

CONDUCTANCES OF SOLUTIONS IN THE REGION
OF MEDIUM CONCENTRATION

A THESIS SUBMITTED TO THE COMMITTEE ON POST-GRADUATE
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by
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To My Parents

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Although I cannot honestly say that I wish to be found in error, yet I do fervently hope that the progress of science in the hands of the many zealous cultivators will be such as, by giving us new and other developments, and laws more and more general in their applications, will make even me think that what is written and illustrated in these experimental researches, belongs to the by-gone parts of science.

Michael Faraday

ABSTRACT

Equivalent conductances, densities, and viscosities of aqueous solutions of ammonium nitrate, silver nitrate, and lithium nitrate were determined at 25°C. and 35°C. at concentrations ranging from 0.01 molar to 1.0 molar.

Experimental equivalent conductances have been compared with those calculated by the Wishaw-Stokes and Falkenhagen-Leist equations. Suitable choice of one parameter, the distance of closest approach, permits reproduction of the experimental data with an error of less than 0.5%. A study of the deviations of calculated from experimental conductances reveals that the distance of closest approach varies appreciably with concentration and temperature.

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THEORETICAL INTRODUCTION

THEORETICAL INTRODUCTION

This introduction will discuss briefly the equations of Debye, Hückel, and Onsager; of Wishaw and Stokes; of Falkenhagen and Leist; and show how the equation of Fuoss and Onsager may be applied to dilute solution data.

1. Early Quantitative Predictions

The equivalent conductance, Λ , of a solution of an electrolyte is defined by

$$\Lambda = \frac{1000 L}{c} \text{ mhos,} \quad (1)$$

where L is called the specific conductance of the solution, and c is the concentration of the electrolyte in moles per liter. L is defined by

$$L = \frac{A}{r} \text{ mho/cm.,} \quad (2)$$

where r is the resistance of the solution measured in a cell of constant A . The specific conductance is the conductance of a one centimeter cube of solution. A is dependent on the geometry of the cell and is invariably determined by measuring the resistance of a solution of known specific conductance.

Λ_0 , the limiting equivalent conductance, is a constant characteristic of the solute in a given solvent at a given temperature. It is obtained by extrapolating some theoretical or semi-theoretical conductance

function to infinite dilution.

It was observed by Kohlrausch¹ in 1916 that conductance data in extremely dilute solutions could be made to fit the equation

$$\Lambda = \Lambda_0 - A\sqrt{c} \quad (3)$$

where A is an empirical constant. The extrapolation of a Λ against \sqrt{c} plot is a time-honoured method for the evaluation of Λ_0 .

In 1923 Debye and Hückel² published a limiting law giving the activity coefficient of an electrolyte in solution as a linear function of the square root of concentration, with slope given by a group of known physical constants. A solution of an electrolyte is assumed to be completely dissociated. This is reasonable, since studies of the structure of electrolyte crystals indicate the occurrence of ordered arrangements of ions rather than molecules in the space lattice. The decrease in equivalent conductance with increase in concentration is attributed to a decrease in the velocity of the ions, a consequence of ion-ion and ion-solvent interactions, rather than to a decrease in the number of ions, as suggested earlier by Arrhenius. Each ion is treated as being surrounded by an atmosphere of ions of opposite charge. Attributing all deviations from ideality to changes in charges on the ionic atmosphere, Debye and Hückel arrived at

the equation

$$\ln \gamma = \frac{-\epsilon^2 K}{2D_0 kT(1 + Ka)}, \quad (4)$$

where

$$K^2 = \frac{\pi \epsilon^2 N}{125 D_0 kT} c, \quad (5)$$

and "a" is the distance of closest approach between ions of opposite charge, one of the boundary conditions imposed in the derivation of equation (4). The meaning of the other symbols will be found in an appendix. $1/K$ is the distance at which the charge density of the ionic atmosphere is a maximum. In very dilute solutions $Ka \ll 1$, and

$$\ln \gamma = \frac{-\epsilon^2 K}{2D_0 kT}. \quad (6)$$

Debye and Hückel³ then applied the ionic atmosphere model to the problem of conductance. They reasoned that the imposition of an external electric field would distort the ionic atmosphere. The central ion will move in one direction while the oppositely-charged ions of the atmosphere will be pulled in the opposite direction. This asymmetry results in a greater force behind the ion than in front of it, retarding the motion of the ion. This is known as the relaxation effect. In addition, the motion of an ion and its atmosphere will be retarded by solvent molecules obstructing its path and by ions of opposite charge. This is the electrophoretic effect.

Onsager⁴ extended the treatment of Debye and Hückel to include the effect of the Brownian motion of the ions, and arrived at the equation

$$\Lambda = \Lambda_o - \left[\frac{82.48}{\eta_o (D_o T)^{1/2}} + \frac{8.204 \times 10^5}{(D_o T)^{3/2}} \Lambda_o \right] \sqrt{c} \quad (7)$$

for uni-univalent electrolytes. This is often written

$$\Lambda = \Lambda_o - (\alpha \Lambda_o + \beta) \sqrt{c} \quad (8)$$

For water at 25°C., $\alpha = 0.2289$, $\beta = 60.49$.

There are several difficulties inherent in the Debye-Hückel theory and its extension by Onsager. The most important are:

(i) The principle of the linear superposition of fields requires that there be a linear relationship between the charge density around an ion and the electrical potential. The charge density, ρ , is given by

$$\rho = \frac{(dn_+ - dn_-)}{dV} e,$$

where dn_+ is the number of positive ions in a volume dV at a distance r from a particular ion. The Boltzmann distribution function predicts that dn_+ will be given by

$$dn_+ = n \exp\left(\frac{-e\psi}{kT}\right) dV,$$

where n is the number of positive or negative ions per unit

volume, $e\psi$ is the work required to bring a unit positive charge from infinity to a distance r from the selected ion, and ψ is the electrical potential. dn_- is defined as

$$dn_- = n \exp\left(\frac{e\psi}{kT}\right) dV .$$

Insertion of the last two equations into the definition of the charge density gives

$$\rho = ne \left[\exp\left(\frac{-e\psi}{kT}\right) - \exp\left(\frac{e\psi}{kT}\right) \right] .$$

If the exponentials be expanded as infinite series, and third and higher order terms be neglected, the equation is reduced to

$$\rho = - \frac{2ne^2\psi}{kT} .$$

This equation has the linear relationship between charge density and electrical potential which the principle of the linear superposition of fields requires. The neglect of third and higher order terms is justifiable only when kT , the thermal energy of the ions, is very much greater than $e\psi$, the interionic potential energy.

(ii) The dielectric constant of the solution is taken to be that of the solvent. This ignores the decrease of the bulk dielectric constant with increase of concentration, as well as the variation of the microscopic dielectric constant with distance from an ion.

(iii) Ions are considered to be so far apart that their size is negligible with respect to the distance between them. In a 0.01 molar solution ions will be of the order of 20 \AA apart, while the diameter of an ion is $2 - 4 \text{ \AA}$.

2. The Wishaw-Stokes Equation

The next significant advance in conductance theory did not occur until 1952 when Falkenhagen, Leist, and Kelbg⁵ modified Onsager's treatment of the relaxation effect to include the effect of finite ion size. Employing a distribution function due to Eigen and Wicke⁶ they arrived at the following expression for the relaxation effect in uni-univalent electrolytes

$$\frac{\Delta X}{X} = \frac{\epsilon^2 K}{3D_0 kT \kappa a(1 + \kappa a)} \left(\exp(0.2929 \kappa a) - 1 \right) \left(1 - \frac{n}{N_1} \right) \left(1 - \frac{n}{N_2} \right) \quad (9)$$

$$\text{where} \quad \kappa^2 = \frac{\pi \epsilon^2 N}{125 D_0 kT} c \left(1 - \frac{n}{2N_1} - \frac{n}{2N_2} \right) \quad (10)$$

n is the number of ions of one kind per cubic centimeter of solution, and N_1 is the reciprocal of the volume of a cation. N_2 is the reciprocal of the volume of an anion. If, as is approximately correct, N_1 is taken as equal to N_2 , equations (9) and (10) become

$$\frac{\Delta X}{X} = \frac{\epsilon^2 K}{3D_0 kT a(1 + a)} \left(\exp(0.2929 a) - 1 \right) \left(1 - \frac{n}{N_1} \right)^2 \quad (11)$$

$$\kappa^2 = \frac{\pi \epsilon^2 N}{125 D_0 kT} c \left(1 - \frac{n}{N_1} \right) \quad (12)$$

Making the approximation $\left(1 - \frac{n}{N_1}\right) = 1$, Wishaw and Stokes⁷ insert the expression for the relaxation effect into their general conductance equation

$$\mathcal{A} = \left(\mathcal{A}_o - \frac{10^7 F^2}{6 \pi \eta_0 N} \cdot \frac{\kappa}{(1 + \kappa a)} \right) \left(1 + \frac{\Delta X}{X} \right) \quad (13)$$

and obtain

$$\mathcal{A} = \left(\mathcal{A}_o - \frac{B_1 \kappa}{(1 + \kappa a)} \right) \left(1 - \frac{B_2 \kappa}{(1 + \kappa a) \kappa a} \exp(0.2929 \kappa a) - 1 \right) \quad (14)$$

where

$$B_1 = \frac{10^7 F^2}{6 \pi \eta_0 N},$$

and

$$B_2 = \frac{\epsilon^2}{3 D_o k T}.$$

Values of B_1 and B_2 are listed in an appendix.

It is useful to determine when the approximation $\left(1 - \frac{n}{N_1}\right) = 1$ is justifiable. Inclusion of this term reduces κ by a factor $\left(1 - \frac{n}{N_1}\right)$, and $\frac{\Delta X}{X}$ by a factor $\left(1 - \frac{n}{N_1}\right)^2$.

Values of these factors are compared with values of the electrophoretic and relaxation terms in the table below. Data are for ammonium nitrate solutions at 25°C., but apply approximately to any uni-univalent electrolyte. N_1 is calculated for an ionic radius of 1.5×10^{-8} cm.

Omission of the factor $\left(1 - \frac{n}{N_1}\right)^2$ would increase the calculated equivalent conductance by 0.01% at 0.1 molar,

by 0.1% at 1 molar, and by 4% at 10 molar.

Concentration	$\left(1 - \frac{n}{N_1}\right)$	$\frac{B_1 \kappa}{1 + \kappa a}$	$\left(1 - \frac{n}{N_1}\right)^2$	$\frac{\Delta X}{X}$
Moles/liter	...	mhos
10^{-3}	1.0000	3	1.0000
10^{-2}	1.0000	6	0.9998	0.021
10^{-1}	0.9996	15	0.9984	0.058
10^0	0.9959	30	0.988	0.13
10^1	0.958	50	0.843	0.30

3. The Falkenhagen-Leist Equation

Consideration of the fact that ions, being impenetrable, must be displaced during migration led Falkenhagen⁸ to evaluate the relaxation effect as

$$\mathcal{A}_I = \frac{0.2929 B_2 \mathcal{A}_0 \kappa}{(1 + \kappa a)(1 + \frac{1}{2}\sqrt{2}\kappa a + \frac{1}{6}\kappa^2 a^2)} \quad (15)$$

and the electrophoretic effect as

$$\mathcal{A}_{II} = \frac{B_3 \kappa}{1 + \kappa a}, \quad (16)$$

where κ is defined by equation (10), but in practice equation (5) is used. The constant B_3 is defined by

$$B_3 = \frac{\epsilon^2 N}{27\pi \eta_0 10^{11}}$$

Falkenhagen's general conductance equation is

$$\mathcal{L} = \mathcal{L}_o - \mathcal{L}_I - \mathcal{L}_{II} \quad (17)$$

instead of the product type proposed by Wishaw and Stokes.

4. The Fuoss-Onsager Equations

Fuoss⁹ has recently summarized the involved theoretical treatment given earlier by Fuoss and Onsager¹⁰ and by Fuoss.¹¹ The simpler case, that in which ion pair formation is negligible, will be considered first.

(i) The Conductance Equation in the Case of Negligible Association.

The conductance of a dilute solution of a strong electrolyte is given by

$$\mathcal{L} = \mathcal{L}_o - Sc^{\frac{1}{2}} + Ec \log c + Jc. \quad (18)$$

In this equation S is defined by

$$S = \alpha \mathcal{L}_o + \beta,$$

where α and β are Debye-Hückel-Onsager constants, defined in equation (8). E is defined through

$$0.4343 E = \frac{\kappa^2 a^2 b^2}{24c} - \frac{\kappa ab\beta}{16c^{\frac{1}{2}}},$$

where
$$b = \frac{\epsilon^2}{aD_0 kT}.$$

"a" is, as usual, the distance of closest approach between two ions. Notice that E and S are independent of concentration

and the value of "a". J is defined by

$$J = \sigma_1 \mathcal{A}_0 + \sigma_2 ,$$

where

$$\sigma_1 = \frac{\kappa^2 a^2 b^2}{12c} \left[h(b) + 0.9074 + \ln \frac{\kappa a}{c^{1/2}} \right] ,$$

and

$$\sigma_2 = a\beta + \frac{11\beta a \kappa}{12c^{1/2}} - \frac{\kappa a \beta}{8c^{1/2}} \left(1.0170 + \ln \frac{\kappa a}{c^{1/2}} \right) .$$

The function $h(b)$ which appears in σ_1 is given by

$$h(b) = \frac{2b^2 + 2b - 1}{b^3} .$$

κ is defined, as before, by equation (5). The functions σ_1 and σ_2 are dependent only on the choice of "a". Thus J depends only on "a" and the limiting equivalent conductance. Experience has shown that "a" and \mathcal{A}_0 depend on the equation used in their evaluation. Consequently they must be determined from experimental data with the Fuoss-Onsager equation. The quantity \mathcal{A}' , defined by

$$\mathcal{A}' = \mathcal{A} + S c^{1/2} - E c \log c , \quad (19)$$

is calculated for each conductance determination. A plot of \mathcal{A}' against concentration will have an intercept \mathcal{A}_0 and slope J, from which "a" may be calculated.

(ii) The Conductance Equation in the Case of Appreciable Association.

Two changes are made in equation (18) to accomodate the new situation. Wherever it appears, the concentration, c , is replaced by $c\gamma_0$, where γ_0 is the degree of dissociation of the electrolyte in solution. The ratio of the conductance to the limiting equivalent conductance is a satisfactory first approximation for γ_0 . An association term is introduced to represent the effect of the ion pairs on the conductance. The new equation is

$$\mathcal{A} = \mathcal{A}_o - S(c\gamma_0)^{\frac{1}{2}} + Ec\gamma_0 \log c\gamma_0 + Jc\gamma_0 - K_a c\gamma_0 f^2 \mathcal{A} \quad (20)$$

where f is the mean ionic activity of the electrolyte, and K_a is the association constant for ion pair formation. All other symbols are defined as before. We are faced with the problem of determining \mathcal{A}_o , " a ", and K_a from experimental data. Three new quantities are defined as shown:

$$\begin{aligned} \mathcal{A}' &= \mathcal{A}_o + Jc\gamma_0 - K_a c\gamma_0 f^2 \mathcal{A} \\ y &= \frac{\mathcal{A}' - \mathcal{A}_o}{c\gamma_0} = J - K_a f^2 \mathcal{A} \\ x &= f^2 \mathcal{A}. \end{aligned}$$

\mathcal{A}_o is chosen so that a plot of y against x is linear, and K_a is obtained from the slope of the plot. J is the y -intercept of this plot. Then " a " may be calculated, since \mathcal{A}_o is already known.

If K_a be known from the extrapolation of a plot of $\log K_a$ against the reciprocal of the dielectric constant, the evaluation of "a" and \mathcal{L}_o is somewhat simpler. The quantity \mathcal{L}_J is defined by

$$\mathcal{L}_J = \mathcal{L}_o + Jc\gamma_o .$$

A plot of \mathcal{L}_J against $c\gamma_o$ will have slope J and intercept \mathcal{L}_o .

On the other hand, if "a" be known, the limiting equivalent conductance and the association constant may be calculated from a plot of \mathcal{L}_K against $c\gamma_o f^2 \mathcal{L}$, where

$$\mathcal{L}_K = \mathcal{L}_o - K_a c\gamma_o f^2 \mathcal{L} .$$

Fuoss and Onsager¹⁰ mention several reasons why their equation can be expected to be in error in linear and higher order terms:

- (1) The change of viscosity with concentration is neglected.
- (2) The change of dielectric constant is neglected.
- (3) Linear superposition of fields is assumed.
- (4) The volume occupied by the ions is neglected.
- (5) The fact that colliding ions have finite velocity is ignored.
- (6) The solvent is assumed to be continuous.

These reasons apply to the Wishaw-Stokes and Falkenhagen-Liest equations as well.

Mathematical approximations restrict the Fuoss-Onsager equation to $\kappa a < 0.3$. (For example, $c < 0.05$ molar for " a " = 3.5 \AA). In developing the theory, all the ions except the reference ion are replaced by a continuous charge distribution. The charge density in the ionic atmosphere is a maximum at $1/\kappa$. The approach of any ion to the reference ion will have a large effect on the potential about the reference ion. It is suggested that the minimum permissible interionic distance is of the order of a few ionic diameters, and $7 \times a$ is arbitrarily chosen. This limits the concentration to which we expect the equation to hold to 0.02 molar.

There is no doubt that some form of viscosity correction is necessary in conductance equations, particularly for concentrations above 0.1 molar. Experience has shown that the factor η/η_0 is probably close to the true correction. The viscosity-corrected equations are, then,

$$\Lambda \eta/\eta_0 = \left(\Lambda_0 - \frac{B_1 \kappa}{1 + \kappa a} \right) \left(1 - \frac{B_2 \kappa}{(1 + \kappa a) \kappa a} \exp(0.2929 \kappa a) \pm 1 \right) \quad (21)$$

and

$$\Lambda \eta/\eta_0 = \Lambda_0 - \Lambda_I - \Lambda_{II} \quad (22)$$

It should be remembered that the theoretical validity of these equations is not being examined. There are good reasons for believing that none of them is applicable to

concentrations higher than 0.02 molar. The question, "Do these equations satisfactorily reproduce experimental data in concentrated solutions ?" is being investigated. It is realized that the "a" values obtained in the range 0.01 molar to 0.1 molar are at best qualitative, and that those calculated for higher concentrations can have little physical significance.

NATURE OF THE PROBLEM

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The applicability of the equations of Robinson and Stokes,

$$\mathcal{L} = \left(\mathcal{L}_o - \frac{B_1 K}{1 + \kappa a} \right) \left(1 - \frac{B_2 K}{(1 + \kappa a) \kappa a} \exp(0.2929 \kappa a) - 1 \right)$$

and of Falkenhagen and Leist,

$$\mathcal{L} = \mathcal{L}_o - \frac{B_2 K}{(1 + \kappa a)(1 + \frac{1}{2}\sqrt{2} \kappa a + \frac{1}{2} \kappa^2 a^2)} - \frac{B_3 K}{1 + \kappa a}$$

to salts previously studied in concentrated solutions in this laboratory has been examined in the region of moderate concentration. The effect of a simple viscosity correction on agreement between calculated and experimental equivalent conductances was investigated.

Densities, viscosities, specific conductances, and equivalent conductances were determined for solutions of ammonium nitrate, silver nitrate, and lithium nitrate at 25°C. and 35°C. at twenty concentrations between 0.01 molar and 1.0 molar.

EXPERIMENTAL PROCEDURE

EXPERIMENTAL PROCEDURE

1. Purity of Materials

Ammonium nitrate, Fisher reagent grade, was recrystallized twice from conductance water, ground in an agate mortar, and stored in an oven at 100°C. until needed.

Silver nitrate from Johnson, Matthey, and Mallory, was fused carefully in a platinum dish, ground in an agate mortar, and stored in darkness over sulphuric acid.

Fisher reagent grade lithium nitrate was fused just before use, as described below.

Potassium chloride, Mallinckrodt analytical reagent, was fused in a platinum dish, ground in an agate mortar, and stored over sulphuric acid.

2. Preparation of the Solutions

Water obtained from a Barnstead still with a block tin condenser had a specific conductance less than 5×10^{-6} mho/cm. at 25°C.

At least 2.5 gm. of the salt was weighed to the nearest 0.2 mg. The salt was washed into a flask, and made up to 0.5 to 2 liters with conductance water. The solution was weighed on a large balance to the nearest 10 mg. A few of the weakest solutions were prepared by weighing as little as 1.5 gm. of the salt to 0.1 mg. on a semi-micro balance.

Lithium nitrate was heated to constant weight in a platinum dish, then dissolved in water and transferred to a flask.

All weights were calibrated, and all weighings were corrected to vacuum. (Density of air: 1.16 mg./ml.). Weight concentrations are, then, known at least within 0.01%.

3. The Conductance Bridge

A model 200C Hewlett-Packard oscillator generated a 1000 c.p.s. alternating current for a bridge composed of a No. 1553 Leeds and Northrup shielded ratio box, a No. 4750 resistance box, and a variable capacitance. The signal from the bridge was amplified by a Heath model EA-2 12 watt amplifier, and detected with a headphone set. The standard resistances were calibrated to the nearest 0.01 ohm with a Jones Bridge. Resistances were measured to the nearest 0.005%. To avoid heating the solution in the cell, it was necessary to limit the output of the oscillator to high voltage over a brief period of time, or low voltage over a longer period. A moment's calculation will show that a resistance of 250 ohms measured at 13 volts (maximum oscillator output) may increase the temperature of the solution in the cell by 0.01°C. per second. It may be pointed out that resistances need not be calibrated with exceptional accuracy, since small errors in cell constant determinations will be compensated by similar errors in specific conductance.

4. The Conductance Cells

Several cells with leads and filling tubes sufficiently separated to avoid stray capacitances (the Parker effect), as recommended by Jones and Bollinger¹² were used. Cell constants determined with the solutions defined by Jones and Bradshaw¹³ were constant within 0.02% throughout this investigation. The cell constants were 0.551, 3.1996, 33.701, 61.237, 101.40, and 135.11.

The variation of cell constant with temperature is given approximately by

$$\Delta A = - \beta A \Delta T$$

for a cell with large electrode separation.¹⁴ β is the linear coefficient of thermal expansion of glass, and A is the cell constant. In this work the cell constant change was not greater than 0.003%, and so was ignored.

5. The Thermostats

Two four-gallon pyrex containers were filled with oil - Marcol GX supplied by Imperial Oil Limited - and lagged with half-inch felt. Each contained two propellor-type stirrers, a mercury-toluene regulator, a 60-watt heating lamp, and a Beckmann thermometer. In addition, the 25°C. bath contained a copper coil for conducting cooling water at roughly constant temperature and constant rate of flow. The thermostats

maintained constant temperature within 0.002°C . Beckmann thermometers were calibrated periodically, with a Müller bridge, against two different platinum resistance thermometers, which ascertained the temperature within 0.005°C .

6. The Viscosity Measurements

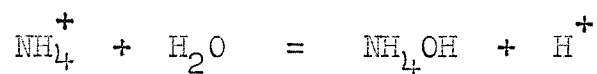
Two viscometers of the Cannon and Fenske¹⁵ type, having negligible drainage and kinetic energy corrections, were calibrated with water at 25°C . and 35°C . They were filled through the capillary arm to a reference mark on the capillary portion by applying suction to the opposite arm. Calibration run times were reproducible to only 0.05%, possibly because of variations in the quantity of air dissolved in the water.

7. The Density Determinations

Two pycnometers of 45 mls. capacity were filled and left in the thermostat for at least 30 minutes. The solution meniscus in one capillary arm was adjusted to a reference mark by withdrawing solution with filter paper through the opposite arm. The pycnometers were rinsed with acetone, dried, and left in the balance case for 1 hour before weighing. Several times during the research the instruments were calibrated with water at both temperatures. Vacuum corrections were applied to all weighings. The mean deviation of a dozen calibrations was 0.002%.

8. The Hydrolysis Correction

Following Campbell and Bock,¹⁶ the equivalent conductances of ammonium nitrate solutions were corrected for hydrolysis, which occurs according to the equation



The hydrolysis constants, taken from Bates and Pinching,¹⁷ are $K_h = 5.689 \times 10^{-10}$ at 25°C. and $K_h = 1.130 \times 10^{-9}$ at 35°C. In no case was the correction greater than 0.11 mho.

Determinations of specific conductance, density, and viscosity were done in duplicate. Measurements at 35°C. were done without refilling the instruments.

EXPERIMENTAL RESULTS

EXPERIMENTAL RESULTS

The experimental results of this research are given in Tables I to VI. Three fundamental properties, the density, the specific conductance, and the relative viscosity are listed for each solution. In addition two derived properties, the molarity and the equivalent conductance are given. The salts described are ammonium nitrate, silver nitrate, and lithium nitrate, each at 25^oC. and 35^oC.

All conductances are corrected for the contribution of water. A hydrolysis correction, described on page 23, has been applied to the data on ammonium nitrate.

TABLE I
DATA FOR AMMONIUM NITRATE SOLUTIONS AT 25.00°C.

Concentration (mole/liter)	Density (gm/ml)	Specific Conductance (mho/cm x 10 ⁶)	Equivalent Conductance (mhos)	Relative Viscosity
0.009893	0.99740	1350.9	136.47	0.999
0.020033	0.99776	2671.8	133.32	0.998
0.029175	0.99801	3830.7	131.26	0.998
0.039970	0.99839	5170.5	129.33	0.997
0.047956	0.99866	6146.3	128.13	0.996
0.050005	0.99872	6395.0	127.86	0.998
0.059997	0.99901	7601.0	126.66	0.996
0.070046	0.99938	8792.6	125.50	0.997
0.079920	0.99967	9955.0	124.54	0.996
0.089968	1.00003	11130.3	123.70	0.996
0.10001	1.00036	12285.5	122.83	0.995
0.15363	1.00208	18344	119.39	0.993
0.20090	1.00360	23529	117.10	0.990
0.30230	1.00684	34302	113.46	0.986
0.39993	1.00990	44327	110.83	0.982
0.49647	1.01296	53993	108.74	0.977
0.59710	1.01613	63832	106.90	0.975
0.69976	1.01937	73640	105.23	0.973
0.79943	1.02246	82966	103.77	0.968
0.90237	1.02566	92510	102.51	0.967
0.99976	1.02868	101306	101.32	0.963

TABLE II
DATA FOR AMMONIUM NITRATE SOLUTIONS AT 35.00°C.

Concentration (mole/liter)	Density (gm/ml)	Specific Conductance ₆ (mho/cm x 10 ⁶)	Equivalent Conductance (mhos)	Relative Viscosity
0.009864	0.99439	1616.0	163.73	0.999
0.019972	0.99472	3192.9	159.78	0.999
0.029087	0.99498	4577.8	157.31	0.999
0.039851	0.99540	6177.1	154.94	0.998
0.047808	0.99559	7341.2	153.49	0.997
0.049851	0.99566	7637.5	153.14	0.998
0.059813	0.99594	9073.9	151.65	0.998
0.069828	0.99628	10495.5	150.28	0.998
0.079673	0.99658	11880.9	149.08	0.997
0.089684	0.99687	13280.7	148.04	0.997
0.099691	0.99720	14661.3	147.03	0.998
0.15315	0.99894	21863	142.73	0.996
0.20026	1.00041	28020	139.89	0.995
0.30130	1.00353	40792	135.37	0.992
0.39859	1.00650	52637	132.04	0.992
0.49478	1.00951	64031	129.40	0.988
0.59498	1.01251	75588	127.03	0.987
0.69727	1.01575	87124	124.94	0.987
0.79655	1.01877	98031	123.06	0.984
0.89905	1.02189	109146	121.39	0.984
0.99578	1.02458	119441	119.94	0.982

TABLE III
DATA FOR SILVER NITRATE SOLUTIONS AT 25.00°C.

Concentration (mole/liter)	Density (gm/ml)	Specific Conductance ($10^6 \times (\text{mhos})/\text{cm}$)	Equivalent Conductance (mhos)	Relative Viscosity
0.010156	0.99842	1267.0	124.75	1.001
0.019394	0.99981	2354.2	121.39	1.003
0.029459	1.00125	3503.3	118.92	1.004
0.039998	1.00278	4675.2	116.89	1.004
0.049659	1.00411	5720.5	115.20	1.003
0.058313	1.00530	6643.7	113.93	1.003
0.068343	1.00675	7691.4	112.54	1.004
0.080014	1.00842	8895.5	111.18	1.005
0.089961	1.00978	9902.2	110.07	1.005
0.10063	1.01127	10968.2	109.00	1.005
0.14915	1.01809	15655	104.96	1.008
0.19797	1.02494	20148	101.77	1.011
0.29997	1.03920	28982	96.62	1.016
0.39976	1.05316	37051	92.68	1.022
0.55271	1.07455	48674	88.06	1.027
0.59523	1.08033	51646	86.77	1.030
0.69928	1.09476	58860	84.17	1.038
0.79154	1.10758	64962	82.07	1.046
0.89509	1.12196	71594	79.99	1.053
1.00088	1.13649	77977	77.91	1.062

TABLE IV
DATA FOR SILVER NITRATE SOLUTIONS AT 35.00°C.

Concentration (mole/liter)	Density (gm/ml)	Specific Conductance (mho/cm x 10 ⁶)	Equivalent Conductance (mhos)	Relative Viscosity
0.010127	0.99549	1519.5	150.04	1.002
0.019335	0.99678	2824.9	146.10	1.003
0.029369	0.99820	4199.9	143.00	1.004
0.039873	0.99966	5604.3	140.55	1.004
0.049507	1.00104	6854.4	138.45	1.005
0.058136	1.00224	7958.2	136.89	1.005
0.068130	1.00362	9212.8	135.22	1.007
0.079763	1.00526	10652.4	133.55	1.007
0.089681	1.00663	11858	132.23	1.008
0.10032	1.00811	13143	131.01	1.008
0.14867	1.01485	18738	126.04	1.011
0.19734	1.02167	24108	122.16	1.0166
0.29899	1.03580	34657	115.91	1.022
0.39842	1.04961	44288	111.16	1.029
0.55080	1.07082	58120	105.52	1.037
0.59314	1.07654	61665	103.96	1.038
0.69679	1.09087	70246	100.81	1.050
0.78873	1.10364	77488	98.24	1.058
0.89180	1.11783	85278	95.63	1.066
0.99713	1.13225	92985	93.25	1.076

TABLE V
DATA FOR LITHIUM NITRATE SOLUTIONS AT 25.00°C.

Concentration (mole/liter)	Density (gm/ml)	Specific Conductance (mho/cm x 10 ⁶)	Equivalent Conductance (mhos)	Relative Viscosity
0.010572	0.99751	1083.0	102.44	1.000
0.021061	0.99793	2105.4	99.97	0.999
0.029775	0.99830	2920.0	98.07	1.004
0.038848	0.99866	3770.3	97.05	1.005
0.050821	0.99897	4861.3	95.66	-
0.059158	0.99949	5604.1	94.73	1.007
0.069872	0.99991	6533.8	93.51	1.009
0.076234	1.00010	7095.4	93.07	1.008
0.091701	1.00079	8448.0	92.13	1.011
0.10111	1.00112	9247.6	91.46	1.011
0.14980	1.00332	13290	88.72	1.017
0.19937	1.00506	17267	86.61	1.022
0.29918	1.00902	24887	83.19	1.043
0.39900	1.01293	32130	80.53	1.042
0.45941	1.01532	36341	79.10	1.050
0.58142	1.02009	44546	76.62	1.060
0.71135	1.02509	52771	74.18	1.0755
0.81640	1.02930	59193	72.51	1.087
0.91191	1.03301	64745	71.00	1.098
1.00464	1.03667	69959	69.64	1.114

TABLE VI
DATA FOR LITHIUM NITRATE SOLUTIONS AT 35.00°C.

Concentration (mole/liter)	Density (gm/ml)	Specific Conductance (mho/cm x 10 ⁶)	Equivalent Conductance (mhos)	Relative Viscosity
0.010540	0.99446	1306.2	123.93	1.000
0.020996	0.99485	2534.9	120.73	1.002
0.029683	0.99523	3525.9	118.79	1.003
0.038729	0.99559	4540.1	117.23	1.006
0.050664	0.99589	5848.5	115.44	-
0.058975	0.99640	6745.2	114.37	1.008
0.069654	0.99679	7861.2	112.86	1.010
0.075999	0.99700	8531.6	112.26	1.008
0.091418	0.99770	10157.2	111.11	1.012
0.10079	0.99798	11114.3	110.27	1.014
0.14929	0.99997	15960	106.91	1.020
0.19874	1.00186	20724	104.28	1.024
0.29821	1.00576	29822	100.00	1.044
0.39769	1.00960	38460	96.71	1.045
0.45787	1.01192	43479	94.96	1.055
0.57945	1.01662	53234	91.87	1.066
0.70889	1.02154	62994	88.86	1.081
0.81351	1.02566	70605	86.79	1.094
0.90862	1.02929	77198	84.96	1.106
1.0010	1.03287	83338	83.26	1.117

DISCUSSION OF THE RESULTS

DISCUSSION OF THE RESULTS

1. Estimation of the Possible Error

Allowing for the same error in density determinations as in calibration, the mean deviation of the densities from their true values should be 0.004%. Data for ammonium nitrate solutions for the equations (obtained by least mean square calculations)

$$d^{25} = 0.997091 + 0.0040552 P$$

and

$$d^{35} = 0.994077 + 0.0039351 P,$$

with a mean deviation of 0.002% at 25°C. and 0.0035% at 35°C. P is the weight percent of ammonium nitrate in the solution. Deviations are random, indicating that the introduction of a term in P^2 is not necessary. The densities for $P = 0$ agree with those of water within 0.0018%. As pointed out earlier, weight percent concentrations are known to at least 0.01%. The molar concentration is then known to at least 0.015%.

The accuracy of the specific conductance determinations depends on the cell constants, which are known to 0.02%, and the resistances, known to 0.01%.

Equivalent conductances may then be expected to be within 0.05% of their true values, making some allowance for the possibility that several errors may operate in the same direction.

2. Comparison with Previous Work

The least mean square equation for the densities of ammonium nitrate solutions at 25°C. is

$$d^{25} = 0.99710 + 0.032364 c - 0.00079 c^2 ,$$

within 0.002% for $0 < c < 1$ mole per liter. Gucker¹⁸ gives the equation

$$d^{25} = 0.997077 + 0.032628 c - 0.000963 c^{3/2} \\ - 0.0000473 c^2 .$$

For $0 < c < 1$ these equations do not differ by more than 0.002% in d^{25} . The differences in the 35°C. data for ammonium nitrate solutions from the data of Campbell and Bock¹⁶ and Campbell, Gray, and Kartzmark¹⁹ are no greater than 0.01%, except in the case of Bock's 0.023 molar solution, which differs by 0.03%, and Gray's 1 molar solution, which differs by 0.14%.

Equivalent conductances for ammonium nitrate solutions at 35°C. were compared with the results of Campbell and Bock¹⁶ by means of a plot of the difference between experimental conductance and that calculated from the Fuoss-Onsager equation, against concentration. In the range checked, 0.01 molar to 0.07 molar, the present results are consistently 0.08 mho, or 0.05% higher than those of Campbell and Bock. This is not unusual, however, when one remembers that a difference of 0.10% would still be within the experimental error. The fact

that one set of results differs consistently from the other may be attributed to a calibration difference, such as might occur in a cell constant determination.

Conductances in the range 0.01 molar to 0.1 molar were fitted to the Shedlovsky extrapolation function²⁰,

$$\mathcal{L} = \mathcal{L}_0 - \frac{(\alpha \mathcal{L}_0 + \beta)\sqrt{c}}{1 - \alpha\sqrt{c}} + \mathcal{A}c$$

where $\alpha = 0.2289$ and $\beta = 60.32$ at 25°C . and $\alpha = 0.2334$ and $\beta = 75.09$ at 35°C . The values of \mathcal{L}_0 obtained are shown below:

		<u>\mathcal{L}_0</u>		
		<u>Shedlovsky</u>	<u>Literature</u>	<u>Reference</u>
Ammonium nitrate	25°C .	145.0	145.01	21
	35°C .	174.2	174.21	16
Silver nitrate	25°C .	133.2	133.36	21
	35°C .	161.0	161.53	22
Lithium nitrate	25°C .	110.5	110.14	21
	35°C .	133.7	133.48	21

In view of the relatively high concentrations to which the extrapolation function was applied, and in view of the effect of association of most nitrates on conductance, agreement may be considered as good.

The viscosity results agree with those of Campbell, Gray, and Kartzmark¹⁹ and of Campbell, Debus, and Kartzmark²³ within 0.2%.

3. Self-consistency of Equivalent Conductances

Plots of the $\overset{\circ}{a}$ values necessary for agreement of experimental conductances with those calculated by one of the conductance equations against a convenient function of concentration are a rigid test of the self-consistency of experimental data. Figure 1 shows such a plot for ammonium nitrate solutions at 25°C., using the Falkenhagen-Leist equation. That none of the points deviates from the average line by more than the experimental error, shown in Figure 2, is evidence that the conductances are self-consistent within 0.05%.

4. Description of the Calculations with the Experimental Results

The Falkenhagen-Leist and Robinson-Stokes equations contain but one parameter, $\overset{\circ}{a}$, the distance of closest approach between ions. This parameter is chosen to give good agreement between experimental and calculated equivalent conductances in dilute solutions. The calculations in this work force $\overset{\circ}{a}$ to absorb all imperfections of the conductance equations.

The data from which the figures were drawn appear in an appendix. In the figures, the size of the circles is not indicative of the experimental error.

(i) Ammonium Nitrate and the Falkenhagen-Leist Equation

Figure 1 shows how $\overset{\circ}{a}$, calculated by the Falkenhagen-Leist equation (17), varies with concentration. The effect of the

viscosity correction, equation (22), is also shown. Above 1 molar the data of Campbell and Kartzmark²⁴ were used.

Note that:

(a) The $\overset{\circ}{a}$ values from the viscosity-corrected equation show greater scatter than those from equation (17).

This is due to the relatively large uncertainty in the viscosity determinations, 0.2%.

(b) The $\overset{\circ}{a}$ values from the viscosity-corrected equation generally lie below those from equation (17). The fact that ammonium nitrate solutions below about 4 molar at 25°C. have viscosities smaller than that of water explains this.

(c) One would expect $\overset{\circ}{a}$ values calculated from equations (17) and (22) to be closer the more dilute the solution. It will be evident from the discussion below that an accuracy of 0.005% or better in the viscosity determinations would be necessary to show that two $\overset{\circ}{a}$ values do not differ by more than 0.2 Å at 0.01 molar.

(d) $\overset{\circ}{a}$ determinations in dilute solutions show greater scatter than in concentrated solutions. The reason lies in the nature of the Falkenhagen-Leist equation. $\overset{\circ}{a}$ is used only in the linear and quadratic divisors, $(1 + \kappa a)$ and $(1 + \frac{1}{2}\sqrt{2}\kappa a + \frac{1}{2}\kappa^2 a^2)$, both of which approach unity with increasing dilution. Consequently the more dilute the solution, the greater the uncertainty in the determination of $\overset{\circ}{a}$ from the conductance.

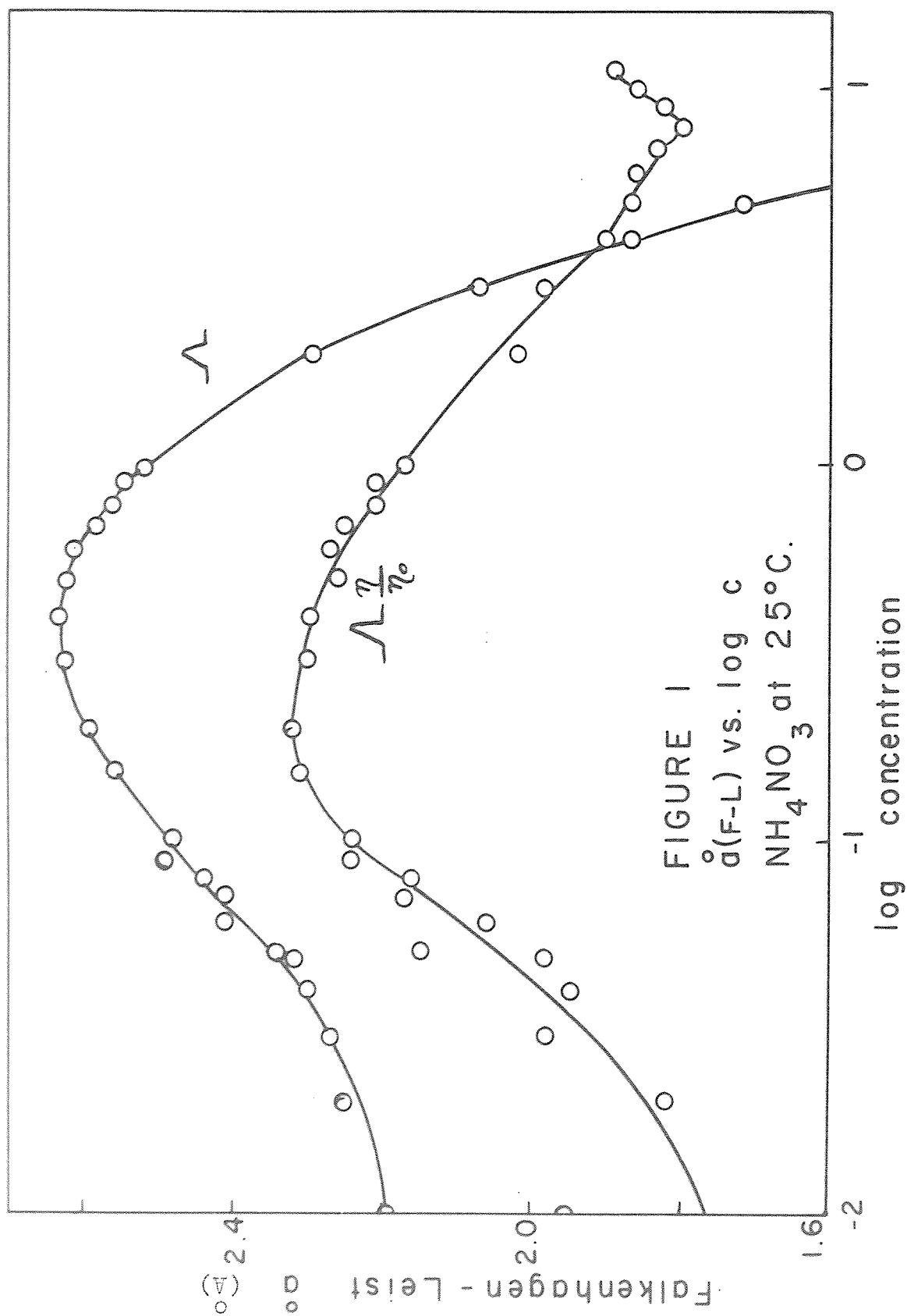
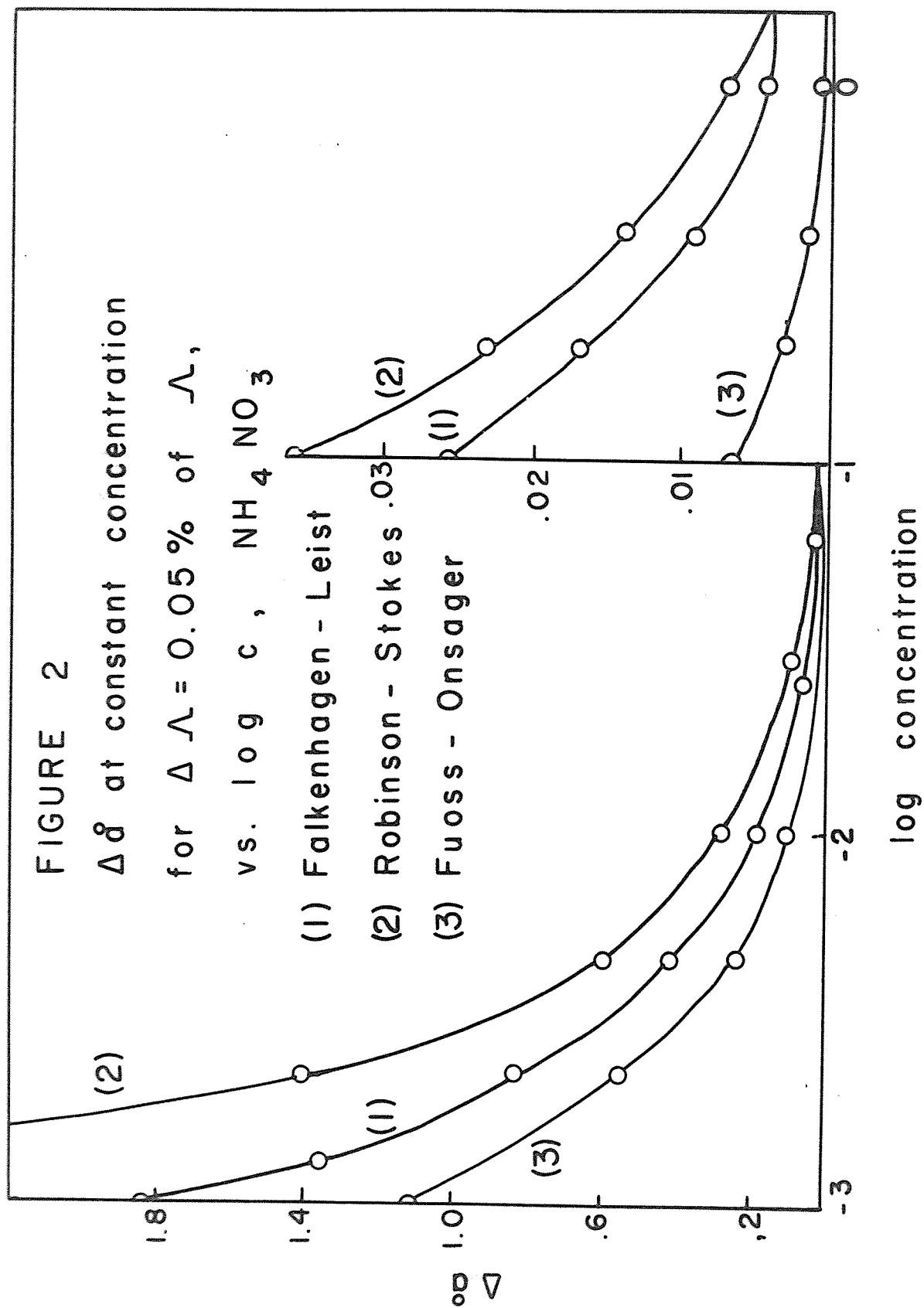


FIGURE 1
 Λ (F-L) vs. $\log c$
 NH_4NO_3 at 25°C .

Figure 2 shows how the uncertainty, $\Delta \bar{a}^0$, in the determination of \bar{a}^0 varies with concentration. There is no significant difference between plots at 25°C. and 35°C. The information given is for ammonium nitrate solutions but also applies, within 20%, to solutions of the two other salts. Points were obtained by calculating the effect of a 0.05% change in conductance on the value of \bar{a}^0 , at constant concentration. Figure 2 is useful for estimating the accuracy of \bar{a}^0 values. For example, if conductance data accurate to 0.01% are available for 0.01 molar solutions, \bar{a}^0 may be calculated to the nearest 0.04 Å, within 2% of most \bar{a}^0 values, and no better. For 0.001 molar solutions \bar{a}^0 may be calculated with a possible error of 0.4 Å, which is about 25% of an \bar{a}^0 value. Data for solutions weaker than 0.001 molar will not contribute to the determination of \bar{a}^0 . It should be remembered that the bulk of conductance data does not meet this criterion of low experimental error.

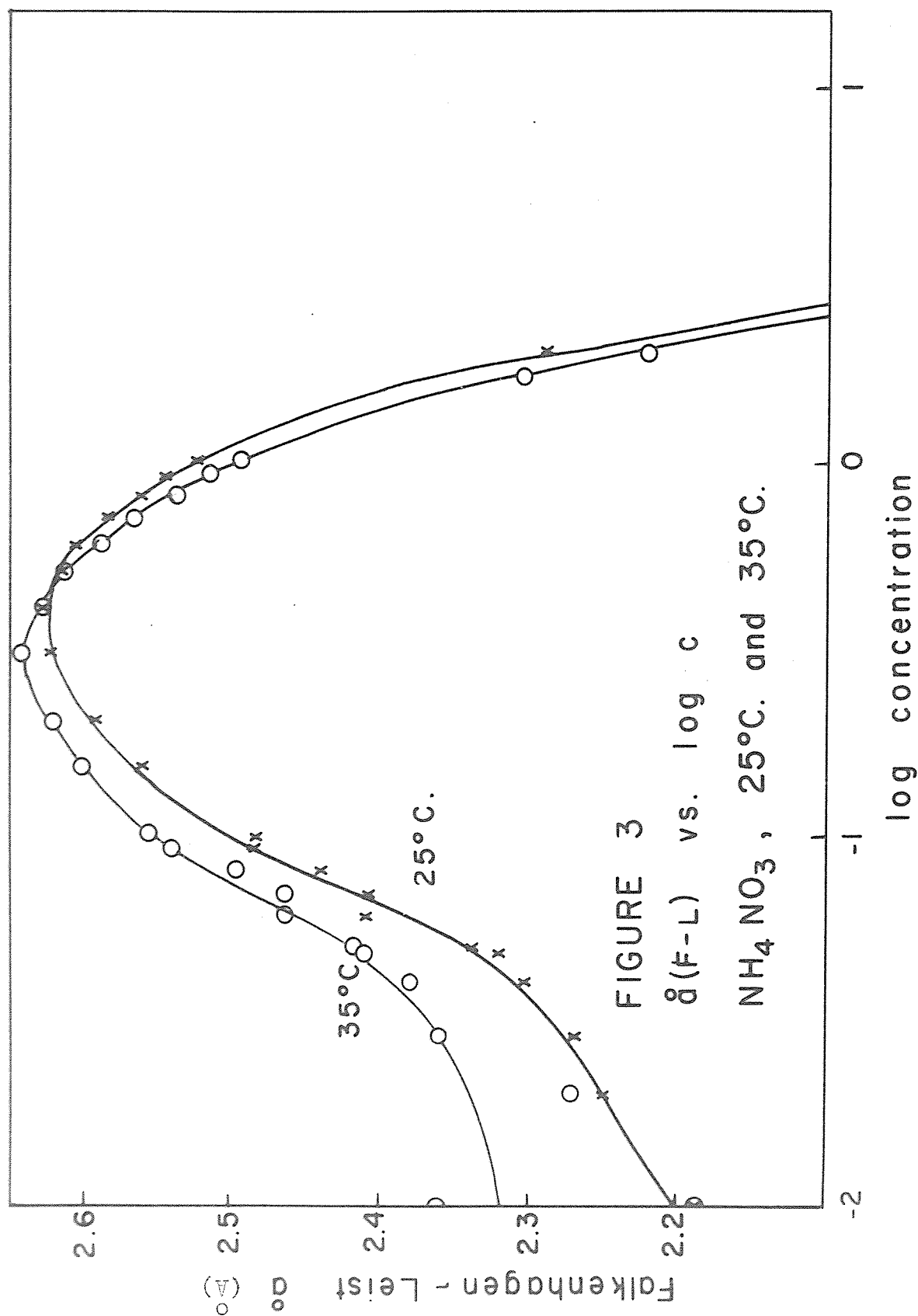
By applying Figure 2 to Figure 1, it may be seen that the change in \bar{a}^0 for a 0.05% change in conductance is 0.20 Å at 0.01 molar, 0.026 Å at 0.1 molar, and 0.005 Å at 1 molar. If \bar{a}^0 were constant the experimentally determined \bar{a}^0 values would lie within a funnel-shaped area opening wider and wider in the direction of lower concentration. Since this is not the case, we may expect the Falkenhagen-Leist equation to show appreciable deviations from the experimental results.

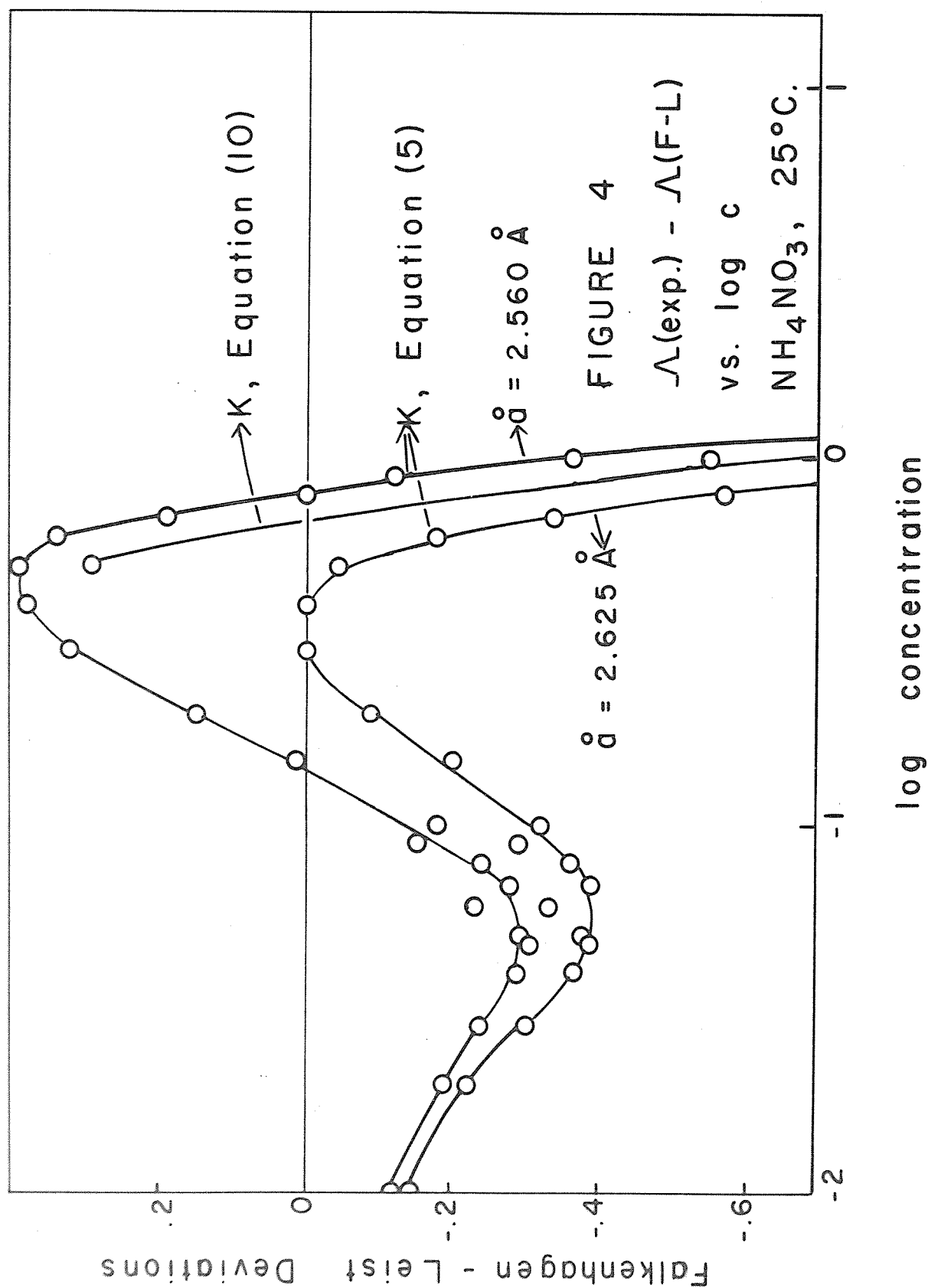


The temperature dependence of $\overset{\circ}{a}$ is examined in Figure 3. Because of the magnitude of the uncertainty in the determination of $\overset{\circ}{a}$, it is not possible to state indisputably whether or not $\overset{\circ}{a}$ varies with temperature for solutions less than 0.1 molar. The fact that $\overset{\circ}{a}$ values at 35°C. are consistently higher than those at 25°C. does, however, suggest that $\overset{\circ}{a}$ varies with temperature. Above 0.1 molar there is no doubt that two different $\overset{\circ}{a}$ values will be necessary to reproduce the experimental results.

The Falkenhagen-Leist equation reproduces the data for ammonium nitrate solutions remarkably well up to 1 molar, in spite of the inconstancy of $\overset{\circ}{a}$ required for a perfect fit. Figure 4 is a plot of the difference between observed and experimental conductances for two arbitrary values of $\overset{\circ}{a}$, 2.560 Å and 2.625 Å. Experimental data are reproduced within 0.4 mho up to 1 molar for $\overset{\circ}{a} = 2.560$ Å, but the deviations are systematic and greater than the experimental error. Most disturbing is the failure of the equation to reproduce dilute solution (< 0.1 molar) data with a physically reasonable $\overset{\circ}{a}$ value. For agreement within 0.1 mho, an $\overset{\circ}{a}$ of about 2.3 Å would be necessary, but the sum of the crystallographic radii of ammonium and nitrate ions is only about 3.5 Å.

The effect of replacing the original Debye-Hückel definition of K , equation (5), by that of Falkenhagen, Leist, and Kelbg, equation (10), is also illustrated in Figure 4. Below 0.1 molar the effect on the calculated conductance is negligible, but at 1 molar the calculated conductance is increased by 0.2 mho by the use of K from equation (10).



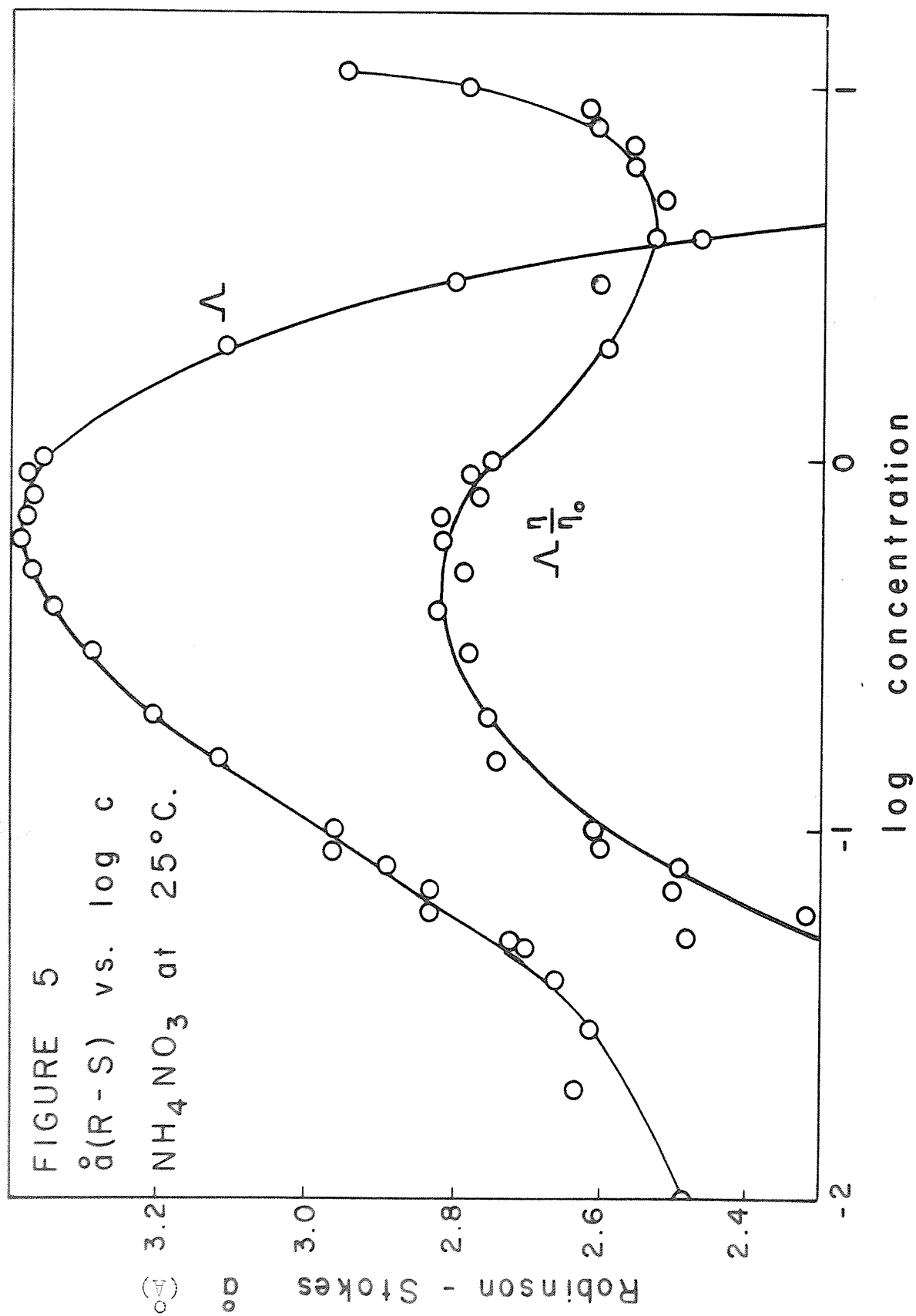


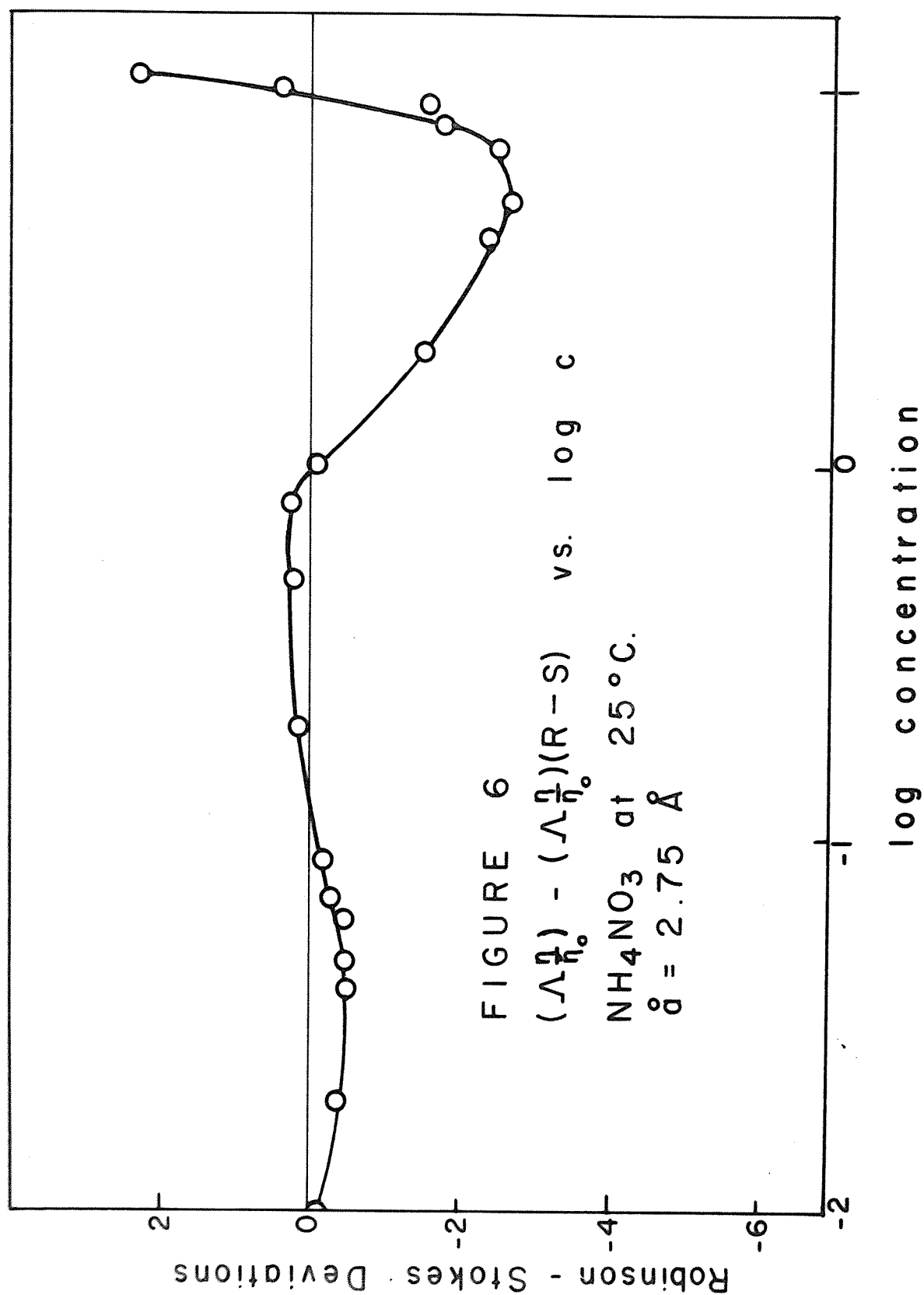
(ii) Ammonium Nitrate and the Robinson-Stokes Equation

The variation of the Robinson-Stokes \bar{a}^0 with concentration, and the effect on \bar{a}^0 of a first power viscosity correction are illustrated in Figure 5. Equations (14) and (21) were used in the calculations. Three observations made earlier in connection with the Falkenhagen-Leist equation (page 37) may be applied to this plot. \bar{a}^0 values calculated with the viscosity-corrected equation show greater scatter than those with equation (14). The \bar{a}^0 values calculated by equations (14) and (21) do not become identical with increasing dilution. The viscosity-corrected \bar{a}^0 values lie below those from equation (14). Examination of Figure 2 will show that the uncertainty in \bar{a}^0 for data accurate to 0.05% is 0.3 Å at 0.01 molar, 0.035 Å at 0.1 molar, and 0.008 Å at 1 molar. The uncertainty in a Robinson-Stokes \bar{a}^0 is about 50% higher than that in a Falkenhagen-Leist \bar{a}^0 .

For equation (14), \bar{a}^0 increases up to 0.6 molar, has the constant value 3.37 Å up to 1 molar, then decreases rapidly. \bar{a}^0 values from equation (21) reach a maximum at 2.82 Å (0.7 molar) and a minimum at 2.53 Å (5 molar). A comparison of Figures 1 and 5 shows that Robinson-Stokes \bar{a}^0 values are higher, and consequently more plausible physically, than Falkenhagen-Leist \bar{a}^0 values.

Experimental conductances and those calculated with the Robinson-Stokes equation corrected for viscosity ($\bar{a}^0 = 2.75$ Å)





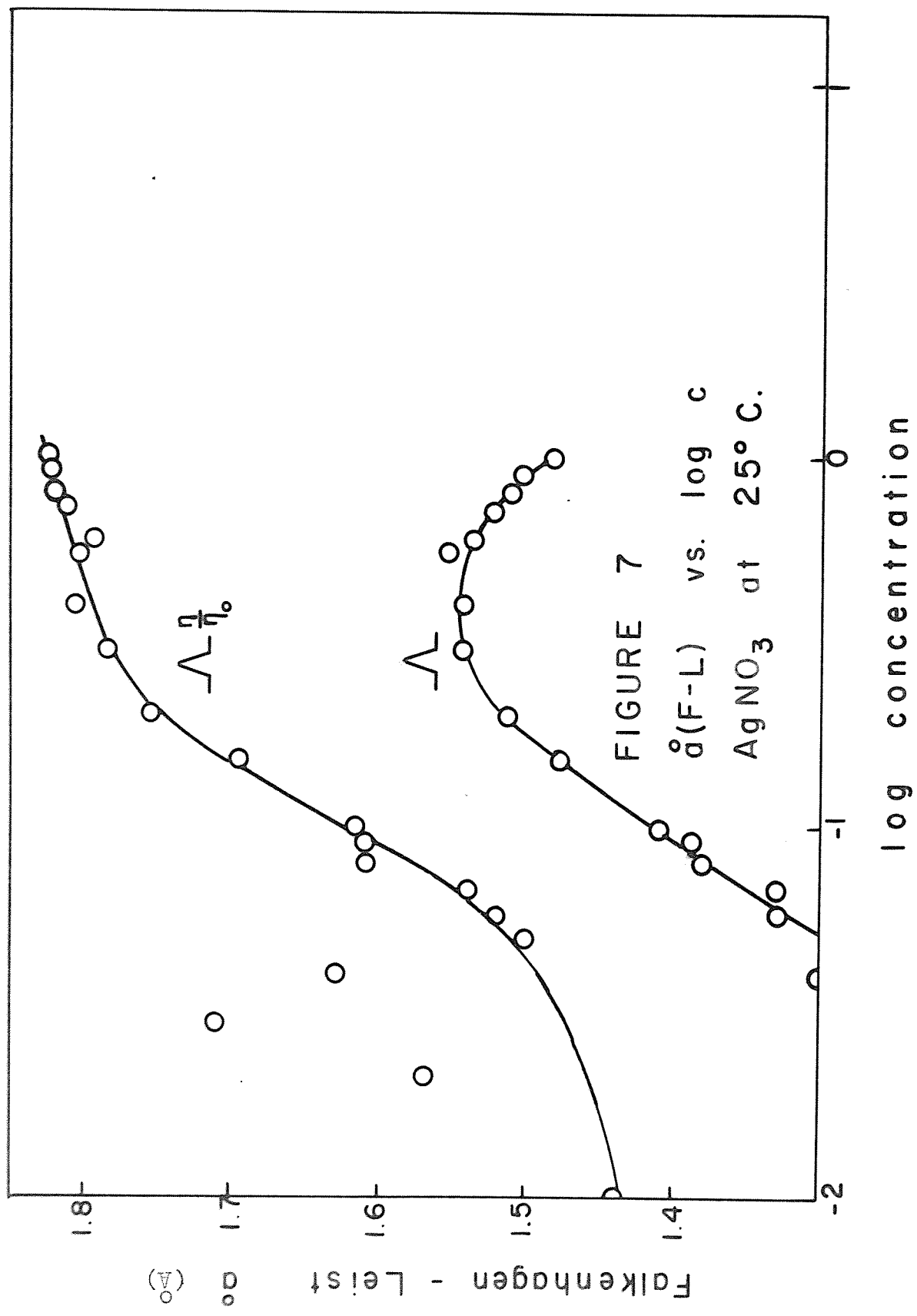
are compared in Figure 6. Up to 1 molar, agreement is within 0.5 mho. Equation (14), with $\overset{\circ}{a} = 3.37 \text{ \AA}$ (not illustrated), reproduces the data equally well below 1 molar.

(iii) Ammonium Nitrate and the Fuoss-Onsager Equation

An attempt was made to fit the 35°C . data of Campbell and Bock¹⁶ and some of the present data to the Fuoss-Onsager equation, (18). The procedure, which neglects ion association, is described on page 11. A plot of the quantity \mathcal{L}' against concentration is linear below 0.003 molar, with intercept $\mathcal{L}_0 = 174.18 \pm 0.08$ mhos and slope $J = 350$. This corresponds to $\overset{\circ}{a} = 4.6 \text{ \AA}$. Data in the range 0.003 molar to 0.04 molar also lie on a straight line but the intercept is 174.5 mhos and the slope 180, corresponding to $\overset{\circ}{a} = 2.2 \text{ \AA}$. The failure of this treatment no doubt lies in the fact that ammonium and nitrate ions form ion pairs; consequently the simpler Fuoss-Onsager equation is not applicable.

(iv) Silver Nitrate and the Falkenhagen-Leist Equation

The Falkenhagen-Leist $\overset{\circ}{a}$ is plotted against log concentration in Figure 7. As with ammonium nitrate, results from the viscosity-corrected equation show considerable scatter because of the uncertainty in viscosity determination, and dilute solution $\overset{\circ}{a}$ values are less certain than calculations in concentrated solutions. A recent review by Stern and Amis²⁵



gives the following radii, determined from ionic constituent mobilities: $\text{Ag}^+ = 1.13 \text{ \AA}$, $\text{NO}_3^- = 2.25 \text{ \AA}$. The sum of the ionic radii obtained in these calculations is always less than 1.9 \AA , no doubt because ion association has been ignored.

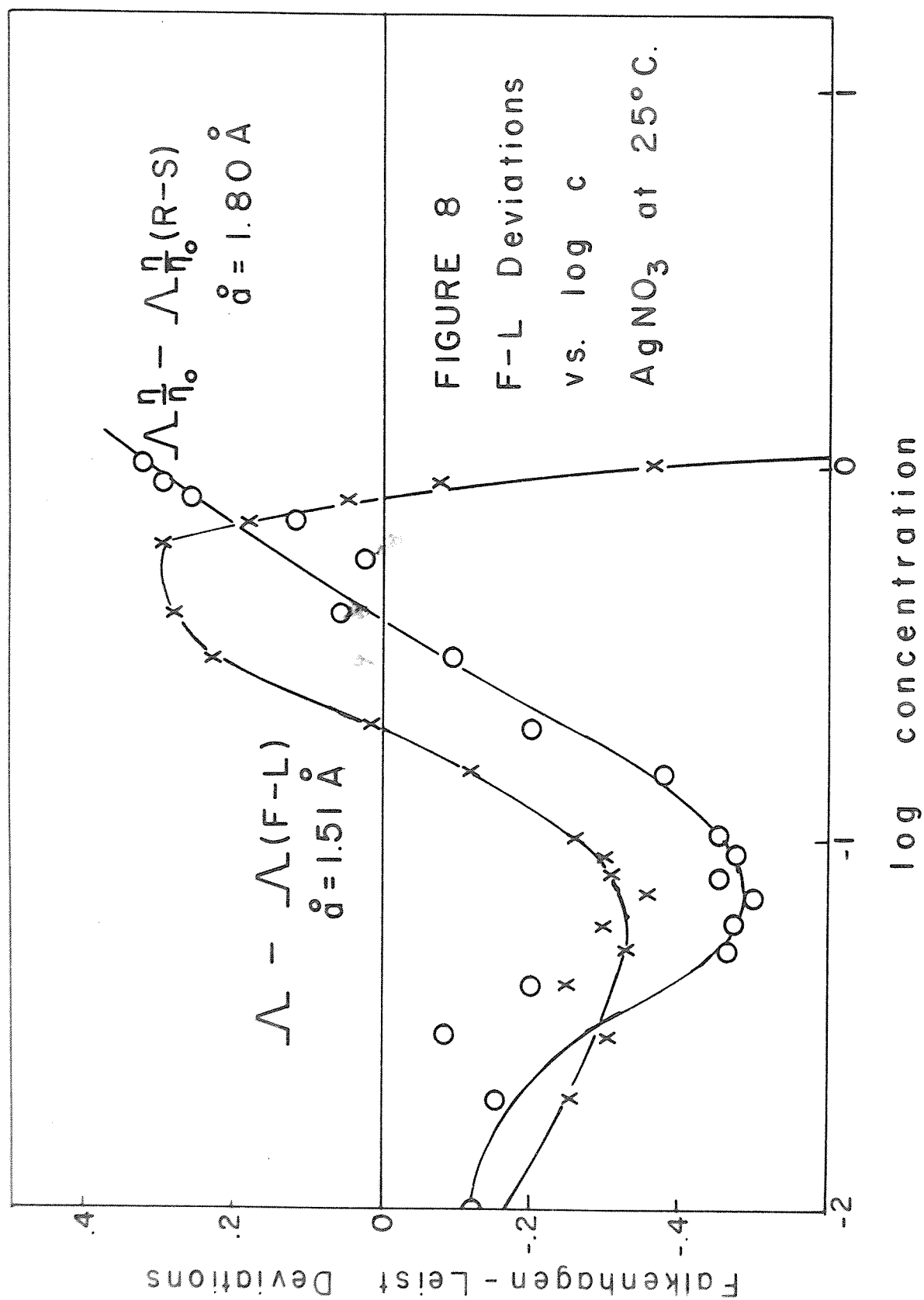
The usefulness of equations (17) and (22) for reproducing experimental data is examined in Figure 8. Either equation will give, at concentrations below 1 molar, a conductance within 0.5 mho of the experimental value if $\overset{\circ}{a}$ is chosen properly.

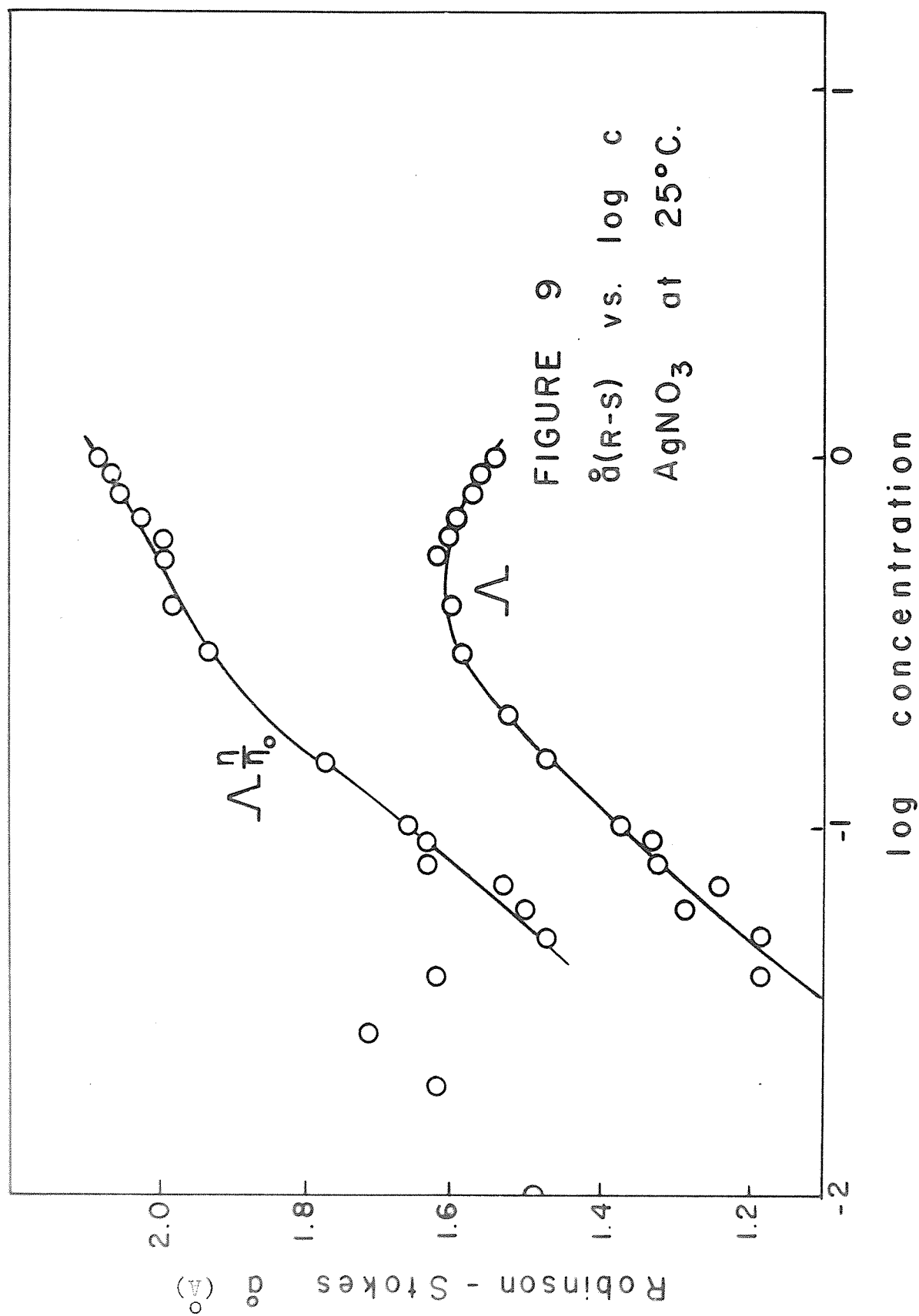
(v) Silver Nitrate and the Robinson-Stokes Equation

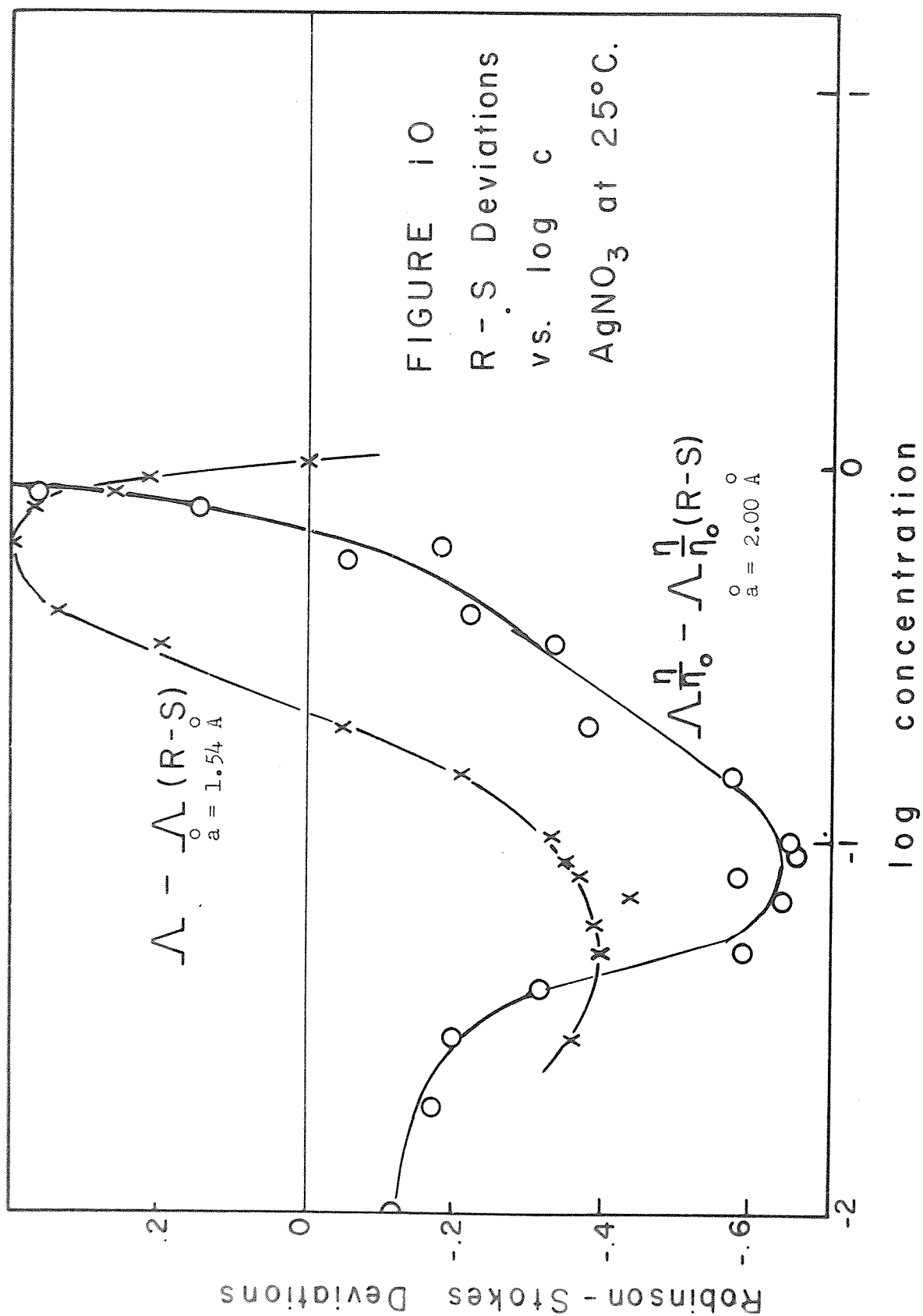
Figure 9 shows the variation of the Robinson-Stokes $\overset{\circ}{a}$ for silver nitrate solutions at 25°C . with concentration. The curves are of the same shape as those for the Falkenhagen-Leist equation, but Robinson-Stokes $\overset{\circ}{a}$ values are higher. Figure 10, a comparison of calculated and experimental conductances, shows that the Robinson-Stokes equation is as satisfactory as the Falkenhagen-Leist equation.

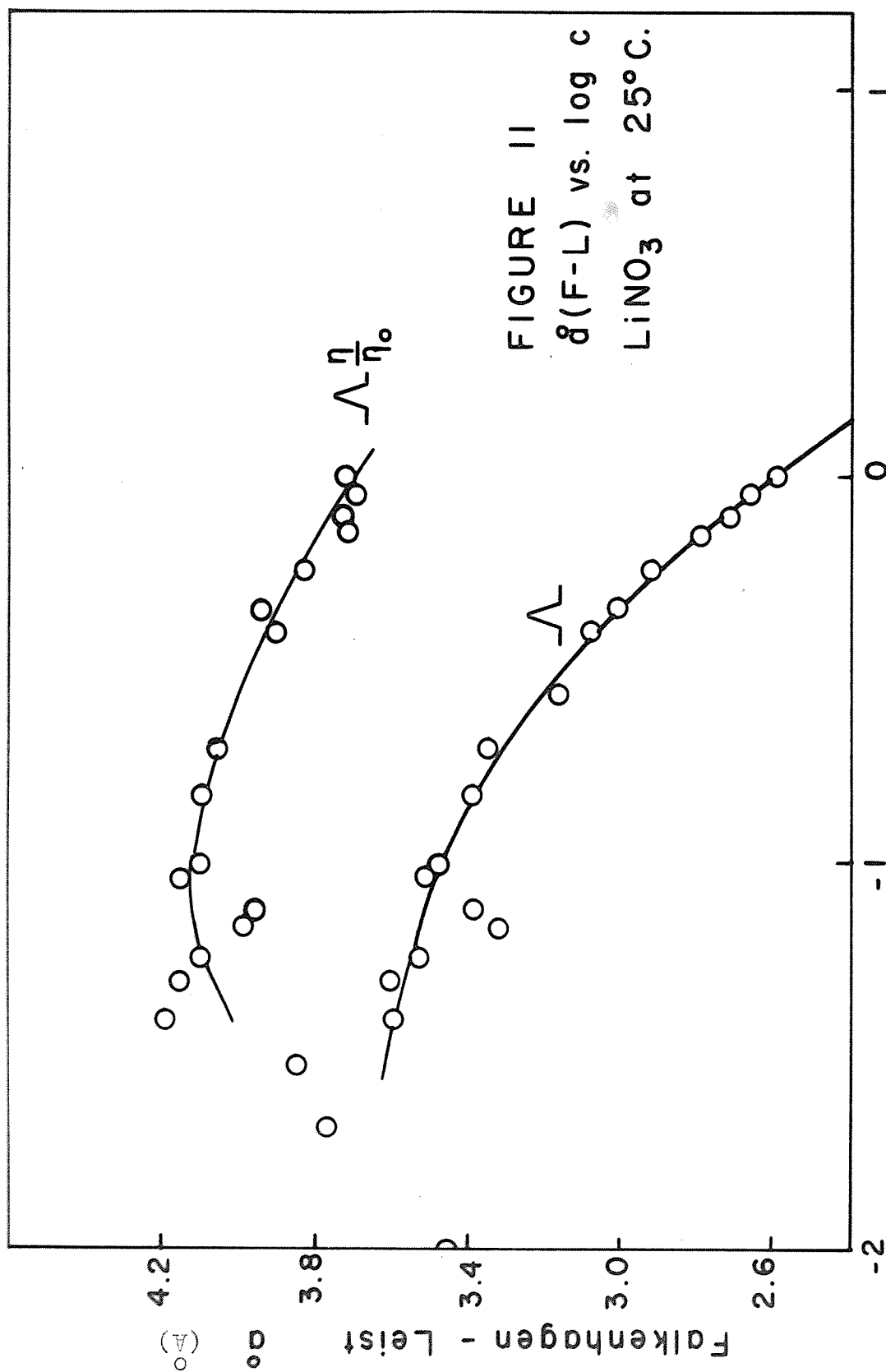
(v) Lithium Nitrate and the Falkenhagen-Leist Equation

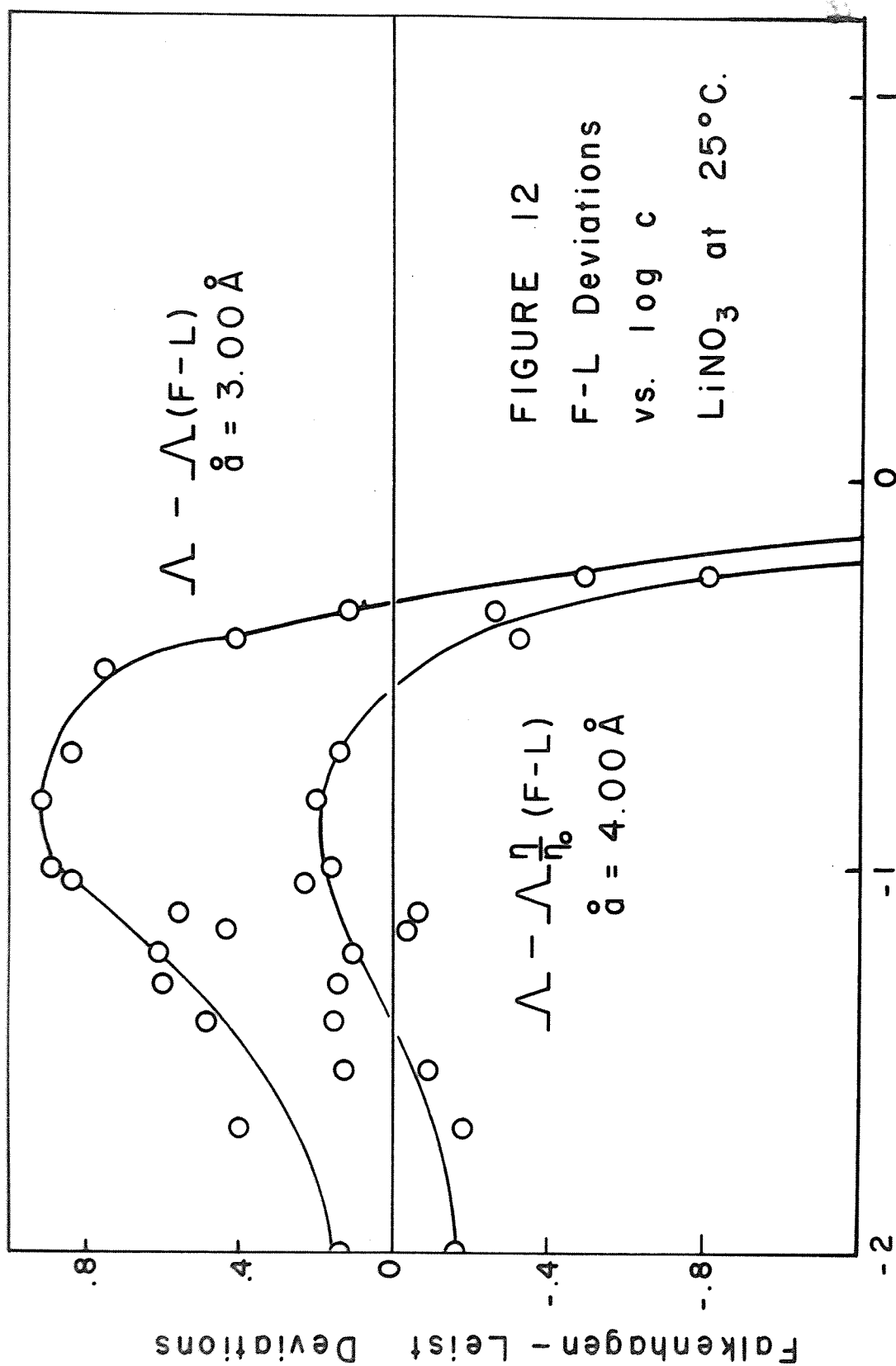
The 25°C . data are summarized in Figure 11. Stern and Amis²⁵ give 3.40 \AA as the radius of the lithium ion. We expect, therefore, that $\overset{\circ}{a}$ will be about 5.7 \AA . All $\overset{\circ}{a}$ values calculated here are less than 4.2 \AA . Once more, the discrepancy may be attributed to ion association. Figure 12 presents the deviation of calculated from experimental conductances. The equation fails rather badly above 0.5 molar.











SUMMARY AND CONCLUSIONS

SUMMARY AND CONCLUSIONS

Equivalent conductances, densities, and viscosities of aqueous solutions of ammonium nitrate, silver nitrate, and lithium nitrate have been determined at 25°C. and 35°C. at concentrations in the range 0.01 molar to 1.0 molar.

Conductance data for solutions weaker than 0.005 molar are of little value for determining the distance of closest approach for an electrolyte in solution.

Ammonium nitrate data up to 1 molar may be reproduced within 0.4 mho by the Falkenhagen-Leist equation if $\overset{\circ}{a}$ is taken as 2.56 Å, or within 0.5 mho by the Robinson-Stokes equation with viscosity correction, using $\overset{\circ}{a} = 2.75$ Å. Silver nitrate data up to 1 molar may be reproduced within 0.4 mho by the Falkenhagen-Leist equation using $\overset{\circ}{a} = 1.51$ Å. The same accuracy is obtainable with the Robinson-Stokes equation and $\overset{\circ}{a} = 1.54$ Å. Lithium nitrate data up to 0.5 molar may be reproduced within 0.3 mho by the Falkenhagen-Leist equation with viscosity correction using $\overset{\circ}{a} = 4.00$ Å.

These distances of closest approach are smaller than we expect, but they increase in the same order as the sizes of the cations: $\text{Ag}^+ > \text{NH}_4^+ > \text{Li}^+$.

APPENDICES

TABLE OF SYMBOLS

a, a^0	Distance of closest approach between ions in solution, in cm. and \AA , respectively
A	Empirical constant
b	Fuoss-Onsager constant, page 10
B_1, B_2, B_3	Theoretical constants, pages 8 and 9
c	Concentration in moles per liter
D_0	Dielectric constant of a pure solvent
E	Fuoss-Onsager constant, page 10
f	Mean ionic activity coefficient of an electrolyte; see Fuoss-Onsager equation, page 12
F	Value of the Faraday, coulombs
J	Fuoss-Onsager constant, page 10
k	Boltzmann's constant in ergs per degree
K_a	Association constant for ion pair formation
L	Specific conductance in mhos per cm.
n	Concentration in ions per unit volume
N	Avogadro's number in molecules per gram mole
N_1, N_2	Reciprocal of the volume of ions 1 and 2
S	Fuoss-Onsager constant, page 10
T	Temperature in degrees absolute
V	Volume in ml.

α, β	Debye-Hückel constants, page 5
γ	Activity coefficient of an electrolyte in solution, Debye-Hückel equation, page 4
γ_0	Degree of dissociation of an electrolyte in solution, Fuoss-Onsager equation, page 12
e	Charge on the electron, e.s.u.
κ	Reciprocal of the "radius" of the ionic atmosphere
Λ, Λ_0	Equivalent conductance and limiting equivalent conductance in mhos
η, η_0	Viscosity of solution and pure solvent, poises
ψ	Electrical potential around an ion
ρ	Charge density around an ion
σ_1, σ_2	Fuoss-Onsager constants, page 11

VALUES OF PHYSICAL CONSTANTS

Faraday, coulombs/equivalent	9.6493×10^3
Charge on the proton, e.s.u.	4.8022×10^{-10}
Boltzmann constant, erg/degree	1.3803×10^{-16}
Avogadro's number	6.0238×10^{23}
π	3.14159
Base of natural logarithms	2.71828
$\log_e 10$	2.30259
Molecular weight, NH_4NO_3	80.048
AgNO_3	169.888
LiNO_3	68.948
KCl	74.557

Constants relevant to water at absolute temperature, degrees,

	298.16	308.16
Density, g/ml.	0.997074	0.994059
Dielectric constant	78.54	75.03
Viscosity, poises	8.910×10^{-3}	7.203×10^{-3}
\mathcal{L}_0 , mhos, NH_4NO_3	145.01	174.21
AgNO_3	133.36	161.53
LiNO_3	110.14	133.48
$\left(\frac{\pi e^2 N}{125 D_0 k T}\right)^{\frac{1}{2}}$ (page 4)	3.2866×10^7	3.3076×10^7
B_1 (page 8)	1.8407×10^{-8}	2.2768×10^{-8}
B_2 (page 8)	2.3783×10^{-8}	2.4087×10^{-8}
B_3 (page 9)	1.8381×10^{-6}	2.2737×10^{-6}

DATA PERTINENT TO THE FIGURES

1. Falkenhagen-Leist α values, NH_4NO_3 , 25°C . and 35°C .

Figures 1 and 3.

(c is in moles per liter, κ in cm^{-1} , α in cm.)

c	$\kappa_{25} \times 10^{-6}$	α_{25}	κ_{25}	α_{25}	$\kappa_{35} \times 10^{-6}$	α_{35}
0.01	3.2691	2.19	136.4	1.95	3.2849	2.36
0.02	4.6519	2.25	133.0	1.82	4.6743	2.27
0.03	5.6135	2.27	131.0	1.98	5.6411	2.36
0.04	6.5706	2.30	128.9	1.95	6.6030	2.38
0.048	7.1973	2.32	127.7	1.98	7.2321	2.41
0.05	7.3495	2.34	127.6	2.15	7.3849	2.42
0.06	8.0502	2.41	126.1	2.06	8.0894	2.47
0.07	8.6983	2.41	125.1	2.17	8.7403	2.47
0.08	9.2912	2.440	124.0	2.16	9.3360	2.500
0.09	9.8582	2.485	123.2	2.24	9.9053	2.542
0.10	10.396	2.482	122.3	2.24	10.443	2.557
0.15	12.882	2.562	118.6	2.31	12.944	2.602
0.2	14.731	2.594	116.0	2.32	14.802	2.623
0.3	18.070	2.624	111.8	2.30	18.156	2.642
0.4	20.785	2.625	108.9	2.30	20.882	2.629
0.5	23.158	2.618	106.2	2.26	23.269	2.615
0.6	25.396	2.605	104.3	2.27	25.513	2.591
0.7	27.493	2.584	102.4	2.25	27.620	2.568

1. (continued)

c	$K_{25} \times 10^{-6}$	a_{25}	$-L\%_{25}$	a_{25}	$K_{35} \times 10^{-6}$	a_{35}
0.8	29.386	2.561	100.5	2.21	29.520	2.538
0.9	31.220	2.547	99.1	2.21	31.362	2.516
1.0	32.862	2.523	97.6	2.17	33.006	2.495
(The following were calculated from data in references 19 and 24.)						
1.0	32.932	2.527	97.3	2.15	33.248	2.47
1.7	-	-	-	-	42.776	2.305
1.9	-	-	-	-	45.427	2.22
2.0	46.400	2.290	87.54	2.016	-	-
3.0	56.756	2.072	82.34	1.976	-	-
4.0	65.897	1.873	77.47	1.900	-	-
5.0	73.594	1.716	74.27	1.865	-	-
6.0	80.745	1.575	72.21	1.856	-	-
7.0	87.049	-	70.00	1.830	-	-
8.0	93.024	-	68.79	1.799	-	-
9.0	98.835	-	67.26	1.816	-	-
10.0	103.952	-	67.83	1.858	-	-
11.3	110.394	-	67.9	1.888	-	-

2. Falkenhagen-Leist Deviations, NH_4NO_3 , $25^\circ C.$, $a = 2.625 \text{ \AA}$
and $a = 2.560 \text{ \AA}$, Figure 4.

2. (continued)

c	$\lambda = 2.625 \text{ \AA}$		$\lambda = 2.560 \text{ \AA}$	
	Calc. Cond.	Deviation	Calc. Cond.	Deviation
0.01	136.61	- 0.14	136.59	- 0.12
0.02	133.54	- 0.22	133.51	- 0.19
0.03	131.56	- 0.30	131.50	- 0.24
0.04	129.69	- 0.36	129.62	- 0.29
0.048	128.52	- 0.39	128.44	- 0.31
0.05	128.24	- 0.38	128.15	- 0.29
0.06	126.99	- 0.33	126.89	- 0.23
0.07	125.89	- 0.39	125.78	- 0.28
0.08	124.90	- 0.36	124.78	- 0.24
0.09	123.99	- 0.29	123.85	- 0.15
0.10	123.15	- 0.32	123.01	- 0.18
0.15	119.59	- 0.20	119.38	+ 0.01
0.2	117.19	- 0.09	116.95	+ 0.15
0.3	113.46	0.00	113.14	+ 0.32
0.4	110.83	0.00	110.45	+ 0.38
0.5	108.78	- 0.04	108.35	+ 0.39
0.6	107.08	- 0.18	106.56	+ 0.34
0.7	105.56	- 0.33	105.04	+ 0.19
0.8	104.34	- 0.57	103.77	0.00
0.9	103.23	- 0.72	102.63	- 0.12
1.0	102.30	- 0.98	101.68	- 0.36

3. Robinson-Stokes \bar{a} values, NH_4NO_3 , 25°C ., Figure 5;
Robinson-Stokes Deviations for $\bar{a} = 2.75 \text{ \AA}$ (with viscosity
correction), Figure 6.

c	\bar{a}	\bar{a}_η	$\bar{a} = 2.75 \text{ \AA}$	
			Calc. Cond.	Deviation
0.01	2.48	2.16	136.5	- 0.1
0.02	2.63	2.07	133.4	- 0.4
0.03	2.61	2.21	-	-
0.04	2.66	2.18	129.4	- 0.5
0.048	2.70	2.19	128.2	- 0.5
0.05	2.72	2.48	-	-
0.06	2.83	2.32	126.6	- 0.5
0.07	2.83	2.50	125.4	- 0.3
0.08	2.89	2.49	-	-
0.09	2.96	2.60	123.4	- 0.2
0.10	2.96	2.61	-	-
0.15	3.114	2.74	-	-
0.2	3.205	2.76	115.9	+ 0.1
0.3	3.293	2.79	-	-
0.4	3.340	2.82	-	-
0.5	3.368	2.78	106.1	+ 0.1
0.6	3.384	2.82	-	-
0.7	3.379	2.82	-	-
0.8	3.366	2.76	100.4	+ 0.1
0.9	3.375	2.78	-	-
1.0	3.357	2.75	97.7	- 0.1

3. (continued)

c	$\overset{\circ}{a}$	$\overset{\circ}{a}_\eta$	$\overset{\circ}{a} = 2.75 \overset{\circ}{A}$	
			Calc. Cond.	Deviation
(The following were calculated from data in reference 24).				
1.0	3.366	2.70	89.0	- 1.5
2.0	3.107	2.591	89.0	- 1.5
3.0	2.798	2.606	83.8	- 1.5
4.0	2.474	2.532	79.9	- 2.4
5.0	2.205	2.518	77.0	- 2.7
6.0	1.953	2.559	73.5	- 1.3
7.0	-	2.555	72.5	- 2.5
8.0	-	2.607	70.6	- 1.8
9.0	-	2.620	68.9	- 1.6
10.0	-	2.785	67.4	+ 0.4
11.3	-	2.95	65.6	+ 2.3

4. Falkenhagen-Leist $\overset{\circ}{a}$ values, AgNO_3 , 25°C ., Figure 7;

Falkenhagen-Leist Deviations for $\overset{\circ}{a} = 1.51 \overset{\circ}{A}$ (no correction)

and $\overset{\circ}{a} = 1.80 \overset{\circ}{A}$ (with viscosity correction), Figure 8.

c	$k \times 10^{-6}$	$\overset{\circ}{a}$	\mathcal{L}/η_0	$\overset{\circ}{a}_\eta$	Deviations	
					$\overset{\circ}{a} = 1.51, 1.80$	
0.01	3.3122	1.7	124.7	1.44	+ 0.55	- 0.1
0.02	4.5769	1.1	121.7	1.57	- 0.26	- 0.2
0.03	5.6411	1.19	119.4	1.71	- 0.30	- 0.3
0.04	6.5729	1.30	117.3	1.63	- 0.25	- 0.3

4. (continued)

c	$\kappa \times 10^{-6}$	\bar{a}	$-L\gamma/\eta_0$	\bar{a}_γ	Deviations	
					$\bar{a} = 1.51,$	1.80
0.05	7.3239	1.29	115.5	1.50	- 0.33	- 0.5
0.06	7.9365	1.33	114.3	1.52	- 0.30	- 0.5
0.07	8.5919	1.33	113.0	1.54	- 0.36	- 0.5
0.08	9.2968	1.38	111.7	1.61	- 0.31	- 0.5
0.09	9.8575	1.39	110.6	1.61	- 0.30	- 0.5
0.10	10.4258	1.41	109.6	1.63	- 0.26	- 0.5
0.15	12.693	1.478	105.8	1.70	- 0.12	- 0.4
0.2	14.623	1.514	102.9	1.76	+ 0.02	- 0.1
0.3	18.001	1.545	98.11	1.79	+ 0.23	- 0.1
0.4	20.780	1.546	94.68	1.81	+ 0.28	+ 0.1
0.55	24.434	1.555	90.42	1.803	+ 0.48	0.0
0.6	25.357	1.538	89.36	1.795	+ 0.30	- 0.1
0.7	27.484	1.526	87.36	1.811	+ 0.18	+ 0.1
0.8	29.241	1.514	85.82	1.823	+ 0.05	+ 0.3
0.9	31.094	1.504	84.20	1.825	- 0.08	+ 0.3
1.0	32.881	1.486	82.71	1.826	- 0.36	+ 0.3

5. Robinson-Stokes \bar{a} values, AgNO_3 , 25°C ., Figure 9;

Robinson-Stokes Deviations for $\bar{a} = 1.54 \text{ \AA}$ (no correction)

and $\bar{a} = 2.00 \text{ \AA}$ (with viscosity correction), Figure 10.

5. (continued)

c	σ_a	$\sigma_{a_{\eta}}$	Deviations	
			$\sigma_a = 1.54,$	2.00
0.01	1.8	1.5	+ 0.07	- 0.1
0.02	-	1.62	-	- 0.2
0.03	1.03	1.71	- 0.36	- 0.2
0.04	1.18	1.62	- 0.31	- 0.3
0.05	1.18	1.47	- 0.40	- 0.6
0.06	1.28	1.50	- 0.39	- 0.8
0.07	1.24	1.53	- 0.43	- 0.6
0.08	1.32	1.63	- 0.36	- 0.6
0.09	1.33	1.63	- 0.35	- 0.7
0.10	1.37	1.66	- 0.33	- 0.7
0.15	1.466	1.77	- 0.21	- 0.6
0.2	1.524	1.87	- 0.05	- 0.4
0.3	1.582	1.93	+ 0.19	- 0.3
0.4	1.595	1.98	+ 0.31	- 0.2
0.55	1.619	1.99	+ 0.53	- 0.1
0.6	1.597	1.99	+ 0.40	- 0.2
0.7	1.586	2.02	+ 0.36	+ 0.2
0.8	1.571	2.049	+ 0.26	+ 0.4
0.9	1.563	2.063	+ 0.20	+ 0.5
1.0	1.540	2.075	0.00	+ 0.7

6. Falkenhagen-Leist \bar{a} values, LiNO_3 , 25°C ., Figure 11;

Falkenhagen-Leist Deviations for $\bar{a} = 3.00 \text{ \AA}$ (no correction)

and $\bar{a} = 4.00 \text{ \AA}$ (with viscosity correction), Figure 12.

c	$\eta \times 10^{-6}$	\bar{a}	$\lambda \%$	\bar{a}_m	Deviations $\frac{\bar{a}}{a} = 3.00, 4.00$	
0.01	3.3793	3.47	102.4	3.44	+ 0.13	- 0.2
0.02	4.7696	3.77	99.90	3.64	+ 0.40	- 0.2
0.03	5.6711	3.19	98.51	3.85	+ 0.13	- 0.1
0.04	6.4778	3.60	97.53	4.19	+ 0.48	+ 0.2
0.05	7.4092	3.61	96.20	4.15	+ 0.61	+ 0.1
0.06	7.9938	3.53	95.37	4.10	+ 0.62	+ 0.1
0.07	8.6876	3.33	94.36	3.98	+ 0.44	0.0
0.08	9.0745	3.39	93.86	3.96	+ 0.56	- 0.1
0.09	9.9525	3.52	93.09	4.15	+ 0.83	+ 0.2
0.10	10.4508	3.48	92.49	4.10	+ 0.89	+ 0.2
0.15	12.7203	3.40	90.25	4.10	+ 0.92	+ 0.2
0.2	14.675	3.348	88.47	4.06	+ 0.85	+ 0.2
0.3	17.977	3.203	86.68	4.27	+ 0.76	+ 0.8
0.4	20.760	3.091	83.90	3.90	+ 0.42	- 0.3
0.5	22.277	3.023	83.07	3.94	+ 0.12	- 0.3
0.6	25.061	2.918	81.22	3.82	- 0.49	- 0.8
0.7	27.720	2.799	79.71	3.71	- 1.30	- 1.5
0.8	29.696	2.728	78.78	3.73	- 2.66	-
0.9	31.385	2.665	77.96	3.69	-	-
1.0	32.942	2.600	77.59	3.73	-	-

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