

# Design and Experimental Evaluation of Cascaded Metasurfaces

by

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# Abstract

Tailoring electromagnetic waves has various applications in wireless communications, medical imaging, and remote sensing. However, to be suitable for a given application, the spatial properties of electromagnetic waves need to be manipulated to fit that application. This is generally done by designing antennas based on the required performance criteria. Designing antennas for complicated performance requirements can be quite time-consuming as it may require extensive full-wave optimization. Alternatively, a metasurface can be used as it can systematically transform electromagnetic waves to what the designer desires. Electromagnetic metasurfaces are sheet-like structures consisting of artificial “atoms” called meta-atoms or unit cells, that are small compared to the operational wavelength. When electromagnetic waves interact with these unit cells, they are transformed based on the properties of the unit cells. The design of lossless and passive metasurfaces requires enforcing total power conservation (TPC) and local power conservation (LPC) to make sure that the power is conserved in the whole metasurface and at each individual unit cell. If a single metasurface cannot satisfy the required constraints, a metasurface pair, also known as cascaded metasurfaces, can be considered. In this thesis, a fully automated design approach is developed for cascaded metasurface system designs with external sources based on an existing design approach. The overall design is divided into macroscopic and microscopic steps. In the macroscopic step, an automated conjugate gradient (CG) optimization algorithm is developed with analytical expressions to arrive at the required surface properties of the cascaded metasurface system. In the microscopic step, the unit cells are physically realized by copper traces determined using a lookup table which was further optimized to achieve the desired forward scattering parameters for each unit cell. The electromagnetic wave transformation is then verified by modeling the cascaded metasurface system in Ansys HFSS software and performing the full-wave simulation. Finally, a cascaded metasurface system is fabricated and



measured. This serves as one of the few fabricated cascaded metasurfaces reported in the literature. Planar near-field antenna range (PNFR) measurements are conducted to experimentally evaluate the performance of the cascaded metasurface system. The successful simulation and experimental verification confirm the working of the fully automated cascaded metasurface design algorithm.

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# List of Abbreviations

| Abbreviation | Description                              |
|--------------|------------------------------------------|
| <b>EM</b>    | Electromagnetic                          |
| <b>2D</b>    | Two-dimensional                          |
| <b>3D</b>    | Three-dimensional                        |
| <b>FF</b>    | Far-field                                |
| <b>NF</b>    | Near-field                               |
| <b>ROI</b>   | Region of Interest                       |
| <b>LPC</b>   | Local Power Conservation                 |
| <b>TPC</b>   | Total Power Conservation                 |
| <b>TE</b>    | Transverse electric                      |
| <b>PEC</b>   | Perfect electric conductor               |
| <b>CG</b>    | Conjugate-gradient                       |
| <b>HPBW</b>  | Half-Power Beam-width                    |
| <b>SLL</b>   | Side-Lobe-Level                          |
| <b>G</b>     | Gain                                     |
| <b>D</b>     | Directivity                              |
| <b>GSTCs</b> | Generalized sheet transitions conditions |
| <b>MS</b>    | Metasurface                              |
| <b>OEW</b>   | Open ended wave guide                    |
| <b>PCB</b>   | Printed circuit board                    |
| <b>PML</b>   | Perfectly matched layer                  |

# List of Symbols

| Symbol                       | Description                         |
|------------------------------|-------------------------------------|
| $\boldsymbol{E}$             | Electric field vector               |
| $\boldsymbol{H}$             | Magnetic field vector               |
| EM                           | Electromagnetic fields              |
| PWS                          | Plane Wave Spectrum                 |
| $c_o$                        | Free space speed of light           |
| $\eta_o$                     | Free space characteristic impedance |
| $\epsilon_o$                 | Free space permittivity             |
| $\mu_o$                      | Free space permeability             |
| $k_o$                        | Free space wavenumber               |
| $\lambda_o$                  | Free space wavelength               |
| $\overline{\overline{\chi}}$ | Susceptibility tensor               |
| $j$                          | Imaginary unit ( $j^2 = -1$ )       |

# Chapter 1

## Introduction

In this introductory chapter, the motivation behind the research is presented followed by the objectives of the research, overall methodology, research contributions, and finally an outline of the thesis.

### 1.1 Motivation

Ever since mankind developed the ability to understand electromagnetic waves, we have been in a continuous race to find ways to further control and manipulate electromagnetic waves for various applications. Electromagnetic waves are an essential part of human lives. For instance, sunlight is an electromagnetic wave in the visible section of the electromagnetic spectrum that enables us to see objects. Sunlight or light emitted from torches, lamps, etc., enables us to mostly see the exterior parts of an object that the light reflects from. Another part of the electromagnetic spectrum belongs to X-rays, which have been widely used in medical imaging to see the internal organs of human beings, e.g., identifying bone fractures. In the past, communication among human beings was mainly limited to human beings being in close proximity

to each other. Now, we can employ radio waves, another part of the electromagnetic spectrum, to communicate with people miles away in almost real-time. Furthermore, in recent decades, the application of electromagnetic waves has become really important in the defense sector; e.g., the military uses radar technology for surveillance. In addition, many remote sensing applications, such as agricultural (e.g., soil moisture detection [1]) and environmental (e.g., oil spill detection [2]) remote sensing applications require illuminating the regions of interest with electromagnetic waves, e.g., in the microwave frequency range, to determine the properties of interest. In all these applications, control and manipulation of electromagnetic waves are necessary. For example, control of the beam direction, beam directivity (i.e., how focused it is), sidelobe levels, and nulls are important.

As can be seen, electromagnetic waves have several beneficial applications. Consequently, there have been extensive research to find new application areas that rely on electromagnetic waves. This typically requires developing methods to tailor the spatial distribution of electromagnetic waves. Different applications require different manipulation of electromagnetic waves, which is typically achieved by an antenna. An antenna is a transition device that converts guided electromagnetic waves to radiating ones (and vice versa) [3] and can therefore transmit and receive electromagnetic waves. For instance, in wireless communications, an electromagnetic wave signal is generated and transmitted from an antenna within the cell phone which can then be received by the base station and the receiving cell phone's antenna. In biomedical microwave imaging, incident electromagnetic waves at microwave frequencies are generated by an antenna that illuminates the body being imaged. These incident electromagnetic waves will then interact with the body being imaged. The scattered electromagnetic waves are then measured using antennas and the collected data is then processed by an imaging algorithm to develop an image of the body.

Antennas are structures typically made from metallic materials like copper and



dielectric materials with different permittivity values. For example, a microstrip antenna can consist of a copper patch and a copper ground plane separated by a dielectric substrate. The design of an antenna depends on various requirements to be achieved such as the power pattern, half-power beam width (HPBW), gain (G), directivity (D), side-lobe-level (SLL), null angles, and polarization. Therefore, the design of an antenna can be simple or it can be complex depending on the requirements. The antenna technology has advanced quite a lot since the first experimental demonstration of the existence of electromagnetic waves by Hertz in 1887. A single antenna element, such as a dipole, monopole, microstrip, horn, or reflector antenna, can be designed and fabricated if the requirements are not too complicated (e.g. if complicated beam shaping is not required). As the requirements begin to get more complicated, we can use an array of antennas, e.g. phased-array antennas, to realize more diverse forms of radiation patterns.

With the development of electromagnetic metamaterials [4], the antenna community was able to use more degrees of freedom in their design approaches. Metamaterials are artificial materials consisting of subwavelength unit cells that can provide dielectric and magnetic properties that do not exist in natural materials. Metamaterials can then be used to further manipulate electromagnetic waves. Metasurfaces [5], which are the focus of this thesis, is a sub-class of metamaterials. These are in fact metamaterials that are thin with respect to the wavelength of operation.

Metasurfaces have gathered lots of attention in recent years because of their ability to systematically transform electromagnetic waves [6, 7]. Just like a transformer in the power system industry that would transform voltage from the primary winding to the secondary winding, a metasurface transforms the incoming electromagnetic wave impinging on the metasurface to a desired electromagnetic wave on the output of the metasurface. A metasurface can be designed to be either a passive or an active device. This is in contrast to phased-array antennas where it is an active device

with components such as phase shifters. Although, a phased-array antenna makes it possible to realize complicated radiation patterns, one of its major drawbacks is the requirement to use active elements. On the other hand, a passive metasurface could be used with a simple antenna (i.e., one radio-frequency excitation port). When the incident wave of this antenna interacts with the metasurface system, phase and amplitude tailoring can be performed by proper design of passive elements (known as unit cells or meta-atoms). This phase and amplitude tailoring can then result in the desired radiation pattern.

In summary, the main motivation for using metasurfaces is the systematic mechanism that they offer to design various types of radiation patterns. In particular, passive metasurfaces provide further practical advantages such as lower fabrication and maintenance costs as compared to active antenna array systems.

## 1.2 Research Objectives

The objective of this research is to develop an automated approach to designing lossless and passive metasurface systems that can transform a given electromagnetic wave into a desired one. In the literature, it is already shown that cascaded metasurfaces (also known as metasurface pairs) can be used for such lossless and passive transformation [8–11].<sup>1</sup> These approaches typically rely on some *ad hoc* choice of parameters in the design optimization.

We have two main objectives. The first one is to avoid this *ad hoc* choice, thus developing an automated design algorithm. The second one is to perform the experimental verification of such cascaded metasurfaces. This is relatively new and rare

---

<sup>1</sup>In this thesis, the cascaded system of metasurfaces refers to the design of a system having two metasurfaces. However, a cascaded system of metasurfaces is not limited to having just two metasurfaces.

in the literature. For example, the author is only aware of the experimental verification for *optical* cascaded metasurfaces reported in [12] and the one in microwave frequencies in [13].

To this end, this research is phased into three stages and thus similarly this thesis is divided into three major sections: (i) theoretical discussion of the cascaded metasurface systems, (ii) design for the physical realization, and (iii) finally an experimental verification of the performance of a cascaded metasurface system designed using the automated algorithm developed herein at microwave frequencies.

The fundamentals of the design process of the cascaded system of metasurfaces considered in this thesis are from [14]. In the work presented in [14], the authors use the space between the two metasurfaces (both the distance between the two metasurfaces and the multiple reflections in this space) to transform the given incident fields into new electromagnetic fields. The new electromagnetic fields serve as the aperture fields for this radiating system. If these aperture fields are synthesized properly, they can create the required radiation pattern. As will be seen, we take the design process used in [14] and turn it into a fully automated conjugate gradient design algorithm.

Once the design algorithm converges to a solution, we know the required tangential fields on both faces of each metasurface. The next stage is then to use these fields to design the subwavelength components of the metasurface, known as unit cells. This can be first performed using *ideal* impedances in full-wave simulations. (Such ideal impedances are implemented using impedance boundary conditions.) We then need to design physically realizable unit cells based on these impedances. The physical design of the utilized metasurfaces is based on three-layer unit cell configurations with copper traces in the shape of dogbones (*H*-shaped copper traces), e.g., see [15–17]. When many of these subwavelength unit cells are put together, they form a

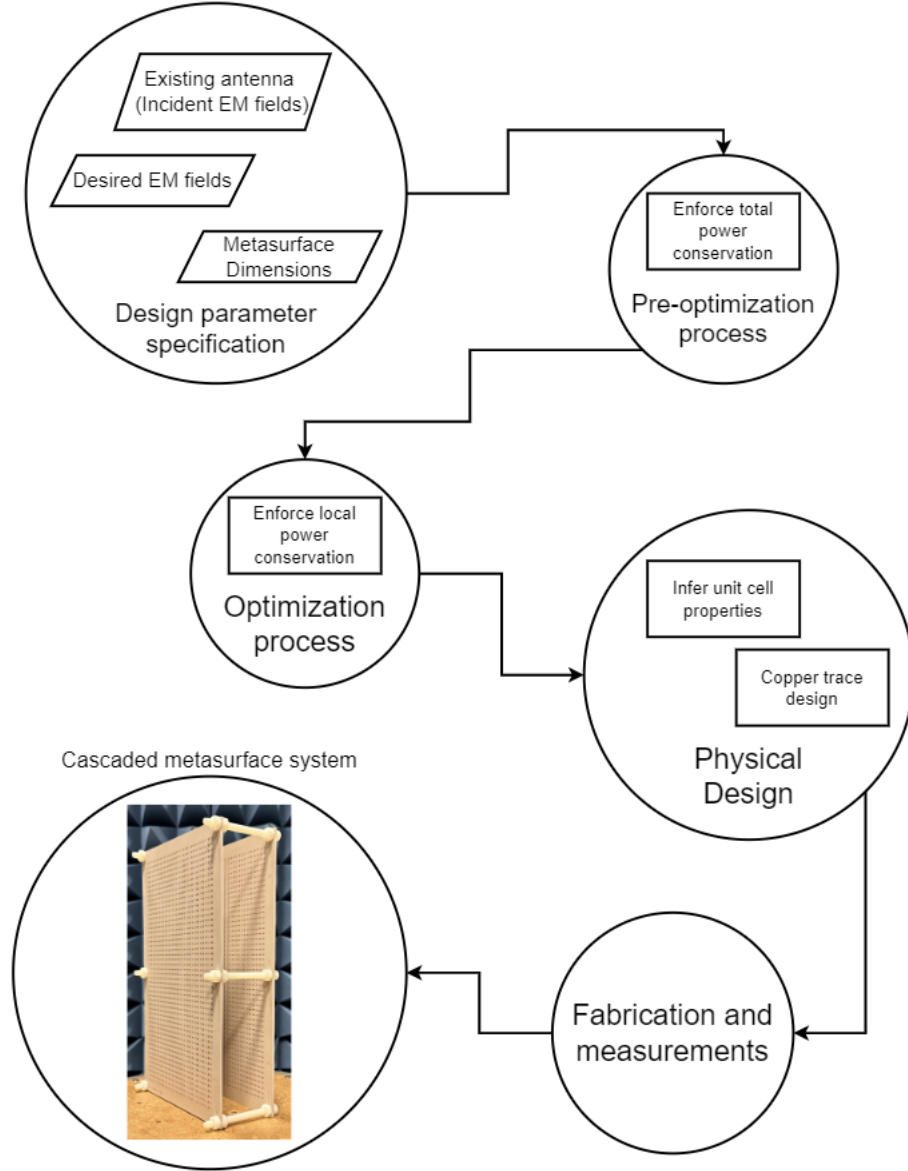


Figure 1.1: Flowchart summary of the thesis

metasurface which collectively transforms the incoming electromagnetic wave to the desired outgoing one. Finally, the physical design is fabricated and experimental tests are performed to evaluate the performance of the cascaded system of metasurfaces.

### 1.3 Methodology Overview

The overall methodology can be seen in Fig. 1.1. As noted earlier, we begin with the formulation presented in [14]. Using this formulation, we then derive analytical expressions needed to develop a conjugate gradient algorithm to design a cascaded system of metasurfaces. This includes the analytical expressions for the gradient vector in the complex domain, and subsequently the conjugate gradient vector in the complex domain, the real-valued step length that changes at each iteration of the algorithm, and an analytical expression for the initial guess of the algorithm. Using this automated algorithm, we focus on some electromagnetic wave transformations such as cylindrical waves to truncated plane waves. The solution of the design algorithm is a set of required tangential fields on the faces of the metasurfaces. These tangential fields are then used to model and simulate the cascaded metasurfaces in ANSYS® HFSS™ software to verify that the cascaded metasurfaces can perform the desired wave transformation. At this stage, the metasurfaces are implemented by ideal surface impedances. Next, we design physically realizable metasurfaces with three-layered unit cells with patterned metallic cladding in the shape of dogbones supported by dielectric substrates. These physically realizable designs are also verified in ANSYS® HFSS™ software. Finally, we fabricate a cascaded metasurface system and test it using a planar near-field antenna range.

### 1.4 Research Contributions

This thesis outlines an end-to-end design of a cascaded system of metasurfaces from analytic expression to the physical realization of the system. Major contributions resulting from this thesis include:

1. Development of the complex-domain analytical expressions for the gradient vec-

tor, Jacobian matrix as well as the development of analytical expressions for the step length and the initial guess for an existing design approach.

2. Design and evaluation of cascaded metasurface systems using physically realizable unit cells at microwave frequencies.
3. Fabrication and experimental evaluation of a cascaded metasurface system at microwave frequencies.

It should be noted that we are only aware of two works that show a fabricated cascaded metasurface system [12, 13]. The work reported in [12] deals with optical frequencies, and the work presented in [13] uses a cascaded metasurface at microwave frequencies to extend the scan angle of a phased array antenna.

The author's publications as a result of this research are the followings.

- **J. Gohel**, Z. Liu, and P. Mojabi, "A Conjugate Gradient Algorithm with Analytical Expressions for Compact Metasurface Pair Design," *IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (AP-S/URSI)*, pp. 73–74, Denver, USA, 2022.
- **J. Gohel**, M. Kelly, and P. Mojabi, "Investigating the Performance of Cascaded Metasurfaces with Physically Realizable Unit Cells," *URSI Int. Symp. Electromag. Theory (URSI EMTS)*, pp. 1-4, Vancouver, Canada, 2023.
- **J. Gohel**, M. Phaneuf, and P. Mojabi, "Design and Experimental Evaluation of Cascaded Metasurface Pairs," *IEEE Conference on Antenna Measurements and Applications (CAMA)*, pp. 1-6, Genoa, Italy, 2023 (Submitted).

## 1.5 Thesis Outline

The remainder of this thesis is outlined as follows. Chapter 2 provides an introduction to metamaterials and metasurfaces. Thereafter, the fundamental formulation of the metasurface is discussed along with the review of related work in the macroscopic and microscopic design of the metasurface.

Chapter 3 elaborates on the macroscopic and microscopic design models of our cascaded metasurface system. In the macroscopic design algorithm, the design formulation in [14] is reviewed in detail. Then an analytical automated conjugate gradient design algorithm is developed in the complex domain. Thereafter, the theoretical and practical model of the 3-layered unit cell design for the microscopic design of the metasurface is discussed.

Chapter 4 discusses the modeling strategy of the cascaded metasurface system in ANSYS® HFSS™ software along with the simulation setup of performing the full-wave simulation. Also, the PNFR experimental setup and measurement technique are discussed to evaluate the performance of the fabricated cascaded metasurface system.

Chapter 5 presents and discusses the full-wave simulation results of examples of cascaded metasurface systems developed using the design algorithm developed herein. Also, an experimental investigation on a fabricated cascaded metasurface system is presented and discussed. Lastly, chapter 6 summarizes the research objectives and areas for consideration in future studies.

# Chapter 2

## Introduction to Metasurfaces

This chapter introduces some general aspects of metasurfaces. In particular, the boundary conditions associated with metasurfaces are considered. As will be seen, these are known as the generalized sheet transition conditions (GSTCs).

### 2.1 Metamaterials

As far as the interaction of electromagnetic waves with objects is concerned, every object can be characterized in terms of permittivity ( $\epsilon$ ), permeability ( $\mu$ ), and conductivity ( $\sigma$ ). (These might be tensors.) The propagation of electromagnetic waves in a medium is affected by these properties. In particular, when the permittivity or permeability of a medium is changed, the electromagnetic fields in the medium also change. This gives the possibility to manipulate electromagnetic waves by changing the dielectric or magnetic properties [18] of materials. This is for example used in the design of dielectric lens antennas [19] or the optical lenses used in eyeglasses. For instance, if an electromagnetic wave goes from free space into a different medium, the wave can go through reflection and refraction. This is due to the fact that the



permittivity ( $\epsilon_0$ ) and/or permeability ( $\mu_0$ ) of free space are different than those of the new medium. Therefore, to satisfy electromagnetic boundary conditions, reflection, and refraction can occur. Many permittivity and permeability values are available to us through natural materials. One way to expand these available permittivity and permeability values is through the mixing of different substances. However, other approaches are needed to create more exotic permittivity and permeability values, e.g., those that lead to a negative refractive index [20]. This led to the realization of artificial materials called metamaterials. (Meta means *beyond* in Greek.) One of the challenges with metamaterials is that they are bulky. Therefore, their fabrication can be costly and due to their bulky structure, they can be lossy as well. Therefore, metasurfaces were considered as an alternative as they are easier to fabricate and they can be less lossy due to their thin structure.

## 2.2 Metasurfaces

A metasurface is a thin artificial material consisting of subwavelength inclusions designed to support the transformation of electromagnetic fields from its input face to its output face [5, 15, 21–25]. In this thesis, we consider transmitting metasurfaces. Thus, we always consider two sides to the metasurface: the input side and the output side. This is demonstrated in Figure 2.1 where the metasurface, placed along the  $x$  axis, is designed such that it transforms the electric and magnetic fields on its input face, denoted by  $E^-$  and  $H^-$  respectively, to the desired fields on its output face, denoted by  $E^+$  and  $H^+$ . As will be seen later in the thesis, the metasurface consists of subwavelength inclusions. These inclusions are responsible for supporting the transformation of the electromagnetic fields across the metasurface. At microwave frequencies, these inclusions are normally copper traces with varying dimensions printed on layers of dielectric substrates. These subwavelength inclusions are referred to as unit cells or

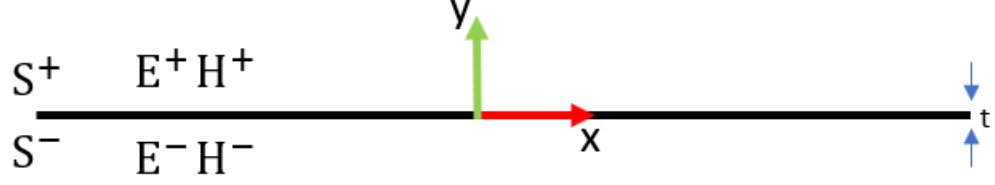


Figure 2.1: A metasurface along the  $x$  axis designed to support the EM field discontinuity between its two faces, i.e.,  $\{E^-, H^-\}$  on its input face and  $\{E^+, H^+\}$  on its output face.

meta-atoms. When an electromagnetic wave impinges on the input side of the metasurface, equivalent electric and magnetic currents are induced on the metasurface [24] as depicted in Figure 2.2. The fields due to these equivalent currents and the incident field result in the fields emanating by the metasurface in its output region. The spatial features of these subwavelength inclusions control these equivalent electric and magnetic currents. Subsequently, these equivalent currents can be used to support the change of the electromagnetic fields between the input face and the output face of the metasurface. This method can be used for radiation pattern control (beam shaping) [26, 27], polarization control [6], impedance matching [23, 28], cloaking [29], absorption [30, 31], etc.

## 2.3 Boundary Conditions of Metasurfaces

The design of metasurfaces can be done based on considering their boundary conditions known as the generalized sheet transition conditions (GSTCs) [32–35], [23]. The GSTCs relate the electromagnetic waves on both sides of the metasurface to the properties of the metasurface. The properties of the metasurface can be described in different ways, e.g., surface susceptibilities [33].

To understand the development of GSTCs, let us begin by considering Figure 2.2

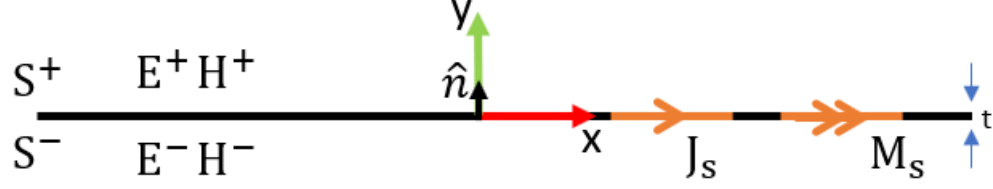


Figure 2.2: When the metasurface is illuminated by an incident electromagnetic wave, equivalent electric  $\mathbf{J}_s$  and magnetic  $\mathbf{M}_s$  currents are induced on the metasurface, which can support the electromagnetic wave transformation.

and the electromagnetic equivalence principle. The electromagnetic equivalence principle states that the discontinuity of the tangential electric and magnetic fields at an interface needs to be supported by surface electric and magnetic currents. This can be written as [36]

$$\hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) = \mathbf{J}_s \quad (2.1a)$$

$$(\mathbf{E}^+ - \mathbf{E}^-) \times \hat{n} = \mathbf{M}_s \quad (2.1b)$$

where the superscripts “+” and “-” indicate both sides of the interface,  $\hat{n}$  is the normal unity vector to the interface pointing from “-” side to “+” side,  $\mathbf{E}$  and  $\mathbf{H}$  are the electric and magnetic fields, and  $\mathbf{J}_s$  and  $\mathbf{M}_s$  denote surface electric and magnetic currents on the interface, respectively. Finally, “ $\times$ ” is the vector cross product.

Next, we look at the mechanism with which a metasurface creates these equivalent surface currents to support the discontinuity of the electromagnetic fields across the interface. A metasurface as mentioned before consists of subwavelength unit cells or meta-atoms. When these unit cells are illuminated by an incident electromagnetic field, electric and magnetic surface polarization densities, denoted by  $\mathbf{P}_s^e$  and  $\mathbf{P}_s^m$  respectively, will be created on the metasurface. (The subscript “s” emphasizes that they are surface quantities.) These surface polarization densities can have both tangential and normal components. The normal components of the surface polariza-

tion densities are ignored which can be justified based on the fact that the thickness of the metasurface is very small compared to the wavelength [37]. Subsequently, the surface polarization densities will only have the tangential components, and can therefore be represented as  $\mathbf{P}_{t,s}^e$  and  $\mathbf{P}_{t,s}^m$  respectively where the subscript “ $t$ ” denotes the tangential component [18].

The required surface currents in (2.1) are provided by these tangential surface polarization densities which result in the GSTCs that govern the operation of a metasurface. The resultant GSTCs are written as as [18, 23, 32, 35, 38]

$$\hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) = j\omega \mathbf{P}_{t,s}^e \quad (2.2a)$$

$$(\mathbf{E}^+ - \mathbf{E}^-) \times \hat{n} = j\omega\mu_0 \mathbf{P}_{t,s}^m \quad (2.2b)$$

where  $j^2 = -1$  and  $\omega$  is the angular frequency. Note that the dimension of  $\mathbf{J}_s$  is  $[\frac{A}{m}]$ , the dimension of  $\mathbf{P}_{t,s}^e$  is  $[\frac{C}{m}]$  ( $[C]$  denotes Coulombs), and therefore  $j\omega \mathbf{P}_{t,s}^e$  will have a dimension of  $[\frac{C}{m.s}] = [\frac{A}{m}]$ , which is the same as that of  $\mathbf{J}_s$ . Similarly, the dimension of  $\mathbf{M}_s$  is  $[\frac{V}{m}]$ , the dimension of  $\mathbf{P}_{t,s}^m$  is  $[A]$ , and therefore  $j\omega\mu_0 \mathbf{P}_{t,s}^m$  will have a dimension of  $[\frac{A.H}{m.s}] = [\frac{V}{m}]$ , which is the same as that of  $\mathbf{M}_s$ .

Having the electromagnetic equivalence principle being represented with tangential surface polarization densities, the next step is to relate these tangential surface polarization densities ( $\mathbf{P}_{t,s}^e$  and  $\mathbf{P}_{t,s}^m$ ) to the properties of the metasurface and the electromagnetic fields. In order to do so, consider a bianisotropic material. In bianisotropic materials, the electric polarization density is related to both electric and magnetic fields. Similarly, the magnetic polarization density is related to both electric and magnetic fields. That is why we use the prefix *bi* to indicate the effect of both electric and magnetic fields on the electric and magnetic polarization densities [38]. Also, the response of anisotropic materials, and therefore bianisotropic materials as well, depends on electromagnetic field orientations. This will make the properties of anisotropic materials into tensor formats, and not simple scalar quantities [39].

One way to represent the properties of the materials is susceptibilities. Similarly, in metasurfaces, we can use surface susceptibilities [33, 38]. In the surface susceptibility model for bianisotropic metasurfaces, we can have electric-to-electric, magnetic-to-magnetic, magnetic-to-electric, and electric-to-magnetic surface susceptibilities denoted by  $\bar{\bar{\chi}}_{ee}$ ,  $\bar{\bar{\chi}}_{mm}$ ,  $\bar{\bar{\chi}}_{em}$ , and  $\bar{\bar{\chi}}_{me}$ , respectively (Note that  $\bar{\bar{\chi}}$  denotes that  $\chi$  is a tensor.). For example, the electric-to-electric surface susceptibility tensor is shown below as

$$\bar{\bar{\chi}}_{ee} = \begin{bmatrix} \chi_{ee}^{xx} & \chi_{ee}^{xy} & \chi_{ee}^{xz} \\ \chi_{ee}^{yx} & \chi_{ee}^{yy} & \chi_{ee}^{yz} \\ \chi_{ee}^{zx} & \chi_{ee}^{zy} & \chi_{ee}^{zz} \end{bmatrix}. \quad (2.3)$$

However, since we are only concerned with tangential surface polarization densities, the above  $3 \times 3$  tensor can be reduced to a  $2 \times 2$  tensor. Assuming that  $\hat{y}$  is normal to the metasurface, the above surface susceptibility tensor can be reduced to

$$\bar{\bar{\chi}}_{ee}|_{2D} = \begin{bmatrix} \chi_{ee}^{xx} & \chi_{ee}^{xz} \\ \chi_{ee}^{zx} & \chi_{ee}^{zz} \end{bmatrix}. \quad (2.4)$$

Note, in our work, the surface susceptibilities are considered to be a function of space which do not change as a function of time. Assuming that the metasurface lies on the  $xz$  plane, (2.4) can be written as

$$\bar{\bar{\chi}}_{ee}|_{2D}(x, z) = \begin{bmatrix} \chi_{ee}^{xx}(x, z) & \chi_{ee}^{xz}(x, z) \\ \chi_{ee}^{zx}(x, z) & \chi_{ee}^{zz}(x, z) \end{bmatrix}. \quad (2.5)$$

Using this surface susceptibility model, the tangential surface polarization densities for a bianisotropic metasurface can be written as [18, 38]

$$\mathbf{P}_{t,s}^e = \epsilon_0 \bar{\bar{\chi}}_{ee}|_{2D} \cdot \mathbf{E}_{t,av} + \frac{1}{c_0} \bar{\bar{\chi}}_{em}|_{2D} \cdot \mathbf{H}_{t,av} \quad (2.6a)$$

$$\mathbf{P}_{t,s}^m = \bar{\bar{\chi}}_{mm}|_{2D} \cdot \mathbf{H}_{t,av} + \frac{1}{\eta_0} \bar{\bar{\chi}}_{me}|_{2D} \cdot \mathbf{E}_{t,av} \quad (2.6b)$$

where  $\epsilon_0$ ,  $c_0$ , and  $\eta_0$  are free space permittivity, speed of light in free space, and free space characteristic impedance, respectively. The free space characteristic impedance ( $\eta_0$ ) is calculated as  $\sqrt{\frac{\mu_0}{\epsilon_0}}$  where  $\mu_0$  is free space permeability. The speed of light ( $c_0$ ) in free space is calculated as  $\frac{1}{\sqrt{\epsilon_0\mu_0}}$ . The average tangential electromagnetic fields ( $\mathbf{E}_{t,\text{av}}$  and  $\mathbf{H}_{t,\text{av}}$ ) can be calculated as follows

$$\mathbf{E}_{t,\text{av}} = \frac{\mathbf{E}_t^+ + \mathbf{E}_t^-}{2}, \quad (2.7a)$$

$$\mathbf{H}_{t,\text{av}} = \frac{\mathbf{H}_t^+ + \mathbf{H}_t^-}{2}. \quad (2.7b)$$

Substituting the surface polarization densities in (2.6) into the resultant GSTCs in (2.2), the GSTCs can now completely be represented with tangential electromagnetic fields and metasurface material properties using the surface susceptibility model as

$$\hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) = j\omega\epsilon_0 \bar{\chi}_{\text{ee}} \Big|_{2\text{D}} \cdot \mathbf{E}_{t,\text{av}} + jk_0 \bar{\chi}_{\text{em}} \Big|_{2\text{D}} \cdot \mathbf{H}_{t,\text{av}}, \quad (2.8a)$$

$$(\mathbf{E}^+ - \mathbf{E}^-) \times \hat{n} = j\omega\mu_0 \bar{\chi}_{\text{mm}} \Big|_{2\text{D}} \cdot \mathbf{H}_{t,\text{av}} + jk_0 \bar{\chi}_{\text{me}} \Big|_{2\text{D}} \cdot \mathbf{E}_{t,\text{av}}, \quad (2.8b)$$

where  $k_0$  is the wavenumber in free space. As noted earlier, our metasurface lies in the  $xz$  plane with  $\hat{n} = \hat{y}$ . Therefore,

$$\hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) = \hat{y} \times (\mathbf{H}^+ - \mathbf{H}^-) = \Delta \mathbf{H}_z \hat{x} - \Delta \mathbf{H}_x \hat{z} \quad (2.9)$$

where  $\Delta$  operator calculates the difference between the  $+$  and  $-$  sides. That is, for example,  $\Delta \mathbf{H}_z = \mathbf{H}_z^+ - \mathbf{H}_z^-$ . Similarly,

$$(\mathbf{E}^+ - \mathbf{E}^-) \times \hat{n} = (\mathbf{E}^+ - \mathbf{E}^-) \times \hat{y} = -\Delta \mathbf{E}_z \hat{x} + \Delta \mathbf{E}_x \hat{z}. \quad (2.10)$$

Using (2.9) and (2.10) in conjunction with (2.8) results in

$$\begin{bmatrix} \Delta H_z \\ -\Delta H_x \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \chi_{\text{ee}}^{xx} & \chi_{\text{ee}}^{xz} \\ \chi_{\text{ee}}^{zx} & \chi_{\text{ee}}^{zz} \end{bmatrix} \begin{bmatrix} E_{x,\text{av}} \\ E_{z,\text{av}} \end{bmatrix} + jk_0 \begin{bmatrix} \chi_{\text{em}}^{xx} & \chi_{\text{em}}^{xz} \\ \chi_{\text{em}}^{zx} & \chi_{\text{em}}^{zz} \end{bmatrix} \begin{bmatrix} H_{x,\text{av}} \\ H_{z,\text{av}} \end{bmatrix} \quad (2.11a)$$

$$\begin{bmatrix} -\Delta E_z \\ \Delta E_x \end{bmatrix} = j\omega\mu_0 \begin{bmatrix} \chi_{\text{mm}}^{xx} & \chi_{\text{mm}}^{xz} \\ \chi_{\text{mm}}^{zx} & \chi_{\text{mm}}^{zz} \end{bmatrix} \begin{bmatrix} H_{x,\text{av}} \\ H_{z,\text{av}} \end{bmatrix} + jk_0 \begin{bmatrix} \chi_{\text{me}}^{xx} & \chi_{\text{me}}^{xz} \\ \chi_{\text{me}}^{zx} & \chi_{\text{me}}^{zz} \end{bmatrix} \begin{bmatrix} E_{x,\text{av}} \\ E_{z,\text{av}} \end{bmatrix}. \quad (2.11b)$$

For a metasurface to be reciprocal, which is the scope of this thesis, the following conditions need to be met [38]

$$\overline{\overline{\chi}}_{ee}^T = \overline{\overline{\chi}}_{ee}, \quad \overline{\overline{\chi}}_{mm}^T = \overline{\overline{\chi}}_{mm}, \quad \overline{\overline{\chi}}_{em}^T = -\overline{\overline{\chi}}_{me} \quad (2.12)$$

where the superscript  $T$  denotes the matrix transpose. This changes (2.11) to

$$\begin{bmatrix} \Delta H_z \\ -\Delta H_x \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \chi_{ee}^{xx} & \chi_{ee}^{xz} \\ \chi_{ee}^{xz} & \chi_{ee}^{zz} \end{bmatrix} \begin{bmatrix} E_{x,av} \\ E_{z,av} \end{bmatrix} - jk_0 \begin{bmatrix} \chi_{me}^{xx} & \chi_{me}^{zx} \\ \chi_{me}^{xz} & \chi_{me}^{zz} \end{bmatrix} \begin{bmatrix} H_{x,av} \\ H_{z,av} \end{bmatrix} \quad (2.13a)$$

$$\begin{bmatrix} -\Delta E_z \\ \Delta E_x \end{bmatrix} = j\omega\mu_0 \begin{bmatrix} \chi_{mm}^{xx} & \chi_{mm}^{xz} \\ \chi_{mm}^{xz} & \chi_{mm}^{zz} \end{bmatrix} \begin{bmatrix} H_{x,av} \\ H_{z,av} \end{bmatrix} + jk_0 \begin{bmatrix} \chi_{me}^{xx} & \chi_{me}^{xz} \\ \chi_{me}^{zx} & \chi_{me}^{zz} \end{bmatrix} \begin{bmatrix} E_{x,av} \\ E_{z,av} \end{bmatrix}. \quad (2.13b)$$

In addition, another criterion that is of interest to us is to have a lossless and passive reciprocal metasurface. As will be seen later in the thesis, this means that the input real power normally incident on each unit cell of the metasurface is equal to the output real power leaving the corresponding unit cell in the normal direction. This is referred to as local power conservation (LPC) [27]. This condition further limits the electric-to-electric and magnetic-to-magnetic surface susceptibilities to be purely real and magnetoelectric surface susceptibility to be purely imaginary [38]. Thus, for a lossless, passive, and reciprocal metasurface, we have

$$\begin{bmatrix} \Delta H_z \\ -\Delta H_x \end{bmatrix} = j\omega\epsilon_0 \overbrace{\begin{bmatrix} \chi_{ee}^{xx} & \chi_{ee}^{xz} \\ \chi_{ee}^{xz} & \chi_{ee}^{zz} \end{bmatrix}}^{\in \mathbb{R}^{2 \times 2}} \begin{bmatrix} E_{x,av} \\ E_{z,av} \end{bmatrix} - jk_0 \overbrace{\begin{bmatrix} \chi_{me}^{xx} & \chi_{me}^{zx} \\ \chi_{me}^{xz} & \chi_{me}^{zz} \end{bmatrix}}^{\in \mathbb{I}^{2 \times 2}} \begin{bmatrix} H_{x,av} \\ H_{z,av} \end{bmatrix} \quad (2.14a)$$

$$\begin{bmatrix} -\Delta E_z \\ \Delta E_x \end{bmatrix} = j\omega\mu_0 \overbrace{\begin{bmatrix} \chi_{mm}^{xx} & \chi_{mm}^{xz} \\ \chi_{mm}^{xz} & \chi_{mm}^{zz} \end{bmatrix}}^{\in \mathbb{R}^{2 \times 2}} \begin{bmatrix} H_{x,av} \\ H_{z,av} \end{bmatrix} + jk_0 \overbrace{\begin{bmatrix} \chi_{me}^{xx} & \chi_{me}^{xz} \\ \chi_{me}^{zx} & \chi_{me}^{zz} \end{bmatrix}}^{\in \mathbb{I}^{2 \times 2}} \begin{bmatrix} E_{x,av} \\ E_{z,av} \end{bmatrix}. \quad (2.14b)$$

Furthermore, we assume that the metasurface is non-gyrotropic [15, 18]. That is, the polarization does not change. This requires that the off-diagonal components of the electric-to-electric and magnetic-to-magnetic surface susceptibility tensors are

zero and the diagonal components of the magnetoelectric surface susceptibility tensor are zero. This will turn (2.14) into

$$\begin{bmatrix} \Delta H_z \\ -\Delta H_x \end{bmatrix} = j\omega\epsilon_0 \overbrace{\begin{bmatrix} \chi_{ee}^{xx} & 0 \\ 0 & \chi_{ee}^{zz} \end{bmatrix}}^{\in \mathbb{R}^{2 \times 2}} \begin{bmatrix} E_{x,\text{av}} \\ E_{z,\text{av}} \end{bmatrix} - jk_0 \overbrace{\begin{bmatrix} 0 & \chi_{me}^{zx} \\ \chi_{me}^{xz} & 0 \end{bmatrix}}^{\in \mathbb{I}^{2 \times 2}} \begin{bmatrix} H_{x,\text{av}} \\ H_{z,\text{av}} \end{bmatrix} \quad (2.15a)$$

$$\begin{bmatrix} -\Delta E_z \\ \Delta E_x \end{bmatrix} = j\omega\mu_0 \overbrace{\begin{bmatrix} \chi_{mm}^{xx} & 0 \\ 0 & \chi_{mm}^{zz} \end{bmatrix}}^{\in \mathbb{R}^{2 \times 2}} \begin{bmatrix} H_{x,\text{av}} \\ H_{z,\text{av}} \end{bmatrix} + jk_0 \overbrace{\begin{bmatrix} 0 & \chi_{me}^{xz} \\ \chi_{me}^{zx} & 0 \end{bmatrix}}^{\in \mathbb{I}^{2 \times 2}} \begin{bmatrix} E_{x,\text{av}} \\ E_{z,\text{av}} \end{bmatrix}. \quad (2.15b)$$

In summary, we have

$$\Delta H_z = j\omega\epsilon_0\chi_{ee}^{xx}E_{x,\text{av}} - jk_0\chi_{me}^{zx}H_{z,\text{av}}, \quad (2.16a)$$

$$-\Delta H_x = j\omega\epsilon_0\chi_{ee}^{zz}E_{z,\text{av}} - jk_0\chi_{me}^{xz}H_{x,\text{av}}, \quad (2.16b)$$

$$-\Delta E_z = j\omega\mu_0\chi_{mm}^{xx}H_{x,\text{av}} + jk_0\chi_{me}^{xz}E_{z,\text{av}}, \quad (2.16c)$$

$$\Delta E_x = j\omega\mu_0\chi_{mm}^{zz}H_{z,\text{av}} + jk_0\chi_{me}^{zx}E_{x,\text{av}}, \quad (2.16d)$$

Herein, we fix the polarization to be two-dimensional (2D) transverse electric (TE) with respect to the  $xy$  propagation plane. Consequently, we will only have  $E_z$ ,  $H_x$ , and  $H_y$  components. As noted earlier, the metasurface lies on the  $xz$  plane. (Since the problem is now 2D, the  $z$  length is assumed to be infinitely long.) Thus, the tangential components on the metasurface will only be  $E_z$  and  $H_x$ . In other words, our metasurface boundary conditions can be explained by (2.16b) and (2.16c) as

$$-\Delta H_x = j\omega\epsilon_0\chi_{ee}^{zz}E_{z,\text{av}} - jk_0\chi_{me}^{xz}H_{x,\text{av}}, \quad (2.17a)$$

$$-\Delta E_z = j\omega\mu_0\chi_{mm}^{xx}H_{x,\text{av}} + jk_0\chi_{me}^{xz}E_{z,\text{av}}. \quad (2.17b)$$

As can be seen in (2.17), to characterize our metasurface in the TE case, only three degrees of freedom exist  $\chi_{ee}^{zz} \in \mathbb{R}$ ,  $\chi_{mm}^{xx} \in \mathbb{R}$ , and  $\chi_{me}^{xz} \in \mathbb{I}$ .



The GSTCs in (2.17) defining the transformation of the electromagnetic fields across the faces of the unit cell in our metasurface can further be represented as a two-port network using the ABCD parameters, also known as the transmission (T) parameters. This is done by first opening (2.17) as follows

$$H_x^- - H_x^+ = O_1(E_z^+ + E_z^-) - O_2(H_x^+ + H_x^-), \quad (2.18a)$$

$$E_z^- - E_z^+ = O_3(H_x^+ + H_x^-) + O_2(E_z^+ + E_z^-), \quad (2.18b)$$

where  $O_1 = \frac{j\omega\epsilon_0\chi_{ee}^{zz}}{2}$ ,  $O_2 = \frac{jk_0\chi_{me}^{xz}}{2}$ , and  $O_3 = \frac{j\omega\mu_0\chi_{mm}^{xx}}{2}$ . We then rearrange (2.18) as follows

$$H_x^- = \frac{O_1 + O_1O_2}{1 + O_2 - O_2^2 - O_1O_3 - O_1O_2O_3}E_z^+ + \frac{O_1O_3 - O_2 - O_2^3 + O_1O_2O_3}{1 + O_2 - O_2^2 - O_1O_3 - O_1O_2O_3}H_x^+, \quad (2.19a)$$

$$E_z^- = \frac{O_1O_3 + O_2 + O_2^2}{1 - O_2^2 - O_1O_3}E_z^+ + \frac{O_3}{1 - O_2^2 - O_1O_3}H_x^+. \quad (2.19b)$$

Thus, (2.19) can be represented as a two-port network in the matrix format as

$$\begin{bmatrix} E_z^- \\ H_x^- \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_z^+ \\ H_x^+ \end{bmatrix} \quad (2.20)$$

where  $A$ ,  $B$ ,  $C$ , and  $D$  are calculated as follows

$$A = \frac{O_1O_3 + O_2 + O_2^2}{1 - O_2^2 - O_1O_3}, \quad (2.21a)$$

$$B = \frac{O_3}{1 - O_2^2 - O_1O_3}, \quad (2.21b)$$

$$C = \frac{O_1 + O_1O_2}{1 + O_2 - O_2^2 - O_1O_3 - O_1O_2O_3}, \quad (2.21c)$$

$$D = \frac{O_1O_3 - O_2 - O_2^3 + O_1O_2O_3}{1 + O_2 - O_2^2 - O_1O_3 - O_1O_2O_3}. \quad (2.21d)$$

Note that  $O_1$ ,  $O_2$ , and  $O_3$  are functions of surface susceptibilities  $\chi_{ee}^{zz}$ ,  $\chi_{me}^{xz}$ , and  $\chi_{mm}^{xx}$ , respectively as mentioned earlier.

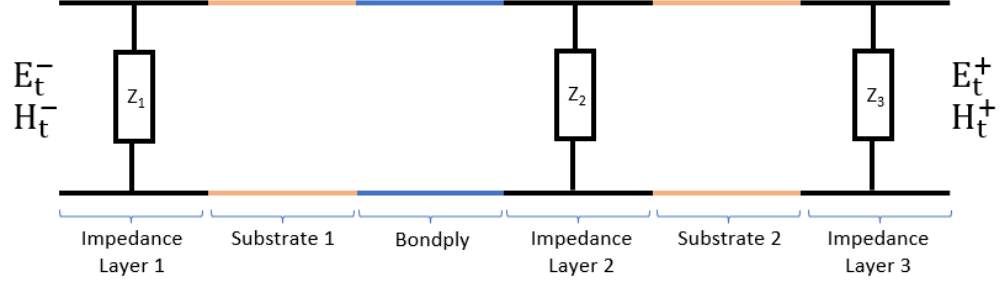


Figure 2.3: Two-port Circuit Representation of a unit cell of metasurface

Due to this limitation in degrees of freedom, a three-layered model is chosen for the design of a unit cell as depicted in Figure 2.3. (The details of this circuit model will be presented later on, but the three reactive impedances can be seen in this figure.)

At this point, the macroscopic design of the unit cell is complete if the surface susceptibilities are known which also marks the end of the metasurface macroscopic design. However, a detailed explanation of the implementation of the three-layered model of the unit cell is provided later in the thesis.

## 2.4 Design Frameworks

Now that we have discussed the boundary conditions associated with the metasurface, let us consider two major design frameworks in the metasurface design process. The design of metasurfaces can be divided into macroscopic and microscopic steps [22]. In the macroscopic metasurface design problem, the intent is to solve for the surface susceptibilities. Once these surface susceptibilities are found, thereafter the microscopic design of the metasurface would commence. In the microscopic design step, the focus is on the physical implementation of surface susceptibilities.

### 2.4.1 Macroscopic Design

As mentioned earlier, a metasurface enables electromagnetic wave transformation which normally entails converting a given electromagnetic wave into a desired wave. Usually, the incident field impinging on the metasurface is known. For example, if a horn antenna illuminates the metasurface, the incident field can be easily calculated given the geometry and the position of the metasurface from the horn antenna.

Depending on the design requirements, single metasurfaces or systems of cascaded metasurfaces can be designed. The macroscopic design of the metasurface can involve solving an electromagnetic inverse problem. For instance, Brown et al. [26, 40, 41] developed an electromagnetic inversion algorithm that is an inverse source problem to perform the macroscopic design of metasurfaces. The basis of this inversion algorithm is based on the electromagnetic surface equivalence principle [36]. The surface equivalence principle establishes equivalent surface electric and magnetic currents on a boundary surrounding a given source. Outside this boundary, the fields produced by these equivalent currents will be identical to those generated by the actual source. This is advantageous for metasurface design since the source and the metasurface can be replaced by equivalent surface currents placed just after the metasurface boundary. Based on this, Brown et al. [40] developed an inverse source algorithm to infer the electric and magnetic surface currents on the output side of the metasurface. Thereafter, from these surface equivalent currents, the Love's condition [42] can be used to calculate the tangential electric and magnetic fields on the output side of the metasurface. Once the tangential fields were found, the GSTCs were used to realize the properties of the metasurface to support the required electromagnetic wave transformation.

Following the above inverse metasurface design work, Qiu et al. extended this approach for three-dimensional power pattern synthesis on two far-field cuts [18, 43].

This approach also solves for the equivalent surface electric and magnetic currents under the Love's condition. The cost function to be minimized is in the form of  $L_2$ -norm augmented by the local power conservation constraint and a total variation regularization term. Once the tangential fields on both sides of the metasurface are calculated, the GSTCs are used to obtain the required parameters for each unit cell. In summary, the approach presented in [43] enabled the design of metasurfaces that have spatially varying unit cells in two directions.

The inversion algorithms reviewed above incorporate auxiliary surface waves [44] to redistribute the power for meeting local power conservation at all the unit cells. However, in practice, it is challenging to excite these surface waves. To not rely on auxiliary surface waves for power redistribution, another methodology was developed that considered a metasurface pair, e.g., see [9, 14]. In these works, the space between the two cascaded metasurfaces can be used for power redistribution. In the work of Ataloglou et al. [14], they used multiple reflections within the region formed by the two metasurfaces to redistribute the power. Then they used the gradient descent method as the optimization tool to find the tangential fields so as to satisfy the local power conservation constraint. Once these tangential fields were obtained, the surface properties of the metasurface can be found. These surface properties were then simulated in Ansys HFSS for verification. No physical unit cell designs were performed in this work [14]. Although accurate arbitrary wave transformation with high efficiency was achieved using the cascaded system of metasurfaces in [14], this design algorithm lacked automation. That is, it required an *ad hoc* specification of the initial guess and the step length associated with the optimization algorithm. It should also be noted that inversion techniques have also been used to design cascaded metasurfaces but the approach was limited to cascaded metasurfaces with no reflections between the two metasurfaces in the case of [10] and required an *ad hoc* choice of parameters in the case of [11].

### 2.4.2 Microscopic Design

Once the metasurface is macroscopically designed, the next objective is to physically realize its unit cells. That is, the microscopic design involves designing the individual unit cells when given their surface susceptibilities or impedance properties. At microwave and mm-wave frequencies, this process typically involves designing copper traces supported by dielectric substrates [6, 15, 45].

The general strategy involves optimizing one unit cell at a time instead of the entire metasurface all at once as the simultaneous optimization of all the unit cells in a metasurface is computationally expensive. In the work presented in this thesis, the unit cells are three-layered structures. These are common structures as used in [15, 16, 27, 46]. This structure provides sufficient degrees of freedom to have a passive, lossless, and reciprocal metasurface. As noted earlier, the design involves converting the surface susceptibility representation of the metasurface to two-port networks and then obtaining the two-port scattering parameters associated with each unit cell. Then, the dimensions of the copper traces in each unit cell can be optimized in full-wave solvers such as ANSYS® HFSS™ so as to match the desired scattering parameters. To this end, a lookup table is usually created for the design of unit cells to reduce the optimization process [22, 46]. Such lookup tables may ignore the mutual coupling between different layers. However, the lookup table formed in [46] took into account the mutual coupling. However, the main error is associated with the fact that the lookup table is formed under the periodic boundary conditions. Therefore, when different unit cells are placed beside each other, the mutual coupling between the neighboring unit cells is not perfectly taken into account by the lookup table.

Another strategy to design unit cells utilizes machine learning approaches, e.g., see [17, 47]. For example, the work presented in [17] assumes a three-layer binary image reconstruction where ‘1’ means the presence of copper and ‘0’ means the ab-

sence of copper. The scattering parameters are then given to the machine learning algorithm and the machine learning algorithm outputs this binary image which can then be fabricated using the printed circuit board (PCB) technology. Unlike the unit cell design methodology in [46, 48] that only considers the transmission scattering parameter ( $S_{21}$ ), the work in [17] used the whole scattering two-port matrix in the design of the unit cells. This enables to design metasurfaces to tailor both reflection and transmission.

## 2.5 Chapter Summary

In summary, this chapter began with some brief notes about metamaterials that can be used to manipulate electromagnetic waves. The development of metamaterials subsequently led to the proposal of metasurfaces, which are thin artificial materials. The design process of cascaded metasurfaces is the focus of this thesis. Therefore, the fundamental formulation of a metasurface from the electromagnetic equivalence principle to the three-layered design of unit cells, also known as meta-atoms, was explained in this chapter. Finally, the macroscopic and microscopic design strategies were defined and some related works were reviewed.

# Chapter 3

## Analytical Derivations for Design

In this chapter, a fully automated end-to-end design procedure for cascaded metasurface systems is derived and then the process of unit cell design is presented. First, the design formulation for cascaded metasurface design is presented. This formulation follows the work of Ataloglou et al. as reported in [14]. This formulation will then be incorporated into the conjugate gradient algorithm with analytical expressions. The three-layer unit cell design process is then discussed. The discussion continues with the way that the unit cells can be implemented using three-layered dogbone-shaped metallic claddings.

### 3.1 Design Formulation – Review

In this section, we review the design formulation developed in [14]. Therefore, our explanation follows what has been noted in [14] and presented for the sake of completeness.

Consider two metasurfaces as shown in Figure 3.1 whose unit cells do not change along the  $z$  direction and the height of the metasurface along the  $z$  direction is as-

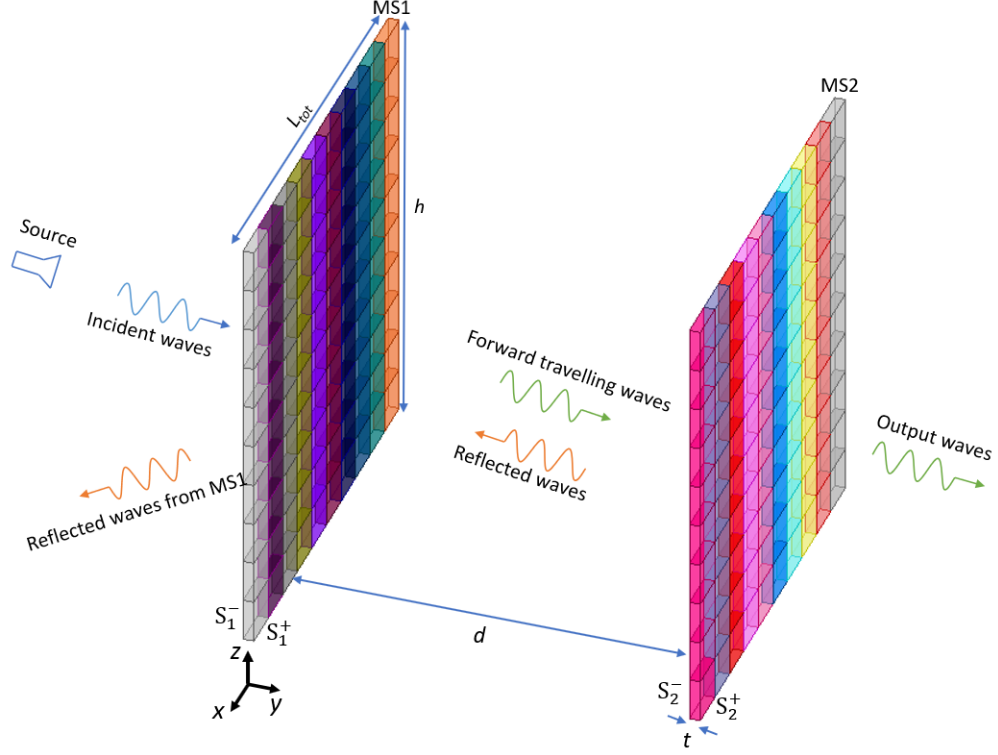


Figure 3.1: A cascaded metasurface system. The first and second metasurfaces are denoted by MS1 and MS2. The separation between the two metasurfaces is  $d$ .

sumed to be very large. Due to this assumption, we consider a two-dimensional (2D) formulation. The antenna that illuminates this system resides external to the two metasurfaces and is assumed to be located on the left side of the first metasurface. The polarization of the illuminating antenna (incident field) is assumed to be transverse electric (TE).

In Figure 3.1,  $MS1$  and  $MS2$  denote the two metasurfaces in the cascaded metasurface system. For our model, metasurface  $MS1$  is placed at the plane  $y = 0$  and the metasurface  $MS2$  is placed at the plane  $y = d$ . The two metasurfaces are separated by a distance  $d$  and form a region between the sides  $S_1^+$  and  $S_2^-$ . The  $S_1^+$  side is the output face of  $MS1$  while the  $S_2^-$  side is the input face of  $MS2$  as shown in Figure 3.1.



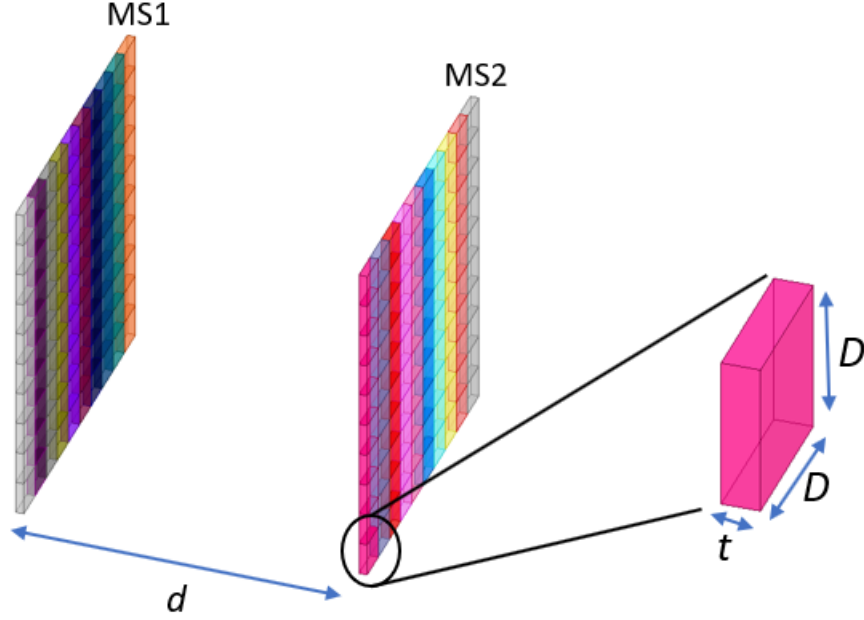


Figure 3.2: Magnified model of the unit cell. The cascaded metasurface system is excited (illuminated) by an external source. In the mathematical model,  $t \rightarrow 0$ .

The total length of each metasurface along  $x$  is  $L_{tot}$ . Although the system is shown as a 3D model, the variation of the metasurfaces in our model is only along one direction (as noted above) while it repeats periodically in the other direction (i.e.,  $z$ -direction). That is, the characteristic of the unit cells varies along the  $x$ -axis and are the same along the  $z$ -axis. This is clearly demonstrated in Figure 3.1 by the color of the unit cells alternating in the  $x$ -direction while the color of the unit cells remains the same in the  $z$ -direction. Thus, the height  $h$  depends on the number of rows repeated along the  $z$ -axis. The number of unit cells repeated along the  $z$ -axis is considered to be large due to the 2D assumption.<sup>1</sup> There are  $2N + 1$  unit cells along the  $x$ -direction where  $N$  defines the number of unit cells on the positive or the negative sides of the  $x$ -axis.

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<sup>1</sup>As will be seen later, in our fabrication, we have used 49 unit cells along the  $z$ -direction.

A magnified image of the unit cell is shown in Figure 3.2. As shown, each unit cell is a cuboid with the dimension of the sides being  $D$  and having a thickness of  $t$ . The dimension  $D$  is calculated as

$$D = \frac{L_{tot}}{2N + 1}, \quad (3.1)$$

and it needs to be subwavelength (e.g., about  $\lambda/6$  where  $\lambda$  is the wavelength of operation). Although the model of the unit cell is shown as one layer, it consists of two substrates, bondply, and copper traces in our case. In the utilized mathematical model, the metasurface is assumed to have no thickness. Thus, thickness  $t$  is zero in the mathematical model used for the design. Considering the unit cell variation in the cascaded metasurface system to be only along one axis ( $x$ -axis) and also considering both the metasurfaces to be of zero thickness, a simpler 2D representation of the system is shown in Figure 3.3. Although in the mathematical model, the thickness  $t$  is assumed to be zero, in our full-wave simulation and fabricated example, the thickness  $t$  is subwavelength and has a finite value. In these cases, the thickness  $t$  is almost twice the thickness of our dielectric substrates ( $t \approx 2.6$  mm at the operational wavelength of 28 mm).

In a cascaded metasurface system having an external source, an incident wave impinges on the  $S_1^-$  side where the  $S_1^-$  side is the input face of  $MS1$  and a desired output wave is to be formed on the  $S_2^+$  side where the  $S_2^+$  side is the output face side of  $MS2$ , see Figure 3.3. When multiple reflections between the two metasurfaces are considered in the system, there exist both forward and reflected electromagnetic waves within the system as depicted in Figure 3.3. In this thesis, the space between the two metasurfaces may be loosely referred to as “*cavity*”. Although this makes it easy to refer to the space between the two metasurfaces, it should be noted that it is not a precise term as two sides of this space are open. When multiple reflections are not considered, then, only forward-propagating electromagnetic waves are present within

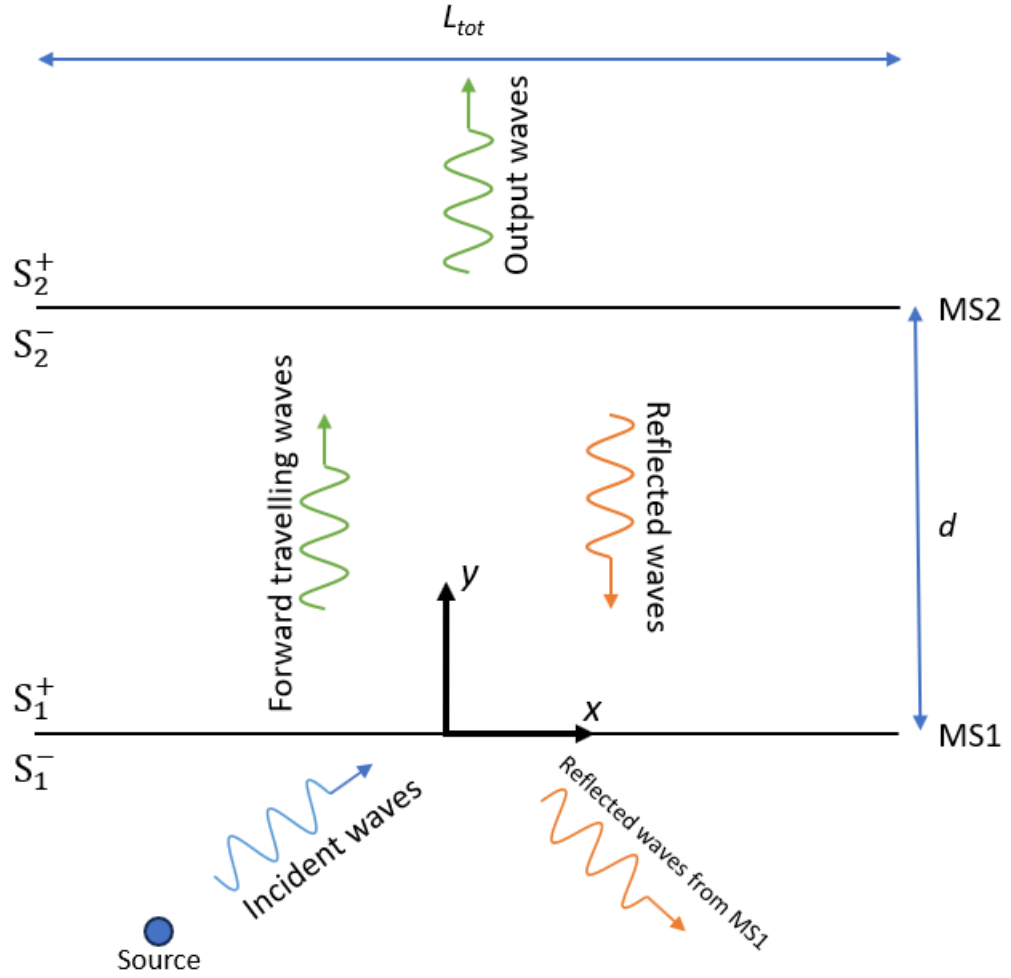


Figure 3.3: 2D model of cascaded metasurfaces system with multiple reflections within the cavity.

the cavity as shown in Figure 3.4, e.g., see [9, 10, 49]. In both cases, depending on the design requirements, some of the incident electromagnetic waves can be tailored to be reflected from the first metasurface.

The macroscopic design of cascaded metasurface systems involves determining the tangential electric and magnetic fields on both sides of the metasurfaces. Thus, the overall process of obtaining these fields is shown in Figure 3.5, and the details of each

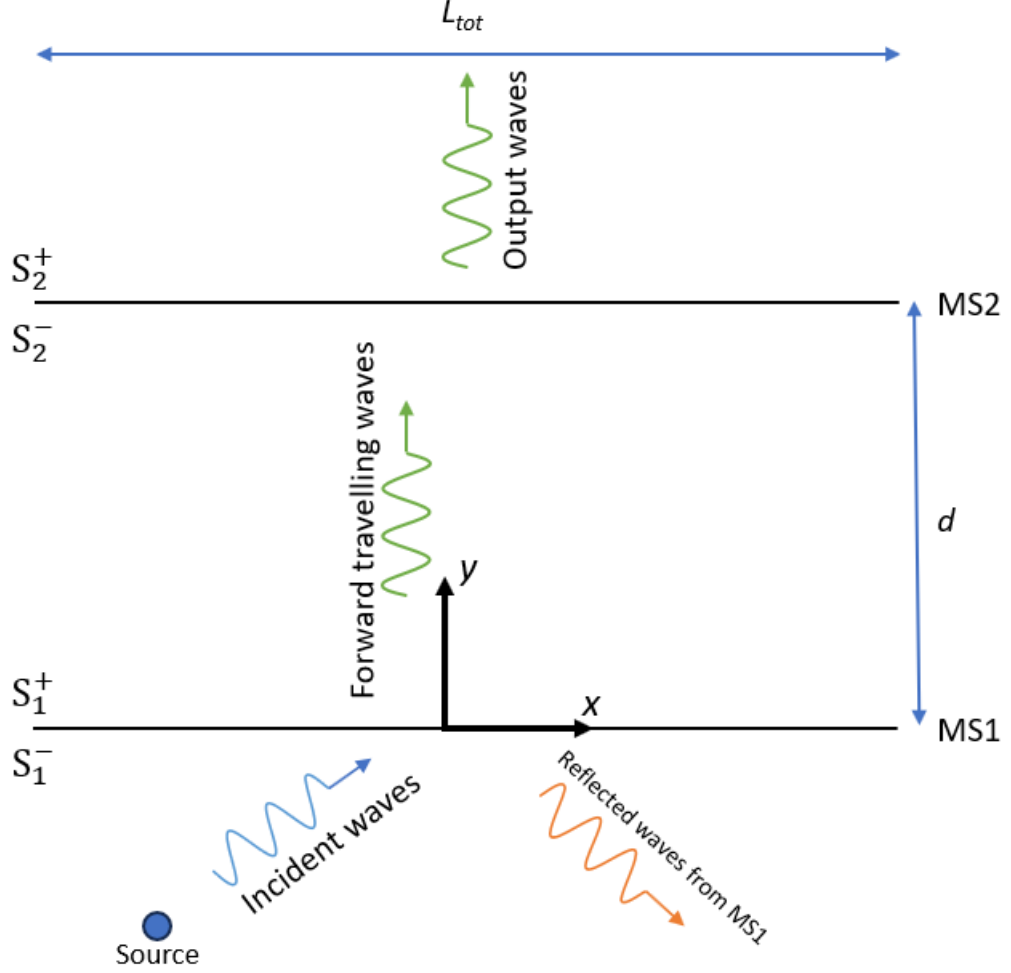


Figure 3.4: 2D model of a cascaded metasurface system without multiple reflections within the cavity.

step are discussed in the following subsections. In [14], the optimization algorithm is a gradient descent method which has an *ad hoc* choice of step length. Herein, we use the conjugate gradient algorithm with analytical expressions for all its components. In addition, as opposed to [14], our optimization is in the complex domain and thus we do not require splitting the unknowns into their real and imaginary parts.

The incident electromagnetic wave impinging on the metasurface  $MS1$  on the  $S_1^-$  side and the desired output electromagnetic wave leaving the metasurface  $MS2$  from

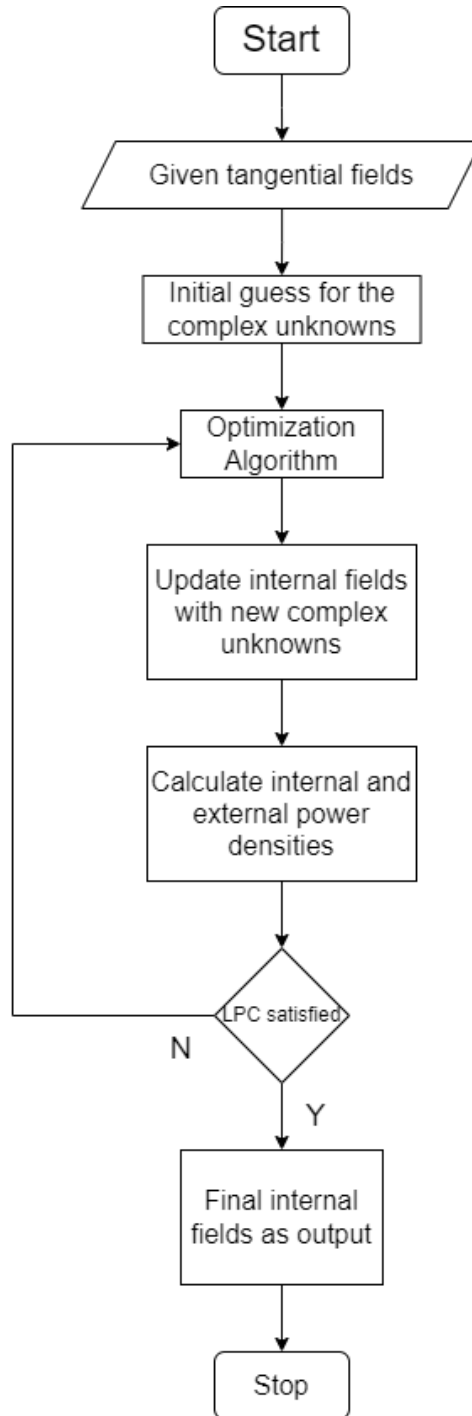


Figure 3.5: Optimization algorithm for the macroscopic design flowchart

the  $S_2^+$  side are the known quantities in the design optimization. Note that in general, the specification of desired tangential fields on  $S_2^+$  may require performing inversion. This is due to the fact that the user typically provides the incident field along with the desired far-field performance criteria such as beam directions, sidelobe levels (SLL), and half-power beamwidth (HPBW). The inversion algorithms reviewed earlier [40] can then be used to infer the tangential fields on  $S_2^+$ .

Since the objective of the metasurface system is to transform the electromagnetic waves, the incident fields can be considered to be generated from a given antenna such as dipole antennas, horn antennas, and microstrip patch antennas for which analytical expressions or numerical values already exist to calculate the fields everywhere. Thus, these analytical expressions can then be used to calculate the electromagnetic fields on the  $S_1^-$  side of the system. Note that since we are considering 2D TE wave propagation, we have limited ourselves to line sources and plane waves to illuminate the metasurface system in this thesis. (As will be seen in the experimental work, since we do not have access to a perfect plane wave, we have used a horn antenna to approximate a plane wave in our measurements.)

As noted earlier, the specification of the desired output fields is more difficult unless the desired output fields are simple such as plane waves or cylindrical waves. When the performance of the cascaded metasurface system is specified in terms of desired far-field power pattern, desired beam directions, desired HPBWs, or desired SLLs, an inversion algorithm [26, 40, 43, 50] can be used to translate the given desired performance criteria to the tangential electric and magnetic fields on the  $S_2^+$  side of the system.

Herein, the metasurface system is developed based on considering the local power conservation so as to ensure having lossless and passive unit cells. This is the real power density that is normal to the unit cell. As noted in Chapter 2, this is called the

local power conservation (LPC) constraint [27]. This establishes a relation between the tangential fields on both sides of the metasurface at each unit cell. Let us consider  $E_{t,1}^-$  and  $H_{t,1}^-$  as the tangential input fields on  $S_1^-$  while considering  $E_{t,2}^+$  and  $H_{t,2}^+$  as the desired output tangential fields at each unit cell on  $S_2^+$ . Due to the 2D TE assumption, only the  $z$ -directed  $E$  component and the  $x$  and  $y$ -directed  $H$  components exist in the solution domain. On the other hand, the tangential fields on the metasurfaces will be  $E_z$  and  $H_x$ . (This particular choice of coordinates allows us to be consistent with the field convention in [14].)

It should be noted that prior to enforcing the LPC, we first need to make sure that total power conservation holds. This is achieved by first calculating the total real power normally incident on the input face of the cascaded metasurfaces system ( $S_1^-$ ) using the provided/calculated input fields at each unit cell as

$$P_y(y = 0^-) = A \frac{1}{2} \text{Re} \left( \sum_{n=-N}^N (E_z(x_n, y = 0^-) \times H_x^*(x_n, y = 0^-)) \right). \quad (3.2)$$

where  $A$  is the surface area of each unit cell, which is considered to be the same among all unit cells. Similarly, the total real power normally leaving the output face of the cascaded metasurface system ( $S_2^+$ ) is calculated as

$$P_y(y = d^+) = A \frac{1}{2} \text{Re} \left( \sum_{n=-N}^N (E_z(x_n, y = d^+) \times H_x^*(x_n, y = d^+)) \right). \quad (3.3)$$

Then, the ratio of total input power to total output power densities is obtained as

$$P_{ratio} = \frac{P_y(y = 0^-)}{P_y(y = d^+)}. \quad (3.4)$$

After obtaining the above power ratio, the output electric and magnetic fields are scaled at each unit cell on  $S_2^+$  to ensure the conservation of the total power. This is done as follows

$$E_z(x_n, y = d^+) |_{scaled} = \sqrt{P_{ratio}} E_z(x_n, y = d^+), \quad (3.5a)$$

$$H_x(x_n, y = d^+) |_{scaled} = \sqrt{P_{ratio}} H_x(x_n, y = d^+). \quad (3.5b)$$

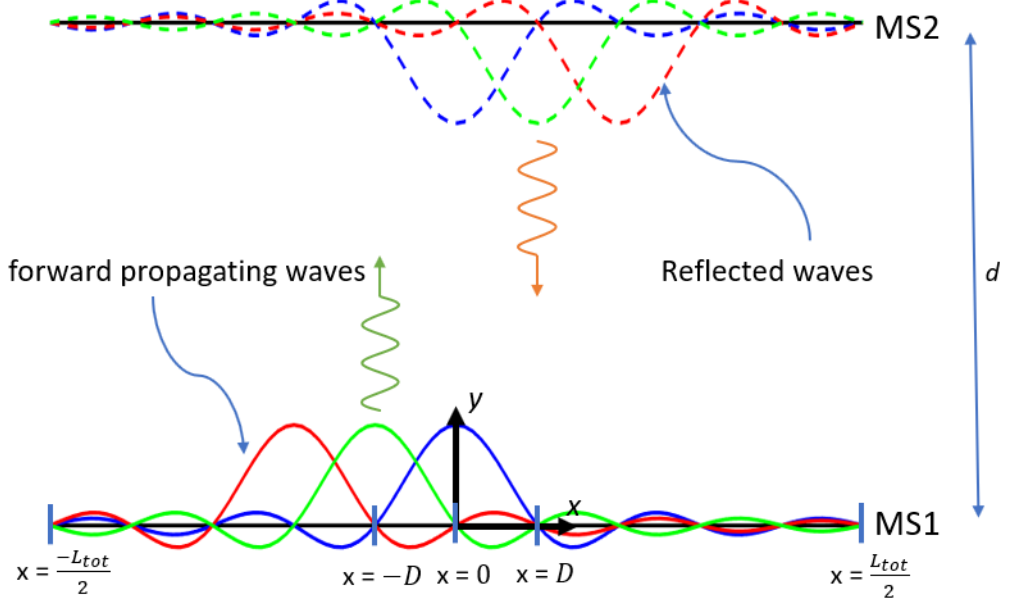


Figure 3.6: Forward and reflected waves within the cavity of the cascaded metasurface system. The figure is based on the discretization used in [14].

The scaled electric and magnetic fields on the output side ( $S_2^+$ ) ensure that the total power on the output end of the cascaded metasurfaces system is equal to the total power from the input side. Thus, the total power is conserved. The scaled output electric and magnetic fields are used in the remaining part of the design algorithm as the aperture fields on  $S_2^+$ .

Let us now focus on the forward and reflected waves in the region between the two metasurfaces. As noted earlier, these are the internal fields for the cascaded metasurface system when multiple reflections between the two metasurfaces are considered. If multiple reflections are ignored, only forward-propagating waves remain in the internal region.

As in [14], the forward-propagating waves are represented by the basis function  $g_n(x)$  and originate from  $S_1^+$  side of  $MS1$  while the reflected waves are defined with



the same basis function  $g_n(x)$  while originating from  $S_2^-$  side of  $MS2$  as shown in Figure 3.6. The basis function  $g_n(x)$  is defined using a ‘sinc’ function whose peak is at the center of the  $n$ th unit cell. Thus, these basis functions can be represented as [14]

$$g_n(x) = \frac{\sin(k_0(x - nD))}{k_0(x - nD)} \quad (3.6)$$

where  $k_0$  is the wavenumber in free space. It is equal to  $2\pi/\lambda_0$  where  $\lambda_0$  is the wavelength in free space,  $D$  is the size of each unit cell, and  $n$  is an integer with a range of  $-N \leq n \leq N$ . The authors of [14] have intentionally chosen (3.6) for their discretization to enforce the absence of evanescent waves in the formulation. This can be understood by noting that the Fourier transform of (3.6) will be  $\frac{\pi}{k_0} \text{rect}(\frac{k_x}{2k_0}) e^{jk_x nD}$  where ‘rect’ denotes rectangular function. This function will be zero for  $|k_x| > k_0$ . Thus, the evanescent spectrum will be absent in the formulation.

Based on the above, the tangential electric field on  $S_1^+$  contributing to the forward propagation waves can be described as [14]

$$E_z^{forw}(x, y = 0^+) = \sum_{n=-N}^N A_{f,n} g_n(x), \quad (3.7)$$

where  $A_{f,n}$  is a complex weight associated with each defined basis function. The subscript ‘ $f$ ’ denotes the ‘forward’ herein, and should *not* be confused with the frequency of operation. As will be seen,  $A_{f,n}$  is an unknown of the problem since we do not know the tangential fields in the internal region. Also, note that the size of  $A_{f,n}$  is  $\mathbb{C}^{2N+1}$ . In the work of Ataloglou et al [14], this unknown is split into its real and imaginary parts whereas we will consider it in the complex domain.

Equation (3.7) only defines the tangential electric field at  $y = 0^+$  along the meta-surface  $MS1$ . Therefore, in order to calculate the electric field everywhere within the internal region due to the forward propagation of these tangential fields, the authors of [14] have used the concept of the plane wave spectrum (PWS) technique [51] to

shift the wave forward in the  $y$ -direction. These waves are called forward propagating waves since they travel in the positive  $y$ -direction. In order to obtain these forward propagating waves, we first take the 2D spatial Fourier transform of (3.7) with respect to  $x$  as

$$\tilde{E}_z^{forw}(k_x, y = 0^+) = \mathcal{F}\{E_z^{forw}(x, y = 0^+)\} = \int_{-\infty}^{\infty} E_z^{forw}(x, y = 0^+) e^{+jk_x x} dx \quad (3.8)$$

where  $\tilde{E}_z$  shows the Fourier domain of  $E_z$ .

The Fourier transform as a function of  $k_x$  will be [14]

$$\tilde{E}_z^{forw}(k_x, y = 0^+) = \begin{cases} \frac{\pi}{k_0} \sum_{n=-N}^N A_{f,n} e^{+jk_x n D} & |k_x| \leq k \\ 0 & \text{Otherwise} \end{cases} \quad (3.9)$$

Based on the plane wave spectrum concept [51], we can take this Fourier domain quantity and forward propagate it through a phase shift and then use an inverse Fourier transform to turn it back into the field value in the spatial domain. That is,

$$E_z^{forw}(x, y) = \mathcal{F}^{-1}\{\tilde{E}_z^{forw}(k_x, y = 0^+) e^{-jk_y y}\} \quad (3.10)$$

where

$$k_y = \sqrt{k_0^2 - k_x^2}. \quad (3.11)$$

Thus, (3.10) becomes

$$E_z^{forw}(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{E}_z^{forw}(k_x, y = 0^+) e^{-jk_x x} e^{-j\sqrt{k_0^2 - k_x^2} y} dk_x \quad (3.12)$$

Using (3.9), (3.12) results in [14]

$$E_z^{forw}(x, y) = \sum_{n=-N}^N \frac{A_{f,n}}{2k_0} \int_{-k_0}^{k_0} e^{-j\sqrt{k_0^2 - k_x^2} y} e^{-jk_x(x-nD)} dk_x \quad (3.13)$$

With (3.13), we can now calculate the forward propagating electric field anywhere within the internal region. We then calculate the tangential magnetic field component

due to the forward propagating wave using the following Maxwell's equation

$$\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H}, \quad (3.14)$$

where ' $\nabla \times$ ' is the curl operator that takes spatial differentiation of  $\mathbf{E}$  with respect to spatial coordinates,  $\omega$  is the angular frequency, and  $\mu_0$  is the permeability in free space. Using (3.14) and (3.13), the tangential component of the magnetic field associated with the forward propagating waves will be [14]

$$H_x^{forw}(x, y) = \sum_{n=-N}^N \frac{A_{f,n}}{2k_0^2\eta_0} \int_{-k_0}^{k_0} \sqrt{k_0^2 - k_x^2} e^{-j\sqrt{k_0^2 - k_x^2}y} e^{-jk_x(x-nD)} dk_x, \quad (3.15)$$

where  $\eta_0$  is the intrinsic impedance of free space ( $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ ).

Similarly, at each unit cell of metasurface  $MS2$  on  $S_2^-$  side, the reflected electric field defined on metasurface  $MS2$  on  $S_2^-$  side is a summation of all the basis functions defined as [14]

$$E_z^{refl}(x, y = d^-) = \sum_{n=-N}^N (A_{r,n}g_n(x)), \quad (3.16)$$

where  $A_{r,n}$  are complex weights associated with basis functions defined at each unit cell on  $S_2^-$  side of metasurface  $MS2$ . The electric fields calculated using (3.16) travel in  $-y$  direction. Thus, they are called reflected waves. Therefore, in order to calculate the electric fields anywhere within the cavity due to these reflected waves, we take the 2D spatial Fourier transform of them defined on the metasurface  $MS2$  on  $S_2^-$  side and then take the 2D spatial inverse Fourier transform of the result to obtain the following equation [14]

$$E_z^{refl}(x, y) = \sum_{n=-N}^N \frac{A_{r,n}}{2k_0} \int_{-k_0}^{k_0} e^{-j\sqrt{k_0^2 - k_x^2}(y-d)} e^{-jk_x(x-nD)} dk_x \quad (3.17)$$

where  $d$  is the separation distance between the metasurfaces  $MS1$  and  $MS2$  along the  $y$ -axis. The tangential magnetic fields are calculated using (3.14) and the resulting

expression is [14]

$$H_x^{refl}(x, y) = - \sum_{n=-N}^N \frac{A_{r,n}}{2k_0^2\eta} \int_{-k_0}^{k_0} \sqrt{k_0^2 - k_x^2} e^{-j\sqrt{k_0^2 - k_x^2}(y-d)} e^{-jk_x(x-nD)} dk_x. \quad (3.18)$$

The expressions presented in (3.13), (3.15), (3.17), and (3.18) enable us to account for multiple reflections within the cavity formed by the two metasurfaces. Therefore, in the case where multiple reflections are considered within the cascaded metasurface system, the electric and magnetic fields at any given point within the cavity are the sum of these forward and reflected traveling waves within the cavity. However, when multiple reflections are not considered within the cascaded metasurface system, then the electric and magnetic fields at any given point within the cavity will just be a result of only the forward propagating waves.

Now, for the unit cell design, we require tangential electric and magnetic fields on both sides of the metasurface  $MS1$  and  $MS2$ . As noted earlier, the tangential electric and magnetic fields external to the system (i.e., on  $S_1^-$  and  $S_2^+$ ) are assumed to be known. The tangential electric and magnetic fields internal to the system, that is on sides  $y = 0^+$  and  $y = d^-$ , are obtained using the following equations [14] when multiple reflections are considered within the cavity of the cascaded metasurface system

$$E_z(x, y = 0^+) = \sum_{n=-N}^N (A_{f,n}I_n(x) + A_{r,n}I'_n(x)), \quad (3.19a)$$

$$H_x(x, y = 0^+) = \sum_{n=-N}^N (A_{f,n}J_n(x) + A_{r,n}J'_n(x)), \quad (3.19b)$$

$$E_z(x, y = d^-) = \sum_{n=-N}^N (A_{f,n}I'_n(x) + A_{r,n}I_n(x)), \quad (3.19c)$$

$$H_x(x, y = d^-) = \sum_{n=-N}^N (A_{f,n}J'_n(x) + A_{r,n}J_n(x)), \quad (3.19d)$$

where  $I_n$ ,  $I'_n$ ,  $J_n$ , and  $J'_n$  are coefficients that depend on the location of the unit cell along the  $x$  axis and are obtained by setting  $y = 0$  in (3.13), (3.15), (3.17), and (3.18) when calculating the electric and magnetic fields on side  $S_1^+$ , and setting  $y = d$  in (3.13), (3.15), (3.17), and (3.18) when calculating the electric and magnetic fields on side  $S_2^-$ . The explicit expressions of  $I_n$ ,  $I'_n$ ,  $J_n$ , and  $J'_n$  are as follows [14]

$$I_n(x) = \frac{1}{2k_0} \int_{-k_0}^{k_0} e^{-jk_x(x-nD)} dk_x, \quad (3.20a)$$

$$I'_n(x) = \frac{1}{2k_0} \int_{-k_0}^{k_0} e^{-j\sqrt{k_0^2 - k_x^2}d} e^{-jk_x(x-nD)} dk_x, \quad (3.20b)$$

$$J_n(x) = \frac{1}{2k_0^2\eta} \int_{-k_0}^{k_0} \sqrt{k_0^2 - k_x^2} e^{-jk_x(x-nD)} dk_x, \quad (3.20c)$$

$$J'_n(x) = \frac{1}{2k_0^2\eta} \int_{-k_0}^{k_0} \sqrt{k_0^2 - k_x^2} e^{-j\sqrt{k_0^2 - k_x^2}d} e^{-jk_x(x-nD)} dk_x, \quad (3.20d)$$

In addition, when multiple reflections are not considered within the cascaded metasurfaces system, then the tangential electric and magnetic fields internal to the system are calculated as follows

$$E_z(x, y = 0^+) = \sum_{n=-N}^N A_{f,n} I_n(x), \quad (3.21a)$$

$$H_x(x, y = 0^+) = \sum_{n=-N}^N A_{f,n} J_n(x), \quad (3.21b)$$

$$E_z(x, y = d^-) = \sum_{n=-N}^N A_{f,n} I'_n(x), \quad (3.21c)$$

$$H_x(x, y = d^-) = \sum_{n=-N}^N A_{f,n} J'_n(x), \quad (3.21d)$$

Note that the argument  $(x)$  will be dropped from  $I_n(x)$ ,  $I'_n(x)$ ,  $J_n(x)$ , and  $J'_n(x)$  for brevity in the later sections. So far, we have reviewed the formulation presented in [14].

## 3.2 Initial Guess

Equations (3.19) and (3.21) provide the analytical expressions to calculate the internal fields for the cascaded metasurface system with and without multiple reflections within the cavity, respectively. We now need to determine the unknown complex coefficient  $A_{f,n}$  and  $A_{r,n}$  in (3.19) and (3.21) to calculate the internal tangential fields of the system. If there existed analytical expressions to determine the correct values for these unknown complex coefficients from the beginning, then determining the values for the internal electric fields would have been simple. Therefore, we have developed an optimization algorithm to determine appropriate values for these unknown complex coefficients.

Since the objective of the algorithm is to optimize the values of the complex coefficients  $A_{f,n}$  and  $A_{r,n}$ , we needed to have a starting value, known as the initial guess, for these complex coefficients. Specifying the initial guess as zero would be an obvious and easy choice, but we cannot do that because the other parts of the optimization algorithm would result in having a value of zero which will fail the optimization algorithm as can be seen in Section 3.3.

Therefore, to come up with an initial value for these unknown complex coefficients  $A_{f,n}$  and  $A_{r,n}$ , we propose and develop the following procedure. To this end, we begin by forming a linear system for the internal fields as follows

$$\begin{bmatrix} \psi_{\text{inner}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{u} \end{bmatrix} \quad (3.22)$$

where  $\psi_{\text{inner}}$  is the concatenation of internal fields of the cascaded metasurfaces sys-

tem into a single column vector as shown

$$\begin{bmatrix} \psi_{\text{inner}} \end{bmatrix} = \begin{bmatrix} E_z(x, y = 0^+) \\ H_x(x, y = 0^+) \\ E_z(x, y = d^-) \\ H_x(x, y = d^-) \end{bmatrix} \quad (3.23)$$

and  $\mathbf{u}_i$  is the concatenation of the unknown complex coefficients  $A_{f,n}$  and  $A_{r,n}$  as

$$\begin{bmatrix} \mathbf{u} \end{bmatrix} = \begin{bmatrix} A_{f,n} \\ A_{r,n} \end{bmatrix} \quad (3.24)$$

$$\begin{bmatrix} \mathbf{u} \end{bmatrix} = \begin{bmatrix} A_{f,n} \end{bmatrix} \quad (3.25)$$

for the cases when we consider or ignore multiple reflections, respectively.

Finally,  $\mathbf{A}$  is a matrix formed by  $\mathbf{I}_n$ ,  $\mathbf{J}_n$ ,  $\mathbf{I}'_n$  and  $\mathbf{J}'_n$  calculated at each unit cell for  $-N \leq n \leq N$  as shown

$$\begin{bmatrix} \mathbf{A} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_n & \mathbf{I}'_n \\ \mathbf{J}_n & -\mathbf{J}'_n \\ \mathbf{I}'_n & \mathbf{I}_n \\ \mathbf{J}'_n & -\mathbf{J}_n \end{bmatrix} \quad (3.26)$$

,

$$\begin{bmatrix} \mathbf{A} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_n \\ \mathbf{J}_n \\ \mathbf{I}'_n \\ \mathbf{J}'_n \end{bmatrix} \quad (3.27)$$

for the cases when multiple reflections are considered or ignored in the formulation respectively.

For the system in (3.22), we assume the fields on both sides of the metasurfaces  $MS1$  and  $MS2$  are equal [52]. That is, we set the internal fields at each unit cell on

metasurfaces  $MS1$  and  $MS2$  are equal to their counterpart external fields as follows

$$\begin{bmatrix} \psi_{\text{inner}} \end{bmatrix} \longleftarrow \begin{bmatrix} \psi_{\text{outer}} \end{bmatrix} \quad (3.28)$$

where  $\psi_{\text{outer}}$  is the concatenation of external fields of the cascaded metasurfaces system into a single-column vector. Thus,

$$\begin{bmatrix} \psi_{\text{inner}} \end{bmatrix} \longleftarrow \begin{bmatrix} E_z(x, y = 0^-) \\ H_x(x, y = 0^-) \\ E_z(x, y = d^+) \\ H_x(x, y = d^+) \end{bmatrix} \quad (3.29)$$

We can calculate the matrix  $A$  using (3.20) [14]. Knowing the matrix  $A$  and setting the internal electric and magnetic fields as the external fields, we then create a cost function as a function of the unknown complex coefficients  $u_i$  as shown

$$C(\mathbf{u}) = \|\psi_{\text{inner}} - A\mathbf{u}\|^2 = \|\psi_{\text{outer}} - A\mathbf{u}\|^2 \quad (3.30)$$

where  $\|\cdot\|$  denotes the  $L_2$  norm.

Expanding the cost function in (3.30) with the expression  $\|\mathbf{x}\|^2 = \mathbf{x}^H \mathbf{x}$  where  $H$  is the Hermitian operator leads the cost function in (3.30) having the following form

$$C(\mathbf{u}) = (\psi_{\text{outer}}^H \psi_{\text{outer}} - \psi_{\text{outer}}^H A\mathbf{u} - \mathbf{u}^H A^H \psi_{\text{outer}} + \mathbf{u}^H A^H A\mathbf{u}) \quad (3.31)$$

The Hermitian operation involves taking the complex conjugate ( $*$ ) and transpose ( $T$ ) as shown

$$\mathbf{x}^H = [\mathbf{x}^*]^T. \quad (3.32)$$

We then minimize (3.31) with respect to  $u^*$  that results in a gradient equation for the cost function in (3.31) as follows

$$\mathbf{g} = A^H(A\mathbf{u} - \psi_{\text{outer}}) \quad (3.33)$$



As it can be seen the gradient in (3.33) is a function of the unknown complex coefficients  $\mathbf{u}$ . For the initial guess optimization algorithm, we perform the conjugate gradient algorithm with only one iteration. This is because the setting of the internal fields being equal to the external fields is obviously wrong for the cascaded metasurface system. Therefore, we only perform the optimization algorithm with only one iteration just to get an initial guess. In order to perform this one iteration, we need to start the optimization algorithm with an initial guess for  $u$ . This is then set to zero ( $u_0 = 0$ ), and thus, the gradient (3.33) will become

$$\mathbf{g}_0 = -\mathbf{A}^H \psi_{outer} \quad (3.34)$$

We then use the gradient in (3.34) in the update equation as shown

$$\mathbf{u}_{\text{initial guess}} = \mathbf{u}_0 + \alpha_0 \mathbf{g}_0 \quad (3.35)$$

where  $\mathbf{u}_{\text{initial guess}}$  will be the initial guess that we will later use in the design process of the cascaded metasurface system. We now need to find the step length  $\alpha_0$  to arrive at our initial guess. To calculate  $\alpha_0$ , we formulate a cost function as a function of  $\alpha_0$  as

$$C(\alpha_0) = \|\psi_{outer} - \mathbf{A}\alpha_0 \mathbf{g}_0\|^2. \quad (3.36)$$

We then expand (3.36) using the expression  $\|\mathbf{x}\|^2 = \mathbf{x}^H \mathbf{x}$  that results in the following expression

$$\begin{aligned} C(\alpha_0) &= \psi_{outer}^H \psi_{outer} - \psi_{outer}^H \mathbf{A} \alpha_0 \mathbf{g}_o - \alpha_0 \mathbf{g}_o^H \mathbf{A}^H \psi_{outer} + \alpha_0^2 \mathbf{g}_o^H \mathbf{A}^H \mathbf{A} \mathbf{g}_o \\ &= \psi_{outer}^H \psi_{outer} - 2\alpha_0 \text{Re}[\psi_{outer}^H \mathbf{A} \mathbf{g}_o] + \alpha_0^2 \mathbf{g}_o^H \mathbf{A}^H \mathbf{A} \mathbf{g}_o \end{aligned} \quad (3.37)$$

We then minimize (3.37) with respect to  $\alpha_0$  to obtain the following expression

$$\frac{dC(\alpha_0)}{d\alpha_0} = -2\text{Re}[\psi_{outer}^H \mathbf{A} \mathbf{g}_o] + 2\alpha_0 \mathbf{g}_o^H \mathbf{A}^H \mathbf{A} \mathbf{g}_o \quad (3.38)$$

Setting (3.38) to zero provides an equation for  $\alpha_0$  as follows

$$\alpha_0 = \frac{\text{Re}[\psi_{outer}^H \mathbf{A} \mathbf{g}_o]}{\|\mathbf{A} \mathbf{g}_o\|^2} \quad (3.39)$$

Now, substituting (3.34) and (3.39) in (3.35) provides the initial guess for the unknown complex coefficients for the optimization algorithm for the internal fields. Thus, the analytical expression for the initial guess is as shown<sup>2</sup>

$$\mathbf{u}_{\text{initial guess}} = \mathbf{u}_1 = \frac{\text{Re}[\psi_{outer}^H \mathbf{A} \mathbf{A}^H \psi_{outer}]}{\|\mathbf{A} \mathbf{A}^H \psi_{outer}\|^2} \mathbf{A}^H \psi_{outer} \quad (3.40)$$

Knowing the external fields, the expression for the internal fields, and the initial guess for the unknown complex coefficients, we now have everything to set up the optimization algorithm for optimizing the unknown complex coefficients  $\mathbf{u}$  to arrive at our final design.

### 3.3 Cascaded Metasurface Design Algorithm

The development of the cost function for the cascaded metasurface system is based on the local power conservation (LPC) [27] at every unit cell for both the metasurfaces. This LPC constraint needs to be met in order to ensure that the metasurfaces are passive and lossless. Therefore, we begin by calculating power densities at every unit cell on  $S_1^-$  and  $S_2^+$  sides of metasurfaces  $MS1$  and  $MS2$ . The power density on the input side, that is,  $S_1^-$  side has already been calculated at every unit cell as shown in (3.2). We then use the scaled electric and magnetic fields shown in (3.5) with the Poynting vector to calculate the power densities at every unit cell on the output side, that is,  $S_2^+$  side as

$$P_y(x, y = d^+) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^N (E_z(x, y = d^+)|_{scaled}) \times H_x^*(x, y = d^+)|_{scaled} \right\} \quad (3.41)$$

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<sup>2</sup>We presented this at [52].

We then concatenate these power densities calculated for every unit cell on the  $S_1^-$  and  $S_2^+$  side into vector  $b$  as shown

$$b = \begin{bmatrix} P_y(x, y = 0^-) \\ P_y(x, y = d^+) \end{bmatrix} \in \mathbb{R}^{4N+2} \quad (3.42)$$

As it can be seen vector  $b$  is composed of  $4N + 2$  real numbers since the real power density at each unit cell is a real value and there are  $2N + 1$  unit cells on each metasurface.

Likewise, we calculate the power densities at every unit cell on the internal surfaces of the cascaded metasurfaces system, that is, on the  $S_1^+$  and  $S_2^-$  sides of metasurfaces  $MS1$  and  $MS2$  using the Poynting vector. We use (3.19) when multiple reflections are considered and (3.21) when multiple reflections are not considered along with the initial guess of the complex coefficient  $A_{f,n}$  and/or  $A_{r,n}$  concatenated in vector  $\mathbf{u}_{\text{initial guess}}$  as shown in Section 3.2 to calculate the initial electric and magnetic fields at every unit cell on the internal surfaces of the cascaded metasurfaces system. We then use the following equations to calculate the power density at each unit cell on the  $S_1^+$  side and  $S_2^-$  side when multiple reflections are considered in the system. This has been provided in [14] as

$$P_y(x, y = 0^+) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^N \sum_{m=-N}^N (I_n J_m^* A_{f,n} A_{f,m}^* + I'_n J_m^* A_{r,n} A_{f,m}^* - I_n J_m^* A_{f,n} A_{r,m}^* - I'_n J_m^* A_{r,n} A_{r,m}^*) \right\} \quad (3.43)$$

$$P_y(x, y = d^-) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^N \sum_{m=-N}^N (I'_n J_m^* A_{f,n} A_{f,m}^* + I_n J_m^* A_{r,n} A_{f,m}^* - I'_n J_m^* A_{f,n} A_{r,m}^* - I_n J_m^* A_{r,n} A_{r,m}^*) \right\} \quad (3.44)$$

When multiple reflections are not considered in the cascaded metasurfaces system, we use the following equations to calculate the power density at each unit cell on the

$S_1^+$  side and  $S_2^-$  side [14]

$$P_y(x, y = 0^+) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^N \sum_{m=-N}^N (I_n J_m^* A_{f,n} A_{f,m}^*) \right\} \quad (3.45)$$

$$P_y(x, y = d^-) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^N \sum_{m=-N}^N (I'_n J'_m{}^* A_{f,n} A_{f,m}^*) \right\} \quad (3.46)$$

The internal power densities calculated using (3.43) and (3.44) or (3.45) and (3.46) can be arranged into a vector form and be represented using an operator notation as shown

$$\begin{bmatrix} P_y(x, y = 0^+) \\ P_y(x, y = d^-) \end{bmatrix} = \mathcal{L} \left( \begin{bmatrix} A_{f,n} \\ A_{r,n} \end{bmatrix} \right) = \mathcal{L}(\mathbf{u}) \quad (3.47)$$

where  $\mathcal{L}$  is an operator that implements the operations in (3.43) and (3.44) or (3.45) and (3.46), and as noted earlier  $\mathbf{u}$  is a complex vector formed by concatenating  $A_{f,n}$  and  $A_{r,n}$  coefficients. Thus,  $\mathbf{u} \in \mathbb{C}^{4N+2}$ .

Now, having both the external and internal power densities calculated for each unit cell and represented as shown in (3.42) and (3.47), we formulate a cost function  $C(\mathbf{u})$  to enforce LPC as shown below

$$C(\mathbf{u}) \triangleq \|b - \mathcal{L}(\mathbf{u})\|^2 \quad (3.48)$$

Thus,  $C(\mathbf{u})$  is a mapping from  $\mathbb{C}^{4N+2}$  to  $\mathbb{R}$ . That is a mapping of a vector of size  $4N + 2$  terms of complex numbers to a single real-valued number. We then formulate the optimization as minimizing the cost function (3.48) over  $\mathbf{u}$  as follows

$$\mathbf{u} = \underset{\mathbf{u}}{\text{argmin}} \{C(\mathbf{u})\}. \quad (3.49)$$

The cost function formulated in (3.48) is nonlinear, and to minimize the cost function over  $\mathbf{u}$ , we use the conjugate gradient (CG) technique to develop the optimization algorithm. The objective of the optimization algorithm is to update the

complex coefficient  $\mathbf{u}$  as

$$\mathbf{u}_{i+1} = \mathbf{u}_i + \alpha_i \mathbf{v}_i \quad (3.50)$$

where  $\mathbf{u}_i$  is a vector of complex coefficients at the current iteration,  $\alpha_i \in \mathbb{R}$  is the step length calculated at the current iteration,  $\mathbf{v}_i \in \mathbb{C}^{4N+2}$  is the conjugate-gradient direction, and  $\mathbf{u}_{i+1}$  is the update of the complex coefficients.

We calculate the conjugate-gradient direction as

$$v_i = \begin{cases} g_i + \beta_i v_{i-1} & i \neq 0 \\ g_0 & i = 0 \end{cases} \quad (3.51)$$

where  $g_i$  denotes the gradient vector at the current iteration. The parameter  $\beta_i$  is found based on the Fletcher-Reeves formulation as

$$\beta_i = \frac{g_i^H g_i}{g_{i-1}^H g_{i-1}} \quad (3.52)$$

where the superscript  $H$  denotes the Hermitian operator,  $g_i$  is the gradient vector at the current iteration and  $g_{i-1}$  is the gradient vector of the previous iteration.

To find the gradient vector  $g$ , we start by finding the variation of  $C$ , denoted by  $\delta C$  when  $\mathbf{u}$  is slightly changed by another vector of the same size, say  $w$  as

$$\begin{aligned} \delta C &= \lim_{\epsilon \rightarrow 0} \frac{C(\mathbf{u} + \epsilon w) - C(\mathbf{u})}{\epsilon} \\ &= \lim_{\epsilon \rightarrow 0} \frac{\|b - \mathcal{L}(\mathbf{u} + \epsilon w)\|^2 - \|b - \mathcal{L}(\mathbf{u})\|^2}{\epsilon} \end{aligned} \quad (3.53)$$

To calculate the limit in (3.53), we first expand  $\mathcal{L}(\mathbf{u} + \epsilon w)$  using its first-order Taylor expansion. The first-order Taylor expansion requires the derivative of  $\mathcal{L}$  with respect to the complex vector  $\mathbf{u}$ . However,  $\mathcal{L}$  is not complex differentiable since it is a mapping from a complex vector to a real-valued vector. One way is to find the derivative of  $\mathcal{L}$  with respect to real and imaginary parts of  $\mathbf{u}$  explicitly, we use Wirtinger calculus where the complex vector  $\mathbf{u}$  and its complex conjugate  $\mathbf{u}^*$  are

treated as two independent vectors [53, 54]. Using this approach, we then write the first-order Taylor expansion of  $\mathcal{L}(\mathbf{u} + \epsilon w)$  as follows

$$\mathcal{L}(u + \epsilon w) \approx \mathcal{L}(u) + \epsilon \frac{\partial \mathcal{L}}{\partial u}(w) + \epsilon \frac{\partial \mathcal{L}}{\partial u^*}(w^*) \quad (3.54)$$

Since the range of  $\mathcal{L}$  is real numbers,  $\frac{\partial \mathcal{L}}{\partial u}(w)$  and  $\frac{\partial \mathcal{L}}{\partial u^*}(w^*)$  are naturally complex conjugate of each other, we therefore re-write (3.54) as

$$\mathcal{L}(u + \epsilon w) \approx \mathcal{L}(u) + 2\epsilon \text{Re}\left\{\frac{\partial \mathcal{L}}{\partial u^*}(w^*)\right\} \quad (3.55)$$

Denoting  $\frac{\partial \mathcal{L}}{\partial u^*}$  as the Jacobian (sensitivity) matrix  $\mathbf{J}$ , and substituting  $\mathcal{L}(\mathbf{u} + \epsilon w)$  described in (3.55) in (3.53), we obtain the variation of  $C$  as

$$\delta C = -4\text{Re}\{\mathbf{J}^H(b - \mathcal{L}(u))\}^H w^* \quad (3.56)$$

We then decompose  $\delta C$  into two components and identify one component being derivative with respect to  $u^*$  and the other being derivative with respect to  $u$  as

$$\begin{aligned} \delta C &= -2\{\mathbf{J}^T(b - \mathcal{L}(u))^*\}^T w^* - 2\{\mathbf{J}^H(b - \mathcal{L}(u))\}^T w \\ &= \frac{\partial C}{\partial u^*}(w^*) + \frac{\partial C}{\partial u}(w) \end{aligned} \quad (3.57)$$

The derivative with respect to  $u^*$  needs to be used, and therefore, the gradient vector  $g$  at the current ( $i$ th) iteration is calculated as follows

$$g_i = -2\mathbf{J}_i^T(b - \mathcal{L}(u_i))^* = -2\mathbf{J}_i^T(b - \mathcal{L}(u_i)) \quad (3.58)$$

The Jacobian matrix  $\mathbf{J}_i$  is the derivative of  $\mathcal{L}$  with respect to the conjugate of the complex coefficients ( $u^*$ ). Thus, It consists of four block matrices each of which with the size of  $\mathbb{C}^{(2N+1) \times (2N+1)}$ , and can be expressed as

$$\mathbf{J}_i = \begin{bmatrix} \frac{\partial P_y(x, y=0^+)}{\partial A_f^*} & \frac{\partial P_y(x, y=0^+)}{\partial A_r^*} \\ \frac{\partial P_y(x, y=d^-)}{\partial A_f^*} & \frac{\partial P_y(x, y=d^-)}{\partial A_r^*} \end{bmatrix} \in \mathbb{C}^{(4N+2) \times (4N+2)} \quad (3.59)$$

Since the Jacobian matrix  $\mathbf{J}_i$  includes the derivatives of the internal power densities with respect to  $u^*$ , that is  $A_f^*$  and  $A_r^*$ , we expand the power densities calculated using (3.43) and (3.44) for the case when multiple reflections are considered and (3.45) and (3.46) for the case when multiple reflections are not considered, by employing

$$\text{Re}\{a\} = \frac{a + a^*}{2} \quad (3.60)$$

where  $\text{Re}\{.\}$  denotes the real part. Therefore, (3.43) and (3.44) can be expressed respectively as

$$\begin{aligned} P_y(x, y = 0^+) = \frac{1}{4} \sum_{n=-N}^N \sum_{m=-N}^N & (I_n J_m^* A_{f,n} A_{f,m}^* + I_n^* J_m A_{f,n}^* A_{f,m} + I_n' J_m^* A_{r,n} A_{f,m}^* \\ & + I_n'^* J_m A_{r,n}^* A_{f,m} - I_n J_m'^* A_{f,n} A_{r,m}^* - I_n^* J_m' A_{f,n}^* A_{r,m} \\ & - I_n' J_m'^* A_{r,n} A_{r,m}^* - I_n'^* J_m' A_{r,n}^* A_{r,m}) \end{aligned} \quad (3.61)$$

$$\begin{aligned} P_y(x, y = d^-) = \frac{1}{4} \sum_{n=-N}^N \sum_{m=-N}^N & (I_n' J_m'^* A_{f,n} A_{f,m}^* + I_n'^* J_m' A_{f,n}^* A_{f,m} + I_n J_m'^* A_{r,n} A_{f,m}^* \\ & + I_n^* J_m' A_{r,n}^* A_{f,m} - I_n' J_m^* A_{f,n} A_{r,m}^* - I_n'^* J_m A_{f,n}^* A_{r,m} \\ & - I_n J_m^* A_{r,n} A_{r,m}^* - I_n^* J_m A_{r,n}^* A_{r,m}) \end{aligned} \quad (3.62)$$

Similarly, for the case of no reflections within the cavity, (3.45) and (3.46) can be expressed respectively as

$$P_y(x, y = 0^+) = \frac{1}{4} \sum_{n=-N}^N \sum_{m=-N}^N (I_n J_m^* A_{f,n} A_{f,m}^* + I_n^* J_m A_{f,n}^* A_{f,m}) \quad (3.63)$$

$$P_y(x, y = d^-) = \frac{1}{4} \sum_{n=-N}^N \sum_{m=-N}^N (I_n' J_m'^* A_{f,n} A_{f,m}^* + I_n'^* J_m' A_{f,n}^* A_{f,m}) \quad (3.64)$$

We then treat  $A_f$ ,  $A_f^*$ ,  $A_r$ , and  $A_r^*$  as independent quantities in the derivative process. Therefore, by performing partial differentiation of (3.61) and (3.62) with

respect to the  $A_f^*$  and  $A_r^*$  for the case considering multiple reflections in the system, we obtain the following four expressions to calculate the blocks of the Jacobian matrix as shown

$$\begin{aligned} \frac{\partial P_y(x, y = 0^+)}{\partial A_f^*} &= \frac{1}{4} \sum_{n=-N}^N (I_n J_p^* A_{(f,n)} + I_n' J_p^* A_{(r,n)}) \\ &\quad + \sum_{m=-N}^N (I_p^* J_m A_{(f,m)} - I_p^* J_m' A_{(r,m)}) \end{aligned} \quad (3.65)$$

$$\begin{aligned} \frac{\partial P_y(x, y = 0^+)}{\partial A_r^*} &= \frac{1}{4} \sum_{n=-N}^N (-I_n J_p^* A_{(f,n)} - I_n' J_p^* A_{(r,n)}) \\ &\quad + \sum_{m=-N}^N (I_p^* J_m A_{(f,m)} - I_p^* J_m' A_{(r,m)}) \end{aligned} \quad (3.66)$$

$$\begin{aligned} \frac{\partial P_y(x, y = d^-)}{\partial A_f^*} &= \frac{1}{4} \sum_{n=-N}^N (I_n' J_p^* A_{(f,n)} + I_n J_p^* A_{(r,n)}) \\ &\quad + \sum_{m=-N}^N (I_p^* J_m' A_{(f,m)} - I_p^* J_m A_{(r,m)}) \end{aligned} \quad (3.67)$$

$$\begin{aligned} \frac{\partial P_y(x, y = d^-)}{\partial A_r^*} &= \frac{1}{4} \sum_{n=-N}^N (-I_n' J_p^* A_{(f,n)} - I_n J_p^* A_{(r,n)}) \\ &\quad + \sum_{m=-N}^N (I_p^* J_m' A_{(f,m)} - I_p^* J_m A_{(r,m)}) \end{aligned} \quad (3.68)$$

Similarly, the partial differentiation of (3.63) and (3.64) with respect to  $A_f^*$  for the case where we ignore multiple reflections in the system are obtained. This results in the following two expressions

$$\frac{\partial P_y(x, y = 0^+)}{\partial A_f^*} = \frac{1}{4} \sum_{n=-N}^N (I_n J_p^* A_{(f,n)}) + \sum_{m=-N}^N (I_p^* J_m A_{(f,m)}) \quad (3.69)$$



$$\frac{\partial P_y(x, y = d^-)}{\partial A_f^*} = \frac{1}{4} \sum_{n=-N}^N (I'_n J_p'^* A_{(f,n)}) + \sum_{m=-N}^N (I_p'^* J'_m A_{(f,m)}) \quad (3.70)$$

Now having the ability to calculate the Jacobian matrix  $\mathbf{J}$  analytically, we can analytically calculate the gradient vector  $g_i$  shown in (3.58). Consequently, we can calculate the parameter  $\beta_i$  and the conjugate-gradient direction  $v_i$  analytically.

Up to this point, everything in the update equation given in (3.50) can be obtained analytically except the step-length  $\alpha_i$ . Therefore to derive the analytical expression for the step-length  $\alpha_i$ , we develop a cost function as a function of  $\alpha_i$  where  $u_i$  is the current estimate of the complex unknowns at the  $i$ th iteration,  $\alpha_i$  is the step length at the  $i$ th iteration, and  $v_i$  is the conjugate gradient direction at the  $i$ th iteration as shown in

$$C(u_i + \alpha_i v_i) \triangleq \|b - \mathcal{L}(u_i + \alpha_i v_i)\|^2 \quad (3.71)$$

We then express the  $\mathcal{L}(u_i + \alpha_i v_i)$  in the cost function in (3.71) using Taylor expansion as shown in (3.54) to obtain the cost function as shown below

$$C(u_i + \alpha_i v_i) = \left\| (b - \mathcal{L}(u_i)) - 2\alpha_i \text{Re}\left(\frac{\partial \mathcal{L}}{\partial u_i^*} v_i^*\right) \right\|^2 \quad (3.72)$$

To handle the  $L_2$  norm of the cost function, we use the following identity

$$\|a + b\|^2 = \|a\|^2 + \|b\|^2 + 2\text{Re}\langle a, b \rangle \quad (3.73)$$

Using (3.73) in conjunction with (3.72) results in

$$\begin{aligned} C(u_i + \alpha_i v_i) = & \|b - \mathcal{L}(u_i)\|^2 + 4\alpha_i^2 \left\| \text{Re}\left(\frac{\partial \mathcal{L}}{\partial u_i^*} v_i^*\right) \right\|^2 \\ & - 4\alpha_i \text{Re}\langle b - \mathcal{L}(u_i), \text{Re}\left(\frac{\partial \mathcal{L}}{\partial u_i^*} v_i^*\right) \rangle \end{aligned} \quad (3.74)$$

We then minimize the above cost function with respect to  $\alpha_i$ . Thus,

$$\frac{\partial C(u_i + \alpha_i v_i)}{\partial \alpha_i} = 0 \quad (3.75)$$

After simplifying the expression associated with (3.75), we obtain the analytical expression for the step-length  $\alpha_i$  computed at every iteration as shown below

$$\alpha_i = \frac{\text{Re}\langle (b - \mathcal{L}(u_i)), \text{Re}(\frac{\partial \mathcal{L}}{\partial u_i^*} v_i^*) \rangle}{2 \left\| \text{Re}(\frac{\partial \mathcal{L}}{\partial u_i^*} v_i^*) \right\|^2} \quad (3.76)$$

where  $\frac{\partial \mathcal{L}}{\partial u_i^*} = \mathbf{J}_i$

Finally, with this analytical expression of the step-length  $\alpha_i$ , all the terms can be finally obtained analytically and in a systematic fashion. The entire process of optimizing for the unknown complex coefficients  $A_{f,n}$  and/or  $A_{r,n}$  can be summarized in the list below.

1. Calculate the initial guess for the unknown complex coefficients  $A_{f,n}$  and/or  $A_{r,n}$  using (3.40).
2. Develop an update equation for the unknown complex coefficients  $A_{f,n}$  and/or  $A_{r,n}$  as shown in (3.50).
3. Calculate the step-length  $\alpha_i$  at every iteration using (3.76), and the conjugate-gradient direction  $v_i$  as shown in (3.51).
4. Iterate until LPC is achieved to the desired extent. That is, minimize (3.48) to an appropriate value.

The termination of the optimization algorithm can happen in three ways:

1. When the  $L_2$  norm of the cost function in (3.48) meets the minimum tolerance error, we terminate the optimization algorithm. When this happens essentially the LPC is satisfied on both metasurface  $MS1$  and  $MS2$  since the error tolerance is set to a small value. For instance, we set the error tolerance to  $10^{-10}$ .

2. The  $L_2$  norm of the cost function in (3.48) is calculated at every iteration. So, when the difference  $L_2$  norm of the two consecutive iterations is smaller than the minimum tolerance error, we terminate the optimization algorithm. This is because the optimization algorithm stagnated.
3. Finally, we set a maximum number of iterations for the optimization algorithm to try and find the solution for the unknown coefficients  $A_{f,n}$  and/or  $A_{r,n}$ . When the maximum number of iterations is reached in the process of optimization, we terminate the algorithm.<sup>3</sup>

Ideally, we should optimize until either termination criteria 1 or 2 in the list above is achieved, and therefore we should let the algorithm run for as many iterations as it requires to achieve either criteria 1 or 2. However, we noticed that too many iterations can lead to a problem of instability of the solution.

Once  $\mathbf{u}$  is obtained from the minimization of the cost function, the complex coefficients  $A_{f,n}$  and/or  $A_{r,n}$  are known, and then the correct internal fields can be obtained using (3.19) or (3.21) depending on the consideration of multiple reflections in the cascaded metasurfaces system. Once these fields are calculated, the macroscopic design of the cascaded metasurfaces system is complete. Thereafter, these fields can be used to design the unit cells.

### 3.4 Unit Cell Design Process

We now need to focus on the design of subwavelength unit cells that serve as meta-atoms for the metasurfaces. Therefore, herein, we begin by discussing a mathematical model for our unit cells. As shown in Chapter 2, we can represent the unit cells with

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<sup>3</sup>We have observed that the solution quality can be sensitive if we perform too many iterations.

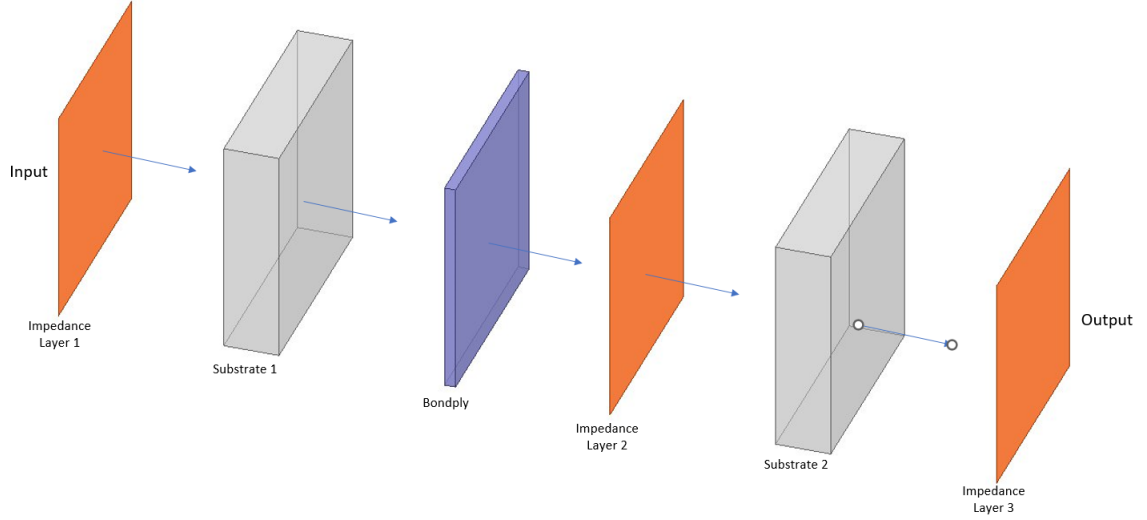


Figure 3.7: 3D stacked unit cell model (not to scale)

three layers of surface impedances. The three-layer unit cells have been previously used in literature, e.g., in [15, 16, 23, 46].

### 3.4.1 Unit Cell Model

In this section, we discuss the mathematical model used to design a unit cell for the cascaded metasurface system. The unit cell design is based on previous works in this area where the unit cell is made up of multiple layers with appropriate characteristics to create a single stacked unit that enables transformation and supports the discontinuity in electromagnetic fields on either side of the unit cell. This stacking of multiple different layers enables the realization of the equivalent surface parameters required for the unit cell design. In our case, the multiple layers include three impedance layers along with two dielectric substrate layers, and a bondply layer for connection as shown in Figure 3.7 [8, 23, 27, 55]. To be more accurate, our unit cell model also includes the bondply layer in the mathematical model.

From the fabrication point of view, the unit cell can be thought of as three sections.

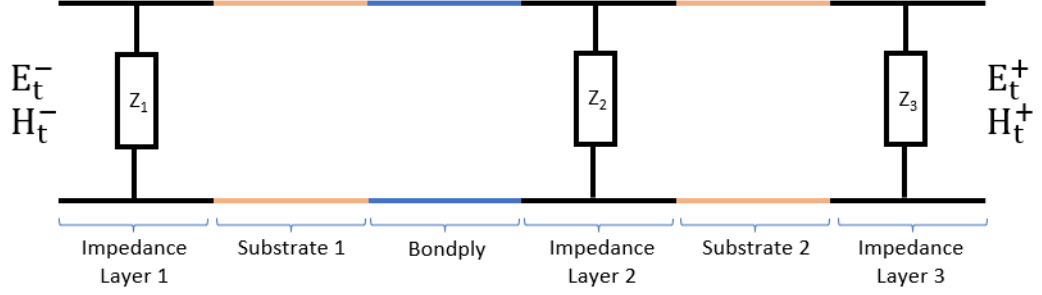


Figure 3.8: Circuit representation of the unit cell [23]

The top section consists of *impedance Layer 1* and *Substrate 1* and the bottom section consists of *impedance Layer 2*, *Substrate 2*, and *impedance Layer 3*. Both the top and bottom sections are connected by the middle section which is a thin bondply layer as shown in Figure 3.7. The arrows in Figure 3.7 indicate the connection of the layers. The length and width of all the layers are the same (The thickness of the substrates can be different). This 3D stacked model of the unit cell can also be represented as a 2-port circuit network as shown in Figure 3.8 [8, 23, 27, 55].

In the circuit model of the unit cell, the impedance layers are represented as shunt elements with impedance  $Z_1$ ,  $Z_2$ , and  $Z_3$  for three impedance layers, while the dielectric substrates and bondply are represented as lossless transmission lines. Following a standard procedure (e.g., see [23]), we then mathematically represent each layer of the 2-Port network in terms of the T parameter matrix and then multiply all of them to get the overall transformation matrix  $T$  as shown [56]

$$[T] = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \times [Sub_1] \times [Bp] \times \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} \times [Sub_2] \times \begin{bmatrix} 1 & 0 \\ Y_3 & 1 \end{bmatrix} \quad (3.77)$$

where  $Y_1$ ,  $Y_2$ , and  $Y_3$  are the admittance of the impedance layers, while the T parameters for the dielectric substrates and the bondply are as follows [23]

$$[Sub_i] = \begin{bmatrix} \cos(\beta_i t_i) & jZ_{sub_i} \sin(\beta_i t_i) \\ \frac{j \sin(\beta_i t_i)}{Z_{sub_i}} & \cos(\beta_i t_i) \end{bmatrix} \quad (3.78)$$

$$\begin{bmatrix} Bp \end{bmatrix} = \begin{bmatrix} \cos(\beta_{Bp}t_{Bp}) & jZ_{sub_{Bp}}\sin(\beta_{Bp}t_{Bp}) \\ \frac{j\sin(\beta_{Bp}t_{Bp})}{Z_{sub_{Bp}}} & \cos(\beta_{Bp}t_{Bp}) \end{bmatrix} \quad (3.79)$$

where  $i \in \{1, 2\}$ ,  $\beta_i$  is the wavenumber in substrate  $i$ ,  $t_i$  is the thickness of substrate  $i$  and  $Z_{sub_i}$  is the characteristic impedance of substrate  $i$ . Similarly,  $\beta_{Bp}$  is the wavenumber of bondply,  $t_{Bp}$  is the thickness of bondply, and  $Z_{sub_{Bp}}$  is the characteristic impedance of bondply. If LPC is met,  $Y_1$ ,  $Y_2$ , and  $Y_3$  can be purely reactive. That is, no loss or gain is needed in the design.

We then use (3.78) and (3.79) to analytically calculate the T parameter matrices for the two dielectric substrates and the bondply. To completely define the transformation matrix in (3.77), we need to specify the admittance  $Y_1$ ,  $Y_2$ , and  $Y_3$  but they are unknown at this point. Therefore, we set up a cost function with the transformation matrix in (3.77) and the tangential fields on both sides of the unit cell as shown in (3.80) and solve for the unknown admittance  $Y_1$ ,  $Y_2$ , and  $Y_3$  from the following equation

$$\begin{bmatrix} E_t^- \\ H_t^- \end{bmatrix} = [T] \times \begin{bmatrix} E_t^+ \\ H_t^+ \end{bmatrix} \quad (3.80)$$

where  $E_t^-$  and  $H_t^-$  are the tangential electric and magnetic fields on the input side of the unit cell and  $E_t^+$  and  $H_t^+$  are the tangential electric and magnetic fields on the output side of the unit cell as shown in Figure 3.9

For demonstration purposes, we modeled the multiple layers of the unit cell as one layer in Figure 3.9 as it can simply be represented as one transformation matrix as in (3.77), and colored the input face in blue and the output face in green with the tangential electromagnetic fields. The tangential fields at every unit cell are obtained from the macroscopic design as discussed earlier. After the cost function in (3.80) is solved, we invert the admittance values to get the impedance values for all the layers

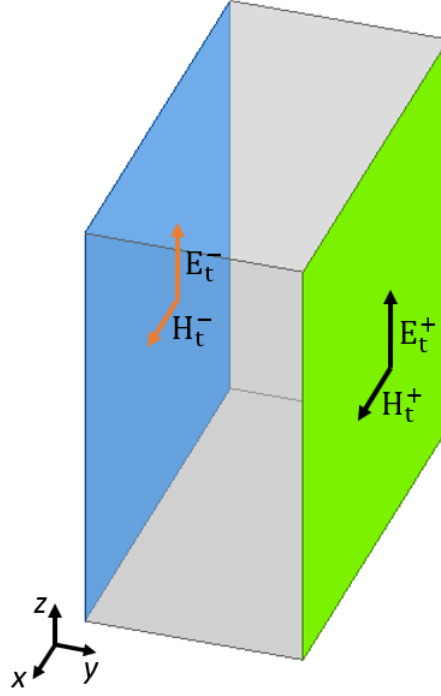


Figure 3.9: Unit cell model with tangential fields

as shown in (3.81) for the unit cell; i.e.,

$$Z_i = \frac{1}{Y_i} \quad (3.81)$$

where  $i \in 1, 2, 3$  for all the impedance layers of the unit cell.

We use the above-described strategy to get the impedance values for all the unit cells in both cascaded metasurfaces and then use them to model a theoretical system in ANSYS® HFSS™ software. The modeling of the theoretical design of the cascaded metasurface system is discussed in Chapter 4. Let us now see how the impedance values for the unit cell can be used in the practical design of the unit cell.

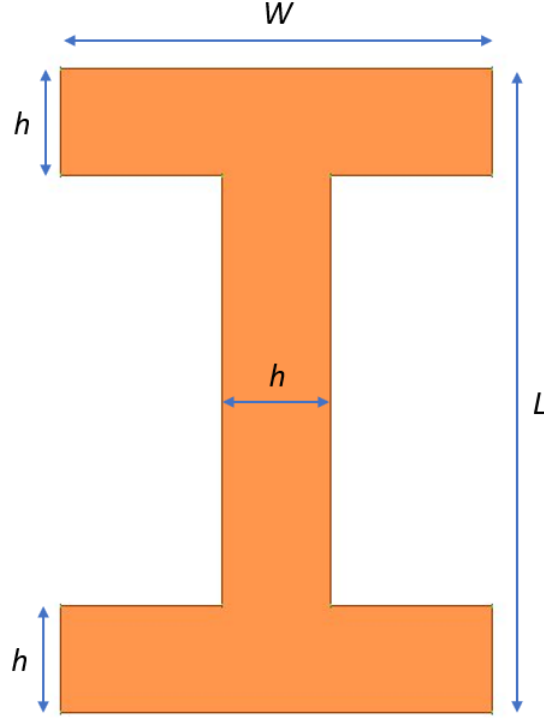


Figure 3.10: Dogbone copper trace dimensions.

### 3.4.2 Physical Unit Cell Design

In this section, we discuss the utilized strategy to convert the impedance values of the impedance layers for the stacked unit cell design as discussed in section 3.4.1 into copper traces for the physical realization of the unit cells. To this end, we briefly discuss the determination of copper trace dimensions based on the look-up table from a data set created by Max Kelly [46].

For the physical realization of the unit cell, we have opted for a “dogbone” shaped, also known as  $H$ -shaped, copper trace design as shown in Figure 3.10. where  $W$  is the width,  $L$  is the length, and  $h$  is the thickness of the copper trace. The cladding thickness of all the copper traces is fixed to 0.017 mm, which is equivalent to 1/2 ounces (oz). The copper trace dimensions are confined within the boundaries of the square



face of the unit cell. The thickness ( $h$ ) of the copper trace is fixed while the width and the length are determined using the strategies explained later in this section.<sup>4</sup>

The orientation of the copper trace depends on the polarization of the electric field. For instance, in Figure 3.10, the length of the copper trace is along the  $z$ -direction while the width is along the  $x$ -direction. Therefore, for this copper trace orientation, the tangential impinging electric field impacting the copper trace will be  $z$  polarized. For a single layer, this  $z$  polarized electric will induce an electric current on the dogbone which is determined by the surface impedance of the copper trace. When we place the three layers together, we can have equivalent surface electric and magnetic currents. A complete image of the physically realizable unit cell is shown in Figure 3.11 where  $D$  is the size of the unit cell,  $T_s$  is the thickness of the substrates and  $T_b$  is the thickness of the bondply. It can be seen in Figure 3.11, that the impedance layers in the theoretical model of the unit cell have been replaced with copper traces with dogbone shapes with different dimensions depending on the impedance values of each layer. This unit cell can now be physically fabricated to perform the desired transformation.

We now discuss the design methodology utilized by Max Kelly [46] to determine the dimensions of the dogbone copper traces. This methodology requires  $S$  parameters to determine the copper trace dimensions. Therefore, we obtain the two-port scattering parameters for each unit cell. To this end, we use the impedance values of each layer of the unit cell with (3.77) to calculate the transmission ( $T$ ) parameters for the unit cell. The transmission parameters also known as  $ABCD$  parameters can be written as

$$\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (3.82)$$

These  $ABCD$  parameters can easily be converted to  $S$  parameters using the trans-

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<sup>4</sup>Note that the dogbone-shaped copper traces have also been used in other works such as [6, 15, 16].

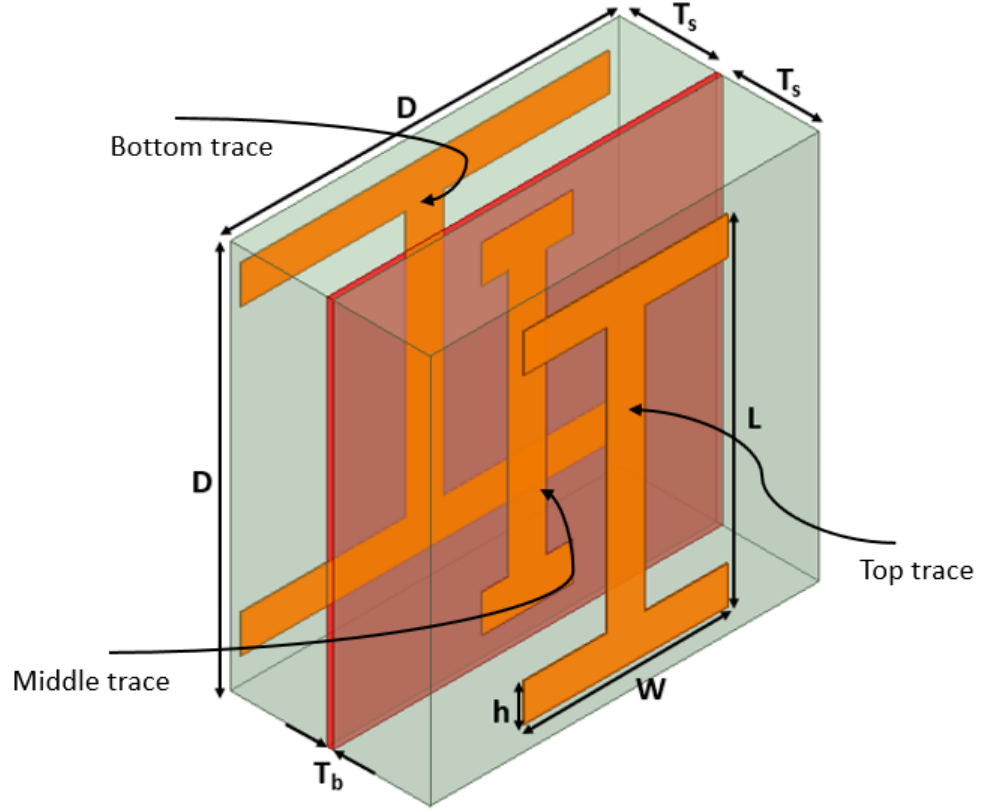


Figure 3.11: A complete physically realizable unit cell [57]

formation equation shown below [56]

$$S_{11_{desired}} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \quad (3.83a)$$

$$S_{12_{desired}} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \quad (3.83b)$$

$$S_{21_{desired}} = \frac{2}{A + B/Z_0 + CZ_0 + D} \quad (3.83c)$$

$$S_{22_{desired}} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \quad (3.83d)$$

where  $Z_0 = \eta_0$  is the characteristic impedance of free space.

After we obtain the magnitude and the phase of the desired transmission coefficient ( $S_{21_{desired}}$ ), then based on Max Kelly's approach, we select the unit cell with the

Table 3.1: Look-up table for Dogbone Copper Trace Dimensions

| Transmission Coeff |                       | $W_{top}$ | $W_{middle}$ | $W_{bottom}$ | $L_{top}$ | $L_{middle}$ | $L_{bottom}$ |
|--------------------|-----------------------|-----------|--------------|--------------|-----------|--------------|--------------|
| $ S_{21} $         | $\angle S_{21}^\circ$ | [mm]      | [mm]         | [mm]         | [mm]      | [mm]         | [mm]         |
| 0.9                | 183                   | 2.09725   | 1.98925      | 2.53225      | 2.32425   | 2.10935      | 2.77899      |

closest  $S_{21}$  magnitude and phase from the look-up table created in [46]. The format of the look-up table is shown in Table 3.1 with an example of random copper trace dimensions for a unit cell.

In this table,  $S_{21}$  magnitude and phase are the simulated values for the copper trace dimensions specified in the same row,  $W_{xyz}$  indicates the width of the copper traces, and  $L_{xyz}$  indicates the length of the copper traces in millimeters of the top, middle, and bottom layers, respectively.

Once we identify the unit cell with the closest magnitude and phase of the desired transmission coefficient based on the look-up table, we model the unit cell in ANSYS® HFSS™ software and further optimize it using the in-built optimizer called “*Sequential Nonlinear Programming (Gradient)*” in the ANSYS® HFSS™ software. The optimization involves minimizing the cost function for the transmission coefficient for the unit cell as

$$C(\overbrace{W_{top}, W_{middle}, W_{bottom}, L_{top}, L_{middle}, L_{bottom}}^x) \triangleq \|S_{21_{HFSS}}(x) - S_{21_{desired}}\|^2 \quad (3.84)$$

where the  $S_{21_{HFSS}}$  is the simulated transmission coefficient in the ANSYS® HFSS™ software based on the copper trace modeled in the software and  $S_{21_{desired}}$  is obtained from (3.83). In the process of optimization for the copper traces width and lengths of the top, middle, and bottom layers, the ANSYS® HFSS™ software optimizer adjusts the widths and length randomly to a value within the range of  $\pm 0.4$  mm of the original dimensions obtained from the look-up table. The optimization is done for one unit cell at a time. Once all the unit cells optimized copper trace dimensions are

obtained, we model both metasurfaces in ANSYS® HFSS™ software to perform a full-wave simulation for the entire cascaded metasurfaces system. If the performance is acceptable, the metasurface is now ready to be fabricated. Otherwise, we may need to perform more fine-tuning on the unit cells. No further fine-tuning was performed in this thesis.

This method of realizing the copper traces suffers from systematic design errors that can result in poor performance. Firstly, since the unit cells are optimized one at a time, the mutual coupling that would exist with the neighboring unit cell is not fully taken into consideration during optimization. Thus, when all the unit cells are modeled to model the complete inhomogeneous metasurfaces, the field interactions that are not accounted for can result in poor performance of the whole metasurface system [46]. Secondly, in this methodology, we only optimize the transmission coefficient of the unit cell while the phase of  $S_{11}$  is ignored. This can work when the forward transmission coefficient is dominant. However, if a design relies heavily on a precise  $\angle S_{11}$ , this look-up table will not be useful. This can be understood by noting that the unit cell is assumed to be lossless, passive, and reciprocal; thus  $S^H S$  becomes the identity matrix. Consequently,  $|S_{11}|^2 + |S_{21}|^2 = 1$ . Therefore, fixing the magnitude of  $S_{21}$  already fixes the magnitude of  $S_{11}$ , but the phase of  $S_{11}$  remains undetermined.

### 3.5 Chapter Summary

To summarize this chapter, we began by describing the macroscopic model of the cascaded metasurface system. We discussed ways to obtain external electromagnetic fields to the cascaded metasurface system followed by discussing ways to compute internal electromagnetic fields with and without reflection as developed by Ataloglou et al. [14] with unknown complex coefficient  $A_{f,n}$  and/or  $A_{r,n}$ . We then derived an

analytical expression to find an initial guess for the optimization algorithm. We then developed an automated optimization algorithm in the complex domain using the conjugate-gradient technique. The key aspect of this conjugate gradient design algorithm is the fact that all of its components are analytically derived without breaking the unknowns into their real and imaginary components. Finally, we discussed the theoretical and practical model of realizing the microscopic design of the metasurface.

## Chapter 4

# Simulation, Fabrication and Experimental Setup

The realization of the impedance values or the copper trace dimensions for the three-layer unit cells marks the end of the design process for the cascaded metasurface system. Up to this point, the only performance criterion achieved for the assessment of the design is how well the LPC constraint is met at every unit cell. However, the achievement of the LPC condition needs to be followed up by the evaluation of the overall performance of the cascaded metasurface system. Therefore, in this chapter, we provide modeling and full-wave simulation procedures for the cascaded metasurface system in ANSYS® HFSS™ software. This is followed by a brief discussion on the fabrication technique and lastly about the experimental setup and measurement technique.



Figure 4.1: A complete setup of a cascaded metasurface system with surface impedance specifications on the three layers of each unit cell. The two dielectric substrates are Rogers RO3010 which are bonded by Rogers 2929 bondply.

## 4.1 Modeling and Full-Wave Simulation

In this section, we discuss the modeling and simulation strategy for the cascaded metasurface system with impedance specifications and dogbone traces for the three-layered unit cells. The modeling and simulation assessment is done using ANSYS® HFSS™ software.

Our modeling procedure for the cascaded metasurface system in ANSYS® HFSS™ software is as follows:

1. Creating a simulation region, i.e. a cuboid, using the “Draw Box” tool as shown in Figure 4.1. The size of the simulation region is such that it encloses the two metasurfaces with absorbers on the ends of each metasurface.
2. A perfectly matched layer (PML) boundary condition is specified as the boundary condition for the side faces of this simulation region while a perfect electrical

conductor (PEC) boundary condition is specified for the top and the bottom faces. This is to enforce the 2D assumption. The simulation region is specified to have air as the medium within the region.

3. The modeling of the metasurface is done by modeling each unit cell at the correct position according to the design. The separation distance from the output face of metasurface  $MS1$  to the input face of metasurface  $MS2$  is  $d$ .
4. The unit cells are modeled by creating cuboids for the top substrate, bottom substrate, and bondply with appropriate dimensions as per the design. The structure of a unit cell is shown in Figure 3.7.
5. For our design, Rogers RO3010 is specified as the material for the top and the bottom substrates while bondply 2929 is specified as the material for the bondply.
6. For the theoretical model of the unit cell, the surface impedance value  $Z_1$ ,  $Z_2$ , and  $Z_3$  calculated using (3.81) are specified as the impedance boundary conditions for the three layers of each unit cell. To be specific, they are respectively assigned to the front face of the top substrate, the front face of the bottom substrate, and the bottom face of the bottom substrate for each unit cell.
7. For the absorbers, two cuboids are created whose materials are specified to be Rogers RO3010 with the middle layer being the PEC. The boundary condition for the front face and the bottom face of the two cuboids are specified to have an impedance boundary condition with the value  $z$  to be determined later. In summary, this is determined to meet the specification for an absorbing metasurface. There is also a tiny gap between the absorbing metasurface and the PML.



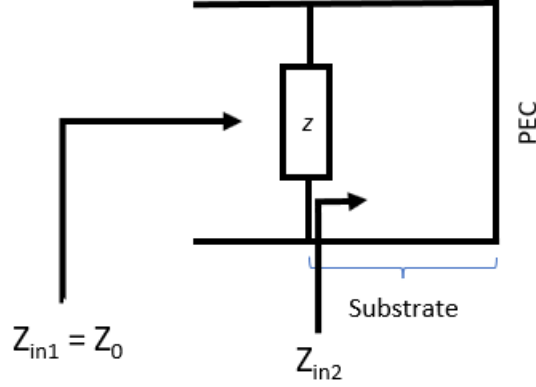


Figure 4.2: A transmission line model for the absorbing metasurface.

8. Depending on the requirement, we add a source to the left side of metasurface *MS1*. Typically, this is either a line source or a plane wave for our 2D problem.

The modeling of the simulation region, the unit cells, the absorbers, and the source marks the end of the modeling phase of the cascaded metasurface system. An example of a cascaded metasurface system with impedance specification for the three-layered unit cells is shown in Figure 4.1. We call this system the theoretical model.

#### 4.1.1 Absorbing Metasurface

The absorbers are designed based on the transmission line theory without reflections for a single frequency and normal plane wave incidence [23, 58]. The circuit model of one side of the substrate in the absorbing metasurface is shown in Figure 4.2 where the input impedance  $Z_{in1}$  is the free space characteristic impedance, the substrate is Roger RO3010 with the thickness of  $t_s + t_{Bp}/2$  followed by a PEC layer, and  $z$  is the unknown surface impedance of the absorber that needs to be determined. For the absorber to be effective, the equivalent impedance as a result of the parallel impedance  $z$  and  $Z_{in2}$  should be equal to impedance  $Z_{in1}$ . Note that  $Z_{in2}$  is the input impedance

as a result of the substrate and the load (PEC) calculated as follows

$$Z_{in2} = Z_c \frac{Z_L + jZ_c \tan(\beta l)}{Z_c + jZ_L \tan(\beta l)}, \quad (4.1)$$

where  $Z_c = \frac{Z_0}{\sqrt{\epsilon_r}}$  is the characteristic impedance of the substrate,  $\beta$  is the wavenumber of the substrate,  $l$  is the length of the substrate,  $\epsilon_r$  is the relative permittivity of the substrate, and  $Z_L$  is the impedance of PEC which is zero. Considering  $Z_L = 0$ , (4.1) simplifies to

$$Z_{in2} = jZ_c \tan(\beta l). \quad (4.2)$$

As mentioned earlier, the equivalent impedance as a result of parallel impedance  $Z_{in2}$  and  $z$  is equal to  $Z_{in1}$  (i.e.,  $Z_{in1} = Z_{in2} \parallel z$ ), the surface impedance  $z$  of the absorber can finally be calculated as follows

$$z = \frac{Z_{in2}Z_{in1}}{Z_{in1} - Z_{in2}}. \quad (4.3)$$

This surface impedance value  $z$  is then specified as the boundary condition for the top face of the top absorber and the bottom face of the bottom absorber.

### 4.1.2 Transmission Power Efficiency

After the modeling of the cascaded metasurface system is complete, a full wave simulation is performed at the operating frequency. The termination of Ansys HFSS is typically based on the so-called  $\Delta S$  parameter with the default value being 0.1. In our work, we enforced a stronger termination criterion which was  $\Delta S = 0.01$ . Finally, to assess the performance of the system, a visual inspection of the transformation of the electromagnetic wave is performed and transmission power efficiency is calculated. The transmission power efficiency is calculated as a result of the ratio of the real power leaving  $MS2$  in the normal direction when both metasurfaces are present to the real power normally incident on the input face of  $MS1$  when both metasurfaces are absent. To calculate this power efficiency, we obtain the input tangential

electric and magnetic fields just before metasurface  $MS1$  when the metasurfaces are not present and we also obtain the output tangential electric and magnetic fields just after metasurface  $MS2$  when the metasurfaces are present in the system. Then using the Poynting vector and the unit cell size, the powers are calculated. Since the unit cell sizes are assumed to be the same, their effect will be canceled out.

### 4.1.3 Copper Trace Model

Once the cascaded metasurface system with surface impedance specifications shows promising results, the cascaded metasurface system is modeled and simulated using copper traces which in our case resemble dogbone shapes. The modeling of the cascaded metasurface system with dogbones remains the same as listed above. However, the surface impedance specification is replaced with a dogbone copper trace. An example of a cascaded metasurface system with dogbones is shown in Figure 4.3. We call this system the practical model. Like the theoretical model, we then perform a full-wave simulation at the designed operating frequency for the practical model and its assessment is performed through visual inspection of the electromagnetic wave transformation and output power efficiency calculation as described for the theoretical model. Once promising results are achieved for the practical model we move to fabrication of the system. As noted in the previous chapter, the cladding thickness of all the copper traces is fixed at 0.017 mm, which is equivalent to 1/2 ounces (oz).

## 4.2 Fabrication Pipeline

The fabrication process of our metasurface system begins by creating Gerber files for the dogbones of each metasurface. To create a Gerber file, we begin by replicating every dogbone in the periodic direction, i.e. the  $z$ -direction in our case, in ANSYS®

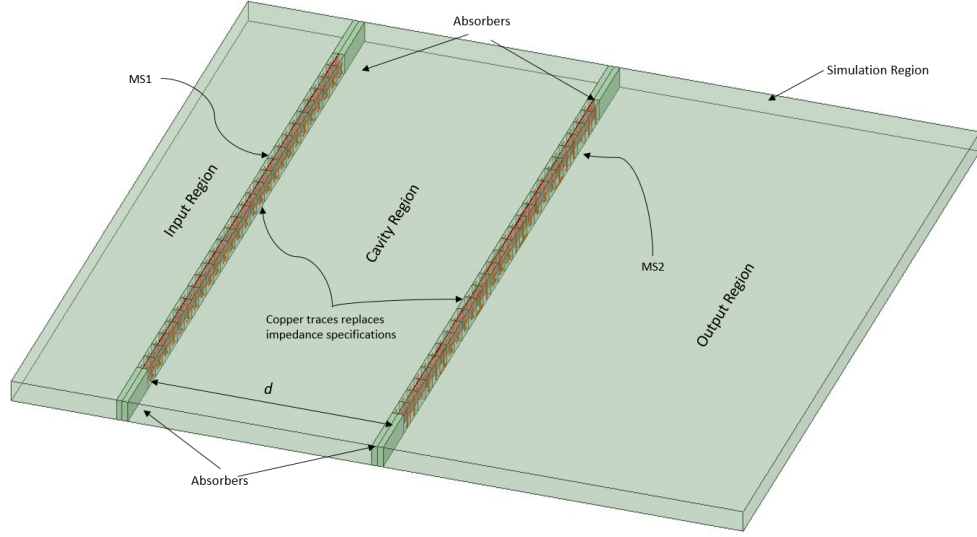


Figure 4.3: A complete setup of cascaded metasurface system with copper traces. Each unit cell consists of three dogbones, each of which with a cladding thickness of 0.5 oz. These dogbones are supported by two dielectric substrates (Rogers RO3010) which are bonded by Rogers 2929 bondply. Note that the difference between Figure 4.1 and this figure lies in the fact that the former used impedance sheets to implement the unit cells whereas this one uses more practical copper traces.

HFSS<sup>TM</sup> software. The copper traces are repeated such that the majority of the substrate board is covered with the copper traces. Figure 4.4 shows the replication of the top layer of a metasurface as an example. Note that this is periodic in the vertical direction, which is our  $z$ -direction. Thereafter, we export every repeated layer in Drawing Exchange Format (dxf) in ANSYS<sup>®</sup> HFSS<sup>TM</sup> software and import the dxf file in the DipTrace PCB Design software to create a Gerber file as shown in Figure 4.5. The purple line indicates the boundary of the substrate board. In summary, we create the Gerber files for all six layers of the copper traces, that is top, middle, and bottom copper traces for both of the metasurfaces. We then send these Gerber files for fabrication with the Roger RO3010 substrates and bondply for PCB

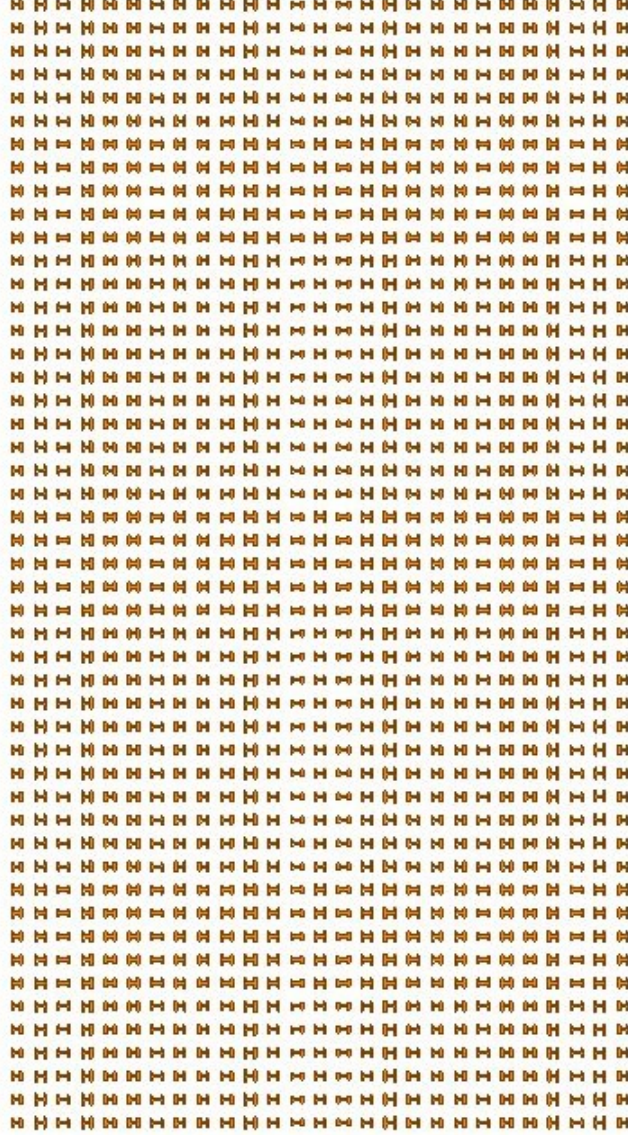


Figure 4.4: Periodic replication of dogbones of the top layer of a metasurface in a cascaded metasurface system. In this case, the periodicity is along the vertical direction which is assumed to be the  $z$ -direction in our setup.

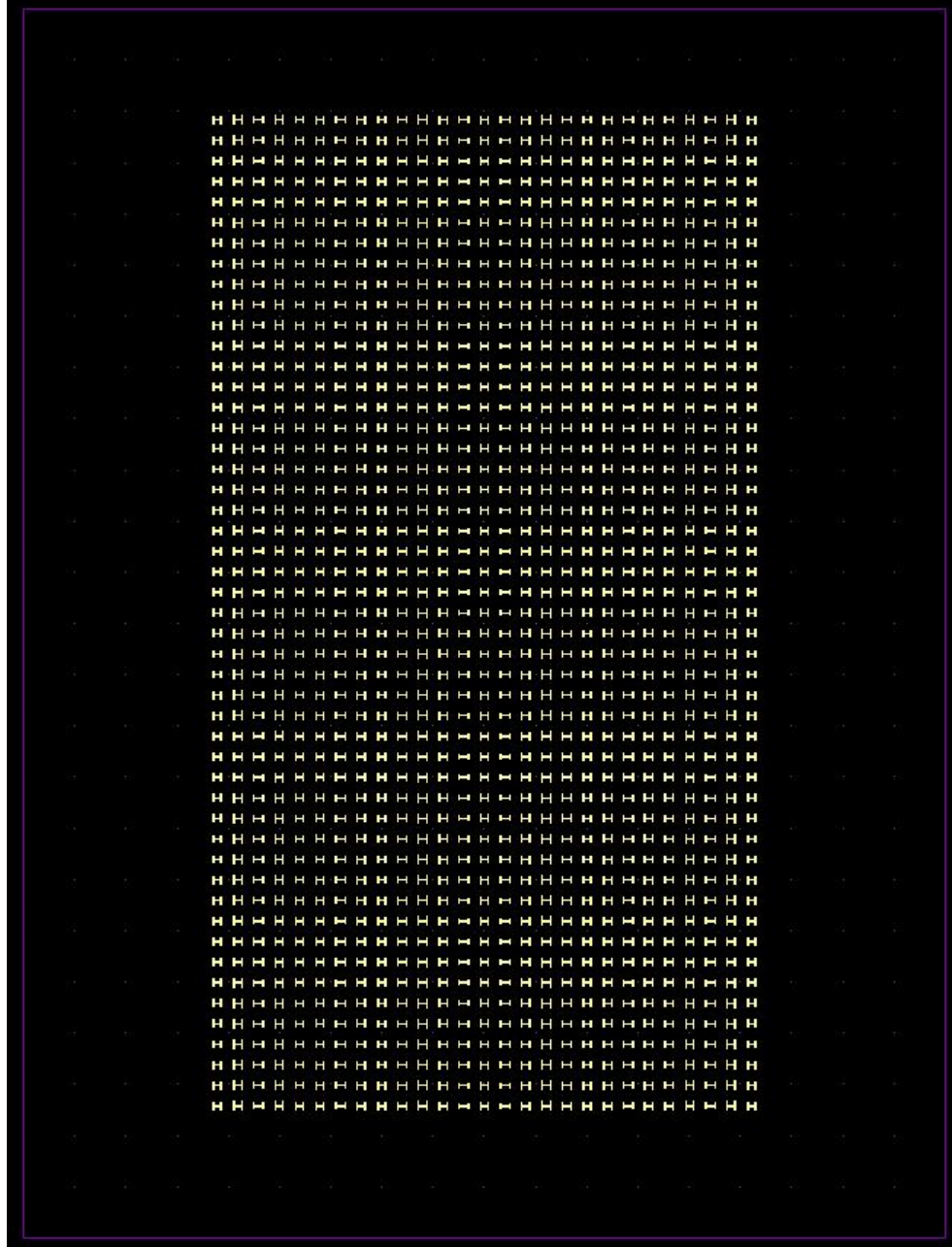


Figure 4.5: Periodic replication of a copper trace layer of the cascaded metasurfaces system as a Gerber file.



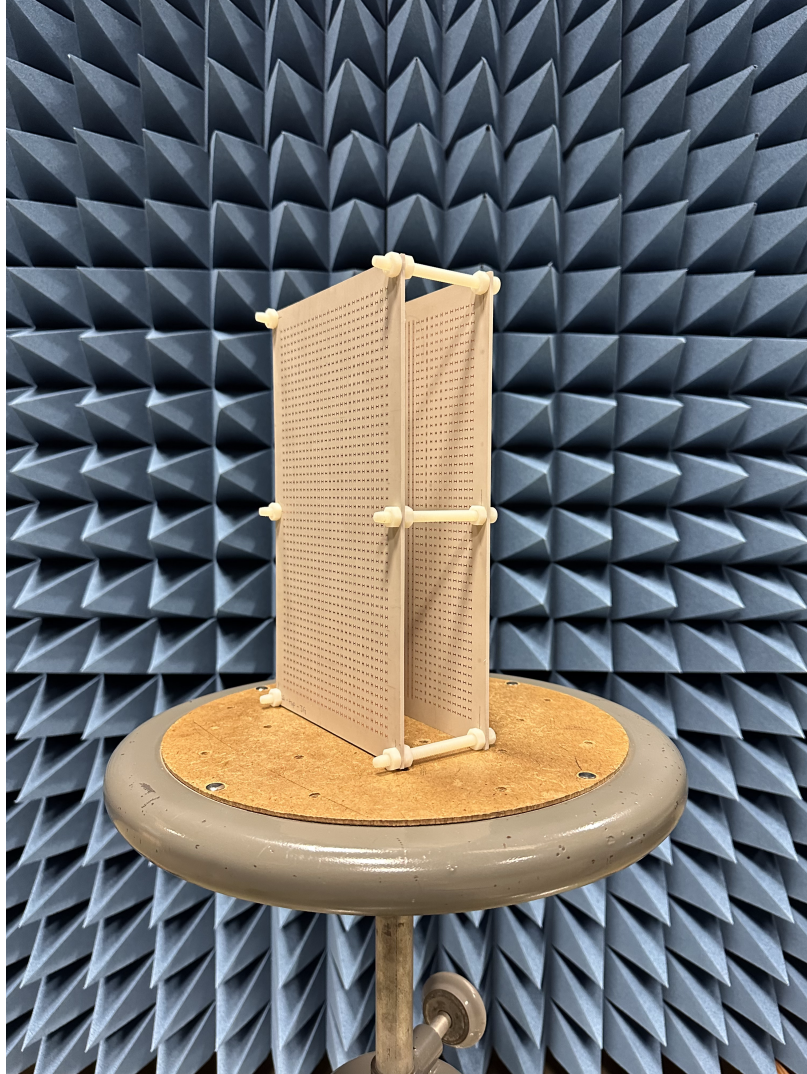


Figure 4.6: A fabricated cascaded metasurface system

fabrication. The copper traces are etched on the appropriate side of the substrates. Finally, the top and the bottom substrates are bonded together using the Rogers 2929 bondply to create one single metasurface. Once we have both metasurfaces, we cascade them using six nylon screws. Figure 4.6 shows a sample of a fabricated cascaded system of metasurfaces.

### 4.3 Experimental Setup

After the fabrication of the metasurfaces, the next step is to experimentally verify the performance of the cascaded metasurface system. In our case, this is done by performing planar near-field antenna range (PNFR) measurements on the cascaded metasurface system in the Antennas Laboratory at the University of Manitoba.<sup>1</sup>

The structure of the PNFR measuring equipment is of an inverted T shape as shown in Figure 4.7. The measurement system includes a horn antenna as shown in Figure 4.8a that illuminates the input face of the cascaded metasurface system. The cascaded metasurface system is mounted on a wooden frame surrounded by the convoluted absorbers as shown in Figure 4.8b. The Open-Ended Waveguide (OEW) probe shown in Figure 4.8c is used as the scanning probe to measure the output tangential electric fields leaving the cascaded metasurface system.

The separation distance between the illuminating horn antenna and the OEW probe is about 69 cm. That is about  $24\lambda$  at 10.5 GHz. The illuminating horn antenna is stationary while the OEW probe moves in two directions, i.e. horizontal and vertical directions to measure the tangential electric fields. It can measure co-pol and cross-pol electric fields but in our experiments, we only measure the co-pol electric fields transmitted from the cascaded metasurface system. The wooden frame with the metasurface system is placed in between the horn antenna and the scanning probe. To perform the measurements, the horn antenna illuminates the input face of the cascaded metasurface system that is to be transformed by the metasurface system, and the output tangential electric fields are measured using the OEW probe. In this experiment, we perform two kinds of tests as listed below.

1. Evaluate the sensitivity to the distance between the OEW probe's aperture and

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<sup>1</sup>The experimental part of this thesis has been reported in the submitted conference paper of the author [59].



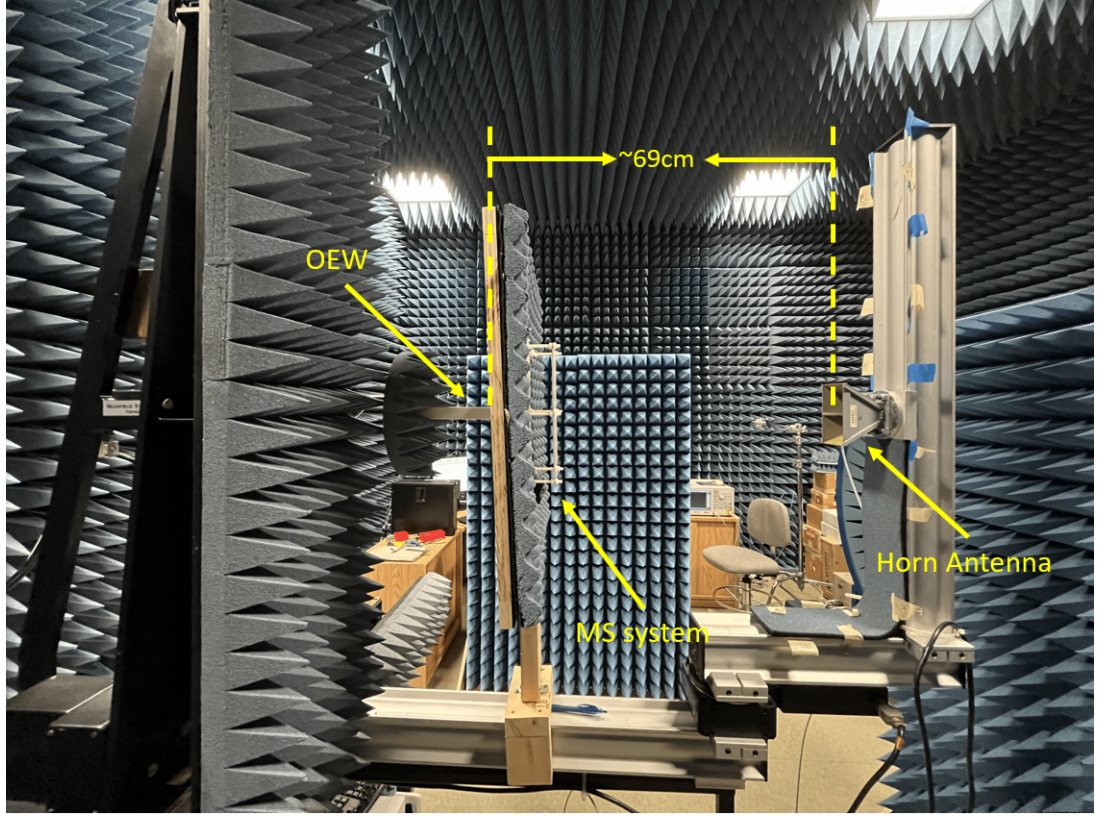
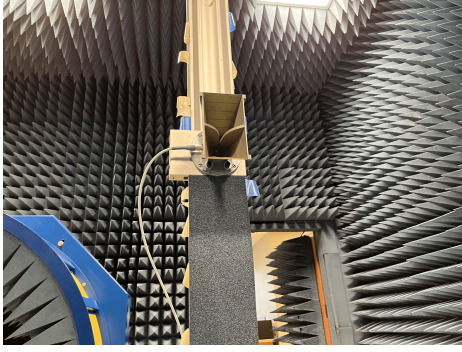


Figure 4.7: Experimental setup for PNFR measurements for a cascaded metasurface system. The horn antenna illuminates the cascaded metasurface system, and the open-ended waveguide (OEW) probe collects the resulting fields.

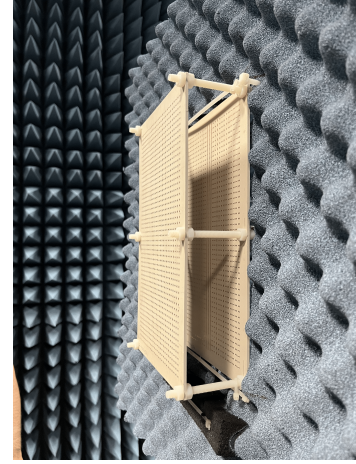
the output face of the cascaded metasurface.

2. Evaluate the sensitivity to the separation between two metasurfaces, i.e.  $d$ .

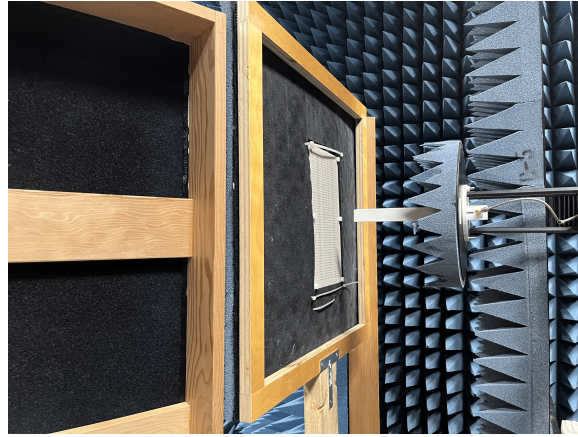
The first sensitivity test is conducted by varying the distance between the output face of metasurface  $MS2$  and the aperture of the OEW probe. This is to evaluate the effect of multiple reflections between the OEW probe and the metasurface. For each distance, we perform measurements and then find the resulting far-field pattern. It should be noted that the separation distance between the horn antenna and the OEW scanning probe is fixed, therefore varying the position of the metasurface system would



(a) Illuminating horn antenna



(b) Metasurface mounted on the wooden frame



(c) Open-ended waveguide scanning probe

Figure 4.8: Components of the PNFR measuring system

also affect the distance from the horn antenna to the input face of the metasurface. The farther the horn antenna is from the metasurface, the better; this is due to the fact that the fabricated metasurface is supposed to be illuminated with a plane wave.

The second sensitivity test is done by varying the separation distance between the two metasurfaces  $MS1$  and  $MS2$  to see its impact on the performance of the metasurface system. This sensitivity test is performed with the OEW probe being

about 3 cm or  $\approx \lambda$  distance away from the output face of the metasurface system. For the separation sensitivity test, the separation distance between the metasurfaces is adjusted by  $\pm 10\%$  and  $\pm 5\%$  from the design separation distance, and the resulting tangential fields are then measured by the OEW probe.

For all the above-mentioned tests, the measurements are collected at 16 different frequencies starting from 9.5 GHz to 11 GHz in steps of 0.1 GHz. The data collection includes the tangential electric fields recorded by the NSI-MI 2000 software and a resulting far-field power pattern that is generated using an in-built function of the NSI-MI 2000 software (near-field to far-field transformation).

To infer the transmission power efficiency of the fabricated metasurface system, the tangential electric fields transmitted through the wooden frame in the absence of the cascaded metasurface system need to be measured. This is shown in Figure 4.9.

The transmission power efficiency is calculated by first transforming the measured tangential near field electric fields to the far field. This will provide us with the total power radiated. This is performed for two scenarios: with and without the cascaded metasurface system. Then, the ratio between these two radiated powers will be the transmission power efficiency of the metasurface. The loss of power can be due to the reflection at the interface of the first metasurface, power leakage from the sides of the cascaded metasurface system, copper loss, and dielectric loss within the substrates.

## 4.4 Chapter Summary

To summarize this chapter, we began by describing the modeling strategy we utilized to create the cascaded metasurface system in ANSYS® HFSS™ software followed by the simulation assessment of the system performance. Then, we briefly discussed the fabrication strategy to manufacture a cascaded metasurface system. Lastly, we



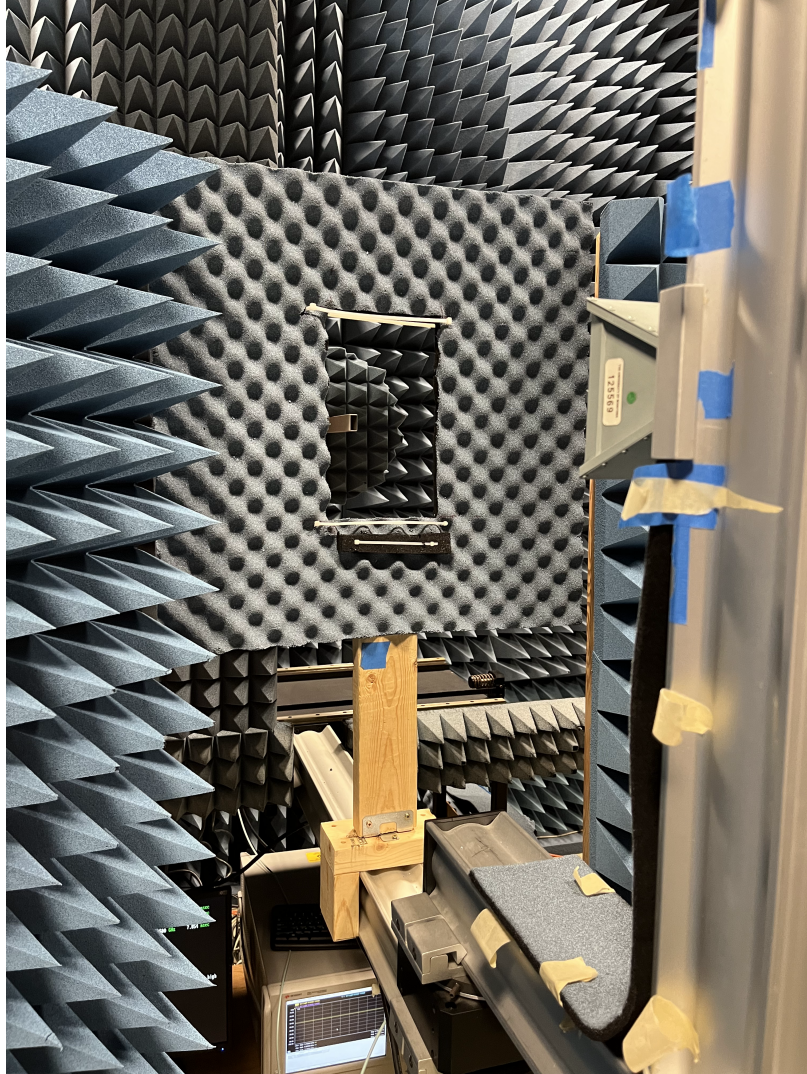


Figure 4.9: Measurements of the tangential near-field electric field without the cascaded metasurface system.

discussed the PNFR measurements to experimentally evaluate the performance of cascaded metasurface systems.

## Chapter 5

# Full-Wave Simulation and Experimental Work

In this chapter, we demonstrate a variety of electromagnetic wave transformations and design specification achievements using cascaded metasurface systems that were designed based on the methodology discussed in Chapter 3. The demonstration was performed through full-wave simulation of the system with impedance specification and copper trace designs for the three-layered unit cells in ANSYS® HFSS™ software. In addition, some experimental results have been shown for a fabricated cascaded metasurface system. Lastly, the impact of the thickness of the substrates on the performance of the cascaded metasurface system is discussed.

## 5.1 Cylindrical to Refracted Plane Wave Transformation

In this section, we discuss the design of a cascaded metasurface system aimed to achieve a transformation of cylindrical electromagnetic waves into truncated plane waves that are refracted by an angle of  $60^\circ$  with respect to the normal vector of the metasurface  $MS2$ . The metasurface system was designed at an operating frequency of 10 GHz with a free space wavelength of about 29.98 mm.

In this design, the two metasurfaces were separated by a distance of  $0.5\lambda_0$ , and the total length of each metasurface is  $6\lambda_0$ . Both metasurfaces have 61 unit cells. Thus, the size of each unit cell ( $D$ ) is about 2.9488 mm. The source (incident field), which is a line source, was placed  $\lambda_0/3$  away from the front face and the center of the metasurface  $MS1$ . The  $\hat{z}$  polarized line source then generates cylindrical waves and the desired output fields are supposed to be a truncated plane wave with a refraction angle of  $60^\circ$  with respect to the normal vector pointing out from  $MS2$ . The incident fields associated with the line source are available analytically. They are the zeroth order Hankel function of the second kind for the electric field and the first order Hankel function of the second kind for the magnetic field. The metasurfaces have spatial modulation along the  $x$  direction and they are invariant along the  $z$  direction. The tangential electric fields on the metasurface are then  $z$ -polarized while the tangential magnetic fields are  $x$ -polarized. The equations to calculate the tangential incident fields are shown in [14]. Since we assume no reflections from the input face of  $MS1$ , the tangential fields on the input face of  $MS1$  will be the same as the tangential incident fields.

We used the following plane wave equations to calculate the tangential output electromagnetic fields on the output face of metasurface  $MS2$  for the desired refracted

truncated output plane wave

$$E_z(x, y = d^+) = e^{-jk_o \sin \phi_{ref}}, \quad (5.1a)$$

$$H_x(x, y = d^+) = \frac{\cos(\phi_{ref})e^{-jk_o \sin \phi_{ref}}}{\eta_o}, \quad (5.1b)$$

where  $k_o$  is the free space wavenumber,  $\phi_{ref}$  is the angle of refraction of the truncated plane waves, and  $\eta_o$  is the free space characteristic impedance. Note that if we use  $e^{+jk_o \sin \phi_{ref}}$  in the above equations, we still get a refracted plane wave at  $\phi_{ref}$  but on the other side of the normal vector to  $MS2$ . These external tangential fields will then need to be considered in conjunction with the incident fields to make sure the total power conservation constraint holds. This requires scaling the magnitude of the external tangential fields. The incident fields and the desired external fields are then used to calculate the internal tangential fields for the cascaded metasurface system using the design algorithm with multiple reflections developed in Chapter 3. The optimization algorithm was terminated after 4000 iterations and the final normalized error was about  $7.485 \times 10^{-4}$ . The normalized input and output real power normal to each unit cell of metasurfaces  $MS1$  and  $MS2$  can be seen in Figure 5.1 and the normalized cost functional value at every iteration of the CG algorithm is shown in Figure 5.2. The local power conservation constraint at each unit cell was successfully achieved as can be seen by the input power and the output power is almost equal at every unit cell in Figure 5.1. The reduction of the error at every iteration and the meeting of the LPC constraint verify the working of the design algorithm.

With the convergence of the design algorithm, we can design our unit cells based on input and output tangential electromagnetic fields. Thus, to confirm that the electromagnetic transformation was successfully achieved, we then model the cascaded metasurface system with impedance specifications for the three-layered unit cells in ANSYS® HFSS™ software to perform full-wave simulation. The impedance values for the unit cells were calculated using the fields obtained above and the methodology

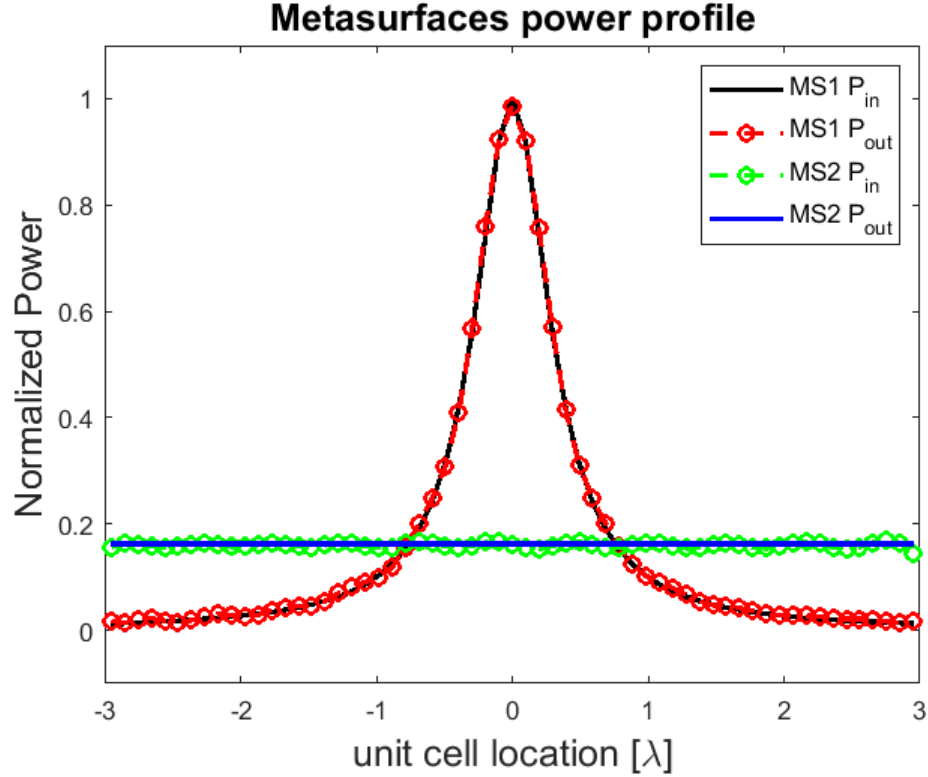


Figure 5.1: Normalized power profile for the cascaded metasurface system designed to achieve cylindrical to refracted truncated plane wave transformation. The horizontal axis corresponds to the locations of the unit cells, and the vertical axis corresponds to the normal real power entering/leaving each unit cell.

discussed in Chapter 3. The specification of the impedance values for the three-layered unit cells in the modeling phase of the metasurfaces was restricted to purely capacitive or inductive values. This was done to have no gain or loss in the system for practical reasons.

For this cascaded metasurface system, we used Rogers RO3010 as the material for the substrates whose thickness was specified to be 0.13 mm and the relative permittivity being 10.2 at 10 GHz. The bondply (Rogers 2929) thickness was specified to be 0.0762 mm and its relative permittivity was specified to be 2.94 at 10 GHz. The



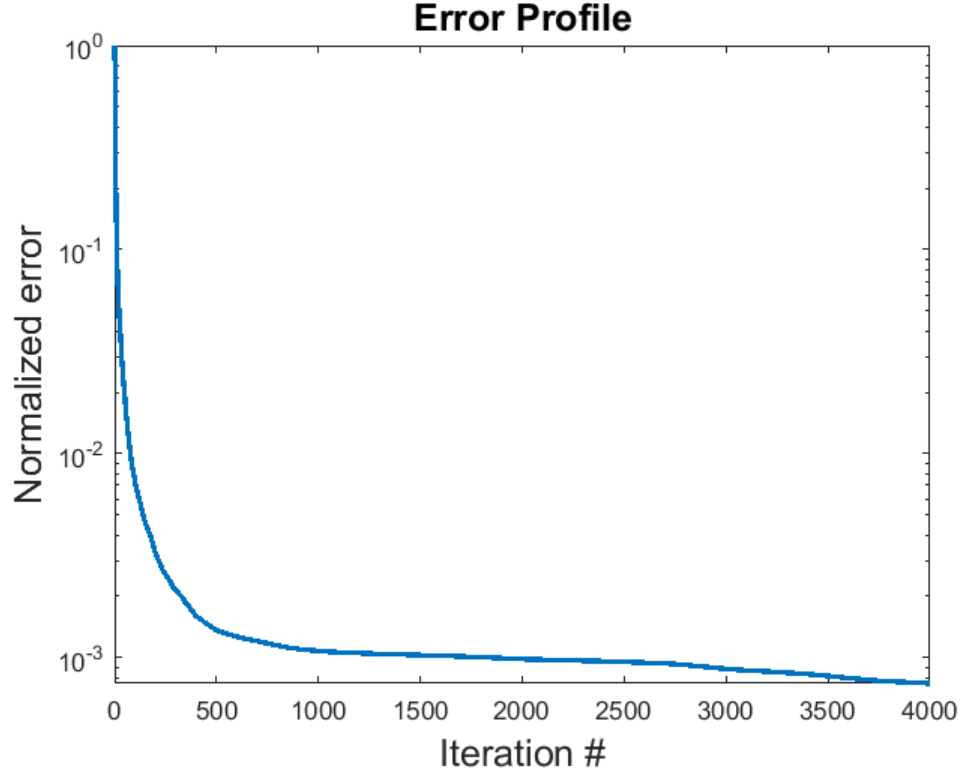


Figure 5.2: Normalized cost functional error for the cascaded metasurface system designed to achieve cylindrical to refracted truncated plane wave transformation.

theoretical model of this cascaded metasurface system modeled in ANSYS® HFSS™ software can be seen in Figure 5.3. The full-wave simulation successfully converged at the specified termination criterion of Ansys HFSS, which was set to  $\Delta S = 0.01$ . The near-field plot of the electric fields for the electromagnetic transformation can be seen in Figure 5.4. It can visually be seen in Figure 5.4 that the cylindrical waves on the left side of the metasurface  $MS1$  are clearly transformed to plane waves that are refracted on the right side of metasurface  $MS2$ . The multiple reflections are also clearly visible within the cavity (i.e., the space between the two metasurfaces) with a hot spot on the top half section of the cavity. This is due to the significant change in the power profile between that of a line source and that of a plane wave. That is, the

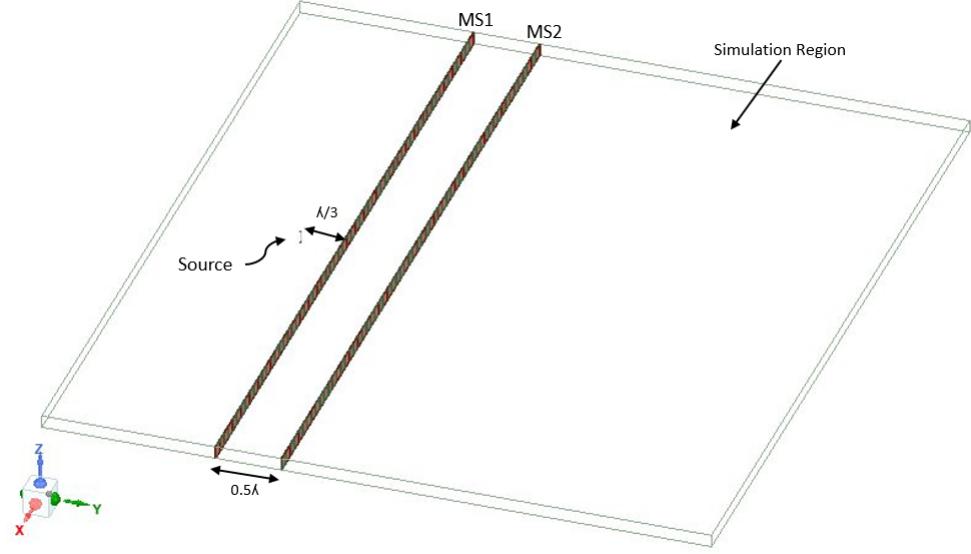


Figure 5.3: Model of the cascaded metasurface system designed for cylindrical to  $60^\circ$  refracted plane wave transformation.

multiple reflections in the cavity of the system redistribute the power to make this aggressive field transformation possible. The confirmation of the refraction being  $60^\circ$  is shown in the far-field normalized power pattern in Figure 5.5. The main lobe is at  $59^\circ$ . That is, the main lobe is offset by  $1^\circ$  from the desired angle of refraction of  $60^\circ$ . This offset is because the theoretical tangential electric and magnetic fields calculated using (5.1) are based on the assumption that the metasurfaces are infinite while in our case the sizes of the metasurfaces in the design are finite, i.e.,  $6\lambda_0$  [16, 17].

Finally, 95.838% of the total input power was transferred to the output, while 0.005% and 1.372% leaked from the top and the bottom openings of the cascaded system respectively. The remaining was lost due to the loss tangent of the dielectric substrates and some small reflections at the input face of  $MS1$ . This shows a successful theoretical lossless and passive design. (Losslessness and passivity are from the point of view of having purely reactive impedance layers.)

From the successful transformation achieved in this example, it can be concluded

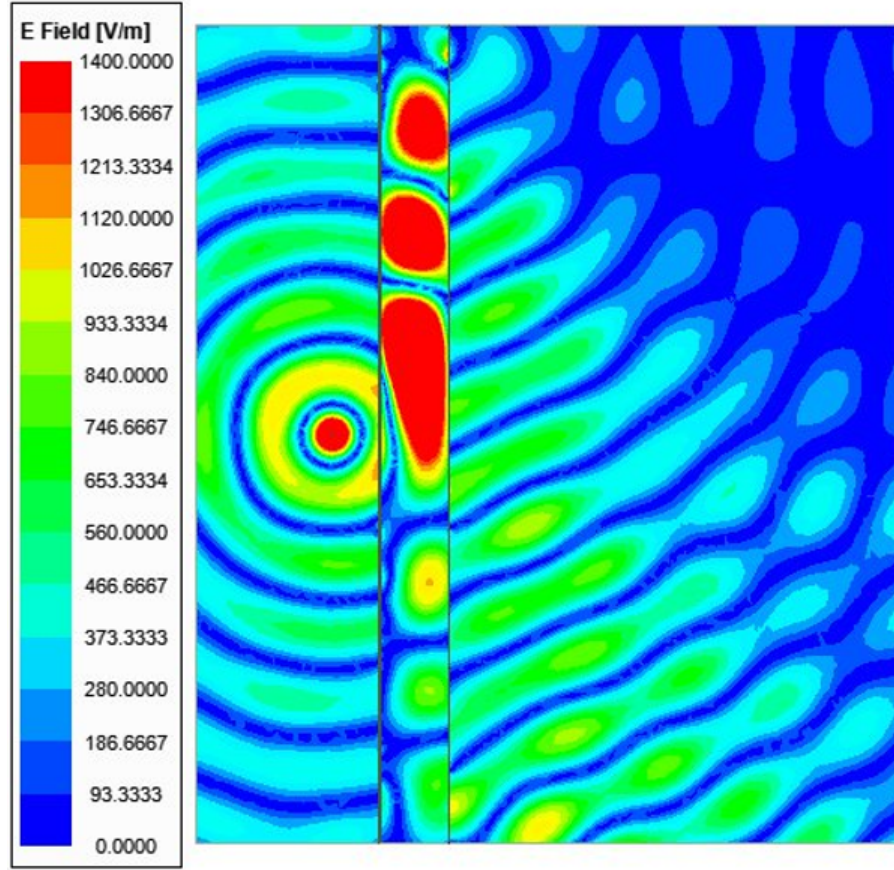


Figure 5.4: Near field simulation results for the cylindrical to  $60^\circ$  refracted plane wave transformation using the cascaded metasurface system. The achieved refracted angle was  $59^\circ$ .

that the design algorithm developed in Chapter 3 can be used to develop a passive, lossless omega-bianisotropic theoretical model of a cascaded metasurface system in an automated fashion.

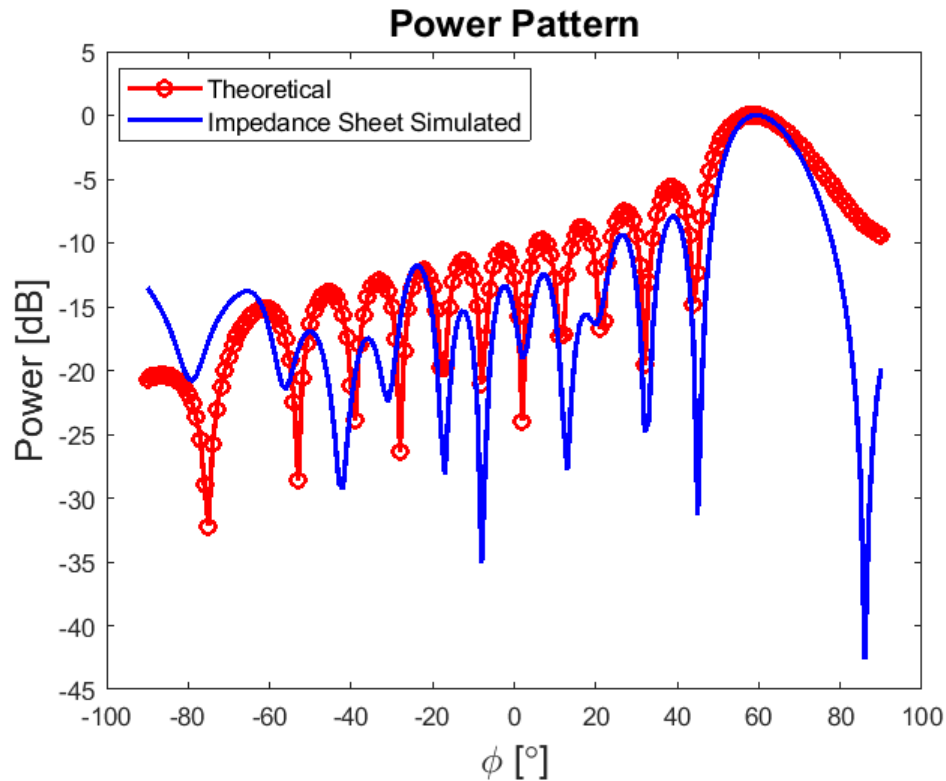


Figure 5.5: Normalized far-field power pattern obtained from Ansys HFSS for the line source to refracted plane wave transformation.

## 5.2 Synthesizing a Desired Power Pattern

In Section 5.1, we showed the algorithm developed in Chapter 3 is capable of performing electromagnetic wave transformation. Herein, we show the promise of this approach to synthesize a desired power pattern [52].<sup>1</sup>

As mentioned earlier, the desired output tangential electromagnetic fields are generally not known. Instead, desired far-field performance criteria such as beam directions, sidelobe levels (SLL), and half-power beamwidth (HPBW) are normally

<sup>1</sup>This work, and the work presented in the previous section, was part of the author's presentation at the IEEE APS conference in Denver, Colorado, USA, 2022.

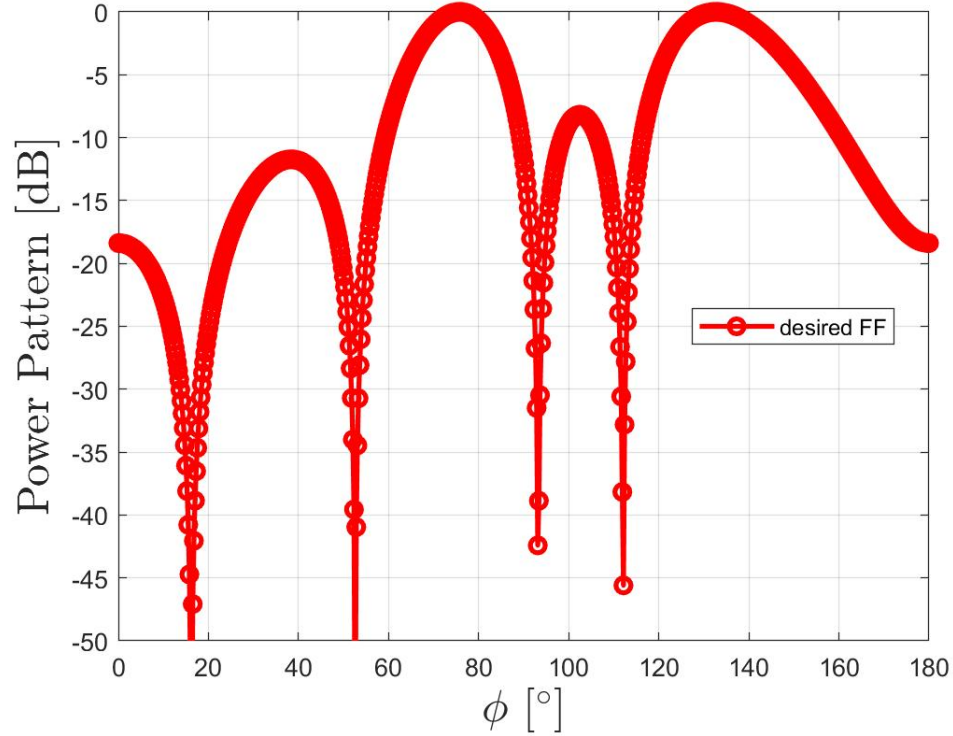


Figure 5.6: Desired far-field power pattern showing two main beams at  $75^\circ$  and  $130^\circ$ .

provided. Therefore, to know the output tangential electromagnetic fields required to design a cascaded metasurface, inversion algorithms mentioned in Chapter 3 are normally used. Once the output tangential electromagnetic fields are found, the design of the cascaded metasurface system can commence. For instance, in this example, the goal is to achieve the far-field power pattern shown in Figure 5.6.

In the power pattern shown in Figure 5.6 there are two main beams at  $75^\circ$  and  $130^\circ$  at an operating frequency of 10 GHz. To achieve this desired power pattern, we design a cascaded metasurface system with a separation distance of  $0.5\lambda_0$ , and the sizes of the metasurfaces  $MS1$  and  $MS2$  being  $6\lambda_0$ . There are 61 unit cells on both metasurfaces. The thickness of the substrate and the bondply for the three-layered unit cell were 0.13 mm and 0.0762 mm, respectively, which are the same as

the previous example.

The design process of the cascaded metasurface system for this example is similar to the cascaded metasurface system design in Section 5.1 once the tangential fields are known from the desired far-field power pattern. The output tangential electromagnetic fields were calculated from the desired power pattern shown in Figure 5.6 using the electromagnetic inversion algorithm [26, 43]<sup>2</sup>. The incident (input) electromagnetic waves were assumed to be cylindrical waves emanating from a line source and therefore the input tangential electric and magnetic fields were calculated using the zeroth order Hankel function of the second kind and the first order Hankel function of the second kind similar to the example shown in Section 5.1. With the desired tangential fields obtained from the inversion algorithms and the known incident fields, we ensure that the total power conservation constraint holds. Consequently, the internal tangential fields for the cascaded metasurface system were obtained using the algorithm developed in Chapter 3. The optimization algorithm successfully converged and finally, the normalized cost functional error was about 0.0015 when the optimization algorithm was terminated after 5000 iterations. The normalized power profile for each unit cell is shown in Figure 5.7 while the normalized cost functional error profile at every iteration is shown in Figure 5.8. The LPC constraint was successfully achieved at each unit cell as shown in Figure 5.7. Therefore, we then proceed to model the cascaded metasurface system to verify the electromagnetic transformation in ANSYS® HFSS<sup>TM</sup> software using purely reactive impedance specifications for the three-layered unit cell. The near-field plot of the electric fields for the system is shown in Figure 5.9. It can clearly be seen that the cylindrical wave on the input side is transformed into two truncated plane waves propagating in two different directions. The far-field power pattern obtained from ANSYS® HFSS<sup>TM</sup> software is shown and compared with the desired power pattern in Figure 5.10.

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<sup>2</sup>The author acknowledges Mr. Tianke Qiu for performing the inversion.

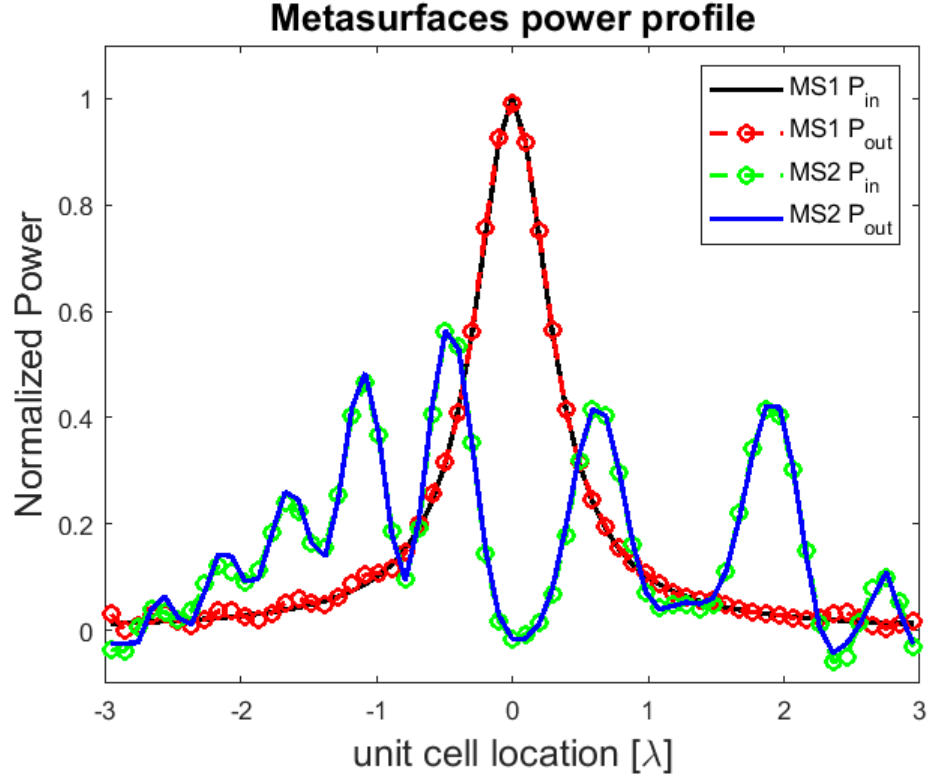


Figure 5.7: Normalized real power entering/leaving each unit cell in the normal direction for the cascaded metasurface system design concerning meeting a desired power pattern with two main beams.

In Figure 5.10, it can be seen the obtained power pattern matches the desired power pattern on the two main beams. However, the side lobes and the nulls are not matched except in the region between the two beams. In general, obtaining an exact side lobe is quite difficult and requires tailoring the magnitude of the aperture fields quite accurately. This can be achieved by *ad hoc* or further tuning of the metasurfaces which have not been pursued here. In addition, most designers care about the side lobe levels and not the actual side lobe geometry. For example, if the designer would like to have a side lobe level that is less than  $-10$  dB for  $\phi \leq 60^\circ$ , the cascaded metasurface system has achieved this specification although it has not met the actual

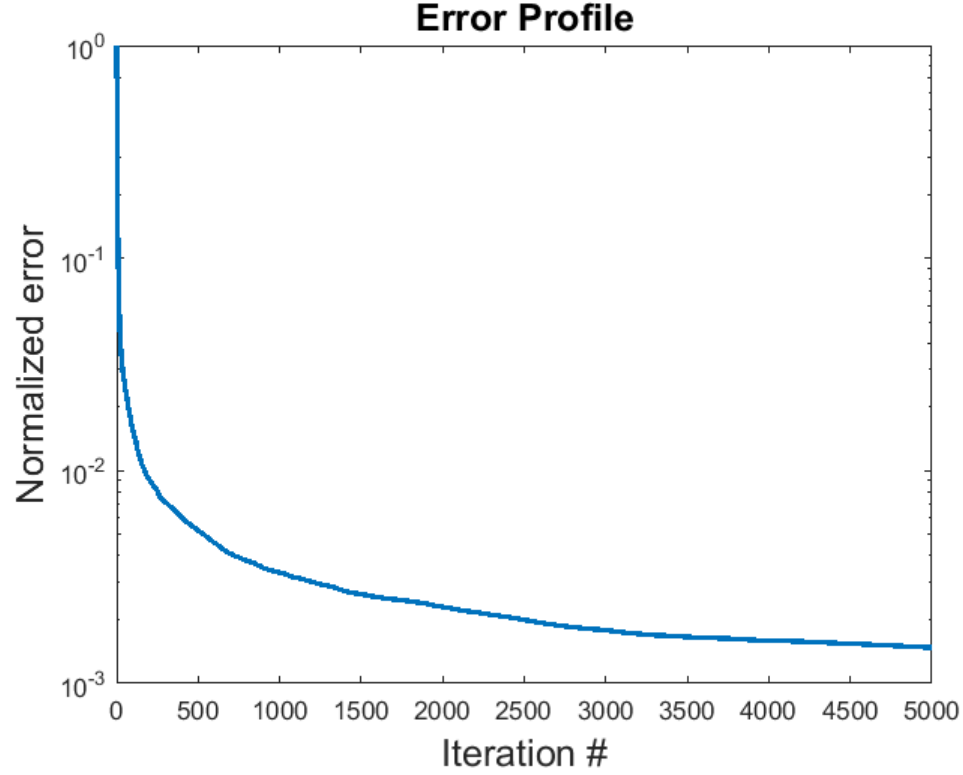


Figure 5.8: Normalized cost functional error profile for the cascaded metasurface system design concerning meeting a desired power pattern with two main beams.

desired side lobe.

In this example, 97.914% of total input real power was transferred to the output, while 0.518% and 0.326% leaked from the top and the bottom openings between the two metasurfaces respectively. The remainder of the power loss is due to the loss tangent of the substrates and the reflection from the input face of the metasurface *MS1*.

With about 98% power transmission efficiency and a reasonable matching of the desired power pattern, it can be concluded that the algorithm developed in Chapter 3 can be used to synthesize desired power patterns using a passive, lossless, and omega-bianisotropic cascaded metasurface system.



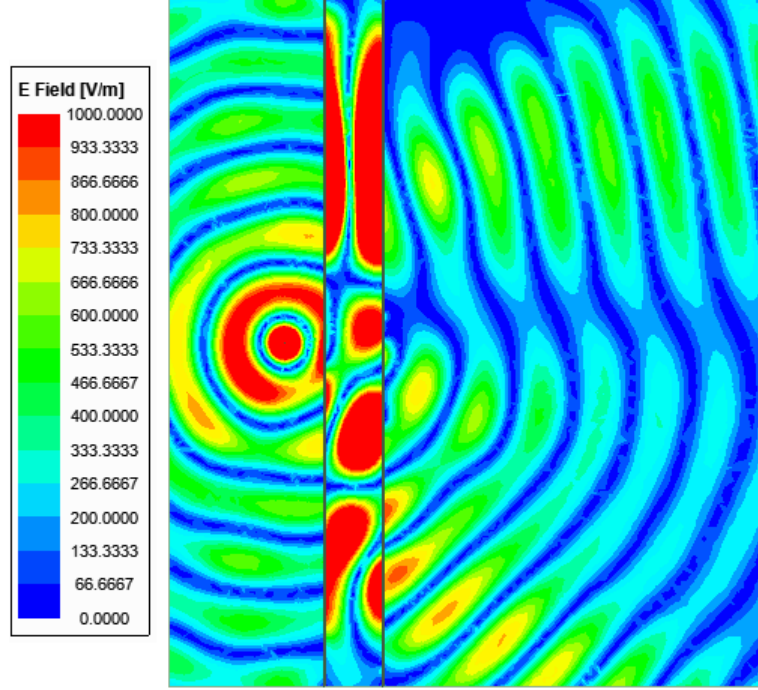


Figure 5.9: Near-field simulation by Ansys HFSS for the example concerning the synthesis of a desired far-field power pattern.

### 5.3 Cylindrical to Plane Wave Transformation with Copper Trace Design

In Sections 5.1 and 5.2, we showed full-wave simulation results of two cascaded metasurface systems modeled using impedance specifications for their three-layered unit cells achieving the desired electromagnetic wave transformations. In both examples, more than 90% transmission power efficiency was obtained. The next step is to realize a copper trace design for the unit cell so that we can physically fabricate it using the printed circuit board (PCB) technology. Therefore, herein, we show full-wave simulation results for the transformation of cylindrical waves to truncated plane waves using a cascaded metasurface system designed with its three-layered unit cells being

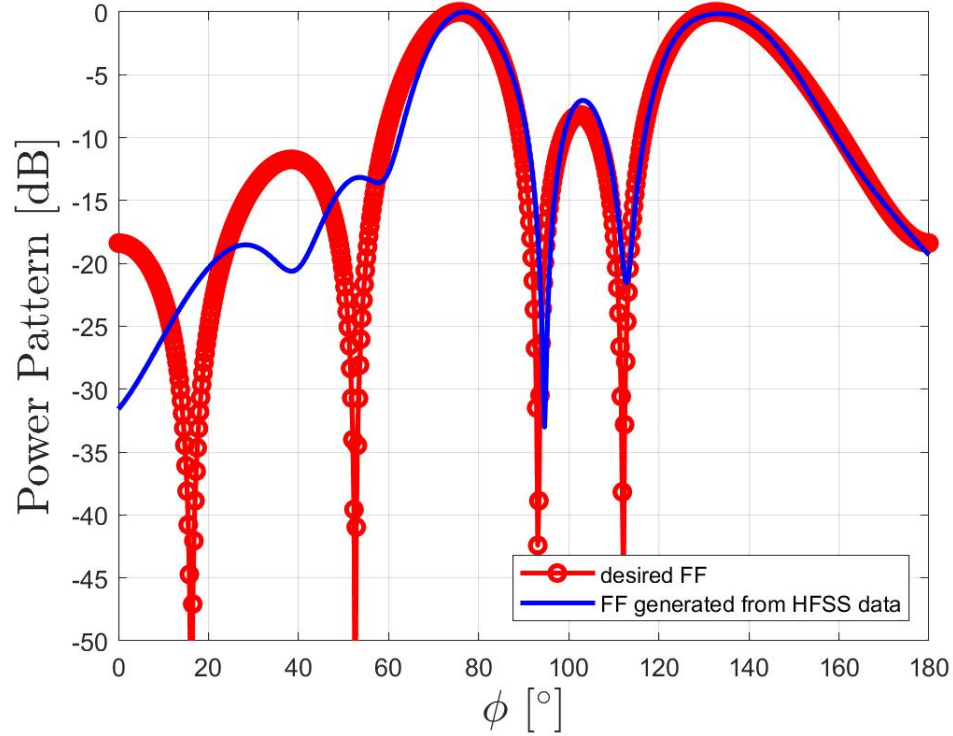


Figure 5.10: Radiation power pattern comparison

specified using copper traces in the shape of a dogbone.<sup>3</sup> The design methodology for obtaining the dimension of the copper traces is discussed in Section 3.4.2.<sup>4</sup>

The cascaded metasurface system was designed at the operating frequency of 10.5 GHz. This is because the lookup library developed by Max Kelly was at 10.5 GHz. Furthermore, since the utilized lookup table was based on the forward scattering parameter  $S_{21}$ , the separation distance between metasurface  $MS1$  and  $MS2$  was set to  $2\lambda_0$  with the hope that multiple reflections between the two metasurfaces are not going to be needed for power redistribution. Increasing the distance to  $2\lambda_0$

<sup>3</sup>This work was part of the author's presentation at the URSI/EMTS 2023 conference in Vancouver, British Columbia, Canada, 2023 [57].

<sup>4</sup>The author acknowledges Mr. Max Kelly for assisting with the copper trace design for the unit cell layers of the cascaded metasurface system designed in this section.

may enable the metasurface system to merely use the separation to redistribute the power [10]. This favors both metasurfaces to be fully transmissive. Each metasurface was designed to have 35 unit cells with their dimensions ( $D$ ) being 5.1 mm. Rogers RO3010 was used as the substrate with a thickness of 1.28 mm and a Rogers bondply 2929 was used to bond the top and bottom substrates. The thickness of Rogers bondply 2929 was 0.0762 mm. The line source generating the incident cylindrical wave was set up similarly as in Section 5.1.

In the cascaded metasurface system's optimization algorithm, the incident tangential electric and magnetic fields were calculated using the zeroth order Hankel function of the second kind and the first order Hankel function of the second kind similar to the example shown in Section 5.1. With the desired tangential fields obtained from the plane wave equations and the known incident fields, we ensure that the total power conservation constraint holds. Consequently, the internal tangential fields for the cascaded metasurface system were obtained using the algorithm developed in Chapter 3. The optimization algorithm successfully converged and finally, the normalized cost functional error was about 0.0209 when the optimization algorithm was terminated after 2500 iterations. (Note that we are not utilizing multiple reflections between the two metasurfaces in this example; thus, it is more difficult to reduce the cost functional error.) The normalized power entering and leaving each unit cell is shown in Figure 5.11 while the normalized cost functional error profile at every iteration is shown in Figure 5.12. A fairly successful LPC constraint was achieved at each unit cell as shown in Figure 5.11. We then proceed to model the cascaded metasurface system to verify the electromagnetic transformation in ANSYS® HFSS™ software using purely reactive impedance specifications for the three-layered unit cell. The near-field plot of the electric fields for the system is shown in Figure 5.13. It can clearly be seen that the cylindrical wave on the input side is transformed into truncated plane waves normally propagating away from  $MS2$ . An output real power

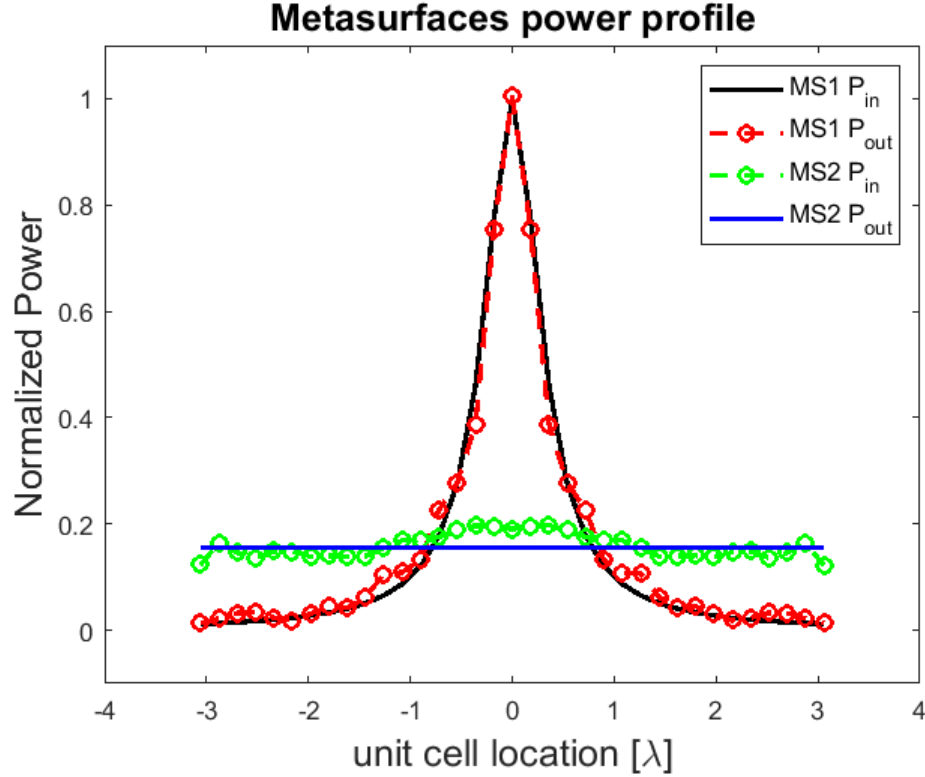


Figure 5.11: Normalized real power entering/leaving each unit cell in the normal direction for the cascaded metasurface system design concerning cylindrical to plane wave transformation to be used for copper trace design.

transmission efficiency of 46.831% was achieved for this designed cascaded metasurface system. About 5.06% and 4.95% of power escaped from the top and bottom openings between the two metasurfaces respectively. The remainder of the power loss is due to the loss tangent of the substrates and the reflection from the input face of the metasurface *MS1*. One of the reasons for high reflections is speculated to be the placement of the line source and the precise separation of metasurfaces. This is due to the fact that the design algorithm assumes that the thickness of the metasurfaces is zero. Once we have created a metasurface with dielectric substrates of appreciable

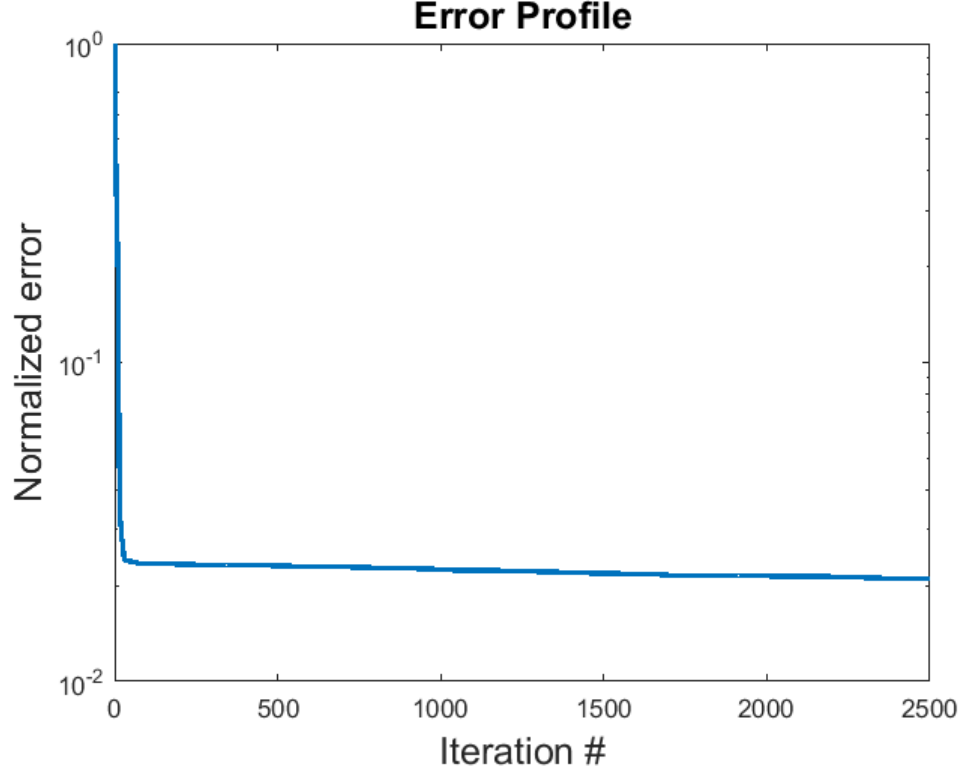


Figure 5.12: Normalized cost functional error profile for the cascaded metasurface system design concerning cylindrical to plane wave transformation to be used for copper trace design.

thickness, there will be ambiguities in how to choose the distances.<sup>5</sup> Note that due to the specifications of the lookup table, the thickness of the metasurface is about 10 times more than that in the previous two examples. Finally, it should be noted that as mentioned in [13, 45], if the phase of the forward scattering parameter is close to zero, increased loss due to resonance can occur.

Next, we obtained the copper traces dimensions for the unit cells using the method-

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<sup>5</sup>In our model, the distance  $(\lambda_0/3)$  between the line source and *MS1* is from the input face of *MS1*. In addition, the separation between *MS1* and *MS2* is from the output face of *MS1* to the input face of *MS2*.

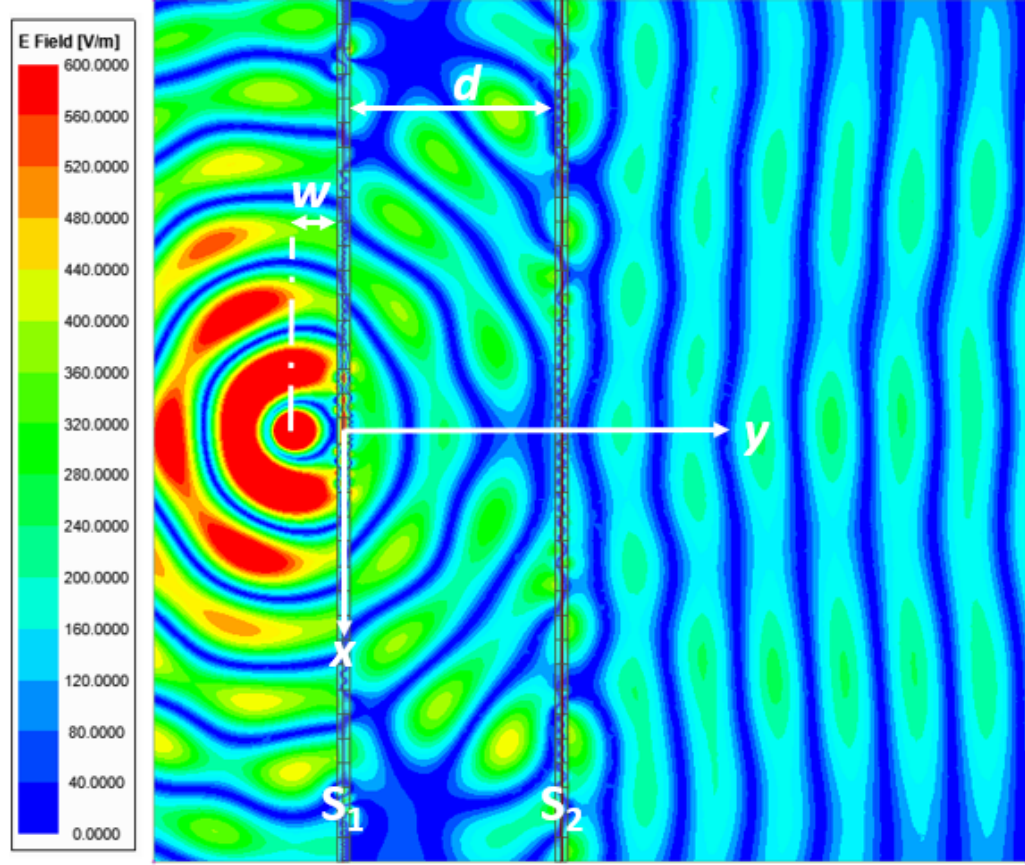


Figure 5.13: Near-field simulation by Ansys HFSS for the example concerning cylindrical to plane wave transformation without multiple reflections using impedance specification for the three-layered unit cells. This is to be used for the copper trace design.

ology discussed in Section 3.4.2 and the dimensions of the copper traces for each unit cell in metasurfaces  $MS1$  and  $MS2$  are shown in Tables 5.1 and 5.2. In these tables, the subscripts “ach” and “des” denote the achieved and desired values respectively. “Top”, “Mid” and “Bot” refer to the top, middle, and bottom layers respectively where the top layer is in the input face.

In Tables 5.1 and 5.2, column 1 indicates the unit cell number, columns 2 through 4 indicate the size  $W$  of the top, middle, and bottom copper traces, columns 5 through

Table 5.1: Metasurface *MS1* Copper Trace dimensions and the corresponding Value of  $S_{21}$  for each unit cell.

| Cell | $W_{Top}$ | $W_{Mid}$ | $W_{Bot}$ | $L_{Top}$ | $L_{Mid}$ | $L_{Bot}$ | $ s_{21} _{ach}$ | $\angle s_{21}_{ach}$ | $ s_{21} _{des}$ | $\angle s_{21}_{des}$ |
|------|-----------|-----------|-----------|-----------|-----------|-----------|------------------|-----------------------|------------------|-----------------------|
| #    | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      |                  | [°]                   |                  | [°]                   |
| 1    | 4.8       | 0.5       | 2.7       | 1.5       | 4.4       | 2.7       | 0.59             | 98.48                 | 0.55             | 98.84                 |
| 2    | 0.8       | 0.8       | 2.8       | 4.4       | 4.0       | 2.7       | 0.80             | 47.42                 | 0.77             | 48.37                 |
| 3    | 2.9       | 0.8       | 2.8       | 4.6       | 2.1       | 4.7       | 0.86             | 10.04                 | 0.86             | 9.95                  |
| 4    | 1.0       | 2.9       | 5.0       | 1.7       | 4.8       | 4.8       | 0.72             | 311.19                | 0.80             | 307.72                |
| 5    | 2.4       | 2.3       | 1.5       | 2.5       | 3.0       | 1.9       | 0.69             | 272.53                | 0.69             | 272.33                |
| 6    | 2.5       | 2.5       | 1.6       | 2.8       | 3.1       | 2.3       | 0.65             | 235.97                | 0.65             | 235.76                |
| 7    | 1.8       | 3.3       | 2.3       | 2.4       | 4.0       | 3.1       | 0.67             | 200.61                | 0.67             | 200.64                |
| 8    | 2.5       | 4.6       | 2.1       | 2.6       | 4.5       | 3.0       | 0.66             | 160.95                | 0.75             | 154.67                |
| 9    | 2.0       | 1.7       | 0.6       | 3.1       | 2.8       | 4.4       | 0.56             | 128.07                | 0.61             | 126.05                |
| 10   | 1.9       | 2.1       | 0.6       | 3.2       | 2.7       | 4.3       | 0.73             | 114.44                | 0.73             | 115.05                |
| 11   | 4.9       | 0.5       | 2.7       | 1.7       | 4.5       | 2.4       | 0.74             | 92.96                 | 0.74             | 92.23                 |
| 12   | 4.4       | 0.7       | 2.4       | 1.8       | 4.3       | 2.7       | 0.80             | 73.88                 | 0.81             | 72.97                 |
| 13   | 0.7       | 0.7       | 2.9       | 4.4       | 4.2       | 2.5       | 0.80             | 61.41                 | 0.89             | 61.09                 |
| 14   | 0.8       | 0.8       | 2.8       | 4.4       | 4.0       | 2.7       | 0.80             | 47.42                 | 0.93             | 46.59                 |
| 15   | 2.5       | 0.9       | 2.9       | 4.8       | 1.8       | 4.7       | 0.80             | 17.90                 | 0.99             | 24.57                 |
| 16   | 3.1       | 0.6       | 2.7       | 4.7       | 1.6       | 4.8       | 0.84             | 13.20                 | 0.98             | 17.42                 |
| 17   | 2.8       | 0.6       | 3.4       | 4.8       | 1.4       | 4.6       | 0.89             | 5.60                  | 1.00             | 8.98                  |
| 18   | 3.4       | 1.0       | 3.2       | 4.6       | 1.4       | 4.7       | 0.95             | 353.89                | 0.99             | 354.64                |
| 19   | 2.8       | 0.6       | 3.4       | 4.8       | 1.4       | 4.6       | 0.89             | 5.60                  | 1.00             | 9.16                  |
| 20   | 3.1       | 0.6       | 2.7       | 4.7       | 1.6       | 4.8       | 0.84             | 13.20                 | 0.98             | 17.73                 |
| 21   | 2.5       | 0.9       | 2.9       | 4.8       | 1.8       | 4.7       | 0.80             | 17.90                 | 0.99             | 24.88                 |
| 22   | 0.8       | 0.8       | 2.8       | 4.4       | 4.0       | 2.7       | 0.80             | 47.42                 | 0.93             | 47.00                 |
| 23   | 0.7       | 0.7       | 2.9       | 4.4       | 4.2       | 2.5       | 0.80             | 61.41                 | 0.89             | 61.63                 |
| 24   | 4.4       | 0.7       | 2.4       | 1.8       | 4.3       | 2.7       | 0.80             | 73.88                 | 0.81             | 73.37                 |
| 25   | 4.9       | 0.5       | 2.7       | 1.7       | 4.5       | 2.4       | 0.74             | 92.96                 | 0.74             | 92.60                 |
| 26   | 1.9       | 2.1       | 0.6       | 3.2       | 2.7       | 4.3       | 0.73             | 114.44                | 0.73             | 115.71                |
| 27   | 2.0       | 1.7       | 0.6       | 3.1       | 2.8       | 4.4       | 0.56             | 128.07                | 0.61             | 126.54                |
| 28   | 2.5       | 4.6       | 2.1       | 2.6       | 4.5       | 3.0       | 0.66             | 160.95                | 0.75             | 154.93                |
| 29   | 1.8       | 3.3       | 2.3       | 2.4       | 4.0       | 3.1       | 0.67             | 200.61                | 0.68             | 200.95                |
| 30   | 2.5       | 2.5       | 1.6       | 2.8       | 3.1       | 2.3       | 0.65             | 235.97                | 0.65             | 236.39                |
| 31   | 2.4       | 2.3       | 1.5       | 2.5       | 3.0       | 1.9       | 0.69             | 272.53                | 0.69             | 273.02                |
| 32   | 1.0       | 2.9       | 5.0       | 1.7       | 4.8       | 4.8       | 0.72             | 311.19                | 0.80             | 308.21                |
| 33   | 2.9       | 0.8       | 2.8       | 4.6       | 2.1       | 4.7       | 0.86             | 10.04                 | 0.86             | 10.97                 |
| 34   | 0.8       | 0.8       | 2.8       | 4.4       | 4.0       | 2.7       | 0.80             | 47.42                 | 0.77             | 50.23                 |
| 35   | 4.8       | 0.5       | 2.7       | 1.5       | 4.4       | 2.7       | 0.59             | 98.48                 | 0.54             | 100.87                |

Table 5.2: Metasurface *MS2* Copper Trace dimensions and the corresponding Value of  $S_{21}$  for each unit cell.

| Cell | $W_{Top}$ | $W_{Mid}$ | $W_{Bot}$ | $L_{Top}$ | $L_{Mid}$ | $L_{Bot}$ | $ s_{21} _{ach}$ | $\angle s_{21}ach$ | $ s_{21} _{des}$ | $\angle s_{21}des$ |
|------|-----------|-----------|-----------|-----------|-----------|-----------|------------------|--------------------|------------------|--------------------|
| #    | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      |                  | [°]                |                  | [°]                |
| 1    | 2.1       | 2.6       | 2.0       | 2.5       | 2.7       | 2.4       | 0.95             | 260.90             | 0.99             | 261.09             |
| 2    | 2.4       | 2.6       | 2.2       | 2.4       | 2.8       | 2.5       | 0.96             | 235.83             | 0.99             | 235.53             |
| 3    | 2.4       | 1.6       | 2.0       | 2.6       | 2.2       | 2.7       | 0.97             | 201.84             | 0.99             | 200.93             |
| 4    | 2.2       | 2.0       | 0.9       | 2.9       | 2.6       | 3.6       | 0.89             | 156.15             | 0.98             | 154.51             |
| 5    | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86             | 0.98             | 111.43             |
| 6    | 4.4       | 0.9       | 2.5       | 1.8       | 4.0       | 2.7       | 0.87             | 80.64              | 0.99             | 70.80              |
| 7    | 0.7       | 0.9       | 3.0       | 4.6       | 3.9       | 2.6       | 0.76             | 32.49              | 1.00             | 27.55              |
| 8    | 4.8       | 0.5       | 4.2       | 4.8       | 1.5       | 3.0       | 0.91             | 345.82             | 0.99             | 346.48             |
| 9    | 1.2       | 2.4       | 0.5       | 1.6       | 3.4       | 1.4       | 0.97             | 305.84             | 1.00             | 305.91             |
| 10   | 2.0       | 2.4       | 2.1       | 2.4       | 3.0       | 2.3       | 0.95             | 266.38             | 1.00             | 266.15             |
| 11   | 2.1       | 2.6       | 2.5       | 2.6       | 3.4       | 2.4       | 0.97             | 228.41             | 1.00             | 229.15             |
| 12   | 2.5       | 3.7       | 2.4       | 2.5       | 3.2       | 2.5       | 0.98             | 209.21             | 0.98             | 209.10             |
| 13   | 2.4       | 2.3       | 1.0       | 2.6       | 2.4       | 3.8       | 0.96             | 165.83             | 1.00             | 168.50             |
| 14   | 2.3       | 2.4       | 1.1       | 2.7       | 2.4       | 3.7       | 0.92             | 142.71             | 1.00             | 141.68             |
| 15   | 1.7       | 2.1       | 0.6       | 3.2       | 2.7       | 4.3       | 0.92             | 125.33             | 0.99             | 119.95             |
| 16   | 1.8       | 2.1       | 0.8       | 3.2       | 2.7       | 4.2       | 0.88             | 105.42             | 0.99             | 106.03             |
| 17   | 4.9       | 0.6       | 2.8       | 1.6       | 4.4       | 2.4       | 0.87             | 93.18              | 0.99             | 98.88              |
| 18   | 4.9       | 0.6       | 2.8       | 1.6       | 4.4       | 2.5       | 0.89             | 96.43              | 0.99             | 96.64              |
| 19   | 4.9       | 0.6       | 2.8       | 1.6       | 4.4       | 2.4       | 0.87             | 93.18              | 0.99             | 98.75              |
| 20   | 1.8       | 2.1       | 0.8       | 3.2       | 2.7       | 4.2       | 0.88             | 105.42             | 0.99             | 105.84             |
| 21   | 1.7       | 2.1       | 0.6       | 3.2       | 2.7       | 4.3       | 0.92             | 125.33             | 0.99             | 119.81             |
| 22   | 2.3       | 2.4       | 1.1       | 2.7       | 2.4       | 3.7       | 0.92             | 142.71             | 1.00             | 141.62             |
| 23   | 2.4       | 2.3       | 1.0       | 2.6       | 2.4       | 3.8       | 0.96             | 165.83             | 1.00             | 168.43             |
| 24   | 2.5       | 3.7       | 2.4       | 2.5       | 3.2       | 2.5       | 0.98             | 209.21             | 0.97             | 211.15             |
| 25   | 2.1       | 2.6       | 2.5       | 2.6       | 3.4       | 2.4       | 0.97             | 228.41             | 1.00             | 228.94             |
| 26   | 2.0       | 2.4       | 2.1       | 2.4       | 3.0       | 2.3       | 0.95             | 266.38             | 1.00             | 266.00             |
| 27   | 1.2       | 2.4       | 0.5       | 1.6       | 3.4       | 1.4       | 0.97             | 305.84             | 1.00             | 305.75             |
| 28   | 4.8       | 0.5       | 4.2       | 4.8       | 1.5       | 3.0       | 0.91             | 345.82             | 0.99             | 345.78             |
| 29   | 0.7       | 0.9       | 3.0       | 4.6       | 3.9       | 2.6       | 0.76             | 32.49              | 1.00             | 27.04              |
| 30   | 4.4       | 0.9       | 2.5       | 1.8       | 4.0       | 2.7       | 0.87             | 80.64              | 0.99             | 70.46              |
| 31   | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86             | 0.98             | 111.17             |
| 32   | 2.2       | 2.0       | 0.9       | 2.9       | 2.6       | 3.6       | 0.89             | 156.15             | 0.98             | 154.08             |
| 33   | 2.4       | 1.6       | 2.0       | 2.6       | 2.2       | 2.7       | 0.97             | 201.84             | 0.99             | 200.42             |
| 34   | 2.4       | 2.6       | 2.2       | 2.4       | 2.8       | 2.5       | 0.96             | 235.83             | 0.99             | 234.78             |
| 35   | 2.1       | 2.6       | 2.0       | 2.5       | 2.7       | 2.4       | 0.95             | 260.90             | 0.99             | 260.45             |



7 indicate the size  $L$  of the top, middle and bottom copper traces, columns 8 and 9 indicate the achieved  $S_{21}$  scattering parameter magnitude and angle for the unit and lastly columns 10 and 11 indicate the desired  $S_{21}$  scattering parameter magnitude and angle for the unit cell.

To model the cascaded metasurface system with copper trace specifications in ANSYS® HFSS™ software, everything in the modeling of the system remained the same, except the impedance sheet specifications for the unit cells were replaced by copper trace designs with the dimensions specified in Tables 5.1 and 5.2. The simulation result of the electromagnetic transformation with the copper traces is shown in Figure 5.14

We achieved the cylindrical wave to plane wave transformation using copper trace designs for the unit cells with a normal output real power transmission efficiency of 44.15%. The sources of power losses remained the same as with the impedance sheet specification with an additional loss source being the copper traces. Also, we speculate that the transformation is not as successful as that of impedance sheets since the lookup tables containing copper traces assume normal incidence whereas the incident wave impinging on the unit cells is not normal in this example. For the central parts of the cascaded metasurface system, the incident waves are relatively normal to the unit cells; that is why the performance in the central region is much better than the unit cells at the edge of the system. However, it can be speculated that better electromagnetic transformation can be achieved using copper trace designs for the three-layered unit cells if further optimization of unit cells is to be performed. This shows the promise of this approach as an electromagnetic wave transformer.

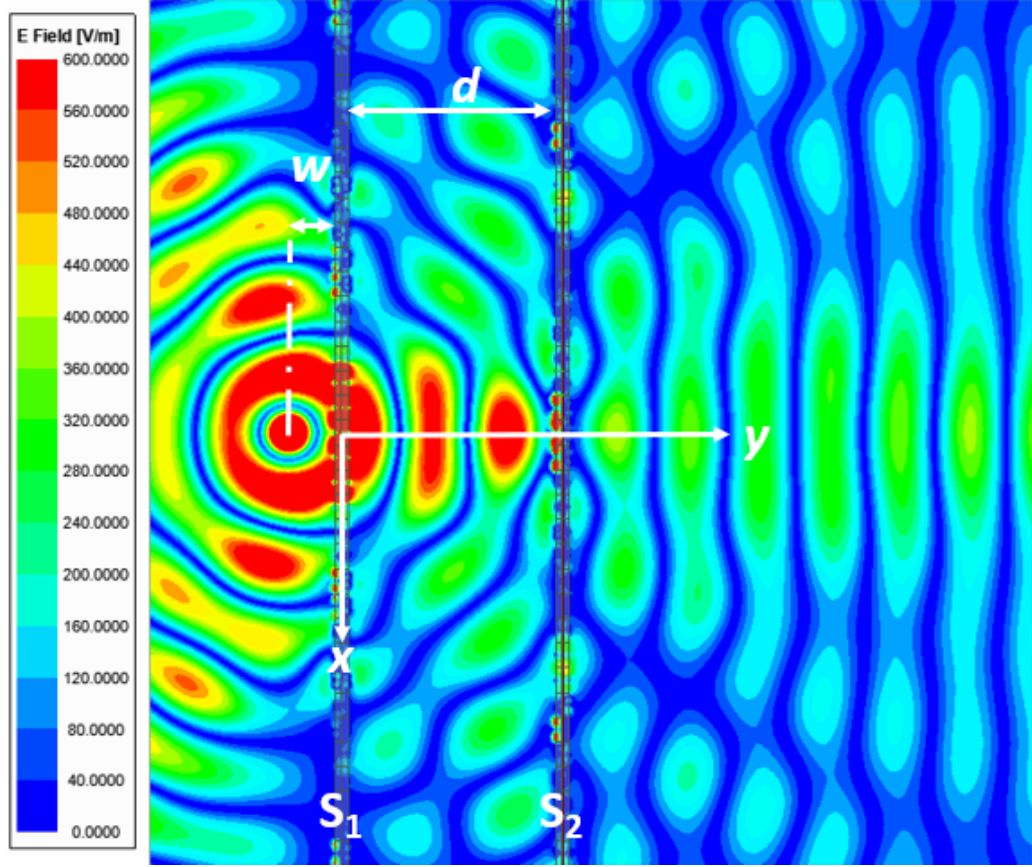


Figure 5.14: Near-field simulation by Ansys HFSS for the example concerning cylindrical to plane wave transformation using copper traces for the three-layered unit cell design.

## 5.4 Design and Fabrication of a Beam Splitter

In Sections 5.1 and 5.2, we demonstrated the successful transformation of electromagnetic waves using the algorithm developed in Chapter 3 with multiple reflections in the cavity and impedance specifications for the three-layered unit cells. In Section 5.3, we investigated the possibility of physically realizing a metasurface using copper traces and demonstrated simulation results of cylindrical wave to plane wave transformation with copper traces for the three-layered unit cells. With the cascaded

metasurface design algorithm and physically realizable unit cell model as discussed in Chapter 3, we have all the tools to fabricate a cascaded metasurface system. Therefore, in this section, we demonstrate the results of a cascaded metasurface system that we fabricated to achieve a beam split transformation.

In this cascaded metasurface system design, we try to achieve a splitting transformation as follows: a normally incident plane wave on  $MS1$  is to be transformed to two plane waves refracted at the angles of  $\pm 30^\circ$  on the output side of  $MS2$ . Hence the name “Beam Splitter”. For this transformation, we design two cascaded metasurface systems; one system excludes multiple reflections within the cavity between  $MS1$  and  $MS2$  and the other system includes multiple reflections within the cavity.

#### 5.4.1 Beam Splitter without Multiple Reflections

In this design, the optimization algorithm does not incorporate multiple reflections in the design process of the cascaded metasurface system.<sup>6</sup> The system was designed at an operating frequency of 10.5 GHz. The size of the metasurfaces  $MS1$  and  $MS2$  was  $4.82\lambda_0$  with 27 unit cells in each metasurface whose size ( $D$ ) was  $\lambda_0/5.6$ , i.e., about 5.1 mm. Rogers RO3010 was used as the substrate with a thickness of 1.28 mm and a Rogers bondply 2929 was used to bond the top and bottom substrates. The thickness of Rogers bondply 2929 was 0.0762 mm. The separation distance  $d$  between the two metasurfaces  $MS1$  and  $MS2$  is  $2\lambda_0$ .

In the cascaded metasurface system’s optimization algorithm, the incident and desired output tangential electric and magnetic fields were calculated using the plane wave equations. With the desired tangential fields and the incident fields, we ensure that the total power conservation constraint holds. Consequently, the internal tan-

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<sup>6</sup>This work was part of the author’s presentation at the URSI/EMTS 2023 conference in Vancouver, British Columbia, Canada, 2023 [57].

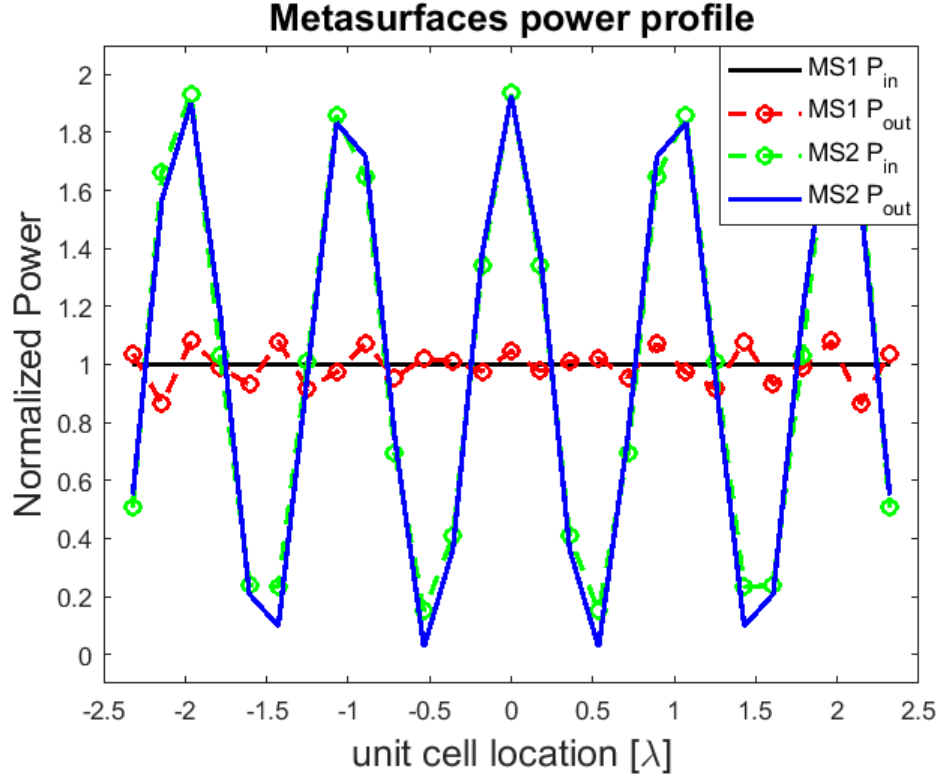


Figure 5.15: Normalized real power entering/leaving each unit cell in the normal direction for the cascaded metasurface system design concerning beam split model.

gential fields for the cascaded metasurface system were obtained using the algorithm developed in Chapter 3. The optimization algorithm successfully converged and finally, the normalized cost functional error was about 0.009 when the optimization algorithm was terminated after 219614 iterations. The normalized power entering and leaving each unit cell is shown in Figure 5.15 while the normalized cost functional error profile at every iteration is shown in Figure 5.16. The LPC constraint was successfully achieved at each unit cell as shown in Figure 5.15. We then proceed to model the cascaded metasurface system to verify the electromagnetic transformation in ANSYS® HFSS<sup>TM</sup> software using purely reactive impedance specifications for the three-layered unit cell. Figure 5.17 shows the near-field simulation results of

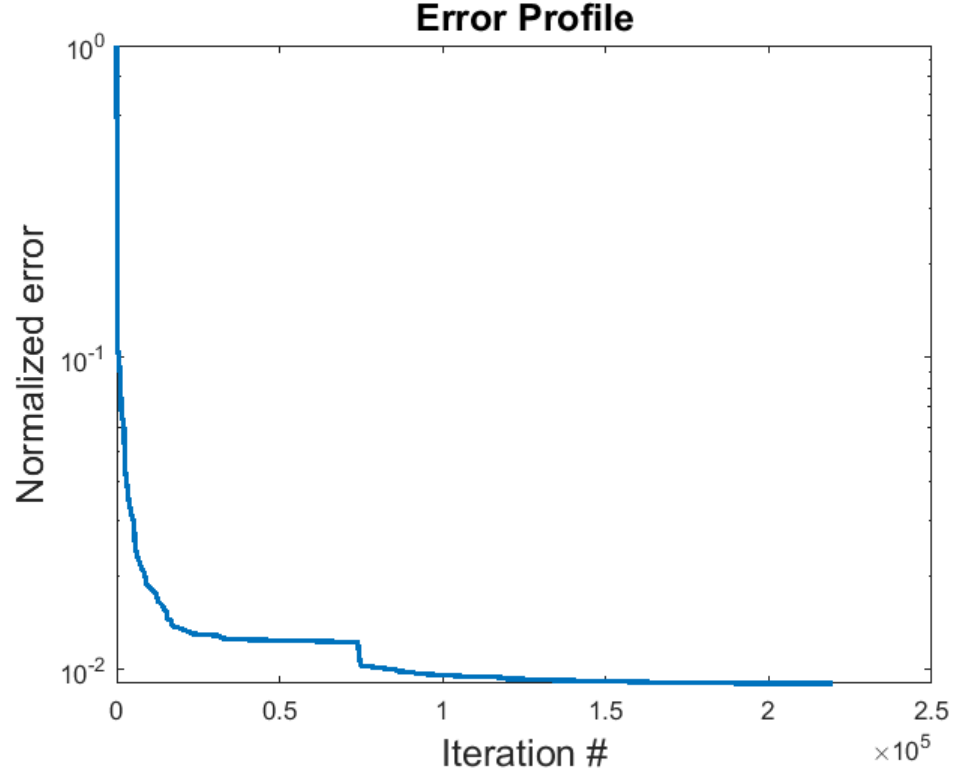


Figure 5.16: Normalized cost functional error profile for the cascaded metasurface system design concerning beam split model.

the transformation obtained in ANSYS® HFSS™ software using impedance specification for the three-layered unit cells. A plane wave source model in ANSYS® HFSS™ software was excited from the left edge of the simulation region.

The symmetric interference pattern can be seen near the output face of metasurface *MS2* and two beams of plane waves can clearly be seen propagating in two directions after a few wavelengths away from the metasurface *MS2* in Figure 5.17 while the input is a single beam plane wave impinging normally to metasurface *MS1*. The transmission power efficiency of the system with the impedance specifications for the three-layered unit cell achieved for this cascaded metasurface system was about 81.053% while the power leaking from the top and bottom opening of the region be-

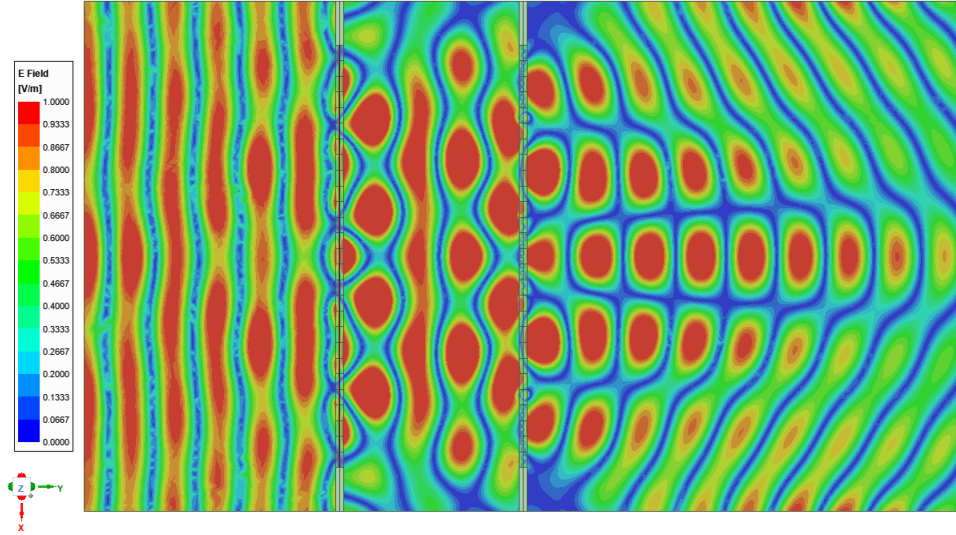


Figure 5.17: Near-field simulation by Ansys HFSS for the example concerning beam splitter with impedance specifications for three-layered unit cells and without multiple reflections in the cavity. The incident wave in this case is a plane wave excited from the left side of the simulation domain.

tween  $MS1$  and  $MS2$  was about 2.155% and 2.113%, respectively. The remaining power was lost due to reflection from metasurface  $MS1$  and  $MS2$ , and some power is dissipated by the substrates. Even though the design of the metasurface system considered zero reflections from metasurface  $MS1$  and  $MS2$ , some reflections still occurred. These reflections are speculated to have arisen due to not considering the thickness of substrates in the design algorithm shown in Chapter 3 while it is considered in modeling in ANSYS® HFSS™ software. In addition, the unit cell design assumes local periodicity which is not fully satisfied. Regardless, an output power efficiency of 81.053% is quite high.

#### 5.4.1.1 Copper trace design

Therefore we proceeded with copper trace design for the unit cells using the lookup table methodology as briefly discussed in Chapter 3. After obtaining the copper trace dimensions for the unit cells from the lookup library, the dimensions were further optimized in ANSYS® HFSS™ software using “Sequential Nonlinear Programming (Gradient)” optimizer within the software. The copper trace dimension optimization was based on the cost function shown in (3.84) to obtain  $S_{21}$  scattering parameter of the unit cells with copper traces as close as possible to the unit cell  $S_{21}$  scattering parameter of the impedance sheet. Tables 5.3 and 5.4 show the optimized copper trace dimensions for the three-layered unit cells for the cascaded metasurface system.

In Tables 5.3 and 5.4, column 1 indicates the unit cell number, columns 2 through 4 indicate the size  $W$  of the top, middle, and bottom copper traces, columns 5 through 7 indicate the size  $L$  of the top, middle and bottom copper traces, columns 8 and 9 indicate the achieved  $S_{21}$  scattering parameter magnitude and angle for the unit cells and lastly columns 10 and 11 indicate the desired  $S_{21}$  scattering parameter magnitude and angle for the unit cells. These dimensions were then used to model the cascaded metasurface system with copper traces for the unit cells. The modeling of the cascaded metasurface system was similar to the impedance specification model of the cascaded metasurface system. However, the impedance specifications were replaced with copper trace designs. The near-field simulated results for the beam splitting example with the copper traces for the three-layered unit cells and without reflections within the region between the metasurfaces  $MS1$  and  $MS2$  are shown in Figure 5.18. The simulation results with the copper traces for the cascaded metasurface system also showed the transformation of a single plane wave beam to two refracted plane wave beams in Figure 5.18. The transmission power efficiency of this system dropped to

Table 5.3: Metasurface  $MS1$  copper trace dimensions and the corresponding value of  $S_{21}$  for each unit cell for the beam splitting transformation without multiple reflections within the cavity.

| Cell | $W_{Top}$ | $W_{Mid}$ | $W_{Bot}$ | $L_{Top}$ | $L_{Mid}$ | $L_{Bot}$ | $ S_{21} _{ach}$ | $\angle S_{21}_{ach}$ | $ S_{21} _{des}$ | $\angle S_{21}_{des}$ |
|------|-----------|-----------|-----------|-----------|-----------|-----------|------------------|-----------------------|------------------|-----------------------|
| #    | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      |                  | [°]                   |                  | [°]                   |
| 1    | 2.1       | 2.0       | 2.5       | 2.3       | 2.1       | 2.8       | 0.93             | 183.43                | 0.92             | 183.32                |
| 2    | 2.4       | 2.5       | 2.2       | 2.4       | 3.1       | 2.4       | 0.95             | 241.32                | 0.94             | 241.39                |
| 3    | 1.9       | 2.3       | 1.9       | 2.7       | 3.0       | 2.2       | 0.86             | 271.74                | 0.84             | 272.55                |
| 4    | 2.5       | 2.8       | 1.7       | 2.3       | 2.4       | 2.7       | 0.96             | 258.43                | 0.98             | 257.80                |
| 5    | 2.0       | 3.1       | 2.0       | 2.5       | 3.4       | 3.0       | 0.91             | 216.33                | 0.91             | 216.48                |
| 6    | 2.1       | 2.9       | 2.1       | 2.4       | 3.8       | 3.0       | 0.88             | 210.69                | 0.88             | 210.82                |
| 7    | 1.9       | 2.3       | 1.9       | 2.8       | 3.1       | 2.6       | 0.97             | 240.29                | 1.00             | 240.33                |
| 8    | 2.2       | 2.4       | 1.9       | 2.3       | 2.9       | 2.2       | 0.93             | 279.05                | 0.89             | 279.38                |
| 9    | 2.0       | 2.9       | 2.0       | 2.4       | 2.5       | 1.9       | 0.90             | 281.47                | 0.90             | 281.57                |
| 10   | 2.0       | 2.6       | 2.2       | 2.6       | 2.6       | 2.5       | 0.97             | 247.79                | 0.99             | 248.27                |
| 11   | 2.5       | 2.6       | 2.4       | 2.7       | 2.8       | 2.3       | 0.88             | 221.37                | 0.88             | 221.31                |
| 12   | 2.1       | 3.2       | 2.3       | 2.6       | 3.1       | 2.4       | 0.94             | 229.70                | 0.94             | 229.86                |
| 13   | 1.6       | 2.4       | 1.7       | 2.8       | 2.8       | 2.6       | 0.96             | 266.74                | 0.97             | 267.30                |
| 14   | 2.1       | 2.7       | 1.7       | 2.5       | 2.5       | 2.4       | 0.87             | 285.66                | 0.87             | 285.40                |
| 15   | 1.6       | 2.4       | 1.7       | 2.8       | 2.8       | 2.6       | 0.96             | 266.74                | 0.97             | 267.30                |
| 16   | 2.1       | 3.2       | 2.3       | 2.6       | 3.1       | 2.4       | 0.94             | 229.70                | 0.94             | 229.86                |
| 17   | 2.5       | 2.6       | 2.4       | 2.7       | 2.8       | 2.3       | 0.88             | 221.37                | 0.88             | 221.31                |
| 18   | 2.0       | 2.6       | 2.2       | 2.6       | 2.6       | 2.5       | 0.97             | 247.79                | 0.99             | 248.27                |
| 19   | 2.0       | 2.9       | 2.0       | 2.4       | 2.5       | 1.9       | 0.90             | 281.47                | 0.90             | 281.57                |
| 20   | 2.2       | 2.4       | 1.9       | 2.3       | 2.9       | 2.2       | 0.93             | 279.05                | 0.89             | 279.38                |
| 21   | 1.9       | 2.3       | 1.9       | 2.8       | 3.1       | 2.6       | 0.97             | 240.29                | 1.00             | 240.33                |
| 22   | 2.1       | 2.9       | 2.1       | 2.4       | 3.8       | 3.0       | 0.88             | 210.69                | 0.88             | 210.82                |
| 23   | 2.0       | 3.1       | 2.0       | 2.5       | 3.4       | 3.0       | 0.91             | 216.33                | 0.91             | 216.48                |
| 24   | 2.5       | 2.8       | 1.7       | 2.3       | 2.4       | 2.7       | 0.96             | 258.43                | 0.98             | 257.80                |
| 25   | 1.9       | 2.3       | 1.9       | 2.7       | 3.0       | 2.2       | 0.86             | 271.74                | 0.84             | 272.55                |
| 26   | 2.4       | 2.5       | 2.2       | 2.4       | 3.1       | 2.4       | 0.95             | 241.32                | 0.94             | 241.39                |
| 27   | 2.1       | 2.0       | 2.5       | 2.3       | 2.1       | 2.8       | 0.93             | 183.43                | 0.92             | 183.32                |



Table 5.4: Metasurface  $MS2$  copper trace dimensions and the corresponding value of  $S_{21}$  for each unit cell for the beam splitting transformation without multiple reflections within the cavity.

| Cell | $W_{Top}$ | $W_{Mid}$ | $W_{Bot}$ | $L_{Top}$ | $L_{Mid}$ | $L_{Bot}$ | $ S_{21} _{ach}$ | $\angle S_{21}_{ach}$ | $ S_{21} _{des}$ | $\angle S_{21}_{des}$ |
|------|-----------|-----------|-----------|-----------|-----------|-----------|------------------|-----------------------|------------------|-----------------------|
| #    | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      | [mm]      |                  | [°]                   |                  | [°]                   |
| 1    | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86                | 1.00             | 103.62                |
| 2    | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86                | 0.96             | 106.86                |
| 3    | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 0.7       | 0.90             | 113.86                | 0.96             | 109.61                |
| 4    | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86                | 1.00             | 115.70                |
| 5    | 2.1       | 2.3       | 1.1       | 2.8       | 2.5       | 3.9       | 0.85             | 129.92                | 0.84             | 129.17                |
| 6    | 1.5       | 2.8       | 5.0       | 2.3       | 4.8       | 4.8       | 0.61             | 299.71                | 0.71             | 292.79                |
| 7    | 2.3       | 2.2       | 2.1       | 2.0       | 3.0       | 2.3       | 0.95             | 286.66                | 0.99             | 288.64                |
| 8    | 1.7       | 2.5       | 1.6       | 2.1       | 3.1       | 2.0       | 0.94             | 285.66                | 0.95             | 285.58                |
| 9    | 1.9       | 2.7       | 1.6       | 2.2       | 2.7       | 2.3       | 0.95             | 286.98                | 0.96             | 287.72                |
| 10   | 2.9       | 2.1       | 1.2       | 1.5       | 3.5       | 2.3       | 0.97             | 297.24                | 1.00             | 297.01                |
| 11   | 1.5       | 2.9       | 4.5       | 2.4       | 4.8       | 4.4       | 0.57             | 298.86                | 0.39             | 293.16                |
| 12   | 1.8       | 2.1       | 0.6       | 3.1       | 2.7       | 4.4       | 0.90             | 114.12                | 0.95             | 113.27                |
| 13   | 5.0       | 0.5       | 2.9       | 1.6       | 4.4       | 2.4       | 0.85             | 102.12                | 0.98             | 100.73                |
| 14   | 4.9       | 0.5       | 2.9       | 1.6       | 4.4       | 2.4       | 0.89             | 100.43                | 0.95             | 98.22                 |
| 15   | 5.0       | 0.5       | 2.9       | 1.6       | 4.4       | 2.4       | 0.85             | 102.12                | 0.98             | 100.73                |
| 16   | 1.8       | 2.1       | 0.6       | 3.1       | 2.7       | 4.4       | 0.90             | 114.12                | 0.95             | 113.27                |
| 17   | 1.5       | 2.9       | 4.5       | 2.4       | 4.8       | 4.4       | 0.57             | 298.86                | 0.39             | 293.16                |
| 18   | 2.9       | 2.1       | 1.2       | 1.5       | 3.5       | 2.3       | 0.97             | 297.24                | 1.00             | 297.01                |
| 19   | 1.9       | 2.7       | 1.6       | 2.2       | 2.7       | 2.3       | 0.95             | 286.98                | 0.96             | 287.72                |
| 20   | 1.7       | 2.5       | 1.6       | 2.1       | 3.1       | 2.0       | 0.94             | 285.66                | 0.95             | 285.58                |
| 21   | 2.3       | 2.2       | 2.1       | 2.0       | 3.0       | 2.3       | 0.95             | 286.66                | 0.99             | 288.64                |
| 22   | 1.5       | 2.8       | 5.0       | 2.3       | 4.8       | 4.8       | 0.61             | 299.71                | 0.71             | 292.79                |
| 23   | 2.1       | 2.3       | 1.1       | 2.8       | 2.5       | 3.9       | 0.85             | 129.92                | 0.84             | 129.17                |
| 24   | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86                | 1.00             | 115.70                |
| 25   | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 0.7       | 0.90             | 113.86                | 0.96             | 109.61                |
| 26   | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86                | 0.96             | 106.86                |
| 27   | 1.8       | 2.1       | 0.7       | 3.1       | 2.8       | 4.2       | 0.90             | 113.86                | 1.00             | 103.62                |

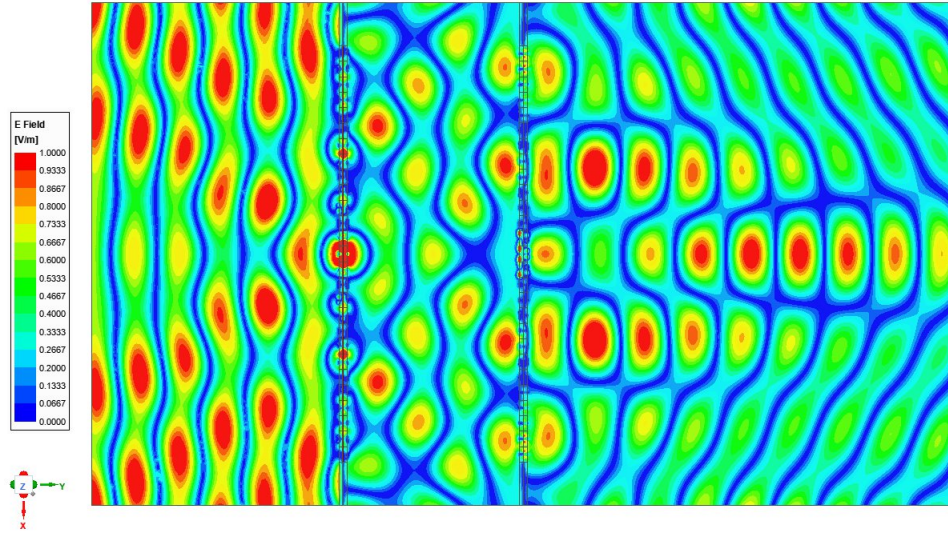


Figure 5.18: Near-field simulation for the beam splitting example with copper traces for the three-layered unit cells without considering multiple reflections in the design process.

about 45.643%. The powers leaking from the top and bottom openings were about 1.854% and 1.854%, respectively. This drop in the efficiency is speculated to be a result of the copper losses<sup>7</sup>, the increased reflections at the input face of *MS1*, and the fact that the lookup table assumes normal incidence.

#### 5.4.1.2 Fabrication

Next, we proceeded to the fabrication of the cascaded metasurface system. To do so, we took copper traces of the top, middle, and bottom layers of the metasurface *MS1* and *MS2*, and created the Gerber files as shown in Figures 5.19 and 5.20. The top, middle, and bottom copper traces were repeated 49 times in the vertical direction (which is  $\hat{z}$  in our coordinates) to occupy the majority of the space in the

<sup>7</sup>As noted earlier, close to resonance, the loss associated with copper traces and dielectric substrates can be significant [45].

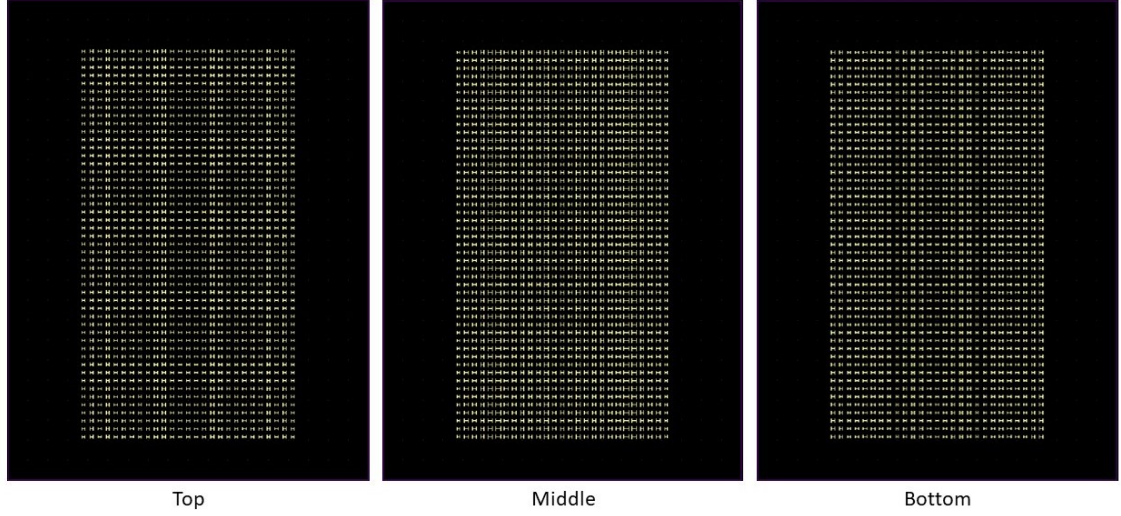


Figure 5.19: Metasurface *MS1*: top, middle, and bottom traces Gerber file representation for beam splitting example.

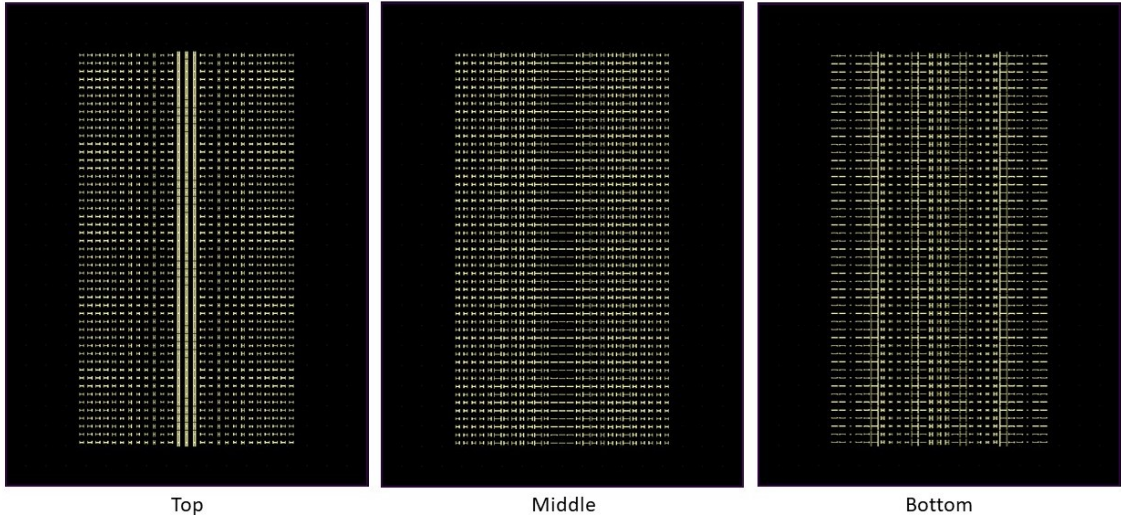


Figure 5.20: Metasurface *MS2*: top, middle, and bottom traces Gerber file representation for beam splitting example.

Roger RO3010 substrate board of size  $12 \times 9$  inch<sup>2</sup> available to us. We left a margin of about 1 inch on all sides around as per the fabrication instructions. The final assembly of the cascaded metasurface system is shown in Figure 5.21.

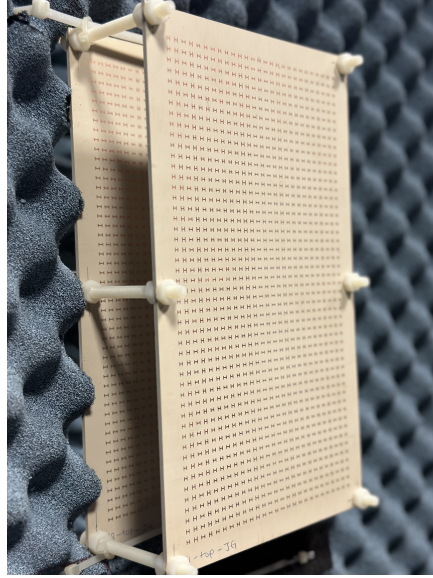


Figure 5.21: The fabricated cascaded metasurface system for the beam splitting example.

#### 5.4.1.3 Near-Field Measurements

Using the cascaded metasurface system shown in Figure 5.21 and the experimental procedure outline in Section 4.3, we then conducted a planar near field range measurement experiment in the Antennas lab at the University of Manitoba. The input plane waves impinging on the metasurface  $MS1$  were illuminated using a horn antenna. The output tangential electric fields were then measured using an OEW probe. Figure 5.22 shows the near field plot of the magnitude of the tangential electric field generated using the NSI-MI 2000 software. The  $x$  and  $y$  axes represent the scanning window for the measurements and the color bar indicates the intensity of the measured electric field in dB. Figure 5.23 shows the far field pattern comparison from the theoretical design through to experimental results. where the  $x$  axis indicates the angle and the  $y$  axis indicates the amplitude of the electric field in dB. In the measured near field plot shown in Figure 5.22, two thick dark red colored vertical lines can be

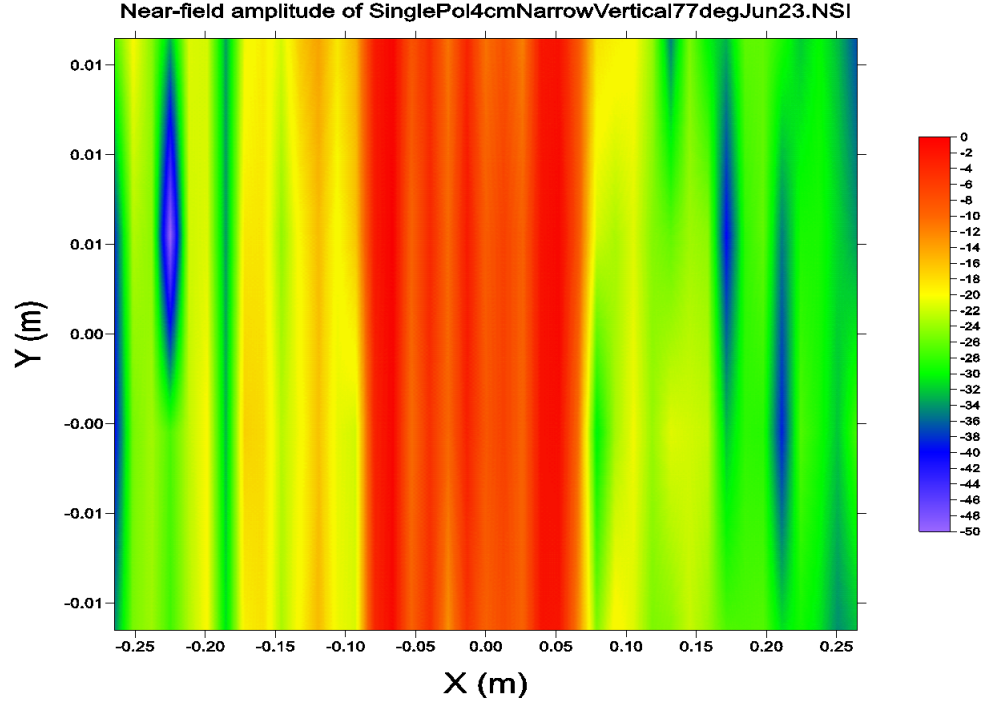


Figure 5.22: Experimental near-field results for the beam splitting example without considering reflections within the region between the metasurfaces in the design process.

seen being separated by a lighter shade of red color in the middle. These thick dark red colored vertical lines may be associated with the two desired beams. These two thick dark red strips are almost symmetric from the center of the  $x$  axis indicating the cascaded metasurface system achieved an almost symmetric transformation. As will be seen later, we have an increased side lobe level between the two main beams. This might be associated with the center section of the measured near-field data which is relatively of high intensity.



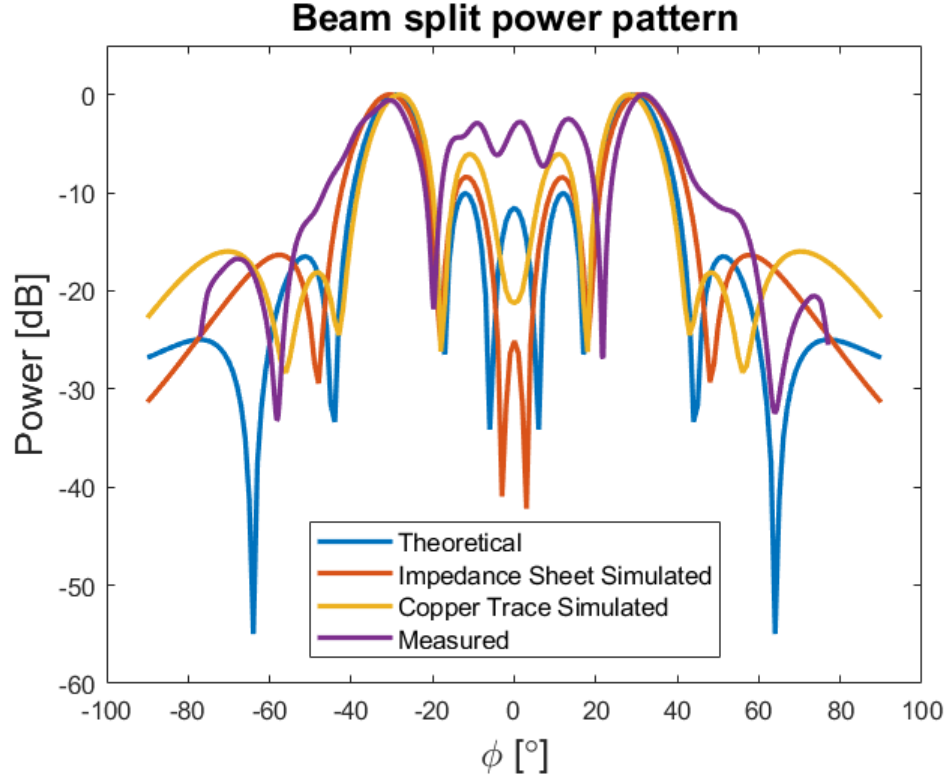


Figure 5.23: Far-field normalized power pattern comparison for the beam splitting example without considering reflections within the region between the metasurfaces in the design process.

#### 5.4.1.4 Measurement Errors

Note that due to the beam steering aspect of this cascaded metasurface system, the distance between the probe and the metasurface needs to be relatively small to capture all the emanating power. To be specific, the distance between the probe aperture and the metasurface was about 4 cm, which is much smaller than what is typically used in near-field antenna measurements ( $3\lambda_0$  to  $5\lambda_0$ ). This small distance increases the multiple reflections between the probe and the metasurface, and that can be a major source of error in the measurements. To avoid this error in the near-

field measurement, we need to use a measurement system with a larger scan plane such that we can afford to have the probe farther from the metasurface. Another source of error in our measurement is the fact that we do not have access to perfect plane wave illumination, and due to the limitation of our system, we could not move the horn antenna farther from the metasurface. Therefore, the incident field is not a perfect plane wave.

#### 5.4.1.5 Measured Near-Field to Far-Field Transformation

The best-case scenario for the far-field pattern can be obtained by transforming the desired output tangential fields to the far-field zone using the near-field to far-field transformation. The simulated far-field power patterns were calculated in ANSYS® HFSS<sup>TM</sup> software. Similarly, the measured far-field power patterns were calculated in NSI-MI 2000 software.

The far-field plot indicates the two beams were achieved as shown in Figure 5.23. At every stage of the design, the side lobe levels (SLLs) at the center kept increasing while the intended two beams were formed almost at the desired split angle with a discrepancy of  $\pm 1^\circ$ . This increment in SLLs and slight deviation in the desired beams in the simulated results can be attributed to issues such as local periodicity assumption and the thickness of the substrates. For the further error in the fabricated example, we have the same reasons and the issues such as multiple reflections between the probe and the metasurface, not having a perfect plane wave illumination, and the alignment issues between the metasurfaces and the horn/probe. Also note that at the theoretical design stage, the desired tangential fields of the refracted beams were calculated based on the infinite size of the metasurfaces but in reality, the sizes of the metasurfaces are finite. Furthermore, note that the dimensions of the copper traces did not achieve a perfect match with the impedance sheet design of

the unit cell due to the limitation of our lookup table. Despite all these errors at every stage in designing the metasurface system for the beam splitting example, the system was able to achieve the main goal of the design. That is, it created two main beams at  $\pm 30^\circ$ . In addition, the metasurface was removed and the incident field measurements were performed as well. Comparing the measurements in the absence and presence of the metasurface, we can obtain the transmission power efficiency. The measured transmission power efficiency was about 59.8%. This higher output efficiency as compared to the simulated value can be regarded as an indication of having multiple reflections between the scanning probe and the metasurface as if some of the emanating powers are considered more than once in the measurements. As noted earlier, the multiple reflections between the OEW probe and the metasurface might also contribute to the increased SLLs in Figure 5.23.

#### 5.4.1.6 Experimental Sensitivity Tests

While performing the experimental work, we conducted three sensitivity tests on the performance of the fabricated cascaded metasurface system. The first sensitivity test was on the sensitivity of the performance with respect to frequency change. The measured far-field result of this sensitivity is shown in Figure 5.24 for three different frequencies. From this test, it can be seen that a drift in the operating frequency occurred. That is, the best result is now at 9.7 GHz as opposed to 10.5 GHz. This was also observed in the design of single metasurfaces [16]. For single metasurfaces, the authors of [16] attributed this shift to fabrication errors and material parameters.

The second sensitivity test was performed on the effect of the separation distance between the scanning OEW probe and the metasurface and its effect on the far-field result as shown in Figure 5.25. The change of this distance showed no significant change in the resulting power pattern except for farther side lobes which are not



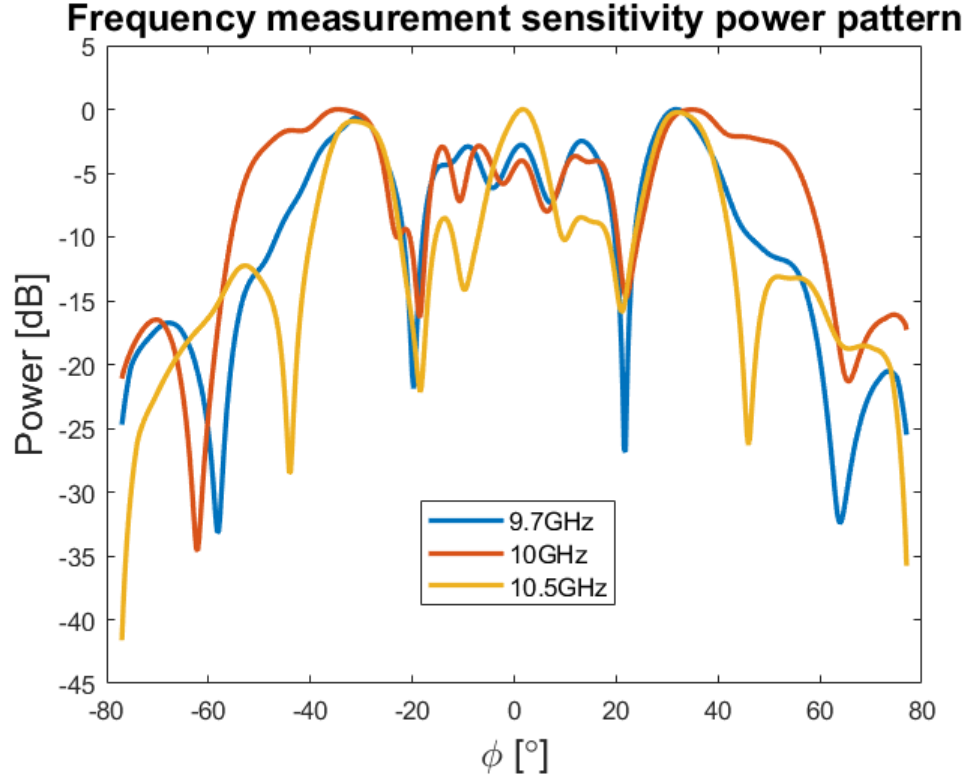


Figure 5.24: Far field pattern at various frequencies for the beam split example without reflections with the region between the metasurfaces

relevant due to the decreased critical angle of the near-field measurements as we get farther from the metasurface. Also, note that as the probe gets farther from the metasurface, the horn antenna becomes closer to the input face of the metasurface, which is itself another source of error (not a perfect plane wave).

The last sensitivity test was performed on the effect of the separation distance between the metasurfaces  $MS1$  and  $MS2$  and their far-field measured results are shown in Figure 5.26. As can be seen, the effect of separation between the two metasurfaces matters, but the main beams remain robust.

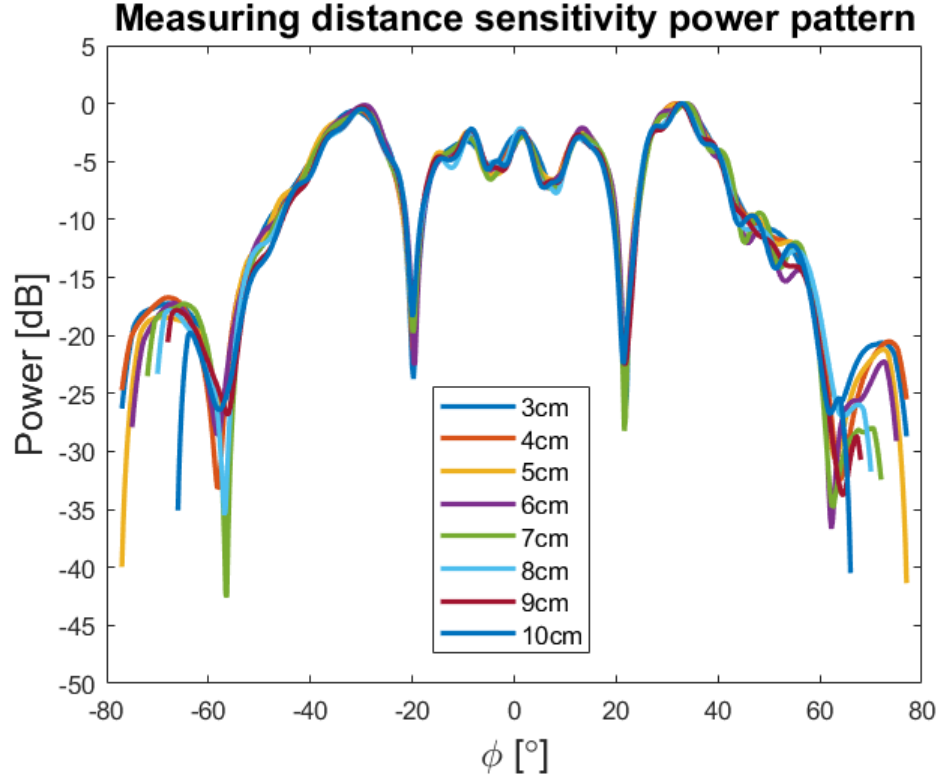


Figure 5.25: Far field power pattern at various measuring distances for the beam splitting example without considering the reflections within the region between the metasurfaces in the design process.

### 5.4.2 Beam Splitter with Multiple Reflections

In this section, the design algorithm takes into account multiple reflections within the region between the metasurfaces to achieve the same beam splitting functionality. The system was designed at an operating frequency of 12.4 GHz. The size of metasurfaces  $MS1$  and  $MS2$  was  $10.17\lambda_0$  with 61 unit cells in each metasurface whose size ( $D$ ) was  $\lambda_0/6$ . That is, about 4 mm. Rogers RO3010 was used as the substrate with a thickness of 1.28 mm and a Rogers bondply 2929 was used to bond the top and bottom substrates. The thickness of Rogers bondply 2929 was 0.0762 mm. The separation

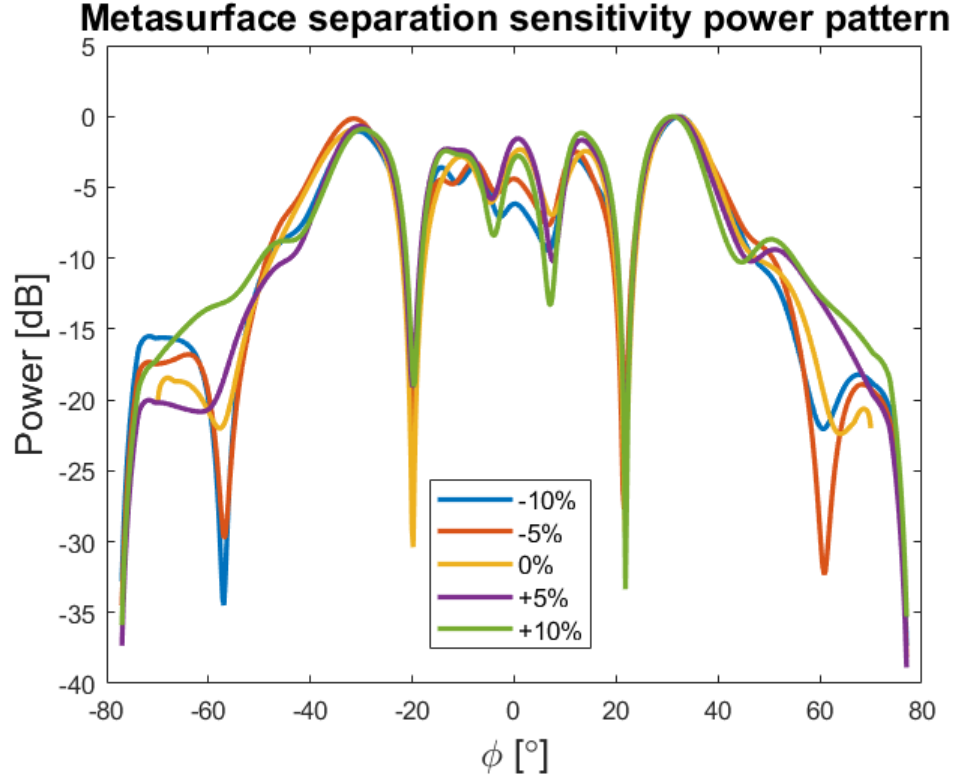


Figure 5.26: Far field power pattern at various separation distances between the metasurfaces for the beam splitting example without considering the reflections with the region between the metasurfaces in the design process.

distance  $d$  between the two metasurfaces  $MS1$  and  $MS2$  is  $0.5\lambda_0$ , which is smaller than the  $2\lambda_0$  separation considered in the previous section.

In the cascaded metasurface system's optimization algorithm, the incident and desired output tangential electric and magnetic fields were calculated using the plane wave equations. With the desired tangential fields and the incident fields, we ensure that the total power conservation constraint holds. Consequently, the internal tangential fields for the cascaded metasurface system were obtained using the algorithm developed in Chapter 3. The optimization algorithm successfully converged and finally, the normalized cost functional error was about  $1.070 \times 10^{-5}$  when the

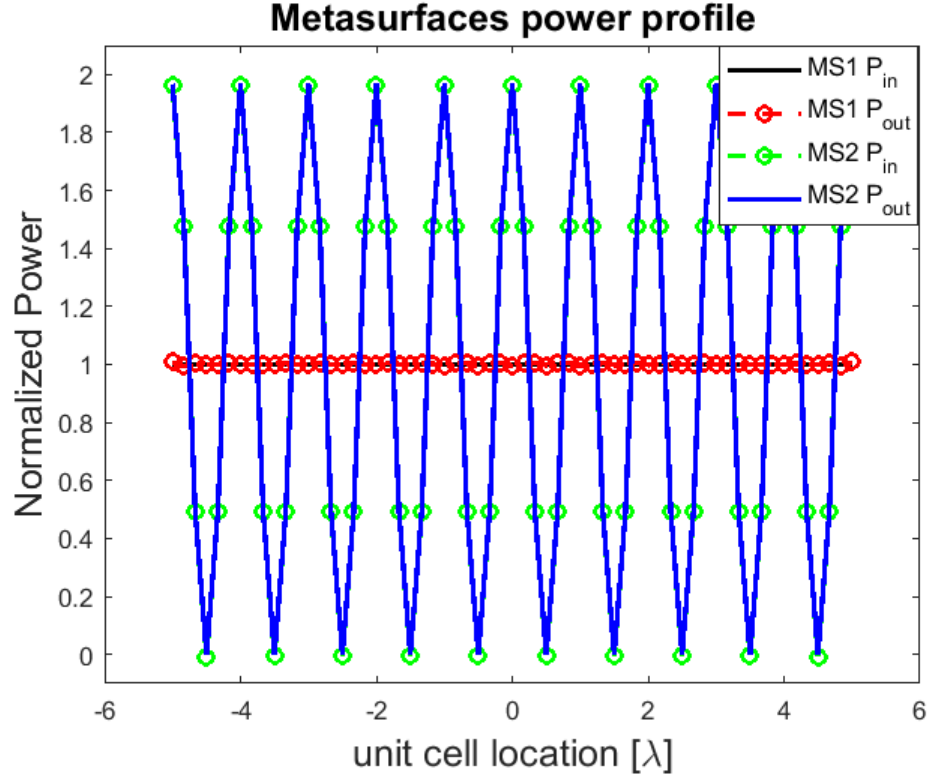


Figure 5.27: Normalized real power entering/leaving each unit cell in the normal direction for the cascaded metasurface system concerning beam splitting example when multiple reflections are considered in the design process.

optimization algorithm was terminated after 5000 iterations. The normalized power entering and leaving each unit cell is shown in Figure 5.27 while the normalized cost functional error at every iteration is shown in Figure 5.28. The LPC constraint was successfully achieved at each unit cell as shown in Figure 5.27. We then proceed to model the cascaded metasurface system to verify the electromagnetic transformation in ANSYS® HFSS™ software using purely reactive impedance specifications for the three-layered unit cell. Figure 5.29 shows the near-field simulation results of the transformation obtained in ANSYS® HFSS™ software using impedance specification for the three-layered unit cells. A plane wave source model in ANSYS®

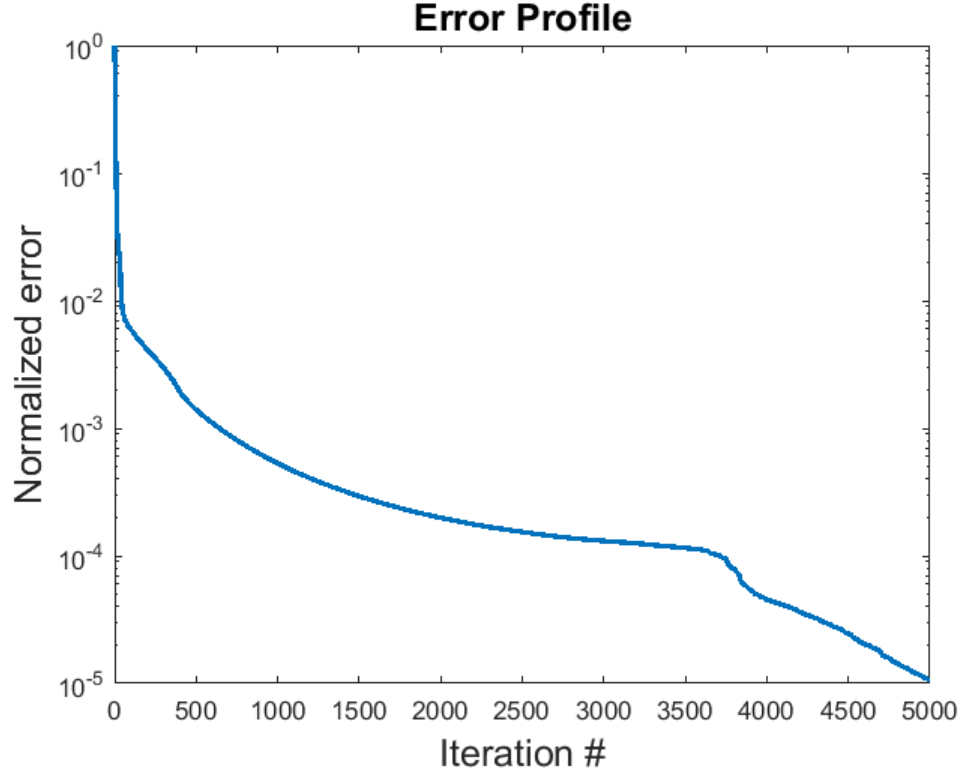


Figure 5.28: Normalized cost functional error profile for the cascaded metasurface system with multiple reflections design concerning beam split model.

HFSS<sup>TM</sup> software was excited from the left edge of the simulation region.

Like the simulation results for the cascaded metasurface system designed without multiple reflections between the metasurfaces  $MS1$  and  $MS2$ , a symmetric interference pattern can be seen near the output face of metasurface  $MS2$  and two beams of plane waves can clearly be seen propagating in two directions after a few wavelengths away from the metasurface  $MS2$  in Figure 5.29 while the input is a single beam plane wave impinging normally to metasurface  $MS1$ . The power transmission efficiency of the system with the impedance specification for the three-layered unit cell achieved for this cascaded metasurface system was about 93.886% while the power leaking from the top and bottom opening of the region between  $MS1$  and  $MS2$  was about

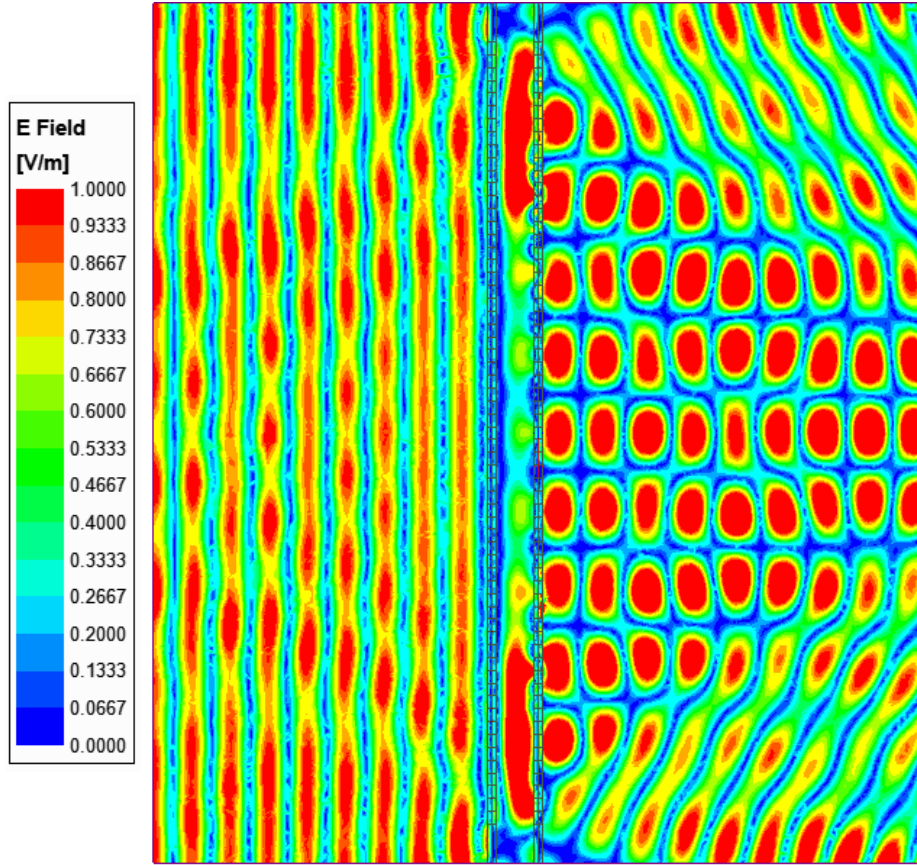


Figure 5.29: Near-field simulation for the beam splitting example with impedance sheet specifications for three-layered unit cells and with considering multiple reflections between the two metasurfaces in the design process.

0.649% and 0.813%, respectively. The remaining power was lost due to reflection from metasurface  $MS1$  and  $MS2$ , and some power is absorbed by the substrates. Even though the design of the metasurface system considered zero reflections from metasurface  $MS1$ , some reflections still occurred. These reflections are speculated to have arisen due to not considering the thickness of substrates in the design algorithm shown in Chapter 3 while it is considered in modeling in ANSYS® HFSS™ software as well as not meeting the local periodicity assumption used in unit cell design. The far-field power pattern shown in Figure 5.30 clearly shows the two beams in  $\pm 30^\circ$

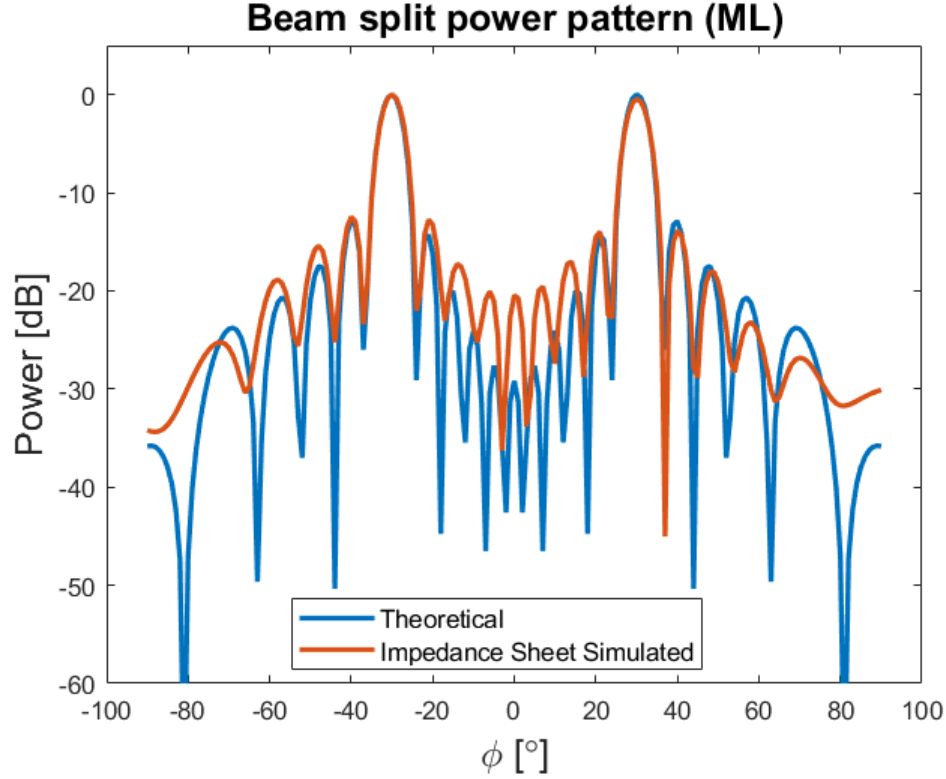


Figure 5.30: Far-field power pattern for the beam splitting example when multiple reflections are considered in the design process.

directions. In summary, considering multiple reflections gave us an extra degree of freedom to arrive at a better design. However, we have found the copper trace design to be much more challenging when significant multiple reflections occur in the space between the two metasurfaces.

## 5.5 Effect of the Dielectric Substrate

In this section, we investigate the effect of the type of material for the substrate and the thickness of the substrate. The design of the cascaded metasurface system we used to perform this investigation is similar to the example discussed in Section 5.4.2.

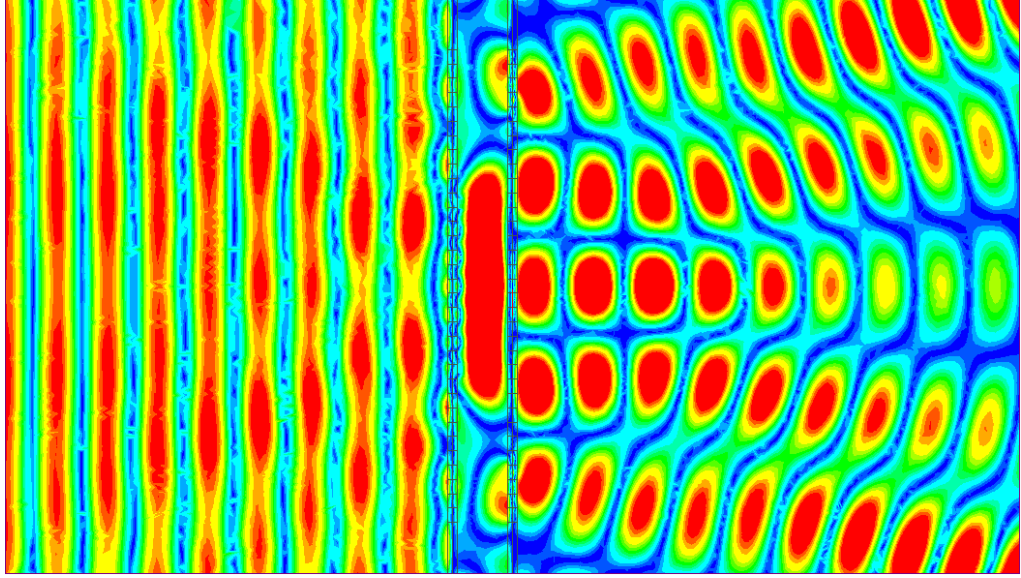
Therefore, the transformation we try to achieve in this investigation is a beam splitter with equal power and a refraction angle of  $\pm 30^\circ$ .

In this section, we use impedance specifications for the three-layered unit cell. The two thickness values of the substrate are 1.28 mm and 0.13 mm. These sizes of substrate are available in the market. For the material, we investigate Rogers RO3010 with a relative permittivity of 10.2 and *Air* with a relative permittivity of 1. In all the designs, the operating frequency is 12.4 GHz and the separation distance between metasurfaces *MS1* and *MS2* is  $0.5\lambda_0$ . There are 33 unit cells on metasurfaces *MS1* and *MS2* for all the designs. The input wave is a plane wave normally incident on *MS1*. In all the designs, Rogers bondply 2929 with a relative permittivity of 2.94 is used to connect the top and the bottom substrates of the metasurfaces. The total length of each metasurface is  $4.6573\lambda_0$  in all the designs.

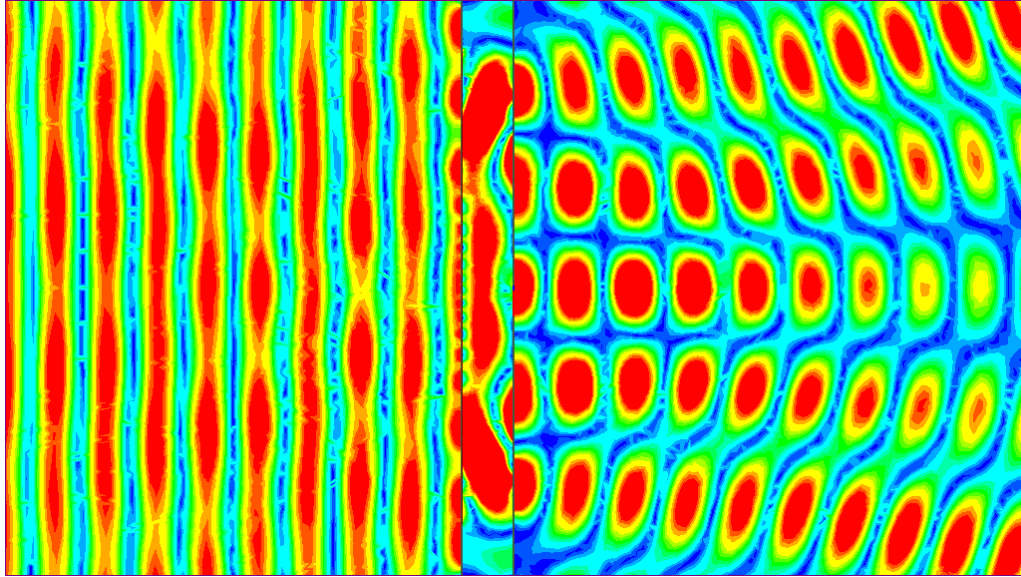
Figures 5.31 and 5.32 show the full-wave simulation results for the beam splitting example of this investigation. Figures 5.31a and 5.31b show the comparison of qualitative performance between the thick and thin Rogers RO3010 substrate and Figures 5.32a and 5.32b show the comparison of qualitative performance between the thick and thin air-filled substrate. In both simulation results, the system with a thinner substrate performs better than the thicker substrate as there is less reflection from metasurface *MS1* as indicated by more vibrant red-colored wavefronts and less destructive interference in the input side of the metasurface system with a thinner substrate as compared to the thicker ones. Furthermore, the performance of the air-filled substrate is better than the Rogers RO3010 substrate as shown in Table 5.5 by the output power efficiency. In addition, for each case, the power efficiency achieved with a thinner substrate is better.

In conclusion, from the experience of the author, having a thinner substrate further improves the transmission power efficiency. This is justified based on the fact that in



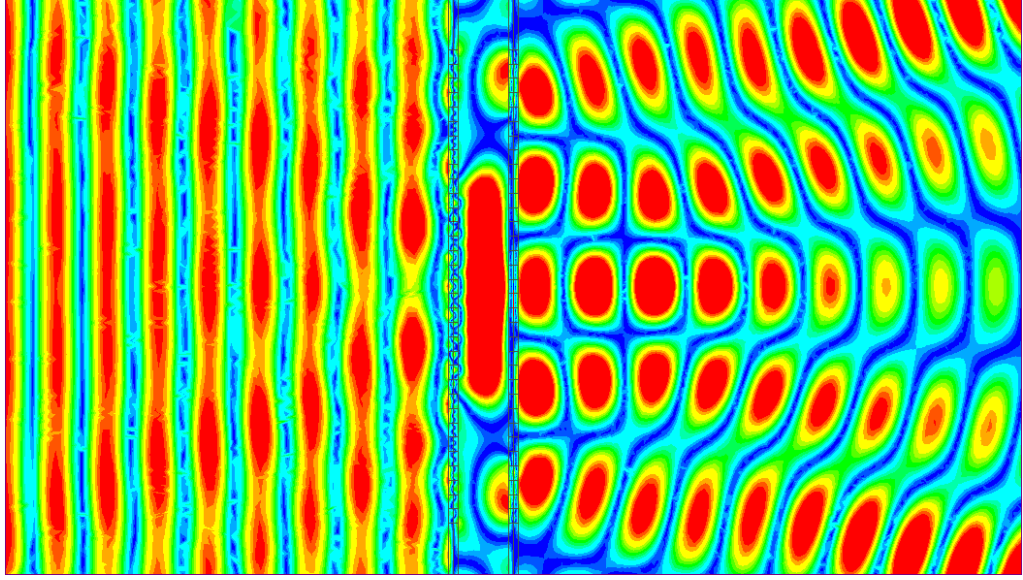


(a) Beam splitting with thicker 1.28 mm substrate

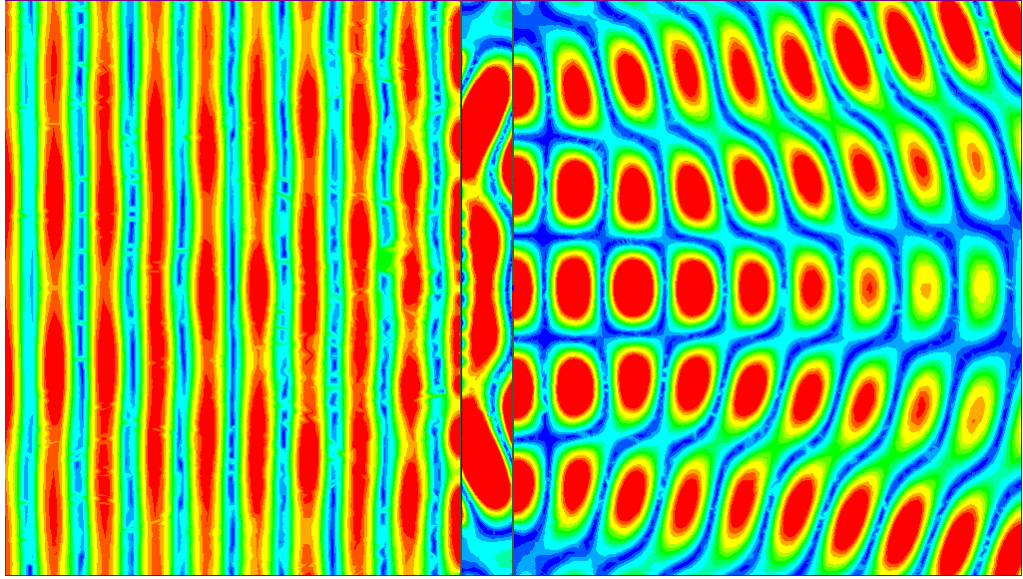


(b) Beam splitting with thinner 0.13 mm substrate

Figure 5.31: Investigation of different thickness values for Rogers RO3010 substrates having a relative permittivity of 10.2.



(a) Beam split with thicker 1.28 mm substrate



(b) Beam split with thinner 0.13 mm substrate

Figure 5.32: Investigation of different thickness values for air-filled substrates

the design optimization algorithm, the thickness of the metasurfaces is not considered. Hence, using a thinner substrate in modeling the cascaded metasurface system results

Table 5.5: Transmission power efficiency for different dielectric substrate materials and thickness values.

| System | Substrate Material Type | Permittivity | Substrate Thickness | Efficiency |
|--------|-------------------------|--------------|---------------------|------------|
| #      |                         |              | [mm]                | [%]        |
| 1      | Rogers RO3010           | 10.2         | 1.28                | 91.80      |
| 2      | Rogers RO3010           | 10.2         | 0.13                | 94.44      |
| 3      | Air                     | 1            | 1.28                | 95.70      |
| 4      | Air                     | 1            | 0.13                | 97.56      |

in much less modeling error. Furthermore, from the experience of the author, keeping the thickness value constant and using a low dielectric substrate improves the result. This can be justified based on the electrical thickness of the substrate. In other words, for example, 1.28 mm is electrically thinner in an air-filled substrate as compared to Rogers 3010 substrate. Thus, in summary, we speculate that the thinner the electrical thickness of the substrate, the better the performance of the overall metasurface based on this zero-thickness design procedure.

## 5.6 Chapter Summary

To summarize this chapter, we began by demonstrating the successful implementation of the design algorithm developed in Section 3 in achieving an electromagnetic wave transformation converting cylindrical waves into refracted plane waves and also synthesizing a desired power pattern using impedance sheet specifications for the three-layered unit cells. Then, we demonstrated the transformation of electromagnetic waves using copper traces for the three-layered unit cells and also the experimental results for one of the examples that was fabricated. Lastly, we demonstrated the investigation of the impact of substrate material type and thickness on the performance of the cascaded metasurface system.

# Chapter 6

## Conclusions and Future Work

This chapter provides overall conclusions of the work performed on cascaded metasurface systems and potential improvements and additions that could be done as future work.

### 6.1 Conclusions

This thesis was aimed at developing a fully automated design procedure for developing a lossless and passive cascaded metasurface system. In particular, it considered cascaded metasurface systems with the source located external to the cascaded metasurface system. This is important as cascaded metasurface systems enable electromagnetic wave transformations with more degrees of freedom compared to single metasurface systems. In particular, we limited ourselves to two planar parallel metasurfaces.

In general, a metasurface is a sheet-like structure divided into smaller segments called unit cells. The design of a metasurface is divided into macroscopic and microscopic phases. A metasurface system with just a single lossless and passive metasur-

face can be used to achieve electromagnetic transformation such as beam refraction. Achieving passive and lossless designs requires enforcement of power conservation at every unit cell in the metasurface system. In practice, for beam shaping, this means that the incident power profile needs to be redistributed so that the input and output powers at each unit cell are equal. In a single metasurface system, the power redistribution can be done by the excitation of surface waves along the surface of the metasurface. In practice, the excitation of surface waves is difficult and requires very small unit cells. The alternative approach considered herein is then to design lossless, passive, omega bi-anisotropic Huygens' cascaded metasurface systems. In this approach, the region enclosed between the two metasurfaces along with multiple reflections between the two metasurfaces can be used for power redistribution.

The first step in performing the macroscopic design is to ensure total power conservation between the input power from the incident electromagnetic wave and the output power for the desired electromagnetic wave holds. Next, the internal tangential electromagnetic fields, which are the unknowns of the problem, were defined as the summation of forward and reflected propagating waves with the coefficients of the expansion being the actual unknowns of the associated mathematical problem. This formulation was based on [14]. We then extended this formulation by developing a conjugate gradient (CG) optimization algorithm in the complex domain to determine the values of these unknowns. For the design of the CG optimization algorithm, the analytical expressions of the initial guess for the unknowns, the gradient vector (and subsequently the conjugate-gradient direction), and the step length were derived to automate the CG optimization algorithm. The CG algorithm then ensures that the local power conservation (LPC) constraint is sufficiently satisfied at every unit cell. After obtaining the tangential fields on either side of the unit cells, the generalized sheet transition conditions (GSTCs) can be used to determine the required surface susceptibility properties of the unit cells. In our work, we used the three-layered

unit cell topology to model the metasurface system. The macroscopic model was then modeled in Ansys HFSS to validate the electromagnetic wave transformation. A successful Ansys HFSS full-wave simulation results of cylindrical to plane wave transformation with and without refraction, and beam splitter examples were presented as examples.

The successful achievement of electromagnetic transformation with the full-wave simulation using surface impedance sheets was then followed by the microscopic design of the cascaded metasurface system for fabrication. In the microscopic design, the surface impedance specifications were transformed to copper trace designs in the shape of dogbones. The dimensions of the copper traces for the three layers of the unit cells were chosen from a lookup table and further optimized in Ansys HFSS to better match the desired forward scattering parameters. A successful full-wave simulation of the electromagnetic wave transformation of cylindrical to plane wave and beam splitting was achieved using cascaded metasurface systems with copper trace specifications. However, the output transmission power efficiency was significantly reduced compared to the impedance specifications for a cascaded metasurface system. We speculate that this is due to the fact that unit cell dimensions in the lookup table were developed under the periodicity assumption, and also the utilized lookup table is not based on the full scattering matrix.

Lastly, a cascaded metasurface system was fabricated using the printed circuit board (PCB) technique to achieve beam splitting. Planar near-field range measurements were performed. In the measurements, the two beams were clearly observed at the desired angle. However, increased side lobe levels were seen between the two desired beams. Also, the experimental output power efficiency was higher than that obtained from Ansys HFSS full-wave simulation using copper traces for the cascaded system. This could be because of the increased side lobe levels, potentially as a result of the occurrence of multiple reflections between the scanning probe and the

metasurface system due to their close proximity.

Overall, the performance of the cascaded metasurface system based on the ideal impedance sheet is very promising. On the other hand, our physical design is not yet automated as the utilized lookup table is based on the periodic boundary condition and is only based on the forward scattering parameter.

## 6.2 Future Work

Currently, in contrast to the macroscopic design step, our microscopic design step is not automated. To this end, we can consider the following:

- The lookup tables associated with the unit cell are typically based on periodicity assumption, but in reality, the neighboring unit cells can be quite different. Therefore, in the future, the unit cell design approach needs to take into account the non-periodicity situation.
- The lookup table needs to be improved to consider the full scattering matrix, and not just the forward scattering parameter. This is particularly important when we have significant multiple reflections between the two metasurfaces.

In addition, the cascaded metasurface system developed herein is limited to the 2D transformation of electromagnetic waves (i.e., pattern synthesis in one cut). As a future work, the design algorithm can be extended to achieve electromagnetic wave transformations in 3D.

Finally, the cascaded metasurface systems considered herein are static. That is, the surface properties of the metasurface do not change as a function of time. In the future, the cascaded metasurface system can be developed to be re-configurable such

that the beam can be changed by altering the status of the reconfigurable elements embedded into the metasurface.



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