

UNIVERSITY OF MANITOBA

PENSION PLANNING AND INFLATION

BY

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the University of Manitoba in partial fulfillment of the requirements
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ABSTRACT

For the last five years, Canadians have been experiencing an average inflation rate of 9% compounded annually. At this rate, the real value of a pension dollar will be halved in roughly 8 years. Over the span of 45 years, from age 20 to age 65, the purchasing power of a dollar will decrease to two cents.

The subject of investigation of this thesis is to develop a computerized model to measure the impact of inflation on pension plans and to devise a plan which would ensure, within defined limits, that the pension benefit would be able to maintain a fixed purchasing power on and after retirement. Then the cost of this plan is examined by simulation of a model plan.

Historical inflation rates are charted.

The effectiveness of the most common methods used to combat inflation is analysed in terms of pre-retirement and post-retirement adjustments. From among these methods, it appears that to ensure an adequate pension benefit an indexed final average plan should be used.

A mathematical model, involving a multiple decrement table, is described and using this model a computer pro-

gram is established. This computer model is a very general one. It can be adapted to calculate the cost of any pension plan.

Using different combinations of interest and inflation rates, the costs of a pension plan are calculated and examined.

The conclusion is reached that the indexed, final-average plan is feasible, subject to reasonable constraints.

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CHAPTER I
EXTENT OF PROBLEM

Introduction

This chapter examines the inflationary trend which has characterized the Canadian economy for the past decade, and focuses on the rate at which inflation is eroding the purchasing power of the pension benefits.

History of Inflation in Canada

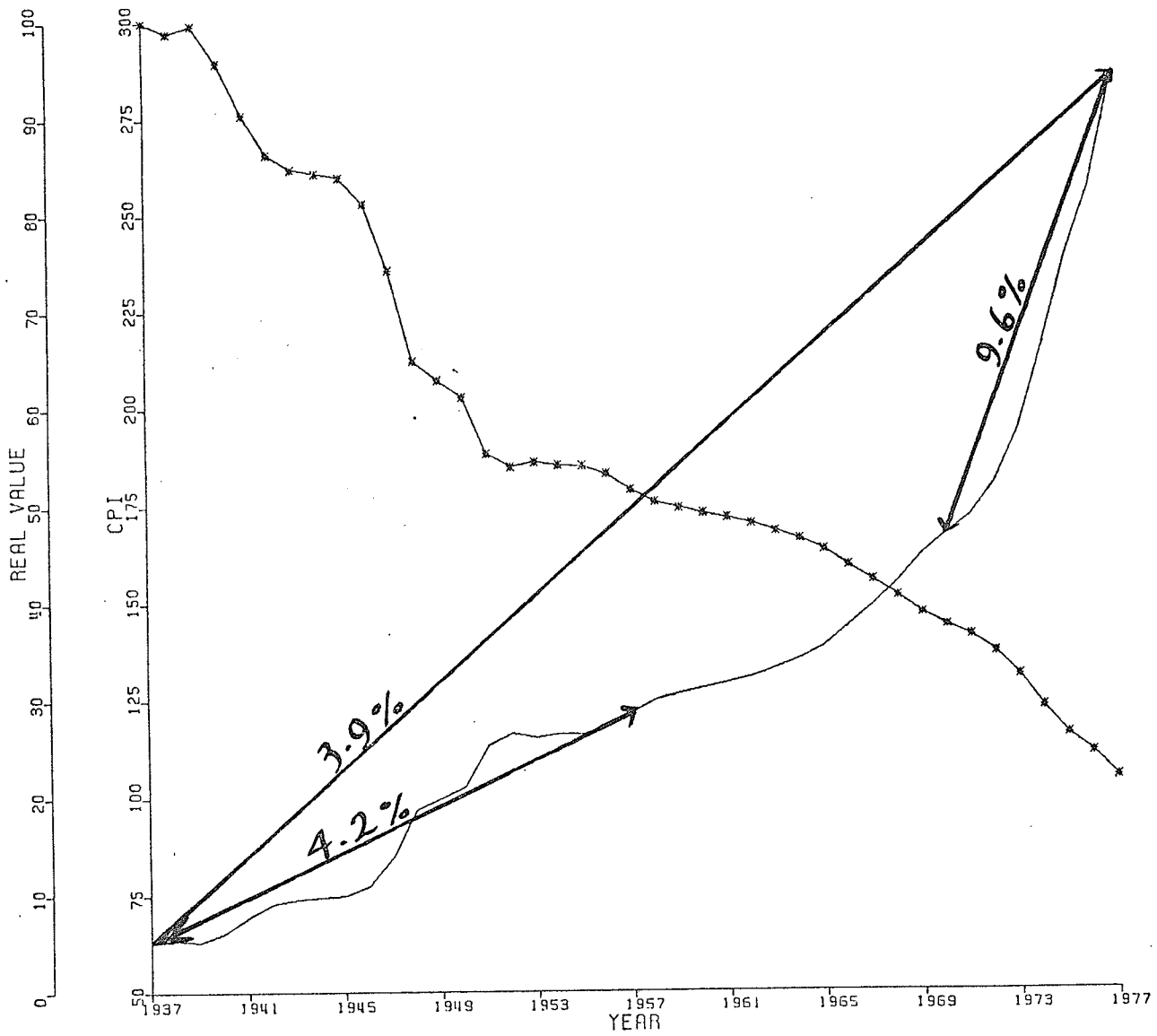
Generally speaking, inflation is measured by tracing a price index. One such example is the Consumer Price Index (CPI). This measure of inflation will be used in this thesis unless otherwise stated.

Figure 1.1 shows the consumer price index during the 1937 - 1977 period. The line with the asterisk (*) shows the purchasing power of a dollar during the same period.

Throughout all these years, inflation averaged 3.9% per year. During the earlier part of this time period inflation rate was slowly increasing and averaged 4.2% a year over a period of twenty years. Only since 1970 has the rate of inflation suddenly accelerated to a feverish 9.6% average.

Taking a succession of periods, all ending in 1977, the

CONSUMER PRICE INDEX, 1937-1977 (1949 = 100)



— CPI
 *— REAL VALUE OF A DOLLAR (1937=100)

SOURCE: CANADA YEAR BOOK, VARIOUS YEARS

average annual rates of inflation are summarized in Table 1.1.

The shorter the period up to the present time, the higher the average rate.

Table 1.1
CHANGES OF CONSUMER PRICE INDEX

<u>Period</u>	<u>Commencing</u>	Average (Compounded) Inflation <u>Rate</u>
1 year	1976	11.5%
5	1972	9.6%
10	1967	6.7%
20	1957	4.3%
30	1947	4.2%
40	1937	3.9%

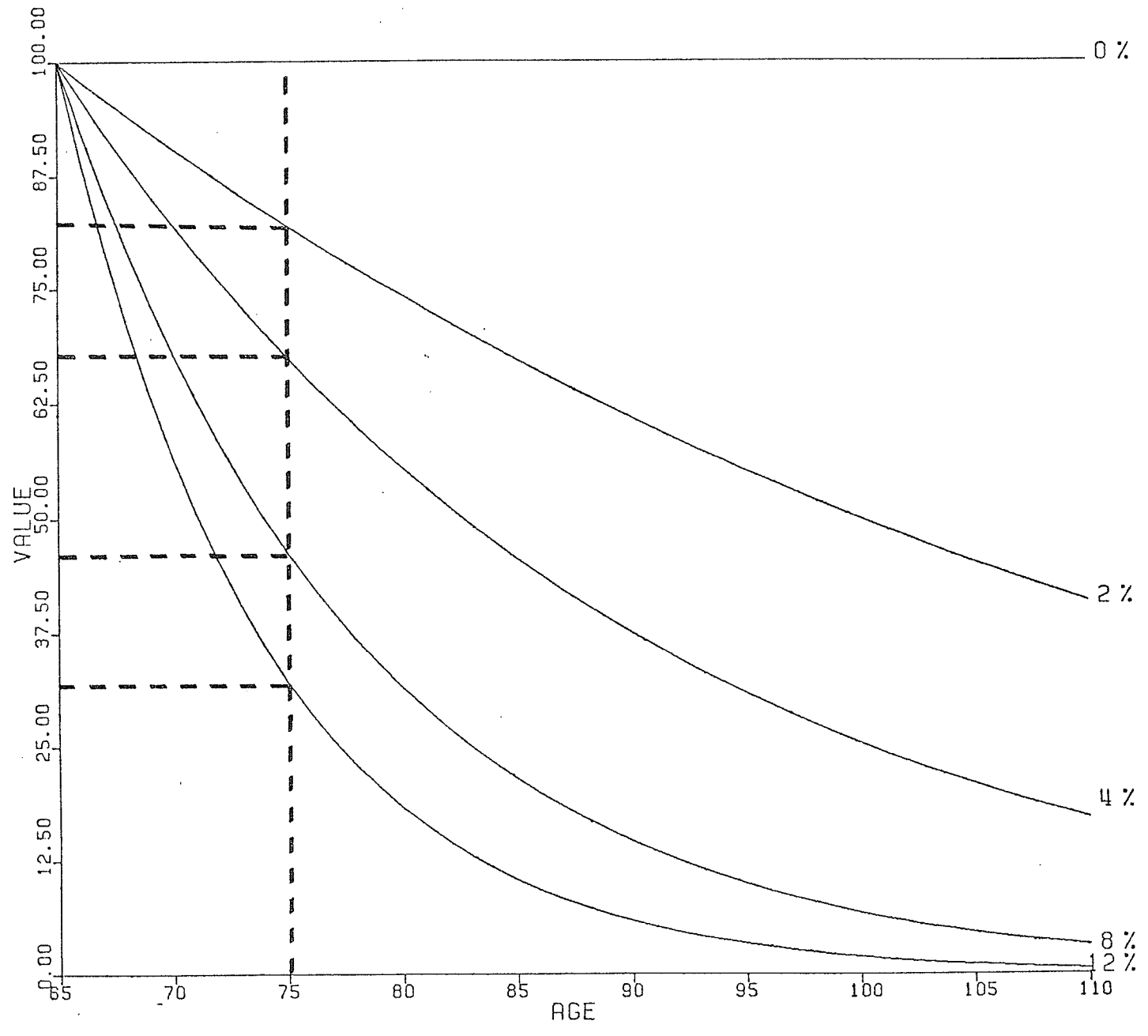
Source: Canada Year Book, various years.

THE EROSION OF PURCHASING POWER OF THE DOLLAR

A history of the purchasing power of the dollar, is shown on Figure 1.1 by the line with the asterisks (*). If one retired in 1937, the real value of his pension would be halved by 1959 (22 years). If he survived another 17-years, he would be left with a pension dollar worth

FIGURE 1.2

VALUE OF PENSION IN REAL DOLLAR
AS PERCENTAGE OF INITIAL VALUE
UNDER INFLATIONARY CONDITIONS



only 20 cents.

Figure 1.2 gives further insight in how the purchasing power of the dollar is eroded. Here inflation rates are taken to be constant and compounded annually.

If there is no inflation, the pension dollar is worth one dollar throughout the retiree's life time. If inflation is 2%, 4%, 8% and 12% annually, in 10 years time, the dollar would be worth only 82 cents, 68 cents, 46 cents and 32 cents respectively.

During an inflationary period money incomes increase at vastly different rates for different groups in our society. Those whose incomes increase faster than the prevailing rate of inflation are able to increase their command over resources and to bid resources away from other groups in periods of scarcity. Those whose money incomes increase in line with the prevailing rate of inflation are at least able to hold their own and are able to maintain their real purchasing power.

Those whose money incomes are fixed, or whose incomes increase less than the prevailing rate of inflation, sustain a steady and continuing erosion of their real purchasing power. In effect, this group will have resources bid away from it by those groups in the community who have had the power to raise their money income above the rate

of inflation.

This means that in an industrial society the groups most vulnerable to the effects of inflation are those who are outside the work force, e.g., the elderly retired members of the population.

If the initial value of the pension dollar represents a fair value the retiree is entitled to at the time of retirement, then there are reasons that this value should not diminish with age as a result of inflation.

Summary and Conclusions

In this chapter the historical inflation rates were first charted and examined. Then the erosion of the purchasing power of the dollar was examined. From these it was pointed out that those on fixed money income were amongst the worst sufferers from the erosive power of inflation.

CHAPTER II
PRESENT CANADIAN SITUATION

Introduction

A pension plan may be established as a result of one or many of the following forces: gratitude of employer, pressure from the union, tax advantage or social pressure. No matter what, the primary function of a pension plan is to provide income to employees in their retirement. Most pension plans can fulfill this basic function since most of them provide retirement income to employees one way or the other. To devise a really efficient plan, it is necessary to extend the basic function. A pension plan should provide adequate income to employees on and after their retirement. In other words, pension benefit should be able to maintain a reasonable and predictable purchasing power.

In the following sections, the present Canadian pension plans are examined in order to find out to what extent these present pension plans are fulfilling their functions and what are being done to improve them.

Survey of Present Pension Plans

Recently, with the rising inflation rate, both employer and employee are reconsidering their pension plans. They

are now taking into account the inflationary force when a new plan is set up or when revising the old ones.

In trying to combat the erosion of the purchasing power of the pension dollar, two inflation risks must be recognized. The employee is faced with the risk of receiving a pension benefit that is not enough to maintain the purchasing power at retirement. On the other hand, the employer is faced with the risk of assuming an unpredictably high pension cost.

A number of adjusting techniques have been developed to give greater assurance that pensions which were deemed adequate when credited would continue to prove adequate at or even after retirement.

These approaches can be differentiated as to whether they pertain to the accruing benefits of active members (pre-retirement adjustment):

- final average salary formula,
- ad-hoc adjustment of salary base,
- variable annuities;

or to the benefit payments to retired members (post-retirement adjustment):

- variable annuities,
- ad-hoc adjustment,
- formula escalator factor,

automatic cost of living adjustment.

Pre-retirement Adjustment¹

Final Average Salary Formula

Under this plan the pension benefits at retirement are computed solely as a function of the employee's average annual compensation over the last few years immediately preceding retirement. Some plans even make the modification of using the average best earnings formula, which eliminates the risk of lowering of pension benefits if one is forced to a lower-paid job during the few years prior to retirement. This plan emphasizes the level of compensation just prior to retirement which reflects increased productivity, greater employee value and inflationary activity during the working years.

A typical final average salary formula would provide an annual retirement income equal to 2% per year of service multiplied by an employee's final average salary.

The logic behind these plans is that the final salary of an employee reflects basically the dollar amount he is earning and which he uses to support his living. Thus, this pension benefit is protected against the erosive

¹ See Appendix A, for a breakdown of plans and members by Type of Benefit. Note the growth in the percentage of members covered by final earnings, final average and average best earnings.

effects of inflation up to the point of retirement.

However, under this approach no future adjustment of benefits is guaranteed. Thus, the benefit dollar that may appear adequate at retirement may, subject to high rates of inflation, soon prove to be inadequate.

Ad-Hoc Adjustments of Salary Base or Flat-Benefit Accruals

A second approach to protecting the accrual benefits of active participants is to make ad-hoc adjustments in the salary base of a career average formula plan or the benefit accruals of a plan that provides a specified dollar benefit for each year of credited service. The rationale is that credits earned by an employee during his early working years are eventually quite out-dated, especially when wages are rising sharply in response to inflationary pressures. Updating the accrued benefit is thus necessary. This can be done by re-calculating the accrued benefit by using an updated salary base which reflects the current economic situation.

The chief characteristic of this approach is that the employer or other plan sponsor does not commit himself in advance to making a given adjustment. Therefore, neither the retirees nor the active members can rely on these

amendments to provide them with adequate pensions.

Variable Annuities

The third approach to protecting the accruing benefits of active participants is by linking the dollar value of benefit accrual with the market value of assets accumulated to pay the benefits. Contributions (usually a fixed flat dollar amount or a fixed percentage of compensation) are invested in a segregated portfolio of equity securities. They are used to establish a fund or an account which, with additional deposit and investment earnings, is used to purchase a life-time annuity at retirement.

The basis underlying the variable annuity concept is the historical long-range relationship between the cost of living and the investment performance of diversified portfolios of common stock. Over the long run, equity prices are expected to rise, reflecting increases in industrial productivity. Dr. George M. Charles¹, in his recent study concludes that "despite the very high rates of inflation of the past five and ten years, the adjusted rates of return for all types of investment were positive during the fifteen year period, 1961-1975." Thus, vari-

¹ George M. Charles, Canadian Securities Industrial Study No. 4: Inflation and Rates of Returns in Selected Investments. The Canadian Experience, 1961 - 1975. Quoted with the kind permission of Dr. Charles.

able annuities not only provide the protection against the erosion of purchasing power, they also provide an opportunity to share in the growth of the economy.

From the standpoint of the employer offering a pension plan, the variable annuity presents a potential disadvantage of relative cost. Under fixed benefit plans, excess investment income earned over a rate guaranteed is used to reduce the employer's cost. Under the variable annuity plan, all investment incomes are passed on to the plan participants.

The disadvantage of this approach is that the amount of pension benefit can never be guaranteed to be adequate, because there is no guarantee that the fund will keep pace with inflation.

Post-retirement adjustment

Variable annuities¹

The same basic principle that is used in pre-retirement adjustment for variable annuities is used here.

At retirement, the accrued contributions (regardless of how they are accrued) are invested. Pension benefits

¹ Situation in Canada as reported by Statistics Canada: There were, in 1974, 71 variable annuity plans with a total membership of 12,708 persons - an increase from 53 plans covering 7,100 individuals that were in operation in 1970.

received by the retirees rise and fall with the stock market. There is no guarantee of a minimum benefit. The participant assumes the investment risk. For the employer, his contributions are fixed in advance. He is relieved of the investment risk. On the other hand the employer now cannot depend on the experience gains (from excess investment earnings) to reduce his future contributions.

The main criticism of this approach is that the expected correlation between equity performance and living standard is a long-term one, whereas the relationship of a retiree's income to his expenses usually is a short-term problem.

The experience of the last few years shows that investment income can lag behind inflation. As Pesando and Rea¹ put it, "over a sufficiently long period of time, the real ex post return on equities should be positive, but this result cannot be used to support the conclusion that the nominal returns on equities rise sufficiently fast in periods of accelerating inflation to ensure that their real returns will not fall." In other words, there are times when inflation and investment return move dif-

¹ James E. Pesando, Samuel A. Rea, Public and Private Pensions in Canada: An Economic Analysis. (Toronto: University of Toronto Press, 1977), p. 161.

ferently. It is especially painful for a retiree to receive a smaller pension as measured by the real value of the dollar when inflation rises. This creates a kind of pressure for the plan sponsor to step in and make ad-hoc adjustments. Therefore these plans could be more expensive in the end, because the situation becomes one of "heads the employee wins, tails the employer loses."

Ad-Hoc Adjustment¹

Another approach to the protection of the purchasing power of benefits is that of ad-hoc adjustments. The employer increases the benefit payments periodically. The amount of increase is usually related to the changes in the consumer price index (CPI).

By using this approach the employer has full control over the cost of the pension plan. He is by no means committed to any further adjustment. On the other hand, there is no way he can predict the change and consequently fund it in advance.

From the employee's view, he cannot be guaranteed that the benefits would be in line with CPI changes. He is "at the mercy" of his plan sponsor. This makes the plan look

¹ It was not possible in the pension data bank system of Statistics Canada to identify and record the ad-hoc adjustments so that there were no empirical data on the extent of this practice.

more like a "welfare plan" than a pension plan. Pension benefits should be regarded as a deferred employee compensation or, in the case of a contributing plan, a right he has paid for. By making the plan look like a welfare plan, the pensioners are subjected to uncertainty. Moreover, the time lag between adjustments may cause some hardship to the retirees.

Formula Escalator Factor

Under this approach the plan stipulates that the benefits of all retired individuals will be automatically adjusted upward each year by a specified amount. Such an increase is built into the plan. The increase is automatic regardless of how much CPI is changing.

Under this plan, when one retires, one is usually given a choice between a fixed pension benefit or a reduced initial benefit with a guaranteed annual increase. Table 2.1 illustrates the relationship between a fixed benefit and a formula escalator plan.

Under a regular fixed pension plan, if one retires with \$100 annual income, this dollar amount is going to remain constant throughout the retirement. If he chooses to have a reduced initial amount, say \$50, the actuarial equivalent escalator factor would be \$7. His pension would start-off at \$50 and increase by \$7 annually. The pension

TABLE 2.1
PENSION BENEFITS

UNDER FORMULA ESCALATOR FACTOR

INITIAL VALUE	ESCALATOR FACTOR					
	50.00	60.00	70.00	80.00	90.00	100.00
ESCALATOR FACTOR	7.21	5.77	4.33	2.88	1.44	0.00
AGE						
66	50.00	60.00	70.00	80.00	90.00	100.00
67	57.21	65.77	74.33	82.88	91.44	100.00
68	64.42	71.53	78.65	85.77	92.88	100.00
69	71.63	77.30	82.98	88.65	94.33	100.00
70	78.84	83.07	87.30	91.53	95.77	100.00
71	86.04	88.84	91.63	94.42	97.21	100.00
72	93.25	94.60	95.95	97.30	98.65	100.00
73	100.46	100.37	100.28	100.19	100.09	100.00
74	107.67	106.14	104.60	103.07	101.53	100.00
75	114.88	111.90	108.93	105.95	102.98	100.00
76	122.09	117.67	113.25	108.84	104.42	100.00
77	129.30	123.44	117.58	111.72	105.86	100.00
78	136.51	129.21	121.90	114.60	107.30	100.00
79	143.72	134.97	126.23	117.49	108.74	100.00
80	150.93	140.74	130.56	120.37	110.19	100.00
81	158.13	146.51	134.88	123.25	111.63	100.00
82	165.34	152.27	139.21	126.14	113.07	100.00
83	172.55	158.04	143.53	129.02	114.51	100.00
84	179.76	163.81	147.86	131.90	115.95	100.00
85	186.97	169.58	152.18	134.79	117.39	100.00
86	194.18	175.34	156.51	137.67	118.84	100.00
87	201.39	181.11	160.83	140.56	120.28	100.00
88	208.60	186.88	165.16	143.44	121.72	100.00
89	215.81	192.65	169.48	146.32	123.16	100.00
90	223.02	198.41	173.81	149.21	124.60	100.00
91	230.22	204.18	178.13	152.09	126.04	100.00
92	237.43	209.95	182.46	154.97	127.49	100.00
93	244.64	215.71	186.79	157.86	128.93	100.00
94	251.85	221.48	191.11	160.74	130.37	100.00
95	259.06	227.25	195.44	163.62	131.81	100.00

dollar would be less than that of the regular fixed pension plan in the first seven years. However, the dollar amount would be more from the eighth year on. If one survives to age 95, the pension amount then would be more than double the regular fixed one.

The present values of the fixed pension and that of the formula escalator factor one are actuarially equivalent. The derivation can be found in Appendix B.

By commitment to a pre-determined rate of increase, the benefit payment may end up to be too much or too little.

Automatic Cost-of Living Adjustment¹

Under this approach, the pension benefit is adjusted according to the changes in a price index (usually the CPI). The benefit changes are guaranteed and automatic.

This is the most direct and responsive method of adjusting pension benefits to changes in price levels. The time lag is minimal.

The main argument against the widespread establishment of these plans is that the cost can go out of control.

¹ Situation in Canada according to Statistics Canada: There were some increases in incidence of indexing in pension plans from 1970 to 1974 (the latest report available). The number of plans increased from 60 with 191,700 members in 1970 to 141 plans with 607,894 members in 1974. Out of the 141 plans 101 were in the private sector, but these tended to be smaller plans interspersed with a few larger ones, so that only 41,560 workers, 2% of the total membership in the private sector, had this benefit.

The cost is unpredictable.

Dr. Charles¹ in his recent study has shown that in periods of very high inflation and declining stock market indexes reinvestment of dividends plays a dominant role in determining the overall rate of investment return. It is reasonable to expect that the pension fund would respond in a similar manner. If we view the cost of living adjustment as the distribution of the excess investment return (over the expected return used in the initial actuarial calculation), then providing cost of living adjustment should be costless.

All of these arguments are still open to debate. The discussion is deferred until later when a model pension plan is structured and costs simulated.

Summary and Conclusions

Of all the different approaches listed above, which one (or what combination of different ones) is the most efficient?

Of the pre-retirement adjustments, the ad-hoc adjustment of salary base and the variable annuities approaches do not guarantee the employees adequate pension benefits. The benefit is under the discretion of management and the

1 Ibid., P. 25.

ups and downs of the stock market respectively. Under the final average salary approach, pension benefit is fixed at a certain percentage of final average salary, employees can calculate, in advance, his pension benefits.

Among the many post-retirement adjustments, the automatic cost of living adjustment approach is the most direct and responsive one. In the ad-hoc adjustment approach, the delay of management to adjust the benefit payments may cause anxiety and a sense of insecurity. Furthermore this puts the retirees in an inferior position by giving the employer full control of the benefit increase.

If the variable annuity is used, hardship may result during periods when CPI and the stock prices move in opposite directions.

The pre-determined rate of benefit increase used in the formula escalator factor approach may end up to be totally unrelated with the general price level.

Only by changing the benefit payment in response to changes of CPI, can the retirees be fully protected.

To give the best protection of the benefits, the best approaches from the pre-retirement and post-retirement adjustment have to be combined, i.e., a plan giving final average benefit at retirement, with benefit adjustment in

response to CPI changes.

In the next chapter, a mathematical model is first established. Then costs of a model pension plan are simulated. By changing the parameters of the model plan, namely the interest rate, inflation rate and cost of living adjustment (COLA) rates, it is hoped that insight could be gained from the study. How costly is a final average plan compared with a cost of living adjustment plan?

CHAPTER III
MATHEMATICAL AND COMPUTER MODEL

Introduction

The principal benefit provided by a pension plan is the pension provided upon retirement of a member. Different benefits may also be claimed upon death, withdrawal, or disability. Due to the difference in benefit provided, members leaving the group are distinguished by their modes of exit from the plan.

The basic mathematical model for the analysis of this is the multiple decrement table. This model is not a new concept. C. W. Jordan¹ gave a very thorough treatment of the model. However, the differential equation approach used in this thesis is quite different. It is adapted from W. S. Bicknell's Ph.D. dissertation².

¹ C. W. Jordan, Life Contingencies, Chicago, The Society of Actuaries, 1975. Chapter 14.

² W. S. Bicknell, Premiums and Reserves in Multiple Decrement Theory. Ph.D. dissertation, University of Michigan, 1957.

Multiple Decrement Table

A multiple decrement table is a mathematical model which assumes a large body of lives subjected to several independent causes of decrement which are operating continuously. The body of lives forms a closed group, there being no new entrants and no re-entrants after the operation of the various decrements.

The multiple decrement table gives not only the number of survivors at any time, but also the number who drop out due to each cause in a given period. Associated with each of the various causes of decrement is a benefit payable on the event of leaving the group, the size of the benefit depending on the particular mode and time of exit from the plan.

Definitions and Notations

$l_{x+t}^{(T)}$ = total number of persons in the group at time t and aged $x+t$.

$d_{x+t}^{(T)}$ = total number of persons who leave the group from cause (k) between ages x and $x+t$.

${}_t p_x^{(T)}$ = probability that a person aged x and subject

to all m causes of decrements will remain in the group after t years.

${}_t q_x^{(T)}$ = probability that a person aged x will drop out of the closed group before t years.

$\mu_{x+t}^{(i)}$ = force of the i th decrement acting on a person aged $x+t$. (This force is assumed known for all t .)

$B_{x+t}^{(i)}$ = the benefit paid when exiting at age $x+t$ due to cause i .

δ = force of interest.

$\bar{P}_t^{(T)}$ = premium paid continuously at time t .

Commutation Functions

$${}_t E_x^{(T)} = v^t {}_t p_x^{(T)}$$

$${}_t F_x^{(T)} = 1/{}_t E_x^{(T)}$$

$$D_{x+t}^{(T)} = v^{x+t} l_{x+t}^{(T)}$$

$$N_{x+t}^{(T)} = \sum_{k=t}^{w-x-1} D_{x+k}^{(T)}$$

$$C_{x+t}^{(i)} = v^{x+t+1} d_{x+t}^{(i)}$$

$$M_{x+t}^{(i)} = \sum_{k=t}^{w-x-1} C_{x+k}^{(i)}$$

Premium-Reserve Equation

Under the usual annuity or insurance policy, there is a different incidence of the payment into the fund and the outgo for the benefits. Payment schemes generally are such that, during the early years of the plan, the premium payments to the fund are in excess of the current benefit outgo. Thus a fund accumulates. The average amount of this fund per member t years after the establishment of the plan, or after the entry into the plan, is called the reserve at time t .

Suppose there are $l_x^{(T)}$ in the original closed group, the total fund at time t would be $l_{x+t}^{(T)} \bar{v}_t^{(T)}$. The change to the reserve can be expressed as follows:

$$\frac{d(l_{x+t}^{(T)} \bar{v}_t^{(T)})}{dt} = l_{x+t}^{(T)} \bar{P}_t^{(T)} + \delta l_{x+t}^{(T)} \bar{v}_t^{(T)} - \sum_{i=1}^m l_{x+t}^{(T)} \mu_{x+t}^{(i)} B_{x+t}^{(i)} \quad (1)$$

The above differential equation can be interpreted as follows: the annual rate of change in the total reserve $(\frac{d(l_{x+t}^{(T)} \bar{v}_t^{(T)})}{dt})$ equals the sum of the annual rate of increase in the fund from premium payment $(l_{x+t}^{(T)} \bar{P}_t^{(T)})$ and the annual rate of increase from interest income $(\delta l_{x+t}^{(T)} \bar{v}_t^{(T)})$, less the annual rate of benefit payments

due to the m modes of exit from the plan

$$\left(\sum_{i=1}^m l_{x+t}^{(T)} \mu_{x+t}^{(i)} B_{x+t}^{(i)} \right).$$

Differentiating the left side,

$${}_t \bar{v}^{(T)} \frac{d}{{}^t \bar{v}^{(T)}} l_{x+t}^{(T)} + l_{x+t}^{(T)} \frac{d}{{}^t \bar{v}^{(T)}} {}_t \bar{v}^{(T)}$$

Rearranging:

$$\frac{d}{{}^t \bar{v}^{(T)}} {}_t \bar{v}^{(T)} = \bar{P}_t^{(T)} + \delta {}_t \bar{v}^{(T)} + {}_t \bar{v}^{(T)} \mu_{x+t}^{(T)} - \sum_{i=1}^m B_{x+t}^{(i)} \mu_{x+t}^{(i)} \quad (2)$$

Once again, the equation can be explained verbally as follows:

the change in individual reserve per active member per year $\left(\frac{d}{{}^t \bar{v}^{(T)}} {}_t \bar{v}^{(T)} \right)$ equals the sum of annual continuous premium paid $\left(\bar{P}_t^{(T)} \right)$ by each member, interest income $\left(\delta {}_t \bar{v}^{(T)} \right)$, and reserve released by other members who have left the group $\left({}_t \bar{v}^{(T)} \mu_{x+t}^{(T)} \right)$, less the benefit payment due to the m modes of exiting $\left(\sum_{i=1}^m B_{x+t}^{(i)} \mu_{x+t}^{(i)} \right)$.

Some interesting relationships can be derived by integrating the above equation.

Rearranging (2) in the form:

$$\frac{d}{{}^t \bar{v}^{(T)}} {}_t \bar{v}^{(T)} - \left(\delta + \mu_{x+t}^{(T)} \right) {}_t \bar{v}^{(T)} = \bar{P}_t^{(T)} - \sum_{i=1}^m B_{x+t}^{(i)} \mu_{x+t}^{(i)}$$

Multiply each side by ${}_tE_x$ ($= e^{-\int_0^t (\delta + \mu_{x+h}^{(T)}) dh}$),

and noting that $\frac{d {}_tE_x}{dt} = (\delta + \mu_{x+t}^{(T)}) {}_tE_x$

$${}_tE_x \frac{d {}_t\bar{V}^{(T)}}{dt} - {}_tE_x (\delta + \mu_{x+t}^{(T)}) {}_t\bar{V}^{(T)} = {}_tE_x (\bar{P}_t^{(T)} - \sum_{i=1}^m B_{x+t}^{(i)} \mu_{x+t}^{(i)})$$

$$\frac{d ({}_tE_x {}_t\bar{V}^{(T)})}{dt} = {}_tE_x (\bar{P}_t^{(T)} - \sum_{i=1}^m B_{x+t}^{(i)} \mu_{x+t}^{(i)}) \quad (3)$$

Let $\delta^s =$ force of salary; $\bar{P}_t^{(T)} = \bar{P}^{(T)} e^{\delta^s t}$; $\bar{P}^{(T)}$ = initial premium.

$$\begin{aligned} \int_0^n \bar{P}_t^{(T)} {}_tE_x dt &= \int_0^n \bar{P}^{(T)} e^{\delta^s t} v^t {}_tP_x^{(T)} dt \\ &= \int_0^n \bar{P}^{(T)} e^{\delta^s t} e^{-\delta t} {}_tP_x^{(T)} dt = \bar{P}^{(T)} \int_0^n e^{(\delta^s - \delta)t} {}_tP_x^{(T)} dt \\ &= \bar{P}^{(T)} s_{\overline{n}|x:\overline{n}|}^{-(\delta^s - \delta)} \end{aligned}$$

From these it can be seen that if the premium is subjected to a force of salary δ^s , then the annuity value would be subjected to a force of interest equal to $-(\delta^s - \delta)$.

Integrating (3) from $t=0$ to $t=n$ (where n is the number of the year after entry that we are interested in):

$${}_n E_x^{(T)} B_{x+t}^{(T)} - {}_0 \bar{V}^{(T)} = \bar{P}^{(T)} s_{\bar{x}:\overline{n}|-(\delta^s-\delta)} - \sum_{i=1}^m \int_0^n B_{x+t}^{(i)} \mu_{x+t}^{(i)} {}_t E_x^{(T)} dt$$

$$\bar{P}^{(T)} = \left\{ \sum_{i=1}^m \int_0^n B_{x+t}^{(i)} \mu_{x+t}^{(i)} {}_t E_x^{(T)} dt + {}_n E_x^{(T)} B_{x+n}^{(T)} - {}_0 \bar{V}^{(T)} \right\} / s_{\bar{x}:\overline{n}|-(\delta^s-\delta)} \quad (4)$$

Formula (4) shows the total initial premium, $\bar{P}^{(T)}$, as the sum of 2 premiums: the first a term premium for the benefits provided due to the m modes of exiting from group, and the second a pure endowment premium for the excess value of the maturity benefit over any initial deposit.

From (3) we can also obtain the related terminal reserve ${}_t \bar{V}^{(T)}$.

Rewrite (3) by replacing t by s :

$$\frac{d {}_s E_x^{(T)} B_s^{(T)}}{ds} = {}_s E_x^{(T)} (\bar{P}_s^{(T)} - \sum_{i=1}^m B_{x+s}^{(i)} \mu_{x+s}^{(i)})$$

Integrating from $s=0$ to $s=t$, we obtain:

$${}_t E_x^{(T)} B_t^{(T)} - {}_0 \bar{V}^{(T)} = \bar{P}^{(T)} s_{\bar{x}:\overline{t}|-(\delta^s-\delta)} - \sum_{i=1}^m \int_0^t B_{x+t}^{(i)} {}_s E_x^{(T)} \mu_{x+s}^{(i)} ds$$

$${}_t \bar{V}^{(T)} = \bar{P}^{(T)} s_{\bar{x}:\overline{t}|-(\delta^s-\delta)} + {}_t E_x^{(T)} {}_0 \bar{V}^{(T)} - \sum_{i=1}^m \int_0^t B_{x+s}^{(i)} {}_{t-s} E_x^{(T)} \mu_{x+s}^{(i)} ds \quad (5)$$

The above can be seen as the usual form of the retrospective reserve formula.

The retrospective reserve can be interpreted as the sum

of the total accumulated value of premiums paid

$(\bar{P}^{(T)} \ddot{s}_{x:\overline{t}|}^{-}(\delta^s - \delta))$, and the accumulated value of the initial reserve $({}_t F_x^{(T)} \bar{V}^{(T)})$, less the total accumulated value of benefit payments paid to members who have left due to all m modes of exiting $(\sum_{i=1}^m \int_0^t {}_s B_{x+s}^{(i)} {}_{t-s} F_{x+s}^{(T)} \mu_{x+s}^{(i)} ds)$.

The usual prospective reserve can also be obtained by integrating (3) from $s=t$ to $s=n$:

$$\begin{aligned} n E_x^{(T)} B_{x+n}^{(T)} - t E_x^{(T)} \bar{V}^{(T)} &= \int_t^n \bar{P}_s^{(T)} {}_s E_x^{(T)} ds - \sum_{i=1}^m \int_t^n {}_t B_{x+s}^{(i)} {}_s E_x^{(T)} \mu_{x+s}^{(i)} ds \\ t \bar{V}^{(T)} &= \sum_{i=1}^m \int_0^t {}_0 B_{x+t+h}^{(i)} {}_h E_{x+t}^{(T)} \mu_{x+t+h}^{(i)} dh + n-t E_{x+t}^{(T)} B_{x+n}^{(T)} - \bar{P}^{(T)} \ddot{s}_{x+t:\overline{n-t}|}^{-}(\delta^s - \delta) \end{aligned} \quad (6)$$

In this derivation, the total reserve is given as the sum of the present value of the future benefits (claims paid to terminating members and the maturity benefits), less the present value of all future contributions.

It is stressed that the number of decrements (m) can take on any positive integer values. The above model can be described as a flexible one. It works regardless of the number of decrements.

The Computer Program

A characteristic of the mathematical model is its flexibility. Decrements are additive. Decrements can be introduced with a minimal change. The algorithm of the computer program was devised following this line of thought.

The program is made up of three distinct and independent levels.

The lowest level consists mainly of subroutines. These subroutines defines the basic actuarial operations. These are the actuarial equivalents of the elementary operations in arithmetic. Take, for example, a call to the subroutine SUM will be equivalent to a summation of some specified entries of an array. With this subroutine established, there is no need to resort to the basic principle of a DO-loop every time a summation is required. A call to the subroutine (with the appropriate parameters) would be all that is needed. In other words, these subroutines would supply the basis for future calculations.

The next level is named the "action" level. It is consisted mainly of the so called remote blocks¹. Each block performs one (and only one) function. For example, in the remote block BDEATH, the present values of the benefits due to the decrement of death are calculated. Similarly, in the remote block CONBUT, the future value of contribu-

¹ REMOTE BLOCK is a feature of structured WATFIV. It is helpful in avoiding duplication of code for a function required in a number of places throughout the program. Executing a REMOTE BLOCK is similar in concept to calling a subroutine, with the advantage that shared variables do not need to be placed in a COMMON block or passed in the argument list. Further reference can be found in the documentation Structured Programming Facilities in WATFIV of the Computer Centre, University of Manitoba.

tions are calculated. The same can be said about BWITH where the present value of benefits due to withdrawal are calculated. In the block RSERVE, the growth of the reserve is traced.

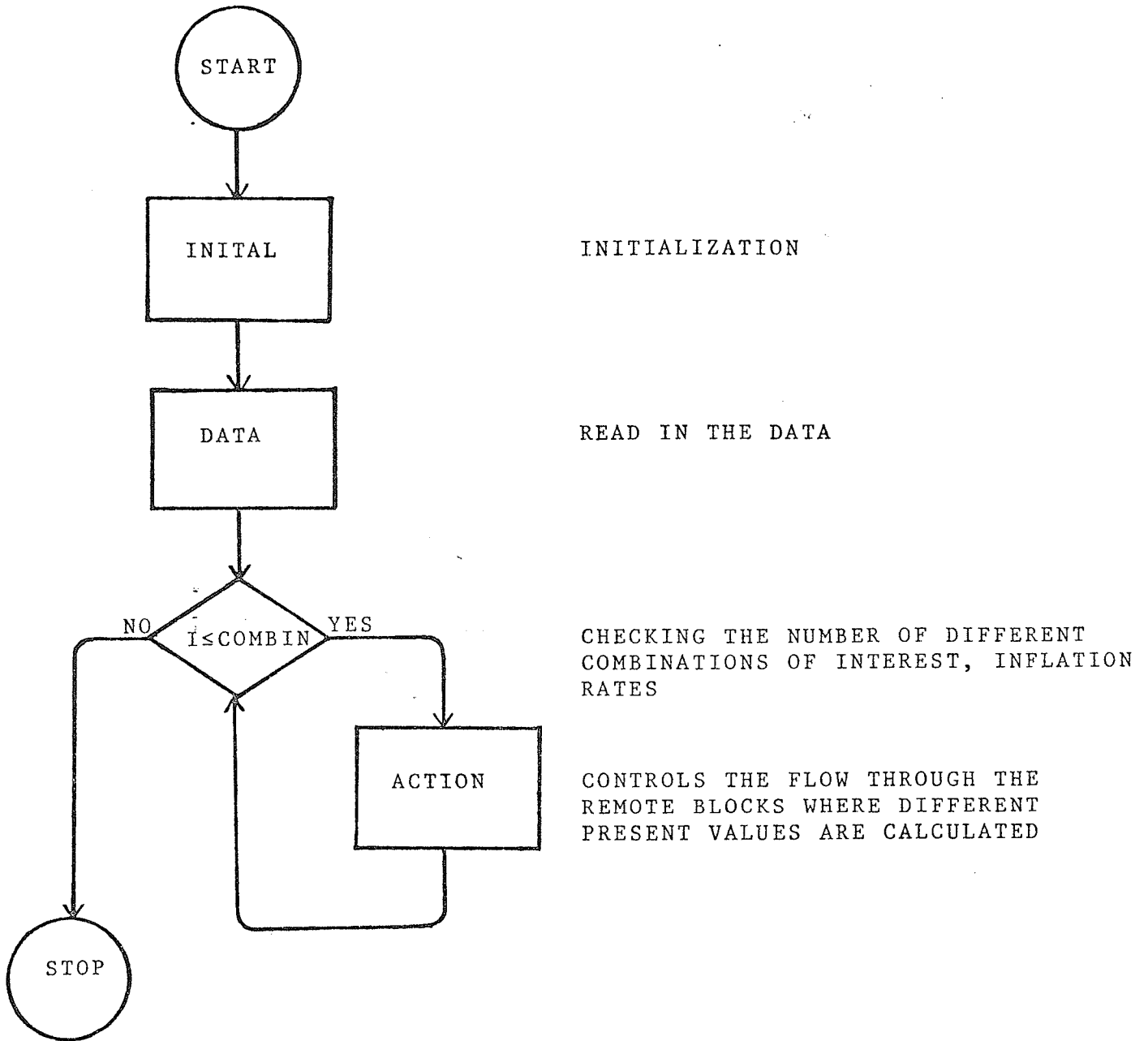
These remote blocks work independently. Blocks can be added or deleted without much changes. Each remote block makes extensive use of the operations defined by the sub-routines in the lowest level.

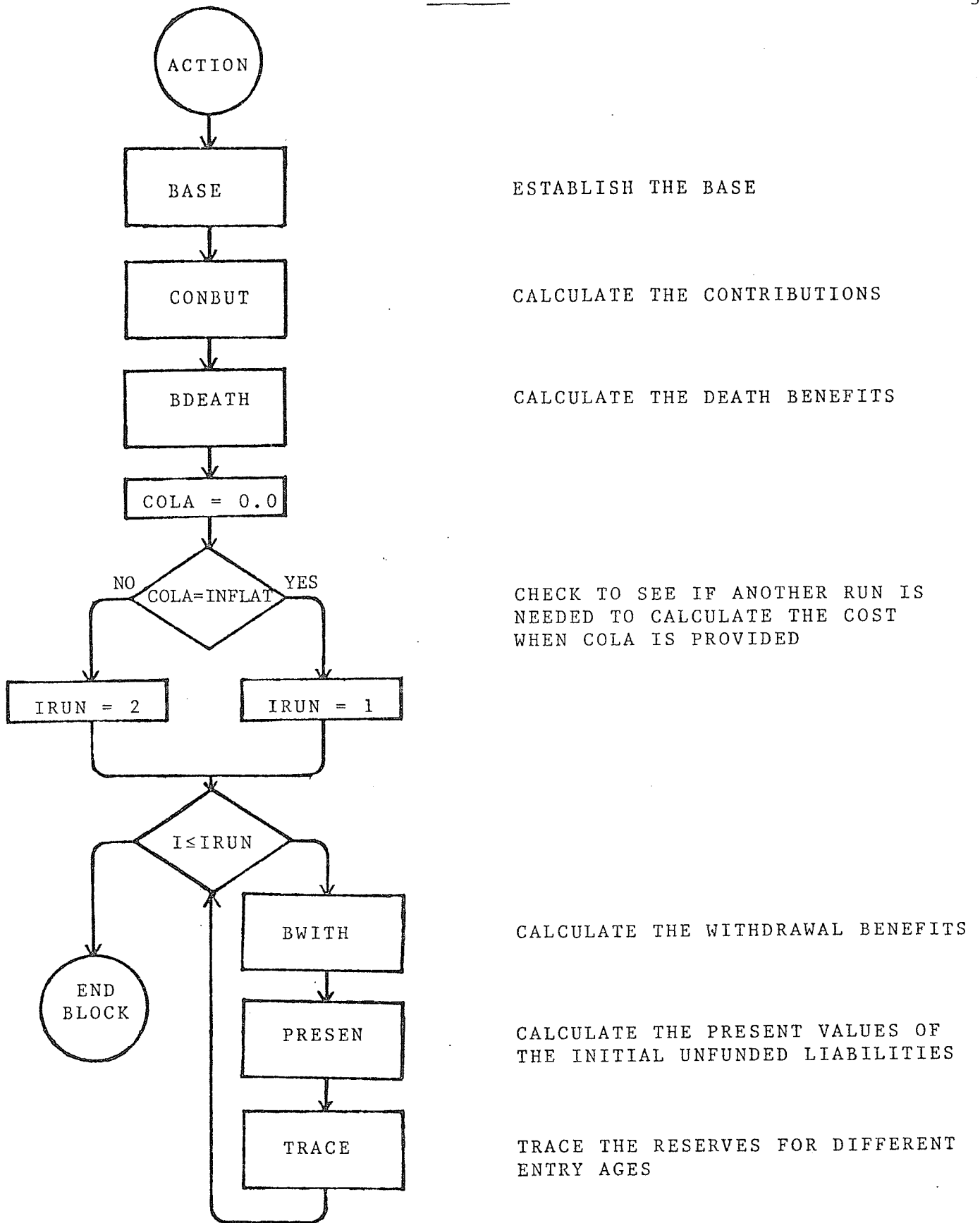
The upper level is called the "control". This controls the flow of logic. The two flow charts on the following pages illustrate briefly how execution flows through the remote blocks.

First of all, variables are initialized (by executing INITAL) and data are read in (by executing DATA). These are going to be the data base. Changes of the data are to be introduced here. If, for example, a new set of interest and inflation rates are required, the program can be re-run after a change in the respective rates in the remote block INITAL. A new set of mortality values, withdrawal rates or salary scales can be used by reading in the new values in stead of the old.

The remote block ACTION controls the flow through the blocks in the middle level where present values are calculated.

THE CONTROL





Remote block BASE is first executed. Discount factors are established. New Salary Scales are set up taking into account of the inflationary force on the salary. Present values of the contributions and death benefits are calculated by passing control to the two remote blocks, CONBUT and BDEATH. The present value of the withdrawal benefits (including the vested pension benefits) changes according to whether COLA is provided. If the inflation rate is zero, there is no need to provide COLA. Thus, before calculating the expected withdrawal benefit, a check is performed to see if 2 withdrawal costs are needed - one for non-indexed and one for indexed pension benefits. Withdrawal benefits and the initial unfunded liabilities are then calculated. Reserves of 3 members, age 30, 40 and 50, are traced by executing TRACE. The trace is performed to see how the reserves grow. This block has no significance in the overall cost evaluations.

All of the above calculations, from BASE to TRACE, would be executed every time ACTION is executed.

As mentioned before, the program is very flexible. If, for example, the trace of the reserve is no longer needed, changes that are required are:

delete 1 statement from ACTION

delete the remote blocks TRACE and RSERVE.

The program can then be run again.

By setting up the program in this way, changes can be made with a minimal effort. Moreover, the flow of logic can be followed easily.

With the above computer program, the cost for a particular pension plan is calculated.

Model Plan Provisions

(a) In the event of retirement at 66, a life annuity of 2% of final annual pay for each year of service is provided beginning at age 66.

(b) In case of death, the total amount of the contribution of the employer and employee, accumulated with interest, is provided.

(c) In case of termination:

(i) within the first 10 years of service, the total amount of the employee contributions accumulated with interest is provided.

(ii) after 10 years of service, a vested pension benefit is provided, i.e., a deferred annuity in the amount of 2% of the final annual pay at time of withdrawal per each year of service is provided at age 66.

Contributions

Employer and employee each contributes 7.5% of the employee's annual pay at the beginning of each year.

Employer assumes all initial unfunded liabilities -- whether or not the total contributions is enough to cover all benefit payments.

In the experiment, for simplicity, it is assumed that every member enters the plan with an annual salary of ten dollars.

Calculations

In this case, the causes of decrement are death and withdrawal. The annual rates of death are taken from the 1971 Group Annuity Mortality Table. The withdrawal rates are taken from Table VI of the non-select employment tables¹.

The premium contributions, $\bar{P}_t^{(T)}$, death benefit, $B_{x+t}^{(d)}$, termination benefits, $B_{x+t}^{(w)}$, and retirement benefits, $B_{x+t}^{(r)}$ are fixed (refund of contributions or a final salary pension benefit). The only variable left for comparison is the initial reserve ${}_0\bar{V}^{(T)}$.

$$\text{From (6) } {}_t\bar{V}^{(T)} = \sum_{i=1}^m \frac{\eta_{x+t+h}^{(i)}}{v^{i-t}} \left[\frac{E_{x+t}^{(T)}}{v^{i-t}} \mu_{x+t+h}^{(i)} dh + \frac{E_{x+t}^{(T)} B_{x+n}^{(T)} - \bar{P}^{(T)} \bar{a}_{x+t:\overline{n-t}|}}{v^{i-t}} \right]_{\delta^s - \delta}$$

¹ Daniel F. McGinn, Indices to the Cost of Vested Pension Benefits, Appendix C, Table C1, Transactions of the Society of Actuaries, Volume XVIII, Part 1, 1966. P. 235.

For $t=0$:

$${}_0\bar{V}^{(T)} = \sum_{i=1}^m \int_0^n B_{x+h}^{(i)} h E_x^{(T)} \mu_{x+h}^{(i)} dh + n E_x^{(T)} B_{x+n}^{(T)} - \bar{P}^{(T)} s_{\overline{x:n}|}^{(\delta^s - \delta)}$$

In this case, since both the premium payments and the claim benefits are payable at the beginning of each year, (6) is adapted to a discrete model with

m = number of decrements = 2

n = number of years after entry to plan when pension

benefit is first paid = $66 - x - 1$

$$\begin{aligned} {}_0\bar{V}^{(T)} &= \sum_{i=1}^2 \sum_{h=0}^{66-x-1} B_{x+h}^{(i)} h E_x^{(T)} d_{x+h}^{(i)} + n E_x^{(T)} B_{x+n}^{(T)} - \bar{P}^{(T)} s_{\overline{x:n}|}^{(\delta^s - \delta)} \\ &= \frac{1}{D_x^{(T)}} \left\{ \sum_{i=1}^2 (B_M^{(i)} - B_{M66}^{(i)}) + B_{D66}^{(T)} - \bar{P}^{(T)} (S_N^{(T)} - S_{N66}^{(T)}) \right\} \end{aligned} \quad (7)$$

In this study, the different initial reserves resulting from different combinations of interest, inflation and Cost of Living Adjustment (COLA) rates are calculated and compared.

In the computer program, (7) is broken up into

$${}_0\bar{V}^{(T)}$$

the initial reserve.

$$= \frac{1}{D_x^{(T)}} (B_M^{(d)} - B_{M66}^{(d)})$$

the total death benefit payable.

+ $\frac{1}{D_x^{(T)}} (B_M^{(w)} - B_{M66}^{(w)} + B_{D66}^{(T)})$ the total withdrawal benefit payable. Those retiring at age 65 are also treated as a withdrawal.

- $\frac{1}{D_x^{(T)}} \{P(S_N^{(T)} - S_{N66}^{(T)})\}$ the total contributions receivable.

The initial reserve mentioned above is the sum of the benefits payable: death, withdrawal and pension benefits, minus the total premium contributions receivable. In other words the initial reserve can be treated as the initial unfunded liability. Since the plan is guaranteed by the employer and, once the employee has made his contributions, he would get all his benefits, the employer has to make a deposit equal to the size of the initial reserve whenever a member enters a plan (on top of the 7.5% annual contribution of employee income). Viewing the initial reserve in this respect, different calculations are made using various combinations of interest, inflation and COLA rates.

In the next chapter these different initial reserves are tabulated and compared.

Summary

In this chapter, a mathematical model was first discussed. Its flexibility was pointed out. Development of the computer program was briefly mentioned. Flexibility was once again the characteristic of the program. Model plan provisions were listed.

In the next chapter, a cost evaluation of the model plan listed above is calculated using the computer program.

CHAPTER IV
SUMMARY AND CONCLUSION

Introduction

The different initial liabilities and contributions are tabulated on the following pages.

Before conclusions are made, attention is drawn to the question of the extent to which one can apply the numerical conclusions from this experiment to other plans. It is stressed that changes of the cost of pension plans are very volatile. A small change in one or some of the parameters, e.g., a change in mortality or withdrawal rates, could bring about a substantial change in cost. Therefore, it is emphasized that the cost so calculated in this study applies to this particular pension plan and under the conditions given.

TABLE 4.3

TOTAL FUTURE WITHDRAWAL BENEFIT
ASSUMING DIFFERENT INTEREST, INFLATION AND COLA RATES

INTEREST RATE	0.03	0.05	0.05	0.07	0.07	0.05	0.05	0.07	0.07
INFLATION RATE	0.00	0.02	0.02	0.04	0.04	0.03	0.03	0.03	0.03
COLA RATE	0.00	0.00	0.02	0.00	0.04	0.00	0.03	0.00	0.03
AGE									
20	2.27	2.28	2.28	2.28	2.28	2.33	2.33	2.23	2.23
21	2.26	2.26	2.26	2.26	2.26	2.31	2.31	2.22	2.22
22	2.23	2.23	2.23	2.24	2.24	2.28	2.28	2.19	2.19
23	2.20	2.20	2.20	2.20	2.20	2.25	2.25	2.16	2.16
24	2.16	2.16	2.16	2.16	2.16	2.21	2.21	2.12	2.12
25	2.11	2.11	2.11	2.12	2.12	2.16	2.16	2.07	2.07
26	2.06	2.07	2.07	2.07	2.07	2.11	2.11	2.02	2.02
27	2.01	2.01	2.01	2.02	2.02	2.06	2.06	1.97	1.97
28	1.96	1.96	1.96	1.96	1.96	2.01	2.01	1.92	1.92
29	1.90	1.90	1.90	1.91	1.91	1.95	1.95	1.86	1.86
30	1.84	1.85	1.85	1.85	1.85	1.89	1.89	1.81	1.81
31	1.77	1.78	1.78	1.78	1.78	1.82	1.82	1.74	1.74
32	1.72	1.72	1.72	1.72	1.72	1.76	1.76	1.68	1.68
33	1.66	1.66	1.66	1.66	1.66	1.70	1.70	1.63	1.63
34	1.60	1.60	1.60	1.61	1.61	1.64	1.64	1.57	1.57
35	1.54	1.54	1.54	1.55	1.55	1.58	1.58	1.51	1.51
36	1.48	1.48	1.48	1.49	1.49	1.52	1.52	1.45	1.45
37	1.42	1.42	1.42	1.42	1.42	1.45	1.45	1.39	1.39
38	1.35	1.35	1.35	1.36	1.36	1.39	1.39	1.33	1.33
39	1.28	1.28	1.28	1.29	1.29	1.31	1.31	1.26	1.26
40	1.20	1.21	1.21	1.21	1.21	1.23	1.23	1.18	1.18
41	1.12	1.13	1.13	1.13	1.13	1.15	1.15	1.10	1.10
42	1.04	1.04	1.04	1.04	1.04	1.06	1.06	1.02	1.02
43	0.94	0.94	0.94	0.94	0.94	0.97	0.97	0.92	0.92
44	0.84	0.84	0.84	0.84	0.84	0.86	0.86	0.82	0.82
45	0.72	0.73	0.73	0.73	0.73	0.74	0.74	0.71	0.71
46	0.60	0.60	0.60	0.60	0.60	0.61	0.61	0.59	0.59
47	0.49	0.49	0.49	0.49	0.49	0.50	0.50	0.48	0.48
48	0.39	0.39	0.39	0.39	0.39	0.40	0.40	0.38	0.38
49	0.31	0.31	0.31	0.31	0.31	0.32	0.32	0.31	0.31
50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.24	0.24
51	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.19	0.19
52	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.15	0.15
53	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
54	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.10	0.10
55	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08
56	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
57	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
58	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.03	0.03
59	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
60	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
61	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
62	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

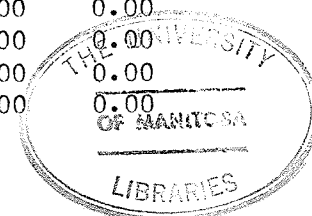


TABLE 4.4

TOTAL FUTURE VESTED PENSION									
ASSUMING DIFFERENT INTEREST, INFLATION AND COLA RATES									
INTEREST RATE	0.03	0.05	0.05	0.07	0.07	0.05	0.05	0.07	0.07
INFLATION RATE	0.00	0.02	0.02	0.04	0.04	0.03	0.03	0.03	0.03
COLA RATE	0.00	0.00	0.02	0.00	0.04	0.00	0.03	0.00	0.03
AGE									
20	7.67	5.63	6.52	4.44	5.87	8.19	10.26	3.03	3.71
21	8.80	6.56	7.60	5.23	6.90	9.50	11.91	3.58	4.39
22	10.00	7.55	8.74	6.07	8.01	10.89	13.64	4.18	5.13
23	11.23	8.59	9.95	6.96	9.19	12.32	15.44	4.82	5.92
24	12.50	9.67	11.20	7.89	10.41	13.80	17.28	5.50	6.75
25	13.80	10.78	12.49	8.86	11.69	15.30	19.16	6.22	7.63
26	15.11	11.92	13.81	9.85	13.00	16.81	21.06	6.96	8.54
27	16.41	13.06	15.13	10.86	14.33	18.31	22.93	7.73	9.48
28	17.69	14.20	16.45	11.86	15.66	19.77	24.76	8.51	10.43
29	18.93	15.31	17.73	12.85	16.96	21.17	26.52	9.28	11.38
30	20.12	16.38	18.98	13.81	18.23	22.50	28.18	10.05	12.32
31	21.24	17.41	20.17	14.74	19.45	23.74	29.73	10.81	13.25
32	22.28	18.37	21.27	15.61	20.60	24.86	31.14	11.53	14.14
33	23.21	19.25	22.30	16.41	21.67	25.86	32.39	12.22	14.99
34	24.06	20.05	23.22	17.15	22.65	26.73	33.48	12.88	15.79
35	24.80	20.77	24.06	17.82	23.53	27.47	34.41	13.49	16.54
36	25.44	21.40	24.79	18.42	24.31	28.09	35.18	14.05	17.23
37	25.99	21.94	25.42	18.93	24.99	28.57	35.78	14.56	17.86
38	26.43	22.39	25.94	19.37	25.57	28.92	36.22	15.02	18.42
39	26.78	22.76	26.36	19.72	26.04	29.14	36.50	15.43	18.92
40	26.95	22.96	26.60	19.94	26.32	29.16	36.52	15.73	19.29
41	27.13	23.17	26.83	20.14	26.59	29.16	36.53	16.03	19.66
42	27.22	23.29	26.98	20.28	26.77	29.06	36.40	16.28	19.97
43	27.24	23.34	27.03	20.34	26.85	28.86	36.15	16.48	20.20
44	27.17	23.30	26.99	20.32	26.82	28.55	35.76	16.61	20.37
45	27.02	23.18	26.85	20.22	26.69	28.14	35.25	16.69	20.46
46	26.79	22.99	26.62	20.04	26.46	27.64	34.62	16.70	20.48
47	26.42	22.66	26.25	19.76	26.09	26.99	33.81	16.62	20.38
48	25.90	22.22	25.73	19.37	25.57	26.21	32.83	16.44	20.16
49	25.23	21.64	25.06	18.86	24.89	25.28	31.66	16.16	19.82
50	24.41	20.93	24.24	18.23	24.07	24.22	30.33	15.78	19.35
51	23.44	20.09	23.27	17.50	23.10	23.03	28.85	15.29	18.75
52	22.33	19.14	22.16	16.66	22.00	21.72	27.21	14.70	18.02
53	21.09	18.07	20.93	15.72	20.76	20.31	25.44	14.00	17.17
54	19.74	16.90	19.57	14.70	19.41	18.81	23.56	13.22	16.21
55	18.28	15.64	18.12	13.60	17.95	17.24	21.60	12.35	15.14
56	16.84	14.40	16.68	12.51	16.52	15.72	19.69	11.47	14.06
57	15.41	13.17	15.26	11.44	15.10	14.24	17.84	10.59	12.98
58	13.99	11.95	13.84	10.37	13.69	12.79	16.02	9.69	11.88
59	12.55	10.71	12.41	9.29	12.27	11.36	14.22	8.77	10.75
60	11.07	9.44	10.94	8.19	10.81	9.92	12.42	7.80	9.57
61	9.53	8.12	9.41	7.04	9.29	8.45	10.58	6.77	8.31
62	7.89	6.73	7.79	5.83	7.69	6.93	8.68	5.66	6.94
63	6.14	5.23	6.06	4.53	5.98	5.33	6.68	4.44	5.44
64	4.25	3.62	4.19	3.13	4.13	3.65	4.58	3.10	3.80
65	2.21	1.88	2.18	1.63	2.15	1.88	2.36	1.63	1.99

TABLE 4.5

TOTAL FUTURE LIABILITIES									
ASSUMING DIFFERENT INTEREST, INFLATION AND COLA RATES									
INTEREST RATE	0.03	0.05	0.05	0.07	0.07	0.05	0.05	0.07	0.07
INFLATION RATE	0.00	0.02	0.02	0.04	0.04	0.03	0.03	0.03	0.03
COLA RATE	0.00	0.00	0.02	0.00	0.04	0.00	0.03	0.00	0.03
AGE									
20	-2.10	-4.21	-3.32	-5.46	-4.03	-2.98	-0.91	-5.81	-5.13
21	-1.83	-4.15	-3.11	-5.55	-3.88	-2.70	-0.30	-6.01	-5.20
22	-1.50	-4.04	-2.84	-5.60	-3.66	-2.36	0.39	-6.17	-5.22
23	-1.14	-3.88	-2.52	-5.60	-3.37	-1.96	1.15	-6.29	-5.19
24	-0.74	-3.68	-2.15	-5.56	-3.03	-1.52	1.96	-6.36	-5.12
25	-0.31	-3.43	-1.73	-5.47	-2.63	-1.04	2.82	-6.40	-5.00
26	0.15	-3.15	-1.27	-5.34	-2.18	-0.53	3.72	-6.40	-4.83
27	0.63	-2.84	-0.77	-5.17	-1.69	0.01	4.63	-6.37	-4.62
28	1.13	-2.50	-0.25	-4.96	-1.17	0.56	5.55	-6.29	-4.36
29	1.63	-2.13	0.29	-4.73	-0.61	1.11	6.46	-6.17	-4.07
30	2.13	-1.75	0.85	-4.46	-0.04	1.66	7.35	-6.02	-3.75
31	2.62	-1.36	1.40	-4.17	0.54	2.20	8.19	-5.84	-3.40
32	3.10	-0.96	1.95	-3.86	1.13	2.71	8.99	-5.62	-3.01
33	3.56	-0.55	2.49	-3.54	1.72	3.20	9.73	-5.38	-2.61
34	4.00	-0.16	3.01	-3.20	2.29	3.66	10.41	-5.12	-2.20
35	4.40	0.22	3.51	-2.87	2.84	4.07	11.01	-4.84	-1.78
36	4.78	0.59	3.97	-2.54	3.36	4.44	11.53	-4.55	-1.37
37	5.13	0.93	4.41	-2.22	3.84	4.76	11.98	-4.25	-0.96
38	5.44	1.26	4.80	-1.91	4.29	5.04	12.34	-3.95	-0.55
39	5.71	1.55	5.15	-1.62	4.69	5.27	12.63	-3.66	-0.17
40	5.94	1.81	5.45	-1.35	5.03	5.43	12.79	-3.36	0.20
41	6.15	2.05	5.72	-1.10	5.35	5.56	12.92	-3.08	0.55
42	6.33	2.27	5.95	-0.88	5.62	5.64	12.98	-2.81	0.87
43	6.47	2.45	6.14	-0.68	5.83	5.68	12.97	-2.56	1.17
44	6.58	2.60	6.28	-0.50	6.00	5.68	12.89	-2.32	1.44
45	6.67	2.72	6.39	-0.35	6.12	5.63	12.74	-2.09	1.68
46	6.73	2.82	6.46	-0.23	6.19	5.55	12.53	-1.88	1.89
47	6.73	2.88	6.47	-0.12	6.21	5.43	12.25	-1.68	2.08
48	6.68	2.91	6.43	-0.02	6.18	5.26	11.88	-1.48	2.23
49	6.58	2.91	6.34	0.05	6.09	5.06	11.44	-1.30	2.36
50	6.43	2.88	6.19	0.11	5.95	4.82	10.94	-1.12	2.44
51	6.23	2.82	6.00	0.16	5.77	4.55	10.37	-0.96	2.50
52	5.99	2.73	5.76	0.20	5.54	4.26	9.75	-0.80	2.53
53	5.71	2.63	5.49	0.24	5.27	3.96	9.09	-0.64	2.52
54	5.39	2.51	5.18	0.27	4.97	3.64	8.39	-0.50	2.49
55	5.05	2.37	4.85	0.30	4.65	3.32	7.68	-0.35	2.44
56	4.72	2.25	4.52	0.33	4.33	3.03	7.00	-0.21	2.38
57	4.39	2.12	4.21	0.37	4.03	2.76	6.36	-0.08	2.32
58	4.06	2.00	3.89	0.40	3.72	2.50	5.73	0.04	2.24
59	3.71	1.86	3.56	0.43	3.40	2.25	5.12	0.15	2.14
60	3.35	1.71	3.21	0.44	3.07	2.00	4.50	0.24	2.00
61	2.95	1.53	2.82	0.44	2.70	1.73	3.86	0.30	1.83
62	2.49	1.32	2.39	0.42	2.28	1.44	3.19	0.33	1.61
63	1.98	1.07	1.90	0.36	1.81	1.13	2.48	0.32	1.32
64	1.40	0.77	1.34	0.28	1.28	0.79	1.71	0.26	0.96
65	0.74	0.41	0.71	0.16	0.68	0.41	0.89	0.16	0.53

Result of the Study¹

Contributions

Table 4.1 lists the total future contribution by entry age, assuming different interest, inflation and COLA rates.

If, for example, there is no inflation and the interest rate is 3% per year, a member entering a plan at age 20 can be expected to contribute \$6.335 during his stay in the plan while the employer would put in an equal amount, making a total of \$12.67. This figure is calculated taking into account inflation, the force of interest, termination rates and the salary scales.

When one examines these figures, two points should be noted.

First, if death is the only decrement, the annuity values (present value of future contributions) decrease by age -- the older one enters the plan, the less contributions one is expected to make. In the case where withdrawal is also introduced, the expected value increases (to about age 40) and then decreases. These reflect the relative magnitude of the two terminating forces -- death

¹ Readers are reminded of the assumptions that every employee enters the plan with a ten dollar annual income. Thus a two dollar initial unfunded liability should be implied to mean a 20% of the employee's annual income at his age of entry.

and withdrawal. At the young ages, the rates of death are much smaller than the withdrawal rates. The single decrement table that appears on Appendix D shows that there is a one in five (0.21595) chance for a person to leave the group by withdrawal, the rate for the same person to die is only one in two thousand (0.00045). If death is the only decrement, it is very unlikely that one would leave the plan before retirement (when all contributions are made). Therefore, it can be expected that the longer one stays in a plan, the more the expected contributions. In other words, the younger one enters, the more his expected contribution.

If withdrawal is allowed, it is very likely that younger members would leave the plan after a short stay. In other words, only a few contributions can be expected from them. However, the older one enters, the less the chance that one is going to leave and so there is more opportunity to make contributions.

On the other hand, since the last contribution is made at age 65, the older one enters the plan, the less contribution one can make.

These two forces are reflected in the present values of the contributions -- the present value increases from age 20 to about age 40 then decreases towards the end.

The second point that should be noted is the relative similarity between the values of the different combinations of interest and inflation rates. The different combinations of rates are chosen taking the interest rate of 3% with no inflation as the "norm" or "control" situation. Then with the introduction of a 2% or 4% inflation rate, an equal amount is added to the interest rate. In other words the differential between the interest rate and inflation rate is roughly kept the same. Changes resulting from a change of this differential are also examined.

From the table, it can be seen that if the differential is kept at 3%, the present values are very closely the same. If the differential is reduced by 1% to 2%, the values get larger. If it is increased by 1% to 4%, the values become smaller.

From these, we can say that even if inflation pushes up the salary scale, thus increasing the dollar amount of the contribution made, provided that the discounting factor keep pace with inflation, the real value of the contribution would remain constant.

As pointed out in chapter III, the total contribution $(\bar{P}^{(T)}) s_{\ddot{x}:\overline{\delta\delta-x}(\delta^s-\delta)}^{(T)}$ is an annuity subject to a force of inter-

est of $(\delta^S - \delta)$

$$\text{Let } \delta' = \delta + k$$

$$\delta^{S'} = \delta^S + h$$

If $h=k$,

$$\delta^{S'} - \delta' = \delta^S + h - \delta - k = \delta^S - \delta$$

If $h < k$

$$\delta^{S'} - \delta' = \delta^S + h - \delta - k < \delta^S - \delta$$

If $h > k$

$$\delta^{S'} - \delta' = \delta^S + h - \delta - k > \delta^S - \delta$$

From these it can be seen that if both forces of interest and salary scale are subject to the same inflatory force, that is $h=k$, then the annuity values would remain the same.

If interest rate increases at a rate less than the rate of increase of the salary scale ($h < k$), then the annuity value would be subjected to a smaller real interest rate. A smaller interest rate would mean a smaller rate of discount. Thus, larger present values result.

By the same reasoning, if the inflationary force works more on the interest rate than on the salary scale ($h > k$), then a higher real interest rate results. Thus the annuity value would be decreased.

In short, though inflation pushes up the salary scale

and thus increases the dollar amount of the contribution, the present values of the contributions remain unchanged, provided that the interest rate is increased by the same amount.

Death Benefit

On the death of a member before retirement the total contributions made, accumulated with interest, would be refunded. Table 4.2 lists the total death benefit assuming different interest, inflation and COLA rates.

If, for example, there is no inflation and interest rate is 3% per year, a member entering at age 30, can expect to get \$2.15 in form of death benefit.

As found in the list of the total contributions, the values increase till around age 40 and then decrease. These reflect both the relative magnitude of the two terminating forces: withdrawal and death rates and the plan provision that age 65 is the last age to die with a death benefit.

A comparison of the values calculated using different combinations of interest and inflation rates shows again that, if the difference between the interest and inflation rates is left constant, no change in the present value results.

Withdrawal Benefit

Table 4.3 shows the total future withdrawal benefits assuming a different interest, inflation and COLA rates.

On the withdrawal of a member from the plan within the first ten years of service, only the member's contributions accumulated with interest are refunded. If termination occurs after 10 years, a vested pension benefit would be provided. The present value of the vested pension benefit is listed in Table 4.4.

In a non-inflationary situation and assuming a 3% interest rate, a member entering the plan at age 20 can expect to receive \$2.27 in withdrawal benefit. The older a member enters the plan the less the present value. These reflect to a large extent the magnitude of the withdrawal rates. For young members the termination rate is very high. Thus, they can expect a lot of withdrawal benefit. For the members entering at a later age, the chance of staying till retirement is high. Thus, little or no withdrawal benefit can be expected.

Since the size of the present value of the withdrawal benefit is rather small, changes in interest or inflation rates do not bring about a drastic change in the present value.

Vested Pension Benefit

Table 4.4 lists the total future vested pension benefits assuming different interest, inflation and COLA rates. In the calculation of the vested pension benefit, the retirement benefit on age 66 is also included.

If, for example, a member enters a plan at age 20, he can expect \$7.67 in vested pension benefit. The present value gets larger as entry age gets larger. This reflects the magnitude of the withdrawal rates -- there are less chances for those entering young to stay for 10 years till vested benefit is available.

If inflation is introduced without the appropriate COLA benefit, the present values of the vested pension are reduced. For a member entering at age 20, the present value of \$7.67 is reduced to \$5.63 if inflation is increased to 2% per year (with the same increase in the interest rate). The present value is further reduced to \$4.44 if a 4% inflation is introduced without the appropriate COLA benefit.

On the other hand, if COLA is introduced, the present values remain roughly the same regardless of what the inflation rates are.

If the real interest rate lags behind inflation, regardless of whether COLA is provided or not, the present

values are increased greatly.

On the other hand, if the interest rate increases more than inflation, the present values are reduced.

The same change was noticed when the present values of contribution were examined. The explanation used there also apply in this case.

If COLA benefits are treated as a means to maintain the purchasing power of the dollar, that is, the real value of the pension benefit is kept constant by providing COLA, then it is just logical that the present value of the vested benefit is kept at a level equal to the non-inflationary case. If no COLA is provided, the real value of the pension benefit is reduced continuously, so the present value is reduced.

Initial Unfunded Liabilities

Table 4.5 shows the total future unfunded liabilities assuming different interest, inflation and COLA rates. The liability is basically the sum of the present values of future benefits -- death, withdrawal and vested pension benefit, less the present value of future contribution. This liability represents the extent to which the contribution (15% of annual salary) is enough to cover all the benefit payments.

Attention is drawn here to the assumption that the emp-

loyer assumes all initial unfunded liabilities. The employer has to make up for the gain (or loss) of the fund if the 15% contribution is more (or less) than is needed in the benefit payment.

In a non-inflationary situation, the table shows negative values for ages before age 26. If, for example, a member enters the plan at age 20, the initial liability is minus \$2.10. This means that employer costs are less than 7.5% of salary. Once again, these can be explained in terms of the plan provisions. If any member withdraws within the first ten years, only the member's contributions are refunded. In this way, no cost is incurred by the employer.

For the young members, the termination rate is very high. Listed below are the probabilities that a young member would stay in the plan for more than 10 years. These probabilities are calculated from the termination rates used in the computer program.

<u>Entry age</u>	<u>Probability</u>
20	0.1406
21	0.1577
22	0.1756
23	0.1942
24	0.2149

From this it can be seen that there is only a one in five chance that a member entering before age 24 would stay in the plan after 10 years. Thus, there is a four in five chance that the employer will recover his full costs (the contributions made accumulated with interest), when an employee withdraws.

If an inflation rate of 2% or 4% is introduced with a corresponding increase in the interest rate, and if no COLA benefit is provided, the initial unfunded liabilities are reduced substantially. If a corresponding COLA benefit is provided, the initial unfunded liabilities remain comparable with those of the non-inflationary situation. From these, it can be concluded that if the differential between the interest rate and inflation (the real interest rate) remains constant, COLA benefit is just a redistribution of the excess investment earning.

Changes of the real rate of interest are also examined. What changes would a decrease or increase of real interest rate bring about?

If there is a decrease from 3% to 2% in the real interest rate, the initial cost, with no COLA benefit, would approximately be the same as that of the non-inflationary case. However, if COLA is also provided the initial cost

would roughly be more than doubled.

On the other hand, if the real rate of interest is increased from 3% to 4%, the initial cost would be reduced in both cases -- with and without COLA benefit. As a matter of interest, if no COLA is provided, all initial unfunded liabilities are negative. The total future contributions are more than enough to cover the benefit payments.

The figures (Figure 4.1 - 4.9) on the following pages present basically the same figures as can be found in Tables 4.1 - 4.5. These charts were drawn up to show the relative magnitude of the present values of contributions and benefits. Under all combinations of interest and inflation rates, withdrawal and death benefits play a rather unimportant part in the total picture. Changes in interest and inflation rate do not bring about much changes.

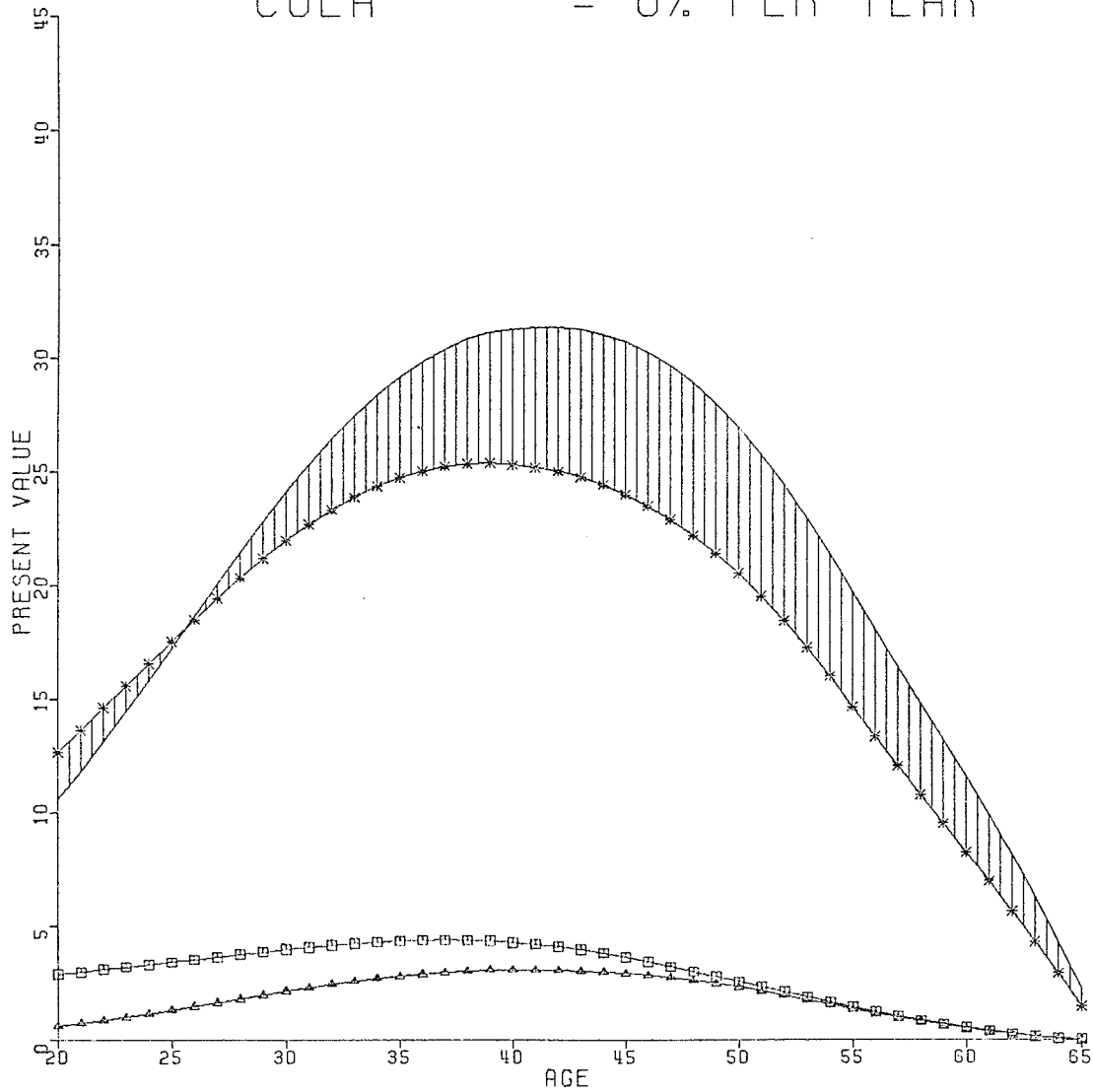
Vesting benefit is the variable that changes drastically in response to changes of interest, inflation and COLA provisions.

If interest and inflation increase at the same rate, no change in the present values results. Decrease in the real interest rate brings about an increase in benefit payment and a decrease in contribution. Thus a net

FIGURE 4.1

INITIAL UNFUNDED LIABILITIES

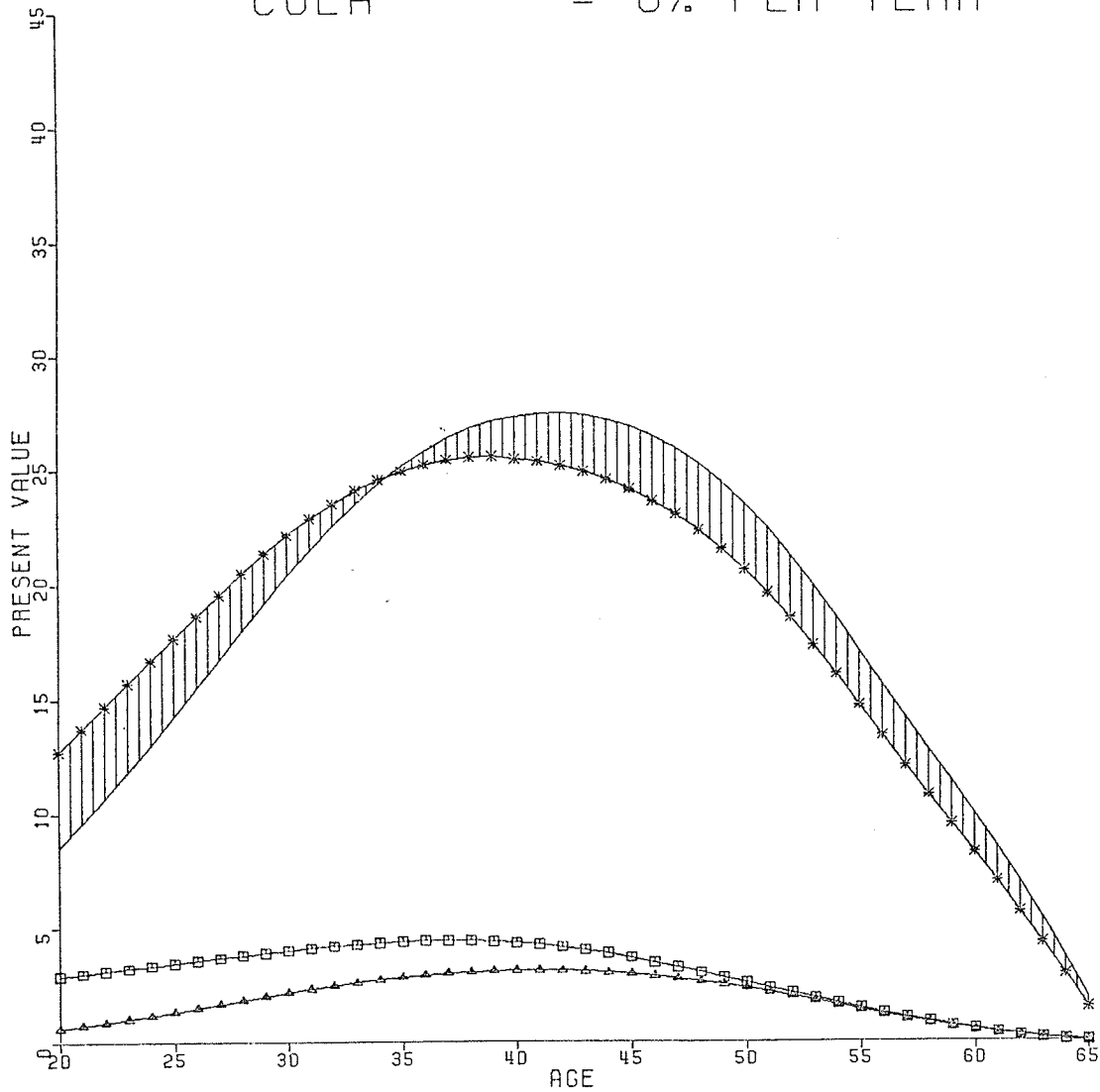
INTEREST = 3% PER YEAR
 INFLATION = 0% PER YEAR
 COLA = 0% PER YEAR



- ▨ INITIAL UNFUNDED LIABILITIES
- *— TOTAL FUTURE CONTRIBUTIONS
- TOTAL VESTED BENEFIT
- TOTAL FUTURE WITHDRAWAL BENEFITS
- △— TOTAL FUTURE DEATH BENEFITS

INITIAL UNFUNDED LIABILITIES

INTEREST = 5% PER YEAR
 INFLATION = 2% PER YEAR
 COLA = 0% PER YEAR

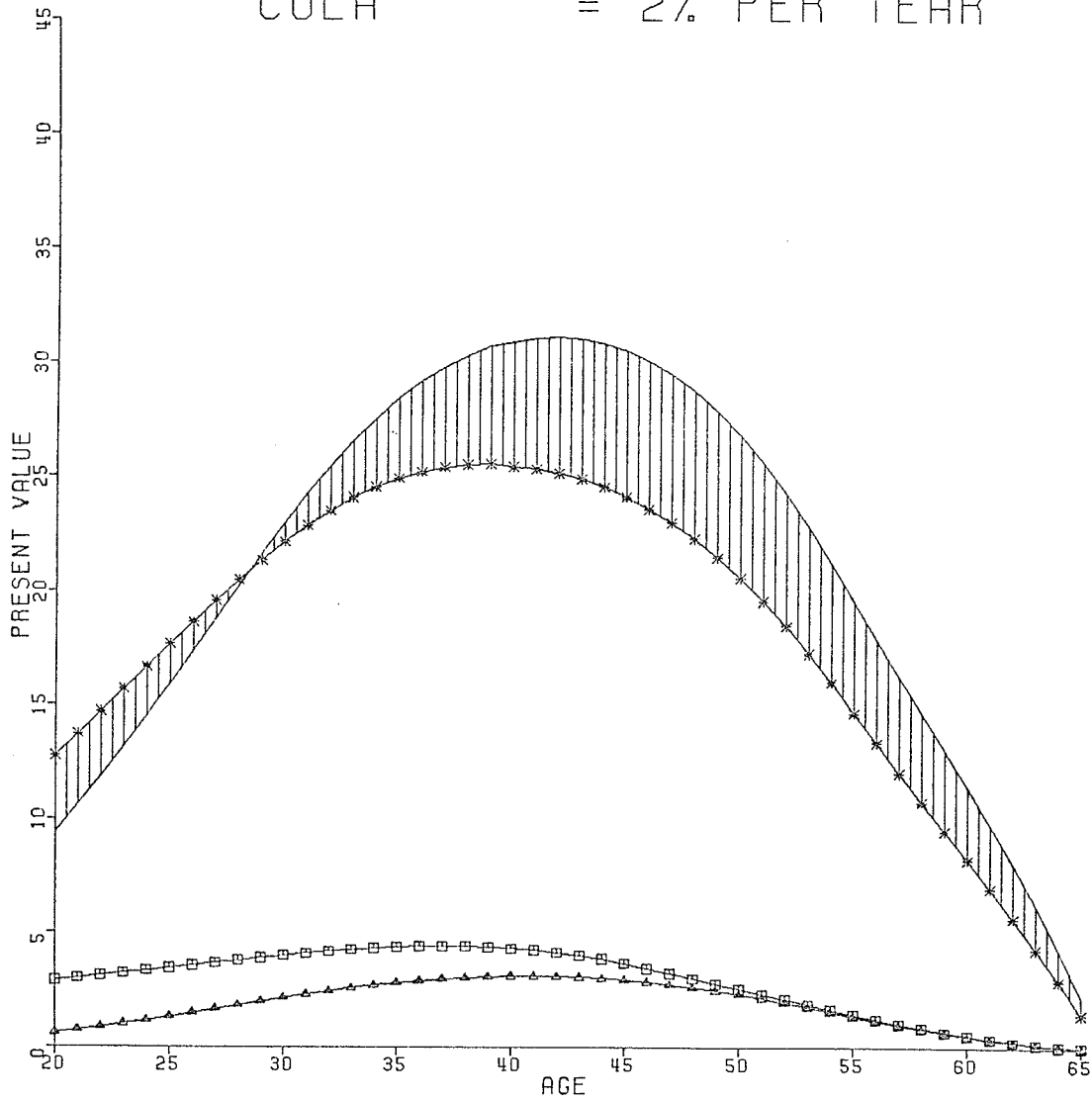





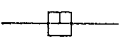

- ▨ INITIAL UNFUNDED LIABILITIES
- *— TOTAL FUTURE CONTRIBUTIONS
- TOTAL VESTED BENEFIT
- TOTAL FUTURE WITHDRAWAL BENEFITS
- △— TOTAL FUTURE DEATH BENEFITS

FIGURE 4.3

INITIAL UNFUNDED LIABILITIES

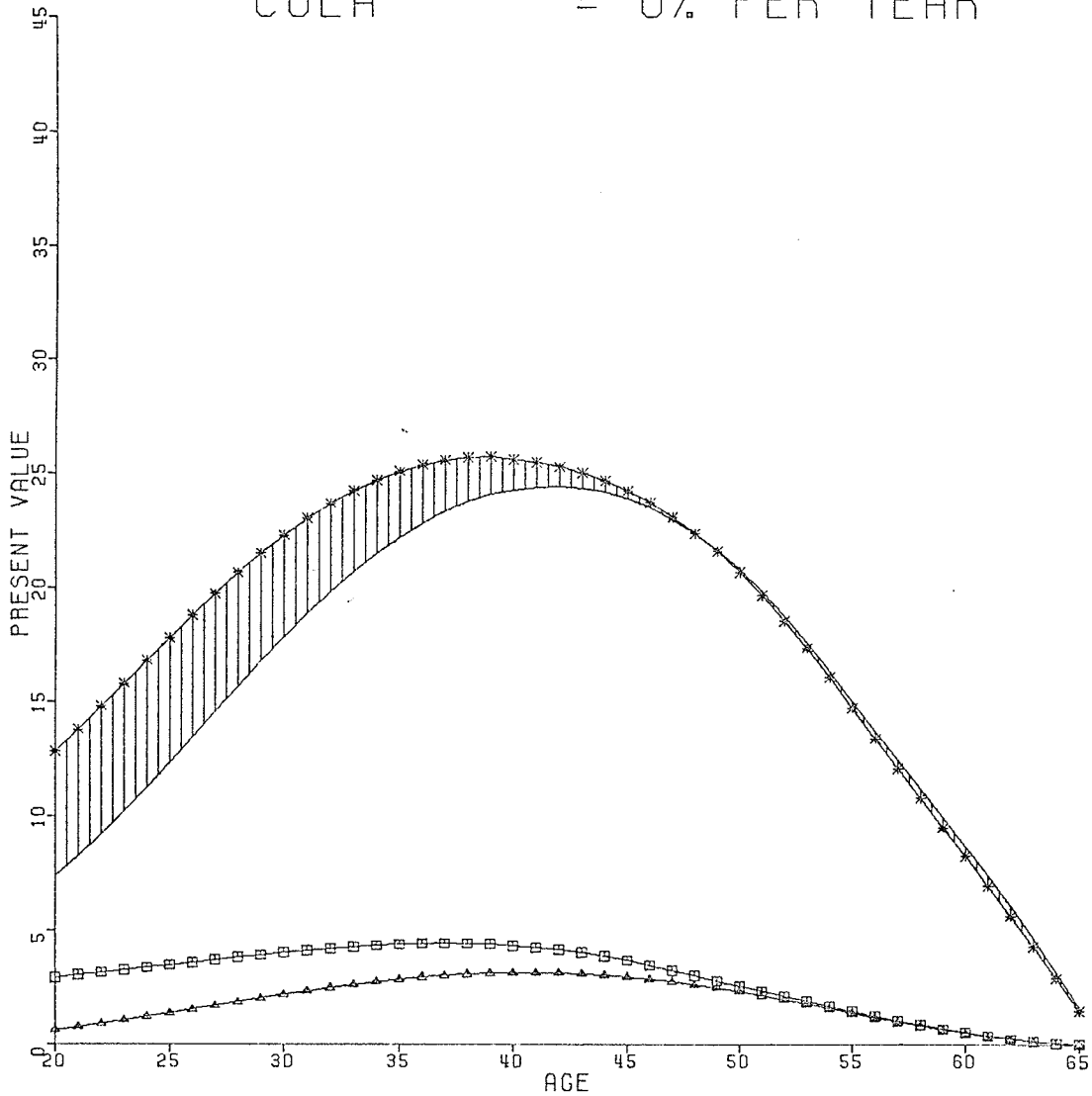
INTEREST = 5% PER YEAR
 INFLATION = 2% PER YEAR
 COLA = 2% PER YEAR



-  INITIAL UNFUNDED LIABILITIES
-  TOTAL FUTURE CONTRIBUTIONS
-  TOTAL VESTED BENEFIT
-  TOTAL FUTURE WITHDRAWAL BENEFITS
-  TOTAL FUTURE DEATH BENEFITS

INITIAL UNFUNDED LIABILITIES

INTEREST = 7% PER YEAR
 INFLATION = 4% PER YEAR
 COLA = 0% PER YEAR

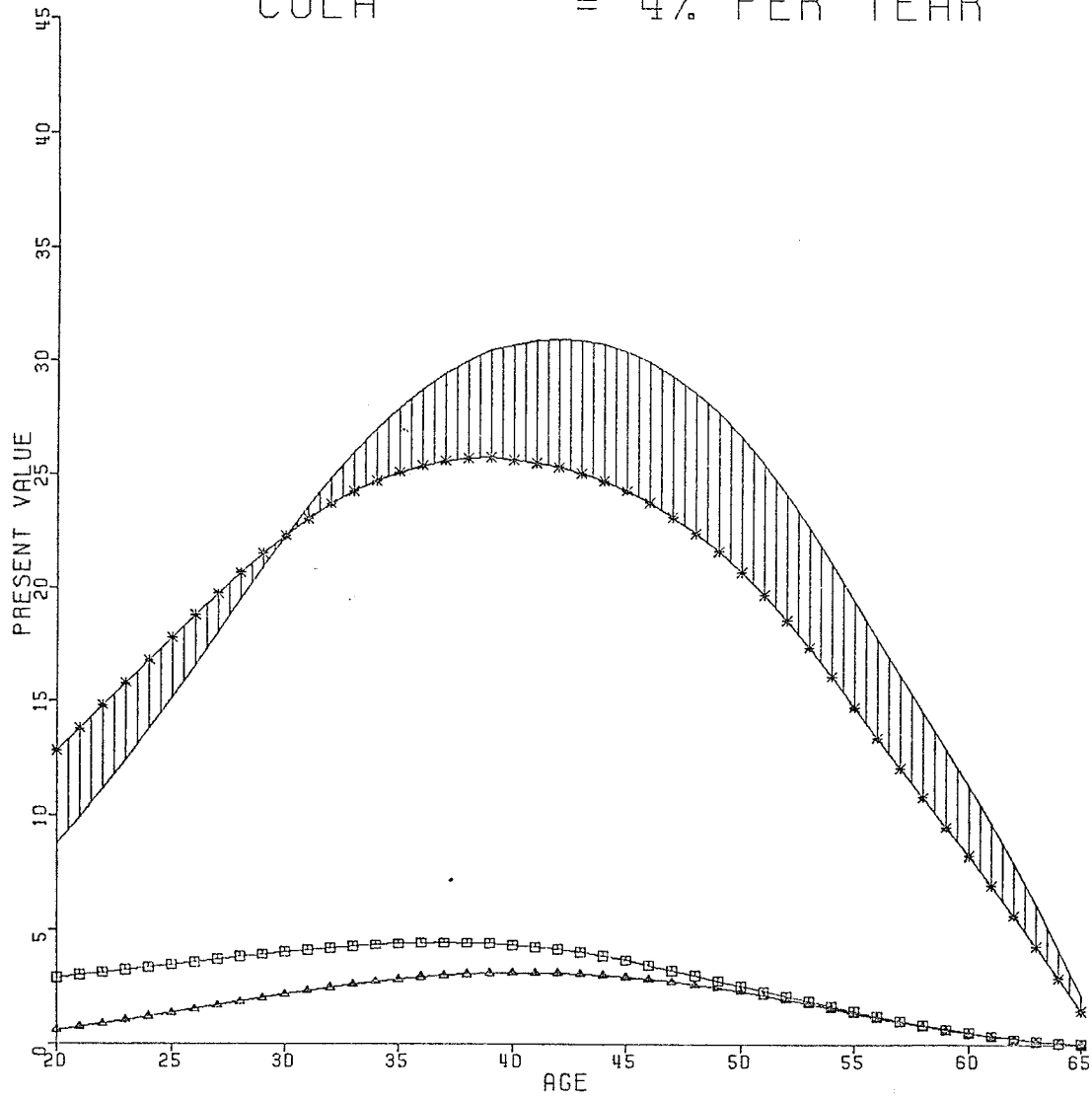





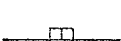
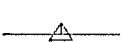
- ▨ INITIAL UNFUNDED LIABILITIES
- *— TOTAL FUTURE CONTRIBUTIONS
- TOTAL VESTED BENEFIT
- TOTAL FUTURE WITHDRAWAL BENEFITS
- △— TOTAL FUTURE DEATH BENEFITS

FIGURE 4.5

INITIAL UNFUNDED LIABILITIES

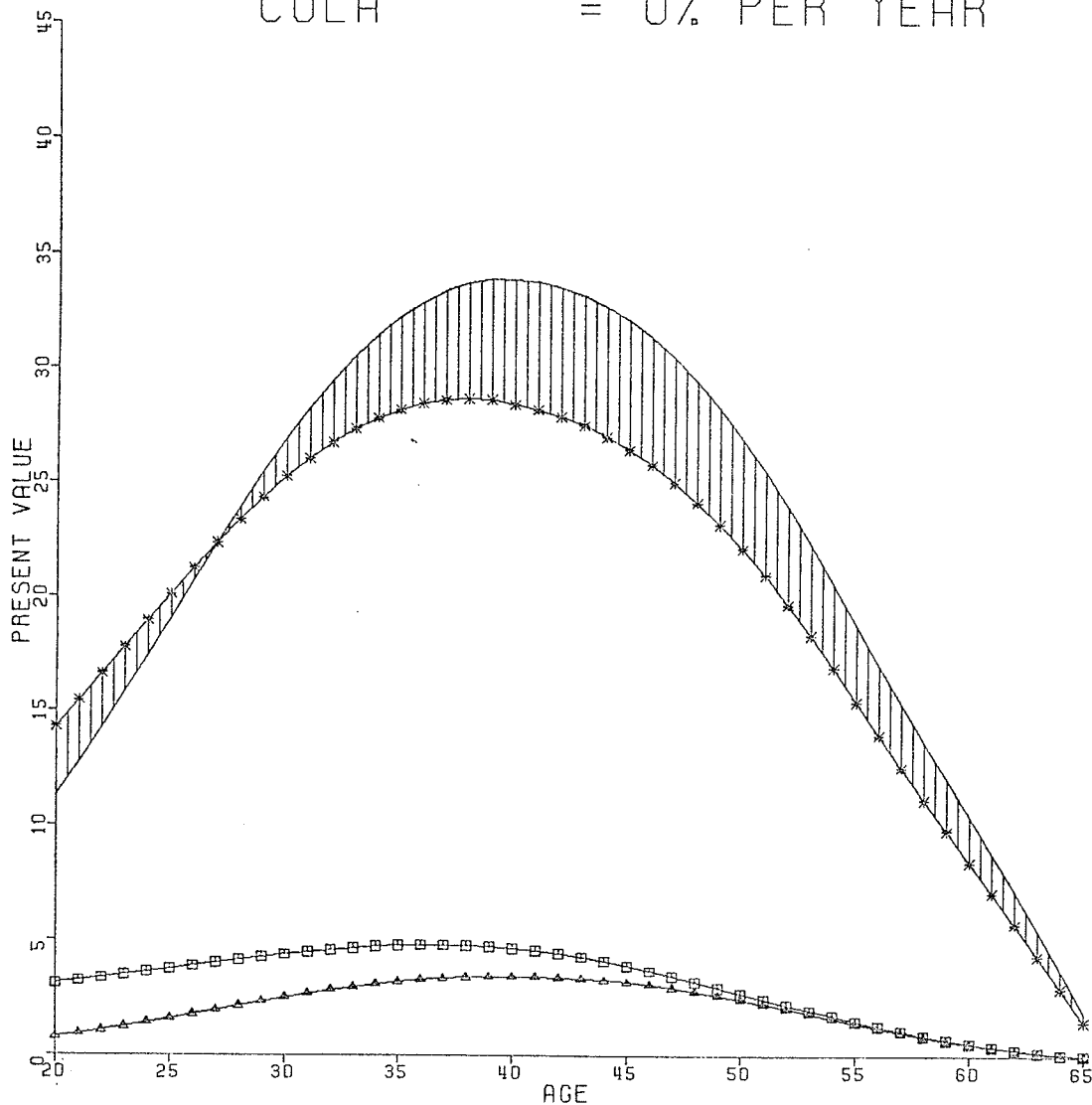
INTEREST = 7% PER YEAR
 INFLATION = 4% PER YEAR
 COLA = 4% PER YEAR



-  INITIAL UNFUNDED LIABILITIES
-  TOTAL FUTURE CONTRIBUTIONS
-  TOTAL VESTED BENEFIT
-  TOTAL FUTURE WITHDRAWAL BENEFITS
-  TOTAL FUTURE DEATH BENEFITS

INITIAL UNFUNDED LIABILITIES

INTEREST = 5% PER YEAR
 INFLATION = 3% PER YEAR
 COLA = 0% PER YEAR



- ▨ INITIAL UNFUNDED LIABILITIES
- *— TOTAL FUTURE CONTRIBUTIONS
- TOTAL VESTED BENEFIT
- TOTAL FUTURE WITHDRAWAL BENEFITS
- △— TOTAL FUTURE DEATH BENEFITS

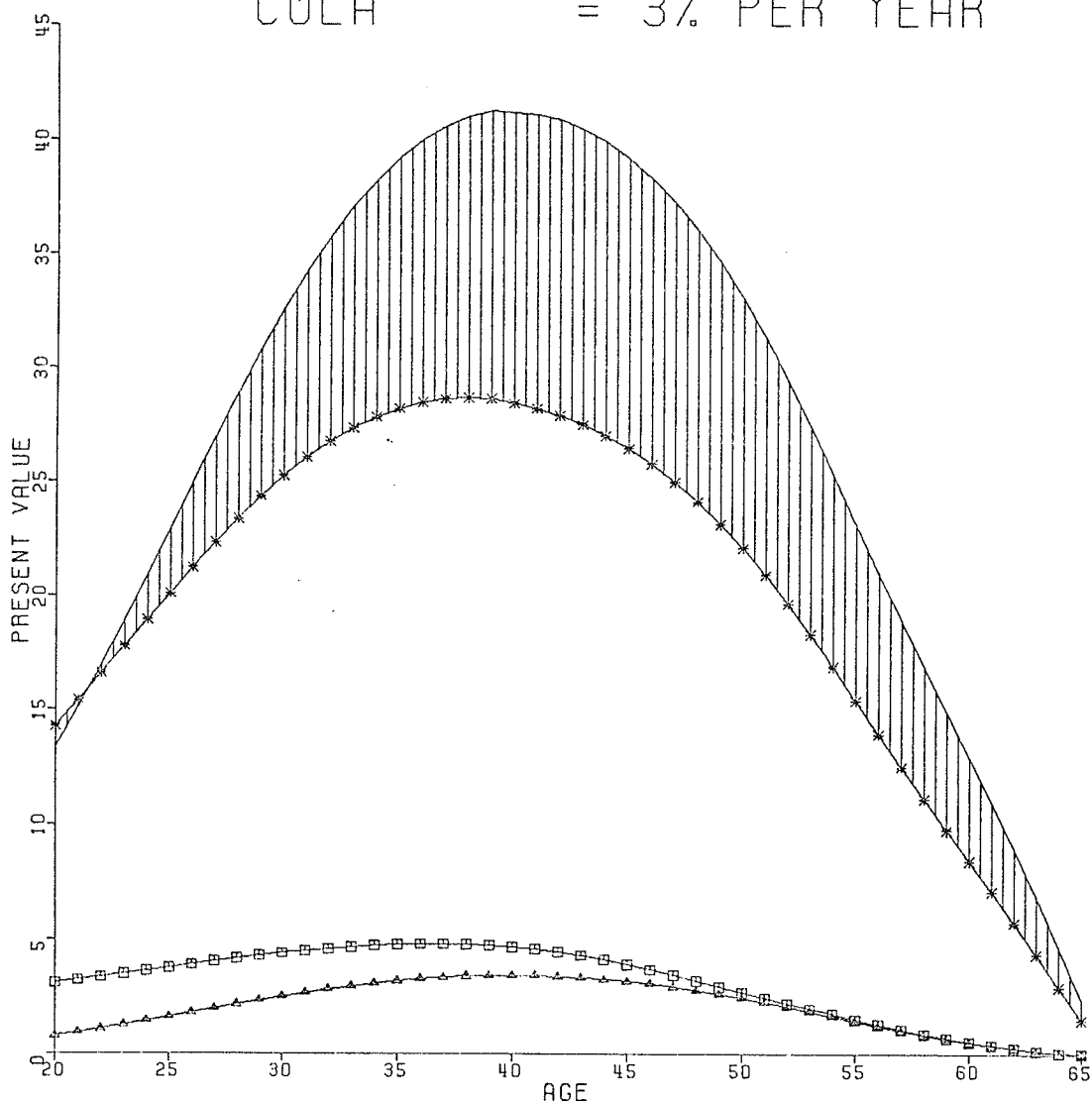
FIGURE 4.7


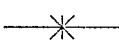

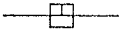
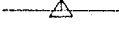
INITIAL UNFUNDED LIABILITIES

INTEREST = 5% PER YEAR

INFLATION = 3% PER YEAR

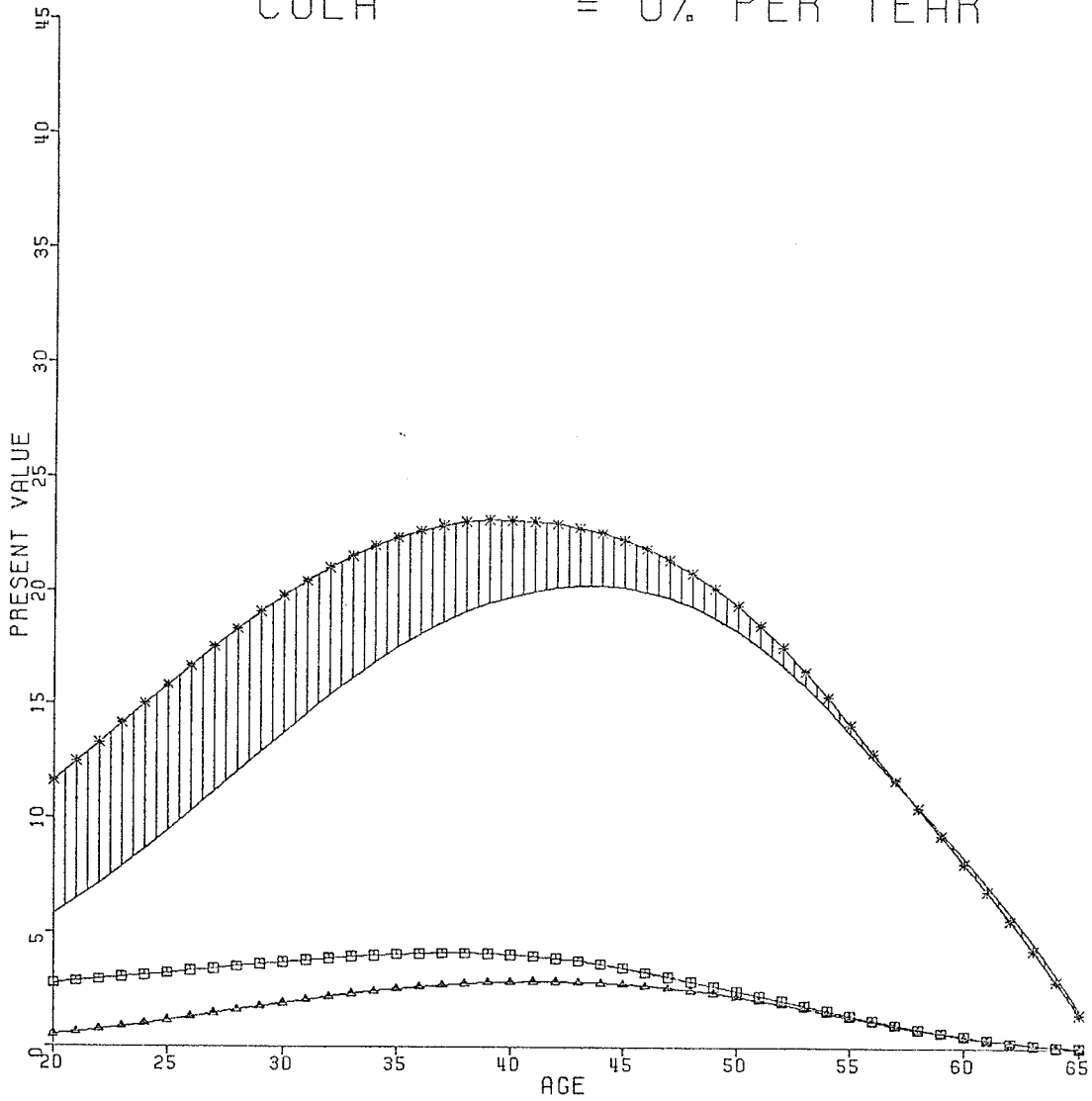
COLA = 3% PER YEAR



-  INITIAL UNFUNDED LIABILITIES
-  TOTAL FUTURE CONTRIBUTIONS
-  TOTAL VESTED BENEFIT
-  TOTAL FUTURE WITHDRAWAL BENEFITS
-  TOTAL FUTURE DEATH BENEFITS

INITIAL UNFUNDED LIABILITIES

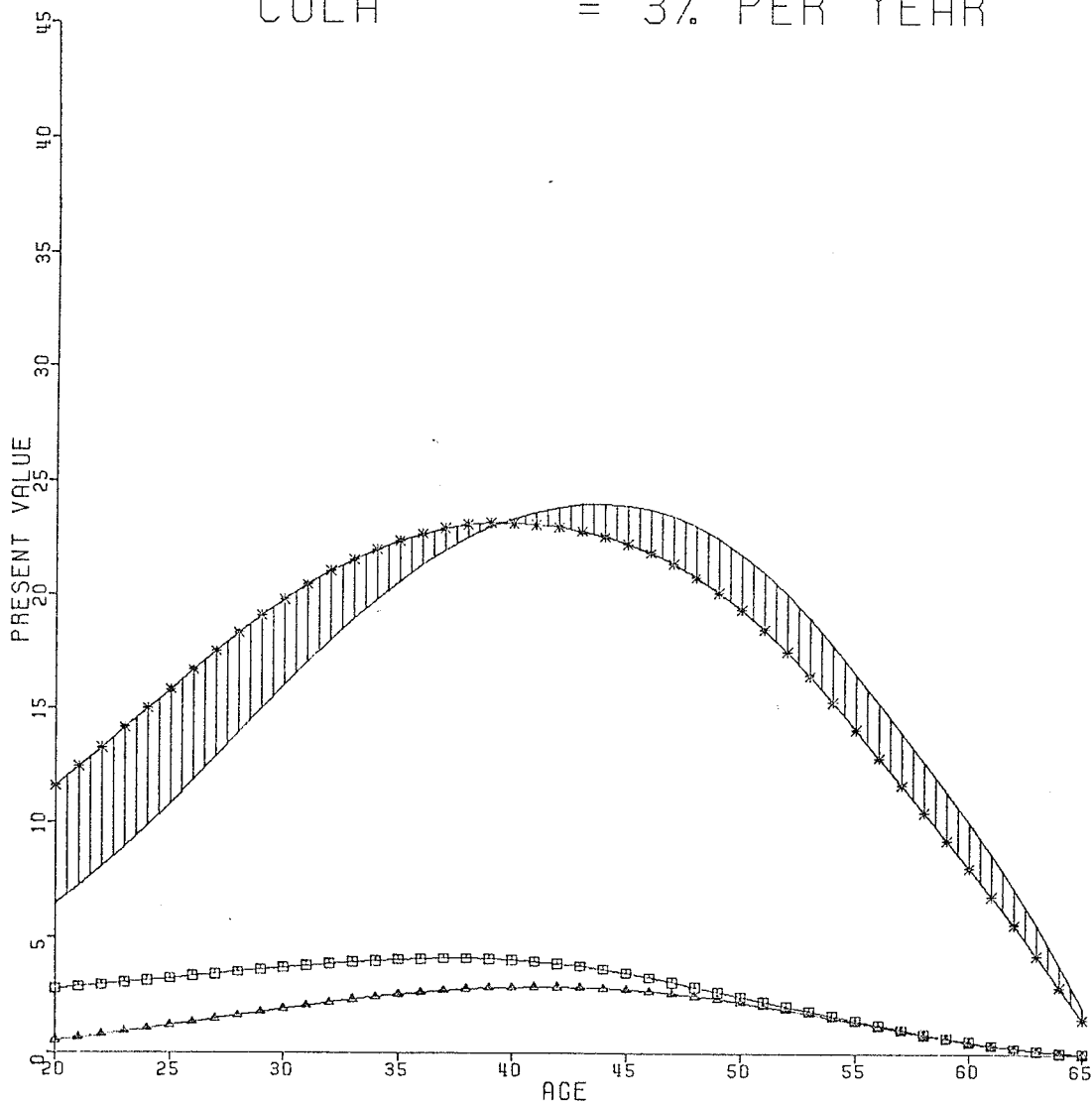
INTEREST = 7% PER YEAR
 INFLATION = 3% PER YEAR
 COLA = 0% PER YEAR



- ▨ INITIAL UNFUNDED LIABILITIES
- *— TOTAL FUTURE CONTRIBUTIONS
- TOTAL VESTED BENEFIT
- TOTAL FUTURE WITHDRAWAL BENEFITS
- △— TOTAL FUTURE DEATH BENEFITS

INITIAL UNFUNDED LIABILITIES

INTEREST = 7% PER YEAR
 INFLATION = 3% PER YEAR
 COLA = 3% PER YEAR



- ▨ INITIAL UNFUNDED LIABILITIES
- *— TOTAL FUTURE CONTRIBUTIONS
- TOTAL VESTED BENEFIT
- TOTAL FUTURE WITHDRAWAL BENEFITS
- △— TOTAL FUTURE DEATH BENEFITS

increase in the initial unfunded liabilities. An increase in the interest rate reduces the present value of the benefit payments and increases the contributions, thus reducing the net initial unfunded liabilities.

The figures (Figure 4.10 - 4.12) on the next 3 pages show the traces of the reserves for members entering at age 30, 40 and 50, under different interest and inflation assumptions. (Note the different vertical scales used.)

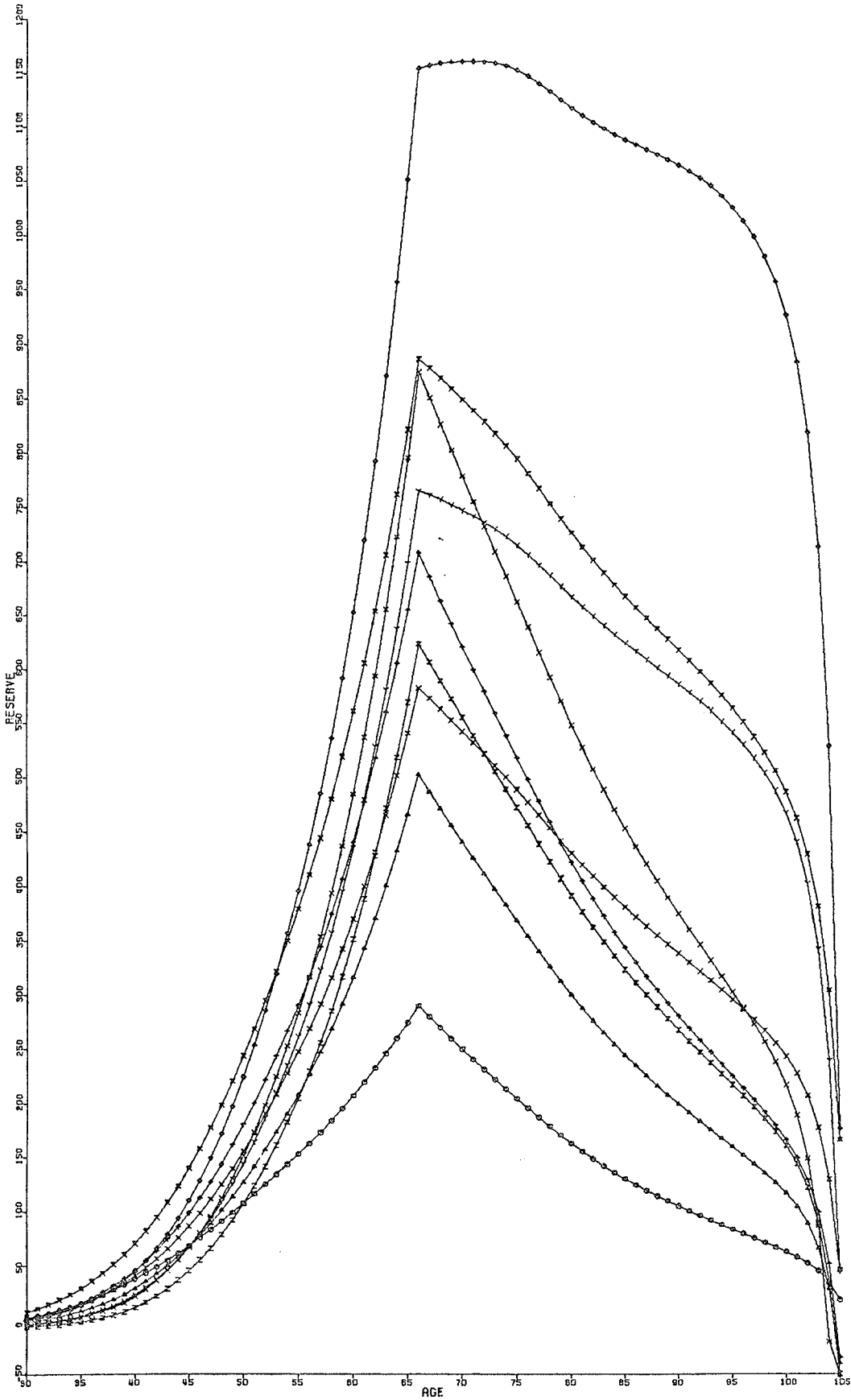
These figures were drawn up by tracing the reserve of a member from entry to the end of the life span. The reserves were calculated taking into account the probability of death and withdrawal.

If, for example, under non-inflationary conditions, the reserve of a member entering at age 30 would follow the path shown by , the reserve would grow to a maximum of around \$300 at age 66 (just before the first pension benefit is paid out). Then the reserve would decrease steadily to a value roughly equal to zero.

Under all interest and inflation assumptions the reserve grows to a maximum at age 66, then decreases to zero at the end of the life span. The rate of decrease differs greatly from one another. However, by comparing the 3 figures it becomes apparent that, given the same assumption about interest, inflation and COLA, the rates

FIGURE 4.10

TRACE OF THE RESERVES
FOR A MEMBER ENTERING AT AGE 30



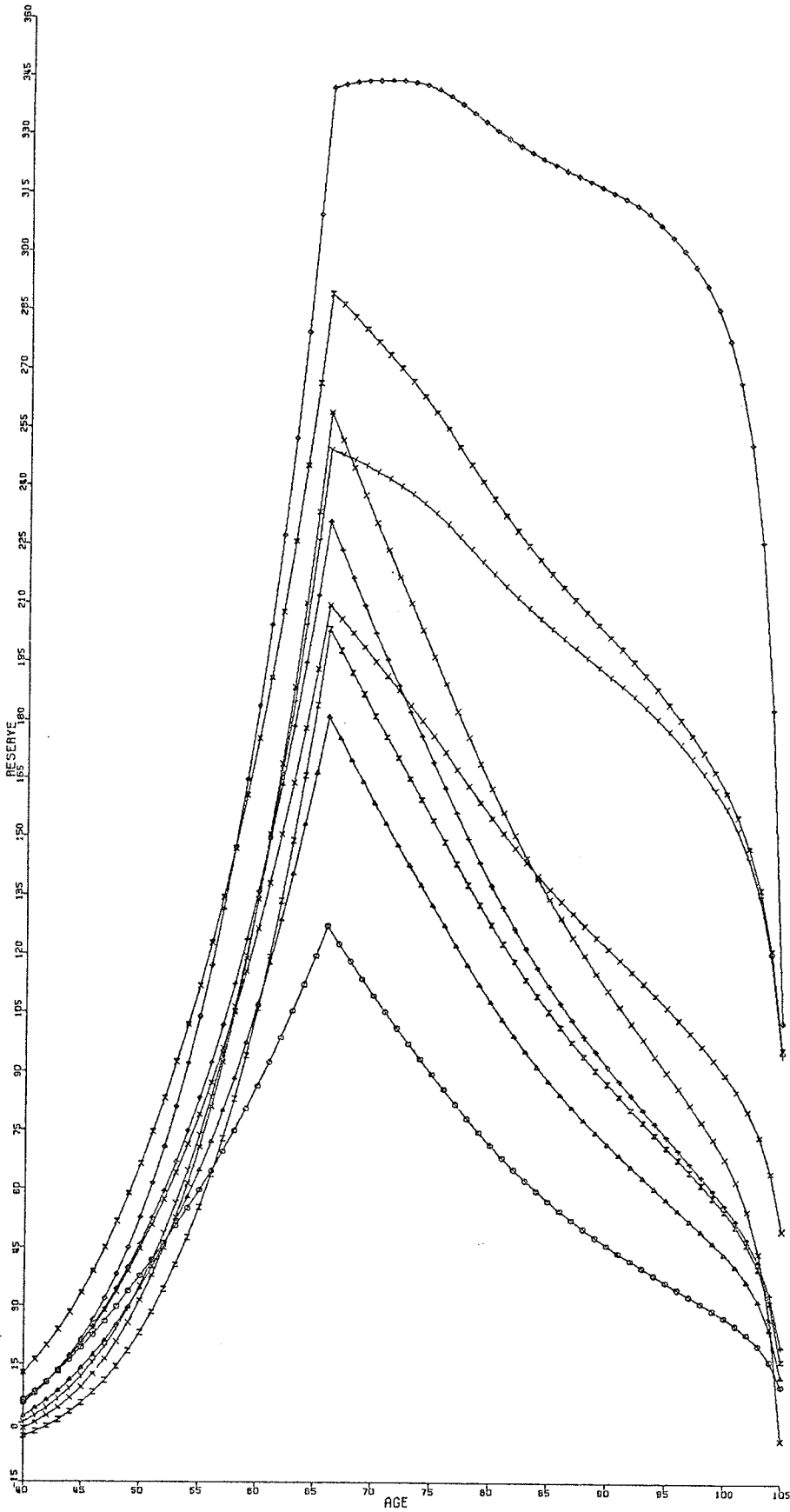
KEY

INTEREST INFLATION COST

- — 3 0 0
- △ — 5 2 0
- × — 5 2 2
- × — 7 4 0
- ◇ — 7 4 4
- ↑ — 5 3 0
- × — 5 3 3
- z — 7 3 0
- Y — 7 3 3

FIGURE 4.11

TRACE OF THE RESERVES
FOR A MEMBER ENTERING AT AGE 40



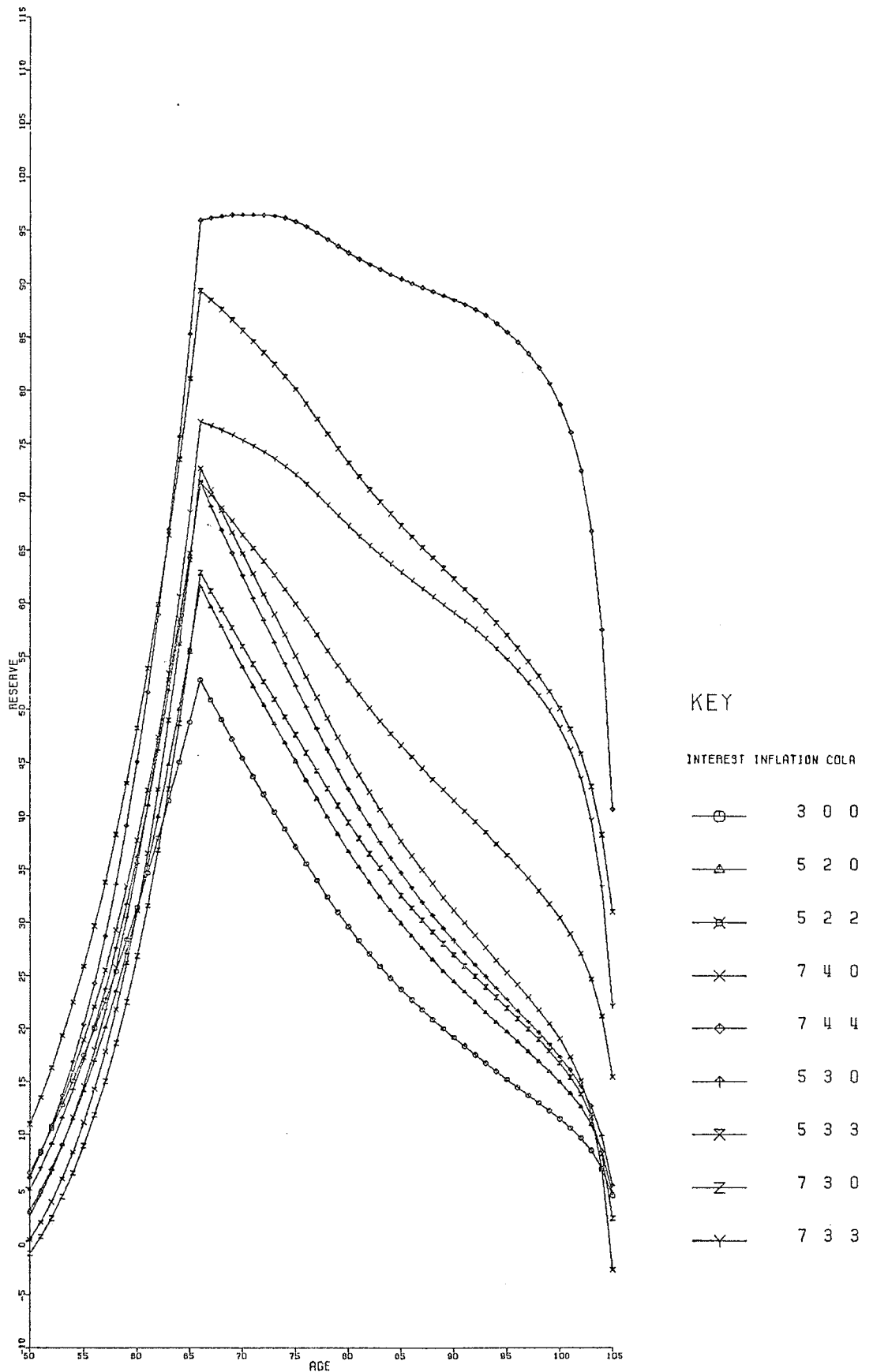
KEY

INTEREST INFLATION COLA

- 3 0 0
- △— 5 2 0
- ×— 5 2 2
- x— 7 4 0
- ◇— 7 4 4
- ▲— 5 3 0
- ▽— 5 3 3
- z— 7 3 0
- y— 7 3 3

FIGURE 4.12

TRACE OF THE RESERVES
FOR A MEMBER ENTERING AT AGE 50



of decrease for all three entry ages are similar.

From these, it can be said that the shape of the reserve is more a function of interest and inflation than of entry age. The size of the reserve is determined by the entry age.

By tracing the reserve more insight can be gained about the relationship between interest, inflation and COLA.

If, for example, inflation and interest rates are 7% and 4% per year respectively, the reserve of a member entering at, say 30, should grow at a rate independent of whether COLA is provided. The difference found in the trace is to be accounted for by the difference in the initial reserve. If COLA is provided the initial reserve would be minus \$0.04. If no COLA is forthcoming, the initial reserve is minus \$4.46. Thus, the initial difference is \$4.42 at age 30. This difference grows, subject to the force of interest, withdrawal and mortality to a value of roughly \$300 (\$1170 - \$870). It is from this \$300 that COLA is to be provided. In the initial pension paying years, the interest income and the reserves released (due to death of members) are enough to pay the pension benefits. There is not much drop in the level of the fund. In the case where no COLA is provided, the fund is smaller, so is the interest income. Therefore the fund

decreases drastically from age 66.

This analysis further points to the idea of providing COLA from interest income.

Summary

The numerical results show that, by providing COLA, no increase in cost would be incurred if real interest rate remains constant. By not providing COLA, the initial cost is reduced. If the employer accepts the cost of the plan under non-inflationary condition, then there is no reason why COLA should not be provided.

Conclusion

In the present inflationary situation, a pension plan is not doing a good job if it only provides a fixed pension income. An income which provides the retirees with a predictable purchasing power should be provided.

From the cost evaluation of a model plan, it was shown that given that real interest rate remains constant -- an increase in the inflation rate results in a comparable increase in the interest rate -- then indexed final average salary formula is feasible.

Suggestions For Further Research

Indexed final average plans work well within limited (and reasonable) constraints. However, theoretically speaking, there are cases when the whole fund could become insolvent. If, for example, all members are of age 64, and inflation suddenly turned wild (salary is increased five times). How could the fund accumulated at the old salary base provide benefits at the new inflated salary? Of course this is a very extreme situation, but how should these situations be tackled? Would a cap on the index help? Should a separate fund be established from which the COLA benefit is to be provided?

If inflation rates continue to be high, the survival of private pensions depends to a large extent on how well it serves its purpose. More research into the above problems and other related ones are needed.

The computer model established in this study can be adapted for further use to evaluate plans with different provisions.

APPENDICES

- A. Plans and Members by Type of Benefit, 1960, 1965, 1970 and 1974.
- B. Derivation of the Annual Rate of Increase in a Formula Escalator Factor Plan.
- C. The Computer Listing.
- D. Single Decrement Tables and Related Rates of Decrements.

APPENDIX A

Plans and Members by Type of Benefit, 1960, 1965, 1970 and 1974

Type of benefit	1960		1965		1970		1974	
	Number	%	Number	%	Number	%	Number	%
(a) By member:								
Unit benefit								
Final earnings	10,793	0.6	23,434	1.0	5,613	0.2	6,691	0.2
Final average earnings	283,720	15.2	261,800	11.2	169,798	6.0	168,500	4.9
Average best earnings	632,295	33.9	859,771	36.7	1,260,917	44.7	1,685,556	49.2
Career average	468,247	25.1	667,224	28.4	679,631	24.1	689,110	20.1
Level % of earnings	-	-	1,034	-	2,721	0.1	576	-
Total	1,395,055	74.9	1,813,263	77.3	2,118,680	75.1	2,550,433	74.5
Money purchase	242,127	13.0	152,738	6.5	137,680	4.9	173,935	5.1
Profit sharing	23,616	1.3	28,253	1.2	21,374	0.8	19,184	0.6
Flat benefit	177,059	9.5	327,932	14.0	424,623	15.0	622,910	18.2
Composite	24,824	1.3	23,185	1.0	26,221	0.9	34,241	1.0
Other	-	-	277	-	93,758	3.3	23,542	0.7
Grand Total	1,862,681	100.0	2,345,648	100.0	2,822,336	100.0	3,424,245	100.0
(b) By plans:								
Unit benefit								
Final earnings	28	0.3	44	0.3	16	0.1	24	0.2
Final average earnings	270	3.0	288	2.1	377	2.3	407	2.6
Average best earnings	117	1.3	238	1.7	970	6.0	1,245	7.9
Career average	2,370	26.6	3,956	29.0	4,753	29.5	5,057	31.9
Level % of earnings	-	-	84	0.6	46	0.3	34	0.2
Total	2,785	31.2	4,610	33.7	6,162	38.2	6,767	42.7
Money purchase	5,392	60.4	7,758	56.8	8,471	52.5	7,532	47.5
Profit sharing	211	2.4	351	2.6	310	1.9	258	1.6
Flat benefit	411	4.6	689	5.0	742	4.6	878	5.5
Composite	121	1.4	227	1.7	265	1.6	230	1.5
Other	-	-	25	0.2	187	1.2	188	1.2
Grand total	8,920	100.0	13,660	100.0	16,137	100.0	15,853	100.0

Source: Statistics Canada, Pension Plans in Canada, 1974, Catalogue No. 74-401.

APPENDIX B

Derivation of the Annual Rate of Increase in a Formula Escalator Factor Plan:

The present values of the following two different annuities should be equal.

(a) An annuity of one dollar payable at the end of each year.

(b) An initial reduced annuity (=k% of one dollar) with a guarantee increase at a fixed rate (factor).

Present Values at age 65:

$$(a) \quad \frac{N_{66}}{D_{65}}$$

$$(b) \quad (0.01)k \frac{N_{66}}{D_{65}} + \text{factor} \cdot \frac{R_{67}}{D_{65}}$$

In order that the present values of the two annuities be equal,

$$N_{66} = (0.01)k \cdot N_{66} + \text{factor} \cdot R_{67}$$

$$\text{factor} = \frac{(1 - (0.01)k) N_{66}}{R_{67}}$$

APPENDIX C
THE COMPUTER LISTING

```

$JOB WATFIV   CHAN,NOEXT
1   REAL INTRST,INFLAT,GATL(110),L(110),W(110),S(110),D(110),WR(110),
   *Q(110),SR(110),QD(110),QW(110),V(110),IF(110),SIF(110),
   *VSIF(110),WSIF(110),VDSIF(110),WVSIF(110),VGATL(110),NVDSIF(110),
   *CON(110),TDEATH(110),NDEATH(110),DDEATH(110),RDEATH(110),
   *PVCON(110),PVWITH(110),PVDTH(110),PVDIFF(110),COLA,COLARY(110),
   *DGATL(110),WDGATL(110),NGATL(110),RGATL(110),WITH(110)
2   REAL NNW,TW(110),DW(110),NW(110),VT(110),TRM(110),PVW(110)
3   REAL RCON,RVALUE,P,OUTDTH,OUTWTH,OUT,IN,BENEFT
   C
4   REAL PREMIUM, INT(10), INF(10), PERCEN
5   INTEGER AGE, FIRST, LAST, RETIRE, COMBIN
6   INTEGER FRTPLS, RTRPLS, RTRMNS, LSTMNS, WAIT
7   COMMON FIRST, RETIRE, LAST
   C
8   EXECUTE INITAL
9   EXECUTE DATA
10  DO 100 III=1, COMBIN
11      INTRST=INT(III)
12      INFLAT=INF(III)
13  EXECUTE ACTION
14  100 CONTINUE
15  WRITE(6,108)
16  STOP
   C
C*****

```

The above is the main program. A detailed description of how the above works can be found in the text.

```

C
17  REMOTE BLOCK INITAL
18  PREMUM=1.5
19  WAIT=10
20  PERCEN=0.2
21  GATL(1)=5.
22  GATL(2)=110.
23  GATL(3)=1.
24  WR(1)=20.
25  WR(2)=54.
26  WR(3)=2.
27  S(1)=20.
28  S(2)=65.
29  S(3)=3.
30  FIRST=20
31  RETIRE=65
32  COMBIN=5
33  INT(1)=0.03
34  INF(1)=0.00
35  INT(2)=0.05
36  INF(2)=0.02
37  INT(3)=0.07
38  INF(3)=0.04
39  INT(4)=0.05
40  INF(4)=0.03
41  INT(5)=0.07
42  INF(5)=0.03
43  LAST=GATL(2)
44  FRTPLS=FIRST+1
45  RTRPLS=RETIRE+1
46  RTRMNS=RETIRE-1
47  LSTMNS=LAST-1
48  END BLOCK

```

```

C
C*****

```

This is the remote block where all the initializations are made. Refer to the Variable Dictionary on the last page for a detailed description of each variable.

```

C*****
C
49     REMOTE BLOCK DATA
50     CALL DREAD(GATL)
51     CALL DREAD(WR)
52     CALL DREAD(S)
53     CALL RATIO(GATL,Q,FIRST,LAST,'Q')
54     CALL RATIO(S,SR,FIRST,RETIRE,'P')
55     CALL PROBAB(Q,WR,QD)
56     CALL PROBAB(WR,Q,QW)
C
C     PRINT OUT THE SINGLE DECREMENT TABLES.
C
57     WRITE(6,101) 'SINGLE DECREMENT TABLES'
58     CALL PRT(GATL,Q,WR,QD,QW,S,SR,FIRST,RETIRE)
59     EXECUTE TABLE
60     END BLOCK
C
C*****

```

The above is the remote block where all data are read in and the related rates of decrements are calculated. Then the multiple decrement table is established.

```

C*****
C
61  REMOTE  BLOCK ACTION
C
C  ESTABLISH THE BASE
C
62  EXECUTE BASE
C
C  CALCULATE THE PRESENT VALUE OF TOTAL FUTURE CONTRIBUTIONS
C
63  EXECUTE CONBUT
C
C  CALCULATE THE PRESENT VALUE OF TOTAL FUTURE DEATH BENEFITS
C
64  EXECUTE BDEATH
C
65  COLA=0.0
C
66  IF (COLA.EQ.INFLAT) THEN DO
67      IRUN=1
68  ELSE DO
69      IRUN=2
70  END IF
71  DO 222 II=1,IRUN
72      EXECUTE BWITH
73      EXECUTE PRESEN
74      EXECUTE TRACE
75      COLA=INFLAT
76  222 CONTINUE
C
77  101  FORMAT ('1', 5X, A50,////)
78  106  FORMAT(5(F13.5))
79  107  FORMAT(I6)
80  108  FORMAT ('1')
81  END BLOCK

```

```

C
C*****

```

A detailed description, together with a flow chart of this remote block can be found in the text. Basically the block controls the flow of logic through the other remote blocks, where the present values of contribution, death benefit and withdrawal benefits are calculated.

```

C*****
C
C
82     REMOTE BLOCK TABLE
83     L(20) = 10000.00
84     DO 1 I = FIRST, RETIRE
85         W(I)=L(I)*QW(I)
86         D(I)=L(I)*QD(I)
87         L(I+1)=L(I)-W(I)-D(I)
88     1  CONTINUE
89     W(RETIRE) = L(RTRPLS)
90     DO 11 I=RTRPLS,LSTMNS
91         W(I)=0.0
92         D(I)=L(I)*QD(I)
93         L(I+1)=L(I)-D(I)
94     11  CONTINUE
95     D(LAST)=L(LAST-1)
96     END BLOCK

C
C
C*****
C
97     REMOTE BLOCK BASE
98     CALL POWER(V,1,LAST,INTRST,-1,1.)
99     CALL POWER(IF,1,LAST,INFLAT,1,1.)
100    CALL WEIGHT(IF,FIRST,RETIRE,S,FIRST,RETIRE,SIF,FIRST,RETIRE)
101    CALL WEIGHT(SIF,FIRST,RETIRE,V,FIRST,RETIRE,VSIF,FIRST,RETIRE)
102    CALL WEIGHT(VSIF,FIRST,RETIRE,L,FIRST,RETIRE,VDSIF,FIRST,RETIRE)
103    CALL WEIGHT(SIF,FIRST,RETIRE,V,FRTPLS,RTRPLS,WSIF,FIRST,RETIRE)
104    CALL WEIGHT(W,FIRST,RETIRE,WSIF,FIRST,RETIRE,WVSIF,FIRST,RETIRE)
105    CALL WEIGHT(GATL,FIRST,LAST,V,FIRST,LAST,VGATL,FIRST,LAST)
106    END BLOCK

C
C*****

```

Remote block TABLE establishes the multiple table using the data read in or calculated in the block DREAD.

In the remote block BASE some commonly used arrays are established.

```

V(x)      = (1+intrst)-x
IF(x)     = (1+inflat)x
SIF(x)    = S(x) * IF(x)
VSIF(x)   = V(x) * SIF(x)
VDSIF(x)  = VSIF(x) * L(x)
WSIF(x)   = SIF(x) * V(x)
WVSIF(x)  = W(x) * WSIF(x)
VGATL(x)  = GATL(x) * V(x)

```

```

C
C*****:
C
C
107      REMOTE BLOCK CONBUT
108      CALL COLSUM(VDSIF, FIRST, RETIRE, NVDSIF, FIRST, RETIRE)
109      CALL MULTI(NVDSIF, CON, FIRST, RETIRE, PREMUM)
110      CALL OVER (CON, VDSIF, PVCON, FIRST, RETIRE)
111      END BLOCK

C
C*****
C
C
112      REMOTE BLOCK BDEATH
113      CALL COLSUM(D, FIRST, RETIRE, TDEATH, FIRST, RETIRE)
114      CALL WEIGHT(TDEATH, FIRST, RETIRE, VSIF, FIRST, RETIRE, DDEATH,
*FIRST, RETIRE)
115      CALL MULTI(DDEATH, NDEATH, FIRST, RETIRE, PREMUM)
116      CALL COLSUM(NDEATH, FIRST, RETIRE, RDEATH, FIRST, RETIRE)
117      CALL OVER(RDEATH, VDSIF, PVDTH, FIRST, RETIRE)
118      END BLOCK

C
C*****

```

In the remote block CONBUT the present values of the contributions are calculated.

$$NVDSIF(x) = \sum_{\text{retire}}^x VDSIF(s)$$

$$CON(x) = NVDSIF(x) * PREMUM$$

$$PVCON(x) = CON(x) / VDSIF(x)$$

In the remote block BDEATH, the present values of the death benefits are calculated.

$$TDEATH(x) = \sum_{\text{first}}^x D(s)$$

$$DDEATH(x) = TDEATH(x) * VSIF(x)$$

$$NDEATH(x) = DDEATH(x) * PREMUM$$

$$RDEATH(x) = \sum_{\text{first}}^x NDEATH(s)$$

$$PVDTH(x) = RDEATH(x) / VDSIF(x)$$

C*****

C
C

```

119      REMOTE BLOCK BWITH
120      COLARY(1) = 1.00
121      IDUMMY=LAST-RETIRE
122      JDUMMY=IDUMMY+1
123      CALL POWER(COLARY, 2, JDUMMY, COLA, 1, 1.)
124      CALL WEIGHT(VGATL, RTRPLS, LAST, COLARY, 1, IDUMMY, DGATL,
    *RTRPLS, LAST)
125      CALL SUM(DGATL, RTRPLS, LAST, ANNUITY)
126      CALL VEST (ANNUITY, VGATL, DGATL, FIRST, RTRPLS)
127      CALL WEIGHT(WVSIF, FIRST, RETIRE, DGATL, FRTPLS, RTRPLS, WDGATL,
    *FIRST, RETIRE)
128      CALL COLSUM(WDGATL, FIRST, RETIRE, NGATL, FIRST, RETIRE)
129      CALL MULTI(NGATL, WITH, FIRST, RETIRE, PERCEN)
130      CALL COLSUM(WITH, FIRST, RETIRE, RGATL, FIRST, RETIRE)
131      WITH(RTRPLS)= RGATL(RTRPLS) = 0.0
132      DO 3 I = FIRST, RTRMNS
133          J=MINO(RTRMNS, (I+WAIT-2))
134          CALL COLSUM(W, I, J, TW, I, J)
135          CALL WEIGHT(TW, I, J, VSIF, I, J, DW, I, J)
136          CALL SUM(DW, I, J, NNW)
137          IF(J.LT.(I+WAIT-2)) THEN DO
138              VT(I)=(RTRPLS-I)*RGATL(RETIRE)
139          ELSE DO
140              VT(I)=RGATL(J+1)+(WAIT-1)*WITH(J+1)
141          END IF
142          TRM(I) = PREMUM*0.5 * NNW
143      3    CONTINUE
144      TRM(RETIRE) = 0.0
145      VT(RETIRE) = RGATL(RETIRE)
146      CALL OVER(TRM, VDSIF, PVWITH, FIRST, RETIRE)
147      CALL OVER(VT, VDSIF, PVW, FIRST, RETIRE)
148      END BLOCK

```

C

C*****

C

C

In this remote block, the present value of the withdrawal benefits is calculated. The block takes into account the number of years (WAIT) when vested pension becomes available.

PVWITH(x) is the present value of the total withdrawal benefits.

PVW(x) is the present value of the vested pension benefits.

```

C
C
149     REMOTE BLOCK PRESEN
150     DO 2 I = FIRST,RETIRE
151     2  PVDIFF(I) = (PVDTH(I) + PVW(I) + PVWITH(I)) - PVCON(I)
152     WRITE(4,106) INTRST,INFLAT,COLA
153     DO 21 I=FIRST,RETIRE
154     21 WRITE(4,106) PVDTH(I),PVW(I),PVWITH(I),PVCON(I),PVDIFF(I)
155     END BLOCK

C
C*****
C
156     REMOTE BLOCK TRACE
157     AGE=40
158     RVALUE=PVDIFF(AGE)
159     ITAPE=9
160     EXECUTE RSERVE
161     AGE=30
162     RVALUE=PVDIFF(AGE)
163     ITAPE=10
164     EXECUTE RSERVE
165     AGE=50
166     RVALUE=PVDIFF(AGE)
167     ITAPE=11
168     EXECUTE RSERVE
169     END BLOCK

C
C*****

```

In the remote block PRESEN, the present value of the initial unfunded liabilities is calculated.

PVDIFF(x) = the initial unfunded liability.
 = (PVDTH(x) + PVW(x) + PVWITH(x)) - PVCON(x)

The remote block TRACE controls the number and age of members, whose reserve is to be traced. Age of the member is first assigned. Then control is passed to the remote block RSERVE which actually carry out the tracing. The assigned value of RVALUE is going to be the initial value of the fund. ITAPE is the number of the tape the calculated reserve values are going to be written on to.

C

C*****

C

```

170     REMOTE BLOCK RSERVE
171     RCON= 0.0
172     EACH = RVALUE
173     RVALUE=RVALUE*L(AGE)
174     WRITE(ITAPE,106) INTRST,INFLAT,COLA
175     WRITE(ITAPE,107) AGE
176     WRITE(ITAPE,106) RVALUE, EACH
177     J=AGE+1
178     K=AGE+WAIT-1
179     DO 4 I= J,K
180         P=PREMUM*SIF(I-1)/SIF(AGE)
181         RCON=(RCON+P)*(1+INTRST)
182         OUTDTH=RCON
183         OUTWTH=RCON*.5
184         OUT=D(I-1)*OUTDTH+W(I-1)*OUTWTH
185         IN =(RVALUE+P*L(I-1))*(1+INTRST)
186         RVALUE=IN-OUT
187         EACH = RVALUE/L(I)
188         WRITE(ITAPE,106) RVALUE, EACH
189     4 CONTINUE
190     J = AGE+WAIT
191     DGATL(RTRPLS)= 0.0
192     DO 5 I = J, RTRPLS
193         P=1.5*SIF(I-1)/SIF(AGE)
194         RCON=(RCON+P)*(1+INTRST)
195         OUTDTH=RCON
196         OUTWTH=(I-AGE)*PERCEN*(SIF(I-1)/SIF(AGE))*DGATL(I)
197         OUT=D(I-1)*OUTDTH+W(I-1)*OUTWTH
198         IN=(RVALUE+P*L(I-1))*(1+INTRST)
199         RVALUE=IN-OUT
200         EACH=RVALUE/L(I)
201         WRITE(ITAPE,106) RVALUE, EACH
202     5 CONTINUE
203     BENEFT= (RTRPLS-AGE)*PERCEN*SIF(RETIRE)/SIF(AGE)
204     P=OUTDTH=OUTWTH=0.0
205     K=RTRPLS+1
206     DO 6 I = K, LAST
207         OUT=L(I-1)*BENEFT*(1+INTRST)
208         IN=RVALUE*(1+INTRST)
209         RVALUE=IN-OUT
210         EACH = RVALUE/L(I)
211         WRITE(ITAPE,106) RVALUE, EACH
212         BENEFT=BENEFT*(1+COLA)
213     6 CONTINUE
214     END BLOCK
215     END

```

C

C*****

This is the remote block where the actual tracing of the reserve of a member is carried out. Reserve for each age is calculated taking into account the probability of death and withdrawal, income from interest and premium contributions, and benefit payments.

C

C*****

C PRINT OUT THE 7 ARRAYS FROM ENTRY IX TO IY.

```

216     SUBROUTINE PRT(A1,A2,A3,A4,A5,A6,A7,IX,IY)
217     REAL A1(110),A2(110),A3(110),A4(110),A5(110),
      *A6(110),A7(110)
218     DO 1 I = IX, IY
219     1  WRITE(6,10)I,A1(I),A2(I),A3(I),A4(I),A5(I),A6(I),A7(I)
220     10  FORMAT (I6, 7(F12.5))
221     RETURN
222     END

```

C

C*****

C FILL IN THE SUBSEQUENT ENTRIES OF AN ARRAY ASSUMING AN ARITH. PROGRESSION.

```

223     SUBROUTINE AP(ARRAY,IY,IZ,VALUE)
224     REAL ARRAY(110),VALUE,INCREM
225     J= IZ-IY
226     IX= IY+1
227     INCREM=(VALUE - ARRAY(IY))/J
228     DO 1 I = IX, IZ
229     1  ARRAY(I) = ARRAY(I-1) + INCREM
230     DO 2 I=IZ,110
231     2  ARRAY(I) = VALUE
232     RETURN
233     END

```

C

C*****

C MULTIPLY EACH ENTRY OF AN ARRAY BY A FACTOR (TIMES).

```

234     SUBROUTINE MULTI(FROM,TO,IX,IY,TIMES)
235     REAL FROM(110),TO(110),TIMES
236     DO 1 I= IX,IY
237     1  TO(I) = FROM(I) * TIMES
238     RETURN
239     END

```

C

C*****

C ESTABLISH AN ARRAY: $ARRAY(I) = (1+RATE)^I \times B$

```

240     SUBROUTINE POWER (ARRAY, X, Y, RATE, N, B)
241     INTEGER X, Y, N
242     REAL ARRAY(110), RATE
243     REAL B
244     ARRAY(X) = B * (1 + RATE) ** N
245     IY = Y - 1
246     DO 1 I = X, IY
247     ARRAY(I+1) = ARRAY(I)*(1 + RATE) ** N
248     1  CONTINUE
249     RETURN
250     END

```

C

```

C*****
C WEIGHT TWO ARRAYS:  END(x+1) = THIS(a+1) x THAT(m+1)
251     SUBROUTINE WEIGHT (THIS,A,B,THAT,M,N,END,X,Y)
252     INTEGER A, B, M, N, X, Y,MINO
253     REAL THIS(110), THAT(110), END(110)
254     I = MINO((B-A+1),MINO((N-M+1),(Y-X+1)))
255     DO 1 J = 1, I
256         END (X+J-1) = THIS(A+J-1)*THAT(M+J-1)
257     1 CONTINUE
258     RETURN
259     END

C
C*****
C SUMMATION OF A SPECIFIED ENTRIES OF THE ARRAY:  RESULT=  $\sum_x^y$  ARRAY(s)
260     SUBROUTINE SUM(ARRAY,X,Y,RESULT)
261     REAL ARRAY(110), RESULT
262     INTEGER X, Y
263     RESULT = 0.
264     I = Y
265     WHILE (I .GE. X) DO
266         RESULT = RESULT + ARRAY(I)
267         I = I - 1
268     END WHILE
269     RETURN
270     END

C
C*****
C ESTABLISH AN ARRAY:  TO(x) =  $\sum_a^x$  FROM(s)
271     SUBROUTINE COLSUM (FROM,A,B,TO,M,N)
272     REAL FROM(110), TO(110)
273     INTEGER A,B,M,N, MINO
274     I = MINO ((B-A),(N-M))
275     IF (I.NE.(B-A).OR.I.NE.(N-M)) THEN DO
276         PRINT, ' WARNING: RANGE FOR COLSUM IS NOT
* THE SAME, SMALLER RANGE ASSUMED'
277     END IF
278     TO(N)=FROM(B)
279     IF (I.LE. 0) THEN DO
280         RETURN
281     END IF
282     DO 1 J = 1, I
283         TO(N-J) = TO (N-J+1) + FROM(B-J)
284     1 CONTINUE
285     RETURN
286     END

C
C*****
C ESTABLISH AN ARRAY:  TO(x) = VALUE / FROM(x)
287     SUBROUTINE VEST(VALUE, FROM,TO,M,N)
288     REAL FROM(110),TO(110),VALUE

```

```

289      DO 1 I = M,N
290      1  TO(I)=VALUE/FROM(I)
291      RETURN
292      END

```

C

C*****

C READ IN THE DATA ACCORDING TO DATA STORED IN THE FIRST 3 ENTRIES.

```

293      SUBROUTINE DREAD(ARRAY)
294      REAL ARRAY(110)
295      INTEGER FIRST,RETIRE,LAST
296      COMMON FIRST,RETIRE,LAST
297      IX=ARRAY(1)
298      IY=ARRAY(2)
299      IZ=ARRAY(3)
300      DO 1 I=IX,IY
301      1  READ(IZ,10) ARRAY(I)
302      IF(IY.LT.RETIRE) THEN DO
303          IF(ARRAY(IY).EQ.1.0) THEN DO
304              CALL AP(ARRAY,IY,RETIRE,1.0)
305          ELSE DO
306              CALL AP(ARRAY,IY,RETIRE,0.0)
307          END IF
308      END IF
309      10  FORMAT(F11.5)
310      RETURN
311      END

```

C

C*****

C CALCULATE THE PROBABILITIES.

```

312      SUBROUTINE PROBAB(ARRAYX,ARRAYY,ARRAYZ)
313      REAL ARRAYX(110),ARRAYY(110),ARRAYZ(110)
314      DO 1 I = 20, 65
315      1  ARRAYZ(I) = ARRAYX(I)*(1-ARRAYY(I)/2)
316      DO 2 I = 66, 110
317      2  ARRAYZ(I) = ARRAYX(I)
318      RETURN
319      END

```

C

C*****

C ESTABLISH AN ARRAY: $TO(x) = FROM1(x) / FROM2(x)$

```

320      SUBROUTINE OVER(FROM1, FROM2, TO, M, N)
321      REAL FROM1(110), FROM2(110), TO(110)
322      DO 1 I=M,N
323      1  TO(I) = FROM1(I)/FROM2(I)
324      RETURN
325      END

```

C

C*****

C

```
C*****
C ESTABLISH AN ARRAY: ARRAYY(x) = ARRAYX(x+1) / ARRAYX(x)
C
326     SUBROUTINE RATIO(ARRAYX,ARRAYY,M,N,CNTL)
327     REAL ARRAYX(110),ARRAYY(110)
328     CHARACTER CNTL
329     J=N-1
330     DO 1 I= M, J
331     1  ARRAYY(I)= ARRAYX(I+1)/ARRAYX(I)
332     ARRAYY(N) = 1.
333     IF (CNTL.EQ.'Q') THEN DO
334     DO 2 I = M, J
335     2  ARRAYY(I) = 1. - ARRAYY(I)
336     END IF
337     RETURN
338     END
C
C*****!
C
$ENTRY
```

Variable Dictionary:

In the following list, only variables found in the remote block INITIAL are listed. These are the only variables that users of the program have to know. Variables are listed in the order found in the block INITIAL.

PREMUM	0.1 x percent of employee's salary to be made as contribution, sum of employer and employee's contributions.
WAIT	number of years of service before vested pension benefit is first available (on termination from the plan).
PERCEN	percentage of final average salary used in calculating the pension benefit.
GATL(1) WR(1) SR(1)	the first data available in each set of data: group annuity table, withdrawal rates, and salary rates respectively.
GATL(2) WR(2) SR(2)	the last data available in each set of data: group annuity table, withdrawal rates, and salary rates respectively.
GATL(3) WR(3) SR(3)	the number assigned in the FTOIF001 statement in the Job Control Language which specifies where the datasets are stored.
FIRST	the first age the study is interested in.
RETIRE	the age of retirement.
COMBIN	the number of combinations of interest rates and inflation rates the study is going to examine.
INT(I)	the ith interest rate that the study is going to use.
INF(I)	the ith inflation rate that the study is going to use.
LAST	the last age of the life span. This is assigned to be equal to the last age found in the mortality table.

APPENDIX D
SINGLE DECREMENT TABLES
AND RELATED RATES OF DECREMENTS

x	l_x	q_x	W_x	$q_x^{(1)} = q_x(1 - \frac{1}{2}W_x)$	$q_x^{(w)} = W_x(1 - \frac{1}{2}q_x)$	S_x	$\frac{S_{x+1}}{S_x}$
20	9936.37100	0.00050	0.21600	0.00045	0.21595	0.15000	1.07673
21	9931.37100	0.00052	0.20360	0.00047	0.20355	0.16151	1.07467
22	9926.18700	0.00054	0.19210	0.00049	0.19205	0.17357	1.07265
23	9920.78900	0.00057	0.18140	0.00051	0.18135	0.18618	1.07063
24	9915.17100	0.00059	0.17150	0.00054	0.17145	0.19933	1.06858
25	9909.31200	0.00062	0.16240	0.00057	0.16235	0.21300	1.06662
26	9903.17900	0.00065	0.15330	0.00060	0.15325	0.22719	1.06475
27	9896.74200	0.00068	0.14460	0.00063	0.14455	0.24190	1.06284
28	9889.97200	0.00072	0.13620	0.00067	0.13615	0.25710	1.06110
29	9882.83200	0.00076	0.12820	0.00071	0.12815	0.27281	1.05935
30	9875.29200	0.00081	0.12060	0.00076	0.12055	0.28900	1.05768
31	9867.30000	0.00086	0.11340	0.00081	0.11335	0.30567	1.05611
32	9858.81600	0.00092	0.10650	0.00087	0.10645	0.32282	1.05458
33	9849.78500	0.00098	0.10010	0.00093	0.10005	0.34044	1.05311
34	9840.15200	0.00105	0.09420	0.00100	0.09415	0.35852	1.05166
35	9829.85900	0.00112	0.08860	0.00107	0.08855	0.37704	1.05031
36	9818.83200	0.00120	0.08340	0.00115	0.08335	0.39601	1.04891
37	9807.00700	0.00129	0.07850	0.00124	0.07845	0.41538	1.04762
38	9794.30800	0.00140	0.07400	0.00135	0.07395	0.43516	1.04630
39	9780.62500	0.00151	0.06990	0.00146	0.06685	0.45531	1.04507
40	9765.86700	0.00163	0.06630	0.00158	0.06625	0.47583	1.04386
41	9749.91700	0.00179	0.06270	0.00173	0.06264	0.49670	1.04274
42	9732.47600	0.00200	0.05900	0.00194	0.05894	0.51793	1.04167
43	9713.01100	0.00226	0.05530	0.00220	0.05524	0.53951	1.04070
44	9691.05800	0.00257	0.05170	0.00250	0.05163	0.56147	1.03982
45	9666.16400	0.00292	0.04800	0.00285	0.04793	0.58383	1.03914
46	9637.91700	0.00332	0.04430	0.00324	0.04423	0.60668	1.03870
47	9605.94100	0.00375	0.04050	0.00368	0.04042	0.63016	1.03839
48	9569.87800	0.00423	0.03650	0.00415	0.03642	0.65435	1.03816
49	9529.41700	0.00474	0.03240	0.00466	0.03232	0.67932	1.03789
50	9484.25000	0.00529	0.02790	0.00521	0.02783	0.70506	1.03746
51	9434.12500	0.00587	0.02310	0.00580	0.02303	0.73147	1.03693
52	9378.77300	0.00648	0.01800	0.00642	0.01794	0.75848	1.03614
53	9318.00000	0.00713	0.01250	0.00708	0.01246	0.78589	1.03488
54	9251.58900	0.00781	0.00650	0.00778	0.00647	0.81330	1.03321
55	9179.37500	0.00852	0.00591	0.00849	0.00588	0.84031	1.03095
56	9101.17500	0.00926	0.00532	0.00924	0.00529	0.86632	1.02771
57	9016.87800	0.01004	0.00473	0.01002	0.00470	0.89033	1.02382
58	8926.35900	0.01089	0.00414	0.01087	0.00411	0.91154	1.01998
59	8829.16000	0.01192	0.00355	0.01190	0.00352	0.92975	1.01636
60	8723.87800	0.01312	0.00295	0.01310	0.00294	0.94496	1.01345
61	8609.43300	0.01444	0.00236	0.01442	0.00235	0.95767	1.01160
62	8485.11300	0.01586	0.00177	0.01585	0.00176	0.96878	1.01090
63	8350.51100	0.01741	0.00118	0.01740	0.00117	0.97934	1.01058
64	8205.10500	0.01918	0.00059	0.01918	0.00059	0.98970	1.01041
65	8047.69100	0.02126	0.00000	0.02126	0.00000	1.00000	1.00000

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