Analytical Extensions for Developing and Testing Urban Location Models

by

Gareth ("Gary") Russell

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Abstract

This paper explores residential and job location decisions in a general microeconomic model, and then specifies the traditional categories of urban location theory within that framework. We examine the vector of housing services including the role of neighbourhood amenities, incorporate search behavior, and investigate a model of out-commuting and cross-commuting in a context of excess commuting. Special quality characteristics of commodities of the general model are examined as special housing needs and job skills as they apply in the traditional model. Data from a Minneapolis survey of 1990-92 are analyzed for their conformity to the price and density gradients of theory, and a set of basic relationships arising from the model are tested. Respondents' adherence to the patterns of the model, the impact of skills and special needs, the influence of family structure, and the incidence of excess commuting are examined in crosstabulations and regression analysis.

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1. OVERVIEW

1.1 Building on Urban Analysis

Urban economics has long been rooted in a simple city structure, where both population and employment are concentrated at the centre. This was not only for reasons of theoretical interest, but also because the process of urbanization indeed produced a high degree of centralization as industrialization brought population to the cities[1]. The centralization phenomenon was never simple or straightforward. Often inter-urban transportation terminals (e.g., railheads) were most economically located at a point removed from the centre, and manufacturing industries which led the industrial revolution were pulled toward them by transportation costs. Nonetheless, agglomeration economies were strong enough at the centre to produce strong central business districts.

The latter half of the 20th century witnessed a rapid population decline in central cities around the industrialized western world, a phenomenon commonly labelled *suburbanization*. Improvements in transportation, along with the growth of incomes, allowed consumers to exercise a greater preference for suburban space (as well as an aversion to deterioration at the centre) – creating a tendency for population to move outward and for jobs to follow.

However, this was characterized by a more rapid decentralization of population than employment, especially as modern transportation made manufacturing less dependent on

[1] See Mills and Hamilton (1994), Chapters 3 and 4.

railheads at the same time as it was declining in relative importance, while the service and information processing sectors were less transportation-dependent overall.

Still, decentralization of employment proceeded as suburban malls and industrial parks developed their own economies of agglomeration and scale. From this mixed picture of contemporary urban structure, we still find that the central business district persists as an important reference point, and the founding model of the city with a significant centre remains as a robust theoretical construct.

The original anchor of urban economics is commonly considered to be (and will be referred to as) the Alonso Model (Alonso, 1964, 1967) – although its genesis can be traced further back through von Thunen (1826) and others, and it developments traced forward through the extensions of Muth (1969) and many others who examined various ramifications of the model in detail[2]. The Alonso Model sets up a basic balance of forces between superior market opportunities found in a specific urban location, balanced by a need to incur greater travel costs in order to take advantage of these opportunities. In considering the decision of where to purchase housing, this balance involves employment opportunities located at city centre in relation to housing prices which peak at city centre and decline with distance in a systematic, downward-sloping *price gradient*. Further analysis of the price gradient will be found in Section 2.5. The result is a spatial optimization, in which the marginal benefits (per unit of distance) of a lower housing price are equated to the marginal costs of travel – which determines how far away the consumer is prepared to travel (or commute) in order to take advantage of a lower price.

In this model, the market under consideration involves housing, that commodity which provides a place of residence for the consumer (or resident). The housing services provided by this commodity were initially specified as land area, an easily quantified category in which more land represented more residential services to the consumer. As the model was developed, land often remained a relevant variable since it was useful as a proxy for the bundle of housing services available to the resident – and its characteristics such as elasticity, productivity, market structure, rental and capital value, transportation costs, accessibility, etc., have been the subjects of a long line of research[3]. Further analysis on the bundle of housing services and its pricing will be discussed in Chapter 3. A key relationship of the model is that residential housing services per unit are assumed to be available along a ray and to decline in price per unit with distance from city centre (as discussed in Section 2.5). At the same time, the necessity to travel arises from the fact that employment is available only at city center (in the original model). This results in commuting costs, which will be greater if the residence is located farther from city center. Hence, as the resident considers housing locations farther from city center, the declining marginal benefit of obtaining lower prices farther from city center is compared to the marginal cost of commuting a greater distance to city center.

Formally the model[4] is one of utility maximization in the form

$$\max U(z,s) \quad \text{subject to} \quad z + R(u)s = Y - T(u) \qquad (1.1)$$

[2] Evans (1973, pages 8-9) gives a good short outline of its genesis.
[3] E.g., Muth (1968), Goldberg (1970), Smith (1969), Brigham (1964, 1965), with roots in Chamberlin (1956, Appendix D).

where R(u) represents rent R at distance from city centre u, T(u) represents transportation cost, s (lot size) represents the quantity of land consumed, and z represents a composite commodity (all other goods). In other words, income minus travel cost is available to spend on land and other goods. However the spatial aspect is better revealed in the alternate form

$$\Psi(u, U_0) = \max\left\{\frac{Y - T(u) - z}{s} \quad s.t. \quad U(z, s) = U_0\right\} . \qquad (1.2)$$

$\Psi(u, U_0)$ is the choice of bid rent for values of fixed utility U_0 – which can be used to derive the bid rent curve, an indifference curve in commodity space (z,s) for each U_i, which was analytical focus of much of the early literature and remains familiar today to facilitate solving for u. But if instead of drawing a set of bid-rent indifference curves for U_i in commodity space, we draw a different but equivalent set of indifference curves in price-distance space (still retaining the label Ψ) – we can construct a preference map of the type introduced by Alan W. Evans (1973, page 75; see also Fujita, 1989, page 24) which can be combined with the price gradient P(u) to illustrate an optimum u^* as shown in Figure 1.1 (which displays only the single indifference curve relevant to tangency).

In this figure, P(u) is a market price gradient and represents the price tradeoff offered by the market. Its slope is assumed to decline in absolute value as u increases. And each Ψ is a price indifference curve which represents the preferred price tradeoff needed to compensate the disutility of travel and a rising c(t) when commuting distance t increases.

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[4] The author makes use of Masahisa Fujita's (1989) invaluable crystallization of the "basic model" (page 12).

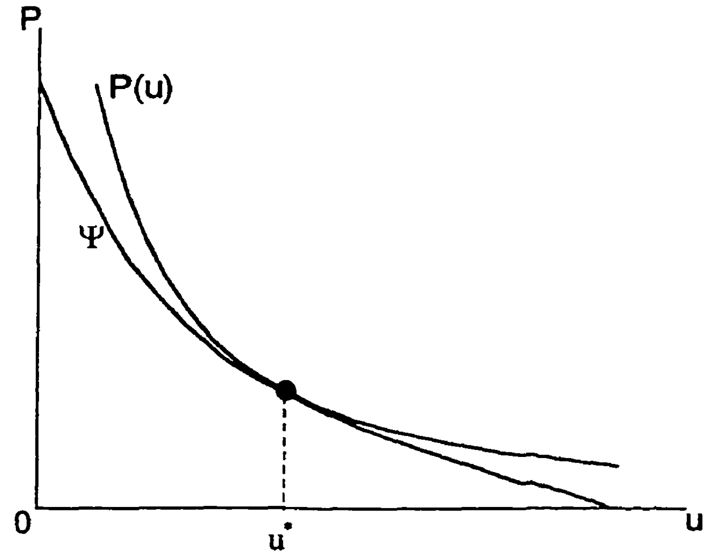

Figure 1.1
Basic Evans (1973, p. 75) Graph

Note that greater utility is realized closer to the origin, given that the axes represent costs to the consumer.

Apart from utility adjustments, greater u means higher travel costs, requiring a lower p to maintain the same utility. The same monotonicity of preferences which applies to the bid-rent curve in commodity space also applies to the choice function in P-u space. It applies in reverse due to the fact of greater utility closer to the origin, but it still results in a negative slope of the indifference relationship Ψ as indicated (e.g., decreasing price in exchange for more commuting).

In addition, the normal assumption of diminishing returns (derived from a strictly quasi-concave utility function which yields a positive second derivative) translates into a diminishing marginal rate of substitution for commodities which also applies to the indifference relationship Ψ. That is – as the consumer moves along Ψ, he or she acquires a lower P in exchange for more commuting time, but the rate at which he/she is willing to sacrifice commuting time for lower P declines.

While a price gradient is simply a vector indicating a relationship between price and distance, there are three possible interpretations, and we should be careful to distinguish them with the following terminology:

P(u)	=	*market price gradient*, a set of options available in the market;
Ψ	=	*compensating price gradient*, a price indifference curve derived from the bid rent curve.

Hence the tangency condition illustrated in Figure 1.1 means:

slope of market price gradient = slope of compensating price gradient.

However there is a third gradient, which might be called the *optimizing price gradient*. This is the locus of (P,u) optima which are observed to occur under various market conditions. It is relevant in that it is the measured price gradient (to the extent that theoretical optima are achieved in practice) when examining market behavior.

Regarding the relationship between the curves in Figure 1.1, it may be described as follows. When u increases from 0 toward the value where the above equality applies, if the decrease in commodity price is greater than the increase in adjusted commuting cost, then it pays to keep moving until they are equal. In other words, the spatial bang for the buck is also maximized. Regarding the relative convexities of the curves, consider a distance u near u = 0. Note that Ψ represents the willingness to trade off more distance for lower price when moving away from city centre. If the resident is going to move away from city centre at all, price must be falling faster than tradeoff preference. The marginal rate of substitution of the gradient must be greater than the marginal rate of substitution of the preference set. Hence p(u) is more convex than Ψ, as they approach tangency.

There are other techniques of graphical analysis, such as a linked multi-graph representation found in Straszheim (1987, page 24). Revealing as that technique may be, the Evans/Fujita indifference graphs, presenting the essential preference relationship in a single quadrant, have proven most useful for the current analysis.

There was no immediate need for early analysts to extend the model to account for decentralized employment (removed from city centre), on the grounds that in a perfect market, wages of decentralized jobs will adjust to make workers indifferent between central and the decentralized jobs. As summarized by Mills (1994, pages 118-119), this "preserves the applicability of the monocentric model in the face of decentralized employment ... [since] ... rent functions are not different if there are suburban jobs than if all jobs are in the CBD [Central Business District]".

While this proposition may be valid in the context of a simple residential decision, it is also useful to focus the analysis on the market adjustment which preserves this indifference, and its implied "wage function, the slope of which equals –t" (Mills, 1994, page 118). By explicitly introducing another price gradient, another downward sloping function called the *wage gradient*, we also facilitate the incorporation of job location into the analysis as a decision variable as well. Thus, in addition, we have a workplace decision which involves balancing the marginal benefit (of higher wages or greater job choice by choosing a workplace closer to city center) against the marginal costs (of further commuting to that workplace). The principal lines of development are found in White (1988b) and Simpson (1980, 1992).

There are two levels of approach to such an analysis. The simpler approach is to assume a decentralized but predetermined workplace, in which case the workplace distance from city centre (v) becomes an additional argument in the residential decision, and its first derivative an additional factor in the optimal choice of location. Figure 1.1, however, can still be applied[5], with the additional derivative affecting the shape of the indifference curve. The same essential method of analysis can be applied to the workplace location decision given a fixed residence, which would produce a second graphical representation similar to Figure 1.1 (call it Figure 1.1a) but which displays an optimal v* instead of u*, where $v^* < u^*$ (since it would be illogical to live at the same commuting distance inward from the workplace and pay more for housing).

What White and Simpson postulated was a simultaneous determination of residence and workplace. At first glance this could simply be represented by adding Figure 1.1a below Figure 1.1, sharing a common x-axis, and displaying the two optimal decisions in their relationship of $v^* < u^*$. But specification of the simultaneous model reveals the hidden assumption which makes this simple determination possible – namely that $\partial p_u/\partial v = 0$,

$\partial p_v / \partial u = 0$ (White, 1988b, pages 139-40), as well as zero values for the cross partials (Simpson, 1992, page 38).

That is, if there is no interference from any u and v interaction, Mills' indifference to a centralized vs decentralized workplace will hold. If there is a positive interaction, it will segregate the population into (a) those who are indifferent between city centre and workplace and prefer to stay within that range, and (b) those who are indifferent to locations beyond the workplace and prefer to stay within that range. If there is a negative interaction, the segregated populations will switch places, resulting in a phenomenon of out-commuting ($v^* > u^*$). The latter is the least likely case according to White, since it would require the reduced commuting costs and lower wages which result from moving the job outward to have the effect of reducing housing consumption. A zero relationship is an unlikely theoretical ideal, while the more likely positive relationship suggests that "households locate in concentric residential rings in order of the centrality of their workers' job locations" (White, page 143).

The analysis to this point has implied a simple family residence which is not essentially different from a single occupancy dwelling. The existence of other family members, and more importantly the existence of others in the family who are employed and who must do their own commuting to their own places of employment, have not been addressed. In addition, the existence of dependent family members (related to the family's

[5] Though neither Evans nor Fujita considered decentralized employment in that manner.

placement on the life cycle) and their effect on the demand for housing, are absent from the analysis.

If we follow Beckmann's (1969, 1973) analysis of family structure in the centralized employment model, in the terms discussed above, we can incorporate two additional parameters into equation 1.1, namely number of earners and number of dependents in the family, simplified by the assumptions that wages and costs are the same for all family members and that all goods are consumed in aggregate. This produces the following results according to Beckmann in terms of the bid rent function of Figure 1.1.

- If there are more dependents (increasing the preference for lot size), the bid rent function becomes steeper in slope, resulting in an equilibrium location further from city centre.
- If there are a fixed number of dependents but more wage-earners (with no non-wage income), the bid rent function becomes shallower is slope, resulting in an equilibrium location closer to city centre.
- If there are no dependents, additional wage earners have no effect on the equilibrium location.

The second item occurs because of the effect of multiple earners on the valuation of leisure time, and the third occurs because of the absence of that factor, along with equal returns to scale in transportation (actually identical transportation costs). Oi (1976) modifies some of Beckmann's conclusions (a) by not accepting the cost neutrality of multiple commuting with the result that multiple earners even without dependents try to economize on increased commuting requirements by moving inward, and (b) by assuming that the existence of dependents would reduce the probability of the female head working such that the family will tend to move even further out. However the models are united in their general suggestion that more dependents place outward pressure on families and more earners may create inward pressure.

Another simplifying assumption which has proven important to relax, in the case of labour markets, is the homegeneity (of skills) of workers. White (1988) considers a two-workplace model with two skill groups in the context of both residence and workplace location decisions. The analysis is based on the assumption that greater skills command higher wages, and proceeds as follows.

For commodities sold, such as labour time, higher wages make time more valuable, resulting in a steeper wage offer curve p_{sk}, as in Figure 1.2

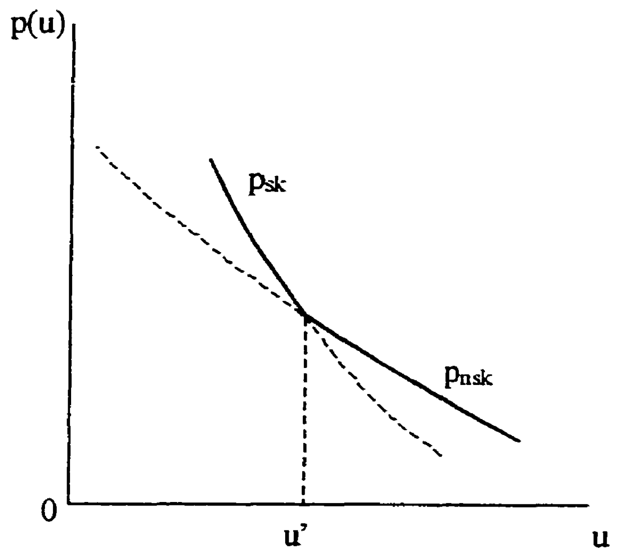

Figure 1.2
M. White (1988) Segregation Model

The result is an envelope curve with non-skilled workers (p_{nsk}) bidding higher for the suburban jobs. White argues however (p. 149-50) that if housing segregation places unskilled workers in a more central location, firms hiring them will find it necessary to locate centrally as well, and the unskilled wage offer curve p_{nsk} will actually fall entirely within the central area where their employers tend to locate, and they will occur at a lower wage level. Figure 1.2 will become Figure 1.3. Unskilled workers will end up segregated centrally, while skilled firms will hire workers throughout the city (which, according to White, is realistic).

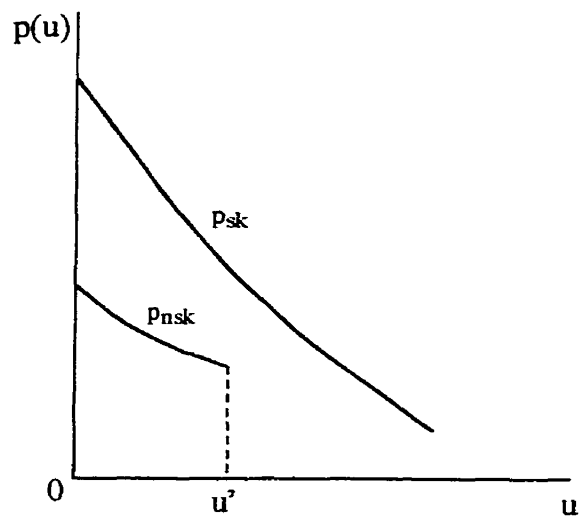

Figure 1.3
Mixed Segregation-Dispersion Model

Simpson (1992) dealt with the skill factor in a more comprehensive manner, combining the island model which stems from Phelps (1970) and Lucas and Prescott (1974) with a full search model. This topic, along with its analytical roots, will be addressed separately in Chapter 4.

There is another simplifying assumption of the original Alonso model which bears further examination. The assumption that any given combination of residence and workplace lies on a ray emanating from city centre has proven quite analytically robust over the years. But it has also required some further refinement in order to deal with questions of excess commuting which have arisen from a paper by Bruce Hamilton (1982). Hamilton and those who contributed to the ensuing debate incorporated the phenomena of cross-commuting (from off-ray locations) and out-commuting (to a job *further* from city centre) into the analysis with a view to explaining those commuting distances which were found to be in excess of the theoretical optimum. The analysis of a variety of placement options (i.e., alternatives to placement along a single ray), and how they relate to the literature on excess commuting, can be found in Chapter 5.

1.2 Nature of the Analysis

This paper will build upon the analysis of urban behavior to meet the following objectives.

A. Introduction of a greater level of generality into the analysis, dealing with generic transaction locations, positive and negative prices, special quality characteristics and some general relationships between different types of transaction locations.

Urban location models are commonly expressed in highly specific terms, such as residence and workplace each in reference to a single defined function relating price to location. It might be useful to explore the roots of such models in a more general theoretical framework where residence and workplace are specific instances of transactions in general, and the price function may take a variety of forms. This is developed in Sections 2.1 to 2.3. In this context, expenditure is associated with purchasing commodities at positive prices, and income (the reverse flow) is associated with selling commodities (including labour time) at negative prices. The result is a set of vectors identifying price and quantity, location and distances, and costs. The manner in which these parameters produce a balance of forces which equilibrate to a unique locational solution is analyzed in relation to positively and negatively priced commodities, including the place of residence and workplace and their relationship to each other, and also in relation to differently sloped gradients.

In addition, specific categories of the model, such as family housing needs and job skills can be treated in a more general manner as special quality characteristics of these commodities – of which special needs and job skills are specific instances. Hence one purpose of this study is to specify the terms of a more general urban framework, and to examine how traditional urban models serve as specific applications giving practical relevance to the model.

B. *Application of some of these categories to preference space analysis, extending a graphical representation introduced by A. Evans, indicating positive and negative prices and the relationships among transaction locations.*

Another purpose addressed in Section 2.4 is to give further development to the intermediate level of analysis, that of identifying choices in preference space – which falls between the highly generalized level of analysis cited above and the simple analysis which employs categories such as marginal benefits and costs. This is not an undeveloped area of theoretical research, but it is useful to consider it in light of the more general framework, and the analysis below is intended to provide a vehicle for new approaches introduced in this study. We adopt the Evans/Fujita graphical representation (Figure 1.1), utilizing a positive quadrant for positive prices and a negative quadrant directly beneath for negative prices (Figure 2.2 in Section 2.4.2), and explore the optimal spatial relationships among various positively-priced and negatively-priced commodities and the place of residence, as well as the conditions required for an equilibrium solution.

C. *Clear articulation of the nature of gradients, as well as the slope and shape they can be expected to take.*

At this point the characteristics of gradients are examined in more detail in Section 2.5, primarily dealing with theoretical support for the various downward-sloping gradients considered in the model. In the process, since the question of upward sloping gradients occasionally arises in this study, the degree to which they find theoretical support is also examined. A clear understanding of the nature of gradients is necessary to interpret the complex data examined in the empirical sections of this paper, but it cannot be fully examined until we consider the placement issues of Chapter 5 and the actual gradients considered in Chapter 6.

D. *Explicit measurement of the quantity of housing, in light of a multi-valued vector of attributes, the role of amenity characteristics in that vector, and the incorporation of these attributes in a hedonic price index.*

While the use of land area to measure the quantity of housing is useful for articulating the core model, it is not adequate to deal with multi-valued housing characteristics. This is the topic of Section 3.1, beginning with the inclusion of a vector of housing characteristics into the general equations developed in Chapter 2. Some of the problems of representing a multi-valued set of characteristics in a single mathematical point in space are discussed, leading to Evans' concept of a space unit, and some of the problems inherent in attempting to quantify optimal housing.

But housing quality characteristics are not limited to the characteristics of the housing unit alone, no matter how multi-faceted. A housing unit exists within a neighbourhood which also has its own vector of characteristics, and the value of the housing is significantly influenced by the latter. A further theoretical exploration examines the incidence of urban amenities, the benefits of which are commonly assumed to be captured by the price function. Their role and impact can be incorporated more explicitly in the model, making use of hedonic pricing techniques, and showing how they are capable of being highly significant factors having their own influence on urban location. Section 3.3 deals in more detail with the effects of amenities on the location decision, including models of segregation by amenity preferences, their relation to public goods, bundling, and implicit pricing – in order to fill in the theoretical background for the empirical examination of amenities in Chapters 7 and 8.

E. Incorporation of uncertainty and search into the model as developed, with their effect on locational choice particularly in relation to quality variations, how search strategy can be analyzed in preference space, and how use of the island model can speak to testable impacts of skill variation.

A major extension of urban analysis has been the adaptation of uncertainty models and search behavior to the urban context, with an emphasis on job search. Section 4.1 begins by returning to the initial generalized analysis, clarifying some definitions, and incorporating market density gradients and search costs, with particular attention to the effect of special quality variables (housing characteristics and skills) to extend the general analysis. This section also considers the generalized constraint of the model, to set the stage for the integral probability analysis which is a major thrust of search theory.

Section 4.2 considers the specifics of search strategy in more detail, outlining the basic model and relating it to the preference space of Figure 2.2. Reservation prices, not normally represented in such analyses, are incorporated into the preference space of Figure 4.1. The analysis allows a clearer examination of when search can be expected to occur. But the development of testable relationships depends on the incorporation of the island model, considered in the remainder of Section 4.2, and the inclusion of skill variations in the manner of Simpson (1992). This sets up a testable opposition of cost effects and search effects as a result of skill variations.

F. *Analysis of alternatives to the simple perfect ray gradient, including perception problems and complexity, in particular the impact of offsets and switches and the methodology of determining their incidence. This leads to analysis of excess commuting – its rationality, measurement, classification, assumptions and definition – leading to a critique and redefinition.*

Section 5.1 considers the question of simple ray-based gradients, and their alternatives in the case of complex urban patterns. These include noisy and poorly-perceived gradients, as well as multiple-centred gradients. Of key importance to the analysis which follows is the consideration of offsets from the ray (where the actual residence or workplace location does not lie directly on the ray itself, but is removed by a small enough amount that the ray is still relevant), and positional switches (which reverse the traditional relationship of residence further out than the workplace), and the inclusion of (possibly separate) amenity centres with their own gradients. The analysis considers impacts on residence and workplace location, and possible theoretical rationales for such behavior.

This leads to the consideration of excess commuting introduced by Hamilton (1982), where related phenomena were studied in specific reference to commuting distances which exceed the theoretical optima, but with a slightly different terminology. The analysis which followed Hamilton explored the circumstances in which it is likely to occur, and where it might be rational, including where it may lead to systematic out-commuting and regional segmentation. The analysis was highly useful for forcing a clarification of terms and theoretical assumptions hidden in the traditional model. This study attempts to clarify the definitions of out-commuting and cross-commuting and further the development of techniques for specifying and measuring their impacts.

G. Examination of a survey dataset which provides gradient information, some unique preference information and a longitudinal component.

Chapter 6 introduces a dataset obtained from a survey conducted in Greater Minneapolis in 1990 and 1992, giving its basic structure and the kinds of data available. Given the apparent complexity of observable gradients in the data, Section 6.2 discuss the question of gradients and take a closer look at the Minneapolis gradients as they appear in the data, as well as making some recommendations as to which assumptions may be realistic. In the end, it is concluded that the traditional city-centred downward-sloping gradient remains robust enough to serve as a starting point for empirical analysis.

The survey has some interesting characteristics, containing not only information about the residential and workplace choices of respondents, but also questions about the reasons for their choices and their methods of search. The data also have a longitudinal component (1990 and 1992) which allows in many instances the determination of the workplace/residence at the time of changing residence/workplace. Direct data on how price and density are related to location are not available from the survey, but some information can be inferred from it to look at the question of how well traditional assumptions about city centre concentration hold up.

The empirical section of this paper employs the 1990-92 Minneapolis survey data, which are used to confirm whether the determinants of behavior arising from the models here developed are reflected in the data – as well as to explore the problems of dealing with noisy and complex data.

H. Analysis of some interesting relationships in crosstabulations, traditional ones such as family structure, but also introducing offsets and switches, some expressed preferences, and examining the extent of excess commuting.

Chapter 7 begins with some of the simpler relationships (the primary examples being relationships between location and travel costs or income) and employing some simple crosstabulations where some preliminary relationships may be discernible before considering regression analysis. Well examined, and evocative of a few discernable relationships, is family structure. But the key relationships studied in this chapter are those revealed by placement data. Survey respondents who are switched in relation to the theoretical norm (e.g. not having a workplace closer to city centre than the workplace) might be expected to show a lesser theoretical consistency if their behavior can be taken to indicate less attention paid to the parameters of theory. These possibilities are checked against the preference questions asked in the survey, as well as some amenity indicators taken from the 1990 U.S. Census, both for residence location and workplace location.

Though related to the question of switching mentioned above, the question of excess commuting merits separate consideration, in Section 7.5. The theory having been discussed in Section 5.2, this section looks at measurements of excess commuting in relation to switching behavior, compares it to estimates from other sources and considers family structure – for both residential and workplace decisions.

I. Estimation of key regression relationships, addressing the question of simultaneity, dealing respectively with residential and workplace choices, focussing particularly on the impact of offsets and switches, and examining the role of excess commuting.

Finally, Chapter 8 conducts several regression analyses, in an attempt to capture more complex interrelationships in the data. But first of all, it is noted that the survey data reveal that only about one-sixth of the respondents can be considered to have made a roughly simultaneous decision as to residential and workplace location. Hence simultaneous regressions are not attempted.

After some discussion of the definition of terms, the residential decision is considered first. Regressors include distances, income and family structure variables. Significant variables and their possible explanations are considered. Switched vs non-switched comparisons are also computed, and relationships they might tentatively reveal are discussed. After that, offsets from the ray are regressed separately, to see if the data reveals any insights into excess commuting.

Then similar tests are performed for the workplace decision. In this case, regressors are distances, skill (broken down by occupational classification) and age – again for switched and non-switched respondents. Similar analyses are performed on these and the offset variables, with their implications for workplace choice.

2. THEORETICAL FRAMEWORK

2.1 The Core Model

2.1.1 The Utility-Maximizing Framework

We begin an analysis of spatial equilibrium with a general model from choice theory, in a manner suggested by Varian (1984, page 217). The consumer problem is one of exchanging an initial endowment of commodities on hand for other commodities available in the market, in order to achieve an optimum consumption bundle.

Let us assume that all commodities are superior goods unless otherwise indicated. Consumer preferences are complete, reflexive, transitive, continuous, strongly monotonic, twice differentiable, additive and locally non-satiated — in order to assume that they can be represented by a smooth continuous well-behaved utility function. We will also assume that they are quasi-concave, to ensure that second order conditions are met.

Consumption has a broad meaning. It refers to commodities obtained now for consumption purposes, but it does not imply their immediate use. It includes commodities obtained now which are held to serve as an endowment for the next period. Current utility derives from either function, if the present utility value of future consumption is considered (at exogenous discount and interest rates). More specifically, more commodities will be retained for future consumption (i.e., endowments for the next period) rather than present consumption as long as, at the margin, the declining PV of future utility exceeds the utility of current consumption (on a per-dollar basis).

We proceed by defining an all-inclusive vector, employing **bold** print in an alternate font to distinguish a vector from a single-valued variable.

Let **V** be the vector of all goods and services entering, remaining within, or exiting the commodity bundle during the optimizing process. The elements of **V** are optimum quantities. Associated with **V** is a fixed-price vector **p**, whose elements are positive or negative depending on whether goods are bought or sold, and which may be considered the second row of a quantity-price matrix. **V** is divided into four sub-vectors — **v**, **w**, **z** and **n** — and **p** is partitioned into the corresponding sub-vectors = 0. Vector **n** refers to exogenous non-market endowments (public goods and externalities or transfers) which are received and entirely consumed during the period, and are replenished for the next period. Those which yield positive utility will in some instances be described as amenities (see Section 3.2.3).

$\mathbf{W}$ = total endowments available at the beginning of the optimizing process

= endowment items sold ($\mathbf{v}$) + endowment items remaining ($\mathbf{w}$) + $\mathbf{n}$

$\mathbf{z}$ = new commodities attained in exchange for the sale of $\mathbf{v}$.

Hence $\mathbf{V} = \mathbf{W} + \mathbf{z}$, and total consumption $\mathbf{X} = \mathbf{w} + \mathbf{z} + \mathbf{n}$. The appropriate sub-price-vector corresponds to each of these sub-vectors. All proceeds from the sale of **v** are spent purchasing **z**. (There is no saving.)

Utility is maximized as follows:

$$\max U(\mathbf{X}) \qquad \text{s.t.} \qquad \mathbf{p_z}\,\mathbf{z} + \mathbf{p_v}\,\mathbf{v} = 0 \qquad (2.1)$$

given that the price values for **v** are negative. In other words, the constraint indicates that while **w** is already available for consumption without cost, any consumption beyond **w** is limited to new goods and services purchased with the proceeds of sold endowments.

The above relationship then may be expressed in the more general form:

$$\max U(\mathbf{V}) \qquad \text{s.t.} \qquad \mathbf{p}\,\mathbf{V} = 0 \qquad (2.2)$$

given that optimum consumption quantities of v are zero, and prices for w are zero.

The solution to Equation (2.2) is determined by the condition that $\mathbf{D}U(\mathbf{V}^*)$ be proportional to **p**, or specifically that

$$\mathbf{D}U(\mathbf{V}^*) = \mathbf{p} \qquad (2.3)$$

2.1.2 Role of Endowments

The vector **n** (exogenous non-market endowments) does not enter into the calculation since its elements are exogenous, constant and priceless as well as entirely received and consumed during the period — there is no allocation problem to solve — except that the existence of **n** will influence the demand for **z**, as well as the supply of **v** offered to the market. There will exist certain vital transfer programs, such as food banks and public shelters. Their technical usefulness to the analysis can be seen by considering an instance where $\mathbf{n} = 0$, in which case we cannot have $\mathbf{v} = 0$ and $\mathbf{w} = 0$ (at least in successive periods) because of the human factors of starvation, exposure, etc. The possibility of $\mathbf{n} > 0$ allows for the normal analysis where all commodities are bounded by a lower consumption limit of zero, and many (or all) market commodities are not consumed at all.

Hence we may realistically investigate a case where the individual does not (or cannot) sell any of his or her endowment items at all, a topic which can be raised when analyzing search behavior[6].

2.1.3 Time

Some models (e.g. Evans, 1973) include time in the utility function, specifically time spent commuting and time spent working, as a disutility. Hekman (1980) points out that time must be separated out of the composite good because (a) the value of commuting time varies with income, and (b) leisure time is substituted for other goods in the case of employment. White (1988b, p. 131ff.) takes it further using a "full income constraint", which in addition incorporates the income that could be earned in the time neither working nor commuting. Treatments following from Hekman by Fujita (1992) and Simpson and van der Veen (1992) incorporate the separate categories of work time, leisure time and commuting time into the utility maximizing model. While these are relevant factors in utility, their inclusion in the analysis does not generally produce results fundamentally different from those below. For one thing, there is a strong correlation between travel cost and travel disutility (if differing modes of transportation are accounted for), such that adopting both makes little substantive difference to the outcome of the model. As well, the only difference between negative prices in relation to work time disutility, and positive prices in relation to consumption time utility, is the aspect of selling time (out of 24 hours) in the wage contract – but this can be captured in the constraint terms of the model. Hence these fuller formulations may be more complete,

[6] This is a possibility which the Varian analysis does not consider.

and may reveal some interesting relationships, but they do not alter the basic results of the model. For example, the indifference curves we shall employ below apply equally to all of them.

2.1.4 Employment

The model can be extended to include employment as a source of income. Consider that the total endowment vector **W** includes among its elements leisure time, which may be consumed as **w** — or sold as **v**, like any other element in the endowment vector (at a negative price more commonly called a wage). Leisure time consumed may simply remain part of **w**, but there must be a provision for treating the sale of leisure time separately. We shall modify **v** by removing a sub-vector **l**, which represents that part of leisure time that is sold as labour. The meaning of **v** now becomes restricted to non-leisure endowments sold. Hence $\mathbf{W} = \mathbf{v} + \mathbf{l} + \mathbf{w}$. It is also useful to identify one element of **l** as the primary source of income, where applicable, and give it the alternative labels of l_0 or y. Equation 2.1 applies with the addition of $\mathbf{p}_l\,\mathbf{l}$ and equation 2.2 still applies.

2.2 Spatial Parameters

2.2.1 The Location Matrix

The analysis employs broader categories than those of the traditional model – which are city centre, residence and workplace. It employs a more general category called transaction location, labeled TL. TL by itself refers to all transaction locations, but it breaks down into TL_+ (transaction locations for goods bought at positive prices) and TL_-

(transaction locations for goods sold at negative prices). In addition, there is a special TL_+ given the label TL_0 or sometimes called the CL (consumption location), which is also referred to as the place of residence (RES). Also, since city centre (CC) is also a possible TL, it receives the special label TL_I. This terminology allows us to set up the analysis in a single matrix of prices, quantities and more.

In the most general context, we consider a basic set of spatial relationships,

$TL_I \rightarrow TL_- \rightarrow TL_0 \rightarrow TL_+$ or $CC \rightarrow TL_- \rightarrow CL \rightarrow TL_+$

which is consistent with traditional urban analysis, but which allows for more categories of transactions[7].

Refining the model further, in order to adapt it to the conditions of urban spatial equilibrium – the introduction of geographical space makes it necessary to adopt spatial reference points. Spatial relationships are represented geometrically in reference to "a flat, featureless plane, no part of which differs from any other part" (Evans, p 18), on which Cartesian co-ordinates are related to a fixed point of geographical origin. It is normally most useful to select city centre (CC) as the geographical origin. That is, each point in space has a pair of Cartesian co-ordinates which specify its location in reference to CC at (0,0).

Economic activity requires a place to conduct business. Each commodity has a transaction location (TL), and a consumption location (CL), all of which are located on a

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[7] Although the position of TL_+ will be questioned in Section 2.4.

single ray emanating from CC. Since each commodity must be purchased or sold at a TL, it is associated with a travel cost related to the distance between CL and TL. In other words, for each commodity, reference must be made to the location co-ordinates of its TL. Hence associated with the commodity vector **V** is a pair of vectors, **x** and **y**, containing the pairs of Cartesian co-ordinates for the TL associated with each commodity. Certain transactions also occur at the CL or in its immediate vicinity, and we designate the CL as TL_0, though we may refer to it as the residence (RES). We also designate CC as TL_1.

Vectors **x** and **y** may be considered the third and fourth rows of the matrix (after quantity and price), which allow us to add a fifth vector **u** containing the straight-line distances between each commodity's transaction location and CC. For the sixth row of the matrix, we can derive a vector of distances actually traveled, **t**, by comparing each element of **u** with the distance between the CL and CC (which is contained in u_0). The seventh row of the matrix, the vector **c**, is derived by applying an invariable travel cost function, **c** = c(**t**). Elements of **c** will be zero for all elements of **w**. Where necessary **u**, **t** and **c** may be partitioned into sub-vectors in the same manner as **p**.

The rows of the commodity matrix which provide the information needed to solve the urban location problem are identified as follows:

Line	Vector	Content
1	**v,z**	Quantities
2	**p**	Prices
3	**x**	X co-ordinates
4	**y**	Y co-ordinates
5	**u**	Distances to CC
6	**t**	Distances traveled
7	**c**	Travel costs

2.2.2 Key Assumptions

We need also adopt additional assumptions regarding the agent who acts at each individual spatial reference point. We can use the category of household common to urban analysis, with an initial assumption of singularity and homogeneity. There is initially a single agent at each location (meaning a single decision-making family head, who is also the sole worker in reference to the wage equations), all of which have identical tastes across households. Alternative specifications will be considered in Section 4.3.

A frictionless model which includes spatial factors must make some additional assumptions. Transportation speeds and per-unit costs are uniform in any direction of travel (which excludes, at least initially, the possibility of roads diverting traffic along specific routes which are longer than the shortest distance between two points). Marginal travel cost may be allowed to increase or decrease with distance traveled, but the rate and sign of change must be the same in any direction. In addition – to the normal set of conditions implied by perfect competition – we must emphasize perfect knowledge (at least until search is considered) and perfect freedom to immediately change the place of residence or the workplace in response to perfectly-competitive market conditions (until actual housing and employment constraints are considered in more detail).

2.2.3 Spatial Optimization

2.2.3 General Analysis

Beyond the optimum choice of commodity mix, the spatial problem is the consumer's choice of his or her single consumption location, plus whatever choices are presented with regards to transaction locations. In other words, it is a problem of choosing the optimal CL and TL (plural) which maximize total utility and minimize the costs of travel which must be incurred to engage in transactions. Travel generates both disutility and cost, though it may simplify the analysis to assume that travel cost serves as a proxy for travel disutility.

Considering travel time as a disutility, utility maximization can be expressed

$$\max U(\mathbf{X},\mathbf{t}) \qquad \text{s.t.} \qquad (\mathbf{p}_z\,\mathbf{z} + \mathbf{c}) + (\mathbf{p}_v\,\mathbf{v} + \mathbf{c}) + (\mathbf{p}_l\,\mathbf{l} + \mathbf{c}) = 0 \qquad (2.4)$$

or more generally, $$\max U(\mathbf{V},\mathbf{t}) \qquad \text{s.t.} \qquad \mathbf{p}\,\mathbf{V} + \mathbf{c} = 0. \qquad (2.5)$$

It is a model of spatial optimization. One may choose which consumption location and which transaction locations will best facilitate utility maximization, by choosing optimum values of u_0 (distance between CL and CC) and $\mathbf{t}$ (distances actually travelled). The solution will take the form of choosing an optimum set of commodities, from a CL which is optimally located with respect to the relevant TL, in order to optimize travel costs. If transaction locations are few, $\mathbf{t}$ will contain blocks of identical values.

The solution contains the same condition as before, that

$$\mathbf{DU}(\mathbf{V}^*) = \mathbf{p}. \qquad (2.3)$$

But it also contains another spatial condition, namely

$$\mathbf{DU}(\mathbf{t}^*) = \mathbf{V}^*\mathbf{D}p(\mathbf{u}^*) + \mathbf{D}c(\mathbf{t}^*). \qquad (2.6)$$

given that the optimum also determines a unique CL which maps $\mathbf{t}^*$ directly to $\mathbf{u}^*$. It means that $\mathbf{DU}(\mathbf{t}^*)$ is proportional to $\mathbf{V}^*\mathbf{D}p(\mathbf{u}^*) + \mathbf{D}c(\mathbf{t}^*)$, or in graphical terms that $\mathbf{DU}(\mathbf{t}^*) - \mathbf{D}c(\mathbf{t}^*) / \mathbf{V}^*$ is tangent to $\mathbf{D}p(\mathbf{u}^*)$.

2.2.4 Unique Solution

The task is to model a unique solution to the location problem. One approach is to introduce production functions for commodities. As shown by Muth (1969), Wheaton (1974), Straszheim (1987) and others, a more comprehensive model of this sort produces market price gradients and price indifference curves with the desired properties, but does not produce a unique solution unless further assumptions are made. This could be solved in theory by assuming that each individual's level of utility is known cardinally. A workaround by Breuckner (1987) uses total differentials to optimize around a single fixed common utility level and a single fixed income level. A less theoretically demanding alternative is to assume identical incomes and preferences for all participants in the market. Unfortunately, this produces a common price indifference curve, and creates a problem whereby every participant prefers precisely the same housing at precisely the same location (unless transportation costs vary across individuals). In other words, everyone would be trying to live in the same house, and no one would want to live anywhere else. One way out of this problem is to create conditions whereby the market price gradient and the price indifference curve are exactly coincident (lying one on top of the other), making all participants indifferent as to location and allowing them to spread out. In this instance, equilibrium becomes a matter of the number of households residing in the urban area in question equaling the number of housing units available. The problem then becomes one of macrospatial structure and inter-regional trade, and a completely different branch of urban economics.

On closer examination, this problem and its solution turn out to be an artifact of the traditional simplified urban model with a limited number of variables and identical incomes. In the broader context considered in this section, income is determined by a vector of negative prices, and the problem of all consumers preferring the same location becomes an improbable coincidence. In the end, allowing a little variance in transportation costs and incomes alleviates the problem of identical choices.

2.3 Optimal Location

The problem is one of determining a unique optimal location. We assume that the demand for each commodity is perfectly elastic with respect to price, to depict the price gradient as a simple function of distance. The following analysis assumes convex price gradients, and calculates marginal costs and benefits in units of distance.

The analysis would be uninteresting if all TL were endogenous, such that all commodities are ubiquitous and continuous (available anywhere, at any mathematical point on the plane). The same commodities could be obtained at any location, with no requirement for travel, and the problem would be reduced to the dimensionless model of equation (2.1). The addition of the appropriate exogenous restrictions, however, creates a spatial choice model.

2.3.1 Residential Location

Consider an example where housing is ubiquitous and continuous but all other commodities are available only at a single TL located at CC. This allows us to examine the simplest instance, where only the CL is to be determined since all commodities are available for purchase and sale only at CC. Given the housing price gradient, the price of housing services varies in a way which counteracts the lower travel costs of moving closer to CC, and provides an incentive to move outward. The resident makes more money available for consumption of housing services and other commodities, as he or she considers locations further from CC, by balancing higher commuting costs against lower housing costs, and may even choose a CL far from CC. This is the basic Alonso locational dynamic which is repeated in various contexts in the analysis which follows.

It is useful to clarify the resolution of forces involved in individual cases before developing the general model further. Considering values of the price gradient, ∂P/∂u, let us begin with a traditional negative price gradient, ∂P/∂u<0. For the choice of a residence, a simple depiction of the resolution of forces (indicated by arrows) where ∂C/∂u>0 is represented in Diagram 2.1 below:

Negatively Sloped Price Gradient

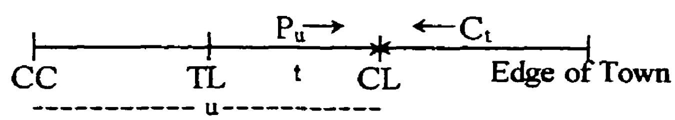

Diagram 2.1
Normal Single Resolution of Forces

TL represents any transaction location, positive or negative, and it may be located anywhere between CC and the edge of town. The basic Alonso model places TL at CC. (Though TL are generally treated as dependent variables, the model will allow for treating some of them as fixed at CC.) Its significance is that the resident at CL must travel to the TL in order to purchase or sell commodities (or commute to a job in the latter case). The problem is to choose a consumption location (CL) which is at an optimum distance from the transaction location (TL). The housing price gradient P_u declines with distance to city centre, and the travel (commuting) cost C_t increases with distance from CC. P_u exerts outward pressure on the location decision in order to minimize housing cost, while C_t exerts inward pressure in order to minimize commuting cost.

The configuration shown is optimal on the following rationale. As the decision-maker considers choices from CC outward toward the TL, both price and travel cost are declining. When the decision-maker continues considering points beyond the TL (where beyond indicates further from CC than the TL), price continues to decline but travel cost begins to rise as the CL moves away from the TL. Hence an optimum may be found at some location beyond the TL as shown.

A general model should theoretically admit of a full range of price gradients, from a vector of negative slopes, through a vector of zero slopes, to a vector of positive slopes. The example above considers a negative slope. A slope of zero would set $P_u = 0$, removing one element from the balance of forces depicted above, and producing a decision to choose a CL precisely at TL.

The less likely case of a positively sloped price gradient acts to reverse the above configuration. As the decision-maker considers choices from CC outward toward the TL, price is increasing while travel cost is declining. If the decision-maker considers points beyond the TL, both price and travel costs would be increasing. Hence an optimum is found at some location between CC and TL, as indicated in Diagram 2.2.

Positively Sloped Price Gradient

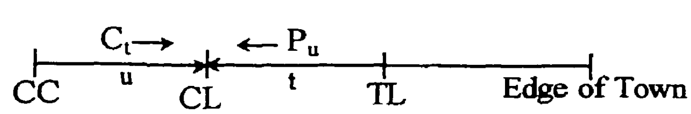

Diagram 2.2
Reverse Resolution

2.3.2 Commodity Locations

Returning to a normal negatively-sloped price gradient, it becomes more interesting when we consider one less exogenous factor and introduce ubiquitous positively-sloped TL to consumption goods **z** (not just ubiquitous housing). That is, TL_+ is now a decision variable. When CL was the only decision variable, any location for TL could be considered. With TL as a decision variable, there are restrictions on its location, as noted below. We assume initially that each TL on the continuum supports the purchase and sale of the entire range of commodities **z**, and that all elements of **z** share a common price gradient. For the moment we continue to assume that nothing is sold by the decision-maker in question, such that all prices are positive. This introduces the question of choosing a single TL for each commodity as well as for the CL.

Looking at a single commodity, a contradiction is apparent, in the case of $\partial P/\partial u<0$, when we examine the balance of forces in Diagram 2.3 below:

Negatively Sloped Price Gradients

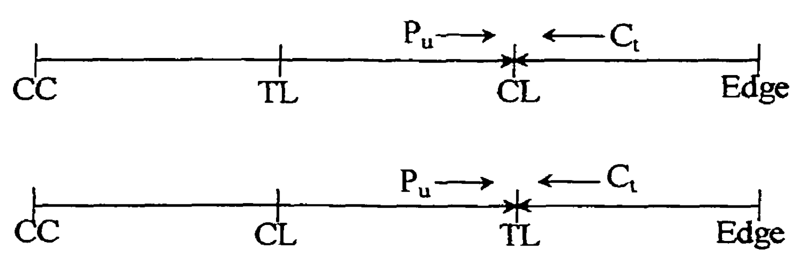

Diagram 2.3
Decentralized TL_+

The first panel reproduces the results of Diagram 2.1. The second panel shows the results of moving away from CC in search of an optimum TL. Moving from CC, both price and travel cost are declining, and hence it pays to move to a point beyond CL where declining price optimizes against increasing travel cost.

The contradiction lies in the relative configuration. The optimum CL lies beyond the TL, while the optimum TL lies beyond the CL. If the decisions are made simultaneously, the system of equations may not converge on a solution. The problem is solved by assuming the $\partial P/\partial u=0$, in a more realistic scenario of ubiquitous shopping centres, making the TL coincident with the CL in the second panel and a matter of indifference with respect to the first panel.

2.3.3 Income Generating Locations

We have been ignoring to this point the fact that the consumer is obtaining the funds to purchase commodities at a positive price by purchasing other commodities at a negative price (i.e., selling), unless he or she is living entirely on the basis of endowments. (We are assuming that social amenities do not take the form of negotiable or barterable commodities, an assumption which would have to be modified if social welfare programs are considered.) If we introduce negative prices, the benefit of moving outward from CC will be negative, since a lower price is obtained for goods sold. Moving TL_{-} from the edge of town toward CL, the received price (or wage) is increasing and travel cost is declining, and hence it pays to move to a point between CL and CC where increasing wage optimizes against increasing travel cost.

The resulting balance of forces is shown in Diagram 2.4. The TL at which commodities are sold at negative prices is labeled TL_, and its location is resolved in the second panel. At TL_, as a source of income the negatively sloped P gradient creates an inward tendency, while transportation cost toward the CL creates an outward tendency. Note that t represents the actual travelling distance resulting from transaction distance u.

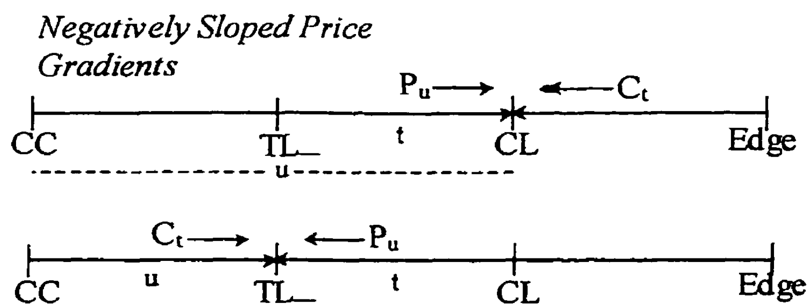

Diagram 2.4
Normal Decentralized TL_

The most significant overall result is that the TL for commodities with negative prices (including employment) are optimized at locations from CC to CL, while the TL for commodities with positive prices are indeterminate (or if they are determinate to any extent they are likely to fall at locations from CL outward). As TL_ moves outward from CC, the wage decline will optimize against a decline in commuting cost. But if TL_ moves beyond CC, the wage decline will be accompanied by an increase in commuting cost, becoming even more sub-optimal. Therefore only the CC – TL_ – CL configuration is optimal. For completeness, note that a reverse conclusion applies in the hypothetical case of positive price gradients, as shown in Diagram 2.5. This is occasionally referred to as the reverse-Alonso case in this study. It would apply with a theoretical consistency if the anomaly of a set of upward-sloping per unit price gradients were encountered.

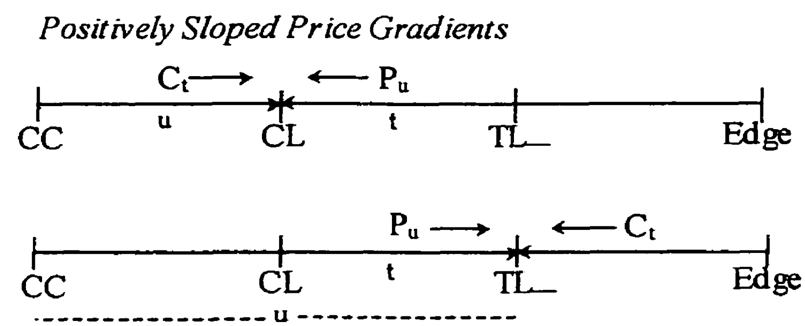

Diagram 2.5
Reverse Decentralized TL_

Suh (1990) provides an interesting variation in a model which can be related to the above analysis, though in less continuous terms (a finite number of unit squares). Postulating that "if jobs are ubiquitous and all workers can be locally employed ... the mean optimal commuting distance is zero" (p. 277), he begins with a situation of complete job centralization at city centre in order to relax that assumption and analyze the results. Introducing variables such as the degree of decentralization, speed (rate or slope) of decentralization, degree of concentration and slope of population density curve – he concludes principally that an increasing decentralization of jobs reduces the "optimum mean commuting distance". Although his mathematics runs in terms of maximizing on local full-employment conditions in each unit square (approaching the concept of islands, see Section 4.2.3), the logic can be intuitively understood simply as the proposition that moving from complete centralization to complete decentralization means approaching the mean optimal commute of zero quoted above.

2.4 Specific Relationships

2.4.1 Preference Space

The analysis of the previous section was cast in highly general terms, employing vector equations to represent the entire system in multiple dimensions. However, many of the most telling relationships are revealed in more specific relationships. This section restates the system in more restricted equilibria, separately for the consumption location and for positive and negative transaction locations, maintaining the critical locational relationships between them. This amounts to imposing restrictions on the generalizing assumptions of the model, so that it can be represented in specific differential and integral equations, and so that it can be depicted in a set of two-dimensional graphical preference spaces.

Considering, then, specific equilibrium relationships which can be drawn from the analysis of Sections 2.1 to 2.3 – for any two commodities i and j consumed:

$$\frac{p_i}{p_j} = \frac{\partial U / \partial z_i}{\partial U / \partial z_j} = -MRS_{z_j, z_i} \qquad (2.7)$$

This can be represented graphically by the usual tangency of indifference curves to budget constraint illustrated in Figure 2.1.

The relationships described in equation (2.6) and Diagram 2.4 are depicted in equations (2.8), (2.9) and (2.10) below. Unbolded symbols (such as u) are specific instances of

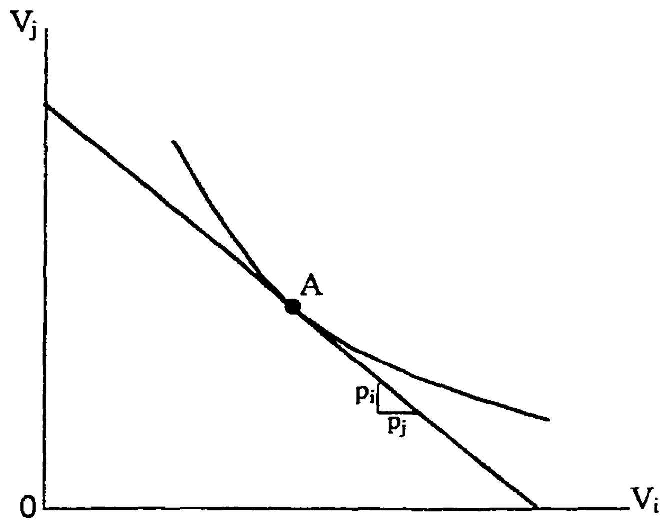

Figure 2.1
Standard Optimum in Commodity Space

bolded vectors (such as **u**). The generality of the model is relaxed to allow for a small number of commodities, a CES utility function with an elasticity of substitution of 1 (to be discussed below), and perfect capital markets under conditions of a simple exogenous rate of interest (to allow a rent-equivalent to represent the price of housing). The following convention applies: the subscript $_0$ represents the consumption location, subscript $_2$ represents commodities purchased at positive prices (and hence identified as Positive Transaction Location in Diagram 3), and subscript $_3$ represents commodities sold at negative prices (and hence identified as Negative Transaction Location in Diagram 2.4).

$$\frac{\partial p_0}{\partial u_0} = -\frac{\dfrac{\partial c}{\partial t_2} + \dfrac{\partial c}{\partial t_3} - p_2(u_2)\left[\dfrac{\partial U/\partial t_2}{\partial U/\partial z_2} + \dfrac{\partial U/\partial t_3}{\partial U/\partial z_2}\right]}{z_0}$$

$$= -\frac{MC_{t_2} + MC_{t_3} + p_2(u_2)\left[MRS_{z_2,t_2} + MRS_{z_2,t_3}\right]}{z_0} < 0 \qquad (2.8)$$

$$= -\frac{MC_0}{z_0} < 0$$

$$\frac{\partial p_2}{\partial u_2} = -\frac{\dfrac{\partial c}{\partial t_2} - p_2(u_2)\left[\dfrac{\partial U/\partial t_2}{\partial U/\partial z_2}\right]}{z_2}$$

$$= -\frac{MC_{t_2} + p_2(u_2)\left[MRS_{z_2,t_2}\right]}{z_2} < 0 \qquad (2.9)$$

$$= -\frac{MC_2}{z_2} < 0$$

$$\frac{\partial p_3}{\partial u_3} = \frac{\dfrac{\partial c}{\partial t_3} - p_2(u_2)\left[\dfrac{\partial U/\partial t_3}{\partial U/\partial z_2}\right]}{z_3}$$

$$= \frac{MC_{t_3} + p_2(u_2)\left[MRS_{z_2,t_3}\right]}{z_3} > 0 \qquad (2.10)$$

$$= \frac{MC_3}{z_3} > 0$$

The left hand terms of (2.8) and (2.9) indicate the negative slope of the market price gradient, while the right hand terms indicate the negatively-sloped price indifference curves. Signs are reversed in (2.10) since p_3 are negative.

Note the use of the MC subscripted with a number alone, to indicate the total marginal cost of geographical distance, incorporating the marginal cost of travel compensated by adjustments in consumption patterns (which the envelope theorem suggests is minimally significant).

2.4.2 Graphical Representation

These relationships in price-distance space are shown in Figure 2.2, extended from Fujita (1973, page 75). It shows the tangency of indifference curves to budget constraint for each of the corresponding elements in $\mathbf{D}U(\mathbf{t}^*)$ and $(\mathbf{V}^*\mathbf{D}p(\mathbf{u}^*) + \mathbf{D}c(\mathbf{t}^*))$. It contains two sets of curves, representing positive prices (commodities purchased) and negative prices (commodities sold).

In Panel 1, p(u) is a convex market price gradient and represents the price tradeoff offered by the market. Its slope is assumed to decline in absolute value as u increases. And each U_i is a price indifference curve which represents the preferred price tradeoff needed to compensate the disutility of travel and a rising c(t) when commuting distance t increases. Apart from utility adjustments, greater t means higher travel costs, requiring a lower positive p or a higher negative p to maintain U. Hence the slope of U as indicated.

Note also that greater utility is realized closer to the origin, given that the axes represent costs to the consumer. The preferred location for the resident is at CL.

In Panel 2 we consider negative prices for goods sold as a source of income. Here the axes represent benefits. Greater price represents greater income. Greater distance of TL_{-} from CC represents a shorter distance from CL and hence lower commuting costs. Hence greater utility is realized further from the origin, and the indifference curves must have a greater degree of concavity than the price gradient for a non-corner solution. The preferred location for the commodity represented in Panel 2 is labeled TL_{-} and indicates that it will occur closer to CC than will the CL, as analyzed in Section 2.3.3 (when explaining Diagram 2.4). A corresponding logic to the above paragraph applies to negative transactions where the price indifference curve is flipped vertically and hence concave.

Following from Hekman (1980), Fujita (1989) and Simpson and van der Veen (1992) incorporate more time variables (work, leisure and commuting). These would create additional derivatives in equations (2.8) to (2.10), affecting how the bid-rent curves are responsive to travel time and market conditions. While they affect the shape of the curves in Figure 2.2 and affect the outcome in that sense, they do not alter the set of basic relationships considered here.

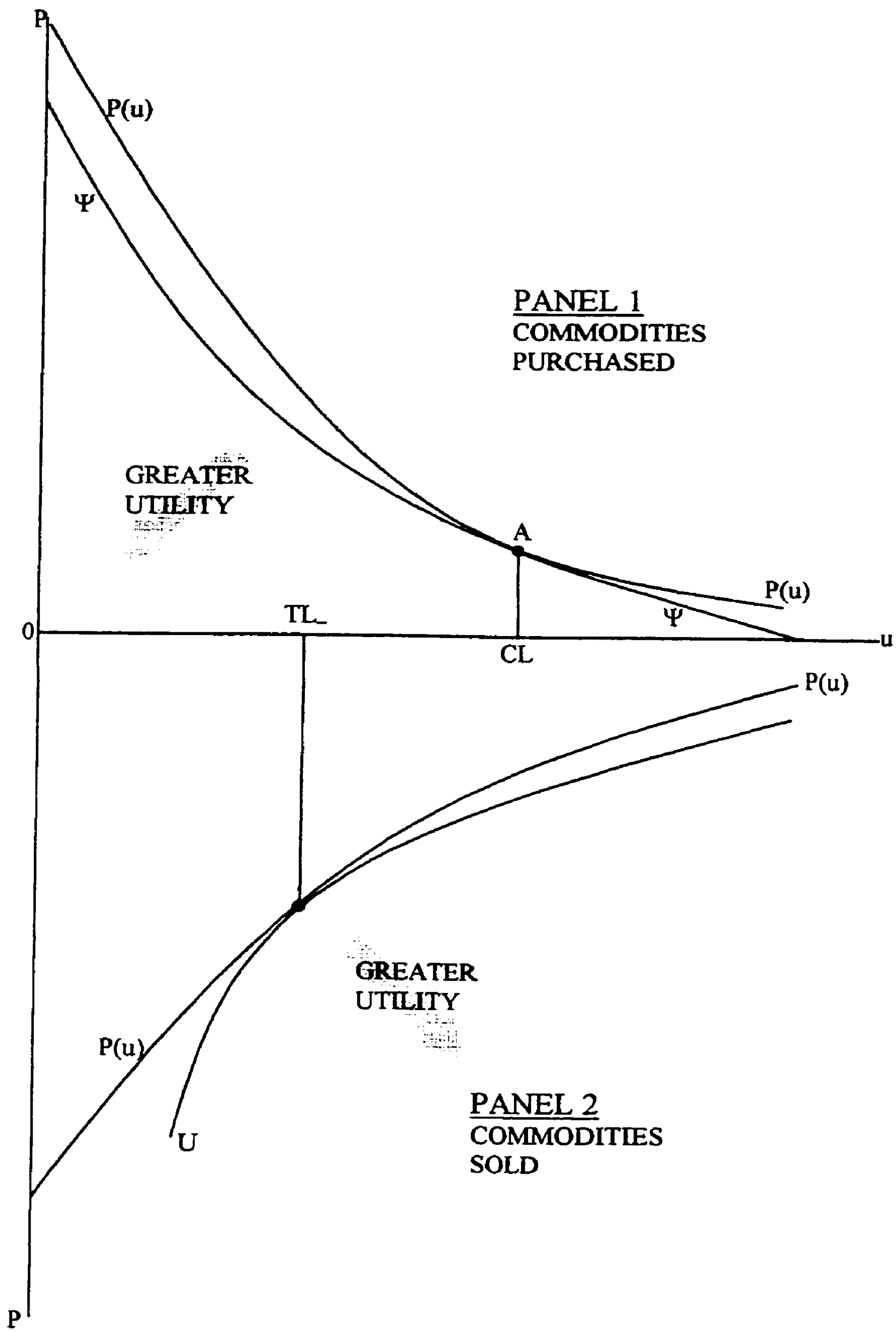

Figure 2.2
Extended Evans Model

2.4.3 Solution Requirements

It is useful to examine this equilibrium more rigorously, in order to be clear on the conditions which can be expected to produce a normal solution. The degree of convexity of the price indifference curves depends upon the transportation function and the marginal rate of substitution between commodities and travelling time.

Let us represent (2.8), (2.9) and (2.10) in the simplified form

$$\frac{\partial p}{\partial u} = \frac{\partial U/\partial t}{\partial U/\partial z} - \frac{\partial c}{\partial t} = \frac{U_t}{U_z} - \frac{\partial c}{\partial t}.$$

Assuming p_2 is unity, or at least is constant because it represents ordinary commodities which are ubiquitous at an invariant price, the second derivative yields

$$\frac{\partial^2 p}{\partial u^2} = -\frac{\partial z/\partial u}{z^2}\left(\frac{U_t}{U_z} - \frac{\partial c}{\partial t}\right) + \frac{1}{z}\left(\frac{\partial (U_r/U_z)}{\partial u} - \frac{\partial^2 c}{\partial u^2}\right). \tag{2.11}$$

Convexity of the price indifference curves requires the above expression to be positive. The first term in parenthesis is negative, as per (2.9). The term $\partial z/\partial u$ might be expected to be positive in the case of positive transactions, since greater distance from CC facilitates the purchase of more commodities. Straszheim (1987, page 722) notes that the existence of t in the utility function further complicates the relationship. A strong aversion to travel may cause a consumer to purchase more of some commodities closer to CC.

The second term is also indeterminate in sign, since it may become negative if the transportation function is not concave or the marginal rate of substitution between z and t turns U_t/U_z sufficiently negative. Hence it is not certain that the price indifference curve is convex unless we can say that the transportation function is concave (a reasonable assumption, though the model usually assumes a constant marginal travel cost) and/or that the disutility of time is not strong enough over the distances considered to reverse the expected signs (a common implied assumption). The latter is not a serious impediment for the following reason. If we remove time from the utility function, the above expression remains the same except that $U_t/U_z = 0$. It is clear that turning that equation negative would require a transportation function in the first term so convex as to overpower the positive second term, which may be considered unlikely, even if some time disutility does come into play. However, this points to the opposite problem, namely that the price indifference curve may be more convex than the market price gradient, producing multiple solutions.

The convexity of curves is measured by their elasticities of substitution, which we will denote σ. The elasticity of substitution in this context represents the degree to which changes in the (u,p) ratio respond to changes in the slope of either p(u) or U_I (Henderson and Quandt, 1958, page 72). A curve of lesser convexity has a higher σ, indicating greater substitutability. A stable optimum is achievable if locally $\sigma_{p(u)} < \sigma_{Ui}$. We may conclude that this conditions is likely to be met unless (1) the marginal cost of transportation declines extremely rapidly, or (2) the aversion to travel is replaced by a preference for travel, both of which may render the price indifference curve excessively convex.

The assumption of independence between different indifference curves, in both panels, is contained in the assumption of a CES utility function with $\sigma = 1$ (Henderson and Quandt, 1958, page 113). In other words, there are neither substitutability or complementarity to destabilize Figure 2.2 by causing an adjustment in the location of point A to shift the other indifference curves. Any relaxation of that assumption, either by allowing for substitutes ($\sigma < 1$) or complements ($\sigma > 1$), or by allowing σ to vary with levels of consumption, would require an accounting for shifting curves as optimization is approached, resulting in either unpredictability or instability. Hence the assumed CES curve is an essential reference point.

2.4.4 Impact of Travel Costs

A decrease in the per-unit cost of travel will decrease the numerical slopes of the indifference curves (flatten them) increasing distance from city centre for all commodities, with an uncertain impact on actual commuting distance. This is because it will also involve an interaction between the TL– and the CL. If for example the location of TL– were pushed further to the right due to lower travel costs, one might think at first glance that the actual commuting distance for any given u would be shorter, which would increase the preference for u in the choice of the CL and push the CL further to the right as well – with an ambiguous impact on commuting. However, that would not necessarily be the case, since it does not take into account the possibility that the benefits of lower travel costs may be employed to increase the consumption of other commodities instead.

This can be illustrated with reference to equation (2.8) in Section 2.4.1. Note that $\partial U/\partial z_2 > 0$ represents the marginal utility of the purchased commodity, which is diminishing as more z is consumed. And $\partial U/\partial t_3 < 0$ represents the marginal disutility of commuting time, which is increasing (becoming more negative because of a diminishing marginal utility of proximity) as commuting increases. When a decrease in travel costs result in more commuting plus increased consumption of z_2, which will be the dominant response?

Define a relatively strong hunger for z_2 in terms of a marginal utility that is high, and slow to diminish such that the elasticity of the marginal utility is weak[8]. If there is a strong hunger for z_2 its marginal utility will not decline as much as the marginal disutility of commuting time will increase. Hence the decisive impact will be that the marginal utility of time will become more negative. Since the marginal utility of time is subtracted from the right-hand side of the equation, the latter will increase in value. The slope of Ψ will become less negative, or shallower, moving the CL rightward. That is, if the hunger for z_2 is relatively strong, there will be a tendency to favour greater commuting because the priority is commodities rather than commuting. A converse logic applied to equation (2.8) suggests that a weak hunger for z_2 encourages less commuting.

[8] Note the occasional practice of using the term weak (or strong) to compare elasticity in the sense of responsiveness, whether positive or negative.

Similar analyses illustrate that a high marginal utility of land has a similar impact to that of a high marginal disutility of commuting. Siegel (1975) defines the elasticity of the marginal utility of land in a manner which makes it similar to the land hunger mentioned above. He shows that if the elasticity of the marginal utility of land is weak, residents will tend to move further out, which is consistent with the above analysis.

Straszheim (1973) considers differentials in travel costs, or more precisely declining marginal travel cost. Although Straszheim's context is focussed more on inter-city or long-term differentials, it is worth examining the impact of lower marginal travel cost within a city in the short run. Specifically, assume a declining congestion factor reduces marginal travel cost further from CC. Such declining travel costs will produce a tendency to move the CL further from CC relative to TL. The result is simply that, ceteris paribus, suburban households will tend to commute further. Testing by Straszheim (1975) tends to verify this in the inter-city context, where locational variations in both price and cost play a role in the final location decision. Other kinds of differential marginal travel costs can be considered, such as those which vary by distance traveled, and those which vary by number of commuters.

2.4.5 Impact of Income

The effects of income changes have long been of interest to urban analysts, though conclusive results are hard to come by. Going back to Wingo (1961), housing consumption at distance m is determined by $p_m q_m = Y_{h+t} - t_m$, where Y_{h+t} represents income available for housing and transportation (as depicted by Richardson, 1971). The result is simply that higher income brings about greater commuting distance m. Later analysts (Harris, 1968; Muth, 1968; Stegman, 1969) stressed how the same relationship may be influenced also (and perhaps dominantly) by other factors such as housing and neighbourhood quality, but the relationship with income remained the same.

Another approach is to determine the response of the price gradient to income changes directly. Note first that, in calculating equations (2.8) to (2.10), it was assumed that the total amount of endowments are constant and all endowments are sold for income, in order to simplify the mathematics. Hence income varies directly with p_3, and we may define a simple income measure as $y = p_3 v_3$.

Consider how the slope of the price gradient varies with income. Since

$$\frac{\partial p_0}{\partial u_0} = -\frac{MC_0}{z_0}$$

at equilibrium, taking its derivative with respect to y yields

$$\begin{aligned} \frac{\partial}{\partial y}\left(\frac{\partial p_0}{\partial u_0}\right) &= -\frac{MC_0}{z_0 y}\left[\frac{\partial MC_0 / MC_0}{\partial y / y} - \frac{\partial z_0 / z_0}{\partial y / y}\right] \\ &= -\frac{MC_0}{z_0 y}\left[\varepsilon_{MC,y} - \varepsilon_{z,y}\right] \\ &> 0 \text{ if } \varepsilon_{MC,y} < \varepsilon_{z,y}. \end{aligned}$$

$\varepsilon_{MC,y}$ and $\varepsilon_{z,y}$ measure how responsive the choices of u_0 and z_0 respectively are to changes in y. Similar to Seigel (1973, page 33), the final inequality holds if the responsiveness to rising income of acquiring commodities is greater than that of incurring travel cost — consistent with the weak $\varepsilon_{MU,z}$ associated with land hunger in the discussion above. It indicates that the negative tradeoff preference, whereby a given reduction in p_0 buys so many units of u_0, increases (becomes less negative and shallower in slope) as income increases. That is, a given reduction in prices will buy more units of u_0. Hence with increased income the terms of trade shift in favour of u_0, resulting in a tendency to choose higher values of u_0.

However, if the increased value of travel time which results from an increase in income becomes significant enough, then $\varepsilon_{MC,y} > \varepsilon_{z,y}$. That would bring the opposite results, reducing the willingness to trade off more u_0 for less p, resulting in a steeper preference and a tendency to choose lower values of u_0.

Comparing the two elasticities — if we now postulate, not terribly unreasonably, that $c(u_0)$ is a linear function while $p(u_0)$ is a convex function, then $\varepsilon_{z,y}$ will be relatively weaker at higher values of u_0 because $p(u_0)$ will be shallower in slope. The rising value of commuting time will play a more influential role in the decision. In other words, in choosing between suburban locations with high values of u_0, as distinct from choosing between suburban and central locations, there will be a greater tendency to choose lower commuting times as incomes rise. That may reduce the positive value of the above equation, or turn it negative. Income may induce more commuting, but only up to a limit.

Evans (1973, p. 120) summarizes a similar analysis by saying, "we cannot predict the pattern of location of households with different incomes in relation both to each other and to the city centre without some knowledge of the income elasticity of demand for space of the households", and goes on to diagram the results of various combinations of values. Nonetheless, he cites some data which suggest that "variations in geographical distribution seem to result from variations in the income distribution between the conurbations". Simpson's data for London and Toronto (1992) are more tentative, stating that simple crosstabulations do not provide enough evidence of a relationship to evaluate the core urban model, but the data also illustrates how "residential location, like most other social phenomena, is complex and multi-faceted" (p. 16).

2.4.6 Impact of Price Changes

Consider an increase in price, in reference to Figure 2.2. In the top panel, a price increase would be represented by an upward shift in p(u). The consumer facing a higher price wishes to lower it, and buys a lower price by reducing travel costs – that is, moves outward away from city centre. In the bottom panel, a higher wage would be represented by a downward shift in p(u). The worker earning a higher wage can afford a reduction in wage and buys lower travel cost by moving outward away from city centre. Hence the overall effect of an increase in price along the whole gradient (for the indifference map that represents normal goods) is an outward tendency for all transactions.

A change in price elasticity over distance will have an ambiguous impact on location, depending on the strength of the income and substitution effects. Consider the distance elasticity of the price gradient, $\varepsilon_{p(u)}$, and the elasticity of substitution of the indifference curve, σ_U , both of which have an impact on the optimal location of any TL. A change in elasticity will have a income effect and a substitution effect. If the positive price gradient becomes shallower in slope and generally of stronger elasticity (meaning more elastic in absolute value, though technically decreasing to a higher negative value), there will be a negative income effect. That is, the income component of the change will include higher commodity costs and a higher cost (including utility cost) of transportation. The consumer, trading off greater commodity price saving for greater travel cost saving (proximity to CC), will have less of these benefits in total to choose, and the final choice will include less of each — greater distance from CC and higher commodity price. In other words, the negative income effect causes the consumer to choose a TL further from CC . At the same time, if the price gradient becomes more strongly elastic, the terms of trade change. Greater proximity to CC becomes relatively cheaper in terms of price increases incurred per unit reduction in travel cost. The substitution effect states that more proximity will be consumed, by choosing a TL closer to CC. The net effect depends on the relative strengths of the income and substitution effects.

This is illustrated in Figure 2.3. The curved arrow results in a stronger elasticity of p(u), as it pivots from p_0p_0 to p_1p_1. The perpendicular shift from p_0p_0 to p_2p_2 indicates the income effect. The movement along U_1 from u_2 to u_1 indicates the substitution effect. The curves happen to be drawn in such a way that the substitution effect exceeds the loss (negative income) effect. Hence the stronger elasticity results in a decrease in the value of u from u_0 to u_1.

The income effect of a strengthening of the elasticity of the price gradient depends on its initial elasticity. Consider again a strengthening of the elasticity of p(u). If initially it is more steeply sloped and therefore $\varepsilon_{p(u)}$ is generally weak, then the parallel outward shift used to measure the income effect will result in a greater increase in u. The substitution effect depends on the elasticity of substitution. If σ_U is high, indicating a low degree of convexity, then a leftward movement along the indifference curve to substitute more proximity for less price reduction will produce a relatively large decrease in u. In other words, the income effect will be strong when the distance-elasticity of the price gradient is relatively weak (supporting a positive relationship between ε and u), and the substitution effect will be strong when the indifference curve exhibits a relatively high elasticity of substitution (supporting a negative relationship between ε and u).

Before examining these effects further, note how the structure of a price gradient dictates the relationships between elasticity, convexity and price differential. An increase in the strength of the overall distance-elasticity will result in a reduction in the convexity (increase in the elasticity of substitution) and a reduction in the price differential between CC and the edge of the city.

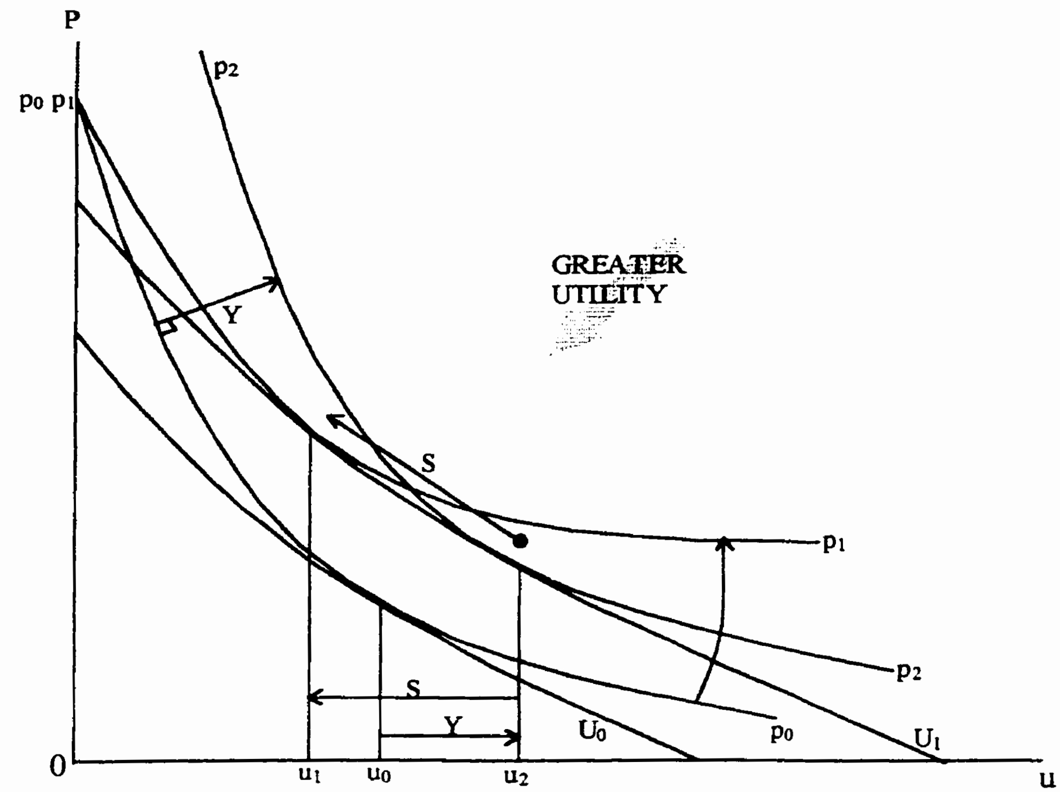

Figure 2.3
Income and Substitution Effects

Consider a case in Figure 2.3 where prices at the edge of town are almost as high as prices at CC. Then p(u) will be shallow in slope and generally strongly elastic. Hence $\varepsilon_{p(u)}$ will be high in value indicating a low degree of price responsiveness to distance. When the strength of distance elasticity increases, we can expect a weak negative income effect, and a tendency for stronger elasticity to be associated with lower values of u. The opposite will occur in the case of large price differentials. We can infer that in this case a weak price gradient produces a tendency to move inward, and a strong price gradient produces a tendency to move outward (which will be weakened if the elasticity of substitution is strong).

Negative prices do not exhibit the same degree of ambiguity. Movement away from the origin yields benefits, namely a higher price (income) for goods sold and lower transportation costs approaching the CL. If the negative price gradient becomes shallower in slope and generally more strongly elastic, the income effect states that the consumer will choose more of each and move further from the CC. But the substitution effect moves in the same direction. One unit closer to the CL costs less units of lower income. The terms of trade shift in favour of distance from CC. Hence there is an unequivocal increase in u in response to an increase in the strength of the elasticity of the negative price gradient.

Let us summarize regarding the basic relative positions of the CL and the various TL.

- Commodities with negative prices are sold at TL between CC and the CL, while commodities with positive prices are assumed ubiquitous for realism and theoretical convenience.
- An increase in per-unit travel cost moves all TL closer to the CL, but the net impact on the location of the CL is uncertain. However, if the effect on CL is dominant, it will tend toward CC.
- Placement of a TL at a greater distance from the CL will tend to move the CL in the same direction if the elasticity of the marginal disutility of commuting and/or the marginal utility of land is weak relative to the elasticity of the marginal utility of other goods

- An increase in price which does not alter the terms of trade will result in all TL (including the CL) further from CC.
- For commodities with substantial positive price gradients and hence large price differentials, those of stronger elasticity will tend to have TL further from CC. For commodities with weak price gradients, those of weaker elasticity will tend to have TL closer to CC.
- For commodities with negative price gradients, an increase in income or its elasticity will move the TL further from CC.

2.5 Gradients

2.5.1 General Nature

We employ the term gradient, to refer to a vector whose variables change in a systematic manner as geographical distance increases or decreases. In the conventional usage of the term in this context, we define two gradients. A density gradient refers to manner in which the quantity or proportion of a commodity available varies as the TL or CL moves further from CC, or $\mathbf{d} = d(\mathbf{u})$ where it is conventionally (but not necessarily) assumed that $\mathbf{D}d(\mathbf{u}) < 0$. A price gradient refers to the manner in which the price of a commodity varies as the TL or CL moves further from CC, or $\mathbf{p} = p(\mathbf{u})$ where it is conventionally (but not necessarily) assumed that $\mathbf{D}p(\mathbf{u}) < 0$.

A general analysis begins with the full range of possibilities, namely that the sign of $\mathbf{D}p(\mathbf{u})$ and $\mathbf{D}d(\mathbf{u})$ are unknown. Of course, only a certain range of values for $\mathbf{D}p(\mathbf{u})$ and $\mathbf{D}d(\mathbf{u})$ produce the kinds of balances of forces which create economic choices. For example, $\mathbf{D}p(\mathbf{u}) = 0$ would remove the spatial aspect of the analysis in its entirety, while $\mathbf{D}p(\mathbf{u}) > 0$ runs contrary to normal urban analysis but is nonetheless capable of producing a reverse balances of forces which may generate economic choices of its own. This hypothetical possibility is touched upon in various sections, though generally not found to be applicable in practice.

2.5.2 Housing Density

The original Alonso model rests on the assumption that housing density is greatest at city centre, and declines with distance from city centre in a smooth exponential gradient. Fujita (1989, as early as page 4) is specific about it, noting that – if "the number of households of each type is assumed to be so large that their distribution throughout the city can safely be represented in terms of a density function", then (page 8) complex mathematical density models ("in which each consumer is assumed to occupy a subset of two-dimensional Euclidian space") can be employed to derive the shape of the density curve. Elsewhere (page 57) Fujita confirms that such density curves are consistent with convex downward-sloping price demand curves ("bid rent curves") in that the latter implies lot sizes which are increasing in u.

Without venturing into mathematical density models, it is usual to take density gradients as given and estimate them using an exponential model such as the following (Mills, 1994): $D(u) = D_0 e^{-\gamma u}$ or, as per Simpson (1992, page 31), $\ln [D(r)] = \ln [D(0)] + \delta r$, where r in this case stands for concentric rings. Both of the above authors find density gradients roughly corresponding with the postulated curve, with indications of suburban concentrations which are significant but do not negate the assumed shape of the gradient (although they provide a rationale for incorporating regional centres into the analysis).

The coefficient δ is commonly assumed to be constant for mathematical tractability, though it is useful (Hoover, 1968) to make it vary quadratically to represent the volcanic scenario of a normal price gradient with a depression right at the centre. The value of δ also reflects transportation costs, and will represent a shallower slope when transport costs are lower. Several attempts (Clark, 1967 and Muth, 1961) have been made to estimate δ empirically. Simpson finds δ suitably negative and significant with quite high explanatory value, despite the presence of significant decentralization. Other studies (see Kain, 1975 and Jackson, 1979) suggest otherwise, based on measured gradients which do not exhibit the postulated negative relationship. Richardson's caution (Richardson, 1971, page 56), that such gradients assume radial symmetry and a singular peaked gradient, should also be noted. Mills and Hamilton (1994, p. 132-4), for purposes of summarizing in an intermediate textbook using a rough calculation consistent with "most estimates", state that "the price of housing declines roughly 4 percent per mile of distance from the CBD", "a 4-percent-per-mile decline in the price of housing must be generated by a 20-percent-per-mile decline in the price of land", "the density function also falls by about 20 percent per mile", and "a wage decline of about 1 percent per mile is expected". They then give a useful summary of empirical studies (p. 134-41) which roughly support those estimates, qualified by a significant incidence of measured gradients which fail to conform to the pattern.

Widespread diffusion on a large city will also tend to make intermediate points feasible, both for residential building and employment location, to serve the more suburban areas.

Hence there may be secondary density peaks as agglomeration economies concentrate diffuse intermediate points into secondary density centres.

2.5.3 Housing Price

Considering housing price gradients, the simplest approach is to assume that high density produces strong competition for every square foot of space, resulting in a land price gradient which also peaks at city centre and declines with distance in a convex manner. Sometimes the reason given is simply that a greater population density at CC, with the accompanying concentration of economic activity and services, makes it desirable in terms of access and results in a greater demand at CC. An alternative rationale is that the declining marginal utility of any given commodity, combined with marginal transportation costs which are non-declining, will endogenously produce a declining demand and a declining price as distance from CC increases. In a similar vein, Evans (1973, Chapters 3 and 4) takes a cost-minimization algorithm and a transportation cost function which is realistically increasing at an increasing rate with distance, to produce standard convex downward-sloping housing density and housing price gradients (see also White, 1988b, where it is also applied to wage gradients). For purposes of comparison below, we shall label this the *static optimizing effect* (for later comparison with other competing effects).

This gradient occurs partially because concentration at the centre produces smaller lot sizes, plus a necessity to build upward, both of which cause the per unit price to rise accordingly. Numerous empirical studies suggest support for this relationship, and they are succinctly summarized by Evans (Ch. 5) up until the 1970's – which find a high degree of variability (and variance) among cities studies, but a good rough confirmation of the both the expected density gradients and the expected price gradients, sufficient to conclude that the convex downward-sloping gradients of theory are reasonably robust.

An earlier approach (going back Burgess, 1925, and at least in implied form to von Thunen, 1826) deals in terms of concentric rings, as a rough and rippled form of downward price gradient. This is based on a different kind of assumption, namely different but competing uses for the land. If each of them has its own price gradient, the effective price gradient will take the form of an inside envelope curve, composed of intercepting points which define bands of preferred uses at that distance from city centre. Taking that model radially, it produces concentric rings. Caution is indicated in that, without employing data which identifies the competing uses, it may not be the same thing just to define arbitrary uniform rings to mimic this phenomenon.

A useful approach is to model price gradients in a monopsony model. As distinct from the either-or demand situation facing a perfectly competitive firm, a monopsony firm is concerned with the relative attractiveness of its product bought or sold, which is a function of price: $g(p)$ where $\partial g/\partial p > 0$. The individual buyer or seller is free to choose a value for $g(p)$ without the risk of flipping between all or nothing implied by perfect competition, allowing for a smooth and manageable adjustment. Simpson (1992) cites Bailey's (1975) work in this area in terms of an optimal wage path[9], and expands the

[9] Note that we are re-applying the analysis, which was cast in terms of wages, to a broader context of all prices including wages.

analysis to specify the relative attractiveness function as: $g(p,t)$ where t is the distance required in order to travel to make the transaction and $\partial g/\partial t < 0$.

The combination of $\partial g/\partial p > 0$ and $\partial g/\partial t < 0$ means that as t increases, less of the commodity is likely to change hands unless the price improves. Hence if density is greatest at city centre, then the seller of positively priced commodities at city centre must offer a lower price to compensate for a greater overall travel required by suburban buyers to participate in the transaction. We shall call this the *attraction effect*.

This effect, however, makes certain assumptions about the relative spatial distributions among markets, and we shall label it the *distribution effect* in order to make the distinction. Consider the case of a commodity which is evenly spread across the city while the purchasing population is relatively centralized. In that case, the quantity required by suburban buyers is likely to be available locally. Hence sellers located at city centre must charge a lower price in order to attract buyers from the suburbs. The result is an upward-sloping component in the price gradient. The same would apply to the commodity housing, but the applicability of the distribution effect to housing is questionable. Almost by definition, housing is spread out to approximately the same degree as the population as a whole. Hence the distribution effect is weak or nonexistent. For other commodities, however, the distribution effect is theoretically relevant, except that shopping centres tend also to be spread out to approximately the same extent as the population. Overall then, it can be argued that the distribution effect has little applicability in the case of positively-priced commodities.

2.5.4 Employment Gradients

The attraction effect applies in the case of negatively-priced commodities – the buyer (or employer) at city centre must offer a higher price (wage) to compensate for a greater overall travel required by suburban sellers (workers) to travel from city centre to participate in the transaction. However in the case of negatively-priced commodities, and jobs in particular, the distribution effect is significant. Consider the case of jobs which are evenly spread across the city while the working population is relatively centralized. In that case, suburban workers are able to find jobs locally. Hence employers located at city centre must offer a higher wage in order to attract workers from the suburbs. The result is a downward-sloping wage gradient. In this case, the set of relative distributions is a practical possibility, and the distribution effect must be considered as a potential determinant of a negatively-priced (wage) gradient.

As we shift the focus from general positive and negative prices toward wages in particular, let us look further into the relevant gradients. When it comes to the location of jobs, the simple extended Alonso model assumes a similar set of employment density and wage gradients – namely convex downward-sloping curves. Simpson's semi-logarithmic linear density gradient, $\ln[D(r)] = \ln[D(0)] + \delta r$, was applied to "survey respondents employed and living in each zone" (page 31) which suggests that it might be considered applicable as a description of job density gradients as well. Implicitly, job decentralization is assumed not to occur to such a great extent that the job density gradient turns upward.

There are theoretical and empirical treatments of how the structures of costs, structures of revenues, and linkages contribute to decentralization of industry. But what is most relevant to our study is the analysis of how production or distribution conditions produce job density and wage gradients. It is difficult to predict a stable relationship, for which reason some analysts prefer just to assume wages constant (Goldberg, 1970). Evans (1973, Ch. 11 and 12) was unable to find the same degree of robustness for job density and wage gradients as he found for price gradients.

Simpson (1992) extends Bailey (1975) with an analysis[10] of how heterogeneous patterns of growth produce higher wages in growth centres, and adds the models of Moses (1962) and Nelson (1973) to support the proposition that employment centres offer a wage premium to recruit commuters – resulting in a wage gradient. Simpson extends his analysis (Section 3.5.1) with a model of monopsony recruitment (implying a degree of control over wage offers) which specifies r, the distance marginal workers must travel to work, as the "catchment area" (page 67). The derivative of workforce growth with respect to r is negative, meaning that recruitment will not be able to reach a larger catchment area unless wages are increased to compensate for increased travel cost (supported by Nelson, 1973). And the derivative of workforce growth with respect to the wage is positive, suggesting that larger firms require a larger catchment area and must pay a higher wage to achieve it. If we look at city centre as the largest collective firm, it must pay the highest wages, and the result is a downward-sloping wage gradient, though this requires an assumption of jobs more centralized than homes (as pointed out by

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[10] Now re-examined in its original wage context.

Moses, 1962).

In summary, the bulk of the theoretical support favours negatively-sloped gradients. Much of the analysis of this study adopts the simple assumption that all exogenous per unit market price gradients are downward sloping and convex – both in order to generate interesting problems, and because evidence to the contrary is not compelling enough to abandon the fundamental building block of traditional urban analysis.

3. LOCATION CHARACTERISTICS

3.1 The Housing Variable

3.1.1 The Geometry of Land

The limitations of employing continuous mathematical functions, where real-life spatial relationships are supposedly represented by mathematical space, are apparent when considering the acquisition of land. Distances between locations are measured from one mathematical point to another, where each location is a infinitesimal point on a Cartesian plane, and they are joined together in a continuous locus which facilitates mathematical differentiation. The problem is that, in order to function as a consumption or transaction location, a finite quantity of physical space is required. Still, if all parcels of land were identical in size, a mathematical transformation may be possible which would allow for the use of continuous and differentiable functions. In fact, this is an implicit assumption of many urban residential location models.

However, the analysis in question is precisely a model of making quantitative choices, including the quantity of land to be purchased, quite contrary to the assumption of identically-sized residential locations. This seemingly negates the possibility of applying smooth mathematical functions to lumpy distributions of land.

It is also problematical to specify single-residence lots, especially when dense housing already exists. The optimizing tradeoff of more commuting for more land will not be possible unless we assume that lot sizes increase with distance from CC in precisely the appropriate degree for each individual consumer in question, to ensure that the optimum choice is available at the optimum location. Given the unlikelihood of this condition being met, the use of land to represent residential services presents some difficulties.

In addition, the assumption of one housing unit at each location runs into practical problems. Perhaps if one is building on vacant land at the edge of town, one might assume that all options are available. But in most cases there are constraints. There is the problem of pre-existing dense housing, where lot sizes are constrained by neighbouring lots. Even in the case of new housing, much new housing is pre-built on fixed lots before being sold. A lot at the desired location may simply not contain the desired bundle of housing characteristics.

An alternative technique would be to assume that every point in space represents an identical parcel of land of size one, on which differing quantities of housing units are available. This is most easily envisioned as a set of identical building blocks, one or more of which constitutes an acceptable bundle of housing services. Then we can speak in terms of a quantity of housing units which are chosen by the consumer at any given location. This approach presents a problem of its own. It speaks as if, at each location, the resident can erect any desired number of building blocks. In practice, this may be realistic if we are dealing with empty lots, upon which new houses are built. Any number of building blocks may be purchased. On the other hand, if most land parcels contain pre-built housing, the choices are constrained, not only as discussed above but also constrained by the difficulty of deconstructing housing to make marginal adjustments in response to continuous preferences.. Nevertheless the building-blocks approach is useful, in order to reduce housing bundles to a single quantitative measure to represent it as a composite commodity representing the weighted vector or attributes. We will refer to them as housing units or simply housing, though sometimes it will be convenient to follow the common practice by simply calling them land. Urban location models often speak as if fixed parcels of land with existing single-home residences were not a problem, which carries an implicit assumption that the prospective resident is staking out virgin land.

Evans (1973) tackles the problem with the concept of the space unit. This addresses a problem of building upward when space is at a premium, such that successive tenants are technically renting diminishing shares of surface land when they may indeed by occupying uniformly large apartment spaces. Evans' space unit is equivalent to Muth's (1969) unit of housing services, as an alternative to Alonso's use of direct ground area, to add some flexibility to the concept. It adds an additional dimension to land, such that a space unit can be defined as a function of ground area, still yielding in principle a single unique measure of housing purchased.

Cropper (1984) noted these limitation in a similar context, referring to the problem of using the mathematics of continuous choice in a case where one may choose from attribute bundles on a locational continuum, but the bundles themselves are not continuous — they each contain a fixed set of attributes. Optimizing conditions such as equation (2.6) cannot be applied. Instead, it is necessary to reverse the process through implicit functions, choosing an set of discrete attributes maximizing utility by optimizing over a continuous location variable. But this is difficult enough over two attributes; with multiple attributes and multiple agents (and, even worse, partially-endogenous attributes), it would require something along the line of simulation models, which are beyond the scope of this paper.

Another approach to these problems is to treat each mathematical point in space as a set of consumption or transaction locations. The fact that the set is located at a single point implies that no further travel costs are required to access any of the locations within the set. For example, CC is commonly chosen as the transaction location and treated as a point in mathematical space, even though in reality it is a geographical area which is collapsed into a single point by assuming that travel costs within the CC are zero. The same technique can be applied to residences. Points in mathematical space represent different districts, or neighbourhoods. Each neighbourhood contains a full range of housing services, and travel costs within the neighbourhood are assumed to be zero. In either of these cases, there would still have to be the assumption of identical-sized districts, in order to map distances between points in mathematical space to actual travelling distances between districts. This technique, involving the assumption of zero intra-district travel costs and positive inter-district travel costs, is addressed in a similar manner by Paelinck and Klaassen (1979), and bears a clear relationship to the island model developed by Phelps (1970), which will be examined in Section 4.2.

Another aspect of housing demand, pointed out by Richardson (1971, pp 24-29), is that standard optimization may be entirely inappropriate for housing. It is more likely that house seekers "tend to select a house as expensive as they can afford", seeking to maximize available housing qualities or neighbourhood amenities rather than optimize for distance. In that context, the only relevant factor is a preferred maximum commuting distance, which only puts bounds on the search area rather than play a role in the precise location of the residence. Richardson cites evidence that residence-seekers are inclined in some circumstances to choose a location on housing-preference grounds which is actually far removed from the place of employment, especially where they exhibit a strong preference for the periphery. The empirical methodology of this study tends to build on the simpler models, and on some occasions refer to these complicating factors when the simple model does not perform adequately. More comprehensive approaches can be found in Arnott (1987) and Olsen (1987).

3.1.2 Capital and Rental Values

The analysis also requires an assumption of competitive markets to produce pricing which reflects true marginal costs, so that all these attributes are accurately capitalized into market housing prices, and rental payments are equal to mortgage payments for equivalent housing (in hopes of avoiding the complications of housing production functions incorporating construction inputs and capital as well as depreciated values of old housing, and complex sets of weighted preferences — a mess well worth avoiding). In other words, given the assumptions of perfect competition plus the assumptions that (1) buildings have infinite lives, (2) rental/maintenance values are independent of building age (Evans, p 97), (3) interest rates account appropriately for inflation, (4) compounding effects are insignificant and (5) taxes play no distorting role in capital allocation – then it can be assumed that rent can be derived from capital value and the rate of interest. If not, it would be necessary to treat property value as a function of time and insert the appropriate integral (Evans, p 99) into the maximizing equations, perhaps also incorporate a function such that rental value is a function of maintenance effort over time (Evans, p 104), along with other adjustments.

3.1.3 Land in the General Context

Returning to the context of general vectors as in equation (2.4), it is useful to modify **X** by removing a sub-vector **q**, which represents those elements of **w** and **z** which constitute the consumption of housing services. Given the convenience of treating housing as a single-valued variable, it is not unusual to simplify housing services by reducing them to the single commodity of land, and measuring land consumption as the amount of land acquired. The more general analysis of this section will make reference to the full bundle of housing services, but will also point out that the inclusion of land in that bundle creates some special analytical problems. In the general analysis, $\mathbf{X} = \mathbf{w} + \mathbf{z}$ now refers to non-housing commodities only. Those elements of **q** drawn from **w** (endowment items remaining) will have prices of zero. Housing services are consumed at the place of residence, which is the CL. Even where the CL is a decision variable, once the decision is made all **q** are consumed at that location, and their travel costs are zero.

Considering these additions, utility maximization now becomes:

$$\max U(\mathbf{X},\mathbf{q},\mathbf{t}) \quad \text{s.t.} \quad (\mathbf{p_z}\,\mathbf{z} + \mathbf{c}) + (\mathbf{p_q}\,\mathbf{q}) + (\mathbf{p_v}\,\mathbf{v} + \mathbf{c}) + (\mathbf{p_l}\,\mathbf{l} + \mathbf{c}) = 0 \qquad (3.1)$$

and more generally, it remains

$$\max U(\mathbf{V},\mathbf{t}) \quad \text{s.t.} \quad \mathbf{p}\,\mathbf{V} + \mathbf{c} = 0. \tag{3.2}$$

with the general solution still represented by equations (2.3) and (2.6).

It is not unusual to reduce **q** and **X** to composite commodities. These are defensible assumptions if the prices of **q** and **X** vary in a constant proportion to each other. Since that is not possible when the prices of all **w** and some **q** are zero, the model makes an implicit assumption that all such consumed endowments do not exist — i.e. the values of all **w** and some **q** are zero. Actually, endowments are simply ignored. Unrealistic though it may be, the implicit assumption that a consumer begins with nothing and consumes only newly purchased commodities is not an uncommon one. An alternative assumption might be that any variations in the prices of **q** and **X** are small and insignificant during the period in question. None of these are any worse than the assumption that prices vary in constant proportion to each other. However it is potentially problematic when we consider housing, and when we introduce the variable training into the analysis, both of which represent significant endowments of accumulated capital.

3.2 The Housing Vector

3.2.1 Quality Attributes

When considering the effect of quality variations, it is often useful to treat any given commodity as if it were composed of elements which all possess a single uniform set of characteristics, and this has been the unstated assumption of much of the analysis to date. However there are often certain commodities containing a subset of members possessing all the defining characteristics of the commodity, but also possessing their own distinctive characteristics — and these specialized qualities play a significant role in marketplace for the commodity. While this possibility could apply to any given commodity, it is especially relevant to **q** (housing services) and **l** (labour services). Specialized housing needs, particularly the need for family housing, are a particularly important quality variation in relation to **q**. And specialized skills are an important quality variation in **l**.

An alternative to including attributes in the vector of housing attributes is to place them into the utility function. The latter might include placing distance to dis-amenities (e.g. a smelly factory, or a racially undesirable neighbourhood) into the utility function (Daniels 1975 and others). This produces another reason why the price indifference curve may become positive in slope, where it begins to border on an undesirable neighbourhood. The result could be a gradient of complex shape indeed, even assuming identical preferences. In addition, any degree of discrimination on the part of landlords, such as exclusion of racial minorities, can produce discontinuities in the market price gradient, while game-theoretic responses may amplify the compartmentalizing impact (Schelling 1972, Courant 1978, in terms of racial segregation). This raises the possibility that no simple price-gradient model can ever be verified empirically.

Another approach is to place the attributes into the vector of housing attributes, but also include locational co-ordinates in that vector, and perhaps also make price a function of those attributes. While mathematically, one can define an optimum which includes longitude and latitude co-ordinates and complex price functions related to attributes, the only remotely tractable approach may be simply to calculate one-dimensional straight-line distances to the city centre or regional centre and fixed market price gradients, as in the traditional Alonso model.

Let us examine some further implications for spatial patterns. Although housing is generally held to be income elastic, it is reasonable to suggest that special qualities contribute to a greater demand for housing. If we are speaking of the special housing quality of family housing, there is obviously a demand for a greater set of housing services. If we are speaking of specialized skills in the labour market, White (1988a, p. 133, 148 ff.) suggests that more "skilled workers have a higher demand for housing", which is not sufficiently counteracted by the higher value of their commuting time to negate the following conclusion. Special quality characteristics imply flatter offer gradients. If we combine greater and lesser need for special housing characteristics, or greater and lesser skills, the result can be a self-segregation of families and skilled workers into the suburbs, in a manner discussed in Figure 1.2 of Section 1.2.

3.2.2 Hedonic Pricing

There still remains the problem of determining what this unit of housing is that we are measuring, and what that measure means. The hedonic technique (Rosen, 1974) would place all amenities, one of them being access (Diamond and Tolley, 1982, page 6), on a set of indifference surfaces of increasing utility, with each of them assigned to an ascending series of numbers representing units of housing (quantity). That is, any given combination of characteristics (if the associated set of indifference surfaces can be taken as representative) can be given an index of quantity by this technique. Then price is determined by dividing property value by this quantity.

Linneman (1992, page 71) cites a series of analyses of the "impacts of a variety of amenities" and "the importance of structural traits, such as rooms and bathrooms" on housing prices in the context of hedonic pricing. They are based on a recognition that a hedonic function is not a demand or supply function for housing. Rather it is a vector of functions which are bundled together, whose elements are traits in the consumer's overall utility function. For each of these individual enclosed elements, an implicit market can reveal the marginal utility for each trait, which can be compared to the appropriate marginal cost. In that way, at least conceptually, the market values of individual amenities can be specified. Each indifference surface of the hedonic function determines the set of market-clearing prices implicit in the bundle of housing services which actually receives a price. But the real market demand function represents only the locus of points produced by the solution of all the individual implicit amenity demands, which lends to the difficulty of estimating the values of individual amenities.

However, the problem does not end there. Following Rosen (1974), Diamond and Tolley (1992a, page 18) point out that we cannot assume linear independence of the housing bundle, because "amenity bundles cannot be divided or repackaged". The result is that the quantity consumed of one amenity is not independent of the implicit price or quantity consumed of another amenity, and hence the arbitrage process which would normally equalize marginal prices across commodities is not capable of performing that function in the case of bundled amenities. Hence even if the indirect values of individual amenities could be determined inside the bundles, they would not be true optimal values. Nonetheless if, under almost experimental conditions, variation in the price of the housing could be observed while the consumption of all other amenities remains unaltered, the result would be the price of the amenity which varies in response.

While we do not have the data to perform a full analysis of all the factors that would enter into a hedonic price function, we can examine amenities as features attached to individual housing units (in spite of their being neighbourhood-based), which contribute to the value of the unit. One possible test we can perform is to determine whether residents show signs of responding to specific neighbourhood amenities, or whether the amenity factor is incorporated into the price of the residence. In the empirical analysis of placement in Chapter 7, there is evidence of the "criterion bundling" which suggests that indeed such hedonic pricing is significant.

Though there have been many estimates of price indices related to hedonic functions (listed in Diamond and Tolley, page 19), any practical survey of housing preferences and the prices of housing purchased cannot be considered to represent the sort of objective indifference set which underlies the concept. Hence we cannot treat available prices per housing bundle as representative of real per-unit housing costs. But we can treat them as a crude measure of total housing unit costs, and seek whatever rough and inadequate measure of housing quantity is available (e.g., number of bedrooms in the survey data examined below). And we must of course be wary of the perfectly-competitive assumptions that are invoked when we treat asset price and monthly rental as comparable or even interchangeable categories, or treat durable housing stock as comparable to monthly rental services.

Housing characteristics can be represented in the general analysis using hedonic pricing. For example, the vector **q** could measure an index of utilities representing an indifference relationship between a set of standard housing characteristics and a the inclusion of, say, an additional family bedroom. Similarly, the vector **l** could indicate an index of productivities representing the impact of hiring workers at a higher skill level. This technique produces a single-valued variable to measure housing quantity.

3.2.3 Amenity Analysis

In *The Economics of Urban Amenities*, Diamond and Tolley (1982, page 5) define an amenity as simply "a location-specific good". It is more realistic to consider housing, as mentioned above, as a vector of attributes (Straszheim 1987, Cropper 1984) and perhaps apply hedonic pricing analysis to that vector (Brown and Rosen 1982). The analysis also calls for a broader vector of attributes, applying to the neighbourhood as well as to the immediate housing. We may measure land price as a weighted index derived from the implicit prices of housing and neighbourhood attributes, and quantity from a weighted average of the same attributes. A more sophisticated theoretical approach is well described by Straszheim: "The price of consuming attribute i will reflect changes in all other attributes which would accompany a change in location increasing consumption of attribute i. The 'price' to a household of any one attribute is the compensating variation needed to restore the household to its former level of utility where, among all possible location changes increasing consumption of the attribute in question one unit, that location change requiring the least compensating variation was selected."

An amenity in this context means a desirable neighbourhood characteristic. Consider a simple amenity readily identified from the census data, such as newer housing. Assume (for theoretical convenience, not realism) it is centred at an amenity centre, it constitutes a gradient diminishing with distance from the amenity centre, and the residence-seeker has a significant preference for new housing. If the marginal utility of money were zero, making amenities the sole deciding factor, then the ray from the residence to the amenity centre would act as a gradient in a similar manner to the ray from the residence to city centre. It would be less easily specified, however, since it does not balance travel costs against another price quantity, but rather against the utility of the amenity. Nonetheless, the existence of amenity centres of varying significance provides an opportunity to examine circumstances where amenities may dominate over housing costs in the location decision (rather than just letting high variance indicate poorly performing gradients and leaving it at that).

A similar relationship follows if there is no amenities gradient – just specific areas of the city which are recognized as desirable, but no reason to expect any systematic decline outwards from them. Then, although there is no gradient, there is still the ray running from the amenity centre through the residence, from which we might infer some basic relationships. If the amenities gradient is removed from the price or wage gradient, it will have some tendency to draw the residence or workplace away from the latter gradient. This could be an effective rationale for the existence of offsets. And while no slope of gradient can be specified, it is possible for the amenity's draw to be sufficient to induce workplace/residence switching behavior. Also there may be major discontinuities to consider, such as slums directly adjoining affluent neighbourhoods, which could result in sharply anomalous relationships. This is especially likely to be a problem if one's analysis ignores major geographical phenomena such as rivers.

Strictly speaking, amenities do not represent a new category, since access to city centre can itself be considered an amenity (when defined as a location-specific good) which is the subject of Alonso's (1964) original analysis. Muth (1964) and Meyer, Kain and Wohl (1965) explicitly recognized non-access amenities, but without relating them to variations in land value. Richards (1963) was an early author who drew attention to the role of amenities ("residential preferences") in affecting land values and residential location, but without relating it to the Alonso model. In the same decade, amenities were included among the independent variables in regressions on land values, by Brigham (1965) and Crecine, Davis and Jackson (1967), but no attention was paid to the distinction between access and non-access amenities. Harris, Tolley and Harrell (1982) estimated amenities on a different basis, calculating their values as the difference between travel savings and residential land values at any given location. Diamond and Tolley's (1982a) *The Economics of Urban Amenities* provides an invaluable consolidation and extension of the research, and it serves as the primary reference for the current analysis.

Diamond and Tolley describe their first major amenity in terms of "environmental goods" which are available at a particular location. These might include "air quality, views, or local public services ... components of the social, physical or legal environment" (page 6) available in that location, all of which are "non-excludable, at least once access to the particular location is obtained". In labour markets they might include regional factors such as infrastructure, which affect marginal productivity. In other words, amenities are a form of public goods or externalities, and are likely to be "non-excludable once access to its location is gained ... [and also] ... not fungible (i.e., subject to transfer or exchange) across space" (Diamond and Tolley). There are those who have argued that externalities, rather than any Alonso optimizing dynamic, are the forces which shape cities – "The economic significance of cities lies in the external economies they provide" (Tsuru, 1963). Lampard (1963) suggests (paraphrased in Richardson, 1971) that "urban growth is the spatial manifestation of increasing returns to scale and external economies", which might equally apply to urban location patterns. Even Alonso (1964, page 12) gives due consideration to the proposition that "the value of land is of minor importance".

Diamond and Tolley proceed to build such amenities into utility functions and develop a model which can account for the dominance of amenities over spatial optimization factors, as do Cropper and Gordon (1991) in a closer examination of Hamilton's "required commute", which will be considered in Section 5.2. This study takes a simpler approach, by considering conditions under which amenities (in the limited sense of locations with independently desirable characteristics) might rival spatial optimization in its influence on location.[11]

Another aspect of the definition of an amenity is that of a public good, the availability of which is a function of location alone. Diamond and Tolley point out that a private good may also be an amenity (using the example of fine restaurants which are a function of location) – but they must still be treated like public goods because it is the quality of the restaurant that is the amenity, and this quality dimension is as non-excludable and non-fungible across space as any public good.

Nonetheless any private or public amenity possesses a marginal rate of substitution in relation to other goods, or other amenities, or to access itself. Diamond and Tolley also define demand shifters as, most significantly, "important determinants of the individual household's marginal valuations of an amenity" (page 9). Hence though the usual concept of demand does not apply to amenities, since consumption can be varied only through relocation, there is an "implicit amenity market" in the necessary purchase of residential property to obtain the amenity – and under proper market conditions the price of the residential property captures the value of the amenities associated with it. That is the basis on which Harris, Tolley and Harrell (1982) can estimate amenities as a form of residual value.

The crucial point is that an amenity does possess a marginal value derived from its

[11] It would fit in with the generalization thrust of this work to incorporate amenities in this manner, but Diamond and Tolley's work falls beyond the scope of this study.

marginal rate of substitution with other goods. However this marginal value is entirely derivative of the values of marketable goods. Including a vector of amenities into the utility-maximizing equation in much the same manner as Equation (3.1) above yields the proposition that "[t]he increment to the price of land times the amount of land purchased **is** the price of the amenity" (emphasis added) when holding the composite good constant, and the following conclusion. "It is not a fixed price. It varies with both the partial of the equilibrium land price function and with the quantity of land consumed", and it is weighted by the partials of each amenity and the location of the residence. These conditions contribute to the difficulty of estimating demand functions for amenities.

3.2.4 Further Analysis of Amenities

Amenities force a closer look at the question of price gradients, as they are associated with a set of indifference curves which may have complex spatial patterns which may be discontinuous or positive in various locations, particularly when multiple centres are combined with multiple amenities. Indeed, examination of spatial patterns may produce the conclusion that empirical analysis is futile at least without greatly more refined data.

In terms of equation 3.2, one way of incorporating housing and amenities would be as follows. If we identify a new vector, **n**, which represents the quantity of the amenity available as a hedonic utility index, and posit that n simply increases relationship with distance from city centre (e.g., fresh air) such that $\mathbf{D}n(\mathbf{u}) > 0$, then utility maximization can be represented as:

$$\max U(\mathbf{X},\mathbf{q},\mathbf{t},\mathbf{n}) \quad \text{s.t.} \quad (\mathbf{p_z}\,\mathbf{z} + \mathbf{c}) + (\mathbf{p_q}\,\mathbf{q}) + (\mathbf{p_v}\,\mathbf{v} + \mathbf{c}) + (\mathbf{p_l}\,\mathbf{l} + \mathbf{c}) = 0 \qquad (3.3)$$

Assume for the sake of a convenient initial simplification that amenities (such as fresh air) increase with distance from CC. We use $n(u_0)$, $\partial n/\partial u_0 > 0$. Amenities contribute to the satisfaction derived from the residence, thereby increasing the quantity of household services consumed. (Implicit in this statement is the assumption that housing constitutes a vector of attributes, and increasing the quantity of one of those attributes increases some measure of housing services as a whole.) Hence consumption of housing services is represented by $z_0(u_0)$, $\partial z/\partial u_0 > 0$. The introduction of this new relationship introduces the expression

$$-\frac{\frac{\partial Z_0}{\partial u_0}\left(p_0(u_0) - p_2(u_2)\frac{\partial U/\partial z_0}{\partial U/\partial z_2}\right)}{z_0} \quad \text{or} \quad -\frac{\frac{\partial Z_0}{\partial u_0}\left(p_0(u_0) + p_2(u_2)\overset{(-)}{MRS_{2,0}}\right)}{z_0} \qquad (2.8a)$$

into the right-hand side of equation (2.8). This accords with the intuitive suggestion that consumers will bid higher for housing which contains greater amenities which are reflected in a hedonic price index. But other possibilities arise. If the consumer is relatively satiated in goods (subscript $_2$), or relatively hungry for housing amenities, the MRS may become sufficiently negative to turn the whole expression positive. This will reduce the negative slope of the price indifference curve, flattening it such that u_0 will tend to be greater. In other words, if $\partial Z_0/\partial u_0 > 0$ meaning that amenities are greater further from CC, and consumers are hungry for amenities, they will move further out. On the other hand, if amenities such as desirable housing developments occur closer to CC, the result would be an inward tendency, again consistent with the mathematical results.

This introduces the possibility of enormous complications. To start with, there is no reason why a major amenity could not make the above term so largely negative that it turns the price indifference curve positive in slope. (See also Polinsky-Shavell, 1976.) One might imagine, for example, a price indifference curve which turns upward at a certain distance from CC and produces a multiple equilibrium. Similarly anomalous results could follow from, say, a rapidly-declining disamenity (negative externality).

In combination, such effects could also produce discontinuities. Imagine a set of two amenity curves, one declining from CC and the second declining from the edge of town, $\partial n_1/\partial u_0 > 0$ and $\partial n_2/\partial u_0 < 0$. Where they meet, the resident moves from the dominating influence of one amenity curve to that of the second. Just as a kinked demand curve will produce a discontinuous marginal revenue curve, this kinked amenities curve may well produce a discontinuous price indifference curve, with the possibility of unpredictable jumps in behavior (added to the previous possibility of multiple equilibria). As if it were not complex enough already, there is also the fact that all TL, both positive and negative, are influenced by the choice of CL, and may exhibit their own corresponding unpredictability.

On the other hand, distinct amenities, such as a major theme park or a parkland beautification project, which would be clearly location-specific, may enhance the predictability of the model as amenity-pull is balanced against transportation cost factors – if distinct data is available on amenities.

All of this rests, incidentally, on the questionable assumption that we are considering a single-valued amenity. Should we later widen the context to consider a full vector of amenities, there would be every reason to expect that various amenities will vary in different combinations as choices are examined, and there is no such thing as a single definable marginal utility.

The relationship between the old and new terms of equation (2.8) and (2.8a) could support some interesting observations. For example, if a consumer is (in a sense) satiated with time, such that the marginal utility of commuting time is relatively low, combined with relatively low levels of transportation costs, all of the terms of the original equation (2.8) would have low values. Hence we might expect the above additional term (2.8a) added to equation (2.8) to be relatively powerful and perhaps dominate the decision. In other words, people working only part-time and travelling by bus may, for example, might tend to respond more to amenities than to the traditional land-price/travel-cost dynamic.

In practice, urban amenities are not likely to resemble any of these simplified patterns, and it is necessary to sketch the rough pattern of amenities within a city, such as regional centres or neighbourhoods, in addition to CC, to determine the influences involved and see how they counteract the forces represented in equations (2.8) to (2.10) and how they interact with each other. It would also be necessary to specify multiple sets of amenities and to define boundaries of regional influence. Possibly with enough controls to account for amenity patterns, the relationships of the core equation (2.6) could be observable.

A graphical representation of the underlying relationship is shown in Figure 3.1 (see Diamond and Tolley, 1982, page 16). As noted above, the existence of an amenity may render city-centre relatively undesirable such that, for people who value the amenity highly, the bid-rent curve may turn downward toward the origin as illustrated in curves U_1 and U_2 (where U_3 is a normal convex bid-rent curve). Disregarding the equilibrium problems that may occur in the case of concave indifference curves, the particular configuration of bid-rent curves in Figure 3.1 result in (for example) the individual of curve U_1 becoming the dominant bidder in the region u_2 to u_3. The overall result is a set of regions in which the holders of specific amenity preferences may be separated out into identifiable regions along the ray from city centre. It would take fairly fortuitous preference sets to produce any practically identifiable regions, but the principle is instructive.

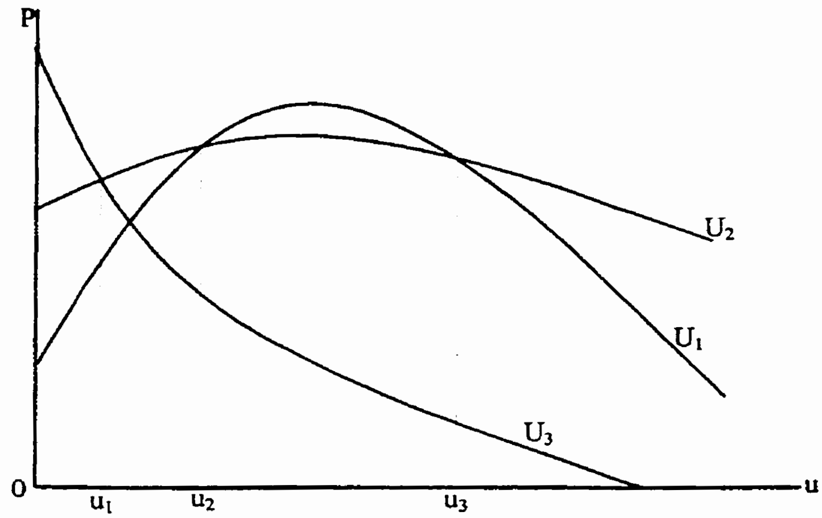

Figure 3.1
Segregation Due to Amenities

In terms of the original Figure 2.2 (allowing the indifference curves to revert back to convexity for simple analytical tractability), this could mean that there may be several alternative U curves representing different amenity preference sets. Hence it might be possible in principle to specify two or more TL positions which may be mappable to different preference groups – or at least some tendency for those with certain preferences to live further from (or closer to) city centre.

3.2.5 Measurement Issues

Measurement of demand functions for a specific amenity have been derived from a function of whatever implicit amenity price vector can be inferred from available data, plus income – commonly on the assumptions that lot sizes are freely variable and there is no interaction between amenities[12]. Harris, Tolley and Harrell (1992) regressed income, family structure and housing characteristics (single vs multiple) on their residual measure of a composite amenity, yielding significant coefficients. Diamond and Tolley present an ad hoc numerical simulation of a two-amenity model (1992a) to produce a consistent set of values and to illustrate some of the effects of demand shifters. Meanwhile, Freeman (1979) pointed out that if care is not taken to ensure that the utility function and the hedonic price function do not take similar functional forms, the result may be a perfect spurious correlation between amenities and their implicit prices with no variation remaining for income and other variables – a problem which remains substantially unresolved to this author's knowledge.

A similar problem could arise if amenity variables are entered into a regression on distance to city centre based on the Alonso model, for two reasons. First, if the value of amenities rise in a systematic manner with distance from city centre (newer housing, better schools, fresh air), while price declines in a systematic manner with distance from city centre, another spurious correlation could result. The relationships among shifting variables which follow from equation (2.8a) may therefore not be testable by this method,

[12] Diamond and Tolley (1992a) list numerous empirical studies on pages 19-21.

and this may be the reason for the lack of research along these lines. Second, if housing price is indeed a hedonic index, then amenities will be effectively captured in the price, and as regressors they would likely produce spurious correlations. Hence any use of regression analysis would require clear evidence that (a) amenities are uncorrelated with distance, and (b) that any amenity considered for regression is not significant in the hedonic price function. These issues will be discussed again in Chapter 8.

Rosen proposes a technique (page 50) for estimating hedonic prices, after a careful examination of the potential identification problem, in a manner essentially as follows.

$$p_i(z) = f^i(z_1,\ldots,z_n, Y) \qquad (3.4)$$

where z represents the commodity whose demand or supply is being estimated, and Y is a vector of characteristics comprising the hedonic price index.

Diamond and Tolley employ the same basic technique, recognizing however the underlying assumption of "freely variable lot sizes"[13]. However, they do acknowledge the impact of relaxing that assumption – namely discontinuities rendering the choice set between amenities and lot size incomplete. However, it still allow ceteris paribus examination of individual amenities (page 20) with all the qualifications mentioned in Section 3.2.2.

[13] Referencing the long line of previous works that have essentially made the same assumption.

The existence of strong amenities or disamenities essentially produce a system of counter-influences to the effects outlined above, and may actually render them irrelevant. One approach is to deal with them as separate research topics. This paper attempts to deal with them within a common analytical model. It would seem relevant to develop a test of whether amenities or market effects dominate the decision, just as Simpson's analysis (1992) tests whether direct effects or search effects dominate the residential vs workplace-choice decision in certain circumstances. Perhaps if poor responsiveness is demonstrated with regard to transaction locations in both the direct and search context (even considering the effect of how strong amenities flatten indifference curves), it can be followed by an analysis to see if their location decisions correlate to amenity-based geographical reference points instead. However, if consumers are responding half-and-half to both of these factors (or to three factors), the empirical results will end up incomprehensible unless there is refined enough data to distinguish the reasons for their choices. The survey may be useful in this regard because of its inclusion of preference questions.

When it comes to testing whether the residence-seeker is responding primarily to price or amenities, the data becomes somewhat difficult to sort out. Simply a lack of indication of movement, in a direction that would suggest price sensitivity, is insufficient evidence – since that can be governed by either a partial response to search criteria (and density), or it can be an indication that a neighbourhood was chosen for its amenities regardless of price or search.

4. UNCERTAINTY AND SEARCH

The question of uncertainty was addressed as far back as Arrow (1959), when he noted that the assumption of perfect knowledge of a known price precluded a process for arriving at that known price through market interactions. There must be at least one agent who is somehow acting as a price setter, or the known price will never be set. That one agent, or subset of agents, must by definition at some point be facing an indeterminate price, i.e. facing uncertainty. Focussing on the bargaining process among price setters, and later (Arrow, 1974) on how this process works itself out in labour markets, Arrow set the stage for the examination of costs of information and raised the spatial implications of searching for an appropriate price.

Stigler (1961, 1962) is credited with carrying forward the formal analysis of equilibrium at the margin where the marginal benefits of search equal the marginal costs, made possible by the assumption of a known probability distribution of unknown prices (or, in this instance, wages). This led to consideration of practical procedures for conducting the search to find such an equilibrium, which took the form of a reservation wage governing an optimum-stopping rule (McCall, 1965), and the sequential search model took shape.

Further research has determined that the distribution of prices need not be fully known (Gastwirth, 1976), and that the optimum-stopping rule must be made a little more sophisticated (reservation prices either flexible or combined with other rules) to produce an equilibrium solution.

There has been some limited exploration of spatial aspects of this search before Simpson (1992). For example, Feinberg (1977) considered the relation between travel costs and search duration. Simpson (pg 54 ff) made the relationships explicit with reference to a known distribution of wage offers and a known offer arrival rate, incorporating search cost and explaining why (1) closer jobs are preferred ceteris paribus and (2) the search distance increases over time until employment is found. He then incorporates the model into the island paradigm which has become well integrated into macroeconomic theory (more detail in Section 4.2), to further specify "the spatial search strategy of workers" and in particular to isolate the impact of skill acquisition on location as a result of search strategy (in contrast to its impact as a result of cost optimization).

In this study, section 4.1 begins by discussing densities, gradients and costs and then extending the initial generalized analysis of Sections 2.1 to 2.3 incorporating market density gradients and search costs into the general model, with particular attention to the effect of special quality variables, producing an extended general analysis. The section also considers the generalized constraint of the model, which provides a direct link to integral probability analysis.

But it is not until section 4.2 that the specifics search strategy are considered in detail, and the role of the reservation price/wage are examined. The purpose is not only to give further examination to Simpson's differential impact of travel costs and search on commuting behavior, but also to link a reservation price analysis which evolved from a standard Alonso model back to the more generalized model of this study – i.e. to represent reservation price analysis in the preference space of Figure 2.2.

4.1 General Relationships

4.1.1 Market Density and Search Costs

Considering the element of uncertainty, if the implicit assumption of perfect knowledge is relaxed, the individual seeking to buy or sell a commodity must devote resources to obtaining sufficient information to make as optimum a decision as possible. We introduce two new but related variables — market density and search costs.

An ordinary commodity may be consumed in varying quantities at any point along the gradient. One may define market density in terms of commodities available relative to the consuming population at each point. But there are an prohibitive number of points involved. It would be more meaningful to divide the urban area into a manageable number of geographical regions, and group clusters of points into neighbourhoods. Then market density is a measure of commodities available in each neighbourhood. Such an approach also requires the simplifying assumption that costs refer to inter-neighbourhood travel, and intra-neighbourhood travel is costless. It is notable that the result provides a theoretical basis for the island model to be discussed in Section 4.2.3.

The precise definition of market density can take a number of forms. It can be the quantity of commodities for purchase or sale in the neighbourhood divided by the population of the neighbourhood. That may defeat the purpose of the analysis, since we are concerned with decisions regarding whether or not to enter the neighbourhood in order to engage in the transaction. One alternative is to measure the proportion of total urban quantity of the commodity which is available in the local neighbourhood. A second alternative is to measure the quantity of the commodity which is available in the local neighbourhood divided by the number of square miles in the neighbourhood. In the case of housing, this is a common perception (and a common measurement) of the meaning of housing density. A third alternative is to examine the extent of market shortfall. Shortages or surpluses may best anticipate the offer rate which the buyer or seller of the commodity can expect to encounter along the density gradient. In fact, it is because the offer rate is the most relevant factor in the search decision that the third alternative is the superior measure of density.

The categories most relevant to search analysis are housing and jobs.

If we are to include housing and other commodities with similar characteristics, we must examine our concept of a density gradient in light of their distinguishing characteristics. An ordinary commodity may be consumed in varying quantities at any location along the gradient. It is meaningful to define density in terms of commodities available relative to the consumers at each location. At first glance, housing is different if single residence is contained at any given location in space. Hence the housing density at each point is precisely one, one residence per location, and is often in fact simplified down to a unit of land.

As discussed above, one way to interpret densities more meaningfully is to allow each location along the ray to represent a set of residences (neighbourhood) each containing a bundle of housing units, each one itself containing a bundles of housing services. A complementary technique is to recognize the hedonic quality of housing prices. Housing quantity represents a hedonic index for the bundle of housing services available, and the density of housing services at each point along the ray will vary accordingly.

Since data on hedonic indices are unavailable, two practical proxies are available. One can use the number of housing units (single-occupancy buildings, or individual apartments in multi-occupancy buildings) available per square mile of land in the neighbourhood in question. Alternately one can use the vacancy rate for housing units in a given neighbourhood. In the present study, data are available for housing units per square mile which, while not as direct a predictor of the offer rate as housing vacancies, still provides a meaningful enough measure of density.

Job density is open to more direct measurement. In fact, jobs per square mile is not particularly meaningful. But data are commonly available to measure jobs per worker available. This links to the employment rate, which is a strong predictor of job offer rate, and hence a strong link to search theory. Simpson (1992, Section 4.3.1) uses a ratio of jobs available to locally available workers.

4.1.2 Search Costs

Let us now consider costs by introducing two kinds of search costs: *activity costs* and *travel costs*. Assuming that the market density gradient declines with distance from city centre, the individual seeking to buy or sell a residence or other commodity can therefore expect to receive a greater number of price quotes closer to city centre, and hence can reduce activity costs (search with less effort) by searching in the direction of city centre. This effect, in its utility aspect, is considered in the next section.

Bringing in the question of special commodity qualities, this constitutes a search for a narrower range of characteristics. Whether it stems from human capital theory, seeking to recoup an investment by restricting the range of offers considered acceptable, or because the narrower boundaries of skill require a further search to achieve the same offer rate – the result is a greater extent of search, in order to achieve the same results. In other words, whether searching for family-specific housing or skilled work, local opportunities are less likely to be sufficient. Even general training is to some degree occupation or industry specific, narrowing the terms of search. Various studies (from Becker, 1964, to Denslow and Eaton, 1984) document the greater market radius associated with more skilled workers. If search activity produces a tendency to search towards CC, then the quality factor extends that search further inward.

If there is a general tendency to search inward ceteris paribus as a result of uncertainty, then the travel cost of search will increase on average as the commodity search extends toward city centre. The greater the extent of search toward city centre, the higher the search travel costs. Hence search travel costs have the opposite impact compared to activity costs. They will have an outward influence. Search travel costs are in fact a primary element of the "costs of search" referred to in the following sections.

4.1.3 The Utility Maximizing Model

Search behaviour can be incorporated into the utility optimizing model, extending equation (3.4) as follows. (As a reminder, note that equation 3.4 introduced the vector $\mathbf{n}$ to represent amenities.) The utility function now incorporates the disutility of search activity (represented by $\mathbf{s}$), the requirement for which increases with $\mathbf{u}$. Travel costs of search for any given commodity are identical to travel costs of purchase, and are already incorporated in $\mathbf{c}$. The maximization problem becomes:

$$\max U(\mathbf{X},\mathbf{q},\mathbf{t},\mathbf{n},\mathbf{s}) \quad \text{s.t.} \quad (\mathbf{p_z}\,\mathbf{z} + \mathbf{c}) + (\mathbf{p_q}\,\mathbf{q}) + (\mathbf{p_v}\,\mathbf{v} + \mathbf{c}) + (\mathbf{p_l}\,\mathbf{l} + \mathbf{c}) = 0. \tag{4.1}$$

and as before it remains

$$\max U(\mathbf{V},\mathbf{t}) \quad \text{s.t.} \quad \mathbf{p}\,\mathbf{V} + \mathbf{c} = 0. \tag{4.2}$$

with the general solution represented by equations (2.3) and (2.6).

However, search activity is a little more complex than just the disutility of search effort. In a spatial context it involves a decision whether or not engage in an additional unit of search by investigating prices at an adjacent location, which depends on the marginal benefits and marginal costs of extending the search in this manner.

Looking at search in terms of the optimizing model in equation (4.1), the solution equation in the manner of (2.6) now becomes

$$\mathbf{DU}(\mathbf{t}^*) + \mathbf{DU}(\mathbf{s}^*)\bullet\mathbf{Ds}(\mathbf{u}^*) = \lambda(\mathbf{V}^*\mathbf{Dp}(\mathbf{u}^*) + \mathbf{Dc}(\mathbf{t}^*)) \tag{4.3}$$

and

$[\mathbf{DU}(\mathbf{t}^*) + \mathbf{DU}(\mathbf{s}^*)\bullet\mathbf{Ds}(\mathbf{u}^*) - \mathbf{Dc}(\mathbf{t}^*)] / \mathbf{V}^*$ is tangent to $\mathbf{Dp}(\mathbf{u}^*)$.

In comparison with equation (2.6) the indifference curve in the positive panel of Figure 2.2 becomes more steeply negative, creating a tendency for u to be smaller with the introduction of search. Considering the negative panel of Figure 2.2, where the most relevant application is job search, the same tendency in this case towards a more steeply positive indifference curve also results in smaller values for u when search is considered.

4.1.4 The Search Constraint

If we consider the utility maximizing equation (4.1), the addition of search activity also requires an equilibrium constraint. The number of commodities found through search must equal the number of commodities available – or what is the same thing expressed in terms relevant to the price gradient, the probability of finding the desired commodity at location u is equal to the market density of the commodity at the same location if the market is to be in equilibrium at all locations. Although in a sense this is just a market-clearing condition and hence implicit in the system represented by equation (4.1) itself, such a constraint reflects search specifically and provides a valuable analytical tool in partial equilibrium to isolate the effect of search activities.

The generalized search constraint can be represented as in Simpson (1980) by

$$\int_0^r Df(p,u)du = N. \qquad (4.4)$$

The integral is calculated over values of u (distance from city centre), but only those values which fall within the distance 0 to r from u, where r represents the maximum distance that the marginal individual at u will search for the commodity in question. Equation (4.4) means (in the context of prices in general) that at equilibrium the number of commodities sold (D) equals the number of commodities bought (N) – because the sum of probabilities that any given commodity will be bought at any given price at any given location, namely f(p,u) where p = price and u = distance from city centre, must equal 1. The context is much simpler in the case of job search, at least if it is assumed that each individual worker has a single job, because then it means that the sum of the probabilities that each job will be filled must equal 1.

At any given location u and price p, if equilibrium occurs at every location, it can be represented by

$$f(p,u) = d_u \qquad (4.5)$$

indicating that the probability of finding the commodity equals d_u (market density at location u).

Or it can be represented by

$$F(p,r) = d \qquad (4.6)$$

for the gradient as a whole, indicating that the marginal individual is the one who must travel the maximum distance, r, to buy or sell the commodity.

4.2 Search Models in Detail

4.2.1 Sequential Search

In order to relate this analysis to modern search analysis, it is necessary to discuss key aspects of sequential search behavior. The search model employs a known probability distribution of prices with an unknown mean, a known price arrival rate which measures the probability of a price quotation arriving in a given time period. The marginal benefit of search is derived from the probability of receiving a higher price offer if search is undertaken, while the marginal cost of search is the transportation cost of obtaining additional prices in an adjacent location.

The sequential search model in its simplest form has the individual selecting an optimal price which represents the equality of the marginal expected benefit of another unit of search to the marginal cost. The analysis begins with Stigler (1961,1962), with a known wage distribution (only) and the ability to calculate probabilities. Search yields successive units of information (wage offers), but the wage improvements over the last offer (marginal benefits) are diminishing. Comparing marginal benefits to marginal costs yields the ability to calculate the number of search required, and the expected best offer. McCall (1965) analyzed how the expected best wage can lead to a reservation wage, from which can be derived an optimal-stopping rule[14].

[14] More realistic complex rules, such as truncation (Gastwirth, 1976), length of search (Stigler, 1962) and job application rules (Morgan, 1983) contribute to realism, but do not essentially alter the analysis below.

At this point we are considering prices in general. Looking more generally at search models – given a knowledge of the price distribution, a price density function f(p) with a variance greater than zero can be employed to determine the optimal stopping point. Search is conducted in units of time and begins by yielding a series of successively lower prices, which at some point falls below the maximum acceptable price (*reservation price*), above which no purchase will be made. For each step in the search, when a lower price is available, the marginal benefit may be measured as the further decrease in price below the reservation price yielded by the last step, which may be compared to the additional cost of making that last step. Marginal benefits are assumed to decline in the relevant range as search progresses, and marginal costs to increase (noting that these conditions arise from the variance of the density function and diminishing returns to search, **not** from travel costs which have not yet been introduced in this introduction to the model). The whole series of steps leading to equilibrium (the first accepted price) may be represented as

$$\int_{p^*}^{0} \left[p^* - p\right] f(p)dp = mc \qquad (4.7)$$

where the marginal cost of search is positive as the searcher seeks a declining price below reservation price p^* (hence $p^* - p$). We may also say

$$\int_{w^*}^{\infty} \left[w - w^*\right] g(w)dw = mc \qquad (4.8)$$

where the marginal cost of search is positive as the searcher seeks a rising wage above reservation wage w^* (hence $w - w^*$). This is applicable because the wage is a negative price.

We now introduce the extended density function of equation (4.5) into the above relationships – namely f(p,u).

$$\int_{p^*}^{0} [p^* - p] f(p,u) dp = mc \qquad (4.9)$$

We assume that a downward-sloping density gradient from city centre exists, such that $\partial f/\partial u < 0$, which also implies that the price arrival rate when engaged in search increases with proximity to city centre. Similarly

$$\int_{w^*}^{\infty} [w - w^*] g(w,u) dw = mc \qquad (4.10)$$

where $\partial g/\partial u < 0$ and the wage arrival rate increases with proximity to city centre.

Extending the relationship to account for economic conditions – if the probability of a price offer arriving is the same as the rate of interest for the same time period, indicating an equal payoff for alternative expenditures, then the above calculations will be optimal. But if the former is higher than the latter, then putting resources into searching for a suitable price yields a proportionally higher return. Hence the marginal benefit of search is higher in the same proportion, and a higher marginal cost must be incurred to restore equilibrium. Let us indicate this ratio – the price arrival rates divided by the rate of interest – as λ. Then equation (4.9) and (4.10) are special cases where $\lambda = 1$, and should be more precisely represented as

$$\lambda \int_{p^*}^{0} [p^* - p] f(p,u) dp = mc \qquad (4.11)$$

and $\lambda \int_{w^*}^{\infty} [w - w^*] g(w,u) dw = mc$. (4.12)

Assume now that search is conducted spatially, and a marginal unit of search is a marginal unit of distance toward city centre (reducing u). Then if density is rising toward city centre at each step, the left-hand sides of these equations will rise as λ rises, and a higher marginal cost will be justified at equilibrium. The search will be extended toward city centre, comparing the diminishing marginal benefit per unit of travel to the rising marginal cost per unit of travel until the optimum is reached.

But the optimum may not be achieved, if the marginal benefit is not initially sufficient to justify search because the reservation price cannot be met. However, there are effects over time which alter the reservation price. It is not uncommon for an individual to begin with certain expectations about relatively local opportunities, and then discover that further search with a less restrictive reservation price will be necessary. Simpson (1992, page 62) refers to this in the job search context in reference to unsuccessful search. Another factor is urgency, to find a house or a job, which may over time require the individual to relax the reservation price requirements. This is more likely to occur as a declining reservation wage in the search for a job, as noted by Mortensen (1986, pg. 859) in reference to a liquidity constraint.

Whatever the source of the increasing reservation price, it increase p* in equation (4.11). The result is to increase the marginal benefits on the left-hand-side. If the reservation price continues to increase, eventually marginal benefits will be sufficient to meet marginal costs, and an optimum will be achieved. In other words, the rising density λ may not be sufficient to ensure an optimum, and a rising reservation price over time may be required for search to be successful. Much of the analysis of search activity has been conducted in terms of job search, so that its most common expression involves a reservation wage which continues to decline until a successful search optimum is achieved.

4.2.2 Searching in Preference Space

This study extends the analysis into the preference space of Figure 2.2 in Section 2.4. If all price quotations are known to be identical, and the probability of another identical price offer arriving in a given time period is known to equal one — then the marginal benefit of search is zero, and the reservation price equals that single price, which is illustrated by point A in Figure 2.2. In other words, point A is optimal in the usual MB = MC sense. Point A is in the positive panel of Figure 2.2, applicable to positive prices, but the analysis will be extended to cover negative prices (specifically wages) when Figure 4.1 is examined in detail below.

If each point on the price gradient contains a distribution of prices rather than a single price, that in itself is insufficient to create movement away from point A, if every other point carries the same expectation of receiving a higher price offer as does point A. In that case, the marginal benefit of search according to equation (4.9) may be positive and justify the marginal costs of search, but that search will occur at point A. The marginal benefit of spatial search, however, remains zero because the offer arrival rate does not increase along the gradient (in spite of the fact of a price distribution). It is no greater than the offer arrival rate obtained by remaining at point A, and hence the optimum remains located at point A. However if the offer arrival rate increases with proximity to CC, then the expected marginal benefits will increase in the direction of CC in accordance with equation (4.9), justifying extension of the search for a residence until cut off by the further increase in search travel costs. In other words, the CL or RES will move inward towards CC.

The analysis around Figure 4.1 attempts to relate the sequential search analysis of this chapter to the preference analysis of Chapter 2 and Figure 2.2. In the upper panel of Figure 4.1, consider an optimum at point A, the tangency of U_0 and p(u). Note the inclusion of a shallower **expected** price gradient $p(u)^e$ to incorporate the effect of greater price offer density closer to CC (i.e., rotated around point A), because search yields lower prices, which produces a new optimum at point B. Point B will occur to the left of point A for normal sets of indifference curves, resulting in a smaller value of u. U_T (lying between U_0 and U_1) is an unusual addition which will be considered below. The same analysis applies to points C and D, representing workplace choice, and the effects occur in the same direction. In this case $w(u)^e$ becomes steeper as it is rotated around point C because search yields higher wages, resulting again in a smaller value of u.

In order to relate the analysis to reservation price, let us consider when transactions may not occur in the immediate location, and when search may or may not be rational.

Normally, a purchase does not occur if the marginal utility is not sufficient to justify the expenditure, which can be depicted as a corner solution in a graph such as Figure 2.1 (e.g., see Varian, 1990, page 76).

An individual consuming none of a commodity will of course have no preferences as to location and hence, considering the top panel of Figure 4.1, we might say that the indifference map which would produce the solution at point A does not exist. However, we cannot represent this by simply specifying a maximum reservation price below the level of point A and say that solution A does not exist. Point B would not be attainable either, because its price is even higher. But point B is part of a different consumption set, with lower commuting costs in the normal configuration, and it is entirely possible that point B is an acceptable optimum lying below a reservation price applicable to that particular location. Each location with its full set of costs and benefits dictates a different reservation price to equate marginal benefits with marginal cost. At the improved terms of trade implied by $p(u)^c$, a higher level of utility (U_1) can be achieved at point B, and hence the consumer is willing to pay a higher price – i.e., the reservation price for that location is higher. One possible illustration of this relationship is shown by the curve U_T in the top panel, describing the set of reservation prices at each distance u. Its shape and location as drawn is fortuitous, but it indicates a situation where point A is not a consumption option but point B is a consumption option. The simple concept of a reservation price must be expanded in the following manner. The shape bounded by the axes and the curve U_T represents the set of acceptable consumption options. Point A lies

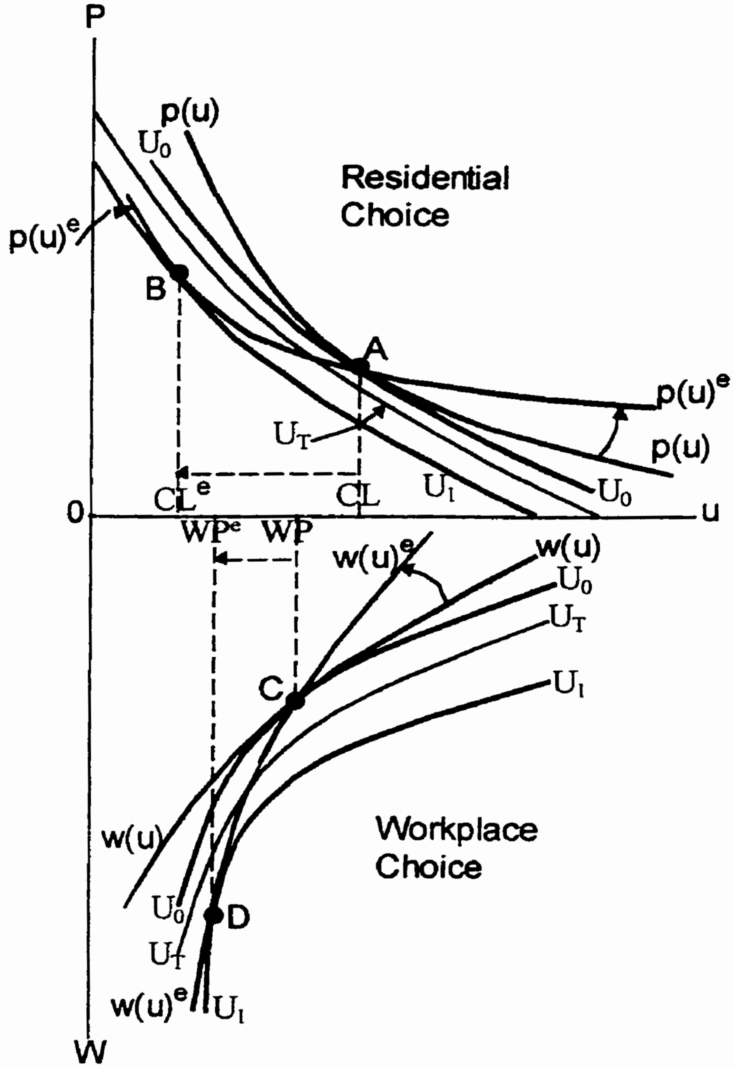

Figure 4.1

Reservation Prices in Price-Distance Space

outside of the acceptable set, but the option of search yields point B which lies inside the acceptable set. It remains possible that U_T, drawn differently, would fall even below point B, rendering it also a zero-consumption option, where it may remain indefinitely (over all values of time t). However time may work to shift U_T upward (it is still useful to say that the reservation price may increase) expanding the acceptable set until a transaction becomes possible at point B.

An alternative way to describe the configuration of Figure 4.1 is to say it represents an individual who is currently not consuming some commodity which is only minimally lacking in marginal utility at point A. That is, there may be sufficient marginal utility to justify consumption if the price were a little lower, or if a more acceptable price were obtainable through search (considering the marginal benefits and costs at all locations). In other words while point A does not exist in Figure 4.1, the existence through search of an expected price gradient $p(u)^e$ creates conditions which may allow point B to exist. The latter is most likely to occur if one or more of the following conditions are present: the deficiency of marginal utility is quite borderline relative to alternative commodities resulting in a high reservation price, the density gradient (determining offer rate) is quite steep, the price gradient is shallow with a significant variance, and/or the travel costs are low.

This allows for the possibility that there may be failure of search for a commodity which is either not available or overpriced locally but has a significant marginal utility, because one or more of the following conditions prevail: the density gradients is not steep enough, the price gradient is steep or exhibits little variance, and/or travel costs are too high – to justify search. As a result no transaction occurs in spite of the apparently strong preference for the commodity.

A similar analysis applies to wages. The reservation wage is represented by U_T in the bottom panel. That is, the set of acceptable jobs is represented by the area which lies beyond the curve U_T. This does not include the point C, on the wage gradient w(u) which does not include search, and hence the job at C is not acceptable in that the reservation wage is not met. However, with search, and the steeper wage gradient representing better terms of trade, the point D is attainable which lies within the acceptable set. However, if U_T were to lie further out from the origin, such that even D is does not meet the reservation wage and is hence not acceptable, then a suitable job will not be found for as long as these conditions attain. However, should the reservation wage fall over time, U_T will shift downward toward the origin, and the job at D will become feasible. The overall result is that search may make finding a suitable job possible in the short run (or over time with a declining reservation wage), and causes the job seeker to find a job closer to city centre than would be the case if a job were findable without search.

Figure 4.1 also allows us to examine the effects of changes in various parameters. There are two kinds of price changes to consider. In the upper panel, a simple rise in prices, say a parallel upward shift in the price gradient, would have no significant effect on the relationship between points A and B.

However, a reduction in the price differential between CL and CC would be associated with a shallower price gradient. This would tend to cause a downward rotation of both p(u) and $p(u)^e$, and move point B further to the left. In other words, search behaviour (in addition to optimizing behavior) will produce a greater move towards CC when the price differential is lower. Also, given the price gradient, a steeper density gradient will increase the expectation of finding a lower price and move $p(u)^e$ even closer to CC. Finally, in examining direct effects on point B, we can expect that lower travel costs will make any improvement in market density more beneficial, and will induce greater search, again moving the optimum towards CC.

4.2.3 Job Search and Islands

Let us now move from the general analysis of commodities with special qualities to the specific category of job skills in the labour market where some practical predictions can be entertained. Considering the choice of workplace, where skill is the primary qualitative factor, some expectations can be summarized about skill levels and location before proceeding. If higher skills bring higher wages, and commuting costs are the greatest priority, then we should expect distance to CC to increase with increasing skill levels by workers who can afford the lower wages that buy lower travel costs. On the other hand, more skilled workers are seeking a narrower range of jobs. Workers who are motivated by search find it necessary to search further in order to find them, and they can be expected to be less sensitive to local labour market conditions. They will look further afield for jobs. In the case of normal placements, that would mean looking further inward toward CC. In other words, where search is the priority, we can expect distance of job from city centre to decrease with increasing skill levels.

The central point is that the requirements of optimizing travel costs and the requirements of search cause skill enhancements to work in opposite directions – a testable proposition.

We now examine applications which arise out of the analysis of housing and neighbourhoods in Chapter 3. When examining housing in that section, it proved useful for the purposes of specifying a housing unit to group residences into neighbourhoods exhibiting zero intra-neighbourhood travel costs and positive inter-neighbourhood travel costs. A similar practice has also proven useful for the purpose of specifying imperfect market conditions. If the assumptions of perfect costless information and a continuous homogeneous locus of market transactions – such that all transactions can be treated as if they occurred at a single point of perfect knowledge – are dropped, then the necessity to consider search requires practical analytical measures to capture realistic conditions. An invaluable and convenient technique, pioneered by Phelps (1970), is to partition the market into distinct areas (called islands) which themselves internally display the ideal characteristics of costless information and perfect surfaces, but between which communication is imperfect and costly.

This bears some relationship to the earlier analysis in which equilibrium is described in terms of the number of households being equal to the availability of housing (where residents are indifferent regarding location). Writers like Wheaton (1974) applies the analysis to an entire city, but it can just as well be applied to neighbourhoods or islands (within which residents are indifferent as to location, often because intra-island travel costs are assumed to be zero). Suh (1990) internalizes the analysis, developing a set of unitary sub-regions, calculating supply and demand equilibrium for each of them, and calculating the commuting matrix which achieves equilibrium while minimizing total commuting. The earlier Alonso-type analysis can only consider discrete locations in terms of discontinuities on the plane, where consumers move to the first available point of greater utility. When the plane is divided into islands, a Wheaton-like analysis can be applied within the island, and an Alonso analysis between the islands. This can be compared to what Simpson (1992) does in a manner similar to the analysis above, as he adds inter-island travel and market density considerations into the analysis. In other words, the current analysis permits greater generality, in that it can be applied equally to continuous gradients or to a discrete set of islands.

Since the island model was developed specifically with labour markets in mind, we shall focus on the negative pricing aspect of the model and refer to wages and job seeking.

Lucas and Prescott (1974) developed a model where even under conditions where expectations adjust to clear the system of markets as a whole continuously, such that no overall deficiency of aggregate demand can arise, the statistical variance of that demand is not distributed uniformly among islands. As a result, islands will exhibit excess or deficient aggregate demand which nets to zero overall. As a result a certain number of islands will receive shocks of deficient aggregate demand, and adjustments are required to transfer the labour supply from islands of excessive to islands of deficient aggregate demand. This creates an incentive for unemployed workers to migrate to another island, but it also creates an incentive for employed workers to seek higher wages. With the additional assumption of autocorrelated demand shocks, the excess demand for labour may be expected to persist over time on a given island, and hence there is also an incentive for employed workers to seek out islands with high demand. The possibility of employed workers engaging in job search were used by Lucas and Prescott to model a natural rate of unemployment for a competitive system as a whole. Our purpose in citing their work is more to model the incentives which are the basis of job search under conditions of imperfect knowledge and significant costs of travel.[15]

We differ from the Lucas and Prescott treatment, following Simpson (1992, Chapter 3) by postulating systematic search, price gradients (which are discrete given the nature of islands) rather than random distributions, density gradients which allow systematic specification of price arrival rates, and a travel cost function which implies a rising marginal cost of search.

First, we may represent the density gradient by the series $\lambda_0 < \lambda_1 < \lambda_2 \ldots$ in equation (4.12) corresponding to job densities in a series of successively distant islands in the direction of city centre. As above, the optimum is achieved when the reservation wage is lowered sufficiently to achieve an optimum on island i with density factor λ_i. However, since the model is based on stochastic variations within the price and density gradients, there cannot be expected to occur a simple smooth tendency to seek higher arrival rates closer to city centre. Rather, especially if workers are aware of the actual wage distributions spread perhaps erratically over the set of islands, there may be a spread of chosen job locations among nearby and central islands with only a vague tendency to follow the gradient.

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[15] Other extensions include Phillips Curve generation (Sanotmero and Seater, 1978), cyclical phenomena (Jovanovic, 1987) and unemployment at the system level (Rowe, 1987).

The most productive avenue of extension is to allow for quality variations, as discussed in general in Chapter 3. In the island model as developed by Simpson (1992, 4.4.1), the quality variable is the general skill level of workers. The impact of skill on search, however, is not straightforward. Considering skill acquisition as an investment in human capital for the purpose of obtaining a higher wage, we would expect more skilled workers to exhibit a higher reservation wage. In addition, people receiving or expecting a higher wage, having generally higher opportunity costs, attach a higher value to commuting time and hence the marginal cost of travel is perceived to be higher (back to Stigler, 1962). This leads to the question of how the direct optimizing effects of the model may be altered and perhaps reversed by the introduction of search. If direct optimizing behavior is the determining factor, then higher marginal costs of travel result in a choice of shorter commuting times, creating an outward tendency toward the residential location and away from city centre (given normal placements). To the extent that they engage in search at all, they engage in less of it because the cost of search time is valued at a higher rate, and any tendency toward the denser city centre to sample more job offers is attenuated. If search behavior is the dominating factor then skilled workers, to the extent that they are seeking jobs within their field of specialization, are restricting themselves to a narrower range of jobs and hence must expect a lower job offer rate as they search – unless they extend their search further inward to find a higher job density. This constitutes a movement away from the place of residence, and greater commuting, in opposition to the direct optimizing effect.

Much evidence on the direction of the effect is not terribly conclusive. Often the research focuses on the income differences that are associated with skill differences. But Duncan (1956) finds more-skilled non-manual workers in Chicago commuted greater distances (in apparent opposition to the direct optimizing effect). This is confirmed, or at least not strongly contradicted, by studies which focus on the reasonably-correlated income factor (Kain, 1985 for Detroit; Simpson, 1987, for London). There is evidence however that skilled workers employ more formal (Macredie, 1972) and more efficient search techniques. Not only are more effective formal job-hunting techniques more normal and available to them, but also their higher valuation of time spent searching serves as an incentive to employ more effective techniques – which counteract the lower density of job offers. It would seem then that the improvements in search techniques may counteract and perhaps nullify the search effect by reducing the need to seek density, except that more formal search techniques also facilitate longer searches at lower per-unit cost. Simpson (1987) analyzes these factors in terms of the "mix" of informal and formal search procedures, concluding that skilled workers alter the mix to facilitate more extensive search. Overall, the evidence cited above may give some tentative support to the search effect as dominant, but the measures are crude and do not account for complicating factors.

There are several approaches to specifying the impact of greater skills on commuting distance. It should be noted that the analysis depends on a normal configuration, where the workplace lies on a ray between residence and city centre. Hence seeking inward toward greater density reduces the distance u, while it reduces actual commuting distance t at the same time. Hence for the current analysis they are interchangeable.

When a relationship similar to equation (4.12) is re-formulated, from the distance which optimizes search to the probability of a worker accepting a wage at any given distance u, we have an equivalent of Simpson's (1992, 4.3.1)[16] equation for the expected commuting distance

$$E(u) = \int_0^{\infty} u\, f(w,u)du \tag{4.13}$$

Underlying that relationship is the equilibrium of probability-of-acceptance to market density, and we are back to equation (4.5), expressed as

$$f(w,u) = d_u. \tag{4.14}$$

Then when skill factor g is incorporated into the equation, we can specify $\partial f/\partial g$ and observe its sign. The search considerations discussed above would suggest that the term tends to be positive due to search effects, in contrast with the expectation for it to be negative due to distance optimizing effects, with the accompanying positive and negative influences on commuting distance.

Simpson extends equation (4.10) above by making marginal cost a function of distance to the furthest relevant island, replacing mc with c(h) to determine the extent of search in the island context. This is further combined with skill, g, such that marginal cost becomes c(h,g). Then if it can be determined that $\partial c/\partial h > 0$ and $\partial c/\partial g > 0$ (given a higher

[16] The principal reference for much of this section

valuation of time by more skilled job seekers) these conditions are consistent with the tendency for skill to shorten the job search, contrary to the direct optimizing effect of quality variations discussed above.

A more precise indicator of the impact of skill would be the effect of skill on the effect of distance. Considering equation (4.13) we can take $\partial f/\partial u$ to represent the effect of distance on the probability of finding a job at wage w. Then $F = \frac{\partial f/\partial u}{\partial g}$ is the effect of skill on the effect of distance. It would follow that if the job seeker is primarily concerned about travel costs and the value or time, more skilled job seekers will be reluctant to search further, and $F < 0$. Conversely, search-oriented job seekers who are more skilled follow a narrower and more extensive search path and will be inclined to seek further, such that $F > 0$. Hence the indicator is the sign of F.

Further, the expected value for distance u is a weighted average of distances, represented by:

$$\int_0^u f(w,u)du = F(w,u) = D_u \qquad (4.15)$$

This can be re-written as:

$$u = H(w,d_u)$$

for which we can expect $\partial H/\partial p < 0$ and $\partial H/\partial d_u < 0$ (when price and density gradients are both downward sloping).

F(w,u) is the probability of selecting a commodity of price p at expected location u. The effect of distance on F(w,u) is $F_u = \partial F/\partial u$. The impact of g on F_u is $\partial F_u/\partial g < 0$. For people seeking to employ their skills, and hence having a need to search further (inward to increase the offer rate and compensate for a more restricted supply), the probability of seeking commodities more distant from CC decreases. This is again the opposite of the earlier conclusion, based on the higher price gradient for skilled workers (combined with the higher valuation of time), that the individual will choose a greater u.

$H(w,d_u)$ is the probability of selecting a commodity where wage is w and density is d_u. The effect of density on $H(w,d_u)$ is $H_d = \partial H/\partial d$. The impact of g on H_d is $\partial H_d/\partial g < 0$. If job density should increase overall, increasing the offer rate, there will be a tendency not to have to search as far (inward toward greater density). But that would not be as true in the case of greater skills, because of the need to extend the search in the face of a more limited demand. That is, the marginal benefits of search will not increase in the same proportion for skilled workers facing a restricted demand, and the inward search will have to extend further to yield adequate marginal benefits. Hence the probability of accepting at any particular location in response to an increase in density is lower as skills increase, producing a greater tendency to search inward, reinforcing the effect of the previous analysis.

4.2.4 Household Structure

Certain key household characteristics have not yet been sufficiently examined. Empirical analyses cannot achieve an adequate degree of significance without introducing family structure variables and life-cycle variables. The most significant aspects of family structure are number of working members (including a distinction between primary and secondary earners) and number of dependent members.

Simply on the grounds of household space requirements, a multi-person household will require more space (except to note that by comparison to living separately they may actually require less space when occupying just one home) and will therefore tend to exhibit a shallower indifference gradient, pushing the household outward (Evans, 1973, p. 147). However, if other (non-head) family members are also employed outside of the home, a simple doubling of commuting costs may create an inward pull. A more analytical approach (e.g., Beckmann, 1969; Oi, 1976 and Hekman, 1980) suggests that the optimizing inward pull of economizing on multiple commuting costs is counterbalanced by the outward pull of larger family space requirements.

Fujita (1989, p. 38) presents a useful standard optimizing analysis to the residential decision to determine that an increase in the number of earners, or a decrease in the proportion of dependents, renders the indifference curves of Figure 2.2 shallower in slope. The analysis can be generalized to all TL. There are counteracting effects on the direction-tendency of TL, but a general outward effect on distance and commuting, enhanced in the case of higher quality commodities.

Another factor related to secondary earners is the question of whether the secondary earner has ended up at a residential location which is unrelated to his or her occupation (since the choice of residence may be more likely dominated by the occupation of the primary earner). If (and only if) as a result, the ratio of jobs to workers in that occupation is relatively low, then there is a likelihood that the job density will climb relatively rapidly with search. In other words, the density gradient may be steep, encouraging greater search. This is in opposition to the search-reducing and commuting-reducing tendencies mentioned above, and the question of which effect is dominant may be answered differently for secondary earners. This is not likely to be a highly discernible effect, especially considering there is not a strong case for assuming any differential in the ratio of jobs to workers. Considering possible extensions of the analysis, the data available for this study enables us to specify whether the residence or workplace was chosen last, and it is possible and perhaps useful (though not attempted here) to distinguish whether the secondary earner was the one to change jobs since the last residential move.

Similar effects are not testable with available data in the case of special housing characteristics. Even though families with children can be identified, and the family status of the respondent (head, spouse, other) can be identified – it is likely that the residential decision was either a decision of the head or the whole family, and the status of the person who responded to the interviewer is irrelevant. On the other hand, when it comes to job and skill related questions, the available data is that relevant to the respondent (whether head or otherwise), and it is possible to relate the respondent's education level to commuting time.

Evans' data for London show that children below working age have an outward pull, while greater female proportion in the family as well as multiple earners produce an inward pull – all of which analysis was conducted on the basis of measuring commuting distance "within each income group" (Evans, 1973, p. 150) Estimates by Kain (1975) suggest no strong denial of the model's expectations but plenty to suggest that all of the other factors are at work. Simpson (1993, pp 110-112) presents a useful summary of various econometric studies which suggest that secondary-earner status has a significant shortening impact on commuting distance, but not without exception. Simpson's own estimates suggest that the general expectations of the model hold well especially for households with small children, but not so clearly as expected for multiple earners overall, and very much affected by levels of income. Tied in with a possible greater space preferences for families with children is the fact that they are also likely to be on the accumulating phase of the life-cycle, and part of the reason for preferring property and perhaps moving further out may also be due to that factor.

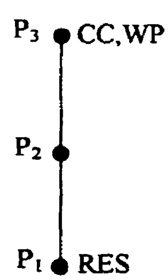
5. PLACEMENT

5.1 Offsets and Switches

5.1.1 Rays and Gradients

We will look at questions such as noisy gradients (if gradients exist at all), but first it is necessary to go back to the ground level spatial relationships which form the foundation for the model.

It should be recalled that the base Alonso model rests on the assumption of residences (RES) located on a ray along a price gradient emanating from city centre (CC). In the centralized-employment version of the model, the workplace (WP) is also located at CC. This is portrayed in Diagram 5.1 below.



where $P_3 > P_2 > P_1 >$ all prices beyond P_1.

Diagram 5.1
Basic Alonso Configuration

In the familiar Alonso analysis, the resident balances lower housing prices against higher travel costs as he or she moves outward from CC.

The direct rationale for the ray assumption in the centralized-employment model is that residences which are radiating out in different directions from CC make no difference.

The analysis is exactly the same as if all RES were on a single ray, since the distance to CC and the resulting travel costs are the same in all instances.

The rationale for the gradient assumption is that it creates a balance of forces between rising price and declining cost over the interval, setting the stage for a unique location decision. However, price gradients in practice may be poorly discerned, or discerned only as scattered price landmarks, with little information about prices in between. Even if that should be the case, some aspects of a price gradient must be preserved, or the simple Alonso model and all built upon it is not useful. The principle imperfections are twofold. First, there are multiple price peaks. Second, the locus joining any given price peak and the set of locational choices is far from smooth and continuous. They tend to be noisy and discontinuous, with a high variance around the hoped-for smooth curve.

The initial problem here considered is how to deal with noisy gradients? The noisy gradients in the data tend to exhibit two levels of variance. There are a few key prominent locations along the ray that exhibit fairly good negatively-sloped gradient characteristics. Say, for sake of argument, every fifth region, if they could be separated out and considered in isolation, would, make a decent gradient. But the four in between each of these widely variant. Hence looking at the locus in the short-interval context relative to differential calculus, the gradient is extremely noisy and seemingly useless. However, looking at the long-interval patterns, there is a useful gradient for those who look to the horizon instead of the immediate neighbourhood. It may be possible to incorporate decision-making based on different levels of perception.

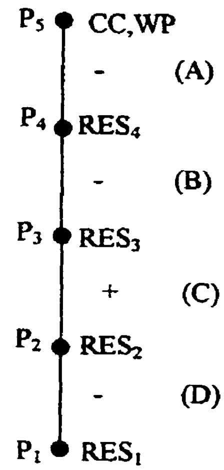


Diagram 5.2
Noisy Gradient

Diagram 5.2 above shows an imperfect price gradient. Namely, the price P_2 increases with distance from CC where it is supposed to decline, but P_1 continues the downward price slope normally. Hence the + sign in interval (C). If the balance of forces would normally have produced an optimum location in the interval from CC to RES_3, the normal analysis applies. Beyond RES_3, up to RES_4, there is no longer a balance of forces. Both cost and price factors move the agent in the direction of CC. Hence region (C) is a null-zone, and any locations within region (C) will definitely not be chosen. Hence we have a discontinuity along the ray, and a greater concentration of residences at RES_2. In other words, gradient reversals will produce a lumpiness in the distribution of residences.

If region (C) did not actually reduce the direction of price change, but the price merely declined at a slower rate than in the other intervals, it would not produce a null zone, but would skew the distribution of residences toward RES_3, depending on the degree of price anomaly. Hence the more general conclusion is that noisy gradients produce a pattern of clusters in the distribution of residences, determined by the degree and direction of variance involved. However, this is an effect which can be accounted for and possibly tested to some degree.

An example of testing for this effect would be the simple proposition that people with higher incomes live further down the price gradient. With a noisy price gradient, agents would have a poor perception of the price gradient relationship, and any regression of income on distance to city centre would produce a weak relationship.

An adjustment could conceivably be made as follows though the technique is not followed up in this study. Calculate either a smooth mathematical gradient, or a detected long-interval gradient as described above. For each survey point, calculate not only the distance from CC, but also the amount of deviation of the price at that point from the chosen gradient. From that can be calculated a coefficient of bias toward CC, as described above. If such coefficients are applied to the actual residential locations, they may produce a more robust regression, and perhaps less distortion in the relationship between price-sensitive and other variables. This of course greatly complicates the regression, and it may be preferable to stick to crosstabs compensated in this manner instead – or resort to fewer cruder regions. In practice, the computational cost would be rather high in any case – since a geographical algorithm to determine through which housing price zones the housing price gradient passes, and which sample points are contained in each, would have to be integrated with the spreadsheet analysis. This would be a possible area of future research if there is some confidence that such coefficients of bias would produce improved results.

5.1.2 Multiple Centres

Turning from poorly perceived rays to multiple rays, the Alonso model treats residence and workplace lying on different rays as suboptimal, in two senses. First, if a residence is chosen such that the workplace sits off the ray from residence to city centre, it will be possible to choose a more optimal residence on the same ray as the workplace and at the same distance from city centre, reducing travel costs for the same benefits. Hence workplaces offset from the residential ray represent unfinished optimizing, or at least impediments to optimizing. Second, if somehow impediments produce a residence with an offset workplace, it will be possible in a fully dense marketplace to find a second family unit on the workplace ray whose own workplace is on the first family unit's residence ray, and swap residences or jobs with mutual benefit. Hence any extra commuting along the hypotenuse to offset workplaces or "circumferential commuting" (Hamilton, 1982) is unnecessary, or "wasteful" in Hamilton's terms. Nonetheless research indicates that it may well exist to a notable degree, with some debate as to its significance, and it will be examined more closely in Section 5.2.

We will also examine multiple centres, which we sometimes call regional centres (RC) and treat them as multiple CC's. Consider two RC's, each with its own gradient (of whatever quality) on a ray, each of which contains a different set of locations. The agent would perceive an option to move along either ray to its optimizing point, and choose the one which offers the best optimum. Essentially, the agent would face two un-identical rays instead of just one, and they would range from parallel to criss-crossed. But they would be independent, and one of them would produce the superior optimum. They cannot, however, be superimposed and treated as a composite, unless they both exhibit exactly the same price gradient, at least in the range where they are indeed superimposed. This makes it difficult to use composite gradients, since all of the observations in the data are independent and there is no reason to believe they represent identical gradients. The only realistic way to analyze the either-ray decision would be to model the two decisions, and observe which ray-optimum is chosen. However, this decision process is a bit of an artifact of the centralized employment model, and the options present themselves differently in the more complex models. It nonetheless needs to be presented here to build the model.

5.1.3 Configurations with Decentralized Workplace

The model gets greatly more complicated by the introduction of a decentralized workplace. For ease of exposition, we will assume price gradients that are perfect in the sense of no deviation from a smooth convex downward-sloping function. We shall also initially make the assumption, in common with simultaneous equation approaches, that the residence and workplace decisions are made simultaneously .

The simple case, once again, is to allow discretionary workplace locations along the same ray as the residential locations. Here tc = travel cost.

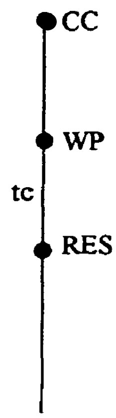


Diagram 5.3
Basic Single Ray

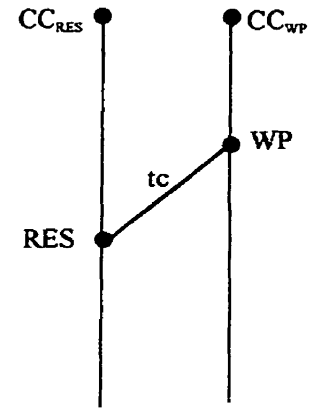


Diagram 5.4
Separate Rays

Diagram 5.3 represents the traditional single-ray model. Diagram 5.4, however, contains two separate rays, one on which the RES decision is made, and one on which the WP decision is made, where CC for RES and CC for WP are not necessarily at the same location. That is, we are considering a multi-peaked gradient in the sense of separate and independent housing price and wage gradients. The same simultaneous-equation regression could be performed on the right, as is commonly performed on the left, except that the travel cost (tc) would have to be triangulated to get the actual diagonal distance traveled. This is a somewhat more complex calculation for regression, especially given that the rays are not likely to be parallel. This, however, is getting one step closer to the kind of configuration observed in the available data.

In practice, an actual urban situation may look more like Diagram 5.5.

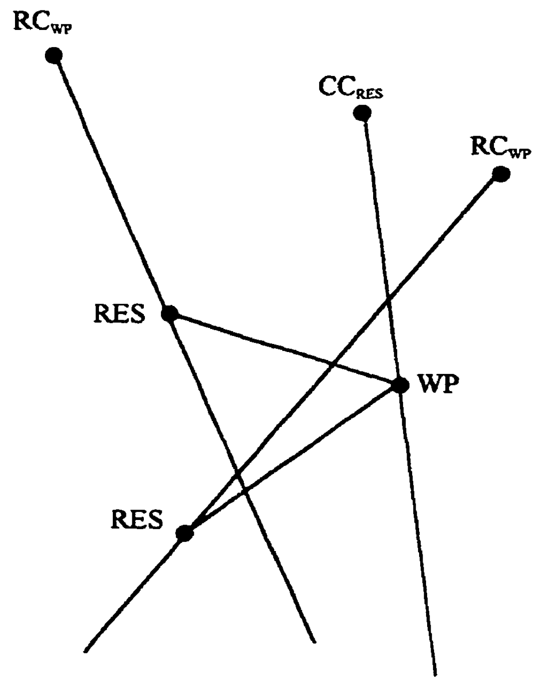


Diagram 5.5
Complex Rays

The assertion that all gradients converge on a single point arises from the commonly-observed concentration of population density at city centre. It follows that housing and jobs will also be concentrated where the population peaks. However, there is considerable evidence of employment decentralization as cities grow, even though population may remain relatively centralized (Mills, 1972; Scott, 1982). Hence we may see configurations like the above, with employment centres split up into suburban or regional centres (RC_{WP} above), though employment at city centre may exist as well. Mills (1994, page 83) summarizes a range of research, noting that from 1950 to 1980 the proportion of jobs located in central locations declined from 70.1 percent to 49.5 percent. The same summary stated that centrally located residences declined from 57.3 percent to 39.9 percent during the same period, which might suggest a swapping of the RES and WP subscripts in the diagram above. The data also indicates that suburbanization occurs in different rings or different areas of the city for jobs than for population. Mills concludes that the "suburbanization of jobs has been modestly faster than that of people, suggesting a slight reduction in city centre jobs per capita". The point of this section is not simply to depict the "anything's possible" scenario, but also to point out that the stability of the per capita job concentration suggests that the traditional assumption of concentration at city centre may remain a viable working hypothesis.

The problem in the above diagram is actually not enormously more complex in theory than the previous examples. The agent chooses the best optimum from the two WP gradients, optimizing it with the simultaneous choice of residence on the RES gradient. But when it comes to calculating it on a spreadsheet, or preparing the data for a regression, the problem becomes significant—especially when we're working with something like say 4 residential regional price centres and 3 workplace wage centres, and trying to model 2 best choices on 7 gradients all compensated for deviations from the smooth gradient. There is something to be said for reverting to a simple normal Alonso gradient as a basic stylized model, and seeing whether any useful empirical performance can be observed. In the data used for this study, to be analyzed below, this emerges as the most realistic option when housing prices are measured on a per-unit-of-land basis.

Given some of the complex configurations noted above, unless one can get some reasonable results out of a crude 3- or 4-region model, it is likely to be impractical to try to run crosstabs or regressions for the simultaneous decision model of Chapter 2. However, there is not a strong case to be made for assuming that residential and workplace decisions are made simultaneously. The analysis of this study retreats from the simultaneous equation model to a much simpler fixed-residence and fixed-workplace analysis.

Diagrammatically, it looks something like Diagrams 5.6 and 5.7.

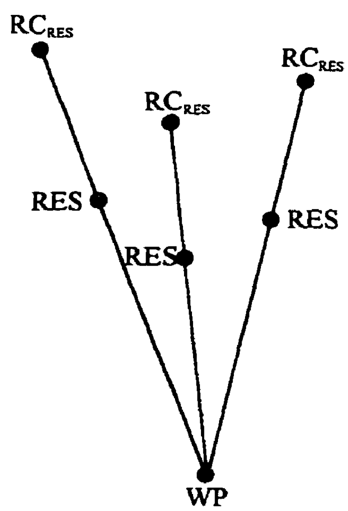


Diagram 5.6
Fixed WP

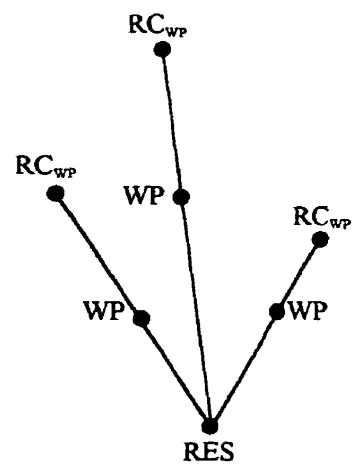


Diagram 5.7
Fixed RES

From a fixed workplace location, a residence is chosen from among several housing price gradients. From a fixed residential location, a workplace is chosen from among several wage gradients. They are separate decisions, subject to separate analyses, based on two different sets of sample points. Because these decisions are now emanating from a single point, the decision point of fixed residence or workplace, the geometry is greatly simplified, and in some analyses can be superimposed onto a single ray for analytical purposes. However, a simple composite remains inapplicable. The single superimposed ray is for the purpose of comparing distances only, but each individual ray has a different price gradient and must be analyzed separately, compensating for differences separately from the hypothetical smooth gradient, unless some of the analysis requires only their negative slopes to be specified.

Nonetheless, model building commonly involves finding the simplest possible configuration which captures the essence of the phenomenon. And a simple city-centre-based single-gradient model which accounts for offset workplaces still qualifies as the best starting point unless proven inadequate.

5.1.4 Offsets and Switches

Let us now consider offsets. In the analysis we have been considering, regarding the choice of residential location, the workplace resides on the same ray as the housing price gradient connecting the residence and city centre, as shown in Diagram 5.8.

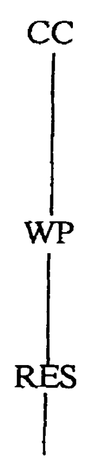


Diagram 5.8
WP on the Ray

While this does not always occur in practice, much of the deviation from this pattern can be accounted for by analyzing offsets from the ray. The configuration that occurs most commonly in practice is a workplace which is offset from the RES-to-CC ray in the manner shown in Diagram 5.9.

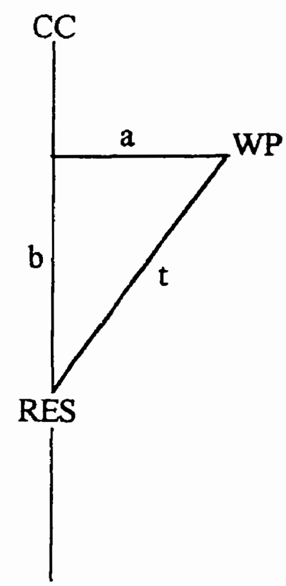


Diagram 5.9
Basic WP Offset

In this instance, the WP is offset. It resides on its own wage gradient, but considering the workplace decision to be exogenous in this instance, the wage gradient is not relevant to the choice of RES.

In the resulting Alonso analysis, the travel cost variable is no longer simply a function of a linear distance along a ray. It is a Pythagorean function indicating the actual travel along the hypotenuse (distance t) to the WP, as we consider movement of the RES along the ray. The offset results in a higher effective travel cost for each unit that the RES moves along the ray. This would have the effect of moving the optimum RES closer to CC, producing a greater bias of the RES toward CC the greater the offset.

The impact of the offset is also affected by placement. Consider first an extreme. If the WP were located perpendicular to RES (i.e., directly to the right of RES), any small marginal movement of RES would result in no significant change in travel cost. Significant movement of the RES towards CC from this point would result in both higher travel costs plus a higher land price, and would not be undertaken. Significant movement away from CC would likely be beneficial, because it would result in higher travel costs affected by the degree of offset but lower land price. The optimum would be at a positive distance b as shown. In other words, the simple ray analysis continues to apply with the inclusion of offset ratios.

This possibility is partially accounted for in equation (2.6), since the travel cost function is a general one, c(t), rather than the commonly employed linear function. Consideration of offsets suggests using a function c(b,a), where a represents the exogenous offset as above. In terms of indifference analysis, such as Figure 2.2 if it were applied to the RES (or CL), the utility curve U in the upper panel would appear steeper because of the offsets, producing the anticipated bias toward CC.

In practice, the ray in question will almost never be truly vertical, and the offset will be better represented by Diagram 5.10 below. Here the distance c represents the difference in latitude between CC and WP, and e represents the difference in longitude between CC and WP.

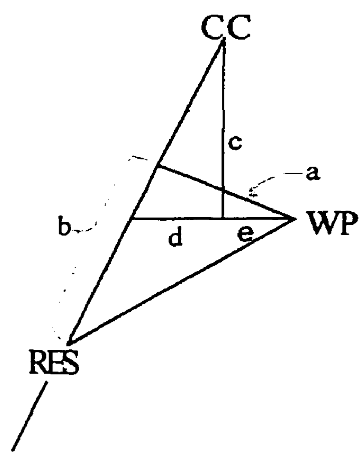


Diagram 5.10
Inclined WP Offset

The offset a can be calculated from the values of c, d and e.

The same basic relationship was stipulated by Simpson (1987, page 125), using a variant of Diagram 5.10. Simpson's diagram referred to the angle Θ between the CC-RES ray and the CC-WP ray (though they are related differently to the vertical line through CC), noting that the regression equation "represents commuting distance only if $\Theta = 0$. Otherwise, commuting distance will increase as Θ increases". This paper carries that suggestion further, relating it in Section 5.2 to the excess commuting discussion.

Essentially the same analysis as in the residential choice instance can be applied to the workplace choice decision. Actual travel costs do not equal travel costs along the ray, but rather are multiplied by an offset factor related to the hypotenuse. Hence commuting is more expensive and the same set of preferences will likely produce a reduction in commuting, and hence an offset-generated bias away from city centre.

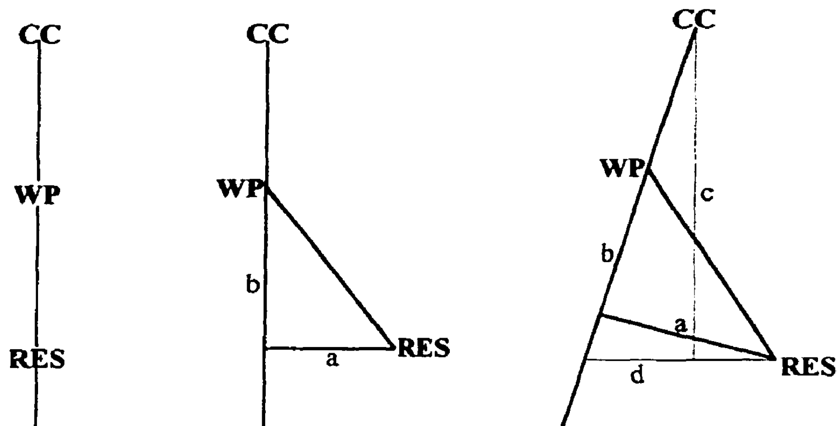


Diagram 5.11
RES Offsets

Again, given the greater travel costs resulting from the offset, the resulting WP location will be relatively biased away from CC.

The calculation of offsets for all sample points yields a new and unique set of variables which can be hypothetically related to other variables and tested. But first, a number of other similar relationships — similar in that they are necessary to account for other ways in which the fundamental Alonso relationship between points must be modified to account for actual spatial relationships — must be considered. Foremost among these is the incidence of switching.

As shown in the Diagram 5.11, the optimal RES location occurs at a greater distance from CC than does the WP. It follows that the optimal WP location occurs between CC and RES. However, in many instances, the data reveals that several of these points are switched. They may be switched in the following ways. First, considering the location of the WP in relation to the choice of RES, we shall apply the following terminology:

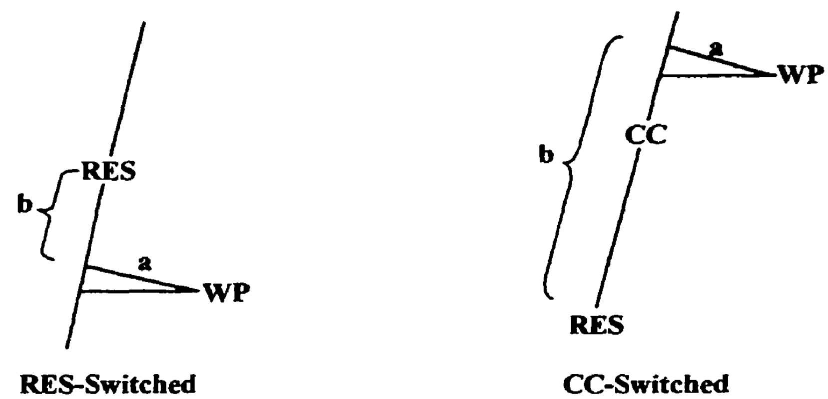


Diagram 5.12
Switching with WP Offset

And similarly, considering the location of the RES in relation to the choice of WP:

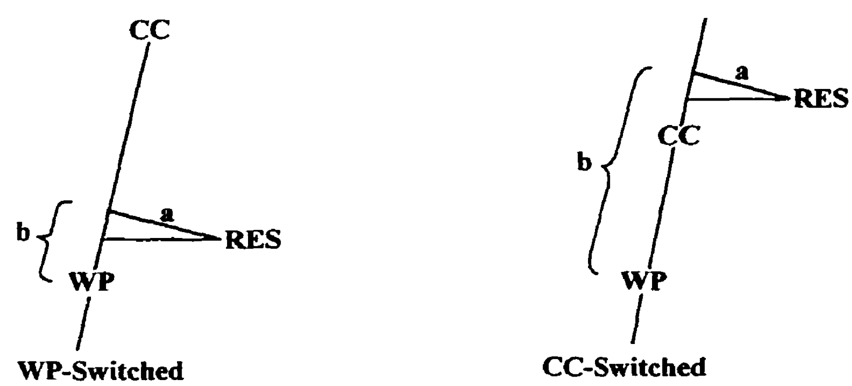


Diagram 5.13
Switching with RES Offset

All usable data points have been analyzed so as to indicate not only the offset for each point, but also what kind of switching has occurred.

The offsets indicated in the diagrams above are drawn to be approximately equal in length. It does not matter how they are switched, the offset still measures no more than the distance to the ray – even though some of the offset residences and workplaces are far removed from the normal optima and others are not – depending on whether and how they are switched. Therefore it would seem that something which simply measures distance to the ray does not capture the full impact of the offset. However, it is the combination of switches (and type of switch) with offsets which produce the full impact, and any examination of the data must consider the various combinations of offsets and switches to capture the full effect.

5.1.5 Multiple Gradients

In the traditional analysis of residential location, the workplace would be located where the workplace (WP) lies along the ray RES-CC_1, such that a = 0 in Diagram 5.14 below.

The empirical data, however, may well produce a set of relationships similar to those as depicted in Diagram 5.14. The workplace is commonly significantly offset from perhaps more than one ray, as indicated by distances a and a_2 (or similarly, RES may be offset from a workplace ray). Locations such as CC_1 and CC_2 may be price peaks which are removed from the density centre DC (which is likely to be at the actual physical city centre). And there will likely be amenity centres like AC above which occur at locations that bear no relationship to any identifiable city centre.

We first consider the case where the WP is offset from the ray by the distance a. It should be kept in mind that, when we move the analysis into two-dimensional space, a wide array of configurations may result. The WP may lie on the other side of RES-CC_1. Also as described above, the WP location may not necessarily obey the laws of Alonso location, and may lie at some point below the RES-CC_1 ray. In addition, the relative

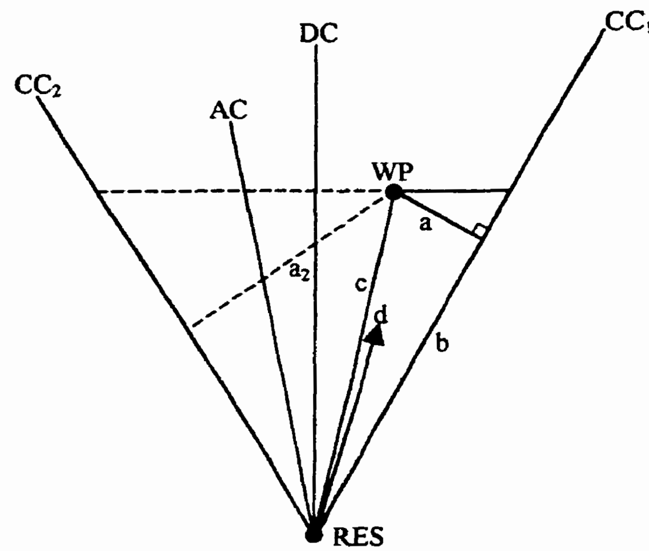


Diagram 5.14
Multiple Centres

vertical positions of CC_1 and RES may be reversed. Any algorithm to determine the distance a must account for all possible combinations of points in all possible directions.

The implication is fairly straightforward. Considering the price gradient RES-CC_1 – if the offset, a, is non-zero, the actual commuting distance traveled is c rather than b. Since $c > b$, that could be represented in Figure 2.2 by a steepening of the indifference curve and a bias of res toward CC_1. In addition, any analysis based on the pull of lower housing cost versus the pull of lower commuting costs is diminished in explanatory power. (This can be readily seen if we consider a case where the offset tangent is equal to the value 10, which occurs in the data.)

AC in Diagram 5.14 stands for amenities centre. The result is a *draw* – that is, an angle of draw signified by the arrow d – showing how much the residence seeker is drawn away from the ray RES-CC_1 toward AC. This would make for a smaller a and perhaps even reverse its direction

A similar d-arrow may be drawn to indicate the draw away from RES-CC_1 produced by RES-CC_2. In this instance, since offset a_2 is somewhat larger and CC_2 is assumed to be less effective, the draw may be relatively insignificant. Hence it is not shown. But this may not always be the case.

DC stands for density centre. The data suggest that Minneapolis downtown is both a housing and job density centre, but definitely not a price or income centre. It may be argued that the individual's perceptions of the market may equate density with price, but we might also allow for a density centre that is not coincident with any price centre. Hence the density gradient RES-DC. The implications of it are that search behavior may be conducted in a different direction than price/cost-seeking behavior, more specifically the element of search that involves seeking a greater density of price quotes. Hence search behavior may produce another draw-arrow d away from the price gradient. The above relationship can be expected to apply in this instance as well. The draw will likely occur in the direction of the workplace, unless there is a strong density gradient to draw the search away, at a cost of greater offsets. And the draw will be enough to reverse the direction of the offset if the draw occurs in the direction of the workplace and there is a strong density gradient involved.

Next let us analyze the impact of multiple price gradients, indicated by the existence of RES-CC_2. This is characteristic of the data being examined, where there are two or three difference housing price peaks. The strongest of them is in a particular western suburb, and city centre is not a price peak at all. In the example portrayed above, CC_2 is assumed to be a ineffective price peak, such that residence seeker's strongest response will be to move along RES-CC_1. In addition, the workplace offset is greater for CC_2, further reducing its decision-making relevance, though it may still remain a factor in the decision. In some cases, CC_1 and CC_2 may lie almost 180° apart, and may possibly have some fairly direct counteracting effect if the workplace is half-way between them. The more general relationship is shown in Figure 5.1.

Suppose utility curve U_A represents the agent's responses to two identical gradients, Gradient A and Gradient B. That is, it depicts two identical superimposed indifference curves. The optimum position is shown as point A. Then consider the effect when one indifference curve becomes steeper because Gradient B has a larger offset. U_A pivots down to U_B, but U_B still represents the same level of utility. The new optimum occurs at point B, but this represents a lower level of utility since utility is maximized towards the origin for positive prices. Hence of the two gradients now considered, the residence seeker will choose to seek along Gradient A. Alternately in the diagram above, CC_2 may represent a lower price at the centre or a different curvature of gradient, for example, which could reposition the indifference curves in such a way that Gradient B is preferable in spite of the greater offset involved.

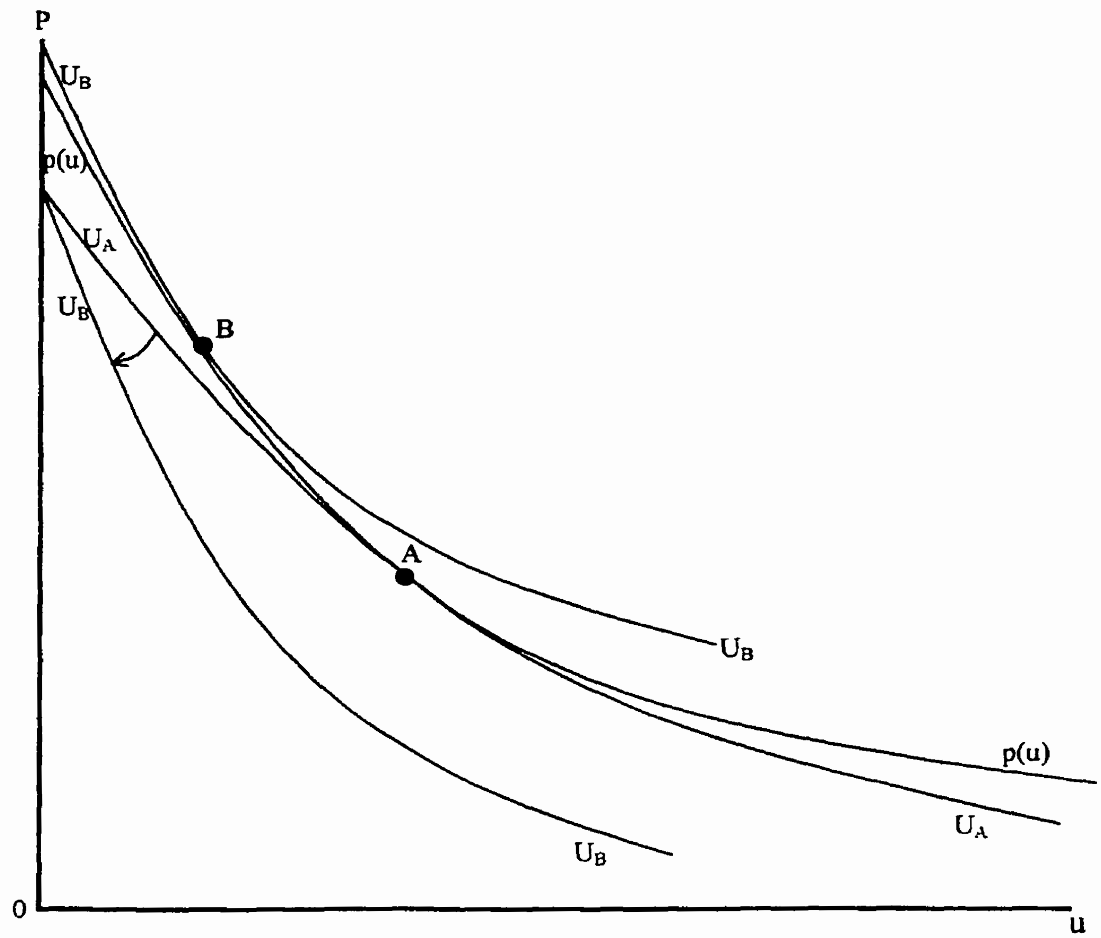


Figure 5.1
Multiple Gradient Case

The situation of multiple price gradients is further analyzed in Figures 5.2 and 5.3. In figure 5.2, two gradients are represented in a three-dimensional graph, showing distances from CC of μ_A and μ_B. The corresponding indifference curves are U_A and U_B along the axes with the same subscripts, while the gradients are simplified by showing only their tangency lines P_A and P_B, as well as the tangency points A and B which represent the same points as in Figure 5.1.

There is another indifference curve, U_{AB}, which represents the indifference relationship which occurs at an intermediate path between Gradient A and Gradient B. In other words, U_A, U_B and U_{AB} are three contours on an indifference gradient surface which fills in the preference set between two divergent gradients. Similarly P_A, P_B and P_{AB} are tangents to a market price gradient surface. If conditions are right for a single point of surface-to-surface tangency, that may occur at any of points A, B or AB, or any number of points in between. The dashed line in Figure 5.2 illustrates what might be considered the locus of all possible tangencies. If that locus is re-plotted into utility space, it may look like either Panel A or Panel B of Figure 5.3. If it looks like Panel B, that is a circumstance in which the optimum location may occur at point AB and hence may be offset from the ray (for example offset from the ray containing the gradient $P\mu_A$).

The outcome of this analysis is a theoretical rationale for an optimum residential location which is offset from the ray. A similar rationale applies to workplace location. A further rationale for offsets, based on the analysis of Michelle White (1988b), will be considered when discussing cross-commuting in Chapter 5.2.

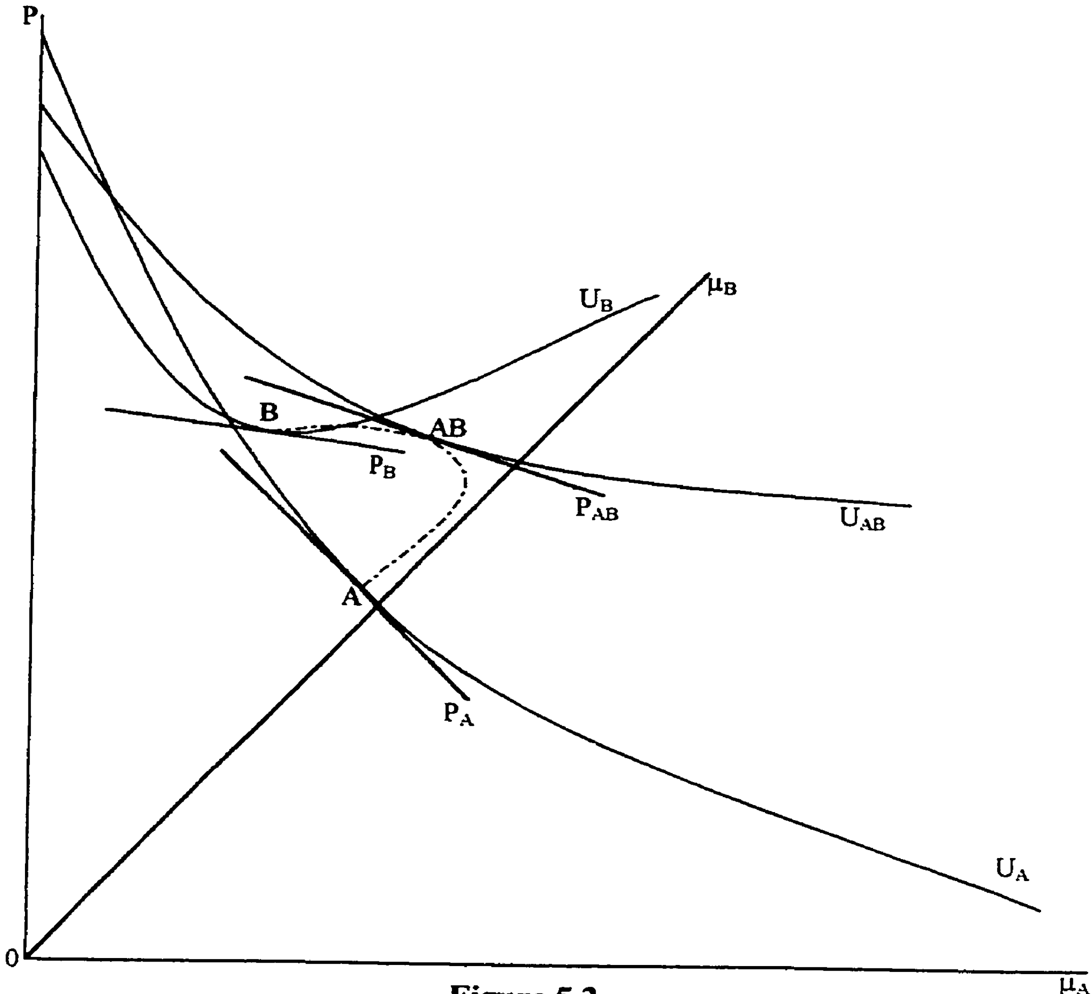


Figure 5.2
Multiple Gradients in 3D

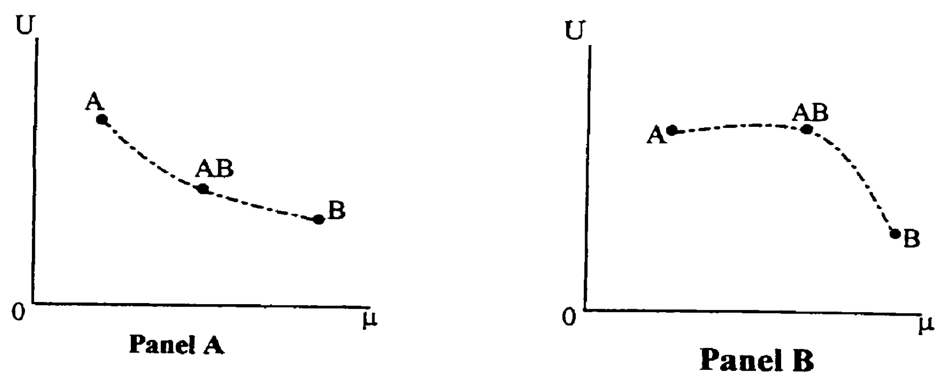


Figure 5.3
Optima in Utility Space

The same situation is depicted in Diagram 5.15. We might consider CC_{AB} to represent the slice in three-dimensional space shown as U_{AB} in Figure 5.15. Under the conditions of Panel B, the decision-maker moves along a virtual gradient, RES-CC_{AB}.

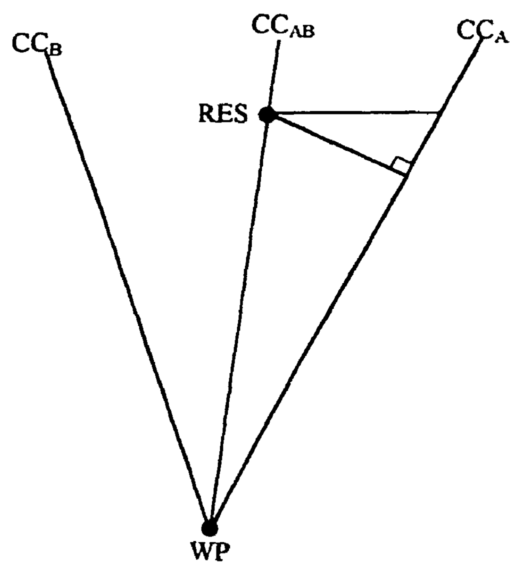


Diagram 5.15
Intermediate Ray

What sort of conditions will generate Panel A? It is convenient first to ask why the decision-maker might not want to move away from optimizing on Gradient A. One possible configuration of market price gradients is where the price gradients along A and B are strong, but drop off quickly in between – resulting in a virtual price gradient surface which takes the shape of a valley in the region of AB. In fact, this is the kind of condition which is most likely to produce a single tangency with the indifference surface. It is also possible for the virtual market price gradient surface in the region of AB to be shallow, or even merely a plane, between the two market price gradients. In this latter case, if one is moving in a direction that increases the offset, and it is not compensated for by a significant enough drop in housing price, the optimum achieved on that intermediate valley virtual gradient will yield a lower total utility, and the decision-maker will stick to Gradient A. Therefore Panel A requires first that the workplace is on the other side of Gradient A from the valley, and that the valley is not sufficiently steep (or its nature is uncertain). It follows that Panel B requires either that the workplace lies in the direction of the valley, or a steep enough valley (clearly enough perceived) compensates for increasing offsets. If price gradients are weak or noisy (high variance), we might expect in general that people locate off the gradient only in the direction of the workplace, or at least that there is a bias in that direction, and that exceptions are more likely to occur off the steeper price peaks. Given the long-range nature of the data (clear major price peaks sticking out of the fog), this relationship may be the only one sufficiently pronounced to detect. Also it may be difficult to distinguish whether poorly perceived gradients are the result of high variance in the gradient itself, or rather from the fact that multiple gradients are interfering with each other.

If there is an amenities gradient, and we can specify an implied price curve for amenities which quantifies the value of the utility of the amenity, then we may make use of Figures 5.2 and 5.3 reapplied to the task of analyzing the relative impacts of the price and amenities gradients. That is, we may consider that the μ_B axis represent the amenities gradient, while μ_A continues to represent a price gradient. Then Panel B represents the circumstance in which the decision-maker may be deflected from the price gradient because of amenities (the draw-arrow, d, in Diagram 5.14).

It would also be possible to draw a Panel AI (an inverted version of Panel A in Figure 5.3), which contains a generally upward sloping curve, indicating that the amenity factor completely dominates the decision – that is, the decision will be made entirely along the amenities gradient, and the price gradient (possibly difficult to discern anyway) is completely ignored. If we consider Panel BI, the original basic relationship obtains. The draw will likely occur in the direction of the workplace (that is, if the amenity gradient lies away from the price gradient in the direction of the workplace, a the decision will be made along a virtual gradient which is offset from the price gradient in the direction of the amenity), unless there is a strong preference for the amenity in question.

In addition, the draw may produce a virtual gradient which lies on the other side of the workplace, turning the offset around (negative). This is most likely to occur if the draw occurs in the direction of the workplace and there is, in this case, a strong preference for the amenity in question.

The data available for this study are not amenable to testing in this regard. Rather than conforming to the structure of Diagram 5.14, where there are multiple gradients they are distributed fairly evenly around the clock, such that a movement in any direction is a movement toward another gradient. In addition, there is no practical way to compare the relative steepnesses of any gradients.

A further factor is the variance of the price gradient itself. This is shown in Figure 5.4 below as a thick and possibly grey price gradient, for which P_H and P_L are boundaries.

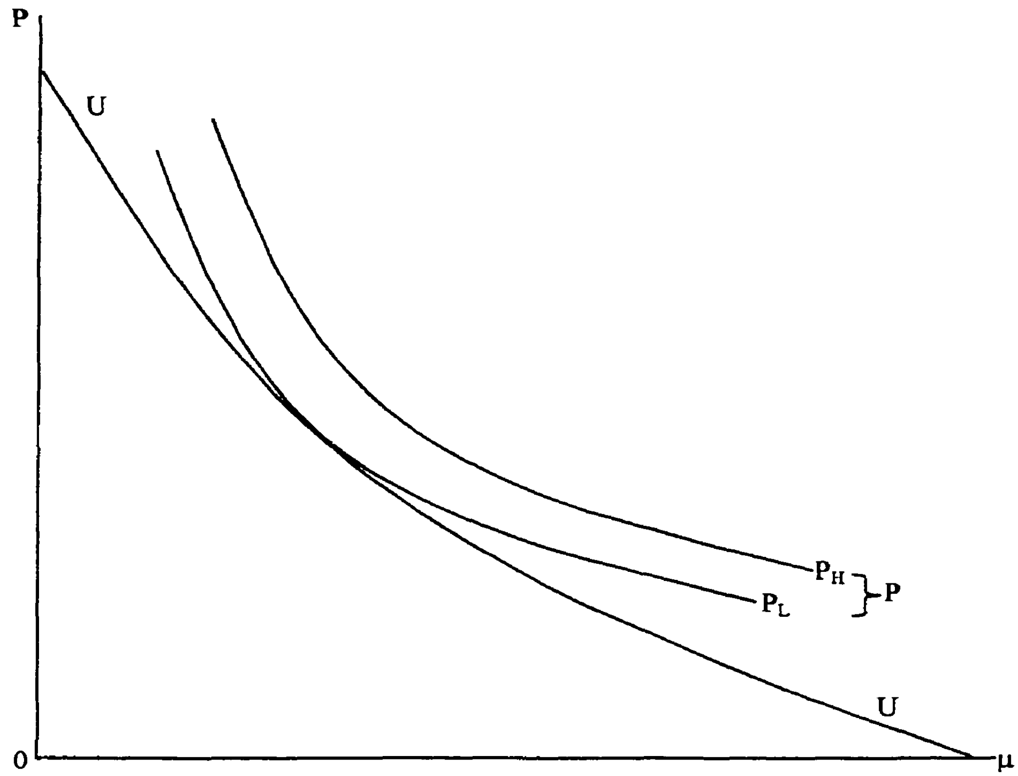


Figure 5.4
Thick Gradient Case

The thickness of the gradient indicates a price gradient with significance variance. Still, if the gradient were perfectly dense, even though thick, the optimum would occur at the tangency of P_L to the appropriate U, and predictability would not be impaired. However if its density is low, meaning the grey of only scattered points between P_H and P_L, then tangency could occur anywhere between the two limiting curves. Should anything tilt the curve for any reason, results could be unpredictable. The residence-seeker may jump to unpredictable discreet points in the band, depending on the relative positions of the few points available, especially if high variance produces a very wide band. In other words, a flattening of the gradient may tend to move the optimum residence outward, but variance produces anomalies.

A high-variance gradient of this sort must be combined with search behavior on the part of the residence-seeker. The searcher will have some knowledge of price peaks, and may expect the gradient to be a grey band, but will have no knowledge of just how high the variance might be. Hence a searcher may economize on knowledge requirements while searching to an expected gradient of uncertain shape and variance. The optimal behavior may be to engage in some systematic sampling to gain a better knowledge of the gradient and reduce the uncertainty. The practical behavior may be to spot check – to sample a few clusters of locations from a few likely areas near desirable tangents. The result might be a set of sample points with small circles around them. A few sample of this sort will be drawn, until the marginal benefits of spot checking in this manner no longer justify the marginal costs. In other words, the gradient band will be grey in the sense of containing a small fixed number of grey circles, and the final choice will be made in the grey circle closest to tangency to U. This may, if the circles are few and far between, increase the likelihood of anomaly. We might expect then that those with a greater incentive to search further, such as those with special needs or those who see a steeper density gradient, will produce fewer anomalies, since they will tend to sample more of the grey circles.

5.2 Excess Commuting

5.2.1 Offsets, Switches and Optimality

In terms of the Alonso model, offsets and switches must be considered sub-optimal behavior. A useful way to articulate this, as employed by Hamilton (1982) when analyzing the concept of excess commuting, is to consider the effects of swapping as a Pareto optimizing process.

First consider offsets. Simply put, if one individual lives on ray A and works at on ray B, while the opposite holds precisely for another individual, they will both reduce their total commuting by swapping one of either residences or jobs with each other. In Hamilton's terms (1982, page 1038), this reduces "circumferential commutes", since they each now have both their residence and job on the same ray. In the terms used in the present analysis, both parties improve their position when they eliminate "offsets" through swapping. Hence offsets are sub-optimal, according to an analysis which could be formalized into a welfare economics analysis, whereby an optimum based on available commodities can be improved upon through trading with others in the same situation thereby achieving a higher level optimum for both. In this case, the individual's existing residence is treated as an initial endowment, which is traded away as described. Of course there are numerous assumptions underlying this analysis: principal among them being (a) ubiquitous housing and jobs which are all equally desirable distributed densely and symmetrically along normal gradients about city centre, and (b) transportation which is smooth and continuous in any direction with no variations in cost.

The importance of this is that it gave Hamilton a means of measuring optimal locations (1982, pages 1039-1040). Every actual location, if it is not an optimal location, will ideally be traded away. This process will continue until the trading stops and everyone has achieved the best possible optimum. Hence, even though such trading may not occur, every actual location is an optimal location for someone at the end of the hypothetical trading. As a result, the average actual distance from city centre is equivalent to the average optimal distance from city centre. This can be subtracted from the average actual workplace distance from city centre to determine the average optimal commuting distance. The average actual commuting distance will be invariably higher (unless there are no offsets), and the difference represents excess commuting.

On the question of "out-commuting" or "switching" (in the terms of this study, in-commuting is normal and hence out-commuting is switched), Michelle White's (1988a, 1988b) graphical analysis is useful. Consider a worker choosing a place of residence in Diagram 5.16, where jobs are located both at CC and WP.

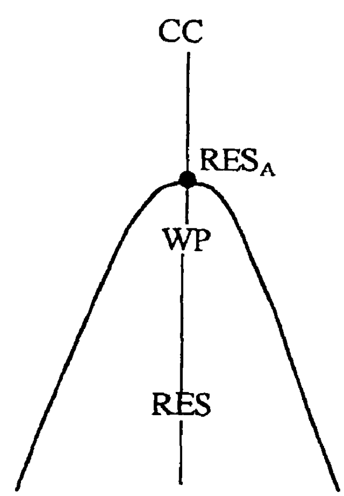


Diagram 5.16
M. White Optimum Set

A worker who a chooses a residence at RES would commute to a job at WP. A firm at WP need only offer a wage which equals the wage at CC minus the savings from not having to commute past WP to CC. Hence a worker who chooses a residence at RES_A will benefit more by commuting to CC – because more is gained from the wage differential than is lost by commuting. But consider if the firm at WP requires more workers than are available beyond WP. Ideally with perfect capital mobility, it will adjust both its wage and its location to find a new optimum WP, at which it does not have to pay a wage premium in order to induce workers who live in the direction of CC to outcommute to WP. This new WP location may be closer to CC to capture more of the labour force without paying a premium, or it may be farther from CC if the lower wage it can pay to all workers is decisive, depending on the nature of its production function. However without the options arising from perfect capital mobility, its only choice is to draw workers from CC with a wage premium. If it requires all the workers that are available outward from RES_A, it must pay a wage premium sufficient to make workers at RES_A indifferent.

In other words the model with perfect mobility, competition and information on both sides of the market, along with fixed transaction costs, produce workplace patterns which preclude out-commuting. But it means that any failure of these assumption may indeed produce rational out-commuting (later in the work to be referred to as res-switching). Indeed, if the mobility and other related assumptions fail badly enough, firms at WP may offer a wage premium sufficient to justify a residence located on the wrong side of CC (to be later referred to as cc-switching).

Allowing for the possibility of circumferential commuting or offsets, White draws a curve through point A to indicate the set of locations which, like A, will benefit from commuting to WP. Hence any offset point of residence within the set will generate commuting to WP, and any offset point of residence outside of the set will generate commuting to CC. In theoretical terms, this seems pointless since swapping is assumed to be available to put the resident on a superior point along the ray. But if there are any effective impediments to mobility, such offset points may well remain, and fixed patterns of activity which inhibit mobility can be related in this manner to both switching behavior and offsets.

This analysis, combined with the rationale for offsets analyzed with reference to Figures 5.2 and 5.3, is consistent with the major thrust of the empirical analysis of this study with its emphasis on offsets and switching.

Switching is to be considered sub-optimal, and can be analyzed in terms of the normal optimization process – whereby individuals will not choose their residence closer to CC than their job since they can reduce both commuting costs and land prices by moving outward, and hence the optimum lies beyond the job location (i.e., in an unswapped position). As such, it lacks the symmetry of the offset case, and the welfare analysis does not produce a straightforward solution. One might think that if we begin with a resident in a swapped position, benefits would accrue from swapping residences with someone who is in an unswapped position. But unlike the case of offsets, where both residents are in the same boat (both with offsets and able to benefit from their removal), there is no unswitched resident to swap with who will gain from becoming switched (if Alonso optimizing is the governing factor), and swapping with other switched residents yields no overall gain. White (1988a, footnote 6) recognizes this when she concludes, "commuting by the two workers together must increase if they switched [in this case] jobs".

In other words, there is no one on the unswitched side of the (let us assume same) workplace who would benefit from such a swap – because the latter would have to move to a less desirable location incurring greater housing costs for no (or insufficient) reduction in commuting costs. At a minimum, there would be limited opportunities to swap, and these would be only with people who are not optimizing (e.g., concerned only with amenities). Given the finitude of the land, we cannot resort to a perfectly elastic supply of housing to escape this dilemma, nor does the model provide a means of allowing the optimizer to bid up the price at the desired location.

Hence in cases which involve switching, the average actual distance from city centre cannot be interpreted as the average optimal distance – because there is no hypothetical swapping process available to optimize locations, at the end of which every actual location has become an optimal location for someone. As a result, the difference between average actual residential distance from city centre and the average actual workplace distance from city centre cannot be construed as the optimal commuting distance, and cannot be used to determine excess commuting. A similar argument treats a job location further out from CC than the residence as suboptimal, with the same basic dilemma.

5.2.2 Inter-dependency and Rings

This analysis pays no direct attention to one aspect of inter-dependency, namely the relationship between the willingness to pay for housing and the location of the householder's job. White analyzes this in detail in a paper of the same year (White, 1998b). The previous analysis assumes by default that the slope ($\partial p_{res}/\partial u_{wp}$) of that relationship is zero. Let us assume (reasonably) that moving the workplace outward (toward the residence) has little impact on disposable income -- as the workplace moves outward, lower wages are balanced by lower commuting distance and the effect on housing consumption is small. But commuting costs are down as a proportion of total expenditure, both in total and per unit distance. This means, according to White (1998b, p. 140), "that households' rent offer curves become flatter as workers' jobs become more suburbanized.". In other words, there are two types of households who differ in terms of the shape of their preferences. That doesn't alter the result, which is still consistent in-commuting. But it does produce two rent offer curves in Figure 5.6, where $p_1(u)$ represents those households who work at CC and $p_2(u)$ represents those households who work at WP. The solid envelope curve is the effective housing offer gradient, representing the fact that CC workers will bid highest for residential locations from CC to WP, and WP workers will bid highest for residential locations beyond WP. (See White, p 146, for the analysis of the same result by means of an internal envelope curve in the case of wages.)

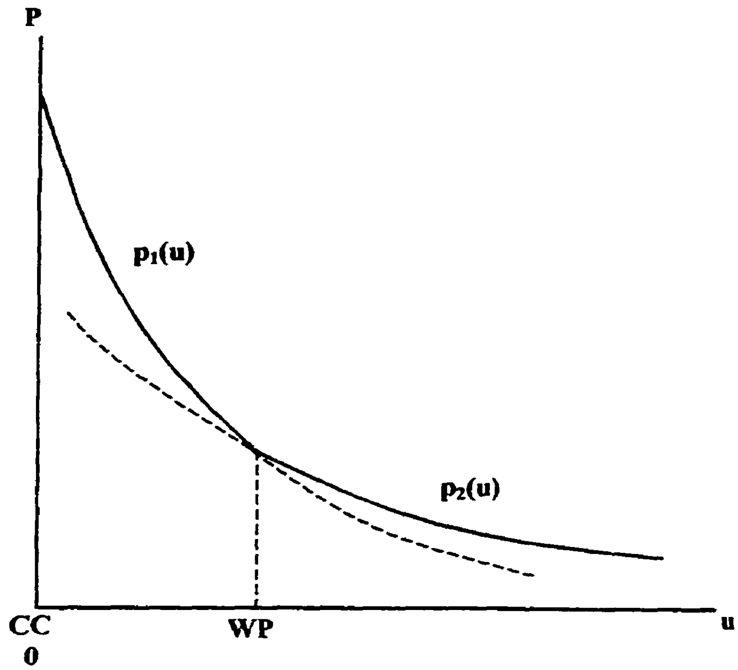


Figure 5.6
Segmentation without Cross Commuting

The result is a clear residential dividing line, providing a starting point for the development of bands or rings of like-minded residents. In particular, it only requires the introduction of some agglomeration economies for residents to cause those on the solid portion of $p_2(u)$ to condense into a ring at the distance at or near WP.

If however $\partial p_{res}/\partial u_{wp} < 0$, the shallower $p_2(u)$ now depicts centrally-located workers instead, who will outbid suburban workers for suburban jobs rather than central jobs. The result is considerable out-commuting – and in fact a tendency for residential groups to cross each other on the way to work. All it takes is a perception (possibly mistaken but possibly accurate if the wage gradient does not conform to the standard slope) in the short run (unless corrected by search) that taking suburban jobs may produce higher rather than lower wages, and the result may be a steeper offer curve relative to suburban jobs, producing this phenomenon of significant out-commuting. However White considers outcommuting in this scenario to be less than realistic and hence unlikely.

5.2.3 Wasteful Commuting

Hamilton's (1982) article generated some more careful definition of optimal (sometimes called "required") commuting, in relation to excess or wasteful commuting. The discussion facilitated tightening up the definition of exactly what an optimal commute entails. The strict definition of optimum would take the idealized conditions of the model as the baseline, assume that the conditions of geographical homogeneity, competition and perfect mobility prevail for purposes of setting a norm, and define the resulting perfect spatial patterns (including required commuting distances) as optimal. Then any commuting in excess of this optimized distance constitutes excess or wasteful commuting. In spite of Small and Song's (1992) preference for a more neutral term, "wasteful" is correct in this context of deviation from perfect efficiency. White (1988a) makes the argument that a more functional definition is needed for real world applications, where for short-term decision-making purposes, perfect mobility is an artificial abstraction and the residence- or job-seeker is dealing with fixed patterns of housing developments and employment centres. In addition, adding to White's point, real-world employers do indeed offer wage premiums to meet specific labour demands in imperfect markets. In that context, Diagram 5.16 suggests that there will likely be a significant amount of both workplace-residence switching and offset locations – all of which are perfectly rational and optimizing decisions in a fixed-configuration environment. Hence "excess commuting" must be calculated to exceed this kind of behavior.

White goes on to wrestle with the question of exactly how to define this imperfect but appropriate commuting, in order to separate out the excess component. She develops three categories (p. 1103). First, there is the category mentioned above, due to "concentrations of employment", which can produce rational out-commuting and circumferential commuting. Second, there is the inefficiency produced by a fixed, often rectangular network of roads which precludes taking the shortest distance between any two points. Third, there is what she labels "cross-commuting", namely "that which could be shortened if workers traded jobs or residences", in the Pareto-optimizing sense outlined at the beginning of Section 5.2.1. Only the third is capable of being eliminated, so only cross-commuting should be counted as "excess commuting". Care must be taken, however, not to lump out-commuting and cross-commuting together in a single diagram like that above without carefully distinguishing them, as will be discussed below.

Now let us examine more closely the model, based on the analysis of Hamilton, in which offsets and switching may result in excess commuting. If excess commuting is commuting that could be eliminated in a Pareto-optimizing sense, then it equals the difference between actual commuting and optimal commuting.

Consider offsets, represented in Diagram 5.17. For agent A, the actual location of the residence is $^{ACT}RES_A$ and the location of the workplace is WP_A. Clearly $^{OPT}RES_A$ would be more optimal, since it is on the same ray as WP_A (avoiding the hypoteneuse). With Hamilton's assumptions of residential density and continuity, there will exist an agent B who lives at $^{OPT}RES_A$ but works at a location such as WP_B. In this circumstance, both agents A and B will benefit from simply swapping residences (in the simple case of identical housing bundles).

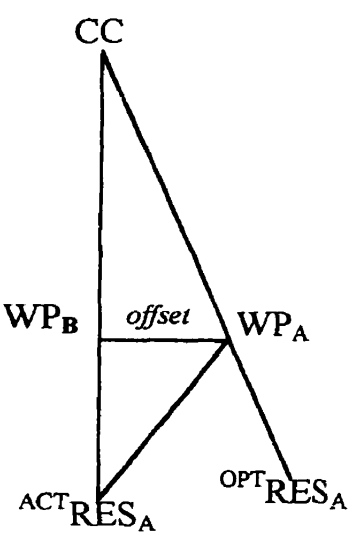


Diagram 5.17
Excess Commuting Offset Example

In other words, the unrealized optimum for agent A could be achieved if Pareto swapping were to place the actual $^{ACT}RES_A$ at $^{OPT}RES_A$ instead. That is, $^{ACT}RES_A$ and $^{OPT}RES_A$ would change places, such that $^{OPT}RES_A$ is on the same ray as WP_A. The resulting optimal commuting distance WP_A-to-$^{OPT}RES_A$ would be smaller than the actual commuting distance WP_A-to-RES_A since the "circumferential element" (offset) has been removed. Hence Hamilton can compute the average optimal commute as the average of all $^{ACT}RES_A$, since each $^{ACT}RES_A$ represents an optimum after all switches have occurred. Actual commuting distance is WP_A-to-$^{ACT}RES_A$ which, when WP_A-to-$^{OPT}RES_A$ is subtracted from it, produces on the average a positive measure of excess commuting.

A similar analysis applies to the case of workplace offsets.

Now consider switching. In the simple, standard one-dimensional linear model of Diagram 5.18, if excess commuting occurs its only possible source is switching – whereby the actual $^{ACT}RES_A$ lies between WP_A and CC, rather than at an optimal position such as $^{OPT}RES_A$. If the Hamilton analysis were applied, swapping would allow all residents to improve their locations and find their most preferred possible locations. That is, every actual location is a preferred location for someone, and in that sense may be considered an optimal location on average. Hence the optimal and therefore expected commuting distance would be the distance WP_A-to-$^{OPT}RES_A$, which can be compared on average to actual commuting distances to determine the average excess.

The problem is that, in the case of switched locations, this optimal swapping process may not occur. The benefits would be one-sided. If agent B is at $^{OPT}RES_A$, how could he/she possibly benefit from swapping to $^{ACT}RES_A$? Only if agent B works at a potentially optimal location such as WP_B. The result is that agents with small RES-WP spreads may potentially be able to exchange only with agents with larger RES-WP spreads. But if a widely-spread agent needs to swap, he/she may not find anyone with whom to swap. There is an asymmetry in the swapping scenario, which means that Hamilton's full optimization to place everyone at preferred locations may not occur. Hence average actual locations cannot be considered to represent average optimal locations for purposes of comparing commuting distances and calculating excess.

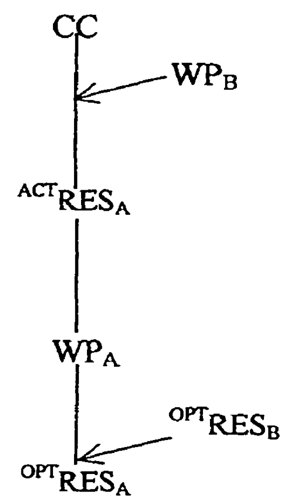


Diagram 5.18
Excess Commuting Switching Example

Summarizing the switching case, the swapping that allows ${}^{ACT}RES_A$ to represent ${}^{OPT}RES_A$ at a higher level of utility does not occur because the benefits would be one-sided. In fact, there is no way of estimating the average value of ${}^{OPT}RES_A$ where switching is involved, and it remains only a theoretical category. The result is that an unidentified proportion of Hamilton's measure of ${}^{OPT}RES_A$ is meaningless, (unless he had explicitly excluded switched data from his analysis) and the same criticism would then apply to his measure of excess commuting. A meaningful measure of excess commuting in this manner would require the isolation of only the offset but unswitched data.

For further clarification, consider Diagram 5.19.

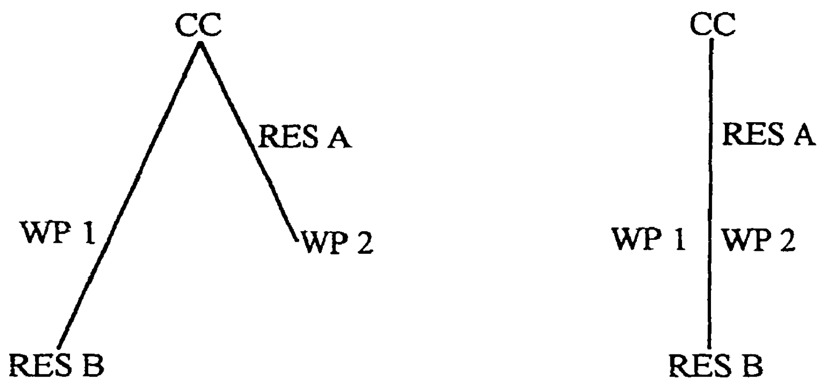


Diagram 5.19
Isolation of Switching Effect

In the left-hand panel, it would appear that Agent A living at RES A and working at WP1 would reduce commuting distance by swapping with Agent B living at RES B and working at WP2. Both parties would benefit. But that is only because of the offsets involved. If the offsets are removed so that the switching aspect of this configuration is isolated, as in the right-hand panel, then it is clear that Agent B at RES B would not benefit by swapping to a sub-optimal position at RES A. Hence mutual-benefit switching derives only from the offsets – and the switching aspect of this configuration may partially counteract the benefits derived from offsets.

In addition, it is not clear that switching produces greater commuting. In the case of res-switching, it is just as likely to produce less commuting since the position of RES_{ACT} in the linear diagram above could be anywhere between CC and WP, and it may therefore lie close to WP. Hence lumping them in with excess commuting due to offsets will bias the results by an unknown amount, but quite possibly significantly downward unless there is sufficient cc-switching to push it upwards. Hamilton essentially acknowledged the existence of "reverse-flow commuting", but he failed to treat them as anything different from "circumferential commuting" because he used a simple measure of average distance to city centre for all sample points in his calculations without determining whether reverse-flow commuting was involved. Even in his discussion of possible biases in the data and determinants of wasteful commuting (1982, pages 1044-1049), he does not acknowledge the role of reverse-flow commuting.

White makes the necessary distinction between out-commuting and cross-commuting in her analysis, but proceeds to employ an analysis which does not account for the distinction. White's analysis (1988a, p. 1104 ff.) is based on urban jurisdictions each of which can be classified as residential or industrial. In the simplest case, someone who lives in an industrial jurisdiction but commutes to another industrial jurisdiction is travelling unnecessarily. White is able to determine the minimum travel necessary to get from each residence to the nearest possible industrial jurisdiction, in order to determine the average required commute. The matrix incorporates travel times between jurisdictions in a way which captures industrial and transportation structures, and all that is left to qualify as excess is cross-commuting. Unfortunately the question of whether some of it may be out-commuting (switching) is not addressed. In his response to White, Hamilton (1989) points out that her optimum figure is a creature of jurisdiction size, since decreasing the size of jurisdictions would likely have the effect of reducing the measured optimal commuting. Small and Song (1992) support this suggestion and produce rather large estimates based on the smallest possible jurisdictions. Merriman, Ohkawara and Suzuki (1995) use a similar methodology to Small and Song, using intermediate jurisdiction sizes, with results similar to White. Suh (1987) generalizes this model, dividing the urban area into standardized unitary blocks as the theoretical counterpart of jurisdictions. In the analysis to follow, Hamilton's results will be the initial reference point for purposes of comparison, followed by consideration of how White, Small and Suh reported somewhat different results.

5.2.4 Estimating Wasteful Commuting

The approach of this paper is to work with cross-commuting, resulting from offsets, directly. This is partly because the required jurisdictional data was not readily available from the survey employed, and more importantly because the data that was available allowed direct calculation of offsets and hence a direct measure of that principal component of excess commuting. It would appear at first glance that this requires us to limit the analysis to those respondents who were not switched. However, using the dataset for the current analysis, it is possible to isolate the offsets for both switched and unswitched respondents, and used them along with the angle of offset to calculate WP_1-to-RES_{ACT} in Diagram 5.20 which represents commuting distance with the portion of WP-to-RES_{ACT} which is due to the offset, removed. To the extent that excess commuting means extra commuting due to offsets, WP_1-to-RES_{ACT} could be considered to represent it.

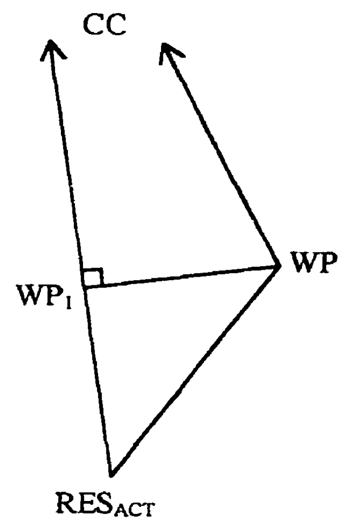


Diagram 5.20
Measure of Excess Commuting

The data show a striking difference from Hamilton's measures of excess commuting. By Hamilton's calculation based on data from 14 American cities (Hamilton, 1982, p. 1041), excess commuting was in the range of 7.6 miles, although he cited a possible upward bias of up to 10 miles (page 1042) when he evaluated average random commute instead of average optimum commute. In percentage terms, 87.4 percent of actual commuting was excessive. He went on to suggest that average excess commuting of 7.6 miles indicates a significant failure of the Alonso model.

Estimates of excess commuting using the data of this study, and their relationships to previous estimates, are contained in Section 7.5. They show excess commuting in the range of 1 mile, or 19 percent.

6. THE SURVEY DATABASE

6.1 The Minneapolis Survey

Data used to evaluate the model are derived from the "1992 Land Use/Transportation Relationship Study" conducted in the greater Minneapolis/St. Paul area in 1990 and 1992. This survey was conducted in two parts, initially in 1990, with follow-up questions in 1992.

The sample of people who participated in both years was 1002 persons. The 102 people who had changed residence in 1992 were asked for their new residence, and the 111 people who changed jobs in the same period were asked about their new jobs. But more important were the extended questions asked of all respondents in 1992, which inquired in detail about the last 4 residences and jobs. This gives the survey a longitudinal aspect, and allowed the author to determine the exact workplace at the time of moving and the exact place of residence at the time of changing jobs.

Each residence and workplace included several forms of locational data. One of these is a set of Traffic Analysis Zones (TAZ), each uniquely numbered and each fairly small, constituting maybe 200 people in 20 city blocks. The author was able to obtain the longitude and latitude of the centroids of each TAZ. However, spot checking suggested that there were possible coding problems in the TAZ, since (for example) certain pairs of locations which were expected to be north/south of each other according to census data were found to be south/north of each other on the TAZ map.

Fortunately, more detailed data were available for most survey points. 1990 Census Blocks were available in many instances, and they cover approximately 4 city blocks. Centroids (in latitude and longitude) for these are available from the U.S. Census, and were used to pinpoint locations for the majority of useful survey points. Failing that, where Census Blocks were not available, the actual addresses were available. Mapping services on the Internet were available to identify the latitude and longitude of individual addresses. These had to be obtained individually and manually with great care (since different municipal subdivisions may have streets with the same name), and in many cases these services gave only the latitude and longitude of the Census Block of the address. However, the effort was well rewarded with a high-quality locational database, from which reasonably precise distances could be calculated.

However, the gaps in the data meant that, while 990 current residential locations and 761 current workplace locations could be pinpointed, when current residential locations were matched with workplace locations at the time of the move and current workplace locations were matched with residence locations at the time of the job change, the result was only 374 useful residential locations and 410 useful workplace locations. After culling a little more unreliable data, we were left with 362 residential locations where the workplace location at the time of the move is known, and 404 workplace locations where the residential location at the time of the change is known.

The importance of obtaining this kind of information is evident when we look at the incidence of simultaneous residential and job changes. Using an arbitrary definition of simultaneity, namely workplace and residential changes which took place within 12 months of each other (or never changed), we find that only 63 of the above-described sample points can be considered simultaneous. Hence if we were to follow the common technique of applying simultaneous equations to combinations of current residence and current workplace, they would only be applicable to at most 17 percent of the data. The remaining 83 percent would involve applying workplace and residential locations which were incorrect at the time of the move or change. (The situation is actually worse than this, since even in the definition of simultaneity described above, it is still possible for the current location to be different from the location which applied at the time of the move.) However, the availability of data which gives the workplace location at the time of the residential change and the residential location at the time of workplace change allows us to examine the workplace and residential decisions separately and more accurately.

Table 6.1 indicates the numbers of valid points obtained from the survey, out of a total of 1002 responses.

Variable Description	**Number Valid Responses**	**Mean**	**Standard Deviation**
Current Distances from City Centre			
Residence	990	11.63	6.34
Workplace	761	8.86	5.77
Commuting Distances			
Current	761	7.61	6.37
Residence from Previous WP	362	7.96	6.30
Workplace from Previous RES	404	7.46	6.51

Table 6.1
Available Survey Points

The bottom two rows are for the categories used in this analysis, where Previous WP means the workplace at the time of making the residence decision, and Previous RES means the residence at the time of making the workplace decision. The first three lines contain current data, meaning that all data are those current at the time of the survey.

The most interesting observation is that commuting distances, to relevant and irrelevant locations alike, are all approximately the same. The relevant workplace location is the the one which was in place at the time when the residential decision was made (and vice versa). Current workplace locations are irrelevant in 83 percent of cases if only 17 percent of residence and workplace decisions are roughly simultaneous. If irrelevant means that current workplace locations are randomly distributed, and commuting distances are minimized, then we might expect distances to current locations to be on average greater than distances to relevant locations – which is not the case. But every sequential residence/workplace move is tied to a previous workplace/residence move, such that there will be a tendency for the set of residence and workplace locations over time for any given household to cluster to some degree. In other words, the current location is not as irrelevant as suggested above. Whatever clustering does occur may partially explain the observed uniformity of commuting distances, even though there is little simultaneity in the decisions.

The are reasons as well for not taking advantage of all available sample. Many individual survey respondents listed a history of up to four residences and a number of jobs during a reported period of several years. However, the mixing of some sample points which contain a history of decisions together with other sample points which contain only current decisions introduces a problematical complexity into the analysis.

The Minneapolis survey is unique in its inclusion of labour market questions along with the standard set of residential location questions. An outline of the most relevant data is listed below:

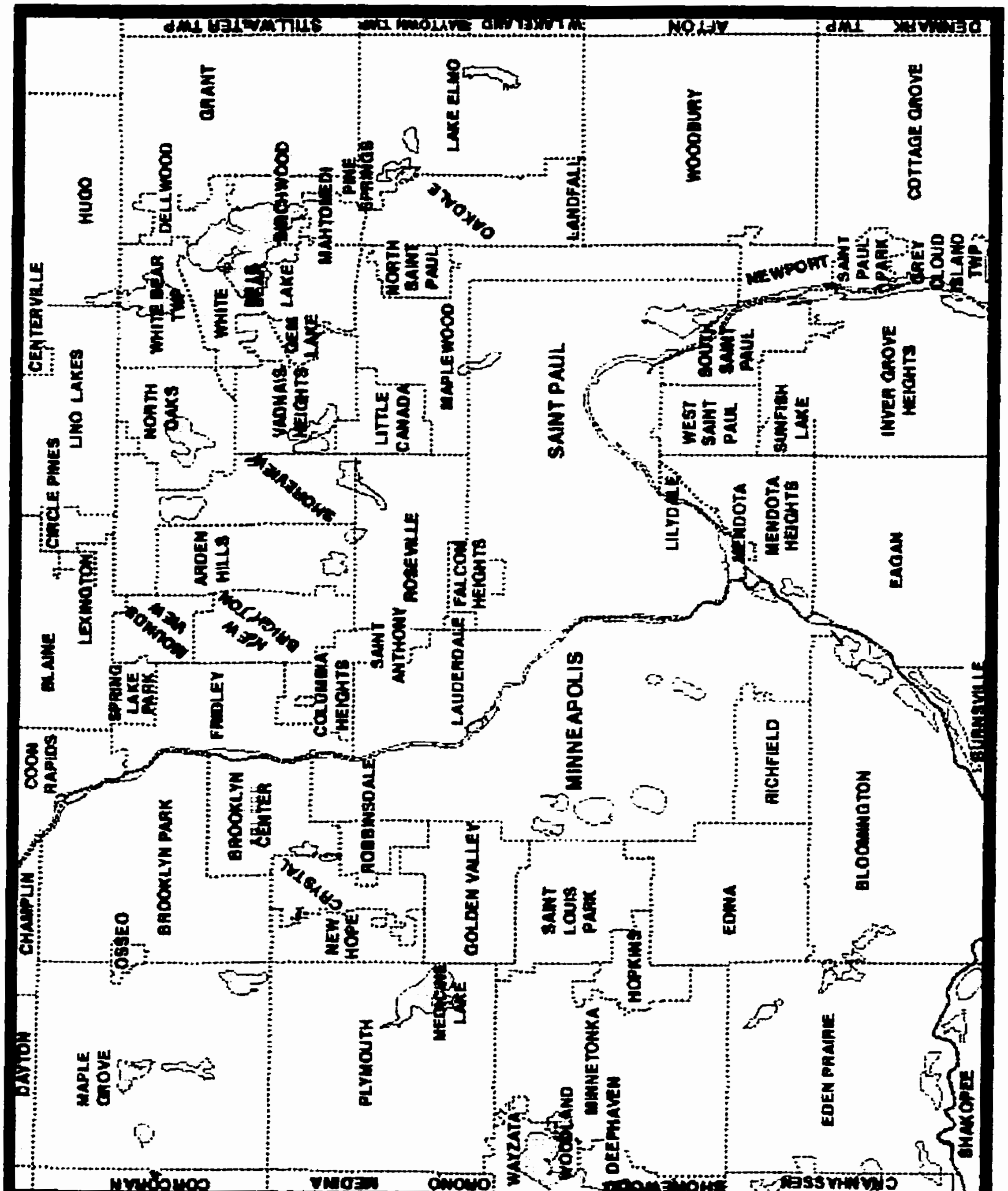
Residential location data:	Job-related data:
Location	Employment status
Search method	Multi-job status
Importance of workplace location	Occupation, industry
Family structure	Transportation mode
Age	*Commuting time*
Ethnicity, gender	Hours of work
Reasons for moving, staying	Job search method
Importance of workplace	Education, experience
What like/dislike about location	Total household income
Rating of location	*Reasons for choosing workplace*

Italics indicate a small number of responses.

Housing prices are not included in the survey. Neither are wages — only family income as a whole. Skill data are limited to education and experience. Perhaps roughly, education could be taken as a measure of general skill, but experience (with its associated on-the-job training) is more likely to be a measure of (or proxy for) general and enterprise-specific training combined. It is hardly adequate. We employ Simpson's terminology (1992, following from Becker, 1964) – where non-enterprise-specific skills are narrower than general skills in that they are general to only certain industries or sectors, while enterprise-specific skills are narrowed to a single employer. Education begins as general training and evolves into non-enterprise-specific training. The benefits of neither are (totally) lost upon leaving a particular employer, so that there may be an investment opportunity available by taking leave of a job for further training. Enterprise-specific training, on the other hand, is more likely to be related to within-job internal promotion, and is hence less relevant to a job-search model. Education level then is relevant to job, but experience is only relevant to the extent that it imparts portable skills. I.e., the latter may be less reliable if it contain an unknown component of enterprise-specific skill acquisition.

6.2 Gradients and the Sample Data

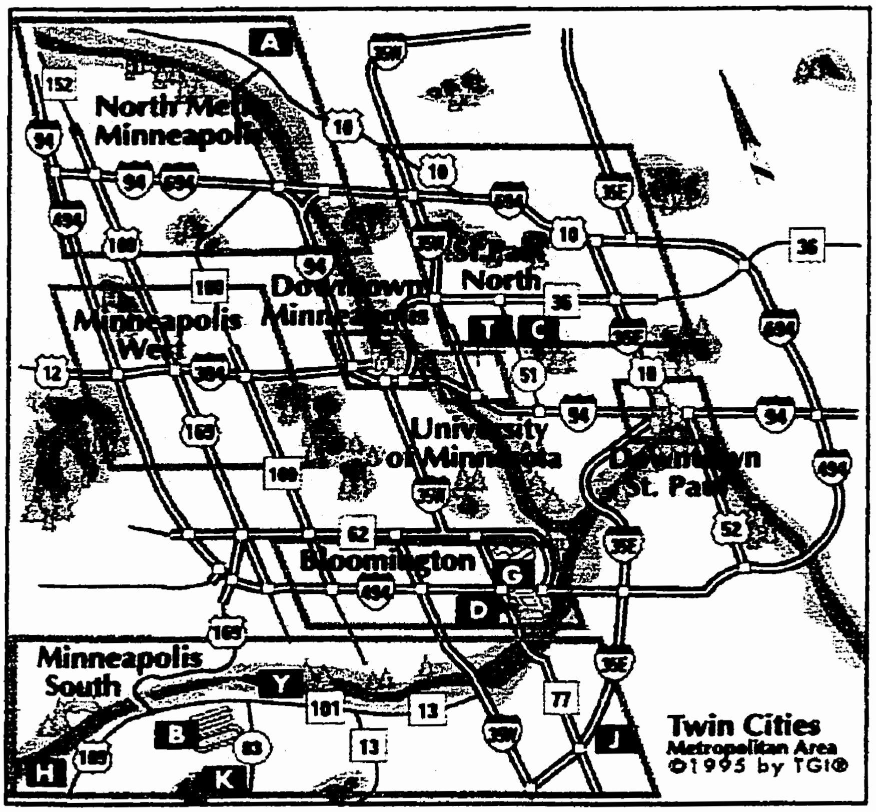
There are no gradients in the survey data. Relationships between housing price and distance, as well as housing density and distance, can be drawn from US Census data. The overall survey area is politically divided into 190 "cities", of which Minneapolis and St. Paul are the two central cities (cities 112 and 166 in an alphabet-based numbering system). See Map 6.1. The set of 190 cities comprises the larger Minneapolis-St. Paul metropolitan area. Although the word "city" is commonly used to refer to a large urban unit in general conversation, we shall follow the Census convention and use the word "city" in lower-case to refer to the 190 small jurisdictions which make up the overall metropolitan area (using the lower-case "metropolitan area" to avoid confusion with the technical Census definition of "Metropolitan Area"). On the other hand, the traditional urban-economics term "city centre" refers to the centre of the overall metropolitan area. Since economic activity tends to be centered more on Minneapolis than St. Paul, the term "city centre" refers to the geographical centroid of Minneapolis city (112) unless



Map 6.1 — Minneapolis-St. Paul Census Divisions

otherwise noted. Housing price and housing density data are available at the city level, as defined above, in the 1990 Census.

Of course, one cannot simply look at a map like Map 6.1 and assume that all regions are equally accessible or equally important. First of all, it is clear from Map 6.2 that the



Map 6.2
Minneapolis Rivers and Transportation

Minneapolis/St. Paul area is divided by major rivers. The initial analysis for this study divided the larger area into broad regions (beginning with three, to conform to the way that rivers divide up the city). This broad regional division, however, was not pursued extensively because (1) it is evident that Minneapolis/St. Paul contains a network of bridges sufficient to ensure that rivers are not a major obstacle to transportation across the city, and (2) because the regional analysis did not produce results appreciably different from the census-division analysis (perhaps confirming the impact of the bridges), except perhaps for the southwestern corner.

An additional structural consideration is the concentration of industry. The historical trend of industrial suburbanization applies also to the Minneapolis area, and there is no shortage of industrial parks and regions removed from the centre. However, as suggested by Grid 8 below, this has not been sufficient to negate the general overall job density gradient. Grid 8 shows a concentration of employment at the centre, with some relatively smaller employment concentrations removed from city centre. But (except perhaps for St. Paul), it would appear that the traditional downward-sloping industrial gradient largely prevails.

It should be noted that any price gradient derived from actual observations will be an optimizing price gradient in the sense that it is theoretically the result of optimizing behavior. That is, the only price gradient we will observe is a locus of optima, which might be interpreted as the result of tangencies of the market price gradient to the indifference curve, but which directly represent neither of the latter. We would have to rely on a crude assumption that measured points on any price gradient are roughly representative in slope and shape of the underlying theoretical price gradients. Given the roughness of the data, and the fact that the market price gradient and indifference curve tend to be of the same convexity as well as nested, that is not an unreasonable starting point.

The Minneapolis data conform roughly to the expected pattern when it comes to densities, but the relationship is much less clear when it comes to price. These relationships are illustrated in Grids 1 to 9. These diagrams, the first seven of which are derived from 1990 U.S. Census data, divide the overall area into regions based on their distance and direction from Minneapolis city (112). For example, in Grid 1, the average housing density index in the third ring in an easterly direction is 0.68. (All locations in the data set have been placed into their respective concentric rings, of 4 miles in width, emanating from Minneapolis city centre.) Clearly there is a rough pattern of housing densities which peak at city centre and decline (with moderate exceptions) with outward distance. A simple exponential regression of housing density on distance from city centre yields an R^2 of 0.61 when distance is measured exactly in miles, and 0.77 when it is measured in rings. Minneapolis proper is also geographically quite central in spite of the proximity of its "twin" city to the east, St. Paul. However, there are also healthy densities to the west, northwest and north, which rival St. Paul to the east, plus the fact that densities are mediocre to the northeast. Hence Minneapolis remains an appropriate choice for overall city centre.

Housing prices are quite another story, as seen in Grids 2 for prices and 3 for rents. There are at least three major price peaks. Most of them are in the third ring and beyond. Several of these price peaks are quite suburban, which would not surprise the average house seeker in many an urban area. They vary greatly in different directions, in both the level of the peak price and its distance, and there are numerous inconsistencies which make it difficult to see a smooth price gradient in many of the directions shown. It comes as no surprise that R^2 measurements from any of the obvious candidates for price peak run no higher than 0.23. R^2 may be higher for subsets of observations restricted to a single direction, but their usefulness may decline with fragmentation.

On the surface it may seem that we can treat the data as indicating a reverse Alonso relationship in terms of price. That is, perhaps we can assume a price gradient which is lowest at city centre. In a sense this does not negate the model – in that it is possible to reproduce the optimizing equations such that the curves of Figure 2.2 look identical, except that the they are switched around in a horizontal mirror image (as illustrated in Diagram 2.2). The result would be to reverse the positions of the optimum residence and workplace in relation to city centre. In other words, it might be considered that the same essential dynamic applies in a different configuration with mirror-image results. However, this not only constitutes a denial of the core of the Alonso model, it fails to account for the role of amenities, discussed in Chapter 3. If the influence of amenities, in the bundle of residential and neighbourhood characteristics which makes up a vector of housing attributes, is factored out of the data – the data give no grounds to abandon the traditional downward-sloping price gradient.

Another alternative is to choose an average ring in which prices are highest, and measure distances from that peak ring, rather than distances from city centre. There might be a reasonable gradient peaking on ring 4, for example, obviously a compromise candidate. Visual perusal of Grids 2 and 3 suggest that it might produce a better R^2 than the crude reverse Alonso model, and may be worth a test model. A far more computationally expensive technique would be to produce a jagged reference ring, where each direction peaks at a different distance. However, the most practical approach is to choose a set of 3 or 4 price peaks and calculate the distance of each sample point from the nearest peak. These price peaks would not necessarily the highest peaks observed, but they should be well positioned in the sense of being at the centre of a general region of top prices. By that criterion, combining prices and rents, the prime candidates for a set of useful housing price peaks are: W5, SE3, NE4 and N7. All of these represent compromise choices, since it is not possible to find a price peak which exhibits a smooth declining gradient in all directions without major discontinuities. The above set, by careful observation of the housing cost diagrams, seems to contain the most promising candidates. The most questionable is N7, since it is far enough north to be considered "off the map" in relation to the Twin Cities metropolitan area, and it is a peak for rent only (not price).

There is a further reason why the normal Alonso gradient cannot be dismissed on the basis of a seemingly upward-sloping relationship. The earlier analysis of housing noted that consumers do not purchase simply a single unit of housing. Rather, they choose various combinations of multiple housing units, defined in several dimensions. Hence simple comparisons of housing purchases (as in Grids 2 and 3) end up comparing total expenditures rather than prices per unit. While the former may sometimes serve as a crude proxy in the absence of per-unit data, it is not sufficient to support the proposition that a reverse Alonso gradient should be substituted for a normal one.

Data on housing units are not directly available from the U.S. Census database. Square footage of housing might be a crude indicator, if it were available. The closest available proxies for units of housing are the number of bedrooms and the total number of rooms in each individual residential unit. The latter would probably bear the closest correlation to square footage. Grids 4 to 7 present per-unit housing prices calculated on this basis.

Grid 4 shows average housing price per bedroom. It does not differ highly from Grid 2, except that the western price peak largely disappears. In other words, residents to the west are opting for more bedrooms, i.e. clearly buying more units of housing for those high prices. Apart from that, few new conclusions about the price gradient can be drawn from Grid 4. The technique of highlighting used in this instance (i.e., the rationale for the boxed figures) is to choose threshold values at a high enough level to reveal the relatively high-priced locations.

Grid 5, average rent per bedroom, is a little more suggestive of change. The rent peaks do not appear to be much different from Grid 4 except that the western peak reappears and the high-rent locations are moved a little further inward toward the centre. Clearly renters to the west are not so inclined to opt for a high numbers of bedrooms (which has no particular interpretive value), and renters in general are moving more in the direction of a normal Alonso gradient, except for the very centre. That is, the cluster of high-rent locations appear to be more grouped around city centre, but fall outside of city centre itself. The significant fact is that the use of crude per-unit housing prices in this instance strongly supports the normal Alonso relationship. Nevertheless, setting the threshold low enough to include city centre would result in too many locations identified to produce a useful relationship.

Grid 6 shows average housing price per room; that is, total number of rooms. It differs little from Grid 4, except that the price peak to the west reappears (which peculiarly seems to represent an especially strong preference for a high ratio of bedrooms to total rooms).

Grid 7 shows average rent per room. This shows the weakest evidence for rejecting the hypothesis of the normal Alonso relationship. Threshold values are clustered more toward city centre than any of the other grids. If the threshold were only one dollar lower, it would include city centre itself (though the number of highlighted locations is already rather high).

Considering Grid 7 in particular then, there is not a strong enough case to support the argument that a normal downward-sloping Alonso gradient should be rejected. Given the normal-shaped density data, the likelihood that we are working with some rather inadequate proxies for numbers of housing units, and the strong possibility that higher prices outside of city centre are the result of purchasing more units of housing (and indeed total number of rooms shows some rough degree of upward slope from city centre) – the Alonso price gradient remains the dominant contender.

Job density presents a daunting problem. Census data give the number of employed people at the place of residence, not at the place of employment. In other words, it gives information only about the residential location of employed people, and nothing about the location of jobs. Similarly the Census gives incomes, not at the place where they are earned, but at the place where the earner resides. The only direct Census information about job location is whether or not they work in the "Central City". There exist maps of the overall area which can give a rough visual indication of where the industrial areas are located, and these are clearly concentrated in the Minneapolis-St. Paul central area. The only desegregated data available at the time of writing is contained in the survey sample dataset itself. It is possible to determine the number of jobs of people surveyed contained in each of the municipal areas. Grid 8 shows the number of job location sample points according to direction and distance from City Centre. The figure confirms the heavy concentration in Minneapolis-St. Paul indicated in the maps, and hence may give some degree of confidence in the technique of using sample point locations to determine job density patterns. However, there are several locations with only a handful of observations, which may perhaps not be considered reliable. In general, Grid 8 suggests that Minneapolis and St. Paul jointly could be considered a job density peak location, but closer examination of a much greater job concentrations south and west of Minneapolis compared to the east side of Minneapolis gives credence to a strategy of sticking to Minneapolis as the effective job density centre.

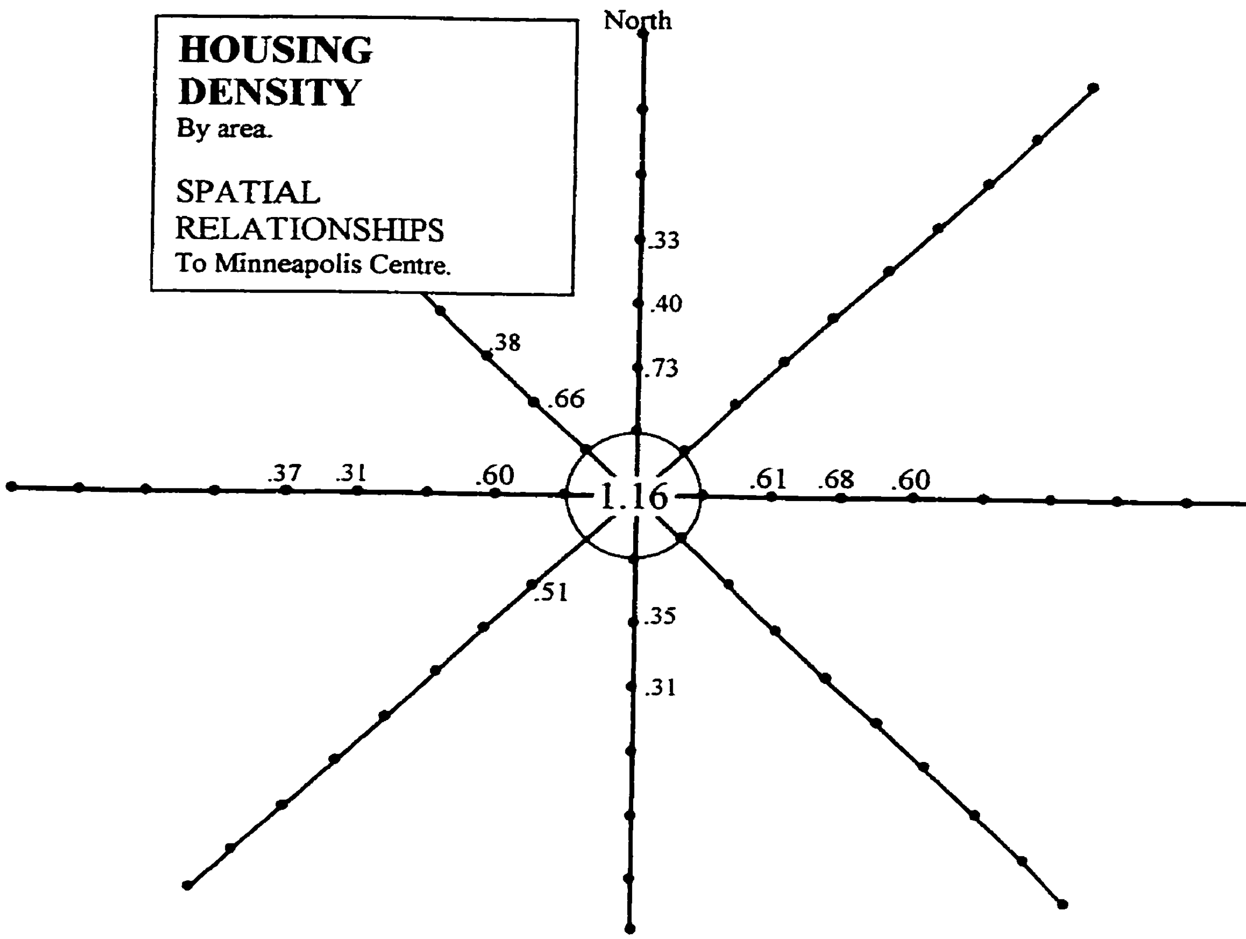
As in the case of residential housing, there is the problem of defining what constitutes a unit of employment. Each job may represent a bundle of job characteristics, and some bundles may be considered a greater number of employment units than others. The kinds of data that would allow such distinctions, however, are not available for this study. In another vein, jobs per square foot may be a better measure of actual job density. But the sample was not designed with the intent of maintaining the ratio of sample jobs to actual jobs in the regions, and hence a precise job density of this type cannot be expected to be very accurate. The data summarized in Grid 8 is adequate to support the suggestion that a job density peak at city centre is a viable starting assumption.

Grid 9 gives indices of family income at job-location sample points. These data are problematical since family income aggregates family members who may be working at entirely different locations, and it may be more appropriate to use education or experience as predictors or individual income. They are also problematical because several of them are indicated at locations where there are very few sample points. These questionable data points are italicized in Grid 9. The above-mentioned analysis, of choosing a small set of well-positioned income peaks and calculating distances to the nearest one, might yield the following peaks: N4, E4 and SW3. Once again, this is an imperfect compromise selection with many imperfections and cannot be expected to produce very clear gradients. A closer examination of the diagram also suggests that ring 4 might qualify as a rough income peak. Given the tentativeness of confidence in the income data (drawn from such a small number of sample points), perhaps a simple peak-income ring in combination with a simple city-centre-based income gradient will give the best results that can be expected for the job-related variables.

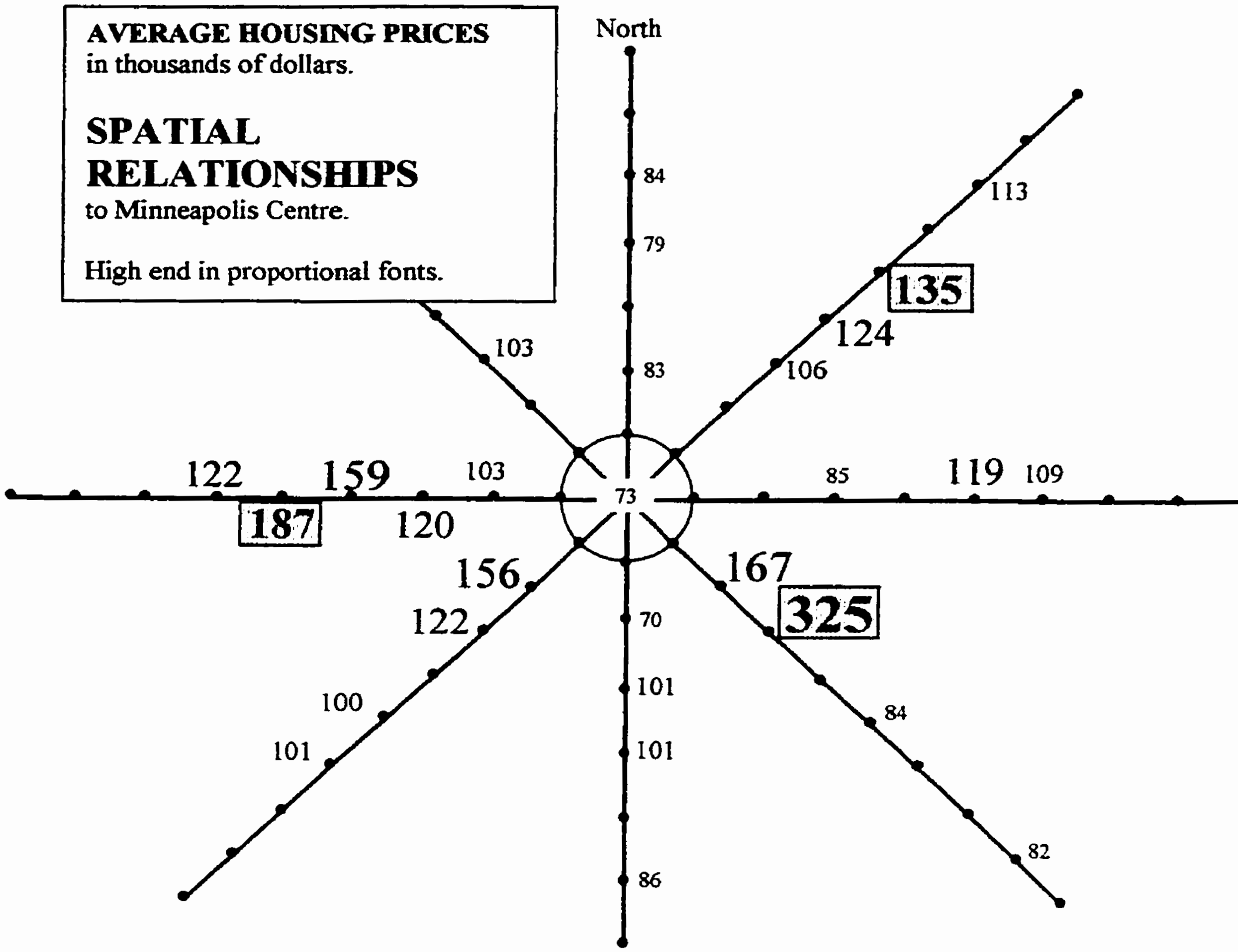
At first glance, there could be some suggestion that a reverse-Alonso Minneapolis-based gradient might show a rough fit in the case of incomes too. In this case, there is no data available to indicate units of employment (comparable to units of housing) even though jobs are clearly no more homogeneous than housing. It should be sufficient, however, to conclude that – just as in the case of housing prices – if per unit data were available and employment amenities were specifiable, we cannot expect to find sufficient grounds to reject a normal Alonso gradient.

In summary, the analysis looks at some of the possibilities:

- Use a simple housing density gradient emanating from Minneapolis city centre (which is city 112).
- Consider a triple-peak nearest-peak gradient based on regions W5, SE3 and NE4. These are best represented respectively by cities 114 (Minnetonka), 128 (North Oaks) and 170 (Sunfish Lake). Alternately, the model may be extended to include distance to the nearest of these three peaks as an additional variable.
- Use a simple job density gradient emanating from Minneapolis city centre.
- Consider a simple Ring-4 income-peak gradient, or a triple-peak nearest-peak gradient based on regions N4, E4 and SW3. These are best represented by cities 34 (Coon Rapids), 133 (Oakdale) and 47 (Eden Prairie). As above, an alternative would be to include distance to the nearest of these peaks as an additional variable.
- Possibly experiment with cruder units in future extensions – such as city centre, immediate surroundings and outskirts – or inside-ring-3, rings-3-4.


North
HOUSING DENSITY
By area.
SPATIAL RELATIONSHIPS
To Minneapolis Centre.
.33
.40
.73
.38
.66
.37
.31
.60
1.16
.61
.68
.60
.51
.35
.31

Grid 1


AVERAGE HOUSING PRICES
in thousands of dollars.
SPATIAL
RELATIONSHIPS
to Minneapolis Centre.
High end in proportional fonts.
North
84
79
83
113
135
124
106
103
122
159
103
187
120
73
85
119
109
156
122
167
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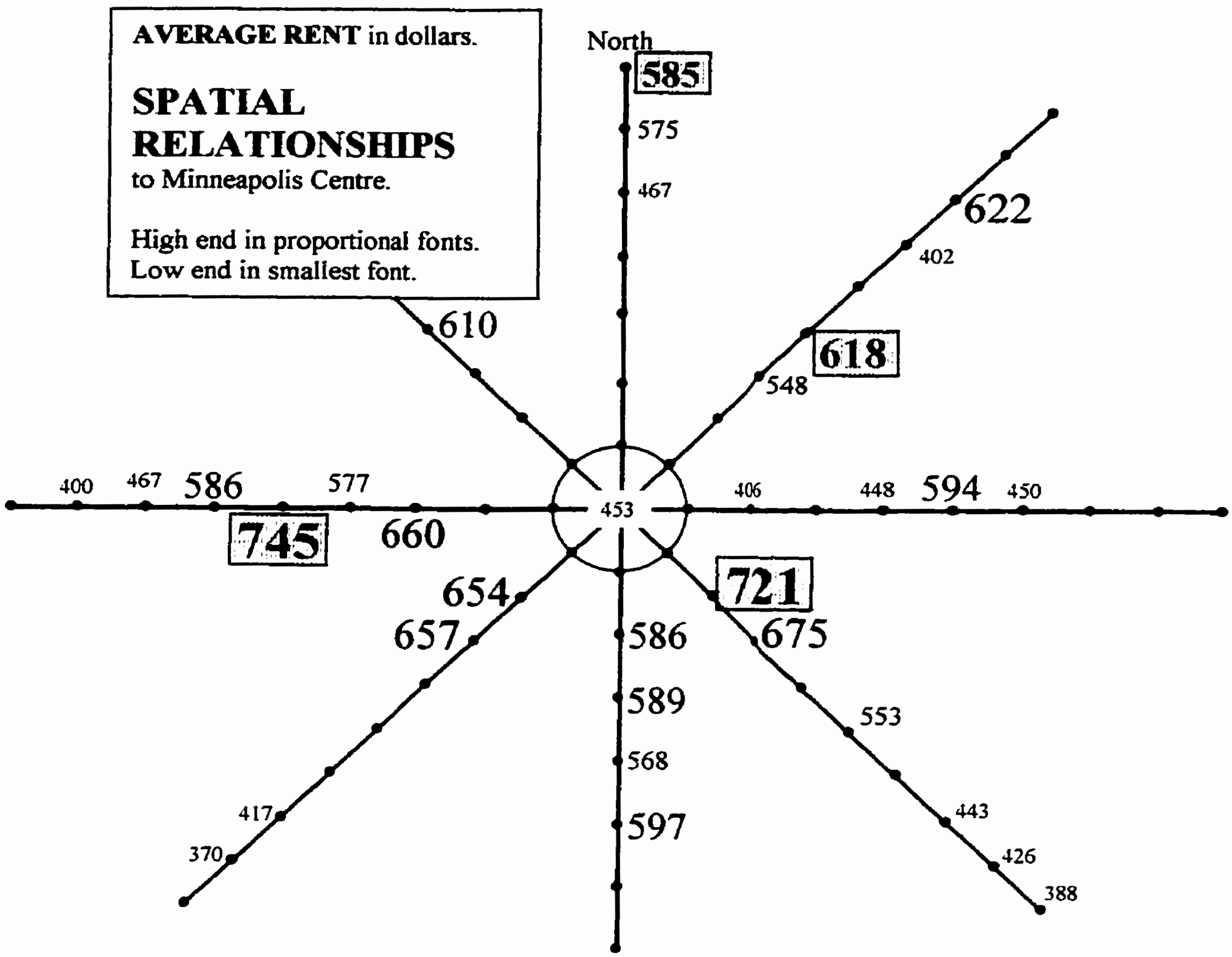
101
101
101
84
82
86

Grid 2



AVERAGE RENT in dollars.
SPATIAL RELATIONSHIPS
to Minneapolis Centre.
High end in proportional fonts.
Low end in smallest font.
North
585
575
467
622
402
618
548
610
400
467
586
577
745
660
453
406
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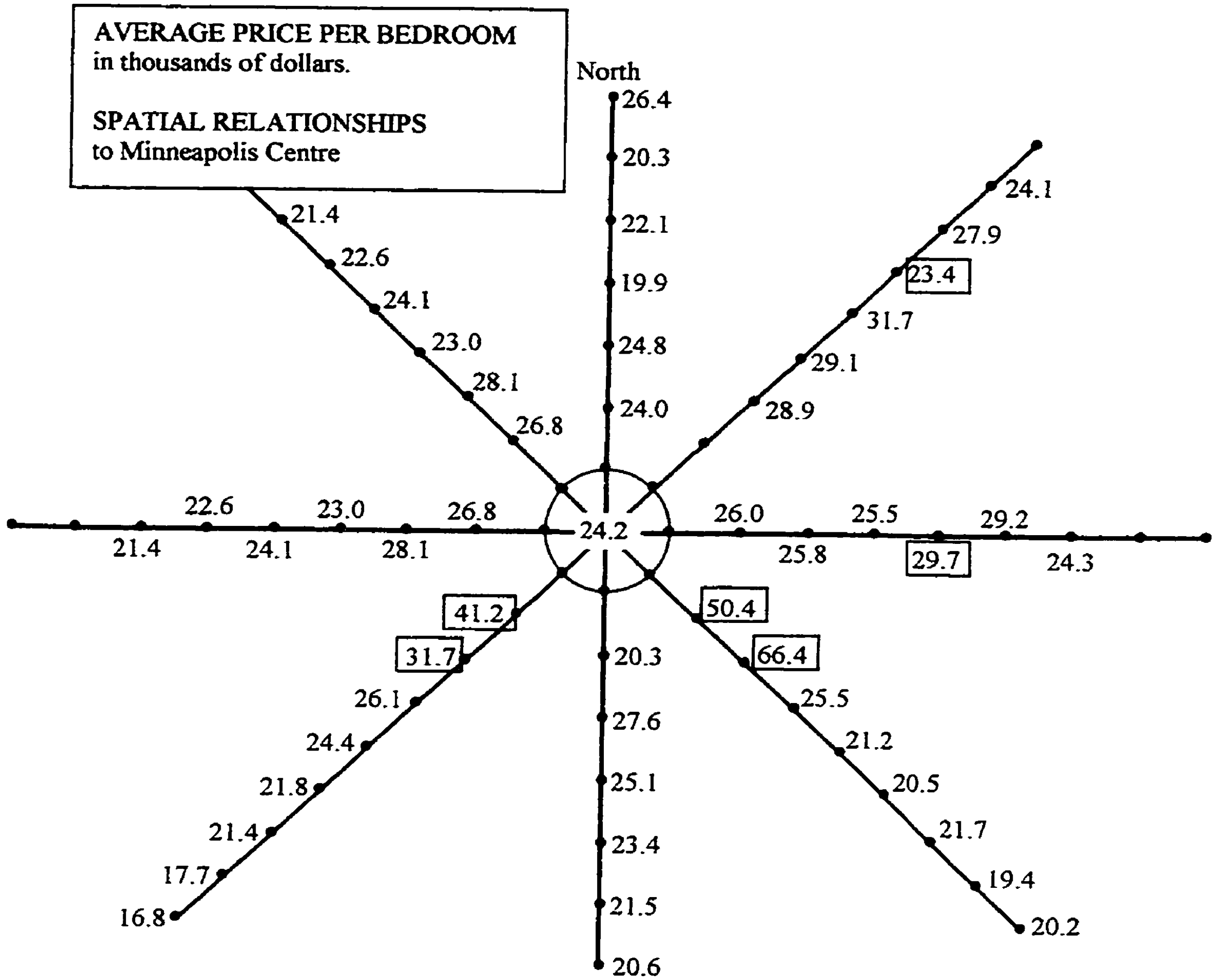
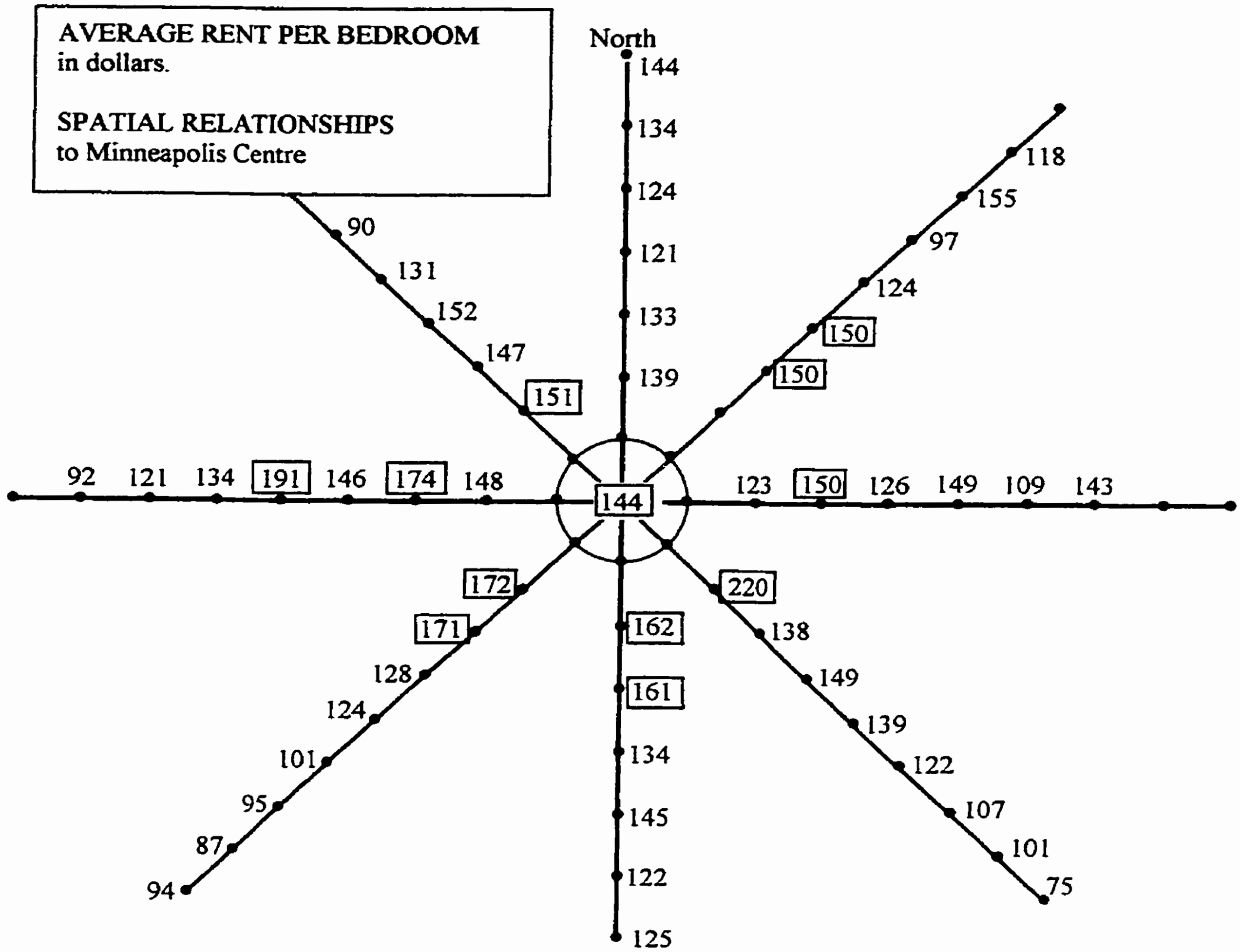
594
450
721
675
654
657
586
589
568
597
553
443
426
388
417
370

Grid 3


AVERAGE PRICE PER BEDROOM
in thousands of dollars.
SPATIAL RELATIONSHIPS
to Minneapolis Centre
North
26.4
20.3
22.1
19.9
24.8
24.0
24.1
27.9
23.4
31.7
29.1
28.9
21.4
22.6
24.1
23.0
28.1
26.8
24.2
22.6
23.0
26.8
21.4
24.1
28.1
26.0
25.5
29.2
25.8
29.7
24.3
41.2
31.7
26.1
24.4
21.8
21.4
17.7
16.8
20.3
27.6
25.1
23.4
21.5
20.6
50.4
66.4
25.5
21.2
20.5
21.7
19.4
20.2

Grid 4


AVERAGE RENT PER BEDROOM
in dollars.
SPATIAL RELATIONSHIPS
to Minneapolis Centre
North
144
134
124
121
133
139
118
155
97
124
150
150
90
131
152
147
151
92
121
134
191
146
174
148
144
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126
149
109
143
172
171
128
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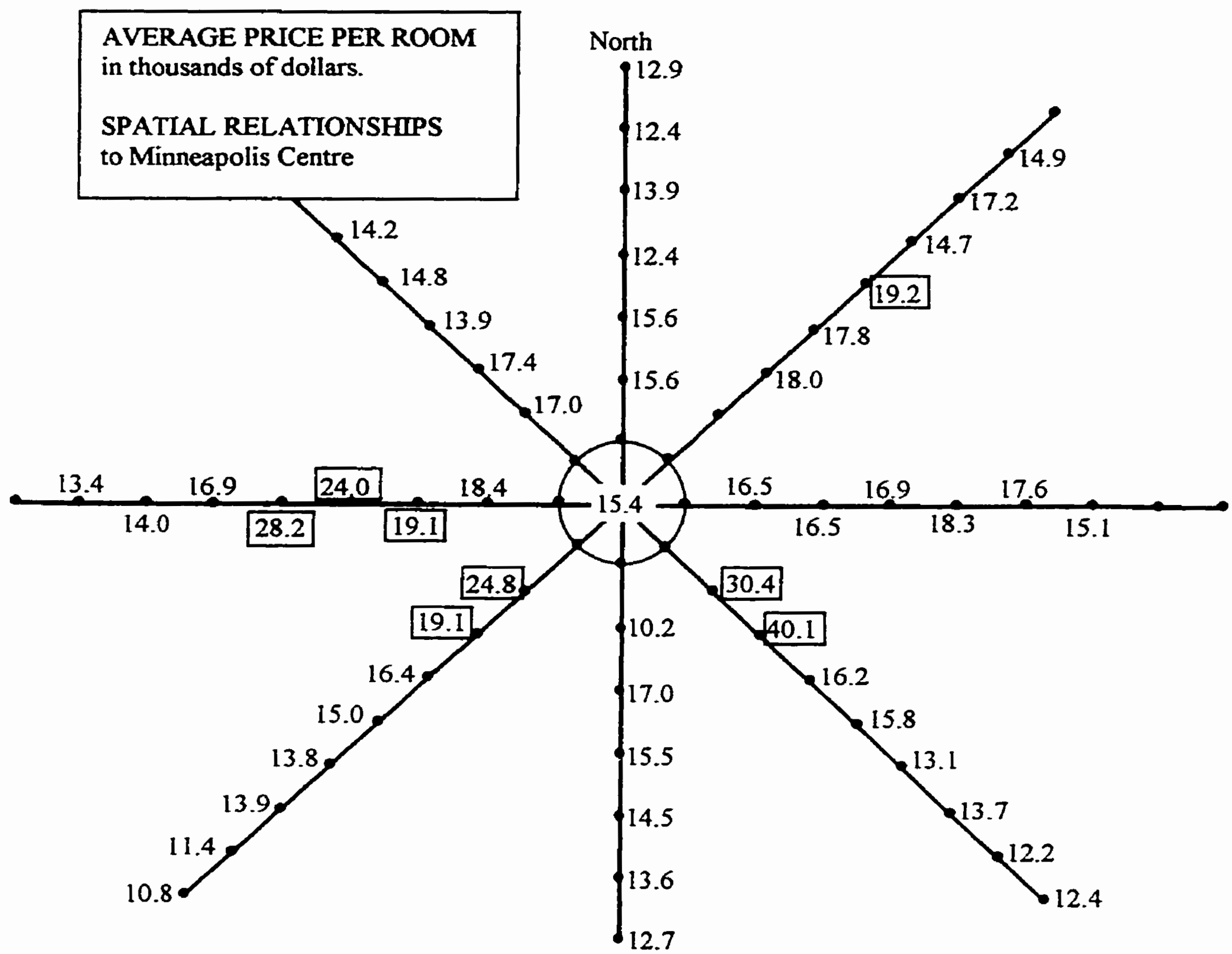
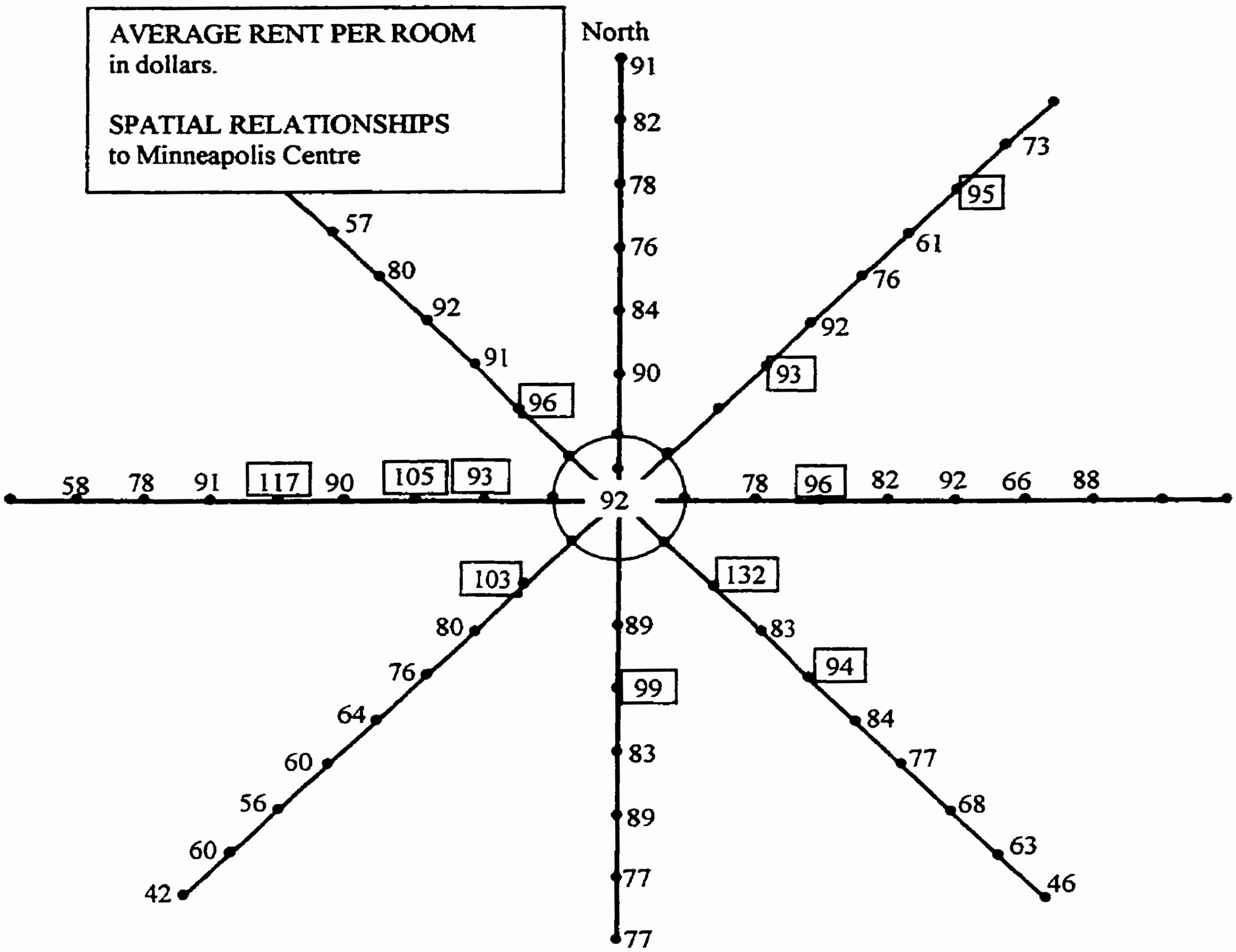
139
122
107
101
75

Grid 5

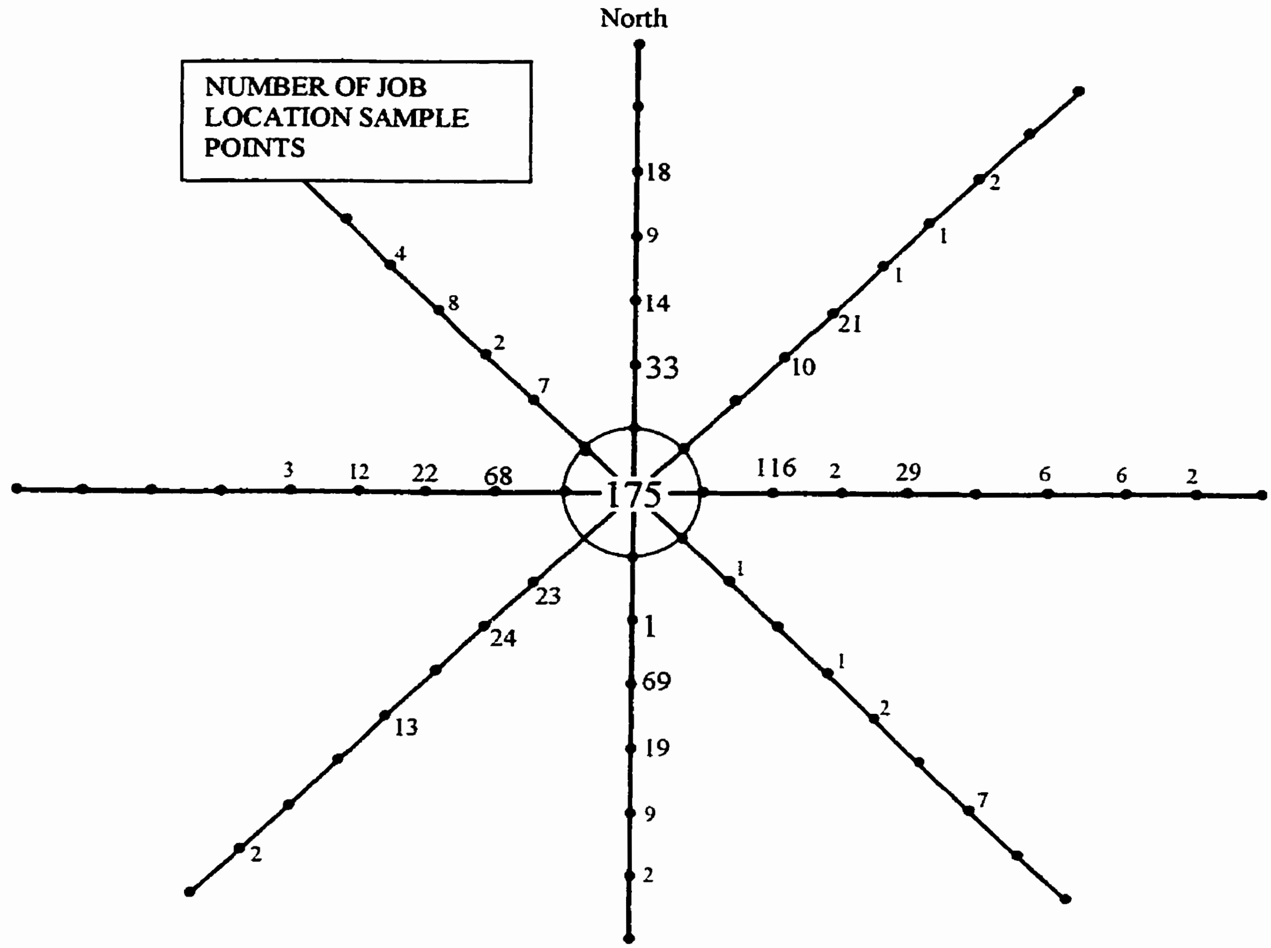

AVERAGE PRICE PER ROOM
in thousands of dollars.
SPATIAL RELATIONSHIPS
to Minneapolis Centre
North
12.9
12.4
13.9
12.4
15.6
15.6
14.9
17.2
14.7
19.2
17.8
18.0
14.2
14.8
13.9
17.4
17.0
13.4
16.9
24.0
18.4
14.0
28.2
19.1
15.4
16.5
16.9
17.6
16.5
18.3
15.1
24.8
19.1
16.4
15.0
13.8
13.9
11.4
10.8
10.2
17.0
15.5
14.5
13.6
12.7
30.4
40.1
16.2
15.8
13.1
13.7
12.2
12.4

Grid 6

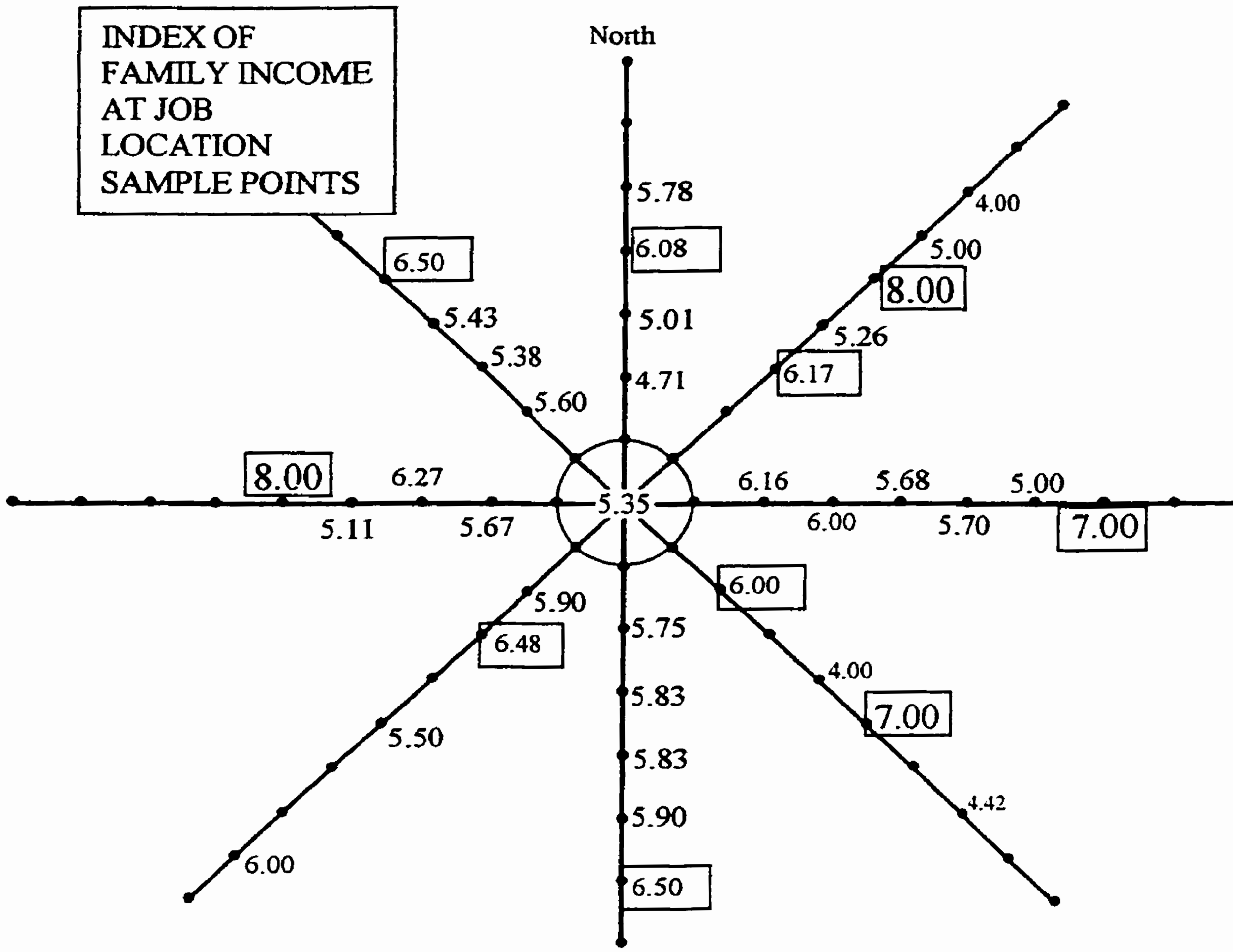


AVERAGE RENT PER ROOM
in dollars.
SPATIAL RELATIONSHIPS
to Minneapolis Centre
North
91
82
78
76
84
90
73
95
61
76
92
93
57
80
92
91
96
58
78
91
117
90
105
93
92
78
96
82
92
66
88
103
80
76
64
60
56
60
42
89
99
83
89
77
77
132
83
94
84
77
68
63
46

Grid 7


North
NUMBER OF JOB LOCATION SAMPLE POINTS
175
18
9
14
33
2
1
1
21
10
4
8
2
7
3
12
22
68
116
2
29
6
6
2
23
24
13
2
1
69
19
9
2
1
1
2
7

Grid 8


INDEX OF FAMILY INCOME AT JOB LOCATION SAMPLE POINTS
North
5.78
6.08
5.01
4.71
4.00
5.00
8.00
5.26
6.17
6.50
5.43
5.38
5.60
8.00
6.27
5.11
5.67
5.35
6.16
5.68
5.00
6.00
5.70
7.00
6.00
5.90
6.48
5.75
5.83
5.83
5.90
6.50
5.50
6.00
4.00
7.00
4.42

Grid 9

7. MEASURABLE DISTANCE RELATIONSHIPS

The analysis of Sections 2.1 to 2.3 extends the usual urban model with its broad basis in transaction locations (TL) in general. The purpose of a general theoretical treatment is to recast the model in a framework which will reveal new aspects of the analysis which can be further developed. Data on TL_+ (apart from the CL) are generally not available in context. This is partly because shopping centres are sufficiently ubiquitous that they play little role in location decisions. There are certain commodities such as major durables which may be available in identifiable locations for which data are available, but they are of limited relevance since their purchase is infrequent and does not involve any regular commuting. There are only two categories of the analysis which lend themselves well to empirical work: the CL because it is the place of residence (RES), and a single TL_- representing the place of employment (WP). Other TL_-, as locations where commodities other than a full commitment of labour time are sold for income, are excluded. In a practical sense, this is not without good reason, since data on selling one's used car (for example) for income are simply not available.

However, the analysis beginning in Section 2.4 develops some more specific testable relationships. This chapter examines various ways in which the functions of the model can change, and the impacts of these changes on location decisions. Section 7.1 below offers some simple crosstabulations to examine the extent to which these relationships are evident in the data.

7.1 Cost and Income

7.1.1 Cost

The simple function which relates travel cost to distance traveled is easily enough testable in theory, but data giving comparable travel costs for each individual transaction are unlikely to be available. However, given some ordinary assumptions about travel technology – the existence of some positive, reasonably continuous, but unspecified travel function is reasonable and sufficient to support many of the conclusions reached.

If we examine the suggestion that suburban locations imply lower travel (congestion) costs, and hence suburban residents may choose to commute further, the data of Table 7.1 gives a degree of confirmation. At first glance the data show a moderate relationship, with a correlation between distance to city centre and distance to work of 0.45.

With the data is on (four-mile) concentric rings, the following chart shows some indication of relationship. It should be noted that these distances are from the RES to the relevant workplace – where the respondent was employed precisely at the time of the move – which this dataset is notably capable of rendering. Hence the residential decision was the last decision made.

Ring	1	2	3	4	5	6	7	8
DistW	3.68	4.75	8.11	9.83	9.01	14.48	10.98	17.87

Table 7.1
Distances from RES to Relevant WP

Recall that Ring 4 appears to be a rough price peak in the census data. If rings 6 and beyond can be considered definitely suburban, then it is fairly clear that suburban households are commuting further. The relationship seems also to apply when ring 4 is taken as a price peak, though the implication that city centre is therefore a price valley confounds the relationship. More importantly, any such simple correlation fails to control for the effect of amenities (lower congestion being one) which can also be implicated in the choice to commute farther.

Evidence on the basis of mode of transportation is more difficult to interpret, since the choice of mode is probably fairly endogenous (part of the package of housing/ neighbourhood characteristics being chosen). First of all, there are only 12 sample points for bus riders. Average distance to work for them is 6.0 miles, compared to 8.0 miles for other modes of transportation. Cheaper commuting does not necessarily induce further commuting. But the cheaper transportation, in dollar terms, may be and likely is the most expensive in utility terms. In addition, there is likely to be sufficient correlation between public transportation and income, that the relationship between income and commuting will further complicate any interpretation. Hence no clear relationship is expected.

The same analysis is applied to the workplace location decision, where Table 7.2 shows distances of workplace to residence (DistH).

Ring	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
DistH	8.58	7.65	7.81	5.58	5.92	5.18	5.02	11.13

Table 7.2
Distances from WP to Relevant RES

Except for Ring 8, these results appear to be opposite to the case of the residence location decision. (Ring 8 has only three sample points.) Should people working in less congested suburban areas should be willing to take jobs further away from the residence? One possible explanation, a factor to be considered later, is that the greater job density toward CC is the dominant factor. The strongest assertion that can be drawn from this data is that transportation cost (related to congestion) is a factor when considering residential location, but is not taken into consideration in the choice of workplace. Perhaps also, less congestion has been mentioned as a residential amenity, for which people will be willing to endure greater commuting – and there is no comparable amenity in the consideration of job location – but this raises a question beyond the realm of transportation costs.

7.1.2 Income

The impact of income on residential location is dependent on elasticity relationships which are difficult or impossible to quantify. Hence one might expect the data to show minimal consistency, as evidenced in Table 7.3.

The top three rows of data refer to residence choice, and the fourth row of data refers to workplace choice – and as before residential choice is the last decision made in the first three rows and workplace choice is the last decision made in the fourth row. Unless the small number of sample points (6) for the lowest income index invalidates it, there is a weak but likely insignificant relationship suggesting that income induces greater distances from city centre 112, and no relationship at all regarding the triple-peak gradient (which appears in general to be a less reliable gradient). However, the suggestion that income may induce more commuting, but only up to a limit, receives a little more support in the third row. By income index 5, income has little further effect. Commuting distance with respect to the workplace choice again in the bottom row shows a relatively inconsistent relationship, suggesting that income has little effect on the choice of workplace.

Income Class	1	2	3	4	5	6	7	8
Distance Res to Centre								
Dist112		4.9	12.2	12.6	11.5	12.5	11.9	13.4
Dist3Peak		11.3	10.4	11.1	11.0	10.4	10.4	10.4
Distance Res to WP		3.9	5.6	6.2	8.3	8.5	8.2	9.5
Distance WP to Res	5.2	2.2	8.0	8.3	6.5	6.6	8.1	9.8

Table 7.3
Impact of Income on RES

7.2 Household Structure

The survey data of this study are able to discern some of the relationships discussed in Section 4.3. Table 7.4 attempts to bring out the relationship discussed above between education levels and family status. It considers average distance from job to residence. When it comes to special quality characteristics which affect the choice of workplace, the obvious candidate is skills. Direct skill measurements are not available, nor is there any clear way to distinguish between enterprise-specific non-specific and general skills (except perhaps that years of education might be considered relatively general). Education is used as a rough proxy.

Status	Years Education								
	11	12	13	14	15	16	17	18	20
Head	9.75	6.38	4.49	5.50	5.82	10.85	16.98	9.89	6.1
Non-Head	3.36	6.74	8.75	9.34	8.96	7.42	8.76	7.63	8.26
Difference	6.39	-0.36	-4.26	-3.84	-3.14	3.43	8.22	2.26	-2.16

Table 7.4
Average Distance of Job from Home

With the exception of the lowest and highest education levels, those of lower education show more commuting for non-heads, while those of higher education show less commuting for non-heads. This may suggest that where the family head is more educated, and there may be a greater drop in education level for secondary earners, the first effect described above may cause secondary earners to commute shorter distances. This is obviously a highly tentative suggestion. Interestingly enough, creating the same table using commuting distance projected onto the ray produces almost identical results.

For the residential decision, the relevant special quality characteristic is specialized housing needs. Distance from Minneapolis 112 should increase for more expensive child-based specialized needs due to the outward effect of a higher price gradient, counteracted by the inward effect of more extensive search toward greater density. If placement is predominantly normal (the residence further from city centre than the workplace), then commuting distances should show the same pattern.

The following evidence gives some limited support to the proposition that the price effect has a greater impact than the search effect. If we examine residential location, showing commuting distance broken down by income index and the existence of children, we find the following crosstabulation in Table 7.5.

Family Size	Income Class						
	2	3	4	5	6	7	8
NoChild	1.8	4.8	6.1	5.6	9.1	7.3	8.6
Children	14.1	7.3	6.4	9.9	7.9	9.0	10.0
Difference	12.3	2.5	0.3	4.3	-1.2	1.7	1.4

Table 7.5
Average Actual Commuting Distance

The outward price effect prevails in general, given that the differences in average commuting distance in the bottom row are almost all positive. If the lowest income index can be ignored because of a small number of sample points (or even if it cannot), there seems to be no consistent relationship with different levels of income. In other words, while both income and the existence of children show a systematic relationship to commuting distance, they bear no systematic relationship to each other in that dimension. It cannot be said, for example, that the higher the income the greater are the distances that people with children will commute – in spite of the effect that the value of commuting time is higher for people with higher incomes, and the price effect which takes that into account appears to prevail. Hence the evidence remains a little inconclusive.

Little difference is shown in Table 7.6 when we consider the distance along the ray from RES to CC, rather than the actual commuting distance. The same basic relationships shown below hold as described above. Hence at least in relationships of this type, any analysis which employs distances along the ray from RES and WP to CC, and interprets commuting distance as the difference between them, may not be far removed from reality.

Family Size	Income Class						
	2	3	4	5	6	7	8
NoChild	1.8	4.0	5.3	4.7	7.9	6.1	7.2
Children	11.2	6.9	5.8	8.7	6.1	7.8	9.1
Difference	9.4	2.9	0.5	4.0	-1.8	1.7	1.9

Table 7.6
Average Commuting Distance Along the Ray

Other commonly-considered family unit variables can be incorporated here as well. For example, position in the life-cycle can be most fundamentally divided up into a building-human-capital phase and a returns-from-human-capital phase. In the former case, with a greater focus on maximizing the returns to building human capital, there is a greater incentive to search. A similar effect can be expected in the case of families with children.

However, crosstabulations relating commuting distance to family size and age (not reproduced) suggest some relation to family size, but none whatsoever in the case of age. The only relationship that can be drawn with any confidence from a complex and inconclusive crosstab is that average commuting distance increases with family size (with a slope of 1.2) as shown in Figure 7.1.

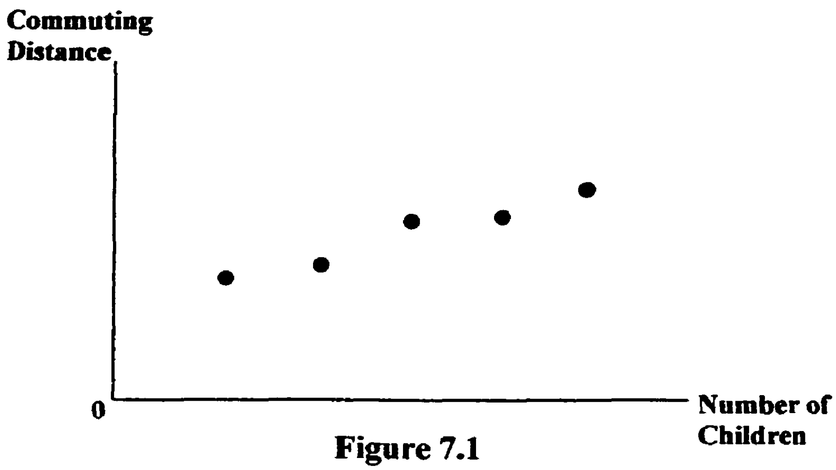


Figure 7.1
Commuting and Family Size

When commuting distance is projected onto the ray, the locus above curves downward again at higher family sizes. In other words, in the event that people were paying attention to distances along the ray instead of direct commuting distances, the relationship would probably not hold.

Overall, the existence of children in the family, requiring special housing characteristics, has some tendency to increase commuting distances, with perhaps some correlation to income – and greater skills (at least as measured by years of education) also tend to induce more commuting, especially for family heads. However, this multiplicity of tenuous relationships suggests that regression analysis is needed to investigate the possibility of more precise conclusions.

7.3 Placement in the Residential Decision

7.3.1 Offsets and Switches

This analysis of the residential location data considers the question of conformity to the theoretical configuration. Most of the sample points involve significant offsets, the greatest exception being those who work at home. Of the 361 survey points of residential locations for which workplace locations at the time of moving are available, 24 work at home, and another 5 show respondents working within one mile of home (suggesting that they work just around the corner, or virtually at home).

Anomalies in the relative placement of residence and workplace are identifiable in the data. The following analysis applies to residential movers, and workplace movers will be considered separately in Section 7.4.

In the Table 7.7, the total set of 361 respondents making residential location decisions is divided up according to how they have positioned themselves along the Alonso ray (disregarding offsets), with one row for the normal gradient based on Minneapolis 112 as city centre, and another row for the triple-peak gradient (distance to nearest of three peaks). The Term "Alonso" refers to the traditional downward-sloping price gradient based on city centre. Throughout the analysis, respondents whose distance to work measures zero are treated as normally placed. Respondents who live less than one mile from work are also considered to live and work at the same location, and are treated as normally placed (even if they are technically switched).

% Respondents	Position		
	Normal	Res-Sw	CC-Sw
Gradient Type			
Alonso 112	61.8%	19.9%	18.3%
Triple-Peak	37.7%	44.6%	17.7%

Table 7.7
RES Placement along Alonso Ray

Choice of residence traditionally places the workplace between residence and city centre. In the case of a single Minneapolis-based city centre, 72 (or 19.9 percent) of the respondents had residences which were out of place in the sense of having a workplace further from city centre than the residence (res-switched). Another 66 (18.3 percent) were out of place in the sense that the workplace was located on the other side of city centre from the residence (cc-switched). Hence 61.8 percent were placed normally. The inapplicability of the reverse-Alonso case (where normal and res-switched are reversed), is here reinforced since "normal" placements in that context are only 19.9 percent. In the triple-peak case, only 37.7 percent are placed normally, and res-switching dominates with 44.6 percent.

Where there is a high percentage of switched respondents, it might suggest that the gradient specification is inappropriate, or that respondents were perceiving a different price gradient or no discernable gradient at all, or that they were responding more to amenities or other factors than to any price gradient. The figures above suggest that a normal downward-sloping price gradient is the most applicable predictor, at least in the limited context of switching behavior. That is, a reasonably high proportion of people are responding in a straightforward traditional way to a Minneapolis-based price normal gradient, even though they may exhibit significant offsets from the ray.

The following tables display average offsets for the different placements considered.

Alonso 112 Gradient	Position			
	Normal	Res-Sw	CC-Sw	Overall
Avg Offset	2.1	4.3	6.2	3.0
Avg Commute	7.0	5.3	15.8	8.0
Offset as %	29.7%	81.1%	39.3%	37.6%

Table 7.8
Average Offsets, Normal Gradient

Triple-Peak Gradient	Position			
	Normal	Res-Sw	CC-Sw	Overall
Avg Offset	3.3	5.8	8.2	5.3
Avg Commute	5.1	7.2	16.1	8.0
Offset as %	65.7%	81.0%	50.75	66.5%

Table 7.9
Average Offsets, Triple-Peak Gradient

Many of these offsets are not huge, with an average offset of 3.0 miles for the Alonso gradient. That figure, however, includes the many instances of working at home or virtually at home (which, if excluded, would produce an average offset of 3.7 miles). It is more relevant for purposes of comparison, however, to record the average offset of 3.0 miles as 37.6 percent of the distance to work. Offsets for normal placements are 29.7 percent, compared to 81.1 percent for res-switched and 39.3 percent for cc-switched respondents.

It is clear that the traditional Alonso gradient best displays the expected results of the model. Those who are sensitive to the price-location dynamic of the model are likely to be the ones who are normally placed, and are more likely to be optimizing in a way which reduces offsets (theoretically to zero) – at least where there is no effective means to test for (out of line) premium wages offered in suburban employment centres which might produce rational out-commuting. Hence the gradient which exhibits the lowest offsets in the normal placement category best represents the behavior of the model. Results are not very ambiguous in this instance, as the traditional Alonso gradient scores a much lower average offset for normally placed respondents. It also scored significantly better in overall placements compared to the triple-peak gradient.

Also, one might expect those who are displaced, and hence may be less concerned about commuting distance, to travel further to work. This is noticeably true only for the more extreme cc-switched respondents, showing the highest actual offsets and averaging 15.8 miles to work. A lack of consistency in the res-switched respondents serves as a reminder of the tenuous nature of such data, and the possibility of distortions caused by fixed industrial patterns.

It should be noted that the greater offsets in the triple-peak case are deceptive, because of the relative sizes of the cities involved. Minneapolis (112) measures up to 12 miles in a north/south direction, and there is a significant high-density/low-price area surrounding Minneapolis to the east and west. By contrast, the cities comprising the triple-peak gradient are relatively tiny. Moreover, the low-price concentration in Minneapolis is spread over a wide area, such that a gradient to the geographic centroid may be quite meaningless. A gradient could be defined through a point far removed from the centroid and still be well within Minneapolis (112), but that could change the offset from 2 miles to 10 miles. Since any residence-workplace combinations located wholly within the broad Minneapolis area could be largely randomly placed around each other, their average distance from city centre or from its immediate gradient may well net to zero or some low value. Hence one might expect the offsets to be lower for the Minneapolis-city-based gradient, as it appears in the data. In other words, one might expect these gradients to produce offsets which are slightly biased in a downward direction because of the shapes of the city jurisdictions in question, but there is no way to determine the extent of this bias. Nonetheless, the offset ratios for normal placements on the Alonso gradient are sufficiently lower to suggest that there is indeed a relationship.

7.3.2 Respondent Preferences

The survey data includes fields on respondent preferences. Reading from the Tables 7.10 and 7.11 which refer to the traditional Alonso gradient for the Minneapolis normal price gradient, figures in the top row accompanying the preference categories refer to the percentage of respondents who identified that category as important. Hence for example, 15.5% of respondents indicated that price was a medium-to-high priority in their selection of a residence. Note that preference categories are grouped for the purpose of this analysis. Residential Quality includes survey questions regarding size, age and general "niceness" of the residence. Neighbourhood Quality includes age, "niceness", property tax bracket, proximity to school/church, and proximity to relatives. Other categories are fairly self-explanatory, except that Neighbourhood Move indicates that the quality of neighbourhood was an important reason for moving from the old residence, and Neighbourhood Quality indicates that the quality of the neighbourhood was an important reason for choosing the new residence.

Preferences	Price	Workplace Location	Residence Quality	Neighbour-hood Move	Other Family Jobs	Family	Neighbour-hood Quality
Preferred by % respondents	15.5%	28.3%	42.9%	54.6%	3.3%	23.4%	88.1%
Percentage Placed ...							
Normally	62.5%	72.6%	73.5%	70.1%	80.4%	69.0%	69.5%
Res-Sw	17.9%	22.6%	12.9%	14.7%	6.5%	13.1%	15.1%
CC-Sw	19.6%	4.9%	13.5%	15.2%	13.0%	17.9%	15.4%
Total	100%	100%	100%	100%	100%	100%	100%

Table 7.10
Preferences and Placements

Preferences	Price	Workplace Location	Residence Quality	Neighbour-hood Move	Other Family Jobs	Family	Neighbour-hood Quality
Preferred by % respondents	15.5%	28.3%	42.9%	54.6%	3.3%	23.4%	88.1%
Offset for ...							
Normal	22.0%	32.7%	29.0%	29.8%	26.5%	30.2%	28.9%
Res-Sw	78.9%	79.0%	87.8%	79.0%	78.2%	72.7%	82.3%
CC-Sw	36.8%	54.7%	42.3%	37.2%	45.5%	42.7%	39.4%
All	34.1%	44.5%	37.1%	37.0%	40.5%	37.9%	37.3%

Table 7.11
Preferences and Offsets

The first table refers to the percentage of respondents in each preference group who were switched or placed normally. For example, for the group of respondents who indicated price as important, the Normal cell in that column indicates that 62.5 percent of respondents in that preference group were placed normally. The second table refers to the average workplace offset as a percentage of distance to work. Using the same cell in the next table as an example, the average offset of normally placed respondents in the price preference group was 22.2 percent. The corresponding offset for all respondents (all placements) together was 34.1 percent.

The first column of each table shows a clear relationship, namely that most respondents in the price preference column were normally placed, and the normally-placed respondents had distinctly lower workplace offsets (22.0 percent), especially in relation to res-switched respondents. This speaks strongly for the normal Alonso relationship, since the Alonso model would predict that most people who pay attention to price would place themselves normally and with lower offsets.

It is important to note that these preferences were not mutually exclusive in the survey. Respondents could express their preferences on up to five categories at once without any suggestion of ranking them.

It is notable that traditional categories score poorly in this set of preferences. Only 15.5 percent of respondents indicated price as a major factor in their move. The highest scoring categories are those related to residential and especially neighbourhood quality, the latter suggesting that the respondents were somewhat amenity oriented.

Taking the first two columns of numbers in these tables as indicating preferences related to the traditional price and location factors, the results appear contradictory at first glance – showing offsets lower than the other preference categories for price, and higher than the other preference categories for location. However, the Alonso model is a price-driven model, and location is the result – suggesting that the first column is most relevant. A stated preference for workplace location (second column), to the extent that it represents a preference for location (which could relate to amenities) over price, might be expected to draw people way from the price gradient and produce larger offsets. This relationship is reinforced in the Normal row, and more weakly discernible in the switched rows where one would not expect that relationship to be as apparent.

It is highly significant that offsets are lower for normally placed respondents in all preference categories, indicating that even when amenity factors play a large role in residential location decisions, the Alonso normal-gradient price relationship is still an important factor. Even with such a large proportion of people indicating neighbourhood quality as a preference, the ratio of normal offsets to res-switched offsets in this category is the same as for those indicating a price preference. Even a strong preference for amenities does not produce a great increase in offsets. There may be some room in these figures for a random-offset hypothesis.

The data may also suggest that it might not reveal much about behavior to separate out preferences for independent analysis. They could indicate that housing prices have a component of hedonic price index in them, where better neighbourhoods and their amenities are captured to a large extent in the price.

Switching behaviour is a little less clear. The price oriented group should show a higher proportion of respondents in the normal placement category compared to other preference groups (if amenities are less likely to draw them away from the normal Alonso relationship), and such is not the case – in fact it shows the lowest (62.5 percent). Workplace location scores higher (72.6 percent), but still not highest. If we interpret the second column as reinforcing the location aspect of the price relationship in the first column, the two columns together show an unclear relationship to the other preferences, with which show only a slightly higher (and hardly significant) priority overall.

In summary, there is good support for the expected theoretical behavior when it comes to offsets, but a less clear picture when it comes to placement. However, the inconsistent behavior with regard to switching is drawn from an examination of switching behavior alone (isolated from offsets). Nevertheless, overall the data are reasonably consistent with the proposition that residence-seekers are perceiving and paying attention to a traditional price gradient even when they give high priority to amenities (i.e., even when 88.1 percent of respondents indicated a strong priority on neighbourhood characteristics when choosing a residence, compared to 15.5 percent for price.) Note, the fact that these percentages overall sum to over 100 percent arise from the crucial fact that the survey did not ask for mutually exclusive preferences, and in fact accepted up to 5 choices at once.

7.3.3 Neighbourhoods

Since the dataset identifies census city divisions, it is possible to obtain characteristics for the locations where respondents reside. Table 7.12 combines preferences, neighbourhood characteristics, and switching behavior in the context of the Alonso 112 gradient. The first column measures an index of average family density in the chosen neighborhoods, and the switching behavior of those respondents who expressed a preference for a neighbourhood amenable to families. The second shows average building year of construction and switching behavior for those who express a preference for housing quality. The following columns show average building year, percentage of non-welfare residents, employment rate and an index of occupational class (from unskilled labour to professional), and switching behavior for those who express a preference for neighbourhood quality.

Preference	Family Dens Family Pref	Building Yr Quality Pref	Building Age Neigh Pref	Non-Welf Neigh Pref	Employment Neigh Pref	Class Neigh Pref
Placement						
Normal	.73	1966.9	43.8	78.7%	95.5%	.66
Res-Sw	.61	1956.6	45.7	71.4%	95.1%	.67
CC-sw	.70	1966.1	44.1	78.9%	95.7%	.67

Table 7.12
Preferences, Neighbourhood Characteristics, and Placement

There is little in this table to confirm the expectations of a model which would suggest that those classified as normally positioned would to focus more on price/location considerations and pay less attention to neighbourhood characteristics such as Family Density, in contrast with those who are res-switched or cc-switched. The latter should be choosing greater Family Density. This is not confirmed in the first column, and it returns us to the suggestion above that housing prices may be capturing the elements of hedonic pricing.

Housing quality preferences confirm the above model no better, as seen in the second column. Switched placements should be characterized by newer housing, as the less price/location oriented opt for quality (at least in the limited dimension of age). There is no evidence of this in the data above. Given a degree of correlation between housing price and year of construction, some consistency between the first two columns might be expected, as respondents may be seeking price and quality at the same time. As well, limitations of the survey must be kept in mind. Price and quality orientations, as well as all others, were not presented in the survey as mutually exclusive options, and respondents were allowed to express several preferences from a long list.

Building age in the context of neighbourhood-quality preferences is no more consistent. Normally placed people should live in older (supposedly inferior) neighborhoods because they are more likely to be price/location oriented (except to the extent that price and age are negatively related to each other). The same relationship should hold when using a lower social-welfare rate as an indication of neighbourhood quality. As well, neither employment rate nor a weighted measure of social class as measures of neighbourhood quality show the expected relationship.

These results suggest a degree of criterion bundling. Residence-seekers view the home as a package – bundling locational advantages, housing quality and neighbourhood quality into a single commodity, which is measured by a single hedonic price. And in that bundle, locational advantages may indeed play a dominant enough role to reduce quality considerations to the relative unimportance indicated by the above data.

7.4 Placement in the Workplace Decision

7.4.1 Offsets and Switches

The same kind of analysis can be applied to the choice of workplace, where a fixed residence is offset from the ray between the workplace being chosen and city centre. Placements favour both gradients in Table 7.13. In the single-peak gradient, 56.8 percent of locations are normally placed, as compared to 66.2 percent in the triple-peak gradient. Normal placements indicate respondents who are sensitive to the gradient, suggesting that both gradients are performing well, as was the case regarding residential decision-making.

% Respondents	Position		
	Normal	Res-Sw	CC-Sw
Gradient Type			
Alonso 112	56.8%	21.3%	21.8%
Triple-Peak	66.2%	21.1%	12.7%

Table 7.13
WP Placement along Alonso Ray

Tables 7.14 and 7.15 bring mixed results when it comes to average offset. Overall, average offsets are 61.0 percent (single-peak) and 64.2 percent (triple-peak) of distance to residence respectively. Broken down by switching behavior, they are:

Alonso 112 Gradient	**Position**			
	Normal	**Res-Sw**	**CC-Sw**	**Overall**
Avg Offset	4.0 mi	2.5	7.8	4.5
Avg Commute	5.5 mi	5.5	13.9	7.5
Offset as %	70.9 %	46.0	56.2	61.0

Table 7.14
Average Offset, Normal Gradient

Triple-Peak Gradient	**Position**			
	Normal	**Res-Sw**	**CC-Sw**	**Overall**
Avg Offset	5.2 mi	2.8	6.0	4.8
Avg Commute	7.2 mi	5.5	12.2	7.5
Offset as %	72.0 %	51.8	49.4	64.2

Table 7.15
Average Offset, Triple-Peak Gradient

Offsets should be higher for switched placements, on the grounds that switched behavior implies less attention to the gradient, which is not evident in these results. Hence the placement criterion and the offset criterion bring opposite results. This contradiction could be suggesting that job seekers are not paying sufficient attention to the wage gradient.

When it comes to correlations (offsets to distance-to-work), the following data are interesting:

Placement	**Correlation**	
	Alonso112	**Triple-Peak**
Normal	.88	.87
Res-Sw	.48	.74
CC-Sw	.69	.50

Table 7.16
Correlations -- Offsets to Distance to WP

As in the case of residential choice, if a simple gradient relationship of further-out/offset-less-significant/offset-may-be-higher relationship holds, it should be truer of normal, more rational placements. In this case, the expected higher correlations for the normal gradients are apparent in the data.

Overall, though there are many inconsistencies in the data, good case can be made for accepting the traditional downward-sloping, city-centre based Alonso gradient for both residential and workplace decision-making – on the basis of better average performance by the various criteria discussed above. The triple-peak gradient occasionally displays better results, but performs relatively poorly in the areas that would most directly suggest an operational gradient.

7.4.2 Respondent Preferences

In the job-seeker case, we do not have any data on workplace location amenities to help focus the data. We do, however, have some indications of preference, namely an attention to locational factors when seeking employment, and an attention to job quality factors. For example, the top row in Table 7.17 indicates that 73.5 percent of respondents considered job location an important factor. While in this instance they are both cited by a high proportion, job quality is cited by more, suggesting that the gradient relationship is of slightly lesser importance.

Preferences	Location 112	Job Quality 112	Location Triple-Peak	Job Quality Triple-Peak
Preferred by % respondents	73.5%	92.8%	73.5%	92.8%
Percentage Placed ...				
Normally	57.4%	57.0%	67.2%	62.3%
Res-Sw	24.3%	21.4%	20.9%	25.1%
CC-Sw	18.2%	21.7%	11.8%	12.6%
Total	100%	100%	100%	100%

Table 7.17
Preferences and Placement

Preferences	Location 112	Job Quality 112	Location Triple-Peak	Job Quality Triple-Peak
Preferred by % respondents	73.5%	92.8%	73.5%	92.8%
Offset for ...				
Normal	45.2%	46.4%	72.6%	80.9%
Res-Sw	69.4%	70.2%	48.9%	51.6%
CC-Sw	57.0%	55.2%	43.0%	94.3%
All	60.0%	60.3%	62.4%	63.8%

Table 7.18
Preferences and Offsets

The Alonso and triple-peak gradients have a high proportion of normal placements, but only the Alonso gradient shows the expected results – higher offsets for the res-switched respondents. However, the expected stronger performance for location-oriented respondents is not strongly evident. Normal placement (previous page) for Alonso 112 is only marginally greater (57.4% > 57.0%), and offsets are not significantly lower (45.2% < 46.4%) in the normal case. The other gradient performs slightly better in this regard.

7.5 Placement and Excess Commuting

7.5.1 Residential Decision Estimates

The current analysis does not attempt to measure optimum commuting in the manner of Hamilton, on the grounds cited in Chapter 3.5 that it incorporates switching in an unpredictable manner and produces an unknown bias in the estimate. Instead, the current analysis estimates excess commuting as the excess of travel along the hypotenuse to an offset workplace or residence – calculated as the difference between the hypotenuse and the adjacent side of the triangle. In other words, the adjacent side represents what the commuting distance would be if the offset component were removed. This calculation produces the tables below, for excess commuting in the context of residential location.

Average Excess (Residential Decision)				
Alonso112	**Position**			
	Normal	**Res-Sw**	**CC-Sw**	**Overall**
% in Group	65.4%	17.0%	17.6%	100%
Excess (mi)	0.7	2.8	0.8	1.0
% Excess	13.2%	55.5%	4.6%	18.7%
Triple-Peak	**Position**			
	Normal	**Res-Sw**	**CC-Sw**	**Overall**
% in Group	38.1%	45.1%	16.8%	100%
Excess (mi)	0.8	1.2	1.2	1.0
% Excess	19.4%	21.6%	9.6%	18.7%

Table 7.19
Average Excess, Residential Decision

Proportional membership in the different placement groups, because only a subset of the data was amenable to excess commuting calculations, is not an issue in these calculations. Since excess commuting is here calculated from a direct measure of hypoteneuse-to-adjacent-side of the commuting triangle itself, the problem – that an offset does not fully measure displacement, because it extends merely from the ray regardless of a perhaps large overall displacement due to switching – does not arise.

The most striking characteristic is that excess commuting by this definition is far smaller than that estimated by Hamilton, averaging only 1.04 miles overall in the residential location context compared to Hamilton's 7.6 miles. That is, the ratio of the hypotenuse to the adjacent side of the offset triangle was small enough on average to produce an excess commuting average barely over one mile in length. In percentage terms, 18.7 percent of actual commuting was excessive, compared to Hamilton's 87.4 percent. The large differential stems partly from the fact the differences in calculation. Hamilton calculated an *ideal* commute of 1.1 miles from the average of actual residential and workplace distances from city centre on the grounds that the actual residential distance could be interpreted as the ideal distance on average given the swapping scenario. Our calculations were based on stripping out the offset component of the commute and calculating its projection onto the ray, which reduced an average commute of 7.9 miles down only to 6.9 miles projected onto the ray. But despite the differences in method, the difference in excess commuting estimates is quite large. It suggests either that Minneapolis commuters in 1992 were behaving differently than the commuters Hamilton examined and are perhaps much less offset, or that Hamilton's aggregation error cited in Section 5.2 concealed a large number of cc-switched residents biasing his estimate upward.

White's (1988a) estimates were substantially lower, for a list of 25 cities. Her estimates were based on minutes traveled, yielding a required commute of 20.0 minutes and an average commute of 22.5 minutes, meaning a rate of wasteful commuting of only 11 percent. Hamilton's (1989) response included another study from Boston, for which he found excess commuting of only 47 percent in this instance (p. 1500). Small and Song's (1992) report on Boston, yielding 33 to 66 percent depending on the size of the jurisdictions (the higher figure for smaller jurisdictions, contrary to Hamilton's suggestion). Merriman, Ohkawara and Suzuki (1995) reported 15 to 29 percent for Tokyo (with greater confidence in the 15 percent estimate) – noting that a high dependence on public transit and a large degree of social homegeneity (producing less segregation) contribute to lower expected commuting times. Suh (1990) modeled a theoretical city and estimated 27 percent excess commuting. He also modeled how greater residential decentralization results in less excess commuting, while greater job decentralization results in more. This study's 18.7 percent is well above White, but low enough to suggest that the attempt to isolate excess commuting due to offsets specifically confirms White's contention that Hamilton's version was considerably over-estimated.

It is interesting to observe the breakdown. In the residential choice data, both the traditional Alonso and the triple peak gradients produce the anticipated behavior – greater excess commuting for those who are at switched locations. Those who are disregarding or less sensitive to Alonso optimizing are engaging in more excess commuting. Indeed, Hamilton (1982, page 1046) suggested that a degree of wasteful commuting may be due to "out of equilibrium" commuting, and the above distinction might be considered one way of measuring that.

A part of the explanation of excess commuting could possibly be two-worker households. If the family head is indeed optimizing the residential location in reference to his or her workplace, the same cannot be said of the secondary earner. It may not be uncommon for two earners to compromise to some degree, and choose a location which is not wholly optimum for either of them in the sense that it is jointly optimal. As a result, excess commuting could arise, and excess commuting interpreted as such is overstated. In other words, we might expect the measured excess on average to be higher in two or more worker families.

Unfortunately the survey data in question does not include adequate data on number of earners (absent from the 1992 survey), which forces us to substitute number of adults as an imperfect proxy for earners. We cannot be certain that two-adult families include two workers, though a proportion of them would. It is more likely that three-adult families contain another worker, though that may not account for adult children at home. Nonetheless we might expect excess commuting to be greater, the greater number of adults in the family. The data below does not support that expectation in any clear manner, calculated for all placements combined.

Percentage Excess Commuting by Number of Adults		
1 adult	**2 adults**	**3 or more adults**
22.7%	17.2%	20.3%

Table 7.20
Percentage Excess Commuting by Number of Adults

There could be all kinds of unspecifiable reasons for this. Single-parent families could have greater specialized needs and are not as free to optimize, but three-adult as compared to two-adult families are more likely to contain a second worker and would exhibit the observed higher excess commuting ratio. This is, of course, just a stab in the dark, and the differences are tiny to begin with. Hence we have to consider these figures as rather inconclusive.

7.5.2 Workplace Decision Estimates

Recall that the data is divided into two parts. The previous analysis (the residential data) applies to the data on residential location, where the workplace at the time of the move is known. The following analysis (the workplace data) refer to workplace location, where the residence at the time of that choice is known. If they were not simultaneously determined, they are separate decisions. In many instance, the same respondent is present in both sets of data because information on both decisions could be ascertained, but they are two separate datasets based on the two ways in which the data could be compiled.

The workplace data shows average excess commuting of 2.5 miles, as shown in the table below.

Average Excess (Workplace Decision)				
Alonso112	**Position**			
	Normal	**Res-Sw**	**CC-Sw**	**Overall**
% in Group	57.1%	21.1%	21.8%	100%
Excess (mi)	2.8	1.1	2.9	2.5
% Excess	38.5%	24.1%	18.8%	31.1%
Triple-Peak	**Position**			
	Normal	**Res-Sw**	**CC-Sw**	**Overall**
% in Group	66.5%	20.8%	12.7%	100%
Excess (mi)	2.3	1.6	4.8	2.5
% Excess	29.4%	29.1%	45.7%	31.1%

Table 7.21
Average Excess, Workplace Decision

These data are less clear cut, and res-switched respondents are shown with lower excess commuting. This could suggest that job-seekers are a little more distracted by job quality factors whether they are switched or not, or it could merely testify to the unreliability of using this specification of excess commuting as an indicator of optimizing behavior.

Overall then – in addition to the existence of offsets and the other conventional explanations of wasteful commuting such as non-equilibrium decision-making, fixed employment patterns, multi-worker households, the concurrent optimization of non-commuting travel and the heterogeneity of housing, jobs and modes of transportation – excess commuting is also partially related in the data above to switching behavior (at least in the case of residential location).

8. REGRESSION ANALYSIS

8.1 Estimation Issues

8.1.1 Simultaneity

It should be noted again that the Minneapolis survey data do not support the hypothesis that residence and workplace are determined simultaneously. Where it has been possible to identify the workplace at the time of changing residence, and the residence at the time of changing workplace, we have found that only 17 percent of the sample respondents had changed workplace and residence within 12 months of each other. One year is a rather arbitrary dividing line – it is not very likely that many people choose a residence in anticipation of a job change even six months into the future. But even on that basis, when only one-sixth of respondents can even loosely described as making the two decisions in concert, it is unrealistic to treat them as one simultaneous decision. Hence we shall treat the two basic decisions, choice of residence and choice of workplace, in turn and independently.

Regression analysis has not uncommonly adopted the simultaneity assumption. Siegel (1973, p. 30) stipulated the assumption that "households simultaneously choose both the location of the home and job along with a bundle of housing characteristics consumed at their home location". His regressions found notable associations between home/job and job/home location, as well as with income and number of rooms in the residence (generally with the expected signs). With R^2 values in the .2 to .4 range, it suggests that commuting distance, income and housing size are still relevant, in spite of many likely instances where home/job changes were made in reference to a different job/home than that held at the time of the survey. Simultaneous regression analyses by Simpson (1988, 1987, 1989, 1992) generally support the conclusions of Siegel, with the addition of family variables showing as quite significant.

The choice not to employ simultaneous equations requires acknowledgement of the possibility of simultaneity bias. However, following from the nature of the sample employed in this instance, the choice of employment was often made years before the current residential choice, and only within one year for 17 percent of the sample. Hence the feedback that would cause simultaneity bias would be minimal.

8.1.2 Choice of Variables

In the theoretical analysis, the symbol u has been used to designate distance from the TL to CC. When moving from the theoretical to the measurable, we employ a common convention of using the symbol h to designate distance from RES to CC, and j for the distance from WP to CC. Actual commuting distance shown below remains designated by the symbol t.

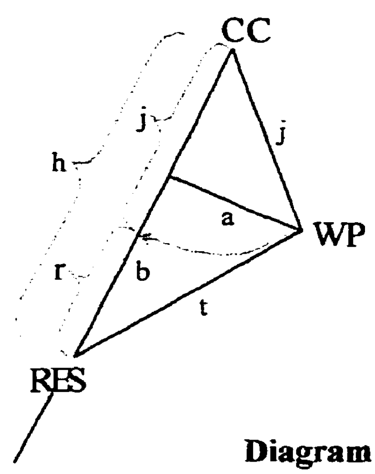


Diagram 8.1
Workplace Offsets

This requires some further interpretation when offsets are considered, as they are described in Section 5.1 and reproduced in Diagram 8.1. Simply taking the difference between h and j produces the distance r — clearly not a measure of t, or even b (the projection of the actual commuting distance onto the ray). This is not directly an issue, when simultaneous regressions of h and j are not employed. Nevertheless, even separate regressions commonly carry the assumption that a reduction in h or an increase in j tends to reduce commuting distance in the same proportion. Regressions which take distances to city centre and interpret them in this simple manner are not adequately taking switches into account.

There are two possible approaches to determining which variable should be included in a regression. Normally, to take the example of regressing on h, the distance j from the workplace to city centre is included as a regressor because j is an indirect measure of commuting time in its relation to h ($j = h - r$). This may be misleading, considering that most workplaces are offset from the ray to some degree, and hence the actual commuting time cannot simply be inferred from the difference between h and j. When calculating the travel cost factor resulting from the offsets in the data (actual distance traveled divided by distance along the ray – which would equal one if the workplace were actually located on the ray), the figures range from approximately 1/1 to 10/1, with a few outliers up to 50/1.

If these offset ratios bear no systematic relationship to h or j, and are often large enough to produce actual commuting distances of an entirely different magnitude from r, then $r = h - j$ may be a meaningless figure. This might suggest that we should substitute t (the actual commuting distance) for j (distance of WP from CC) in the regression on h. It can be argued that this is in fact the more meaningful variable, if it is the one more likely to be taken into account in choosing a residence. Who in SW3 cares how far as job in S4 is from CC? It's a short drive from home.

On the other hand, this relationship still means that workers may live farther from CC because their workplace is farther from CC, even if there are offsets involved. Hence the basic linear relationship, $r = h - j$, may still be the decision-making variable, even if the variance of the (perhaps random) offsets makes it a noisy variable. It may (likely) be no noisier than the gradients themselves. This approach also carries the advantage of yielding a consistent anchor point for the analysis, because it uses location co-ordinates which refer to a common geographic origin such as city centre – i.e. a consistent reference point, which a variable based on commuting distance lacks.

Offsets need not be an issue at all if they are random. Then offsets become simply a component of the random error term, and impart no bias to the regression. In terms of simply whether the offsets are evenly distributed on either side of the ray, that is clearly the case since the average of the signed offsets cancel out to near zero (0.28 miles for residential decisions and 0.26 miles for workplace decisions). More to the point, if the offsets are random and uncorrelated with h, then the influence on h will be distributed randomly around a mean value – and in any regression on h, they will merely add an increment to the constant term. Hence the relevance of the offset to regression analysis depends on the absence of any correlation between offsets and commuting distances. An examination of the data reveals that such correlations are weak. For residential decisions, the correlation coefficient between offsets and h is 0.113, and for workplace decisions it is -0.156, giving some support to the proposition of random offsets.

Both approaches have been tested. The regression results given below indicate that regressions based on j perform as well as those based on t, but have the crucial advantage of providing a common geographical reference point. Nonetheless observations from the regressions based on t are included, because they may support a suggestion that a sample which contains a large number of respondents who are residentially switched and hence less responsive to urban structure in the traditional sense may be a little more responsive to the structurally footloose direct commuting variable.

8.2 Residential Choice

8.2.1 Regression by Placement

We begin the analysis of the residential decision by identifying its likely determinants, which are commonly considered to be workplace distance from CC, income, children, number employed, and age. However, the Minneapolis data is not well structured to isolate the number employed in the family. The question was directly asked in the 1990 part of the survey, but not in the 1992 part. Limiting the analysis to those who have not changed between 1990 and 1992 would reduce an already sparse number of sample points (sparse because only 361 to 404 sample points could be found which identify workplace/residence location at the time of residence/workplace choice). Hence the closest practical proxy for number of employed in the family is family size, which is used below. The sample is comprised of the 362 most recent residential choices where the workplace at the time of residential choice could be identified.

The initial regression equation is specified as follows.

$$h = a + b_1 j + b_2 y + b_3 childr + b_4 adults + b_5 age + \varepsilon_h \qquad (8.1)$$

The following data are for linear and single log estimates of the dependent variable h based on the gradient from Minneapolis Centre (Dist112), and based also on the triple-peak gradient identified in Chapter 5 as a possible alternative. The latter is based on the minimum distance from the nearest of the three price peaks identified. The mean and standard deviation of h are given in Table 6.1 as 7.96 and 6.3 respectively.

Simply regressing these variables on h yields little in the way of solid relationships, as evidenced in Table 8.1 below.

TABLE 8.1. Regression estimates on Distance to Minneapolis (112)
All respondents

Regressors	Linear ($R^2 = 0.236$)		Logarithmic ($R^2 = 0.194$)	
Residential Location Equation (7.1)				
constant	7.86		1.95	
j	0.54	(0.05)	0.05	(0.006)
y	-0.07	(0.15)	-0.003	(0.02)
childr	-0.01	(0.03)	-0.001	(0.003)
adults	0.002	(0.08)	0.001	(0.01)
age	-0.01	(0.03)	-0.001	(0.003)

Notes: j = distance of workplace to 112; y = household income class; Childr = number of pre-school children present; Adults = number of adults present; Age = age of head.
Notes for all tables: *Source is 1990-92 Greater Minneapolis Survey; travelling distances are in miles; standard errors in parentheses; Logarithmic transformations are applied to the dependent variable only.*
All pairs of columns take the form: Coefficient (Standard Error)

Siegel (1975, page 37) used two-stage least-squares to estimate similar variables in San Francisco, broken down by rent/owner status and by race. Looking at the data for whites, and acknowledging the limitations of such a comparison – we note that Siegel's log coefficients for j ranged from .68 for owners and to .99 for renters. Blacks showed similar coefficients (except for 1.31 for young homeowners). His coefficients for log income were .07 to .14 (noting that only blacks showed negative coefficients). For number of employed (related but not identical to *adults* above), linear coefficients ranged from -.05 to +.01. Simpson's (1980, 1987) two-stage least-squares estimates were based on data from Greater London/Toronto. He found coefficients for j (.68/1.65), y (.75/.04), number employed (-.63/-.59), and children (.17/-.14)

It might be thought that restricting the regression to those who are not switched (in the normal Alonso relationship to CC and WP) might produce better results. These are, after all, the people who are more likely to be paying attention to the price gradient. The data in Table 8.2 yield some improvement.

TABLE 8.2. Regression estimates on Distance to Minneapolis (112)
Unswitched Respondents Only

Regressors	Linear ($R^2 = 0.428$)		Logarithmic ($R^2 = 0.318$)	
Residential Location Equation (7.1)				
constant	9.45		2.15	
j	0.72	(0.05)	0.07	(0.01)
y	-0.08	(0.17)	-0.02	(0.02)
childr	-0.10	(0.19)	-0.01	(0.02)
adults	0.15	(0.38)	0.02	(0.05)
age	-0.04	(0.03)	-0.004	(0.004)

Notes: See Table 8.1 notes.

A little more of the overall variance is explained in this case. A notable feature in the linear estimate is a strong positive coefficient for distance to city centre, j, indicating that for every mile the residence is further from city centre, commuting increases by 0.72 mile. Another is a negative coefficient for income which is quite consistent throughout the data analyzed below but rarely significant – which seems weakly to suggest that higher income people have a counter-tendency more to choose residences inward toward city-centre and their jobs (noting that this is the non-switched sample), possibly priorizing commuting costs or job offer densities. It seems puzzling to suggest that higher-income earners prefer central locations, but the low coefficients and usual insignificance may simply suggest that the data do not reveal much in the way of income relationships. The 0.15 coefficient for number of adults is an imperfect (but best available) proxy for number of earners, and seems to suggest some weak degree of priorization of space (over multiplied commuting costs) by moving outward, but the errors are too high. There does not appear to be any useful additional data to be derived from the logarithmic regressions.

TABLE 8.3. Regression estimates on Distance to Minneapolis (112)
Linear regression

Regressors	Unswitched ($R^2 = 0.196$)		Switched ($R^2 = 0.281$)	
Residential Location Equation (7.1)				
constant	9.86		11.62	
t	0.44	(0.05)	0.61	(0.06)
y	-0.11	(0.16)	-0.30	(0.19)
childr	-0.02	(0.03)	0.12	(0.21)
adults	0.04	(0.08)	0.23	(0.43)
age	-0.01	(0.03)	0.03	(0.03)

Notes: t = distance from residence to workplace; See Table 8.1 notes.

Replacing j with the actual commuting distance t makes little difference, as shown in Table 8.3 above. The same basic relationships apply in a similar manner. This supports the suggestion that – even if it can be argued that residence-seekers pay more attention to direct commuting distance than they pay to the workplace distance from city centre – an estimation technique which conforms to the city-centre-based structure of the model, and performs at least as well, should be considered preferable.

It appears that the more urban-structured model (measuring distances in reference to city centre) displays a stronger set of relationships for the people who are optimizing normally (i.e., non-switched). But, as may (perhaps) be expected, the behavior of the people who are switched and hence not responding as rationally to the simple model of urban structure is reflected to some extent in the model based on direct commuting distances.

Overall the apparent relationships are not compelling at best, and we should also be open to different explanations of behavior that do not rely on Alonso optimizing behavior as a strong behavioral trait – for example that people may be choosing a residential location in a fashionable suburb which bears little relationship to any ray which may extend to CC, or they could be responding to the amenity structure of the city and choosing by rough areas in relation to city centre even if the actual commuting distance is not being seriously minimized. Available data is not quite refined enough to make clear distinctions which could support any such hypotheses. The only hypothesis supported overall is that workplace location in relation to city centre is a significant variable, though direct commuting distance provides a useful model as well especially for res-switched respondents, and it appears that family and amenity factors can be discerned to some extent in respondent behavior.

The same analysis with respect to the three-peak gradient brings poor results. For non-switched respondents, they display weak coefficients with only family structure variables showing relatively low standard errors.

8.2.2 Amenities

In Chapter 3 it was suggested that, in cases where amenities are (a) not correlated with location and (b) not implicated in hedonic pricing, it could be fruitful to include amenities in a regression analysis. The kinds of data which would test for correlation by location in any precise manner are not available for this study. A casual perusal of various residential patterns within Greater Minneapolis/St. Paul would seem to support a suggestion that ring 4 is a crude peak for neighbourhood quality variables, and hence in a model which employs a normal downward-sloping price gradient it is not clear that amenities and housing prices are well correlated along the gradient – or at least it can be argued that the question of correlation is unresolved.

The second criterion presents a more serious obstacle. Theoretically, we should expect housing prices to incorporate neighbourhood amenities in a manner suggested by hedonic price theory – and who fails to examine the neighbourhood when assessing the value of a prospective residence? In addition, the crosstabulations of Section 7.3 support the suggestion that considerable "criterion bundling" is occurring. Hence there is some reason to believe that it the inclusion of amenity variables in the regression would be misleading in terms of spurious correlations. This is supported by a quick and dirty application of Rosen's regression technique in equation 3.4.

There are several types of amenity measures available by region from the 1990 U.S. Census – but only a handful of them can be selected. For example, average family income in the neighbourhood and percentage of families not receiving welfare payments are likely to be sufficiently correlated to bias the regression coefficients. It turns out that only three of them – housing construction date, percentage non-welfare, percentage family residences – might be considered sufficiently independent of each other to include (along with distance from city centre) in the regression. The following regressions produced the results in Table 8.4.

$$\text{housing price} = a + b1\text{distance} + b2\text{houseyear} + b3\text{nonwelf} + b4\text{family} + \varepsilon_h \quad (8.2)$$

$$\text{housing rent} = a + b1\text{distance} + b2\text{houseyear} + b3\text{nonwelf} + b4\text{family} + \varepsilon_h \quad (8.2a)$$

Table 8.4 Regression estimates on housing prices
(Linear regression for all city units in Greater Minneapolis

Regressors	Housing Price ($R^2 = 0.173$)		Rent ($R^2 = 0.325$)	
Hedonic Price Equation (8.2)			*(8.2a)*	
constant	-83177		-544	
distance	40.08	(21.63)	-0.08	(0.08)
houseyear	-61.25	(50.52)	-0.27	(0.18)
nonwelf	53.89	(38.49)	0.34	(0.13)
family	-255.08	(43.20)	-1.09	(0.15)

The regression is probably too crude to attempt any interpretation of the coefficients, except perhaps to note that they are not wildly improbable (contrary to regressions which include all possible amenities). The most important observation is simply that the regression does not disprove the hypothesis that amenities are significantly present in hedonic price indices. Hence it would be misleading to incorporate them into the regressions of equation (8.1).

8.2.3 Regressing on Offsets

With data available on offsets, it is useful to examine the predictive power in relation to offsets. The strongest theoretical explanation of offsets involves the impact of pre-existing fixed patterns of housing and industry, and the way they may induce cross-commuting. However, without sufficient data in the present survey to account for these patterns, any empirical investigation may end up taking the form of a fishing expedition, to see if some patterns emerge which might provide hints about systematic behavior.

One approach might be to include offsets as an additional independent variable in the above regressions. The data reveal a correlation of 0.54 between offsets and commuting distances. In addition, the F statistics on all the relevant regressions are sufficient to reject the null hypothesis. Interpreting this as suggesting that neither random nor non-random offsets are a foregone conclusion, it opens the possibility for investigating the impact of offsets in a regression on h. If offsets are indeed random, we might expect them to have no significant coefficient. Results of the regression are shown in Table 8.5, which includes only those variables which exhibit significant coefficients.

TABLE 8.5. Regression estimates on Distance to Minneapolis (112)

Linear regression for all respondents

Regressors	Dist112 ($R^2 = 0.338$)	
Residential Location Equation (7.2)		
j	0.56	(0.05)
off112	-0.14	(0.07)

Notes: off112 = offset from ray to 112.

There is very little change in the original variables – but the offset variable is significant and negative, though not strong. In particular, those who live further out have smaller offsets, suggesting perhaps that the commuting disadvantage is significant. Explanations may contain an element of guesswork, but it is not unreasonable to suggest that suburban living makes travel time a relatively larger expense, and there is a greater incentive to economize by minimizing circumferential commuting.

A more direct approach is to regress on offsets. The result is shown in Table 8.6. In this case, amenity variables are included because there is no problem of them being correlated with the dependent variable.

TABLE 8.6. Regression estimates on Offsets to I12 Ray

Linear regression for all respondents

Regressors	Offset ($R^2 = 0.146$)	
Dependent variable is the offset from ray to Minneapolis (I12)		
constant	-75.90	
j	0.23	(0.03)
t	-0.08	(0.04)
y	-0.01	(0.08)
childr	0.01	(0.02)
adults	-0.04	(0.04)
age	0.001	(0.02)
houseyear	0.04	(0.03)
nonwelf	-0.04	(0.04)
fam	0.01	(0.03)

houseyear is year of construction. nonwelf is percentage of non-welfare recipients. fam is percentage of families as distinct from single-family residences.

The explanatory power is weak. The most significant coefficient (0.23) is distance from workplace to city centre, yielding a moderate suggestion that offset workplaces move out a little (toward the residence) as offsets increase, which is not surprising. Generally, the lack of information in this table supports the suggestion that offsets are perhaps an irrational (in the Alonso sense) phenomenon, and the resulting excess commuting is difficult to relate to any discernible causal factors. This does not necessarily contradict our earlier statement that the non-switched respondents show more rational behavior than switched respondents.

8.3 Workplace Choice

8.3.1 Regression by Placement

The focus regarding the choice of workplace is skill levels. Initially, a simple measure of skill based on years of education is employed. While this is a rather crude measure, and relates only to the most general and non-enterprise-specific levels of skill, other possible measures have their own problems. Given the later need to match skill levels attained with skill levels demanded, education has been represented as a three-level skill index which incorporates regional job density for the skill in question. Also included is age, to capture some of the effects of life-cycle differences. Job experience data are available, but it is not clear whether they might measure distinct enterprise-specific skills or a wide range of variegated job-switching experience. It is not included in the regressions shown because test regressions which included experience yielded no useful inferences and had no effect on other coefficients, and there is a possible (though unexamined) risk of correlation with the skills variable.

The employment rate in the neighbourhood of the job location was initially considered for inclusion as an independent variable, since the employment rate may represent a rough job-related amenity measure. That is, while employment is not the same as job density, it may be taken to represent finding a job in an economically successful area of the city which might be perceived as an amenity. The unemployment rate is derived from the 1990 U.S. Census, and hence could be considered a lagged variable, measured in 1989 – suggesting that there should be no problem of an independent variable which is determined by the dependent variable. However, test regressions including the unemployment rate produced some wildly different coefficients for all parameters, compared to those shown below (without unemployment), suggesting that spurious correlations may be a problem after all. Hence unemployment is excluded.

The sample is comprised of the 404 most recent workplace choices where the residence at the time of workplace choice could be identified. The mean and standard deviation of the dependent variable j are given in Table 6.1 as 7.46 and 6.51 respectively.

Initial regression simply included an index of education as an explanatory variable. They are not summarized here because of insignificant results. The next level of regression included all the variables mentioned above, according to the following regression equations.

$$j = b_0 + b_1 h + b_2 skill + b_3 age + \varepsilon_j \qquad (8.3)$$

$$j = b_0 + b_1 t + b_2 skill + b_3 age + \varepsilon_j \qquad (8.3a)$$

Results are summarized in Tables 8.6 to 8.9.

TABLE 8.7. Regression estimates on Distance to Minneapolis (112)
*Linear regression for **all respondents***

Regressors	Dist-to-CC ($R^2 = 0.224$)		Dist-to-res ($R^2 = 0.027$)	
Workplace Location Equation (7.3)			*(7.3a)*	
constant	4.72		10.49	
h	0.44	(0.04)		
t			-0.11	(0.04)
skill	-0.60	(0.41)	-0.79	(0.45)
age	0.01	(0.02)	0.03	(0.03)

Notes: exper = years employed; emplrate = employment rate at job location; Dist-CC is the distance from the workplace to city centre at Minneapolis (112); Dist-to-res is actual commuting distance.

TABLE 8.8. Regression estimates on Distance to Minneapolis (112)
*Linear regression for **non-switched respondents***

Regressors	Dist-to-CC ($R^2 = 0.228$)		Dist-to-res ($R^2 = 0.064$)	
Workplace Location Equation (7.3)			*(7.3a)*	
constant	5.10		10.64	
h	0.40	(0.05)		
t			-0.19	(0.06)
skill	-0.68	(0.52)	-0.54	(0.58)
age	0.01	(0.03)	0.02	(0.04)

See notes for Table 8.7.

TABLE 8.9. Regression estimates on Distance to Minneapolis (112)
*Linear regression for **res-switched respondents***

Regressors	Dist-to-CC ($R^2 = 0.507$)		Dist-to-res ($R^2 = 0.051$)	
Workplace Location Equation (7.3)			*(7.3a)*	
constant	-1.16		11.14	
h	0.90	(0.09)		
t			-0.33	(0.17)
skill	-0.40	(0.72)	-0.86	(1.00)
age	0.02	(0.04)	0.01	(0.06)

See notes for Table 8.7.

TABLE 8.10. Regression estimates on Distance to Minneapolis (112)
*Linear regression for **cc-switched respondents***

Regressors	Dist-to-CC ($R^2 = 0.056$)		Dist-to-res ($R^2 = 0.007$)	
Workplace Location Equation (7.3)			*(7.3a)*	
constant	7.24		9.46	
h	0.18	(0.10)		
t			-0.003	(0.08)
skill	-0.07	(1.02)	-0.23	(1.04)
age	0.03	(0.06)	0.04	(0.06)

See notes for Table 8.7.

A striking result is the extremely low R^2 values for dist-to-res[17] (i.e., actual commuting distances). It appears clear that job-seekers are primarily focussed on the traditional Alonso variable, distance from city centre. Housing distance coefficients are highly significant. Skill factor coefficients are strongly negative, suggesting at this level of analysis that more highly-skilled job-seekers search inward toward greater job (and offer) density. For unswitched respondents that means reduced commuting, but for switched respondents it means greater commuting. This suggests that, for more highly skilled workers in the res-switched group, search is important enough to justify job locations which are less optimal in terms of commuting.

[17] Sufficiently different to elicit a re-examination of data calculations to ensure they are not in error.

8.3.2 Refining Skill

Census data is available to allow a distinction between job density rates according to skills. While the data does not allow for fine distinctions, the regions have been rated according to the job densities in the categories of labour, technical and trades, and professional. Actually, various middle-income occupations which do not belong in the labour or professional categories have been included in technical and trades. This procedure allows a direct mapping of people with education levels 1, 2 and 3 to the three job density categories above.

Following Simpson (1002, page 100), we employ the job density of the respondent's skill category as the regressor for skill (though with a simpler methodology). Letting E_i stand for three levels of education, and s_i stand for three occupational groupings (labour, tech/trades, professional), the composite coefficient $E_i s_i$ (or Ei in the tables) represents job density for the respondent's skill level.

The regression equations are:

$$j = b_0 + b_1 h + b_{21} E_1 s_1 + b_{22} E_2 s_2 + b_{33} E_3 s_3 + b_3 exp + b_4 age + b_5 empl + \varepsilon_j \qquad (8.4)$$

$$j = b_0 + b_1 t + b_{21} E_1 s_1 + b_{22} E_2 s_2 + b_{33} E_3 s_3 + b_3 exp + b_4 age + b_5 empl + \varepsilon_j \qquad (8.4a)$$

and the regression results are summarized below.

TABLE 8.11. Regression estimates on Distance to Minneapolis (112)
*Linear regression for **all respondents***

Regressors	Dist-to-CC ($R^2 = 0.245$)		Dist-to-res ($R^2 = 0.049$)	
Workplace Location Equation (7.4)			*(7.4a)*	
constant	-0.65		5.15	
h	0.44	(0.04)		
t			-0.10	(0.04)
E1	0.16	(0.05)	0.15	(0.05)
E2	0.12	(0.04)	0.10	(0.05)
E3	0.11	(0.05)	0.09	(0.05)
age	0.01	(0.02)	0.02	(0.03)

See notes for Table 8.7. E1, E2, E3 defined above.

TABLE 8.12. Regression estimates on Distance to Minneapolis (112)
*Linear regression for **non-switched respondents***

Regressors	Dist-to-CC ($R^2 = 0.249$)		Dist-to-res ($R^2 = 0.080$)	
Workplace Location Equation (7.4)			*(7.4a)*	
constant	0.11		7.28	
h	0.40	(0.05)		
t			-0.18	(0.06)
E1	0.15	(0.06)	0.11	(0.07)
E2	0.10	(0.06)	0.06	(0.06)
E3	0.09	(0.06)	0.05	(0.07)
age	0.01	(0.03)	0.02	(0.04)

See notes for Table 8.10.

TABLE 8.13. Regression estimates on Distance to Minneapolis (112)
*Linear regression for **res-switched respondents***

Regressors	Dist-to-CC ($R^2 = 0.509$)		Dist-to-res ($R^2 = 0.086$)	
Workplace Location Equation (7.4)			*(7.4a)*	
constant	-2.89		2.54	
h	0.89	(0.09)		
t			-0.33	(0.17)
E1	0.05	(0.09)	0.24	(0.12)
E2	0.03	(0.09)	0.18	(0.12)
E3	0.02	(0.10)	0.20	(0.14)
age	0.02	(0.10)	0.01	(0.06)

See notes for Table 8.11.

TABLE 8.14. Regression estimates on Distance to Minneapolis (112)

*Linear regression for **cc-switched respondents***

Regressors	Dist-to-CC ($R^2 = 0.067$)		Dist-to-res ($R^2 = 0.026$)	
Workplace Location Equation (7.4)			*(7.4a)*	
constant	4.29		4.76	
h	0.18	(0.10)		
t			0.01	(0.08)
E1	0.16	(0.12)	0.15	(0.12)
E2	0.13	(0.11)	0.11	(0.11)
E3	0.12	(0.12)	0.10	(0.12)
age	0.02	(0.06)	0.04	(0.06)

See notes for Table 8.11.

TABLE 8.15. Regression estimates on Distance to Minneapolis (112)

*Linear regression for **all respondents** with ownership included*

Regressors	Dist-to-CC ($R^2 = 0.248$)		Dist-to-res ($R^2 = 0.058$)	
Workplace Location Equation				
constant	0.78		7.32	
h	0.43	(0.04)		
t			-0.10	(0.04)
E1	0.15	(0.05)	0.15	(0.05)
E2	0.11	(0.04)	0.10	(0.05)
E3	0.11	(0.05)	0.08	(0.05)
age	0.004	(0.02)	0.01	(0.03)
ownership	-0.93	(0.70)	-1.45	(0.79)

See notes for Table 8.11. Ownership means home ownership status.

These results differ from the earlier regression by the use of a three-level skill index mapped to a three-level density index. The expectation, as discussed on Section 4.2, is that j will decrease or increase with increasing skill levels depending on whether search or commuting cost respectively is the dominant factor. The evidence seems largely supportive of the search emphasis in most cases. Coefficients (of the impact on j) are decreasing with increasing skill levels with a strong consistency (the higher the skill, the less j moves outward). The data also support the suggestion that switched respondents show less consistency, since the skill variables may still decline with level of skill, but they are relatively weak compared to standard errors. None of these are strong responses, and cannot support any definitive conclusions about skills and search behavior, except that a degree of consistency of the relationship across all regressions is moderately suggestive.

It should be noted that the skill index coefficients are consistently much lower than the single skill coefficient of the first set of workplace regressions (and different in sign, which is due only to the different manner of specification). It may be that the particular skill index employed does not well capture the skill/density structure of the sample, and the seeming relationships noted above must be viewed with caution as perhaps a spurious result of fairly low R^2 values and small coefficients.

Comparisons can be made only with Table 8.11, for all respondents. Siegel's (1975) results for Los Angeles showed log coefficients for skill in the vicinity of -.01, but (since he was not focussing on the issue) it was left unclear whether his definition of skill drawn from Standard Industrial Classifications was such that negative coefficients might confirm the search emphasis as discussed above. Simpson's (1980, 1987; see also 1992, page 104) analysis shows a coefficient for h of .68. But more interestingly, it also shows coefficients for skill (though negative) decreasing in value for increasing skill levels – consistent with the analysis above and supportive of the search emphasis.

Table 7.18 adds home ownership status into the mix. It is argued that home ownership implies greater transaction costs and a greater commitment to the neighbourhood, suggesting a possible greater responsiveness to local labour market conditions. By an analogous argument to that above, homeowners will tend to have a lower j. The data suggests that a declining index from non-owner (index 2) to owner (index 1) induces a 0.93 mile increase in distance of the WP from CC, confirming the suggested relationship if enough respondents are unswitched. In all, it's a tenuous confirmation of a weak relationship.

8.3.3 Regressing on Offsets

Finally, we consider again whether any causal factors may be associated with offsets and switching, but this time in the context of the workplace decision. However, the switching data must be disregarded in this case, because both res-switching and cc-switching occur in the same direction, and so positive and negative numbers for them are completely misleading. Including offsets in the regression on distance from residence to city centre yields Table 8.16 (again including only the significant coefficients).

TABLE 8.16. Regression estimates on Distance to Minneapolis (112)

Linear regression for all respondents

Regressors	Dist112 ($R^2 = 0.294$)	
Workplace Location Equation		
h	0.50	(0.04)
skill	-0.55	(0.33)
ownership	-0.67	(0.57)
off112	-0.50	(0.05)

Notes: Off112 = offset from ray to 112;

Since the skill/density index performed relatively poorly in terms of explanatory power in the previous tables, we have reverted to the single measure of skill level. As before, the previous variables display similar coefficients as before, and the offset itself appears to be quite significant. When the workplace is further away (closer to the residence in non-switched cases), the incidence of excess commuting decreases in this context.

The regression on offsets produces Table 8.17.

TABLE 8.17. Regression estimates on Offsets to 112 Ray

Linear regression for all respondents		
Regressors	Offset ($R^2 = 0.377$)	
Dependent variable is the offset from ray to Minneapolis (112)		
h	0.51	(0.03)
t	-0.42	(0.04)
skill	0.44	(0.30)
age	-0.02	(0.02)
emplrate	0.39	(0.14)
ownership	0.20	(0.53)

The negative relationship of work distance from city centre with offsets noted above is confirmed, as well as a strong positive relationship with residential distance from city centre. When the workplace moves further out toward the residence, offsets decline – and when the residence moves further out, offsets increase. Notable are the significant coefficients for skill and employment rate (ownership has too high an error rate). There may be some causal relationships involved, where more skilled workers are more inclined to engage in wasteful commuting to match employment to skills, and those seeking employment in more successful neighbourhoods will also pay the cost of wasteful commuting to satisfy their preference.

9. CONCLUSION

9.1 Closer Examinations, Contributions and Possible Further Work

This paper contributes to the development of urban economic analysis in the following areas:

A. Greater Generality of Treatment

This study takes some beginning steps toward placing urban analysis into a more general framework, in hopes of discerning some of the underlying broad microeconomic relationships and their mathematical specifications inherent in urban theory. It is particularly relevant regarding (a) the broad similarity between the cases of positive and negative prices, and the set of relationships between them, and (b) the common framework of quality which underlies the two key categories of specialized housing needs and skills.

It is unclear how useful it may be to develop the more generalized treatment further. It is considered desirable in microeconomics to understand the most powerful model – because it brings the disparate individual relationships into a common framework which reveals new relationships and shows old models to be special cases of a more general phenomenon. The treatment of specialized housing needs and employment skills in the common framework of commodity qualities is useful, and there may be theoretical room for integrating a model of this sort with hedonic pricing methods – because this study's general quality factors and Rosen's indifference surface of housing characteristics have a point of intersection worth exploring.

B. Extension of the Preference-space Model

This study extends a little further the analysis in preference space, in particular extending the Evans/Fujita graphic into positive/negative space, articulating the meaning of normal positioning in these terms. Aided by a balance-of-forces representation, it allows for alternative models – such as models with positive price gradients which, though no use of them was made in this instance, remain as theoretical possibilities which may find application in further analysis.

Regarding the Evans/Fujita model, it is only a minor extension to extend it down into the negative quadrant to include negative prices as income. But it does help articulate the meaning of normal positioning in more rigorous terms. This might be worth exploring as an element of offset and switching analysis, to be discussed below.

C. Further Articulation of Search-based Models

The analysis articulates key basic relationships which are to be tested in this context. This is facilitated by setting up the island model, especially the distinction between opposing cost and search effects. But more original to this study is a degree of integration of the search model with the preference model, most notably in order to develop an expanded concept of reservation price/wage, which in its greater generality can cover a wider range of theoretical possibilities. It is significant that the single-valued *reservation price* or *reservation wage* is a special case of a more general relationship, identified as U^T in Figure 4.1. Perhaps this can be modelled on its own, and perhaps a wider range of responses to labour market conditions can be modelled. This may indeed carry policy implications. Consider Simpson's (1992, Chapter 7) argument for a policy focus on local labour market conditions to combat spatially restricted job search. A model which employs a reservation wage function (rather than level) which is responsive to location (and by extension local labour market conditions) may be able to provide a further theoretical differentiation of labour market behavior which varies across space.

D. Closer Look at the Housing Unit and its Pricing

This thesis takes a closer look at the problems of housing unit specification. In particular, it focuses on the use of hedonic pricing techniques for specifying both inherent housing characteristics, *and* neighbourhood characteristics – or amenities – including the effects of amenities and their relation to hedonic bundling. But it also looks at cases where amenities not included in hedonic index, and the resulting draw of the location decision away from the traditional ray. From a point of view of urban development policy, when analyzing the dynamics of neighbourhood development and its market externalities (self-perpetuating, inward-looking slums, to take a strong example), the externality of an amenity which fails to integrate with the hedonic pricing process and hence draws location decisions inward or outward should be developed as a component of that analysis.

This study only partially addressed the question of whether all significant housing or neighbourhood characteristics are present in a hedonic price index. When are they, and when aren't they? If important characteristics are not represented in the price, how can we model the way in which they may draw the residence-seeker away from a price-determined location? It's worth a closer look to see how much the literature which flowed from Rosen's and from Diamond & Tolley's work has addressed this question. And the question of how hedonic pricing applies to wages is relatively unexplored.

E. Development of Offset and Switching Analysis,

Here the thesis has been useful in specifying clearly the exact meaning and form of offsets from the ray and switched-from-normal positioning along the ray. Offsets and switches are classified, and precise methods of definition and calculation are developed. These clear definitions are useful for the analysis of rational commuting, as contrasted with excess commuting – and their definitions from Hamilton on. In addition, this paper has clarified some of the shortcomings of earlier measures of excess commuting, and proposed an alternative.

It might be useful to incorporate White's analysis of rational as an extension of this analysis of offsets and switches. The additional analytical power, to be able to specify and calculate precisely the incidence of offsets and switches, is worth further exploration. The analysis facilitated empirical tests of these relationships. It might be worthwhile to formalize that analysis a little more, and work on integrating it more carefully with the core theory. That would also useful for developing a rational theory or switched or offset commuting – that is, specifying when it's rational to do so in terms of an extended Alonso model. White develops this theme separately, and it could be better integrated. It also would provide an opportunity to give a little more rigorous analysis to exactly how excess commuting is measured, and to present further evidence based on this model.

From a policy perspective, the improved analysis could be used to specify the extent to which Hamilton's suggestion that higher fuel prices could solve the problem by providing a disincentive for excess commuting, or the traditional policy prescriptions of human-capital based employment programs, stack up against Simpson's suggestion that measures to improve mobility and the effectiveness of search for lower-skilled workers are required.

F Investigation of Background Patterns from Census

A technique was developed for depicting patterns of prices, wages and densities across the urban landscape, in order to appraise patterns and determine if the traditional assumptions appear to be reasonably present. The grids that were used, to make a visual assessment of how the gradients look in the data, were quite useful for that purpose. However, they are not highly sophisticated, and it is uncertain how much effort might usefully go into developing them further.

G. Extension of Crosstab Evidence Using Offsets & Switches

While the analysis used crosstabs to pick out some of the usual relationships suggested by urban analysis – more importantly it was able to examine some relationships not commonly considered by breaking down the data by normal-vs-switched status, as well as offset status. In other words, this opened up the possibility of new types of testing. In several instances these new tests were able to confirm that normally placed respondents were more inclined to exhibit Alonso-like behavior, backed up in some instances by the behavior of less-offset respondents. Also examined were preference data combined with some crude amenity indicators in crosstabs, which tended to be consistent with the proposition that housing prices are hedonic indices which include amenities.

Since crosstabulations by switching and offset status are not elsewhere attempted, it could be worthwhile to present these results – as much for the methodology as for the results – since they basically constitute some new tests. In addition, though the current results just tend to confirm the hedonic pricing hypothesis, that may not always be the case, and the offset and switching analysis may be useful in that context as well.

In addition, the analysis used offsets and switches to specify a more direct measure of excess commuting. This produced new estimates, in the lower range, for the dataset that was here employed. Analysis again supported the suggestion that normally placed respondents in this context were more inclined to conform to Alonso-like behavior, which may merit some further testing.

H. Regression Estimates for Residential and Workplace Choice

Chapter 8 used regression to demonstrate some of the usual relationships suggested by urban analysis. As with crosstabs, it was able to examine some relationships not commonly considered by breaking down the data by normal-vs-switched status, as well as offset status. In other words, new tests were developed in the area of regression analysis as well. In several instances they were able to confirm that normally placed respondents were more inclined to conform to Alonso-like behavior. backed up in some instances by the behavior of less-offset respondents. They also showed that workplace location behavior display noticably less consistent behavior in this regard.

As with the crosstabs, it may be useful to present to a wider audience the results of breaking down the data by switching and offset behavior – in the context of these new tests being applied. It may be also be useful to take a closer look at why the workplace location decisions performed more poorly than the residential decisions in the regressions. – whether it may arise from the nature of the data, or may indicate that further analytical work is required.

I. Implications for Data Requirements

Although the data used in this study was from a well-designed survey – and it included questions which were extremely useful for the current analysis – it was to facilitate certain specific purposes, in a way which was perhaps not considered when designing the survey. For example, this paper's emphasis on determining the residential location precisely at the time when an employment decision was made, may not have been on the minds of the survey designers – and some further investigation may be able to suggest what kinds of questions may yield a greater number of those kinds of sample points. Another example is income, which needs greater clarity in distinguishing between respondent income and family head income, as well as specific breakdowns of other income. Overall, a treatment on data requirements for surveys of this type might be useful.

9.2 Commentary

The articulation of a model which employs general economic categories, whereby residence and housing costs, or workplace and wages, are specific instances of general transaction locations and prices, is instructive in that it brings out some of the abstract relationships between these diverse market activities. Apart from the theoretical pursuit of examining the theoretical roots of specific applications, its combination with the balance of forces diagrams of Section 2.1 offers a technique for clarifying the full set of relationships impinging on the urban decision-making environment. Its examination of the full range of options for urban price and density gradients, such as positive or negative slopes, gives occasion to clarify the reasons which underlie the traditional downward-sloping gradients. And it leads to developing some of the theoretical basis for understanding why some seemingly upward-sloping gradients in the sample data need nonetheless be interpreted with the tools of traditional gradient analysis.

A significant shortcoming of this generalized analysis is that it is really not as general as one might think. There is some arbitrariness in selecting which variables should be subject to the full range of possibilities. Why gradient slopes and not production functions (to take one of many possible examples)? One must be careful not to treat such analysis as if it has produced an ultra-general equilibrium analysis, when it is not even a full general equilibrium system at all. Nonetheless, opening the doors of generality a crack can be useful for broadening one's understanding of the fuller context of familiar models.

The graphical tool taken and extended from Evans/Fujita for representing urban decision-making in individual single quadrants (Figure 2.2) proves useful for laying out the full set of relationships, and to assist in articulating the impacts of changing parameters – and clarifying when parameters in different states (e.g., elasticities) produce different or opposing results. This is illustrated in Figure 2.3, laying out the conditions in which a different combination of price elasticities will yield a different direction of movement in urban space.

Place of residence plays a special role in urban analysis, and presents special problems. Much of the mathematical analysis of urban location works in differential and integral calculus on a spatial plane, as if housing were consumed at a single unique point in space. Analysts have wrestled with this, and ultimately applied a technique of pricing analysis drawn from commodity markets in general – hedonic pricing models. This has allowed the capture of not only specific housing attributes including access and spatial quantity, but also including attributes of individual housing locations or neighbourhoods – namely amenities – into a model of residential pricing. This technique is drawn into the current analysis, to further refine an understanding of how economic forces affect location, and to help specify the model for later testing. The drawback of this approach is that hedonic functions can be specified only in the abstract. Their implicit prices cannot simply be represented by the crude quantity of some amenity which happens to be available from census data (without unknowable distortions) – and yet this turns out to be the only available application.

A key development in urban modeling has been the incorporation of search analysis. The contributions of White and Simpson have added the power of sequential search analysis to urban location models, giving further power to predictions of urban location patterns. To that end, Simpson has incorporated the concepts of the island economy, pioneered by Phelps – and developed a specification of how the impact of search behavior acts in opposition to the traditional impact of commuting costs allows for the explanation of a wider range of behavior, and affords the opportunity to test for the relative strengths of these opposing effects. This study has used these tools in several ways: further articulating the model here being developed, combining search with the analytical tools of preference analysis to refine further the concept of reservation wage, and setting up the theoretical base for an empirical look at some of these phenomena.

A major focus of this paper has been the original assumption that all economic activity occurs along a single ray emanating from city centre, and some of the alternative specifications of urban placement. Poorly perceived gradients, multiple gradients, and the separation of price, density and amenity gradients may produce complex configurations, which may have to be taken into account when interpreting urban data, and this paper gives some theoretical analysis of their impact on urban location. Even more significant theoretically is the occurrence of offsets from the ray, and positional switches in relation to optimal placement along the ray. They occur in all urban data, but greater attention has been paid to them since Hamilton raised questions about excess commuting. Along with Hamilton, White and others, this paper contributes to the theoretical rationale for such location patterns (i.e., when offsets and switches may be rational), and to the precise distinction between rational and non-rational (excess) commuting. After some commentary on perceived shortcomings of the methodology of measuring excess commuting, an alternative technique is proposed (which is employed later in the empirical analysis).

Empirical analysis was based on a Minneapolis survey described in Chapter 6. Underlying the analysis presented in Chapters 7 and 8 are a variety of techniques which have been employed to render the data amenable to analysis. Particularly fortunate is the fact that the survey contains a longitudinal aspect which allows a more precise determination of the workplace/residence location at the precise time of a given residence/workplace decision – plus a number of preference questions which allow the examiner to separate out respondents according to their personal priorities. To make best use of such data, techniques were developed to calculate the precise incidence of offsets and switching in the sample, using latitude and longitude data in the survey. Techniques were also developed for mapping out the price, wage and density patterns of the Minneapolis area, in order to evaluate the realism of applying theoretical gradients.

In the empirical section, crosstabulations were used to study some simple relationships, such as impacts of income, family structure, etc., with mixed results. The more interesting crosstabs were those which took advantage of the offsets and switching data, for both the residential and the workplace decision. It was possible not only to determine the extent to which these were occurring, but also to bring in preference and neighbourhood data – in mixed support of theoretical expectations, and in the expectation that normally placed residence are more likely to conform to theoretical expectations. But probably more significant was the evidence that neighbourhood amenities seem to be well captured by housing prices, as suggested by the hedonic pricing models. Crosstabs were also run on excess commuting, indicating that its incidence is much less than Hamilton originally suggested, and that it also is sensitive to placement status in that normally placed respondents exhibit less excess commuting.

Regression analysis was also related to placement, supporting the earlier conclusions that normally placed respondents behaved more normally, but the relationships were not compelling. It was difficult to make any judgments on amenities, except perhaps that they may be effectively captured by prices, since they were found to be sufficiently correlated to prices to allow that possibility. Regressions on offsets were also a little weak in explanatory power, with a mild suggestion that offsets and commuting are correlated.

When analyzing the workplace decision, there were similar conclusions but one important addition. It was possible to develop a rough skill index and test for whether increased skill is associated with shorter/longer workplace distance from city centre (and roughly greater/lesser commuting in the unswitched cases) because search/cost is the dominant determinant of location choices. In this manner, it was possible to test the same proposition Simpson proposed – that more skilled workers tend more to priorize search factors, which may cause them to commute further to their jobs. The consistently declining skill coefficient as the skill index increased supported this conclusion, though the coefficients were not strong.

One possible area of improvement may be in the data. The surveys gave data on residence and workplace locations, employing a few different measures. One measure of location was the Traffic Analysis Zone (TAZ), which is comparable to 20 city blocks. Perhaps the author was not conversant enough in GIS (geographical) software to interpret them correctly, but they appeared to give inconsistent data. For that reason, street addresses were used instead. The problem is that attaching longitude and longitude information to street addresses was a manual process via the Internet, with all the potential for error inherent in coding 1003 addresses (though every entry was double checked).

Since that time, an affordable CD database of latitude and longitude by street address was discovered ("Microsoft Streets 98"), but too late to incorporate in this study. In addition, it has been drawn to the author's attention that the Census Transportation Planning package available through the U.S. Department of Transport provides extensive origin-destination information about household commuting behavior[18]. This, as well, was discovered too late in the research process to employ here. It is an open question whether the ability to use TAZ data, or the use of a more reliable address location database (in terms of possibly less coding errors), might have produced stronger results in the crosstabs or regressions. Perhaps more importantly, it might have yielded better results when determining workplace at the time of residential move and residence at the time of workplace change – better than the 362 and 404 useful data points that were employed. This precise specification of workplace-relevant residences and residence-relevant workplaces is a contribution worth examining in more detail. In addition, it might have given better (or worse) support for the decision not to employ simultaneous equations because so few of the decisions appeared to be simultaneous. In retrospect, it possibly should have been attempted anyway, to compare it to other studies which knew of no reason not to adopt the simultaneity assumption.

[18] I thank Professor David Merriman for pointing this out.

10. GLOSSARY

New Terms Introduced

Amenity Centre	A location where a particularly amenity is available, such that surrounding locations either lack the amenity or offer significantly less of it (though it may not form a gradient). Hence it may be attractive to some housing consumers and, if it is not incorporated into the price, may draw them away from the price gradient.
Consumption Location	The location where goods and services are consumed, normally considered identical with the place of residence (hence sometimes referred to as RES).
Draw	The extent and direction to which an Amenity Centre attracts a residence-seeker away from the price gradient, if the value of the amenity is not represented in the housing price.
Effect, Attraction	The effect which produces a downward sloping price gradient as a result of offering lower prices to compensate for greater travel requirements.
Effect, Distribution	The manner in which differences in commodity and population distributions produce an *Attraction Effect.*
Effect, Static Optimizing	The effect which produces a downward sloping price gradient as a result of cost and price optimizing behavior.
Gradient	A vector in which a variable change in a systematic manner as geographical distance increases or decreases. The normal gradient in urban analysis is convex and downward-sloping from the origin at city centre.
Gradient, Noisy	A gradient which is poorly discernible to the decision-maker because of missing or extraneous information.
Gradient, Reverse	A gradient which is upward-sloping from the origin at city centre.
Gradient, Thick	A gradient with significance variance, such that it would be represented by a band of values rather than a single locus.

Gradient, Triple-peak — A gradient which is calculated as the distance to the nearest of three price or density peaks (each of which, in themselves, are gradients).

Graph, Evans/Fujita — A graphical representation, derived from bid-rent curves, representing the same relationship in price-distance space.

Multiple Centres — The circumstance where there is not a single price or density peak, but rather two or more competing peaks. See *Regional Centre* and *Triple-peak Gradient*. (Not necessarily gradients – see *Amenity Centre*.)

Normal Configuration — The traditional placement of CC – WP – RES.

Offset — The perpendicular distance from the RES from the ray connecting CC to the WP, or the perpendicular distance of the WP from the ray connecting CC to the RES.

Placement — The relative positions of CC, WP and RES – which may or may not involve offsets and switches.

Regional Centre — One of several price or density peaks.

Reservation Price Set — The locus of reservation prices in price-distance space which represent the reservation price at that particular location.

Resolution of Forces Diagram — A diagrammatic representation of the forces working in opposition to each other which resolve the optimal location of RES and/or WP.

Special Quality Characteristics — Characteristics of a residence or workplace which distinguish them from the ordinary, and hence add an additional dimension to the commodity. In the case of residence, it refers to family housing attributes; and in the case of employment, it refers to employee skills.

Switched — A reversal of the normal configuration, where the RES lies between WP and CC (hence *Res-Switched*), or where RES/WP lies on the opposite side of CC to the WP/RES (hence *CC-Switched*).

Transaction Location (TL)	The location where any type of transaction occurs – whether they be the sale or purchase of housing, other commodities, or labour. A positive TL (TL_+) is a location where goods are bought. A negative TL (TL_-) is a location where goods are sold (i.e., at negative prices), which is usually a reference to employment (the sale of labour time).

Mathematical and Graphical Symbols

CC	City centre
CL	Consumption Location, or Residence
C_t	Travel cost function, showing its impact in a Balance of Forces diagram
d(u)	Density gradient
d_u	Density at location u
DC	Density Centre
Dist112	Distance to City Centre (Minneapolis city code 112)
Dist3Peak	Distance to the nearest of three identified price peaks
DistH	Distance to residence
DistW	Distance to workplace
$E_i s_l$, Ei	Job density at the respondent's skill level
F_u	Effect of distance on the probability of selecting a commodity
g	Quantitative measure or index of worker skill
h	Measured distance of residence to city centre
H_d	Effect of density on the probability of selecting a commodity
j	Measured distance of workplace to city centre
λ_i	Job density at island i

p, p^*	Price, reservation price
P_u	Housing price gradient, showing its impact in a Balance of Forces diagram
$p(u)$	Market price gradient, price at location u.
$p(u)^e$	Expected price gradient
Ψ	Price indifference curve in (P,u) space.
RC	Regional Centre
RES	Place of residence
$^{ACT}RES_A$	The actual place of residence for agent A.
$^{OPT}RES_A$	The optimal place of residence for agent A.
r	Measured commuting distance
s	Size of residential lot
t	Distance from residence to transaction location
tc	Travel cost
TL	Transaction Location, which may be positive (TL_+) or negative (TL_-)
u	Distance of residence from city centre
U^T	Set of reservation prices at each distance u
v	Distance of workplace from city centre
w, w^*	Wage, reservation wage
$w(u)$	Wage gradient, wage at location u
$w(u)^e$	Expected wage gradient

Vectors

c	Cost of travel, C(**t**)
$\mathbf{d}_u$	Market density at location u
l	That part of leisure time sold as labour
n	Quantity available of an amenity, measured as a hedonic index
p	Fixed prices, which may be positive or negative, and which may be divided into sub-vectors $\mathbf{p}_v$, $\mathbf{p}_w$, $\mathbf{p}_z$, $\mathbf{p}_n$, $\mathbf{p}_l$, **and** $\mathbf{p}_q$
q	Consumption of housing services
s	Disutility of search activity
t	Distance travelled to reach transaction location
u	Distance of transaction location to city centre
v	Endowment items sold
V	All goods and services in the commodity bundle
w	Endowment items remaining
W	Total endowments available at the beginning of the optimizing process
x	x-coordinate of a transaction location
X	Total consumption
y	y-coordinate of a transaction location
z	New commodities attained in exchange for the sale of **v**

BIBLIOGRAPHY

Alonso, William (1964), *Location and Land Use.* Cambridge, MA: Harvard University Press.

Alonso, W. (1967), "A Reformulation of Classical Location Theory and its Relation to Rent Theory", in *Papers and Proceedings of the Regional Science Association,* Vol. 19: 23-44.

Arnott, Richard (1987), "Economic Theory and Housing" in Mills (1987): 959-988.

Arrow, Kenneth (1974), "Limited Knowledge and Economic Analysis", *American Economic Review*, 64: 1-10.

Bailey, Martin (1975), "Dynamic Monopsony and Structural Change", in *American Economic Review*, 65: 338-49.

Becker, Gary (1964), *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education.* New York: Columbia University Press.

Beckmann, M. J. (1968), *Location Theory.* New York: Random House.

Beckmann, M. J. (1969), "On the Distribution of Urban Rent and Residential Density", *Journal of Economic Theory*, 1: 60-7

Beckmann, M. J. (1973), "Equilibrium Models of Residential Land Use", *Regional and Urban Economics*, 3:361-8

Blair, John P. (1991), *Urban and Regional Economics.* Homewood: Irwin.

Bloomquist, Glenn and Worley, Lawrence (1992), "Specifying the Demand for Housing Characteristics: The Exogeneity Issue", in Diamond and Tolley (1982).

Brigham, E.F. (1964). *A Model of Residential Land Values*, RAND Corporation.

Brigham, E.F. (1965), "The Determinants of Residential Land Values", in *Land Economics*, volume 41: 325-34.

Brown, J. and Rosen, H. (1982), "On the Estimation of Structural Hedonic Price Models", in *Econometrica*, 50: 765-768.

Brueckner, J. K. (1987), "The Structure of Urban Equilibria" in Mills (1987): 821-845.

Burgess, Ernest W. (1925), "The Growth of the City: an Introduction to a Research Project", in *The City*, ed. R. E. Park, E. W. Burgess, and R. A. McKenzie, Chicago: University of Chicago Press.

Chamberlin, E.H., (1956), *Theory of Monopolistic Competition*, 8th ed., Harvard University Press.

Clark, Colin (1967), *Population Growth and Land Use*, London: MacMillan.

Crecine, J. P., Davis, O. A., and Jackson, J. E. (1967), "Urban Property Markets: Some Empirical Results and Their Implications for Municipal Zoning", in *Journal of Law and Economics*, 10 (October, 1967): 79-100.

Cropper, M. (1984), "Should the Rosen Model be Used to Value Environmental Amenities?", in *University of Maryland Sloan Workshop in Urban Public Economics*, mimeo. College Park, Md. Cited in Straszheim (1987).

Cropper, Maureen L. and Gordon, Patrice L. (1991), "Wasteful Commuting: A Re-examination", in *Journal of Urban Economics*, 29: 2-13.

Courant, P. (1978), "Racial Prejudice in Search Models of the Urban Housing Market", in *Journal of Urban Economics*, 5: 329-45.

Daniels, C. (1975), "The Influence of Racial Segregation on Housing Prices", in *Journal of Urban Economics*, 2: 102-122.

Denslow, D., and Eaton, P. (1984), "Migration and Intervening Opportunities", *Southern Economic Journal*, 51: 369-87.

Diamond, Douglas B. Jr. and Tolley, George S., eds. (1982), *The Economics of Urban Amenities*, New York: Academic Press.

Diamond, Douglas B. Jr. and Tolley, George S. (1982a), "The Economic Role of Urban Amenities", in Diamond and Tolley (1982).

Duncan, Beverly (1956), "Factors in Work-Residence Separation, Wage and Salary Workers, Chicago, 1951", in *American Sociological Review*, 21: 48-56.

Evans, A. (1973), *The Economics of Residential Location*. London: Macmillan Press.

Evans, A. (1985), *Urban Economics*. London: Blackwell.

Feinberg, Robert (1977), "Risk-Aversion, Risk and the Duration of Unemployment", *Review of Economics and Statistics*, 59:264-71.

Freeman, A. (1979), "The Hedonic Price Approach in Measuring Demand for Neighbourhood Characteristics", in *Studies in Urban Economics*, edited by D. Segal. New York: Academic Press, 1979.

Fujita, Masahisa (1989), *Urban Economic Analysis*. Cambridge University Press.

Gastwirth, Joseph (1976), "On Probabilistic Models of Consumer Search for Information", *Quarterly Journal of Economics*, 90: 38-50.

Goldberg, M. A. (1970), "Transportation, Urban Land Values, and Rent: A Synthesis", in *Land Economics*, volume 46: 153-162.

Goodall, Brian (1972), *The Economics of Urban Areas.* Oxford: Pergammon.

Hamilton, Bruce (1982), "Wasteful Commuting", in *Journal of Political Economy*, 90: 1035-1053.

Hamilton, Bruce (1989), "Wasteful Commuting Again", in *Journal of Political Economy*, 97: 1497-1504.

Handlin, O. and Burchard, J. eds. (1963), *The Historian and the City*, MIT Press.

Harris, Britton (1968), "Quantitative Models of Urban Development", in *Issues in Urban Economics*, ed. Harvey S. Perloff and Lowdon Wingo, Jr., Baltimore: Johns Hopkins Press.

Harris, R. N. S., Tolley, George S., and Harrell, C. (1982), "The Residential Site Choice", in Diamond and Tolley (1982).

Hekman, John (1980), "Income, Labor Supply, and Urban Residence", in *American Economic Review*, 70: 805-11

Henderson, James M. and Quandt, Richard E. (1958), *Microeconomic Theory.* McGraw-Hill: New York.

Hoover, E. M. (1968), "The Evolving Form and Organization of the Metropolis", in H. S. Perloff and L. Wingo Jr., *Issues in Urban Economics*, Johns Hopkins: Baltimore: 237-84.

Jackson, Jerry (1979), "Interurban Variation in the Price of Housing", in *Journal of Urban Economics*, 6: 464-79.

Kain, John (1975), *Essays on Urban Spatial Structure.* Cambridge, Mass.: Ballinger.

Kohlhase, Janet E. (1986), "Labor Supply and Housing Demand for One- and Two-Earner Households", in *Review of Economics and Statistics*, 68: 48-57.

Krugman, Paul (1991), *Geography and Trade.* Cambridge, Mass.: MIT Press.

Lampard, E. E. (1963), "Urbanization and Social Change" in Handlin and Burchard (1963): 225-247.

Linneman, Peter (1992), "Hedonic Prices and Residential Location", in Diamond and Tolley (1982).

Lucas, Robert and Prescott, Edward (1974), "Equilibrium Search and Unemployment", in *Quarterly Journal of Economics*, 84: 113-26.

McCall, John (1965), "The Economics of Information and Optimal Stopping Rules", *Journal of Business*, 38: 300-17.

Merriman, David, Ohkawara, Toru and Suzuki, Tsutomu (1995), "Excess Commuting in the Tokyo Metropolitan Area: Measurement and Policy Simulations". Urban Studies 32(1):69-85.

Meyer, J. P., Kain, J. F. and Wohl, M. (1965), *The Urban Transportation Problem.* Cambridge, Mass.: Harvard University Press.

Mills, Edwin (1972), *Studies in the Structure of the Urban Economy*, Baltimore: Johns Hopkins Press.

Mills, Edwin (1987), ed. *Handbook of Regional and Urban Economics*, Volumes I and II, North Holland.

Mills, Edwin S. and Hamilton, Bruce W. (1994), *Urban Economics.* New York: Harper Collins.

Morgan, Peter (1983), "Search and Optimal Sample Sizes", in *Review of Economic Studies*, 50: 659-75.

Mortenstern, Dale (1986), "Job Search and Labor Market Analysis", in Orley Ashenfelter and Richard Layard (eds.), *Handbook of Labor Economics: II*, Amsterdam: North Holland: 849-919.

Moses, Leon (1962), "Towards a Theory of Intra-urban Wage Differentials and their Influence on Travel Patterns", in *Papers and Proceedings of the Regional Science Association*, 9: 53-63.

Muth, R. F. (1961), "Economic Change and Rural-urban Land Conversion", *Econometrica*, 29: 1-23.

Muth, R. F. (1968). "Urban Residential Land and Housing Markets", in Perloff and Wingo (1968): 285-330.

Muth, R. F. (1969), *Cities and Housing*. University of Chicago Press.

Nelson, Philip (1973), "The Elasticity of Labor Supply to the Individual Firm", *Econometrica*, 41: 853-66.

Oi, Walter (1976), "Residential Location and Labor Supply", in *Journal of Political Economy*, 84: S221-238.

Olsen, Edgar O. (1987), "The Demand and Supply of Housing Services: A Critical Survey of the Empirical Literature", in *Handbook of Regional and Urban Economics* (1987): 989-1022.

Paelinck, J. H. P. and Klassen, L. H. (1979), *Spatial Econometrics*, Farnborough: Saxon House.

Perloff, H.S. and Wingo, L. Jr. (eds.), (1968), *Issues in Urban Economics*, Johns Hopkins Press.

Phelps et al. (1970), *Microeconomic Foundations of Employment and Inflation Theory*. New York: W. W. Norton.

Polinsky-Shavell, 1976, "Amenities and Property Values in a Model of an Urban Area", in *Journal of Public Economics*. 5: 199-299.

Rees, Albert and Schultz, George (1970), *Workers and Wages in an Urban Labor Market*. Chicago: University of Chicago Press.

Richards, J. M. (1963), "The Significance of Residential Land Use in Cities", in *Human Resources in the Urban Economy*. Baltimore, Md.: Johns Hopkins: 123-150.

Richardson, Harry W. (1971), *Urban Economics*. Penguin Books.

Richardson, Harry W. (1978), *Regional Economics*. Urbana: University of Illinois Press.

Rosen, Sherwin (1974), "Hedonic Prices and Implicit Markets", in *Journal of Political Economy*, 82: 34-55.

Schelling, T. (1972), "Neighbourhood Tipping", in A. Pascal, ed. *Racial Discrimination in Economic Life*, Lexington, Mass.: D.C. Health.

Siegel, Jay (1975), "Intrametropolitan Migration: A Simultaneous Model of Employment and Residential Location of White and Black Households", in *Journal of Urban Economics*, 2: 29-47.

Simpson, Wayne (1980), "A Simultaneous Model of Workplace and Residential Location Incorporating Job Search", in *Journal of Urban Economics*, 8: 330-349.

Simpson, Wayne (1986), "Workplace Location, Residential Location and Urban Commuting", in *Urban Studies*, 24: 119-128.

Simpson, Wayne (1992), *Urban Structure and the Labour Market*. Clarendon Press.

Simpson, Wayne and van derVeen, Anne (1992), "The Economics of Commuting and The Urban Labour Market", in *Journal of Economic Surveys*, 6: 45-62.

Small, Kenneth A. and Song, Shunfeng (1992), "Wasteful Commuting: A Resolution", in *Journal of Political Economy*, 100: 888-98.

Smith, W.F. (1969), "Optimal Lot Size When Rent is a Transfer Payment", in *Annals of Regional Science*, volume 3: 8-15.

Stegman, M. A. (1969), "Accessibility Models and Residential Location", in *Journal of the American Institute of Planners*, 35: 22-39.

Stigler, George (1961). "The Economics of Information", in *Journal of Political Economy*, 69: 213-25.

Stigler, George (1962), "Information in the Labor Market", in *Journal of Political Economy*, 70: S94-105.

Strazheim, Mahlon (1973), "Estimation of the Demand for Urban Housing Services from Household Interview Data", in *Review of Economics and Statistics*, 55: 1-8.

Strazheim, Mahlon (1975), *An Econometric Analysis of the Urban Housing Market*, New York: Columbia University Press.

Strazheim, Mahlon (1987), "The Theory of Urban Residential Location" in Edwin Mills (1987): 717-757.

Suh, Seoung Hwan (1990), "Wasteful Commuting: an Alternative Approach", in *Journal of Urban Economics*, 28: 277-86.

Tsuru, S. (1963), "The Economic Significance of Cities" in Handlin and Burchard (1963): 44-55.

Varian, H. R. (1984), *Microeconomic Analysis*, 2nd Ed. New York: Norton.

Varian, H. R. (1990), *Intermediate Microeconomics: A Modern Approach,* 2nd Ed. New York: Norton.

von Thunen, J. H. (1826), *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalekonomie*. Hamburg.

Wheaton, W. C. (1974), "A Comparative Static Analysis of Urban Spatial Structure". in *Journal of Economic Theory*, 9: 223-237.

White, Michelle (1988a), "Urban Commuting Journeys are not Wasteful", in *Journal of Political Economy*, 96: 1097-1110.

White, Michelle (1988b), "Location Choice and Commuting Behavior in Cities with Decentralized Employment", in *Journal of Urban Economics* 24: 129-152.

Wingo, Lowdon, Jr. (1961), *Transportation and Urban Land*, Washington, D. C.: Resources for the Future.