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RECURSIVE PROGRAMMING ANALYSIS OF PRAIRIE LAND UTILIZATION PATTERNS

by

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ABSTRACT

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To operate agricultural supply management programs efficiently in Canada, production response information is needed. This information will assist farmers in adjusting their crop acreages each year in order to bring supply of individual crops in line with demands.

The general objective of the study was to explain historical prairie land utilization patterns as well as to predict future crop acreages for alternative levels of policy variables. To deal with these objectives, a recursive programming model was developed. Six grain crops and summerfallow were included in the model. The crops were wheat, oats, barley, rye, flaxseed and rapeseed.

Several programming restraints were used in the study. The major restrictions were flexibility restraints which are the upper and lower bounds on allowable year-to-year changes in the acreages of each crop in the model. These restraints add predictive and recursive quality to the ordinary programming model.

The flexibility restraints were estimated on the basis of flexibility coefficients. In most previous recursive programming studies, the coefficients were estimated

such that they were immune to year-to-year changes in economic and non-economic conditions. However, in this study, an attempt was made to develop a multiple regression model which could estimate the coefficients such that they could vary from year to year, depending upon the levels of exogeneous explanatory variables. This method was considered to be conceptually superior to previous ones. Observations for years 1953 to 1967 were used to estimate the upper and lower flexibility coefficients.

The basic recursive programming model was utilized to construct twenty-four individual models: one was based on the prairie data, three employed provincial observations, and twenty models were developed using crop districts of Saskatchewan as units of analysis. These models were structurally the same, differing only in terms of coefficient values.

The explanatory power of the models was tested by solving the prairie and provincial recursive programs for each individual year from 1958 through 1967 and then comparing the estimated acreages against actual observations. The results indicated that the models explained the land utilization patterns of each province and the prairies as a whole with reasonable accuracy; the average deviation for all land use was less than seven percent.

The ability of the recursive model to predict land use <u>outside</u> the period used for construction was also established in this study by estimating acreages for 1968 and

1969. Comparison of the actual and predicted acreages showed that land utilization patterns for the prairies and most provinces were predicted with moderate precision.

A few sizeable errors, however, occurred in both explanatory and predictive analyses. A number of explanations can be provided for these errors. First, the use of inappropriate expectation models for prices, costs, yields and/or quota levels is likely to be a source of errors. Secondly, relatively too wide or too narrow flexibility bounds on year-to-year changes in crop acreages might have caused errors in the estimates for some crops. Thirdly, excessively dry weather conditions perhaps caused discrepancies between the estimated and actual acreages of some crops in a few years.

In this study, the relative performance of the aggregate and disaggregate models was also examined. It was found that none is clearly superior in explaining and predicting the crop acreages. However, a crop-by-crop comparison of the errors demonstrated that the aggregate models produced less accurate results than the disaggregate ones for the relatively more profitable crops (i.e., wheat, flaxseed and rapeseed). Another conclusion was that sizeable changes in crop acreages were explained or predicted more accurately by the disaggregate models.

After performing the above tests, the provincial recursive programming models were utilized to estimate the impact of the Operation LIFT (Lower Inventory for Tomorrow) on the 1970 prairie land utilization patterns. The analysis suggested that the program reduced wheat acreage by ten million acres and increased summerfallow acreage by 6.6 million acres. The study also indicated that the program did not have any significant impact on acreages of other crops.

The provincial models were further applied to forecast the 1971 land use for each province and the prairies as a whole. The study projected 20.9 million acres of wheat, 9.9 million acres of barley, 4.5 million acres of rapeseed and 27.3 million acres of summerfallow in 1971. However, the actual acreages turned out to be somewhat different from the estimated, perhaps because inaccurate levels of exogeneous variables, such as exports, stocks and prices, were assumed in the analysis.

The impact of changes in barley prices and quota levels on the 1971 forecasts was also analyzed. While identical land utilization patterns were estimated for both 20 and 25 bushels barley quota, the patterns changed with respect to increases in barley prices. The results indicated that as the price rose from $0.66 to $0.86 per bushel, barley acreage increased from 9.9 million to 11.8 million acres in the prairies.

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INTRODUCTION

Purpose and Need of the Study

During the 1960's, Canada experienced large fluctuations in the exports of farm products. Wheat exports were only 331 million bushels in the 1962-63 crop year, but increased to an unprecedented level of 595 million bushels in 1963-64, and dropped again to 306 million bushels in 1968-69. Fluctuations were also sharp for other crops such as oats, barley and rapeseed. This situation has created untenable inventories of several crops in certain years. There are at least three possible solutions to this problem. One is to create a high and sustainable demand for these products. The second is to facilitate adjustments by farmers in their acreages of individual crops quickly and substantially so that production is brought into line with demand. The third solution is a combination of the above two (i.e., expanding demand and aligning supply with demand). Since examination of demand expansion is beyond the scope of this study, attention is focussed here only on supply adjustment aspects.

In order to develop policies for supply adjustments, information concerning farmers' production response is needed. Answers are required to questions such as: Why has production changed as it has in the past? How is production expected to change next year? How may production respond in the next year or so to alternative agricultural policies contemplated by government? At the present time, quantitative estimates of production response in the prairies are in short supply. Empirical explanations of historical production patterns and models which would enable the prediction of future crop acreages are essential to a policy of rapid adjustments in production to market demands.

Production response research would also be useful in other directions. Precise production forecasts in the prairies can help the Government of Canada in formulating effective and consistent policies directed towards greater stability in farm prices and incomes, and in developing export markets, storage, price and auxiliary mechanisms which contribute to this end. It may also be such that future production patterns predicted on the basis of current programs do not satisfy agricultural policy goals. Therefore, it might be desirable to change these future production patterns through farm policies and programs. In order to determine the direction and magnitude of policy measures that can be applied to bring the outcome currently anticipated in line with that desired, information on historical production response to changes in economic and non-economic variables is required. Quantitative estimates of the historical supply response as well as the prediction of the future production are, therefore, required for developing intelligent agricultural policies.

Accurate production forecasts can assist agribusiness firms in their investment and planning decisions. Firms supplying farm inputs can utilize information about the potential intensity of crop production since the levels of certain inputs vary by crops. Estimates of future crop acreages would enable these firms to foresee the future demands for their inputs.

Agribusiness firms processing farm products can also utilize production forecasts for efficient planning of their businesses. Better estimates about future production could be of value in their investment and planning decisions.

Production response studies are also of significance to farmers. Because production is a major determinant of price and sales quota, crop production forecasts would be of value to farmers in allocating their resources among alternative enterprises. Lack of accurate forecasts hampers farm planning.

Improved knowledge of farmers' production response is, therefore, essential to the entire agricultural industry. As mentioned above, at the present time, there is a lack of agricultural production response studies in Canada. A study is, therefore, needed to quantitatively estimate the production response in the prairies.

Objectives of the Study

The general objective of this study is to develop and apply a recursive programming model to analyze year-to-year changes in the prairie land utilization patterns. More specific objectives are:

(1) to develop a theoretical recursive programming model to explain the historical crop acreages and to predict

the future land utilization patterns;

(2) to quantify the model and test its explanatory and predictive powers by estimating acreages of major crops in Manitoba, Saskatchewan and Alberta, and in the prairies as a whole, during the period 1958 to 1967;

(3) to verify its predictive ability by estimating land utilization patterns of each prairie province and the prairies as a whole for 1968 and 1969 (years outside the period used for model quantification);

(4) to determine the performance of aggregate models relative to the disaggregate ones[a] for explaining and predicting the land utilization patterns of Saskatchewan and the prairies, using 1958 to 1969 data;

(5) to use the recursive models to estimate the impact of the Lower Inventory for Tomorrow (LIFT) program on the 1970 prairie land use; and

(6) to forecast the 1971 crop acreages for each prairie province and for the prairies as a whole, using the models developed.

Organization of the Study

Chapter II studies the theory of production response and examines a number of approaches for deriving aggregate supply response. Merits and demerits of each approach are discussed with the objective of selecting one as a tool for

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[a]The aggregate and disaggregate models are explained on pages 102 to 102a.

the present study. In Chapter III, four production response studies utilizing the recursive programming technique are reviewed and problems encountered in these studies are noted.

Chapter IV is concerned with the formulation of the model utilized in this study. This chapter is divided into two sections. In the first section, the recursive programming model is developed and the procedure used for estimating flexibility coefficients[a] is discussed in detail. The second and final portion of this chapter describes three levels of aggregation (i.e., prairie, provincial and crop district) used in the analysis.

In Chapter V empirical results are presented. In the first section of this chapter, the explanatory test of the recursive model is undertaken. In the second section, the predictive test is discussed. The third section describes the performance of aggregate models relative to the disaggregate ones. In the fourth section, the impact of Operation LIFT on the 1970 prairie land utilization pattern is examined. The fifth and the last portion of this chapter presents the 1971 forecast of acreages of major crops in the prairie provinces.

The sixth chapter summarizes the study and conclusions drawn therefrom. Limitations of the present study and suggestions for future ones are also described in this chapter.

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[a]The concept of flexibility coefficients is discussed on pages 31 to 33.

CHAPTER II

PRODUCTION RESPONSE: THEORY AND APPROACHES

Several approaches have been used to analyze production response. Each has advantages and disadvantages. With this thought in mind, the purpose of this chapter is to evaluate some of these approaches in light of the objectives of the present study. The chapter is divided into three sections. The first presents the theory of production response, while the second describes methods for estimating aggregate supply functions as well as discusses aggregation problems usually encountered in estimation. The third and final section of the chapter draws on the theoretical scaffolding to critically evaluate methods of analyzing production response.

Theory of Production Response

The basic theory of production response hinges on the static production function of the individual firm. But an operational theory cannot omit the dynamic elements involved in the production process. Risk and uncertainty, fixity of factors, and technological change must be related to a dynamic theory of production response. However, a convenient starting point remains the elementary theory of the firm and the static supply function.

Static Supply Function

A generalized production function can be expressed as an implicit functional relationship between all outputs and all variable inputs:

$$F(Y_1,\ldots,Y_n;\ X_1,\ldots,X_m) = 0 \qquad (1)$$

where:

$Y_1,\ldots,Y_n$ = quantities of n outputs, and

$X_1,\ldots,X_m$ = quantities of m inputs.

A simple production function for one output, Y, in terms of the m inputs can be obtained from relation (1) above. This relationship is expressed in an explicit functional form as:

$$Y = f(X_1,\ldots,X_m) \qquad (2)$$

Relation (2) can take any functional form. Some of the frequently utilized forms are linear, Cobb-Douglas, Spillman, quadratic, square root, cubic and logistic.

The supply function of a firm can be derived from the production function, making certain assumptions about the nature of the factor and product markets, and behaviour of the entrepreneur.[a] The supply curve describes the quantity that a perfectly competitive firm will offer for sale in response to changes in market price of the product, _ceteris paribus_.

The relevant short-run supply function of a firm is

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[a]A derivation is demonstrated on pages 18-25 of this chapter. Important assumptions are listed there.

identical with the rising portion of the short-run marginal cost curve which lies above the average variable cost curve (Figure 1). The function is not defined for outputs less than the abscissa of the intersection of the marginal cost and average variable cost curves (31, p. 90). At all prices less than the intersection point, the quantity supplied is zero because the price does not cover the average variable cost. Figure 1 illustrates that a firm's supply curve consists of the segments OR and ST.

In the long-run when there are no fixed factors of production, the supply function of a firm consists of that portion of its long-run marginal cost curve which is above the average total cost curve. Thus, the supply function is not defined for output levels where marginal cost is less than the average total cost.

The aggregate supply functions in both the short-run and the long-run are obtained by horizontal summation of individual functions of all firms in the industry, other things being equal. Two *ceteris paribus* conditions are: (1) the changes in quantities of factors demanded by the firms do not affect their prices. Implicit in this statement is also the condition that factor prices do not change in response to industry output; and (2) the number of firms in the industry is known (38, pp. 140-141). If these conditions are not satisfied, the marginal cost functions of the individual firms will not sum to the industry supply function.

The nature of the static supply function depends upon a variety of factors. Some of the important factors are:

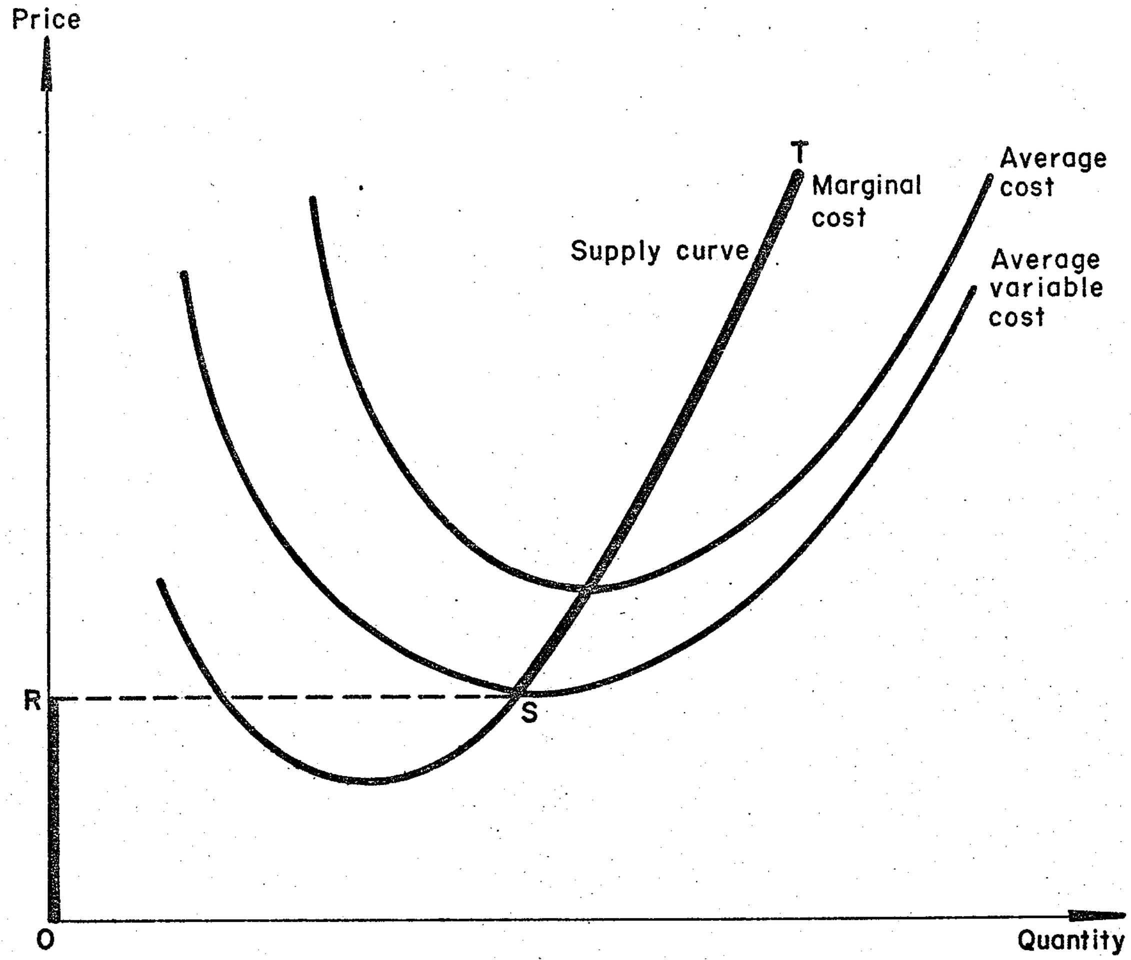

Figure 1. Short-run supply curve for a perfectly competitive firm.

(1) the nature of the physical production functions,

(2) the nature of the supply functions for inputs, and

(3) fixed costs as proportion of total costs. These factors affect slope and location of both short-run and the long-run supply functions.

Dynamic Forces Affecting Supply Response

An analysis of dynamic supply response entails modifying the static function to account for the effects of uncertainty, flexibility of the "fixed" factors and technological change on production response. Some of the effects are discussed below. Also, attempts by other researchers to integrate static supply functions with theories relating to the dynamic forces are outlined, but methods for combining them are discussed in detail in the next section.

Expectation and Uncertainty

In the production process, some parts of inputs are usually committed long before output is realized. A farmer therefore bases his investment and planning decisions not on current prices, but on uncertain expected future prices. Thus, a study of farmers' expectation formation about prices seems to be a necessary ingredient of production response analysis.

Little known research has been carried out to date to relate farmers' expectations and uncertainty with supply analysis. Nerlove (50, pp. 24-26) proposed a price expectation model and combined this with a static supply function to produce a distributed lag model of production response.

Flexibility of "Fixed" Factors

Nerlove and Bachman (53, p. 538) stated that fixed factors of production, which form the basis for the traditional distinction between short-run and long-run supply functions, are in reality not fixed for all times but can be varied in response to product prices.[a] In the shortest of all short-runs, most or all factors of production are fixed; but as time passes, successively more of these factors become variable. Therefore, the longer the time or "run" allowed for adjustment, the closer is the short-run curve to the long-run. Thus, there is a fan of short-run supply curves, and each is appropriate for a different interval of time.

There are no well-developed theories of investment which can be used to explain the variations in output due to changes in the so-called fixed factors of production. Nerlove (51, pp. 308-11) developed a distributed lag model by integrating the supply function with an output adjustment equation. However, Glenn Johnson (35, pp. 25-28) considered that this model is inadequate to study dynamic production response because it is based on an output adjustment equation which is too simple to represent complex production adjustments made by farmers. Nerlove's adjustment equation indicates that next year's production is equal to this year's output plus some proportion, γ, of the difference between

[a] In contrast to this statement, Glenn Johnson (36, pp. 442-51) stated that imperfect second-hand markets and high relocation costs for durable factors of production make some factors fixed in the short-run.

this year's actual and planned output, regardless of the levels of economic and non-economic variables in the year. His model hinges on a simple coefficient of adjustment, γ, which does not respond to year-to-year variation in economic and physical conditions of an area.

The second limitation of the Nerlove distributed lag model is the problem in identifying the estimated coefficient. There is no theoretical basis to determine whether the estimated coefficient in distributed lag models is estimate of β (the coefficient of price expectation) or estimate of γ (the coefficient of adjustment). Attempts to produce separate estimates of β and γ using an identifying variable were unsuccessful.

Technological Change

A given technology underlies all the micro production functions from which aggregate static supply functions are derived. Technological change clearly violates the theoretical assumptions on which the static supply response is based and points to the need for a theory of response which accounts for the essentially dynamic nature of technology.

For developing a dynamic theory of response, a knowledge of rates of adoption of new technologies and their effects on the production process is essential. Much work has been done for examining the effects of technological changes on output (6, 47). But barring a few notable exceptions, such as Griliches (25, pp. 501-22), very little research has been carried out to study the rate of adoption of technologies and to investigate how this rate relates to the changes in prices.

Methods for Estimating Supply Response

This section briefly introduces a number of methods for estimating supply response and consequently discusses, in detail, four of the more frequently used or promising ones. Then, aggregation problems in supply analysis are described. No attempt at point-by-point comparative analysis between methods is made in this section. Rather, this is left to the third section on "Evaluation" which follows.

General Description of Alternative Methods

A wide range of methods for estimating aggregate supply functions have been proposed and applied. These methods have been classified in several ways. Schaller (59, pp. 98-109) proposed one classification based on differences in types of data and economic theory (i.e., micro or macro) utilized by the models. He classified all methods for estimating supply response into four categories: pure micro, micro-oriented, macro-oriented and pure macro approach.

As Figure 2 indicates, the first group consists of models which use the theory of the firm and micro data to derive supply functions of individual firms. The main feature of the micro-oriented approach is that a sample of farms are selected to derive group supply functions and a weighted sum of these functions produces the aggregate supply function. By contrast, in a macro-oriented approach, the unit of analysis is a sub-aggregate or a region as a whole. In this approach, as illustrated in Figure 2 through two zig-zag lines, both micro and macro data are utilized to derive

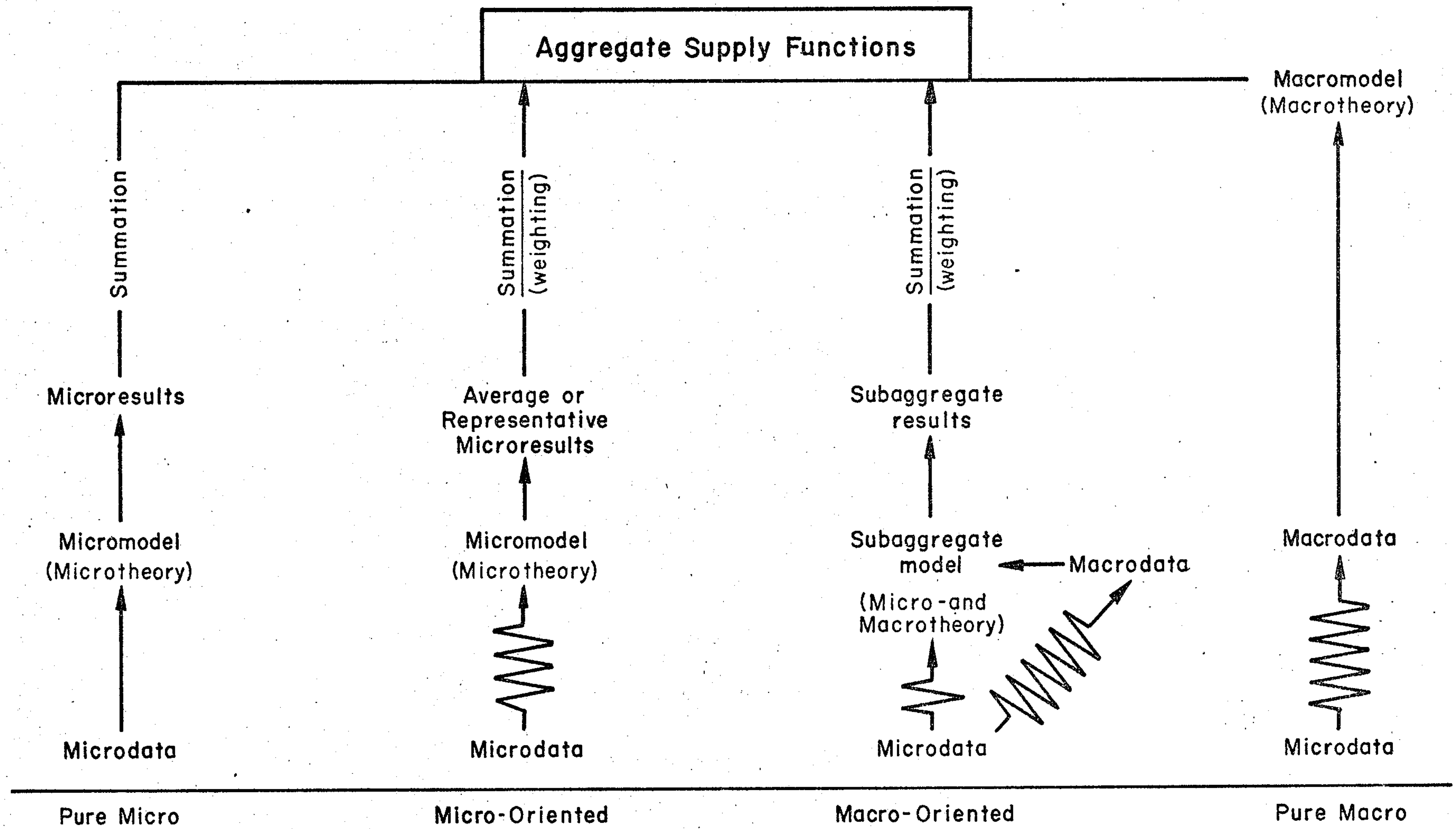

Figure 2. Alternative approaches for estimating aggregate supply functions.

supply functions. The pure macro approach is very aggregative. It uses the entire industry (i.e., aggregate data) as a unit of analysis.

The pure micro approach of supply analysis includes the estimation of production functions, cost relationships estimated from the financial records, budgeting, linear programming and production surveys. The first method, as the name indicates, requires the estimation of production functions to derive the aggregate supply function. However, in the cost function approach, supply curves are estimated directly through the analysis of the firm's accounting data, rather than starting with the underlying production function. It is conceivable, in this case, that firms operating with the same production functions may not have the same marginal cost curves because their levels of fixed capital may differ substantially.

In budgeting, the supply functions are constructed by developing a series of production plans, each corresponding to a level of price. The major limitation of this approach is that, in a complex production problem where there are many restrictions and production alternatives, it is unlikely that all of them will be fully considered.

Linear programming can be used to do precisely the same job, employing a better analytical framework than budgeting. Relatively more restrictions and production alternatives can be considered. Using this framework, optimum supply functions are estimated and information such as shadow prices,[a] which are not forthcoming from budgeting, are obtained.

The production survey is another pure micro method of supply analysis. Farmers are questioned firstly to find out what production changes are taking place and secondly, to identify the price and non-price factors causing these changes. Questions range from those of inventory nature to those which require the farmer to describe the causes of production adjustments.

In some industries, there are so many firms that estimation of individual supply functions is economically not feasible. Therefore, in order to estimate aggregate supply response, the foregoing five methods are used to derive supply functions of a sample of firms. Thus, insofar as application is concerned, the above methods are micro-oriented type.

The macro-oriented method includes recursive programming. In this method, farm sub-aggregates (usually geographic regions) are used as units of analysis. Aggregate data are utilized to specify restraints in the model, but input-output coefficients are estimated using micro observations (i.e., from representative farms).

Regression analysis of aggregate time-series data is an example of the pure macro approach. With this method,

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[a]A shadow price represents marginal value product of a scarce resource. In other words, it is the amount added to the total net revenue by one unit increase in the resource.

the supply function is estimated by fitting regression models to aggregate time-series observations. These models can also be applied at the regional level. An aggregate function can be obtained by summing the regional supply functions.

Of the seven methods for estimating supply response outlined above, only four are discussed in detail in the remaining portion of this chapter. These methods are estimation through production functions, linear programming, recursive programming and regression analysis of time-series data.

Derivation of Aggregate Supply Function from Production Functions

The production function approach can be used in several ways to derive an aggregate supply function. Two of the more common methods are: (1) Estimate the production function of individual firms and from these derive the individual supply functions. These micro-derived supply functions are then summed to obtain the aggregate function. (2) Estimate the aggregate production function directly from aggregate data with the supply function derived from it. Since the derivation of a supply curve from either a micro production function or an aggregate production function is fundamentally similar, a derivation of only the micro level one is discussed below.

Derivation of Firm's Supply Function

The derivation of a static supply function from a production function is a commonly discussed topic in textbooks on micro-economic theory. While supply functions can be derived from production functions for both perfect and certain types of imperfect factor and product markets, in order to simplify the illustration of the derivation process, attention here is focussed on perfectly competitive markets. However, the methodology used here can, with certain appropriate modifications, be extended to the case of imperfect markets. Specifically, in deriving a firm's supply function, it is assumed that:

(1) the form of the production function is perfectly known;

(2) prices are known with certainty;

(3) perfect competition exists in the product market;

(4) supplies of resources are perfectly elastic; and

(5) the goals of the entrepreneurs are to maximize profits.

Production function of any form such as the Cobb-Douglas, quadratic, square root or cubic can be employed to derive a supply function. However, a supply function can be derived in concept without necessarily dealing with a particular form of production function. This general method of deriving a supply function is demonstrated in several economics textbooks (56, p. 41). Here, a more specific method is chosen. That is, the Cobb-Douglas production function is used to illustrate the derivation.

Consider the following production function in which a single product, Y, is an exponential function of inputs X_1

and X_2:

$$Y = aX_1^{b_1}X_2^{b_2} \tag{3}$$

Treating X_2 as a fixed factor, the following short-run production function can be obtained from equation (3):

$$Y = kX_1^{b_1} \tag{4}$$

where:

$$K = aX_2^{b_2}$$

The short-run total cost function in terms of variable input X_1 is presented in equation (5) below:

$$C = c + P_1X_1 \tag{5}$$

where:

C = total cost of production,

P_1 = price per unit of variable input X_1, and

c = total fixed cost.[a]

A short-run supply function can be derived using equations (4) and (5). Equation (4) is utilized to express input as a function of output. This relationship is then substituted into equation (5) to obtain the short-run total cost function in terms of Y. The first derivative of this cost function with respect to Y gives the marginal cost function. Finally, a firm's supply function is obtained from the first-order condition for profit maximization by equating marginal cost with price of the product and solving for Y such that output Y becomes a function of its own price (31, p. 90).

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[a]Fixed cost is defined as that portion of production expense which must be paid regardless of how much the firm produces, or whether it produces at all.

Thus, the first step in the derivation of a supply function is to express input as a function of output, utilizing the short-run production function. Equation (4) is employed to express X_1 in terms of Y as:

$$X_1 = (k^{-1}Y)^{b_1^{-1}} \tag{6}$$

After substituting the above expression for X_1 in equation (5), the following short-run total cost function, in terms of Y, can be obtained:

$$C = c + P_1(k^{-1}Y)^{b_1^{-1}} \tag{7}$$

The first derivative of this function with respect to Y produces the marginal cost function as:

$$\frac{dC}{dY} = b_1^{-1} P_1 k^{-b_1^{-1}} Y^{(1-b_1)/b_1} \tag{8}$$

The following first-order condition for profit maximization is then employed to obtain the supply function:

$$\frac{dC}{dY} = b_1^{-1} P_1 k^{-b_1^{-1}} Y^{(1-b_1)/b_1} = MR = P_Y \tag{9}$$

In the above equation, price of a product has been equated to marginal revenue. This relationship is based on the assumption made on page 18 that perfect competition exists in the product market. In this event, marginal revenue and price of the product are equal.

The first-order condition is not sufficient for profit maximization, but it must be supplemented by a second-order condition which is expressed below:

$$\frac{d^2C}{dY^2} > o \qquad (10)$$

This condition implies that the marginal cost curve must be rising at the profit-maximizing output level (31, p. 57).

In order to obtain the supply function, equation (9) is solved for Y and expressed as follows:

$$Y = (b_1 k^{b_1^{-1}} \frac{Py}{P_1})^{b_1/(1-b_1)} \qquad (11)$$

By substituting the expression for k in equation (11), the supply function can be expressed as below:

$$Y = (b_1 a^{b_1^{-1}} X_2^{b_2 b_1^{-1}} \frac{Py}{P_1})^{b_1/(1-b_1)} \qquad (12)$$

Equation (12) indicates that output depends upon the parameters a, b_1, and b_2 of the production function, the level of X_2 and the prices of the input and output. However, for any given level of the fixed variable X_2, output becomes a function of its own price.

In order to derive a long-run supply function, long-run production and cost functions are employed. As expressed in equation (13) below, the long-run total cost ($\bar{C}$) is a function of two variable inputs, X_1 and X_2, whereas, the short-run total cost (C) is a function of only one input X_1, since X_2 was treated fixed:

$$\overline{C} = P_1X_1 + P_2X_2 \tag{13}$$

The procedure for deriving the long-run supply function is basically the same as that employed for the short-run function. The difference exists because of the fact that, for this specific example, the long-run production and cost functions include two variable inputs (X_1 and X_2), whereas, the short-run functions consist of only one variable input. The first step in the derivation of the long-run supply function is to express X_1 as a function of X_2, using the first-order condition of the cost minimization for any given output. The second step is to substitute the value of X_1 in terms of X_2 to both the long-run production and cost functions in order to make them functions of only one input--X_2. Then, the procedure outlined above for deriving the short-run supply function is also used for the long-run function. The complete procedure is presented below.

The first step is to express X_1 as a function of X_2 using the first-order condition for the minimization of cost. This condition requires that in order to minimize the cost of production for any given output, the marginal productivity of the last dollar must be equal in every use (56, pp. 60-61). For the two input case, the condition can be expressed in the following mathematical form:

$$\frac{MPP_{x1}}{P_1} = \frac{MPP_{x2}}{P_2} \tag{14}$$

where:

MPP_{x1} = marginal physical product of X_1,

MPP_{x2} = marginal physical product of X_2,

with other variables as previously defined.

After substituting the expressions for the marginal products of X_1 and X_2 into equation (14), the following relationship can be obtained:

$$\frac{b_1 Y/X_1}{P_1} = \frac{b_2 Y/X_2}{P_2} \tag{15}$$

In addition to the first-order condition, the second-order condition must also be satisfied for cost minimization. The latter condition requires that isoquants[a] are convex from below. If both conditions are met, a rational entrepreneur should select only those combinations of X_1 and X_2 which lie on his expansion path.[b]

Equation (15) can be solved to obtain X_1 as the following function of X_2:

$$X_1 = \frac{b_1 P_2}{b_2 P_1} X_2 \tag{16}$$

After substituting equation (16) into equations (3) and (13), the long-run production and cost functions are

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[a]An isoquant is defined as the locus of all combinations of X_1 and X_2 which yield a specified level of output.

[b]An expansion path is defined as a locus of points of minimum costs for different levels of output.

expressed in terms of only one input X_2 as:

$$Y = a\left(\frac{b_1P_2}{b_2P_1}\right)^{b_1} X_2^{b_1+b_2} \tag{17}$$

$$\bar{C} = \left(\frac{b_1P_2}{b_2} + P_2\right) X_2 \tag{18}$$

Equation (17) can now be used to express X_2 as the following function of Y:

$$X_2 = \left[a^{-1}\left(\frac{b_2P_1}{b_1P_2}\right)^{b_1} Y\right]^{1/(b_1+b_2)} \tag{19}$$

The value of X_2 from the above equation is then substituted into equation (18) in order to express the following long-run total cost function in terms of output:

$$\bar{C} = \left(\frac{b_1P_2}{b_2} + P_2\right) \left[a^{-1}\left(\frac{b_2P_1}{b_1P_2}\right)^{b_1} Y\right]^{1/(b_1+b_2)} \tag{20}$$

The first derivative of this equation with respect to Y is the marginal cost function which is presented below as equation (21):

$$\frac{d\bar{C}}{dY} = \frac{1}{b_1+b_2}\left(\frac{b_1P_2}{b_2} + P_2\right) \left[a^{-1}\left(\frac{b_2P_1}{b_1P_2}\right)^{b_1}\right]^{1/(b_1+b_2)}$$

$$Y^{(1-b_1-b_2)/(b_1+b_2)} \tag{21}$$

Given that the second-order condition for profit maximization is satisfied (i.e., $d^2\overline{C} / dY^2 > 0$), the long-run supply function can be derived from the first-order condition by equating marginal cost to output price (P_y) and then by solving for Y. The supply function so derived is presented below:

$$Y = a^{1/(1-b_1-b_2)} \left(\frac{b_1}{P_1}\right)^{b_1/(1-b_1-b_2)} \left(\frac{b_2}{P_2}\right)^{b_2/(1-b_1-b_2)} P_y^{(b_1+b_2)/(1-b_1-b_2)} \qquad (22)$$

Equation (22) is the long-run supply function in which output, Y, depends upon a, b_1, b_2, P_1, P_2 and P_y. However, for given prices of the inputs, the output becomes a function of its own price.

Derivation of Aggregate Supply Function

Potentially, a different supply function exists for every firm in the industry. If sufficient funds, computing facilities and data were available, separate function for every firm could be estimated. But the fact is that we never have this happy combination of research resources. Therefore, the aggregate is stratified into groups of relatively homogeneous firms. A production function is then estimated for each group and the supply function is derived from this. The aggregate function is obtained by summing appropriately weighted group supply functions.

Linear Programming Approach to Aggregate Supply Response

General Description

Linear programming is a normative approach.[a] It consists of optimizing a linear objective function subject to linear constraints. A common objective function in agricultural production problems is one which maximizes net revenue generated by the activities included in the model. The constraints are typically the amounts of available resources and various accounting equations. A programming problem can be stated algebraically as follows:

$$\text{Optimize} \qquad Z = \text{opt.} \left[C_1X_1 + C_2X_2 + \ldots + C_jX_j + \ldots + C_nX_n\right]$$

$$\begin{aligned}
\text{Subject to:} \qquad & a_{11}X_1 + a_{12}X_2 + \ldots + a_{1j}X_j + \ldots + a_{1n}X_n \leq b_1 \\
& a_{21}X_1 + a_{22}X_2 + \ldots + a_{2j}X_j + \ldots + a_{2n}X_n \leq b_2 \\
& \vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
& a_{i1}X_1 + a_{i2}X_2 + \ldots + a_{ij}X_j + \ldots + a_{in}X_n \leq b_i \\
& \vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
& a_{m1}X_1 + a_{m2}X_2 + \ldots + a_{mj}X_j + \ldots + a_{mn}X_n \leq b_m
\end{aligned}$$

$$\text{and} \qquad X_j \geq 0 \qquad (j = 1, \ldots, n)$$

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[a]Normative approaches are used to determine how firms

where:

Z = objective function value to be optimized (in this case, maximized),

X_j = level of the j th activity,

C_j = price or cost per unit of the j th activity,

a_{ij} = amount of the i th restraint required per unit of the j th activity,

b_i = amount of the i th restraint available,

n = total number of activities, and

m = total number of restraints.

In developing a linear programming model, the selection of the type and level of constraints, activities, production coefficients and prices to be used depend upon the objectives of the study and the choice of planning horizon.

Derivation of Firm's Supply Function

Linear programming can be utilized to derive normative supply functions for individual firms. Using parametric

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ought to behave given certain assumptions. More precisely, these models are used to answer such questions as: (1) How much should entrepreneurs produce to maximize their income at each set of prices? (2) How much should they change their production patterns with respect to changes in product prices? In contrast to normative, positive approaches are employed to describe how entrepreneurs do behave or to predict how they will behave. These methods are used to explain the production patterns as they actually exist.

programming,[a] various optimum solutions are obtained over a range of prices for a product (29, pp. 265-307). These solutions indicate the quantity of output at each price level and thus provide a stepped supply function (Figure 3).

This supply function describes the quantities of output which the firm <u>should</u> produce to maximize income at various price levels, <u>ceteris paribus</u>. The function becomes stepped because in response to price changes optimum production pattern shifts from one corner point to another of the multi-dimensional production frontier. Changes in price, therefore, bring a series of discrete shifts in the production plans. That is, one plan is stable over a range of prices, and a discrete shift between plans takes place when price exceeds a critical level. This phenomenon produces the stepped supply function. In the production function

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[a]Two types of parametric programming are used in supply response studies. Using linear programming, one method calculates the limit of the range of price changes which causes a plan to become sub-optimum. The method also determines new optimum plan for the increased level of price. This procedure is repeated and a number of optimum plans and ranges of prices over which each of these plans remain optimum are estimated. The plans indicate quantities of output for different price levels and thereby provide a supply function. In the second method, one could estimate a supply function through linear programming by specifying output prices in discrete intervals and determining optimum plans for these prices. For example, one might start out with a zero price and increase it to $5.00, then to $10.00 and $15.00 and so on. The limitation of this discrete method is that we may not catch a corner point. That is, it may be such that the critical point is at $16.00 and yet using this method we may fail to catch it. However, major computer programs often utilize this method to derive supply functions, not the first one (e.g., MPS-360).

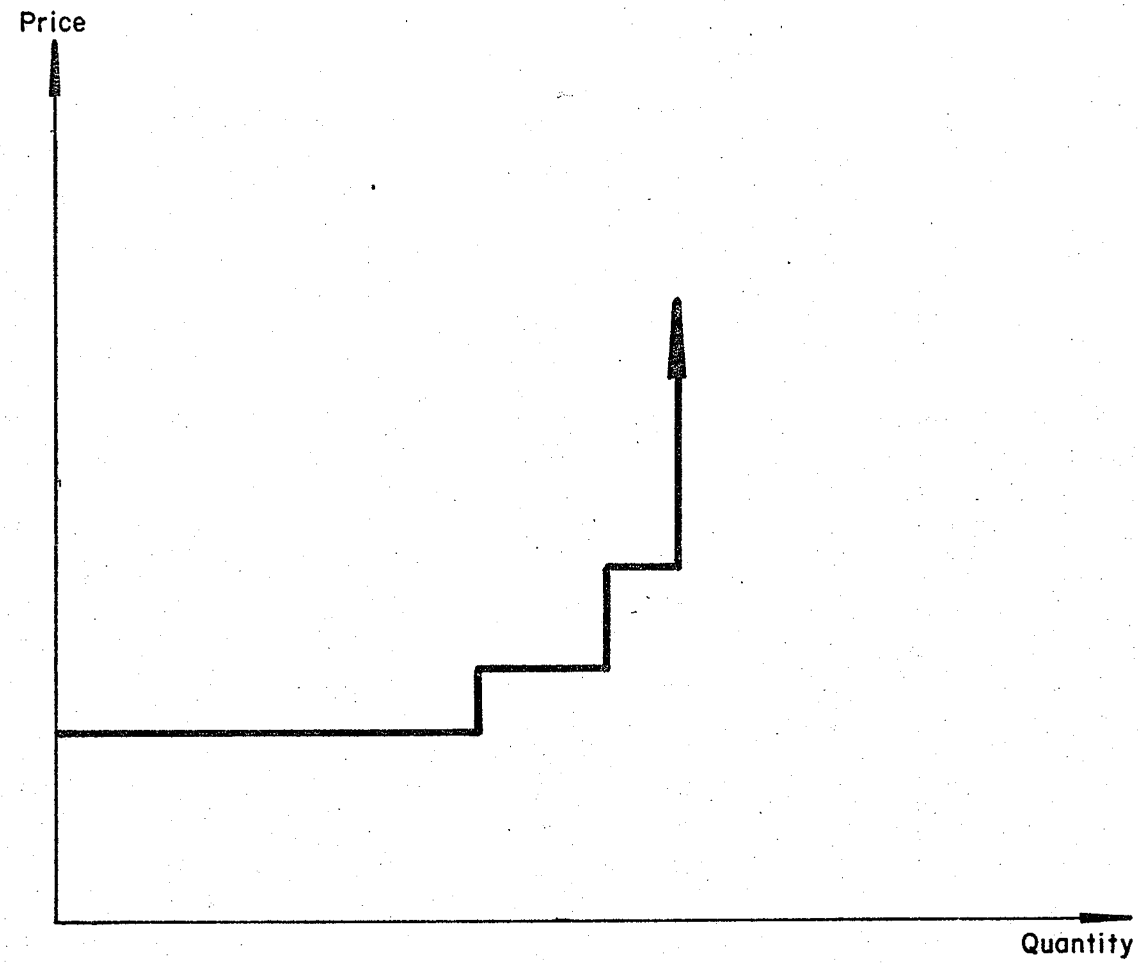

Figure 3. A stepped supply curve from linear programming analysis.

approach, since the production frontier is continuous, a small price change results in a marginal change in production. Thus, a continuous supply function is obtained.

Estimating Aggregate Supply Function

An aggregate supply function for an industry can be obtained through the horizontal summation of the functions of individual firms, *ceteris paribus*. However, the number of firms in some industries is so large that the derivation of supply function is economically not feasible. In this situation, other approaches are utilized. One approach is to stratify the aggregate into categories of homogeneous firms. A supply function could then be derived for a firm, representative of each category. By attaching appropriate weights, firm supply functions could be summed to obtain the aggregate function.

Alternatively, the aggregate supply function can be estimated directly by treating the industry as a whole as one decision-making unit and developing a single aggregate linear programming model for that. Methods and procedures which are used to derive a micro-level function can then be applied for estimating the aggregate function (i.e., parametric programming). However, in order to derive an unbiased supply function, certain technical requirements for using an aggregate model should be met. These requirements are discussed on pages 43 to 45.

Recursive Programming and Supply Response

Definition and Concepts

Recursive programming was developed and used by J. M. Henderson (30, pp. 242-59) in 1959 for predicting land utilization patterns in the United States. Richard H. Day (11) gave it a rigorous mathematical orientation and defined it as a sequence of mathematical programming in which the parameters of a given problem are functionally related to the optimal variables of the preceding problems in the sequence.

In terms of solution procedures, recursive programming is the same as linear programming. Both are mathematical techniques which can be employed to optimize a linear objective function subject to linear constraints. The difference between these models is, however, of a conceptual nature. The recursive programming is capable of predicting the actual behaviour of firms, whereas the linear programming is designed to estimate an optimum behaviour. This feature of the recursive model is acquired through the use of flexibility constraints in addition to the ordinary linear programming restraints. The upper and lower flexibility constraints could represent, for example, the maximum and minimum limits, respectively, which farmers in aggregate change production of a crop from one year to the next. Also, with respect to inputs, similar restraints (referred to as capacity constraints) can be used to specify the maximum potential investment in any "fixed" factor in a given year.

Limited year-to-year change in production and investment patterns may result from producers' inability or unwillingness to make profitable adjustments due to insufficient knowledge, risk and uncertainty, institutional restrictions, personal preferences, and goals other than short-run profit maximization. The following is an algebraic formulation of a recursive programming model:

Optimize $Z_t = \text{Opt.}\ (C_{1t}X_{1t} + \ldots + C_{nt}X_{nt})$

Subject to:

$$X_{jt} \leq (1+\bar{\beta}_{jt})X_{jt-1}$$

$$X_{jt} \geq (1-\underline{\beta}_{jt})X_{jt-1} \qquad (j = 1,\ldots,n)$$

$$\sum_{j=1}^{n} a_{ijt}X_{jt} \leq S_{it} \qquad (i = 1,\ldots,m)$$

and $X_{jt} \geq 0 \qquad (j = 1,\ldots,n)$

where:

Z_t = objective function value to be optimized in the t th year,

X_{jt} = level of the j th activity to be determined in the t th year,

X_{jt-1} = level of the j th activity in the t-1 th year,

S_{it} = level of the i th restraint in the t th year,

C_{jt} = price or cost per unit of the j th activity in the t th year,

a_{ijt} = amount of the i th restraint required per unit of the j th activity in the t th year,

$\bar{\beta}_{jt}$, $\underline{\beta}_{jt}$ = maximum allowable proportionate increase and decrease, respectively, in the t th year level of the j th activity from that in the t-1 th year. These are known as upper and lower flexibility coefficients.[a]

n = total number of restraints, and

m = total number of activities.

As shown in the above formulation, the solution of the first time period determines the flexibility restraints for the second time period. Likewise, the second period solution determines the constraints for the third period, and so on. Therefore, the constraints can be generated in a recursive manner, and a distributed lag response to a policy variable can be traced out through the changes in the levels of constraints in the successive time periods. In this sense, the model is self-generating and is, therefore, dynamic. It is a dynamic model not only in the Hicks' sense, as are most so-called "dynamic programming problems", but also in the Frish-Samuelson sense (11, pp. vii-viii).

Derivation of Supply Function

Recursive programming can be used to derive positive

[a]A variety of methods can be used to estimate the flexibility coefficients. Some of them will be discussed in Chapter III.

supply functions. For a given year, a number of "optimum" solutions of a recursive problem are estimated over a range of prices using parametric programming. The solutions indicate quantities of output for different price levels and, thereby, provide a stepped supply function for that particular year. Using the same procedure, another function can be estimated from recursive problem of the next year. Thus, a series of supply functions can be estimated; one relating to each time period (Figure 4).

The reasoning behind obtaining a separate supply function for each year is that in recursive models the levels of "optimum" output for a given year are determined by the flexibility restraints which are estimated using the preceding year level of crop production. The supply curves are, thus, dependent upon the past production; they have a dynamic context.

Regression Analysis of Time-Series Data and Supply Response

Estimation of Aggregate Supply Function

Regression analysis of time-series data has been a major method for estimating a supply response because of its simplicity and because of the relative ease with which its highly aggregate data requirements are met. In this approach, the aggregate supply function is estimated directly by regression techniques, rather than via production function estimation. Exogeneous variables affecting supply of a

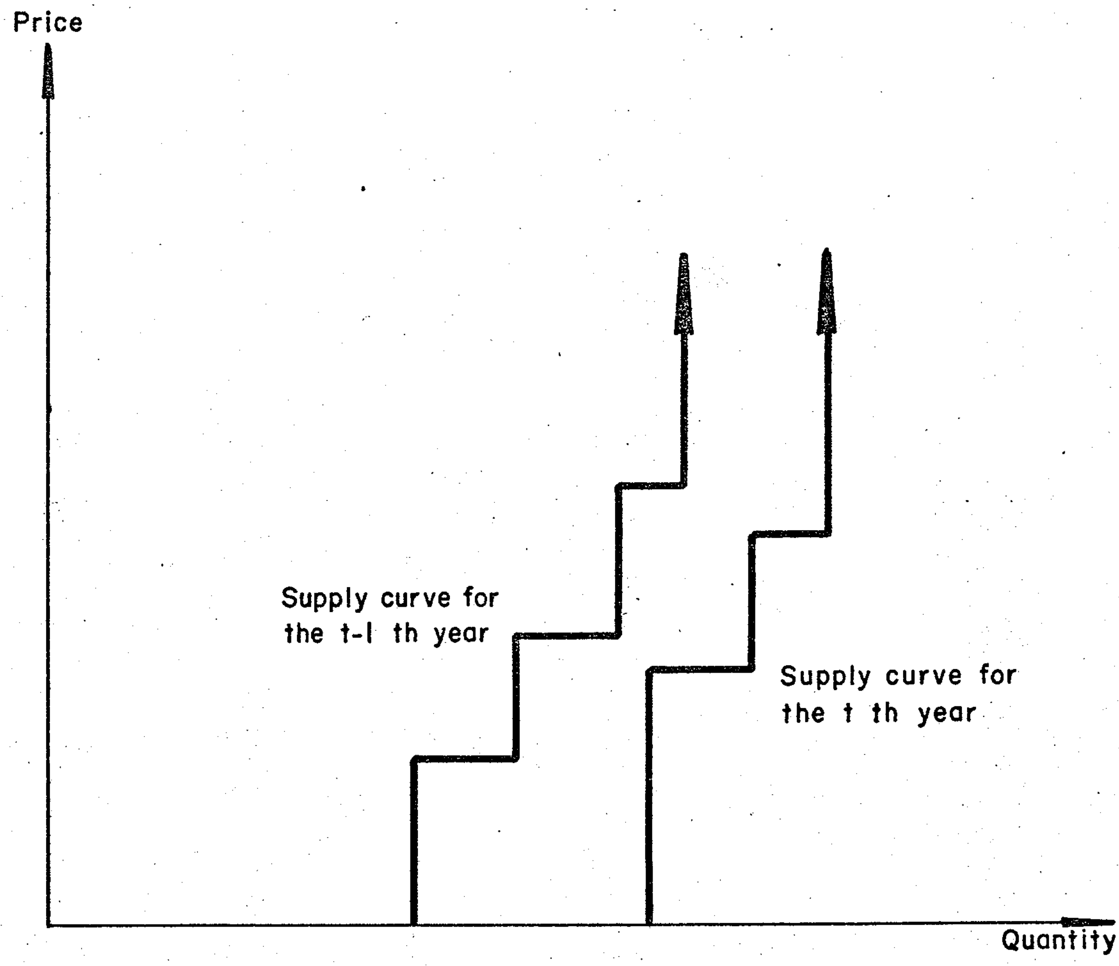

Figure 4. Stepped supply curves from recursive programming analysis.

product are singled out using economic logic and knowledge of agronomic practices. The form of the function is then specified, and assumptions about the error term and about the independent variables are made (37, p. 107). Finally, the supply response is estimated by fitting the function to the aggregate time-series data. The aggregate supply function is thus obtained directly (i.e., without estimating the underlying production functions).

Introduction of Dynamic Forces in Regression Models

Several methods for introducing dynamic forces into supply analysis have been proposed. Some of them are discussed below. Problems encountered in introducing these forces are also described briefly.

Expectation and Uncertainty

Uncertainty can be accounted for in supply analysis by replacing uncertain prices by their certainty equivalents. As Nerlove (53, p. 540) has stated, expectation models can be utilized to arrive at certainty equivalents. Two expectation models are presented below. One of them is utilized to demonstrate how an expectation model is integrated with supply response.

Extrapolative Model. This model was initially proposed by Goodwin (24, pp. 181-204). This is one of the most common price expectation models utilized in agricultural supply analysis. In this model, expected price of the t th year is the actual price in the t-1 th year plus or minus a fraction of the change in price from year t-2 to t-1:

$$P^*_t = P_{t-1} + \alpha(P_{t-1} - P_{t-2}) \tag{23}$$

where:

P^*_t = expected price in the t th year,

P_{t-1} = actual price in the t-1 th year, and

α = proportion of change in price from the t-2 th to t-1 th year, which influences the t th year price.

This model is likely to be conceptually unsound because it uses only two observations of a price variable and neglects other information which is normally utilized by firms in forming price expectations. The empirical performance of this model has also not been satisfactory. In a number of studies based on this model, projected agricultural production has been substantially different from the observed values (49, p. 46).

Adaptive Model. This model was developed by Cagan (2, pp. 25-117). It has greater intuitive appeal than the extrapolative one. As expressed below, the model regards the current year's expected price as last year's expected price plus some proportion, β, of the difference between last year's actual and last year's expected prices:

$$P^*_t = P^*_{t-1} + \beta(P_{t-1} - P^*_{t-1}) \tag{24}$$

$$0 < \beta \leq 1$$

where:

β = coefficient of expectations,

and other variables are defined as before.

If last year's expected price can be considered to be a similar function of the same variables for the preceding year, the model makes current expected price a function of the average of all past prices, the most recent prices receiving the largest weight according to the size of β.

When $\beta = 1$, the adaptive model becomes the naive model:

$$P^*_t = P_{t-1} \tag{25}$$

which indicates that the expected price for the current year is equal to the preceding year price.

Nerlove has developed a distributed lag model of supply response based on adaptive expectations. He hypothesized that output in the t th year is a linear function of the same year's expected price. This hypothesis can be expressed through the following equation:

$$Y_t = a_o + a_1 P^*_t + U_t \tag{26}$$

where:

Y_t = level of output in the t th year,

P^*_t = expected price for the t th year, and

U_t = random error in the t th year.

Using the adaptive model for the expected price (P^*_t), equation (26) can be solved in terms of observable variables as follows: (a) substitute equation (24) into

(26) in order to express Y_t as a function of P_{t-1} and P^*_{t-1}; (b) to eliminate P^*_{t-1}, lag equation (26) by one period and derive P^*_{t-1} as a function of Y_{t-1}, and (c) then use the relationships obtained in (a) and (b) to express Y_t in terms of observable variables. The following equation is thus derived:

$$Y_t = a_o\beta + a_1\beta P_{t-1} + (1-\beta)Y_{t-1} + U_t - (1-\beta)U_{t-1} \qquad (27)$$

This distributed lag model has some advantage over the traditional static models. For instance, in a survey of major agricultural supply response studies, Nerlove (51, p. 301) found that a greater proportion of year-to-year variations in production was explained by the distributed lag models than by static models. The coefficients estimated through the former models were also more reasonable in terms of sign and magnitude. Also, the calculated residuals indicated a lesser degree of serial correlation.

Flexibility of "Fixed" Factors

Nerlove (49, pp. 36-42) found that the year-to-year variations in the so-called fixed factors produce a lagged adjustment in supply. He proposed an output adjustment equation and combined it with a static supply function to develop a distributed lag model. As expressed below, the adjustment equation indicates that the output in the t th year is equal to the preceding year output plus some proportion of the difference between the planned and actual output:

$$Y_t = Y_{t-1} + \gamma(\overline{Y}_t - Y_{t-1}) \tag{28}$$

$$0 < \gamma \leq 1$$

where:

$\overline{Y}_t$ = planned long-run equilibrium output for the t th year,

γ = coefficient of adjustment, and

other variables are defined as above.

Using the above adjustment equation, a distributed lag model can be developed from the following supply function:

$$\overline{Y}_t = \alpha P_{t-1} + e_t \tag{29}$$

By substituting equation (28) into (29) and rearranging terms, the following distributed lag model can be obtained:

$$Y_t = \alpha\gamma P_{t-1} + (1-\gamma)Y_{t-1} + \gamma e_t \tag{30}$$

This is a dynamic model of supply response which accounts for the flexibility of the so-called fixed factors of production.

Cassels believed that allowing for flexibility in the factors not only produces lagged adjustment in supply response, but generates asymetry in the response to increase and decrease in output prices (Figure 5). He stated:

> Capital once fixed in a specialized form cannot quickly be withdrawn, and entrepreneurs committed to a particular line of production will commonly continue to produce even when the price they receive does little more than cover the direct costs of operation. If producers have alternative

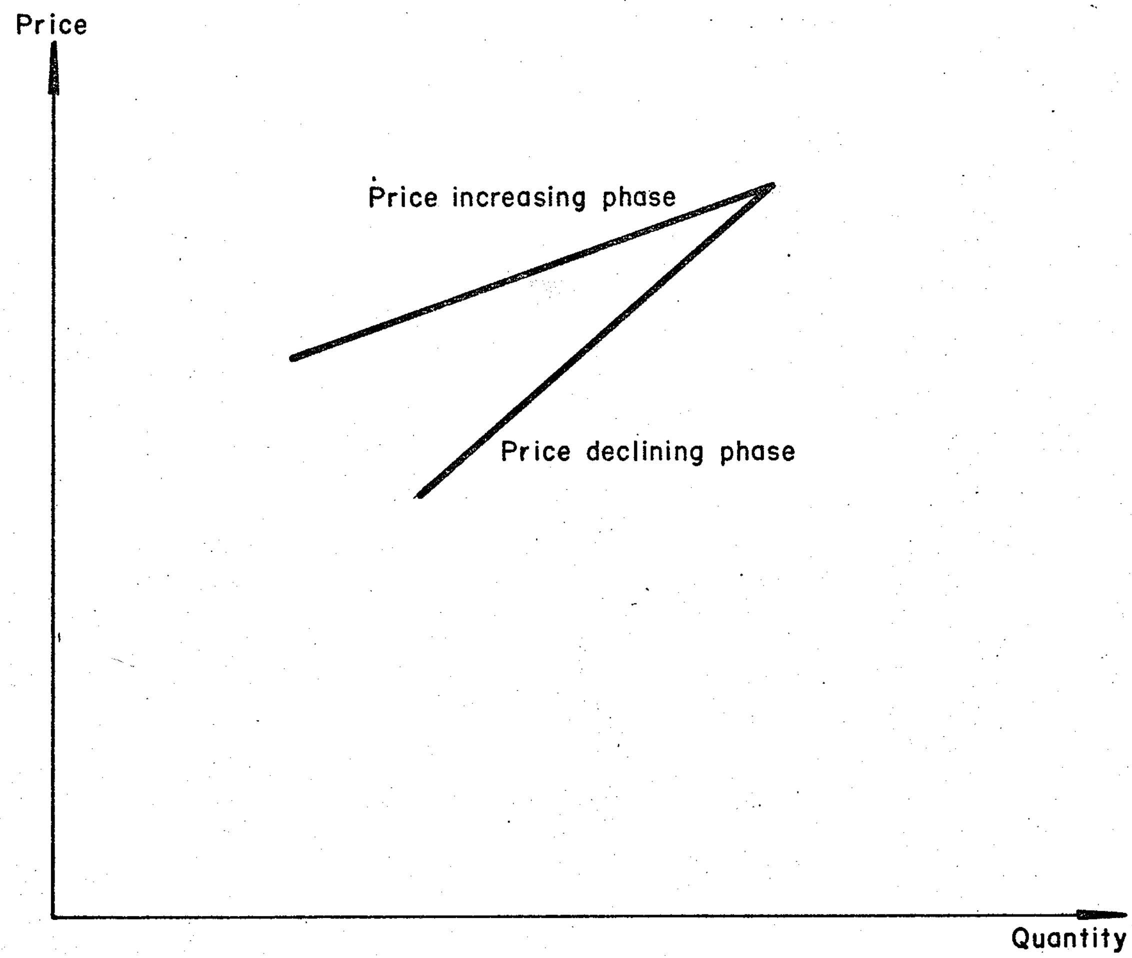

Figure 5. Asymetric supply curves for increases and decreases in output prices.

> products to which they can turn, . . . , the supply will be more sensitive to price declines but even in these circumstances there is no reason to suppose that the process of contraction will be an exact reversal of the process of expansion. It seems to me, therefore, that each supply curve must be regarded as relating to an established level of output and should be recognized to have two distinct parts, one representing expansion beyond that output and the other representing contraction below it (5, p. 384).

In other words, he suggested that the supply function should be considered irreversible. It implies that supply elasticity is not likely to be equal in the phases of price increases and decreases. In order to examine this irreversibility, one would have to stratify time-series observations by the direction of price changes, thus separating observations by increases and decreases in prices. In analyzing agricultural supply response, Tweeten and Quance (67, pp. 342-52) estimated supply elasticities separately for the rising and falling phases of price changes. They found significant differences between the elasticities of these two phases.

Technological Change

Technological change is another major dynamic force that should be incorporated in supply analysis. Usually, time is introduced into regression models as a proxy for technological change. However, it appears that this is not a truly representative proxy, because the time trend assumes that the technological change occurs at a uniform monotonically increasing or decreasing manner. And it is unlikely that any technological change does in fact take place in this manner.

Schrnooker (61, pp. 214-32) and Leontief (46, pp. 27-35) suggested that changes in total input per unit of output could be taken as a proxy for technological change. This suggestion is based on the reasoning that if input and output are corrected for price changes, a ratio of year-to-year changes in the input index and changes in the output index measures the contribution of technological change. However, this procedure measures an exact contribution only if the following restrictive conditions are met: (1) the industry must be operating under equilibrium conditions, (2) the prices of the factors of production relative to each other, and the prices of the products of the industry relative to each other must remain unchanged, and (3) technological progress must be neutral.[a]

Aggregation Problems in Supply Analysis

Aggregation problems lead to a major difficulty in supply analysis. The appropriate level of analysis has to be selected from a spectrum of alternatives ranging from the most disaggregate to the most aggregate. On one extreme, individual firms can be analyzed to obtain an

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[a]Neutral technological change, as defined by J. R. Hicks (32), shifts the production function for all factors upward but leaves the marginal rates of substitution unchanged.

aggregate supply function. But, such an approach is of academic interest only because the number of firms in some industries is so large that it is not economically feasible to collect data for each firm. Even if data are obtained, enough resources may not be available to estimate supply functions for individual firms.

At another end of the micro-macro spectrum, the unit may be as aggregate as a country. In this case, problems of data collection and analysis are greatly simplified. But the estimates will produce aggregation bias[a] if certain technical conditions are not met. Richard H. Day described some of these conditions as:

> . . . under suitable conditions a single linear programming model for the aggregate is equivalent to a direct aggregation of the solutions of a set of individual firm models. Conditions sufficient for this equivalence are proportional variations of resources and behavioral "bounds", proportional variation of net return expectations among all firms in the aggregate; and, finally, common technical coefficients which appear in the constraints on the firm's decision (12, p. 797).

However, Day (12, p. 812) believed that in a large number of industries, variations among firms are such that the foregoing sufficient conditions for using a single programming model are difficult to obtain. Thus, for

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[a]Aggregation bias can be defined as the difference between (1) the aggregate supply function as developed by summing the linear programming solutions of each individual firm in the industry, and (2) the function estimated using an aggregate model.

reliable results the aggregate should be stratified into homogeneous groups.

Several criteria have been proposed to stratify farm firms. The selection of any specific one depends to a great extent upon the objectives of the study and on the type of model employed. It may be desirable to stratify farm firms on the basis of size of a limiting factor (e.g., land or capital), resource mix or resource ratios (23, pp. 696-700; 45, pp. 681-695). But, in most supply response studies, data voids normally prevent stratification using any one of these criteria. Usually stratifying the industry (i.e., aggregate) by regions is the only alternative available to researchers. Moreover, Day provided a theoretical justification for using this method of stratification. He stated:

> . . . imitation of prominent producers' decisions by surrounding firms may lead to a considerable degree of proportional variation in farm activities, more than the linear programming behavior based on wide technical dissimilarities would predict. The idea is that individual farmers in a given area tend to imitate "management leaders" or prominent producers, and, as a consequence, behave as if they were much more homogeneous in input-output and resource structure than they really are (13, p. 673).

Hence, it may be appropriate to aggregate farms at a given location into one model even though they are somewhat heterogeneous with respect to their resource levels. Regional disaggregation, therefore, appears to be a practical and valid criterion for stratification. With this thought in mind, this method has been utilized in the present study.

Evaluation of Methods for Estimating Supply Response

In this section, a point-by-point comparison between the four earlier discussed methods for estimating supply response has been undertaken. The purpose of this comparison is to select one method which seems to be the most appropriate for the present study. The methods are evaluated in the light of their capability to deal with the following problems: reference to the technical structure of production, multi-product farms, large number of production inputs, uncertainty, investments in "fixed" factors, technological change, structural change, and goals other than short-run profit maximization.

Comparison of Methods

1. In the production function approach, and linear and recursive programming models of supply response, the technical structure of production is first estimated. The supply function is then derived from it. But, in regression analysis of time-series data, the aggregate supply function is estimated without explicit reference to the technical structure of production.

2. In agriculture, production typically takes place on multi-product farms. If crops on these farms are independent in resource use (i.e., they do not use and compete for the same resource), production function estimation of multi-product farms is similar to that of single product farms. Even if many crops draw inputs from the same stock,

there is no problem in estimating the production function, provided the amount of inputs utilized by each crop are recorded correctly. However, this information is not normally recorded by crops. This is particularly true for durable assets. Therefore, production function for a crop cannot be accurately estimated on multi-product farms. As a result, the supply function derived from it is likely to be inaccurate. This is a major limitation of the production function approach.

In contrast, programming models can be effectively employed to estimate supply functions on multi-product farms. Any number of production activities can be introduced in the models. Providing data are available, the number is restricted only by computational capacity.

Regression analysis of time-series data, however, encounters two major problems in estimating supply response on multi-product farms. One is due to strong correlation (i.e., multicollinearity) among exogeneous explanatory variables such as prices of different crops. The second problem is the inadequate number of observations for reliable estimates of agricultural supply functions.

3. In agriculture, usually a large number of inputs are required for the production of any one crop. But each individual input cannot be used as variable in the production function analysis because of insufficient degrees of freedom. Therefore, farm inputs are aggregated into broad categories. But this aggregation may cause biased estimates

of production parameters. Also, intercorrelations between inputs are usually too high to produce reliable estimates of the parameters. These problems constitute further limitations of the production function approach.

However, large number of inputs do not pose any serious problem to linear and recursive programming models. Greatly disaggregated input categories can be used in the programming framework. For example, capacity of each distinct type of machine can be designated as a separate restraint. Also, different types of labor restraints can be incorporated.

4. In estimating supply response, the production function approach and ordinary linear programming technique employ the assumption that future prices are known with certainty. Therefore, these methods cannot be utilized to study the effects of price uncertainty on supply response.

Also, uncertainty can not explicitly be accounted for in recursive programming and regression analysis because prices utilized in these models are assumed to be known with certainty. However, by limiting the year-to-year changes in production patterns, the flexibility restraints of recursive models indirectly incorporate the effects of risk and uncertainty in supply functions. Also, using distributed lag models, regression analysis of time-series data can indirectly examine the impact of uncertainty on supply response.

5. Effects of investments in "fixed" factors such as machinery on supply response cannot be examined using

the production function approach. However, the capacity constraints can be utilized in recursive programming models to examine the effects of investments on supply response. These constraints account for any unwillingness and inability of farmers to invest in fixed factors. Also, utilizing the regression framework, the effects of investments can be incorporated into supply analysis through the use of distributed lag models.

6. Technological change is primarily an intertemporal phenomena. Therefore, under the production function approach, which uses cross-section data, the relationship between technological change and supply response cannot be accurately analyzed. Ordinary linear programming is also inappropriate for studying the relationship because this framework is employed to estimate only short-run and timeless supply functions (33, pp. 179-80).

With regression models, a simple time trend is sometimes introduced as a proxy for technological change. But this procedure is very crude. A sound model of production response should explicitly include the rates of adoption of technology and their effects on the production process. Because diffusion of technology is an investment process, the diffusion can be examined in recursive programming models through capacity constraints (14, pp. 117-19). In this approach, rates of adoption of new technologies and of abandonment of the old ones are determined within the system. However, the effects of technological uncertainty are not examined through this approach.

7. The effects of structural change on supply response can be studied within the linear and recursive programming frameworks. By changing net income coefficients and/or resource levels, the programming models are used to estimate the impact of new structures on production response. However, the regression approach cannot be used to predict supply response in light of new structures because equations are fitted to historical data.

8. In production function and linear programming approaches, short-run profit maximization is normally considered as the only goal of an entrepreneur, whereas, in reality firms have multiple goals. Therefore, these approaches cannot be used as predictive tools of supply analysis.

As the supply function is estimated using historical data, the regression model indirectly accounts for farmers' likes, dislikes, non-economic goals and other considerations. Also, in recursive programming framework, flexibility restraints indirectly account for the effects of non-economic consideration and norms other than short-run profit maximization.

Summary

On the basis of the above discussion, it appears that recursive programming has a great promise as a tool of supply analysis. However, there are two major limitations to this approach. First, when structural changes occur, the adequacy of the flexibility restraints to predict supply response is greatly reduced because they are usually estimated from historical time-series data. Secondly, the research resource required to estimate a supply function is considerably greater for the recursive model than for regression approach. However, on the whole, the former seems to be superior than the latter for studying production response in agriculture. With these points in view, recursive model is selected as the method of analysis for the present study.

CHAPTER III

A REVIEW OF PRODUCTION RESPONSE STUDIES USING RECURSIVE PROGRAMMING

Recursive programming has received limited application in production response studies. This chapter reviews some of the more significant studies with the primary focus on methodology.[a] The purpose of this review is to point out their limitations and to suggest appropriate modifications for the development of an improved supply response model. The works of four different authors are reviewed below in chronological order.

Henderson

James M. Henderson (30) developed and applied a recursive programming model to predict land utilization patterns of the United States. Acreages of a dozen major field crops were predicted for the 1955-56 crop year. Henderson hypothesized that farmers' decision process could be treated as a recursive programming problem. That is, farmers of a region, as a group, select a land utilization pattern which maximizes an objective function subject to some linear constraints. The objective function was the total expected net revenue generated by the activities of the model. The constraints comprised an overall land restriction and a number of

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[a]Recursive programming has not been applied in Canada to study farmers' production response. Two Canadian studies based on regression analysis of time-series data are reviewed in Appendix A.

flexibility restraints. The former constraint limited the amount of land that could be allocated to crops. The latter were upper and lower bounds on allowable year-to-year acreage changes of each crop in the solution from the preceding year. These restraints were intended to account for farmers' desire for diversity and reluctance to depart from an established land use pattern.

The Henderson recursive programming model may be expressed through the following mathematical notation:

$$\text{Maximize } \Pi_t = \sum_{j=1}^{m} (P_{jt}Y_{jt} - C_{jt})\, X_{jt} \qquad (1)$$

Subject to:

$$\sum_{j=1}^{m} X_{jt} \leq \overline{X}_t \qquad (2)$$

$$X_{jt} \leq (1+\overline{\beta}_j)\, X_{jt-1} \qquad (3)$$

$$X_{jt} \geq (1-\underline{\beta}_j)\, X_{jt-1} \qquad (4)$$

$$\text{and} \quad X_{jt} \geq 0 \qquad (5)$$

$$j = 1,\ldots,m$$

where:

Π_t = total expected net revenue to be maximized in the t th year,

X_{jt} = solution acreage of the j th crop in the t th year,

X_{jt-1} = acreage planted for the j th crop in the t-1 th year,

$\overline{X}_t$ = total improved land available for cultivation in the t th year,

C_{jt} = expected cost of production per acre of the j th crop in the t th year,

P_{jt} = expected price of the j th crop in the t th year,

Y_{jt} = expected yield for the j th crop in the t th year, and

$\overline{\beta}_j$, $\underline{\beta}_j$ = maximum allowable year-to-year proportionate increase and decrease, respectively, in the acreage of the j th crop.

In this study, the expected yield was taken as an average of the preceding five years (1949 to 1953). The expected prices for crops were equal to the announced support prices. And the current year cost of production was assumed at the previous year level. The preceding year acreages of all crops included in the model were summed to obtain the total improved land restraint ($\overline{X}$).

Method for Estimating Flexibility Coefficients

In order to estimate flexibility coefficients of a crop, the year-to-year proportionate changes in acreages were calculated from 1946 to 1954. The proportionate changes were then stratified by direction of change, thus separating increases and decreases in acreages.

Henderson observed an inverse relationship between the levels of year-to-year proportionate changes and the base year acreage of a crop as percentage of the total improved land. He hypothesized that this relationship arose from the fact that as acreage of a crop increases farmers become more reluctant to expand that acreage at the same rate. To account for this hypothesis, the proportionate changes of a crop which were first classified by the direction of change, were further classified into two or three groups on the basis of proportion of the total improved land devoted to the crop in the base year.

Therefore, the observations were stratified in total into four or six groups, depending upon the crop. The average proportion for each group was computed to obtain flexibility coefficient for that class. Thus, when a high proportion of the total improved land was devoted to a crop in the preceding year, Henderson estimated a lower value of the upper flexibility coefficient. On the other hand, when a small proportion of the total improved land was allocated to the crop, he computed a higher value of the coefficient. Hence, two upper flexibility coefficients were estimated in this study. Using similar reasoning, two lower coefficients were computed.

Results

Henderson divided the United States into 160 geographical regions on the basis of soil types, climate and methods of farming. The recursive model was applied separately to each region for predicting the land utilization patterns for the 1955-56 crop year. The regional estimates were then aggregated to obtain the national results. Results of this study as well as the estimates of the Crop Reporting Board of the United States Department of Agriculture were compared with the actual acreages of each crop. On the average, the results of this model were more precise than the Board estimates.

The relative accuracy of the recursive model verified Henderson's hypothesis that a profit-maximizing model with "flexibility" restraints on year-to-year changes in crop acreages could be used as a predictive device.

A less disaggregated analysis was also performed in the Henderson study. He stratified the United States into 55 regions, solved the recursive programming model for each region and summed the results to obtain national estimates. These estimates were compared against the results obtained through more disaggregated analysis (160 regions). It was found that the accuracy of the estimates increased with an increase in disaggregation.

Henderson discovered that the average error in the estimated acreages of all crops would be large, if only a few major crops of a region were included in the model. He contends that the usual practice of including only a few crops in the model and leaving out some of the enterprises of the region is not a valid procedure because such separation is based upon a false premise that the decisions to plant the included crops[a] are independent of the decisions

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[a]Crops for which acreage was predicted in his study were referred to as included crops.

to raise the excluded enterprises[a] (30, p. 247). Therefore, he suggested that all major crops of the region should be included in the analysis.

Day

Richard H. Day (11) developed a recursive programming model for explaining and predicting production of eight major crops in the Mississippi Delta. The study is presented in his monograph "Recursive Programming and Production Response". Day applied a detailed recursive programming model in which four technological "stages", three soil classes, and four fertilizer levels were included. The model was used to estimate annual acreages and production of eight major crops during 1940 to 1957.

The model used in this study is basically similar to that employed by Henderson. However, a single input-output matrix was not employed for the entire period of analysis. Rather, for each new year, a different matrix was used. This method was utilized to introduce in the model the productivity changes which are of continuous nature. However, to account for sporadic technological changes, another technique was employed. Production activities corresponding to new technology were introduced in the analysis at various points in time.

In contrast to Henderson's model, Day estimated only

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[a]Excluded enterprises are those which were operated by farmers but not analyzed in his study.

one upper flexibility coefficient for each crop regardless of the size of the base year acreage. Similarly, only one lower coefficient was computed.

The upper and lower flexibility restraints for the first year of the analysis were calculated by multiplying the preceding year actual acreage by the flexibility coefficients. For other years, the bounds were estimated by multiplying the coefficients with the preceding year solution acreage. As recognized by Day (11, pp. 106-07), the use of actual acreage rather than solution values for estimating flexibility restraints has some merits. For example, if actual data are used, errors in the estimates of restraints do not accumulate over time. But lack of annual acreage data for some crops precluded the application of this approach.

Methods for Estimating Flexibility Coefficients

In order to calculate flexibility coefficients, observations were stratified into two groups on the basis of positive or negative year-to-year changes in acreages of a crop. These two groups of data were then used to estimate upper and lower flexibility coefficients, employing any one of the three methods discussed below. Selection of the method for a crop depended upon the type of the data available for that enterprise.

Regression Model. Regression equations, treating current acreage as the dependent variable and lagged acreage as the independent variable were used to estimate flexibility coefficients. More specifically, the following equation was employed:

$$X_{it} = (1+\beta_i)\ X_{it-1} + e_{it} \qquad (6)$$

$$i = 1,\ldots,n$$

In order to estimate equation (6), it was transformed as:

$$X_{it} = \alpha_i X_{it-1} + e_{it} \qquad (7)$$

where:

α_i = $1+\beta_i$,

e_{it} = random error for the i th crop in the t th year, and

other variables are defined as before.

Method of Select Point. This technique was applied when acreage data of a crop was available for only two years, for example, census years. The following equation was solved to estimate flexibility coefficients for given values of X(t) and X(o):

$$X_i(t) = (1+\beta_i)^t\ X_i(o) \qquad (8)$$

where:

$X_i(t)$ = acreage of the i th crop in the t th year,

$X_i(o)$ = acreage of the i th crop in the initial year,

with β_i defined as before.

Method of Average Rates. In this method, year-to-year proportionate changes in acreages of a crop were classified into two groups on the basis of direction of change, and then the average was computed for each group to estimate flexibility coefficients. The average of positive changes was defined as the upper flexibility coefficient, and the average of negative changes gave the lower coefficient.

Results

Day applied his model to explain production, acreage and yield of eight major crops during 1940 through 1954. After establishing the explanatory power of the model, he used it for prediction purposes during 1955 to 1959. The model results were compared with estimates of the Crop Reporting Board of the USDA. Day's predictions turned out to be reasonably close to the Board's estimates for the changes in acreage patterns, but the difference was considerable for changes in yield patterns.

Incorrect estimation of the turning points of changes in crop acreages was a problem in Day's model. He considered that this error was caused by the inaccuracy in per acre expected net returns which was brought about by the use of inappropriate price and yield expectation models. Aggregation bias was also considered as a source of errors in the model's estimates.

Schaller and Dean

W. N. Schaller and G. W. Dean (60) used a recursive programming model to study the year-to-year changes in the production of twelve crops in Fresno county of California

State. In addition, they also carried out methodological research by examining the characteristics of a recursive model and evaluating it empirically in relation to regression analysis. Three different tests (viz. explanatory, predictive and projection) were undertaken to compare the efficiency of recursive programming and regression analysis for explaining and predicting production as well as acreages of crops during the period 1951 to 1965.

In this study, two levels of analysis for estimating aggregate response were considered: (1) establish representative farms as units of analysis and then aggregate the results on the basis of numbers in each category, and (2) estimate directly from aggregated farm data. Although the second method was likely to produce aggregation bias, it was selected for the study. However, to reduce the bias, the Fresno county region was stratified into two subregions. This step was a compromise between analysis through representative farms and through an aggregate approach.

The method used to estimate flexibility constraints in this study was different from that employed by Day. The constraints were computed by multiplying the preceding year <u>actual</u> acreages by the flexibility coefficients, whereas, in Day's model, the preceding year <u>solution</u> acreages were used. However, Day's method was used in the projection phase of this analysis (i.e., during 1962 to 1965) because actual acreages were not available during this period.

Methods for Estimating Flexibility Coefficients

In order to estimate flexibility coefficients, Schaller and Dean undertook regression analysis of time-series data. The observations were stratified into two groups on the basis of positive or negative year-to-year changes in crop acreages. The following equation was then fitted separately to both data sets for estimating upper and lower flexibility coefficients:

$$\frac{X_{it}}{\sum_{i=1}^{n} X_{it}} = (1+\beta_i) \frac{X_{it-1}}{\sum_{i=1}^{n} X_{it-1}} + e_{it} \qquad (9)$$

$$i = 1,\ldots,n$$

For the purpose of estimating equation (9), it was transformed as:

$$X^*_{it} = \alpha_i X^*_{it-1} + e_{it} \qquad (10)$$

where:

$X^*_{it} = X_{it} / \sum_{i=1}^{n} X_{it}$,

$X^*_{it-1} = X_{it-1} / \sum_{i=1}^{n} X_{it-1}$, and

other variables are defined as before.

In this equation, acreages were converted into percentages to account for year-to-year changes in the total land base.

Results

In the explanatory test, results of both regression and recursive programming analyses were compared in relation to actual acreages of individual crops during 1951 to 1958. The recursive model usually overestimated relatively small changes in crop acreages. On the average, acreages and production were explained more accurately by the regression model than by the recursive programming. However, Schaller and Dean (60, p. 40) observed that the recursive model was more effective in estimating acreages in years of sharp structural changes.

On the basis of the explanatory test, the complementary and supplementary role of programming and regression analyses were quite evident for improving the performance of both techniques in estimating crop acreages. For example, a refined regression analysis could be used to estimate components of a recursive model such as the flexibility coefficients. Similarly, in periods of structural changes, recursive programming could be used to adjust independent variables of regression models.

A predictive test of the recursive model was also undertaken. Predicted acreages of recursive programming and regression analysis were compared in relation to actual data for each year during 1959 to 1961. The regression approach was still superior than the recursive programming, but the latter had improved its relative performance.

Schaller and Dean found that the relatively greater error in the recursive model's estimates was a result of: (1) excessively wide flexibility bounds, and (2) use of a very limited number of resource restraints. Therefore, the authors recommended that the method for estimating flexibility coefficients should be improved and more restraints should be used in the model to more adequately reflect the production environment.

The land utilization patterns were also projected in this study for each year during 1962 to 1965. Based upon this projection, Schaller and Dean observed that the recursive model: (1) gave more stable results than the regression analysis, and (2) was less likely to provide extreme values.

In this study, one of the sources of errors in the acreage estimates was likely to arise from application of an aggregate model which had not accounted for most of the inter-farm variations in yields and costs. Therefore, Schaller and Dean (60, p. 25) suggested that a breakdown of the region into farm type groups could result in more accurate estimates.

Sharples and Schaller

J. A. Sharples and W. N. Schaller (63) estimated the short-run impact of alternative government programs on land utilization patterns of the United States in 1968. They developed a national model consisting of ninety profit-maximizing, linear programming sub-models.

In order to create homogeneous areas where all farmers could be assumed to respond in the same way to given economic stimuli, the United States was divided into seven regions which were further stratified into ninety resource situations on the basis of soil conditions, production alternatives, and resource combinations. Separate programming models were then developed for each individual area. A number of activities were included in each model to embrace the major techniques of producing a crop. Enterprises unique to each area were introduced in the regional models. Due to the problems of quantification and aggregation, capital constraint was not introduced in any model and labor was included in only a few. In order to estimate flexibility coefficients, Sharples and Schaller used a variety of methods. But none was found suitable for all regions.

The explanatory power of the model was tested by comparing the estimated acreages with actual data during 1960 to 1964. A predictive test was undertaken for the North Central region for the year 1968. The errors in the predicted acreages were reasonably small for some crops, but substantially large for other crops, for example, oats. The Treasury cost to the government for the alternative support policies were also estimated in this study.

Major Limitations of the Studies and Measures for Improvements

The review of the above studies shows that recursive programming has not been very successful as an empirical model of supply response. It appears that the poor performance of this model was a result of some major limitations of the studies. Some of the limitations and needed improvements are discussed below.

Addition of Resource Restraints to the Model

In most of the earlier studies, only one physical restraint (i.e., total improved land) was used. However, theoretically, all possible restraints affecting farmers' decisions should be incorporated in a recursive programming model. These restraints could be different types of land and labor, and various kinds of machinery and fertilizers. The requirements of crops for these resources are different. Therefore, inclusion of such restraints would likely increase explanatory and predictive powers of the model.

Usually, two major problems are encountered in introducing physical resource restrictions. First, there is a lack of data on these restraints. The second is the overestimation of available amounts of these resources due to aggregation bias. For example, many fixed or quasi-fixed factors of production, such as tractors, combines, etc., are owned by specific farm units. Even though these are not used to their capacities on those farms, they may not be available to others. Thus, these resources are likely to be overestimated in the aggregate. This argument casts some doubt on the validity of using the total physical resources as restrictions in the programming model. Therefore, attempts should be made to estimate the unused capacities of these resources and to adjust the constraints. The adjusted constraints could then be incorporated in recursive models.

Estimation of Flexibility Coefficients

In most of the studies examined, simple techniques were used to estimate flexibility coefficients. Among them, the application of a simple regression model was the most common. Upper and lower flexibility coefficients were estimated by fitting the following equation separately to acreage data stratified into two groups on the basis of positive or negative year-to-year changes:

$$X_t = (1+\beta)X_{t-1} + e_t \tag{11}$$

The estimation of this equation provides a pair of flexibility coefficients ($\overline{\beta}$, $\underline{\beta}$) which are used in the analysis of each year regardless of the levels of economic and non-economic variables in that year. Thus the coefficients so estimated were immune to year-to-year changes in these variables. This is likely to be an unreasonable assumption and reduces the reliability of results.

Bawden (1a, pp. 1549-51), Doll (18, p. 126) and King (40, pp. 1536-38) are among many who have recognized that the crux of recursive models lies in the estimation of the flexibility coefficients and, therefore, they have recommended improvements in this direction. Some of the methods for attaining improvements are described below.

(1) A linear equation estimates a constant percentage change, for example 120 percent, in year-to-year acreages of a crop. This constant change implies that the

larger the base year acreage, the greater is the potential for absolute expansion. But, in reality, this situation is not likely to happen due to resource restrictions and uncertainty attached to specialization. In contrast, as the base year acreage of a crop increases, farmers become more reluctant to expand the acreage at the same rate. Thus, instead of a linear equation, a non-linear function appears to be a better construct for approximating farmers' behavior.

(2) Schaller and Dean (60, p. 68) found that the bounds in some years allowed too much flexibility in the solution and, thereby, resulted in an overestimation of the crop acreages. In other years, bounds were too narrow. Thus, it appears that the flexibility coefficients should be more adaptable to the conditions of each new year. Schaller and Dean have recommended that the bounds should be estimated using more information than just the preceding year's acreages. Day (11, pp. 87-88) has also suggested that flexibility coefficients should be related to such variables as (i) the elasticities of demand, and (ii) variations in the yields of crops.

In order to carry out the above suggestions, perhaps a multiple regression equation needs to be employed for estimating flexibility coefficients. Both economic as well as non-economic variables can be used as independent variables. Lagged acreage may be included in the equation in a non-linear form.

Aggregation Problems and Units of Analysis

In the previous studies, regional data representing several thousand farms have been used to develop recursive programming models. The models assumed that all farmers in a region respond equally to economic stimuli. But, in reality, some farmers might respond more and others less. Therefore, the models' estimates might differ from the actual response. This discrepancy could be attributed to the application of aggregate models.

The most accurate method of supply response is to derive the aggregate estimates using individual farms as units of analysis. However, it is usually not economically feasible. Therefore, the practical method is to stratify farms into homogeneous groups, estimate supply function for each class, and then sum them to obtain the aggregate results.

Normally, data are not available to stratify farms on the basis of farm size and/or resource combination. Usually regional stratification is the only expedient. Therefore, in order to obtain accurate estimates of aggregate supply response, relatively small and homogeneous regions should be used as units of analysis.

CHAPTER IV

ANALYTICAL FRAMEWORK

The analytical framework of a study is normally developed in the light of the objectives of the inquiry. In this chapter, a recursive programming model is constructed to explain the historical crop acreages, and to predict the 1971 land utilization pattern of each prairie province. The chapter is divided into two sections. The first presents a formulation of a recursive programming model and discusses, in detail, a framework developed for estimating flexibility coefficients. The second and final section describes three levels of aggregation (i.e., prairie, provincial and crop district) used in the analysis.

Formulation of a Recursive Programming Model

A recursive programming model has four basic components: objective function, activities, input-output matrix and constraints. This section describes these components only in a general way. Specific details such as describing the activities and constraints, and input-output coefficients are presented in the appendix of this manuscript.

A set of assumptions is normally required for developing a model. The following are major assumptions utilized in this study:

(1) perfect competition exists in factor and product markets;

(2) farmers aim to maximize their total net farm income;

(3) the current year land utilization pattern is not determined by the current year output prices, but depends upon the

preceding year prices;

(4) there is no interregional dependence in the land utilization decisions; and

(5) farmers do not adjust instantaneously to changes in economic and non-economic conditions. Rather, their adjustments are distributed over time.

The above five assumptions are fairly realistic in describing the behavior of prairie farmers. Samuelson (57, p. 60), Bach (1, pp. 444-45) and Day (16, p. 137) stated that agriculture is one of the very few industrial and commercial sectors that resemble the market structure assumed by theory of perfect competition. For major farm commodities such as those included in this study, large numbers of producers grow and market substantially homogeneous products. None has the power to influence appreciably the product prices. Input prices are also not affected to any significant degree by the action of any one producer. Therefore, assumption of perfect competition appears to be a realistic one in this study.

Regarding assumption of profit maximization, Bach (1, p. 439) stated that although it is unreasonable to assume that every firm is striving exclusively to maximize profits, there is impressive evidence that the desire for profits is a dominant motive in most businesses. Even though firms may not be very conscious profit maximizers as individuals, a competitive market will force them to become so. For example, if producers do not respond to price changes, they will be driven to bankruptcy before long. In order to verify this assumption for farmers, Richard Day (16, p. 135) made detailed discussions with farmers of the Mississippi Delta and Iowa State. He found that they make their production plans in order to maximize incomes with due regard to uncertain prices and yields. The profit maximization can therefore be considered as a valid approximation of the goal of prairie farmers.

The third assumption (i.e., the current year land utilization pattern depends upon the preceding year price) also appears to be realistic enough to describe the behavior of prairie farmers. Since at the time of making land allocation decisions, farmers do not know the price at which the crop will be sold, they commonly use the preceding year's price as a basis to allocate acreages among crops. This assumption is commonly made in economics textbooks for describing cobweb models (31, p. 117).

The assumption on regional independence in the land utilization decisions is required to simplify analysis. Using this assumption, acreage response can be examined separately for each region. However, in its absence, all regions should be analyzed simultaneously. This assumption is also realistic. It is a corollary of the previous assumption which states that the current year land use depends upon the preceding year's prices. Since these prices are predetermined variables for the current year land allocation decisions, regions become independent with respect to product prices. In other words, a simultaneous increase in production of a crop in all regions of the prairies cannot influence the price utilized in that year's analysis, rather it will affect future prices. Thus with respect to product prices, regions are independent in their production decisions. Similar arguments can be provided to justify regional independence with respect to input prices, exports etc.

A number of justifications can be provided for the validity of the last assumption. In response to changes in economic and non-economic variables, farmers do not fully adjust their crop acreages in one year's time. Rather, they change their land use only partly because of uncertainty, lack of knowledge and quasi-fixed factors. Another explanation for distributed lag is that, in response to changes in economic variables, farmers adjust their land utilization patterns with different rates. In one year's time, a few make complete adjustment, others do not adjust at all and some change only partly. Therefore, the total response for the area is distributed over time.

Objective Function

The type of objective function to be maximized is usually determined by the behavior of the producing units, objectives of the study and underlying assumptions. In a recursive programming model, it is assumed that farmers determine economic plans by a sequence of optimizing decisions. Therefore, the objective function maximizes the total expected net income from production activities of only one year at a time.

In the present study, a number of producing and selling activities were included in the recursive model. Alternative methods were allowed to produce each crop and to sell each product. The crop activities were employed only to produce and not to sell the output. The product was disposed of through selling activities. Therefore, variable costs per acre were used as C_j values of the crop activities and the output prices were utilized as the C_j's for selling activities. The objective function is expressed below in mathematical notation. The components of this function will be discussed under a separate subsection, namely "Activities" which immediately follows:

$$\text{Maximize } Z_t = \underbrace{\sum_{i=1}^{I} \sum_{k=1}^{K} P_{ikt}\, Q_{ikt}}_{\text{(Gross Revenue)}} - \underbrace{\sum_{i=1}^{I+1} \sum_{j=1}^{J} C_{ijt}\, X_{ijt}}_{\text{(Total Cost)}} \qquad (1)$$

where:

Z_t = total net revenue to be maximized in the t th year,

Q_{ikt} = quantity of the i th crop sold by the k th method in the t th year,

X_{ijt} = solution acreage of the i th crop produced by the j th method in the t th year,

P_{ikt} = expected price of the i th crop sold by the k th method in the t th year,

C_{ijt} = expected cost of production per acre of the i th crop produced by the j th method in the t th year,

I = total number of grain and oilseed crops, 1 is added to I on cost side in order to recognize a summerfallow activity which adds to total cost, but not to gross revenue,

K = total number of selling activities for each product, and

J = total number of producing activities for each crop.

Activities

The six major crops of the prairies were included in the recursive model. These crops are wheat, oats, barley, rye, flaxseed and rapeseed. Rapeseed was a nominal crop in the 1950's, but its acreage has substantially increased in the 1960's. Therefore, inclusion of rapeseed in the model was a necessity for the purpose of making a precise forecast of acreages of other crops. Summerfallow was also included in the model as an activity.

The total area of these six crops and summerfallow comprises a major proportion of the total improved land of the prairies--84 percent during 1958 to 1967. Tame hay, pasture and a few speciality crops such as mustard and sugar-beets were omitted from this study.

A considerable difference was found between yields of a crop sown on summerfallow and on stubble land (19). Production costs also differ for summerfallow and stubble seeded crops (7, pp. 161-63). Therefore, two producing activities were employed for each crop: seeding on summerfallow and on stubble land. These activities are listed in detail in Appendix Table 11.

In order to bring more realism in the analysis, the quota system of the Canadian Wheat Board was incorporated into the model. Four selling activities were used for each of wheat, oats, barley and rye. The activities are: (1) selling through unit quota; (2) specified acreage quota sales; (3) selling through supplementary quotas; and (4) non-quota sales and/or discounted values of production not sold. Sales of flaxseed and rapeseed have been restricted in the beginning of each crop year, but have usually been declared open by the middle of the year. Therefore, only one selling activity was employed for each of these two crops. All selling activities are listed in Appendix Table 11.

Most prairie farmers make land utilization decisions for the current crop year towards the end of the preceding crop year. Therefore, the expected prices of cereal and oilseed crops for the t th period are anticipated in the t-1 th period. A simple price expectation model was utilized in this study. The preceding year prices of cereal crops were used as C_j values for all selling activities except the fourth type (i.e., non-quota sales and/or discounted values of excess production). A price discounted by twenty percent was utilized for this activity.[a] For rapeseed, a one year lagged price was used as its C_j value. In the case of flaxseed, the preceding year price was first tried, but later on it was discovered that the crop was considerably overestimated in most years. Therefore, in order to improve the

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[a]The twenty percent discount rate was chosen because prices for non-quota grain sales as proportions to quota sales in the prairies seem to be about eighty percent for wheat, oats, barley and rye. In order to verify this proportion, the monthly percentages during August 1962 to January 1965, given in a study by Kerr (39, p. 61), were utilized. The averages of these percentages were computed by crops. It was found that prices for non-quota grain sales as proportions to quota sales were 77.80 percent for wheat, 74.23 percent for oats and 76.73 percent for barley. Eighty percent was, therefore, selected as an approximate figure.

model estimates, a price discounted by twenty percent was used for this crop. The idea of discounting was prompted by the writing of Theil (65, pp. 33-36) which describes that if the model consistently underestimates or overestimates, it is possible to improve the results by either inflating or deflating the estimates by some constant.

Cost of production figures of each crop were required in this study for each year from 1958 through 1971. A number of studies have been carried out to estimate production costs of different crops in the prairies. However, in most of the studies significant differences often exist in the procedures and assumptions used to estimate cost items. As a result, these cost figures have been considered inappropriate for the present study.

In a research report entitled "Interregional Competition in Canadian Cereal Production" by W. J. Craddock (7, pp. 161-63), cost of producing wheat, oats, barley and rye separately on summerfallow and on stubble land are available only for the year 1966. Since yearly cost data for 1958 to 1971 were not available, and to estimate these values would have been a major research undertaking, Craddock's 1966 figures were used for each of the fourteen years of the analysis. Obviously costs have changed significantly during this period. However, the cost of producing one crop relative to another has probably not changed too much. In this study, it is not the absolute level of cost which is important, but the relative cost between crops. Further justifications for this procedure are given in Appendix B.

Cost of producing flaxseed and rapeseed were not estimated in the publication from which the cost estimates for other crops were obtained. Comparable estimates were not available from other studies. Therefore, cost of producing oats was used for flaxseed. The rationale for selecting oats is that production cost for flaxseed seems to be higher than for most grain crops because of its requirements for greater tillage operations. Therefore, the costs of production of a high-cost crop are likely to appropriate figures for this crop. Since, in the Craddock study, oats had the highest cost of production, its cost figures were utilized for flaxseed.

For rapeseed, the cost of production of wheat was utilized because cost figures for both crops were found to be somewhat equal. This observation was based on cost figures in other studies. For example, Porter and McBain (54, pp. 16-19) found that during 1961 to 1963, the average costs of production of wheat and rapeseed per acre were \$11.64 and \$10.30 respectively, in the Peace River area and \$14.27 and \$12.43, respectively, in central Alberta.

Input-Output Matrix

A recursive programming model requires an input-output matrix for each year of the analysis. The matrix for a single year is determined by the activities and constraints included in the model. However, whether the same matrix should be used for each period of analysis or different matrices should be utilized depends upon the objectives of the study, and intertemporal changes occurring in production and marketing conditions of the study area. A distinct input-output matrix was utilized in this model for each year of analysis.

Yield per acre was the first coefficient to vary between years in the recursive model. In this study, trend yields were used as expected values. This expectation model was selected because it was assumed that farmers' decisions are not governed by random fluctuation in yields, especially due to weather variations. But the decisions are based on long-run yields which are perhaps equal to trend yields.

The use of trend yield in the model can be justified on other grounds as well. During the last two decades or so, changes in cultivation techniques, fertilizer use and seed varieties have created an upward trend of a few bushels in yields in the prairies. Therefore, use of the trend yields in the model can be considered as a method to introduce that part of technological change which is continuous.

Trend yields for only rapeseed were estimated in this study and for other crops these were taken from Craddock's study.[a] The following equation was estimated to obtain trend yields for rapeseed:

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[a]Trend yields of wheat, oats, barley, rye and flaxseed were estimated for the period 1939 to 1965 by Craddock but were not published in his monograph (7).

$$Y = a + bT + U \tag{2}$$

where:

Y = yield per acre,

T = time trend; 1956 = 1, and

U = random error.

The above equation was fitted to actual yield data for the period 1956 to 1965 only, because data for the earlier years were not available.

As described earlier, two producing activities were used for each group, viz. seeding on summerfallow and on stubble. Therefore, two separate trend yield series were required. However, published data were available only since 1963 on the basis of summerfallow and stubble crop yields, and a time period of this length was considered insufficient to estimate reliable trend yields. Therefore, a method presented in Appendix C was utilized to compute separate trend yields for summerfallow and stubble crops.

Quota levels have varied over time. In this study, in order to estimate future quota levels, a simple expectation model was employed. The preceding year quotas were taken as the expected levels for the current year.

Restraints

A variety of restraints are required in the formulation of programming models. Some of the restraints are determined by the actual restrictions on production and marketing such as limited amount of resources and regulation of sales quota. Another group of constraints depend upon the type of programming technique employed, for example, in recursive programming, flexibility restraints are used. The following types of restraints were utilized in this study:

(1) physical resource restraints;

(2) flexibility restraints;

(3) absolute minimum and maximum acreage restraints;

(4) sales quota restraints; and

(5) supply restraints to relate producing and selling activities.

Physical Resource Restraints

Only two physical resource restraints were included in the model: the current year total improved land and the preceding year summerfallow acreage. The first restraint is expressed below in mathematical notation as:

$$TL_t = \sum_{i=1}^{7} \sum_{j=1}^{2} X_{ijt} \quad (3)$$

where:

TL_t = total improved land in the t th year,

X_{ijt} = solution acreage of the i th crop produced by the j th method in the t th year,

i = index ranging from 1 to 7 for crops ordered as: wheat, oats, barley, rye, flaxseed, rapeseed and summerfallow, and

j = index taking value 1 for crop sown on summerfallow and 2 for crop seeded on stubble. However, when i=7, j takes only value 1.

The above restraint implies that all the available land must be completely utilized by the activities of the model. There are two reasons for forcing this constraint to equality. First, it is unreasonable to assume that crop land would be left idle; rather it is likely that land not cropped would be summerfallowed. Secondly, in the absence of an equality sign for this restraint, the solution acreage of summerfallow will always be equal to the lower flexibility restraint because this activity has a negative C_j value (i.e., cost of working summerfallow). However, this activity should not necessarily appear at this low level because summerfallow is maintained from the point of view of long-run objectives with the return being obtained in future years.

The total improved land restraint for each year from 1958 through 1970 was set equal to the sum of the acreages of the six crops and summerfallow for the respective year. The 1969 acreage was used for the 1971 crop year.

The second physical resource restraint is the preceding year summerfallow acreage. This restraint requires that the total area of summerfallow crops should not exceed the preceding year summerfallow acreage. The constraint can be expressed as follows:

$$SF_{t-1} \geq \sum_{i=1}^{6} X_{i1t} \qquad (4)$$

where:

SF_{t-1} = summerfallow acreage in the t-1 th year, and all other variables are defined as before.

Flexibility Restraints

These restraints are dynamic in nature. They relate land utilization patterns of one year with crop acreages of the preceding year. Specifically, the flexibility restraints are upper and lower bounds on the allowable year-to-year changes in the solution acreage of each crop in the model. In this study, these restraints are utilized for the six crops and summerfallow and stubble land. The restraints may be expressed in the following dynamic notation:

$$(1-\underline{\beta}_{it})X_{it-1} \leq X_{it} \leq (1+\overline{\beta}_{it})X_{it-1}$$

$$\text{or} \quad X_{it} \leq (1+\overline{\beta}_{it})X_{it-1} \tag{5}$$

$$\text{and} \quad X_{it} \geq (1-\underline{\beta}_{it})X_{it-1} \tag{6}$$

$$i = 1,\ldots,8$$

where:

X_{it} = solution acreage of the i th crop in the t th year,

X_{it-1} = actual or solution acreage of the i th crop in the t-1 th year,

$\overline{\beta}_{it}$, $\underline{\beta}_{it}$ = maximum allowable proportionate increase and decrease, respectively, of the t th year acreage from that of the t-1 th year. These are known as upper and lower flexibility coefficients.

In order to estimate the flexibility coefficients which are needed for computing the flexibility restraints, relation (5) or (6) can be rewritten as follows:

$$X_{it} = (1+\beta_{it})X_{it-1} \qquad (7)$$

$$i = 1,\ldots,8$$

Estimation of this regression equation produces a flexibility coefficient which does not vary between years and is not affected by the economic and non-economic conditions prevailing in any one year. This constancy of the coefficient has been considered as a major short-coming of the above regression model. An attempt is made in this study to develop an alternative model which can overcome this weakness.

Development of a Model for Estimating Flexibility Coefficients

It can be hypothesized that year-to-year proportionate changes in acreage of a crop depend upon two major factors. The first factor is the effect of the base year acreage of a crop on its future acreage. This hypothesis is based upon a premise that as acreage of a crop increases, farmers become more reluctant to expand that acreage at the same rate due to resource restriction and risk of specialization. The second factor consists of a group of economic and non-economic variables which affect farmers' decisions about land utilization patterns.

In this study, two regression models were developed. One was related to farmers' reluctance to increase crop acreage at a constant rate. The second model was developed using economic and non-economic forces as exogeneous explanatory variables. For estimation purpose, both models were combined into one.

In order to develop the first model, the following non-linear equation can be specified to express the effects of the preceding year acreage on the current year land utilization patterns:

$$X_{it} = c_{i1}X_{it-1} + c_{i2}X^2_{it-1} \quad (8)$$

$$i = 1,\ldots,8$$

Equation (8) can be rewritten as:

$$X_{it} = (c_{i1} + c_{i2}X_{it-1})X_{it-1} \quad (9)$$

By rearranging equation (9), the following expression can be obtained:

$$\frac{X_{it}}{X_{it-1}} = c_{i1} + c_{i2}X_{it-1} \quad (10)$$

$$i = 1,\ldots,8$$

Equation (7) can be also expressed as:

$$\frac{X_{it}}{X_{it-1}} = 1 + \beta_{it} \quad (11)$$

Using equations (10) and (11), the following relationship between flexibility coefficient and the preceding year acreage can be obtained:

$$\frac{X_{it}}{X_{it-1}} = 1 + \beta_{it} = c_{i1} + c_{i2}X_{it-1} \quad (12)$$

The flexibility coefficients of a crop estimated from equation (12) would not be equal in different years.

Rather, it would vary depending upon the levels of the preceding year acreage. Since it was hypothesized that $c_{i2} < 0$, a large β_{it} would be estimated if the preceding year acreage of a crop was small. And, a small β_{it} would be produced if the base year acreage was large.

The development of the second regression model is based upon the premise that farmers do <u>not</u> increase acreage of a crop between years by the same proportion. But the proportion varies depending upon the levels of certain exogeneous explanatory variables in the year.

Now a question arises: What are the exogeneous variables affecting flexibility coefficients? From economic theory, we know that supply of a product (in the present context flexibility coefficient) is a function of its own price and that of its major competitor. Sometimes, grain stocks and exports are also considered as variables affecting output. But they can be ignored if the following assumptions are satisfied:

(1) the government does not interfere in the marketing of agricultural products,

(2) prices are solely determined by the market forces (i.e., demand and supply), and

(3) price acts as a force to allocate resources to alternative crops.

However, in conducting research on "The Pricing Structure of Wheat at the Country Elevator Level," Farris observed:

> The results show substantial departures from perfect competition and the kind of pricing imperfections that are most serious. The findings have implications for any market in which perfect or nearly perfect competition is assumed. . . . There is broader question. We commonly assume that the pricing process in most agricultural markets tends to yield results not greatly different from results under our theoretical ideal of perfect competition. This means, for one thing, that price can be relied upon to allocate resources. But if the empirical existence of a high or low price fails to indicate that more or less of a commodity of particular quality is demanded, the price signal to producers is not likely to call forth the quantities and qualities of products that consumers and the trade really want (22, pp. 607-24).

Thus, heavy reliance cannot be attached to prices for allocating resources to different crops. Hence, it is hypothesized that flexibility coefficients depend not only upon farmers' expected prices, but also on stocks and exports for the same crop and its major competitor.

The flexibility coefficient is also affected by weather. Moisture at the time of seeding modifies farmers' intentions about land utilization patterns and thereby creates discrepancy between planned and actual acreages of crops. Therefore, precipitation was used as an independent variable in this study.

Technological change has been a dynamic force in Canadian agriculture. It has shifted supply functions to the right. If the impact of technology would have been even on all crops, perhaps technology could have been omitted from the present study on land utilization pattern. But the impact has not been even. It has been more favourable to some and less to others. For example, during 1939 to 1966, wheat yield in the prairies trended upward by about half of a percent per year, whereas yields of oats, rye and rapeseed rose by more than one percent a year. Assuming cost of production per acre of wheat relative to other crops did not change over this period, uneven rates of increases in yields might have caused a smaller reduction in per bushel production cost of wheat and larger reductions for other crops. Thus, the effect of technological change has been uneven in the prairies. Therefore, it is included as an independent variable in this study.

The relationship between the above exogeneous explanatory variables and flexibility coefficients can be expressed in the following functional form:

$$(1+\beta_{it}) = f(P^*_{it}, P^*_{jt}, S^*_{it}, S^*_{jt}, E^*_{it}, E^*_{jt}, M_t, T_t) \quad (13)$$

$$i = 1,...,8$$

where:

P^*_{it}, P^*_{jt} = expected prices of the i th and j th crops, respectively, for the t th year,

S^*_{it}, S^*_{jt} = expected grain stocks of the i th and j th crops, respectively, for the t th year,

E^*_{it}, E^*_{jt} = expected exports for the i th and j th crops, respectively, for the t th year,

M_t = total rainfall in April and May of the t th year, and

T_t = time trend variable to account for technological change (1953=1).

Substitution of relation (13) into equation (7) produces the following function:

$$X_{it} = \{f(P^*_{it}, P^*_{jt}, S^*_{it}, S^*_{jt}, E^*_{it}, E^*_{jt}, M_t, T_t)\} X_{it-1} \quad (14)$$

Equation (14) can be also written as:

$$\frac{X_{it}}{X_{it-1}} = f(P^*_{it},\ldots,T_t) \quad (15)$$

In order to transform the expected variables of equation (15) into observable variables, simple expectation models were employed for exports, prices and stocks. Current year exports were assumed at the preceding year level. The expected prices and stocks were taken at the preceding year values. Because precipitation before seeding creates a gap between actual and intended land use, total rainfall during April and May was treated as the relevant observation for the rainfall variable.

Using the above expectation models, the function (15) can be expressed as:

$$\frac{X_{it}}{X_{it-1}} = f(P_{it-1}, P_{jt-1}, S_{it-1}, S_{jt-1}, E_{it-1}, E_{jt-1}, M_t, T_t) \quad (16)$$

The variables of this function are not identified because basically the same variables have been described in equation (13). In the case of equation (13), a star indicates expected values, whereas in function (16), lack of star shows actual values.

Assuming linear relationships between dependent and independent variables, relation (16) is expressed below in the form of a mathematical model:

$$\frac{X_{it}}{X_{it-1}} = a_{0i} + a_{1i}P_{it-1} + a_{2i}P_{jt-1} + a_{3i}S_{it-1} + a_{4i}S_{jt-1} + a_{5i}E_{it-1} + a_{6i}E_{jt-1} + a_{7i}M_t + a_{8i}T_t \quad (17)$$

$$i = 1,\ldots,8$$

Equation (17) indicates that the year-to-year proportionate changes in crop acreages are dependent upon the economic and non-economic conditions prevailing during the year. Equations (12) and (17) can be combined to obtain a complete model which encompasses both sources of year-to-year variations in flexibility coefficients as: (1) changes in the base year acreage, and (2) variations in the levels of economic and non-economic variables. The complete model is presented below:

$$\frac{X_{it}}{X_{it-1}} = b_{0i} + b_{1i}X_{it-1} + b_{2i}P_{it-1} + b_{3i}P_{jt-1} + b_{4i}S_{it-1} + b_{5i}S_{jt-1} + b_{6i}E_{it-1} + b_{7i}E_{jt-1} + b_{8i}M_t + b_{9i}T_t \quad (18)$$

For the purpose of estimating equation (18), an error term was added:

$$\frac{X_{it}}{X_{it-1}} = b_{0i} + b_{1i}X_{it-1} + \cdots + b_{9it}T_t + U_{it} \qquad (19)$$

Using economic logic and knowledge of agronomic practices, the following a priori expectations about the signs of regression coefficients were anticipated: $b_{1i}<0$, $b_{2i}>0$, $b_{3i}<0$, $b_{4i}<0$, $b_{5i}>0$, $b_{6i}>0$, $b_{7i}<0$ and $b_{9i}>0$. The sign for rainfall variable was not specified because it is hypothesized to depend upon the crop under consideration.

Because a sizeable porportion of the total improved land of the prairies is usually seeded to wheat, this crop was assumed to be the major resource competitor of all non-wheat crops in the regression models. That is, when equation (19) was estimated for oats, barley, rye, flaxseed or rapeseed; price, stocks and exports of wheat were used as the competitive crop variables. For estimating the wheat equation, flaxseed was used as a major competitor, but empirical results did not verify this specification. Therefore, this crop was dropped from the wheat equation and no other crop was used in its place.

Problems in Estimation

The multiple regression model developed above can be utilized to estimate upper and lower flexibility coefficients. However, certain econometric and statistical problems normally arising in estimation must be taken care of before the analysis is finalized. Only three major problems are discussed below. These are: autocorrelation, multicollinearity and insufficient degrees of freedom.

Autocorrelation. Time-series regression equations often have autocorrelated error terms. While estimating a number of equations, Ladd (43) found that 26 to 66 percent of equations had significant levels of autocorrelation, depending on the test used. This error normally arises due to incomplete specification of the model. A relevant variable might be omitted from the equation because its significance as independent variable could not be recognized. Sometimes, a few variables are dropped because of non-availability of data and/or lack of enough degrees of freedom. If omitted variables are serially correlated, the residuals of the estimated equation are also autocorrelated.

If the lagged dependent variable is not included in the equation as an independent variable, the presence of serial correlation in the error term does not lead to bias and inconsistency in the estimates of the regression parameters. It does, however, lead to biased estimates of the standard errors of the regression coefficients. In the presence of lagged dependent variable, serial correlation causes bias and inconsistency in the estimates of parameters. However, Nerlove (52, pp. 866-76) found a significant decline in the magnitude of the serial correlation coefficient if the lagged dependent variable is included in supply equations.

In this study, the lagged dependent variable was not included in the regression model, but the numerator of the dependent variable lagged one year appears as an independent variable. Even though the Durbin-Watson test of serial correlation becomes inappropriate when an equation includes lagged dependent variable (26, pp. 65-75), using this test, substantial variations in the levels of autocorrelation were observed in this study. For some crops, autocorrelation was insignificant, but for others highly significant. On average, it was not pronounced and the problem was ignored in the analysis.

Multicollinearity. Multicollinearity between the explanatory variables is frequently observed in econometric studies based on time-series data. Candler (4, pp. 1735-38) and Johnson (34) encountered high degrees of intercorrelations in their analyses. In the presence of multicollinearity, the detection and quantification of the effect of changes in a single independent variable on the dependent variable becomes difficult. The intercorrelation biases the regression coefficients towards zero (66, p. 348), and may produce their signs inconsistent with that expected (21, p. 77). Therefore, their estimates are highly unreliable. Moreover, the sampling variances of the least squares coefficients are expected to be large (65, p. 216).

In this study, a few independent variables in a number of equations were found highly intercorrelated. For example, strong correlation was observed between prices, between grain stocks, and among price, stocks and export of a single crop.

In order to deal with multicollinearity, independent variables of some equations were transformed in such a way that price ratio was used in the place of individual prices. Likewise, stock and export ratios were used in some equations. Another merit of using ratio variables is that it saves a few degrees of freedom. However, the use of ratio variables has three major limitations. First, the estimate of the regression parameter of this variable appears conceptually inconsistent. For example, the regression coefficient of the ratio variable of wheat and flaxseed prices indicates that both prices have almost equal effects on wheat acreage. This would probably not be the case. The second limitation is that the use of ratio variable requires an erroneous premise that the levels of individual prices are unimportant. A third reason for avoiding ratio variable is the difficulty in interpreting the estimated coefficients. Learn and Cochrane (45, p. 67), therefore, suggested that it is probably worth the price of one degree of freedom to let the data determine the separate effects of individual prices.

In this study, only those independent variables which were strongly intercorrelated were transformed into ratios, and uncorrelated variables were used in linear form.

<u>Problems of Degrees of Freedom</u>. Because the analysis was

related to annual acreages, there were only fifteen observations over the period 1953 through 1967. For the separate estimation of upper and lower flexibility coefficients, observations were stratified into two groups on the basis of positive or negative changes in year-to-year acreages of a crop. This procedure resulted in as few as six observations for estimating some equations. Therefore, it was possible to include only a few variables in those equations. The selection of variables was made on the basis of: (1) multicollinearity between independent variables; (2) inconsistency of coefficient signs with a priori expectations; (3) the statistical significance of the regression coefficients, and (4) the level of R^2.

Lagged acreage and price or price ratio variables were included in every equation. The reason for using the former variables arises from the fact that the signs of its coefficients were consistent in almost all cases. The coefficients were also highly significant. Inclusion of the price variable can be justified on two grounds. First, this variable was included in order to examine the impact of price changes on land utilization patterns. Secondly, the empirical performance of this variable, based upon the above four criteria, was also reasonably good.

Method of Estimation

Ordinary least squares regression was used to estimate separate equations for upper and lower flexibility coefficients of six crops, summerfallow and stubble land. That is, a set of eight equations was developed for the purpose of estimating upper flexibility coefficients. Likewise, another set of eight equations was estimated for lower coefficients. These equations are presented in Appendix Table 14. Both sets of equations were used to compute $\overline{\beta}_i$ and $\underline{\beta}_i$ for each year during the period 1958 to 1967. Also, these equations were utilized to estimate ex-ante flexibility coefficients for years 1968 through 1971.

A few observations were omitted from the analysis of some crops because year-to-year proportionate changes in acreages were excessively high. These extreme changes are likely to be a reflection of some unique and non-recurring forces affecting the crop in those years.

For most of the regions, rapeseed acreage data were not available for the entire period of analysis (1953-1967). Therefore, in these cases, a different procedure was adopted to estimate flexibility coefficients. Observations were not stratified into two groups, but all data were utilized to estimate only one equation. And the estimated values of the dependent variable were increased and decreased by ten percent in order to obtain upper and lower flexibility coefficients.

In some years, in contrast to a priori expectations, negative values of the upper and lower flexibility coefficients were estimated for a few crops. Perhaps these values resulted because a very limited number of observations were utilized in the multiple regression analysis. Inadequate degrees of freedom cause instability in the estimates of regression parameters. Hence, the flexibility coefficients based upon the regression parameters could have been unstable and taken extreme values for years not included in the fit.

Therefore, equations were also fitted for each crop using all observations and flexibility coefficients were computed from these. These coefficients were used to replace negative coefficients estimated by the earlier method.

The flexibility coefficients estimated through the multiple regression model vary from year-to-year depending upon the levels of exogeneous explanatory variables. The coefficients are presented in Appendix Table 16. The flexibility restraints were computed using these coefficients. The upper flexibility restraint for a crop was estimated by multiplying $(1+\overline{\beta}_{it})$ by the acreage in the preceding year. Similarly, the lower restraint for a crop was estimated by multiplying the lagged acreage by $(1-\underline{\beta}_{it})$. Upper and lower flexibility restraints were estimated in this manner for each crop. In addition, similar restraints were computed for summerfallow and stubble land.

Absolute Minimum and Maximum Acreage Constraints

Absolute minimum and maximum acreages of each of the six crops, summerfallow and stubble land were other restraints included in the recursive model. The rationale for introducing such constraints is that a region maintains a certain minimum acreage of a crop (and, therefore, a certain maximum acreage of other crops) for the purpose of diversifying cropping pattern and/or raising feed for livestock.

There is also an empirical significance in introducing these absolute constraints into the recursive model. These constraints reduce aggregation bias. Since an aggregate programming model assumes that all farms in a region respond in a similar way to economic stimuli, (i.e., all farms respond to the same extent or do not respond at all), an all-or-nothing type of solution can be obtained (63, pp. 1531-32). However, in reality farmers respond at different rates. Therefore, a region maintains at least some minimum acreages of a few crops and some maximum acreages of others. This feature was introduced in the recursive model by adding absolute minimum and maximum acreage restraints for each of the six crops, summerfallow and stubble land. The absolute restraints are presented as follows:

$$\text{Max } X^*_i \geq \sum_{j=1}^{2} X_{ijt} \qquad (20)$$

$$\text{Max } X^*_8 \geq \sum_{i=1}^{6} X_{i2t} \qquad (21)$$

$$\text{Min } X^*_i \leq \sum_{i=1}^{2} X_{ijt} \qquad (22)$$

$$\text{Min } X^*_8 \leq \sum_{i=1}^{6} X_{i2t} \tag{23}$$

$$i = 1,2,\ldots,7$$

where:

Max X^*_i = maximum acreage of the i th crop during a period,

Min X^*_i = minimum acreage of the i th crop during a period, and

other variables are defined as before.

In order to carry out programming analysis during 1958 to 1967, minimum and maximum acreages observed over the period of fit (1953-1967) were used as the absolute acreage restraints.[a] For analysis beyond 1967, these restraints were updated for each new year using the preceding year crop acreage. For example, the 1968 actual acreages were used to update the absolute minimum and maximum acreage constraints for the 1969 analysis.

Quota Restraints

Sales of some cereal grains in the prairies have been regulated by the Canadian Wheat Board. Prairie farmers have sold grains under: (1) unit quota, (2) specified acreage quota, and (3) supplementary quotas. In this study,

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[a]Acreage data prior to 1960 were not available for rapeseed. Therefore, minimum and maximum acreages observed during the period 1960 to 1967 were used as absolute constraints for this crop.

all three quota restraints were included in order to account for the restrictions put on by the Wheat Board on the sale of individual crops, and to examine the impact of changes in quota levels on land utilization patterns.

Under the unit quota system, the Wheat Board allowed every prairie grain producer to deliver a maximum of one hundred units of grains. Each unit consisted of some specified number of bushels of wheat, oats, barley and rye. For instance, in 1969, one unit of quota was equivalent to three bushels of wheat, ten bushels of oats, eight bushels of barley or sixteen bushels of rye. A farmer could deliver one hundred units of any one of these products or any combination thereof. This restraint can be expressed through the following mathematical notation:

$$UQ_t \geq \sum_{i=1}^{4} Q_{ilt} / q_{ilt} \qquad (24)$$

where:

Q_{ilt} = quantity of the i th crop sold through the unit quota in the t th year,

UQ_t = total amount of unit quota in the t th year, and

q_{ilt} = number of bushels of the i th crop per unit of quota in the t th year.

Total unit quota constraint (UQ) was estimated by multiplying the number of commercial farms in a region by one hundred.

The specified acreage quota restraint, which is presented in equation (25), indicates that the total sales of wheat, oats, barley and rye under this constraint cannot exceed the number of bushels of quota generated by the specified acreage (i.e., area of wheat, oats, barley, rye, tame hay and summerfallow). Tame hay was not included in the analysis. Therefore, the number of bushels of the specified quota generated by this crop was determined by multiplying hay acreage by the quota level. This amount was then introduced in the restraint as follows:

$$QH_t \geq \sum_{i=1}^{4} Q_{i2t} - \sum_{i=1}^{4} \sum_{j=1}^{2} q_{i2t}X_{ijt} - q_{72t}X_{71t} \qquad (25)$$

and $q_{12t} = \ldots = q_{42} = q_{72t}$

where:

Q_{i2t} = quantity of the i th crop sold through the specified quota in the t th year,

QH_t = total amount of specified acreage quota generated by tame hay in the t th year,

q_{i2t} = specified quota level per acre of the i th crop in the t th year, and

all other variables are as previously defined.

The supplementary quota has been designed for the sale of a specific crop. In this model, separate quota restraint was introduced for each of wheat, oats, barley and rye. These restraints are presented below:

$$Q_{i3t} \geq \sum_{j=1}^{2} q_{i3t}X_{ijt} \qquad (26)$$

$$i = 1,2,3,4$$

or

$$0 \leq Q_{i3t} - \sum_{j=1}^{2} q_{i3t}X_{ijt}$$

where:

Q_{i3t} = quantity of the i th crop sold through the supplementary quota in the t th year,

q_{i3t} = bushels of supplementary quota per acre of the i th crop in the t th year, and

all other variables are as defined before.

Supply Restraints

These restraints were utilized to relate producing and selling activities of crops. They specified that the total quantity of a product sold should not exceed the amount produced. In the present study, a separate constraint was used for each of wheat, oats, barley, rye, flaxseed and rapeseed. The constraints can be expressed as follows:

$$\sum_{k=1}^{4} Q_{ikt} \leq \sum_{j=1}^{2} Y_{ijt}X_{ijt} \qquad (27)$$

or

$$0 \geq \sum_{k=1}^{4} Q_{ikt} - \sum_{j=1}^{2} Y_{ijt}X_{ijt}$$

$$i = 1,2,\ldots,6$$

where:

Q_{ikt} = quantity of the i th crop sold through the k th method in the t th year,

Y_{ijt} = per acre yield of the i th crop produced by the j th method in the t th year,

with X_{ijt} defined as before.

Summary of the Model

The recursive programming model developed above is presented in detail in Appendex Table 13. A summary is given below. Some of the equations expressed here are very specific, whereas, in the earlier portion of this chapter, they were presented in a more general form and then explained in the text. The specific forms taken by the objective function and related constraints are as follows:

$$\text{Maximize} \quad Z_t = \sum_{i=1}^{4} \sum_{k=1}^{4} P_{ikt} Q_{ikt} + \sum_{i=5}^{6} P_{i1t} Q_{i1t}$$

(Gross Revenue)

$$- \sum_{i=1}^{6} \sum_{j=1}^{2} C_{ijt} X_{ijt} - C_{71t} X_{71t}$$

(Total Cost)

Subject to:

$$TL_t = \sum_{i=1}^{6} \sum_{j=1}^{2} X_{ijt} + X_{71t}$$

$$SF_{t-1} \geq \sum_{i=1}^{6} X_{i1t}$$

$$(1+\overline{\beta}_{it}) X_{it-1} \geq \sum_{j=1}^{2} X_{ijt} \qquad i = 1,\ldots,6$$

$$(1+\overline{\beta}_{7t}) X_{7t-1} \geq X_{71t}$$

$$(1+\overline{\beta}_{8t})\ X_{8t-1} \geq \sum_{i=1}^{6} X_{i2t}$$

$$(1-\underline{\beta}_{it})\ X_{it-1} \leq \sum_{j=1}^{2} X_{ijt} \qquad i = 1,\ldots,6$$

$$(1-\underline{\beta}_{7t})\ X_{7t-1} \leq X_{71t}$$

$$(1-\underline{\beta}_{8t})\ X_{8t-1} \leq \sum_{i=1}^{6} X_{i2t}$$

$$\text{Max*}_i \geq \sum_{j=1}^{2} X_{ijt} \qquad i = 1,\ldots,6$$

$$\text{Max*}_7 \geq X_{71t}$$

$$\text{Max*}_8 \geq \sum_{i=1}^{6} X_{i2t}$$

$$\text{Min*}_i \leq \sum_{j=1}^{2} X_{ijt} \qquad i = 1,\ldots,6$$

$$\text{Min*}_7 \leq X_{71t}$$

$$\text{Min*}_8 \leq \sum_{i=1}^{6} X_{i2t}$$

$$UQ_t \geq \sum_{i=1}^{4} Q_{i1t}/q_{i1t}$$

$$QH_t \geq \sum_{i=1}^{4} Q_{i2t} - \sum_{i=1}^{4} \sum_{j=1}^{2} q_{i2t}\ X_{ijt} - q_{72t}\ X_{71t}$$

$$0 \leq Q_{i3t} - \sum_{j=1}^{2} q_{i3t}\ X_{ijt} \qquad i = 1,2,3,4$$

$$0 \geq \sum_{k=1}^{4} Q_{ikt} - \sum_{j=1}^{2} Y_{ijt}\ X_{ijt} \qquad i = 1,2,3,4$$

$$0 \geq Q_{i1t} - \sum_{j=1}^{2} Y_{ijt} X_{ijt} \qquad i= 5,6$$

and $X_{ijt} \geq 0$; $Q_{ikt} \geq 0$

All variables are defined as before.

Levels of Aggregation in the Analysis

Because the major objectives of this study are to explain the historical acreages of crops and to forecast the 1971 land utilization patterns for each of the three prairie provinces, and for the prairies as a whole, the analysis was carried out separately for Manitoba, Saskatchewan and Alberta, and for the prairies treated as one unit. Thus, four models were constructed, which were similar in structure, but different with respect to numerical values of coefficients and constraints.

The analysis was also carried out separately for each of the twenty crop districts of Saskatchewan. Land utilization patterns of each district were estimated, and the provincial results were obtained by summing the crop district estimates. This disaggregate analysis was undertaken for the purpose of testing a pypothesis: the provincial estimates obtained through the aggregate analysis are less accurate than the results obtained by summing the solutions of the disaggregate models. These twenty models were structually the same, different only in terms of coefficient values.

In recapitulation, twenty-four models, in all, were developed. Twenty of them used crop districts as levels of aggregation in analysis. Three provincial models were constructed based on provincial data. The remaining model was developed utilizing prairie level data.

CHAPTER V

EMPIRICAL RESULTS

The purpose of this chapter is to present the results based on the analytical framework described in the previous chapters. Solution acreages of the recursive programming analysis are presented in tabular form in the Appendix. In this chapter, a graphic comparison is made between the estimated and actual acreages of crops, in order to evaluate the performance of the recursive model.

The chapter is divided into five sections. The first tests the explanatory power of the model. The second evaluates the predictive ability. In the third section, the performance of the aggregate models relative to the disaggregate ones are judged. The fourth section evaluates the effects of the "Lower Inventory for Tomorrow" program on the 1970 prairie land utilization pattern. The fifth and the last portion of this chapter presents the 1971 projected land use for each prairie province and for the prairies as a whole.

The Explanatory Test

This test was conducted to determine whether the model was capable of explaining farmers' behaviour during the period 1958 through 1967. Data for the entire period were used to explain land utilization patterns of any one year. For example, flexibility coefficients for each single year were estimated by fitting the multiple regression equations to the data for the entire period. Estimated results of the recursive programming analysis were compared against actual land utilization patterns in order to test the explanatory power of the model.

In this study, acreages were estimated separately for Manitoba, Saskatchewan and Alberta, and for the entire prairie region treated as a unit. The results from the aggregate prairie model are presented first, and the provincial estimates then follow. While presenting the results for individual crops, no attempt has been made to simultaneously discuss the causes of discrepancy between the actual and estimated acreages. Rather, the sources of errors are explained in the last portion of this section which follows the presentation of the results. The reasoning behind this organization arises from the fact that a number of factors which explained the errors in the acreage estimates were identical for most of the crops.

Prairies

The prairie land utilization patterns were explained with reasonable accuracy by the aggregate prairie model. In Table 1, the results are illustrated by crops and by years, in terms of percentage difference between actual and estimated acreage. The overall weighted average[a] of absolute

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[a]Actual acreages were used as weights to calculate weighted average of absolute differences.

TABLE 1

DIFFERENCE[a] BETWEEN ACTUAL PRAIRIE LAND USE AND ESTIMATES FROM THE AGGREGATE PRAIRIE MODEL, 1958-1967

Year	Wheat	Oats	Barley	Rye	Flax-seed	Rape-seed	Summer-fallow	Weighted average of absolute differences
				(percent)				
1958	-3.13	13.01	0.26	0.00	-37.05	-100.06	5.51	6.74
1959	4.26	1.43	- 4.86	4.00	-35.34	-212.16	1.59	4.63
1960	-2.67	9.27	- 0.41	0.42	- 2.97	- 6.34	0.74	2.34
1961	-0.65	-31.09	2.91	5.82	15.38	- 7.46	4.69	5.46
1962	-0.52	14.91	- 9.16	7.42	-29.89	- 25.59	0.02	3.26
1963	-0.97	0.94	4.36	1.20	14.91	- 12.03	-0.91	1.64
1964	1.06	0.00	-13.38	2.65	37.14	15.65	-1.75	3.36
1965	-6.07	10.47	8.50	1.72	35.28	- 1.53	-0.68	5.39
1966	0.09	- 4.14	11.84	1.31	3.45	- 6.23	-2.45	2.64
1967	1.18	-10.95	2.28	-2.15	-13.07	5.86	0.32	1.96
Weighted average of absolute differences	2.00	9.59	5.34	2.63	22.99	19.55	1.87	3.74

[a]A positive difference indicates an underestimate (i.e., estimated acreage less than actual), and a negative error represents an overestimate.

differences for all land use was 3.74 percent. The error was more than ten percent for only two crops (viz., flaxseed and rapeseed). During the period 1958 to 1967, the weighted average of absolute deviations for any one year was less than seven percent.

Table 1 shows that wheat acreages were closely explained through the model; the error was less than seven percent in any one year. Estimated and actual acreages of this crop are shown in Figure 6. While the model performance was remarkable in estimating year-to-year increases in wheat acreages, it was not as satisfactory in explaining the decreases. For example, a decline of 1.5 million acres in 1965 from 1964 was completely unexplained.

The estimates of oats acreages in the prairies were not satisfactory. The weighted average of absolute deviations during 1958 to 1967 was ten percent. Figure 7 illustrates a moderate proximity between model solution and actual acreages in only five years out of ten. However, the model estimates agree more closely with the actual observations during the period 1963 to 1967.

In spite of sizeable variations in prairie barley acreages during 1958 to 1967, the model estimates, graphed in Figure 8, were considerably similar to the actual observations. While barley acreage declined from 9.1 million in 1958 to 5.4 million in 1961, year-to-year decreases in this period were estimated with remarkable accuracy. However, the model had a tendency to underestimate this crop during 1965 to 1967.

Rye acreage was estimated with pronounced accuracy. Table 1 shows that the average error was only 2.63 percent. However, from Figure 9 it is apparent that the acreages were underestimated in almost all years. While the model was very successful in estimating sharp declines in acreages, it had estimated only a part of year-to-year increases.

Performance of the model was poor for explaining flaxseed acreages. The average error was twenty-three percent. Figure 10 illustrates that the direction of year-to-year changes in actual acreages were incorrectly estimated in four out of ten years (1958 to 1967). This crop acreage was substantially overestimated in the early years, but considerably underestimated during the period 1963 to 1965.

Table 1 shows that the weighted average of the absolute percentage differences between actual and estimated acreages of rapeseed was very high; the average error was twenty percent. However, as illustrated in Figure 11, the solution acreages were reasonably close to actual in all years except 1958 and 1959. Increases of rapeseed acreages by 250 percent in 1960 and 80 percent in 1965 were fully explained. However, considering the overall performance, the model had a tendency to overestimate this crop.

Summerfallow acreages were estimated with striking precision; the average error was less than two percent. From Figure 12, it seems that the model explained sharp acreage declines with reasonable accuracy, but it was not very successful in estimating year-to-year increases. For

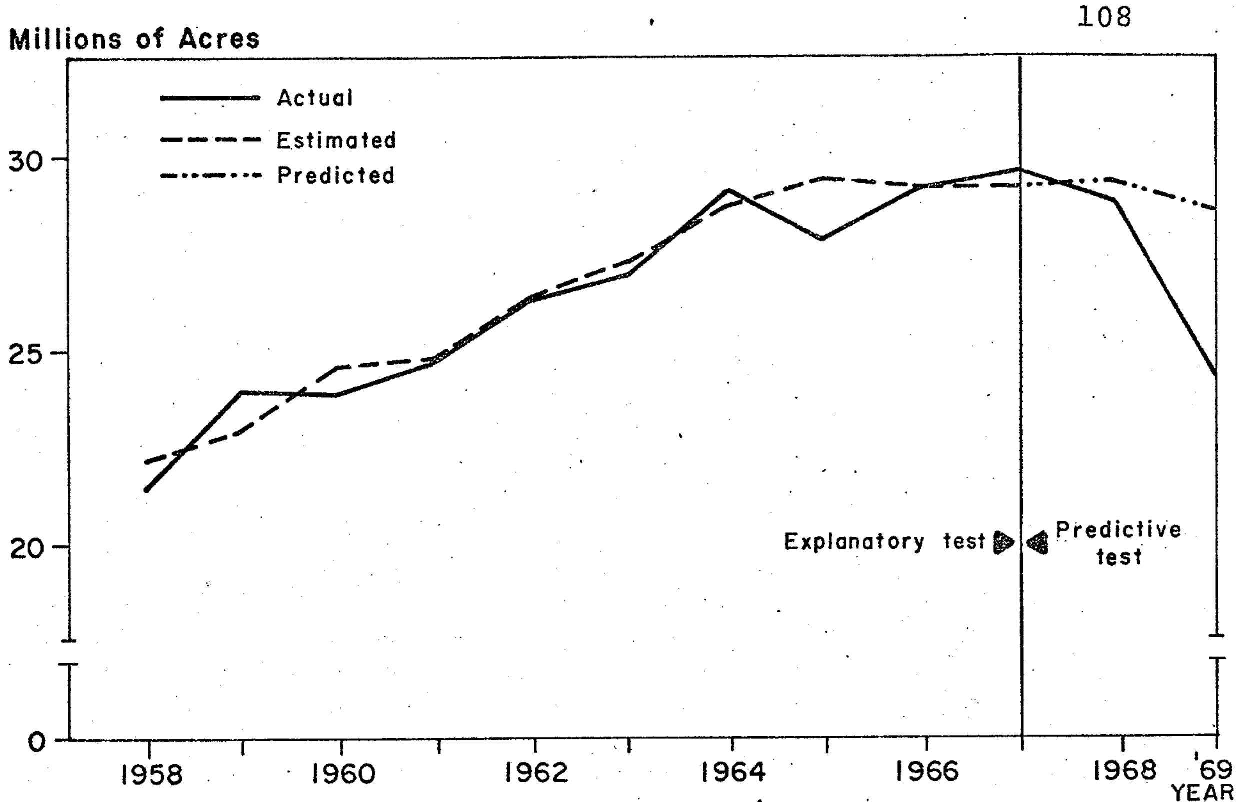

Figure 6. Explanatory and predictive tests[a] of prairie wheat acreage estimated by the aggregate model, 1958-1969.

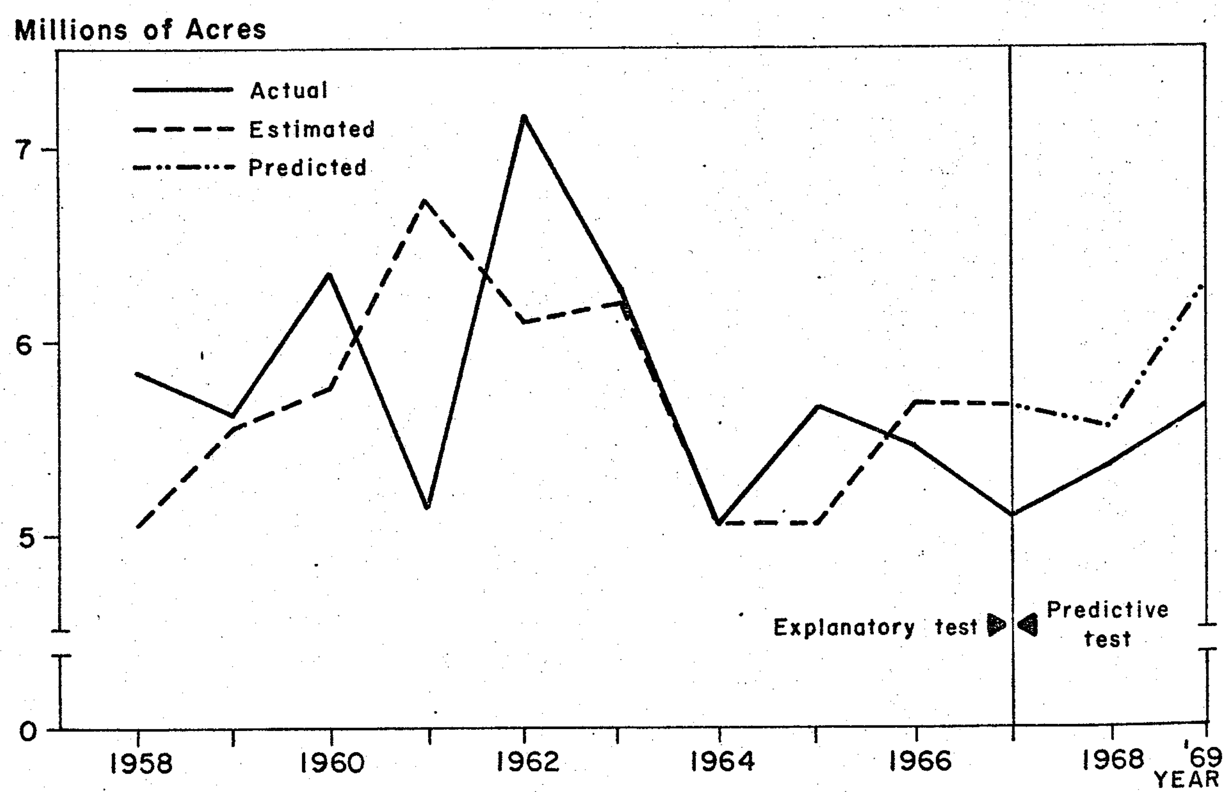

Figure 7. Explanatory and predictive tests of prairie oats acreage estimated by the aggregate model, 1958-1969.

a/ *The predictive test is described on pages 127 to 130 of this chapter.*

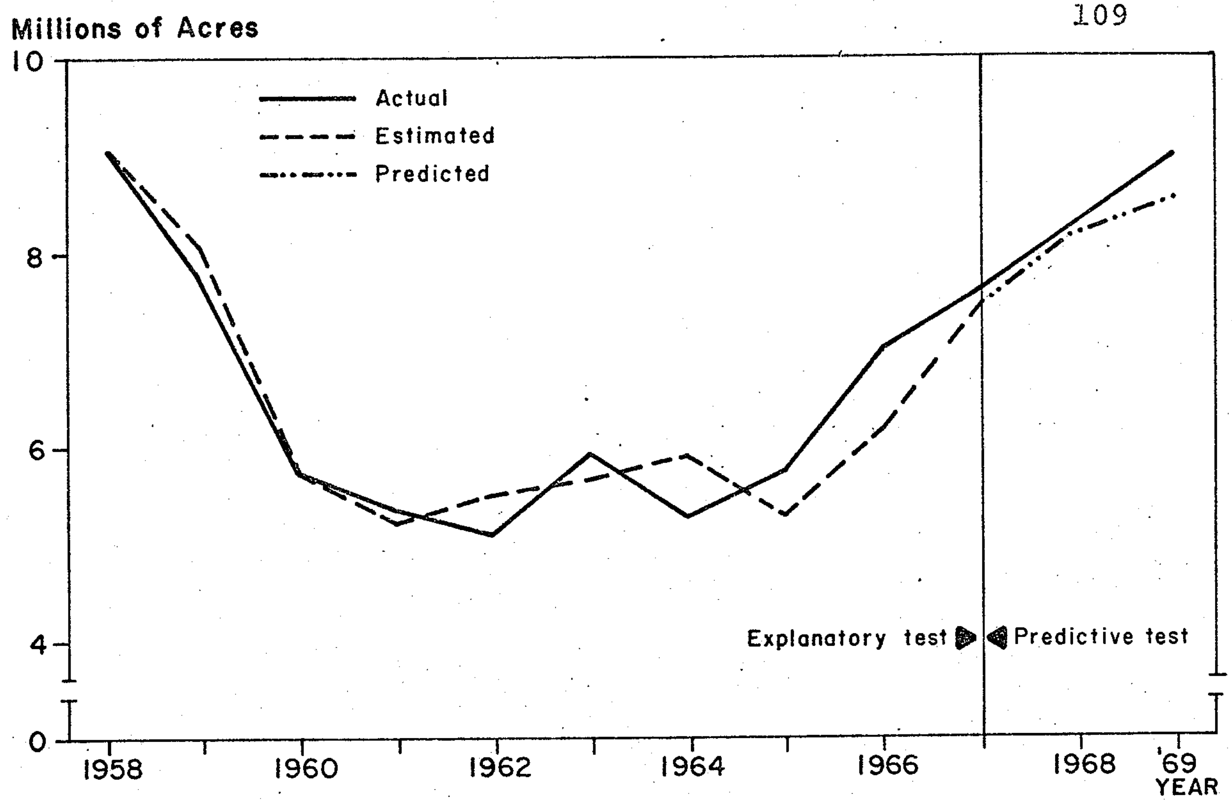

Figure 8. Explanatory and predictive tests of prairie barley acreage estimated by the aggregate model, 1958-1969.

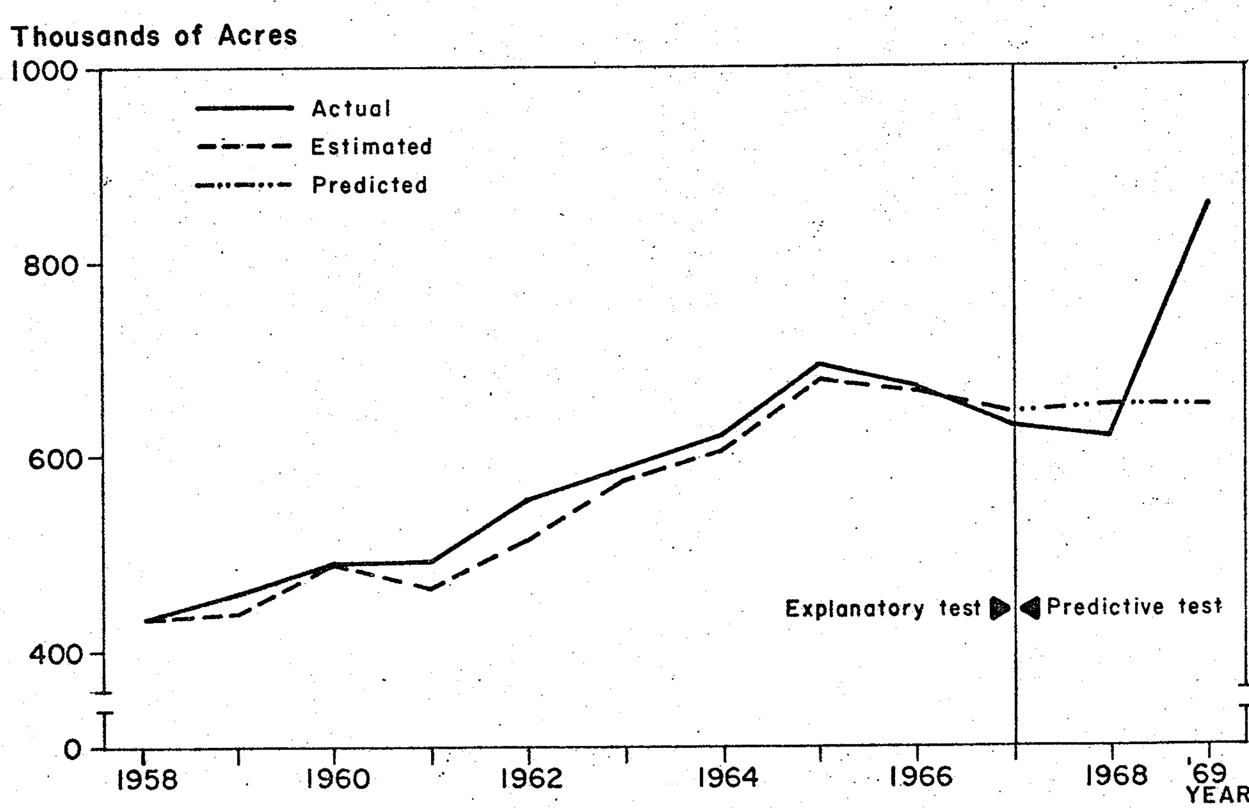

Figure 9. Explanatory and predictive tests of prairie rye acreage estimated by the aggregate model, 1958-1969.

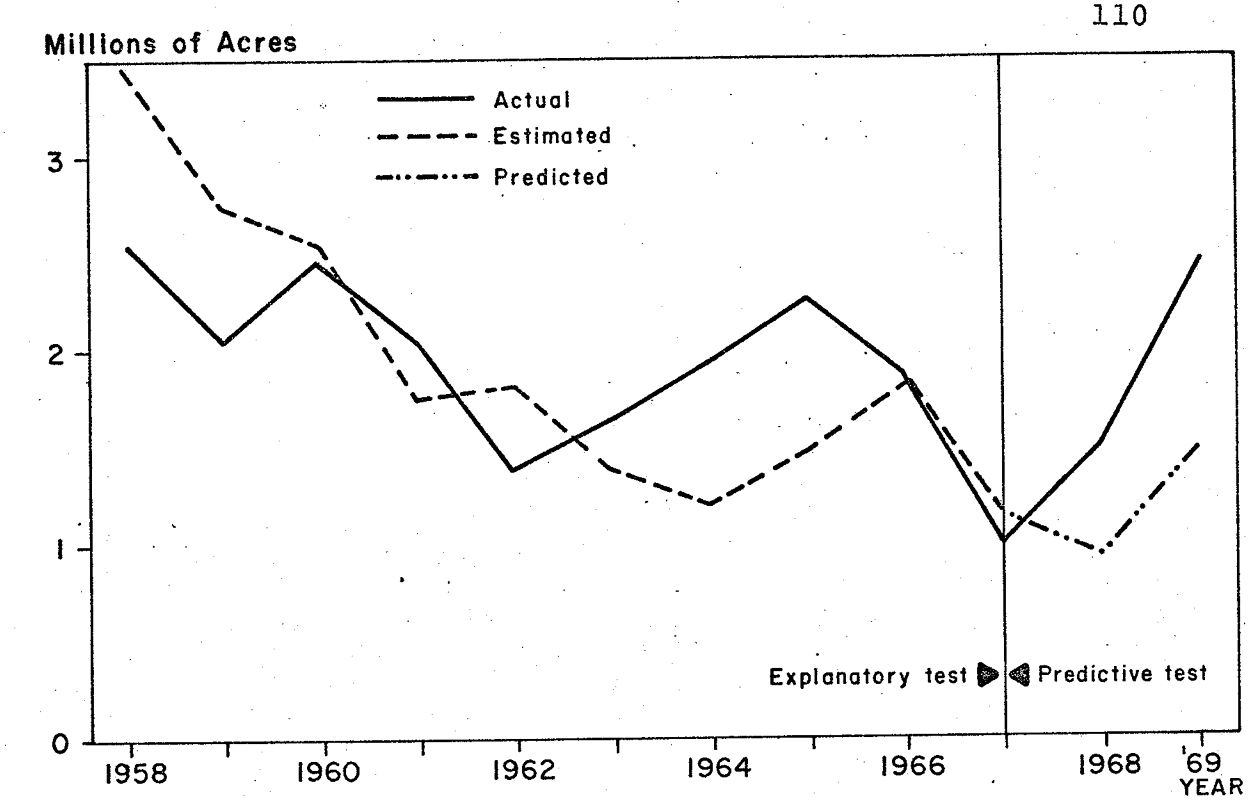

Figure 10. Explanatory and predictive tests of prairie flaxseed acreage estimated by the aggregate model, 1958-1969.

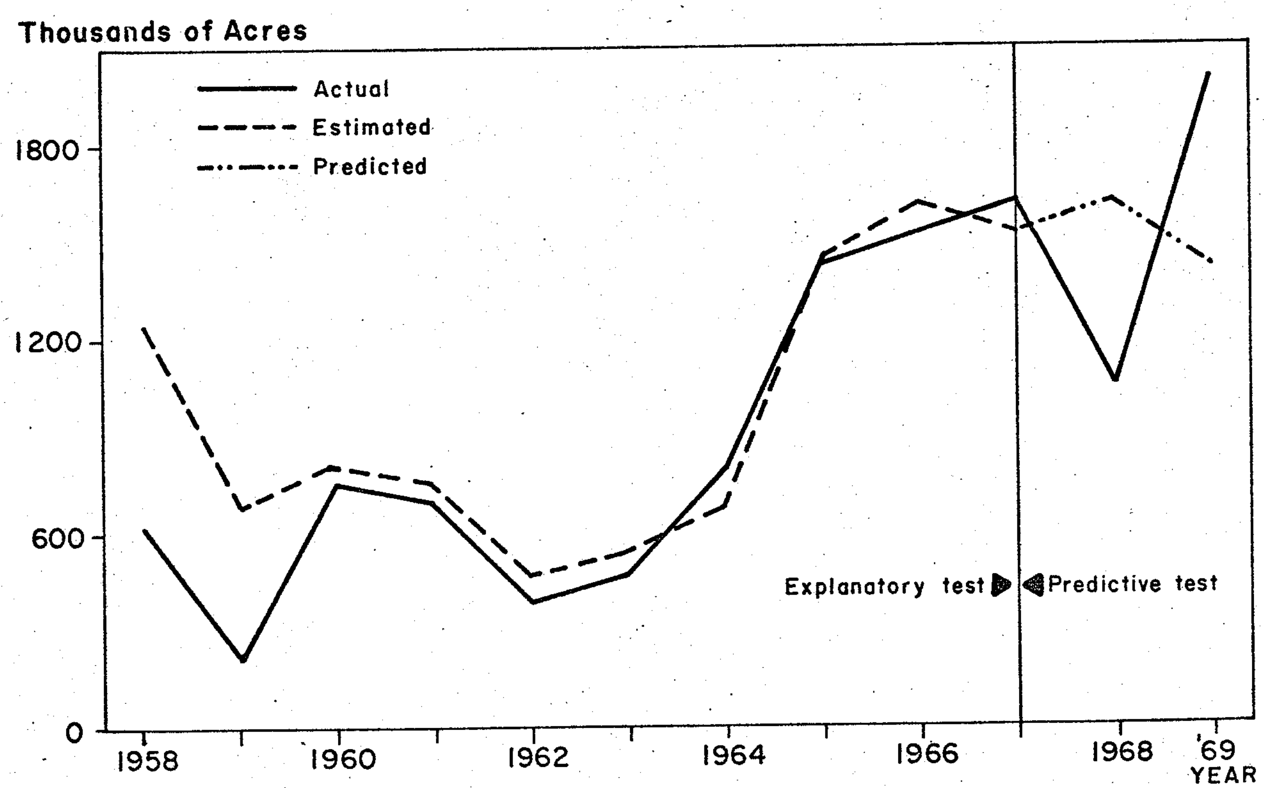

Figure 11. Explanatory and predictive tests of prairie rapeseed acreage estimated by the aggregate model, 1958-1969.

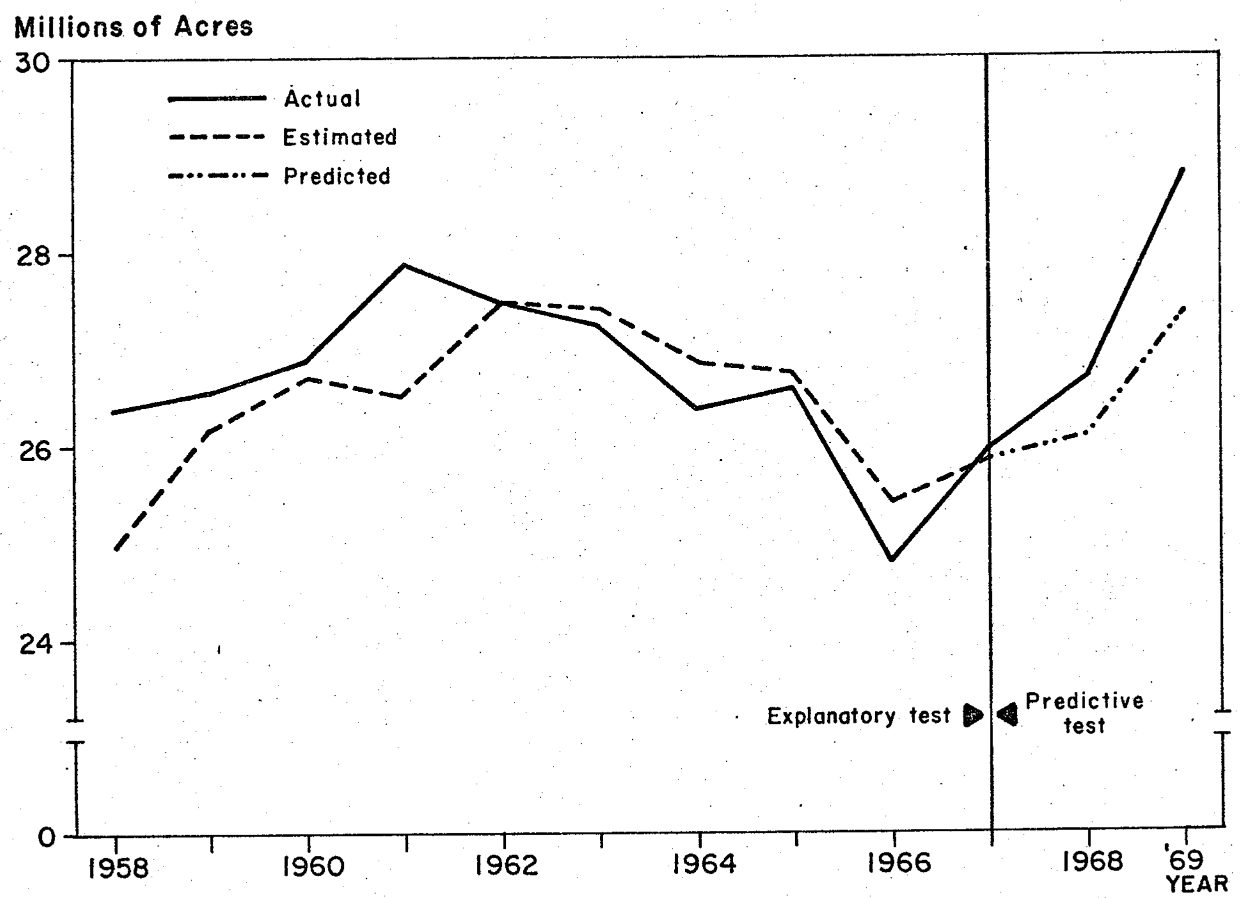

Figure 12. Explanatory and predictive tests of prairie summerfallow acreage estimated by the aggregate model, 1958 - 1969.

instance, the rise in summerfallow acreages in 1958 and 1961 were completely unexplained and, therefore, an error of 1.5 million acres was produced in each year.

It can be concluded from the above discussion that rye is the only crop that is consistently underestimated during the entire period of analysis (1958 to 1967). Other crops were explained with very small discrepancy between estimated and actual acreages in some years but with gross errors[a] in other years. However, in general, the recursive programming model has performed reasonably well.

Provinces

The land utilization patterns for each of the prairie provinces were explained with reasonable accuracy by the provincial models. In Table 2, the weighted average of absolute percentage deviations for the entire land use is presented by province and year. The model estimates for Manitoba, Saskatchewan and Alberta were moderately similar to actual acreages in each year during 1958 to 1967. The average error was less than ten percent in any one year for Saskatchewan and Alberta and was greater than ten percent for Manitoba in only 1958.

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[a]Causes of errors are explained in the last portion of this section.

TABLE 2

WEIGHTED AVERAGE OF ABSOLUTE PERCENTAGE DEVIATIONS FOR THE ENTIRE LAND USE, BY PROVINCE AND YEAR

Year	Manitoba	Saskatchewan	Alberta
		(percent)	
1958	12.67	6.47	5.49
1959	7.47	4.74	1.62
1960	9.71	3.17	1.57
1961	2.95	7.39	4.27
1962	3.82	4.99	6.49
1963	2.24	1.50	3.37
1964	8.43	3.48	3.77
1965	4.94	5.01	5.03
1966	5.82	2.30	1.74
1967	4.41	2.68	4.48
Weighted average	6.25	4.17	3.78

Table 3 shows that in each province the estimates of wheat acreages were considerably similar to actual values. The weighted average of differences was less than three percent in each case. Figure 13 reveals that, in each province, an increase of thirteen percent in wheat acreage between 1958 and 1959 was estimated with pronounced accuracy. However, a decline in 1965, which was the sharpest during the period 1958 to 1967, was unexplained in Manitoba and Saskatchewan. And, in Alberta, it was only partly estimated. It can thus be concluded that the model explained sizeable increases in acreages of this crop with remarkable accuracy; whereas, its performance was not satisfactory in estimating sharp declines.

TABLE 3

DIFFERENCE[a] BETWEEN PRAIRIE PROVINCES' ACTUAL LAND USE AND ESTIMATES FROM THE PROVINCIAL MODELS, 1958-1967

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	Weighted average of absolute differences
				(percent)				
				Manitoba				
1958	1.95	-12.02	-16.62	-20.77	- 39.09	-106.23	13.98	12.67
1959	1.07	2.04	-19.53	- 2.84	- 17.94	-384.93	7.27	7.47
1960	-0.83	- 2.99	-55.75	- 1.37	- 2.70	- 0.80	9.14	9.71
1961	-0.73	- 6.86	-12.81	0.12	- 2.86	-124.25	- 0.38	2.95
1962	0.47	1.15	- 7.79	0.25	- 29.91	- 4.12	- 2.38	3.82
1963	0.53	- 6.76	-10.22	-20.38	0.48	3.47	0.13	2.24
1964	0.74	- 2.39	-65.58	1.30	- 1.93	2.12	-13.88	8.43
1965	-6.44	4.01	10.08	1.89	8.72	- 1.59	- 1.11	4.94
1966	-8.14	- 5.20	22.87	- 2.79	0.31	0.51	0.49	5.82
1967	0.00	12.75	-14.96	0.00	- 4.00	- 17.24	- 0.72	4.41
Weighted average of absolute differences	2.14	5.57	22.73	4.53	8.88	19.22	5.08	6.25
				Saskatchewan				
1958	-0.54	-20.15	- 1.21	0.02	- 42.61	- 15.89	7.79	6.47
1959	3.28	-14.30	- 1.04	5.91	- 65.19	- 19.41	2.28	4.74
1960	4.64	- 6.89	3.75	3.45	4.45	- 2.02	- 1.01	3.17
1961	-1.00	-71.48	0.73	- 2.29	15.32	- 47.06	7.29	7.39
1962	-1.42	29.10	- 8.59	14.34	-141.90	- 30.48	0.94	4.99
1963	0.70	- 0.27	6.39	2.39	8.89	- 22.65	- 1.46	1.50
1964	1.49	4.24	-29.92	15.84	43.96	23.88	- 1.65	3.48
1965	-4.67	24.76	- 7.77	20.01	60.94	- 2.48	0.76	5.01
1966	0.00	- 5.59	19.03	- 1.11	8.72	0.00	- 2.32	2.30
1967	-0.15	-31.50	4.83	19.00	- 21.51	40.99	0.79	2.68
Weighted average of absolute differences	1.97	21.21	6.90	9.11	37.10	18.01	2.62	4.17
				Alberta				
1958	-8.32	15.22	1.60	7.63	- 3.18	-147.62	1.38	5.49
1959	0.16	- 3.14	- 1.67	9.38	3.03	- 71.18	1.61	1.62
1960	0.17	- 1.39	- 0.43	19.17	11.58	27.29	- 1.41	1.57
1961	0.15	- 6.23	- 1.57	19.99	- 60.22	25.61	3.85	4.27
1962	-0.95	15.22	-17.60	9.50	- 20.24	68.01	1.22	6.49
1963	0.85	2.18	5.10	19.09	4.59	- 59.02	- 2.80	3.37
1964	3.23	- 1.28	2.44	-10.49	13.48	6.08	4.81	3.77
1965	-2.44	15.28	- 5.96	13.89	- 2.34	- 19.05	1.95	5.03
1966	0.00	- 1.01	1.62	5.39	1.14	19.46	- 2.37	1.74
1967	-1.97	-15.03	4.62	4.72	- 25.66	6.16	2.99	4.48
Weighted average of absolute differences	1.74	7.69	4.11	12.26	12.80	23.32	2.42	3.78

[a]A positive difference indicates an underestimate and a negative error represents an overestimate.

Table 3 illustrates moderately close agreement between the actual and solution acreages of oats in Manitoba and Alberta, but shows gross dissimilarity in Saskatchewan. In this province, the weighted average of absolute differences was twenty-one percent. Also, Figure 14 shows that the model estimates for Saskatchewan were considerably greater than the actual acreages during the period 1958 to 1961. In 1962, however, an eighty-two percent rise in the crop acreage of this province was estimated with reasonable accuracy. In Alberta, the model estimates were moderately close to actual acreages in all years except 1958, 1962 and 1965. It can, however, be concluded that estimates of oats acreages in the prairie provinces were not satisfactory.

The estimated barley acreages for Manitoba were widely different from actual observations. From Figure 15, it can be seen that the crop was overestimated for most of the period. By contrast, the error was reasonably small for Saskatchewan and Alberta. In these provinces, barley acreages were continuously declining through 1962, but these declines were correctly estimated in all years except 1962.

As shown in Table 3, the weighted average of absolute deviations for rye was less than five percent in Manitoba, but more than nine percent in Saskatchewan and Alberta. In the latter two provinces, Figure 16 also illustrates that the solution acreages were substantially lower than the actual data. A general conclusion drawn from the analyses of Saskatchewan and Alberta is that the model accurately estimated year-to-year decreases in rye acreages, but failed to explain the sharp increases.

Table 3 and Figure 17 present the discrepancies between the model estimates and actual acreages of flaxseed. The average error was as large as thirty-seven percent. In Saskatchewan, the model overestimated the crop in the early years and substantially underestimated in 1964 and 1965. In Manitoba, the model did not explain declines of flaxseed acreages in 1958 and 1962. Rather it estimated increases in these years over the previous year acreages. In other words, the turning points in the changes in flaxseed acreages were not correctly estimated. Also, in Alberta, the direction of change in the acreage of this crop was incorrectly estimated in 1961. Based upon these results, it can be concluded that the models for Manitoba, Saskatchewan and Alberta did not accurately estimate the changes in flaxseed acreages, but, rather produced many turning point errors.

The model estimates and actual acreages of rapeseed are graphed in Figure 18. In Manitoba, changes in rapeseed acreages were strikingly well explained for all years except 1958, 1959 and 1961. In this province, a sharp increase of 175 percent in 1960 was closely estimated. However, in Saskatchewan and Alberta solution acreages were quite different from actual data. In some years, changes were underestimated and in other years these were overestimated. Therefore, it can be concluded that changes in rapeseed acreages were, in general, poorly estimated in the individual prairie provinces.

Summerfallow acreages in each province were closely approximated by the models. The average error during 1958 to 1967 for any one province was less than seven percent (Table 3). However, Figure 19 shows that in Manitoba an increase in summerfallow acreage between 1957 and 1958 was not fully estimated. In Saskatchewan, an increase of one million acres in 1958 and 1961 each was completely unexplained. However, in 1966, a drop of the same magnitude was closely estimated. In Alberta, only fifty percent of the increase in summerfallow acreage in 1965 was explained. Therefore, it can be concluded that the models were successful in estimating sharp declines in acreages, but their performance was not very satisfactory in explaining sizeable increases.

Summarizing the performance of the provincial models in explaining the land utilization patterns, it can be concluded that although there were large errors in the acreage estimates for some crops, the solution acreages were on the average reasonably close to the actual values. On the basis of a subjective evaluation of these results, it appears that the recursive programming models have satisfactory explanatory powers. This good performance of the recursive models was perhaps a result of the precision in the estimates of the flexibility restraints. This follows from the fact that the land utilization patterns were basically determined by these restraints. In a typical solution, the most profitable crops would reach their upper flexibility bounds, the less profitable crops would go to

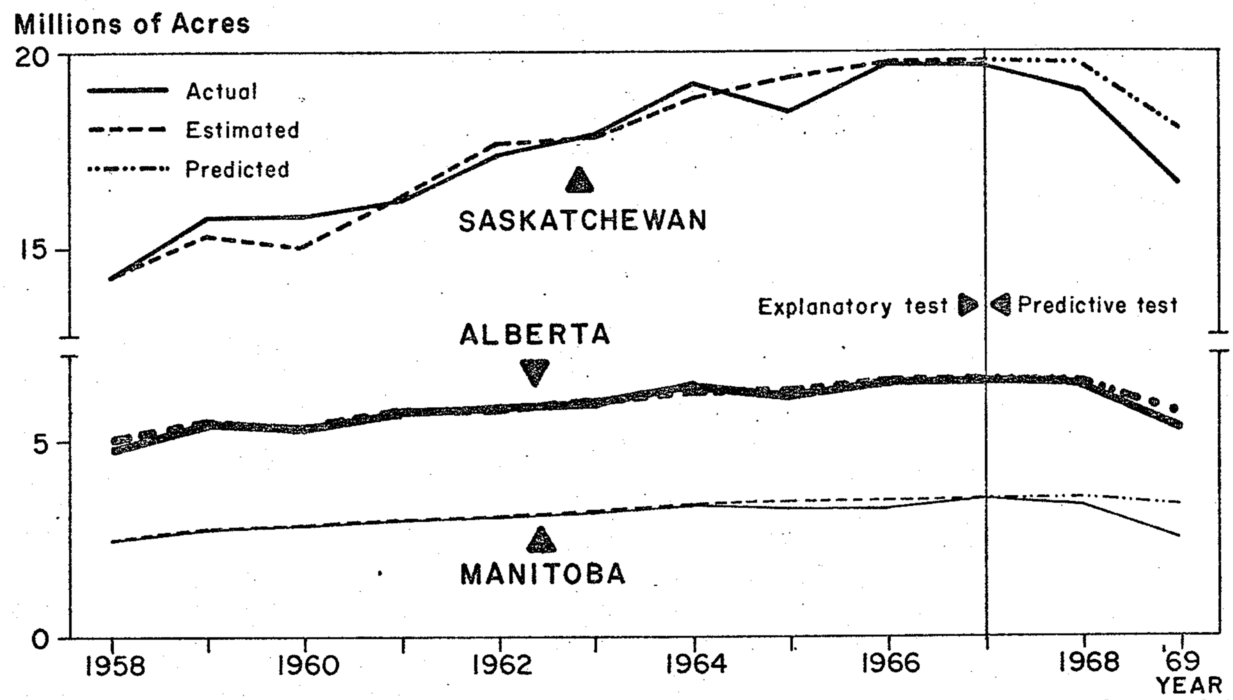

Figure 13. Explanatory and predictive tests of wheat acreage estimated by the provincial models, 1958 - 1969.

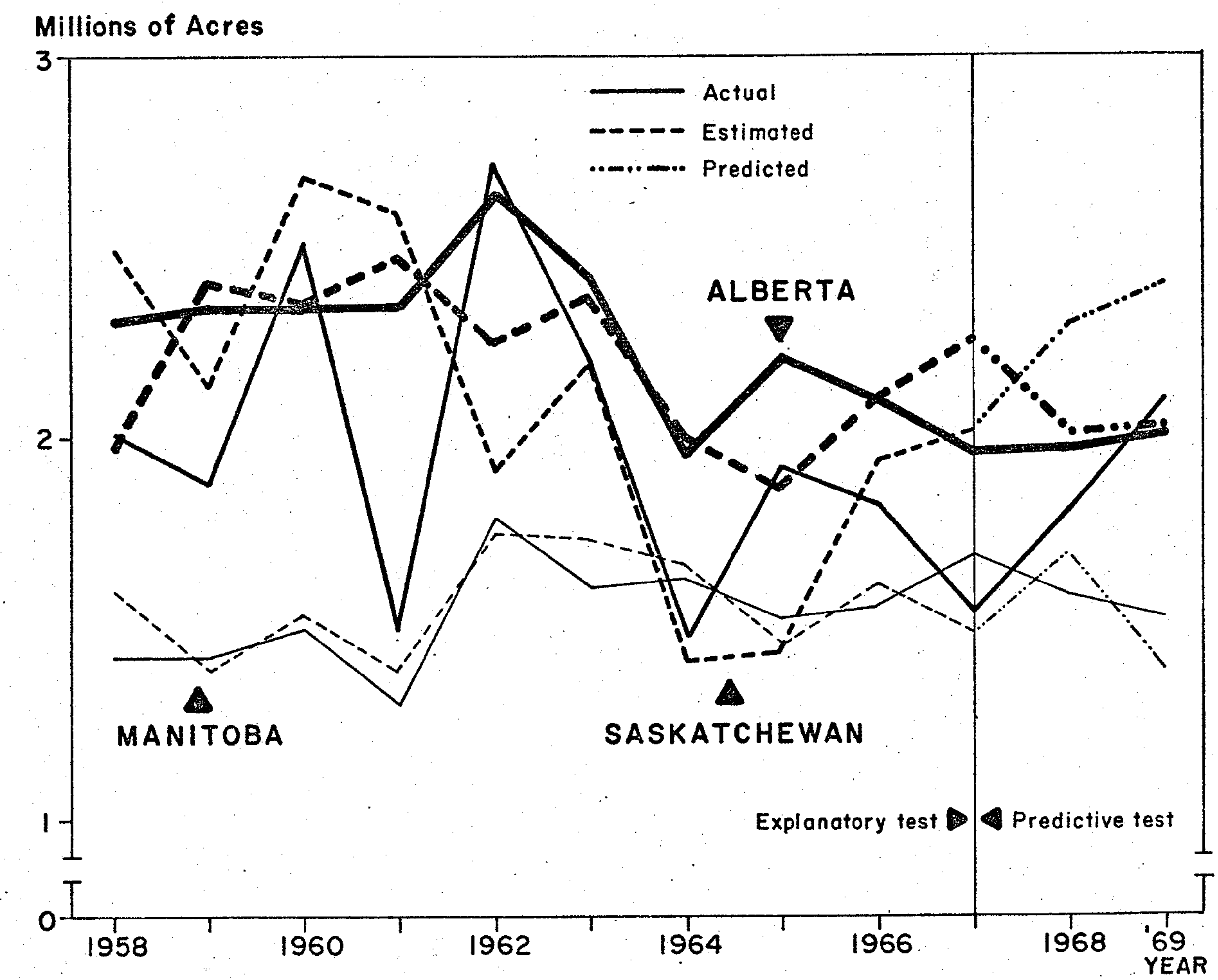

Figure 14. Explanatory and predictive tests of oats acreage estimated by the provincial models, 1958 - 1969.

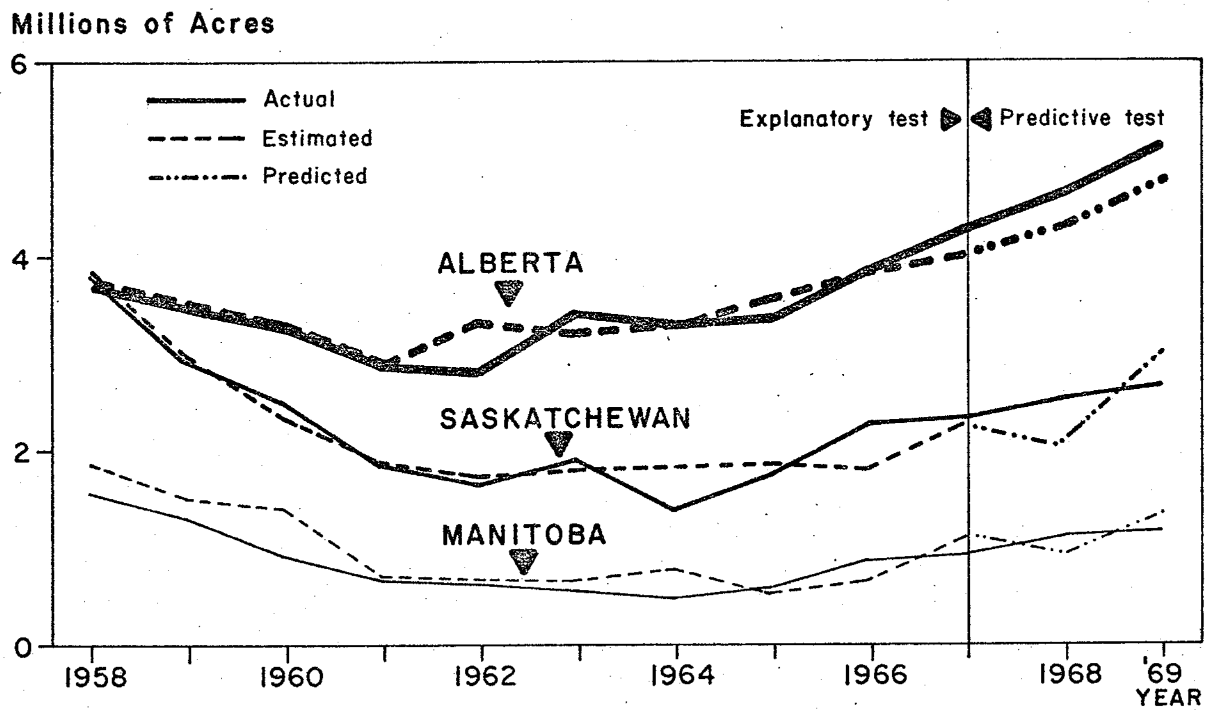

Figure 15. Explanatory and predictive tests of barley acreage estimated by the provincial models, 1958-1969.

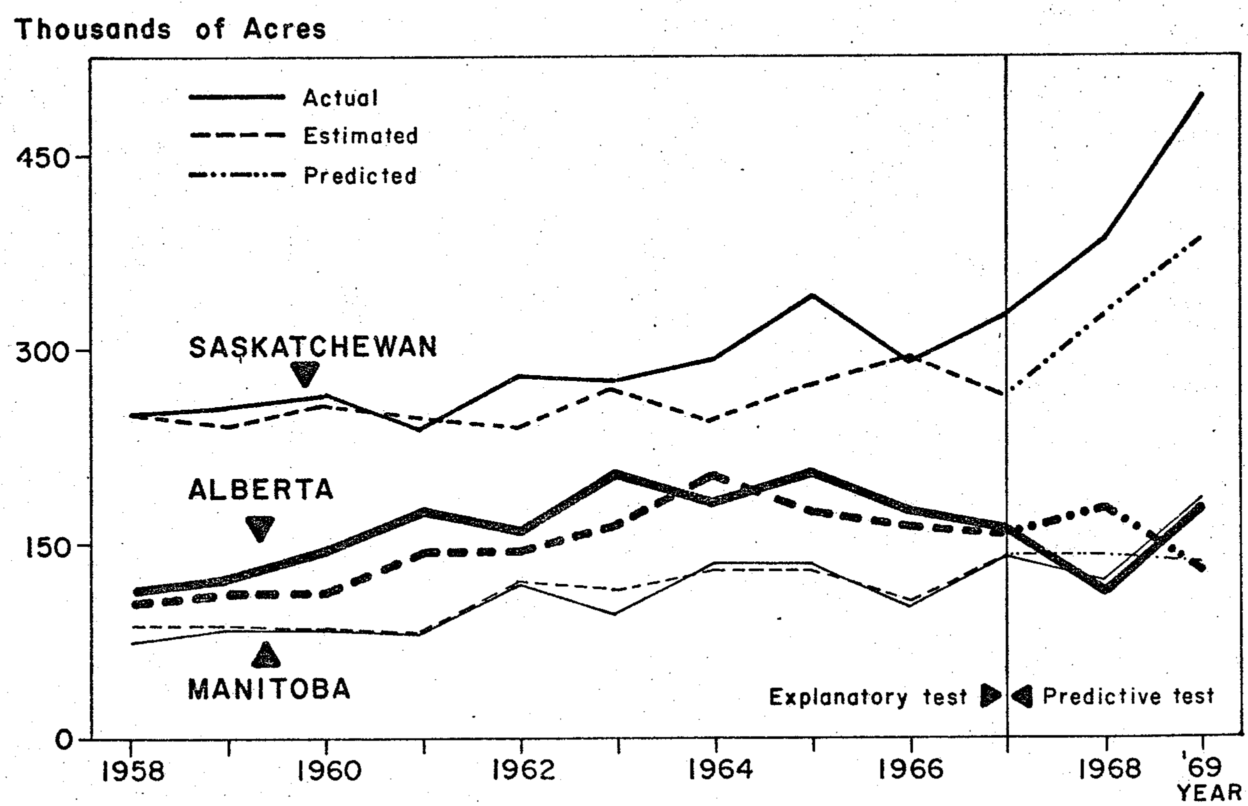

Figure 16. Explanatory and predictive tests of rye acreage estimated by the provincial models, 1958-1969.

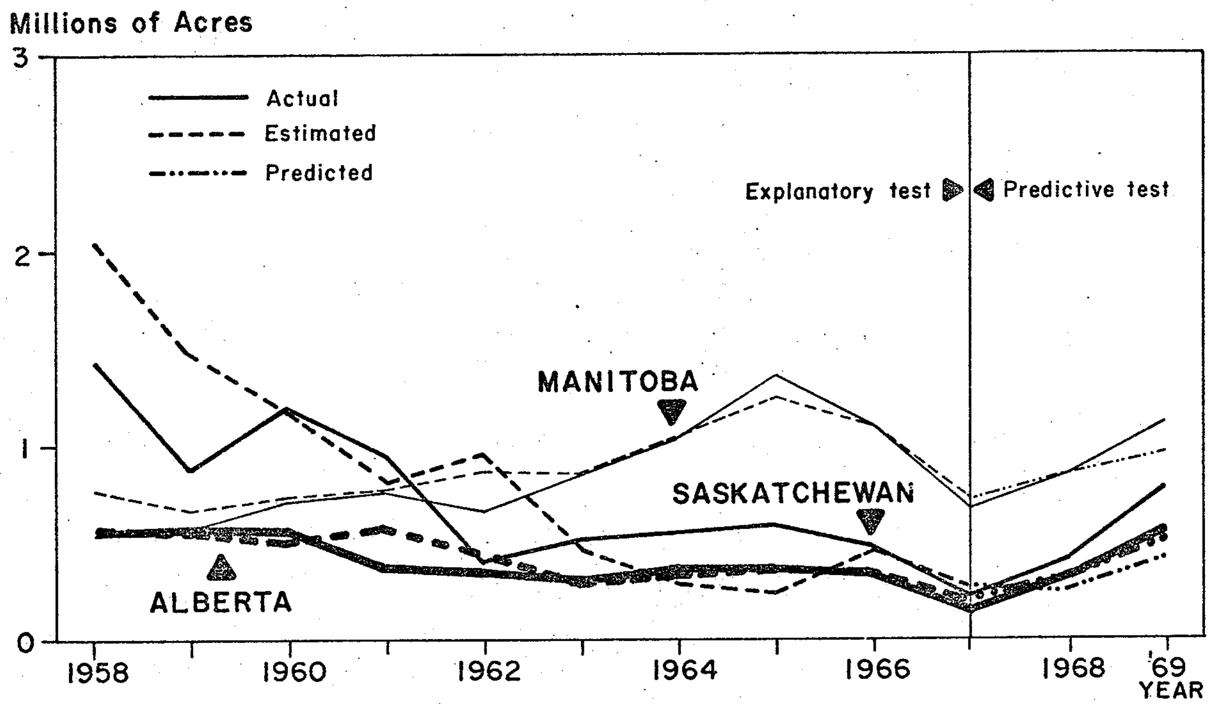

Figure 17. Explanatory and predictive tests of flaxseed acreage estimated by the provincial models, 1958-1969.

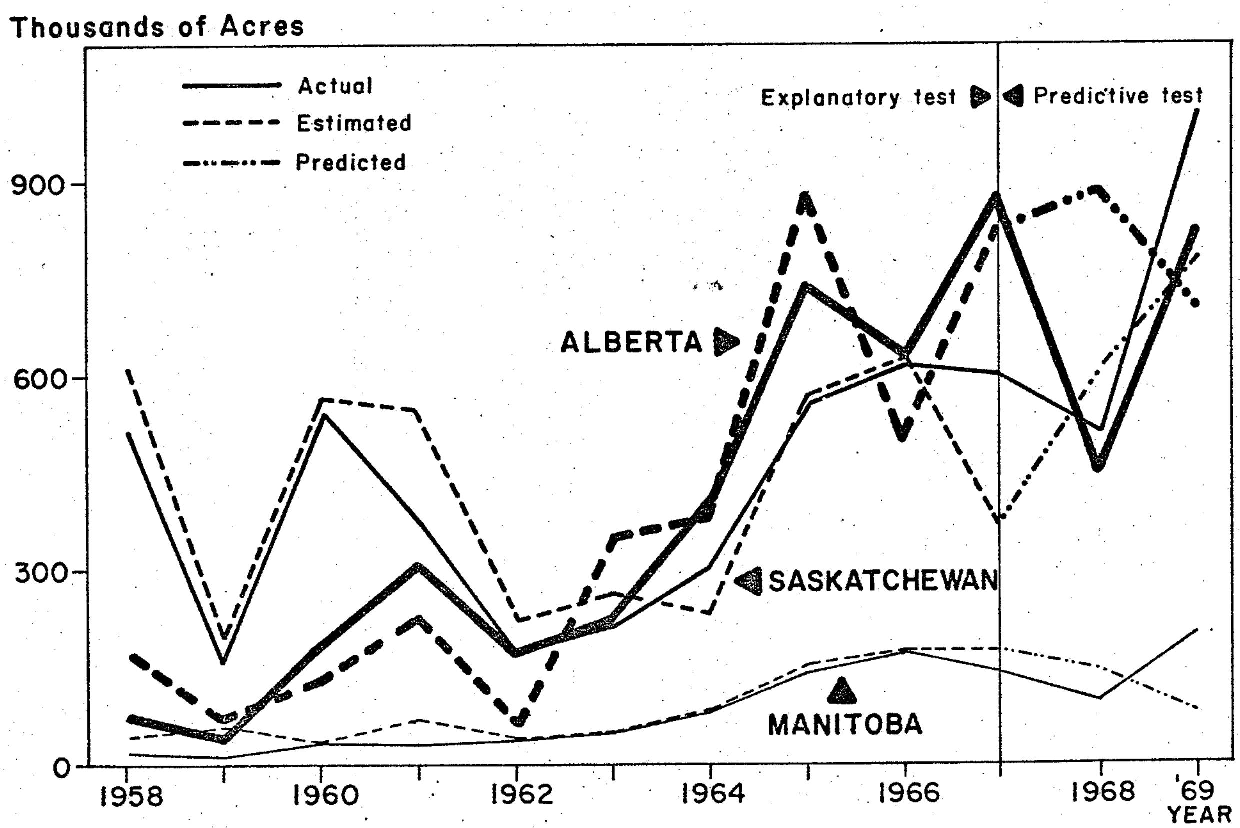

Figure 18. Explanatory and predictive tests of rapeseed acreage estimated by the provincial models, 1958-1969.

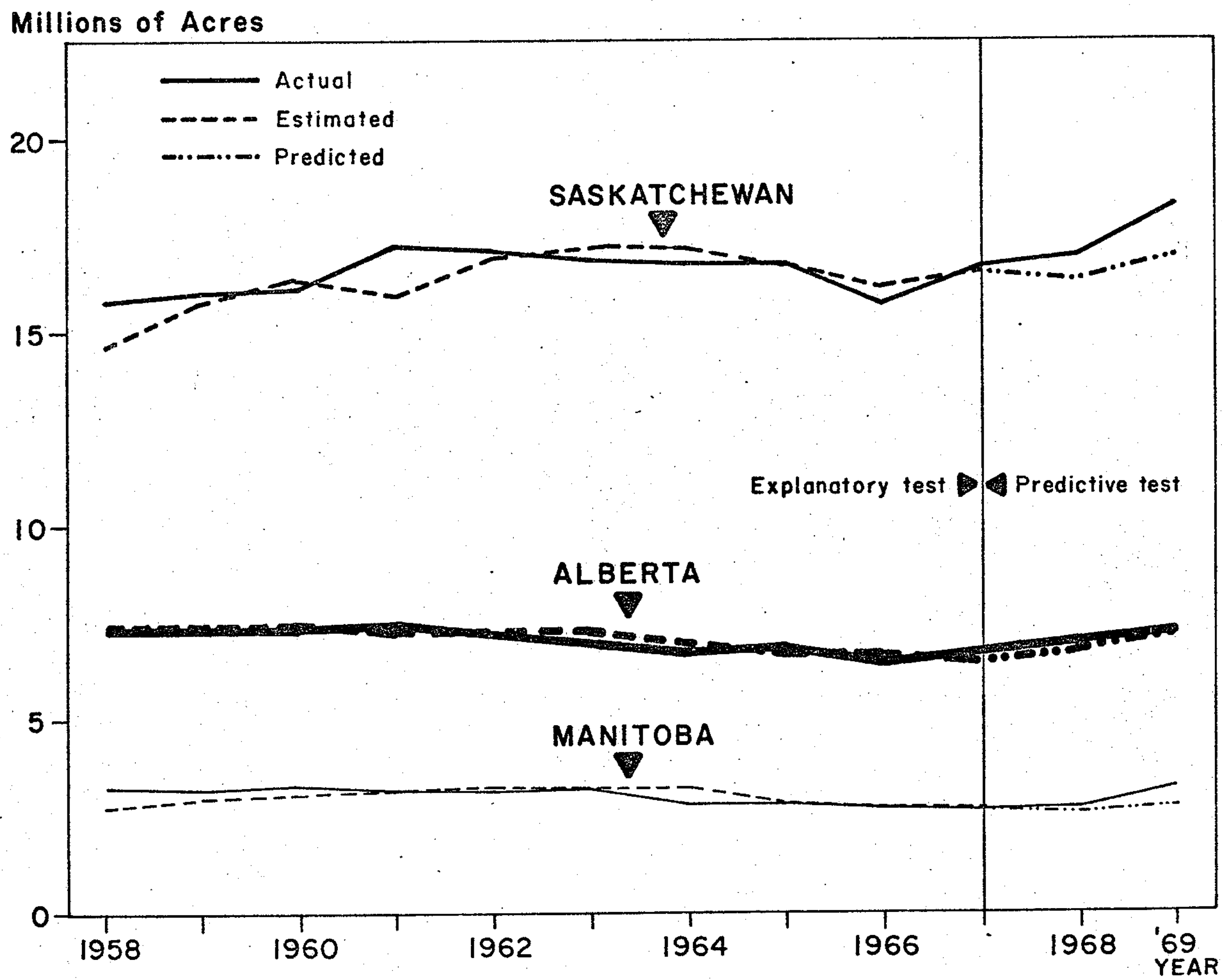

Figure 19. Explanatory and predictive tests of summerfallow acreage estimated by the provincial models, 1958-1969.

lower bounds, and one crop would be between the bounds, restricted by the total crop land constraint. Absolute acreage constraints would affect the model solutions only in a few years.

Sources of Errors in the Model Estimates

Before examining the sources of errors, it must be pointed out that the data which were utilized as norms were estimates published by the Dominion Bureau of Statistics. This source is not of impeccable accuracy. The data are subject to errors of observation. In treating them as norms for evaluating the performance of our analysis, we may attribute errors to the model which actually occur in the published data. However, in the absence of better norms, it is necessary to use these data for the models' evaluation.

During the period 1958 to 1967, one of the significant discrepancies between the estimated and actual acreages in each of the four models (i.e., Manitoba, Saskatchewan, Alberta and the prairies) existed for wheat in 1965. As indicated earlier, wheat acreages in these models declined in 1965 from the preceding year levels, but solution acreages showed increases. Hence, the models did not estimate the directions of change and, therefore, produced turning point errors. Two plausible hypotheses can be put forward to explain these errors. One is that errors result from the fact that land utilization patterns were estimated using highly aggregate models. These models estimate either full change or no change at all in crop acreages. This situation arises because the models presuppose perfect homogeneity in the response of individual farm firms. But this assumption does not hold in the real world because there are many interfarm differences which create variations in response to price and other economic variables.

The second hypothesis for explaining the turning point errors is that the expected net returns per acre for crops included in the model are not accurate. This inaccuracy could be a result of the application of inappropriate expectation models for estimating prices, costs, yields and/or quotas. In other words, the expectation model utilized in this study (i.e., the current year expected values equal to the preceding year actual data) does not appear to be the true model used by the prairie grain producers.

These two hypotheses are not necessarily independent; together they make the programming model in some years insensitive to price changes. The price of wheat utilized for the 1965 analysis was eight to eighteen cents per bushel lower than that used for 1964, depending on the provincial model considered. Perhaps farmers reacted to this price decline and <u>reduced</u> wheat acreage in 1965. But, in the recursive programming analysis, net income per acre of wheat was still higher than other crops and, therefore, the 1965 solution acreage reached the upper flexibility bound which was more than the actual acreage of the previous year.

Hence, the decline in the wheat acreage was not anticipated by the models. Rather, the models estimated increases.

Another major discrepancy between actual and estimated acreages occurred in the case of rapeseed. The area of this crop declined substantially in 1958 and 1959. But the crop was considerably overestimated in all four models due to the turning point errors. These errors were also caused by the application of highly aggregate recursive models which did not respond to declines in rapeseed prices.

In 1962, the solution acreages of flaxseed in all four models grossly exceeded the actual values because the programming models were non-responsive to declines in flaxseed prices. As a result, the turning point errors in changes in the acreages of this crop were produced.

A thorough examination of the solution of models, during the period 1958 to 1967, shows that most of the discrepancies between actual and estimated acreages took place as a result of turning point errors. Table 4 illustrates that the frequency of turning point errors in the estimates of crop acreages was very high in all provinces. Moreover, considering the high level of performance of the recursive programming model in explaining the crop acreages, the frequency of the errors is particularly large.

TABLE 4

FREQUENCY OF TURNING POINT ERRORS IN THE ESTIMATES OF THE CROP ACREAGES IN THE PRAIRIES, 1958 TO 1967

Crops	Total number of turning points in each model	Number of turning point errors			
		Prairies	Manitoba	Saskatchewan	Alberta
Wheat	10	4	1	3	0
Oats	10	6	2	5	6
Barley	10	4	3	2	3
Rye	10	2	4	6	4
Flaxseed	10	6	4	7	4
Rapeseed	10	1	5	1	0
Summer-fallow	10	3	3	3	4

As described earlier, the turning point errors were mainly brought about by the insensitivity of the aggregate programming models for changes in the price of a crop. In the two hypotheses mentioned above, the insensitivity could have been caused by the use of: (1) highly aggregate data and (2) inaccurate net income coefficients in the programming models. Hence, the techniques and expectation models utilized to derive the net income components (such as price, costs, yield and/or quota levels) might not be those used by prairie farmers and, therefore, might have produced bias in the estimates of the net return per acre. No comment is made here about the errors resulting from the application of aggregate models. Rather, this is discussed in section three.

There are some discrepancies between estimated and actual acreages, which cannot be explained through the sources discussed above and, therefore, other explanations have to be sought. The increases of wheat acreages between 1958 and 1959 in Saskatchewan and for the prairies were only partly explained by the recursive model. This underestimation of acreage was caused by the relatively narrow upper flexibility bound for this crop. On the other hand, too wide upper flexibility restraint was responsible for the overestimation of wheat in Manitoba by eight percent in 1966. In Alberta, during the period 1958 through 1967, rapeseed acreages were grossly underestimated in some years and highly overestimated in other years. This was brought about by relatively too wide or too narrow flexibility bounds on acreages of this crop.

Many more examples of the errors created by the inappropriate levels of the flexibility restraints can be cited from the results of the analysis. Thus, the inappropriate estimates of the restraints can be considered as a source of errors. However, this source caused errors only in years where sharp changes in the acreages of a crop had occurred.

Another source of errors in acreage estimates was abnormal weather conditions. In Saskatchewan and the prairies, errors in the estimated acreages of flaxseed in 1958, 1959 and 1962 could be attributed to excessively dry weather conditions. April and May precipitation in these years were only sixty percent of the ten year average (1958-1967). Even though precipitation was considered as a variable in the basic regression model developed for estimating flexibility coefficients, it was omitted from most of the equations because of inadequate degrees of freedom. Moreover, a regression model cannot completely account for abnormal fluctuations in moisture.

The above four were the major sources of errors in the provincial and the prairie models. It was observed that the errors in the estimates were additive: an error in the solution acreage of one crop creates discrepancy between the estimated and actual acreages of other crops. Furthermore, even a small error in the estimates of wheat or summerfallow causes a sizeable error in the solution acreages of the more minor crops.

The Predictive Test

This test is more rigorous than the previous explanatory one because the ability of the model to make predictions outside the period used for its construction is examined, and permanence and completeness of the structure are thereby evaluated. The model which was developed on the basis of the 1958 through 1967 data[a], was used to make predictions for 1968 and 1969. Comparison was then made between the

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[a]Observations during 1953 to 1967 were utilized to estimate the flexibility coefficients.

predicted and actual acreages.

This section is organized also on the lines of the previous one. Results from the aggregate prairie model are discussed first. A presentation of the provincial estimates then follows. In the final portion of this section, sources of the errors in the acreage predictions are examined.

Prairies

The actual and predicted acreages for the years involved in the predictive test are illustrated in Figures 6 through 12 which have been utilized earlier to present the results of the explanatory test. The percentage deviations of predicted from actual acreages are given in Table 5. The overall weighted average of the absolute deviations for 1968 and 1969 was only 7.39 percent. The predicted acreages were reasonably similar to actual observation in 1968. But, in 1969, the average error was as high as eleven percent. However, considering the abnormal situation which existed in the prairie agriculture in that year, the error was moderate.[a]

Figure 6 illustrates that the predicted and actual acreages of wheat were strikingly close in 1968, but the error was substantially large in 1969. The results for oats

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[a]During the 1968 crop year, wheat supplies were at a very high level, exports were lower than the ten year average (1958-1959 to 1967-1968), and the foreign demand outlook for Canadian wheat was poor.

TABLE 5

DIFFERENCE[a] BETWEEN ACTUAL AND PREDICTED LAND USE IN THE PRAIRIES BASED ON THE AGGREGATE MODELS, 1968-1969

Crop	1968	1969	Weighted average of absolute differences	1968	1969	Weighted average of absolute differences
			(percent)			
		Prairies			Manitoba	
Wheat	- 1.70	-16.90	8.66	- 3.53	-34.43	16.62
Oats	- 3.78	-11.29	7.63	- 6.96	8.50	7.72
Barley	1.62	4.51	3.12	17.85	-13.73	15.77
Rye	- 5.33	24.04	16.20	-17.50	26.16	22.73
Flaxseed	39.55	39.70	39.64	0.50	15.10	8.86
Rapeseed	-53.99	29.22	32.89	-59.34	62.02	61.17
Summerfallow	2.11	4.96	3.59	3.40	15.31	9.85
Weighted average of absolute differences	3.57	11.10	7.34	6.17	19.99	13.08
		Saskatchewan			Alberta	
Wheat	- 3.68	- 8.57	5.96	- 0.71	- 7.33	3.69
Oats	-27.42	-13.97	20.17	- 5.09	- 4.42	4.75
Barley	18.08	-10.95	14.39	7.53	6.01	6.74
Rye	15.06	22.38	19.18	-56.20	26.32	37.91
Flaxseed	42.85	49.81	47.44	- 4.65	8.13	6.94
Rapeseed	-17.42	22.26	20.62	-94.44	14.04	42.62
Summerfallow	3.54	7.11	5.39	4.26	- 0.50	2.32
Weighted average of absolute differences	6.17	9.61	7.89	6.20	4.81	5.51

[a]A positive difference indicates an underestimate and a negative error represents an overestimate.

are presented in Figure 7. The prediction of acreage was found remarkably accurate in 1968, but grossly disparate in 1969. Table 5 and Figure 8 show that the model predicted barley acreages very closely to actual values in both years. The discrepancy between the solution and actual acreages of rye is illustrated in Figure 9. The 1968 acreage was closely predicted, but in 1969, the actual acreage was twenty-four percent greater than the predicted values. Flaxseed results are presented graphically in Figure 10. The model predicted forty percent larger acreage for this crop in each year. Figure 11 shows that rapeseed acreages fluctuated violently during 1968 to 1969. The actual acreage declined by thirty-five percent (0.6 million acres) between 1967 and 1968, but the model predicted an increase in acreage. Therefore, the predicted acreage was considerably more than the actual. In 1969, the actual acreage of this crop doubled from the preceding year level, but a decline in acreage was predicted by the recursive model. Therefore, actual acreage was substantially greater than that predicted. Figure 12 illustrates accurate predictions of summerfallow acreage in both years.

Provinces

The percentage deviations of predicted from actual acreages for each of the prairie provinces are presented in Table 5. Overall average errors for the entire land use were reasonably small in Saskatchewan and Alberta, but very large in Manitoba--thirteen percent. Year-wise, the quality of the prediction was quite variable. In 1968, the average error for any one province was only six percent. However, in 1969, the acreages were predicted with reasonable accuracy only in Saskatchewan and Alberta. In Manitoba, the average error was a high of twenty percent.

Figure 13 shows that the predicted acreages of wheat were strikingly close to actual observations in each province in 1968. But, in 1969, the model over-predicted this crop by one million acres in Manitoba and by 1.5 million acres in Saskatchewan. In Alberta, acreages were satisfactorily predicted in both years. As shown in Figure 14, the predicted acreages of oats were significantly similar to actual values in both years in every province except Saskatchewan. In this province, the solution acreage exceeded actual in both 1968 and 1969. In the case of barley, the graphic representation of Figure 15 shows that the actual acreages were greater than predicted in 1968 in each province. By contrast, in 1969, the predicted acreages were greater than actual. Figure 16 presents the results for rye. Errors were large in both years, especially in 1969. The discrepancies between actual and predicted acreages for flaxseed are illustrated in Figure 17. The differences were moderate in Manitoba and Alberta. But, in Saskatchewan, the predicted acreages were only half of the actual observations in each year. Figure 18 presents results for rapeseed. In every province, the 1968 predicted acreages were considerably greater than the actual. A maximum error of ninety-five percent occurred in Alberta. In 1969, the model predictions of acreages were substantially lower than the actual data of each province. The results for summerfallow are shown in Figure 19. The predicted values were fairly accurate for all provinces. The only exception was for Manitoba in 1969.

On the basis of the above observations, it can be concluded that the 1968 land utilization patterns were predicted with reasonable accuracy in all provinces, and the recursive model had a satisfactory predictive power. But the performance of this model in 1969 was poor. However, this year was considerably abnormal for prairie agriculture. Therefore, the 1969 results cannot be viewed as conclusive evidence of the predictive power of the model.

Sources of Errors in the Predictive Analysis

One of the largest errors in the acreage predictions of each model, provincial as well as prairie, was for rapeseed in 1968. The predicted acreages were substantially greater than the actual. These discrepancies were caused due to the inaccurate prediction of turning points in changes in rapeseed acreages. Acreage of this crop declined by more than thirty-five percent between 1967 and 1968. Perhaps this decline was brought about by farmers' response to a major decrease in the expected prices of rapeseed for 1968.

But the programming models were not sensitive to the lower price and, therefore, did not predict the accurate direction of changes in rapeseed acreages.

In 1968, the prediction of flaxseed acreage for the prairies was not very precise. Also, barley acreages in Manitoba and Saskatchewan were not predicted with a satisfactory degree of accuracy. These discrepancies were also a result of turning point errors in the models. The actual acreages of these crops increased, perhaps because of increase in their relative prices. But the models were insensitive to the product prices, and therefore, predicted small acreages of these crops in 1968.

Other important errors to be explained were associated with wheat, flaxseed and rapeseed in 1969. In each model, predicted acreage of wheat was considerably more than the actual observation. But the solution acreages of flaxseed and rapeseed were substantially less than the actual performance. There exists three possible causes of these errors: (1) the lower flexibility bound for wheat was not low enough; (2) the upper flexibility bounds for flaxseed and rapeseed were not sufficiently high to the extent that the unprecedented increases in their acreages could not be fully predicted; and (3) the upper restraint on stubble land was so high that it allocated more land to crops and less acreage to summerfallow than that required for accurate predictions.

This evidence on flexibility bounds leads us to conclude that the inaccurate estimation of flexibility coefficients causes considerable errors in the solution. The method employed for estimating the coefficients, in this study, does not appear appropriate in years, such as 1969 when very abnormal crop production conditions occur.

The above discussion on errors in the predictions of wheat, flaxseed and rapeseed also confirms the observation made in the explanatory test that errors in the model are additive in nature: a discrepancy between predicted and actual acreages in one crop produces errors in others. Furthermore, it can be observed that a small error in the predicted acreage of a major crop, such as wheat, causes considerable errors in the results for more minor crops.

Testing the Relative Performance of Aggregate Versus Disaggregate Models

In the previous sections, it was hypothesized that the more aggregate the data utilized in the recursive programming models, the larger would be the possible errors in the estimates of crop acreages. In the present section, an analysis is undertaken to examine this hypothesis. Two sets of comparisons are made. In one set, the aggregate estimates obtained directly by utilizing the prairie data were compared with the results derived by summing the estimates of Manitoba, Saskatchewan and Alberta provincial models. In the second set of comparisons, estimates for Saskatchewan obtained by utilizing the provincial data were compared with results derived by totaling the outcome of an analysis for each of the twenty crop districts of the province.

For the sake of clarity, these four methods are referred to as follows:

(1) Model P-1 = prairie model utilizing aggregate data,

(2) Model P-2 = prairie model based upon the summation of the provincial results,

(3) Model S-1 = model for Saskatchewan using provincial data, and

(4) Model S-2 = model for Saskatchewan based upon the summation of the results for the twenty crop districts of the province.

The percentage deviations of estimated from actual acreages were calculated by crops and by years during the period 1958 to 1969. The results for the prairies are presented in Table 6 and for Saskatchewan in Table 7. The solution and actual acreages of selected crops (viz., wheat, barley and flaxseed) are illustrated in Figures 20 to 22 for the prairies and in Figures 23 to 25 for Saskatchewan. The overall weighted averages of percentage deviations in model P-1 were reasonably close to those in model P-2 for both periods 1958 to 1967 and 1968 to 1969. Also, the overall average errors in model S-1 were almost equal to those in model S-2 for both periods.

A crop-by-crop comparison of the errors in the estimates from models P-1 and P-2 shows that the performance of

TABLE 6

DEVIATION[a] OF ESTIMATED FROM ACTUAL PRAIRIE LAND USE, BY MODELS, 1958-1969

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Weighted average of absolute deviations	
	P-1	P-2	P-1	P-2	P-1	P-2	P-1	P-2	P-1	P-2	P-1	P-2	P-1	P-2	P-1	P-2
	(percent)															
1958	-3.13	-1.99	13.01	-4.15	0.26	-2.79	0.00	-1.71	-37.05	-33.16	-100.06	-33.65	5.51	6.76	6.74	5.66
1959	4.26	2.32	1.43	-5.56	-4.86	-4.37	4.00	5.24	-35.34	-32.25	-212.16	-48.75	1.59	2.70	4.63	4.06
1960	-2.67	3.01	9.27	-3.96	-0.41	-6.60	0.92	7.19	-2.97	4.04	-6.34	4.95	0.74	0.16	2.34	2.41
1961	-0.65	-0.71	-31.09	-25.40	2.91	-2.16	5.82	5.96	15.38	-4.64	-7.46	-18.79	4.69	5.48	5.46	5.10
1962	-0.52	-1.10	14.91	16.96	-9.16	-13.51	7.42	9.95	-29.89	-58.76	-25.59	17.46	0.02	0.63	3.26	4.84
1963	-0.97	0.71	0.94	-1.00	4.36	4.01	1.20	6.58	14.91	3.86	-12.03	-37.16	-0.91	-1.63	1.64	1.76
1964	1.06	1.79	0.00	-0.03	-13.36	-12.73	2.65	7.16	37.14	13.52	15.65	12.48	-1.75	-3.79	3.36	3.76
1965	-6.07	-4.39	10.47	15.46	8.50	-4.83	1.72	16.84	35.28	19.90	-1.53	-10.88	-0.68	1.25	5.39	4.88
1966	0.09	-1.92	-4.14	-4.17	11.84	9.60	1.31	16.88	3.45	0.83	-6.23	16.04	-2.45	-2.32	2.64	3.42
1967	1.18	-0.53	-10.95	-12.79	2.28	2.18	-2.15	11.10	-13.07	-10.53	5.86	16.97	0.32	1.20	1.96	2.43
1968	-1.70	-3.00	-3.78	-13.17	1.62	12.16	-5.33	-4.37	39.55	10.72	-53.99	-53.99	2.11	3.72	3.57	5.98
1969	-16.90	-10.95	-11.29	13.51	4.54	-1.71	24.04	24.01	39.70	24.56	29.22	22.80	4.96	6.07	11.42	9.09
Weighted average of absolute deviations (1958-1967)	2.00	1.83	9.59	8.92	5.34	5.88	2.63	9.49	22.99	17.74	19.55	17.61	1.87	2.59	3.74	3.83
Weighted average of absolute deviations (1968-1969)	6.66	6.64	7.63	13.36	3.12	6.73	16.20	15.79	39.64	19.26	37.72	33.51	3.59	4.94	7.50	7.54

[a]A positive difference indicates an underestimate and a negative error represents an overestimate.

TABLE 7

DEVIATION[a] OF ESTIMATED FROM ACTUAL LAND USE IN SASKATCHEWAN, BY MODELS, 1958-1969

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Weighted average of absolute deviations	
	S-1	S-2	S-1	S-2	S-1	S-2	S-1	S-2	S-1	S-2	S-1	S-2	S-1	S-2	S-1	S-2
								(percent)								
1958	-0.54	-0.05	-20.15	-24.39	- 1.21	-10.52	0.02	5.97	- 42.61	- 27.40	-15.89	..	7.79	8.32	6.47	7.01
1959	3.28	7.08	-14.30	-18.66	- 1.04	-24.91	5.91	19.90	- 65.19	- 77.60	-19.41	..	2.28	8.11	4.74	11.20
1960	4.64	2.53	- 6.89	8.31	3.75	-14.49	3.45	19.81	4.45	- 12.47	- 2.02	- 4.92	-1.01	3.25	3.17	4.41
1961	-1.00	0.22	-71.48	-93.35	0.73	-45.33	- 2.29	10.58	15.32	- 32.66	-47.06	-15.04	7.29	5.64	7.39	9.49
1962	-1.42	1.00	29.10	- 0.21	- 8.59	-31.96	14.34	40.68	-141.90	-121.32	-30.48	- 6.06	0.94	3.58	4.99	4.81
1963	0.70	-0.59	- 0.27	0.08	6.39	- 6.76	2.39	29.58	8.89	6.72	-22.65	- 8.32	-1.46	-0.43	1.50	1.11
1964	1.49	1.23	4.24	20.41	-29.92	-33.64	15.84	10.87	43.96	6.62	23.88	10.15	-1.65	0.91	3.48	3.38
1965	-4.67	-2.14	24.76	32.70	- 7.77	-15.58	20.01	28.08	60.94	- 1.88	- 2.48	-10.54	0.76	0.26	5.01	4.02
1966	0.00	2.11	- 5.59	15.85	19.03	2.25	- 1.11	6.00	8.72	- 2.45	0.00	-18.84	-2.32	-3.50	2.30	3.66
1967	-0.15	-0.34	-31.50	4.81	4.83	- 2.44	19.00	22.94	- 21.51	- 77.73	40.99	- 4.89	0.79	1.06	2.68	1.55
1968	-2.68	-2.01	-27.42	-20.94	18.08	2.31	15.06	11.68	42.85	20.94	-17.42	-32.58	3.54	2.35	6.17	3.63
1969	6.57	-8.47	-13.97	- 3.26	-10.95	5.03	22.38	16.72	49.81	10.63	22.26	-34.74	7.11	6.75	9.61	8.00
Weighted average of absolute deviations (1958-1967)	1.97	1.70	21.21	19.09	6.90	17.12	9.11	19.87	37.10	31.56	18.01	10.25	2.62	3.46	4.17	5.06
Weighted average of absolute deviations (1968-1969)	5.96	5.02	20.17	11.42	14.39	3.72	19.18	14.52	47.44	14.14	20.62	34.01	5.39	4.63	7.89	8.17

[a]A positive difference indicates an underestimate and a negative error represents an overestimate.

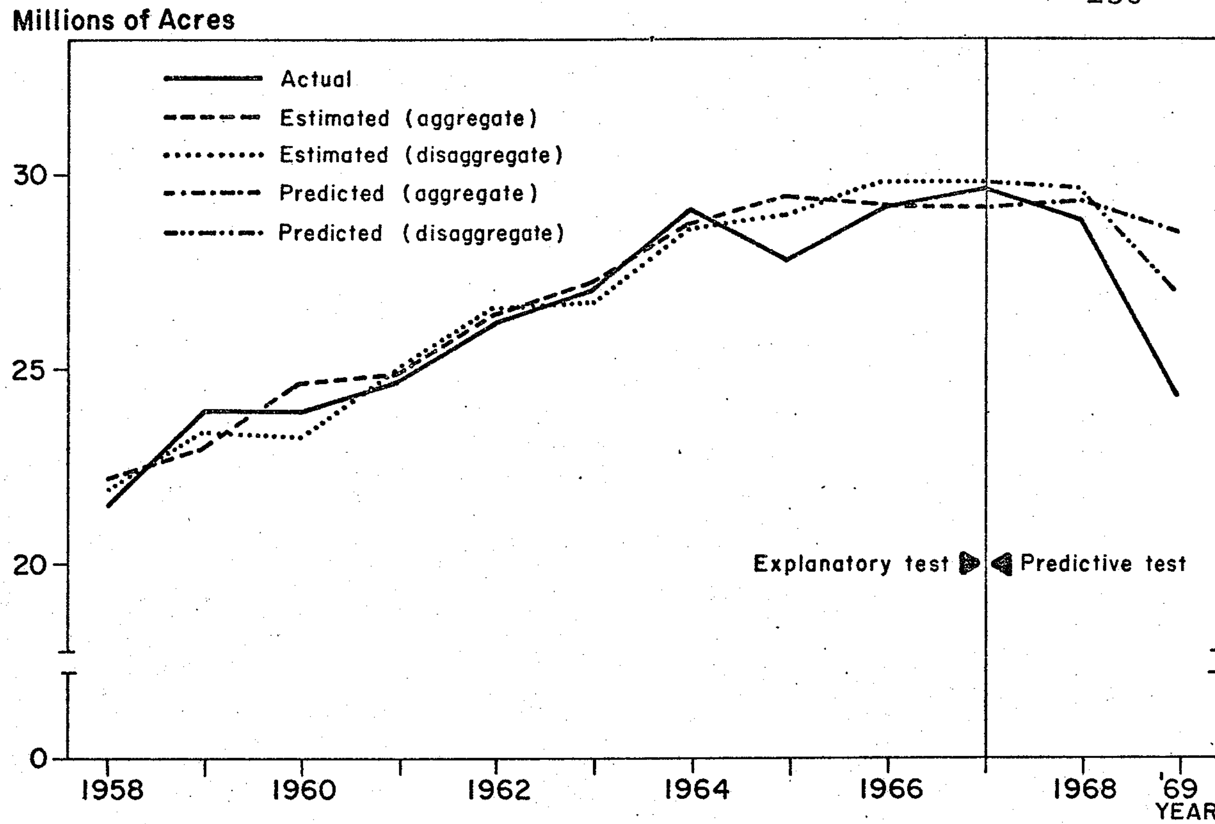

Figure 20 Comparison of results from aggregate and disaggregate models for wheat acreage in the prairies, 1958-1969.

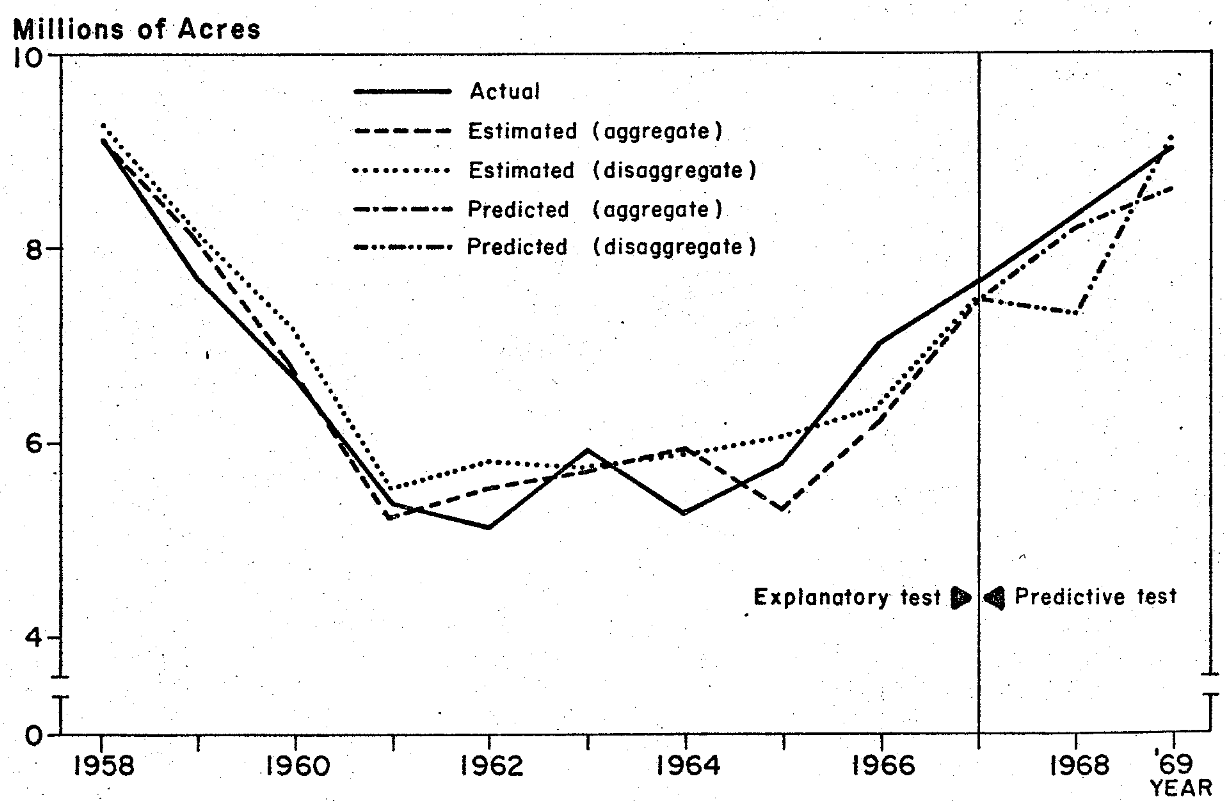

Figure 21. Comparison of results from aggregate and disaggregate models for barley acreage in the prairies, 1958-1969.

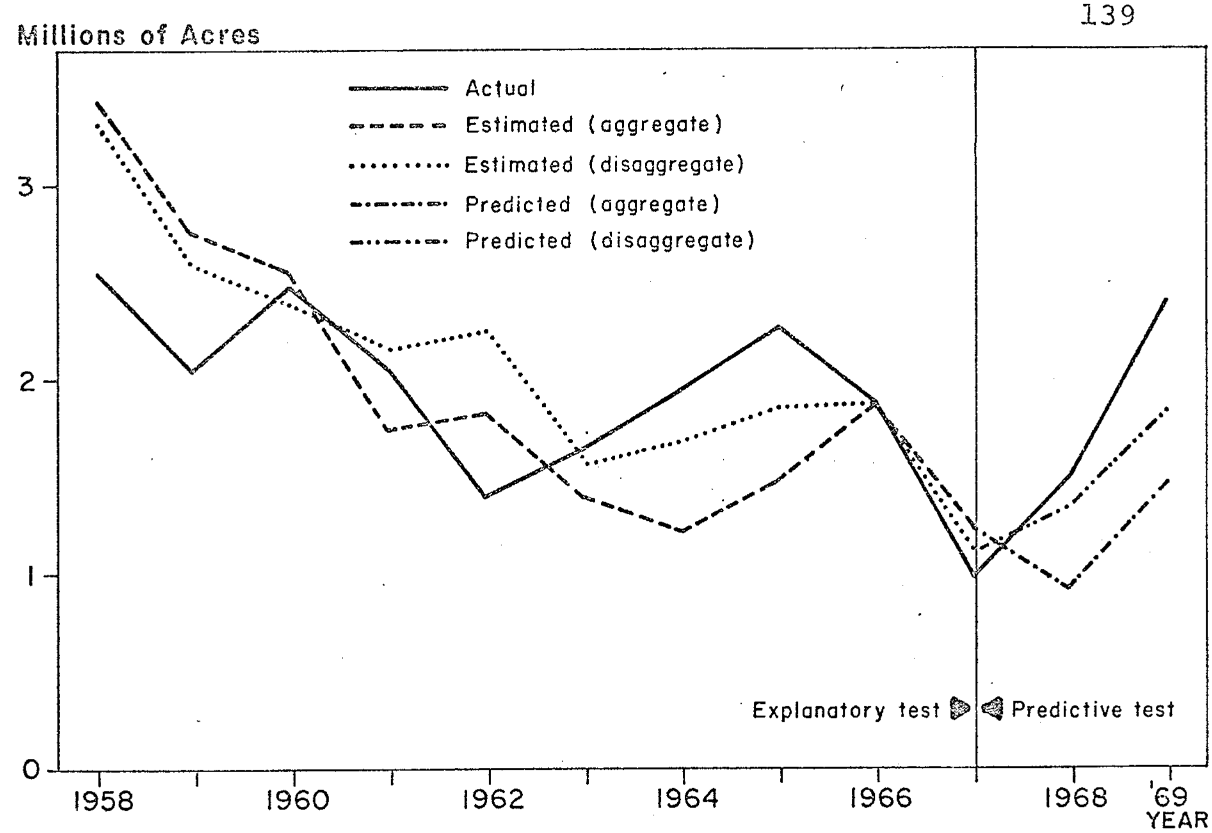

Figure 22. Comparison of results from aggregate and disaggregate models for flaxseed acreage in the prairies, 1958-1969.

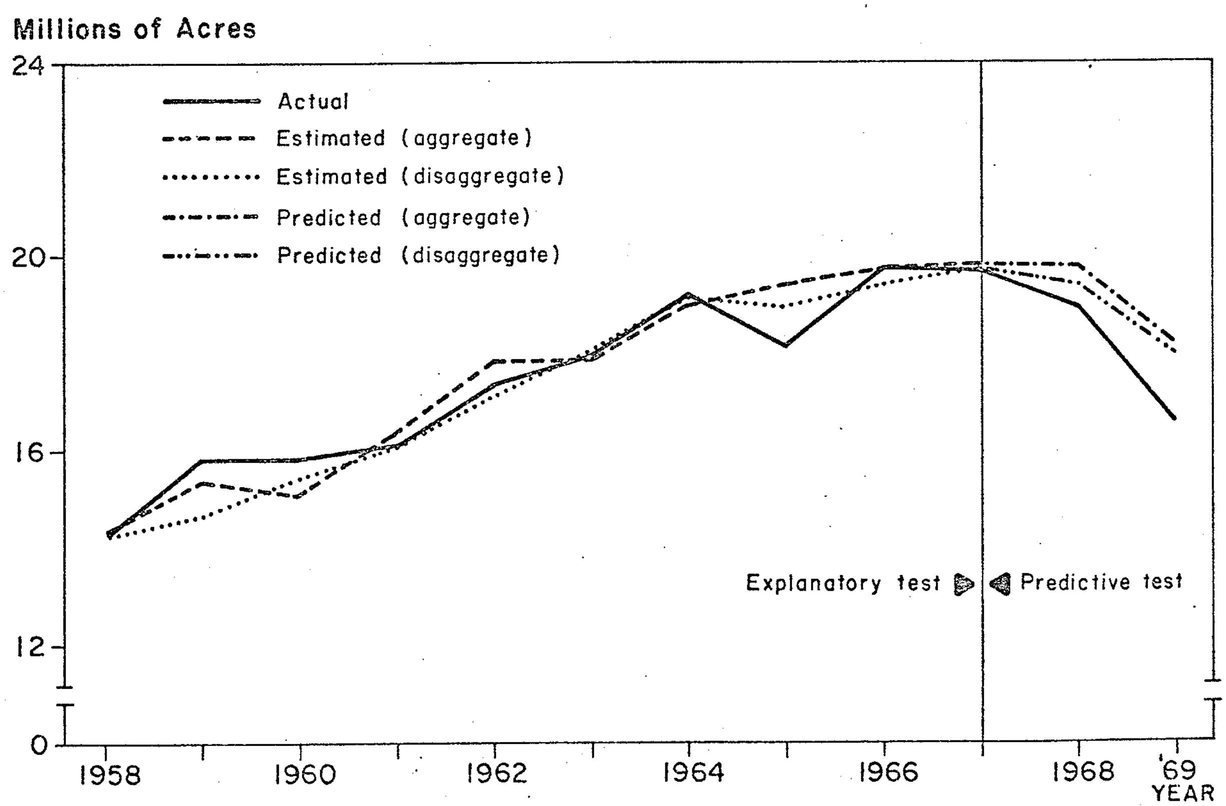

Figure 23. Comparison of results from aggregate and disaggregate models for wheat acreage in Saskatchewan, 1958-1969.

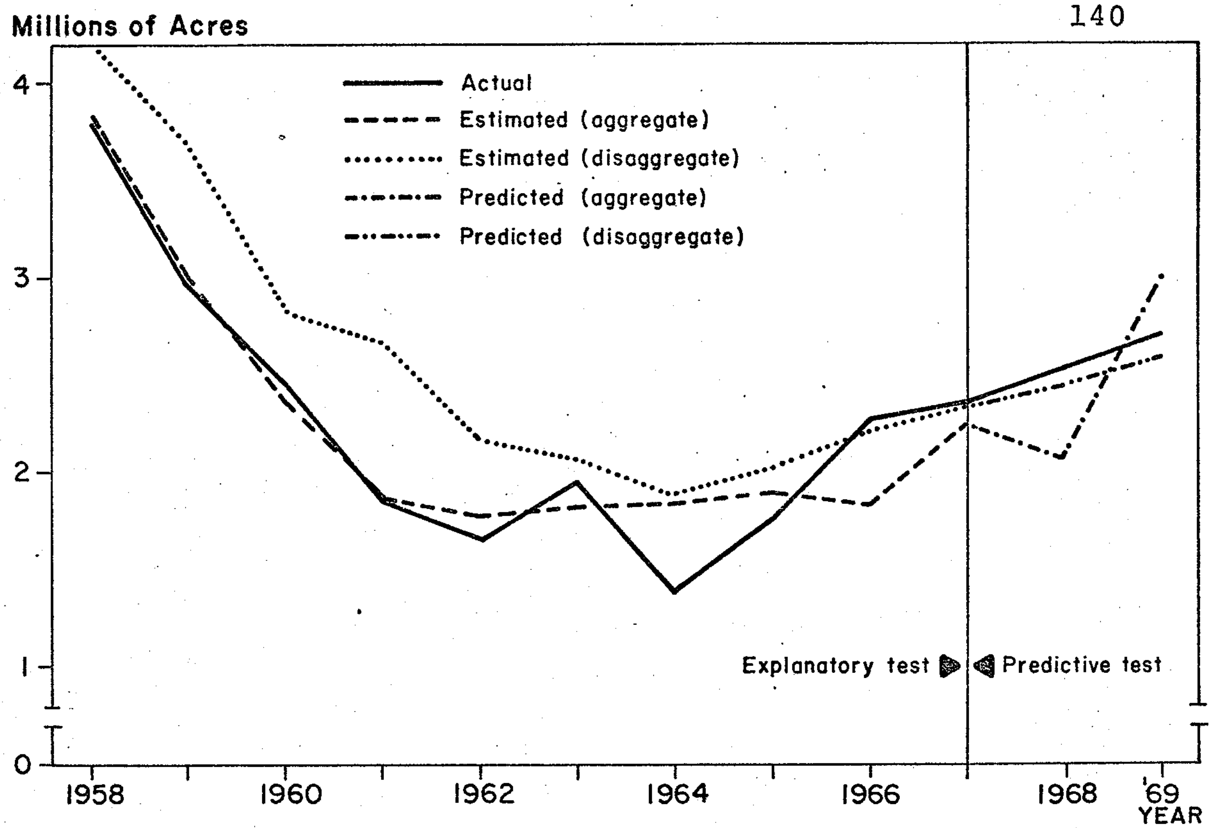

Figure 24. Comparison of results from aggregate and disaggregate models for barley acreage in Saskatchewan, 1958 - 1969.

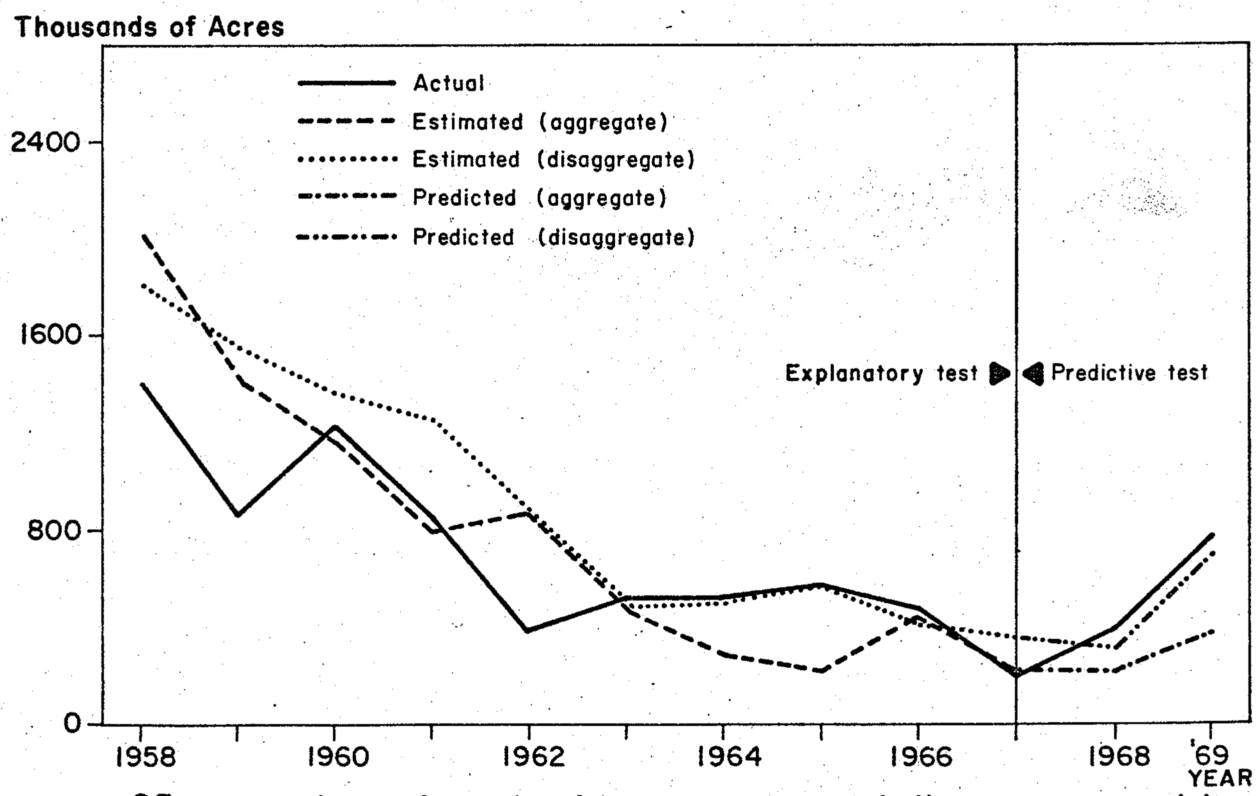

Figure 25. Comparison of results from aggregate and disaggregate models for flaxseed acreage in Saskatchewan, 1958 - 1969.

both models was nearly similar. But for some crops such as wheat, flaxseed and rapeseed, the errors were greater with model P-1 than with the model P-2. And for some other crops, such as rye and barley, model P-1 produced relatively more accurate results.

A similar conclusion can be drawn from the comparison of the models for Saskatchewan. Results of both models S-1 and S-2 are approximately similar. Model S-1 provided more accurate results for some crops and S-2 for others. No definite conclusion can be drawn about the superiority of any one model. However, a detailed and critical examination of Tables 6 and 7 and Figures 20 to 25 reveals the following two conclusions:

(1) The aggregate models (P-1 and S-1) produced less accurate results than the disaggregate models (P-2 and S-2) for relatively more profitable crops.

(2) Sizeable changes in crop acreages were explained or predicted more accurately by the disaggregate models than through the aggregate ones. The following evidence can be cited to support this statement:

(a) The disaggregate models (P-2 and S-2) provided relatively more accurate estimates of wheat acreages in 1965. This was the year when wheat acreage declined the sharpest during the 1958 to 1967 period.

(b) The disaggregate models had improved their relative performance in 1968 and 1969 (abnormal years) compared to during 1958 to 1967 (normal period). Moreover, in the very abnormal year of 1969, the disaggregate models produced more accurate results than the aggregate ones for all crops except rapeseed. For instance, the 1969 wheat acreage in the prairies was overestimated by seventeen percent by model P-1, but by only eleven percent by model P-2. Furthermore, the 1969 flaxseed acreage was underestimated by forty percent in the prairies and by fifty percent in Saskatchewan through the aggregate models, but by only twenty-five percent and eleven percent through the disaggregate models.

The first conclusion that the aggregate models produced comparatively less accurate results for relatively more profitable crops can be verified through inter-crop comparisons of errors in acreage estimates. Such a comparison reveals that models P-2 and S-2 estimated acreages of wheat, flaxseed and rapeseed (relatively more profitable crops in the prairies) with smaller errors than did models P-1 and S-1. For less profitable crops such as barley and rye, the opposite result was observed.

These findings were a result of the fact that the aggregate models estimated the acreages of the more profitable crops to be mostly equal to their upper flexibility bounds and, thereby, caused gross errors. In the case of the disaggregate models, a breakdown of the aggregate into the smaller regions perhaps increased the competition among crops because the models became more representative to their respective areas due to isolation of regional variations in yields, prices and costs. Therefore, solution acreages of the more profitable crops did not reach the upper flexibility bounds that often. And the acreages were thus estimated more closely.

Evaluating the Effects of Operation LIFT on the 1970 Prairie Land Utilization Pattern

The Honourable Otto E. Lang, Minister in charge of the Canadian Wheat Board, presented to the House of Commons on February 27, 1970, the program known as Lower Inventory for Tomorrow (LIFT). The following statement was made by the Minister regarding the program:

> Producers in the Wheat Board designated region who reduce wheat acreage below 1969 levels and increase summerfallow or perennial forage by the same amount will receive federal compensation payments of $6.00 per acre for additions to summerfallow or $10.00 per acre for additions to perennial forage acreage (44, p. 3).

The stated purpose of this program was to reduce wheat acreage because supplies of this crop were at an unprecedented level during the 1969-70 crop year. Carryover of wheat was 987 million bushels at July 31, 1970, seventy-seven percent above the ten-year average (1960-1961 to 1969-1970) of 557 million bushels. This mounting inventory was equivalent to about two years normal disappearance.

Prairie farmers have been highly specialized in wheat production. Hence, the lower wheat marketings, which in part explained the large carryover, resulted in acute cash shortages among prairie grain producers and, therefore, required a special program to improve the situation.

The LIFT program was also aimed at discouraging a wholesale switch of wheat acreage to feed grains and oilseeds. However, the stock position for feed grains was very large and it was unlikely that significant switches to these crops would have taken place. In the winter of 1969-70, it was expected that a major increase in rapeseed and flaxseed production would have resulted in unreasonably low prices and large carryovers at the end of the 1970-71 crop year.

In this study, the recursive programming model was utilized to quantitatively estimate the impact of Operation LIFT on the 1970 acreages of the principal crops in the prairies. The model was used to predict 1970 acreages of wheat, oats, barley, rye, flaxseed, rapeseed and summerfallow that would have resulted in the absence of the LIFT program. A comparison of the predicted against actual acreages would show the effectiveness of the program in determining the 1970 prairie land utilization pattern.

In estimating the 1970 acreages, the aggregate prairie model was not used, but the prairie results were obtained by totaling the provincial estimates. The selection of this approach was governed by the findings of the third section of this chapter, which describes the superiority of the disaggregative models over the aggregate ones in estimating the land utilization patterns in years of sizeable changes.

It was also discovered in the earlier sections that some flexibility bounds did not fully account for the abnormal situations prevailing in a year and, therefore, caused errors in the acreage estimates. In order to deal with this limitation, the following changes were made to the basic model: (1) the lower flexibility restraint and absolute minimum acreage constraint for wheat were dropped, and (2) for flaxseed and rapeseed, the greater of the upper flexibility restraint and absolute maximum acreage constraint was kept. The constraints for wheat were omitted from the analysis in order to account for particularly abnormal situation for this crop in 1969-70 crop year. The flaxseed and rapeseed constraints were modified because acreages of these crops had maintained upward trends in recent years, and had frequently exceeded their absolute maximum acreage constraints, which were defined on the basis of 1953 to 1967 data.

After making these modifications, the recursive programming model was utilized to predict the 1970 land utilization pattern. Results are presented in Table 8. Wheat acreage showed a decline from 24.4 million in 1969 to a predicted 22.0 million in 1970. However, the actual acreage was 12.0 million acres, less than half of the 1969 acreage. This study attributes the difference between the predicted 22.0 million acres and the actual 12.0 million acres in 1970 (forty-five percent) to the LIFT program. This would indicate that the program was highly effective in reducing wheat acreage. Since the differences between actual and predicted acreages of other crops except rapeseed were not particularly large, it appears that the program did not have any considerable impact on these crops.

TABLE 8

THE 1970 ACTUAL PRAIRIE LAND UTILIZATION PATTERN AND ESTIMATES BASED ON SUMMATION OF PROVINCIAL RESULTS

Crops	1969 Actual Acreage	1970 Actual Acreage	1970 Estimated Acreage
	(thousand acres)		
Wheat	24,400	12,000	21,954
Oats	5,630	5,390	5,342
Barley	9,000	9,500	8,276
Rye	859	944	896
Flaxseed	2,420	3,350	3,505
Rapeseed	2,012	3,950	3,289
Summerfallow	28,800	36,900	30,335

In the case of rapeseed, even though the difference between actual and predicted acreages was large (twenty percent), the LIFT program might not have caused this difference. Rather, it might be explained by the insensitivity of the model to the dynamic situation surrounding rapeseed production.

It was estimated that, without the LIFT program, the summerfallow acreage would have increased from 28.8 million in 1969 to 30.3 million in 1970. However, the actual acreage was 36.9 million acres. On the basis of these figures, it could be concluded that the program resulted in an additional 6.6 million acres in summerfallow.

Thus, this analysis suggests that the LIFT program had a considerable impact on the prairie land utilization pattern in 1970; the major effects being the reduction in wheat acreage and increase in summerfallow.

Forecasting the 1971 Land Utilization Patterns

One of the specific objectives of this study was to forecast the 1971 land utilization patterns for each prairie province and for the prairies as a whole. The recursive programming models were utilized for this purpose, but certain changes were made in the basic construct of the models in the light of the proposed price policy for 1971.

The Government of Canada outlined a new price policy on October 29, 1970 for the crop year 1971-72. In this policy, the initial prices of wheat, oats, and barley were announced on March 1, 1971; whereas, no such announcements prior to seeding were made in the past. To incorporate this policy change into the recursive programming analysis, the price expectation model utilized earlier was replaced by a new one. The announced initial prices were taken as the expected values. However, the prices for the 1971-72 crop year were not available at the time of the analysis. Initial prices for 1970-71 crop year were, therefore, taken as the expected values and were utilized in the recursive model.[a]

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[a]Since undertaking the analyses the initial prices have been announced by the Canadian Wheat Board. The initial price of wheat for the top grade is $1.46 per

Other modifications were also made to the model in order to account for changes in grain marketing structure. The Government of Canada has announced a new quota system for the year 1971. Unit, specified and supplementary quotas will be abandoned and an assignable quota system will be introduced. In this system, farmers will have the option to assign the total cultivated land to any crops before the crop year begins. A separate quota will be announced for each crop by the Canadian Wheat Board.

To introduce this structure into the model, some of the activities and restraints were changed or augmented. Three selling activities for each crop (viz., sale on unit quota, selling through specified quota, and sale on supplementary quota) were replaced by one activity (i.e., sale through assignable quota). Similarly, restraints such as unit, specified and supplementary quota constraints were dropped and an assignable quota restraint was introduced.

Both upper and lower flexibility restraints of wheat and summerfallow were omitted from this analysis because they were estimated on the basis of the abnormal acreages of 1970. Only their absolute minimum and maximum acreage constraints were retained. For flaxseed and

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bushel. The average of the two top grades under the new grading system is the same as the initial price for Northern 2 last year. Hence, it appears that basically 1970 and 1971 initial prices for wheat are at the same level. Also, the initial prices of oats and barley for 1971 were announced at their 1970 levels.

rapeseed, the greater of the upper flexibility restraint and absolute maximum acreage constraint was kept because these crops had maintained upward trends in recent years, thus breaking the previously determined absolute maximum.

The 1971 projected land utilization patterns for each prairie province and for the prairies as a whole are presented in Table 9. The prairie forecast is the summation of the provincial results. The actual acreages, which were published by Statistics Canada long after completing the analysis, are also presented in Table 9 in order to examine the accuracy of the forecasts of this study.

Table 9 shows that the recursive models projected wheat, rye and summerfallow acreages with reasonable accuracy--the deviations of projected from actual acreages were less than ten percent. However, the differences were large in the case of barley, flaxseed and rapeseed. There are several causes for these differences. One might be the inaccurate assumptions about the levels of exogeneous variables utilized in the forecasting analysis. For example, barley acreages were underestimated perhaps because bigger barley stocks as a proportion to wheat stocks were assumed than that observed. The proportion utilized in this study was .18, whereas, the actual ratio turned out to be .15. The second reason for the underestimation of barley acreages might be the surge in the exports of this crop in the 1970-71 crop year. However, this variable was omitted from the analysis because of the multicollinearity problem. Both

TABLE 9

THE 1971 PROJECTED AND ACTUAL PRAIRIE LAND UTILIZATION PATTERNS, BY PROVINCE

Crops	Prairies		Manitoba		Saskatchewan		Alberta	
	Projected	Actual	Projected	Actual	Projected	Actual	Projected	Actual
	(thousand acres)							
Wheat	20,858	18,700	2,258	2,400	13,800	12,200	4,800	3,500
Oats	6,059	5,177	1,621	1,472	2,488	1,960	1,950	1,756
Barley	9,898	14,600	1,693	2,200	3,237	6,300	4,968	6,100
Rye	987	1,029	194	184	535	620	258	225
Flaxseed	3,477	2,000	1,150	570	1,627	1,030	700	400
Rapeseed	4,544	5,475	350	625	2,594	2,750	1,600	2,100
Summerfallow	27,345	26,000	2,713	2,700	17,432	16,600	7,200	6,700

these factors together could be responsible for the substantial underestimation of barley acreages in 1971.

It appears that flaxseed acreages were overestimated due to errors in the assumed level of flaxseed stocks as a proportion to wheat stocks. The observed proportion was .026, whereas the assumed ratio was only half as much (.014).

The major cause for the underestimation of rapeseed acreages was the assumption of a lower price for this crop ($2.10 per bushel) than that prevailed ($2.75 per bushel) during the crop year 1970-71. This source perhaps resulted not only in the underestimation of rapeseed acreage but in the overestimation of flaxseed as well.

Other sources of errors in the forecasts could be defects in the models themselves. The models were never very successful in estimating flaxseed and rapeseed acreages. As indicated earlier in the explanatory and predictive tests, errors were substantially large for both crops.

The structure of prairie agriculture (e.g., quota system, price policy, etc.) has also considerably changed since 1969. Therefore, it is unlikely that the recursive models, especially regression equations which were developed on the basis of 1953 to 1967 data, could forecast the 1971 crop acreages with a high level of accuracy.

Effects of Changes in Barley Quota and Price Levels on the 1971 Land Utilization Patterns

The impact of changes in the levels of a number of variables on the 1971 forecast can be examined. However, in this study, only barley price and quota level were varied. The impact of barley prices of $.76 and $.86 per bushel were analyzed. The effects of barley quota levels of twenty and twenty-five bushels per acre assigned to this crop were also examined. Other variables such as previous year acreage, stocks, exports, etc., were assumed at the levels utilized in the forecasting analysis.

The flexibility coefficients and restraints were re-estimated for each different level of barley prices. The absolute maximum acreage bound was removed from this crop because maintenance of this bound appeared inconsistent with the increase in barley price. No other change was made to the model utilized for the forecasting analysis.

The model was run separately for each of the four alternatives: two price levels and two quota levels. The estimated land utilization patterns for both twenty and twenty-five bushels quota were found identical. Therefore, results are not presented separately by quota levels.

Along with the forecast results (based upon $.66 barley price and twenty bushels quota), two land utilization patterns corresponding to two levels of barley prices ($.76 and $.86 per bushel) are presented in Table 10. The results show that as the price rose from $.66 to $.86, barley

TABLE 10

PRAIRIE LAND UTILIZATION PATTERNS, BY PROVINCE, FOR DIFFERENT LEVELS OF BARLEY PRICES, 1971[a]

Crop	Barley price per bushel			Barley price per bushel		
	$.66	$.76	$.86	$.66	$.76	$.86
	(thousands of acres)					
	Prairies			Manitoba		
Wheat	20,858	20,858	20,858	2,258	2,258	2,258
Oats	6,059	6,059	6,013	1,621	1,621	1,575
Barley	9,898	10,907	11,799	1,693	1,822	1,950
Rye	983	983	931	194	194	194
Flaxseed	3,477	3,477	3,204	1,150	1,150	1,133
Rapeseed	4,544	4,544	4,544	350	350	350
Summerfallow	27,345	27,115	25,814	2,713	2,584	2,519
	Saskatchewan			Alberta		
Wheat	13,800	13,800	13,800	4,800	4,800	4,800
Oats	2,488	2,488	2,488	1,950	1,950	1,950
Barley	3,237	3,464	3,691	4,968	5,621	6,158
Rye	535	530	530	258	258	206
Flaxseed	1,627	1,627	1,500	700	700	571
Rapeseed	2,594	2,594	2,594	1,600	1,600	1,600
Summerfallow	17,432	17,205	17,105	7,200	6,547	6,190

[a]Same land utilization pattern was estimated with both 20 and 25 bushels barley quota.

acreage increased in Manitoba from 1.7 million to 2.0 million acres, in Saskatchewan from 3.2 million to 3.7 million, and in Alberta from 5.0 million acres to 6.2 million acres. The result for the prairie land utilization patterns, obtained by totaling the provincial estimates, indicated that barley increased from 9.9 million acres to 11.8 million acres due to the price rise. This expansion of barley acreage was brought about by the transfer of land from many crops (viz., oats, rye, flaxseed and summerfallow) but the major acreage (1.5 million acres) was obtained by a transfer from summer-fallow.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Statement of the Problem

In the 1960's, there were prodigious fluctuations in the exports of Canadian farm products, which created colossal inventories of several crops in certain years. One possible solution to the inventory problem is to assist adjustments by farmers in their acreages of individual crops so that supply can be brought in line with demand. In order to follow this course of action, production response information is required by people involved in Canadian agriculture. These people need an explanation of historical production patterns as well as prediction of the future crop acreages for alternative levels of policy variables.

The production response research would be useful in other directions as well. For example, precise production forecast would assist farmers and agribusiness firms in their investment and planning decisions. Thus, the response information is needed by the entire Canadian agricultural industry. However, very little is known to date about the quantitative estimates of production response in the prairies.

The general objective of the study was to develop and apply a recursive programming model to analyze acreage response of the major crops in the prairies. More specific objectives were: (1) to evaluate the power of the recursive model to explain and predict the land utilization patterns of Manitoba, Saskatchewan, Alberta and the prairies as a whole; (2) to examine the relative performance of the aggregate versus disaggregate analysis of the land utilization patterns, and (3) to apply the recursive models to: (a) estimate the impact of the LIFT program on the 1970 crop acreages, and (b) forecast the 1971 land utilization patterns for each prairie province and for the prairies as a whole.

Methodology

It was postulated, in this study, that decision process of farmers of a region could be expressed as a recursive programming problem. It was assumed that farmers would select a land utilization pattern which would maximize their expected net returns from all crops included in the model, subject to certain constraints. Only the six major cereal and oilseed crops of the prairies were included in the analysis. These crops are: wheat, oats, barley, rye, flaxseed and rapeseed. Summerfallow was also introduced in the model. Two producing activities were utilized for each crop: sowing on summerfallow and on stubble land. In order to incorporate three kinds of quotas (viz., unit, specified acreage and supplementary) into the model, a large number of selling activities were used.

A wide variety of restraints were utilized in this study. The crucial restrictions of the model were flexibility restraints. They are dynamic in nature and relate the land utilization pattern of the current year with the crop acreages of the preceding year. Specifically, these restraints are the upper and lower bounds on the allowable year-to-year changes in the solution acreages of each crop in the model.

The flexibility restraints were estimated on the basis of flexibility coefficients. The upper and lower flexibility coefficients represent the maximum allowable proportionate increase and decrease, respectively, in the acreage of a crop from one year to the next. In previous studies, a variety of methods have been utilized to estimate these coefficients. Some of these have been discussed in Chapter III. Most of these studies used simple regression models. This method estimates a pair of flexibility coefficients for each crop, irrespective of year-to-year changes in economic and non-economic conditions. This approach is likely to be unreasonable and reduce the reliability of the results.

In the present study, a multiple regression model of time-series data was developed. Observations during 1953 to 1967 were used to estimate upper and lower flexibility coefficients. The coefficients estimated through this model vary from year to year, depending upon the levels of exogeneous explanatory variables. This method was considered to be conceptually superior to the previous one.

The flexibility restraints were computed from these coefficients. The upper flexibility restraint was estimated by multiplying one plus the upper flexibility coefficient for that year by the preceding year acreage. Similarly, the lower restraint was estimated by multiplying the lagged acreage by one minus the lower coefficient. Upper and lower flexibility restraints were estimated in this manner for each of the six crops included in the analysis. Also, similar restraints were computed for summerfallow and stubble land.

The basic recursive programming model was applied to twenty-four data sets. Thus, twenty-four individual models were constructed: one utilizing the prairie data, three using the provincial observations and twenty models were developed treating crop districts of Saskatchewan as units of analysis. These models were similar in structure, but different with respect to numerical values of coefficients and constraints.

Summary of the Findings

The explanatory power of the recursive programming model was tested by examining its ability to explain the land utilization patterns during the period 1958 through 1967. Estimated results of the model were compared against actual acreages for Manitoba, Saskatchewan, Alberta and the prairies. The results indicated that the land utilization patterns of the provinces and the prairies were explained with reasonable accuracy. The overall weighted average of absolute deviations of estimated from actual acreages for all crops was less than seven percent in any one model.

The solution and actual acreages of wheat were strikingly close in each of the four models (i.e., Manitoba, Saskatchewan, Alberta and the prairies). The maximum average error was only three percent. The performance of the models for explaining oats acreages was not very satisfactory. In spite of sizeable variations in barley acreages during 1958 to 1967, this crop was estimated with moderate accuracy in all models except Manitoba. In this province, acreages were grossly overestimated in five out of ten years. The recursive model was successful in estimating the sharp declines in rye acreage, but it explained only a part of year-to-year increases. However, on the whole, acreage of this crop was estimated with good precision.

Flaxseed acreages were poorly explained. In a number of years, the direction of change (i.e., turning point) in actual acreages was incorrectly estimated. Rapeseed acreages were estimated with large <u>average</u> errors in each model. However, examination of results of individual years shows that in Manitoba and the prairies, errors were large in only three years. In other years, errors were reasonably small. Summerfallow acreages were estimated in all models with remarkable accuracy. The maximum average error was only six percent.

On the basis of these results, it is concluded that although there were large errors in the model estimates for some crops, in general, the solution acreages were reasonably close to the actual observations. Therefore, the recursive programming model is accepted as having satisfactory explanatory power.

The ability of the recursive model to predict crop acreages outside the period used for construction was also evaluated in this study. The model, which was developed on the basis of the 1958 through 1967 data, was used to make predictions for 1968 and 1969. The solution acreages were then compared with actual data, in order to test the predictive power of the model. The comparison suggests that the models predicted with reasonable accuracy the land utilization patterns of the prairies and all provinces except Manitoba. In this province, the overall average error was thirteen percent.

The quality of prediction was quite different between 1968 and 1969. In 1968, wheat acreages were predicted with remarkable accuracy in each model. But, in 1969, predicted acreages exceeded actual observations by one million acres in Manitoba, by 1.5 million acres in Saskatchewan and by as much as 4 million acres in the prairies. In Alberta, the prediction of wheat acreage was reasonably close to actual values. Predicted acreages of oats in both years were fairly close to actual observations for all models except Saskatchewan. Barley acreages were predicted with moderate precision in Alberta and the prairies in both years. But the solution acreages in Manitoba and Saskatchewan were considerably different from the actual. The models were not successful in predicting rye acreages, even though they had satisfactorily explained their historical movements. Errors were very large in each year.

In both years, flaxseed acreages were predicted with small errors in Alberta, but with large errors in other models. The models were also not successful in predicting rapeseed acreages. In each model, solution acreage was considerably greater than the actual observation in 1968, but substantially smaller in 1969. However, phenomenal year-to-year fluctuations in rapeseed acreages must be considered in evaluating the performance of the recursive model. Summerfallow acreages were predicted in both years with remarkable accuracy in all models. The only exception was in Manitoba in 1969, when the error was about sixteen percent.

On the basis of the above results, it is concluded that the 1968 land utilization patterns were predicted with reasonable accuracy, and the recursive model had a satisfactory predictive power. But the performance of this model in 1969 was poor. However, this year was very abnormal for prairie agriculture and, therefore, the results cannot be used as a basis to draw a sound conclusion about the predictability of the recursive models.

Many explanations can be provided for a number of sizeable errors which occurred in the explanatory and predictive analyses. The most important error was the inaccurate estimation of the direction of change in crop acreages. In other words, while actual acreage of a crop had declined, the solution estimates showed increases and vice versa. Two plausible hypotheses can be put forward to explain this type of error. One is that it results from the application of the models to highly aggregate observations such as prairie and provincial data. Such a model predicts either full change or no change at all in crop acreages. The second hypothesis is that the net income coefficients utilized in this study are inaccurate. This inaccuracy could be due to use of inappropriate expectation models for prices, costs, yields and/or quota levels. These two hypotheses are not independent, and together they could make the programming models insensitive to price changes, and thereby could cause turning point errors. For example, wheat acreage declined in 1965 from the 1964 level, perhaps because farmers reacted to a significant decline in the expected price of this crop for 1965. But, in the recursive programming analysis, net income per acre for wheat was still higher than for other crops. As a result, turning point error was caused and the crop was overestimated.

Some discrepancies between the estimated and actual acreages could not be accounted for through the above explanations. These errors were brought about by relatively too wide or too narrow flexibility bounds on acreages of a crop. For example, in 1969, wheat acreage was overestimated and flaxseed and rapeseed acreages were underestimated because: (1) the lower flexibility bound of wheat was not sufficiently low, and (2) the upper flexibility bounds of flaxseed and rapeseed were not adequately wide.

Thus, it appears that the multiple regression model, which was employed in this study, did not produce appropriate flexibility coefficients for 1969 when the economic conditions affecting prairie land use were abnormal. However, the performance of the methods employed in previous studies (such as simple regression models and averaging the proportionate changes) would not have been better if applied to an abnormal year such as 1969, because coefficients estimated through these methods are, in some sense, the averages of the year-to-year proportionate changes in the historical acreages of crops. Therefore, the appropriate flexibility coefficients would not have been generated for the year when the unprecedented changes took place in wheat, flaxseed and rapeseed acreages.

In some years, excessively dry weather conditions have caused discrepancies between the estimated and actual acreages of some crops. For example, errors in the solution acreages of flaxseed for Saskatchewan and the prairies in 1958, 1959 and 1962 could be attributed to weather.

As mentioned earlier, the application of the recursive model to very aggregate data can be considered as a reason for errors in the acreage estimates. In order to verify this hypothesis, land utilization patterns for Saskatchewan and the prairies each were estimated through aggregate as well as disaggregate models. That is, the prairie results were obtained: (1) by using the recursive model to the prairie data, and (2) by totaling the acreage estimates for all three prairie provinces. Results for Saskatchewan were obtained: (1) by analyzing the provincial data, and (2) by summing the acreage estimates of the twenty crop districts of the province.

The percentage deviations of estimated from actual acreages were calculated for each model and results of both aggregate and disaggregate models were then compared. The results suggested that none is clearly superior in explaining and predicting the crop acreages. However, a crop-by-crop comparison of the errors demonstrated that the aggregate models produced less accurate results than the disaggregate ones for the relatively more profitable crops. For example, the disaggregate models for both the prairies and Saskatchewan produced relatively small average errors for each of wheat, flaxseed and rapeseed (comparatively more profitable crops in the prairies).

The reasoning behind this superiority of the disaggregate models over the aggregate ones arises from the fact that the latter models, in general, allocated land to the relatively more profitable crops to be equal to their upper flexibility bounds, even in years when their prices had declined substantially. Thus, many turning point errors were produced and thereby, large errors were created in the aggregate models. In the case of the disaggregate models, perhaps a breakdown of aggregate into smaller regions made these crops more competitive with relatively less profitable ones because regional variations in yields, costs, and prices were indirectly taken into account in these models. Therefore, relatively accurate estimates were produced by the disaggregate models.

Another interesting conclusion drawn from the comparison of the aggregate versus disaggregate model results was that the sizeable changes in crop acreages were explained or predicted more accurately by the latter model. For example, declines in wheat acreages in 1965 (this was the sharpest decline during 1958 to 1967) in both Saskatchewan and the prairies were estimated by the disaggregate models with relatively greater accuracy. Furthermore, in the really abnormal year of 1969, these models produced more accurate results for all crops except rapeseed.

After being tested for its explanatory and predictive powers, the recursive model was utilized to estimate the short-run impact of an agricultural policy on crop acreages. A recent agricultural policy, Lower Inventory for Tomorrow (LIFT), was selected for this purpose.[a] The

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[a]In this study, the purpose of evaluating the effects of the LIFT program was not to provide answers to federal government questions, but to show an application of the recursive model in estimating the impact of a policy measure.

purpose of the program was to reduce wheat acreage and to encourage farmers in the Wheat Board designated region to hold this land out of production of any crop in 1970.

The recursive programming model was utilized to estimate the 1970 acreages of wheat, oats, barley, rye, flaxseed, rapeseed and summerfallow that would have resulted in the absence of the LIFT program. The comparison of the estimated against actual acreages would show the impact of the program on the 1970 prairie land utilization pattern. In order to estimate the crop acreages, the aggregate prairie model was not utilized, but the prairie results were obtained by summing the provincial estimates.

The solution of the recursive programming analysis showed that, without the LIFT program, 22.0 million acres of wheat would have been seeded in 1970. However, the actual acreage was only 12.0 million, less than half of the 24.4 million acres in 1969. This study attributes the difference between the estimated 22.0 million acres and the actual 12.0 million acres in 1970 to the LIFT program. This would indicate that the program was considerably effective in reducing wheat acreage. The results also showed that the program did not have any significant impact on other crops. However, without the LIFT program, summerfallow acreage would have increased from 28.8 million acres in 1969 to 30.3 million in 1970. But the actual area was 36.9 million acres. Thus, the program resulted in an additional 6.6 million acres of summerfallow.

The provincial recursive programming models were also utilized to forecast the 1971 land utilization patterns for each prairie province and for the prairies as a whole. Then the effects of changes in barley prices and quota levels on the 1971 forecasts were examined.

In light of the new marketing structure for 1971, some modifications were made to the models. Because the Government of Canada has announced a new quota system and a price policy for the crop year 1971-72, changes were made to the quota restraints and the price expectation model.

The forecast shows that wheat was expected to be seeded on 20.9 million acres, barley on 9.9 million acres and rapeseed on 4.5 million acres in 1971. Summerfallow was estimated to be 27.3 million acres in the prairies. However, the 1971 actual acreages turned out to be 18.7 million acres of wheat, 14.6 million acres of barley, 5.5 million acres of rapeseed and 26.0 million acres of summerfallow. The actual and projected acreages differed perhaps because wheat exports and barley stocks happened to be smaller than that anticipated. Also, the actual rapeseed price was found to be greater than that utilized in the analysis--$2.75 per bushel compared to $2.10.

For estimating the effects of changes in barley prices and quota levels on 1971 forecasts, three levels of barley prices were assumed: $0.66, $0.76 and $0.86 per bushel. Quota level was set at 20 and 25 bushels per acre assigned to barley. It was found that the estimated land utilization patterns for both 20 and 25 bushel quotas were identical, while the patterns changed with respect to increases in barley prices. The results indicated that as the price rises from $0.66 to $0.86, barley acreage in Manitoba increases from 1.7 million to 2.0 million acres; in Saskatchewan from 3.2 million to 3.7 million; in Alberta from 5.0 million to 6.2 million, and in the prairies from 9.9 million to 11.8 million acres. These increases in barley acreages are brought about by declines in areas of oats, rye, flaxseed and summerfallow.

Conclusions

A number of economic conclusions can be drawn from the results of the present study. Six major conclusions are described below. These are related to: (1) factors affecting the prairie land utilization patterns, (2) farmers' response to agricultural policies and programs, (3) interrelationships among crops and agricultural policy development, (4) the effects of the Canadian Wheat Board's quota system, (5) success of the LIFT program, and (6) the Government of Canada's target of sixteen million acres of barley in 1971.

1. The first conclusion, as mentioned above, is concerned with the identification of significant variables affecting the prairie land utilization patterns during 1958 to 1969. In the estimation of the flexibility coefficients, while expected values of prices, stocks and exports, and precipitation and the preceding

year acreage were found as significant variables for most of the crops, a time variable turned out to be insignificant in the case of a large number of products. It was observed in the programming phase of the analysis that the relatively accurate results indicating the prairie land utilization patterns were produced mainly by the flexibility restraints.[a] It can, therefore, be concluded that price, stocks, exports, precipitation and the preceding year crop acreage which affected the flexibility coefficients (and thereby restraints), in turn, affected the prairie land utilization patterns.

Quota variables were not utilized to estimate flexibility coefficients, but were used as restraints in the recursive programming models. Because of substantially large quota levels per acre during 1958 to 1967, the restraints did not affect a large number of programming solutions. During this period, this variable does not, therefore, appear to be a restrictive force affecting the prairie land utilization patterns, especially wheat acreage. However, in years of relatively low quota levels (i.e., 1968 and 1969), the restraints became effective in the programming solutions; and wheat, oats and barley acreages were thereby affected.

2. The level of flexibility coefficients has been used here as a basis to draw conclusions about farmers'

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[a]In a typical recursive programming solution, the most profitable crops would reach their upper flexibility bounds, the less profitable crops would go to the lower bounds, and one crop would be between the bounds and be governed by the total crop land constraint.

ability to adjust or change crop acreages in the prairies. For example, a large upper flexibility coefficient was interpreted as reflecting farmers' ability to make substantial increases in acreages. Similarly, a large lower flexibility coefficient was taken as indicating an ability to reduce acreage considerably. This is likely to be a reasonable assumption because, as mentioned above, the flexibility restraints (and in turn coefficients) were responsible for the accurate estimation of the prairie land utilization patterns. Thus the flexibility coefficients can be considered as close approximations of the actual rates of year-to-year adjustments by the prairie farmers in their crop acreages.

This study suggests that in response to agricultural policy, the prairie farmers can substantially change their land utilization patterns even in a year's time. This is obvious from the large flexibility coefficients for most crops. For example, in the prairie model the upper flexibility coefficient of rapeseed was as high as 2.80. Similarly, the lower coefficient was as large as 0.62. The levels of the flexibility coefficients of other crops were also immense. Thus, it indicates that the prairie grain growers have the ability to change their land utilization patterns considerably. Hence, in response to an agricultural policy designed for changing land use, they are likely to make substantial changes in their acreages. This conclusion is further supported by the evidence that the LIFT program reduced wheat acreage substantially in the prairies in 1970.

The above conclusion on response to an agricultural policy can be extended to indicate the variations in the expected response for individual crops. A considerable difference between flexibility coefficients of different crops was observed in this study. For example, both upper and lower flexibility coefficients of wheat were, on the average, smaller than those of rapeseed. This evidence indicates that the prairie farmers are likely to change oilseeds acreages proportionately more than cereal acreages in response to an identical policy for each crop. In other words, in order to obtain the same level of proportionate change in the acreage of each crop, a stronger policy measure is required for cereals than for oilseeds.[a] That is, for example, in order to increase acreage of each of wheat and rapeseed by twenty-five percent, the price of the former should be increased by forty percent and price of the latter by only twenty percent.

Another conclusion which is related to the previous one is that a policy measure required to reduce wheat acreage should be stronger than to increase the acreage. This conclusion is based upon an observation that the lower flexibility coefficients of this crop were, on the average,

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[a]It must be noted that most of the above conclusions are based upon the underlying structure of the prairie agricultural industry during 1953 to 1967. The conclusions could be different when the structure has changed.

smaller than the upper coefficients. Thus, it appears that the prairie farmers have a tendency to make reasonably large increases, but only small decreases in the acreages of this crop. As a result, a lower response can be anticipated from farmers if a policy is directed towards reducing wheat acreage than towards increasing it.

3. In order to develop a policy for changing the land utilization patterns, this study suggests that the interrelationships among crops should be taken into account. This conclusion is based upon the evidence that prices of some crops increased in a few years, but their acreages did not expand because price of some other crop had also risen. For example, barley price increased substantially in 1961, but its acreage declined in 1962, probably due to an increase in wheat price. Thus, returns from alternative crops should be considered for ascertaining the effectiveness of agricultural policies for changing the prairie land use. Results of this study also indicate that an increase (or decrease) in acreage of one crop does not affect all other crops equally (in absolute or proportionate sense). But, some crops are considerably affected and others are only marginally influenced. For developing agricultural policies, the consequences of changes in acreages of one crop on others should be evaluated.

4. This study indicates that the Canadian Wheat Board's quota policy did not restrict wheat acreage during 1958 through 1967, but rather promoted it. The specified

quota constraint rarely affected the acreage of this crop. There exist two reasons for this. First, the quota system was so designed that substitutability of quota among crops was permitted. Secondly, net income per acre for wheat was greater than for other crops. Both these reasons together allowed the utilization of total specified quota first for wheat and then for other crops, if some portion was left over. Another interesting observation related to the above one is that the recursive programming models showed a strong pressure to increase wheat acreage--the shadow price for upper flexibility bound of this crop was greater than for other crops. On the basis of these two observations, it can be concluded that if the Government of Canada wants to have a small wheat acreage (e.g., below the 1968 level) a policy measure should be adopted to reduce the relative advantage of wheat, especially by changing the quota system.

5. This study has shown that the LIFT program was very successful in reducing the prairie wheat acreage in 1970. The recursive models predicted that, in the absence of the program, the 1970 wheat acreage would have been 22.0 million, whereas, the actual acreage was only 12.0 million. If the models are correct, it can be concluded that the program resulted in a reduction of 10.0 million acres of this crop.

6. As estimated in this study, a fairly significant increase in barley price from $0.66 to $0.86 per bushel for 1971 produced an increase in the acreage of this crop from

9.9 million to only 11.8 million acres. The recursive programming analysis indicates that, if the models are reliable, the Government of Canada's target of sixteen million acres of barley in 1971 will not be achieved.[a]

Suggestions for Future Studies

The ability of the recursive programming model to estimate and forecast crop acreages can be improved through a large number of ways. Some of the methods are suggested below.

The results of the present study suggested inaccuracy in the net income per acre as a possible reason of errors in the estimates of crop acreages. The inaccuracy could be a result of application of inappropriate expectation models for prices, costs, yields and/or quota levels. More research should be directed towards studying farmers' expectations about these variables, and for investigating how farmers use this information to make their plans.

In the absence of reliable cost of production data for each year during 1958 to 1971, the 1966 cost figures were used in this study for every year. Also, with some adjustments, the cost of production figures for cereals were utilized for oilseeds. Errors in the acreage estimates could therefore be a result of utilization of inappropriate

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[a]The September survey figures released by the Dominion Bureau of Statistics show that 14.6 million acres of barley have been seeded in the prairies in 1971.

cost figures. Research needs to be undertaken to estimate cost of production of major crops on an annual basis by regions in the prairies.

It was observed that the flexibility coefficients did not sufficiently vary between years in order to account for the structural changes or abnormal conditions occurring in some years, and therefore caused errors in the acreage estimates. Perhaps the errors arose from the fact that regression models, in principle, cannot effectively be employed for estimation and prediction purposes, when the underlying structure has changed. In this situation, some other methods such as farm surveys should be employed to estimate flexibility coefficients.

In the present study, tame hay was not included in the model. Similarly, livestock activities such as cow-calf, feeder, etc. were omitted. However, these are major users of the land resource of the prairies, and changes in their levels may influence acreages of the crops included in the model. Therefore, tame hay as well as livestock activities should be incorporated in future studies. With the introduction of these activities, labor and capital constraints should also be added to the model because the requirements of crop and livestock enterprises for these resources are grossly different.

Thus, much further research needs to be undertaken in order to improve estimates of supply response in the prairies. Recursive programming can provide the necessary framework for this purpose. This study is a first attempt in Canada to apply a recursive programming model to the estimation and prediction of agricultural supply response.

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APPENDIX A

A REVIEW OF PRODUCTION RESPONSE STUDIES IN CANADA

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A REVIEW OF PRODUCTION RESPONSE STUDIES IN CANADA

A limited number of agricultural supply response studies have been carried out in Canada. Two studies are reviewed here with the primary focus on methodology. Major findings of these studies are also presented below.

Schmitz

Andrew Schmitz (60a, pp. 79-86) developed regression models of time-series data to determine factors causing yearly fluctuations in Canadian wheat acreage. Regression equations were fitted to annual observations during 1947 to 1966. Schmitz hypothesized that wheat, barley, flaxseed, and livestock prices were relevant variables affecting Canadian wheat acreage. Some non-price variables were also included in the model. These were total rainfall in the month of April, farm stocks and exports of wheat, technology and capital availability.

Schmitz estimated twenty-five multiple regression equations, of which six were distributed lag models. The results indicated that R^2's of these equations were in the range of .76 to .89. The study revealed that wheat and flaxseed prices, wheat stocks and exports, and time trend were statistically significant variables in most of the equations, whereas, the previous year acreage, barley price and April rainfall were statistically insignificant variables.

Capel

Richard E. Capel (3, pp. 87-89) used distributed lag models to forecast 1968 wheat acreage in the prairies. He postulated that the preceding year acreage, price and supplies of wheat were exogeneous explanatory variables for the current year acreage of this crop. The regression equations were estimated using annual data over the period 1950 through 1967.

In this study, about 82 percent of the year-to-year variations in wheat acreage was explained. The previous year acreage, price and stocks were found to be statistically significant variables. The models predicted the 1968 prairie wheat acreage to be about 28.5 million acres which turned out to be very close to the actual observation.

Evaluation of the Studies

Both studies are highly limited in scope--acreage response of only wheat was estimated. Therefore, they neither provide production response information for all major crops in the prairies nor supply a detailed model to estimate them. Furthermore, interrelationships among crops are not adequately analyzed in these studies. While Schmitz recognized in his model the interdependence between crops by including prices of competitive crops, Capel completely ignored it. However, this information is very essential for developing intelligent agricultural policies.

The second limitation of the studies is the high level of aggregation used in the analysis. Schmitz utilized Canada data to estimate regression equations, whereas, Capel used the prairies as a unit of analysis. However, for agricultural policy formulation, supply response information is normally required by province.

Thirdly, the empirical performance of these studies was not very impressive. In both studies, only some eighty percent of the year-to-year variations in the prairie wheat acreage was explained. Schmitz claimed that he accurately predicted wheat acreage during 1962 to 1966. However, he did not actually predict acreages for these years (in the true sense of prediction). Rather he presented the estimated acreages for the years used in the model estimation.

Therefore, a study is needed to estimate the acreage response of all major crops in the prairies. Interrelationships between crops need to be also examined. The present study is an attempt in this direction.

APPENDIX B

LIMITATIONS AND RATIONALIZATION OF PRODUCTION COST COEFFICIENTS

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LIMITATIONS AND RATIONALIZATION OF PRODUCTION COST COEFFICIENTS

The cost of production data of six crops were required in this study for each year from 1958 through 1971. In a research report entitled "Interregional Competition in Canadian Cereal Production" by W. J. Craddock (7, pp. 169-71), cost of producing one acre of wheat, oats, barley and rye separately on summerfallow and on stubble land are presented by farm size and by crop district for the year 1966. Cost of working an acre of summerfallow is also published in this report. Since appropriate published cost of production data are not available for each year during 1958 to 1971, Craddock's 1966 figures were used in the recursive programming analyses of the entire period. Furthermore, average total cost of production for only large farm size group was utilized in this study. The use of these cost figures can be questioned on three grounds.

Firstly, the application of the 1966 cost figures to each of the fourteen years (1958-1971) of analyses appears conceptually unsound because per acre input requirements of most crops have considerably changed during this period. Input prices have also changed.

Secondly, in this study, while average total cost was used as C_j value, average variable cost should have been utilized because short-run economic decisions are governed by only variable cost. Fixed costs such as interest and depreciation are germane only for decisions to invest in land or machinery, but not for utilization of given stocks of these inputs. Therefore, for this study, average variable costs would have been more appropriate C_j values.

Thirdly, production cost figures for only large farm size group were utilized in this study. But there are many small farms in any one crop district of the prairies. Therefore, the cost data of large farmers are not likely to be representative figures for a crop district.

A number of arguments are put forward below which indicate that the above limitations do not affect the optimal land utilization pattern as much as it appears. In a linear programming framework, the optimal solution (i.e., optimum land allocation pattern) depends on the cost relationship rather than on production costs of individual crops. Hence, optimum allocation pattern would not change if all cost coefficients would move up or down at the same rate. In this study, it was postulated that year-to-year proportionate changes in costs of production would be the same for all crops during the period 1958 to 1971. As a result, the cost relationship between different crops would be unchanged. This postulation was based upon a premise that each crop was almost equally affected by the rise in input prices.

Cost of production is only one of the three components of net income (i.e., cost, price and yield) which determines the optimum land utilization pattern. Therefore, a less than exact cost figure for any one year is not likely to have as serious an affect on optimum land allocation pattern.

The use of the 1966 cost data for each year of the analysis also became a necessity, considering the problems and difficulties involved in the collection of data. It is very difficult to get reliable information on cost items for the last fourteen years by surveying farmers today. Many have not kept adequate records and the time span is too great for memory recall. Moreover, it is a stupendous job to collect data and compute production cost coefficients of six crops for fourteen years and for twenty-four models.[a]

Also, it was not possible to compute the weighted average of the production costs of large and small farm size groups because published historical acreage data were not available by crops and by farm size. Therefore, cost figures of only large representative farms were used in this study. However, the size of the so-called large farms utilized in the Craddock study was not 2,000 or 3,000 acres, but was only 650 acres for Manitoba, and 850 acres for Saskatchewan and Alberta.

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[a]Description of the twenty-four models is provided on pages 102 and 102a.

APPENDIX C

ESTIMATION OF CROP YIELD COEFFICIENTS

APPENDIX C

ESTIMATION OF CROP YIELD COEFFICIENTS[a]

In the prairies, considerable variations often exist between summerfallow and stubble yields of every crop (19). In order to identify these variations, two producing activities for each crop (viz., seeding on summerfallow and on stubble) were introduced in the recursive programming model. But published historical yield data were available separately for summerfallow and stubble crop only since 1963. For estimating reliable trend yields, a time series of this length was considered insufficient. Therefore, the yield data during 1939 to 1965 which included crops grown on both summerfallow and stubble were utilized to estimate trend yields for a composite acre. Then, ratios of stubble yields to summerfallow yields were calculated for each year during the period 1963-64 to 1969-70 for all crops. These ratios, along with the estimated composite trend yields, and stubble and summerfallow crop acreages, were used to compute yield of summerfallow crop for each year during 1963-64 to 1970-71 based upon the following equation:

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[a]Methodology presented here draws heavily on a study--"Interregional Competition in Canadian Cereal Production" by W. J. Craddock (7). However, some modifications were made to the Craddock methodology to meet the particular need of this study.

$$\hat{Y}_{i1t} = \frac{\hat{Y}_{it} \ (A_{i1t} + A_{i2t})}{(A_{i1t} + P_{it} \ A_{i2t})} \qquad (1)$$

$$i = 1, \ldots, 6$$

where:

$\hat{Y}_{i1t}$ = summerfallow yield to be estimated for the i th crop in the t th year,

$\hat{Y}_{it}$ = trend yield for a composite acre of the i th crop in the t th year,

A_{i1t} = acreage of the i th crop sown on summerfallow in the t th year,

A_{i2t} = acreage of the i th crop sown on stubble in the t th year, and

P_{it} = (stubble yield of the i th crop in the t th year) / (summerfallow yield of the i th crop in the t th year).

Trend yields of stubble crop were then calculated using the following equation:

$$\hat{Y}_{i2t} = P_{it} \ \hat{Y}_{i1t} \qquad (2)$$

where:

$\hat{Y}_{i2t}$ = stubble yield to be estimated for the i th crop in the t th year, and

other terms are defined as before.

Historical acreage data are not available prior to crop year 1963-64 on the basis of summerfallow and stubble crops. Therefore, in order to estimate the trend yields during 1957-58 to 1962-63, the following procedure was adopted:

(1) The average proportion of summerfallow and stubble crop acreages in the total acreage of each crop were estimated over the period 1963-64 to 1969-70.

(2) Summerfallow and stubble crop acreage variables (i.e., A_{i1t} and A_{i2t}) of equation (1) were then replaced by their respective average proportions.

(3) An average was also computed of yield ratios (P_{it}) during the period 1963-64 to 1969-70.

(4) This average ($\overline{P}_i$) was then substituted for yield ratio variable (P_{it}).

After introducing these changes, equations (1) and (2) become:

$$\hat{Y}_{i1t} = \frac{\hat{Y}_{it} \, (A^*_{i1} + A^*_{i2})}{(A^*_{i1} + \overline{P}_i \, A^*_{i2})} \qquad (3)$$

$$\hat{Y}_{i2t} = \overline{P}_i \, \hat{Y}_{i1t} \qquad (4)$$

where:

A^*_{i1t} = average proportion of the i th summerfallow crop acreage to the combined acreage over the period 1963-64 to 1969-70,

A^*_{i2t} = average proportion of the i th stubble crop acreage to the combined acreage over the period 1963-64 to 1969-70,

$\overline{P}_i$ = average of the i th stubble crop yields as a proportion to summerfallow yields over the period 1963-64 to 1969-70,

with the other variables defined as before.

APPENDIX D

MODEL DESCRIPTION AND SOLUTION DATA

TABLE 11

DESCRIPTION OF ACTIVITIES USED IN EACH RECURSIVE PROGRAMMING MODEL FOR EVERY YEAR

Act. No.	Activity name	Code
1	Work summerfallow	WSMFL
2	Grow wheat, summerfallow	GWHTSF
3	Grow wheat, stubble	GWHTSTB
4	Grow oats, summerfallow	GOATSSF
5	Grow oats, stubble	GOATSSTB
6	Grow barley, summerfallow	GBARSF
7	Grow barley, stubble	GBARSTB
8	Grow rye, summerfallow	GRYESF
9	Grow rye, stubble	GRYESTB
10	Grow flaxseed, summerfallow	GFLAXSF
11	Grow flaxseed, stubble	GFLAXSTB
12	Grow rapeseed, summerfallow	GRAPESF
13	Grow rapeseed, stubble	GRAPESTB
14	Sell wheat, unit quota	SWTUQTA
15	Sell wheat, specified acreage quota	SWTSQTA
16	Sell wheat, supplementary quota	SWTSUPQ
17	Sell wheat, non quota, discount price	SWTDIS
18	Sell oats, unit quota	SOTUQTA
19	Sell oats, specified acreage quota	SOTSQTA
20	Sell oats, supplementary quota	SOTSUPQ
21	Sell oats, non quota, discount price	SOTDIS
22	Sell barley, unit quota	SBARUQTA
23	Sell barley, specified acreage quota	SBARSQTA
24	Sell barley, supplementary quota	SBARSUPQ
25	Sell barley, non quota, discount price	SBARDIS
26	Sell rye, unit quota	SRYEUQTA
27	Sell rye, specified acreage quota	SRYESQTA
28	Sell rye, supplementary quota	SRYESUPQ
29	Sell rye, non quota, discount price	SRYEDIS
30	Sell flaxseed	SFLAX
31	Sell Rapeseed	SRAPE

TABLE 12

DESCRIPTION OF CONSTRAINTS USED IN EACH RECURSIVE PROGRAMMING MODEL FOR EVERY YEAR

Constraint No.	Name of constraint	Code
1	Total improved land	TLIMLD
2	Preceding year summerfallow	PRECYRSF
3	Wheat, upper flexibility restraint	WTUPFLEX
4	Wheat, lower flexibility restraint	WTLOFLEX
5	Oats, upper flexibility restraint	OTUPFLEX
6	Oats, lower flexibility restraint	OTLOFLEX
7	Barley, upper flexibility restraint	BLUPFLEX
8	Barley, lower flexibility restraint	BLLOFLEX
9	Rye, upper flexibility restraint	RYUPFLEX
10	Rye, lower flexibility restraint	RYLOFLEX
11	Flaxseed, upper flexibility restraint	FLUPFLEX
12	Flaxseed, lower flexibility restraint	FLLOFLEX
13	Rapeseed, upper flexibility restraint	RAUPFLEX
14	Rapeseed, lower flexibility restraint	RALOFLEX
15	Summerfallow, upper flexibility restraint	SFUPFLEX
16	Summerfallow, lower flexibility restraint	SFLOFLEX
17	Stubble land, upper flexibility restraint	STUPFLEX
18	Stubble land, lower flexibility restraint	STLOFLEX
19	Wheat, absolute maximum acreage	WTABSMAX
20	Wheat, absolute minimum acreage	WTABSMIN
21	Oats, absolute maximum acreage	OTABSMAX
22	Oats, absolute minimum acreage	OTABSMIN
23	Barley, absolute maximum acreage	BLABSMAX
24	Barley, absolute minimum acreage	BLABSMIN
25	Rye, absolute maximum acreage	RYABSMAX
26	Rye, absolute minimum acreage	RYABSMIN
27	Flaxseed, absolute maximum acreage	FLABSMAX
28	Flaxseed, absolute minimum acreage	FLABSMIN
29	Rapeseed, absolute maximum acreage	RAABSMAX
30	Rapeseed, absolute minimum acreage	RAABSMIN
31	Summerfallow, absolute maximum acreage	SFABSMAX
32	Summerfallow, absolute minimum acreage	SFABSMIN
33	Stubble land, absolute maximum acreage	STABSMAX
34	Stubble land, absolute minimum acreage	STABSMIN
35	Wheat supply	WHTSUP
36	Oats supply	OATSSUP
37	Barley supply	BARSUP
38	Rye supply	RYESUP
39	Flaxseed supply	FLAXSUP
40	Rapeseed supply	RAPESUP
41	Unit quota	UNITQTA
42	Specified acreage quota	SPECQTA
43	Supplementary quota, wheat	SUPPQTAW
44	Supplementary quota, oats	SUPPQTAO
45	Supplementary quota, barley	SUPPQTAB
46	Supplementary quota, rye	SUPPQTAR

TABLE 13

SCHEMATIC REPRESENTATION OF STRUCTURAL MATRIX OF EACH RECURSIVE PROGRAMMING MODEL FOR EVERY YEAR

Rest. No.	Restraint Name	Unit	Sign	Level	Activity							
					WSMFL	GWHTSF	GWHTSTB	GOATSSF	GOATSSTB	GBARSF	GBARSTB	GRYESF
1	TLIMLD	ac.	=	b_1	1	1	1	1	1	1	1	1
2	PRECYRSF	ac.	≤	b_2		1		1		1		1
3	WTUPFLEX	ac.	≤	b_3		1	1					
4	WTLOFLEX	ac.	≥	b_4		1	1					
5	OTUPFLEX	ac.	≤	b_5				1	1			
6	OTLOFLEX	ac.	≥	b_6				1	1			
7	BLUPFLEX	ac.	≤	b_7						1	1	
8	BLLOFLEX	ac.	≥	b_8						1	1	
9	RYUPFLEX	ac.	≤	b_9								1
10	RYLOFLEX	ac.	≥	b_{10}								1
11	FLUPFLEX	ac.	≤	b_{11}								
12	FLLOFLEX	ac.	≥	b_{12}								
13	RAUPFLEX	ac.	≤	b_{13}								
14	RALOFLEX	ac.	≥	b_{14}								
15	SFUPFLEX	ac.	≤	b_{15}	1							
16	SFLOFLEX	ac.	≥	b_{16}	1							
17	STUPFLEX	ac.	≤	b_{17}			1		1		1	
18	STLOFLEX	ac.	≥	b_{18}			1		1		1	
19	WTABSMAX	ac.	≤	b_{19}		1	1					
20	WTABSMIN	ac.	≥	b_{20}		1	1					
21	OTABSMAX	ac.	≤	b_{21}				1	1			
22	OTABSMIN	ac.	≥	b_{22}				1	1			
23	BLABSMAX	ac.	≤	b_{23}						1	1	
24	BLABSMIN	ac.	≥	b_{24}						1	1	
25	RYABSMAX	ac.	≤	b_{25}								1
26	RYABSMIN	ac.	≥	b_{26}								1
27	FLABSMAX	ac.	≤	b_{27}								
28	FLABSMIN	ac.	≥	b_{28}								
29	RAABSMAX	ac.	≤	b_{29}								
30	RAABSMIN	ac.	≥	b_{30}								
31	SFABSMAX	ac.	≤	b_{31}	1							
32	SFABSMIN	ac.	≥	b_{32}	1							
33	STABSMAX	ac.	≤	b_{33}			1		1		1	
34	STABSMIN	ac.	≥	b_{34}			1		1		1	
35	WHTSUP	bu.	≤	b_{35}		$-Y_w$	$-Y_w$					
36	OATSSUP	bu.	≤	b_{36}				$-Y_o$	$-Y_o$			
37	BARSUP	bu.	≤	b_{37}						$-Y_{ba}$	$-Y_{ba}$	
38	RYESUP	bu.	≤	b_{38}								$-Y_{ry}$
39	FLAXSUP	bu.	≤	b_{39}								
40	RAPESUP	bu.	≤	b_{40}								
41	UNITQTA	bu.	≤	b_{41}								
42	SPECQTA	bu.	≤	b_{42}	$-s$	$-s$	$-s$	$-s$	$-s$	$-s$	$-s$	$-s$
43	SUPPQTAW	bu.	≤	b_{43}		$-S_w$	$-S_w$					
44	SUPPQTAO	bu.	≤	b_{44}				$-S_o$	$-S_o$			
45	SUPPQTAB	bu.	≤	b_{45}						$-S_{ba}$	$-S_{ba}$	
46	SUPPQTAR	bu.	≤	b_{46}								$-S_{ry}$
47	Net Price	$			$-C_s$	$-C_w$	$-c_w$	$-C_o$	$-c_o$	$-C_{ba}$	$-c_{ba}$	$-C_{ry}$

The activities and constraints have been described in Appendix Tables 11 and 12. The input-output coefficients are identified on page 199.

TABLE 13 -- (continued)

Restr. No.	Activity										
	GRYESTB	GFLAXSF	GFLAXSTB	GRAPESF	GRAPESTB	SWTUQTA	SWTSQTA	SWTSUPQ	SWTDIS	SOTUQTA	SOTSQTA
1	1	1	1	1	1						
2		1		1							
3											
4											
5											
6											
7											
8											
9	1										
10	1										
11		1	1								
12		1	1								
13				1	1						
14				1	1						
15											
16											
17	1		1		1						
18	1		1		1						
19											
20											
21											
22											
23											
24											
25	1										
26	1										
27		1	1								
28		1	1								
29				1	1						
30				1	1						
31											
32											
33	1		1		1						
34	1		1		1						
35						1	1	1	1		
36										1	1
37											
38	$-Y_{ry}$										
39		$-Y_f$	$-Y_f$								
40				$-Y_{ra}$	$-Y_{ra}$						
41						$1/u_w$				$1/u_o$	
42	$-s$						1				1
43								1			
44											
45											
46	$-S_{ry}$										
47	$-c_{ry}$	$-C_f$	$-c_f$	$-C_{ra}$	$-c_{ra}$	P_w	P_w	P_w	P_w	P_o	P_o

TABLE 13 -- (continued)

Restr. No.	Activity											
	SOTSUPQ	SOTDIS	SBARUQTA	SBARSQTA	SBARSUPQ	SBARDIS	SRYEUQTA	SRYESQTA	SRYESUPQ	SRYEDIS	SFLAX	SRAPE
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												
13												
14												
15												
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25												
26												
27												
28												
29												
30												
31												
32												
33												
34												
35												
36	1	1										
37			1	1	1	1						
38							1	1	1	1		
39											1	
40												1
41			$1/u_{ba}$				$1/u_{ry}$					
42				1				1				
43												
44	1											
45					1							
46									1			
47	P_o	P_o	P_{ba}	P_{ba}	P_{ba}	P_{ba}	P_{ry}	P_{ry}	P_{ry}	P_{ry}	P_f	P_{ra}

TABLE 13 -- (continued)

The symbols utilized in Table 13 can be explained as follows:

b = a vector of constraint levels,

Y = yield per acre of a summerfallow crop,

y = yield per acre of a stubble crop,

u = number of bushels per unit quota for a crop,

s = specified acreage quota per acre,

S = supplementary quota per acre,

C = cost of production per acre of a summerfallow crop,

c = cost of production per acre of a stubble crop,

P = price per bushel (undiscounted), and

p = price per bushel discounted by 20 percent.

Subscript:

w = wheat,

o = oats,

ba = barley,

ry = rye,

f = flaxseed, and

ra = rapeseed.

Description of activities and constraints have been presented in Appendix Tables 11 and 12.

TABLE 14

REGRESSION EQUATIONS FOR FLEXIBILITY COEFFICIENTS ESTIMATED FOR EACH RECURSIVE PROGRAMMING MODEL USING STRATIFIED OBSERVATIONS

Crop	Equation
	Upper flexibility coefficient
Wheat	$(X_{1t}/X_{1t-1}) = b_{10} + b_{11}X_{1t-1} + b_{12}P_{1t-1} + b_{15}E_{1t-1} + b_{17}T + e_{1t}$
Oats	$(X_{2t}/X_{2t-1}) = b_{20} + b_{21}X_{2t-1} + b_{22}P_2{}^*{}_{t-1} + b_{26}M_{t-1} + e_{2t}$
Barley	$(X_{3t}/X_{3t-1}) = b_{30} + b_{31}X_{3t-1} + b_{32}P_3{}^*{}_{t-1} + b_{34}S_3{}^*{}_{t-1} + e_{3t}$
Rye	$(X_{4t}/X_{4t-1}) = b_{40} + b_{41}X_{4t-1} + b_{42}P_{4t-1} + b_{45}E_{1t-1} + e_{4t}$
Flaxseed	$(X_{5t}/X_{5t-1}) = b_{50} + b_{51}X_{5t-1} + b_{52}P_{5t-1} + b_{53}P_{1t-1} + b_{56}M_{t-1} + e_{5t}$
Rapeseed[a]	$(X_{6t}/X_{6t-1}) = b_{60} + b_{61}X_{6t-1} + b_{62}P_{6t-1} + b_{63}P_{1t-1} + b_{66}M_{t-1} + e_{6t}$
Summerfallow	$(X_{7t}/X_{7t-1}) = b_{70} + b_{71}X_{7t-1} + b_{73}P_{1t-1} + b_{75}E_{1t-1} + b_{76}M_{t-1} + e_{7t}$
Stubble land	$(X_{8t}/X_{8t-1}) = b_{80} + b_{81}X_{8t-1} + b_{83}P_{1t-1} + b_{84}S_{1t-1} + b_{86}M_{t-1} + e_{8t}$
	Lower flexibility coefficient
Wheat	$(X_{1t}/X_{1t-1}) = b_{10} + b_{11}X_{1t-1} + b_{12}P_{1t-1} + b_{17}T + e_{1t}$
Oats	$(X_{2t}/X_{2t-1}) = b_{20} + b_{21}X_{2t-1} + b_{22}P_{2t-1} + b_{23}P_{1t-1} + b_{27}T + e_{2t}$
Barley	$(X_{3t}/X_{3t-1}) = b_{30} + b_{31}X_{3t-1} + b_{32}P_3{}^*{}_{t-1} + b_{34}S_3{}^*{}_{t-1} + b_{36}M_{t-1} + e_{3t}$
Rye	$(X_{4t}/X_{4t-1}) = b_{40} + b_{41}X_{4t-1} + b_{42}P_4{}^*{}_{t-1} + b_{44}S_4{}^*{}_{t-1} + b_{45}E_{1t-1} + e_{4t}$
Flaxseed	$(X_{5t}/X_{5t-1}) = b_{50} + b_{51}X_{5t-1} + b_{52}P_{5t-1} + b_{54}S_5{}^*{}_{t-1} + e_{5t}$
Rapeseed[a]	$(X_{6t}/X_{6t-1}) = b_{60} + b_{61}X_{6t-1} + b_{62}P_6{}^*{}_{t-1} + b_{66}M_{t-1} + e_{6t}$
Summerfallow	$(X_{7t}/X_{7t-1}) = b_{70} + b_{71}X_{7t-1} + b_{73}P_{1t-1} + b_{76}E_{1t-1} + e_{7t}$
Stubble land	$(X_{8t}/X_{8t-1}) = b_{80} + b_{81}X_{8t-1} + b_{83}P_{1t-1} + b_{85}E_{1t-1} + e_{8t}$

[a]Because of lack of degrees of freedom, these equations were not estimated for Alberta and crop districts of Saskatchewan. Rather, rapeseed equation given in Table 15 was estimated.

TABLE 15

REGRESSION EQUATIONS USED TO ESTIMATE FLEXIBILITY COEFFICIENTS FOR EACH MODEL USING ALL (UNSTRATIFIED) OBSERVATIONS

Crop	Equation
Wheat	$(X_{1t}/X_{1t-1}) = b_{10} + b_{11}X_{1t-1} + b_{12}P_{1t-1} + b_{14}S_{1t-1} + b_{16}M_{t-1} + b_{17}T + e_{1t}$
Oats	$(X_{2t}/X_{2t-1}) = b_{20} + b_{21}X_{2t-1} + b_{22}P_{2t-1} + b_{23}P_{1t-1} + b_{25}E_{1t-1} + b_{27}T + e_{2t}$
Barley	$(X_{3t}/X_{3t-1}) = b_{30} + b_{31}X_{3t-1} + b_{32}P_{3t-1} + b_{33}P_{1t-1} + b_{34}S_{3}^{*}{}_{t-1} + b_{36}M_{t-1} + e_{3t}$
Rye	$(X_{4t}/X_{4t-1}) = b_{40} + b_{41}X_{4t-1} + b_{42}P_{4}^{*}{}_{t-1} + b_{44}S_{4}^{*}{}_{t-1} + b_{45}E_{1t-1} + b_{47}T + e_{4t}$
Flaxseed	$(X_{5t}/X_{5t-1}) = b_{50} + b_{51}X_{5t-1} + b_{52}P_{5}^{*}{}_{t-1} + b_{54}S_{5}^{*}{}_{t-1} + b_{65}E_{5t-1} + b_{56}M_{t-1} + e_{5t}$
Rapeseed	$(X_{6t}/X_{6t-1}) = b_{60} + b_{61}X_{6t-1} + b_{62}P_{6t-1} + b_{63}P_{1t-1} + b_{66}M_{t-1} + e_{6t}$
Summerfallow	$(X_{7t}/X_{7t-1}) = b_{70} + b_{71}X_{7t-1} + b_{73}P_{1t-1} + b_{74}S_{1t-1} + b_{76}M_{t-1} + b_{77}T + e_{7t}$
Stubble land	$(X_{8t}/X_{8t-1}) = b_{80} + b_{81}X_{8t-1} + b_{83}P_{1t-1} + b_{85}E_{1t-1} + b_{86}M_{t-1} + b_{87}T + e_{8t}$

Variables are explained on page 15.

The variables utilized in Tables 14 and 15 can be expressed as follows:

X_t = acreage of a crop in the t th year,

X_{t-1} = acreage of a crop in the t-1 th year,

P_{t-1} = price of a product in the t-1 th year,

P^*_{t-1} = price of a crop as a proportion to wheat price in the t-1 th year,

S_{t-1} = total stocks of a product in the t-1 th year,

S^*_{t-1} = total stocks of a product as a proportion to wheat stocks in the t-1 th year,

E_{t-1} = exports of a crop in the t-1 th year,

M_{t-1} = total precipitation during April and May of the t-1 th year, and

T = time trend (1953=1).

Crops are identified by subscripts 1 to 8 as follows:

Subscript	Crop
1	= wheat
2	= oats
3	= barley
4	= rye
5	= flaxseed
6	= rapeseed
7	= summerfallow
8	= stubble land

TABLE 16

ESTIMATED FLEXIBILITY COEFFICIENTS BY CROPS, YEARS AND MODELS

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$
								Prairies								
1958	0.997	1.061	0.877	1.127	0.961	1.043	0.942	1.014	0.808	1.000	0.989	2.023	0.987	1.044	0.927	1.013
1959	0.975	1.068	0.911	1.134	0.887	0.933	1.020	1.030	0.753	1.086	0.380	1.062	0.991	0.994	0.915	1.024
1960	0.986	1.058	1.023	1.129	0.871	1.009	1.065	1.094	0.562	1.261	1.016	3.809	0.979	1.016	0.934	1.049
1961	0.993	1.037	0.867	1.093	0.779	0.985	0.948	1.027	0.700	0.858	0.908	1.000	0.987	1.043	0.813	1.038
1962	0.991	1.071	0.926	1.201	0.923	1.038	1.044	1.100	0.720	0.884	0.510	0.656	0.965	1.021	1.040	1.129
1963	0.964	1.039	0.867	1.072	0.926	1.111	1.036	1.046	0.655	1.159	0.612	1.443	0.983	0.999	1.013	1.091
1964	0.982	1.066	0.781	1.160	0.898	1.008	1.035	1.057	0.739	1.266	0.968	1.396	0.986	1.033	1.036	1.087
1965	0.960	1.014	0.965	1.230	0.995	1.164	1.095	1.125	0.765	1.066	1.352	1.842	1.001	1.015	0.997	1.100
1966	1.005	1.062	1.005	1.138	1.076	1.233	0.958	1.025	0.803	0.815	1.129	1.195	0.956	1.050	1.062	1.121
1967	0.961	1.002	0.918	1.171	1.044	1.059	0.956	1.196	0.599	1.118	0.983	1.000	1.022	1.043	1.006	1.087
1968	0.967	0.993	1.003	1.187	1.078	1.192	1.038	1.062	0.538	1.603	0.702	1.000	1.006	1.012	0.727	1.078
1969	0.980	0.988	0.852	1.173	0.964	1.032	0.979	1.054	0.553	0.972	0.934	1.354	0.912	1.033	0.876	1.087
1970	0.920	1.038	0.833	1.178	0.996	1.100	1.023	1.079	1.187	1.312	1.244	1.375	1.032	1.050	0.969	1.044
1971	0.869	1.740	0.874	1.196	0.754	1.892	0.934	1.295	1.045	1.155	1.398	2.801	0.620	1.000	0.645	1.301
								Manitoba								
1958	1.049	1.105	0.970	1.068	0.793	1.084	0.989	1.191	0.771	0.884	0.772	1.547	0.907	1.099	0.950	1.124
1959	0.981	1.065	0.973	1.032	0.849	1.176	1.186	1.213	0.939	1.233	0.652	2.771	0.929	1.021	0.924	1.077
1960	0.987	1.057	0.960	1.088	0.822	1.235	1.014	1.120	0.677	1.263	0.778	2.772	0.956	1.033	0.936	1.134
1961	1.028	1.048	0.926	1.292	0.794	1.275	0.963	1.398	0.775	1.160	0.858	1.971	0.956	1.016	0.959	1.151
1962	0.981	1.039	1.140	1.364	0.775	1.035	0.933	1.484	0.892	1.158	0.652	1.149	0.956	1.065	0.936	1.111
1963	0.941	1.031	0.914	0.964	0.834	1.023	0.784	0.961	0.999	1.223	0.629	1.357	1.005	1.030	1.025	1.105
1964	1.019	1.066	0.956	1.033	0.833	1.409	1.017	1.382	0.907	1.274	0.821	1.827	0.883	1.088	0.922	1.021
1965	0.957	1.019	0.895	0.977	0.867	1.087	0.981	1.006	1.010	1.223	0.854	1.754	0.983	1.038	1.008	1.118
1966	1.007	1.091	1.042	1.071	0.902	1.350	0.781	0.804	0.817	1.000	0.849	1.166	0.949	1.036	0.985	1.048
1967	0.991	1.094	0.930	0.949	0.997	1.274	0.936	1.442	0.580	0.870	0.772	1.373	1.000	1.044	0.983	1.073
1968	0.978	1.073	1.000	1.000	0.991	1.068	0.970	1.225	0.775	1.444	0.695	1.000	0.988	1.040	0.965	1.067
1969	0.988	1.114	0.886	1.138	1.166	1.263	1.126	1.236	0.722	1.366	0.795	0.818	1.000	1.027	0.844	1.151
1970	0.878	1.000	1.000	1.000	0.997	1.066	1.000	1.000	0.631	1.000	0.883	1.000	1.142	1.262	0.853	1.016
1971	0.898	1.613	1.250	1.287	0.804	1.300	1.000	1.000	0.666	1.000	0.880	1.000	0.821	0.984	0.927	1.342

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$
								Saskatchewan								
1958	0.931	1.035	0.727	1.296	0.798	1.058	0.925	0.941	0.801	2.031	0.910	1.655	0.978	1.041	0.892	1.108
1959	0.978	1.076	0.847	1.311	0.787	0.847	0.925	1.030	0.511	1.138	0.368	0.981	0.990	1.025	0.858	1.112
1960	0.945	0.954	1.106	1.438	0.780	0.797	0.912	1.007	1.286	1.326	1.231	3.401	0.958	1.016	0.855	1.083
1961	0.953	1.028	0.679	1.114	0.745	0.855	0.923	1.132	0.659	1.089	0.686	1.000	0.989	1.073	0.704	1.083
1962	0.977	1.097	0.786	1.444	0.835	0.962	0.871	1.170	0.493	1.000	0.421	0.583	0.971	0.985	1.008	1.139
1963	0.976	1.023	0.809	1.007	0.817	1.109	0.962	1.075	1.000	1.185	0.780	1.542	1.004	1.044	0.990	1.135
1964	0.952	1.056	0.635	1.116	0.782	1.000	0.884	1.106	0.577	1.133	1.000	1.098	0.982	1.012	1.043	1.102
1965	0.960	1.008	0.972	1.443	0.874	1.347	0.941	1.154	0.420	1.051	1.671	1.877	0.991	1.029	1.002	1.096
1966	0.941	1.075	1.011	1.357	0.846	1.308	0.856	1.130	0.774	1.000	1.117	1.259	0.956	1.003	1.130	1.194
1967	0.999	1.074	0.786	1.356	0.811	0.992	0.920	1.300	0.494	0.514	0.381	0.571	0.996	1.057	1.006	1.090
1968	0.945	1.016	0.849	1.499	0.783	0.875	0.969	1.000	0.809	1.176	0.529	1.000	0.946	1.014	0.850	1.087
1969	0.931	0.949	0.616	1.330	0.827	1.194	0.983	1.000	0.922	0.973	0.764	1.521	0.949	1.003	0.891	1.134
1970	0.860	0.877	1.000	1.202	0.734	0.770	1.000	1.000	1.239	2.436	1.392	1.633	0.907	1.009	0.573	0.633
1971	0.655	1.725	1.000	1.244	0.634	0.981	0.991	1.000	1.000	1.085	1.126	1.297	0.725	0.825	1.050	1.151
								Alberta								
1958	0.951	1.065	0.913	1.055	0.979	1.053	0.907	1.242	1.003	1.076	1.651	2.476	0.950	1.004	0.961	1.011
1959	0.926	1.144	0.905	1.045	0.939	1.058	0.979	1.183	0.847	1.012	0.880	1.320	0.981	1.028	0.984	1.109
1960	0.962	1.043	1.010	1.126	0.955	1.086	0.949	1.095	0.861	2.038	2.559	3.839	1.018	1.038	0.967	1.000
1961	0.923	1.061	0.883	1.067	0.877	1.147	0.980	1.170	0.844	1.175	0.846	1.269	0.968	0.986	0.965	1.143
1962	0.937	1.041	0.895	1.078	1.011	1.164	0.822	1.296	0.811	1.129	0.179	0.269	0.961	0.978	0.976	1.119
1963	0.984	1.013	0.896	1.027	0.930	1.139	1.024	1.250	0.843	1.267	1.374	2.062	0.966	1.010	0.972	1.118
1964	1.043	1.089	0.815	0.832	0.950	1.032	1.005	1.157	1.056	1.267	1.701	2.552	0.981	1.084	0.899	1.171
1965	0.954	1.016	0.956	1.147	1.095	1.137	0.946	1.149	0.982	1.398	2.166	3.406	1.010	1.017	0.826	1.082
1966	1.031	1.079	0.956	1.028	1.031	1.133	0.814	1.021	0.930	1.000	0.446	0.668	0.938	0.967	0.798	1.046
1967	0.970	1.038	0.928	1.116	0.948	1.052	0.886	1.083	0.546	1.512	0.897	1.346	0.992	1.032	1.003	1.109
1968	0.908	1.125	0.924	1.051	0.753	1.068	1.113	1.259	1.091	2.057	0.529	1.000	1.008	1.051	0.820	1.069
1969	0.867	0.881	0.912	1.065	0.944	1.080	1.163	1.195	1.089	1.773	1.275	1.559	1.062	1.124	0.784	1.164
1970	0.898	0.945	0.860	1.050	0.880	0.973	0.909	1.321	1.033	1.428	1.464	1.789	1.052	1.079	0.718	1.211
1971	0.955	1.846	0.790	0.915	0.956	1.057	0.960	1.201	0.816	1.000	1.000	1.000	0.769	0.922	0.950	1.426

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$
	Saskatchewan -- crop district 1A															
1958	1.027	1.138	0.759	0.997	0.647	1.070	0.971	2.500	0.324	0.687	..	..	0.989	1.110	0.733	1.139
1959	0.954	1.142	0.702	1.097	0.677	0.785	0.914	1.247	0.650	1.064	..	..	0.972	1.087	0.626	1.218
1960	0.892	1.040	1.188	1.259	0.718	1.000	0.869	1.480	0.960	1.004	..	..	1.021	1.062	0.902	1.263
1961	1.031	1.095	0.777	1.062	0.514	1.247	0.657	0.873	0.796	0.973	..	..	0.975	1.128	0.799	1.096
1962	1.021	1.135	0.759	1.047	0.754	1.018	0.725	1.478	0.627	2.022	..	..	0.917	1.092	0.973	1.025
1963	0.861	1.093	0.830	1.108	0.932	1.000	0.673	1.695	1.000	1.107	..	..	0.935	1.007	0.881	1.281
1964	1.010	1.048	0.723	1.017	0.640	0.881	1.372	1.439	0.886	1.096	..	..	0.979	1.149	0.992	1.125
1965	0.952	1.006	0.802	1.215	1.019	1.110	0.667	0.932	1.158	1.582	..	..	0.984	1.033	0.870	1.188
1966	1.028	1.100	0.922	1.104	0.714	1.779	0.733	2.144	0.700	1.089	..	..	0.940	1.145	1.031	1.083
1967	1.015	1.030	1.020	1.071	0.611	1.145	0.714	4.492	0.503	1.179	..	..	1.020	1.174	1.082	1.248
1968	0.985	1.059	1.060	1.106	0.548	1.000	0.995	1.000	0.616	1.000	..	..	0.967	1.144	1.047	1.057
1969	0.968	1.014	0.777	1.026	0.624	1.081	1.000	1.000	0.569	1.000	..	..	1.011	1.151	1.058	1.108
1970	0.918	1.021	1.074	1.132	0.664	1.000	1.000	1.000	1.000	1.365	..	..	1.048	1.062	0.385	0.892
	Saskatchewan -- crop district 1B															
1958	1.131	1.171	0.884	1.040	0.741	0.949	0.843	1.650	0.521	0.670	..	..	1.009	1.072	0.930	1.000
1959	1.024	1.184	0.630	1.148	0.846	0.848	0.694	0.938	0.929	0.961	..	..	0.974	1.076	0.769	0.970
1960	0.974	0.997	1.270	1.318	0.934	1.000	0.950	0.952	1.167	1.384	..	..	1.059	1.068	1.064	1.086
1961	1.047	1.136	0.691	1.266	0.730	0.787	0.647	1.166	0.901	1.000	..	..	0.928	1.061	0.847	1.166
1962	1.040	1.181	0.852	1.036	0.849	1.326	1.000	1.292	0.630	1.000	..	..	0.931	1.097	0.852	1.327
1963	0.953	1.089	0.835	1.152	0.523	1.208	1.648	1.818	1.000	1.183	..	..	1.028	1.093	1.120	1.350
1964	1.006	1.085	0.717	1.288	0.759	0.786	1.000	1.245	1.000	1.220	..	..	0.966	1.050	0.911	1.226
1965	0.998	1.000	0.775	1.312	1.045	1.295	0.687	1.192	1.519	1.549	..	..	0.995	1.053	1.013	1.056
1966	1.017	1.100	0.972	1.151	0.748	1.026	0.863	2.334	0.751	0.914	..	..	0.937	1.142	1.023	1.152
1967	0.983	1.007	1.107	1.165	0.242	0.598	0.811	2.907	1.007	1.726	..	..	0.989	1.072	0.765	0.877
1968	0.990	1.080	1.000	1.000	0.878	1.000	1.000	1.589	1.230	1.700	..	..	0.949	0.978	0.976	1.043
1969	0.923	1.043	0.901	1.060	1.000	1.000	1.000	1.180	1.252	1.597	..	..	0.940	0.947	1.000	1.128
1970	0.807	1.040	0.890	1.050	1.525	1.686	0.688	1.000	1.618	2.035	..	..	0.900	1.001	1.000	1.528

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	1-$\underline{\beta}$	1+$\bar{\beta}$	1-$\underline{\beta}$	1+$\bar{\beta}$	1-$\underline{\beta}$	1+$\bar{\beta}$	1-$\underline{\beta}$	1+$\bar{\beta}$	1-$\underline{\beta}$	1+$\bar{\beta}$	1-$\underline{\beta}$	1+$\bar{\beta}$	1-$\underline{\beta}$	1+$\bar{\beta}$	1-$\underline{\beta}$	1+$\bar{\beta}$
							Saskatchewan -- crop district 2A									
1958	0.898	1.065	0.787	1.116	0.859	2.096	0.819	1.122	0.417	2.088	..	..	1.049	1.139	0.804	1.178
1959	0.845	1.127	0.818	1.131	0.695	2.205	0.860	1.110	0.793	1.320	..	..	1.006	1.099	0.714	1.119
1960	0.877	1.031	0.843	1.313	0.797	2.339	0.899	1.190	0.667	1.536	..	..	0.977	1.003	0.621	1.230
1961	0.872	1.165	0.861	1.049	0.713	2.013	0.771	0.952	1.005	1.243	..	..	0.968	1.090	0.790	1.109
1962	0.833	1.151	0.834	0.997	0.547	1.506	0.911	1.308	0.266	0.330	..	..	0.941	1.239	0.904	1.314
1963	0.953	1.188	0.780	1.042	0.889	1.475	0.947	1.276	0.932	1.469	..	..	0.984	1.021	0.840	1.064
1964	1.006	1.027	0.812	1.000	0.653	1.657	0.848	1.370	0.905	1.186	..	..	0.950	1.042	1.000	1.006
1965	0.929	1.163	0.896	1.044	1.249	1.615	0.874	1.118	1.000	1.258	..	..	1.010	1.029	0.931	0.977
1966	0.986	1.155	0.930	1.007	1.000	1.126	0.722	1.454	0.606	0.643	..	..	0.939	0.947	1.144	1.377
1967	1.006	1.164	0.986	1.051	1.263	1.859	0.834	1.613	0.459	0.470	..	..	1.050	1.081	1.132	1.298
1968	0.853	1.045	1.036	1.130	1.247	2.150	0.564	1.099	1.113	1.230	..	..	0.963	1.000	1.000	1.000
1969	0.907	1.148	0.939	1.038	1.446	2.146	0.523	1.150	1.325	1.465	..	..	0.929	1.000	0.942	1.000
1970	0.830	1.140	0.788	0.921	2.012	2.170	0.491	0.897	1.509	2.060	..	..	0.841	1.000	0.599	1.000
							Saskatchewan -- crop district 2B									
1958	0.994	1.035	0.878	0.971	0.426	1.252	0.843	1.076	0.673	1.029	..	..	1.006	1.060	0.879	1.317
1959	0.963	1.077	0.871	1.092	0.611	0.943	0.836	1.038	0.468	1.390	..	..	0.972	1.039	0.811	1.215
1960	0.936	1.065	0.825	1.288	0.884	0.915	0.990	1.167	1.126	1.793	..	..	1.034	1.039	0.767	1.098
1961	0.988	1.061	0.882	1.299	0.636	0.703	0.695	0.994	0.903	1.195	..	..	0.959	1.030	0.841	1.373
1962	0.993	1.223	0.739	0.782	0.308	0.398	1.021	1.257	0.469	0.794	..	..	0.931	1.020	0.900	1.226
1963	0.961	1.029	0.732	0.852	0.933	1.043	1.278	1.381	1.574	1.583	..	..	0.982	1.022	0.793	1.042
1964	0.998	1.036	0.708	1.196	0.695	1.000	1.031	1.282	0.736	1.361	..	..	1.002	1.061	0.838	1.281
1965	0.946	1.039	0.720	1.268	0.533	1.988	1.095	1.151	1.000	1.458	..	..	0.973	1.023	0.824	1.004
1966	0.992	1.091	0.963	0.997	1.411	1.570	0.842	1.374	0.463	1.152	..	..	0.895	1.055	0.886	1.097
1967	0.987	1.015	0.857	1.601	1.246	1.315	0.956	1.527	0.401	1.457	..	..	1.008	1.073	0.802	1.197
1968	0.989	1.133	0.886	1.420	1.577	1.812	0.521	1.088	1.049	1.924	..	..	1.039	1.050	0.815	0.995
1969	0.950	0.995	0.932	1.538	1.951	2.156	0.649	1.098	0.248	2.066	..	..	1.000	1.000	1.122	1.326
1970	0.908	1.426	0.998	1.000	0.727	1.000	1.009	1.091	0.998	1.950	..	..	1.000	1.000	1.747	1.931

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	1-β	1+β̄	1-β	1+β̄	1-β	1+β̄	1-β	1+β̄	1-β	1+β̄	1-β	1+β̄	1-β	1+β̄	1-β	1+β̄
	Saskatchewan -- crop district 3AS															
1958	0.964	1.033	0.789	1.218	0.740	0.989	0.899	0.933	0.377	1.755	..	..	0.974	1.069	0.817	1.042
1959	0.960	1.070	0.713	1.384	0.753	0.781	0.808	0.954	0.254	1.364	..	..	0.987	1.070	0.724	1.058
1960	0.953	1.016	1.391	1.441	0.773	1.000	0.992	1.090	0.531	3.682	..	..	0.996	1.084	1.053	1.218
1961	0.968	1.004	0.743	1.227	0.814	0.905	0.722	0.724	0.431	1.766	..	..	0.962	1.073	0.951	1.091
1962	0.962	1.082	0.814	1.338	0.524	0.918	0.561	1.833	0.283	0.625	..	..	0.973	1.077	0.690	0.908
1963	0.962	1.039	0.523	1.341	0.575	1.198	0.961	2.181	0.625	2.386	..	..	0.984	1.069	0.930	1.113
1964	0.988	1.012	0.833	0.871	0.682	1.000	1.115	1.287	0.783	1.000	..	..	0.966	1.017	0.923	1.098
1965	0.967	1.033	0.842	1.372	0.483	1.765	1.000	1.093	0.968	3.146	..	..	0.969	1.045	0.984	1.226
1966	0.988	1.083	0.951	1.178	1.015	1.250	0.921	1.000	0.489	0.981	..	..	0.958	1.045	0.700	1.089
1967	0.982	1.032	0.747	1.082	0.569	1.022	0.634	0.909	0.627	2.184	..	..	0.962	1.060	1.052	1.226
1968	0.957	1.047	0.816	1.206	0.589	1.020	0.628	1.860	0.494	4.937	..	..	0.991	1.061	0.964	0.972
1969	0.950	1.064	0.874	1.129	1.000	1.133	1.185	1.730	0.796	3.969	..	..	0.946	1.036	0.876	1.264
1970	0.953	1.113	0.985	1.221	0.886	1.631	1.000	1.000	0.908	4.253	..	..	0.955	1.106	0.868	1.681
	Saskatchewan -- crop district 3AN															
1958	0.969	1.099	0.907	0.967	0.886	1.448	0.866	0.920	0.343	2.895	..	..	0.972	1.038	0.903	1.211
1959	0.958	1.112	0.807	1.105	0.723	1.365	0.976	1.046	0.449	2.321	..	..	0.979	1.044	0.840	1.188
1960	0.948	1.013	1.111	1.394	0.756	1.297	0.988	1.151	2.203	2.293	..	..	0.992	1.064	0.886	0.938
1961	0.960	1.014	0.558	1.254	0.612	0.845	0.829	0.940	0.778	2.161	..	..	0.979	1.046	0.772	1.073
1962	0.969	1.051	0.778	1.000	0.532	1.479	0.651	1.544	0.236	0.500	..	..	0.964	1.029	0.556	1.054
1963	0.954	1.039	0.722	1.229	1.059	1.235	0.957	1.683	0.980	0.984	..	..	0.969	1.012	0.922	1.206
1964	0.996	1.022	0.536	1.276	0.568	0.649	0.972	1.229	0.762	1.054	..	..	1.019	1.048	0.904	1.268
1965	0.950	1.043	0.646	1.786	0.721	1.618	0.933	1.153	1.042	1.361	..	..	1.005	1.029	1.110	1.172
1966	0.989	1.094	0.771	1.310	0.683	1.124	0.842	0.859	0.726	1.017	..	..	0.956	1.054	0.735	1.254
1967	0.983	1.035	0.847	1.357	0.776	1.100	0.674	1.044	0.421	1.408	..	..	1.029	1.067	0.995	1.000
1968	0.955	1.067	0.595	1.283	0.740	1.140	0.812	1.558	0.986	1.443	..	..	0.855	1.005	0.778	0.916
1969	0.932	1.034	1.000	1.396	0.755	1.384	0.922	1.492	1.309	1.600	..	..	0.905	1.019	1.000	1.036
1970	0.917	1.039	1.000	1.650	1.000	1.025	1.000	1.296	1.348	2.437	..	..	0.852	1.000	1.266	1.302

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$
Saskatchewan -- crop district 3BS																
1958	0.927	1.014	0.712	1.004	1.207	1.280	0.781	1.236	0.732	2.421	..	..	1.000	1.029	0.718	1.184
1959	0.968	1.141	0.762	1.435	0.684	1.328	0.628	1.508	0.529	2.238	..	..	0.961	1.063	0.885	1.353
1960	0.947	1.010	1.015	1.295	0.903	1.356	1.015	1.565	0.812	2.610	..	..	0.981	1.078	0.998	1.497
1961	0.992	1.081	0.706	1.607	0.824	1.205	0.616	1.163	0.382	2.022	..	..	1.013	1.023	0.773	1.360
1962	0.940	1.051	0.596	1.622	0.822	1.299	0.605	1.962	0.167	1.935	..	..	0.961	1.065	0.726	1.562
1963	1.025	1.055	0.948	1.656	1.187	1.423	0.798	2.270	0.437	2.250	..	..	0.968	1.079	1.145	1.743
1964	0.964	1.007	0.593	1.253	0.663	1.287	0.874	1.020	0.787	0.857	..	..	0.953	1.027	1.152	1.202
1965	0.991	1.085	0.971	1.451	0.988	1.184	0.992	1.464	0.850	1.558	..	..	0.982	1.069	0.839	1.505
1966	1.004	1.074	0.639	1.754	0.883	1.196	0.768	0.820	0.643	0.790	..	..	0.956	1.044	1.154	1.274
1967	0.983	0.999	0.895	1.425	0.858	1.196	0.778	1.053	0.501	0.700	..	..	0.945	1.067	1.003	1.237
1968	0.936	1.035	0.706	1.351	0.851	1.365	0.716	2.026	0.846	1.822	..	..	0.985	1.092	0.728	1.346
1969	1.021	1.152	0.774	1.532	0.796	1.226	1.084	1.985	1.211	2.006	..	..	0.976	1.070	1.176	1.559
1970	1.000	1.000	0.752	1.793	1.000	1.392	0.680	1.000	0.606	2.512	..	..	0.974	0.998	1.085	1.504
Saskatchewan -- crop district 3BN																
1958	0.928	1.154	0.659	1.091	1.056	1.275	0.793	2.019	0.338	1.038	..	..	1.028	1.048	0.853	1.241
1959	0.931	1.158	0.845	1.176	0.694	1.197	0.973	1.153	0.591	1.847	..	..	0.951	1.060	0.865	1.083
1960	0.952	1.120	0.954	1.830	0.818	1.136	0.921	1.455	1.280	1.476	..	..	1.052	1.081	0.972	1.021
1961	0.919	1.083	0.946	1.373	0.692	1.030	0.745	1.000	0.553	0.658	..	..	0.986	1.026	0.771	1.170
1962	0.920	1.074	0.506	1.000	0.695	1.367	0.889	2.271	0.418	2.417	..	..	0.937	1.045	0.787	1.335
1963	0.930	1.001	0.769	1.256	0.680	1.217	0.792	2.418	0.405	1.533	..	..	0.962	1.026	0.918	0.938
1964	0.981	1.066	0.544	0.715	0.737	1.258	0.838	1.000	0.741	0.900	..	..	0.963	1.028	0.968	1.225
1965	0.911	1.031	1.050	1.898	1.044	1.480	0.975	1.077	0.632	1.457	..	..	0.970	0.990	0.831	1.234
1966	0.970	1.014	0.822	0.897	0.968	1.501	0.854	1.000	0.738	2.486	..	..	0.946	1.042	0.790	1.254
1967	0.878	1.021	0.940	1.124	0.937	1.365	0.768	1.447	0.543	0.660	..	..	1.033	1.074	0.806	1.361
1968	0.928	1.047	0.840	1.000	0.955	1.144	0.812	2.668	0.597	1.000	..	..	0.958	1.048	0.781	0.863
1969	0.933	1.155	0.943	1.042	1.054	1.188	0.740	2.804	0.927	1.675	..	..	0.870	1.000	1.173	1.182
1970	0.918	1.015	0.617	0.682	1.000	1.100	0.891	1.353	1.037	2.128	..	..	0.908	1.000	1.000	1.000

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$
							Saskatchewan -- crop district 4A									
1958	0.987	1.026	0.885	1.132	0.993	1.112	0.837	0.951	0.534	1.109	..	..	0.969	1.091	0.813	1.041
1959	0.984	1.091	0.803	1.148	0.894	1.049	0.858	1.249	0.629	1.603	..	..	0.954	1.107	0.711	1.366
1960	0.958	1.045	0.759	1.627	0.785	1.000	0.916	1.383	0.597	1.641	..	..	1.072	1.083	0.784	1.086
1961	0.955	1.021	0.877	2.355	0.761	0.978	0.786	0.883	0.634	1.000	..	..	0.984	1.080	0.747	1.414
1962	0.993	1.115	0.479	1.000	0.869	1.461	0.699	1.809	0.157	0.173	..	..	0.974	0.993	0.615	1.500
1963	0.972	1.100	0.933	1.721	0.952	1.017	0.766	2.032	0.816	1.211	..	..	0.995	1.008	0.645	1.745
1964	0.969	1.024	0.747	0.855	0.775	0.829	0.862	1.270	0.689	0.884	..	..	0.950	1.027	0.841	1.030
1965	0.976	1.065	0.807	1.237	0.791	1.217	0.898	1.256	0.588	1.298	..	..	1.002	1.019	0.742	1.255
1966	1.036	1.058	0.774	1.242	0.882	1.234	0.806	0.928	0.778	1.547	..	..	0.931	0.938	0.768	1.505
1967	0.980	1.030	0.622	0.774	0.789	0.801	0.885	0.942	0.377	0.589	..	..	1.007	1.047	0.917	1.000
1968	0.910	1.020	0.802	1.887	0.895	1.060	0.856	1.797	0.375	2.465	..	..	0.936	0.970	0.780	1.107
1969	0.975	1.012	0.745	1.807	0.862	0.918	0.844	1.758	1.911	1.934	..	..	0.899	0.994	0.963	1.000
1970	0.870	1.007	0.860	1.775	0.821	1.102	0.813	1.118	2.219	2.704	..	..	1.072	1.100	0.517	1.091
							Saskatchewan -- crop district 4B									
1958	0.897	0.943	0.937	1.054	1.095	1.527	0.820	1.233	0.504	1.343	..	..	0.966	1.064	0.836	1.028
1959	0.962	1.100	0.595	1.337	0.624	0.906	0.966	1.183	0.534	1.401	..	..	0.938	1.042	0.661	1.135
1960	0.929	1.036	0.807	2.168	0.833	1.000	0.924	1.155	1.449	1.711	2.070	2.531	0.997	1.071	0.741	1.139
1961	0.971	1.076	0.499	2.571	0.737	0.883	0.942	1.015	0.386	1.419	1.794	2.192	0.943	1.020	0.695	1.138
1962	0.920	1.213	0.615	1.202	0.671	0.828	0.955	1.136	0.356	0.748	0.217	0.265	0.978	1.076	1.021	1.224
1963	0.989	1.046	0.615	1.584	0.704	1.130	0.944	1.113	0.523	1.672	1.476	1.805	1.017	1.130	1.007	1.165
1964	0.853	1.025	1.842	1.923	0.804	1.130	0.904	1.179	0.520	1.369	0.965	1.179	0.983	1.035	1.142	1.208
1965	0.951	1.036	0.946	1.484	1.003	1.421	0.977	1.230	0.399	0.992	1.640	2.005	0.974	1.113	0.985	1.110
1966	0.826	1.072	0.454	1.660	0.862	1.238	0.849	0.962	1.000	1.000	0.630	0.770	0.955	1.104	1.146	1.261
1967	0.985	1.038	0.664	1.641	0.793	1.098	0.892	0.935	0.374	0.447	1.101	1.345	1.009	1.049	0.816	1.019
1968	1.008	1.099	0.800	0.875	0.900	1.584	0.991	1.125	1.325	1.464	0.994	1.214	0.970	1.019	0.790	0.912
1969	0.911	1.007	0.716	1.017	1.074	1.197	0.883	1.112	1.582	1.748	1.460	1.784	0.838	0.967	0.994	1.031
1970	0.974	1.000	0.389	0.714	1.491	1.591	0.816	1.285	0.170	2.009	1.470	1.796	0.919	1.057	0.449	0.545

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$
								Saskatchewan -- crop district 5A								
1958	1.051	1.090	0.846	1.108	0.858	1.044	0.941	0.977	0.519	1.000	..	..	0.994	1.052	0.964	1.541
1959	0.991	1.133	0.862	1.115	0.847	0.876	0.926	1.137	0.610	1.773	..	..	0.983	1.064	0.934	1.356
1960	0.931	1.019	1.034	1.058	0.842	0.874	0.893	1.189	1.457	1.531	4.167	5.093	1.011	1.117	0.838	1.278
1961	1.004	1.057	0.798	1.267	0.827	1.000	0.897	1.070	0.501	2.152	0.562	0.688	0.953	1.034	0.945	1.323
1962	1.079	1.238	0.757	1.182	0.635	1.059	0.831	1.477	0.463	0.735	0.570	0.697	0.970	1.109	0.935	0.991
1963	0.962	1.186	0.889	0.914	0.604	1.116	0.667	0.826	0.897	1.268	0.900	1.100	0.997	1.096	0.728	1.027
1964	1.016	1.083	0.770	0.807	0.743	0.825	0.779	1.037	0.872	2.059	1.277	1.560	0.972	1.061	0.782	1.205
1965	1.011	1.201	0.967	1.092	0.556	1.444	0.746	1.102	0.705	1.188	2.244	2.743	0.964	1.013	0.624	1.146
1966	1.013	1.079	0.940	1.242	0.557	1.264	0.731	0.922	0.994	1.098	2.036	2.489	0.951	1.014	0.907	1.113
1967	1.019	1.041	0.930	1.223	0.652	1.021	0.818	1.269	0.658	0.685	0.678	0.828	0.944	1.086	1.111	1.345
1968	0.995	1.009	0.935	1.231	0.676	1.018	0.835	1.911	1.000	2.035	1.000	1.000	0.934	1.043	0.790	1.320
1969	0.816	0.902	0.840	1.281	1.000	1.011	0.742	1.423	1.000	1.620	3.354	4.099	0.884	1.046	1.098	1.685
1970	1.000	1.000	0.770	1.417	0.873	1.174	0.769	1.181	1.000	1.000	2.659	3.250	0.873	1.133	0.958	1.000
								Saskatchewan -- crop district 5B								
1958	1.009	1.147	0.752	1.172	0.973	1.303	0.951	1.149	0.498	1.250	..	..	0.980	1.061	0.987	1.169
1959	1.048	1.214	0.755	1.036	0.800	1.292	1.111	1.228	0.565	1.107	..	..	0.935	1.144	0.884	1.197
1960	0.863	1.172	0.909	1.361	0.874	1.287	0.934	1.093	1.603	2.250	3.628	4.434	1.008	1.206	0.816	1.088
1961	1.094	1.145	0.836	1.328	0.790	1.085	0.885	1.136	0.725	1.000	0.559	0.683	0.902	1.094	0.947	1.032
1962	1.180	1.292	0.755	0.919	0.769	1.248	0.704	1.842	0.377	1.000	0.612	0.747	0.969	1.136	0.778	1.172
1963	1.001	1.032	0.770	0.976	0.972	1.170	0.496	0.719	0.768	1.726	1.832	2.239	0.970	1.205	1.047	1.134
1964	1.049	1.085	0.717	1.127	0.866	0.923	0.928	0.952	0.728	0.851	1.048	1.280	0.980	1.084	1.132	1.187
1965	0.932	0.971	0.916	1.139	1.290	1.293	0.804	1.101	0.660	1.343	2.248	2.748	0.937	1.036	0.821	1.061
1966	0.985	1.088	0.851	1.197	1.026	1.033	0.760	0.937	0.604	1.471	1.191	1.456	0.928	1.012	0.755	1.026
1967	0.967	1.008	1.023	1.251	0.909	1.113	0.856	0.916	0.246	2.944	0.816	0.998	0.916	1.057	1.053	1.223
1968	1.023	1.174	1.058	1.064	0.780	1.201	0.914	1.144	0.274	5.102	1.000	1.000	0.848	1.314	0.704	1.140
1969	1.025	1.281	0.807	1.024	0.867	1.356	0.764	1.105	0.297	4.590	2.417	2.954	0.824	1.369	1.043	1.068
1970	0.930	1.047	0.837	1.085	1.098	1.162	1.172	1.176	0.967	4.026	1.000	1.000	0.778	0.952	1.156	1.278

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$	1-$\underline{\beta}$	1+$\overline{\beta}$
								Saskatchewan -- crop district 6A								
1958	0.948	1.153	0.594	1.151	0.657	1.158	1.290	1.577	0.620	1.906	..	..	0.965	1.055	0.784	1.609
1959	1.011	1.158	0.685	1.009	0.659	1.143	0.687	1.122	0.540	1.255	..	..	0.963	1.052	0.935	1.490
1960	0.960	1.130	0.731	1.313	0.770	1.230	0.303	1.502	0.814	1.962	8.501	10.390	1.020	1.071	0.776	1.153
1961	0.985	1.113	0.748	1.139	0.804	0.827	0.854	1.262	0.688	1.029	0.275	0.336	0.928	1.048	0.830	1.461
1962	0.975	1.153	0.785	0.798	0.689	1.287	2.013	2.752	0.366	1.000	0.223	0.284	0.854	1.026	0.861	1.356
1963	1.003	1.089	0.698	1.133	0.970	0.989	1.516	1.928	0.801	1.466	1.543	1.886	0.978	1.044	0.710	1.093
1964	0.970	1.085	0.618	1.198	0.758	1.123	1.350	1.402	0.689	1.518	0.518	0.633	0.951	1.019	0.753	1.306
1965	0.926	1.063	0.652	1.627	0.899	1.551	0.901	1.735	0.900	1.283	2.129	2.602	0.985	1.053	1.035	1.143
1966	0.839	1.075	0.756	1.103	0.630	1.218	0.854	1.424	0.408	1.000	0.765	0.935	0.925	1.010	0.840	1.176
1967	1.000	1.039	0.843	1.046	0.911	1.137	1.000	1.595	0.432	1.421	1.004	1.228	0.970	1.015	0.922	1.201
1968	0.835	0.923	0.866	1.212	0.687	0.892	1.000	1.000	1.343	1.484	2.615	3.196	1.024	1.056	0.823	1.061
1969	0.779	0.861	0.643	1.075	0.611	1.102	1.000	1.000	0.587	2.371	8.467	10.348	0.944	1.039	1.050	1.225
1970	0.548	0.606	0.550	1.251	1.609	1.779	1.000	1.000	0.809	1.883	8.447	9.386	0.891	1.027	1.154	1.653
								Saskatchewan -- crop district 6B								
1958	0.965	1.177	0.623	0.980	0.777	1.000	0.598	1.130	0.672	1.029	..	..	1.010	1.056	0.899	0.983
1959	0.967	1.188	0.620	0.947	0.775	1.000	0.593	1.181	0.624	1.382	..	..	0.982	1.057	0.886	0.995
1960	0.964	1.116	1.345	1.418	0.789	1.000	0.451	1.409	0.948	1.282	5.044	6.166	1.027	1.077	0.976	1.010
1961	0.956	1.110	0.599	1.374	0.799	0.997	0.924	3.051	0.771	1.323	0.404	0.493	0.980	1.078	0.847	0.961
1962	0.981	1.154	0.803	0.864	0.681	1.000	1.287	4.541	0.276	1.000	0.236	0.325	0.953	1.045	0.862	0.880
1963	0.953	1.067	0.794	1.232	1.101	1.117	0.798	2.716	0.863	0.936	1.489	1.820	0.984	1.052	0.937	1.006
1964	0.978	1.060	0.798	1.245	0.812	1.000	0.942	1.545	0.923	1.018	0.776	0.948	0.978	1.051	1.082	1.118
1965	0.952	1.051	0.721	1.378	0.684	1.330	0.640	2.291	0.962	1.338	2.181	2.666	0.975	1.047	0.984	1.135
1966	0.970	1.077	0.868	1.026	0.729	1.278	0.830	1.351	0.725	1.000	0.497	0.607	0.935	1.045	1.116	1.220
1967	0.950	1.070	0.874	1.001	0.858	0.944	0.876	1.366	0.420	2.010	0.841	1.028	1.024	1.058	0.930	1.013
1968	0.950	1.046	0.779	1.279	0.870	1.353	1.000	1.000	1.287	2.104	2.012	2.459	0.996	1.091	0.879	0.881
1969	0.962	1.092	0.733	1.124	0.778	1.225	1.000	1.000	1.670	2.279	4.915	5.461	1.062	1.071	0.966	1.000
1970	0.782	1.145	0.580	1.178	1.102	1.143	1.000	1.000	1.935	2.142	4.297	5.252	0.868	1.038	0.756	0.836

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$
								Saskatchewan -- crop district 7A								
1958	0.841	1.219	0.748	1.368	0.943	1.288	1.080	1.254	0.504	0.895	..	..	0.990	1.069	0.740	0.909
1959	0.957	1.400	0.766	1.348	0.908	1.346	1.404	1.923	0.613	0.746	..	..	0.926	1.049	0.709	1.409
1960	0.970	1.252	0.689	1.076	0.797	0.812	1.266	2.392	0.880	1.266	2.962	3.621	1.044	1.122	1.186	1.224
1961	1.009	1.131	0.714	1.704	0.703	1.523	0.899	1.000	0.835	0.882	0.155	0.189	0.969	1.017	0.812	0.903
1962	0.885	1.295	0.566	0.572	0.526	1.490	0.159	3.013	0.382	0.499	0.381	0.465	0.919	1.005	0.927	1.285
1963	0.982	1.000	0.568	0.718	1.333	1.680	0.882	6.503	0.992	1.393	1.170	1.433	0.966	1.064	1.018	1.155
1964	1.018	1.048	0.658	0.667	0.555	1.000	1.000	1.000	0.780	1.131	1.075	1.315	0.946	1.011	0.826	1.074
1965	1.017	1.039	0.893	1.199	1.469	1.679	0.934	1.648	0.910	1.170	2.164	2.646	0.942	1.037	0.900	0.987
1966	0.950	1.029	0.967	1.134	0.642	1.056	0.857	1.000	0.506	0.541	0.838	1.024	0.924	1.063	0.903	1.233
1967	0.917	0.958	0.894	1.235	0.929	0.936	0.641	1.984	0.749	1.273	1.974	2.413	1.003	1.137	0.893	1.000
1968	0.868	1.013	1.132	1.295	0.706	0.997	0.881	4.311	1.435	1.961	1.893	2.314	0.978	1.163	0.731	0.747
1969	0.859	0.902	0.798	1.753	1.322	1.461	1.260	3.710	1.810	2.751	1.000	1.000	1.031	1.196	0.580	0.641
1970	0.730	0.767	0.598	1.801	1.000	1.363	1.200	1.326	1.664	2.342	1.000	1.000	1.128	1.198	0.265	0.294
								Saskatchewan -- crop district 7B								
1958	0.967	1.067	0.991	1.034	0.665	1.172	0.953	1.858	0.812	1.206	..	..	0.989	1.090	0.943	1.122
1959	0.934	1.132	0.625	0.975	0.763	1.709	0.785	1.796	0.518	0.839	..	..	0.995	1.099	0.904	1.132
1960	0.976	1.062	0.835	1.435	0.759	1.191	0.476	2.184	0.680	1.885	3.034	3.708	0.914	1.092	0.867	1.145
1961	0.956	1.031	0.982	1.131	0.829	1.454	0.907	1.000	0.622	1.173	0.250	0.306	0.950	1.007	0.813	1.127
1962	0.966	1.149	0.861	0.908	0.776	1.558	0.397	0.635	0.262	0.991	0.272	0.429	0.983	1.077	0.904	1.141
1963	1.020	1.023	0.812	1.313	0.771	1.245	0.459	2.581	1.377	1.789	1.290	1.577	0.974	1.007	0.758	1.133
1964	1.061	1.106	0.556	1.203	0.607	1.000	0.678	0.991	1.198	1.476	1.025	1.253	0.926	1.014	1.003	1.115
1965	0.987	1.106	1.201	1.433	1.503	1.569	0.500	1.025	0.970	1.189	1.894	2.315	0.989	1.021	1.025	1.148
1966	0.958	1.013	0.978	1.150	0.682	1.179	0.884	1.000	0.403	1.664	0.576	0.704	0.958	1.027	0.917	1.142
1967	0.948	1.023	0.856	1.172	0.902	1.041	0.678	0.769	0.406	1.901	0.853	1.043	1.018	1.050	0.867	1.138
1968	0.986	0.986	0.841	1.474	1.023	1.274	0.536	1.556	1.043	2.585	1.598	1.953	0.913	1.085	0.812	0.872
1969	0.997	1.000	0.822	1.214	1.208	1.374	0.949	1.170	1.555	2.538	3.173	3.526	0.830	1.036	0.820	1.128
1970	0.962	1.590	1.000	1.000	1.684	1.861	1.000	1.000	1.236	1.909	2.911	3.234	1.000	1.046	0.847	1.000

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\overline{\beta}$	$1-\underline{\beta}$	$1+\overline{\beta}$	$1-\underline{\beta}$	$1+\overline{\beta}$	$1-\underline{\beta}$	$1+\overline{\beta}$	$1-\underline{\beta}$	$1+\overline{\beta}$	$1-\underline{\beta}$	$1+\overline{\beta}$	$1-\underline{\beta}$	$1+\overline{\beta}$	$1-\underline{\beta}$	$1+\overline{\beta}$
								Saskatchewan -- crop district 8A								
1958	0.939	1.107	0.766	0.911	0.808	1.055	0.989	1.647	0.684	1.382	..	..	0.900	1.086	0.921	1.209
1959	0.788	1.203	0.719	1.333	0.989	1.111	0.831	1.667	0.594	1.000	..	..	0.931	1.130	0.979	1.280
1960	0.791	1.045	0.818	1.134	0.857	0.965	1.237	1.735	0.648	1.566	2.276	2.782	0.993	1.263	0.888	1.298
1961	0.805	1.335	0.689	1.350	0.887	1.137	0.842	1.101	0.604	1.220	0.682	0.833	0.825	1.008	0.943	1.082
1962	0.868	1.299	0.879	1.416	0.780	1.100	0.739	0.772	0.850	1.028	0.488	0.597	0.969	1.002	0.991	1.159
1963	0.977	1.054	0.782	1.386	1.070	1.117	0.937	1.206	0.659	1.000	1.101	1.346	0.995	1.017	0.987	1.134
1964	1.008	1.047	0.761	1.000	0.739	0.925	1.075	1.188	0.805	1.099	1.173	1.434	0.948	1.031	0.947	1.025
1965	0.822	0.849	0.851	1.300	1.287	1.319	1.000	1.022	0.870	1.255	1.963	2.399	0.973	1.069	0.903	1.148
1966	0.983	1.153	0.761	1.149	0.764	1.299	0.718	0.981	0.764	0.986	1.224	1.496	0.915	1.004	0.850	1.088
1967	0.932	1.049	0.882	1.202	0.964	1.214	1.234	1.300	0.597	1.090	0.670	0.819	0.942	1.067	0.738	1.173
1968	0.985	1.089	0.838	1.308	0.756	0.835	0.594	1.237	0.761	2.236	0.758	0.926	0.891	1.022	0.625	1.072
1969	1.000	1.000	0.788	1.258	0.794	1.281	0.676	0.695	1.000	1.189	1.805	2.207	1.032	1.141	0.772	0.853
1970	0.801	0.885	0.685	1.428	1.084	1.150	1.000	1.000	0.574	1.755	1.000	1.000	0.998	1.103	0.632	1.251
								Saskatchewan -- crop district 8B								
1958	0.959	1.085	0.948	1.177	0.900	1.068	1.023	1.358	0.676	1.207	..	..	0.968	1.023	0.942	1.106
1959	0.961	1.214	0.580	0.970	0.971	1.000	1.000	1.044	0.757	1.000	..	..	0.970	1.033	0.977	1.155
1960	0.855	0.985	0.695	1.471	0.767	0.874	1.421	1.756	0.790	1.574	3.581	4.377	1.014	1.112	0.939	1.069
1961	1.023	1.085	0.563	1.277	0.945	0.952	0.713	0.898	0.635	1.492	0.601	0.734	0.942	1.041	0.880	0.974
1962	0.954	1.169	0.774	0.781	0.916	1.276	0.530	0.547	0.721	1.029	0.196	0.240	0.953	1.017	0.916	1.159
1963	0.973	0.997	0.907	1.000	1.096	1.103	1.446	2.408	0.783	1.300	1.326	1.621	0.976	1.097	0.956	1.043
1964	1.060	1.120	0.726	0.946	0.815	0.816	0.965	1.000	0.608	1.005	1.168	1.427	0.965	1.016	0.909	1.125
1965	0.983	1.106	0.964	1.204	1.178	1.361	1.088	1.348	0.763	1.103	2.016	2.464	0.978	1.105	0.923	1.046
1966	0.977	0.990	0.879	1.002	1.096	1.261	0.570	0.795	0.662	1.000	0.814	0.995	0.950	1.042	0.886	1.116
1967	0.966	1.068	0.992	1.001	0.962	1.001	1.018	1.223	0.427	1.616	0.702	0.858	0.990	1.062	0.975	1.146
1968	0.950	0.957	1.117	1.235	0.837	0.851	1.281	1.626	0.423	2.795	1.434	1.752	0.993	1.129	0.770	1.087
1969	0.900	0.903	0.867	0.923	0.761	0.842	0.723	0.799	0.334	2.382	3.518	3.909	0.993	1.090	0.823	1.079
1970	0.904	0.953	0.958	1.350	0.966	1.068	1.044	1.468	0.526	1.527	3.422	3.803	0.909	1.001	0.849	1.249

TABLE 16 -- (continued)

Year	Wheat		Oats		Barley		Rye		Flaxseed		Rapeseed		Summerfallow		Stubble	
	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$	$1-\underline{\beta}$	$1+\bar{\beta}$
Saskatchewan -- crop district 9A																
1958	0.904	1.092	0.777	0.901	0.888	1.067	0.588	1.134	1.063	1.564	..	..	0.966	1.030	0.931	0.951
1959	0.949	1.109	0.526	1.187	0.927	1.001	0.899	1.142	0.693	1.124	..	..	0.947	0.967	0.902	1.166
1960	0.892	1.077	1.873	2.024	0.858	1.000	0.973	1.118	0.525	1.033	2.698	3.297	0.924	1.085	1.070	1.252
1961	0.943	1.085	0.794	1.355	0.852	0.960	0.931	1.152	1.101	1.434	0.684	0.836	0.956	1.054	0.953	0.967
1962	0.912	1.176	1.052	1.086	0.826	1.179	0.900	1.101	0.463	1.000	0.460	0.562	0.934	0.972	0.875	1.180
1963	0.928	1.046	0.628	0.986	0.928	1.076	0.965	1.085	0.502	1.232	0.836	1.021	0.923	1.064	1.020	1.058
1964	0.993	1.075	0.573	1.110	0.794	0.820	1.094	1.095	0.736	0.876	0.683	0.835	0.975	1.107	0.846	1.021
1965	0.914	0.946	1.517	1.852	1.189	1.251	1.046	1.115	0.522	1.223	2.180	2.664	0.926	1.012	0.850	1.180
1966	0.887	1.047	0.893	1.572	0.844	1.256	0.893	1.156	0.726	0.733	1.617	1.976	0.983	1.078	0.939	1.200
1967	0.927	1.016	0.883	1.538	0.908	1.041	0.797	1.124	0.723	1.097	0.700	0.856	0.949	1.163	0.880	0.963
1968	0.896	0.974	0.818	1.515	0.978	1.080	0.864	1.138	2.247	2.483	1.000	1.000	0.904	1.061	0.933	1.000
1969	0.929	0.949	1.138	1.424	0.770	0.972	1.000	1.179	1.000	1.000	2.453	2.998	0.938	1.041	0.944	1.004
1970	0.930	0.941	1.062	1.400	1.043	1.081	1.172	1.277	1.153	1.515	1.000	1.000	0.901	0.995	1.028	1.940
Saskatchewan -- crop district 9B																
1958	0.907	1.073	0.621	1.084	0.846	1.412	1.068	1.107	0.785	0.995	..	..	0.988	1.076	0.804	0.930
1959	0.974	1.112	0.617	1.204	0.812	1.186	0.865	1.027	0.618	2.607	..	..	0.954	1.073	0.752	0.926
1960	0.909	1.010	1.934	2.243	0.822	0.873	1.103	1.239	1.022	1.683	2.758	3.371	0.990	1.158	1.144	1.315
1961	0.979	1.080	1.061	1.366	0.789	1.156	0.972	1.000	0.860	1.100	1.034	1.264	0.987	1.091	0.855	1.009
1962	1.016	1.113	0.709	1.000	0.845	1.174	0.806	1.032	0.162	0.318	0.545	0.666	0.955	1.046	0.836	0.871
1963	0.971	1.069	0.536	0.799	0.866	1.211	0.978	1.512	1.273	1.530	1.174	1.435	0.935	1.061	1.035	1.108
1964	1.011	1.085	0.704	1.000	0.794	0.843	1.000	1.040	0.804	1.324	1.318	1.611	0.985	1.030	0.876	1.209
1965	0.957	0.993	0.479	1.373	0.857	1.155	1.132	1.183	0.684	0.955	1.977	2.416	0.961	1.125	1.099	1.386
1966	0.965	1.067	0.794	1.098	0.797	1.208	0.776	0.935	0.681	1.970	1.070	1.308	0.937	1.079	1.192	1.220
1967	0.962	0.978	0.654	1.426	0.991	1.002	0.894	1.138	0.304	0.928	0.632	0.772	1.036	1.111	1.033	1.218
1968	0.925	0.993	0.679	0.944	0.854	0.962	1.171	1.326	0.446	1.202	1.076	1.316	0.954	1.140	0.978	1.095
1969	0.920	0.932	0.659	0.683	0.942	0.959	1.264	1.398	1.054	3.607	2.243	2.742	0.925	1.161	0.866	1.285
1970	0.980	1.045	0.728	1.189	1.000	1.225	1.026	1.102	2.461	6.062	2.778	3.087	0.856	1.178	0.624	1.176

TABLE 17

ESTIMATED LAND UTILIZATION PATTERNS, BY PROVINCE, IN PRAIRIES, 1958-1970

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summerfallow
			Prairies -- using prairie model				
1958	22,153,279	5,054,000	9,079,998	431,000	3,462,000	1,252,355	24,943,368
1959	22,949,447	5,545,336	8,074,247	439,663	2,741,973	664,893	26,171,441
1960	24,539,218	5,755,848	6,707,547	487,921	2,554,664	811,355	26,694,447
1961	24,790,275	6,714,302	5,204,856	464,324	1,735,633	763,000	26,553,610
1962	26,372,733	6,085,424	5,563,646	514,727	1,813,289	465,952	27,488,229
1963	27,257,619	6,201,070	5,663,939	576,016	1,386,154	535,490	27,458,712
1964	28,771,527	5,054,000	5,915,148	603,580	1,204,417	667,207	26,837,121
1965	29,477,814	5,054,000	5,253,043	679,086	1,465,893	1,456,919	26,760,245
1966	29,140,816	5,675,878	6,179,670	662,234	1,818,048	1,620,000	25,408,354
1967	29,220,540	5,647,529	7,426,815	641,516	1,128,463	1,525,000	25,866,137
1968	29,349,408	5,541,632	8,194,852	651,971	908,000	1,620,000	26,097,137
1969	28,524,358	6,265,422	8,594,228	652,525	1,459,373	1,424,124	27,370,970
1970	23,048,458	5,349,084	8,959,590	878,791	2,871,957	2,766,520	29,721,600
			Prairies -- Summation of the provincial estimates				
1958	21,907,321	6,050,894	9,353,642	438,360	3,363,630	836,635	24,615,518
1959	23,414,022	5,938,599	8,036,848	434,005	2,679,339	316,845	25,875,342
1960	23,181,324	6,595,152	7,120,632	454,765	2,380,865	725,227	26,849,131
1961	24,803,294	6,422,759	5,476,641	463,603	2,146,233	843,410	26,334,060
1962	26,524,705	5,939,327	5,785,465	500,692	2,216,308	306,235	27,321,268
1963	26,804,015	6,322,776	5,684,528	544,613	1,566,152	655,621	27,655,399
1964	28,558,177	5,055,755	5,880,881	575,586	1,656,866	692,299	27,373,436
1965	29,009,401	4,772,412	6,018,566	574,667	1,814,311	1,591,069	26,248,574
1966	29,726,000	5,677,422	6,337,271	557,758	1,867,351	1,280,439	25,374,759
1967	29,726,000	5,740,936	7,433,954	558,309	1,103,136	1,345,130	25,638,535
1968	29,726,000	6,043,234	7,317,216	646,069	1,341,060	1,620,000	25,669,421
1969	27,071,312	5,881,548	9,153,775	652,743	1,825,736	1,553,297	27,052,589
1970	21,953,606	5,341,961	8,275,531	896,197	3,504,654	3,289,004	30,335,047

TABLE 17 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summerfallow
			Manitoba -- using provincial model				
1958	2,431,704	1,601,850	1,847,221	86,955	764,971	43,308	2,768,991
1959	2,641,448	1,391,061	1,518,036	85,356	678,133	58,191	2,989,775
1960	2,823,231	1,544,889	1,448,445	84,137	726,116	33,264	3,082,918
1961	2,935,268	1,389,180	738,885	79,902	769,381	65,033	3,242,351
1962	3,027,733	1,773,330	678,004	118,704	866,483	33,318	3,240,428
1963	3,136,150	1,729,542	643,656	114,365	816,054	43,437	3,260,900
1964	3,360,058	1,674,108	822,932	131,274	1,044,754	82,220	3,279,654
1965	3,448,672	1,463,799	540,393	130,492	1,232,293	147,311	2,831,040
1966	3,520,000	1,633,687	674,896	103,820	1,103,585	169,128	2,655,884
1967	3,520,000	1,474,465	1,115,091	141,000	686,417	170,000	2,669,027
1968	3,520,000	1,690,000	961,134	141,000	815,904	145,000	2,617,962
1969	3,360,764	1,399,927	1,364,793	135,121	933,955	74,440	2,710,000
1970	2,378,772	1,600,000	1,196,508	183,000	694,320	196,000	3,654,400
			Saskatchewan -- using provincial model				
1958	14,276,376	2,499,044	3,846,000	247,956	2,025,000	620,000	14,568,624
1959	15,281,614	2,144,360	2,990,790	239,000	1,438,790	197,030	15,634,416
1960	15,067,038	2,697,988	2,358,025	255,852	1,155,172	561,089	16,261,932
1961	16,243,348	2,558,464	1,825,642	244,481	796,852	550,000	15,928,213
1962	17,634,878	1,922,759	1,768,897	239,000	941,000	217,896	16,920,570
1963	17,785,316	2,222,021	1,806,626	268,434	460,992	257,561	17,144,050
1964	18,913,139	1,406,717	1,818,849	243,224	291,992	230,645	17,092,434
1965	19,363,200	1,444,666	1,885,968	271,961	218,711	568,758	16,671,736
1966	19,700,000	1,940,755	1,825,808	291,210	433,563	620,000	16,064,664
1967	19,700,000	2,011,908	2,236,599	264,862	234,512	354,070	16,518,049
1968	19,700,000	2,293,470	2,056,250	327,000	226,891	600,000	16,349,389
1969	18,022,260	2,393,280	2,995,710	385,000	386,476	777,415	16,905,859
1970	14,566,334	1,640,226	2,590,309	496,000	2,025,000	1,633,360	18,897,771

TABLE 17 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summerfallow
Saskatchewan -- Summation of the crop districts' results							
1958	14,207,630	2,587,241	4,199,148	233,204	1,809,070	..	14,485,610
1959	14,680,586	2,226,118	3,697,305	203,450	1,546,921	..	14,702,920
1960	15,400,295	2,314,148	2,804,958	212,504	1,359,743	577,086	15,577,554
1961	16,046,621	2,884,804	2,672,619	213,709	1,248,311	430,261	16,210,435
1962	17,214,379	2,717,756	2,149,659	165,500	860,917	177,121	16,469,548
1963	18,016,077	2,214,230	2,060,410	193,667	471,982	227,469	16,970,215
1964	18,963,962	1,623,717	1,870,942	257,600	486,502	272,247	16,662,130
1965	18,896,383	1,568,030	2,022,570	244,523	570,543	613,505	16,755,846
1966	19,284,863	1,841,300	2,204,281	270,727	486,645	736,790	16,249,842
1967	19,737,808	1,846,719	2,407,254	251,984	343,020	629,315	16,474,016
1968	19,381,914	2,176,898	2,451,921	340,019	313,874	677,477	16,551,302
1969	18,005,547	2,168,452	2,564,056	413,087	688,171	1,347,435	16,970,941
Alberta -- using provincial model							
1958	5,199,241	1,950,000	3,660,421	103,449	573,659	173,327	7,277,903
1959	5,490,960	2,403,178	3,528,022	109,649	562,416	661,624	7,251,151
1960	5,291,055	2,352,275	3,314,162	114,776	499,577	130,874	7,504,281
1961	5,624,678	2,475,115	2,912,114	139,220	580,000	228,377	7,163,496
1962	5,862,094	2,243,238	3,338,564	142,988	408,825	55,021	7,160,270
1963	5,882,549	2,371,213	3,234,246	161,814	289,106	354,623	7,250,449
1964	6,284,980	1,974,930	3,239,100	201,088	320,120	379,434	7,001,348
1965	6,197,529	1,863,947	3,592,205	172,214	363,307	875,000	6,745,798
1966	6,506,000	2,102,980	3,836,567	162,728	330,203	491,311	6,654,211
1967	6,506,000	2,254,563	4,082,264	152,447	182,207	821,060	6,451,459
1968	6,506,000	2,059,764	4,299,832	178,069	298,265	875,000	6,702,070
1969	5,688,288	2,088,341	4,793,272	132,622	505,305	701,442	7,436,730
1970	5,008,500	2,101,735	4,488,714	217,197	785,334	1,459,644	7,782,876

TABLE 18

DEVIATION OF ESTIMATED FROM ACTUAL LAND USE, BY CROP DISTRICT, IN SASKATCHEWAN, 1958-1969

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 1A				
1958	- 0.73	- 6.38	- 0.91	- 2.37	-108.32	..	13.34	11.27
1959	5.65	- 5.90	- 2.91	11.33	- 27.94	..	- 1.27	4.46
1960	- 0.11	0.97	-14.74	3.00	5.39	..	1.26	1.69
1961	- 8.45	2.91	-69.06	21.38	2.99	..	9.89	10.15
1962	0.67	26.39	0.24	48.63	-167.86	..	1.63	7.85
1963	0.44	- 4.68	0.77	- 0.79	4.20	..	0.10	0.72
1964	2.50	- 6.25	-39.71	-33.60	16.38	..	- 1.25	3.43
1965	- 3.33	17.65	- 0.55	28.57	- 3.38	..	0.82	3.32
1966	- 4.41	2.04	50.00	0.05	36.59	..	- 3.41	6.79
1967	2.45	- 0.77	2.81	36.01	- 76.48	..	- 1.72	3.78
1968	- 2.22	- 1.26	26.67	8.74	3.30	..	0.49	2.70
1969	-11.99	1.89	6.46	67.44	38.16	..	- 1.15	10.40
Wt. av. of abs. devs. (1958-1967)	2.86	7.77	12.75	23.94	39.93	..	3.48	5.35
Wt. av. of abs. devs. (1968-1969)	6.86	1.58	15.83	53.02	24.84	..	.83	6.55

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 1B				
1958	- 4.13	- 3.35	- 3.98	- 0.57	-20.78	..	8.57	5.66
1959	7.58	- 3.97	- 6.42	- 0.05	- 3.96	..	- 0.67	4.58
1960	0.30	3.87	- 8.80	37.89	2.18	..	3.54	3.94
1961	- 7.98	15.18	- 0.28	0.51	- 2.69	..	0.66	4.78
1962	2.70	- 0.67	- 3.63	21.57	-21.41	..	- 0.76	2.07
1963	- 2.73	- 9.97	7.51	53.49	- 1.99	..	1.28	3.96
1964	2.04	4.92	- 14.64	4.40	4.12	..	2.15	4.08
1965	3.46	17.46	- 9.28	6.84	- 3.93	..	- 4.11	5.86
1966	- 2.33	-18.27	11.85	0.05	7.07	..	1.22	4.98
1967	1.26	0.98	-163.51	61.68	-19.14	..	11.42	12.70
1968	-10.61	6.36	11.76	17.62	-24.76	..	7.82	9.46
1969	-21.43	22.83	12.82	-226.97	-23.53	..	14.41	19.80
Wt. av. of abs. devs. (1958-1967)	3.24	7,85	10.26	28.20	8.31	..	3.46	5.26
Wt. av. of abs. devs. (1968-1969)	15.47	15.23	12.33	66.66	24.06	..	11.27	14.63

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 2A				
1958	2.95	0.01	- 6.13	4.41	-139.41	..	9.87	10.85
1959	5.69	0.24	- 7.28	24.69	- 92.57	..	0.17	5.50
1960	- 0.07	- 0.07	1.47	34.07	- 0.96	..	- 0.08	0.21
1961	-13.17	13.80	- 3.72	- 2.58	- 0.92	..	15.14	13.26
1962	3.03	8.09	0.00	27.84	-154.08	..	- 0.26	4.04
1963	- 0.81	0.08	30.13	1.91	0.29	..	- 0.32	1.06
1964	0.56	- 1.69	0.00	27.00	- 17.40	..	0.03	0.70
1965	- 1.34	4.76	6.26	0.26	0.68	..	0.76	1.28
1966	7.62	0.03	- 0.47	- 3.13	-186.21	..	- 4.22	8.14
1967	0.53	0.00	9.32	21.73	-157.26	..	0.59	1.78
1968	- 1.75	-11.12	-20.36	- 0.74	19.95	..	4.55	4.07
1969	- 5.88	11.76	-29.89	54.05	16.79	..	6.90	7.88
Wt. av. of abs. devs. (1958-1967)	3.46	3.33	6.56	15.49	54.98	..	3.18	4.68
Wt. av. of abs. devs. (1968-1969)	3.73	11.46	25.38	29.11	17.93	..	5.76	5.98

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 2B				
1958	- 3.45	12.16	23.18	1.21	- 56.30	..	5.15	9.02
1959	8.20	- 9.77	- 6.69	7.16	-147.31	..	3.71	10.70
1960	- 4.70	20.84	8.81	13.03	- 0.80	..	1.29	4.07
1961	-11.69	9.37	28.22	- 4.19	- 9.07	..	7.84	10.35
1962	- 0.96	- 1.80	- 2.91	-15.89	-173.58	..	8.07	7.10
1963	- 0.03	0.38	21.85	13.25	7.19	..	-1.46	1.27
1964	1.52	0.00	-14.92	19.18	- 9.99	..	-0.75	1.57
1965	- 1.73	20.51	40.91	1.43	- 3.65	..	-1.04	2.85
1966	1.19	6.11	5.93	3.80	28.22	..	-4.49	3.46
1967	0.00	0.67	14.09	36.41	-131.63	..	0.33	1.63
1968	4.95	21.52	-51.69	52.26	47.55	..	-4.21	7.86
1969	- 5.27	29.23	-79.70	30.26	74.75	..	6.87	12.24
Wt. av. of abs. devs. (1958-1967)	3.05	8.94	16.98	13.27	38.45	..	3.43	5.20
Wt. av. of abs. devs. (1968-1969)	5.10	25.90	66.27	41.66	65.62	..	5.59	10.05

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 3AS				
1958	-3.29	29.03	9.45	- 4.48	-318.27	..	9.80	12.85
1959	3.61	-17.13	-18.97	19.53	- 77.67	..	0.36	4.02
1960	0.45	1.66	6.36	3.99	9.43	..	-1.99	1.62
1961	-3.45	7.58	- 0.59	10.06	- 24.37	..	3.73	4.26
1962	1.85	- 1.33	-90.67	- 4.49	- 20.72	..	1.91	3.48
1963	-0.69	-43.79	0.55	54.32	72.79	..	0.51	3.04
1964	1.36	0.00	-80.77	- 0.53	- 19.29	..	1.00	2.26
1965	-6.97	19.49	0.23	9.18	- 2.92	..	6.17	6.85
1966	1.47	- 3.65	- 0.85	- 4.98	- 0.81	..	-1.44	1.53
1967	0.48	23.29	-57.49	- 1.39	0.00	..	-0.16	1.90
1968	1.25	-20.50	12.23	44.39	56.29	..	-2.14	2.97
1969	-1.61	-17.92	-16.00	18.65	-118.75	..	6.64	6.08
Wt. av. of abs. devs. (1958-1967)	2.31	13.81	17.34	11.33	62.60	..	2.69	4.18
Wt. av. of abs. devs. (1968-1969)	1.43	19.24	14.09	26.88	96.56	..	4.45	4.53

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 3AN				
1958	6.85	2.85	27.65	4.05	-241.64	..	4.74	15.23
1959	17.08	-13.03	- 35.84	- 3.69	-355.45	..	0.15	16.44
1960	0.76	12.87	8.32	4.31	2.73	..	-3.16	2.68
1961	- 2.39	32.60	15.54	- 3.32	-149.27	..	7.51	11.19
1962	5.21	- 7.94	-211.19	0.00	- 94.53	..	6.78	11.16
1963	- 0.94	-30.70	- 2.34	42.98	17.46	..	2.16	3.04
1964	1.35	-13.49	- 8.17	23.31	4.83	..	-1.28	1.91
1965	- 3.99	51.02	- 2.95	13.65	- 20.77	..	0.42	4.27
1966	0.99	0.56	2.35	5.24	28.73	..	-2.23	1.92
1967	- 2.72	8.02	1.72	- 4.60	-124.10	..	2.71	3.11
1968	- 4.25	-15.17	41.55	28.10	50.29	..	1.75	4.93
1969	- 1.14	7.14	- 28.25	-85.69	32.70	..	2.02	3.33
Wt. av. of abs. devs. (1958-1967)	4.15	18.60	27.96	11.39	116.89	..	3.12	7.10
Wt. av. of abs. devs. (1968-1969)	2.74	11.01	34.65	47.21	37.90	..	1.89	4.13

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 3BS				
1958	-6.40	35.75	5.77	2.90	- 4.52	..	2.97	5.66
1959	7.49	- 54.78	-68.60	44.46	- 2.38	..	1.97	9.14
1960	-2.91	- 15.51	-51.46	1.90	-30.83	..	10.03	9.31
1961	-3.00	60.44	-32.54	23.17	23.10	..	0.00	5.26
1962	1.56	1.70	-71.55	-16.40	-40.60	..	3.23	4.60
1963	-0.32	5.19	- 1.94	64.29	72.73	..	- 0.82	1.21
1964	1.73	0.00	-94.30	2.06	- 1.88	..	3.92	5.33
1965	-1.76	- 2.44	- 0.01	22.53	25.28	..	1.59	1.79
1966	6.70	-147.80	- 0.54	8.96	9.05	..	- 3.46	7.03
1967	0.25	31.43	-28.98	-21.46	-14.11	..	0.62	2.04
1968	-1.31	- 12.58	-38.83	36.59	3.56	..	4.66	4.36
1969	-4.99	- 43.21	- 9.36	82.53	- 3.35	..	5.50	7.21
Wt. av. of abs. devs. (1958-1967)	3.17	34.37	32.17	21.80	18.66	..	2.87	5.14
Wt. av. of abs. devs. (1968-1969)	3.12	27.33	23.26	76.16	3.42	..	5.09	5.79

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 3BN				
1958	- 6.18	36.57	12.03	1.94	7.30	..	- 0.77	5.50
1959	10.28	0.53	-54.26	19.69	- 93.22	..	2.80	12.28
1960	0.40	-46.72	7.34	10.86	- 8.48	..	2.49	3.35
1961	- 5.18	17.65	-36.96	7.15	- 1.00	..	6.21	7.09
1962	7.06	0.88	-96.29	24.95	-584.98	..	9.29	15.73
1963	- 0.93	21.39	- 4.61	28.59	2.16	..	- 0.65	1.71
1964	- 3.45	- 0.76	-22.29	5.09	- 48.04	..	5.40	5.07
1965	- 2.44	38.78	- 2.64	22.23	- 24.89	..	0.66	3.03
1966	6.31	- 8.65	- 0.07	- 1.88	-154.45	..	- 3.42	6.43
1967	2.87	0.22	-44.26	- 3.54	- 59.47	..	0.73	3.57
1968	- 3.74	3.92	- 8.25	28.12	52.66	..	3.35	4.30
1969	-13.97	27.21	15.14	-695.93	29.80	..	11.24	14.32
Wt. av. of abs. devs. (1958-1967)	4.46	22.54	25.81	13.45	52.21	..	3.30	6.38
Wt. av. of abs. devs. (1968-1969)	8.61	17.63	12.27	128.53	36.55	..	7.37	9.31

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 4A				
1958	-4.98	13.64	2.98	-3.63	0.00	..	2.15	3.68
1959	3.59	-47.07	-9.56	22.03	-131.76	..	3.67	7.28
1960	1.70	-39.81	8.93	2.12	13.36	..	-1.40	3.35
1961	-5.41	5.05	-7.57	18.46	-11.81	..	4.30	5.42
1962	-0.43	-27.44	-2.29	-1.66	-107.58	..	3.01	2.82
1963	-2.29	0.69	5.37	55.95	9.19	..	-3.20	4.57
1964	1.46	0.35	-15.00	2.14	11.63	..	0.63	1.98
1965	-1.36	0.78	-1.93	29.67	-15.39	..	-0.92	2.25
1966	3.06	-1.65	-0.33	-11.22	-114.23	..	-1.63	2.56
1967	-1.45	-29.02	0.20	8.22	-91.50	..	4.47	3.84
1968	0.71	-63.16	-26.86	-6.42	-23.25	..	9.32	8.71
1969	3.48	-76.07	-1.82	-49.73	3.31	..	5.65	8.06
Wt. av. of abs. devs. (1958-1967)	2.52	12.87	5.09	17.21	28.23	..	2.55	3.78
Wt. av. of abs. devs. (1968-1969)	2.11	69.70	14.99	22.04	9.96	..	7.49	8.39

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 4B				
1958	-5.98	6.72	0.00	-52.48	10.69	..	4.76	6.71
1959	7.53	-138.46	-36.85	14.17	- 85.97	..	9.78	15,84
1960	-2.23	- 82.25	- 3.68	1.28	- 15.90	-19.12	6.72	6.25
1961	-1.46	74.19	- 9.74	2.08	-100.83	4.46	8.29	11.38
1962	5.46	- 5.97	-51.80	8.42	-178.36	20.54	0.39	5.98
1963	1.38	- 38.46	-45.35	20.94	- 8.49	-38.00	-0.79	2.47
1964	-1.02	7.90	- 5.44	- 1.75	14.90	8.86	0.34	1.20
1965	-5.46	- 13.07	7.33	22.55	25.97	6.15	2.52	5.45
1966	6.20	-138.46	1.83	- 1.28	2.01	-20.67	-4.68	6.49
1967	0.67	60.79	-14.07	1.21	-142.54	-55.19	-0.67	2.66
1968	-6.73	20.03	- 1.14	13.84	26.27	-43.50	5.24	6.58
1969	-3.35	- 6.50	1.69	27.21	- 11.17	18.23	5.26	5.32
Wt. av. of abs. devs. (1958-1967)	3.67	51.20	14.03	12.61	38.44	21.69	3.82	6.44
Wt. av. of abs. devs. (1968-1969)	5.06	13.42	1.44	21.92	17.04	26.17	5.25	5.95

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 5A				
1958	-1.93	3.30	0.00	0.85	- 92.38	..	2.59	3.01
1959	8.79	-10.57	-30.62	10.17	1.48	..	-0.33	7.19
1960	-9.77	- 0.21	4.05	-32.78	- 4.74	- 9.14	7.89	7.54
1961	-2.66	5.54	-22.40	3.61	- 94.23	-20.31	4.87	5.88
1962	0.38	- 0.71	-16.72	45.68	- 3.61	-82.40	0.32	1.16
1963	-0.52	0.20	17.50	0.13	34.33	28.57	-1.33	1.60
1964	2.28	- 8.78	0.00	3.71	-135.57	- 9.20	0.31	2.45
1965	-5.62	2.15	26.67	25.36	37.88	-24.68	2.84	5.27
1966	-0.29	0.00	38.32	0.27	5.12	0.00	-4.01	3.42
1967	0.67	- 2.59	8.20	-55.64	34.92	0.25	-0.59	1.43
1968	-1.46	- 5.98	3.07	-21.45	- 37.32	-18.92	3.71	3.16
1969	-2.80	-11.54	16.32	-73.66	17.79	-89.56	4.89	5.97
Wt. av. of abs. devs. (1958-1967)	3.03	3.48	15.04	18.13	47.84	9.94	2.51	3.90
Wt. av. of abs. devs. (1968-1969)	2.08	8.95	10.32	44.96	24.36	67.22	4.33	4.57

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 5B				
1958	-13.64	10.61	1.73	3.79	3.99	..	4.85	7.24
1959	9.72	- 6.87	- 7.70	-10.98	- 60.74	..	- 2.77	7.64
1960	-11.66	30.19	0.53	- 4.29	- 2.81	-10.37	0.98	7.72
1961	- 2.17	1.06	-25.40	20.29	28.48	19.66	4.98	6.44
1962	0.14	- 0.25	0.00	62.60	- 34.56	-74.08	0.38	1.19
1963	0.32	- 1.62	4.74	0.18	- 43.48	1.38	- 0.21	1.20
1964	5.30	-30.03	- 7.08	10.41	25.53	0.42	- 1.29	5.49
1965	- 0.16	13.69	-15.28	19.55	48.59	- 0.42	- 0.16	3.05
1966	- 0.91	0.00	7.93	0.68	23.08	5.40	2.38	2.49
1967	0.59	2.97	6.82	- 6.95	-322.45	-16.96	0.12	2.54
1968	- 2.50	4.32	-41.98	27.28	-123.46	-37.90	16.54	13.25
1969	-23.22	32.85	18.59	- 8.96	- 88.52	-18.25	12.49	19.42
Wt. av. of abs. devs. (1958-1967)	3.65	9.17	7.22	18.94	30.15	9.90	1.78	4.50
Wt. av. of abs. devs. (1968-1969)	11.91	19.89	29.91	19.73	101.89	23.86	14.42	16.34

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 6A				
1958	-16.52	45.92	2.56	- 8.71	1.33	..	7.27	12.79
1959	4.76	0.74	-11.31	17.57	-110.93	..	1.75	6.16
1960	- 4.15	8.69	13.59	-70.84	- 34.60	0.00	4.52	5.96
1961	-13.62	8.15	0.26	-36.85	8.75	8.65	10.35	11.17
1962	0.20	- 6.33	-64.11	18.93	-298.58	14.81	9.16	8.61
1963	- 0.70	12.55	4.22	-24.84	38.80	- 22.56	- 1.34	1.99
1964	- 1.36	- 9.05	-20.14	-29.25	21.01	47.73	2.35	2.55
1965	- 4.51	37.39	- 7.57	26.46	33.26	- 19.26	1.60	4.90
1966	1.89	- 1.04	6.80	- 1.79	27.46	- 19.68	- 3.29	2.78
1967	- 0.95	7.22	0.52	33.70	-367.81	- 46.13	2.26	2.61
1968	2.25	- 4.01	24.89	16.18	55.40	-279.85	- 3.76	4.69
1969	- 2.20	20.09	10.05	27.03	18.40	-372.81	5.01	6.45
Wt. av. of abs. devs. (1958-1967)	4.29	15.14	10.66	22.72	47.14	15.94	4.43	5.95
Wt. av. of abs. devs. (1968-1969)	2.23	13.23	16.72	22.45	27.87	343.66	4.41	5.57

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 6B				
1958	- 9.03	37.66	4.59	45.41	- 53.89	..	3.81	11.12
1959	9.21	-14.12	-14.44	41.84	-122.25	..	-1.17	9.76
1960	- 0.85	2.16	0.00	-154.12	13.97	0.00	0.23	1.37
1961	-16.91	49.05	- 9.14	0.00	20.72	6.68	5.74	14.53
1962	1.46	- 2.00	-43.24	73.97	- 32.62	0.00	0.62	3.13
1963	- 1.08	14.40	- 0.61	- 4.55	- 10.48	- 12.79	-0.53	1.70
1964	2.06	-27.59	-19.63	11.06	13.91	27.21	0.44	2.87
1965	- 3.55	29.20	-10.82	43.08	- 19.37	- 20.66	0.88	4.40
1966	0.02	6.38	10.29	11.66	19.24	- 4.84	-2.22	1.67
1967	- 3.93	15.88	7.30	65.31	- 7.32	- 22.93	-0.50	4.43
1968	- 6.13	32.80	30.52	16.18	75.78	-197.68	-2.26	8.19
1969	-14.95	34.46	33.78	9.90	30.36	- 27.31	3.04	12.34
Wt. av. of abs. devs. (1958-1967)	3.84	21.49	10.29	44.53	42.95	9.76	1.60	5.50
Wt. av. of abs. devs. (1968-1969)	10.17	33.68	32.28	12.88	40.99	59.52	2.67	10.27

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 7A				
1958	-43.34	40.87	21.32	9.96	50.55	..	4.85	23.85
1959	3.58	- 3.28	1.68	0.46	-15.33	..	-1.21	3.23
1960	0.57	8.33	-21.69	60.35	-22.75	11.57	4.41	5.99
1961	- 6.76	59.84	-38.99	3.18	8.30	- 3.80	3.46	8.98
1962	- 3.19	- 0.62	-74.11	-38.65	- 7.46	- 27.00	7.88	7.20
1963	0.70	-25.07	1.10	- 1.06	4.48	- 7.50	-0.47	1.04
1964	- 1.00	- 1.95	-24.59	2.63	29.50	- 5.20	-0.14	2.19
1965	- 0.57	16.67	9.24	28.32	- 5.69	1.64	-0.57	1.40
1966	4.24	-18.36	4.09	-26.92	4.27	6.17	-4.69	4.80
1967	0.21	26.53	3.44	19.08	- 8.18	- 7.74	-1.21	1.27
1968	1.70	21.16	32.38	-76.23	28.71	-3009.50	-6.62	7.59
1969	2.77	51.08	0.51	-38.56	1.14	77.78	-5.40	5.38
Wt. av. of abs. devs. (1958-1967)	4.61	25.79	17.92	20.85	22.40	9.00	2.93	6.00
Wt. av. of abs. devs. (1968-1969)	2.18	39.70	14.20	58.28	10.88	610.82	5.99	6.49

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 7B				
1958	- 7.69	- 1.16	23.91	14.95	6.59	..	0.17	5.74
1959	5.10	-13.77	- 5.40	- 0.97	-104.11	..	- 0.46	4.75
1960	- 1.33	5.81	- 0.51	71.42	-154.20	0.00	0.63	3.34
1961	- 3.09	11.70	-75.39	- 7.70	4.42	- 3.49	8.46	9.93
1962	2.01	- 5.71	- 3.64	-28.97	-227.61	0.00	1.05	2.66
1963	0.29	-11.64	4.93	55.56	7.72	-25.21	0.12	1.78
1964	- 1.41	- 0.44	-54.71	35.05	9.63	3.18	4.64	4.67
1965	- 0.27	11.07	-14.21	60.05	- 2.72	13.18	- 1.38	2.53
1966	2.30	- 3.18	10.71	- 7.19	2.18	-35.22	- 4.32	3.70
1967	0.20	- 1.68	4.61	18.92	-390.26	-21.28	0.08	1.02
1968	- 5.85	31.92	10.13	55.03	77.99	-90.89	1.81	6.95
1969	-22.83	- 7.77	- 9.77	11.76	13.80	27.05	19.94	19.24
Wt. av. of abs. devs. (1958-1967)	2.15	6.81	18.02	34.42	46.55	9.05	2.19	4.01
Wt. av. of abs. devs. (1968-1969)	13.47	19.12	9.93	30.37	30.07	37.99	11.28	13.10

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 8A				
1958	- 8.45	10.89	-21.49	- 2.06	19.20	..	9.96	12.09
1959	6.06	-48.93	- 3.53	0.00	-77.94	..	- 0.23	8.02
1960	-39.27	29.59	- 3.38	33.50	-32.29	- 7.65	20.49	22.19
1961	2.50	9.36	-23.95	19.69	25.81	-16.67	3.09	7.71
1962	- 1.74	- 0.29	-18.18	- 2.72	- 2.99	6.79	5.60	4.98
1963	0.41	3.76	6.72	15.95	-17.08	-17.07	- 0.65	2.44
1964	8.77	- 6.97	-29.68	0.51	24.66	- 4.17	- 3.50	8.40
1965	3.07	0.00	- 1.57	0.00	7.05	-13.31	0.99	3.17
1966	0.71	2.82	32.58	0.68	0.23	0.00	-12.18	7.98
1967	-13.06	11.54	27.27	- 9.82	4.75	- 7.04	1.49	10.24
1968	- 4.60	-12.10	9.98	21.37	15.91	- 2.17	1.90	4.87
1969	- 4.90	21.23	2.81	70.15	13.39	-16.72	3.34	7.55
Wt. av. of abs. devs. (1958-1967)	7.02	11.71	16.54	9.88	22.97	7.75	5.77	8.72
Wt. av. of abs. devs. (1968-1969)	4.73	16.67	6.76	55.22	14.56	11.01	2.65	6.21

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
					(percent)			
					Crop district 8B			
1958	- 9.53	14.42	1.10	1.48	19.18	..	4.08	7.16
1959	3.55	-14.47	- 7.41	7.02	- 53.41	..	- 2.76	5.07
1960	-10.20	35.16	8.77	23.59	- 69.17	0.00	3.86	9.66
1961	- 3.27	5.54	- 4.44	31.83	9.97	- 13.34	3.52	4.29
1962	- 1.66	5.09	-10.52	-50.55	- 46.46	- 50.00	5.33	4.93
1963	0.07	-13.49	6.85	43.27	17.16	- 8.07	- 0.04	1.99
1964	- 0.78	- 4.07	- 6.28	8.85	38.22	- 16.17	1.98	2.54
1965	0.66	12.63	-10.05	- 8.83	35.42	10.62	- 3.19	3.82
1966	3.71	- 8.15	1.25	- 2.26	- 16.33	- 10.18	- 2.63	3.72
1967	0.00	3.24	11.05	- 7.55	-373.73	- 4.71	- 0.52	2.51
1968	- 4.59	6.74	22.94	18.63	37.24	-105.26	3.82	9.12
1969	-15.23	19.18	9.67	-77.41	16.61	- 48.78	14.55	16.86
Wt. av. of abs. devs. (1958-1967)	2.99	11.94	6.62	18.46	38.58	9.36	2.79	4.57
Wt. av. of abs. devs. (1968-1969)	9.26	13.18	16.70	35.65	21.96	64.35	9.59	12.99

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 9A				
1958	-5.00	14.51	-13.38	13.60	0.00	..	3.40	7.01
1959	6.38	-68.65	- 6.28	- 1.42	- 38.60	..	1.42	8.23
1960	-5.62	6.34	- 0.61	15.96	11.72	- 8.18	3.52	4.62
1961	-0.48	10.62	-13.31	0.99	23.79	-21.09	1.05	4.29
1962	-2.27	- 9.09	11.95	- 0.73	- 0.60	20.77	0.78	3.98
1963	0.38	7.24	1.48	5.52	-148.13	-14.53	-0.93	2.13
1964	4.09	-27.73	- 2.27	- 3.18	13.57	- 8.08	-0.11	3.72
1965	2.44	19.47	- 1.22	10.41	48.55	-10.99	-7.72	6.26
1966	5.14	3.03	4.30	0.31	- 1.88	0.00	-8.99	5.72
1967	-3.38	5.83	-14.29	26.46	- 1.59	- 3.44	6.48	6.34
1968	-1.02	-39.47	5.77	24.28	- 20.89	-13.06	8.81	8.73
1969	-4.33	- 7.25	18.82	22.69	39.20	-45.58	5.24	10.30
Wt. av. of abs. devs. (1958-1967)	3.46	13.82	7.01	8.46	23.13	8.92	3.37	5.23
Wt. av. of abs. devs. (1968-1969)	2.53	22.88	12.12	23.39	32.28	33.47	6.94	9.52

TABLE 18 -- (continued)

Year	Wheat	Oats	Barley	Rye	Flaxseed	Rapeseed	Summer-fallow	[a]Wt. av. of abs. devs.
				(percent)				
				Crop district 9B				
1958	- 7.30	29.95	-22.06	5.04	23.68	..	0.11	10.39
1959	8.56	-74.00	- 4.30	16.14	- 74.73	..	1.70	9.83
1960	- 1.43	11.69	- 5.02	4.83	- 64.70	- 4.52	0.00	2.95
1961	- 3.78	30.53	-42.61	- 3.35	3.52	-32.02	- 0.44	10.85
1962	- 0.18	2.27	17.46	-24.02	- 87.28	-26.18	- 2.52	3.59
1963	-10.00	23.77	0.26	34.75	- 10.16	7.74	1.25	6.95
1964	3.42	-17.39	- 1.57	4.73	- 43.91	- 3.70	0.08	2.83
1965	- 3.63	34.88	1.27	7.98	15.54	-26.53	0.55	6.67
1966	- 3.17	21.21	- 2.24	8.35	-102.78	0.00	- 0.94	3.64
1967	3.41	- 0.22	2.08	- 2.29	-234.20	11.33	- 0.03	2.38
1968	- 6.50	19.87	8.08	-11.11	39.90	-35.34	2.92	9.00
1969	-26.53	36.64	12.80	- 7.39	-112.18	-38.28	11.79	21.84
Wt. av. of abs. devs. (1958-1967)	4.41	22.07	9.14	9.85	41.16	12.39	0.75	6.01
Wt. av. of abs. devs. (1968-1969)	15.00	28.57	10.55	9.00	85.41	37.63	7.48	15.42

[a]Wt. av. of abs. devs. stands for weighted average of absolute deviations.

TABLE 19

DATA ASSUMPTIONS FOR 1971 CROP ACREAGE FORECASTS, BY PROVINCE, IN PRAIRIES

Items		Manitoba	Saskatchewan	Alberta	Prairies
[a]Total improved land available (thous. acres)		9,909.00	41,866.00	21,346.00	73,121.00
Acreage in 1970 (thous. acres)					
	Wheat	1,400.00	8,000.00	2,600.00	12,000.00
	Oats	1,260.00	2,000.00	2,130.00	5,390.00
	Barley	1,500.00	3,300.00	4,700.00	9,500.00
	Rye	194.00	535.00	215.00	944.00
	Flaxseed	1,150.00	1,500.00	700.00	3,350.00
	Rapeseed	350.00	2,000.00	1,600.00	3,950.00
	Summerfallow	4,000.00	24,000.00	8,900.00	36,900.00
	Stubble land	2,654.00	0.00	4,545.00	6,334.00
[b]Expected farm level price for 1971 ($ per bushel)					
	Wheat	1.28	1.28	1.24	1.27
	Oats	.45	.45	.44	.45
	Barley	.66	.66	.66	.66
	Rye	.80	.80	.79	.80
	Flaxseed	2.30	2.30	2.32	2.30
	Rapeseed	2.10	2.10	2.11	2.10
[c]Expected grain stocks on farms, July 31, 1971 (thous. bushels)					
	Wheat	20,349.00	185,220.00	53,928.00	259,497.00
	Barley	4,842.00	16,659.00	24,759.00	46,251.00
	Rye	757.00	2,513.00	1,162.00	4,377.00
	Flaxseed	559.00	2,171.00	813.00	3,576.00
[d]Quota per assigned acre (bushels)					
	Wheat	10.00	10.00	10.00	10.00
	Oats	10.00	10.00	10.00	10.00
	Barley	20.00	20.00	20.00	20.00
	Rye	8.00	8.00	8.00	8.00
	Flaxseed	7.00	7.00	7.00	7.00
	Rapeseed	9.00	9.00	9.00	9.00
[e]Expected total precipitation in April and May 1971 (inch per acre)		3.94	2.27	2.42	2.87
[f]Expected wheat exports from Canada in crop year 1970-71 (thous. bushels)		480,000.00	480,000.00	480,000.00	480,000.00

[a]Total improved land available for 1971 was set at the 1969 level.

[b]To account for regional differences in transportation costs, a different price was used for each province. The price differentials between provinces were calculated for each year during 1961 to 1970. The average of these differentials were then used to adjust the 1971 provincial prices.

[c]Grain stocks at farms on July 31, 1971, for each crop were computed utilizing (a) Canadian total stocks (farms and non-farms) estimated by W. J. Craddock, and (b) average ratio of farm stocks of a province to the Canadian total stocks over 1961 to 1970.

[d]Quota levels were anticipated by W. J. Craddock.

[e]The ten-year average precipitation (1961-1970) was used as the 1971 figure.

[f]Wheat exports for 1971 was the guestimate made by W.J. Craddock.