TORSIONAL LOAD ON A TEN-STOREY BUILDING FRAME MODEL

A Thesis

Presented to

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By

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CONTENTS

CHAPTER I

INTRODUCTION

This paper presents the torsional rigidity, the angle of twist and stresses analysis, of a ten-storey building model subjected to a torsional load, M_t.

This ten-storey building was built by Mr. Robert Petri, according to ACI Building Code (ACI 318-56) in May 1964. The model is ten-storeys high with two shear walls for lateral support in one direction only, 3 bays 3 x 12" in length in the short side direction and 4 bays 4 x 12" in length in the long side direction. Its foundation was also perfectly rigid and there was no possibility of sliding. The detail of this 1/16 scale prototype is shown in Fig. (1), Fig. (2), Fig. (3), Fig. (4), Fig. (5), Fig. (6), and Fig. (7).

The unit stresses are as follows:

$f'_c = 3{,}000$ psi

$f_y = 40{,}000$ psi

$E_s = 30 \times 10^6$ psi

$E_c = 3 \times 10^6$ psi

$n = E_s / E_c = 10$

in which f'_c = compressive strength of concrete

f_y = yield strength of reinforcing

E_s = modulus of elasticity of steel

E_c = modulus of elasticity of concrete

n = modulus ratio

Concrete :

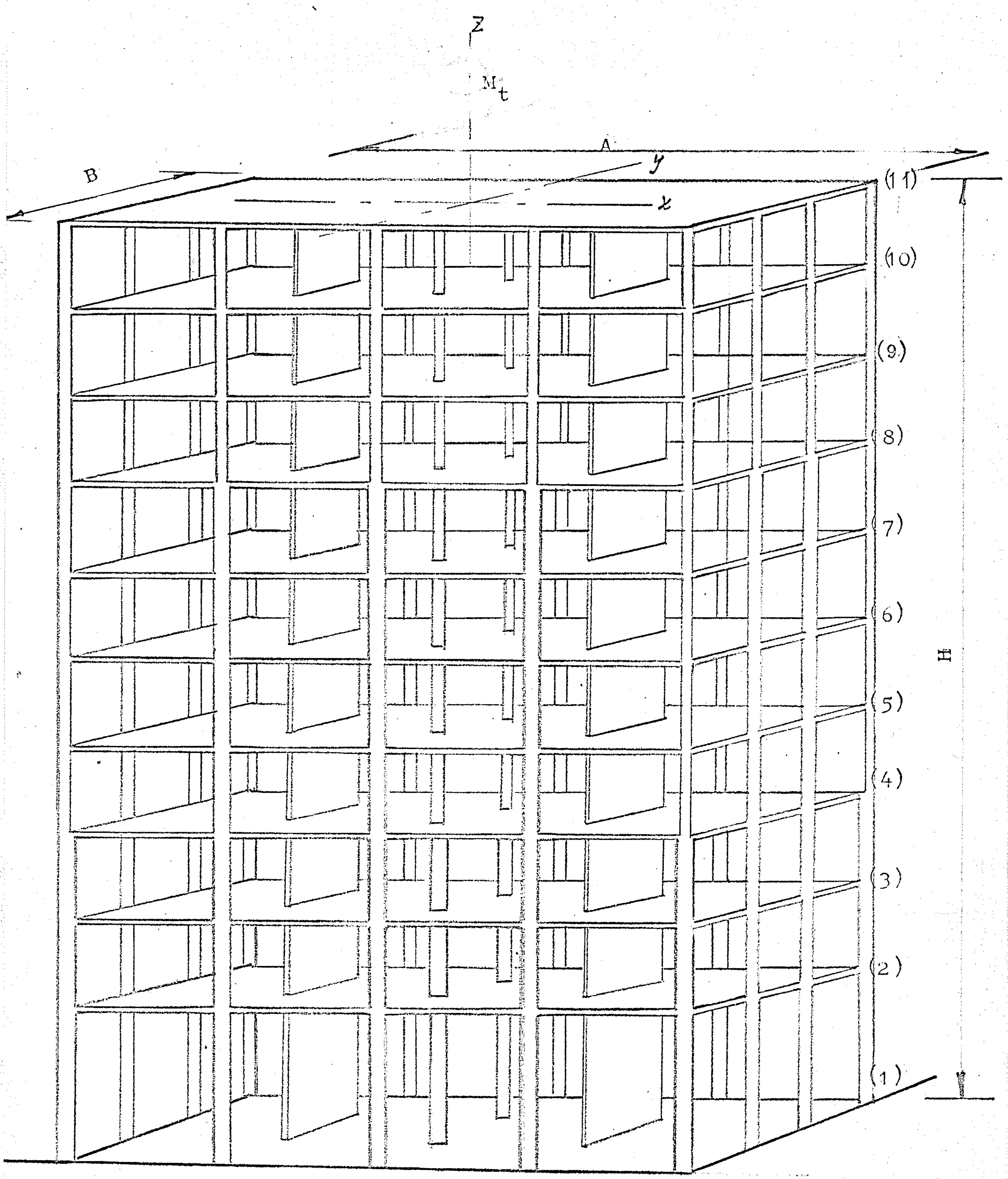
The model aggregate and sand were obtained from the Selkirk Silica Co. Ltd., which was taken from Black Island, Lake Winnipeg, Manitoba. The sand particles are rounded, the percentage of moisture content is zero, the specific gravity of sand is 2.65 and fineness modulus of the complete aggregate is 2.20. The cement is used High-Early-Stremgth Bortland cement. The water used in the concrete is from the city supply.

Reinforcement :

There are three types of model reinforcing steel; one type for the columns and shear walls reinforcing steel; one for the slab steel; and another for the lateral column ties. All three types were black annealled wire which can be obtained in any hardware store, as shown in following table.

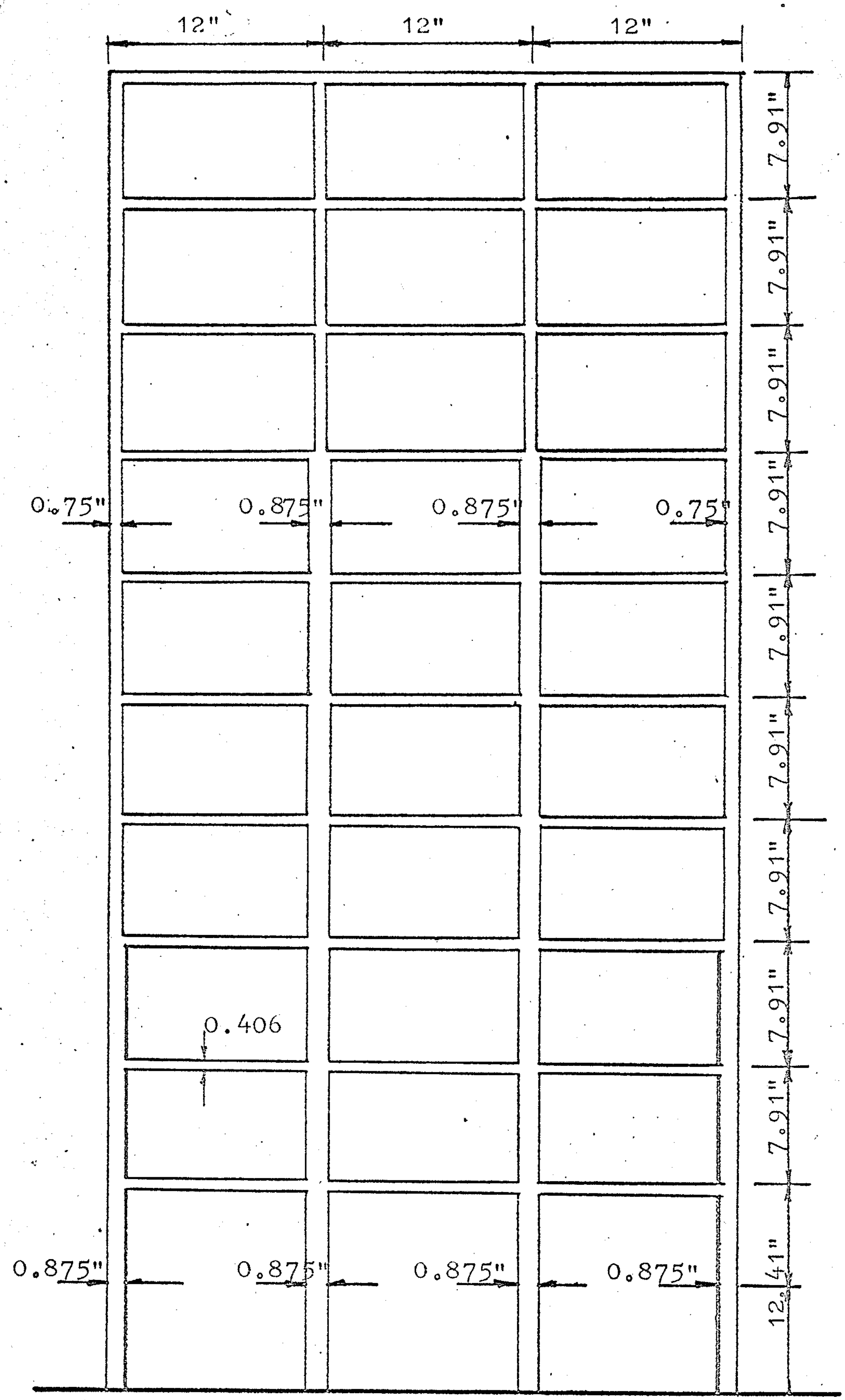
REINFORCEMENT DIAMETER OF PROTOTYPE AND MODEL

PROTOTYPE(dia.)	MODEL(dia.)
3#(0.375")	0.0234"
4#(0.500")	0.0312"
5#(0.625")	0.0390"
6#(0.750")	0.0469"
7#(0.875")	0.0547"
8#(1.000")	0.0625"
9#(1.128")	0.0705"
10#(1.270")	0.0795"



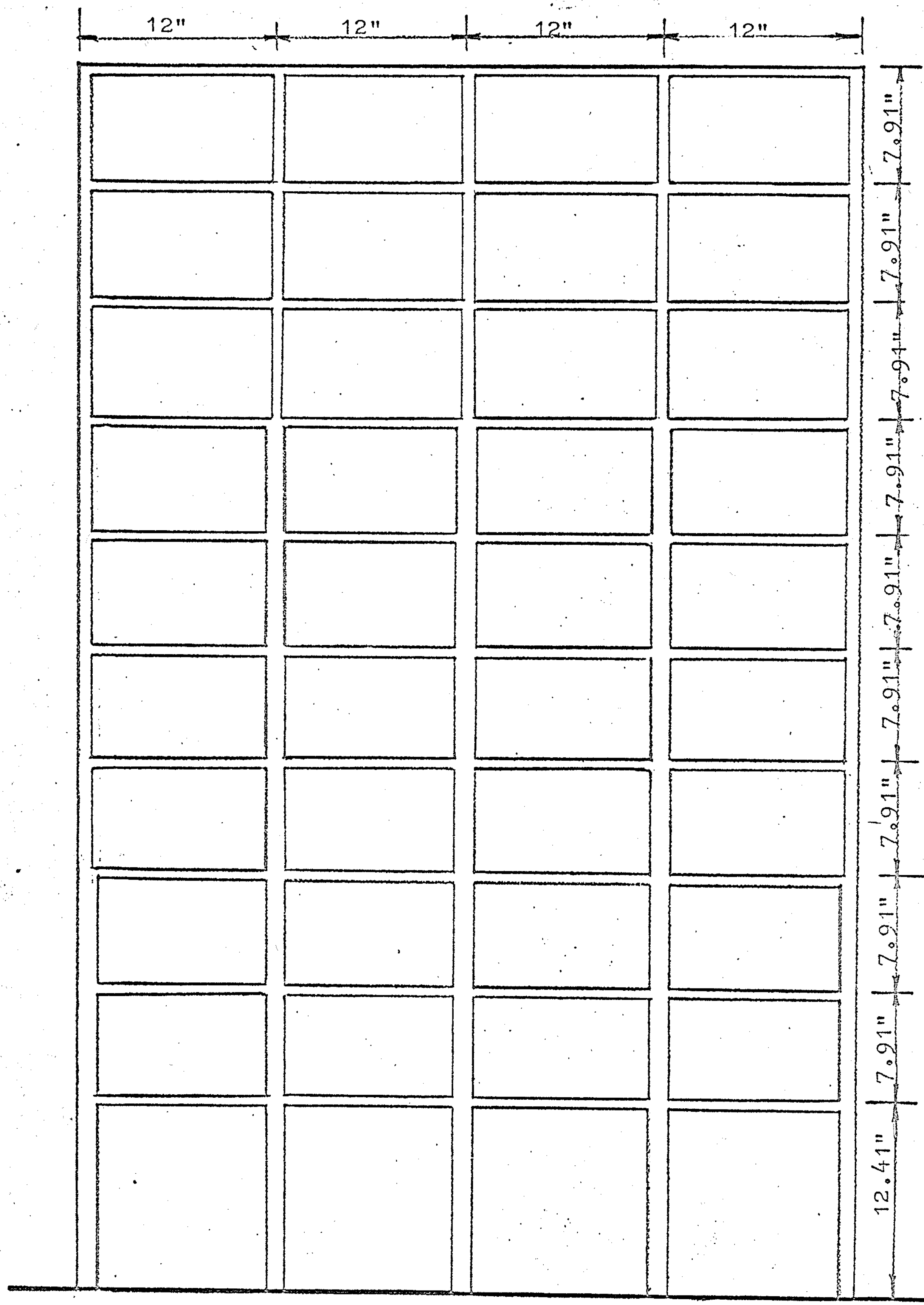
Finished Model

Figure 1.



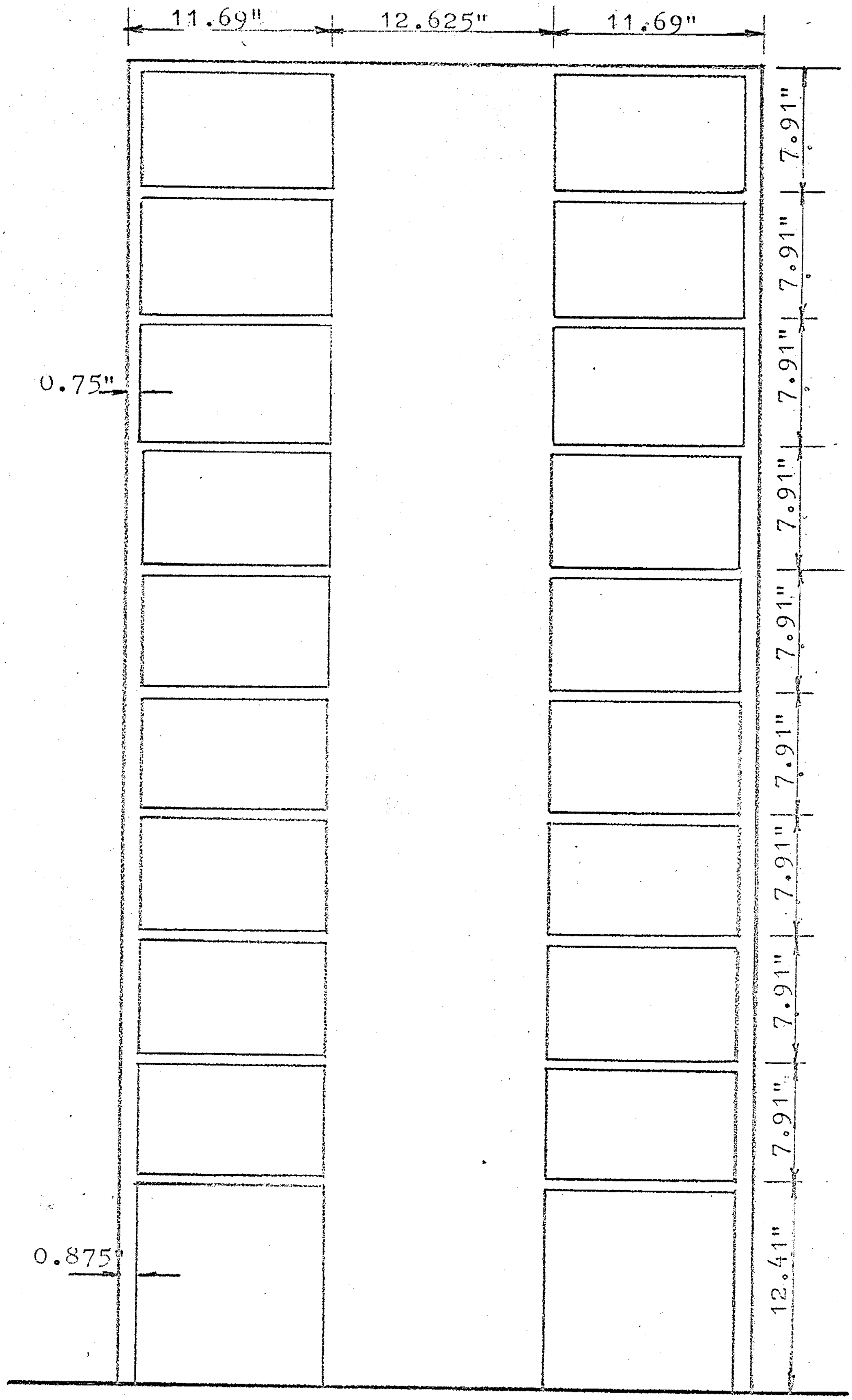
Section Through an Exterior Frame(Model)

Figure 2.



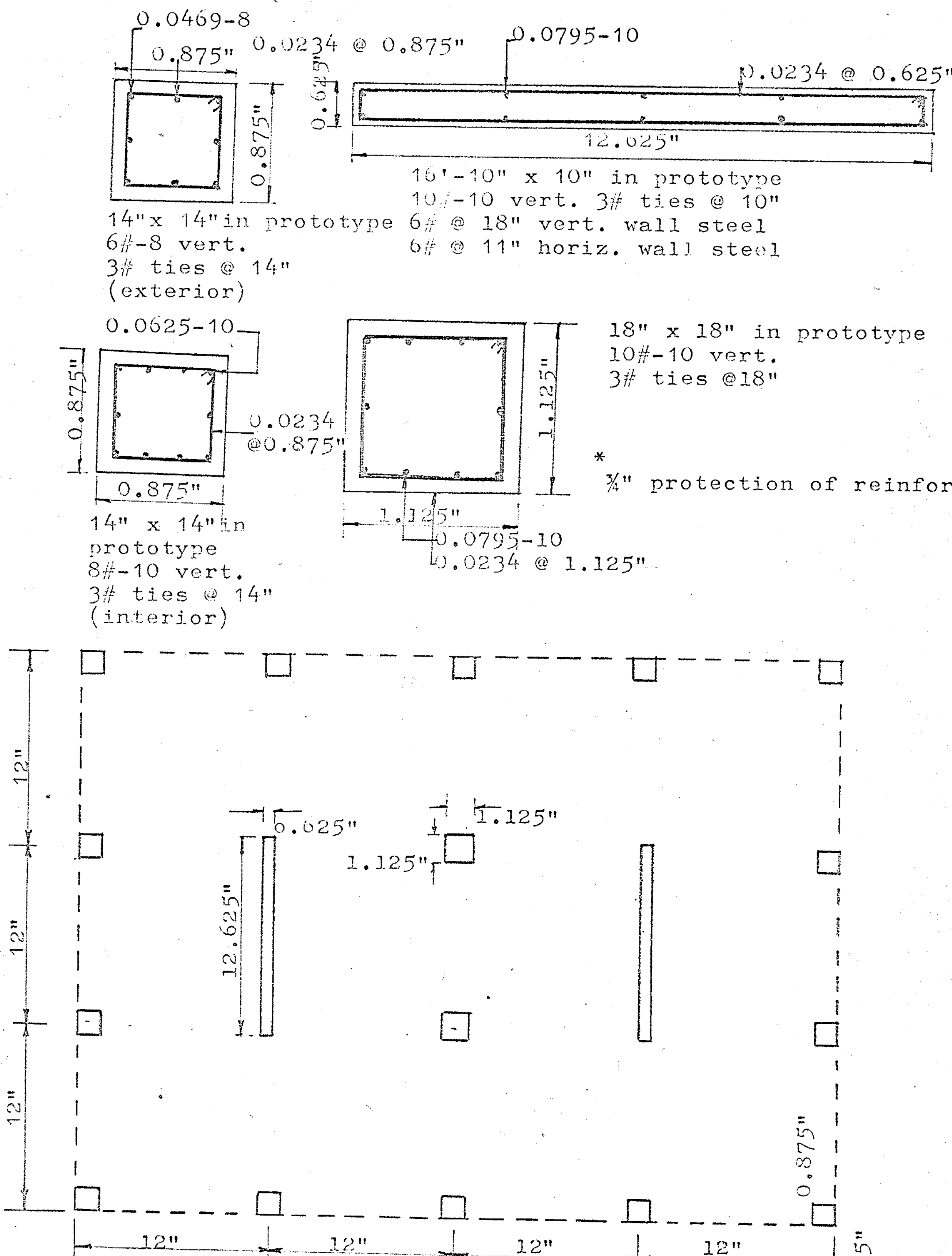
Section Through an Exterior Frame(Model)

Figure 3.

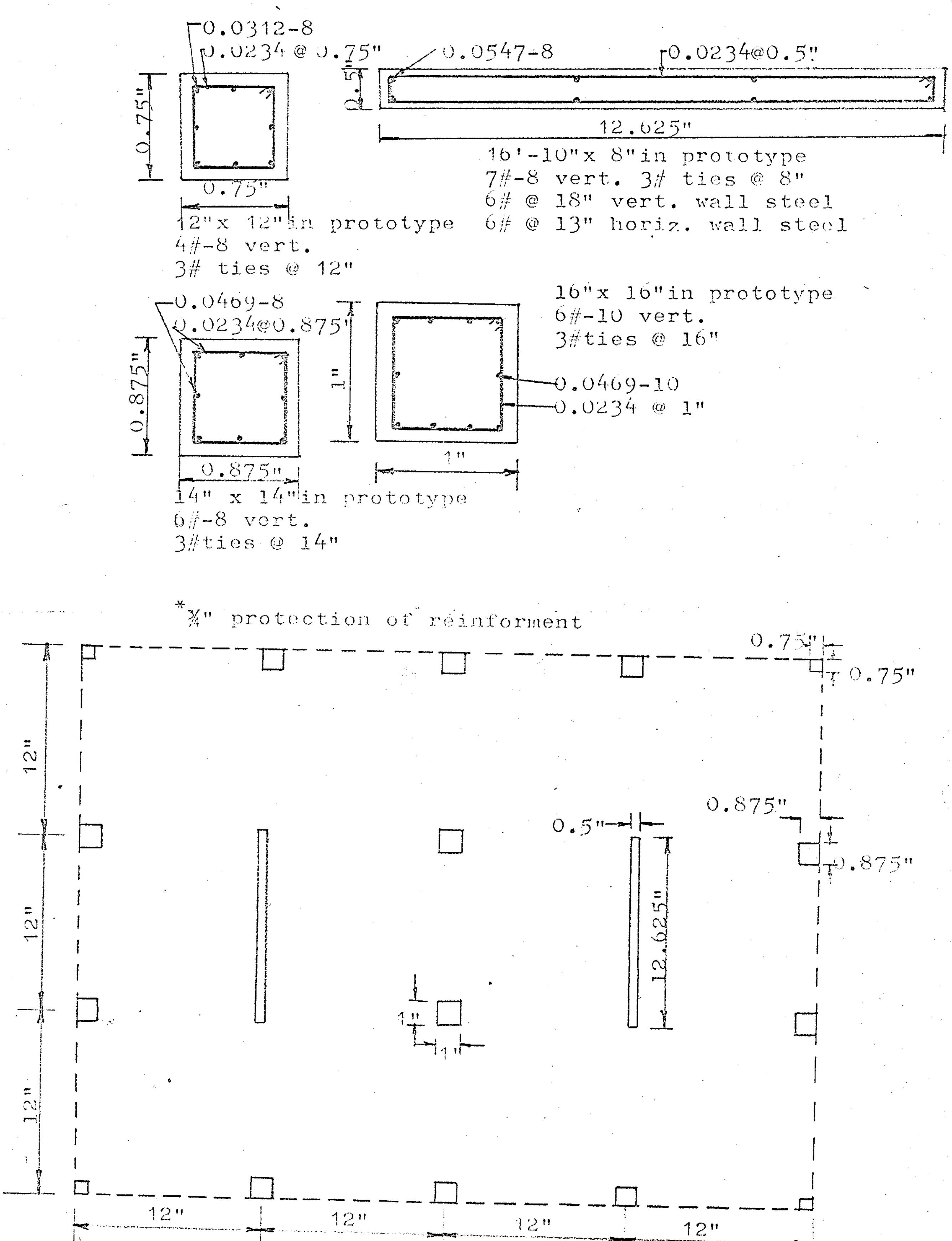


Section Through a Shear Wall (Model)

Figure 4.

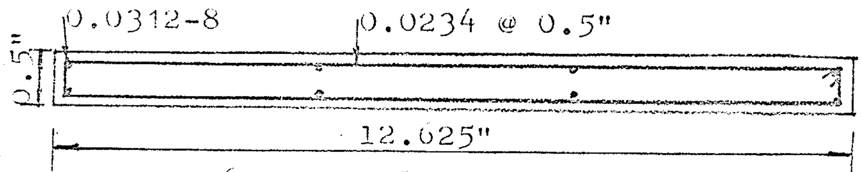


Floor Plan for First, Second and Third Floor(Model)
Figure 5.

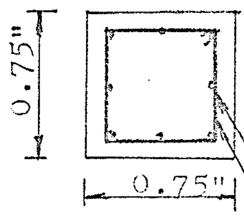


Floor Plan for Fourth, Fifth, Sixth,& Seventh Floor(Model)

Figure 6.

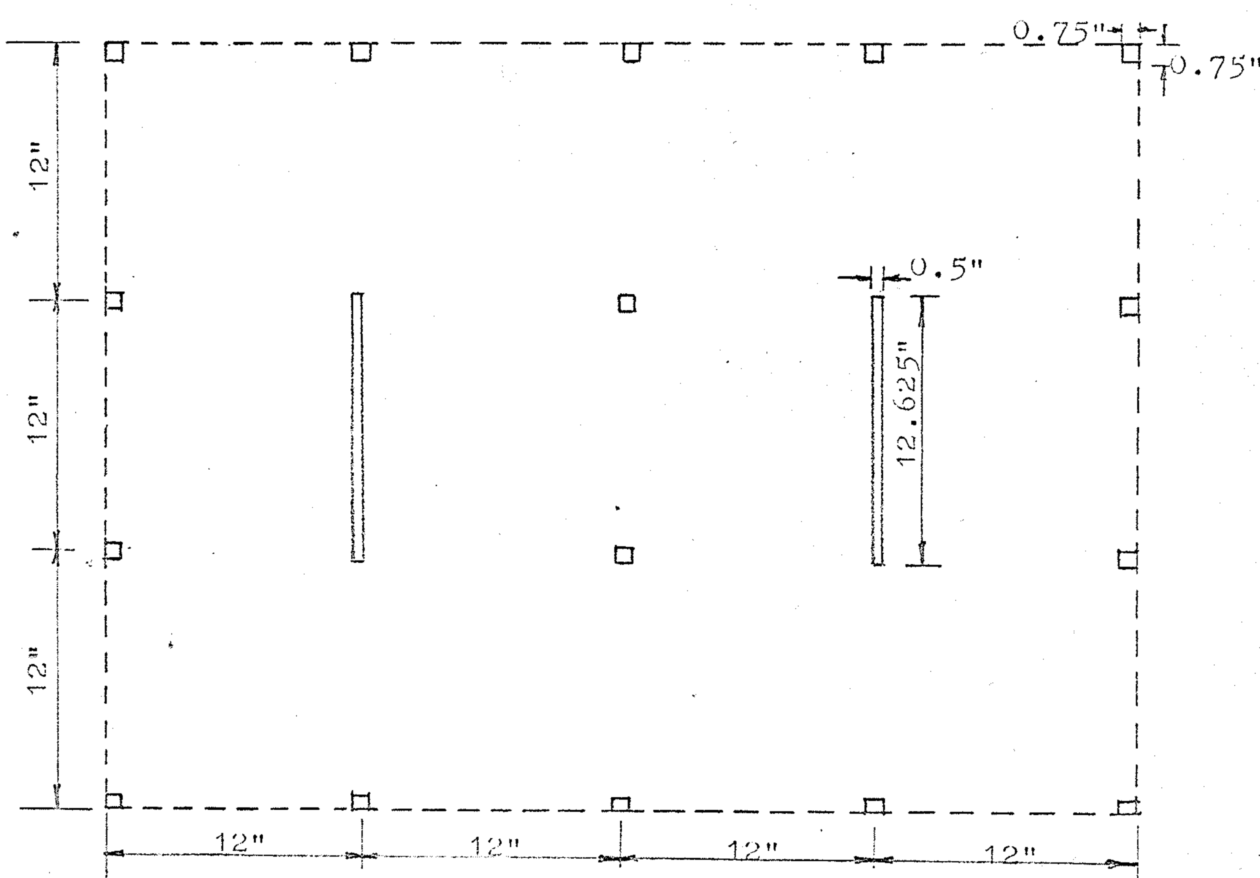


16'-10" x 8"in prototype
4#-8 vert. 3# ties @ 8"
6# @ 18" vert. wall steel
6# @ 13" horiz. wa;; steel



12" x 12" in prototype
4#-8 vert.
3#ties @ 12"

*¾" for protection reinforcement



Floor Plan for Eighth, Ninth,& Tenth Floors(Model)

Figure 7.

CHAPTER II

LABORATORY STUDIES

In this chapter the torsional rigidity of the ten-storey model is analysed by elastic theory. If a fixed end column is a rectangular cross section with L in length, with homogeneous and isotropic properties, its free end is subjected to a torsional load, M_t. The torsional moment curve is a vertical line paralled to the centre line of the column as shown in Fig. (8). The torsional moment at any cross section is same magnitude, and hence the torsional shearing stress is $M_t/\beta ab^2$, "βab^2" is torsional rigidity of cross section. "β" is a function of the ratio a/b given in table I. "a" is the long side of rectangular cross section, "b" is the short side of rectangular cross section. The torsional shearing stresses are nil at the corners and at the centroid of the cross section. Along diagonals this shearing stress increases to a maximum at some point intermediate between zero stress at the centroid and zero stress

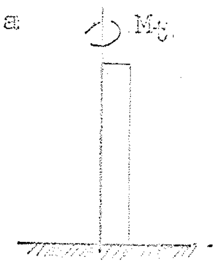


Fig. (8)

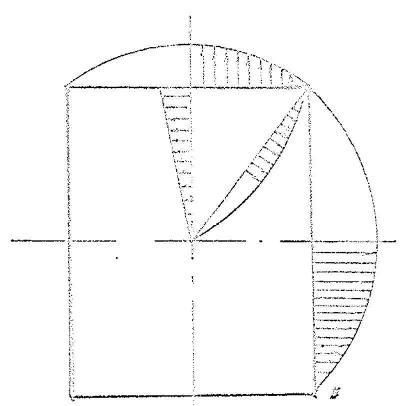


Fig. (9)

at the corner approximately as the ordinates of a parabola. The maximum torsional shearing stresses occur at the center of the long side, and the torsional stress at the center of the short side also attains a maximum which is less than that occurring at the center of the long side. This shearing stress at any point on a transverse cross section varies directly as **approximately the distance** from the centroid of the cross section, and along each side of rectangular cross section the torsional shearing stress varies from zero stress at the corner to the maximum at the center of the long side approximately as **a parabola as shown in Fig.**(9).

TABLE I

COEFFICIENTS FOR TORSION OF RECTANGULAR SECTION

a/b	α	β	γ
1	0.208	0.141	0.675
1.5	0.231	0.196	0.869
2	0.246	0.229	0.930
3	0.267	0.263	0.985
4	0.283	0.281	0.988
5	0.299	0.291	0.999
10	0.312	0.312	1.000
	0.333	0.333	1.000

If this fixed end column with rectangular cross section was made of reinforced concrete, from its previous investigations are:

1. Failure of concrete in torsion is due to diagonal tension, the maximum diagonal tension tends to run along a 45 degree angle, hence the cracks along this 45 degree angle spiral on the surface layer as shown in Fig. (10).

2. The most effective reinforcement is a series of 45 degree spirals; in practice, however, spiral reinforcement; is hardly ever used in rectangular section because of the difficulty of manufacture.

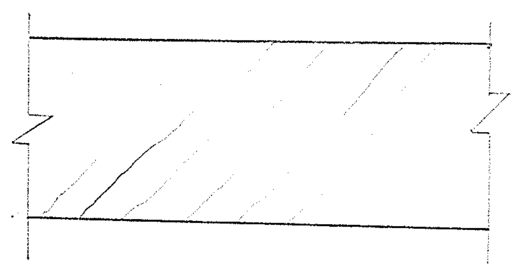


Fig. (10)

3. The combined effect of longitudinal and lateral reinforcement (in the form of stirrups) can be used for resisting torsion. The longitudinal bars resist the horizontal component of the diagonal tension stress and the stirrups take the vertical component of the diagonal tension stress as shown in Fig. (11).

This ten-storey building model with H in height can be considered as a short rectangular column with "A" in the long side and "B" in short side. Its foundation is completely restrained against warping. In other words, the foundation lies in a horizontal plane and is perpendicular to the longitudinal axis as shown in Fig. (1). Comparing this ten-storey model with the fixed end rectangular cross section column as shown in Fig. (8), the only difference is the torsional rigidity.

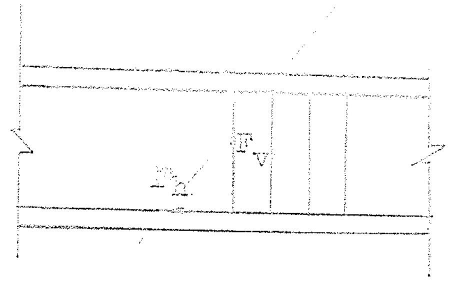


Fig. (11)

The torsional rigidity, K, may be defined as that the torsional moment, M_t, required to develop a unit angle of twist in unit length, $\frac{M_t}{\phi/L}$, where ϕ is the total angle of twist of L in length as shown in Fig. (12). For a rectangular cross section member, the torsional rigidity is dependent upon a torsional stiffness factor C, which is a function of the dimensions of cross section and the modulus of elasticity in shear, G. It may be expressed:

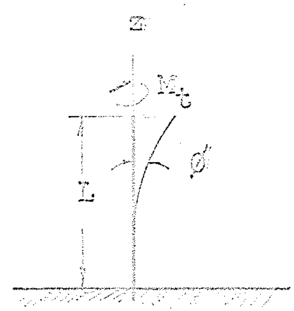


Fig. (12)

$$K = CG_c = G_c \beta ab^3.$$

"β" is the function of the ratio a/b given in Table I.

The twist angle of a rectangular cross section can be expressed:

$$\phi = \frac{m_{tc} \times z}{K}$$

differentiation above Equation:

$$\frac{d\phi}{dz} = \frac{m_{tc}}{K}, \quad \left(\frac{dz}{dz} = 1\right)$$

The torsional rigidity is:

$$K = \frac{m_{tc}}{\frac{d\phi}{dz}}$$

in which, m_{tc} is the torsional moment in concrete

a is the long side of retangular cross section

b is the short side of rectangular cross section

z is the longitudinal axis length

G_c is the modulus of elasticity in shear

The torsional rigidity of columns and shear walls of the ten-storey building model between the first and second floor in plain concrete can be expressed:

The torsional rigidity of columns are:

$$K'_{1-2} = K'_{2-3} = K'_{3-4} = \frac{m_{tc}}{\frac{d\phi}{dz}} = G_c \beta a b^3$$

$$= 0.141\ G_c (14 \times 0.875 \times 0.875^3 + 2 \times 1.125 \times 1.125^3) = 1.61 G_c \ldots\ldots\ldots (1')$$

The torsional rigidity of shear walls are:

$$K''_{1-2} = K''_{2-3} = K''_{3-4} = \frac{m_{tc}}{\frac{d\phi}{dz}} = G_c \beta a b^3$$

$$=0.333\ G_c(2 \times 12.625 \times 0.625^3)$$

$$=2.06\ G_c \quad \ldots\ldots\ldots\ldots\ldots\ldots (2')$$

The total torsional rigidity of columns and shear walls equal equation (1') plus (2'):

$$K_{1-2}=K_{2-3}=K_{3-4}=(K'_{1-2}+K''_{1-2})=(K'_{2-3}+K''_{2-3})$$

$$=(K'_{3-4}+K''_{3-4})=(1.61+2.06)G_c=3.67\ G_c \quad \ldots (1)$$

The torsional rigidity of columns and shear walls of the ten-story building model between 4th to 5th floor can be expressed:

$$K_{4-5}=K_{5-6}=K_{6-7}=\frac{m_{tc}}{\frac{d\phi}{dz}}=G_c\beta ab^3$$

$$=0.141\ G_c(10 \times 0.875 \times 0.875^3\ 2 \times 1 \times 1^3$$

$$+4 \times 0.75^4)+0.333G_c(2 \times 12.625 \times 0.5^3)$$

$$=2.336\ G_c \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The torsional rigidity of columns and shear walls of the ten-storey building model between 8th and 9th floor can be expressed:

$$K_{8-9}=K_{9-10}=K_{10-11}=\frac{m_{tc}}{\frac{d\phi}{dz}}=G_c\beta ab^3$$

$= 0.141 G_c (16 \times 0.75^4) + 0.333\ G_c (2 \times 12.625 \times 0.5^3)$

$= 1.765\ G_c$ . . . . (3)

Comparing with equation (1),(2) and (3),the equation (3) is the smallest torsional rigidity of the ten-storey building model.

The additional torsional rigidity of reinforcement of the columns and shear walls of the ten-storey building model can be expressed as follows:

The torsional moment, m_{tr}, due to longitudinal reinforcing bars with co-ordinates $(\pm\frac{a'}{2}, \pm\frac{a'}{2})$ in the square cross section of columns is given by:

$$m_{tr} = 2\ A_s a' \left(1 - \frac{G_c}{G_s}\right)(S_{xz} + S_{yz}) \quad \ldots\ldots\ldots (4)$$

in which

A_s is the area of one bar of the longitudinal reinforcing

G_s is the modulus of rigidity in reinforcement

S_{xz} and S_{yz} is the component shearing stresses at the center of the longitudinal bar

$$S_{xz} = G_s\ \rho\left(y + \frac{\partial\varphi}{\partial x}\right)_{x=y=a'/2}$$

$$S_{yz} = G_s\ \rho\left(\frac{\partial\varphi}{\partial y} - x\right)_{x=y=a'/2}$$

where

$$\psi = -x\,y + \frac{8\,a^2 \sinh \pi y/2}{\pi^3 \cosh \pi/2} \sin \pi x/a$$

ψ is the torsional function for the rectangular section

differentiating

$$\frac{\partial \psi}{\partial x} = -y + \frac{8\,a \sinh \pi y/a}{\pi^2 \cosh \pi/2} \cos \pi x/a$$

$$\frac{\partial \psi}{\partial x} = -x - \frac{8\,a \sinh \pi y/a}{\pi^2 \cosh \pi/2} \sin \pi x/a$$

substituting into equation (4)

$$\frac{m_{tr}}{\frac{\phi}{L}} = 2\,A_s\,a'\,L\,(G_s - G_c)\left(\frac{8\,a \sinh \pi y/a}{\pi^2 \cosh \pi/2} \cos \pi x/a - \frac{8\,a \sinh \pi y/a}{\pi^2 \cosh \pi/2} \sin \pi x/a - 2x\right) \quad . \; . \; . \; . \; . \; (5)$$

in which the term

$$\left(\frac{8\,a \sinh \pi y/a}{\pi^2 \cosh \pi/2} \cos \pi x/a - \frac{8\,a \sinh \pi y/a}{\pi^2 \cosh \pi/2} \sin \pi x/a - 2x\right)$$

is very small, **and** can be neg**lected.**

Another torsional rigidity, $\frac{m_{ts}}{\phi/L}$, due to horizontal stirrups is given by:

$$\frac{m_{ts}}{\phi/L} = A_v \; E_s \; ab^2 \lambda/p \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (6)$$

in which

A_v is the area of stirrups steel

λ is the function of ratio of cross section

p is the pitch of stirrups

a/b	:	1.0	1.2	1.4	1.6
λ	:	1.67	1.62	1.60	1.59

substituting λ = 1.67 (square) into equation (6)

$$\frac{m_{tr}}{\phi/L} = 1.67 \; A_v \; E_s \; ab^2 \lambda/p \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (7)$$

the right side of equation (7) is very small.

Comparing with

$$\frac{m_{tr}}{\frac{d\phi}{dz}} = Gc\beta ab^3$$

the above equation can be negligible.

The reason of negligible torsional rigidity of reinforcing and stirrups, equation (5) and (7), can be verified from previous investigation of various specimens.

1. Marshall, W. T. and Tembe, N. R. said, "The tests on the reinforced specimens showed that the torsional rigidity of a reinforced rectangular specimen is the same as that for a plain specimen of same dimensions........," in the experiments on plain and reinforced concrete in torsion, Structural Engineering, November, 1941, Volume 19/20, pp190.

2. Cowan, H. J. said, "The longitudinal reinforcement increased the torsional stiffness at no load by 18 percent, which is within 1 percent of the value calculated by the elastic theory, the continuous binding did not result in further increase of the torsional stiffness at no load, whereas increases ranging from 2 percent to 4 per cent, depending on the pitch of the binding, are obtained from the elastic theory, this suggests that the stiffening effect of the binding is not brought into action until the concrete cracks...........," in the Tests of the Torsional Strength and Deformation of Rectangular Reinforced Concrete Beams, Concrete and Constructional Engineering, February, 1951, pp. 54.

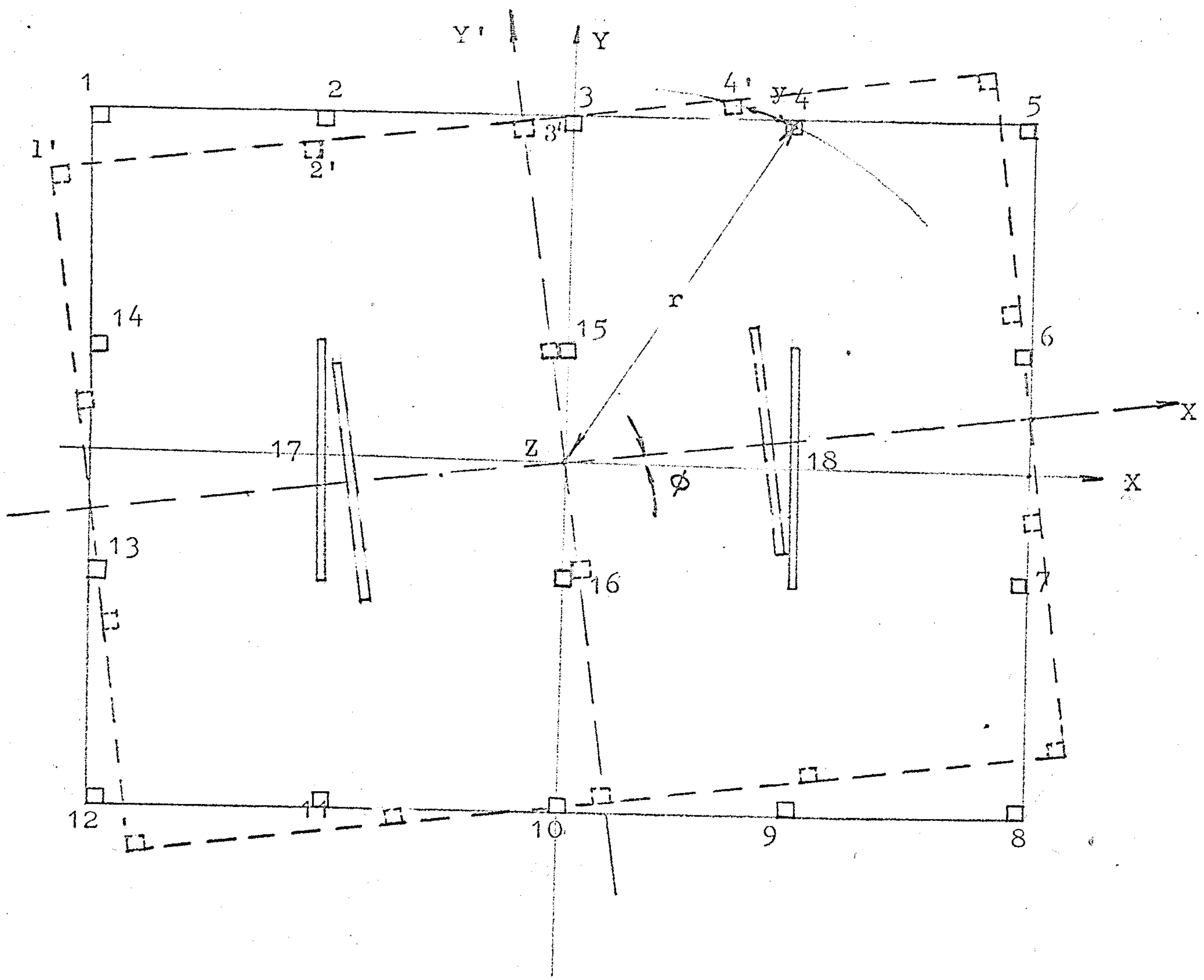
3. Young, C. R. , Sagar, W. L. and Hughes,

C. A., said,"The rigidity of a specimen of any given size is but little increased by the addition of reinforcement, whether it be longitudinal or spiral, such deformation as can occur before the breakdown of the specimen is apparently not affected by reinforcement of the type and quantity used in the specimens", in the Torsional Strength of Rectangular Section of Concrete, Plain and Reinforced, Engineering Research Bulletin No.3, University of Toronto, 1922, pp.168.

The contribution to the total torsional moment developed by the bending moment, m_{tb}, of each of the columns and shear walls of xth floor, a pure torsional moment is applied at the top of (x+1)th floor causing the cross section between xth floor and (x+1)th floor to rotate through an angle ϕ into a position 1', 2', 3', 4',........, as shown in Fig.(13). In other words, a point, for example, 4, originally parallel to the longitudinal axis Z of the cross section after distorting into a helix 4→4' through a distance (arc) y as shown in Fig. (13).

From Fig. (13).

$$dy = r d\phi$$



The typical cross section(before & after twisting a very small angle ϕ)

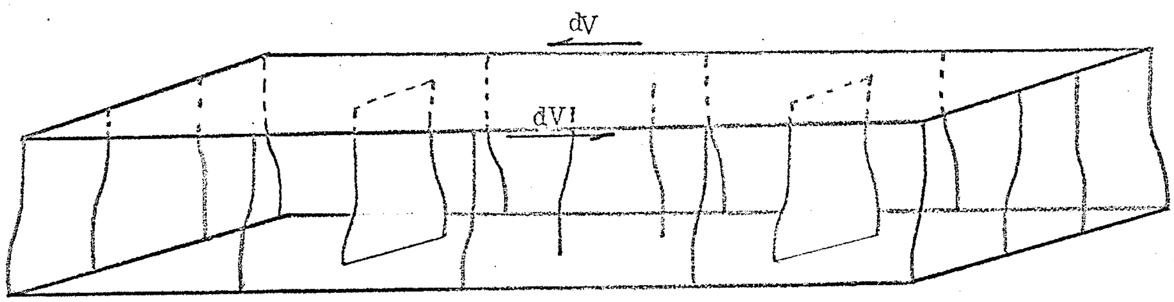


Figure 13

in which

r is a radius from the center of the cross section of ten-storey building model to the center of columns or shear walls

ϕ is the angle of twist

when the angle of twist, ϕ, is very small

$$r_3 = r_{10} = 18''$$

$$r_2 = r_{11} = r_9 = r_4 = \sqrt{12^2 + 18^2} = \sqrt{468} = 21.63''$$

$$r_1 = r_{12} = r_8 = r_5 = \sqrt{18^2 + 24^2} = \sqrt{900} = 30''$$

$$r_6 = r_{14} = r_7 = r_{13} = \sqrt{6^2 + 24^2} = \sqrt{612} = 24.74''$$

$$r_{17} = r_{18} = 16''$$

The torsional moment developed by the bending is $dm_{tb} = rdV$, m_{tb} is bending moment, which obtained from Timoshenko, the Elements of Strength of Materials, 1949, pp. 165.

$$V = -E_c I \frac{d^3y}{dx^3}$$

or

$$V = -E_c I\, r \frac{d^3\phi}{dx^3}, \qquad dy = rd\phi$$

in which

E_c is modulus of elasticity in concrete

I is moment of inertia of ecah column or shear wall, for square column, $I = \frac{a^4}{12}$, for shear wall, $I = \frac{ab^3}{12}$.

$$dm_{tb} = rdV$$

Integrating

$$\sum \Delta m_{tb} = \sum \gamma \Delta V$$

$$m_{tb} = rV$$

$$= r\left(-E_c I r \frac{d^3\phi}{dx^3}\right)$$

$$= -E_c r^2 \frac{a^4 d^3\phi}{12 dx^3}, \quad \left(\text{if } I = \frac{a^4}{12}\right)$$

$$dx = dz$$

$$m_{tb} = -E_c r^2 \frac{a^4 d^3\phi}{12 dz^3}$$

Or

$$\frac{d^3\phi}{dz^3} = -12 \frac{m_{tb}}{a^4 r^2 E_c} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

Integrating and solving for ϕ

$$\frac{d^2\phi}{dz^2} = \frac{-12 m_{tb}}{a^4 r^2 E_c} z + C_1$$

Integrating

$$\frac{d\phi}{dz} = \frac{-12 m_{tb}}{2a^4 r^2 E_c} Z^2 + C_1 z + C_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (9)$$

Integrating again

$$\phi = -\frac{12\ m_{tb}}{3\cdot 2a^4r^2E_c}z^3 + \tfrac{1}{2}C_1z^2 + C_2z + C_3$$

$$= -\frac{2\ m_{tb}}{a^4r^2E_c}z^3 + \tfrac{1}{2}C_1z^2 + C_2z + C_3 \quad . . . (10)$$

solve constant C_1, C_2 and C_3 by boundary condition

(1) $\phi = 0$, $Z = 0$

substituting into equation (10)

$$C_3 = 0$$

(2)

$$\frac{d\phi}{dz} = 0, \quad Z = 0$$

substituting into equation (9)

$$C_2 = 0$$

(3) $\frac{d\phi}{dz} = 0, \quad Z = L$

substituting into equation (9) again

$$-\frac{12\ m_{tb}\ L^2}{2\cdot a^4r^2E_c} + C_1L + C_2 = 0$$

$$C_2 = 0$$

$$C_1 = \frac{6\ m_{tb}\ L}{a^4r^2E_c}$$

substituting C_1, C_2 and C_3 into equation (10)

$$\phi = \frac{-2\ m_{tb}}{a^4r^2E_c}z^3 + \tfrac{1}{2}\,\frac{6\ m_{tb}L}{a^4r^2E_c}z^2$$

for $z = L$

$$\phi = \frac{m_{tb}}{a^4 r^2 E_c}(3L^3 - 2L^3)$$

$$= \frac{m_{tb}}{a^4 r^2 E_c} L^3$$

Therefore the torsional rigidity is:

$$\frac{m_{tb}}{\phi/L} = \sum \frac{a^4 r^2 E_c}{L^2} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

The torsional rigidity of columns and shear walls of the ten-storey building model between 2nd and 3rd floor can be expressed:

substituting a, r, L and E_c value into equation (11)

$$K_{2-3} = K_{3-4} = \frac{m_{tb}}{\phi/L} = \sum \frac{a^4 r^2 E_c}{L^2}$$

$$= \left[\frac{4 \times 0.875^4 \times \frac{30^2}{2}}{7.504} + \frac{4 \times 0.875^4 \times \frac{21.63^2}{2}}{7.504} \right.$$

$$+ \frac{4 \times 0.875^4 \times \frac{24.74^2}{2}}{7.504} + \frac{2 \times 0.875^4 \times \frac{18^2}{2}}{7.504}$$

$$\left. + \frac{2 \times 1.125^4 \times \frac{6^2}{2}}{7.504} + \frac{2 \times 12.625 \times 0.625^3 \times \frac{12^2}{2}}{7.504} \right]$$

$$= \frac{6018.1 \times 3 \times 10^6}{56.31}$$

$$= 321 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots (12)$$

The torsional rigidity of columns and shear walls of the ten-storey building model between the 4th and 5th floor can be expressed:

$$K_{5-6}=K_{6-7}=K_{7-8}=\sum\frac{a^4 r^2 E_c}{L^2}$$

$$=\left(\frac{\frac{4\times 0.75^4\times 30^2}{2}}{7.504}+\frac{\frac{4\times 0.875^4\times 21.63^2}{2}}{7.504}+\frac{\frac{4\times 0.875^4\times 24.74^2}{2}}{7.504}+\frac{\frac{2\times 0.875^4\times 18^2}{2}}{7.504}+\frac{\frac{2\times 1^4\times 6^2}{2}}{7.504}+\frac{2\times 12.625\times 0.5^3\times 12^2}{7.504^2}\right)$$

$$=\frac{4573.8\times 3\times 10^3}{56.31}$$

$$=243.8\times 10^6\text{ lb-in}^2. \qquad (13)$$

The torsional rigidity of columns and shear walls of the ten-storey building model between the 8th and 9th floor can be expressed:

$$K_{9-10}=K_{8-9}=K_{10-11}=\sum\frac{a^4 r^2 E_c}{L^2}$$

$$=\left(\frac{\frac{4\times 0.75^4\times 30^2}{2}}{7.504}+\frac{\frac{4\times 0.75^4\times 21.63^2}{2}}{7.504}+\frac{\frac{4\times 0.75^4\times 24.74^2}{2}}{7.504}+\frac{\frac{2\times 0.75^4\times 18^2}{2}}{7.504}+\frac{\frac{2\times 12.625\times 0.5^3\times 12^2}{2}}{7.504}\right)$$

$$= \frac{3203.5 \times 3 \times 10^6}{56.31}$$

$$= 171 \times 10^6 \text{lb-in}^2 \quad \ldots\ldots\ldots\ldots \quad (14)$$

The torsional rigidity of columns and shear walls of ten-storoy building model between the 1st and 2nd floor can be expressed:

$$K_{1-2} = \frac{\sum a^4 r^2 E_c}{L^2}$$

$$= \frac{E_c}{12.004} \left(4 \times 0.875^4 \times 30^2 + 4 \times 0.875^4 \times 21.63^2 \right.$$

$$+ 4 \times 0.875^4 \times 24.74^2 + 2 \times 0.875^4 \times 18^2$$

$$\left. + 2 \times 1.125^4 \times 6^2 + 2 \times 12.625 \times 0.625^3 \times 12^2 \right)$$

$$= \frac{6018.1 \times 3 \times 10^6}{144.096}$$

$$= 125 \times 10^6 \text{lb-in}^2 \quad \ldots\ldots\ldots\ldots \quad (15)$$

Comparing equations (12), (13), (14) and (15), equation (15) is the smallest of torsional rigidity.

The total torsional rigidity of columns and shear walls between the 1st and 2nd floor is equal to equation (1) Plus equation (15), which is the smallest of torsional rigidity. Therefore the torsional crack will be occurred along a 45 degree spiral on the surface layer of the 1st and 2nd floor, because this section is the weakest torsional rigidity.

equation (1) + (15) is:

$$3.67 \times G_c + 125 \times G_c$$

$$=3.67 \times 1.5 \times 10^6 + 125 \times 10^6$$

$$=130.5 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots (16)$$

The above torsional rigidity can be checked by another method using the 1620 Computer.

The torsional moment developed by columns and shear walls when the cross section are permitted to warp a angle ϕ. The torsional moment can be expressed :

$$m_{tc}=G_cC\frac{d\phi}{dz}$$

$$=G\beta ab^3\frac{d\phi}{dz} \quad \ldots\ldots\ldots\ldots (17)$$

A further moment is developed by bending of each column and shear wall. The torsional moment can be expressed :

$$m_{tb}=-E_c r^2\frac{a^4 d^3\phi}{12\, dz^3} \quad \ldots\ldots\ldots\ldots (18)$$

The total resisting torsional moment at any cross section xth and (x+1)th floor is equal to equation (17)+(18).

$$M_t=m_{tc}+m_{tb}$$

$$=G_cC\frac{d\phi}{dz}-\frac{E_c r^2 a^4 d^3\phi}{12 dz^3} \quad \ldots\ldots\ldots\ldots (19)$$

solving the above differential equation for ϕ

let

$$M_t = -n_3 \quad , \quad G_c C = n_2 \quad , \quad \frac{E_c r^2 a^4}{12} = n_1$$

equation (19) becomes

$$n_1 \frac{d^3\phi}{dz^3} - n_2 \frac{d\phi}{dz} = n_3 \qquad \ldots\ldots\ldots\ldots (20)$$

the complimentary solution, ϕ_c, is :

$$n_1 z^3 - n_2 z = 0 \qquad \ldots\ldots\ldots\ldots (21)$$

solving equation (21)

$$z = 0$$

Or

$$z = -\frac{n_2}{n_1}$$

therefore

$$\phi_c = C_1 + C_2 e^{\sqrt{\frac{n_2}{n_1}} z} + C_3 e^{-\sqrt{\frac{n_2}{n_1}} z}$$

the particular solution, ϕ_p, is :

$$\phi_p = E_1 Z + E_2 z^2 + E_3 z^3 \qquad \ldots\ldots\ldots (22)$$

differentiating equation (22)

$$\frac{d\phi}{dz} = E_1 + 2E_2 z + 3E_3 z^2$$

$$\frac{d^2\phi}{dz^2} = 2E_2 + 6E_3 z$$

$$\frac{d^3\phi}{dz^3} = 6E_3$$

substituting $\frac{d\phi}{dz}$ and $\frac{d^3\phi}{dz^3}$ into equation (20)

$$n_1(6E_3)-n_2(E_1+2E_2z+3E_3z^2) = n_3$$

Comparing constant E_1, E_2 and E_3 with right side of equation (20), obtained

$$E_2 = 0 \ , \ E_3 = 0 \ , \ E_1 = -\frac{n_3}{n_2}$$

therefore

$$\phi_p = -\frac{n_3}{n_2}z$$

adding up the complimentary solution and the the particular solution

$$\phi = \phi_c + \phi_p = C_1 + C_2 e^{\sqrt{\frac{n_2}{n_1}}z} + C_3 e^{-\sqrt{\frac{n_2}{n_1}}z} - \frac{n_3}{n_2}z \quad \ldots\ldots\ldots\ldots(23)$$

solve the constant C_1, C_2 and C_3 with the boundary condition

(1) $\phi = 0 \ , \quad z = 0$

substitution into equation (23)

$$C_1 + C_2 + C_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

(2)

$$\frac{d\phi}{dz} = 0 \ , \quad z = 0$$

substituting into the derivative equation of (23)

$$\frac{d\phi}{dz} = \sqrt{\frac{n_2}{n_1}}\, C_2\, e^{\sqrt{\frac{n_2}{n_1}}z} - \sqrt{\frac{n_2}{n_1}}\, C_3\, e^{-\sqrt{\frac{n_2}{n_1}}z} - \frac{n_3}{n_2}$$

$$0 = \frac{n_2}{n_1}C_2 - C_3\sqrt{\frac{n_2}{n_1}} - \frac{n_3}{n_1} \quad \ldots\ldots\ldots(25)$$

(3)

$$\frac{d\phi}{dz} = 0 \ , \quad z = L$$

substituting into the derivative of equation (23)

$$\sqrt{\frac{n_2}{n_1}}\, C_2 e^{\sqrt{\frac{n_2}{n_1}}L} - \sqrt{\frac{n_2}{n_1}}\, C_3 e^{-\sqrt{\frac{n_2}{n_1}}L} - \frac{n_3}{n_2} = 0 \quad \ldots\ldots\ldots(26)$$

solve equations (24),(25) and (26) for C_1, C_2 and C_3

$$C_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 0 & 1 & 1 \\ \frac{n_3}{n_2} & \sqrt{\frac{n_2}{n_1}} & -\sqrt{\frac{n_2}{n_1}} \\ \frac{n_3}{n_2} & \sqrt{\frac{n_2}{n_1}}\, e^{\sqrt{\frac{n_2}{n_1}}L} & -\sqrt{\frac{n_2}{n_1}}\, e^{-\sqrt{\frac{n_2}{n_1}}L} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 0 & \sqrt{\frac{n_2}{n_1}} & -\sqrt{\frac{n_2}{n_1}} \\ 0 & \sqrt{\frac{n_2}{n_1}}\, e^{\sqrt{\frac{n_2}{n_1}}L} & -\sqrt{\frac{n_2}{n_1}}\, e^{-\sqrt{\frac{n_2}{n_1}}L} \end{vmatrix}}$$

$$= \frac{n_3\sqrt{n_1}\ (\cosh\sqrt{\frac{n_2}{n_1}}L - 1)}{n_2\sqrt{n_2}\sinh\sqrt{\frac{n_2}{n_1}}L}$$

$$C_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 1 & 0 & 1 \\ 0 & \frac{n_3}{n_2} & -\sqrt{\frac{n_2}{n_1}} \\ 0 & \frac{n_3}{n_2} & \sqrt{\frac{n_2}{n_1}}\, e^{-\sqrt{\frac{n_2}{n_1}}L} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 0 & \sqrt{\frac{n_2}{n_1}} & -\sqrt{\frac{n_2}{n_1}} \\ 0 & \sqrt{\frac{n_2}{n_1}}\, e^{\sqrt{\frac{n_2}{n_1}}L} & \sqrt{\frac{n_2}{n_1}}\, e^{-\sqrt{\frac{n_2}{n_1}}L} \end{vmatrix}}$$

$$= \frac{n_3 \sqrt{n_1}\ \left(1 - e^{-\sqrt{\frac{n_2}{n_1}}L}\right)}{2 n_2 \sqrt{n_2} \sinh \sqrt{\frac{n_2}{n_1}}\ L}$$

$$C_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 0 & \sqrt{\frac{n_2}{n_1}} & \frac{n_3}{n_2} \\ 0 & \sqrt{\frac{n_2}{n_1}}\, e^{\sqrt{\frac{n_2}{n_1}}L} & \frac{n_3}{n_2} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 0 & \sqrt{\frac{n_2}{n_1}} & -\sqrt{\frac{n_2}{n_1}} \\ 0 & \sqrt{\frac{n_2}{n_1}}\, e^{\sqrt{\frac{n_2}{n_1}}L} & \sqrt{\frac{n_2}{n_1}}\, e^{-\sqrt{\frac{n_2}{n_1}}L} \end{vmatrix}}$$

$$= \frac{n_3 \sqrt{n_1}}{2n_2\sqrt{n_2}} \frac{(1 - e^{\sqrt{\frac{n_2}{n_1}}L})}{\sinh\sqrt{\frac{n_2}{n_1}}L}$$

substituting C_1, C_2 and C_3 into equation (23)

$$\phi\Big|_{z=L} = \frac{n_3\sqrt{n_1}\ (\cosh\sqrt{\frac{n_2}{n_1}}L - 1)}{n_2\sqrt{n_2}\ \sin\sqrt{\frac{n_2}{n_1}}L} + \frac{n_3\sqrt{n_1}(1 - e^{-\sqrt{\frac{n_2}{n_1}}L})e^{\sqrt{\frac{n_2}{n_1}}L}}{2\ n_2\sqrt{n_2}\ \sinh\sqrt{\frac{n_2}{n_1}}L}$$

$$+ \frac{n_3\sqrt{n_1}\ (1 - e^{\sqrt{\frac{n_2}{n_1}}L})e^{-\sqrt{\frac{n_2}{n_1}}L}}{2\ n_2\sqrt{n_2}\ \sinh\sqrt{\frac{n_2}{n_1}}L} - \frac{n_3}{n_2}L$$

simplifying

$$\phi = \frac{2\ n_2\sqrt{n_1}(\cosh\sqrt{\frac{n_2}{n_1}}L - 1)}{n_2\sqrt{n_2}\ \sinh\sqrt{\frac{n_2}{n_1}}\ L} - \frac{n_3}{n_2}L \quad \ldots\ldots (27)$$

where

$$\frac{n_3}{n_2} = -\frac{M_t}{G_c C} \quad \ldots\ldots\ldots\ldots (28)$$

assumed $u = 0$, $G_c = E_c/2(1+u)$

u is Poisson's ratio

substituting $u = 0$ into equation (28)

$$\frac{n_3}{n_2} = -\frac{M_t}{\frac{1}{2}E_c C} = \frac{-2M_t}{E_c\beta a^4} \quad \ldots\ldots\ldots\ldots (29)$$

$$\sqrt{\frac{n_2}{n_1}} = \sqrt{\frac{\frac{1}{2}E_c\beta a^4}{\frac{a^4 r^2 E_c}{12}}} = \sqrt{\frac{6\beta}{r}} \quad \ldots\ldots\ldots\ldots (30)$$

$$\sqrt{\frac{n_1}{n_2}} = \sqrt{\frac{a^4 r^2 E_c}{\frac{12}{2E_c \beta a^4}}} = \frac{r}{\sqrt{6\beta}} \quad \ldots\ldots\ldots\ldots (31)$$

substituting equations (29),(30) and (30) into equation (27)

$$\phi = -\frac{4 M_t r (\cosh\sqrt{\frac{6\beta}{r}}L - 1)}{E_c a^4 \sqrt{6\beta} \sinh\sqrt{\frac{6}{r}}L} + \frac{2 M_t L \sqrt{6\beta} \sinh\sqrt{\frac{6\beta}{r}}L}{E_c a^4 \sqrt{6\beta} \sinh\sqrt{\frac{6\beta}{r}}L}$$

or

$$\frac{M_t}{\phi} = \frac{E_c a^4 \sqrt{6\beta} \sinh\sqrt{\frac{6}{r}}L}{2(2r+2r \cosh\sqrt{\frac{6\beta}{r}}L + L\sqrt{6\beta} \sinh\sqrt{\frac{6}{r}}L} \quad \ldots (32)$$

the above equation (32) is the torsional rigidity of each of the columns and shear walls in xth floor to (x+1)th floor, hence the total rigidity is equal to the sum of equation (32) of each columns and shear walls.

using the 1620 computer and substituting r, a, E_c, L and β into equation (32), the 2nd floor to 3rd floor torsional rigidity is:

$\beta = 0.141$

$$\sum \frac{M_t}{\phi/L} = \sum \frac{0.1297 \times E_c \times a^4 \times 7.91 \times \sinh \frac{0.92}{r} 7.91}{2(2 \times r + 2r \cosh \frac{0.92}{r} 7.91 + 0.92 \times 7.91 \sinh \frac{0.92}{r} 7.91)}$$

simplifying

$$= 4 \times 28.22 \times 10^6 + 4 \times 14.75 \times 10^6 + 2 \times 10.26 \times 10^6$$
$$+ 2 \times 3.48 \times 10^6$$
$$= 276.36 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots (33)$$

$\beta = 0.333$

$$\sum \frac{M_t}{\phi/L} = \sum \frac{0.4714 \times E_c \times ab^3 \times 7.91 \times \sinh \frac{1.414}{r} 7.91}{2(2r + 2r\cosh \frac{1.414}{r} 7.91 + 1.414 \times 7.91 \sinh \frac{1.414}{r} 7.91)}$$

simplifying

$$= 51.02 \times 10^6 \text{ lb-in}^2 \ldots\ldots\ldots\ldots (34)$$

adding up equation (33) and (34)

$$\frac{M_t}{\phi/L} = 276.36 \times 10^6 + 51.02 \times 10^6$$

$$= 327.38 \times 10^6 \text{ lb-in}^2 \ldots\ldots\ldots\ldots (35)$$

The 4th floor to 5th floor, 5th to 6th floor, 6th to 7th floor, and 7th to 8th floor, torsional rigidity is:

$\beta = 0.141$

$$\sum \frac{M_t}{\phi/L} = \sum \frac{0.1297 \times E_c \times a^4 \times 7.91 \sinh \frac{0.92}{r} 7.91}{2(2r + 2r\cosh \frac{0.92}{r} 7.91 + 0.92 \times 7.91 \sinh \frac{0.92}{r} 7.91)}$$

simplifying

$$= 4 \times 15.23 \times 10^6 + 4 \times 14.75 \times 10^6 + 4 \times 19.25$$

$$+ 2 \times 10.26 \times 10^6 + 2 \times 2.17 \times 10^6$$

$$= 22.78 \times 10^6 \text{ lb-in}^2 \ldots\ldots\ldots\ldots (36)$$

$\beta = 0.333$

$$\sum \frac{M_t}{\phi/L} = \sum \frac{0.714 \times E_c \times ab^3 \times 7.91 \sinh \frac{1.414}{r} 7.91}{2(2r + 2r\cosh \frac{1.414}{r} 7.91 + 1.414 \times 7.91 \sinh \frac{1.414}{r} 7.91)}$$

simplifying

$$= 26.12 \times 10^6 \text{ lb-in}^2 \ldots\ldots\ldots\ldots (37)$$

adding up equations (36) and (37)

$$\frac{M_t}{\phi/L} = (221.78 + 26.12) \times 10^6$$

$$= 247.9 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots \quad (38)$$

The 8th to 9th floor, 9th to 10th floor, and 10th to 11th floor torsional rigidity is:

$\beta = 0.141$

$$\sum \frac{M_t}{\phi/L} = \sum \frac{0.129 \times E_c \times a^4 \times 7.91 \sinh \frac{0.92}{r} 7.91}{2(2r + 2r\cosh \frac{0.92}{r} 7.91 + 0.92 \times 7.91 \sinh \frac{0.92}{r} 7.91)}$$

simplifying

$$= 4 \times 1523 \times 10^6 + 4 \times 7.96 \times 10^6 + 4 \times 10.39 \times 10^6$$

$$+ 2 \times 1.4 \times 10^6 + 2 \times 0.68 \times 10^6$$

$$= 146.76 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots \quad (39)$$

$\beta = 0.333$

$$\sum \frac{M_t}{\phi/L} = \sum \frac{0.4714 \times E_c \times ab^3 \times 7.91 \sinh \frac{1.414}{r} 7.91}{2(2r + 2r \cosh \frac{1.414}{r} 7.91 + 1.414 \times 7.91 \sinh \frac{1.414}{r} 7.91)}$$

$$= 26.12 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots \quad (40)$$

adding up equations (39) and (40)

$$\frac{M_t}{\phi/L} = 172.88 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots \quad (41)$$

The 1st and 2nd floor torsional rigidity is:

$\beta = 0.141$

$$\sum \frac{M_t}{\phi/L} = \sum \frac{0.1297 \times E_c \times a^4 \times 12.004 \sinh \frac{0.92}{r} 12.004}{2(2r + 2r\cosh \frac{0.92}{r} 12.004 + 0.92 \times 12.004 \sinh \frac{0.92}{r} 12.004)}$$

simplifying

$$= 4 \times 11.12 \times 10^6 + 4 \times 5.85 \times 10^6 + 4 \times 7.6 \times 10^6$$

$$+ 2 \times 1.6 \times 10^6 + 2 \times 4.1 \times 10^6$$

$$= 109.72 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots (42)$$

$\beta = 0.333$

$$\frac{M_t}{\phi/L} = \frac{0.4714 \times E_c \times ab^3 \times 12.004 \sinh \frac{1.414}{r} 12.004}{2(2r + 2r\cosh \frac{1.414}{r} 12.004 + 1.414 \times 12.004 \sinh \frac{0.92}{r} 12.004)}$$

simplifying

$$= 22.18 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots (43)$$

adding up equations (42) and (43)

$$\frac{M_t}{\phi/L} = 109.72 \times 10^6 + 22.18 \times 10^6$$

$$= 131.9 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots\ldots (44)$$

Comparing equation (44) and (15), the answer is same, 131.9×10^6 is correct and more accurate torsional rigidity.

CHAPTER III

TORSIONAL ANGLE AND STRESSES ANALYSIS

This chapter consists of two parts, in part I, the angle of twist is derived from torsional rigidity. In part II the torsional stresses are analyzed by elastic theory.

In equation (44) or (15), the angle of twist and total torsional moment relationship is a straight line as shown in Fig.(14). M_t is plotted against ϕ, ϕ/L is a unit angle of twist, θ,.

from equation (44) or (15)

$$\frac{M_t}{\phi/L} = 131.9 \times 10^6 \text{ lb-in}^2 \quad \ldots\ldots\ldots (44)$$

from Fig.(13) and Fig. (5), the M_t and V can be expressed:

$$M_t = 18 \times V \quad , \text{(moment arm = 18")}$$

$$L = 12.004"$$

substituting into equation (44)

$$\phi = \frac{M_t \times L}{131.9 \times 10^6} = \frac{V \times 18 \times 12.004}{131.9 \times 10^6}$$

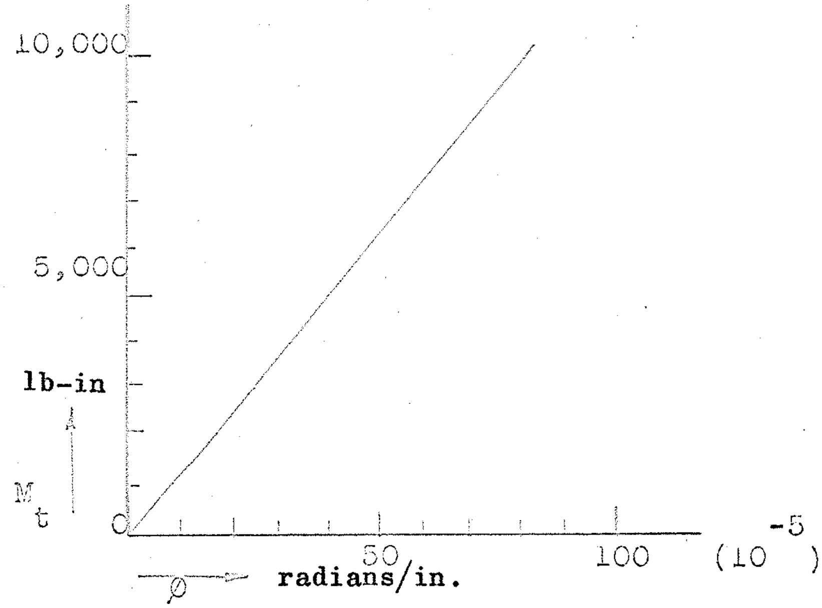
$$= 0.164 \text{ V} \times 10^{-5}$$

The angle of twist, ϕ, torsional moment, M_t, and load V is given in table II

TABLE II

V lb	M_t lb-in	$\emptyset$ (10^{-5}) radians
0	0	0
50	900	8.2
100	1800	16.4
150	2700	24.6
200	3600	32.8
250	4500	41.0
300	5400	49.2
350	6300	57.4
400	7200	65.6
450	8100	73.8
500	9000	82.0

from table II , Fig. (14) is ploted



M_t and $\emptyset$ diagram

Fig. (14)

The above digram is the angle of twist for the 1st floor to 2nd floor correspound with torsional moment, M_t.

A typical torque and twist curve is shown in

Fig. (15). It is seem that it consists of three distinct stages:

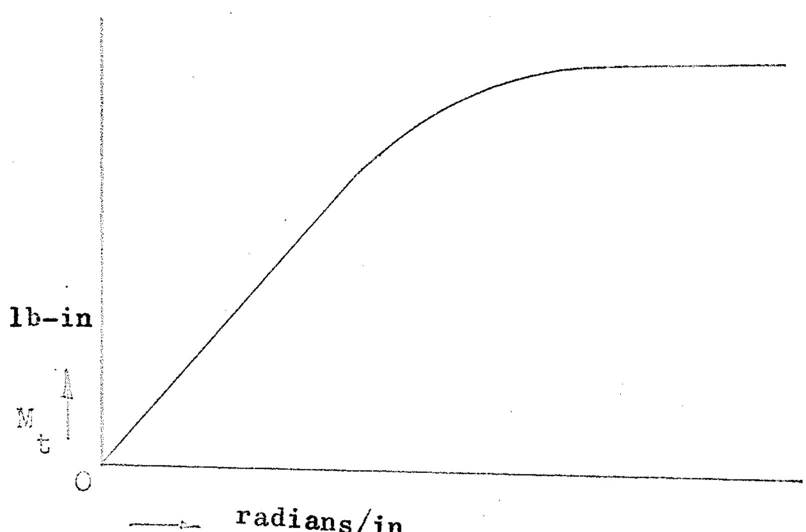
1. A straight line from zero up to about half of the failing load; it also shows that during the early part the concrete alone is resisting the torsional moment, therefore, the maximum torsional rigidity of a reinforced concrete section is the same as that of a plain concrete of the same dimensions.

2. A curved portion joins the two straight lines.

3. A straight line which started just failing load and continued as far as it can possible.

It also shows that the torsional rigidity is very small taken by reinforcement.

The straight line of Fig. (14) is same as to stage 1. of the typical torque-twist curve. Its meaning is in the elastic state.



Typical M_t and $\varnothing$ diagram

Fig. (15)

In part II the torsional stresses are analyzed by elastic theory. From Fig. (16) it is realized that point 5 moves to 5' due to a force V_5 which is developed by bending, twisting and shear.

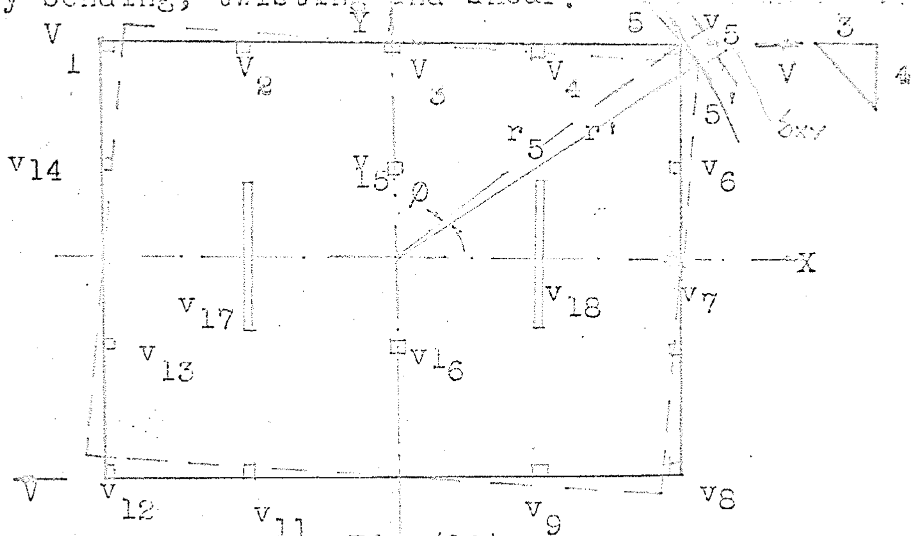


Fig.(16)

The torsional stress is developed by bending moment, when the torque, V x 18, is applied at the cross section of the ten-storey building model, and permitted to warp at an angle $\emptyset$.

$$\delta_{xy} = r' \emptyset$$

when $\emptyset$ is very small

$$r_5 = r'$$

from table II $\emptyset$ 820×10^{-6} when load V=500

$$\delta_{xy} = r\emptyset$$

$$= 30 \times 820 \times 10^{-6} \text{ in,}$$

$$\delta x = (30 \times 820 \times 10^{-6})\ 3/5 \text{ in}$$

$$\delta y = (30 \times 820 \times 10^{-6})\ 4/5 \text{ in}$$

on column "5" the top is free end and the bottom is fixed end as shown in Fig. (17). Its deflection is:

$$\delta = \frac{V L^3}{3EI}$$

or $$V = \frac{3EI\delta}{L^3}$$

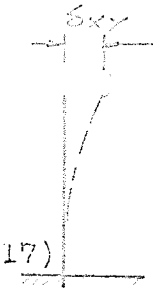


Fig.(17)

therefore

$$V_{5x} = \frac{3EI\delta_x}{L^3}$$

$$= \frac{3 \times 3 \times 10^6 \times \frac{0.875^4}{12} \times 30 \times 820 \times 10^{-6} \times 3/5}{12.004^3}$$

$=3.75$ **lb**

$$V_{5y} = \frac{3EI\delta_y}{L^3}$$

$$= \frac{3 \times 3 \times 10^6 \times \frac{0.875^4}{12} \times 30 \times 820 \times 10^{-6} \times 3/5}{12.004^3}$$

$=5.0$ **lb**

Using the same method, V_6, V_4, V_3, V_{15} and V_{17} can be found:

$$\delta_6 = r_6\,\phi$$

$$= 24.74 \times 820 \times 10^{-6}\,''$$

$$V_{6x} = \frac{3 \times 3 \times 10^6 \times \frac{0.875^4}{12} \times 24.74 \times 820 \times 10^{-6} \times 3/5}{12.004^3}$$

$=3.09$ **lb**

$$V_{6y} = \frac{3 \times 3 \times 10^6 \times \frac{0.875^4}{12} \times 24.74 \times 820 \times 10^{-6} \times 4/5}{12.004^3}$$

$=4.12$ **lb**

$$\delta_4 = r_4\phi$$

$$= 21.63 \times 820 \times 10^{-6}$$

$$V_{4x} = \frac{3 \times 3 \times 10^{6} \times \frac{0.875^{4}}{12} \times 21.63 \times 820 \times 10^{-6} \times 3/5}{12.004^{3}}$$

$$= 2.712 \text{ lb}$$

$$V_{4y} = \frac{3 \times 3 \times 10^{6} \times \frac{0.875^{4}}{12} \times 21.63 \times 820 \times 10^{-6} \times 4/5}{12.004^{3}}$$

$$= 3.612 \text{ lb}$$

$$\delta_3 = r_3 \phi$$

$$= 18 \times 820 \times 10^{-6} \text{"}$$

$$V_{3x} = \frac{3 \times 3 \times 10^{6} \times \frac{0.875^{4}}{12} \times 18 \times 820 \times 10^{-6} \times 3/5}{12.004^{3}}$$

$$= 2.25 \text{ lb}$$

$$V_{3y} = \frac{3 \times 3 \times 10^{6} \times \frac{0.875^{4}}{12} \times 18 \times 820 \times 10^{-6} \times 4/5}{12.004^{3}}$$

$$= 3.00 \text{ lb}$$

$$\delta_{15} = r_{15} \phi$$

$$= 6 \times 820 \times 10^{-6} \text{"}$$

$$V_{15x} = \frac{3 \times 3\ 10^{6} \times \frac{1.125^{4}}{12} \times 6 \times 820 \times 10^{-6} \times 3/5}{12.004^{3}}$$

$$= 2.04 \text{ lb}$$

$$V_{15y} = \frac{3 \times 3 \times 10^{6} \frac{1.125^{4}}{12} \times 6 \times 820 \times 10^{-6} \times 4/5}{12.004^{3}}$$

$$= 2.72 \text{ lb}$$

$$\delta_{17} = r_{17} \phi$$

$$= 12 \times 820 \times 10^{-6} \text{"}$$

$$V_{17x} = \frac{3 \times 3 \times 10^{6} \times 12.625 \times 0.625^{3} \times 12 \times 820 \times 10^{-6} \times 3/5}{12.004^{3}}$$

$$= 7.92 \text{ lb}$$

$$V_{17y} = \frac{3 \times 3 \times 10^6 \times 12.625 \times 0.625^3 \times 12 \times 820 \times 10^{-6} \times 4/5}{12.004^3}$$

$= 10.56$ **lb**

From Fig.(16), it is realized that V_1, V_{12}, and V_8 are equal to V_5; V_2, V_{11}, and V_9 are equal to V_4; V_3 is equal to V_{10}; V_{14}, V_{13} and V_{17} are equal to V_6; V_{15} is equal to V_{16}; V_{17} is equal to V_{18}.

The stresses are developed by bending moment, for example, column "5" of 1st to 2nd floor of short side. At 10" from the top of column "5"

$V_{5x} = 3.75$ **lb**

$M_{5x} = 3.75 \times 10$ **lb-in**

$$\sigma_{5x} = \frac{M_{5x} \cdot C}{I}$$

$$= \frac{37.5 \times 0.875/2}{0.875^4/12}$$

$= 335$ psi

At 8" from the top of column "5"

$$\sigma_{5x} = \frac{3.75 \times 8 \times 0.437 \times 12}{0.875^4}$$

$= 268$ psi

At 6" from the top of column "5"

$$\sigma_{5x} = \frac{3.75 \times 6 \times \frac{0.875}{2}}{0.875^4/12}$$

$= 201$ psi

At 4" from the top of the column "5"

$$\sigma_{5x} = \frac{3.75 \times 4 \times \frac{1}{2} 0.875}{0.875^4/12}$$

$=201$ psi

At 4" from the top of the column "5"

$$\sigma_{5x} = \frac{3.75 \times 4 \times \frac{1}{2} 0.875}{\frac{0.875^4}{12}}$$

$= 134$ psi

At 2" from the top of the column "5"

$$\sigma_{5x} = \frac{3.75 \times 2 \times \frac{1}{2} 0.875}{\frac{0.875^4}{12}}$$

$=67$ psi

The stresses developed by bending moment at column "5" of the long side are as follows:

At 10" from the top of the column "5"

$$V_{5y} = 5.00$$

$$M_{5y} = 5 \times 10 = 50 \text{ lb-in}$$

$$\sigma_{5y} = \frac{50 \times \frac{1}{2} 0.875}{\frac{0.875^4}{12}}$$

$=446$ psi

At 8" from the top of the column "5"

$$\sigma_{5y} = \frac{5 \times 8 \times \frac{1}{2} 0.875}{\frac{0.875^4}{12}}$$

$=357$ psi

At 6" from the top of the column "5"

$$\sigma_{5y} = \frac{5 \times 6 \times \frac{1}{2} 0.875}{\frac{0.875^4}{12}}$$

$=260$ psi

At 4" from the top of the column "5"

$$\sigma_{5y} = \frac{5 \times 4 \times \frac{1}{2} 0.875}{0.875^4/12}$$

$$= 178.4 \text{ psi}$$

At 2" from the top of the column "5"

$$\sigma_{5y} = \frac{5 \times 2 \times \frac{1}{2} 0.875}{0.875^4/12}$$

$$= 89.2 \text{ psi}$$

The torsional stresses developed by torque are:

The unit angle of twist, θ.

$$\theta = \frac{\phi}{L} \frac{820 \times 10^{-6}}{12.004}$$

$$= 68.3 \times 10^{-6}$$

The maximum shearing stresses occur on the middle of the longer side of the column "5"

$$\tau = \gamma a \, G_c \theta$$

in which

γ is the constant, depending on the proportion of the rectangular cross section given in Table I

G_c is the shear modulus of the concrete

a is the dimension of the rectangle

therefore

$$\tau = 0.765 \times 0.875 \times 1.5 \times 10^6 \times 68.3 \times 10^{-6}$$

$$= 60.6 \text{ psi}$$

Another shearing stresses developed by torque can be expressed:

$$v = \frac{3V}{2A}$$

in which

v is the shearing stresses

A is the cross section area of column "5"

when $V_y = 5$ lb , $V_x = 3.75$ lb

$$v_x = \frac{3 \times 3.75}{2 \times 0.875 \times 0.875} = 7.35 \text{ psi}$$

$$v_y = \frac{3 \times 5}{2 \times 0.875 \times 0.875} = 9.8 \text{ psi}$$

From the above analysis the bending, torsional and shearing stresses are developed by M_t load. Using the Mohr's Circle, the 45 degree angle stresses on the surface of the column "5" can be obtained;

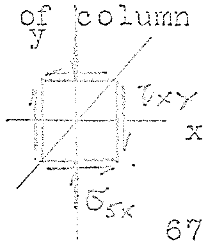
At 10" from the top of column "5" of short side

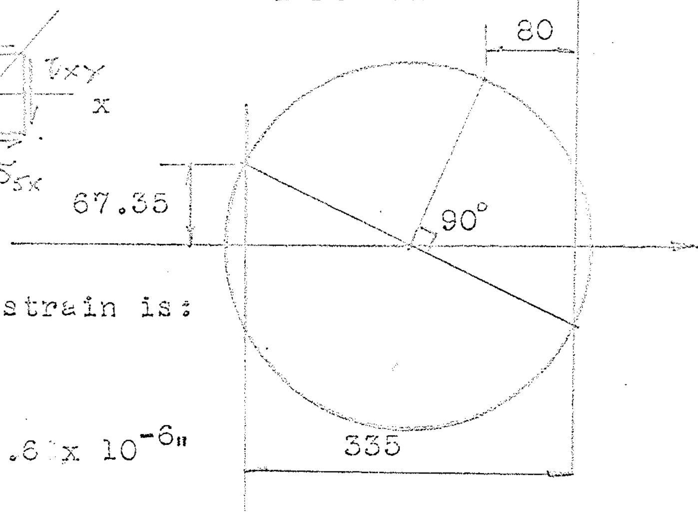
$\tau = 60.6$ psi

$v_x = 7.35$ psi

$\tau_{xy} = 67.35$ psi

$\sigma_{5x} = -335$ psi





from Mohr's Circle, the strain is:

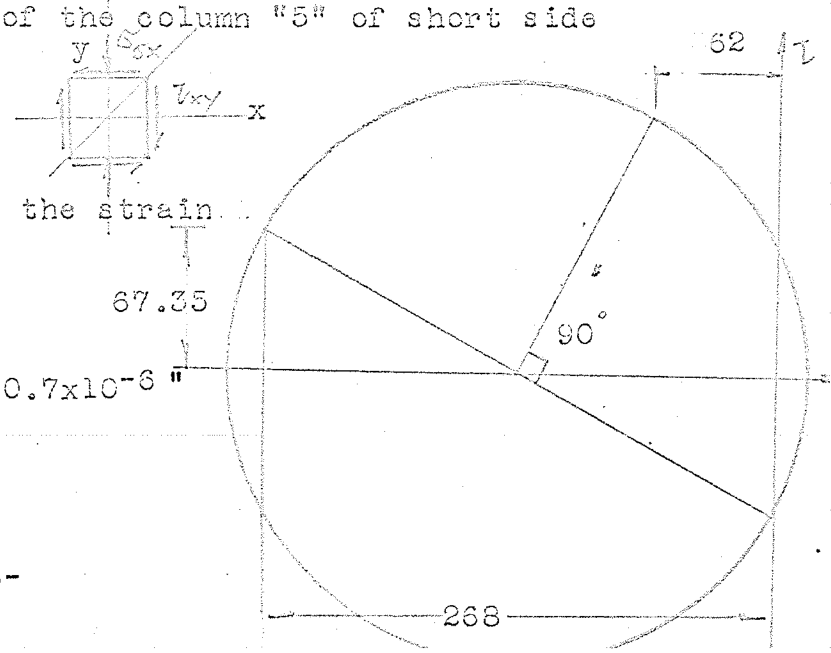
$\sigma_{10} = -80$ psi

$$\epsilon_{10} = \frac{\sigma_{10}}{E} = \frac{-80}{3 \times 10^6} = -26.6 \times 10^{-6} \text{"}$$

At 8" from the top of the column "5" of short side

$\tau_{xy} = 67.35$ psi

$\sigma_{5x} = -268$ psi



from Mohr's Circle, the strain is:

$$\epsilon_8 = \frac{\sigma_8}{E}$$

$$= \frac{-62}{3 \times 10^{-6}} = -20.7 \times 10^{-6} \text{"}$$

At 6" from the top of the column "5" of short side

$\tau_{xy} = 67.35$ psi

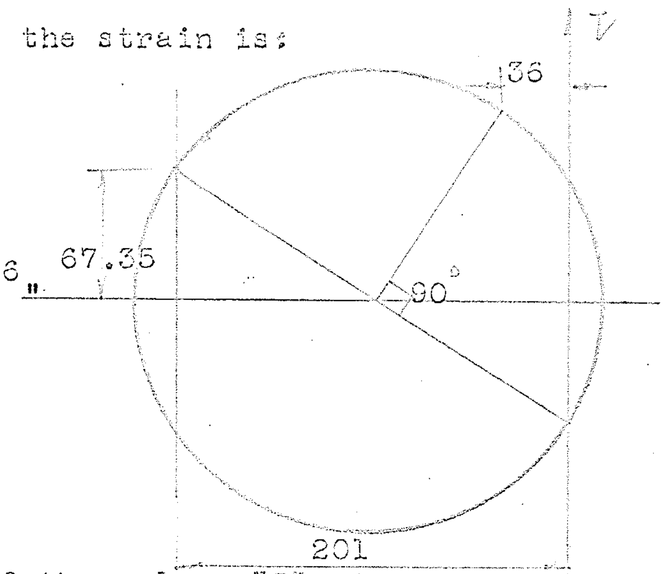
$\sigma_{5x} = -201$ psi

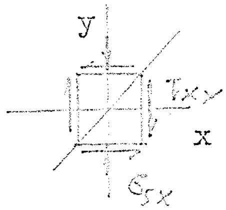
from Mohr's Circle, the strain is:

$$\epsilon_6 = \frac{\sigma_6}{3\times10^6}$$

$$= \frac{-36}{3\times10^6}$$

$$= -12\times10^{-6}\ "$$





At 4" from the top of the column "5" of short side

$\tau_{xy} = 67.35$ psi

$\sigma_{5x} = -134$ psi

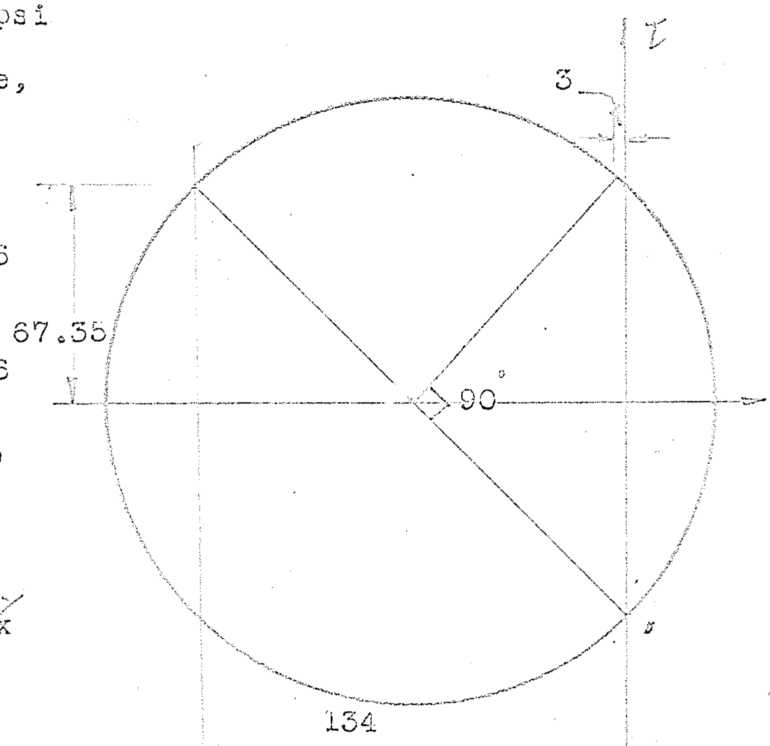
from Mohr's Circle,

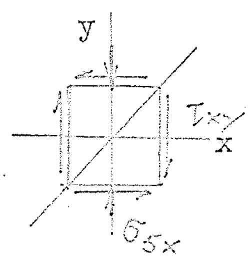
the strain is :

$$\epsilon_4 = \frac{\sigma_4}{3\times10^6}$$

$$= \frac{-3}{3\times10^6}$$

$$= -10^{-6}\ "$$





At 2" from the top of the column "5" of short side

$\tau_{xy} = 67.35$ psi

$\sigma_{5x} = -67$ psi

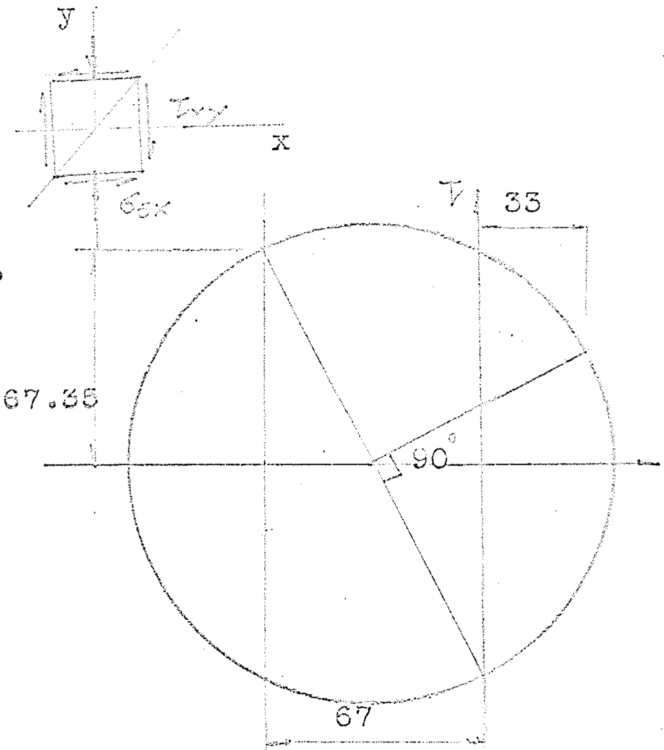
from Mohr's Circle,

the strain is:

$$\epsilon_2 = \frac{\sigma_2}{3 \times 10^6}$$

$$= \frac{33}{3 \times 10^6}$$

$$= 11 \times 10^{-6}\ "$$



At 10" from the top of the column "5" of long side

$\tau = 60.6$ psi

$v_x = 9.8$ psi

$\tau_{xy} = 70.4$ psi

$\sigma_{5y} = 446$ psi

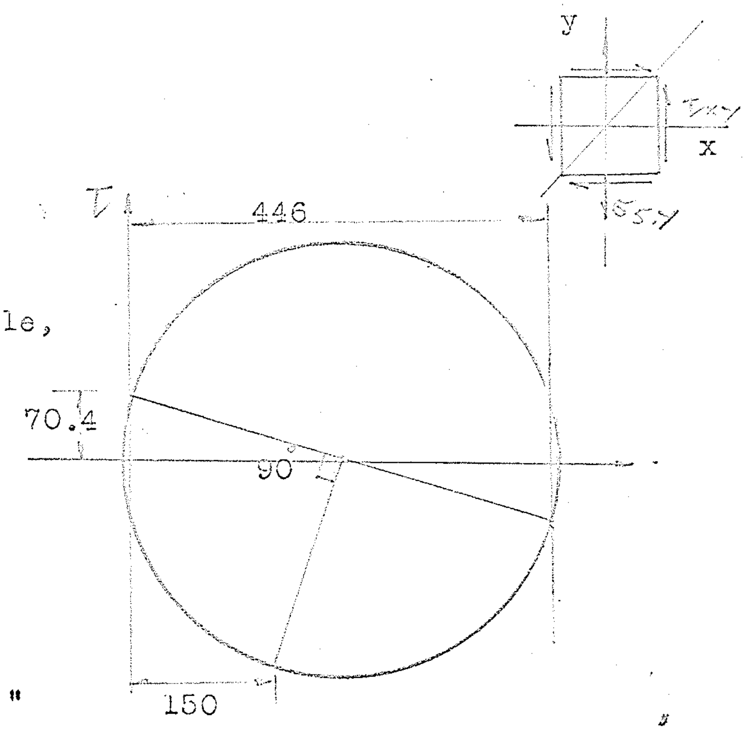
from Mohr's Circle,

the strain is:

$$\epsilon_{10} = \frac{\sigma_{10}}{3 \times 10^6}$$

$$= \frac{150}{3 \times 10^6}$$

$$= 50 \times 10^{-6}\ "$$



At 8" from the top of the column "5" of long side

$\tau_{xy} = 70.4$ psi

$\sigma_{5x} = 357$ psi

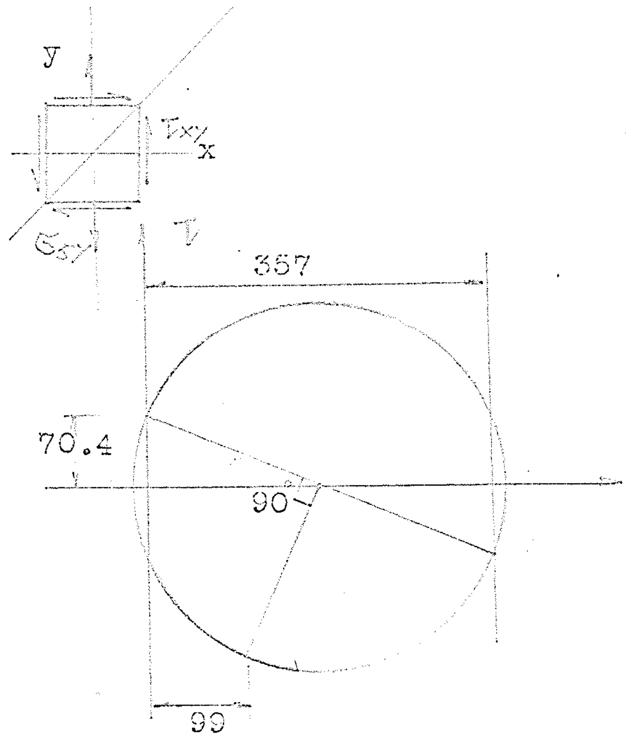
from Mohr's Circle,

the strain is:

$$\epsilon_8 = \frac{\sigma_8}{3 \times 10^6}$$

$$= \frac{99}{3 \times 10^6}$$

$$= 33 \times 10^{-6} \text{ "}$$



At 6" from the top of the column "5" of long side

$\tau_{xy} = 70.4$ psi

$\sigma_{5y} = 268$ psi

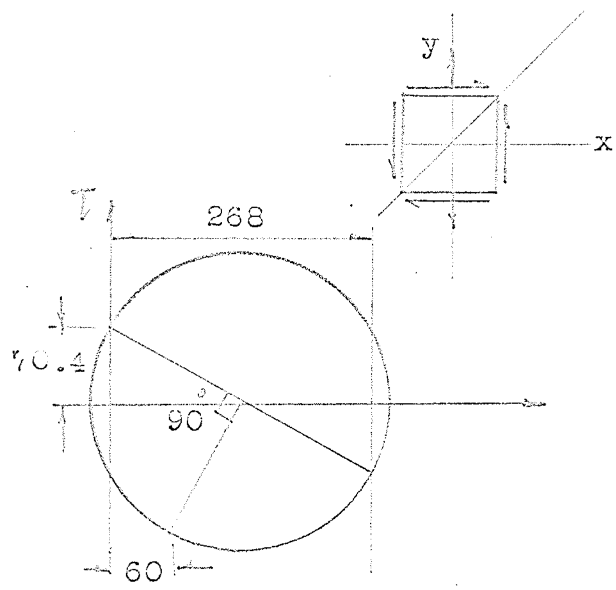
from Mohr's Circle,

the strain is:

$$\epsilon_6 = \frac{\sigma_6}{3 \times 10^6}$$

$$= \frac{60}{3 \times 10^6}$$

$$= 20 \times 10^{-6} \text{ "}$$



At 4" from the top of the column "5" of long side

$\tau_{xy} = 70.4$ psi

$\sigma_{5y} = 178.4$ psi

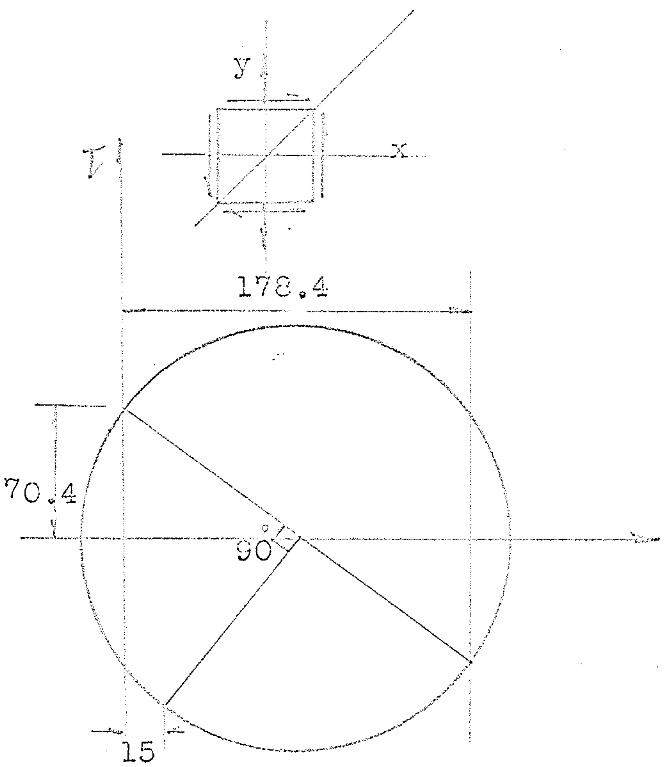
from Mohr's Circle,

the strain is:

$$\epsilon_4 = \frac{\sigma_4}{3 \times 10^6}$$

$$= \frac{15}{3 \times 10^6}$$

$$= 5 \times 10^{-6} \text{"}$$



At 2" from the top of the column "5" of long side

$\tau_{xy} = 70.4$ psi

$\sigma_{5y} = 89.2$ psi

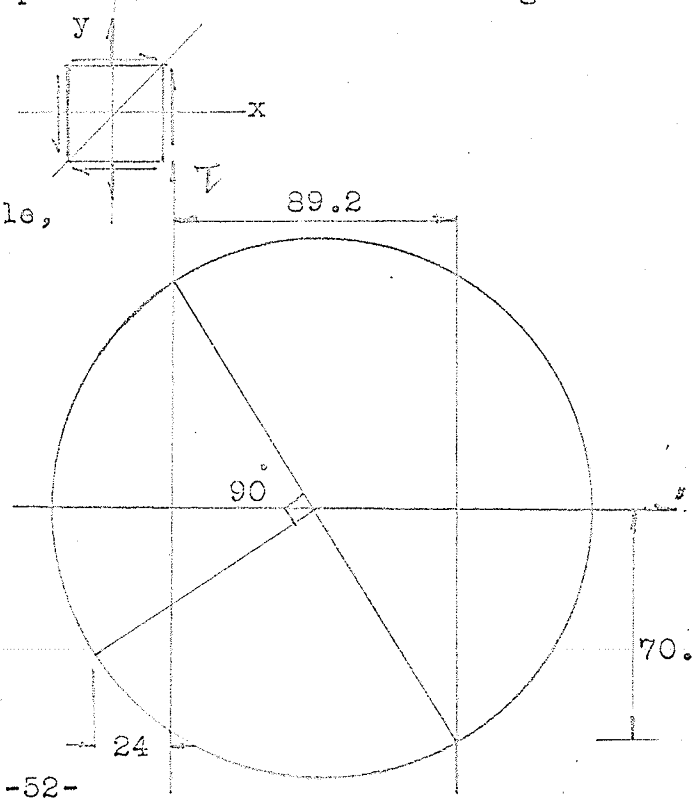
from Mohr's Circle,

the strain is:

$$\epsilon_2 = \frac{\sigma_2}{3 \times 10^6}$$

$$= \frac{-24}{3 \times 10^6}$$

$$= -8 \times 10^{-6} \text{"}$$



CHAPTER IV

EXPERIMENTAL VERIFICATION

In this chapter the angle of twist and stresses is checked by experimental verification.

1. Loading frame -- The beams were 4" in width 6" in depth and 4" feet long as shown in Fig. (18). Two beams were connected by two 5/8" mild steel bars and one pulley. The loading system was as shown in Fig. (18). Another beam with 4" in width 6" in depth and 4.5 feet long were connected to two stranded metal wires and another stranded metal wire to the dynamometer. The dynamometer was adjusted such as, 100 lb, 200 lb, 300 lb ...., therefore, the loading is 100 lb, 200 lb, 300 lb.. ......, R is the distance from center line of the cross section of the ten-storey building model to the stranded metal wire, hence the pure torsional moment is M_t which equal to V x R. This test is in the range of elasticity. The angle of twist was measured by dial gage corresponding to the loading, 100 lb, 200 lb, 300 lb ....

2. Strain gage -- Each column and shear wall will produced strain when the torsional moment is applied at the ten-storey building model. Sr-4

strain gages were attached to the columns and shear walls of the ten-storey building model. Each strain gage was separated 2" at the columns and shear walls. The SR-4 strain gage was made in U. S. A. with gage factor 1.98, resistance 120.3 OHMS. The surface preparation and cleaning instructions are as follows:

a. Surfaces to which gages are to be bonded must be smooth and even, free of pits, deep tool marks, or deep scratches, but not polished. Grinding or filing may sometimes be necessary on sand

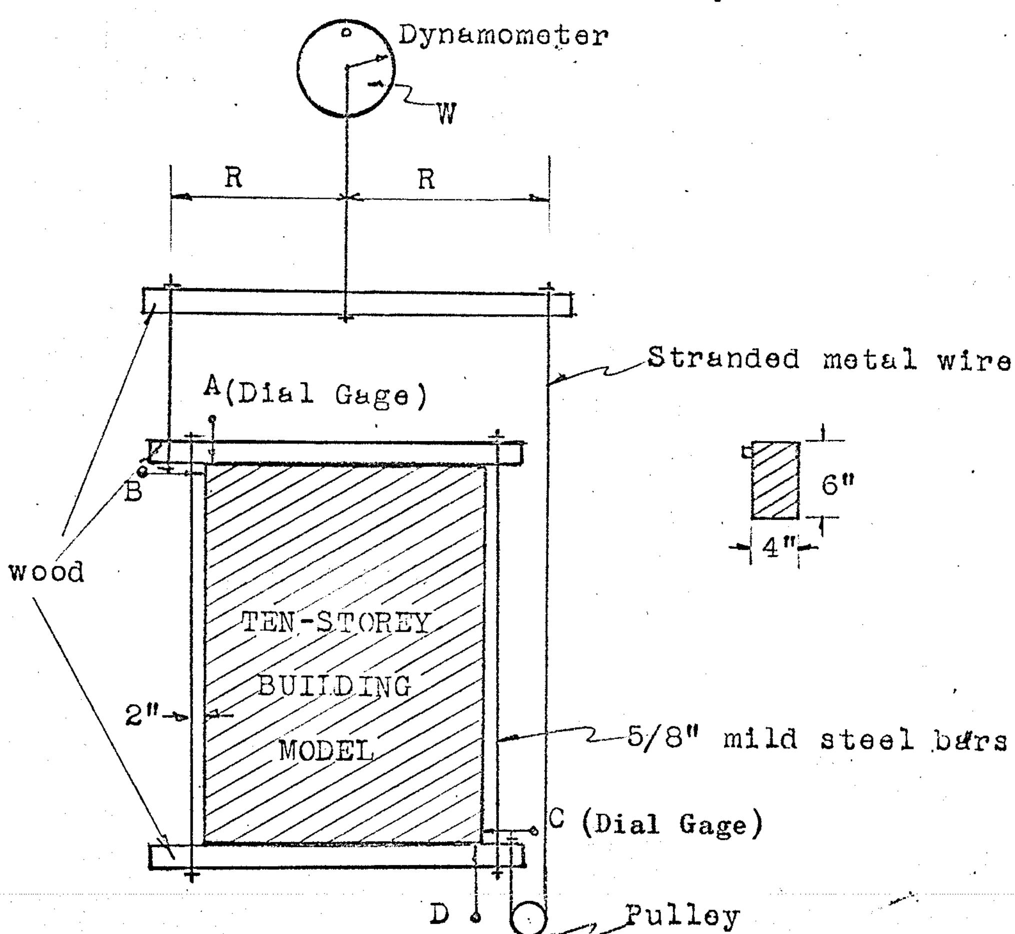


Fig.(18)

castings or similar rough surfaces. Scale rust and paint must be removed from metals.

b. Remove grease and heavy oils, if present, by washing with carbon tretrachloride, trichlorethylene, or methyl chloroform.

c. Scrub suface area considerably larger than gage with clean cotton or gauze pads saturated in methyl ethylktone (SR-4 solvent). Unsterilized cotton is satisfactory. Discard each pad after using once. Repeat with fresh saturated pads until cotton or gauze shows no trace of dirt after use. Clean slightly decreasing areas with successive pads so as not to contaminate gage bonding area with oil grease from edge of cleaned area. Absolute cleanliness is essential.

The installation is:

(a).Apply liberal coat of Duce cement to area gage is to cover.

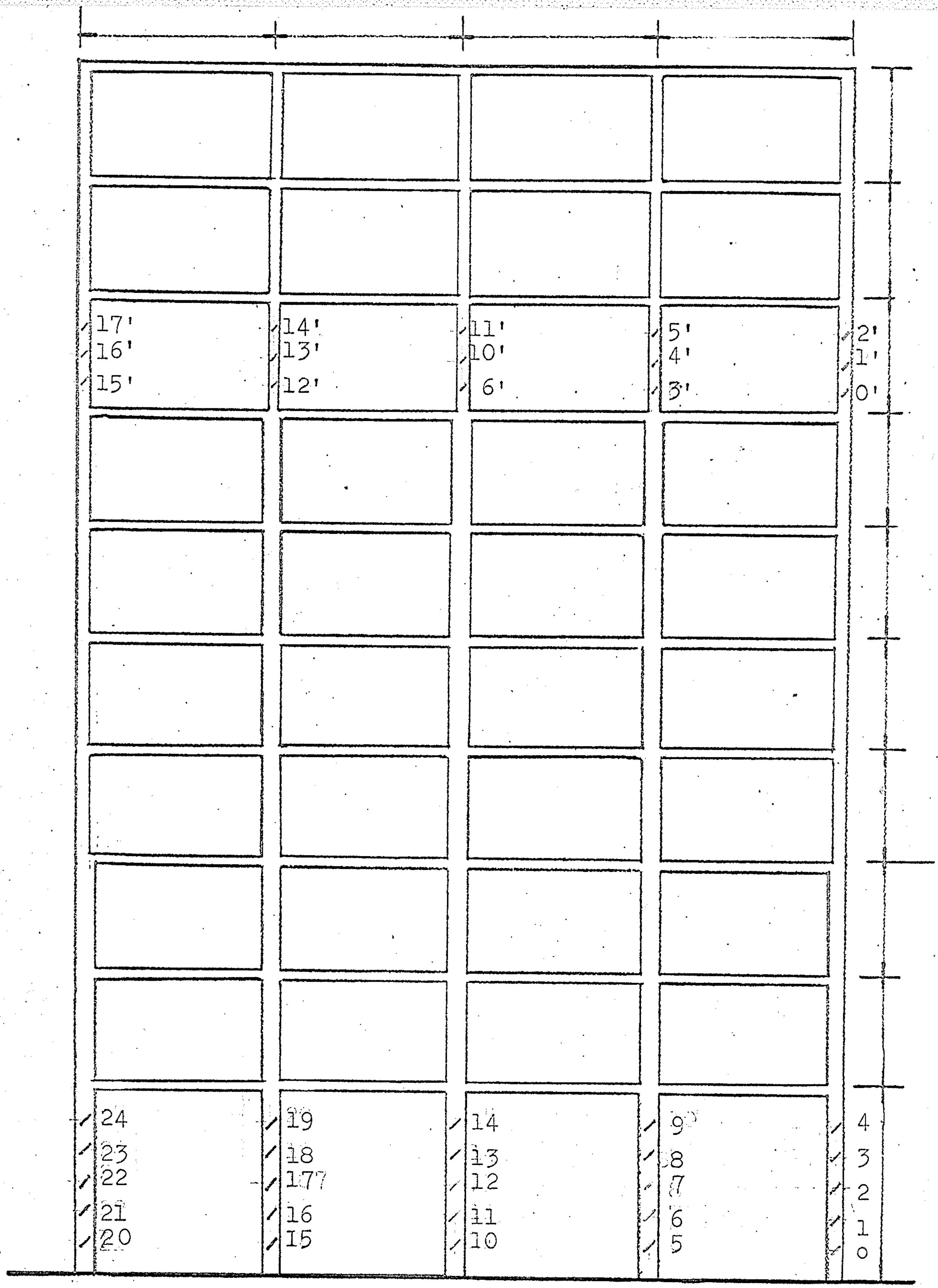
(b). Place gage on cement and position correctly. Press gage gently into cement.

(c). Hold gage in place with a force of approximately one pound applied through metal or wood block and a sponge rubber or felt pad. Force may be applied by a weight, light spring, or rubber bonds. Hold in place for 20 to 30 minutes. Drying time depends on temperature and humidity. For best results, allow 24 hours drying time under average ambient conditions, longer time is required if atmosphere is exceptionally humid or cold. 30 minutes at room temperature after removed of clamps. Too rapid application of heat will cause bubbing of the cement.

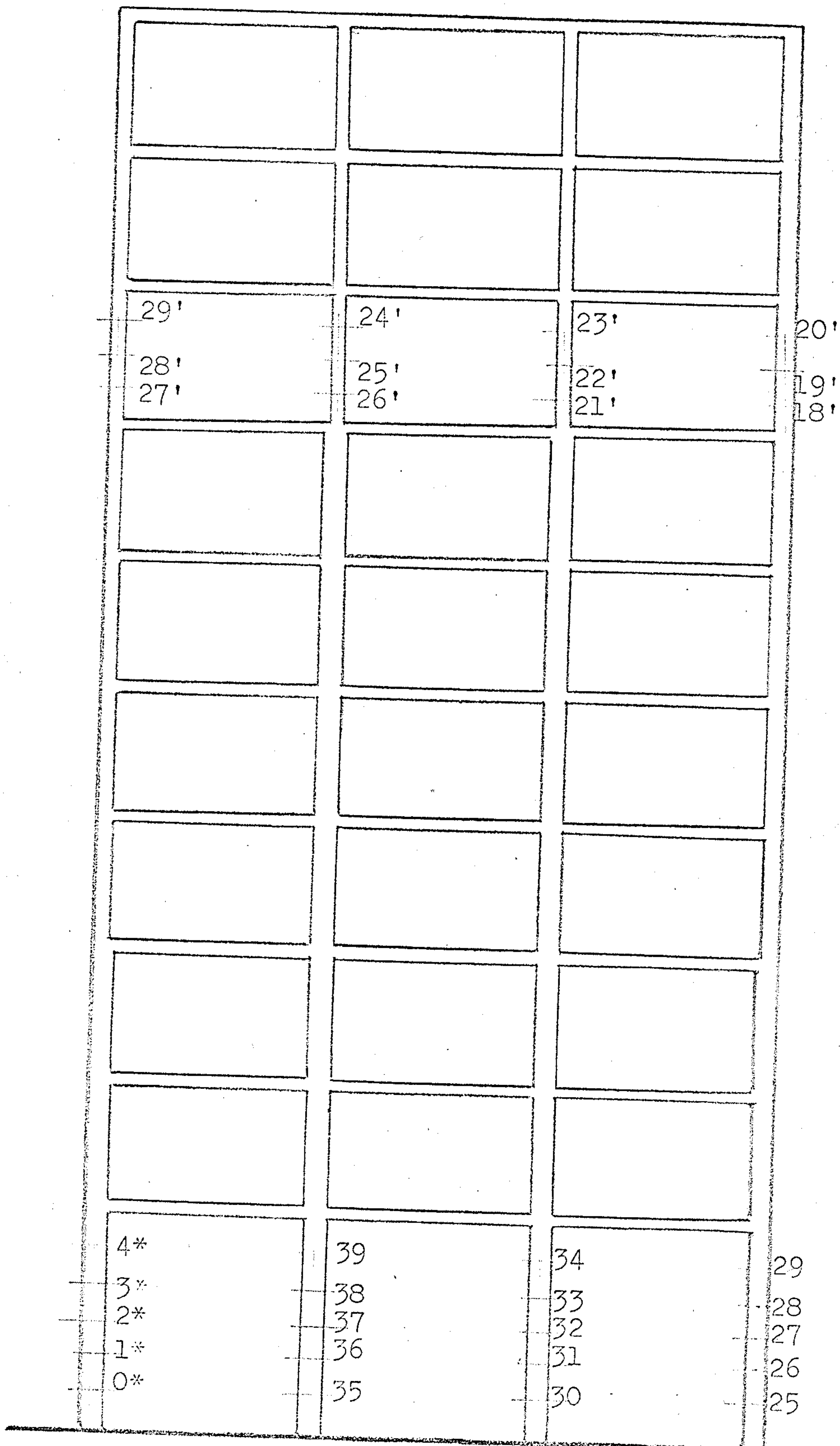
The locations of strain gages are shown in Fig. (19), (20), (21) and (22). There are 120 strain gages. Each column and shear walls were attached 5 strain gages. Each strain gage was separated 2".

3. Testing procedure -- The Dynamometer was adjusted shch as 100 lb, 200lb, 300lb..... as the strain gages connected with wires to the Datran Strain Indication. The Advanced button of the Datran Stran Indication was adjusted and pushed, then the automatic typewriter presented the readings of each strain gage. A full set of strain gages reading was taken at each loading increament in each test.

4. Test result -- The result unit is " 10^{-6}". These results are shown in the following Table.

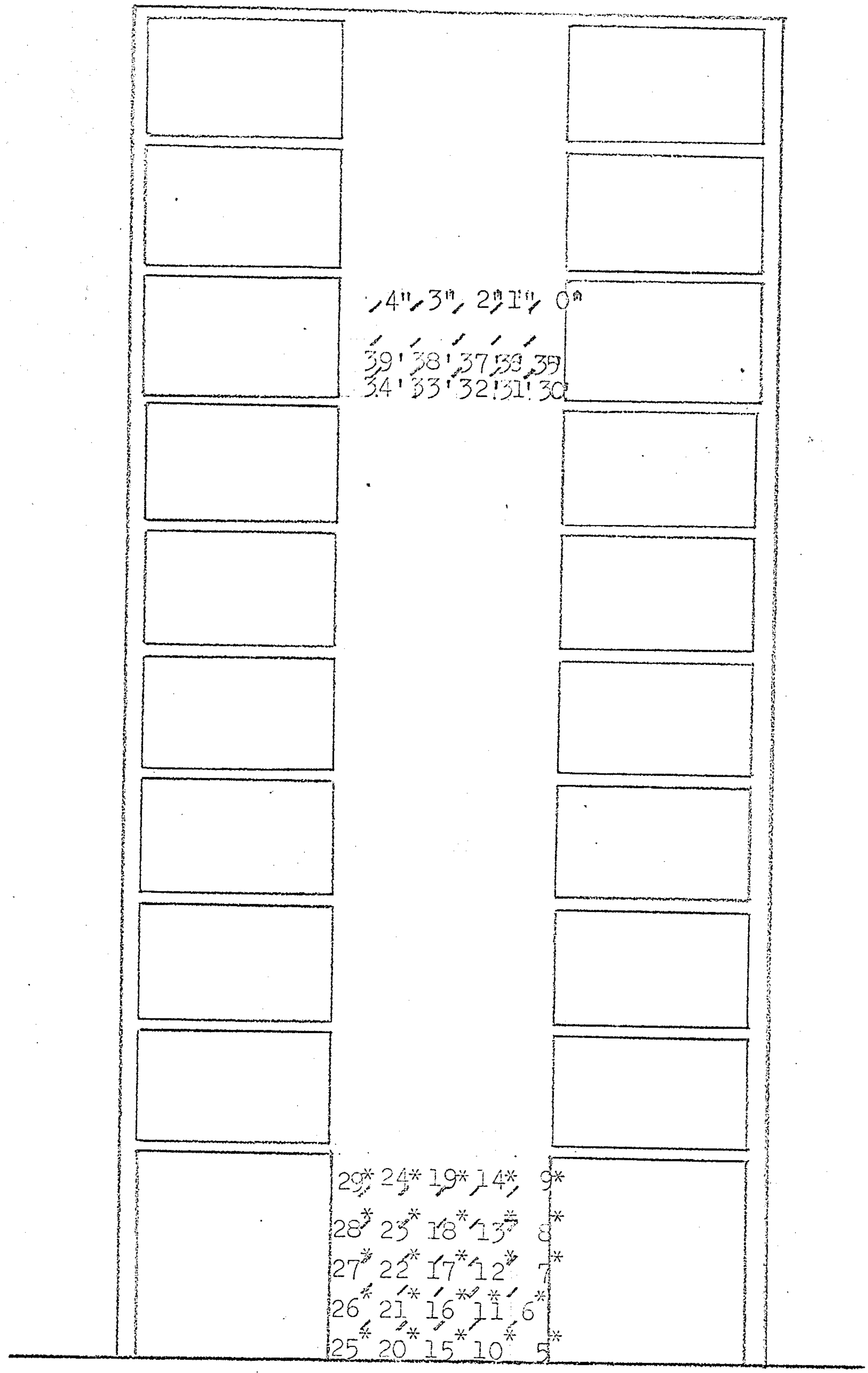


Location and designation of strain gages
on columns of model
in long side direction
Fig. (19)



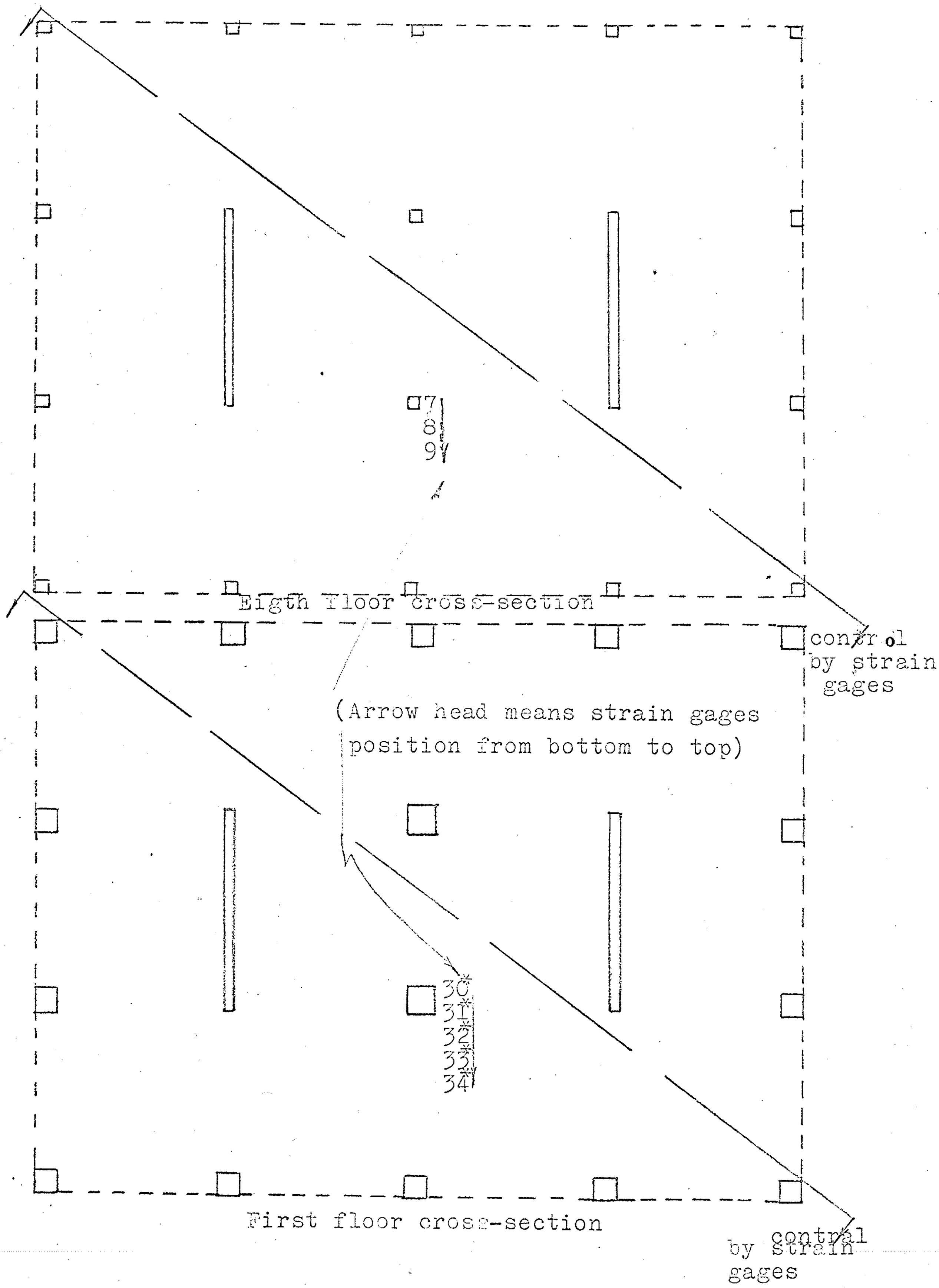
Location and designation of strain gages on column of model in short side

Fig. (20)



Location and designation of strain gages on shear wall of model

Fig. (21)



Location and designation of strain gages of model

Fig. (22)

In this part the angle of twist was checked by experiment.

From the experiment the Δy and Δx is component of Δ_{xy} as shown in Fig. (23). This Δx and Δy could be converted into the angle of twist, ϕ.

from Fig. (23)

$$\Delta_{xy} = r' \phi$$

when ϕ is very small

$$r' = r$$

or

$$\Delta_{xy} = r \phi$$

$$\phi = \frac{\Delta_{xy}}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

from the proportion of triangle

$$\frac{\Delta_{xy}}{5} = \frac{\Delta y}{4}$$

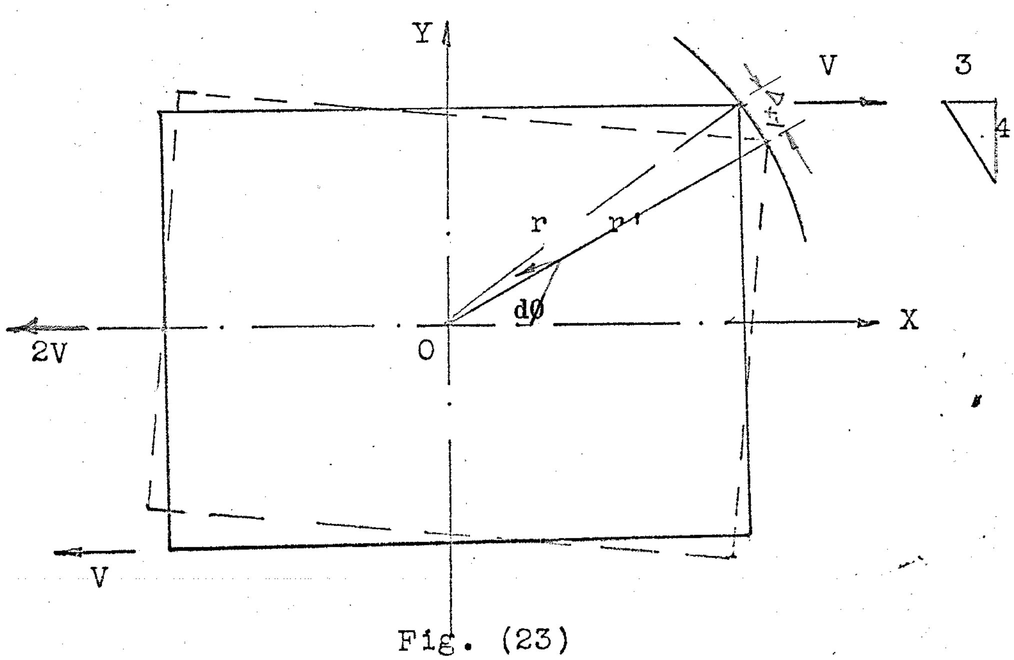


Fig. (23)

or

$$\Delta_{xy} = \frac{5}{4}\Delta y \qquad \ldots\ldots\ldots\ldots\ldots \text{(b)}$$

substituting équation (b) into equation (a)

$$\phi = \frac{\Delta xy}{r}$$

$$= \frac{5\Delta y}{4\ r}$$

when $r = 30$

$$\phi = \frac{\Delta y}{24} \qquad \ldots\ldots\ldots\ldots\ldots \text{(c)}$$

in equation (c) Δ_y is the Δ_{xy} component which can be directly obtained from experiment.

from Fig. (23), the proportion of triangle

$$\frac{\Delta xy}{5} = \frac{\Delta x}{3}$$

or $\Delta_{xy} = \frac{5\Delta x}{3}$ $\qquad \ldots\ldots\ldots\ldots\ldots$ (d)

substituting equation (d) into equation (a)

$$\phi = \frac{\Delta xy}{r}$$

$$= \frac{5\Delta_x}{3\ r}$$

when $r = 30$

$$\phi = \frac{\Delta x}{18} \qquad \ldots\ldots\ldots\ldots\ldots \text{(e)}$$

Equation (c) and (e) are the angle of twist which can be obtained directly from experiment. The following Tables were shown this torsional angle.

Load 2V	V_{lb}	$M_{t\,lb-in}$	ΔX_a	ΔX_d	ϕ_a (10^{-5})	ϕ_d (10^{-5})
0	0	0	0	0	0	0
100	50	900	0.0020	0.0018		
			0.0018	0.0015		
			0.0017	0.0019		
ave.			0.0018	0.0017	10	9.5
200	0	1800	0.0050	0.0048		
			0.0035	0.0040		
			0.0042	0.0041		
ave.			0.0043	0.0043	24	24
300	150	2700	0.0068	0.0069		
			0.0052	0.0062		
			0.0060	0.0060		
ave.			0.0060	0.0063	33	35
400	200	3600	0.0078	0.0082		
			0.0082	0.0080		
			0.0087	0.0083		
ave.			0.0083	0.0083	46	46
500	250	4500	0.0101	0.0106		
			0.0085	0.0103		
			0.0094	0.0092		
ave.			0.0090	0.0100	50	55.5
600	300	5400	0.0116	0.0120		
			0.0105	0.0118		
			0.0108	0.0111		
ave.			0.0109	0.0116	60.5	64.5

700	350	6300	0.0128	0.0138		
			0.0131	0.0130		
			0.0125	0.0130		
ave.			0.0128	0.0133	71	74
800	400	7200	0.0146	0.0146		
			0.0140	0.0142		
			0.0141	0.0148		
ave.			0.0142	0.0145	79	80.5
900	450	8100	0.0158	0.0162		
			0.0157	0.0160		
			0.0160	0.0162		
ave.			0.0158	0.0161	88	89.5
1000	500	9000	0.0172	0.0170		
			0.0170	0.0168		
			0.0170	0.0180		
ave.			0.0171	0.0172	95	95

2V Load	V lb	M_t lb-in	ΔY_c	ΔY_b	$\phi_c (10^{-5})$	$\phi_b (10^{-5})$
0	0	0	0	0	0	0
100	50	900	0.0018	0.0016		
			0.0015	0.0021		
			0.0020	0.0018		
ave.			0.0018	0.0019	7.5	7.9
200	100	1800	0.0040	0.0045		
			0.0045	0.0046		
			0.0042	0.0048		
ave.			0.0043	0.0046	17.9	19.1
300	150	2700	0.0050	0.0070		
			0.0044	0.0076		
			0.0070	0.0073		
ave.			0.0055	0.0073	23	30.04
400	200	3600	0.0060	0.0189		
			0.0092	0.0094		
			0.0080	0.0102		
ave.			0.0077	0.0095	32	39.4
500	250	4500	0.0110	0.0126		
			0.0103	0.0124		
			0.0118	0.0120		
ave.			0.0110	0.0123	46	51.2
600	300	5400	0.0110	0.0142		
			0.0118	0.0142		
			0.0140	0.0140		
ave.			0.0123	0.0141	51.2	58.6

700	350	6300	0.0130	0.0164		
			0.0125	0.0165		
			0.0165	0.0163		
ave.			0.0140	0.0164	58.4	68.4
800	400	7200	0.0170	0.0183		
			0.0148	0.0185		
			0.0184	0.0182		
ave.			0.0167	0.0183	69.7	76.2
900	450	8100	0.0202	0.0202		
			0.0180	0.0201		
			0.0180	0.0200		
ave.			0.0187	0.0201	78	83.7
1000	500	9000	0.0190	0.0220		
			0.0221	0.0220		
			0.0180	0.0223		
ave.			0.0197	0.0221	82	92

** Δ_{xa}, Δ_{xd} are movement in X direction shown in Fig.(18) due to A,D gage

yb, yc are movement in Y direction shown in Fig.(18) due to B,C gage

ϕ_a, ϕ_d are twisting angle in X direction due to gage A and D

ϕ_b, ϕ_c are twisting angle in Y direction due to gage Band C

CHAPTER V

CONCLUSIONS

In this chapter the torsional rigidity, the angle of twist, and the torsional stresses of the theoretical analysis will be compared with the experimental results. Logical conclusions then will be given.

In the first part the torsional rigidity is dealt with. From equation (44) the total torsional rigidity is 131.9 x 10^6 lb-in^2, in which the rigidity developed by twist is 4.18% (3.67 x 1.5/131.9), and the rigidity developed by bending moment is 95.2% (125/131.9). The torsional rigidity developed by bending moment plays a most important role in the torsional rigidity of the strength in this ten-storey building model frame.

In the second part the angle of twist of the theoretical analysis is compared with the experiment results as shown in the following Table.

ANGLE OF TWISTING OF THEORETICAL AND EXPERIMENTAL RESULTS

Load	Theory	Experimental Results (10^{-5})							
2V	ϕ (10^{-5})	ϕ_a	ϕ_a/ϕ	ϕ_d	ϕ_d/ϕ	ϕ_b	ϕ_b/ϕ	ϕ_c	ϕ_c/ϕ
100	8.2	10.0	1.22	9.50	1.16	7.5	0.92	7.90	0.97
200	16.4	24.0	1.46	24.0	1.46	17.9	1.08	19.1	1.16
300	24.6	33.0	1.34	35.0	1.42	23.0	0.94	30.4	1.28
400	32.8	46.0	1.40	46.0	1.40	32.0	0.98	39.4	1.20
500	41.0	50.0	1.22	55.5	1.35	46.0	1.12	51.2	1.29
600	49.2	60.5	1.23	64.5	1.31	51.2	1.04	58.6	1.20
700	57.4	71.0	1.24	74.0	1.30	58.4	1.02	68.4	1.20
800	65.6	79.0	1.20	80.5	1.22	69.7	1.06	76.2	1.16
900	73.8	88.0	1.20	89.5	1.21	78.0	1.06	83.7	1.13
000	82.0	95.0	1.16	95.0	1.16	82.0	1.00	92.0	1.12

Table II

From the above Table the accuracy of the angle of twist is from 0.92 to 1.46. The theoretical results and the experimental results can be ploted as shown in Fig. (24) and (25) . The torsional rigidity of main reinforcement and stirrups has very little influence on the anlge of twist . Therefore, at the early stages the concrete alone is resisting the torsional moment. The main reinforcement and stirrups do not provide any resistance. In a theoretical analysis the torsional rigidity of steel and stirrups is neglected (see Chapter II). In an experimental analysis the angle of twist was still obtained, generally, equal to the theoretical twisting angle.

In the third part the torsional strains of the theoretical analysis are compared with the experimental results as shown in Table III.

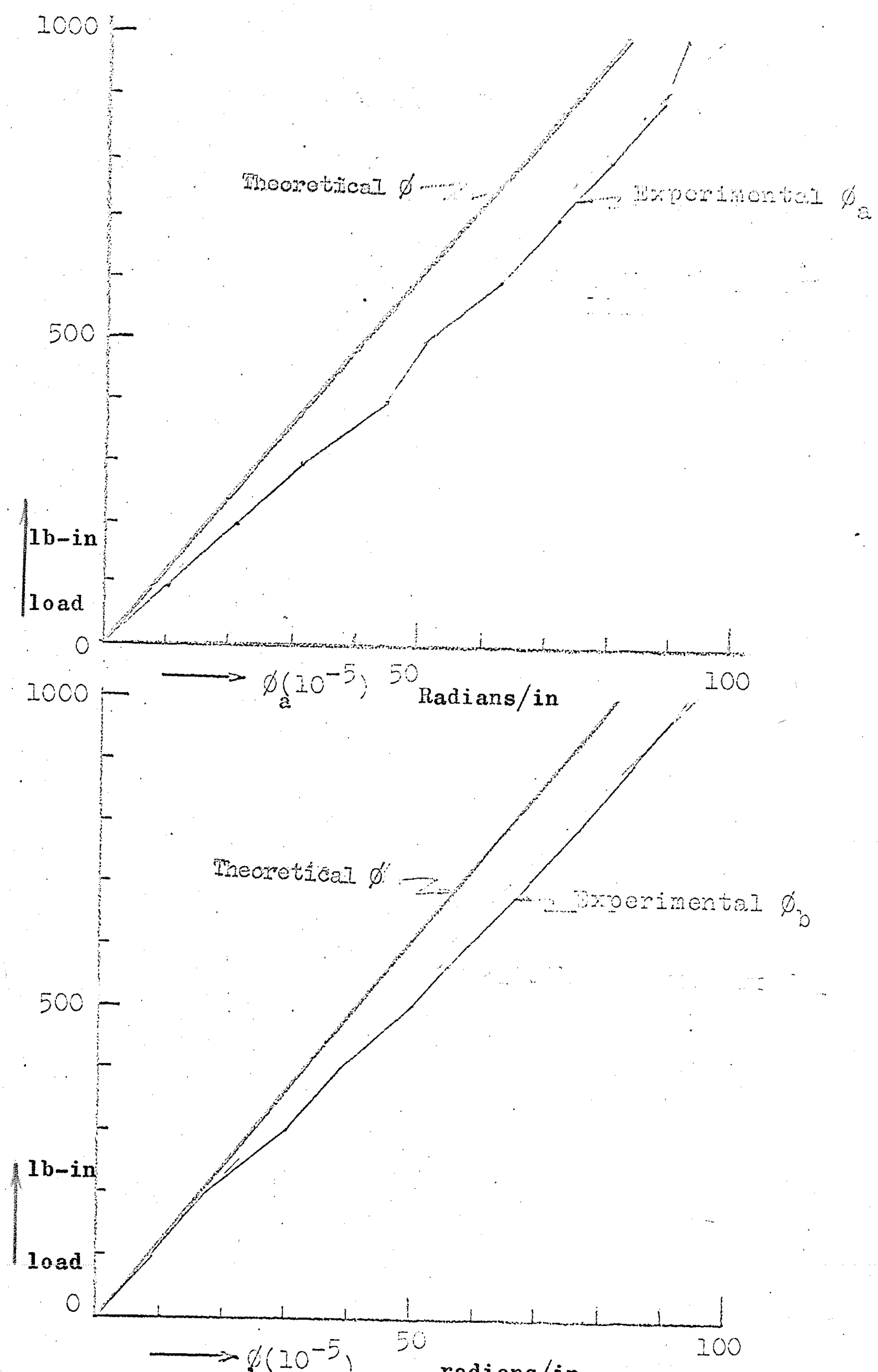
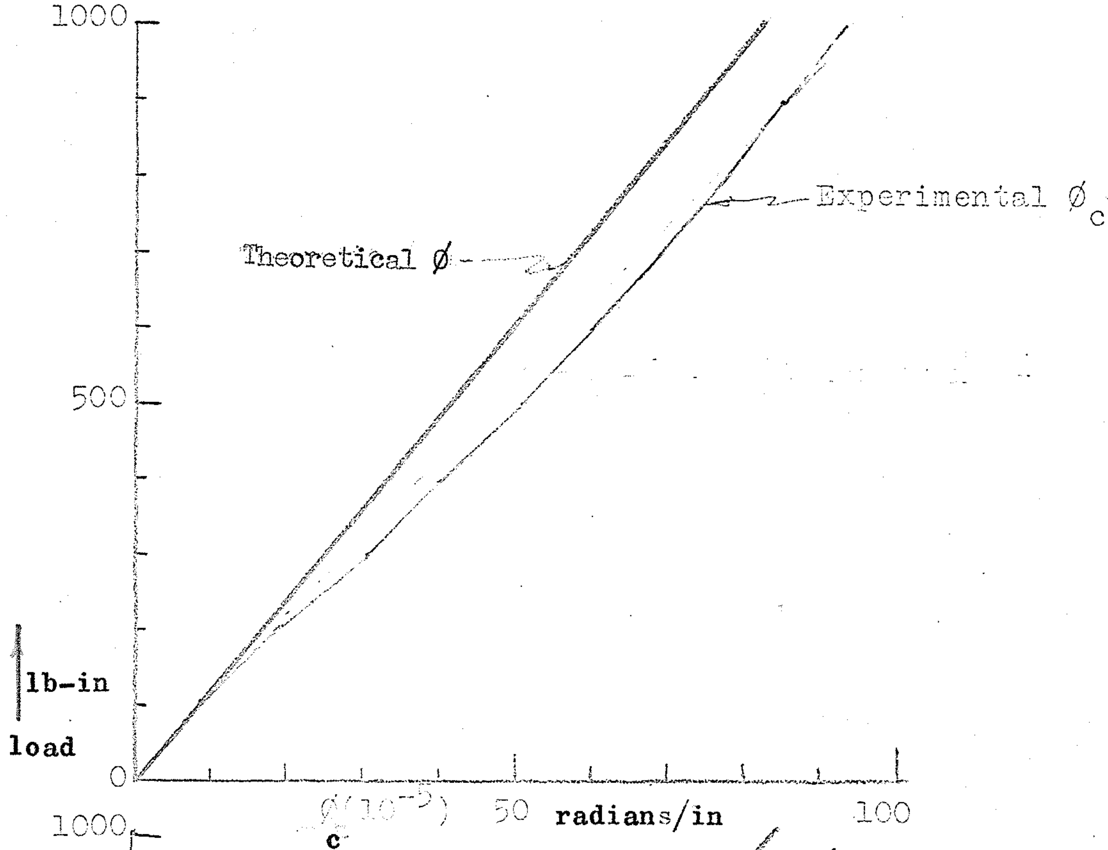
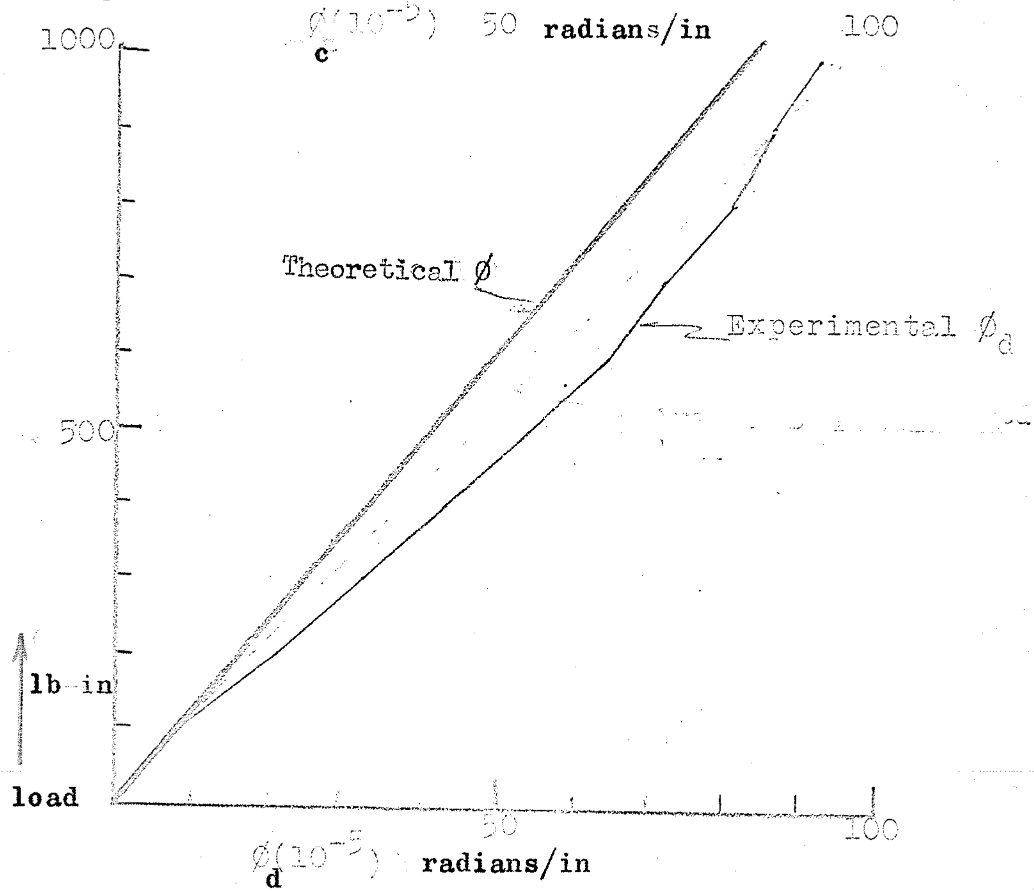

1000
500
0
Theoretical ø
Experimental $ø_a$
lb-in
load
$ø_a(10^{-5})$ Radians/in
50
100
1000
500
0
Theoretical ø
Experimental $ø_b$
lb-in
load
$ø_b(10^{-5})$ radians/in
50
100

Fig. (24)



1000
500
0
lb-in
load
Theoretical ∅
Experimental ∅c
∅c(10⁻⁵) radians/in
50
100



1000
500
lb-in
load
Theoretical ∅
Experimental ∅d
∅d(10⁻⁵) radians/in
50
100

Fig. (25)

Distance from the top of col. "5"	Theoretical strain of col. "5" in direction		Experimental Results in direction	
	X(10^{-6})	Y(10^{-6})	X(10^{-6})	Y(10^{-6})
10"	50.0	-26.6	10.0	-24.0
8"	33.0	-20.7	27.0	-14.0
6"	20.0	-12.0	9.0	-20.0
4"	5.0	-10.0	- 8.0	-17.0
2"	- 8.0	11.0	-18.0	- 7.0

Table III

Results from the above table shown that measured strains at 1o" from top of column "5" in Y direction and 8" from top of column "5" in X direction are identical to theoretical values. All results indicate a tendency for higher strains at the bottom of the column and low strains at top of the column. These experimental results are all in the range of 10^{-6} in. Absolute values are so small that in the labolatory they may be affected by

a. Room temperature influence causing surface tension or compression.

b. Neiboring construction.

c. Human technical error.

d. concrete itself is not a real elastic homogeneous isotropic material.

For the purpose of analysis "free end" conditions at the top of the column are more suitable than "fixed end" conditions. This is by substituting "fixed end" conditions into Chapter III analysis.

APPENDIX

PART A -- REFERENCES

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3. Marshall, W.T. and Tembe, N.R., "Experiments on Plain and Reinforced Concrete in Torsion", Structural Engineer (London), V 19, 1949, p. 171-191.

4. Cowan, H.J., and Armstrong, S., "Experiments on the Strength of Reinforced and Prestressed Concrete beam and of concrete-encased steel joists in combined bending and Torsion". Magazine of Concrete Research (London) V, 7, 1955, p. 3-20.

5. Cowan, H.J., "Tests of the Torsional Strength and Deformation of Rectangular Reinforced Concrete Beams". Magazine of Concrete Research (London), Feb. 1951.

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9. Frank Barron, and Anthony G. Arioto, "Torsional Behavior of Suspension Bridge Towers", ASCE, August, 1961, p. 1-29.

10. J. Sterling Kinney, "Indeterminate Structural Analysis", Chapter 8, p. 308.

APPENDIX

PART B--COMPUTER

```
ZZJOB 5
ZZFORX52
C       MINGS CIVIL ENGINEERING CALC. NO. 1
      1 FORMAT (F6.3,F5.2,F5.3)
      2 FORMAT (F7.3,FXF6.2,5XF6.3,5XF12.1)
      3 READ  1,XL,G,A
        Z = 0.92 *XL/G
        CH = (EXPF(Z) +EXPF(-Z))/2.0
        SH = (EXPF(Z)-EXPF(-Z))/2.0
        XK= (194550.0*A **4.0*XL*SH)/(2.0*G-2.0*G*CH+0.92*XL*SH)
        PUNCH 2,XL,G,A,XK
        GO TO 3
        END

10.00003.001.000
12.00430.000.875
12.00421.630.875
12.00424.740.875
12.00406.001.125
12.00418.000.875
19.50830.000.875
19.50821.630.875
19.50824.740.875
19.50806.001.125
```

19.50818.000.875

34.51630.000.750

34.51621.630.875

34.51624.740.875

34.51606.001.000

34.51618.000.875

27.01230.000.750

27.01221.630.875

27.01224.740.875

27.01206.001.000

27.01218.000.875

64.53230.000.750

64.53221.630.750

64.53224.740.750

64.53206.000.750

64.53218.000.750

57.02830.000.750

57.02821.630.750

57.02824.740.750

57.02806.000.750

57.02818.000.750

10.00003.001.000

7.50430.00 .875

77.50421.63 .875

7.50424.74 .875

7.504.6.001.125

7.50430.00 .750

7.504 6.001.000

7.504 6.00 .750

```
 7.50421.63 .750
 7.50424.74 .750
 7.50418.00 .875
 7.50418.00 .075

ZZZZ
ZZJOB 5
ZZFORX52
C      MINGS CIVIL ENGINEERING CALC NO. 2
     1 FORMAT(F6.3,F5.2,F6.3,F5.3)
     2 FORMAT(F7.3,5XF6.2,5XF7.3,5XF6.3,5XF12.1)
     3 READ 1,XL, G,A,B
       Z =1.414*XL/G
       CH = (EXPF(Z)+EXPF(-Z))/2.0
       SH = (EXPF(Z)-EXPF(-Z))/2.0
       XK = (707100.0*A*B** 3.0*XL*SH)/(2.0*G-2.0*G*CH+1.414*
                                                      XL*SH)
       PUNCH 2,XL,G, A,B,XK
       GO TO 3
       END

12.00412.0012.625 .625
19.50312.0012.625 .625
34.51612.0012.625 .500
27.01212.0012.625 .500
64.53212.0012.625 .500
57.02812.0012.625 .500
 7.50412.0012.625 .625
 7.50412.0012.625 .500
```

12.004	30.00	.875	11124196.0
12.004	21.63	.875	5854656.4
12.004	24.74	.875	7613159.6
12.004	6.00	1.125	1604697.6
12.004	18.00	.875	4100233.6
19.508	30.00	.875	4304807.4
19.508	21.63	.875	2309188.0
19.508	24.74	.875	2975193.5
19.508	6.00	1.125	856827.8
19.508	18.00	.875	1644810.3
34.516	30.00	.750	796815.6
34.516	21.63	.875	838542.3
34.516	24.74	.875	1051362.2
34.516	6.00	1.000	337869.0
34.516	18.00	.875	626169.9
27.012	30.00	.750	1250322.8
27.012	21.63	.875	1275406.0
27.012	24.74	.875	1622807.0
27.012	6.00	1.000	397321.8
27.012	18.00	.875	928810.9
64.532	30.00	.750	284948.0
64.532	21.63	.750	186202.0
64.532	24.74	.750	219199.8
64.532	6.00	.750	83857.5
64.532	18.00	.750	153168.3
57.028	30.00	.750	342527.4
57.028	21.63	.750	216231.3
57.028	24.74	.750	258414.4
57.028	6.00	.750	86743.2

57.028	18.00	.750	174055.7
7.504	30.00	.875	28220396.0
7.504	21.63	.875	14746292.0
7.504	24.74	.875	19245716.0
7.504	6.00	1.125	3476107.9
7.504	30.00	.750	15232668.0
7.504	6.00	1.000	2170117.1
7.504	6.00	.750	686638.6
7.504	21.63	.750	7959682.0
7.504	24.74	.750	10388359.0
7.504	18.00	.875	10259997.0
7.504	18.00	.750	5538091.4

12.004	12.00	12.625	.625	11090076.0
19.503	12.00	12.625	.625	5340871.1
34.516	12.00	12.625	.500	1503722.8
27.012	12.00	12.625	.500	1871502.4
64.532	12.00	12.625	.500	1070438.9
57.028	12.00	12.625	.500	1122437.4
7.504	12.00	12.625	.625	25505033.0
7.504	12.00	12.625	.500	13058576.0

Appendix

Part -- C **Experimental results**

Short Side & Long Side Column

Gage						Load					
No.	0	100	200	300	400	500	600	700	800	900	1000
00	-002	-018	-030	-040	-050	-059	-070	-078	-088	-100	-110
	-004	-016	-030	-039	-050	-058	-069	-088	-088	-099	-109
	-002	-016	-028	-039	-049	-059	-070	-078	-087	-098	-110
ave.	-003	-017	-029	-039	-050	-059	-070	-078	-088	-099	-110
F.	0	-014	-026	-036	-048	-056	-068	-075	-085	-096	-107
01	+001	-008	-012	-015	-018	-022	-024	-029	-030	-034	-037
	+000	-006	-011	-014	-018	-020	-024	-028	-031	-035	-038
	-000	-007	-011	-014	-018	-022	-024	-028	-030	-034	-037
ave.	-000	-007	-011	-014	-018	-022	-024	-028	-030	-034	-037
F.	0	-007	-011	-014	-018	-022	-024	-028	-030	-034	-037
02	-000	-006	-010	-013	-016	-018	-020	-022	-023	-026	-030
	+001	-006	-010	-012	-014	-017	-020	-022	-023	-026	-030
	+001	-005	-010	-012	-015	-017	-020	-024	-023	-027	-030
ave.	+001	-006	-010	-012	-015	-017	-020	-023	-023	-026	-030
F.	0	-007	-011	-013	-016	-018	-021	-024	-024	-026	-031
03	-002	-006	-008	-010	-010	-011	-012	-012	-015	-016	-018
	-001	-006	-008	-010	-010	-011	-012	-013	-014	-016	-017
	-002	-006	-008	-010	-010	-011	-012	-013	-014	-016	-018
ave.	-002	-006	-008	-010	-010	-011	-012	-013	-014	-006	-018
F.	0	-004	-006	-008	-008	-009	-010	-010	-012	-014	-016

Gage No.	0	100	200	300	400	500	600	700	800	900	1000
					Load						
04	-000	-002	-004	-004	-002	-002	-000	-002	-000	-000	-000
	+001	-002	-003	-003	-002	-000	-002	-000	-000	-000	-000
	-000	-002	-004	-003	-002	-002	-001	-001	-000	-000	-000
ave.	-000	-002	-004	-003	-002	-002	-001	-001	-000	-000	-000
F.	0	-002	-004	-003	-002	-002	-001	-001	-000	-000	-000
05	-001	-004	-006	-010	-012	-014	-018	-021	-024	-027	-030
	-000	-003	-006	-010	-012	-014	-016	-021	-024	-026	-030
	-000	-004	-006	-008	-012	-015	-018	-020	-023	-026	-031
ave.	-000	-004	-006	-009	-012	-014	-018	-021	-024	-026	-030
F.	0	-004	-006	-009	-012	-014	-018	-021	-024	-026	-030
06	+002	+001	-003	-005	-008	-010	-012	-015	-018	-022	-022
	+002	-000	-002	-005	-008	-010	-013	-016	-019	-020	-022
	+002	+002	-002	-005	-008	-010	-012	-016	-018	-021	-022
ave.	+002	-001	-002	-005	-008	-010	-012	-016	-018	-021	-022
F.	0	-001	-004	-007	-010	-012	-014	-018	-020	-023	-024
07	+001	-000	-002	-005	-008	-010	-011	-014	-018	-018	-022
	+001	-000	-002	-005	-007	-010	-012	-014	-015	-018	-022
	+002	-000	-002	-006	-008	-010	-012	-014	-017	-018	-022
ave.	-001	-000	-002	-005	-008	-010	-012	-014	-017	-018	-022
F.	0	-001	-003	-004	-009	-011	-013	-015	-018	-019	-023

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
08	-001	-000	-002	-004	-004	-006	-008	-008	-008	-010	-010
	-000	-000	-002	-004	-004	-006	-008	-008	-009	-010	-010
	-000	-000	-002	-003	-005	-006	-008	-008	-009	-010	-010
ave.	-000	-000	-002	-004	-004	-006	-008	-008	-009	-010	-010
F.	0	-000	-002	-004	-004	-006	-008	-008	-009	-010	-010
09	-000	-000	-003	-005	-005	-006	-007	-010	-010	-010	-011
	-000	-002	-002	-004	-005	-007	-008	-008	-009	-010	-011
	-000	-000	-002	-004	-005	-006	-008	-008	-010	-010	-011
ave.	-000	-001	-002	-004	-005	-006	-008	-009	-010	-010	-011
F.	0	-001	-002	-004	-005	-006	-008	-009	-010	-010	-011
10	-002	-002	-002	-002	-004	-005	-006	-006	-009	-010	-010
	-000	-002	-002	-004	-004	-005	-006	-007	-008	-010	-010
	-002	-000	-002	-003	-004	-005	-006	-006	-008	-010	-010
ave.	-001	-002	-002	-003	-004	-005	-006	-006	-008	-010	-010
F.	0	-001	-001	-002	-004	-005	-006	-006	-008	-010	-010
11	-000	-000	-002	-002	-005	-006	-006	-006	-010	-010	-012
	-000	-000	-002	-003	-004	-006	-006	-008	-010	-011	-012
	-000	-001	-001	-002	-004	-006	-007	-008	-010	-010	-012
ave.	-000	-001	-002	-002	-004	-006	-006	-007	-010	-010	-012
F.	0	-001	-002	-002	-002	-004	-006	-006	-007	-010	-012

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
12	+002	-001	-004	-006	-009	-010	-012	-014	-016	-018	-020
	-000	-001	-004	-006	-010	-010	-012	-014	-016	-019	-020
	+002	-002	-004	-005	-008	-010	-012	-014	-016	-018	-021
ave.	+002	-001	-004	-006	-009	-010	-012	-014	-016	-018	-021
F.	0	-003	-006	-008	-011	-012	-014	-016	-018	-020	-023
13	-003	-004	-004	-006	-008	-009	-010	-010	-010	-012	-012
	-003	-002	-004	-006	-007	-006	-009	-010	-011	-012	-012
	-003	-002	-004	-006	-006	-008	-009	-010	-010	-012	-014
ave.	-003	-003	-004	-006	-007	-008	-009	-010	-010	-012	-014
F.	0	-000	-001	-002	-004	-005	-006	-007	-007	-009	-009
14	-000	-002	-005	-006	-009	-010	-012	-013	-014	-016	-018
	-000	-002	-005	-006	-008	-010	-012	-012	-014	-016	-017
	-000	-002	-005	-006	-008	-010	-012	-013	-014	-016	-016
ave.	-000	-002	-005	-006	-008	-010	-012	-013	-014	-016	-017
F.	0	-002	-005	-006	-008	-010	-012	-013	-014	-016	-017
15	-002	-000	-000	-000	-000	-002	-002	-002	-002	-002	-002
	-001	-000	-000	-000	-000	-002	-002	-002	-002	-002	-002
	-001	-000	-000	-000	-000	-000	-002	-002	-002	-002	-002
ave.	-001	-000	-000	-000	-000	-001	-002	-002	-002	-002	-002
F.	0	+001	+001	+001	+001	-000	-001	-001	-001	-001	-001

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
16	+000	+002	+007	+009	+012	+014	+016	+017	+018	+019	+020
	+000	+004	+006	+009	+011	+014	+015	+016	+018	+019	+020
	+000	+004	+006	+009	+012	+012	+016	+016	+018	+019	+020
ave.	+000	+003	+006	+009	+012	+013	+016	+016	+018	+019	+020
F.	0	+003	+006	+009	+012	+013	+016	+016	+018	+019	+020
17	+002	-000	-002	-002	-002	-002	-005	-004	-007	-006	-010
	+000	-000	-000	-002	-002	-004	-002	-005	-006	-008	-008
	+002	-001	-001	-001	-002	-002	-003	-006	-007	-006	-010
ave.	+001	-000	-001	-002	-002	-003	-003	-005	-007	-007	-009
F.	0	-001	-002	-003	-003	-004	-004	-005	-008	-008	-010
18	+001	-001	-003	-005	-006	-008	-010	-010	-012	-014	-016
	+001	-002	-003	-005	-006	-008	-008	-010	-012	-014	-015
	+001	-000	-002	-005	-006	-008	-010	-011	-012	-014	-015
ave.	+001	-001	-003	-005	-006	-008	-009	-010	-012	-014	-015
F.	0	-002	-004	-006	-007	-009	-010	-011	-013	-015	-016
19	-002	-004	-007	-010	-013	-014	-016	-019	-020	-024	-024
	-000	-004	-008	-010	-012	-014	-016	-019	-020	-022	-025
	-001	-004	-006	-010	-012	-016	-018	-018	-020	-022	-024
ave.	-001	-004	-007	-010	-012	-015	-017	-019	-020	-023	-024
F.	0	-003	-006	-009	-011	-014	-016	-018	-020	-022	-023

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
20	-000	+002	+004	+004	+006	+006	+008	+008	+009	+010	+010
	-001	+003	+004	+004	+006	+007	+006	+008	+008	+010	+010
	-000	+002	+003	+005	+005	+006	+007	+006	+008	+008	+010
ave.	-000	+003	+004	+004	+006	+006	+007	+008	+009	+010	+010
F.	0	+003	+004	+004	+006	+006	+007	+008	+009	+010	+010
21	+000	+005	+008	+010	+014	+018	+020	+022	+024	+026	+027
	+001	+004	+009	+012	+014	+017	+019	+022	+024	+025	+028
	+001	+006	+008	+010	+016	+018	+019	+021	+023	+024	+028
ave.	+001	+002	+002	+004	+005	+004	+006	+008	+008	+009	+010
F.	0	+004	+007	+010	+014	+017	+019	+021	+023	+024	+027
22	+001	+002	+002	+004	+005	+004	+006	+008	+008	+009	+010
	+001	+002	+003	+002	+004	+005	+006	+006	+008	+008	+008
	+000	+002	+003	+004	+004	+005	+006	+008	+008	+010	+010
ave.	+001	+002	+003	+003	+004	+005	+006	+007	+008	+009	+010
F.	0	+001	+002	+002	+003	+004	+005	+006	+007	+008	+009
23	-000	-001	-002	-003	-004	-005	-006	-006	-006	-007	-008
	-000	-000	-002	-003	-004	-005	-005	-006	-006	-006	-008
	-000	-001	-002	-003	-004	-005	-006	-005	-006	-006	-007
ave.	-000	-001	-002	-003	-004	-005	-006	-006	-006	-006	-008
F.	0	-001	-002	-003	-004	-005	-006	-006	-006	-006	-008

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
24	+003	-000	-004	-006	-008	-008	-010	-012	-014	-014	-016
	+002	-000	-004	-006	-008	-010	-011	-012	-014	-015	-016
	+001	-001	-004	-005	-008	-010	-010	-012	-012	-014	-016
ave.	+002	-000	-004	-006	-008	-009	-010	-012	-013	-014	-016
F.	0	-002	-006	-008	-010	-011	-012	-014	-015	-016	-018
25	-002	-005	-009	-012	-014	-008	-019	-020	-022	-024	-026
	-002	-006	-010	-011	-013	-016	-019	-020	-022	-026	-026
	-002	-006	-010	-011	-014	-016	-019	-021	-022	-024	-027
ave.	-002	-006	-010	-011	-014	-016	-019	-020	-022	-025	-026
F.	0	-004	-008	-009	-012	-014	-017	-018	-018	-023	-024
26	+004	-001	-004	-006	-006	-008	-009	-010	-011	-012	-013
	-000	-002	-004	-005	-006	-008	-008	-010	-011	-012	-013
	-000	-002	-004	-005	-006	-008	-008	-010	-010	-012	-013
ave.	+001	-002	-004	-005	-006	-008	-008	-010	-011	-012	-013
F.	0	-003	-005	-006	-006	-009	-009	-011	-012	-013	-014
27.	+002	-002	-005	-007	-010	-012	-015	-017	-018	-020	-022
	+002	-002	-005	-008	-010	-012	-014	-016	-018	-020	-023
	+002	-002	-005	-006	-010	-011	-014	-016	-018	-021	-022
ave.	+002	-002	-005	-007	-010	-012	-014	-017	-018	-020	-022
F.	0	-000	-003	-005	-008	-010	-012	-015	-016	-018	-020

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
28	-000	-002	-003	-004	-005	-005	-006	-006	-003	-019	-018
	-000	-002	-002	-004	-004	-006	-006	-006	-008	-019	-018
	-002	-002	-003	-004	-004	-006	-006	-007	-008	-018	-018
ave.	-001	-002	-003	-004	-004	-006	-006	-006	-008	-018	-018
F.	0	-001	-002	-003	-003	-005	-005	-005	-007	-017	-017
29	-000	-000	-000	-002	-002	-004	-004	-004	-005	-006	-007
	-000	-000	-001	-002	-003	-005	-004	-006	-006	-006	-006
	-000	-001	-000	-002	-002	-004	-004	-004	-006	-006	-008
	-000	-000	-000	-002	-002	-004	-004	-004	-006	-006	-007
ave.	-000	-000	-000	-002	-002	-004	-004	-004	-006	-006	-007
F.	0	-000	-000	-002	-002	-004	-004	-004	-006	-006	-007
30	+001	-000	-002	-004	-005	-005	-006	-008	-009	-010	-010
	+001	-001	-002	-004	-004	-006	-006	-008	-009	-010	-010
	+002	-000	-002	-004	-005	-005	-006	-008	-008	-010	-010
ave.	+001	-000	-002	-004	-005	-005	-006	-008	-009	-010	-010
F.	0	-001	-003	-005	-006	-006	-007	-009	-011	-011	-011
31	-000	-002	-005	-006	-008	-010	-011	-012	-014	-014	-016
	-000	-002	-005	-006	-008	-008	-011	-012	-014	-014	-016
	-000	-002	-005	-006	-008	-010	-010	-012	-012	-014	-016
ave.	-000	-002	-005	-006	-008	-009	-011	-012	-013	-014	-016
F.	0	-002	-002	-006	-008	-009	-011	-012	-013	-014	-016

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
32	+002	-002	-006	-008	-011	-014	-017	-018	-020	-024	-026
	+002	-002	-006	-010	-011	-014	-016	-019	-020	-024	-026
	+002	-002	-006	-010	-011	-014	-016	-018	-021	-023	-026
ave.	+002	-002	-006	-009	-011	-014	-016	-018	-020	-024	-026
F.	0	-004	-008	-011	-013	-016	-018	-020	-022	-026	-028
33	-000	-002	-003	-005	-006	-008	-010	-010	-012	-014	-016
	-000	-002	-004	-005	-006	-008	-010	-010	-012	-014	-014
	-000	-002	-003	-004	-006	-008	-010	-010	-012	-014	-015
ave.	-000	-002	-003	-005	-006	-008	-010	-010	-012	-014	-015
F.	0	-002	-003	-005	-006	-008	-010	-010	-012	-014	-015
34	-000	-002	-002	-004	-006	-006	-008	-010	-010	-011	-016
	-000	-002	-002	-004	-006	-008	-008	-010	-010	-011	-012
	-000	-002	-002	-004	-006	-006	-008	-010	-010	-011	-012
ave.	-000	-002	-002	-004	-006	-006	-008	-010	-010	-011	-012
F.	0	-002	-002	-004	-006	-006	-008	-010	-010	-011	-012
35	-002	-002	-002	-000	+002	+002	+002	+003	+006	+008	+007
	-003	-000	-000	-000	+003	+002	+004	+003	+006	+008	+007
	-002	-002	-000	-002	+002	+002	+004	+004	+005	+008	+008
ave.	-002	-001	-001	-001	+001	+002	+003	+003	+006	+008	+007
F.	0	+001	+001	+001	+003	+004	+005	+005	+009	+010	+009

Gage No.	0	100	200	300	400	500	600	700	800	900	1000
36	-00	-000	-002	-002	-002	-003	-005	-005	-005	-006	-008
	-000	-000	-002	-002	-003	-003	-005	-005	-006	-006	-006
	-000	-000	-002	-002	-002	-003	-004	-005	-006	-006	-006
ave.	-000	-000	-002	-002	-002	-003	-005	-005	-006	-006	-007
F.	0	0	-002	-002	-002	-003	-005	-005	-006	-006	-007
37	-002	-000	-000	-005	-003	-005	-006	-008	-008	-010	-010
	-004	-000	-000	-005	-002	-005	-007	-006	-008	-010	-011
	-005	-000	-000	-005	-003	-005	-006	-007	-008	-010	-010
ave.	-004	-000	-000	-005	-003	-005	-006	-007	-008	-010	-010
F.	0	+004	+004	-001	-001	-001	-002	-003	-004	-006	-006
38	+002	-000	-002	-004	-006	-006	-010	-010	-012	-014	-016
	+001	-000	-002	-003	-006	-007	-009	-010	-012	-014	-016
	+001	-000	-001	-004	-006	-006	-008	-010	-012	-014	-016
ave.	+001	-000	-002	-005	-006	-006	-009	-010	-012	-014	-016
F.	0	-001	-003	-006	-007	-007	-010	-011	-013	-015	-017
39	-000	-002	-003	-005	-006	-008	-009	-010	-012	-014	-015
	-000	-002	-002	-005	-006	-008	-009	-010	-012	-014	-015
	-000	-002	-003	-005	-006	-008	-010	-010	-012	-013	-015
ave.	-000	-002	-003	-005	-006	-008	-009	-010	-012	-014	-015
F.	0	-002	-003	-005	-006	-008	-009	-010	-012	-014	-015

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
00*	-002	+012	+032	+047	+066	+087	+108	+126	+142	+164	+188
	-002	+014	+032	+048	+068	+090	+011	+128	+144	+166	+188
	-002	+014	+033	+048	+068	+090	+110	+128	+144	+168	+188
ave.	-002	+013	+032	+048	+067	+089	+109	+127	+143	+166	+188
F.	0	+011	+030	+046	+065	+087	+107	+125	+141	+164	+186
01*	-002	-001	-000	-002	-000	+002	+002	+004	+005	+008	+010
	-003	-002	-001	-000	-001	+002	+003	+005	+006	+008	+010
	-002	-000	-001	-002	+002	+002	+004	+005	+006	+008	+012
ave.	-002	-001	-001	-001	+001	+002	+003	+005	+006	+008	+011
F.	0	-001	-001	-001	+003	+000	+005	+007	+008	+010	+013
02*	-002	-004	-005	-006	-008	-008	-009	-011	-012	-012	-013
	-003	-004	-006	-007	-008	-009	-010	-010	-012	-012	-012
	-004	-006	-005	-006	-007	-009	-010	-010	-012	-012	-012
ave.	-003	-005	-005	-006	-008	-009	-010	-010	-012	-012	-012
F.	0	-002	-002	-003	-005	-006	-007	-007	-009	-009	-009
03*	-000	-010	-016	-024	-032	-040	-046	-052	-058	-062	-070
	-000	-012	-017	-024	-032	-040	-046	-052	-058	-062	-070
	-000	-012	-017	-024	-032	-040	-046	-052	-058	-062	-070
ave.	-000	-012	-017	-024	-032	-040	-046	-052	-058	-062	-070
F.	0	-012	-017	-024	-032	-040	-046	-052	-058	-068	-070

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
04*	+004	-000	-004	-007	-011	-012	-015	-018	-019	-022	-024
	+004	-000	-003	-007	-010	-012	-014	-017	-019	-022	-024
	+004	-000	-004	-006	-010	-012	-015	-020	-020	-021	-024
ave.	+004	-000	-004	-007	-010	-012	-015	-019	-019	-022	-024
F.	0	-004	-008	-011	-014	-016	-019	-023	-023	-026	-028
05*	-002	-008	-012	-020	-026	-032	-039	-042	-046	-051	-054
	-000	-006	-014	-019	-026	-031	-038	-040	-045	-050	-052
	-001	-009	-012	-020	-024	-031	-037	-040	-045	-048	-052
ave.	-001	-008	-013	-020	-025	-031	-038	-041	-045	-050	-053
F.	0	-007	-012	-019	-024	-030	-037	-040	-044	-049	-052
06*	-001	-010	-018	-024	-034	-040	-038	-054	-060	-067	-072
	-000	-010	-018	-024	-034	-040	-038	-054	-060	-067	-072
	-000	-010	-016	-026	-032	-040	-037	-056	-059	-068	-071
ave.	-000	-010	-017	-025	-033	-040	-038	-055	-060	-068	-071
F.	0	-010	-017	-025	-033	-040	-038	-055	-060	-068	-071
07*	-001	-006	-012	-019	-024	-032	-040	-046	-050	-054	-060
	-000	-006	-012	-019	-024	-032	-038	-044	-049	-054	-059
	-002	-006	-012	-020	-024	-031	-040	-044	-050	-054	-058
ave.	-001	-006	-012	-019	-024	-032	-039	-045	-050	-054	-059
F.	0	-005	-011	-018	-023	-031	-038	-044	-049	-053	-058

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
08*	-002	-006	-019	-014	-018	-022	-028	-034	-036	-040	-045
	-000	-006	-018	-015	-019	-023	-030	-032	-036	-040	-045
	-001	-005	-018	-015	-019	-023	-029	-033	-036	-040	-045
ave.	-001	-005	-018	-015	-019	-023	-029	-033	-036	-040	-045
F.	0	-004	-017	-014	-018	-020	-028	-032	-035	-039	-044
09*	+001	-002	-005	-010	-012	-016	-020	-024	-026	-030	-032
	-000	-002	-006	-010	-012	-016	-021	-024	-026	-028	-032
	+001	-002	-005	-010	-015	-016	-020	-023	-026	-029	-032
ave.	+001	-002	-005	-010	-013	-016	-020	-024	-026	-029	-032
F.	0	-003	-006	-011	-014	-017	-021	-025	-027	-030	-033
10*	+001	-010	-010	-016	-027	-033	-042	-050	-056	-064	-069
	-003	-006	-010	-022	-023	-034	-040	-052	-056	-064	-069
	-002	-006	-010	-022	-023	-034	-040	-052	-056	-064	-069
ave.	-001	-008	-010	-019	-026	-034	-041	-048	-057	-062	-069
F.	0	-007	-009	-018	-025	-033	-040	-047	-056	-061	-068
11*	-002	-007	-011	-016	-024	-028	-036	-040	-046	-048	-053
	-002	-007	-012	-017	-023	-027	-034	-041	-045	-050	-052
	-002	-006	-012	-016	-022	-027	-034	-039	-045	-050	-052
ave.	-002	-007	-012	-016	-023	-027	-035	-040	-045	-049	-052
F.	0	-005	-010	-014	-021	-025	-033	-038	-043	-047	-050

Gage No.	0	100	200	300	400	500	600	700	800	900	1000
					Load						
12*	-000	-006	-012	-022	-024	-030	-038	-042	-048	-054	-057
	-001	-006	-011	-018	-026	-030	-033	-042	-048	-052	-056
	-000	-008	-010	-018	-023	-030	-038	-042	-046	-052	-057
ave.	-000	-007	-011	-019	-024	-030	-036	-042	-047	-053	-057
F.	0	-007	-011	-019	-024	-030	-036	-042	-047	-053	-057
13*	+002	-003	-008	-013	-016	-022	-026	-030	-035	-040	-044
	+002	-003	-006	-012	-018	-023	-026	-031	-035	-040	-044
	+002	-002	-007	-012	-016	-022	-026	-030	-034	-039	-039
ave.	+002	-003	-007	-012	-017	-022	-026	-030	-035	-040	-042
F.	0	-005	-009	-014	-019	-024	-028	-032	-037	-042	-044
14*	-000	-006	-010	-014	-020	-024	-030	-034	-038	-042	-048
	-000	-005	-010	-013	-018	-026	-030	-034	-031	-040	-047
	+001	-006	-010	-014	-018	-025	-029	-036	-039	-042	-042
ave.	-000	-006	-010	-014	-018	-025	-030	-035	-039	-041	-046
F.	0	-006	-010	-014	-019	-025	-030	-035	-039	-041	-046
15*	-002	-008	-012	-016	-022	-028	-034	-037	-042	-047	-048
	-002	-008	-012	-017	-022	-027	-034	-037	-042	-046	-050
	-002	-007	-012	-018	-022	-027	-034	-037	-042	-046	-050
ave.	-002	-008	-012	-017	-022	-027	-034	-037	-042	-046	-049
F.	0	-006	-010	-015	-020	-025	-032	-035	-040	-044	-047

Gage	Load										
No.	0	100	200	300	400	500	600	700	800	900	1000
16*	-003	-008	-012	-018	-024	-028	-034	-038	-042	-048	-052
	-002	-009	-012	-018	-024	-028	-035	-038	-042	-048	-051
	-002	-008	-013	-018	-023	-028	-034	-038	-042	-047	-052
ave.	-002	-008	-012	-018	-024	-028	-034	-038	-042	-048	-052
F.	0	-006	-012	-016	-002	-026	-032	-036	-040	-046	-050
17*	-001	-008	-015	-022	-028	-036	-043	-050	-056	-060	-066
	-002	-008	-015	-022	-028	-036	-044	-048	-054	-060	-066
	-002	-008	-015	-021	-028	-036	-044	-050	-054	-060	-066
ave.	-002	-008	-015	-022	-028	-036	-044	-049	-054	-060	-066
F.	0	-006	-015	-020	-026	-034	-042	-047	-052	-058	-064
18*	-000	-006	-011	-017	-022	-030	-035	-040	-044	-050	-055
	-002	-006	-011	-016	-023	-028	-034	-040	-045	-048	-053
	-000	-006	-011	-017	-022	-029	-035	-038	-042	-049	-054
ave.	-001	-006	-011	-017	-022	-029	-035	-039	-044	-049	-054
F.	0	-005	-010	-016	-021	-028	-034	-038	-043	-048	-053
19*	-000	-005	-010	-016	-021	-025	-032	-038	-042	-046	-050
	-000	-006	-010	-016	-020	-025	-032	-036	-042	-048	-050
	-000	-006	-011	-016	-020	-026	-032	-036	-042	-044	-050
ave.	-000	-006	-010	-016	-020	-026	-032	-037	-042	-046	-050
F.	0	-006	-010	-016	-020	-025	-032	-036	-042	-046	-050

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
20*	+002	-002	-006	-010	-014	-018	-023	-026	-030	-034	-037
	+002	-003	-006	-010	-013	-018	-022	-026	-029	-032	-036
	+001	-002	-006	-010	-014	-018	-022	-026	-029	-032	-036
ave.	+002	-002	-006	-010	-014	-018	-022	-026	-029	-032	-036
F.	0	-004	-008	-012	-016	-020	-024	-028	-031	-034	-038
21*	+002	-003	-008	-012	-017	-022	-026	-030	-035	-039	-044
	+002	-002	-008	-012	-017	-022	-027	-030	-034	-038	-042
	+002	-003	-006	-011	-017	-022	-027	-030	-034	-042	-042
ave.	+002	-003	-008	-012	-017	-022	-027	-030	-034	-040	-042
F.	0	-005	-010	-014	-019	-024	-029	-032	-036	-042	-044
22*	+000	-004	-008	-014	-019	-026	-030	-036	-040	-046	-050
	+002	-005	-010	-014	-020	-023	-033	-036	-042	-046	-051
	+001	-006	-009	-015	-020	-026	-030	-036	-040	-046	-048
ave.	+001	-005	-009	-014	-020	-025	-031	-036	-041	-046	-050
F.	0	-006	-010	-015	-021	-026	-032	-037	-042	-047	-051
23*	-002	-008	-012	-020	-026	-031	-040	-044	-050	-055	-061
	-001	-007	-012	-019	-026	-032	-036	-044	-048	-056	-060
	-000	-006	-012	-020	-026	-031	-039	-044	-050	-056	-060
ave.	-001	-007	-012	-020	-026	-032	-038	-044	-049	-056	-060
F.	0	-006	-011	-019	-025	-031	-037	-043	-048	-055	-059

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
24*	+002	-006	-010	-015	-021	-026	-032	-030	-042	-046	-050
	-000	-006	-010	-016	-021	-027	-034	-037	-042	-046	-050
	-001	-006	-010	-016	-020	-026	-032	-038	-040	-046	-050
ave.	-000	-006	-010	-016	-021	-026	-032	-035	-041	-046	-050
F.	0	-006	-010	-016	-021	-026	-032	-035	-041	-046	-050
25*	+001	-000	-002	-003	-003	-004	-006	-006	-010	-010	-010
	+001	-000	-002	-002	-003	-004	-006	-008	-008	-010	-010
	+002	-000	-002	-003	-004	-005	-006	-006	-009	-010	-011
Ave.	+001	-000	-002	-003	-003	-004	-006	-006	-009	-010	-010
F.	0	-001	-003	-004	-004	-005	-006	-006	-010	-011	-011
26*	+001	-005	-010	-016	-020	-027	-034	-039	-044	-048	-052
	+001	-005	-011	-016	-021	-027	-034	-038	-044	-048	-052
	+001	-005	-011	-016	-022	-027	-034	-038	-042	-048	-052
ave.	+001	-005	-011	-016	-021	-027	-034	-038	-043	-048	-052
F.	0	-006	-012	-017	-022	-027	-035	-038	-043	-048	-053
27*	+001	-004	-008	-013	-018	-022	-027	-031	-035	-040	-044
	+001	-004	-008	-013	-018	-022	-027	-031	-034	-040	-043
	+001	-004	-008	-014	-016	-022	-027	-031	-034	-040	-042
ave.	+001	-004	-008	-013	-017	-022	-027	-031	-034	-040	-043
F.	0	-005	-009	-014	-018	-023	-028	-032	-035	-041	-044

Gage No.	Load 0	100	200	300	400	500	600	700	800	900	1000
28*	+004	-004	-010	-016	-023	-028	-036	-042	-047	-052	-056
	+002	-004	-010	-016	-022	-030	-036	-042	-046	-052	-056
	+003	-003	-010	-016	-022	-030	-036	-042	-048	-052	-057
ave.	+003	-004	-010	-016	-022	-039	-036	-042	-047	-052	-056
F.	0	-007	-013	-019	-025	-042	-039	-045	-050	-055	-059
29*	-000	-006	-010	-016	-021	-027	-032	-037	-040	-044	-052
	-000	-006	-010	-016	-021	-028	-032	-037	-040	-045	-050
	-000	-006	-011	-016	-020	-026	-032	-036	-040	-045	-050
ave.	-000	-006	-010	-016	-021	-027	-032	-037	-040	-045	-050
F.	0	-006	-010	-016	-021	-027	-032	-037	-040	-045	-050

Internal Column

30*	-000	-002	-004	-005	-006	-006	-009	-010	-011	-012	-014
	-000	-002	-002	-004	-006	-007	-008	-010	-011	-013	-014
	-002	-002	-003	-005	-006	-006	-009	-010	-011	-012	-014
ave.	-001	-002	-003	-005	-006	-006	-009	-010	-011	-012	-014
F.	0	-001	-002	-004	-005	-005	-008	-009	-010	-011	-013

Gage No.	0	100	200	300	400	500	600	700	800	900	1000
31*	-002	-005	-006	-008	-010	-012	-013	-014	-017	-018	-019
	-004	-005	-006	-010	-010	-011	-013	-015	-017	-018	-019
	-004	-005	-006	-008	-010	-012	-013	-014	-018	-018	-019
ave.	-003	-005	-006	-009	-010	-012	-013	-014	-018	-018	-019
F.	0	-002	-003	-006	-007	-009	-010	-011	-014	-015	-016
32*	-002	-002	-005	-006	-007	-009	-011	-011	-014	-014	-018
	-000	-004	-005	-006	-008	-009	-010	-011	-014	-015	-017
	-000	-002	-004	-006	-008	-010	-010	-012	-014	-015	-016
ave.	-001	-003	-005	-006	-008	-009	-010	-011	-014	-015	-017
F.	0	-002	-004	-005	-007	-008	-009	-010	-013	-014	-016
33*	+001	-004	-006	-008	-010	-012	-014	-016	-019	-019	-021
	-000	-004	-006	-008	-010	-012	-014	-015	-019	-020	-021
	-000	-003	-005	-008	-010	-012	-014	-016	-019	-019	-022
ave.	-000	-004	-006	-008	-010	-012	-014	-016	-019	-019	-021
F.	0	-004	-006	-008	-010	-012	-014	-016	-019	-019	-021
34*	-000	-004	-006	-010	-011	-014	-015	-016	-019	-020	-022
	+002	-005	-006	-010	-011	-013	-015	-017	-020	-020	-023
	-000	-004	-006	-010	-011	-013	-015	-017	-020	-019	-022
ave.	+001	-004	-006	-010	-011	-013	-015	-017	-020	-019	-022
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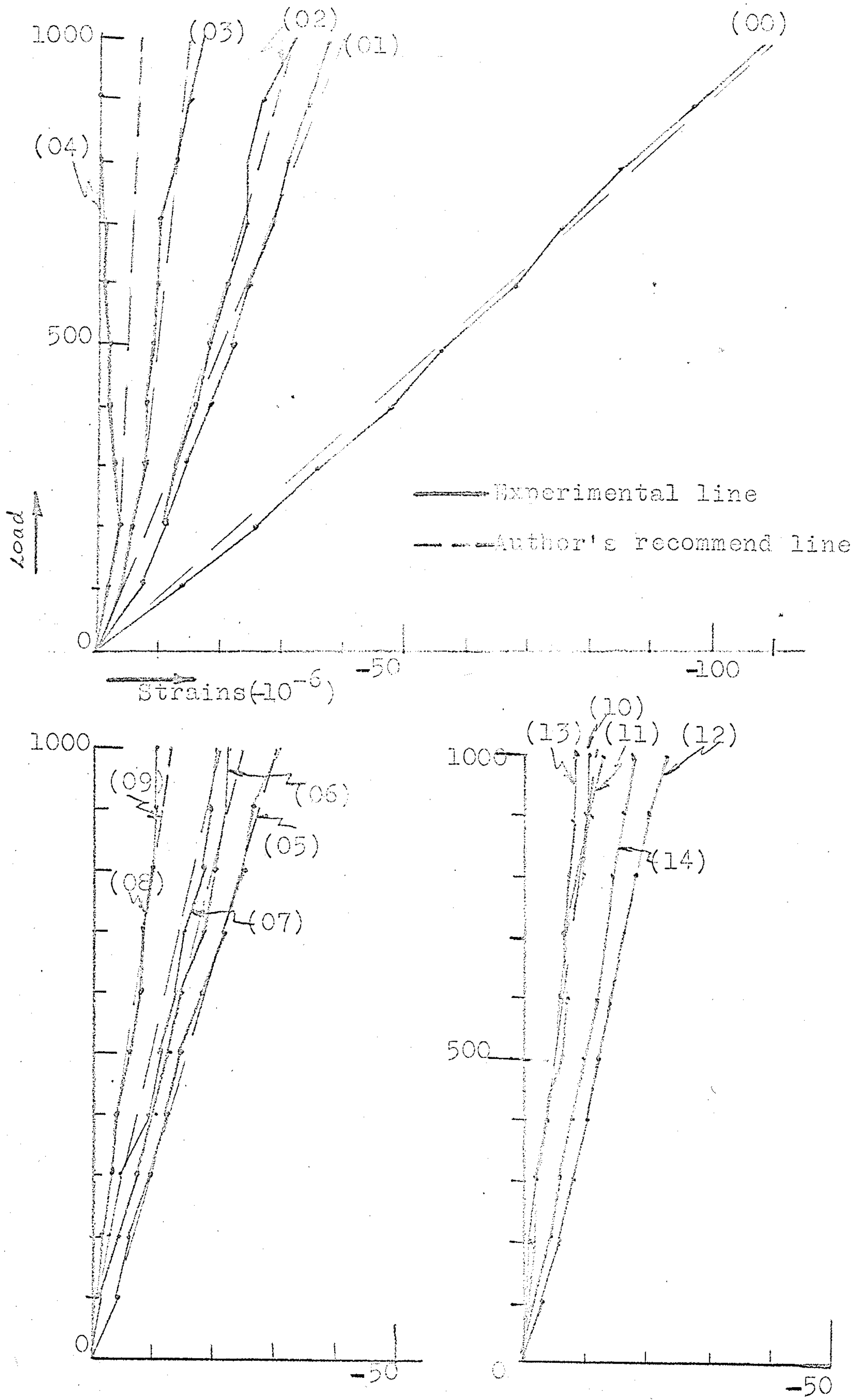
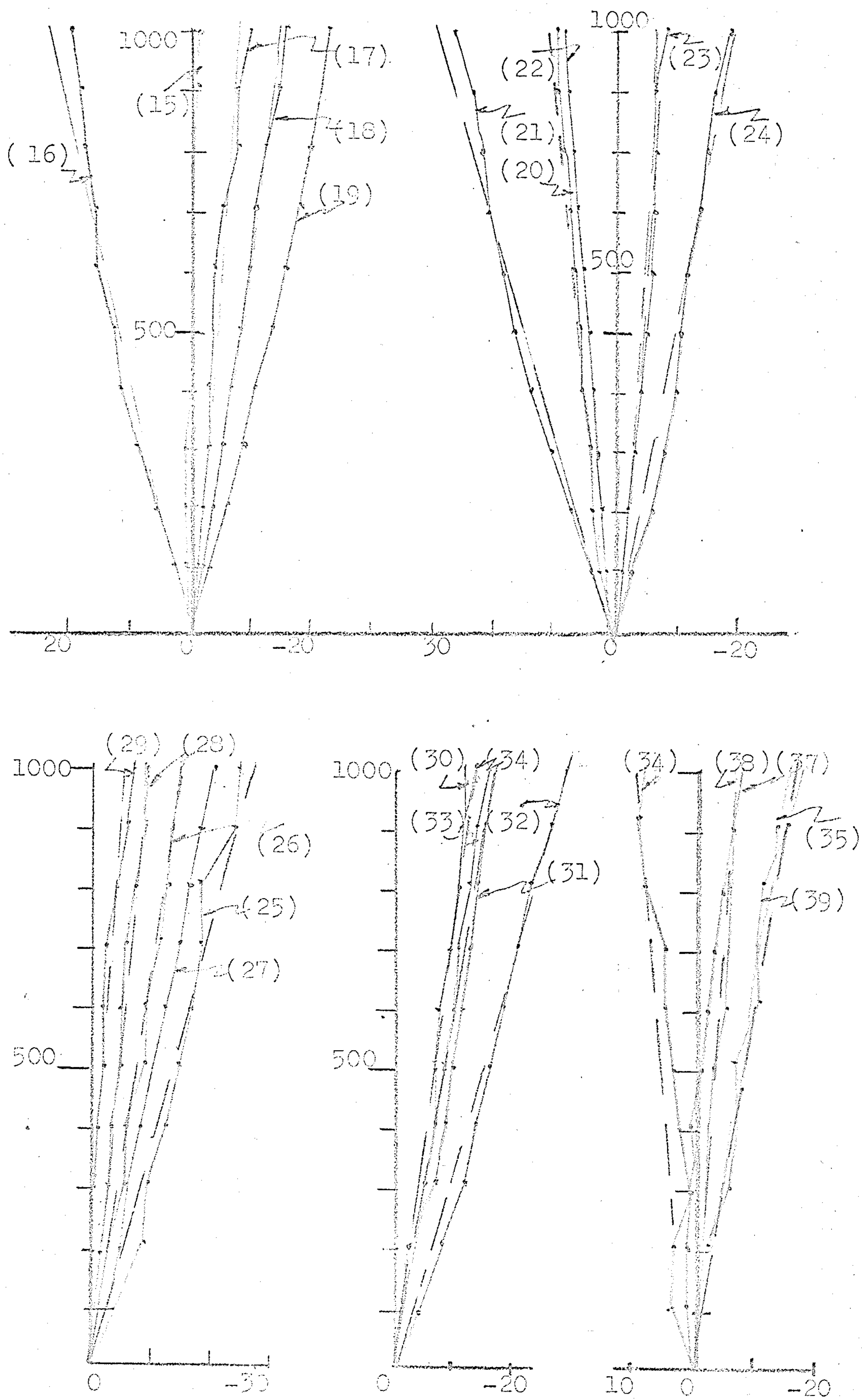
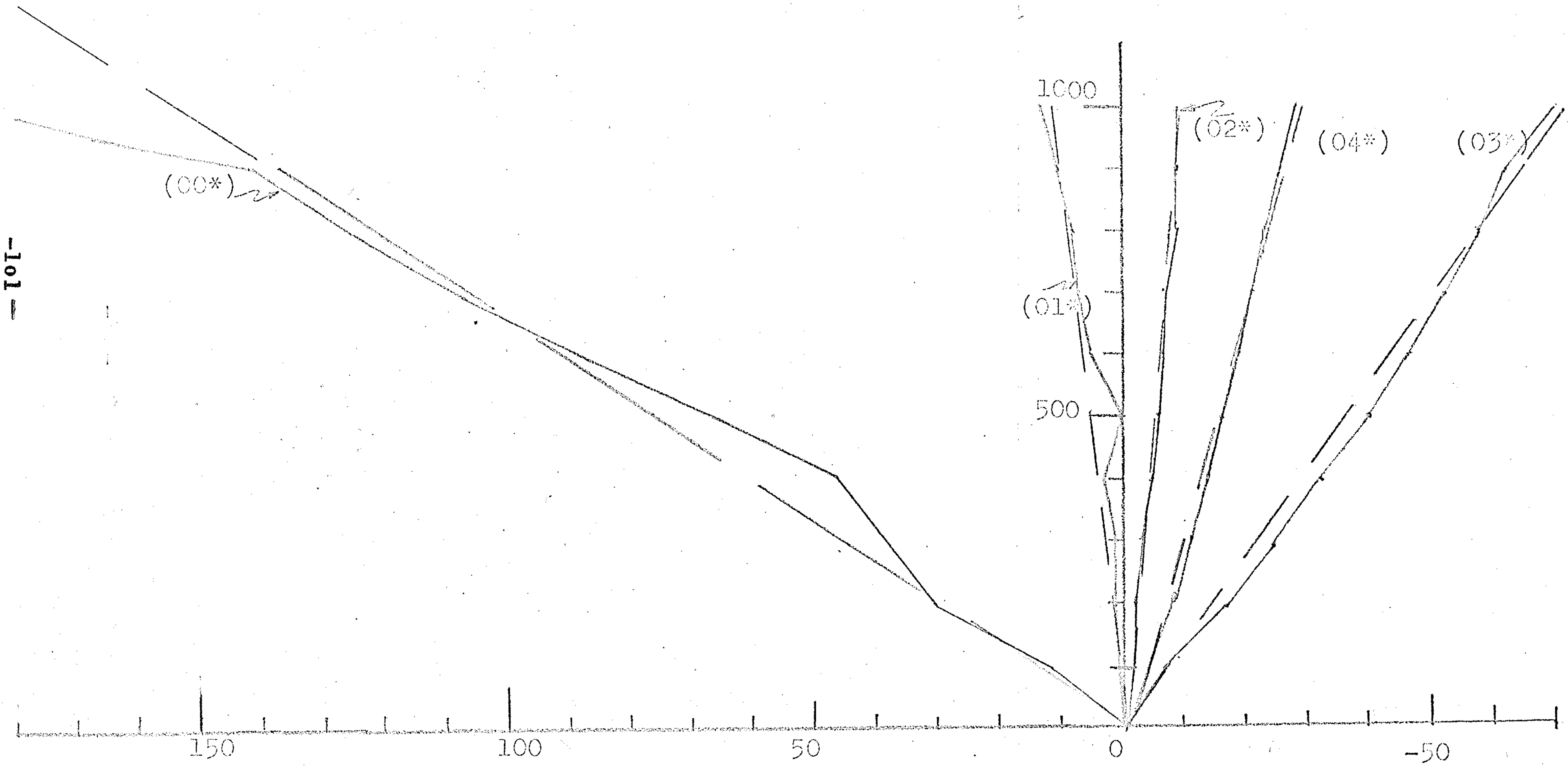
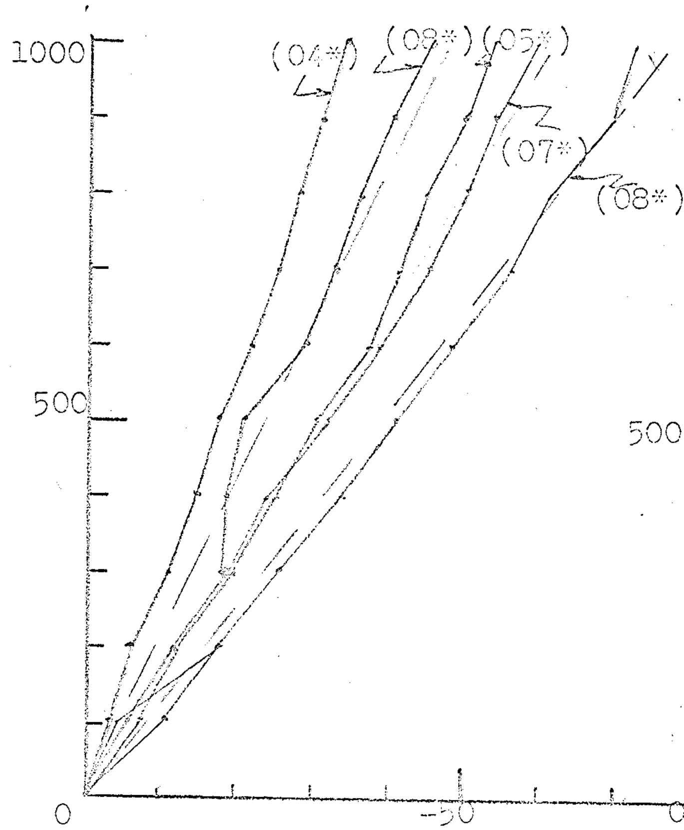
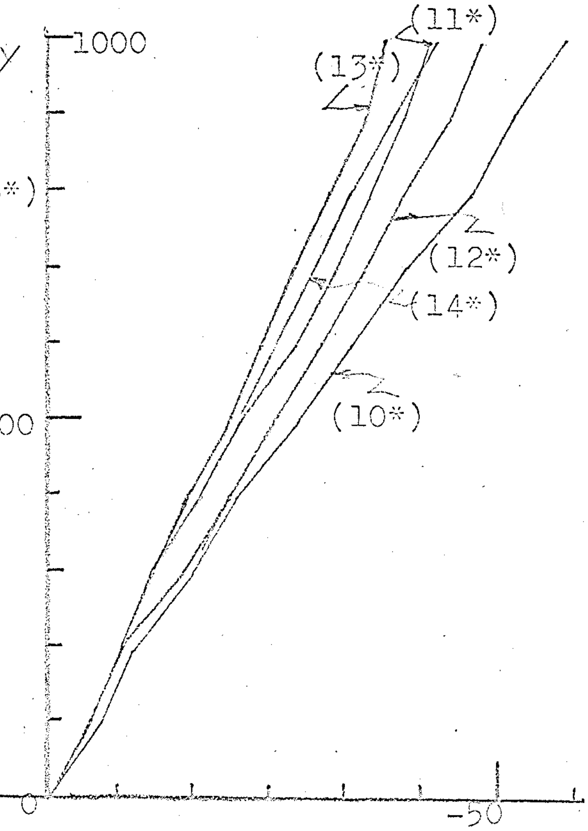


1000
500
0
(00)
(01)
(02)
(03)
(04)
Load
Experimental line
Author's recommend line
-50
-100
Strains(-10^-6)
(05)
(06)
(07)
(08)
(09)
(10)
(11)
(12)
(13)
(14)


1000
500
(15)
(16)
(17)
(18)
(19)
20
0
-20
1000
500
(20)
(21)
(22)
(23)
(24)
30
0
-20
1000
500
(25)
(26)
(27)
(28)
(29)
0
1000
500
(30)
(31)
(32)
(33)
(34)
0
-20
(34)
(35)
(37)
(38)
(39)
10
0
-20


1000
500
(00*)
(01*)
(02*)
(04*)
(03*)
150
100
50
0
-50


1000
500
0
-50
(04*)
(08*)
(05*)
(07*)
(08*)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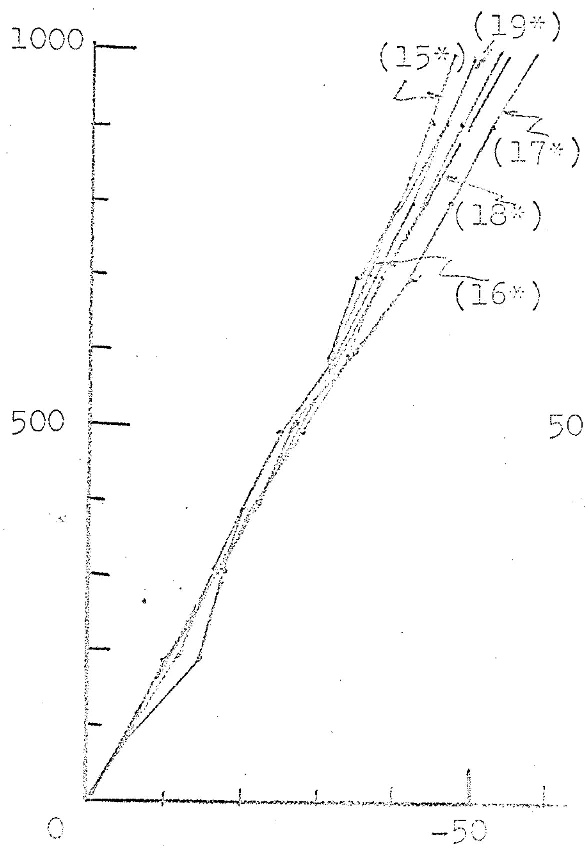
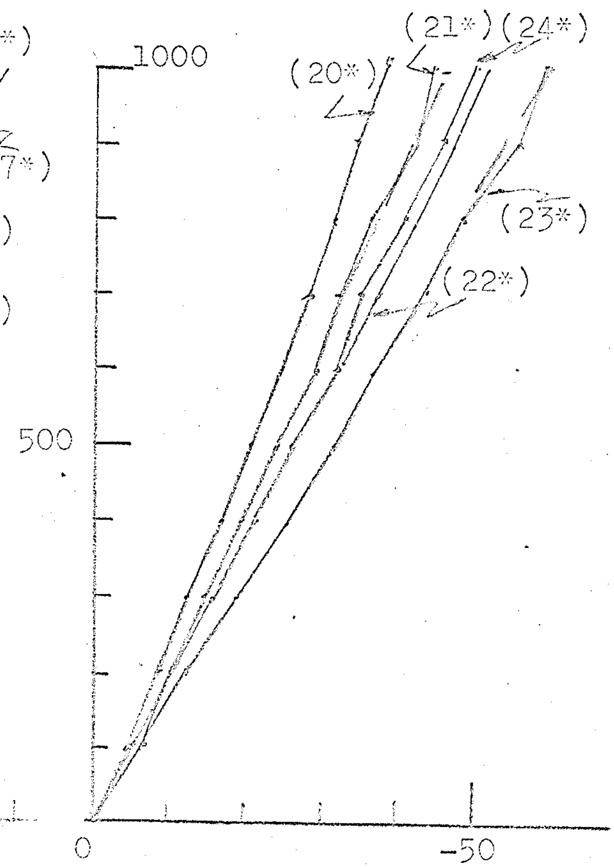
500
0
-50
(11*)
(13*)
(12*)
(14*)
(10*)


1000
500
0
-50
(19*)
(15*)
(17*)
(18*)
(16*)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500
0
-50
(21*)
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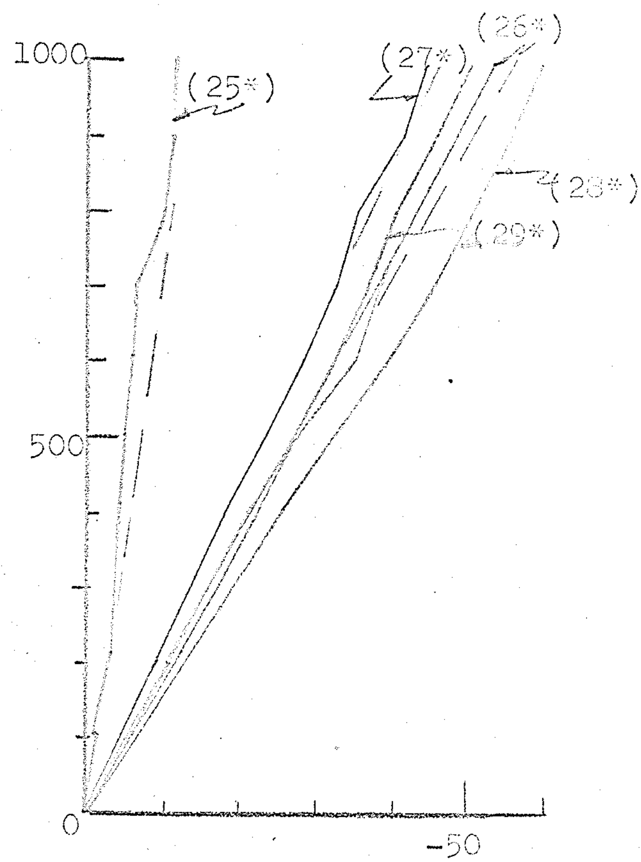
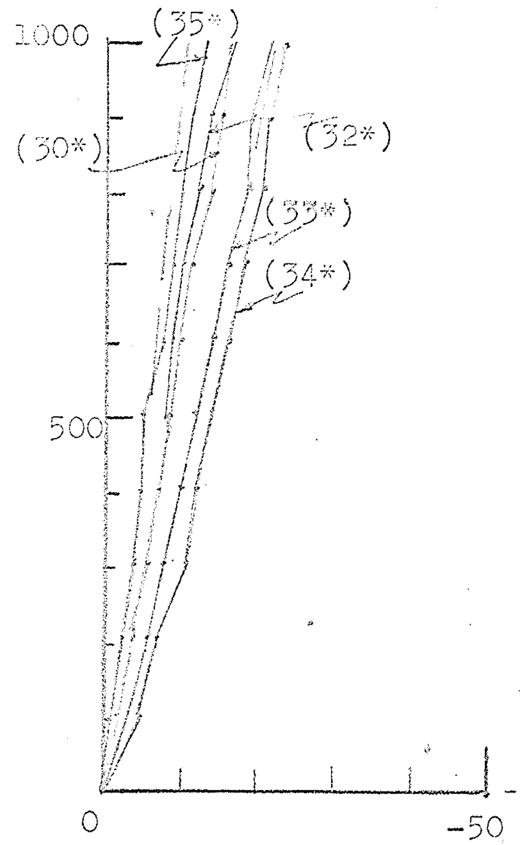
(20*)
(23*)
(22*)



1000
500
0
-50
(25*)
(27*)
(26*)
(28*)
(29*)



1000
500
0
-50
(35*)
(30*)
(32*)
(33*)
(34*)