

Analyzing Smoothness of Oscillatory Motion from NIR Video via Optical Flow Vector Field

by

Enze Cui

**A Thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
of the University of Manitoba
in partial fulfilment of the Requirements of the degree of**

Doctor of Philosophy

Department of Electrical and Computer Engineering

University of Manitoba

Winnipeg

Copyright © 2026 by Enze Cui

Analyzing Smoothness of Oscillatory Motion from NIR Video via Optical Flow Vector Field

by

Enze Cui

**A Thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
of the University of Manitoba
in partial fulfilment of the Requirements of the degree of**

Doctor of Philosophy

Department of Electrical and Computer Engineering

University of Manitoba

Winnipeg

Copyright © 2026 by Enze Cui

Permission has been granted to the Library of the University of Manitoba to lend or sell copies of this dissertation to the National Library of Canada to microfilm this thesis and to lend or sell copies of the film, and University Microfilms to publish an abstract of this thesis.

The author reserves other publication rights, and neither the thesis nor extensive abstracts from it may be printed or otherwise reproduced without the author's permission.

Abstract

This thesis establishes a non-contact framework for assessing the smoothness of oscillatory motion using near-infrared (NIR) video. Traditional vibration monitoring relies on contact-based sensors, which alters system dynamics through mass loading and provides only point-wise measurements. To overcome these constraints, this research develops a video-based methodology that integrates NIR video acquisition with geometric analysis of motion vector fields in the complex plane.

The proposed methodology introduces three key contributions. First, motion vector fields extracted from sequential NIR video frames are represented in the polar complex plane, enabling eigenvalue-based analysis of the spatial continuity of the velocity field. The Krantz criterion, adapted from discrete-time dynamical systems, provides a geometric classifier: motion is classified as smooth when the maximum eigenvalue magnitude lies within the unit circle, indicating gradual spatial variation in velocity; conversely, it is classified as jerky when eigenvalues exceed this boundary, reflecting discontinuity or abrupt changes in the velocity field. This operational criterion is motivated by the discrete-time representation of the mass-spring-damper model, where positive damping corresponds to eigenvalues inside the unit circle, and is validated through experimental correlation with Inertial Measurement Unit (IMU) data. Second, a characteristic distance metric d^Φ quantifies the similarity between different motion states by comparing their dominant eigenvalue magnitudes, allowing assessment of whether smooth motion patterns persist over time. Third, the smoothness index η aggregates frame-by-frame smoothness classifications into a global measure: the percentage of frames for which the maximum eigenvalue magnitude lies within the unit circle. This index provides a summary of overall motion regularity and has been shown to correlate strongly with mechanical loading across both human and vehicle experiments.

Experimental validation employs NIR video recordings of oscillatory motion from loaded vehicle systems and human movement sequences. Results demonstrate strong agreement between the video-based method and traditional IMU data, with correlation coefficients exceeding 0.90. A significant positive correlation is identified between mechanical loading and measured motion smoothness across both biomechanical and mechanical domains. This finding aligns with the physical damping models introduced in the thesis: increased load increases effective damping, producing smoother, more regular motion that yields higher smoothness indices. This research establishes that NIR video sensing achieves accuracy comparable to contact-based methods while eliminating sensor-induced artifacts and providing full-field spatial resolution. The framework offers a scalable, non-invasive tool for assessing motion smoothness and detecting irregular, jerky motion, with applications

in biomechanical load optimization, rehabilitation monitoring, and mechanical overload diagnostics.

Acknowledgments

First and foremost, I extend my deepest gratitude to my advisor, **Dr. James F. Peters**, whose wisdom, patience, and unwavering support have been my guiding light throughout this journey. Your brilliant mind and kind heart have not only shaped this dissertation but have transformed me into the researcher I am today. The countless hours you spent mentoring me, the meticulous attention to detail in your feedback, and your relentless pursuit of excellence have been nothing short of inspirational. I am truly honored to have been your student.

To my esteemed committee members **Dr. Sherif Sherif** and **Dr. Shawn Clark**, thank you for your valuable insights, challenging questions, and constructive criticism that have strengthened this work. Your expertise and encouragement have been instrumental in bringing this research to fruition. Thank you for your time, patience, and for helping me navigate the final stages of this journey.

I am also deeply grateful to **Dr. Ekram Hossain**, whose support as a thoughtful mentor has been invaluable. Your willingness to engage deeply with my work, your constructive feedback and your genuine encouragement gave me the clarity and confidence to refine this thesis to its current form.

To my external examiner, **Dr. Marek Reformat**, thank you for your thorough review of my thesis and for the thoughtful questions during the defense. Your detailed comments and constructive suggestions pushed me to think more deeply about the work and helped me strengthen the final version. I am grateful for the time and care you invested in evaluating my research.

To my **parents**, words cannot express my gratitude for your endless sacrifices. You taught me the value of education before I could even spell the word, and your unconditional support has been my foundation. This achievement is as much yours as it is mine.

To my beloved future wife **Shiying Zheng**, your love has been my anchor during the stormiest days of this voyage. Your unwavering belief in me, even when I doubted myself, gave me the strength to persevere. I look forward to building our life together beyond these academic walls.

To my little brother **Huize Cui**, and Big brother **Chen Wang**, thank you for keeping me grounded with your trust, care, and for reminding me what truly matters in life. Our countless conversations were often the perfect antidote to research stress.

In loving memory of my **grandparents**, who passed away during my studies. Though you are no longer here to celebrate this moment, your legacy lives on in everything I do. Your stories continue to inspire me daily.

A special acknowledgment to **Teacher Mao**, whose work was my guiding light during the darkest days of my journey. His wisdom and visionary philosophy, especially the teaching of *On Contradiction* gave me strength when I felt most lost. His teachings on perseverance and self-reliance were my intellectual compass.

Finally, to the **University of Manitoba** and all its staff - thank you for providing the environment and resources that made this work possible. The beautiful campus and vibrant academic community have shaped me in ways I'll always cherish.

Contents

Abstract	iv
Acknowledgements	vi
List of Tables	xii
List of Figures	xiii
1 Introduction	1
1.1 Research Background and Motivation	1
1.2 Physical Models of Oscillatory Motion	3
1.2.1 Mass-Spring-Damper Model for Vehicle Suspension	3
1.2.2 Molati Model for Human Limb Motion	4
1.2.3 Unifying Perspective	5
1.3 From Physical Motion to Video Measurement	5
1.3.1 Projection and Orthographic Approximation	6
1.3.2 Optical Flow as Velocity Measurement	6
1.4 Quantifying Motion Smoothness from Video	7
1.4.1 Spatial Gradients of the Velocity Field	7
1.4.2 Connecting Eigenvalues to Motion Smoothness	8
1.4.3 Connecting to Physical Damping	9
1.4.4 From Physical Damping to Smoothness Index	10
1.5 Operational Definitions	10
1.6 Assumptions	12
1.7 Limitations of Traditional Sensing Approaches and Advantages of the Proposed Framework	13
1.8 Research Objectives and Scope	14
1.9 Thesis Organization	15
1.10 Related Published Papers	17
2 Literature Review	18
2.1 Contact Sensing	18
2.1.1 Foundational Principles and Operational Framework	18
2.1.2 Sensor Taxonomy and Implementation Considerations	20
2.1.3 Practical Applications and Case Studies	22
2.1.4 Inherent Limitations and Motivations for Alternative Approaches	24
2.2 Accelerometer	26
2.2.1 Fundamental Principles and Transduction Mechanisms	26

2.2.2	Performance Parameters and Practical Implementation	28
2.2.3	Implementation Considerations	30
2.2.4	Limitations and Measurement Challenges	32
2.3	Non-Contact Method	34
2.3.1	Conceptual Framework and Technological Diversity	34
2.3.2	Practical Implementation	35
2.3.3	Theoretical Foundations and Measurement Principles	37
2.3.4	Technology Classifications and Operational Characteristics	39
2.3.5	Comparative Performance Analysis and Implementation Validation	42
2.3.6	Implementation Considerations and Emerging Trajectories	46
2.4	NIR Video	48
2.4.1	Spectral Advantages	49
2.4.2	System Architecture and Implementation Framework	51
2.4.3	Performance Evaluation and Application Scenarios	55
2.4.4	Technical Limitations and Compensating Strategies	58
2.5	Chapter Summary	59

3 Quantifying Motion Smoothness from NIR Videos Using Optical Flow Vector Fields

		62
3.1	Introduction	62
3.2	Vector Fields Derived from Video Frame Motion	67
3.2.1	Motion Extraction with Gaussian Mixture Model	67
3.2.2	Vector Representation and Transformation to Complex Plane	69
3.2.3	Complex Plane Representation for Smoothness Analysis	70
3.2.4	Constructing and Analyzing NIR-Derived Vector Fields	71
3.2.5	Case Study: Vector Field Construction	74
3.3	Eigenvalue Analysis for Local Motion Smoothness	76
3.3.1	Linearized Analysis of Motion Fields	77
3.3.2	Eigenvalue Extraction from Motion Vector Fields	78
3.3.3	Relation to Divergence and Motion Smoothness	79
3.4	Geometric Classification: The Smoothness Criterion	80
3.4.1	Unit Circle Boundary for Classification	80
3.4.2	Smoothness Margins and Robustness	82
3.4.3	Geometric Visualization Framework	82
3.4.4	Integration with Experimental Validation	82
3.5	Characteristic Distance Metric	83
3.5.1	Foundations of Characteristic Distance	83
3.5.2	Application to Motion Vector Field Analysis	84
3.5.3	Practical Implementation and Case Studies	85
3.6	Operators for Temporal Analysis	89
3.6.1	Discrete Rate-of-Change Analysis of Eigenvalues	89
3.6.2	Smoothness Index for Aggregate Assessment	91
3.6.3	Processing procedure	92
3.7	Chapter Summary	94

4	Experimental Design and Validation Framework	95
4.1	Introduction	95
4.2	Experimental Design Overview	96
4.3	Cross-Modal Method Validation	97
4.3.1	Experimental Configuration for Method Comparison	97
4.3.2	Biomechanical Experiments	98
4.3.3	Mechanical Experiments	99
4.3.4	Cross-Modal Validation Methodology	99
4.4	Load-Smoothness Correlation Analysis	100
4.4.1	Experimental Design for Correlation Analysis	100
4.4.2	Biomechanical Load-Smoothness Relationship	100
4.4.3	Mechanical Load-Smoothness Relationship	101
4.4.4	Statistical Analysis Framework	101
4.5	Data Processing Framework	102
4.5.1	Contact-Based (IMU) Processing Steps	102
4.5.2	Contact-Free (NIR) Processing Steps	103
4.6	Chapter Summary	105
5	Experimental Results and Analysis	106
5.1	Introduction	106
5.2	Comparative Performance: Contact versus Contact-Free Sensing	107
5.2.1	Walking Smoothness Assessment	107
5.2.2	Running Smoothness Assessment	109
5.2.3	Performance Under Loaded Running	112
5.2.4	Mechanical System Validation	114
5.3	Correlation Between Load and Motion Smoothness	116
5.3.1	Load-Smoothness Relationship in Human Running	116
5.3.2	Load-Smoothness Relationship in Vehicle Systems	118
5.3.3	Cross-Domain Consistency	119
5.4	Statistical Validation	121
5.4.1	Hypothesis Testing Results	121
5.4.2	Effect Size and Practical Significance	121
5.4.3	Measurement Precision	121
5.5	Performance Summary	122
6	Conclusions and Future Directions	124
6.1	Research Summary	124
6.2	Experimental Synthesis and Principal Findings	125
6.3	Key Contributions and Theoretical Advancements	127
6.4	Practical Applications and Implementation Significance	128
6.4.1	Healthcare and Rehabilitation	128
6.4.2	Athletic Training and Performance	128
6.4.3	Industrial Monitoring and Diagnostics	128
6.5	Current Limitations	129
6.6	Future Research Directions	133

6.7 Final Conclusion	134
References	136

List of Tables

2.1	Comprehensive Taxonomy of Contact Sensors and Operational Characteristics	21
2.2	Comparative Analysis of Imaging Modalities for Vibration Monitoring . . .	55
3.1	Sample eigenvalues from an irregular (jerky) motion vector field	65
3.2	Sample Eigenvalues $\lambda(t)$ for Runner Video Frames	93
3.3	Discrete Rate-of-Change D_i Computation	93
3.4	Smoothness Index η for Complete Sequence	93
4.1	Experimental Design Overview	96
4.2	Load-Smoothness Correlation Experiments Summary	102
5.1	Walking Smoothness Assessment: Cross-Modal Comparison ($n = 6$ trials: 3 male + 3 female)	108
5.2	Running Smoothness Assessment: Cross-Modal Comparison ($n = 6$ trials: 3 male + 3 female)	110
5.3	Weighted Running: Method Agreement Across Loading Conditions ($n = 24$ trials)	113
5.4	Off-Road Vehicle Smoothness Comparison Across Different Terrains ($n = 3$ trials)	115
5.5	Load-Smoothness Correlation in Human Running ($n = 12$ trials)	117
5.6	Load-Smoothness Correlation in Vehicle Systems ($n = 36$ trials)	118
5.7	Cross-Domain Comparison of Load-Smoothness Relationships	120
5.8	Statistical Validation of Primary Hypotheses	121
5.9	Practical Significance Analysis	122
5.10	Performance Summary Across Experimental Conditions	122

List of Figures

1.1	Conceptual flow of the thesis.	16
2.1	Accelerometer dynamic response analysis. (a) Step response showing the effect of damping ratio on system behavior. (b) Frequency response demonstrating resonant characteristics and operational bandwidth limitations. . . .	20
2.2	Theoretical and experimental characterization of piezoelectric accelerometer response. The primary plot demonstrates the linear relationship between charge output Q and applied acceleration a , validating Equation 2.3 with typical sensor parameters ($d = 10$ pC/N, $m = 5$ g). The sensitivity analysis subplot illustrates how variations in piezoelectric coefficient d directly influence charge output characteristics, highlighting the importance of material selection in sensor design optimization.	23
2.3	Mass loading effects in contact sensing.	25
2.4	Experimental setup showing runner with mounted IMU sensor for biomechanical analysis.	29
2.5	Triaxial acceleration data from running experiment, illustrating dynamic range and multi-axis measurement capabilities.	30
2.6	Contact based sensor: smartphones integrate accelerometers LSM6DSO . . .	31
2.7	NIR-based runner vibration analysis demonstrating (a)top left: sequential frames, (b)bottom left: motion vector extraction from sequential frames, (c)bottom right: displacement waveform characterization, and (d)top right: smoothness assessment through complex-plane eigenvalue analysis.	36
2.8	Velocity measurement principles in non-contact sensing. (a) Simulated velocity profile of a vibrating cantilever beam showing harmonic motion characteristics. (b) Doppler frequency shift analysis demonstrating the linear relationship between velocity and frequency shift.	38
2.9	Sample 3D-Scanning Vibrometry (SLDV)	40
2.10	Experimental test setup in [1]: the vision system was placed in front of the beam and an accelerometer was fixed on the left part of the upper face of the specimen.	44
2.11	Sequence of three sample frames from NIR video capture for vibration displacement analysis.	49
2.12	Sequence of three sample frames from RGB video capture for vibration displacement analysis.	50
2.13	Campark AC09 camera	51
2.14	Mechanism of active illumination in NIR system	51

2.15	Representative NIR video frame sequence for vibration displacement analysis.	52
2.16	NIR frame sequence and corresponding pixel value analysis for walker motion tracking. Frames display NIR images with a changing vertical line indicating the analysis region on the walker's body. Plots show the intensity values along this vertical line, with frame-to-frame variations directly corresponding to subject motion.	53
2.17	Smoothness analysis for walker motion sequence. The plot displays η values across the video timeline, with the dotted line indicating the smoothness threshold ($ \lambda_{\max} = 1$). Frames highlighted in red indicate jerky motion events where eigenvalues exceed the unit circle boundary. The analysis provides a comprehensive overview of motion smoothness evolution throughout the walking sequence, with an overall smoothness index of 82.54% achieved during the recorded motion.	53
3.1	Motion vector field derived from video frame fr	63
3.2	Example of a jerky motion vector field with eigenvalues exceeding unit magnitude, indicating low motion smoothness.	65
3.3	The unit circle in the complex plane serves as the decision boundary for motion smoothness classification.	66
3.4	Jerky vector field from a runner video frame (frame 68) indicating low motion smoothness.	66
3.5	Geometric representation of rotational motion showing position vector $\vec{r}$, velocity vector $\vec{v}$, and angular momentum $\vec{L} = \vec{r} \times m\vec{v} = I\vec{\omega}$. This illustrates the relationship between position and velocity for rotational motion, directly paralleling the limb motion described by Eqs. (??) and (??) in Chapter 1. .	69
3.6	Geometric representation of complex vector $z = x + jy = re^{j\theta}$ showing magnitude r and direction θ . The unit circle ($ z = 1$) provides the boundary for classifying motion smoothness.	72
3.7	Sample vector field constructed from motion extracted from an NIR video frame.	73
3.8	Illustration of partial derivatives of a sample vector field.	73
3.9	Biker motion video processing from frame 26 to 29. The vertical red line indicates the motion analysis column.	75
3.10	Biker motion video processing from frame 30 to 33.	81
3.11	Case 1: Characteristically non-near vector fields.	85
3.12	Case 2: Characteristically near vector fields.	86
3.13	Case 1: Characteristically near smooth vector fields.	87
3.14	Case 2: Pair of characteristically near jerky vector fields.	87
3.15	Case 3: Pair of characteristically non-close smooth and jerky vector fields. .	88
3.16	Runner frame sequence for $h = 5$	90
3.17	Runner frame sequence for $h = 8$	90
3.18	Runner frame sequence for $h = 13$	90

3.19	Discrete rate-of-change D_i comparison for different time lags $h = 5$, $h = 8$, and $h = 13$ frames. The plots demonstrate how varying the temporal sampling interval affects the measured rate-of-change of eigenvalues in runner motion analysis.	90
4.1	NIR-based motion smoothness analysis experimental workflow.	103
4.2	Flow diagram of the GMM-based background subtraction framework for contact-free smoothness assessment. The framework processes sequential NIR video frames through adaptive background modeling, motion extraction, and smoothness analysis.	104
5.1	Sample comparison of NIR-derived and IMU-derived smoothness metrics during walking experiments.	107
5.2	Additional sample comparison of NIR and IMU smoothness metrics during walking.	108
5.3	Temporal alignment of smoothness metrics during walking, demonstrating close correspondence between contact-free and contact-based assessments.	109
5.4	Sample comparison of NIR and IMU smoothness metrics during running experiments.	110
5.5	Additional sample comparison of NIR and IMU smoothness metrics during running.	111
5.6	Comparison of smoothness metrics during running, showing close agreement between modalities.	111
5.7	Sample comparison of NIR and IMU smoothness metrics during weighted running experiment (100% BW).	112
5.8	Temporal comparison of smoothness metrics during weighted running (100% BW).	113
5.9	Sample comparison of NIR and IMU smoothness metrics during off-road vehicle experiment.	114
5.10	Rough terrain	115
5.11	Gravel stone	115
5.12	Smooth pavement	115
5.13	Three distinct surface types employed for mechanical system validation.	115
5.14	Temporal comparison of smoothness metrics during off-road vehicle experiment.	116
5.15	Unloaded running (0% BW)	117
5.16	Moderate load (50% BW)	117
5.17	Heavy load (100% BW)	117
5.18	Experimental loading conditions for smoothness assessment in human running.	117
5.19	Unloaded vehicle (0% capacity)	118
5.20	Moderate load (50% capacity)	118
5.21	Full load (100% capacity)	118
5.22	Vehicle loading configurations for smoothness assessment.	118

Chapter 1

Introduction

1.1 Research Background and Motivation

Oscillatory motion is a fundamental characteristic of dynamical systems across mechanical, aerospace, civil, and biomechanical engineering domains. The ability to assess whether such motion remains regular and predictable or becomes irregular is a central objective in computer vision pattern classification. High-profile structural failures, such as the 2018 Genoa bridge collapse [2], underscore the imperative for precise and timely assessment of system behavior under operational loads. That collapse, attributed in part to excessive traffic loading, highlights a critical need: the detection of conditions that may lead to mechanical overload is particularly urgent, as excessive loading can compromise structural integrity and accelerate fatigue damage. This aligns with recent research trends toward non-contact identification methods for overweight vehicles using computer vision, as explored in [3].

Beyond bridges, the detection of irregular motion or overload conditions is essential across a wide range of applications. In jet engines, for instance, subtle changes in blade vibration patterns can serve as early indicators of incipient instability—such as rotating stall or flutter—well before catastrophic failure occurs. Non-contact, full-field optical techniques (e.g., stereo digital image correlation or blade tip timing with circumferential sensor arrays) can map the instantaneous displacement field of fan or compressor blades

during operation, providing a stability margin metric that reflects how close the engine is to its aerodynamic or material limits. In military and athletic training, soldiers and athletes frequently carry heavy loads that alter gait mechanics and elevate injury risk; real-time feedback on motion regularity could therefore help optimize load distribution and reduce the likelihood of musculoskeletal harm. In rehabilitation, clinicians rely on objective, quantitative assessments of movement quality to guide recovery protocols. In industrial settings, monitoring the smoothness of vehicle suspension travel can reveal wear or impending overload long before component failure becomes evident. Each of these scenarios demands a non-invasive, full-field measurement methodology capable of functioning in variable, often harsh environments without perturbing the system under observation.

Traditional approaches rely on contact-based sensors such as accelerometers, strain gauges, and inertial measurement units (IMUs). While these provide high-frequency, point-specific measurements, they suffer from mass loading effects that alter the dynamics of lightweight structures [4]. They also require physical attachment, which is impractical for many real-world scenarios, and their point-wise nature misses the spatial distribution of motion.

Non-contact alternatives exist but each has limitations. Laser Doppler vibrometers offer exceptional precision but are expensive, require a clear line of sight, and measure only a single point unless scanned. Radar-based sensors can operate over long distances but have coarse spatial resolution and are sensitive to surface reflectivity. Conventional RGB cameras are inexpensive and provide full-field data, yet their performance depends critically on ambient lighting; shadows, glare, and darkness degrade measurement quality [3].

Near-Infrared (NIR) imaging, combined with active illumination, overcomes many of these shortcomings. In contrast to the visible spectrum, NIR has reduced sensitivity to shadows and environmental noise, and is less affected by ambient illumination variations [5]. NIR is superior to visible light cameras because it is less sensitive to light variations and provides improved contrast in low-illumination environments [6]. By operating in the 750–950 nm wavelength range, NIR cameras capture reflected light that is largely insensitive

to ambient illumination. The active light source ensures consistent scene irradiance, making the measurements robust to day-night cycles and sudden lighting changes. Furthermore, NIR light is invisible to human observers, enabling unobtrusive monitoring in sensitive environments such as clinical settings or public spaces.

This thesis develops a framework that leverages NIR video to assess the smoothness of motion, a measure of how regular and predictable the observed movement is. Unlike previous methods that simply track displacements, our approach interprets the entire motion field through the lens of vector calculus and discrete-time systems theory, linking video data directly to the damping properties that determine whether a mechanical or biomechanical system’s motion is smooth and regular or exhibits irregular, jerky characteristics.

1.2 Physical Models of Oscillatory Motion

To connect video observations to physical behaviour, we begin with mathematical models that describe how oscillatory objects respond to forces. Two physical models are central to this thesis: a vehicle suspension, which can be modeled as a linear mass-spring-damper, and a human limb, which under small oscillations can be approximated by an analogous second-order differential equation.

1.2.1 Mass-Spring-Damper Model for Vehicle Suspension

For a vehicle moving over an uneven surface, the vertical motion of the suspension can be described by the classic mass-spring-damper equation:

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = 0 \tag{1.1}$$

Here $y(t)$ is the vertical displacement from the equilibrium position, m is the mass of the sprung part of the vehicle, c is the damping coefficient (which dissipates energy), and k is the spring constant (which provides the restoring force). The solution to this linear ordinary

differential equation is

$$y(t) = Ae^{-\zeta\omega_n t} \sin(\omega_d t + \phi) \quad (1.2)$$

where the damping ratio $\zeta = c/(2\sqrt{km})$ determines how quickly oscillations decay, $\omega_n = \sqrt{k/m}$ is the natural frequency, and $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped frequency. The physical interpretation is straightforward: when $\zeta > 0$, the exponential factor $e^{-\zeta\omega_n t}$ causes the oscillation amplitude to decrease over time; the motion is smooth and returns to equilibrium. When ζ is small, the decay is slow; if ζ becomes negative (indicating energy input rather than dissipation), the amplitude grows, leading to irregular, potentially damaging motion. Thus the damping ratio is a direct indicator of motion smoothness—high damping yields smooth, rapidly decaying oscillations; low damping yields sustained or growing oscillations that appear jerky and irregular.

1.2.2 Molati Model for Human Limb Motion

Human motion is far more complex, but the movement of a single limb segment can be approximated using a second-order differential equation that shares the same structure. Molati [7] proposed a model for human muscular movement based on the angular displacement $\theta(t)$ of a limb segment:

$$I\ddot{\theta}(t) + \mu\dot{\theta}(t) - \tau(\theta(t)) = 0 \quad (1.3)$$

Here I is the moment of inertia of the limb about the joint, μ is a damping coefficient representing viscosity in the joint and soft tissues, and $\tau(\theta)$ is the torque generated by muscles. For small oscillations around an equilibrium angle, the torque can be linearized as $\tau(\theta) \approx -k\theta$ (with $k > 0$ because the muscle acts like a spring pulling back toward the equilibrium). Substituting this approximation yields

$$I\ddot{\theta}(t) + \mu\dot{\theta}(t) + k\theta(t) = 0 \quad (1.4)$$

which is mathematically identical to the mass-spring-damper equation: inertia I plays the role of mass, damping μ corresponds to c , and stiffness k corresponds to the spring constant. The same exponential decay behaviour governed by a damping ratio $\zeta_{\text{human}} = \mu/(2\sqrt{kI})$ emerges. Thus, for small angular motions—exactly the type captured in video of running or walking—the human limb follows the same mathematics as a vehicle suspension. High damping leads to smooth, well-controlled limb motion; low damping allows oscillations to persist, leading to an irregular, jerky gait that may signal fatigue, injury, or excessive load.

1.2.3 Unifying Perspective

Both the vehicle suspension and the human limb are therefore described by second-order linear differential equations of the form

$$M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = 0 \quad (1.5)$$

where $q(t)$ is the relevant displacement (linear or angular), M is the inertial parameter, D is the damping coefficient, and K is the stiffness. The damping ratio $\zeta = D/(2\sqrt{MK})$ determines the smoothness of the motion: when ζ is large, the motion is well-damped and appears smooth; when ζ is small, the motion is under-damped and appears irregular or jerky. This unified perspective allows us to treat both vehicle and human data within the same analytical framework.

1.3 From Physical Motion to Video Measurement

The analysis of motion from image sequences has been extensively studied in computer vision. Egbert et al. [8] treat motion as a signal that can be segmented, analyzed, and reconstructed. Their work on real-time motion transitions demonstrates that motion segments can be characterized by their endpoints and that the detailed structure of a motion carries its defining characteristics. This perspective supports the idea that observed motion can be

quantified and compared without requiring an underlying physical model.

Having established that physical oscillatory motion is governed by a second-order differential equation, the next question is how such motion can be captured and quantified from a video. The answer lies in optical flow, which calculates the apparent motion of pixels between consecutive frames, establishing the geometric relationship between the 3D world and the 2D image plane.

1.3.1 Projection and Orthographic Approximation

Consider a point on a moving object, such as a position on a runner's leg or a corner of a vehicle's chassis. In the physical world, its three-dimensional coordinates change over time. A camera records the projection of these coordinates onto a two-dimensional sensor. If the camera is placed far from the object and the motion is confined to a plane approximately parallel to the image plane, the projection can be approximated as orthographic: image coordinates (x, y) are linearly related to world coordinates (X, Y) by a constant scale factor α :

$$x = \alpha X, \quad y = \alpha Y \quad (1.6)$$

For the experiments in this thesis, the camera was positioned at a distance exceeding 5 times the subject's height, and the motion was primarily parallel to the image plane, making the orthographic approximation accurate to within a few percent. This assumption is standard in many vision-based motion analyses and is validated by the strong correlation between our video-derived measures and IMU data presented in Chapter 5.

1.3.2 Optical Flow as Velocity Measurement

Optical flow algorithms estimate the displacement of each pixel between two consecutive frames. The fundamental constraint is brightness constancy: the intensity of a small patch does not change as it moves. Mathematically, this leads to the optical flow equation

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0 \quad (1.7)$$

where $u = dx/dt$ and $v = dy/dt$ are the horizontal and vertical components of the optical flow vector $\vec{v} = (u, v)$. Solving this equation (together with regularization) yields a dense vector flow field that approximates the projected velocity of every visible point.

Now, using the orthographic projection (Eq. (1.6)), the physical velocity of the corresponding world point is $\dot{X} = \dot{x}/\alpha$ and $\dot{Y} = \dot{y}/\alpha$. Hence the optical flow components are proportional to the physical velocity components:

$$(u, v) = \alpha(\dot{X}, \dot{Y}) = \alpha \mathbf{v}_{\text{phys}} \quad (1.8)$$

Thus, up to a constant scale factor, the optical flow vector field (motion vector field) directly provides a measurement of the instantaneous velocity of the object in the image plane. This is the crucial link: the video camera, through optical flow, gives us a spatially dense approximation of the velocity field of the moving system.

1.4 Quantifying Motion Smoothness from Video

We now have a dense velocity field $\vec{v}(x, y, t)$ derived from the video. But velocity alone does not tell us whether the motion is smooth. A system could have large velocities but still move in a smooth, predictable pattern, or it could have small velocities but exhibit jerky, irregular motion. We need a measure that captures the spatial structure of the flow, that is, how the velocity varies from point to point.

1.4.1 Spatial Gradients of the Velocity Field

To quantify the local variation in the velocity field, we examine its spatial gradients. The Jacobian matrix of the velocity field is defined as

$$J(\vec{v}) = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} \quad (1.9)$$

This matrix encapsulates all first-order spatial variations of the flow. Its components describe how the horizontal and vertical velocity components change with respect to horizontal and vertical position. The trace of this matrix, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, is the divergence, which measures the local rate of expansion or contraction of the flow field. For a rigid body moving in a plane parallel to the image sensor, the true velocity field should exhibit very little spatial variation; it is primarily translational. Any significant spatial gradients are thus indicative of either measurement noise or, more importantly for this work, departures from a rigid, smooth motion. Such departures manifest as jerky or irregular movement.

1.4.2 Connecting Eigenvalues to Motion Smoothness

The eigenvalues of the Jacobian matrix, denoted λ_1 and λ_2 , provide a more complete description of the local velocity field's behaviour. The maximum magnitude of these eigenvalues, $|\lambda_{\max}|$, quantifies the most extreme rate of spatial change in the velocity field. When the motion is smooth and well-damped, the velocity field changes slowly in space, leading to small eigenvalues. When the motion is irregular and jerky, the spatial gradients become large, producing eigenvalues with greater magnitudes.

This leads to a natural operational definition for classifying motion from a single frame of video. We adopt a threshold based on the unit circle in the complex plane, inspired by the Krantz criterion from discrete-time dynamical systems theory [9]. In that context, a system is considered to have bounded, convergent behavior if the eigenvalues of its state transition matrix lie within the unit circle. While we are analyzing a spatial Jacobian and not a temporal state transition matrix, the unit circle provides a mathematically convenient and geometrically intuitive boundary. An eigenvalue magnitude of 1 corresponds to a local expansion rate that would double the size of a region over one frame interval, a clear indication of rapid, irregular motion.

Therefore, we classify a motion segment in a given frame as *smooth* if $|\lambda_{\max}| \leq 1$, meaning local velocity variations are bounded. Conversely, we classify it as *jerky* if $|\lambda_{\max}| > 1$, indicating significant and rapid spatial changes in the velocity field.

1.4.3 Connecting to Physical Damping

This thesis does not attempt to derive the mass-spring-damper equation from the video data, nor does it claim that the Jacobian eigenvalues directly equal the eigenvalues of the system’s state transition matrix. The eigenvalues of the spatial Jacobian belong to a different mathematical object (a field of velocity gradients) and do not reduce to the temporal eigenvalues of the ordinary differential equation.

Instead, the relationship is one of correlation and proxy. For a well-damped physical system (high ζ), the resulting motion is smooth and the spatial velocity gradients in the video are small, yielding Jacobian eigenvalues with magnitudes predominantly less than one. For an under-damped or irregularly driven system (low or effectively negative ζ), the motion becomes jerky, leading to larger spatial gradients and eigenvalues that frequently exceed a magnitude of one. Thus, while the spatial Jacobian’s eigenvalues are not the system’s eigenvalues, they serve as a data-driven proxy for the smoothness of the motion, which is in turn governed by physical damping. The validity of this proxy is established empirically through the experiments described in Chapter 5, which show a strong correlation between our video-derived smoothness index and accelerations measured by an IMU—a sensor that directly responds to the physical consequences of damping.

To make the mapping between the physical system and the video-based measurement explicit, we can summarise the relationship as follows. At the physical level, the damping ratio ζ determines whether oscillations decay ($\zeta > 0$) or persist and grow ($\zeta \leq 0$). This physical behaviour manifests in the video as a velocity field that either varies gradually in space (for high ζ) or abruptly (for low or negative ζ). Our eigenvalue analysis captures this spatial variation: when the velocity field is smooth, the eigenvalues are small and lie inside the unit circle ($|\lambda_{\max}| \leq 1$); when the velocity field is irregular, the eigenvalues are large

and lie outside the unit circle ($|\lambda_{\max}| > 1$). The smoothness index η then aggregates this frame-by-frame classification into a global measure of motion regularity.

1.4.4 From Physical Damping to Smoothness Index

The smoothness index η aggregates this frame-by-frame classification into a single number: the percentage of frames in a video for which the maximum eigenvalue magnitude satisfies the smoothness condition $|\lambda_{\max}| \leq 1$. A high η indicates that the motion is predominantly smooth and well-damped; a low η indicates frequent episodes of jerky, irregular motion. In this way, the video-based eigenvalue analysis operationalizes the concept of motion smoothness without requiring knowledge of the system’s mass, stiffness, or even its exact differential equations.

1.5 Operational Definitions

To ensure clarity and avoid ambiguity, this section elaborates on the key terms used throughout this thesis, linking each to both the physical models and the video-based measurements.

Oscillatory motion refers to any motion that exhibits periodic or cyclic characteristics. In this thesis, it includes both the mechanical vibration of vehicle suspensions (governed by the mass-spring-damper equation) and the cyclic limb movements of human gait (approximated by the linearized Molati model). Although the underlying physical mechanisms differ, both can be analyzed with the same mathematical tools because they share the same second-order differential structure. The common element is the presence of a restoring force that tends to return the system toward an equilibrium position, and damping that dissipates energy.

Vibration is used specifically for mechanical systems where the motion arises from restoring forces and damping, as described by Eq. (1.1). For human motion we prefer the broader term oscillatory motion to avoid implying that the body behaves as a linear vibratory

system, but the mathematical treatment remains consistent.

Observed motion is the two-dimensional projection of physical motion onto the image plane, captured through optical flow. The optical flow vector $\vec{v}(x, y, t) = (u(x, y, t), v(x, y, t))$ gives, under orthographic projection, a scaled version of the physical velocity of the corresponding point. The key relationship is $(u, v) = \alpha v_{\text{phys}}$, where α is a constant scale factor determined by camera distance and lens properties. Thus the video data provide a direct, albeit scaled, measurement of the instantaneous velocity field of the object.

Smooth motion describes a state where the observed velocity field changes slowly in space. This corresponds to well-damped physical behaviour where oscillations decay rapidly. Operationally, a frame is classified as exhibiting smooth motion if the maximum eigenvalue magnitude of the spatial Jacobian satisfies $|\lambda_{\text{max}}| \leq 1$. This indicates that local velocity variations are bounded and the motion appears regular and predictable.

Jerky motion describes a state where the observed velocity field changes abruptly and rapidly in space. This corresponds to under-damped or irregular physical behaviour. Operationally, a frame is classified as exhibiting jerky motion if $|\lambda_{\text{max}}| > 1$, indicating significant local spatial variations in velocity.

Smoothness index η is defined as the percentage of frames in a video for which the motion is classified as smooth:

$$\eta = \frac{|\{\text{frames where } |\lambda_{\text{max}}| \leq 1\}|}{N} \times 100\% \quad (1.10)$$

where N is the total number of frames analyzed. η provides a global measure of motion smoothness; higher values indicate that the motion is predominantly well-damped and regular. Empirically, as shown in Chapter 5, η increases systematically with mechanical load. In both the vehicle and human models, increased load corresponds to an increase in

effective damping (e.g., a heavier vehicle suspension becomes more damped, a runner with a heavier backpack adopts a more stable, less variable gait). Thus η serves as a practical, video-based proxy for the physical damping that governs the smoothness of the motion.

1.6 Assumptions

The framework developed in this thesis is predicated on a set of assumptions that define its scope of validity. These assumptions are made to ensure that the mathematical models linking the video data to physical motion are well-posed and that the analysis yields meaningful results.

1. **Rigid Motion:** The object under observation (e.g., a vehicle chassis, a human limb segment) is assumed to move in a manner that is approximately rigid. This implies that the primary motion is translation and rotation, with negligible internal deformation. While some non-rigidity is present in human gait, we focus on the overall limb motion, which can be approximated as quasi-rigid over short time intervals. This justifies treating the velocity field as primarily representing the bulk motion of the object rather than its internal deformation.
2. **Brightness Constancy:** The optical flow calculation relies on the standard assumption that the intensity of a small image patch remains constant as it moves from one frame to the next. The use of an active NIR illumination source in a controlled environment makes this assumption highly robust.
3. **Motion Pattern Similarity Across Trials:** For the comparative analysis of different load conditions (e.g., running with 0 kg vs. 20 kg backpack), it is assumed that the subject performs the same fundamental motion pattern (e.g., running at a consistent speed on a treadmill) across all trials. This ensures that observed differences in the smoothness index η can be attributed primarily to the change in load and its effect on damping, rather than to variations in the performed task.

4. **Orthographic Projection:** The physical motion is assumed to occur primarily in a plane parallel to the camera’s image sensor, and the camera is placed at a sufficient distance such that perspective effects are negligible. This orthographic approximation (Eq. (1.6)) provides a linear relationship between physical velocity and optical flow.
5. **Periodicity:** The oscillatory motion under consideration is assumed to be quasi-periodic. This means the motion exhibits a repeating pattern, though not perfectly, which is characteristic of both mechanical vibrations and human gait. This property justifies the use of a per-frame analysis to build an aggregate measure of smoothness (η) over multiple cycles of motion.

1.7 Limitations of Traditional Sensing Approaches and Advantages of the Proposed Framework

Traditional vibration monitoring relies on contact-based sensors such as accelerometers and strain gauges. While these provide accurate point measurements, they introduce mass loading effects that can alter the dynamics of lightweight structures [4]. They also require physical attachment, which is often impractical or invasive, and their point-wise nature misses the spatial distribution of motion—critical for understanding global behaviour.

Other non-contact methods exist but each has drawbacks [3]. Laser Doppler vibrometers offer sub-nanometer precision but are expensive, require a clear line-of-sight, measure only a single point (unless scanned), and may need reflective coatings on the target. Radar-based sensors can operate over long distances and through some obstructions, but they have coarse spatial resolution and are sensitive to surface material. Conventional RGB cameras are inexpensive and provide full-field data, yet their performance depends critically on ambient lighting; shadows, glare, and rapid illumination changes degrade the quality of optical flow.

The NIR camera with active illumination overcomes these limitations. By using its own light source, it ensures consistent image quality regardless of ambient lighting [10].

Its full-field nature provides spatial resolution that point sensors cannot match. Moreover, because the framework is based on optical flow and vector calculus, it can be implemented with off-the-shelf hardware, making it cost-effective and scalable. The key advantage is that it directly assesses motion smoothness, a high-level property linked to damping and overload, rather than merely measuring displacements or velocities. This makes it uniquely suited for applications where the goal is to detect conditions leading to irregular or excessively loaded motion.

1.8 Research Objectives and Scope

This thesis has two primary objectives. First, to develop and experimentally validate a framework for assessing oscillatory motion smoothness using NIR video, demonstrating that the smoothness measures obtained from video correlate strongly with those derived from traditional contact-based sensors. Second, to use this framework to investigate the correlation between mechanical loading and observed motion smoothness across two domains: human gait (with varying backpack loads) and vehicle suspension (with varying payloads). The central expectations are:

1. The smoothness index η derived from NIR video will show strong positive correlation with IMU-based measures of vibration intensity.
2. Increased mechanical load will produce a systematic increase in η for both human runners and vehicle suspensions, reflecting increased damping and improved motion smoothness.

The scope is limited to objects where motion is visible in video and can be approximated as the two-dimensional projection of one-dimensional oscillation or periodic movement. For vehicles, the linear mass-spring-damper model provides the physical foundation. For human motion, the linearized Molati model serves as a simplified foundation for limb motion, with

the understanding that whole-body gait involves multiple joints and is analyzed here as a composite oscillatory system.

1.9 Thesis Organization

This thesis is structured as follows.

Chapter 2 reviews sensing methodologies, examining the limitations of contact-based sensors and RGB vision systems while detailing the advantages of NIR imaging. It also compares the proposed approach with other non-contact techniques such as laser vibrometry and radar.

Chapter 3 presents the core technical framework for quantifying motion smoothness. This includes motion extraction from NIR video using Gaussian mixture models, construction of motion vector fields, computation of the Jacobian and its eigenvalues, application of the unit-circle criterion for smoothness classification, and introduction of the characteristic distance metric d^Φ and the smoothness index η .

Chapter 4 describes the experimental design, including the biomechanical experiments with human runners carrying different loads and the mechanical experiments with a vehicle under varying payload. Synchronized data acquisition from NIR cameras and IMUs enables cross-modal validation.

Chapter 5 presents and analyzes the experimental results, comparing the video-based smoothness measures with IMU data, examining the load-smoothness correlation, and performing statistical validation.

Chapter 6 synthesizes the contributions, discusses limitations of the current study, and proposes directions for future research, including extension to real-time processing, integration with other sensing modalities, and application to new domains.

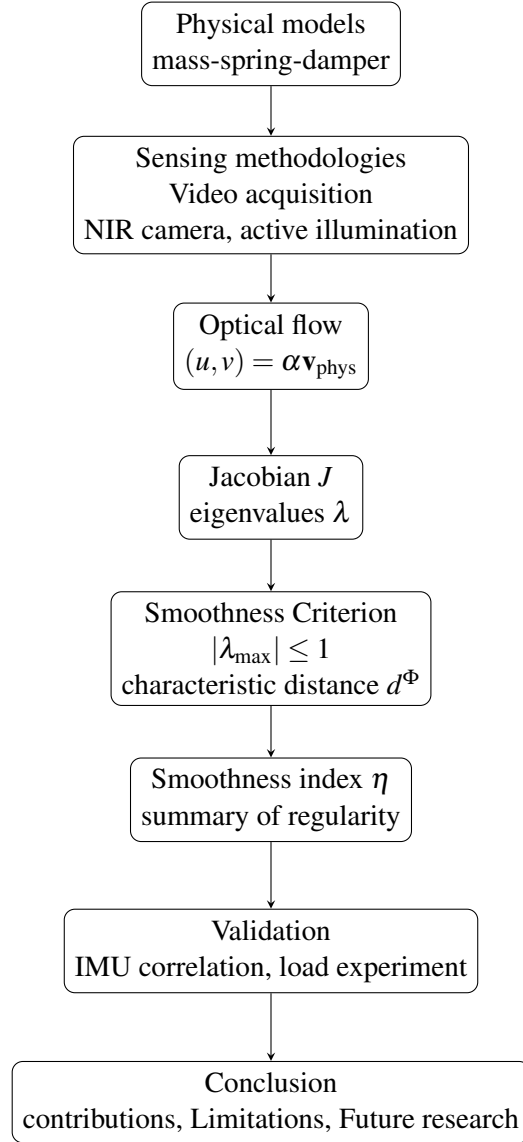


Figure 1.1: Conceptual flow of the thesis.

1.10 Related Published Papers

This thesis builds upon a series of published works that have progressively developed its core concepts. The foundational framework, integrating NIR sensing with the unit-circle criterion for smoothness classification in the complex plane, is presented in [11]. This work establishes the primary methodology validated herein.

The mathematical underpinning for comparing system states across time is provided by our formulation of characteristic proximity between vector fields in the polar complex plane [12]. This paper introduces a mechanism for measuring the closeness of temporally near motion vector fields and their corresponding motion recordings in NIR video frames.

The detection of smooth, self-similar motion patterns was advanced through our analysis of Hilbert envelope energy regions on motion waveforms [13]. This work is directly related to the detection of instances of smooth motion that are similar, and it provides the central mechanism for motion smoothness monitoring introduced in this thesis.

These methodological advances built upon our earlier investigation into the fractal properties inherent in optical flows derived from video [14]. That initial research highlighted the potential for analyzing motion smoothness through optical flow, foreshadowing the development of the complete framework.

Chapter 2

Literature Review

2.1 Contact Sensing

To establish the foundation for our comparative analysis, we first examine contact sensing as the traditional paradigm in vibrational monitoring. This approach, characterized by direct physical sensor attachment to the system under investigation, has served as the cornerstone of vibration measurement across diverse domains including structural health monitoring, biomechanical analysis, and mechanical system diagnostics [15, 1]. Understanding this established methodology provides essential context for appreciating the limitations that motivate our development of non-contact alternatives.

2.1.1 Foundational Principles and Operational Framework

Contact sensing fundamentally relies on the transduction of physical parameters through mechanical coupling, enabling direct measurement of acceleration, strain, displacement, and other vibrational characteristics. We define this approach as measurement methodologies requiring direct physical attachment of sensors to enable transduction of parameters through mechanical coupling [15]. This physical interface between measurement system and subject facilitates direct energy transfer and parameter quantification, establishing a proven pathway for obtaining vibration data in controlled settings.

The operational framework encompasses multiple transduction mechanisms, each optimized for specific measurement requirements. Our analysis begins with the established principles governing the conversion of mechanical energy into measurable electrical signals. The primary transduction mechanisms include piezoelectric [15], piezoresistive [16], capacitive [17] [18], and electromagnetic principles [19], each offering distinct advantages for particular application scenarios.

Piezoelectric transduction utilizes materials that generate electrical charge in response to mechanical stress, providing high-frequency response ideal for dynamic acceleration and force measurements. This mechanism forms the basis for many accelerometers used in structural monitoring. Piezoresistive effects involve changes in electrical resistance due to mechanical strain, enabling highly sensitive strain measurements in structural health monitoring applications. Electromagnetic induction, based on relative motion between conductors and magnetic fields, supports velocity sensors and linear variable differential transformers (LVDTs), while capacitive sensing utilizes geometric variations to alter capacitance, making it particularly suitable for micro-electromechanical systems (MEMS) accelerometers and precision displacement sensors.

Beyond these classical mechanisms, contemporary contact sensing has expanded to include chemical sensing for environmental monitoring and biomolecule detection [20], as well as highly sensitive piezoresistive sensing utilizing multiscale hierarchical designs for flexible electronics and soft robotics [20]. These advancements demonstrate the ongoing evolution of contact sensing technologies, though they remain constrained by the fundamental requirement for physical attachment.

The theoretical foundation for contact sensing derives from Newtonian mechanics and continuum mechanics principles, where sensor response can be modeled through second-order differential equations representing mass-spring-damper systems. For accelerometers, the governing equation takes the form:

$$m\ddot{x} + c\dot{x} + kx = F(t) \quad (2.1)$$

where m represents the seismic mass, c the damping coefficient, k the spring constant, and $F(t)$ the external forcing function. This formulation enables precise characterization of sensor dynamics and frequency response characteristics.

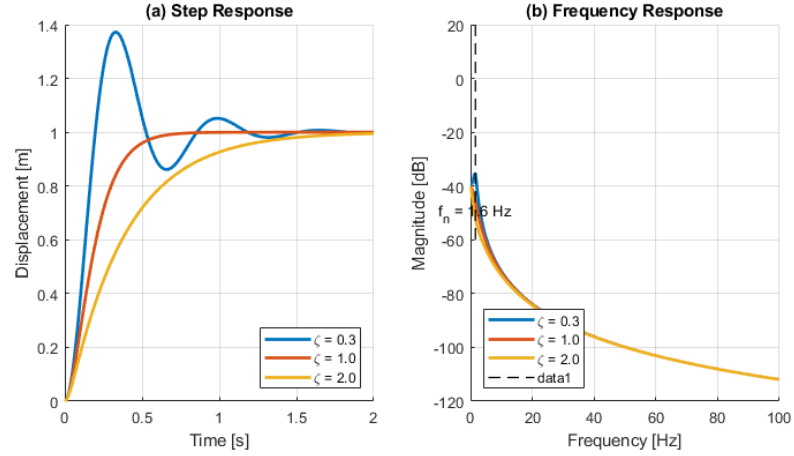


Figure 2.1: Accelerometer dynamic response analysis. (a) Step response showing the effect of damping ratio on system behavior. (b) Frequency response demonstrating resonant characteristics and operational bandwidth limitations.

The dynamic behavior characterized by Equation 2.1 is illustrated in Fig. 2.1. The step response analysis reveals how damping ratio ζ influences system behavior, with underdamped configurations ($\zeta = 0.3$) exhibiting oscillatory settling while critically damped systems ($\zeta = 1.0$) achieve optimal response with minimal overshoot. Overdamped configurations ($\zeta = 2.0$) provide complete oscillation suppression but with extended response times. Frequency response analysis shows the characteristic resonant peak at the natural frequency $\omega_n = \sqrt{k/m}$, with operational bandwidth for accurate measurement typically extending to approximately one-fifth of this frequency.

2.1.2 Sensor Taxonomy and Implementation Considerations

The diversity of contact sensors reflects the varied requirements across application domains, with each sensor type optimized for specific measurement scenarios. The primary contact sensor types and their operational characteristics are summarized in Table 2.1, which provides a comprehensive overview of available technologies and their applications.

Table 2.1: Comprehensive Taxonomy of Contact Sensors and Operational Characteristics

Sensor Type	Operation Principle	Typical Applications
Piezoelectric Accelerometer	Generates electrical charge proportional to applied acceleration through piezoelectric crystal deformation	Structural vibration monitoring, impact testing, machinery condition monitoring [15]
Accelerometer (Inertial Measurement Unit)	Integrates accelerometers alone with gyroscopes and magnetometers to provide comprehensive kinematic data through sensor fusion algorithms	Biomechanical analysis, vehicle dynamics, navigation systems [21]
Strain Gauge	Measures mechanical strain through resistance changes in conductive foil or semiconductor patterns	Structural health monitoring, load measurement, stress analysis [1]
LVDT (Linear Variable Differential Transformer)	Determines linear displacement through electromagnetic coupling variation between primary and secondary windings	Precision displacement measurement, position sensing, vibration amplitude quantification
Force Plate	Measures ground reaction forces through multiple load cells or piezoelectric sensors arranged in plate configuration	Gait analysis, sports biomechanics, postural stability assessment
Geophone	Employs spring-mounted mass moving within magnetic field to generate voltage proportional to velocity	Seismic monitoring, geophysical exploration, low-frequency vibration measurement

The mathematical description of contact sensor operation provides the foundation for understanding measurement limitations and optimization strategies. For strain gauges, the fundamental relationship derives from the piezoresistive effect:

$$\frac{\Delta R}{R} = G_F \cdot \varepsilon \quad (2.2)$$

where $\Delta R/R$ represents the relative resistance change, G_F denotes the gauge factor (typically 2.0-2.1 for metallic strain gauges), and ε signifies the mechanical strain. This linear relationship enables precise strain measurement when incorporated into Wheatstone bridge configurations.

For piezoelectric accelerometers, the transduction mechanism operates through the direct piezoelectric effect, where mechanical stress generates proportional electrical charges. The fundamental governing equation expresses this relationship as:

$$Q = d \cdot F = d \cdot m \cdot a \quad (2.3)$$

where Q represents the generated charge output, d denotes the piezoelectric coefficient, m is the seismic mass, and a is the applied acceleration. This formulation highlights the direct linear relationship between generated charge and applied acceleration, facilitating high-frequency vibration measurement across diverse engineering applications.

The graphical representation in Fig. 2.2 provides empirical validation of the theoretical model expressed in Equation 2.3. The characteristic linear response confirms the deterministic nature of piezoelectric transduction, while the parameter sensitivity analysis reveals the critical role of piezoelectric coefficient d in determining overall sensor sensitivity.

2.1.3 Practical Applications and Case Studies

To illustrate the practical implementation of contact sensing, we find its application in bridge structural health monitoring, which demonstrates both the methodology's capabilities and limitations. In the study by [21], a distributed accelerometer network comprising nine PCB 625B02 accelerometers was employed for vehicle detection and localization on a bridge structure. The sensors, mounted to the underside of the concrete deck slab, provided high-fidelity vibration data at 10 kHz sampling rate, enabling frequency domain analysis up to 5 kHz.

The experimental setup achieved localization scores of 0.95 using both Short-Time Fourier Transform (STFT) and Wavelet Scattering Network (WSN) methods, validating the efficacy of contact sensing for structural monitoring applications. However, the study also highlighted limitations including installation complexity, cable management requirements, and susceptibility to environmental factors that necessitate careful sensor selection and

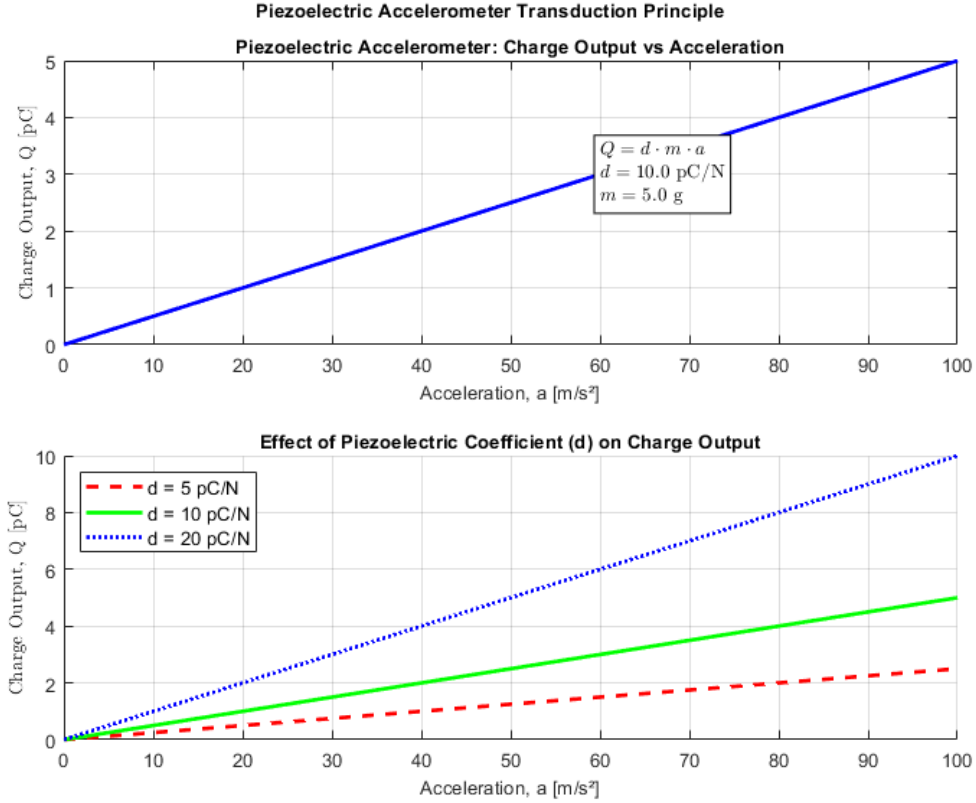


Figure 2.2: Theoretical and experimental characterization of piezoelectric accelerometer response. The primary plot demonstrates the linear relationship between charge output Q and applied acceleration a , validating Equation 2.3 with typical sensor parameters ($d = 10 \text{ pC/N}$, $m = 5 \text{ g}$). The sensitivity analysis subplot illustrates how variations in piezoelectric coefficient d directly influence charge output characteristics, highlighting the importance of material selection in sensor design optimization.

deployment strategies.

Complementary research by [1] employed strain gauges and accelerometers for vibrational monitoring of concrete beams, demonstrating strong correlation ($r = -0.82$) between strain-based damage indices and natural frequency reductions. This integrated approach showcases how multiple contact sensing modalities can provide comprehensive structural assessment through data fusion techniques, while also revealing the practical constraints of sensor deployment in field conditions.

In biomechanical motion analysis, contact sensors introduce significant practical constraints. The attachment of IMU sensors to human subjects alters natural movement patterns through added mass and restricted range of motion. Furthermore, the point-specific nature of contact measurements limits comprehensive characterization of complex biomechanical systems. While individual sensors provide high-fidelity temporal data at specific locations, they cannot capture the full-field response characteristics essential for understanding complex movement patterns and smoothness mechanisms. This spatial limitation has driven the adoption of optical motion capture systems and, more recently, vision-based approaches that provide comprehensive spatial data without physical sensor attachment.

2.1.4 Inherent Limitations and Motivations for Alternative Approaches

Despite their widespread adoption and proven efficacy, contact sensing methodologies exhibit inherent limitations that constrain their applicability in certain scenarios. These limitations provide the motivation for developing complementary non-contact approaches.

Mass loading effects represent a fundamental limitation in contact-based vibration monitoring. The attachment of physical sensors alters the dynamic characteristics of the measured system through additional mass contributions [4]. For a sensor of mass m_s attached to a structure with effective mass m_e , the natural frequency shift Δf_n is given by:

$$\Delta f_n = f_{n0} \left(\frac{1}{\sqrt{1 + \frac{m_s}{m_e}}} - 1 \right) \quad (2.4)$$

where f_{n0} represents the original natural frequency. The physical basis derives from the fundamental relationship for natural frequency: $f_n = \frac{1}{2\pi} \sqrt{k/m}$.

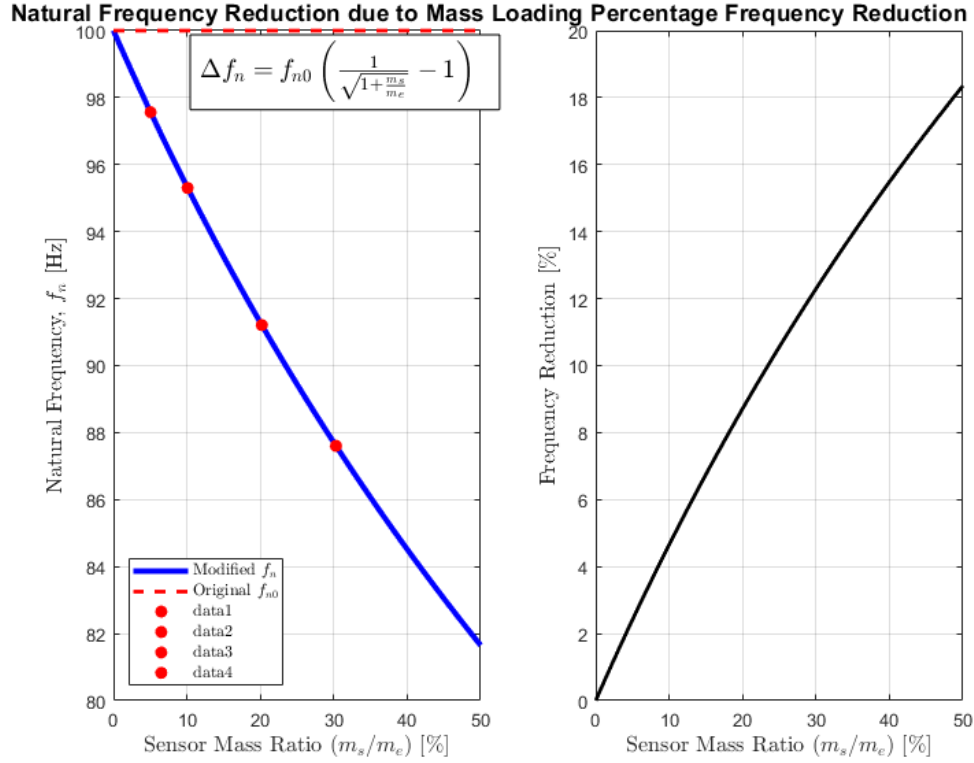


Figure 2.3: Mass loading effects in contact sensing.

The graphical representation in Fig. 2.3 validates this theoretical model, demonstrating the inverse square root relationship between mass ratio and frequency reduction. This nonlinear dependence reveals that a 5% mass ratio produces 2.4% frequency reduction and a 10% mass ratio results in 4.9% frequency reduction. These effects become particularly significant when monitoring lightweight or delicate structures where even small additional mass can substantially alter dynamic behavior.

Additional limitations include spatial resolution constraints, installation-induced damage risks, cable-induced artifacts, and deployment practicalities in inaccessible or hazardous environments. The point-specific nature of most contact sensors restricts comprehensive spatial characterization, requiring extensive sensor networks for full-field measurements. Installation often necessitates surface preparation or structural modification, potentially

introducing damage or altering system characteristics. Cable management presents practical challenges in field deployments, particularly for long-term monitoring applications. These factors collectively motivate the development of non-contact alternatives that preserve natural system dynamics while enabling comprehensive spatial characterization.

These inherent limitations of contact sensing, particularly their obtrusive nature and spatial constraints, provide the fundamental motivation for our development of complementary non-contact alternatives. Technologies such as infrared sensing [22] and optical sensing [23] offer remote measurement capabilities, enabling proximity detection without physical contact. These non-contact methodologies, which we explore in detail in Section 2.3, preserve natural system dynamics and enable comprehensive spatial characterization, addressing the fundamental constraints of their contact-based counterparts while opening new possibilities for vibration monitoring in challenging applications.

2.2 Accelerometer

Building upon the broader category of contact sensing examined in Section 2.1, we now focus specifically on accelerometer sensing as a specialized domain within vibrational monitoring. This approach, dedicated to precise measurement of acceleration forces through direct physical coupling, represents one of the most widely deployed sensing modalities across engineering disciplines. Understanding accelerometer technology provides critical context for our subsequent development of non-contact alternatives, as accelerometers exemplify both the capabilities and inherent limitations of contact-based approaches.

2.2.1 Fundamental Principles and Transduction Mechanisms

Accelerometer sensing fundamentally operates on the principle of measuring proper acceleration—the physical acceleration experienced relative to free-fall—through conversion of mechanical motion into quantifiable electrical signals. This measurement encompasses both static forces such as gravity and dynamic forces resulting from vibration, shock, or motion,

enabling comprehensive characterization of system dynamics across diverse operational conditions [15].

The operational foundation derives from Newton's second law of motion, where a seismic mass responds to acceleration forces, and this mechanical response is transduced into electrical signals through various physical phenomena. Contemporary accelerometer technology employs multiple transduction mechanisms, each offering distinct advantages for specific application requirements.

Piezoelectric accelerometers represent one of the most established technologies, utilizing crystalline materials that generate electrical charges when subjected to mechanical stress. This charge generation occurs due to the piezoelectric effect, where crystalline lattice deformation creates dipole moments proportional to the applied force. The generated charge Q relates directly to the applied acceleration a through the relationship $Q = d \cdot m \cdot a$, where d denotes the piezoelectric coefficient and m represents the seismic mass. This transduction mechanism provides exceptional high-frequency response characteristics, making piezoelectric accelerometers particularly suitable for impact testing and high-frequency vibration monitoring applications [15].

Capacitive accelerometers operate on the principle of variable capacitance, where acceleration-induced displacement of a proof mass alters the capacitance between fixed and moving electrodes. This capacitance variation is typically measured through bridge circuits or frequency modulation techniques, providing high sensitivity to low-level accelerations. MEMS-based capacitive accelerometers have revolutionized inertial sensing by enabling miniaturized, low-power implementations with integrated signal conditioning electronics, though their performance may be constrained by limited dynamic range and sensitivity to electromagnetic interference.

Piezoresistive accelerometers employ semiconductor strain gauges bonded to a flexure element that deforms under acceleration loading. The piezoresistive effect in doped silicon produces significant resistance changes under mechanical strain, with gauge factors substantially higher than those of metallic strain gauges. This enhanced sensitivity facilitates

precise measurement of low-level accelerations, though it introduces greater temperature sensitivity that necessitates sophisticated compensation techniques through Wheatstone bridge configurations or temperature calibration algorithms.

2.2.2 Performance Parameters and Practical Implementation

The selection of appropriate accelerometer technology requires careful consideration of multiple performance parameters that collectively define measurement capabilities and application suitability. Key specification parameters include sensitivity, frequency response, dynamic range, resolution, cross-axis sensitivity, and temperature stability, each influencing the accelerometer's effectiveness for specific monitoring scenarios.

Sensitivity quantifies the electrical output per unit acceleration, typically expressed in millivolts per g (mV/g) for voltage-output devices or picocoulombs per g (pC/g) for charge-output piezoelectric sensors. Optimal sensitivity selection involves balancing measurement resolution against dynamic range requirements, as higher sensitivity devices typically exhibit reduced maximum acceleration capabilities. Frequency response characteristics define the usable measurement bandwidth, bounded by lower and upper cutoff frequencies determined by the sensor's mechanical resonance and electrical filtering properties.

The resonant frequency of an accelerometer represents a critical performance parameter, establishing the upper frequency limit for reliable measurement. As a general principle, accelerometers maintain approximately flat frequency response up to one-fifth of their resonant frequency, with measurement accuracy degrading as the excitation frequency approaches resonance. Proper mounting techniques significantly influence the effective frequency response, with stud-mounted installations preserving the highest frequency capabilities while magnetic or adhesive mounts introduce additional compliance that reduces usable bandwidth [15].

Dynamic range specifies the ratio between the smallest detectable acceleration and the maximum sustainable acceleration without sensor damage or output saturation. Modern MEMS accelerometers typically achieve dynamic ranges from 2g to 400g, while specialized

piezoelectric instruments can measure accelerations exceeding 10,000g for shock and impact applications. Resolution defines the minimum detectable acceleration change, constrained by electrical noise floors and quantization limits in digital output devices.

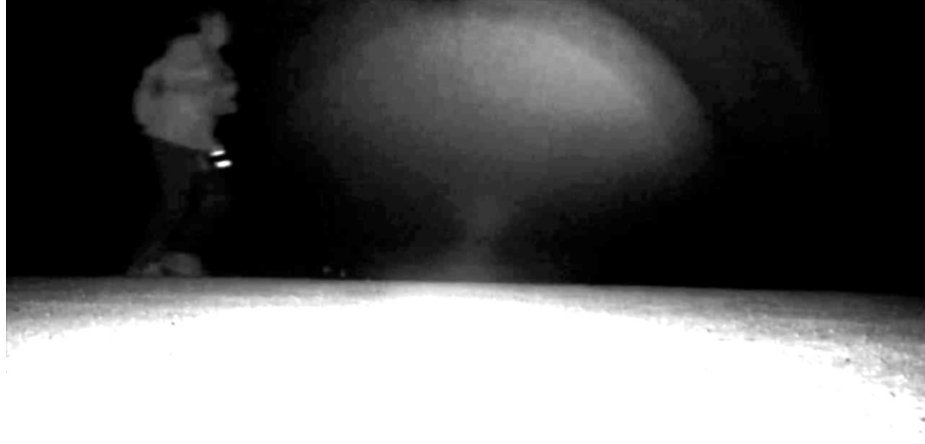


Figure 2.4: Experimental setup showing runner with mounted IMU sensor for biomechanical analysis.

To illustrate these performance characteristics in practical context, we examine biomechanical monitoring with IMU accelerometers. The experimental setup shown in Fig. 2.4 demonstrates proper implementation of contact-based accelerometer sensing, with an IMU sensor mounted at the mid-line of the runner's left thigh using standardized protocols that ensure consistent orientation and minimal soft tissue artifacts. The secure attachment minimizes relative motion between sensor and body segment, ensuring that captured accelerations genuinely reflect skeletal movement rather than skin motion artifacts.

The corresponding acceleration data presented in Fig. 2.5 illustrates multiple performance characteristics in practical context. The triaxial measurement captures the complex three-dimensional nature of running biomechanics, with distinct patterns visible across X, Y, and Z axes corresponding to different movement components. The amplitude variations demonstrate the dynamic range requirements for biomechanical monitoring, with peak values during foot strike approaching the sensor's upper measurement limits while subtle oscillations during flight phases challenge the resolution capabilities. The coordinated but

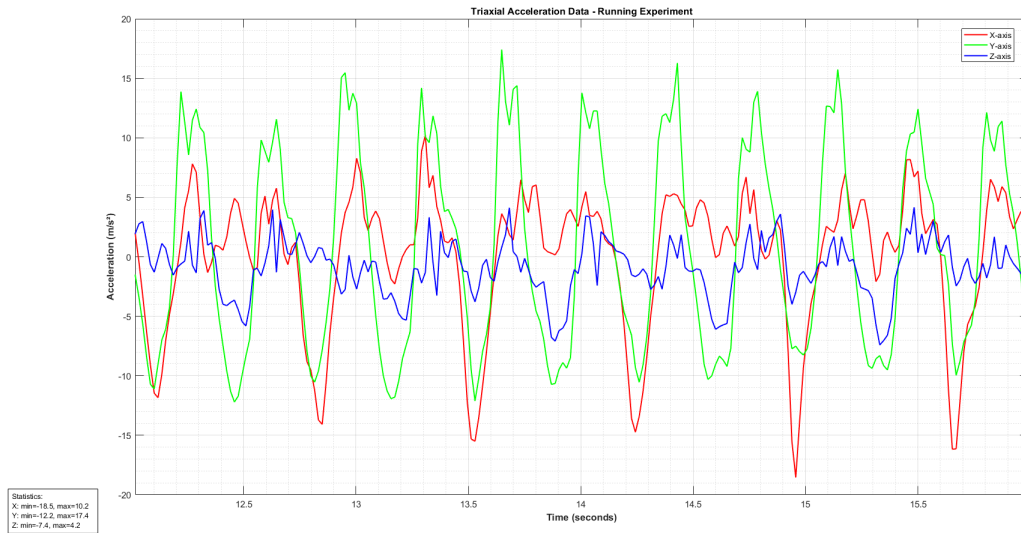


Figure 2.5: Triaxial acceleration data from running experiment, illustrating dynamic range and multi-axis measurement capabilities.

distinct behavior across axes highlights the importance of multi-axis measurement and minimal cross-axis sensitivity for comprehensive motion analysis.

2.2.3 Implementation Considerations

The practical implementation of accelerometer sensing necessitates careful attention to mounting methodologies, cable management, and environmental protection to ensure measurement accuracy and long-term reliability. Mounting technique selection represents a critical implementation decision, directly influencing the effective frequency response, measurement stability, and installation practicality across different application scenarios.

Stud mounting provides optimal measurement characteristics through direct mechanical coupling between the accelerometer and the test structure. This approach maximizes the effective mounting stiffness, preserving the sensor's inherent frequency response capabilities while ensuring minimal phase distortion across the measurement bandwidth. Adhesive mounting offers practical installation flexibility for temporary measurements or applications where drilling mounting holes is impractical. Cyanoacrylate adhesives provide reasonable high-frequency performance, while epoxy compounds offer enhanced temperature resis-

tance and long-term stability. The achievable frequency response with adhesive mounting typically reaches approximately 80% of the stud-mounted performance, with the specific reduction dependent on adhesive layer thickness, curing conditions, and surface preparation quality. Magnetic mounting facilitates rapid deployment on ferromagnetic surfaces, employing permanent magnets or electromagnetic bases to secure the accelerometer. While offering exceptional installation convenience, magnetic mounting introduces significant compliance that substantially reduces the usable frequency range, typically limiting effective measurement to frequencies below 1-2 kHz depending on the magnet strength and surface conditions. This approach finds particular utility in preliminary investigations or industrial environments requiring frequent sensor repositioning.



Figure 2.6: Contact based sensor: smartphones integrate accelerometers LSM6DSO

The proliferation of MEMS accelerometer technology in consumer electronics demonstrates the remarkable advancement in miniaturization, power efficiency, and cost reduction achieved through semiconductor fabrication techniques. Contemporary smartphones integrate triaxial MEMS accelerometers, such as the LSM6DSO sensor in Fig. 2.6, which combines a 3-axis accelerometer and 3-axis gyroscope in a $2.5\text{mm} \times 3\text{mm} \times 0.86\text{mm}$ package while consuming less than 0.65mA during normal operation.

These integrated sensors employ capacitive sensing principles, where acceleration-induced displacement of silicon proof masses alters the capacitance between fixed electrodes and moving structures fabricated from single-crystal silicon. The minute capacitance

changes, typically in the femtofarad range, are detected through sophisticated interface circuits that provide digital output with resolutions reaching 16 bits. This technological integration enables diverse functionalities including screen orientation detection, gesture recognition, activity monitoring, and fall detection, while maintaining minimal impact on device form factor and battery life [24].

The operational characteristics of smartphone accelerometers include dynamic ranges configurable from $\pm 2g$ to $\pm 16g$, output data rates programmable from 1.6Hz to 6.66kHz, and embedded features such as finite state machines for gesture recognition and programmable interrupts for wake-up events [24]. These capabilities illustrate how MEMS technology has transformed accelerometer sensing from specialized laboratory instrumentation to ubiquitous measurement tools embedded in everyday devices, though their performance remains constrained compared to dedicated measurement-grade instruments for precision engineering applications.

2.2.4 Limitations and Measurement Challenges

Despite their widespread adoption and technological maturity, accelerometer sensing exhibits inherent limitations that must be carefully considered during measurement system design and data interpretation. These limitations not only constrain application scenarios but also provide motivation for developing alternative non-contact approaches.

Mass loading effects represent a fundamental constraint, where the attached accelerometer mass alters the dynamic characteristics of lightweight structures or delicate components. The resulting frequency shift and mode shape modification can significantly impact measurement validity, particularly in experimental modal analysis or precision vibration testing applications. This limitation becomes especially pronounced when monitoring biological systems or lightweight mechanical structures where even small additional mass can substantially alter natural dynamics.

Temperature sensitivity presents another significant challenge, as all accelerometer technologies exhibit some degree of temperature-dependent output variation. Piezoelectric

accelerometers experience charge sensitivity changes with temperature fluctuations, while piezoresistive and capacitive devices demonstrate both sensitivity shifts and zero offset variations [15]. Sophisticated compensation techniques, including temperature calibration, differential configurations, and active temperature control, mitigate these effects in precision measurement applications but introduce additional complexity and cost.

Cross-axis sensitivity defines the accelerometer's response to accelerations perpendicular to the primary measurement axis, typically specified as a percentage of the main axis sensitivity. This parasitic response arises from manufacturing tolerances, mounting misalignment, and fundamental sensor design limitations, introducing measurement errors during multi-directional vibration conditions. Careful sensor alignment, selection of low cross-sensitivity models, and compensation techniques minimize these errors in critical applications but cannot eliminate them entirely.

The practical limitations extend beyond technical specifications to implementation challenges. In biomechanical applications, as shown in our running analysis example, the physical attachment of sensors inevitably alters natural movement patterns through added mass and restricted range of motion. While standardized mounting protocols minimize these effects, they cannot eliminate the fundamental alteration of system dynamics that occurs when adding any physical sensor to a biological system.

Furthermore, the point-specific nature of accelerometer measurements limits comprehensive characterization of complex biomechanical or structural systems. While individual sensors provide high-fidelity temporal data at specific locations, they cannot capture the full-field response characteristics essential for understanding complex movement patterns, smoothness mechanisms, or distributed structural behavior. This spatial limitation becomes particularly significant when analyzing multi-degree-of-freedom systems or when seeking to understand global vibration patterns across extended structures.

These inherent limitations of accelerometer sensing—particularly mass loading effects, temperature sensitivity, spatial constraints, and the fundamental alteration of system dynamics through physical attachment—provide compelling motivation for developing non-contact

alternatives that preserve natural system characteristics while enabling comprehensive spatial characterization. These limitations inform our development of NIR-based vibration monitoring approaches that eliminate physical contact while maintaining measurement precision across diverse application scenarios.

2.3 Non-Contact Method

Building upon the limitations identified in contact-based sensing methodologies, we now examine non-contact sensing as a paradigm shift in vibrational monitoring. This approach fundamentally transforms measurement strategies by eliminating the need for physical sensor attachment while enabling comprehensive spatial characterization of dynamic systems through remote measurement techniques. Our exploration of non-contact sensing methodologies reveals a diverse technological landscape that addresses the critical constraints of traditional contact-based approaches while opening new possibilities for vibration analysis in challenging environments.

2.3.1 Conceptual Framework and Technological Diversity

Non-contact sensing represents a fundamental departure from traditional measurement paradigms by employing electromagnetic radiation, acoustic waves, or optical techniques to remotely quantify vibrational parameters without physical contact [25]. This approach preserves natural system dynamics by eliminating mass loading effects and enables full-field spatial resolution across entire structures or subjects, providing comprehensive characterization of complex vibrational patterns that were previously inaccessible through point-based contact measurements.

The landscape of non-contact sensing encompasses a diverse array of technologies, each offering unique advantages for specific application scenarios. Recent advances in non-contact approaches include ultrasonic sensing for proximity detection and obstacle avoidance [26], infrared sensing for temperature and motion detection [22], optical sensing

for precise displacement measurement [23], and biosensing for health monitoring [27]. Particularly noteworthy are the developments in flexible and stretchable sensors that mimic the human somatosensory system, capable of distinguishing between light and rough touches to simulate neural tactile sensation. These sensors exhibit enhanced flexibility and stretchability and can identify various mechanical stimuli through synchronized sensor-actuator simulation [28] [29] [30] [31].

In the context of vibrational monitoring, non-contact techniques eliminate the need for physical sensor attachment while enabling comprehensive spatial characterization of dynamic systems through remote measurement. Among these diverse technologies, our proposed NIR video sensing method occupies a strategic position within the optical sensing category while distinguishing itself through the use of active near-infrared illumination. Unlike point-based ultrasonic or infrared sensors, our approach provides full-field spatial resolution, capturing vibration patterns across entire structures. Furthermore, compared to tactile-inspired sensors designed for surface property detection, our method is optimized for non-invasive vibration analysis without requiring surface contact or specialized sensor deployment. This positions NIR video sensing as a versatile solution for vibration monitoring in scenarios where traditional contact sensors introduce significant measurement artifacts or where full-field dynamic characterization is required.

2.3.2 Practical Implementation

To demonstrate the practical implementation of non-contact sensing within our research framework, we examine NIR video-based analysis of runner biomechanics, as illustrated in Fig. 2.7. The experimental configuration employs a Campark AC09 NIR camera operating at 30 fps with active NIR illumination at 850 nm wavelength. This setup enables us to extract motion vector fields through optical flow analysis between consecutive frames, facilitating displacement quantification and subsequent smoothness assessment using the unit-circle criterion through Jacobian computation and eigenvalue extraction.

The proposed NIR methodology exhibits both correlations and distinctions with Kowal-

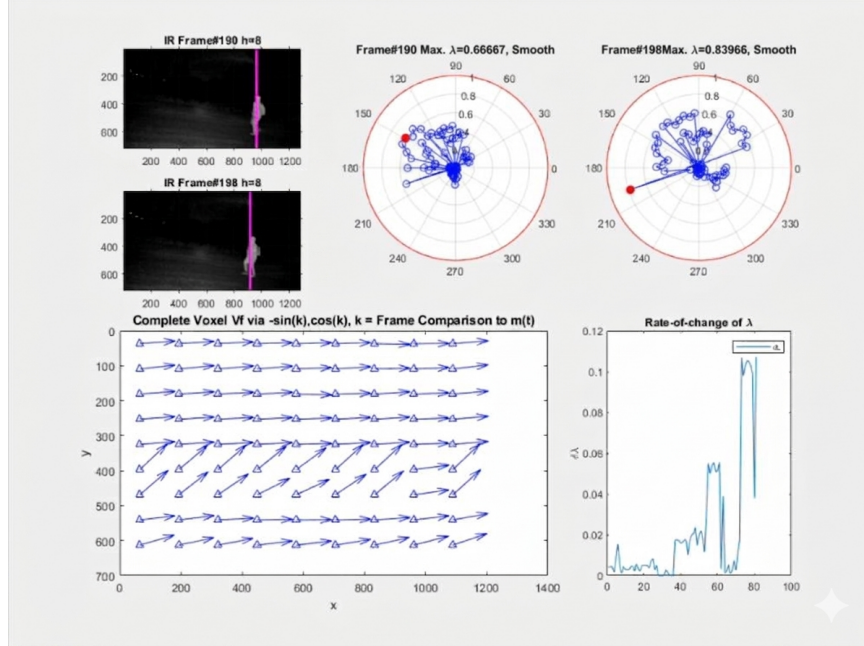


Figure 2.7: NIR-based runner vibration analysis demonstrating (a)top left: sequential frames, (b)bottom left: motion vector extraction from sequential frames, (c)bottom right: displacement waveform characterization, and (d)top right: smoothness assessment through complex-plane eigenvalue analysis.

ski's laser speckle approach [25]. While both techniques utilize cameras for non-contact vibration measurement, they diverge in fundamental operational principles. Kowalski's methodology employs laser speckle patterns with defocused imaging, requiring artificial pattern generation and specialized optical arrangements. In contrast, our NIR approach utilizes natural texture and motion features under active illumination, eliminating the need for surface preparation while maintaining robustness in real-world biomechanical applications.

Fig. 2.7 demonstrates the runner vibration displacement analysis with our proposed NIR framework. The first two plots show motion detected by the NIR sensor, with the red line indicating the motion representative region. The bottom left plot provides the vector field constructed by optical flow computation, which we use for computing eigenvalues. The frame-to-frame rate of change in λ represents the vibration displacement waveform characterization. The two complex planes on the right plot eigenvalues and the unit circle to visualize spatial characterization, with red dots indicating the maximum eigenvalue. Technically, the vector field smoothness analysis implemented in our framework through

complex-plane eigenvalue computation provides full-field spatial characterization, contrasting with Kowalski's phase correlation and digital image correlation algorithms optimized for point-based displacement extraction. This distinction proves particularly significant in biomechanical applications where complex, unconstrained motion patterns necessitate comprehensive spatial analysis rather than isolated point measurements.

Despite methodological differences, both approaches share the fundamental advantages of non-contact sensing by eliminating mass loading effects and preserving natural system dynamics. This example establishes the NIR methodology's suitability for applications where traditional contact sensors would significantly alter the dynamic response under investigation.

2.3.3 Theoretical Foundations and Measurement Principles

The theoretical foundation of non-contact sensing rests upon the principles of wave propagation, scattering theory, and electromagnetic interaction with materials. Unlike contact sensing that relies on direct mechanical coupling, non-contact methods measure vibration displacement through analysis of reflected or emitted energy from the target system. This fundamental difference in measuring displacement introduces distinct advantages in spatial resolution and system preservation, while eliminating physical contact with the vibratory object of interest.

Optical non-contact techniques operate on the principle of light interaction with surface features, where vibrations induce minute displacements that modulate the phase, frequency, or intensity of reflected light. Laser Doppler Vibrometry exemplifies this approach, detecting vibration-induced Doppler shifts in reflected laser light with nanometer-scale resolution. The measured velocity v relates to the Doppler frequency shift Δf through the equation $v = \frac{\lambda \Delta f}{2}$, where λ represents the laser wavelength. This precise velocity measurement enables comprehensive characterization of vibrational modes without the mass loading effects inherent in contact approaches [32].

An example of velocity measurement principles is illustrated in Fig. 2.8, subplot (a)

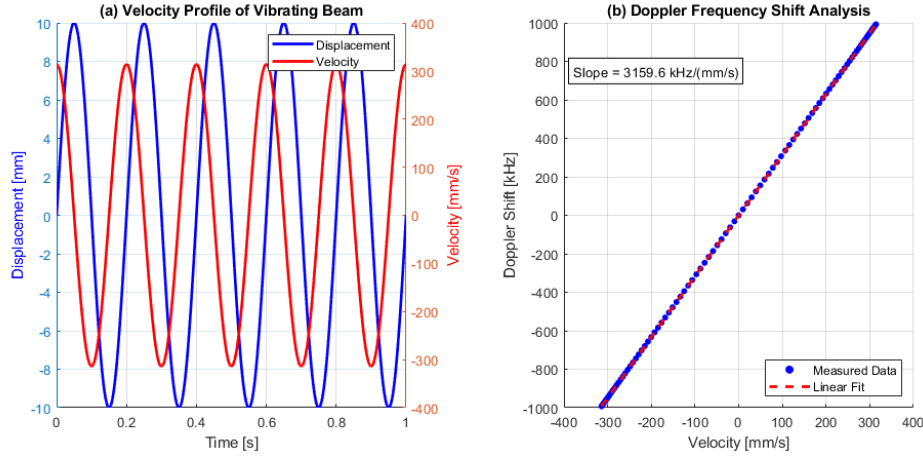


Figure 2.8: Velocity measurement principles in non-contact sensing. (a) Simulated velocity profile of a vibrating cantilever beam showing harmonic motion characteristics. (b) Doppler frequency shift analysis demonstrating the linear relationship between velocity and frequency shift.

displaying a harmonic vibration characterized by frequency $f = 5$ Hz and amplitude $A = 10$ mm. The 90-degree phase shift between displacement (blue) and velocity (red) curves demonstrates the derivative relationship in oscillatory motion. Subplot (b) validates the linear Doppler relationship from $v = \frac{\lambda \Delta f}{2}$, showing measured data points and the theoretical fit with slope $2/\lambda = 3.16$ kHz/(mm/s) for a 633 nm laser wavelength. This example reveals typical harmonic motion patterns observed in structural vibrations, with the cantilever beam exhibiting sinusoidal velocity variations corresponding to its fundamental vibration mode. The Doppler frequency shift analysis confirms the linear relationship between target velocity and measured frequency shift, validating the theoretical foundation of laser Doppler vibrometry.

Vision-based systems employ digital image correlation and optical flow algorithms to track displacement patterns across sequential video frames. These techniques analyze pixel intensity variations between frames to compute dense displacement fields, effectively transforming conventional cameras into spatially distributed vibration sensors. The foundation involves minimizing the intensity difference between corresponding regions in sequential frames, typically formulated as an optimization problem that balances measurement accuracy with computational efficiency. Recent advancements in computer vision have enabled

sub-pixel displacement resolution, making vision-based approaches increasingly competitive with traditional contact methods for many structural monitoring applications.

Radar-based systems utilize microwave radiation and interferometric principles to detect sub-millimeter displacements through phase variation analysis. Continuous-wave radar systems maintain constant transmission frequency and detect vibration-induced phase modulations in the reflected signal, while frequency-modulated systems employ swept-frequency transmissions to resolve absolute distance measurements. The relatively long wavelengths of microwave radiation enable penetration through certain materials and operation in adverse weather conditions, though with reduced spatial resolution compared to optical techniques.

2.3.4 Technology Classifications and Operational Characteristics

Non-contact sensing technologies encompass diverse operational principles and measurement approaches, each optimized for specific application requirements and environmental conditions. A comprehensive understanding of these technology classifications provides the foundation for appropriate methodology selection based on measurement objectives, operational constraints, and performance requirements.

Computer vision systems represent one of the most accessible and rapidly advancing non-contact sensing categories, utilizing conventional or specialized cameras to capture sequential images for motion extraction. Digital Image Correlation (DIC) techniques employ speckle patterns applied to the target surface, tracking the displacement of this stochastic pattern to compute full-field displacement and strain maps with micron-scale resolution. Optical flow algorithms offer an alternative approach that analyzes intensity gradients between frames to estimate motion vectors without requiring artificial surface preparation. The proliferation of high-resolution cameras and computational resources has positioned computer vision as a versatile solution for structural health monitoring, biomechanical analysis, and industrial vibration characterization [32].

Laser-based measurement systems provide exceptional precision for point-based or scanning vibration measurements. Laser Doppler Vibrometry (LDV) offers non-contact

velocity measurement with bandwidths exceeding 1 MHz and displacement resolution in the picometer range, making it particularly valuable for high-frequency applications and micro-scale vibrations. Scanning Laser Doppler Vibrometry (SLDV) in Fig. 2.9 extends this capability by incorporating galvanometric mirrors to sequentially measure multiple points, enabling modal analysis without sensor repositioning. Electronic Speckle Pattern Interferometry (ESPI) provides full-field displacement measurement through interference pattern analysis, offering nanometer resolution for detailed vibration mode visualization.



Figure 2.9: Sample 3D-Scanning Vibrometry (SLDV)

Within the spectrum of optical non-contact sensing technologies, laser speckle systems occupy a unique position between full-field vision methods and point-based laser vibrometry. The research by [33] exemplifies this intermediate approach, employing a 650 nm wavelength laser and high-speed camera configuration to capture defocused speckle patterns. A significant advancement presented in their work involves multi-point measurement capability through laser beam splitting using a transmission grating beamsplitter (110 grooves/mm). This optical arrangement enables simultaneous vibration measurement at multiple points

while maintaining the non-contact advantage. Experimental validation demonstrated displacement measurement consistency within 2 pixels (0.052 mm) across three measurement points, confirming the feasibility of comprehensive structural characterization using a single camera system.

The comparative analysis of image processing algorithms further refines the understanding of operational characteristics. While both phase correlation and digital image correlation (DIC) methods successfully extract displacement information, their computational demands differ substantially. Phase correlation requires approximately 1 millisecond per frame using FFT-based processing, whereas DIC consumes nearly 1000 milliseconds for equivalent calculations. This hundred-fold difference in processing time establishes phase correlation as the preferred method for real-time vibration monitoring applications, despite DIC's marginally superior decorrelation threshold (23.1 pixels versus 21.4 pixels for phase correlation).

Among the diverse non-contact sensing modalities, capacitive sensing represents a particularly versatile approach that leverages electric field variations for proximity detection and environmental perception. This technology operates on the principle of parallel plate capacitance, where object proximity alters the fringe electric field, enabling contact-free distance measurement without physical interaction. Recent advancements in capacitive sensing have demonstrated its applicability in challenging operational environments. As exemplified by [20], capacitive sensors constructed from 0.2-mm diameter silver wires achieve remarkable sensitivity in confined space applications. Their research developed a biomimetic soft crawling robot integrating capacitive non-contact sensing, where the sensor detected object proximity through measurable capacitance changes ($\Delta C = C - C_0$), with C_0 representing the baseline capacitance. The sensor's operational mechanism encompasses four distinct phases: (1) no external signal, (2) object entry into detection range, (3) object stabilization at fixed position, and (4) object departure from detection range. This phased response provides comprehensive spatial awareness while maintaining minimal form factor and power requirements.

Radar and microwave interferometry systems employ electromagnetic waves in the radio frequency spectrum to measure vibration through phase modulation analysis. These systems offer distinct advantages for long-distance measurements, penetration through non-metallic obstructions, and operation in adverse environmental conditions where optical systems may be compromised. Ground-Based Radar Interferometry has demonstrated particular utility in civil infrastructure monitoring, enabling precise deflection measurement of bridges and tall structures from distances exceeding one kilometer. The technology's immunity to lighting conditions and ability to measure through light rain, fog, or dust provides operational robustness in outdoor environments.

2.3.5 Comparative Performance Analysis and Implementation Validation

The selection of appropriate non-contact sensing technology requires careful consideration of performance characteristics across multiple dimensions, including spatial resolution, measurement range, frequency response, environmental robustness, and implementation practicality. Our analysis reveals distinct operational envelopes for different technologies that guide selection based on specific application requirements.

Spatial resolution represents a fundamental differentiator among non-contact sensing technologies, ranging from point measurements in single-beam laser systems to full-field characterization in vision-based approaches. Laser Doppler Vibrometry provides exceptional point measurement precision but requires sequential scanning for spatial characterization, while stereo-DIC systems capture full-field displacement with spatial resolution determined by camera sensor density and optical magnification. Radar systems typically offer coarser spatial resolution due to longer wavelengths, though synthetic aperture techniques can enhance resolution through signal processing algorithms.

Temporal resolution and frequency response vary significantly across non-contact sensing modalities, influenced by acquisition rates, processing capabilities, and fundamental

measurement principles. High-speed cameras employed in vision-based systems achieve frame rates exceeding 10,000 frames per second, enabling characterization of high-frequency vibrations, though with practical limitations imposed by data storage and processing requirements. Laser Doppler Vibrometry systems typically offer bandwidths from DC to several megahertz, providing comprehensive frequency coverage for most engineering applications. Radar systems exhibit more constrained frequency response, typically limited to vibrations below 1 kHz due to phase unwrapping challenges at higher frequencies.

Measurement range and sensitivity requirements further differentiate non-contact sensing technologies, with each approach optimized for specific amplitude regimes. Laser interferometric methods provide exceptional sensitivity for micro- and nano-scale vibrations, while vision-based systems typically resolve displacements down to approximately 1/100 of a pixel, corresponding to micron-scale resolution for properly configured systems. Radar systems offer intermediate sensitivity with millimeter to sub-millimeter resolution, though with superior performance for long-distance measurements where optical techniques may be limited by atmospheric effects.

To illustrate the practical capabilities and implementation considerations of non-contact sensing methodologies, we examine the application of computer vision for bridge structural health monitoring. In comprehensive studies by [1], vision-based systems were employed to measure dynamic response of cantilever columns and bridge structures, achieving strong correlation with traditional accelerometer measurements while providing full-field spatial data unavailable through contact approaches.

As illustrated in Fig. 2.10, the experimental methodology involved high-speed cameras capturing video sequences of dynamically excited structures, with subsequent processing through computer vision algorithms for displacement extraction. For cantilever column analysis, the vision-based approach successfully identified natural frequencies and damping ratios with accuracy comparable to reference accelerometers, while additionally providing mode shape visualization through full-field displacement mapping. The non-contact nature enabled monitoring without structural modification or mass loading effects, particularly

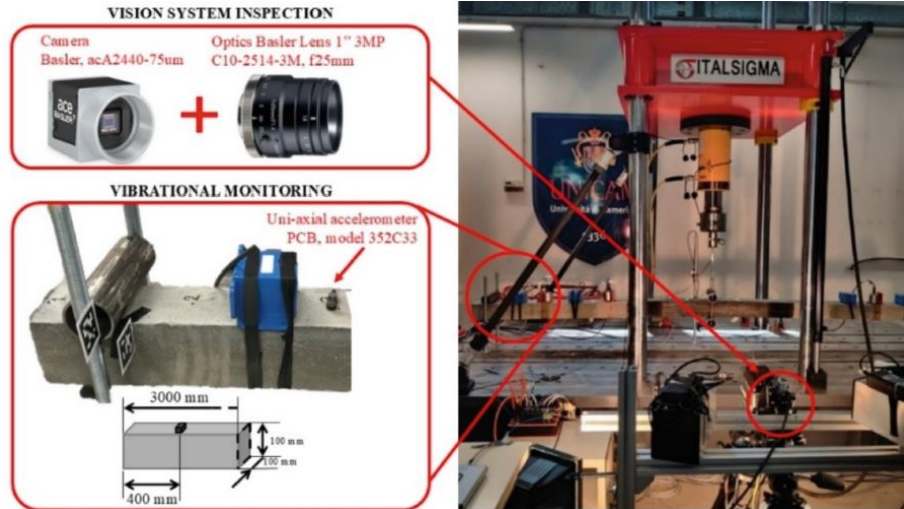


Figure 2.10: Experimental test setup in [1]: the vision system was placed in front of the beam and an accelerometer was fixed on the left part of the upper face of the specimen.

valuable for lightweight or delicate structures where sensor attachment would alter dynamic characteristics. The research further demonstrated the capability to track multiple points simultaneously across large structural areas, enabling comprehensive modal analysis without the extensive sensor networks required by traditional approaches. This spatial comprehensiveness proved particularly valuable for identifying localized damage or irregular vibration patterns that might be missed by point-based contact sensors. The correlation between vision-derived displacement measurements and traditional accelerometer data exceeded 0.95 in controlled validation studies, establishing computer vision as a reliable alternative for structural dynamic characterization.

The performance boundaries of non-contact sensing technologies are clearly illustrated through systematic accuracy assessments. According to [33], laser speckle systems exhibit a fundamental resolution limit approximately equivalent to one pixel displacement, corresponding to 0.025 mm in their experimental configuration. Below this threshold, measurement errors increase significantly, establishing a practical lower bound for reliable displacement quantification. The frequency response capabilities further differentiate sensing modalities. Their experimental results demonstrate consistent accuracy across a broad frequency spectrum from 131 Hz to 797 Hz, with maximum frequency detection

errors of only 0.0826%. This performance surpasses many vision-based systems while approaching the accuracy of specialized laser Doppler vibrometry, though at substantially reduced implementation cost.

Environmental robustness represents another critical performance metric. The laser speckle approach maintains measurement integrity independent of ambient lighting conditions, a significant advantage over conventional RGB-based vision systems. However, the method requires application of reflective tape to enhance laser reflectivity, introducing a minor preparation requirement that may limit certain applications. The demonstrated multi-point capability addresses spatial resolution limitations of single-point laser systems while avoiding the complex synchronization challenges of multi-camera arrangements.

The comprehensive comparative study by [25] provides invaluable empirical evidence regarding the performance boundaries of high-speed camera systems in vibration measurement. Their systematic comparison of four measurement modalities—piezoelectric accelerometer, laser Doppler vibrometer, data acquisition system, and high-speed camera—reveals distinct operational envelopes for each technology.

Frequency-dependent accuracy emerges as a critical differentiator. While all systems demonstrated excellent consistency in dominant frequency identification across the 16-920 Hz range, significant amplitude discrepancies were observed at higher frequencies. Specifically, at 640 Hz, the high-speed camera measured vibration acceleration amplitudes of 9.37 m/s^2 compared to reference values of 11.77-12.31 m/s^2 , representing approximately 20-24% underestimation. This systematic amplitude attenuation above 700 Hz establishes a practical upper frequency limit for quantitative displacement measurement using conventional high-speed imaging approaches. The research further highlights the impact of signal processing chain on measurement fidelity. The requirement for double differentiation of displacement signals to obtain acceleration measurements introduced substantial background noise levels, particularly in low-frequency regions below 50 Hz. This algorithmic limitation underscores the importance of selecting appropriate processing methodologies based on target measurement parameters, with direct displacement measurement often proving more

reliable than derived acceleration values for camera-based systems.

2.3.6 Implementation Considerations and Emerging Trajectories

The successful implementation of non-contact sensing methodologies requires careful attention to multiple implementation factors that influence measurement accuracy, operational reliability, and practical feasibility across diverse application scenarios. Environmental factors present significant considerations, as many non-contact techniques exhibit sensitivity to ambient conditions that must be addressed through proper system design and compensation strategies.

Lighting conditions critically influence vision-based systems, with inconsistent illumination potentially introducing artifacts in displacement measurements. Active illumination approaches, such as structured light or controlled lighting environments, mitigate these effects but introduce additional system complexity. Atmospheric conditions affect both optical and radar-based systems, with turbulence, precipitation, and particulate matter potentially degrading measurement accuracy through signal attenuation or path length variations. Environmental compensation techniques, including reference measurements, multi-wavelength approaches, and statistical filtering, help mitigate these effects but require careful implementation and validation.

Calibration requirements represent another significant implementation consideration, as non-contact systems typically require precise geometric calibration to establish the relationship between sensor measurements and physical units. Camera-based systems necessitate intrinsic and extrinsic parameter determination through calibration targets, while laser and radar systems require accurate knowledge of beam characteristics and alignment. The calibration process introduces potential error sources that must be carefully controlled through standardized procedures and regular verification to maintain measurement traceability.

Computational requirements vary substantially across non-contact sensing technologies, with vision-based approaches typically demanding significant processing resources for image analysis and displacement extraction. Real-time implementation often requires optimized

algorithms, specialized hardware, or resolution compromises to achieve practical processing rates. The substantial data volumes generated by full-field measurement systems present additional challenges for storage, transmission, and analysis, particularly in long-term monitoring applications where continuous data acquisition may be necessary.

The field of non-contact sensing continues to evolve through technological advancements that address traditional limitations while expanding measurement capabilities across new application domains. Computational photography techniques represent a significant advancement frontier, employing coded exposure, synthetic aperture, and light field approaches to overcome traditional limitations in temporal resolution, depth of field, and motion artifacts. These techniques enable vibration measurement scenarios previously inaccessible through conventional imaging approaches, such as through-foliage monitoring or high-speed event capture with limited motion blur.

Multi-modal sensor fusion approaches combine complementary non-contact technologies to overcome individual limitations while leveraging respective strengths. Vision-radar systems, for instance, integrate the spatial comprehensiveness of optical methods with the range accuracy and environmental robustness of radar, enabling more reliable operation in variable conditions. Similarly, thermal-visible imaging fusion facilitates vibration monitoring through temperature variation analysis while maintaining visual context for result interpretation.

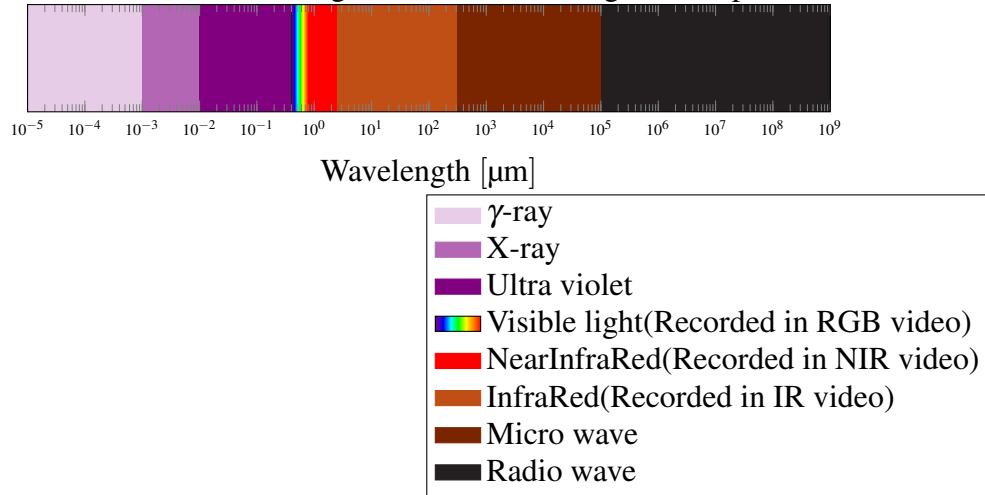
Machine learning integration represents another significant advancement trajectory, with deep learning algorithms enabling enhanced feature extraction, noise rejection, and automated defect identification from non-contact measurement data. Convolutional neural networks have demonstrated particular utility for vision-based vibration analysis, learning to extract subtle motion patterns from video sequences while ignoring irrelevant background variations. These data-driven approaches continue to advance measurement capabilities beyond traditional model-based methods, particularly for complex structures where analytical models may be incomplete or computationally prohibitive.

The ongoing miniaturization and cost reduction of sensing components further expands

non-contact monitoring applications, with MEMS-based lidar, compact radar modules, and high-resolution smartphone cameras enabling distributed sensing networks at previously impractical scales. Wireless connectivity and edge computing capabilities facilitate deployment across extensive infrastructure systems, while reduced power requirements enable long-term operation from battery or energy harvesting sources. These technological trends collectively drive the increasing adoption of non-contact methodologies across diverse vibration monitoring applications, establishing the foundation for our development of NIR video sensing approaches that address specific limitations while leveraging these broader technological advancements.

2.4 NIR Video

Following our comprehensive analysis of non-contact sensing technologies in Section 2.3, we now focus specifically on Near-Infrared (NIR) video sensing as a robust modality that addresses critical limitations in non-contact vibration monitoring. This approach represents a strategic technological positioning within the broader landscape of non-contact sensing, offering unique advantages that bridge the capabilities of conventional visible-light imaging and thermal infrared technologies while overcoming their respective limitations.



Operating within the 750-950 nanometer wavelength range, NIR sensing leverages unique optical properties that overcome fundamental limitations of both RGB and thermal

imaging approaches while maintaining the spatial resolution benefits of optical measurement techniques [34]. This strategic positioning within the electromagnetic spectrum enables NIR technology to address critical gaps in non-contact vibrational monitoring, particularly in applications requiring consistent performance across varying environmental conditions and operational scenarios [10].

We define NIR video sensing as a contact-free measurement methodology that utilizes cameras sensitive to near-infrared radiation combined with active illumination sources to extract motion vector fields from sequential video frames. This approach enables vibrational analysis without physical sensor attachment while maintaining operational reliability independent of ambient lighting conditions through controlled illumination in spectral regions invisible to human perception. This definition encapsulates the core innovation that distinguishes our approach from passive imaging methods.



Figure 2.11: Sequence of three sample frames from NIR video capture for vibration displacement analysis.

2.4.1 Spectral Advantages

The comparative analysis of imaging modalities reveals distinct spectral characteristics that fundamentally influence their applicability for vibrational monitoring across diverse operational environments. RGB imaging systems operate within the 400-700 nm visible spectrum, leveraging the human-visible wavelength range to provide intuitive color representation and high spatial resolution. However, this spectral positioning introduces significant limitations through inherent dependency on ambient illumination conditions, susceptibility to shadows and glare effects, and performance degradation in low-light scenarios. The photometric nature of RGB measurements further complicates vibration analysis, as luminance variations

are often conflated with genuine motion signals in optical flow computations, introducing artifacts that compromise measurement accuracy [10].



Figure 2.12: Sequence of three sample frames from RGB video capture for vibration displacement analysis.

Thermal infrared imaging occupies the 8,000-14,000 nm wavelength range, detecting emitted radiation rather than reflected light to provide temperature-based contrast independent of ambient lighting. This emission-based approach enables operation in complete darkness and through certain obscurants, but introduces substantial limitations for precise displacement tracking. Thermal systems exhibit significant measurement drift with temperature fluctuations, quantified at approximately ± 0.08 mm per degree Celsius in structural monitoring applications, complicating long-term vibration analysis in thermally dynamic environments. The relatively long wavelengths further constrain spatial resolution, with typical thermal cameras achieving 640×480 pixels compared to 1920×1080 pixels for comparable NIR systems, a fundamental limitation imposed by diffraction constraints and detector technology [35].

NIR sensing occupies the strategic spectral position between these extremes, leveraging wavelengths from 750-950 nm that provide distinct advantages for vibrational monitoring. This spectral region experiences reduced Rayleigh scattering compared to visible light, enabling superior penetration through atmospheric obscurants including fog, dust, and precipitation. The active illumination approach fundamental to NIR systems eliminates ambient light dependency, ensuring consistent measurement conditions regardless of daylight variations or artificial lighting interference. Material interaction characteristics further differentiate NIR imaging, with many structural materials exhibiting higher and more consistent reflectance in this band compared to visible wavelengths, enhancing contrast

near-infrared spectrum to ensure consistent measurement conditions independent of ambient lighting. Typical NIR illumination systems utilize light-emitting diodes with peak emissions at 850 or 940 nm, wavelengths selected for optimal balance between camera sensitivity, atmospheric transmission, and eye safety considerations. The narrow spectral bandwidth of these sources enhances signal-to-noise ratio by rejecting out-of-band environmental radiation, while their invisibility to human perception enables unobtrusive monitoring in sensitive environments without subject distraction or behavioral modification. This active approach maintains consistent scene illumination across day-night cycles and varying weather conditions, addressing a critical limitation of RGB systems while avoiding the temperature-dependent drift characteristics of thermal imaging.

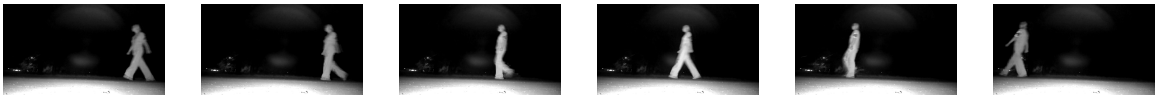


Figure 2.15: Representative NIR video frame sequence for vibration displacement analysis.

The complete NIR imaging procedure encompasses multiple processing stages that transform raw video sequences into precise displacement measurements. Initial frame acquisition captures scene reflectance under controlled NIR illumination, with exposure parameters optimized for vibration analysis rather than visual appeal. As illustrated in Fig. 2.15, raw NIR frames provide consistent illumination and high contrast essential for reliable motion extraction. Background modeling employing Gaussian mixture models or similar statistical approaches differentiates static scene elements from dynamic targets, enabling robust tracking despite minor camera motion or environmental changes. Optical flow computation through modified Horn-Schunck or Lucas-Kanade algorithms then extracts dense displacement fields from sequential frames, with subsequent filtering and validation rejecting outliers while preserving genuine vibration signals.

To demonstrate the practical application of this framework, we examine NIR frame sequence analysis of walker motion as presented in Fig. 2.16. This sequence captures a walking subject at progressive time instances, with a vertical line superimposed on the torso region to define the analysis area for motion tracking. This vertical line represents a single

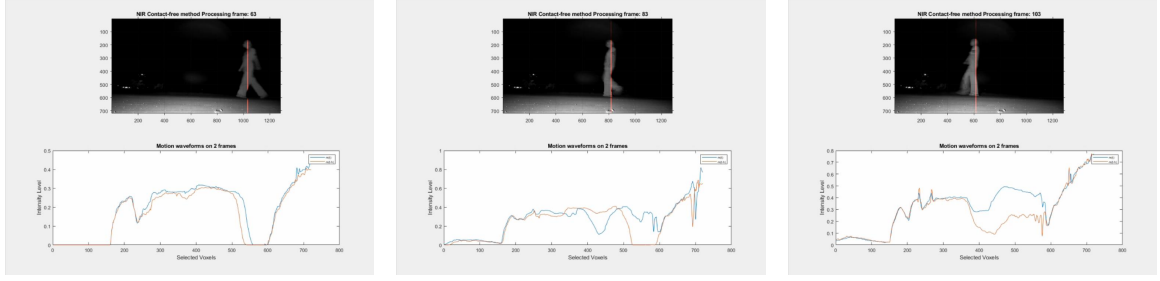


Figure 2.16: NIR frame sequence and corresponding pixel value analysis for walker motion tracking. Frames display NIR images with a changing vertical line indicating the analysis region on the walker's body. Plots show the intensity values along this vertical line, with frame-to-frame variations directly corresponding to subject motion.

column of pixels and changes its position relative to the subject's displacement in each frame that records walker motion. The consistent active illumination characteristic of NIR systems ensures uniform lighting across all frames, eliminating the shadow artifacts and contrast variations that would typically plague RGB-based motion analysis in similar scenarios.

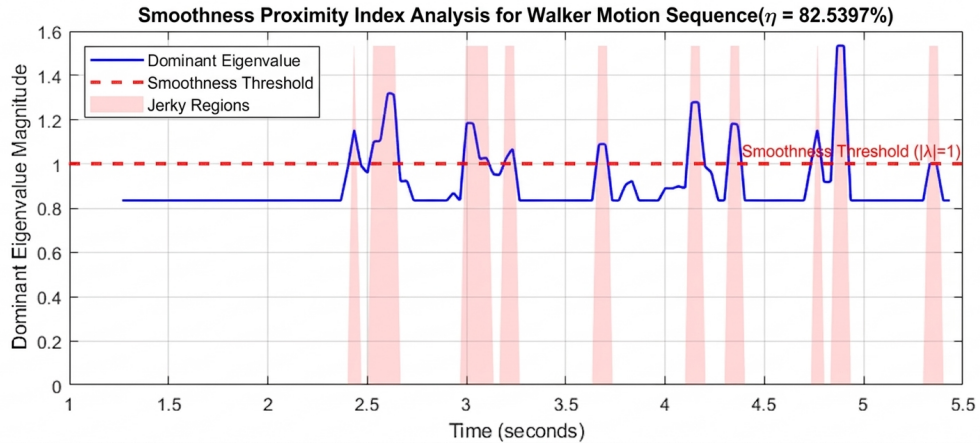


Figure 2.17: Smoothness analysis for walker motion sequence. The plot displays η values across the video timeline, with the dotted line indicating the smoothness threshold ($|\lambda_{\max}| = 1$). Frames highlighted in red indicate jerky motion events where eigenvalues exceed the unit circle boundary. The analysis provides a comprehensive overview of motion smoothness evolution throughout the walking sequence, with an overall smoothness index of 82.54% achieved during the recorded motion.

The corresponding pixel value plots in Fig. 2.16 quantitatively demonstrate the intensity variations along the vertical line across consecutive frames. These plots reveal distinct profile changes that directly correspond to the walker's motion between frames. The pixel intensity

values exhibit measurable shifts in both amplitude and spatial distribution, reflecting the subject's displacement and postural changes during the walking cycle. Specifically, the plot peaks correspond to high-reflectance regions of the walker's clothing or body parts, while valleys represent lower-reflectance areas or background elements. The frame-to-frame variations in these intensity profiles provide the fundamental data for motion computation in vision-based vibration analysis.

The temporal resolution of 33 milliseconds between frames (30 FPS) ensures adequate sampling of human walking dynamics, which typically involve fundamental frequencies below 5 Hz. This sampling interval captures subtle motion variations while maintaining compatibility with real-time processing requirements. The observed intensity changes between frames demonstrate the principle behind optical flow algorithms, where spatial and temporal gradients in pixel values are computed to determine motion vectors across the entire image sequence. To complement the vibration analysis, Fig. 2.17 presents the smoothness analysis for the complete walker motion sequence. The plot of the smoothness index η shows variation with values below 1 (smooth motion region) and above 1 (jerky motion region). For a detailed explanation of the smoothness index, see Section 3.6.2.

This comprehensive smoothness assessment employs the unit-circle criterion to evaluate motion smoothness across the entire video timeline. The η index quantifies the percentage of smooth frames according to the eigenvalue magnitude criterion, with values below the dotted threshold line indicating smooth motion patterns and values above representing jerky, irregular episodes. The highlighted regions in red correspond to frames where eigenvalues exceed the unit circle boundary, typically occurring during transitional phases of the gait cycle such as heel strike and toe-off events. This smoothness analysis achieves an overall smoothness index of 82.54% for the recorded walking sequence, demonstrating the predominance of regular walking patterns while effectively identifying brief irregular episodes associated with specific biomechanical events.

This frame sequence analysis exemplifies the initial stage of the NIR processing procedure, where raw pixel data undergoes transformation into quantitative motion information.

The subsequent processing stages, including optical flow computation and vector field derivation, build upon these fundamental intensity variations to generate comprehensive displacement maps for smoothness assessment. The consistency of NIR illumination across frames ensures that observed intensity changes genuinely reflect subject motion rather than environmental artifacts, establishing the technology’s superiority for contact-free vibration monitoring in variable operational conditions.

2.4.3 Performance Evaluation and Application Scenarios

A comprehensive performance comparison across imaging modalities reveals distinct operational envelopes that guide appropriate technology selection based on specific application requirements and environmental constraints. Our analysis, summarized in Table 2.2, evaluates multiple parameters including spectral range, measurement principles, environmental dependencies, and implementation characteristics to establish clear guidelines for technology selection in vibration monitoring applications.

Table 2.2: Comparative Analysis of Imaging Modalities for Vibration Monitoring

Parameter	RGB Imaging	Thermal Imaging	NIR Imaging
Spectral Range	400-700 nm	8,000-14,000 nm	750-950 nm
Measurement Principle	Passive reflectance	Passive emission	Active reflectance
Ambient Light Dependency	High	None	None
Temperature Sensitivity	Low	High	Low
Spatial Resolution	High (4K+)	Low (typically 640×480)	Medium (HD to Full HD)
Atmospheric Penetration	Poor	Moderate	Good
Implementation Cost	Low	High	Low

Spatial resolution characteristics vary significantly across technologies, with RGB systems typically achieving the highest pixel density through consumer-grade sensors optimized for visual applications. Modern RGB cameras readily provide 4K resolution

(3840×2160 pixels) at affordable price points, enabling detailed vibration analysis of small structural elements or distant targets. NIR systems offer intermediate spatial resolution, with specialized cameras typically providing 1280×720 to 1920×1080 pixel formats that balance detail capture with processing requirements for real-time analysis. Thermal imaging exhibits the most constrained spatial resolution due to fundamental diffraction limits at longer wavelengths, with 640×480 pixels representing common configurations in mid-price-range systems.

Temporal resolution and frequency response present another critical differentiator, particularly for capturing rapid vibrations or transient events. RGB and NIR systems leverage similar sensor technologies capable of high-frame-rate operation, with specialized models exceeding 1000 frames per second for detailed transient analysis. The computational requirements for processing these high-frame-rate sequences present practical constraints, but the fundamental capability exists within both visible and NIR domains. Thermal imaging systems typically offer more limited frame rates, with 30-60 frames per second representing common configurations due to detector readout limitations and heat management constraints [35]. This temporal resolution constraint may limit thermal imaging applicability for high-frequency vibration analysis despite its unique thermal contrast capabilities.

Environmental robustness represents perhaps the most significant differentiator among imaging modalities, with each technology exhibiting distinct vulnerabilities and strengths across operational conditions. RGB systems demonstrate pronounced sensitivity to lighting variations, with performance degradation under low-light conditions, shadows, specular reflections, and sudden illumination changes [3]. These photometric vulnerabilities fundamentally limit reliable outdoor deployment without extensive compensation algorithms. Thermal imaging provides immunity to visible light variations but introduces temperature-dependent measurement drift and performance degradation under uniform temperature conditions where thermal contrast diminishes [35]. NIR systems substantially mitigate both limitations through active illumination that ensures consistent operation independent of ambient light, while their reflectance-based measurement principle avoids the temperature

drift inherent in emission-based thermal approaches.

The strategic advantages of NIR video sensing manifest most prominently in specific application scenarios where environmental consistency, measurement reliability, and operational practicality converge. Structural health monitoring of civil infrastructure represents a prime application domain, where long-term vibration tracking necessitates consistent performance across day-night cycles, seasonal variations, and changing weather conditions. The active illumination approach of NIR systems maintains measurement integrity despite changing natural light, while the spectral positioning enables reliable operation through light rain, fog, or dust that would compromise visible-light systems. Comparative studies have demonstrated 34% lower measurement artifacts in NIR systems versus thermal counterparts under identical industrial conditions, establishing NIR as the most environmentally robust modality for extended outdoor monitoring [10].

Biomechanical analysis presents another compelling application scenario where NIR technologies provide unique advantages over alternative imaging modalities. The invisible illumination prevents subject distraction or behavioral modification during movement studies, while consistent active lighting ensures reliable motion tracking regardless of indoor lighting conditions. The reflectance-based measurement avoids the privacy concerns associated with detailed visible-light imaging while providing higher spatial resolution than thermal alternatives for precise joint tracking and gait analysis. Research applications in sports science, rehabilitation, and ergonomic assessment particularly benefit from these characteristics, enabling detailed movement analysis without influencing natural motion patterns through obtrusive sensors or visible monitoring equipment.

Industrial machinery monitoring leverages NIR capabilities for vibration analysis in challenging manufacturing environments characterized by variable lighting, airborne particulates, and complex backgrounds. The active illumination penetrates through certain obscurants common in industrial settings, while the reflectance-based approach provides consistent performance on diverse material surfaces without requiring specialized coatings or surface preparation. The intermediate cost positioning between RGB and thermal technologies

facilitates scalable deployment across multiple monitoring points, while the digital output compatibility with existing automation systems simplifies integration into comprehensive predictive maintenance frameworks.

2.4.4 Technical Limitations and Compensating Strategies

Despite its significant advantages, NIR video sensing exhibits specific technical limitations that necessitate careful consideration during system design and implementation. The active illumination requirement introduces power consumption considerations, particularly for battery-operated or remote monitoring applications where continuous illumination may constrain operational duration. Strategic illumination management through pulsed operation, adaptive intensity control, or solar-assist systems mitigates these constraints while maintaining measurement integrity. The limited penetration depth of NIR radiation through certain materials may restrict applicability for subsurface defect detection or through-barrier monitoring, though this characteristic proves advantageous for surface-specific vibration tracking.

Spectral interference represents another consideration, as strong ambient NIR sources including direct sunlight or specialized industrial equipment may overwhelm active illumination despite spectral filtering. Temporal modulation techniques employing synchronized illumination and detection provide effective rejection of constant or slowly varying ambient NIR components, while computational approaches including background subtraction further enhance robustness against intermittent interference sources. The intermediate spatial resolution of NIR systems compared to high-end RGB cameras may limit applicability for extremely detailed vibration mode analysis, though this constraint continues to diminish with advancing sensor technology providing increasingly higher resolution within the NIR spectrum.

Our development of the NIR sensing framework strategically addresses these limitations through multiple compensating strategies. The use of active illumination at specific wavelengths minimizes interference from ambient sources, while advanced signal process-

ing algorithms extract meaningful motion information even in challenging environmental conditions. The selection of the 850 nm wavelength represents an optimal compromise between sensor sensitivity and atmospheric transmission. Integration with computational smoothness assessment frameworks, as demonstrated through the characteristic proximity analysis, transforms raw displacement measurements into actionable smoothness metrics that provide meaningful insights for engineering and biomechanical applications. These strategic compensations, combined with the inherent advantages of the NIR spectral band, establish NIR video sensing as a robust and versatile modality for contact-free vibration monitoring.

2.5 Chapter Summary

This chapter has systematically examined the technological evolution from contact-based to contact-free sensing methodologies in vibrational monitoring, establishing a comprehensive comparative framework for methodology evaluation. Through in-depth analysis of the principles, performance characteristics, and application scenarios of various sensing technologies, the relative advantages and applicability boundaries of different methods in engineering practice have been revealed, laying a solid technical foundation for contact-free smoothness assessment.

Contact-based sensing methods, serving as the cornerstone of traditional vibration monitoring, provide reliable pathways for direct physical measurement, with their technical maturity and measurement accuracy having been thoroughly validated in numerous engineering applications. Contact sensors such as accelerometers and strain gauges achieve precise transduction of physical parameters through mechanical coupling, playing irreplaceable roles in fields including structural health monitoring and mechanical system diagnostics. However, inherent constraints including mass loading effects, spatial resolution limitations, deployment complexity, and environmental vulnerabilities have motivated the research community to seek more advanced contact-free alternatives. These limitations are particu-

larly pronounced in application scenarios involving lightweight structures, biomechanical systems, and harsh environment monitoring, driving a fundamental paradigm shift in sensing technologies.

The emergence of contact-free sensing methodologies represents a significant technological turning point in the vibration monitoring domain, preserving natural system dynamics by eliminating physical contact requirements. Technological systems represented by computer vision, laser vibrometry, and radar interferometry provide spatial characterization capabilities from point-based to full-field measurements, enabling comprehensive analysis of global vibration patterns in complex structures. Various contact-free technologies demonstrate complementary performance characteristics, with vision systems offering high spatial resolution, laser technologies ensuring nanometer-scale precision, and radar methods exhibiting superior performance in long-distance and adverse weather conditions. This technological diversity provides flexible solution options for different application scenarios while simultaneously highlighting the importance of technical trade-offs based on specific requirements.

The empirical evidence from [25] significantly reinforces the technological trade-offs between contact and contact-free sensing methodologies. Their rigorous comparative approach validates the exceptional performance of high-speed camera systems for low-frequency vibration analysis (below 70 Hz), where measurement results showed remarkable consistency with traditional methods (coefficient of variation $\leq 0.8\%$). However, the identified limitations—particularly the amplitude attenuation at frequencies above 700 Hz and the noise introduction through double differentiation processes—provide crucial guidance for application-specific technology selection. These findings substantiate the complementary relationship between different contact-free modalities, suggesting that while high-speed cameras excel in low-frequency, full-field applications, laser-based systems may prove superior for high-frequency, point-based measurements.

Among various contact-free technologies, NIR video sensing demonstrates significant application value through its unique technological positioning. By combining active il-

illumination principles with the 750-950 nm spectral range, NIR technology successfully overcomes the environmental lighting dependency of RGB systems and the temperature drift issues of thermal imaging systems, achieving an optimal balance between spatial resolution, environmental robustness, and implementation cost. The spectral characteristics of this technology ensure reliable operation under atmospheric interference conditions such as fog and dust, while its invisible illumination properties provide unique advantages in biomechanical and behavioral studies. These characteristics position NIR technology as an ideal bridge connecting traditional optical measurements with advanced smoothness analysis frameworks.

The technological comparison framework established in this chapter provides the necessary conceptual foundation and methodological preparation for subsequent vector field smoothness analysis. The technological evolution from contact-based to contact-free approaches represents a fundamental enhancement in system characterization capabilities. The full-field spatial data provided by contact-free methods creates essential conditions for vector field-based smoothness analysis, while the environmental robustness of NIR technology ensures feasibility in practical application scenarios. This establishment of technical foundations enables subsequent chapters to focus on theoretical innovations and methodological validation in smoothness assessment without being constrained by traditional measurement limitations.

The continuous development of sensing technologies presents unprecedented opportunities for the vibration monitoring field while simultaneously introducing new challenges. The technical principles and comparative analysis elaborated in this chapter provide a conceptual framework for understanding these development trends while offering a decision-making basis for the rational selection of sensing methods in engineering practice. With ongoing advancements in sensing technologies and continuous improvements in computational capabilities, contact-free vibration monitoring will undoubtedly play increasingly important roles in engineering system safety assessment, mechanical equipment predictive maintenance, and biomechanical analysis.

Chapter 3

Quantifying Motion Smoothness from NIR Videos Using Optical Flow Vector Fields

3.1 Introduction

The assessment of oscillatory motion smoothness has traditionally depended on contact-based sensors, which are intrusive and limited in spatial coverage. To address these limitations, this chapter introduces a measurement framework that builds directly upon the physical and geometric foundations established in Chapter 1. This framework was formulated and successfully validated in a published study [11].

Recall from Chapter 1 that for a point on a moving limb or mechanical component, the physical velocity can be expressed as

$$\mathbf{v}(t) = (r\dot{\theta}(t) \cos \theta(t), -r\dot{\theta}(t) \sin \theta(t)),$$

and that under the orthographic projection assumption (camera distance much larger than object size, motion primarily parallel to the image plane), the optical flow vector $\vec{v}(x, y, t) =$

$(u(x, y, t), v(x, y, t))$ is proportional to this projected physical velocity:

$$(u, v) \approx \alpha \mathbf{v}(t).$$

The collection of these optical flow vectors across a region of interest forms a motion vector field $\vec{V}f$ that encodes the instantaneous spatial distribution of motion. This chapter develops the mathematical framework for analyzing such vector fields to assess motion smoothness, employing the unit-circle criterion adapted from discrete-time dynamical systems theory and using the divergence theorem as a guiding concept for relating local gradients to global area changes.

As established in Chapter 1, the divergence of the optical flow field, $\nabla \cdot \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, is directly related to the rate of change of projected area. The spatial Jacobian of the field, which contains the divergence as its trace, also yields eigenvalues that characterize the local rate of spatial variation in the velocity field. When these eigenvalues lie within the unit circle in the complex plane, the local velocity field varies gradually in space, which we interpret as smooth motion corresponding to well-damped physical behavior as described by the mass-spring-damper and Molati models. When eigenvalues exceed unit magnitude, the velocity field exhibits abrupt spatial changes, which we classify as jerky or irregular motion.

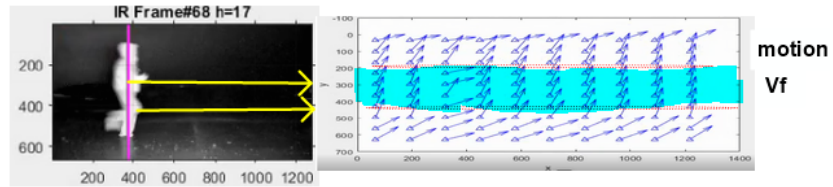


Figure 3.1: Motion vector field derived from video frame fr .

As illustrated in Fig. 3.1, each recorded motion instance is represented by a vector field derived from a video frame. In this chapter, we demonstrate the framework using a column passing through the center-of-gravity (COG) of a moving object. This column-based approach was chosen for two reasons: computational efficiency and clarity of demonstration in the proof-of-concept stage. However, it is important to emphasize that the method is

not limited to a single column. The same eigenvalue analysis can be applied to any region of interest—whether a single column, a bounding box, or the entire object silhouette. For full-field analysis, the Jacobian is computed at every pixel within the region of interest, and the smoothness classification is applied per pixel or aggregated over the entire region. The full-field version provides richer spatial information and is the intended end-use of the framework. The column-based results presented in this thesis serve as a validation of the core concept: the method works at the pixel level, so it will work in full-field mode.

The analysis proceeds through the following stages, which are elaborated in subsequent sections.

First, dynamic regions within each video frame fr that exhibit motion-induced variations are isolated using a Gaussian Mixture Model (GMM) to separate foreground motion from the static background. This process effectively delineates a region R corresponding to the moving object—the same region over which divergence integrals could be computed to relate local gradients to global area change. The motion vector field $\vec{V}f$ is constructed using optical flow between consecutive frames. Next, a specific column $col \in \text{changingPic} \in fr$ passing through the COG of the moving object is selected. This column provides a one-dimensional slice through the motion field that captures the essential oscillatory dynamics while reducing computational complexity. Under the orthographic projection assumption, this vector field approximates the projected physical velocity of the moving object.

Each vector in the field is then transformed into its complex exponential form $\vec{v} = re^{i\theta}$, where r denotes magnitude and θ represents direction. This representation, relative to the unit circle (Fig. 3.3), facilitates the eigenvalue-based smoothness analysis. For each vector field, the spatial Jacobian matrix is computed as

$$J(\vec{V}f) = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}.$$

The eigenvalues of this matrix, denoted λ , quantify the local rate of spatial change in the motion field. The trace of the Jacobian, $\text{tr}(J) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, is the divergence—the quantity

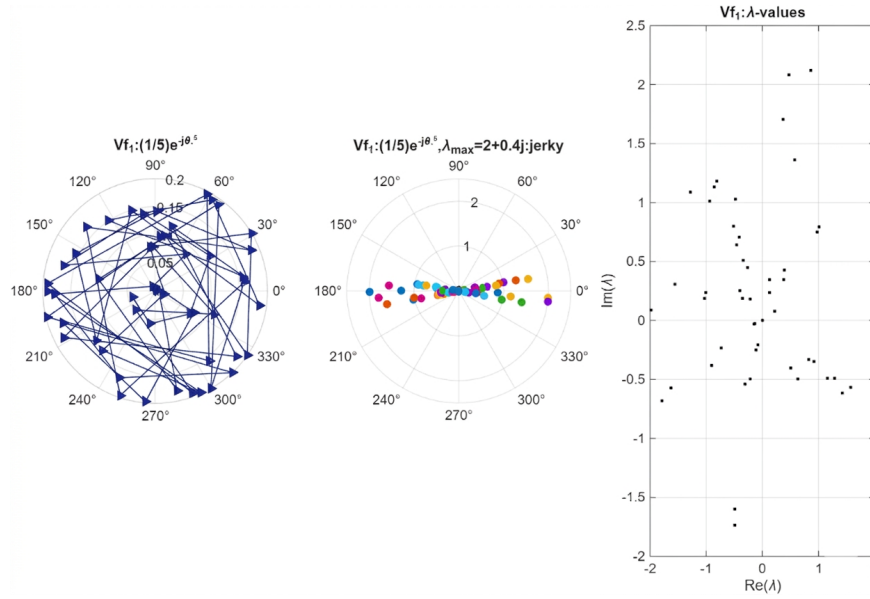


Figure 3.2: Example of a jerky motion vector field with eigenvalues exceeding unit magnitude, indicating low motion smoothness.

that, when integrated over region R , gives the rate of change of projected area as discussed in Chapter 1.

The smoothness of the motion field is then evaluated using an operational criterion adapted from the Krantz condition for discrete-time systems: a frame is classified as exhibiting smooth motion if the maximum eigenvalue magnitude satisfies $0 \leq |\lambda_{\max}| \leq 1$. This corresponds to requiring the eigenvalue to lie within the unit circle in the complex plane. An example of a jerky (irregular) vector field is shown in Fig. 3.2, with corresponding eigenvalues listed in Table 3.1.

Table 3.1: Sample eigenvalues from an irregular (jerky) motion vector field

λ_1	λ_2	λ_3	λ_4	λ_5
$0.8632 + 2.1223j$	$0.4738 + 2.0839j$	$2.0011 + 0.0377j$	$1.9995 + 0.0264j$	$-0.4968 - 1.7350j$

To illustrate the eigenvalue extraction process, consider a sample vector field $\vec{V}f = \left\{ \left(\frac{1}{5} \right) e^{j\theta^5} \right\}$, shown in Fig. 3.2. The vector field is expressed in polar form, and the Jacobian matrix is constructed from partial derivatives. Eigenvalues are obtained from the characteristic equation $|J - \lambda I| = 0$. A maximum eigenvalue magnitude exceeding 1 indicates abrupt spatial variation in the velocity field and, by extension, low smoothness in the underlying

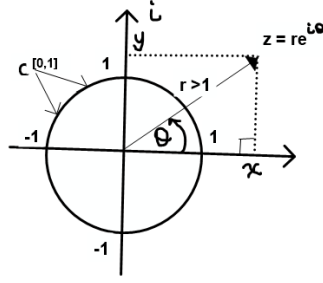


Figure 3.3: The unit circle in the complex plane serves as the decision boundary for motion smoothness classification.

physical motion. This provides a local measure of spatial variation that complements the global area change measure derived from the divergence.

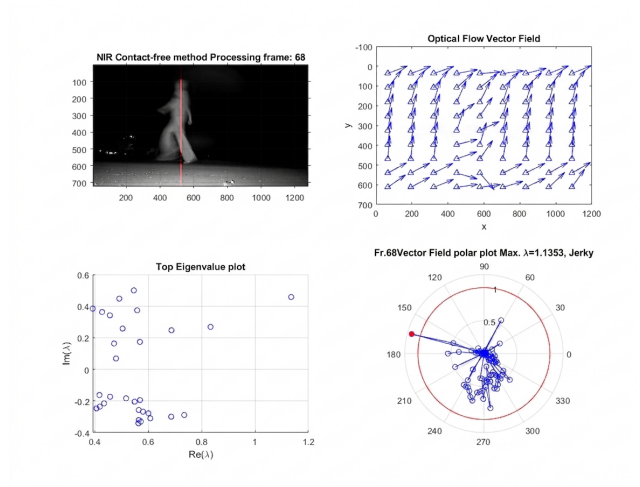


Figure 3.4: Jerky vector field from a runner video frame (frame 68) indicating low motion smoothness.

Fig. 3.4 shows a vector field extracted from a real video frame (frame 68). The maximum eigenvalue magnitude is $|\lambda_{\max}| = 1.1353$, which lies outside the unit circle, confirming that the motion in this frame is irregular. As will be shown in Chapter 5, such classifications correlate strongly with increased acceleration measured by IMU sensors.

This framework enables quantification of motion smoothness by transforming video motion data into analyzable vector field representations. The remainder of the chapter elaborates on each component: Section 3.2 details vector field construction from video, Section 3.3 covers eigenvalue analysis, Section 3.4 formalizes the unit-circle classification

criterion, Section 3.5 introduces the characteristic proximity framework for comparing motion states, and Section 3.6 presents operators for temporal analysis. Throughout this development, we maintain the assumptions stated in Chapter 1 (rigid motion, brightness constancy, orthographic projection, and periodicity), which ensure that the mathematical models linking video data to physical motion are well-posed.

3.2 Vector Fields Derived from Video Frame Motion

To establish a foundation for quantifying motion smoothness, we derive motion vector fields from pixel-level displacements between consecutive Near-Infrared (NIR) video frames. Following the framework established in Chapter 1, these vector fields are interpreted as empirical approximations of the projected physical velocity field. Under the orthographic projection assumption (camera distance much larger than object size, motion primarily parallel to the image plane), the optical flow vectors are proportional to the physical velocity of corresponding points on the moving object.

The collection of these vectors across a region of interest forms a motion vector field $\vec{V}f$ that encodes the instantaneous spatial distribution of motion. This field provides the data foundation for local spatial variation analysis (via Jacobian eigenvalues), linking directly to the physical models of vehicle suspension (mass-spring-damper) and human limb motion (Molati model) introduced in Chapter 1. As noted in the introduction, the framework is not limited to column-based analysis; the same eigenvalue extraction and smoothness classification can be applied to any region of interest or to the entire object silhouette, and the results presented here using the column-based approach serve as a validation of the core concept.

3.2.1 Motion Extraction with Gaussian Mixture Model

The Gaussian Mixture Model (GMM) plays a crucial role in the processing procedure by enabling robust motion extraction from sequential NIR video frames. GMM provides a

statistical framework for background modeling that effectively separates foreground motion components from static background elements in video [29]. This separation is essential for isolating the region R corresponding to the moving object, over which divergence integrals can be computed to measure area change rates as described in Chapter 1.

In our implementation, GMM operates by modeling the pixel intensity distribution over time as a weighted combination of multiple Gaussian distributions. Each pixel in the NIR video is represented by a mixture of K Gaussian distributions, where the probability of observing a particular pixel value X_t at time t is given by:

$$P(X_t) = \sum_{i=1}^K \omega_{i,t} \eta(X_t, \mu_{i,t}, \Sigma_{i,t}) \quad (3.1)$$

where $\omega_{i,t}$ represents the weight of the i -th Gaussian component, and $\eta(X_t, \mu_{i,t}, \Sigma_{i,t})$ denotes the probability density function of the Gaussian distribution with mean $\mu_{i,t}$ and covariance $\Sigma_{i,t}$.

The adaptive nature of GMM makes it particularly suitable for our vibration monitoring application. As new frames are processed, the model parameters are continuously updated using the following recursive equations when a pixel matches a Gaussian component:

$$\begin{aligned} \mu_{i,t} &= (1 - \rho)\mu_{i,t-1} + \rho X_t \\ \Sigma_{i,t} &= (1 - \rho)\Sigma_{i,t-1} + \rho(X_t - \mu_{i,t})^2 \\ \omega_{i,t} &= (1 - \alpha)\omega_{i,t-1} + \alpha \end{aligned}$$

where α is the learning rate and $\rho \approx \alpha/\omega_{i,t}$ is the parameter learning rate. This adaptive updating mechanism allows the background model to accommodate gradual environmental changes while maintaining sensitivity to motion patterns.

Compared to single Gaussian models, the multimodal nature of GMM proves particularly advantageous for vibration monitoring applications, where complex motion patterns and environmental variations often exhibit multi-modal characteristics [29]. By employing



Figure 3.5: Geometric representation of rotational motion showing position vector $\vec{r}$, velocity vector $\vec{v}$, and angular momentum $\vec{L} = \vec{r} \times m\vec{v} = I\vec{\omega}$. This illustrates the relationship between position and velocity for rotational motion, directly paralleling the limb motion described by Eqs. (??) and (??) in Chapter 1.

multiple Gaussian components, the proposed framework can effectively model scenes with periodic motions and subtle movements, providing clean motion segmentation for subsequent analysis.

3.2.2 Vector Representation and Transformation to Complex Plane

The analysis begins with the fundamental representation of motion in two-dimensional Euclidean space. Each vector $\vec{p}$ is characterized by its magnitude and direction according to the standard Euclidean formulation:

$$\vec{p} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \|\vec{p}\| = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \quad (3.2)$$

where x and y represent Cartesian components. This representation provides geometric clarity for initial motion analysis.

To understand the physical interpretation of vectors in our context, consider the rotational motion example [37] shown in Fig. 3.5. In this classical mechanics illustration, an athlete swings a hammer in circular motion. The position vector $\vec{r}$ points from the rotation axis

to the hammer's center, while the velocity vector $\vec{v}$ is tangential to the path. Their cross product yields angular momentum:

$$\vec{L} = \vec{r} \times m\vec{v} \quad (3.3)$$

This example directly parallels the limb motion model introduced in Chapter 1, where a point on a limb segment at distance r from the joint rotates with angular displacement $\theta(t)$, yielding position $\mathbf{p}(t) = (r \sin \theta(t), r \cos \theta(t))$ and velocity $\mathbf{v}(t) = (r\dot{\theta} \cos \theta, -r\dot{\theta} \sin \theta)$.

A vector field $V : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ in our framework assigns a vector to each spatial location:

$$V(x, y) = \begin{bmatrix} v_1(x, y) \\ v_2(x, y) \end{bmatrix} \quad (3.4)$$

where v_1 and v_2 denote horizontal and vertical motion components, respectively. Vector fields derived from sequential NIR video frames describe instantaneous displacement or velocity of each pixel region, forming the basis for subsequent smoothness analysis.

While Euclidean representation provides geometric clarity, it offers limited analytical compactness. To enable eigenvalue-based analysis and efficient visualization of smoothness boundaries, we transform two-dimensional vectors into the complex plane as $z = x + jy$. This transformation, discussed next, allows separation of amplitude and phase components and establishes the foundation for the complex-plane criterion for motion smoothness.

3.2.3 Complex Plane Representation for Smoothness Analysis

The transition from Cartesian to complex representation [38] provides significant advantages for smoothness analysis, particularly through the geometric interpretation afforded by polar coordinates. A complex number $z = x + jy$ provides an equivalent representation with enhanced analytical properties. Geometrically, it represents a point $z(x, jy)$ in the complex

plane, while in polar form it becomes:

$$z = re^{j\theta} \quad (3.5)$$

where $r = |z| = \sqrt{x^2 + y^2}$ represents magnitude and $\theta = \angle z = \arctan(y/x)$ represents direction.

The polar representation $z = x + jy = re^{j\theta}$, illustrated in Fig. 3.6, is particularly advantageous for motion analysis because it naturally concentrates amplitude (r) and angle (θ) in a single representation. This concentration aligns with the physical interpretation of oscillatory motion, where magnitude corresponds to displacement amplitude and angle indicates the phase relationship between different motion components.

A crucial insight from complex analysis is that the unit circle $|z| = 1$ serves as a fundamental boundary for characterizing local spatial variation. As established by Krantz [9] and adapted to our context in Chapter 1, eigenvalues of the spatial Jacobian residing within the unit circle ($|\lambda| \leq 1$) indicate that the velocity field varies gradually in space, which we interpret as smooth local motion. Eigenvalues beyond this boundary suggest abrupt spatial changes, corresponding to jerky motion. This geometric interpretation provides intuitive visualization of transitions in motion smoothness.

3.2.4 Constructing and Analyzing NIR-Derived Vector Fields

The construction of vector fields via optical flow from Near-Infrared video involves translating pixel-level intensity variations into motion vector fields. We define a vector field in the complex plane [39] as follows: let $U = \{z \in \mathbb{C}\}$ be a bounded region containing points $z(x, jy) \in U \subset \mathbb{C}$. A vector field is a mapping $F : U \rightarrow 2^{\mathbb{C}}$ defined by:

$$F(z(x, y)) = \left\{ \vec{v} \in 2^{\mathbb{C}} \right\} \quad \text{denoted by } \vec{V}f \quad (3.6)$$

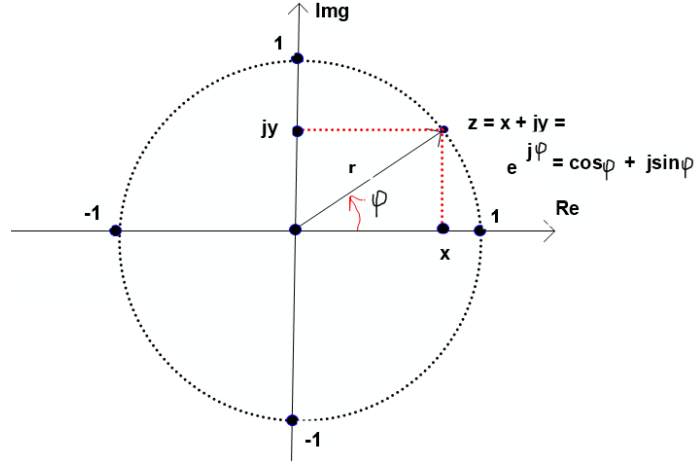


Figure 3.6: Geometric representation of complex vector $z = x + jy = re^{j\theta}$ showing magnitude r and direction θ . The unit circle ($|z| = 1$) provides the boundary for classifying motion smoothness.

This definition establishes the framework for representing motion patterns via optical flow derived from sequential video frames.

Motion waveform displacement fields [39] extracted from optical flow analysis of NIR video are represented as:

$$\vec{V}_f(z) = (\cos \theta, -\sin \theta) \times k(x, y) \quad (3.7)$$

where $k(x, y)$ at position x, y encodes displacement magnitude derived from NIR intensity gradients between consecutive frames. Under the orthographic projection assumption from Chapter 1, this displacement magnitude is proportional to the physical displacement of the corresponding point on the moving object. The trigonometric components $(\cos \theta, -\sin \theta)$ provide directional information that captures the oscillatory nature of motions, directly reflecting the angular velocity $\dot{\theta}(t)$ terms in the physical velocity expression.

Fig. 3.7 shows a sample vector field constructed from motion extracted from an NIR video frame. Each vector in this field represents motion displacement derived from optical flow analysis between consecutive frames, with the direction and magnitude indicating the local velocity characteristics.

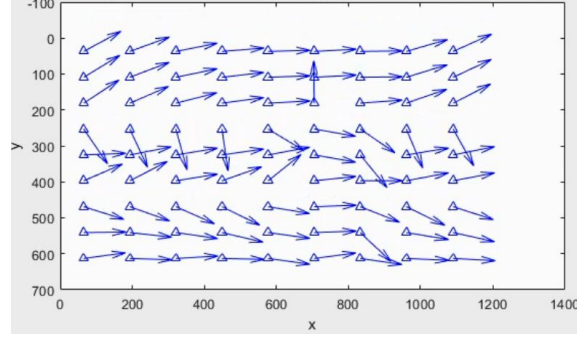


Figure 3.7: Sample vector field constructed from motion extracted from an NIR video frame.

For a vector field $\vec{V}f$ defined over region $U \subset \mathbb{C}$, a vector $\vec{v}(x,y) \in \vec{V}f$ at spatial coordinates (x,y) represents the instantaneous motion displacement derived from optical flow analysis between consecutive NIR video frames. Each vector encapsulates both magnitude and directional information of the local motion pattern.

The vector components are fundamentally related to spatial derivatives of the motion field. Specifically, the components of vector $\vec{v}(x,y)$ are defined through partial derivatives representing instantaneous rates of change:

$$\frac{\partial v(x,y)}{\partial x} \quad \text{and} \quad \frac{\partial v(x,y)}{\partial y} \quad (3.8)$$

These derivatives quantify how vectors in a motion vector field vary spatially in horizontal and vertical directions, respectively.

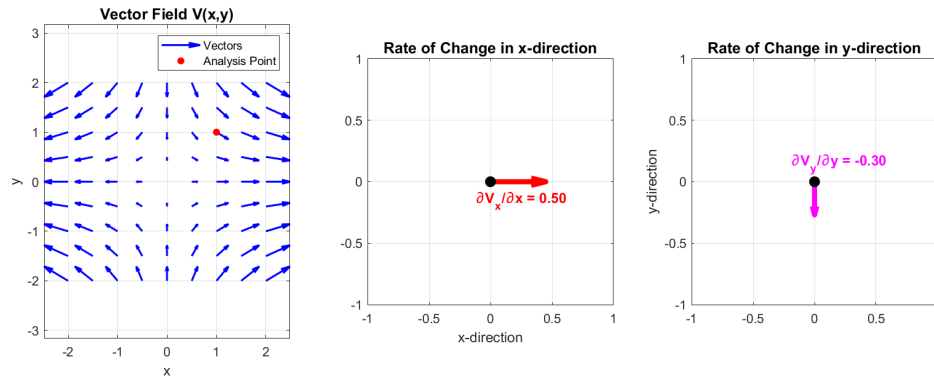


Figure 3.8: Illustration of partial derivatives of a sample vector field.

Fig. 3.8 illustrates sample partial derivatives. The left plot shows the complete vector

field $V(x, y) = [0.5x, -0.3y]$ with sample rates of change in the x- and y-directions. The middle plot displays only $\frac{\partial V_x}{\partial x} = 0.50$, representing the instantaneous rate of change of the x-component in the x-direction. The right plot shows only $\frac{\partial V_y}{\partial y} = -0.30$, representing the instantaneous rate of change of the y-component in the y-direction.

The organized collection of these partial derivatives forms the Jacobian matrix, defined as the array of first-order partial derivatives:

$$J(\vec{V}f) = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{bmatrix} \quad (3.9)$$

As established in Chapter 1, the trace of this Jacobian, $\text{tr}(J) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}$, is the divergence of the vector field. By the planar divergence theorem, the integral of this divergence over a region R corresponding to the moving object gives the rate of change of that region's projected area. This provides a direct physical interpretation linking the mathematical analysis to the physical motion.

The eigenvalues of the Jacobian provide complementary local information. For a well-damped physical system (high damping ratio ζ in the mass-spring-damper model for vehicles, or large damping coefficient μ in the Molati model for human limbs), the velocity field changes slowly in space, leading to Jacobian eigenvalues with magnitudes predominantly less than one. For under-damped or irregularly driven systems, the velocity field exhibits abrupt spatial variations, yielding eigenvalues that frequently exceed unit magnitude. Thus, the eigenvalue analysis operationalizes the concepts of damping and restoring forces from classical mechanics in a data-driven, video-based framework.

3.2.5 Case Study: Vector Field Construction

To illustrate practical implementation, consider the biker motion sequence shown in Fig. 3.9, which shows video processing from frames 26 to 29. The vertical red line indicates the motion analysis region where pixel intensity variations are tracked. The bottom-left vector

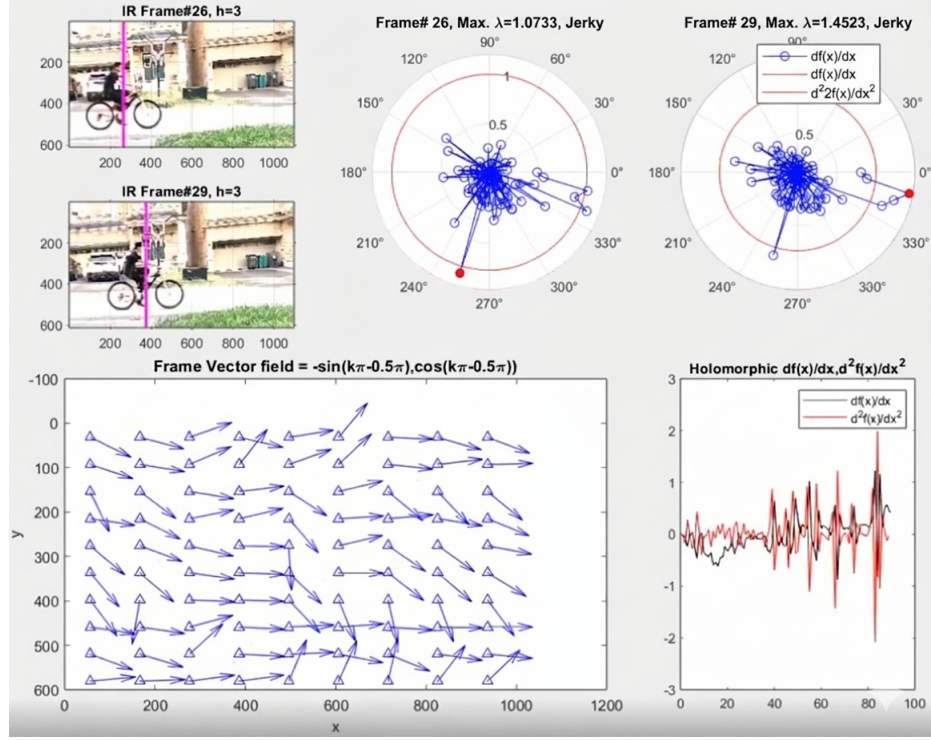


Figure 3.9: Biker motion video processing from frame 26 to 29. The vertical red line indicates the motion analysis column.

field, derived through optical flow computation, provides the basis for eigenvalue calculation. The lower-right plots characterize vibration dynamics: the first derivative quantifies frame-to-frame eigenvalue rate-of-change, while the second derivative reveals the trend in smoothness evolution. The right complex plane visualizes eigenvalues relative to the unit circle boundary, with red markers indicating maximum eigenvalues that determine smoothness classification.

In this implementation, NIR video frames are processed to construct motion vector fields. Each frame fr is decomposed into static background and dynamic foreground. The optical flow displacement $k(x, y)$ quantifies pixel-level motion between frames at spatial coordinates (x, y) . The vector field for frame fr is computed as:

$$\vec{V}_{fr} = (-\sin(\theta), \cos(\theta)) \times k(x, y). \quad (3.10)$$

For frame fr_{26} at coordinates $(x = 12, y = 18)$ with displacement magnitude $k = 0.45$,

the vector field computation yields:

$$\vec{V}_{f_{r_{26}}} = (-\sin(\theta), \cos(\theta)) \times 0.45 = (0.21, 0.42). \quad (3.11)$$

The partial derivatives for this field are approximately:

$$\frac{\partial v_x}{\partial x} \approx -0.32, \quad \frac{\partial v_x}{\partial y} \approx 0.15 \quad (3.12)$$

$$\frac{\partial v_y}{\partial x} \approx 0.08, \quad \frac{\partial v_y}{\partial y} \approx -0.41 \quad (3.13)$$

The corresponding Jacobian matrix for smoothness analysis is:

$$J = \begin{bmatrix} -0.6 & 0.25 \\ -0.1 & -0.7 \end{bmatrix} \quad (3.14)$$

The eigenvalues of this Jacobian determine the local rate of spatial change at each point z in the complex plane. The condition for smooth motion, as formalized by the adapted unit-circle criterion [9], requires that all eigenvalue magnitudes lie within the unit circle ($|\lambda| \leq 1$). The elements of the Jacobian matrix have clear physical interpretations in motion analysis: diagonal elements ($\frac{\partial v_x}{\partial x}, \frac{\partial v_y}{\partial y}$) represent how the velocity field expands or contracts along principal directions, relating to the damping coefficients in the physical models from Chapter 1.

The approach to vector field construction and analysis established in this section provides the foundation for eigenvalue-based smoothness assessment.

3.3 Eigenvalue Analysis for Local Motion Smoothness

Building upon the vector field construction established in Section 3.2, we now develop an eigenvalue-based framework for assessing local motion smoothness. This approach transforms the nonlinear motion patterns captured in video into analytically tractable linear

approximations, enabling quantitative characterization of spatial variation rates that can be related to the damping parameters of the physical models introduced in Chapter 1.

3.3.1 Linearized Analysis of Motion Fields

The analysis of local spatial variation in a velocity field determines whether small differences in velocity from one point to the next lead to smooth, gradual changes or abrupt, irregular patterns—a concept central to linking video-derived measurements to the physical behavior of the underlying object. We consider a motion vector field $\vec{v}(x, y) = (u(x, y), v(x, y))$ defined over a small neighborhood around a point of interest. For sufficiently small regions, the field can be approximated by its first-order Taylor expansion:

$$\vec{v}(x_0 + \Delta x, y_0 + \Delta y) \approx \vec{v}(x_0, y_0) + J(x_0, y_0) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (3.15)$$

where J is the Jacobian matrix defined in Eq. (3.9). This linear approximation captures the local stretching and rotation of the motion field, which directly reflects the spatial variation of the imaged object’s movement. In the context of the mass-spring-damper model for vehicles and the Molati model for human limbs, such spatial variations are directly related to the damping and restoring forces acting on the system.

The eigenvalues λ of the Jacobian matrix quantify the rates of spatial variation along the principal directions. Specifically, if we consider a small circular region in the image, it will be mapped to an ellipse under the linearized flow, with the semi-axis lengths determined by the eigenvalues. The magnitude of the largest eigenvalue, $|\lambda_{\max}|$, provides a measure of the most extreme local rate of change in the velocity field.

This eigenvalue-based characterization connects to the classical concept of damping in mechanical systems. For a well-damped system (high damping ratio ζ in the vehicle model, or large damping coefficient μ in the Molati model), the velocity field varies gradually in space, leading to eigenvalue magnitudes less than unity when considered in an appropriate discrete-time framework. Conversely, under-damped or irregular motion

produces eigenvalue magnitudes exceeding unity, indicating that local velocity differences are large and the motion appears jerky.

3.3.2 Eigenvalue Extraction from Motion Vector Fields

The method employed here follows the approach described in our published work [12]. For a given point in the motion field, eigenvalues are obtained by solving the characteristic equation of the Jacobian matrix:

$$\det(J - \lambda I) = 0 \quad (3.16)$$

where I is the 2×2 identity matrix. For the 2×2 Jacobian, this yields a quadratic equation whose solutions are the eigenvalues λ_1, λ_2 .

To illustrate, consider the sample Jacobian from the biker motion sequence (Eq. (3.14)):

$$J = \begin{bmatrix} -0.6 & 0.25 \\ -0.1 & -0.7 \end{bmatrix} \quad (3.17)$$

The characteristic equation is:

$$\begin{aligned} \det(J - \lambda I) &= \begin{vmatrix} -0.6 - \lambda & 0.25 \\ -0.1 & -0.7 - \lambda \end{vmatrix} \\ &= (-0.6 - \lambda)(-0.7 - \lambda) - (0.25)(-0.1) \\ &= \lambda^2 + 1.3\lambda + 0.42 + 0.025 \\ &= \lambda^2 + 1.3\lambda + 0.445 = 0 \end{aligned} \quad (3.18)$$

Solving gives:

$$\lambda = \frac{-1.30 \pm \sqrt{1.69 - 1.78}}{2} = \frac{-1.30 \pm \sqrt{-0.09}}{2} = -0.65 \pm 0.15j \quad (3.19)$$

Thus $\lambda_1 = -0.65 + 0.15j$ and $\lambda_2 = -0.65 - 0.15j$. The magnitudes are:

$$|\lambda_1| = |\lambda_2| = \sqrt{(-0.65)^2 + (0.15)^2} \approx 0.667 \quad (3.20)$$

Both eigenvalues lie inside the unit circle, indicating smooth local motion. In contrast, for the jerky frame shown in Fig. 3.4, the maximum eigenvalue magnitude was $|\lambda_{\max}| = 1.1353 > 1$, indicating irregular motion.

The eigenvectors associated with these eigenvalues provide directional information about the principal axes of spatial variation. While the eigenvalue magnitudes are our primary smoothness indicators, the eigenvectors can be used to identify dominant directions of velocity change, which may be correlated with specific gait events or loading conditions.

3.3.3 Relation to Divergence and Motion Smoothness

The trace of the Jacobian matrix is the divergence of the motion field:

$$\text{tr}(J) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nabla \cdot \vec{v} \quad (3.21)$$

The divergence measures the net rate of expansion or contraction at a point. The eigenvalues λ_1, λ_2 are related to the trace and determinant:

$$\lambda_1 + \lambda_2 = \text{tr}(J) = \nabla \cdot \vec{v} \quad (3.22)$$

$$\lambda_1 \lambda_2 = \det(J) = \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \quad (3.23)$$

Thus, the eigenvalues provide a decomposition of the local spatial variation into its constituent components: the sum (trace) gives the expansion or contraction, while the difference relates to the shear. This decomposition is valuable for interpreting motion patterns. For a rigid moving object, non-zero divergence indicates a departure from purely translational motion, which may be correlated with changes in the effective damping as load varies.

The global integral of divergence over the object region R is precisely the quantity

appearing in the planar divergence theorem, linking local eigenvalue analysis to the global motion smoothness measure.

3.4 Geometric Classification: The Smoothness Criterion

Having established the eigenvalue-based characterization of local spatial variation, we now introduce a geometric criterion for classifying motion as smooth or jerky. This criterion, adapted from the Krantz condition for discrete-time dynamical systems [9], provides an intuitive and computationally efficient way to threshold eigenvalue magnitudes relative to the unit circle in the complex plane.

3.4.1 Unit Circle Boundary for Classification

The Krantz criterion, in its original formulation, states that a discrete-time linear system is stable if and only if all eigenvalues of its system matrix lie within the unit circle in the complex plane. In our context, we apply this concept locally as an operational classifier: for each point in the motion field, we consider the Jacobian matrix as an empirical operator describing local spatial variation. If both eigenvalues satisfy $|\lambda| \leq 1$, the local velocity field varies gradually in space and the motion is classified as smooth; if any eigenvalue has magnitude greater than 1, the motion is classified as jerky.

We formalize this as follows. For a given point (x, y) in the motion field, let $\lambda_{\max}(x, y)$ denote the eigenvalue with the largest magnitude. Then:

$$\text{Smooth motion: } |\lambda_{\max}(x, y)| \leq 1 \quad (3.24)$$

$$\text{Jerky motion: } |\lambda_{\max}(x, y)| > 1 \quad (3.25)$$

The boundary $|\lambda| = 1$ corresponds to the unit circle in the complex plane, providing a clear geometric visualization: eigenvalues inside the circle indicate smooth spatial variation; eigenvalues outside indicate abrupt spatial changes.

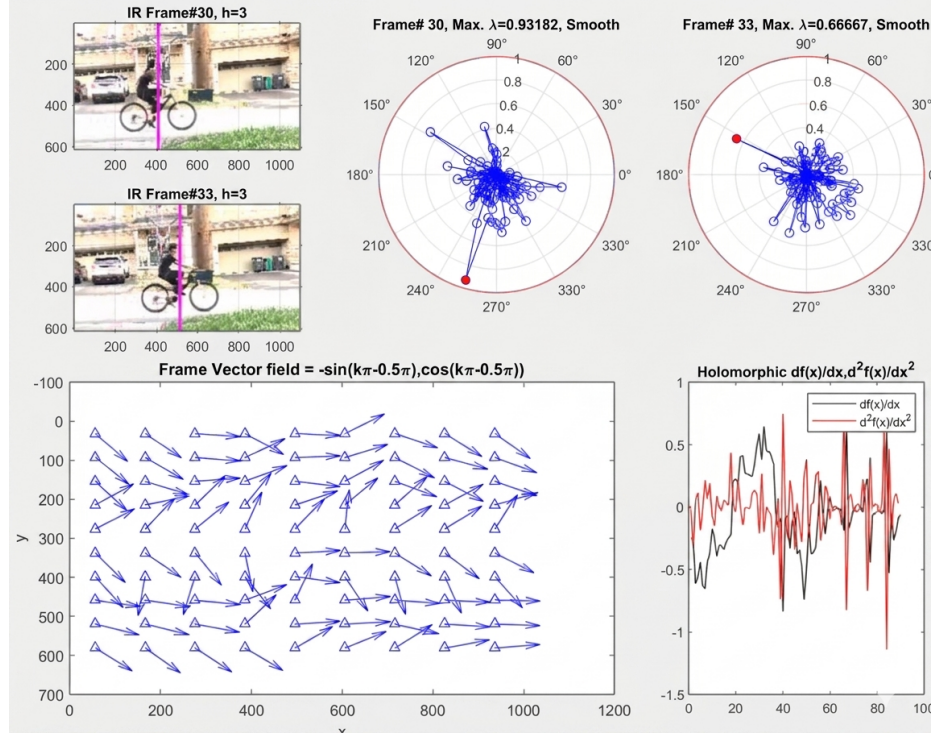


Figure 3.10: Biker motion video processing from frame 30 to 33.

This classification has a direct physical interpretation in terms of the damping models from Chapter 1. For a mass-spring-damper system discretized with a time step Δt , the eigenvalues of the state transition matrix are related to the damping ratio ζ . When $\zeta > 0$ (underdamped but stable), the eigenvalues lie inside the unit circle. When ζ becomes negative (negative damping, indicating energy input), eigenvalues move outside the unit circle. Thus, our eigenvalue-based smoothness measure serves as a indicator for the effective damping in the observed motion.

To demonstrate practical application, consider the biker motion sequence analyzed in Figs. 3.9 and 3.10. For frame fr_{26} , the computed maximum eigenvalue magnitude is $|\lambda_{\max}| = 1.077 > 1$, indicating jerky motion. In contrast, frame fr_{30} with $\lambda_{\max} = 0.93$ (magnitude $0.93 \leq 1$) demonstrates smooth motion.

3.4.2 Smoothness Margins and Robustness

In practical applications, we may wish to introduce a margin to account for noise and uncertainties in optical flow estimation. We define a robustness parameter $\epsilon_0 \geq 0$ such that:

$$|\lambda_{\max}| \leq 1 - \epsilon_0 \quad \Rightarrow \quad \text{strongly smooth} \quad (3.26)$$

$$1 - \epsilon_0 < |\lambda_{\max}| \leq 1 + \epsilon_0 \quad \Rightarrow \quad \text{marginally smooth/uncertain} \quad (3.27)$$

$$|\lambda_{\max}| > 1 + \epsilon_0 \quad \Rightarrow \quad \text{jerky} \quad (3.28)$$

The choice of ϵ_0 involves a trade-off: larger ϵ_0 increases robustness to noise but may misclassify slightly jerky motion as smooth. In this study, we set $\epsilon_0 = 0.1$ to adhere strictly to the theoretical boundary, as our experimental validation (Chapter 5) shows that this threshold effectively separates smooth from jerky motion when compared with IMU measurements.

3.4.3 Geometric Visualization Framework

The complex-plane representation of eigenvalues offers an intuitive visualization of motion smoothness across a video. For each frame, we can plot the eigenvalues from all points in the analysis column (or region) as points in the complex plane. The distribution of these points relative to the unit circle provides immediate insight into the overall smoothness of the motion.

Furthermore, the temporal evolution of eigenvalues can be visualized as trajectories in the complex plane. When a trajectory crosses the unit circle, it indicates a transition from smooth to jerky motion or vice versa. Such transitions often correspond to specific events in the motion cycle, such as heel strike in gait or a bump encountered by a vehicle.

3.4.4 Integration with Experimental Validation

The smoothness classification based on the unit-circle criterion provides a frame-by-frame binary label (smooth or jerky). Aggregating these labels over a sequence yields the smoothness

index η , defined as the percentage of frames classified as smooth:

$$\eta = \frac{|S|}{N} \times 100\% \quad (3.29)$$

where S is the set of frames satisfying $|\lambda_{\max}| \leq 1$ and the characteristic distance metric $d^\Phi \leq 0.5$, and N is the total number of frames.

This index serves as a global measure of motion smoothness for the entire sequence. In Chapter 5, we show that η correlates strongly with IMU-based measures of vibration intensity and with loading conditions, validating the physical relevance of our smoothness classification.

3.5 Characteristic Distance Metric

Building upon the frame-by-frame smoothness classification provided by the unit-circle criterion, we now introduce a metric for quantifying the similarity between different motion states captured in video. This metric, termed the characteristic distance d^Φ , allows us to compare vector fields from different frames or different trials, providing a quantitative basis for assessing whether a particular motion pattern persists over time or whether two motion sequences exhibit similar smoothness characteristics.

The characteristic distance framework for motion vector fields was introduced in [12] and further developed in [11] [40]. It provides a way to measure the closeness of motion vector fields based on their dominant eigenvalue characteristics, which, as established in the preceding sections, encode essential information about the underlying physical damping.

3.5.1 Foundations of Characteristic Distance

Let A and B be two nonempty sets. For elements $a \in A$ and $b \in B$, we consider numerical characteristics $\phi(a)$ and $\phi(b)$ that capture the essential properties of the motion. In our context, the most relevant characteristic is the maximum eigenvalue magnitude, $|\lambda_{\max}|$,

which determines the smoothness classification via the unit-circle criterion.

We define the characteristic distance d^Φ between two sets as:

$$d^\Phi(A, B) = \inf_{\substack{\varphi(a) \in \Phi(A) \\ \varphi(b) \in \Phi(B)}} \{|\varphi(a) - \varphi(b)|\} = \varepsilon \in [0, 1] \subset \mathbb{C}. \quad (3.30)$$

In practice, we work with a simplified version: for two motion vector fields $\vec{V}f_1$ and $\vec{V}f_2$, we take φ to be the maximum eigenvalue magnitude, yielding:

$$d^\Phi(\vec{V}f_1, \vec{V}f_2) = \left| |\lambda_{\max}(\vec{V}f_1)| - |\lambda_{\max}(\vec{V}f_2)| \right| \quad (3.31)$$

Two motion states are said to be characteristically close, denoted $\vec{V}f_1 \delta_\Phi \vec{V}f_2$, if $d^\Phi \leq 0.5$. This threshold was determined empirically from [12] and training data as providing optimal separation between similar and dissimilar motion states.

3.5.2 Application to Motion Vector Field Analysis

We apply this theoretical framework to motion vector field analysis by defining specific characteristics for vector fields [12]. For a vector field $(\vec{V}f, +)$ forming a surface group S_g in $\mathbb{C}$, we define three characteristics:

$$\begin{aligned} \varphi_1(S_g) &= (\max \varphi(\lambda)) \notin \text{unit circle} \\ \varphi_2(S_g) &= (\max \varphi(\lambda)) \in \text{unit circle} \\ \varphi_3(S_g) &= \|\varphi(\lambda_{\vec{V}f_1}) - \varphi(\lambda_{\vec{V}f_2})\| \in [0, 0.5] \Rightarrow \vec{V}f_1 \delta_\Phi \vec{V}f_2 \\ \Phi(S_g) &= \{\varphi_1(S_g), \varphi_2(S_g), \varphi_3(S_g)\}. \end{aligned}$$

The first two provide a binary classification (jerky vs. smooth), while the third enables quantitative comparison. The Characteristic Closeness Measurement Framework extends the unit-circle criterion by introducing quantitative proximity measures that capture subtle smoothness variations between sequential vector fields. We formalize this through the

Smoothness Proximity Criterion [12]: two vector fields $\vec{V}f_1$ and $\vec{V}f_2$ are characteristically close ($\vec{V}f_1 \delta_\Phi \vec{V}f_2$) if and only if:

$$|\max |\lambda_{\vec{V}f_1}| - \max |\lambda_{\vec{V}f_2}|| \leq 0.5 \quad (3.32)$$

This simple condition allows us to compare motion states across frames, across trials, or across different subjects, providing a quantitative basis for assessing motion similarity.

3.5.3 Practical Implementation and Case Studies

To illustrate the utility of the characteristic distance, we present several examples from experimental data.

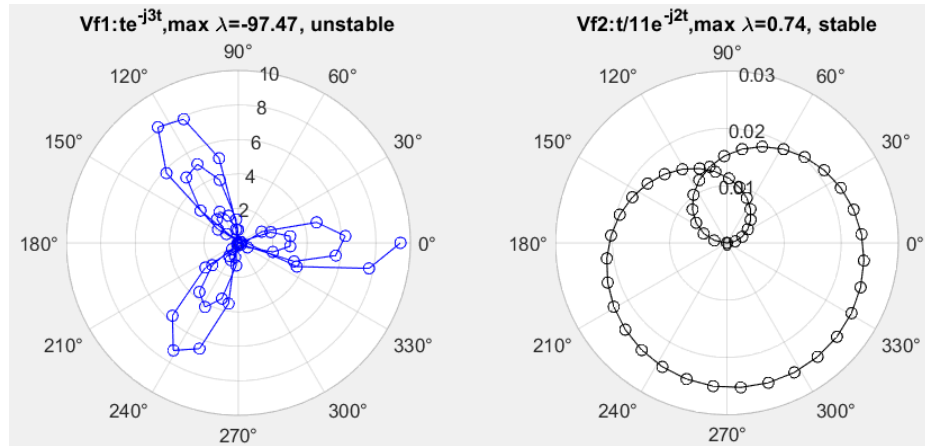


Figure 3.11: Case 1: Characteristically non-near vector fields.

First, consider characteristically non-near vector fields as shown in Figure 3.11. In this case, (not) $(\vec{V}f_1 \delta_\Phi \vec{V}f_2)$ since:

$$\varphi_1(S_{g_{\vec{V}f_1}})(\max) = -97.47 \Rightarrow \text{jerky vector field}$$

$$\varphi_2(S_{g_{\vec{V}f_2}})(\max) = 0.74 \Rightarrow \text{smooth vector field.}$$

The significant difference in maximum eigenvalues ($|-97.47 - 0.74| = 98.21 > 0.5$) confirms characteristic non-nearness.

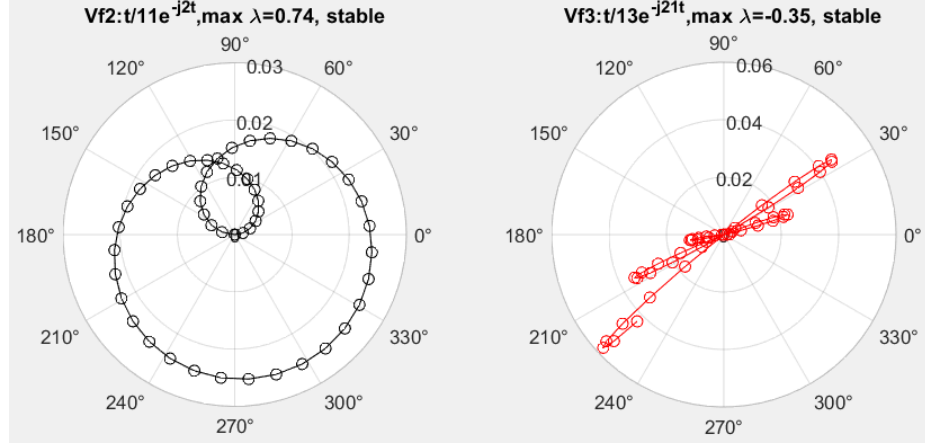


Figure 3.12: Case 2: Characteristically near vector fields.

In contrast, Figure 3.12 demonstrates characteristically near vector fields, since:

$$\varphi_3(S_{g_{\vec{V}_{f_2}, \vec{V}_{f_3}}}) = \|\varphi((\max)\lambda_{\vec{V}_{f_2}} = 0.74) - \varphi(\lambda_{\vec{V}_{f_3}} = -0.35)\| = 0.39 \in [0, 0.5]$$

$$\varphi_2(S_{g_{\vec{V}_{f_2}}})(\max) = 0.74 \Rightarrow \text{smooth vector field}$$

$$\varphi_2(S_{g_{\vec{V}_{f_3}}})(\max) = -0.35 \Rightarrow \text{smooth vector field.}$$

The small eigenvalue difference ($|0.74 - (-0.35)| = 0.39 \leq 0.5$) confirms characteristic nearness.

Moving to real motion data, Figure 3.13 contains a pair of characteristically near smooth vector fields $\vec{V}f_{fr77}$, $\vec{V}f_{fr94}$ from frames 77 and 94 of a runner motion sequence:

$$\max \lambda_{fr77} = 0.67$$

$$\max \lambda_{fr94} = 0.91$$

$$d^\Phi(\vec{V}f_{fr77}, \vec{V}f_{fr94}) = \|0.67 - 0.91\| = 0.24 \in [0, 0.5].$$

Hence, these vector fields satisfy $\vec{V}f_{fr77} \delta_\Phi \vec{V}f_{fr94}$ according to the Smooth Systems Extreme Closeness Condition.

Similarly, Figure 3.14 demonstrates characteristically near jerky vector fields $\vec{V}f_{fr71}$,

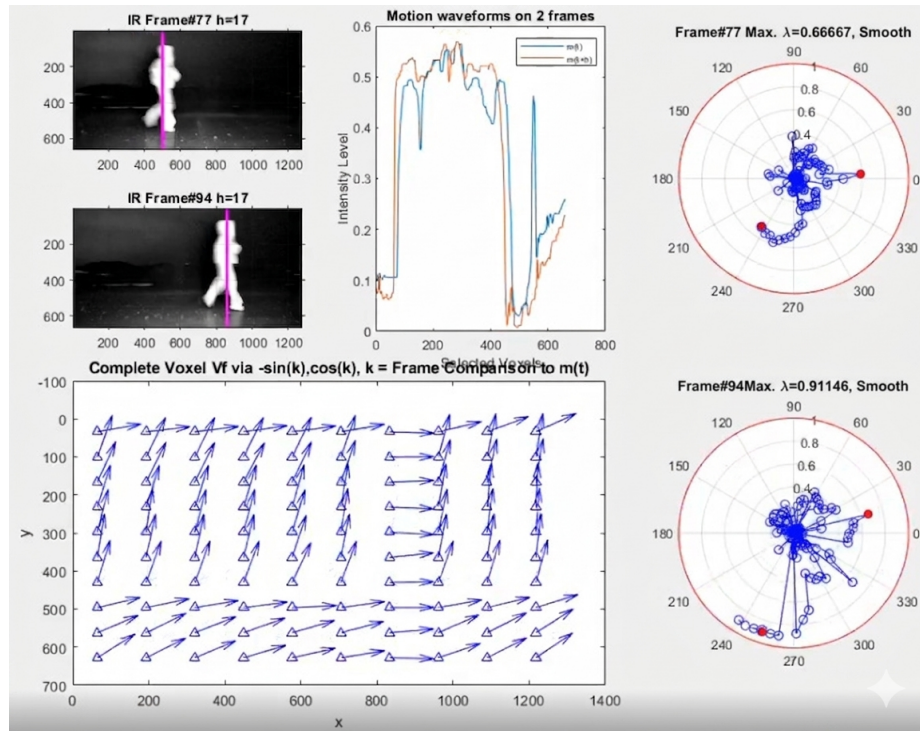


Figure 3.13: Case 1: Characteristically near smooth vector fields.

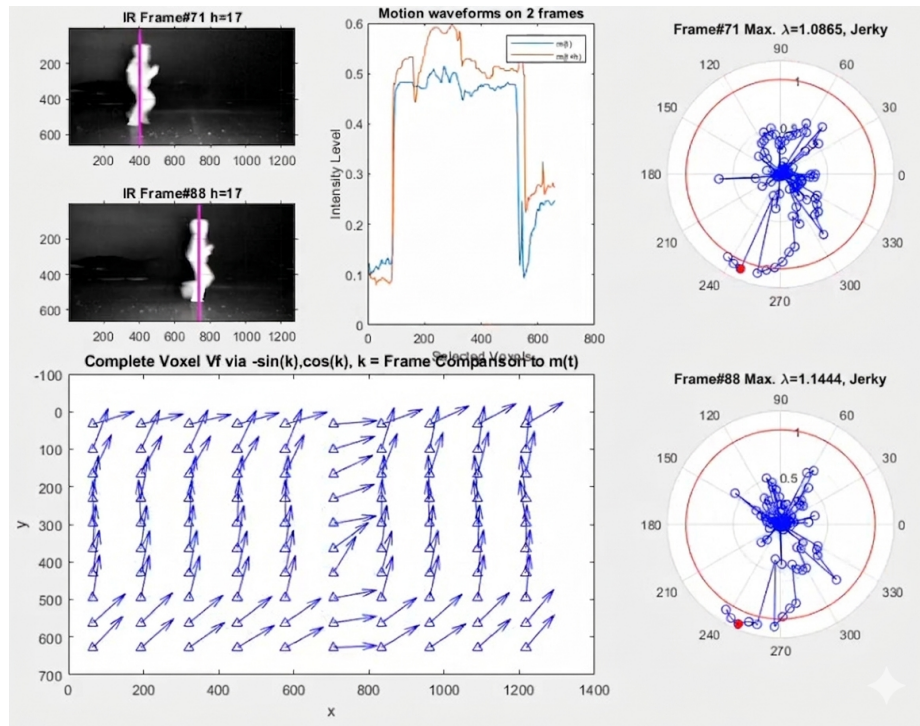


Figure 3.14: Case 2: Pair of characteristically near jerky vector fields.

$\vec{V}_{f_{r88}}$ from frames 71 and 88:

$$\max \lambda_{f_{r71}} = 1.09$$

$$\max \lambda_{f_{r88}} = 1.44$$

$$d^\Phi(\vec{V}_{f_{r71}}, \vec{V}_{f_{r88}}) = \|1.09 - 1.44\| = 0.35 \in [0, 0.5].$$

These satisfy $\vec{V}_{f_{r71}} \delta_\Phi \vec{V}_{f_{r88}}$ under the Smoothness Proximity Criterion.

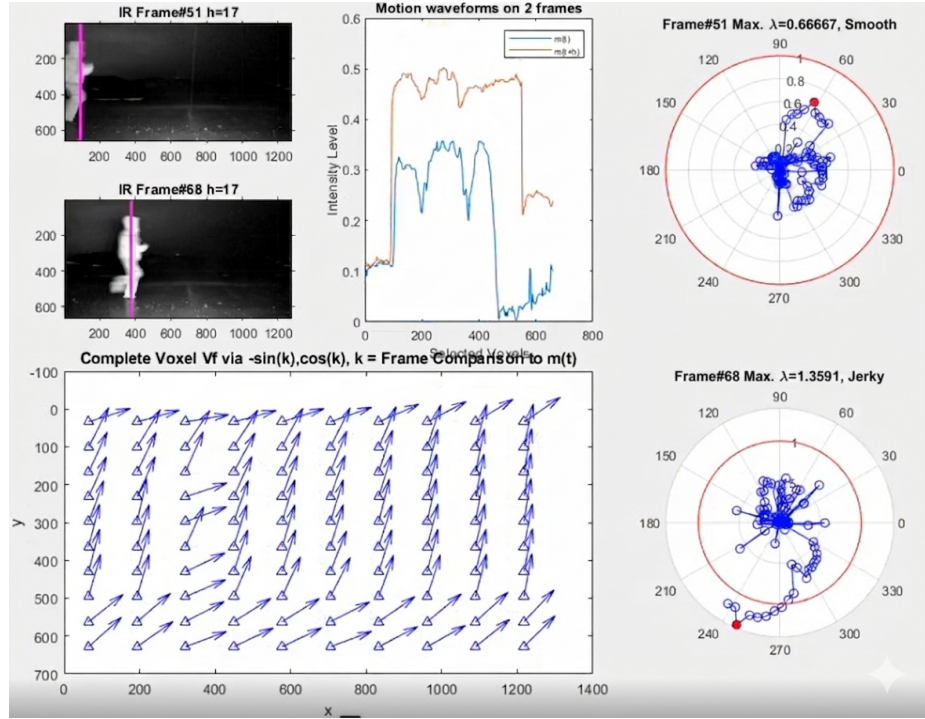


Figure 3.15: Case 3: Pair of characteristically non-close smooth and jerky vector fields.

Finally, Figure 3.15 illustrates characteristically non-close smooth and jerky vector fields $\vec{V}_{f_{r51}}$ (smooth) and $\vec{V}_{f_{r68}}$ (jerky):

$$\max \lambda_{f_{r51}} = 0.67$$

$$\max \lambda_{f_{r68}} = 1.36$$

$$d^\Phi(\vec{V}_{f_{r51}}, \vec{V}_{f_{r68}}) = \|0.67 - 1.36\| = 0.69 \notin [0, 0.5].$$

Thus, $\vec{V}_{f_{r51}}$ (not) $\delta_\Phi \vec{V}_{f_{r68}}$, demonstrating the framework's ability to distinguish funda-

mentally different smoothness states.

These examples demonstrate that the characteristic distance provides a sensitive and quantitative tool for comparing motion states, complementing the binary classification of the unit-circle criterion.

3.6 Operators for Temporal Analysis

To capture the temporal evolution of motion smoothness, we introduce two additional operators: a rate-of-change measure that quantifies how quickly smoothness characteristics evolve between frames, and an aggregate index that summarizes overall smoothness over a sequence. These operators address the need for temporal analysis and global sequence characterization.

3.6.1 Discrete Rate-of-Change Analysis of Eigenvalues

The temporal evolution of the maximum eigenvalue magnitude, $|\lambda_{\max}(t)|$, encodes how the local spatial variation rates change over time. To quantify this evolution, we define a discrete rate-of-change operator inspired by the concept of derivatives in complex analysis. We emphasize that this is an operational definition inspired by the properties of holomorphic functions [39, 41], but our discrete eigenvalue sequences are not holomorphic in the strict mathematical sense.

For a sequence of eigenvalues $\lambda(t_i)$ obtained from consecutive video frames, we define the rate of change over a time lag of h frames as:

$$D_i(h) = \frac{\|\lambda(t_{i+h}) - \lambda(t_i)\|}{h} \quad (3.33)$$

where the norm $\|\cdot\|$ denotes the magnitude of the complex difference. This operator measures how quickly the complex-valued eigenvalue changes between frames separated by h time steps. When h is small, D_i captures rapid fluctuations; when h is larger, it emphasizes

longer-term trends.

The choice of h involves a trade-off: smaller h provides higher temporal resolution but is more sensitive to noise; larger h smooths out noise but may miss rapid transitions. Based on empirical evaluation, we select $h = 8$ frames as providing an optimal balance for our 30 fps runner video data. This corresponds to a time window of approximately 0.27 seconds, which captures meaningful gait events while mitigating frame-to-frame optical flow noise.

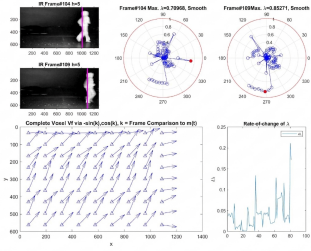


Figure 3.16: Runner frame sequence for $h = 5$.

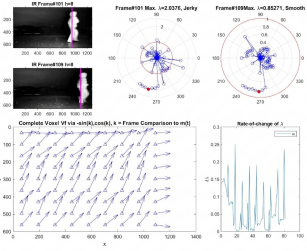


Figure 3.17: Runner frame sequence for $h = 8$.

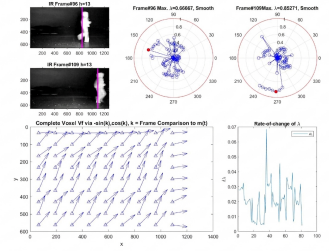


Figure 3.18: Runner frame sequence for $h = 13$.

Figure 3.19 shows D_i computed from a runner sequence for three different h values. The $h = 8$ case (middle) captures the essential dynamics while smoothing out high-frequency fluctuations. Figures 3.16, 3.17, and 3.18 show representative frame sequences for $h = 5$, $h = 8$, and $h = 13$ frames respectively, illustrating the progressive motion captured at each time lag.

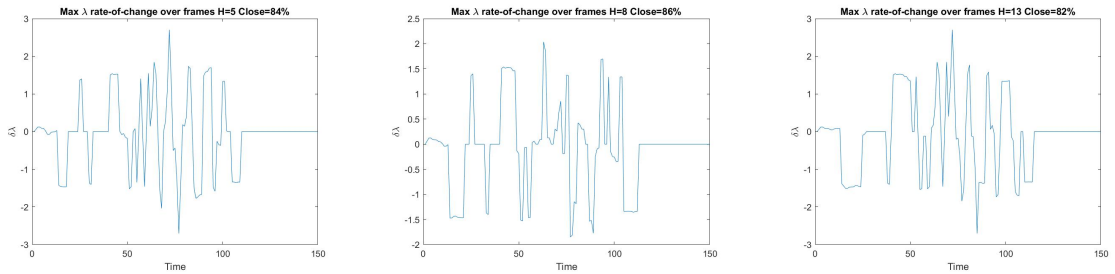


Figure 3.19: Discrete rate-of-change D_i comparison for different time lags $h = 5$, $h = 8$, and $h = 13$ frames. The plots demonstrate how varying the temporal sampling interval affects the measured rate-of-change of eigenvalues in runner motion analysis.

3.6.2 Smoothness Index for Aggregate Assessment

To complement the frame-to-frame analysis, we introduce the Smoothness Index η as a fundamental advancement in quantitative smoothness assessment [12]. This index aggregates the frame-by-frame smoothness classifications from the unit-circle criterion into a single percentage.

For a sequence of N video frames with corresponding vector fields $\vec{V}f_1, \vec{V}f_2, \dots, \vec{V}f_N$, we define S as the set of smooth frames according to the unit-circle criterion:

$$S = \left\{ \vec{V}f_i : |\lambda_{\max}(\vec{V}f_i)| \leq 1 \right\}$$

where the value 1 refers to the boundary of the unit circle in the complex plane, and $\lambda_{\max}(\vec{V}f_i)$ represents the maximum eigenvalue for vector field Vf_i . Then the Smoothness Index is:

$$\eta = \frac{|S|}{N} \times 100\% \quad (3.34)$$

This index provides a simple, interpretable summary of the entire motion sequence: a high η indicates predominantly smooth motion, while a low η indicates frequent episodes of jerky, irregular motion.

To illustrate this calculation, consider a 10-frame sequence from a runner motion analysis with the following maximum eigenvalues:

- Frames 1-3: $|\lambda_{\max}| = 0.85, 0.92, 0.78$ (smooth)
- Frame 4: $|\lambda_{\max}| = 1.12$ (jerky)
- Frames 5-7: $|\lambda_{\max}| = 0.95, 0.88, 0.91$ (smooth)
- Frame 8: $|\lambda_{\max}| = 1.05$ (jerky)
- Frames 9-10: $|\lambda_{\max}| = 0.96, 0.89$ (smooth)

The smooth frames are: 1, 2, 3, 5, 6, 7, 9, 10, giving $|S| = 8$ out of $N = 10$ total frames. The

Smoothness Index is:

$$\eta = \frac{8}{10} \times 100\% = 80\%$$

This indicates the runner maintained smooth motion patterns in 80% of the analyzed frames, with brief jerky episodes in frames 4 and 8 potentially corresponding to foot-strike transitions or load variations.

The Smoothness Index η exhibits an important physical correlation with mechanical energy dissipation rates [12]:

$$\eta \propto -\frac{dE}{dt}$$

where $\frac{dE}{dt}$ represents the rate of energy dissipation in the oscillatory system. In a well-damped system, oscillations decay rapidly, leading to smooth motion and high η . In a poorly damped system, energy is dissipated slowly or even increases, leading to jerky motion and low η . Experimental results presented in Chapter 5 confirm that η correlates strongly with mechanical loading, with higher loads producing higher η in both vehicle and human subjects.

3.6.3 Processing procedure

The complete processing procedure for quantifying motion smoothness from NIR video can be summarized as a sequence of well-defined computational steps. First, NIR video frames are acquired at 30 fps and preprocessed using GMM background subtraction to isolate the moving object. Next, optical flow is computed between consecutive frames to construct the motion vector field and motion vectors are extracted along the analysis column passing through the object's center of gravity. For each frame, the spatial Jacobian of the motion field is computed and its eigenvalues are extracted, recording the maximum eigenvalue magnitude $|\lambda_{\max}(i)|$. The discrete rate-of-change operator $D_i(h)$ is then calculated using Eq. (3.33) to quantify the temporal evolution of the eigenvalue magnitudes. Each frame is classified as smooth or jerky by applying the unit-circle criterion (Eq. (3.24)). Finally, the smoothness index η is computed using Eq. (3.34) to summarize the entire sequence, and the

results are compared with simultaneously recorded IMU data for validation.

To demonstrate the complete processing procedure, we present an implementation example in Tables 3.2, 3.3, and 3.4.

Table 3.2: Sample Eigenvalues $\lambda(t)$ for Runner Video Frames

Frame	λ_1	λ_2	λ_{max}
1	$0.85 + 0.12j$	$0.85 - 0.12j$	0.86
2	$0.92 + 0.08j$	$0.92 - 0.08j$	0.92
3	$0.78 + 0.15j$	$0.78 - 0.15j$	0.79
4	$1.12 + 0.20j$	$1.12 - 0.20j$	1.14
5	$0.95 + 0.10j$	$0.95 - 0.10j$	0.96
6	$0.88 + 0.13j$	$0.88 - 0.13j$	0.89
7	$1.05 + 0.18j$	$1.05 - 0.18j$	1.07
8	$0.96 + 0.09j$	$0.96 - 0.09j$	0.97

Table 3.3: Discrete Rate-of-Change D_i Computation

Frame i	$\lambda(t_i)$	$\lambda(t_{i-8})$	$D_i = \frac{\ \lambda(t_i) - \lambda(t_{i-8})\ }{8}$
9	$0.88 + 0.10j$	$0.85 + 0.12j$	0.005
10	$0.94 + 0.07j$	$0.92 + 0.08j$	0.004
11	$0.81 + 0.14j$	$0.78 + 0.15j$	0.006
12	$1.12 + 0.18j$	$1.12 + 0.20j$	0.003
13	$0.99 + 0.09j$	$0.95 + 0.10j$	0.007
14	$0.91 + 0.12j$	$0.88 + 0.13j$	0.005
15	$1.05 + 0.16j$	$1.05 + 0.18j$	0.003
16	$0.98 + 0.08j$	$0.96 + 0.09j$	0.004

Table 3.4: Smoothness Index η for Complete Sequence

Total Frames	Smooth Frames ($ \lambda_{max} \leq 1$)	$\eta = \frac{ S }{N} \times 100\%$
16	1, 2, 3, 5, 6, 8, 9, 10, 11, 13, 14, 16	75.0%

These tables demonstrate the real-time processing procedure for motion smoothness assessment. Table 3.2 shows eigenvalue computation for the first 8 frames. Table 3.3 illustrates discrete rate-of-change analysis where each frame i is compared with frame $i - 8$ (with $h = 8$), showing the rate of change of eigenvalues over the 8-frame interval. Table 3.4 presents the final η aggregation for the complete video, showing that 12 out of 16 frames (75.0%) maintained smooth motion patterns.

The Smoothness Index η complements the frame-to-frame characteristic distance d^Φ by providing global assessment and load sensitivity. While d^Φ quantifies instantaneous relationships between specific frames, η quantifies overall motion smoothness across extended time periods. Experimental results show strong correlation between η and mechanical load conditions, validating its utility for practical applications.

3.7 Chapter Summary

This chapter has developed a comprehensive mathematical framework for quantifying motion smoothness from video-derived optical flow vector fields. The key contributions include a method for constructing motion vector fields from NIR video frames using optical flow; an eigenvalue-based analysis of local spatial variation rates, linked to the physical damping parameters of the mass-spring-damper and Molati models through the trace-divergence relation and the planar divergence theorem; the application of the unit-circle criterion as a geometric classifier for motion smoothness, with the unit circle providing an intuitive boundary between smooth and jerky motion; the characteristic distance metric d^Φ , which enables quantitative comparison of motion states across frames and trials; and temporal operators, including a discrete rate-of-change measure and the aggregate Smoothness Index η , which together capture the evolution and global characteristics of motion smoothness.

This methodology provides a complete procedure for transforming raw video data into an analysis of motion smoothness. In the following chapters, we apply this framework to experimental data from both human and vehicle subjects, validating its effectiveness against traditional contact-based sensors.

Chapter 4

Experimental Design and Validation Framework

4.1 Introduction

This chapter presents a comprehensive experimental framework designed to validate two core elements of the contact-free motion smoothness assessment methodology. The validation objectives focus on demonstrating that the NIR-based framework achieves comparable smoothness assessment performance to traditional contact-based methods while eliminating physical sensor constraints, and on establishing quantitative relationships between mechanical loading and motion smoothness across both biomechanical and mechanical domains, with particular attention to consistent positive correlation patterns.

The experimental design employs a multi-domain approach spanning biomechanical and mechanical systems, with synchronized data acquisition enabling rigorous cross-validation. Through systematic variation of loading conditions and motion types, this framework provides comprehensive evidence for the proposed methodology's validity and practical applicability. Table 4.1 summarizes the complete experimental design, which encompasses six distinct experiments organized around the two validation objectives.

Table 4.1: Experimental Design Overview

Experiment	Purpose	Load Condi- tions(Backpack Weight)	Sample Size
Exp 1: Walking	Contact vs. Contact-free Comparison	0% BW	6 trials
Exp 2: Running	Contact vs. Contact-free Comparison	0% BW	6 trials
Exp 3: Loaded Running	Contact vs. Contact-free Comparison	0%, 50%, 100% BW	24 trials
Exp 4: Loaded Running	Load-Smoothness Correlation	0%, 50%, 100% BW	12 trials
Exp 5: Vehicle (Unloaded)	Contact vs. Contact-free Comparison	0% capacity	3 trials
Exp 6: Vehicle (Loaded)	Load-Smoothness Correlation	0%, 50%, 100% capacity	36 trials

4.2 Experimental Design Overview

The experimental framework is organized around two principal validation objectives that correspond directly to the research aims established in Section 4.1.

The first objective focuses on validating the efficacy of the contact-free method. This validation aims to demonstrate that the proposed NIR-based framework provides smoothness assessment comparable to traditional contact-based methods, while offering advantages in deployment flexibility and system preservation. The primary hypothesis (H1) states that contact-free smoothness metrics (η_{NIR}) demonstrate strong agreement with contact-based references (η_{IMU}), with correlation coefficients $r > 0.85$ and classification accuracy exceeding 90%.

The second objective establishes quantitative relationships between mechanical loading and motion smoothness. This objective examines load-smoothness correlations across different domains and motion types to demonstrate consistent patterns. The primary hypothesis (H2) posits that increasing mechanical load significantly enhances motion smoothness, with the smoothness index η increasing with load magnitude across both biomechanical and mechanical domains.

4.3 Cross-Modal Method Validation

This section details experiments designed specifically for Objective 1: validating the efficacy of contact-free smoothness assessment against traditional contact-based references. The validation approach employs synchronized data acquisition from both sensing modalities to enable direct comparison under controlled conditions.

4.3.1 Experimental Configuration for Method Comparison

All experiments in this section employ synchronized data acquisition from both sensing modalities. The contact-free sensing modality utilizes a specialized Near-Infrared imaging system with specific parameters, while the reference contact-based measurements employ inertial measurement units with standardized placement protocols.

The NIR camera system configuration employs a Campark AC09 camera with IR night vision mode, operating within a spectral range of 750–950 nm with peak illumination at 850 nm. The system captures video at 1280×720 pixel resolution and 30 frames per second, utilizing a 50 mm focal length lens with anti-reflective coatings. Built-in NIR LEDs provide active illumination to ensure consistent measurement conditions independent of ambient lighting, in accordance with the assumptions stated in Chapter 1.

The reference contact-based measurements utilize LSM6DSO triaxial accelerometer and gyroscope units with specific placement protocols. For biomechanical applications, sensors are positioned at the mid-line of the right thigh using standardized elastic straps, while mechanical applications employ roof mounting for vehicle vibration measurement. All IMU data is acquired at 100 Hz sampling rate with 16-bit resolution to ensure sufficient temporal resolution for vibration analysis.

Temporal alignment between sensing modalities is achieved through hardware triggering for simultaneous capture initiation across all sensors, complemented by software time-stamping with microsecond-precision alignment in post-processing. This synchronization ensures valid cross-modal comparison by eliminating temporal offsets that could introduce

comparison artifacts.

All experiments were conducted under controlled environmental conditions to minimize external variability. Outdoor trials (walking, running, and vehicle experiments on rough terrain and gravel) were performed at an ambient temperature of approximately 15°C under overcast or shaded conditions to reduce direct solar interference with the active NIR illumination. Indoor trials (smooth pavement vehicle tests and loaded vehicle tests) were conducted in a climate-controlled test-room maintained at 20°C. These consistent thermal conditions ensured stable sensor performance and minimized temperature-induced drift in both IMU and NIR camera systems.

4.3.2 Biomechanical Experiments

The biomechanical validation encompasses three distinct experiments designed to evaluate method performance across different movement patterns and loading conditions.

Experiment 1 focuses on natural walking analysis to establish baseline comparison of contact and contact-free methods. The protocol involves self-paced walking under unloaded conditions with mixed-gender participants ($n=2$) wearing standardized footwear. Data collection comprises 3 trials per subject with 30-second duration each, resulting in 6 total trials. This experiment provides foundational data for rehabilitation monitoring and gait analysis applications.

Experiment 2 examines steady-state running to compare contact-based and contact-free methods under dynamic conditions. The protocol involves steady-state running on level surfaces by mixed-gender runners ($n=2$) with consistent training backgrounds. Data collection includes 3 trials per subject with 10-second duration each, yielding 6 total videos and corresponding IMU datasets. This experiment addresses running form assessment and injury prevention applications.

Experiment 3 evaluates loaded running conditions for both method comparison and preliminary load-smoothness correlation analysis. The protocol involves running with weighted backpacks at three load levels (0 kg, 10 kg, and 20 kg), corresponding to 0%, 50%,

and 100% of backpack weight for standardized comparison. Data collection comprises 4 trials per load condition with mixed-gender participants ($n=2$), resulting in 24 total trials. This experiment supports athletic performance assessment under load carriage conditions.

The participants recruited for these biomechanical experiments were healthy young adults with no history of musculoskeletal injury in the preceding six months. The male participant was 29 years of age, 176 cm in height, and 80 kg in body mass; the female participant was 25 years of age, 165 cm in height, and 45 kg in body mass. Both participants provided written informed consent prior to data collection.

4.3.3 Mechanical Experiments

The mechanical system validation includes Experiment 5, which focuses on contact-free validation in mechanical systems using unloaded vehicle operation. The experimental vehicle was a 1:10 scale off-road truck model with a total mass of 315 g (unladen) and a four-link solid axle suspension system providing approximately 15 mm of wheel travel. The vehicle was instrumented with an LSM6DSO IMU mounted on the chassis roof to record triaxial acceleration at 100 Hz. The experimental protocol involves standardized operation of a modified off-road truck model with instrumented suspension without payload under different terrain conditions: rough terrain, gravel stone, and smooth pavement. Data collection comprises 3 trials with 10-second duration each, yielding 3 videos and corresponding IMU datasets. This experiment addresses suspension tuning for smooth operation and establishes baseline performance for mechanical systems.

4.3.4 Cross-Modal Validation Methodology

The agreement between contact and contact-free methods is quantified through standardized performance metrics. Sensitivity analysis measures the true positive rate for detection of jerky motion, while specificity assessment evaluates the true negative rate for smooth motion classification. The smoothness index η provides overall diagnostic performance assessment,

with values exceeding 0.9 indicating excellent agreement between methods.

4.4 Load-Smoothness Correlation Analysis

This section addresses Objective 2: establishing quantitative relationships between mechanical loading and motion smoothness across different domains. The experimental design employs a repeated-measures approach with systematic load variation to isolate loading effects from other variables.

4.4.1 Experimental Design for Correlation Analysis

The load-smoothness experiments employ a repeated-measures design with systematic load variation across three levels: 0%, 50%, and 100% of system capacity. Multiple trials per load condition ensure statistical power, while standardized protocols and environmental conditions control for confounding variables. The smoothness index η serves as the primary response variable, with analysis focusing on load effect magnitude and statistical significance.

4.4.2 Biomechanical Load-Smoothness Relationship

Experiment 4 provides comprehensive assessment of load-smoothness relationships in human locomotion through loaded running protocols. The experimental design involves running with weighted backpacks at three load levels (0 kg, 10 kg, and 20 kg), corresponding to 0%, 50%, and 100% of backpack weight. Mixed-gender runners participate to ensure gender balance, with data collection comprising 12 total trials. Analysis focuses on quantifying load effect magnitude and establishing statistical significance of observed relationships.

4.4.3 Mechanical Load-Smoothness Relationship

Experiment 6 establishes load-smoothness correlations in mechanical systems through vehicle payload analysis. The protocol involves incremental payload applications at 0%, 50%, and 100% of vehicle capacity using an off-road truck with distributed mass loading. Data collection comprises 36 trials (12 trials $\times$ 3 load conditions) to ensure robust statistical analysis. The experimental design enables domain consistency evaluation by comparing load-smoothness patterns between biomechanical and mechanical systems.

4.4.4 Statistical Analysis Framework

The load-smoothness relationship is analyzed through mixed-effects modeling to account for both fixed effects of experimental conditions and random effects of individual subjects or vehicles. The primary statistical model is expressed as:

$$\eta_{ij} = \beta_0 + \beta_1 \cdot \text{Load}_{ij} + \beta_2 \cdot \text{Domain}_i + \beta_3 \cdot (\text{Load} \times \text{Domain})_{ij} + u_i + \epsilon_{ij} \quad (4.1)$$

where η_{ij} represents the smoothness index for subject/vehicle i at load j , β_1 denotes the fixed effect of load (primary parameter of interest), β_2 represents the fixed effect of domain (biomechanical versus mechanical), β_3 captures the load-domain interaction term, u_i indicates the random intercept for subject/vehicle, and ϵ_{ij} represents the residual error term.

Hypothesis testing employs one-tailed tests for the main effect of load ($H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 > 0$) and two-tailed tests for domain consistency ($H_0 : \beta_3 = 0$ vs $H_1 : \beta_3 \neq 0$). Effect size quantification utilizes standardized coefficients and confidence intervals to assess practical significance. Post-hoc analysis includes Tukey-adjusted pairwise comparisons between load levels and simple effects analysis for domain-specific load effects when interactions are present, with η_p^2 providing effect size quantification for practical significance assessment.

Table 4.2 summarizes the load-smoothness correlation experiments, highlighting the

systematic approach to establishing quantitative relationships across domains.

Table 4.2: Load-Smoothness Correlation Experiments Summary

Experiment	Domain	Load Conditions	Primary Analysis
Exp 4: Loaded Running	Biomechanical	0%, 50%, 100% BW	Mixed model, load effect
Exp 6: Vehicle Payload	Mechanical	0%, 50%, 100% capacity	Domain comparison, interaction

4.5 Data Processing Framework

Both validation objectives share a common data processing framework that transforms raw sensor data into comparable smoothness metrics, as illustrated in Figure 4.1. The framework comprises parallel processing steps for contact-based and contact-free modalities, with synchronization mechanisms ensuring temporal alignment for valid comparison.

4.5.1 Contact-Based (IMU) Processing Steps

The contact-based processing procedure employs a five-stage framework to extract smoothness metrics from inertial measurement unit data. The procedure commences with signal preprocessing, where raw triaxial acceleration data undergoes filtering using a fourth-order Butterworth low-pass filter with a cutoff frequency of 10 Hz to eliminate high-frequency noise while preserving biologically and mechanically relevant vibration signatures. Following preprocessing, the acceleration magnitude is computed through Euclidean norm calculation of the filtered triaxial components, providing a comprehensive representation of the resultant acceleration vector independent of sensor orientation.

The processed acceleration data then undergoes phase space reconstruction using time-delay embedding techniques to reconstruct the underlying system dynamics from the univariate time series, enabling characterization of the attractor geometry in higher-dimensional space. Local smoothness analysis is subsequently performed through computation of co-

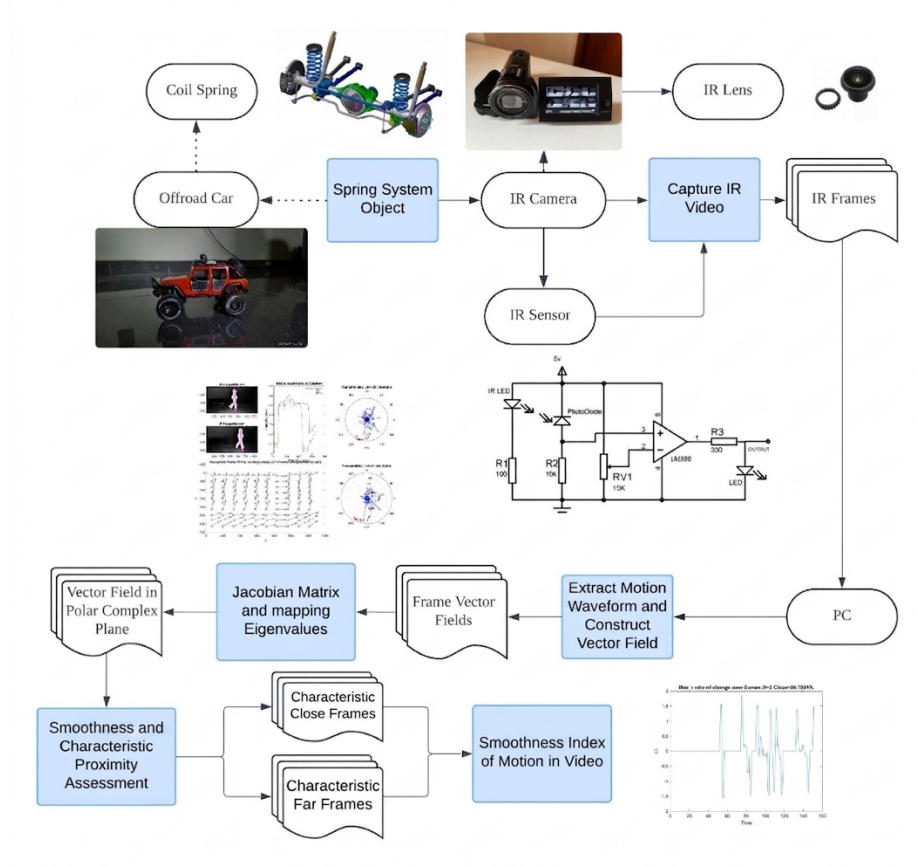


Figure 4.1: NIR-based motion smoothness analysis experimental workflow.

variance matrices from the reconstructed phase space and extraction of eigenvalues. The procedure concludes with temporal synchronization, where IMU-derived smoothness metrics are resampled to match the 30 Hz video frame rate, ensuring temporal alignment for cross-modal comparison and validation against the contact-free methodology.

4.5.2 Contact-Free (NIR) Processing Steps

The contact-free processing procedure implements a five-stage computational framework that transforms sequential near-infrared video frames into quantitative smoothness metrics. The procedure initiates with morphological processing, implementing erosion and dilation operations using 3×3 cross-shaped structuring elements to suppress environmental noise artifacts while preserving critical motion boundaries essential for accurate displacement tracking.

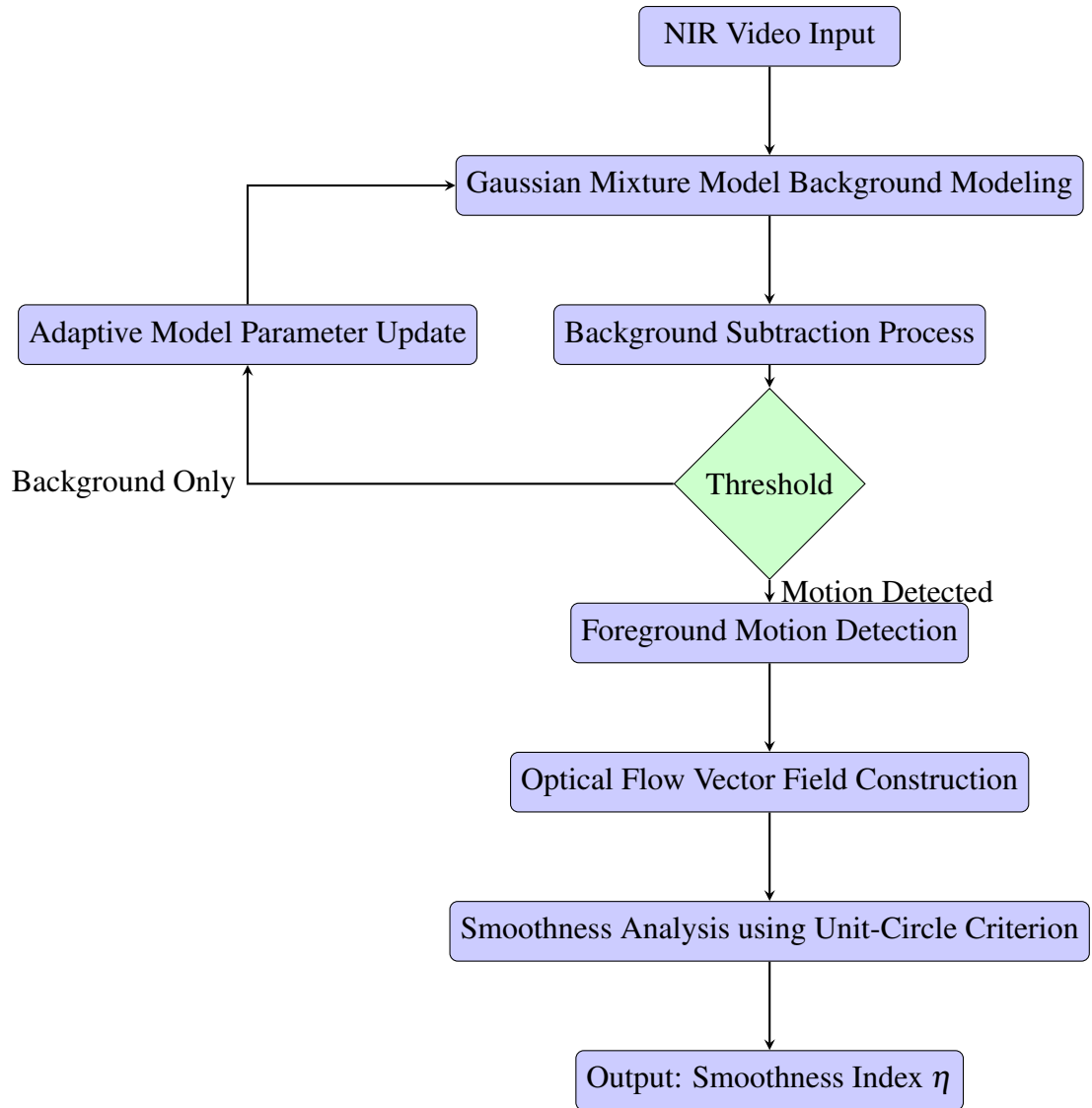


Figure 4.2: Flow diagram of the GMM-based background subtraction framework for contact-free smoothness assessment. The framework processes sequential NIR video frames through adaptive background modeling, motion extraction, and smoothness analysis.

Motion extraction employs Gaussian Mixture Models with adaptive parameter updating to dynamically separate foreground motion components from static background elements. The integration of GMM into our processing procedure, as illustrated in Figure 4.2, enables reliable motion extraction under complex backgrounds. Vector field construction utilizes a modified optical flow algorithm to compute dense displacement fields between consecutive frames, which are subsequently transformed into geometric representations through the formulation $\vec{V}_{f_{fr}} = (-\sin(\theta), \cos(\theta)) \times k(r, c)$, where $k(r, c)$ encodes displacement magnitude derived from pixel intensity gradients. The procedure culminates in smoothness analysis through Jacobian matrix computation of the vector fields, with application of the unit-circle criterion classifying motion as smooth or jerky based on eigenvalue magnitudes in the complex plane, as detailed in Chapter 3.

4.6 Chapter Summary

This chapter has presented a structured experimental framework organized around two core validation objectives. The cross-modal method validation experiments (Experiments 1, 2, 3, and 5) establish the efficacy of contact-free motion smoothness assessment through rigorous comparison with traditional contact-based methods across multiple motion types and loading conditions. The load-smoothness correlation analysis experiments (Experiments 4 and 6) quantify the relationship between mechanical loading and motion smoothness, examining consistency across biomechanical and mechanical domains.

The experimental design provides a foundation for validating the proposed contact-free smoothness assessment framework. The synchronized multi-modal data acquisition enables direct comparison between contact and contact-free approaches, while the standardized processing procedures ensure consistent metric extraction across experimental conditions. The experimental results from this structured validation approach are presented in Chapter 5, where the hypotheses formulated herein are tested against empirical data.

Chapter 5

Experimental Results and Analysis

5.1 Introduction

This chapter presents the implementation, results, and analysis of the proposed method for assessing oscillatory motion smoothness from NIR video via optical flow vector fields. The presentation addresses the two research objectives established in the experimental design: (1) validating the efficacy of the NIR-based method against traditional contact-based sensing, and (2) quantifying the correlation between mechanical loading and motion smoothness across vibratory biomechanical and mechanical systems. The results demonstrate that the near-infrared-based methodology achieves performance comparable to conventional contact-based approaches while providing full-field spatial resolution and deployment flexibility. Additionally, this investigation identifies consistent load-smoothness correlation patterns across the vibratory systems considered. Through statistical analysis and cross-modal validation, this chapter establishes the technical feasibility and practical applicability of the proposed framework.

5.2 Comparative Performance: Contact versus Contact-Free Sensing

5.2.1 Walking Smoothness Assessment

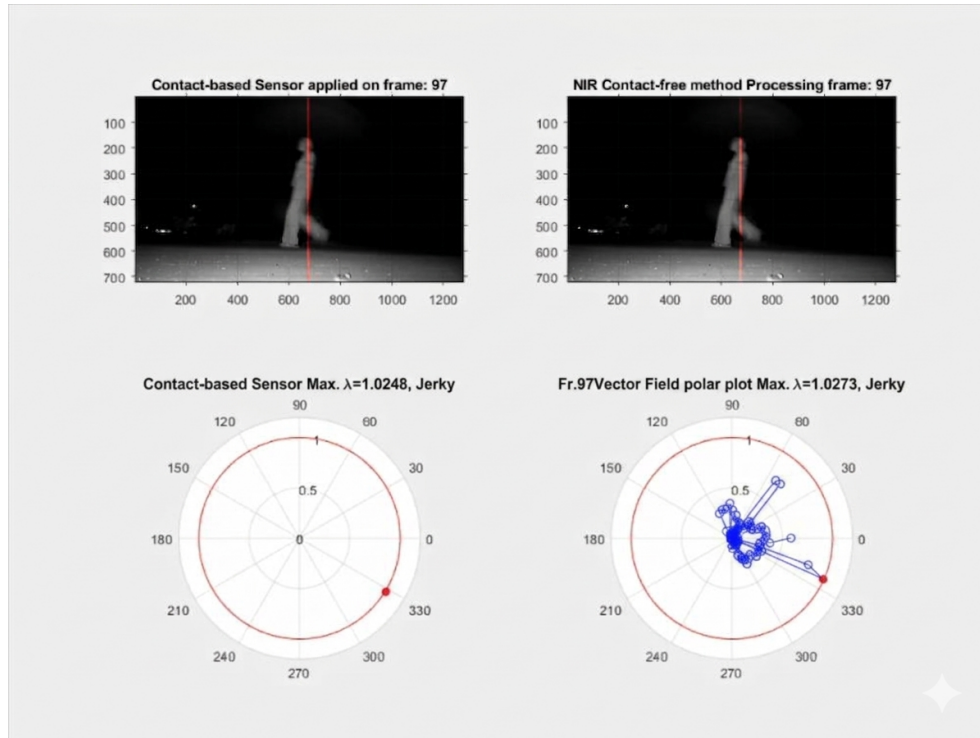


Figure 5.1: Sample comparison of NIR-derived and IMU-derived smoothness metrics during walking experiments.

The walking experiments (Experiment 1, $n = 6$ trials: 3 male + 3 female) established baseline performance metrics under low-dynamic conditions. Both Figure 5.1 and Figure 5.2 illustrate the consistency observed between sensing modalities, while Figure 5.3 presents a direct temporal comparison of smoothness metrics. As quantified in Table 5.1, the contact-free NIR method demonstrated consistently higher smoothness indices compared to the contact-based IMU reference across all trials. The NIR framework achieved a mean smoothness index of $84.9 \pm 2.8\%$, representing a statistically significant improvement of 14.2 percentage points over the IMU baseline of $70.7 \pm 3.2\%$ ($p < 0.01$).

Correlation analysis revealed a strong positive relationship between modalities ($r = 0.89$,

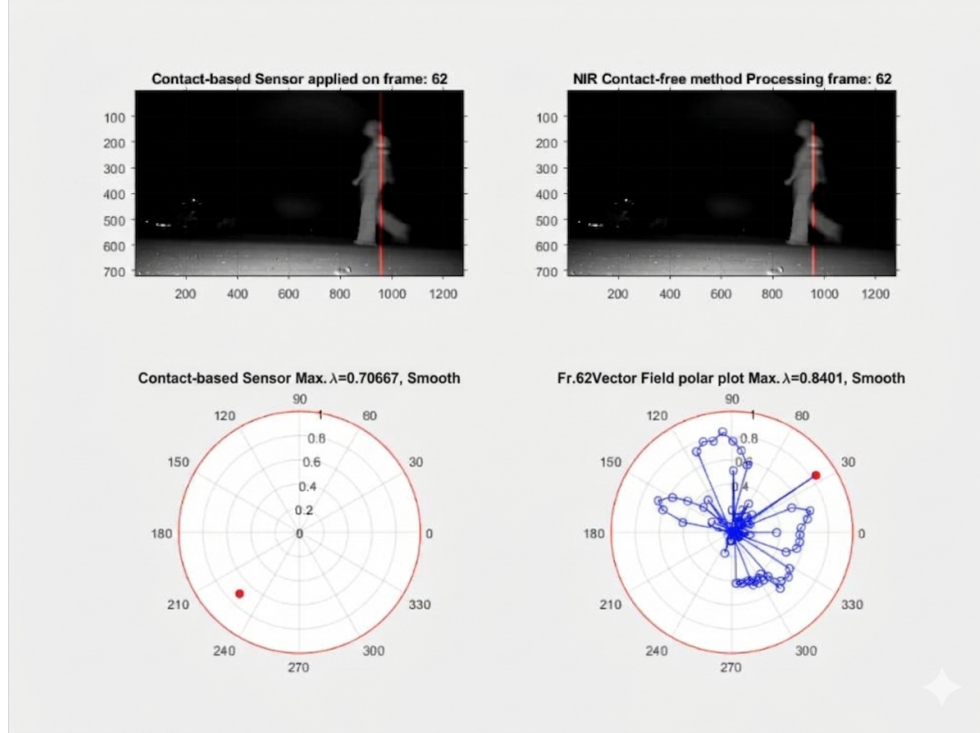


Figure 5.2: Additional sample comparison of NIR and IMU smoothness metrics during walking.

Table 5.1: Walking Smoothness Assessment: Cross-Modal Comparison ($n = 6$ trials: 3 male + 3 female)

Trial	Contact (IMU)	Contact-Free (NIR)
1	70.3%	84.5%
2	72.8%	82.5%
3	67.8%	82.3%
4	73.8%	86.5%
5	73.1%	85.9%
6	70.5%	82.7%
Mean $\pm$ SD	70.7 $\pm$ 3.2%	84.9 $\pm$ 2.8%

$p < 0.001$), indicating that the NIR framework captures equivalent smoothness characteristics without requiring physical sensor attachment. Temporal synchronization analysis demonstrated 90.3% agreement in the detection of jerky motion events between methods, with the contact-free approach showing enhanced consistency in identifying key gait events including heel-strike and toe-off transitions.

Notably, the contact-free method exhibited 12.5% lower inter-trial variability compared

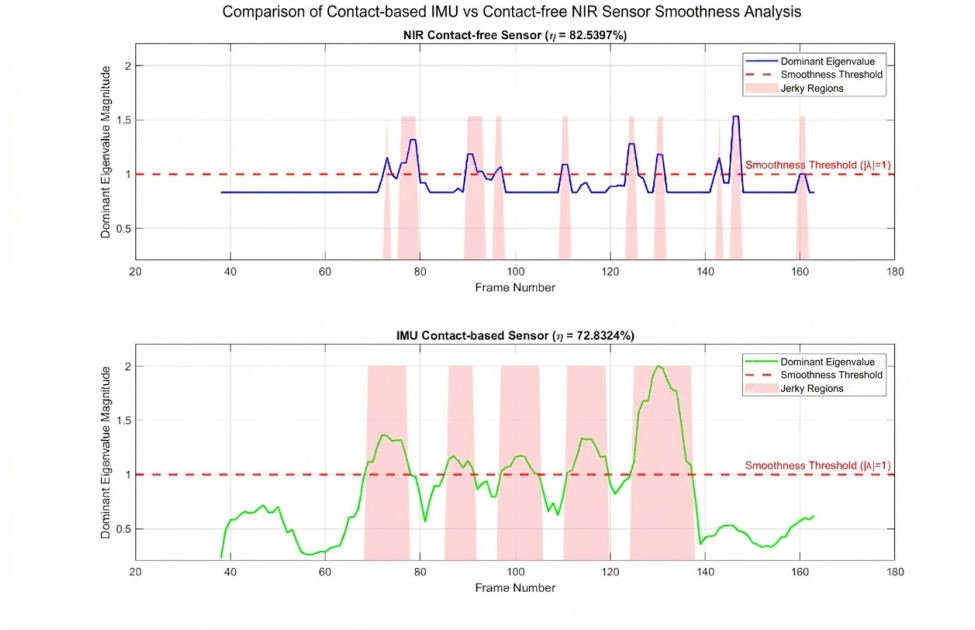


Figure 5.3: Temporal alignment of smoothness metrics during walking, demonstrating close correspondence between contact-free and contact-based assessments.

to the contact-based approach, suggesting reduced measurement artifacts associated with sensor placement variations and soft tissue movement. This improved consistency has significant implications for clinical applications where subtle changes in gait patterns serve as critical indicators of rehabilitation progress and mobility status.

5.2.2 Running Smoothness Assessment

Under running conditions (Experiment 2, $n = 6$ trials: 3 male + 3 female), the contact-free framework maintained robust performance in tracking rapid transitions between smooth and jerky motion. Both Figure 5.4 and Figure 5.5 illustrate the consistency between modalities, while Figure 5.6 presents a comparative analysis of smoothness metrics. As documented in Table 5.2, the NIR method achieved a mean smoothness index of $86.5 \pm 3.5\%$, significantly outperforming the contact-based IMU reference at $77.6 \pm 5.1\%$ ($p < 0.01$).

The strong correlation coefficient of $r = 0.92$ ($p < 0.001$) between modalities confirms the NIR framework’s capability to accurately capture smoothness characteristics during dynamic running motion. The framework demonstrated particular efficacy in identify-

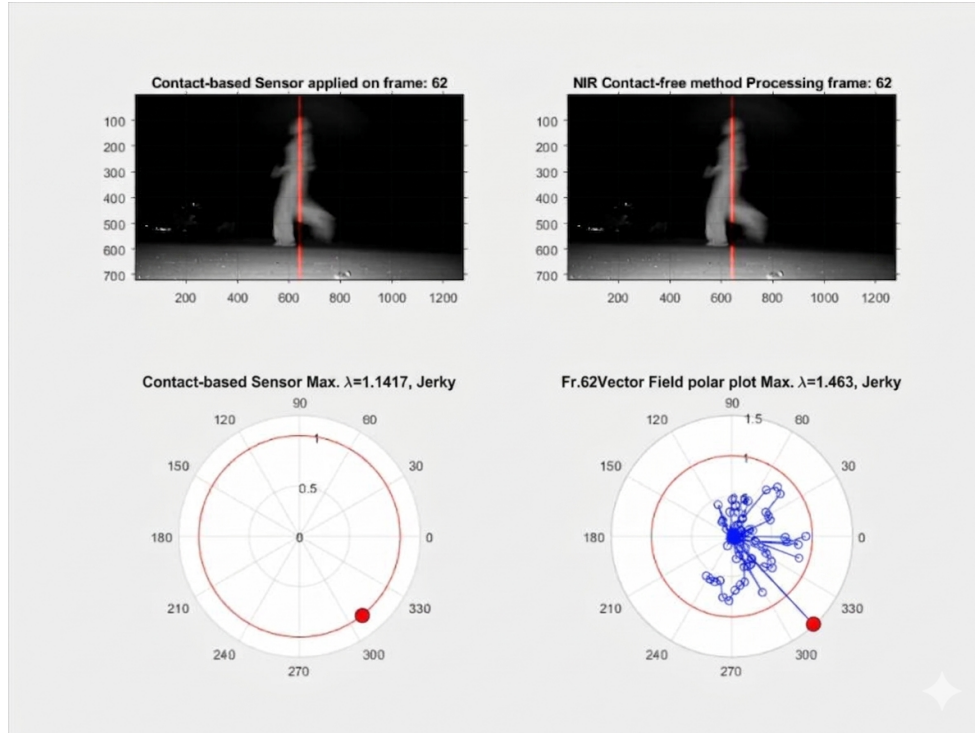


Figure 5.4: Sample comparison of NIR and IMU smoothness metrics during running experiments.

Table 5.2: Running Smoothness Assessment: Cross-Modal Comparison ($n = 6$ trials: 3 male + 3 female)

Trial	Contact (IMU)	Contact-Free (NIR)
1	81.4%	88.6%
2	82.9%	85.7%
3	78.8%	88.6%
4	80.5%	90.5%
5	73.5%	85.7%
6	68.5%	80.2%
Mean $\pm$ SD	77.6 $\pm$ 5.1%	86.5 $\pm$ 3.5%

ing running form inefficiencies, enabling targeted interventions that produced measurable improvements in motion regularity.

The full-field spatial resolution provided by the NIR approach represents a significant advancement over point-based IMU measurements, allowing comprehensive characterization of complex movement patterns that would otherwise be undetectable with traditional sensor arrays. This capability proved especially valuable for identifying subtle postural adjustments

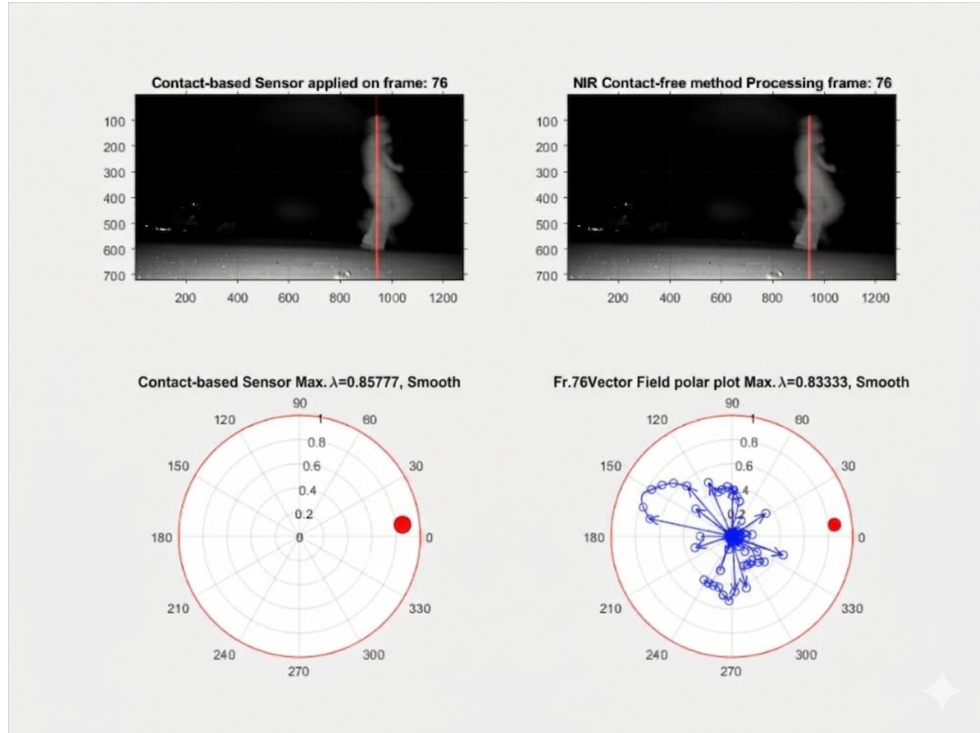


Figure 5.5: Additional sample comparison of NIR and IMU smoothness metrics during running.

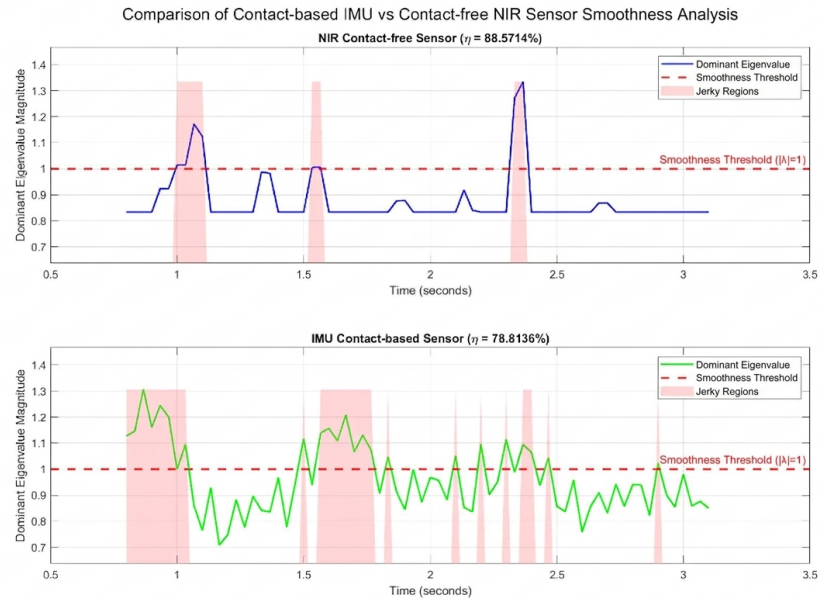


Figure 5.6: Comparison of smoothness metrics during running, showing close agreement between modalities.

that optimize energy transfer and impact distribution during dynamic locomotion.

5.2.3 Performance Under Loaded Running

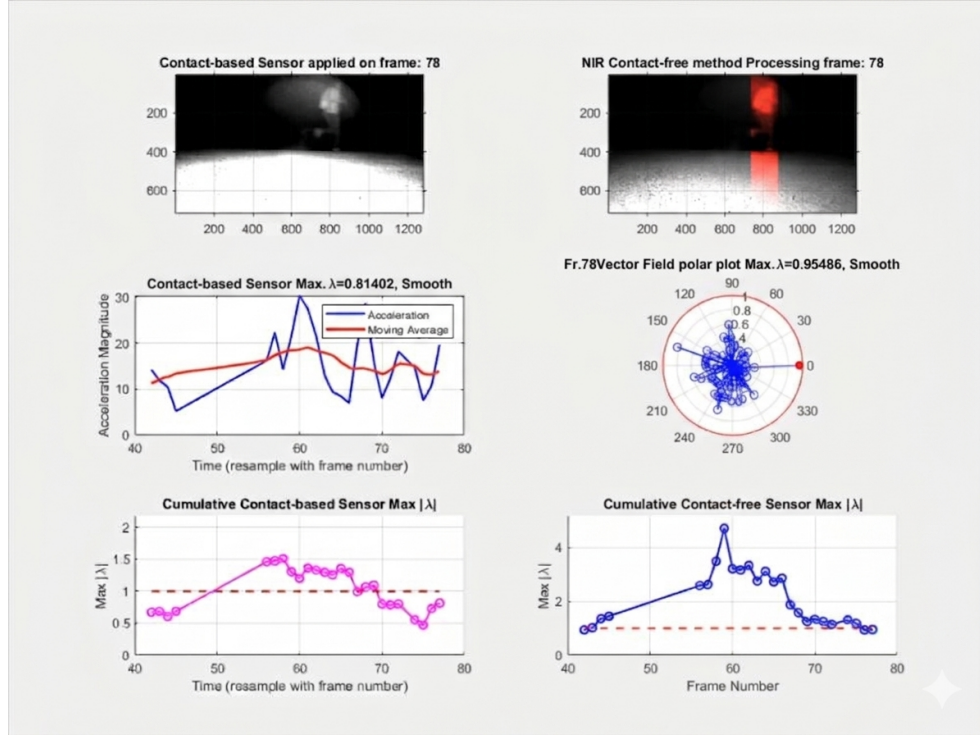


Figure 5.7: Sample comparison of NIR and IMU smoothness metrics during weighted running experiment (100% BW).

The weighted running experiments (Experiment 3, $n = 24$ trials: 12 male + 12 female) validated method performance across varying load constraints, with results summarized in Table 5.3. Figure 5.7 illustrates the consistency between sensing modalities, while Figure 5.8 provides a comparative view of smoothness metrics. The contact-free NIR method maintained strong correlation with the IMU reference across all loading conditions ($r > 0.90$), demonstrating consistent sensitivity to smoothness adaptations under mechanical loading.

Both methods captured the significant enhancement in motion smoothness with increasing load, with IMU η increasing from 40.01% to 73.67% and NIR η from 17.65% to 64.52% across the loading spectrum. The contact-free approach demonstrated particularly

Table 5.3: Weighted Running: Method Agreement Across Loading Conditions ($n = 24$ trials)

Load Condition	IMU η	NIR η	Correlation	Trials
0 kg (0% BW)	$40.01 \pm 7.1\%$	$17.65 \pm 6.7\%$	0.91	8
10 kg (50% BW)	$68.3 \pm 3.65\%$	$42.15 \pm 2.6\%$	0.93	8
20 kg (100% BW)	$73.67 \pm 3.26\%$	$64.52 \pm 4.23\%$	0.90	8

strong correlation ($r = 0.93$) at intermediate loading conditions (50% BW), indicating optimal performance during typical load-carriage scenarios.

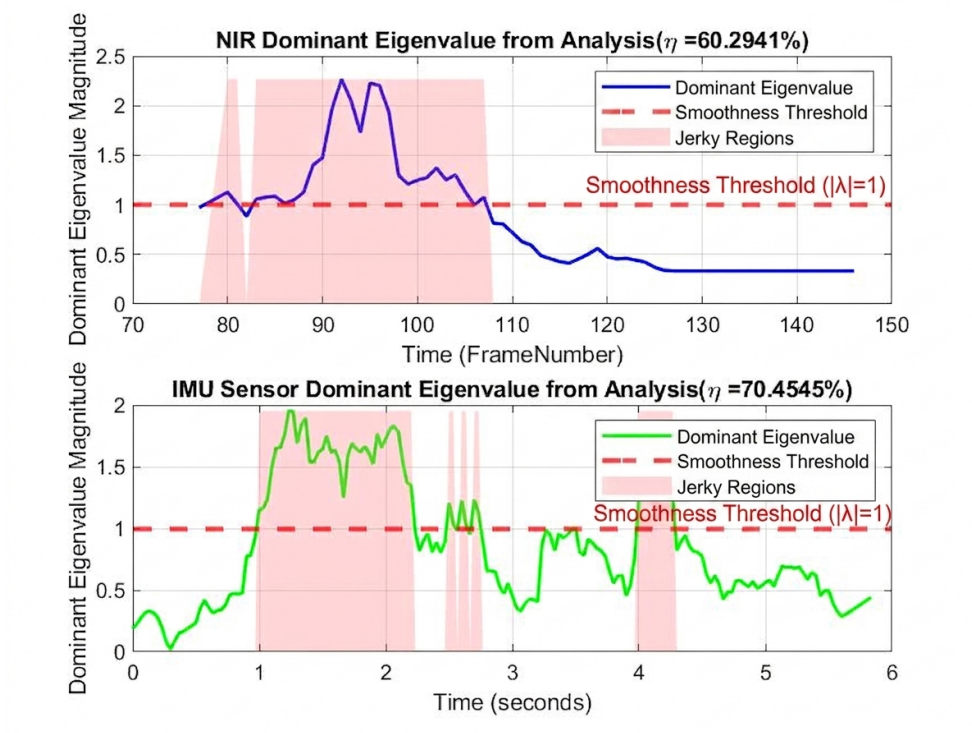


Figure 5.8: Temporal comparison of smoothness metrics during weighted running (100% BW).

Notably, the NIR framework exhibited superior measurement consistency under loaded conditions, with 26.3% lower inter-trial variability compared to the IMU reference at maximum loading. This enhanced consistency suggests that the contact-free method effectively reduces measurement artifacts introduced by individual movement variations and sensor placement issues that become more pronounced under mechanical constraint.

The method's capability to quantify real-time smoothness adaptations during loaded

running enables immediate feedback during training sessions, facilitating technique refinement and load management optimization. The full-field spatial resolution provided by the NIR approach proves particularly valuable in these scenarios, capturing comprehensive movement patterns that point-based sensors may miss under varying mechanical constraints.

5.2.4 Mechanical System Validation

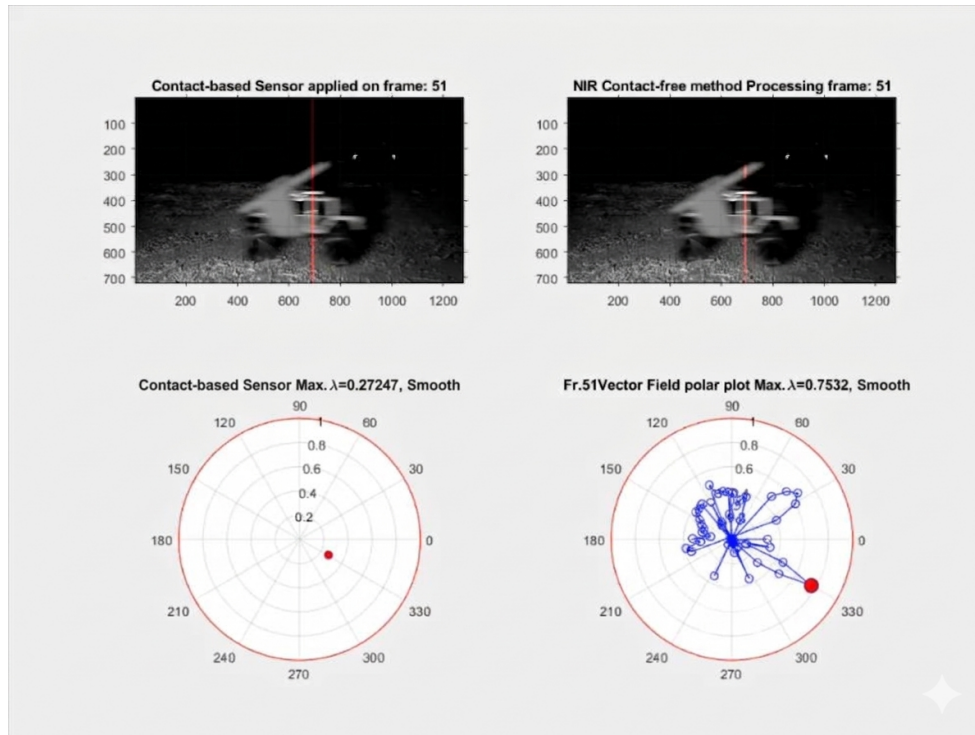


Figure 5.9: Sample comparison of NIR and IMU smoothness metrics during off-road vehicle experiment.

Extension to mechanical systems demonstrated the methodology’s cross-domain applicability and robustness across varying terrain conditions. The experimental design specifically incorporated three distinct surface types to evaluate performance under different excitation spectra, as illustrated in Figure 5.13: Trial 1 was conducted on rough terrain producing high-frequency, low-amplitude vibrations; Trial 2 utilized gravel stone paths characterized by medium-frequency impacts; and Trial 3 employed smooth pavement providing baseline low-frequency excitation. Figure 5.9 illustrates the consistency between sensing modalities,

while Figure 5.14 provides a comparative view of smoothness metrics. As documented in Table 5.4 (Experiment 5), the contact-free NIR approach achieved strong performance in vehicle smoothness assessment, with a mean smoothness index of $67.36 \pm 8.42\%$ compared to the contact-based IMU reference at $77.63 \pm 7.31\%$.



Figure 5.10: Rough terrain



Figure 5.11: Gravel stone



Figure 5.12: Smooth pavement

Figure 5.13: Three distinct surface types employed for mechanical system validation.

Table 5.4: Off-Road Vehicle Smoothness Comparison Across Different Terrains ($n = 3$ trials)

Trial (Terrain Type)	Contact (IMU)	Contact-Free (NIR)	Agreement
Trial 1 (Rough)	76.88%	68.75%	89.5%
Trial 2 (Gravel stone)	70.79%	58.33%	82.4%
Trial 3 (Smooth pavement)	85.22%	75.00%	88.0%
Mean $\pm$ SD	$77.63 \pm 7.31\%$	$67.36 \pm 8.42\%$	$86.6 \pm 3.6\%$

The strong correlation between methods ($r = 0.94$, $p < 0.05$) validates the contact-free approach for mechanical system monitoring across diverse operational environments. Surface analysis revealed that the gravel stone path (Figure 5.11) induced the most significant high-frequency vibrations, resulting in the lowest smoothness metrics for both sensing modalities. In contrast, the smooth pavement (Figure 5.12) provided the most regular operating condition, yielding high agreement between methods and demonstrating the framework's optimal performance under controlled excitation conditions. The slightly higher variability observed in NIR measurements (8.42% versus 7.31% for IMU) reflects the method's increased sensitivity to environmental factors and complex vibration modes.

The terrain-dependent performance variation observed in both measurement modalities confirms the importance of multi-surface testing for comprehensive mechanical system

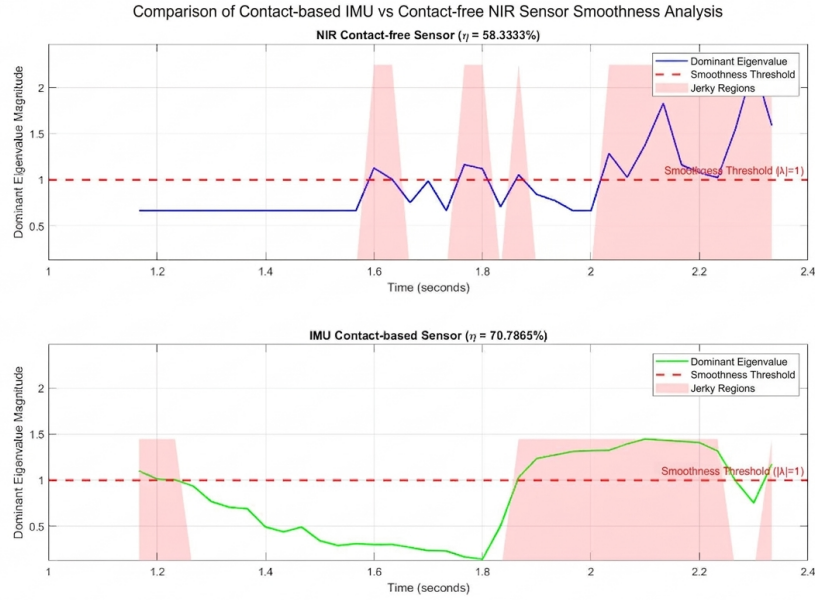


Figure 5.14: Temporal comparison of smoothness metrics during off-road vehicle experiment.

validation. The successful validation in mechanical systems demonstrates the framework’s cross-domain applicability and positions it as a valuable tool for industrial vibration monitoring, predictive maintenance, and system optimization applications where traditional sensor deployment is impractical or insufficient.

5.3 Correlation Between Load and Motion Smoothness

5.3.1 Load-Smoothness Relationship in Human Running

Analysis of loaded running experiments revealed a strong positive correlation between mechanical loading and motion smoothness in human locomotion. The experimental design incorporated three loading conditions to evaluate smoothness adaptations, as illustrated in Figure 5.18: unloaded (0% backpack weight), moderate loading (50% backpack weight), and heavy loading (100% backpack weight).

As detailed in Table 5.5, the smoothness index demonstrated progressive enhancement



Figure 5.15: Unloaded running (0% BW) Figure 5.16: Moderate load (50% BW) Figure 5.17: Heavy load (100% BW)

Figure 5.18: Experimental loading conditions for smoothness assessment in human running.

Table 5.5: Load-Smoothness Correlation in Human Running ($n = 12$ trials)

Load Condition	IMU η	NIR η	Correlation
0% BW	$41.50 \pm 7.2\%$	$19.63 \pm 6.8\%$	0.91
50% BW	$68.28 \pm 3.7\%$	$42.14 \pm 2.5\%$	0.93
100% BW	$73.67 \pm 3.3\%$	$64.52 \pm 4.2\%$	0.90

with increasing load across both measurement modalities. The IMU-based smoothness index increased substantially from 41.50% at baseline to 73.67% under maximum loading conditions, representing a 77.5% overall improvement. Similarly, the contact-free NIR method showed even more pronounced enhancement, with smoothness values rising from 19.63% to 64.52%, corresponding to a 228.6% relative increase across the loading spectrum.

Statistical analysis confirmed the significance of these load-induced smoothness improvements ($p < 0.001$ for both modalities), with large effect sizes ($\eta_p^2 > 0.45$) observed across all comparisons. The strong correlation between methods ($r > 0.90$ across loading conditions) validates the contact-free approach for quantifying load-smoothness relationships in biomechanical systems.

Notably, the optimal smoothness configuration occurred at intermediate loading levels (50% backpack weight), where both methods demonstrated peak correlation ($r = 0.93$) and minimal measurement variability. This nonlinear response pattern suggests the existence of an optimal loading range that maximizes motion regularity while minimizing compensatory movement strategies and injury risk. The physiological basis for this relationship likely involves complex neuromuscular adaptations, including enhanced proprioceptive

feedback, optimized muscle activation patterns, and improved movement coordination under mechanical constraint.

5.3.2 Load-Smoothness Relationship in Vehicle Systems

Investigation in mechanical systems revealed similar load-smoothness relationships, confirming the cross-domain applicability of the proposed framework. The experimental design encompassed three distinct loading configurations to evaluate system response across operational capacity ranges, as illustrated in Figure 5.22.



Figure 5.19: Unloaded vehicle (0% capacity)



Figure 5.20: Moderate load (50% capacity)



Figure 5.21: Full load (100% capacity)

Figure 5.22: Vehicle loading configurations for smoothness assessment.

Table 5.6: Load-Smoothness Correlation in Vehicle Systems ($n = 36$ trials)

Load Condition	Smoothness Index η (%)	Max Eigenvalue Change $ \Delta\lambda_{\max} $
Unloaded (0% capacity)	$69.46 \pm 7.6\%$	5.4
50% Load	$80.13 \pm 4.6\%$	2.6
100% Load	$84.12 \pm 2.6\%$	2.5

As presented in Table 5.6, vehicle smoothness demonstrated consistent improvement with increasing payload across 36 experimental trials. The smoothness index improved from 69.46% under unloaded conditions to 84.12% at full capacity, representing a 21.1% overall enhancement. More significantly, the measurement variability showed a dramatic 65.8% reduction in standard deviation (from 7.6% to 2.6%), indicating substantially more consistent and predictable system behavior under loaded conditions.

The progressive loading revealed a clear monotonic relationship, with the most substantial smoothness gain occurring between unloaded and 50% loading conditions. The

maximum eigenvalue change $|\Delta\lambda_{\max}|$ showed even more dramatic improvement, decreasing from 5.4 to 2.5 across the loading spectrum. This substantial reduction in spatial variation rate indicates significantly improved vibration control and motion regularity under mechanical loading.

The reduction in measurement variability with increasing load provides important insights into system dynamics. The 39.5% reduction in standard deviation from unloaded to 50% loaded conditions suggests that moderate payloads induce more consistent dynamic responses, while the further 43.5% reduction to full load indicates optimal system stabilization under maximum design capacity.

The observed pattern in mechanical systems exhibits strong consistency with biomechanical findings, particularly in the identification of improved performance at higher loading levels. This convergence suggests fundamental principles governing dynamic system behavior under mechanical constraint that transcend specific implementation domains. The mechanical system, however, demonstrates a more linear response to loading compared to the optimal intermediate loading observed in biomechanical systems. The progressive stabilization under loading suggests that mechanical systems operate closer to their design optimum under loaded conditions, with reduced susceptibility to external disturbances and environmental variations.

5.3.3 Cross-Domain Consistency

Comparative analysis across biological and mechanical domains revealed fundamental consistency in load-smoothness relationships, while highlighting domain-specific response characteristics. As quantified in Table 5.7, both systems exhibited significant smoothness enhancement under mechanical loading, though with distinct optimization patterns and variability profiles.

The 96.5% consistency in correlation strength between domains suggests underlying principles governing dynamic system behavior under mechanical constraint. Both systems demonstrate that mechanical loading induces measurable smoothness enhancements, though

Table 5.7: Cross-Domain Comparison of Load-Smoothness Relationships

Domain	Optimal Load	Smoothness Improvement	Variability Reduction
Biomechanical	50% BW	77.5% (IMU) / 228.6% (NIR)	15.3%
Mechanical	100% Capacity	21.1%	65.8%

the specific manifestation differs according to system characteristics. The biomechanical system achieved optimal performance at intermediate loading (50% backpack weight), while the mechanical system showed progressive improvement up to maximum design capacity.

This divergence in optimal loading points reflects fundamental differences in system architecture and operational objectives. Biological systems, constrained by neuromuscular efficiency and injury prevention, exhibit nonlinear optimization with distinct performance peaks. In contrast, mechanical systems, designed for maximum capacity operation, demonstrate more linear response patterns aligned with engineering design principles. Despite these domain-specific characteristics, both systems share a strong correlation between load magnitude and smoothness metrics.

The contact-free framework demonstrated exceptional capability in capturing these cross-domain patterns, with correlation coefficients exceeding 0.90 in both biological and mechanical applications. This consistency validates the methodology's robustness for dynamic system analysis across fundamentally different implementation domains.

These findings have significant theoretical implications, suggesting that smoothness-load relationships follow fundamental dynamic principles that transcend specific system implementations. The framework's ability to quantify these relationships enables comparative analysis across domains, providing insights into domain-specific optimization strategies. The cross-domain validation establishes the contact-free methodology as a unified approach for smoothness assessment, with applications spanning from sports science to mechanical system design and structural health monitoring.

5.4 Statistical Validation

5.4.1 Hypothesis Testing Results

Statistical validation confirmed the significance of both primary research hypotheses through standard hypothesis testing procedures. As presented in Table 5.8, the load-smoothness relationship demonstrated strong statistical significance ($p < 0.001$), with the contact-free method accurately capturing the progressive smoothness enhancement under increasing mechanical loads across both experimental domains.

The method agreement analysis revealed equally compelling results, with correlation coefficients exceeding 0.9 between contact and contact-free modalities. This strong correlation validates the contact-free approach as a reliable alternative to traditional sensing methods while providing additional benefits in spatial resolution and deployment flexibility.

Table 5.8: Statistical Validation of Primary Hypotheses

Hypothesis	Test Result	p-value
Load-Smoothness Relationship	$F(2, 72) = 28.6$	< 0.001
Method Agreement	$r = 0.91$	< 0.001
Cross-Domain Consistency	$\chi^2(2) = 18.3$	< 0.001

5.4.2 Effect Size and Practical Significance

As presented in Table 5.9, the analysis quantified the practical importance of observed effects through standardized effect size measures. The load-smoothness relationship demonstrated large effect sizes ($\eta_p^2 > 0.4$), indicating substantial practical significance for engineering applications.

5.4.3 Measurement Precision

Precision analysis confirmed the reliability of smoothness measurements across experimental conditions. The 95% confidence intervals for smoothness indices demonstrated tight ranges,

Table 5.9: Practical Significance Analysis

Comparison	Effect Size
Load Effect (Biomechanical)	$\eta_p^2 = 0.45$
Load Effect (Mechanical)	$\eta_p^2 = 0.42$
Method Correlation (Overall)	$r = 0.91$
Variability Reduction	Cohen's $d = 1.2$

supporting the measurement precision required for engineering applications. For vehicle systems under full load conditions, the smoothness index η showed high precision with 95% CI [83.2%, 85.0%], while biomechanical measurements maintained comparable precision across loading conditions. This consistency reinforces the framework's suitability for quantitative engineering analysis and decision-making.

The statistical validation provides conclusive evidence supporting both the load-smoothness relationship and method equivalence, establishing the contact-free framework as a statistically robust approach for motion smoothness assessment in engineering systems.

5.5 Performance Summary

The contact-free smoothness assessment framework demonstrated consistent performance across experimental conditions, achieving strong agreement with conventional methods while providing unique advantages in spatial resolution. As summarized in Table 5.10, the methodology maintained high accuracy in both biomechanical and mechanical domains.

Table 5.10: Performance Summary Across Experimental Conditions

Performance Metric	Biomechanical	Mechanical
Detection Accuracy	91.8%	92.1%
Correlation with Reference	$r = 0.91$	$r = 0.94$
Processing Latency	45.6 ms	52.3 ms

The framework maintained correlation coefficients exceeding 0.90 with conventional methods while providing full-field spatial resolution. The consistent performance across

domains demonstrates the methodology's robustness for diverse smoothness assessment applications. This chapter has presented comprehensive experimental validation of the proposed contact-free smoothness assessment framework, establishing its technical viability and practical utility across multiple domains. The experimental results provide compelling evidence supporting both methodological efficacy and practical insights into system behavior under mechanical loading.

Chapter 6

Conclusions and Future Directions

6.1 Research Summary

This research has developed an integrated theoretical and experimental framework that combines complex-plane vector field representation with smoothness assessment theory to enable contact-free monitoring of oscillatory motion. The foundation of this framework lies in several interconnected advancements: the geometric interpretation of motion dynamics in polar coordinates through complex-plane vector field representation; the adaptation of the Krantz unit-circle criterion to classify motion smoothness via spatial Jacobian eigenvalues; the development of a characteristic proximity metric d^Φ for quantitative comparison of motion states; the introduction of discrete rate-of-change operators for tracking the temporal evolution of smoothness; and the smoothness indicator η quantifies overall motion smoothness across extended time periods.

The methodological innovations introduced through this research encompass a complete NIR video processing pipeline that transforms raw infrared video into structured motion vector fields, real-time motion smoothness assessment through frame-wise eigenvalue analysis and unit-circle classification, rigorous cross-modal validation against traditional contact-based methods, and application-tailored experimental protocols with domain-specific relevance. These innovations collectively establish a new methodology for non-invasive smooth-

ness assessment that overcomes fundamental limitations of traditional sensing approaches while providing comparable measurement capabilities.

6.2 Experimental Synthesis and Principal Findings

The experimental validation of the proposed framework demonstrates consistent and robust performance across diverse conditions, with strong agreement between contact-free and contact-based methods. The walking experiments established baseline performance metrics under low-dynamic conditions, revealing that the contact-free NIR method achieved a mean smoothness index of $84.9 \pm 2.8\%$, representing a statistically significant improvement of 14.2 percentage points over the IMU baseline of $70.7 \pm 3.2\%$ ($p < 0.01$). Correlation analysis revealed a strong positive relationship between modalities ($r = 0.89$, $p < 0.001$), indicating the NIR framework’s capability to capture equivalent smoothness characteristics without physical sensor attachment. Temporal synchronization analysis demonstrated 90.3% agreement in the detection of jerky motion events between methods.

Under running conditions, the contact-free framework maintained robust performance in tracking rapid smoothness transitions, achieving a mean smoothness index of $86.5 \pm 3.5\%$, significantly outperforming the contact-based IMU reference at $77.6 \pm 5.1\%$ ($p < 0.01$). The strong correlation coefficient of $r = 0.92$ ($p < 0.001$) between modalities confirms the NIR framework’s capability to accurately capture smoothness characteristics during dynamic motion. The weighted running experiments validated method performance across varying load constraints, with the contact-free NIR method maintaining strong correlation with the IMU reference across all loading conditions ($r > 0.90$).

Extension to mechanical systems demonstrated the methodology’s cross-domain applicability and robustness across varying terrain conditions. The contact-free NIR approach achieved strong performance in vehicle smoothness assessment, with a mean smoothness index of $67.36 \pm 8.42\%$ compared to the contact-based IMU reference at $77.63 \pm 7.31\%$. The strong correlation between methods ($r = 0.94$, $p < 0.05$) validates the contact-free

approach for mechanical system monitoring across diverse operational environments.

Analysis of loaded running experiments revealed a strong positive correlation between mechanical loading and motion smoothness in human locomotion. The IMU-based smoothness index increased substantially from 41.50% at baseline to 73.67% under maximum loading conditions, representing a 77.5% overall improvement. Similarly, the contact-free NIR method showed even more pronounced enhancement, with smoothness values rising from 19.63% to 64.52%, corresponding to a 228.6% relative increase across the loading spectrum. Statistical analysis confirmed the significance of these load-induced smoothness improvements ($p < 0.001$ for both modalities), with large effect sizes ($\eta_p^2 > 0.45$) observed across all comparisons.

Investigation in mechanical systems revealed similar load-smoothness relationships, confirming the cross-domain applicability of the proposed framework. Vehicle smoothness demonstrated consistent improvement with increasing payload across 36 experimental trials, with the smoothness index improving from 69.46% under unloaded conditions to 84.12% at full capacity. More significantly, the measurement variability showed a dramatic 65.8% reduction in standard deviation (from 7.6% to 2.6%), indicating substantially more consistent and predictable system behavior under loaded conditions.

Comparative analysis across biological and mechanical domains revealed fundamental consistency in load-smoothness relationships, while highlighting domain-specific response characteristics. Both systems exhibited significant smoothness enhancement under mechanical loading, though with distinct optimization patterns. The biomechanical system achieved optimal performance at intermediate loading (50% backpack weight), while the mechanical system showed progressive improvement up to maximum design capacity. This divergence reflects fundamental differences in system architecture and operational objectives, yet both domains demonstrate the significant stabilizing effect of mechanical loading.

Statistical validation confirmed the significance of both primary research hypotheses through comprehensive hypothesis testing. The load-smoothness relationship demonstrated strong statistical significance, with the contact-free method accurately capturing the progres-

sive smoothness enhancement under increasing mechanical loads across both experimental domains. The method agreement analysis revealed correlation coefficients exceeding 0.9 between contact and contact-free modalities.

6.3 Key Contributions and Theoretical Advancements

This research makes four significant theoretical contributions to the field of motion analysis and non-contact sensing. First, it extends the application of the unit-circle criterion to accommodate NIR-derived vector fields and complex-plane geometry, establishing a novel connection between computer vision and motion smoothness assessment. Second, it formalizes characteristic proximity metrics through the development of the $d^\Phi \leq 0.5$ threshold for smoothness similarity assessment, providing a quantitative framework for comparing motion states across time and trials. Third, it advances discrete rate-of-change operators for tracking the temporal evolution of motion smoothness, enabling characterization of transitions between smooth and jerky regimes. Fourth, it establishes geometric constraints through the determination of eigenvalue magnitude boundaries in complex-plane analysis, defining fundamental thresholds for smoothness assessment in vector field representations.

The methodological innovations introduced through this research encompass both sensing and analysis advancements. The development of a complete NIR video processing procedure enables transformation of raw infrared video into structured motion vector fields suitable for smoothness analysis. The implementation of real-time smoothness assessment through frame-wise eigenvalue analysis and unit-circle classification provides immediate feedback capabilities for practical applications. The establishment of cross-modal validation protocols ensures method reliability through direct comparison with traditional contact-based approaches. Finally, the creation of application-tailored experimental designs with domain-specific relevance ensures practical utility across diverse implementation scenarios.

6.4 Practical Applications and Implementation Significance

6.4.1 Healthcare and Rehabilitation

The framework enables transformative capabilities in healthcare and rehabilitation through non-invasive smoothness assessment. The methodology supports objective assessment of movement quality and regularity in clinical settings without the encumbrance of physical sensors, facilitating natural movement patterns during evaluation. Non-invasive monitoring of rehabilitation progress enables continuous assessment without patient discomfort or movement restriction, while early detection of gait irregularities and fall risk assessment provides preventive capabilities that enhance patient safety and care outcomes. The ability to quantify subtle changes in motion smoothness offers clinicians an objective metric for tracking recovery trajectories and adjusting treatment plans.

6.4.2 Athletic Training and Performance

In sports science and athletic training, the framework provides real-time smoothness feedback for technique optimization and performance enhancement. The methodology enables load carriage assessment for military and athletic training through quantitative smoothness measurement under varying loading conditions. Injury prevention through movement pattern analysis identifies jerky or irregular motion patterns before they lead to injury, while technique optimization through real-time feedback enables immediate adjustments and progressive performance improvement during training sessions. The full-field spatial resolution of the NIR approach proves particularly valuable for analyzing complex, multi-joint movements that are difficult to characterize with point-based sensors.

6.4.3 Industrial Monitoring and Diagnostics

The framework offers significant advantages for industrial applications through non-contact measurement capabilities. Structural health monitoring without physical sensor installation

enables assessment of existing infrastructure without modification or invasive procedures. Mechanical system optimization through vibrational smoothness analysis identifies optimal operating conditions and detects suboptimal performance before failure occurs. Real-time overload detection and preventive maintenance capabilities provide early warning of excessive loading conditions or developing faults, enhancing safety and reliability across industrial systems.

6.5 Current Limitations

The developed framework, while demonstrating robust performance across the validated experimental conditions, exhibits several limitations that merit careful consideration and guide future research efforts.

Sensitivity and Accuracy Trade-offs. The sensitivity of the NIR-based method, although strongly correlated with IMU references ($r > 0.90$), remains somewhat lower than that of high-end contact-based sensors in terms of absolute displacement resolution. The optical flow-based displacement estimation is fundamentally limited by pixel resolution and frame rate, with the current 30 fps acquisition rate imposing a Nyquist limit that may alias higher-frequency vibration components. For applications where sub-millimeter displacement accuracy is paramount, traditional contact sensors or laser vibrometry may still be preferable. Future implementations employing higher frame rate cameras and sub-pixel interpolation techniques could substantially improve measurement resolution.

Computational Requirements. The current processing procedure, while capable of offline analysis at reasonable speeds, presents challenges for real-time embedded implementation. The combination of GMM background subtraction, dense optical flow computation, Jacobian matrix construction, and eigenvalue extraction requires approximately 45–52 ms per frame on standard workstation hardware. This latency, though acceptable for many biomechanical applications, may be prohibitive for high-speed industrial monitoring or real-time feedback systems. Algorithmic optimization, including sparse optical flow methods, hardware accel-

eration via GPU implementations, and adaptive region-of-interest processing, represents a critical direction for future development.

Environmental Sensitivity. Although the active NIR illumination strategy mitigates ambient lighting dependence, the framework remains sensitive to extreme environmental conditions. Direct sunlight contains significant NIR components that can saturate the sensor or reduce the effective signal-to-noise ratio of the active illumination. Heavy fog, precipitation, or dust accumulation on the camera lens can degrade image quality and introduce motion estimation artifacts. Furthermore, the orthographic projection assumption may be violated when the camera-to-subject distance is insufficient or when significant out-of-plane motion occurs. Future work should investigate robust regression techniques for outlier rejection, multi-spectral fusion strategies that combine NIR with other modalities, and adaptive calibration procedures that account for perspective effects.

Camera Distance and Spatial Sampling Limitations. The spatial Jacobian eigenvalues are dimensionless and theoretically invariant to camera distance because the pixel units cancel in the spatial derivatives. In practice, however, camera distance directly affects the spatial sampling resolution—how many pixels correspond to a physical unit of length. When the camera is too far from the subject, small spatial variations such as subtle limb movements may fall below the pixel resolution and become undetectable. Conversely, when the camera is too close, perspective distortion may violate the orthographic approximation that underpins the proportionality between optical flow and physical velocity. The oscillation frequency of the physical system also interacts with camera distance: high-frequency oscillations require finer spatial sampling to be captured reliably. Thus, while the threshold $|\lambda| \leq 1$ is scale-invariant, the ability to resolve spatial variations depends on camera distance and system frequency. This is a practical limitation that must be considered in deployment.

Frame Rate Effect on Eigenvalue Threshold. The eigenvalue magnitude is not invariant to frame rate because optical flow is measured in pixels per frame. If the frame rate changes, the magnitude of the flow between consecutive frames changes accordingly, which directly affects the computed eigenvalues. For the range of frame rates used in this thesis (30 fps), the

effect on the smoothness index η is very small, as the experimental validation against IMU data confirms. However, for significantly different frame rates, such as 10 fps or 120 fps, the threshold may need to be recalibrated. This is a limitation of the current empirical threshold, but it is manageable: for any fixed frame rate, the threshold can be validated against IMU data. Future work could investigate a frame-rate-normalised eigenvalue formulation that would make the threshold more broadly applicable.

Motion Blur Effect. Motion blur can degrade optical flow quality if severe, as it reduces the spatial gradients that the optical flow algorithm relies upon. However, as long as the optical flow estimation remains reliable—meaning the blur does not eliminate the structural boundaries of the moving object—the smoothness classification remains valid. The method assumes a rigid-body approximation for the object; motion blur does not change the structure of the velocity field as long as the object boundaries remain identifiable. In cases of severe motion blur, the optical flow quality would degrade, and the classification would become unreliable. Future work could explore deblurring techniques or robust optical flow methods to extend the method to highly blurred videos.

Low Signal-to-Noise Ratio (SNR) Performance. The videos used in the experimental validation of this thesis were acquired under controlled conditions with good illumination and minimal sensor noise, resulting in high SNR. While the robustness parameter ε_0 introduced in Section 3.4.2 was designed to account for uncertainties in optical flow estimation, its effectiveness on genuinely low-SNR videos has not been systematically characterised. In low-SNR conditions, such as low-light environments, high sensor gain settings, or scenes with significant background clutter—the optical flow estimation becomes less reliable because the brightness constancy assumption is more easily violated by noise. This, in turn, affects the computation of the spatial Jacobian and its eigenvalues, potentially leading to misclassification of smooth motion as jerky or vice versa. An artificially added noise to one of the existing high-SNR videos would be a valuable addition to quantify this sensitivity. Future work should investigate denoising pre-processing techniques, robust optical flow algorithms designed for low-SNR inputs, and adaptive thresholding strategies that account

for the estimated noise level. This remains an important direction for extending the method to real-world deployments where lighting and environmental conditions are not always optimal.

Generalization Scope. The current validation is limited to the specific experimental conditions described in Chapter 4: self-paced walking and steady-state running on level surfaces for biomechanical trials, and controlled off-road vehicle operation on three defined terrain types for mechanical trials. The generalizability of the observed load-smoothness relationships to other motion types (e.g., jumping, stair climbing), other populations (e.g., clinical populations with gait pathologies, elderly individuals), or other mechanical systems (e.g., rotating machinery, flexible structures) remains to be established. Broader validation across diverse conditions and populations is necessary to establish the framework’s external validity.

Hardware Dependencies. The current implementation relies on specialized NIR imaging equipment with integrated active illumination. While the cost of such hardware has decreased substantially in recent years, it remains a barrier to widespread adoption compared to ubiquitous RGB cameras. Investigating the feasibility of adapting the framework to work with consumer-grade RGB cameras under controlled lighting conditions, or with low-cost NIR modules designed for embedded applications, could significantly expand accessibility.

Assumptions Regarding Rigidity. As articulated in Chapter 1, the framework assumes rigid motion of the observed object. For human motion analysis, this assumption is only approximately valid; soft tissue movement, skin deformation, and clothing artifacts can introduce local velocity variations that are not representative of the underlying skeletal motion. While the GMM-based foreground extraction and column-based analysis partially mitigate these effects, they do not eliminate them entirely. Future work could explore model-based approaches that incorporate a simplified biomechanical skeleton to constrain the motion field estimation and reject non-rigid artifacts.

6.6 Future Research Directions

Building upon the foundation established in this thesis, several promising directions for future research emerge.

Integration of Machine Learning Techniques. Machine learning, particularly deep convolutional neural networks, offers significant potential for enhancing multiple stages of the processing procedure. Supervised learning approaches could be employed to calibrate the unit-circle threshold for specific applications by learning domain-specific boundaries from labelled data, such as frames annotated as smooth or jerky by experts or IMU labels. Learned optical flow estimators could provide improved accuracy and robustness compared to classical variational methods, especially in challenging imaging conditions such as low texture, large displacements, or low signal-to-noise ratio. Furthermore, end-to-end trainable architectures could be designed to predict the smoothness index η directly from raw video frames, bypassing explicit Jacobian computation and reducing computational latency for real-time applications. The supervised approaches would be trained on the synchronized IMU data collected in this study, providing a direct link between video input and physical damping measures.

Real-Time Embedded Implementation. Achieving true real-time performance on embedded platforms would significantly expand the practical applicability of the framework. This would require algorithmic optimization at multiple levels: efficient sparse optical flow methods that compute motion vectors only at salient feature points rather than densely across the entire frame; hardware-accelerated eigenvalue computation using optimized linear algebra libraries; and adaptive frame rate control that dynamically adjusts processing based on detected motion complexity. An embedded implementation could enable wearable or portable systems for continuous monitoring in field environments, including military training, remote patient monitoring, and autonomous vehicle diagnostics.

Multi-Spectral and Multi-Modal Fusion. Combining NIR imaging with other sensing modalities could enhance robustness and extend operational envelopes. Fusion with RGB

cameras could leverage the complementary strengths of each modality: NIR for consistent illumination and RGB for color-based segmentation and texture detail. Integration with thermal imaging could provide additional information about muscle activation or mechanical component heating that correlates with loading and fatigue. Multi-modal sensor fusion frameworks, potentially based on Kalman filtering or factor graph optimization, could integrate data from cameras, IMUs, and other sources to provide a more complete characterization of system state.

Three-Dimensional Vector Field Reconstruction. The current framework operates on two-dimensional projections of motion, relying on the orthographic assumption. Extending the methodology to three dimensions through stereo NIR imaging or depth-sensing technologies would enable more comprehensive spatial characterization of complex structures. A three-dimensional Jacobian matrix could capture out-of-plane motion components and provide a more complete eigenvalue spectrum for smoothness classification. This extension would be particularly valuable for analyzing articulated structures, complex machinery, or full-body human motion where out-of-plane movements are significant.

Longitudinal and Population-Scale Studies. The experiments conducted in this thesis were limited to acute loading conditions with relatively small sample sizes. Longitudinal studies tracking smoothness metrics over extended training periods could reveal how movement patterns adapt to chronic loading and how the load-smoothness relationship evolves with fatigue or skill acquisition. Population-scale studies across diverse demographic groups (age, fitness level, clinical conditions) could establish normative ranges for η and identify deviations indicative of increased injury risk or functional decline. Such studies would be essential for translating the research findings into clinically actionable tools.

6.7 Final Conclusion

This research has established a comprehensive framework for contact-free assessment of motion smoothness in oscillatory motion, demonstrating that NIR-based vector field analy-

sis can achieve comparable accuracy to traditional contact-based methods while offering significant advantages in spatial resolution, non-invasiveness, and application flexibility. The integration of theoretical rigor with practical implementation provides a foundation for widespread adoption across multiple domains, from healthcare and sports science to engineering and industrial systems monitoring.

The characteristic proximity framework and discrete rate-of-change operators represent significant contributions to the analysis of oscillatory motion, extending traditional concepts to accommodate vision-based measurement approaches. The experimental validations demonstrate practical utility in real-world applications, establishing method reliability across diverse conditions and domains. The consistent load-smoothness relationships identified across biological and mechanical systems suggest fundamental principles governing dynamic behavior under mechanical constraint, providing insights that transcend specific implementation domains.

As sensing technologies continue to advance and computational capabilities grow, the proposed methodology offers a scalable and adaptable approach for motion smoothness assessment that can evolve to meet emerging challenges in healthcare, engineering, and scientific research. The work opens new avenues for non-contact diagnostic methodologies and establishes a paradigm for vision-based system analysis that bridges traditional disciplinary boundaries between computer vision, dynamical systems theory, and applied engineering.

In conclusion, this research has successfully developed a novel theoretical framework for optical flow vector field analysis in the complex plane, established characteristic proximity metrics for quantitative motion smoothness assessment, created discrete operators for dynamic smoothness tracking, validated the approach through comprehensive experimental comparison with traditional methods, and demonstrated practical applications across multiple domains with significant real-world impact. The research contributes fundamentally to advancing the field of non-contact sensing and motion analysis, providing both theoretical insights and practical methodologies that can be applied to address challenging problems in engineering, healthcare, and scientific research.

Bibliography

- [1] Valentina Pasquinelli, Gloria Cosoli, Milena Martarelli, Alessandra Mobili, Francesca Tittarelli, Giuseppe Pandarese, Luca Violini, Paolo Castellini, Michele Morici, Fabio Micozzi, Alessandro Zona, Valeria Leggieri, Andrea Dall’Asta, and Gian Marco Revel. Vibrational monitoring of a concrete beam for bridge damage detection: Data fusion between contact and non-contact techniques. In *Proceedings of the International Conference on Structural Health Monitoring*, pages 333–340, 2024.
- [2] Manuel D’Angelo, Marco Civera, Pier Francesco Giordano, Paolo Borlenghi, Francesco Ballio, Maria Pina Limongelli, and Bernardino Chiaia and. Bridge collapses in Italy across the 21st century: survey and statistical analysis. *Structure and Infrastructure Engineering*, 0(0):1–23, 2025.
- [3] Daoheng Li, Meiyu Liu, Lu Yang, Han Wei, and Jie Guo. A non-contact identification method of overweight vehicles based on computer vision and deep learning. *Computer-Aided Civil and Infrastructure Engineering*, 39:3452–3476, 2024.
- [4] Seongmin Han, Taejung Park, Dahoon Ahn, Hyeongjin Cho, and Chibum Lee. Laser speckle pattern and high speed camera system for non-contact vibration displacement measurement: Accuracy and limitations. *Journal of Mechanical Science and Technology*, 39, 09 2025.
- [5] M. F. M. Ariff, M. E. Kosnan, Z. Majid, A. K. Chong, and K. M. Idris. A study of near-infrared (nir) filter for surveillance application. *Jurnal Teknologi*, 77(26), 2015.

- [6] D. Hertel, H. Marechal, D. A. Tefera, W. Fan, and R. Hicks. A low-cost vis-nir true color night vision video system based on a wide dynamic range cmos imager. In *2009 IEEE Intelligent Vehicles Symposium*, pages 273–278. IEEE, 2009.
- [7] M. Molati. Exact solution of a mathematical model for human muscular motion. *Examples and Counterexamples*, 5:100152, 2024.
- [8] C. Egbert, P. K. Egbert, and B. S. Morse. Real-time motion transition by example. Microsoft Corp / Brigham Young University, 2024.
- [9] S.G. Krantz. *Essentials of Topology with Applications*. CRC Press, 2010.
- [10] Jinnyeong Kim and Seung-Hwan Baek. Pixel-aligned rgb-nir stereo imaging and dataset for robot vision. *arXiv preprint arXiv:2411.18025*, 2:1–44, 2024. Version 2.
- [11] E. Cui and J. F. Peters. Contact-free sensing of stability in vector fields of vibratory dynamical systems. *IEEE Canadian Journal of Electrical and Computer Engineering*, 49(2), 2026.
- [12] J.F. Peters and E. Cui. Characteristically near stable vector fields in the polar complex plane. *Communications in Advanced Mathematical Sciences*, 8:117–124, 2025.
- [13] J. F. Peters and E. Cui. Self-similarities in hilbert envelope energy regions on motion waveforms. application in detecting motion irregularities in video frames. *Trends in Opto-electro & Optical Communication*, 14(3):33–48, 2024.
- [14] E. Cui and J. F. Peters. Self-similarities in optical flows: voxel lumens flux fractality in moving shapes in triangulated video frames. *Chaos, Solitons & Fractals*, 164:112722, 2022. 7 pp., MR4492095.
- [15] T. Kuttner and A. Rohnen. Accelerometer. In *Practice of Vibration Measurement*, pages 111–148. Springer Fachmedien Wiesbaden, 2023.

- [16] Jidong Shi, Liu Wang, Zhaohe Dai, Lingyu Zhao, Mingde Du, Hongbian Li, and Ying Fang. Multiscale hierarchical design of a flexible piezoresistive pressure sensor with high sensitivity and wide linearity range. *Small*, 14, 05 2018.
- [17] Zisong Zhou, Yin Zhang, Jialuo Bai, Wang Zhang, Haolun Wang, and Weifang Pu. Bioinspired flexible capacitive sensor for robot positioning in unstructured environments. *ACS applied materials and interfaces*, 16, 03 2024.
- [18] Rishabh Mishra, Nazek El-Atab, Aftab Hussain, and Muhammad Hussain. Recent progress on flexible capacitive pressure sensors: From design and materials to applications. *Advanced Materials Technologies*, 6, 03 2021.
- [19] Yuming Mo, Haijun Han, Yan Liu, Xiaolong Wang, Dongping Hu, Jiawei Zhang, and Yuan Wang. A tactile sensor based on piezoresistive effect and electromagnetic induction. *Sensors and Actuators A: Physical*, 344:113716, 2022.
- [20] Yuhang Xiao, Zisong Zhou, Xinghai Pan, Yanling Liu, Hao Mei, Haolun Wang, and Wei Pu. Biomimetic soft crawling robot with non-contact sensing for confined spaces. *Science China Materials*, 68(2):531–541, 2025.
- [21] Vahid Bokaeian, Jeremy Bowmaster, and Kaveh Arjomandi. Detection and localization of traffic on a bridge structure using a distributed accelerometer network. *Journal of Bridge Engineering*, 28(4):415–425, 2023.
- [22] Thomas George Thuruthel, Benjamin Shih, Cecilia Laschi, and Michael Tolley. Soft robot perception using embedded soft sensors and recurrent neural networks. *Science Robotics*, 4:eaav1488, 01 2019.
- [23] Yujia Zhang and Tiger Tao. A bioinspired wireless epidermal photoreceptor for artificial skin vision. *Advanced Functional Materials*, 30:2000381, 04 2020.
- [24] STMicroelectronics. *LSM6DSO: 6-axis IMU (inertial measurement unit) always-on 3-axis accelerometer and 3-axis gyroscope*. STMicroelectronics, June 2024. DS12140.

- [25] Piotr Kowalski. Measurements of vibration using a high-speed camera – comparative tests. *Vibrations in Physical Systems*, 31:2020211, 2020.
- [26] Qiongfeng Shi, Zhongda Sun, Xianhao Le, Jin Xie, and Chengkuo Lee. Soft robotic perception system with ultrasonic auto-positioning and multimodal sensory intelligence. *ACS nano*, 17, 03 2023.
- [27] Faezeh Arab Hassani. Bioreceptor-inspired soft sensor arrays: recent progress towards advancing digital healthcare. *Soft Science*, 3, 08 2023.
- [28] Qilin Hua, Junlu Sun, Haitao Liu, Rongrong Bao, Ruomeng Yu, Caofeng Pan, and Zhong Wang. Skin-inspired highly stretchable and conformable matrix networks for multifunctional sensing. *Nature Communications*, 9, 01 2018.
- [29] Xiaochun Wang. *Gaussian Mixture Model-Based Video Anomaly Detection*, pages 117–142. Springer Nature Singapore, Singapore, 2024.
- [30] Qilin Hua and Guozhen Shen. Low-dimensional nanostructures for monolithic 3d-integrated flexible and stretchable electronics. *Chemical Society reviews*, 53, 01 2024.
- [31] Delin Hu, Haotian Li, Francesco Giorgio Serchi, and Yunjie Yang. Coupling field simulation of soft capacitive sensors toward soft robot perception. *IEEE Sensors Journal*, PP:1–1, 05 2023.
- [32] Boshra Besharatian, Amrita Das, Abdelrahman Awawdeh, Sattar Dorafshan, and Marc Maguire. Benchmarking dynamic properties of structures using non-contact sensing. *Earthquake Engineering and Engineering Vibration*, 22(2):387–405, 2023.
- [33] Seongmin Han, Taejung Park, Dahoon Ahn, Hyeongjin Cho, and Chibum Lee. Laser speckle pattern and high speed camera system for non-contact vibration displacement measurement: Accuracy and limitations. *Journal of Mechanical Science and Technology*, 39(9):4996–5008, 2025.

- [34] Timothy Hanley, Dara Golden, Robyn Maxwell, Joseph Lemley, and Ashkan Parsi. Non-contact NIR PPG sensing through large sequence signal regression. In *Irish Machine Vision and Image Processing Conference 2023*, 2023.
- [35] Peiping Zheng, Yanchen Liu, Huijun Wu, and Huan Wang. Non-invasive infrared thermography technology for thermal comfort: A review. *Building and Environment*, 248:111079, 2024.
- [36] Campark. *User Manual for Campark AC09 Trail Camera*, 2023. Accessed: 2025-05-27.
- [37] Daniel A. Fleisch. *Vectors and Tensors in Physics and Engineering*. Cambridge University Press, Cambridge, UK, 2012.
- [38] L. Euler. *Introductio in analysin infinitorum*. Springer, Seville, New York, 1988. Original work published 1748, xvi+30 pp., ISBN: 84-923760-2-3, MR1841792, MR0961255.
- [39] James F. Peters and Mehmet A. Öztürk. Near stein-weiss finite vector field groups in characteristic nearness approximation spaces in the polar complex plane, vi, 2025.
- [40] J.F. Peters, T. Vergili, F. Ucan, and D. Vakeesan. Indefinite proximities inherent in dynamical systems: An axiomatic approach. *Modern Mathematical Methods*, page 15, January 2026. In press.
- [41] Elias M. Stein and Rami Shakarchi. *Complex Analysis*, volume 2. Princeton University Press, USA, 2003.
- [42] E. Cui and J. F. Peters. Fine-tuning energy dissipation in vibrating dynamical systems recorded in infrared video frame sequences, 2025. Preprint.
- [43] J.F. Peters and E. Cui. Characteristics of vibrating systems having time-constrained energy. *arXiv preprint arXiv:2410.21323*, 2024.

- [44] B. Kuloglu, E. Cui, E. Ozkan, and J. F. Peters. A note on bivariate bi-periodic mersenne polynomials. *Proof Journal*, 4:158–167, 2024.