REPRESENTATION OF INDUCTION MOTORS DURING LARGE FREQUENCY SWINGS

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ABSTRACT

Large frequency swings of 1± 0.30 p.u. frequency are predicted to occur in isolated power stations connected through a H.V.D.C. transmission line to an A.C. system. This would have drastic effects on station service induction motors. An equivalent representation of a group of induction motors is proposed for large frequency swing analysis. For two induction motors having the same parameters, if the total moment of inertia of one motor does not exceed ±40% of the other motor, then the maximum percentage errors in active power and reactive power will not exceed 10%, using the proposed model. Accuracy of this representation is expected to be higher for frequency swings of lesser magnitudes than those considered.

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NOMENCLATURE

Symbols

A,B,C	load torque constants
e	axis voltage, p.u.
E	r.m.s. value of voltage, p.u.
E_1	voltage on secondary side of transformer, p.u.
E_g	voltage of the generator bus, p.u.
f	frequency, p.u.
h	step size of integration, sec.
H	total inertia constant, sec.
hp	horse power rating, h.p.
i	current, p.u.
I	magnitude of current, p.u.
j	complex operator
p	differential operator $\frac{d}{d(\omega_0 t)}$
P	active power, p.u.
Q	reactive power, p.u.
R	resistance, p.u.
s	slip, p.u.
t	time, sec.
T_E	electrically developed torque, p.u.

T_M	mechanical load torque, p.u.
v	voltage representing the equivalent flux linkage of the rotor, p.u.
V	peak value of supply voltage,p.u.
X	reactance, p.u.
X'	locked-rotor short-circuit reactance, p.u.
Δ	small changes in a variable
θ	electrical angle turned through by rotor, rad.
θ_S	electrical angle turned through by synchronous-speed axes relative to the rotor, rad.
τ	time, p.u.
τ_0	open circuit time constant
ϕ	phase angle, rad.
ω	angular frequency, rad./sec.
ω_0	synchronous speed of equivalent two-pole machine at rated frequency, rad./sec.
ν	speed of motor, p.u.

Subscripts

a,b,c	stator phase quantities
α,β	variable speed axes quantities
d,q	stationary axes quantities
o	quantities of o coil
	open-circuit quantities

s,r	stator and rotor quantities respectively
e	thevenin equivalent circuit quantities
l	leakage quantities
m	magnetizing quantities

Superscripts

$\hat{}$	complex quantities
$\bar{}$	phasor quantities

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

Large frequency swings in the order of 1 ± 0.30 p.u. are predicted [1,2] to occur in isolated power stations connected through a H.V.D.C. transmission line to an A.C. system. This frequency swing is caused by the overspeeding of the generators. When the generators are running loaded and the transmission of power through the D.C. link is suddenly disrupted, the hydraulic energy will cause the generators to overspeed before the governors can close the turbine wicket gates. This overspeeding will last for about 8 seconds and then generators will run underspeed. This kind of swing will occur for more than 30 seconds under the worst condition of total load rejection, and it will not only affect the system reactance, but also the regulation of the bus and motor terminal voltages. The induction motors may run at an extra high slip which will cause the motors to draw a much higher current and in turn will cause a higher drop in terminal voltage. If the torque developed by the motor is lower than the load torque, the motors may stall. Therefore it is important to know the behaviour of the individual induction motors driving vital loads as well as the voltage level and power flows in the station service buses.

In the past, it has always been assumed that the change in frequency in a power system is so small that the effect of the induction motors on the system is negligible. Little was written about the effect of frequency on the induction motor. But with the introduction of static frequency convertors, the induction motor has become more important in the field of speed control; the effects of frequency change on induction motors thus become of interest. However there is hardly anything written on the subject of large frequency swings. Work has been done by Akhtar [3] concerning small frequency changes in the order of 1 ± 0.05 p.u. frequency; larger frequency changes have not been examined. Several authors [4,5,6] had investigated the effects of frequency changes on induction motors; the frequency changes may be large or small. But they dealt only with the transient nature(including all transients) of the induction machine and the methods are not suitabel for representation in power system load flow studies.

Therefore it is the purpose of this thesis to study the effects of large frequency swings on the performance of station service induction motors and on the representation of these motors for load flow studies. A system of two induction motors connected to the same generator or transmission bus through a transformer of 0.1 p.u. is considered. The system is shown in fig. 1.1. It is assumed that the voltage on the generator or transmission bus remains constant at 1.0 p.u. during the frequency swing due to the fast action of the excitation control.

Only the worst type of frequency swing is investigated

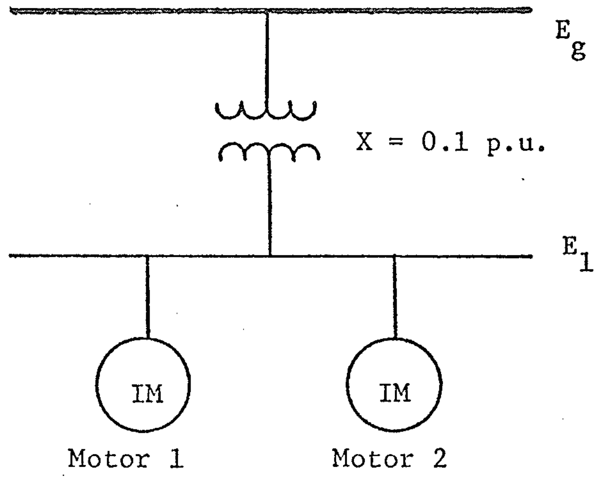


Fig.1.1. A System of Two Induction Motors

here (Fig. 1.2); the first frequency peak is 1.35 p.u., 5 seconds after load rejection; the second peak is 1.1 p.u. at 22.5 seconds. It is assumed that if the analysis holds for large frequency swings, the analysis will also hold for smaller frequency swings.

In Chapter II, the various methods of representing an induction motor are reviewed and adapted for frequency studies where possible.

In Chapter III, the validity of the transient representation including the mechanical transients only is investigated using a digital computer.

In Chapter IV, a simplified representation of combining two induction motors is proposed. Certain types of combination using this equivalent representation have attained reasonable accuracy.

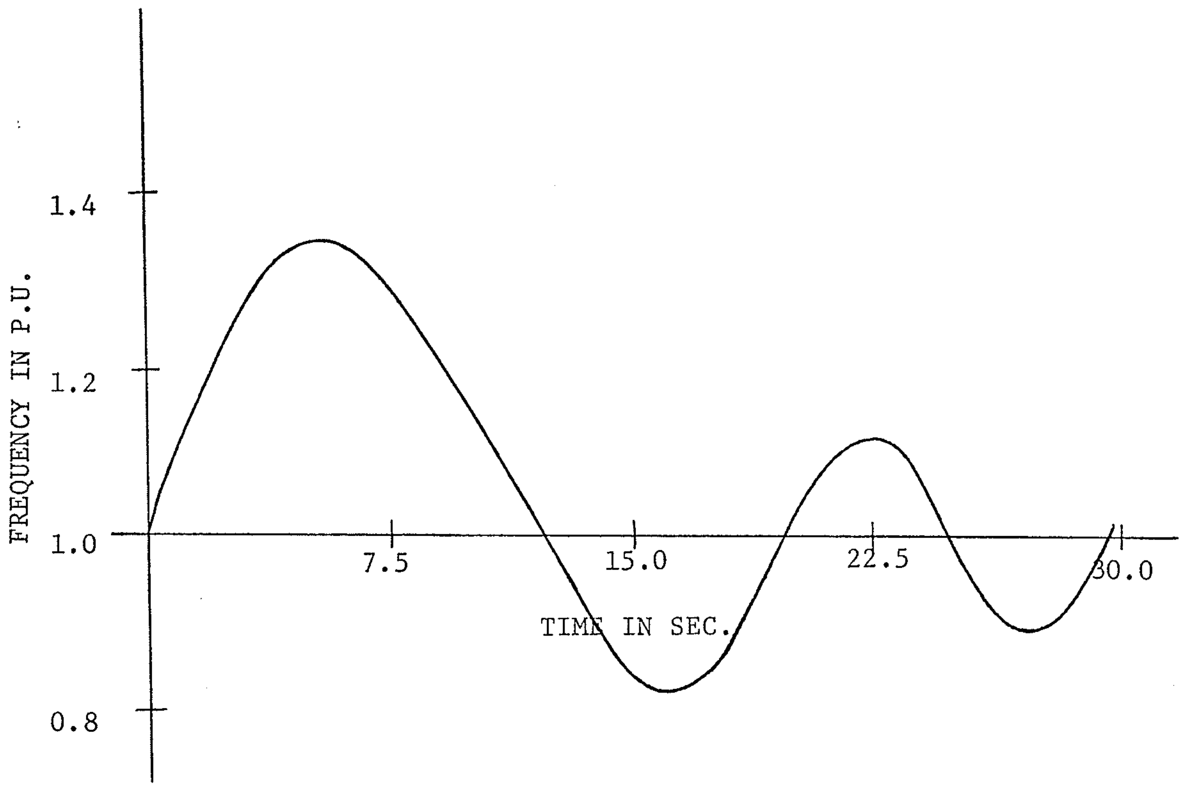


Fig.1.2. The Worst Type of Frequency Excursion.

CHAPTER II

METHODS OF REPRESENTING INDUCTION MOTORS

There are several methods of representing an induction motor either for load flow studies or stability studies. Some conventional methods have been used to represent induction motor loads as constant impedances, constant current sinks, or nonlinear loads. However, their inadequacy rests in the fact that the induction motor loads are dynamic and frequency dependent. It is the objective of this chapter to review the different kinds of representations and examine how they can be adapted for the purpose of large frequency swing analysis. The methods are:

(1) The transient representation including all transients (stator transients, rotor transients and mechanical transients)

(2) The transient representation neglecting stator transients.

(3) The Steady state equivalent circuit representation.

(4) The transient representation including the mechanical transients only.

(5) Linearized approximation

(6) Representation as simple loads

2.1. THE TRANSIENT SOLUTION INCLUDING ALL TRANSIENTS (STATOR TRANSIENTS, ROTOR TRANSIENTS AND MECHANICAL TRANSIENTS)

The equations of performance of an ideal polyphase induction machine can be obtained in a simple form, first suggested by Park[7]. Instead of the actual phase voltages and currents i_{sa}, i_{sb}, i_{sc}, i_{ra}, i_{rb}, i_{rc}, of a three phases machine, new fictitious quantities i_{sd} i_{sq}, i_{so}, i_{rd}, i_{rq}, i_{ro}, related to the original quantities are used. The changing from one set of variables to another is done by means of transformation matrix and the process is called tranformation. The transformation is more than a mathematical process. The d,q quantities can be treated as fictitious coils located on the direct and quadrature axes. Stanley[8] has given a more general analysis on induction motors. The reference frame of the axes is stationary with respect to the stator. The derivation of the Stanley equations are based on the following assumptions:

(1) Balanced windings are assumed on both rotor and stator.

(2) The mutual inductance between any stator and rotor windings is assumed to be a sinusoidal function of the electrical angle between the axes of the two windings.

(3) It is assumed that the rotor is smooth and that the self-inductance of any windings is independent of rotor position.

(4) The effects of saturation, hysteresis, and eddy currents are neglected.

(5) The machine impedances are assumed to be independent of rotor speed.

(6) The machine electrical neutral is assumed to be ungrounded.

(7) In all cases, the motor is considered connected to an infinite bus through negligible impedance. The effect of system impedance can be taken into account by increasing the stator impedance, that is, by moving the infinite bus from the terminals of the machine and place it further back in the system.

The derivation of the Stanley equations can be found in his paper and is not repeated here. The Stanley equations written in per-unit form are:

$$\left.\begin{aligned} e_{rd} &= p\psi_{rd} + R_r\, i_{rd} + \psi_{rd}\,[p\theta] \\ e_{rq} &= p\psi_{rq} + R_r\, i_{rq} - \psi_{rq}\,[p\theta] \\ e_{sd} &= p\psi_{sd} + R_s\, i_{sd} \\ e_{sq} &= p\psi_{sq} + R_s\, i_{sq} \end{aligned}\right\} \quad \ldots(2.1)$$

where

$$\left.\begin{aligned} \psi_{rd} &= X_r\, i_{rd} + X_{sr}\, i_{sd} \\ \psi_{rq} &= X_r\, i_{rq} + X_{sr}\, i_{sq} \\ \psi_{sd} &= X_s\, i_{sd} + X_{sr}\, i_{rd} \\ \psi_{sq} &= X_s\, i_{sq} + X_{sr}\, i_{rq} \end{aligned}\right\} \ldots(2.2)$$

For the electrical torque

$$T_E = \psi_{rd}\, i_{rq} - \psi_{rq}\, i_{rd} \qquad \ldots(2.3)$$

and for the acceleration

$$p\dot{\theta} = \frac{1}{2\omega_0 H} (T_E - T_M) \qquad \ldots(2.4)$$

where all quantities are listed in the nomenclature.

In most induction motor applications, the coils on the rotating parts are short-circuited, that is, $e_{rd} = e_{rq} = 0$ at all instances. The stator terminals are usually connected to a constant frequency source.

$$\left.\begin{aligned} e_{sd} &= V \, \mathrm{Cos}(\omega t + \phi) \\ e_{sq} &= V \, \mathrm{Sin}(\omega t + \phi) \end{aligned}\right\} \qquad \ldots(2.5)$$

However, for the case in which the source frequency is not a constant, The stator voltages are:

$$\left.\begin{aligned} e_{sd} &= V(\tau) \, \mathrm{Cos} \left(\int_0^\tau p\theta \, d\tau + \phi \right) \\ e_{sq} &= V(\tau) \, \mathrm{Sin} \left(\int_0^\tau p\theta \, d\tau + \phi \right) \end{aligned}\right\} \qquad \ldots(2.6)$$

here V indicates a voltage which can be constant or time dependent, and the angular frequency $p\,\theta$ varies with the frequency change as desired. In usual machine practice, the air gap flux is required to be constant therefore the terminal voltage is taken to be proportional to the frequency. Of course, in the case when the frequency change is unexpected, the voltage change is also at the mercy of system changes.

These equations can be either solved by an analog computer or a digital computer. The methods will be mentioned in a later chapter.

Either method has its own limitations. The representation is in terms of per-unit time and the analysis will require great amount of computer time. This representation would be useful if the minute detail is required.

2.2 THE TRANSIENT REPRESENTATION NEGLECTING STATOR TRANSIENTS

It is possible to simplify the transient equations if the stator transients can be neglected. The neglecting of the stator transients means that the d.c. component in the armature phase current and pulsating electrical torque are also neglected. This prospect was first pioneered by Lewis and Marsh [9] who obtained a new set of transient equations by neglecting the $p\psi_s$ terms. The work was further carried out by Brereton, Lewis and Young [10], who, by combining the simplified equations, obtained a set of equations which can be represented by an equivalent circuit. The procedures of simplification are outlined as follows:

The Stanley equations can be rewritten, referring them to axes rotating at unit or synchronous speed.

$$\left.\begin{aligned} e_{s\alpha} &= p\psi_{s\alpha} + R_s\, i_{s\alpha} - \psi_{s\beta} \\ e_{s\beta} &= p\psi_{s\beta} + R_s\, i_{s\beta} + \psi_{s\alpha} \\ 0 &= p\psi_{r\alpha} + R_r\, i_{r\alpha} + \psi_{r\beta} p\theta_S \\ 0 &= p\psi_{r\beta} + R_r\, i_{r\beta} - \psi_{r\alpha} p\theta_S \end{aligned}\right\} \quad \ldots(2.7)$$

here $p\theta_S$ is the negative of the rotor slip.

$$\left.\begin{aligned}
\psi_{s\alpha} &= X_s\, i_{s\alpha} + X_{sr}\, i_{r\alpha} \\
\psi_{s\beta} &= X_s\, i_{s\beta} + X_{sr}\, i_{r\beta} \\
\psi_{r\alpha} &= X_r\, i_{r\alpha} + X_{sr}\, i_{r\alpha} \\
\psi_{r\beta} &= X_r\, i_{r\beta} + X_{sr}\, i_{s\beta}
\end{aligned}\right\} \quad \ldots(2.8)$$

$$T_E = \psi_{r\beta}\, i_{r\alpha} - \psi_{r\alpha}\, i_{r\beta} \quad \ldots\ (2.9)$$

$$p^2\theta_S = \frac{1}{2\ \omega_0 H}\ (\,T_E - T_M\,) \quad \ldots\ (2.10)$$

$$\left.\begin{aligned}
e_{s\alpha} &= V \operatorname{Cos} \phi \\
e_{s\beta} &= V \operatorname{Sin} \phi
\end{aligned}\right\} \quad \ldots(2.11)$$

The above equations include the effects of the stator and the rotor transients. If the stator transients are neglected, $p\psi_{s\alpha}$ and $p\psi_{s\beta}$ can be set equal to zero in the equations. The equations can be simplified as

$$\left.\begin{aligned}
e_{s\alpha} &= R_s\, i_{s\alpha} - X'\, i_{s\beta} - \frac{X_{sr}}{X_r}\, \psi_{r\beta} \\
e_{s\beta} &= R_s\, i_{s\beta} + X'\, i_{s\alpha} + \frac{X_{sr}}{X_r}\, \psi_{r\alpha} \\
e_{r\alpha} &= 0 = p\psi_{r\alpha} + \frac{R_r}{X_r}\, \psi_{r\alpha} + \psi_{r\beta} p\theta_S - \frac{R_r}{X_r}\, X_{sr} i_{s\alpha} \\
e_{r\beta} &= 0 = p\psi_{r\beta} + \frac{R_r}{X_r}\, \psi_{r\beta} - \psi_{r\alpha} p\theta_S - \frac{R_r}{X_r}\, X_{sr} i_{s\beta}
\end{aligned}\right\} \quad (2.12)$$

where $\quad X' = X_s - X_{sr}^2 / X_r$

By further manipulation, the resulting equations are:

$$p\hat{v} = -\frac{1}{\tau_0}(\hat{v} - j(X_0 - X')\hat{i}_s) + j\hat{v}(p\,\theta_s) \qquad \ldots(2.13)$$

$$\hat{i}_s = \frac{(\hat{e}_t - \hat{v})}{R_s + jX'} \qquad \ldots(2.14)$$

and

$$\tau_0 = \frac{X_r}{R_r} \qquad \ldots(2.15)$$

The equations can be represented by an equivalent circuit:

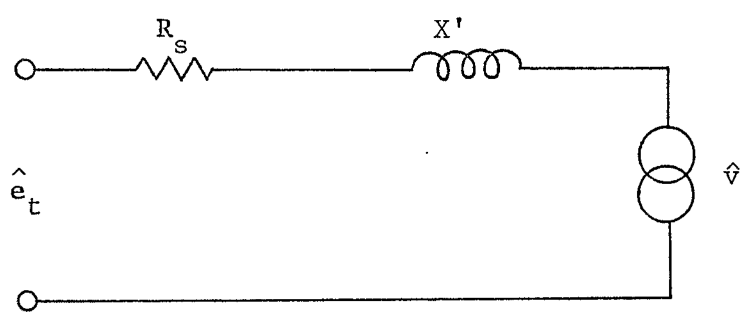


Fig.2.1. Circuit Representation of Eq. 2.14.

Further work has yet to be done to develop this representation for frequency studies.

2.3 THE STEADY STATE APPROXIMATION

When all the transients(electrical and mechanical) are neglected, the induction motor can be approximately represented by the steady state equivalent circuit(Fig.2.2).

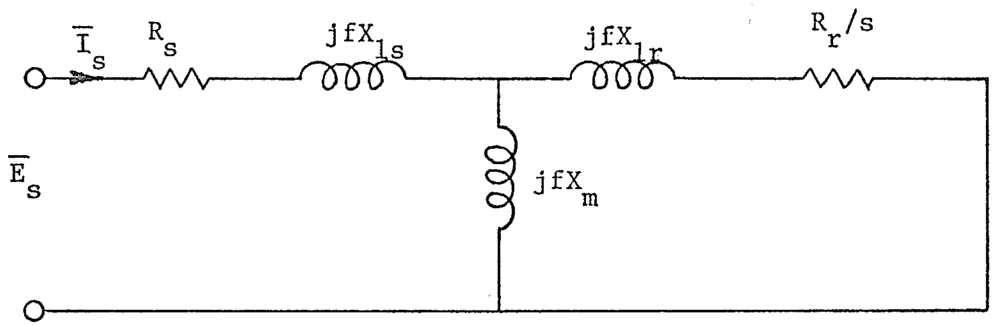


Fig.2.2. The Steady State Representation of an Induction Motor

The steady state equations are:

$$T_E = \frac{R_r}{sf} \frac{E_e^2}{(R_e + R_r/s)^2 + (X_e + X_{1r})^2 f^2} \tag{2.16}$$

where
$$\bar{E}_e = \frac{jfX_m}{R_s + jf\ (X_{1s} + X_m)} \bar{E}_s \qquad \text{...(2.17)}$$

and
$$R_e + jfX_e = \frac{jfX_m(R_s + jfX_{1s})}{R_s + jf(X_{1s} + X_m)} \qquad \text{...(2.18)}$$

in which X_e, X_{1r}, X_{1s}, X_m are quantities at the base frequency. The steady state current equation is:

$$\bar{I}_s = \frac{\bar{E}_s}{j(X_{1s} + X_m)f} + \left(\frac{X_m}{X_{1s} + X_m}\right)^2 \frac{\bar{E}_s}{R_r/s + jf(X_e + X_{1r})} \qquad \text{...(2.19)}$$

The steady state active power and reactive power are:

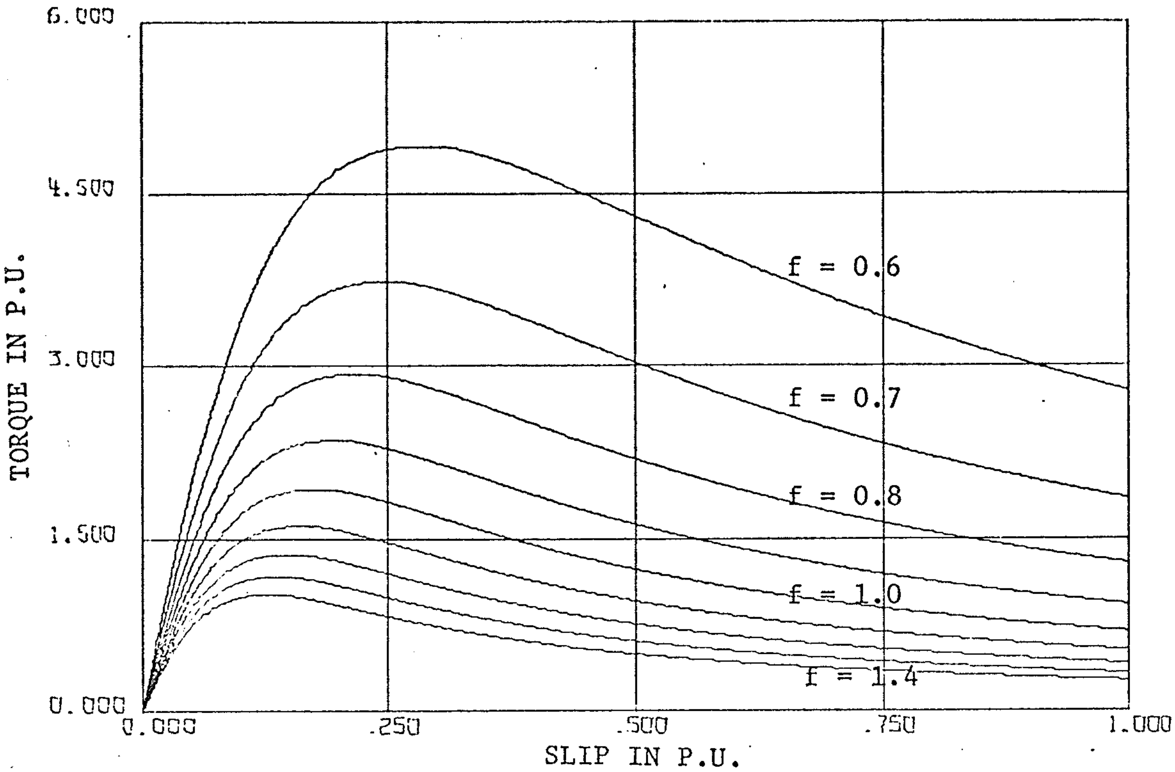


Fig. 2.3. The Torque-slip Characteristics of an Induction Motor With the Supply Frequency as a Running Parameter (E = 1.0 p.u.)

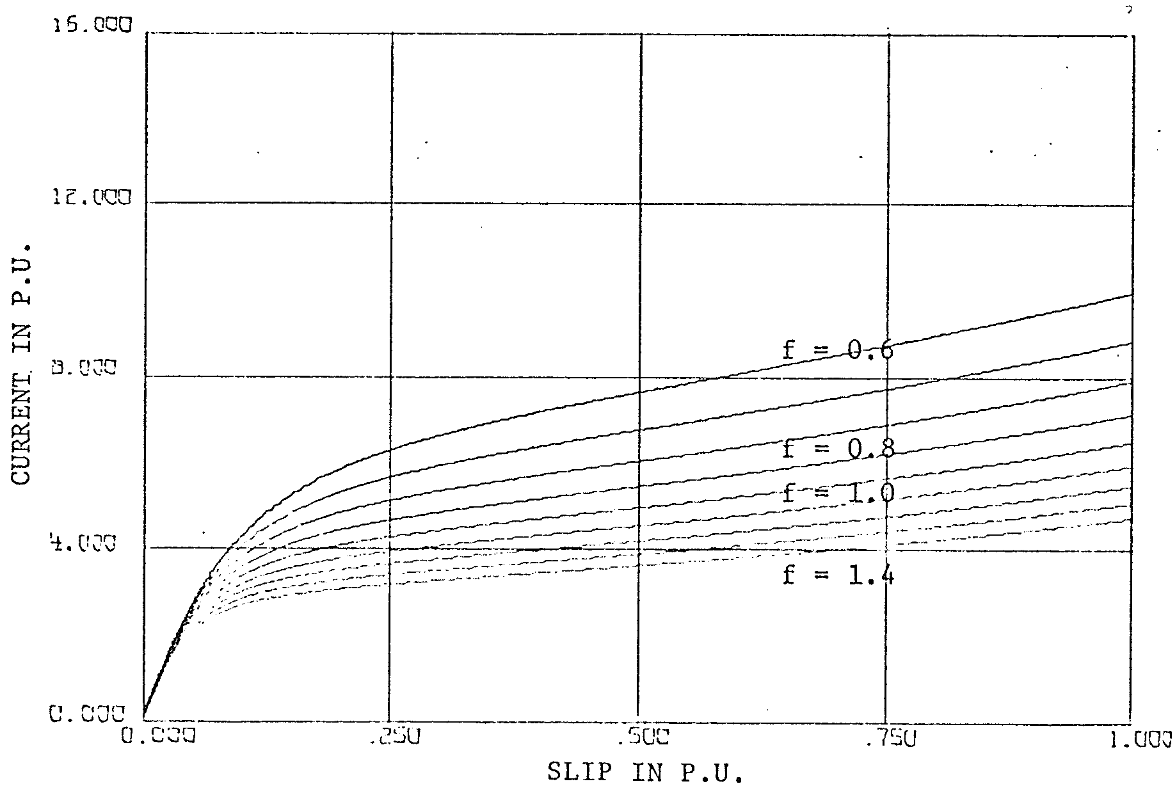


Fig. 2.4. The Current-slip Characteristics of an Induction Motor With the Supply Frequency as A Running Parameter (E = 1.0 p.u.)

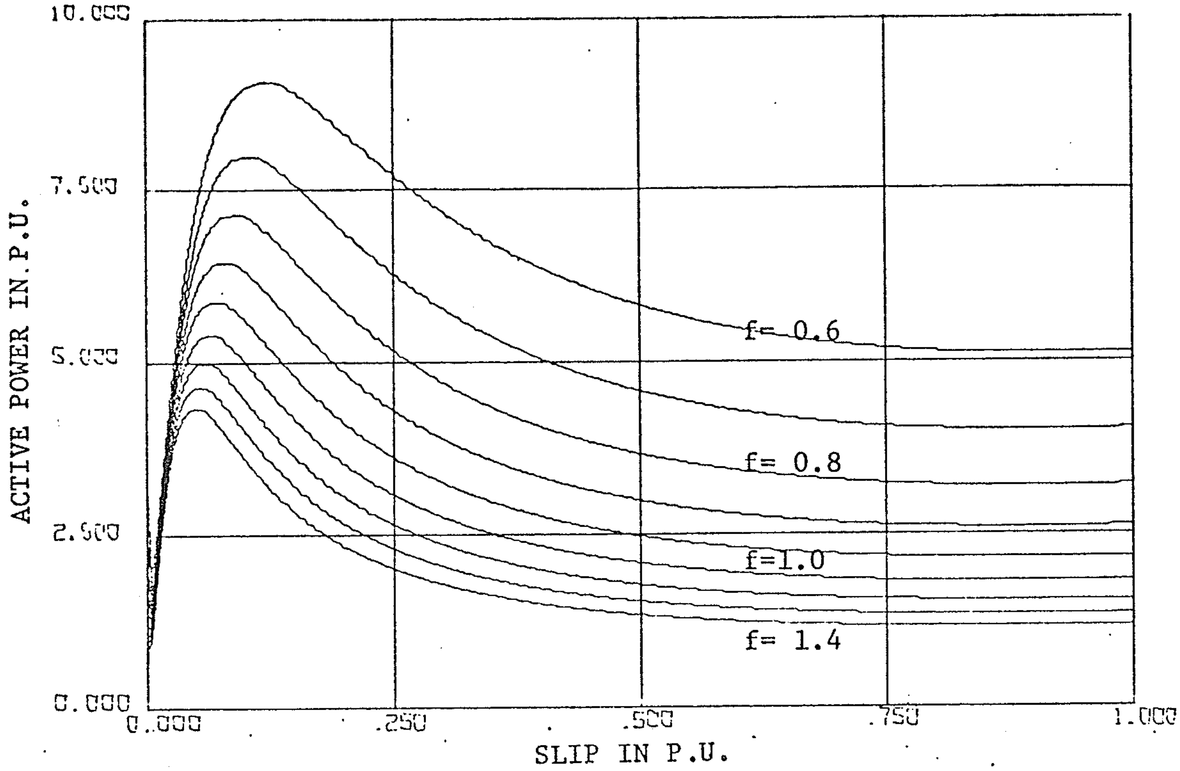


Fig. 2.5. The Active Power Slip Characteristics of an Induction Motor With the Supply Frequency As a Running Parameter (E = 1.0 p.u.)

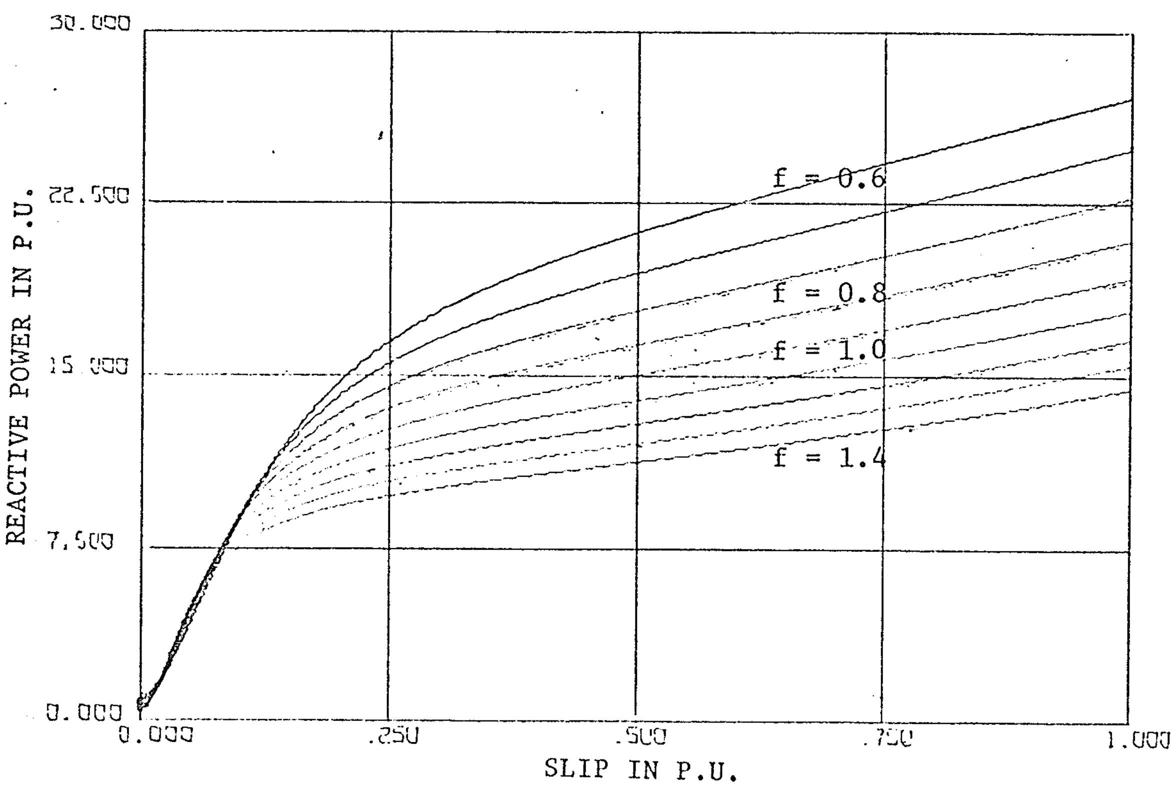


Fig. 2.6. The Reactive Power Slip Characteristics of an Induction Motor With Supply Frequency As a Running Parameter (E = 1.0 p.u.)

$$P = \mathcal{Re}\,[\,\bar{I}_s^{\,*}\,\bar{E}_s\,]$$

and
$$Q = \mathcal{Im}\,[\,\bar{I}_s^{\,*}\,\bar{E}_s\,]$$

where $\bar{I}_s^{\,*}$ is the complex conjugate of $\bar{I}_s$. At different frequencies and different slips, the characteristics of the induction motor are shown from Fig.2.3. to Fig.2.6. At each frequency, at each slip, and at each terminal voltage, there are corresponding values of input and output quantities. These quantities can be obtained easily by using the above equivalent circuit.

2.4. THE REPRESENTATION USING THE STEADY STATE EQUATIONS BUT INCLUDING ONLY THE MECHANICAL TRANSIENTS

Usually it is erroneous to represent something of a transient nature by a steady state representation. Unless the disturbance is negligible, the mechanical transient must be taken into account. When the frequency at the terminals of an induction motor changes, it will produce an accelerating torque on the motor. Under steady state frequency f_0 the motor operates at ν_0 . As the frequency increases to f_1, the new steady state operating speed would be ν_2 but because of the finite inertia of the motor and the connected load, the motor does not reach the steady state operating speed of ν_2 but only say ν_1 before the frequency is again increased to f_2. At the end of time $2\Delta t$ the frequency changes to f_2. Again, the accelerating

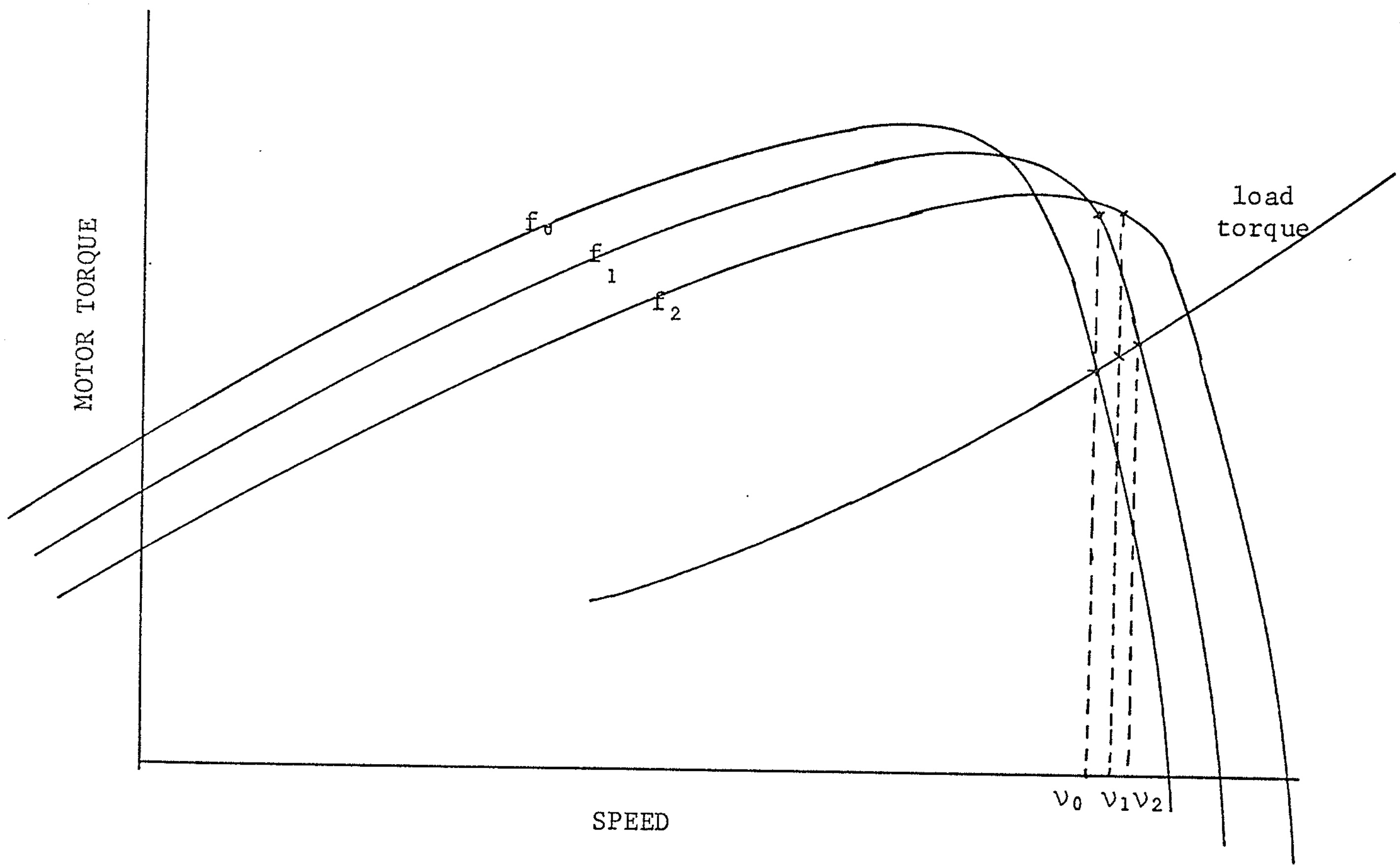


Fig.2.7. The Acceleration of an Induction Motor When the Frequency Changes.

torque is that given by the difference between the torque produced at f_2 and ν_2 and the load torque. In this way, the speed variation of the induction motor can be determined.

Here the equations are similar to those given in section 2.3. except there is an additional equation for the acceleration.

$$\frac{d\nu}{dt} = (T_E - T_M)/2H \qquad \text{...(2.20)}$$

This method has been used by different authors [3, 10] and it has always been assumed without proof that this method of representation is valid for load flow studies or stability studies. The question of its accuracy in representation has never been raised. In Chapter III, the validity of this method is investigated. It is found that the representation neglecting the stator transients and rotor transients but including the mechanical transients is valid for this type of large frequency excursion. The stator and rotor transients can in fact be neglected.

2.5. LINEARIZED APPROXIMATION

It is possible to observe from Fig. 2.3. to Fig. 2.6. inclusively that the steady state equations can be linearized at s = 0. The range of the approximated linearity is determined by the characteristics of the induction motor. In most cases, the range of linearity is small and the approximation is only good in that region. For large frequency swings, the slip usually is large and linearity does not apply.

The torque equation is

$$T_E = \frac{R_r}{sf} \frac{E_e^2}{(R_e + R_r/s)^2 + (X_e + X_{1r})^2 f^2} \qquad \ldots(2.16)$$

At s=0, the torque-slip curve is linear and the slope of the torque-slip curve is given by

$$\frac{\partial T_E}{\partial s}(s = 0) = \frac{E_e^2}{fR_r} \qquad \ldots(2.21)$$

Therefore
$$T_E = \frac{E_e^2}{f\ R_r}\, s \qquad \ldots(2.22)$$

The current equation is given by:

$$\overline{I}_s = \frac{\overline{E}_s}{j(X_{1s}+X_m)f} + \left(\frac{X_m}{X_{1s}+X_m}\right)^2 \frac{\overline{E}_s}{R_r/s + jf(X_e+X_{1r})} \qquad \ldots(2.19)$$

$\overline{I}_s$ can be linearized at s = 0 :

$$\frac{\partial\ \overline{I}_s}{\partial\ s}(s=0) = \left(\frac{X_m}{X_{1s}+X_m}\right)^2 \frac{\overline{E}_s}{R_r} \qquad \ldots(2.23)$$

Hence $\overline{I}_s$ can be linearized as

$$I_s = \frac{\overline{E}_s}{j(X_{1s}+X_m)f} + \left(\frac{X_m}{X_{1s}+X_m}\right)^2 \frac{\overline{E}_s}{R_r}\, s \qquad \ldots(2.24)$$

If the values of T_E and I_s are known, the procedure of analysis is the same as that of the representation using the steady state equation by including the mechanical transients(section 2.4.).

2.6. REPRESENTATION AS SIMPLE LOADS

Mainly due to the fact that convenient methods of assessing induction motors were not available, induction motor loads had previously been represented as simple loads [11, 12],

(1) Constant impedances, giving active and reactive power proportional to the square of the terminal voltage but independent of the system frequency.

(2) Constant-current sinks, where the active and reactive power vary as the system voltage.

From Fig.2.3. to Fig.2.6., it is possible to see that this method is far from sufficient to represent an induction motor during large frequency excursions, the active and reactive powers are functions of the frequency and the slip of the motor.

CHAPTER III

THE VALIDITY OF THE REPRESENTATION INCLUDING THE MECHANICAL TRANSIENTS ONLY

The representation including the mechanical transients only will be the basis for analysis in the following chapter. Therefore it is important to investigate the validity of this representation. It has been always assumed without proof that this representation of induction motor is valid for load flow studies or stability studies. The question of its accuracy in representation has never been raised. Maginniss and Schultz [13] in the conclusion of their paper stated that large error may be introduced by using the steady-state speed-torque curve for the calculation of transients. Berg and DeSarkar[5] assumed that maximum or breakdown torque applies during transition when the change of frequency is linear and their assumption was supported by results of solving the set of differential equations such as eq. 2.1. Naturally, it is only safe to test the validity of the presentation before the method is used. In this chapter the program including the mechanical transients only is compared with the transient program which includes all transients. If the results of the two programs compare favourably with each other, then it implies that the representation including the mechanical transients only is valid.

The solution of both representation programs involves differential equations. It is important that some methods must be available for the solution of these two. In the past, authors have tried to solve the equations of an induction motor either by the analog computer [14, 15, 16, 17, 18,] or the digital computer [19, 20]. It is noted that the analog computer has the following advantages:-

(1) When a set of equations has been modelled on the analog computer, any change in the parameters, coefficients, initial values and given functions can be adjusted conveniently by dials. For a program in which there are a lot of changes in the parameters, the total solution time would be much reduced.

(2) Results are readily displayed as a continuous plot of the variable quantities. But for the digital computer method, it is necessary to write an additional program to plot graphs.

The digital computer has the following advantages:-

(1) With the improvements in software and hardware the modern computer is fast and efficient and has a large memory capacity. It is able to solve a practical power system problem without much difficulty while the analog computer is limited by the number of components available.

(2) The accuracy of the digital computer is high and independent of the nonlinear characteristics of the components. But the accuracy of the analog computer is about four significant figures.

(3) The storage of the program and the result is easily done

by means of tape or disk.

With the rapid improvement in digital computers, the present-day digital computer proves to be a convenient, efficient and powerful tool for solving differential equations. Here in this thesis, emphasis is placed on the solution by digital means. In solving differential equations, there are two major kinds of numerical methods, namely, (1) the Runge-Kutta methods and (2) the predictor-corrector methods. In order to achieve efficiency in solution, the first kind is used in solving the performance of an induction motor including all transients, and the second kind is employed in the solution of the performance of induction motors including the mechanical transients only.

3.1. THE SOLUTION OF THE PERFORMANCE OF AN INDUCTION MOTOR INCLUDING ALL TRANSIENTS

The solution involves the simultaneous solution of a set of six differential equations. Jordan[19] favours the solution using the predictor-corrector methods. Lawrenson and Stephenson[4] prefer the solution using the Runge-Kutta-Merson method. The Runge-Kutta-Merson method is essentially a variable step-length method; the maximum truncation error can be calculated and the step-length can be varied to attain a certain range of accuracy. It was found[21] that this method is twenty percent faster than the standard fixed step-length procedure; the number of steps of integration as well as the round-off error can be minimized. The solution of a set of differential

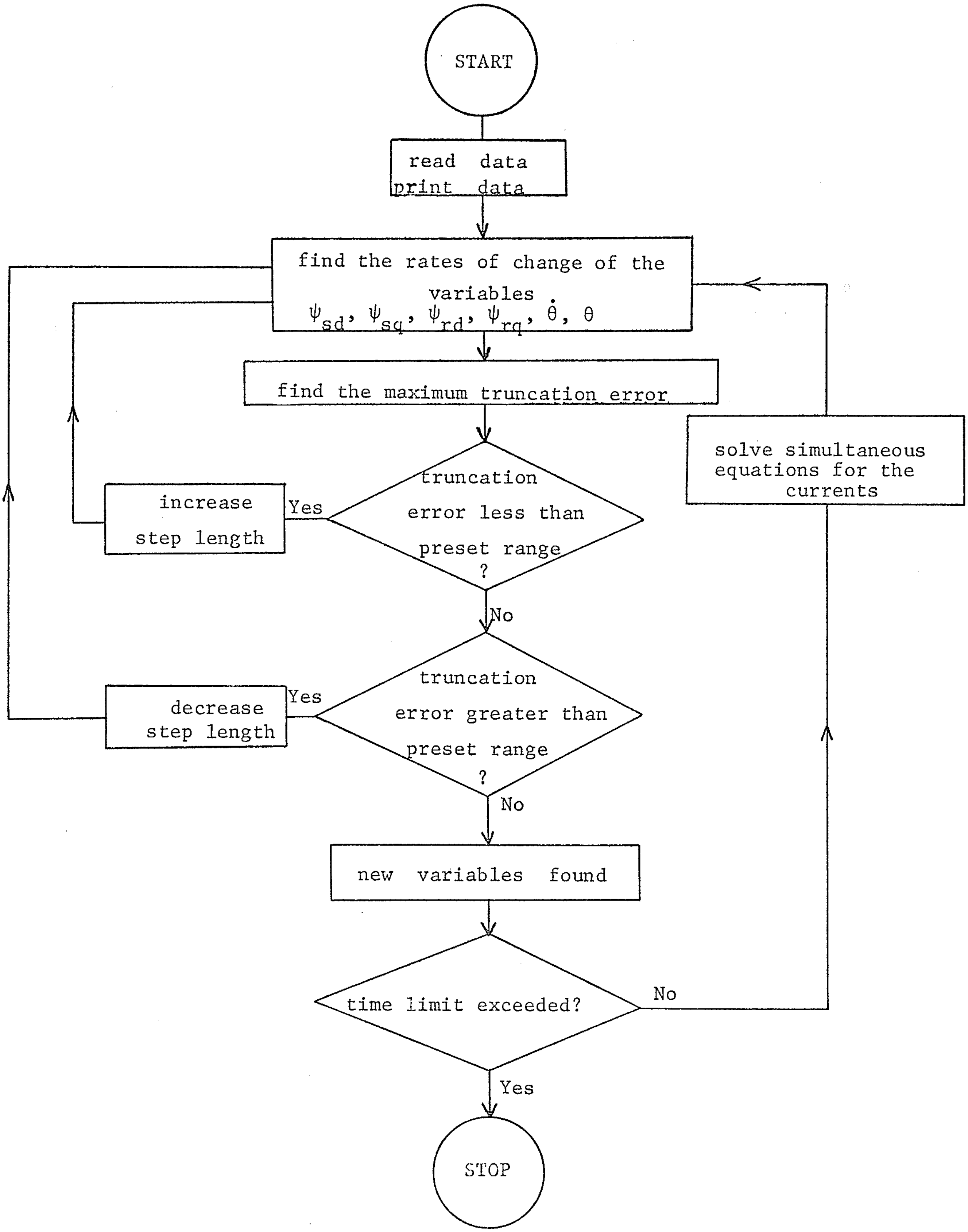


Fig.3.1. Flow Chart For Solving a set of differential equations by The Runge-Kutta-Merson Method

equations involves the evaluation of the rates of change of variables five times during each step of integration. The Runge-Kutta methods in general are described in Appendix A and the formulae for the Runge-Kutta-Merson method can be found in literature by Cadzow and Martens[22]. The procedures of solution are summarized in Fig.3.1.

3.2. THE SOLUTION OF THE PERFORMANCE OF AN INDUCTION MOTOR INCLUDING ONLY THE MECHANICAL TRANSIENTS

The Euler-Cauchy predictor-corrector method is used in this case because of its simplicity. Accuracy is of the order h^2 and is sufficient for this analysis. The method consists of predicting a value y^*_{i+1} by linear extrapolation.

The new value y^*_{i+1} is used in correction and a still better value of y^*_{i+1} is found. Therefore by iteration procedure, a value y'_{i+1} is found which is closer to the 'true' value y_{i+1}. The iteration procedure is terminated when the displacement of y^*_{i+1} is less than a preset accuracy. The Euler-Cauchy method is outlined in Appendix B. There are three major steps in this program:

(1) To read in parameters and print out parameters.

(2) To calculate the initial speeds of the machines and to calculate the voltage on the secondary side of the transformer.

(3) To use the predictor-corrector method to find the new speed

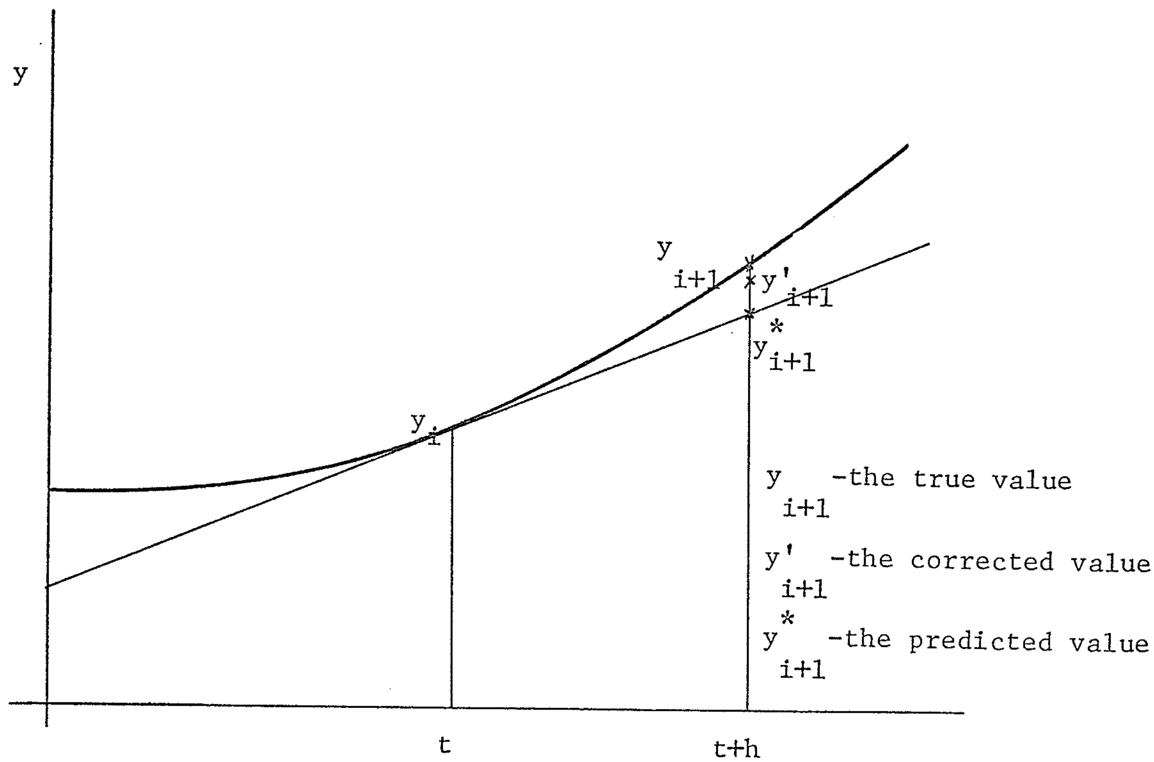


Fig.3.2. A Graphical Illustration of the Predictor-Corrector Method

in each step. The input power and the voltage on the secondary side of the transformer are found at the same time.

The details of the second and third step are described in the flow-charts in Fig. 3.3. and Fig. 3.4. repectively, and the subprograms are:

SUBROUTINE VSTART- A program to calculate the steady state speed of an induction motor. To start with a machine speed of 0.9 p.u. and an increasing interval of 0.0005 p.u. the torque developed by the motor is compared with the torque developed by the load. If the load torque is just greater than the motor torque, then, that speed is the operating speed.

FUNCTION F- A function program which describes the frequency excursion at different instances of time.

M = 1   is for large frequency excursion.

M = 2   is a medium frequency excursion.

M = 3   is for small frequency excursion.

FUNCTION TORL- A function program which calculates the load torque at a certain value of speed. The load torque is given by

$$T_M = A\nu^2 + B\nu + C$$

where A, B and C are constants. With $A \neq 0$, $B = 0$, and $C = 0$, the load represents a centrifugal or fan type. With $A = 0$, $B = 0$, and $C \neq 0$, the load is a constant load type.

FUNCTION CURR- A function program to calculate the current at different frequencies and speeds.

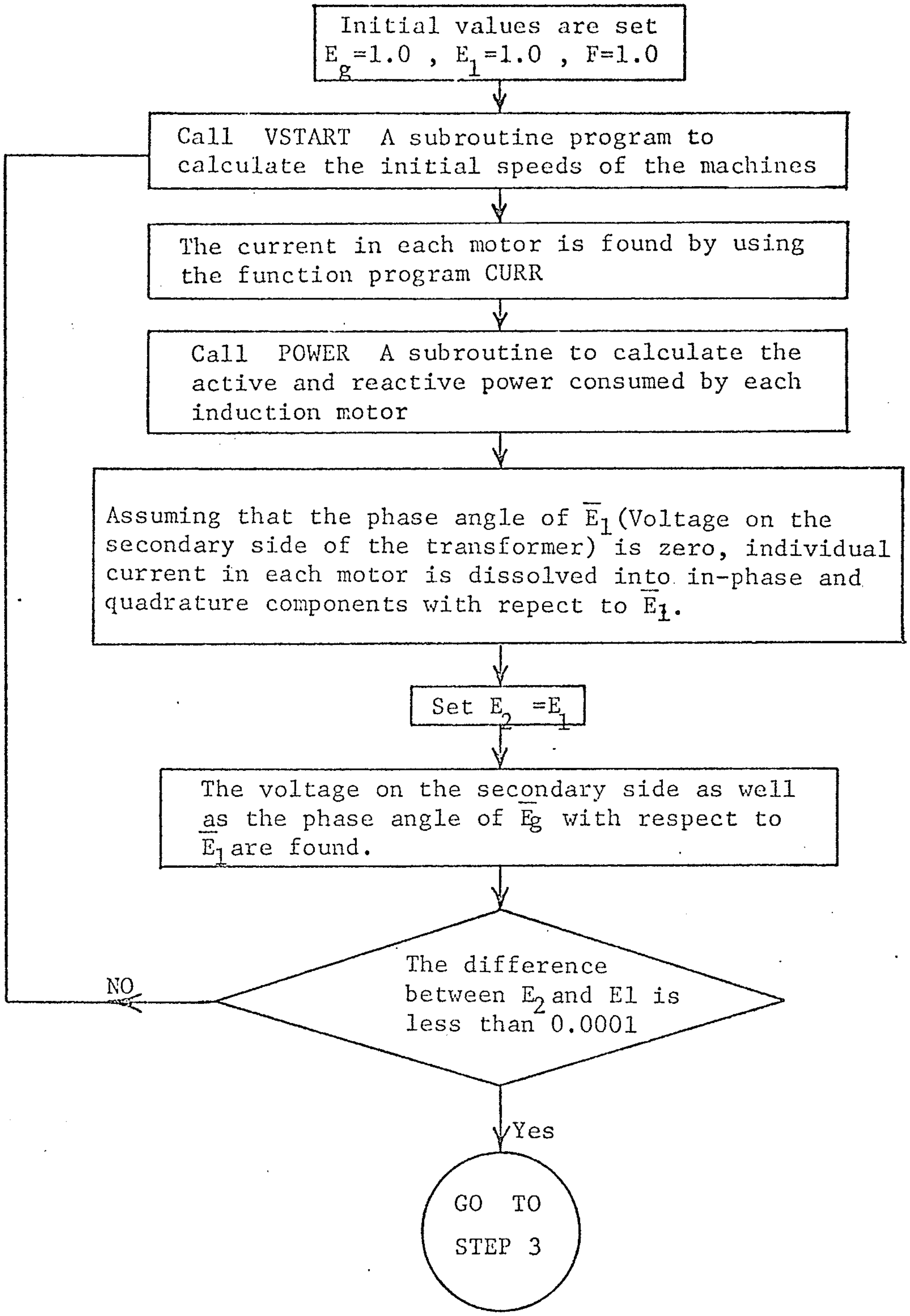


Fig. 3.3. A Flow Chart for Step Two: To Calculate The Initial Speed of the Machines and to Calculate the Voltage on the Secondary Side of the Transformer.

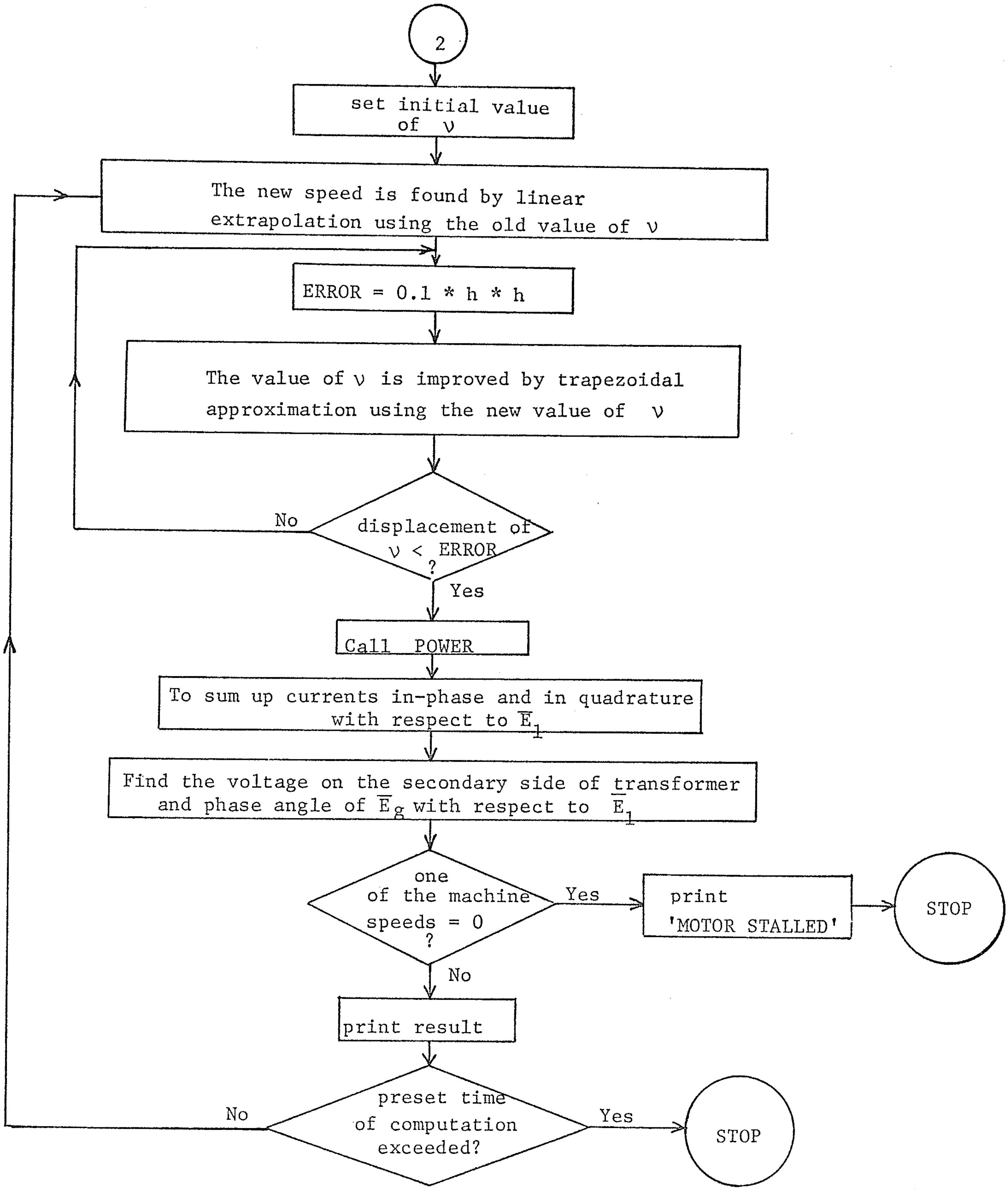


Fig.3.4. A Flow Chart For Step Three: The Predictor-Corrector Method to Find the New Speeds of the Machines.

FUNCTION TORM - A function program to calculate the motor torque at different frequencies and speeds.

FUNCTION ACCEL - A function program to calculate the acceleration of the motor.

SUBROUTINE POWER - A subroutine program to calculate the active and reactive power when the current, speed and frequency are given.

3.3. RESULT AND ANALYSIS

The following graphs are the comparisons of the motor speeds and motor currents by the methods outlined in the previous two sections. A typical machine used by Blom[17] was analyzed by the two methods. The data of the machine are:

10 H.P. 4-pole, 50 Hz, 380V, 15.6A

$R_s = 0.03057$ p.u., $R_r = 0.03596$ p.u.,

$X_s = 3.5065$ p.u., $X_r = 3.506$ p.u.,

$X_{sr} = 3.399$ p.u., $H = 0.252$ sec.

Base voltage = peak value of rated phase voltage = 310V

Base current = peak value of rated phase current = 22A

$$\text{Base impedance} = \frac{\text{base voltage}}{\text{base current}} = 14.1 \text{ ohm}$$

Base power = rated apparent power = 10.3 KW

$$\text{Base torque} = \frac{\text{base power}}{\text{base mechanical speed}} = 65.5 \text{ N-m}$$

Base frequency = 50 Hz

Base angular frequency = 314 rad/sec.

$$\text{Base time} = \frac{1}{\text{rated angular frequency}} = 3.19 \times 10^{-3} \text{ sec.}$$

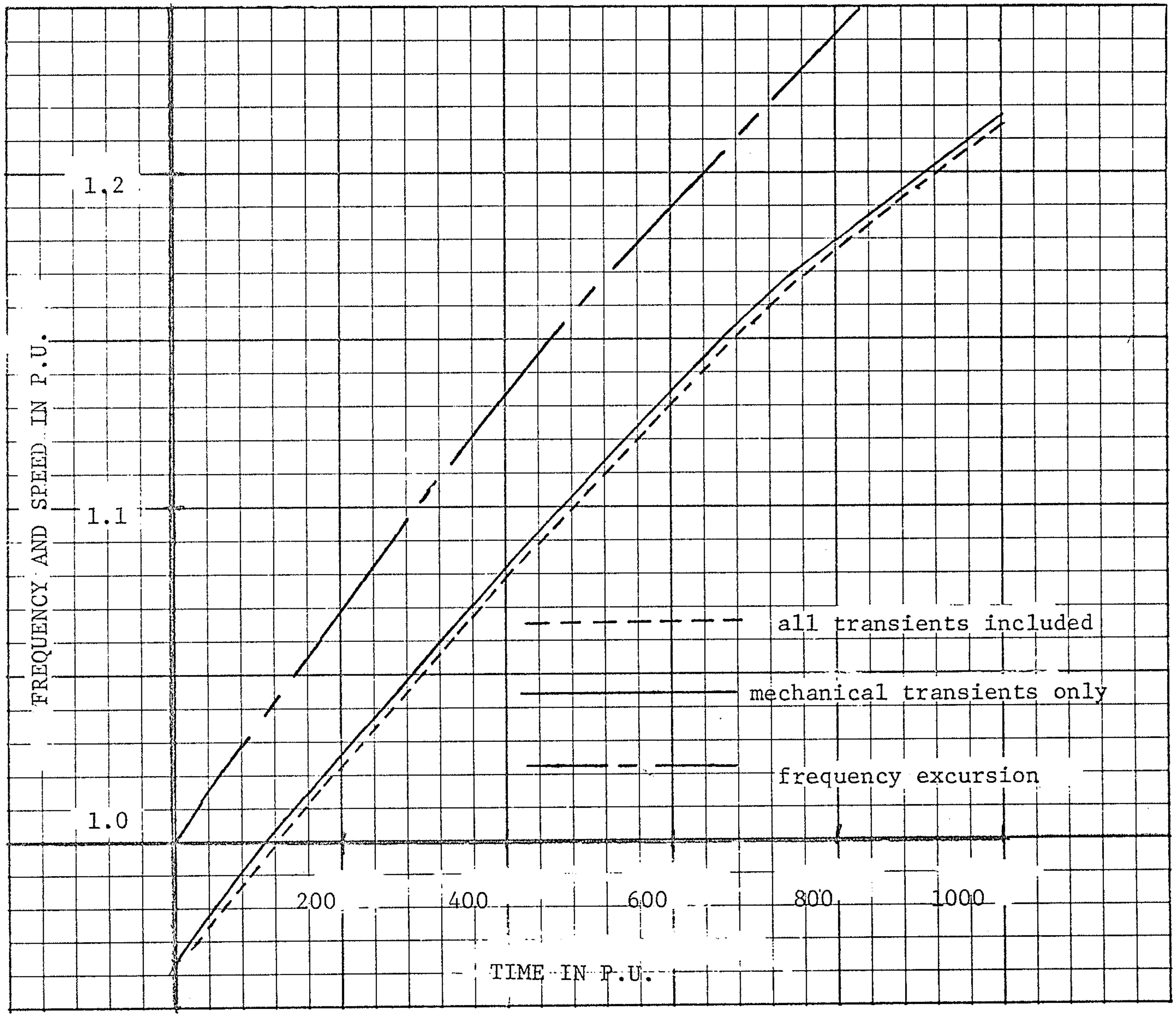


Fig.3.5. A Comparison of the Motor Speed by Different Methods

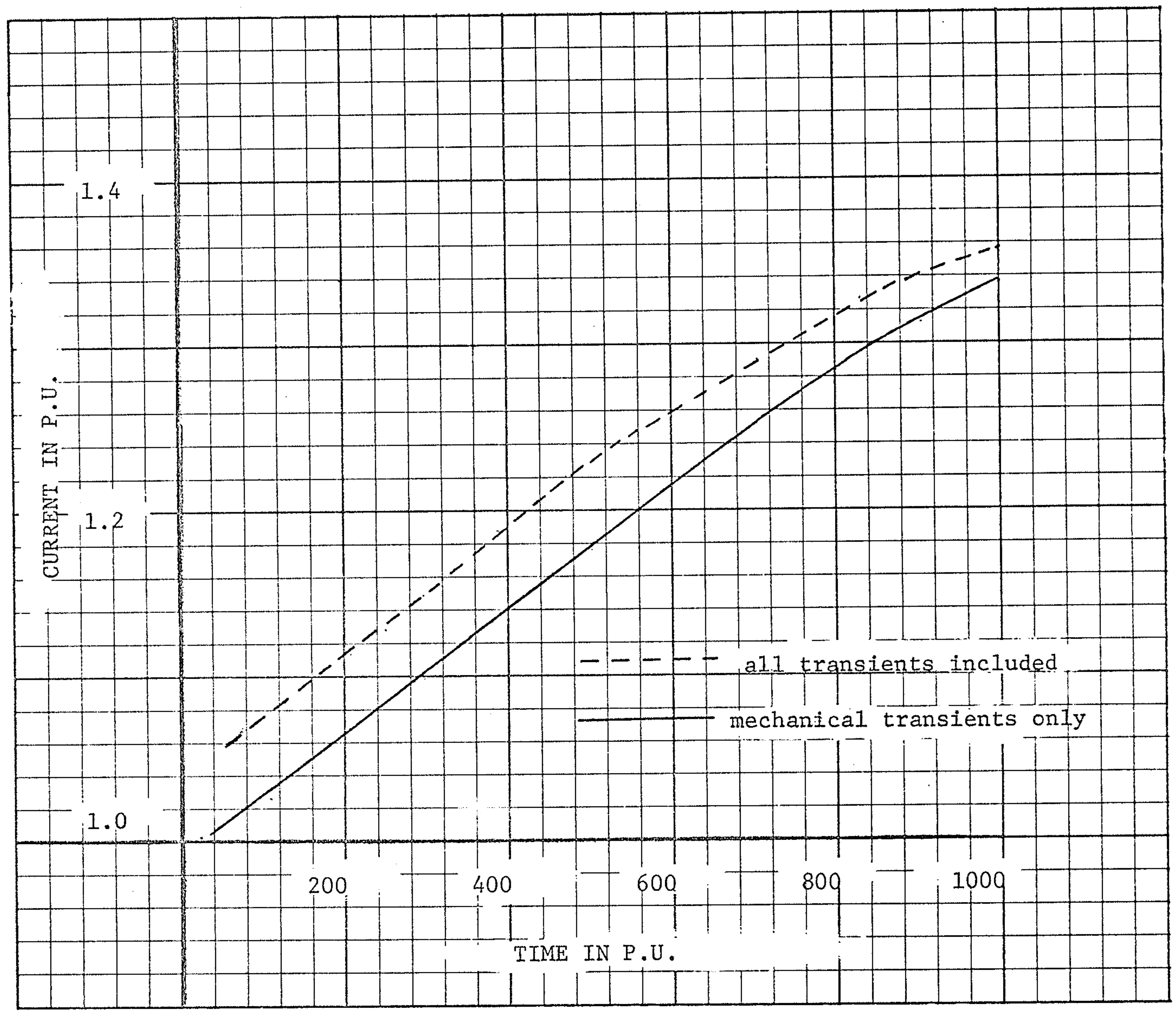


Fig.3.6. A Comparison of the Motor Current by Two Different Methods

In the program in which all transients are included, a preset error limit per step of 0.5×10^{-5} is used, while in the representation including only the mechanical transients, a step length of 0.05 second was used.

The results come striking close especially for the speeds of the motors. The currents seem to differ by a constant value all through. This could be due to the approximation made in the equivalent circuit. For the current by the transient representation, the initial current goes up to 1.85 p.u. This transient current only lasts for a cycle and is neither large nor long enough to cause any significant change in speed. It can be concluded that the dynamic representation in which only the mechanical transients are included is valid for accurate representation during large frequency excursions.

CHAPTER IV

THE SIMPLIFIED METHOD OF REPRESENTING A GROUP OF INDUCTION MOTORS

The induction motors not only contribute to the station loads as pump and fan drives, they also comprise the majority of the loads in industry. The merit is simplicity in analysis if some method exists to combine the induction motors, connected to the same node, to form one equivalent motor such that the effects of the equivalent motor on the system is the same as the group of induction motors on the system. Padiyar and Ramshaw [23] derived an induction machine equivalent network together with three differential equations and a torque equation. This representation is accurate but demands special techniques in solving the equations and requires a lot of computer time. Lewis and Marsh [9] proposed an equivalent machine by combining the parameters in a certain way, however the validity and limitations of the model are not clearly stated. Prabhashankar and Janischewsyj [24] stated a similar equivalent for synchronous machines. In this chapter, the difficulties of representing a group of induction motors are stated and an equivalent model is proposed for large frequency excursions. The accuracy of this representation for two induction motors having the same parameters is examined.

4.1 THE DIFFICULTIES OF THE PROBLEM

It is a difficult task to represent two induction motors as an equivalent one. The performance of an induction motor is extremely nonlinear and sensitive to many changes such as :

(1) frequency The parameters are functions of the frequency. When the frequency changes, the electrical torque and the current will change which in turn will cause the motor to change its speed.

(2) terminal voltage The torque is approximately proportional to the square of the terminal voltage. When the motor draws a large amount of current, the voltage drop across the transformer reactance is great and in turn the electrical torque produced by the motor is tremendously reduced.

(3) speed of the motor itself The torque and current are nonlinear functions of speed. Only when the motor is operating near its synchronous speed, the torque-speed and current-speed curves can be linearized.

(4) parameters of the motor For different types and different sizes of motors, the parameters have different values. Aktar[3] gives a comprehensive list of machine parameters.

(5) types of loads There are, in general, many types of induction motor loads; the two extreme types being:

(i) the constant load torque

and, (ii) the load torque varies as the square of the speed.

(6) total moment of inertia The total moment of inertia is the inertia of the motor and connected load. This, of course, will affect the acceleration of the motor.

In Fig.4.1. to Fig.4.6., the performance of one single induction motor connected to a variable frequency source through a transformer reactance is illustrated. The complexity of the problem can be easily realized. An equivalent model to represent a group of induction motors is no easy task.

4.2. THE PROPOSED MODEL

A model is proposed to represent a group of induction motors. The procedures in combining the motors are as follows:

(1) The reactances and resistances of the steady state equivalent circuits of the machines are expressed in per-unit values based on the sum of the mva ratings of the motors.

(2) The load torque constants A_i, B_i, and C_i and the inertia constant H_i of the individual machine are multiplied by the ratio:

$$\frac{\text{horse-power of individual motor}}{\text{total horse-power of the group of motors}}$$

(3) The resistances or reactances of the equivalent circuit are found by combining corresponding resistances or reactances in parallel.

(4) The equivalent load torque constants and the equivalent

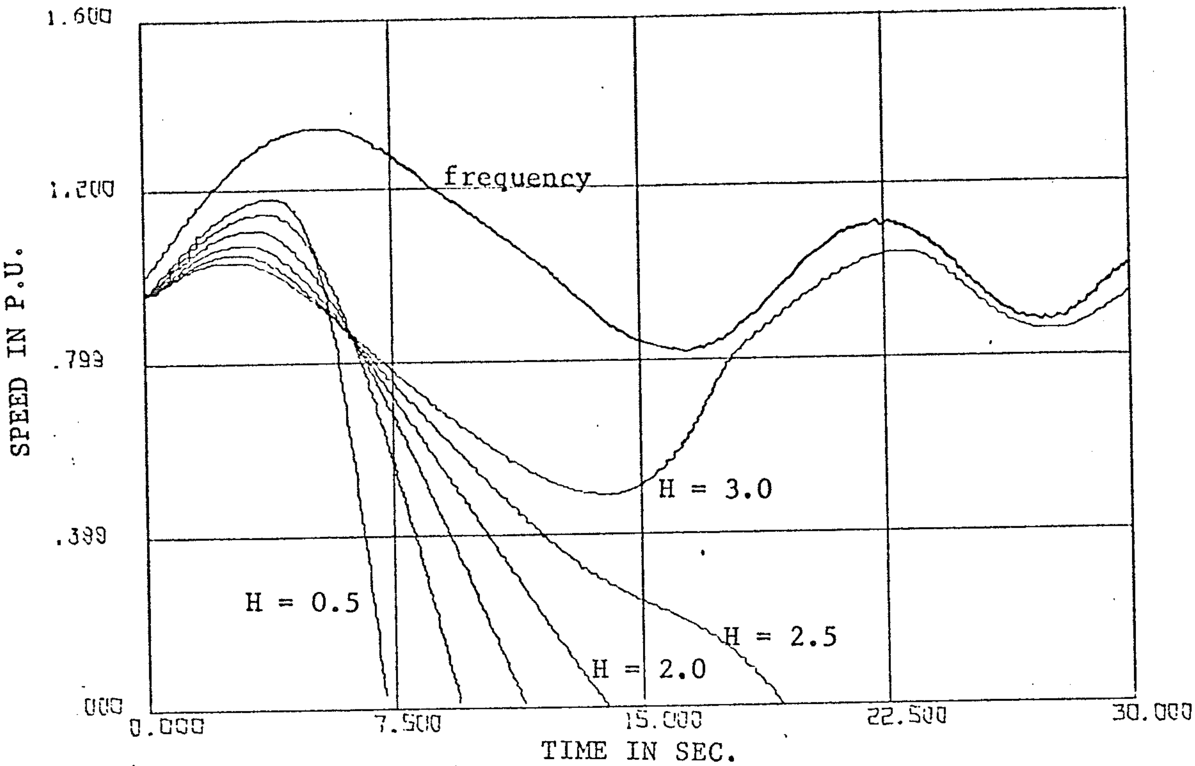


Fig.4.1. Speed of an Induction Motor with Constant Load Torque When the Frequency on the Generater Bus is Under Large Frequency Excursion.

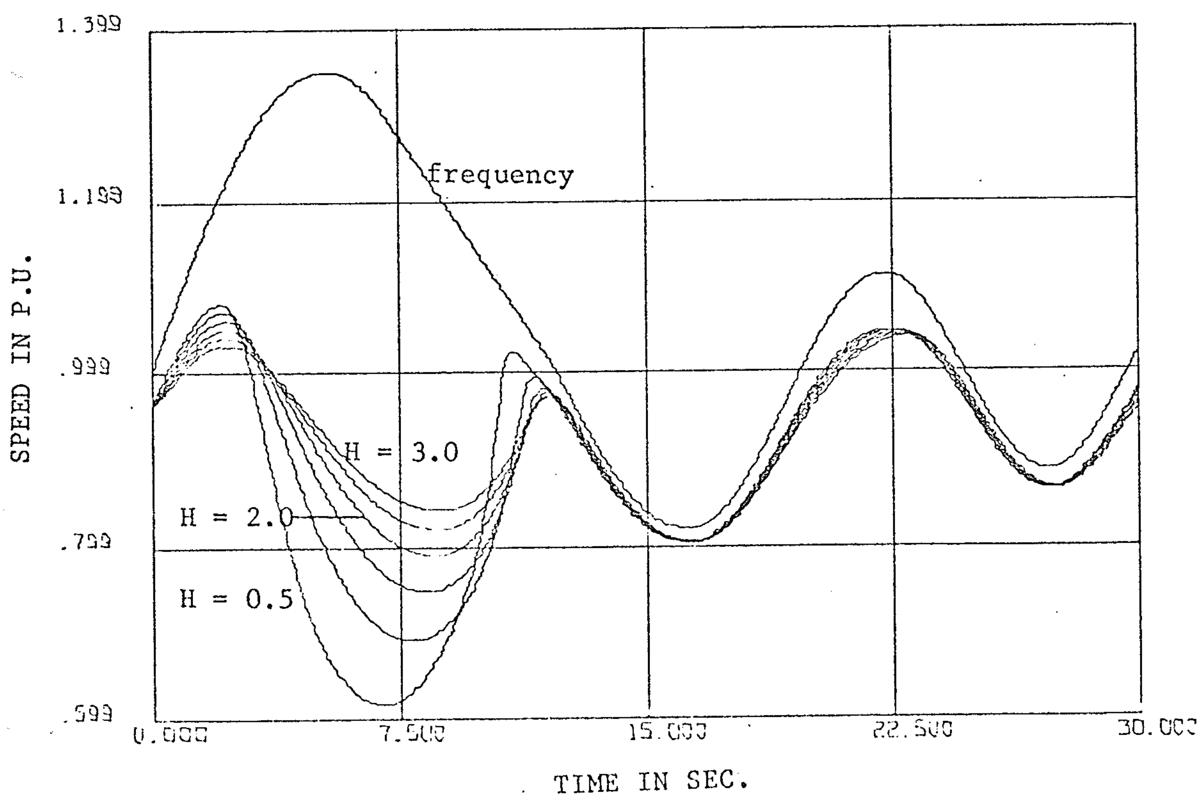


Fig.4.2. Speed of an Induction motor with Load Torque Varying as the Square of the Motor Speed When the Frequency on the Gererator Bus is Under Large Frequencv Excursion

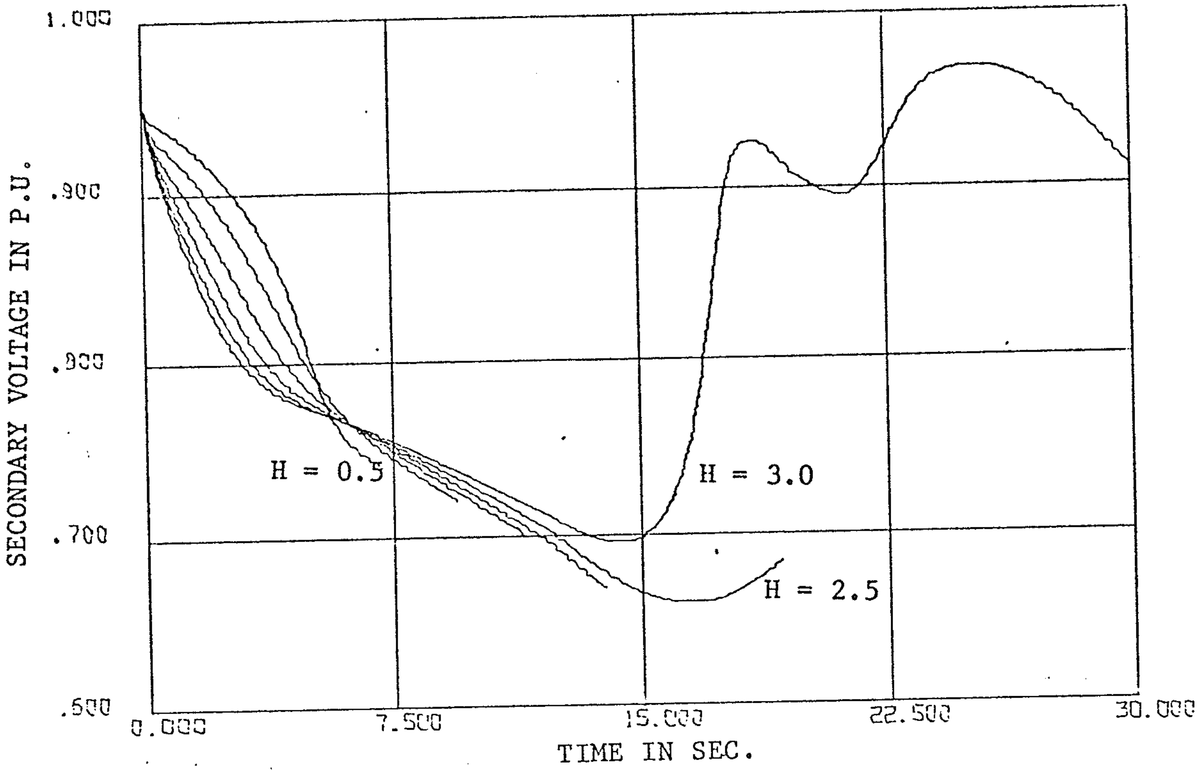


Fig. 4.3. Secondary Voltage When An Induction Motor is Connected to A Generator Bus Which is under Large Frequency Excursion. The Load torque on the Motor is Constant. $E_g = 1.0$ p.u.

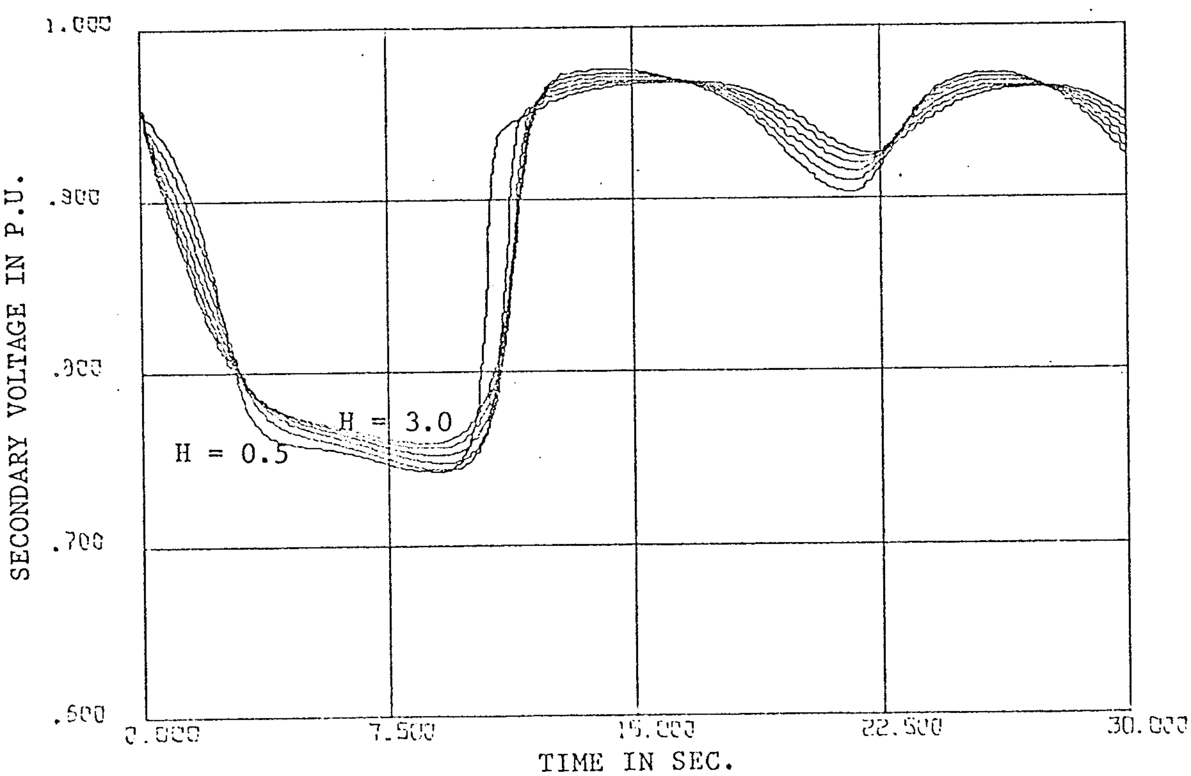


Fig. 4.4. Secondary Voltage When An Induction Motor is Connected to A Generator Bus Which is under Large Frequency Excursion. The Load torque Varies as the Square of the Motor Speed. E_g=1.0p.u.

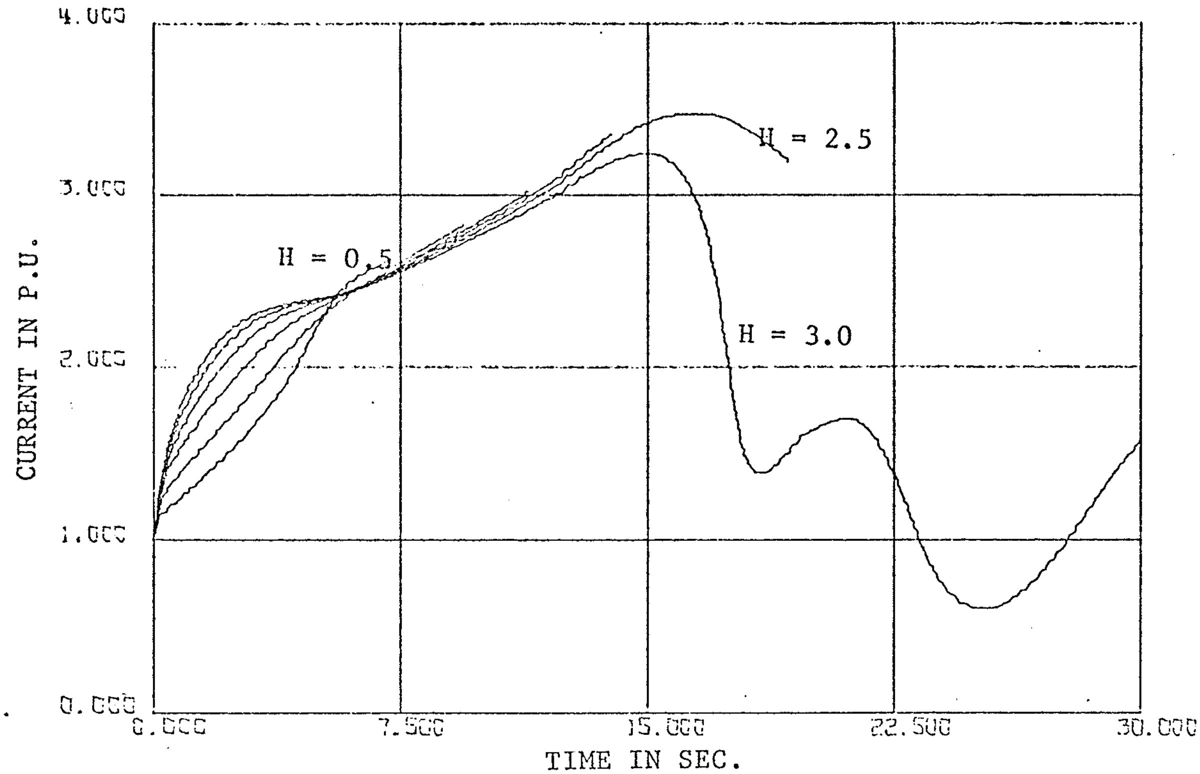


Fig. 4.5. Current of An Induction Motor When the Motor is Connected to A Generator Bus which is under Large Frequency Excursion. The Load torque on the Motor is Constant.

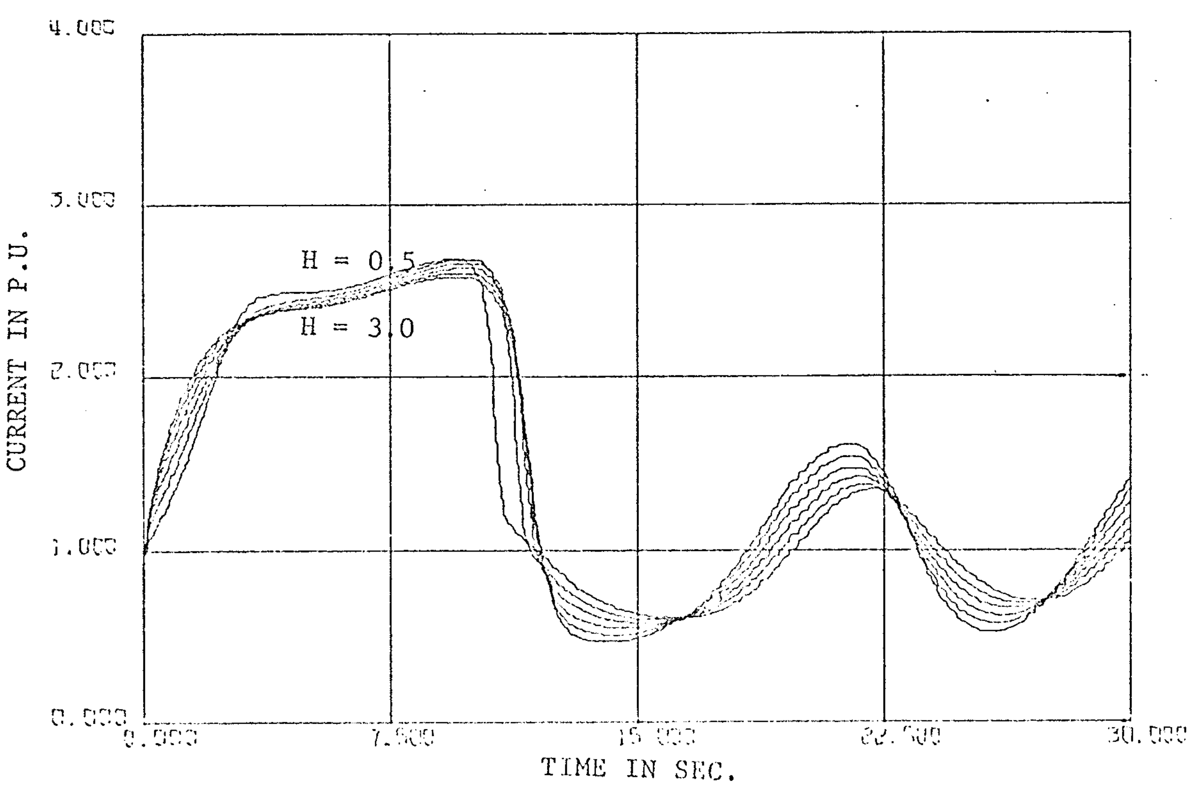
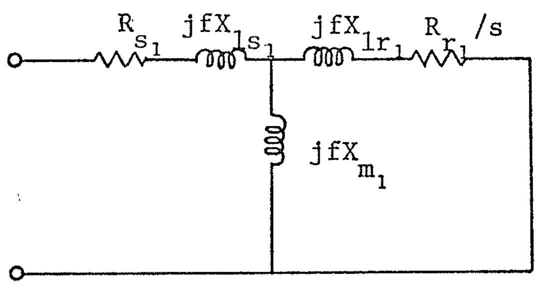
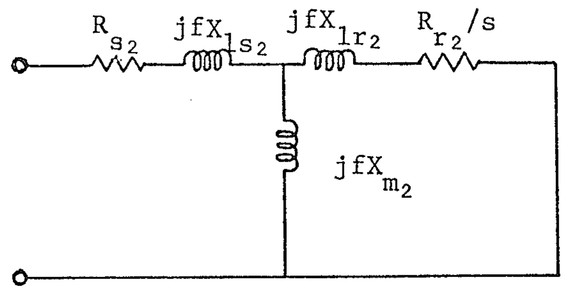


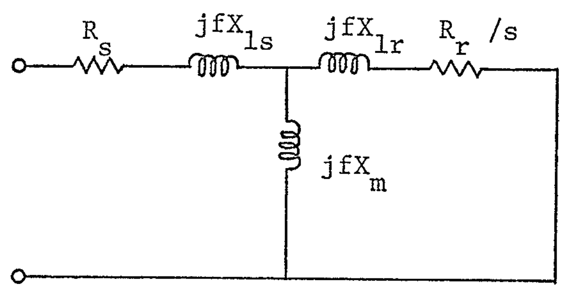
Fig. 4.6. Current of An Induction Motor When the Motor is Connected to A Generator Bus Which is under Large Frequency Excursion. The



(a) Motor 1



(b) Motor 2



$R_s = R_{s1} \,//\, R_{s2}$, $\qquad X_{1s} = X_{1s_1} \,//\, X_{1s_2}$,

$R_r = R_{r1} \,//\, R_{r2}$, $\qquad X_{1r} = X_{1r_1} \,//\, X_{1r_2}$,

and $X_m = X_{m1} \,//\, X_{m2}$

(c) The equivalent model

Fig.4.7. A Proposed Model For Two Induction Motors.

inertia constant are found by adding up corresponding new load torque constants and new inertia constants of all the motors in the group. The new constants are:

$$A = \frac{\sum_{i=1}^{n} A_i \; hp_i}{\sum_{i=1}^{n} hp_i}, \qquad B = \frac{\sum_{i=1}^{n} B_i \; hp_i}{\sum_{i=1}^{n} hp_i},$$

$$C = \frac{\sum_{i=1}^{n} C_i \; hp_i}{\sum_{i=1}^{n} hp_i}, \qquad H = \frac{\sum_{i=1}^{n} H_i \; hp_i}{\sum_{i=1}^{n} hp_i}$$

where n is the number of motors in the group.

4.3. RESULT AND ANALYSIS

4.3.1. The procedure of the analysis

Analysis has been done on two motors which have identical parameters but different total moments of inertia and drive same or different types of loads. The motors are connected a transformer of total leakage reactance of 0.1 p.u. based on the sum of the mva ratings of the motors. The frequency swing is the worst type as shown in Chapter I. The parameters of the two motors in per unit based on the sum of the mva ratings of the motors are:

	MOTOR 1	MOTOR 2
Stator resistance	0.06114	same
Stator leakage reactance	0.2150	same
Magnetising reactance	6.7980	same
Referred rotor leakage reactance	0.2140	same
Referred rotor resistance	0.07192	same

The parameters of the equivalent motor in per unit are:

Stator resistance	0.03057
Stator leakage reactance	0.1075
Magnetising reactance	3.3990
Referred rotor leakage reactance	0.1070
Referred rotor resistance	0.03596

The computation procedures were as follows:

(1) To compute the results by considering individual motors up to the time when one of the motors reaches zero speed. The results were stored in tape.

(2) To compute the results of the equivalent representation and store results in tape.

(3) To repeat steps one and two for different combinations of total moments of inertia.

(4) To compare the accuracy of the two methods for each combination of total moments of inertia at each time interval by using the tape. The maximum percentage error was found for each combination of moments of inertia.

(5) Each maximum percentage error was assigned a grid location (a,b) where a is the total moment of inertia of the first motor and b is the total moment of inertia of the second motor. A built-in cimputer program was used to plot contours joining all points which have the same percentage error. In

this way the accuracy can be easily read out from the contour plots.

There are two types of load torques considered: the constant load torque and the load torque which varies as the square of the motor speed. From here on, type 1 load torque refers to the constant load torque and type 2 load torque refers to the load torque which varies as the square of the motor speed. There are three different combinations of load torques:-

(1) motor 1 with type 1 load torque and motor 2 with type 1 load torque,

(2) motor 1 with type 1 load torque and motor 2 with type 2 load torque,

(3) motor 1 with type 2 load torque and motor 2 with type 2 load torque.

4.3.2. Result of analysis

The results are plotted in the following figures:-

Fig.4.8. Contour plot of maximum percentage error in active power.

motor 1 - type 1 load torque

motor 2 - type 1 load torque

Fig.4.9.. Contour plot of maximum percentage error in reactive power

motor 1 - type 1 load torque

motor 2 - type 1 load torque

Fig.4.10. Contour plot of maximum percentage error in active power.

motor 1 - type 1 load torque

motor 2 - type 2 load torque

Fig.4.11 Contour plot of maximum percentage error in reactive power.

motor 1 - type 1 load torque

motor 2 - type 2 load torque

Fig.4.12. Contour plot of maximum percentage error in active power.

motor 1 - type 2 load torque

motor 2 - tupe 2 load torque

Fig.4.13. Contour plot of maximum percentage error in reactive power.

motor 1 - type 2 load torque

motor 2 - type 2 load torque

4.3.3. Error analysis

Fig.4.8. The symmetry of the contours with respect to the main diagonal is due to the fact that the motors are identical except for their total moments of inertia. The percentage error in the active power may be significant; as high as

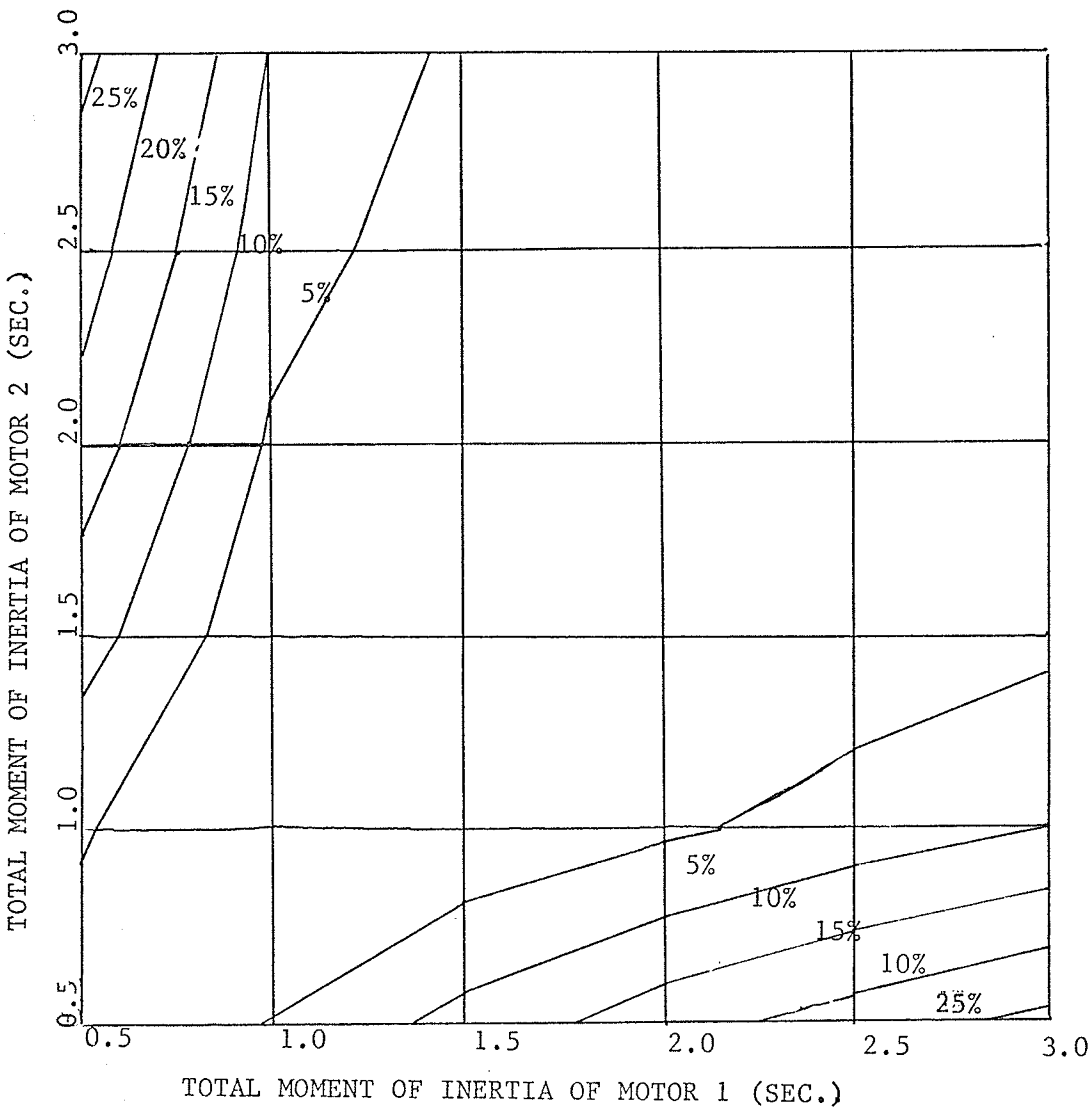


Fig. 4.8. Contour Plot of Maximum Percentage Error in Active Power.

Motor 1 - Constant Load Torque

Motor 2 - Constant Load Torque

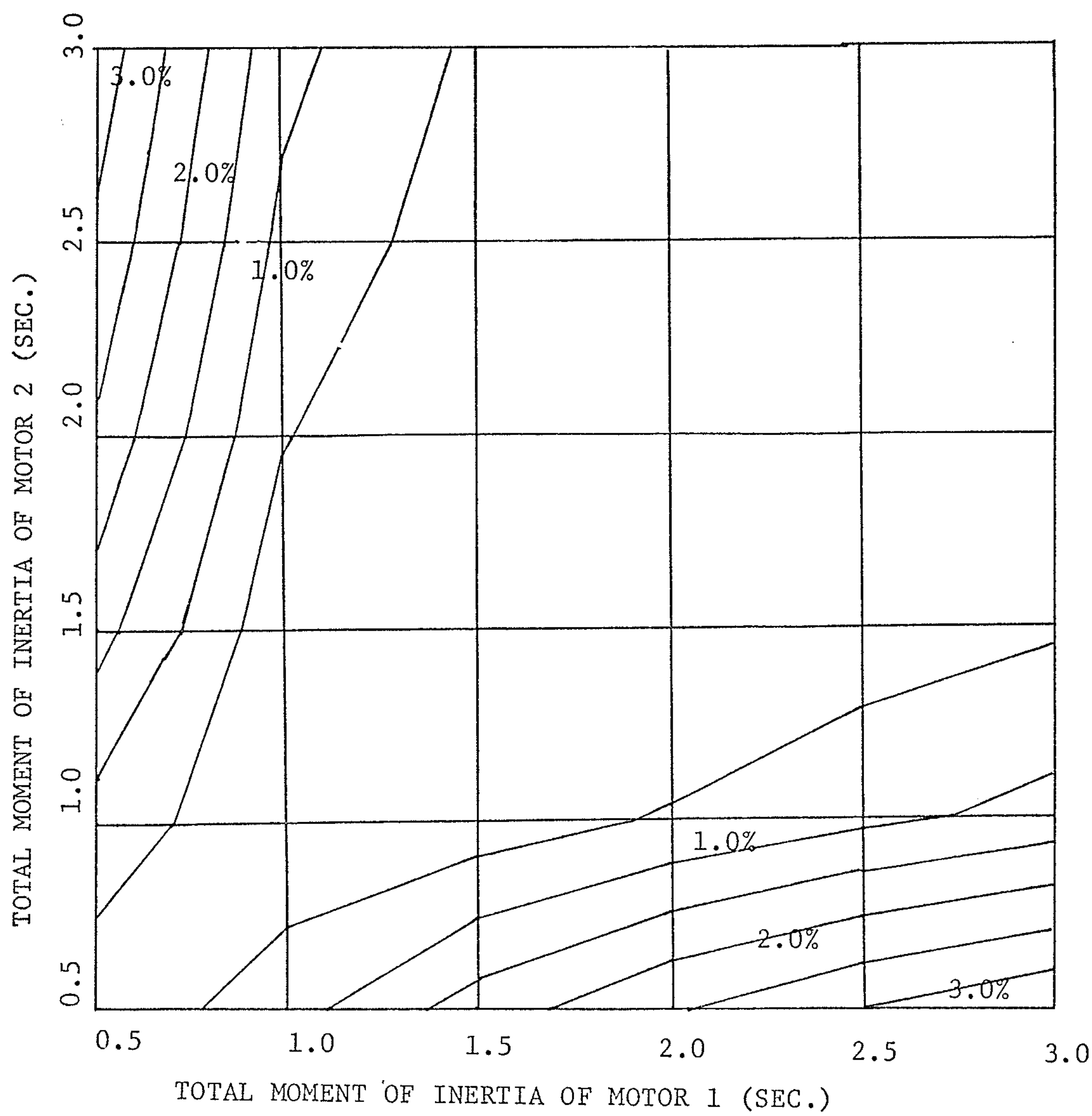


Fig. 4. 9. Contour Plot of Maximum Percentage Error in Reactive Power

Motor 1 - Constant Load Torque

Motor 2 - Constant Load Torque

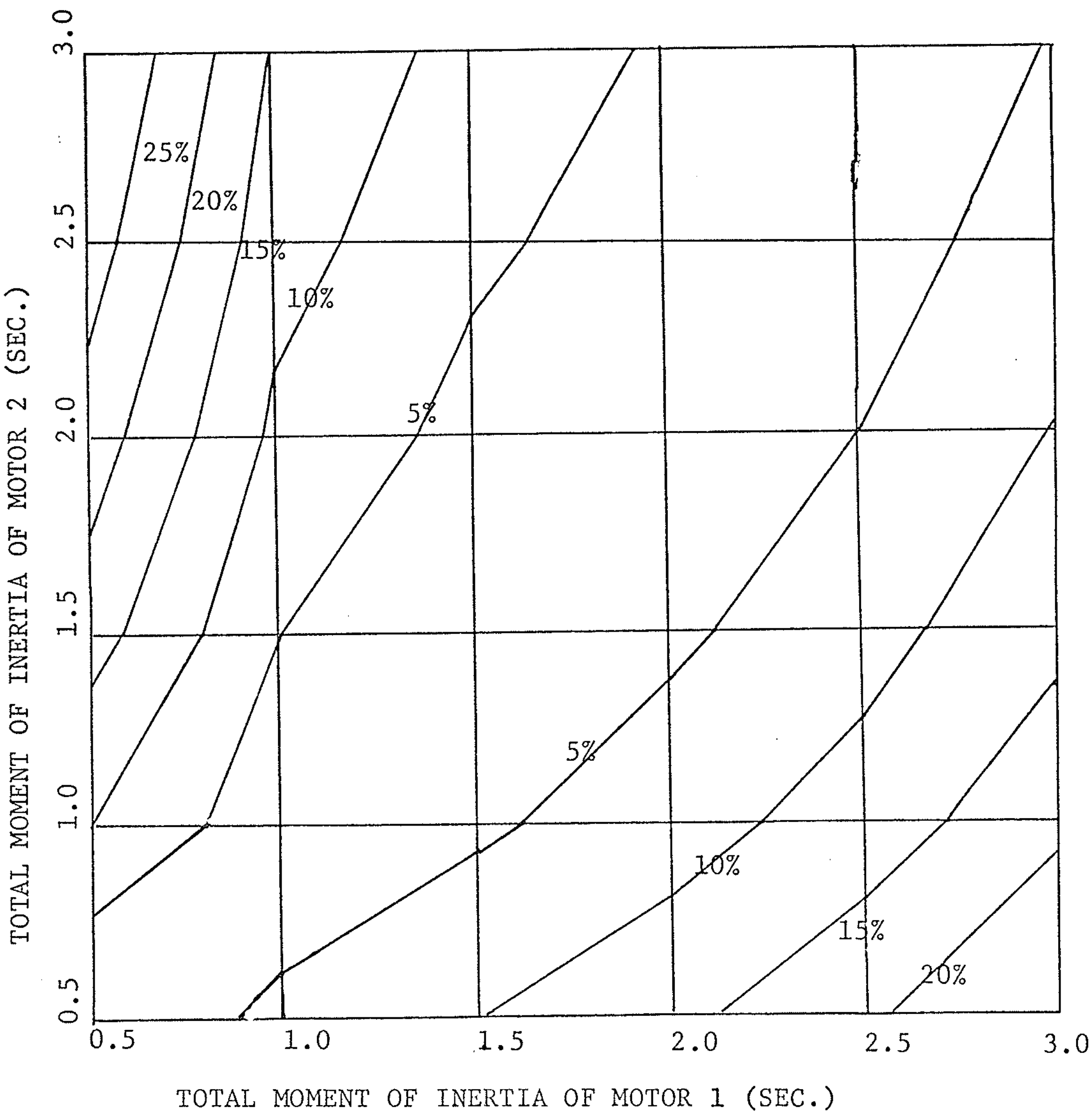


Fig. 4.10.Contour Plot of Maximum Percentage Error in Active Power

Motor 1 - Constant Load Torque

Motor 2 - Load Torque Varies as the Square of Motor Speed

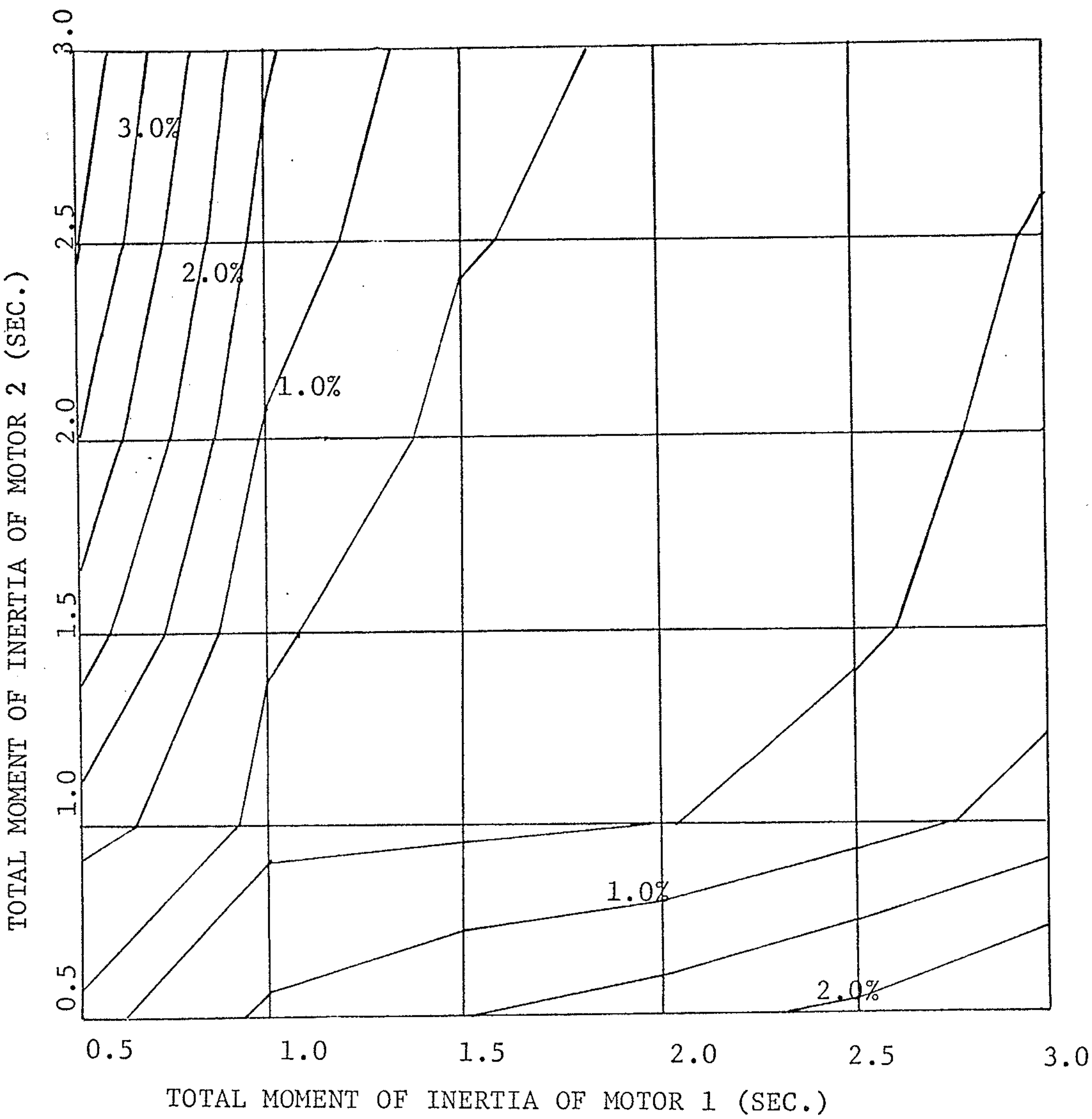


Fig. 4.11. Contour Plot of Maximum Percentage Error in Reactive Power.

Motor 1 - Constant Load Torque

Motor 2 - Load Torque Varies as the Square of Motor Speed

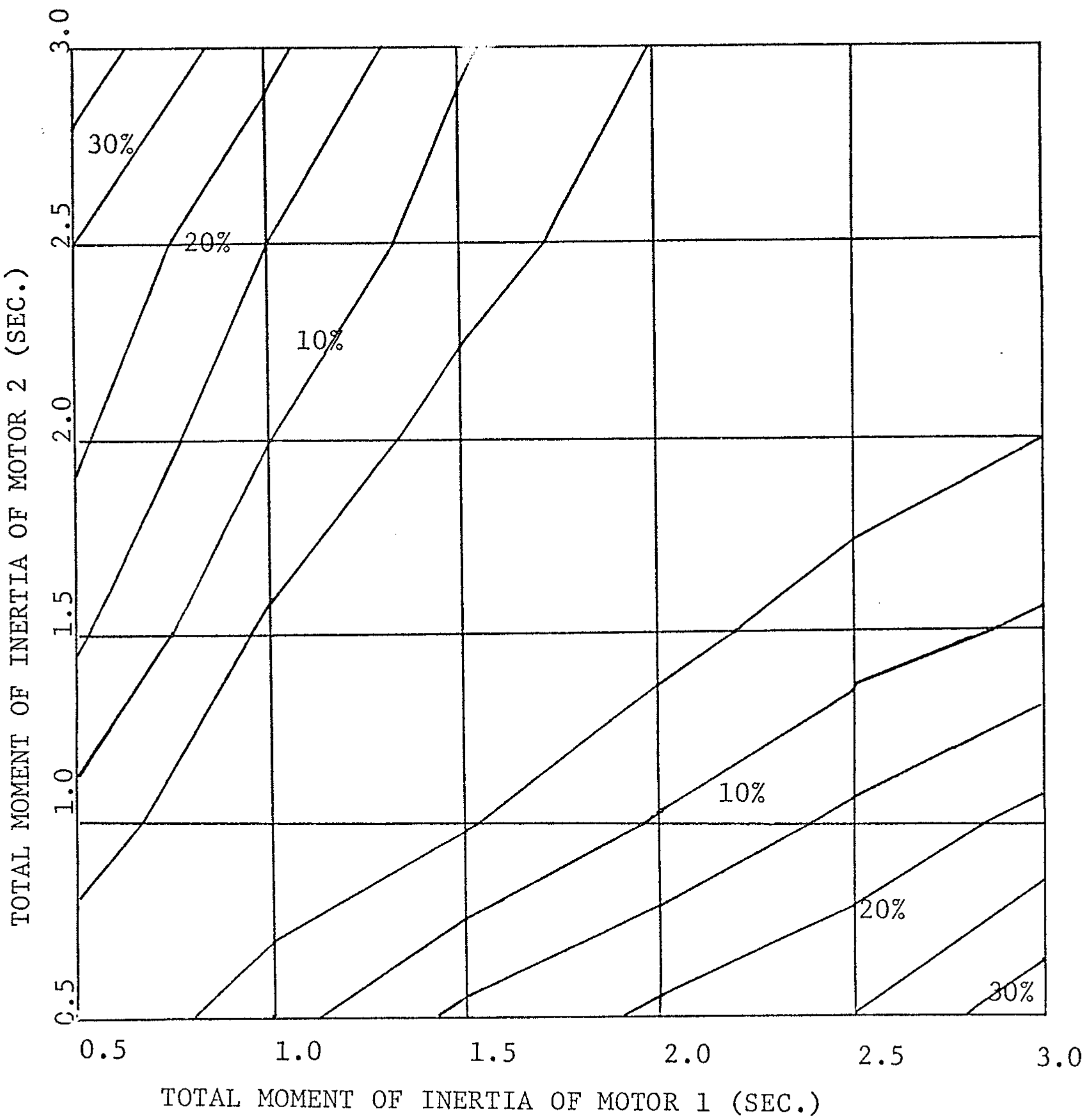


Fig. 4.12. Contour Plot of Maximum Percentage Error in Active Power

Motor 1 - Load Torque Varies as the Square of Motor Speed

Motor 2 - Load Torque Varies as the Square of Motor Speed

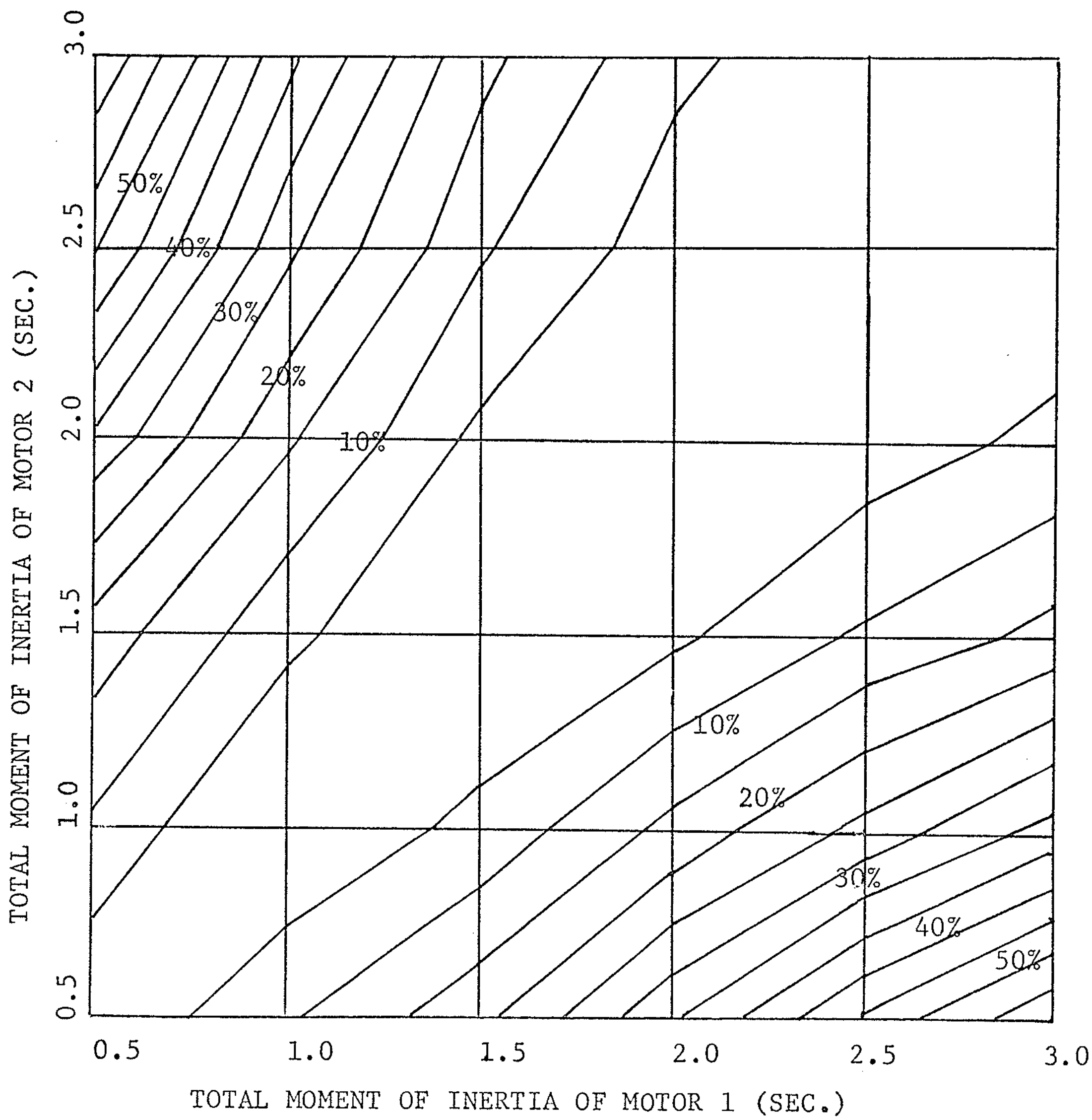


Fig. 4.13. Contour Plot of Maximum Percentage Error in Reactive Power

Motor 1 - Load Torque Varies as the Square of Motor Speed

Motor 2 - Load Torque Varies as the Square of Motor Speed

25% for extreme differences in inertia loads on the two motors. The magnitude of the difference in inertia load becomes less significant for larger inertia loads. From the contour plot, it can be concluded that if the total moment of inertia of one motor does not exceed ±50% of the other motor, then the maximum percentage error in the active power will not exceed 10%.

Fig.4.9. The symmetry is again observed. However, it can be seen that the maximum percentage error in the reactive power is insignificant as compared to the maximum percentage error in the active power. Therefore the consideration of the active power will be sufficient for this particular case.

Fig.4.10. The symmetry of the contours is distorted due to the different types of load torques. There is an anticlockwise shift of the contours. However, the conclusion can still be drawn that if the total moment of inertia of each motor does not exceed ± 50% of the other motor, then the maximum percentage error in the active power will not exceed 10%.

Fig.4.11. There is no symmetry in the plot. It is interesting to note that motor 1 with a high total moment of inertia, combined with motor 2 with a low total moment of inertia will give a higher accuracy than the combination of

motor 1 with a low total moment of inertia with motor 2 with a high total moment of inertia. In the whole region, the error does not exceed 4% and is insignificant compared to the error in the active power.

Fig.4.12. The area enclosed by the 10% maximum percentage error contour is much less than that in Fig.4.8. This is becoming more obvious when the moments of inertia of both motrs are high.

Fig.4.13. The error here is large compared with all other plots. The maximum percentage error goes up to 65%. The error enclosed by the maximum percentage error contour is the smallest compared with other plots. The large error is due to the fact that neither motor will stall with this type of load torque characteristic at the completion of the frequency swing; and in that frequency swing period, one of the motors has a much higher slip than the other and the equivalent motor is unable to give accuracy prediction. However if the total moment of inertia of one motor does not exceed $\pm$ 40% (compared with 50% previously) of the other motor, then the maximum percentage error in the reactive power will not exceed 10%.

4.3.4. General conclusions from analysis

From the analysis of the above plots, it is possible to make some generalizations.

(1) The accuracy of the representation is high when the motors have the same total moment of inertia.

(2) If the total moment of inertia of one motor does not exceed $\pm 40\%$ of the other motor, then the maximum percentage error in both the active and reactive power will not exceed 10%, using the proposed model.

CHAPTER V

CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDIES

Different representations of induction motors as dynamic and static loads have been reviewed. The representation neglecting the stator transients and rotor transients but including the mechanical transients has been considered in detail. This representation was compared with the one which includes all transients. It is found that the results of the methods compared favourably with each other. Therefore the representation including only the mechanical transients was used in the analysis of induction motors during large frequency excursions.

A model is proposed to represent induction motors by an equivalent motor such the the effect of the equivalent motor on the power system is the same as the individual motors on the system. For two induction motors which have the same parameters, if the moment of inertia of one motor does not exceed $\pm$ 40% of the other motor, the maximum percentage error in both the active power and reactive power will not exceed 10%, using the proposed model. For frequency swings which are of smaller magnitudes, this model is expected to be even more accurate.

This model may be used to combine many induction motors which are connected to the same node following the procedures outlined in section 4.3. In this way, the representation of a multi-machine system is much simplified. More investigation must be carried out to study the combination of motors of different parameters and different horse-powers. The model proposed in this thesis may be the basis for further investigations.

BIBLIOGRAPHY

1. Shearer, D.A., 'Frequency Control of Kettle Generating Station For the Nelson River HVDC Project', National Research Coucil, Associate Committee on Automatic Control, Western Subcommittee Meeting, Winnipeg, Oct.1970.

2. Radley, R.G., and Chin, L.S., 'Nelson River Conversion Station Auxilliaries and Power Supplies', Manitoba Power Conference EHV-DC, June 1971.

3. Akhtar, M.Y., 'Frequency-dependent Dynamic Representation Of In Induction-Motor Loads', Proc. IEE, Vol. 115, No. 6, June 1968, pp.802-812.

4. Lawrenson, P.J. and Stephenson, J.M., 'Note On Induction-Machine Performance With A Variable-Frequency Supply', Proc. IEE, Vol. 113, No. 10, Oct.1966, pp.1617-1623.

5. Berg G.J., and DeSarkar, A.K., 'Speed Change of Induction Motors With Variable Frequency Supply', IEEE Trans., Vol. PAS-90, No. 2, 1971, pp.500-508.

6. Jordan, H.E., 'Analysis of Induction Machines in Dynamic Systems', IEEE Trans., Vol. PAS-84, No. 11, Nov.1965, pp.1080-1085.

7. Park, R.H., 'Two Reaction Theory of Synchronous Machines Part I', AIEE Trans., Vol. 48, 1929, pp.716-730.

8. Stanley, H.C., 'An Analysis of The Induction Machine', AIEE Trans., Vol. 57, 1938, pp.751-757.

9. Lewis, D.G. and Marsh, W.D., 'Transfer Of Steam-Electric Generating-

Station Auxiliary Buses', AIEE Trans., Vol. 74, June 1955, pp.322-327.

10. Brereton, D.S., Lewis, D.G., and Young, C.C., 'Representation of Induction-Motor Loads During Power-System Stability Studies', AIEE Trans., Vol. 76, Aug.1957, pp.451-461.

11. Hore, R.A., 'Advanced Studies in Electrical Power System Design', Chapman and Hall,1966.

12. Huening, W.C. Jr., 'Time Variation of Industrial System Short-Circuit Currents and Induction Motor Contributuons', AIEE Trans., Vol. 74, May 1955, pp.90-101.

13. Maginniss, F.J. and Schultz, N.R., 'Transient Performance of Induction Motors', AIEE Trans., Vol. 63, Sept.1944, pp.641-646.

14. Krause, P.C. and Thomas, C.H., 'Simulation of Symmetrical Induction Machinery', IEEE Trans., Vol. PAS-84, No. 11, Nov.1965, pp.1038-1053.

15. Krause, P.C., 'Simulation of Unsymmetrical 2-phase Induction Machines' IEEE Trans., Vol. PAS-84, No. 11, Nov.1965, pp.1025-1037.

16. Huges, F.M. and Aldred, A.S., 'Transient Characteristics and Simulation Of Induction Motors', Proc. IEE, Vol. 111, No. 12, Dec.1964, pp.2041-1964.

17. Blom, J.F., 'The Transient Behaviour of a Three Phase Induction Motor Studied With An Analog Computer', INGINIEUR, Vol. 71, 1959, pp. E61-E70.

18. Black, K.G. and Noorda, R.J., 'Analog Computer Study of Wind-Tunnel Drive', AIEE Trans., Vol. 77, Jan.1958, pp745-750.

19. Jordan, H.E., 'Digital Computer Analysis of Induction Machines in Dynamic Systems', IEEE Trans., Vol. PAS-86, No. 6, June 1967, pp.722-728.

20. Smith, I.R., and Sriharan, S., 'Transient Performance of the Induction Motor', Proc. IEE. Vol. 113, No. 7, July 1966.

21. Lance, G.N.,'Numerical Methods for Highspeed Computers', Iliffe, 1960.

22. Cadzow, J.C. and Martens, H.R., 'Discrete-Time and Computer Control Systems', Prentice-Hall E.E.Series 1970.

23. Padiyar, K.R. and Ramshaw, R.S., 'Dynamic Analysis of Multi-Machine Power Systems', IEEE Summer Meeting and International Symposium on High Power Testing, Portland, Ore., July 1971.

24. Prabhashanker K. and Janischewskyj, 'Digital Simulation of Multi-Machine Power Systems For Stability Studies', IEEE Trans., Vol. PAS-87, Jan.1968, pp.78-80.

APPENDIX A

THE RUNGE-KUTTA METHODS

Let the first order differential equation be

$$\frac{dy}{dx} = f\,(x,y) \qquad \ldots(1)$$

y(x+h) can be expressed in Taylor series expansion:

$$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2!}\,y''(x) + \frac{h^3}{3!}\,y'''(x) + \ldots \qquad \ldots(2)$$

where the following are defined:

$$\begin{aligned} y' &= f \\ y'' &= f_x + f_y f \\ y''' &= f_{xx} + 2f_{xy}f + f_{yy}f^2 + f_y(f_x + f_y f) \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \qquad \ldots(3)$$

$$f_x = \frac{\partial f}{\partial x}$$

and

$$f_y = \frac{\partial f}{\partial y}$$

Δy is defined as $y(x+h) - y(x)$

$$\Delta y = hy'(x) + \frac{h^2}{2!}\,y''(x) + \frac{h^3}{3!}\,y'''(x) + \ldots$$

i.e., $$\Delta y = hf + \frac{h^2}{2}(f_x + f_y f) + \frac{h^3}{6}[f_{xx} + 2f_{xy}f + f_{yy}f^2 + f_y(f_x + f_y f)]$$
$$+ \frac{h^4}{24}[f_{xxx} + 3\, f_{xxy}f + 3\, f_{xyy}f^2 + f_{yyy}f^3$$
$$+ f_y(f_{xx} + 2f_{xy}f + f_{yy}f^2) + 3(f_x + f_y f)\ (f_{xy} + f_{yy}f)$$
$$+ f_{yy}^2\ (f_x + f_y f)] + \ldots \qquad \ldots(4)$$

assume that Δy can be expressed as

$$\Delta y = ak_1 + bk_2 + ck_3 + dk_4 \qquad \ldots(5)$$

where

$$\begin{aligned} k_1 &= h[f(x,y)] \\ k_2 &= h[f(x + mh,\ y + mk_1)] \\ k_3 &= h[f(x + nh,\ y + nk_2)] \\ k_4 &= h[f(x + ph,\ y + pk_3)] \end{aligned} \qquad \ldots(6)$$

each of these k_i can also be expanded in Taylor series:

$$\begin{aligned} f(x+\alpha,y+\beta) &= f(x,y) + \alpha f_x(x,y) + \beta f_y(x,y) \\ &\quad + \frac{1}{2!}[\alpha^2 f_{xx}(x,y) + 2\alpha\beta f_{xy}(x,y) + \beta^2 f_{yy}(x,y)] \\ &= f + \alpha f_x + \beta f_y + \frac{1}{2!}\ [\alpha^2 f_{xx} + 2\alpha\beta f_{xy} + \beta^2 f_{yy}] + \ldots \end{aligned} \qquad \ldots(7)$$

It is the primary purpose of the Runge-Kutta methods to establish the interrelationship between the constants a,b,c,d,m,n,and p such that equations (6) and (7) correspond to the degree of accuracy desired. Here the solution is not unique and the many possible solutions give rise to many methods.

APPENDIX B

THE EULER-CAUCHY PREDICTOR-CORRECTOR METHOD

For the first order differential equation:

$$\frac{dy}{dx} = f(x,y) \qquad \ldots(1)$$

the solution may be expressed mathematically as:

$$y(x) = y(0) + \int_0^x f(x',y)\,dx' \qquad \ldots(2)$$

where y(0) is the initial condition. The difficulty of solution lies in the fact that the integrand contains the unknown y(x). Equation (2) can be written as:

$$y_{i+1} = y_i + \int_{x_i}^{x_{i+1}} f(x',y)\,dx' \qquad \ldots(3)$$

The numerical method is to start with an initial condition y_o, compute y_1 then y_2 and so on until a sufficient range has been covered.

Assume that f(x,y) is constant over the interval and that h, the interval between x_i and x_{i+1}, is sufficiently small,(3) may be written as:

$$y_{i+1} = y_i + h\,f(x_i,y_i) \qquad \ldots(4)$$

This technique of predicting the value y_{i+1} is known as Euler's method and it suffers from two principal disadvantages:

(1) The prediction of y_{i+1} is not accurate as it is accomplished by simple linear extrapolation; and

(2) The integration scheme is abysmal.

Since y_{i+1} is found approximately, let it be called y^*_{i+1}. Then a better approximation is found by invoking the trapezoidal rule in the integration of (3).

$$y_{i+1} = y_i + \frac{h}{2}(f(x_i,y_i) + f(x_{i+1},y^*_{i+1})) \qquad \ldots(5)$$

This is known as the Euler-Cauchy method. In this way, by using equation (5) repeatedly the value of y^*_{i+1} may be brought within a certain range of the true value of y_{i+1}.

APPENDIX C

COMPUTER PROGRAM FOR THE ANALYSIS OF AN INDUCTION MOTOR INCLUDING ALL TRANSIENTS

```
      LOAD FLOW STUDY OF INDUCTICN MOTORS
      A,B,C ARE THE COEFFICIENTS OF THE EQUATION TORL=A*V*X+B*V+C
      RS IS THE STATOR RESISTANCE
      XS IS THE STATOR LEAKAGE REACTANCE AT A PARTICULAR FREQUENCY.
         AND XSO IS THAT AT THE BASE FREQUENCY
      XM  IS THE MAGNETISING REACTANCE AT A PARTICULAR FREQUENCY
         AND XMO IS THAT AT THE BASE FREQUENCY
      XR IS THE ROTOR LEAKAGE REACTANCE REFERRED TO THE STATOR SIDE.
         XR VARIES WITH FREQUENCY AS WELL AS THE SLIP.
         XR=XRO*(1.0-DELXR*S)*F WHERE XRO IS XR AT BASE FREQUENCY AND
         AT SYNCHRONOUS SPEED
      RR IS THE ROTOR RESISTANCE FEFERRED TO THE STATOR SIDE. THE QUANTITY ALSC
         CHANGES WITH SLIP.  RR=RRO*(1.0+DELRR*S)
         WHERE RRO IS RR AT SYNCHRONOUS SPEED
      PMI IS THE POLAR MOMENT OF INERTIA OF THE ROTOR
      TT IS THE TOTAL TIME IN FREQUENCY EXCURSION
      M REFERS TO THE TYPE OF FREQUENCY EXCURSION
      XLINE=TRANSMISSION LINE REACTANCE AND/OR TRANSFORMER LEAKAGE REACTANCE
      RLINE=TRANSMISSION LINE RESISTANCE AND/OR TRANSFORMER RESISTANCE
      NM=NO. OF INDUCTION MOTORS
*****************************************************************************

       STEP 1:     INPUT AND OUTPUT OF PARAMETER DATA

*****************************************************************************
      DIMENSIONVE(20)
      DIMENSICN A(10),B(10),C(10),RS(10),XS(10),XSO(10),XMO(10),
     1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10),V(10),CURRN(10),
     2VOLD(10),SLIP(10),       RR(10),XM(10),XR(10)
      COMMON A,B,C,RS,XSC,XMO,XRC,RRO,PMI,DELRR,DELXR,NM,M
      CCMPLEX Z,CMPLX
  210 READ(5,12,END=201)VE
   12 FORMAT(20A4)
      WRITE(6,13)
   13 FORMAT('1',14X,'A',10X,'B',8X,'C',9X,'RS',7X,'XSO',7X,'XMO',
     17X,'XRO',7X,'RRO',7X,'PMI',6X,'DELRR',6X,'DELXR')
      READ(5,11,END=201)NM,XLINE,RLINE,TT,M
   11 FORMAT(I10,3F10.3,I10)
      READ(5,VE)(A(I),I=1,NM),(B(I),I=1,NM),(C(I),I=1,NM),
     1(RS(I),I=1,NM),(XSO(I),I=1,NM),(XMO(I),I=1,NM),(XRO(I),I=1,NM),
     2(RRO(I),I=1,NM),(PMI(I),I=1,NM),(DELRR(I),I=1,NM),
     3(DELXR(I),I=1,NM)
      WRITE(6,14) (I,A(I),B(I),C(I),RS(I),XSO(I),XMO(I),XRO(I),RRO(I),
     1PMI(I),DELRR(I),DELXR(I) ,I=1,NM)
   14 FORMAT(1X,'MOTOR',I2,2X,11F10.3)
      WRITE(1,18)(PMI(I),I=1,NM)
   18 FORMAT('MOMENT OF INERTIA',10F7.4)
*****************************************************************************

       STEP 2:     TO CALCULATE THE INITIAL SPEEDS OF THE MACHINES AND
         TO CALCULATE THE VOLTAGE ON THE SECONDARY SIDE OF THE
         TRANSFORMER

*****************************************************************************
      ES=1.0
      E=1.0
      F=1.0
      CREAL=0.0
      CIMAG=0.0
```

```
      Q1=0.0
      DO 60 I=1,1000
      DO 70 NB=1,NM
      CALL VSTART(V(NB),E,F,NB)
      CURRN(NB)=CURR(F,E,V(NB),NB)
      THIS CALCULATES THE TOTAL CURRENT
      CALL POWER(PP,QQ,NB,CURRN(NB),F,E,V(NB))
      PHI=ATAN(QQ/PP)
      CREAL=CREAL+CURRN(NB)*COS(PHI)
      CIMAG=CIMAG+CURRN(NB)*SIN(PHI)
      P1=P1+PP
      Q1=Q1+QQ
   70 CONTINUE
      CURRNT=SQRT(CREAL**2.0+CIMAG**2.0)
      E1=E
      THETA=ARSIN((CREAL*XLINE-CIMAG*RLINE)/ES)
      E=ES*COS(THETA)-CREAL*RLINE-CIMAG*XLINE
      IF(ABS(E-E1).LT.0.0001) GO TO 80
   60 CONTINUE
   80 ICOUNT=1
      WRITE(6,15)(V(I),I=1,NM)
   15 FORMAT('0','THE INITIAL SPEEDS OF THE MACHINES ARE',10F8.3)
*************************************************************************************

       STEP 3:      THE PREDICTOR-CORRECTOR METHOD TO FIND THE NEW
         SPEEDS OF THE MACHINES

*************************************************************************************
      H=0.05
      MT=TT/H
      DO 100 II=1,NM
      VOLD(II)=V(II)
  100 CONTINUE
      DO 20 J=1,MT
      T1=(J-1)*H
      T2=J*H
      F=FREQ(T2)
      P1=0.0
      Q1=0.0
      CREAL=0.0
      CIMAG=0.0
      DO 90 NC=1,NM
      ACC1=ACCEL(F,E,V(NC),NC)
      V(NC)=VOLD(NC)+H*ACC1
      DO 10 K=1,1000
      VTEMP=V(NC)
      ACC2=ACCEL(F,E,V(NC),NC)
      V(NC)=VOLD(NC)+0.5*H*(ACC1+ACC2)
      ERROR=0.1*H*H
      IF(ABS(VTEMP-V(NC)).LT.ERROR) GO TO 30
   10 CONTINUE
   30 CURRN(NC)=CURR(F,E,V(NC),NC)
      CALL POWER(PP,QQ,NC,CURRN(NC),F,E,V(NC))
      P1=P1+PP
      Q1=Q1+QQ
      PHI=ATAN(QQ/PP)
      CREAL=CREAL+CURRN(NC)*COS(PHI)
      CIMAG=CIMAG+CURRN(NC)*SIN(PHI)
      VOLD(NC)=V(NC)
```

```
 90 CONTINUE
    CURRNT=SQRT(CREAL**2.0+CIMAG**2.0)
    THETA=ARSIN((CREAL*XLINE-CIMAG*RLINE)/ES)
    E=ES*COS(THETA)-CREAL*RLINE-CIMAG*XLINE
    TIME=T2
    DO 91 I=1,NM
    IF(V(I).GT.0.0) GO TO 91
    WRITE(6,92)I
    GO TO 210
 91 CONTINUE
 92 FORMAT(10X,'MOTOR',I2,' STALLED')
    WRITE(1,17)TIME,F,E,P1,Q1,V(1),CURRN(1)
 17 FORMAT(8F10.5)
    WRITE(6,16)TIME,F,E,THETA,P1,Q1,V(1),CURRN(1)
 16 FORMAT(10X,10F10.4)
 20 CONTINUE
    GO TO 210
201 STOP
    END
C*****************************************************************************
C
C            A SUBROUTINE TO CALCULATE THE STEADY STATE OPERATING
C        SPEEDS OF INDUCTION MOTORS
C
C*****************************************************************************

    SUBROUTINEVSTART(VO,E,F,N)
    COMMON A,B,C,RS,XSO,XMO,XRO,RRO,PMI,DELRR,DELXR,NM,M
    DIMENSIONA(10),B(10),C(10),RS(10),XSO(10),XMO(10),
   1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10)
    DIMENSIONV(201)
    X=0.90
    DO 10 K=1,200
    V(K)=X
    TM=TORM(F,E,V(K),N)
    TL=TORL(V(K),N)
    IF(TM.LT.TL) GO TO 11
    X=X+0.0005
 10 CONTINUE
 11 VO=V(K-1)
    RETURN
    END
C*****************************************************************************
C
C            THIS FUNCTION IS GIVEN DEPENDING ON THE FREQUENCY
C        EXCURSION FUNCTION FREQ(T)
C
C*****************************************************************************

    FUNCTION FREQ(T)
    COMMON A,B,C,RS,XSO,XMO,XRO,RRO,PMI,DELRR,DELXR,NM,M
    DIMENSIONA(10),B(10),C(10),RS(10),XSO(10),XMO(10),
   1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10)
    PI=3.14159
    IF(M.EQ.2) GO TO 66
    IF(M.EQ.3) GO TO 77
    IF (T.GT.7.0) GO TO 50
    F     =1.+0.35*SIN(PI/10.5*T)
    GO TO 20
 50 IF (T.GT.12.3) GO TO 51
```

```
      GO TO 20
   51 IF (T.GT.16.2) GO TO 52
      F     =1.-.18*SIN(PI*(T-12.3)/7.5)
      GO TO 20
   52 IF(T.GT.19.7)GOTO 53
      F     =1.-.18*COS(PI*(T-16.2)/7.0)
      GO TO 20
   53 F     =1.+.113*SIN(PI*(T-19.7)/5.)
      GO TO 20
   66 IF(T.GT.4.7) GO TO60
      F=1.0+0.237*SIN(PI/9.4*T)
      GO TO 20
   60 IF (T.GT.12.9) GO TO 61
      F=1.001+0.236*COS(PI/8.2*(T-4.7))
      GO TO 20
   61 IF(T.GT.19.0) GO TO 62
      F=1.0-0.235*COS(PI*(T-12.9)/12.2)
      GO TO 20
   62 F=1.0 +0.263*SIN(PI*(T-19.0)/10.2)
      GO TO 20
   77 IF(T.GT.3.8) GO TO 70
      F=1.0+0.086*SIN(PI*T/7.6)
      GO TO 20
   70 IF(T.GT.9.0) GO TO 71
      F=1.015+0.071*COS(PI*(T-3.8)/5.2)
      GO TO 20
   71 IF(T.GT.12.9) GO TO 72
      F=1.0-0.056*COS(PI*(T-9.0)/7.8)
      GO TO 20
   72 IF (T.GT.16.1) GO TO 73
      F=1.0+0.066*SIN(PI*(T-12.9)/6.4)
      GO TO 20
   73 IF(T.GT.18.5) GO TO 74
      F=1.0+0.066*COS(PI*(T-16.1)/4.8)
      GO TO 20
   74 F=1.0-0.05*SIN(PI*(T-18.5)/4.0)
   20 FREQ = F
      RETURN
      END
C***********************************************************************
C
C              TORL  IS THE LOAD TORQUE WHICH DEPENDS ON THE OPERATING
C        SPEED
C
C***********************************************************************

      FUNCTION TORL(V,N)
      COMMON A,B,C,RS,XSO,XMO,XRO,RRO,PMI,DELRR,DELXR,NM,M
      DIMENSIONA(10),B(10),C(10),RS(10),XSO(10),XMO(10),
     1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10)
      TORL=A(N)*V*V+B(N)*V+C(N)
      RETURN
      END
C***********************************************************************
C
C              A SUBROUTINE TO CALCULATE THE CURRENT AT DIFFERENT
C        FREQUENCIES AND AT DIFFERENT SPEEDS
C
C***********************************************************************
```

```
      FUNCTION CURR(F,E,V,N)
      COMMON A,B,C,RS,XSC,XMC,XRC,RRO,PMI,DELRR,DELXR,NM,M
      DIMENSICNA(10),B(10),C(10),RS(10),XSO(10),XMO(10),
     1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10)
      CCMPLEX ZM,ZS,ZSM,ZE,ZR,EE,AR,AS,CMPLX,CONJG
      XS=XSO(N)*F
      XM=XMO(N)*F
      ZM=CMPLX(0.0,XM)
      ZS=CMPLX(RS(N),XS)
      ZSM=ZS+ZM
      EE=ZM*E/ZSM
      ZE=ZM*ZS/ZSM
      S=(F-V)/F
      RR=RRO(N)*(1.0+DELRR(N)*S)
      XR=XRO(N)*(1.0-DELXR(N)*S)*F
      ZR=CMPLX(RR/S,XR)
      AR=EE/(ZE+ZR)
      AS=E /ZSM+ZM/ZSM*AR
      ASS=AS*CONJG(AS)
      CURR =SQRT(ASS)
      RETURN
      END
C************************************************************************
C
C               A FUNCTION SUBPROGRAM TO CALCULATE THE MOTOR TORQUE
C          DEVELOPED AT DIFFERENT FREQUENCY AND AT DIFFERENT SPEED
C
C************************************************************************

      FUNCTION TORM(F,E,V,N)
      CCMMON A,B,C,RS,XSC,XMO,XRC,RRC,PMI,DELRR,DELXR,NM,M
      DIMENSIONA(10),B(10),C(10),RS(10),XSO(10),XMO(10),
     1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10)
      CCMPLEX ZM,ZS,ZSM,ZE,ZR,EE,AR,AS,CMPLX,CONJG
      XS=XSO(N)*F
      XM=XMO(N)*F
      ZM=CMPLX(0.0,XM)
      ZS=CMPLX(RS(N),XS)
      ZSM=ZS+ZM
      EE=ZM*E/ZSM
      ZE=ZM*ZS/ZSM
      S=(F-V)/F
      RR=RRO(N)*(1.0+DELRR(N)*S)
      XR=XRO(N)*(1.0-DELXR(N)*S)*F
      ZR=CMPLX(RR/S,XR)
      AR=EE/(ZE+ZR)
      SQCR=AR*CONJG(AR)
      TORM=RR*SQCR/S/F
      RETURN
      END
C************************************************************************
C
C               A FUNCTION SUBPROGRAM TO CALCULATE THE ACCELERATION
C
C************************************************************************

      FUNCTION ACCEL(F,E,V,N)
      COMMON A,B,C,RS,XSC,XMC,XRO,RRO,PMI,DELRR,DELXR,NM,M
      DIMENSICNA(10),B(10),C(10),RS(10),XSO(10),XMO(10),
     1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10)
```

```
      ACCEL=(TORM(F,E,V,N)-TORL(V,N))/PMI(N)
      RETURN
      END
C*****************************************************************************
C
C                A SUBROUTINE SUBPROGRAM TO CALCULATE THE REAL AND
C          IMAGINARY POWER WHEN THE CURRENT, SPEED AND FREQUENCY ARE
C          GIVEN
C
C*****************************************************************************

      SUBROUTINE POWER(P,Q,N,CURRN,F,E,V)
      COMMON A,B,C,RS,XSC,XMC,XRC,RRO,PMI,DELRR,DELXR,NM,M
      DIMENSICNA(10),B(10),C(10),RS(10),XSO(10),XMO(10),
     1XRO(10),RRO(10),PMI(10),DELRR(10),DELXR(10)
      DIMENSICN XS(10),XM(10),RR(10), XR(10)
      S=(F-V)/F
      XS(N)=XSO(N)*F
      XM(N)=XMO(N)*F
      RR(N)=RRO(N)*(1.0+DELRR(N)*S)
      XR(N)=XRO(N)*(1.0-DELXR(N)*S)*F
      P=3.0*CURRN**2.0*RR(N)/S
      Q=3.0*E**2.0/XM(N)+3.0*CURRN    **2.0*(XS(N)+XR(N))
      RETURN
      END

$ENTRY
```

APPENDIX D

COMPUTER PROGRAM FOR THE ANALYSIS OF A GROUP OF INDUCTION MOTORS CONNECTED TO THE SAME NODE INCLUDING ONLY THE MECHANICAL TRANSIENTS

```
$JOB   WATFIV    STEPHENCT
C-----A PROGRAM TO CALCULATE THE STARTING PERFORMANCE OF A SYNCHRONOUS
C     MACHINE.RUNGE CUTTA MERSON METHOD IS USED TO SOLVE THE SET OF
C     DIFFERENTIAL EQUATIONS. THE ADVANTAGE OF THIS METHOD IS THAT THE
C     TRUNCATIONERROR IS KNOWN SO THAT THE STEP  LEHGTH CAN BE VARIED
C     TO ATTAINED THE REQUIRED ACCURACY. THE ACCURACY IS OF
C     THE ORDER H**5.
C-----X(1) = STATOR DIRECT AXIS FLUX LINKAGE .
C     X(2)=STATOR DIRECT AXIS FLUX LINKAGE
C     X(3)= ROTOR COIL DAMPER DIRECT AXIS FLUX LINKAGE
C     X(4)= ROTOR COIL DAMPER QUADRATURE AXIS FLUX LINKAGE
C     X(5)=VELOCITY OF THE ROTOR
C     X(6) ANGLE OF THE ROTOR
C----- CURR(I) REPRESENT THE CURRENT IN  RESPECTIVE COILS
C-----R(I) REPRESENT THE COIL RESISTANCE OF RESPECTIVE COILS
C----- TORM IS THE TORQUE DEVELOPED BY THE MACHINE
C-----TORL IS THE LOAD TORQUE
      REAL K,L
      DIMENSION X(6),XTEMP(6), CURR(4),R(4),K(5,6),
     1          L(4,4),A(4,4),B(4)
      COMMON R,CURR,EM,PMI,TORL,TORM,G
      READ(5,15)((L(I,J),I=1,4),J=1,4)
   15 FORMAT(4F10.4)
      READ(5,16)(X(I),I=1,6),(R(I),I=1,4),(CURR(I),I=1,4)
   16 FORMAT(6F10.4/4F10.4/4F10.4)
      READ(5,17)EM,PMI,TORL,H,ACCUR,G
   17 FORMAT(4F10.4,F10.6,F10.6)
      WRITE (6,201)
  201 FORMAT ('1',10X,'THE PARAMETERS OF THE MACHINE ARE:')
      WRITE (6,202)
  202 FORMAT ('0',20X,'THE INDUCTANCE MATRIX IS:')
      WRITE (6,203)((L(I,J),I=1,4),J=1,4)
  203 FORMAT ('0',25X,4F10.4)
      WRITE (6,204)
  204 FORMAT ('0',20X,'THE RESISTANCES ARE')
      WRITE (6,205)(R(I),I=1,4)
  205 FORMAT('0',25X,4F10.4)
      WRITE (6,206)
  206 FORMAT ('0',10X,'INITIAL CONDITIONS ARE')
      WRITE (6,207)
  207 FORMAT ('0',20X,'INITIAL CURRENT')
      WRITE (6,208)(CURR(I),I=1,4)
  208 FORMAT ('0',25X,4F10.4)
      WRITE (6,209)
  209 FORMAT ('0',20X,'INITIAL FLUX LINKAGES:')
      WRITE (6,210)(X(I),I=1,4)
  210 FORMAT ('0',25X,4F10.4)
      WRITE (6,211)X(5),X(6)
  211 FORMAT ('0',20X,'INITIAL SPEED=',F8.4//
     121X,'INITIAL ANGLE=',F8.4)
      WRITE (6,212)EM,PMI,TORL,H,ACCUR
  212 FORMAT ('0',10X,'OTHER DATA:'//
     121X,'TERMINAL VOLTAGE=',F10.4//
     221X,'MOMENT OF INERTIA=',F10.4//
     321X,'LOAD TORQUE=',F10.4//
     421X,'INITIAL STEP LENGTH=',F10.4//
     521X,'ACCURACY=',F10.6)
      WRITE(6,213)G
  213 FORMAT('0'20X,'RATE OF FREQUENCY CHANGE=',F10.6)
      III=0
```

```
      PRINT 91
      II=0
      T1=0.1
      H=0.01
      TPRNT=0.2
      N=4
      NDIM=4
      GO TO 11
   12 DMMLL=L(1,3)*L(1,3)-L(1,1)*L(3,3)
C-----TO SOLVE SIMULTANEOUS EQUATIONS FOR THE CURRENTS
      CURR(4)=(L(1,3)*X(2)-L(1,1)*X(4))/DMMLL
      CURR(3)=(L(1,3)*X(1)-L(1,1)*X(3))/DMMLL
      CURR(2)=(X(2)-L(1,3)*CURR(4))/L(1,1)
      CURR(1)=(X(1)-L(1,3)*CURR(3))/L(1,1)
C-----TO SET THE K(I,J) VALUES
   11 DO 10 JJ=1,6
      K(1,JJ)=H*F(JJ,X,T1)/3.0
      XTEMP(JJ)=X(JJ)+K(1,JJ)
   10 CONTINUE
      T=T1+H/3.0
      DO 20 JJ=1,6
      K(2,JJ)=          H*F(JJ,XTEMP,T)/3.0
      XTEMP(JJ)=X(JJ)+0.5*K(1,JJ)+0.5*K(2,JJ)
   20 CONTINUE
      DO 30 JJ=1,6
      K(3,JJ)=          H*F(JJ,XTEMP,T)/3.0
      XTEMP(JJ)=X(JJ)+0.375*K(1,JJ)+1.125*K(3,JJ)
   30 CONTINUE
      T=T1+H/2.0
      DO 40 JJ=1,6
      K(4,JJ)=          H*F(JJ,XTEMP,T)/3.0
      XTEMP(JJ)=X(JJ)+1.5*K(1,JJ)-4.5*K(3,JJ)+6.0*K(4,JJ)
   40 CONTINUE
      T=T1+H
      DO 50 JJ=1,6
      K(5,JJ)=          H*F(JJ,XTEMP,T)/3.0
   50 CONTINUE
C---- TO FIND THE MAXIMUM TRUNCATION ERROR
      ERR1=0.0
      DO 63 I=1,6
      ERR=ABS(K(1,I)-4.5*K(3,I)+4.0*K(4,I)-0.5*K(5,I))
      IF(ERR.GT.ERR1)ERR1=ERR
   63 CONTINUE
      DO 60 I=1,6
      X(I)=X(I)+0.5*(K(1,I)+4.0*K(4,I)+K(5,I))
   60 CONTINUE
      IF(T1.LT.TPRNT) GO TO 65
      CURRNT=CURR(1)*COS(X(6))+ CURR(2)*SIN(X(6))
      WRITE(6,64)T1,H,TORM,X(5),CURR(1),CURR(2),CURRNT,CURR(3),CURR(4)
      TPRNT=TPRNT+0.2
   64 FORMAT(2X,10F10.5)
   65 IF(T1.GT.1000.0)GOTO 75
      T1=T1+H
      IF(ERR1.GT.ACCUR) GO TO 62
      IF(ERR1.LT.0.8*ACCUR) GO TO 61
      GO TO 12
   61 H=1.1*H
      GO TO 12
   62 H=0.9*H
      GO TO 12
```

```
 91 FORMAT ('1',       2X,    'TIME          STEP          TORQUE      SPEED
   1  I(D)          I(Q)          I(A)          I(DR)          I(QR)')
 75 WRITE(6,92)(X(I),I=1,6)
 92 FORMAT('0','THE LAST VALUES OF X ARE:',6F10.4)
    STOP
    END

    FUNCTION F(JJ,X,T)
    DIMENSICN X(6),R(4),CURR(4)
    COMMON R,CURR,EM,PMI,TORL,TORM,G
    W=T+368.0-368.0*COS(T/1050.0)
    GO TO (1,2,3,4,5,6),JJ
  1 F=EM*SIN(W-X(6))-R(1)*CURR(1)- X(5)*X(2)
    GO TO 8
  2 F=EM*COS(W-X(6))-R(2)*CURR(2)+X(5)*X(1)
    GO TO 8
  3 F=-R(3)*CURR(3)
    GO TO 8
  4 F=-R(4)*CURR(4)
    GO TO 8
  5 TORM=-0.5*(X(1)*CURR(2)-X(2)*CURR(1))*2.0
    F=(TORM-TORL)/PMI
    IF (X(5) .LT.0.0) F=(TORM+TORL)/PMI
    GO TO 8
  6 F=X(5)
  8 RETURN
    END

$ENTRY
```