

Prototyping Passive and Active Cavity Magnonic Devices

by

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Abstract

Microwave technologies are fundamental to modern communication, radar, navigation, and sensing systems. Magnetic materials, such as ferrites, have served as the core components of many microwave devices including circulators, filters, and oscillators. Their properties such as low loss and frequency tunability are indispensable for the control and manipulation of microwave signals. The ability to utilize and optimize ferrite devices has propelled significant advancements in modern microwave engineering.

Among many innovations, cavity magnonics have emerged as a promising platform, attracting considerable attention in recent years. A cavity magnonic system harnesses the interaction between microwave cavity photon mode and magnetic magnon mode, leading to hybridized states that combine the high mobility of photons with the long coherence time of spins. Although initially recognized for their potential in spintronics and quantum information processing devices, subsequent researches have also uncovered novel phenomena such as unidirectional invisibility and coherent microwave emission. As a result, increasing attention is being given to their potential as alternatives to conventional microwave components.

This thesis seeks to study the aforementioned non-reciprocal transmission and coherent microwave emission phenomena through passive and active cavity magnonic devices, with an emphasis on improving their practicality for real-world applications. It begins with a classical model of coupled RLC resonators to illustrate the frequency and linewidth evolution of hybridized states, establishing a connection to the quantum mechanical description of cavity magnonics and shedding light on the physical origins of coherent and dissipative coupling.

The first experiment introduces a passive cavity magnonic isolator that combines a mi-

crostrip cavity with a ferrite material, achieving non-reciprocal signal transmission with isolation up to 40 dB. Coupling and damping control mechanisms are introduced to reduce insertion loss to below 1 dB—an important step toward practical integration.

As a pre-requisite for developing an active cavity magnonic system, the second experiment focuses on realizing a transistor embedded microstrip oscillator. The design process was then applied to an interdigital capacitive sensor, resulting in the first demonstration of an active, high quality factor resonant sensor. Key performance aspects such as stability and sensitivity are also evaluated. The sensor is capable of detecting biochemical reactions at the nucleic acid level, up to 2.8×10^6 nucleic base pairs per $0.2\mu\text{L}$ sample.

Finally, coupling the active cavity to a ferrite material reveals a novel phase noise reduction effect. As the hybridized photon-magnon mode becomes increasingly magnon-like, sideband noise power of the active cavity is suppressed, leading to a sharp emission spectrum. The degree of reduction surpasses that of conventional methods, offering a new approach to designing low-phase-noise oscillators. Both fixed and tunable oscillators are demonstrated, with phase noise levels as low as -117 dBc/Hz at 10 kHz offset.

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- D.2 Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 1 nM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature. . . . 127
- D.3 Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 100 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature. . . . 128

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- D.4 Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 10 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature. . . . 129
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- D.11 Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 100 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature. . . . 136
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- D.13 Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 1 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature. . . . 138
- D.14 Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 100 fM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature. . . . 139
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List of Publications

As First Author

- (1) **Kim, M.**, Yang, Y., Gui, Y. S., & Hu, C. M. (2022). Visualization of synchronization zone on the Bloch sphere through an anti-PT-symmetric electrical circuit. *AIP Advances*, 12(3).
- (2) **Kim, M.**, Rao, J., Wang, Y., Gui, Y., Bridges, G. E., & Hu, C. M. (2022). Prototyping of novel isolator design based on cavity magnonics. *IEEE Transactions on Microwave Theory and Techniques*, 70(6), 3020-3028.
- (3) **Kim, M.**, Tabesh, A., Zegray, T., Barzanjeh, S., & Hu, C. M. (2024). Nonreciprocity in cavity magnonics at millikelvin temperature. *Journal of Applied Physics*, 135(6).
- (4) **Kim, M.**, Zhang, C., Lu, C., & Hu, C. M. (2024). Low phase noise microwave oscillator based on gain driven polariton. *Applied Physics Letters*, 124(11).
- (5) **Kim, M.**, Zhang, C., Lu, C., Burgess, J., & Hu, C. M. (2025). A two stage injection locking amplifier based on a cavity magnonic oscillator. *Chinese Physics B*, 34(6), 067104.
- (6) **Kim, M.**, Ogunsola, S., Zhang, C., Tomsa, D., Xie, J. Y., & Hu, C. M. (2025). Detection of RNA hybridization using a transistor integrated interdigital capacitor sensor. *IEEE Sensors Journal*, In Press.

Collaborative Works

- (7) Lu, C., **Kim, M.**, Yang, Y., Gui, Y. S., & Hu, C. M. (2023). Synchronization of dissipatively coupled oscillators. *Journal of Applied Physics*, 134(22).
- (8) Zhang, C., **Kim, M.**, Wang, J., & Hu, C. M. (2024). Gain-driven synchronization and asymmetric bound states in the continuum within non-Hermitian circuit model. *AIP Advances*, 14(6).
- (9) Lu, C., **Kim, M.**, Zhang, C., & Hu, C. M. (2024). Solid-state injection locking microwave amplifier. *Journal of Applied Physics*, 136(5).
- (10) Zhang, C., **Kim, M.**, Wang, J., & Hu, C. M. (2024). Van der Pol–Duffing oscillator and its application to gain-driven light-matter interaction. *Physical Review Applied*, 22(1), 014034.
- (11) Zhang, C., **Kim, M.**, Zhang, Y. H., Wang, Y. P., Trivedi, D., Krasnok, A., ... & Hu, C. M. (2025). Gain-Loss Coupled Systems. *APL Quantum*, 2(1).
- (12) Lu, C., Turner, B., Yao, J., Zhong, L., **Kim, M.**, Yang, Y., & Hu, C.-M. (2025). Temporal evolution of indirectly coupled resonators in open systems. *Phys. Rev. B*, 112(13), 134309.

Patent

- (1) Hu, C. M., Wang, Y. P., Gui, Y., & **Kim, M.** (2024). Device and related method for providing unidirectional microwave propagation. *U.S. Patent No. 11,946,878*. Washington, DC: U.S. Patent and Trademark Office.

Contribution of Authors

The thesis comprises three types of experimental work presented in Chapters 3, 4, and 5. These studies were conducted in collaboration with the colleagues listed below, and their individual contributions are detailed accordingly.

Chapter 3: Non-reciprocal Microwave Transmission: Isolator

This chapter includes two experimental components: room-temperature and low-temperature studies. The room-temperature experiment was primarily conducted by Mun Kim, covering experimental design, isolator fabrication, troubleshooting and manuscript drafting. Dr. Jinwei Rao and Dr. Yipu Wang contributed to idea development and manuscript editing, and Dr. Yongsheng Gui assisted with proofreading. Dr. Greg Bridges and Dr. Can-Ming Hu served as supervisors, offering conceptual guidance and feedback throughout the research and manuscript preparation [see List of Publications (2)].

The low-temperature experiment was designed and conceptualized by Mun Kim, with experimental work performed by Armin Tabesh and Tyler Zegray at the University of Calgary. Dr. Shabir Barzanjeh provided access to facilities and, together with Dr. Can-Ming Hu, supervised the work, offering conceptual guidance and feedback [see List of Publications (3)].

Chapter 4: Gain Embedded Cavity Oscillator: Sensor

The work presented in this chapter was primarily conducted by Mun Kim, including experimental and sensor design, troubleshooting, data verification, and manuscript drafting.

Chunlei Zhang, Sammuel Ogunsola and Dimitru Tomsa contributed to material synthesis, manuscript editing, and proofreading. Dr. Jiuyong Xie provided access to experimental facilities and, together with Dr. Can-Ming Hu, supervised the project, offering conceptual guidance and feedback throughout the research and manuscript preparation [see List of Publications (6)].

Chapter 5: Coherent Microwave Emission: Oscillator

The work presented in this chapter was jointly performed by Mun Kim and Chunlei Zhang, with equal overall contributions. Their work included experimental design, oscillator design and fabrication, troubleshooting, and data verification. Chunlei Zhang primarily contributed to the theoretical development, while Mun Kim focused on experimental implementation and manuscript drafting. Chenyang Lu made minor contributions to oscillator design, manuscript editing, and proofreading. Dr. Can-Ming Hu served as the supervisor, providing conceptual guidance and critical feedback throughout the research and manuscript preparation [see List of Publications (4), (5), (9), and (10)].

List of Abbreviations

Abbreviations	Expanded Forms
3D, 2D	Three Dimensional, Two Dimensional
Au-IDC	Gold - Interdigital Capacitor
BJT	Bipolar Junction Transistor
BW	Bandwidth
CMOS	Complementary Metal Oxide Semiconductor
DC	Direct Current
DTT	Dithiothreitol
DUT	Device Under Test
EM field	Electromagnetic Field
ESD	Electrostatic Discharge
FET	Field Effect Transistor
FLL	Frequency Locked Loop
FMR	Ferromagnetic Resonance
FOM	Figure of Merit
FWHM	Full Width at Half Maximum
GdIG	Gadolinium Iron Garnet
GMR	Giant Magnetoresistance
HDD	Hard Disk Drive
HEMT	High Electron Mobility Transistor
High/Low Q	High/Low Quality Factor

List of Variables

Variable	Definition
$a(t)$	Time-dependent oscillation amplitude
$\alpha_{1,2,3\dots}$	Intrinsic damping of electrical resonators
$\alpha_{c,m}$	Intrinsic damping of cavity and magnon modes, respectively
$\Delta f_{\text{Hybrid.}}$	Change in sensor's frequency due to RNA hybridization
$\Delta\omega$	Injection locking range
$\epsilon_{0,r}$	Vacuum and relative electric permittivity
$f_{0,t}, H_{0,t}, T_{0,t}$	Frequency, relative humidity, and temperature at $t = 0$ and t
F	Noise factor of negative resistance circuit
$\vec{H}$	Static bias magnetic field strength
$\vec{H}_A$	Magnetic anisotropy field
Γ	Dissipative coupling strength
G	Gain factor of negative resistance circuit
γ	Non-linear damping of negative resistance circuit
γ_e	Gyromagnetic ratio
$\hat{c}^\dagger(\hat{c}), \hat{m}^\dagger(\hat{m})$	Creation (Annihilation) operators for photon and magnon modes, respectively
$\hat{O}$	Generic quantum mechanical operator
η	Mode overlapping factor between cavity and magnon

Variable	Definition
Θ	$= \pi$ or 0 , sign convention term
$I_{1,2,\dots}$	Current
J	Coherent coupling strength
K	Duffing (Kerr) non-linearity of negative resistance circuit (Chapter 2)
	Stability factor of negative resistance circuit (Chapter 3)
K_B	Boltzmann constant
$\kappa_{1,2,3,\dots}$	Extrinsic damping of electrical resonators
$\kappa_{c,m}$	Extrinsic damping of cavity and magnon modes
$\kappa(L, C, Cg)$	Injection locking amplifier performance indicator (function of circuit parameters)
k	Wave vector
λ	Wavelength
$L_{\Delta f}$	Phase noise measured at offset frequency Δf
V_{cc}	Oscillator circuit power supply voltage
$V_{1,2,3,\dots}$	Voltage of resonant circuits
V_c	Cavity mode volume
$\mu_{0,r}$	Vacuum and relative magnetic permeability
N	Number of spins
P_s	Available power at negative resistance circuit input
P_{DC}	DC Power consumption by oscillator circuit
Q_0	Unloaded quality factor
Q_L	Loaded quality factor

Variable	Definition
$R_{1,2,3...}, L_{1,2,3...}, C_{1,2,3...}$	Resistors, inductors, and capacitors
T	Temperature
$\tilde{\omega}_{\pm, I, II...}$	Hybridized frequencies
$\tilde{\omega}_{c, m}$	Complex frequencies of uncoupled cavity and magnon modes Real part represents frequencies; imaginary part represents damping
X	Reactance
ω	Driving frequency, provided by instruments
$\omega_{1,2,3...}$	Resonant frequencies of electrical resonators
$\omega_{c, m}$	Resonant frequencies of cavity and magnon modes (Real)
$\omega_m - \omega_c$	Field detuning condition
Z	Impedance
$\zeta_{1,2...}$	Frequency shift due to resonator–port coupling
E, B	Vector form of electric and magnetic fields

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THANK YOU!

To My Wife

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Chapter 1

Introduction

1.1 Background

Microwave electronics are fundamental to many modern technologies, supporting a wide range of applications across both civilian and military domains [1]. Operating within the frequency range of 300 MHz to 300 GHz, microwave electronics are integral to systems such as radar [2], mobile and satellite communication [3], as well as consumer products including automatic door openers [4], microwave ovens [5], and wireless devices [6]. In addition, the precision and efficiency of microwave circuits are also beneficial for advanced applications in remote sensing [7], medical imaging [8], spectroscopic analysis [9], and many others. Looking ahead, the continued evolution of microwave electronics is expected to bring greater integration with internet of things (IoT) and quantum information science [10]. Accordingly, state-of-the-art devices for generating, detecting, rerouting, and amplifying microwave signals will remain indispensable in fostering innovation.

Magnetism is an important part of microwave engineering. While the use of iron alloys is limited at high frequencies due to eddy current losses, ferrites —insulating materials made from iron oxide combined with metals such as nickel, zinc, or manganese, offer a viable alternative. These ferrimagnetic ceramics exhibit magnetic properties similar to ferromagnetic materials but with a distinct arrangement of magnetic domains [11]. Ferrites are characterized by high magnetic permeability, enabling effective interaction with magnetic fields, and

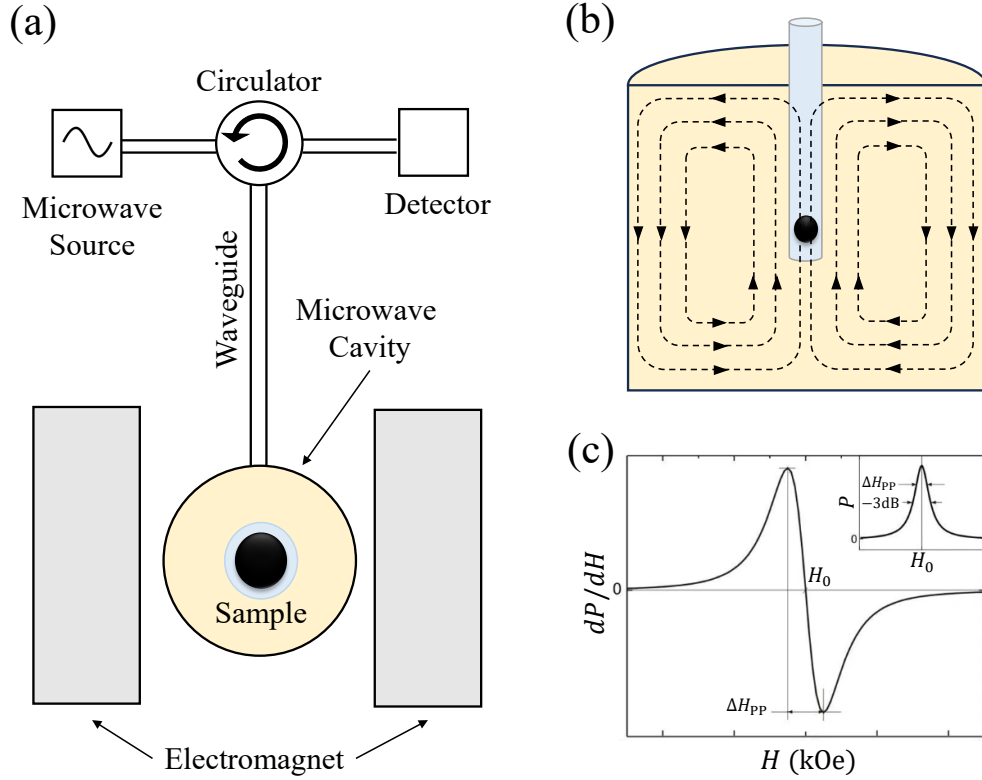


Figure 1.1: Overview of cavity based FMR experiment. (a) Experiment setup, the sample is held inside a microwave cavity, while the bias magnetic field is swept. A lock-in detector is used to measure the reflected microwave signal from the cavity. (b) Schematic illustration of microwave magnetic field (dashed line) inside the cavity. (c) Measured first derivative of microwave power with respect to bias magnetic field. Inset: Absorption spectrum. H_0 is the resonant field, the half power (-3 dB) linewidth of resonance is $\sqrt{3}\Delta H_{pp}$.

low electrical conductivity that can significantly reduce eddy current losses. Additionally, the resonant frequency of a ferrite can be tuned across a wide frequency range by an external magnetic field. As a result, they are commonly found in inductors, transformers, filters, phase shifters, and non-reciprocal devices [12]. Moreover, polished ferrimagnetic single crystals are increasingly sought after for use in narrow-band filters and power limiters due to their exceptionally low losses as resonating elements and the ease with which they couple to microwave circuits [13].

The interaction between microwave and magnetic materials has been harnessed since the early 20th century, through a phenomenon known as ferromagnetic resonance (FMR) [15].

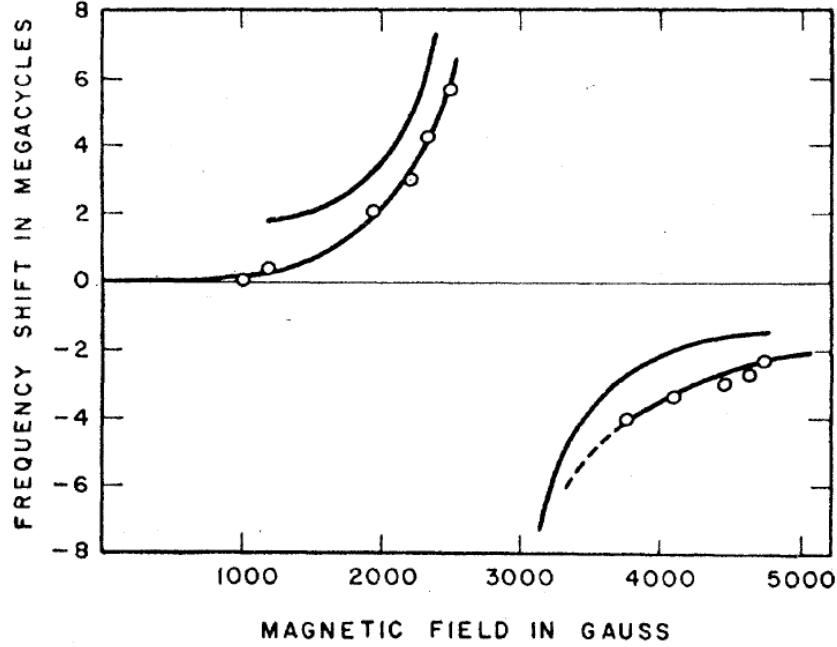


Figure 1.2: Plot of frequency shift versus magnetic field by Artman *et al.* [14], showing mode splitting phenomenon found in the microwave cavity and ferrimagnetic sample setup. The solid line represents their calculation.

FMR is especially useful for characterizing magnetic susceptibility of samples. Shown in Fig. 1.1, FMR is achieved by placing the magnetic sample inside a microwave cavity while varying the magnetic field to scan through the resonant condition. The sample introduces additional lossy inductance to the cavity, which affects both the frequency and the linewidth of the cavity's resonance profile [16]. By analyzing these changes, the complex magnetic susceptibility of the sample can be extracted.

In 1953, Artman *et al.* made a notable discovery while investigating a nickel-zinc ferrite sample during an FMR experiment [14]. Contrary to the changes in frequency and linewidth of cavity resonance, an unexpected mode splitting was observed as the magnetic field was swept [Fig. 1.2]. The phenomenon revealed a unique interaction between the ferrite sample and the microwave cavity. Their analysis later showed that the gap between the split mode frequencies was influenced by both the cavity's geometry and the sample volume [17]. The discovery highlighted two key factors that affect the strength of interaction between

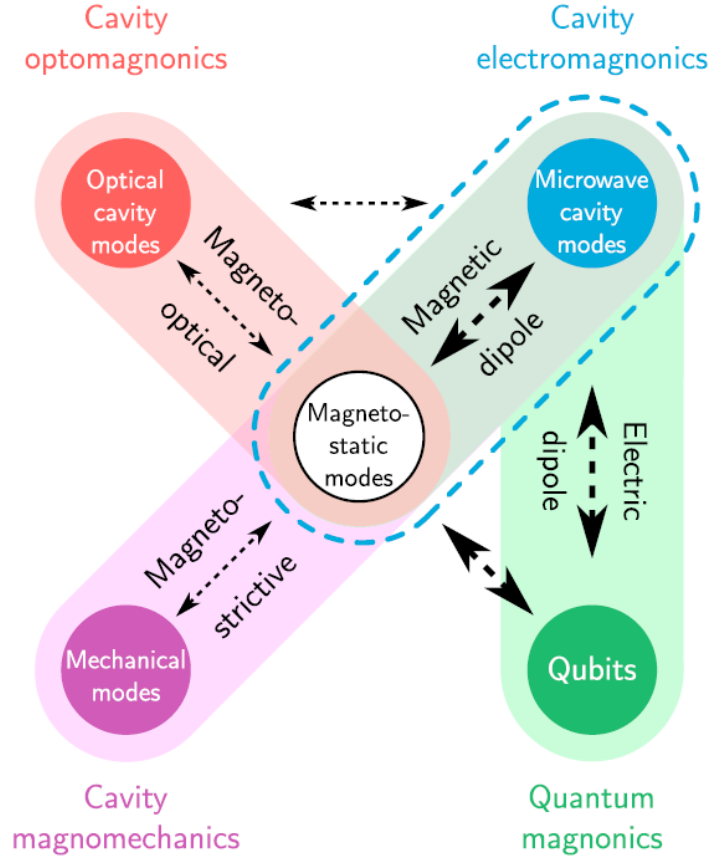


Figure 1.3: Schematic diagram illustrating various interactions in hybrid systems centered around magnons. In particular, magnons can interact with optical photons, microwave photons, and mechanical phonons through magneto-optical, magnetic dipole, and magnetostrictive interactions, respectively [18].

microwave cavity and magnetic material. Although initially overlooked, it has since gained renewed interest and is now being widely applied in many branches of hybrid system research.

A hybrid system essentially combines different subsystems for unique properties and functions that can not be attained by any single subsystem alone. Optomechanical systems, for example, combine optical and mechanical elements, leveraging the interaction between light and mechanical vibrations through radiation pressure. They are used in precision sensing applications, including displacement and mass sensors, accelerometers, and scanning probe microscopy [19]. Micro-electro-mechanical systems (MEMS) are miniature devices that integrate mechanical and electronic components on a micro scale. MEMS devices are

commonly found in radiowave frequency (RF) applications such as filters and oscillators, as well as in inkjet printer heads, automotive safety systems, biomedical testing, accelerometers, and gyroscopes [20]. Spintronics combine spin-based electronics with microwave or photonic systems, harnessing the electron's spin rather than its charge to address the limitations of traditional electronics, such as resistive heating. It has enabled the creation of spin-based transistors and memory devices [21]. A worth mentioning breakthrough in this field came in 1988 with the discovery of the giant magnetoresistance (GMR) effect by A. Fert and P. Grünberg [22], which is now used in hard disk drive (HDD) read heads.

Magnonics is a sub-field of hybrid systems that combines magnons (quanta of collective spin excitations) with other excitations, such as optical photons, microwave photons, mechanical phonons, and superconducting qubits [Fig. 1.3]. Due to their high tunability and ability to easily couple with a variety of dynamic platforms, magnons have gained interest as a promising basis for information processing devices that can bridge disparate technologies. For example, modern communication networks need to integrate fiber-optic and wireless systems for seamless data transfer. However, these signals are often weakly coupled due to their significantly different frequencies [23].

The foundation of hybrid systems involving magnons lies in cavity electromagnonics (or cavity magnonics in short), which studies the interaction between magnons and microwave photons. The interaction leads to the formation of hybridized modes: Superposed states having both magnonic and photonic characteristics. Cavity magnonics also allows for the manipulation of hybridized modes using conventional microwave circuit technologies. The concept was experimentally demonstrated in 2013 by Huebl *et al.*, using a setup where a yttrium iron garnet (YIG) plate was coupled to a planar superconducting resonator at millikelvin temperature [24]. The experiment not only confirmed that the coupling strength depends on number of spins, but also revealed a notable difference from the previous work by Artman *et al.*: mode splitting obtained in Huebl *et al.*'s setup was order of magnitude greater than spin relaxation rate and resonator decay rate, a phenomenon known as

strong coherent coupling. It was made possible by the large, high-quality sample and the confined microwave fields, which increased the filling factor. Although the experiment was conducted at cryogenic environment, researchers soon realized that strong coupling could also be achieved under relaxed conditions, enabling room temperature experiments [25], in various microwave cavities with controlled coupling strength [26, 27, 28, 29, 30, 31, 32]. To this day, YIG has remained the preferred material for cavity magnonics experiments due to its low damping and high spin density, but strong coupling can also be observed in many other materials, such as Gadolinium Iron Garnet (GdIG) [33], lithium ferrite [34]. As a result, the flexibility to harness photon-magnon interactions has spurred many interests in this field.

A remarkable breakthrough was the discovery of dissipative coupling. Unlike mode splitting (or level repulsion) seen in the coherent case, dissipative coupling causes two hybridized modes to merge, resulting in a level attraction phenomenon. The first experimental evidence was reported by Harder *et al.* in 2018 [35], using a YIG sphere placed inside a cylindrical cavity [Fig. 1.4], and was subsequently realized by other researchers with different setups [36, 37, 38, 39, 40]. Another key aspect of Harder's work is its demonstration that both coherent and dissipative coupling can be realized within the same setup, depending on whether the YIG sphere is positioned at the cavity field node or antinode. The finding has sparked the exploration of simultaneously achieving both coherent and dissipative interactions [41].

From a theoretical standpoint, understanding a cavity magnonic system requires Maxwell's equations to describe the electrodynamics of the cavity and the Landau-Lifshitz-Gilbert (LLG) equation to describe magnetization dynamics. Alternatively, a more intuitive approach is the coupled resonator model, which employs RLC circuit analogies [Fig. 1.5]. The model consists of a fixed-frequency resonator representing the microwave cavity and a tunable-frequency resonator representing the YIG, enabling frequency sweep experiments as a function of their detuning, analogous to those experiments conducted in cavity magnonic

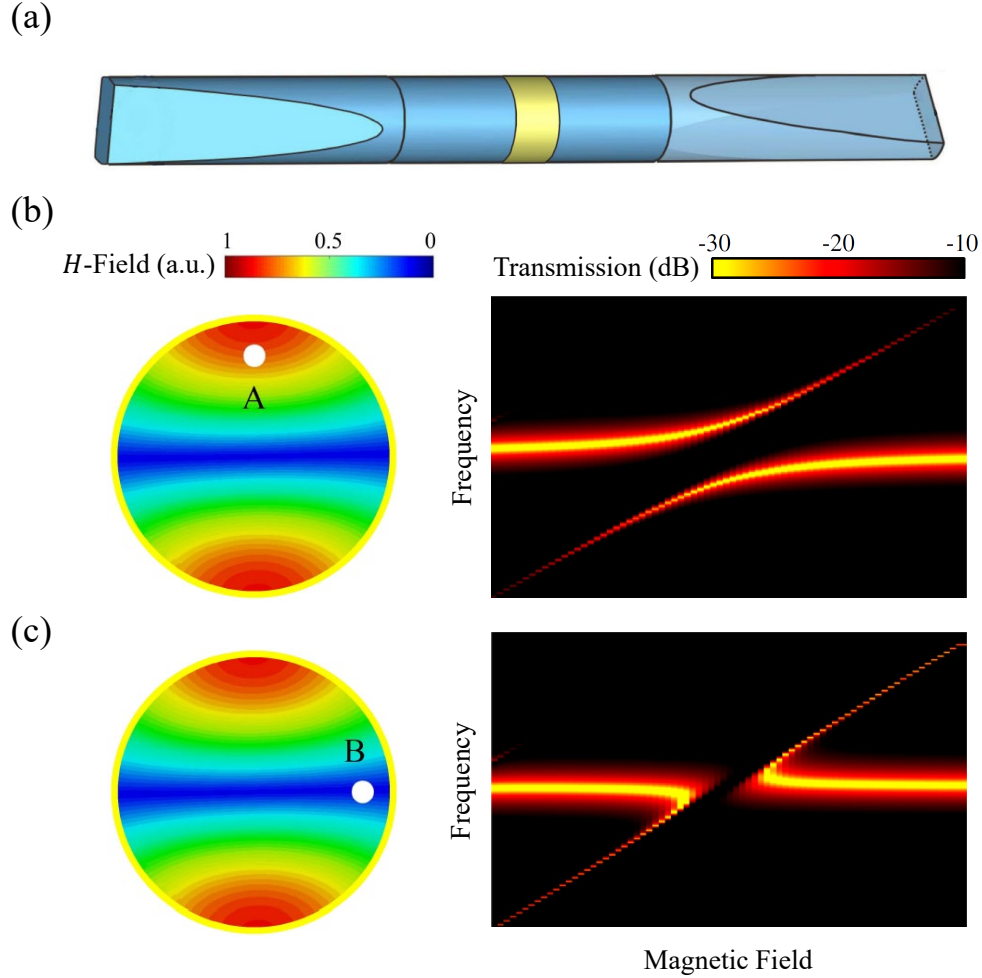


Figure 1.4: (a) Setup by Harder *et al.* [35] that can realize either coherent or dissipative coupling. (b) When the YIG sample is positioned at point A: the antinode of the magnetic field, coherent coupling becomes dominant, leading to level repulsion in the transmission spectrum. The H-field distribution shown in left represents the cross-section of the gold ring portion of the cavity depicted in (a). (c) When the YIG sample is positioned at point B: the node of the magnetic field, dissipative coupling becomes dominant, resulting in level attraction.

systems. Despite the microscopic differences between the coupled subsystems, the RLC circuit model successfully reproduces level attraction and level repulsion phenomena. Moreover, the model provides valuable insights into the coherent and dissipative interactions: The former, coherent coupling is a lossless energy exchange process in which resonators transfer energy via a reactive element (a capacitor or an inductor). Whereas dissipative coupling is inherently lossy, with energy exchange occurring through a common dissipation channel (a

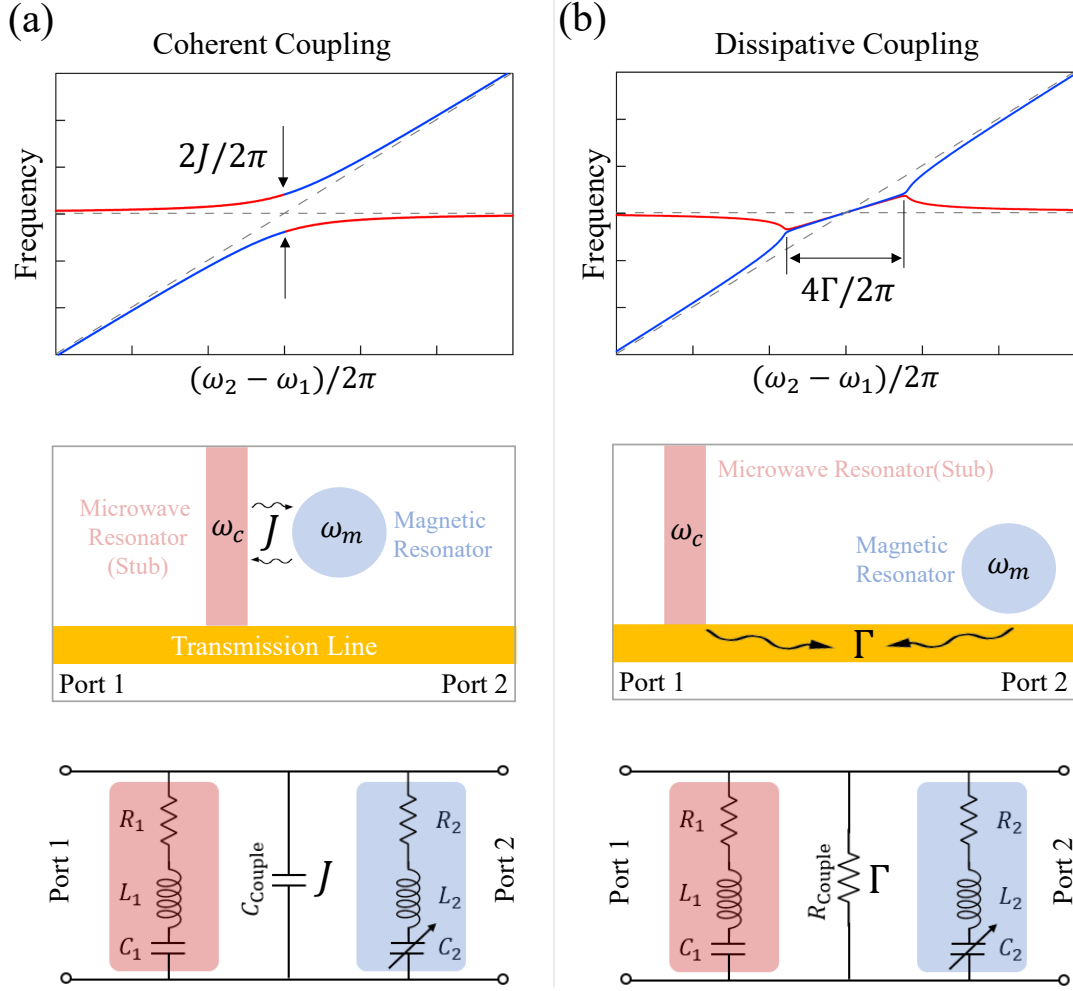


Figure 1.5: Illustration of (a) coherent and (b) dissipative coupling between a planar microwave resonator and a magnetic resonator, along with their corresponding equivalent circuit models. In coherent coupling, the interaction is direct and lossless, characterized by a coupling strength J . The corresponding circuit model consists of two RLC resonators coupled via a capacitor. In dissipative coupling, the interaction is indirect and lossy, mediated through a shared reservoir, with coupling strength denoted by Γ . Dissipatively coupled systems are modeled using a resistor-coupled configuration.

resistor). The strength of coherent coupling affects the degeneracy of hybridized states, while the strength of dissipative coupling determines the detuning range within which degeneracy can be established [Fig. 1.5 Top Panels]. However, the coupled resonator model has certain limitations. For instance, it does not capture the chirality of spin precession, a topic that will be addressed in later chapters.

From the coupled resonator model, one may recognize that a cavity magnonic device requires only three components: (1) a microwave cavity - the first resonator, (2) a magnetic resonator - the second resonator, and (3) a path for energy exchange between the two. Such inherent simplicity, combined with the flexibility of microwave circuit design and the tunability of magnon modes, has led to the development of a wide range of cavity magnonic devices, which operate on principles entirely distinct from those of conventional counterparts. One promising direction is the microwave signal processing electronics, including isolators [41, 47, 48, 49], circulators [50, 51], logic gates [52], oscillators [53], unidirectional amplifiers [54], injection locking amplifiers [53, 55], and perfect absorbers [56]. More recently, research into cavity optomagnonics has also emerged, bridging microwave and optical frequencies [57]. Other advancements include devices for memory [58], and speech recognition [59]. On-chip integration is another emerging field with the potential to advance cavity magnonic technologies toward miniaturization [16].

1.2 Motivation

Many microwave circuits use planar structures, such as those ones shown in Fig. 1.6(a)-(c). Commonly known as microstrips, they consist of a top conductor, a ground plane, and a dielectric substrate sandwiched in between. Microstrips are favored for their compact size, ease of integration with other electronics, and cost-effective fabrication [60]. They also offer more flexible control over wave propagation. However, compared to waveguides [Fig. 1.6(d)], planar designs face certain limitations [61]:

- **Radiation Loss:** A portion of microwave electromagnetic field (EM field) propagates in the air above the microstrip, leading to radiation losses.
- **Dielectric Loss:** The other portion of EM field propagates through the substrate. The imperfect polarization response of electric dipoles in the substrate to a fast alternating electric field leads to signal attenuation.

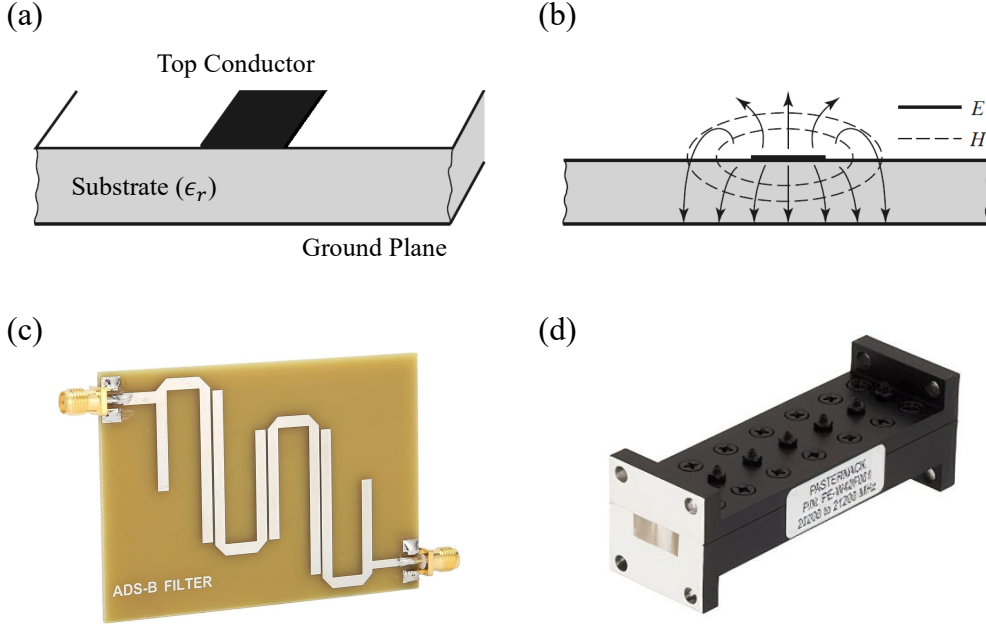


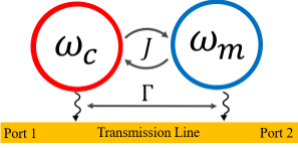
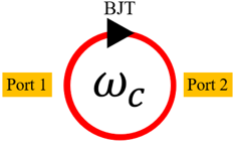
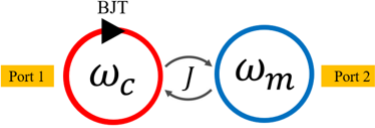
Figure 1.6: (a) Microstrip geometry. The top conductor (signal line) is separated from the bottom ground plane by a dielectric substrate. (b) Cross-sectional view of the microstrip, illustrating the distribution of electric and magnetic field lines. (c) A commercial microstrip bandpass filter. (d) A commercial waveguide bandpass filter.

- Skin Effect (Thin Conductor Loss): The skin effect causes current to concentrate near the surface of the conductor, increasing resistance and resulting in additional losses.

The loss mechanisms described above become more pronounced at higher frequencies and over longer propagation distances, posing difficulties in designing filters and oscillators with high spectral selectivity. The challenge can be fundamentally attributed to the low quality factor (Q-factor) of resonant elements. Past efforts to improve the Q-factor have focused on optimizing materials, geometries, circuit topologies, and layouts [62].

This thesis investigates complex resonant phenomena in planar cavity magnonic systems. By harnessing unique interactions between microwave cavity mode and magnetic magnon mode, the research aims to develop high-Q resonant devices, both passive and active, capable of overcoming the limitations discussed above. The key challenges and the proposed approach are summarized in Table. 1.1.

Table 1.1: Motivation and Approach Summary for Prototypes

Working Principle	Description
	Chapter 3: Isolator Challenge: Limited isolation due to intrinsic losses in ferrite material or microstrip resonators. Solution: Use coupling-induced zero-dissipation in hybrid magnon–cavity modes. Advantage: Enables theoretically infinite signal attenuation.
	Chapter 4: Sensor Challenge: Limited resolution caused by low Q-factor in planar resonant structures. Solution: Integrate a negative resistance circuit to compensate losses. Advantage: Improved resolution due to sharpened resonant profile.
	Chapter 5: Oscillator Challenge: Phase noise bottleneck stemming from low-Q resonator and transistor noise. Solution: Hybridize both carrier and sideband spectrum toward the single magnon mode frequency. Advantage: Suppresses phase noise while retaining transistor-based gain.

For detailed circuit configurations and performance parameters, please refer to Table 1.2.

The primary objective of this thesis is to design, construct, and optimize prototypes that validate the proposed solutions. Specifically, it focuses on exploring coupling mechanisms in cavity magnonic devices and developing optimization strategies to demonstrate feasible pathways for enhancing the performance of functional prototypes beyond that of conventional counterparts.

1.3 Thesis Outline

Chapter 1. Introduction provides an overview of hybrid systems, gradually narrowing

the focus to cavity magnonic systems. It then explains how a cavity magnonic system can be understood using coupled oscillator models, emphasizing the key components, interaction mechanisms, and notable advantages—their simplicity in components and compatibility with established microwave circuit technology. The chapter also touches on recent advancements in novel signal processing devices enabled by cavity magnonics. Finally, it presents the motivation: overcoming the low Q-factor limitations of planar microwave circuits through innovative coupling strategies.

Chapter 2. Resonant Circuits and Principles of Coupled Resonant Circuits introduces the fundamental concepts and principles of cavity magnonic devices. It begins by classifying passive and active resonant circuits, along with key performance metrics such as damping and phase noise. It then explains coherent and dissipative coupling mechanisms using coupled RLC circuits. The discussion subsequently transitions to a quantum mechanical description of microwave cavity photons and magnons, showing the connection between cavity magnonic systems and coupled harmonic oscillator systems. This chapter lays the theoretical foundation for the topics discussed in the following chapters.

Chapter 3. Non-reciprocal Microwave Transmission: Isolator presents passive cavity–YIG coupled device: isolator. The prototypes leverage the interplay between coherent and dissipative coupling to achieve non-reciprocal transmission, where isolation can theoretically become infinite under certain bias conditions. Non-reciprocal behavior is first demonstrated at millikelvin temperatures using a proof-of-concept prototype. Further research focuses on key performance metrics, including isolation, bandwidth, and insertion loss. To improve insertion loss and reduce the device size, two additional prototypes are developed and tested at room temperature.

Chapter 4: Gain Embedded Cavity Oscillator: Sensor focuses on the design of an active cavity oscillator, where a transistor circuit is integrated with a resonator to generate a sustained microwave signal. It begins by outlining design methods, such as the negative resistance concept and the evaluation of stability factors. A transistor-powered

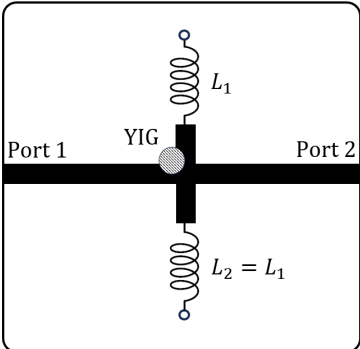
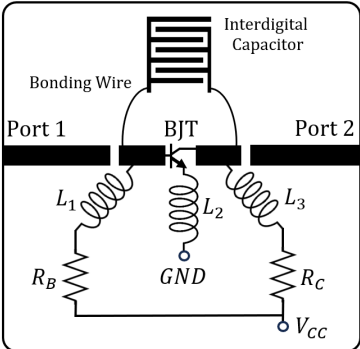
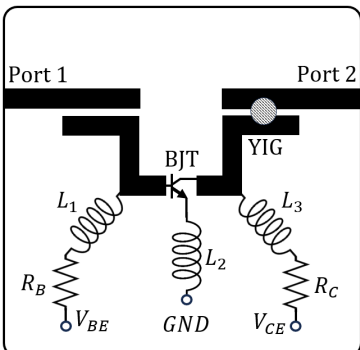
active interdigital capacitor sensor is then introduced. The chapter proceeds to detail the optimization process, emphasizing how various circuit parameters influence the output signal characteristics. Both short-term and long-term frequency stability are analyzed. The chapter concludes with a sensing experiment, demonstrating the sensor's ability to detect the hybridization of a target nucleic acid with a complementary sequence.

Chapter 5: Coherent Microwave Emission: Oscillator builds on the limitations found in Chapter 4: While the phase noise of an active oscillator can be reduced by fine tuning the transistor circuit, it remains fundamentally limited by electrical noise. To address the issue, the first part of this chapter introduces a low-phase-noise oscillator by coupling an active cavity to a YIG sphere. It reviews a new phase noise reduction mechanism and presents two low phase noise oscillator prototypes: one with a fixed frequency and another with tunable frequency. The second part of the chapter tackles the challenge of the narrow lock range observed when low-phase-noise cavity magnonic oscillators are used in injection locking amplification. Since the lock range is proportional to input power, a pre-amplification injection locking stage is placed ahead of the cavity magnonic oscillator to extend the lock range while preserving the input signal quality.

Chapter 6. Conclusion serves as the conclusion, highlighting the contributions and device performances demonstrated throughout this thesis.

A summary of the experiments and contributions can be found in Table. 1.2.

Table 1.2: Main Thesis Content at a Glance

Device	Description
	Chapter 3: Isolator Components: Passive Microstrip Cavity and YIG. Coupling Mechanism: Coherent and Dissipative Coupling. Experiment#1: Proof of concept isolator showing non-reciprocal transmission at milikelvin temperature. Experiment#2: Two additional prototype isolators to show improved insertion loss and reduced device size. Contribution: Isolation > 40 dB; Insertion Loss < 1 dB; small form factor.
	Chapter 4: Sensor Components: Active Microstrip Cavity. Coupling Mechanism: None. Experiment#1: Optimized transistor powered gold interdigital capacitor to address the short-term and long-term frequency stability. Then the sensor is used to detect RNA hybridization events. Contribution: Sensor detection limit: 2.8×10^6 nucleic base pairs per $0.2 \mu\text{L}$ sample.
	Chapter 5: Oscillator Components: Active Microstrip Cavity and YIG Coupling Mechanism: Coherent Coupling. Experiment#1: Reveals phase noise reduction mechanism in a coherently coupled active cavity-YIG device. Two prototype low phase noise oscillators are presented one with fixed frequency and one with tunable frequency. Experiment#2: Improves the lock range of a cavity magnonic injection locking amplifier by cascading a pre-injection locking amplification stage. Contribution: In Experiment #1, the lowest phase noise levels achieved were -117 dBc/Hz at 10 kHz offset and -132 dBc/Hz at 100 kHz offset by the fixed frequency oscillator; The tunable oscillator provides a tuning range of 600 MHz; In Experiment #2, the lock range tripled when the two stage injection locking amplifier gain > 20 dB.

Chapter 2

Resonant Circuits and Principles of Coupled Resonant Circuits

2.1 Introduction

Resonant circuits are fundamental building blocks in many microwave systems and can be classified into two types: Passive and Active. Passive resonant circuits, also known as resonators, are designed to store electromagnetic energy at a specific frequency or within a frequency band. They are used for frequency selection purposes such as filters [63], antennas [64], and sensors [65, 66]. In contrast, active resonant circuits, known as oscillators, generate microwave signals at desired frequencies. This is typically achieved by integrating an amplifier with a frequency-selective element (the resonator) to sustain oscillations. Unlike resonators, oscillators require an external DC power supply to maintain operation. Oscillators are commonly used as low noise signal sources, clocks and sensors [67, 68, 69, 70].

On the other hand, many advanced systems involve coupled resonant circuits, where resonators and (or) oscillators are interconnected to allow energy exchange between them. Coupled systems can realize functions that are not possible with a single resonant circuit. For instance, in near-field wireless charging, energy can be transmitted over a short distance via magnetic fields between charging coils [71]. In communication systems, coupled resonators are used to achieve certain filter responses, such as sharp transitions and multiple stop

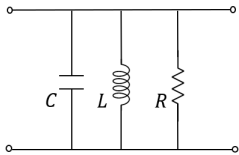
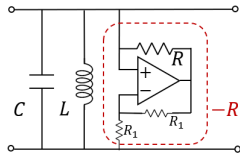
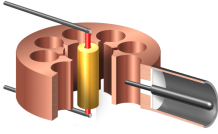
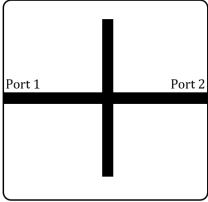
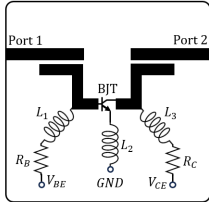
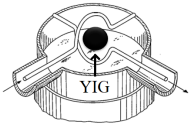
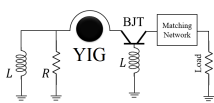
bands [72]. Additionally, in multi-mode antenna arrays, multiple elements can interact to produce improved radiation patterns and multi-band performance [73].

This chapter introduces the fundamentals of resonant circuits and coupling, first framed within the context of electrical RLC models. The electrical concepts are then extended to the realm of cavity magnonic systems where quantum mechanical description is also provided. As previously noted in Chapter 1.1, the dynamics of a cavity magnonic system can be understood using a coupled harmonic oscillator model, effectively capturing hybridization phenomena such as changes in resonance frequency and linewidth. The goal of this chapter is thus, to establish a conceptual framework for understanding cavity magnonic devices, thereby setting a stage for later chapters, where design strategies and parameter control will be explored in greater detail and become more intuitive to readers.

2.2 Classification of Single Resonant Circuit

In addition to being classified by power as either passive or active, resonant circuits can also be further categorized based on the types of elements they incorporate. For instance, RLC circuits offer basic frequency selection but are limited by losses and parasitic effects that hinder their widespread use in microwave frequencies [74]. 3D Cavities utilize metallic enclosures to confine electromagnetic waves, providing low loss and are widely applicable in high performance systems. However, its resonance frequency is a function of geometric shape, making them relatively bulky at low frequencies. Microstrips (2D Cavities), are a compact, cost-effective solution that falls in between [75]. Ferrites, which utilize magnetically tunable materials, provide frequency agility and are commonly used in non-reciprocal devices, filters, phase shifters and oscillators [11, 76]. A summary of classification method is provided in Table 2.1.

Table 2.1: Overview of Resonant Circuit Classification Methods.

Elements	Passive	Active	Features
RLC			Straight forward design, inherent loss and parasitic effects at high frequencies.
3D Cavity		 Cavity Magnetron	Low loss but bulky at lower frequencies.
Microstrip			Compact, easy to integrate, relatively lossy compared to 3D Cavities
Ferrite			Highly tunable, but needs external magnetic field

Many other resonant structures also exist, such as dielectric resonators and substrate-integrated waveguides (SIW); however, they are not directly relevant to the present discussion and are therefore omitted.

2.3 Performance Measure for Resonant Circuits

Performance measures for resonant circuits depend on whether they are passive or active. Passive resonators, which do not amplify signals and rely on external excitation, are characterized by their loss properties. Depending on whether the circuit is series or parallel, their impedance characteristics vary, resulting in a peak or dip shaped frequency response.

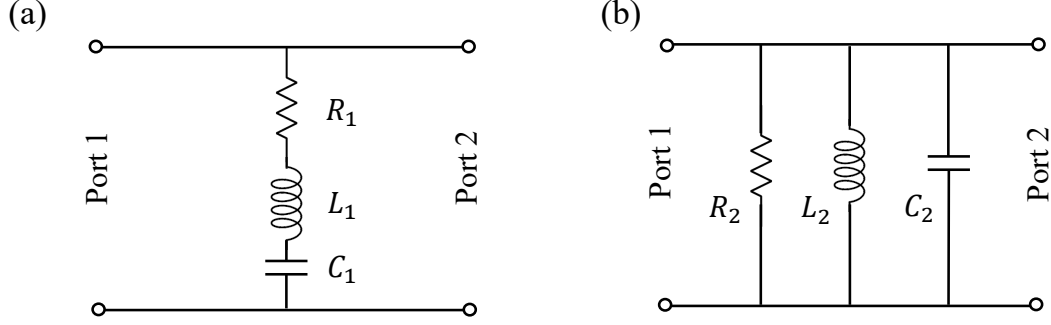


Figure 2.1: (a) A series RLC resonator circuit connected in a shunt configuration. (b) A parallel RLC resonator circuit connected in a shunt configuration.

In contrast, oscillators generate a continuous output signal with a more complex spectrum, which includes noise and harmonics. Therefore, characterization of oscillator performance revolves around spectral purity.

2.3.1 Passive Resonators

Consider the two RLC circuits connected in shunt configuration shown above in Fig. 2.1. Their resonant frequencies are given by $\omega_{1,2} = 1/\sqrt{L_{1,2}C_{1,2}}$, and their total impedance can be expressed as:

$$Z_{\text{Series}} = R_1 + i\omega L_1 + \frac{1}{i\omega C_1} \approx -i2L_1 \left((\omega_1 - \omega) + i\frac{R_1}{2L_1} \right) \quad (2.1a)$$

$$Z_{\text{Parallel}} = \left(\frac{1}{R_2} + \frac{1}{i\omega L_2} + i\omega C_2 \right)^{-1} \approx \left[-i2C_2 \left((\omega_2 - \omega) + i\frac{1}{2R_2C_2} \right) \right]^{-1} \quad (2.1b)$$

where ω denotes the driving frequency. In the final step, the approximation $\omega + \omega_{1,2} \approx 2\omega$ is applied, assuming that the driving frequency is close to their resonant frequency.

To analyze the response of the circuits under external excitation, the $ABCD$ matrix formalism is first applied, followed by a conversion to S-parameters (scattering parameters).

As illustrated in Fig. 2.2(a), an $ABCD$ matrix defines a linear relationship between the

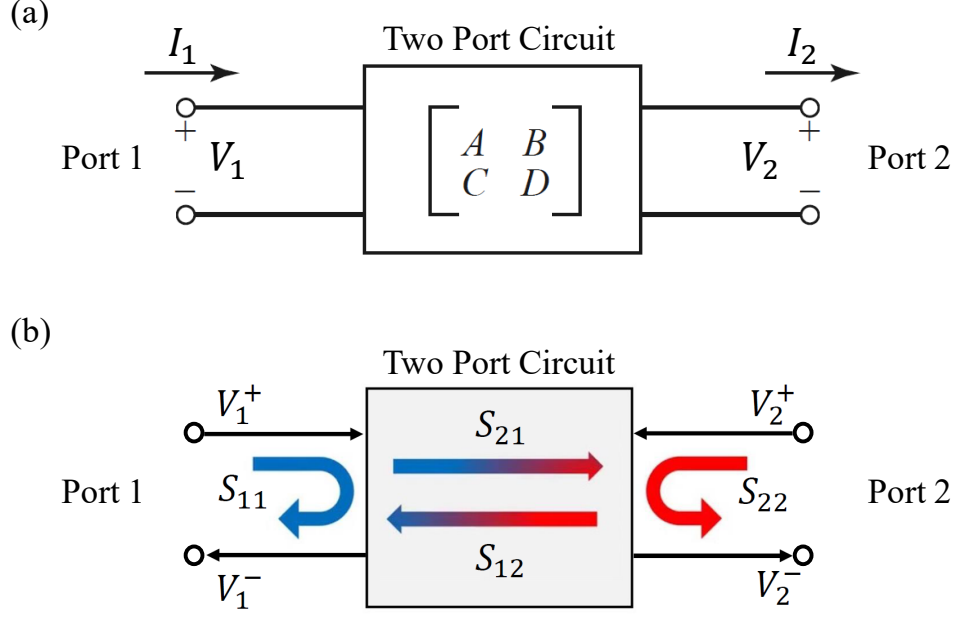


Figure 2.2: A two port network showing the implications of entries in (a) the $ABCD$ matrix, and (b) the S -parameter matrix. S_{11} : input reflection. S_{21} : forward transmission. S_{12} : backward transmission. S_{22} : output reflection.

input voltage and current at Port 1, $[V_1, I_1]^T$, and the output voltage and current at Port 2, $[V_2, I_2]^T$:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad (2.2)$$

For a shunt element with total impedance $Z_{\text{Series/Parallel}}$ found earlier in Eqs. (2.1), their corresponding $ABCD$ matrices can be written as:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{Series}} = \begin{bmatrix} 1 & 0 \\ 1/Z_{\text{Series}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -i2L_1 \left((\omega_1 - \omega) + i\frac{R_1}{2L_1} \right) & 1 \end{bmatrix}^{-1} \quad (2.3a)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{Parallel}} = \begin{bmatrix} 1 & 0 \\ 1/Z_{\text{Parallel}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -i2C_2 \left((\omega_2 - \omega) + i\frac{1}{2R_2C_2} \right) & 1 \end{bmatrix} \quad (2.3b)$$

While the $ABCD$ matrix is a useful analytical tool, microwave components are commonly characterized by their S -parameters, which describe the relationship between incident and

reflected voltages (or power) at each port [Fig. 2.2(b)]. Besides, S-parameter descriptions are more convenient in practice, as they can be directly obtained using a vector network analyzer (VNA). The S-parameters of an arbitrary two port circuit are given by:

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \quad (2.4)$$

Then, the conversion from $ABCD$ matrix to S-parameters can be carried out using the following expression, assuming a characteristic impedance of Z_0 :

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D} & \frac{2(AD-BC)}{A+B/Z_0+CZ_0+D} \\ \frac{2}{A+B/Z_0+CZ_0+D} & \frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D} \end{bmatrix} \quad (2.5)$$

Of particular interest is the forward transmission coefficient (S_{21}), which quantifies how much of the input signal incident at Port 1 is transmitted to Port 2, as illustrated in Fig. 2.2(b). By substituting Eqs. (2.3) into the S-parameter conversion formula in Eq. (2.5), the transmission responses of the series and parallel RLC circuits can be derived.

Shunt Series RLC

$$\begin{aligned} S_{21}^{\text{Series}} &= \frac{(\omega - \omega_1) + i(R_1/2L_1)}{(\omega - \omega_1) + i(R_1/2L_1) + i(Z_0/4L_1)} \\ &= \frac{(\omega - \omega_1) + i\alpha_1}{(\omega - \omega_1) + i\alpha_1 + i\kappa_1} \end{aligned} \quad (2.6)$$

where $R_1/2L_1 = \alpha_1$ and $Z_0/4L_1 = \kappa_1$, are defined as intrinsic and extrinsic dampings, respectively. The implication of α_1 can be understood by examining the unloaded Q-factor of a series RLC circuit, Q_0^{Series} [77]:

$$Q_0^{\text{Series}} = \frac{\omega_1 L_1}{R_1} = \frac{\omega_1}{2\alpha_1} \quad (2.7)$$

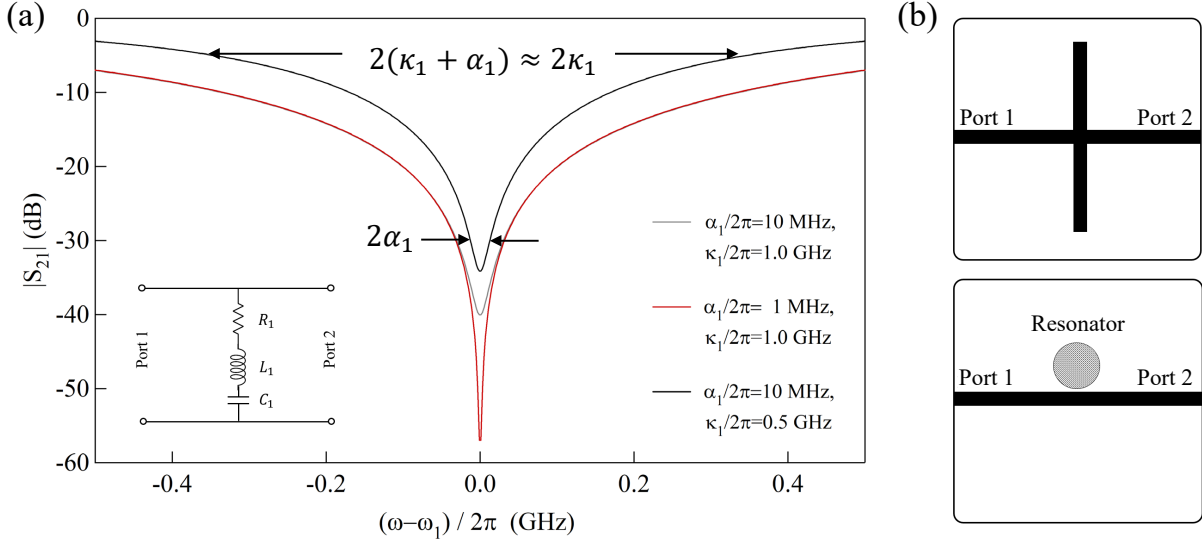


Figure 2.3: (a) Different resonant profiles of series shunt RLC circuit with various α_1 and κ_1 values. α_1 and κ_1 governs 3 dB width measured from bottom up and top down, respectively. (b) A shunt series RLC circuit can be used to model stub resonators or aperture coupled resonators. For details on modeling aperture coupled resonators, see Appendix A.

which shows α_1 is the Half-Width-Half-Maximum (HWHM) of the resonant profile, as shown in Fig. 2.3(a). It characterizes the rate of energy dissipation within the resonator due to intrinsic loss mechanisms, such as resistive and dielectric losses.

In contrast, κ_1 depends on the external load impedance, Z_0 , and quantifies the amount of energy that couples out of the resonator into the external circuit. It can be seen from Fig. 2.3(a) that κ_1 governs the broadening of the resonance background, measured from 3 dB top down. A larger value of κ_1 indicates stronger coupling to the load, resulting in greater signal leakage from the resonator to the external circuitry.

Regarding the overall resonance shape, the impedance of a series shunt RLC circuit reaches its minimum at resonance, resulting in maximum current being drawn from the transmission line, which manifests as a dip in the forward transmission coefficient. Such behavior makes Eq. (2.6) applicable for modeling stub loaded or aperture coupled resonators [Fig. 2.3(b)]. In the absence of a resonator, nearly all of the microwave power passes directly from port 1 to port 2 through the transmission line. The presence of resonator, thus,

extracts energy from the transmission path, causing a frequency-selective attenuation.

Shunt Parallel RLC

Following the same steps as those applied in obtaining Eq. (2.6), the transmission coefficient for a shunt parallel RLC circuit can be expressed as:

$$\begin{aligned} S_{21}^{\text{Parallel}} &= \frac{i/(Z_0 C_2)}{(\omega - \omega_2) + i/(2R_2 C_2) + i/(Z_0 C_2)} \\ &= \frac{i\kappa_2}{(\omega - \omega_2) + i\alpha_2 + i\kappa_2} \end{aligned} \quad (2.8)$$

where $\alpha_2 = 1/(2R_2 C_2)$ and $\kappa_2 = 1/(Z_0 C_2)$ represent the intrinsic and extrinsic damping rates, respectively. As with the previous case, comparing α_2 to the unloaded Q-factor of a parallel RLC circuit (Q_0^{Parallel}) yields:

$$Q_0^{\text{Parallel}} = \omega_2 R_2 C_2 = \frac{\omega_2}{2\alpha_2} \quad (2.9)$$

The spectral response of a parallel RLC circuit is graphed in Fig. 2.4(a). Notably, an increase in the extrinsic damping κ_2 raises the overall signal level near resonance. Such behavior can be understood by noting that Z_0 is loaded in parallel with R_2 . A large κ_2 corresponds to a small Z_0 , which results in more power being drawn by the load, thereby leading to a higher measured signal level. In addition, $\kappa_2 + \alpha_2$ is the HWHM.

The impedance reaches maximum at resonance for a parallel RLC circuit, effectively blocking current flow from the top signal line through the resonator branch. As a result, large portion of power continues along the transmission line path, producing a peak in the S_{21}^{Parallel} . Equation (2.8) can therefore be used to describe gap coupled resonators such as those shown in Fig. 2.4(b), where input signals from port 1 excite a resonant mode that stores energy and releases it to port 2.

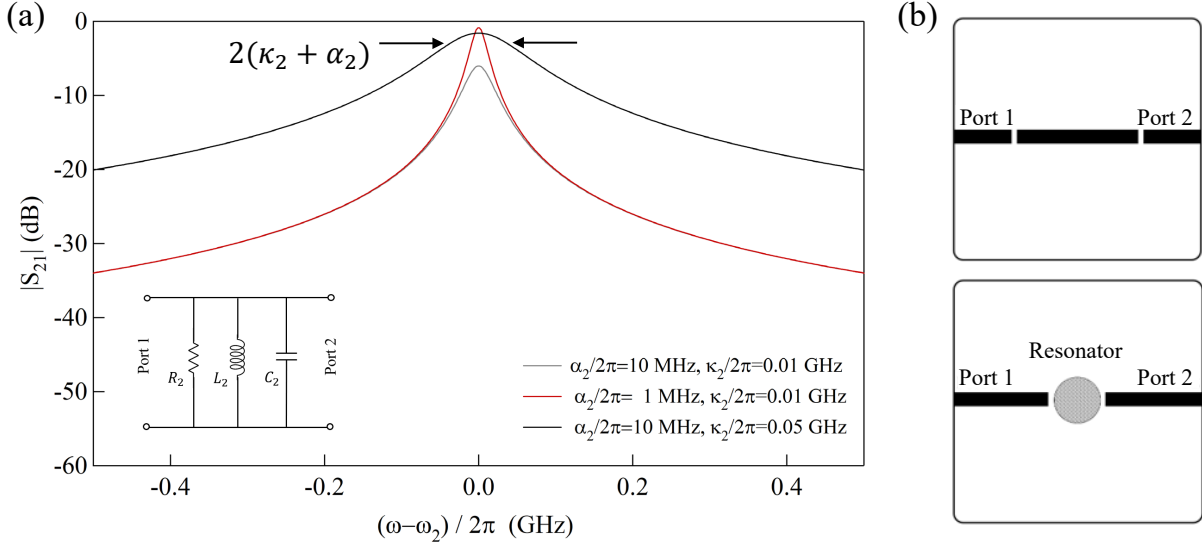


Figure 2.4: (a) Different resonant profile of parallel shunt RLC circuit with various α_2 and κ_2 values. α_2 and κ_2 together govern the 3 dB width measured from top down (b) A shunt parallel RLC circuit can be used to model gap coupled resonators.

2.3.2 Active Oscillators

Following the characterization of passive resonators in terms of damping rates and their ability to filter energy, attention now shifts to active oscillators—devices that incorporate gain elements to offset internal losses and support self-sustained oscillation. Oscillation commences when the loop gain exceeds unity, allowing small perturbations from thermal or electronic noise to grow progressively. As the signal amplitude increases, nonlinearities in the active components cause gain compression, leading to a steady state where gain and loss are balanced. Once stabilized, the oscillator operates at a defined frequency, though noise can introduce frequency instability and degrade spectral purity. The following discussion covers oscillation initiation, stabilization, and noise influences.

The Van der Pol-Duffing Oscillator

The active oscillation process was originally formulated by Van der Pol, using a negative resistance oscillator whose amplitude progressively increases until it stabilizes at a constant value [78]. The model describes a nonlinear oscillator in which damping is not a constant

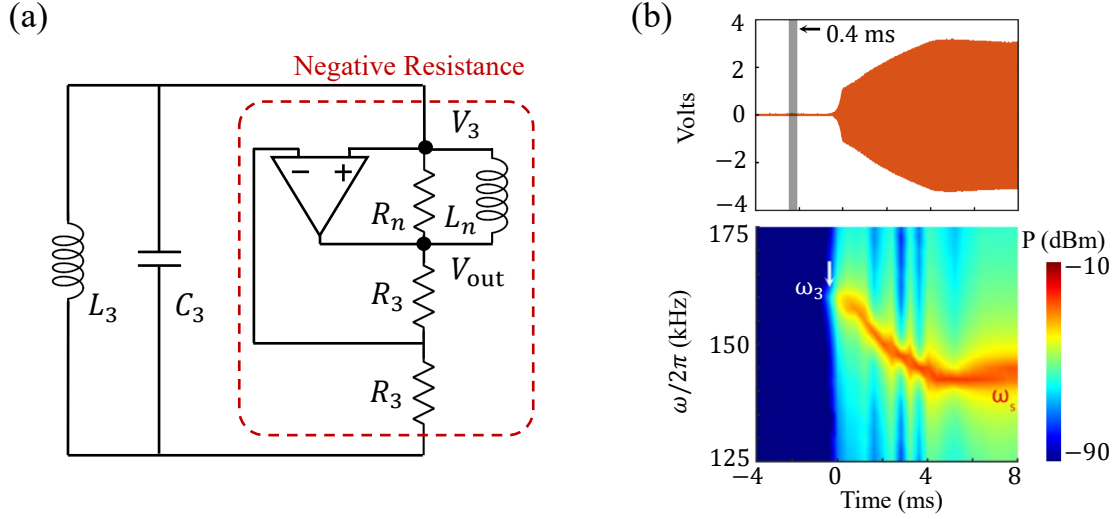


Figure 2.5: (a) Circuit schematic of a negative resistance oscillator. (b) Experimentally obtained time-domain dynamics of the output voltage V_3 , illustrating the amplitude stabilization process described by the van der Pol model. The spectrogram is obtained by applying a Fourier transform to each consecutive time-domain signal within the window (shaded in gray) and then concatenating the results. It illustrates the frequency shift during the stabilization process, as described by the Duffing model [80].

but varies with the oscillation amplitude. Another notable nonlinear oscillator model is proposed by Duffing, which introduces frequency as a variable that depends on amplitude, known as Kerr nonlinearity [79]. In simple terms, the Van der Pol model describes amplitude-dependent damping, while the Duffing model describes amplitude-dependent frequency shift.

Consider the circuit shown in Fig. 2.5(a), consisting of a parallel LC circuit and a negative resistance block. The op-amp output voltage V_{out} is twice of the voltage at the non-inverting input V_3 ¹. As a result, current flows upward through the resistance R_n and inductance L_n , giving rise to the term “negative resistance” [80].

In the most general case, the output response of a nonlinear circuit with input voltage V_3 and output voltage V_{out} can be modeled as a Taylor series expansion [6]:

$$V_{out} = \rho_0 + \rho_1 V_3 + \rho_2 V_3^2 + \dots \quad (2.10)$$

¹Op-amp Gain: $V_{out}/V_3 = 1 + R_3/R_3 = 2$

where ρ_n represents nonlinear coefficients. The first DC term (ρ_0), along with all even-powered terms can be neglected for Eq. (2.10) to hold true in non-inverting configuration. Therefore, the expansion of V_{out} simplifies to:

$$V_{\text{out}} = \rho_0 V_3 - \frac{\rho_1}{3} V_3^3 \quad (2.11)$$

Applying Kirchoff's current law at junction V_3 yields the following expression:

$$\int \frac{V_3}{L_3} dt + C_3 \frac{dV_3}{dt} + \left(\frac{V_3 - V_{\text{out}}}{R_n} + \int \frac{V_3 - V_{\text{out}}}{L_n} \right) = 0 \quad (2.12)$$

the first, second, and third terms represent the currents flowing through the inductor, capacitor, and negative resistance branches, respectively. Setting $G = (\rho_0 - 1)/2R_n C_3$, $\gamma = \rho_1/8R_n C_3$, and $\beta = \rho_1/3L_n C_3$, then the oscillator can be described by a second order differential equation:

$$\frac{d^2 V_3}{dt^2} + 2(-G + 4\gamma V_3^2) \frac{dV_3}{dt} + (\omega_3^2 + \beta V_3^2) V_3 = 0 \quad (2.13)$$

where $\omega_3 = 1/\sqrt{L_3 C_3}$. Assuming a general solution of the form $V(t) = \frac{1}{2}[a(t)e^{i\omega_3 t} + a^*(t)e^{-i\omega_3 t}]$, Eq. (2.13) can be simplified further to

$$\frac{da}{dt} = (G - \gamma|a|^2)a - iK|a|^2 a \quad (2.14)$$

the variable a denotes time-dependent oscillation amplitude, the first term in bracket describes amplitude-dependent damping (Van der Pol term), and the second term $K = -3\beta/8\omega_3$ is the Kerr nonlinearity (Duffing term). Using the ansatz $a(t) = A(t)e^{i\Delta f t}$, the corresponding steady state amplitude and frequency shift (Δf) can be found to be:

$$\frac{dA}{dt} = (G - \gamma|A|^2)A, \quad \text{and} \quad \Delta f = -K|A|^2 \quad (2.15)$$

when oscillator's amplitude is small ($|A| \approx 0$), the nonlinear damping term ($\gamma|A|^2$) becomes negligible, making the system behaves like a linear oscillator with gradually increasing amplitude driven by the gain factor, G . As the amplitude grows to $|A| = \sqrt{G/\gamma}$, the gain is balanced by the nonlinear damping, and oscillator enters steady state with frequency $\omega_s = \omega_3 - K|A|^2$.

Temporal evolution of the oscillation signal is illustrated in Fig. 2.5(b), showing an initial increase in amplitude, accompanied by a shift in frequency, and eventual stabilization. Thus, a negative resistance oscillator can be described using a Van der Pol-Duffing model [80].

Phase Noise

The Van der Pol-Duffing oscillator model effectively captures both the startup and steady-state oscillation behavior. However, it assumes the output signal is an ideal sinusoidal waveform, neglecting an important aspect of oscillators: noise.

Figure. 2.6(a) shows the output of an ideal sinusoidal oscillator: $V(t) = A \sin(2\pi f_0 t + \phi)$. Its amplitude (A), frequency (f_0), and phase (ϕ) are all constant. Viewed from frequency domain, $V(t)$ therefore appears as a single spectral line, with all of its power concentrated at f_0 . Yet, real-world oscillators inevitably exhibit variations in their output that can never be completely eliminated.

A real-world oscillator can be described by the expression: $V'(t) = A(t) \sin(2\pi f_0 t + \phi(t))$, where amplitude and phase vary over time, usually in an unpredictable manner. The output of a real-world oscillator is illustrated in Fig. 2.6(b). In the time domain, amplitude fluctuations manifest as changes in the signal's peak to peak values, while phase variations appear as horizontal shifts in the waveform². Amplitude fluctuations are significantly smaller than phase variations, as nonlinearities in the gain element act as stabilizers. However, phase deviations are not actively corrected and can accumulate gradually over time. As a result, real-world oscillators almost exclusively exhibit phase variations. When viewed in the

²A phenomenon often referred to as jitter

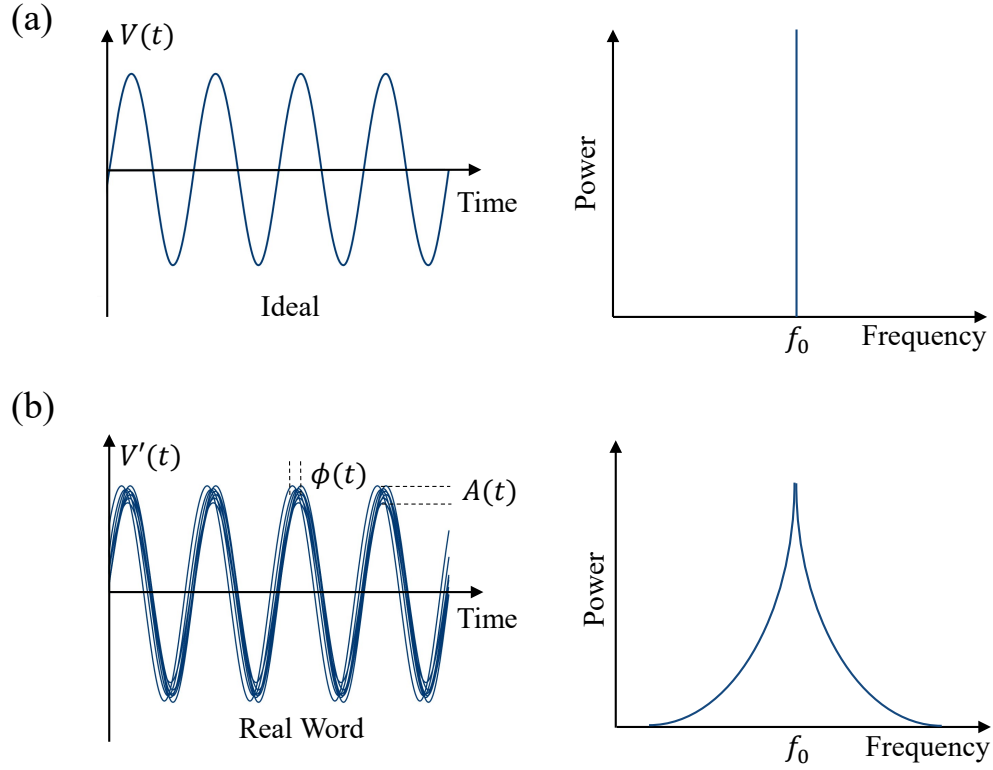


Figure 2.6: (a) Time and frequency domain representation of an ideal sinusoidal oscillator output: $V(t) = A \sin(2\pi f_0 t + \phi)$. Its amplitude (A), frequency (f_0), and phase offset (ϕ) are all constant. (b) Time and frequency domain representations of a real-world oscillator output: $V'(t) = A(t) \sin(2\pi f_0 t + \phi(t))$, showing variations in phase, $\phi(t)$ and amplitude, $A(t)$.

frequency domain, phase variations create sidebands extending out from both sides of the carrier.

Depicted in Fig. 2.7(a), phase noise (L) is used to quantify the noise power at the offset frequency (Δf) from the carrier within a 1 Hz bandwidth:

$$L_{\Delta f} = \frac{\text{Noise Power at Offset Frequency}}{\text{Carrier Power}} \quad (2.16)$$

Since phase noise reduces spectral purity, lower phase noise values result in a cleaner output spectrum. A common way to describe oscillator phase noise is by measuring spot phase noise defined at specific frequency offsets—typically at decade intervals: $\Delta f = 1$ kHz,

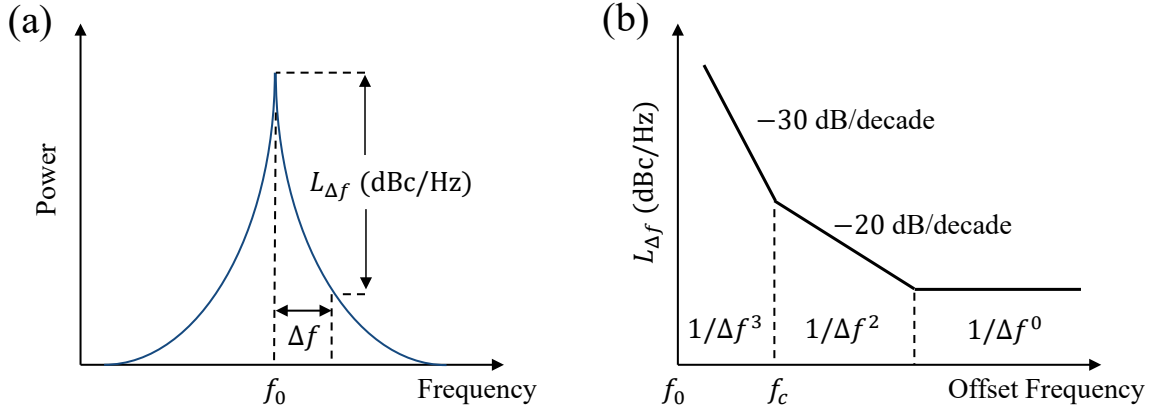


Figure 2.7: Illustration of oscillator power spectrum and phase noise characteristics. (a) Phase noise ($L_{\Delta f}$) quantifies the amount of phase instability per unit frequency offset from the carrier (f_0). (b) Phase noise profile of an oscillator as a function of offset frequency (Δf) predicted by Leeson's Model.

10 kHz, and 100 kHz, etc.

Oscillators are inherently nonlinear systems, making phase noise analysis a complex and long-standing area of research [81, 82]. Among the various models developed, Leeson's model is the most well known and remains highly relevant in practical design [83].

$$L_{(\Delta f)} = \frac{FK_B T}{P_s} \left[1 + \left(\frac{f_0}{2Q_L \Delta f} \right)^2 \left(1 + \frac{f_c}{\Delta f} \right) \right] \quad (2.17)$$

here, F is the noise factor of the amplifier—an empirical fitting parameter, and K_B is Boltzmann's constant. P_s represents the available power at the input of the amplifier, and T denotes the absolute temperature. f_0 is the carrier frequency and Q_L is the loaded quality factor of the resonator. f_c is the corner frequency, which marks the transition between the 30 dB/decade and 20 dB/decade slope regions in the phase-noise spectrum. The predicted phase noise spectral density by Leeson's model is illustrated in Fig. 2.7(b), where three distinct regions are identified in Table. 2.2.

Leeson's model not only offers a theoretical framework for analyzing phase noise but also provides practical design insights for minimizing it. Key strategies include selecting

Table 2.2: Characteristic Regions of Oscillator Phase Noise According to Leeson's Model

Region	Dominant Noise Source	Typical Offset Range
$1/f^3$	Transistor (Flicker) Noise	<1 kHz
$1/f^2$	Thermal and Shot Noise	1 kHz–100 kHz
$1/f^0$	Additive White Noise	>100 kHz

low-noise active devices, maximizing output power, and using high-Q resonators.

Apart from phase noise, the figure of merit (FOM) allows evaluating and comparing oscillator performance by incorporating both phase noise and power efficiency [67]. It enables fair comparisons across oscillators operating at different frequencies, with varying noise characteristics and power consumption. A lower (more negative) FOM indicates better noise performance relative to power usage, making it a critical benchmark in oscillator design:

$$\text{FOM} = L_{\Delta f} - 20 \log_{10} \left(\frac{f_0}{\Delta f} \right) + 10 \log_{10} \left(\frac{P_{\text{DC}}}{1 \text{ mW}} \right) \quad (2.18)$$

where P_{DC} is the DC power consumed by an oscillator .

2.4 Coupling Resonant Circuits

Following the discussion on single resonant circuits, attention is now directed towards coupled scenario. Interactions between resonant circuits can occur through three primary ways: coherent coupling, dissipative coupling, and a combination of the former two. This section explores the fundamental concepts and behaviors associated with different coupling methods using passive circuit models.

Consider the coupled two shunt-series resonator circuit shown in Fig. 2.8, where each colored branch consists of a resistor, an inductor and a capacitor denoted by $R_{1,2}$, $L_{1,2}$, and $C_{1,2}$, respectively. The resonant frequency of the left (red) branch is kept fixed at $\omega_1 = 1/\sqrt{L_1 C_1}$, while the resonant frequency of the right (blue) branch $\omega_2 = 1/\sqrt{L_2 C_2}$, is adjustable via a tunable capacitor. Energy exchange between the two resonators take place

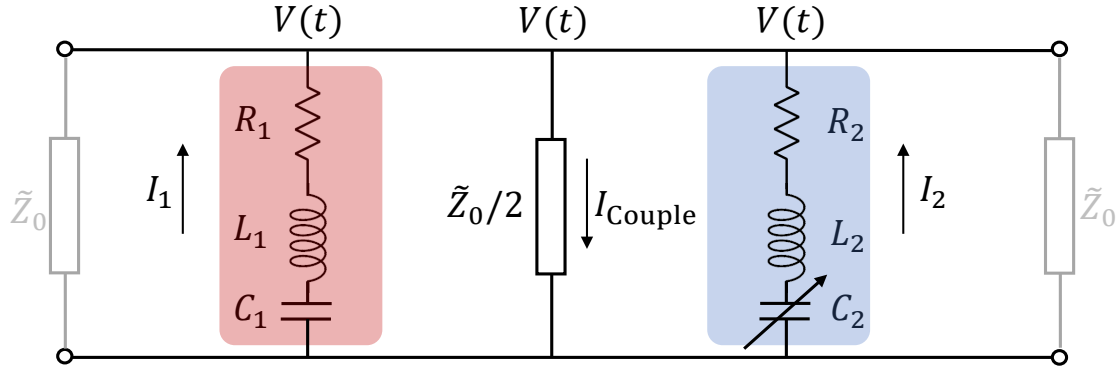


Figure 2.8: Schematic of a coupled RLC circuit. The resonance frequency $\omega_1 = 1/\sqrt{L_1 C_1}$ is fixed, while $\omega_2 = 1/\sqrt{L_2 C_2}$ is tunable via the adjustable capacitor C_2 , enabling frequency sweep measurements against their detuning. Energy exchange between ω_1 and ω_2 is realized through the middle branch, which is parallel combination of the left and right complex port impedances $\tilde{Z}_0 = Z_0 + iX_0$.

through the complex port impedances $\tilde{Z}_0 = Z_0 + iX_0$, which combine in parallel to form an effective impedance of $\tilde{Z}_0/2$ in the middle³. Under such configuration, the voltage at top junction, $V(t)$, can be expressed in the following three forms:

$$R_1 I_1 + L_1 \frac{dI_1}{dt} + \frac{1}{C_1} \int I_1 dt = V(t) \quad (2.19a)$$

$$I_{\text{Couple}}(\tilde{Z}_0/2) = V(t) \quad (2.19b)$$

$$R_2 I_2 + L_2 \frac{dI_2}{dt} + \frac{1}{C_2} \int I_2 dt = V(t) \quad (2.19c)$$

where I_1 , I_{Couple} , and I_2 denote the currents flowing through each of the shunt branches, as indicated by the arrows in Fig. 2.8. According to Kirchhoff's current law, I_{Couple} is related to I_1 and I_2 by:

$$I_{\text{Couple}} = -(I_1 + I_2) \quad (2.20)$$

Substituting Eq. (2.20) to Eqs. (2.19), then differentiating both sides, and expressing the

³For more details regarding a single-shunt series RLC circuit with complex port impedance, please see the Appendix. A

resulting equations in terms of the intrinsic and extrinsic damping rates defined earlier in Eq. (2.6), where $\alpha_{1,2} = R_{1,2}/(2L_{1,2})$, $\kappa_{1,2} = Z_0/(4L_{1,2})$, and $\zeta_{1,2} = X_0/(4L_{1,2})$:

$$\ddot{I}_1 = -\omega_1^2 I_1 - 2(\alpha_1 + \kappa_1 + i\zeta_1)\dot{I}_1 - 2(\kappa_1 + i\zeta_1)\dot{I}_2 \quad (2.21a)$$

$$\ddot{I}_2 = -\omega_2^2 I_2 - 2(\alpha_2 + \kappa_2 + i\zeta_2)\dot{I}_2 - 2(\kappa_2 + i\zeta_2)\dot{I}_1 \quad (2.21b)$$

Finally, assume a solution of the form $I_{1,2} = a_{1,2}(t)e^{-i\omega_{\text{ref}}t}$, where $a_{1,2}(t)$ are time dependent amplitudes and $\omega_{\text{ref}} = \frac{1}{2}(\omega_1 + \omega_2)$. The dynamics of the coupled resonators can then be summarized as:

$$i \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix} = \begin{bmatrix} \omega_1 - i(\alpha_1 + \kappa_1) & J - i\Gamma \\ J - i\Gamma & \omega_2 - i(\alpha_2 + \kappa_2) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.22)$$

where real coherent coupling strength is given by $J = \sqrt{\zeta_1 \zeta_2} = X_0/(4L)$ and the dissipative coupling strength is $\Gamma = \sqrt{\kappa_1 \kappa_2} = Z_0/(4L)$, assuming $L_1 \approx L_2 = L$. It can be observed that the diagonal terms of the 2×2 matrix in Eq. (2.22) correspond to the uncoupled complex frequencies of the two resonators $\tilde{\omega}_{1,2} = \omega_{1,2} - i(\alpha_{1,2} + \kappa_{1,2})$, and off-diagonal terms represent the coupling strength:

$$i \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix} = \begin{bmatrix} \text{Uncoupled Resonator 1} & \text{Coupling Strength} \\ \text{Coupling Strength} & \text{Uncoupled Resonator 2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.23)$$

Case #1: No Coupling

If coupling is absent, the two resonators remain completely isolated, meaning that variations in ω_2 have no influence on ω_1 . Intuitively, when ω_2 is swept across ω_1 , the dispersion should show a simple crossing behavior. The damping rates also remain constant for both circuits. In this case, Eqs. (2.22) can be simplified into:

$$i \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix} = \begin{bmatrix} \omega_1 - i(\alpha_1 + \kappa_1) & 0 \\ 0 & \omega_2 - i(\alpha_2 + \kappa_2) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.24)$$

The eigen-frequencies ($\tilde{\omega}_{\pm}$) in this case are simply the diagonal elements of the matrix. As shown in Fig. 2.9(a), the result agrees with the intuitive expectation.

Case #2: Coherent Coupling through X_0

Consider a second case where the coherent interaction dominates (i.e., $X_0 \gg Z_0$ such that $J \gg \Gamma$). In this scenario, the dynamics of the reactively coupled resonators can be approximated as:

$$i \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix} = \begin{bmatrix} \omega_1 - i(\alpha_1 + \kappa_1) & J \\ J & \omega_2 - i(\alpha_2 + \kappa_2) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.25)$$

The coupling introduces real non-zero off-diagonal terms, which represent lossless energy exchange between the two resonators. The eigenfrequencies corresponding to hybridized states are:

$$\tilde{\omega}_{\pm} = \frac{\omega_1 + \omega_2}{2} - i \frac{(\alpha_1 + \alpha_2 + \kappa_1 + \kappa_2)}{2} \pm \frac{1}{2} \sqrt{[\omega_1 - i(\alpha_1 + \kappa_1) - \omega_2 + i(\alpha_2 + \kappa_2)]^2 + 4J^2} \quad (2.26)$$

The hybridized frequencies are plotted in the Fig. 2.9(b). When the resonators are coupled through a reactive component, mode splitting occurs, leading to a frequency dispersion with a repulsive behavior accompanied by linewidth attraction. The mode splitting gap at zero detuning condition $(\omega_2 - \omega_1)/2\pi = 0$, is a function of J .

In coherent coupling, a time-varying current from one resonator induces a current through the coupling element X_0 , which in turn affects the current in the second resonator. Since X_0 is a reactive component, it introduces a 90° phase shift (either leading or lagging, depending on its nature). Consequently, the induced current in the second resonator can either reinforce or oppose its native resonance. Such phase dependent interaction ultimately gives rise to two distinct eigenmodes: an in-phase mode and an anti-phase mode, each associated with a different resonant frequency.

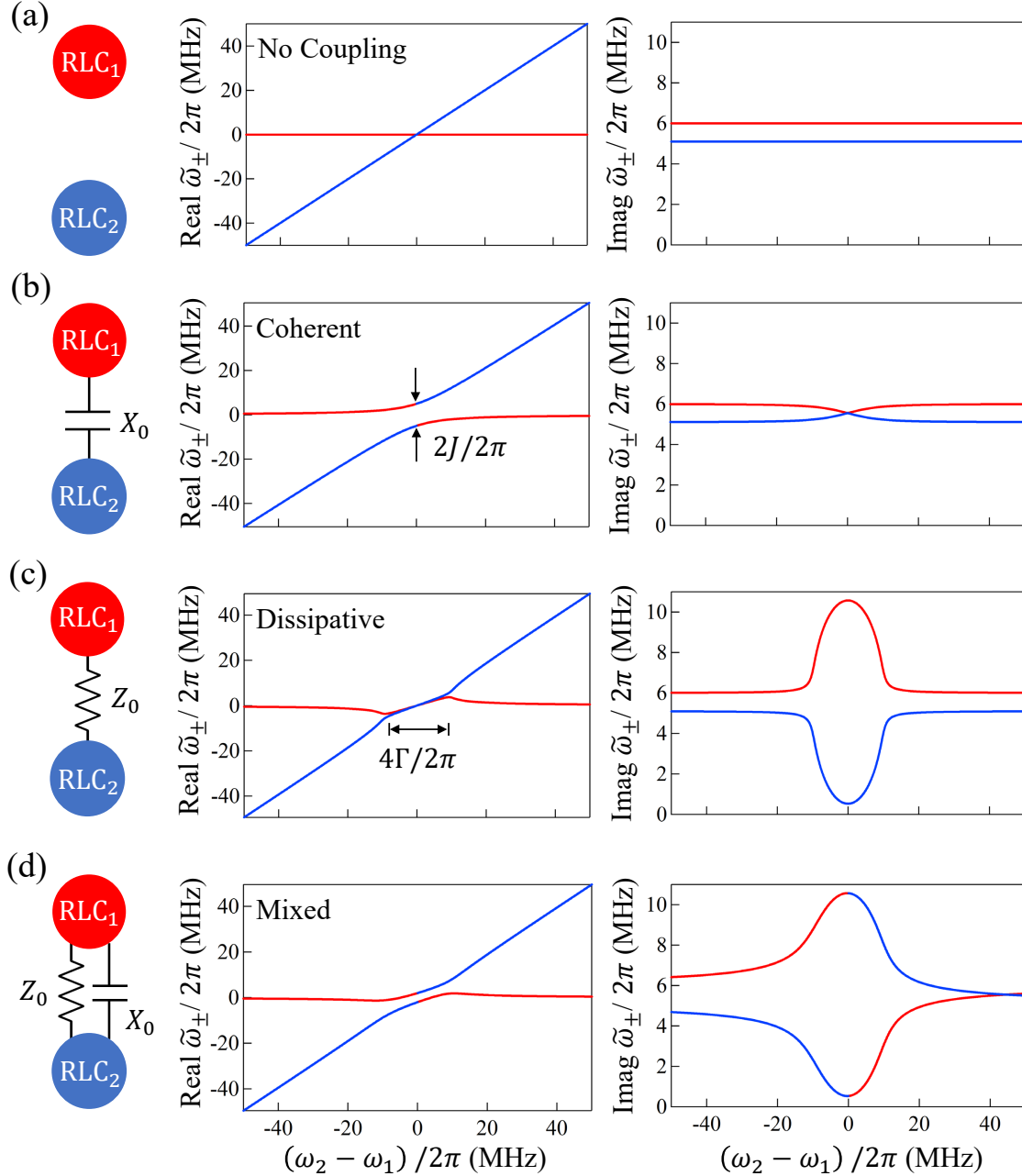


Figure 2.9: Hybridized frequencies of coupled RLC resonators. $\text{Real}(\tilde{\omega}_{\pm})$ are offset by $-\omega_1$. $\omega_1/2\pi = 100$ MHz, $\omega_2/2\pi = 50 - 150$ MHz. $\alpha_1/2\pi = 1.0$ MHz, $\alpha_2/2\pi = 0.1$ MHz, $\kappa_{1,2} = 5$ MHz. A capacitor is used to represent the reactive element X_0 . (a) No coupling, showing a cross over in frequency and constant in linewidth. (b) Coherent coupling, showing level repulsion, and linewidth exchange. $J/2\pi = 5$ MHz. (c) Dissipative coupling, showing level attraction, and linewidth repulsion. $\Gamma/2\pi = 5$ MHz. (d) Combination of coherent and dissipative coupling demonstrating features found in both coherent and dissipative coupled scenario. $J/2\pi = 2$ MHz, $\Gamma/2\pi = 5$ MHz.

Case #3: Dissipative Coupling through Z_0

In contrast to coherent coupling, where energy is exchanged between resonators via lossless, reactive elements, dissipative coupling involves interaction through a shared reservoir (the real part of port impedance Z_0). In the absence of the reactive impedance X_0 , the dynamics of the resistor only coupled circuits can be described by:

$$i \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix} = \begin{bmatrix} \omega_1 - i(\alpha_1 + \kappa_1) & -i\Gamma \\ -i\Gamma & \omega_2 - i(\alpha_2 + \kappa_2) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.27)$$

Then, the hybridized frequencies are:

$$\tilde{\omega}_{\pm} = \frac{\omega_1 + \omega_2}{2} - i \frac{(\alpha_1 + \alpha_2 + \kappa_1 + \kappa_2)}{2} \pm \frac{1}{2} \sqrt{[\omega_1 - i(\alpha_1 + \kappa_1) - \omega_2 + i(\alpha_2 + \kappa_2)]^2 - 4\Gamma^2} \quad (2.28)$$

The real and imaginary part of Eq. (2.28) are displayed in Fig. 2.9(c). In contrast to coherently coupled case, dissipative coupling causes frequencies to merge, a phenomenon known as level attraction, and linewidth to become repulsive. The detuning range of which hybridized frequencies tend to merge is a function of Γ .

The coupling resistor $Z_0/2$, allows current to flow between the two resonators. At the extreme, when two resonators' currents are out of phase, I_{Couple} becomes zero and there is no current passes through $Z_0/2$ branch [Eq. (2.20)]. The hybridized mode appears less damped. In contrast, when the currents $I_{1,2}$ are in phase, maximum power is transferred to $Z_0/2$, and this mode appears to be heavily damped.

Case #4: Coherent and Dissipative Coupling through X_0 and Z_0

Connecting two resonators through X_0 and Z_0 , then both coherent and dissipative coupling mechanisms will be present, with their respective strengths competing to shape the overall dispersion behavior. The dynamics are governed by Eq. (2.22) and the following hybridized

frequencies are shown in Fig. 2.9(d).

$$\tilde{\omega}_{\pm} = \frac{\omega_1 + \omega_2}{2} - i \frac{(\alpha_1 + \alpha_2 + \kappa_1 + \kappa_2)}{2} \pm \frac{1}{2} \sqrt{[\omega_1 - i(\alpha_1 + \kappa_1) - \omega_2 + i(\alpha_2 + \kappa_2)]^2 + 4(J - i\Gamma)^2} \quad (2.29)$$

2.5 Cavity Magnonics as Coupled Harmonic Oscillators

2.5.1 Cavity Photon

Instead of describing the electric and magnetic fields as classical continuous entities, quantum electrodynamics treats them as quantized fields, with the electromagnetic (EM) field carrying energy in discrete units called photons. Microwave cavities are fabricated using metallic and (or) dielectric materials. Impedance mismatches, caused by interfaces between different materials or structural discontinuities, lead to the reflection within the cavity. These reflections give rise to standing waves at specific wavelengths, known as cavity modes.

The Hamiltonian of the EM field in a cavity is given by the sum of the electric and magnetic energy densities:

$$\hat{H} = \int_V \left(\frac{1}{2} \epsilon_0 \mathbf{E}^2 + \frac{1}{2\mu_0} \mathbf{B}^2 \right) d^3r \quad (2.30)$$

where the integration is carried out over the entire volume of the cavity. ϵ_0 and μ_0 denote the vacuum permittivity and permeability, respectively. For a one-dimensional cavity of length L . The electric field can be expanded as a sum of standing wave modes:

$$\mathbf{E}(x, t) = \sum_n [\mathbf{E}_n e^{i(k_n x - \omega_n t)} + \text{c.c.}] \quad (2.31)$$

where $k_n = \frac{n\pi}{L}$ and $\omega_n = ck_n$ and *c.c.* represent complex conjugate.

The quantized electric field operator can then be expressed as:

$$\hat{\mathbf{E}}(x, t) = i \sum_n \left[\sqrt{\frac{\hbar \omega_n}{2 \epsilon_0 L}} (\hat{c}_n e^{i(k_n x - \omega_n t)} - \hat{c}_n^\dagger e^{-i(k_n x - \omega_n t)}) \right] \quad (2.32)$$

here, $\hat{c}_n$ and $\hat{c}_n^\dagger$ are the annihilation and creation operators, for the n -th mode, respectively.

These operators obey the commutation relations:

$$[\hat{c}_n, \hat{c}_m^\dagger] = \delta_{nm}, \quad [\hat{c}_n, \hat{c}_m] = 0, \quad [\hat{c}_n^\dagger, \hat{c}_m^\dagger] = 0 \quad (2.33)$$

For simplicity, consider only the energy associated with the electric field, and substitute Eq. (2.32) to the Hamiltonian (a similar result will be obtained for the magnetic field). The integral over the volume V can be simplified by noting that the standing modes are independent and the integration of the product of different modes will be zero. Thus:

$$\hat{H}_{\text{Cavity}} = \sum_n \hbar \omega_n \left(\hat{c}_n^\dagger \hat{c}_n + \frac{1}{2} \right) \quad (2.34)$$

$\hat{c}_n^\dagger \hat{c}_n$ is the number operator $\hat{n}_n$, which counts the number of photons in the n -th mode. $\frac{1}{2} \hbar \omega_n$ term represents the zero-point energy, a residual energy arising from quantum fluctuations due to the uncertainty principle. Eq. (2.34) is identical to that of a collection of harmonic oscillators, each with frequency ω_n , and energy $n \hbar \omega_n$.

2.5.2 Magnon

In magnetic materials, atomic spins interact through two primary mechanisms: the short-range exchange interaction and the long-range dipole-dipole interaction. The exchange interaction is significantly stronger and acts predominantly between neighboring spins that are in close proximity. It energetically favors specific spin alignments, thereby establishing magnetic order (parallel in ferromagnets and antiparallel in antiferromagnets). The dipole-dipole interaction is much weaker and contributes only modestly to the overall spin orientation in

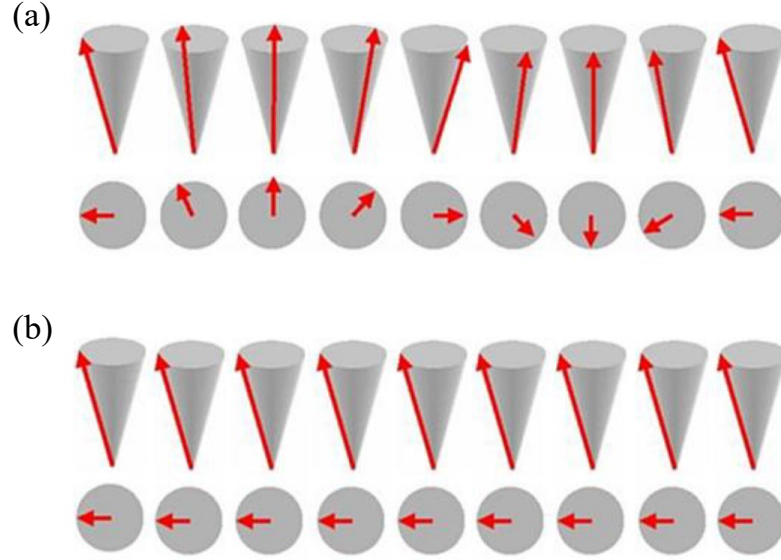


Figure 2.10: Side and top view of spin precession around an equilibrium magnetization pointing upward. (a) The precession phase varies across the magnetic material, defining a certain wavelength. (b) Kittel mode ($k = 0$), where all the spin precessions are in phase synchronized.

bulk material [84].

When all spins are perfectly aligned, the system is in its ground state. However, external perturbations can cause deviations from the perfect alignment. Due to the interplay of exchange and dipole-dipole interactions, deviations do not remain localized, instead, they propagate through the lattice as spin waves [Fig. 2.10(a)]. The phase difference among precessional angles determines the wavelength and wavenumber.

For short-wavelength spin waves, where the wavelength is comparable to the distance between adjacent spins, exchange interaction dominates. At longer wavelengths, the dipole-dipole interaction becomes more significant, leading to the formation of magnetostatic modes. An extreme case is the Kittel mode, where all spins precess in phase, and the spin ensemble behaves collectively as a single macrospin [Fig. 2.10(b)].

A magnon is the quantum of a spin wave, similar to a photon is the quantum of an EM wave. The harmonic oscillator analogy for magnons can be inferred from the fact that during precession, deviations from spin alignments are small in the low energy excitation limit, the

system's energy varies quadratically with the tilt angle of precession, in analogy with mass on a spring, whose energy varies quadratically with displacement.

The Hamiltonian of a magnon is given by:

$$\hat{H}_{\text{Magnon}} = \sum_k \hbar\omega_k \hat{m}_k^\dagger \hat{m}_k \quad (2.35)$$

where $\hat{m}_k^\dagger$ and $\hat{m}_k$ are the creation and annihilation operators, and $\hbar\omega_k$ is the energy of a single magnon with wave vector k .

2.5.3 Coupling in Cavity Magnonics

A typical cavity magnonic system consists of a microwave cavity and a magnetic material. From a classical perspective, the system's dynamics are governed by a two-way interaction. First, the electric current in the microwave circuit generates a magnetic field that exerts torque on the magnetic spins, causing them to precess. In response, the precessing spins modify the magnetic flux through the circuit, inducing a back-action current. This way, energy stored in the resonant modes can be exchanged with the surrounding at a steady rate, making the relative positioning of the cavity and the magnetic material crucial in determining types of coupling mechanism.

In the same way as coupled RLC circuits, coherent coupling occurs when energy is exchanged between cavity and magnon modes in a lossless manner. This can be achieved by placing the magnetic material inside the cavity antinode, creating a direct mode overlap, as demonstrated in Fig. 2.11(a). The coherent coupling strength, J is found to be proportional to spatial mode overlapping factor (η) between the microwave field and magnetic material, the total number of spins in the magnetic sample (N), and following parameters [25]:

$$J = \frac{\eta}{2} \gamma_e \sqrt{\frac{2NS\hbar\omega\mu_0}{V_c}} \quad (2.36)$$

where $S = 5/2$ is the spin number of the ground state of Fe^{3+} ion in YIG, ω is the driving

frequency, μ_0 is the vacuum magnetic permeability, and V_c is the cavity mode volume.

Dissipative coupling occurs when energy is exchanged irreversibly between the magnon and cavity modes. In this case, the energy is lost to the environment through a radiative channel such as a transmission line (the microstrip connects port 1 and port 2 in Fig. 2.11). Therefore, dissipative coupling requires both the magnon and cavity modes to discharge energy simultaneously to the common transmission line [Fig. 2.11(b)]. A process similar to a dissipatively coupled RLC circuits, where energy is discharged from the resonators through a resistor that connects them. The dissipative coupling strength is a function of cavity (and magnon) modes' extrinsic dissipation rates, $\kappa_{c(m)}$:

$$i\Gamma = i\sqrt{\kappa_c\kappa_m} \quad (2.37)$$

Lastly, both dissipative and coherent coupling can be observed when the magnon mode overlaps with not only the cavity mode but also transmission line, shown in Fig. 2.11(c).

To describe a generic cavity magnonic system that includes both coherent and dissipative interactions, the Hamiltonian can be written as [42, 41]:

$$H/\hbar = \tilde{\omega}_c \hat{c}^\dagger \hat{c} + \tilde{\omega}_m \hat{m}^\dagger \hat{m} + (J - i\Gamma)(\hat{c}^\dagger \hat{m} + \hat{m}^\dagger \hat{c}) \quad (2.38)$$

where $\tilde{\omega}_c$ and $\tilde{\omega}_m$ represent the uncoupled cavity and magnon modes frequencies: $\tilde{\omega}_{c,m} = \omega_{c,m} - i\alpha_{c,m}$, with $\alpha_{c,m}$ representing their respective intrinsic dampings. The cavity mode's intrinsic damping arises from ohmic loss, dielectric loss and radiation losses. α_m stems from Gilbert damping, determined by the properties of magnetic materials, such as impurities and lattice structures. The first term describes cavity photon energy and second term describes magnon energy. And the last term characterizes both coherent and dissipative interactions.

Applying Heisenberg equation of motion for an operator $\hat{O}$: $d\hat{O}/dt = i/\hbar[H, \hat{O}]$, a set of coupled differential equations can be obtained.

$$i\frac{d}{dt} \begin{bmatrix} \hat{c} \\ \hat{m} \end{bmatrix} = \begin{bmatrix} \tilde{\omega}_c & J - i\Gamma \\ J - i\Gamma & \tilde{\omega}_m \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{m} \end{bmatrix} \quad (2.39)$$

Then, the hybridized eigenfrequencies of a generic cavity magnonic system can be expressed as:

$$\tilde{\omega}_{\pm} = \frac{1}{2} \left[\tilde{\omega}_c + \tilde{\omega}_m \pm \sqrt{(\tilde{\omega}_c - \tilde{\omega}_m)^2 + 4(J - i\Gamma)^2} \right] \quad (2.40)$$

It can be seen that Eq. (2.39) and Eq. (2.40) are analogous to the coupled resonator case [Eq. (2.22) and Eq. (2.29)].

2.6 Summary

This chapter provides an overview of resonant circuits and coupled resonant circuits, beginning with descriptions for single resonant circuits, contrasting passive resonators—characterized by damping—with active oscillators, which incorporate gain element and are evaluated by phase noise. The discussion then shifts to coupled resonant circuits, outlining three main coupling mechanisms: coherent (via reactive elements), dissipative (via resistive components), and mixed coupling. A coupled circuit model is presented to illustrate different interaction mechanisms and effect. The chapter concludes by exploring cavity magnonics, introducing the quantum mechanical descriptions of microwave photons and magnons, their interaction Hamiltonian, and the resulting hybridized mode characteristics, drawing parallels with the coupled circuit model.

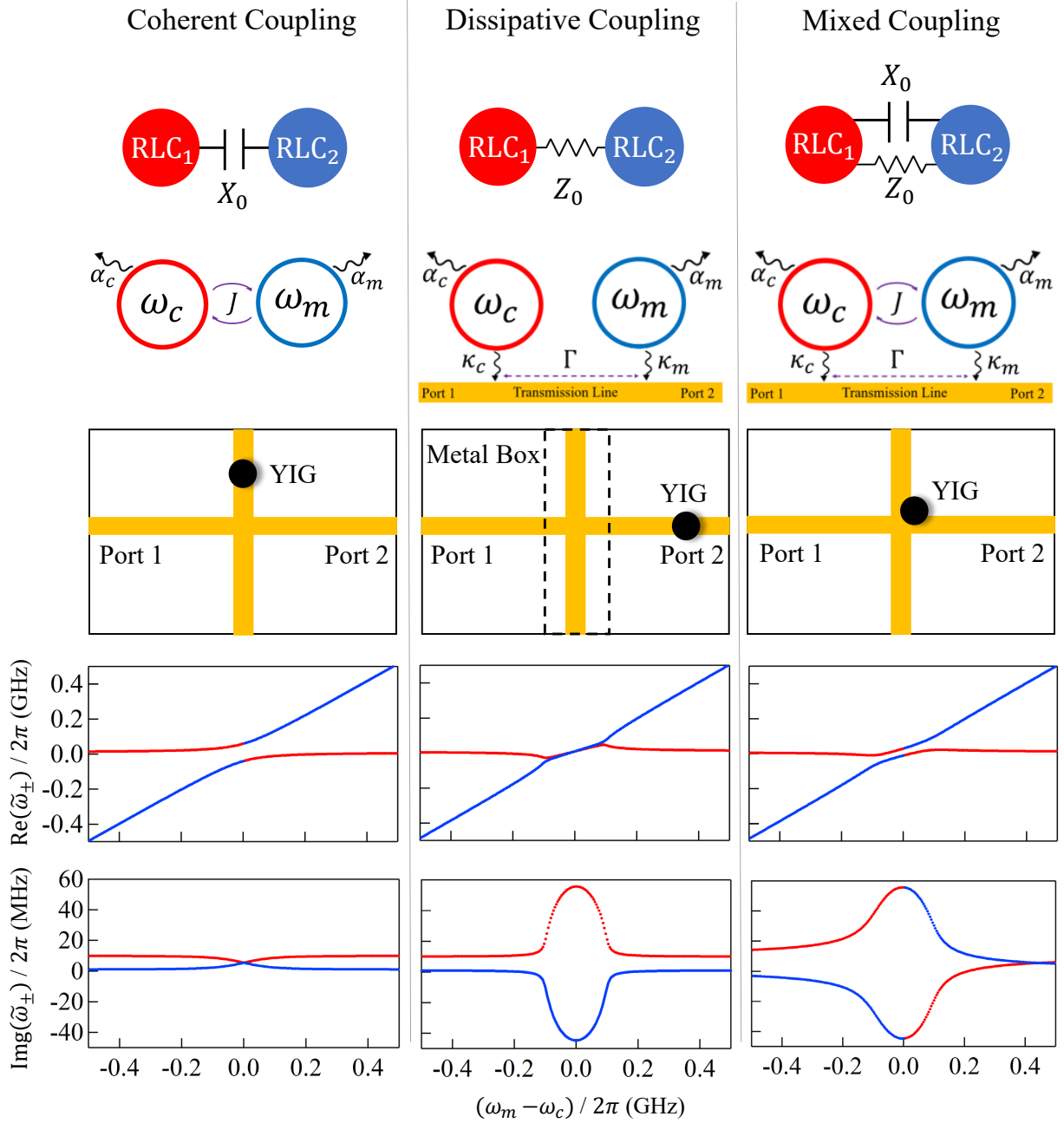


Figure 2.11: Top to bottom: Phenomenological interaction diagram of coupled electrical resonators and coupled cavity and magnon. Cavity magnonic device layout, and calculated dispersion and linewidth of hybridized frequencies ($\tilde{\omega}_{\pm}$) plotted against field detuning, $\Delta_m = \omega_m - \omega_c$. The vertical axis is normalized to cavity resonant frequency. $\alpha_c/2\pi = 10$ MHz, $\alpha_m/2\pi = 1$ MHz represent intrinsic damping rates of cavity and magnon modes, respectively. First Column: Coherent coupling. Magnetic material is placed at the antinode of resonant stub (cavity mode). $J/2\pi = 50$ MHz. Middle Column: Dissipative Coupling. The magnetic material is placed far away from the cavity mode. A metal box shields the EM field of the stub to minimize direct mode overlap. $\Gamma/2\pi = 50$ MHz. Last Column: A mix of coherent and dissipative coupling. The magnetic material is placed at the junction so that it interacts with both stub and transmission line. $J/2\pi = 20$ MHz and $\Gamma/2\pi = 50$ MHz.

Chapter 3

Non-Reciprocal Microwave Transmission: Isolator

3.1 Introduction

Numerous technologies, rely on non-reciprocal components such as isolators and circulators. In test and measurement circuits, isolators protect sensitive components by preventing unwanted reflections and minimizing signal degradation, while in simultaneous transmit-receive modules, circulators route signals between transmitters and receivers, enabling them to share the same antenna [Fig. 3.1] [85]. Additionally, cryogenic quantum experiments utilize non-reciprocal components to shield delicate signals from backscattering noise [86, 87].

Traditionally, isolators and circulators are realized using magnetically biased ferrites, which rely on non-reciprocal effects such as resonance absorption, field displacement, and Faraday rotation [77]. However, these ferrite-based devices typically operate at scales comparable to, or larger than the signal wavelength, making them bulky and challenging to integrate into compact microwave systems. Y-junction circulators can be considered a refined design that leverages interference between counter-rotating modes to direct input signal to an adjacent port [88]. Although this approach helps reduce the overall size, ferrite devices still face the physical limitation: wavelength of the operating frequency.

Efforts have been made to develop alternative non-reciprocal technologies aimed at creating cost-efficient and small-form-factor devices with improved performance. On the one hand, there are magnet-free approaches that provide considerable isolation (> 20 dB). Tran-

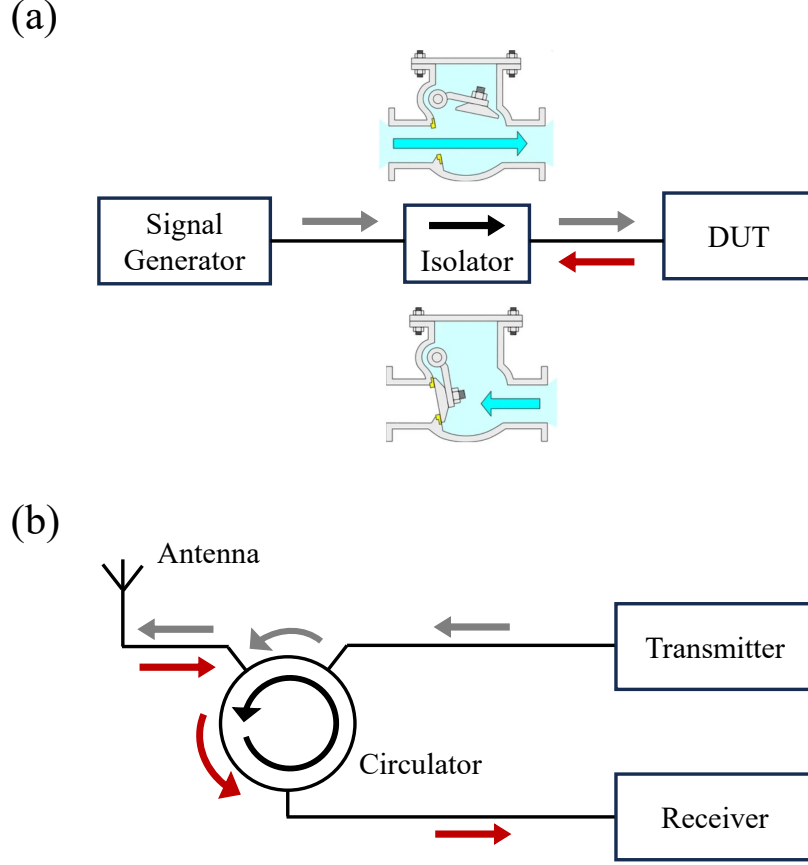


Figure 3.1: Diagrams illustrating working principles of non-reciprocal devices. (a) An isolator is used to protect instrument by suppressing reflections from the Device Under Test (DUT). Its function is similar to that of a check valve found in plumbing systems. (b) A circulator operates as a duplexer, isolating the transmitter from the receiver while allowing both to share the same antenna.

sistors and temporal modulation are the two categories. Even though the former is readily suitable for scalability and compactness, the transistor's added noise and nonlinear distortion have prevented them from being widely deployed [89]. Reciprocity can also be violated through spatio-temporally modulated waveguides [90]. However, in practice, the material being modulated usually exhibits large loss at high frequencies.

Non-reciprocity has also been observed in cavity magnonics [41], where the interplay between coherent and dissipative coupling of a YIG-planar microstrip resonator demonstrated exceptional isolation and flexible controllability. Experimental results have shown isola-

tion levels of up to 80 dB. Shortly thereafter, a circulator based on the same principle was developed [51].

A cavity magnonic isolator can provide high isolation while maintaining a compact profile, making it well-suited for protecting sensitive signals from unwanted reflections, for example in cryogenic experiments operating at the quantum limit [86, 87]. These experiments make use of dilution refrigerators, where the internal space is limited. In this regard, this chapter focuses on advancing the design of cavity magnonic isolators. It begins with a proof-of-concept device that demonstrates isolator functionality at millikelvin temperatures, followed by two additional prototypes aimed at reducing insertion loss and further minimizing device size for improved scalability.

3.2 Proof-of-Concept Cavity Magnonic Isolator at Millikelvin Temperature

The proof-of-concept isolator consists of a single 1 mm diameter YIG sphere positioned near the intersection of a cross-shaped microstrip resonator, as shown in Fig. 3.2(a). A permanent magnet, mounted at the end of an actuator, is used to control the magnetic field in $\hat{z}$ direction, as illustrated in Fig. 3.2(b). Such configuration ensures that the YIG is saturated, with its magnetization frequency ($\omega_m/2\pi$) tunable by varying the actuator's position. The entire setup is enclosed inside a dilution refrigerator, with the device anchored to the mixing plate.

The microstrip resonator is fabricated using a 0.81mm thick ROGERS laminate (RO4003C with $\epsilon_r = 3.55$). A 50Ω transmission line connects two ports for bi-directional transmission measurements. At the midpoint, the transmission line splits into two open-ended stubs, each 6.6 mm in length, creating a junction where direction dependent electromagnetic field chirality can be observed [Fig. 3.2(c)]. The resonant frequency and damping rates of the cavity can be found to be: $\omega_c/2\pi = 6.354$ GHz, $\alpha_c/2\pi = 11$ MHz, and $\kappa_c/2\pi = 3$ GHz.

For the setup described, the tunable parameter of interest is the Kittel mode frequency, which follows the dispersion relation $\omega_m = \gamma_e(\vec{H}_A + \vec{H}_z)$, where $\gamma_e = 2\pi \times 28 \mu_0 \text{GHz/T}$ is

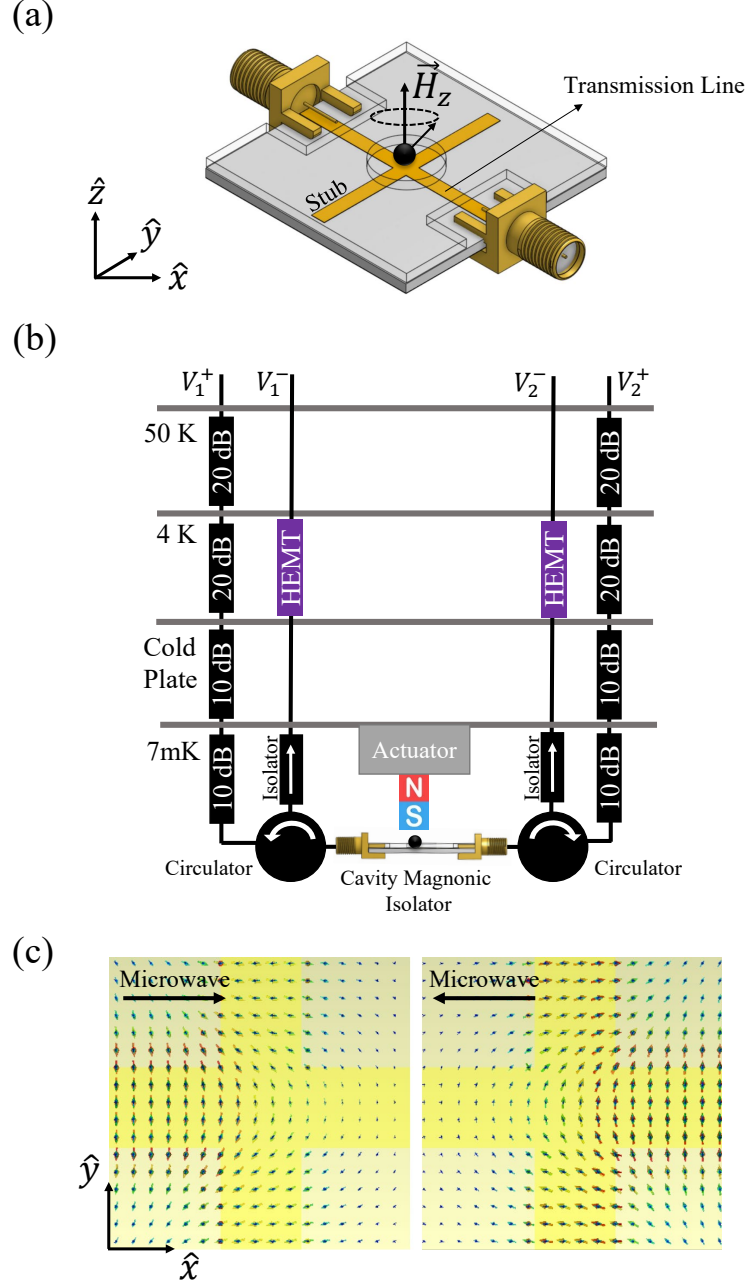


Figure 3.2: (a) Isolator prototype. The microstrip circuit supports both standing and traveling waves. A 1 mm diameter YIG sphere is cemented onto the sample holder and placed nearby the cross junction. (b) Schematic of the measurement apparatus. The cavity magnonic isolator is cooled to 7 mK inside a dilution fridge. A tunable static magnetic field ($\vec{H}_z$) is applied perpendicularly to the cavity board by the actuator. (c) $\vec{H}$ field of the unloaded microstrip cavity near the cross junction, showing opposite chirality for microwave traveling in the opposite direction.

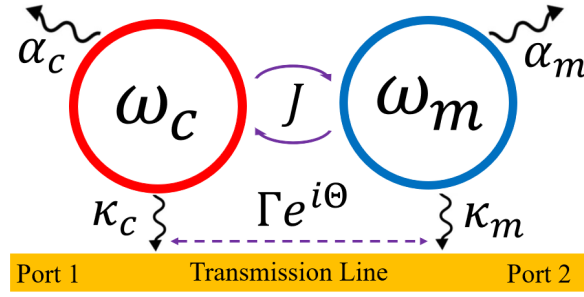


Figure 3.3: Schematic diagram showing interactions inside a cavity magnonic isolator. $\omega_{c(m)}$ represent the cavity and magnon modes, respectively. $\alpha_{c(m)}$ and $\kappa_{c(m)}$ are their intrinsic and extrinsic damping rates. J is the coherent coupling strength, and $\Gamma e^{i\Theta}$ is the direction dependent dissipative coupling strength.

the gyromagnetic ratio, $\vec{H}_z$ is the static bias magnetic field, and $\vec{H}_A$ is the magnetocrystalline anisotropy field. During the experiment, $\omega_m/2\pi$ ranged from 4.570 to 8.890 GHz, corresponding to field detuning range of $(\omega_m - \omega_c)/2\pi = -1.784$ to 2.536 GHz. The intrinsic and extrinsic damping rates for the magnon mode are found to be $\alpha_m/2\pi = 1$ MHz, and $\kappa_m/2\pi = 0.16$ MHz, respectively.

The Hamiltonian of the system takes the following form [41]:

$$\hat{H}/\hbar = \tilde{\omega}_c \hat{c}^\dagger \hat{c} + \tilde{\omega}_m \hat{m}^\dagger \hat{m} + (J - i\Gamma e^{i\Theta_{1(2)}})(\hat{c}^\dagger \hat{m} + \hat{m}^\dagger \hat{c}) \quad (3.1)$$

where $\hat{c}^\dagger(\hat{c})$ and $\hat{m}^\dagger(\hat{m})$ represent the creation (annihilation) operators for the cavity and magnon modes, respectively. Their uncoupled complex frequencies are given by $\tilde{\omega}_{c(m)} = \omega_{c(m)} - i\alpha_{c(m)}$. The spatial overlap between the magnon and cavity modes leads to coherent coupling, with strength J . And dissipative coupling arises as both modes also interact through the common transmission line, described by an imaginary term $i\Gamma = i\sqrt{\kappa_c \kappa_m}$. $\Theta_{1(2)} = 0$ or π such that the $e^{i\Theta_{1(2)}}$ establishes a sign convention for Γ in relation to the direction of traveling wave. It essentially implies the chirality of the microwave field near the junction causes YIG to couple differently with waves traveling in opposite directions, as shown earlier in Fig. 3.2(c). A graphical representation of interactions inside the cavity magnonic isolator is provided in Fig. 3.3.

The scattering parameters of the system can be expressed as [41]:

$$S_{21(12)} = 1 + \frac{\kappa_c}{[i(\omega - \omega_c) - (\alpha_c + \kappa_c)] - \frac{[iJ + \Gamma e^{i\Theta_{1(2)}}]^2}{i(\omega - \omega_m) - (\alpha_m + \kappa_m)}} \quad (3.2)$$

where the forward ($|S_{21}|$) and backward ($|S_{12}|$) transmission spectra are obtained through scalar network analysis: $|S_{21(12)}| = 20 \log_{10}(V_{2(1)}^-/V_{1(2)}^+)$. $\Theta_1 = \pi$ and $\Theta_2 = 0$ correspond to S_{21} and S_{12} , respectively. As one may notice, non-reciprocity¹ arises from the $2iJ\Gamma e^{i\Theta_{1(2)}} = \pm 2iJ\Gamma$ term. This direction-dependent cooperative coupling term requires the magnetic field of the traveling wave on the transmission line to interact with the standing wave on the stub, such that the net field chirality promotes spin precession in one measurement direction while suppressing it in the other. In the case of a pure interaction (J or $\Gamma = 0$), the S-parameters are reciprocal.

Two way transmission profiles of the device are measured first, and their difference: Isolation (ISO) = $|S_{21}| - |S_{12}|$ is subsequently calculated. Figs. 3.4(a) and (c) show the $|S_{21(12)}|$ against field detuning. At $(\omega_m - \omega_c)/2\pi = 0$, a normal mode splitting of 110 MHz is observed, which is order of magnitude greater than the intrinsic damping rates of cavity and magnon modes, indicating a strong coupling between the magnon and cavity modes.

The coupling strengths are determined by fitting each transmission spectrum to Eq. (3.2). The average of coherent coupling strength is $J/2\pi = 63$ MHz, and the dissipative coupling strength is $\Gamma/2\pi = 22$ MHz. A good agreement between the fitting results and the raw measurement data is observed by comparing Fig. 3.4(a) with (b), and (c) with (d). Notably, the mode splitting is smaller than that of a pure coherent coupling case ($110\text{MHz} < 2J/2\pi$), suggesting the presence of dissipative interaction responsible for the coalescence of the hybridized mode frequencies. Nevertheless, the transmission mappings still exhibit level repulsion behavior since $J > \Gamma$. The two bright peaks located at $(\omega_m - \omega_c)/2\pi = -0.26, 0.21$ GHz in Fig. 3.4(e) reveal a non-reciprocal property, further confirming that the two subsystems are not interacting in a pure coherent manner.

¹ $|S_{21}| \neq |S_{12}|$

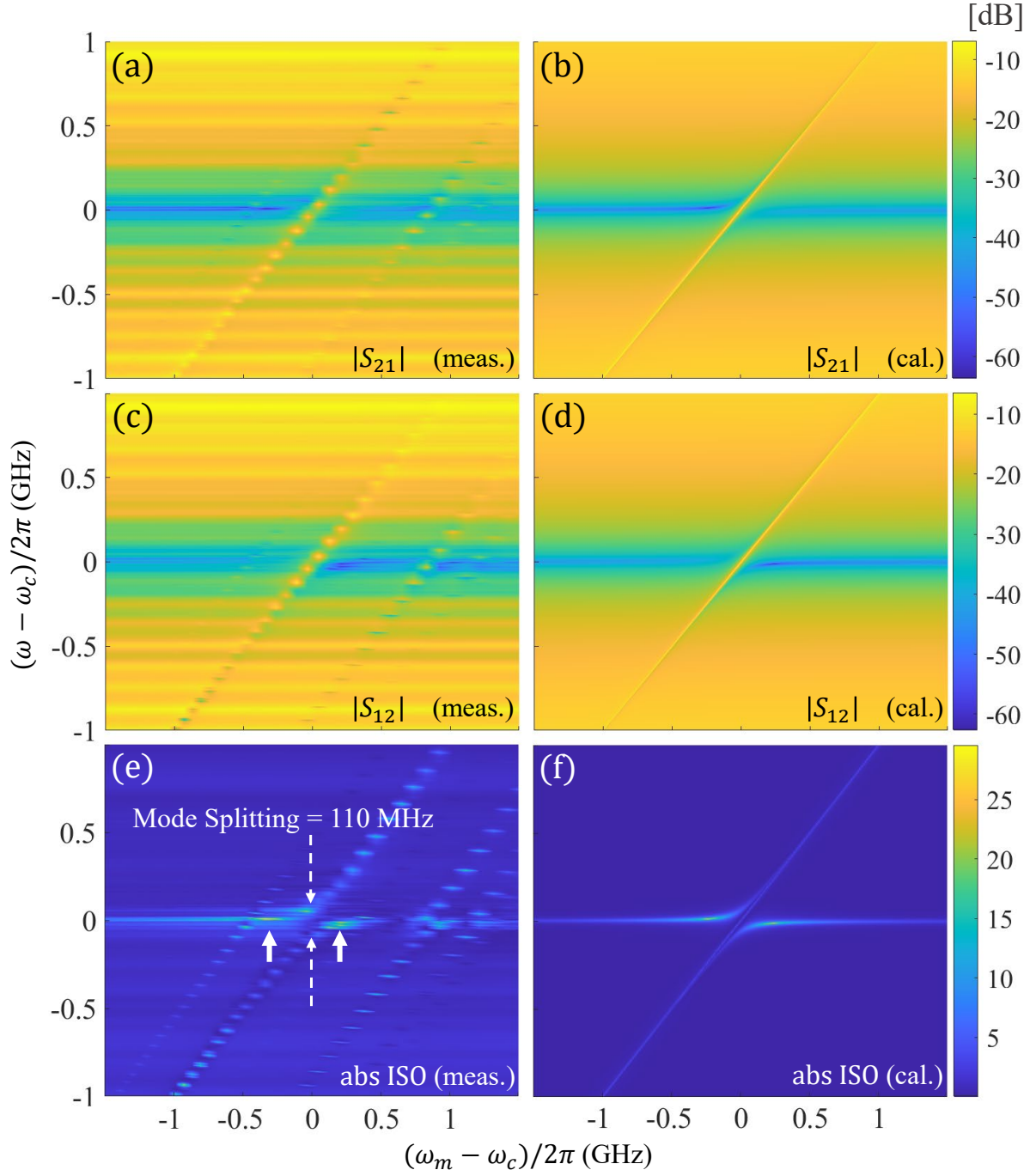


Figure 3.4: Transmission profile of the cavity magnonic isolator. The probing microwave power is 0 dBm. (a) Forward transmission amplitude ($|S_{21}|$) as a function of field detuning. (c) Backward transmission amplitude ($|S_{12}|$) as a function of field detuning. (e) The absolute difference between forward and backward transmission, $|ISO|$, showing two bright peaks near zero field detuning, marked by white arrows. Panel (b), (d) and (f) are corresponding calculation results based on fitting parameters.

3.3 Characteristics of the Non-reciprocal Band

3.3.1 Formation

The complex eigenfrequencies can be derived from Eq. (3.1), which leads to the following:

$$\tilde{\omega}_{\pm} = \frac{1}{2} \left[\omega_c + \omega_m - i(\alpha_c + \alpha_m) \pm \sqrt{[(\omega_c - \omega_m) - i(\alpha_c - \alpha_m)]^2 + 4[J - i\Gamma e^{i\Theta_{1(2)}}]^2} \right] \quad (3.3)$$

The real part of $\tilde{\omega}_{\pm}$ governs the frequency dispersion, whereas the imaginary part controls linewidth behavior. By carefully examining Eq. (3.3), it can be seen that $\text{Re}(\tilde{\omega}_{\pm})$ are the same regardless of the chosen $\Theta_{1(2)}$ value, meaning the system exhibits an identical frequency dispersion for both forward and backward transmission measurement. The imaginary part is different due to the $[J - i\Gamma e^{i\Theta_{1(2)}}]^2$ term, implying asymmetrical linewidth behavior that depends on the direction of traveling wave.

The real part of hybridized frequencies are presented in Fig. 3.5(a), while the imaginary parts are presented in Figs. 3.5(b) and (c). The vertical dashed line marks the field detuning condition where the linewidth of the hybridized modes crosses zero, referred to as the zero damping condition (ZDC). At ZDC, one of the hybridized modes effectively becomes zero bandwidth resonator and thus zero loss, allowing it to absorb all the energy from the transmission line and produce maximum signal attenuation [Fig. 3.5(b)]. In contrast, the hybridized modes propagating in the opposite direction retain a non-zero linewidth at the same field detuning [Fig. 3.5(c)]. Theoretical infinite isolation is achieved this way.

The two transmission spectra corresponding to the two black arrows in Fig. 3.5 are presented in Fig. 3.6. They are the closest detuning points to the theoretical ZDC obtained experimentally, and also correspond to the two bright peaks observed in the isolation mapping, highlighted earlier in Fig. 3.4(e).

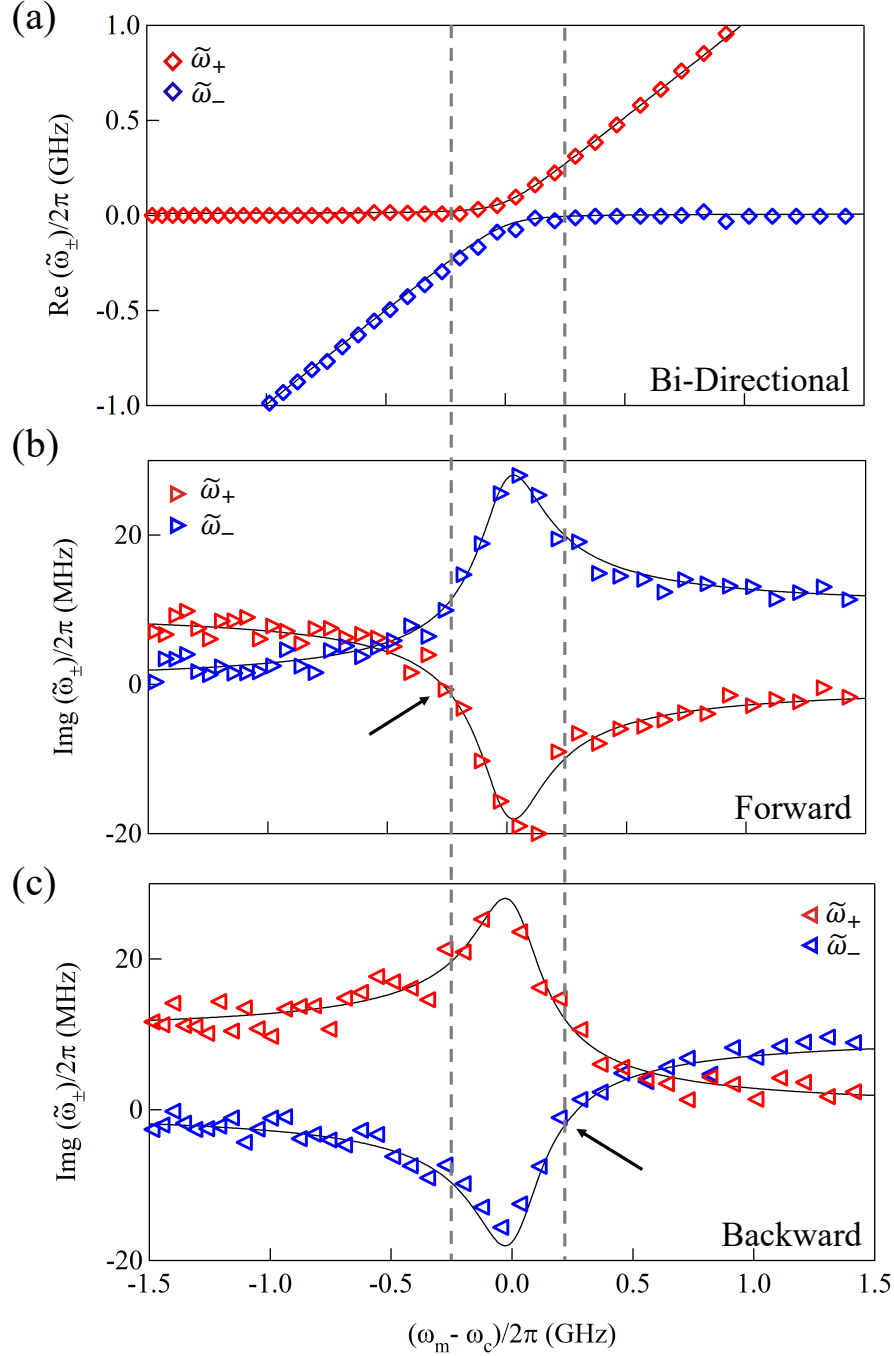


Figure 3.5: (a) Real part of $\tilde{\omega}_{\pm}$ against field detuning. (b) and (c) are the imaginary part of $\tilde{\omega}_{\pm}$ corresponding to forward and backward traveling waves, respectively. The black solid lines represent the results of calculations using Eq. (3.3) with $\Theta = \pi$ and $\Theta = 0$, corresponding to forward and backward traveling waves, respectively. Dashed lines mark where the hybridized mode's linewidth crosses zero. Spectra corresponding to the black arrows can be found in Fig. 3.6.

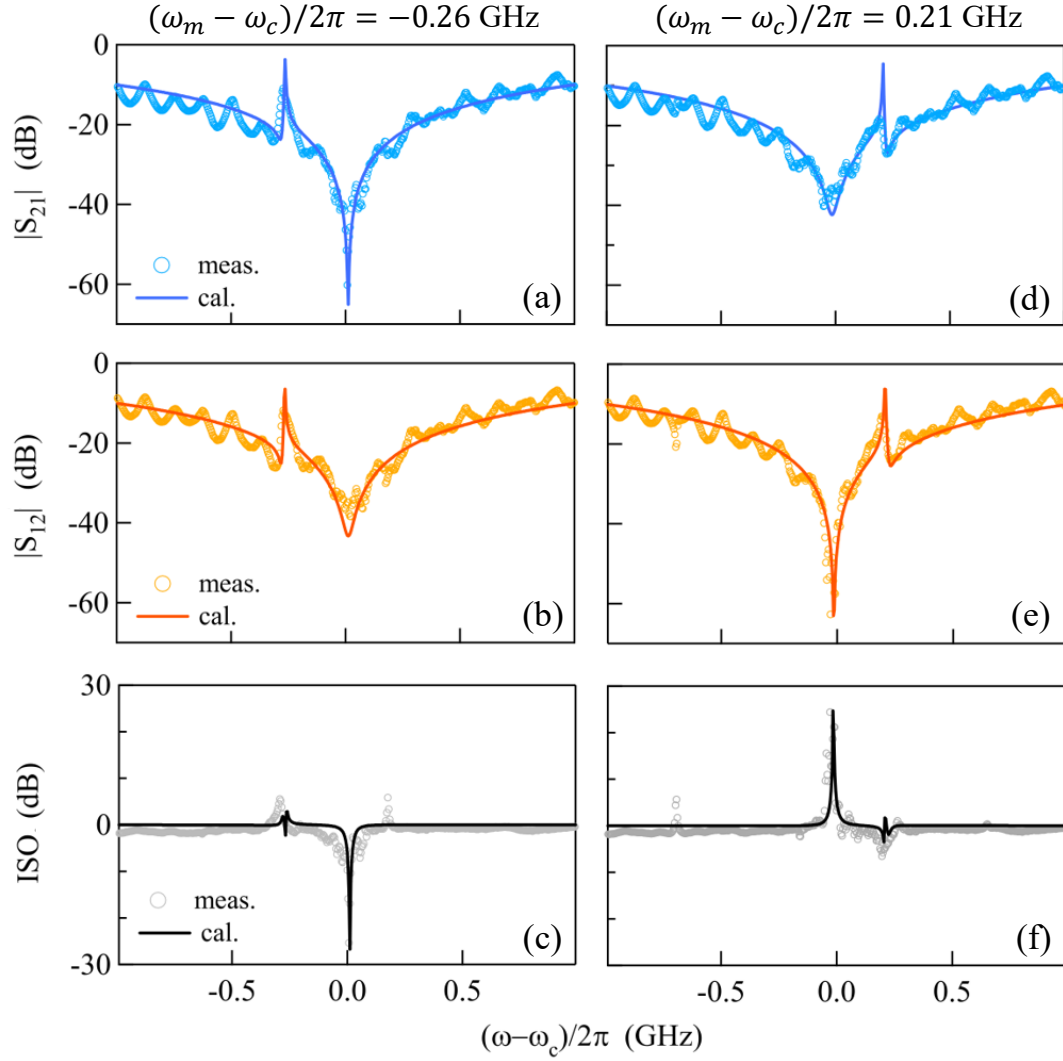


Figure 3.6: Transmission spectra correspond to maximum ISO. (a) $|S_{21}|$ (b) $|S_{12}|$ and (c) $\text{ISO} = -26.9 \text{ dB}$ are obtained at the field detuning of -0.26 GHz . (d) $|S_{21}|$ (e) $|S_{12}|$ and (f) $\text{ISO} = 25.9 \text{ dB}$ are obtained at the field detuning of 0.21 GHz .

3.3.2 Isolation, Insertion Loss, and Bandwidth

To characterize the non-reciprocal band, it is useful to define insertion loss (IL) and bandwidth (BW) in addition to isolation. As shown in Fig. 3.7, IL quantifies the signal attenuation as it propagates through the device, specifically in the direction with relatively smaller attenuation ($IL = |S_{12}|$). According to the previous definition², when $|S_{12}| > |S_{21}|$, isolation is negative. However, since the device is a two-port system that can be rotated

² $\text{ISO} = |S_{21}| - |S_{12}|$

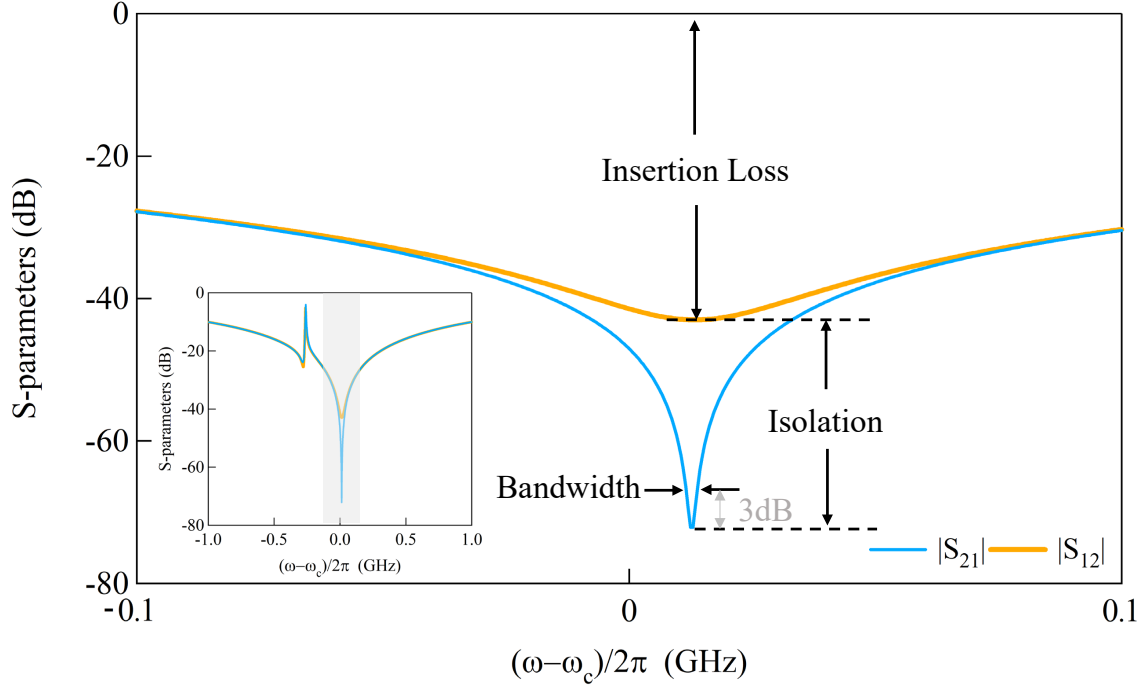


Figure 3.7: Illustration of how isolation (ISO), insertion loss (IL), and bandwidth (BW) are defined. Figure is the zoomed-in non-reciprocal band, corresponding to the shaded region in the inset.

Table 3.1: Properties of Non-reciprocal Bands Shown in Fig. 3.6

Field Detuning (GHz)	-0.26	0.21
Transmission	$ S_{21} < S_{12} $	$ S_{21} > S_{12} $
ISO (dB)	-26.9	25.9
IL (dB)	-41.0	-40.8
BW (MHz)	2.5	2.7

180 degrees and reconnected, the sign of isolation is not of importance-it simply indicates which direction experiences greater attenuation. Bandwidth (BW) refers to the frequency range over which isolation is reduced by half (-3 dB). The non-reciprocal properties of transmission identified previously in Figs. 3.6 are analyzed in Table 3.1.

To better understand the device performance, ISO, BW, IL were analyzed across a range of field detuning. As shown in Fig. 3.8(a), calculated isolation reach their maximum at $(\omega_m - \omega_c)/2\pi = \pm 0.23$ GHz (at ZDCs, marked by the dashed lines), indicating mirror-symmetric non-reciprocity relative to field detuning. The left peak results from the attenuation of

the forward traveling wave [Fig. 3.6(c)], while the right peak is due the attenuation of the backward traveling wave [Fig. 3.6(f)]. Away from the two detuning conditions, isolation decreases drastically.

Figure 3.8(b) shows the bandwidth of isolation. A comparison between Figs. 3.8(a) and (b) reveals a trade-off between isolation and bandwidth: Maximum isolation corresponds to minimum bandwidth. Moving away from the ZDCs, isolation gradually decreases while bandwidth increases. Eventually, isolation drops below 3 dB, at which point the bandwidth becomes undefined.

Insertion loss is illustrated in Fig. 3.8(c) , defined as $IL = |S_{12}|$ for negative field detuning and $IL = |S_{21}|$ for positive field detuning. The IL is large (>30 dB), since non-receiprocal bands are located near the cavity's resonance dip. However, there is no apparent trade-off between ISO and IL, meaning that the device can be biased nearby but not at the ZDCs for a balanced performance. As an example, by setting $(\omega_m - \omega_c)/2\pi = -0.19$ GHz, the ISO, IL, BW are 19.7 dB, -38.4 dB and 12 MHz, respectively. Alternatively, the cavity design and system parameters can be fine-tuned to improve non-reciprocal performance, as discussed below.

3.3.3 Insertion Loss Reduction and Miniaturization Strategy

Based on the previous findings, two possible improvements are: (1) to reduce the device's insertion loss and (2) to miniaturize its size. The rationale and proposed strategies are outlined below.

The operating frequency of a cavity magnonic isolator is primarily determined by the resonance frequency of the uncoupled cavity as the non-reciprocal band emerges near the cavity's resonance, which is ultimately dictated by the length of the stubs. For the isolator to operate in a lower frequency regime, a longer stub is required. However, this contradicts the goal of miniaturization. Therefore, a revised stub design is necessary.

To improve insertion loss, it is beneficial to position the ZDC further away from the

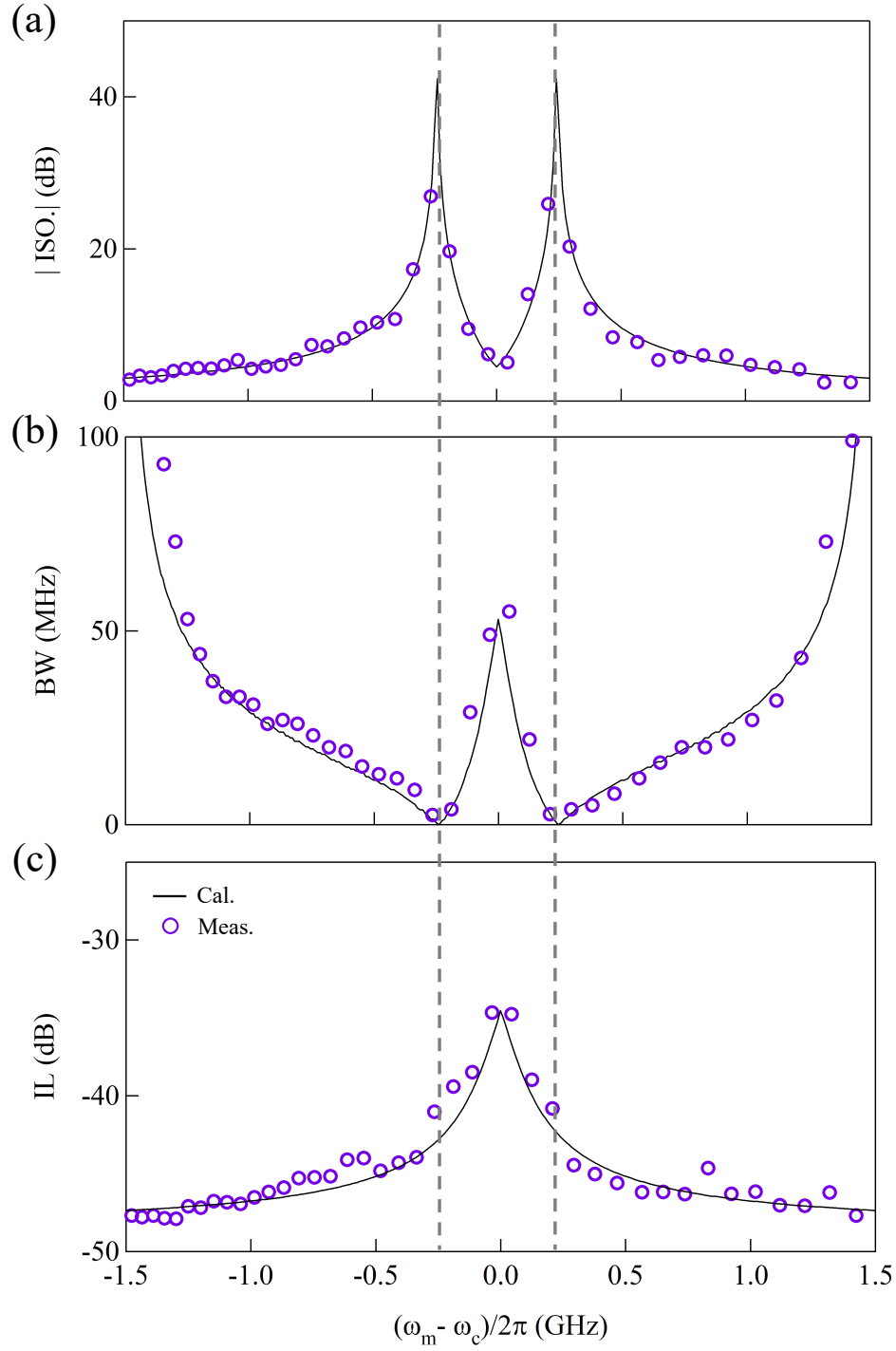


Figure 3.8: (a) Absolute value of isolation, (b) bandwidth, and (c) insertion loss against field detuning. Two ZDCs are marked by dashed lines. There exists a trade-off between isolation and bandwidth but not insertion loss.

cavity's resonance, which can be achieved by increasing coherent coupling strength J . The idea is illustrated in Figs. 3.9(a) and (b). All else being equal, a large J corresponds to a large detuning between the position of ZDC and the cavity like mode frequency. According to Eq. (2.36):

$$J \propto \eta \sqrt{N/V_c} \quad (3.4)$$

where η is the spatial mode overlapping factor between the microwave field and magnetic material, N is the total number of spins in the magnetic sample, and V_c is the cavity mode volume.

Thus, two approaches can be pursued from the above analysis:

- **Replace the stubs with surface mount inductors**

Benefits of this approach is threefold. First, it reduces the overall size of microstrip cavity. Second it transforms the frequency response of cavity into a notch filter with reduced signal attenuation near its resonance [Fig. 3.9(c)]. The rationale is based on the circuit model presented earlier in Chapter 2.3.1, where the extrinsic damping of a series shunt resonator is inversely proportional to resonator inductance ($\kappa = Z_0/(2L)$). Since ZDC appear near the cavity resonance, lowering the overall insertion loss of cavity also reduces the insertion loss of non-reciprocal band. Lastly, the use of surface mount element decreases the mode volume term (V_c), thereby increasing the coherent coupling strength J . The effect arises from the localized propagation achieved with surface mount components.

- **Increase the Number of Spins**

Introducing an additional YIG sphere increases the number of spins N and enhances spatial mode overlap η . As a result, the coherent coupling strength can be further increased.

Building on the analysis above, the next section presents two cavity magnonic isolator

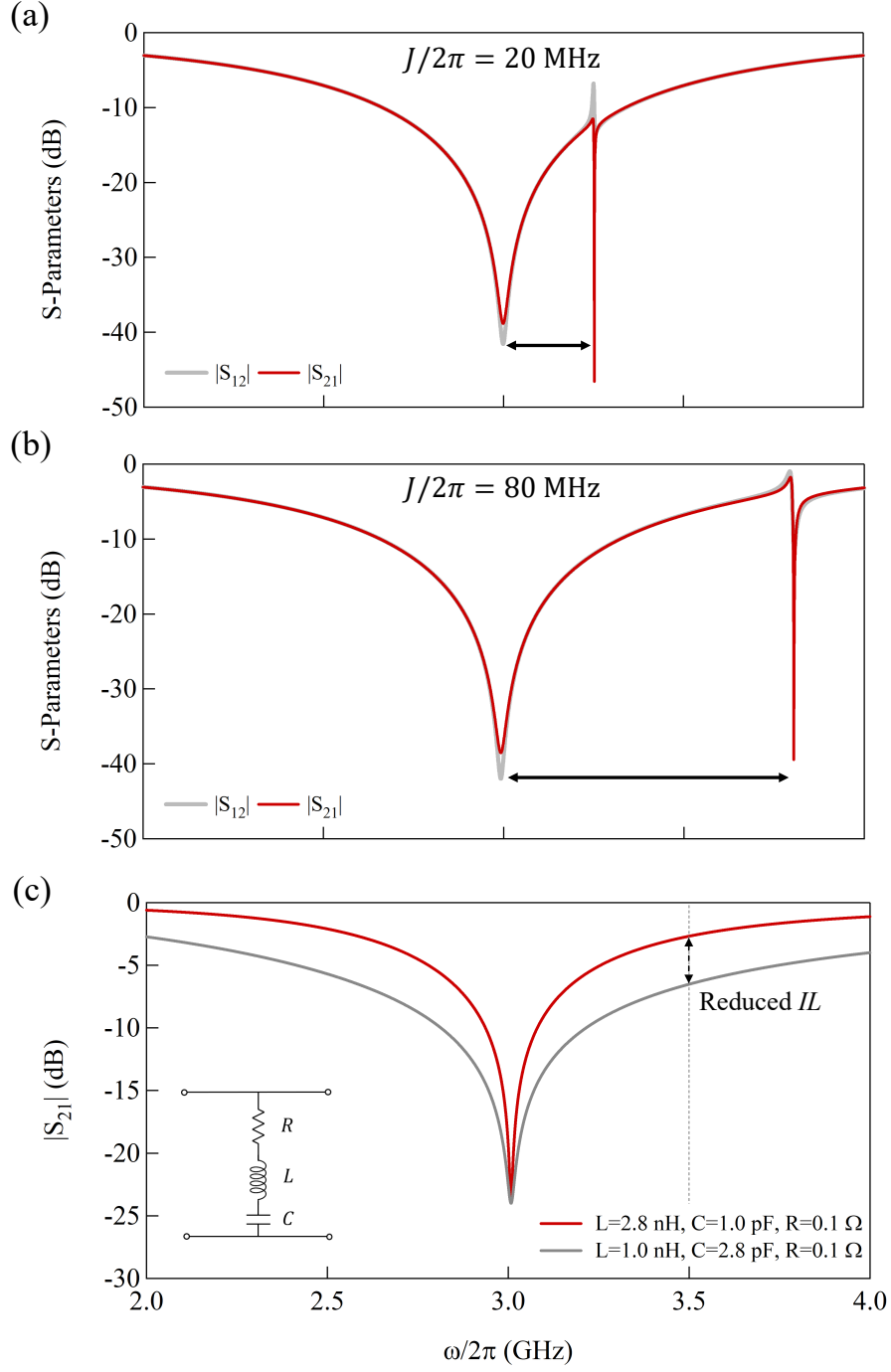


Figure 3.9: (a) ZDC occur at $(\omega_m - \omega_c)/2\pi = 0.25$ GHz for $J/2\pi = 20$ MHz (b) All else equal, ZDC occur at $(\omega_m - \omega_c)/2\pi = 0.80$ GHz for $J/2\pi = 80$ MHz. (c) The calculated $|S_{21}|$ for a series shunt resonator with different combinations of L and C values show that the configuration with large inductance exhibits low extrinsic damping for a given resonant frequency, making it the preferred design to minimize insertion loss near the cavity resonance. The black arrow illustrates reduced insertion loss at $\omega/2\pi = 3.5$ GHz.

prototypes, each incorporating the proposed improvements. The first device features a cavity with surface-mount inductors, while the second device uses microstrip meander inductors, offering an alternative cavity design. Both devices leverage the benefits offered by two YIG spheres. Detailed implementations and performance of each design are discussed in the following.

3.4 Prototype: Room Temperature Isolators

Figure 3.10(a) shows a 3-D diagram of the second prototype. Two improvements were implemented to enhance the coherent coupling strength: surface mount inductors (4.7 nH) were used to replace the stubs, while the rest of the cross cavity design remained unchanged from the previous version. Additionally, a second YIG sphere was incorporated into the assembly. Neodymium magnets were used to provide the necessary biasing conditions.

Two 2-mm-tall panel supports were placed between the cavity board and the 0.9-mm-thick acrylic panel, resulting in a total elevation of 3.7 mm from the top magnet to the bottom one. The increased magnetic field of 0.26 T allows the magnon mode frequency to be tuned further away from the cavity resonance. Combined with the increased number of spins, ZDC appears farther from the cavity resonance, thereby forming a non-reciprocal band with reduced insertion loss.

Ideally, two YIG spheres need to be placed symmetrically with respect to the cavity center so that the coupling strength between each sphere and the cavity is the same. However, the magnetic field penetrating the board was not precisely uniform. Therefore, a fine adjustment to the second YIG's position was necessary. Consequently, two non-reciprocal bands appear at the same frequency. Picture of the fabricated device is provided in Fig. 3.10(a).

The increased coherent coupling strength is depicted as enlarged detuning between non-reciprocal band and cavity frequency, as shown in Fig. 3.10(b). Fig. 3.10(c) and (d) present zoomed-in measurement results for this prototype. At the frequency of 7.3538 GHz, the device achieved 40.5 dB of isolation with an insertion loss of just 0.7 dB. Compared to the

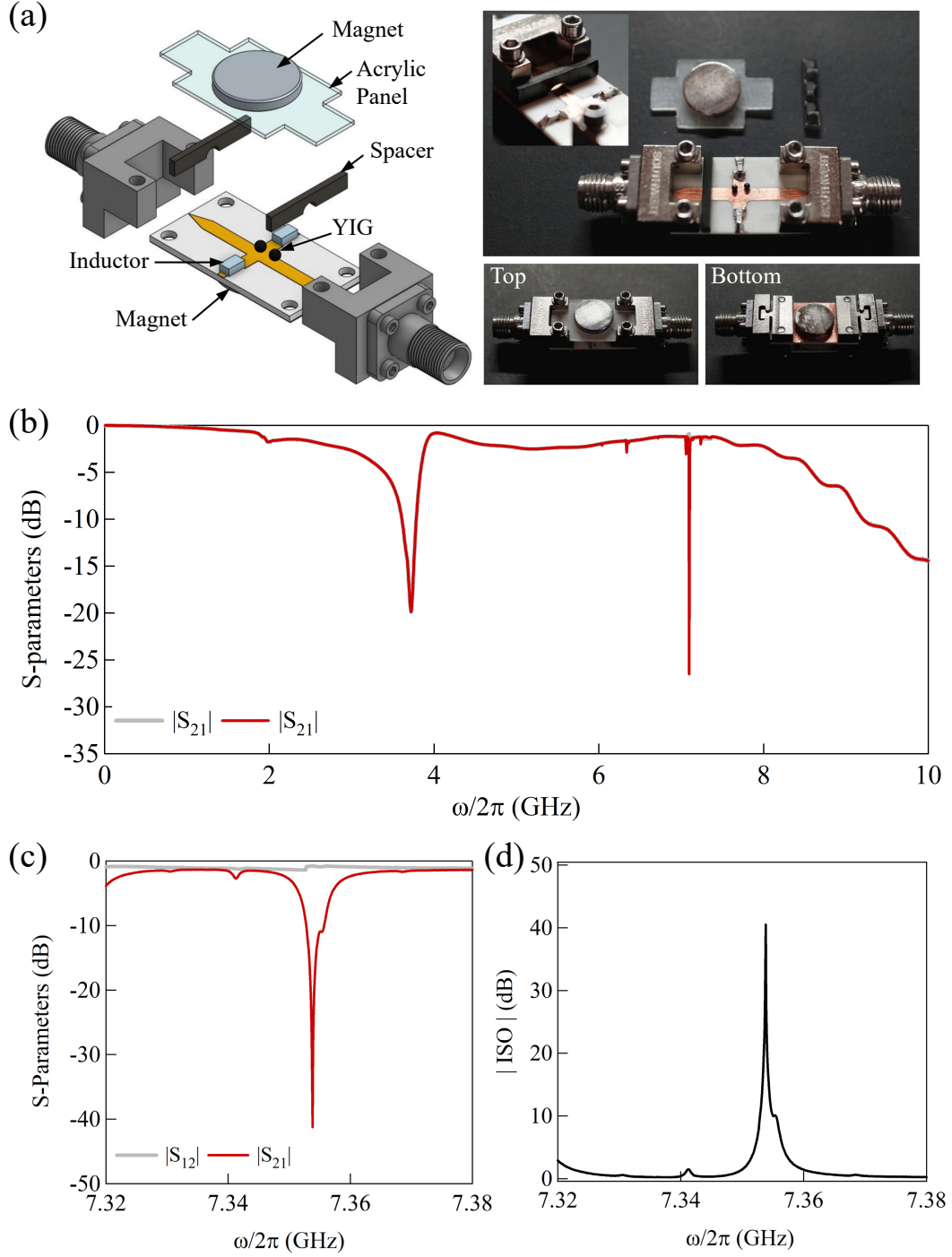


Figure 3.10: (a) Assembly diagram of second prototype and its picture, disassembled at top and assembled at bottom. The device size is 50 mm $\times$ 13 mm $\times$ 11 mm. (b) Measured transmission spectrum. Non-reciprocal band is located at 7.354 GHz. Uncoupled cavity frequency is $\omega_c/2\pi = 3.741$ GHz. (c) Magnified view of the non-reciprocal band and (d) Extracted isolation for the second prototype. IL=0.7 dB, ISO=40.5 dB.

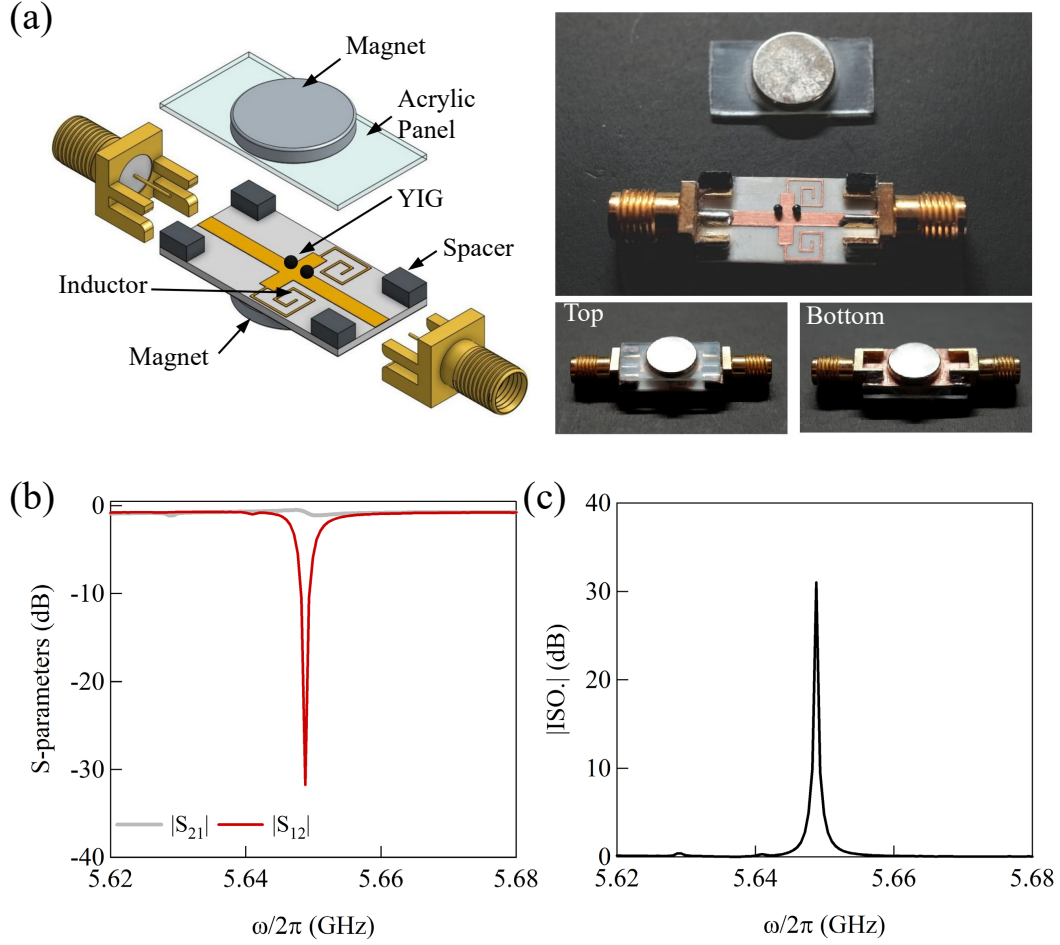


Figure 3.11: (a) Assembly diagram of the third prototype and picture of the fabricated third prototype, disassemble at top and assembled at bottom. The device size is $39 \text{ mm} \times 10 \text{ mm} \times 7 \text{ mm}$. (b) Measured transmission spectrum near ZDC. (c) Extracted isolation for the third prototype. $IL=0.7 \text{ dB}$, $ISO=31.0 \text{ dB}$

proof of concept device used in millikelvin temperature experiment, this second prototype manifested approximately 40 dB of less insertion loss.

Figure. 3.11 (a) depicts the third prototype. It is merely an alternative design approach to the second, replacing surface-mount inductors with meander shaped inductors. The characteristic impedance is determined strongly by the ratio of substrate height to microstrip linewidth. Consequently, for a narrow transmission line, its negligible capacitance and high impedance result in an overall inductive behavior. Spiral configuration was chosen for miniaturization purposes. Similar to previous design, third prototype contains two neodymium

Table 3.2: Comparison of prototypes with state-of-the-art isolators

Reference (Year)	Technology	Frequency Range (GHz)	Insertion Loss (dB)	Isolation (dB)	Return Loss (dB)	Size [†] (λ_0)
[91] (2012)	Ferrite Substrate Coplanar Waveguide	9.95–11.73	1	17	–	$0.36 \times 0.14 \times 0.04$
[92] (2012)	Ferrite Loaded Serrated Microstripline	13.50–14.10	3.5	19	10	$0.97 \times 0.60 \times 0.02$
[93] (2014)	Substrate Integrated Waveguide	9.81–10.24	2.2	55	12.5	$0.76 \times 0.27 \times 0.02$
		9.79–10.40	3.2	42	20	$0.97 \times 0.22 \times 0.05$
[94] (2014)	Inkjet Printed Ferrite Waveguide	9.95–11.73	4.0	69	15	$1.37 \times 0.29 \times 0.01$
[95] (2020)	High Impedance Wire	13.56–13.64	0.8	57	20	0.50×0.18
This Work	Proof of Concept	6.093–6.095	41	26.9	–	$0.81 \times 0.20 \times 0.13$
	Second Prototype	7.3535–7.3541	0.7	40.5	10.3	$1.23 \times 0.32 \times 0.27$
	Third Prototype	5.6485–5.6491	0.7	31.0	17.3	$0.73 \times 0.19 \times 0.13$

† Normalized to the free-space wavelength.

magnets and two YIG spheres. Strength of biasing magnetic field at the position of YIG is 0.2 T.

Fig. 3.11(c) illustrates the measurement results for this unit. It exhibits an insertion loss of 0.7 dB, an isolation of 31.0 dB at 5.6488 GHz.

3.5 Summary and Future Work

To conclude, this work provides microstrip line designs that are simpler and more versatile than traditional ferromagnetic alternatives, which often require complex geometries or machining processes. The cavity component only requires a resonance shaped like a notch filter. After determining the operating frequency from the unloaded cavity spectrum and applying the desired magnetic field, the placement of the YIG sphere becomes critical. However, as long as ZDC is satisfied in this nonreciprocal scheme, the physical dimensions and shapes of both the cavity and ferrites are not the primary factors influencing device performance.

Undeniably, the most noticeable drawback of a cavity magnnoic isolator is the narrow bandwidth that originated from the near-zero linewidth of the hybridized mode. At 20 dB isolation, the maximum bandwidth among three prototypes are associates with the low temperature unit, being 21 MHz. For Best performance, the system can be biased nearby but not at the ZDC for a performance that balances between insertion loss, isolation and bandwidth.

The proposed units, with their high isolation and narrow bandwidth, are well-suited for applications that require frequency-selective isolation. For example, they can be used to isolate qubits from their environment [96] or to separate signals from a common port in a multiplexer [97]. Additionally, a cascaded system can be employed to achieve a broader bandwidth. Specifically, single transmission line with multiple stub resonators covered by thin-film YIG plates could represent the next promising iteration of this technology.

Chapter 4

Gain Embedded Cavity Oscillator: Sensor

4.1 Introduction

Expanding upon the concepts of passive cavity magnonic isolator introduced in Chapter 3, this chapter shifts focus to the design and implementation of an active cavity oscillator¹. While passive cavity magnonic devices primarily function as filters, they cannot sustain oscillations. The distinction highlights the need to explore another important aspect of microwave electronics: generating a continuous output signal. In this regard, the following sections present the design of an active cavity and its potential application as a sensor.

As shown in Fig. 4.1, microwave oscillators can be implemented in two different ways: feedback loop type and reflection type [6, 75]. In a feedback loop configuration [Fig. 4.1(a)], a portion of the signal from the amplifier output passes through the resonator and is then fed back to the input, creating a regenerative loop. In contrast, reflection type oscillators use a transmission line terminated by a frequency-selective element, such as a resonator. Oscillations are sustained through back-and-forth reflections between the resonator and the amplifier, with the latter delivering the amplified oscillation signal to the load [Fig 4.1(b)].

Popular choices for microwave amplifiers include tunnel diodes, Gunn diodes, bipolar junction transistors (BJTs), and field effect transistors (FETs). Among these, transistors are particularly advantageous due to their compactness and ease of integration with microstrip

¹Without coupling to any secondary resonator, such as YIG.

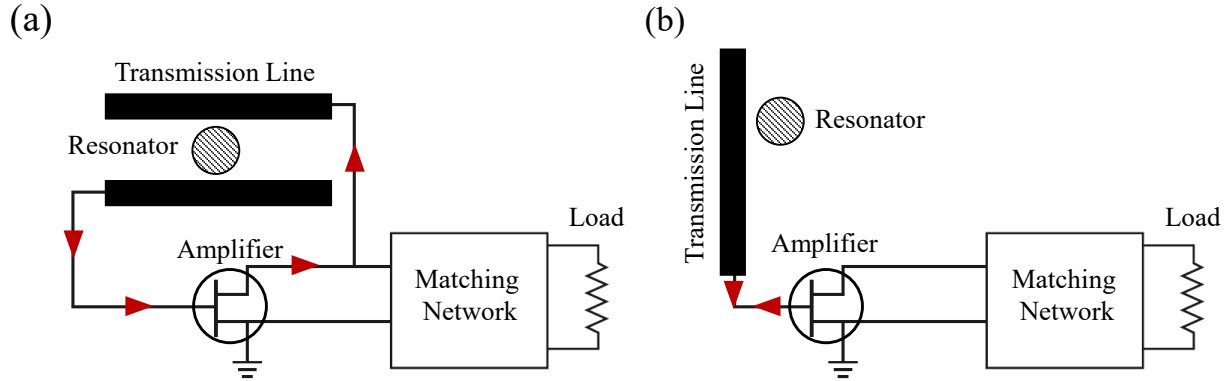


Figure 4.1: Two different approaches to realize a microwave oscillator are illustrated. Red arrows indicate the direction of signal flow. (a) In a feedback loop approach, a portion of the signal from the amplifier output passes through the resonator and is fed back to the amplifier input. (b) In a reflection approach, a transmission line is terminated with a resonator on one end and an amplifier on the other, enabling oscillations to be sustained through back-and-forth reflections. The amplifier provides the necessary gain to the oscillations and delivers the amplified signal to the load.

circuits. In particular, FETs are the most commonly used devices for both low-noise and power amplification in the 4 to 60 GHz range [75]; however, they are susceptible to parasitic effects and are vulnerable to electrostatic discharge (ESD) and oxide breakdown [98, 99]. The millimeter-wave region is largely dominated by high electron mobility transistors (HEMTs), a subclass of FETs. BJTs are sensitive to variations in biasing conditions, which can degrade phase noise and frequency stability when used in oscillator design [100]. They also face challenges such as limited frequency range and risk of thermal runaway [101]. Nevertheless, BJTs remain the simplest and most effective, and have historically been the dominant choice below 4 GHz [75].

The first half of this chapter focuses on the principles and design considerations of a gain embedded microstrip line oscillator. The second half presents various adaption strategies, including resonance stabilization and sensitivity enhancement to tailor the active cavity for sensing applications. Specifically, a transistor powered interdigital capacitor sensor is developed to detect dual strand ribonucleic acid (RNA) hybridization events, with a sensitivity of 2.8×10^6 nucleic base pairs per $0.2 \mu\text{L}$ sample. The methodology for incorporating gain into

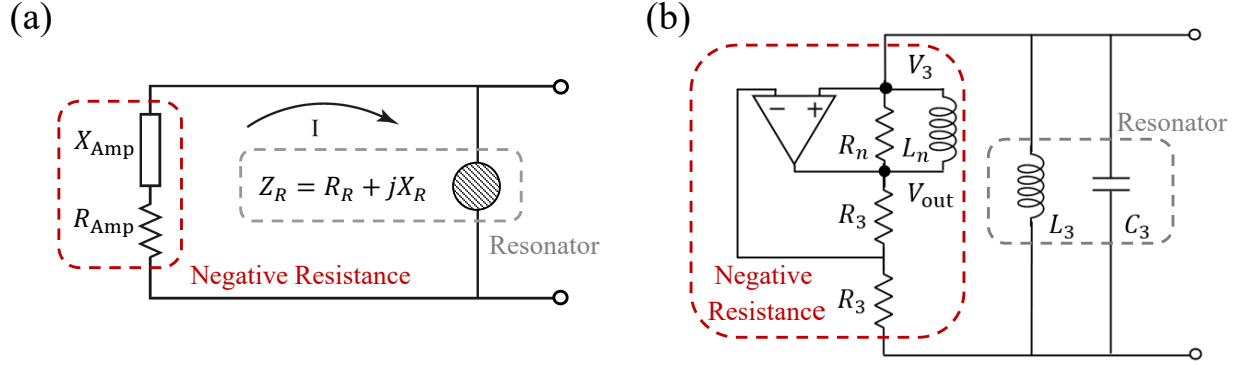


Figure 4.2: (a) A generic circuit for a one-port negative resistance oscillator. (b) Circuit model for the Van der Pol-Duffing oscillator, highlighting features conceptually identical to those in (a), reproduced from Fig. 2.5(a).

the cavity presented here lays the groundwork for more advanced coupling schemes involving both active cavity and passive YIG elements, which will be discussed in Chapter 5.

4.2 Working Principle of an Active Cavity Oscillator

4.2.1 Negative Resistance Oscillator Circuit

BJT powered oscillators employ common-emitter, common-base, or common-collector configurations. Different configurations form the basis for several well-known oscillator designs, such as the Hartley and Colpitts [102]. Despite their topological differences, oscillators can be broadly represented by the general negative resistance circuit shown in Fig. 4.2(a). It is helpful to draw a parallel to the van der Pol-Duffing oscillator introduced in Chapter 2.3.2. As depicted in Fig. 4.2(b), the van der Pol-Duffing oscillator is conceptually analogous to the general negative resistance circuit.

Referring back to Fig. 4.2(a), the one-port oscillator features an amplifier with impedance expressed as $Z_{\text{Amp}} = R_{\text{Amp}} + jX_{\text{Amp}}$, where Z_{Amp} depends on both current and frequency. The amplifier drives a resonator with impedance given by $Z_R = R_R + jX_R$. Applying

Kirchhoff's voltage law around the loop yields:

$$(Z_{\text{Amp}} + Z_R)I = 0 \quad (4.1)$$

Under the steady state oscillation ($I \neq 0$, then $Z_{\text{Amp}} + Z_R = 0$), the following two conditions must be met:

$$R_{\text{Amp}} + R_R = 0 \quad (4.2a)$$

$$X_{\text{Amp}} + X_R = 0 \quad (4.2b)$$

Since the load is a passive resonator, $R_R > 0$, which implies that the amplifier resistance must be negative, i.e., $R_{\text{Amp}} < 0$. In other words, an oscillator can employ any device and configuration, as long as the amplifier circuit exhibits negative resistance. Putting it in another way, the active device must supply energy equal to the amount dissipated by the resonator's losses, while the resonator must compensate for the imaginary part of the active device's impedance at the desired oscillation frequency.

4.2.2 Stability Factor of an Oscillator Circuit

To initiate oscillation, the circuit must initially be unstable over a certain frequency range, allowing ambient noise to induce a current in the loop. The condition requires that $R_{\text{Amp}} + R_L < 0$. The noise current passes through the resonator, where it is filtered and then fed back into the amplifier. The process repeats iteratively, with the noise signal being continuously amplified till gain compression sets in. During the iteration, R_{Amp} gradually becomes less negative until it reaches a value that satisfies Eq. (4.2). As previously discussed in Chapter 2.3.2, the final oscillation frequency will differ from the initial startup frequency because Z_{Amp} depends on both frequency and current.

In practice, the K-factor is used to evaluate the stability of a transistor-based oscillator circuit [103].

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} < 1 \quad (4.3)$$

where $|\Delta| = |S_{11}||S_{22}| - |S_{12}||S_{21}|$ represents the determinant of the S-parameter matrix. Equation (4.3) is the opposite of Rollet's Stability Criterion for transistor amplifiers [6]. According to the criterion, a transistor is unconditionally stable at any frequency where $K > 1$, meaning it will not oscillate or become unstable under any passive termination, a desirable property for amplification purposes. However, if the goal is to design an oscillator, it is preferable for the transistor to operate in a unstable condition where $K < 1$. Stability analysis can be performed using simulation tools or small-signal S-parameters provided by the transistor manufacturer.

4.2.3 Active Cavity Oscillator

Based on the previous discussion, the steps for designing a transistor powered fixed frequency oscillator can be summarized as follows.

- (1) Determine the working frequency ($\omega_s/2\pi$) and select/design the resonator.
- (2) Select a transistor and evaluate the stability factor, K under various biasing conditions.
Ensure that $K < 1$ over the frequency range that includes the operating frequency $\omega_s/2\pi$.
- (3) Based on the analysis, design the transistor bias circuit.
- (4) Fine tune the completed design, by modifying either the resonator, the transistor bias circuit, or both, until the desired working frequency is achieved.

Figure 4.3(a) illustrates a BJT-embedded half-wavelength transmission line oscillator, and the bill of material is provided in Table 4.1 [80]. The total effective length of the transmission line resonator is 29.0 mm, and the substrate is FR-4 material with a thickness of 1.57 mm and 35 μm copper cladding on both sides. As shown in Fig. 4.3(b), the spectrum

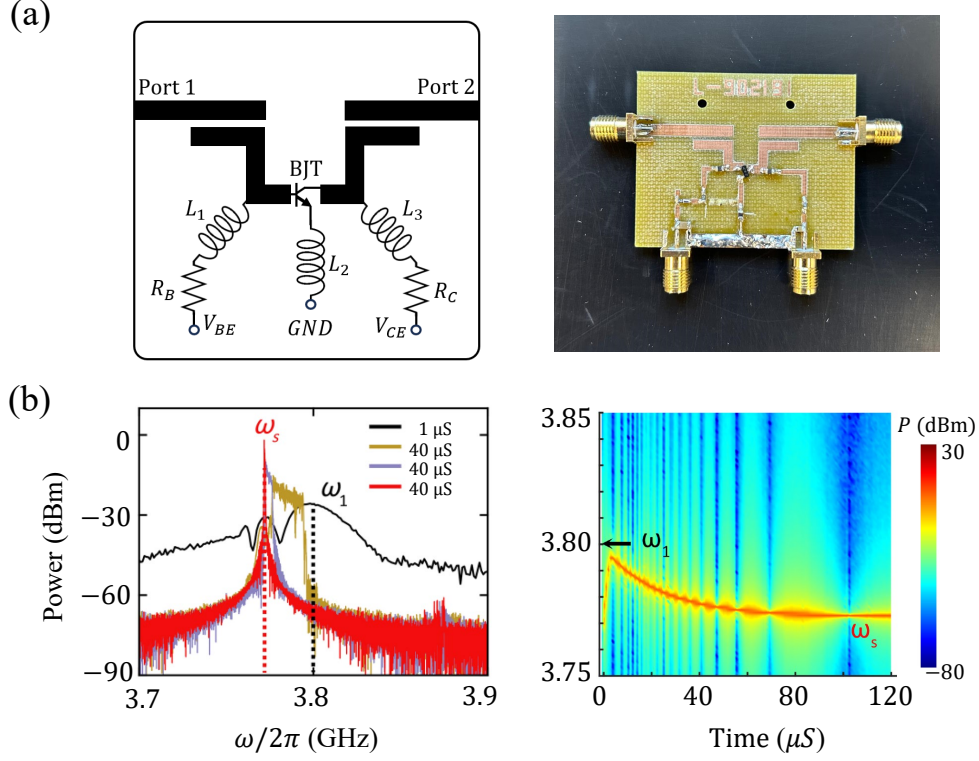


Figure 4.3: (a) Circuit diagram of the microstrip active cavity is shown on the left, with a corresponding photograph on the right. (b) Left: Spectrum illustrating the frequency-domain dynamics over the first four time intervals. The black, gold, purple, and red curves represent the resonance profiles during the first 1 μ s, the first 40 μ s, the second 40 μ s, the third 40 μ s, and the fourth 40 μ s after powered on. Right: Spectrogram showing the evolution of frequency over time [80].

Table 4.1: List of Transistor Circuit Parameters

Symbol	Description	Value
$L_{1,2,3}$	RF Chokes	$1 \mu H$
R_B	Base Resistor	30Ω
R_C	Collector Resistor	100Ω
V_{BE}	Base Terminal Voltage	$0.8 V$
V_{CE}	Collector Terminal Voltage	$8.0 V$
BJT	BFU550AR	N.A.

gradually evolves from the start up frequency $\omega_1/2\pi = 3.80$ GHz to a sharp, high amplitude peak at steady-state frequency $\omega_s/2\pi = 3.77$ GHz. Such transition mirrors the behavior observed previously in a Van der Pol-Duffing oscillator [Fig. 2.5(b)].

4.3 Prototype: Active Interdigital Electrode Sensor

Microstrip oscillators offer several advantages, including a low profile, simple design, layout flexibility, and real-time signal readout—features that are also highly desirable in resonant sensors [66]. The Q-factor is a critical parameter in any resonant sensor, as it directly affects sensor resolution, hence the minimum detectable signal [104]. For this reason, the principles of negative resistance and the amplifier’s ability to compensate for losses offer a compelling basis for extending oscillator architectures to sensing applications. In fact, active sensors have shown significantly improved resolution over their passive counterparts [69, 105, 69, 106, 107, 108, 70, 109]. Moreover, the enhanced performance can be attained without a substantial increase in design or fabrication complexity.

When an oscillator is configured for sensing purposes, the resonator functions as a signal transducer. In capacitive-type sensors, the resonator concentrates electric field around specially designed structures. Consequently, changes in the electrical properties of the surrounding medium, such as the dielectric constant, alter the resonator’s equivalent capacitance, leading to measurable shifts in resonant frequency. The sensitivity of capacitive-type sensors can be improved by maximizing the electric field intensity within narrow confined regions [110]. The role of amplifier circuit is to compensate for resonator losses and sustaining a high-Q signal output, as previously discussed. Active capacitive sensors have been used in a wide range of applications, such as food quality monitoring [111], as well as material characterization across—liquid, solid, and vapor [69, 106, 105, 109]. However, their use in biochemical reaction sensing remains relatively underexplored, despite some progress achieved with passive sensors [112, 113, 114]. Given the advantages of active sensors and the well-established strategies for increasing sensitivity in capacitive-type designs [110], exploring active capacitive-type sensors for nucleic acid level sensing appears to be a worthwhile attempt.

The remainder of this chapter focuses on an active interdigital capacitor sensor for RNA hybridization detection. It begins by outlining the sensor design, as well as optimization

strategies for both the sensor and the sensing procedure, followed by an analysis of detection limits.

4.3.1 Sensor Components

As illustrated in Fig. 4.4(a) and (b), the active sensor consists of three main components: (1) a gold interdigital capacitor (Au-IDC), (2) a transistor-based negative resistance circuit connected to the Au-IDC through bonding wires, and (3) a $50\ \Omega$ matched microstrip line for signal readout, which is gap-coupled to the active circuit.

The bipolar junction transistor is integrated with the Au-IDC in a common-emitter configuration, and the circuit parameters are listed in Table. 4.2. The DC bias voltage V_{cc} is important here, as it controls many transistor parameters such as noise figure, gain and collector base capacitance [115]. The design procedure follows a similar approach to that described previously in section 4.2. A slot in the substrate allows the Au-IDC to be removed and reinserted. The bonding wire is then pressed against the Au-IDC electrodes by a top cover, which is not shown in the figure.

The Au-IDC is commercially fabricated using a quartz glass substrate, with a 100 nm thick gold film as the top conductor. The resonant properties of Au-IDC depends on its geometry. The Au-IDC used in this study features 100 fingers and a finger length of $l = 4.6\text{ mm}$, with a gap of $g = 10\ \mu\text{m}$ between adjacent fingers. The dimensions are illustrated in Fig. 4.4(c).

Figure 4.4(d) presents the equivalent circuit model of the Au-IDC consists of a series RLC with two shunt capacitors to ground [116, 117]. The inductance (L_{IDC}), sensing capacitance (C_{IDC}), and shunt capacitance (C_s) are determined by the Au-IDC's physical properties, including number of fingers, finger length l , finger gap g and effective dielectric constant of the substrate [118]. The sensing capacitance, C_{IDC} is estimated to be 7.2 pF. The bonding wire is modeled as a series inductance (L_w) [119].

Figure 4.5 illustrates the working principle of the active sensor. The sensing process be-

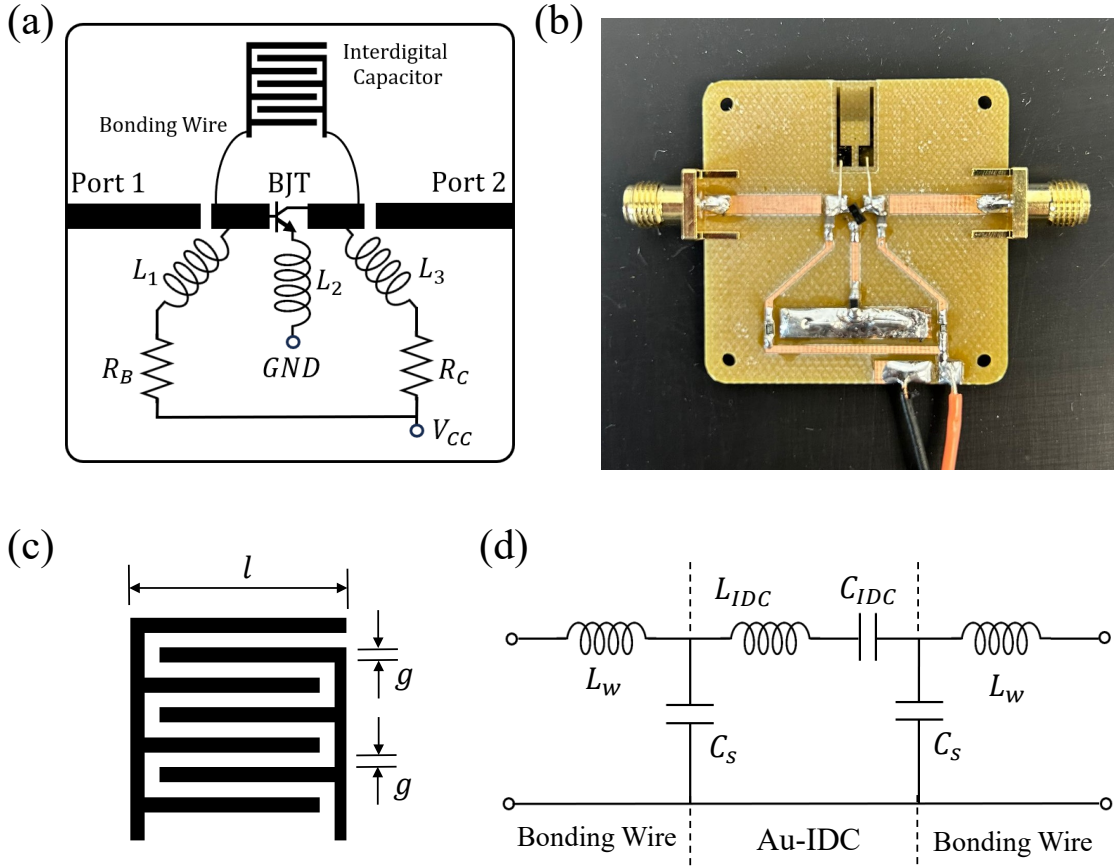


Figure 4.4: (a) Schematic and (b) picture of active Au-IDC sensor consisting of transistor feedback circuit and read out ports. (c) Dimension of Au-IDC, l is the finger length, and g is the finger gap. (d) Equivalent circuit model of resonant structure of sensor.

Table 4.2: List of Circuit Parameters for Active Au-IDC Sensor

Symbol	Description	Value
$L_{1,2,3}$	RF Chokes	$1 \mu H$
R_B	Base Resistor	$19.3 k\Omega$
R_C	Collector Resistor	100Ω
V_{cc}	Base Terminal Voltage	$1.5 - 8 V$
BJT	PBR-941	N.A.

gins with the immobilization of single-stranded nCOV2 RNA (ssRNA-A)² oligonucleotides onto clean, bare gold electrodes inside a separate chamber, using covalent bonding for im-

²Immobilized, A :5'-5DTPA-GGA CCA CGU CUG CCG AAA GCU UG-3' (Methylated)
Hybridized Complementary, A': 5'-GUA ACA CAA GCU UUC GGC AGA CGU GGU CC-3'
Hybridized Non-Complementary B': 5'-GAC ACA UGG UUU AGC CAG CGU GGU GGU AG-3'

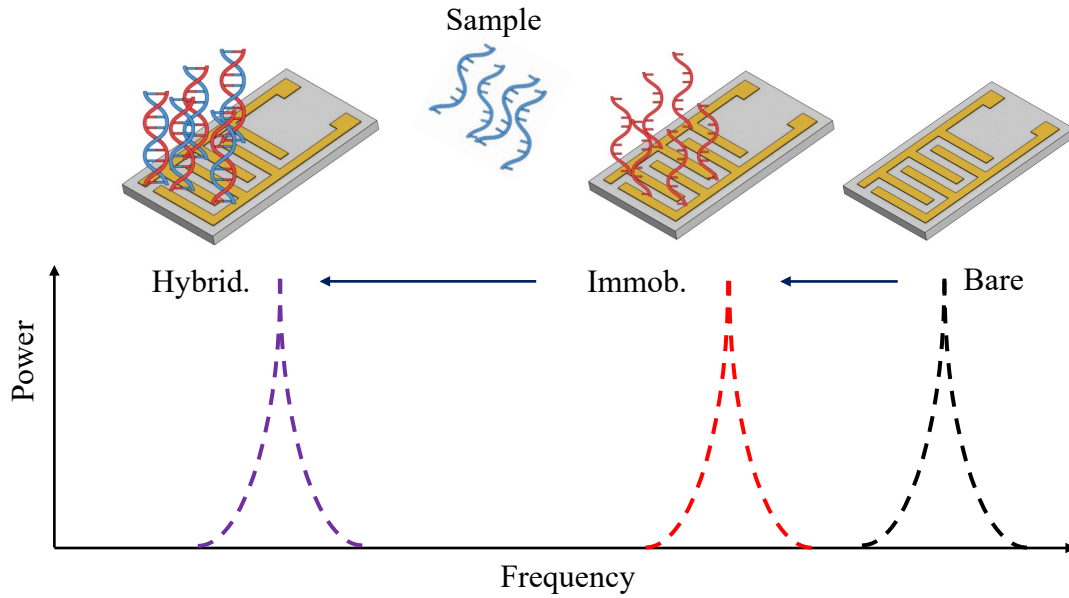


Figure 4.5: Working principle of sensor. Two consecutive frequency shifts occur due to the immobilization and hybridization processes.

proved stability and strength compared to alternative methods [120]. After immobilization, the electrode is reinserted back into the sensor and powered on, generating a resonant signal indicated by the red dashed peak labeled “Immob”. When a sample containing complementary sequence (ssRNA-A’) is introduced, it hybridizes with the immobilized ssRNA, forming a dual strand structure, leading to a shift in resonant frequency, shown by the purple peak. In contrast, non-complementary targets (ssRNA-B’) do not bind and can be washed away, resulting in no significant frequency change during hybridization.

4.3.2 Sensor Fine Tuning

The sensor’s performance can be characterized by four categories: long-term frequency stability, short-term frequency stability, resolution, and sensitivity. In this section, each of these factors will be defined and discussed. An optimization strategy will then be presented before moving on to the sensing experiments. All measurements are performed using bare Au-IDC, that is, no chemical reaction is involved for this part.

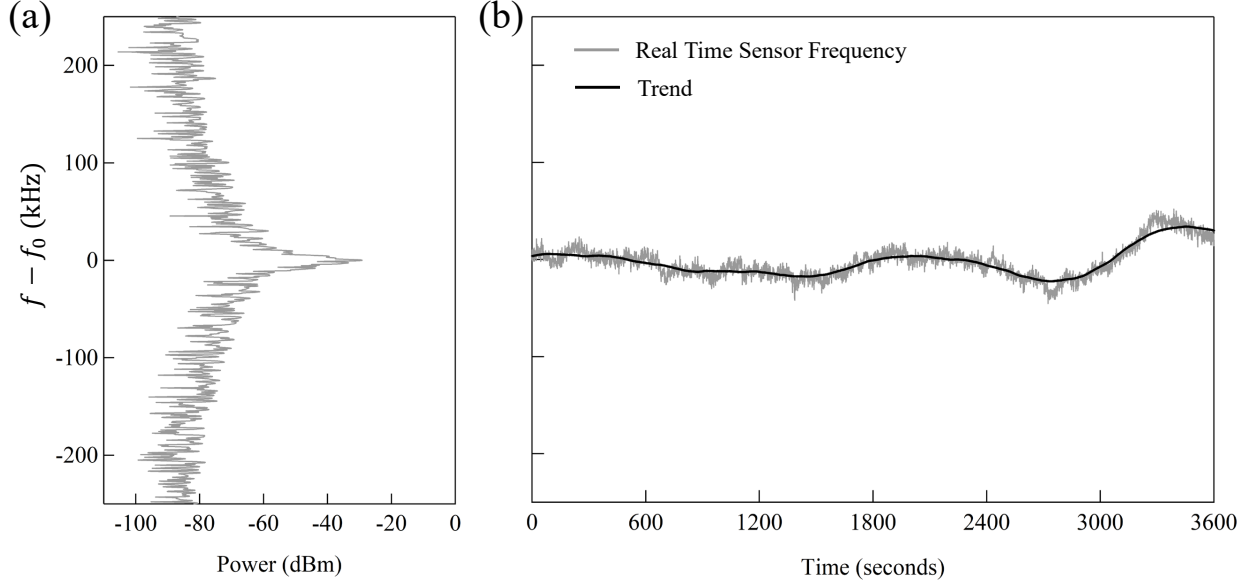


Figure 4.6: (a) Normalized resonant frequency of the bare sensor at $t = 0$ (f_0) over a 500 kHz span. (b) Normalized resonant frequency ($f_t - f_0$) recorded over a one-hour period, showing short-term fluctuations (gray) and long-term drift (black). $V_{cc} = 5.0$ V and $f_0 = 1.2115$ GHz.

Long-Term Stability

Figure 4.6 shows the bare sensor's resonant frequency (f_t), recorded over a one-hour period. Although the supply voltage remains constant, f_t displays unavoidable fluctuations caused by electrical noise, seen as rapid, zigzagging variations (gray). Beyond those fast fluctuations, a slower, gradual drift is also evident (black), indicating long-term instability. Among all performance metrics, long-term stability is the most important, as it not only results in larger frequency deviations than short-term noise, but also necessary for accurately interpreting sensor output during hybridization experiments, which require extended measurement durations. Thus, to ensure reliable sensing, long-term stability should be quantified, have its causes identified, and its effects compensated for.

$$\Delta_{\text{Long Term}} = \text{STDEV}(f_t - f_0) \quad (4.4)$$

where f_t is the bare sensor's resonant frequency at time t , and f_0 is the initial resonant frequency at $t = 0$. $\Delta_{\text{Long Term}}$ represents the standard deviation of the change in resonant

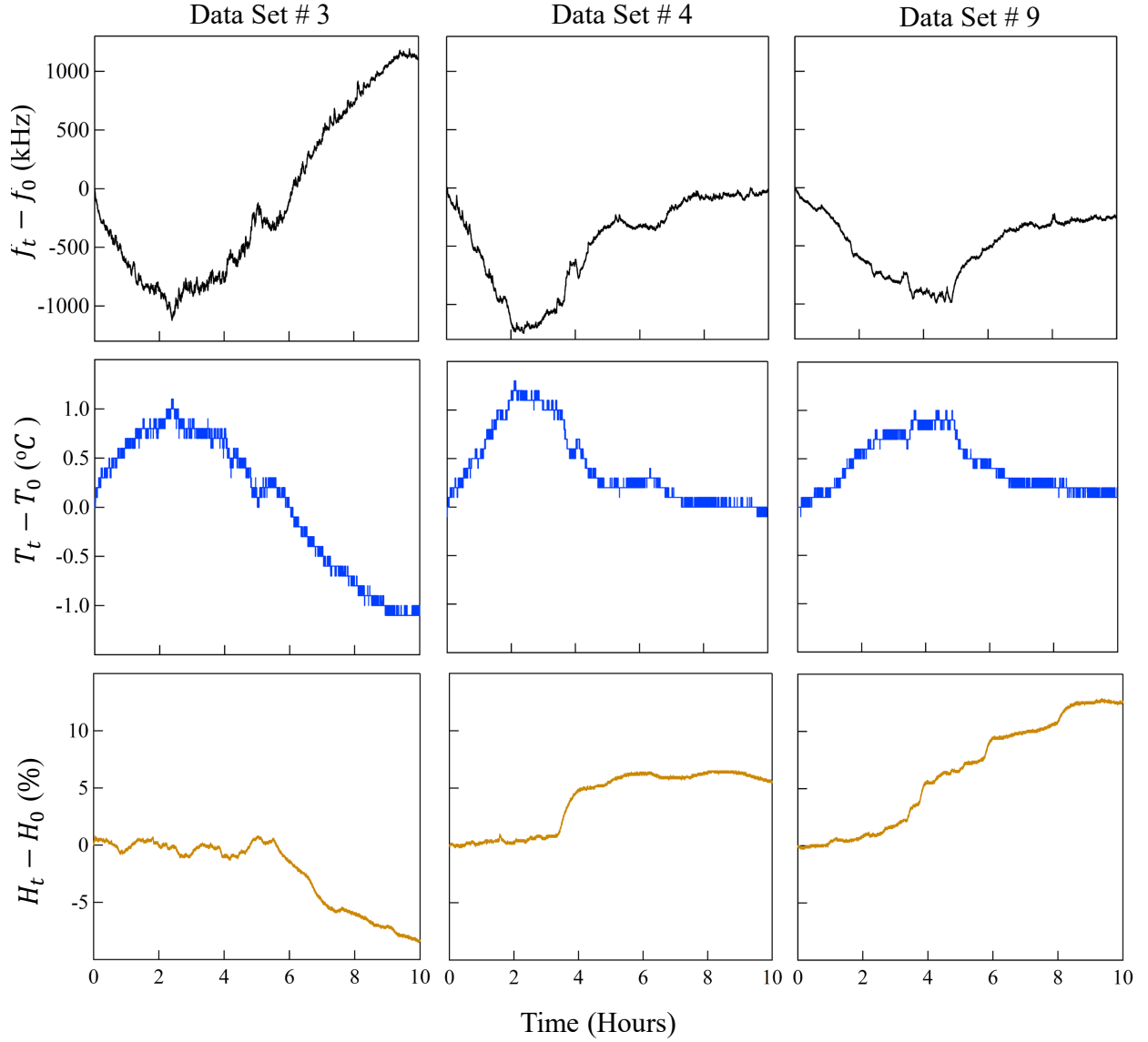


Figure 4.7: Three datasets from ten 10-hour measurements, including simultaneous recordings of the bare sensor's resonant frequency (f_t), temperature (T_t), and relative humidity (H_t). Each vertical column presents data from a single 10-hour measurement.

frequency over the measurement period.

It has been reported that resonant sensors are prone to respond to ambient environmental factors, such as humidity and temperature [121]. To study the effect, f_t is measured along with temperature (T_t), and relative humidity (H_t) readings obtained from a DHT22 module. 10 hour measurements were performed 10 times and the selected data sets are shown in

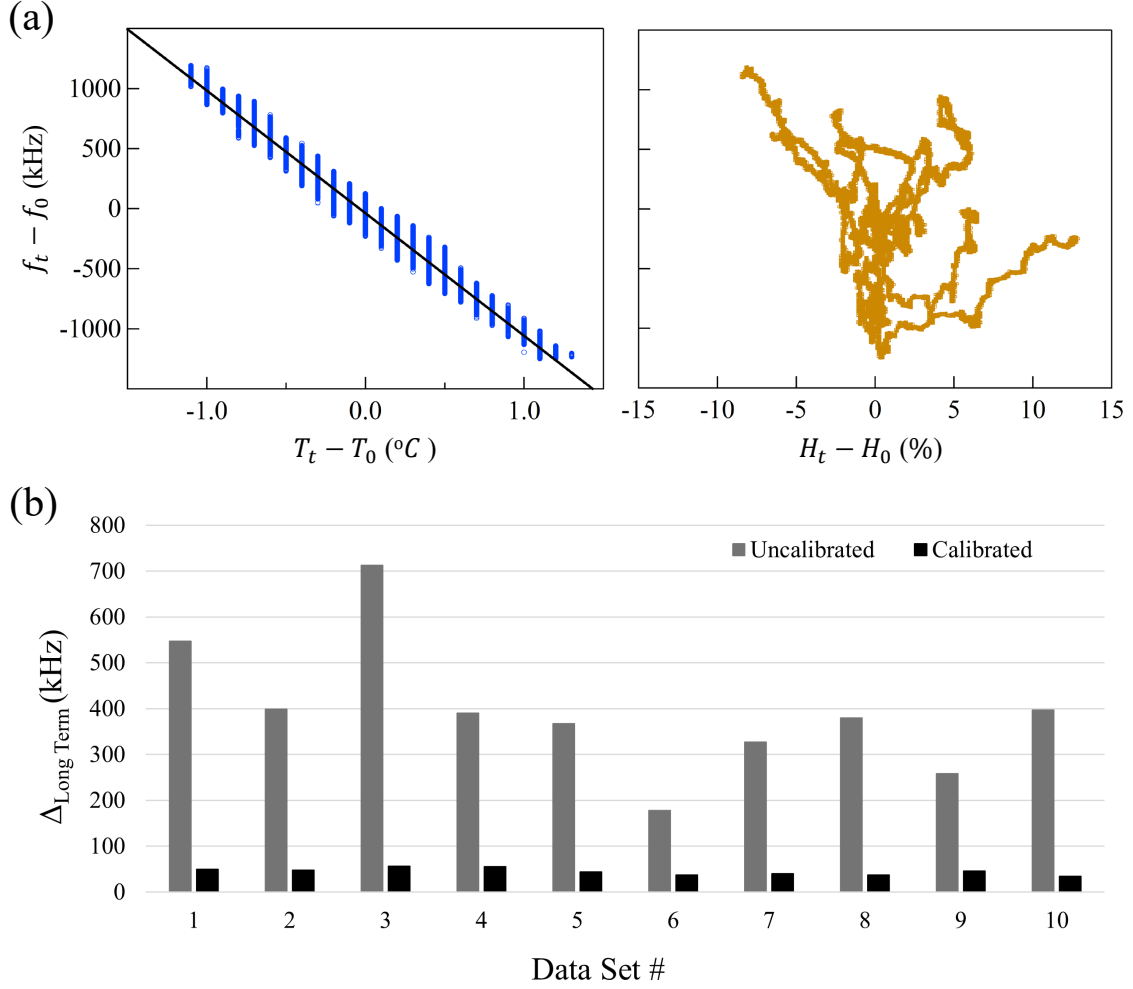


Figure 4.8: (a) Left Panel: Scatter plot of concatenated ten 10-hour measurements between change in sensor's frequency and change in temperature, showing a strong negative linear correlation. The black solid line is the best fit line with slope $-1023.3 \text{ kHz}/^{\circ}\text{C}$. Right Panel: Scatter plot of concatenated ten 10-hour measurements between change in sensor's frequency and change in humidity, showing no correlation. (b) Long-term stability comparison between raw and calibrated sensor frequencies, demonstrating improved frequency consistency due to drift elimination.

Fig. 4.7.

By comparing the first and second rows in Fig. 4.7, a strong negative correlation between change in sensor frequency and change in temperature is evident. The latter appears to inversely mirror the former. In addition, no apparent relationship is observed between sensor frequency and humidity [Fig. 4.8(a)]. The correlation coefficients for all measurements are listed in Appendix C, Table C.1, with an average value of -0.97 between $f_t - f_0$ and $T_t - T_0$,

further support these findings. Therefore, sensor frequency can be effectively calibrated using temperature data alone. Specifically, a linear model can be assumed, with the slope representing the frequency shift per unit change in temperature. Using the concatenated data from all ten measurements, the calibrated sensor frequency at any given time is:

$$(f_t - f_0)_{\text{cal}} = (f_t - f_0) + 1023.3 [\text{kHz}/^{\circ}\text{C}] \times (T_t - T_0) \quad (4.5)$$

Figure 4.8(b) demonstrates an improvement in long-term stability, with the maximum calibrated frequency standard deviation observed in data 4, being $\Delta_{\text{Long Term}} = 55 \text{ kHz}$. It is also worth noting that $\Delta_{\text{Long Term}}$ is positively correlated with measurement time; as time increases, the likelihood of observing temperature variation also increases.

Resolution and Short Term Stability

Resolution is defined as the smallest detectable change in resonant frequency. Enhancing resolution requires minimizing the stochastic variations near the signal peak. This can be achieved by reducing phase noise, which sharpens the signal profile and improves the accuracy of readout. Phase noise also influences the sensor's short-term stability—the rapid fluctuations shown previously in Fig. 4.6(b).

Figure 4.9(a) compares the oscillator's resonance profiles at two different bias voltages: $V_{\text{CC}} = 1.5 \text{ V}$ and $V_{\text{CC}} = 8.0 \text{ V}$. At the lower bias voltage, the resonance peak appears broader and less distinct, indicating a lower signal-to-noise ratio and reduced signal resolution. In contrast, at $V_{\text{CC}} = 8.0 \text{ V}$, the resonance becomes sharper and more well-defined. Fig 4.9(b) shows that the total output power of the sensor increases by nearly 20 dB as the bias voltage is raised from 1.5 V to 8.0 V. Since the increase in carrier power exceeds that of the sideband, increasing V_{CC} indeed can improve the sensor resolution without compromising phase noise. The substantial gain in output power can be attributed to the increase in collector current with higher supply voltage.

Variations in phase noise levels at 10 kHz and 100 kHz offset frequencies are shown in

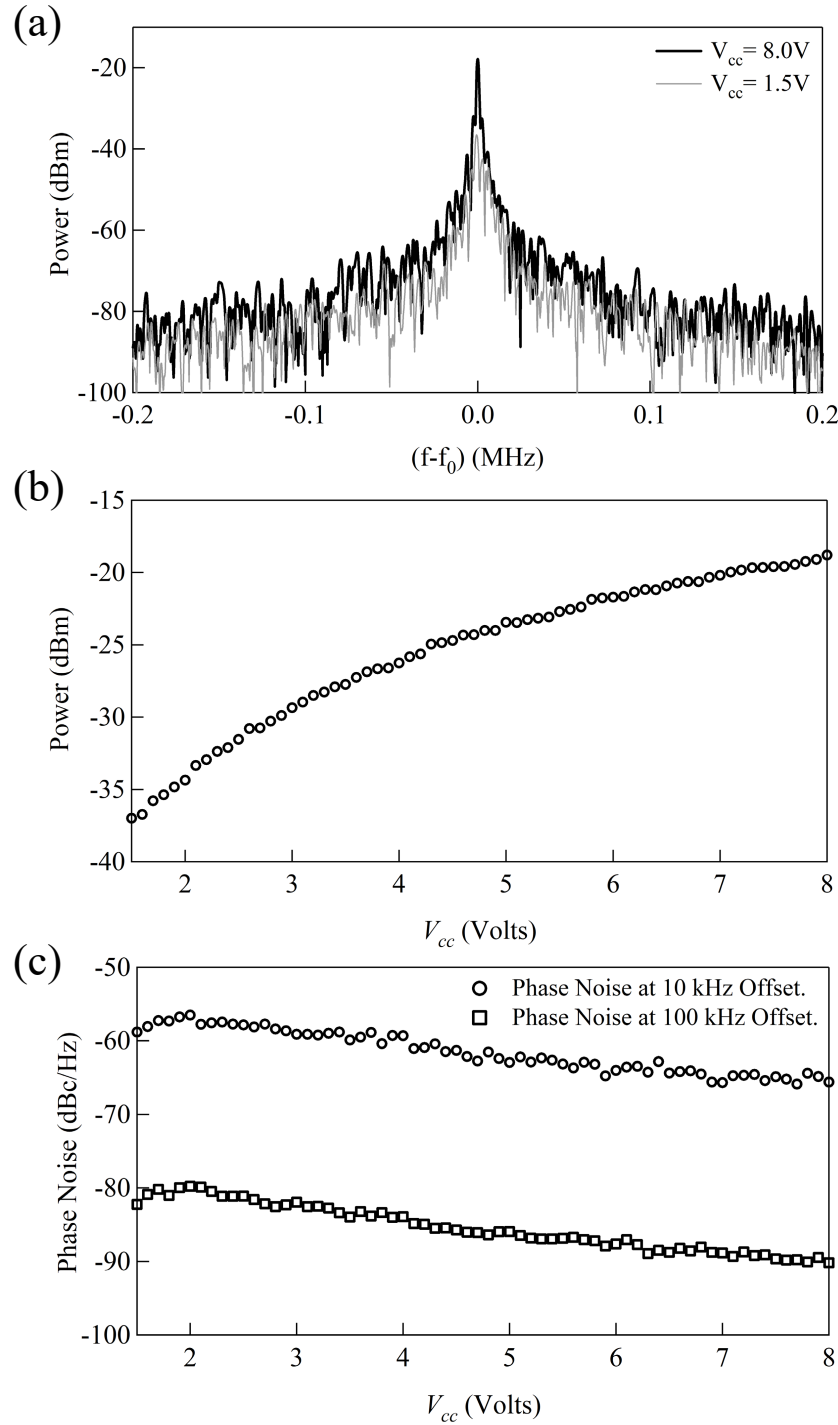


Figure 4.9: Resolution and Short-term Stability Analysis. (a) Sensor under two bias conditions, $V_{cc} = 1.5$ and $8.0V$ showing an increase in signal to noise ratio at higher V_{cc} . (b) Sensor output power versus V_{cc} . (c) 10 kHz, 100 kHz offset phase noise level versus V_{cc}

Fig. 4.9(c) as a function of the supply voltage. Both curves exhibit a similar trend: phase noise initially increases, peaking around $V_{CC} = 2.0V$, followed by a gradual decline and

eventual flattening beyond $V_{CC} = 7.0\text{ V}$. The behavior reflects an interplay between the transistor's biasing conditions and its intrinsic noise generation mechanisms. At lower bias voltages, near $V_{CC} = 2.0\text{ V}$, the transistor operates with limited collector current, resulting in reduced transconductance and a lower signal-to-noise ratio. These conditions increase the oscillator's susceptibility to flicker noise ($1/f$ noise) and other low-frequency disturbances, thereby elevating the phase noise. The phase noise eventually levels off, likely due to gain compression or thermal effects. Across the full sweep range of V_{CC} from 1.5 V to 8.0 V , a net reduction of approximately 5 dB in phase noise is observed at both offset frequencies.

Although the “side skirt” noise increases with V_{CC} , the carrier power grows at a faster rate, resulting in an overall improvement in phase noise performance. In summary, increasing the supply voltage enhances both the sensor's resolution and its phase noise characteristics (short term stability).

Sensitivity

The sensitivity of the sensor is defined as the change in resonant frequency per unit change in the resonator's capacitance. It is influenced not only by the geometry of the Au-IDC, but also by the transistor's nonlinear behavior and intrinsic capacitances, such as the collector–base capacitance.

To evaluate the sensor's sensitivity, the Au-IDC was replaced with a varactor, as shown in Fig. 4.10(a) and (b). The varactor circuit parameters are listed in Table. 4.3, and the transistor bias circuits are the same as those in Table. 4.2, except $V_{cc} = 8.0\text{ V}$. The varactor circuit enables a controlled and quantifiable simulation of a sensing event by introducing a known capacitance change. At each supply voltage, the varactor's reverse bias is toggled ON and OFF to create a capacitance variation of approximately 0.11 pF , and the corresponding shift in the oscillator's resonant frequency, Δf is measured. The results shown in Fig. 4.10(c) illustrate change in sensor's frequency per change in capacitance for each V_{CC} . The data clearly demonstrate that the sensor's sensitivity improves with increasing supply voltage.

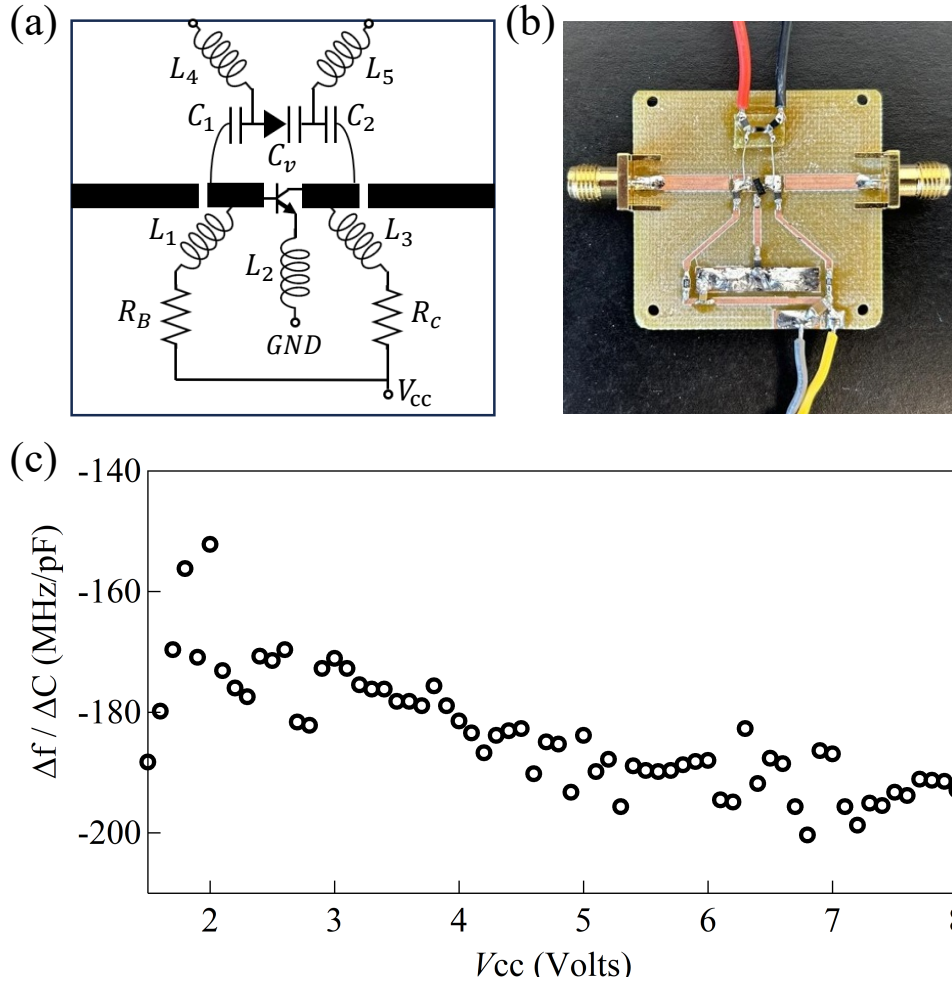


Figure 4.10: (a) The varactor circuit and (b) photo, used to quantitatively characterize the sensor sensitivity. (c) Change in sensor frequency given a same amount of change in capacitance, showing a gradual increase in sensitivity.

Table 4.3: List of Circuit Parameters for Sensitivity Optimization Circuit

Symbol	Description	Value
$L_{4,5}$	RF Chokes	$1 \mu H$
$C_{1,2}$	DC Blocking Capacitor	1.2 pF
C_v	Varactor (SMV1705)	$31.5 \text{ pF @ } 0 \text{ V} - 5.2 \text{ pF @ } 5 \text{ V}$

The improved sensitivity can be attributed to the reduction in base-collector junction capacitance as V_{CC} increases [115]. Since the junction capacitance loads in parallel with the sensing element [Fig. 4.4(d)], a decrease in its value effectively lowers the total capacitance. As a result, a unit change in the external sensing capacitance induces a larger relative

frequency shift, thereby enhancing the frequency response and overall sensitivity.

To summarize the fine-tuning measurements, there are two key takeaways

- Calibrate the sensor’s resonant frequency to account for temperature variations.
- Increase the transistor bias voltage to improve resolution, phase noise, and sensitivity.

4.3.3 Sensing Experiment

Pretreatment of the Au-IDC Surface.

During manufacturing and packaging, the gold electrode surface may be exposed to ambient and uncontrolled conditions, making it susceptible to contaminants. These contaminants can significantly affect binding kinetics of ssRNA—the efficiency at which thiol groups (–SH) of chemically modified ssRNA bind to gold surfaces (immobilization). It is the thiolation that leads to covalent Gold–Sulfur (Au–S) bonds, enabling the attachment of thiol-modified nucleotides to gold surface.

To prepare the gold surface, the Au-IDC was first cleaned with rubbing alcohol to remove any dust and surface contaminants. It was then immersed in a cleaning solution containing 50 mM Potassium Hydroxide (KOH) and 25% Hydrogen Peroxide (H_2O_2) for 10 minutes. Following the treatment, the electrode was thoroughly rinsed with Milli-Q water ($18.2 \text{ M}\Omega \cdot \text{cm}^3$) and left to dry in a clean chamber [122].

Thiol Group Modification

In order for thiol bond formation between the oligo and the gold surface to proceed as expected, the 5′ thiol-modified oligomers must undergo reduction of the disulfide bonds before application. To begin, 10 μL of stock-*A* solutions were mixed with 10 μL of 1 M DTT (Dithiothreitol) and 30 μL of Milli-Q water in a tube. The mixture was vortexed and allowed to sit on the bench for 10 minutes to facilitate the reduction of disulfide bonds.

³The resistance between two electrodes spaced 1 cm apart, immersed in the Milli-Q water, serving as an indicator of purity.

Next, 600 μL of G-25 resin was added to a column, and the column was centrifuged at 3000 rpm for 1 minute. The flow-through liquid was discarded, and each column was rinsed thoroughly with Milli-Q water and centrifuged to remove residual ethanol. Next, 50 μL of the reduced ssRNA were added to the column and centrifuged at 3000 rpm for 1 minute. The flow-through, containing the purified thiol-reduced ssRNA, was collected. To stabilize the RNA, 1/40th the volume of 20 $\times$ SSC (Saline-Sodium Citrate) buffer was added to the tube.

Immobilization

In addition to surface cleanliness, immobilization efficiency is also determined by time the thiol modified ssRNA is in contact with the gold surface, with overnight immobilization being very common [120, 114]. 10 μL of the ssRNA solutions were applied to the electrode, and the electrode was covered with a lab film to prevent evaporation during the immobilization process, and left inside a clean chamber for overnight. Afterward, the electrode was rinsed thoroughly with Milli-Q water and dried. This completes the immobilization process.

Hybridization

After immobilization, the gold electrode was inserted back into the sensor, and a pipette guide was mounted. The hybridization procedure is illustrated in Fig. 4.11(a) and includes the following four phases: ① the immobilized electrode was left to idle for 150 seconds, during which its frequency was recorded to serve as the baseline [Fig. 4.4(e), “Immob.”]. ②: 0.2 μL of target ssRNA was introduced onto the sensor. Then, the sensor surface was sealed with a lab film and incubated to allow hybridization. ③: After incubation, the film was removed, and the sensor was thoroughly rinsed with Milli-Q water. If non-complementary ssRNA was introduced in phase ②, it will be washed away during this step. ④: After rinsing, the sensor was left to dry and stabilize with the Au-IDC now containing hybridized double-stranded RNA. Finally, the hybridization-induced frequency change (Δf_{Hybrid}) was calculated as the

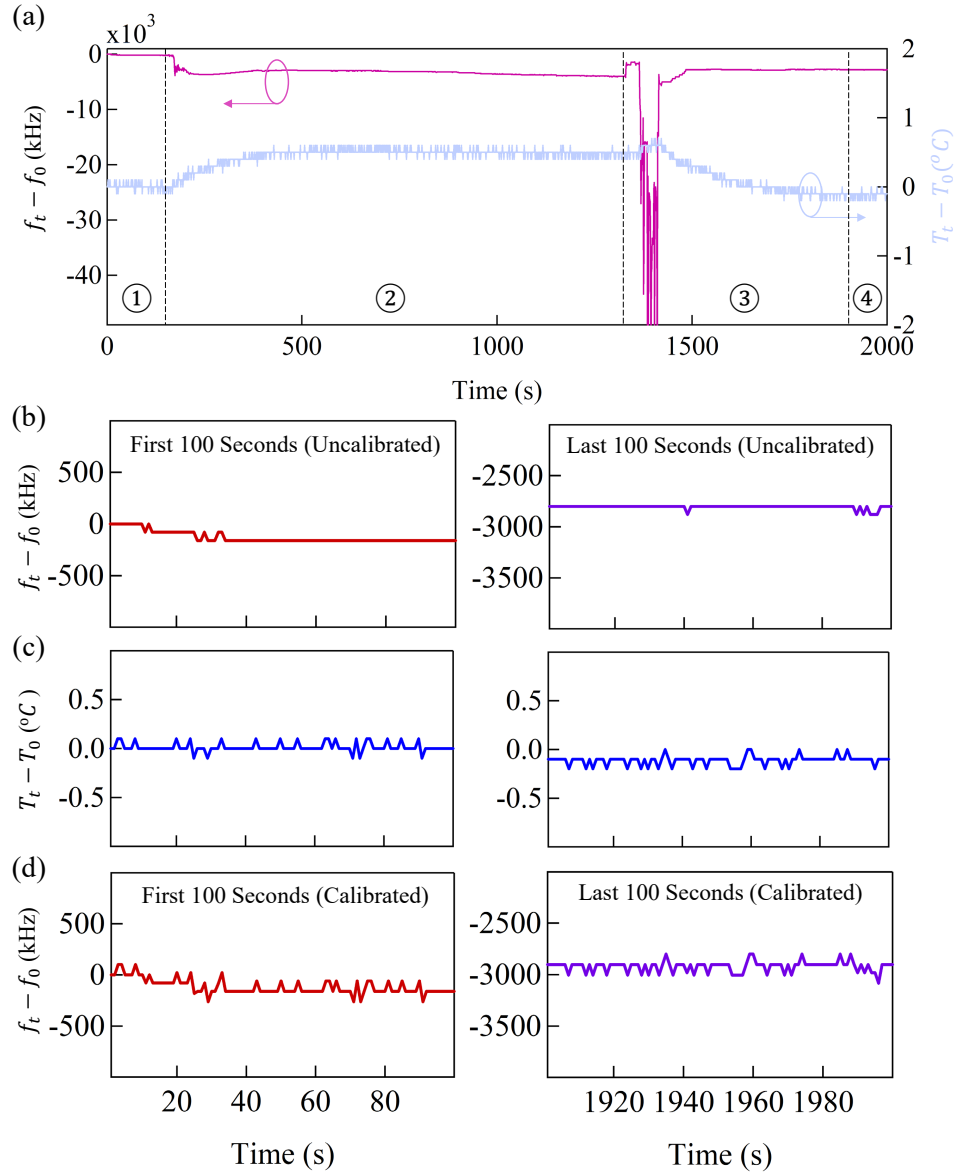


Figure 4.11: Illustration of data capture process for hybridization experiment. Time for hybridization is 20 minutes. Target RNA concentration is 10 nM. (a) Raw sensor frequency and temperature data captured during the entire process. ①: Immobilized sensor idle. ②: Complementary oligonucleotide, A' is introduced to the sensor and covered with film. ③: removal of film, washing, and evaporation. ④: stabilization. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) First 100 seconds of sensor frequency calibrated against temperature. Right: last 100 seconds of sensor frequency calibrated against temperature. The difference between average of left and right panel data are interpreted as hybridization signal.

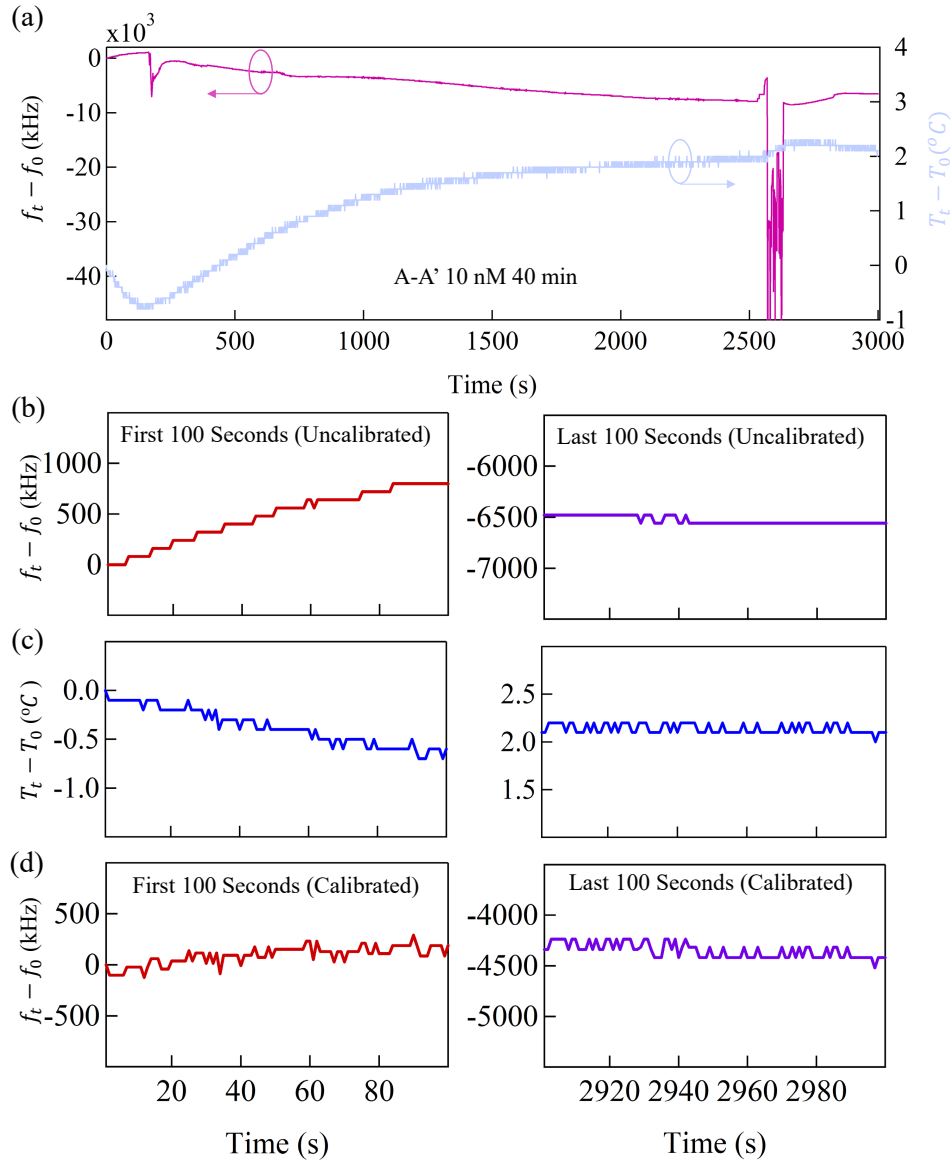


Figure 4.12: Illustration of data capture process for hybridization experiment. Time for hybridization is 40 minutes. Target RNA concentration is 10 nM. (a) Raw sensor frequency and temperature data captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) First 100 seconds of sensor frequency calibrated against temperature. Right: last 100 seconds of sensor frequency calibrated against temperature. The difference between average of left and right panel data are interpreted as hybridization signal.

difference between the average frequency during the first 100 seconds of phase ① and last 100 seconds of phase ④. Throughout the entire process, temperature data was also recorded

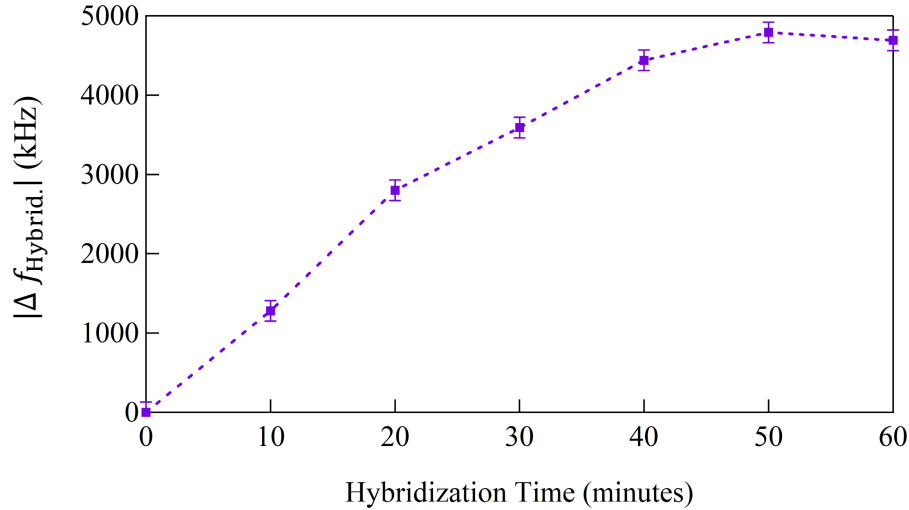


Figure 4.13: Change of sensor frequency for different time of hybridization. Sample volume is $0.2 \mu\text{L}$ and concentration is 10 nM . Based on the finding, a 60 minute hybridization time was chosen for subsequent measurements.

to calibrate the sensor readings, as shown in Fig. 4.11(a) and (c). The raw measurement data corresponding to phases ① and ④, along with temperature calibrated data can be found in Fig. 4.11(b) and (d), respectively. For twenty minutes of hybridization with complementary ssRNA, the hybridization induced frequency change is found to be $\Delta f_{\text{Hybrid}} = -2800 \text{ kHz}$.

The rate of hybridization depends on RNA length and time, with shorter duplex regions exhibiting faster hybridization kinetics [123]. To determine the optimal (shortest) hybridization time, $0.2 \mu\text{L}$ 10 nM of target ssRNA was applied to the sensor and monitored over varying time intervals. A forty minute hybridization yields a similar measurement trend as shown in Fig. 4.12, though the extended duration results in a greater frequency drop of $\Delta f_{\text{Hybrid}} = -4440 \text{ kHz}$. It can be seen from Fig. 4.13 that the resonant frequency showed statistically indistinguishable change after 50 minutes, suggesting that hybridization had reached saturation. Based on these results, a 60 minute hybridization time was selected for subsequent experiments. The 60 minute interval is also appropriate for lower target RNA concentrations, since the target samples contain fewer oligonucleotides.

Following probe immobilization, specificity tests were conducted using both complementary and non-complementary ssRNA targets. To evaluate the sensor's detection limit, vary-

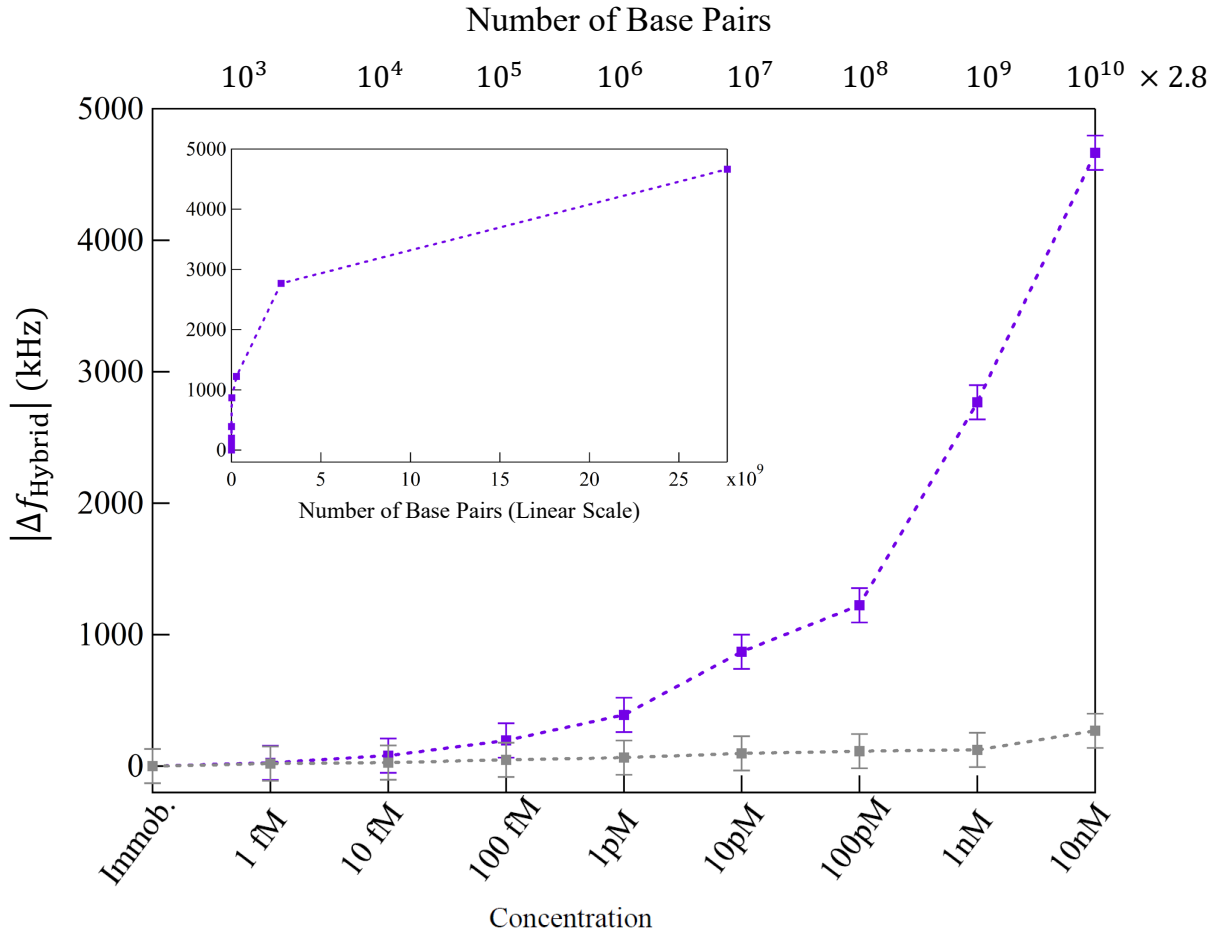


Figure 4.14: Variation of hybridization frequency against target sample concentration, for both complementary and non-complementary molecules. The findings suggest that the label-free capacitive detection limit for complementary RNA targets is at 2.8×10^6 number of base pairs or 1 pM. The inset shows data plotted with a linear horizontal axis. All data can be found in Appendix. D

ing concentrations of targets—ranging from 1 fM (2800 base pairs⁴) to 10 nM (2.8×10^{10} base pairs) were incubated on the sensor, and the corresponding changes in resonant frequency were measured. As shown in Fig. 4.14, the complementary RNA target produced a noticeable shift in resonant frequency with as few as 2.8×10^6 base pairs, indicating a strong binding response. In contrast, non-complementary RNA caused no statistically significant change in frequency, even at higher concentrations. The specificity became more pronounced

⁴A nucleic base pair refers to the bases that are connected to form the double helix structure. For RNA, Adenine (A) pairs with Uracil (U) and Cytosine (C) still pairs with Guanine (G).

as the concentration increased. Increasing the concentration of the complementary target resulted in progressively greater changes in capacitance, whereas non-complementary RNA showed no meaningful response regardless of concentration.

At low concentrations (below 1 pM), distinguishing between the responses of complementary and non-complementary targets becomes challenging. Even among complementary targets, the difference between 1 pM and 100 fM is difficult to discern. Thus, the detection limit for the sensor for nCoV2 virus is 1 pM for a 0.2 μ L sample.

Two main factors contribute to the error bars, thereby affecting system noise and detection limits. The first is the digitization of the recorded sensor frequency, which has a resolution of 80 kHz. This is constrained by the necessity to capture the full resonance spectrum within one second. The second factor is the resolution of the temperature sensor, which is 0.1 $^{\circ}$ C. Given a calibration constant of 1023 kHz/ $^{\circ}$ C, resulting in an additional fluctuation of approximately 102 kHz in the calibrated sensor frequency. Assuming the two factors are independent, the size of error bar is:

$$\text{Error Bar for } |\Delta f_{\text{Hybrid}}| = \sqrt{(80 \text{ kHz})^2 + (102 \text{ kHz})^2} = 130 \text{ kHz} \quad (4.6)$$

4.4 Summary and Future Work

This work presents the first demonstration of active sensing for nucleic acid reaction, establishing a framework for optimizing sensor electronics, environmental calibration, and data acquisition strategies. The preparation protocol—including probe immobilization and hybridization—was carefully designed to balance ease of functionalization, target specificity, and signal quality for such active capacitance-based sensors. Moreover, the amplification circuit developed is universally compatible with a variety of resonant structures or geometries, offering flexibility for integration across various platforms.

Compared to conventional resonant-type sensors [Table 4.4: [124, 114, 128]], the presented design demonstrates superior sensitivity, approaching levels achieved by more fine structures

Table 4.4: Comparison of Nucleic Acid Sensors Working in Microwave Frequency Regime

Reference (Year)	Technology	Working Frequency (GHz)	$ \Delta f_{\text{Hybrid}} $ (MHz)	Corresponding Concentration	Sensitivity (Hz/Nucleobase Pair per Liter)
[124] (2017)	Split Ring Resonator	4.910	10	1 μM	6.4×10^{-13}
[125] (2004)	Microstrip Spiral Inductor CMOS [†]	22.500	5600	5 nM	5.0×10^{-9}
[114] (2010)	Split Ring Resonator	12.350	60	100 nM	–
[126] (2010)	Gated Fiber Microwave Photonics	7.000	100	10 μM	6.4×10^{-13}
[127] (2023)	Graphene Modified CPW ^{††}	0.050-50.000	-1 dB	1 aM	–
[128] (2012)	Band Gap Resonator CPW	16.070	2670	1 μM	3.4×10^{-11}
[129] (2012)	Transmission Line Shaped Polymeric Resonator	6.400	600	1 pM	3.7×10^{-5}
This Work (2025)	Active Interdigital Capacitor	1.810	0.383	1 pM	2.8×10^{-8}

[†] Complementary Metal Oxide Semiconductor

^{††} Coplanar Waveguide

with complex materials such as CMOS technology or polymer-based modifications [125, 129], yet maintaining relative simplicity. The technology shows potential for cost efficient, real-time, electrochemical detection, laying the groundwork for further development into a point-of-care (POC) diagnostic device for both clinical and field environments.

There have been ultra high sensitive approaches reported which achieved limits of detection in single molecular level. Most of them rely on amplification methods using nano-materials [130], quantum dots [131], carbon nanotubes to control/read sensors conduction state [132]. However, these approaches incur additional complexity related to fabrication. The sensor presented in this chapter serve as an intermediate approach which balances sensitivity, ease of fabrication and operation.

Future development can continue focus on improving the sensitivity and integration. First, using interdigital capacitors with narrower finger spacing, on the nanometer scale for example, can strengthen the electric field between adjacent fingers, thereby increasing sensitivity. Additionally, selecting a transistor with lower base collector capacitance can be beneficial, as the capacitance loads in parallel with the sensing capacitance and unnecessarily increases the total capacitance, reducing the sensor's responsiveness. Moreover, translating the sensing circuitry into a monolithic CMOS or system-on-chip implementation can not

only reduce sensor size but also enable the design of single-use, disposable units. Given the small sample volume requirement (only $0.2\ \mu\text{L}$), expanding the on-chip sensors into arrays allow simultaneous analysis of multiple identical targets, improving detection accuracy.

Chapter 5

Coherent Microwave Emission: Oscillator

5.1 Introduction

Beyond their role in sensing, oscillators, in particular low phase noise oscillators stand as key components in many high-performance microwave systems. For example, in radars, reduced source phase noise enhances resolution and detection sensitivity, since the received signal can undergo minuscule Doppler shift from the source frequency [133, 134]. In communication modules, lower phase noise helps mitigate intercarrier interference, especially beneficial for systems with densely packed subcarriers [135, 136]. Low phase noise oscillators can also be found in many test and measurement equipment.

The oscillator design presented in Chapter 4 employs tuning of the transistor bias circuitry to reduce phase noise. Although the approach was effective to some extent, phase noise performance is ultimately limited by the inherent noise characteristics of the transistor itself.

Beyond the methods guided by Leeson's phase noise model (Eq. 2.17) such as selecting high-Q resonators and low-noise amplifiers, phase noise reduction is commonly accomplished using feedback control loops. Namely, phase locked loops (PLLs) and frequency locked loops (FLLs). The former, PLLs are servo systems that continuously adjust their output to minimize the phase error relative to a reference signal [137], whereas the latter, FLLs aim to minimize frequency error [138]. Additional techniques including environmental isolation,

shielding, and minimization of circuit crosstalk can further contribute to phase noise reduction [139].

This chapter presents a new approach for realizing low phase noise oscillators by coherently coupling an active microstrip oscillator with a YIG resonator. The resulting hybrid system combines the strengths of both components: the passive YIG resonator provides high frequency selectivity, while the active cavity offsets intrinsic losses and sustains oscillation. As a result, the output signal benefits from both elements, producing an exceptionally sharp emission spectrum.

The rest of the chapter is organized as follows: Section 5.2 presents the working principle of a cavity magnonic oscillator and describes phase noise reduction mechanism found therein. Section 5.3 details its implementation as a low-phase-noise signal source, including both fixed and tunable frequency prototypes. Section 5.4 explores its operation as an injection locking amplifier. Finally, key findings are summarized in Section 5.5

5.2 Working Principle of a Cavity Magnonic Oscillator

Proof of Concept Cavity Magnonic Oscillator Components

A proof of concept cavity magnonic oscillator is shown in Fig. 5.1. It consists of (1) a gain-embedded, half-wavelength microstrip oscillator, with symmetrically positioned readout ports on both sides, and (2) A 1 mm diameter YIG sphere placed at the right side of the coupling gap, held in position by a T-shaped structure, 0.5 mm above the cavity board.

The entire device is placed inside an electromagnet, with the magnetic field $\vec{H}$ applied along the $\hat{x}$ direction, as shown by the photo in Fig. 5.1. The microwave within the coupling gap generates a strong, uniform electromagnetic field that is perpendicular to $\hat{x}$, establishing a coherent interaction between the cavity and magnon modes. The T-structure is constrained to slide only along the x-direction, minimizing variations in device performance due to YIG sphere misalignment. Bill of material for the device can be found in Table. 5.1.

When the electromagnet is turned off ($\vec{H} = 0$), no magnon resonance is present in the

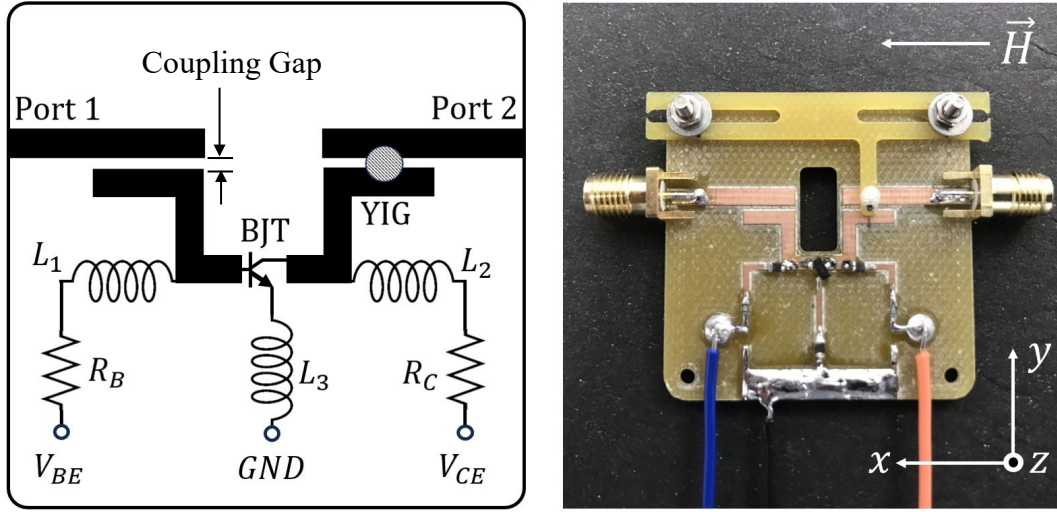


Figure 5.1: Schematic diagram and photo of the proof of concept cavity magnonic oscillator, consisting of an active cavity and a YIG sphere. List of circuit parameters are provided in Table. 5.1. The magnetic field $\vec{H}$ applied by the electromagnet is directed along the $\hat{x}$ direction.

Table 5.1: List of Circuit Parameters for the Proof of Concept Cavity Magnonic Oscillator

Symbol	Description	Value
$L_{1,2,3}$	RF Chokes	$1\ \mu\text{H}$
R_B	Base Resistor	$28\ \Omega$
R_C	Collector Resistor	$108\ \Omega$
V_{BE}	Base Terminal Voltage	$0.8\ \text{V}$
V_{CE}	Collector Terminal Voltage	$7.0\ \text{V}$
BJT	BFU550AR	N.A.

system, and the output spectrum reflects that of an uncoupled cavity. As shown in Fig. 5.2(a) and (b), the uncoupled cavity oscillates at $\omega_c/2\pi = 3.619\ \text{GHz}$, with phase noise levels of $L_{10\text{kHz}} = -49\ \text{dBc/Hz}$ at 10 kHz offset and $L_{100\text{kHz}} = -79\ \text{dBc/Hz}$ at 100 kHz offset.

Hybridized Frequency

Increasing the magnetic field $\vec{H}$ brings the magnon mode frequency close to cavity frequency so that coherent interactions occurs. The focus of this work is on the Kittel mode. For a YIG sample with axial magnetization and spherical symmetry around $\hat{x}$, the magnon

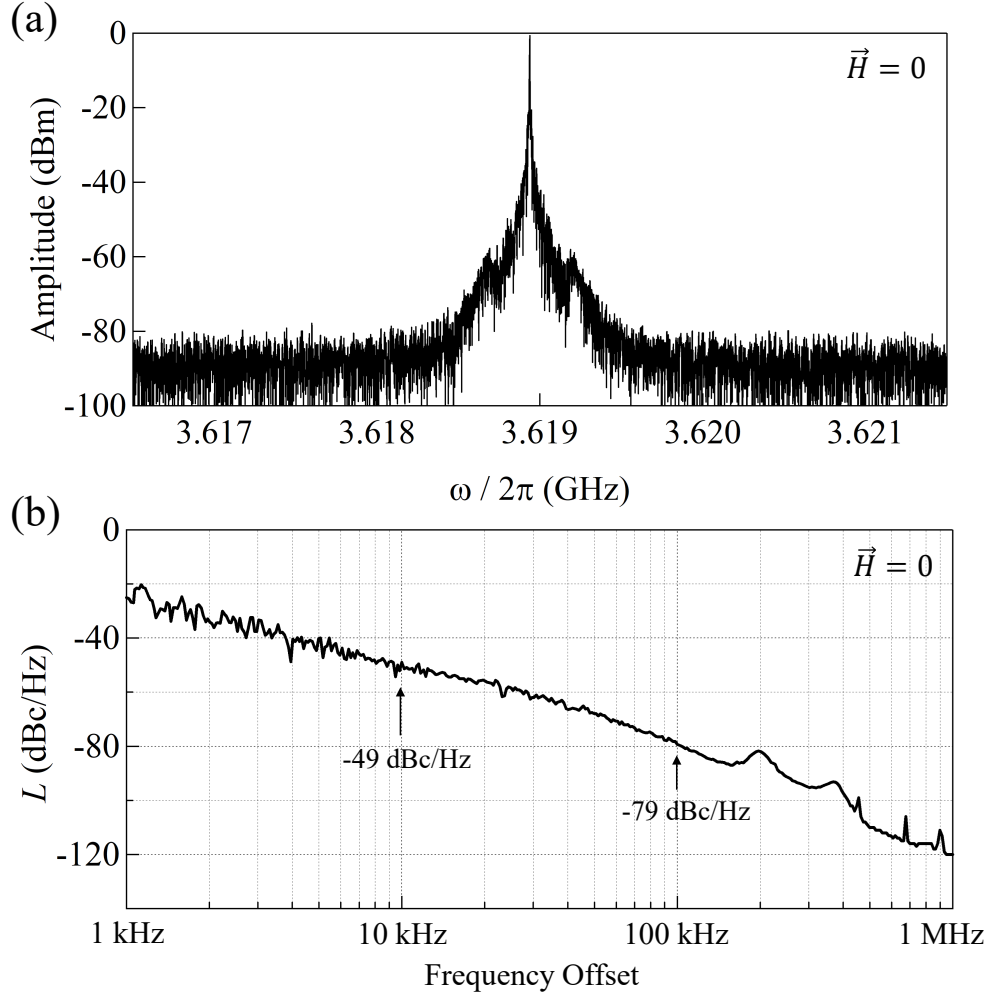


Figure 5.2: (a) Resonant profile of uncoupled cavity, $\omega_c/2\pi = 3.619$ GHz. $\omega/2\pi$ is the sweep frequency of the signal analyzer. (b) Phase noise profile of the uncoupled cavity. $L_{10\text{ kHz}} = -49$ dBc/Hz, and $L_{100\text{ kHz}} = -79$ dBc/Hz.

mode frequency is given by $\omega_m = \gamma_e(H + H_A)$, where $\gamma_e = 28 \times 2\pi\mu_0$ GHz/T denotes the gyro-magnetic ratio, μ_0 is the vacuum permeability, H is the field supplied by the electromagnet, and H_A is the magnetocrystalline anisotropy field.

In Fig. 5.3, the hybridized frequency ($\tilde{\omega}/2\pi$) is plotted against field detuning, with left and right panels representing field upswep (gradually increase $\vec{H}$) and downswep (gradually decrease $\vec{H}$), respectively. Contrary to characteristic level repulsion, only one hybridized frequency is observed, undergoing a sudden change at $(\omega_m - \omega_c)/2\pi = 64$ MHz for upswep and 42 MHz for downswep. The sudden change in frequency arises from the nonlinear

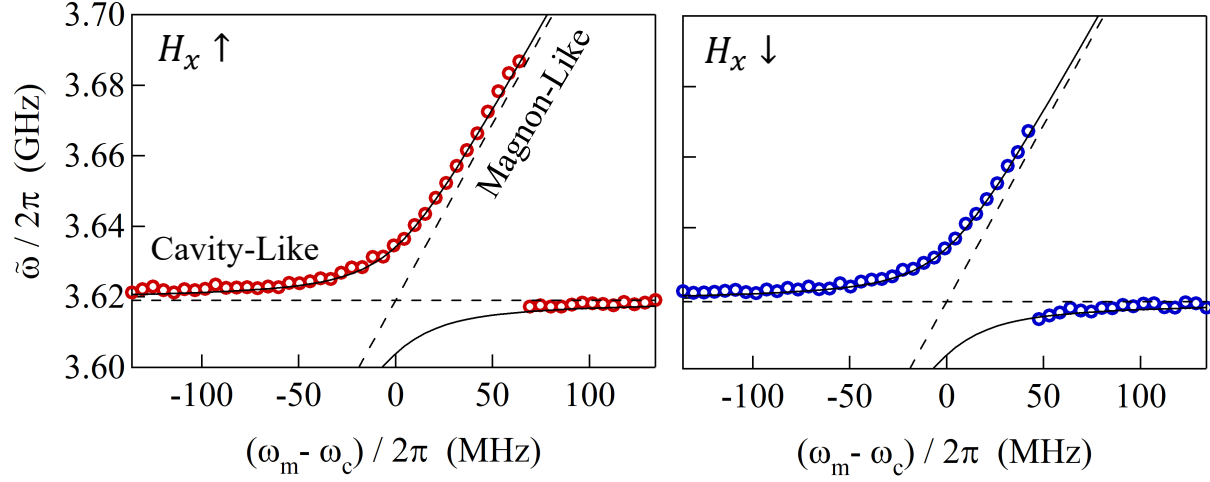


Figure 5.3: The variation of hybridized frequency $\tilde{\omega}/2\pi$ against field detuning. Left panel: increasing magnetic field (upsweep). Right panel: decreasing magnetic field (downsweep). Black dashed lines represent uncoupled cavity and magnon mode frequencies, and solid line represent coherently coupled mode frequencies, coupling strength is $J/2\pi = 15$ MHz.

phenomenon involving coupled Van der Pol-Duffing oscillator, which exhibits bistability: the ability to oscillate at any of two distinct, stable eigenfrequencies depending on the detuning path and the system's initial conditions [80]. The bistable behavior manifests as different frequency trajectories and sudden “jump” points when the detuning is slowly varied in either direction. For example, during the upsweep experiment, $\tilde{\omega}$ initially follows a stable path toward the magnon-like frequency. As the detuning increases, this path eventually becomes unstable at $(\omega_m - \omega_c)/2\pi = 64$ MHz, prompting the system to abruptly jump to another stable branch: the cavity-like frequency.

Phase Noise Reduction Mechanism

Figure 5.4(a) and (b) show the spot phase noise levels at 10 kHz and 100 kHz offsets, respectively, plotted against field detuning. At 10 kHz offset, the phase noise is reduced by 32 dB during upsweep measurement and 20 dB during downsweep measurements. At 100 kHz offset, the reductions are 30 dB and 19 dB, respectively.

Fig. 5.4(c) illustrates phase noise variation with field detuning for upsweep measurement. A noticeable decreasing trend is observed as detuning increases, reaching the minimum device

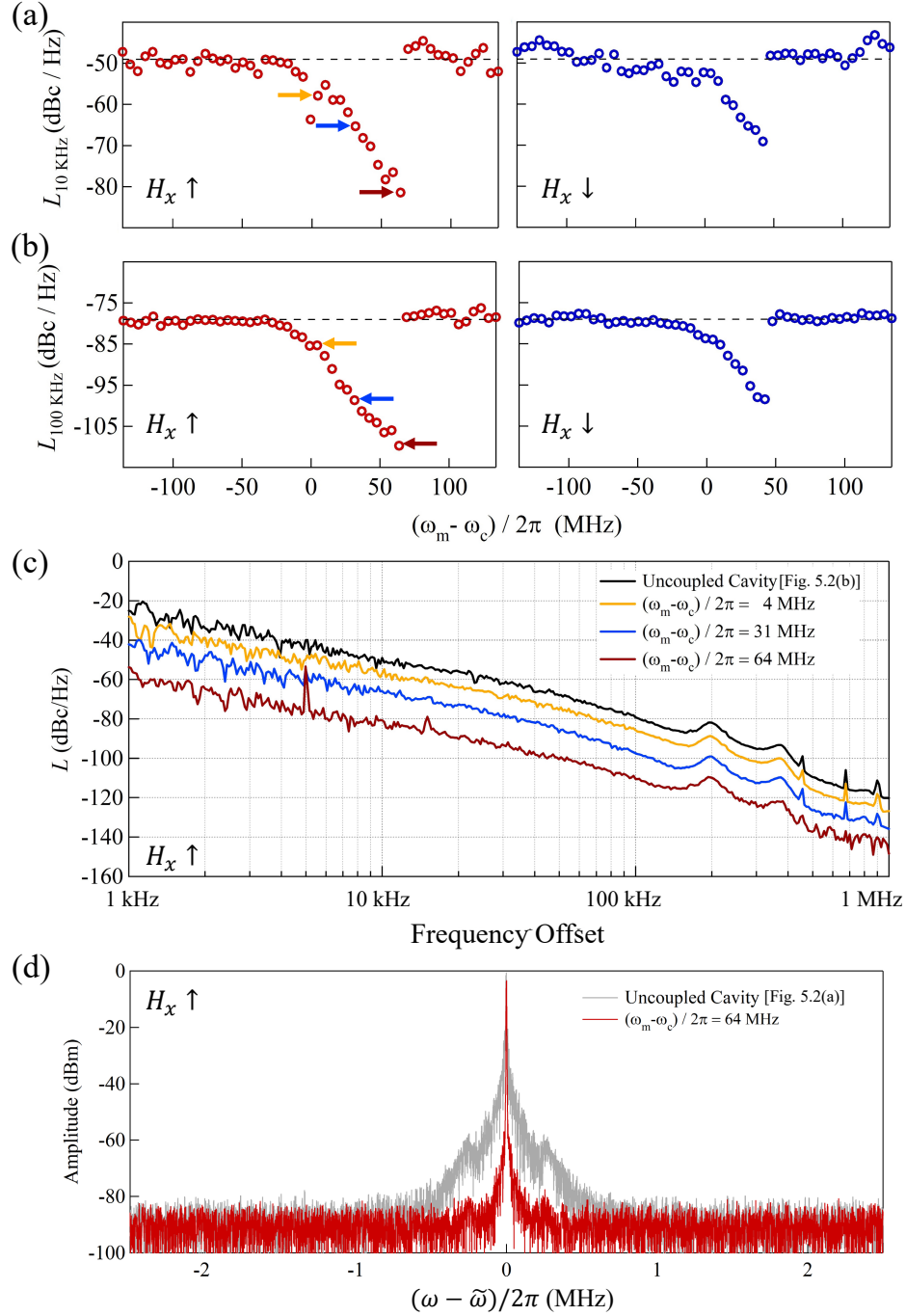


Figure 5.4: The cavity magnonic oscillator shows variations in phase noise depending on the field detuning, with the lowest phase noise occurring right before the sudden frequency jump. (a) and (b) illustrate how $L_{10 \text{ kHz}}$ and $L_{100 \text{ kHz}}$ change with detuning. The colored arrows correspond to the phase noise plots at different detuning conditions shown in (c). (d) presents the oscillator spectrum when biased at $(\omega_m - \omega_c) = 64 \text{ MHz}$ for upswEEP measurement, revealing a sharper spectral line compared to the uncoupled cavity.

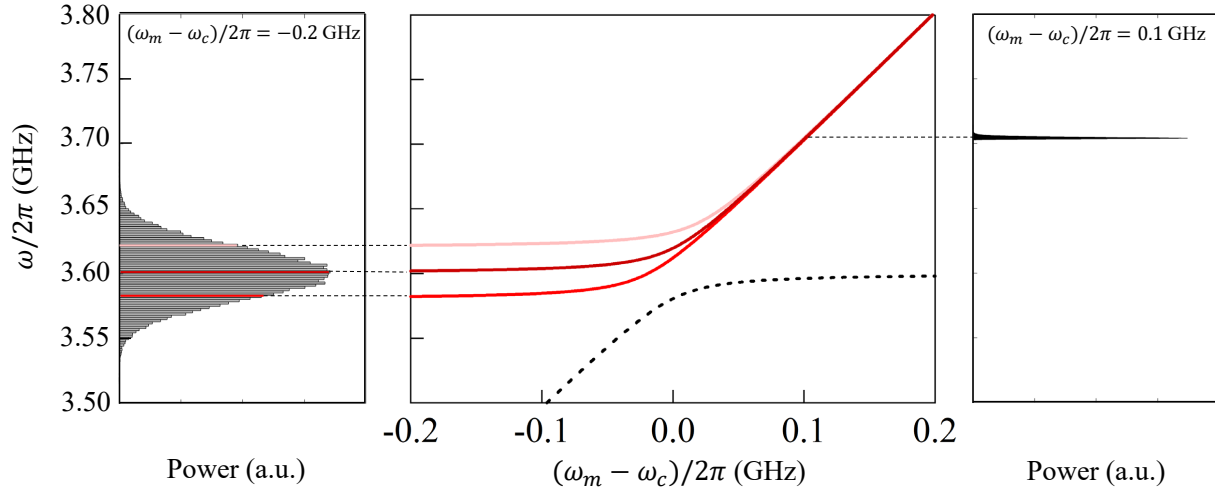


Figure 5.5: Numerical calculation demonstrating the phase noise reduction scheme. The phase noise around the carrier frequency $\omega_c/2\pi = 3.6$ GHz is modeled using a Gaussian distribution. Light Red: $\omega_c/2\pi = 3.62$ GHz (noise), Dark Red: $\omega_c/2\pi = 3.60$ GHz (carrier), Red: $\omega_c/2\pi = 3.58$ GHz (noise). As the detuning increases, all of the uncoupled frequencies evolve toward the single magnon-like frequency, resulting in a narrowing of oscillation linewidth.

phase noise level at $(\omega_m - \omega_c)/2\pi = 64$ MHz for up-sweep measurement. In Fig 5.4(d), the spectrum at $(\omega_m - \omega_c)/2\pi = 64$ MHz is presented, together with the uncoupled cavity [shown in Fig. 5.2(a)]. The decrease in phase noise manifests as sharpened resonance.

From a phenomenological perspective, the hybridization effect drives the noisy, cavity-like mode toward the noiseless, magnon-like mode. For instance, at detuning of $(\omega_m - \omega_c)/2\pi = -100$ MHz [Fig. 5.4(a) and (b)], the hybridized mode remains largely cavity-like and exhibits phase noise levels nearly identical to those of the uncoupled cavity. As the field detuning increases, the mode gradually acquires more magnon-like characteristics and begins to display a linear dependence on the applied magnetic field strength $\vec{H}$.

Above rationale is illustrated in Fig. 5.5, where the cavity spectrum is modeled as a Gaussian distribution centered around the carrier frequency [left panel]. As detuning increases, the frequency distribution progressively concentrates around the magnon-like frequency [middle panel], leading to a reduction in phase noise [right panel]. In essence, not only the carrier, but also the noise power is hybridized onto the magnon-like frequency.

5.3 Prototype: Low Phase Noise Microwave Oscillators

Based on the phase noise reduction effects observed in Figs. 5.4, it can be inferred that the phase noise of the uncoupled cavity establishes the baseline for the oscillator's overall phase noise performance. Additionally, it was previously found that in a similar device, the detuning value at which the frequency “jump” occurs is inversely related to the height of the YIG sphere with respect to the coupling gap (see supplementary material of [53]). Reducing the YIG sphere height enhances the mode overlapping factor, strengthens the coherent coupling, delays the frequency jump, and consequently can make the hybridized mode more magnon-like, leading to greater phase noise suppression. Therefore, to develop a low phase noise oscillator, the following two improvements can be made to the proof of concept device.

- Reduce the phase noise of the uncoupled cavity to lower the baseline noise level.
- Increase the coherent coupling strength, enabling that the hybridized mode incorporates a greater magnon-like component, which in turn improves noise reduction.

Fixed Frequency Microwave Source

For the design of a fixed-frequency oscillator (Prototype I), R_B is raised to 32Ω and V_{BE} is lowered to $0.77V$, ensuring that the noise power from the DC supply entering the base terminal of transistor is minimized¹. Simultaneously, R_c was reduced to 96Ω and V_{CE} was raised to $8.0V$ to increase the gain of the transistor circuit. Rest of the circuits are identical to that shown in Fig. 5.1 and Table. 5.1. Moreover, bypass capacitors were added to both the V_{BE} and V_{CE} terminals, effectively filtering out power supply noise, minimizing frequency pushing and output power fluctuation.

Increasing the collector current of the transistor offers an additional benefit: it increases the cavity power and strengthens the electromagnetic field within the coupling gap, thereby

¹ $\Delta I_B = \Delta V_{BE}/R_B$

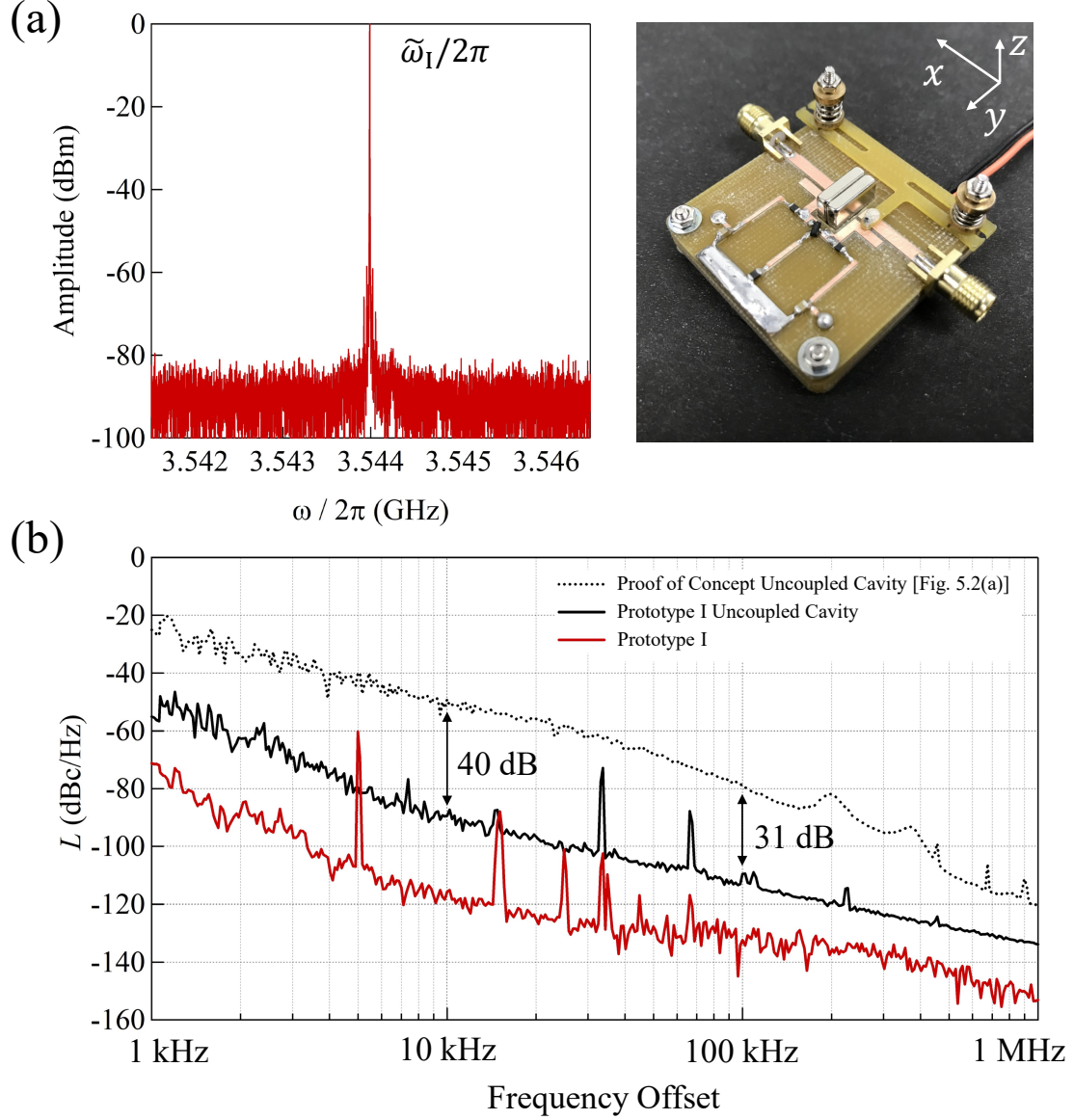


Figure 5.6: The fixed frequency cavity magnonic oscillator. (a) The left panel shows the resonant profile of Prototype I. Its working frequency is $\tilde{\omega}_I/2\pi = 3.544$ GHz. Right panel shows the fabricated assembly. (b) Uncoupled cavity phase noise of proof of concept device and Prototype I (black solid line). Phase noise of Prototype I at working frequency (red solid line). The spikes are spurious phase noises caused by power supply noise entering the non ideal resonator-transistor feedback path due to parasitic effects [140].

increasing the coherent coupling strength due to improved mode overlapping factor.

The size of the coupling gaps remain unchanged due to a trade-off: narrowing the gap benefits power delivery to the ports, but also confines the electromagnetic field more tightly

within the gap. Since the YIG sphere is positioned above the gap, reduced field distribution leads to a lower mode overlap, weakening the coherent interaction.

Finally, neodymium magnets were used to replace the electromagnet. Since the YIG sphere can slide horizontally along the $\hat{x}$ direction—where the electromagnetic field is uniform—adjusting its position has a negligible effect on the cavity frequency but causes a significant change in $\vec{H}$, allowing the field detuning condition to be manually controlled.

Figure 5.6(a) provides the output profile and picture of fixed frequency oscillator, biased at $\tilde{\omega}_I/2\pi = 3.544$ GHz. In Fig. 5.6(b), the uncoupled cavity phase noise of prototype I (black solid line) is compared to that of proof of concept device (black dashed line). Prominent reductions in phase noise can be observed, reading -40 and -31 dB at offset frequencies of 10 and 100 kHz, respectively. Following a careful adjustment to field detuning, the fixed frequency cavity magnonic oscillator demonstrated phase noise levels of $L_{10\text{kHz}} = -117$ dBc/kHz and $L_{100\text{kHz}} = -132$ dBc/kHz, respectively (red solid line).

The spikes shown in Fig. 5.6(b) represent spurious oscillations in the system. These can occur if the power supply is not ideal; for example, any oscillations on the supply can be injected into the resonator-transistor feedback path or other unwanted feedback paths arising from non-ideal circuit components [140]. Such noise sources may not be affected by the mode hybridization (i.e., unlike the active cavity, they are not strongly coupled to the magnon mode) and could persist despite efforts to reduce phase noise.

Tunable Frequency Microwave Source

Table 5.2: List of Circuit Parameters for Tunable Cavity Magnonic Oscillator

Symbol	Description	Value
$L_{4,5,6,7}$	RF Chokes	$1\ \mu\text{H}$
$C_{1,2,3,4}$	DC Blocking Capacitor	$9\ \text{pF}$
$C_{v1,v2}$	Varactor (SMV2019)	$0.3 - 2.2\ \text{pF}$
V_{Tune}	Varactor Bias Voltage	$0 - 10\ \text{V}$

In this part, the feasibility of converting the fixed-frequency cavity magnonic oscillator

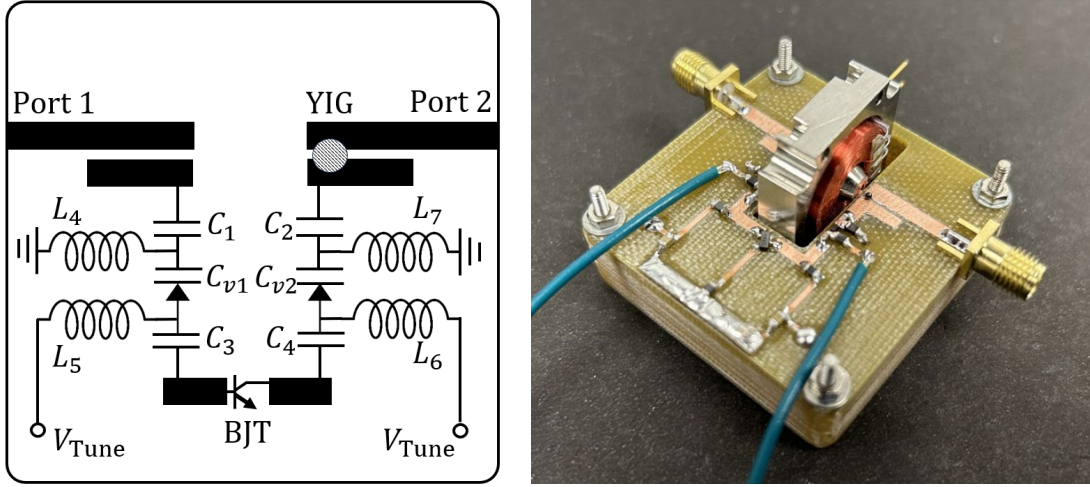


Figure 5.7: Schematic and photo of cavity magnonic oscillator with tunable working frequency (Prototype II).

into a tunable-frequency device (Prototype II) will be demonstrated. To achieve frequency tunability, the uncoupled cavity frequency is first adjusted, followed by tuning the bias magnetic field (H) to establish the optimal detuning condition for maximum phase noise reduction. Thus, implementation requires a frequency-adjustable microstrip oscillator, a YIG sphere, and an electromagnet.

As depicted in Fig. 5.7, vertical sections of microstrip is replaced by varactor ($C_{v1,v2}$) circuits to control the cavity frequency. List of circuit parameters are provided in Table. 5.2 The function of capacitors C_{1-4} is to prevent varactor bias voltages from reaching the transistor junctions. Values of C_{1-4} should be small enough to maintain Q-factor of resonator circuit, yet large enough to not deteriorate the tuning capability of varactor when used in series. Finally, an electromagnet is inserted to control the bias magnetic field.

Figure 5.8(a) shows the resonance profiles of prorotype II at various operating frequencies, ranging from $\tilde{\omega}_{II}/2\pi = 2.1$ to 2.7 GHz. The cavity was initially tuned for target operating frequency, followed by gradually increasing the electromagnet current from zero until appropriate detuning condition was met. This way, reduction in phase noise can be achieved across the entire tuning range. The minimum working frequency is primarily constrained by the

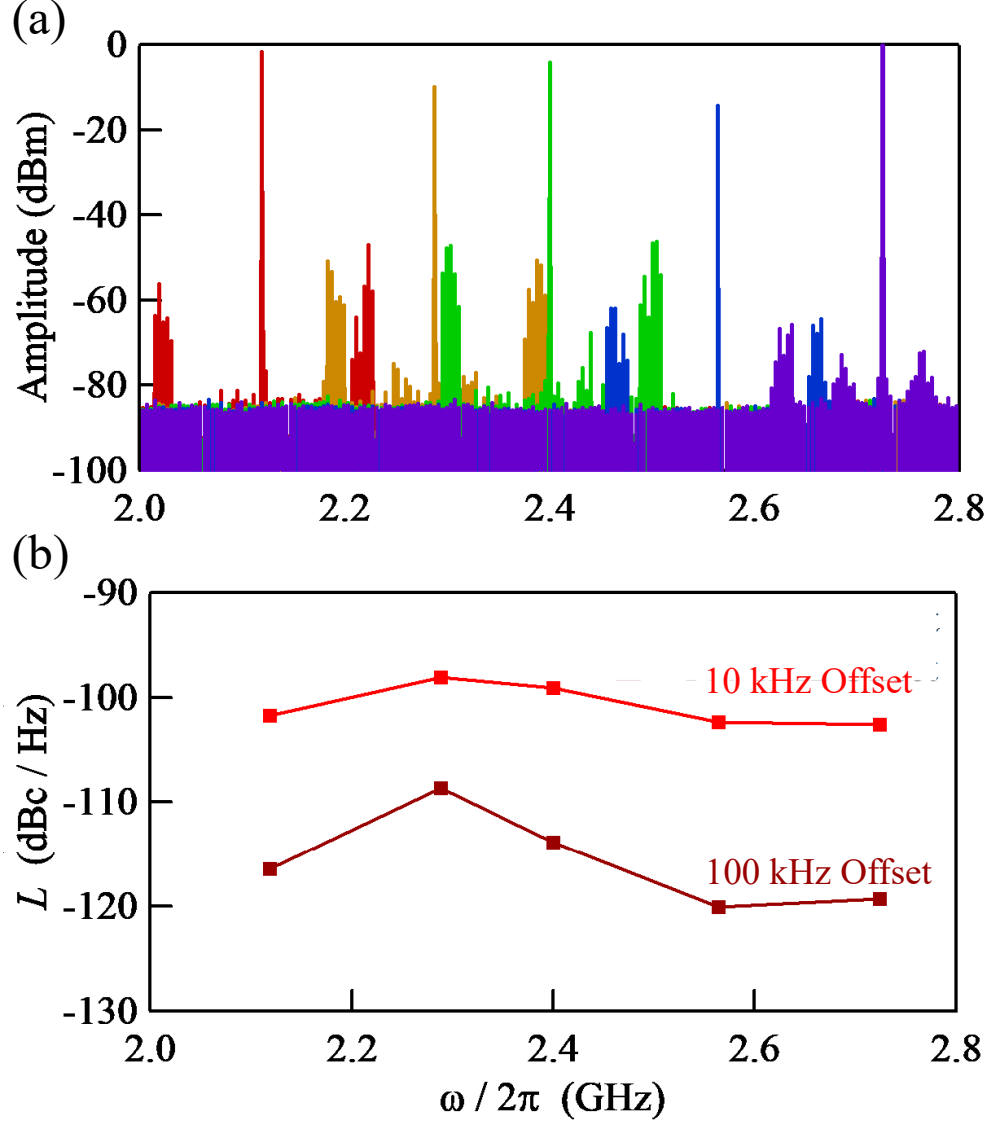


Figure 5.8: (a) Resonant profiles and (b) phase noise levels of cavity magnonic oscillator with tunable working frequency.

varactor's range, while the maximum is limited by the current allowed in the electromagnet. According to Fig. 5.8(b), prototype II was able to maintain good phase noise level across a 600 MHz span.

In summary, low phase noise oscillators based on cavity magnonics are introduced, leveraging coherent coupling between a gain driven cavity mode and a magnon mode. The energy exchange induces mode hybridization and suppresses frequency distribution around

Table 5.3: Comparison with YIG Oscillator Technologies

Year	Technology	Tuning Range (GHz)	$L_{10 \text{ kHz}}$ (dBc/Hz)	$L_{100 \text{ kHz}}$ (dBc/Hz)
1996 [141]	NPN Transistor and Phase Locked Loop	4.5 - 12	-110	N.A.
2017 [142]	SiGe hetero bipolar transistor (HBT)	6 - 12	-110	-133
2019 [143]	SiGe HBT, Differential Amplifier	19 - 41	-100	-127
2019 [143]	SiGe HBT, Differential Amplifier	32 - 48	-97	-99
Prototype II	Cavity Magnonics	2.1 - 2.7	-98	-109
Prototype I	Cavity Magnonics	N.A.	-117	-132
Commercial [144]	N.A.	2 - 4	-100	-125

the carrier. The fixed frequency oscillator achieved the best reported phase noise levels of $L_{10 \text{ kHz}} = -117 \text{ dBc/kHz}$ and $L_{100 \text{ kHz}} = -132 \text{ dBc/kHz}$. An comparison with existing YIG Tuned Oscillators (TYO) is given in Table. 5.3. It can be seen that prototype I exhibits superior phase noise level at 10 kHz offset. Existing YTOs (YIG-Tuned Oscillators) employ transistor-based amplification circuits, effectively functioning as single negative resistance oscillators. As a result, the overall device performance is largely dictated by the noise characteristics of the transistors. The phase noise reduction method presented here offers a complementary approach to those outlined in Table. 5.3.

Although phase noise can be further decreased through refined cavity design and increased coupling strength, achieving complete output stability may not be possible. Because (1) mode hybridization inevitably introduces frequency instability from the active cavity, and (2) the magnon mode frequency also exhibits instability, likely due to ambient temperature fluctuations or coil current noise in the electromagnet. The frequency stability of YIG resonance determines the ultimate device performance. As a final point, the FOM² of prototype I is -205 dBc/Hz, which is the second lowest among those been compared in [67].

5.4 Prototype: Two Stage Injection Locking Amplifier

Building upon the discussion of phase noise reduction techniques in previous section, at-

²Eq. (2.18): Figure of Merit (FOM) = $L_{\Delta f} - 20 \log_{10}(f_0/\Delta f) + 10 \log_{10}(P_{\text{DC}}/1 \text{ mW})$ where $L_{\Delta f}$ is the phase noise measured at frequency offset Δf , f_0 is the operating frequency, and P_{DC} is the power drawn by the oscillator.

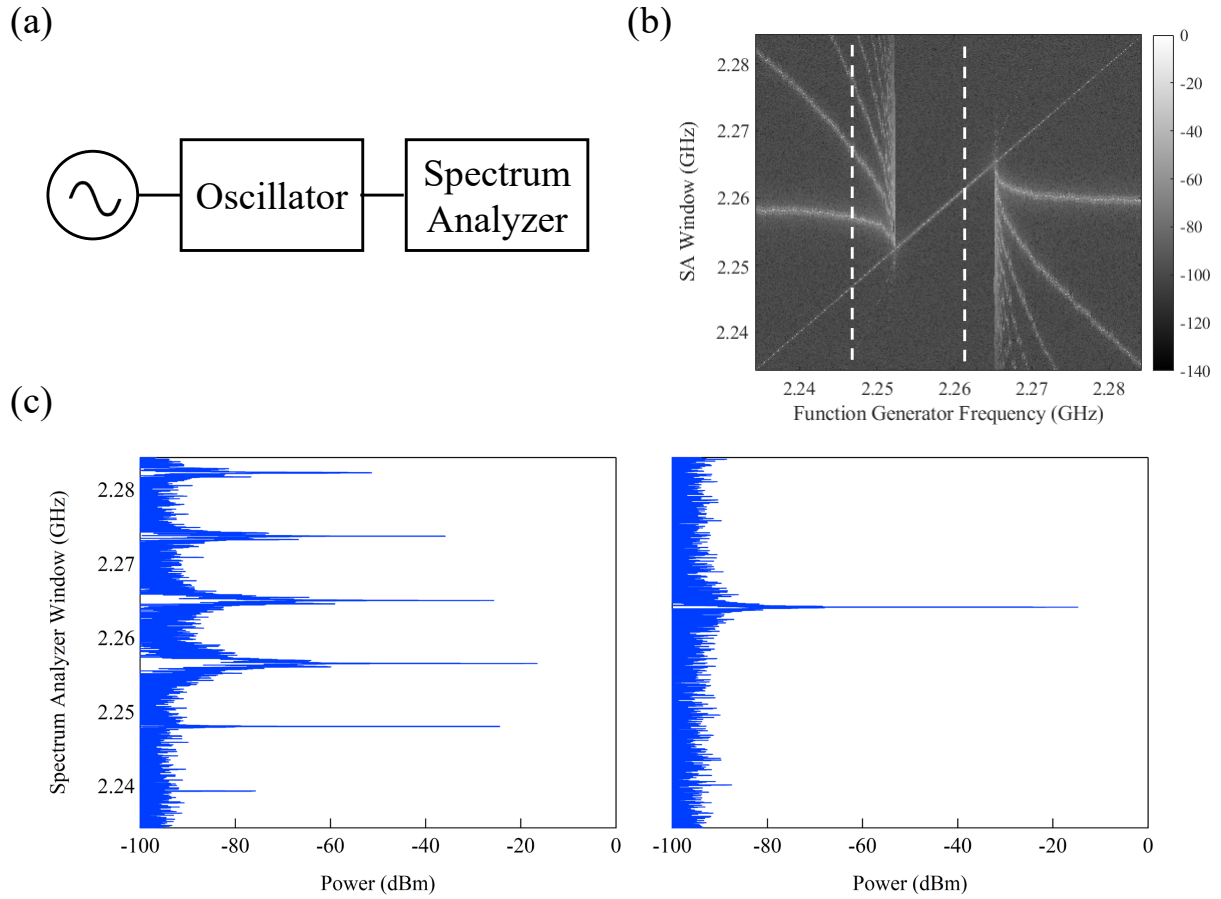


Figure 5.9: Illustration of the Injection Locking Experiment: (a) A signal generated by the function generator is first sent to the oscillator. The resulting output is then captured using a spectrum analyzer. (b) Spectrogram of the injection locking experiment, highlighting a locking region characterized by a single resonance, flanked by two pulling regions. (c) A cross-sectional slice of the spectrogram, where the left and right panels correspond to the white dashed lines indicated in (b).

tention for the rest part of this chapter turns to another important aspect of signal control: amplifying low power signals with minimal added noise while maintaining coherence. Of particular interest is the potential to use a cavity magnonic oscillator as an amplifier through the mechanism of injection locking.

Referring to Fig. 5.9, when an external signal is injected into a free-running oscillator with a nearby frequency, two phenomena can occur: injection locking and injection pulling. During injection locking, the oscillator adjusts its output frequency and phase to synchronize with the injected signal [145]. The range of which the two signals stay synchronized is termed

lock range. On the other hand, injection pulling causes the oscillator's frequency to shift toward the frequency of the injected signal, which is generally undesirable as frequency deviations and harmonics will be generated [146].

The injection locking phenomenon can be leveraged to create injection locking amplifiers (ILAs). An ILA is essentially a type of oscillator in which its output signal is synchronized to the input signal. Since the output power of an ILA is derived from self-oscillation, the input amplitude can be much lower than what is typically required by conventional amplifiers to achieve the same output level. Additionally, ILAs offer the advantage of generating minimal spurious emissions and harmonic distortions, making them particularly well-suited for amplification purposes [147, 148, 149].

However, there are trade-offs to consider. The lock range, $\Delta\omega$ of an ILA is inversely related to both the power gain and the oscillator's Q-factor [145, 146]:

$$\frac{\Delta\omega}{\omega_{\text{OSC}}} = \frac{1}{2Q} \times \sqrt{\frac{P_{\text{in}}}{P_{\text{OSC}}}} \quad (5.1)$$

where ω_{OSC} denotes the oscillator's free running frequency, making $\Delta\omega/\omega_{\text{OSC}}$ the normalized lock range. P_{in} and P_{OSC} denote the input and output power, respectively, with the latter being the power of free running oscillator.

Equation (5.1) indicates that for an ILA, aiming for high gain with a low input power results in a narrow lock range, limiting its practical use. While decreasing the oscillator's Q-factor can broaden the lock range, it comes at the cost of reduced free-running stability, potentially leading to unpredictable operating frequencies.

That being said, when a cavity magnonic oscillator is used as an ILA, its low phase noise becomes a double-edged sword. On one hand, low phase noise is associated with a high quality factor (Q), leading to better frequency stability and a more reliable operating frequency and lock range. On the other hand, because the lock range is inversely proportional to the oscillator's Q-factor, cavity magnonic oscillators tend to exhibit narrow lock ranges. In fact, their function as ILA has been demonstrated, albeit with a very limited lock range

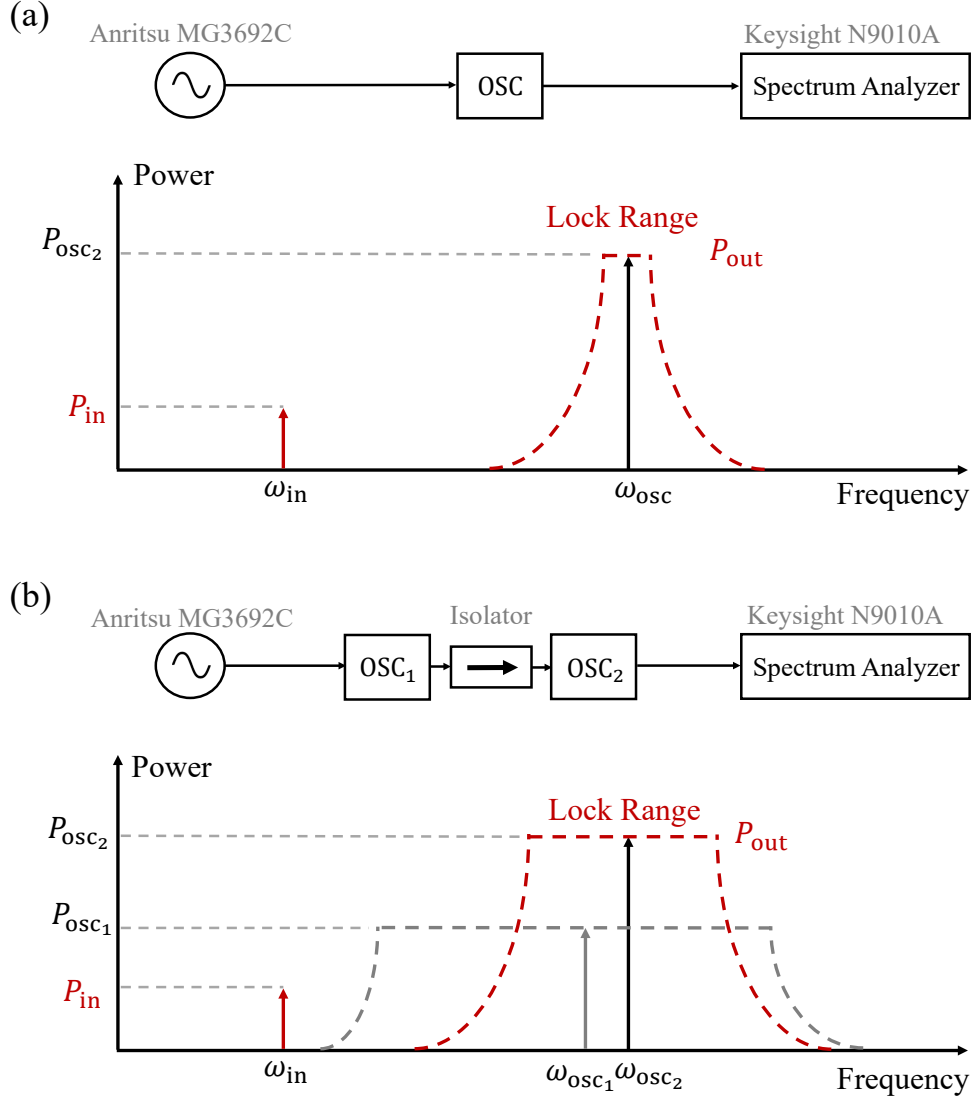


Figure 5.10: Schematic representation of injection locking amplification for (a) a single stage (b) dual stage setup. The red dashed line represents the output power profile of each, over a range of ω_{in} .

(< 1 MHz for 30 dB gain) [53, 55] .

Incorporating a preamplification stage can elevate input signal level, effectively reducing the power gain, and thus, improve the lock rang [150]. However, transistor components inherently increase the noise floor and introduce distortion, which presents challenges for low-power and sensitive input signal amplification [151]. In this regard, rest of the section aims to (1) expand the lock range of the cavity magnonic oscillator by adding another ILA

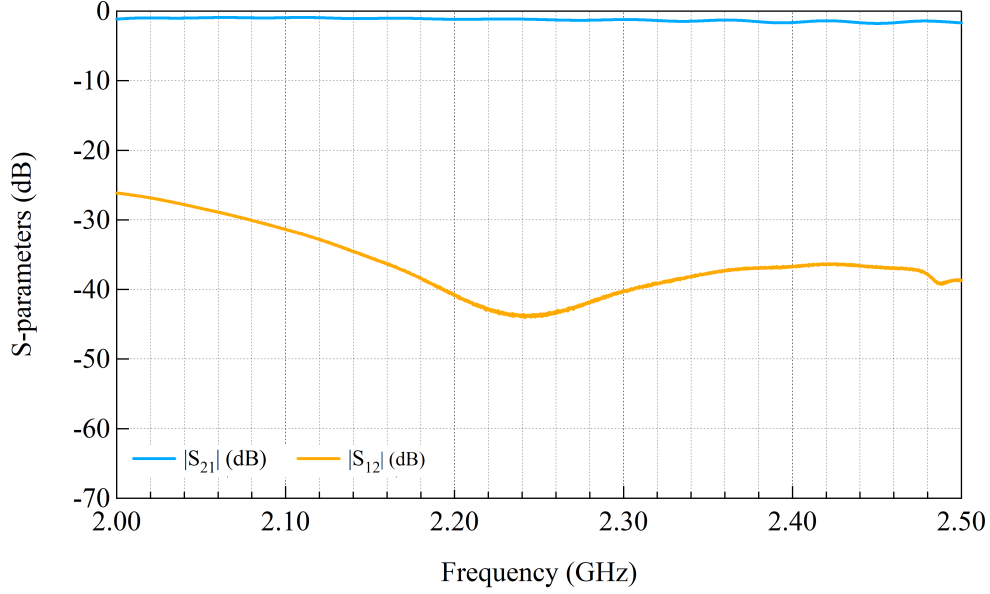


Figure 5.11: S-parameters of the isolator.

in front as a pre-amplifier, (2) optimize both stages for lock ranges.

A Two Stage Injection Locking Amplifier

Figure. 5.10(b) shows the proposed two-stage system consisting of a pre-amplifier (OSC_1), an isolator, and a cavity magnonic oscillator (OSC_2). The input signal is first amplified by OSC_1 , then passes through the isolator before being further amplified by OSC_2 . The output from OSC_1 raises the input power level seen by OSC_2 , effectively lowering the gain at OSC_2 and increasing the lock range. A comparison with single-stage amplification is provided in Fig. 5.10(a).

For a two-stage configuration, the isolator shall be the first component being selected. Its function is to maximize isolation between the two oscillators, consequently preventing backward locking. The isolator provides a maximum isolation of 44 dB at 2.24 GHz, with an insertion loss of -1 dB, and maintains over 40 dB of isolation from 2.20 to 2.29 GHz, as shown in Fig. 5.11. Therefore, the frequencies of the two oscillators should be set within the range of 2.20 GHz to 2.29 GHz.

A modified Eq. (5.1) will be used to evaluate various optimization options for the two oscillator stages:

$$\frac{\Delta\omega}{\omega_{\text{OSC}}} = \kappa(L, C, C_g) \times \sqrt{\frac{P_{\text{in}}}{P_{\text{OSC}}}} \quad (5.2)$$

For a transistor powered oscillator shown in Fig. 5.12, its Q-factor is determined by the equivalent circuit characteristics (L, C) , and its coupling to the external circuit (C_g) . Under a fixed gain, $\kappa(L, C, C_g)$ is the focal parameter for lock range optimization. Specific methodologies for increasing κ in both OSC_1 and OSC_2 , will be addressed separately. The two oscillators are then cascaded, and the overall performance as an assembly will be examined.

Pre-amplification (OSC_1)

As shown in Fig. 5.12, the pre-amplifier is a transistor-integrated RLC oscillator. Since the lock range is inversely proportional to the tank's Q-Factor, the oscillator can be implemented without an explicit capacitor component, relying instead on the parasitic capacitance of the inductor (L) to oscillate [152]. Thus, for OSC_1 , κ_1 can be controlled by two components: the oscillator inductance, L and the gap capacitance, C_g . List of circuit parameters can be found in Table. 5.4.

First, the lock ranges were measured for different values of inductance, while keeping the gap capacitance fixed at $C_g = 0.8 \text{ pF}$. Fig. 5.13(a) portrays the linear relationship described in Eq. (5.2). The slope term κ_1 increases with L , leading to a larger lock range for a fixed gain.

Table 5.4: List of Circuit Parameters for Pre-amplifier (OSC_1)

Symbol	Description	Value
$L_{1,2}$	RF Chokes	$1 \mu\text{H}$
C_g	Coupling Capacitor	$0.5 - 100 \text{ pF}$
L	Oscillator Inductance	$1.0, 1.8, 3.3 \text{ nH}$
C	Oscillator Capacitance (Parasitic)	N.A.
R_1	DC Bias Resistance	100Ω
V_{cc}	Supply Voltage	0.9 V

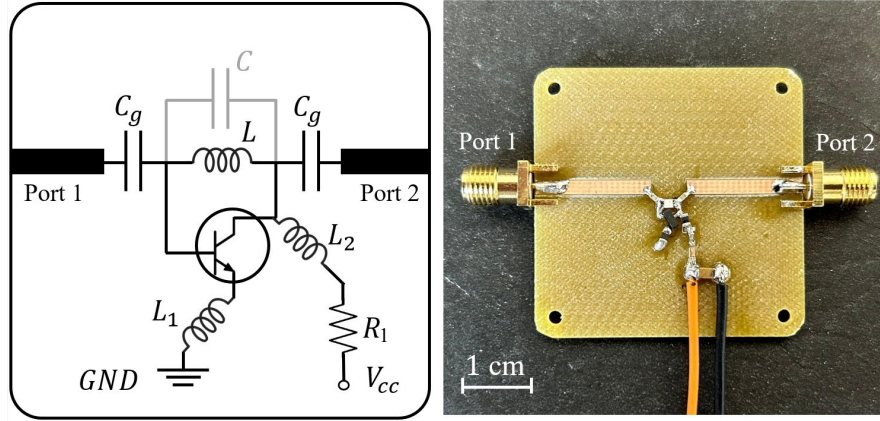


Figure 5.12: Schematic diagram of the pre-amplifier (OSC_1) on the left and corresponding photograph on the right.

In Fig. 5.13(b), oscillator inductance was held constant at $L = 1.8 \text{ nH}$, and corresponding κ_1 values were extracted from the lock ranges obtained with different C_g , ranging from 0.5 to 100 pF. Notably, κ_1 plateaus for $C_g > 1 \text{ pF}$. The plateau occurs because signal entering Port 1 passes through C_g before reaching the resonator. When C_g is large, it presents a lower impedance at a given frequency, such that P_{in} seen by OSC_1 is greater. This, in turn, results in a lower gain and wider lock range.

Overall, to improve the lock range for OSC_1 , maximizing the oscillator inductance is desirable, and the coupling gap capacitors should be sufficiently large to avoid attenuation of the input signal. These two principles are carried forward for the cavity magnonic ILA stage (OSC_2).

Cavity Magnonic Amplifier (OSC_2)

Refer to Fig. 5.14, the cavity magnonic oscillator consists of a transistor integrated microstrip oscillator, which is coherently coupled to a 1mm diameter YIG sphere via the overlap of their electromagnetic modes. The frequency of the cavity mode is set by its geometric properties, whereas the frequency of magnon mode is linearly related to the strength of bias magnetic field and tuned by adjusting the distance between YIG and the magnet. When

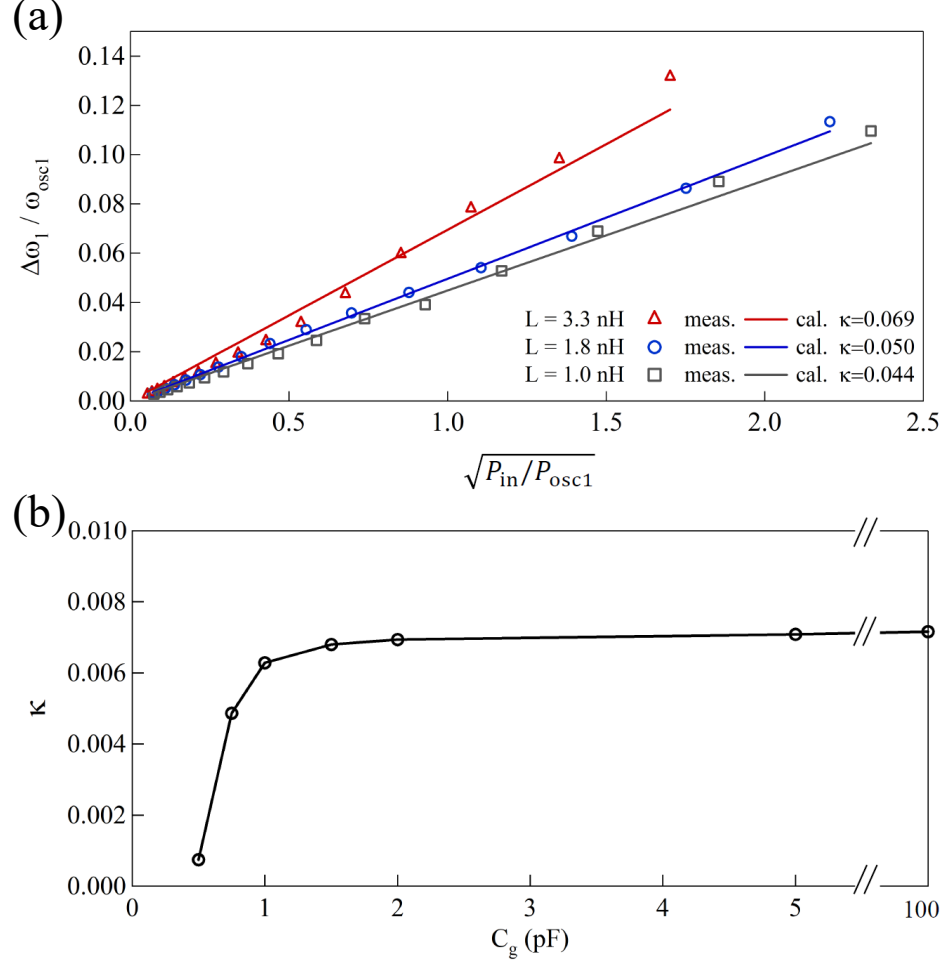


Figure 5.13: (a) Normalized lock range for OSC₁ plotted against gain, defined as P_{in}/P_{OSC1} . The slope term κ raises as oscillator inductance L is increased. (b) κ as a function of C_g for fixed $L = 1.8$ nH.

powered on, the cavity magnonic oscillator generates a single hybridized frequency $\omega_{OSC2}/2\pi$.

Table 5.5: List of Circuit Parameters for Cavity Magnonic Amplifier (OSC₂)

Symbol	Description	Value
$L_{3,4,5}$	RF Chokes	$1\ \mu H$
R_2	Base Resistor	$19\ k\Omega$
R_3	Collector Resistor	$100\ \Omega$
V_{cc2}	Supply Voltage	$7.0\ V$

The coupling gap geometry is defined by the gap length (l) and gap distance (g), with variations in these parameters analogous to adjusting C_g in OSC₁ [153]. The transmission line

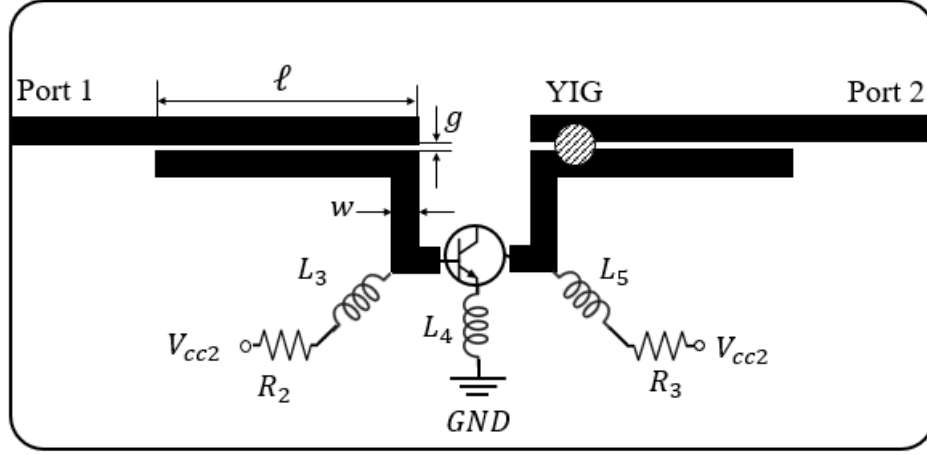


Figure 5.14: Schematic diagram of OSC₂.

width (w) determines the resonator's equivalent inductance and capacitance [154]: reducing w increases the inductance and decreases the capacitance, and vice versa. Rest of the circuit parameters are listed in Table. 5.5.

Results for lock range optimization are provided in Fig. 5.15. With w and g held constant, halving l significantly reduces κ_2 and, consequently, the lock range (lines 2 and 5). The effect can be attributed to a decrease in coupling gap capacitance, which drops from $C_g = 1.4$ to 0.7 pF. A similar trend is observed when comparing $g = 0.5$ mm and $g = 1.0$ mm (lines 2 and 4), where C_g decreases from 1.4 to 0.9 pF. Therefore, the coupling of OSC₂ to the external circuit must remain sufficiently strong.

For the OSC₁, increasing the inductance increases the lock range. However, this does not hold for OSC₂, as shown in Fig. 5.15(lines 1 to 3). The reason is due to mode hybridization, meaning $\omega_{\text{OSC } 2}/2\pi$ mimics that of the magnon mode, and the resonance profile of OSC₂ is not governed by equivalent circuit parameters of cavity.

In summary, to increase the lock range OSC₂, the coupling to the external signal lines should be sufficiently strong. Narrowing w offers little benefit to the lock range.

Two Stage Amplifier

Figure 5.16(a) shows the assembled two stage amplifier that incorporates optimal design

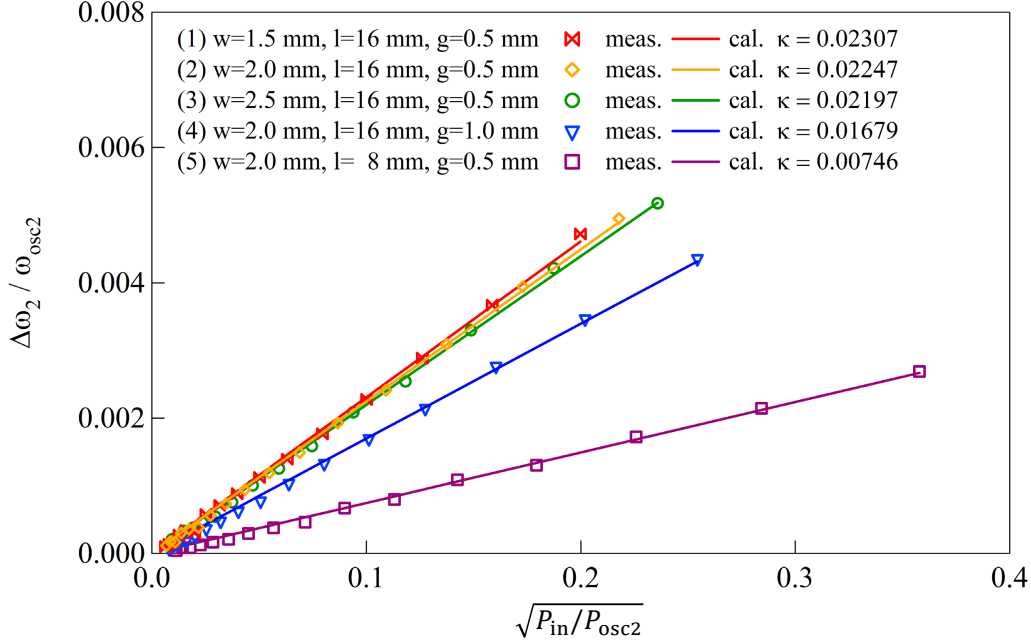


Figure 5.15: Normalized lock range for OSC₂ with various transmission line width w , coupling gap length l , and coupling gap distance g , plotted against $\sqrt{P_{in}/P_{OSC2}}$.

strategies identified previously. The input signal enters from bottom port, first passing through the pre-amplifier (OSC₁) and then the isolator before being amplified again by cavity magnonic stage (OSC₂). Output spectrum is measured from the top port.

Figure 5.16(b) displays the lock range of two stage amplifier in comparison to that of OSC₂. An increase in lock range is observed when the input power is below $P_{in} < -12$ dBm. For input power above this level, OSC₁ will cease to function as a pre-amplifier. This is due to the fact that injection-locked amplification cannot produce output power exceeding that of the free-running oscillator. As a result signal power after OSC₁ is limited to $P_{OSC1} = -12$ dBm, which constrains the lock range of OSC₂ and, by extension, the overall performance of the two-stage configuration.

In Fig. 5.16(c), the phase noise levels (L) of the output signal are compared to those of the input. Two distinct input signals, each having the same power, but different phase noise characteristics are used: One with a higher phase noise of $L_{10\text{kHz}} = -40.3$ dBc/Hz (blue), and $L_{100\text{kHz}} = -58.0$ dBc/Hz, followed by another one with lower phase noise $L_{10\text{kHz}} =$

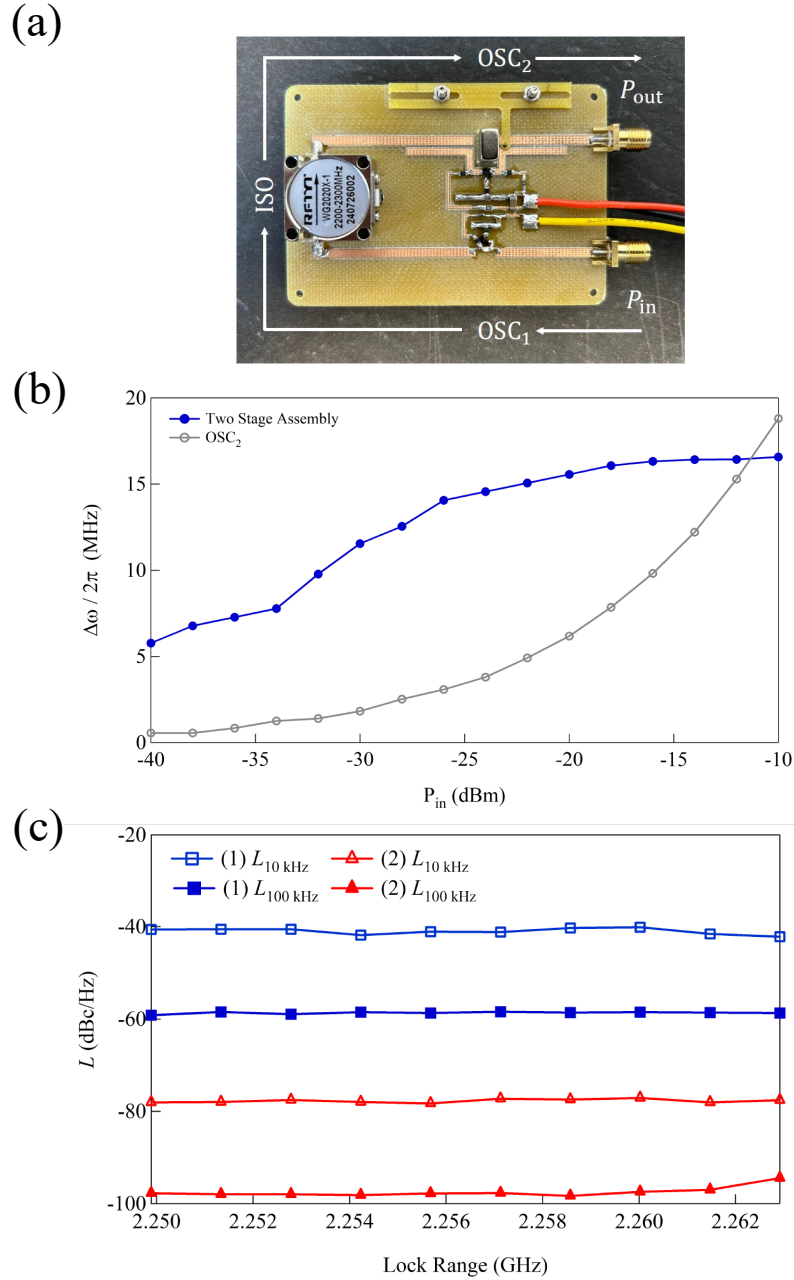


Figure 5.16: (a) Two stage amplifier assembly. (b) Lock range of two stage assembly and OSC_2 against various input power levels. (c) Phase noise levels of output signal sampled across the lock range. Blue: Phase noise level of P_{out} corresponding to input with $L_{10\text{ kHz}} = -40.3\text{ dBc/Hz}$, and $L_{100\text{ kHz}} = -58.0\text{ dBc/Hz}$. Red: $L_{10\text{ kHz}} = -77.6\text{ dBc/Hz}$ and $L_{100\text{ kHz}} = -97.6\text{ dBc/Hz}$

-77.6 dBc/Hz , $L_{100\text{ kHz}} = -97.6\text{ dBc/Hz}$ (red). The amplifier assembly replicates the phase noise of the input, ensuring no degradation in the output spectral purity across the entire

Table 5.6: Comparison of Injection-Locked Amplifier Technologies

Ref	Technology	Stages	ω_{osc} (GHz)	$\Delta\omega$ (MHz)	$\Delta\omega/\omega_{\text{osc}}$	Gain (dB)	Output (dBm)	κ
[156]	Power Combining	2	41	250	0.006	32	40	0.243
[157]	Class-E, CMOS	2	3.4	400	0.118	28	28	2.955
[148]	Positive Feedback	1	2.4	4.5	0.002	16	36	0.012
[147]	Buffered Input/Output, CMOS	3	54	2000	0.037	30	17	1.171
[149]	Class-E, CMOS	2	2.3	300	0.130	13	10	0.583
[158]	Tapped Cascode, CMOS	2	2	500	0.025	21	29	2.806
[159]	CMOS	1	53.5	2000	0.037	20	10	0.374
[53]	Cavity Magnonic	1	3.6	9	0.003	20	0	0.025
[55]	Cavity Magnonic	1	4.2	2.5	0.001	15	5	0.003
This Work	Cavity Magnonic	2	2.3	15	0.007	25	0	0.118

Reported numbers correspond to maximum lock range $\Delta\omega$.

lock range. Moreover, benefiting from the low phase noise nature of the two oscillators, any phase noise degradation at the edges of the lock range is negligible [155].

A list of ILAs are provided in Table.5.6. The lock range achieved in this study shows an improvement over its predecessors, though it remains one order of magnitude smaller than most entries. A two-stage cavity magnonic ILA offers several advantages, include: it eliminates noise from the pre-amplification stage, requires no minimum input power. These features make it particularly suitable for narrow bandwidth, low signal level and sensitive front end receiver applications. The main drawbacks are the size and complexity of the cavity magnonic oscillator. Recent approaches to overcoming such challenge include replacing bulk YIG material with thin films or alternative high spin density materials [160, 50], which benefit circuit integrability and compactness. From another perspective, OSC_2 essentially functions as a coherently coupled active and passive oscillator, and there exists a low-frequency circuit analog that can serve as the foundation for developing a high-frequency electric counterpart of OSC_2 [80].

5.5 Summary and Future Work

Low phase noise oscillators are fundamental components in a wide range of microwave systems, where precise frequency control and signal purity are paramount. This chapter

begins by introducing a phase noise reduction mechanism that leverages a coherently coupled active cavity and YIG platform. The design methodology for achieving low phase noise performance is detailed using three prototypes. The first prototype showcases the operational principle behind the phase noise reduction mechanism, while the second prototype, operating at a fixed frequency, exhibits phase noise levels of that outperforms many contemporary technology. The third prototype further advances the design by providing a method to implement a tunable frequency device.

Despite the inherent advantages of cavity magnonic oscillators, such as their superior frequency stability and low phase noise, they present challenges for injection locking amplification, particularly due to their high-Q resonance. To address this limitation, a two-stage architecture is proposed, combining a pre-amplification oscillator with a cavity magnonic oscillator. This design, separated by an isolator to prevent backward locking, resulted an order of magnitude increase compared to previous cavity magnonic ILAs. Moreover, the two-stage assembly preserves the phase noise characteristics of the input signal, making it ideally suited for narrowband, sensitive signal amplification applications. The work also demonstrates the potential for utilizing oscillators with high spectral purity as injection locking amplifiers.

One of the most promising directions for future research is the integration of phase noise reduction schemes into oscillator technologies. While the work presented here focuses on cavity magnonics, the underlying approach can be extended to any negative resistance oscillator. By incorporating a tunable passive resonator and a coherently coupled channel, an inductor or capacitor for example, many oscillator designs can potentially achieve lower phase noise levels than those currently attainable.

Chapter 6

Conclusion

Ferrimagnetic materials can interact with microwave photons to create hybridized states, which are essentially coupled electric and magnetic resonances. The coupling blends their individual properties, allowing the system to transition between cavity-like and magnon-like modes through the control of field detuning. Hybridization also leads to unconventional resonance phenomena, such as zero dissipation, making cavity magnonic devices an intriguing platform for microwave signal processing applications.

As detailed in Chapter 2, the working principle of a cavity magnonic device can be understood by modeling it as a pair of coupled harmonic oscillators. Such method successfully captures the resonance and dissipation characteristics of hybridized states that arise from two distinct interaction mechanisms: coherent coupling results from the direct exchange of energy, achieved by overlapping the electromagnetic field of microwave cavity with the magnetic material. Thus, the strength of coherent coupling is influenced, for example, by the size of the magnetic material and its position relative to the electromagnetic field of the resonator. In contrast, dissipative coupling is an indirect, lossy interaction whose strength is determined by the extrinsic damping rates of the cavity and magnon modes, mediated through a shared reservoir. The coupled oscillator model forms the basis for understanding the working principles of two key devices presented in this thesis: a passive high-attenuation isolator and an active low-phase-noise oscillator.

For the passive cavity magnonic isolator described in Chapter 3, it was found that the interplay between coherent and dissipative coupling can reduce the linewidth of one of the hybridized modes to zero, thereby creating a complete stopband. When the chirality of spin precession is taken into account, magnetic spins can interact asymmetrically with microwaves traveling in the opposite direction, enabling non-reciprocal transmission. By controlling the coupling strengths within the device, one can reshape the dispersion and tune the frequency at which the linewidth vanishes. This, in turn, allows for control over the non-reciprocal characteristics of the isolator, including operating frequency, insertion loss, isolation, and bandwidth. A prototype tested at millikelvin temperatures further shows that the non-reciprocal scheme remains effective across a wide temperature range, suggesting its potential for use in cryogenic environments where sensitive signals must be protected from unwanted reflections.

Incorporating a transistor amplifier into a resonator creates an active cavity. As demonstrated in Chapter 4, this approach is broadly applicable as long as the transistor circuit functions as a negative resistance element and is biased into an initially unstable regime. Upon powering up, broadband oscillation spontaneously develops and enters a feedback loop where the signal is continuously filtered by the resonator and amplified by the transistor until amplitude saturates. Once stabilized, the amplifier's role is to compensate for losses in the circuit, while the resonator ensures necessary phase shift for constructive summation in each cycle. The documented method can be generalized to convert any passive resonator into an oscillator, and transform passive sensors into active ones, with high-Q resonance and improved resolution, as demonstrated in the detection of nucleic acid-level reactions.

Finally, in Chapter 5, coupling an active cavity to a magnetic material forms an active cavity magnonic oscillator. Unlike passive systems, an active cavity magnonic system exhibits a single hybridized mode frequency. By varying the field detuning, the oscillation mode can be controlled to evolve from an electrically noisy, cavity-like mode to a more stable magnon-like mode. Such transition effectively suppresses the contribution of electrical noise

from the cavity, enabling a sharp emission spectrum with phase noise levels comparable to state-of-the-art oscillator designs. Beyond conventional methods for phase noise reduction, such as transistor circuit optimization and resonator selection, the methods presented in Chapter 5 introduce an additional technique that offers further noise suppression, which can be used alongside or complement existing noise reduction methods.

This thesis demonstrates that cavity magnonic devices hold strong potential as functional counterparts for existing microwave-frequency electronics. The use of a coupled harmonic oscillator model, combined with an analysis of coupling mechanisms and control strategies, establishes a framework for device optimization. The presented approach is applicable not only to planar cavity magnonic systems but also to emerging on-chip cavity magnonic technologies. Furthermore, since the underlying principles have already been validated in classical RLC circuits, the findings presented here may also inspire the development of purely electrical analogs that operate on similar functional principles.

Appendix A

Single Shunt Series RLC circuit with Complex Port Impedance

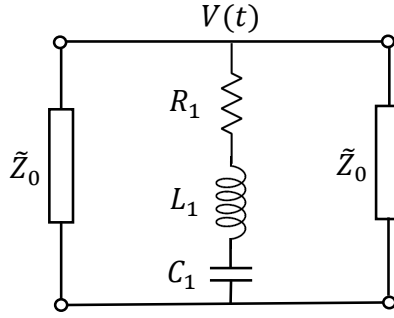


Figure A.1: A series RLC resonator circuit is connected in a shunt configuration, with the port impedance modeled as a complex value: $\tilde{Z}_0 = Z_0 + iX_0$.

To determine the uncoupled mode frequency of the circuit shown in Fig. A.1, the voltage at the top junction, $V(t)$ is expressed as:

$$R_1 I_1 + L_1 \frac{dI_1}{dt} + \frac{1}{C_1} \int I_1 dt = V(t) \quad (\text{A.1a})$$

$$-\frac{1}{2}(Z_0 + iX_0)I_1 = V(t) \quad (\text{A.1b})$$

Using the same approach outlined in Chapter 2.4, the differential equation governing the

current dynamics of the resonator can be obtained:

$$\ddot{I}_1 = -\omega_1^2 I_1 - 2(\alpha_1 + \kappa_1 + i\zeta_1)\dot{I}_1 = 0 \quad (\text{A.2})$$

where $\alpha_1 = R_1/(2L_1)$, $\kappa_1 = Z_0/(4L_1)$, and $\zeta_1 = X_0/(4L_1)$. Assume a general solution of the form $I_1 = a(t)e^{-i\omega t}$, the resonator dynamics reduce to:

$$i(\dot{a}) = \left[(\omega_1 - \zeta_1) - i(\alpha_1 + \kappa_1) \right] a \quad (\text{A.3})$$

In other words, the uncoupled frequency of a shunt series RLC circuit connected to complex port impedance is given by:

$$\tilde{\omega}_1 = (\omega_1 - \zeta_1) - i(\alpha_1 + \kappa_1) \quad (\text{A.4})$$

Interpretation of ζ_1

Compared to the case of a purely real port impedance ($\tilde{\omega}_1 = \omega_1 - i(\alpha_1 + \kappa_1)$), introducing a reactive component X_0 modifies the resonant frequency by a shift of ζ_1 . In this context, X_0 provides an additional degree of freedom in modeling the resonator, which is useful in configurations where the resonant frequency depends on the spatial relationship between the resonator and the transmission line. A notable example is the aperture-coupled dielectric resonator, where the resonant frequency has been shown to vary with the distance between the resonator and the transmission line [161]. The reactive component captures the coupling distance effect, which may behave as a capacitive or inductive reactance.

However, when deriving the coupled circuit equations, the approximation: $\tilde{\omega}_1 \approx \omega_1 - i(\alpha_1 + \kappa_1)$ is used under the following assumptions:

- Although $\zeta_1 = \frac{X_0(\omega)}{4L_1}$ is a reactive quantity that depends on the driving frequency ω , ω is typically close to resonance. As a result, small variations in ω cause only minor changes in X_0 , allowing ζ_1 to be approximated as constant.

- Also, in most cavity magnonic experiments, the resonator is typically fixed in position during measurements, such that ζ_1 remains constant.
- As established in Chapter. 2.4, the coherent coupling strength is defined as $J = \sqrt{\zeta_1 \zeta_2}$, indicating that ζ_1 is of the same order of magnitude as J and therefore, significantly smaller than ω_1 ($\zeta_1 \ll \omega_1$). For example, in the millikelvin temperature experiment presented in Chapter. 3, the resonant frequency was $\omega_1/2\pi = 6.354$ GHz, while the coupling strength was $J/2\pi = 63$ MHz (approximately 1% of ω_1). Similarly, in the circuit model presented in Chapter 2.4, the ratio $J/\omega_1 \approx 2\%$.

Given the above considerations, ζ_1 can be effectively absorbed into ω_1 , allowing Eq. (A.4) to be approximated as:

$$\tilde{\omega}_1 \approx \omega_1 - i(\alpha_1 + \kappa_1) \quad (\text{A.5})$$

Scattering parameter S_{21} , S_{11} , and Absorption

Since the impedance of a network is independent of the port impedance, the ABCD matrix for the circuit in Fig. A.1 is identical to that of the circuit with a real port impedance, as given in Eq. (2.1a):

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{Series}} = \begin{bmatrix} 1 & 0 \\ 1/Z_{\text{Series}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \left[-i2L_1 \left((\omega_1 - \omega) + i\frac{R_1}{2L_1} \right) \right]^{-1} & 1 \end{bmatrix} \quad (\text{A.6})$$

Consequently, the reflection and transmission coefficients of the circuit are, respectively:

$$S_{11} = \frac{-i\kappa_1 + \zeta_1}{(\omega - \omega_1 - \zeta_1) + i\alpha_1 + i\kappa_1} \approx \frac{-i\kappa_1 + \zeta_1}{(\omega - \omega_1) + i\alpha_1 + i\kappa_1} \quad (\text{A.7a})$$

$$S_{21} = \frac{\omega - \omega_1 + i\alpha_1}{(\omega - \omega_1 - \zeta_1) + i\alpha_1 + i\kappa_1} \approx \frac{\omega - \omega_1 + i\alpha_1}{(\omega - \omega_1) + i\alpha_1 + i\kappa_1} \quad (\text{A.7b})$$

where the same approximation used in Eq.(A.5) is applied. By setting $\zeta_1 = 0$, Eqs.(A.7) reduce to the form of a resonator loaded by real port impedance [Eq.(2.6)].

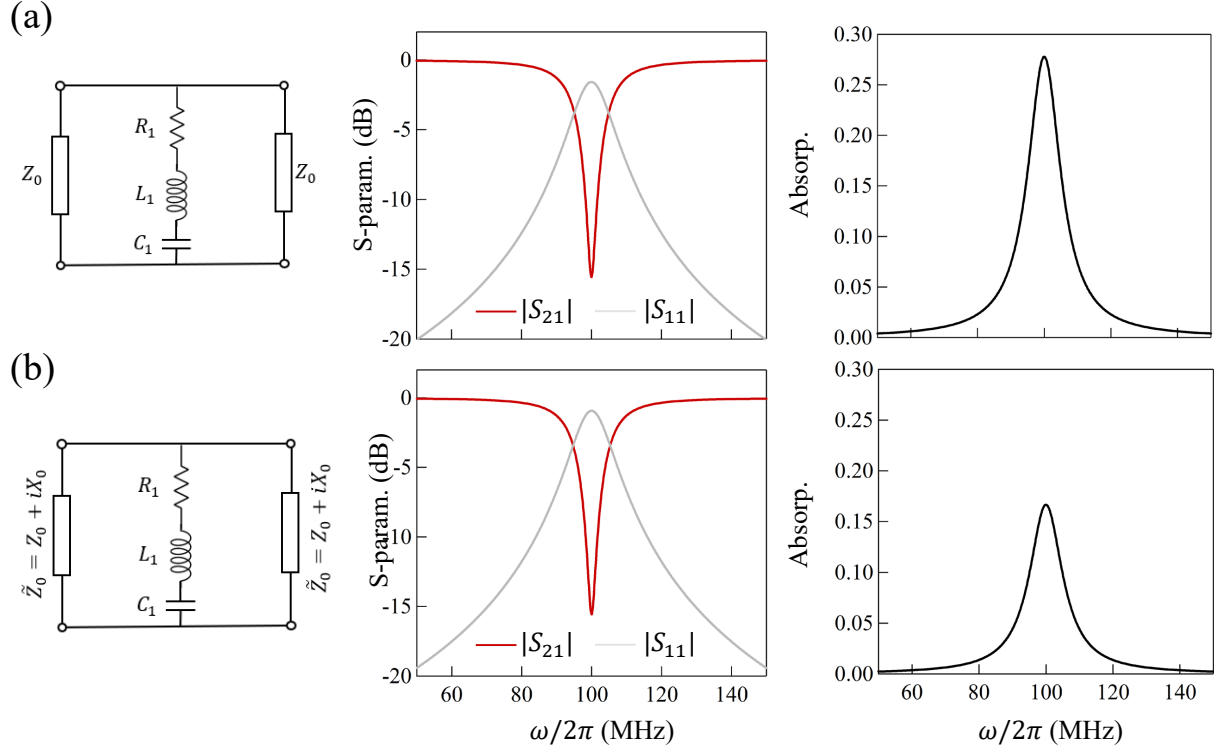


Figure A.2: (a) S_{21} , S_{11} , and Absorp. of a series shunt RLC circuit connected to real port impedance. The circuit parameters are $\omega_1/2\pi = 100$ MHz, $\alpha_1/2\pi = 1$ MHz, $\kappa_1/2\pi = 5$ MHz, (b) Corresponding results for the same circuit connected to a complex port impedance with, $\zeta_1 = 2$ MHz. All other circuit parameters are identical to those in (a).

In addition, the resonator absorption is defined as:

$$\text{Absorp.} = 1 - |S_{21}|^2 - |S_{11}|^2 \quad (\text{A.8})$$

which quantifies the portion of input energy that is neither transmitted to port 2 nor reflected back to port 1, but is instead absorbed by the resonators and dissipated through intrinsic losses. As a result, absorption directly reflects the net energy drawn by a resonator and offers a more accurate representation of the eigenmodes, as it reaches maximum values at the eigenfrequencies. Absorption can become a useful tool when analyzing coupled resonators, as discussed in Appendix. B.

Fig. A.2 presents the $|S_{21}|$, $|S_{11}|$, and the absorption for an RLC resonator with either real or complex port loading.

Appendix B

Scattering Parameters of Coupled Resonant Circuit

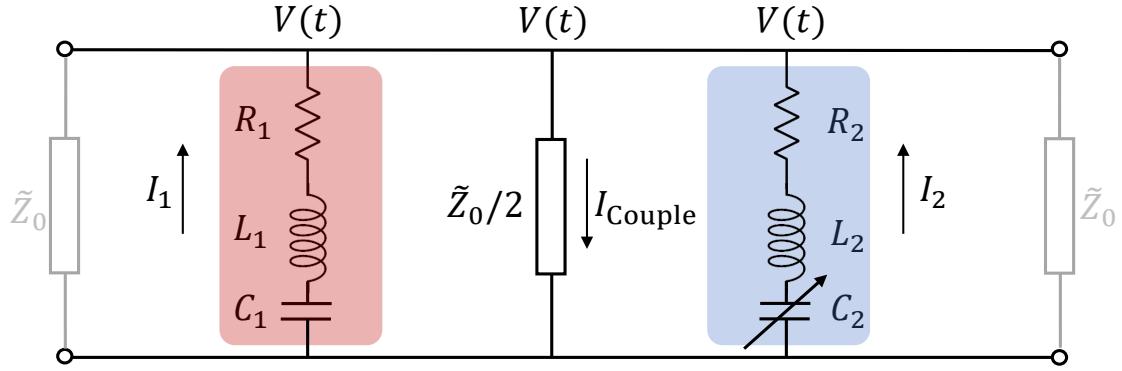


Figure B.1: Schematic of a coupled RLC circuit. The resonance frequency $\omega_1 = 1/\sqrt{L_1 C_1}$ is fixed, while $\omega_2 = 1/\sqrt{L_2 C_2}$ is tunable via the adjustable capacitor C_2 , enabling frequency sweep measurements against their detuning. Energy exchange between ω_1 and ω_2 is realized through the middle branch, which is parallel combination of the left and right complex port impedances $\tilde{Z}_0 = Z_0 + iX_0$. This figure replicates Fig. 2.8 from the main text.

Again, using the ABCD matrix to S-parameter conversion approach, the reflection and transmission of a mixed coupled circuit can be expressed as follows and are shown in Fig. B.2:

$$S_{11} = \frac{-(i\kappa_1 - \zeta_1)(\omega - \omega_2 + i\alpha_2 + i\kappa_2) - (i\kappa_2 - \zeta_2)(\omega - \omega_1 + i\alpha_1 + i\kappa_1) + 2(i\kappa_1 - \zeta_1)(i\kappa_2 - \zeta_2)}{(\omega - \omega_1 + i\alpha_1 + i\kappa_1)(\omega - \omega_2 + i\alpha_2 + i\kappa_2) - (i\kappa_1 - \zeta_1)(i\kappa_2 - \zeta_2)} \quad (\text{B.1a})$$

$$S_{21} = \frac{(\omega - \omega_1 + i\alpha_1)(\omega - \omega_2 + i\alpha_2)}{(\omega - \omega_1 + i\alpha_1 + i\kappa_1)(\omega - \omega_2 + i\alpha_2 + i\kappa_2) - (i\kappa_1 - \zeta_1)(i\kappa_2 - \zeta_2)} \quad (\text{B.1b})$$

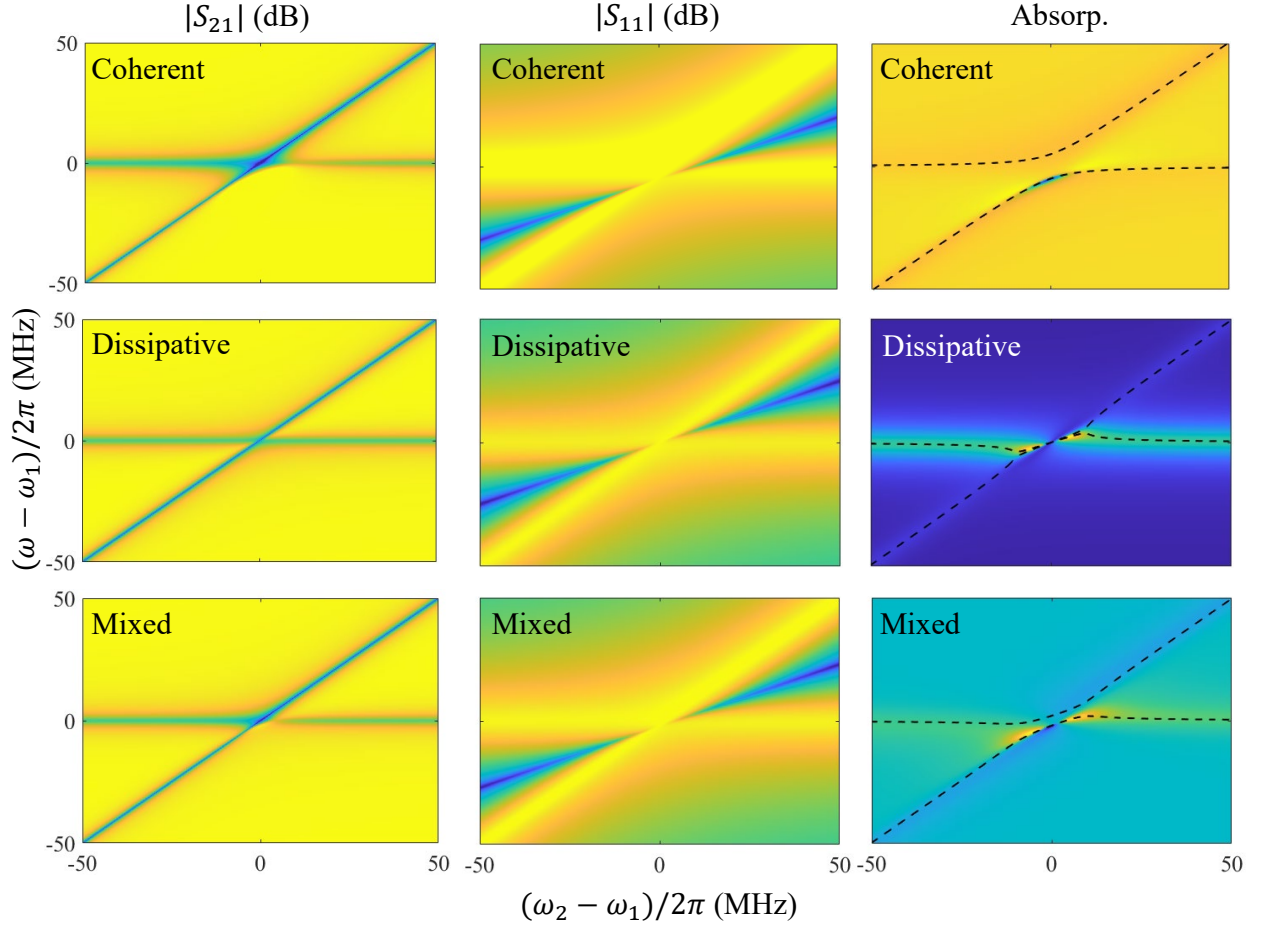


Figure B.2: Transmission, reflection, and absorption spectra for resonant circuits with different coupling mechanisms. Each row corresponds to a distinct coupling type: coherent (top), dissipative (middle), and mixed (bottom). Each column shows a different observable: from left to right, $|S_{21}|$, $|S_{11}|$, and Absorp. All calculation parameters are the same as those used in Fig. 2.9. The black dashed lines indicate the hybridized eigenfrequencies calculated from Eq. (2.29).

Discrepancy between $|S_{21}|$ of coupled circuits and cavity magnonic system.

The $|S_{21}|$ equation used to determine the parameters of a cavity magnonic isolator is given in Eq. (3.2), as rewritten below:

$$S_{21} = 1 - \frac{i\kappa_c}{[(\omega - \omega_c) + i(\alpha_c + \kappa_c)] + \frac{(J - i\Gamma)^2}{(\omega - \omega_m) + i(\alpha_m + \kappa_m)}} \quad (\text{B.2})$$

where $\omega_{c,m}$, $\alpha_{c,m}$, and $\kappa_{c,m}$ are the uncoupled frequencies, intrinsic and extrinsic damping

rates of cavity and magnon mods respectively. J is the coherent coupling strength and $\Gamma = \sqrt{\kappa_c, \kappa_m}$ is the dissipative coupling strength.

The above equation differs from the circuit equation shown in Eq. (B.1b). The distinction arises because the circuit model is derived from a more rigorous approach: resonators and their coupling are modeled using standard circuit theory, meaning the expressions for the scattering parameters directly result from the system's topology and the energy exchanges within the network.

The cavity magnonic $|S_{21}|$ is phenomenological in nature, generalized from a purely coherent coupled case [162]. The expression describes the coupled system based on observed phenomena: parameters such as the dissipative coupling strength are introduced to capture the essential features of the interaction, rather than being strictly originated (determined) from the system's microscopic fundamentals.

The phenomenological model served as a practical tool for analyzing the performance of the isolators. Since the prototypes are based on pioneering work by Yipu *et al.*, where Eq. (B.2) was first introduced [41], and the same equation was later adapted to analyze both the tunability of the isolation band [163], and the feasibility of a circulator [51]. Eq. (B.2) provides a consistent and straightforward approach to compare and analyze results across different works, thus aiding the device optimization process.

For reference, Fig. B.3 compares the absorption mapping calculated from Eqs. (B.1) with the transmission mapping derived from Eq. (B.2), using the same set of parameters.

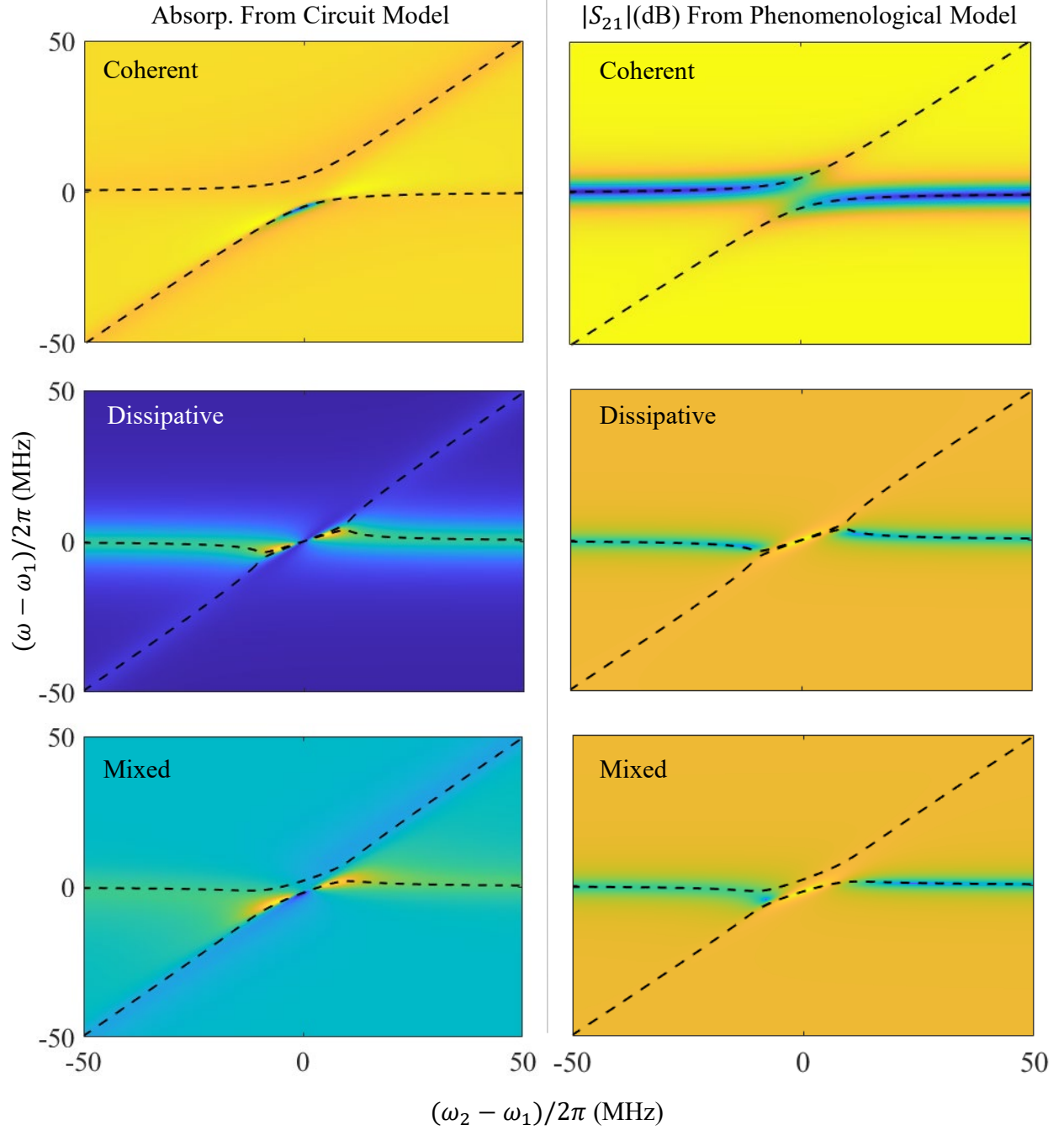


Figure B.3: A comparison is shown between the absorption maps (left column), obtained from the coupled circuit model, and the transmission maps (right column), derived from the phenomenological model. Each row represents a different coupling regime: coherent (top), dissipative (middle), and mixed (bottom). All calculation parameters are consistent with those used in Fig. 2.9 and Fig. B.2. The black dashed lines indicate the hybridized eigenfrequencies, computed using Eq.(2.29) for the circuit model and Eq. (2.40) for the cavity magnonics phenomenological model.

Appendix C

Long Term Stability Test Analysis for Au-IDC Sensor.

Table C.1: Correlation Coefficient between Bare Sensor Frequency (f_t), Temperature (T_t), and Relative Humidity(H_t)

Data Set	$f_t - f_0$ and $T_t - T_0$	$f_t - f_0$ and $H_t - H_0$	$H_t - H_0$ and $T_t - T_0$
#1	-0.94	-0.70	0.49
#2	-0.99	0.79	-0.79
#3	-0.99	-0.94	0.93
#4	-0.99	0.71	-0.77
#5	-0.99	-0.87	0.86
#6	-0.98	-0.60	0.58
#7	-0.99	-0.58	0.57
#8	-0.99	0.89	-0.88
#9	-0.98	0.23	-0.34
#10	-0.84	0.20	0.15
Average	-0.97	-0.09	0.08

Appendix D

Hybridization Experiment Data Sets

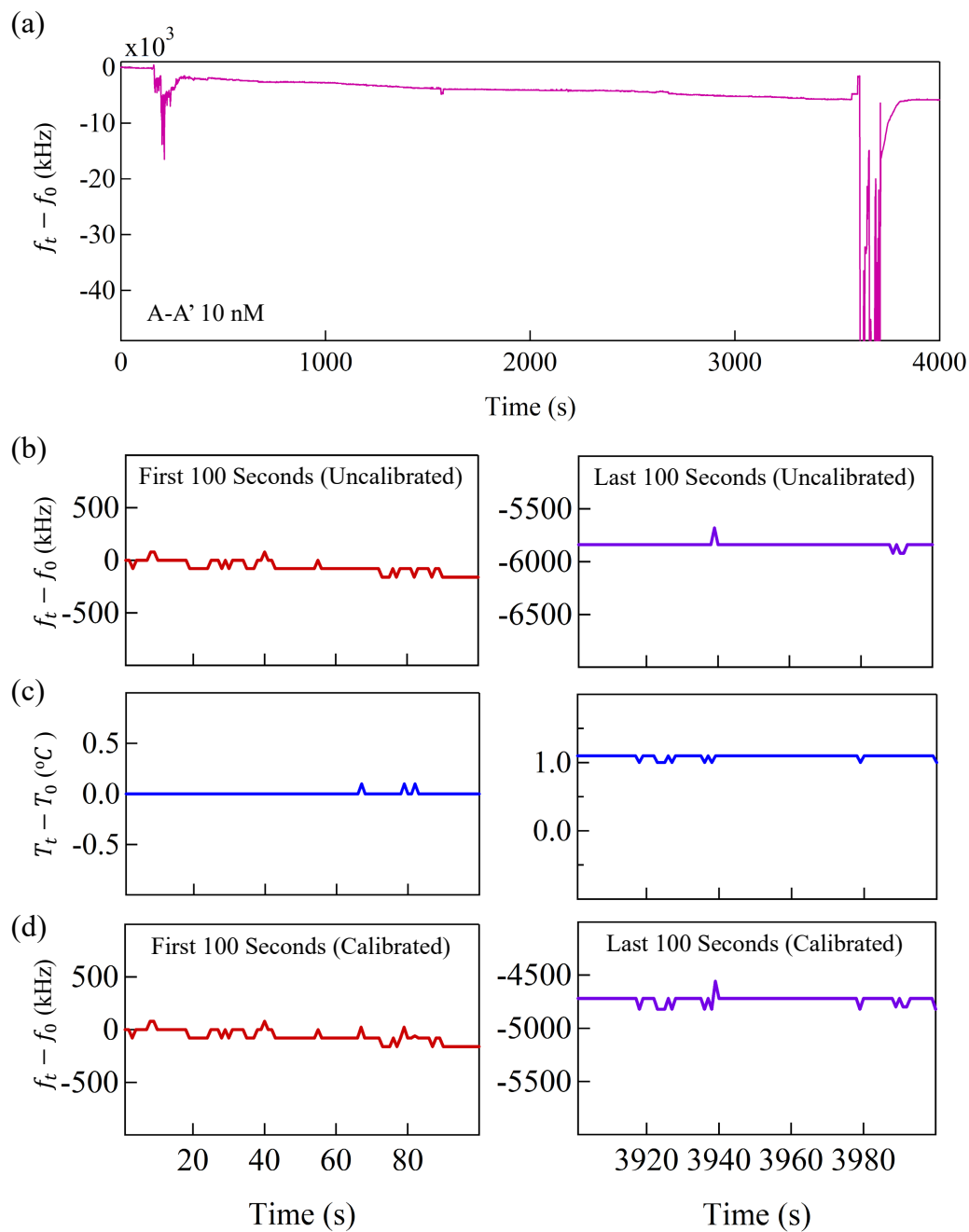


Figure D.1: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 10 nM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

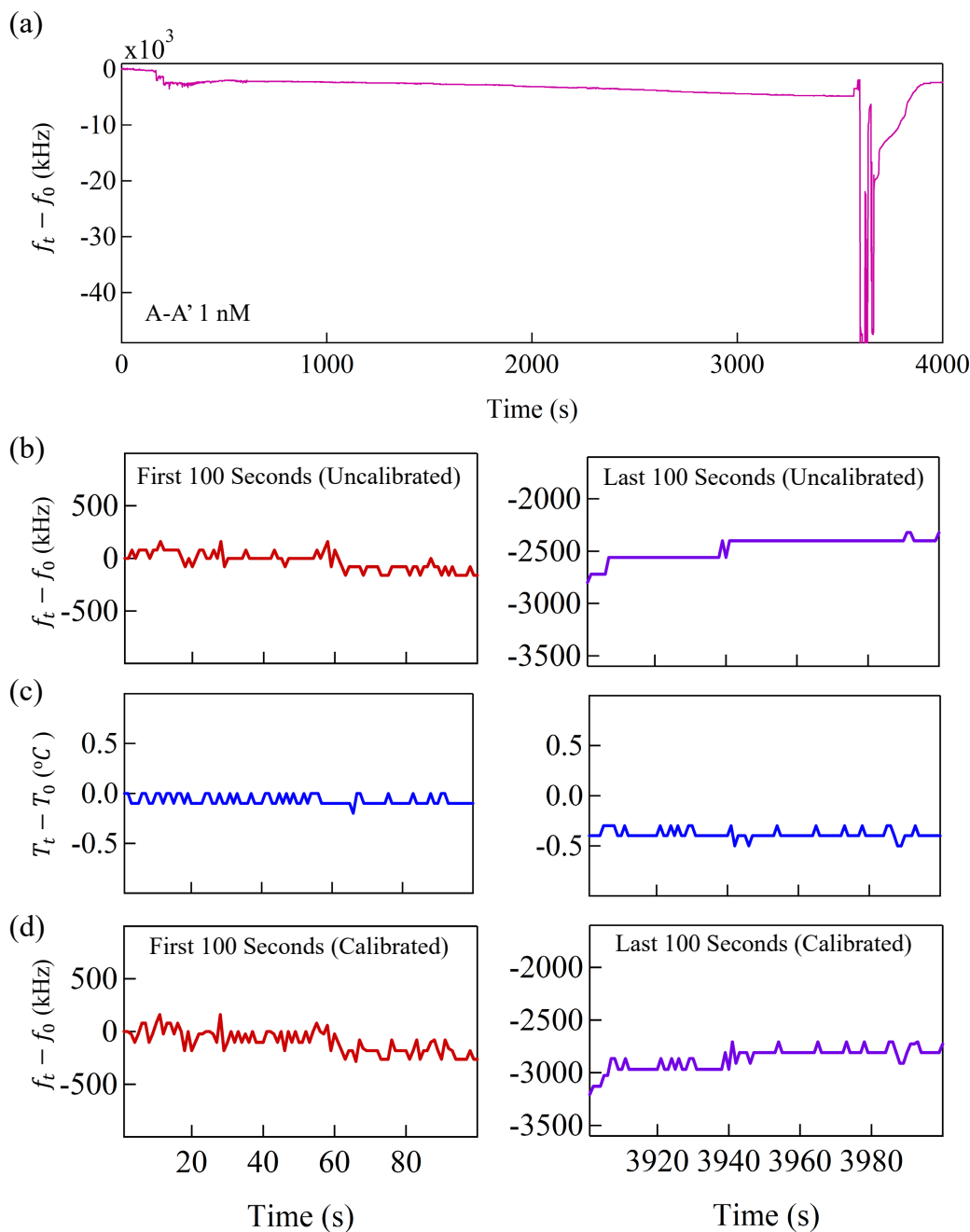


Figure D.2: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 1 nM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

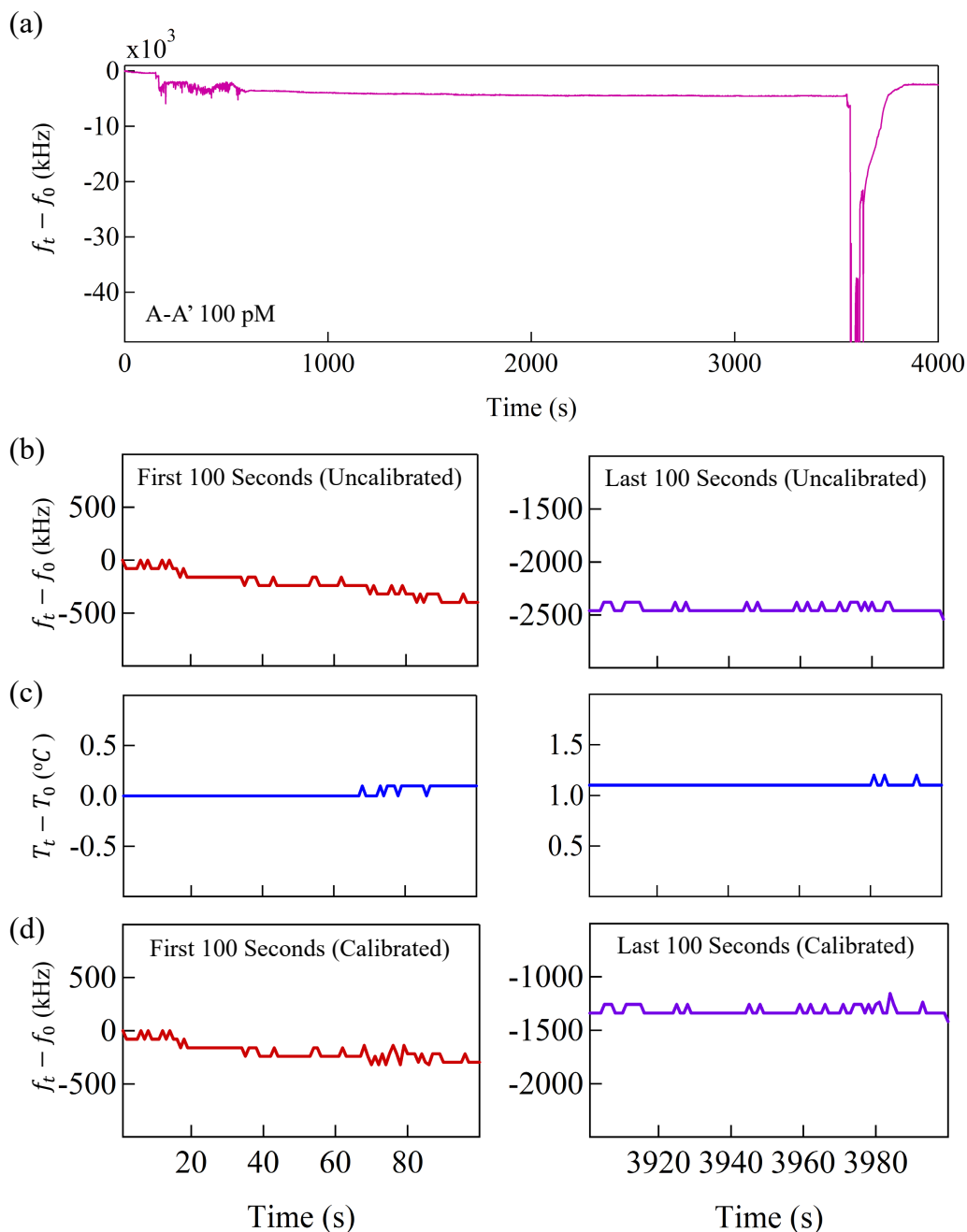


Figure D.3: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 100 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

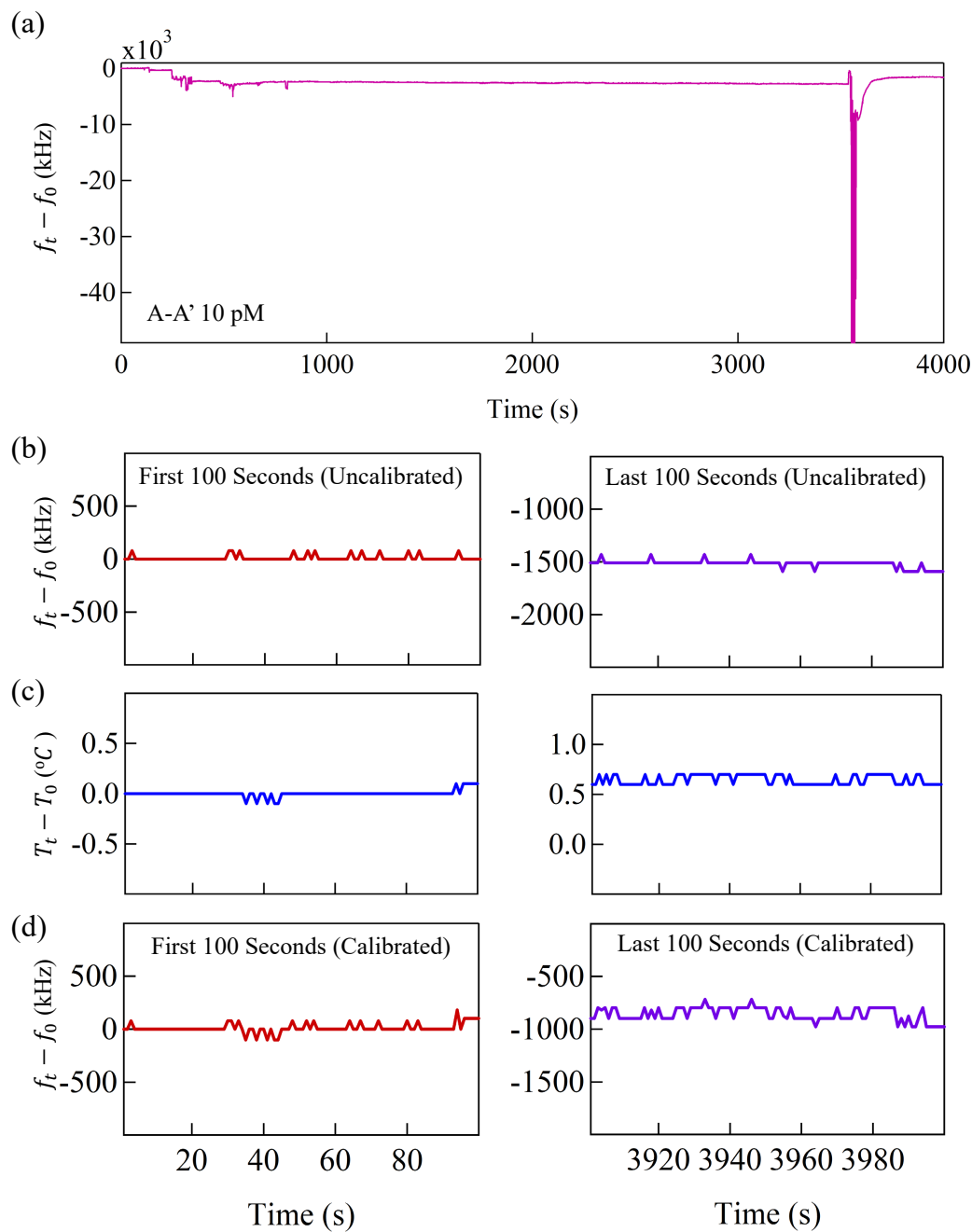


Figure D.4: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 10 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

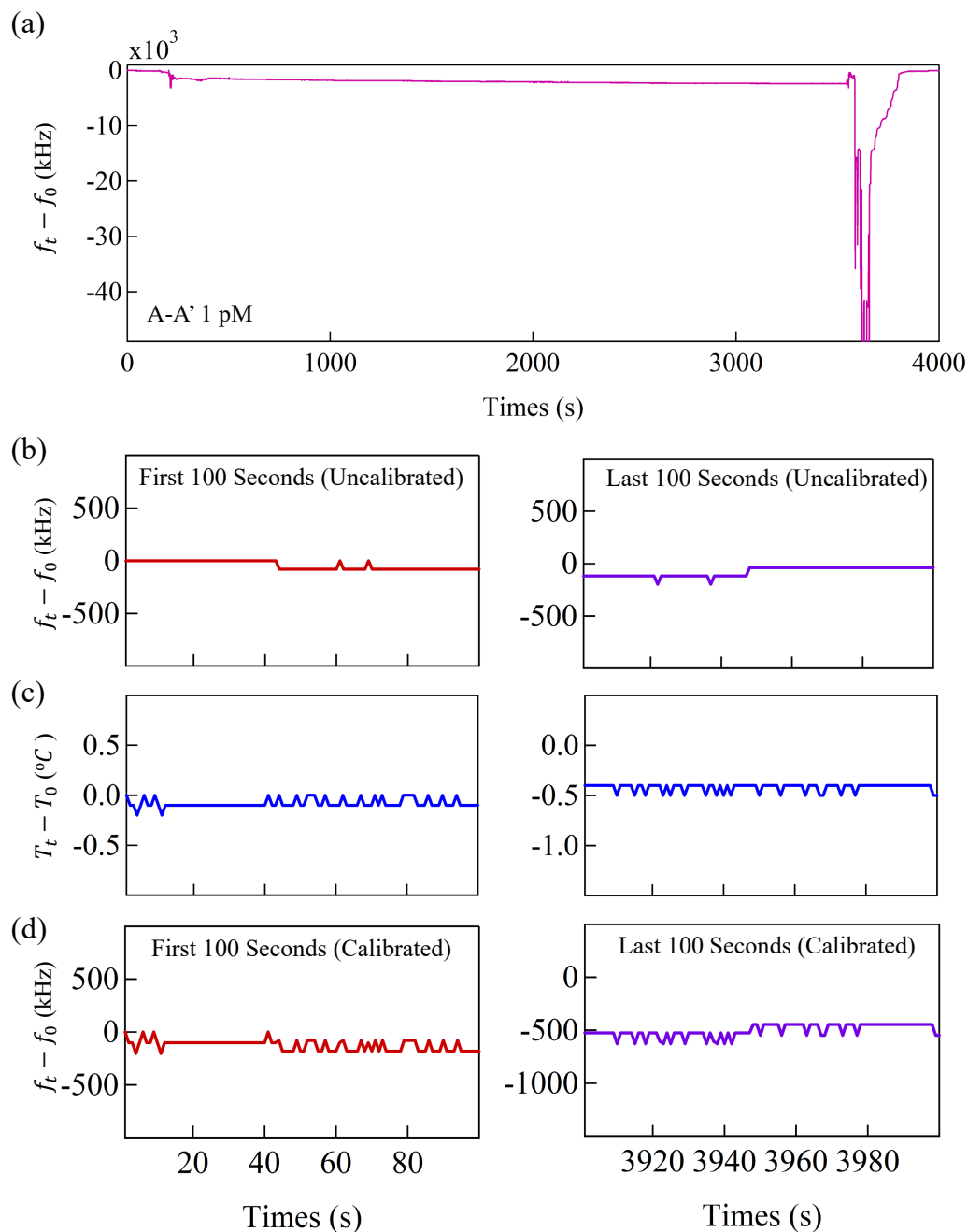


Figure D.5: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 1 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

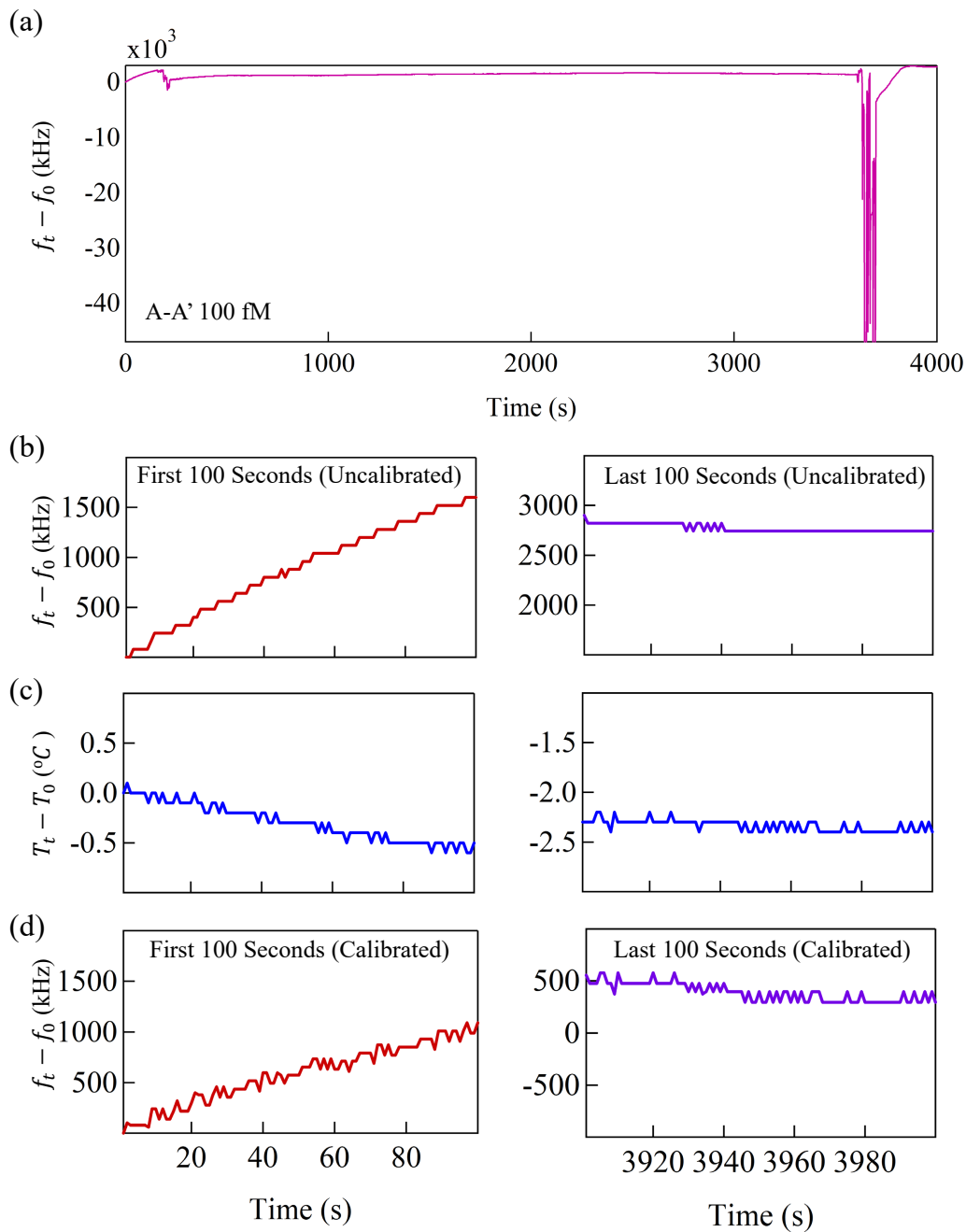


Figure D.6: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 100 fM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

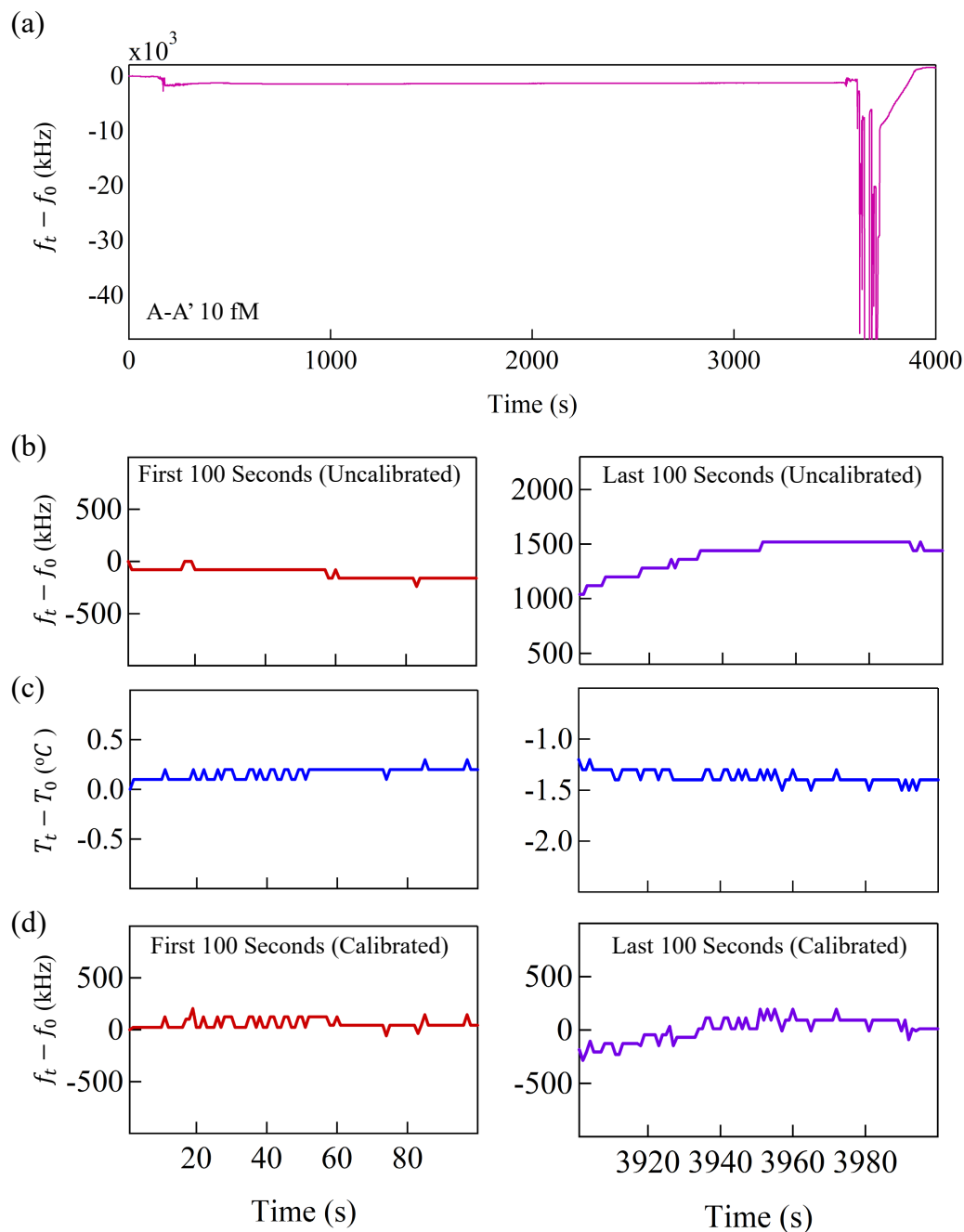


Figure D.7: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 10 fM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

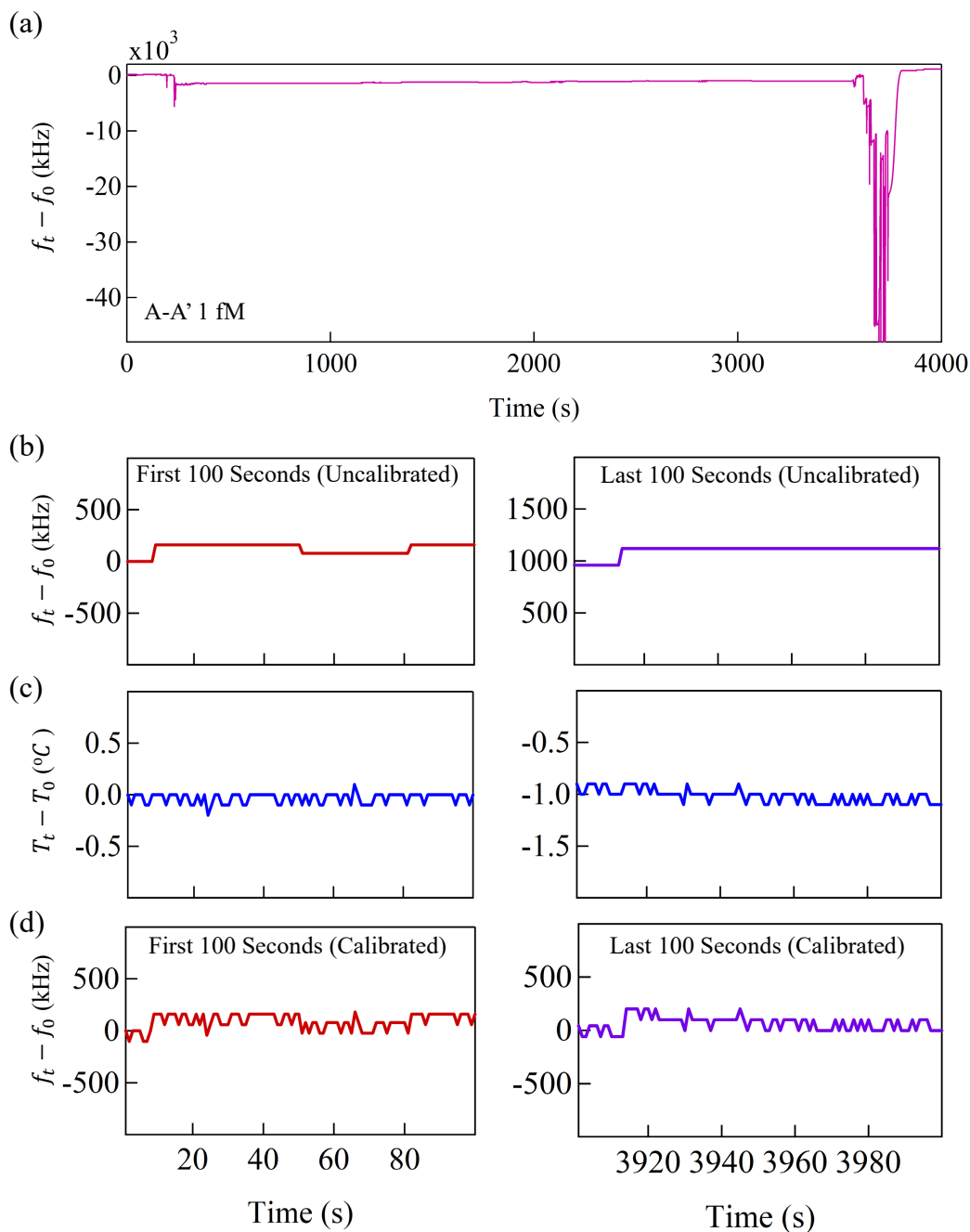


Figure D.8: Data for hybridization experiment (A-A'). Time for hybridization is 60 minutes. Target RNA concentration is 1 fM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

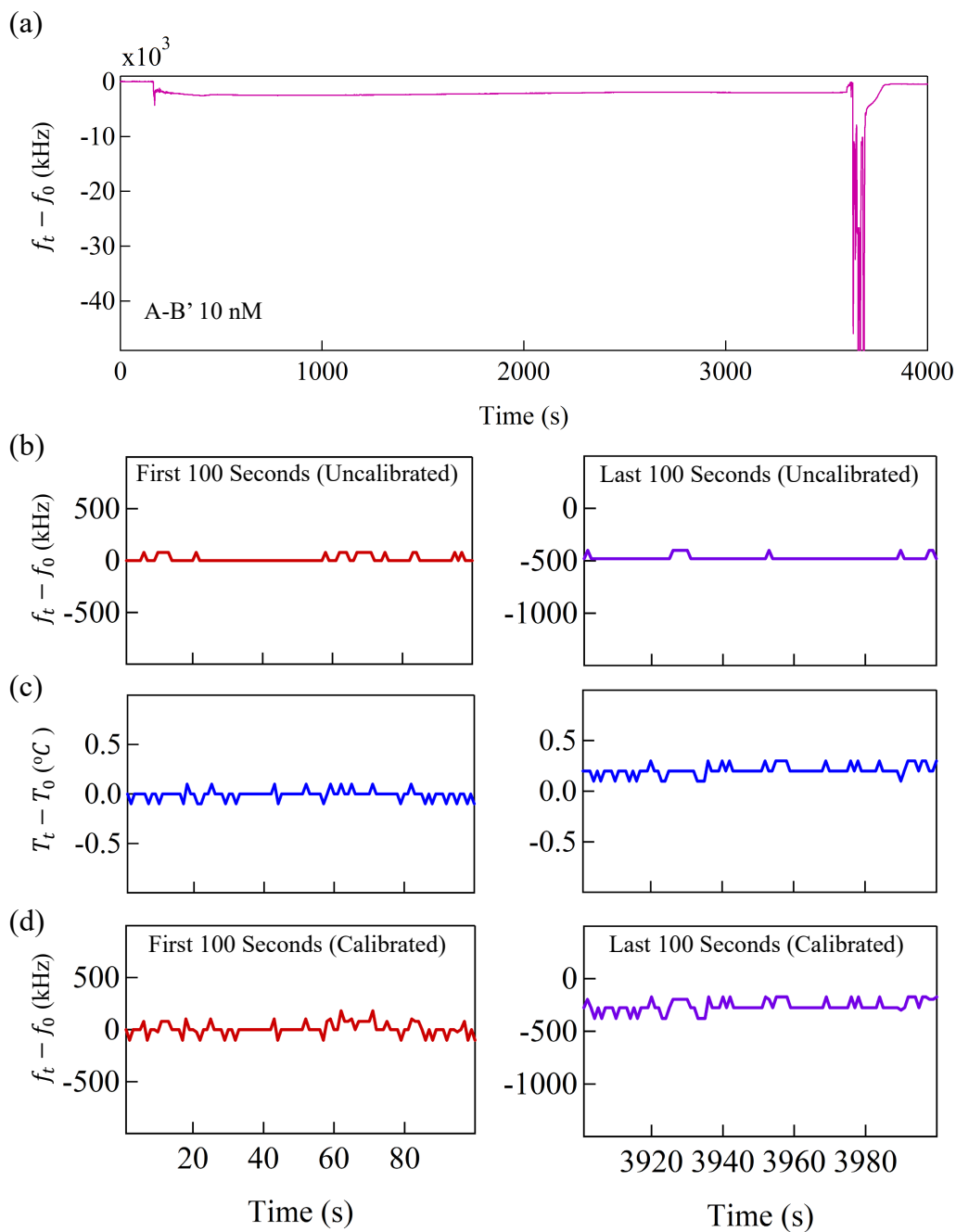


Figure D.9: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 10 nM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

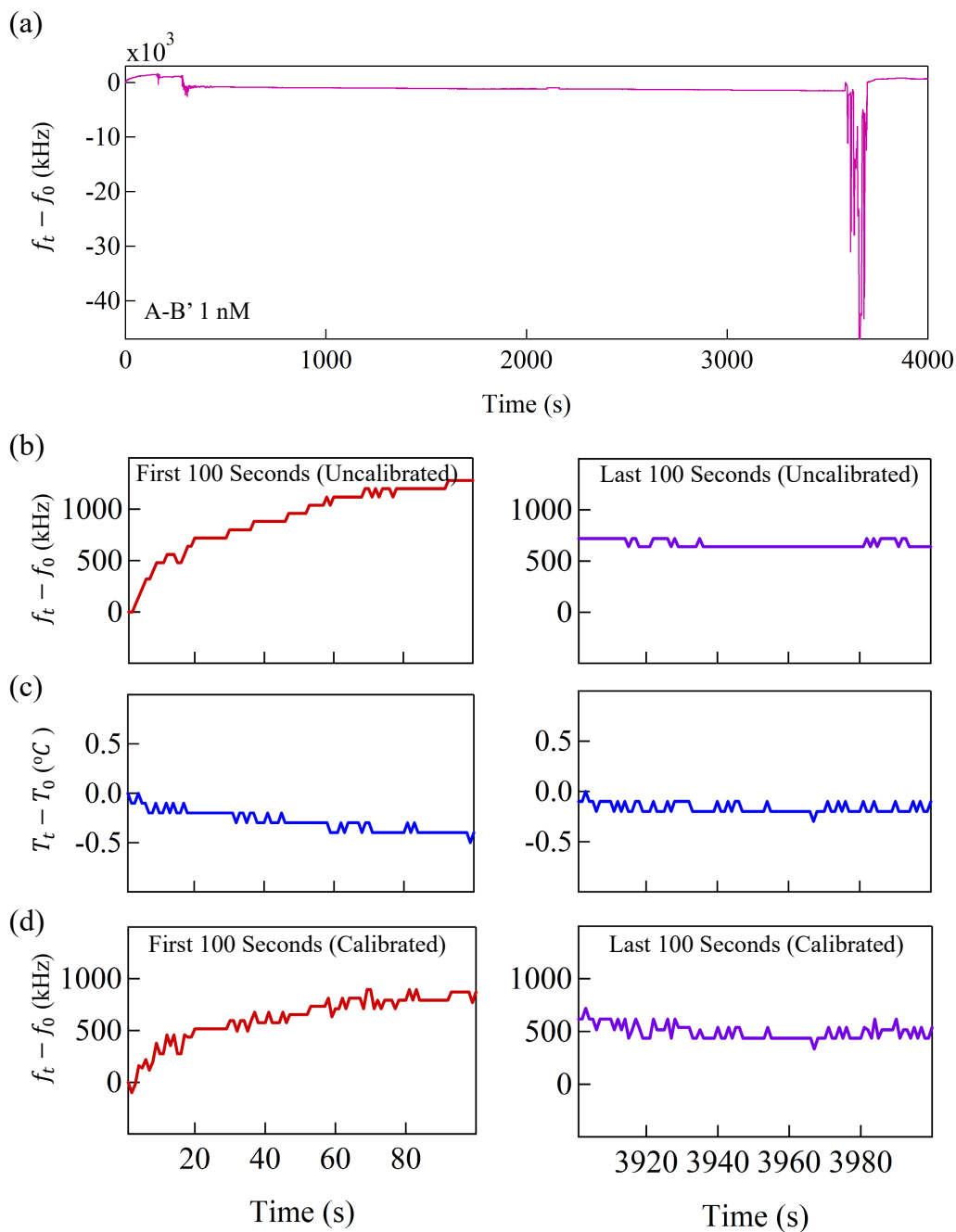


Figure D.10: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 1 nM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

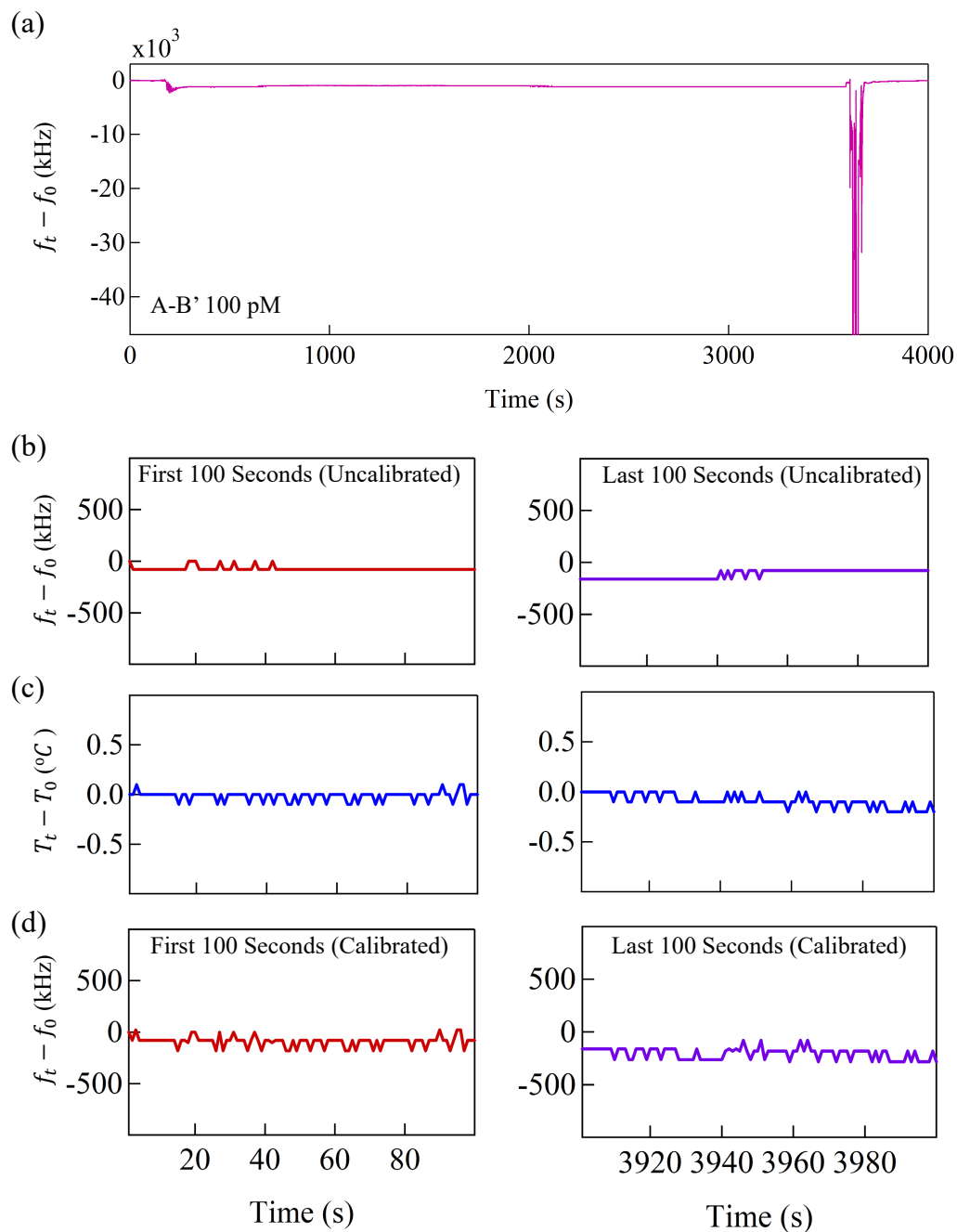


Figure D.11: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 100 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

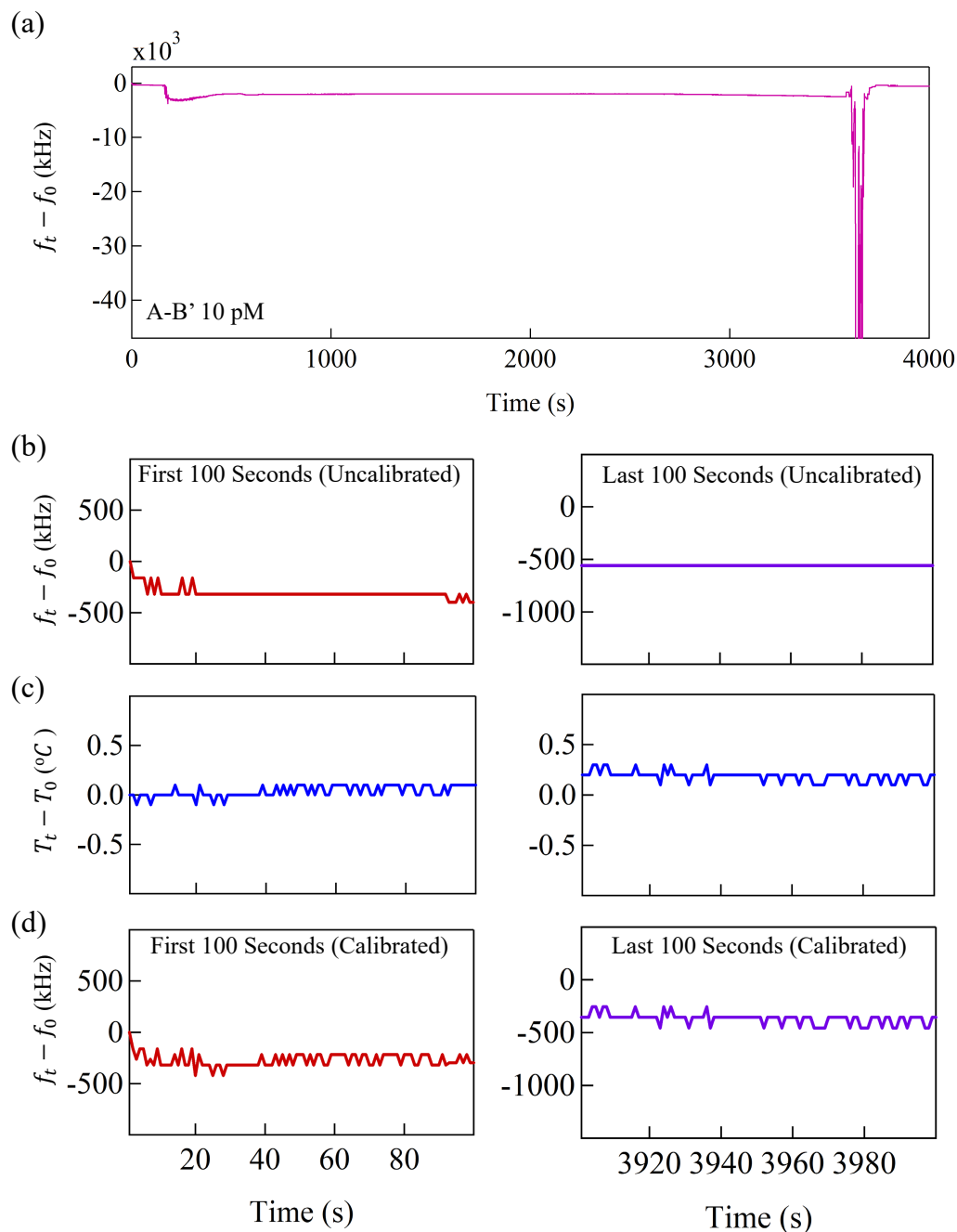


Figure D.12: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 10 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

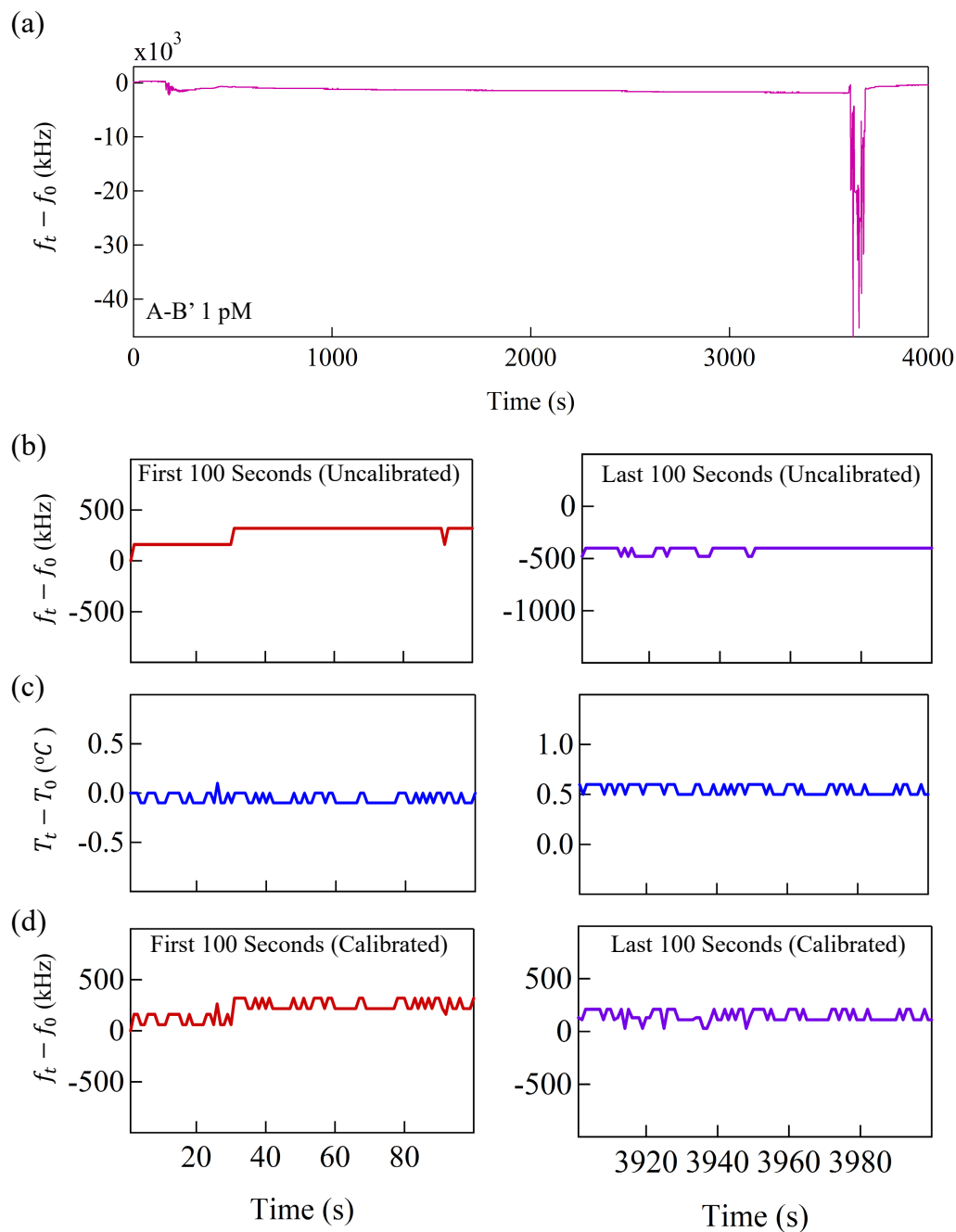


Figure D.13: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 1 pM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

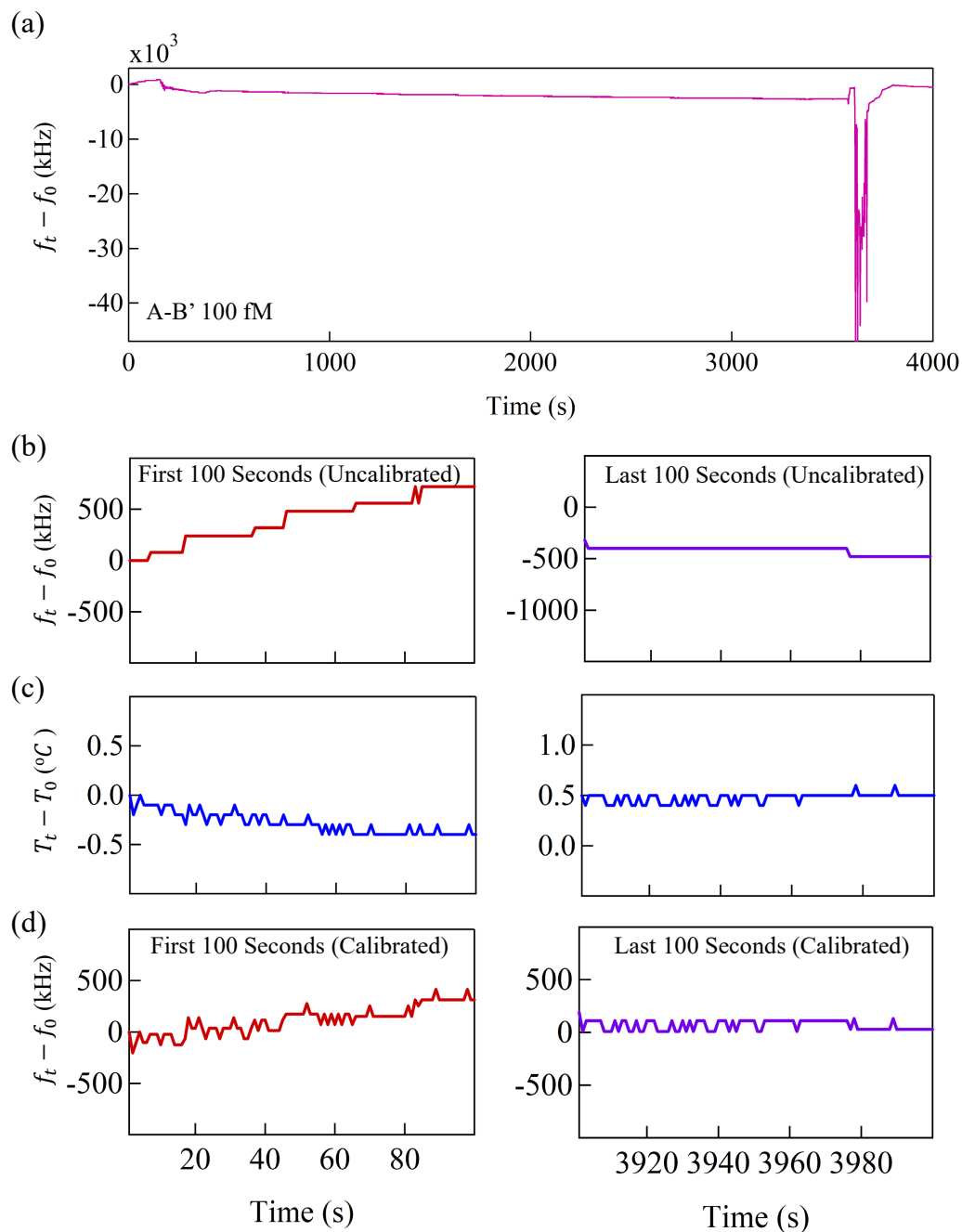


Figure D.14: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 100 fM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

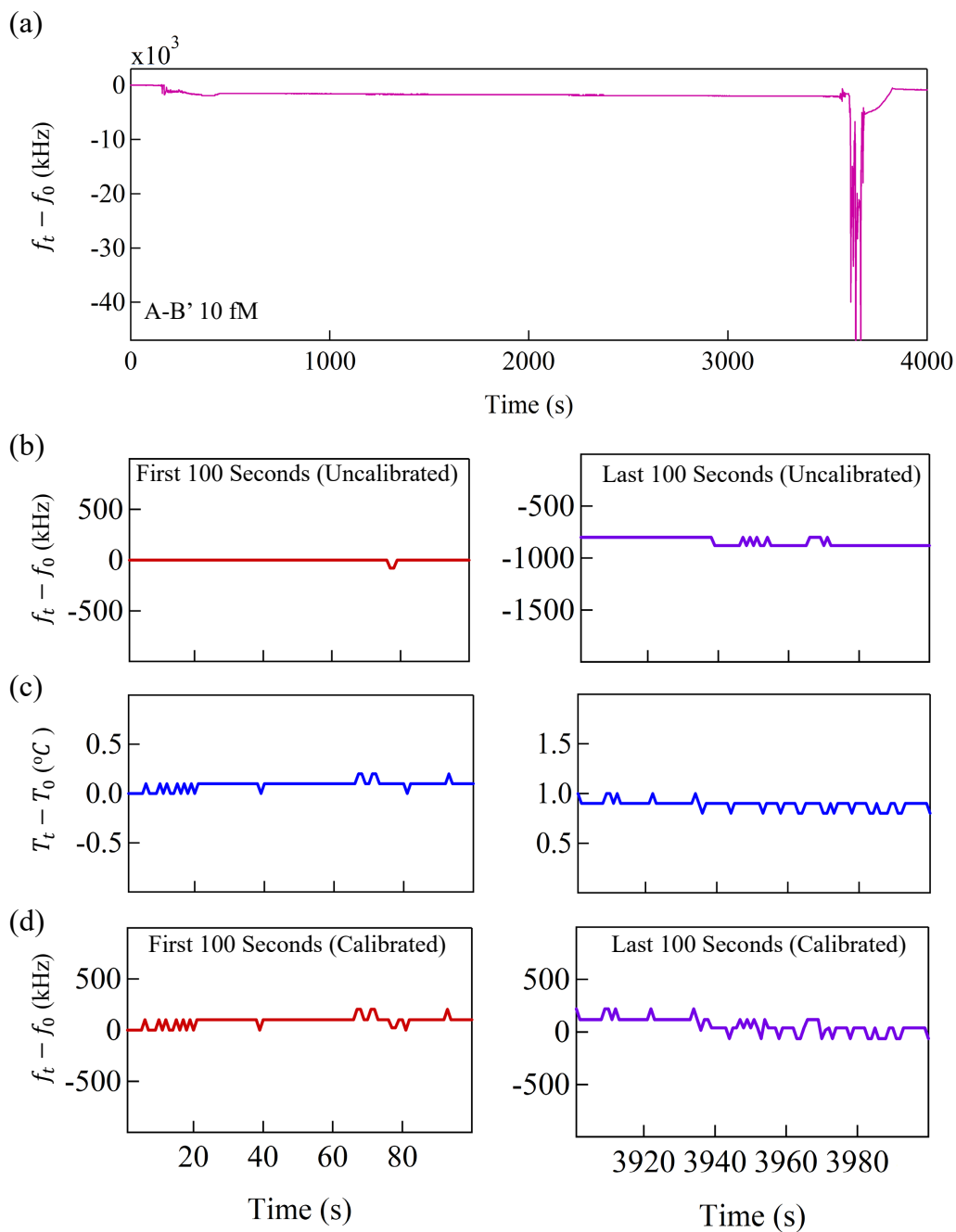


Figure D.15: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 10 fM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

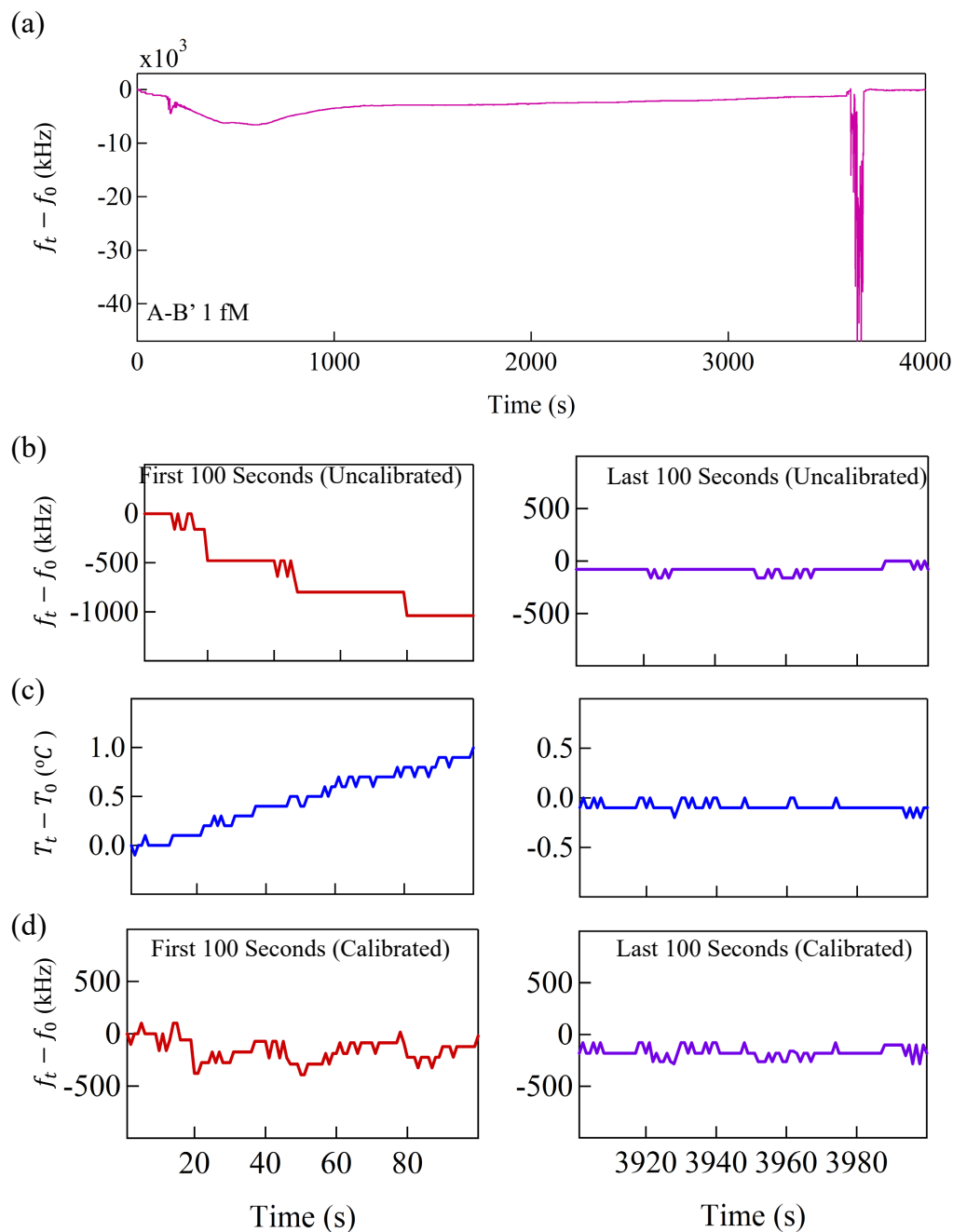


Figure D.16: Data for hybridization experiment (A-B'). Time for hybridization is 60 minutes. Target RNA concentration is 1 fM. (a) Uncalibrated sensor frequency captured during the entire process. (b) Left: First 100 seconds of sensor frequency, uncalibrated. Right: Last 100 seconds of sensor frequency, uncalibrated. (c) First and last 100 seconds of change in temperature. (d) Left: First 100 seconds of sensor frequency calibrated against temperature. Right: Last 100 seconds of sensor frequency calibrated against temperature.

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