

# AN OPTIMIZATION METHOD FOR MODELING AC POWER SYSTEMS WITH TWO-PORT NETWORKS

by

Xiaobai Liang

A thesis presented to the Faculty of Graduate Studies

University of Manitoba

in partial fulfillment of the requirement for the degree

Doctor of Philosophy

June 1996, Winnipeg, Manitoba



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**AN OPTIMIZATION METHOD FOR  
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WITH TWO-PORT NETWORKS**

**BY**

**XIAOBAI LIANG**

**A Thesis/Practicum submitted to the Faculty of Graduate Studies of the University of Manitoba in partial  
fulfillment of the requirements for the degree of**

**DOCTOR OF PHILOSOPHY**

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## ABSTRACT

In this thesis, a new method for modeling power systems with equivalent two-port networks is presented. This new method—the three-step approach is based on a multiple-step strategy, a modified pattern search optimization technique and circuits with good move abilities. The pattern search technique has been modified to overcome the local minimum problem. The move abilities of zeros and poles of transfer functions of RLC circuits were studied. Some circuits with good move ability and pairs of circuits which are complementary with respect to realizability limitations have been found. The basic idea of a multiple-step strategy is illustrated. A complex problem is broken up into small parts, and each part is solved and finally the overall problem is solved by optimization. A very effective technique using the group delay to find the starting point for the optimization was developed. Computer programs were developed to implement the three-step-approach method. A specific modeling problem of a two-port power system and other examples were solved by the new method. The results are satisfactory and encouraging.

## ACKNOWLEDGMENTS

The author wishes to thank to his supervisor, Prof. G. O. Martens, for his patient supervision and invaluable suggestions throughout this research.

The author is grateful to G. Mazur of Manitoba Hydro, Prof. A. Gole, X. Hua and S. Elez of the University of Manitoba for providing data for some of the examples used in this thesis.

The author is also thankful for the assistance he received from his friends.

On a personal note, the author is greatly indebted to his mother and father for their love and to his sister, Mrs. Yu Ling Liang, and his wife, Yingying Yang, for their support.

# TABLE OF CONTENTS

Abstract .....	iv
Acknowledgments .....	v
Chapter 1 Introduction .....	1
1.1 Outline of Dissertation .....	6
Chapter 2 Two-Port Network .....	7
2.1 Two-Port Networks and Network Functions .....	7
2.2 Positive Real Properties of Two-Ports .....	12
Chapter 3 Pattern Search Optimization Method .....	13
3.1 Introduction .....	13
3.2 The Formulation of Optimization for Two-Port Network Modeling.....	14
3.3 The Pattern Search Optimization Technique .....	15
3.4 Pattern Search Computer Program .....	19
3.5 Example.....	27
3.6 Discussion.....	31
Chapter 4 Modeling of An AC System .....	32
4.1 Introduction .....	32
4.2 The Strategy of Three-Step Optimization .....	36
4.3 Step 1: Degrees of $z_{11}$ , $z_{12}$ and $z_{22}$ .....	40
4.3.1 Degrees and Types of Zeros and Poles of $z_{12}$ .....	41
4.3.1-a The Algorithm .....	41
4.3.1-b Finding the starting point for the optimization.....	47
4.3.1-c Tests and Results.....	51
4.3.2 Degrees and Zeros and Poles of $z_{11}$ .....	53



4.3.2-a The Algorithm .....	53
4.3.2-b Tests and Results .....	55
4.3.3 Degrees and Zeros and Poles of $z_{22}$ .....	57
4.3.3-a The Algorithm .....	57
4.3.3-b Tests and Results .....	58
4.4 Step 2: Structures for the Two-Port Network .....	60
4.4.1 The Main Structure of the Network .....	60
4.4.2 Micro Structure of $z_{12}$ .....	62
4.4.3 Micro Structure of $Z_1$ .....	72
4.4.4 Micro Structure of $Z_3$ .....	74
4.5 Step 3: Elemental Circuits for the Network .....	76
4.5.1 Elemental Circuits for $z_{12}$ .....	77
4.5.2 Elemental Circuits for $Z_1$ .....	83
4.5.3 Elemental Circuits for $Z_3$ .....	87
4.6 Results of $z_{12}$ , $z_{11}$ and $z_{22}$ .....	90
4.7 Improving the Frequency Responses of $z_{12}$ and $z_{22}$ .....	94
4.7.1 Error Analysis for $z_{12}$ .....	94
4.7.2 Error Analysis for $z_{22}$ .....	94
4.7.2 Tests and Results .....	100
4.8 Discussion .....	104
Chapter 5 More Examples .....	107
5.1 The Second Example .....	107
5.2 Step 1: The degrees and types of zeros and poles of $z_{12}$ , $z_{11}$ and $z_{22}$ .....	109
5.3 Step 2: The Network Structures .....	113
5.4 Step 3: The Elemental Circuits .....	116
5.5 Results and Discussion .....	118

5.6 The Third Example .....	122
5.6.1 Step 1 .....	124
5.6.2 Step 2 .....	129
5.6.3 Step 3 .....	134
5.6.4 Results and discussion .....	137
5.7 Example Four.....	140
Chapter 6 Further Study of the Three-Step Approach .....	155
6.1 Introduction .....	155
6.2 Further Study of the basic elemental circuits .....	156
6.3 The convergence .....	166
6.4 Accuracy .....	170
Chapter 7 Summary and Conclusions.....	174
Appendix A Computer Program for the Three-Step Approach Optimization	
Method .....	177
Appendix B Hydro Data .....	197
Appendix C Elemental Circuits and their Impedance Functions .....	200
Appendix D Circuit Diagram for Hydro Example .....	210
References .....	214

# Chapter 1

## INTRODUCTION

Studies of transient phenomena on electrical power systems can be carried out by real time simulation facilities [15], numerical programs on a computer [14], and equivalent networks [5, 7, 8, 9]. Research using equivalent electric networks for the simulation or modeling of power systems including both ac only and ac with dc tie, has been carried out for many years. One-port equivalent networks have been studied since 1970 [8, 9]. The results of one-port equivalent networks are very good for a number of different examples.

However for certain cases, a two-port model has advantages over a one-port model. Fig. 1.1 shows an ac system with a dc tie. Although in reality there are three phases for an ac system, a single phase representation is adequate for most cases. For the modeling of such a power system there are two formulations. One is the one-port formulation displayed in Fig. 1.2. The other is a two-port formulation that is illustrated in Fig. 1.3. The two-port formulation is better than the one-port formulation, since it takes the cross coupling of port 1 and port 2 through the ac system into account and the one-port model does not.

A multi-port equivalent network has been studied by Do and Garrilovic [7]. However, the network functions of their equivalent network are not guaranteed to be positive real. A positive real two-port equivalent network has been studied and the advantages of a two-port equivalent network were described [5]. There are relatively large errors in the realized frequency response, therefore, the possibility of finding a better solution for a positive real two-port equivalent network is considered.

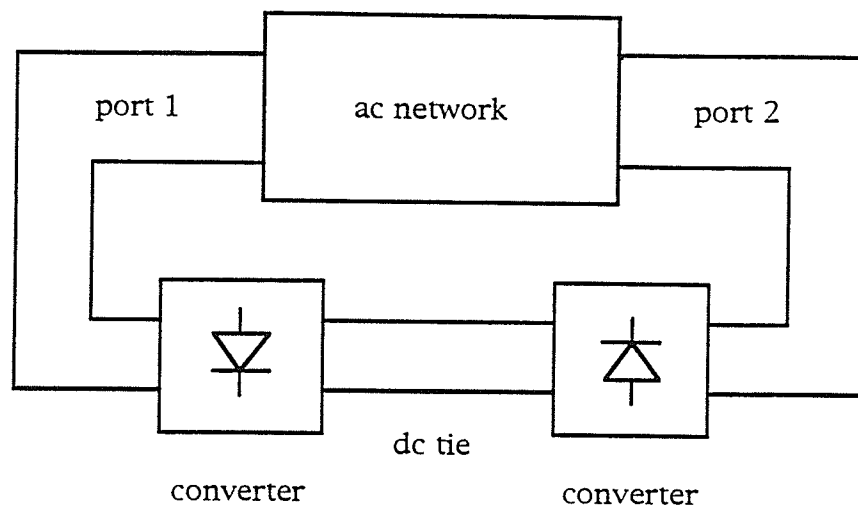


Fig. 1.1 An ac system with dc tie

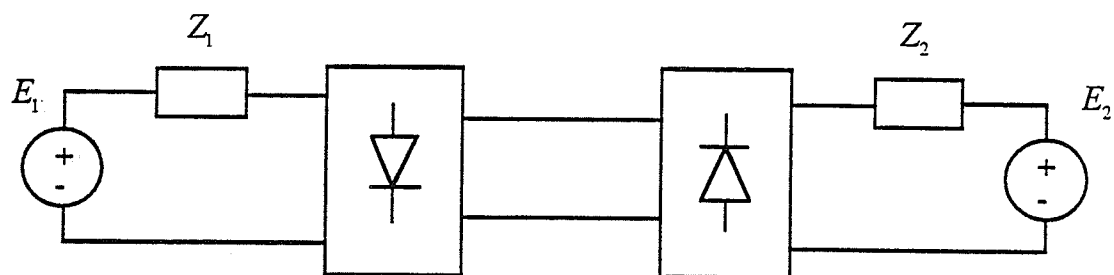


Fig. 1.2 One-port equivalent circuit

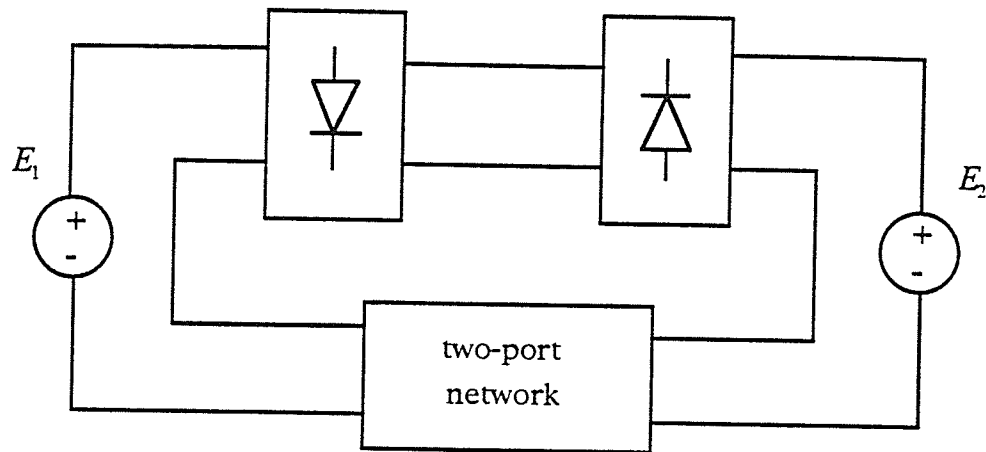


Fig. 1.3 Two-port model

Although a power system network is complicated, the types of components are relatively simple, i.e. resistors, capacitors, inductors and transformers. We believe that:

1. For the modeling of an ac power network, a positive real network is much better than a non-positive real one, because the ac network and the model contain the same types of positive real elements.
2. A network containing resistors, capacitors, inductors and transformers should be able to represent the ac network well.

In order to obtain a good positive real equivalent two-port network, a new method has to be developed.

The goal of this thesis is to develop a new method that can guarantee a positive real two-port equivalent network. As well, this positive real two-port equivalent network should have frequency responses very close to the frequency responses of the electrical power system, i.e., the difference between the frequency responses of the found equivalent two-port network

and the actual frequency responses measured in terms of a norm should be a minimum.

The procedure for finding a minimum can be seen as an optimization procedure. Optimization methods can be applied to a broad scope of problems in network analysis, network modeling and network synthesis. Many examples have demonstrated the effectiveness of the optimization technique [10]. However, the application of optimization does not automatically guarantee a good result. If the formulation of the optimization—the way the optimization is used—is not appropriate, then satisfactory results will not be achieved. For example, a parameter-optimization technique was applied in [7] that did not give a positive real network. A Simplex optimization method was used in [5] that did not yield frequency responses very close to the given (required) frequency responses.

There are several different optimization techniques that can be used to solve minimization problems. The Pattern Search Technique is a simple yet effective direct search method [10], [12], [20], [21], [26], [27], [28]. To apply the Pattern Search Optimization Technique to the modeling of a power system, a computer program that implements the Pattern Search Technique is essential.

This thesis presents a novel method for the modeling of ac power systems. This method is based on the way that the optimization technique is used, on a modified pattern search optimization technique and on elemental circuits with good move ability. (The definition of good move ability is stated in Chapter 3). The way in which the optimization is applied, is referred to as a strategy. The strategy—a multiple-step approach—is developed for solving the problem of modeling ac power systems. The basic idea of the multiple-step approach is to divide a complicated and difficult problem into small parts correctly, and to use the Pattern Search Optimization method throughout every

step, solving each small part, step by step. The Pattern Search Optimization Technique can also be applied to handle the movement of the zeros and poles of the elemental circuits by changing the values of the circuit elements, and thus the poles and zeros of the two-port network functions.

The procedure for modeling an ac power system with an equivalent two-port network is divided into three parts. In part one, the optimal impedance functions of the equivalent network are determined by an optimization process. Part two deals with the structures of the equivalent network. The structures are sorted into a main structure and a micro structure which are determined by optimization. In part three, the Pattern Search Optimization Technique is used to find the elemental circuits of the two-port network.

The synthesis problem is to find the (approximate) functional expression (network function), according to the required frequency response, and then, according to this network function, to realize the network. The final network might be realized or not, subject to the network function and to the characteristics of the network elements. In the new method, every step uses optimization. In the last step, that of finding the elemental circuits, the objective function is still the norm of the difference between the found and the required frequency response. Therefore, there is only one error. The objective function of the optimization is always the frequency response, this gives the synthesis procedure great flexibility and avoids the limitation of having a fixed network function as the objective function, because a frequency response can be approximated by many network functions.

The equivalent network consists of only positive resistors, positive capacitors, positive inductors and transformers. Therefore, the associated

impedance function must satisfy the positive real conditions. In the optimization procedure, the developed computer program monitors every movement of the optimization process and makes sure that the positive real conditions are always satisfied.

## 1.1 Outline of Dissertation

Chapter 2 describes the basic two-port network and the positive real properties of a two-port. Chapter 3 presents the pattern search optimization technique and the implemented computer program. In Chapter 4, the basic idea of the multiple-step-approach strategy is introduced. Also, a modeling problem of an ac power system is solved along with a detailed introduction of the three-step-approach. In Chapter 5, more examples and more main network structures are studied. Chapter 6 further studies the basic elemental circuits, the convergence and accuracy for the three-step-approach method. Chapter 7 gives conclusions regarding the successful new method and suggests work for further study.



# Chapter 2

## TWO-PORT NETWORK

### 2.1 Two-Port Networks and Network Functions

Regardless how simple or how complex a circuit is, as long as it has two ports to connect to outside sources or electrical circuits (input or output), and the currents of the circuit satisfy the following conditions:

$$\begin{aligned} I_1 &= I_1' \\ I_2 &= I_2' \end{aligned} \quad (2.1.1)$$

the circuit is called two-port network, otherwise, it is called four-terminal network. A two-port network is shown in Fig. 2.1.1.

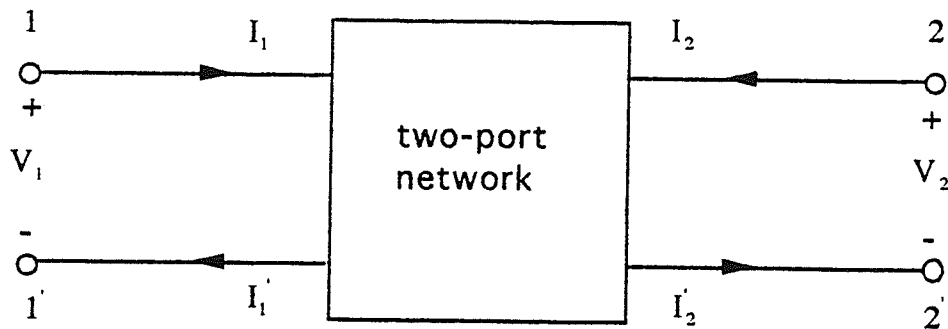


Fig. 2.1.1 A two-port network

According to the circuit structure, a two-port network can be classified as T,  $\Pi$ , Ladder, T-bridge, bridge and other basic types of networks. Fig. 2.1.2 shows the T two-port network. Fig. 2.1.3 illustrates the  $\Pi$  two-port

network. The ladder two-port network is presented in Fig. 2.1.4. The series connected two-port network is demonstrated in Fig. 2.1.5. Fig. 2.1.6 presents the parallel connected two-port network. These network structures are simple yet useful. We will use them for the modeling of an ac power system.

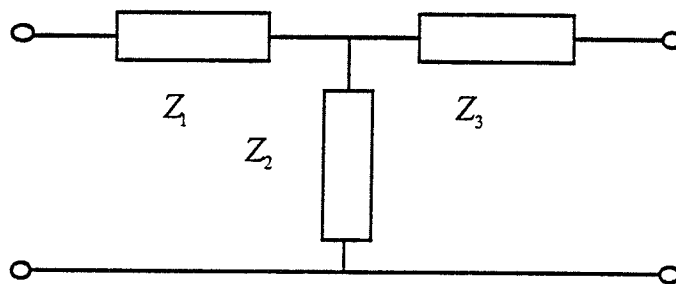


Fig. 2.1.2 T two-port network

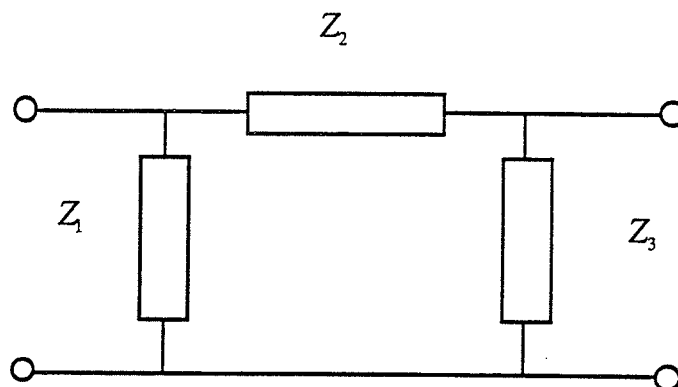


Fig. 2.1.3 Π two-port network

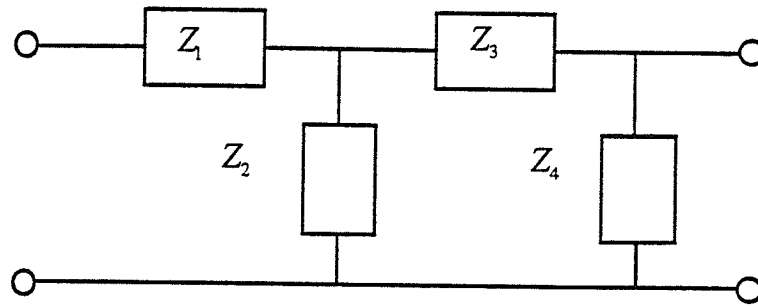


Fig. 2.1.4 A ladder two-port network

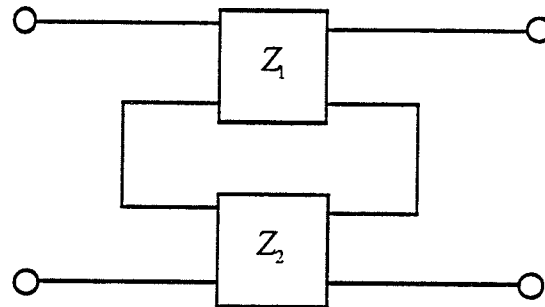


Fig. 2.1.5 Series connection of two-port networks

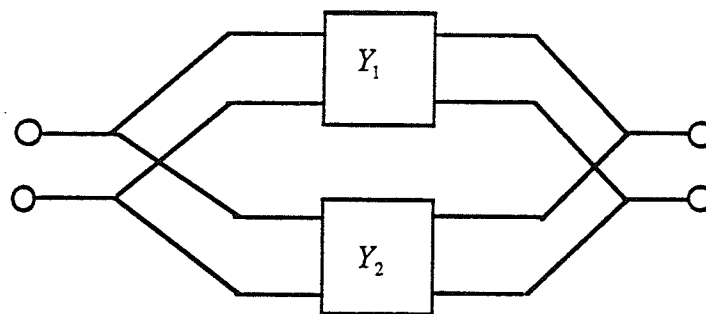


Fig. 2.1.6 Parallel connection of two-port networks

Two-port network theory studies the relations between the currents and voltages at the input port and at the output port. The functions relating the voltages and currents of the two port are called two-port network functions. There are several forms of the two-port network functions. In this thesis, we only use the open-circuit impedance and the short-circuit admittance functions. Also, we only study two-port networks that contain resistors, capacitors, inductors and transformers. This type of two-port network is also described as a passive, lumped, linear, time-invariant, reciprocal two-port network. The open-circuit impedance matrix

$$\mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \quad (2.1.2)$$

where  $z_{11}$  is the driving point function of port 1,  $z_{22}$  is the driving point function of port 2, and  $z_{12}$  is the transfer function of the network. Because the two-port network is a reciprocal network,  $z_{12}$  is equal to  $z_{21}$ . The parameters of  $\mathbf{Z}$  are defined as follows:

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad (2.1.3)$$

$$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad (2.1.4)$$

$$z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} \quad (2.1.5)$$

## 2.2 Positive Real Properties of Two-Ports

The impedance matrix  $Z$  has the following form:

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{d} \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \quad (2.2.1)$$

where  $n_{11}$ ,  $n_{12}$ ,  $n_{22}$  and  $d$  are polynomials in the complex frequency variable  $s$ .

It is known that impedance and admittance matrices of a passive, lumped, linear, time-invariant, reciprocal two-port network are positive real matrices [34]. Consequently, the following conditions hold:

a. Conditions for each function:

- (1)  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are real for  $s$  real.
- (2) Poles of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are all in the left- half  $s$ -plane and on the  $j\omega$  axis, Poles of  $z_{11}$  and  $z_{22}$  on the  $j\omega$  axis are simple with real positive residues, Poles  $z_{12}$  on the  $j\omega$  axis are simple.
- (3) The real parts of  $z_{11}$  and  $z_{22}$  satisfy the following inequalities:

$$0 \leq \operatorname{Re}(z_{11}), \operatorname{Re}(z_{22}) < \infty \quad s = j\omega \quad -\infty \leq \omega \leq \infty \quad (2.2.2)$$

where  $\operatorname{Re}$  denotes the real part.

b. Collective conditions:

- (1) When  $s = j\omega$ , the real parts of impedance functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  satisfy the following inequality:

$$\operatorname{Re}(z_{11})\operatorname{Re}(z_{22}) - \operatorname{Re}^2(z_{12}) \geq 0 \quad s = j\omega \quad -\infty \leq \omega \leq \infty \quad (2.2.3)$$

(2) Poles of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  on the  $j\omega$  axis satisfy the following residue condition:

$$\begin{aligned} k_{11} &\geq 0 & k_{22} &\geq 0 \\ k_{11}k_{22} - k_{12}^2 &\geq 0 \end{aligned} \tag{2.2.4}$$

where  $k_{11}$ ,  $k_{12}$  and  $k_{22}$  stand for the residues of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ , respectively.

In the optimization process, the positive real conditions play an important role, i.e., the PR (positive real) conditions must be satisfied to guarantee finding an RLC network (including transformers) for the modeling of an ac power system.

## Chapter 3

# PATTERN SEARCH OPTIMIZATION METHOD

### 3.1 Introduction

Optimization techniques can be applied to a wide range of problems in network analysis, network modeling and network synthesis. The conventional network synthesis methods for two-port networks, except for simple cases, have disadvantages. For example, passive RLC filters, usually require the network capacitors and inductors to be ideal. As well, the procedures are complicated[10]. In this chapter, an optimization method for the two-port network synthesis is introduced which focuses on the transfer function of a two-port network. This new method is based on the fact that zeros and poles of the transfer function of an RLC network can be moved over the (left-half) complex plane by changing the values of the network elements—resistors, capacitors, inductors and transformer ratios. A pattern search optimization process can be used to move the zeros and poles of a two-port network transfer function. As an example, consider a sixth-order network transfer function. To determine the passive element values so that required zeros and poles are achieved is a challenging problem but is solved by the developed pattern search optimization method.

### 3.2 The Formulation of Optimization for Two-Port Network Modeling

The basic parts of an optimization are the input variables  $\mathbf{x} = (x_1, x_2, \dots, x_n)$ , the objective function  $F$ , the target function  $G(\mathbf{x})$  (the performance of a network or a network function as a function of the input variables), and the (given) requirements  $\mathbf{R} = (R_1, R_2, \dots, R_m)$ . Without loss of generality, we define the objective function of the optimization:

$$F = \sum_{i=1}^m |R_i - G_i|$$

where  $R_i$  is a required (given) value and  $G_i$  is the corresponding value of the target function  $G$ . If the values of the variables need to satisfy some conditions, e.g.

$$x_j \geq 0 \text{ (or } x_j \leq 0) \quad j = 1, 2, 3, \dots, n$$

then the optimization is called a constrained optimization.

For example, an optimization process is used to find a driving point function which has a frequency response very close to the required frequency response. The given frequency responses at some specified frequencies are the required values, the constant factor, the zeros and the poles of the impedance function are the input variables, and the relation between the frequency response and the constant factor, the zeros and poles of the impedance function is the target function. The values of the constant factor must be non negative, and the real part of the zeros and poles must be negative. Another example is that the optimization process is used to find a network such that the impedance function of this network has some specified zeros and poles, these specified zeros and poles are the required values; the elements of this network—resistors, inductors, capacitors and transformers are



the input variables; the relation between the required zeros and poles and the network elements are the target function. the values of the network elements resistors, inductors and capacitors must be non negative.

### 3.3 The Pattern Search Optimization Technique

There are several different optimization methods that can be used to solve minimization problems. These optimization methods are classified into direct search methods and non-direct search methods. The direct search methods do not require smooth functions or their derivatives. Among these direct search optimization methods, the pattern search method is the most widely used one [10],[12]. The theory is largely geometric. Despite its mathematical simplicity, the pattern search method works well in a wide range of applications[10], [12], [20], [21],[26],[27],[28].

The process of pattern search optimization contains two major steps. The first one is an exploration search which includes a sequence of single variable searches and the second one is a pattern search. The starting point of the exploration search is also called a base point of the exploration search. The point reached by an exploration search is called the exploration search expansion point. The starting point of the pattern search is called a base point of the pattern search. The point reached by the pattern search is called the pattern search expansion point. At the beginning of the optimization process, a starting point,  $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$ , is chosen. A step size (step increment)  $\Delta x_i$  for each independent variable is also selected.

## 1) The exploration search

The purpose of an exploration search is to explore the vicinity of the current base point. An exploration search includes a sequence of single variable searches. The procedure for an exploration search is as follows:

(1) The variable  $x_1$  is given a positive increment, i.e.  $x_1 + \Delta x$  and a new point is reached:

$$\mathbf{x}^{(1)} \longrightarrow \mathbf{x}^{(1)} + \Delta x \mathbf{e}_1$$

where  $\mathbf{x}^{(1)}$  is a vector and  $\mathbf{e}_1$  is a unit vector along coordinate axis 1.

Then the objective function  $F$  is evaluated. If the move is a successful move, i.e.  $F(\mathbf{x}^{(1)} + \Delta x \mathbf{e}_1) < F(\mathbf{x}^{(1)})$ , then the new point replaces the original one. If the move is a failure, then a negative move i.e.  $x_1 - \Delta x$  is taken and a new point is reached:

$$\mathbf{x}^{(1)} \longrightarrow \mathbf{x}^{(1)} - \Delta x \mathbf{e}_1$$

Next the objective function  $F$  is evaluated. If the move is a successful move, i.e. the value of the objective function decreases, the new point replaces the original one. If the move is a failure, then the original point is maintained.

(2). Step (1) is repeated for all other variables, i.e. for  $x_i, i = 2, 3, 4, \dots, n$ . Finally an exploration search expansion point is reached. This point is denoted as  $\mathbf{x}^{(2)}$  and will be used as the pattern search base point.

## 2) Pattern search and subsequent search

The pattern search is based on a successful move in a single variable search. A move attempts to speed up the search while following the direction of the successful single variable search move i.e. in the direction  $\mathbf{x}^{(2)} - \mathbf{x}^{(1)}$ . Since a move of a single variable search in this direction has already led to a

decrease in the value of the objective function  $F$ , the pattern search procedure move starting from point  $\mathbf{x}^{(2)}$  is as follows:

- (1) Move from point  $\mathbf{x}^{(2)}$  to  $\mathbf{x}^{(3)}$ :  $\mathbf{x}^{(3)} = \mathbf{x}^{(2)} + (\mathbf{x}^{(2)} - \mathbf{x}^{(1)})$ .
- (2) Check if the value of the objective function  $F$  at the pattern search expansion point  $\mathbf{x}^{(3)}$ , is better than that of the starting point  $\mathbf{x}^{(2)}$ . If there is improvement, then point  $\mathbf{x}^{(3)}$  will be taken as a new base point for the next sequence of single variable moves of the exploration search. If no improvement is made, then  $\mathbf{x}^{(3)}$  is abandoned and point  $\mathbf{x}^{(2)}$  becomes a new base point for the next exploration search, and a new sequence of single variable moves are made.
- (3) Repeat the sequence of single variable moves and a pattern move in turn, until a preset limit is reached.

A two dimensional example of an exploration search and pattern search is demonstrated in Fig. 3.3.1. The starting point is  $\mathbf{x}^{(1)}$ . The first single variable move is successful in the positive direction. The second single variable move is successful in the negative direction and point  $\mathbf{x}^{(2)}$  is reached.  $\mathbf{x}^{(2)}$  becomes the base point for the pattern search. The first pattern move in the direction  $\mathbf{x}^{(2)} - \mathbf{x}^{(1)}$  is successful and  $\mathbf{x}^{(3)}$  becomes the pattern search expansion point and the base point for a new round of exploration searches and pattern searches. The first single variable move starts at point  $\mathbf{x}^{(3)}$ . The step size has to be reduced before the first single variable move is successful, in the negative direction. The second single variable move is successful in the negative direction and  $\mathbf{x}^{(4)}$  becomes the base point for the pattern search. However, the pattern search move fails, so that  $\mathbf{x}^{(4)}$  becomes a base point for the next exploration search. The next round of exploration searches and pattern searches is successful and the final optimal point  $\mathbf{x}^{(6)}$  is reached.

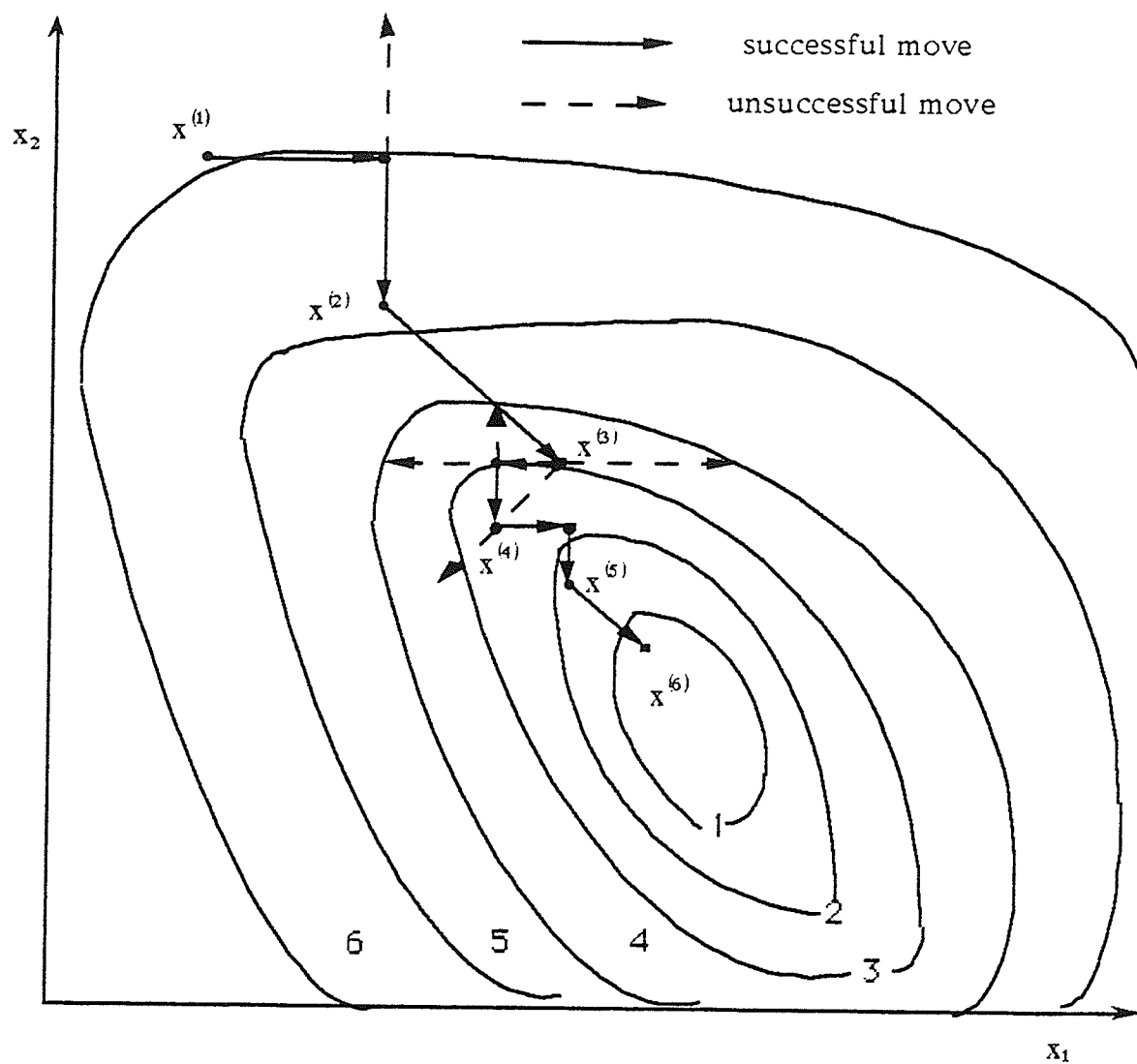


Fig. 3.3.1 An example of the single variable search and the pattern search

### 3.4 Pattern Search Computer Program

A computer program is developed for the pattern search optimization. In the computer program two kinds of step increments are used. The first kind of step increment is a constant:

$$\Delta x = c$$

The second kind of step increment is a constant scalar factor  $k$  times the vector component, i.e.,

$$\Delta x_i = k \cdot x_i$$

The second step increment is called percentage step increment. In this thesis, it is found that when the optimal values of variables differ greatly, the percentage step increment leads to a faster convergence of the optimization process than that with a constant step increment. Since the single variable search moves in both  $+\Delta x$  and  $-\Delta x$  directions, there is no loss of generality in choosing  $\Delta x$  to be positive. The computer program is written in PASCAL and is shown in Appendix A. A flowchart of this computer program is presented in Fig. 3.4.1.

In the flowchart,  $x[i,1]$  is the starting point of the exploration search,  $x[i,2]$  is the exploration expansion point and the starting point of the pattern search, and  $x[i,3]$  is the pattern search expansion point.  $A$  represents an initial error limit and  $\beta$  represents the final error limit.  $A_1$  is a variable error limit.  $B_1$  represents the value of the objective function, i.e. the error of the exploration search, and  $B_2$  represents the value of the objective function, i.e. the error of the pattern search. In the exploration search, if the error  $B_1$  is less than the present error limit  $A_1$ , then the value of  $B_1$  is set to be the updated error limit  $A_1$  for the pattern search and a starting point of the pattern search— $x[i,2]$  is reached.

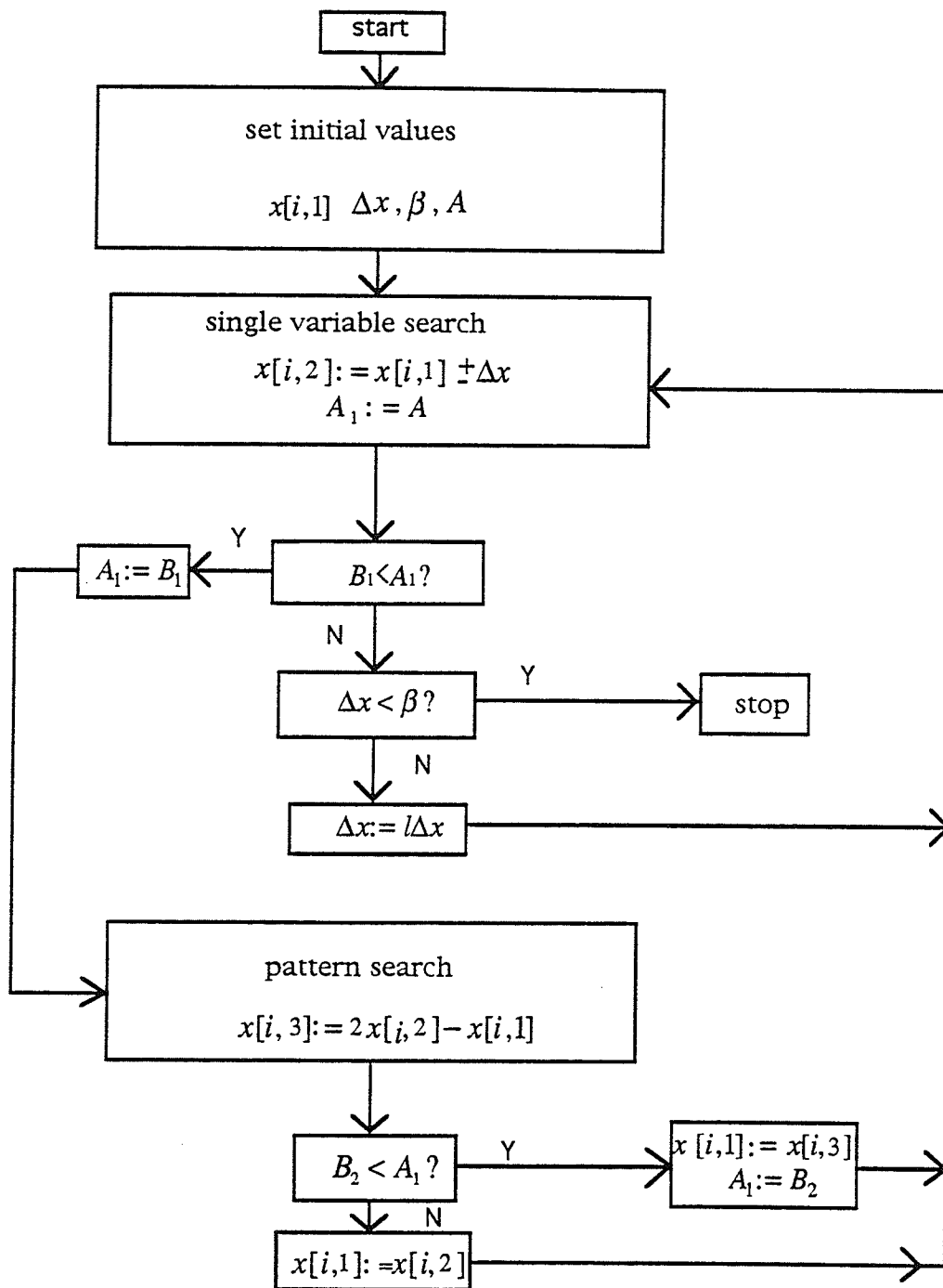


Fig. 3.4.1 Flowchart of a procedure

In the exploration search, if the error  $B_1$  is not less than the updated error limit  $A_1$ , then the exploration move fails and the step size must be reduced. The step size reduction shown in the flowchart is a linear one:

$$\Delta x_2 = l\Delta x_1$$

where  $l$  is a constant. The step size reduction can also be nonlinear:

$$\Delta x_2 = f(\Delta x_1)$$

where  $f(\Delta x_1)$  is a nonlinear function of  $\Delta x_1$ , e. g. the square function. In some cases nonlinear step size reduction leads to a faster convergence of the optimization process.

In the pattern search, if the error  $B_2$  is less than the present error limit  $A_1$ , then the value of error  $B_2$  is set to be the updated limit  $A_1$ , and a pattern search expansion point  $x[i,3]$  is reached and it is set to be the next starting point of an exploration search; if the error  $B_2$  is larger than the present error limit  $A_1$ , then the pattern search fails and the present starting point of the pattern search is set to be the next starting point of an exploration search.

A problem which arises in the optimization process is how to prevent the program from stopping at a local minimum (premature termination). Figure 3.4.2 shows a curve where point C is the global minimum of the curve; both points B and D are local minima. If the program of the flowchart 1, shown in Fig. 3.4.1, is used to minimize the function of this curve, starting at point A or point E, it could stop at either point B or point D for some step increments. Fig. 3.4.3 displays in more detail why the search will stop at the local minimum. From Fig. 3.4.3, it can be seen that as the search attempts to find a lower point, it reduces the step increment  $\Delta x$ . Eventually the search reaches the local minimum point and the step increment size decreases to a minimum. Therefore, while the overall error of the optimization is still

large, the step size has become a minimum, i.e., a local minimum has been (possibly) reached. It is clear that without overcoming the local minimum problem, the pattern search is useless.

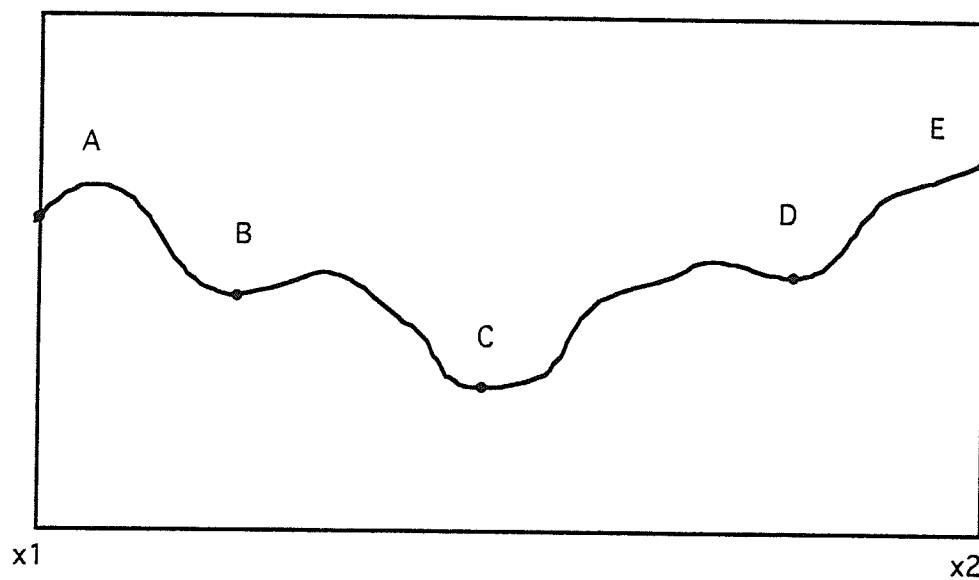


Fig. 3.4.2

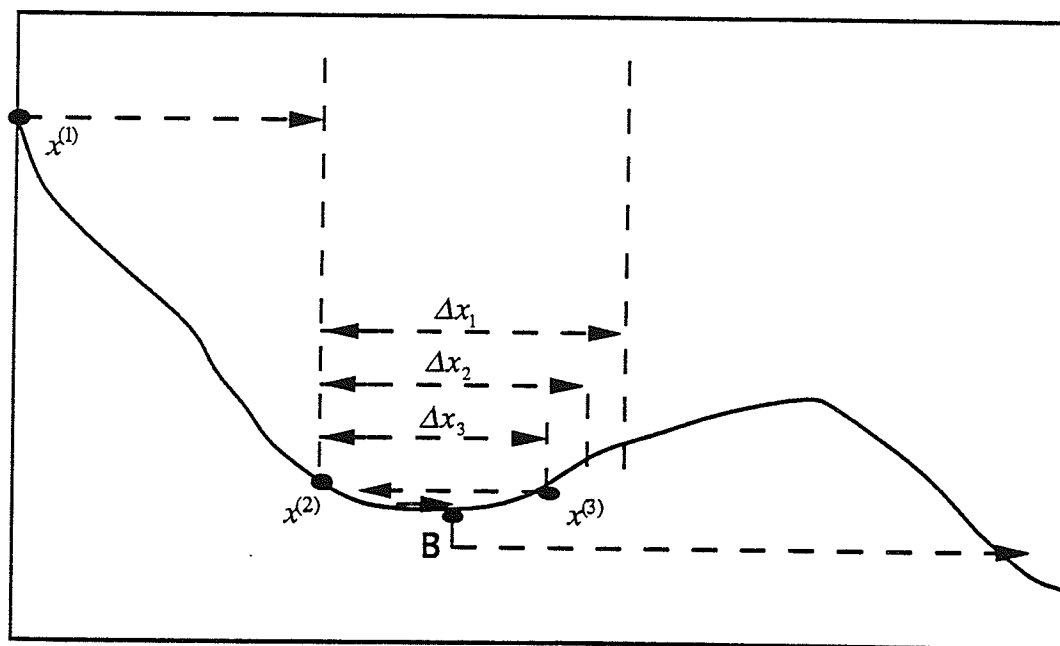


Fig. 3.4.3



In order to leap over the local minima and search toward the global minimum, one useful way is to increase the step size. Fig. 3.4.4 gives a birds-eye view. From Fig. 3.4.4, it is clear that increasing the step size is an effective way to overcome the local minimum problem. Therefore, to overcome the local minimum problem, another flowchart shown in Fig. 3.4.6 must be added to the flowchart 1 (flowchart of a procedure). The program of flowchart 2 makes it possible to leap over the local minima. When a local minimum is detected, the program increases the step size dramatically to leap over the neighborhood of the local minimum to search for a new direction to the global minimum. An execution of the program of flowchart 1 is called a procedure. During the optimization process, if  $n$  local minima are met, then  $n+1$  procedures are needed to reach the global minimum.

There are some other techniques for solving the local minimum problem. Fig. 3.4.5 shows that moving along one direction to change the starting point and then searching in the other direction also helps to move over a local minimum. Other techniques such as using different step sizes and using different starting points (different in all directions) can also be applied to overcome the difficulty of a local minimum.

We will discuss how to deal with the constraints of the optimization in Chapter 4 along with the PR requirement of the impedance function.

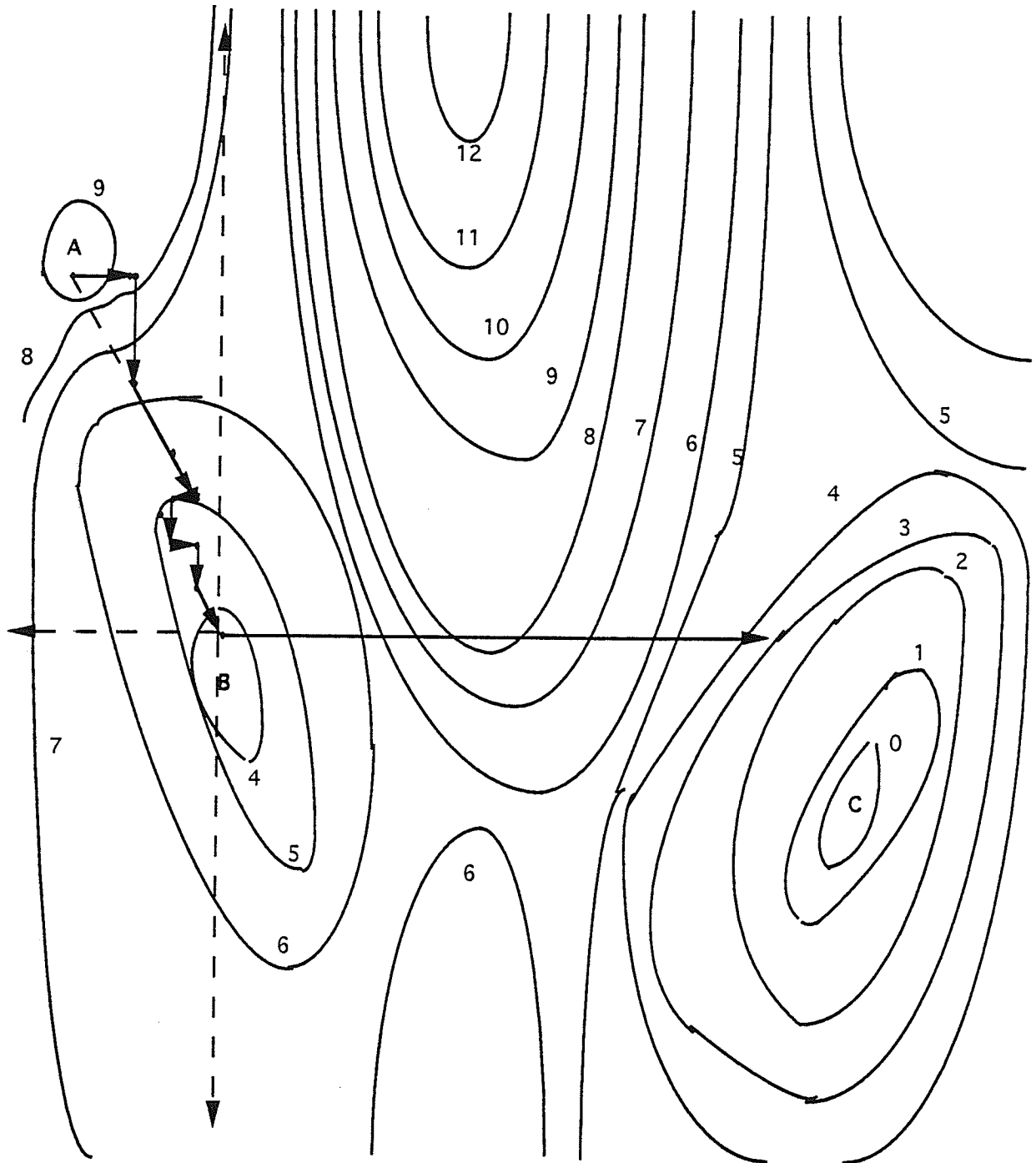


Fig. 3.4.4

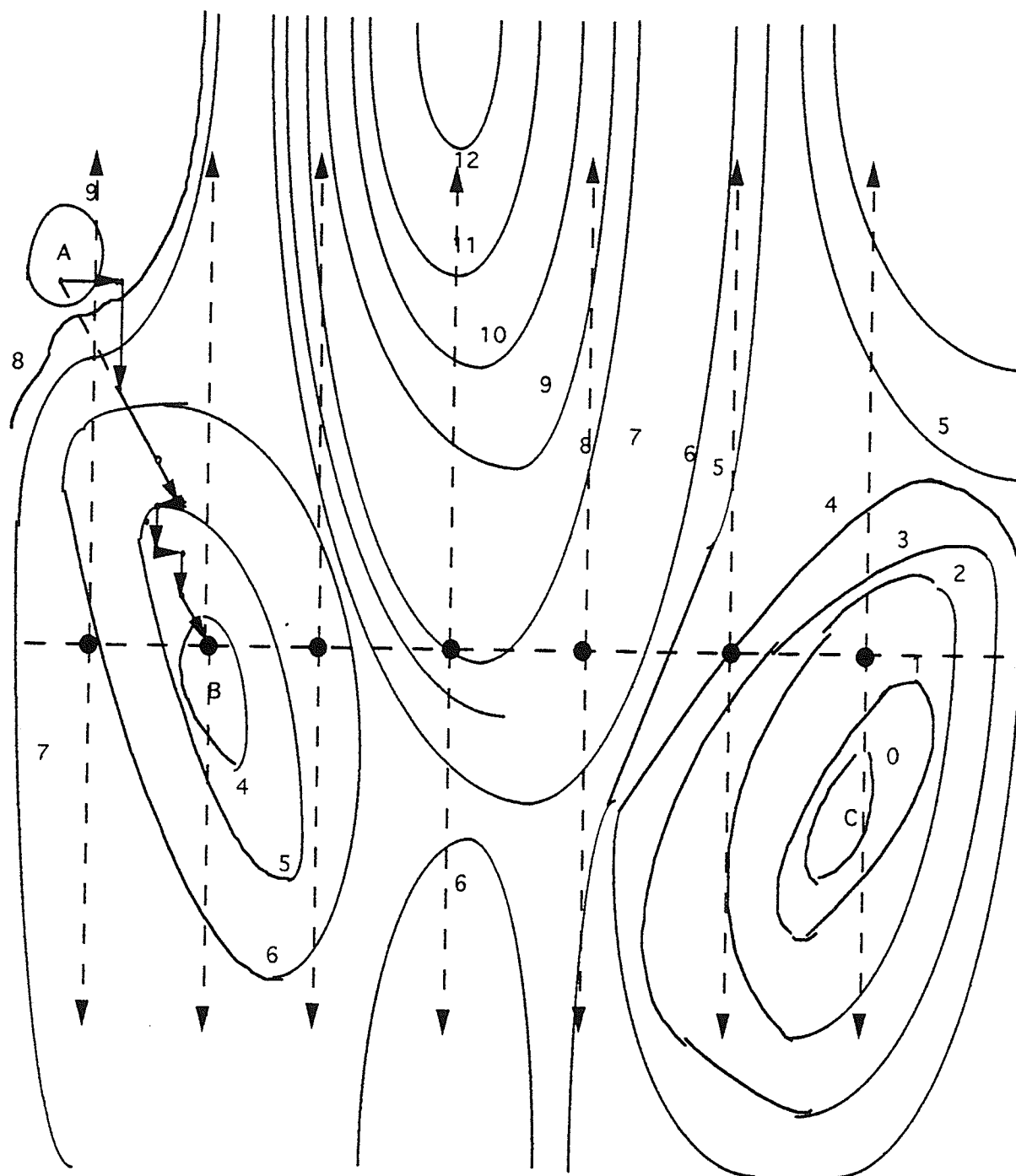


Fig. 3.4.5

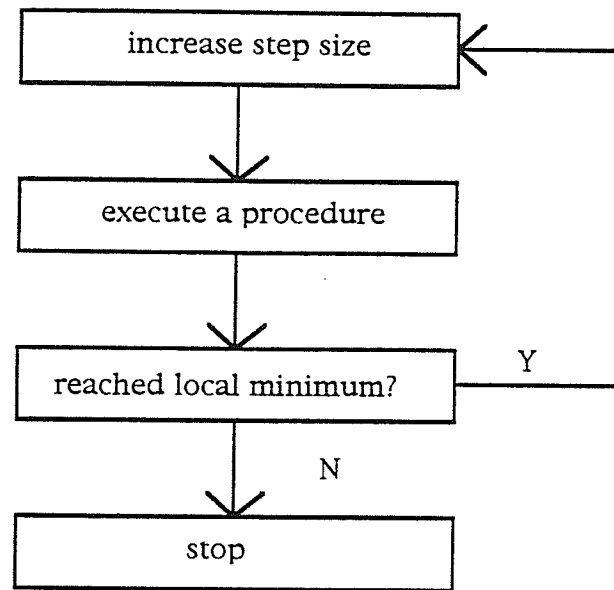


Fig. 3.4.6 Flowchart 2

### 3.5 Example

The following example is used to demonstrate the method discuss in Section 3.4. The zeros and poles of a transfer function  $z_{12}$  are given as follows:

Table 3.5.1 zeros and poles of given  $z_{12}$

Numerator Degree:

5

Constant factor followed by Zeros:

real part	imaginary part
1.8162721865407291370e+2	0.0000000000000000000e+0
-3.8237749779750948910e+2	1.1220919662871389730e+3
-3.8237749779750948910e+2	-1.1220919662871389730e+3
-2.7521457411148106060e+2	1.6318360713505543120e+3
-2.7521457411148106060e+2	-1.6318360713505543120e+3
-4.4622888403015500970e+1	0.0000000000000000000e+0

Denominator Degree:

6

Constant factor followed by Poles:

real part	imaginary part
1.0000000000000000000e+0	0.0000000000000000000e+0
-2.4490476161442084880e+2	-2.1630087493962056980e+3
-2.4490476161442084880e+2	2.1630087493962056980e+3
-2.6728612608663766060e+2	-1.3032530067106722280e+3
-2.6728612608663766060e+2	1.3032530067106722280e+3
-2.7047454246291536250e+2	5.5586014706930690560e+2
-2.7047454246291536260e+2	-5.5586014706930690560e+2

In order to determine what kind of network structure can be used to realize this transfer function, we analyze the given transfer function. The degree of the numerator is 5 and the degree of the denominator is 6. The numerator has one real zero and two complex conjugate pairs. The denominator has three complex conjugate pairs of zeros. There are many kinds of network structures that can be used to realize this transfer function. We choose the following  $\Pi$  structure to carry out the synthesis process.

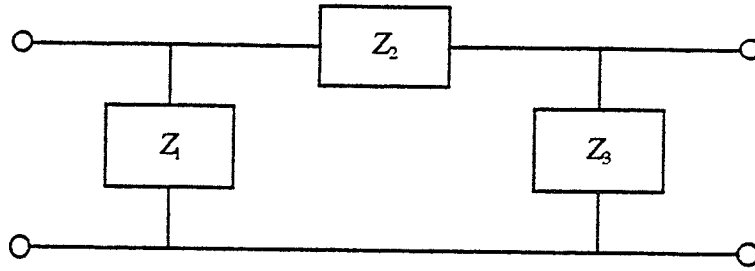


Fig. 3.5.1

The next step is to find the micro structure—the elemental circuit. The relation between  $z_{12}$  and  $z_1 = \frac{n_1}{d_1}$ ,  $z_2 = \frac{n_2}{d_2}$ ,  $z_3 = \frac{n_3}{d_3}$  is:

$$z_{12} = \frac{n_1 d_2 n_3}{n_1 d_2 d_3 + n_2 d_1 d_3 + n_3 d_1 d_2} \quad (3.5.1)$$

There are many combinations of  $z_1$ ,  $z_2$  and  $z_3$  which can satisfy the degree requirements of  $z_{12}$ . One choice is the degree of the numerator of  $z_1$  equal 1 and the degree of its denominator equal 2, and the degree of the numerators and denominators of both  $z_2$  and  $z_3$  equal 2. With some basic knowledge of network theory, some circuits which satisfy the degree requirements can easily be found. (Some elemental circuits are shown in appendix C.) The next step is to run the optimization program for different combinations of these circuits—the elemental structures. A good combination that has poles and zeros very close to the required poles and zeros is found. The network is shown in Fig. 3.5.2. The zeros and poles are listed in Table 3.5.2.

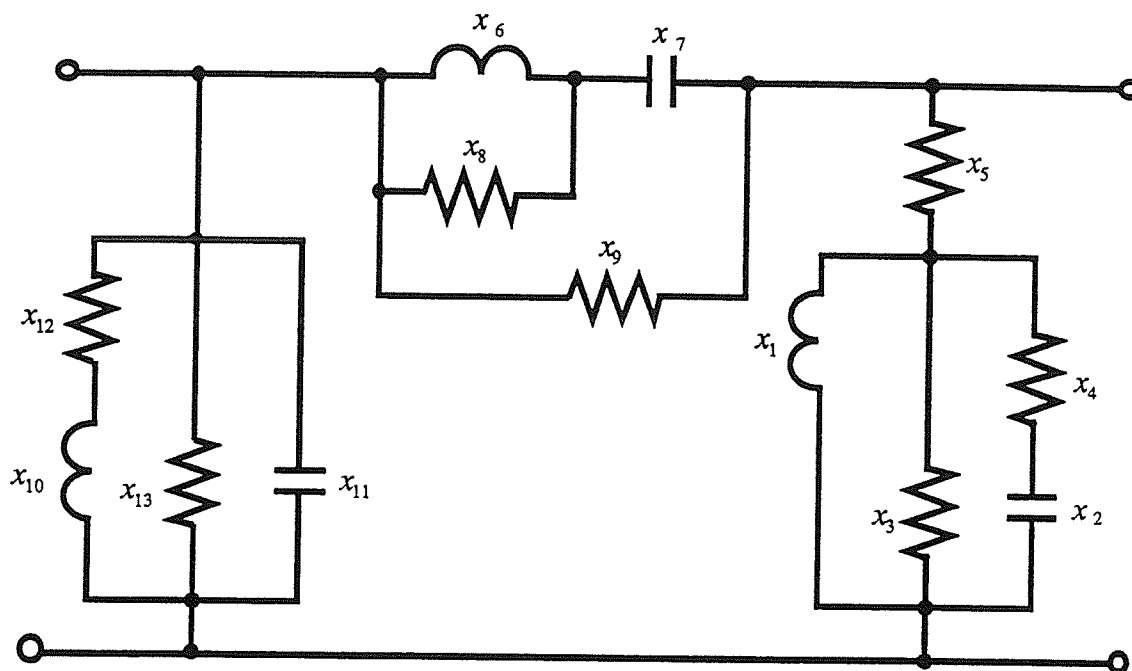


Fig. 3.5.2

Table 3.5.2 zeros and poles of found  $z_{12}$

Numerator Degree:

5

Constant factor followed by Zeros:

real part

imaginary part

1.8162700069998530240e+2	0.0000000000000000000e+0
-2.7521457397329092710e+2	-1.6318360608542535030e+3
-2.7521457397329092710e+2	1.6318360608542535030e+3
-3.8661525095396568610e+2	-1.1320615687626235780e+3
-3.8661525095396568610e+2	1.1320615687626235780e+3
-4.4622889388678210170e+1	-0.0000000000000000000e+0

Denominator Degree:

6

Constant factor followed by Poles:

real part

imaginary part

1.0000000000000000000e+0	0.0000000000000000000e+0
-2.6712364758515397920e+2	5.5411513465699502140e+2
-2.6712364758515397920e+2	-5.5411513465699502140e+2
-2.6728612543130950140e+2	1.3032530000334810330e+3
-2.6728612543130950140e+2	-1.3032530000334810330e+3
-2.4490476326542253280e+2	2.1630087606343870050e+3
-2.4490476326542253280e+2	-2.1630087606343870050e+3

### 3.6 Discussion

In the introduction we introduced an optimization method that is based on the fact that zeros and poles of an impedance function of an RLC network can be moved over a region in the (left-half) complex plane by changing the values of the network elements—resistors, capacitors, inductors and transformers. It is found that the poles and zeros of different network impedance functions have different capabilities for moving over the (left-half) complex plane, because of inherent restrictions. For ease of description, we use the term move ability to discuss the movement of the zeros and poles of a network function. For an impedance function of a network with the number of zeros and poles greater or equal to 1, the movements of the zeros and poles in the (left-half)  $s$ -plane are not independent. That is, the movement of a pole (zero) is restricted by the location of the zero (pole). For certain locations of zeros (poles), the poles (zeros) of some network functions can move over a larger region in the (left-half) complex plane than some other network functions can. If the poles and zeros of a network function can move over a large region of the (left-half) complex plane, we simply say that the elemental network has good move ability; otherwise we say it has poor move ability.

Now we can use the results of the example to further describe the movement of the zeros and poles of the network function. For an RLC network, we find that:

(1) Zeros and poles of some networks (for example elemental circuit No. F1, Appendix C) are always real, regardless of the values of the elements. Zeros and poles of some networks have fixed relations. We say these have poor move ability.



(2) An elemental network's move ability will affect the main network's move ability. For example, if elemental circuit No. F4, shown in Appendix C, is used as  $Z_1$  of the network shown in Fig. 3.5.2 then the errors in the zeros and poles of this network function become very large. Elemental circuit No. F4 is the kind of network described in (1). Its real zero is always equal to one half of the real part of its poles.

In this chapter, we presented the pattern search optimization method for the realization of a two-port network. The move ability of the impedance functions of elemental circuits was studied. Some elemental circuits with good move ability were found and presented in this Chapter. The pattern search optimization technique and the elemental circuits with good move ability are the basis of the new method for the modeling of an ac system. In Chapter 4, we will carry out more studies for applying this basis to the modeling of an ac power system.

## Chapter 4

# MODELING OF AN AC SYSTEM

### 4.1 Introduction

In this chapter, we will develop a new method to solve a modeling problem—the synthesis of an equivalent two-port network for an ac power system. Figs. 4.1.1, 4.1.2 and 4.1.3 demonstrate the frequency responses of  $z_{12}$ ,  $z_{11}$  and  $z_{22}$ , respectively. (Data were obtained from G. Mazur of Manitoba Hydro and are listed in Appendix B.) We will see that the pattern search optimization technique can be applied to the development of the new method for generating a positive real equivalent two-port network. Also, as described in Chapter 1, the formulation of the optimization (the way we apply the optimization technique) plays a very important role in the new two-port synthesis method. To implement this basic idea, a multi-step strategy is developed in the following section. In Chapter 3 we found that a poor move ability of an elemental circuit will cause a poor move ability of the two-port network. This will lead to a failure to generate the required frequency responses with the two-port network. Therefore, we must always try to use elemental circuits with good move abilities. This finding will also be applied to the development of the new synthesis method to guarantee the success of the optimization process. We shall apply the new method in detail through the example.

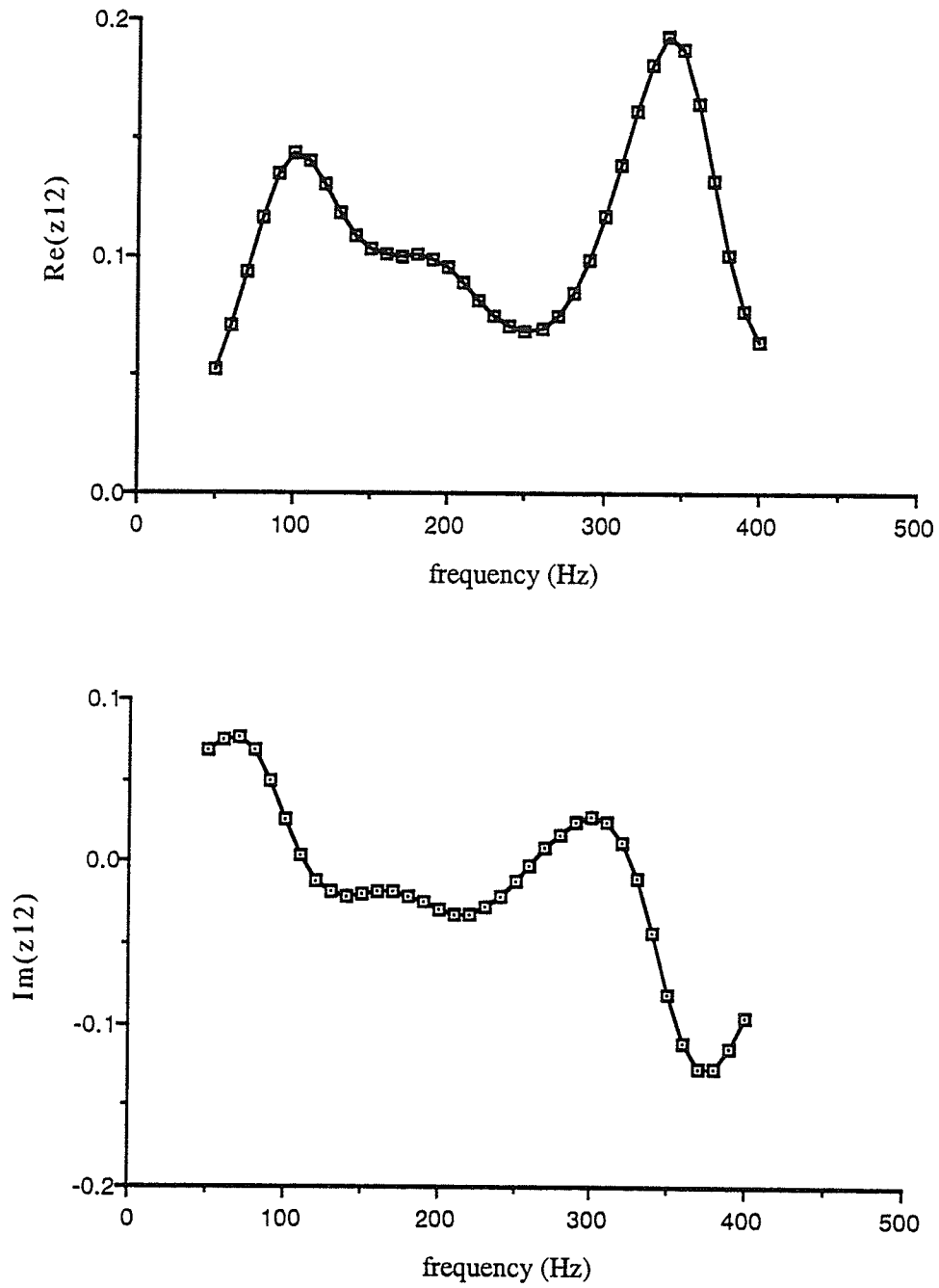


Fig. 4.1.1 Given frequency response of  $z_{12}$

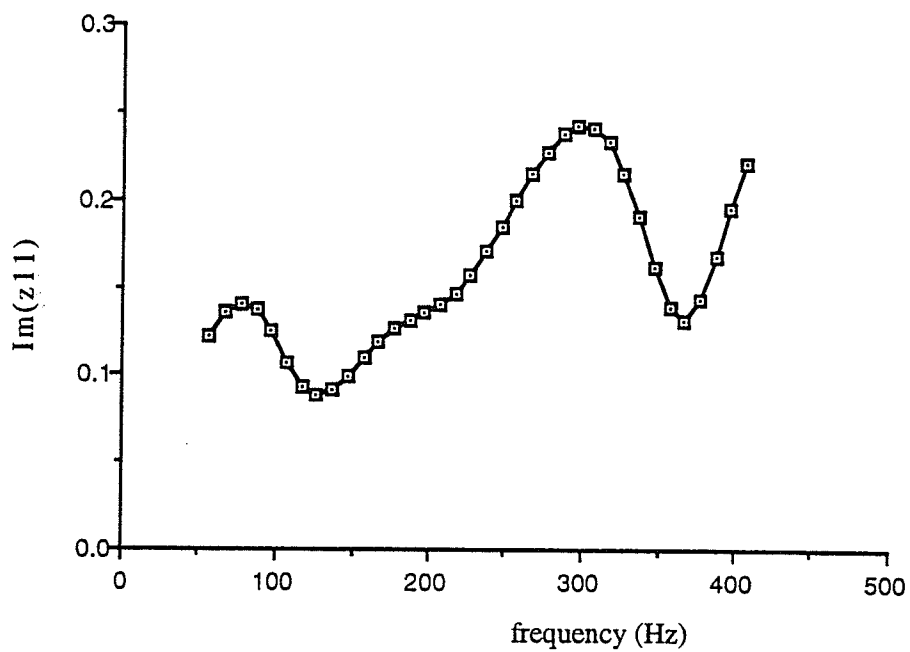
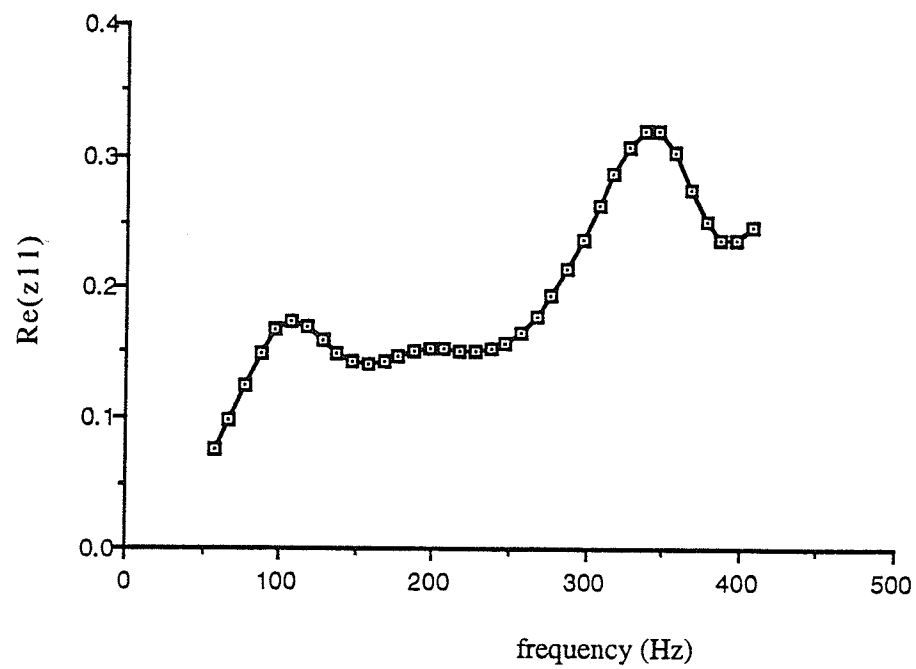


Fig. 4.1.2 Given frequency response of  $z_{11}$

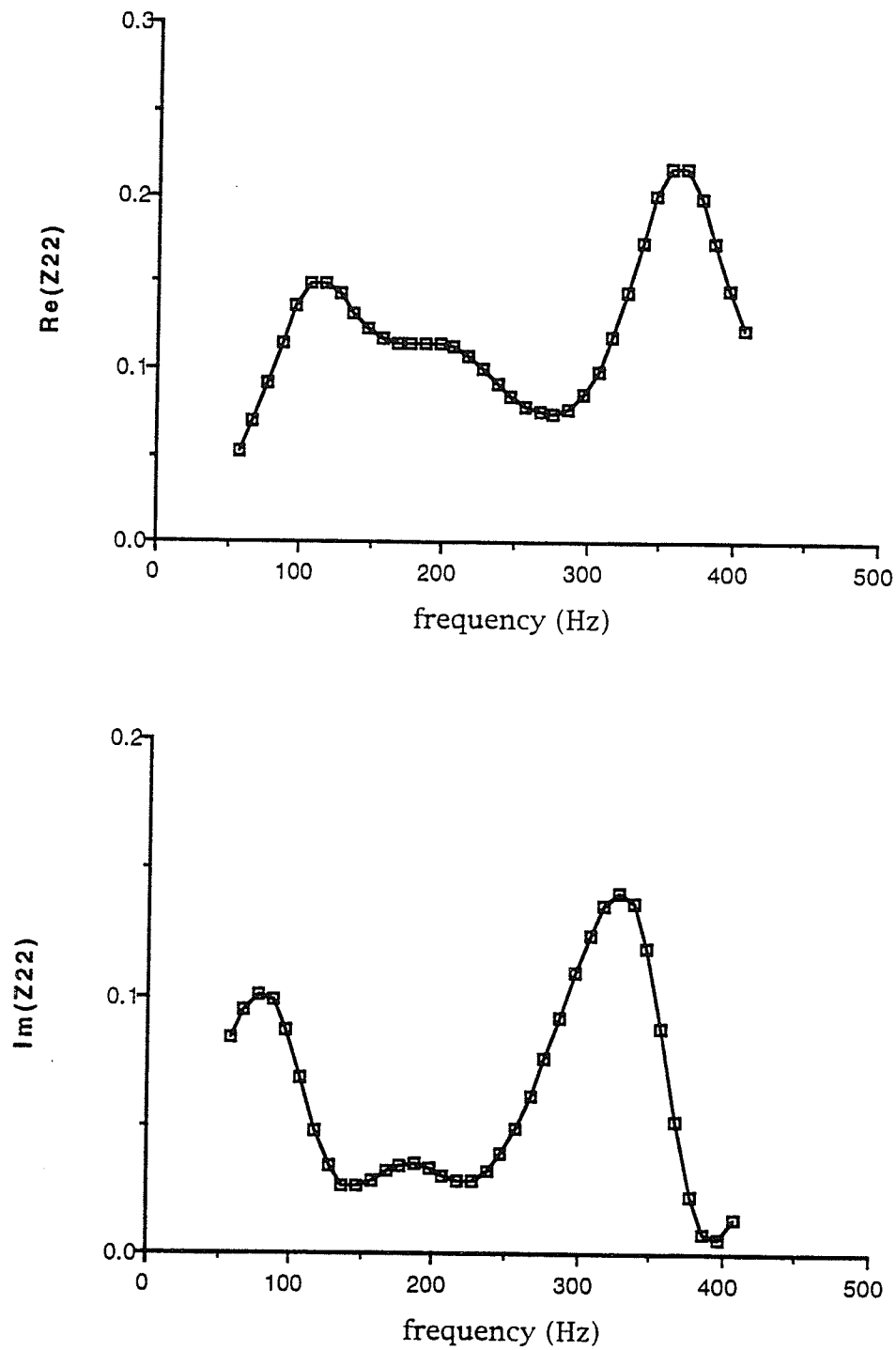


Fig. 4.1.3 Given frequency response of  $z_{22}$

## 4.2 The Strategy of Three-Step Optimization

The goal in modeling an ac system is to find a positive real equivalent two-port network that can give frequency responses close to the frequency responses of the ac system. This is a typical real modeling and synthesis problem. Now we will find a strategy for a new modeling and synthesis method by using the pattern search optimization technique. The new method must be able to guarantee a satisfactory solution with a positive real equivalent two-port network.

Looking back at Chapter 3, we find that the difficulty in satisfying the requirement for a set of zeros and poles is similar to that of satisfying one frequency response. In comparison to the problem we face right now, the example in Chapter 3 is a much simpler case. It can be solved by using optimization once—we call it the one-step approach. Now the synthesis task is to find a two-port network that has frequency responses very close to the given frequency responses of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ . Although the requirements are to satisfy only two more frequency responses, the difficulty for the latter is much more than three fold the former. We can see that the pattern search technique is a good tool. But to use a good tool to do a good job, we still need to use it in an appropriate way—an optimal way. The appropriate way is the right strategy.

The frequency response of an electric network depends on the structure of the electric network and the values of the network elements such as resistors, capacitors, inductors and transformers. This dependence is equivalent to dependence on the zeros and poles of the impedance functions of this network. Since all the good move elemental circuits have only relatively

good move ability, every elemental circuit has its own limitations. If the one-step approach is used, the result will most likely not be satisfactory. Then we will be practically unable to analyze what is wrong because there would be so many combinations of different elemental circuits and network structures. It will be very time-consuming to find the optimal combination. This raises the questions: Is the degree of the polynomial of the numerator of the impedance function of the two-port network not right? Is it the wrong type of elemental circuit? It is very likely to happen that, until most of the possible combinations have been tried, a good result would not be found. (From now on, for simplifying the description, we use 'degree  $m/n$ ' to represent the number of zeros  $m$  and the number of poles  $n$  of an impedance function of an electrical network, and call this electrical network a degree  $m/n$  network.) For example, from degree  $4/4$  to  $10/10$ , there are 19 different degree combinations. There are at least 30 different elemental circuits with degrees of  $1/2$  and  $2/2$ , therefore, the possible circuit combinations for each degree combination will be large and the total number of combination will be very large. To use the optimization program to calculate the values of all circuit components for all combinations would be very time consuming. Therefore, using the one-step approach to solve the problem of finding a circuit to realize  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  is not practical.

To solve this complex and difficult problem, we must find a strategy—a systematic approach. Like all other kinds of difficult problem solving, we can not solve the whole problem at once. We must try a step-by-step approach and in each step solve a piece of the overall problem. With this in mind, we see that the logical way to solve the complicated and difficult synthesis problem is to divide the whole problem into several parts and then to solve each part. We will find that the pattern search optimization method has to be used in

every step in which we attempt to solve a piece of the problem. In comparison with the previous one-step approach, this is called a multiple-step approach.

With some basic knowledge of modeling and synthesis of electric networks, we find that by using the modified pattern search optimization technique the whole procedure of synthesis can be divided into three parts which can be solved one part at a time. We call this the three-step approach. The three-step approach procedure is described as follows:

Step 1, determine the degrees of the numerator and denominator polynomials of the impedance function of the electrical network. For the whole synthesis procedure, the first thing we have to do is to find how high the degree of the electric network is. It includes determinations of the optimal degree of the numerator and the optimal degree of the denominator with certain types of zeros and poles of network functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ .

Step 2, find a network structure which can guarantee that the network will be able to give frequency responses very close to the required frequency responses.

Step 3, determine the elemental values of all network elements such as resistors, capacitors and inductors. As the network elements are determined, the impedance functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are also determined.

Fig. 4.2.1 shows a flowchart of the three-step approach. In the three-step approach, the number of tests is significantly reduced. The total number of tests is the sum of the numbers in each step. The total number of tests for the one-step approach is equal to the product of all three steps. For example, if the possible number of degree combinations is 19, the number of possible network structures is 20 and the number of elemental circuits is 5, then the sum of these numbers is 44 and the product of these numbers is 1900. From the example, we can see that the three-step approach has many fewer tests than



the one-step approach does. Comparing the three-step approach with the traditional network synthesis, we find that the three-step approach procedure is a complete reversal of the traditional network synthesis procedure. We will discuss the details of this new approach in the following sections.

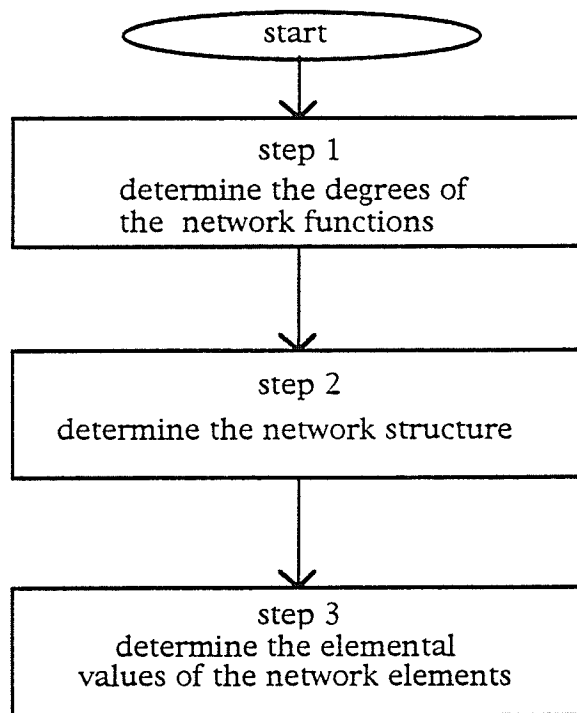


Fig. 4.2.1 Three-step approach

### 4.3 Step 1: Degrees of $z_{11}$ , $z_{12}$ and $z_{22}$

Now we will try to use the pattern search optimization method developed in Chapter 3 to determine the optimal degrees of an electric network. More specifically, it includes the degrees of the numerator polynomials and the denominator polynomials of the network functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  with certain types of poles and zeros. In order to apply the pattern search optimization technique, we must first determine the input variables and output target functions. Since the goal of modeling is to find a positive real equivalent two-port network that gives frequency responses close to the frequency responses of an ac system, the target functions to be found must be the frequency responses of the to be found  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  and the input variables of the optimization are the poles and zeros of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ . The optimization is to change the values of the poles and the zeros of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  to obtain the required frequency responses. There are two types of poles and zeros. One kind is real and the other kind is a complex conjugate pair. At the beginning of Section 2, we described two major reasons why we do not use a one-step approach. The first one is that every elemental circuit has limitations, i.e., its zeros and poles can not move over the entire (left-half)  $s$ -plane. The second one is that there are so many combinations of different network structures and elements that it would be very difficult to find the optimal combinations. In order to determine: 1) the optimal degrees of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ , 2) the types of zeros and poles of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  associated with 1), we must test combinations of different degrees and different types of zeros and poles of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ . Since we only want to find the optimal numbers of the poles and zeros of the impedance functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ , at this stage, we carry out separate optimization processes for  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ . We will start with  $z_{12}$ .

### 4.3.1 Degrees and Types of Zeros and Poles of $z_{12}$

#### 4.3.1-a The Algorithm

In order to apply the pattern search optimization technique to the optimization, we first determine the input variables and the target functions. Since the task is to find a transfer function  $z_{12}$  that has a frequency response very close to the given frequency response, we consider the following objective function:

$$F = \sum_{i=1}^{m'} |z_{12}(j\omega_i) - z'_{12}(j\omega_i)| \quad (4.3.1-1)$$

where  $m'$  is the number of given frequency responses points,  $z'_{12}(j\omega_i)$  is the given frequency response and  $z_{12}(j\omega_i)$  is the frequency response of transfer function  $z_{12}$  to be found.  $z_{12}(j\omega_i)$  is a function of zeros, poles and a constant factor, which are to be found i.e.,

$$z_{12} = \frac{K_{12} \prod_{k=1}^m (s - z_k)}{\prod_{l=1}^n (s - p_l)} \quad (4.3.1-2)$$

where  $z_k$  are the zeros,  $p_l$  are the poles,  $K_{12}$  is the constant factor,  $m$  is the number of the zeros and  $n$  is the number of the poles of  $z_{12}$ .

In the optimization,  $K_{12}$  and the zeros and poles of  $z_{12}$  are the variables. Now we consider some of the details of the computer program. As described above, we only need to deal with two kinds of zeros and poles: real and complex conjugate pairs. For a complex zero, two variables are needed.

$$x_k = \text{Re}(z_k) \quad k = 1, 2, \dots, m \quad (4.3.1-3)$$

$$y_k = \text{Im}(z_k) \quad k = 1, 2, \dots, m \quad (4.3.1-4)$$

where  $\text{Re}(z_k)$  and  $\text{Im}(z_k)$  denote the real and imaginary parts of a complex variable  $z_k$ . Similarly for a complex pole, two variables are needed as well.

$$u_k = \text{Re}(p_k) \quad k = 1, 2, \dots, n \quad (4.3.1-5)$$

$$v_k = \text{Im}(p_k) \quad k = 1, 2, \dots, n \quad (4.3.1-6)$$

For a real pole or a real zero, one variable is sufficient for the program. Hence we have:

$$x_k = z_k \quad k = 1, 2, \dots, m_1 \quad (4.3.1-3)$$

$$u_k = p_k \quad i = 1, 2, \dots, n_1 \quad (4.3.1-4)$$

where  $m_1$  is the number of real zeros and  $n_1$  is the number of real poles.

Fig. 4.3.1-1 gives a flowchart of the optimization procedure for finding the optimal degrees of the numerator and the denominator for different types of zeros and poles of  $z_{12}$ . The flowchart is very similar to the one shown in Fig. 3.4.1. Since  $z_{12}$  is not necessarily positive real, the flowchart shown in Fig. 4.3.1-1 is appropriate for many cases. However, for certain cases  $z_{12}$  may need to satisfy the PR conditions. Then the real parts of the zeros and poles of  $z_{12}$  must satisfy:

$$\text{Re}(z_k) \leq 0 \quad k = 1, 2, \dots, m \quad (4.3.1-9)$$

$$\text{Re}(p_k) \leq 0 \quad l = 1, 2, \dots, n \quad (4.3.1-10)$$

where  $\text{Re}(z_k)$  is the real part of a zero and  $\text{Re}(p_k)$  is the real part of a pole.

In Chapter 3, the input variables are the element values of the elemental circuits. By keeping the values of elemental circuit components positive, the PR (positive real) conditions are satisfied, i.e., the constraint of the optimization is satisfied. However, in the process of the first step of the three-step optimization, keeping the real parts of all the zeros and the real parts of all poles negative is not sufficient to satisfy the PR requirements.

For a rational function  $Z(s)$  to be positive real, necessary and sufficient conditions are [3]:

- (1)  $Z(s) = N(s)/D(s)$  is real for  $s$  real
- (2)  $N(s) + D(s)$  is strictly Hurwitz
- (3) The real even polynomial

$$E(s) = ND_* + N_*D$$

is non-negative along the entire  $j\omega$ -axis, i.e.

$$\{E(s)\}_{s=j\omega} = \hat{E}(\omega^2) \geq 0 \quad -\infty \leq \omega \leq \infty$$

The above conditions can be used for testing if a rational function is positive real. Condition (1) is satisfied if the coefficient of  $Z(s)$  are real or equivalently if the zeros are real or occur in complex conjugate pairs. Condition (2) is satisfied if the zeros are all in the open left-half  $s$ -plane. In order for  $\hat{E}(\omega^2)$  to be non-negative for all  $\omega$ , the necessary and sufficient condition is that the real positive  $\omega^2$  zeros of  $\hat{E}(\omega^2)$  must be of even multiplicity. Therefore, by determining the multiplicity of the positive  $\omega^2$  zeros of the real even polynomial  $\hat{E}(\omega^2)$ , condition (3) is easily checked. A computer program [30] for checking the above positive real conditions is used in the optimization program.

To retain only those  $z_{12}$  which are PR, the flowchart of Fig. 4.3.1-1 must be modified by the addition of the flowchart shown in Fig. 4.3.1-2. The computer program of flowchart Fig. 4.3.1-2 implements the above checks for the positive real conditions and determines if the transfer function found during the single variable search is PR or not. If the program detects a movement of a pole or a zero which causes the transfer function to become non PR, then the move is declared a failure and the pole or the zero has to remain in its former PR position. Fig. 4.3.1-3 shows the flowchart for checking if  $z_{12}$  is PR during the pattern search. If the program detects a movement of a pole or a zero causing the transfer function to become non PR, then the move is declared a failure and the pole or the zero has to remain its former PR position. In order to maintain  $z_{12}$  always PR, a PR starting point is required. By moving the zeros or the poles of  $z_{12}$  further to the left of the complex plane, a PR starting point can always be obtained. In this way, a positive real transfer function  $z_{12}$  is always guaranteed.

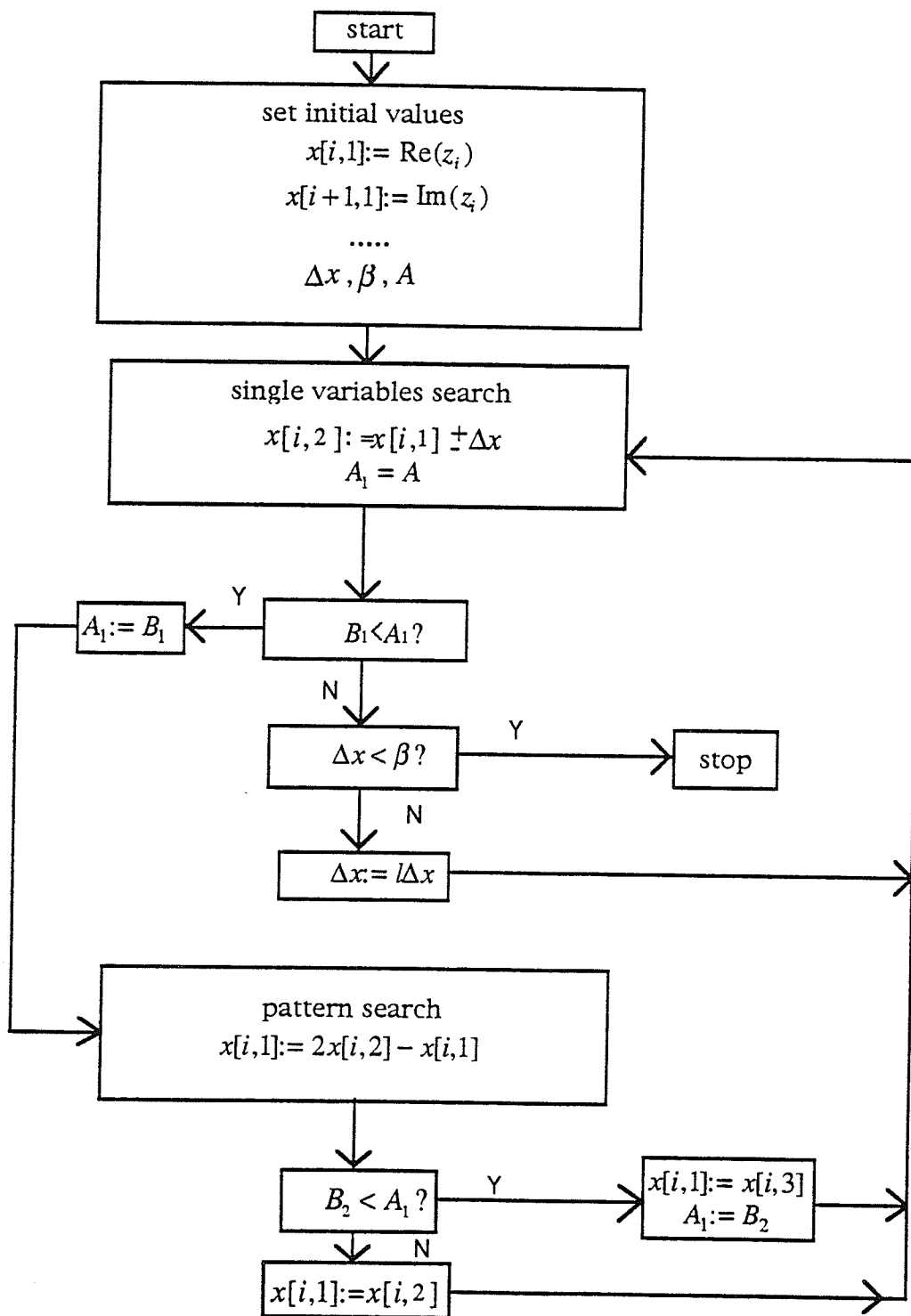


Fig. 4.3.1-1 Flowchart for determining degree of  $z_{12}$

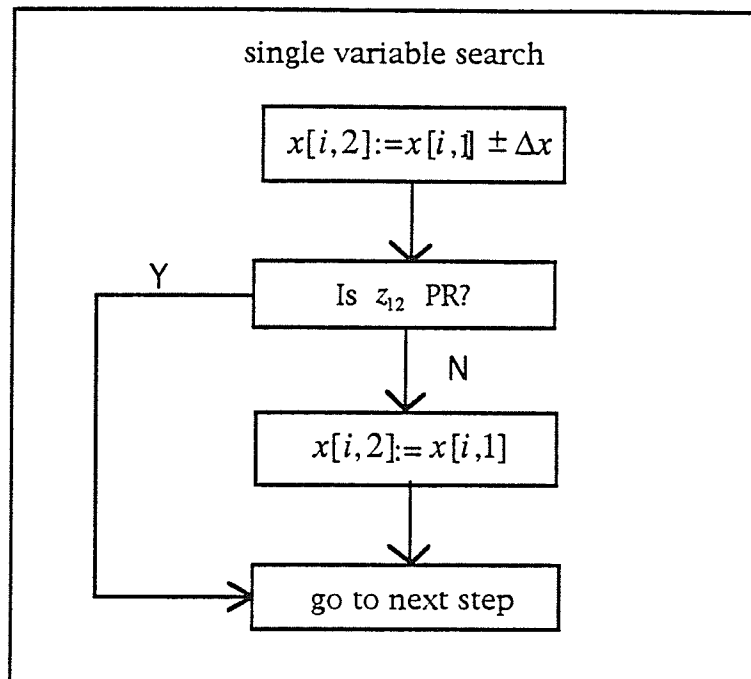


Fig. 4.3.1-2 Flowchart for PR checking in single variable search

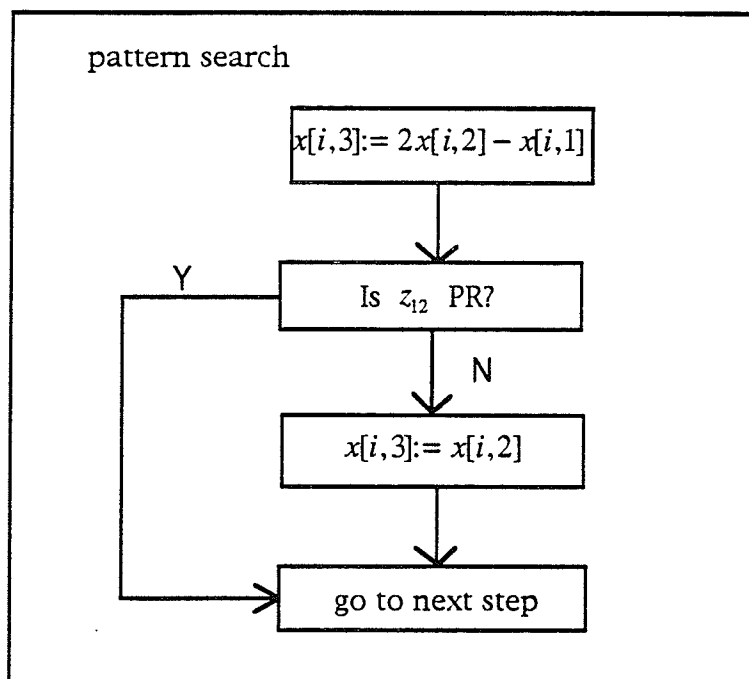


Fig. 4.3.1-3 Flowchart for PR checking in pattern search



#### 4.3.1-b Finding the starting point for the optimization

A starting point for the optimization can be found by using the group delay [30]. This method is based on the relation between the group delay and the frequency response of the real and imaginary parts of the impedance function and the zeros and poles of the impedance function. Two expressions for the group delay are used. One is in terms of the real and imaginary parts of the impedance function and their derivatives. The other expression for group delay is in terms of the zeros and poles of the impedance function. The group delay curve is determined from the real part and the imaginary part of the frequency response data. The group delay  $\tau$  is defined by

$$\tau(j\omega) = -\frac{d\theta(j\omega)}{d\omega}$$

where  $\theta(j\omega)$  is the phase of  $Z(j\omega)$  and

$$\tan \theta(j\omega) = \frac{\text{Im}(Z)}{\text{Re}(Z)}$$

where  $\text{Im}(Z)$  is the imaginary part and  $\text{Re}(Z)$  is the real part of the impedance function  $Z(s)$ . Let  $X = \text{Im}(Z)$  and  $R = \text{Re}(Z)$  to simplify the notation, then we have

$$\theta = \tan^{-1}\left(\frac{X}{R}\right)$$

$$\frac{d\theta}{d\omega} = \frac{d}{d\omega} \left( \tan^{-1}\left(\frac{X}{R}\right) \right)$$

$$= \frac{-X'R + R'X}{R^2 + X^2}$$

where the prime denotes differentiation. Thus the expression for the group delay in terms of the real and the imaginary parts and their derivations is

$$\tau(j\omega) = \frac{X'R - R'X}{R^2 + X^2} \quad (4.3.1-b1)$$

The values of the derivatives are calculated by using Lagrange's interpolation formula [40].

We can obtain another expression for the group delay  $\tau(j\omega)$  in terms of the zeros and poles of the impedance function  $Z(s)$  using the following steps.

Let

$$Z(s) = \frac{N(s)}{D(s)} = \frac{k \prod_{i=1}^m (s - z_i)}{\prod_{i=1}^n (s - p_i)}$$

where  $z_i$  is a zero and  $p_i$  is a pole. We have

$$\ln(Z(s)) = \ln N(s) - \ln D(s)$$

$$\frac{d \ln Z(s)}{ds} = - \left( \frac{D'(s)}{D(s)} - \frac{N'(s)}{N(s)} \right)$$

Also

$$\ln Z(s) = \ln k + \sum_{i=1}^m \ln(s - z_i) - \sum_{i=1}^n \ln(s - p_i)$$

and

$$\frac{d \ln Z(s)}{ds} = \sum_{i=1}^m \frac{1}{s - z_i} - \sum_{i=1}^n \frac{1}{s - p_i}$$

The group delay is also given by [39]:

$$\tau(j\omega) = \operatorname{Re} \left\{ \frac{D'(s)}{D(s)} - \frac{N'(s)}{N(s)} \right\} \Big|_{s=j\omega}$$

Therefore, we have

$$\begin{aligned}
\tau(j\omega) &= \operatorname{Re} \left\{ -\frac{d \ln Z(s)}{ds} \right\} \Big|_{s=j\omega} \\
&= \sum_{i=1}^n \operatorname{Re} \left( \frac{1}{s-p_i} \right) \Big|_{s=j\omega} - \sum_{i=1}^m \operatorname{Re} \left( \frac{1}{s-z_i} \right) \Big|_{s=j\omega} \\
&= \sum_{i=1}^n \left( \frac{\alpha_i}{\alpha_i^2 + (\omega - \beta_i)^2} \right) - \sum_{i=1}^m \left( \frac{\gamma_i}{\gamma_i^2 + (\omega - \delta_i)^2} \right) \quad (4.3.1b-2)
\end{aligned}$$

where  $p_i = -\alpha_i + j\beta_i$  and  $z_i = -\gamma_i + \delta_i$ . Using equation (4.3.1b-1) we can find the curve of the group delay  $\tau(j\omega)$ . Using equation (4.3.1b-2) we can estimate the real parts of the poles which are the negatives of the inverses of the peak values of the curve and the imaginary parts of the poles, which are the frequencies corresponding to the peaks of the curve. Also, we can estimate the real parts of the zeros from the inverses of the negative values at the valleys of the group delay curve and the imaginary parts of the zeros from the frequencies corresponding to the valleys of the curve. Fig. 4.3.1-4 shows a curve of group delay and the estimates of the zeros and poles. The above method can be justified as follows: when the poles and zeros are not very close together, one term of (2) dominates in the neighborhood of a pole or a zero. For example, for a dominant pole, we have

$$\tau(j\omega) \approx \frac{\alpha_i}{\alpha_i^2 + (\omega - \beta_i)^2}$$

At  $\omega = \beta_i$ , we have a maximum and

$$\tau(j\beta_i) \approx \frac{1}{\alpha_i}$$

Therefore, the above method for estimating the starting point is readily applied. However, if the poles and zeros are close together, one term of (2) no

longer clearly dominates in the neighborhood of a pole or a zero. The starting point then can not be estimated precisely.

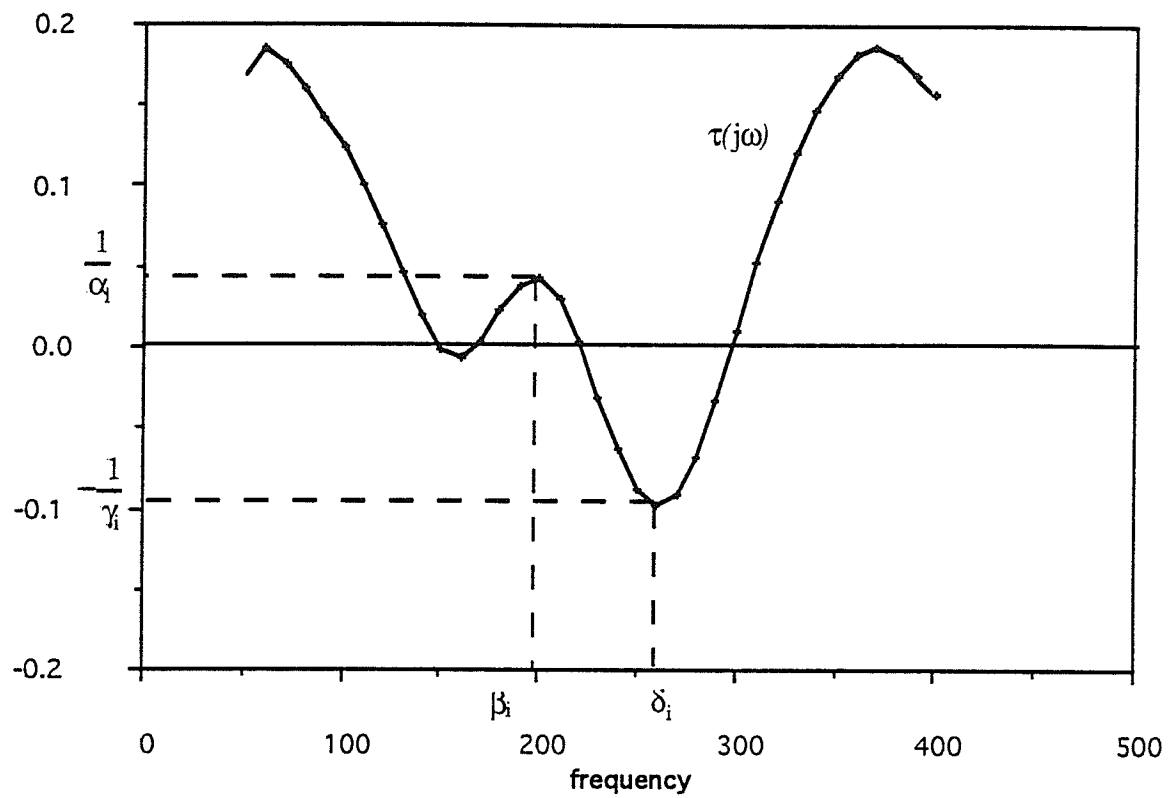


Fig. 4.3.1-4

### 4.3.1c Tests and Results

Using the method described above for finding the starting point, the values of the real and imaginary parts of the poles and zeros of the starting point for  $z_{12}$  are found from the given data (see Appendix B). Table 4.3.1-1 displays the estimated values for the starting point of  $z_{12}$ . (In all the following tables the absolute value of the real part is given. Also, in the following sections, only absolute error is used.)

Table 4.3.1-1 Starting point for  $z_{12}$

zeros:

#	real part	imaginary part
1	1060	870
2	1570	1380

poles:

#	real part	imaginary part
1	950	2250
2	1550	1250
3	850	650

Since the group may not give all zeros and poles, some additional combinations of zeros and poles are tested. Table 4.3.1-2 displays the calculated results for different combinations. The number of iterations is 600 for all combinations. For combination #1 (5/5 ) the error is 2.958. The error of

combination #2 (5/6) is 0.4308. The higher degree combination #3 (6/6) yields a higher error (2.723) than that of #2. However, the highest degree combination #4 (7/8) has the lowest error 0.4303. From these results it can be seen that the optimal degree difference between the numerator and the denominator is one, i.e., the degree of the numerator is one less than that of the denominator. Therefore, the degree difference between numerator and denominator plays a very important role for finding the optimal degree of the impedance function.

Table 4.3.1-2 The test results of different combinations

#	degree of numerator	degree of denominator	number and types of zeros	number and types of poles	error
1	5	5	2 complex conjugate pairs 1 real	2 complex conjugate pairs 1 real	2.958
2	5	6	2 complex conjugate pairs 1 real	3 complex conjugate pairs	0.4308
3	6	6	3 complex conjugate pairs	3 complex conjugate pairs	2.725
4	7	8	3 complex conjugate pairs 1 real	4 complex conjugate pairs	0.4303

## 4.3.2 Degrees and Zeros and Poles of $z_{11}$

### 4.3.2-a The Algorithm

In a way similar to (4.3.1), we determine the variables and target function first. We have the following objective function:

$$F = \sum_{i=1}^{m'} |z_{11}(j\omega_i) - z'_{11}(j\omega_i)| \quad (4.3.2-1)$$

where  $m'$  is the number of given frequency responses points,  $z'_{11}(j\omega_i)$  is the given frequency response and  $z_{11}(j\omega_i)$  is the frequency response of impedance function  $z_{11}$  which is to be found.  $z_{11}(j\omega_i)$  is the target function and a function of the zeros, poles and constant factor, which are to be found:

$$z_{11} = \frac{K_{11} \prod_{k=1}^m (s - z_k)}{\prod_{l=1}^n (s - p_l)} \quad (4.3.2-2)$$

where  $z_k$  denotes the zeros of  $z_{11}$ .  $p_l$  represents the poles of  $z_{11}$ .  $m$  is the number of the zeros and  $n$  is the number of the poles of  $z_{11}$ ,  $K_{11}$  is a constant factor.  $K_{11}$  and the zeros and poles of  $z_{11}$  are the variables of the optimization.

The impedance function  $z_{11}$  must satisfy the positive real conditions. Therefore, during the optimization process, the PR conditions must be checked, after every move of either a single variable search move or a pattern search move. The technique for PR checking described in Section 4.3.1 is used. If the program finds the PR conditions are violated by a move, then the move is a failure and the pole or the zero has to retain its former PR position. In this way, a positive real impedance function  $z_{11}$  is always guaranteed. Fig. 4.3.2-1 displays a flowchart of the optimization procedure for finding the optimal

degree of the numerator and the denominator and the types of zeros and poles of  $z_{i1}$ . The flowchart is similar to that of  $z_{i2}$  shown in Figs. 4.3.1-1, 4.3.1-2 and 4.3.1-3.

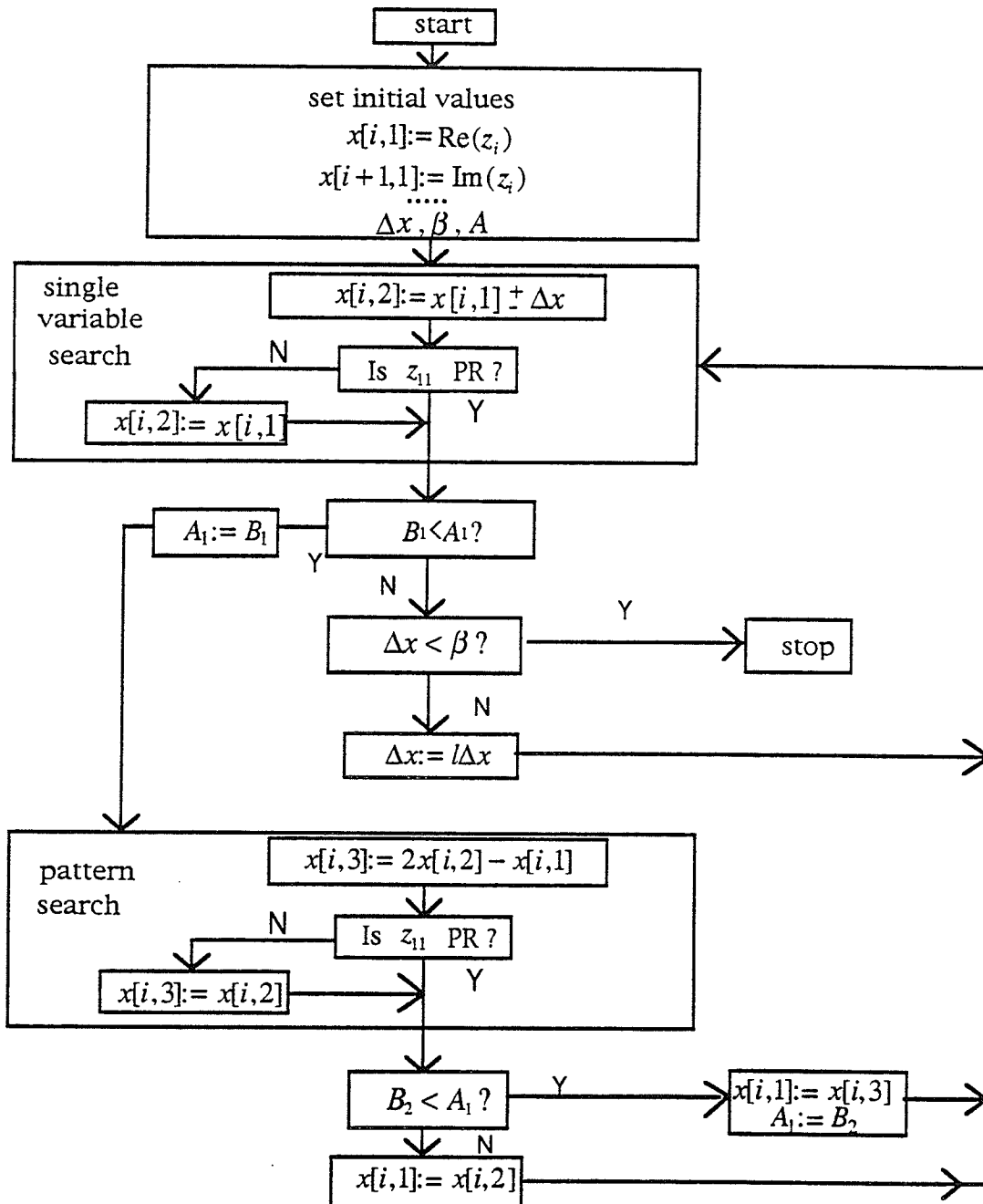


Fig. 4.3.2-1 Flowchart for determining degree of  $z_{i1}$



#### 4.3.2-b Tests and Results

To find the optimal combination of degrees and types of zeros and poles of the driving point function  $z_{11}$ , various examples were tested. Similar to  $z_{12}$ , to begin the test for the optimal degree of  $z_{11}$ , a good starting point is needed. The method of using the group delay to find a starting point is applied here as well. Table 4.3.2-1 gives the values for the starting point determined by group delay.

Table 4.3.2-1

zeros

#	real part	imaginary part
1	900	850
2	1440	1320
3	680	2242

poles

#	real part	imaginary part
1	600	480
2	2100	1100
3	650	2000

Some different combinations of degrees and zeros and poles are tested.  
The test results are listed in Table 4.3.2-2.

Table 4.3.2-2

#	degree of numerator	degree of denominator	number & types of zeros	number & types of poles	error
1	7	6	3 complex conjugate pairs 1 real	3 complex conjugate pairs	1.03
2	7	7	3 complex conjugate pairs 1 real	3 complex conjugate pairs 1 real	1.85
3	8	7	4 complex conjugate pairs	3 complex conjugate pairs 1 real	0.794
4	9	8	4 complex conjugate pairs 1 real	4 complex conjugate pairs	0.763

From Table 4.3.2-2, it can be seen that the optimal degree difference between the numerator and the denominator is that the degree of the numerator is one larger than that of the denominator. With this optimal degree difference, #4 gave the lowest error.

### 4.3.3 Degrees and Zeros and Poles of $z_{22}$

#### 4.3.3-a The Algorithm

In a way similar to (4.3.2), we determine the variables and target function for the optimization first. The objective function is

$$F = \sum_{i=1}^{m'} |z_{22}(j\omega_i) - z'_{22}(j\omega_i)| \quad (4.3.3-1)$$

where  $z'_{22}(j\omega_i)$  is the given frequency response and  $z_{22}(j\omega_i)$  is the frequency response of impedance function  $z_{22}$  which is to be found.  $z_{22}(j\omega_i)$  is a function of the zeros, the poles and the constant factor which are to be found:

$$z_{22} = \frac{K_{22} \prod_{k=1}^m (s - z_k)}{\prod_{l=1}^n (s - p_l)} \quad (4.3.3-2)$$

where  $z_k$  denotes the zeros of  $z_{22}$ ,  $p_l$  represents the poles and  $K_{22}$  is the constant factor; and these are the variables of the optimization. The real parts of the zeros and poles must always satisfy the PR conditions. During the optimization process, the PR conditions are always checked by the PR checking program to guarantee that the computed impedance function  $z_{22}$  is PR. The flowchart for this optimization is the same as the flowchart shown in Fig. 4.3.2.-1 except for the name of the impedance function, therefore, it is not given.

### 4.3.3-b Tests and Results

The starting point for the optimization is calculated by the method of Section 4.3.1-b. The real parts and imaginary parts of the starting point determined by the group delay method are listed in Table 4.3.3-1.

Table 4.3.3-1 The starting point of  $z_{22}$   
zeros

#	real part	imaginary part
1	2500	880
2	600	1400
3	900	2500

poles

#	real part	imaginary part
1	780	530
2	3000	1200
3	700	2100

To find the optimal combination of degrees and types of zeros and poles of the transfer function  $z_{22}$ , different combinations were tested. Table 4.3.3-2 gives the test results of three combinations of different degrees and types of zeros and poles of  $z_{22}$ . For all examples, the number of iterations is 1000. From

Table 4.3.3-2, it is clear that the optimal difference in degree between numerator and denominator is one.

Table 4.3.3-2 Test result of  $z_{22}$

#	degree of numerator	degree of denominator	number & types of zeros	number & types of poles	error
1	8	8	4 complex conjugate pairs	4 complex conjugate pairs	2.3
2	9	8	4 complex conjugate pairs 1 real	4 complex conjugate pairs	0.18
3	9	9	4 complex conjugate pairs 1 real	4 complex conjugate pairs 1 real	1.9

## 4.4 Step 2: Structures of the Two-Port Network

In the preceding study, we have found some optimal combinations of degrees and types of zeros and poles for  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ . Now we will try to find the network structures. Just as in the arguments we made in Section 4.2 of this chapter, we foresee that a complicated network structure of a high degree ( $>6$ ) network is very difficult to achieve at once. Therefore, we divide the whole network structure into two levels. We call the first one the main structure and the second one the micro structure. We will see that this classification gives us a much more convenient way to find the optimal network structure by using the pattern search optimization technique.

### 4.4.1 The Main Structure for the Network

In Chapter one, five main structures are presented. The T main structure is used for the example under consideration.

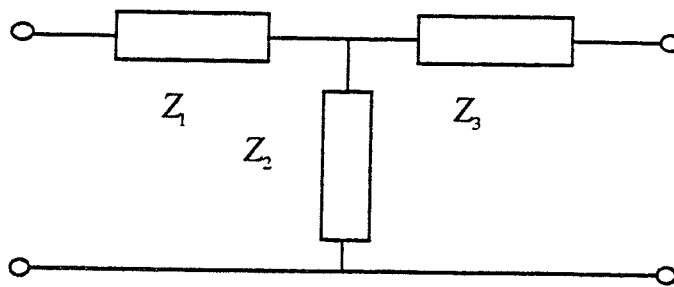


Fig. 4.4.1 T two-port network

For the T two-port network, the relationships between the impedance functions  $Z_1$ ,  $Z_2$  and  $Z_3$  and  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are very simple, i.e.,

$$z_{12} = Z_2 \quad (4.4.1-1a)$$

$$z_{11} = Z_1 + Z_2 = Z_1 + z_{12} \quad (4.4.1-1b)$$

$$z_{22} = Z_3 + Z_2 = Z_3 + z_{12} \quad (4.4.1-1c)$$

In Step 1, since we only try to find the optimal degrees of impedances  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  with certain types of zeros and poles, we can compute either  $z_{11}$ , or  $z_{12}$  or  $z_{22}$  first. However, in Step 2, we calculate  $z_{12}(Z_2)$  first and then  $z_{11}$  and  $z_{22}$ . From the above equations (4.4.1-1a-c), it is clear that  $z_{12}$  is a part of  $z_{11}$  and  $z_{22}$  for the T structure. Therefore,  $z_{11}$  and  $z_{22}$  can have private poles. In other words, there is less freedom in the choice of poles for  $z_{12}$  than for  $z_{11}$  and  $z_{22}$ . If necessary, poles and zeros can be added to both  $z_{11}$  and  $z_{22}$  without affecting  $z_{12}$ . In Step 1, we observed that  $z_{12}$  is not generally required to be PR, however, for certain cases it must be. Clearly, this is one of those cases. There are some other general requirements for the realization of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ , but we will discuss these requirements later.

For the optimization process, not only must the positive real conditions of the individual impedance functions, but also the two-port positive real conditions be satisfied. For the T structure, however, as long as each impedance function is positive real, then the two-port PR conditions are automatically satisfied. This feature is an advantage, since checking the two-port PR conditions for each movement of the optimization would be very time consuming.

#### 4.4.2 Micro Structure of $z_{12}(Z_2)$

After the main network structure is chosen, the next step is to find the micro structures of the networks. We must also determine the impedance functions of the elements of the micro structures. From Step 1, a good degree configuration for  $z_{12}$  is found to be five for the numerator and six for the denominator for the example under consideration. In order to satisfy the requirement for the types of zeros and poles of  $z_{12}$ , we must choose the appropriate elemental impedance functions. The chosen elemental impedance functions must be realizable, that is, they must be realized by a circuit containing R, L, C and transformer components only. There are several types of elemental impedance functions which can be used for the micro structures, and of these, we select six. These six elemental impedance functions are:

1. A 0/1 circuit, where the degree of the numerator is zero, the degree of the denominator is one, and the pole is a real number. The impedance function is

$$z = \frac{a}{s+b}$$

2. A 1/0 circuit, where the degree of the numerator is one, the degree of the denominator is zero, and the zero is real:

$$z = as + b$$

3. A 1/1 elemental circuit, where the degrees of both the numerator and denominator are 1, and both the pole and zero are real:

$$z = \frac{as+b}{s+c}$$



4. A 1/2 elemental circuit, the degree of the numerator is one and the degree of the denominator is two, the zero is real, and the poles are real or a complex conjugated pair:

$$z = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0}$$

5. A 2/1 elemental circuit, the degree of the numerator is two and the degree of the denominator is one, the pole is real, and the zeros are real or a complex conjugate pair:

$$z = \frac{a_2 s^2 + a_1 s + a_0}{s + b_0}$$

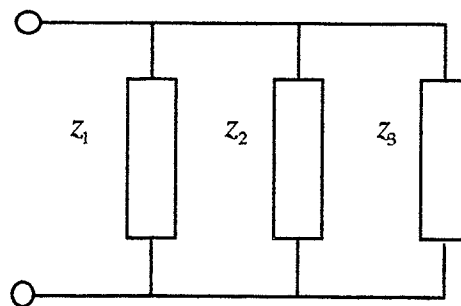
6. A 2/2 elemental circuit, the degrees of both the numerator and denominator are two, and both zeros and poles are real or complex conjugate pairs:

$$z = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

All six kinds of elemental impedance functions must satisfy the positive real requirement. Other kinds of impedance functions can be constructed by using these six kinds of elemental impedance functions.

There are many micro-structures which satisfy the requirements of degrees and types of the zeros and poles of  $z_{12}$ . Fig. 4.4.2-1 demonstrates three micro-structures.

(a) parallel



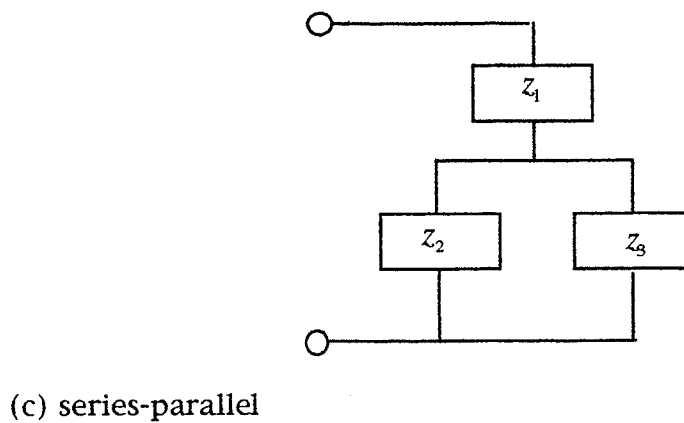
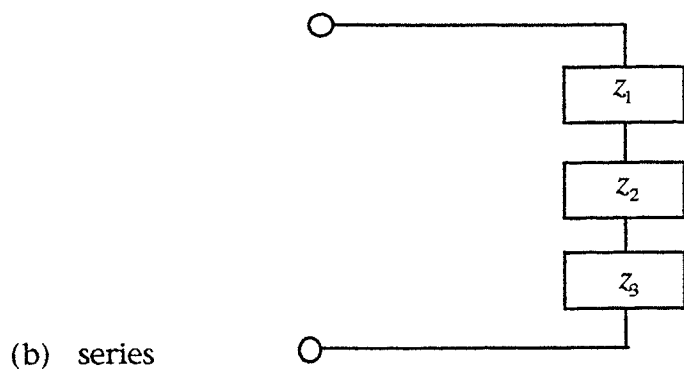


Fig. 4.4.2-1 Three Micro structures

For the parallel micro structure, the relation between  $z_{12}$  and  $z_1 = \frac{n_1}{d_1}$ ,  $z_2 = \frac{n_2}{d_2}$ ,  $z_3 = \frac{n_3}{d_3}$  is

$$z_{12} = z_1 // z_2 // z_3$$

$$= \frac{n_1 n_2 n_3}{n_1 n_2 d_3 + n_1 n_3 d_2 + n_2 n_3 d_1} \quad (4.4.2-1)$$

For the example being considered, the sum of the degrees of  $n_1$ ,  $n_2$  and  $n_3$  must be five. Thus if two of them are two, then the third must be 1 and if the degrees of  $d_1$ ,  $d_2$  and  $d_3$  are all two, the degree of the denominator of  $z_{12}$  will be six.

For the series micro structure, the relation between  $z_{12}$  and  $z_1, z_2, z_3$  is

$$z_{12} = z_1 + z_2 + z_3$$

$$= \frac{n_1 d_2 d_3 + n_2 d_1 d_3 + n_3 d_1 d_2}{d_1 d_2 d_3} \quad (4.4.2-2)$$

A possible combination of  $z_1, z_2$  and  $z_3$  is that all the degrees of the numerators are 1, and all the degrees of the denominators are 2 to give 5/6.

For the series-parallel micro structure, we have:

$$z_{12} = z_1 + z_2 // z_3$$

$$= \frac{n_1 n_2 d_3 + n_1 n_3 d_2 + n_2 n_3 d_1}{n_2 d_1 d_3 + n_3 d_1 d_2} \quad (4.4.2-3)$$

The requirement that the numerator degree equals 5 and the denominator degree equals 6 is satisfied if  $z_2$  or  $z_3$  has a numerator of degree 2 and a denominator of degree 2, and the other two have a numerator of degree 1 and a denominator of degree 2.

After selecting the degrees of the zeros and poles for  $z_1, z_2$  and  $z_3$ , the next step is to select the types of zeros and poles of the impedance functions of the network elements. For the optimization process, the input variables are the constant factor, the zeros and poles of the elemental impedance functions, and the target function  $z_{12}(j\omega_i)$  is the frequency response of transfer function  $z_{12}$  which is to be found:

$$z_{12}(j\omega_i) = f(n_1, d_1, n_2, d_2, n_3, d_3) \quad (4.4.2-4)$$

where  $f$  denotes the function given in (4.4.2-1, (4.4.2-2) or (4.4.2-3);  $n_1, n_2$  and  $n_3$  are the numerators of  $z_1, z_2$  and  $z_3$ , respectively;  $d_1, d_2$  and  $d_3$  are the denominators of  $z_1, z_2$  and  $z_3$ , respectively. Thus we have the objective function:

$$F = \sum_{i=1}^m |z_{12}(j\omega_i) - z'_{12}(j\omega_i)| \quad (4.4.2-5)$$

where  $F$  is the objective function of the optimization, and  $z'_{12}(j\omega_i)$  is the given frequency response and  $z_{12}(j\omega_i)$  is defined above.

If the series connection is adopted, the poles of three elemental impedances are already known, since they are the poles of  $z_{12}$ . This makes the starting point easy to determine which is an advantage of the series connection. In order to compare the series connection with the other two connections being considered, the known poles were not chosen as the starting point. However, the convergence of the optimization in Step 2 with the series connection is observed to be still faster than with the other two connections.

A flowchart of this optimization procedure is shown in Fig. 4.4.2-2. The starting point of the optimization for the network structure is selected as follows: (1) Select poles and zeros according to the zeros and poles shown in Table 4.3.1-1 of Step 1. (2) For the zeros and poles not determined from Table 4.3.1-1, select poles and zeros of the same orders, as those listed in Table 4.3.1-1. For example, for the parallel connection, we choose two zeros for two elemental impedances according to Table 4.3.1-1, and the real zero of the 1/2 elemental impedance is chosen to be 900—the same order of magnitude as those in Table 4.3.1-1. The poles of  $z_1, z_2$  and  $z_3$  are also chosen to be the same order of magnitude as the poles shown in Table 4.3.1-1. Also, zeros and poles of the series connection and the series-parallel connections are selected in the same

way. During the optimization process, the program monitors each elemental impedance function  $z_i$  ( $i=1,2,3$ ). If a move of a pole or a zero of  $z_i$  causes the impedance function to become non PR, then the move is abandoned. Similar to Section 4.3.1-a, in order to maintain  $z_i$  PR, a PR starting point is required. By moving the zeros or the poles of  $z_i$  further to the left in the complex plane, a PR starting point can always be obtained. In this way, a positive real  $z_i$  is guaranteed.

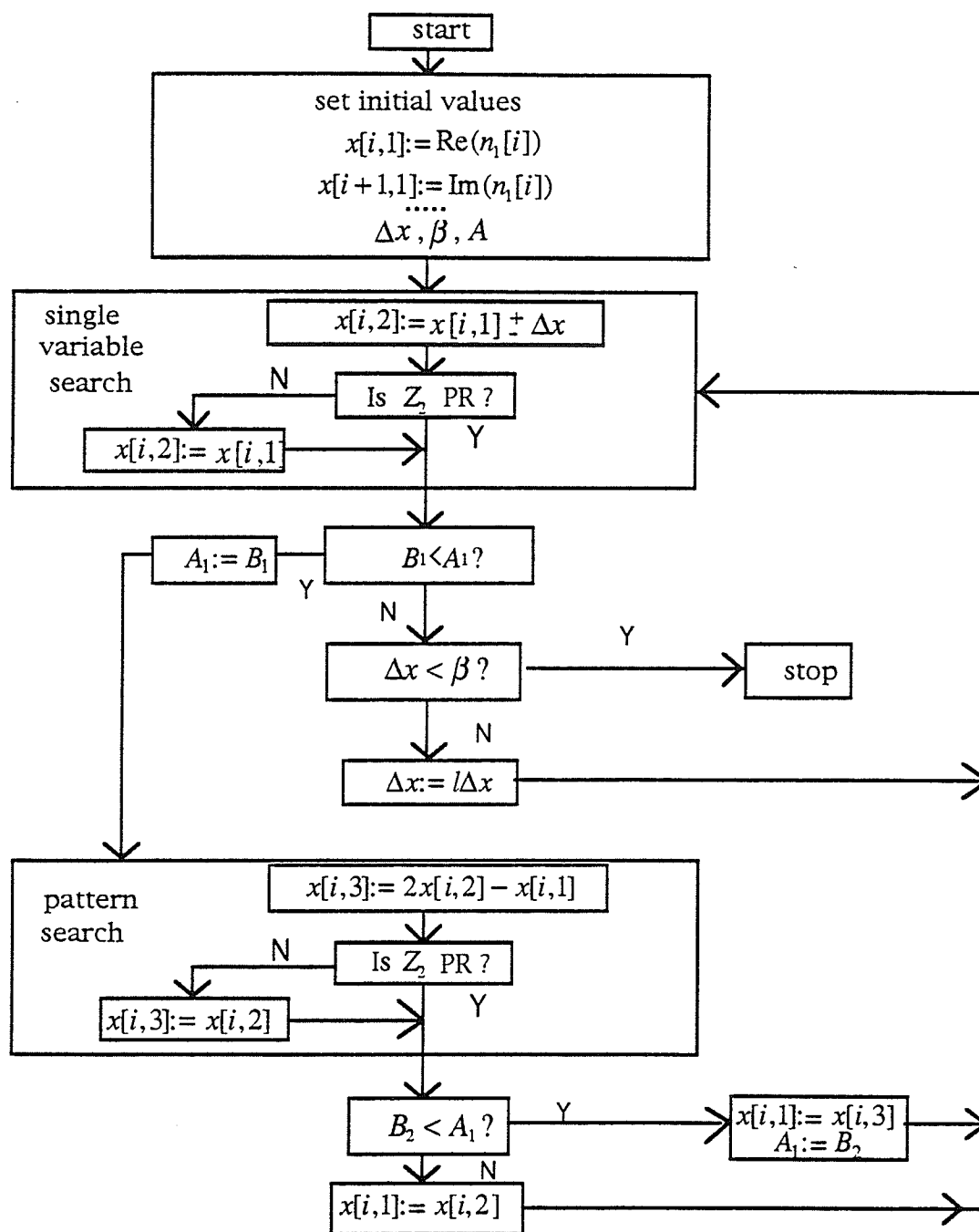


Fig. 4.4.2-2 Flowchart for determining micro structure

Table 4.4.2-1 displays the results of the three different configurations of the micro structure. From Table 4.4.2-1, it is easy to see that the results from both the parallel and the series networks are very close and they are better than that from the series-parallel connection.

Table 4.4.2-1 Results of different micro structures  
(iteration number = 1000)

#	configuration	error
1	parallel	1.048
2	series	1.047
3	series-parallel	1.260

Because using a series connection makes it easy to determine the starting point and also gives the best result, we use the series connection for the micro structure and carry out the computation for the elemental impedance functions. The zeros and the poles of the three elemental impedances  $z_1$ ,  $z_2$  and  $z_3$  are shown in Tables 4.4.2-2, 4.4.2-3 and 4.4.2-4, respectively.

Table 4.4.2-2 Zeros and poles of  $z_1$

Rational Function  $z_1$

polyRep= prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part

imaginary part

1.0605295293157858580e+2 0.000000000000000000e+0

-1.1872930102968954970e+2 -0.000000000000000000e+0

Denominator Degree:

2

Constant factor followed by Poles:

real part

imaginary part

1.000000000000000000e+0 0.000000000000000000e+0

-2.9708677431520540790e+2 -2.1376625660685373100e+3

-2.9708677431520540790e+2 2.1376625660685373100e+3

Table 4.4.2-3 Zeros and poles of  $z_2$

Rational Function  $z_2$

polyRep=

prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part

imaginary part

2.0425336951208768150e+1 0.000000000000000000e+0

-8.1084471165217919340e+1 -0.000000000000000000e+0

Denominator Degree:

2

Constant factor followed by Poles:

real part

imaginary part

1.000000000000000000e+0 0.000000000000000000e+0

-2.2339044966524438980e+2 -1.1930567158458860050e+3

-2.2339044966524438980e+2 1.1930567158458860050e+3



Table 4.4.2-4 Zeros and poles of  $z_3$

Rational Function  $z_3$

polyRep=

prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part	imaginary part
8.1208750924862936100e+1	0.0000000000000000000e+0
-2.8321994628851924380e+1	-0.0000000000000000000e+0

Denominator Degree:

2

Constant factor followed by Poles:

real part	imaginary part
1.0000000000000000000e+0	0.0000000000000000000e+0
-3.0390086832620016970e+2	-5.6261174795654029820e+2
-3.0390086832620016970e+2	5.6261174795654029820e+2

Table 4.4.2-2 Zeros and poles of  $z_{12}$

Rational Function  $z_{12}$

polyRep=

prodRep

Numerator Degree:

5

Constant factor followed by Zeros:

real part	imaginary part
2.0768704080765029000e+2	0.0000000000000000000e+0
-3.9685372419290754310e+1	0.0000000000000000000e+0
-2.5901673952745514000e+2	1.0678575332142838200e+3
-2.5901673952745514000e+2	-1.0678575332142838200e+3
-2.9285349110317893030e+2	1.6060583174442002430e+3
-2.9285349110317893030e+2	-1.6060583174442002430e+3

Denominator Degree:

6

Constant factor followed by Poles:

real part	imaginary part
1.0000000000000000000e+0	0.0000000000000000000e+0
-2.9708677431520540790e+2	-2.1376625660685373100e+3
-2.9708677431520540790e+2	2.1376625660685373100e+3
-2.2339044966524438980e+2	-1.1930567158458860050e+3
-2.2339044966524438980e+2	1.1930567158458860050e+3
-3.0390086832620016970e+2	-5.6261174795654029820e+2
-3.0390086832620016970e+2	5.6261174795654029820e+2

### 4.4.3 Micro Structure of $Z_1$

For the chosen T main structure, the impedance

$$z_{11} = Z_1 + z_{12} \quad (4.4.3-1)$$

Similar to the micro structure of  $z_{12}$ , the micro structures considered for  $Z_1$  are the parallel, series, and series-parallel configurations. Since the series configuration makes it easy to determine the starting point, it gives a better result for  $z_{12}$ , and it is simple to express, therefore we use the series configuration for  $Z_1$ , i.e.,

$$\begin{aligned} Z_1 &= z_1 + z_2 + \dots + z_m \\ &= \frac{n_1}{d_1} + \frac{n_2}{d_2} + \dots + \frac{n_m}{d_m} \end{aligned} \quad (4.4.3-2)$$

Fig. 4.4.3-1 shows the series connection for  $Z_1$ .

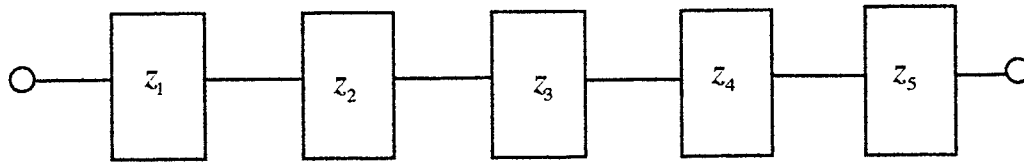


Fig. 4.4.3-1

After selecting the micro structure, the next step is to find the optimal combination of different degrees of numerator polynomial and denominator polynomial, and the types of zeros and poles of  $Z_1$ , i.e., the impedance functions of the network elements. Since the optimal degree of  $z_{11}$  is 9/8 from Step 1, it is reasonable to use a degree of 9/8 for the optimization. The zeros and poles of  $z_{11}$  from the results of Step 1 are adopted as the starting point.

Combinations of different types of elemental impedance functions are tested by optimization. Since the optimal degree difference between the numerator and denominator has been found to be one, the degree differences for all the tests are set to one.

The test results are listed in Table 4.4.3-1. Combination No. 1 and combination No. 2 have the same type of impedance functions, but combination No. 1 has a fixed denominator that is identical to the denominator of  $z_{12}$ .

The results of different combinations of elemental impedances are listed in Table 4.4.3-1. From Table 4.4.3-1 the following is found:

1. Combination No. 2 gives the best result.
2. No. 1 gives a poorer result than No. 2, because its poles are fixed.
3. The 2/2 combination does not have advantages over the 1/2 combination.

From these test results, it is clear that combination No. 2 is the best configuration.

Table 4.4.3-1 Test results of elemental impedance for  $z_{11}$

#	#1 elemental impedance 1/2	#2 elemental impedance 2/2	#3 elemental impedance 1/0	error
1	4		1	1.55
2	4		1	0.763
3		4	1	1.06

#### 4.4.4 Micro Structure of $Z_3$

For the T main structure, we have

$$z_{22} = Z_3 + z_{12} \quad (4.4.4-1)$$

Similar to the micro structure of  $z_{12}$ , the micro structures considered for  $Z_3$  are the parallel, series, or series-parallel configurations. Since the series configuration is good for  $z_{12}$  and  $Z_1$ , and is also simple to express, we use the series configuration for  $Z_3$ , i.e.,

$$\begin{aligned} Z_3 &= z_1 + z_2 + \dots + z_m \\ &= \frac{n_1}{d_1} + \frac{n_2}{d_2} + \dots + \frac{n_m}{d_m} \end{aligned} \quad (4.4.4-2)$$

The series connection of  $Z_3$ , is shown in Fig. 4.4.4-1.

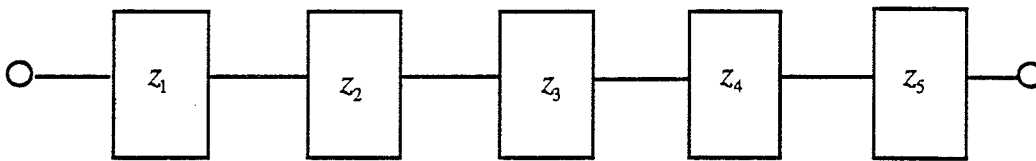


Fig. 4.4.4-1 Series connection of  $Z_3$

In a way similar to Section 4.4.3, we carry out optimization for the micro structure of  $Z_3$ . Combinations of different types of elemental impedance

functions are tested. Since the results of Step 1 indicate that the optimal degree difference is that the numerator is one degree larger than the denominator, all combinations are set according to this result. Once again, combination No. 1 has the same type of zeros and poles as that of No. 2, but its poles are fixed, i.e. equal to the poles of  $z_{12}$ . Test results of 3 of these combinations are presented in Table 4.4.4-1.

From Table 4.4.4-1, it is found that:

1. No. 2 combination gives the best result.
2. The 2/2 combination gives a poorer result than the 1/2 combination.
4. No. 1 with fixed poles gives a poorer result than No. 2, with non- fixed poles.

Table 4.4.4-1 Test results of elemental impedance for  $z_{22}$

#	#1 elemental impedance 1/2	#2 elemental impedance 2/2	#3 elemental impedance 1/0	error
1	4		1	1.64
2	4		1	0.69
3		4	1	0.86

## 4.5 Step 3: Elemental Circuits

The last step of the three-step approach is to find the elemental circuits of the two-port network. First we select some elemental circuits on which to run the optimization test, and then we pick the most satisfactory one. To run the optimization program, we also need to determine the variables and the target functions. In Step 1 and Step 2, the target functions are always the frequency responses of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ , and we only need to run the optimization program once for each impedance function. In the optimization of Step 3, the input variables are the values of the elemental circuit components and the target functions are the frequency responses of the elemental impedance functions. The frequency responses of the elemental impedance functions are the results from Step 2. If the errors of all these individual impedance functions are very small, then the frequency responses of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  also have relative small error, because of the simple relationship between the impedances. The elemental circuits are determined as part of the optimization process. In the optimization of Step 3, a percentage step increment less than 100% is used, so that if the initial elemental values are all set to be positive, then the minimum elemental values will be greater than zero. Therefore, the PR requirement for the elemental circuits is automatically satisfied. This is an advantage of a percentage step increment.

#### 4.5.1 Elemental Circuits for $Z_2$ ( $z_{12}$ )

In Step 2, the optimal micro structure was determined—three 1/2 circuits in a series connection—we call them circuits 1, 2, and 3. The corresponding elemental impedances  $z_1$ ,  $z_2$  and  $z_3$  were found as well. For the optimization procedure for the elemental circuits, we have the objective function

$$F_n = \sum_{i=1}^m |z_n(j\omega_i) - z'_n(j\omega_i)| \quad n = 1, 2, 3 \quad (4.5.1-1)$$

where  $F_n$  is the objective function of the optimization for the elemental circuit  $n$ ,  $z'_n(j\omega_i)$  is the known frequency response of elemental impedance  $z'_n$  from the optimization of Step 2, and  $z_n(j\omega_i)$  is the frequency response of elemental impedance  $z_n$  of the elemental circuit  $n$  which is to be found.

The input variables for the optimization are the values of the resistors, capacitors and inductors of the elemental circuits 1, 2 and 3. The target functions are the frequency responses of  $z_1$ ,  $z_2$  and  $z_3$  which are the impedances of the elemental circuits 1, 2, and 3, respectively. The target functions are functions of the element values in addition to frequency:

$$z_1(j\omega_i) = f_1(\omega_i, x_k) \quad (4.5.1-2)$$

$$z_2(j\omega_i) = f_2(\omega_i, x_k) \quad (4.5.1-3)$$

$$z_3(j\omega_i) = f_3(\omega_i, x_k) \quad (4.5.1-4)$$

where the  $x_k$  denote the values of resistors, inductors and capacitors. The exact functional dependencies  $f_1$ ,  $f_2$  and  $f_3$  are determined by the specific circuits and are listed in Appendix C.

In order to satisfy the PR requirement, the values of the circuit components must always be positive. Fig. 4.5.1-1 shows the flowchart of the optimization procedure for finding the elemental circuit values of circuits 1, 2 and 3. Since the number of variables is much smaller than that in the optimizations of Step 1 and Step 2, the optimization for the elemental circuits usually converges very quickly. The choice of starting point does not have a great effect on the optimization process, so the initial values can always be one.



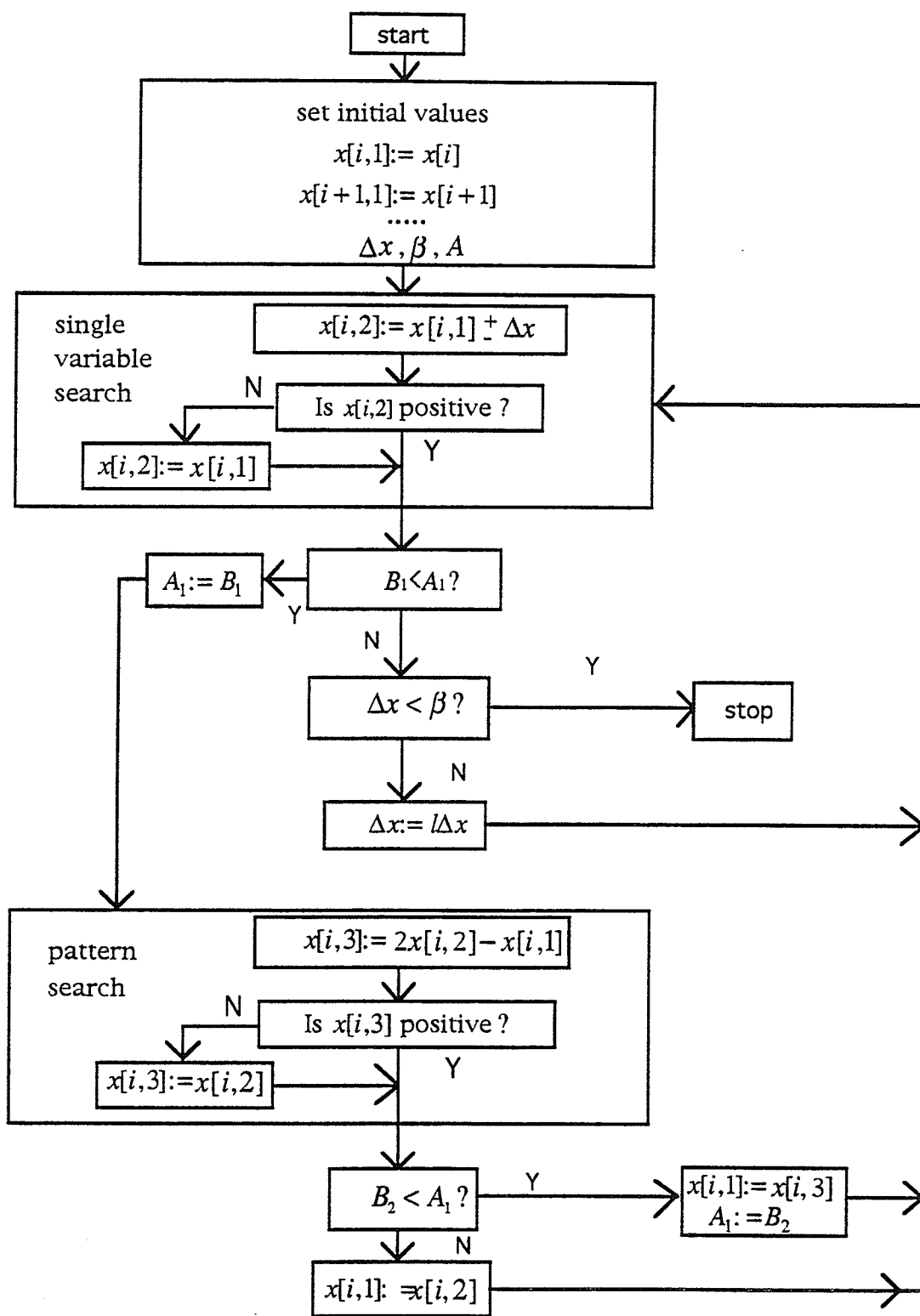


Fig. 4.5.1-1 Flowchart for finding the circuit

Some elemental circuits were selected for testing. The test results for  $z_1$  are displayed in Table 4.5.1-1. It can be seen that elemental circuit No. A2 is the best one. The test results of  $z_2$  are listed in Table 4.5.1-2. It is clear that, again the best circuit is elemental circuit No. A2. Table 4.5.1-3 gives the results for the elemental circuits for  $z_3$ . The best one is again elemental circuit No. A2 .

Table 4.5.1-1 Elemental circuits for  $z_1$

#	elemental circuit number	error
1	A2	1e-10
2	D4	231
3	D8	1254
4	A7	654

Table 4.5.1-2 Elemental circuits for  $z_2$

#	elemental circuit number	error
1	A2	1e-10
2	D4	351
3	D8	1291
4	A7	137

Table 4.5.1-3 Elemental circuits for  $z_3$

#	elemental circuit number	error
1	A2	1e-10
2	D4	536
3	D8	1026
4	A7	253

Since the errors of the elemental circuits  $z_1$ ,  $z_2$ , and  $z_3$  are all very small, the frequency response of  $z_{12}$  is satisfactory. No further optimization procedure is needed. The elemental circuit No. A2 is used for  $z_1$ ,  $z_2$ , and  $z_3$ . Fig. 4.5.1-2 shows the circuit diagram of circuit No. A2. The results for the circuit components of  $z_1$ ,  $z_2$ , and  $z_3$  are given in Tables 4.5.1-4, 4.5.1-5 and 4.5.1-6, respectively. The complete circuit diagram of  $z_{12}$  is shown in Fig. 4.5.1-3.

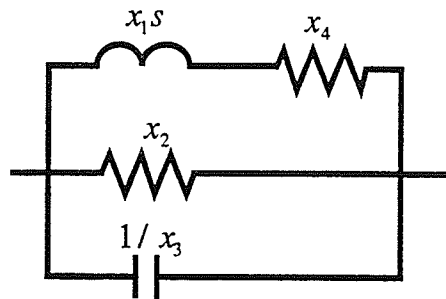


Fig. 4.5.1-2 The elemental circuit No. A2

Table 4.5.1-4 Component values of  $z_1$  (1/2 degree)

component number	value of component	
$x_1$	2.30479118810222895e-5	H
$x_2$	2.230607552131790681e-1	$\Omega$
$x_3$	9.429251825219451879e-3	F
$x_4$	2.736462467827653723e-3	$\Omega$

Table 4.5.1-5 Component values of  $z_2$  (1/2 degree)

component number	value of component	
$x_1$	1.414854339322355025e-5	H
$x_2$	5.585325799785458158e-2	$\Omega$
$x_3$	4.895880064983800236e-2	F
$x_4$	1.147227158797669458e-3	$\Omega$

Table 4.5.1-6 Component values of  $z_3$  (1/2 degree)

component number	value of component	
$x_1$	2.069140893283955348e-4	H
$x_2$	1.401407936044170506e-1	$\Omega$
$x_3$	1.231394386209970877e-2	F
$x_4$	5.860219726592605622e-3	$\Omega$

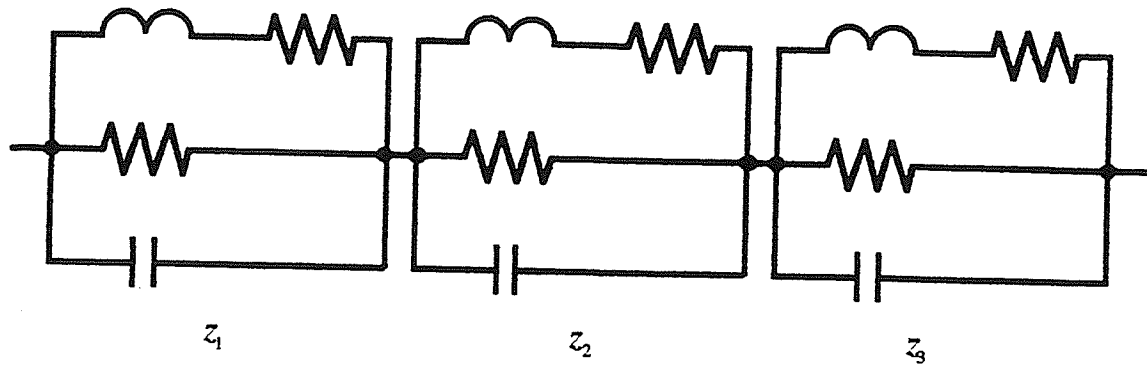


Fig. 4.5.1-3 The circuit diagram for  $Z_2$  ( $z_{12}$ )

#### 4.5.2 Elemental Circuits for $Z_1$

First, we adopt the best combination from Step 2, i.e. combination No. 5 for the optimization of Step 3. The elemental circuits for combination No. 5 are four 1/2 circuits and a circuit consisting of one inductor. In a way similar to that for  $Z_2$ , we conduct the optimizations to find elemental circuits for  $Z_1$ . Different elemental circuits were tested for each elemental impedance. The elemental circuit No. A2 turned out to be the best for  $z_1$ ,  $z_2$ ,  $z_3$ , and  $z_4$ . The circuit for  $z_5$  is an inductor. The final results of the values of the circuit components are shown in Tables 4.5.2-1 to 4.5.2-4. The value of the inductor  $z_5$  is  $6.1912 \times 10^{-5}$  H. The circuit diagram for  $Z_1$  is presented in Fig. 4.5.2-1. The zeros and poles of impedance  $Z_1$  are displayed in Table 4.5.2-5.

Table 4.5.2-1 Component values of  $z_1$

component number	value of component	
$x_1$	3.638536419913212044e-5	H
$x_2$	4.050886005793401554e+0	$\Omega$
$x_3$	2.991261563497029355e-3	F
$x_4$	1.089965697218452937e-2	$\Omega$

Table 4.5.2-2 Component values of  $z_2$

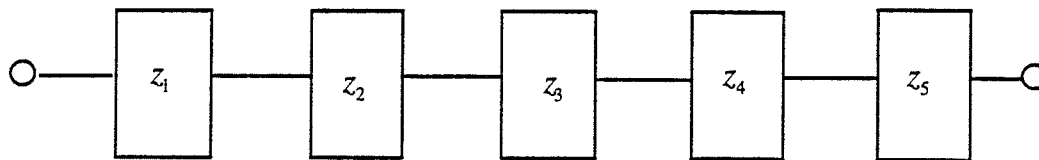
component number	value of component	
$x_1$	1.236760952639927769e-5	H
$x_2$	1.522737129152054306e-1	$\Omega$
$x_3$	2.144919332752727387e-2	F
$x_4$	2.673188700317238956e-3	$\Omega$

Table 4.5.2-3 Component values of  $z_3$

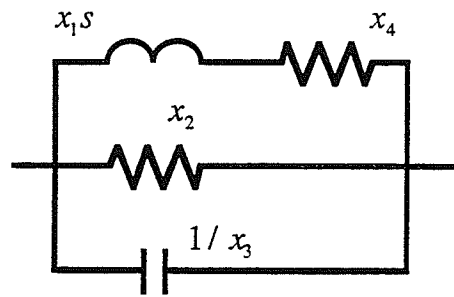
component number	value of component	
$x_1$	4.491453690298754601e-6	H
$x_2$	9.300862490738797453e-2	$\Omega$
$x_3$	7.881359162694534314e-2	F
$x_4$	1.785724238715612934e-3	$\Omega$

Table 4.5.2-4 Component values of  $z_4$

component number	value of component	
$x_1$	$7.777478478977115902\text{e-}6$	H
$x_2$	$3.725628495069809140\text{e-}2$	$\Omega$
$x_3$	$6.280941730610968224\text{e-}2$	F
$x_4$	$3.587668789185950443\text{e-}3$	$\Omega$



(a) series connection for  $Z_1$



(b) No. A2 elemental circuit for  $z_1, z_2, z_3$  and  $z_4$



(c)  $z_5$

Fig. 4.5.2-1 The circuit for  $Z_1$

Table 4.5.2-5

Rational Function  $Z_1$ polyRep=  
prodRep

Numerator Degree:

9

Constant factor followed by Zeros:

real part	imaginary part
6.1912670994743365300e-5	0.000000000000000000e+0
-1.5339149057720703410e+2	0.000000000000000000e+0
-4.3250294370763409250e+2	1.4647032610630773100e+3
-4.3250294370763409250e+2	-1.4647032610630773100e+3
-2.6435949705414706530e+2	1.6999319747855593160e+3
-2.6435949705414706530e+2	-1.6999319747855593160e+3
-2.5560313652409132830e+2	2.0446224512078876440e+3
-2.5560313652409132830e+2	-2.0446224512078876440e+3
-1.3435735262727006290e+2	3.8956876982562191470e+3
-1.3435735262727006290e+2	-3.8956876982562191470e+3

Denominator Degree:

8

Constant factor followed by Poles:

real part	imaginary part
1.000000000000000000e+0	0.000000000000000000e+0
-1.9104424231591091030e+2	-3.0292209377216785780e+3
-1.9104424231591091030e+2	3.0292209377216785780e+3
-2.6115768236518365710e+2	-1.9410417972831678370e+3
-2.6115768236518365710e+2	1.9410417972831678370e+3
-2.6700096977491941620e+2	-1.6756808016697022000e+3
-2.6700096977491941620e+2	1.6756808016697022000e+3
-4.4431578074573208250e+2	-1.4306639881822750010e+3
-4.4431578074573208250e+2	1.4306639881822750010e+3



### 4.5.3 Elemental Circuits for $Z_3$

In a way similar to Section 4.5.2, we conduct the optimization to find elemental circuits for  $Z_3$ . Different elemental circuits were tested for each elemental impedance. The elemental circuit No. A2 turned out to be the best for  $z_1$ ,  $z_2$ ,  $z_3$ , and  $z_4$ . The circuit for  $z_5$  is an inductor. The value for  $z_5$  is  $3.7400\text{e-}5$  H. The results of the values of the circuit components are shown in Tables 4.5.3-1 to 4.5.3-4. The circuit diagram of  $Z_3$  is presented in Fig. 4.5.3-1. The zeros and poles of impedance  $Z_3$  are displayed in Table 4.5.3-5.

Table 4.5.3-1 Component values of  $z_1$

component number	value of component	
$x_1$	$7.447990601552763502\text{e-}6$	H
$x_2$	$1.232245238132263403\text{e-}1$	$\Omega$
$x_3$	$1.730405086120167709\text{e-}2$	F
$x_4$	$2.851501617919038618\text{e-}7$	$\Omega$

Table 4.5.3-2 Component values of  $z_2$

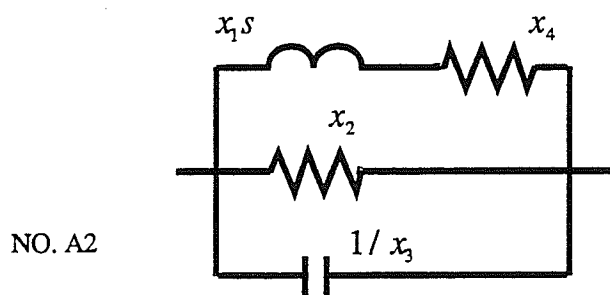
component number	value of component	
$x_1$	$3.730842928895938333\text{e-}6$	H
$x_2$	$9.750509954647770271\text{e-}3$	$\Omega$
$x_3$	$3.052976626601924284\text{e-}1$	F
$x_4$	$7.874581866735870868\text{e-}14$	$\Omega$

Table 4.5.3-3 Component values of  $z_3$

component number	value of component	
$x_1$	1.652949684957860686e-6	H
$x_2$	8.817033252766961848e-1	$\Omega$
$x_3$	3.227125187256059007e-1	F
$x_4$	4.074174560117270216e-4	$\Omega$

Table 4.5.3-4 Component values of  $z_4$

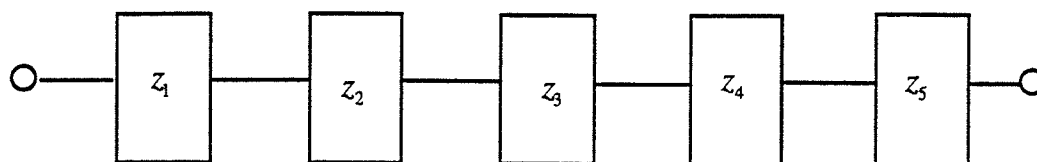
component number	value of component	
$x_1$	2.198608994795953348e-22	H
$x_2$	1.554179451801474000e-9	$\Omega$
$x_3$	1.206025440000000000e+8	F
$x_4$	1.031826514155010000e-23	$\Omega$



(a) elemental circuit for  $z_1, z_2, z_3$  and  $z_4$



(b)  $z_5$



(b) series connection of  $Z_3$

Fig. 4.5.3-1 The circuit of  $Z_3$

Table 4.5.3-5

Rational Function  $Z_3$

polyRep= prodRep

Numerator Degree: 9

Constant factor followed by Zeros:

real part	imaginary part
3.7421213880250026430e-5	0.000000000000000000e+0
-2.6910142362370667200e+0	-6.1411225675451271040e+6
-2.6910142362370667200e+0	6.1411225675451271040e+6
-8.1151844031964627780e+0	0.000000000000000000e+0
-1.6845970417455414220e+2	9.5730041715100390550e+2
-1.6845970417455414220e+2	-9.5730041715100390550e+2
-1.2107540840141110950e+2	1.3888398596763832070e+3
-1.2107540840141110950e+2	-1.3888398596763832070e+3
-2.3387876001588049560e+2	3.0466477098774588880e+3
-2.3387876001588049560e+2	-3.0466477098774588880e+3

Denominator Degree: 8

Constant factor followed by Poles:

real part	imaginary part
1.000000000000000000e+0	0.000000000000000000e+0
-2.3450954591347313820e+2	-2.7756358654829539070e+3
-2.3450954591347313820e+2	2.7756358654829539070e+3
-1.6796515496225390520e+2	-9.2181193638803314720e+2
-1.6796515496225390520e+2	9.2181193638803314720e+2
-1.2499676391771693530e+2	-1.3637764702943150340e+3
-1.2499676391771693530e+2	1.3637764702943150340e+3
-2.6910142362370667200e+0	-6.1411225675451271040e+6
-2.6910142362370667200e+0	6.1411225675451271040e+6

## 4.6 Results of $z_{11}$ , $z_{12}$ and $z_{22}$

The frequency response of  $z_{12}$  found in Section 4.5 i.e.  $z_{12}(j\omega_i)$  is shown in Fig. 4.6.1. The frequency response of  $z_{11}$  i.e.  $z_{11}(j\omega_i)$  is presented in Fig. 4.6.2.  $z_{11}(j\omega_i)$  is calculated using Equation (4.6.1):

$$z_{11}(j\omega_i) = Z_1(j\omega_i) + Z_2(j\omega_i) \quad (4.6.1)$$

Fig. 4.6.3 displays  $z_{22}(j\omega_i)$ —the frequency response of  $z_{22}$ .  $z_{22}(j\omega_i)$  is calculated using Equation (4.6.2):

$$z_{22}(j\omega_i) = Z_3(j\omega_i) + Z_2(j\omega_i) \quad (4.6.2)$$

The frequency response of  $z_{11}$  is the best one of the three frequency responses. The frequency response of  $z_{12}$  is quite good except at the higher frequencies, and the frequency response of  $z_{22}$  is not as good as  $z_{12}$ . In the next section, we will analyze the source of the errors and how to improve the accuracy of the frequency responses.

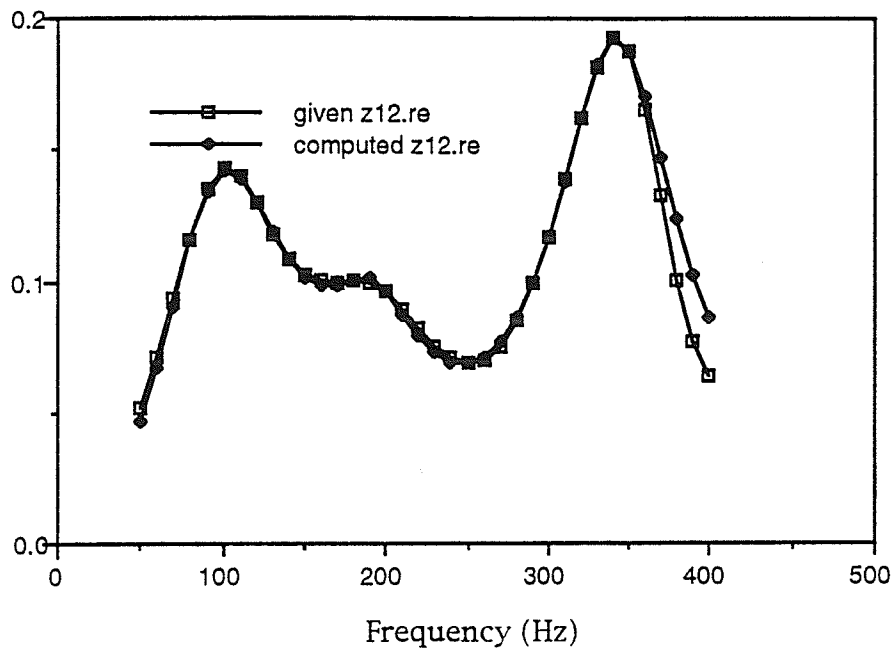


Fig. 4.6.1a. Frequency response of the real part of  $z_{12}$

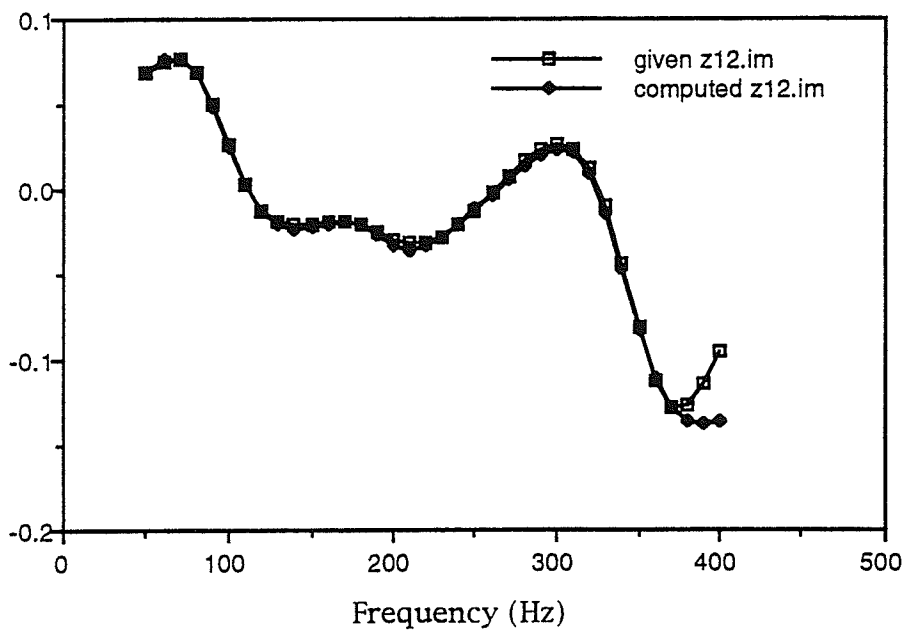


Fig. 4.6.1b. Frequency response of the imaginary part of  $z_{12}$

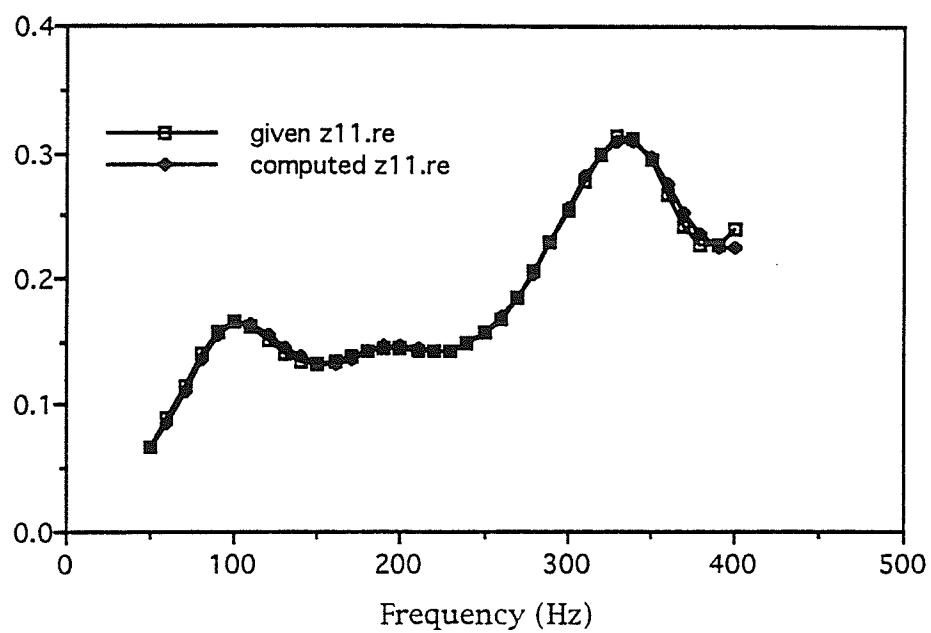


Fig. 4.6.2a. Frequency response of the real part of  $z_{11}$

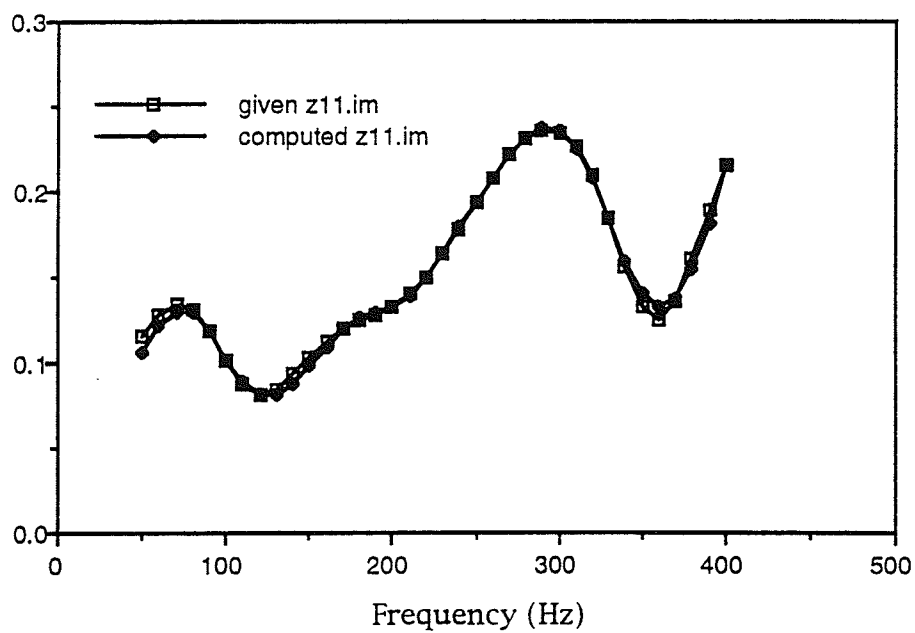


Fig. 4.6.2b. Frequency response of the imaginary part of  $z_{11}$

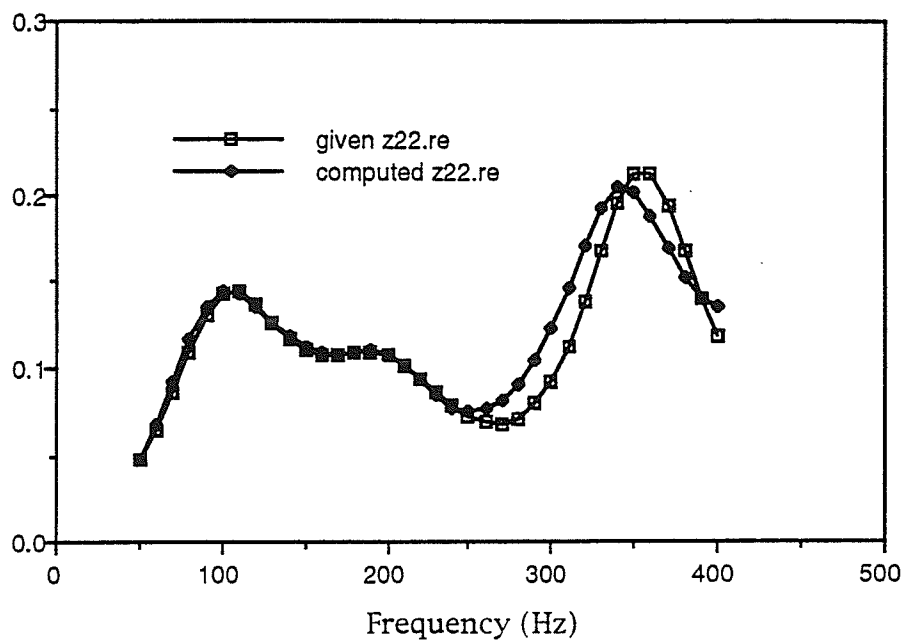


Fig. 4.6.3a. Frequency response of the real part of  $z_{22}$

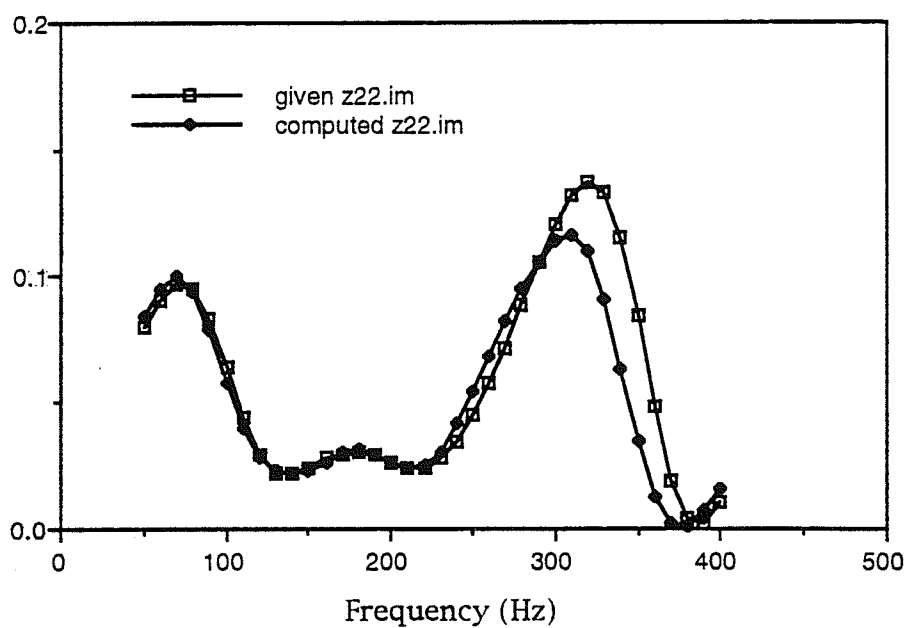


Fig. 4.6.3b. Frequency response of the imaginary part of  $z_{22}$

## 4.7 Improving the frequency responses of $z_{12}$ and $z_{22}$

In this section, we will first find the reasons for the poor frequency responses of impedance functions  $z_{12}$  and  $z_{22}$ . Then we will try to find solutions for these problems to improve the accuracy of the frequency responses. Initially, we will work on the frequency response of  $z_{12}$ .

### 4.7.1 Error analysis for $z_{12}$

The error of the frequency response of  $z_{12}$  occurs at the higher frequencies due to the fact that five zeros and six poles of  $z_{12}$  can not generate three valleys in the frequency response curve. In order to improve the frequency response at the higher frequencies, the number of zeros of the numerator and the denominator must be increased. Table 4.3.2, shows the degree combination 7/8 is a good choice, therefore, this 7/8 degree combination is adopted. The starting point is chosen as the follows: the imaginary part of the new zero and pole are chosen equal to the highest frequency of the frequency response curve and the real part of the new zero and pole are chosen equal to the nearest existing pole(zero). In this way, the initial error is equal to the error of the previous optimization and the optimization process will move the poles and zeros smoothly starting from the chosen point, this leads to fast convergence.



#### 4.7.2 Error analysis for $z_{22}$

To find the source of the large error in the frequency response of  $z_{22}$ , we analyze the optimization process and find that the error is not due to the optimization program, but due to the fact that the given real part of  $z_{22}$  at some frequencies is smaller than the real part of  $z_{12}$ . An analysis of the T structure given below shows that it can not realized responses with  $\text{Re}(z_{22}) < \text{Re}(z_{12})$ . However, if a transformer is cascade with the T structure, it may be possible to realize the response. The T main structure is shown in Fig. 4.7.1.

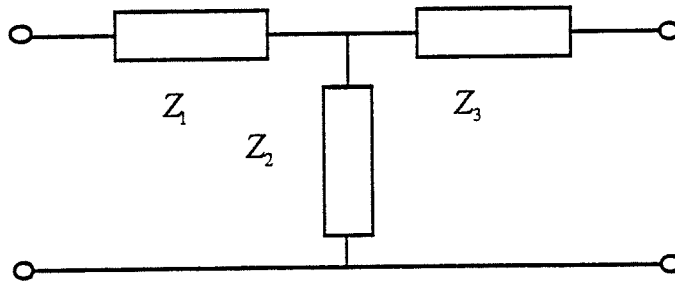


Fig. 4.7.1

From Fig. 4.7.1, we have:

$$z_{11} = Z_1 + Z_2$$

$$z_{12} = z_{21} = Z_2$$

$$z_{22} = Z_3 + Z_2$$

and therefore

$$Z_1 = z_{11} - z_{22}$$

$$Z_2 = z_{12}$$

$$Z_3 = z_{22} - z_{12}$$

If  $Z_1$ ,  $Z_3$  and  $Z_2$  are PR, then we have:

$$\text{Re}(z_{11}) \geq \text{Re}(z_{12}) \geq 0 \quad \forall \omega \quad (4.7.1-1)$$

$$\text{Re}(z_{22}) \geq \text{Re}(z_{12}) \geq 0 \quad \forall \omega \quad (4.7.1-2)$$

Equations (4.7.1-1) and (4.7.1-2) imply :

$$\operatorname{Re}(z_{11})\operatorname{Re}(z_{22}) \geq \operatorname{Re}^2(z_{12}) \quad \forall \omega \quad (4.7.1-3)$$

If given  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  satisfy the PR conditions of Section 2.2 and equations (4.7.1-1) and (4.7.1-2), then  $Z_1$ ,  $Z_2$  and  $Z_3$  are PR. Therefore, equations (4.7.1-1), (4.7.1-2) together with the PR condition of Section 2.2 are the necessary and sufficient PR conditions for  $Z_1$ ,  $Z_2$  and  $Z_3$  to be PR.

Since the given  $\operatorname{Re}(z_{12})$  and  $\operatorname{Re}(z_{22})$  of the Hydro Example do not satisfy equations (4.7.1-1) and (4.7.1-2),  $z_{22}$  can not be realized by a PR  $Z_3$ . To solve this problem, a transformer must be added to the T network.

Fig. 4.7.2 shows a T network with a transformer cascaded.

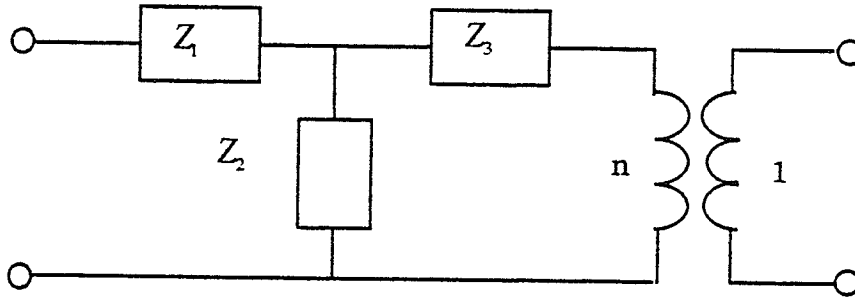


Fig. 4.7.2

From Fig. 4.7.2, we have

$$z_{11} = Z_1 + Z_2$$

$$z_{12} = z_{21} = \frac{Z_2}{n}$$

$$z_{22} = \frac{Z_2 + Z_3}{n^2}$$

and

$$Z_2 = nz_{12}$$

$$Z_1 = z_{11} - nz_{12}$$

$$Z_3 = n^2 z_{22} - nz_{12} = n(n z_{22} - z_{12})$$

$$\text{If } Z_2 \text{ is PR, then } \operatorname{Re}(nz_{12}) = n\operatorname{Re}(z_{12}) \geq 0 \quad \forall \omega \quad (4.7.1-4)$$

$$\text{if } Z_1 \text{ is PR, then } \operatorname{Re}(z_{11}) \geq n\operatorname{Re}(z_{12}) \geq 0 \quad \forall \omega \quad (4.7.1-5)$$

$$\text{if } Z_3 \text{ is PR, then } n^2 \operatorname{Re}(z_{22}) \geq n\operatorname{Re}(z_{12}) \geq 0 \quad \forall \omega \quad (4.7.1-6)$$

From equation (4.7.1-6), we have

$$|n|\operatorname{Re}(z_{22}) \geq |\operatorname{Re}(z_{12})| \quad \forall \omega$$

For equation (4.7.1-4), if  $n > 0$ , then

$$\operatorname{Re}(z_{12}) \geq 0 \quad \forall \omega$$

if  $n < 0$  then

$$\operatorname{Re}(z_{12}) < 0 \quad \forall \omega$$

From equation (4.7.1-5), we have

$$\frac{1}{|n|} \geq \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{11})} \quad \forall \omega$$

and therefore

$$\frac{1}{|n|} \geq \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{11})}$$

Similarly from equation (4.7.1-6), we have

$$|n| \geq \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{22})}$$

Therefore, we have

$$\frac{1}{\max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{11})}} \geq |n| \geq \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{22})}$$

Which implies

$$1 \geq \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{11})} \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{22})}$$

Vice versa, for given  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ , if

a)  $\operatorname{Re}(z_{12}) \geq 0 \quad \forall \omega$ , or

b)  $\operatorname{Re}(z_{12}) \leq 0 \quad \forall \omega$

and c)

$$1 \geq \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{11})} \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{22})}$$

then, if a) holds, choose  $n > 0$ , and if b) holds, choose  $n < 0$ , then in both cases

$$n \operatorname{Re}(z_{12}) \geq 0 \quad \forall \omega$$

Now choose  $|n|$  so that

$$\frac{1}{\max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{11})}} \geq |n| \geq \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{22})}$$

and then

$$|\operatorname{Re}(z_{12})| \leq |n| \operatorname{Re}(z_{22}) \quad \forall \omega$$

$$n \operatorname{Re}(z_{12}) \leq n^2 \operatorname{Re}(z_{22}) \quad \forall \omega$$

Similarly, we have

$$|n| |\operatorname{Re}(z_{12})| \leq \operatorname{Re}(z_{11}) \quad \forall \omega$$

and then

$$n \operatorname{Re}(z_{12}) \leq \operatorname{Re}(z_{11}) \quad \forall \omega$$

Therefore, the necessary and sufficient conditions for the realization of a T network cascaded with an ideal transformer are [30]:

$$\operatorname{Re}(z_{12}) \leq 0 \quad \text{or} \quad \operatorname{Re}(z_{12}) \geq 0 \quad \forall \omega \quad (4.7.1-7 \text{ a,b})$$

and

$$\max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{11})} \max_{\omega} \frac{|\operatorname{Re}(z_{12})|}{\operatorname{Re}(z_{22})} \leq 1 \quad (4.7.1-7c)$$

Summarizing: two network structures for realizing positive real two-port functions have been discussed: 1) the simple T structure, and 2) a T structure cascaded with a transformer. For practical realizations, usually the simplest possible network structure is adopted. For the Hydro Example, there are only five frequency points at which Equation (4.7.1-7c) is not satisfied. However, it is very close to the requirement of Equation (4.7.1-7c). We will use the structure with a transformer to approximate the positive real two port function for the Hydro data.

### 4.7.3 Tests and Results

The T main structure with a transformer used for the optimization is shown in Fig. 4.7. 3. The transformer ratio  $n=0.82$  is found to be a good choice.

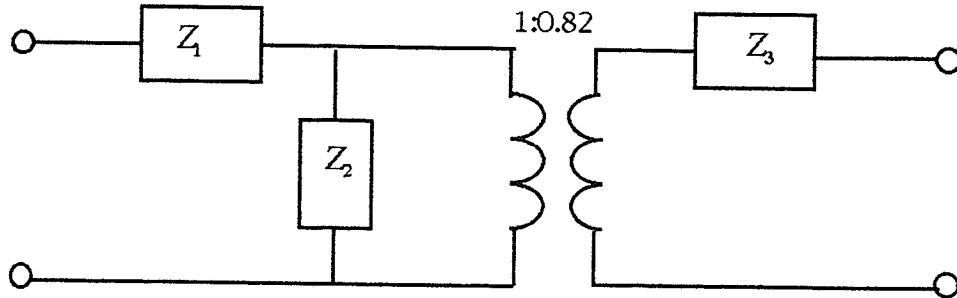


Fig. 4.7.3

The series connected micro structure is used for the realization of the two-port impedance function. The degree combination of  $7/8$  is used for the optimization. We apply the group delay method to find a starting point for the degree combination of  $7/8$ . Then we obtain a better result for the frequency response of  $z_{12}$ . Fig. 4.7.4 shows the frequency responses of the real part and the imaginary part of  $z_{12}$ . Following this, we obtain a better result for the frequency response of  $z_{22}$  as well. Fig. 4.7.5 shows the frequency responses of the real part and the imaginary part of  $z_{22}$ . The frequency response of  $z_{11}$  is shown in Fig. 4.7.6. The circuit diagram is shown in Appendix D.

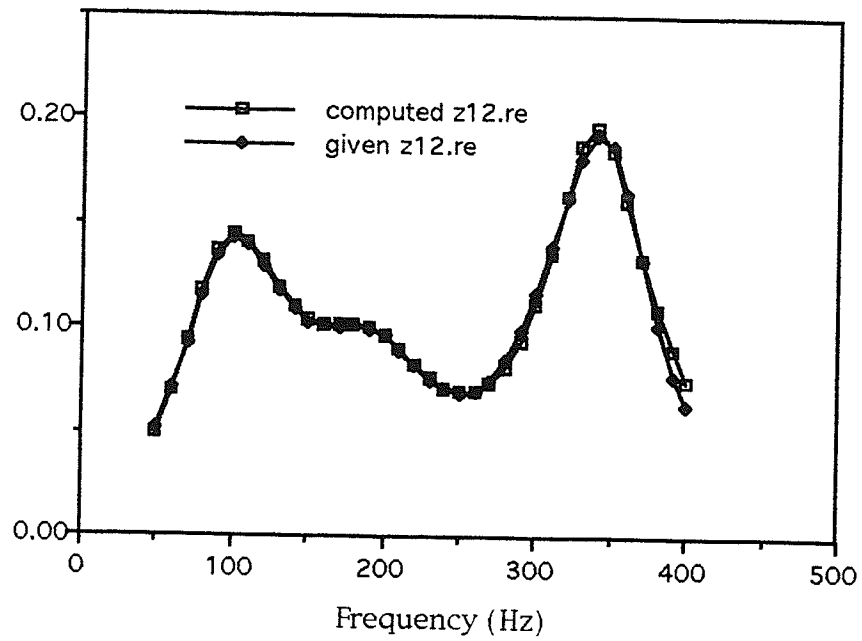


Fig. 4.7.4a Frequency response of the real part of  $z_{12}$

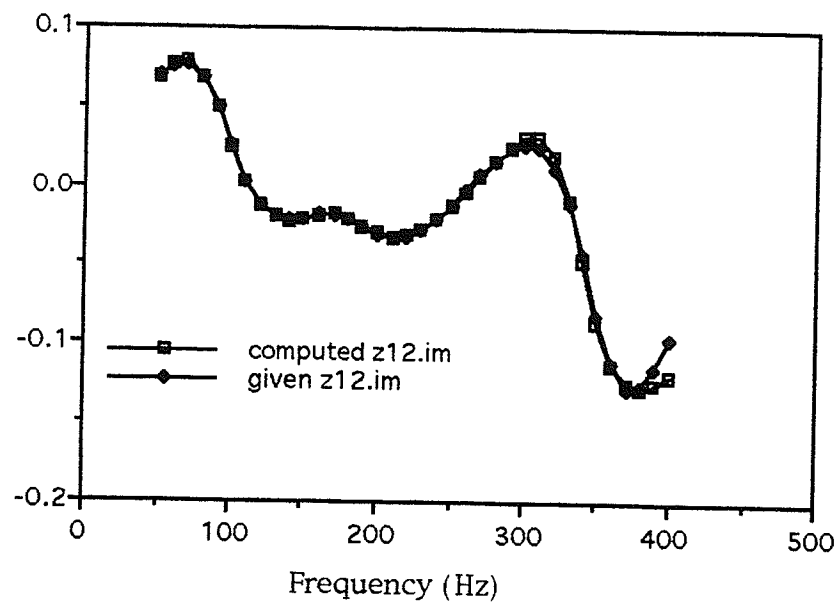


Fig. 4.7.4b Frequency response of the imaginary part of  $z_{12}$

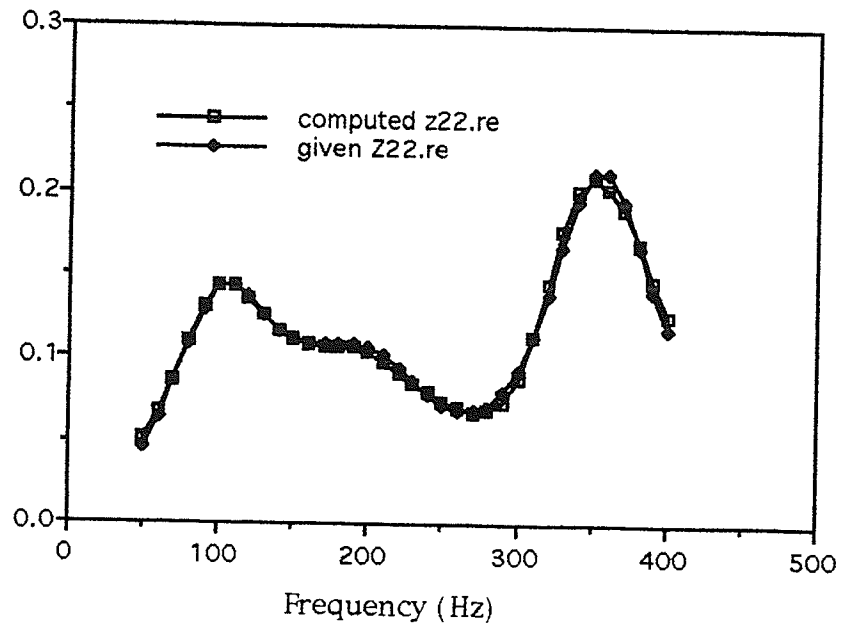


Fig. 4.7.5a Frequency response of the real part of  $z_{22}$

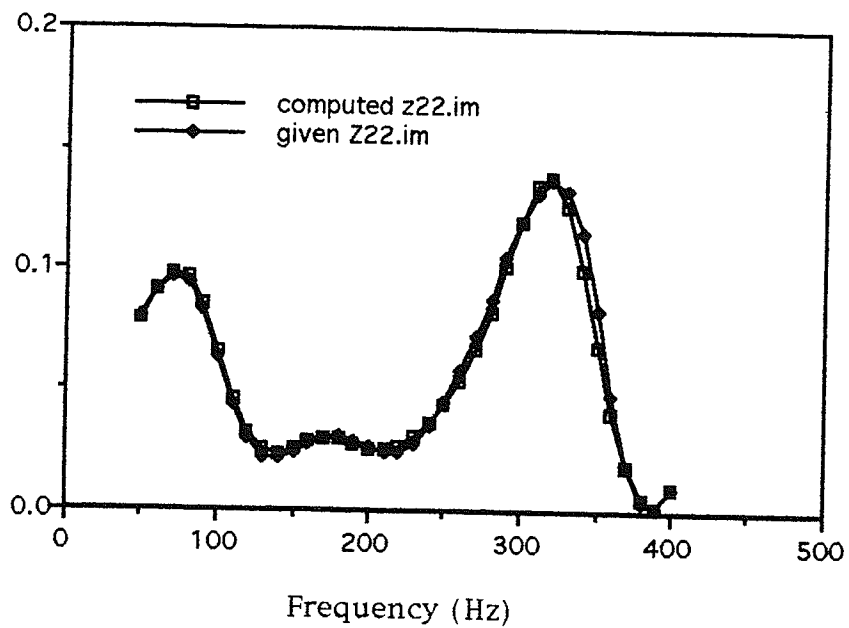


Fig. 4.7.5b Frequency response of the imaginary part of  $z_{22}$



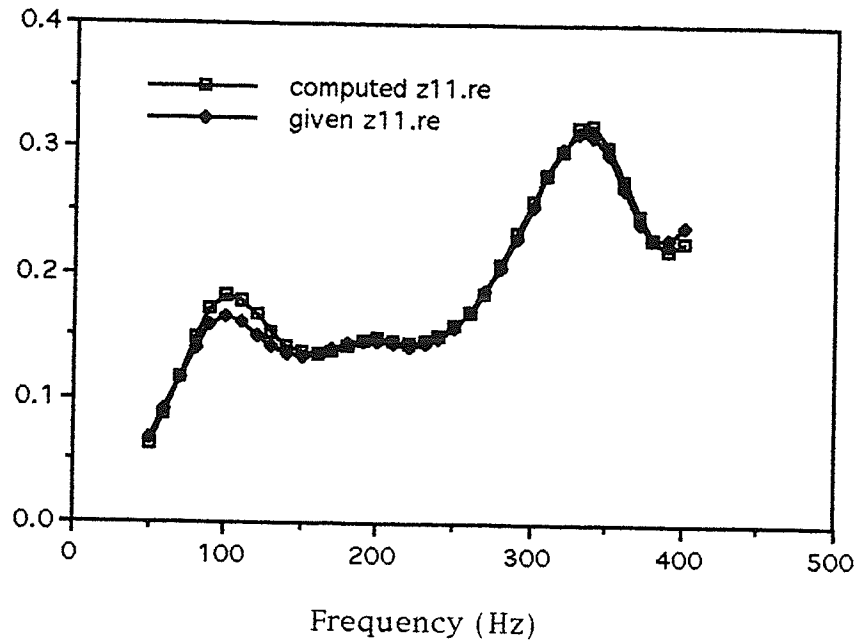


Fig. 4.7.6a Frequency response of the real part of  $z_{11}$

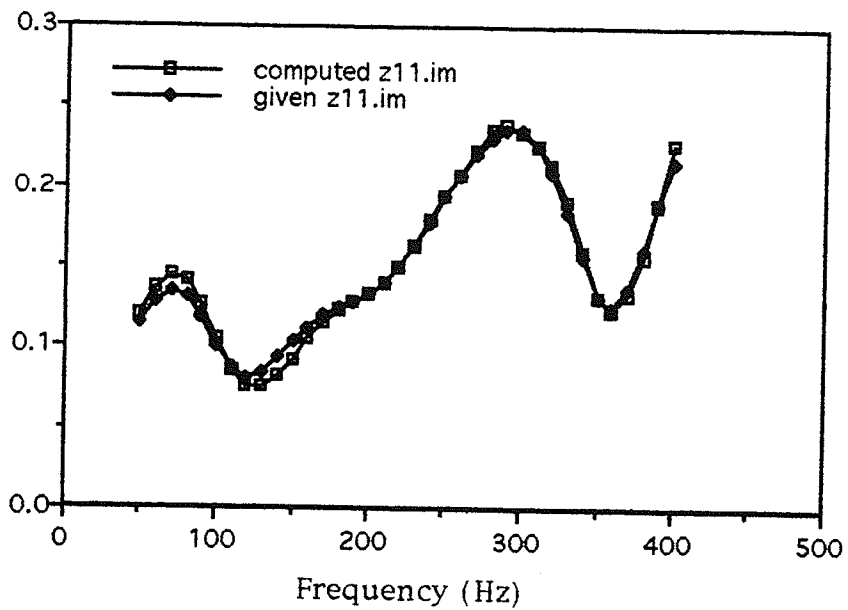


Fig. 4.7.6b Frequency response of the imaginary part of  $z_{11}$

## 4.8 Discussion

First, we summarize the basic procedures of the three-step optimization approach as follows:

(1) Step 1: determine the optimal degree and types of zeros and poles of the transfer functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ .

For the optimization process, the input variables are the zeros, the poles and the constant factor of the impedance function to be found. The target function is the frequency response of the impedance function to be found. To begin the optimization process, a starting point is needed. In general, the group delay method can provide a quite accurate starting point. When the given frequency response data does not contain enough information for the group delay method, then an optimization process may be required to determine some additional poles or zeros for the starting point. For each degree combination, a choice as to the type of zero and pole needs to be made. We can assign as many conjugate pairs as possible, and then a real zero or a real pole. For the degree of the numerator to be larger than that of the denominator, a zero at 0 must be assigned to the numerator. The optimization procedures are the same for  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ .

(2) Step 2: determine the network structure and the elemental impedance.

For the optimization process, the input variables are the zeros, the poles and the constant factor of the elemental impedance function which is to be found. The target function is the frequency response of the impedance function. First we choose the main structure, then the micro structure. Second, we determine the types of zeros and poles and the degree of the elemental functions according to the chosen network structure and the impedance

function found in Step 1. In the above Hydro Example, with chosen T main structure and series micro structure, with one real zero and two complex conjugate pairs of zeros and three complex conjugate pairs of poles of  $z_{12}$ , found in Step 1, we find the form of the elemental impedance function—three impedance functions with one real zero and a complex conjugate pair of zeros. The initial point of the optimization for  $z_{12}$  can easily be determined according to the found zeros and poles of  $z_{12}$ . In a similar way, we can determine the network structures and elemental impedance functions of  $z_{11}$  and  $z_{22}$ .

(3) Step 3: find the values of the components of the elemental circuits.

For the optimization process, the input variables are the values of the components of the elemental circuits. The target functions are the frequency responses of the elemental impedance functions. To start the optimization process, some initial values of the elemental circuit need to be assigned. The initial values can all be 1.

From the above example of a two-port equivalent network, we can conclude that the developed three-step approach can yield a good result for modeling a power system. Also, we find:

1. The three-step approach divides the whole modeling problem into small parts making it easier to solve each small part and then finally the whole problem.

2. The modified pattern search is a useful and powerful direct search optimization technique. For the Hydro Example, this method was able to find the optimal values at each step of the optimization procedures.

3. In Step 3, to find the values of the network elements, the elemental circuits with good moving ability play an important role in the optimization process.

4. In step 1, the solutions for the impedance functions are not unique i.e., there are several impedance functions that have frequency responses close to the required one.

5. In the optimization process of Step 2, the target functions are always the frequency responses, therefore there is only one error. This has advantages over using the zeros and poles of the impedance function as the target function. Since the problem is to find a network that has a frequency response very close to the given one, if zeros and poles of an impedance are set to be the target function an extra error will be introduced. This is due to the fact that, for a given frequency response, a perfectly matched impedance function is not achievable in general.

6. It is noted that it is very important to have a good fit at 60 Hz and if necessary this can be achieved by introducing a weighting function in the objective function.

# Chapter 5

## More Examples

### 5.1 The Second Example

In Chapter 4, the T network structure is used for the Hydro Example and a good result is obtained. In order to prove the effectiveness of the proposed optimization method, some more examples need to be done. In this Chapter, the  $\Pi$  two-port network will be used for the realization of two-port networks for two more examples (the data was obtained from A. Gole, X. Hua and S. Elez) and a ladder two-port network (the data was obtained from G. O. Martens) will be used for one example. For the two examples using the  $\Pi$  two-port network, the impedance functions are used in one example and admittance functions are used in another example.

Frequency responses of an ac system model [41] are shown in Figs. 5.1.1, 5.1.2 and 5.1.3:

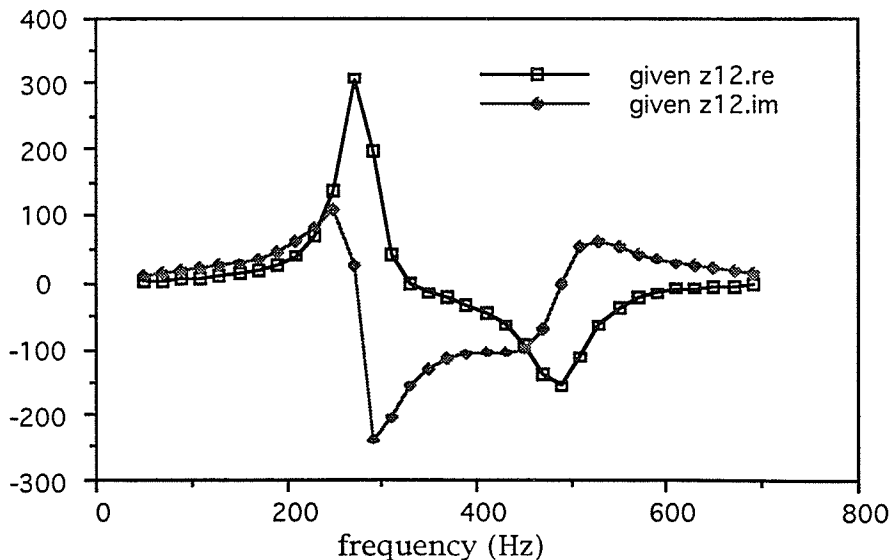


Fig. 5.1.1

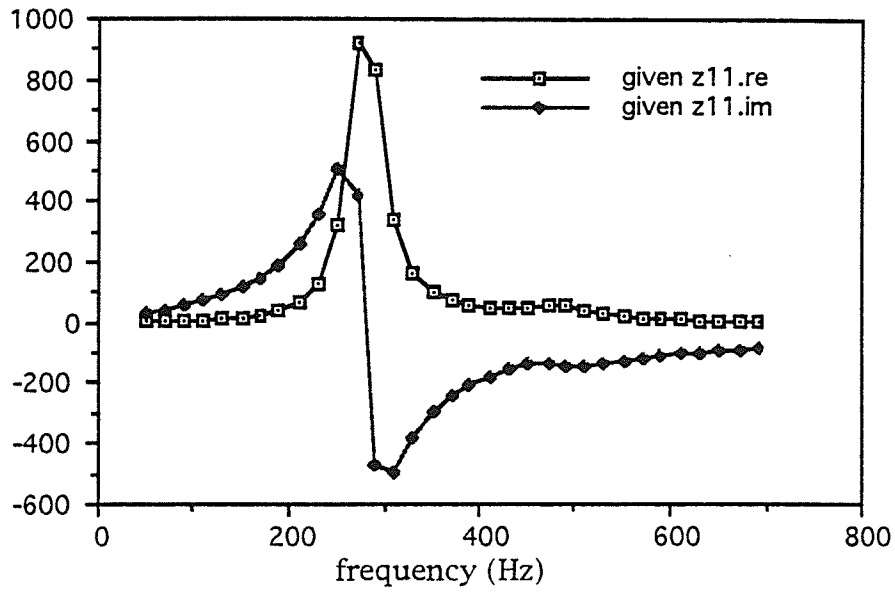


Fig. 5.1.2

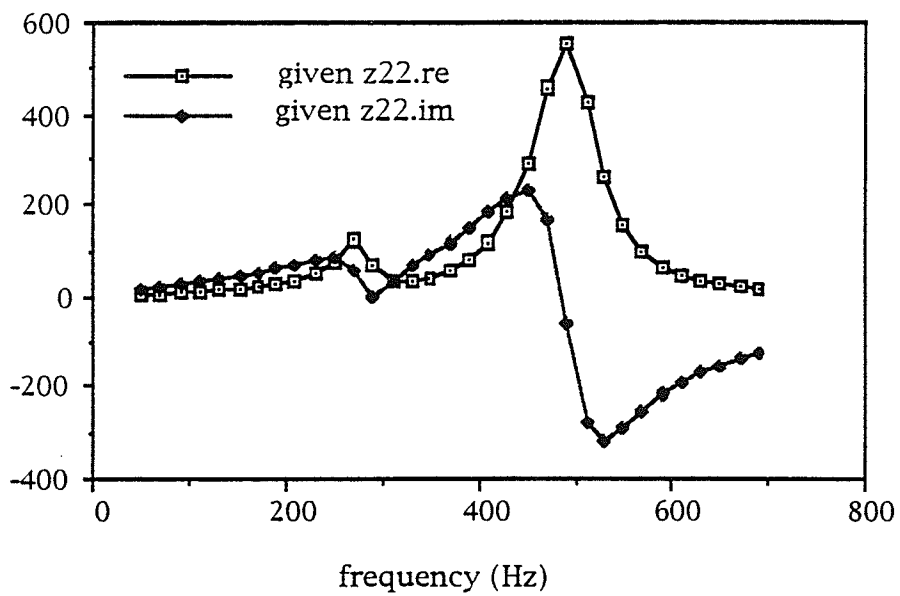


Fig. 5.1.3

We will follow the optimization procedures described in Chapter 4 to carry out the optimization for the degrees of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  in the next section.

## 5.2 Step 1: The degrees and types of zeros and poles of $z_{11}$ , $z_{12}$ and $z_{22}$

In order to find the starting point the group delay of  $z_{12}$  is calculated. The group delay curve of  $z_{12}$  is shown in Fig. 5.2.1. It is easy to see that  $z_{12}$  has two complex conjugate pairs of poles. The estimated values of these poles of  $z_{12}$  are listed in Table 5.2.1. However, no complex conjugate pairs of zeros are found from the group delay. Therefore, the zeros of  $z_{12}$  must all be real.

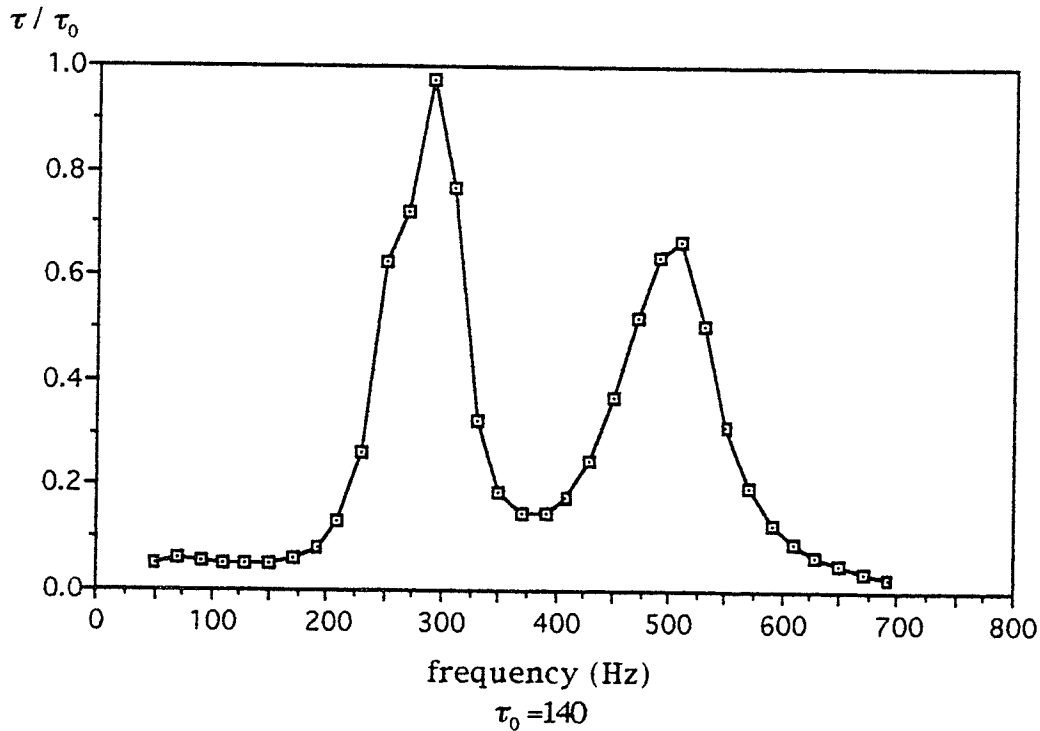


Fig. 5.2.1

Table 5.2.1 poles of the starting point

#	real part	imaginary part
1	120	1700
2	180	3000

Since group delay can not be used to find the number and magnitude of the real zeros and poles, we must apply the optimization program to test possible real zeros and poles. We test degree combinations of 2/4, 3/4, 4/4, 3/6, 4/6 and 5/6. The best result comes from the degree combination 3/6. Since the given  $z_{12}$  is not PR, the optimization procedure for  $z_{12}$  does not have PR constraints.

The starting point for  $z_{11}$  is also calculated using the group delay method. The group delay curve is shown in Fig. 5.2.2. The zeros and poles for the starting point are listed in Table 5.2.2. One complex conjugate pair of zeros is found. The poles of  $z_{11}$  are very close to those of  $z_{12}$ .

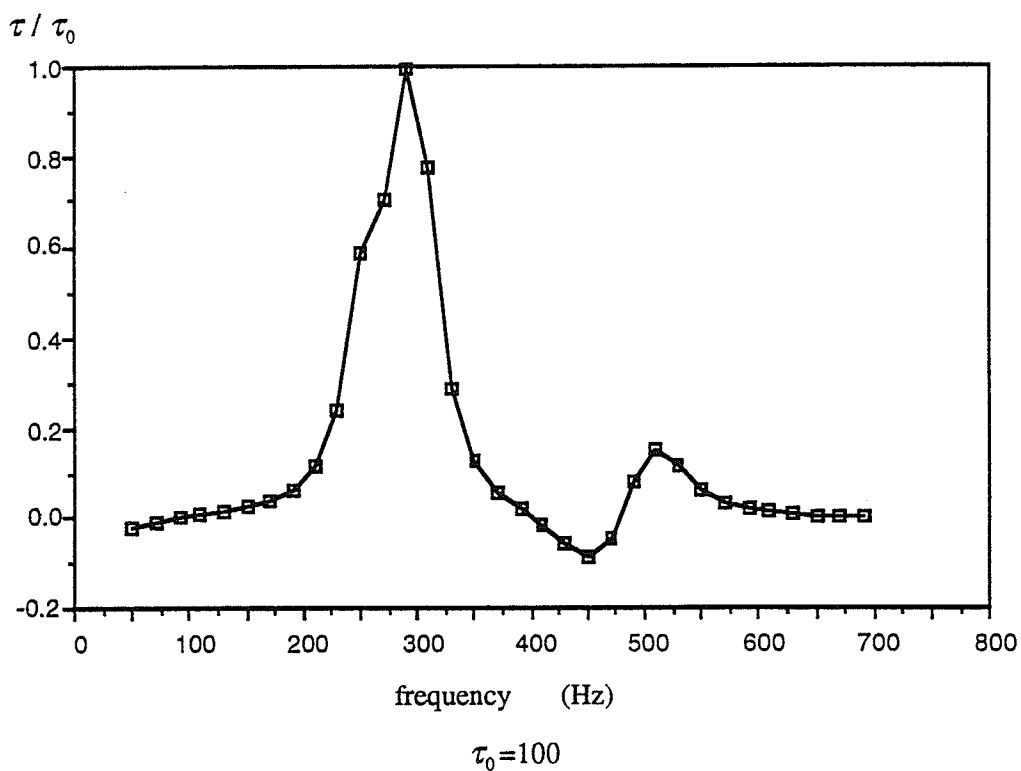


Fig. 5.2.2



Table 5.2.2 Zeros and poles for the starting point

zeros

#	real part	imaginary part
1	450	2650

poles

#	real part	imaginary part
1	100	1750
2	500	3200

In order to find the optimal degree combination, we use the optimization program to test degree combinations of 5/6, 6/6 and 7/6. The best combination is found to be 5/6. For the optimization of  $z_{11}$ , the PR conditions must be satisfied.

The starting point for  $z_{22}$  is also calculated using the group delay method. The group delay curve is shown in Fig. 5.2.3. The zeros and poles for the starting point are listed in Table 5.2.3.

Using the optimization to test 5/6, 6/6 and 7/6 degree combinations, we find the optimal degree configuration for  $z_{22}$  is 5/6. The PR constraint is also applied to the optimization process for  $z_{22}$ .

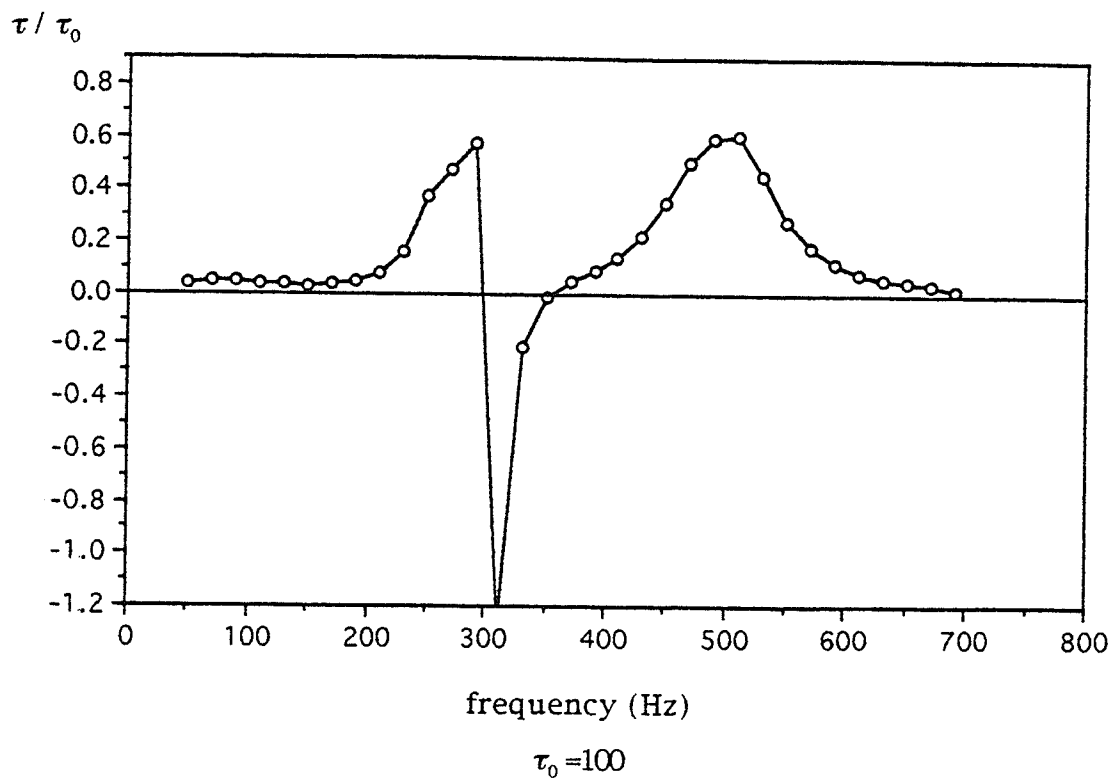


Fig. 5.2.3

Table 5.2.3

zeros

#	real part	imaginary part
1	80	1900

poles

#	real part	imaginary part
1	140	1700
2	150	3200

## 5.3 Step 2: The network structure

Fig. 5.3.1 shows a  $\Pi$  two-port network. For the  $\Pi$  two-port network, the relation between  $z_{12}$  and  $Z_1$ ,  $Z_2$  and  $Z_3$  is

$$z_{12} = \frac{n_1 d_2 n_3}{n_1 d_2 d_3 + n_2 d_1 d_3 + n_3 d_1 d_2} \quad (5.3.1)$$

We also have

$$z_{11} = \frac{n_1 (d_2 n_3 + n_2 d_3)}{n_1 d_2 d_3 + n_2 d_1 d_3 + n_3 d_1 d_2} \quad (5.3.2)$$

and

$$z_{22} = \frac{n_3 (n_1 d_2 + d_1 n_2)}{n_1 d_2 d_3 + n_2 d_1 d_3 + n_3 d_1 d_2} \quad (5.3.3)$$

From (5.3.1) to (5.3.3) it can be seen that  $z_{12}$ ,  $z_{11}$  and  $z_{22}$  have the same poles, i.e.,  $z_{11}$  and  $z_{22}$  do not have private poles, and this is necessary for using a  $\Pi$  two-port network to realize the two-port impedance functions.

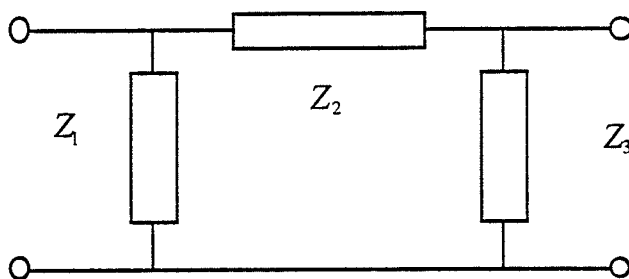


Fig. 5.3,1  $\Pi$  two-port network

From Section 5.2, it is found that the complex conjugate pole pairs of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are very close, therefore, it is possible to use the  $\Pi$  two-port network to solve the two-port network modeling problem.

The optimization process for the  $\Pi$  main network structure is different from that for the T main structure. When the T structure is used, the optimization for  $z_{12}$  can be carry out alone, but when  $\Pi$  main network structure is used, the optimizations for  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are interdependent. That is,  $Z_1$ ,  $Z_2$  and  $Z_3$  have to be determined together. Also, to start the optimization process, the number of poles and zeros of  $Z_1$ ,  $Z_2$  and  $Z_3$  need to be determined first. Equations 5.3.1, 5.3.2 and 5.3.3 impose some restrictions on the degrees of  $Z_1$ ,  $Z_2$  and  $Z_3$ . A combination of degrees of  $Z_1$ ,  $Z_2$  and  $Z_3$  which satisfies the imposed degree restrictions is: 1/2 for  $Z_1$ , 1/0 for  $Z_2$  and 2/3 for  $Z_3$ . For the optimization of step 2, the objective function includes the frequency responses of  $Z_1$ ,  $Z_2$  and  $Z_3$ . The optimization process of step 2 takes a much longer time than that of step 1, to reach the optimal point. The found impedance functions are listed in Tables 5.3.1, 5.3.2 and 5.3.3.

Table 5.3.1

Rational Function  $Z_1$

polyRep= prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part	imaginary part
2.9151186053158045200e+5	0.000000000000000000e+0
-1.5869992763430197680e+1	0.000000000000000000e+0

Denominator Degree:

2

Constant factor followed by Poles:

real part	imaginary part
1.000000000000000000e+0	0.000000000000000000e+0
-7.6292071001056810060e+1	-1.3658223110397025240e+3
-7.6292071001056810060e+1	1.3658223110397025240e+3

Table 5.3.2

Rational Function  $Z_2$ 

polyRep= prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part	imaginary part
1.7360363258562693040e-1	0.0000000000000000000e+0
-2.6719906779697277930e+1	0.0000000000000000000e+0

Denominator Degree:

0

Constant factor followed by Poles:

real part	imaginary part
1.0000000000000000000e+0	0.0000000000000000000e+0

Table 5.3.3.

Rational Function  $Z_3$ 

polyRep= prodRep

Numerator Degree:

2

Constant factor followed by Zeros:

real part	imaginary part
2.9208658087403923000e+5	0.0000000000000000000e+0
-2.8607785684514154450e+1	0.0000000000000000000e+0
-1.1489912592040089900e+3	0.0000000000000000000e+0

Denominator Degree:

3

Constant factor followed by Poles:

real part	imaginary part
1.0000000000000000000e+0	0.0000000000000000000e+0
-6.0819844947882877440e+2	0.0000000000000000000e+0
-3.2451037959160165060e+2	-2.6893858443781487090e+3
-3.2451037959160165060e+2	2.6893858443781487090e+3

## 5.4 Step 3: The elemental Circuits

The elemental circuit for  $Z_1$  is easily found. Elemental circuit No. A2 matches the frequency response of  $Z_1$  perfectly.  $Z_2$  also has a perfectly matching elemental circuit. Since the degree of  $Z_3$  is larger than 2 (the largest degree of an elemental circuit), an optimization is carried out to find the elemental impedance functions for  $Z_3$ . The possible combinations for  $Z_3$  are: 1)  $1/2$  in parallel with  $1/1$ , 2)  $1/2$  in series with  $0/1$ , 3)  $2/1$  in parallel with  $0/1$  and 4)  $2/2$  in parallel with  $0/1$ . After 800 iterations, the errors of combination 1 and combination 4 are greater than 3000, the error of combination 2 is very close to the error of combination 3. However, further test tells that the combination 3 is the best. The two-port network for the combination 3 is shown in Fig. 5.4.1. The element values are listed in Table 5.4.1.

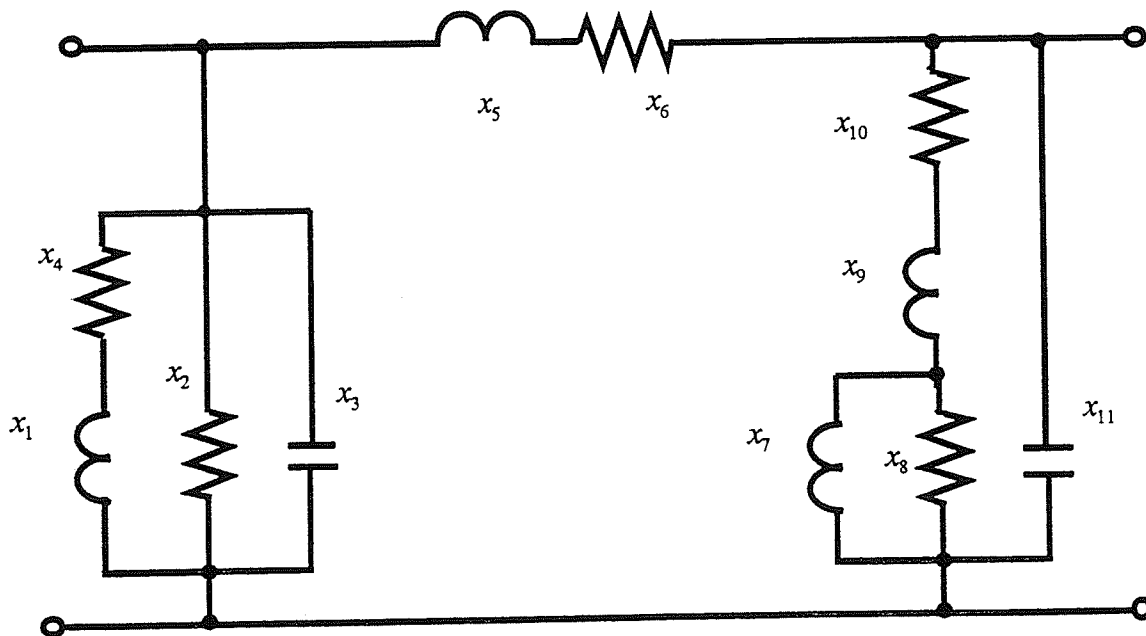


Fig. 5.4.1

Table 5.4.1 Values of elements

#	elemental values	unit
$x_1$	1.559619754552841e-1	H
$x_2$	2.132272705078125e+3	$\Omega$
$x_3$	3.430392098380253e-6	F
$x_4$	2.475115299224854e+0	$\Omega$
$x_5$	1.736036390066147e-1	H
$x_6$	4.638672828674316e+0	$\Omega$
$x_7$	2.031643502414227e-2	H
$x_8$	2.363301277160645e+1	$\Omega$
$x_9$	3.546867519617081e-2	H
$x_{10}$	3.884041070938110e+0	$\Omega$
$x_{11}$	3.499000000000000e-6	F

## 5.5 Results and discussion

The frequency responses of the impedance functions are displayed in Figs. 5.5.1, 5.5.2 and 5.5.3.

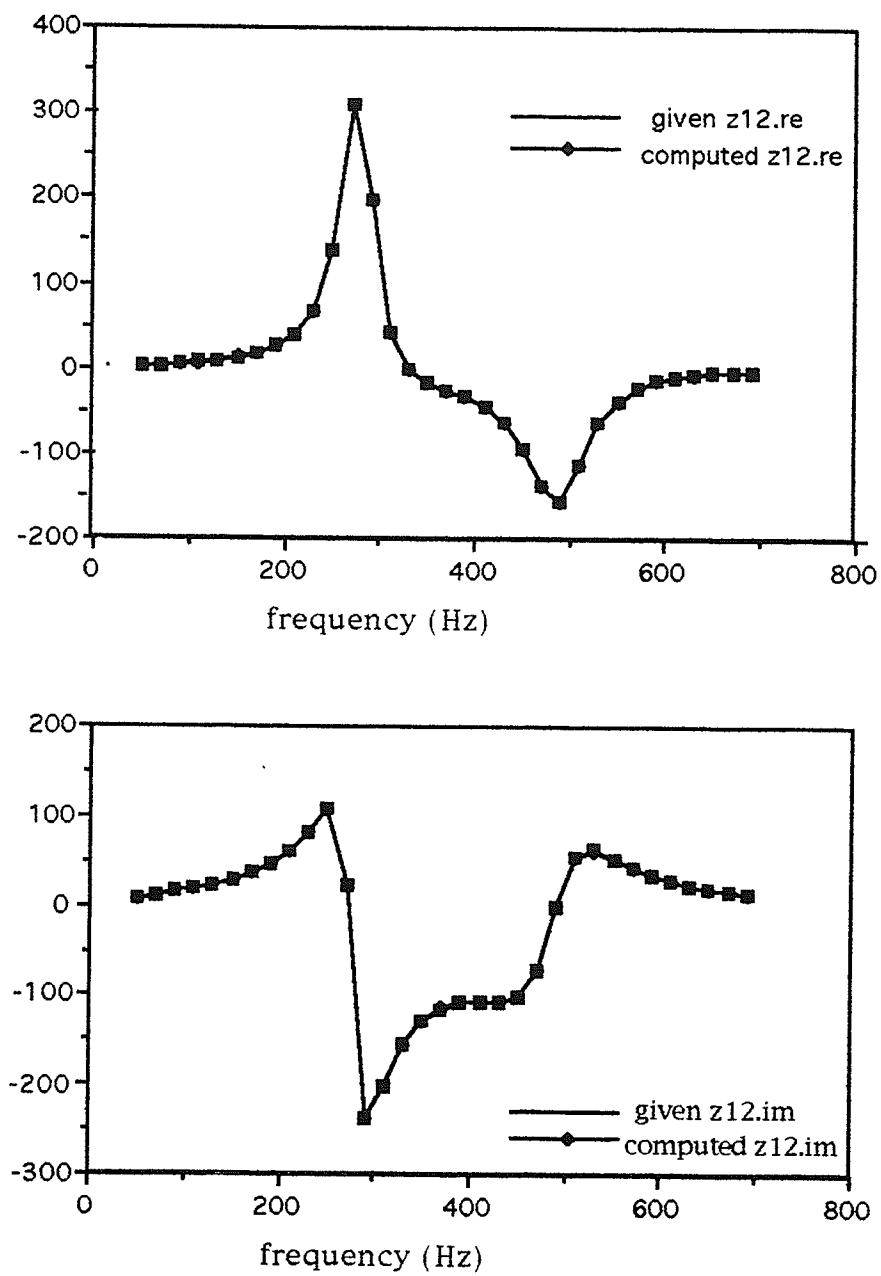


Fig. 5.5.1



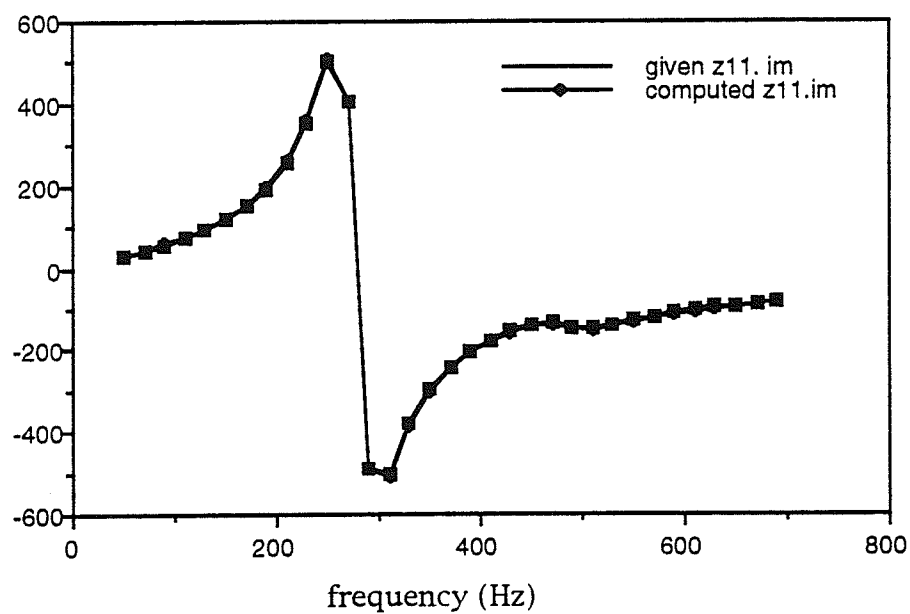
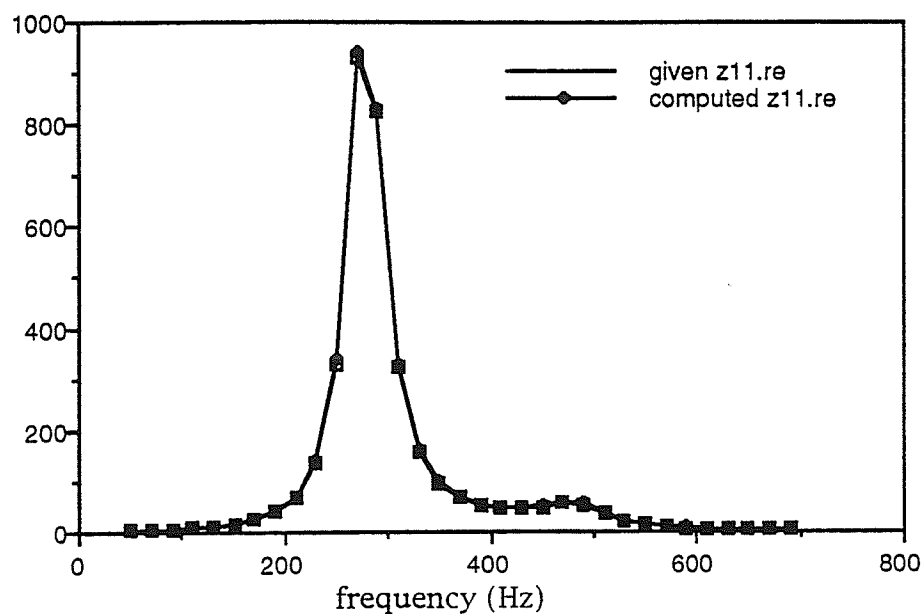


Fig. 5.5.2

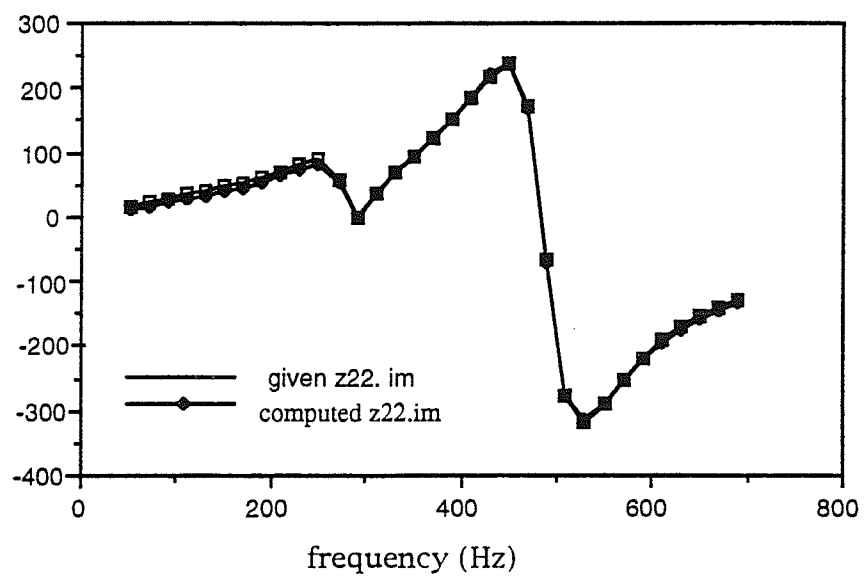
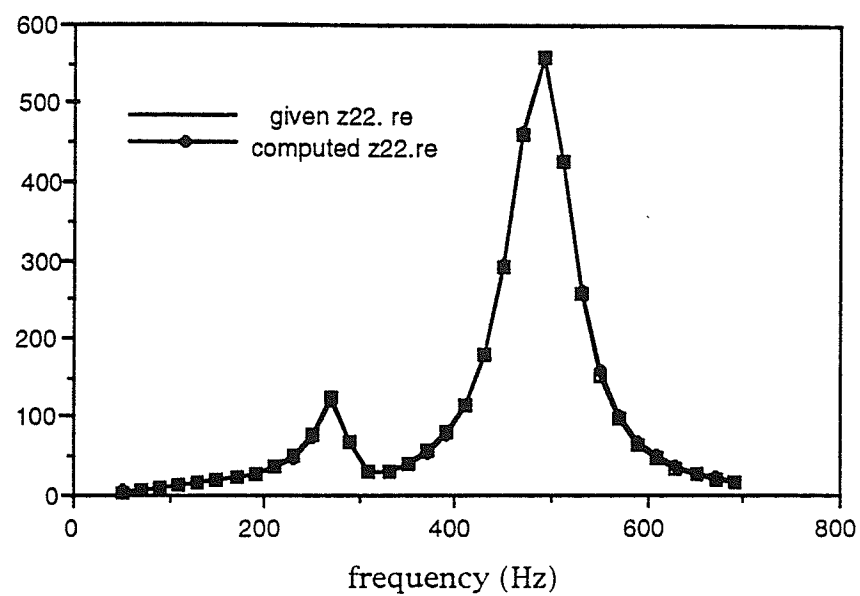


Fig. 5.5.3

From the above example, we find that the  $\Pi$  network structure can be used to realize impedance functions with the real part of  $z_{12}$  not either positive or negative for all frequencies. For these cases, the T structure is not realizable. The computation time of the optimization for the above example in this chapter is much longer than that required for the Hydro Example. This is due to the fact that the impedance functions have many more real poles and zeros that can not be found by the group delay method than the impedance functions of the Hydro Example has and therefore, the optimization starts at a point far from the optimal point. For the Hydro Example, the optimization only requires increasing the step size to overcome local minima. However, for the example of this section, changing the starting point and increasing the step size—the techniques for overcoming a local minimum—both have to be used intensively. On the other hand, this also demonstrates the effectiveness of the optimization method.

## 5.6 The third example

The given frequency responses of the impedance functions for the third example are shown in Figs. 5.6a, 5.6b and 5.6c. The optimization procedures for this example will be similar to those for the second example. However, some novel modifications will be developed to make the optimization procedure more efficient.

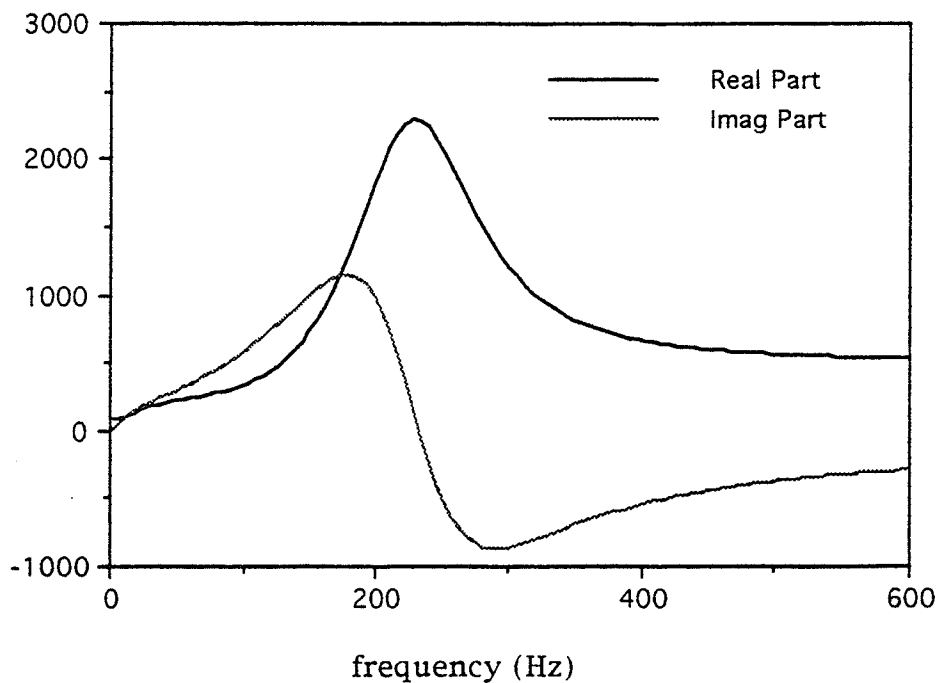


Fig. 5.6 a Frequency response of  $z_{11}$

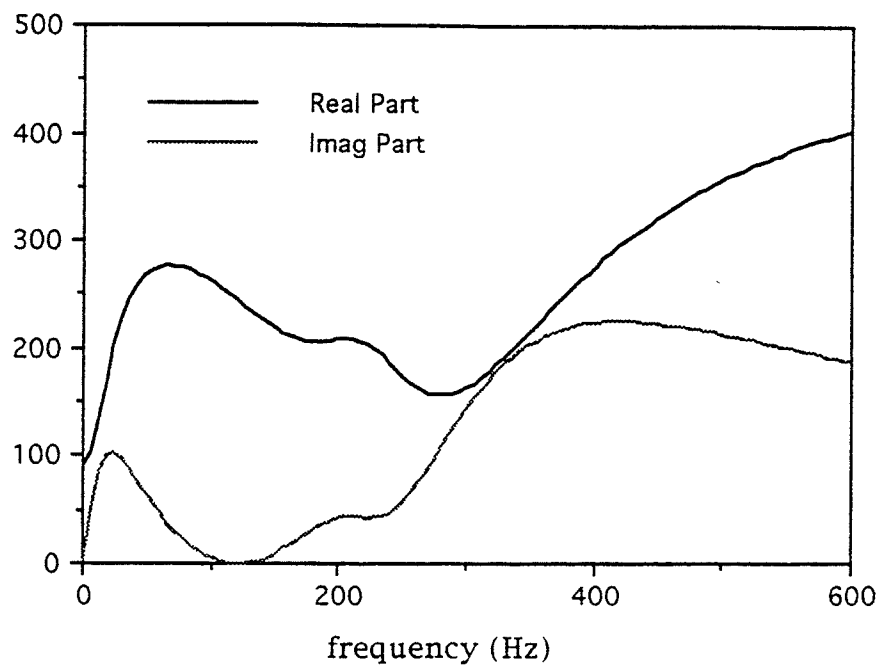


Fig. 5.6 b Frequency response of  $z_{12}$

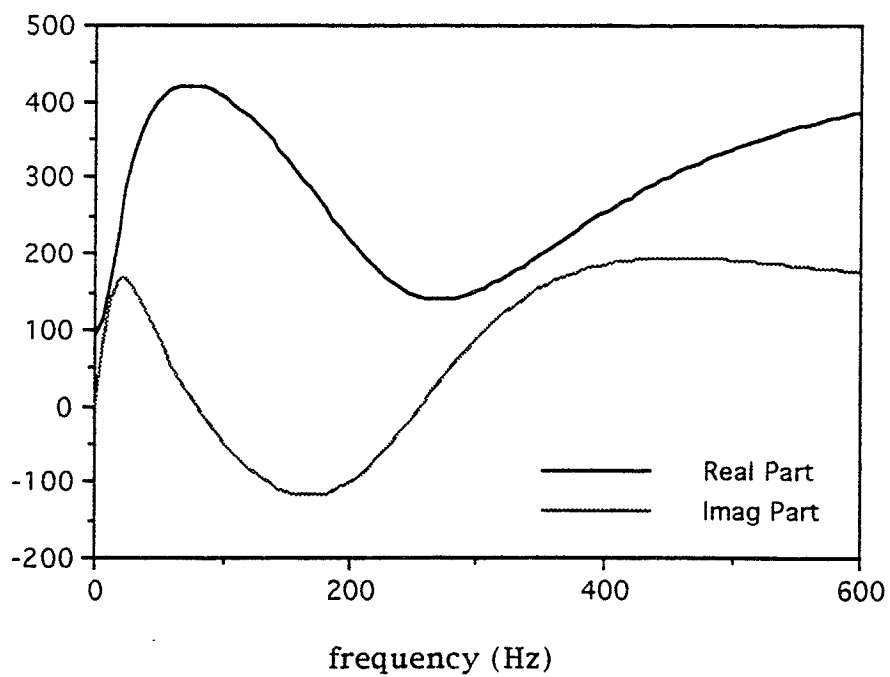


Fig. 5.6 c Frequency response of  $z_{22}$

### 5.6.1 Step 1

From the previous examples, it is found that the group delay is useful for estimating the starting points of the optimization of Step 1. There are some limitations in using the group delay to estimate the starting point:

1. It is difficult to find the numbers of real zeros and poles and their magnitudes.
2. In general, when poles or zeros are close together and poles and zeros are close to each other, then it is very difficult to estimate these poles and zeros.

In the previous examples, the way to deal with this shortcoming was to test some combinations of poles and zeros in addition to the poles and zeros found by the group delay method and then to pick the best one. In this example, a new way, which applies optimization directly to the group delay—the group delay is the target function—is adopted. In this new technique, some combinations of zeros and poles in addition to those estimated from the group delay are also tested. However, because the target function is the group delay, no complex-number operations are required, the latter optimization process is much faster than the former.

Fig. 5.6.1a displays the given group delay of  $z_4$ . From this delay curve it is difficult to estimate the real poles and zeros. To apply the optimization, we set the group delay as the target function of the optimization and the poles and zeros of the impedance functions as the input variables. Rewriting (4.3.1b-2) we have

$$\begin{aligned}
\tau(j\omega) &= \sum_{i=1}^n \operatorname{Re} \left( \frac{1}{s - p_i} \right) \Big|_{s=j\omega} - \sum_{i=1}^m \operatorname{Re} \left( \frac{1}{s - z_i} \right) \Big|_{s=j\omega} \\
&= \sum_{i=1}^{n1} \left( \frac{\alpha_i}{\alpha_i^2 + (\omega - \beta_i)^2} + \frac{\alpha_i}{\alpha_i^2 + (\omega + \beta_i)^2} \right) + \sum_{i=1}^{n2} \frac{\alpha_i}{\alpha_i^2 + \omega^2} \\
&\quad - \sum_{i=1}^{m1} \left( \frac{\gamma_i}{\gamma_i^2 + (\omega - \delta_i)^2} + \frac{\gamma_i}{\gamma_i^2 + (\omega + \delta_i)^2} \right) - \sum_{i=1}^{m2} \frac{\gamma_i}{\gamma_i^2 + \omega^2}
\end{aligned}$$

where  $p_i = -\alpha_i + j\beta_i$ , and  $z_i = -\gamma_i + j\delta_i$ , and  $\frac{\alpha_i}{\alpha_i^2 + (\omega - \beta_i)^2} + \frac{\alpha_i}{\alpha_i^2 + (\omega + \beta_i)^2}$  represents a pair of complex conjugate poles,  $\frac{\alpha_i}{\alpha_i^2 + \omega^2}$  presents a real pole,  $\frac{\gamma_i}{\gamma_i^2 + (\omega - \delta_i)^2} + \frac{\gamma_i}{\gamma_i^2 + (\omega + \delta_i)^2}$  represents a pair of complex conjugate zeros, and  $\frac{\gamma_i}{\gamma_i^2 + \omega^2}$  represents a real zero.  $n1$  and  $m1$  are the numbers of pairs of complex conjugate poles and zeros, respectively, and  $n2$  and  $m2$  are the numbers of real poles and zeros, respectively.

We also select several different combinations of poles and zeros for testing. For  $z_{12}$ , with 40 iterations, the difference of objective function values between the best combination of zeros and poles and the other combinations is great, so that we can easily pick out the best combination. Table 5.6.1a shows the errors for two different degree combinations.

Table 5.6.1a

degree combination	5/5	4/5
error	0.0018	0.0256

poles. The found group delay is shown in Fig. 5.6.1a. The unknown constant factor is found by an optimization—a single variable optimization using the above found poles and zeros. Thus the impedance function  $z_{12}$  is found. Fig. 5.6.1b displays the given and found frequency responses of  $z_{12}$ . It can be seen that the error is quite small. In a similar way, we find that there are one real and two complex conjugate pairs of zeros for  $z_{22}$ , and there are three real zeros and one complex conjugate pair of zeros for  $z_{11}$ . The poles of both  $z_{11}$  and  $z_{22}$  are the same as those of  $z_{12}$ . The group delay of  $z_{11}$  is shown in Fig. 5.6.1c and the frequency response of  $z_{11}$  is shown in Fig. 5.6.1d. The computed group delay of  $z_{22}$  is illustrated in Fig. 5.6.1e and the computed frequency response of  $z_{22}$  is displayed in Fig. 5.6.1f. The errors of both  $z_{11}$  and  $z_{22}$  are very small as well.

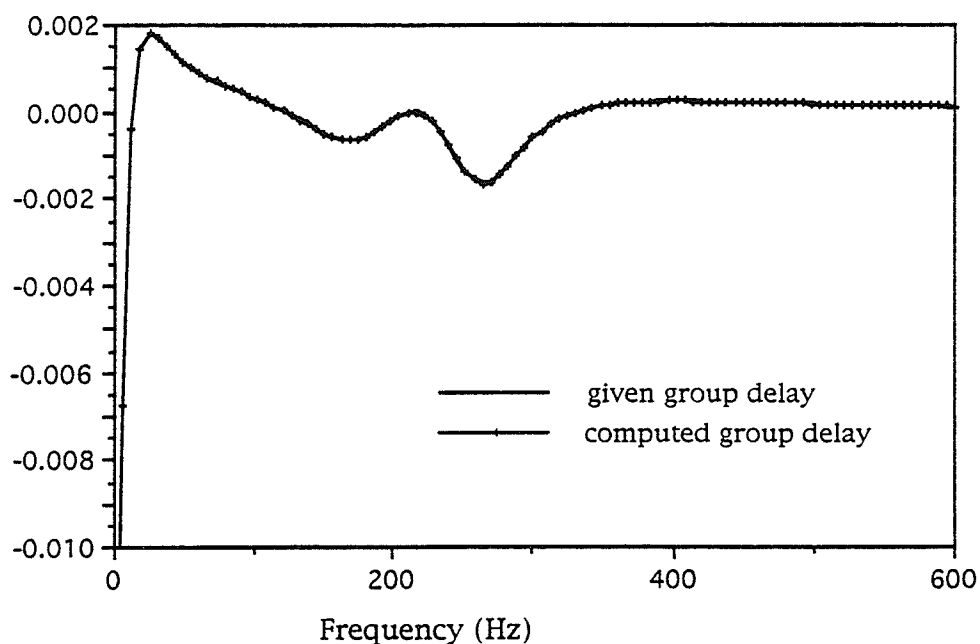


Fig. 5.6.1a Group delay of  $z_{12}$



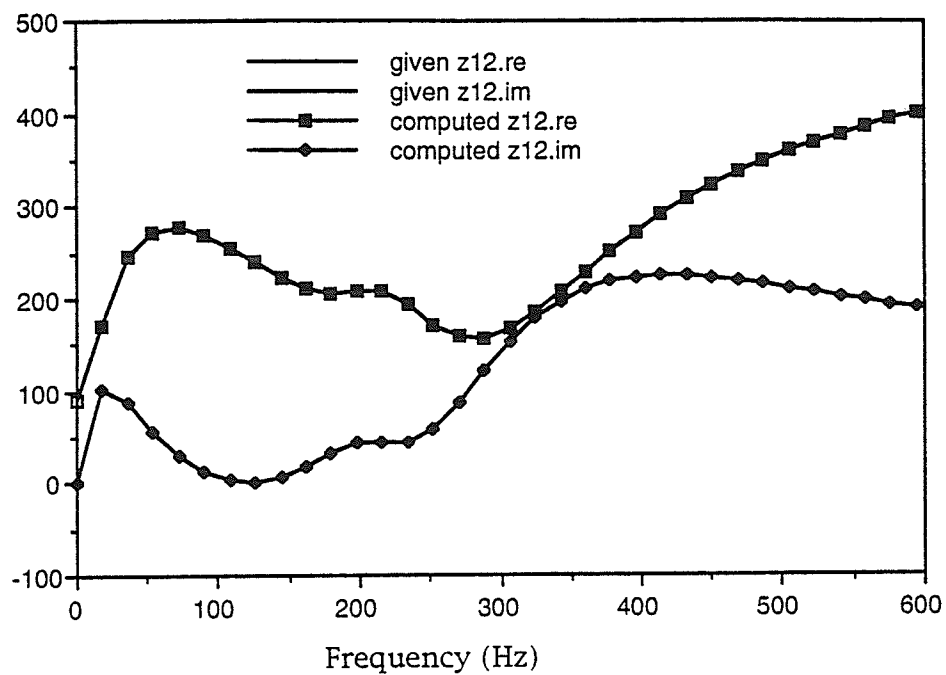


Fig. 5.6.1b Frequency response of  $z_{12}$

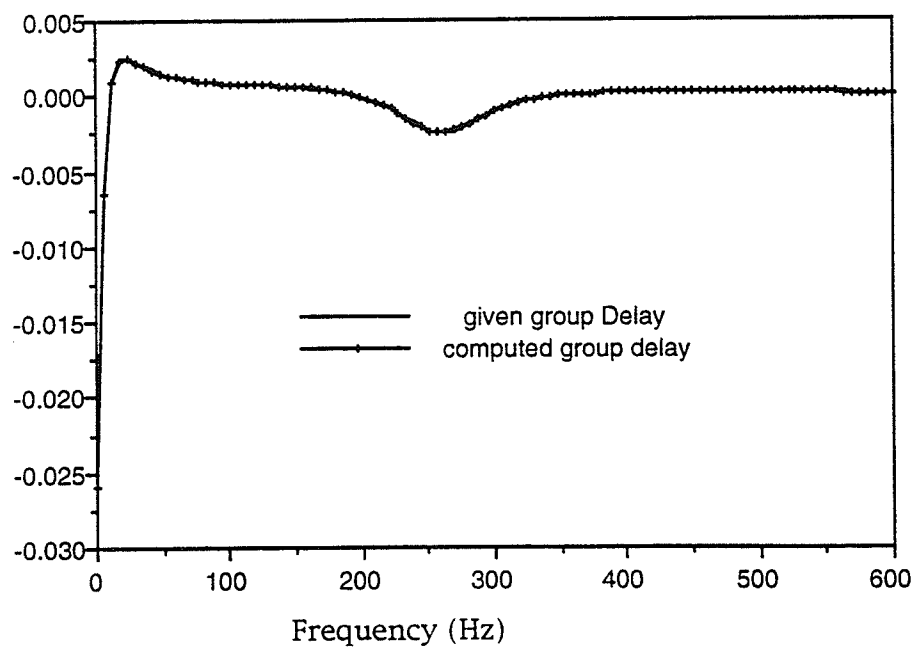


Fig. 5.6.1c Group delay of  $z_{22}$

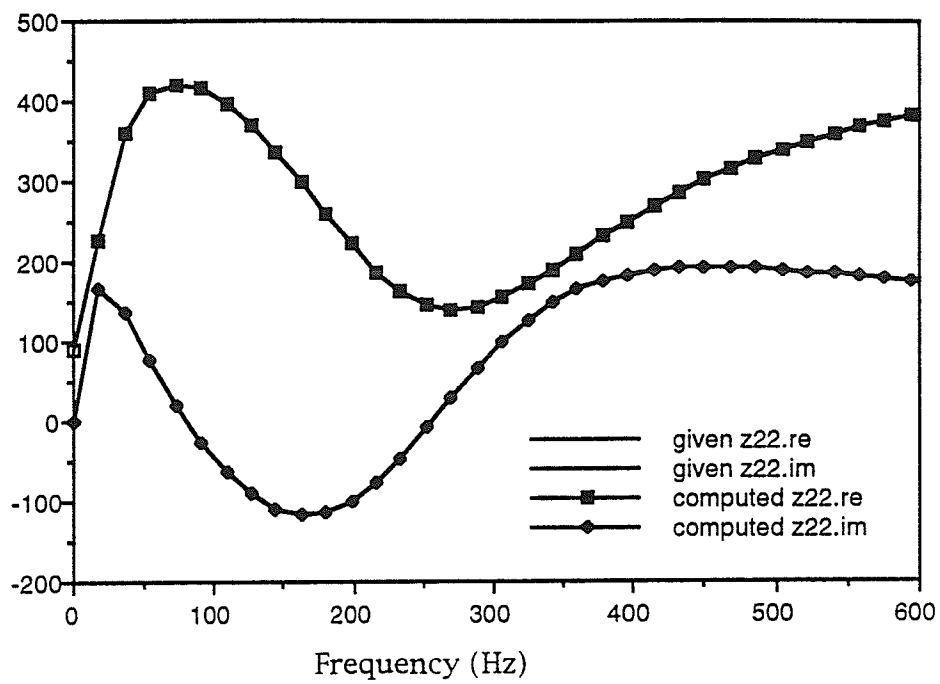


Fig. 5.6.1d Frequency response of  $z_{22}$

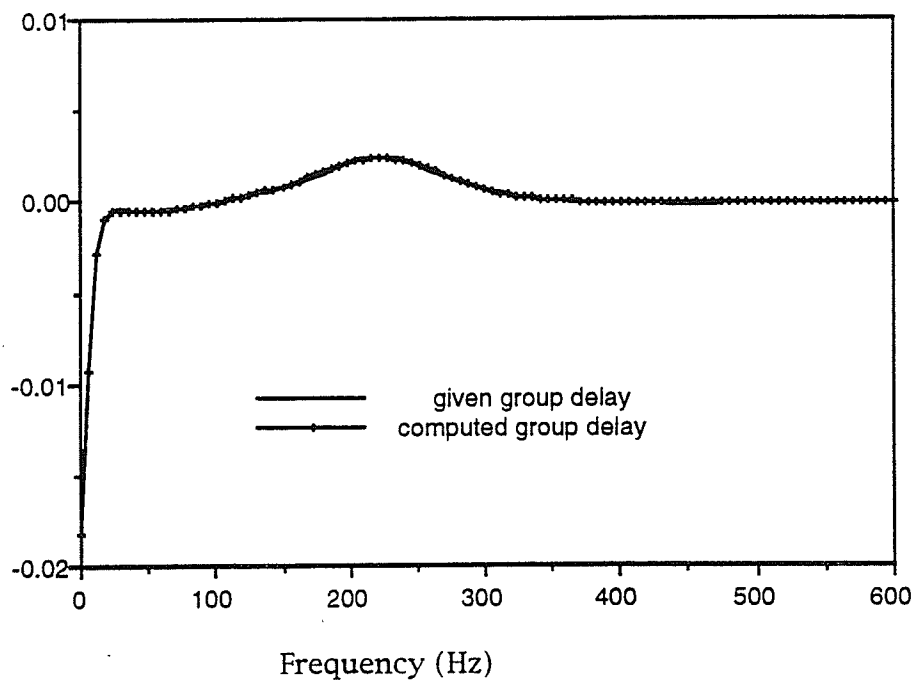


Fig. 5.6.1e Group delay of  $z_{11}$

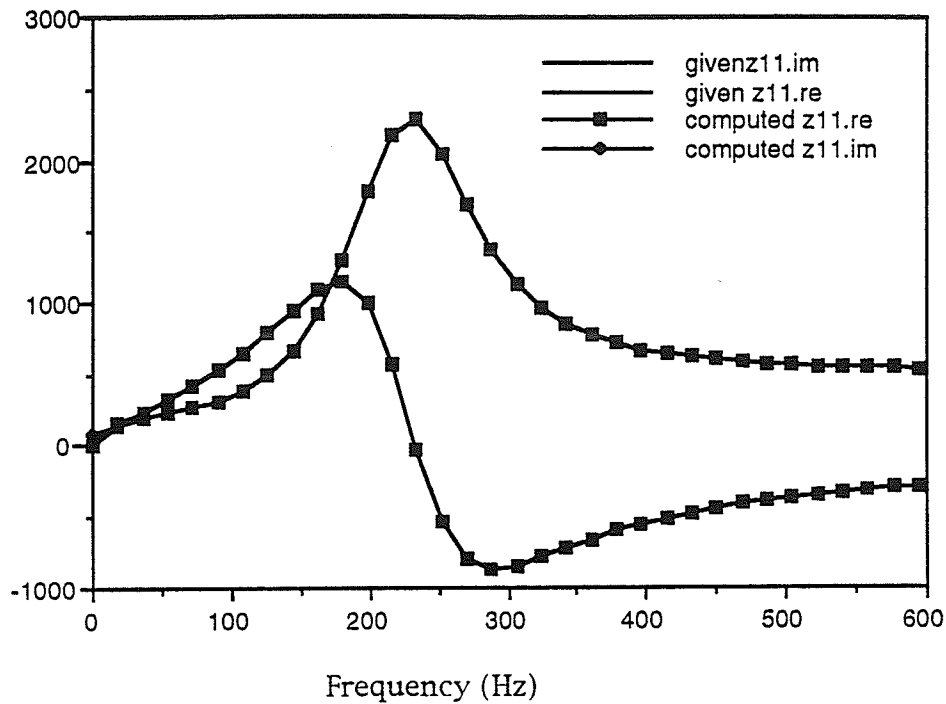


Fig. 5.6.1f Frequency response of  $z_{11}$

## 5.6.2 Step 2

From the given frequency responses of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  shown in Figs. 5.6a, 5.6b and 5.6c, it can be seen that the real parts of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  do not satisfy the condition for the T structure, therefore, the  $\Pi$  structure is adopted. In Example 2, the computation time is much longer than that required for the Hydro Example. The reason for the longer computation time is that the calculations for  $Z_1$ ,  $Z_2$  and  $Z_3$  in the Hydro Example are separate, however, the calculations for Example 2 are not separate and this leads to more variables and more constraints on the optimization. For the optimization, the computation time increases exponential with the number of the input variables. Also, the more constraints there are, the slower the convergence is. To accelerate the

optimization process, we need to find a way to enable the optimization for  $Z_1$ ,  $Z_2$  and  $Z_3$  to be carried out separately. The relationship of network element impedance functions  $Z_1$ ,  $Z_2$  and  $Z_3$  and the corresponding short-circuit admittance functions are as follows.

$$\begin{aligned} y_{12} &= -\frac{1}{Z_2} \\ y_{11} &= \frac{1}{Z_2} + \frac{1}{Z_1} \\ y_{22} &= \frac{1}{Z_3} + \frac{1}{Z_2} \\ Z_1 &= \frac{1}{y_{11} + y_{12}} \\ Z_2 &= -\frac{1}{y_{12}} \\ Z_3 &= \frac{1}{y_{12} + y_{22}} \end{aligned}$$

It is clear that by using the admittance functions, the optimizations for  $Z_1$ ,  $Z_2$  and  $Z_3$  can be carried out separately. The admittance functions can be calculated from the impedance functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ . The calculation of the frequency responses of the admittance functions using the given frequency responses of the impedance functions is as follows:

$$y_{11}(j\omega_i) = \frac{z_{22}(j\omega_i)}{\Delta z(j\omega_i)}$$

$$y_{12}(j\omega_i) = \frac{z_{12}(j\omega_i)}{\Delta z(j\omega_i)}$$

$$y_{22}(j\omega_i) = \frac{z_{11}(j\omega_i)}{\Delta z(j\omega_i)}$$

where  $\Delta z(j\omega_i) = z_{11}(j\omega_i) \cdot z_{22}(j\omega_i) - (z_{12}(j\omega_i))^2$ .

In the optimization processes for  $Z_2$ , the input variables are the zeros and poles of  $Z_2$  and the objective function value is the difference between the

frequency response of  $-\frac{1}{y_{12}}$  and the frequency response of the calculated  $Z_2$ .

The frequency response found for  $Z_2$  is shown in Fig. 5.6.2a. Next an optimization process is carried out to find the impedance function  $Z_2$ . Several different combinations of zeros and poles are chosen to be tested by optimization. It is found that the best degree combination for  $Z_2$  is 1/2. The zeros and poles of  $Z_2$  are listed in Table 5.6.2a.

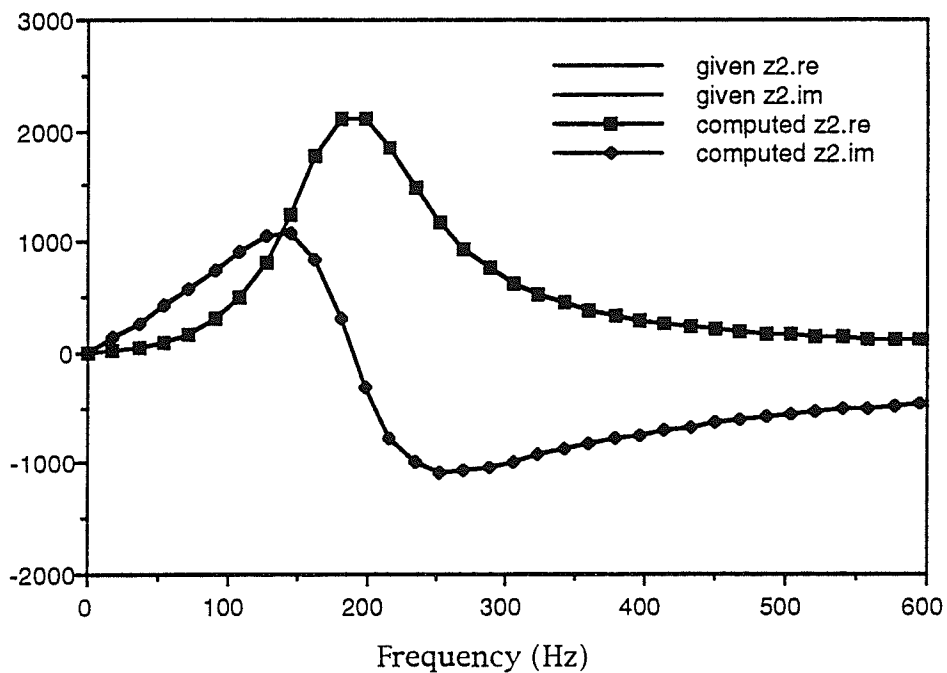


Fig. 5.6.2 a frequency response of  $Z_2$

Table 5.6.2a

Rational Function  $Z_2$

polyRep=

prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part      imaginary part

1.650 e+6      0.000 e+0

0.000 e+0      0.000 e+0

Denominator Degree:

2

Constant factor followed by Poles:

real part      imaginary part

1.000 e+0      0.000 e+0

-3.820 e+2      -1.122 e+3

-3.820 e+2      1.122 e+3

In a similar way, we find the frequency response of  $Z_1$  which is displayed in Fig. 5.6.2b. From the frequency response we find the impedance function shown in Table 5.6.2b by optimization (Step 2).

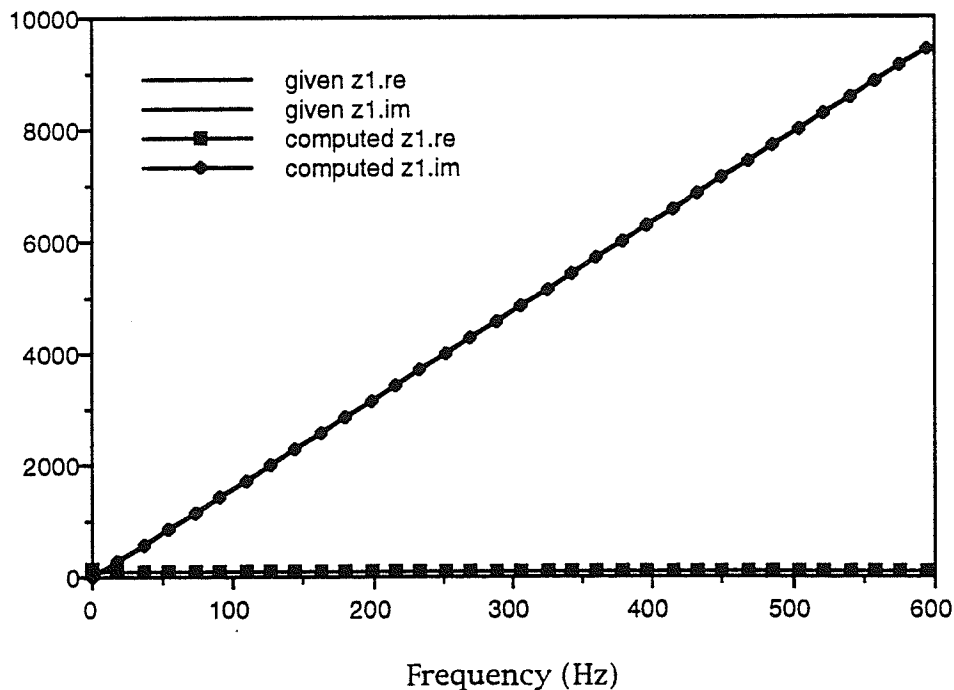


Fig. 5.6.2 b Frequency response of  $Z_1$

Table 5.6.2b

Rational Function  $Z_1$

polyRep=

prodRep

Numerator Degree:

1

constant factor followed by zeros

real part	imaginary part
2.526e+0	0.000e+0
4.500e+0	0.000e+0

Denominator Degree:

0

constant factor followed by poles

real part	imaginary part
1.000 e+0	0.000 e+0

The frequency response of  $Z_3$  is calculated in a manner similar to the above calculation of  $Z_1$  and is shown in Fig. 5.6.2c. The zeros and poles of  $Z_3$  are found by an optimization process and the result is listed in Table 5.6.2c.

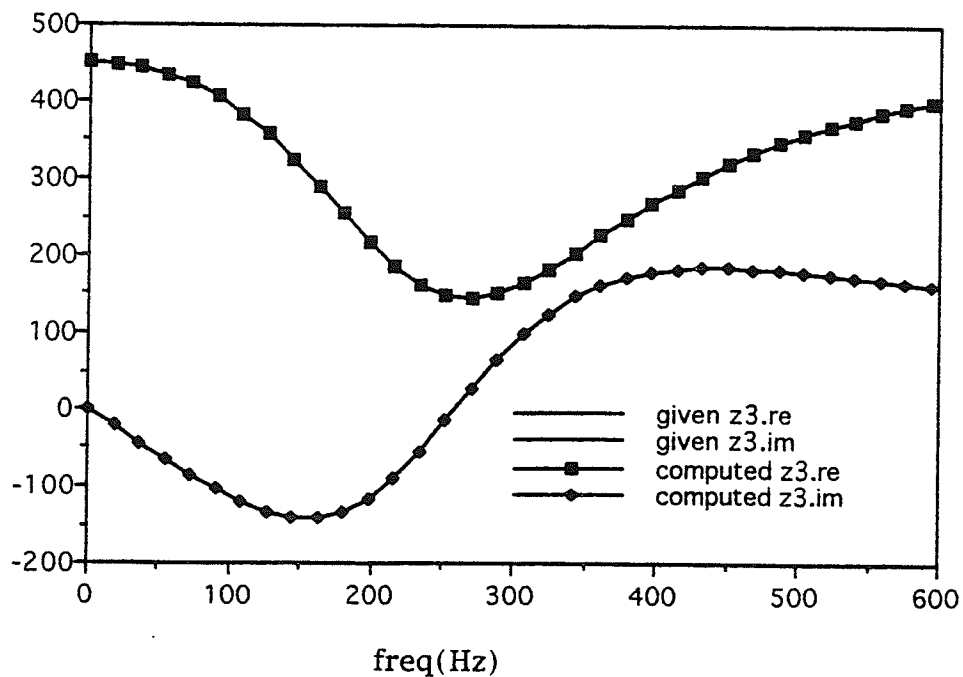


Fig. 5.6.2c Frequency response of  $Z_3$

Table 5.6.2c

Rational Function  $Z_3$

polyRep=

prodRep

Numerator Degree:

2

Constant factor followed by Zeros:

real part	imaginary part
-----------	----------------

4.919e+2	0.000e+0
----------	----------

-2.750e+2	-1.632e+3
-----------	-----------

-2.750e+2	1.632e+3
-----------	----------

Denominator Degree:

2

Constant factor followed by poles:

real part	imaginary part
-----------	----------------

1.000e+0	0.000e+0
----------	----------

-9.344e+2	-1.453e+3
-----------	-----------

-9.344e+2	1.453e+3
-----------	----------

### 5.6.3 Step 3

From the impedance function  $Z_1$ , the circuit for  $Z_1$  is easily found, it has only two elements, an inductor in series connection with a resistor. Fig. 5.6.3a shows the circuit. The value of the inductor is 2.526439879427041535H and the value of the resistor is 113.68979457421687 $\Omega$ .



Fig. 5.6.3a



The circuit for  $Z_2$  is readily to found by using optimization (Step 3). The circuit is elemental circuit No. A1 which perfectly matches the frequency response of impedance function  $Z_2$ . The circuit of  $Z_2$  is illustrated in Fig. 5.6.3b and the values of the circuit components are listed in Table 5.6.3b.

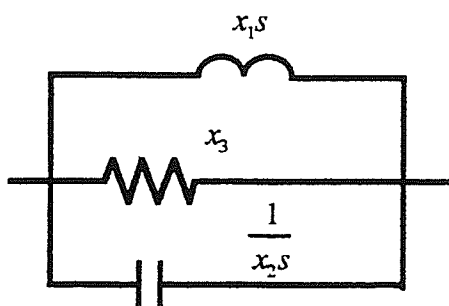


Fig. 5.6.3b Circuit for  $Z_2$

Table 5.6.3a

component number	value of component	
$x_1$	1.175213694572449e+0	H
$x_2$	6.057119890101603e-7	F
$x_4$	2.160928710937500e+3	$\Omega$

From the impedance function  $Z_3$  in Step 2, it is found that after a parallel resistor  $x_5$  is removed from the reciprocal of  $Z_3$ —admittance  $Y_3$ , then the remaining impedance function is a 2/1 impedance function which can be realized by the basic elemental circuit No. B2 (see Chapter 6). The circuit

realizing  $Z_3$  is displayed in Fig. 5.6.3b. The values of the circuit components are listed in Table 5.6.3b.

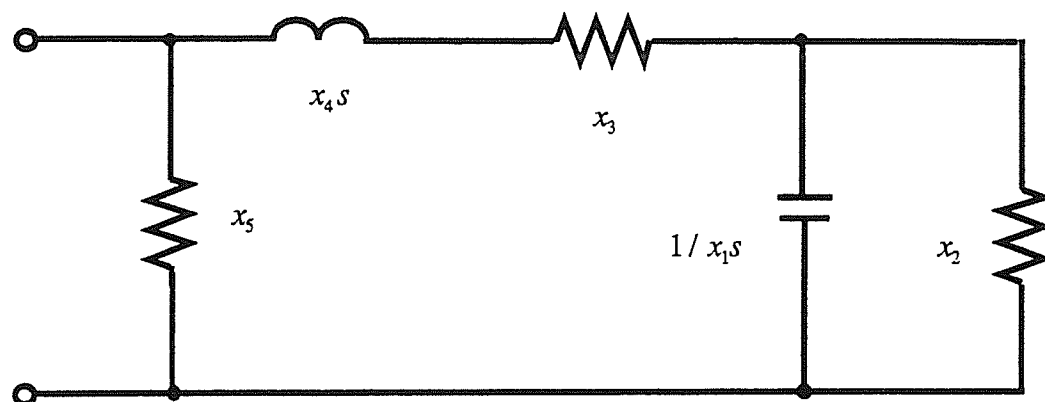


Fig. 5.6.3b

Table 5.6.3b

component number	value of component	
$x_1$	1.003731881610292e-6	F
$x_2$	5.354114746093750e+3	$\Omega$
$x_3$	1.357267608642578e+2	$\Omega$
$x_4$	3.7295358528327080e-1	H
$x_5$	4.9188428578299663e+2	$\Omega$

## 5.6.4 Results and discussion

Fig. 5.6.4a shows the complete circuit realizing  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ . The frequency responses of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are displayed in Figs. 5.6.4b, 5.6.4c and 5.6.4d, respectively. It is clear that the results are very good.

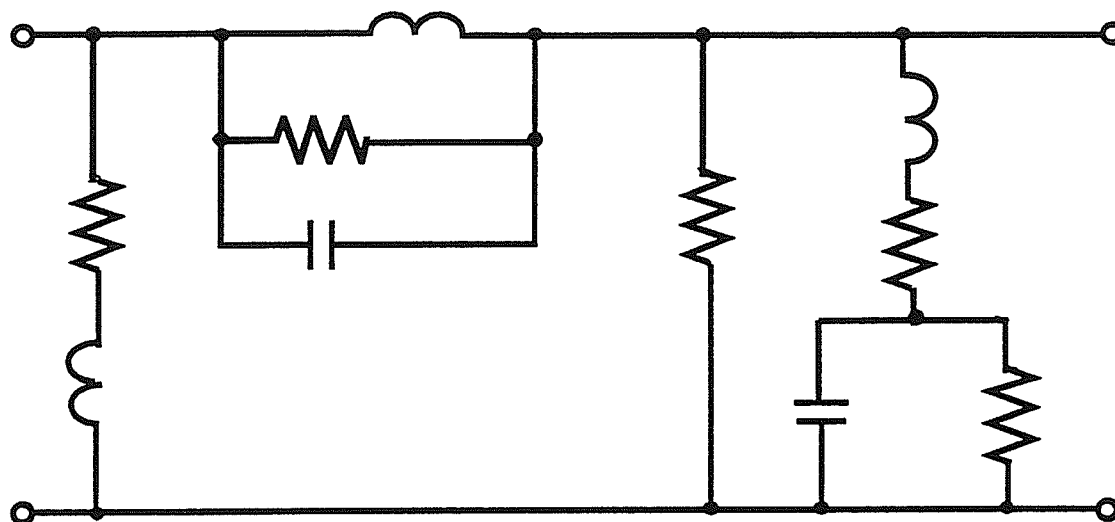


Fig. 5.6.4a

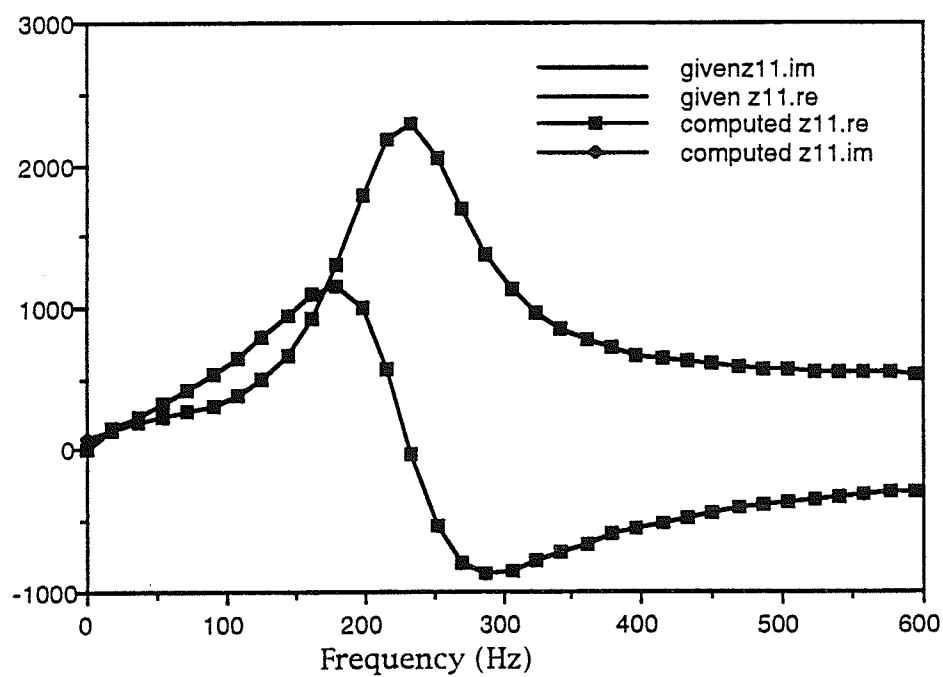


Fig. 5.6.4b

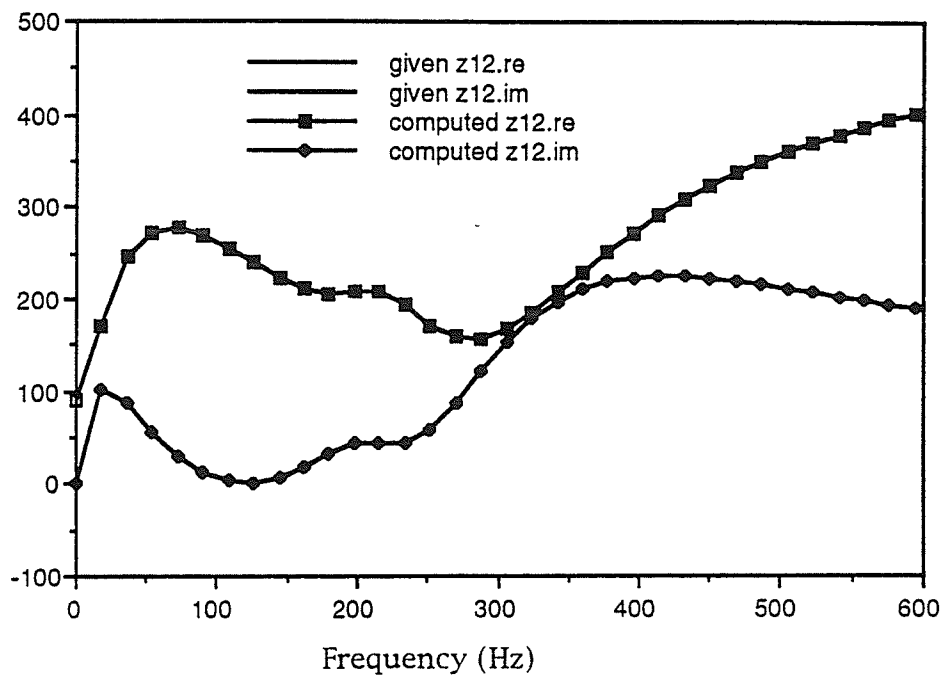


Fig. 5.6.4c

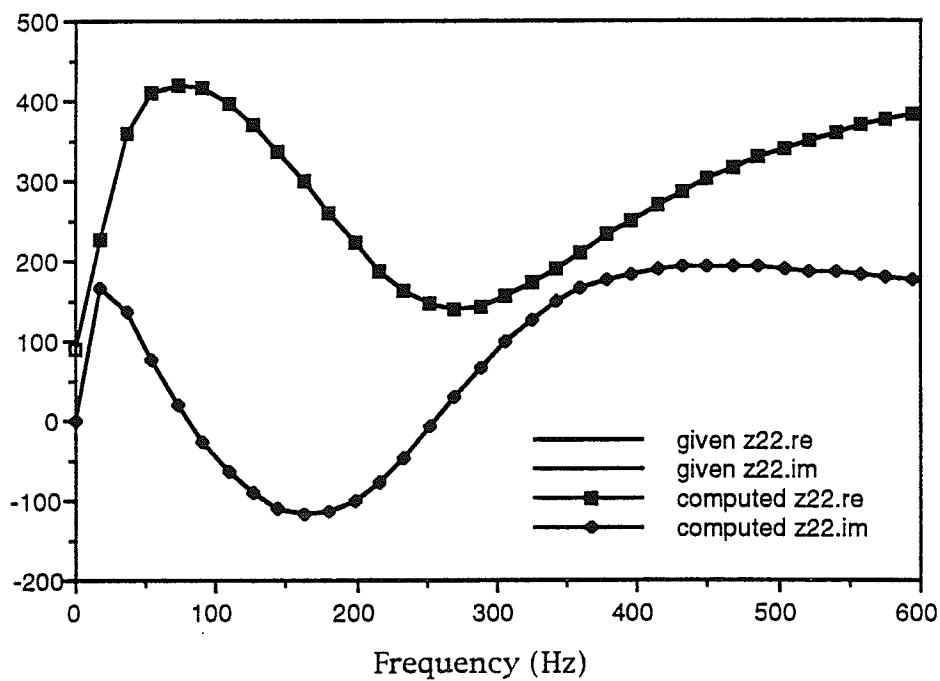


Fig. 5.6.4d

In Step 1, applying an optimization to the group delay determines not only the numbers and types of zeros and poles, but also the zeros and poles of the impedance functions quite accurately. In addition, it speeds up the optimization process of Step 1 more than 10 times, since the optimization only deals with real numbers instead of complex numbers and polynomial operations. Using an optimization process on the group delay to find the zeros and poles is also applied to Example 2, with similar results.

In Step 2, when the  $\Pi$  two-port structure is required, using admittance functions to realized the network is much faster than using impedance functions, because by using admittance functions, the optimizations can be carried out separately for  $Z_1$ ,  $Z_2$  and  $Z_3$  and the computation time of optimization is related exponentially to the number of input variables. The same approach of using admittance functions for the  $\Pi$  two-port structure is applied to Example 2, the optimization process is also much faster than that using impedance functions.

## 5.7 Example Four

The given frequency responses of the impedance functions for Example 4 are shown in Figs. 5.7.1, 5.7.2 and 5.7.3.

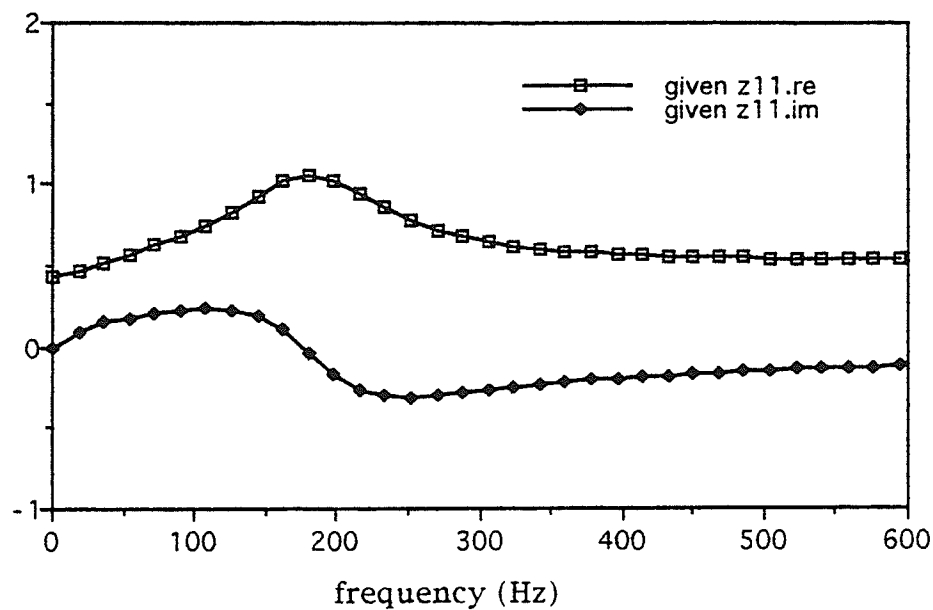


Fig. 5.7.1

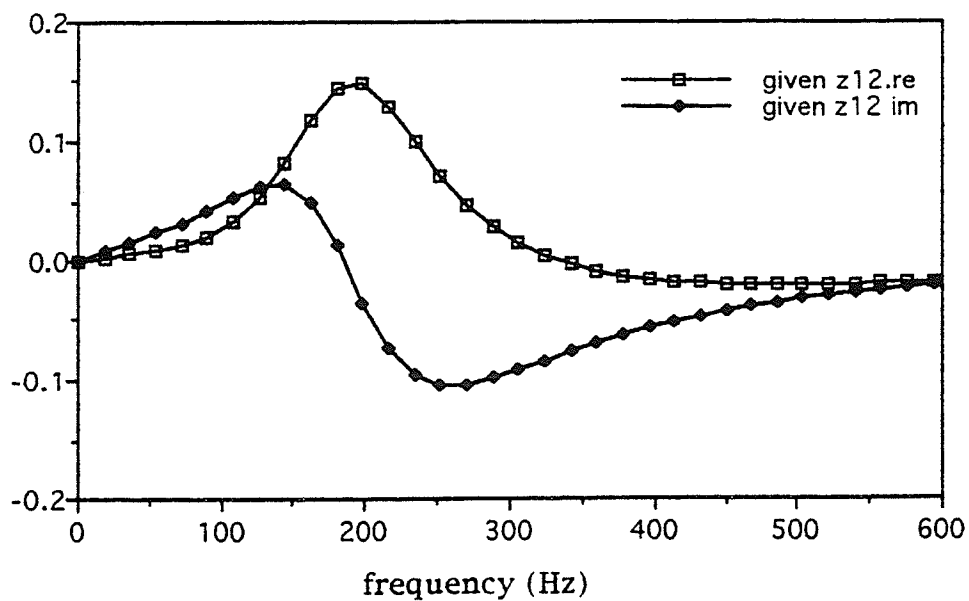


Fig. 5.7.2

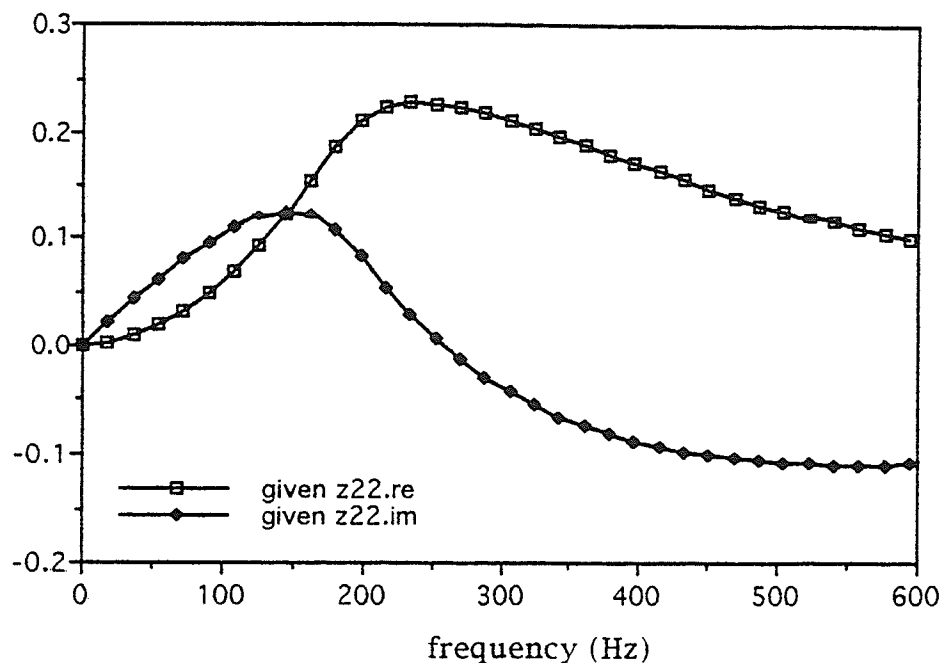


Fig. 5.7.3

We follow the three-step approach to carry out the optimization. The found zeros and poles of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are shown in Tables 5.7.1, 5.7.2 and 5.7.3.

Table 5.7.1

Rational Function  $z_{11}$

polyRep=

prodRep

Numerator Degree:

6

Constant factor followed by Zeros:

real part	imaginary part
5.000000000 e-1	0.000000000e+0
-2.234337589 e+2	0.000000000 e+0
-1.638061539 e+2	0.000000000 e+0
-8.696307808 e+2	1.135577665 e+3
-8.696307808 e+2	-1.135577665 e+3
-1.549249262 e+3	8.703214287 e+2
-1.549249262 e+3	-8.703214287 e+2

Denominator Degree:

6

Constant factor followed by Poles:

real part	imaginary part
1.000000000 e+0	0.000000000 e+0
-1.840367338 e+2	0.000000000 e+0
-4.237360698 e+2	1.136401723 e+3
-4.237360698 e+2	-1.136401723 e+3
-1.496745563 e+3	1.068883960 e+3
-1.496745563 e+3	-1.068883960 e+3
-3.000000000 e+2	0.000000000 e+0

Table 5.7.2

Rational Function  $z_{12}$

polyRep=

prodRep

Numerator Degree:

3

Constant factor followed by Zeros:

real part	imaginary part
3.750000000e+5	0.000000000e+0
-4.000000000e+2	0.000000000e+0
-6.000000000e+2	0.000000000e+0
0.000000000e+0	0.000000000e+0

Denominator Degree:

5

Constant factor followed by Poles:

real part	imaginary part
1.000000000e+0	0.000000000e+0
-1.840367338e+2	0.000000000e+0
-4.237360698e+2	1.136401723e+3
-4.237360698e+2	-1.136401723e+3
-1.496745563e+3	1.068883960e+3
-1.496745563e+3	-1.068883960e+3

Table 5.7.3

Rational Function  $z_{22}$

polyRep=

prodRep

Numerator Degree:

4

Constant factor followed by Zeros:

real part	imaginary part
6.000000000e+2	0.000000000e+0
-1.000000000e+0	0.000000000e+0
-1.999089293e+2	0.000000000e+0
-5.375455353e+2	1.121884349e+3
-5.375455353e+2	-1.121884349e+3



Denominator Degree:

5

Constant factor followed by Poles:

real part	imaginary part
1.000000000e+0	0.000000000e+0
-1.840367338e+2	0.000000000e+0
-4.237360698e+2	1.136401723 e+3
-4.237360698e+2	-1.136401723 e+3
-1.496745563e+3	1.0688839603e+3
-1.496745563e+3	-1.0688839603e+3

From Tables 5.7.1, 5.7.2 and 5.7.3, the degree of  $z_{11}$  is 6/6, the degree of  $z_{12}$  is 3/5, and the degree of  $z_{22}$  is 4/5. Since  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  have five identical poles, and  $z_{11}$  has a private pole, a ladder structure is adopted. Fig. 5.7.1 shows the ladder network.

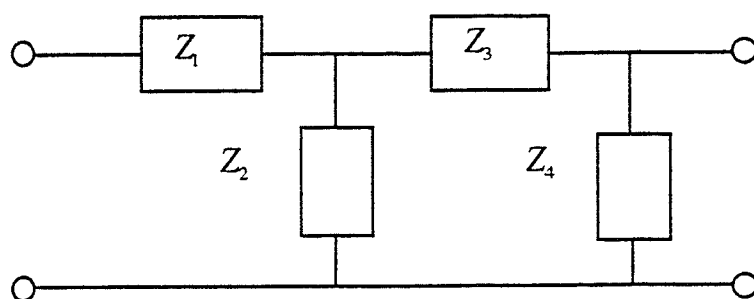


Fig. 5.7.4 A ladder two-port network

This ladder network has a series element added to the  $\Pi$  two-port structure and the higher degree of  $z_{11}$  is generated by this series element. Therefore, if we can determine  $Z_1$ , then we can easily find  $Z_2$ ,  $Z_3$  and  $Z_4$  of the remaining  $\Pi$  network by using the method developed in Example 3. Let  $z'_{11}$  be the driving point function of the remaining  $\Pi$  network, we have:

$$\begin{aligned}
z_{11} &= Z_1 + z'_{11} \\
&= \frac{n_1}{d_1} + \frac{n'_{11}}{d} \\
&= \frac{n_1 d + n'_{11} d_1}{d_1 d}
\end{aligned} \tag{5.7.1}$$

where  $d$  is known from  $z_{12}$  and  $d_1$  is also known by comparing the poles of  $z_{11}$  with the poles of  $z_{12}$ .  $n$  and  $n'_{11}$  can be obtained by an optimization process. To start the optimization process, the degree of  $n$  and  $n'_{11}$  must be known, or all possible degree combinations of both  $n$  and  $n'_{11}$  have to be determined. For a driving function the degree difference between the numerator and the denominator can not be larger than one, and the degree of  $d$  is 5, hence the degree of  $n'_{11}$  can only be 4, 5 or 6. The possible degrees of  $n_1$  are 0, 1 and 2, because the degree of  $d_1$  is 1. From the above possible degrees of  $n_1$  and  $n'_{11}$ , there are 9 degree combinations of  $n_1$  and  $n'_{11}$ . However, since the degree of the numerator of  $z_{11}$  is 6, i.e. the maximum degree of  $n'_{11} d_1$  is 6, hence the degree of  $n'_{11}$  must be less than 6. Similarly, the maximum degree of  $n_1 d$  is 6, hence the degree of  $n_1$  can not be 2. If the degree of  $n_1$  equals 0 and the degree of  $n'_{11}$  equals 4, then the degree of the numerator can not equal 6. Therefore, the possible degree combinations are down to three:

1. degree of  $n_1$  equal to 0 and degree of  $n'_{11}$  equal to 5,
2. degree of  $n_1$  equal to 1 and degree of  $n'_{11}$  equal to 5,
3. degree of  $n_1$  equal to 1 and degree of  $n'_{11}$  equal to 4.

Optimizations are conducted for these three combinations. When the iteration number equals 200, the difference of the objective function values clearly shows that the right choice is the #3 degree combination. Therefore, the #3 degree combination is adopted. For the # 3 degree combination, there are three types of zeros for  $n'_{11}$ , one is two complex conjugate zero pairs, the other is one complex conjugate pair and two real zeros and four real zeros.

Further tests tell us that the second one is a good choice, hence the optimization continues for the second one only. The starting points of the optimization, the zeros of  $n_1$  and  $n'_{11}$  and the constant factor of  $n'_{11}$  are chosen to be the same order as the zeros of  $z_{11}$ . After more than 3000 iterations, both  $n_1$  and  $n'_{11}$  are found. Table 5.7.4 and Table 5.7.5 gives the zeros of  $n_1$  and  $n'_{11}$ , respectively.

Table 5.7.4

Rational Function  $Z_1$   
polyRep=  
prodRep  
Numerator Degree:  
1  
Constant factor followed by Zeros:  

real part	imaginary part
5.0000e-1	0.0000e+0
-2.0000e+2	0.0000e+0

Denominator Degree:  
1  
Constant factor followed by poles:  

real part	imaginary part
1.0000e+0	0.0000e+0
-3.0000e+2	0.0000e+0

Table 5.7.5

Rational Function  $z'_{11}$   
polyRep=  
prodRep  
Numerator Degree:  
4  
Constant factor followed by Zeros:  

real part	imaginary part
5.0000e+2	0.0000e+0
-4.0000e+2	0.0000e+0
-1.3283e+2	0.0000e+0
-1.3835e+3	1.1950e+3
-1.3835e+3	-1.1950e+3

Denominator Degree:

5

Constant factor followed by poles:

real part	imaginary part
1.0000 e+0	0.0000 e+0
-1.8403 e+2	0.0000 e+0
-4.2373 e+2	1.1364 e+3
-4.2373 e+2	-1.1364 e+3
-1.4967 e+3	1.0688 e+3
-1.4967 e+3	-1.0688 e+3

After  $Z_1$  and  $z'_{11}$  are determined, the next step is to determine the remaining  $\Pi$  network. The remaining procedures are exactly the same as that developed in the third example. The frequency responses of the admittance functions  $y_{11}$ ,  $y_{12}$  and  $y_{22}$  are calculated from the frequency responses of impedance functions  $z'_{11}$ ,  $z_{12}$  and  $z_{22}$ .

$$y_{11}(j\omega_i) = \frac{z_{22}(j\omega_i)}{\Delta z(j\omega_i)}$$

$$y_{12}(j\omega_i) = -\frac{z_{12}(j\omega_i)}{\Delta z(j\omega_i)}$$

$$y_{22}(j\omega_i) = \frac{z'_{11}(j\omega_i)}{\Delta z(j\omega_i)}$$

where  $\Delta z(j\omega_i) = z'_{11}(j\omega_i) \cdot z_{22}(j\omega_i) - (z_{12}(j\omega_i))^2$ .  $y_{12}(j\omega_i)$ ,  $y_{11}(j\omega_i)$  and  $y_{22}(j\omega_i)$  are the frequency responses of  $y_{12}$ ,  $y_{11}$  and  $y_{22}$ , respectively.  $z'_{11}(j\omega_i)$ ,  $z_{12}(j\omega_i)$  and  $z_{22}(j\omega_i)$  are the frequency responses of  $z'_{11}$ ,  $z_{12}$  and  $z_{22}$ , respectively.

The frequency responses of impedance functions  $Z_2$ ,  $Z_3$  and  $Z_4$  are calculated as the follows

$$Z_3(j\omega_i) = -\frac{1}{y_{12}(j\omega_i)}$$

$$Z_2(j\omega_i) = \frac{1}{y_{11}(j\omega_i) + y_{12}(j\omega_i)}$$

$$Z_4(j\omega_i) = \frac{1}{y_{22}(j\omega_i) + y_{12}(j\omega_i)}$$

where  $Z_2(j\omega_i)$ ,  $Z_3(j\omega_i)$  and  $Z_4(j\omega_i)$  are the frequency responses of  $Z_2$ ,  $Z_3$  and  $Z_4$ , respectively.

The impedance function  $Z_3$  is determined by using optimization. In the optimization processes for  $Z_3$ , the input variables are the zeros and poles of  $Z_3$  and the objective function value is the difference between the frequency response of  $-\frac{1}{y_{12}}$  and the frequency response of the calculated  $Z_3$ . The impedance functions  $Z_2$  and  $Z_4$  are obtained by optimization in a similar way to that for  $Z_3$ . The resulting impedance functions  $Z_2$ ,  $Z_3$  and  $Z_4$  are illustrated in Tables 5.7.6, 5.7.7 and 5.7.8, respectively.

Table 5.7.6

Rational Function  $Z_2$

polyRep=

prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part	imaginary part
-----------	----------------

5.0000e+2	0.0000e+0
-----------	-----------

-4.0000e+2	0.0000e+0
------------	-----------

Denominator Degree:

2

Constant factor followed by poles:

real part	imaginary part
-----------	----------------

1.0000e+0	0.0000e+0
-----------	-----------

-2.5000e+2	-1.0000e+3
------------	------------

-2.5000e+2	1.0000e+3
------------	-----------

Table 5.7.7

Rational Function  $Z_3$

polyRep=

prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part	imaginary part
8.0000e-1	0.0000e+0
-1.5000e+2	0.0000e+0

Denominator Degree:

1

Constant factor followed by Poles:

real part	imaginary part
1.0000e+0	0.0000e+0
-6.0000e+2	0.0000e+0

Table 5.7.8

Rational Function  $Z_4$

polyRep=

prodRep

Numerator Degree:

1

Constant factor followed by Zeros:

real part	imaginary part
6.0000 e+2	0.0000 e+0
0.0000 e+0	0.00000 e+0

Denominator Degree:

2

Constant factor followed by Poles:

real part	imaginary part
1.0000 e+0	0.0000e+0
-1.0000 e+3	-1.4000e+3
-1.0000 e+3	1.4000e+3

The circuits of  $Z_1$ ,  $Z_2$ ,  $Z_3$  and  $Z_4$  are determined and shown in Figs.

5.7.5, 5.7.6, 5.7.7 and 5.7.8 and the values of the circuit elements are listed in

Tables 5.7.9, 5.7.10, 5.7.11 and 5.7.12, respectively.

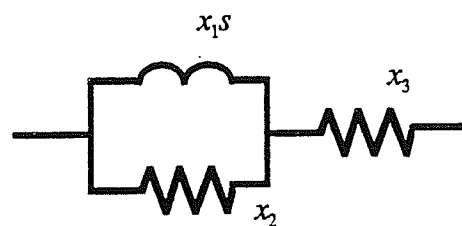


Fig. 5.7.5

Table 5.7.9

component number	value of component	
$x_1$	5.0000e+1	H
$x_2$	1.6667e-1	$\Omega$
$x_3$	3.3333e-1	$\Omega$

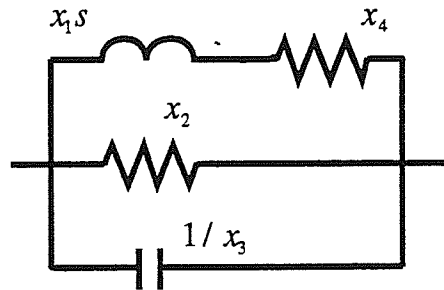


Fig. 5.7.6

Table 5.7.10

component number	value of component	
$x_1$	4.8900e-4	H
$x_2$	5.0000e+0	$\Omega$
$x_3$	2.0000e-3	F
$x_4$	1.9560e-1	$\Omega$

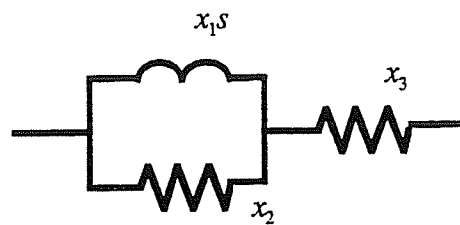


Fig. 5.7.7

Table 5.7.11

component number	value of component	
$x_1$	3.6000e+2	H
$x_2$	6.0000e-1	$\Omega$
$x_3$	2.0000e-1	$\Omega$

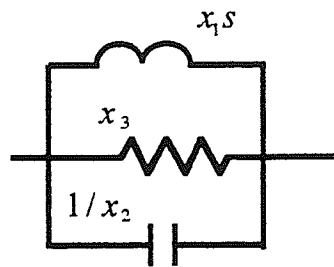


Fig. 5.7.8

Table 5.7.12

component number	value of component	
$x_1$	4.7059e-4	H
$x_2$	2.0000e-3	F
$x_3$	1.0000e+0	$\Omega$

The complete ladder network is shown in Fig. 5.7.13. The resulting frequency responses of  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are illustrated in Figs. 5.7.14, 5.7.15 and 5.7.16.



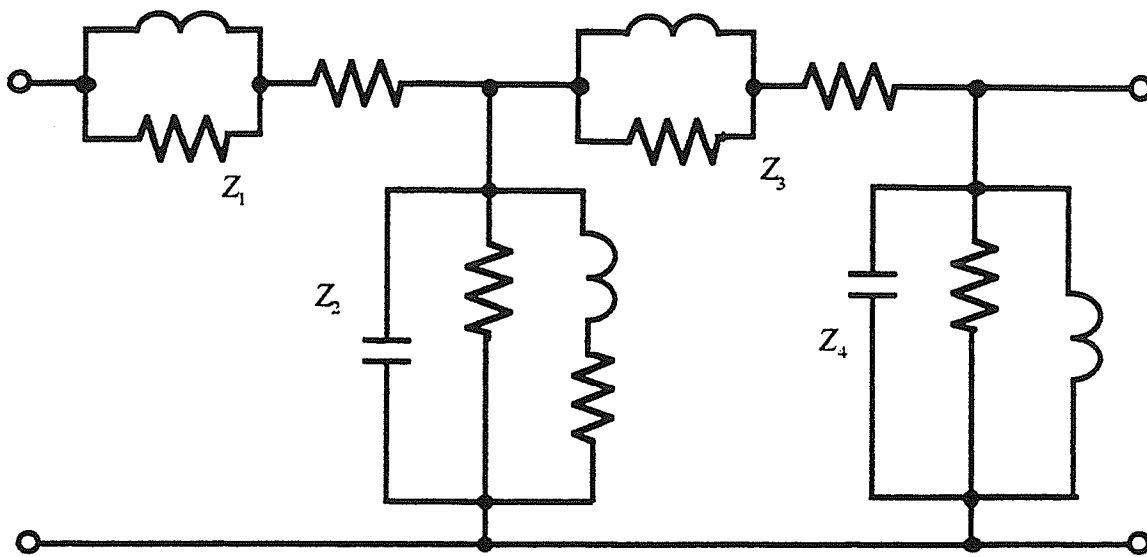


Fig. 5.7.13 The ladder network

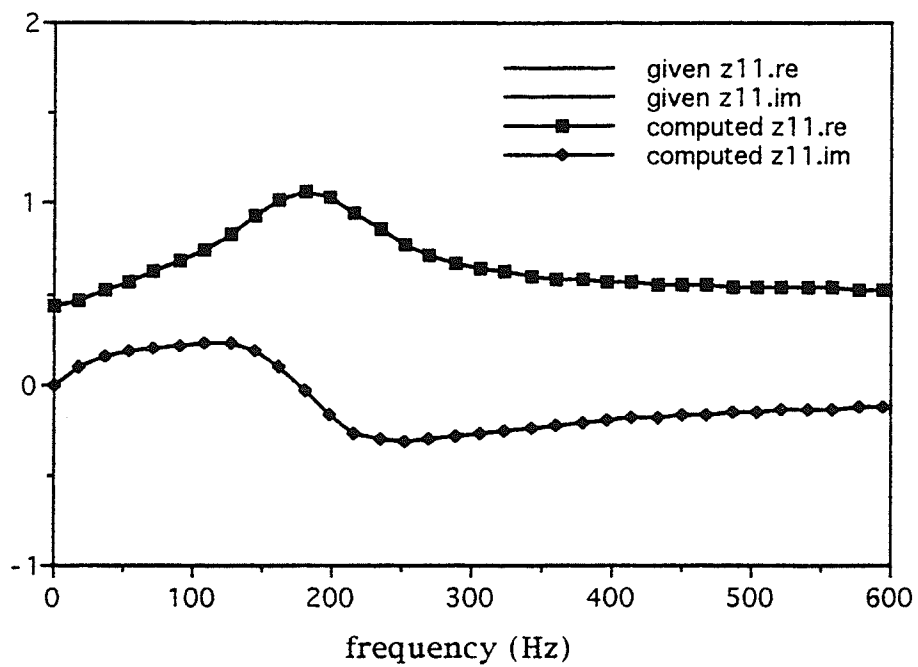


Fig. 5.7.14

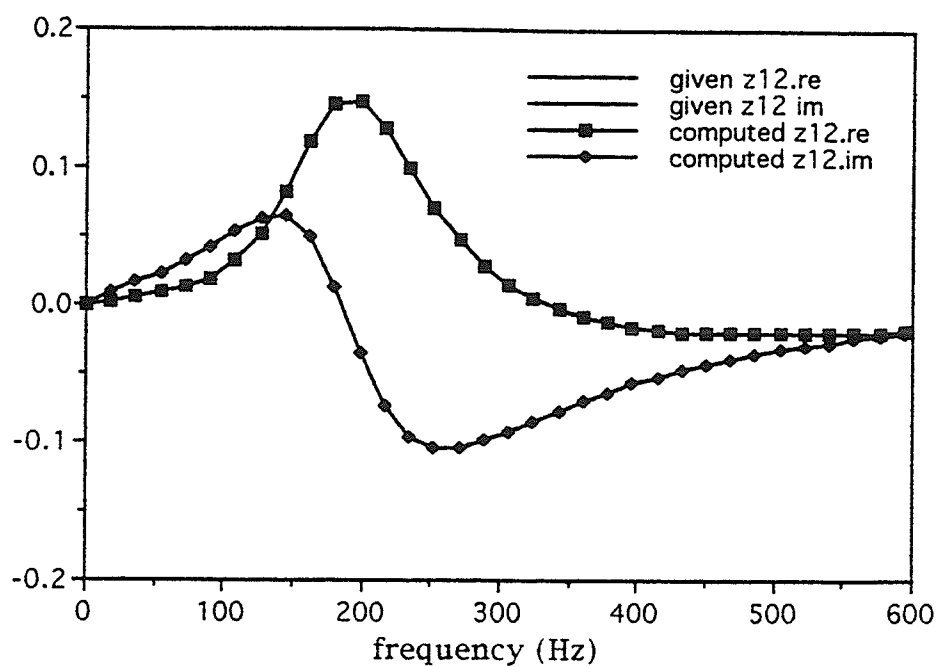


Fig. 5.7.15

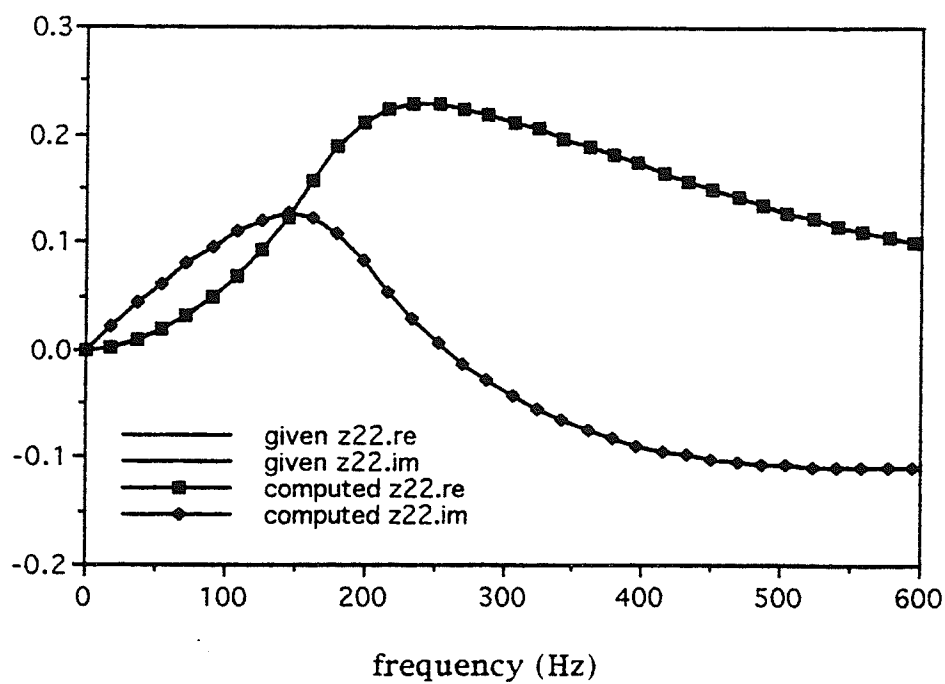


Fig. 5.7.16

In Example 4, an optimization is conducted to determine the impedance function  $Z_1$ , then the remaining  $\Pi$  network is found by using the method developed in the third example. In this way, the optimization has only six input variables and three possible degree combinations. The other way to apply the optimization is to find  $Z_1$ ,  $Z_2$ ,  $Z_3$  and  $Z_4$  at once. In this case, there are ten unknown variables and many more possible degree combinations than in the former case, therefore, the optimization will be much more difficult and time consuming.

## 5.8 Discussion

In this chapter, three examples which were realized in a  $\Pi$  and a ladder two-port network were discussed. The results of these three examples demonstrate the effectiveness of the three-step-approach optimization method. To make the optimization method more efficient three techniques/procedures were used:

1. An optimization process was applied to the group delay to find the degree, the zeros and poles of the impedance function. This speeds up the optimization process of Step 1 by more than ten times. This improvement is due to the fact that the number of input variables using the group delay is much smaller than that of an optimization involving complex numbers and polynomial operations.

2. The realization of a  $\Pi$  two-port network using admittance functions, leads to a much faster convergence than using impedance functions, because it does not require the optimization to determine  $Z_1$ ,  $Z_2$  and  $Z_3$  jointly and this leads to fewer input variables in the optimization.

3. For the ladder two-port network, a series impedance  $Z_1$  is first determined and then the remaining  $\Pi$  two-port network, leads to a faster convergence, because of fewer input variables and fewer possible degree combinations.

The basic idea for using these techniques/procedures is to find a way to reduce the number of input variables for an optimization, because the computation time required for an optimization process is related exponentially to the number of input variables. This is similar to the idea of using a multi-step optimization approach.

## Chapter 6

# FURTHER STUDY OF THE THREE-STEP APPROACH

### 6.1 Introduction

In Chapters 4 and 5, we developed a new method—the three-step-approach for finding an equivalent two-port network for modeling an ac power system. Also, a specific modeling problem of a power system and three other examples are solved by applying the method. The results of the Hydro Example gave a better fit for the frequency responses than those reported in [5] using a different method. The other three examples gave very accurate results. These examples involve four different network structures: T, T with a transformer,  $\Pi$  and a ladder. These examples demonstrate the effectiveness of the three-step approach method and the possibility of the three-step-approach becoming a general approach for the modeling of ac power systems.

In this chapter we will further discuss some of the features of the three-step-approach method. First we present the basic elemental circuits. Then we analyze the convergence and the accuracy of the optimization process of the three-step-approach method in terms of the results of the four examples.

## 6.2 Basic elemental circuits

The basic elemental circuits play an important role in the optimizations of Steps 2 and 3. That is, for any one impedance function required in Step 2, a circuit which can satisfy this function must be found by optimization in Step 3. The degrees of the basic elemental circuits range from the lowest 0/1 to the highest 2/2. The lowest degree circuit is considered first.

The 0/1 elemental circuit is shown in Fig. 6.2.1.

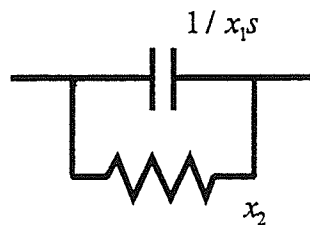


Fig. 6.2.1 A 0/1 circuit

The impedance function of this circuit is

$$z = \frac{1/x_1}{s + 1/(x_1x_2)}$$

It is easy to see that there are only two variables for the impedance function. One is the numerator, i.e.  $1/x_1$ , and the other is the zero of denominator, i.e.  $-1/(x_1x_2)$ . The amplitude of the numerator can be any positive real number and it is not affected by the denominator, so that full movement is possible. For the denominator, the zero can also be any negative real number and it is not affected by the numerator. Thus full movement of the denominator is also

achieved. Therefore, this circuit can realize any 0/1 PR driving point function.

Second, we consider the 1/0 circuit shown in Fig. 6.2.2.

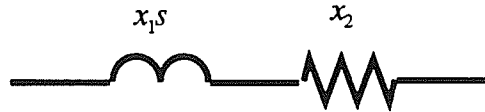


Fig. 6.2.2

The impedance function of the circuit is

$$\begin{aligned} z &= x_1 s + x_2 \\ &= \frac{s + x_2 / x_1}{1 / x_1} \end{aligned}$$

There are two variables. One is the zero of the numerator and the other is the constant factor of the denominator. It is clear that the zero of the numerator can be any negative real number and the constant factor of the denominator can be any positive real number. Also, the numerator and the denominator are independent of each other. Thus full movement is achieved. Therefore, this circuit can realize any 1/0 PR driving point function.

Third, we analyze a 1/1 circuit shown in Fig. 6.2.3.

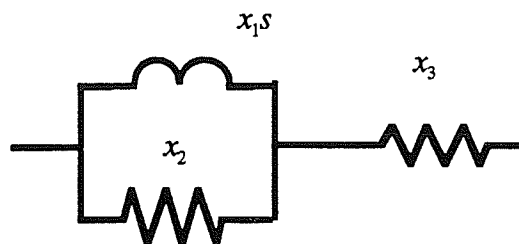


Fig. 6.2.3 A 1/1 circuit (No. E4)

The impedance function of this circuit is

$$\begin{aligned}
 z &= \frac{(x_1 x_2 + x_1 x_3)s + x_2 x_3}{x_1 s + x_2} \\
 &= \frac{(x_2 + x_3)s + \frac{x_2 x_3}{x_1}}{s + \frac{x_2}{x_1}} \\
 &= \frac{a_1 s + a_0}{s + b_0}
 \end{aligned}$$

where  $\frac{a_1 s + a_0}{s + b_0}$  is the function that is to be realized by the 1/1 circuit. There are three variables:  $x_1$ ,  $x_2$  and  $x_3$ . We have

$$x_1 = \frac{a_1}{b_0} - \frac{a_0}{b_0^2}$$

$$x_2 = a_1 - \frac{a_0}{b_0}$$

$$x_3 = \frac{a_0}{b_0}$$

Therefore, the limitation of this circuit is  $a_1 b_0 - a_0 \geq 0$ .

Another 1/1 circuit (No. C4) is shown in Fig. 6.2.4.

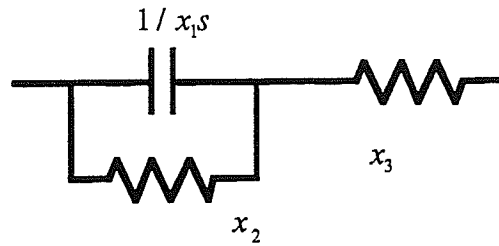


Fig. 6.2.4 A 1/1 circuit (No. C4)

The impedance function of this circuit is

$$z = \frac{x_1 x_2 x_3 s + x_2 + x_3}{x_1 x_2 s + 1}$$



$$= \frac{x_3 s + \frac{x_2 + x_3}{x_1 x_2}}{s + \frac{1}{x_1 x_2}}$$

$$= \frac{a_1 s + a_0}{s + b_0}$$

we also have

$$x_1 = \frac{1}{a_0 - a_1 b_0}$$

$$x_2 = \frac{a_0}{b_0} - a_1$$

$$x_3 = a_1$$

The limitation of this 1/1 circuit is  $a_0 - a_1 b_0 \geq 0$ . It is clear that circuit No. C4 and circuit No. E4 complement each other with respect to limitations. Therefore, together these two circuits can satisfy the required full move ability of 1/1 circuits, i.e., any 1/1 PR function can be realized by either circuit No. C4 or by circuit No. E4.

Fourth, we consider 1/2 circuits; 1/2 circuits are more complicated than the above three cases. However, similar to the above, two complementary circuits (with respect to restrictions) are obtained. A 1/2 circuit—elemental circuit No. A2 is shown in Fig. 6.2.5.

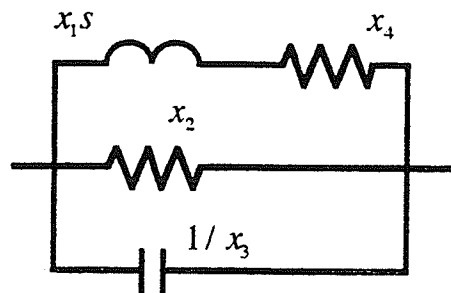


Fig. 6.2.5 A 1/2 circuit

The impedance function of the circuit in Fig. 6.2.5 is

$$\begin{aligned}
z &= \frac{x_1 x_2 s + x_2 x_4}{x_1 x_2 x_3 s^2 + (x_1 + x_2 x_3 x_4) s + x_2 + x_4} \\
&= \frac{\frac{1}{x_3} s + \frac{x_4}{x_1 x_3}}{s^2 + \frac{x_1 + x_2 x_3 x_4}{x_2 x_3 x_1} s + \frac{x_2 + x_4}{x_1 x_2 x_3}} \\
&= \frac{a_1 s + a_0}{s^2 + b_1 s + b_0} \tag{6.2.1}
\end{aligned}$$

where  $\frac{a_1 s + a_0}{s^2 + b_1 s + b_0}$  is the equation that is to be realized by the 1/2 circuit—

circuit No. A2. We have

$$a_0 = \frac{x_4}{x_1 x_3} \tag{6.2.2-1}$$

$$a_1 = \frac{1}{x_3} \tag{6.2.2-2}$$

$$b_0 = \frac{x_2 + x_4}{x_1 x_2 x_3} \tag{6.2.2-3}$$

$$b_1 = \frac{x_1 + x_2 x_3 x_4}{x_2 x_3 x_1} \tag{6.2.2-4}$$

Solving equations 6.2.2, we have

$$x_1 = \frac{a_1^3}{a_1^2 b_0 - a_0 a_1 b_1 + a_0^2} \tag{6.2.3-1}$$

$$x_2 = \frac{a_1^2}{a_1 b_1 - a_0} \tag{6.2.3-2}$$

$$x_4 = \frac{a_0 a_1^2}{a_1^2 b_1 - a_0 a_1 b_1 + a_0^2} \tag{6.2.3-3}$$

$$x_3 = \frac{1}{a_1} \quad (6.2.3-4)$$

Since  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  must be positive, we have the following constraints:

$$a_1 b_1 - a_0 \geq 0 \quad (6.2.4-1)$$

$$a_1^2 b_1 + a_0^2 - a_0 a_1 b_1 \geq 0 \quad (6.2.4-2)$$

Constraint (6.2.4-1) is a part of the PR conditions on the impedance function. (Some other PR conditions are  $a_0 \geq 0$ ,  $a_1 \geq 0$ ,  $b_0 \geq 0$  and  $b_1 \geq 0$ .) Constraint (6.2.4-2) demonstrates the limitation of the elemental circuit No. A2.

Fig. 6.2.6 presents the other 1/2 circuit—elemental circuit No. D8 .

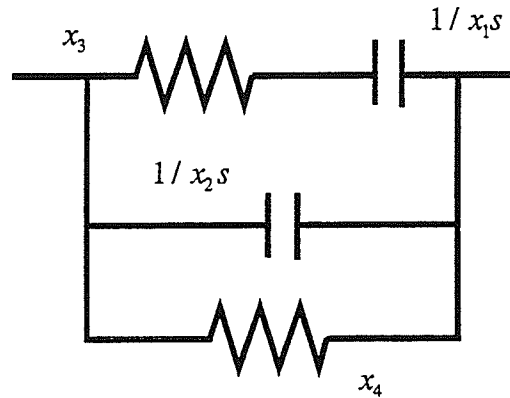


Fig. 6.2.6

The impedance function of this circuit is:

$$z = \frac{x_1 x_3 x_4 s + x_4}{x_1 x_2 x_3 x_4 s^2 + (x_2 x_4 + x_1 x_4 + x_1 x_3) s + 1} \quad (6.2.5)$$

In a way similar to that of elemental circuit No. A2, we find the limitation of circuit No. D8:

$$a_0 a_1 b_1 - a_1^2 b_1 - a_0^2 \geq 0 \quad (6.2.6)$$

constraints (6.2.4-2) and (6.2.6) clearly show that circuits No. A2 and No. D8 complement each other with respect to constraints. Hence these two 1/2 circuits can be used to realize any 1/2 PR driving point function.

Fifth, we consider the 2/1 circuits shown in Fig. 6.2.7.

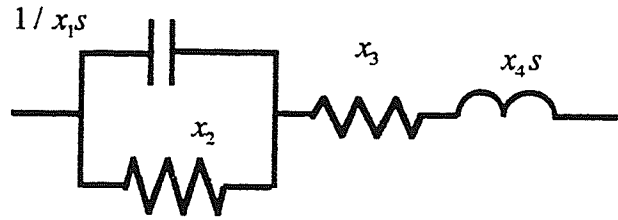


Fig. 6.2.7 No. B2 circuit

The impedance function of this circuit is

$$\begin{aligned} z &= \frac{x_1 x_2 x_4 s^2 + (x_4 + x_1 x_2 x_3) s + x_2 + x_3}{x_1 x_2 s + 1} \\ &= \frac{x_4 s^2 + \frac{x_4 + x_1 x_2 x_3}{x_1 x_2} s + \frac{x_2 + x_3}{x_1 x_2}}{s + \frac{1}{x_1 x_2}} \\ &= \frac{a_2 s^2 + a_1 s + a_0}{s + b_0} \end{aligned}$$

where  $\frac{a_2 s^2 + a_1 s + a_0}{s + b_0}$  is the equation that is to be realized by the 2/1 circuit. We have

$$a_2 = x_4 \quad (6.2.7a)$$

$$a_1 = \frac{x_4 + x_1 x_2 x_3}{x_1 x_2} \quad (6.2.7b)$$

$$a_0 = \frac{x_2 + x_3}{x_1 x_2} \quad (6.2.7.c)$$

$$b_0 = \frac{1}{x_1 x_2} \quad (6.2.7d)$$

Solving equations 6.2.7), we have:

$$x_1 = \frac{1}{a_0 - a_1 b_0 + a_2 b_0^2}$$

$$x_2 = \frac{a_0}{b_0} - a_1 + a_2 b_0$$

$$x_3 = a_1 - a_2 b_0$$

$$x_4 = a_2$$

Since  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  must be positive, we have the following constraints:

$$a_1 - a_2 b_0 \geq 0 \quad (6.2.7g)$$

$$a_0 + a_2 b_0^2 - a_1 b_0 \geq 0 \quad (6.2.7h)$$

Constraint (6.2.7g) is a part of the PR conditions of the impedance function. (Some other PR conditions are:  $a_0 \geq 0$ ,  $a_1 \geq 0$ ,  $a_2 \geq 0$ , and  $b_0 \geq 0$ .) Condition (6.2.7h) represents the limitation of the elemental circuit No. B2.

Fig. 6.2.8 shows the other 2/1 circuit—elemental circuit No. E6.

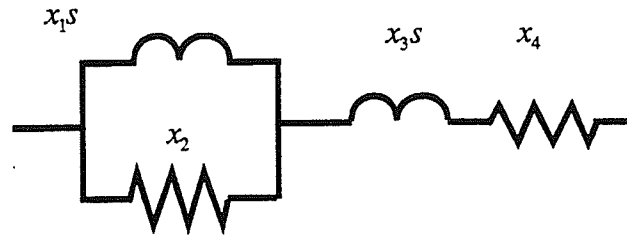


Fig. 6.2.8

The impedance function of the circuit is:

$$z = \frac{x_1 x_3 s^2 + (x_1 x_2 + x_2 x_3 + x_4 x_1) s + x_2 x_4}{x_1 s + x_2}$$

In a way similar to that of circuit No. B2 , we find the limitation of circuit No. E6:

$$a_1 b_0 - a_0 + a_2 b_0^2 \geq 0 \quad (6.2.7i)$$

From constraints (6.2.7h) and (6.2.7i), it is clear that circuit No. B2 and circuit No. E6 complement each other with respect to the limitation. Therefore, these two circuits can achieve the realization of any 2/1 PR driving point function.

Sixth, we consider the 2/2 circuits. In many cases the 2/2 circuit can be realized by combinations of 0/1, 1/0, 1/1, 2/1 and 1/2 basic elemental circuits and R, L, C elements. For example, in Example 3, a 2/2 circuit is achieved by a resistor in parallel with the No. B2 circuit. However, when the 2/2 impedance function is a minimum function, then the combination of these basic elemental circuits can no longer realize the required 2/2 minimum impedance function. In this case the Brune circuit can be used. The derivation of the Brune circuit is given in many text books [2], [39]. Hence, it is not given here. A Brune circuit is shown in Fig. 6.2.9 and its equivalent circuit is shown in Fig. 6.2.10. For the realization of a 2/2 minimum function, if a positive(negative) value is given to  $L_1$  then a negative(positive) value must be given to  $L_3$ . The relations between  $L_1$ ,  $L_2$ ,  $L_3$ ,  $L_a$  and  $L_b$  are:

$$L_a = L_1 + L_2$$

$$L_b = L_2 + L_3$$

Also, an optimization process is used to carry out the calculation for the element values of the Brune circuit for the realization of a  $2/2$  minimum function.

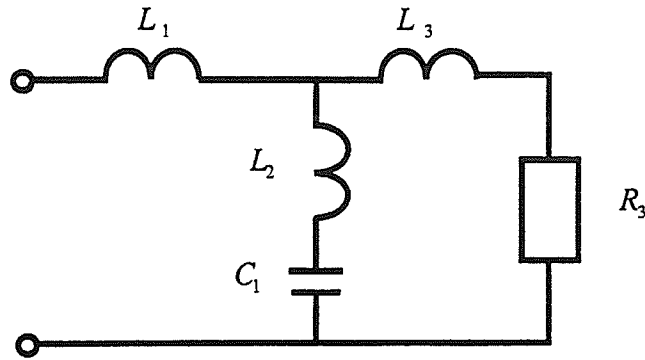


Fig. 6.2.9

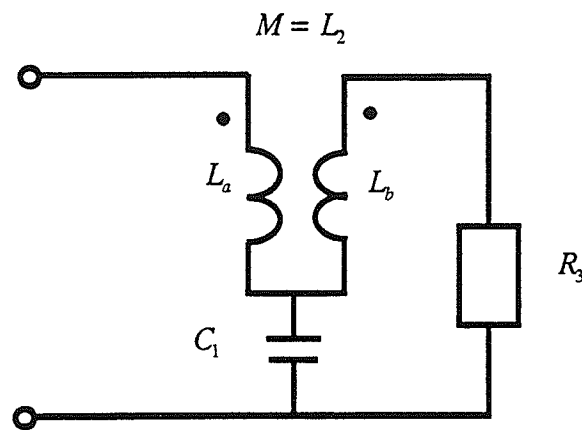


Fig. 6.2.10

Now we have the basic elemental circuits of degrees  $0/1$  to  $2/2$  which can guarantee that all required impedance functions of degrees  $0/1$  to  $2/2$  can be achieved. For degrees higher than  $2/2$  PR functions, the required networks can also be achieved by using these basic elemental circuits. For example, in Example 2 the  $2/3$  impedance function  $Z_3$  is realized by the No. E6 circuit in parallel with a capacitor. Therefore, the optimizations in Step 2 and Step 3 are guaranteed to be successful by using these basic elemental circuits.

## 6.3 The convergence

The convergence of the optimization is the most important problem in the entire optimization process. The convergence of the optimization depends on two factors, one is the optimization method and the other is the setting of the optimization—the formulation at every step of the optimization procedure. It is apparent that in order to obtain convergence of the three-step-approach, the optimization method itself must be capable of converging. We will use examples to describe the importance of the techniques for overcoming the local minimum problem in the optimization process for the modified pattern search optimization method.

From previous study, it is known that overcoming local minima is the key problem which needs to be solved for the convergence of the optimization. Several techniques are developed to modify the pattern search optimization method to overcome the local minimum problem. Two of these techniques that have proven to be very effective are the following: 1) repeatedly varying the step size in both the single variable search phase and the pattern search phase, 2) changing the starting points. From the examples of our study, it become very clear that without these modifications for overcoming the local minimum problem, the pattern search optimization had little chance to succeed. For example, in Step 1 of Example 3, finding the degrees and types of poles and zeros without continually increasing the step size, the optimization process will stop in a very early phase with a large error. This is also true for all other examples we have studied—without repeatedly increasing the step size, the pattern search optimization stops prematurely. An example for illustrating the importance of changing the starting points is the optimization



process for finding the network structures and the elemental impedance functions in Step 2 of Example 2. If the technique of changing the starting points is not applied, then the optimization process stops very early with a very large error ( $>1000$ ) and far away from the final optimal point.

To demonstrate the effectiveness of the modified pattern search optimization method, we apply the optimization to a non-optimal combination of the zeros and poles in Step 1 of Example 3—the degrees of both the numerator and denominator equal 6. (The optimal degree combination is the degree of both the numerator and the denominator equal to 5.) After more than a 1000 iterations, we obtain a result which is shown in Fig. 6.3.1. The impedance function corresponding to the frequency response is listed in Table 6.3.1. It is clear that the result is acceptable. For the optimal degree combination of the numerator and the denominator, the optimization takes 400 iterations to reach a result very close to the optimal point (see Fig. 5.6.4b). Comparing the results and number of iterations, we can see that although the optimization for the non-optimal degree combination takes a much longer time, the optimization eventually approaches a point close to the optimal point. The importance of this example is that it illustrates the ability of the developed optimization method to find a solution close to the optimal point.

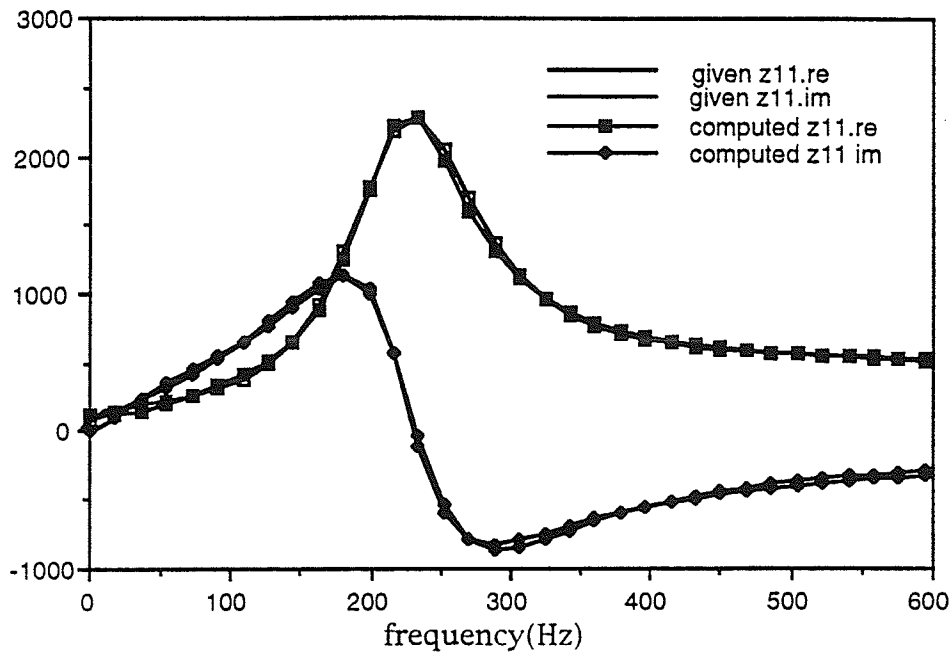


Fig. 6.3.1

Table 6.3.1

Rational Function  $z_{11}$

polyRep=

prodRep

Numerator Degree:

6

Constant factor followed by Zeros:

real part	imaginary part
4.4776808646673164120e+2	0.0000000000000000000e+0
-1.6332368267630182640e+2	-9.7406490647989810730e-8
-1.6332368267630182640e+2	9.7406490647989810730e-8
-8.0088324072499939180e+3	-2.0163814594122669120e+3
-8.0088324072499939180e+3	2.0163814594122669120e+3
-1.6654111967378977340e+3	-2.2209744114314249660e-7
-1.6654111967378977340e+3	2.2209744114314249660e-7

Denominator Degree:

6

Constant factor followed by Poles:

real part	imaginary part
1.0000000000000000000e+0	0.0000000000000000000e+0
-3.1308364445306449420e+2	-1.3805725543337763610e+3
-3.1308364445306449420e+2	1.3805725543337763610e+3
-8.0088324072499939180e+3	-2.0163814594122669120e+3
-8.0088324072499939180e+3	2.0163814594122669120e+3
-3.4801527298610167770e+2	-1.3890832848126365720e+2
-3.4801527298610167770e+2	1.3890832848126365720e+2

From the above four examples, it is found that:

1. In all four examples, the technique of changing the starting points is not required for the optimizations in Step 1.

2. For the optimization process of Step 2, when the T structure is used with the impedance functions (Hydro Example) and the  $\Pi$  structure is used with the admittance functions (Example 3), the technique of changing starting points is not required. However, when the admittance functions are not used for the  $\Pi$  structure (Example 2), the technique of changing starting points has to be used extensively. This is mainly due to the fact that the number of the input variables in this case is much larger. Also, more variables means more constraints that contribute to a longer computation time.

3. In the optimization of Step 3, the optimization process always converges much faster than in Step 1 and Step 2. This is due to the fact that there are fewer input variables in Step 3, and fewer constraints—only the input variable values are required to be positive.

## 6.4 Accuracy

To demonstrate that accurate results can be obtained from the optimization method, we use the results of Example 3. Table 6.4.1 lists the given frequency response and the computed frequency response of  $z_{12}$  in Step 3. From Table 6.4.1, we can see that the given frequency response has seven digits of precision and the computed frequency response also has an accuracy up to 7 digits. Accuracy of more than 15 digits is also feasible if the input precision is 15 digits. The result of step 2 also has an accuracy similar to that of Step 3.

Table 6.4.2 displays a frequency response of the impedance function  $z_{12}$  which is obtained by applying an optimization process to the group delay in Step 1. The given group delay data are shown in Table 6.4.3. From this table, it can be seen that the given group delay data have only two to three digits of precision. Comparing the frequency response computed by using the group delay with the given one, we see that the calculated frequency response of  $z_{12}$  is accurate up to 3 digits under the condition of low precision given group delay data.

Here only the results for  $z_{12}$  are presented, however, the results for  $z_{11}$  and  $z_{22}$  are very similar to that for  $z_{12}$ . Moreover, the results of the other examples also have similar accuracy.

Table 6.4.1

Freq(Hz)	given z12.re	given z12.im	computed z12.re	computed z12.im
1.8e+1	1.709196e+2	1.015658e+2	1.70919576e+2	1.0156581e+2
3.6e+1	2.460213e+2	8.832054e+1	2.46021317e+2	8.83205402e+1
5.4e+1	2.729728e+2	5.636776e+1	2.72972841e+2	5.63677573e+1
7.2e+1	2.769090e+2	3.041876e+1	2.76908968e+2	3.04187645e+1
9e+1	2.695834e+2	1.248524e+1	2.69583365e+2	1.24852393e+1
1.08e+2	2.561071e+2	2.429570e+1	2.5610708e+2	2.42956936e+0
1.26e+2	2.397512e+2	3.210150e-1	2.39751154e+2	3.2101497e-1
1.44e+2	2.237589e+2	6.065860e+0	2.23758859e+2	6.06585865e+0
1.62e+2	2.118409e+2	1.831203e+1	2.11840927e+2	1.83120277e+1
1.8e+2	2.071805e+2	3.294295e+1	2.07180504e+2	3.2942945e+1
1.98e+2	2.088382e+2	4.281742e+1	2.0883818e+2	4.28174178e+1
2.16e+2	2.076395e+2	4.365522e+1	2.07639481e+2	4.36552238e+1
2.34e+2	1.935698e+2	4.439291e+1	1.93569842e+2	4.43929048e+1
2.52e+2	1.723661e+2	5.926387e+1	1.72366088e+2	5.92638594e+1
2.7e+2	1.584107e+2	8.852718e+1	1.58410664e+2	8.85271758e+1
2.88e+2	1.577667e+2	1.227015e+2	1.57766647e+2	1.22701453e+2
3.06e+2	1.684207e+2	1.541626e+2	1.68420687e+2	1.54162607e+2
3.24e+2	1.861721e+2	1.796242e+2	1.86172108e+2	1.79624172e+2
3.42e+2	2.074215e+2	1.985399e+2	2.07421523e+2	1.98539848e+2
3.6e+2	2.297139e+2	2.115835e+2	2.29713868e+2	2.11583502e+2
3.78e+2	2.515631e+2	2.198152e+2	2.51563078e+2	2.19815165e+2
3.96e+2	2.721607e+2	2.243006e+2	2.72160677e+2	2.2430064e+2
4.14e+2	2.911312e+2	2.259655e+2	2.9113115e+2	2.25965487e+2
4.32e+2	3.083587e+2	2.255588e+2	3.08358703e+2	2.2555879e+2
4.5e+2	3.238739e+2	2.236633e+2	3.23873905e+2	2.23663326e+2
4.68e+2	3.377832e+2	2.207217e+2	3.37783156e+2	2.20721724e+2
4.86e+2	3.502265e+2	2.170650e+2	3.50226535e+2	2.17065008e+2
5.04e+2	3.613536e+2	2.129381e+2	3.61353633e+2	2.1293811e+2
5.22e+2	3.713104e+2	2.085208e+2	3.71310405e+2	2.08520786e+2
5.4e+2	3.802326e+2	2.039440e+2	3.80232595e+2	2.03944019e+2
5.58e+2	3.882430e+2	1.993025e+2	3.88242975e+2	1.99302534e+2
5.76e+2	3.954507e+2	1.946642e+2	3.95450706e+2	1.94664213e+2
5.94e+2	4.019518e+2	1.900771e+2	4.01951841e+2	1.90077117e+2

Table 6.4.2

Freq(Hz)	given z12.re	given z12.im	computed z12.re	computed z12.im
0e+0	9.081813e+1	0e+0	9.1188137e+1	-1.4169597e-18
1.8e+1	1.709196e+2	1.015658e+2	1.7172346e+2	1.0196084e+2
3.6e+1	2.460213e+2	8.832054e+1	2.4722774e+2	8.8406669e+1
5.4e+1	2.729728e+2	5.636776e+1	2.7416894e+2	5.6029485e+1
7.2e+1	2.769090e+2	3.041876e+1	2.7785875e+2	2.9797526e+1
9e+1	2.695834e+2	1.248524e+1	2.7019837e+2	1.1779671e+1
1.08e+2	2.561071e+2	2.429570e+0	2.5642093e+2	1.8376311e+0
1.26e+2	2.397512e+2	3.210201e-1	2.3991042e+2	3.2309120e-1
1.44e+2	2.237589e+2	6.065860e+0	2.2397819e+2	5.9866601e+0
1.62e+2	2.118409e+2	1.831203e+1	2.1227304e+2	1.8315771e+1
1.8e+2	2.071805e+2	3.294295e+1	2.0773311e+2	3.2790392e+1
1.98e+2	2.088382e+2	4.281742e+1	2.092283e+2	4.2548098e+1
2.16e+2	2.076395e+2	4.365522e+1	2.0792088e+2	4.3584545e+1
2.34e+2	1.935698e+2	4.439291e+1	1.9405629e+2	4.4401177e+1
2.52e+2	1.723661e+2	5.926387e+1	1.7288352e+2	5.9107709e+1
2.7e+2	1.584107e+2	8.852718e+1	1.5881017e+2	8.8382971e+1
2.88e+2	1.577667e+2	1.227015e+2	1.5819801e+2	1.2266932e+2
3.06e+2	1.684207e+2	1.541626e+2	1.6899048e+2	1.5413545e+2
3.24e+2	1.861721e+2	1.796242e+2	1.8684485e+2	1.7947855e+2
3.42e+2	2.074215e+2	1.985399e+2	2.081007e+2	1.9822807e+2
3.6e+2	2.297139e+2	2.115835e+2	2.3031235e+2	2.1112441e+2
3.78e+2	2.515631e+2	2.198152e+2	2.5202825e+2	2.192599e+2
3.96e+2	2.721607e+2	2.243006e+2	2.7247319e+2	2.2370634e+2
4.14e+2	2.911312e+2	2.259655e+2	2.91295e+2	2.253817e+2
4.32e+2	3.083587e+2	2.255588e+2	3.0839126e+2	2.2502283e+2
4.5e+2	3.238739e+2	2.236633e+2	3.2379845e+2	2.2320029e+2
4.68e+2	3.377832e+2	2.207217e+2	3.3762419e+2	2.203465e+2
4.86e+2	3.502265e+2	2.17065e+2	3.5000715e+2	2.1678468e+2
5.04e+2	3.613536e+2	2.129381e+2	3.6109424e+2	2.1275421e+2
5.22e+2	3.713104e+2	2.085208e+2	3.7102833e+2	2.0843111e+2
5.4e+2	3.802326e+2	2.039440e+2	3.7994213e+2	2.03944e+2
5.58e+2	3.882430e+2	1.993025e+2	3.879556e+2	1.9938621e+2
5.76e+2	3.954507e+2	1.946642e+2	3.951755e+2	1.9482496e+2
5.94e+2	4.019518e+2	1.900771e+2	4.0169583e+2	1.9030808e+2

Table 6.4.3 Given group delay of  $z_{12}$ 

Freq(Hz)	Delay	Freq(Hz)	Delay	Freq(Hz)	Delay
0e+0	-1.559e-2	2.04e+2	-7e-5	4.08e+2	2.6e-4
6e+0	-6.76e-3	2.1e+2	1e-5	4.14e+2	2.6e-4
1.2e+1	-3.7e-4	2.16e+2	2e-5	4.2e+2	2.5e-4
1.8e+1	1.45e-3	2.22e+2	-6e-5	4.26e+2	2.5e-4
2.4e+1	1.8e-3	2.28e+2	-2.2e-4	4.32e+2	2.5e-4
3e+1	1.71e-3	2.34e+2	-4.7e-4	4.38e+2	2.4e-4
3.6e+1	1.52e-3	2.4e+2	-7.8e-4	4.44e+2	2.4e-4
4.2e+1	1.33e-3	2.46e+2	-1.1e-3	4.5e+2	2.3e-4
4.8e+1	1.15e-3	2.52e+2	-1.38e-3	4.56e+2	2.3e-4
5.4e+1	1.01e-3	2.58e+2	-1.57e-3	4.62e+2	2.2e-4
6e+1	8.8e-4	2.64e+2	-1.64e-3	4.68e+2	2.2e-4
6.6e+1	7.8e-4	2.7e+2	-1.6e-3	4.74e+2	2.1e-4
7.2e+1	6.8e-4	2.76e+2	-1.45e-3	4.8e+2	2.1e-4
7.8e+1	6e-4	2.82e+2	-1.25e-3	4.86e+2	2e-4
8.4e+1	5.2e-4	2.88e+2	-1.02e-3	4.92e+2	2e-4
9e+1	4.4e-4	2.94e+2	-8e-4	4.98e+2	2e-4
9.6e+1	3.6e-4	3e+2	-6e-4	5.04e+2	1.9e-4
1.02e+2	2.9e-4	3.06e+2	-4.3e-4	5.1e+2	1.9e-4
1.08e+2	2.1e-4	3.12e+2	-2.9e-4	5.16e+2	1.8e-4
1.14e+2	1.2e-4	3.18e+2	-1.7e-4	5.22e+2	1.8e-4
1.2e+2	3e-5	3.24e+2	-7e-5	5.28e+2	1.7e-4
1.26e+2	-7e-5	3.3e+2	1e-5	5.34e+2	1.7e-4
1.32e+2	-1.7e-4	3.36e+2	7e-5	5.4e+2	1.7e-4
1.38e+2	-2.8e-4	3.42e+2	1.2e-4	5.46e+2	1.6e-4
1.44e+2	-3.9e-4	3.48e+2	1.6e-4	5.52e+2	1.6e-4
1.5e+2	-4.9e-4	3.54e+2	1.9e-4	5.58e+2	1.5e-4
1.56e+2	-5.7e-4	3.6e+2	2.1e-4	5.64e+2	1.5e-4
1.62e+2	-6.3e-4	3.66e+2	2.3e-4	5.7e+2	1.5e-4
1.68e+2	-6.5e-4	3.72e+2	2.4e-4	5.76e+2	1.4e-4
1.74e+2	-6.3e-4	3.78e+2	2.5e-4	5.82e+2	1.4e-4
1.8e+2	-5.7e-4	3.84e+2	2.5e-4	5.88e+2	1.4e-4
1.86e+2	-4.6e-4	3.9e+2	2.6e-4	5.94e+2	1.3e-4
1.92e+2	-3.3e-4	3.96e+2	2.6e-4	6e+2	1.3e-4
1.98e+2	-1.9e-4	4.02e+2	2.6e-4		

# Chapter 7

## SUMMARY AND CONCLUSIONS

In this thesis a new method for obtaining an equivalent two-port network for the modeling of an ac power system has been developed. This new method is called the three-step-approach optimization method and contains three parts—three steps. The procedure for this method is as follows:

Step 1: The delay curve is calculated from the frequency response data for the real and imaginary parts of an impedance function. Next the locations of poles and zeros are estimated from this curve. Then these estimated poles and zeros are used as a starting point for an optimization that determines if additional poles and zeros will improve the fit of the delay curve. In this optimization the target function is the delay and the resulting zeros and poles will be used as the starting point for the second optimization. Finally, the second optimization process is carried out to determine the constant factor, and the final position of the zeros and poles of the impedance function. The procedures for  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  are essentially the same.

Step 2: The characteristics of the impedance functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$  found in Step 1 are analyzed and the main network structure is determined accordingly. Then the micro structures and elemental impedance functions are determined from the main network structure and the impedance functions  $z_{11}$ ,  $z_{12}$  and  $z_{22}$ .

Step 3: The elemental circuits and the element values are determined from the micro structure and the elemental impedance functions found in Step 2, by using optimization.



The multiple-step strategy, the modified pattern search optimization technique and elemental circuits with good move ability are the basis of the developed method. The multiple-step strategy divides the whole modeling problem into small parts which are much easier to solve, thereby simplifying the solution of the overall problem. The modified pattern search method proved to be effective for the problems considered in this thesis. It was able to find the optimal value in each case. The circuits with good move ability presented in Chapter 6 also play an important role in making the optimization process in Step 3 successful.

The group delay is very effective for finding the starting point of the optimization. When an optimization is applied to the group delay not only a starting point but also the zeros and poles of the impedance function can be determined, moreover it speeds up the optimization for the frequency response in Step 1 more than 10 times. Different network structures such as T, T with a transformer,  $\Pi$  and ladder two-port networks are studied for Step 2. Techniques to make the optimizations more efficient for  $\Pi$  and ladder two-port networks are developed. Computer programs that apply the three-step approach for different network structures are developed and implemented in PASCAL. These programs are simple yet effective.

Data for an ac power system and data for three other examples have been used to demonstrate the versatility and power of the three-step approach method as a useful tool to obtain equivalent two-port networks.

### **Suggestions for further work:**

1. In the above studies, the three-step-approach optimization method used the modified pattern search optimization technique and gave very good

results. However, the three-step-approach optimization method could also use other direct search and non direct search optimization techniques that may be even more effective. Since digital computers were developed and tedious computations can be carried out easily, different optimization methods have been developed. Since the seventies, many computer programs that implement optimization methods have been available. It would be interesting to see which optimization method is the most effective one for the modeling of ac power systems using the three-step-approach method.

2. The programs of step 2 could be modified to include additional network structures.

3. Potentially the three-step-approach method could be applied to multi-port equivalent networks for modeling ac power systems.

## APPENDIX A

### COMPUTER PROGRAM FOR THE THREE-STEP APPROACH OPTIMIZATION METHOD

The following computer program contains three main parts:

1. The function 'absv' carries out the calculation of the values of objective function. The PR condition is also checked in this function. If the PR condition is violated, then a huge value is given to the objective function. The function 'absv' has to be changed for different impedance functions in the different steps. The function 'absv' shown here is for finding the impedance function  $z_{12}$  in Step 2, Example 3.

2. The procedure 'search' conducts the modified pattern search optimization. The parameters that need to be altered are the step size, the step size reduction and the iteration number. The normal step size ranges from 1% to 10% and the step size reduction factor ranges from 0.8 to 0.95. The iteration numbers are usually from 80 to 150. These three parameters are controlled in the main program.

3. The main program controls the subprograms. For overcoming the local minimum, the techniques of continually varying the step size and changing the starting points are implemented in the main program. The technique of continually varying the step size is always executed. However, the technique of changing the starting points is executed only when the technique of continually varying the step size alone can no long provide a satisfactory result.

Beside these three main parts, there are several procedures for computing the values of the elemental components from the elemental impedance functions.

{-----computer program-----}

```

program step2;
uses
  Types, Complex, Arithmetic, Polynomial, Rational;
var
  i, j, k, n, o, j1, k1, oo2: integer;
  xx2, dxx, xx, y1, y2, DX, Q: EXTENDEDARRAY;
  l1, d1, d2, ab, ac, aa1, l3, l2, o2, F, b3, L, m, A, aa, A1, a3, a4, B, B1, B2, p, x1, x2, x3,
d: EXTENDED;
  ya, d11, x4, x5, x6, x11: EXTENDED;
  X: EXTENDEDTABLEPtr;
{s1, s2: complex;}
  p1, P11: POLYNOMIAL;
  z11, z12, z1, z2, z3, z4: rational;
  PRINT: boolean;
  PRINT2: boolean;
  en, PR: boolean;
label
  1;
function NoOddOrderPosZeros (var p: polynomial): boolean;
var
  x: extended;
  i, k: integer;
begin
  NoOddOrderPosZeros := TRUE;
  x := -1;
  k := 0;
  for i := 1 to p.deg do
    begin
      if (p.coef[i].Re > 0) and (p.coef[i].Im = 0) then
        begin
          if not REqualityTol(p.coef[i].Re, x, 1e-16) then
            begin
              k := 1;
              NoOddOrderPosZeros := FALSE;
              x := p.coef[i].Re
            end
          else
            begin
              k := (k + 1) mod 2;
              if k = 0 then
                NoOddOrderPosZeros := TRUE
              else
                NoOddOrderPosZeros := FALSE;
            end;
          end; {end if}
        end;
      end; {end if}
    end;
  end;

```

```

    end; {end for}
end;
function Rational_pr (var r: rational): boolean;
var
    p, q: polynomial;
    i: integer;
    Rationalispr: boolean;
begin
    r := PolysToRational(r.num, r.den); {makes den monic}
    Rationalispr := TRUE;
    if (r.num.coef[0].Re < 0) or (abs(r.num.deg - r.den.deg) > 1) then
        Rationalispr := FALSE;
    if Rationalispr then
        begin
            p := AddPoly(r.num, r.den);
            {WritePolynomial2(p, 'r.num + r.den');   Writes polynomial to Text window }
            for i := 1 to p.deg do
                if (p.coef[i].Re > 0) then
                    Rationalispr := FALSE;
            end;
            if Rationalispr then
                begin
                    q := HurwitzConjPoly(r.den);
                    p := MulPoly(r.num, q);
                    {WritePolynomial2(p, 'nd*');   Writes polynomial to Text window }
                    p := EvenPartPoly(p);
                    {WritePolynomial2(p, 'EvenPartPoly(p)');   Writes polynomial to Text window }
                    Rationalispr := NoOddOrderPosZeros(p); {pr if there are no odd order
positive zeros}
                    if (p.coef[0].Re < 0) then
                        Rationalispr := FALSE;
                    end;
                    Rational_pr := Rationalispr;
                end; {end function}
            end;
        end;
    procedure computinga2 (var x: extendedarray; z: rational);
    var
        cc1, cco: complex;
        cx12, cx, sx, cz11, cz12: complexarray;
        i, j, k, j1, k1, n: integer;
        y: realarray;
        a, a0, a1, a2, b0, b1: extended;
        z1, zia, z2: rational;

    begin
        CC1.RE := 1;
        CC1.IM := 0;
        cco.re := 0;
        cco.im := 0;
        writerrational3(z, 'z');
        a0 := z.num.coef[0].re;
        a1 := z.num.coef[1].re;

        b0 := z.den.coef[0].re;

```

```

b1 := z.den.coef[1].re;

a := a0 * a1 * b1;
writeln('a', a);
y[1] := (sqr(a1) * a1) / ((sqr(a1) * b0 + sqr(a0) - a0 * a1 * b1));

y[2] := sqr(a1) / (a1 * b1 - a0);

y[3] := 1 / a1;
y[4] := a0 * sqr(a1) / (sqr(a1) * b0 - a0 * a1 * b1 + sqr(a0));

for i := 1 to 4 do
begin
  x[i] := y[i];
  writeln('x[' , i, ']:= ', y[i] : 24, ');
end;
a := sqr(a1) * b0 + sqr(a0) - a0 * a1 * b1;
writeln('condition1', a);
a := a1 * b1 - a0;
writeln('condition2', a);
end;

procedure computingzi2 (var zi: rational; x: EXTENDEDARRAY);

{*****z= (x1s+x4)//(1/x3s)//x2 *****}

var
  cc1, C2, cco, zz, c1, z1a, z1b, z2a, z2b, z2c, z3a, z3b, z4a, z4b, z4c: complex;
  cx12, cx, sx, cz11, cz12: complexarray;
  i, j, k, j1, k1, n: integer;
  y1, y2: realarray;
{x: extendedarray;}
  z3, z1, zia, z2: rational;
  cz11a, c3, C11, C11B, C12, C12A, C12B: complex;

begin
  CC1.RE := 1;
  CC1.IM := 0;
  cco.re := 0;
  cco.im := 0;

  for i := 1 to 4 do
    begin
      sx[i].re := x[i];
    {writeln(' x ', x[i]);}
      sx[i].im := 0.0;
    end;

  Z3.NUM.REP := SUMREP;
  Z3.DEN.REP := SUMREP;
  z3.num.deg := 1;

```

```

z3.num.coef[0] := cmul(sx[2], sx[4]);
z3.num.coef[1] := cmul(sx[1], sx[2]);
z3.den.deg := 2;
z3.den.coef[0] := cadd(sx[2], sx[4]);

z3.den.coef[1] := cmul(sx[2], SX[3]);
z3.den.coef[1] := cmul(z3.den.coef[1], SX[4]);
z3.den.coef[1] := cadd(z3.den.coef[1], sx[1]);
z3.den.coef[2] := cmul(sx[1], SX[2]);
z3.den.coef[2] := cmul(z3.den.coef[2], SX[3]);
z3.num := sumreptoprodrep(z3.num);
z3.den := sumreptoprodrep(z3.den);
C1 := Z3.NUM.COEF[0];
C2 := Z3.DEN.COEF[0];
Z3.NUM.COEF[0] := CDIV(C1, C2);
Z3.DEN.COEF[0] := CC1;
{writerrational3(z3, 'z3');}

zi := z3;
end;

function absv (var xx: EXTENDEDARRAY; z11, z12: rational): EXTENDED;
var
    npr, Rational_is_pr: boolean;
    cz11bn, cz11bd, cz22n, cz11n, cz11d, C2, c3, sqp11, cco, zz, c1, z1a, z1b, z2a, z2b,
    z2c, z3a, z3b, z4a, z4b, z4c: complex;
    cz1b, cz1bn, cz1bd, cz11an, cz11ad, cz11a, c22, c4, c12n, c12d, sqp12, cz1n:
    complex;
    cz1d, cz2, cz2n, cz2d, cz3n, cz3d, cz4, cz4n, cz4d, cc, c1a, c1b, c2a, c2b, c3a, c3b,
    cc1: complex;
    cz22bn, cz22bd, cz22a, cz22an, cz22ad, C11, C11A, C11B, C12, C12A, C12B:
    complex;
    cz11b, cz11, cz1, p11, cy1112, cx22, cx1: complexarray;
    cx11b, cx11, cz3, cx, cx2, cx3, cx4: complexarray;
    i, j, k, j1, k1, n: integer;

    v1, v2, v3, v4, v5, v6, ya1, ya2, a1, a2, a3, Y11, Y11A, Y11B, Y12, Y12A, Y12B, a,
    yz11: extended;
    b, y22a, yz11a, yz12, yz12a, re1, y, a11, a12, ac1, ac2: extended;

    z22, z1, z2: rational;
    n1, n11: POLYNOMIAL;
    label
    11;

begin
    {-----z11---- hydro }
    k := 101;
    npr := FALSE;
    n := 13;
    CC1.RE := 1;
    CC1.IM := 0;

```

```

cco.re := 0;
cco.im := 0;
cz11b[1].re := 4.30303030303030303e-1;
cz11b[2].re := 4.60316369227122845e-1;
cz11b[3].re := 5.17125082669379499e-1;
cz11b[4].re := 5.71670646205569614e-1;
cz11b[5].re := 6.23131247778895924e-1;
cz11b[6].re := 6.78537783727140339e-1;
cz11b[7].re := 7.45154761552867819e-1;
cz11b[8].re := 8.28077270479268029e-1;
cz11b[9].re := 9.24904918718386051e-1;
cz11b[10].re := 1.01507975872409643e+0;
cz11b[11].re := 1.05808937540621121e+0;
cz11b[12].re := 1.02777541897702056e+0;
cz11b[13].re := 9.46308829743770162e-1;
cz11b[14].re := 8.55175362194503796e-1;
cz11b[15].re := 7.77768170139416451e-1;
cz11b[16].re := 7.18778118566379592e-1;
cz11b[17].re := 6.75366542862004491e-1;
cz11b[18].re := 6.43465710357306503e-1;
cz11b[19].re := 6.19710827954512977e-1;
cz11b[20].re := 6.01681347442958323e-1;
cz11b[21].re := 5.87711973392648118e-1;
cz11b[22].re := 5.76667644771962014e-1;
cz11b[23].re := 5.67770091744686375e-1;
cz11b[24].re := 5.60478736556360374e-1;
cz11b[25].re := 5.54412010958210416e-1;
cz11b[26].re := 5.49295877180035966e-1;
cz11b[27].re := 5.44930040747530256e-1;
cz11b[28].re := 5.41165566883715165e-1;
cz11b[29].re := 5.37889866636394894e-1;
cz11b[30].re := 5.35016485510068811e-1;
cz11b[31].re := 5.32478056853598015e-1;
cz11b[32].re := 5.30221367265326227e-1;
cz11b[33].re := 5.28203850346569642e-1;
cz11b[34].re := 5.26391059618974276e-1;
cz11b[1].im := 1.42187642187642188e-20;
cz11b[2].im := 9.47329448885904056e-2;
cz11b[3].im := 1.50739625820787770e-1;
cz11b[4].im := 1.81781226417626795e-1;
cz11b[5].im := 2.03751913203556902e-1;
cz11b[6].im := 2.21751453808594722e-1;
cz11b[7].im := 2.33298214052089177e-1;
cz11b[8].im := 2.29090367387024078e-1;
cz11b[9].im := 1.92254013718027787e-1;
cz11b[10].im := 1.04512050549451424e-1;
cz11b[11].im := -3.01388330473715258e-2;
cz11b[12].im := -1.67807803154497039e-1;
cz11b[13].im := -2.61716162296737500e-1;
cz11b[14].im := -3.03902500706754059e-1;
cz11b[15].im := -3.11567194751505277e-1;
cz11b[16].im := -3.02127186424691031e-1;
cz11b[17].im := -2.86005180124014475e-1;
cz11b[18].im := -2.68270565757133739e-1;

```



```

cz11b[19].im := -2.51101066434368383e-1;
cz11b[20].im := -2.35301571245223627e-1;
cz11b[21].im := -2.21068299358505941e-1;
cz11b[22].im := -2.08345661865898877e-1;
cz11b[23].im := -1.96987012589331061e-1;
cz11b[24].im := -1.86824651622647027e-1;
cz11b[25].im := -1.77698520757974946e-1;
cz11b[26].im := -1.69466373684232983e-1;
cz11b[27].im := -1.62005876124889523e-1;
cz11b[28].im := -1.55213415641128482e-1;
cz11b[29].im := -1.49001778586514544e-1;
cz11b[30].im := -1.43297636399742580e-1;
cz11b[31].im := -1.38039222031541169e-1;
cz11b[32].im := -1.33174322112160330e-1;
cz11b[33].im := -1.28658599021572157e-1;
cz11b[34].im := -1.24454212635047515e-1;
npr := FALSE;
k := 34;
for i := 1 to k do
  begin
    p11[i].im := 18 * (i - 1) * 2 * 3.1415926;
    {p11[i].im := 6 * (i - 1) * 2 * 3.1415926;}
    p11[i].re := 0;
    {writeln(p11.coef[i].im);}

  end;

{-----z1 -----}
z1.num.rep := prodrep;
z1.den.rep := prodrep;

z1.den.coef[0].re := 1;
z1.den.coef[0].im := 0;
z1.num.deg := 1;
z1.den.deg := 1;

z1.num.coef[0].re := 0.5;
z1.num.coef[0].im := 0;
z1.num.coef[1].re := xx[1];
z1.num.coef[1].im := 0;

z1.den.coef[1].re := -3.0e+2;
z1.den.coef[1].im := 0;
{writerational3(z1, 'z1');}
if PRINT then
  begin
    Rational_is_pr := Rational_pr(z1);
    if Rational_is_pr then
      writeln('z1 is pr')
    else
      begin
        writeln('z1 is NOT pr');
      end;
  end;
end;

```

end;

z11.num.rep := prodrep;  
z11.den.rep := prodrep;

z11.den.coef[0].re := 1;  
z11.den.coef[0].im := 0;  
z11.num.deg := 4;  
z11.den.deg := 5;

z11.num.coef[0].re := xx[2];  
z11.num.coef[0].im := 0;

z11.num.coef[1].re := xx[3];  
z11.num.coef[1].im := 0;  
z11.num.coef[2].re := xx[4];  
z11.num.coef[2].im := 0;  
z11.num.coef[3].re := xx[5];  
z11.num.coef[3].im := xx[6];  
z11.num.coef[4].re := xx[5];  
z11.num.coef[4].im := -xx[6];

z11.den.coef[1].re := -4.2373606985503662710e+2;  
z11.den.coef[1].im := 1.1364017233713318040e+3;  
z11.den.coef[2].re := -4.2373606985503662710e+2;  
z11.den.coef[2].im := -1.1364017233713318040e+3;  
z11.den.coef[3].re := -1.4967455632335248200e+3;  
z11.den.coef[3].im := 1.0688839603868126390e+3;  
z11.den.coef[4].re := -1.4967455632335248200e+3;  
z11.den.coef[4].im := -1.0688839603868126390e+3;  
z11.den.coef[5].re := -1.8403673382287710530e+2;  
z11.den.coef[5].im := 0;

if print2 then

begin

writerrational3(z11, 'z11-1');  
writerrational3(z1, 'z1');

z22 := addrational(z11, z1);  
writerrational3(z22, 'z11-2');  
z22.num := mulpoly(z11.num, z1.den);  
n1 := mulpoly(z1.num, z11.den);  
z22.num := addpoly(z22.num, n1);  
z22.den := mulpoly(z11.den, z1.den);  
Z22.NUM.COEFF[0] := CDIV(z22.num.coef[0], Z22.DEN.COEFF[0]);  
Z22.DEN.COEFF[0] := CC1;

writerrational3(z22, 'z11-3');

```

end;

if PR then
begin

    Rational_is_pr := Rational_pr(z11);
    if Rational_is_pr then
{writeln('z11 is pr')}
    else
    begin
        writeln('z11 is NOT pr');
        en := true;
        goto 11;
    end;
end;

for l := 1 to k do
begin
{-----cz1-----}
    cz1n := cc1;
    for j := 1 to 1 do
    begin
        c1 := csub(p11[j], z1.num.coef[j]);
        cz1n := cmul(cz1n, c1);
{writeln('cz1n', cz1n.re : 20, cz1n.im : 20);}
    end;
    cz1n := cmul(z1.num.coef[0], cz1n);
{writeln('cz1n', cz1n.re : 20, cz1n.im : 20);}

    cz1d := cc1;
    for j := 1 to 1 do
    begin
        cz1d := cmul(cz1d, csub(p11[j], z1.den.coef[j]));
{writeln('cz1d', cz1d.re : 20, cz1d.im : 20);}

    end;
{cz1d := cmul(z1.den.coef[0], cz1d);}
{writeln('cz1d', cz1d.re : 25, cz1d.im : 25)}

    CX1[l] := cdiv(cz1n, cz1d);
{writeln(i, 'cz1-' || l, ' ', cz1.re : 16, cz1.im : 16);}

{-----cz11-----}
    cz11bn := cc1;
    for j := 1 to 4 do
    begin
        c1 := csub(p11[j], z11.num.coef[j]);
        cz11bn := cmul(cz11bn, c1);
{writeln('cz11bn', cz11bn.re : 20, cz11bn.im : 20);}

```

```

    end;
    cz11bn := cmul(z11.num.coef[0], cz11bn);
{writeln('cz11bn', cz11bn.re : 20, cz11bn.im : 20);}

    cz11bd := ccl;
    for j := 1 to 5 do
        begin
            cz11bd := cmul(cz11bd, csub(p11[i], z11.den.coef[j]));
{writeln('cz11bd', cz11bd.re : 20, cz11bd.im : 20);}

        end;
{writeln('cz11bd', cz11bd.re : 20, cz11bd.im : 20);}

    cx11[i] := cdiv(cz11bn, cz11bd);
{writeln('cz11b ', cz11b.re : 20, cz11b.im : 20);}
{-----cz11b-----}

    cx11b[i] := cadd(cx1[i], cx11[i]);
{writeln('cz11b ', cz11b.re : 20, cz11b.im : 20);}

    cx[i] := csub(cz11b[i], cx11b[i]);
end;

if print then
begin
{-----z1-----}
    for i := 1 to 34 do
        begin
            writeln('cz1[' , i, '].re:=', cx1[i].re : 25, ');');
        end;
    for i := 1 to 34 do
        begin
            writeln('cz1[' , i, '].im:=', cx1[i].im : 25, ');');
        end;

{-----cz11-----}
    for i := 1 to 34 do
        begin
            writeln('cz11[' , i, '].re:=', cx11[i].re : 25, ');');
        end;
    for i := 1 to 34 do
        begin
            writeln('cz11[' , i, '].im:=', cx11[i].im : 25, ');');
        end;
{-----cz11b-----}
    for i := 1 to 34 do

```

```

begin
  writeln('cz11b[' , i , '].re:=', cx11b[i].re : 25, ');
end;
for i := 1 to 34 do
begin
  writeln('cz11b[' , i , '].im:=', cx11b[i].im : 25, ');
end;

end;

YA1 := 0;
YA2 := 0;
for i := 2 to 34 do
begin
{writeln('cx=', i, '= ', cx[i].re, cx[i].im);}
{YA1 := YA1 + ABS(CX[I].RE);}

{YA2 := YA2 + ABS(CX[I].IM);}
  YA1 := YA1 + sqr(CX[I].RE);

  YA2 := YA2 + sqr(CX[I].IM);

end;

YA1 := ya1 + YA2;

11:
if npr then
begin
  a := 1e5;
  absv := a;
{writeln('a', a : 16);}
end
else
begin
  ABSV := YA1;
{writeln('b', ya1 : 16);}
end;
npr := FALSE;

end;

procedure search (var k1, n: integer; a, d11: EXTENDED; x: EnTENDEDtableptr);
var
  i, j, k, o, j1, oo2: integer;
  xx2, dxx, xx, y1, y2, DX, Q: EXTENDEDARRAY;
  k2, l3, l2, o2, F, b3, bb, L, m, aa, A1, a3, a4, B, B1, B2, p, x1, x2, x3, d: EXTENDED;

```

```

dd, ya, x4, x5, x6, x11: EXTENDED;
{X: EnTENDEDTABLEPtr;}
{s1, s2: complex;}
p1, P11: POLYNOMIAL;
z11, z12, z1, z2, z3, z4: rational;
PRINT: boolean;
PRINT2: boolean;
PR: boolean;
label
2, 4, 5, 7, 9, 10;
{SEARCH 1 _____}
begin
begin

k := 0;
l := 0.85;

{d11 := 1e-8;}
{WRITELN('d11', d11);}
for i := 1 to n do
begin
xx[i] := x^[i, 0];
{WRITELN(' X [i,0]', i, '1= ', X^[i, 0] : 10 : 8);}
dx[i] := d11
{WRITELN('dx[i]', dx[i]);}
end;
end;
begin
2:
begin
{writeln('sigle search ');}
K := K + 1;
{writeln(' k1=', k1, ' k = ', k);}
k2 := 1e-11;
dd := dx[1];
if K > k1 then
begin
X^[0, 0] := a;
goto 10;
end;
if dd < k2 then
begin
X^[0, 0] := a;
goto 10;
end;

A1 := A;

for i := 1 to n do
begin
q[i] := x^[i, 0];
end;
end;
for i := 1 to n do

```

```

begin

  X^[I, 1] := X^[I, 0] + DX[I] * x^[i, 0];
  {X[I, 1] := X[I, 0] + DX[I];}

  Q[I] := X^[I, 1];
  xx[i] := q[i];

  {writeln('xx[j]', j, '=', xx[j] : 10 : 8);}
  {Writeln('x^[i,1]');}
  {WRITELN(' X [i,1]', i, '1= ', X^[i, 1] : 25);}

  b1 := absv(xx, z12, z11);

  if i = 1 then
    begin
      WRITELN('dx', dx[1], 'k=', k);
    end;
  if B1 < A1 then
    begin
      A1 := B1;
      {WRITELN(' +dx B1=', i, ' ', B1 : 12, 'dx', dx[1], 'k=', k);}
      {computingz11z12(xx, z11, z12);}
      for j := 1 to n do
        begin
          xx2[j] := x^[j, 1];
          {writeln('+DX xx2', j, '--', xx2[j] : 30);}

          end;
        end
      else
        begin
          X^[I, 1] := X^[I, 0] - DX[I] * x^[i, 0];
          {X[I, 1] := X[I, 0] - DX[I];}

          Q[I] := X^[I, 1];

          xx[i] := q[i];

          {WRITELN('DX[i]', i, '=', DX[I] : 20 : 18);}
          {WRITELN('X[i,0]', i, '=', X[I, 0] : 10 : 8);}

          {WRITELN(' X [i,1]', i, '1= ', X[I, 1] : 10 : 8);}
          b1 := absv(xx, z11, z12);

          {WRITELN('SEARCH1 -dx B1=', i, '--', B1 : 20);}

          if B1 < A1 then
            begin
              A1 := B1;
              {WRITELN(' -dx B1=', i, '--', B1 : 30);}
            end
          end

```

```

{computingz11z12(xx, z11, z12);}
  for j := 1 to n do
    begin
      xx2[j] := x^[j, 1];
    {writeln('-DX  xx2', j, '-', xx2[j] : 30);}

      end;

    end
  else
    begin
      x^[i, 1] := x^[i, 0];
      xx[i] := x^[i, 1];
    end;
  {WRITELN('Xx[i]', i, Xx[i] : 15);}

    end;
  end;
  for i := 1 to n do
    begin
      for j := 0 to 1 do
        begin
          {writeln('x[i,j]', i, j, '=      ', x[i, j] : 10 : 6)}

            end;
          end;
        {  WRITELN('A=', A);}
        if a1 < a then
          begin
            a := a1;
          {if i > 100 then}
          {begin}
            WRITELN('A=', A : 12);
          {end;}
          goto 5;
        end
      else
        begin
          {  WRITELN('A=', A);}
          end;
        end;
      {writeln('X[1,0]', X[1, 0]);}
      {writeln('step2');}
      {DX<D ?*****}
      4:
      {writeln('-----step4, reduce dx-----');}
      begin

        {DX3 := DX11 + DX22 + dx33 + dx44 + dx55 + dx66;}
        {TELN('1----DX4=', DX4);}
        {WRITELN('D=', D);}
        {if DX4 < D then}
        {goto 10}

```



```

{else}
  begin
    {WRITELN('l=', l : 18 : 16);}
    for j := 1 to n do
      begin
        dx[j] := dx[j] * l;
        if dx[j] < 1.0e-16 then
          {writeln('dx[j]', dx[j]);}
        end;
        {DX[1] := DX[1] * L;}
        {DX[2] := DX[2] * L;}
        for J := 1 to n do
          begin
            {writeln('dx[J]', J, '=', dx[J] : 10 : 8);}
          end;
        {l := l * l;}
        end;
        { writeln('-----X[1,0]', X[1, 0]);}
        goto 2;
      end;
      {writeln('step4');}

      { PATTEN SEARC -----}
      5:
      {writeln('    ---patten search-----');}

      begin
        for j := 1 to n do
          begin
            x^[j, 2] := x^[j, 1] * 2 - x^[j, 0];

            xx[j] := x^[j, 2];
            {writeln('x[j,2]', j, x[j, 2] : 28);}

            end;
            b2 := absv(xx, z11, z12);

            {WRITEln(' PA    -B2=', B2 : 30);}
            {if b2 > a then}
            {begin}
            {for j := 1 to n do}
            {begin}
            {x[j, 0] := x[j, 1];}
            {goto 2}
            {end;}
            {end;}
            if b2 > a then
              begin
                p := 0.95;
                j := 0;
                while B2 > A do
                  9:
                    begin

```

```

for j := 1 to n do
begin
x^[j, 2] := x^[j, 1] + p * (x^[j, 1] - x^[j, 0]);

end;
for j := 1 to n do
begin
xx[j] := x^[j, 2];
{writeln('xx=', xx[j] : 25)}
end;
b2 := absv(xx, z11, z12);

{WRITEln(' PA  p for b2<a ', ' p= ', p, ' B2=', B2 : 28, 'k=', k);}
{writeln('a', a : 40 : 36);}
if b2 < a then
goto 7;
if b2 = a then
goto 7;

p := p * p;
if P < 1e-100 then
begin
writeln('b2>a');
writeln('a= ', a : 28);
for j := 1 to n do
begin
x^[j, 0] := x^[j, 1];

writeln(' XX[' , J, ']:= ', x^[j, 2] : 30, ');');
end;
goto 2;
end;
{writeln('a', a : 40 : 36);}
{writeln('b2', b2 : 40 : 36);}
if b2 > a then
goto 9
else
goto 7;

end;
end
else

7:
begin
for j := 1 to n do
begin
x^[j, 0] := x^[j, 2];
xx2[j] := x^[j, 2];
end;
if k > 100 then
begin

```

```

        for j := 1 to n do
        begin
{writeln(' XX[' , J, ' ]:=', x^[j, 2] : 30, ' ');}
            end;
        end;
        A := B2;
{if k > 100 then}
{begin}
{writeln('PS a= ', a : 12);}
{end;}

{WRITEln(' patten search b2<a ', ' dx', dx[1], '-1-B2=', B2 : 15, 'k=', k);}
{writeln('dx', dx[1] : 28 : 24, dx[1]);}
        goto 2;
        end;
    end;
end;

```

10:

```

        for j := 1 to n do
        begin
            writeln('xx[ ', j, ' ]:=', x^[j, 0] : 30, ' ');
        end;

    end;

{-----main program-----}

begin
{ReadpolyFromDataFile(p, 'p');}
New(X);
OpenTextWindow(15, 40, 630, 470);
DateTime;
PRINT := FALSE;
{PRINT := true;}

    PRINT2 := FALSE;
{PRINT2 := true;}
{pr := true;}
{readrationalfromdatafile(z11, 'choose z11 ');}
{readrationalfromdatafile(z12, 'choose z12 ');}
{writerational3(z11, 'z11');}
{writerational3(z12, 'z12');}
    begin
        D := 1.0E-80;
        L2 := 1.0;
        m := 0.9;
        N := 6;

        xx[1] := 100;
    end
end

```

```

xx[2] := -100;

xx[3] := 100;
xx[4] := -100;
xx[5] := -100;
XX[6] := -100;

for i := 1 to n do
begin
  writeln(' XX[' , i, ']:= ', xx[i] : 27, ');');
  {x^[i, 0] := xx[i];}
end;

A := absv(xx, z11, z12);
writeln('3   A= ', A : 40);
aa1 := a;
o := 0;
end;

for i := 1 to n do
begin
  y1[i] := xx[i];
  {write('xx', i, xx[i] : 20);}
end;

for o := 0 to 57 do

1:
begin
  o := o + 1;

  k1 := 100;
  d11 := 0.001;
  writeln('o= ', o);
  writeln(' search 1 input aa1= ', aa1 : 20);

  for i := 1 to n do
  begin
    x^[i, 0] := y1[i];
    {writeln(' xX [' , i, ']:= ', y1[i] : 25, ');');}
    {writeln(' X [' , i, ',0]:= ', X^[i, 0] : 25, ');');}
  end;
  k := 100;
  d := 1e-8;
  search(k, n, aa1, d, x);
  a := x^[0, 0];
  writeln(' search 1 output a= ', a : 25);

```

```

if a < aa1 then
begin
  ac := aa1;
  aa1 := a;
  writeln(' search 1 a<aa1', aa1 : 20);

  for i := 1 to n do
  begin
    y1[i] := x^[i, 0];
    xx[i] := y1[i];
    WRITELN(' xX [' , i , ']:=      ', Xx[i] : 27, ';');

  end;
  ab := (ac - a) / ac;
  if ab > 0.01 then

    goto 1;
  end;
  d1 := 1.01;
  L1 := 0.98;

  {-----changing starting point-----}
  for j := 1 to 10 do
  begin
    d1 := d1 * L1;
    writeln('d1=', j, ' ', d1 : 20);

    begin
      writeln('- change input aa1=', aa1 : 20);
      for i := 1 to n do
      begin
        {x^[i, 0] := 0.98 * y1[i];}
        X^[i, 0] := y1[i] * D1;

        xx[i] := x^[i, 0];
        WRITELN(' X [' , i , ',0]:=      ', X^[i, 0] : 25, ';');
        {WRITELN(' xX [' , i , ']:=      ', Xx[i] : 25, ';');}

      end;
    end;

    A := absv(xx, z11, z12);
    writeln('_ change input  A=', A : 15);

    search(k1, n, a, d11, x);
    a := x^[0, 0];
    writeln(' - change output a=', a : 15, 'aa1=', aa1 : 15);
    if a < aa1 then
    begin
      aa1 := a;
      {writeln(' - change ', 'aa1=', aa1 : 15);}

      for i := 1 to n do
      begin

```

```

    y1[i] := x^[i, 0];
    writeln(' XX[' , i, ']:= ', y1[i] : 27, ');');
    end;
    goto 1;
end;
writeln('+ change');
d2 := 2 - d1;
writeln('d2=', d2 : 15);
for i := 1 to n do
    begin
        {x^[i, 0] := 1.02 * y1[i];}
        X^[i, 0] := y1[i] * (2 - D1);

        WRITELN(' X [' , i, ',0]:= ', X^[i, 0] : 25, ');');

    end;

A := absv(xx, z11, z12);
writeln('+ change input  A=', A : 15);

search(k1, n, a, d11, x);
a := x^[0, 0];
writeln(' + change output a=', a : 15, 'aa1=', aa1 : 15);

if a < aa1 then
    begin
        aa1 := a;
        {writeln(' + change output', 'aa1=', aa1 : 15);}

        for i := 1 to n do
            begin
                y1[i] := x^[i, 0];
                writeln(' XX[' , i, ']:= ', y1[i] : 27, ');');
            end;
            goto 1;
        end;
    end;
end;
end.

```

## APPDENDIX B

### HYDRO DATA

Given z11(scaled) Frequency Response

Freq.(Hz)	Real Part	Imag. Part
50.00000	0.06822	0.11569
60.00000	0.09059	0.12886
70.00000	0.11571	0.13475
80.00000	0.14025	0.13114
90.00000	0.15864	0.11823
100.00000	0.16572	0.10083
110.00000	0.16133	0.08686
120.00000	0.15095	0.08155
130.00000	0.14093	0.08475
140.00000	0.13483	0.09315
150.00000	0.13328	0.10326
160.00000	0.13531	0.11261
170.00000	0.13921	0.11992
180.00000	0.14306	0.12499
190.00000	0.14527	0.12886
200.00000	0.14523	0.13334
210.00000	0.14377	0.14031
220.00000	0.14274	0.15061
230.00000	0.14399	0.16379
240.00000	0.14864	0.17863
250.00000	0.15709	0.19387
260.00000	0.16943	0.20845
270.00000	0.18570	0.22130
280.00000	0.20580	0.23108
290.00000	0.22907	0.23617
300.00000	0.25411	0.23502
310.00000	0.27880	0.22631
320.00000	0.29993	0.20922
330.00000	0.31286	0.18433
340.00000	0.31213	0.15580
350.00000	0.29531	0.13269
360.00000	0.26796	0.12523
370.00000	0.24212	0.13668
380.00000	0.22774	0.16113
390.00000	0.22762	0.18958
400.00000	0.23953	0.21533

Given z12(scaled) Frequency Response

Freq(Hz)	Real Part	Imag Part
50.000	0.052	0.069
60.000	0.071	0.075
70.000	0.093	0.076
80.000	0.116	0.068
90.000	0.135	0.050
100.000	0.143	0.026
110.000	0.140	0.003
120.000	0.130	-0.012
130.000	0.118	-0.019
140.000	0.109	-0.021
150.000	0.103	-0.020
160.000	0.101	-0.018
170.000	0.100	-0.019
180.000	0.101	-0.021
190.000	0.099	-0.025
200.000	0.096	-0.030
210.000	0.089	-0.032
220.000	0.082	-0.032
230.000	0.075	-0.028
240.000	0.071	-0.021
250.000	0.069	-0.012
260.000	0.070	-0.002
270.000	0.075	0.008
280.000	0.085	0.017
290.000	0.099	0.024
300.000	0.117	0.027
310.000	0.139	0.024
320.000	0.162	0.012
330.000	0.182	-0.010
340.000	0.193	-0.043
350.000	0.188	-0.081
360.000	0.165	-0.112
370.000	0.133	-0.128
380.000	0.101	-0.127
390.000	0.077	-0.114
400.000	0.064	-0.096



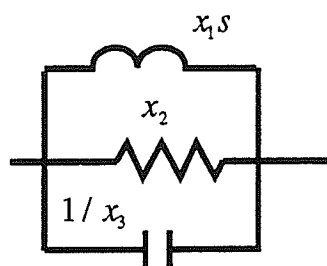
Given z22(scaled) Frequency Response

Freq(Hz)	Real Part	Imag Part
50.000	0.047	0.080
60.000	0.064	0.091
70.000	0.086	0.097
80.000	0.109	0.095
90.000	0.130	0.083
100.000	0.143	0.064
110.000	0.144	0.044
120.000	0.137	0.030
130.000	0.126	0.022
140.000	0.117	0.022
150.000	0.111	0.024
160.000	0.108	0.028
170.000	0.108	0.030
180.000	0.109	0.031
190.000	0.109	0.029
200.000	0.107	0.026
210.000	0.102	0.024
220.000	0.094	0.024
230.000	0.086	0.028
240.000	0.078	0.035
250.000	0.072	0.045
260.000	0.069	0.058
270.000	0.068	0.072
280.000	0.071	0.088
290.000	0.080	0.105
300.000	0.093	0.120
310.000	0.113	0.132
320.000	0.139	0.137
330.000	0.168	0.133
340.000	0.195	0.115
350.000	0.212	0.084
360.000	0.212	0.048
370.000	0.194	0.019
380.000	0.168	0.004
390.000	0.140	0.002
400.000	0.118	0.010

## APPENDIX C

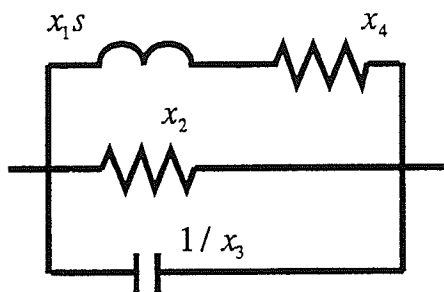
### ELEMENTAL CIRCUITS AND THEIR IMPEDANCE FUNCTIONS

1. No. A1



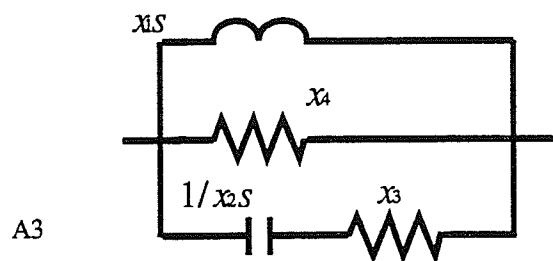
$$z = \frac{x_1 x_3 s}{x_1 x_2 x_3 s^2 + x_1 s + x_3}$$

2. No. A2



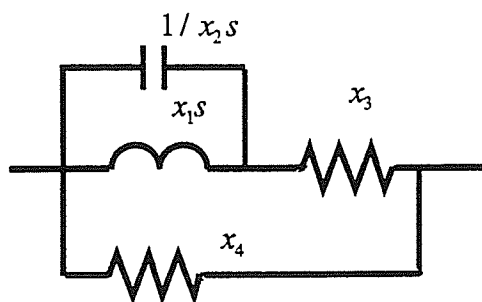
$$z = \frac{x_1 x_3 s + x_3 x_4}{x_1 x_2 x_3 s^2 + (x_2 x_3 x_4 + x_1) s + x_3 + x_4}$$

3. No. A3



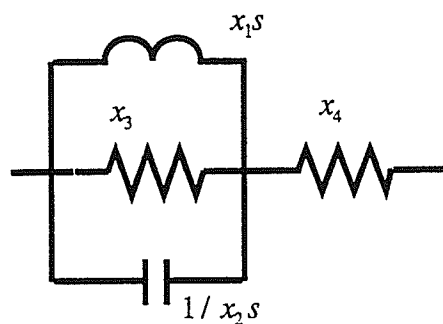
$$z = \frac{x_1 x_2 x_3 x_4 s^2 + x_1 x_4 s}{x_1 x_2 x_4 s^2 + x_1 x_2 x_3 s^2 + (x_2 x_3 x_4 + x_1) s + x_4}$$

4. No. A4



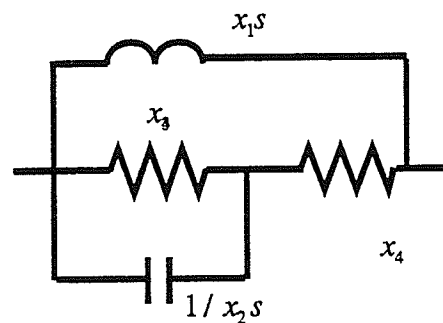
$$z = \frac{x_1 x_2 x_3 x_4 s^2 + x_1 x_4 s + x_3 x_4}{x_1 x_2 x_3 s^2 + x_1 x_2 x_4 s^2 + x_1 s + x_3 + x_4}$$

5. No. A5



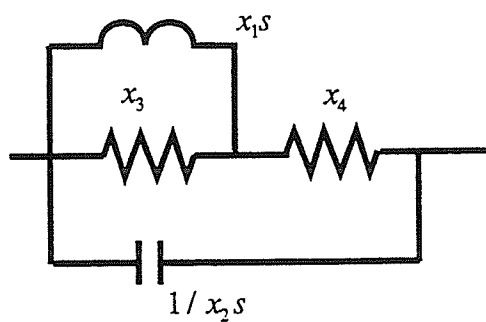
$$z = \frac{x_1 x_2 x_3 x_4 s^2 + (x_1 x_4 + x_1 x_3) s + x_3 x_4}{x_1 x_2 x_3 s^2 + x_1 s + x_3}$$

6. No. A6



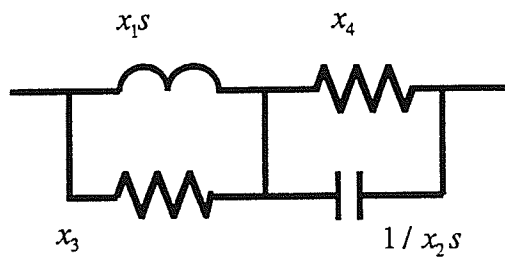
$$z = \frac{x_1 x_2 x_3 x_4 s^2 + x_1 x_4 s + x_1 x_3 s}{x_4 x_2 x_3 s^2 + x_1 x_2 x_4 s^2 + x_1 s + x_4}$$

7. No. A7



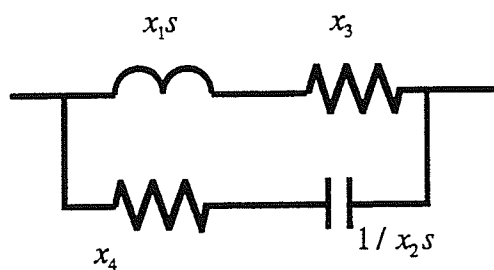
$$z = \frac{x_1(x_3 + x_4)s + x_3x_4}{(x_1x_2x_3 + x_1x_2x_4)s^2 + (x_2x_3x_4 + x_1)s + x_3}$$

8. No. A8



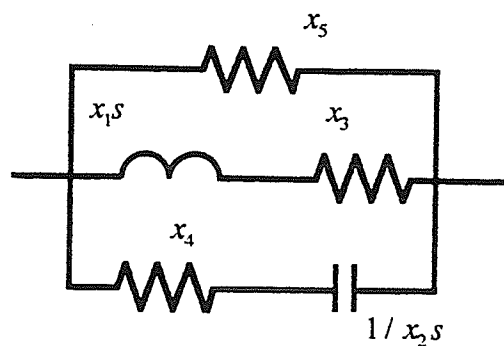
$$z = \frac{x_1x_2x_3x_4s^2 + x_1(x_3 + x_4)s + x_3x_4}{x_1x_2x_4s^2 + (x_2x_3x_4 + x_1)s + x_3}$$

9. No. A9



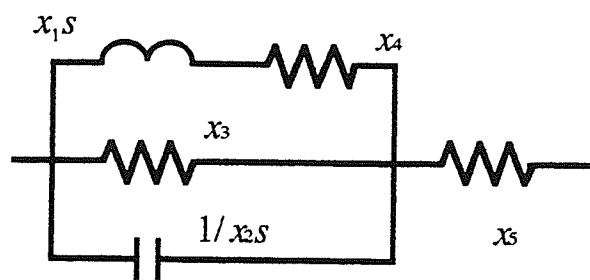
$$z = \frac{x_1x_2x_4s^2 + (x_2x_3x_4 + x_1)s + x_3}{x_1x_2s^2 + (x_2x_3 + x_2x_4)s + 1}$$

10. No. A10



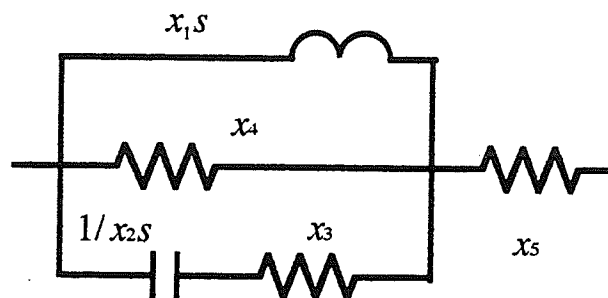
$$Z = \frac{x_1 x_2 x_4 x_5 s^2 + x_5 (x_2 x_3 x_4 + x_1) s + x_3 x_5}{(x_1 x_2 x_4 + x_1 x_2 x_5) s^2 + (x_2 x_3 x_4 + x_2 x_3 x_5 + x_2 x_5 x_4 + x_1) s + x_3 + x_5}$$

11. No. A11



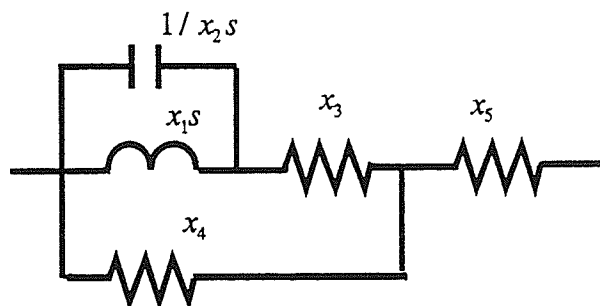
$$Z = \frac{x_1 x_2 x_3 x_5 s^2 + x_5 (x_2 x_3 x_4 + x_1) s + x_1 x_3 s + x_3 x_4 + x_3 x_5 + x_4 x_5}{x_1 x_2 x_3 s^2 + (x_2 x_3 x_4 + x_1) s + x_3 + x_4}$$

12. No. A12



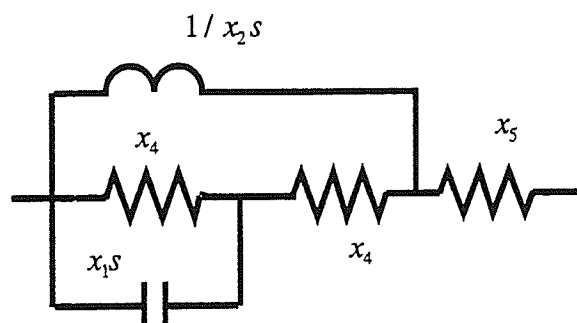
$$Z = \frac{(x_1 x_2 x_3 x_4 + x_1 x_2 x_3 x_5 + x_1 x_2 x_4 x_5) s^2 + x_5 (x_2 x_3 x_4 + x_1) s + x_1 x_4 s + x_4 x_5}{x_1 x_2 x_4 + x_1 x_2 x_3 s^2 + (x_2 x_3 x_4 + x_1) s + x_4}$$

13. No. A13



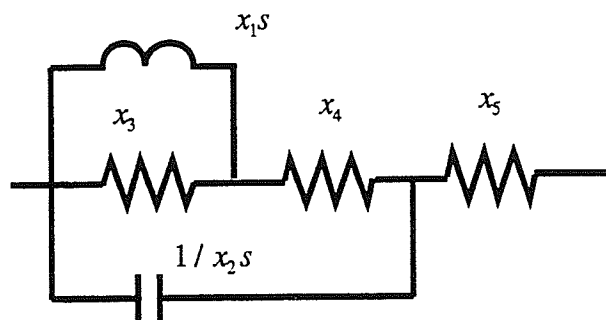
$$z = \frac{(x_1 x_2 x_3 x_4 + x_1 x_2 x_3 + x_5 x_1 x_2 x_4 x_5) s^2 + x_1 x_4 s + x_1 x_5 s + x_3 x_4 + x_3 x_5 + x_4 x_5}{(x_1 x_2 x_4 + x_1 x_2 x_3 s)^2 + x_1 s + x_3 + x_4}$$

14. No. A14



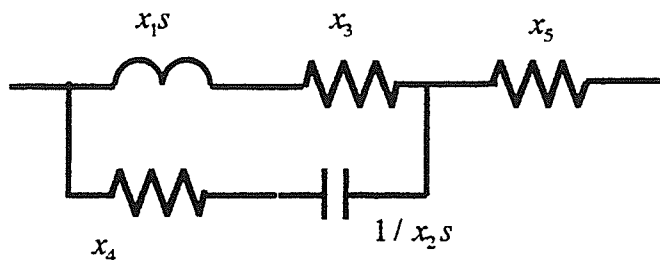
$$z = \frac{(x_1 x_2 x_3 x_4 + x_1 x_2 x_4 x_5) s^2 + (x_1 x_3 + x_1 x_4 + x_1 x_5 + x_2 x_3 x_4 x_5) s + x_4 x_5}{x_1 x_2 x_4^2 + x_2 x_3 x_4 s + x_1 s + x_4}$$

15. No. A15



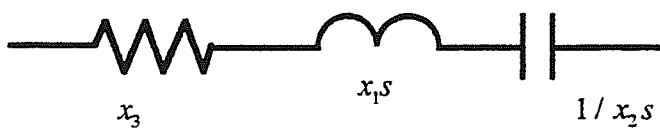
$$z = \frac{(x_1 x_2 x_3 x_5 + x_1 x_2 x_4 x_5) s^2 + (x_1 x_3 + x_1 x_4 + x_1 x_5 + x_2 x_3 x_4 x_5) s + x_4 x_5 + x_3 x_4}{(x_1 x_2 x_3 + x_1 x_2 x_4) s^2 + x_2 x_3 x_4 s + x_1 s + x_3}$$

16. No. A16



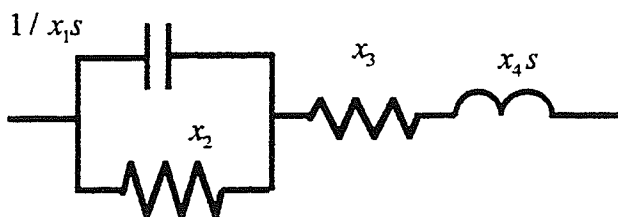
$$z = \frac{(x_1 x_2 x_5 + x_1 x_2 x_4) s^2 + (x_3 x_2 x_5 + x_4 x_2 x_5 + x_3 x_2 x_4) s + x_5 + x_3}{x_1 x_2 s^2 + (x_3 x_2 + x_4 x_2) s + 1}$$

17. No. B1



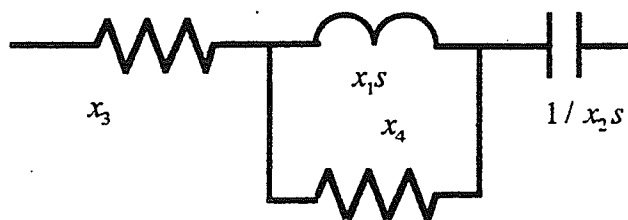
$$z = \frac{x_1 x_2 s^2 + x_3 x_2 s + 1}{x_2 s}$$

18. No. B2



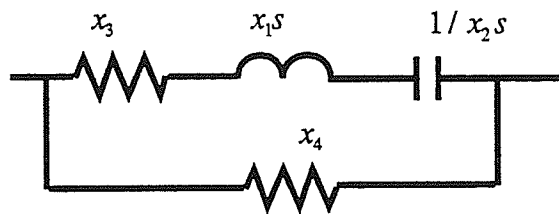
$$z = \frac{x_1 x_2 x_4 s^2 + (x_4 + x_1 x_2 x_3) s + x_2 + x_3}{x_1 x_2 s + 1}$$

19. No. B3



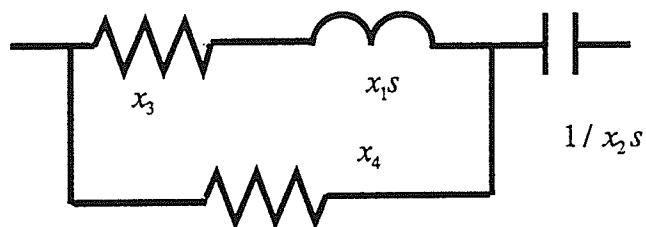
$$z = \frac{(x_1 x_2 x_3 + x_1 x_2 x_4) s^2 + (x_3 x_2 x_4 + x_1) s + x_4}{x_1 x_2 s^2 + x_4 x_2 s}$$

20. No. B4



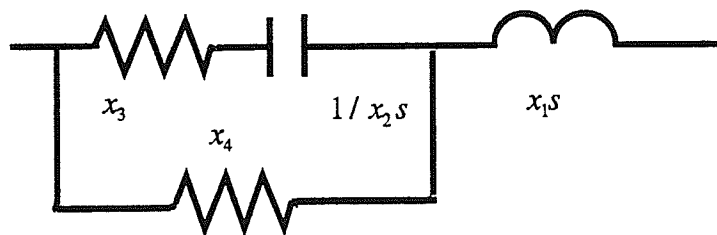
$$z = \frac{x_1 x_2 x_4 s^2 + x_2 x_3 x_4 s + x_1 s + x_3 + x_4}{x_1 x_2 s^2 + (x_2 x_3 + x_2 x_4) s}$$

21. No. B5



$$z = \frac{x_1 x_2 x_4 s^2 + (x_3 x_2 x_4 + x_1) s + x_3 + x_4}{x_1 x_2 s^2 + (x_4 x_2 + x_3 x_2) s}$$

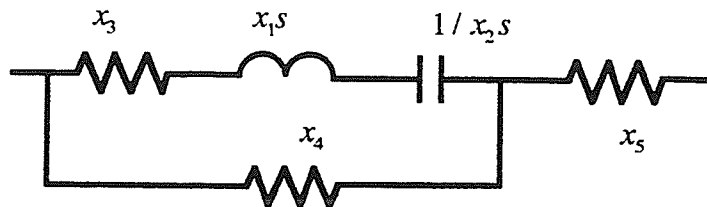
22. No. B6



$$z = \frac{(x_1 x_2 + x_1 x_3) s^2 + (x_4 + x_1 x_2 x_3) s + x_3}{(x_1 x_2 + x_1 x_3) s + 1}$$

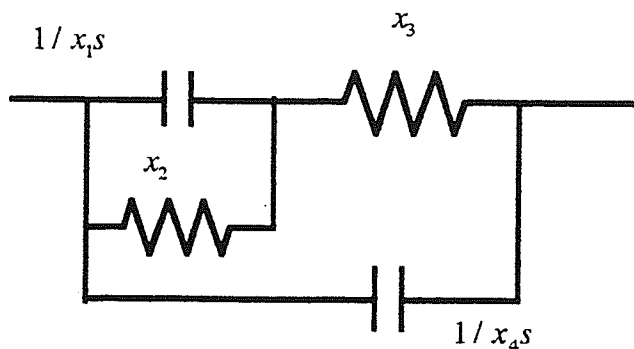


23. No. B13



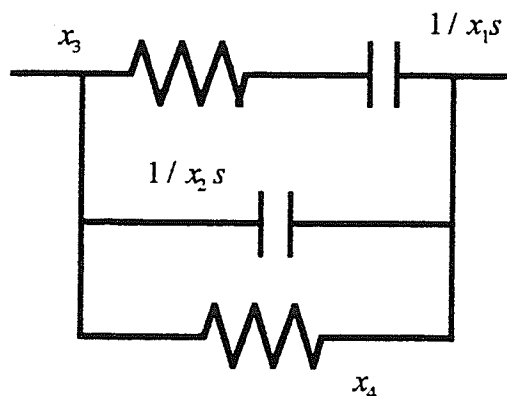
$$z = \frac{(x_1 x_2 x_4 + x_1 x_2 x_5) s^2 + (x_2 x_3 x_4 + x_2 x_3 x_5 + x_2 x_4 x_5) s + x_4 + x_5}{x_1 x_2 s^2 + (x_2 x_3 + x_2 x_4) s + 1}$$

24. No. D4



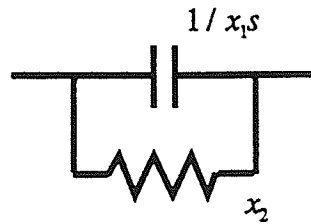
$$z = \frac{x_1 x_2 x_3 s + x_2 + x_3}{x_1 x_2 x_3 x_4 s^2 + (x_2 + x_3 + x_1 x_2) s + 1}$$

25. No. D8



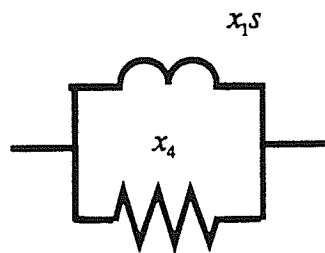
$$z = \frac{x_1 x_3 x_4 s + x_4}{x_1 x_2 x_3 x_4 s^2 + (x_2 x_4 + x_1 x_4 + x_1 x_3) s + 1}$$

26. No. C2



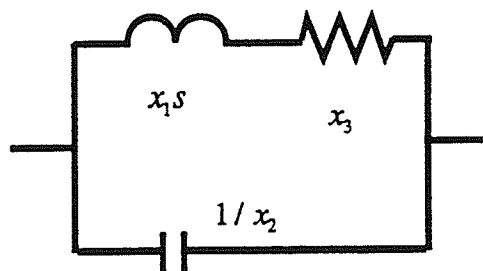
$$z = \frac{x_2}{x_2 x_1 s + 1}$$

27. No. E1



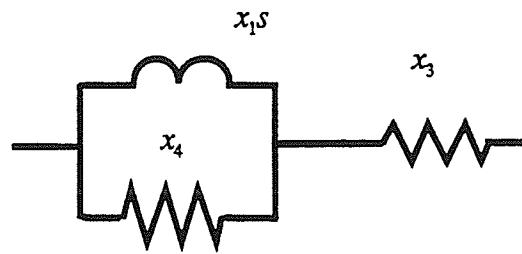
$$z = \frac{x_1 x_2 s}{x_1 s + x_2}$$

28. No. F4



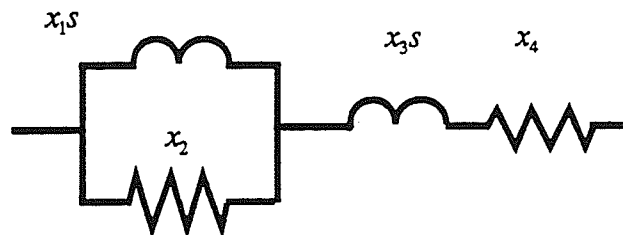
$$z = \frac{x_1 s + x_3}{x_1 x_2 s^2 + x_2 x_3 + 1}$$

29. No. E4



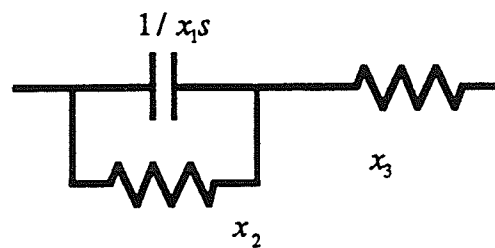
$$z = \frac{(x_1x_2 + x_1x_3)s + x_2x_3}{x_1s + x_2}$$

30. No. E6



$$z = \frac{x_1x_3s^2 + (x_1x_2 + x_2x_3 + x_4x_1)s + x_2x_4}{x_1s + x_2}$$

31. No. C4

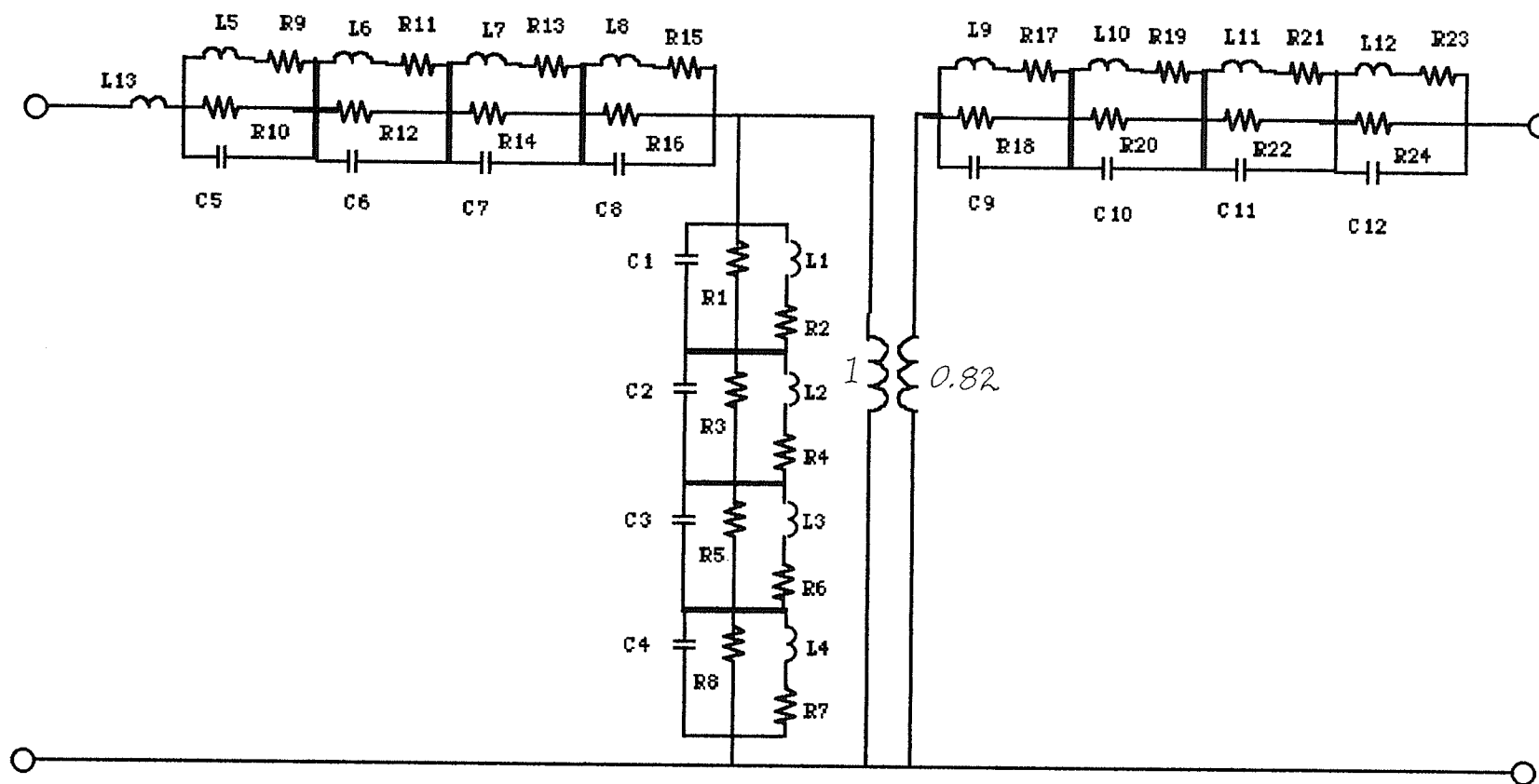


$$z = \frac{x_1x_2x_3s + x_2 + x_3}{x_1x_2s + 1}$$

## APPENDIX D

### CIRCUIT DIAGRAM FOR HYDRO EXAMPLE

The circuit diagram of Hydro example is shown in page 211. The element Values are presented in page 212.



Circuit diagram for Hydro Example

## Table of Element values

Resistor:  $\Omega$ , Inductor: H, Capacitor: F

L1=2.407223837508354e-5  
R1= 2.389173954725266e-1  
C1=9.129871614277363e-3  
R2= 1.192208495922387e-3  
L2= 2.425556158414111e-5  
R3=6.999243795871735e-2  
C2=2.827802859246731e-2  
R4=1.646707067266107e-3  
L3=2.590678632259369e-4  
R5=1.806209683418274e-1  
C3=1.013956591486931e-2  
R6=1.307248324155807e-2  
L4= 6.131077952886699e-6  
R7=6.372318416833878e-2  
C4= 2.305218484252691e-3  
R8=1.494958560215309e-4  
L5=3.629102511977675346000e-5  
R9=7.134397957352106690000e-1  
C5= 3.121618437731108759000e-3  
R10= 6.568243254519025206000e-8  
L6=1.229532202301360569000e-5  
R11=1.154105174990523624000e-1  
C6= 2.331057611916097339000e-2  
R12= 1.055507198898414074000e-3  
L7=4.097762480898779068000e-6  
R13= 6.195869481839603960000e-2  
C7= 9.608930459911617805000e-2  
R14= 8.594009219747512460000e-4  
L8=2.700770205352986795000e-6  
R15= 1.220887737359677343000e-2  
C8=1.974014201208384302000e-1  
R16=3.789801016956668489000e-5

L9=5.540265978075893726000e-6  
R17=8.951448933725363738000e-2  
C9=3.194859710540290076000e-2  
R18= 2.849292089493381096000e-5  
L10=3.536820940625267341000e-5  
R19= 3.260416899336399593000e-2  
C10=4.167479119939041582000e-2  
R20=2.492626151390433293000e-3  
L11=3.473803759175370541000e+3  
R21= 4.486535665028298477000e-11  
C11=2.228891335236002735000e-8  
R22=3.743686883839680938000e-6  
L12=1.819720667837585479000e-7  
R23=8.666173195282263630000e+14  
C12=8.614177703857422000000e-1  
R24=1.980995579354740723000e-22  
L13= 5.939292623767285908000e-5

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