5	2	90	2	2	997	793	0	0	0	0	0	983	518	0	0	0	0	0
3	10	5	2	90	2	3	997	925	0	0	0	0	0	983	718	0	0	0	0	0
3	10	5	2	90	2	4	998	967	42	0	0	0	0	988	835	5	0	0	0	0
3	10	5	2	90	3	1	1000	854	37	0	0	0	0	999	618	6	0	0	0	0
3	10	5	2	90	3	2	1000	667	2	0	0	0	0	994	396	0	0	0	0	0
3	10	5	2	90	3	3	1000	835	2	0	0	0	0	994	601	0	0	0	0	0
3	10	5	2	90	3	4	1000	908	40	0	0	0	0	997	724	8	0	0	0	0
3	10	5	3	10	2	1	320	30	1	0	0	0	0	127	5	0	0	0	0	0
3	10	5	3	10	2	2	188	0	0	0	0	0	0	59	0	0	0	0	0	0
3	10	5	3	10	2	3	214	8	0	0	0	0	0	67	0	0	0	0	0	0
3	10	5	3	10	2	4	229	30	1	0	0	0	0	64	0	0	0	0	0	0
3	10	5	3	10	3	1	335	43	2	0	0	0	0	133	4	0	0	0	0	0
3	10	5	3	10	3	2	182	0	0	0	0	0	0	65	0	0	0	0	0	0
3	10	5	3	10	3	3	213	7	0	0	0	0	0	84	0	0	0	0	0	0
3	10	5	3	10	3	4	234	25	0	0	0	0	0	70	2	0	0	0	0	0
3	10	5	3	10	4	1	282	29	0	0	0	0	0	111	4	0	0	0	0	0
3	10	5	3	10	4	2	165	2	0	0	0	0	0	52	0	0	0	0	0	0
3	10	5	3	10	4	3	196	9	0	0	0	0	0	66	0	0	0	0	0	0
3	10	5	3	10	4	4	209	20	0	0	0	0	0	55	2	0	0	0	0	0
3	10	5	3	40	2	1	953	596	126	0	0	0	0	836	297	15	0	0	0	0
3	10	5	3	40	2	2	784	102	1	0	0	0	0	471	16	0	0	0	0	0
3	10	5	3	40	2	3	785	269	1	0	0	0	0	473	58	0	0	0	0	0
3	10	5	3	40	2	4	861	452	120	0	0	0	0	529	126	11	0	0	0	0
3	10	5	3	40	3	1	948	548	111	0	0	0	0	836	253	16	0	0	0	0
3	10	5	3	40	3	2	803	113	4	0	0	0	0	514	20	0	0	0	0	0
3	10	5	3	40	3	3	808	258	4	0	0	0	0	518	65	0	0	0	0	0
3	10	5	3	40	3	4	870	420	110	0	0	0	0	567	136	20	0	0	0	0
3	10	5	3	40	4	1	929	461	85	0	0	0	0	818	220	11	0	0	0	0
3	10	5	3	40	4	2	822	103	2	0	0	0	0	539	20	0	0	0	0	0
3	10	5	3	40	4	3	827	243	2	0	0	0	0	549	58	0	0	0	0	0
3	10	5	3	40	4	4	873	399	82	0	0	0	0	589	107	10	0	0	0	0
3	10	5	3	90	2	1	1000	993	760	0	0	0	0	1000	952	467	0	0	0	0
3	10	5	3	90	2	2	997	777	233	0	0	0	0	957	449	2	0	0	0	0
3	10	5	3	90	2	3	997	919	233	0	0	0	0	957	702	2	0	0	0	0
3	10	5	3	90	2	4	998	969	785	0	0	0	0	970	823	500	0	0	0	0
3	10	5	3	90	3	1	1000	982	588	0	0	0	0	999	920	265	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
3	10	5	3	90	3	2	997	729	101	0	0	0	0	968	390	2	0	0	0	0
3	10	5	3	90	3	3	997	900	101	0	0	0	0	968	632	2	0	0	0	0
3	10	5	3	90	3	4	1000	956	619	0	0	0	0	984	784	303	0	0	0	0
3	10	5	3	90	4	1	1000	959	506	0	0	0	0	1000	854	199	0	0	0	0
3	10	5	3	90	4	2	999	652	67	0	0	0	0	976	333	0	0	0	0	0
3	10	5	3	90	4	3	999	840	67	0	0	0	0	976	552	0	0	0	0	0
3	10	5	3	90	4	4	1000	923	521	0	0	0	0	982	704	236	0	0	0	0
3	10	10	1	10	1	1	280	20	1	0	0	0	0	110	3	0	0	0	0	0
3	10	10	1	10	1	2	193	0	0	0	0	0	0	71	0	0	0	0	0	0
3	10	10	1	10	1	3	273	7	0	0	0	0	0	117	0	0	0	0	0	0
3	10	10	1	10	1	4	235	19	0	0	0	0	0	76	1	0	0	0	0	0
3	10	10	2	10	2	1	334	37	1	0	0	0	0	133	5	0	0	0	0	0
3	10	10	2	10	2	2	192	0	0	0	0	0	0	62	0	0	0	0	0	0
3	10	10	2	10	2	3	235	7	0	0	0	0	0	81	0	0	0	0	0	0
3	10	10	2	10	2	4	245	30	1	0	0	0	0	75	1	0	0	0	0	0
3	10	10	2	10	3	1	298	31	2	0	0	0	0	129	7	0	0	0	0	0
3	10	10	2	10	3	2	197	6	0	0	0	0	0	72	0	0	0	0	0	0
3	10	10	2	10	3	3	248	14	0	0	0	0	0	102	2	0	0	0	0	0
3	10	10	2	10	3	4	244	29	1	0	0	0	0	78	6	0	0	0	0	0
3	10	10	1	40	1	1	952	44	0	0	0	0	0	850	6	0	0	0	0	0
3	10	10	1	40	1	2	950	5	0	0	0	0	0	833	0	0	0	0	0	0
3	10	10	1	40	1	3	978	23	0	0	0	0	0	909	5	0	0	0	0	0
3	10	10	1	40	1	4	952	54	1	0	0	0	0	837	9	0	0	0	0	0
3	10	10	2	40	2	1	973	467	21	0	0	0	0	878	217	4	0	0	0	0
3	10	10	2	40	2	2	916	212	0	0	0	0	0	711	55	0	0	0	0	0
3	10	10	2	40	2	3	917	409	0	0	0	0	0	720	170	0	0	0	0	0
3	10	10	2	40	2	4	949	553	22	0	0	0	0	748	257	3	0	0	0	0
3	10	10	2	40	3	1	973	397	14	0	0	0	0	890	149	1	0	0	0	0
3	10	10	2	40	3	2	907	148	0	0	0	0	0	737	39	0	0	0	0	0
3	10	10	2	40	3	3	911	325	0	0	0	0	0	748	120	0	0	0	0	0
3	10	10	2	40	3	4	943	469	11	0	0	0	0	764	176	1	0	0	0	0
3	10	10	1	90	1	1	1000	52	1	0	0	0	0	1000	8	1	0	0	0	0
3	10	10	1	90	1	2	1000	6	0	0	0	0	0	1000	1	0	0	0	0	0
3	10	10	1	90	1	3	1000	30	0	0	0	0	0	1000	5	0	0	0	0	0
3	10	10	1	90	1	4	1000	54	1	0	0	0	0	1000	15	0	0	0	0	0
3	10	10	2	90	2	1	1000	976	54	0	0	0	0	1000	887	11	0	0	0	0
3	10	10	2	90	2	2	1000	919	0	0	0	0	0	999	770	0	0	0	0	0
3	10	10	2	90	2	3	1000	972	0	0	0	0	0	999	903	0	0	0	0	0
3	10	10	2	90	2	4	1000	985	56	0	0	0	0	999	936	12	0	0	0	0
3	10	10	2	90	3	1	1000	922	32	0	0	0	0	1000	748	4	0	0	0	0
3	10	10	2	90	3	2	1000	789	0	0	0	0	0	1000	562	0	0	0	0	0
3	10	10	2	90	3	3	1000	906	0	0	0	0	0	1000	753	0	0	0	0	0
3	10	10	2	90	3	4	1000	946	36	0	0	0	0	1000	833	7	0	0	0	0
3	10	10	3	10	2	1	327	39	2	0	0	0	0	143	13	0	0	0	0	0
3	10	10	3	10	2	2	195	6	0	0	0	0	0	62	0	0	0	0	0	0
3	10	10	3	10	2	3	231	14	0	0	0	0	0	75	1	0	0	0	0	0
3	10	10	3	10	2	4	233	32	0	0	0	0	0	73	7	0	0	0	0	0
3	10	10	3	10	3	1	376	47	2	0	0	0	0	163	9	0	0	0	0	0
3	10	10	3	10	3	2	215	5	0	0	0	0	0	74	0	0	0	0	0	0
3	10	10	3	10	3	3	253	14	0	0	0	0	0	95	2	0	0	0	0	0
3	10	10	3	10	3	4	276	33	1	0	0	0	0	89	6	0	0	0	0	0
3	10	10	3	10	4	1	318	30	1	0	0	0	0	141	5	0	0	0	0	0
3	10	10	3	10	4	2	195	1	0	0	0	0	0	60	1	0	0	0	0	0
3	10	10	3	10	4	3	233	7	0	0	0	0	0	79	1	0	0	0	0	0
3	10	10	3	10	4	4	246	33	1	0	0	0	0	66	3	1	0	0	0	0
3	10	10	3	40	2	1	968	658	177	0	0	0	0	885	398	39	0	0	0	0
3	10	10	3	40	2	2	833	174	4	0	0	0	0	603	34	0	0	0	0	0
3	10	10	3	40	2	3	834	394	4	0	0	0	0	605	131	0	0	0	0	0
3	10	10	3	40	2	4	896	548	160	0	0	0	0	641	211	33	0	0	0	0
3	10	10	3	40	3	1	971	598	124	0	0	0	0	893	320	17	0	0	0	0
3	10	10	3	40	3	2	858	153	1	0	0	0	0	648	44	0	0	0	0	0
3	10	10	3	40	3	3	862	348	1	0	0	0	0	651	115	0	0	0	0	0
3	10	10	3	40	3	4	920	513	121	0	0	0	0	684	186	15	0	0	0	0
3	10	10	3	40	4	1	977	550	95	0	0	0	0	896	270	16	0	0	0	0
3	10	10	3	40	4	2	889	145	2	0	0	0	0	677	40	0	0	0	0	0
3	10	10	3	40	4	3	891	332	2	0	0	0	0	684	108	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
3	10	10	3	40	4	4	928	495	94	0	0	0	0	718	185	15	0	0	0	0
3	10	10	3	90	2	1	1000	996	856	0	0	0	0	999	982	574	0	0	0	0
3	10	10	3	90	2	2	998	890	231	0	0	0	0	992	636	0	0	0	0	0
3	10	10	3	90	2	3	998	964	231	0	0	0	0	992	849	0	0	0	0	0
3	10	10	3	90	2	4	993	987	860	0	0	0	0	993	908	625	0	0	0	0
3	10	10	3	90	3	1	1000	994	655	0	0	0	0	1000	948	362	0	0	0	0
3	10	10	3	90	3	2	1000	831	101	0	0	0	0	995	590	0	0	0	0	0
3	10	10	3	90	3	3	1000	946	101	0	0	0	0	995	794	0	0	0	0	0
3	10	10	3	90	3	4	1000	982	670	0	0	0	0	997	873	390	0	0	0	0
3	10	10	3	90	4	1	1000	975	540	0	0	0	0	1000	911	249	0	0	0	0
3	10	10	3	90	4	2	1000	777	55	0	0	0	0	998	548	0	0	0	0	0
3	10	10	3	90	4	3	1000	912	55	0	0	0	0	998	740	0	0	0	0	0
3	10	10	3	90	4	4	1000	953	552	0	0	0	0	999	816	280	0	0	0	0
3	10	50	1	10	1	1	337	13	0	0	0	0	0	129	2	0	0	0	0	0
3	10	50	1	10	1	2	264	2	0	0	0	0	0	98	0	0	0	0	0	0
3	10	50	1	10	1	3	346	13	0	0	0	0	0	158	2	0	0	0	0	0
3	10	50	1	10	1	4	293	19	0	0	0	0	0	107	2	0	0	0	0	0
3	10	50	2	10	2	1	341	29	1	0	0	0	0	162	5	0	0	0	0	0
3	10	50	2	10	2	2	244	1	0	0	0	0	0	86	1	0	0	0	0	0
3	10	50	2	10	2	3	290	19	0	0	0	0	0	117	1	0	0	0	0	0
3	10	50	2	10	2	4	276	29	0	0	0	0	0	96	1	0	0	0	0	0
3	10	50	2	10	3	1	373	33	2	0	0	0	0	172	6	0	0	0	0	0
3	10	50	2	10	3	2	265	3	0	0	0	0	0	97	1	0	0	0	0	0
3	10	50	2	10	3	3	302	19	0	0	0	0	0	124	2	0	0	0	0	0
3	10	50	2	10	3	4	297	23	1	0	0	0	0	104	4	0	0	0	0	0
3	10	50	1	40	1	1	965	43	2	0	0	0	0	895	14	0	0	0	0	0
3	10	50	1	40	1	2	961	9	0	0	0	0	0	888	1	0	0	0	0	0
3	10	50	1	40	1	3	985	41	0	0	0	0	0	947	9	0	0	0	0	0
3	10	50	1	40	1	4	965	51	2	0	0	0	0	888	9	0	0	0	0	0
3	10	50	2	40	2	1	971	524	20	0	0	0	0	908	257	0	0	0	0	0
3	10	50	2	40	2	2	924	281	0	0	0	0	0	794	108	0	0	0	0	0
3	10	50	2	40	2	3	930	535	0	0	0	0	0	801	277	0	0	0	0	0
3	10	50	2	40	2	4	949	577	18	0	0	0	0	811	299	0	0	0	0	0
3	10	50	2	40	3	1	977	445	20	0	0	0	0	915	192	2	0	0	0	0
3	10	50	2	40	3	2	941	219	0	0	0	0	0	841	69	0	0	0	0	0
3	10	50	2	40	3	3	948	473	0	0	0	0	0	858	214	0	0	0	0	0
3	10	50	2	40	3	4	959	507	15	0	0	0	0	861	243	2	0	0	0	0
3	10	50	1	90	1	1	1000	42	0	0	0	0	0	1000	1	0	0	0	0	0
3	10	50	1	90	1	2	1000	4	0	0	0	0	0	1000	0	0	0	0	0	0
3	10	50	1	90	1	3	1000	47	0	0	0	0	0	1000	4	0	0	0	0	0
3	10	50	1	90	1	4	1000	59	0	0	0	0	0	1000	4	0	0	0	0	0
3	10	50	2	90	2	1	1000	994	32	0	0	0	0	1000	952	7	0	0	0	0
3	10	50	2	90	2	2	1000	963	1	0	0	0	0	1000	855	0	0	0	0	0
3	10	50	2	90	2	3	1000	998	1	0	0	0	0	1000	964	0	0	0	0	0
3	10	50	2	90	2	4	1000	998	34	0	0	0	0	1000	972	7	0	0	0	0
3	10	50	2	90	3	1	1000	946	32	0	0	0	0	1000	821	8	0	0	0	0
3	10	50	2	90	3	2	1000	871	0	0	0	0	0	999	707	0	0	0	0	0
3	10	50	2	90	3	3	1000	957	0	0	0	0	0	999	872	0	0	0	0	0
3	10	50	2	90	3	4	1000	963	34	0	0	0	0	1000	886	8	0	0	0	0
3	10	50	3	10	2	1	350	50	4	0	0	0	0	156	8	0	0	0	0	0
3	10	50	3	10	2	2	207	1	0	0	0	0	0	73	0	0	0	0	0	0
3	10	50	3	10	2	3	236	17	0	0	0	0	0	91	2	0	0	0	0	0
3	10	50	3	10	2	4	261	29	2	0	0	0	0	82	2	0	0	0	0	0
3	10	50	3	10	3	1	405	50	5	0	0	0	0	170	8	0	0	0	0	0
3	10	50	3	10	3	2	247	4	0	0	0	0	0	91	0	0	0	0	0	0
3	10	50	3	10	3	3	284	19	0	0	0	0	0	122	3	0	0	0	0	0
3	10	50	3	10	3	4	307	25	0	0	0	0	0	99	4	0	0	0	0	0
3	10	50	3	10	4	1	337	36	3	0	0	0	0	157	5	0	0	0	0	0
3	10	50	3	10	4	2	232	2	0	0	0	0	0	77	0	0	0	0	0	0
3	10	50	3	10	4	3	277	20	0	0	0	0	0	98	2	0	0	0	0	0
3	10	50	3	10	4	4	273	29	1	0	0	0	0	86	3	0	0	0	0	0
3	10	50	3	40	2	1	977	696	199	0	0	0	0	921	429	46	0	0	0	0
3	10	50	3	40	2	2	885	244	1	0	0	0	0	687	82	0	0	0	0	0
3	10	50	3	40	2	3	887	323	1	0	0	0	0	689	235	0	0	0	0	0
3	10	50	3	40	2	4	929	570	182	0	0	0	0	715	261	40	0	0	0	0
3	10	50	3	40	3	1	975	616	138	0	0	0	0	904	358	23	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
3	10	50	3	40	3	2	905	214	1	0	0	0	0	718	71	0	0	0	0	0
3	10	50	3	40	3	3	907	477	1	0	0	0	0	725	209	0	0	0	0	0
3	10	50	3	40	3	4	939	531	126	0	0	0	0	738	232	20	0	0	0	0
3	10	50	3	40	4	1	983	588	104	0	0	0	0	925	326	21	0	0	0	0
3	10	50	3	40	4	2	932	194	0	0	0	0	0	765	59	0	0	0	0	0
3	10	50	3	40	4	3	933	460	0	0	0	0	0	772	193	0	0	0	0	0
3	10	50	3	40	4	4	964	503	90	0	0	0	0	783	209	20	0	0	0	0
3	10	50	3	90	2	1	1000	1000	903	0	0	0	0	1000	995	712	0	0	0	0
3	10	50	3	90	2	2	1000	961	237	0	0	0	0	998	808	0	0	0	0	0
3	10	50	3	90	2	3	1000	993	237	0	0	0	0	998	964	0	0	0	0	0
3	10	50	3	90	2	4	1000	995	904	0	0	0	0	999	968	729	0	0	0	0
3	10	50	3	90	3	1	1000	994	680	0	0	0	0	1000	970	441	0	0	0	0
3	10	50	3	90	3	2	1000	911	99	0	0	0	0	997	756	0	0	0	0	0
3	10	50	3	90	3	3	1000	980	99	0	0	0	0	997	914	0	0	0	0	0
3	10	50	3	90	3	4	1000	984	631	0	0	0	0	998	923	467	0	0	0	0
3	10	50	3	90	4	1	1000	988	590	0	0	0	0	1000	950	322	0	0	0	0
3	10	50	3	90	4	2	1000	878	59	0	0	0	0	1000	685	0	0	0	0	0
3	10	50	3	90	4	3	1000	961	59	0	0	0	0	1000	879	0	0	0	0	0
3	10	50	3	90	4	4	1000	969	587	0	0	0	0	1000	893	337	0	0	0	0
6	3	5	1	10	1	1	272	20	0	0	0	0	0	84	2	0	0	0	0	0
6	3	5	1	10	1	2	191	5	0	0	0	0	0	34	0	0	0	0	0	0
6	3	5	1	10	1	3	226	5	0	0	0	0	0	43	0	0	0	0	0	0
6	3	5	1	10	1	4	232	21	0	0	0	0	0	57	1	0	0	0	0	0
6	3	5	2	10	2	1	318	41	0	0	0	0	0	110	4	0	0	0	0	0
6	3	5	2	10	2	2	184	6	0	0	0	0	0	35	0	0	0	0	0	0
6	3	5	2	10	2	3	195	6	0	0	0	0	0	35	0	0	0	0	0	0
6	3	5	2	10	2	4	241	34	0	0	0	0	0	70	3	0	0	0	0	0
6	3	5	2	10	3	1	315	47	0	0	0	0	0	125	6	0	0	0	0	0
6	3	5	2	10	3	2	209	11	0	0	0	0	0	46	0	0	0	0	0	0
6	3	5	2	10	3	3	222	11	0	0	0	0	0	51	0	0	0	0	0	0
6	3	5	2	10	3	4	249	46	0	0	0	0	0	94	8	0	0	0	0	0
6	3	5	1	40	1	1	754	46	0	0	0	0	0	434	6	0	0	0	0	0
6	3	5	1	40	1	2	705	15	0	0	0	0	0	329	1	0	0	0	0	0
6	3	5	1	40	1	3	785	15	0	0	0	0	0	362	1	0	0	0	0	0
6	3	5	1	40	1	4	730	56	0	0	0	0	0	412	10	0	0	0	0	0
6	3	5	2	40	2	1	914	477	0	0	0	0	0	671	151	0	0	0	0	0
6	3	5	2	40	2	2	687	235	0	0	0	0	0	262	21	0	0	0	0	0
6	3	5	2	40	2	3	689	235	0	0	0	0	0	262	21	0	0	0	0	0
6	3	5	2	40	2	4	777	483	0	0	0	0	0	466	161	0	0	0	0	0
6	3	5	2	40	3	1	904	396	0	0	0	0	0	666	114	0	0	0	0	0
6	3	5	2	40	3	2	712	185	0	0	0	0	0	285	14	0	0	0	0	0
6	3	5	2	40	3	3	714	185	0	0	0	0	0	287	14	0	0	0	0	0
6	3	5	2	40	3	4	792	400	0	0	0	0	0	503	123	0	0	0	0	0
6	3	5	1	90	1	1	982	73	0	0	0	0	0	825	15	0	0	0	0	0
6	3	5	1	90	1	2	978	27	0	0	0	0	0	754	5	0	0	0	0	0
6	3	5	1	90	1	3	989	27	0	0	0	0	0	794	5	0	0	0	0	0
6	3	5	1	90	1	4	980	95	0	0	0	0	0	817	19	0	0	0	0	0
6	3	5	2	90	2	1	1000	946	0	0	0	0	0	991	643	0	0	0	0	0
6	3	5	2	90	2	2	965	789	0	0	0	0	0	666	222	0	0	0	0	0
6	3	5	2	90	2	3	965	789	0	0	0	0	0	666	222	0	0	0	0	0
6	3	5	2	90	2	4	989	955	0	0	0	0	0	891	673	0	0	0	0	0
6	3	5	2	90	3	1	999	886	0	0	0	0	0	987	531	0	0	0	0	0
6	3	5	2	90	3	2	981	688	0	0	0	0	0	703	179	0	0	0	0	0
6	3	5	2	90	3	3	981	688	0	0	0	0	0	703	179	0	0	0	0	0
6	3	5	2	90	3	4	993	909	0	0	0	0	0	905	593	0	0	0	0	0
6	3	10	1	10	1	1	388	21	0	0	0	0	0	152	2	0	0	0	0	0
6	3	10	1	10	1	2	322	2	0	0	0	0	0	115	0	0	0	0	0	0
6	3	10	1	10	1	3	379	2	0	0	0	0	0	149	0	0	0	0	0	0
6	3	10	1	10	1	4	339	18	0	0	0	0	0	125	0	0	0	0	0	0
6	3	10	2	10	2	1	428	52	0	0	0	0	0	179	7	0	0	0	0	0
6	3	10	2	10	2	2	304	5	0	0	0	0	0	93	0	0	0	0	0	0
6	3	10	2	10	2	3	321	5	0	0	0	0	0	112	0	0	0	0	0	0
6	3	10	2	10	2	4	323	50	0	0	0	0	0	123	7	0	0	0	0	0
6	3	10	2	10	3	1	414	53	0	0	0	0	0	183	8	0	0	0	0	0
6	3	10	2	10	3	2	294	7	0	0	0	0	0	98	0	0	0	0	0	0
6	3	10	2	10	3	3	325	7	0	0	0	0	0	112	0	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	3	10	2	10	3	4	328	47	0	0	0	0	0	128	9	0	0	0	0	0
6	3	10	1	40	1	1	965	40	0	0	0	0	0	855	8	0	0	0	0	0
6	3	10	1	40	1	2	956	5	0	0	0	0	0	832	0	0	0	0	0	0
6	3	10	1	40	1	3	970	5	0	0	0	0	0	878	0	0	0	0	0	0
6	3	10	1	40	1	4	958	48	0	0	0	0	0	847	8	0	0	0	0	0
6	3	10	2	40	2	1	980	690	0	0	0	0	0	917	352	0	0	0	0	0
6	3	10	2	40	2	2	925	293	0	0	0	0	0	704	19	0	0	0	0	0
6	3	10	2	40	2	3	925	293	0	0	0	0	0	704	19	0	0	0	0	0
6	3	10	2	40	2	4	941	698	0	0	0	0	0	786	351	0	0	0	0	0
6	3	10	2	40	3	1	987	617	0	0	0	0	0	923	309	0	0	0	0	0
6	3	10	2	40	3	2	955	252	0	0	0	0	0	742	10	0	0	0	0	0
6	3	10	2	40	3	3	956	252	0	0	0	0	0	745	10	0	0	0	0	0
6	3	10	2	40	3	4	964	640	0	0	0	0	0	824	316	0	0	0	0	0
6	3	10	1	90	1	1	1000	63	0	0	0	0	0	999	16	0	0	0	0	0
6	3	10	1	90	1	2	1000	13	0	0	0	0	0	999	0	0	0	0	0	0
6	3	10	1	90	1	3	1000	13	0	0	0	0	0	999	0	0	0	0	0	0
6	3	10	1	90	1	4	1000	71	0	0	0	0	0	999	18	0	0	0	0	0
6	3	10	2	90	2	1	1000	999	0	0	0	0	0	1000	952	0	0	0	0	0
6	3	10	2	90	2	2	1000	935	0	0	0	0	0	997	452	0	0	0	0	0
6	3	10	2	90	2	3	1000	935	0	0	0	0	0	997	452	0	0	0	0	0
6	3	10	2	90	2	4	1000	999	0	0	0	0	0	999	963	0	0	0	0	0
6	3	10	2	90	3	1	1000	976	0	0	0	0	0	1000	861	0	0	0	0	0
6	3	10	2	90	3	2	1000	823	0	0	0	0	0	995	302	0	0	0	0	0
6	3	10	2	90	3	3	1000	823	0	0	0	0	0	995	302	0	0	0	0	0
6	3	10	2	90	3	4	1000	978	0	0	0	0	0	997	879	0	0	0	0	0
6	3	50	1	10	1	1	478	26	0	0	0	0	0	234	2	0	0	0	0	0
6	3	50	1	10	1	2	425	0	0	0	0	0	0	192	0	0	0	0	0	0
6	3	50	1	10	1	3	511	0	0	0	0	0	0	246	0	0	0	0	0	0
6	3	50	1	10	1	4	432	26	0	0	0	0	0	203	2	0	0	0	0	0
6	3	50	2	10	2	1	510	83	0	0	0	0	0	274	17	0	0	0	0	0
6	3	50	2	10	2	2	414	2	0	0	0	0	0	179	0	0	0	0	0	0
6	3	50	2	10	2	3	445	2	0	0	0	0	0	202	0	0	0	0	0	0
6	3	50	2	10	2	4	424	75	0	0	0	0	0	195	14	0	0	0	0	0
6	3	50	2	10	3	1	488	76	0	0	0	0	0	246	11	0	0	0	0	0
6	3	50	2	10	3	2	369	2	0	0	0	0	0	172	0	0	0	0	0	0
6	3	50	2	10	3	3	405	2	0	0	0	0	0	190	0	0	0	0	0	0
6	3	50	2	10	3	4	381	64	0	0	0	0	0	190	10	0	0	0	0	0
6	3	50	1	40	1	1	997	40	0	0	0	0	0	982	9	0	0	0	0	0
6	3	50	1	40	1	2	997	3	0	0	0	0	0	982	0	0	0	0	0	0
6	3	50	1	40	1	3	998	3	0	0	0	0	0	994	0	0	0	0	0	0
6	3	50	1	40	1	4	997	43	0	0	0	0	0	982	10	0	0	0	0	0
6	3	50	2	40	2	1	989	806	0	0	0	0	0	973	570	0	0	0	0	0
6	3	50	2	40	2	2	972	352	0	0	0	0	0	926	16	0	0	0	0	0
6	3	50	2	40	2	3	972	352	0	0	0	0	0	926	16	0	0	0	0	0
6	3	50	2	40	2	4	974	811	0	0	0	0	0	940	568	0	0	0	0	0
6	3	50	2	40	3	1	996	722	0	0	0	0	0	984	458	0	0	0	0	0
6	3	50	2	40	3	2	990	247	0	0	0	0	0	939	12	0	0	0	0	0
6	3	50	2	40	3	3	990	247	0	0	0	0	0	939	12	0	0	0	0	0
6	3	50	2	40	3	4	991	728	0	0	0	0	0	947	461	0	0	0	0	0
6	3	50	1	90	1	1	1000	57	0	0	0	0	0	1000	11	0	0	0	0	0
6	3	50	1	90	1	2	1000	3	0	0	0	0	0	1000	0	0	0	0	0	0
6	3	50	1	90	1	3	1000	3	0	0	0	0	0	1000	0	0	0	0	0	0
6	3	50	1	90	1	4	1000	58	0	0	0	0	0	1000	11	0	0	0	0	0
6	3	50	2	90	2	1	1000	1000	0	0	0	0	0	1000	995	0	0	0	0	0
6	3	50	2	90	2	2	1000	972	0	0	0	0	0	1000	633	0	0	0	0	0
6	3	50	2	90	2	3	1000	972	0	0	0	0	0	1000	633	0	0	0	0	0
6	3	50	2	90	2	4	1000	1000	0	0	0	0	0	1000	995	0	0	0	0	0
6	3	50	2	90	3	1	1000	991	0	0	0	0	0	1000	964	0	0	0	0	0
6	3	50	2	90	3	2	1000	898	0	0	0	0	0	1000	395	0	0	0	0	0
6	3	50	2	90	3	3	1000	898	0	0	0	0	0	1000	395	0	0	0	0	0
6	3	50	2	90	3	4	1000	992	0	0	0	0	0	1000	968	0	0	0	0	0
6	6	10	3	90	2	1	999	991	673	24	1	0	0	999	946	332	5	0	0	0
6	6	10	3	90	2	2	995	794	348	2	1	0	0	971	491	107	0	0	0	0
6	6	10	3	90	2	3	995	941	663	13	0	0	0	971	760	324	2	0	0	0
6	6	10	3	90	2	4	996	968	776	37	2	0	0	980	815	461	3	0	0	0
6	6	10	3	90	3	1	1000	979	465	15	1	0	0	1000	865	193	[illegible]	[illegible]	[illegible]	[illegible]

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	6	10	3	90	3	2	998	791	190	0	0	0	0	980	489	55	0	0	0	0
6	6	10	3	90	3	3	998	939	450	10	0	0	0	980	749	166	0	0	0	0
6	6	10	3	90	3	4	999	969	563	29	1	0	0	986	804	271	1	0	0	0
6	6	10	3	90	4	1	1000	957	349	16	2	0	0	999	825	125	2	0	0	0
6	6	10	3	90	4	2	998	703	128	0	0	0	0	985	422	27	0	0	0	0
6	6	10	3	90	4	3	998	889	328	5	1	0	0	985	672	111	0	0	0	0
6	6	10	3	90	4	4	1000	936	448	17	1	0	0	988	722	184	2	0	0	0
6	6	10	5	10	2	1	285	26	1	1	0	0	0	114	4	0	0	0	0	0
6	6	10	5	10	2	2	164	0	0	0	0	0	0	50	0	0	0	0	0	0
6	6	10	5	10	2	3	202	5	0	0	0	0	0	69	0	0	0	0	0	0
6	6	10	5	10	2	4	194	11	0	0	0	0	0	64	0	0	0	0	0	0
6	6	10	5	10	3	1	281	36	2	0	0	0	0	108	5	0	0	0	0	0
6	6	10	5	10	3	2	148	3	0	0	0	0	0	40	0	0	0	0	0	0
6	6	10	5	10	3	3	187	5	0	0	0	0	0	65	2	0	0	0	0	0
6	6	10	5	10	3	4	182	23	0	0	0	0	0	47	2	0	0	0	0	0
6	6	10	5	10	4	1	269	19	1	0	0	0	0	101	2	0	0	0	0	0
6	6	10	5	10	4	2	144	0	0	0	0	0	0	48	0	0	0	0	0	0
6	6	10	5	10	4	3	197	8	0	0	0	0	0	66	0	0	0	0	0	0
6	6	10	5	10	4	4	175	21	0	0	0	0	0	59	0	0	0	0	0	0
6	6	10	5	40	2	1	955	667	290	62	16	0	0	839	405	73	17	2	0	0
6	6	10	5	40	2	2	717	107	7	1	0	0	0	419	22	0	0	0	0	0
6	6	10	5	40	2	3	719	269	53	8	4	0	0	421	73	6	0	0	0	0
6	6	10	5	40	2	4	779	375	129	26	11	0	0	475	110	12	2	1	0	0
6	6	10	5	40	3	1	944	628	226	39	7	0	0	826	354	62	3	0	0	0
6	6	10	5	40	3	2	740	84	5	0	0	0	0	447	22	1	0	0	0	0
6	6	10	5	40	3	3	741	276	55	3	0	0	0	453	68	4	0	0	0	0
6	6	10	5	40	3	4	801	373	108	24	2	0	0	502	89	11	0	0	0	0
6	6	10	5	40	4	1	933	497	110	14	2	0	0	811	248	18	2	0	0	0
6	6	10	5	40	4	2	797	103	1	0	0	0	0	567	25	0	0	0	0	0
6	6	10	5	40	4	3	804	287	34	3	0	0	0	576	78	0	0	0	0	0
6	6	10	5	40	4	4	829	378	72	9	1	0	0	600	110	7	2	0	0	0
6	6	10	5	90	2	1	1000	998	963	788	404	0	0	1000	991	879	490	83	0	0
6	6	10	5	90	2	2	990	690	336	114	11	0	0	929	357	90	9	1	0	0
6	6	10	5	90	2	3	990	913	699	422	198	0	0	929	642	285	98	10	0	0
6	6	10	5	90	2	4	994	956	813	628	450	0	0	952	722	452	215	70	0	0
6	6	10	5	90	3	1	1000	999	918	548	161	0	0	1000	983	745	248	25	0	0
6	6	10	5	90	3	2	995	723	288	65	7	0	0	940	380	76	4	3	0	0
6	6	10	5	90	3	3	995	924	654	295	74	0	0	940	675	261	53	1	0	0
6	6	10	5	90	3	4	996	956	765	462	203	0	0	953	748	385	137	15	0	0
6	6	10	5	90	4	1	1000	979	654	210	30	0	0	1000	906	385	56	2	0	0
6	6	10	5	90	4	2	997	674	158	15	12	0	0	975	335	27	2	0	0	0
6	6	10	5	90	4	3	997	874	417	101	7	0	0	975	622	137	11	0	0	0
6	6	10	5	90	4	4	997	924	530	176	36	0	0	980	696	215	31	1	0	0
6	6	10	1	10	1	1	254	7	1	0	0	0	0	84	2	0	0	0	0	0
6	6	10	1	10	1	2	186	0	0	0	0	0	0	47	0	0	0	0	0	0
6	6	10	1	10	1	3	270	4	0	0	0	0	0	106	0	0	0	0	0	0
6	6	10	1	10	1	4	205	9	1	0	0	0	0	57	0	0	0	0	0	0
6	6	10	2	10	2	1	267	33	2	0	0	0	0	107	0	0	0	0	0	0
6	6	10	2	10	2	2	169	3	0	0	0	0	0	43	0	0	0	0	0	0
6	6	10	2	10	2	3	225	12	0	0	0	0	0	65	2	0	0	0	0	0
6	6	10	2	10	2	4	196	25	0	0	0	0	0	54	3	0	0	0	0	0
6	6	10	2	10	3	1	288	16	0	0	0	0	0	97	1	0	0	0	0	0
6	6	10	2	10	3	2	167	1	0	0	0	0	0	55	0	0	0	0	0	0
6	6	10	2	10	3	3	216	7	0	0	0	0	0	86	0	0	0	0	0	0
6	6	10	2	10	3	4	193	15	0	0	0	0	0	60	2	0	0	0	0	0
6	6	10	1	40	1	1	905	38	1	0	0	0	0	717	5	0	0	0	0	0
6	6	10	1	40	1	2	894	4	0	0	0	0	0	697	0	0	0	0	0	0
6	6	10	1	40	1	3	968	32	0	0	0	0	0	853	4	0	0	0	0	0
6	6	10	1	40	1	4	897	52	1	0	0	0	0	703	5	0	0	0	0	0
6	6	10	2	40	2	1	940	348	22	2	0	0	0	805	135	5	0	0	0	0
6	6	10	2	40	2	2	851	154	2	0	0	0	0	634	39	0	0	0	0	0
6	6	10	2	40	2	3	859	362	14	0	0	0	0	659	133	1	0	0	0	0
6	6	10	2	40	2	4	876	459	25	0	0	0	0	672	174	2	0	0	0	0
6	6	10	2	40	3	1	933	316	19	1	0	0	0	802	117	3	0	0	0	0
6	6	10	2	40	3	2	876	113	0	0	0	0	0	659	31	0	0	0	0	0
6	6	10	2	40	3	3	889	299	11	1	0	0	0	695	97	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	6	10	2	40	3	4	901	391	18	1	0	0	0	684	136	2	0	0	0	0
6	6	10	1	90	1	1	1000	29	1	0	0	0	0	997	4	0	0	0	0	0
6	6	10	1	90	1	2	1000	2	0	0	0	0	0	997	0	0	0	0	0	0
6	6	10	1	90	1	3	1000	24	0	0	0	0	0	1000	1	0	0	0	0	0
6	6	10	1	90	1	4	1000	43	1	0	0	0	0	997	6	0	0	0	0	0
6	6	10	2	90	2	1	1000	923	30	2	0	0	0	1000	724	7	0	0	0	0
6	6	10	2	90	2	2	1000	859	6	0	0	0	0	999	604	0	0	0	0	0
6	6	10	2	90	2	3	1000	961	22	0	0	0	0	999	852	5	0	0	0	0
6	6	10	2	90	2	4	1000	971	47	1	0	0	0	999	889	11	0	0	0	0
6	6	10	2	90	3	1	1000	838	31	1	0	0	0	1000	584	7	0	0	0	0
6	6	10	2	90	3	2	1000	706	5	0	0	0	0	999	439	0	0	0	0	0
6	6	10	2	90	3	3	1000	974	28	1	0	0	0	999	680	4	0	0	0	0
6	6	10	2	90	3	4	1000	912	48	1	0	0	0	999	748	7	0	0	0	0
6	6	10	3	10	2	1	307	32	3	1	0	0	0	118	5	0	0	0	0	0
6	6	10	3	10	2	2	185	0	0	0	0	0	0	50	0	0	0	0	0	0
6	6	10	3	10	2	3	215	12	0	0	0	0	0	72	0	0	0	0	0	0
6	6	10	3	10	2	4	207	21	1	0	0	0	0	62	0	0	0	0	0	0
6	6	10	3	10	3	1	316	30	2	0	0	0	0	125	3	0	0	0	0	0
6	6	10	3	10	3	2	190	0	0	0	0	0	0	53	0	0	0	0	0	0
6	6	10	3	10	3	3	227	11	0	0	0	0	0	68	0	0	0	0	0	0
6	6	10	3	10	3	4	221	24	0	0	0	0	0	64	1	0	0	0	0	0
6	6	10	3	10	4	1	274	39	2	0	0	0	0	113	5	0	0	0	0	0
6	6	10	3	10	4	2	165	1	0	0	0	0	0	52	0	0	0	0	0	0
6	6	10	3	10	4	3	205	12	0	0	0	0	0	69	1	0	0	0	0	0
6	6	10	3	10	4	4	196	22	0	0	0	0	0	59	2	0	0	0	0	0
6	6	10	3	40	2	1	943	526	88	4	0	0	0	819	253	19	0	0	0	0
6	6	10	3	40	2	2	794	121	12	0	0	0	0	536	19	1	0	0	0	0
6	6	10	3	40	2	3	799	320	51	1	0	0	0	543	82	7	0	0	0	0
6	6	10	3	40	2	4	833	418	92	6	0	0	0	572	134	13	0	0	0	0
6	6	10	3	40	3	1	934	475	66	4	0	0	0	804	213	10	0	0	0	0
6	6	10	3	40	3	2	804	109	4	0	0	0	0	553	24	0	0	0	0	0
6	6	10	3	40	3	3	812	325	29	0	0	0	0	566	90	4	0	0	0	0
6	6	10	3	40	3	4	847	414	57	4	0	0	0	600	132	9	0	0	0	0
6	6	10	3	40	4	1	935	456	53	4	0	0	0	795	199	6	0	0	0	0
6	6	10	3	40	4	2	819	121	5	0	0	0	0	587	18	0	0	0	0	0
6	6	10	3	40	4	3	828	299	27	1	0	0	0	605	98	3	0	0	0	0
6	6	10	3	40	4	4	850	402	64	3	0	0	0	625	132	8	0	0	0	0
6	6	5	1	10	1	1	204	16	0	0	0	0	0	72	2	0	0	0	0	0
6	6	5	1	10	1	2	127	0	0	0	0	0	0	34	0	0	0	0	0	0
6	6	5	1	10	1	3	201	4	0	0	0	0	0	63	0	0	0	0	0	0
6	6	5	1	10	1	4	147	9	0	0	0	0	0	38	1	0	0	0	0	0
6	6	5	2	10	2	1	221	16	0	0	0	0	0	78	2	0	0	0	0	0
6	6	5	2	10	2	2	132	1	0	0	0	0	0	29	0	0	0	0	0	0
6	6	5	2	10	2	3	173	7	0	0	0	0	0	47	1	0	0	0	0	0
6	6	5	2	10	2	4	158	23	1	0	0	0	0	39	2	0	0	0	0	0
6	6	5	2	10	3	1	220	23	1	0	0	0	0	87	2	0	0	0	0	0
6	6	5	2	10	3	2	122	1	0	0	0	0	0	34	0	0	0	0	0	0
6	6	5	2	10	3	3	168	8	0	0	0	0	0	46	0	0	0	0	0	0
6	6	5	2	10	3	4	143	22	0	0	0	0	0	40	3	0	0	0	0	0
6	6	5	1	40	1	1	709	29	3	0	0	0	0	439	9	0	0	0	0	0
6	6	5	1	40	1	2	674	3	0	0	0	0	0	377	0	0	0	0	0	0
6	6	5	1	40	1	3	819	24	0	0	0	0	0	544	2	0	0	0	0	0
6	6	5	1	40	1	4	691	64	5	1	0	0	0	401	3	0	0	0	0	0
6	6	5	2	40	2	1	834	233	17	0	0	0	0	612	64	1	0	0	0	0
6	6	5	2	40	2	2	657	53	0	0	0	0	0	356	3	0	0	0	0	0
6	6	5	2	40	2	3	675	161	5	0	0	0	0	378	29	0	0	0	0	0
6	6	5	2	40	2	4	731	324	23	0	0	0	0	418	77	1	0	0	0	0
6	6	5	2	40	3	1	859	195	11	1	0	0	0	582	57	1	0	0	0	0
6	6	5	2	40	3	2	694	42	0	0	0	0	0	347	4	0	0	0	0	0
6	6	5	2	40	3	3	719	131	1	0	0	0	0	392	26	0	0	0	0	0
6	6	5	2	40	3	4	767	254	14	1	0	0	0	429	71	1	0	0	0	0
6	6	5	1	90	1	1	983	40	2	0	0	0	0	887	7	0	0	0	0	0
6	6	5	1	90	1	2	982	5	0	0	0	0	0	882	2	0	0	0	0	0
6	6	5	1	90	1	3	999	32	0	0	0	0	0	982	4	0	0	0	0	0
6	6	5	1	90	1	4	983	65	2	0	0	0	0	886	10	0	0	0	0	0
6	6	5	2	90	2	1	999	750	32	2	0	0	0	987	416	5	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	6	5	2	90	2	2	988	517	3	0	0	0	0	907	175	2	0	0	0	0
6	6	5	2	90	2	3	988	771	17	0	0	0	0	908	430	2	0	0	0	0
6	6	5	2	90	2	4	999	904	58	2	0	0	0	950	640	13	0	0	0	0
6	6	5	2	90	3	1	998	668	30	0	0	0	0	979	338	3	0	0	0	0
6	6	5	2	90	3	2	988	400	1	0	0	0	0	902	116	0	0	0	0	0
6	6	5	2	90	3	3	989	672	15	0	0	0	0	904	326	1	0	0	0	0
6	6	5	2	90	3	4	995	825	49	1	0	0	0	936	504	10	0	0	0	0
6	6	5	3	10	2	1	247	20	1	0	0	0	0	84	3	0	0	0	0	0
6	6	5	3	10	2	2	117	0	0	0	0	0	0	28	0	0	0	0	0	0
6	6	5	3	10	2	3	148	4	0	0	0	0	0	36	0	0	0	0	0	0
6	6	5	3	10	2	4	144	16	0	0	0	0	0	41	1	0	0	0	0	0
6	6	5	3	10	3	1	235	23	0	0	0	0	0	76	2	0	0	0	0	0
6	6	5	3	10	3	2	108	1	0	0	0	0	0	21	0	0	0	0	0	0
6	6	5	3	10	3	3	140	6	0	0	0	0	0	32	0	0	0	0	0	0
6	6	5	3	10	3	4	146	14	0	0	0	0	0	31	0	0	0	0	0	0
6	6	5	3	10	4	1	230	23	0	0	0	0	0	74	3	0	0	0	0	0
6	6	5	3	10	4	2	122	0	0	0	0	0	0	30	0	0	0	0	0	0
6	6	5	3	10	4	3	153	4	0	0	0	0	0	40	0	0	0	0	0	0
6	6	5	3	10	4	4	151	20	0	0	0	0	0	41	1	0	0	0	0	0
6	6	5	3	40	2	1	884	423	42	7	0	0	0	695	157	14	0	0	0	0
6	6	5	3	40	2	2	622	45	0	0	0	0	0	279	6	0	0	0	0	0
6	6	5	3	40	2	3	627	162	8	0	0	0	0	285	22	0	0	0	0	0
6	6	5	3	40	2	4	714	300	62	6	0	0	0	375	66	3	0	0	0	0
6	6	5	3	40	3	1	860	346	33	0	0	0	0	659	103	5	0	0	0	0
6	6	5	3	40	3	2	620	41	1	0	0	0	0	306	3	0	0	0	0	0
6	6	5	3	40	3	3	631	141	11	0	0	0	0	313	23	1	0	0	0	0
6	6	5	3	40	3	4	706	297	42	3	0	0	0	391	60	9	0	0	0	0
6	6	5	3	40	4	1	876	294	25	0	0	0	0	637	84	2	0	0	0	0
6	6	5	3	40	4	2	655	26	0	0	0	0	0	291	3	0	0	0	0	0
6	6	5	3	40	4	3	668	121	5	0	0	0	0	302	15	0	0	0	0	0
6	6	5	3	40	4	4	737	232	28	0	0	0	0	385	49	3	0	0	0	0
6	6	5	3	90	2	1	1000	969	464	27	1	0	0	998	835	155	2	0	0	0
6	6	5	3	90	2	2	977	468	100	0	0	0	0	819	129	8	0	0	0	0
6	6	5	3	90	2	3	977	752	351	7	0	0	0	819	347	54	0	0	0	0
6	6	5	3	90	2	4	993	915	661	57	2	0	0	888	563	257	5	0	0	0
6	6	5	3	90	3	1	999	915	317	9	2	0	0	994	726	93	1	0	0	0
6	6	5	3	90	3	2	977	443	61	0	0	0	0	849	129	5	0	0	0	0
6	6	5	3	90	3	3	977	736	223	0	0	0	0	849	341	35	0	0	0	0
6	6	5	3	90	3	4	990	985	448	23	1	0	0	899	552	159	0	0	0	0
6	6	5	3	90	4	1	999	868	239	14	0	0	0	993	628	66	0	0	0	0
6	6	5	3	90	4	2	981	375	33	0	0	0	0	850	107	1	0	0	0	0
6	6	5	3	90	4	3	982	666	134	1	0	0	0	850	294	24	0	0	0	0
6	6	5	3	90	4	4	992	841	348	25	1	0	0	911	474	96	1	0	0	0
6	6	5	5	10	2	1	222	30	2	0	0	0	0	83	2	0	0	0	0	0
6	6	5	5	10	2	2	96	0	0	0	0	0	0	21	0	0	0	0	0	0
6	6	5	5	10	2	3	125	4	0	0	0	0	0	30	0	0	0	0	0	0
6	6	5	5	10	2	4	129	18	0	0	0	0	0	34	0	0	0	0	0	0
6	6	5	5	10	3	1	240	23	1	0	0	0	0	85	3	0	0	0	0	0
6	6	5	5	10	3	2	121	0	0	0	0	0	0	30	0	0	0	0	0	0
6	6	5	5	10	3	3	147	3	0	0	0	0	0	36	0	0	0	0	0	0
6	6	5	5	10	3	4	163	20	2	0	0	0	0	40	0	0	0	0	0	0
6	6	5	5	10	4	1	229	26	3	0	0	0	0	83	7	1	0	0	0	0
6	6	5	5	10	4	2	113	0	0	0	0	0	0	24	0	0	0	0	0	0
6	6	5	5	10	4	3	150	3	0	0	0	0	0	34	0	0	0	0	0	0
6	6	5	5	10	4	4	143	18	0	0	0	0	0	33	0	0	0	0	0	0
6	6	5	5	40	2	1	912	578	237	58	10	0	0	744	297	56	9	0	0	0
6	6	5	5	40	2	2	548	33	2	0	0	0	0	242	0	0	0	0	0	0
6	6	5	5	40	2	3	550	108	19	5	1	0	0	247	16	2	0	0	0	0
6	6	5	5	40	2	4	645	247	76	25	7	0	0	334	40	6	3	1	0	0
6	6	5	5	40	3	1	884	523	157	38	7	0	0	706	231	39	5	0	0	0
6	6	5	5	40	3	2	545	28	0	0	0	0	0	226	3	0	0	0	0	0
6	6	5	5	40	3	3	552	116	14	0	0	0	0	231	15	0	0	0	0	0
6	6	5	5	40	3	4	637	276	69	19	3	0	0	317	42	7	0	0	0	0
6	6	5	5	40	4	1	855	398	75	12	0	0	0	673	150	9	0	0	0	0
6	6	5	5	40	4	2	628	32	1	0	0	0	0	297	2	0	0	0	0	0
6	6	5	5	40	4	3	642	126	7	0	0	0	0	304	14	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
6	6	5	5	40	4	4	710	269	34	2	0	0	0	398	52	5	0	0	0	0
6	6	5	5	90	2	1	1000	997	930	685	306	0	0	996	952	748	349	53	0	0
6	10	50	2	10	3	1	222	17	1	0	0	0	0	83	2	0	0	0	0	0
6	10	50	2	10	3	2	141	0	0	0	0	0	0	39	0	0	0	0	0	0
6	10	50	2	10	3	3	227	11	0	0	0	0	0	84	0	0	0	0	0	0
6	10	50	2	10	3	4	149	13	1	0	0	0	0	38	0	0	0	0	0	0
6	10	50	1	40	1	1	901	28	0	0	0	0	0	751	4	0	0	0	0	0
6	10	50	1	40	1	2	893	6	0	0	0	0	0	743	2	0	0	0	0	0
6	10	50	1	40	1	3	967	45	0	0	0	0	0	908	8	0	0	0	0	0
6	10	50	1	40	1	4	894	54	0	0	0	0	0	743	9	0	0	0	0	0
6	6	50	5	40	2	1	965	732	343	89	13	0	0	897	483	123	16	1	0	0
6	6	50	5	40	2	2	796	145	13	2	0	0	0	555	33	4	0	0	0	0
6	6	50	5	40	2	3	797	314	68	14	1	0	0	557	80	11	2	0	0	0
6	6	50	5	40	2	4	833	473	143	37	10	0	0	596	136	14	3	0	0	0
6	6	50	5	40	3	1	972	693	273	57	9	0	0	998	431	81	11	1	0	0
6	6	50	5	40	3	2	844	143	8	1	0	0	0	609	27	1	0	0	0	0
6	6	50	5	40	3	3	849	300	59	6	1	0	0	616	77	5	0	0	0	0
6	6	50	5	40	3	4	860	437	128	21	6	0	0	641	136	25	2	0	0	0
6	6	50	5	40	4	1	976	605	122	15	3	0	0	902	326	32	1	0	0	0
6	6	50	5	40	4	2	897	164	4	0	0	0	0	700	50	0	0	0	0	0
6	6	50	5	40	4	3	900	339	32	4	0	0	0	710	102	3	0	0	0	0
6	6	50	5	40	4	4	913	453	65	6	0	0	0	739	180	10	1	0	0	0
6	6	50	5	90	2	1	1000	999	979	843	433	0	0	1000	997	927	599	120	0	0
6	6	50	5	90	2	2	999	860	455	158	16	0	0	992	621	151	33	21	0	0
6	6	50	5	90	2	3	999	954	773	508	194	0	0	992	785	438	170	14	0	0
6	6	50	5	90	2	4	1000	978	880	714	482	0	0	993	858	596	306	94	0	0
6	6	50	5	90	3	1	1000	999	952	617	180	0	0	1000	993	840	314	43	0	0
6	6	50	5	90	3	2	998	873	373	70	7	0	0	990	655	119	7	6	0	0
6	6	50	5	90	3	3	999	958	722	331	63	0	0	990	777	355	79	5	0	0
6	6	50	5	90	3	4	998	977	835	522	212	0	0	991	870	529	157	29	0	0
6	6	50	5	90	4	1	1000	985	728	218	40	0	0	1000	952	455	79	3	0	0
6	6	50	5	90	4	2	1000	815	200	19	10	0	0	997	597	47	0	0	0	0
6	6	50	5	90	4	3	1000	915	460	105	8	0	0	997	730	166	17	0	0	0
6	6	50	5	90	4	4	1000	953	591	180	48	0	0	997	813	302	40	1	0	0
6	10	50	1	10	1	1	235	13	0	0	0	0	0	94	1	0	0	0	0	0
6	10	50	1	10	1	2	173	1	0	0	0	0	0	59	0	0	0	0	0	0
6	10	50	1	10	1	3	278	12	0	0	0	0	0	115	1	0	0	0	0	0
6	10	50	1	10	1	4	177	15	0	0	0	0	0	59	1	0	0	0	0	0
6	10	50	2	10	2	1	251	14	0	0	0	0	0	93	2	0	0	0	0	0
6	10	50	2	10	2	2	159	2	0	0	0	0	0	42	0	0	0	0	0	0
6	10	50	2	10	2	3	227	19	0	0	0	0	0	86	1	0	0	0	0	0
6	10	50	2	10	2	4	162	24	0	0	0	0	0	41	2	0	0	0	0	0
3	3	5	1	10	1	1	411	26	0	0	0	0	0	188	2	0	0	0	0	0
3	3	5	1	10	1	2	383	13	0	0	0	0	0	165	0	0	0	0	0	0
3	3	5	1	10	1	3	445	13	0	0	0	0	0	194	0	0	0	0	0	0
3	3	5	1	10	1	4	395	41	0	0	0	0	0	170	2	0	0	0	0	0
3	3	5	2	10	2	1	455	83	0	0	0	0	0	188	7	0	0	0	0	0
3	3	5	2	10	2	2	334	45	0	0	0	0	0	90	4	0	0	0	0	0
3	3	5	2	10	2	3	361	45	0	0	0	0	0	100	4	0	0	0	0	0
3	3	5	2	10	2	4	372	132	0	0	0	0	0	116	9	0	0	0	0	0
3	3	5	2	10	3	1	460	86	0	0	0	0	0	206	13	0	0	0	0	0
3	3	5	2	10	3	2	355	49	0	0	0	0	0	128	3	0	0	0	0	0
3	3	5	2	10	3	3	394	49	0	0	0	0	0	146	3	0	0	0	0	0
3	3	5	2	10	3	4	376	118	0	0	0	0	0	140	11	0	0	0	0	0
3	3	5	1	40	1	1	955	50	0	0	0	0	0	789	11	0	0	0	0	0
3	3	5	1	40	1	2	952	46	0	0	0	0	0	772	4	0	0	0	0	0
3	3	5	1	40	1	3	970	46	0	0	0	0	0	836	4	0	0	0	0	0
3	3	5	1	40	1	4	954	96	0	0	0	0	0	780	17	0	0	0	0	0
3	3	5	2	40	2	1	992	835	0	0	0	0	0	934	448	0	0	0	0	0
3	3	5	2	40	2	2	930	772	0	0	0	0	0	706	300	0	0	0	0	0
3	3	5	2	40	2	3	950	772	0	0	0	0	0	707	300	0	0	0	0	0
3	3	5	2	40	2	4	969	903	0	0	0	0	0	752	500	0	0	0	0	0
3	3	5	2	40	3	1	993	754	0	0	0	0	0	943	379	0	0	0	0	0
3	3	5	2	40	3	2	966	692	0	0	0	0	0	729	251	0	0	0	0	0
3	3	5	2	40	3	3	966	672	0	0	0	0	0	730	251	0	0	0	0	0
3	3	5	2	40	3	4	982	876	0	0	0	0	0	785	426	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
3	3	5	1	90	1	1	1000	44	0	0	0	0	0	993	8	0	0	0	0	0
3	3	5	1	90	1	2	1000	33	0	0	0	0	0	993	5	0	0	0	0	0
3	3	5	1	90	1	3	1000	33	0	0	0	0	0	998	5	0	0	0	0	0
3	3	5	1	90	1	4	1000	99	0	0	0	0	0	993	14	0	0	0	0	0
3	3	5	2	90	2	1	1000	1000	0	0	0	0	0	1000	968	0	0	0	0	0
3	3	5	2	90	2	2	1000	1000	0	0	0	0	0	987	932	0	0	0	0	0
3	3	5	2	90	2	3	1000	1000	0	0	0	0	0	987	932	0	0	0	0	0
3	3	5	2	90	2	4	1000	1000	0	0	0	0	0	997	985	0	0	0	0	0
3	3	5	2	90	3	1	1000	985	0	0	0	0	0	1000	894	0	0	0	0	0
3	3	5	2	90	3	2	1000	983	0	0	0	0	0	994	821	0	0	0	0	0
3	3	5	2	90	3	3	1000	983	0	0	0	0	0	994	821	0	0	0	0	0
3	3	5	2	90	3	4	1000	994	0	0	0	0	0	995	929	0	0	0	0	0
3	3	10	1	10	1	1	537	25	0	0	0	0	0	248	4	0	0	0	0	0
3	3	10	1	10	1	2	498	13	0	0	0	0	0	207	0	0	0	0	0	0
3	3	10	1	10	1	3	553	13	0	0	0	0	0	246	0	0	0	0	0	0
3	3	10	1	10	1	4	518	46	0	0	0	0	0	216	0	0	0	0	0	0
3	3	10	2	10	2	1	572	164	0	0	0	0	0	330	38	0	0	0	0	0
3	3	10	2	10	2	2	465	95	0	0	0	0	0	193	14	0	0	0	0	0
3	3	10	2	10	2	3	480	95	0	0	0	0	0	200	14	0	0	0	0	0
3	3	10	2	10	2	4	504	214	0	0	0	0	0	225	37	0	0	0	0	0
3	3	10	2	10	3	1	568	129	0	0	0	0	0	290	26	0	0	0	0	0
3	3	10	2	10	3	2	455	73	0	0	0	0	0	185	3	0	0	0	0	0
3	3	10	2	10	3	3	468	73	0	0	0	0	0	197	3	0	0	0	0	0
3	3	10	2	10	3	4	494	162	0	0	0	0	0	209	24	0	0	0	0	0
3	3	10	1	40	1	1	995	46	0	0	0	0	0	968	5	0	0	0	0	0
3	3	10	1	40	1	2	994	26	0	0	0	0	0	961	1	0	0	0	0	0
3	3	10	1	40	1	3	994	26	0	0	0	0	0	974	1	0	0	0	0	0
3	3	10	1	40	1	4	995	90	0	0	0	0	0	964	6	0	0	0	0	0
3	3	10	2	40	2	1	999	892	0	0	0	0	0	980	658	0	0	0	0	0
3	3	10	2	40	2	2	988	821	0	0	0	0	0	895	452	0	0	0	0	0
3	3	10	2	40	2	3	988	821	0	0	0	0	0	895	452	0	0	0	0	0
3	3	10	2	40	2	4	995	940	0	0	0	0	0	924	679	0	0	0	0	0
3	3	10	2	40	3	1	999	808	0	0	0	0	0	984	546	0	0	0	0	0
3	3	10	2	40	3	2	988	718	0	0	0	0	0	906	361	0	0	0	0	0
3	3	10	2	40	3	3	988	718	0	0	0	0	0	907	361	0	0	0	0	0
3	3	10	2	40	3	4	993	885	0	0	0	0	0	926	574	0	0	0	0	0
3	3	10	1	90	1	1	1000	39	0	0	0	0	0	1000	10	0	0	0	0	0
3	3	10	1	90	1	2	1000	21	0	0	0	0	0	1000	2	0	0	0	0	0
3	3	10	1	90	1	3	1000	21	0	0	0	0	0	1000	2	0	0	0	0	0
3	3	10	1	90	1	4	1000	77	0	0	0	0	0	1000	11	0	0	0	0	0
3	3	10	2	90	2	1	1000	1000	0	0	0	0	0	1000	991	0	0	0	0	0
3	3	10	2	90	2	2	1000	999	0	0	0	0	0	999	973	0	0	0	0	0
3	3	10	2	90	2	3	1000	999	0	0	0	0	0	999	973	0	0	0	0	0
3	3	10	2	90	2	4	1000	1000	0	0	0	0	0	1000	994	0	0	0	0	0
3	3	10	2	90	3	1	1000	997	0	0	0	0	0	1000	979	0	0	0	0	0
3	3	10	2	90	3	2	1000	992	0	0	0	0	0	1000	932	0	0	0	0	0
3	3	10	2	90	3	3	1000	992	0	0	0	0	0	1000	932	0	0	0	0	0
3	3	10	2	90	3	4	1000	998	0	0	0	0	0	1000	985	0	0	0	0	0
3	3	50	1	10	1	1	637	28	0	0	0	0	0	374	3	0	0	0	0	0
3	3	50	1	10	1	2	611	8	0	0	0	0	0	354	0	0	0	0	0	0
3	3	50	1	10	1	3	660	8	0	0	0	0	0	427	0	0	0	0	0	0
3	3	50	1	10	1	4	619	42	0	0	0	0	0	358	3	0	0	0	0	0
3	3	50	2	10	2	1	594	160	0	0	0	0	0	363	44	0	0	0	0	0
3	3	50	2	10	2	2	524	77	0	0	0	0	0	274	6	0	0	0	0	0
3	3	50	2	10	2	3	546	77	0	0	0	0	0	307	6	0	0	0	0	0
3	3	50	2	10	2	4	545	199	0	0	0	0	0	280	38	0	0	0	0	0
3	3	50	2	10	3	1	633	161	0	0	0	0	0	380	28	0	0	0	0	0
3	3	50	2	10	3	2	549	66	0	0	0	0	0	274	2	0	0	0	0	0
3	3	50	2	10	3	3	561	66	0	0	0	0	0	296	2	0	0	0	0	0
3	3	50	2	10	3	4	579	211	0	0	0	0	0	281	21	0	0	0	0	0
3	3	50	1	40	1	1	1000	50	0	0	0	0	0	992	7	0	0	0	0	0
3	3	50	1	40	1	2	1000	17	0	0	0	0	0	992	0	0	0	0	0	0
3	3	50	1	40	1	3	1000	17	0	0	0	0	0	997	0	0	0	0	0	0
3	3	50	1	40	1	4	1000	90	0	0	0	0	0	992	8	0	0	0	0	0
3	3	50	2	40	2	1	1000	949	0	0	0	0	0	997	807	0	0	0	0	0
3	3	50	2	40	2	2	996	898	0	0	0	0	0	974	533	0	0	0	0	0

p	q	n	r	trh	pat	t	alpha=.05 1	2	3	4	5	6	7	alpha=.01 1	2	3	4	5	6	7
3	3	50	2	40	2	3	996	898	0	0	0	0	0	974	533	0	0	0	0	0
3	3	50	2	40	2	4	996	967	0	0	0	0	0	976	820	0	0	0	0	0
3	3	50	2	40	3	1	1000	865	0	0	0	0	0	998	668	0	0	0	0	0
3	3	50	2	40	3	2	1000	783	0	0	0	0	0	980	421	0	0	0	0	0
3	3	50	2	40	3	3	1000	783	0	0	0	0	0	980	421	0	0	0	0	0
3	3	50	2	40	3	4	1000	906	0	0	0	0	0	982	682	0	0	0	0	0
3	3	50	1	90	1	1	1000	45	0	0	0	0	0	1000	6	0	0	0	0	0
3	3	50	1	90	1	2	1000	14	0	0	0	0	0	1000	0	0	0	0	0	0
3	3	50	1	90	1	3	1000	14	0	0	0	0	0	1000	0	0	0	0	0	0
3	3	50	1	90	1	4	1000	75	0	0	0	0	0	1000	7	0	0	0	0	0
3	3	50	2	90	2	1	1000	1000	0	0	0	0	0	1000	999	0	0	0	0	0
3	3	50	2	90	2	2	1000	1000	0	0	0	0	0	1000	994	0	0	0	0	0
3	3	50	2	90	2	3	1000	1000	0	0	0	0	0	1000	994	0	0	0	0	0
3	3	50	2	90	2	4	1000	1000	0	0	0	0	0	1000	999	0	0	0	0	0
3	3	50	2	90	3	1	1000	999	0	0	0	0	0	1000	987	0	0	0	0	0
3	3	50	2	90	3	2	1000	992	0	0	0	0	0	1000	962	0	0	0	0	0
3	3	50	2	90	3	3	1000	992	0	0	0	0	0	1000	962	0	0	0	0	0
3	3	50	2	90	3	4	1000	1000	0	0	0	0	0	1000	988	0	0	0	0	0
3	6	5	1	10	1	1	344	23	3	0	0	0	0	139	3	0	0	0	0	0
3	6	5	1	10	1	2	269	1	1	0	0	0	0	91	0	0	0	0	0	0
3	6	5	1	10	1	3	329	9	1	0	0	0	0	133	1	0	0	0	0	0
3	6	5	1	10	1	4	306	27	2	0	0	0	0	103	4	1	0	0	0	0
3	6	5	2	10	2	1	363	43	1	0	0	0	0	145	6	0	0	0	0	0
3	6	5	2	10	2	2	235	2	0	0	0	0	0	72	0	0	0	0	0	0
3	6	5	2	10	2	3	271	17	0	0	0	0	0	81	1	0	0	0	0	0
3	6	5	2	10	2	4	290	43	1	0	0	0	0	92	5	0	0	0	0	0
3	6	5	2	10	3	1	413	46	2	0	0	0	0	168	5	0	0	0	0	0
3	6	5	2	10	3	2	265	2	0	0	0	0	0	84	0	0	0	0	0	0
3	6	5	2	10	3	3	296	20	0	0	0	0	0	105	0	0	0	0	0	0
3	6	5	2	10	3	4	340	48	1	0	0	0	0	105	3	0	0	0	0	0
3	6	5	1	40	1	1	942	43	3	0	0	0	0	786	10	0	0	0	0	0
3	6	5	1	40	1	2	933	3	0	0	0	0	0	760	1	0	0	0	0	0
3	6	5	1	40	1	3	963	28	0	0	0	0	0	860	3	0	0	0	0	0
3	6	5	1	40	1	4	940	65	2	0	0	0	0	771	10	0	0	0	0	0
3	6	5	2	40	2	1	969	529	25	0	0	0	0	857	223	1	0	0	0	0
3	6	5	2	40	2	2	888	178	4	0	0	0	0	642	35	0	0	0	0	0
3	6	5	2	40	2	3	891	480	4	0	0	0	0	643	153	0	0	0	0	0
3	6	5	2	40	2	4	935	635	33	0	0	0	0	706	301	1	0	0	0	0
3	6	5	2	40	3	1	970	481	17	0	0	0	0	873	198	1	0	0	0	0
3	6	5	2	40	3	2	903	166	2	0	0	0	0	683	36	0	0	0	0	0
3	6	5	2	40	3	3	907	431	2	0	0	0	0	690	145	0	0	0	0	0
3	6	5	2	40	3	4	942	584	39	0	0	0	0	745	261	1	0	0	0	0
3	6	5	1	90	1	1	1000	46	3	0	0	0	0	999	7	0	0	0	0	0
3	6	5	1	90	1	2	1000	6	0	0	0	0	0	999	2	0	0	0	0	0
3	6	5	1	90	1	3	1000	32	0	0	0	0	0	1000	6	0	0	0	0	0
3	6	5	1	90	1	4	1000	71	1	0	0	0	0	999	17	0	0	0	0	0
3	6	5	2	90	2	1	1000	981	46	0	0	0	0	1000	878	7	0	0	0	0
3	6	5	2	90	2	2	1000	972	15	0	0	0	0	986	585	0	0	0	0	0
3	6	5	2	90	2	3	1000	982	15	0	0	0	0	986	863	0	0	0	0	0
3	6	5	2	90	2	4	1000	993	77	0	0	0	0	993	940	12	0	0	0	0
3	6	5	2	90	3	1	1000	939	45	0	0	0	0	1000	740	6	0	0	0	0
3	6	5	2	90	3	2	1000	733	18	0	0	0	0	995	466	1	0	0	0	0
3	6	5	2	90	3	3	1000	924	18	0	0	0	0	995	723	1	0	0	0	0
3	6	5	2	90	3	4	1000	969	75	0	0	0	0	998	849	11	0	0	0	0
3	6	5	3	10	2	1	407	73	8	0	0	0	0	178	15	0	0	0	0	0
3	6	5	3	10	2	2	250	2	0	0	0	0	0	66	0	0	0	0	0	0
3	6	5	3	10	2	3	271	22	0	0	0	0	0	71	1	0	0	0	0	0
3	6	5	3	10	2	4	312	55	6	0	0	0	0	94	7	0	0	0	0	0
3	6	5	3	10	3	1	400	80	12	0	0	0	0	186	14	1	0	0	0	0
3	6	5	3	10	3	2	237	3	0	0	0	0	0	79	0	0	0	0	0	0
3	6	5	3	10	3	3	258	31	0	0	0	0	0	91	2	0	0	0	0	0
3	6	5	3	10	3	4	307	64	6	0	0	0	0	94	12	0	0	0	0	0
3	6	5	3	10	4	1	382	47	4	0	0	0	0	164	6	0	0	0	0	0
3	6	5	3	10	4	2	232	2	0	0	0	0	0	75	1	0	0	0	0	0
3	6	5	3	10	4	3	256	11	0	0	0	0	0	84	2	0	0	0	0	0
3	6	5	3	10	4	4	301	41	1	0	0	0	0	89	3	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
3	6	5	3	40	2	1	987	799	310	0	0	0	0	931	488	66	0	0	0	0
3	6	5	3	40	2	2	860	161	123	0	0	0	0	571	35	5	0	0	0	0
3	6	5	3	40	2	3	860	462	103	0	0	0	0	574	136	5	0	0	0	0
3	6	5	3	40	2	4	926	639	342	0	0	0	0	669	264	63	0	0	0	0
3	6	5	3	40	3	1	975	725	216	0	0	0	0	895	417	40	0	0	0	0
3	6	5	3	40	3	2	859	171	84	0	0	0	0	612	31	2	0	0	0	0
3	6	5	3	40	3	3	860	462	76	0	0	0	0	613	151	2	0	0	0	0
3	6	5	3	40	3	4	913	622	270	0	0	0	0	657	285	49	0	0	0	0
3	6	5	3	40	4	1	977	664	130	0	0	0	0	893	358	23	0	0	0	0
3	6	5	3	40	4	2	883	151	61	0	0	0	0	623	23	0	0	0	0	0
3	6	5	3	40	4	3	886	417	54	0	0	0	0	627	137	0	0	0	0	0
3	6	5	3	40	4	4	939	592	216	0	0	0	0	710	238	30	0	0	0	0
3	6	5	3	90	2	1	1000	998	922	0	0	0	0	1000	988	706	0	0	0	0
3	6	5	3	90	2	2	999	857	812	0	0	0	0	981	496	283	0	0	0	0
3	6	5	3	90	2	3	999	982	810	0	0	0	0	981	838	280	0	0	0	0
3	6	5	3	90	2	4	999	994	959	0	0	0	0	989	941	757	0	0	0	0
3	6	5	3	90	3	1	1000	996	792	0	0	0	0	1000	969	488	0	0	0	0
3	6	5	3	90	3	2	1000	803	630	0	0	0	0	983	459	151	0	0	0	0
3	6	5	3	90	3	3	1000	970	622	0	0	0	0	983	788	148	0	0	0	0
3	6	5	3	90	3	4	1000	990	868	0	0	0	0	988	896	552	0	0	0	0
3	6	5	3	90	4	1	1000	937	690	0	0	0	0	1000	931	374	0	0	0	0
3	6	5	3	90	4	2	998	740	508	0	0	0	0	979	403	94	0	0	0	0
3	6	5	3	90	4	3	998	939	501	0	0	0	0	979	726	94	0	0	0	0
3	6	5	3	90	4	4	1000	972	779	0	0	0	0	993	844	439	0	0	0	0
3	6	10	1	10	1	1	373	15	0	0	0	0	0	172	0	0	0	0	0	0
3	6	10	1	10	1	2	315	1	0	0	0	0	0	122	0	0	0	0	0	0
3	6	10	1	10	1	3	398	11	0	0	0	0	0	129	1	0	0	0	0	0
3	6	10	1	10	1	4	348	17	0	0	0	0	0	138	3	0	0	0	0	0
3	6	10	2	10	2	1	421	53	5	0	0	0	0	192	7	0	0	0	0	0
3	6	10	2	10	2	2	306	2	1	0	0	0	0	98	0	0	0	0	0	0
3	6	10	2	10	2	3	341	23	1	0	0	0	0	77	1	0	0	0	0	0
3	6	10	2	10	2	4	359	51	2	0	0	0	0	116	6	1	0	0	0	0
3	6	10	2	10	3	1	398	44	0	0	0	0	0	192	7	0	0	0	0	0
3	6	10	2	10	3	2	278	5	0	0	0	0	0	106	0	0	0	0	0	0
3	6	10	2	10	3	3	322	27	0	0	0	0	0	89	2	0	0	0	0	0
3	6	10	2	10	3	4	331	57	2	0	0	0	0	125	4	0	0	0	0	0
3	6	10	1	40	1	1	987	42	0	0	0	0	0	917	9	0	0	0	0	0
3	6	10	1	40	1	2	980	7	0	0	0	0	0	912	0	0	0	0	0	0
3	6	10	1	40	1	3	990	40	0	0	0	0	0	933	8	0	0	0	0	0
3	6	10	1	40	1	4	985	60	1	0	0	0	0	914	14	0	0	0	0	0
3	6	10	2	40	2	1	987	665	25	0	0	0	0	941	354	2	0	0	0	0
3	6	10	2	40	2	2	955	360	3	0	0	0	0	822	70	0	0	0	0	0
3	6	10	2	40	2	3	959	655	3	0	0	0	0	760	335	0	0	0	0	0
3	6	10	2	40	2	4	975	738	41	0	0	0	0	860	448	1	0	0	0	0
3	6	10	2	40	3	1	991	559	23	0	0	0	0	945	287	4	0	0	0	0
3	6	10	2	40	3	2	961	265	5	0	0	0	0	849	63	0	0	0	0	0
3	6	10	2	40	3	3	962	567	5	0	0	0	0	803	259	0	0	0	0	0
3	6	10	2	40	3	4	981	652	34	0	0	0	0	876	348	4	0	0	0	0
3	6	10	1	90	1	1	1000	49	2	0	0	0	0	1000	10	1	0	0	0	0
3	6	10	1	90	1	2	1000	8	1	0	0	0	0	1000	0	0	0	0	0	0
3	6	10	1	90	1	3	1000	45	1	0	0	0	0	1000	10	0	0	0	0	0
3	6	10	1	90	1	4	1000	64	4	0	0	0	0	1000	13	1	0	0	0	0
3	6	10	2	90	2	1	1000	997	36	0	0	0	0	1000	964	4	0	0	0	0
3	6	10	2	90	2	2	1000	964	6	0	0	0	0	1000	792	0	0	0	0	0
3	6	10	2	90	2	3	1000	997	6	0	0	0	0	997	967	0	0	0	0	0
3	6	10	2	90	2	4	1000	999	62	0	0	0	0	1000	985	6	0	0	0	0
3	6	10	2	90	3	1	1000	977	35	0	0	0	0	1000	876	8	0	0	0	0
3	6	10	2	90	3	2	1000	886	9	0	0	0	0	999	601	0	0	0	0	0
3	6	10	2	90	3	3	1000	974	9	0	0	0	0	999	891	0	0	0	0	0
3	6	10	2	90	3	4	1000	983	59	0	0	0	0	1000	927	8	0	0	0	0
3	6	10	3	10	2	1	390	52	9	0	0	0	0	167	13	0	0	0	0	0
3	6	10	3	10	2	2	251	2	0	0	0	0	0	73	0	0	0	0	0	0
3	6	10	3	10	2	3	265	26	0	0	0	0	0	65	2	0	0	0	0	0
3	6	10	3	10	2	4	270	47	8	0	0	0	0	92	4	0	0	0	0	0
3	6	10	3	10	3	1	470	100	13	0	0	0	0	258	17	0	0	0	0	0
3	6	10	3	10	3	2	315	5	2	0	0	0	0	126	0	0	0	0	0	0

							alpha=.05							alpha=.01						
p	q	n	r	trh	pat	t	1	2	3	4	5	6	7	1	2	3	4	5	6	7
3	6	10	3	10	3	3	343	47	2	0	0	0	0	90	4	0	0	0	0	0
3	6	10	3	10	3	4	375	92	13	0	0	0	0	143	7	0	0	0	0	0
3	6	10	3	10	4	1	430	63	5	0	0	0	0	196	11	1	0	0	0	0
3	6	10	3	10	4	2	306	2	1	0	0	0	0	105	0	0	0	0	0	0
3	6	10	3	10	4	3	333	40	1	0	0	0	0	76	2	0	0	0	0	0
3	6	10	3	10	4	4	360	65	5	0	0	0	0	126	6	1	0	0	0	0
3	6	10	3	40	2	1	990	822	375	0	0	0	0	963	581	118	0	0	0	0
3	6	10	3	40	2	2	930	299	146	0	0	0	0	740	45	3	0	0	0	0
3	6	10	3	40	2	3	931	628	143	0	0	0	0	630	271	3	0	0	0	0
3	6	10	3	40	2	4	969	742	436	0	0	0	0	788	393	111	0	0	0	0
3	6	10	3	40	3	1	986	769	264	0	0	0	0	950	523	61	0	0	0	0
3	6	10	3	40	3	2	932	279	82	0	0	0	0	788	43	2	0	0	0	0
3	6	10	3	40	3	3	933	602	90	0	0	0	0	704	265	2	0	0	0	0
3	6	10	3	40	3	4	960	708	308	0	0	0	0	826	376	65	0	0	0	0
3	6	10	3	40	4	1	985	721	203	0	0	0	0	951	449	48	0	0	0	0
3	6	10	3	40	4	2	945	254	64	0	0	0	0	789	46	3	0	0	0	0
3	6	10	3	40	4	3	949	557	62	0	0	0	0	729	245	3	0	0	0	0
3	6	10	3	40	4	4	965	655	247	0	0	0	0	834	328	44	0	0	0	0
3	6	10	3	90	2	1	1000	1000	961	0	0	0	0	1000	996	805	0	0	0	0
3	6	10	3	90	2	2	999	959	847	0	0	0	0	998	715	352	0	0	0	0
3	6	10	3	90	2	3	999	994	847	0	0	0	0	995	960	352	0	0	0	0
3	6	10	3	90	2	4	1000	997	975	0	0	0	0	998	978	839	0	0	0	0
3	6	10	3	90	3	1	1000	998	831	0	0	0	0	1000	990	572	0	0	0	0
3	6	10	3	90	3	2	1000	925	629	0	0	0	0	999	650	173	0	0	0	0
3	6	10	3	90	3	3	1000	990	629	0	0	0	0	997	929	173	0	0	0	0
3	6	10	3	90	3	4	1000	996	881	0	0	0	0	999	961	618	0	0	0	0
3	6	10	3	90	4	1	1000	991	736	0	0	0	0	1000	963	443	0	0	0	0
3	6	10	3	90	4	2	1000	871	500	0	0	0	0	998	552	89	0	0	0	0
3	6	10	3	90	4	3	1000	970	497	0	0	0	0	995	874	89	0	0	0	0
3	6	10	3	90	4	4	1000	987	794	0	0	0	0	999	912	489	0	0	0	0
3	6	50	1	10	1	1	461	27	0	0	0	0	0	233	2	0	0	0	0	0
3	6	50	1	10	1	2	416	1	0	0	0	0	0	192	0	0	0	0	0	0
3	6	50	1	10	1	3	516	45	0	0	0	0	0	268	4	0	0	0	0	0
3	6	50	1	10	1	4	433	28	0	0	0	0	0	197	3	0	0	0	0	0
3	6	50	2	10	2	1	430	53	5	0	0	0	0	215	11	0	0	0	0	0
3	6	50	2	10	2	2	338	5	0	0	0	0	0	120	1	0	0	0	0	0
3	6	50	2	10	2	3	382	84	0	0	0	0	0	158	5	0	0	0	0	0
3	6	50	2	10	2	4	366	53	9	0	0	0	0	125	3	0	0	0	0	0
3	6	50	2	10	3	1	419	54	2	0	0	0	0	205	8	0	0	0	0	0
3	6	50	2	10	3	2	342	6	0	0	0	0	0	134	2	0	0	0	0	0
3	6	50	2	10	3	3	406	84	0	0	0	0	0	169	12	0	0	0	0	0
3	6	50	2	10	3	4	366	60	2	0	0	0	0	140	7	0	0	0	0	0
3	6	50	1	40	1	1	991	44	0	0	0	0	0	970	7	0	0	0	0	0
3	6	50	1	40	1	2	990	7	0	0	0	0	0	969	2	0	0	0	0	0
3	6	50	1	40	1	3	996	86	0	0	0	0	0	986	23	0	0	0	0	0
3	6	50	1	40	1	4	990	53	0	0	0	0	0	970	12	0	0	0	0	0
3	6	50	2	40	2	1	994	745	24	0	0	0	0	976	450	3	0	0	0	0
3	6	50	2	40	2	2	985	524	3	0	0	0	0	933	236	0	0	0	0	0
3	6	50	2	40	2	3	935	850	3	0	0	0	0	937	658	0	0	0	0	0
3	6	50	2	40	2	4	989	806	33	0	0	0	0	938	566	2	0	0	0	0
3	6	50	2	40	3	1	992	635	32	0	0	0	0	969	348	2	0	0	0	0
3	6	50	2	40	3	2	986	403	2	0	0	0	0	938	185	0	0	0	0	0
3	6	50	2	40	3	3	989	754	2	0	0	0	0	941	520	0	0	0	0	0
3	6	50	2	40	3	4	987	678	45	0	0	0	0	942	442	1	0	0	0	0
3	6	50	1	90	1	1	1000	52	1	0	0	0	0	1000	10	0	0	0	0	0
3	6	50	1	90	1	2	1000	9	0	0	0	0	0	1000	2	0	0	0	0	0
3	6	50	1	90	1	3	1000	110	0	0	0	0	0	1000	27	0	0	0	0	0
3	6	50	1	90	1	4	1000	69	0	0	0	0	0	1000	13	0	0	0	0	0
3	6	50	2	90	2	1	1000	997	35	0	0	0	0	1000	982	2	0	0	0	0
3	6	50	2	90	2	2	1000	985	1	0	0	0	0	1000	956	0	0	0	0	0
3	6	50	2	90	2	3	1000	1000	1	0	0	0	0	1000	999	0	0	0	0	0
3	6	50	2	90	2	4	1000	1000	55	0	0	0	0	1000	991	2	0	0	0	0
3	6	50	2	90	3	1	1000	982	40	0	0	0	0	1000	933	11	0	0	0	0
3	6	50	2	90	3	2	1000	944	11	0	0	0	0	1000	832	0	0	0	0	0
3	6	50	2	90	3	3	1000	995	11	0	0	0	0	1000	976	0	0	0	0	0
3	6	50	2	90	3	4	1000	988	66	0	0	0	0	1000	960	12	0	0	0	0

APPENDIX 7

ANALYSIS OF VARIANCE

Table M gives the results of the analysis of variance which was used in the text (page 17) to obtain an estimate of the variability inherent in the raw data. The data analyzed were the percentage exceedances for the tests of the first root in the null case at a nominal significance level of .05.

Table M

Analysis of variance

Source	SS	df	MS
Main effects			
Number of variates (p)	2.56	3	0.85
Number of groups (q)	40.51	2	20.23
Group size (n)	4.38	2	2.19
Test (t)	103.63	3	34.54
Two-way interactions			
p x q	6.89	6	1.15
p x n	4.04	6	0.67
p x t	6.67	9	0.74
q x n	3.13	4	0.78
q x t	7.45	6	1.24
n x t	2.57	6	0.43
Three-way interactions			
p x q x n	5.01	12	0.42
p x q x t	4.55	18	0.25
p x n x t	2.86	18	0.16
q x n x t	1.07	12	0.09
Four-way interaction			
p x q x n x t	4.61	36	0.128