THE UNIVERSITY OF MANITOBA TURBULENT DIFFUSION IN AN OPEN CHANNEL

by

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A THESIS

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SUMMARY

The present study was undertaken to develop a method of measuring turbulent diffusion coefficients in the fully developed flow of water in a rectangular open channel.

A sodium chloride solution was injected at some point in the cross section of the open channel and the concentration profiles in the lateral and vertical directions at three cross sections downstream of the source were obtained. From these profiles lateral and vertical turbulent diffusion coefficients were determined. This procedure was repeated at various points in the cross section to determine the variation of the diffusion coefficients in the lateral and vertical directions.

The channel used in the experiments was one foot square in cross section and 35 feet in length. The slope of the channel was set at 0.001. Experiments were performed at flow depths ranging from 7 cm. to 15 cm. and flow velocities ranging from 45 cm/sec. to 75 cm/sec.

Results indicated that the diffusion coefficients were 1.0±0.5 cm²/sec. in magnitude and that they increased towards the boundaries. Results also showed that as the depth to width ratio of the flow decreased the ratio of the vertical diffusion coefficient to the lateral diffusion coefficients became greater.

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CHAPTER I

INTRODUCTION

The study of the dispersion of mass and heat in rivers has gained considerably in importance with the increasing emphasis on the study of pollution of rivers. However, even the much simpler case of dispersion in open channels has not yet been completely analysed. This thesis is a study of the turbulent diffusion of mass emitted from a continuous point source placed in the fully developed flow of water in a rectangular open channel.

The study of turbulent diffusion has progressed in two directions. The statistical approach considers the properties of turbulence^{*}, and attempts to predict the diffusion process from these properties. The pioneering work in this field was done by Taylor [T2]^{**}; who in 1920 postulated the theory of diffusion by continuous movements. Kalinske and Pien [K1] combined this theory with experimental observations, in analysing the turbulent diffusion from a continuous source in an open channel.

The second approach to the study of turbulent diffusion is based on the conservation of mass principle. The application of this principle to an elemental volume in the

*Examples of these properties are the fluctuating velocities and functions based upon the spatial and time variation of these velocities.

**Numbers in square brackets refer to the bibliography.

flow field yields [D1]:

$$\frac{\delta c}{\delta t} + u \frac{\delta c}{\delta x} + v \frac{\delta c}{\delta y} + w \frac{\delta c}{\delta z} = \frac{\delta}{\delta x} \left(\varepsilon_{x} \frac{\delta c}{\delta x} \right) + \frac{\delta}{\delta y} \left(\varepsilon_{y} \frac{\delta c}{\delta y} \right) + \frac{\delta}{\delta z} \left(\varepsilon_{z} \frac{\delta c}{\delta z} \right)$$
(2.1)

where

С	is	the	time	averaged	concentration	of	dif-
	fusing		mass				

u,v,w are the time averaged velocities in the x,y,z directions and

 $\epsilon_x, \epsilon_y, \epsilon_z$ are the turbulent diffusion coefficients in the x,y,z directions.

Taylor [T4] solved equation (1.1) to obtain the distribution of cross sectional average concentration downstream of an instantaneous point source in turbulent pipe flow. Elder [E2] and Fischer [F2], extended Taylor's analysis to the instantaneous point source in two and three dimensional open channel flow.

The question of how the diffusion coefficients vary in the lateral and vertical directions remains largely unanswered. Although Kalinske and Pien [K1] examined this question experimentally in the case of a steady source placed in a rectangular open channel, their results were not independently verified. Furthermore, it is shown in chapter V, of this report, that if the experimental concentration profiles in the lateral and vertical directions have small errors in the relatively unimportant tail section of the profile,*

*Concentration profiles are normal.

the analysis of Kalinske and Pien may lead to large errors in the diffusion coefficients.

The present study has examined the same question by a different method. This method is based on the analytical solution of equation (1.1), for the turbulent diffusion of mass from a steady and continuous point source. The solution was first obtained by Roberts [R1], who modified it to the following form:

$$C = \frac{q}{4\pi x (\epsilon_x \epsilon_y)^{\frac{1}{2}}} e^{-\frac{U}{4x} \left(\frac{y^2}{\epsilon_y} + \frac{z^2}{\epsilon_z}\right)}$$
(1.2)

where

- q is the rate of injection of mass from the point source,
- x is the downstream direction (see figure 1
 on page 23),

y and z are the vertical and lateral directions The assumptions made in obtaining this solution are:

- The time averaged velocity in the direction of flow is constant across the cross section and this velocity does not vary from one cross section to another.
- (2) The time averaged velocities in the lateral and vertical directions are zero.
- (3) Transfer of mass by turbulent diffusion in the direction of flow is much smaller than the convective transport. This assumption has been verified by Elder, [E2].

In the present experiments a sodium chloride solution was injected at a point in the flow, and concentrations were obtained at three cross sections downstream of the source, in a small region around the line of injection.* There were eight sampling points in each cross section. The vertical and the horizontal turbulent diffusion coefficients, \mathcal{E}_x and \mathcal{E}_y , were then determined by fitting a slightly modified form of equation (1.2) to the twenty four data points. This procedure was repeated with the source located at various positions in the cross section of the channel.

The experiments were performed at three flow depths (7,10,15 centimeters) and bulk flow velocities ranging from 45 cm/sec. to 75 cm/sec.

The channel used in the experiments was a one foot square channel, 35 feet in length. The slope of the channel was set at a constant value of 0.001 throughout the experiments. With this slope setting, the depth of the flow did not vary by more than 0.5 centimeters in the 8 feet long test section.

*By considering a small region around the line of injection assumption (1) was satisfied. This region was 2.5 centimeters in radius.

CHAPTER II

LITERATURE REVIEW

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A. STATEMENT OF THE PROBLEM

One of the main questions asked in turbulent diffusion experiments is: What is the distribution of concentration of the diffusing quantity downstream of a source placed in a steady turbulent flow?

An example of the turbulent diffusion process is the dispersion of smoke from a chimney. The main characteristics of this example are the rapid and random fluctuation of the smoke cloud and the expansion of the cloud downstream of the chimney. The turbulent diffusion in an open channel is similar to the dispersion of smoke from a chimney, although the boundary conditions and the fluid are different.

The rapid fluctuation of the smoke cloud is due to the rapid, random fluctuation of the wind velocity. This suggests an analysis which predicts the bulk diffusion process by examining the velocity fluctuations and their variations over space and time. These ideas form the basis of the statistical theories of turbulence. A detailed review of these theories is given by Taylor et al. [T5]. In even the simplest cases however, such analyses become very complex mathematically. Furthermore, determination of fluctuating velocities require complex instrumentation. A characteristic of the example given above is that the concentration of smoke at a point can be divided into mean and fluctuating components. The division into mean and fluctuating components, although not exact, becomes very nearly true as the observation time increases. Therefore a more practical method is to apply the principle of conservation of mass to the flow field; and after time averaging obtain the standard hydrodynamic equation: [D1].

 $\frac{\delta \mathbf{c}}{\delta \mathbf{t}} + \mathbf{u} \frac{\delta \mathbf{c}}{\delta \mathbf{x}} + \mathbf{v} \frac{\delta \mathbf{c}}{\delta \mathbf{y}} + \mathbf{w} \frac{\delta \mathbf{c}}{\delta \mathbf{z}} = \frac{\delta}{\delta \mathbf{x}} \left(\varepsilon_{\mathbf{x}} \frac{\delta \mathbf{c}}{\delta \mathbf{x}} \right) + \frac{\delta}{\delta \mathbf{y}} \left(\varepsilon_{\mathbf{y}} \frac{\delta \mathbf{c}}{\delta \mathbf{y}} \right) + \frac{\delta}{\delta \mathbf{z}} \left(\varepsilon_{\mathbf{z}} \frac{\delta \mathbf{c}}{\delta \mathbf{z}} \right)$ (2.1)

where

c is the time averaged concentration u,v,w are the time averaged velocities in the

x,y,z directions.

 $\boldsymbol{\epsilon}_{\rm X},\boldsymbol{\epsilon}_{\rm Y},\boldsymbol{\epsilon}_{\rm Z}$ are the eddy diffusion coefficients.

In the derivation of equation (2.1) the velocity fluctuations are dismissed from the analysis by expressing their effects by means of the turbulent diffusion coefficients, ε .

The above equation can be adopted and solved for various cases. These cases may be divided according to the type of source (instantaneous, continuous, point, line, etc.), the type of channel geometry (pipe, open channel or an infinite medium), flow directions (one, two, or three dimensional), and the character of the dispersion process (steady or unsteady). In the following paragraphs, the literature review will be divided into two sections according to whether the statistical formulation or the conservation of mass equation is used as the starting point.

B. TAYLOR'S THEORY OF "DIFFUSION BY CONTINUOUS MOVEMENTS" AND STUDIES BASED ON THIS THEORY

A turbulent flow field is characterized by rapid velocity fluctuations. Taylor [T2]*, considering the velocity fluctuations and statistical correlations between them, derived a law which governs the turbulent diffusion process. The theory of diffusion by continuous movements given in reference [T2], forms the basis of this law. A brief interpretation of this law is given in the succeeding paragraphs.

The theory of diffusion by continuous movements examines "the characteristic properties of the variations of some quantity which varies continuously, but which appears to have no very definite law of variation." [T2]. A velocity component in steady turbulent flow can be represented as

u(t) = U + u'(t)

where

U is the mean velocity

u'is the fluctuating velocity

*Symbols in square brackets refer to the references in bibliography.

The fluctuating velocity component u'is one quantity that exhibits the above characteristic. Taylor, [T2], showed that provided $\left[u_t^2\right]^*$ is constant, an even correlation function $R(\xi)$ exists, such that:

$$R(\boldsymbol{\xi}) = \frac{\begin{bmatrix} u_{t}^{\prime} & u_{t+\boldsymbol{\xi}}^{\prime} \end{bmatrix}}{\begin{bmatrix} u_{t}^{\prime} & ^{2} \end{bmatrix}}$$
(2.2)

where

 ξ is a finite interval of time.

Denoting by X the distance traversed by the particle in time T as the result of eddy motion - i.e. the distance traversed by convection excluded - ; and using the definition of $R(\xi)$ above, Taylor proceeded as follows:

$$\frac{d}{dt} \begin{bmatrix} X^{2} \end{bmatrix} = 2 \begin{bmatrix} X \frac{dX}{dt} \end{bmatrix} = 2 \begin{bmatrix} X u' \end{bmatrix} ,$$

$$2 \begin{bmatrix} X u' \end{bmatrix} = 2 \int_{0}^{T} \begin{bmatrix} u'(t) \cdot u'(t+\xi) \end{bmatrix} d\xi ,$$

$$= 2 \begin{bmatrix} u'_{t}^{2} \end{bmatrix} \int_{0}^{T} R(\xi) d\xi ,$$
and
$$\begin{bmatrix} X^{2} \end{bmatrix} = 2 \begin{bmatrix} u'_{t}^{2} \end{bmatrix} \int_{0}^{T} \int_{0}^{t} R(\xi) d\xi dT$$
(2.3)

*Square bracket indicates the mean value of $u_t^{\prime 2}$ over a long period of time. In his analysis Taylor considered a statistically uniform field of turbulence. It follows from this assumption that $\lceil u_t^{\prime 2} \rceil$ is constant. Two conclusions can be drawn from this equation:

- (1) Determination of $\left[u_t'^2\right]$ and $R(\xi)$ is sufficient to describe the dispersion process.
- (2) It is reasonable to assume that for large values of ξ , R(ξ) will become zero.^{*} The integral in equation (2.3) is then constant, therefore equation (2.3) becomes:

$$\left[X^{2}\right] = 2 \cdot D_{X} \cdot T \qquad (2.4)$$

$$D_{X} = \left[u_{t}^{\prime}\right] \cdot \int_{0}^{1} R(\xi) d\xi \qquad (2.5)$$

where

T, is the time large enough for $R(\xi) = 0$.

Subscript x denotes diffusion in the x direction.

The similarity of the form of D_x as given above to the form of the eddy diffusion coefficients derived in mixing length theories led to the naming of D_x as a diffusion coefficient. It must be emphasized however, that the similarity is in form and not in character.

Kalinske and Pien [K1], applied equations (2.3) and (2.4) to the dispersion of sodium chloride transverse to the line of flow in an open channel. The flow in the channel was steady and a continuous point source was used. They

*Physically this means that $u'_{t+\xi}$ is independent of u'_t .

applied equation (2.3) in a direction normal to the axis of the flow to obtain:

$$\frac{d}{dt} \left[Y^2 \right] = 2 \left[v^{*2} \right] \int_0^t R(\xi) \cdot d\xi \qquad (2.6)$$

where

Y is the vertical distance travelled by a particle above or below the line of injection. v' is the fluctuating vertical velocity.

$$R(\xi) = \left[\underbrace{V_t' \cdot V_{t+\xi}}_{\left[V' \right]} \right]$$

For large ξ , $\int_{0}^{t} R(\xi) \cdot d\xi$ becomes constant and the definition of the turbulent diffusion coefficient from equation (2.5) yields:

$$D_{y} = \left[v'^{2}\right] \int_{0}^{t} R(\xi) d\xi \qquad (2.7)$$

 $\frac{d\left[Y^{2}\right]}{dt} = 2 \cdot D_{Y}$ (2.8)

Making the substitution x = Ut in equation (2.8) and integrating, one obtains:

$$\begin{bmatrix} Y^2 \end{bmatrix} = \frac{2 \cdot D \cdot X}{U}$$
(2.9)

where

x is the distance along the line of flow.

U is the stream velocity in the direction of flow.

It can be experimentally verified that the concentration distribution of the injected material transverse to the line of flow follows the normal distribution. Therefore

$$\frac{C_y}{C_0} = e^{-\frac{y^2}{2[y^2]}}$$
 for any x (2.10)

where

 C_V is the concentration at Y,

Co is the concentration at the centerline.

By measuring the concentration in the vertical direction, one can determine $|Y^2|$ from the slope of the straight line:

 $Y^{2} = -2.0 \left[Y^{2}\right] \ln \left(\frac{C_{Y}}{C_{O}}\right)$ (2.11)

*Kalinske and Pien give $[Y^2] = 2D (x-x_0)$ where $x_0 = \int_0^x Rdx$, for values of x when R has become zero. This in the present writer's opinion is impossible, since Y must be zero for x = 0on physical grounds. However the effect of this on the following analysis is negligible.

After $[Y^2]$ has been determined at several cross sections, equation (2.9) can be used to determine the value of D.

A similar argument can be applied to dispersion in the lateral direction and diffusion coefficients D_Z can be obtained in a similar manner.

In addition to the model proposed in this thesis, the present experimental data was also analysed by the above method. The results of this analysis and of comparisons with Kalinske and Pien's results are given in chapter V.

C. STUDIES BASED ON THE CONSERVATION OF MASS EQUATION

It was Taylor |Tl| who first suggested that the diffusion of matter in turbulent flow could be described by the conservation of mass equation:

 $\frac{\delta c}{\delta t} + u \frac{\delta c}{\delta x} + v \frac{\delta c}{\delta y} + w \frac{\delta c}{\delta z} = \frac{\delta}{\delta x} \left(\varepsilon_{\mathbf{x}} \frac{\delta c}{\delta x} \right) + \frac{\delta}{\delta y} \left(\varepsilon_{\mathbf{y}} \frac{\delta c}{\delta y} \right) + \frac{\delta}{\delta z} \left(\varepsilon_{\mathbf{z}} \frac{\delta c}{\delta z} \right)$ (2.1)

where

c is the time averaged concentration at the point (x,y,z),

u, v, w are the time averaged velocities in the

x,y,z directions,

 $\epsilon_X,\ \epsilon_Y,\ \epsilon_Z$ are the eddy diffusion coefficients.

Roberts [R1], obtained the solutions of equation (2.1) for three boundary conditions, assuming constant ε_x , ε_y , ε_z and constant u,v,w. They were:

(1) Instantaneous generation of matter at a point,

- (2) Continuous and uniform generation of matter at a fixed point in a uniform wind,*
- (3) Continuous and uniform generation of matter along an infinite straight line perpendicular to the direction of a uniform wind.

In 1954 Taylor [T4], studied the dispersion of matter in turbulent flow through a pipe, and arrived at the conclusion that this process could be described by a one dimensional equation of the form**

$$\frac{\delta c}{\delta t} + U \frac{\delta c}{\delta x} = K \frac{\delta^2 c}{\delta x^2}$$
(2.12)

where

K is a virtual coefficient of diffusion.

Since this work forms the basis of most of the subsequent studies discussed in this section, it will be described in some detail.

In cylindrical co-ordinates, the conservation of mass equation can be written as:

*This case will be analyzed in detail in chapter III.

**(1) This analysis applies only to instantaneous point
generation of mass as will be discussed later.
(2) The coefficients of diffusion should not be

equated to each other.

$$\frac{1}{a^{2}z} \frac{\delta}{\delta z} \left(\varepsilon_{z} z \frac{\delta C}{\delta z} \right) = u \frac{\delta C}{\delta x} + \frac{\delta C}{\delta t}$$
(2.13)

where

z = r/a
r is the radial distance from the center of
the pipe,

a is the radius of the pipe.

Taylor seperated the axial velocity into two components such that:

$$u(z) = \overline{U} + u_{\uparrow}(z) \tag{2.14}$$

where

 \overline{U} is the cross sectional mean velocity u_†(z) is the deviation from \overline{U} , at radius r = a·z

Making the transformation

 $\xi = x - \overline{U} t \tag{2.15}$

and letting

$$\overline{U} \frac{\delta C}{\delta \xi} + \frac{\delta C}{\delta t} = 0$$
(2.16)

* Elder |E2|writes that this is a good approximation since the fluid elements are greatly extended in the x-direction.

equation (2.13) becomes:

$$\frac{1}{a^2 z} \frac{\delta}{\delta z} \left(\varepsilon_z z \frac{\delta C}{\delta z} \right) = u_{\dagger} \frac{\delta C}{\delta \xi}$$
(2.17)

Taylor then divided C into two functions such that:

 $C = C_{\xi} + C_{z}$ (2.18) $\frac{\delta C_{\xi}}{\delta z} = 0 , \frac{\delta C_{z}}{\delta \xi} = 0 \text{ and } \frac{\delta C_{\xi}}{\delta \xi} = \text{constant}$

equation (2.17) can then be integrated giving

$$C_{z} = a^{2} \frac{\delta C_{\xi}}{\delta \xi} \int_{0}^{z} \left(\int_{0}^{z} u_{\dagger} z dz \right) dz \qquad (2.19)$$

The rate of transfer of C accross a section at $\boldsymbol{\xi}$ is given by

$$\int_{A} u_{\dagger} C_{z} dA = - K A \frac{\delta C_{\xi}}{\delta \xi}$$
(2.20)

where

A is the cross sectional area

Comparison with equation (2.19) yields:

$$K = -2 a^{2} \int_{0}^{1} z u_{\dagger} \left\{ \int_{0}^{z} \frac{1}{\varepsilon_{z}^{z}} \left(\int_{0}^{z} u_{\dagger} z dz \right) dz \right\} dz \qquad (2.21)$$

Taylor used the universal velocity distribution in a pipe and the Reynold's analogy to integrate the above expression. His final result was:

$$K = 10.06 \text{ a.v}_{f}$$
 (2.22)

where

 v_{f} is the friction velocity The solution of equation (2.12) is:

$$C = M t^{-\frac{1}{2}} e^{-\frac{(x - Ut)^2}{4Kt}}$$
 (2.23)

Taylor then performed experiments by instantaneously injecting a salt solution at some point on the centerline of the pipe and measuring salt concentration downstream. His results indicated that the distribution after approximately 100 pipe diameters was as indicated by equation (2.12) and the K values obtained were very close to that predicted by equation (2.21). He, therefore concluded that turbulent dispersion from an instantaneous source in pipe flow could be described by equation (2.12) with K determined by equation (2.21).

Elder |E2| extended Taylor's analysis to flow in a two dimensional open channel. The expression Elder obtained for the virtual coefficient of diffusion was:

$$K = -h^{2} \int_{0}^{1} u_{\dagger} \left\{ \int_{0}^{w} \frac{1}{\varepsilon_{y}} \left(\int_{0}^{w} u_{\dagger} dw \right) dw \right\} dw \quad (2.24)$$

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where

h is the depth of flow

y is the vertical distance from the free surface w = y/h

Employing the logarithmic velocity profile and the Reynold's analogy to determine u_{\dagger} and ε_{y} , Elder obtained

 $K = 5.86 \, hv_{f}$

Experiments in rivers and canals however, indicated that K/hv_f was as much as two orders of magnitude higher than that predicted by equation (2.24). One such experiment was performed by Godfrey and Frederick [G1].

Fischer [F2] offered an explanation for this phenomenon. He suggested that the reason for high values of K/hv_f was the significant variation of the axial velocity in the lateral direction in rivers. Using Taylor's analysis Fischer developed the following formula for the determination of K.

$$K = -\frac{1}{A} \int_{0}^{0} q(z) dz \int_{0}^{z} \frac{dz}{\varepsilon_{z} d(z)} \int_{0}^{z} q(z) dz$$
(2.25)

where

A is the cross sectional area, $q(z) = \int_{0}^{d(z)} u_{\dagger} dy$ and the rest of the variables are illustrated in the sketch below.



It will be noticed that whereas Elder's formula is integrated in the y-direction alone, Fischer's formula is integrated over the whole cross section.

A second contribution of Fischer was an analysis for the prediction of the period after which equation (2.12) can be applied. [F2].

Before leaving the discussion of the works of Taylor, Elder, and Fischer a brief reference to their experimental work will be made. Taylor and Fischer's experimental method consisted of placing a probe at a fixed cross section and measuring the response as the salt cloud passes the probe. The response of the probe i.e. c(t), is not symmetrical. This can be seen from equation (2.23). However, if the probe is placed at a position x, sufficiently for downstream, such that $(K/\bar{U}x_0) << 1$, the response becomes nearly gaussian with a mean value at (x_0/\overline{U}) . Under these circumstances the standard deviation of the response can be used to determine K. As pointed out by Chatwin [C1], both Taylor and Fischer made this approximation. Chatwin also argues that some of Fischer's data was not taken sufficiently downstream, for equation (2.12) to be applicable. The measurements must be made sufficiently far downstream for another reason. In equation (2.12), c refers to the cross sectional average concentration. If only one probe is used it must be sufficiently far downstream so that the concentration is approximately constant accross the cross section.

In Elder's method, a dye was injected into the stream and the photograph of the cloud taken at a later time. The film was then analyzed by a microdensitometer to determine the dye concentration. Since the time is "frozen" equation (2.23) predicts a Gaussian response, therefore the above approximation need not be made. Furthermore, a better cross sectional average may be obtained by Elder's method. Clearly, Elder's method is superior, however its application to large channels may be impractical.

A disadvantage of the instantaneous point source method is that the experiments must be repeated many times so that the effect of fluctuating velocities and concentrations is averaged out.

Church [C2], applied the above methods to study

dispersion in the Lewis River in Baffin Island. He obtained lateral diffusion coefficients ranging from 84 to 1000 cm²/sec. He also observed that the overall pattern of diffusion depended on the flow and the channel geometry of the stream.

Evans [E3] studied the dispersion of a dye from a continuous source in a pipe. Two of his results are of interest.

(1) Equation (2.1) was solved numerically using a constant $\varepsilon_{_{\rm Z}}$, and a parabolic $\varepsilon_{_{\rm Z}}$. The results did not differ very much.

(2) The turbulent Schmidt number was 0.65 for a Reynold's number of 10,000 and 1.0 for Reynold's numbers between 50,000 and 100,000.

Finally, Elata [E1] extended Taylor's analysis to elliptical open channels by a numerical method which made use of conformal transformations. He found that the diffusion coefficients depended heavily on the shape of the constant velocity lines in the cross section.

CHAPTER III

THE MODEL

Roberts [R1] has shown that if the conservation of mass equation (equation 2.1) is solved with the boundary conditions:

(1) $c \rightarrow 0$ as $x \rightarrow \infty$ (2) $c \rightarrow \infty$ as $x \rightarrow 0$ and nowhere else (3.1) (3) $\iint_{-\infty}^{\infty} cu dy dz = Q$ where

> Q is the rate of injection of matter, the result for the steady state is:

$$C = \frac{U}{2\sqrt{x}} \left\{ \left(\frac{x^2}{\epsilon_x} + \frac{y^2}{\epsilon_y} + \frac{z^2}{\epsilon_z} \right)^{\frac{1}{2}} - \frac{x}{\sqrt{\epsilon_x}} \right\}$$

$$C = \frac{Q e}{4\pi \left(\epsilon_x \epsilon_y \epsilon_z \right)^{\frac{1}{2}} \left\{ \frac{x^2}{\epsilon_x} + \frac{y^2}{\epsilon_y} + \frac{z^2}{\epsilon_z} \right\}^{\frac{1}{2}}}$$

$$(3.2)$$

Furthermore if
$$\frac{\gamma_{\ell_y}^2 + z_{\ell_z}^2}{x^2/c_x}$$
 is very small equation (3.2)

becomes:

$$C = \frac{Q}{4 \pi (\epsilon_y \epsilon_z)^{\frac{1}{2}} \times} e^{-\frac{U}{4 \times} \left(\frac{y^2}{\epsilon_y} + \frac{z^2}{\epsilon_z}\right)}$$
(3.3)

Convective transport in the x-direction is much greater than the turbulent transport; therefore ε_x may be assumed to approach zero. In addition, in the present experiments x^2 was at least 36 times as large as $(y^2 + z^2)$. For these reasons the above approximation may be applied in this case.

The experiments described in this thesis consisted of the repetition of a simple procedure over a range of parameters.* This procedure consisted of two parts: (See figure 1, on the next page)

- A 2% by weight salt solution was injected steadily and continuously at a point A in the flow cross section. Origin of the co-ordinates used in equation (3.3) coincided with this point.
- (2) Samples of concentration were taken at three downstream locations and at eight positions at each location, such that the sampling points were within a 2.5 cm radius of the line of injection. The central point of sampling at each cross section coincided with the line of injection. The 24 concentrations measured were then used to determine the turbulent diffusion coefficients ε_{y} and ε_{z} , as well as three correction lengths x_{0} , y_{0} , z_{0} from a slightly modified form of

*The parameters are: position in the cross section, flow depth, and bulk velocity of flow. The method will be described in detail in the next chapter.



equation (3.3).

$$C = \frac{U}{4 \operatorname{Tr} \left(\epsilon_{y} \ \epsilon_{z}\right)^{\frac{1}{2}} \left(x - x_{o}\right)} e^{-\frac{U}{4\left(x - x_{o}\right)} \left\{\frac{\left(y - y_{o}\right)^{2} + \left(z - z_{o}\right)^{2}}{\epsilon_{z}}\right\}} (3.4)$$

The criterion used for the determination of $\epsilon_y^{}, \ \epsilon_z^{}, \ x_0^{}, \ y_0^{}, \ and \ z_0^{}$ is that

$$\sum_{i=1}^{24} \left[c_{i}' - c_{i} \left(\epsilon_{y}, \epsilon_{z}, x_{0}, y_{0}, z_{0} \right) \right]^{2}$$
(3.5)

be minimized. Here c' refers to the experimentally determined concentration at point i, the x,y,z co-ordinates of which are known.

The following assumptions are inherent in the analysis: (1) There is a negligible difference between the densities of the injected fluid and that flowing in the channel. (Specific gravity of the salt solution is 1.02)

 (2) The flow in the channel is steady and fully developed.
 (3) Presence of the injection probe or the jet does not affect the flow.

"These three correction lengths were included later when it was found that this improved the fit of the data considerably. Reasons as to why this happened are given later.

** This was verified by measuring the velocity profiles in different cross sections and checking that they were the same. Test section was twenty channel widths downstream of the flow straighteners. No references on the length of the developing region in a rectangular open channel could be found in the standard textbooks. However the length of the developing region in turbulent pipe flow is given as 10 to 25 diameters. (Schlicting [S2], Kays [K2])

- (4) Velocity within a 2.5 cm radius of the injection probe is constant.Finally, some of the properties of equation (3.3) will be listed:
- Equation (3.3) predicts Gaussian distribution of concentration in the y and z directions.
- (2) Increase in the flow velocity U, does not effect the maximum concentration in a cross section, but the standard deviation of the concentration distribution is reduced.



Y OR Z

(3) Decrease of ε_y has the effect of increasing the maximum concentration, and decreasing the standard deviation in the y-direction. Standard deviation in the z-direction is unaffected.



(4) The effect of the change in parameters decrease as the downstream distance increases.

CHAPTER IV

APPARATUS AND METHOD

The experiments were performed in a flume one foot square in cross section and 35 feet in length. The flume, located at the hydraulics laboratory of the University of Manitoba, was connected to a small head tank at one end; and could be supplied with up to 80 cfm of water by the pumps which recirculated the water in a large sump. The flow straighteners, situated at the junction of the flume and the head tank consisted of a two inch thick horsehair net followed by a honeycomb of two inch diameter galvanized pipe two feet long. The channel boundaries were made of smooth plywood, with a Manning roughness factor of 0.010 to 0.014 ft. $\frac{1}{6}$ The slope of the flume was set at approximately 0.001 and the depth of water in the flume could be regulated by gates located near the exit. The depth of the flow was very nearly constant along the test section in all cases. The experiments were performed approximately 23 feet downstream of the flow straighteners.

The function of the injection system was to inject the salt solution at a steady rate. For this purpose two tanks were joined together as shown in figure 2, on the next page. The flow from the reservoir tank to the supply tank was adjusted to remain slightly greater than the injection



rate. The extra fluid could overflow through an opening at the side, thus keeping the level of the solution in the supply tank constant. The needle valve between the injection probe and the supply tank was set to the desired injection rate and the injection was turned on or off by a clamp on the flexible tubing.

The sampling system had to satisfy two criteria:

- (1) The sample had to be taken over a period long enough for the turbulent fluctuations in concentration to be averaged out.
- (2) The sample had to be drawn in through the probes at about the same velocity as the flow.

Preliminary tests indicated that criterion (2) required a longer time of sampling, therefore criterion (1) was automatically satisfied. The sampling times ranged from 3 to 4.5 minutes.

The sampling system is illustrated in figure 3, on the next page. By adjusting the vacuum produced by the pump the sampling rates could be altered.

Velocities in the channel were measured by the Kent miniflo velocity kit. This kit consisted of a vane anemometer and an electronic system that converted the rotation of the propeller to velocities. Velocities from 2.5 to 300 cm per second could be measured with this kit.

The tests conducted consisted of the repetition of a simple procedure over a range of parameters. This basic



procedure consisted of two parts: (See figure 1, on page 23)

- A 2% by weight salt solution was injected steadily and continuously at a point A in the flow cross section.
- (2) Samples of concentration were taken at three downstream locations. The central point of sampling at each cross section coincided with the line of injection. Samples upstream of the test section were taken at regular intervals to determine the background concentration of sodium chloride. The test samples were then corrected accordingly.

Tests were performed at three flow depths and three flow velocities. These were approximately 7, 10, 15 cm and 45, 60, 75 cm/sec. respectively. For the lowest velocity the procedure mentioned above was repeated at various positions in the cross section as shown in figure 4, on the next page. For the two high velocities diffusion measurements were made only at the first and last positions shown in figure 4.

For the three cases where measurements were made over the whole cross section, complete velocity profiles were obtained for the three depths of flow. For the remaining cases, the velocity was measured only at the points of injection.

The distances between the sampling probes were measured to within 0.05 cm by a cathetometer several times during the tests.










(c) \approx 15cm Deep Flow

FIGURE 4 POSITIONS OF INJECTION AND VELOCITY MEASUREMENTS The preliminary tests indicated that the velocity of injection had to be approximately 88 cm/sec. for the injection to be steady. At lower velocities the injection rate steadily became smaller. This is possible due to the salt particles settling at the needle valve. The error that the high velocity injection might introduce is discussed further in chapter V.

33

The analysis of the samples was made by the Argenteometric Method, described in the Standard Methods for the Examination of Water, [S1].

In this method, one milliliter of potassium chromate was added to 100 milliliters of the sample. The resulting solution was titrated with 0.01 normal silver nitrate until a pinkish orange end point was reached. Knowing the amount of titration the salt concentration could be calculated from the formula:

parts per million salt = 1.65 - 3.54 (A-R)

where

A is the milliliters of titration R is a constant dependent on the normality of silver nitrate.

CHAPTER V

RESULTS AND DISCUSSION

Concentration profiles obtained at various cross sectional positions, with different flow depths and velocities are given in figures 5 to 34. For each data set*, a standard deviation indicating the scatter of the data points from the best fitting binormal distribution was computed. This computation is described in detail in appendix I. The standard deviations varied from 3.4 to 14.9 ppm, (parts per million sodium chloride). These figures may be compared with average centerline concentrations of 80, 120 and 375 ppm at x equal to 40, 30, and 15 centimeters respectively.

One of the physical principles that the data must satisfy is that the mass flux at each cross section be the same and equal to the injection rate. Therefore

$$\iint_{A} U c dA = Q$$
 (5.1)

where

U is the flow velocity along the line of injection (constant since a small region around the line of injection is considered.) C is the concentration Q is the rate of injection.

*A data set refers to the twenty four concentrations determined per injection.

The theoretical model represented by equation (3.4) automatically satisfies equation (5.1). Furthermore the closeness of the experimental concentrations to the theoretical curves indicates that the experimental data also satisfies the physical criterion for steady state turbulent diffusion.

Turbulent diffusion coefficients computed at various cross sectional positions with different flow depths and velocities are given in table 1. Computations showed that the turbulent diffusion coefficients were in the range $1.0 \pm 0.5 \text{ cm}^2/\text{ sec.}$

Variation of the turbulent diffusion coefficients across the cross section is shown in figures 35 and 36. Turbulent diffusion coefficients increased towards the boundaries indicating that the sodium chloride solution spread faster when injected close to the boundaries. It must be indicated however, that very close to the boundaries this trend may reverse. In the present experiments, the injection closest to the boundaries was at 3 cm from the bottom and 5.0 cm from the side wall. The diffusion coefficients in the lateral direction were in general smaller than the diffusion coefficients in the vertical direction. Furthermore the ratios of the vertical diffusion coefficients increased as the width to depth ratio of the flow increased. Variation of the diffusion coefficients with the bulk flow

velocity did not exhibit a regular pattern.

The experiments of Kalinske and Pien [K1] , performed at bulk flow velocities of approximately 90 cm/sec., and similar width to depth ratios, gave results considerable different from the above trends. Their turbulent diffusion coefficients were approximately three times higher than the present ones. This may be due to the use of channels with different wall roughnesses. They also found that the diffusion coefficients had maximum values near mid depth and midway between the centerline and the wall. In the present data only the variation of ε_{z} in the lateral direction indicates a similar trend. Otherwise the general trend is an increase in the diffusion coefficients towards the solid boundaries. Furthermore, Kalinske and Pien observed that the lateral diffusion coefficients were in general greater than the vertical diffusion coefficients. This is in contrast to the present results.

The data of the present experiments was also analysed using Kalinske and Pien's method, which was described in detail in chapter II. It was found that if hypothetical data coinciding with the best fitting binormal distribution was used, the two methods gave identical results. This can

be proved theoretically.* However, if the experimental points were used, results differed considerably, despite the fact that the two sets of values were very close.

A possible explanation is that in Kalinske and Pien's method the standard deviation of the concentration distribution was determined by plotting the logarithm of concentration against the square of the distance. Therefore the small concentrations near the tails of the approximately binormal distribution were overemphasized. It must be pointed out that Kalinske and Pien used a "mean curve" through the data points in their analysis.

$$\frac{c}{c_0} = e^{-\frac{U}{4x}\left(\frac{y^2}{\epsilon_y} + \frac{Z^2}{\epsilon_z}\right)}$$
 and
$$\ln \frac{c}{c_0} = \frac{-U}{4x}\left(\frac{y^2}{\epsilon_y} + \frac{z^2}{\epsilon_z}\right)$$
 let $z = 0$, then
$$\ln \frac{c}{c_0} = -\frac{U \cdot y^2}{4x \cdot \epsilon_y}$$

$$y^{2} = \left(-\frac{4x\varepsilon_{y}}{U}\right) \ln \frac{c_{y}}{c_{o}}$$

Comparing this with equation (2.11), one obtains

 $\begin{bmatrix} Y^2 \end{bmatrix} = \begin{array}{c} 2\varepsilon_y x \\ U \end{array}$ which is Kalinske and Pien's result.

It was found during the analysis stage that the inclusion of the correction lengths x_0 , y_0 and z_0 greatly improved the fit of the data to the model. These three lengths along with the two diffusion coefficients were determined by a least square error optimization process which was described in chapter III. Correction lengths y_0 and z_0 indicate a shift of the point of maximum concentration from the line of injection. This could be explained by the existence of the secondary flows.

The correction length in the axial direction x_0 , varied between 7 and 9 centimeters. One possible explanation for the existence of this factor may be that since the sodium chloride solution was injected at a velocity of 88 cm per second, a certain distance would be traversed before the jet slows down to the flow velocity. However, x_0 did not decrease as the flow velocity increased. The flow of the sodium chloride solution in the injector probe is laminar. Therefore a more likely explanation is that a length x_0 is traversed before turbulence in the flow penetrates into the laminar jet.

Velocity contours obtained for the three flow depths, at the bulk velocity of 45 cm/sec., are given in figure 37. Figure 38 shows several velocity profiles measured at different depths.

It may be possible to modify the present experimental technique for application in rivers. However, the results

obtained from such an experiment will not be general, because the variation of cross section and the presence of bends in rivers render the applicability of results obtained in one section to another section unlikely. Furthermore, nonuniformity of the flow boundaries cause large secondary flows. Therefore a technique which also measures the secondary flows would be more useful. However, as a preliminary step towards the study of turbulent diffusion in rivers, the present method may be applied to channels of different cross sections and boundary roughnesses.

The present experiments did not investigate the dispersion of a diffusing quantity other than the sodium chloride solution. Turbulent diffusion of heat is of special interest. Despite the fact that the molecular Prandtl number of water is three orders of magnitude higher than the Schmidt number of the sodium chloride solution, the ratio of the turbulent Prandtl number to the turbulent Schmidt number is expected to be approximately one, with either value being slightly less than one.* This estimate is based on the results of Evans, [E3] and results of Ludwieg reparted by Schlicting [S2].

*Reynolds number range is 20,000 to 100,000.

CONCLUSIONS

Concentration profiles downstream of a continuous point source in steady turbulent open channel flow were obtained at various cross sectional positions, depths of flow and flow velocities.

Diffusion coefficients in the lateral and vertical directions were obtained by fitting the concentration profiles to a binormal distribution by a least square error optimization process.

Diffusion coefficients were found to be in the range 1.0 ± 0.5 and in general they increased towards the boundaries. It was also found that as the width to depth ratio of the flow became smaller, the ratio of the vertical diffusion coefficient to the lateral diffusion coefficient became larger. No regular variation of the diffusion coefficients with the flow velocity was observed.

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APPENDIX I

ERROR ANALYSIS

A. Experimental Errors

Experimental errors occured in three processes. They were: (1) Collection of the samples (2) Analysis of the samples and (3) Measurement of Velocity.

(1) Collection of the samples:

Preliminary experiments indicated that provided the sampling time was over 2.5 minutes, the reproducibility of the sample concentration was within 3 parts per million.* The sampling time in all experiments was over three minutes but was varied so that the velocity of the fluid in the sampling probe was approximately equal to the stream velocity.

The main source of error in the experiments was due to the positioning of the sampling probes with respect to the injection probe in the downstream direction. Since the concentration changed rapidly in the downstream direction, even small errors in the positioning could cause a discernible change in the observed concentration. The uncertainty in the distance between the injection probe and the sampling probes was approximately 0.25 cm. From a study of the experimental concentration variation in the x direction, this error was

*This figure may be compared with average centerline concentrations of 80, 120 375 ppm at x = 40, 30, 15 cm respectively.

estimated to cause 2 to 3 parts per million change in the concentration of the samples.

The uncertainty in the lateral and vertical distances - y and z directions - between the injection probe and the sampling probes was negligible. This is because the injection probe could be brought very close to the sampling probes for alignment in the y and z directions. Furthermore the distances between the sampling probes were determined to within 0.01 cm by a cathetometer before every set of tests.

(2) Analysis of samples

The samples were analysed using the Argenteometric method described in the Standard Methods for the Examination of Water. [S1] After one milliliter of the potassium chromate indicator solution was added to 100 milliliters of the sample, the resulting solution was titrated with 0.01 normal silver nitrate solution until an orange end point was reached.* Although reference solution of the desired color was prepared, there was an uncertainty of approximately 0.5 milliliters of titrant in reaching the end point. The concentration of the sodium chloride in a solution is given in the Standard Methods for the Examination of Water as:

> *This solution could be very accurately prepared by using standardized ampules supplied by the Canadian Laboratory Supplies Ltd.

$$ppm NaC1 = 3.54 1.65 (A-R)$$
(A1.1)

where

A is the milliliters of titrant required

to reach the end point,

R is a constant.

The uncertainty in the analysis of the sample concentrations was therefore approximately 3 parts per million sodium chloride.

(3) Measurement of Velocity:

The accuracy of the mini flow meter used in measuring the velocities in this experiment was stated to be 1.5% by the manufacturer.* For the velocities used in the present experiment this error ranges from ±0.5 to 1.0 cm/sec.

B. Confidence Limits of the Partial Regression Coefficients

The twenty four samples taken per test were used to determine five parameters - two diffusion coefficients and three correction lengths - which established a unique binormal distribution for that set. These parameters were influenced by the uncertainty at each of the 24 data points. Therefore, it was desirable to establish confidence limits for each of the five parameters. The method used to determine the confidence limits is described in detail by Bevington [B1].

*Kent-Lea Instruments Ltd., Lution Bedfordshire, England.

$$\sigma_{z}^{2} = \sum_{i=1}^{24} \left[\sigma_{i}^{2} \left(\frac{\delta z}{\delta c_{i}} \right)^{2} \right]$$
(Al.2)

where

 σ_{i} is the standard deviation for the data point c_{i} .

The standard deviation of all data points in a set were assumed to be equal, and calculated by:

$$\sigma_{i}^{2} = \sigma^{2} = \frac{1}{24 - N} \sum (c_{i}' - c_{i})^{2}$$
 (A1.3)

where

 $\mathbf{c}_i^{\,\prime}$ is the calculated concentration,

 $\boldsymbol{c}_{\,i}$ is the measured concentration,

N is the number of parameters determined

by the method of least squares.

The derivatives in equation (Al.2) were calculated numerically.

The 95% confidence interval \mathbb{A}_{Z} for a parameter z was computed by:

$$\Delta_{r} = \pm t \cdot \sigma_{r} \tag{A1.4}$$

where

t is 2.093 for 19 degrees of freedom, as given by Neville and Kennedy [N1].

Results are given in table 2. It is seen that the 95% confidence limits for the diffusion coefficients and the x correction length are from 10 to 20% of the mean values, whereas the confidence limits for the y and z crossflow lengths were as high or higher than the mean values.

APPENDIX II IMPROVEMENTS TO THE APPARATUS AND METHOD

1. If the concentrations are to be measured by sampling, it is suggested that the injection and sampling systems be constructed as a single unit and that the positioning of the injection and sampling probes be made by a rack and pinion arrangement.

2. Two less time consuming but more elaborate methods of measuring concentrations are suggested:

- A set of portable conductivity probes which could be moved,
- (2) A dye injection and the subsequent filming of the dye. This method is described by Elder [E2].

3. In order to obtain more reliable results the number of sampling points in each cross section and the number of cross sections for sampling could be increased.

			DIST.	DIST.	VERTICAL	HORIZANTAL	
		11		CENITED		OFFEICIENT	σ
JERIES	(cm)		(cm)	(cm)	coefficient	coerrorent	(mag)
		'sec'			(<u>sec</u>)	(Sec)	
A1	7.0	36.0	3.5	0.0	1.29	1.04	8.6
A2	7.0	42.5	3.5	7.5	1.13	1.15	4.6
A3	7.0.	36.0	3.5	12.5	1.19	0.82	4.2
B1	10.3	49.0	7.0	0.0	1.45	1.09	8.9
B2	10.3	42.5	3.0	0.0	1.33	1.30	8.4
B3	10.3	46.0	7.0	5.0	1.12	0.84	6.3
B4	10.3	42.5	3.0	5.0	1.67	1.46	6.1
B5	10.3	42.5	7.0	10.0	1.46	0.90	9.1
B6	10.3	38.0	3 <i>.</i> 0	10.0	1.90	1.35	11.1
<u>C1</u>	15.0 .	45.0.	12.0	0.0	0.89	0.81	6.3
C2	15.0	47.0	8.0	0.0	0.90	0.51	10.5
· C3	15.0	45.0	4.0	0.0	1.26	1.08	6.7
C4	15.0	46.5	12.0	5.0	1.09	0.92	5.7
C5	15.0	48.0	8.0	5.0	1.06	0.60	9.8
C6	15.0	45.0	4.0	5.0	1.61	1.24	7.6
C7	15.0	46.5	12.0	10.0	1.21	0.91	8.6
C8	15.0	46.5	8.0	10.0	1.24	0.63	8.9
C9	15.0	45.0	4.0	10.0	1.31	0.94 .	7.3
• D1 ·	8.0	71.0.	3.5	0.0	1.03	1.12	4.5
D2	8.0	66.0	3.5	12.5	1.33	1.29	6.2
E1	8.0	57.0	3 <i>.</i> 5	0.0	1.01	1.11	5.2
E2	8.0	52.5	3.5	12.5	1.11	1.27	4.7
F1	10.8	70.5	7.0	0.0	1.29	1.04	3.0
F2	10.8	66.0	3.0	10.0	1.56	1.69	3.8
G1	10.8	60.0	7.0	0.0	0.76	0.60	3.7
G2	10.8	51.5	3.0	10.0	1.46	1.18	6.1
H1	15.0	71.0	12.0	0.0	1.17	0.75	4.4
H2	15.0	69.5	4.0	10.0	2.10	1.97	4.0
I1	15.0	58.0	12.0	0.0	0.89	0.68	4.5
12	15.0	53.5	4.0	10.0	1.35	1.24	3.8

 σ is the standard deviation of the data points from the model.

TABLE 1

TURBULENT DIFFUSION COEFFICIENTS

	VERTICAL DIFFUSION	HORIZANTAL	X CORRECTION	Y CROSSFLOW	Z CROSSFLOW
			LENGTH	LENGTH	I ENGTH
JENICO					LLINOTT
	(cm / sec)	(cm/sec)	(cm)	(cm)	(cm)
Al	1.286 ± 0.169	1.042 ± 0.132	8.429 ± 0.658	-0.063 ± 0.071	0.034 ± 0.072
A2	1.131 ± 0.093	1.153 ± 0.070	8.056 ± 0.315	0.164 ± 0.034	0.003 ± 0.042
A3	1.190 ± 0.180	0.820 ± 0.203	8.764 ± 0.709	-0.135 ± 0.102	-0.229 ± 0.106
Bl	1.450 ± 0.223	1.090 ± 0.169	8.476 ± 0.731	-0.260 ± 0.089	0.010 ± 0.105
B2	1.331 ± 0.186	1.299 ± 0.180	7.465 ± 0.748	-0.039 ± 0.088	-0.038 ± 0.078
, B3	1.118 ± 0.087	0.841 ± 0.089	7.794 ± 0.405	-0.067 ± 0.052	-0.127 ± 0.066
В4	1.674 ± 0.168	1.458 ± 0.151	8.597 ± 0.421	-0.013 ± 0.066	-0.086 ± 0.060
B5	1.465 ± 0.192	0.900 ± 0.130	7.264 ± 0.683	-0.056 ± 0.088	-0.056 ± 0.104
B6	1.904 ± 0.378	1.355 ± 0.248	8.477 ± 0.874	-0.012 ± 0.128	-0.126 ± 0.109
Cl	0.890 ± 0.080	0.813 ± 0.076	7.116 ± 0.572	-0.164 ± 0.052	-0.031 ± 0.057
C2	0.990 ± 0.178	0.512 ± 0.138	3.421 ± 2.690	-0.163 ± 0.081	0.024 ± 0.126
C3	1.259 ± 0.096	1.081 ± 0.120	8.533 ± 0.354	-0.056 ± 0.064	0.001 ± 0.071
C4	1.087 ± 0.102	0.923 ± 0.081	7.650 ± 0.525	-0.053 ± 0.049	-0.002 ± 0.057
C5	1.064 ± 0.142	0.600 ± 0.115	7.573 ± 0.478	-0.007 ± 0.072	-0.086 ± 0.139
C6	1.609 ± 0.196	1.244 ± 0.143	8.931 ± 0.424	0.006 ± 0.074	0.005 ± 0.084
C7	1.215 ± 0.213	0.911 ± 0.157	8.221 ± 1.001	-0.022 ± 0.072	-0.118 ± 0.100
C8	1.243 ± 0.283	0.633 ± 0.114	7.815 ± 2.585	-0.102 ± 0.119	-0.219 ± 0.200
C9	1.311 ± 0.130	0.947 ± 0.113	8.486 ± 0.448	-0.150 ± 0.066	-0.091 ± 0.083
Dl	1.032 ± 0.105	1.122 ± 0.129	9.806 ± 1.688	0.425 ± 0.025	-0.084 ± 0.064
D2	1.326 ± 0.261	1.287 ± 0.203	8.628 ± 0.749	0.237 ± 0.080	-0.074 ± 0.117
El .	1.012 ± 0.173	1.115 ± 0.134	8.903 ± 0.550	0.185 ± 0.084	-0.086 ± 0.071 `
E2	1.108 ± 0.118	1.273 ± 0.121	9.428 ± 0.325	0.161 ± 0.056	-0.172 ± 0.048
Fl	1.288 ± 0.094	1.045 ± 0.054	6.667 ± 0.308	0.222 ± 0.024	0.059 ± 0.048
F2	1.557 ± 0.157	1.692 ± 0.173	8.751 ± 0.334	0.162 ± 0.050	-0.112 ± 0.059
Gl	0.758 ± 0.044	0.601 ± 0.064	6.155 ± 0.713	0.084 ± 0.025	-0.083 ± 0.040
G2	1.459 ± 0.155	1.180 ± 0.137	9.307 ± 0.296	0.045 ± 0.061	-0.137 ± 0.080
ні	1.174 ± 0.067	0.752 ± 0.057	7.617 ± 0.311	-0.055 ± 0.039	0.048 ± 0.078
H2	2.103 ± 0.219	1.968 ± 0.226	7.526 ± 0.491	0.087 ± 0.062	-0.115 ± 0.066
. I l	0.887 ± 0.072	0.679 ± 0.059	7.200 ± 0.544	0.122 ± 0.024	-0.083 ± 0.055
12	1.355 ± 0.082	1.240 ± 0.093	9.143 ± 0.186	-0.061 ± 0.043	-0.105 ± 0.049

TABLE 2CONFIDENCEREGRESSION

LIMITS OF THE PARTIAL COEFFICIENTS (95 %)

50





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FIGURE



يريد(مار) . مريد









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an the Second



14 FIGURE .





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FIGURE 21


FIGURE: 22







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FIGURE



and a



FIGURE 35 CROSS-SECTIONAL VARIATION OF DIFFUSION COEFFICIENTS



FIGURE 37 VELOCITY CONTOURS



