ALGEBRA PROGNOSIS: PREDICTING FIRST YEAR ALGEBRA ACHIEVEMENT IN THE ST. JAMES-ASSINIBOIA SCHOOL DIVISION NO. 2, ST. JAMES-ASSINIBOIA, MANITOBA

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ABSTRACT

The purpose of this study was to establish effective methods of predicting success in grade nine algebra for students in the St. James-Assiniboia School Division No. 2. The five independent variables selected as predictors were the grade eight mathematics grade, the Verbal Reasoning and Numerical Ability sub-tests of the Differential Aptitude Test, the Iowa Algebra Aptitude Test, and the Math Attitude Scale.

A review of the relevant literature revealed that numerous tests of aptitude, academic performance, mental ability, and attitude towards mathematics were used with varying degrees of success for predicting students' achievement in an elementary algebra course.

The empirical analysis consisted of two major investigations. In the first investigation, stepwise regression was applied to the data. For the complete population sample, the five independent variables accounted for 68 per cent of the variance of the ninth grade algebra grades. Regression weights of the independent variables selected as the best predictors were obtained and multiple regression equations were used to predict ninth grade algebra grades. The predicted grades were relatively accurate when compared with the actual grades.

The stepwise regression investigation demonstrated that the five selected independent variables, which were measures

of cognitive and affective areas, did account for a sufficient amount of the variance in ninth grade algebra grades to make possible an accurate prediction by regression equations.

A sub-study involving all grade nine algebra students from one junior high school in the St. James-Assiniboia School Division revealed that the regression equation for the School Division sample predicted grades with accuracy equal to the regression equation developed for the one school. On the basis of this evidence, the conclusion was made that the regression equation derived from the School Division population sample data was as good a predictor of grades as was the equation derived from the one-school population sample data for that school's students.

In the second part of the study, expectancy tables were constructed to predict the probability of a student's attaining a grade as great as, or greater than, a specific grade nine algebra grade. For a student who sought counseling, expectancy tables could have provided a less restricting predictive picture than that presented by the regression technique; however, no concrete conclusion could be made regarding the superiority of either of these predictive devices.

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THE OBJECTIVES AND SIGNIFICANCE OF THE STUDY

INTRODUCTION

At the time of inception of this study, in the province of Manitoba, special mathematics courses existed for grade nine students enrolled in either Occupational Entrance or Special Education programs. All remaining grade nine students, without respect to ability or past performance, studied the same mathematics course. When this study was begun in August of 1970, the grade nine mathematics course consisted mainly of algebra and a short section of geometry. The underlying assumption was that all students upon completion of grade nine were academically prepared to undertake the Mathematics 100 program in grade 10.

Due to its abstract nature, algebra has proved to be a difficult study. From the teacher's standpoint, it was difficult to motivate the student to study a subject for which he had neither the ability to comprehend nor the likelihood of need for the mathematical concepts studied.

Although in many areas outside of Manitoba, the student has been free to choose, upon entering grade nine, from algebra and general mathematics courses, course selection as such within a specific subject field has not been afforded the Manitoba student, until the grade 10 level. Then, he has been free to choose

either the university preparatory course or one from a series of general mathematics courses.

Commencing September, 1971, a new course with a concept unique to Manitoba will be instituted in grade nine mathematics. This course will still allow, on its successful completion, a student to enter Mathematics 100 or any other grade 10 mathematics program at the grade 10 level. The course has a mandatory section and an elective section. All students study the mandatory section which should at most consume one-half of the school year. The remainder of the school year will then be free for the student to study the portions of the elective section which he finds of interest and use. The elective section contains an extension of algebra topics and geometry along with numerous topics in applied mathematics. Essentially, the decision to choose the Mathematics 100 or another mathematics program will be made mid-year in grade nine. At this point, adequate and judicious counseling will need to be carried out by the mathematics teacher in order for the student to undertake the program of studies best suited to his interests and needs. It should be pointed out that although the choice of Mathematics 100, 101, or vocational courses, technically, will still be open at the end of the school year, the student should make his decision at mid-year when the study of the elective topics is begun. Where teachers and counselors have in such circumstances constantly attempted to give advice, they have been limited to the subjective interpretation of attitudes and empirical evidence.

The basic purpose of this study was to develop objective instruments which would prove valuable as guidance tools to assist the student in deciding whether or not to select algebra topics for study from the elective section of the new grade nine mathematics course. The student's selection could then be made with considerable confidence in its relation to his area of interest.

THE OBJECTIVES OF THE STUDY

Statement of Objectives

As has already been stated, the purpose of this study was to develop prediction instruments to assist the student in deciding whether or not to select algebra topics for study from the elective section of the new grade nine mathematics course.

More specifically, the major objectives were:

(1) To establish regression equations for use in predicting performance in grade nine algebra based on the grade eight mathematics grade, the Verbal Reasoning and Numerical Ability sub-tests of the Differential Aptitude Test, the Iowa Algebra Aptitude Test, and the Math Attitude Scale.

(2) To develop expectancy tables based on the variables listed in number one above.

(3) To compare regression equations and expectancy tables as techniques of prediction in order to determine which device would be more effective in predicting success in grade nine algebra.

Delineation of the Study

The limitations of the study were found in the predictors

tested and in the sampling technique.

Additional predictors and a larger sample would have explained more completely the variance between the variables, but the resources necessary to carry this out were not available to the writer, resulting in delimitations. Nevertheless, significant and useful predictive devices were developed using the sample and predictors studied.

Since the entire population is not used in sampling, the ability to draw inferences is limited. For the writer's study, the population was defined as all students in grade nine classes of mathematics in full junior high schools from September, 1970 to June, 1971, in the St. James-Assiniboia School Division No. 2, of the province of Manitoba. The sample used was one class selected on a random basis from each of these schools.

The study considered the complete population sample, as well as all males and females separately. The purpose of considering the males and females separately was to determine if one sex was more predictable than the other. In addition, a special study was made using all the grade nine mathematics students at Stevenson Junior High School. This special study was made to determine if the prediction devices based on an entire grade of a school would predict significantly better than those based on the School Division population sample.

The study considered only the following independent variables as predictors of success in grade nine algebra: measures of scholastic aptitude, attitude, school marks, and mathematical aptitude. The data for these independent variables were avail-

able either from cumulative record folders in the schools, from tests conducted annually by the School Division, or from tests conducted by the writer. The reasons for the selection of specific variables are discussed in greater detail in Chapter III (Design of the Study).

Data Sources

In carrying out this study, the following data were acquired:

(1) the grade eight grade point for the students included in the study sample;

(2) the Verbal Reasoning sub-test percentile score from the Differential Aptitude Test for the students included in the study sample;

(3) the Numerical Ability sub-test percentile score from the Differential Aptitude Test for the students included in the study sample;

(4) the Iowa Algebra Aptitude Test raw score for the students included in the study sample;

(5) the Math Attitude Scale score for each student in the study sample;

(6) the final grade nine mathematics grade point for each student in the study sample.

All of the above data were also utilized for the remainder of the grade nine students not included in the main part of the study from Stevenson Junior High School.

THE SIGNIFICANCE OF THE STUDY

A Need for Investigation

Education today, as never before, is being pressed to a place of accountability. Mathematics is no exception. Johnson (1966) expressed his position as follows:

> We have reached a crucial time in mathematics education. Questions are being asked and criticism made of school mathematics programs. In mathematics we emphasize logic and proof, deductive and inductive reasoning. We teach probability and statistics, game theory and problem solving, the tools of the research worker. Isn't it strange that at the same time we have not made intensive and organized use of these tools to put our own house in order? (p. 419)*

In seeking to assist a student in assessing his probability of success in pursuing a particular course of study, the counselor must choose certain independent variables which he considers to be the best predictors of performance in the subject under consideration. This selection of variables has usually been based on little, if any, experimental evidence. If a proven set of variables were established, not only would the counselor be more confident in giving the advice, but also the student would be more receptive, realizing the basis of the counsel.

Statement of Importance

Numerous studies have been carried out attempting to establish a predictive device to insure effective counseling to aid a student in choosing between algebra or general mathematics

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*Refer to Bibliography for all references.

for grade nine. With the revision of the curriculum in Manitoba to be implemented commencing September, 1971, the usefulness of such a predictive device might appear questionable. Since a choice of whether to continue with algebra topics from the mandatory section of the course will have to be made at mid-year, some predictive device would in fact be valuable. Test results based on the mandatory section of the revised curriculum will not be available until early in 1972, so these could not be used in this study. However, a prediction study of the traditional variety for grade nine algebra was carried out with students who studied an algebra program with a short section of geometry. Basically, this could be considered an algebra course.

Predictive devices were developed, and it is hoped, will prove useful in assisting the student to decide whether to continue with algebra or to seek other areas of study.

No relevant studies done in the past have used the same combination of predictors as outlined later in this study. It is felt that since the studies previously carried out have used neither predictors nor samples from which inferences could be drawn beyond the experimental situation, the need existed for a local study to be undertaken. This study therefore has relevance to the immediate needs of the local schools.

SUMMARY

This study attempted to establish prediction devices based on regression techniques and expectancy tables, for use in pre-

dicting performance in grade nine algebra. The effectiveness of two techniques of prediction was also compared.

Commencing September, 1971, the province of Manitoba will institute a new mathematics course for grade nine students. One-half of this course contains elective topics while the mandatory section contains topics of basic algebra. At mid-year, the student must decide whether to continue with algebra topics or to select other areas of study which might be more useful to him in his future studies.

The predictive instruments developed through this study, it is hoped, will prove of some benefit to teacher and student involved in the grade nine mathematics program in that it will give them some concrete basis for deciding which topics should be studied in the elective portion of the new grade nine mathematics course.

Chapter II contains a review of some of the literature relevant to this study, where mathematics aptitudes, specific independent variables and methods of prediction were reviewed in order to aid in establishing the independent variables to be tested in this study. A description of the variables, the population samples, and a detailed description of the techniques of analysis used in the study appear in Chapter III. In Chapter IV, the finding of the investigations are presented. Chapter V consists of a summary of the study and the implications of the findings.

Chapter II

THE REVIEW OF THE LITERATURE

Literature relative to prediction of success in elementary algebra was reviewed for pertinent studies. From this review, a variety of predictors was found in certain studies. This led to a review of the literature dealing with a factor analysis of mathematical aptitudes. From this, certain specific variables were selected and reviewed in detail. This was followed by a summary of pertinent methods of prediction.

The studies reviewed are presented with emphasis placed on the factors of significance to the writer's research. This significance will be detailed following the review of the studies.

The following abbreviations will be used throughout the remainder of the thesis:

DAT - Differential Aptitude Tests

SCAT - Cooperative School and College Ability Test

I.Q. - Intelligence Quotient

PREDICTION OF SUCCESS IN ELEMENTARY ALGEBRA

Tempero and Ivanoff (1960) carried out a prediction study using the SCAT as a predictor. The independent variable was the final grade in beginning algebra. Only the portion of the study dealing with prediction in mathematics was found

to be relevant to the writer's research. The SCAT gave a verbal, quantitative, and total score. The correlation between the total score and the end-of-year achievement score in beginning algebra was .66 for males and .67 for females. This one test (SCAT) accounted for approximately 44 per cent of the variance in mathematics achievement.

Osburn and Melton (1963) carried out a study predicting proficiency in modern and traditional courses in beginning algebra. The research involved 155 students and three algebra teachers representing one school. The Iowa Algebra Aptitude Test, the Orleans Algebra Prognosis Test, the SRA Primary Mental Abilities Test, and the DAT were used as predictors. The dependent variables were a series of proficiency tests developed by the investigators along with the Cooperative Algebra Test and a final examination. Results of only the experimental group (modern course) were found to be useful in the writer's study. For this group, the correlation coefficient between the Iowa Algebra Aptitude Test and the total scores on the proficiency tests was .69. Between the Orleans Algebra Prognosis Test and the total scores on the proficiency tests, a correlation of .67 was found. The correlations involving the SRA Primary Mental Abilities Test were as low as .33 and no higher than .56, but were generally around .45. In reporting the results of the study, the DAT was listed separately. The correlations varied from .17 to .71 for the various sub-tests, but the best correlation was found using the Verbal Reasoning and Numerical Ability together as predictors. This correlation coefficient was .72. The sum of the Verbal

Reasoning and Numerical Ability scales from the DAT predicted proficiency with equal validity to that of the Iowa Algebra Aptitude Test and the Orleans Algebra Prognosis Test. The latter two tests were designed specifically for the purpose of algebra success prediction.

In both the modern and traditional courses, there were no significant differences in the proficiency of predictors by the aptitude tests. The coefficients of correlation between aptitude and achievement tests for both groups ranged from .62 to .66. Spatial and Mechanical Reasoning tests were more valid for the modern course than for the traditional course. The DAT Spelling tests gave characteristically higher validities in the traditional courses.

An algebra aptitude test, along with the Verbal Reasoning and Numerical Ability sub-tests of DAT were used as predictors in the writer's research on the basis of the significant correlation coefficients presented above.

Barnes and Asher (1962) attempted to predict success in first year algebra by means of ten predictors. The subjects were 192 students with six different algebra teachers from two junior high schools in the community. The criterion was the grade nine algebra grade. Among the predictors were previous mathematics grades, reading tests, algebra aptitude tests, arithmetic achievement tests of previous grades, and an I.Q. test. The best single predictor of success in algebra in this school system was the eighth grade mathematics grade with a correlation of .59. The only other factor which greatly raised the multiple

correlation was the grade equivalent on the arithmetic part of the achievement test given near the end of the seventh grade. This raised the multiple correlation coefficient to .62. From this study, we have evidence of the grade eight mathematics grade being a rather valid predictor of success in a beginning algebra course.

Dinkel (1956) carried out a study to predict success in introductory algebra. The subjects were 148 students in one California school with the same teacher. The predictors tests were I.Q. (California Test of Mental Maturity), Survey Test of Algebra Aptitude, grades given by previous teachers, and arithmetic competency (Cooperative Mathematics Test for Grades 7, 8, and 9). The criterion measures were the Seattle Algebra Test and the Cooperative Elementary Algebra Test. From the above predictors, a multiple correlation of .86 resulted. No individual correlations were reported except for a .66 correlation between the aptitude test and the algebra achievement test. It was noted that the independent variables used for prediction in the study regarding prediction instruments appeared to be those most frequently used in algebra prognosis.

Callicutt (1961) worked with the problem of predicting success in algebra. His subjects were 150 algebra students, taken from a total of 360 grade nine students from three schools in different sections of the city of Memphis, Tennessee. Attention was given neither to the various teaching techniques being employed, nor to the various social and psychological factors that might have influenced the student's prognosis. Callicutt stated that

an earlier study found the correlation between chronological age and achievement in algebra to be negative and of negligible importance in prediction. For this reason, chronological ages of the students selected were not included among the criteria for predicting success in algebra.

In this study, four independent variable factors were selected as criteria. They were: (1) intelligence quotients, (2) achievement tests, (3) grade eight composite grades, and (4) grade eight mathematics grades. The dependent variable was the final grade in grade nine algebra. The conclusion reached was that grade eight mathematics grades were found to be the best criterion for the selection of students ($r = .58$). Since the arithmetic section of the achievement test correlated at .56 with achievement in algebra, a combination of achievement test scores in arithmetic and eighth grade mathematics grades have tended to offset errors in either criterion, as the correlation coefficients are clustered together.

As a result of the evidence presented in Callicutt's (1961) study, the grade eight mathematics grade was included in the writer's research as an independent variable. On the basis of Seagoe's (1938) study, a general intelligence test was not included in the independent variables selected. Seagoe compared the predictive validity of intelligence, aptitude, and achievement tests, for general mathematics and algebra. She found that for both groups, the Stanford Arithmetic Achievement Test was superior to both the aptitude test and general intelligence tests.

Lee and Hughes (1934) studied prediction of algebra and geometry achievement. They found that for both algebra and geometry, the aptitude test predicted scores on the respective achievement tests better than the test predicted final grades. They also found that teacher ratings of student ability provided a better prediction of final grades than did final achievement test scores. On the other hand, the prediction of achievement scores from aptitude tests was more meaningful than prediction of final grades from teacher ratings. This would imply a higher correlation between aptitude and achievement tests of which there is little actual difference, than between teachers' ratings and final grades. It is not too unlikely that the teachers based their ratings on final grades. Therefore, an algebra aptitude test was selected for use as an independent variable by the writer in preference to teacher ratings.

Cain (1964) tested to discover the relationship of verbal reasoning and numerical ability to achievement in first year algebra. Two groups of students were involved, one in a traditional biology program and one in an experimental biology program. The students in the study were enrolled in five high schools in three school systems. Cain used the Numerical Ability and Verbal Reasoning sub-tests of the DAT. For both the traditional and experimental groups, the degree of relationship between scores on the DAT Numerical Ability Test and grades in first year algebra were significantly greater than the degree of relationship between scores on the DAT Verbal Reasoning Test and grades in first year algebra. Cain concluded that since the DAT Numerical

Ability Test is reported to be primarily a measure of computational skill, and the DAT Verbal Reasoning Test a measure, at least in part, of deductive reasoning ability, the first year algebra courses probably placed more emphasis upon computational skills than upon reasoning ability. This study was reviewed in an attempt to determine whether computational skills or reasoning ability is of greater importance in learning algebra.

Whereas all prediction studies reviewed thus far employed the technique of multiple regression in the analysis of the experiment, Ivanoff and DeWane (1965) carried out a study in the use of discriminant analysis for selection of students for ninth grade algebra or general mathematics. The significance of this study to the writer was based on the variation in technique of analysis.

In the Ivanoff and DeWane study, the subjects were 448 freshman males at one large midwestern United States high school. There was a total of 286 in the algebra group and 162 in the general mathematics group. The dependent variable used in this study was defined in terms of whether the student successfully completed ninth grade algebra, or whether he completed the general mathematics course. Using this criterion of algebra versus general mathematics, an attempt was made to determine whether the predictor variables could differentiate between the two groups.

The independent variables included the I.Q. rating, reading, arithmetic, language, and composite scores from the High School Placement Test, published by Science Research Associates, and the eighth grade mathematics mark.

Since the criterion--whether the student was in the algebra or general mathematics group--was considered a dichotomous variable, an adaption of Fisher's discriminant analysis was made.

To test the hypothesis that there is no significant relationship between the algebra and general mathematics groups, discriminant equations were computed in a stepwise manner. The composite score was selected as the best single predictor of the dichotomy. The generalized optimizing discriminant equation used the reading, arithmetic, language, composite, and eighth grade mathematics mark. It was noted that Ivanoff did not find I.Q. to make a significant contribution to the equation.

> Although four variables were significant to the discrimination scheme, for all practical purposes the school system should probably use a simpler equation rather than the optimizing, four-variable combination. Such an approach could well be used of a combination of the HSPT composite and arithmetic scores. The addition of the eighth grade mathematics mark or reading score, although statistically significant contributors in a practical sense, may not be worth the time and expenditures required in processing these data.
>
> The foregoing illustrates a common phenomenon in prediction of scholastic behavior. Typically, little practical value is gained in having a combination of more than two or three intellective predictors in such schemes.

ANALYSIS OF MATHEMATICAL APTITUDES

In her factor analysis of mathematics ability, Sister M. Canisia's (1962) oblique solution produced 12 factors. In second order analysis, she obtained four factors.

> The findings of this study suggest that mathematical thought processes appear to be mainly processes of education, organization, and manipulation of relations. Mathematical thinking seems to be characterized by a fluency and flexibility of thought material under restricting conditions such as are often imposed by the assumptions, postulates,

and definitions of a mathematical problem. The number factor appears to be quite unrelated to the other factors in terms of which mathematical ability was described in this study.

The following study gave further evidence to support the use of the DAT in the writer's research. It also introduced evidence to justify the use of the Iowa Algebra Aptitude Test. Guildford, Hoepfner, and Peterson (1965), in their study of ninth grade mathematics, compared the predictive validities of standard academic aptitudes, e.g., Iowa Algebra Aptitude Test and DAT, with the predictive validities of measures of structure-of-intellect. The ninth grade mathematics courses studied included basic mathematics, non-college algebra, regular algebra, and accelerated algebra. Guildford concluded that 12 different factors, primarily from the symbolic category of the structure-of-intellect and from the products of relationships and implications, were most necessary for ninth grade mathematics. Some of these factors were also represented in Canisia's study.

Leton (1966) conducted a study with two major objectives: the first was to identify the types of intellectual abilities which define the mathematical aptitudes of ninth grade students; and the second was to determine how these abilities were related to achievement in specific mathematics courses.

The factor analysis supported his first hypothesis that mathematical ability is composed of a number of aptitudes; however, the nature of these aptitudes as specific and independent, or as correlated traits was still found in question. His hypothesis that mathematical aptitude scores were superior to general aptitude scores for the prediction of mathematics achievement was supported.

It was also hypothesized that algebra aptitude and abstract reasoning would show greater predictive validity for algebra achievement than would other aptitude and achievement predictors. The algebra aptitude portion of the hypothesis was supported but the related hypothesis, that abstract reasoning abilities would be an important predictor of algebra success was not supported. Another hypothesis was that numerical ability would hold greater predictive validity for arithmetic achievement than would other aptitude and achievement predictors. The data for the two arithmetic sections of the study tended to support this hypothesis.

These findings proved to be important in that they further supported the use of the Iowa Algebra Aptitude Test as an independent variable in the writer's study.

REVIEW OF SPECIFIC INDEPENDENT VARIABLES

General Intelligence Tests as Predictors of Scholastic Achievement

Of the studies reviewed, none which included I.Q. as an independent variable found it to contribute significantly to the variance in the dependent variable. In the following reviews, conflicting reports were found.

Frost (1965) stated that a reasonable amount of the variance in scholastic achievement could be predicted from I.Q., but rarely more than 50 per cent. He found, however, that school marks were better predictors of scholastic achievement in the middle school than were intelligence tests.

Frost concluded his report with a timely remark: "The more identical the predictor item with the material actually taught in the classroom, the better the prediction".

This accounts to some degree for the frequency of use of the grade eight mathematics grade as a variable in prediction studies involving grade nine mathematics.

The usefulness of I.Q. as an independent variable was found to be questionable, as shown by the four following studies reviewed by the writer.

Bloom (1963) observed that I.Q., or some other averaged type of generalized aptitude test performance was useful for predicting performance on a great range of intellectual tasks. The relationship between a general index of intelligence and overall performance on a battery of achievement tests approached unity at the elementary level and dropped to about .50 at the college level.

Lipsett and Wilson (1954) tested the hypothesis that a person would have greatest job satisfaction if his measured mental ability were within the same range as that of most of the people employed in his field of work. They found that only 44 per cent of the participants in the study had levels of mental ability which were "suitable" according to the Minnesota Occupational Rating Scales. There was a slight positive relationship between mental ability and job satisfaction, but this relationship was not statistically significant.

Coleman and Cureton (1954) attempted to show that a group intelligence test and selected sub-tests from a school achievement battery measured substantially identical functions. They found that a good school achievement test of reading and arithmetic measured essentially the same combination of functions as a typical group intelligence test, the overlap being approximately 95 per cent.

Bennett, Seashore, and Wesman (1966) said it was superfluous to give an intelligence test in the same grade in which scores from the DAT were available. The test authors stated that the Verbal Reasoning and Numerical Ability scores served the same purposes for which general mental ability tests were used.

Attitude

In studies by Dutton (1954), Aiken and Dreger (1961), Ellingson (1962), and Dutton (1956), evidence was cited both for the inclusion of attitude, and for the exclusion of personality as independent variables. Based on this evidence, the writer included attitude, a non-intellective variable, in addition to the intellective factors presented previously.

Dutton (1954) studied the attitudes toward arithmetic of prospective teachers as determined by an objective evaluation instrument. The main finding of the study was that prospective teachers' attitudes toward arithmetic may be measured objectively.

Aiken and Dreger (1961) carried out a study to test the applicability of high school mathematics average, mathematics attitude, DAT Verbal Reasoning, and DAT Numerical Ability as predictors of final grades in general mathematics at the freshman college level. Tests of significance of the partial regression coefficients showed that, for the males, all variables except mathematics attitude contributed significantly to the prediction. Only mathematics attitude and DAT Numerical Ability were significant in the prediction equations for the females. The authors suggested that for females, individual differences in Verbal Reasoning and Abstract Reasoning made important contributions to the determination of attitude towards mathematics. The particular significance of this study to the writer was the incongruance between the male and female resulting prediction equations. This led the writer to attempt a study in male and female attitudes toward mathematics.

Ellingson (1962) constructed a mathematics attitude inventory and administered the inventory to 755 pupils in 31 mathematics classes in nine Oregon junior and senior high schools. From an analysis of the results, Ellingson found that the attitude inventory was as valid a predictor of attitudes toward mathematics as were the judgments of teachers who had eight months of experience with the students.

Dutton (1956) administered an attitudes scale and an opinion questionnaire to 226 boys and 233 girls in junior high school in order to determine the students' attitudes toward arithmetic. His findings were incongruous, in that 66 per cent

said arithmetic was fun, but only seven per cent maintained they had satisfaction in the working of problems.

Edholm (1953) was not able to increase the prediction of success in algebra when a personality test was added to previous achievement and intelligence scores and an algebra prognostic test. Kuykendall (1956), after testing 398 grade seven students, could not establish any significant differences in the personality and adjustment factors of over- and underachievers in arithmetic. On the basis of the evidence presented in these two studies, personality as an independent variable was excluded from the writer's research.

School Marks

The evidence cited from the studies of Marks and Murray (1965), Klugh and Bierley (1959), Scannell (1960), and Boney (1966) justified the writer's including school marks as an independent variable in his study.

Marks and Murray (1965) stated that from their research, the high school average provided the most information about future academic performance at the college level as measured by college grade point average.

Klugh and Bierley (1959) did a study to determine: (1) the validity of the School and College Ability Test against a criterion of first semester freshman liberal arts college grade point average at Alma College, and (2) whether a significant improvement in prediction would be gained by using the student's high school grade point average in addition to the above test.

The criterion measure was the student's grade point average computed at the end of the first semester of college work. The results of the study indicated that the SCAT was a valid test, and this test also was about as valid as was high school average in predicting first semester college grades. It also indicated that the inclusion of the student's high school average with the SCAT score increased the correlation coefficient significantly over the SCAT score alone.

Scannell(1960) studied the prediction of college success from elementary and high school performance. He found that the accuracy with which general academic success was predicted from achievement test scores increased year by year from grade four through high school, and that high school grade point average was the best single predictor of college success. This conclusion was based on correlations of .67 and .57 with freshmen and four-year grade point average respectively.

Boney (1966) attempted to predict the high school achievement of Negro students. The predictor variables used were: the DAT, the California Test of Mental Maturity, the Cooperative Ability Tests, the Sequential Tests of Educational Progress, junior high school grade point average, and social status ratings. The junior high school grade point average contributed most significantly to the multiple correlation coefficient, while the social status variable did not contribute significantly.

Measures of Aptitude

Jacobs (1959) attempted a study to evaluate the effectiveness of certain measures of aptitude and achievement in predicting

high school academic success in three representative Cincinnati public high schools. The predictors used were: four sub-tests from the DAT, i.e., Verbal Reasoning, Numerical Ability, Mechanical Reasoning, and Language Usage; an English and arithmetic proficiency test; and the Terman-McNemar Tests of Mental Ability. Prediction was attempted for general academic success and in some specific subject areas. The criteria of success used were: grade point average and the Essential High School Content Battery.

The findings of the study led to the following significant conclusions:

(1) In predicting academic success, the arithmetic proficiency test proved to be the best predictor of total grade point average, while the Terman-McNemar Test was the best single predictor of the composite score of the Essential High School Content Battery.

(2) For the most part, the girls were a more predictable group than the boys.

(3) For the purpose of predicting academic success, the inclusion of the four DAT sub-tests did not appear to be justified.

Layton and Swanson (1958) attempted to determine the relationship of grade nine DAT scores to the grade 11 test scores and the high school rank of the students. They hypothesized that the DAT scores were most suitable as a measure of both academic ability and special aptitudes, and that these tests would indicate to what degree the student possesses special

aptitudes which he may capitalize upon in making a vocational choice other than college. They found that the Verbal Reasoning Test was the best single predictor of grade 11 test scores. They also found that high school rank and the combination of Verbal Reasoning with Numerical Ability Tests gave a slightly higher correlation with the grade 11 scores than did the Verbal Reasoning Test alone. These investigations also found the girls to be a more predictable group than the boys, in that the correlation coefficients were on the average higher for the girls.

The studies of Jacobs (1959) and Layton and Swanson (1958) gave evidence to justify the inclusion of a measure of aptitude in the writer's research.

METHODS OF PREDICTION

Regression

The object of this study was to establish a linear regression equation for a particular response in terms of "independent" or predictor variables X_1 $X_2 \ldots X_k$.

In selecting the best regression equation, a number of procedures were considered, such as (a) all possible regressions, (b) backward elimination, (c) forward selection, and (d) stepwise regression. A brief outline of each follows.

(a) All possible regressions: The examination of all possible regressions would not provide a clear-cut answer to the problem. Other information, such as knowledge of the characteristics of the final grade and the role of the X- variables, must

always be added to enable a decision to be made. Draper (1968) said analysis of all regressions was quite unwarranted. All possibilities would have been looked at; however, a large number of regression equations which intelligent thought would often reject would have had to be examined. Some sort of selection procedure which shortens this task was preferable.

(b) The backward elimination procedure: The backward elimination method is an improvement on the "all regressions" method in that it attempts to permit the examination of not all regression but of only the "best" regression containing a certain number of variables.

(c) The forward selection procedure: The backward elimination method begins with the largest regression, using all variables, and subsequently reduces the number of variables in the equation until a decision is reached on the equation to use. The forward selection procedure is an attempt to achieve a similar conclusion working from the other direction. The order of insertion is determined by using the partial correlation coefficient as a measure of the importance of variables not yet in the equation.

The forward selection procedure is more economical than the methods previously discussed, and it avoids working with more X's than necessary while improving the equation at every stage. One of its drawbacks is that it makes no effort to explore the effect that the introduction of a new variable may have on the role played by a variable which entered at an earlier stage. This deficiency is overcome by the stepwise procedure which is

an improvement on the forward selection procedure.

(d) The stepwise regression procedure: This procedure is an improved version of the forward selection procedure. The improvements involve the re-examination at every stage of the regression variables incorporated into the model in previous stages. A variable which may have been the best single variable to enter at an early stage may, at a later stage, be superfluous because of the relationships between it and other variables now in the regression. To check on this, the partial F criterion for each variable in the regression at any stage of calculation is evaluated and compared with the pre-selected percentage point of the appropriate F distribution. This provides a judgment on the contribution made by each variable as though it had been the most recent variable entered, irrespective of its actual point of entry into the model. Any variable which provides a non-significant contribution is removed from the model. This process is continued until no more variables will be admitted to the equation and no more rejected.

The superiority of the stepwise regression procedure has been revealed in the above discussion. As a result of this superiority, the stepwise regression procedure was utilized by the writer in his study.

Expectancy Tables

The expectancy table is a helpful device which the counselor can use as a means of communicating results of tests to other people. This device translates test scores into "chances of

success" statements. They are called expectancy tables on the assumption that the best guess to be made as to a student's probable success is what students with similar scores have done before.

It should also be noted that confidence in prediction depends directly on the numbers of students involved. Large numbers of cases permit more precise predictions without the loss of reliability.

Bennett, Seashore, and Wesman (1966) stated the following with regard to expectancy tables:

> Expectancy tables are not intended to replace correlation coefficients as evidence of validity. They present in more detailed form the same data which are summarized in a correlation coefficient. The chief advantages of using the correlation coefficient are that it expresses by means of a single number the relationship between the test and the criterion and that it may be used in statistical analysis of the data. On the other hand, the expectancy table has the advantage of making easily apparent even to non-statisticians the specific relationship which exists between the test and the course, thus facilitating the counselor's task of explaining the meaning of scores to students, teachers, and parents.

Blair (1966) found that expectancy tables were of particular value in the selection of students for admission to college, since it indicated the student's chance or probability of achieving a satisfactory grade point average.

Anderson and Stiles (1968) constructed expectancy tables using the grade nine and grade 12 stanine grades and the comparable rankings in the Scholastic Ability Test. They found the tables useful in helping the students assess their future chances of success rather than to see them drop out of school without any awareness of the probabilities of success or failure involved.

The above studies presented the usefulness of expectancy tables as an aid in prediction. This tool was therefore included as a technique of prediction in the writer's study.

SUMMARY

The material presented in this chapter gave justification to the writer for selecting the variables included in his study. Evidence was also presented to justify the exclusion of certain other variables from the study. Substantial discussion was presented to support the selection of stepwise regression and expectancy tables as prediction techniques.

The review of the literature was carried out in four parts: (1) predicting success in elementary algebra, (2) analyzing mathematical aptitudes, (3) reviewing specific independent variables, and (4) examining methods of prediction. In reviewing specific independent variables, general intelligence, attitude, school marks, and measures of aptitude were looked at for their validity in predicting success in grade nine algebra.

The review was begun by examining studies which attempted to predict success in grade nine algebra. Numerous variables were found to have been used and no general consensus was obvious. This led to a review of the literature pertaining to aptitudes necessary for the study of mathematics. This gave some direction and certain specific variables were then selected and reviewed in detail. This was followed by a review of literature dealing with regression and expectancy tables as prediction techniques.

To gather data for the variables selected from the research discussed, the writer used test instruments which are presented in Chapter III.

Chapter III

THE DESIGN OF THE STUDY

The purpose of this chapter is to outline and describe the study in detail. The independent and dependent variables selected for use in this study are presented. The description of the population and population sample is followed by an outline of the data sources and by the methods of collecting the data. The analysis of the data consisted of a series of applications of the stepwise regression techniques followed by cross-validation of the equations developed. The cross-validation attempted to determine the effectiveness of the prediction equations. Also in the analysis, expectancy tables were constructed.

The two instruments of prediction were compared using students randomly selected with predicted grades in grade nine mathematics. Comparisons were also made between the complete population sample and the entire grade nine population of Stevenson Junior High School. The comparisons were presented to establish which method of prediction would be more suitable for use by the student and counselor.

THE VARIABLES USED IN THE STUDY

In this study, six variables were selected for use. There were five independent variables and one dependent variable. These

are stated and described in the following two sections of this chapter.

Independent Variables

After reviewing the relevant literature, five independent variables were selected for use in this study. In three cases, this resulted in the need to select an appropriate test to quantify the variable. The selection of the method of acquiring the empirical data was to some extent influenced by the testing services carried out by the St. James-Assiniboia School Division, and by the resources available to the investigator.

(a) Scholastic Aptitude: Since the DAT battery was administered to every grade nine student in the St. James-Assiniboia School Division, this test seemed worthy of consideration as a source of data for this study. As well, this battery of tests has received considerable use in prediction studies of a general nature and more specifically as a predictive index of achievement in mathematics. The use of this battery of tests was discussed in Chapter II. On examining the reviews closely, it was found that the Verbal Reasoning and Numerical Ability sub-tests were valid predictors of mathematics achievement. Consequently, these two sub-tests were included as independent variables in this study.

Cain (1966) reported there was general agreement that both reasoning ability and computational skill are important tools for the study of mathematics. The modern mathematics program claims to emphasize understanding and reasoning. This did not

rule out the need for computational skills, but further justified the inclusion of both the Verbal Reasoning and Numerical Ability (DAT) tests as predictors.

The DAT Verbal Reasoning and Numerical Ability are two tests from an eight-test battery designed to be administered in one sitting, but Buros (1959) reported that each test in the battery is considered to be valid in itself. The tests are power tests rather than speed tests and are considered to be excellent in construction, standardization, validity, and reliability.

The DAT Verbal Reasoning Test is a 50-item multiple choice test with 16 choices per item. In the following statement from a review of the test, Buros (1959) indicates the characteristics of the test.

> Verbal Reasoning presents a series of verbal analogies items which probably measure a combination of the 'verbal ability' and 'deduction reasoning' factors. In any event, it is a good measure of the student's ability to handle complex logical relationships which can be stated in verbal terms and, in this sense, it is largely a measure of 'intelligence' as this is ordinarily conceived.

The DAT Numerical Ability Test is a 40-item multiple choice test with five choices per item. A description of the test is contained in the following statements excerpted from Buros' (1959) reviews of the test.

> Numerical Ability presents a series of relatively simple numerical problems requiring a minimum of arithmetic reasoning. It is chiefly a measure of mental computational skill.
>
> Some of the items are straight computational items involving the adding, subtraction, multiplying, and dividing of whole numbers, decimals and fractions. Other items seem to require reasoning, although the form of the items resembles the purely computational type.

The authors of the DAT, Bennett, Seashore, and Wesman (1966), reported that the Verbal Reasoning plus Numerical Ability combination or the Numerical Ability Test alone showed the greatest predictive power for mathematics grades.

The above reports gave further evidence for the inclusion of the DAT Verbal Reasoning and Numerical Ability Tests as test instruments to quantify scholastic aptitude in this study.

(b) Attitude Towards Mathematics: From the reviews in Chapter II, attitude towards academic subjects was found to have differing validities in the prediction of subject achievement. At the local level, many conjectures have been made with regard to the effects of attitude on subject performance. In an attempt to answer the question of its validity, attitude was included as an independent variable in this study.

The Math Attitude Scale was selected to provide an estimate of the student's attitude toward mathematics as a subject. The Math Attitude Scale was developed by Aiken and Dreger (1961) for use in a study to determine the effect of attitudes on performance in mathematics. It has since been deposited with the American Documentation Institute; from this source the scale was secured and reproduced for testing the population sample. The Math Attitude Scale consists of 10 items connoting negative attitudes and 10 connoting positive attitudes. Investigations by the originators found its reliability to be $r = .94$, for test-retest. In addition, a test of independence between the scores on the attitude scale and scores on four items designed to measure attitudes toward

academic subjects, suggested that attitudes specific to mathematics were being measured (X^2 = .80, d.f. = 1). In the study reported by Aiken and Dreger (1961), the hypothesis of significant contribution of mathematics attitudes to prediction of achievement was borne out for females but not for males.

The Math Attitude Scale was scored according to the Likert Method described by Ahmann and Glock (1969). Using this method, the student reacted to each statement on a five point scale indicating that he strongly disagreed, disagreed, neither agreed nor disagreed, agreed or strongly agreed. The score was computed by weighting the responses from one to five for a favorable statement beginning with strong disagreement and summing the weights for all favorable statements on the Math Attitude Scale. Values were assigned in reverse order for unfavorable statements and were similarly summed.

The total score on the scale was the sum of all weighted items with a maximum favorable score of 100, a maximum unfavorable score of 20, and a total neutral score of 60. This scoring was done by the writer.

(c) Algebra Aptitude: The third edition of the Iowa Algebra Aptitude Test was selected to give an estimate of algebra aptitude. This test consists of four parts. Part one includes 34 items of the type usually referred to as a number series. The student must infer the rule of formation of a sequence by supplying the next term. Part two involves four lessons on mathematical

concepts which the student has probably never encountered in previous work. In writing this part, the student must first study the material on these unfamiliar concepts and then answer a number of questions on the material. The authors included this part in the test to duplicate one aspect of the tasks the student faces in the study of modern mathematics. Part three is entitled "Open Phrases", and requires the student to translate 20 quantitative ideas into symbolic statements. Part four presents 10 statements about counting numbers which are represented by letters. The student is required to decide how one of these numbers changes if certain specified changes occur in the others. For the purpose of this study, only the total raw score was used.

In their study, Sabers and Feldt (1968) found the validity of the Iowa Algebra Aptitude Test to be .78 in correlation with achievement tests and .71 in correlation with the teachers' marks. They suggested the correlation with marks may have been lower than those with the achievement test scores as a result of the lower reliability of the teachers' marks. They also estimated the reliability of the Iowa Algebra Aptitude Test at .93 by the Kuder-Richardson procedure.

(d) Grade Eight Mathematics: The fifth and final independent variable selected was the student's final grade in grade eight mathematics. Callicutt (1961) found, in a study already reviewed in Chapter II, that eighth grade mathematics grades were found to be a better predictor than intelligence quotients, achievement test scores, or eighth grade composite averages. This report supports the writer's selecting the final grade as

an independent variable for his study.

Dependent Variable

Grade Nine Mathematics: Consideration was given to setting the criterion variable as a standardized achievement test, but after considering the fact that numerous different methods of instruction would likely be given by different teachers to the students participating in the study and that objectives of the teachers would probably not match those of the authors of the standardized test, the grade nine final grade was selected as the dependent variable.

Summary

The variables of the investigation are summarized as follows:

Independent Variables

(a) Scholastic aptitude measured by:
    (1) Verbal Reasoning sub-test (DAT)
    (2) Numerical Ability sub-test (DAT).
(b) Attitude towards mathematics measured by Math Attitude Scale.
(c) Algebra aptitude measured by Iowa Algebra Aptitude Test.
(d) Grade eight mathematics measured by grade eight mathematics final grade.

Dependent Variable

Grade nine mathematics final grade.

POPULATION AND THE POPULATION SAMPLE

The population was defined as all students enrolled in the regular grade nine mathematics program (excluding occupational entrance and special education students) in the St. James-Assiniboia School Division No. 2, from September, 1970 to June, 1971. Ideally, all these students should have been assigned numbers and then using random number tables, subjects selected for the population sample. This, however, was found impractical as tests could not be administered to individual students throughout the School Division. As a compromise, all seven full junior high schools in the Division were selected (i.e., those schools offering grades seven, eight, and nine only). This eliminated two schools--one with two classes of grade nine comprising 60 students, the other with four classes comprising 75 grade nine students. From the participating schools, one complete homeroom class was selected at random by lot, giving seven classes and a total of 173 students for the population sample.

A special sub-study involving all the students of Stevenson Junior High School was carried out. The data collected for the main study was also collected for all the Stevenson students. From this group of students, one complete class was randomly selected by lot for the sample for the main part of the study. The purpose and further details of this sub-study are found later in this chapter.

Table 1(a) lists the participating schools and relevant information concerning each. Table 1(b) lists the randomly selected

classes, the number of students in each class, and the general ability structure of the class.

Table 1(a)

The Population

School	Number of Grade Nine Classes	Total Grade Nine Students	Description of Class Structure
Bruce Junior High School	5	162	Heterogeneous
Deer Lodge Junior High School	6	149	Homogeneous
Golden Gate Junior High School	10	265	Top and bottom students homogeneous; middle heterogeneous--about one-half homogeneous, one-half heterogeneous
Hedges Junior High School	4	120	Heterogeneous
Ness Junior High School	7	189	Heterogeneous
Sansome Junior High School	10	299	Heterogeneous
Stevenson Junior High School	4	84	Heterogeneous
Total	46	1268	

Total grade nine classes in the Division: 52

Total grade nine students in the Division: 1403

Table 1(b)

THE POPULATION SAMPLE

School	Number of Grade Nine Classes Selected	Number of Students in Class Selected	Description of Class Structure by Ability
Bruce Junior High School	1	32	Heterogeneous
Deer Lodge Junior High School	1	22	Average
Golden Gate Junior High School	1	15	Low
Hedges Junior High School	1	23	Heterogeneous
Ness Junior High School	1	29	Heterogeneous
Sansome Junior High School	1	30	Heterogeneous
Stevenson Junior High School	1	18	Heterogeneous
Total	7	169	

DATA SOURCES AND COLLECTION

Once the classes to be involved in the study were selected, the collecting of the data was carried out. The Iowa Algebra Aptitude Test and the Math Attitude Scale were administered to the population sample during the early part of October, 1970. Ideally, this testing should have been done in June of 1970 while the students were still in grade eight. However, this was not found possible and early October was chosen as an appropriate time for the testing. September was intentionally avoided due to the probability of the students being in a somewhat restless state of mind following the summer recess. This made it necessary to review skills and concepts learned during the eighth grade as is usually done in the beginning of the grade nine mathematics program.

The scores for the above tests were recorded as raw scores since no local norms were available. These tests had not been administered in the School Division prior to this study.

The Iowa Algebra Aptitude Test and the Math Attitude Scale were administered by the local classroom teacher. For the Iowa Algebra Aptitude Test, the instructions were prepared by the writer, using the accompanying manual for the test, compiled by Greene and Sabers (1969). These instructions were duplicated and distributed to each teacher concerned and each was asked to follow the instructions strictly. The Math Attitude Scale was duplicated with instructions for the students printed at the beginning of the test. No instructions were needed for the super-

vising teachers. It was hoped these precautions would approach identical test administration conditions for all students of the population sample. Copies of the above tests are included in the appendix.

The St. James-Assiniboia School Division annually administers the complete battery of the DAT to all grade nine students in the Division. This was done during the month of November, 1970. Since the test is considered an aptitude test and not an achievement test, it was felt this time of administering the test would not alter the validity as a predictor of success in algebra. This test was administered in the gymnasium of each school to all students at the same time, and this operation was supervised by the guidance counselor of each school. This person had received the same instructions for administering the tests from the supervisor of guidance for the School Division. The tests were scored by computer and then returned to the schools for translating raw scores to percentile ranks. The percentile ranks were taken from norms developed by the local Division for its own students. The percentile ranks were used as the measures of the independent variables of Verbal Reasoning and Numerical Ability.

The grade eight mathematics grade for each student was taken from the record files of the students in the population sample and recorded as A, B, C, D, and E. These were then converted to numbers according to the scale A = 5, B = 4, C = 3, D = 2, and E = 1.

The dependent variable of the final grade point in grade nine mathematics was also taken from the students' record files. This grade was a composite of the entire year's work and was composed of 20 per cent of the first term together with 40 per cent of each of the remaining two terms of the school year. This grade was also expressed as A, B, C, D, and E, and then translated to numbers according to the same scale stated for the grade eight mathematics grade.

This data along with a coded school, room, student number, and sex was then punched into computer cards by the writer.

ANALYSIS TECHNIQUES

Stepwise Regression

The basis for selecting this technique was presented in Chapter II. The actual analysis was performed with the aid of a computer program and thus only a general outline of the steps involved will be given.

Step 1. The stepwise regression procedure started with the simple correlation matrix and entered into regression, the independent variable most highly correlated with the criterion variable.

Step 2. Using a partial correlation coefficient as before, the next variable with the highest partial correlation coefficient with the criterion was selected.

Step 3. Given the regression equation from Step 2, the method now examined the contribution of the last variable entered as if it had been entered first. (It is worth noting that the

forward selection procedure does not do this.) If the value of the partial F was statistically significant, the first variable was retained. Next, the stepwise method selected the variable most highly partially correlated with the criterion.

Step 4. A new regression equation was determined by least squares. The next variables entered and the partial F tests for the two previously entered variables were made to determine if they should remain in the regression equation.

Step 5. The above procedure was carried on until all independent variables had been considered.

From the procedure outlined, a prediction equation of the form $Y = ax_1 + bx_2 + - - - - + M$ resulted where Y represents the dependent variable final grade nine grade and x_1 x_2 represent the independent variables as selected through the regression analysis. The coefficients a, b, c...represent the regression weights of the independent variables and M represents the constant of the equation.

Prediction of Dependent Variable Grades by Regression

This validation of the stepwise regression procedure was designed to determine how effectively the academic performance of a grade nine algebra student could be predicted from the general regression equation developed for the total population sample. Students were randomly selected by lot from the complete population sample that served as the source of the data for the investigation.

The validation procedure was begun by selecting seven students by lot from the population sample. Each student's grade nine algebra grade was predicted by the general stepwise regression equation. The differences between the predicted and actual grades in algebra in grade nine were compared.

The results and findings of the validation procedure are reported in Chapter IV.

Expectancy Table Analysis

The second technique used in the analysis of the data was an investigation of the establishing and interpreting of expectancy tables.

The purpose of including the expectancy tables was stated by Anderson and Stiles (1968).

"The expectancy table is a useful test interpretation instrument and educational guidance device even in the absence of a trained specialist."

In constructing the expectancy tables, a tally mark was placed vertically, showing the student's score on the independent variable and horizontally, showing the subsequent achievement on the dependent variable. After compiling raw totals for each cell in the table, these were converted to percentages by row. This was done in order to facilitate interpretation and to allow the data to be viewed in a more practical fashion. To aid the practicality further, the median score of the dependent variable of each row was placed in the farthest right hand column of the table. A sample of the expectancy tables, based on the population

sample, is found on page 74.

COMPARISON OF THE TWO TECHNIQUES OF PREDICTION

The comparison of the findings based on the two methods of prediction took the form, not of a statistical analysis, but of a general descriptive analysis. This comparison is discussed in the findings of the study in Chapter IV.

COMPARISONS BASED ON THE STEPWISE REGRESSION PROCEDURE

General Comparisons

A number of general comparisons were made using the regression equation developed for the population sample and examined against the equations developed for:

(a) all male students in the population sample;

(b) all female students in the population sample;

(c) the complete population sample.

Prediction for Entire Grade Nine Population of Stevenson Junior High School

With the cooperation of the principal of Stevenson Junior High School, all grade nine students from this school were tested and the data was made available for the variables considered in the study. This group of students constituted four classes and from these, one was randomly selected by lot to be included in the sample for the main part of the study. There were two purposes for including all grade nine students in the study:

first, to compare the prediction equation for success in algebra achievement for the entire school to that of a single class randomly selected from the same school; and second, to compare the prediction equation developed for the entire school population of grade nine students to the prediction equation developed for the population sample from the entire School Division.

These comparisons were carried out by examining the prediction equations and the predictions of a few students randomly selected by lot with each pair of equations. Examining the percentage of the variance which could be accounted for and the Standard Error of Estimate provided a statistical comparison.

SUMMARY

In this chapter, the five independent variables and the one dependent variable, selected for use in this study, were presented. The descriptions of the population and the population sample were followed by an outline of the data sources and then by the methods of collecting the data. The analysis consisted of a series of applications of the technique stepwise regression, followed by the cross-validation of the regression equations established by the stepwise regression procedure. The cross-validation was done by randomly selecting students by lot and predicting their grade nine algebra grade utilizing the regression equation developed. The predicted scores were compared with the actual scores to evaluate the effectiveness of the prediction equation. The second part of the analysis involved the construction of a series of expectancy tables to predict, on the basis of a

student's score on an independent variable, the probability of a student's obtaining a given grade or better on the dependent variable.

The two instruments of prediction were compared using students randomly selected by lot from the population sample with the predicted grades in grade nine algebra. Comparisons were also made based on the entire population sample and on the entire grade nine population of Stevenson Junior High School.

The results of the investigation and comparisons of these findings are reported in detail in Chapter IV.

Chapter IV

THE FINDINGS OF THE PREDICTION STUDY

In this chapter, the findings of the stepwise regression investigations for the complete population sample, males and females, and the students of Stevenson Junior High School, are presented. These include the correlation coefficients, and means and regression equations for the groups listed. Second, the findings of the expectancy table investigation for the complete population sample and the Stevenson Junior High students are summarized. This chapter is concluded with the findings based on a comparison of the regression equations and expectancy tables.

FINDINGS OF THE STEPWISE REGRESSION INVESTIGATION

The findings of the analysis based on stepwise regression are presented in the following order: those of the total population sample, those of the female segment of the sample, those of the male segment of the sample. A comparison of the latter two follows. Finally, there is a presentation of findings of the analysis based on stepwise regression for the entire grade nine population of Stevenson Junior High School.

Simple Correlation Coefficients and Means for the Complete Population Sample

In the total population sample, there were 169 students, and the means for the six variables quantified for each student

were calculated and are presented in Table 2.

Table 2

Population Sample Means and Scales for the Five Independent Variables and the One Dependent Variable

Variable No.	Variable	Scale	Mean
1	Grade 8 mathematics grade	1-5	3.373
2	DAT Verbal Reasoning	0-100	53.473
3	DAT Numerical Ability	0-100	45.284
4	Iowa Algebra Aptitude Test	0-80	51.834
5	Math Attitude Scale	0-100	65.639
6	Grade 9 mathematics grade	1-5	3.219

Independent variables numbers 1-5

Dependent variable number 6

For the complete population sample, the simple correlations coefficients between all of the variables are presented in Table 3 on page 52.

The simple intercorrelation between grade nine mathematics and grade eight mathematics was 0.754, followed by 0.677 for grade nine mathematics with the DAT Verbal Reasoning Test, and the Iowa Algebra Aptitude Test of 0.650. If a logical continuity exists in basic content of a mathematics course from one grade to the next, one would expect similar areas of evaluation from one grade to the next. The high correlation between grades eight and nine mathematics grades could be attributed to the similar quantities being measured in each grade.

Table 3 demonstrates quite significant intercorrelations between the results in grade eight mathematics, the DAT Numerical Ability Test, the Iowa Algebra Aptitude Test, and grade nine mathematics. Being aware of the subject matter of these variables individually, one now presumes the content to be common to all.

Table 3

Simple Correlation Coefficients Between All Variables

	Grade 8 Maths.	DAT Verbal Reas.	DAT Num. Ability	Iowa Alg. Apt. Test	Math Attitude Scale	Grade 9 Maths.
Grade 8 Maths.	1.000					
DAT Verbal Reas.	0.458	1.000				
DAT Num. Ability	0.668	0.500	1.000			
Iowa Alg. Aptitude Test	0.621	0.558	0.679	1.000		
Math Attitude Scale	0.460	0.138	0.335	0.346	1.000	
Grade 9 Maths.	0.754	0.460	0.677	0.650	0.519	1.000

The stepwise regression analysis findings are presented in Table 4.

Table 4

Stepwise Regression Findings for Complete Population Sample

Step No.	Variable Entered	Per Cent Variance Accounted For	Standard Error of Predicted Variable
1	Grade 8 Mathematics	56.829	0.955
2	DAT Numerical Ability	62.236	0.896
3	Math Attitude Scale	65.641	0.857
4	Iowa Algebra Aptitude Test	67.290	0.840
5	DAT Verbal Reasoning	67.490	0.840

From the results of the stepwise regression procedure, the increases in per cent variance accounted for through the addition of variables were tested for statistical significance at the .01 level. In the procedure, as the variables were added, the only one which did not contribute significantly to the variance accounted for was the DAT Verbal Reasoning Test.

Letting the grade nine mathematics grade = Y, the grade eight mathematics score = X_1, the DAT Numerical Ability = X_3, the Iowa Algebra Aptitude Test = X_4, the the Math Attitude Scale = X_5, the prediction equation for the complete population sample was

$$(1)\quad Y = 0.461X_1 + 0.011X_3 + 0.024X_4 + 0.018X_5 - 1.239.$$

This equation accounted for 67.3 per cent of the variance with a standard error of 0.840. The regression weights were derived from the stepwise regression program as carried out by the computer.

In the tables, all per cent variance accounted for was given to five significant figures while, for convenience in the discussion, only three are used.

Although the use of four variables in the prediction equation was justified statistically, in a practical sense this may not be feasible. The Iowa Algebra Aptitude Test and the Math Attitude Scale were not administered as part of the St. James-Assiniboia School Division's annual testing program. These tests were given specially for this study. As a result, these scores would not be readily available. The equation predicting grade

nine mathematics grades with two variables is:

$$(2)\ Y = 0.619X_1 + 0.015X_3 + 0.431.$$

This equation accounted for 62.2 per cent of the variance with a standard error of 0.896. The variables are as in equation (1).

If, at the local level, it were considered important enough to account for an additional 3.4 per cent of the variance by administering the Math Attitude Scale at the beginning of the school year, and adding it to the predictors of equation (2), 65.6 per cent of the variance could be accounted for with a standard error of 0.857. The following equation could be used:

$$(3)\ Y = 0.518X_1 + 0.015X_3 + 0.019X_5 - 0.453.$$

Table 4 shows the grade eight mathematics mark as the best single predictor of grade nine mathematics. As reported in Chapter II, Callicutt (1961) found the same result in his study.

Table 4 also demonstrates an increase of approximately 10 per cent in the variance accounted for. This occurred from Step 1 to Step 4 in the regression procedure. The addition of the DAT Verbal Reasoning and the Iowa Algebra Aptitude Test should be used in counseling not only due to their statistically significant contribution but also since these are standardized tests which are not subjectively influenced by the teacher. Whereas all other independent variables represent cognitive areas, the Math Attitude Scale measures a non-cognitive area. The inclusion of the above variables as predictors in addition to the grade eight mathematics grade is warranted. The student would no doubt have

greater confidence in the prediction knowing it was based on more than one variable.

Despite the statistical significance, it is up to the individual wishing to make use of the regression equations to determine the number of independent variables to be used. For this reason, three regression equations were presented for consideration.

Prediction of Dependent Variable Scores for the Complete Sample

Seven predicted and reported students' scores for the dependent variable are reported in Table 5. The students whose scores were predicted were selected at random by lot from the complete population sample, which consisted of one randomly selected class from each of the seven full junior high schools in the St. James-Assiniboia School Division. The average difference between the reported and predicted scores for equation (1) was 0.441 while for equation (2), it was 0.614.

Table 5 lists the per cent grade and the reported grade point for the selected students. Accuracy is lost in converting a per cent grade to a reported grade, but the latter is the only grade available to the student. For this reason, the reported grade and predicted grade were used for comparisons in this study.

It would be up to the user of the equations to decide whether the increase in predictive accuracy was important enough to warrant the use of equation (1) over equation (2) with the extra time and effort necessary to collect the additional data.

Table 5

Prediction of Grade Nine Mathematics Grades

Student No.	Sex	Per Cent Grade	Reported Grade Point	Predicted Grade Point Equation(1)	Predicted Grade Point Equation(2)
231	F	83	5	4.714	4.426
159	M	90	5	4.621	4.651
121	F	Failed	1	1.040	1.350
268	M	89	5	4.718	4.032
169	F	52	2	2.072	2.438
195	F	56	2	3.738	3.582
237	M	63	3	3.292	3.038

Interpretation of Scores Predicted by Regression Equations

The person employing regression equations in a counseling situation must be extremely careful to interpret the predicted scores correctly. For example, in the case of student number 231 where equation (1) predicted his score to be 4.714, the counselor must tell the student that of the students who received the grades and scores that he achieved on the independent variables, 68 per cent of these students would achieve a grade nine mathematics grade point of 4.714 ± 0.839, taking into account the standard error of measurement. Although the standard error in this case was rather high, the student should be told that in the range of scores resulting (4.714 ± 0.839), the one most likely to occur is 4.714. If this point were stressed, this prediction would be meaningful and significant to the student. Unless the predicted scores are interpreted correctly, their value is in jeopardy.

Simple Correlation Coefficients and Means for Males and Females of the Entire Population Sample

In the total population sample of 169 students, there were 81 females and 88 males. The means for each group are presented in Table 6 on page 59, and the correlation matrices are presented in Tables 7 and 8 on pages 60 and 61. The correlation matrices and other relevant statistical information were derived from the stepwise regression procedure as carried out by the computer. All means are lower for females than for males for corresponding variables, but the difference of 10.906 between the DAT Numerical Ability scores was the only statistically

Table 6

Means of the Independent and Dependent Variables for Males and Females of the Complete Population Sample

Variable	Mean	
	Female	Male
Grade 8 Mathematics Grade	3.309	3.432
DAT Verbal Reasoning	51.790	55.023
DAT Numerical Ability	39.605	50.511
Iowa Algebra Aptitude Test	50.716	52.864
Math Attitude Scale	64.222	66.943
Grade 9 Mathematics Grade	3.049	3.376

Table 7

Simple Correlation Coefficient Matrix for Females of the Complete Population Sample

	Grade 8 Maths.	DAT Verbal Reas.	DAT Num. Ability	Iowa Alg. Apt. Test	Math Attitude Scale	Grade 9 Maths.
Grade 8 Maths.	1.000					
DAT Verbal Reas.	0.472	1.000				
DAT Num. Ability	0.708	0.558	1.000			
Iowa Alg. Aptitude Test	0.619	0.581	0.721	1.000		
Math Attitude Scale	0.564	0.167	0.443	0.476	1.000	
Grade 9 Maths.	0.748	0.472	0.730	0.697	0.615	1.000

Table 8

Simple Correlation Coefficient Matrix for Males of the Complete Population Sample

	Grade 8 Maths.	DAT Verbal Reas.	DAT Num. Ability	Iowa Alg. Apt. Test	Math Attitude Scale	Grade 9 Maths.
Grade 8 Maths.	1.000					
DAT Verbal Reas.	0.446	1.000				
DAT Num. Abil.	0.638	0.439	1.000			
Iowa Alg. Apt. Test	0.620	0.534	0.636	1.000		
Math Att. Scale	0.337	0.088	0.196	0.192	1.000	
Grade 9 Maths.	0.761	0.439	0.616	0.599	0.392	1.000

significant one at the .01 level. When comparing the female and male correlation coefficients, it was noted that the majority of corresponding correlations were very close, but considerable differences were seen for attitude relating to the other variables. The following correlations were also observed: female Math Attitude--eighth grade mathematics = 0.564 while males Math Attitude--eighth grade mathematics = 0.337, female Math Attitude--DAT Numerical Ability = 0.443 while male Math Attitude--DAT Numerical Ability = 0.196, female Math Attitude--Iowa Algebra Aptitude Test = 0.476 while male Math Attitude--Iowa Algebra Aptitude Test = 0.192, and female Math Attitude--ninth grade mathematics = 0.615 while male Math Attitude--ninth grade mathematics = 0.392. From these correlations, the conclusion made was that attitude towards mathematics was much more highly correlated to achievement in the dependent variable, as well as to the independent variable, for females than for males. Aiken and Dreger (1961) arrived at the similar conclusion in their study, as reported in Chapter II.

Selection of Independent Variables as Predictors for the Females and Males of the Total Population Sample

The stepwise regression analysis findings for females are reported in Table 9 on page 63, and for males in Table 10 on page 64. While testing the increase in variance for statistical significance through entering additional variables, only the grade eight mathematics score contributed significantly for males. For females, the three variables, grade eight mathematics, Iowa Algebra Aptitude Test, and Math Attitude Scale, contributed

Table 9

Stepwise Regression Findings for Females From the Complete Population Sample

Step No.	Variable Entered	Per Cent Variance Accounted For	Standard Error of Predicted Variable
1	Grade 8 Mathematics	55.922	1.002
2	Iowa Algebra Aptitude Test	64.750	0.901
3	Math Attitude Scale	67.980	0.865
4	DAT Numerical Ability	70.507	0.835
5	DAT Verbal Reasoning	70.596	0.840

Table 10

Stepwise Regression Findings for Males From the Complete Population Sample

Step No.	Variable Entered	Per Cent Variance Accounted For	Standard Error of Predicted Variable
1	Grade 8 Mathematics	57.907	0.908
2	DAT Numerical Ability	60.764	0.882
3	Math Attitude Scale	62.959	0.862
4	Iowa Algebra Aptitude Test	64.065	0.854
5	DAT Verbal Reasoning	64.378	0.856

significantly to the variance. The tests for significance were all conducted at the .01 level.

Another interesting observation was that for males with the five variables used, 64.4 per cent of the variance could be accounted for; for the females, 70.6 per cent of the variance could be accounted for. This supported Jacobs' (1959) finding that for high school students, females were, for the most part, a more predictable group than males.

Selection of the Independent Variables as Predictors for Males and Females From the Population Sample

Since the grade eight mathematics grade was the only variable that contributed significantly to the variance for males, the regression equation contained only this one variable in equation (4) with a standard error of 0.908 and 57.9 per cent of the variance accounted for.

(4) $Y = 0.836X_1 + 0.505$.

In equation (4), Y = grade nine mathematics grade and X_1 = grade eight mathematics grade.

The best prediction equation for females developed from the stepwise regression technique was equation (5) with a standard error of 0.865 and 67.8 per cent of the variance accounted for.

(5) $Y = 0.484X_1 + 0.045X_4 + 0.019X_5 - 2.061$.

In this equation, Y = grade nine mathematics grade, X_1 = grade eight mathematics grade, X_4 = Iowa Algebra Aptitude Test, and X_5 = Math Attitude Scale.

The above equations are presented for comparison purposes only with equation (1) for the complete sample. No predictions

were attempted using the above equations since for males, equation (4) accounted for less variance than equation (1) and in the case of females, equation (5) accounted for 67.8 per cent, while equation (1) accounted for 67.3 per cent of the variance; this was not a significant difference.

Simple Correlation Coefficients and Means for the Entire Grade Nine Population of Stevenson Junior High School

At Stevenson Junior High School, there were 84 grade nine students during the 1970-71 school year. Of this total number, some were absent during testing days for this study and some students transferred during the school year which allowed only 71 students to participate in this study. The stepwise regression technique was applied to the data from the 71 students, and the means are presented in Table 11 on page 67. The correlation matrix is found in Table 12 on page 68.

As principals of schools participating in this study were assured that no comparisons between schools or between a particular school's and the Division's means would be made, none are presented here with reference to Tables 2 or 11.

On comparing the correlation matrix for the complete population sample and that for Stevenson Junior High students (Tables 3 and 12), some interesting observations were found. First, except for the Math Attitude Scale, all correlation coefficients were lower for the Stevenson students in comparison to the complete sample. This could have accounted for the Stevenson students'

Table 11

Means of the Independent and Dependent Variables for All Grade Nine Mathematics Students of Stevenson Junior High

Variable	Mean
Grade 8 Mathematics	3.141
DAT Verbal Reasoning	51.775
DAT Numerical Ability	35.225
Iowa Algebra Aptitude Test	48.268
Math Attitude Scale	60.732
Grade 9 Mathematics	2.887

Table 12

Simple Correlation Matrix for All Grade Nine Mathematics Students of Stevenson Junior High School

	Grade 8 Maths.	DAT Verbal Reas.	DAT Num. Ability	Iowa Alg. Apt. Test	Math Attitude Scale	Grade 9 Maths.
Grade 8 Maths.	1.000					
DAT Verbal Reas.	0.387	1.000				
DAT Num. Ability	0.668	0.492	1.000			
Iowa Alg. Aptitude Test	0.505	0.478	0.587	1.000		
Math Attitude Scale	0.540	0.262	0.555	0.388	1.000	
Grade 9 Maths.	0.649	0.402	0.691	0.595	0.596	1.000

being less predictable (as seen in Table 13), than the complete sample.

Also in comparing Tables 3 and 12, it was seen that the correlation coefficients involving attitude were consistently higher for the Stevenson students than for the complete sample. Therefore, the conclusion was made that the attitude towards mathematics of the students of Stevenson Junior High School affected their performance in mathematics and the related aptitudes tested, more than for students from the complete population sample.

Table 12 demonstrated that the correlation coefficients for DAT Numerical Ability and grade eight mathematics with grade nine mathematics were highest and almost the equal, while the coefficients for the Iowa Algebra Aptitude Test and Math Attitude Scale with grade nine mathematics were almost equal, and differed only slightly from the two mentioned above.

Selection of Independent Variables as Predictors--All Grade Nine Mathematics Students in Stevenson Junior High School

The stepwise regression analysis findings for the students of Stevenson Junior High School are found in Table 13 on page 70

In comparing Tables 4 and 13, it was seen that, for the complete population sample, the grade eight mathematics grade accounted for the greatest amount of the variance in the grade nine mathematics grade. However, for the students in Stevenson Junior High School, the DAT Numerical Ability Test accounted for the greatest amount of the variance. For those students, the correlations of these variables with the grade nine mathematics

Table 13

Stepwise Regression Findings for All Grade Nine Mathematics Students of Stevenson Junior High School

Step No.	Variable Entered	Per Cent Variance Accounted For	Standard Error of Predicted Variable
1	DAT Numerical Ability	47.719	0.793
2	Math Attitude Scale	54.241	0.748
3	Iowa Algebra Aptitude Test	58.723	0.716
4	Grade 8 Mathematics	61.164	0.699
5	DAT Verbal Reasoning	61.173	0.705

grades were the highest of any of the independent with dependent variable correlations.

The only variables which contributed significantly to the variance accounted for, from the stepwise regression analysis, were the DAT Numerical Ability, Iowa Algebra Aptitude Test, and the Math Attitude Scale. The resulting equation was equation (6) with a standard error of 0.716, while 58.7 per cent of the variance was accounted for.

$$(6)\quad Y = 0.015X_3 + 0.025X_4 + 0.019X_5 - 0.040.$$

In the above equation, Y represented the grade nine mathematics grade, X_3 represented the DAT Numerical Ability Test, X_4 represented the Iowa Algebra Aptitude Test, and X_5 represented the Math Attitude Scale.

Four students from the grade nine Stevenson Junior High School population were selected at random, and equation (1) from page 54 and equation (6) were used to predict their grade nine mathematics grade points. Table 14 on page 72 gives the results and the average difference from the reported grade point was found to be 0.220 using equation (6), and 0.292 using equation (1). These differences were very small and from this sample, it would appear that the equation for the complete population sample was very nearly as good a predictive instrument as was the equation developed for Stevenson Junior High School students, some of which were included in the complete population sample.

Table 14

Predicted Grade Nine Mathematics Grades for Students of Stevenson Junior High School Using Equations(1) and (2)

Student No.	Sex	Per Cent Grade	Reported Grade Point	Predicted Grade Point	
				Equation(6)	Equation(2)
200	F	65	3	2.923	2.836
211	M	73	4	3.479	3.300
310	F	51	2	2.220	1.814
292	M	61	2	2.063	1.882

FINDINGS OF THE EXPECTANCY TABLE INVESTIGATION

Separate expectancy tables were constructed for the complete population sample, and for the students of Stevenson Junior High School. The writer only constructed those tables which he considered to be useful as predictive devices, based on their correlation coefficients. The tables for the complete population sample are presented first, followed by those for the Stevenson Junior High School students.

Prediction Based on the Grade Eight Mathematics Grade for the Complete Population Sample with Expectancy Tables

Since the single correlation coefficient for the grade eight mathematics grade to grade nine mathematics was found to be the largest ($r = 0.754$), it was expected that Table 17 on page 78 would be the most reliable in predicting a student's chances of success in grade nine mathematics (algebra). The prediction of performance based on all the expectancy tables to be discussed is presented in terms of the probability, expressed as a number of chances in a hundred, of a student obtaining a score as great as, or greater than, a specific dependent variable score. All predictions were based on the population sample for which the expectancy tables were constructed.

An example using Table 15 could be the case where a student received a grade of D in grade eight mathematics. The student could then be told that on the basis of this study, he has a 61 per cent chance of achieving a D, a 30 per cent chance of achieving a C, and a 12 per cent chance of achieving a B grade in an all-

Table 15

Complete Population Sample

Grade Nine Mathematics Expectancy Table Based on DAT Numerical Ability Test

DAT Num. Ability Percentile	N	Grade 9 Math. Grade D	C	B	A	Median Grade
90-99	16	94	94	81	81	A
80-89	15	93	93	87	67	A
70-79	17	100	100	100	65	A
60-69	13	100	85	77	23	B
50-59	15	100	80	53	20	B
40-49	21	100	81	33	5	C
30-39	15	53	53	40	7	C
20-29	18	67	33	22	11	D
10-19	18	72	33	17	6	D
0-9	21	61	28			D
Total N=	169					

algebra program. The student could also be told of the extremes where it is very unlikely he would achieve an A and, he could even fail the all-algebra course. This type of a prediction is very wide open and leaves a great many possibilities for the student. If he chooses to work hard, the student referred to could even achieve a B grade. Numerous other examples could be cited, using the same table.

Prediction Based on DAT Numerical Ability Test for the Complete Population Sample with Expectancy Tables

It is worth mentioning to the student that a prediction based on one expectancy table should not be considered alone, but in conjunction with as many other tables as are available. For example, from Table 15 based on the population sample studied, if a student achieved a percentile rating between 40 and 79, he would have a 100 per cent chance of passing the grade nine mathematics course (algebra), while if any other rating were achieved, there would be a possibility the student could fail the grade nine course. This seems a rather strange prediction, but it must be stressed to the student that, based on the sample considered, this is what could be expected. This example shows the need for more than one table to be used in counseling and points out one advantage of the regression equations developed in this study, where usually the prediction was based on more than one independent variable.

Table 15 demonstrates that a student with a high percentile rating could fail the grade nine algebra course; also, a student

with a low rating could pass the course.

Prediction Based on the Iowa Algebra Aptitude Test From the Complete Population Sample

Table 16 on page 77 shows that, if a student received a raw score of 40 or better on the Iowa Algebra Aptitude Test, and 142 or 169 students in the study achieved this score, that student would have some chance of achieving an A grade in grade nine algebra, although the chances would diminish as the raw score on the Iowa Algebra Aptitude Test becomes lower. In this table, as one proceeds in any row from left to right, the numbers decrease except for the 20-29 row. This could be due to the small number of students found in this row.

Table 16

Complete Population Sample

Grade Nine Mathematics Expectancy Table Based on Iowa Algebra Aptitude Test

Iowa Raw Score	N	Grade 9 Math. Grade D	C	B	A	Median Grade
70-79	6	100	100	100	83	A
60-69	39	100	98	87	64	A
50-59	52	89	75	46	19	C+
40-49	43	74	37	30	9	D
30-39	26	58	31	12		D
20-29	3	33	33			E
10-19	0					
0-9	0					
Total N=	169					

Table 17

Complete Population Sample

Grade Nine Mathematics Expectancy Table Based on Grade Eight Mathematics Grade Point

Grade 8 Maths. Grade	N	Grade 9 Math. Grade D	C	B	A	Median Grade
A	42	100	100	88	67	A
B	40	93	88	75	40	B
C	40	93	58	25		C
D	33	61	30	12		D
E	14	21	1			E
Total N=	169					

FINDINGS OF THE STEVENSON JUNIOR HIGH SCHOOL INVESTIGATION

Prediction Based on Expectancy Tables From the Stevenson Junior High School Students

Tables 18 through 21 consist of four expectancy tables derived from the results for the students of Stevenson Junior High School. As was reported on page 68, the correlation coefficients relating the independent and dependent variables showed two pairs clustered together. These were the DAT Numerical Ability with grade eight mathematics, and the Iowa Algebra Aptitude Test with the Math Attitude Scale. Therefore, these pairs of tables will have approximately equal reliability, with the DAT Numerical Ability to grade eight mathematics being slightly more reliable. These expectancy tables should be better predictive instruments for Stevenson Junior High School in that they were derived from results of students from that school alone. As a result, they should reflect any traits which might be unique to this school as a part of the St. James-Assiniboia School Division.

Once again, it should be stressed that these tables should not be used singly, but in conjunction with other expectancy tables or regression equations.

Table 18

Stevenson Junior High School

Grade Nine Mathematics Expectancy Table Based on Iowa Algebra Aptitude Test

Iowa Algebra Aptitude Test Raw Score	N	Grade 9 Math. Grade D	C	B	A	Median Grade
70-79	1	100	100	100	100	A
60-69	10	100	100	60	30	B
50-59	26	92	77	31	12	C
40-49	16	100	44	6		D
30-39	15	80	33	13		D
20-29	3	66	33			D
10-19						
0-9						
Total N=	71					

Table 19

Stevenson Junior High School

Grade Nine Mathematics Expectancy Table Based on Math Attitude Scale

Math Attitude Scale Raw Score	N	Grade 9 Math. Grade D	C	B	A	Median Grade
90-99	4	100	100	100	50	A
80-89	4	100	100	100	25	B
70-79	13	100	77	39	8	C
60-69	19	89	74	26	16	C
50-59	12	100	58			C
40-49	12	75	25			D
30-39	4	100	25			D
20-29	3	100	25			D
10-19						
0-9						
Total N=	71					

Table 20

Stevenson Junior High School

Grade Nine Mathematics Expectancy Table Based on Eighth Grade Mathematics Grade

Eighth Grade Mathematics Grade	N	Grade 9 Math. Grade D	C	B	A	Median Grade
A	13	100	100	77	31	B
B	12	100	92	33	17	C
C	26	92	62	15	4	D
D	12	75	17			D
E	8	88	25			D
Total N=	71					

Table 21

Stevenson Junior High School

Grade Nine Mathematics Expectancy Table Based on DAT Numerical Ability Test

DAT Numerical Ability Percentile Rank	N	Grade 9 Math. Grade D	C	B	A	Median Grade
90-99	3	100	100	100	33	B
80-89	5	100	100	100	60	A
70-79	3	100	100	100	33	B
60-69	4	100	75	50		B
50-59	6	100	100	17		C
40-49	8	100	75	13		C
30-39	9	78	56	22	11	C
20-29	10	100	50			D
10-19	11	91	45			D
0-9	12	75	25			D
Total N=	71					

FINDINGS BASED ON A COMPARISON OF THE TWO PREDICTION TECHNIQUES FOR THE COMPLETE POPULATION SAMPLE

The comparison of the two techniques of regression equations and expectancy tables makes reference only to that part of the study dealing with the complete population sample. Any conclusions with respect to the sub-studies carried out would be repetitious.

Using equation (1) from page 54 as the predictive instrument, the predicted grade points were found to be quite close to the actual students' grades. The average difference between the two scores was found to be 0.441, which was considerably less than the standard error of estimate, 0.839 predicted. However, in the long run, the standard error must be considered. The standard error implied that given a set of independent variables, grades and scores, a grade point of X plus or minus 0.839 would be predicted as an actual mark for 68 per cent of the students. Therefore, a regression equation would not predict an actual mark but a mark within a band of marks or scores. In the case cited above, the range would be 1.678 grade points, with the predicted score most likely to fall in the middle of the range.

The object of regression prediction is to account for the greatest amount of the variance possible with the smallest standard error. However, in dealing with human beings, a very small standard error would be difficult to achieve. Thus, in a counseling situation, we would do well to predict a mark within a range rather than a specific mark. The usefulness of regression equations would be quite dependent on the interpretation given to the student,

and a counselor using this device should be well trained in this area.

The expectancy tables do not give quite as restricted a picture for counseling. For example, refering to Table 17 on page 78, a student who, in grade eight mathematics received a final grade of B could be advised of having a 93 per cent chance of passing the grade nine mathematics course based on mainly algebra content. He could also be told he has an 88 per cent chance of achieving a C grade, a 75 per cent chance of achieving a B grade, and a 40 per cent chance of achieving a grade of A. This situation should show the student the increasing amount of work necessary as he sets his goal from the D, C, B, or A possibilities predicted based on the sample studied. It may be more advantageous for the student to choose less theoretical topics as his electives, rather than those which are strictly algebra.

As Tables 18, 19, 20, and 21 were examined, differences between these tables and those for the complete population sample were found. In the latter tables, the numbers generally decreased as one read down and from left to right. In the former, this pattern was not strictly adhered to. This phenomenon could be accounted for by the Stevenson students' being found less predictable than the complete sample as has already been reported. It is suggested that if at all possible, the counselor should not restrict himself to regression equations or expectancy tables, but rather, he should use both devices in order to establish which is preferred by the student. Once this has been determined,

he then could give the student further information on this type of prediction device and thus make the predictions more meaningful to the student.

SUMMARY

In this chapter, the findings of the investigation were reported. First, the findings of the stepwise regression procedures were reported for the complete population sample, males and females of the population sample, and the entire grade nine population of Stevenson Junior High School. Second, the findings of the expectancy table investigation were reported for the complete population sample and for the special Stevenson Junior High School study. This chapter was concluded with a comparison of the relative usefulness of the two predictive devices in the situation where a student is counseled as to whether in the elective section of his grade nine mathematics program, he should continue with algebra topics or seek other areas of study in which the chance and success would appear more promising.

In Chapter V, a summary of the findings will be presented and the implications of the findings discussed.

Chapter V

SUMMARY AND CONCLUSIONS

This chapter contains a review of the study, followed by a presentation of the major findings based on the information as found in Tables 2 to 21, Chapter IV. The conclusions based on this data are then presented to complement the objectives and major findings of the study.

Lastly, recommendations are made for future study in order to explain minor discrepancies not already accounted for, and to include other variables that possibly affect grade nine mathematics achievement.

REVIEW OF THE STUDY

This study was concerned with the prediction of students' academic performance in a basic algebra course. The significance of the study is based on the change in the grade nine mathematics curriculum being instituted by the Department of Education in Manitoba schools in September, 1971. At this time, a grade nine mathematics course will be authorized consisting of a mandatory elementary algebra section involving one-half of the school year and an elective section of many varied topics from which the student and teacher can choose topics for study during the latter half of the school year. The main purpose of this study was to develop prediction instruments which would be useful to the student

and teacher in helping the student decide whether to continue with algebra topics as the electives, thus preparing himself for Mathematics 100 in grade 10 or selecting more general topics in applied mathematics, thus preparing himself for Mathematics 101 or some other less theoretical mathematics course.

At mid-year of the student's grade nine program, the student does not have to decide on his mathematics program for grade 10, but since one-half of his grade nine course consists of options to prepare for further study, it is hoped the student will be able to decide generally his program for grade 10 based on reliable predictive instruments developed in this study.

The study developed prediction equations from the stepwise regression procedure and expectancy tables based on the independent variables with the highest correlation coefficients to the dependent variable. Through the study, regression equations were developed for the complete population sample, males and females of the population sample, and all the grade nine mathematics students of Stevenson Junior High School. Students' actual and predicted grade points were compared in order to establish the usefulness of the instruments developed.

The second method of prediction developed was that of expectancy tables. Through this method, the object was to develop an instrument which would predict academic achievement in a less restricting way than the regression equations. The expectancy table expresses the student's probability to achieve a specific grade or better as a number of chances out of a hundred.

The study concluded with a comparison of the two types of predictive instruments developed in an attempt to establish their relative usefulness in a counseling situation.

MAJOR FINDINGS

The major findings of the study will be presented in relation to prediction based on the stepwise regression procedure and to prediction based on the expectancy tables.

Prediction Based on Stepwise Regression

The stepwise regression procedure gave correlation coefficients and regression equations. These findings are summarized as follows:

(1) With the independent variables of grade eight mathematics, DAT Numerical Ability, Math Attitude Scale, and the Iowa Algebra Aptitude Test as predictors, 67.3 per cent of the variance of the dependent variable, grade nine mathematics, was accounted for. (See Table 4, page 53.)

(2) In the main part of the study involving the complete population sample, the eighth grade mathematics grade alone accounted for the greatest portion of the variance.

(3) The mean for females for the DAT Numerical Ability Test was 39.605, while the same mean for males was 50.511. The difference in these two means was statistically significant.

(4) When the males and females from the complete population sample were considered separately and the simple correlation coefficients with the variables examined, the correlation for females of

attitude towards mathematics with grade nine mathematics achievement was found to be 0.615. The correlation for males with the same variable was found to be 0.392.

(5) When the males and females from the complete population sample were considered separately with the five independent variables, 70.6 per cent of the variance in the dependent variable was accounted for in the case of females, while 64.4 per cent of the variance was accounted for in the case of males.

(6) In the special sub-study involving all grade nine mathematics students from Stevenson Junior High School, the equation developed predicted grade nine mathematics achievement for students from this school only slightly better than by the equation developed from the complete population sample. The average difference between actual and predicted grade points for these two equations was 0.292 for the complete population sample equation and 0.220 for the special one-school equation.

(7) The DAT Verbal Reasoning score was found to have the lowest correlations with the dependent variable for all groups considered and did not contribute significantly as the stepwise regression procedure developed the regression equations.

Caution should be exercised in comparing the predicted to the actual grade points in that the actual is given only to one significant figure while the predicted is expressed to four significant figures.

Prediction Based on the Expectancy Tables

The major findings from the expectancy tables investigation are summarized as follows:

(1) The expectancy table, developed on the basis of grade eight mathematics achievement for the complete population sample, should be of the greatest value as a predictive device since the eighth grade mathematics correlation to ninth grade mathematics was the highest of any independent to dependent variable at 0.754. (See Table 3, page 52).

(2) The expectancy tables most reliable for predicting grade nine mathematics achievement for the Stevenson Junior High School students were found to be those based on the DAT Numerical Ability Test and eighth grade mathematics achievement resulting from correlations of 0.691 and 0.649 respectively, with the dependent variable.

CONCLUSIONS

The conclusions based on the findings are summarized as follows:

(1) Functional regression equations were established based on the grade eight mathematics grades, DAT Numerical Ability Test, Iowa Algebra Aptitude Test, and Math Attitude Scale. The DAT Verbal Reasoning Test did not contribute significantly to the equations. These equations will be a valuable tool for use in predicting performance in grade nine algebra.

(2) Expectancy tables were developed, based on the variables listed in Number(1)above. Since the DAT Verbal Reasoning Test did not contribute significantly in the analysis, an expectancy table was not constructed for this variable.

(3) In comparing regression equations and expectancy tables as techniques of prediction, it could not be established whether one prediction instrument was more effective than the other.

Using stepwise regression, a single variable never accounted for as much of the variance in the dependent variable as a combination of independent variables did. Therefore, an expectancy table should not be used singly, but in conjunction with other tables, equations, or both, predicting success in the same variable.

The counselor may find the student prefers the information supplied by one of the prediction devices over the other. Both devices have advantages and the student in conjunction with the counselor could decide which instrument yields more helpful information.

(4) The grade eight mathematics grade is the best single predictor of grade nine algebra achievement.

(5) The females' attitudes toward mathematics is more significant in predicting achievement in grade nine algebra than is the males' attitudes.

(6) In predicting achievement in grade nine algebra, females are more predictable than males.

(7) It is not necessary to establish regression equations from a school individually; a random sample chosen from each school in a division will form a basis for prediction almost as well.

RECOMMENDATIONS FOR FURTHER STUDY

Based on the findings of this study, the following recom-

mendations are made for further study in the prediction field:

(1) Although in the case of the complete population sample, 67.5 per cent of the variance was accounted for in the dependent variable with a standard error of 0.840 using the five independent variables of this study, perhaps further testing could be done with non-cognitive measures of academic performance. Areas looked into might include measures of study habits, interests, and motivation.

(2) Since the per cent of variance accounted for in the female group was larger than that of the males of the population sample, further study could be done to determine if this phenomenon is more generally true in academic endeavors and why this might be so.

(3) A significant difference between male and female scores on the DAT Numerical Ability Test was found. To discover why this occurred and if it is true beyond this experimental situation, further study is recommended.

(4) Further study should be attempted to explain the difference found in the correlation of Math Attitude Scale to the numerous variables for males versus females.

(5) In all parts of the regression study, the DAT Verbal Reasoning Test did not contribute significantly to the dependent variable variance. In further studies, this variable could quite justifiably be omitted in preference to some of the variables suggested in Statement (1) of this section.

(6) On the basis of the investigations carried out in this study, no firm conclusion could be drawn as to whether regression

equations or expectancy tables were more suitable in a counseling situation. It is recommended that further study be carried out in an attempt to establish which predictive device is more useful to the student and counselor.

SUMMARY

The ultimate goal of this study was to provide information of a concrete nature that would enable school counselors or the classroom teacher to offer guidance to students with respect to their choice of elective topics from the grade nine mathematics course in order to prepare themselves better for their grade 10 mathematics program. This goal has been met in that information valuable to the student, classroom teacher, and counselor has evolved from this study to aid the student in making appropriate decisions during his grade nine year.

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APPENDIXES

A. Correspondence Relevant to the Study

July 31, 1970

Bureau of Educational Research and Service
State University if Iowa
Iowa City, Iowa
U. S. A.

Gentlemen:

Please rush one specimen set of the Iowa Algebra Aptitude Test. This is to be considered for use in a research study to begin in September of this year. Your immediate attention to this would be very much appreciated.

Yours sincerely,

Donald J. Timmerman

September 17, 1970

Mr. R. A. MacIntosh, Superintendent
St. James-Assiniboia School Division Number 2
2000 Portage Avenue
Winnipeg 12, Manitoba

Dear Mr. MacIntosh:

My purpose in writing is to seek your permission to undertake a Master of Education thesis study in our school division.

I have submitted a formal proposal to my advisor, Mr. M. McPherson at the University of Manitoba Faculty of Education, and he has approved the plan. A copy of this proposal is enclosed for your perusal.

Briefly, the proposal can be summarized as follows:

Purpose - To attempt to establish a prediction instrument for success in grade 9 algebra.

Significance of the Study - In the future, there is a strong possibility that two mathematics courses will be offered at the grade 9 level, i.e., algebra and general mathematics. Some concrete guidance instrument would then be helpful to aid the students in selecting his course.

Predictors to be Considered -

(a) Grade 8 mathematics grade point average.

(b) Iowa Algebra Aptitude Test.

(c) Differential Aptitude Test (Verbal Reasoning subtest).

(d) Differential Aptitude Test (Numerical Ability subtest).

(e) Math Attitude Scale.

The Population - One class will be selected at random from each of the full junior high schools in the St. James-Assiniboia School Division, namely Bruce Junior High, Deer Lodge Junior High, Golden Gate Junior High, Hedges Junior High, Sansome Junior High, and Stevenson Junior High.

Analysis of Data - Analysis of the data will be done by using a Step-wise regression program at the Department of Education Research Branch.

In connection with this proposal, I would request the following:

(a) Your permission to carry out this study in the St. James-Assiniboia School Division Number 2.

(b) Your permission to contact the principals of the schools mentioned for their approval in including his/her school in the study.

(c) Your permission to ask the principals' secretaries concerned to record the following data:

(1) Scores of the students' grade 8 mathematics G.P.A.

(2) Scores of the students' DAT (Verbal Reasoning and Numerical Ability).

I feel certain that the last request would utilize no more than one hour of the secretary's time.

Mr. Goebel is fully aware of this proposal as it was discussed with him sometime ago.

Your approval of these requests would be greatly appreciated and I look forward to hearing from you in this regard at your earliest convenience.

Yours truly,

Donald J. Timmerman

SUPERINTENDENT
R. A. MACINTOSH
SECRETARY-TREASURER
T. C. MACGREGOR
MAINT. SUPERVISOR
L. D. BESTER

The St. James-Assiniboia School Division No. 2
BOARD OFFICE–2000 PORTAGE AVENUE • ST. JAMES 12 • MANITOBA • PHONE 888-7951

October 1, 1970

Mr. D. Timmerman,
810 - 1305 Taylor Ave.,
Winnipeg 9, Man.

Sir:

Re: Your letter of September 17th

Please consider this permission to undertake the Master of Education thesis study in our school division. I understand that the purpose of the study is to attempt to establish a prediction instrument for success in grade nine algebra and that it has significance for this division inasmuch as there is a possibility that two grade nine mathematics courses will be developed.

You herewith have permission to contact principals of the schools for the purpose of arranging to select, at random, classes for use in the study and to arrange to have the necessary data typed out in the schools.

It is understood that you will make the results of your study available to the division.

Best wishes,

Yours truly,

R.A. MacIntosh,
Superintendent of Schools

RAM/mc

October 2, 1970

A.D.I. Auxiliary Publications Project
Photo Duplication Service
Library of Congress
Washington 25, D. C.

Dear Sirs:

As I am presently engaged in work on a thesis towards a Master of Education degree at the University of Manitoba, I would request the following:

(1) a copy of the Math Attitude Scale (Document No. 6545), for which I enclose $1.25;

(2) permission to use the above in the study being undertaken in connection with the aforementioned thesis.

Thank you.

Yours truly,

Donald J. Timmerman

Sent to:

Mr. D. Stefanson
Bruce Junior High School
330 Booth Drive
Winnipeg 12, Manitoba

Mr. E. Witoski
Hedges Junior High School
Fairlane and Knox
Winnipeg 12, Manitoba

Mr. J. Pemberton
Deer Lodge Junior High School
1970 Ness Avenue
Winnipeg 12, Manitoba

Mr. J. Atamanchuk
Ness Junior High School
3300 Ness Avenue
Winnipeg 12, Manitoba

Mr. P. Lewis
Golden Gate Junior High School
330 Bruce Avenue
Winnipeg 12, Manitoba

Mr. D. McKinnon
Sansome Junior High School
181 Sansome Avenue
Winnipeg 12, Manitoba

Mr. G. Jones
Stevenson Junior High School
Silver Avenue
Winnipeg 12, Manitoba

March 8, 1971

Re: Algebra Prognosis: Predicting First Year Algebra Achievement in the St. James-Assiniboia School Division No. 2

Dear_____:

Your cooperation with regard to the above study has already been much appreciated.

Further data is now needed in order to complete the predictor variables. The following scores are required from the recently administered Differential Aptitude Test, along with the students' Grade VIII Mathematics final mark.

Verbal Reasoning, expressed as percentile rank
Numerical Ability, expressed as percentile rank
Grade VIII Mathematics final mark, expressed as letter grade.

The above items are needed for each student in room____of your school.

I would appreciate receiving this information at your earliest convenience.

Thank you once again.

Yours truly,

Donald J. Timmerman

April 19, 1971

Mr. T. M. Carlyle,
Secondary Consultant
St. James-Assiniboia School Division No. 2
2000 Portage Avenue
Winnipeg 12, Manitoba

Dear Tom:

In connection with my thesis study, the following statistics would be most useful:

(1) Number of Grade IX students at each of the schools in the Division, excluding O.E. and Special Education students;

(2) The number of Grade IX classes at each school.

I would greatly appreciate receiving this information at your convenience and thank you in anticipation.

Yours truly,

Donald J. Timmerman

Sent to:

Mr. D. Stefanson
Bruce Junior High School
330 Booth Drive
Winnipeg 12, Manitoba

Mr. E. Witoski
Hedges Junior High School
Fairlane and Knox
Winnipeg 12, Manitoba

Mr. J. Pemberton
Deer Lodge Junior High School
1970 Ness Avenue
Winnipeg 12, Manitoba

Mr. J. Atamanchuk
Ness Junior High School
3300 Ness Avenue
Winnipeg 12, Manitoba

Mr. P. Lewis
Golden Gate Junior High School
330 Bruce Avenue
Winnipeg 12, Manitoba

Mr. D. McKinnon
Sansome Junior High School
181 Sansome Avenue
Winnipeg 12, Manitoba

Mr. G. Jones
Stevenson Junior High School
Silver Avenue
Winnipeg 12, Manitoba

April 19, 1971

Dear_____:

Please accept my sincere thanks for your immediate response to my request for DAT results and Grade VIII final mathematics scores.

Now I am in need of a general description of the way your Grade IX classes are structured, that is, whether they are heterogeneous or homogeneous, etc.

I would appreciate if you would give this description in the space provided at the bottom of this page, and return to me at Silver Heights Collegiate.

Thank you in anticipation.

Yours truly,

Donald J. Timmerman

- -

School:_____

Number of regular Grade IX classes_____

Description of Grade IX class structure:_____

B. Directions for Administration of Iowa Algebra Aptitude Test

DIRECTIONS FOR ADMINISTRATION OF IOWA ALGEBRA APTITUDE TEST

To the Supervising Teacher:

The Test. The test you will be administering today is the Iowa Algebra Aptitude Test. This is to be part of a thesis study on Prediction of Achievement in Grade Nine Mathematics.

The class in the school chosen to participate in the study was done on a random basis. Please be assured that the results of this test will be kept strictly confidential and no comparisons between schools will be made.

Thank you for your cooperation in this endeavor.

To the Students:

Today you will be writing the Iowa Algebra Aptitude Test. You are to do the best you can, but you are assured that the mark attained on this test will not affect your mathematics grade in any way. The results of the test will be used for research purposes only and will be kept strictly confidential. Also, no comparisons between schools will be made.

Directions for Administration

Please note: No marks should be made on the booklets.

General Information:

If the scores on standardized tests are to be correctly interpreted, the test must be administered under carefully controlled conditions. If the directions are not followed precisely, the test norms will not be applicable and much of the value of

the results will be lost.

A class period of 45 to 50 minutes is required for administering the test. Forty minutes of this time is actual working time for the test, leaving 5 to 10 minutes for preliminary activities, distributing materials, and the reading of directions for the separate parts of the test. To time the test, use a regular watch with a second hand, or a stop-watch, if one is available.

The following suggestions may be useful to the teacher:

(1) Have the pupils' desks cleared and see that each pupil is supplied with a sheet of scratch paper and at least two pencils.

(2) Follow the directions for each test carefully. See that all pupils start and stop in accordance with your signals. Each test is timed separately. Do not allow the pupils to work ahead and do not permit them to return to an unfinished subtest. If they finish before time is called, they are to wait quietly until directed to take the next test.

(3) The time limits for each test-part should be followed exactly:

Part 1 - 14 minutes

Part 2 - 12 minutes

Part 3 - 10 minutes

Part 4 - 4 minutes

As the test booklets and answer sheets are being distributed, say to the pupils:

"Do not write on the test booklets. Do your work on scratch paper and record your answers on the special answer sheet.

On the lines at the top of the answer sheet, write your name, the name of our school, and the other information requested. DO THIS NOW!" (Pause)

"Now listen carefully and follow directions. Do not begin work until I say 'GO!' This test consists of four parts. You will be told when to start and when to stop on each part. When you come to the end of each part you will see the words 'Do not turn the page until told to do so.' If you finish work on a part before time is called, you may look over your work on that part only. You may answer an exercise even though you are not absolutely sure that your answer is correct. But if you come to a problem you cannot solve, omit it and come back to it later if you have time.

"Now look at the special directions on your answer sheet. Read silently as I read these directions to you. They say: 'To mark your answer to an exercise, find the set of parallel lines numbered the same as the problem. Then blacken the space between the pair of lines which corresponds to the answer you think is correct. Fill in the space between the lines by moving your pencil point back and forth, making the mark heavy and dark. If you wish to change an answer, erase your first mark completely. Do not make stray marks on the answer sheet.'

"Now place your separate answer sheet on your desk and look at the directions for Part 1 of the test booklet. Read silently as I read these directions to you."

Part 1

"Each of the following sequences is made up according to some rule. Addition, subtraction, multiplication, division, or various combinations of these operations are used in forming each sequence. Discover the rule for each sequence and decide which of the four suggested answers is correct as the next term.

"The answers to this part are to be recorded under Part 1 on the answer sheet. The sample is answered correctly. Look at the sample and see how it is marked. (Pause for a few seconds.) You will have 14 minutes to work 34 exercises. Are there any questions? If not, ready, GO!"

At the end of fourteen (14) minutes, say "STOP. Turn to Part 2 on page 4."

Part 2

When the students have turned to Part 2, say:

"This section presents four short lessons. After each lesson is a set of exercises. Read the first lesson and work the exercises which follow before going on to the next lesson. Work each exercise and compare your answer with the suggested answers. If your answer is not among those suggested, choose answer D, 'Not Given'--meaning that the correct answer is not given.

"The answers to this part are to be recorded under Part 2 on the answer sheet. Study the sample. (Pause for a few seconds.) You will have 12 minutes to work the 16 exercises. Ready? GO!"

At the end of twelve (12) minutes, say: "STOP. Turn to Part 3 on page 6."

Part 3

When the students have turned to Part 3, say:

"The answer to each of these problems is an indicated addition, subtraction, multiplication, division, or combination of these operations. Study each problem carefully and decide what the answer is. If none of the suggested answers is equivalent to yours, choose answer D, 'Not Given'--meaning that the correct answer is not given.

"You will have 10 minutes to work the 20 problems in Part 3. Ready? GO!"

At the end of ten (10) minutes, say "STOP. Turn to Part 4 on page 8."

Part 4

When the students have turned to Part 4, say:

"Each of these exercises presents a true statement about counting numbers which are represented by the letters X, y, k, m, or s. You are to tell what must happen to X if certain changes are made in y, k, m, or s and the statement is to remain true. If there is not enough information given to decide what happens to X, choose answer D, 'Cannot Tell.'

"You will have four (4) minutes to answer the 10 questions. Ready? GO!"

At the end of four (4) minutes, say "STOP. This is the end of the test."

COLLECT THE ANSWER SHEETS AND TEST BOOKLETS AT ONCE.

C. Test Instruments

MATH ATTITUDE SCALE

Name_____ Sex_____ Class_____

School_____ Grade_____

GENERAL INSTRUCTIONS

Please fill in the information above before reading the instructions.

This questionnaire contains a set of statements about the mathematics subjects you may be taking this year. You are to read carefully the directions for the following statements on page 2, and answer them according to the directions given. This questionnaire asks about your own attitudes and judgements. It is not a test, so it is very important that you answer the questions according to your own feelings and judgements. Remember, this questionnaire is concerned with your attitudes, and it is important that you answer according to your own feelings.

IMPORTANT--Feel free to answer honestly and frankly, as your answers will be kept confidential and will not be used by anyone in your school.

NOW TURN THE PAGE AND READ THE INSTRUCTIONS FOR THE SET OF STATEMENTS WHICH FOLLOW.

Below are a number of statements pupils have made about mathematics. Indicate how much you agree or disagree with each of these by circling the letter which represents one of the following expressions.

Strongly Disagree (SD) Strongly Agree (SA)

Neither Agree nor Disagree (N)

Disagree (D) Agree (A)

1. I am always under a terrific strain in a math class.	SD	D	N	A	SA
2. I do not like mathematics, and it scares me to have to take it.	SD	D	N	A	SA
3. Mathematics is very interesting to me, and I enjoy math courses.	SD	D	N	A	SA
4. Mathematics is fascinating and fun.	SD	D	N	A	SA
5. Mathematics makes me feel secure, and at the same time it is stimulating.	SD	D	N	A	SA
6. My mind goes blank, and I am unable to think clearly when working math.	SD	D	N	A	SA
7. I feel a sense of insecurity when attempting mathematics.	SD	D	N	A	SA
8. Mathematics makes me feel uncomfortable, restless, irritable and impatient.	SD	D	N	A	SA
9. The feeling that I have toward mathematics is a good feeling.	SD	D	N	A	SA
10. Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way.	SD	D	N	A	SA
11. Mathematics is something which I enjoy a great deal.	SD	D	N	A	SA
12. When I hear the word math, I have a feeling of dislike.	SD	D	N	A	SA
13. I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.	SD	D	N	A	SA

14. I really like mathematics.	SD	D	N	A	SA
15. Mathematics is a course in school which I have always enjoyed studying.	SD	D	N	A	SA
16. It makes me nervous to even think about having to do a math problem.	SD	D	N	A	SA
17. I have never liked math, and it is my most dreaded subject.	SD	D	N	A	SA
18. I am happier in a math class than in any other class.	SD	D	N	A	SA
19. I feel at ease in mathematics, and I like it very much.	SD	D	N	A	SA
20. I feel a definite positive reaction to mathematics; it is enjoyable.	SD	D	N	A	SA

Iowa Algebra Aptitude Test

Third Edition

by H. A. Greene and Darrell Sabers

Published by the Bureau of Educational Research and Service,
Division of Extension and University Services
The University of Iowa, Iowa City

Name_____

School_____

Date_____ Grade_____

Scores

Part 1	Part 2	Part 3	Part 4	Total	PR

art 1 Sequences

ime allowance—14 minutes)

irections: Each of the following sequences is ıade up according to some rule. Addition, ubtraction, multiplication, division, or various ombinations of these operations are used in rming each sequence. Discover the rule for each equence and decide which of the four suggested nswers is correct as the next term.

Sample: 1 3 5 7 ____

A) 7
B) 8
C) 9
D) 10

The correct answer is C, since 9 is the next term in the sequence.

12 0 10 0 8 0 ____

A) 0
B) 6
C) 7
D) 10

42 37 32 27 ____

A) 22
B) 23
C) 24
D) 25

r^6 6r r^5 5r r^4 ____

A) r^3
B) 3r
C) $4r^3$
D) 4r

4. 2 4 8 16 32 ____

A) 46
B) 48
C) 54
D) 64

5. 8 11 14 17 ____

A) 18
B) 19
C) 20
D) 21

6. 1 4 7 10 13 ____

A) 17
B) 16
C) 15
D) 14

Turn to page 2 and go right on working.

7. 4 5 7 8 10 ____

A) 14
B) 13
C) 12
D) 11

8. 17 17 13 9 9 ____

A) 4
B) 5
C) 6
D) 7

9. ab 4589 2ab 458 3ab ____

A) 45
B) 58
C) 48
D) 4ab

10. $\frac{a+4}{2a}$ $\frac{a+10}{5a}$ $\frac{a+16}{8a}$ _____

A) $\frac{a+18}{9a}$
B) $\frac{a+20}{11a}$
C) $\frac{a+22}{11a}$
D) $\frac{a+22}{12a}$

11. 64 32 16 8 ____

A) 0
B) 2
C) 4
D) 6

12. 56342 xy^2 5634 x^2y^3 563 x^3y^4 ____

A) x^4y^5
B) y^4y^5
C) 63
D) 56

13. 48.08 24.04 12.02 ____

A) 6.06
B) 6.02
C) 6.01
D) 6.1

14. 2 6 10 11 15 19 ____

A) 20
B) 21
C) 22
D) 23

15. $2y^3$-10 $6y^8$-16 $10y^{13}$-22 ____

A) $16y^{18}$ -28
B) $16y^{16}$ -24
C) $14y^{16}$ -24
D) $14y^{18}$ -28

16. $4y^8$ $5y^7$ $4y^6$ $7y^5$ $4y^4$ ____

A) $5y^7$
B) $9y^3$
C) $4y^2$
D) $5y^3$

17. 2 5 9 14 20 ____

A) 27
B) 26
C) 25
D) 24

18. 3 2 4 3 5 4 ____

A) 6
B) 5
C) 4
D) 3

19. 2 4 6 7 9 11 ____

A) 15
B) 14
C) 12
D) 11

20. 3 8 13 18 23 ____

A) 31
B) 29
C) 28
D) 27

Turn to page 3 and go right on working.

21. 1 3 6 10 15 ____

A) 17
B) 19
C) 20
D) 21

22. $\frac{3}{5}$ $1\frac{1}{5}$ $1\frac{4}{5}$ $2\frac{2}{5}$ ____

A) $2\frac{1}{2}$

B) $2\frac{4}{5}$

C) 3

D) $3\frac{1}{5}$

23. 2 4 8 14 22 ____

A) 30
B) 32
C) 34
D) 36

24. 4 5 10 11 22 23 ____

A) 24
B) 44
C) 45
D) 46

25. 1 3 7 15 ____

A) 31
B) 29
C) 27
D) 22

26. 24 23 22 20 19 18 ____

A) 13
B) 14
C) 15
D) 16

27. $\frac{5}{32}$ $\frac{5}{16}$ $\frac{5}{8}$ $1\frac{1}{4}$ $2\frac{1}{2}$ ____

A) 5

B) $3\frac{3}{4}$

C) $3\frac{1}{2}$

D) $3\frac{1}{4}$

28. $\frac{5}{108}$ $\frac{5}{36}$ $\frac{5}{12}$ $\frac{5}{4}$ $3\frac{3}{4}$ ____

A) $11\frac{1}{4}$

B) $7\frac{1}{4}$

C) $6\frac{3}{4}$

D) $6\frac{1}{4}$

29. 1 1 2 6 24 ____

A) 124
B) 120
C) 96
D) 42

30. 1 4 9 16 ____

A) 23
B) 24
C) 25
D) 27

31. h j l n ____

A) o
B) p
C) q
D) r

32. a c f j ____

A) m
B) n
C) o
D) p

33. a cd ghi mnop ____

A) uvwx
B) vwxyz
C) wxyz
D) uvwxy

34. a^1 c^3 g^5 m^7 ____

A) s^9

B) t^9

C) u^9

D) v^9

End of Part 1.

Do not turn to next page until told to do so.

art 2. Lessons
'me allowance—12 minutes)

irections:
his section presents four short lessons. After ach lesson is a set of exercises. Read the first sson and work the exercises which follow it before oing on to the next lesson. Work each exercise nd compare your answer with the suggested nswers. If your answer is not among those uggested, choose answer D, "Not Given"— ieaning that the correct answer is not given.

Sample:
The symbol "·" represents multiplication; A · B means multiply A by B.

2 · 3 equals

A) 2
B) 3
C) 5
D) Not Given

Since 2 times 3 equals 6, and 6 is not listed as a suggested answer, the correct answer is D, "Not Given"

---

ou are already familiar with the common symbols sed in arithmetic for the operations of addition (+), ubtraction (—), multiplication (×), and division (÷).

n other branches of mathematics there is a need for ymbols which define new operations—operations ot often needed to solve everyday problems. For example, one such operation involves the addition of integers 1,2,3, . . . up to some final integer A. This operation we might represent by the special symbol A*. From this definition of A*, we can see that $4^* = 1 + 2 + 3 + 4 = 10$.

Use the above definition to answer questions 1 - 4.

1. What is the value of 3*?

A) 3
B) 5
C) 7
D) Not Given

2. What is the sum of 5* and 7*?

A) 24
B) 42
C) 43
D) Not Given

3. If $B^* - 5^* = 13$, what does B* equal?

A) 28
B) 18
C) 8
D) Not Given

4. What is another way of representing $8 + 7 + 6$?

A) 8*
B) 8* — 5*
C) 8* — 6*
D) Not Given

The symbol $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$ means A times D minus B times C. That is, the product of B and C is subtracted from the product of A and D.

Use this definition to answer questions 5-8.

5. What is the value of $\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix}$?

A) 2
B) 3
C) 7
D) Not Given

6. If $\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 3 & A \end{vmatrix}$, what does A equal?

A) 3
B) 5
C) 6
D) Not Given

7. If $\begin{vmatrix} 4 & D \\ D & 5 \end{vmatrix} = 11$, what does D equal?

A) 3
B) 6
C) 9
D) Not Given

8. What is the value of $\begin{vmatrix} 3 & H \\ 2 & H \end{vmatrix}$?

A) 1
B) 5
C) H
D) Not Given

Turn to page 5 and go right on working.

Part 2. Lessons—*(Continued)*

A vector may be represented by a row of numbers inside parentheses. An example of a vector is (1,3,7). The numbers 1,3, and 7 are called the elements of the vector. A vector may have any number of elements.

The following is an example of vector addition: (4,5,6) + (1,2,3) = (5,7,9). The first elements of the added vectors are 4 and 1, and the first element of the sum vector is 4 + 1 =5. The other elements of the sum vector are found in the same manner. Vector subtraction is performed by subtracting comparable elements.

Use this definition to answer questions 9-11.

9. What vector is obtained by subtracting (14,42,18) from (67,59,83)?

A) (53,17,65)
B) (53,17,75)
C) (53,27,75)
D) Not Given

10. Which of these operations is not defined in the introductory material?

A) (7,8,3) — (7,8,3)
B) (4,10,3) — (0,0,0)
C) (9,5,2,1) — (3,4,1,1)
D) (4,6,2) — (5,3,1,7)

11. Which of these defines in symbolic form the addition of vectors with three elements?

A) (a,b,c) + (d,e,f) =
(a + b + c, d + e + f)

B) (a,b,c) + (d,e,f) =
(a,b,c,d,e,f)

C) (a,b,c) + (d,e,f) =
(a + b + c + d + e + f)

D) (a,b,c) + (d,e,f) =
(a + d, b + e, c + f)

Vector multiplication is shown by an example.
(1,2,3) × (4,6,5) = 1 × 4 + 2 × 6 + 3 × 5 =
4 + 12 + 15 = 31

Another example is (1,1) × (2,2) = 4.
Questions 12-16 are based on vector multiplication.

12. What is the product of (3,7,11) and (1,2,2)?

A) 28
B) 39
C) 40
D) Not Given

13. What is the product of (3,3,3,3) and (4,5,4,3)?

A) 38
B) 48
C) 53
D) Not Given

14. What is the product of (17,4) and (0,0)?

A) 21
B) 1
C) 0
D) Not Given

15. When D = 4, what does (D,7) × (1,D) equal?

A) 4
B) 28
C) 32
D) Not Given

16. If (2,4) × (3,C) = 18, what does C equal?

A) 3
B) 6
C) 9
D) Not Given

End of Part 2.

Do not turn to next page until told to do so.

'art 3. Open Phrases
'ime allowance—10 minutes)

)irections: The answer to each of these problems is n indicated addition, subtraction, multiplication, ivision, or combination of these operations. Study ach problem carefully and decide what the answer ɜ. If none of the suggested answers is equivalent ɔ yours, choose answer D, "Not Given"—meaning ıat the correct answer is not given.

Sample: How many pencils would I have if I bought k pencils and someone gave me m more pencils?

A) k — m
B) k + m
C) $\frac{k}{m}$
D) Not Given

The answer is found by adding k and m, so the correct choice is B.

---

1. Monday I read a pages of a book, Tuesday I read b pages, Wednesday I read c pages. How many pages did I read altogether?

A) abc
B) a + b + c
C) 3abc
D) Not Given

2. If the number of days in March is represented by d, indicate the number of days in January.

A) d — 1
B) d — 2
C) d — 3
D) Not Given

3. A man 45 years old has a son who is y years younger. Indicate the age of the son.

A) y — 45
B) 45 — y
C) 45y
D) Not Given

4. Ray wants to buy a gun which costs x dollars. He has y dollars, but that is not enough. How many dollars more does he need?

A) y — x
B) xy
C) x — y
D) Not Given

5. Represent the integer next greater than the one represented by m.

A) m
B) m + 1
C) 1 — m
D) Not Given

6. What is the volume of a room which is r feet wide, s feet long and t feet high?

A) rst
B) r + s + t
C) r + st
D) Not Given

7. One day a boy caught f fish, then gave away a of them. The next day he caught b more, but sold half of them. How many fish had he left?

A) fa — b
B) fa — ½b
C) f — a + ½b
D) Not Given

8. Indicate the number of days in r weeks.

A) 7r
B) $\frac{r}{7}$
C) r + 7
D) Not Given

9. John and James have x marbles between them, and each has as many as the other. How many has each?

A) 2x
B) $\frac{2}{x}$
C) $\frac{x}{2}$
D) Not Given

10. If a certain number be represented by x and we give x a value of 5, what would be the value of x + 2?

A) 10
B) 5x + 2
C) 7
D) Not Given

Turn to page 7 and go right on working.

Part 3. Open Phrases—*(Continued)*

11. If y represents an even number, represent the next greater even number.

A) $y + 2$
B) $y + 1$
C) $2y + 1$
D) Not Given

12. If one load of coal costs c dollars, what will be the cost of b loads of such coal?

A) $b - c$
B) $b + c$
C) $\frac{b}{c}$
D) Not Given

13. How much money is in Joe's bank account when y dollars are deposited and x dollars withdrawn if the previous balance was k dollars?

A) $y - xk$
B) $k + y - x$
C) $y - k + x$
D) Not Given

14. Three boys decide to run a refreshment stand. The equipment costs r dollars. The father of one of the boys gave them $20.00. Indicate how many dollars each boy will have to pay, if the remaining expense is to be divided evenly among them.

A) $\frac{r - 20}{3}$
B) $\frac{r + 20}{3}$
C) $\frac{20r}{3}$
D) Not Given

15. How many hours will it take a person to go m miles, if he goes r miles in one hour?

A) mr
B) $\frac{r}{m}$
C) $\frac{m}{r}$
D) Not Given

16. If the product of two numbers is 24, and one of the numbers is m, what is the other number?

A) $24m$
B) $24 + m$
C) $24 - m$
D) Not Given

17. If the quotient of two numbers is y and the divisor is m, represent the dividend.

A) $\frac{y}{m}$
B) my
C) $\frac{m}{y}$
D) Not Given

18. Indicate the number of cents in m dimes and 2 pennies.

A) 12
B) $m + 2$
C) $10m$
D) Not Given

19. A book, a pencil, and a tablet together cost y cents. If the pencil cost m cents, and the tablet cost twice as much as the pencil, what did the book cost?

A) $y - 3m$
B) $2m - y$
C) $3m$
D) Not Given

20. One roll of telephone wire will reach f feet. How many rolls of such wire are needed to reach t feet?

A) $t - f$
B) $\frac{t}{f}$
C) tf
D) Not Given

End of Part 3.

Do not turn to next page until told to do so.

art 4. Dependence and Variation
ime allowance—4 minutes)

irections: Each of these exercises presents a ue statement about counting numbers which are epresented by the letters X, y, k, m, or s. You are) tell what must happen to X if certain changes re made in y, k, m, or s and the statement is to emain true. If there is not enough information iven to decide what happens to X, choose answer D, "cannot tell."

Sample: $X + y = 10$
If y were made less, what must happen to X so that the answer 10 would still be correct?

A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

The correct answer is B, since X must become greater.

---

1. $X = y + 8$
If y were made greater, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

2. $X = y + m$
If m were to be made 1 greater, and y were to be made 1 less, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

3. $y = X + m$
If m were made greater, what must happen to X so that y would remain the same?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

4. $X = ym$
If y were made greater and m were made less, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

5. $X = y + \frac{m}{s}$
If m were made less and y and s did not change, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

6. $k = Xm$
If m were made less and k remained the same, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

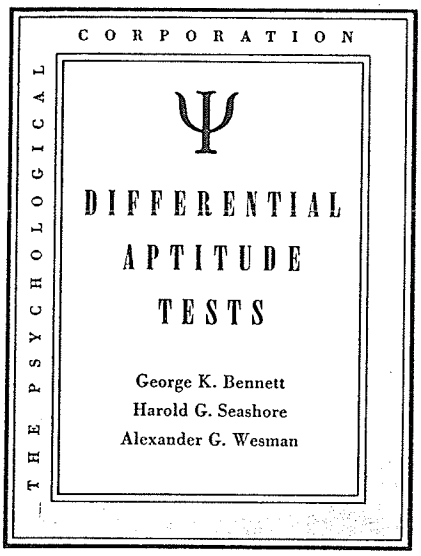
7. $X = y - m$
If m were made greater and y remained the same, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

8. $X = y + m - k$
If y, m, and k were made equal to each other, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

9. $X = y + \frac{m}{s}$
If s were made less and y and m did not change, what must happen to X?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

10. $X = \frac{y}{m}$
Given that y and m are always equal, what must happen to X if y and m are made greater in the same proportion?
A) Remain the same
B) Become greater
C) Become less
D) Cannot tell

End of test; close your paper.



FORM L

BOOKLET 1

Do not open this booklet until you are told to do so. Wait for the examiner's instructions. Then:

On the SEPARATE ANSWER SHEET, print your name and fill in the other requested information in the proper spaces.

In the space after **Form,** blacken the space for the letter **L.**

Then wait for further instructions.

DO NOT MAKE ANY MARKS IN THIS BOOKLET

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How to Mark an Answer Sheet

The *Differential Aptitude Tests* have been carefully constructed to help you to learn about your abilities. Careless marking of your answer sheets may lower the scores you earn on these tests. It is therefore important that you follow the requirements for proper marking of your answers.

1. **Use the proper pencil.** When answer sheets are scored by machine, only special lead will register properly. Do not use ordinary pencils.

2. **Make neat, solid marks.** A mark which is not solid may be missed by the scoring machine. A mark which extends outside the proper space may be read as an answer to another question. Fill in the space completely.

3. **Erase cleanly.** Poorly erased marks may be picked up as intended answers. If you wish to change an answer, make sure you erase completely the mark you want to change.

4. **Do not rest your pencil on the answer sheet.** This may cause a stray mark—and stray marks of any kind may be picked up by the machine as intended answers.

Remember: Careless marks may lower your scores. Correct scoring requires proper marking of your answer sheet.

Do not make any marks in this booklet

VERBAL REASONING

Mark your answers on the separate Answer Sheet

DIRECTIONS

Find the space for Verbal Reasoning on the Answer Sheet.

Each of the fifty sentences in this test has the first word and the last word left out. You are to pick out words which will fill the blanks so that the sentence will be true and sensible.

For each sentence you are to choose from among five pairs of words to fill the blanks. The first word of the pair you choose goes in the blank space at the beginning of the sentence; the second word of the pair goes in the blank at the end of the sentence. When you have picked the pair to fill in the blanks, mark the letter of that pair on the separate Answer Sheet after the number of the sentence you are working on.

Example X. ..... is to water as eat is to .....

A. continue —— drive
B. foot —— enemy
C. drink —— food
D. girl —— industry
E. drink —— enemy

Drink is to water as eat is to **food. Drink** is the first word of pair C and **food** is the second word of pair C, so the space under C has been filled in on line X on the sample Answer Sheet below.

Now look at the next example.

Example Y. ..... is to night as breakfast is to .....

A. supper —— corner
B. gentle —— morning
C. door —— corner
D. flow —— enjoy
E. supper —— morning

Supper is to night as breakfast is to **morning.** Pair E has both **supper** and **morning; supper** fits in the blank at the beginning of the sentence and **morning** fits in the blank at the end. On the sample Answer Sheet, the space under E has been blackened on line Y to show that pair E is the right one.

Now look at the next example.

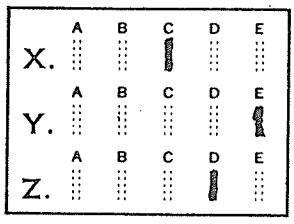
Example Z. ..... is to one as second is to .....

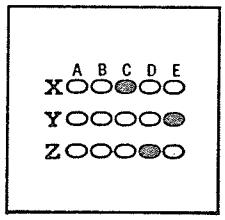
A. two —— middle
B. first —— fire
C. queen —— hill
D. first —— two
E. rain —— fire

First is to one as second is to **two. First** fits in the blank at the beginning of the sentence, and **two** belongs in the blank at the end. **First** and **two** make up pair D, so the space under D is filled in on line Z of the sample Answer Sheet.

Fill in **only one** space for each sentence.

SAMPLES OF ANSWER SHEETS





You will have 30 minutes for this test. Work as rapidly and as accurately as you can. If you are not sure of an answer, mark the choice which is your best guess.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. ..... is to street as rd. is to .....

A. st. —— city
B. lo. —— city
C. st. —— road
D. ma. —— road
E. st. —— France

2. ..... is to cavalry as foot is to .....

A. horse —— travel
B. horse —— infantry
C. horse —— yard
D. cemetery —— yard
E. horse —— armory

3. ..... is to wide as thin is to .....

A. narrow —— weight
B. store —— present
C. narrow —— fat
D. nothing —— fat
E. street —— weight

4. ..... is to masculine as woman is to .....

A. man —— madame
B. malicious —— feminine
C. malicious —— girl
D. man —— feminine
E. man —— girl

5. ..... is to dispute as endure is to .....

A. argue —— invert
B. repute —— verdure
C. impute —— verdure
D. impute —— last
E. argue —— last

6. ..... is to verse as sculptor is to .....

A. poet —— artist
B. poet —— statue
C. music —— statue
D. reverse —— statue
E. reverse —— artist

7. ..... is to chain as bead is to .....

A. link —— pearl
B. watch —— pearl
C. iron —— necklace
D. pull —— necklace
E. link —— necklace

8. ..... is to animal as rind is to .....

A. husk —— melon
B. skin —— nut
C. skin —— melon
D. man —— hard
E. husk —— nut

9. ..... is to cork as box is to .....

A. bottle —— lid
B. bottle —— crate
C. bottle —— hat
D. bottle —— fight
E. brittle —— crate

10. ..... is to tusk as deer is to .....

A. elephant —— doe
B. ivory —— doe
C. elephant —— antler
D. ivory —— antler
E. ivory —— hunt

11. ..... is to contralto as tenor is to .....

A. singer —— song
B. sonata —— baritone
C. solo —— song
D. solo —— baritone
E. soprano —— baritone

12. ..... is to hang as guillotine is to .....

A. gallows —— behead
B. criminal —— behead
C. picture —— capitulate
D. picture —— behead
E. punish —— citizen

13. ..... is to tree as melon is to .....

A. apple —— vine
B. bush —— vine
C. elm —— water
D. elm —— ripe
E. bush —— sweet

14. ..... is to pea as shell is to .....

A. green —— nut
B. pod —— crack
C. green —— peel
D. green —— crack
E. pod —— nut

GO ON TO THE NEXT PAGE.

15. ..... is to steer as pork is to .....

A. cow —— pig
B. beef —— pig
C. bull —— pig
D. beef —— chop
E. bull —— chop

16. ..... is to sentence as sentence is to .....

A. jail —— phrase
B. word —— paragraph
C. word —— phrase
D. jail —— paragraph
E. jail —— fine

17. ..... is to Dick as Margaret is to .....

A. Richard —— Mary
B. Francis —— Frances
C. William —— Peggy
D. Richard —— Peggy
E. Richard —— Joan

18. .....is to childhood as adolescence is to.....

A. infantry —— adultery
B. infancy —— maturity
C. infantry —— intelligence
D. infancy —— adultery
E. health —— intelligence

19. ..... is to potato as beater is to .....

A. mashed —— egg
B. skin —— steak
C. skin —— egg
D. masher —— winner
E. masher —— egg

20. ..... is to dog as Guernsey is to .....

A. terrier —— cow
B. bark —— cow
C. tail —— cow
D. tail —— Jersey
E. bark —— Jersey

21. ..... is to top as base is to .....

A. spin —— bottom
B. side —— bottom
C. spin —— ball
D. apex —— bottom
E. ibex —— bottom

22. ..... is to eagle as Pekinese is to .....

A. sparrow —— collie
B. sparrow —— Chinese
C. flag —— Chinese
D. vulture —— Chinese
E. vulture —— crow

23. ..... is to river as coast is to .....

A. flood —— beach
B. tide —— sea
C. bank —— sea
D. flood —— sea
E. tide —— beach

24. ..... is to foot as elbow is to .....

A. toe —— shoulder
B. toe —— hand
C. knee —— hand
D. man —— hand
E. knee —— shoulder

25. ..... is to day as calendar is to .....

A. noon —— year
B. sun —— year
C. night —— year
D. sun —— March
E. clock —— year

26. ..... is to constitution as prologue is to .....

A. preamble —— play
B. independence —— epilogue
C. independence —— play
D. law —— epilogue
E. amendment —— epilogue

27. ..... is to proceed as stop is to .....

A. recede —— prevent
B. intercede —— prevent
C. halt —— go
D. profit —— go
E. intercede —— go

28. ..... is to horse as bray is to .....

A. neigh —— donkey
B. hoof —— donkey
C. saddle —— wagon
D. hoof —— wagon
E. hoof —— pony

GO ON TO THE NEXT PAGE.

29. ..... is to sea as rebellion is to .....

A. navy —— war
B. mutiny —— land
C. sailor —— war
D. sailor —— soldier
E. river —— revolting

30. ..... is to distance as pound is to .....

A. far —— ounce
B. far —— weight
C. travel —— ounce
D. rod —— ounce
E. rod —— weight

31. ..... is to door as pane is to .....

A. lock —— window
B. panel —— window
C. home —— window
D. lock —— glass
E. wood —— ache

32. ..... is to never as all is to .....

A. seldom —— whole
B. seldom —— every
C. always —— every
D. seldom —— none
E. always —— none

33. ..... is to future as regret is to .....

A. ahead —— past
B. ahead —— sins
C. hope —— past
D. ahead —— atone
E. forecast —— atone

34. ..... is to rain as levee is to .....

A. water —— departure
B. water —— rise
C. water —— wash
D. umbrella —— flood
E. cloud —— rise

35. ..... is to fish as gun is to .....

A. bait —— shot
B. cod —— trigger
C. rod —— shot
D. cod —— bullet
E. rod —— hunt

36. ..... is to pacifist as religion is to .....

A. atlantis —— minister
B. object —— minister
C. atlantis —— sacred
D. war —— atheist
E. conscience —— minister

37. ..... is to deft as awkward is to .....

A. clumsy —— stupid
B. hearing —— stupid
C. hearing —— ugly
D. clumsy —— skillful
E. blindness —— skillful

38. ..... is to nut as hook is to .....

A. bolt —— eyehole
B. fruit —— pitch
C. fruit —— bend
D. bolt —— bend
E. hazel —— bend

39. ..... is to land as knot is to .....

A. acre —— rope
B. mile —— sea
C. desert —— rope
D. mile —— meter
E. farm —— rope

40. ..... is to bird as shed is to .....

A. fly —— barn
B. fly —— dog
C. fly —— hay
D. moult —— dog
E. migrate —— barn

41. ..... is to physician as secretary is to .....

A. doctor —— office
B. nurse —— executive
C. doctor —— stenographer
D. medicine —— office
E. medicine —— executive

42. ..... is to England as lira is to .....

A. pound —— Italy
B. London —— money
C. London —— Mexico
D. London —— mandolin
E. London —— Italy

GO ON TO THE NEXT PAGE.

43. ..... is to city as national is to .....

A. mayor —— government
B. municipal —— country
C. Chicago —— government
D. mayor —— country
E. Chicago —— international

44. ..... is to prison as Louvre is to .....

A. warden —— paramour
B. warden —— museum
C. warden —— France
D. Bastille —— museum
E. crime —— artist

45. ..... is to Canada as Havana is to .....

A. Nome —— Cuba
B. Detroit —— Cuba
C. Toronto —— Cuba
D. Alberta —— Cuba
E. Alberta —— Florida

46. ..... is to opera as lyric is to .....

A. baritone —— music
B. baritone —— poem
C. composer —— music
D. composer —— song
E. drama —— song

47. ..... is to bleach as flushed is to .....

A. color —— drained
B. color —— truffle
C. color —— blushed
D. color —— wan
E. gay —— drained

48. ..... is to static as dynamic is to .....

A. inert —— active
B. radio —— active
C. radio —— speaker
D. inert —— speaker
E. radio —— motor

49. ..... is to all as part is to .....

A. full —— separate
B. each —— separate
C. each —— many
D. full —— many
E. each —— whole

50. ..... is to diamond as circle is to .....

A. gold —— round
B. square —— oval
C. shape —— round
D. cube —— round
E. square —— round

STOP. YOU MAY CHECK YOUR WORK ON THIS TEST. DO NOT TURN TO ANY OTHER TEST.

Do not make any marks in this booklet

NUMERICAL ABILITY

Mark your answers on the separate Answer Sheet

DIRECTIONS

Find the space for Numerical Ability on the Answer Sheet.

This test consists of forty numerical problems. Next to each problem there are five answers. You are to pick out the correct answer and fill in the space under its letter on the separate Answer Sheet. If you do not find a correct answer among the first four choices, blacken the space under E as your answer. Choice E for every problem is "none of these" which means that a correct answer is not among the first four choices. Only one answer should be marked for each problem. Do your figuring on the scratch paper you have been given, and reduce fractions to lowest terms.

The following are examples of problems in the test. The samples of the Answer Sheets show how you are to mark your answers.

Example X.

Add 13
12

A **14**
B **25**
C **16**
D **59**
E **none of these**

In Example X, 25 is the correct answer, so the space under the letter for 25—B—has been filled in.

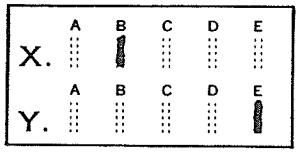
Example Y.

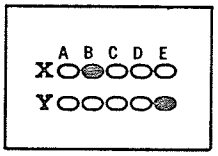
Subtract 30
20

A **15**
B **26**
C **16**
D **8**
E **none of these**

In Example Y, the correct answer has not been given, so the space under the letter for "none of these"—E—has been blackened.

SAMPLES OF ANSWER SHEETS





Remember, each answer must be reduced to its simplest terms. For example, if two choices are **1 ½** and **1 ¾**, only the **1 ½** is correct.

DO ALL YOUR FIGURING ON THE SEPARATE SHEET OF SCRATCH PAPER.

You will have 30 minutes for this test. Work as rapidly and as accurately as you can. Do not spend a long time on any one problem. If you are not sure of an answer, mark the choice which is your best guess.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

ANSWER

1. **Add**

393
4658
3790
67

A 7908
B 8608
C 8898
D 8908
E none of these

2. **Subtract**

5473
2987

A 2485
B 2486
C 2496
D 3486
E none of these

3. **Multiply**

484
25

A 10900
B 11100
C 11900
D 11700
E none of these

4. **Multiply**

2.04
.75

A 1.5300
B 153.0
C 1530
D 15300
E none of these

5. **Multiply**

4.50
22

A .99
B 98.40
C 99.00
D 9900
E none of these

6. **Multiply**

.025
.025

A .001375
B .00625
C .625
D 1.375
E none of these

7. **Multiply**

.016
.016

A 256
B 25.6
C .00256
D .000256
E none of these

ANSWER

8. **Divide**

$46\overline{)69}$

A $1\,13/46$
B $1\,23/46$
C 1.5
D 15
E none of these

9. **Divide**

$.75\overline{)2.25}$

A .0003
B .03
C .3
D 3
E none of these

10. **Divide**

$3.6\overline{).72}$

A .02
B .2
C 2
D 20
E none of these

11. **Divide**

$64.7\overline{)304.09}$

A .47
B 4.07
C 4.7
D 47
E none of these

12. **Divide**

$.04\overline{)4.036}$

A 1.009
B 10.9
C 10.09
D 100.9
E none of these

13. $\frac{1}{4} \div \frac{1}{8} =$

A $1/32$
B $1/8$
C $1/2$
D 2
E none of these

14. $\frac{2}{7} \times \frac{3}{7} =$

A $6/49$
B $3/7$
C $2/3$
D $6/7$
E none of these

GO ON TO THE NEXT PAGE.

15.

$\frac{3 \times 10}{5 \times 9} =$

ANSWER

A $\frac{27}{50}$
B $1\frac{1}{2}$
C $\frac{30}{45}$
D $\frac{2}{3}$
E none of these

16. Add

$4\frac{3}{4}$
$9\frac{1}{2}$
$13\frac{7}{8}$

A $26\frac{11}{14}$
B $27\frac{1}{8}$
C $28\frac{1}{2}$
D $28\frac{11}{14}$
E none of these

17. Add

2 ft. 3 in.
28 ft. 11 ½ in.
17 ft. 5 in.
4 ½ in.

A 49 ft.
B 48 ft. 2 in.
C 47 ft. 24 in.
D 48 ft.
E none of these

18. Add

3 lbs. 3 oz.
6 lbs. 7 oz.
7 lbs. 5 oz.
11 lbs. 1 oz.

A 28 lbs. 16 oz.
B 28 lbs.
C 27 lbs. 16 oz.
D 18 lbs.
E none of these

19. Square root

$\sqrt{169}$

A 13
B 43
C 84 ½
D 169
E none of these

20. Square root

$\sqrt{.09}$

A .03
B .3
C 3
D 9
E none of these

21. Square root

$\sqrt{\frac{4}{9} \times \frac{25}{36}}$

A $\frac{25}{81}$
B $\frac{25}{36}$
C $\frac{5}{9}$
D $2\frac{7}{9}$
E none of these

22.

$? = 33\frac{1}{3}\%$ of 963

ANSWER

A 32.19
B 231
C 321
D 32100
E none of these

23.

$? = 12\frac{1}{2}\%$ of 816

A .12
B 12
C 102
D 104
E none of these

24.

$? = \frac{4}{9}$ of 648

A 14.58
B 72
C 218
D 1458
E none of these

25.

15 = 75% of ?

A .20
B 10.25
C 20
D 22.5
E none of these

26.

25 = ? % of 125

A $\frac{1}{5}$
B 5
C 20
D 31.25
E none of these

27.

2.5 = ? % of 2

A 5
B 8
C 80
D 125
E none of these

28.

$\frac{?}{8} = \frac{3}{24}$

A $\frac{1}{8}$
B 1
C 3
D 4
E none of these

GO ON TO THE NEXT PAGE.

29.

$\frac{5}{9} = \frac{55}{?}$

ANSWER

A $55/99$
B 11
C 45
D 99
E none of these

30.

$\frac{11}{4} = \frac{77}{?}$

A $77/28$
B 28
C 44
D 308
E none of these

31. Cube root

$\sqrt[3]{32 \times 2}$

A 4
B 8
C 21 1/3
D 192
E none of these

32. Cube root

$\sqrt[3]{.000729}$

A .000243
B .009
C .027
D .09
E none of these

33. Cube root

$\sqrt[3]{\frac{1}{8} \times \frac{125}{64}}$

A 5/8
B $375/512$
C 2 1/2
D 15 5/8
E none of these

34.

List price = \$75.00
Discounts = $33\frac{1}{3}\%$; 2%
Net price = \$?

A 25
B 48.50
C 49.50
D 50
E none of these

ANSWER

35. What **one** number can replace **both** question marks?

$\frac{2}{?} = \frac{?}{50}$

A 1
B 10
C 25
D 100
E none of these

36. What **one** number can replace **both** question marks?

$\frac{1}{?} = \frac{?}{36}$

A 6
B 12
C 35
D 36
E none of these

37. What **one** number can replace **both** question marks?

$\frac{4}{?} = \frac{?}{100}$

A 1
B 20
C 25
D 200
E none of these

38. What **one** number can replace **both** question marks?

$\frac{8}{?} = \frac{?}{12\frac{1}{2}}$

A 1 1/2
B 4
C 64
D 100
E none of these

39. What **one** number can replace **both** question marks?

$\frac{6.25}{?} = \frac{?}{16}$

A 4
B 10
C 16
D 50
E none of these

40.

$\frac{9 + 1 \times 6 - 3}{4 + 2 \times 7 - 6} =$

A $57/50$
B $1\ 7/12$
C 1
D $57/36$
E none of these

STOP.

YOU MAY CHECK YOUR WORK ON THIS TEST. DO NOT TURN TO ANY OTHER TEST.