

THE UNIVERSITY OF MANITOBA

IMPEDANCE CALCULATION OF A FAULTED LINE USING  
DIGITAL TECHNIQUES

by

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A THESIS

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### ABSTRACT

The object of this thesis was to develop and to investigate a method of impedance calculation using digital techniques with a view such that it could be used for distance protection.

The impedance of a faulted single phase line as seen from the relay point was calculated. A theoretical study was made using the digital computer in real time and simulating the sampling of current and voltage signals using an A/D convertor.

The transient behaviour of a faulted power system was studied and an attempt was made to eliminate the effect of these transients on the impedance calculation. At the same time the advantages of fast fault detection should be retained in order to alleviate the high cost of Digital Protection by offering higher load transfers from the same transmission lines without affecting the stability and the reliability.

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## CHAPTER I

### INTRODUCTION

Protective relays constantly monitor power systems to assure maximum continuity of electrical service with minimum damage to life and property. They are on guard throughout from the generation through transmission into distribution and utilisation. To perform these multifarious functions many type of relays must be used.

In the present text we are mainly concerned with distance protection, i.e. with impedance relays. Phase distance relays have come into use specifically because they offer higher selectivity than other types of relays for transmission protection.

The conventional electromagnetic types of impedance relays have been in use for many years and during this period improvements have been made to better their performance. The advent of electronic relays and more recently solid state relays has led to faster and more reliable methods of protection. However the conventional relays are still being used extensively.

Solid state relays have reduced the operating time to within 1 cycle. However recent developments in the computer

field have suggested that small real time special purpose computers can further reduce relay time. The main hoped for advantages of such a scheme are-

- a) The scheme could be a part of a larger scheme for complete automatic control of a substation, resulting in substantial economy.
- b) Since relays work only for infinitesimal periods during faults, they leave doubt as to whether they are capable of operation. Computers can be monitoring throughout.
- c) The relay action may be performed faster and in a more selective way, thereby decreasing the damage and increasing the reliability of supply.

Thus there is a great impetus for developing a fast and accurate method by which a digital computer can perform the function of relays. A few methods have already been suggested(15,18,19)\* ,but they all have serious limitations when dealing with transients just after the fault. In this thesis some new approaches have been suggested and critically examined for the measurement of impedance.

\* Refer to BIBLIOGRAPHY

## CHAPTER II

### METHODS OF POWER SYSTEM PROTECTION

2.1 Conventional Impedance Relays:- Conventional impedance relays are of the electromagnetic type. The torque produced by a current element is balanced against the torque of a voltage element. The current element produces positive (pickup) torque, whereas the voltage element produces negative (reset) torque. If the control spring torque is  $-k_3$ , then the torque equation is given by-

$$T = k_1 I^2 - k_2 V^2 - k_3$$

where  $I$  and  $V$  are the rms values of the current and voltage respectively. At the balance point, the net torque is zero. Therefore

$$k_2 V^2 = k_1 I^2 - k_3$$

or  $V/I = Z = \sqrt{\frac{k_1}{k_2} - \frac{k_3}{k_2 I^2}}$

If  $k_3$  is negligible, then we find that the relay is on the verge of operating when the ratio of  $V/I$  equals a fixed preset value of the impedance,  $Z$ .

However as the relay should trip in only one direction, it is made directional by the addition of a directional element. The construction of these relays and their

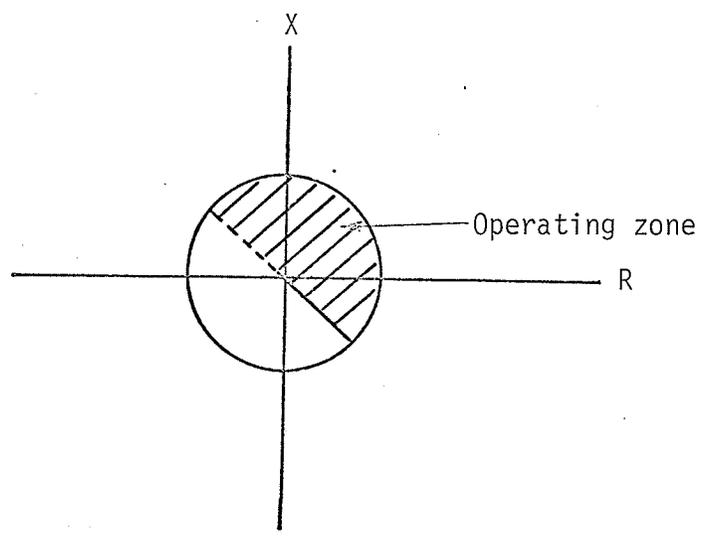


Figure 1: Relay characteristics on an R-X plane.

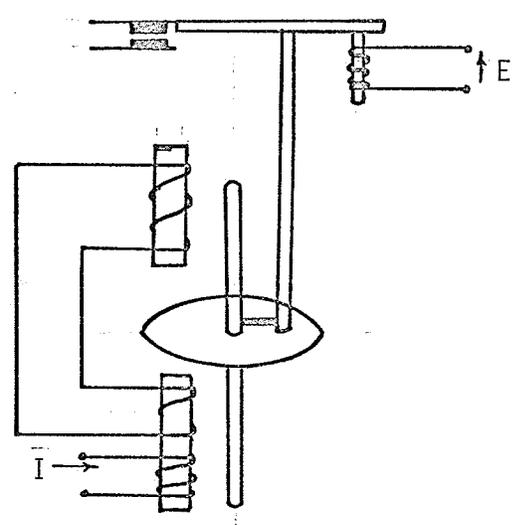


Figure 2: Induction units<sup>1</sup>

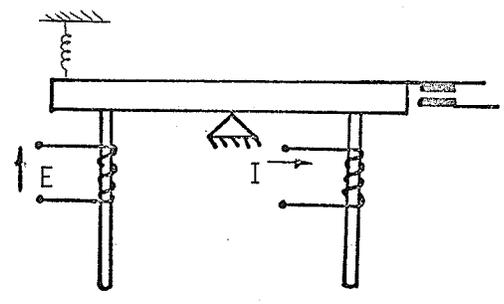


Figure 3: Magnetic attraction<sup>1</sup>  
type units

characteristics on an R-X diagram are shown in Figures 1, 2 and 3.

To use these relays for different zones of protection timing units are used, in conjunction with usual targets, seal in units, and other auxiliaries. The operating characteristics of each unit is independently adjustable.

These relays can operate within 1 or 2 cycles.

Recently reed relays<sup>20</sup> have also come into use as auxiliary relays. These are about 10 times faster than the conventional electromagnetic relay once they get the indication to operate. They have two accurately positioned flexible nickel-iron strips (reed) sealed into a closed glass capsule in an inert atmosphere. The inner ends of the reeds overlap and are separated by a gap of 0.01 inch. or less. A coil surrounds one or more of the reed contact units and produces a magnetic field, which makes the reed come together, thus operating the contacts. These contacts consist of wafers of royal metal welded to the inner ends of the reed.

Relays which have offset impedance characteristics are more commonly used. These are known as mho relays and have a torque equation-

$$T = k_1 VI \cos (\theta - \tau) - k_2 V^2 - k_3$$

Therefore at balance and neglecting  $k_3$

$$Z = k_1/k_2 \cdot \cos (\theta - \tau)$$

2.2 Static Relays:- These relays were initially developed

using tube circuits which were later replaced by transistorised circuits. Figures 4, 5a and 5b show the principle of both types. ( These diagrams show the earliest relays and since then many modifications to these circuits have taken place. )

Some properties of the different relays described so far can be compared as follows

Function	Electromagnetic	Reed	Semiconductor
Delay	10ms	1-2ms	20 $\mu$ s
Operations	10 <sup>7</sup>	10 <sup>7</sup>	no limit
Ambient Temp. Range	-5 <sup>o</sup> C to 70 <sup>o</sup> C	-5 <sup>o</sup> C to 55 <sup>o</sup> C	-20 <sup>o</sup> C to 100 <sup>o</sup> C

Some of the main advantages of static relays are as follows:-

a) Fast operation , long life and high resistance to shock and vibration.

b) A high resetting value and the absence of overshoot are easy to obtain in static relays because of the absence of mechanical inertia and thermal storage.

c) With the absence of bearing friction and contact troubles (corrosions bouncing and wear) better characteristics can be obtained and there is less necessity

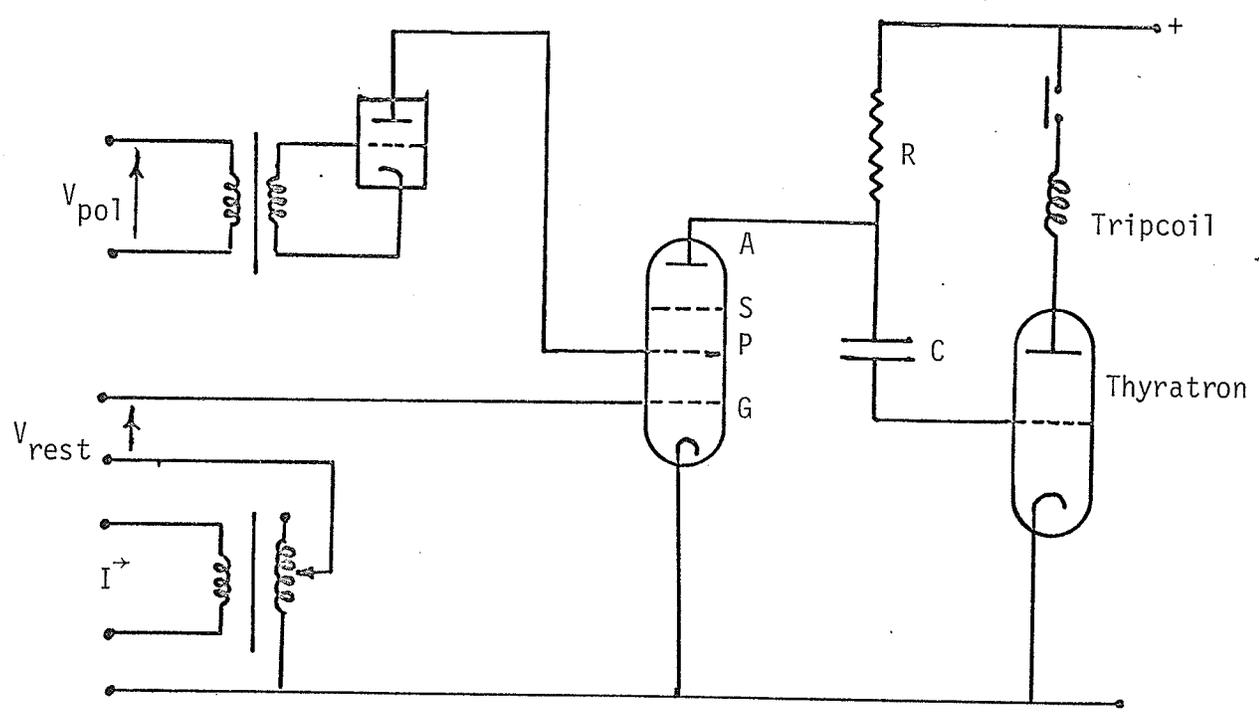


Figure 4: Tube version of an impedance relay<sup>16</sup>

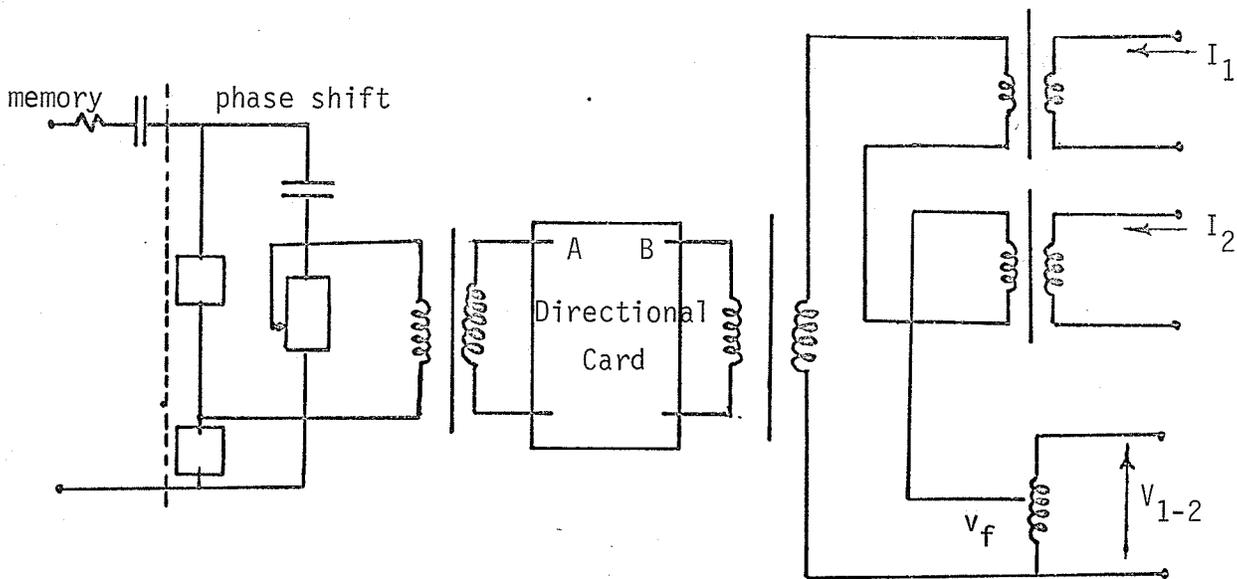


Figure 5a: Input circuit for block spike mho unit.<sup>7</sup>

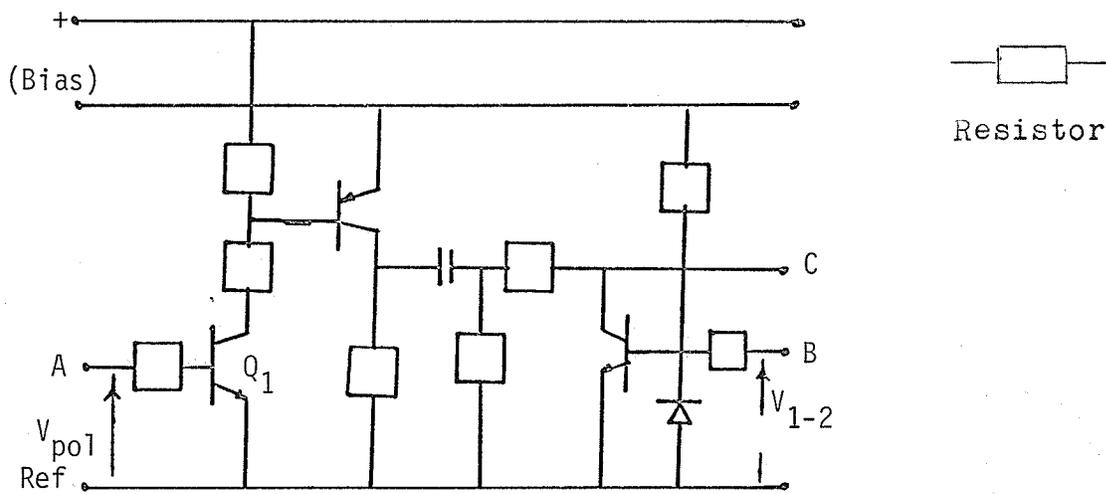


Figure 5b: Directional card of mho unit.<sup>7</sup>

for maintenance.

d) Very frequent operation causes no deterioration.

e) The ease of providing amplification enables greater sensitivity to be obtained.

f) The low energy levels in the measurement circuits permits miniaturisation of equipment and minimizes c.t. inaccuracies.

2.3 Protection by Digital Computer:- With the possibility of the development of fast vacuum type breakers within a few years, pressure will be exerted upon protection engineers to produce even faster relays.

To achieve this end digital computers seem to provide the answer. Hence new methods are being suggested to programme the Computer to perform the function of a relay. We now discuss some of the methods which can be used to perform the function of an impedance relay.

In all the methods described the current and voltage signals are sampled at a predetermined rate and converted into digital signals by an analog to digital<sup>al</sup> convertor as shown in Figure 7a.

2.31 The RMS Method:- In this method the sampled values are collected over one cycle and their rms value is

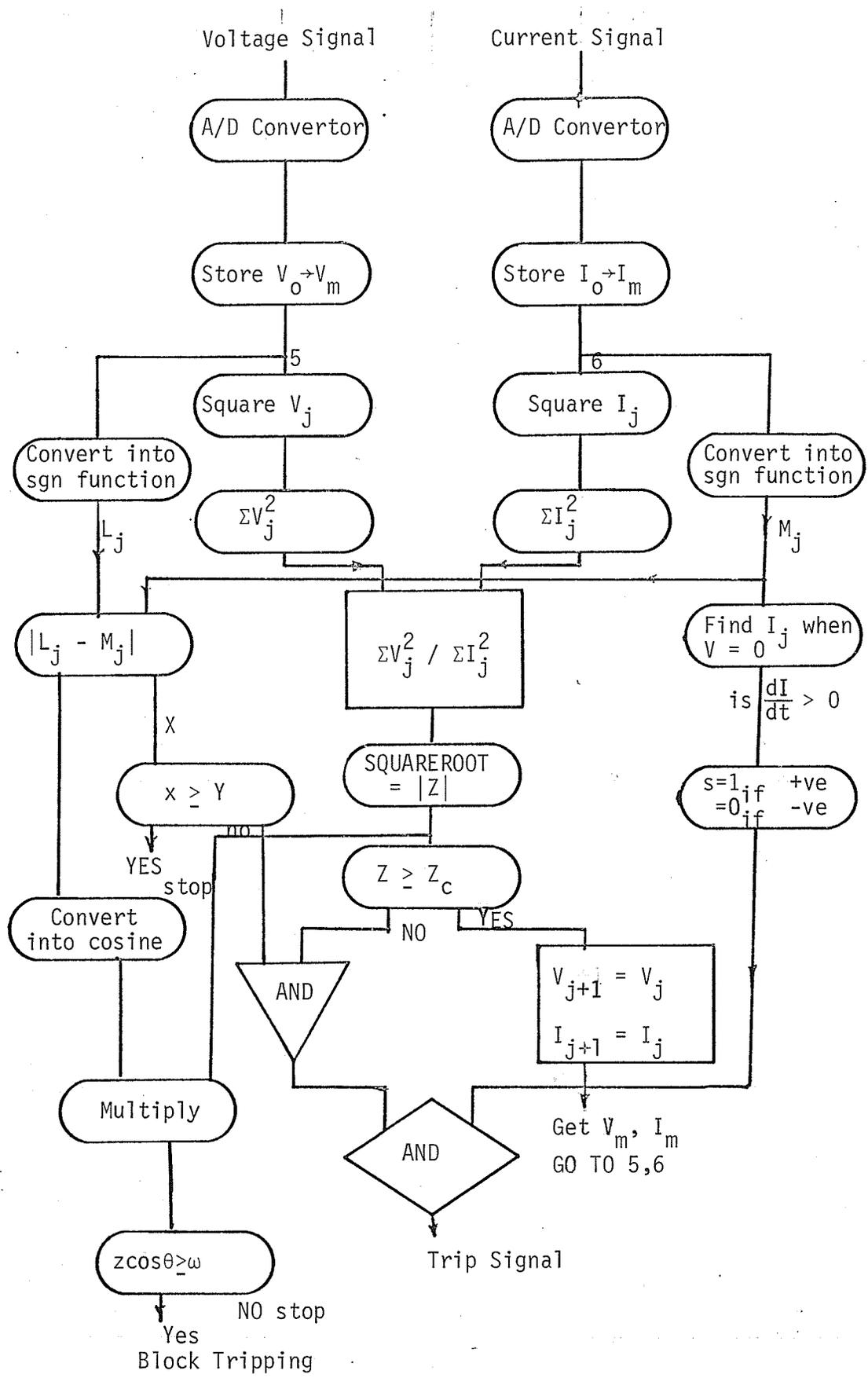


Figure 6: Block diagram for RMS digital relay

calculated. The zero crossover for both signals is determined and this provides an indication of the phase angle. The block diagram representation of the method is shown in Figure 6.

2.32 First Difference Method:- This method is very similar to the method described in 2.31 except that instead of finding the rms values of the current and voltage signals their first difference is used. Thus the effect of d.c. transients in the current and voltage signals is minimized as shown in Figure 7b & 7c.

2.33 Fourier Analysis Method:- The input signals of current and voltage are sampled and stored over one period of the fundamental frequency. Assuming the input waveform to be repetitive over this period, fourier analysis techniques are applied to the sampled values of the input to obtain the fundamental amplitude and phase. The fundamental quantity  $F_1(t)$  is given by:-

$$F_1(t) = (a_1^2 + b_1^2)^{1/2} \sin(\omega t - \tan^{-1} a_1/b_1)$$

where  $\omega$  is the fundamental angular frequency, and

$$a_1 = x/2\pi (f_0 + 2f_1 \cos x + \dots + 2f_{n-1} \cos(n-1)x + f_n \cos nx)$$

$$b_1 = x/2\pi (2f_1 \sin x + 2f_2 \sin 2x + \dots + 2f_{n-1} \sin(n-1)x)$$

where  $x$  is the sampling interval in radians,  $n=2\pi/x$ , and  $f_0, f_1, \dots$  are the amplitudes of the input wave form at the

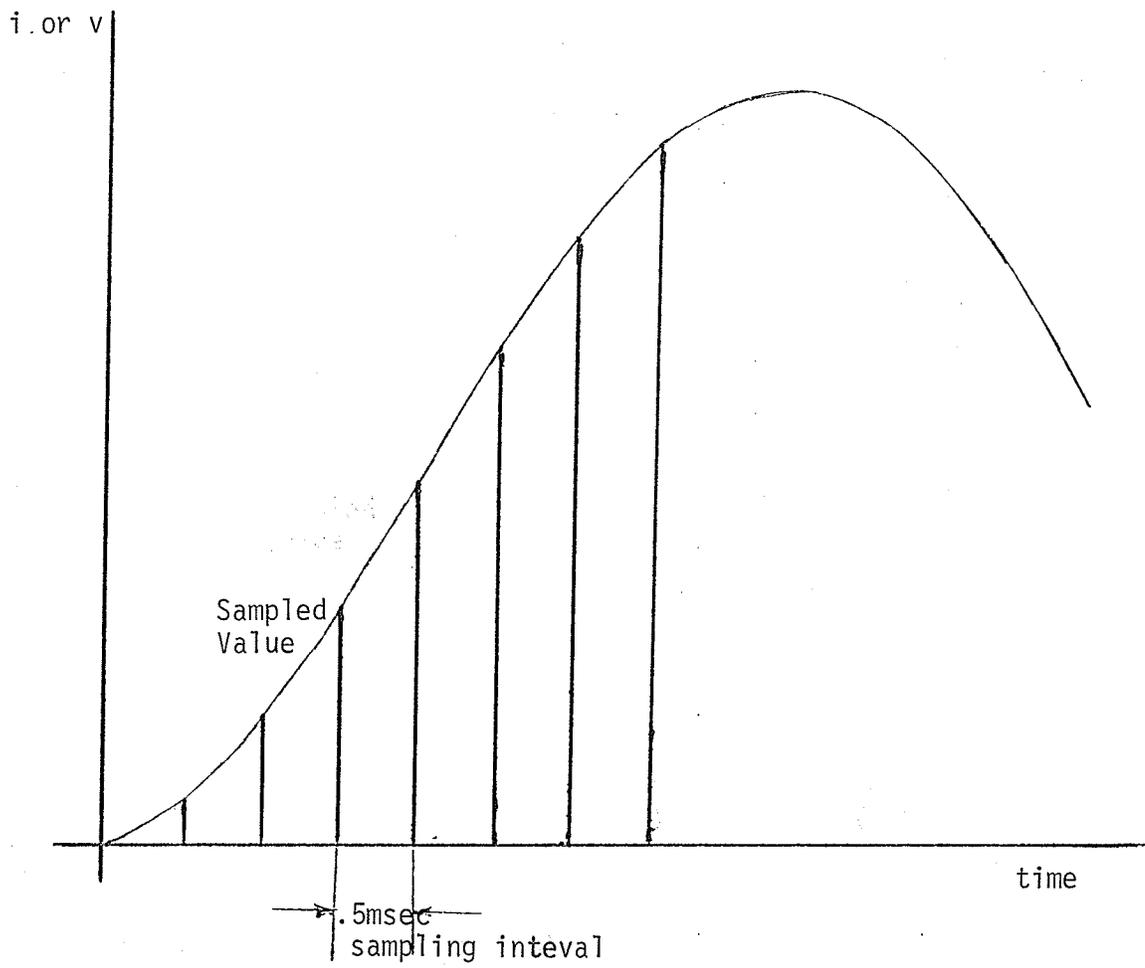


Figure 7a: Analog to digital conversion.

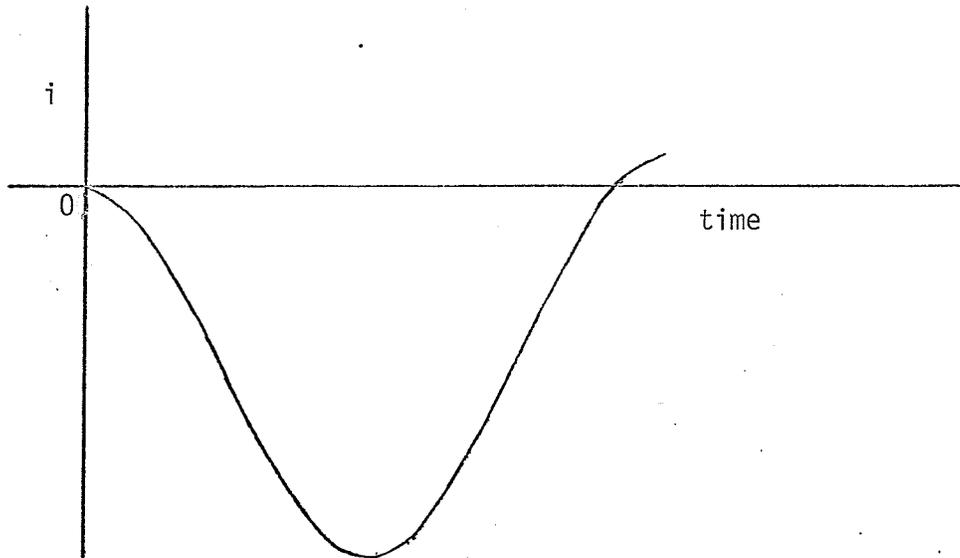


Figure 7b: A completely offset wave.

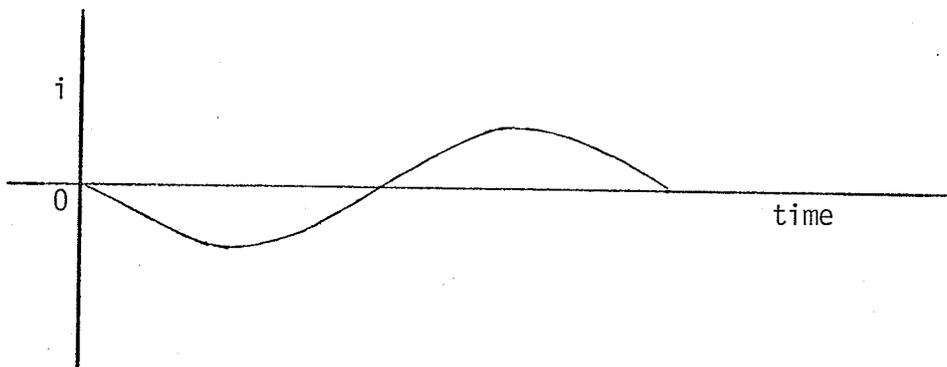


Figure 7c: First difference of the offset wave.

sampling instants

2.34 Mann-Morrison Method:- In this method 2 samples of each signal are used to obtain the values of  $v, v', i, i'$ . This is shown in Figure 7 d. The process continues for 3 successive samples. These values are then averaged and the central difference expression for derivatives can be used to find the peak value of current and voltage. The necessary formulae are-

$$\text{Peak Voltage: } V_{\max}^2 = V^2 + \left(\frac{v'}{\omega}\right)^2$$

$$\text{Peak Current: } I_{\max}^2 = i^2 + \left(\frac{i'}{\omega}\right)^2$$

Point on the cycle of voltage sample:-

$$\theta_v = \arctan \frac{\omega V}{v'}$$

Point on the cycle of current sample:-

$$\theta_i = \arctan \frac{\omega i}{i'}$$

Impedance Modulus:  $Z^2 = V_{\max}^2 / I_{\max}^2$

Impedance Argument:  $\theta_Z = \theta_v - \theta_i$

Therefore both the magnitude and the phase can be determined.

2.35 3 Sample Method:- This method which is introduced in this thesis also uses 3 samples of both signals at any time.

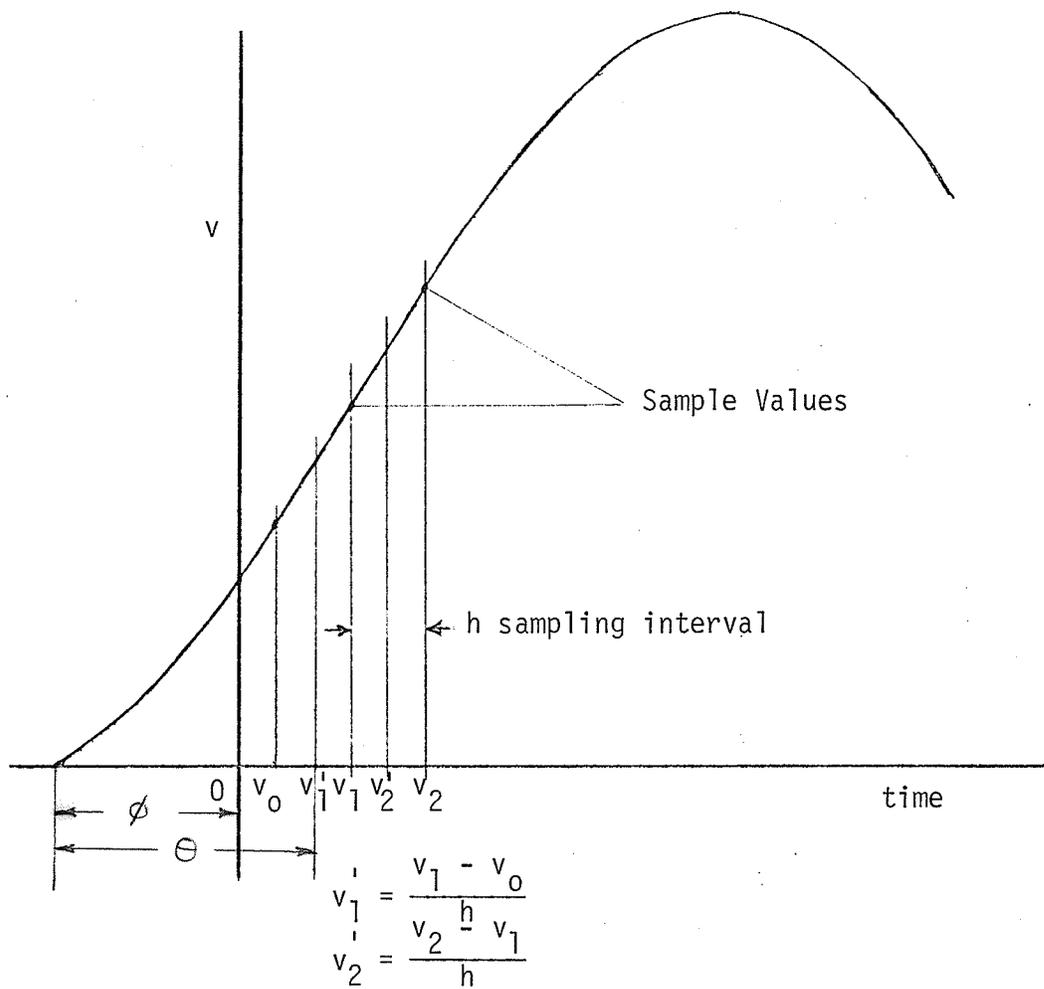


Figure 7d: Evaluation of first differential.

From these 3 samples the successive derivatives are calculated and then by using the following equations the magnitude and phase of the individual signals are obtained

$$v_1' = (v_1 - v_0)/h = \omega V_{\max} \cos(\omega 2h/2 + \phi)$$

$$v_2' = (v_2 - v_1)/h = \omega V_{\max} \cos(\omega \cdot 4h/2 + \phi)$$

where  $h$  is the sampling interval. (It has been assumed that sampling starts  $h/2$  sec. after the initiation of the fault. The validity of this assumption has been checked in section 3.45.) Then the magnitude and phase of a signal is calculated as follows:-

$$\tan \theta = \tan(\phi + h\omega) = \frac{-(v_2 - v_1)/(v_1 - v_0) - \cos h\omega}{\sin h\omega}$$

$$\text{Phase} = \theta - h\omega = \phi$$

$$V_{\max} = (v_1 - v_0)/\cos \theta \cdot \omega$$

Similar calculations are also performed with the current samples. The magnitude and phase of impedance is calculated by-

$$\theta = \text{Phase}(V) - \text{Phase}(I)$$

Thus  $Z = V_{\max}/I_{\max}$  and by using the following equations-

## CHAPTER III

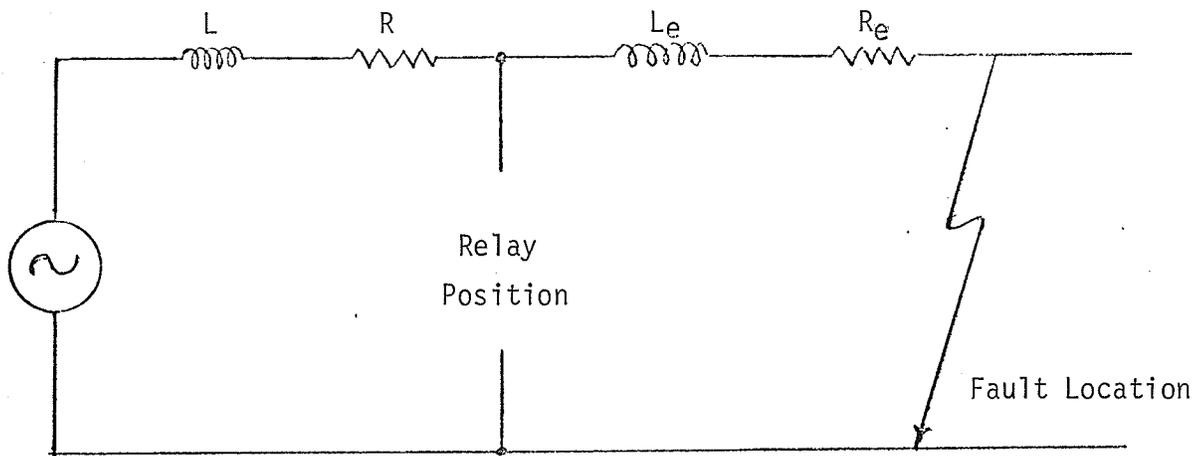
### TRANSIENT BEHAVIOUR OF RELAYS

#### 3.1 Behaviour of Currents and Voltages in Power Systems

during Post fault Conditions:- In order that the effect of these transients on the operation of relays may be predicted a detailed study of the post fault conditions is a necessity. This has become very important because the faster relays may be more affected by these short lived transients.

Consider the circuit of Figure 8. When a fault occurs, the current and voltage at the relay point undergo a sudden change in amplitude or phase angle or both. This sudden change causes the sinusoidal current to be offset by a decaying dc component whose amplitude depends upon the moment in the cycle at which the fault occurs and whose duration increases with  $L/R$  ratio (time constant) of the circuit as shown in Figure 9a. If the system impedance is not homogeneous there will be a separate transient in the potential due to the sudden change in phase angle of  $V$  to the angle of  $I Z$  when the fault occurs. However the presence of load current before the fault also modifies the transients and the effect is shown in Figure.9b.

Thus for the theoretical transients we can write the following equations-



$$\phi_L = \tan^{-1} (L+L_e)/(R+R_e)$$

$$\phi = \tan^{-1} (L_e/R_e)$$

Figure 8: Schematic diagram of basic system.

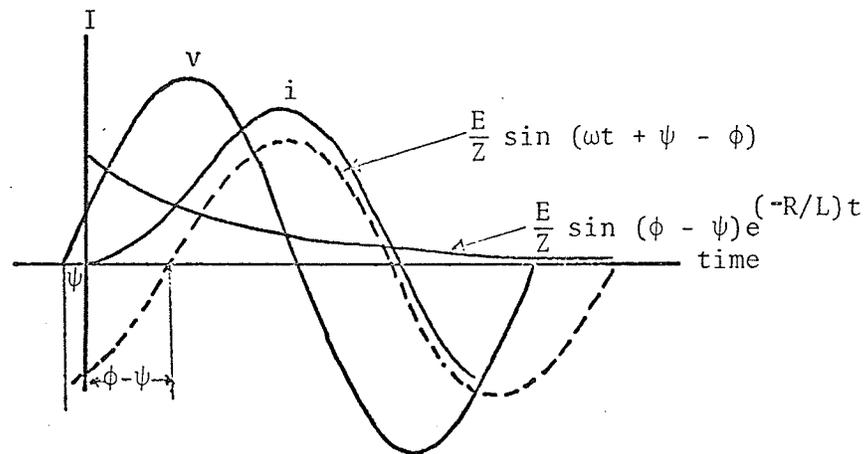


Figure 9a: Fault current transient.<sup>20</sup>

$$i = \frac{E}{Z+Z_e} \{ \sin(\omega t + \psi - \phi) \} + V \left\{ \frac{\sin(\psi + \theta)}{Z + Z_e + Z_L} - \frac{\sin(\psi - \theta)}{Z + Z_e} \right\} e^{-(R/L)t}$$

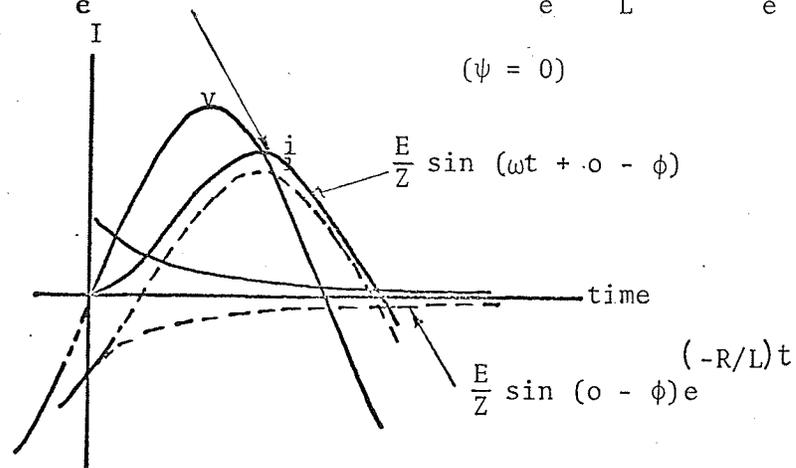


Figure 9b: Effect of previous load current.<sup>20</sup>

1. Current Transients:- Disregarding shunt loads and capacitances the instantaneous value of the current after the fault is given by-

$$i = I_{\max} (\sin(\omega t + \psi - \phi) + e^{-(R/L)t} \sin(\psi - \phi))$$

2. Potential Transient:- The voltage at the relay point after the fault can be written as:-

$$v = V_{\max} \cdot Z/Z_S (\sin(\omega t + \psi + \phi_L - \phi) - \frac{\sin(\phi - \phi_L) \cdot \sin \psi e^{-(R/L)t}}{\sin \phi})$$

Apart from the above theoretical discussion, staged fault tests on the Manitoba Hydro ( see Appendix B ) system have shown that current and voltages also contain some high frequency transients. These die out quite rapidly, as can be seen from the recordings . These high frequency transients were also obtained when improved models were used for power system analysis in a study by Slemon et al. (19).

Therefore the study of the effect of transients on the various impedance relays described so far is divided into two parts, one dealing with simple dc transients and the other dealing with the actual recordings. In the latter case the study has been limited to only the cases for which the test results were available.

In dealing with the ideal dc transients the following assumptions were made:-

1. The ct's and pt's transform the effects of dc transients exactly.

2. The time constant is such that for high speed relaying there is no ac decrement and in this case the magnitude of the current and voltage is given by the subtransient reactances. This assumption is made only when analysing the conventional relays.

Only the worst case was considered for studying the effects of these transients, i.e. the current wave contains a full dc offset while the voltage wave has no offset.

3.2 Behaviour of Conventional Relays to Transients:- The conventional relays sense the rms values of current and voltage and from these sense the value of impedance. Thus with the help of the assumptions made above we can calculate the rms values of transient current and voltages mathematically. The second assumption made above can be defended by the fact that the operating time of modern conventional relays is of the order of 20 ms where as the time constant of a modern power system is of the order of 100 ms.

The maximum offset of the current wave occurs when

$$(\psi - \phi) = \pi/2$$

$$i = V_{\max}/Z (\cos \omega t - e^{-t/\tau})$$

and for minimum voltage offset  $\phi = \phi_L$ , hence

$$v = V_{\max}/Z_s (\sin(\omega t + \phi_L))$$

Integrating these values over one cycle and then calculating the impedance we observe that the maximum overreach possible is 73%.

However the effect of these relays with actual staged faults cannot be studied as it was not possible to accurately carry out the integration of the recorded values since the scale of the recordings was too small.

3.3 Behaviour of Static Relays with Transients:<sup>10</sup> Some of the static relays employ rectifiers for instantaneous comparison and some use the block spike principle. In either case they do not lend themselves to an analytical approach because of the difficulty of formulating an analytical expression.

However it is easy to see how dc transients will affect the relay operation. Consider an extreme case where the current wave is completely offset. The rectified wave will have a much higher average value. Similarly in the phase comparison technique, the pulse will be generated at a different instant and also may be generated when it should not.

3.4 Behaviour of Digital Methods with Transients: The behaviour of the five methods previously discussed was studied for ideal transients and also for the cases for

Which the staged fault recordings were available. The response of these methods is shown graphically in Figures 10 to 25. Only two representative cases are shown. The sampling rate of the system is taken to be .5 msec\* and in all 40 samples have been considered in this study. The main characteristics of each method are discussed below.

3.41 RMS Method:- In this method the result of fault impedance will be available only after 33 samples, i.e. the value of impedance will be calculated after 1 cycle. The value for 8 consecutive samples are shown in Figures 10 to 12. It is observed that the value of the impedance does not vary much from sample to sample but the value differs from the true value by a considerable amount. The value calculated for the actual staged faults is almost constant over the samples (fig.12) and also the variation from true value is not much. This is mainly due to the fact that dc transients are not of very high magnitude. For this method the phase was not calculated because the calculation would have been based on the zero crossover principle and therefore the calculated phase would always be 180 degrees.

3.42 First Difference Method:- This method is actually a refinement of the first method in an effort to reduce the error due to dc transients. As expected the impedance calculation is affected very little by the long time constants; however small time constants produce

\* throughout this study

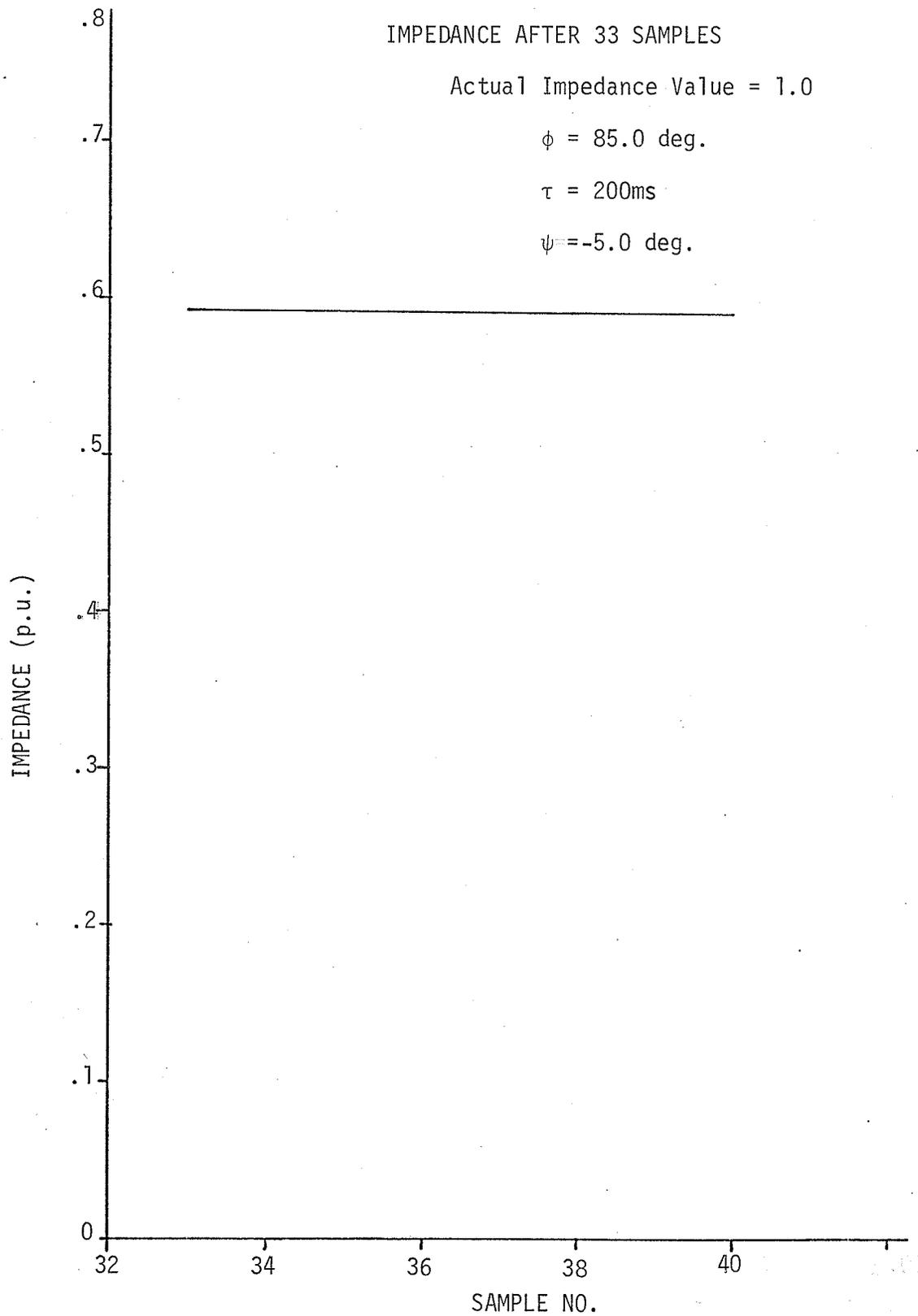


Figure 10: RMS value method.

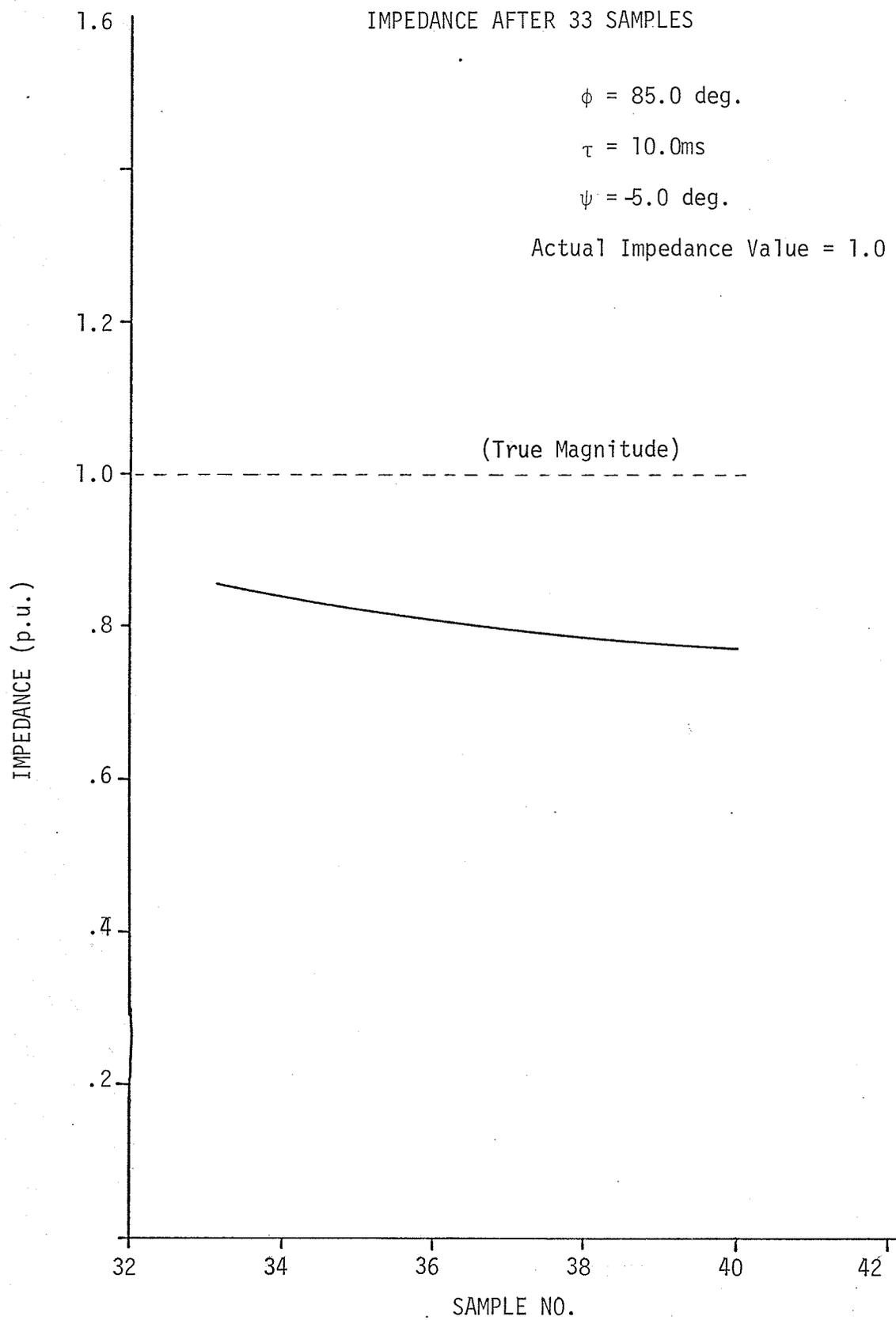


Figure 11: RMS value method.

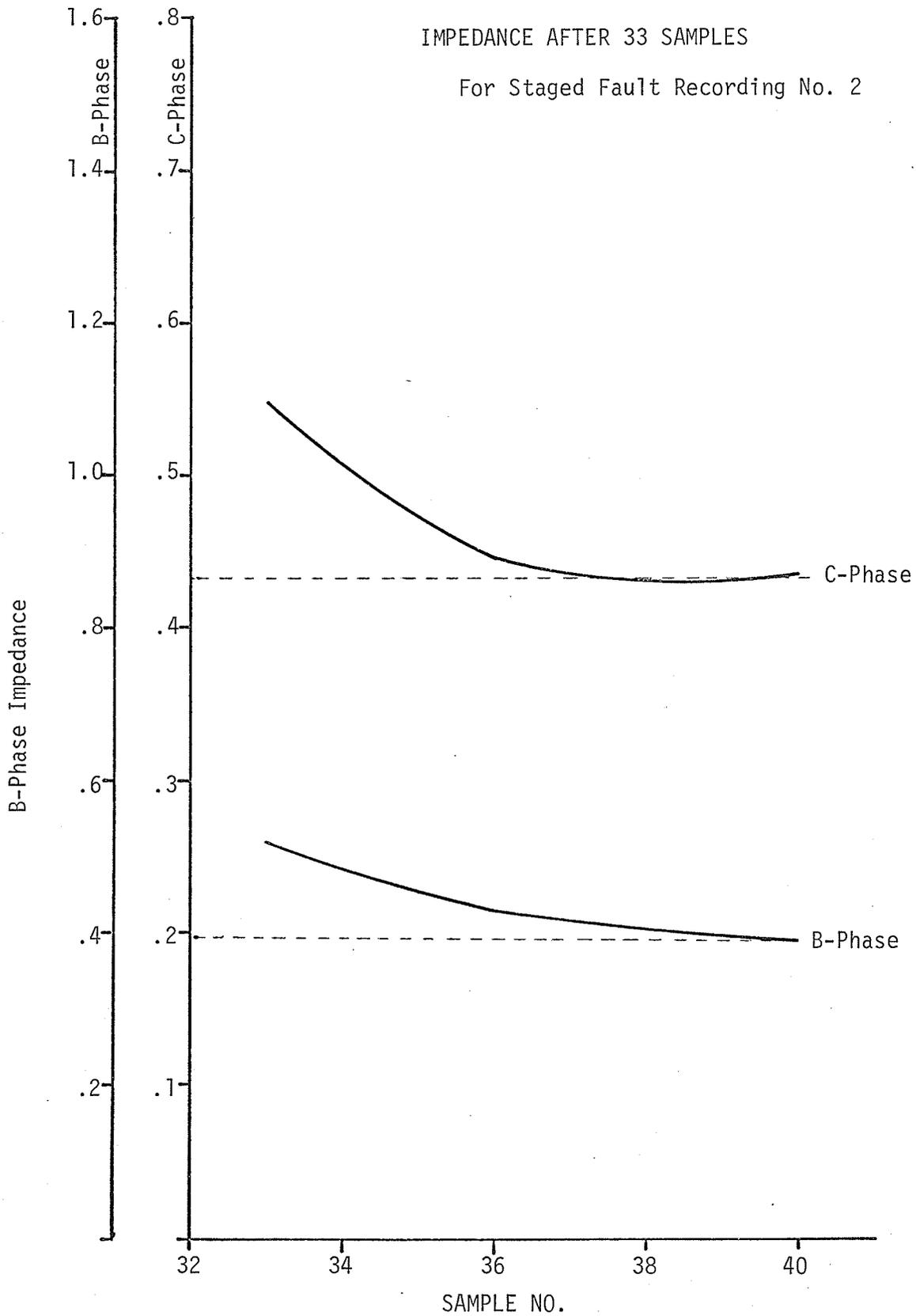


Figure 12: RMS value method.

significant errors. The impedance values are available after 17 samples and are shown in figures 13 to 15. The method is found to be of good accuracy in comparison with the other methods as far as the calculation of impedance magnitude is considered but gives incorrect values of phase angle with high frequency transients as expected.

3.43 Fourier Analysis Method:- This Method(19) was suggested to combat the transient errors for high speed protection of power systems. The method seemed very promising however detailed results show that though the impedance calculated is almost constant over the samples it is very much different from the actual value as shown in figures 16 to 18. However the method seems to be good for high frequency transients.

One major disadvantage of the method is that it requires extensive calculations for each set of samples and hence is very slow..

The methods discussed so far require either 17 or 33 samples before a calculation of the fault impedance is made, and since 1 cycle relaying is now quite common little advantage would be gained in using these methods. Therefore the following two methods which require only 3 samples before they start calculating the impedance of the faulted line seem more promising.

3.44Mann-Morrison Method:-This method is more inaccurate with long time constants than with short time constants.

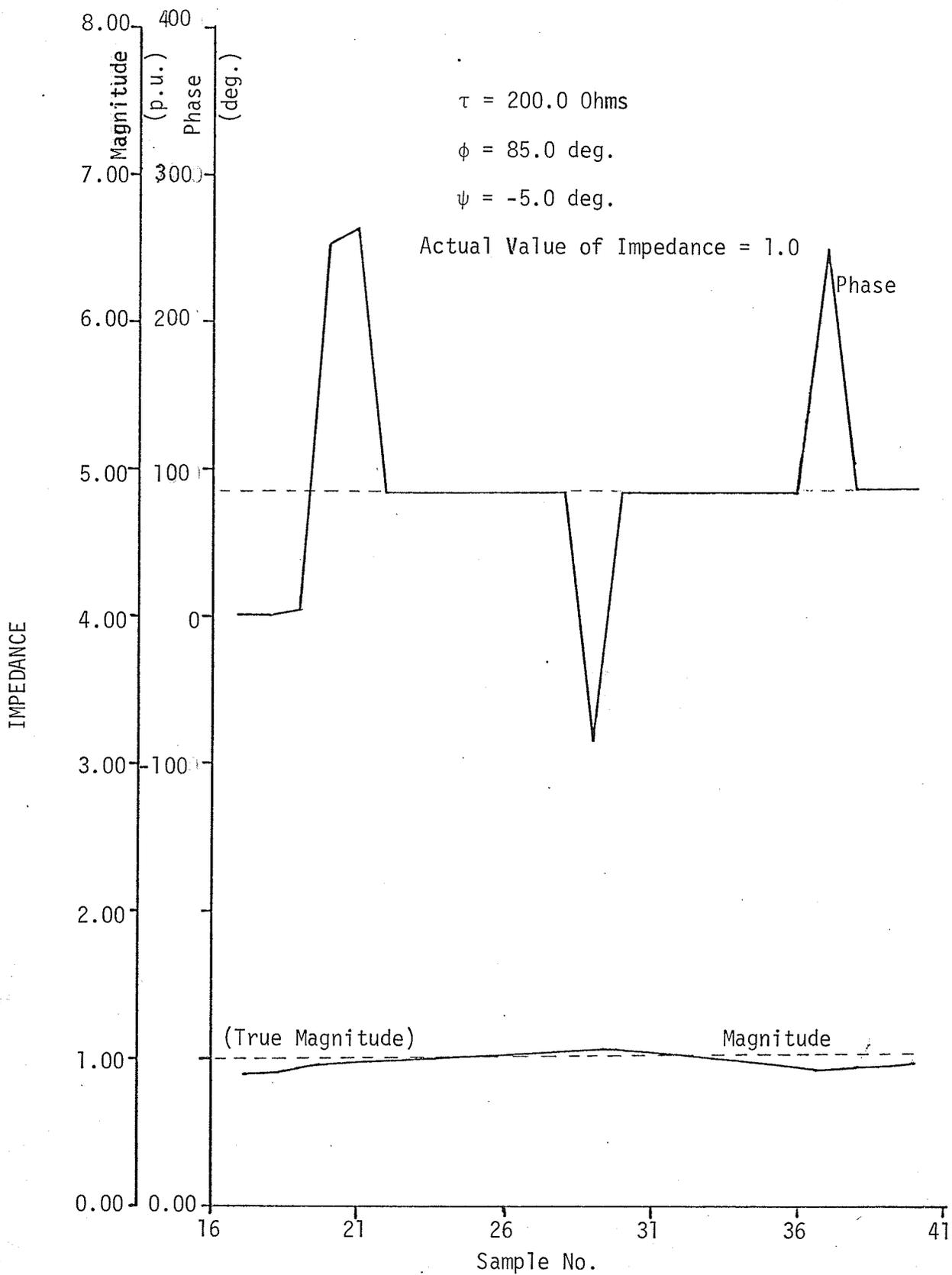


Figure 13: First difference method.

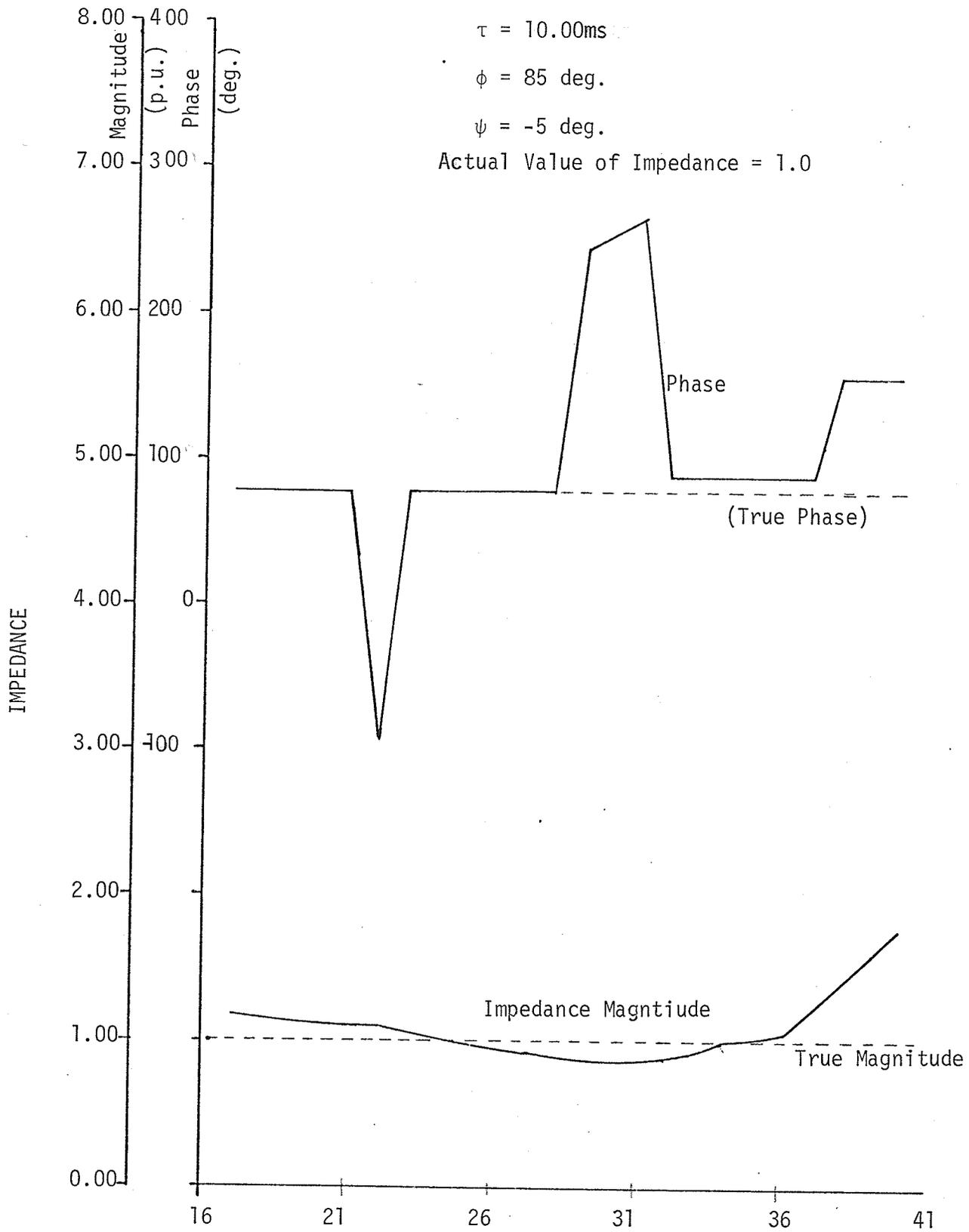


Figure 14: First difference method.

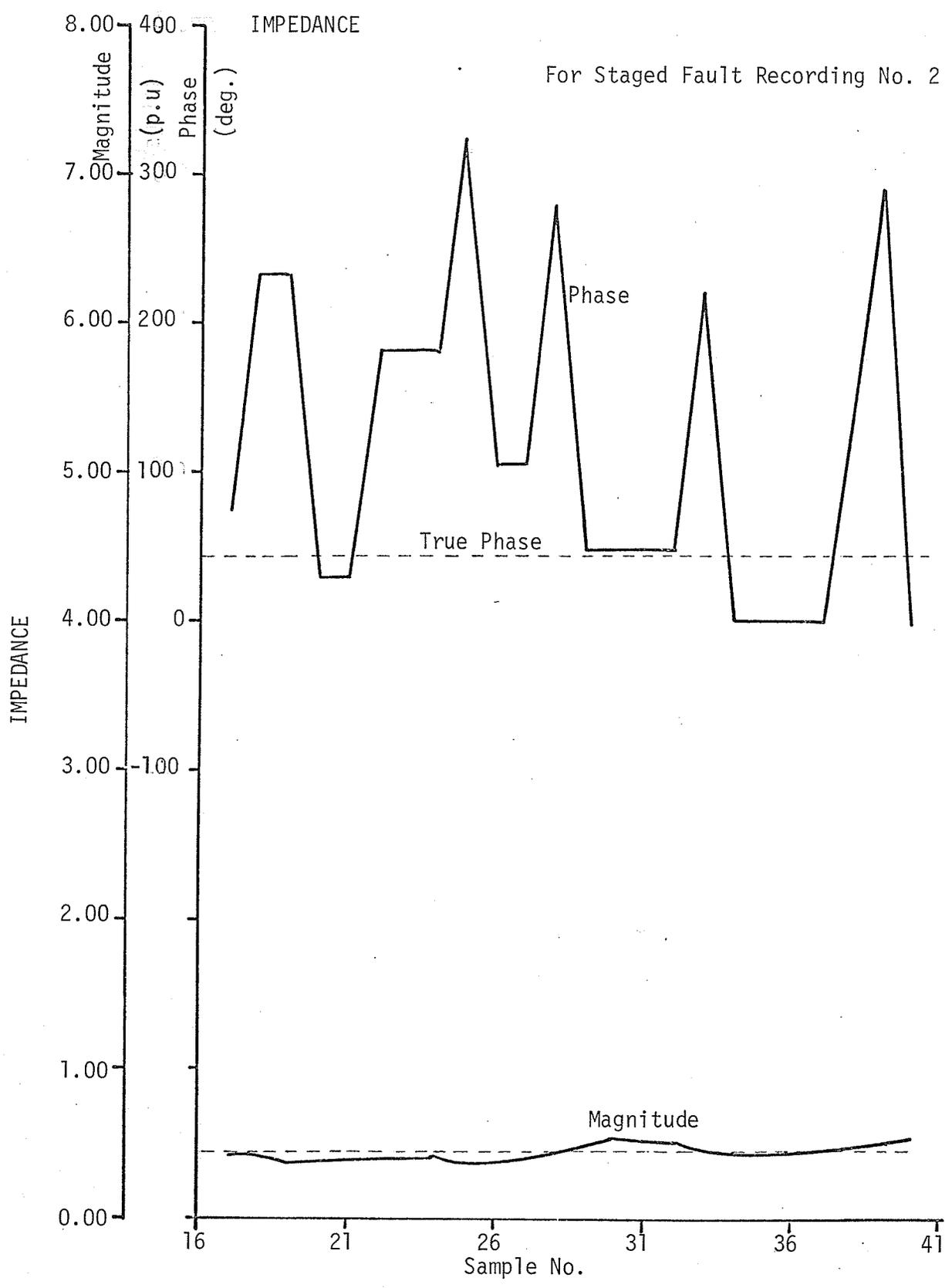


Figure 15: First difference method.

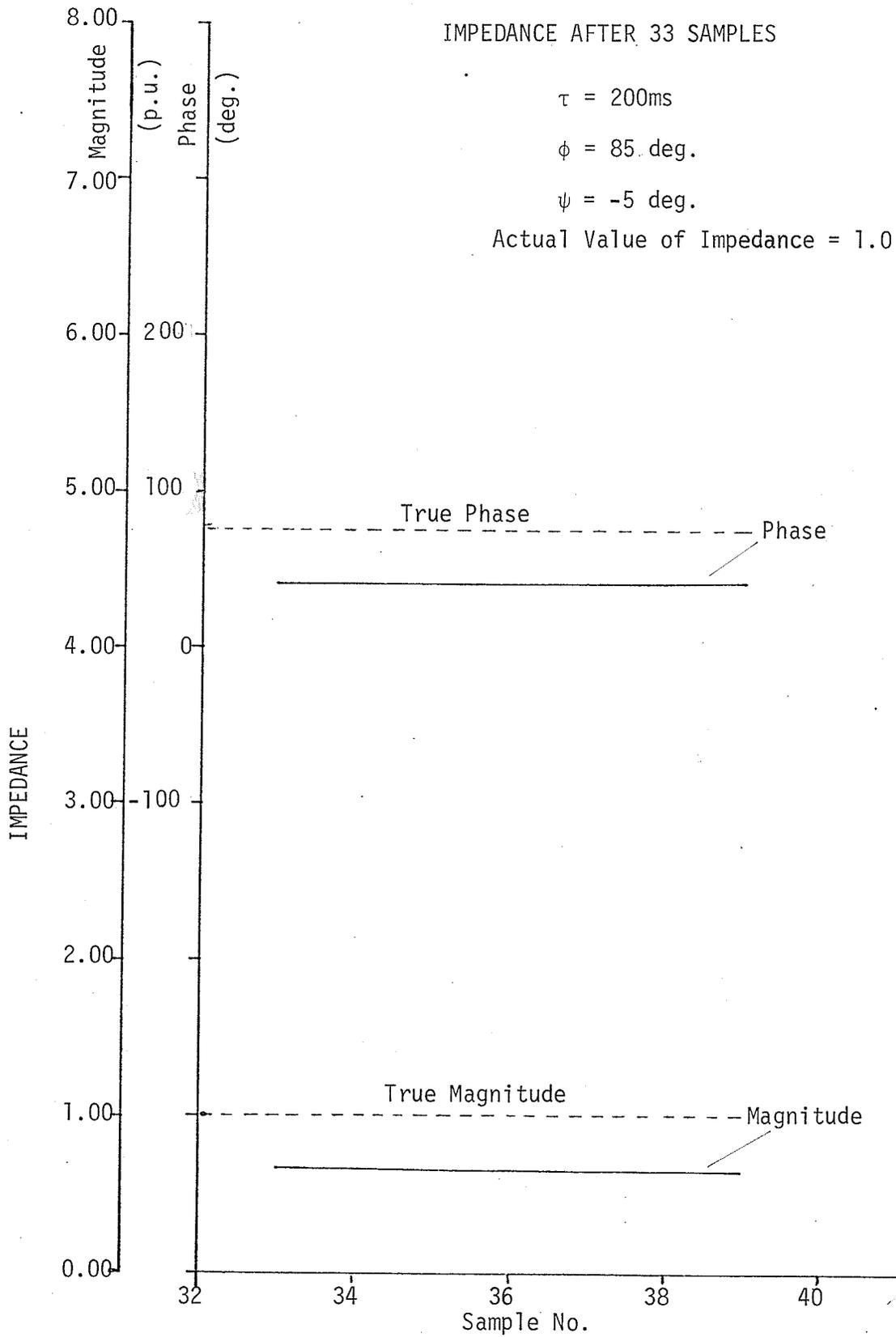


Figure 16: Fourier analysis method.

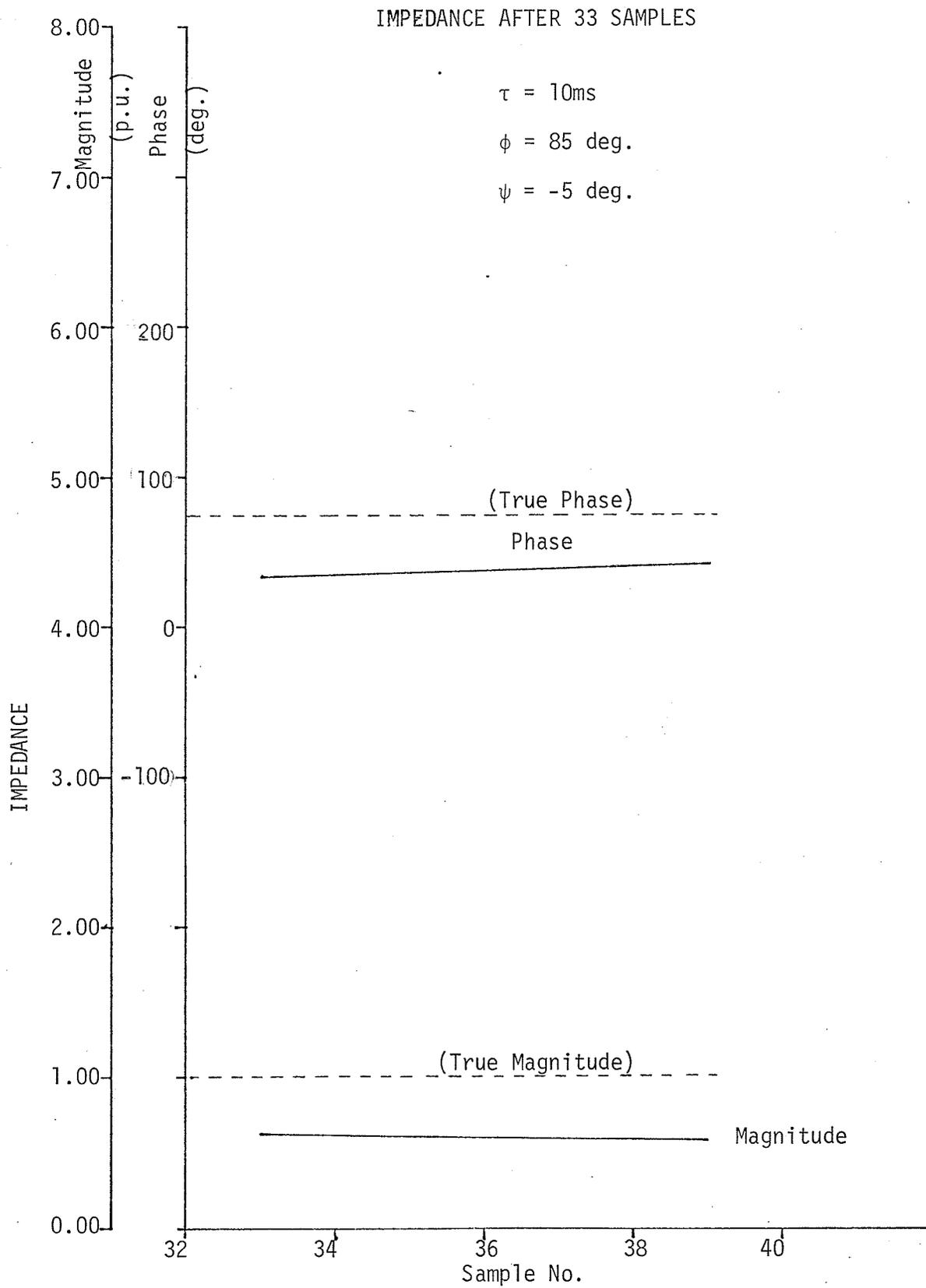


Figure 17: Fourier analysis method.

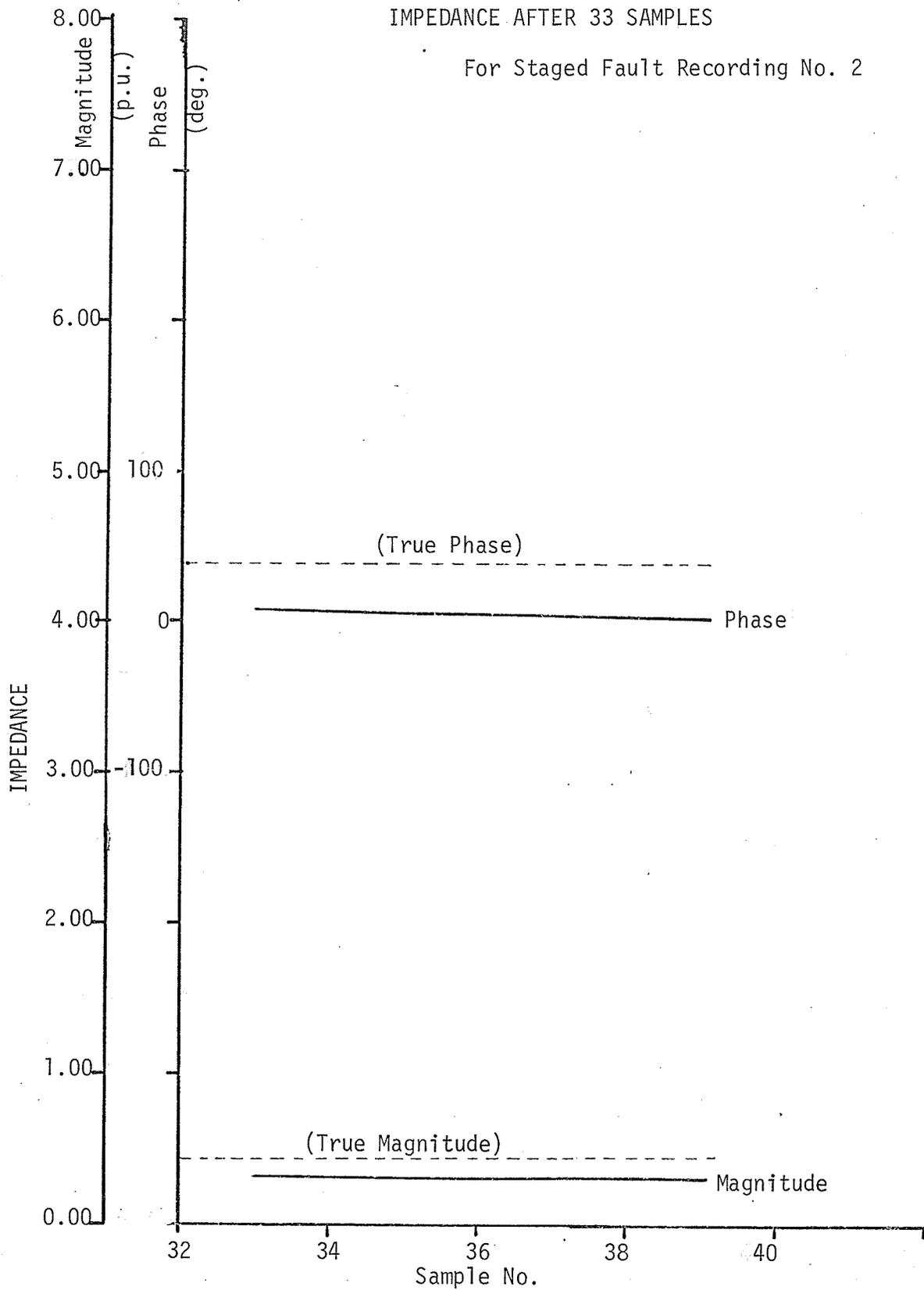


Figure 18: Fourier analysis method.

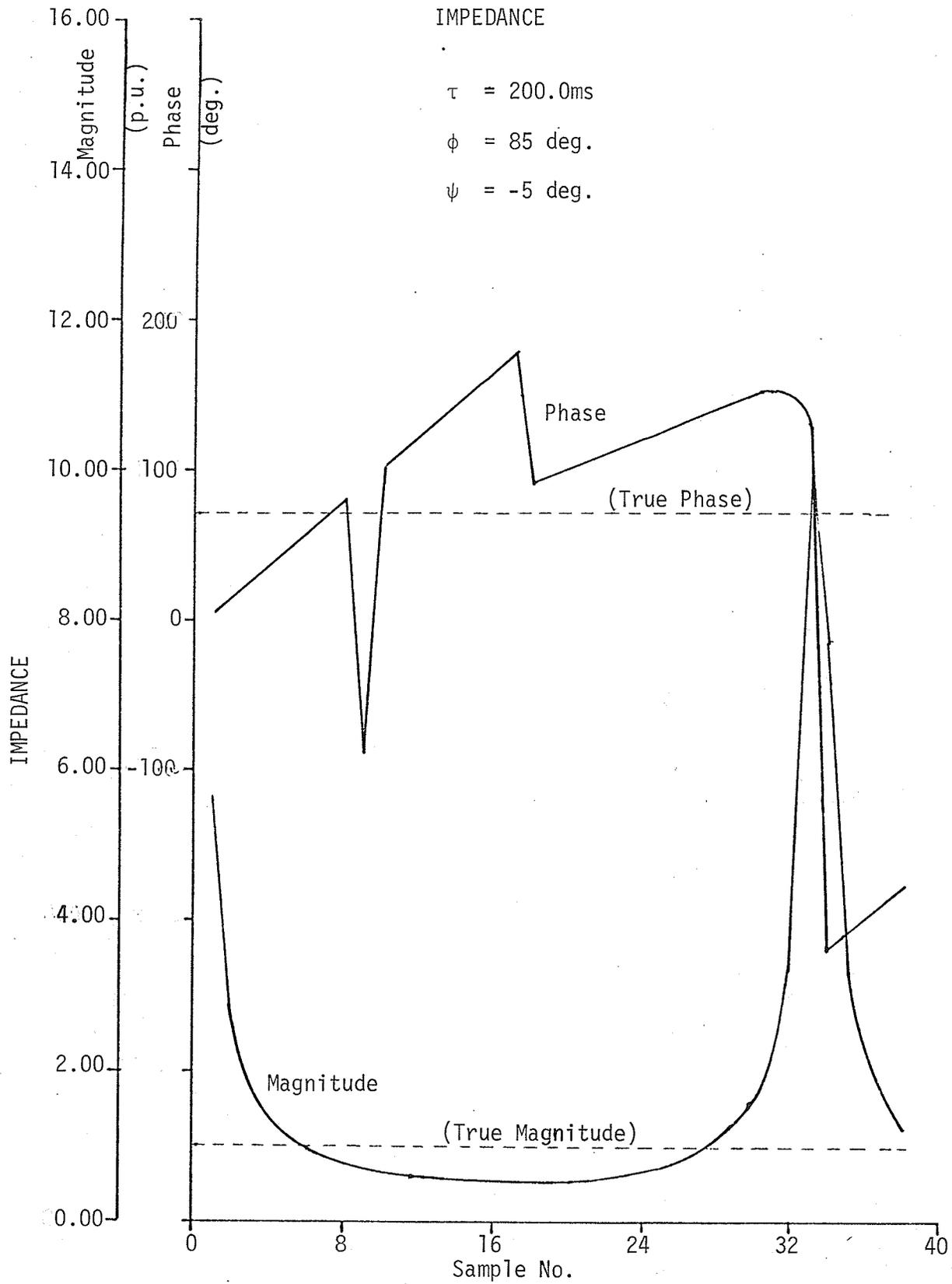


Figure 19: Mann-Morrison method.

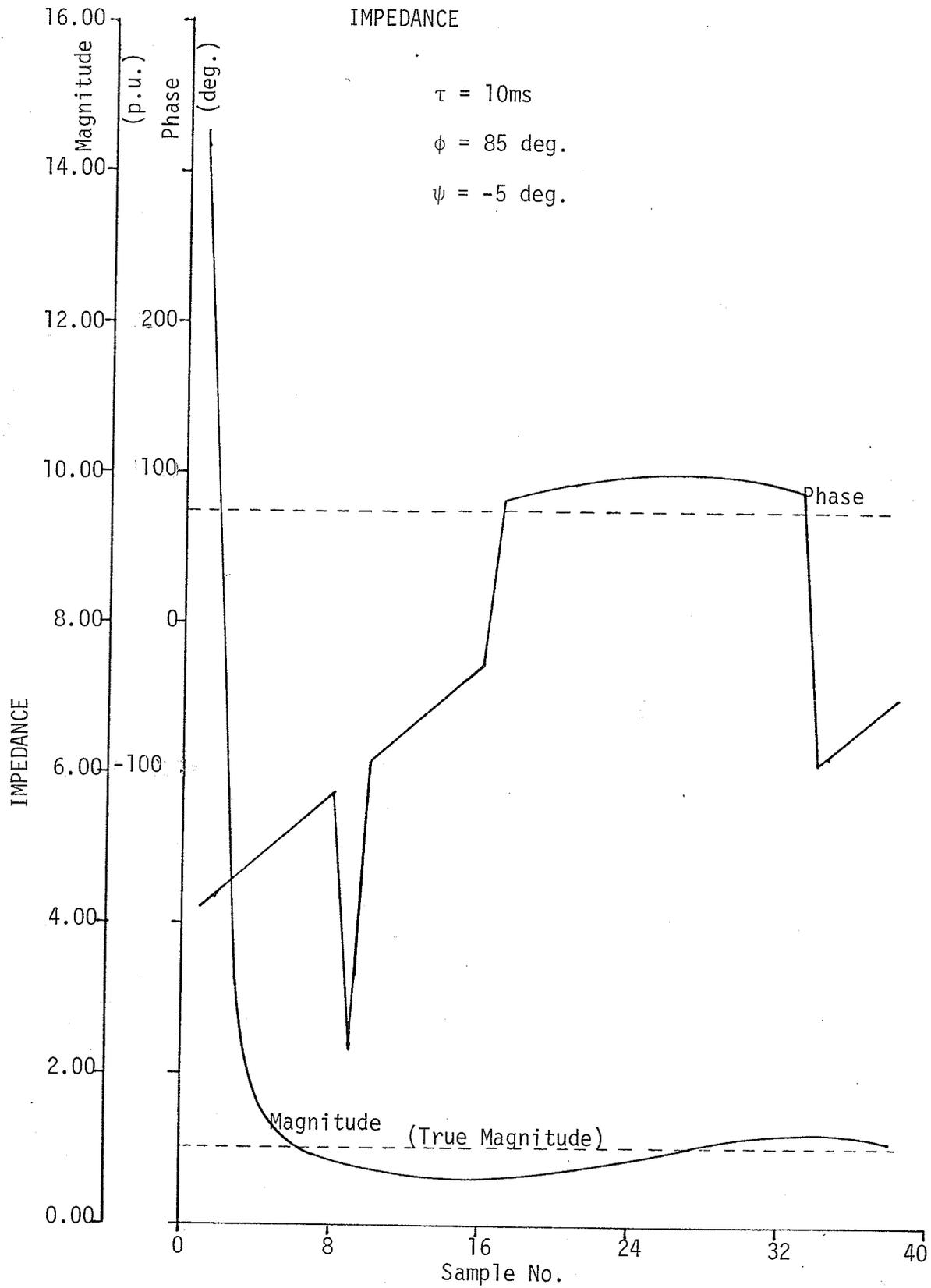


Figure 20: Mann-Morrison method.

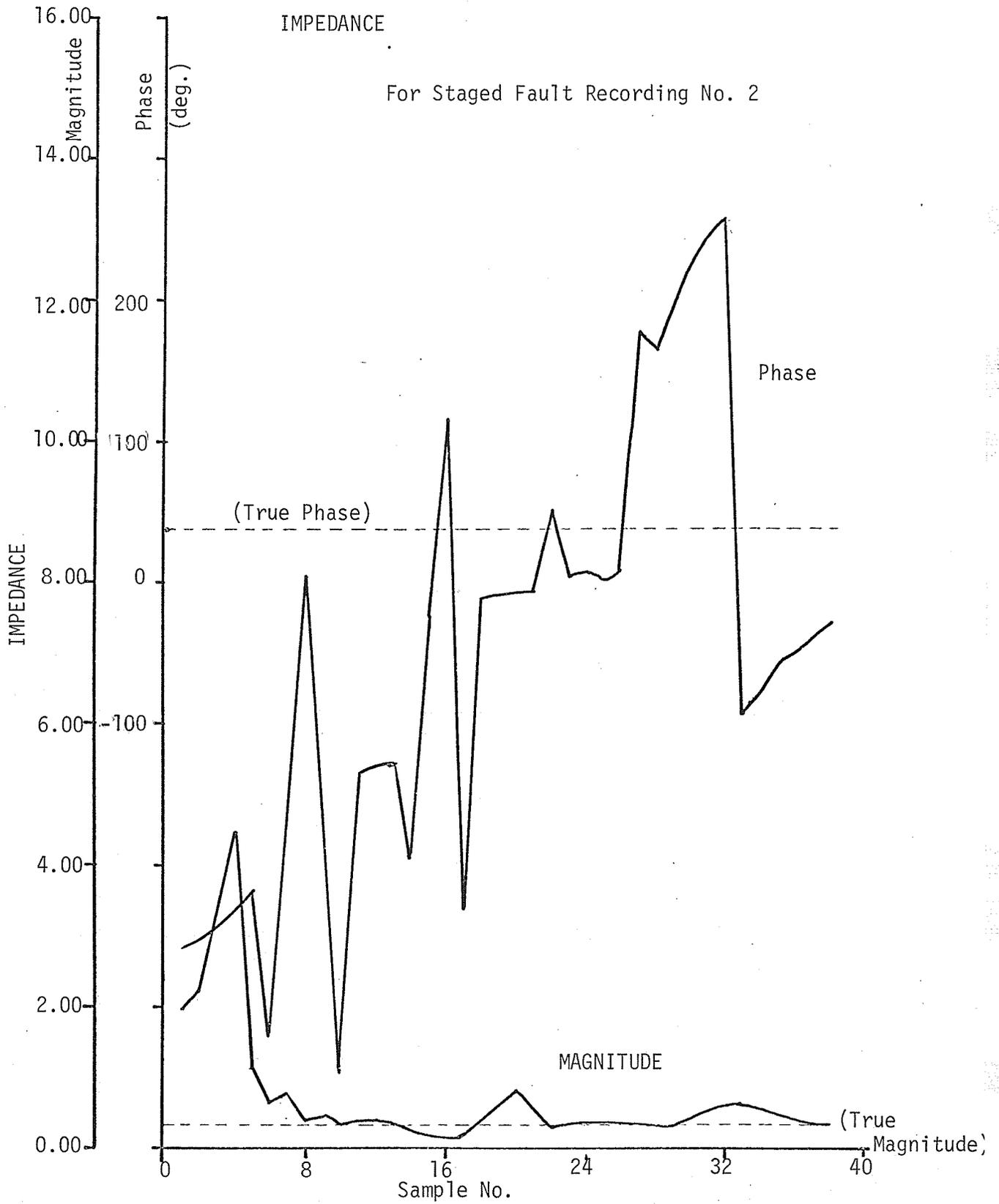


Figure 21: Mann-Morrison method.

The calculation of impedance values is shown Figures 19 to 21. Apart from overreach the impedance settles down to a constant value after about 8 samples; however the value again jumps up near the zero crossover, which is not surprising, but this may delay the tripping for a few more samples. The calculation of phase angle is not very accurate and varies quite considerably. The response of the method to the actual staged fault recordings is very poor and the value of both magnitude and phase are erratic.

3.45 Three Sample Method:- This method is introduced in this thesis. The variation of calculated values is shown in figures 22 to 24. It is obvious that the method is affected very little for ideal dc transients except when the time constant is very small. However the method does not give accurate results anywhere near actual values for the staged fault recordings. This is because in this method effectively we are differentiating the signals and therefore the effect of noise is more prominent.

As it was assumed that the sampling takes place  $h/2$  seconds after the fault initiation, the effect of non-synchronisation is also studied and a typical case is shown in figure 25. It may be observed that it has very little effect on the calculation of the impedance which is expected, however the phase angle undergoes a small deviation.

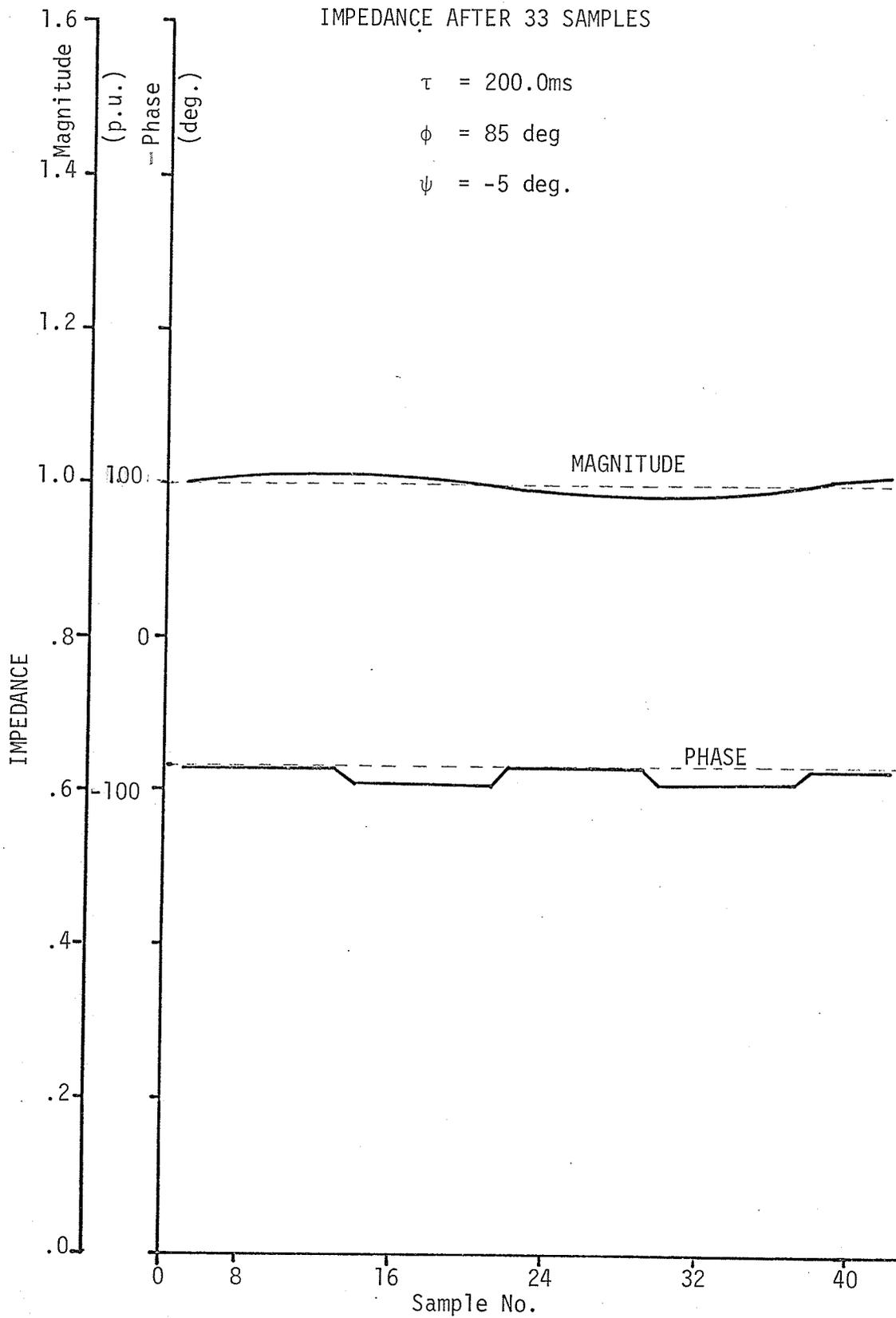


Figure 22: Three sample method.

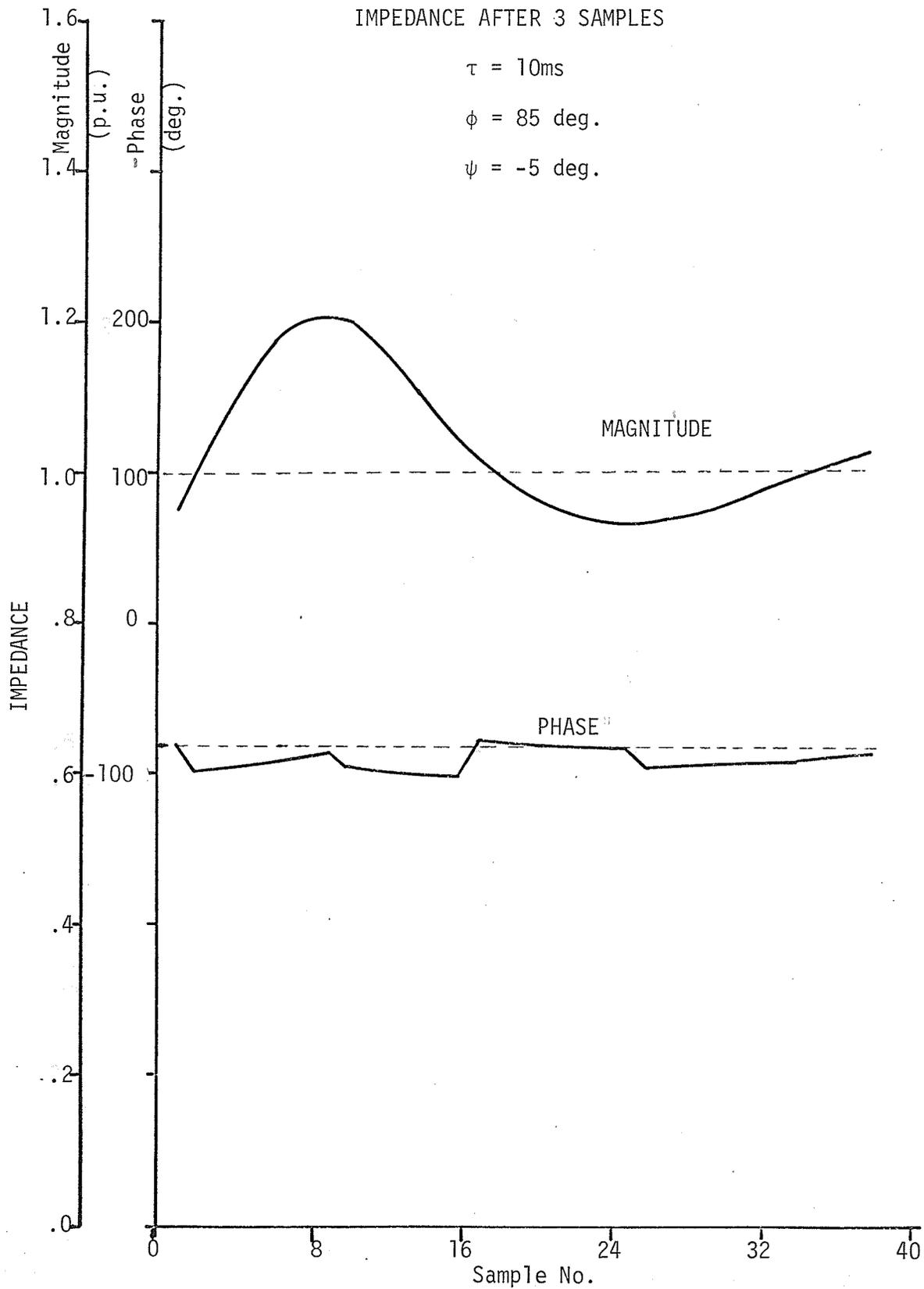


Figure 23: Three sample method.

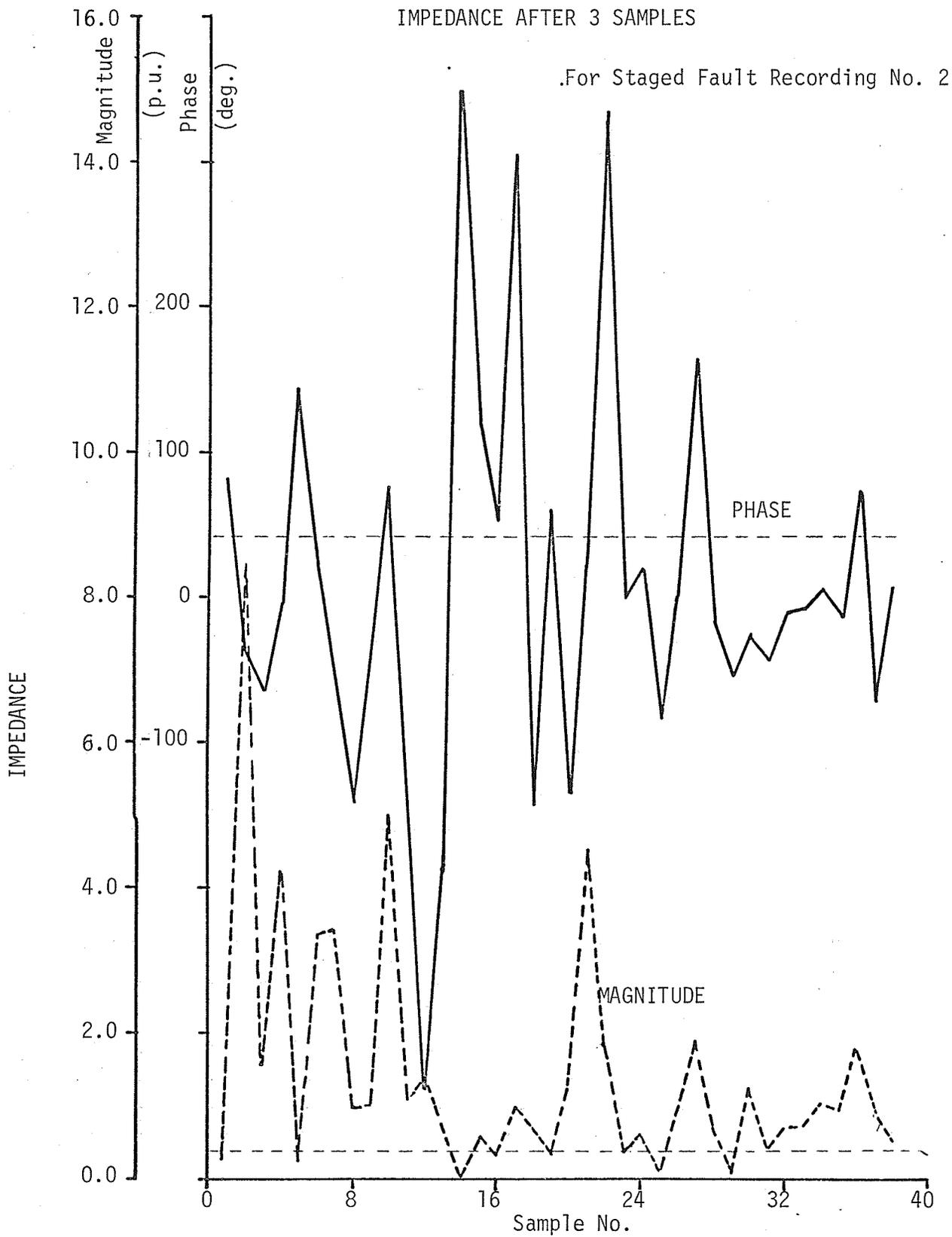


Figure 24: Three sample method

EFFECT ON NON SYNCHRONISATION OF SAMPLING FOR TIME CONSTANT  
OF 10 & 20 MSEC

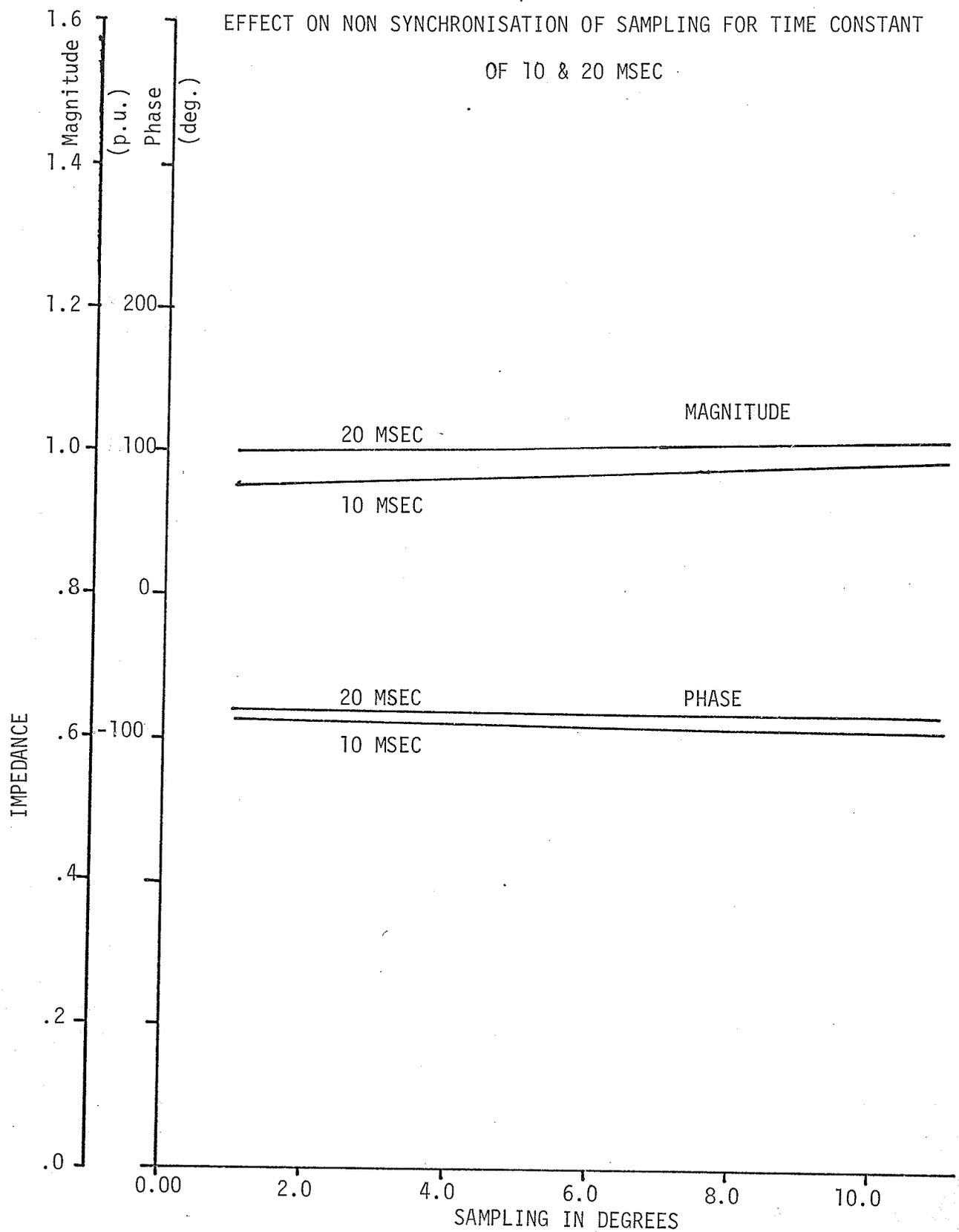


Figure 25: Three sample method.

## CHAPTER IV

### METHODS OF IMPROVING IMPEDANCE CALCULATIONS

4.1 Selecting the Best Method:-The relative performance of each of the methods discussed so far is summed up in Table 1. At a first glance it is not possible to decide which method is the best. It should be stated that a large deviation <sup>was</sup> is not necessarily a disadvantage; it all depends upon the duration of this error. The table along with Figures 10 to 24 allows us to take this into account, and to critically evaluate the performance of each of the methods.

Two methods are clearly better. They are the First Difference Method and the Three Sample Method. The main disadvantage with the first difference method is that it takes a lot of time before fault impedance can be calculated. Therefore only the Three Sample Method was considered inspite of its obvious susceptibility to high frequency transients. Therefore it was suggested to use filtering techniques to get rid of these high frequency transients. In the following sections the various filtering techniques used and the results obtained are described.

4.2 Analog Techniques used to Improve the Behaviour:<sup>22</sup>- Simple low pass filters were designed with an upper cut off frequency of 120 Hz. Two of these filters were considered.

TABLE I

	RMS VALUE Method	FIRST DIFFERENCE Method	FOURIER ANALYSIS Method	MANN MORRISON Method	THREE SAMPLE Method
Max. deviation in Impedance for ideal dc transient.	69.5%	175%	49%	4300%	12%
Max. deviation in phase for ideal dc transients.	-	500%	143%	500%	17%
Max. deviation in Impedance for staged fault tests.	65%	171%	210%	3000%	27600%
Max. deviation in phase for staged fault tests.	-	300%	160%	100%	400%
Relative CPU Time	1.64	.72	2.69	.35	.60
Values required to be stored at a time.	64	34	66	6	6
Relative Program size	952	1978	1584	1468	1424

Their transfer functions are-

$$T_1(s) = (s+500)/(.0000051 s^3 + .00255 s^2 + 2.0s + 500)$$

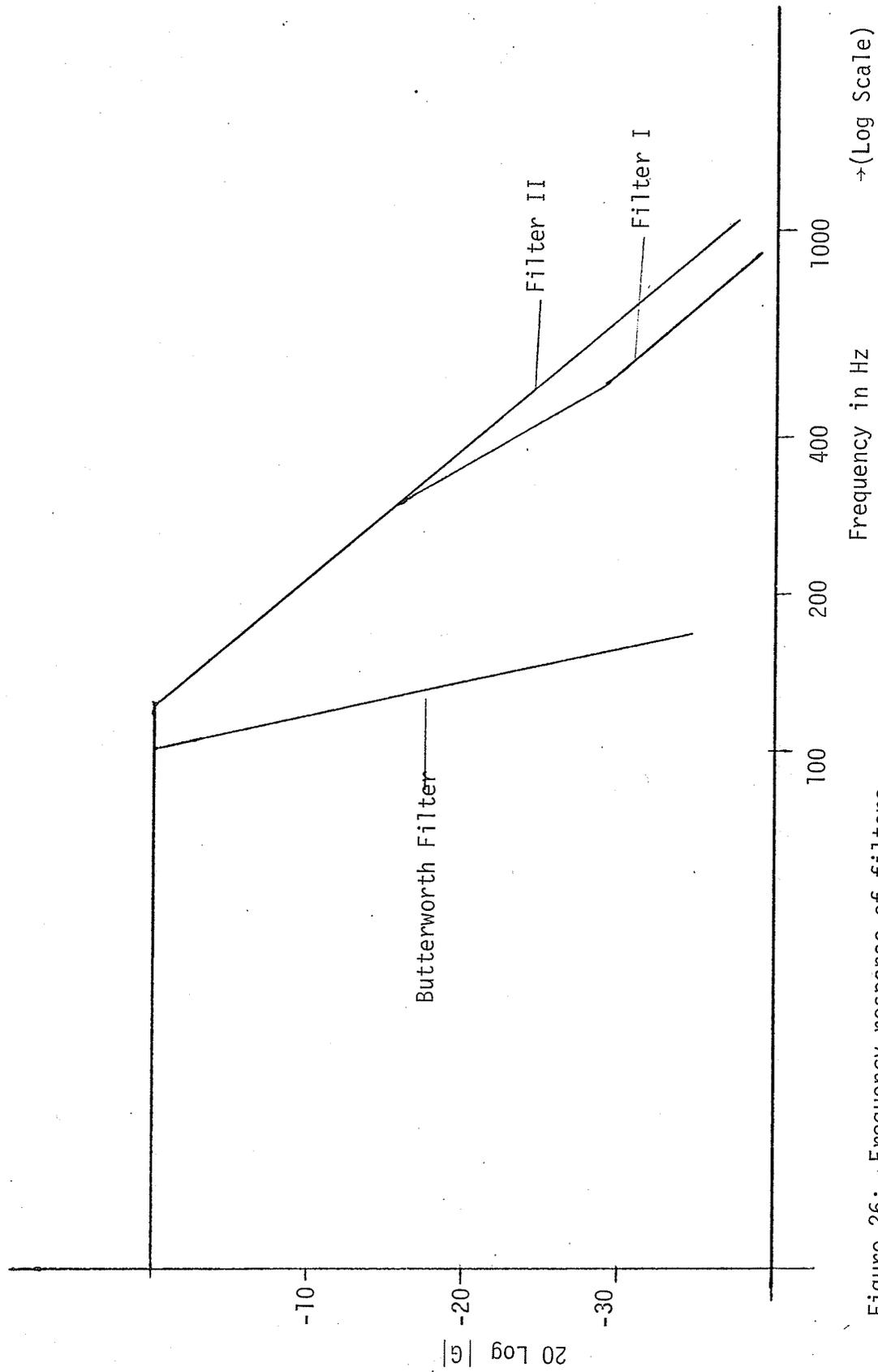
$$T_2(s) = 2.0/ (.0000102 s^2 + .0051 s + 2.0)$$

The amplitude frequency response curves are given in Fig.26

The impedance variation for staged fault tests when these filters were in use is shown in Figures 27 & 28. The filter response was evaluated by using the convolution integral, which presupposes zero initial conditions and therefore the calculations may not be very exact. The use of these filters allows the variation in the impedance magnitude to be reduced however the phase calculation is still very erratic. It may also be noted that the filtering also introduces a time delay of about 8 samples.

As the two simple filters designed above did not give a smooth output, an 8th order Butterworth maximally flat filter<sup>7</sup> was designed. The cut off frequency was made very near to 100 Hz. The frequency response of the filter is shown in Figure 26. The transfer function of the filter for the normalised frequency of  $\omega = 1$  is-

$$T_3(s) = 1.0/(s^8 + 5.126s^7 + 13.138s^6 + 21.848s^5 + 25.691s^4 + 21.848s^3 + 13.138s^2 + 5.126s + 1)$$

Figure 26: Frequency response of filters.

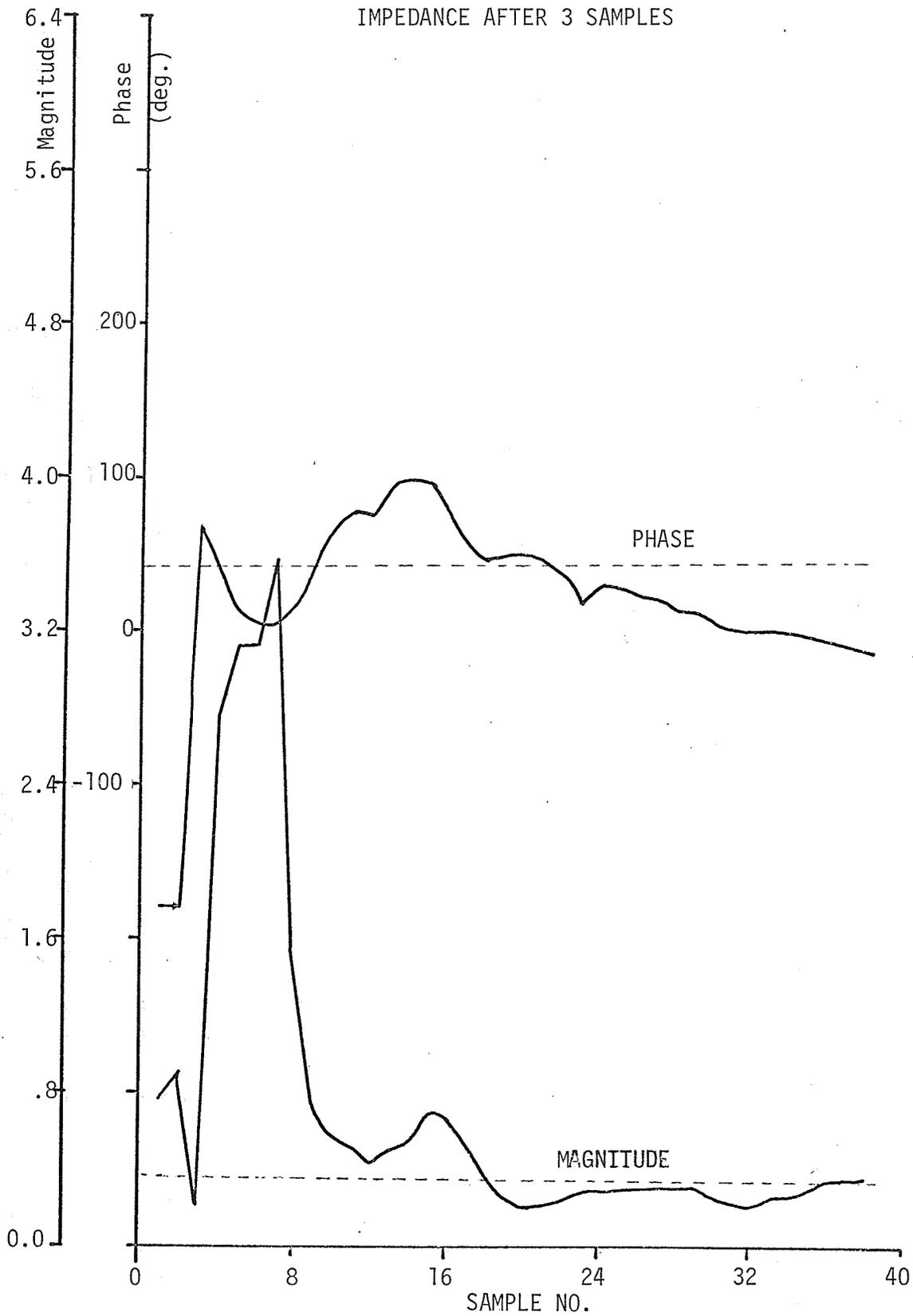


Figure 27: Three sample method after using filter I.

## IMPEDANCE AFTER 3 SAMPLES

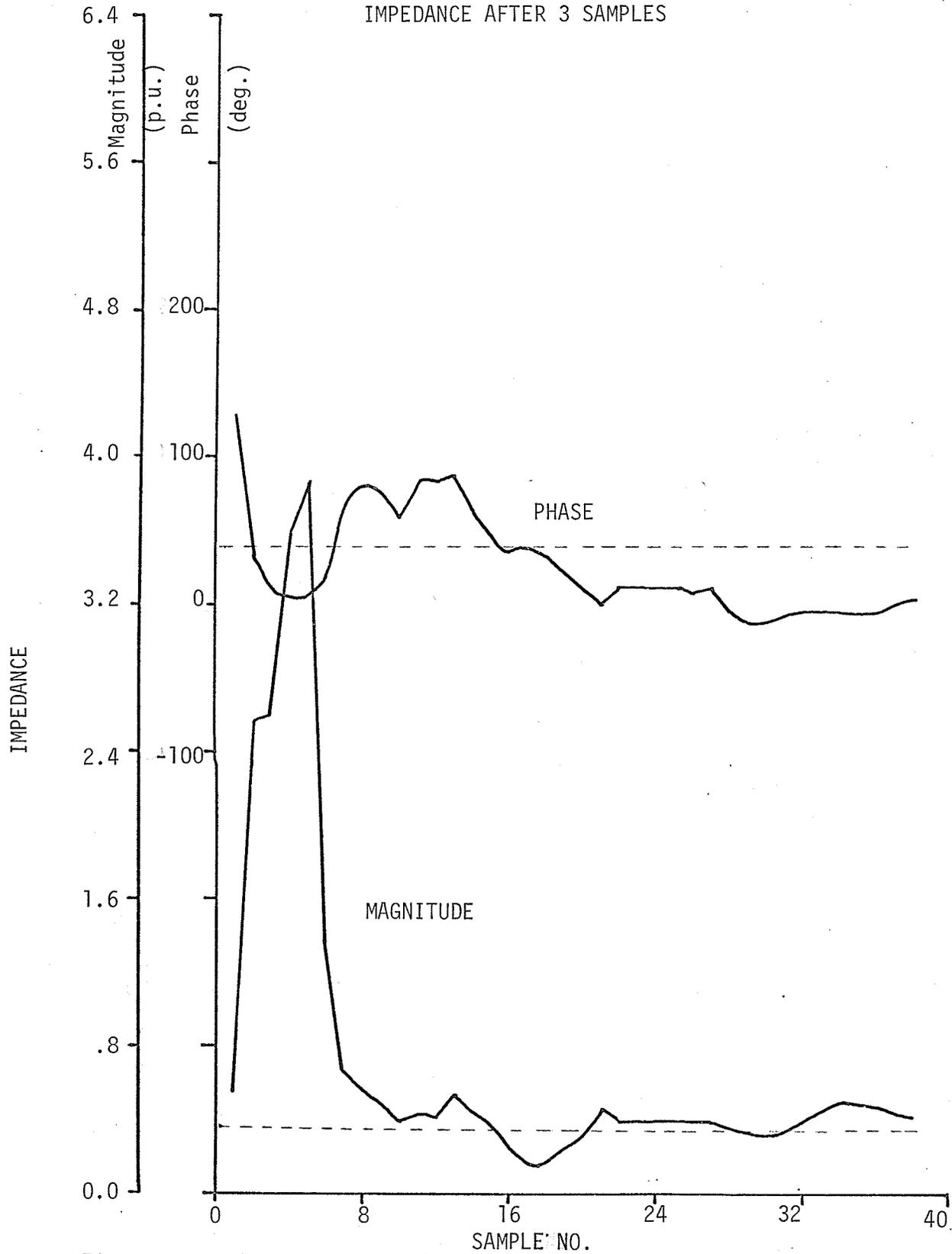


Figure 28: Three sample method after using filter II

The response of the filter to actual recordings was computed using the Convolution Integral. The variation of the magnitude and phase of the impedance for our particular case is shown in Figure 29.

The results are once again not very encouraging.

4.3 Some Other Techniques by Which the Methods may be Improved:- .As experienced in the foregoing sections the use of filters does not help very much, and has the added disadvantage of introducing a large time delay. Therefore if by some means we can reduce the high frequency transients in the voltages and current, or at least make them equal, then we may be able get better results. To realize this aim we can use the method of REPLICA IMPEDANCE in the p.t. circuits. This may reduce the high frequency transients in the voltage to the same level as those in the current. ( It has been noted that the voltage is much more susceptible to transients in the higher frequencies than the current.)

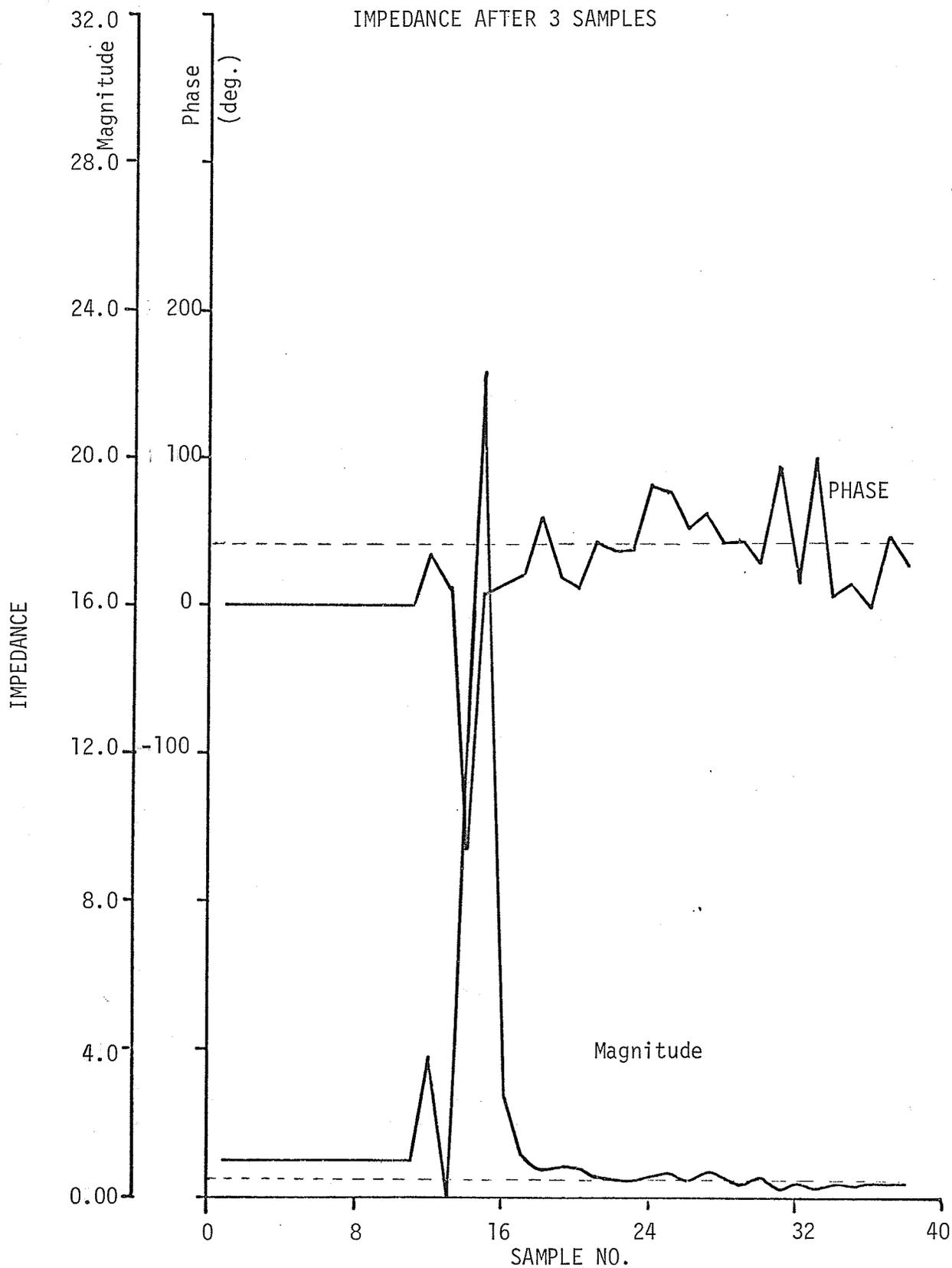


Figure 29: Three sample method after using Butterworth maximally flat filter.

## CHAPTER V

### THE RELAYING SCHEME USING THE SUGGESTED DIGITAL METHOD

It should be obvious by now that digital techniques can be employed for relaying in an electric power system. A suggested scheme of protection is as follows:-

A small special purpose minicomputer is installed and the sampled values of current and voltage are fed into it at a rate of 1 sample per .5 msec. The computer calculates the impedance magnitude and phase and if the value computed is less than a predetermined value, it sends this computed value to the main computer, which performs more logic to determine in which zone the fault has occurred and then takes remedial action.

Since it has been shown that the nonsynchronisation of signals does not drastically affect the calculation, the complete synchronisation of the sampling of current and voltage signals may not be necessary even if it is possible.

The special purpose computer may possibly use hardware only, instead of software which may reduce the cost and increase the speed of computation.

The amount of calculation required with each sample takes a little less than .3 msec. and hence the computer will be able to compute the values before the next sample arrives. The block diagram representation of the scheme is

shown in Figure 30.

The main supervising computer will in general be doing other control and record keeping, but as soon as it receives an input from one of these small computers the system protection will have top priority. In this way there will be little delay in fault isolation and at the same time the main computer will be utilised to its full extent.

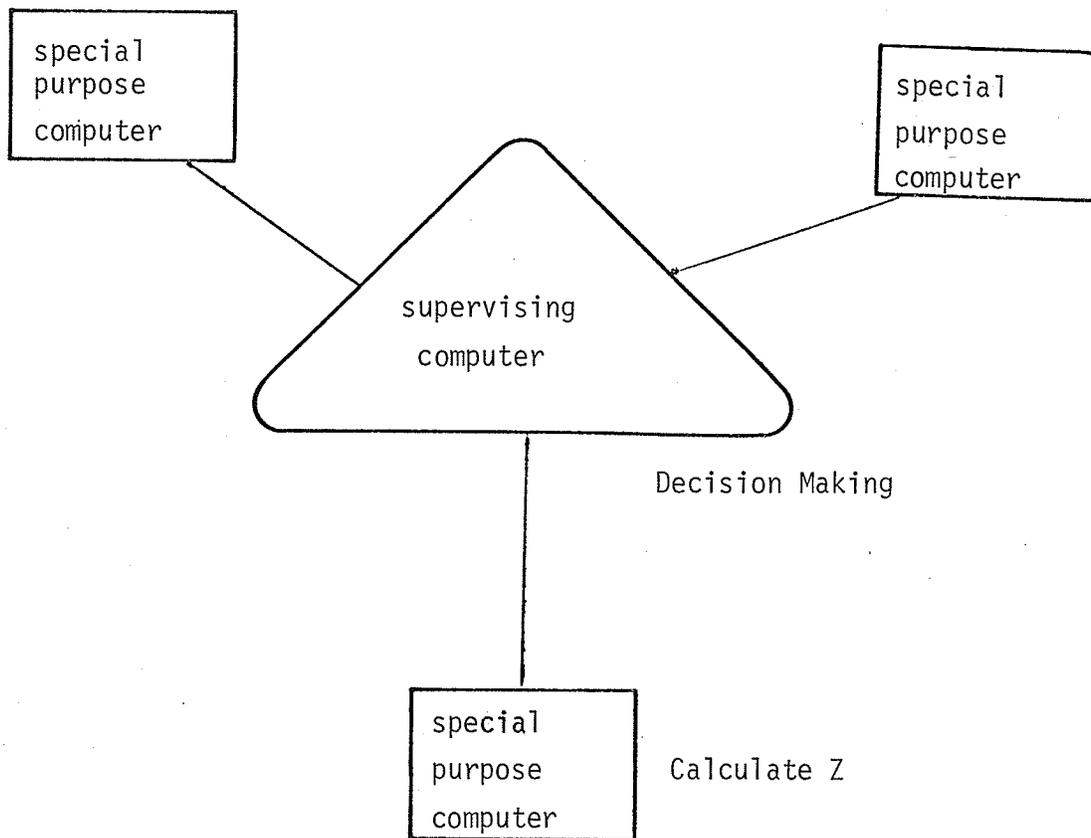


Figure 30 Block diagram representation of the scheme.

## CHAPTER VI

### CONCLUSIONS

From the foregoing chapters the following can be concluded.

1. For the theoretical current and voltage samples the Three Sample Method is by far the best.
2. For the actual fault recordings the Three Sample Method gives erratic results even after filtering and thus the First Difference Method seems to be the best in this case.
3. If filtering or digital smoothing is resorted to, it results in a delay of about 8 samples.
4. The fastest fault detection using any of the methods discussed, may not be possible before at least 10 samples have elapsed.

Even though this thesis seems to indicate that the Three Sample method gives erratic results for actual fault recordings, it is suggested that before the method is completely discarded, it should be tested with actual signals that have been filtered. The use of Replica

Impedance in the p.t. circuit may also be investigated.

## APPENDIX A: Computer Programs for Digital Methods

## 1. RMS Relay

```
      C   RMS RELAY
0001      DIMENSION C(40),V(40)
0002      9 DO2I=1,40
0003      READ(5,10) V(I),C(I)
0004      IF(V(I).GE.10.) GO TO 99
0005      10 FORMAT(2F6.0)
0006      2 CONTINUE
0007      DO12K=1,8
0008      X=0.0
0009      Y=0.0
0010      M=32+K
0011      DO11I=K,M
0012      X=X+(C(I))**2
0013      11 Y=Y+(V(I))**2
0014      Z=SQRT(Y/X)
0015      12 WRITE(6,13) Z
0016      13 FORMAT(F30.4)
0017      GO TO 9
0018      99 RETURN
0019      END
```

```

C FIRST DIFFERENCE RELAY
C CALCULATION OF IMPEDANCE AND PHASE ANGLE
0001 DIMENSION V(17),C(17)
0002 9 DO10I=1,17
0003 READ(5,11) V(I),C(I)
0004 IF(V(I).GE.10.) GO TO 99
0005 10 CONTINUE
0006 11 FORMAT(2F6.0)

C FIRST DIFFERENCE OF SAMPLED VALUES
0007 DO12I=1,16
0008 V(I)=V(I+1)-V(I)
0009 12 C(I)=C(I+1)-C(I)

C CALCULATION OF IMPEDANCE MAGNITUDE
0010 K=10
0011 DO2L=1,24
0012 50 X=0.0
0013 Y=0.0
0014 DO13I=1,16
0015 X=X+ABS(V(I))
0016 13 Y=Y+ABS(C(I))
0017 Z=X/Y

C CALC. OF ZERO CROSSOVER
0018 IF(V(1))20,21,22
0019 21 THETA=0.0
0020 IF(V(2))23,28,29
0021 20 X=-1.00
0022 GO TO 100
0023 22 X=+1.00
0024 100 DO26J=1,15
0025 IF(V(J+1))23,24,25
0026 24 A=J
0027 THETA=A*10.8
0028 GO TO 16
0029 23 Y=-1.00
0030 GO TO 101
0031 25 Y=+1.00
0032 101 IF(X+Y)26,27,26
0033 26 CONTINUE
0034 27 A=J
0035 THETA=A*10.8+(V(J)*10.8/(V(J)-V(J+1)))
0036 16 IF(V(J+1)-V(J))28,28,29
0037 28 DV=-1.00
0038 GO TO 102
0039 29 DV=+1.00
0040 102 IF(C(1))30,31,22
0041 31 BETA=0.0
0042 IF(C(2))33,38,39
0043 30 X=-1.00
0044 GO TO 103
0045 32 X=+1.00
0046 103 DO36J=1,15
0047 IF(C(J+1))33,34,25
0048 34 A=J
0049 BETA=A*10.8
0050 GO TO 17
0051 33 Y=-1.00
0052 GO TO 104
0053 35 Y=+1.00

```

First difference relay continued.

```

0054      104 IF (X+Y)36,37,36
0055      36 CONTINUE
0056      37 A=J
0057      BETA=A*10.8+(C(J)*10.8/(C(J)-C(J+1)))
0058      17 IF (C(J+1)-C(J))38,39,29
0059      38 DC=-1.00
0060      GO TO 105
0061      39 DC=+1.00
0062      C CALC. OF PHASE ANGLE
0063      105 IF (DC+DV)41,40,41
0064      40 PHI=180.0+BETA-THETA
0065      GO TO 106
0066      41 PHI=BETA-THETA
0067      106 WRITE(6,18) Z,PHI
0068      18 FORMAT(F8.3,F8.2)
0069      DO19I=1,16
0070      V(I)=V(I+1)
0071      19 C(I)=C(I+1)
0072      K=K+1
0073      IF (K.GE.40) GO TO 9
0074      READ(5,11) V(17),C(17)
0075      V(16)=V(17)-V(16)
0076      C(16)=C(17)-C(16)
0077      2 CONTINUE
0078      GO TO 9
0079      99 CALL EXIT
0080      END

```

## 3. Fourier series relay

```

C      FOURIER SERIES RELAY
0001      DIMENSION C(40),V(40)
0002      S DO2I=1,40
0003      READ(5,10) V(I),C(I)
0004      IF(V(I).GE.10.) GO TO 99
0005      10 FORMAT(2F6.0)
0006      2 CONTINUE
0007      DO3K=1,7
0008      X=0.0
0009      Y=0.0
0010      M=K+1
0011      N=K+32
0012      DO13I=M,N
0013      W=I-K
0014      X=X+2.0*SIN(W*10.8/57.3)*V(I)
0015      13 Y=Y+2.0*COS(W*10.8/57.3)*V(I)
0016      Y=Y+V(K)+V(N+1)
0017      AV=Y/33.0
0018      BV=X/33.0
0019      DO14I=M,N
0020      W=I-K
0021      X=X+2.0*SIN(W*10.8/57.3)*C(I)
0022      14 Y=Y+2.0*COS(W*10.8/57.3)*C(I)
0023      Y=Y+C(K)+C(N+1)
0024      AC=Y/33.0
0025      BC=X/33.0
0026      Z=SQRT(AV**2+BV**2)/SQRT(AC**2+BC**2)
0027      THET1=ATAN(AV/BV)
0028      THET2=ATAN(AC/BC)
0029      P=(THET1-THET2)*57.3
0030      3 WRITE(6,15) Z,P
0031      15 FORMAT(2F15.4)
0032      GO TO 9
0033      99 RETURN
0034      END

```

## 4. Mann Morrison relay

```

      C MANN -- MORRISON RELAY
0001      DIMENSION C(40),V(40)
0002      S DD2I=1,40
0003      READ(5,10) V(I),C(I)
0004      IF(V(I).GE.10.) GO TO 99
0005      10 FORMAT(2F5.0)
0006      2 CONTINUE
0007      DD14I=1,39
0008      D1=(C(I+1)-C(I))/0.1884
0009      D2=(C(I+2)-C(I+1))/0.1884
0010      CA=(C(I)+C(I+1)+C(I+2))/3.0
0011      CD=(D1+D2)/2.0
0012      CPS=CA**2+CD**2
0013      CP=SQRT(CPS)
0014      CD=ATAN(CA/CD)
0015      IF(D1.GE.0.0) GO TO 400
0016      PHI=57.3*CD+180.0
0017      GO TO 200
0018      400 IF(CA.GE.0.0) GO TO 201
0019      PHI=57.3*CD+360.0
0020      GO TO 200
0021      201 PHI=57.3*CD
0022      200 D1=(V(I+1)-V(I))/0.1884
0023      D2=(V(I+2)-V(I+1))/0.1884
0024      VA=(V(I)+V(I+1)+V(I+2))/3.0
0025      VD=(D1+D2)/2.0
0026      VPS=VA**2+VD**2
0027      VP=SQRT(VPS)
0028      VD=ATAN(VA/VD)
0029      IF(D1.GE.0.0) GO TO 500
0030      PH2=57.3*VD+180.0
0031      GO TO 100
0032      500 IF(VA.GE.0.0) GO TO 101
0033      PH2=57.3*VD+360.0
0034      GO TO 100
0035      101 PH2=57.3*VD
0036      100 Z=5.0*(VP/CP)
0037      P=(PH2-PHI)/100.0+4.00
0038      14 WRITE(6,15) Z,P
0039      15 FORMAT(2F15.4)
0040      GO TO 9
0041      99 RETURN
0042      END

```

## 5. Three sample relay

```

0001          DIMENSION C(40),V(40)
0002          9 DO2I=1,40
0003          READ(5,10) V(I),C(I)
0004          IF(V(I).GE.10.) GO TO 99
0005          10 FORMAT(2F6.0)
0006          2 CONTINUE
0007          DO14I=1,38
0008          D1=(C(I+1)-C(I))/0.1884
0009          D2=(C(I+2)-C(I+1))/0.1884
0010          X=D2/(D1+.001)
0011          Y=-(X-0.9823)/0.1873
0012          THETA=ATAN(Y)
0013          IF(D1.GE.0.0) GO TO 400
0014          PH1=57.3*(THETA-.1884)+180.0
0015          GO TO 200
0016          400 PH1=57.3*(THETA-.1884)+360.0
0017          200 CREN=D1/COS(THETA)
0018          D1=(V(I+1)-V(I))/0.1884
0019          D2=(V(I+2)-V(I+1))/0.1884
0020          X=D2/(D1+.001)
0021          Y=-(X-0.9823)/0.1873
0022          THETA=ATAN(Y)
0023          IF(D1.GE.0.0) GO TO 500
0024          PH2=57.3*(THETA-.1884)+180.0
0025          GO TO 100
0026          500 PH2=57.3*(THETA-.1884)+360.0
0027          100 VOLT=D1/COS(THETA)
0028          Z=5.0*ABS(VOLT/CREN)
0029          P=(PH2-PH1)
0030          IF(ABS(P).LE.180.0) GO TO 17
0031          IF(P.GE.0.0) GO TO 16
0032          P=P+360.0
0033          GO TO 17
0034          16 P=P-360.0
0035          17 P=P/100.0+4.0
0036          14 WRITE(6,15) Z,P
0037          15 FORMAT(2F15.4)
0038          GO TO 9
0039          99 RETURN
0040          END

```

APPENDIX B: Actual Fault Recordings  
Figure No. 31

7 Staged Fault Recording No. 1  
E OF FAULT.

15. R.M.S.  $\sqrt{515^2 + 88.5^2} = 522 \text{ AMP.}$

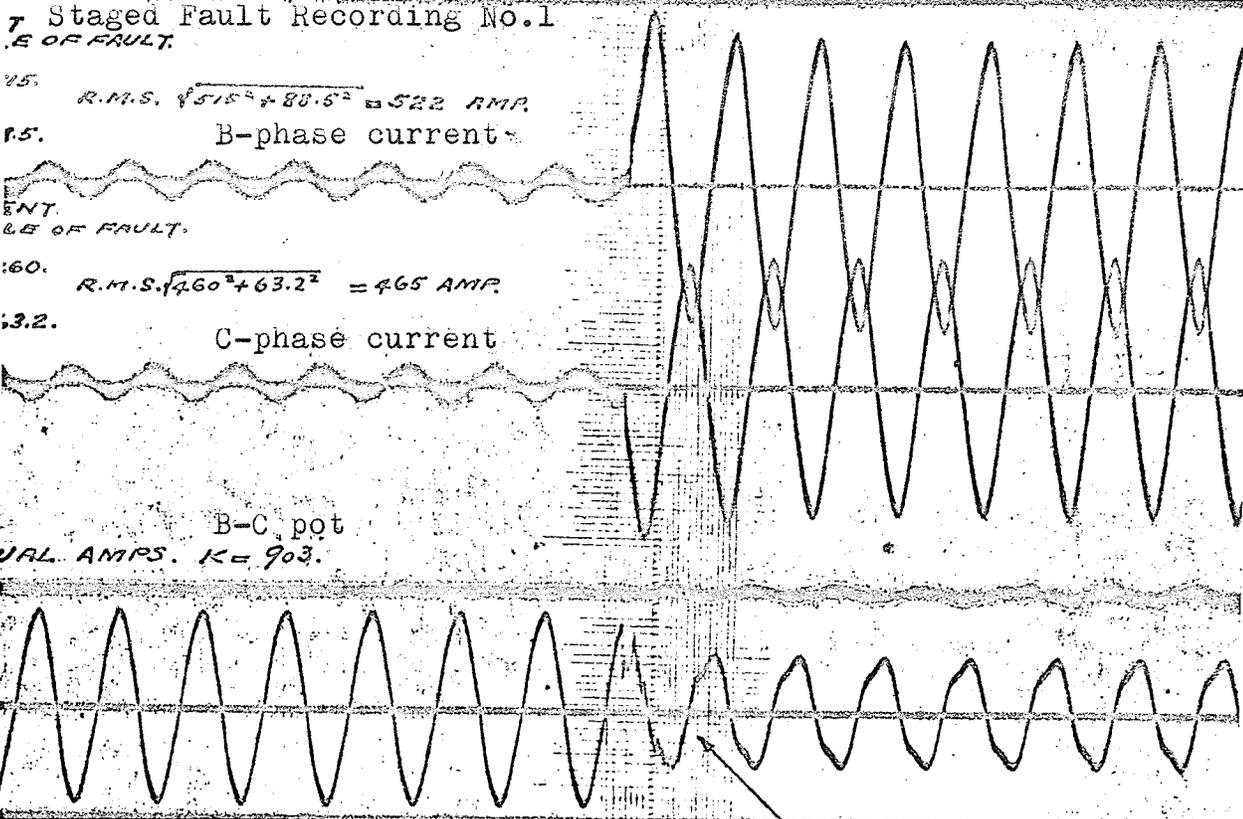
15. B-phase current

16. C-phase current

16. R.M.S.  $\sqrt{460^2 + 63.2^2} = 465 \text{ AMP.}$

13.2. B-C pot

VAL. AMPS.  $K = 903.$



Staged Fault Recording No. 2

R.M.S.  $\sqrt{523^2 + 16^2} = 523 \text{ AMP.}$

B-phase current

C-phase current

R.M.S.  $\sqrt{485^2 + 3.2^2} = 485 \text{ AMP.}$

B-C pot

B-C pot

500 V.

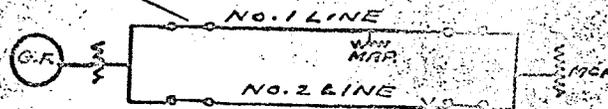
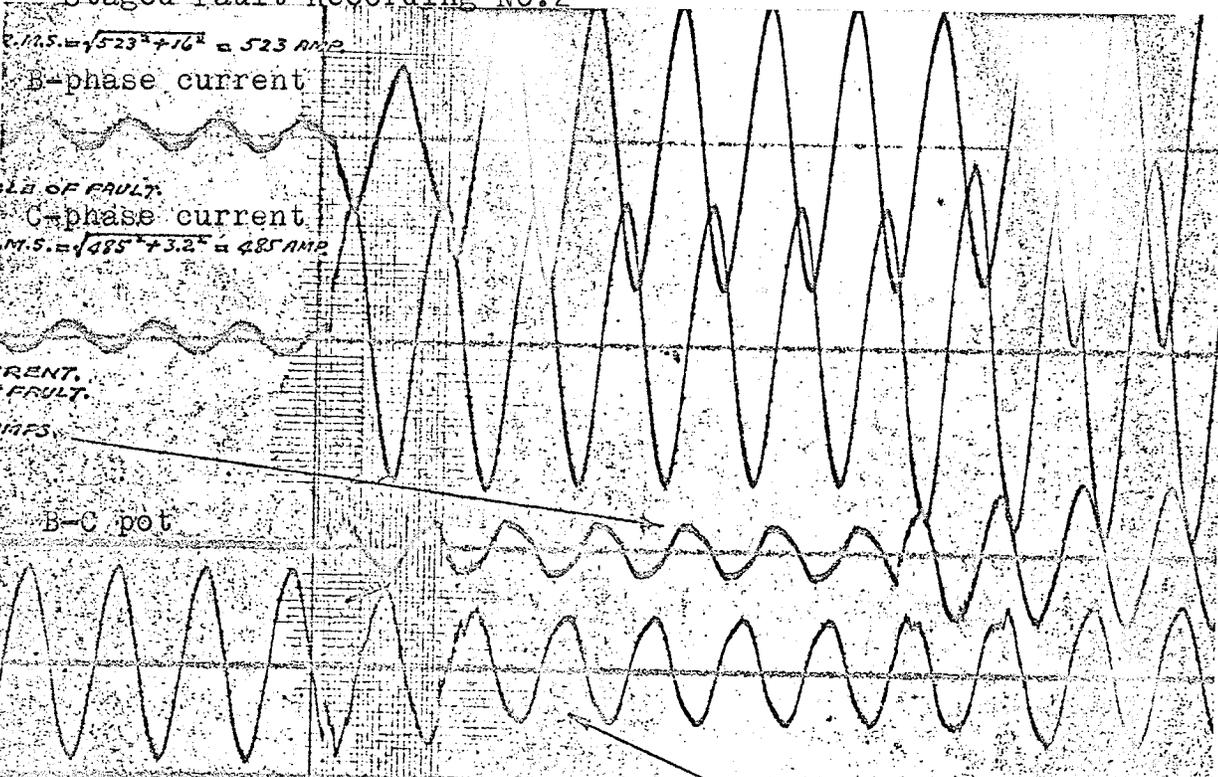


Figure No.32

Staged Fault Recording No.3

B φ CURRENT.  
 EYE CYCLE OF FAULT.  
 $\frac{.62 \times 903}{\sqrt{2}} = 503 \text{ AMP.}$   
 $\frac{.650 \times 903}{\sqrt{2}} = 138 \text{ AMP.}$

$R.M.S. = \sqrt{503^2 + 138^2} = 527 \text{ AMP.}$

C φ CURRENT.  
 EYE CYCLE OF FAULT.  
 $\frac{.58 \times 903}{\sqrt{2}} = 472 \text{ AMP.}$   
 $\frac{.58 \times 903}{\sqrt{2}} = 143 \text{ AMP.}$

$R.M.S. = \sqrt{472^2 + 143^2} = 490 \text{ AMP.}$

E RESIDUAL AMPS K-903.

Staged fault recording No.4

$908 = 540.$

$98 = 75.$

$R.M.S. = \sqrt{540^2 + 75^2} = 546 \text{ AMP.}$

$103 = 552.$

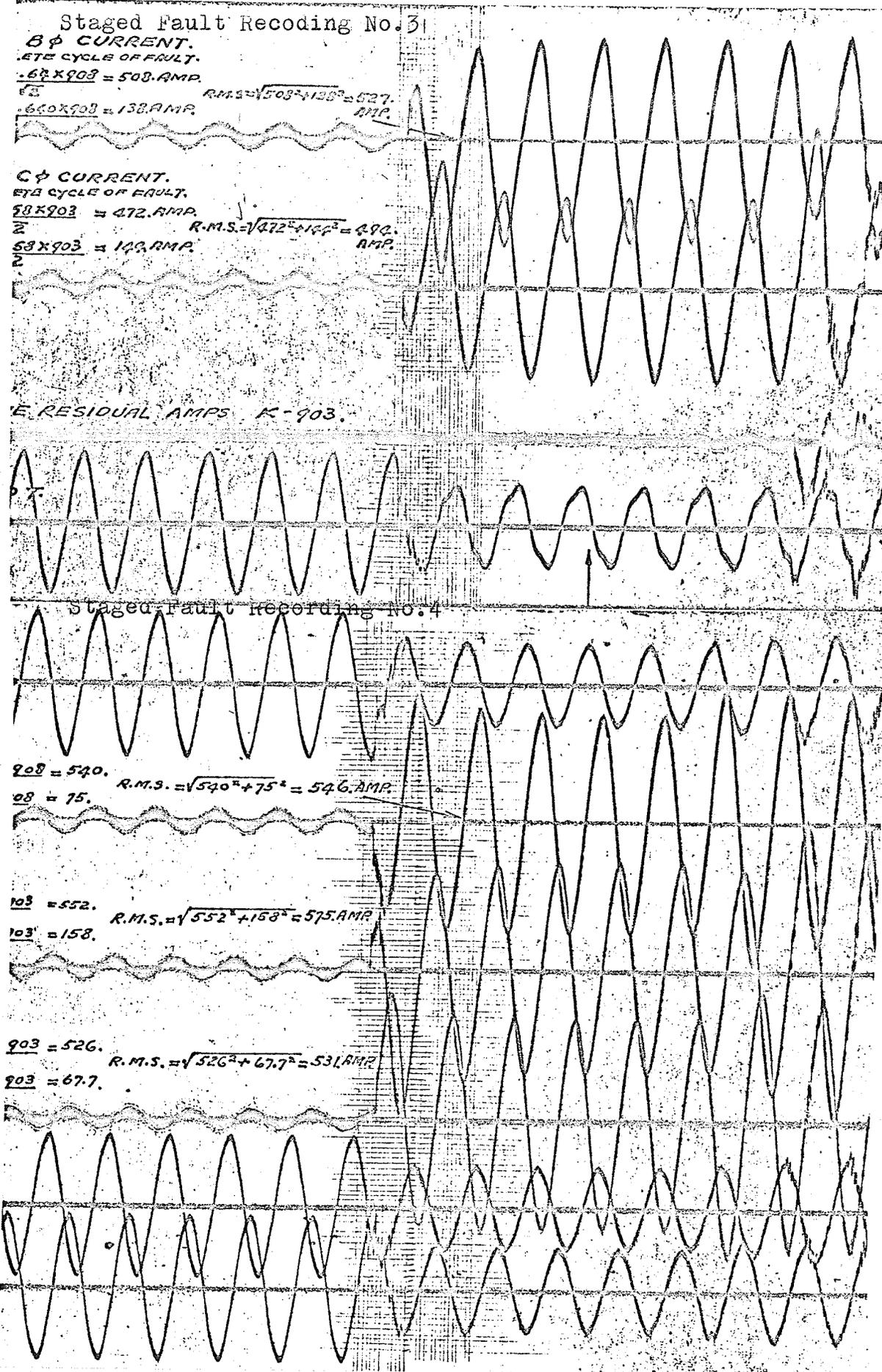
$103 = 158.$

$R.M.S. = \sqrt{552^2 + 158^2} = 575 \text{ AMP.}$

$903 = 526.$

$903 = 67.7.$

$R.M.S. = \sqrt{526^2 + 67.7^2} = 531 \text{ AMP.}$



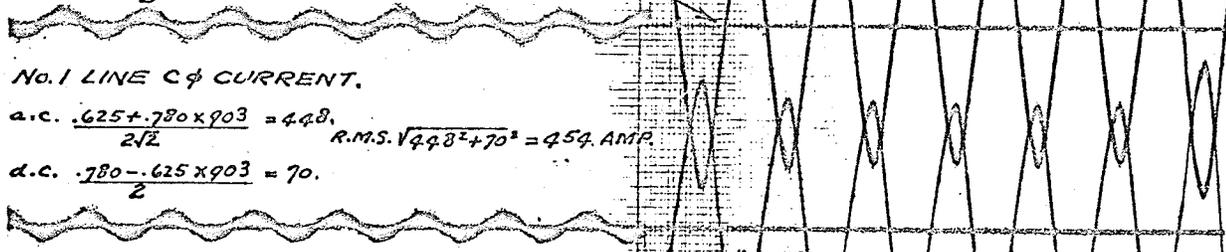
Staged Fault Recording No. 5

No. 1 LINE B φ CURRENT.

$$\text{a.c. } \frac{.860 + .66 \times 908}{2\sqrt{2}} = 488.$$

$$\text{R.M.S. } \sqrt{488^2 + 90.8^2} = 496 \text{ AMP.}$$

$$\text{d.c. } \frac{.86 - .66 \times 908}{2} = 90.8.$$

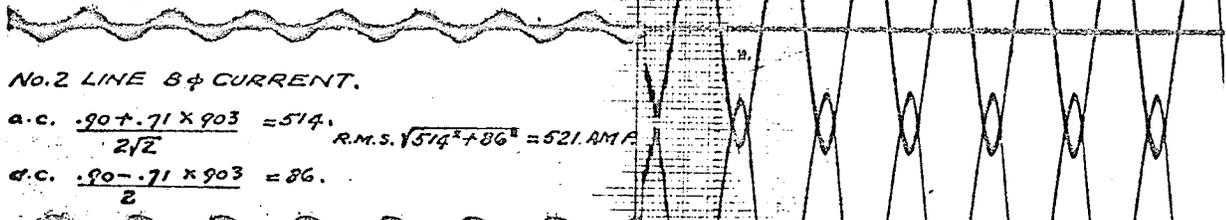


No. 1 LINE C φ CURRENT.

$$\text{a.c. } \frac{.625 + .780 \times 903}{2\sqrt{2}} = 448.$$

$$\text{R.M.S. } \sqrt{448^2 + 70^2} = 454 \text{ AMP.}$$

$$\text{d.c. } \frac{.780 - .625 \times 903}{2} = 70.$$

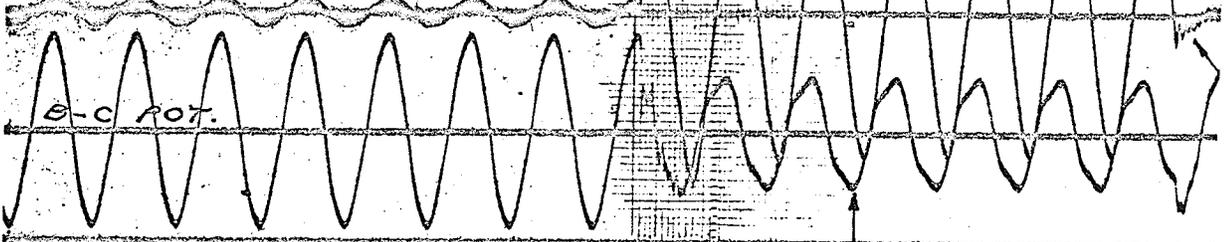


No. 2 LINE B φ CURRENT.

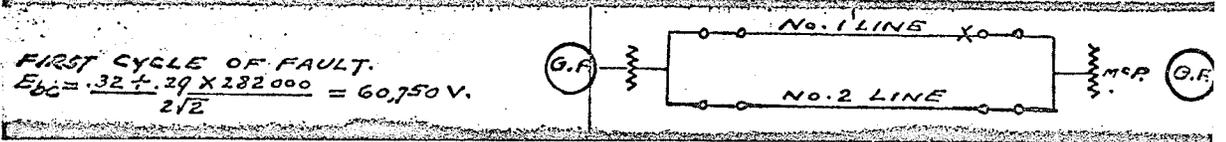
$$\text{a.c. } \frac{.90 + .71 \times 903}{2\sqrt{2}} = 514.$$

$$\text{R.M.S. } \sqrt{514^2 + 86^2} = 521 \text{ AMP.}$$

$$\text{d.c. } \frac{.90 - .71 \times 903}{2} = 86.$$



B-C POT.



FIRST CYCLE OF FAULT.

$$E_{bc} = \frac{.32 + .29 \times 182000}{2\sqrt{2}} = 60,750 \text{ V.}$$

Figure No. 33

## APPENDIX C: Table of Symbols

TABLE OF SYMBOLS

$i$ =instantaneous value of current

$I$ =RMS value of current

$I_{\max}$ =Peak value of current

$i'$ =first differential of current

$v$ =instantaneous value of voltage

$V$ =RMS value of voltage

$V_{\max}$ =Peak value of voltage

$v'$ =first differential of voltage

$x$ =sampling interval in radians

$h$ =sampling interval in millisecs.

$\omega$ =angular frequency in radians

$\phi$ =phase angle of the impedance

$L/R=\tau$ = time constant of the primary circuit

$\psi$ =is the angle after voltage zero at which the fault occurs

$\phi_L$ = the phase angle of the primary circuit

$Z$ =the total impedance of the primary circuit

$s$ = Laplace operator

$\theta$ =angle after the zero crossover of the sinusoid at the instant of calculation

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