Profitability of Coalition Programs: An Analytical Investigation

by

Siming “Simon” Jiang

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Department of Supply Chain Management

University of Manitoba

Winnipeg

Table of Contents

Abstract

This paper investigates the external and internal conditions that sustain a favored framework of loyalty programs for customer acquisition and retention among numerous industries – coalition. After reviewing pertinent literature, a stylized economic model is generated for both stand-alone and coalition cases, with the goal of profit maximization. Each case is then solved analytically to obtain parameterized optimal solutions, subject to necessary constraints. With the help of 3-D plots and enumerations of the optimized solutions, we are able to identify the profit-maximizing conditions for each case and recognize critical thresholds for a profitable coalition. We find that a coalition would not generate a superior total profit if the optimal reward value is far less than the sum of product prices. On the contrary, when the auxiliary product's price is distributed around its median, a coalition could be more profitable. Additionally, we discover that the optimal reward of a coalition will increase as the product prices rise. This research contributes to academia by offering a preliminary study on coalition regarding customer loyalty, paving the way for future research to advance for more sophisticated study on collaborative business partnerships. Meanwhile, this paper guides the practitioners within the coalition by identifying break-even points, facilitating the generation of operational and marketing plans, and securing the coalition's success and continuance in the long run.

Keywords: Loyalty Program; Coalition Program

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Dedication

For my beloved parents and grandparents.

List of Figures

List of Plots

Chapter 1. Introduction

Nearly four decades have passed after the introduction of AAdvantage by American Airlines in May 1981 (Shelper, 2020), the world's first loyalty program using miles after stamps (1891) and coupons (1921) (Shelper, 2020). Ever since then, LPs have been growing in size and scope on a global scale. Specifically, in 2016, there were "3.8 billion individual loyalty memberships in the United States" (Fruend, 2017, p. 5), a 15% increase from 2014. Initially invented as a marketing tool for customer retention, LPs have helped firms in hospitality, airline, car rental, and retailing increase sales revenue by inducing repeated purchases (Fruend, 2017; SAS & Loyalty 360, n.d.). However, there are so many active LPs nowadays that competition among LPs within the same industry has diminished their effectiveness in achieving the sales targets. For example, among those LP memberships in the US in 2016, about 54% were inactive (Fruend, 2017), while more than half of the registered memberships were unused/abandoned. The stand-alone LPs are not as attractive as they used to be.

Alternatively, firms have been collaborating across different business sectors to promote their products jointly: they form coalitions to lessen competition and enhance profitability for each member firm. There are quite a few major coalitions in North America. In the US, Chase Bank collaborates with United Airlines, Marriott International forms a coalition with both American Express (Amex) and Chase Bank, and Safeway teams up with Chevron (Fruend, 2017). Besides, the AIR MILES Reward Program is the most remarkable coalition in Canada, counting "more than two-thirds of all Canadian households as members" (Fruend, 2017). Among numerous coalitions, airline companies have been a prime member and beneficiary. Specifically, airlines generate revenue by selling rewards points/miles to their coalition partners, mainly the credit card issuers. The coalition between Amex and Delta is an excellent example of getting along well in the partnership. They started their coalition in 1996 and signed an agreement in April of 2019, extending their portfolio through the end of 2029 (Delta Airlines, 2019). On the one hand, the Delta CEO thinks that their partnership with Amex "provides a competitive advantage as we deliver substantial value to our customers and owners" (Delta Airlines, 2019). Indeed, Delta's revenue increased by 3.4 billion in 2018 due to the coalition and expects to reach almost $7 billion by 2023 (Delta Airlines, 2019). On the other hand, Amex said that the Delta portfolio represents 8 percent of spending on Amex's cards and more than 20 percent of its loans (Surane & Schlangenstein, 2019). Thus, such a coalition sustains a long-term partnership and benefits both firms financially.

Nevertheless, sometimes the close collaboration can also cause issues that undermine the profitability and stability of a coalition. Take the coalition between Chase Bank and United Airlines, for example; tensions have been rising between the two after the launch of Chase Sapphire Reserve (CSR) credit card in 2016 (Andriotis & Benoit, 2019). The United executives were unpleasant as the United cards' applications slowed because of the "more generous sign-up bonus" the CSR card offered. Furthermore, United cardholders started to cancel the co-branded card due to the CSR card's highly overlapped benefits (Andriotis & Benoit, 2019). In a word, the CSR card has been competing with the co-branded one, reducing United's cash flow and its brand loyalty. While the partnership contract lasts for another five years, such competition runs counter to the coalition's purpose, threatening its existence since 1987 (Andriotis & Benoit, 2019). Another unsuccessful example comes from Plenti, a coalition program created by American Express in 2015, and its members included AT&T, Exxon Mobil, Macy's, Rite Aid, Enterprise, Hulu, and so on. (Fruend, 2017). Plenti was expected to be a huge success back then, bringing benefits for all the merchants. However, due to the lack of a central focal point and different shopping patterns among various industries, the coalition never really boosted the members' sales. As a result, the program ended in 2018.

With the observation of both successful and unsuccessful coalition programs, it is of interest to find out the reasons why coalitions face disparate fates. Thus, this research utilizes an analytical model to simulate a simplified real-world situation. To facilitate the research progress, four specific research questions that have so far been either partially or entirely left unanswered are generated and stated:

Research Question:

1. How do product prices affect the optimal rewarding decisions?

2. How do product prices affect the optimal profit for the coalition?

3. Under what conditions is a cooperative coalition sustainable/profitable?

4. Is a coalition more socially efficient? If yes, when?

The contribution of this research is multi-fold as it is the initial attempt to study the profitability of a coalition loyalty program but not a stand-alone one. Specifically, we try to solve the coalition's profit maximization problem with a stylized economic model. Furthermore, the model was designed to be multi-period to capture the dynamic decision-making process by the customers. Consequently, solving the model and producing parameterized optimal solutions provide an explanation and rationale for the in-demand marketing/operational scheme in various industries. In short, the theoretical insights generated by the model is a significant advancement to academia because it serves as a pioneer for future research avenues. Moreover, practitioners can treat this research as a paradigm when drafting their coalition proposal, and the research solutions could catalyze preferable operational and marketing decisions, ensuring the coalition's longevity and prosperity.

Following this introduction, the paper continues with a literature review section to brief the existing literature and identify the knowledge gaps. Section 3 sets up and solves the non-coalition case, serving as the benchmark and facilitating comparison with the coalition case. Consequently, section 4 identifies and addresses two scenarios of the coalition case, along with comparative statics analysis and enumeration of the total profits. Finally, section 5 summarizes the findings and contributions, highlights limitations, and suggests future research avenues.

Chapter 2. Literature review

The major methodologies utilized by the relevant literature are empirical and analytical; marketing journals prefer the empirical approach, while journals in operations management usually favor an analytical one. This literature review categorizes and synthesizes papers using either or both methods since our research lies at the intersection between marketing and operations management. We elaborate on the analytical subgroup as our paper addresses the research questions with an analytical model.

Empirical Approach (Marketing)

Fundamentally, the practice of loyalty programs (LPs) would affect customers' purchase intensity by a significant degree (Meyer-Waarden, 2008); LPs improve sales by promoting more frequent purchases. Specifically, there are two main themes of studying consumers' behavior towards LPs. On the one hand, researchers probed how a particular LP design affects consumers' behavior and the firm's sales.

Kivetz and Simonson (2002) conducted empirical studies and found out that "when program requirements are higher, consumers are (1) more likely to prefer luxury over necessity rewards and (2) more likely to join an FP that offers a luxury reward" (Kivetz & Simonson, 2002, p. 155).

Also, Zhang and Breugelmans (2012) conducted an empirical investigation to probe how switching from a price discounts scheme to an item-based loyalty program would affect the retailer's sales revenue as well as consumers' purchase behavior. They found that consumers would be more interested in hearing about a loyalty program that is item-based but not a loyalty program that merely offers a price discount. Also, if a retailer intends to improve sales, it should rely on several mechanisms rather than just one because some loyal members may stop to purchase from the seller. Meanwhile, the firm faces the chance of obtaining new loyal customers after the switch (Zhang & Breugelmans, 2012).

Lewis (2004) utilized both dynamic model and empirical testing to show that short-term loyalty promotion would effectively increase repeat-purchase rates.

Taylor and Neslin (2005) conducted an empirical study of the short-term and long-term impacts of how a retail loyalty program that offers rewards based on frequency would affect sales in a store setting. After analyzing the data sets, they found that frequency reward program would tend to generate additional revenue in both short term and long term for a store thanks to two effects, namely the rewarded behavior and point pressure (Taylor & Neslin, 2005).

Furthermore, McCaughey and Behrens (2011) utilized an empirical approach to calculate the premium that the customers would be willing to pay, solely because of the existence of the frequent flier programs. As a result, they claimed that the premium effect should apply even when there is competition or other exogenous factors (McCaughey & Behrens, 2011). This research offers much guidance to the managers of airlines on how to exploit more consumer surplus by modifying their frequent flier programs (McCaughey & Behrens, 2011).

Dorotic et al. (2014) conducted an empirical study using longitudinal data to probe how a linear loyalty program with points that never expire would affect consumers' behavior: the redemption rate. They found that the redemption effects have a significant influence on both pre-reward and post-reward behavior of the consumers, stimulating purchases (Dorotic et al., 2014).

On the other hand, researchers have studied how consumers' attributes shape their responses to a given LP setting. For instance, Liu (2007) conducted an empirical study to examine how a loyalty program in a setting of a convenience store would influence the consumers' usage in the long run. As a result, he found that a loyalty program can exhibit disparate influences on consumers with different usage before the loyalty program is introduced, suggesting that the seller needs to offer diverse loyalty programs to accommodate various shopping patterns and characteristics among different consumer groups to enhance marketing effectiveness (Liu, 2007).

Melnyk and van Osselaer (2012) used a field study along with three experiments to test if the two opposite genders would respond to loyalty programs with a different attitude. The reward aspects are status rewards and personalized offers. They found that "men are more responsive than women to status rewards that are highly visible to others. Meanwhile, women are more sensitive than men to personalized offers in private settings" (Melnyk & van Osselaer, 2012, p. 558).

It is commonly recognized that the expiration date for points/miles is paramount in loyalty programs designing, as it may have a disparate effect on sales generation. For instance, Breugelmans and Liu-Thompkins (2017) investigated empirically how expiration policy would affect consumers' behavior towards a loyalty program; they compared consumer responses to loyalty programs with and without a finite expiration policy. As a result, they found that consumers would react to expiration policy positively or negatively based on their flexibility to adjust for the policy, such factors as multi-store shopping and usage level (Breugelmans & Liu-Thompkins, 2017).

Concurrently, Kopalle et al. (2012) generated an empirical model to test how the frequency of the reward as well as the tier of the customer would affect sales in the hoteling industry jointly. The findings indicate that 1) both programs increase hotel's sales, and the two combined would create synergy; 2) customers tend to purchase more for either type of reward when they get closer to the qualification; 3) heterogeneous customers prefer different reward types (Kopalle et al., 2012).

Cao, Nsakanda, Diaby, and Armstrong (2015) tried to study the buyer-supplier management in a setting where there is one seller that is assumed to be the focal firm and the seller of points and many coalition partners/buyers, who would sign option contracts with the focal firm rather than any type of wholesale-price contracts. With proper empirical research methods, they found that their numerical simulations demonstrate that an option contract can act as a useful tool to fight against acute fluctuations in redemption demand, and such effect is even more obvious when the number of buyers in the network is relatively large (Cao, Nsakanda, Diaby, and Armstrong, 2015).

Sayman and J. Hoch (2014) developed a conceptual model and tested it empirically, aiming to probe the consumers' willingness to pay for a premium of the price when joining a loyalty program. They found that in the model, as consumers accumulate purchases, the maximal price premium would tend to increase. In the empirical test, they discovered that consumers' maximal premiums are less than those calculated in the model, but the price premium would still rise, especially when the reward is immediately redeemable (Sayman & J. Hoch, 2014).

Last but not least, Palmeira et al. (2016) conducted three online studies to see how communication framing would affect customers' responses to loyalty programs. Their findings are quite interesting and insightful: 1) the low-frequency customers are likely to reject a loyalty program that emphasizes status. In contrast, they tend to accept a loyalty program that advertises concrete rewards; 2) High-frequency customers would tend to accept both communication frames (Palmeira et al., 2016).

Yi et al. (2013) conducted three experiments, trying to find out how perceived uncertainty and reward frame (linear VS non-linear) would affect consumers' evaluation towrads loyalty programs. They found that consumers would tend to choose non-linear reward scheme when perceived uncertainty is low and would prefer a linear reward scheme when perceived uncertainty is high (Yi et al., 2013).

Schumann et al. (2014) conducted a few empirical tests to see if special treatment would mitigate dissatisfaction from consumers towards the coalition after the occurrence of severe service failures. As a result, they found that a failure in service that is due to one partner's fault in the coalition would harm the firm itself and damage the reputation, reducing consumers' loyalty toward the program. Also, if customers are satisfied with the coalition and its products as well as services, then it would be really hard to reduce customers' loyalty towards the coalition's loyalty program even though service failure occurs. Last but not least, benefits offered by special treatments would mitigate the negative effect of a failure in service that is due to one of the partners' fault in the coalition (Schumann et al., 2014).

Danaher et al. (2016) intended to compare price of one point in a loyalty program with a corresponding market price to see if the issuing firm awards rewards to the member firms of the loyalty program fairly for their efforts of earning points. As a result, they found that member firms of the loyalty program would tend to have a stronger sense of inequity when the market prices of the items are high. Also, those customers who purchase from the store/firm more often are less likely to be negatively impacted by the relatively higher redemption requirements (Danaher et al., 2016). These findings echo with Sayman and J. Hoch (2014) in a way that firms use a loyalty program as an identifier to practice price discrimination, luring the willing customers to pay a premium and alienating the other unwilling customers. Will the price discrimination generate more profits for the firm/coalition? The answer depends on a few factors, such as customers' income distribution, market competition, and customers valuation of the rewards.

Indeed, research in marketing has provided empirical evidence about the contribution a loyalty program can make to a single firm's sales. However, our paper simulates how consumers respond to a coalition's loyalty program in an analytical model and treats the profit as the critical measurement of a successful loyalty program, a significant advancement from the previous literature.

Analytical Approach (Operations Management)

Researchers in OM have intended to find the optimal loyalty program setting for profit maximization. A few papers indicate a loyalty program may not be necessary. For instance, Gandomi and Zolfaghari (2013) generated an analytical model to exam how customers' post-purchase satisfaction could affect a loyalty program's profitability. To solve the model, they considered three types of consumers' valuation settings: uniform distribution, normal distribution, and deterministic valuation. As a result, they found that if a firm could keep its customers satisfied through regular purchases, it would be optimal not to offer a loyalty program. If a firm does offer a loyalty program, then it has to increase the sales price to cover redemption costs as well as opportunity costs for obtaining a higher profit (Gandomi & Zolfaghari, 2013).

Alternatively, Chung et al. (2019) developed a dynamic pricing model to probe how redemptions by customers at a seller's store would affect its pricing and inventory decisions. In the model, the point issuer would compensate or reimburse a seller by an amount of money if a customer redeems points when purchasing the good. As a result, the sellers would have two sources of revenues: cash and sales of rewards. The customers would then decide to use cash or points according to their point balance, perceived value of a point, and reservation price. After deduction of the model, the authors found that adjusting prices can reduce its fluctuations. What is more, if a seller has enough authority, it can forbid the redemption activity due to insufficient reimbursement. Last but not least, a seller can either offer a discount or add a premium to influence the redemption rate (Chung et al., 2019).

Similarly, Calmon et al. (2018) developed an analytical model to explore a revenue management scenario in which a firm/coalition interacts with customers over a number of periods rather than just one. In the model, customers would decide the amount of money they would allocate to the firm's products according to the experience of past interactions. As a result, a firm faces a trade-off: it can either myopically maximize short-term revenues or enjoy higher long-term revenues by improving its reputation among customers. The authors found that a firm should practice myopic policies under specific conditions because it would be optimal to overlook the customers' budget variations. On the contrary, without those conditions, myopic policies can lead to sub-optimal financial outcomes in this repeated setting (Calmon et al., 2018).

Nevertheless, other studies demonstrate the necessity and effectiveness of a loyalty program. Singh et al. (2008) used a Hoteling's Game framework to test if a market would hold up an equilibrium for the competition between two firms: one offers a loyalty program while the other one tries to win the market share by using a strategy with low prices. As a result, they found the conclusion to be confirmative; the firm that offers a loyalty program can even achieve a higher profitability in this asymmetric setting (Singh et al., 2008). And the maximal price premium would increase as consumers accumulate purchases (Sayman & J. Hoch, 2014).

Especially, Sun and Zhang (2019) developed a conceptual model to simulate a loyalty program that would annul the rewards once past the expiration date. They found that a firm would be able to increase its profit by strategically setting expiration date on the issued rewards to consumers. In this way, the firm could subtract more consumer surplus from the shoppers with less-frequent visits to the store for repeated purchases over an extended time horizon (Sun & Zhang, 2019).

Alternatively, Cao, Nsakanda, and Diaby (2015) found that an option mechanism is effective for reducing demand uncertainties for rewards and later offered a mathematical model that considers a cooperative relationship between the focal firm and its partners on purchasing decision of rewards (Cao, Nsakanda, and Diaby, 2015).

What is more, a multi-tier loyalty program can facilitate price discrimination to achieve higher profits (Gandomi & Zolfaghari, 2018; Sayman & Usman, 2016). Specifically, Sayman and Usman

(2016) used an analytical approach to demonstrate that if a firm offers a loyalty program with a multi-level rewarding scheme rather than a single-reward loyalty program, it would be able to practice price discrimination to different groups of customers (light VS heavy users) to obtain higher profits (Sayman & Usman, 2016). Also, Gandomi and Zolfaghari (2018) intended to probe the conditions that would yield optimal profits for a firm that offers a multi-tier loyalty program. They found that the optimal structure of a loyalty program would mainly depend on its customers' characteristics, such as sensitivity to physical distance, price, and reward, rather than a competitor's pricing (Gandomi & Zolfaghari, 2018).

Finally, Chun and Ovchinnikov (2019) focused on the status reward aspect of a loyalty program; they compared and contrasted different loyalty program designs (Quantity only, Spending only, Quantity or Spending, and Quantity & Spending) in order to measure the disparate impacts they can exhibit, assuming a firm is shifting from quantity- to spending-based designs as well as customers who seek status maximize their total surplus through purchases and status rewards. As a result, there are three major findings: 1) all four designs would somehow promote additional spending among a sub-group of customers; 2) if the customers are strategic, then the firms would enjoy a higher profit by switching "from a quantity-based design to a spending-based design" (Chun & Ovchinnikov, 2019, p. 3969); 3) the analytical model designed by Chun and Ovchinnikov shows that all three designs with the Spending element would yield the same profits at optimal conditions, which are higher than the one in the Quantity-only design (Chun & Ovchinnikov, 2019).

Scholars have also broadened the spectrum of research on operations management. Especially, Kim et al. (2004) developed an analytical model, trying to find a firm's optimal pricing strategy in a competitive market setting if that firm has limited capacity. As a result, they found that a firm should offer excess capacity as rewards when the market demand is low in order to reduce competition. In this way, firms would be able to increase prices and enjoy enhanced profits (Kim et al., 2004).

Besides, Chen and Simchi-Levi (2004) intended to find a pricing strategy as well as an inventory policy that operates within a limited time horizon, trying to achieve a maximized profit. This is an early attempt of the application of dynamic programming on firm's optimization problem (Chen & Simchi-Levi, 2004).

Moreover, Ovchinnikov et al. (2014) utilized an analytical approach to probe how a firm with limited capacity can retain its customers by improving two management aspects: revenue and customer relationship. As a result, they found that the optimal spending for the firm is constant as well as depends on a customer's lifetime value for the firm when it has unlimited capacity. Meanwhile, such results can change dynamically and is generally irrelevant to a customer's lifetime value when the firm has limited capacity (Ovchinnikov et al., 2014).

Lastly, the points/miles that carry monetary value can substantially impact a firm's accounting record. Thus, Nsakanda et al. (2011) generated an analytical model to predict how accumulation of points/miles affect customers' redemption decisions and the firm's liability status under certain accounting principles.

Moreover, Chun et al. (2020) tried to answer the question of how to decide the points' value in regard to their corresponding liabilities in the accounting aspect. They developed a conceptual model, using dynamic programming method and comparative statics analysis. As a result, they found that a firm which operates a loyalty program should set the points' value based on the sum of the outstanding deferred revenue and realized cash flows, which they called "profit potential" (Chun et al., 2020). Also, a firm should set a target for the total value of points, and the target should increase as the "profit potential" as well as uncertainty does. Last but not least, good operating performance should lead to an increased total value of points (Chun et al., 2020).

Buchinger et al. (2013) used a case study to study the questions of interest. They stated that the virtual currency issuer is enabled to build a network, consisting of third parties, thanks to the move from loyalty points to virtual currencies. In this way, competition among the joining firms would be intensified, and it would be easy to attract new customers. On the one hand, the virtual currency issuer will be able to sell more reward points at an increased price with the expansion of the network due to imbalanced relationship between buyers and suppliers. On the other hand, the 3rd parties would want to join the coalition since they can expand their customer base and tap into new customer groups (Buchinger et al., 2013).

The past literature has discussed numerous aspects of stand-alone loyalty programs as well as Operations Management and Revenue Management. However, our model extends the knowledge base by analyzing a coalition program in a multi-period setting, offering state-of-the-art academic contributions and cutting-edge managerial suggestions.

Chapter 3. Non-Coalition Case: two firms in different industries with independent LPs

Consider there are two firms, *A* and *B*, in different industries, selling a good or service through two periods. We assume the two firms have their independent loyalty program (LP) in the Non-Coalition Case, and consumers would receive a reward valued at "R" if they buy from firm *A* or *B* in both periods. Also, consumers are strategic in making the purchase decision; they consider potential gains or losses in period two when purchasing in period one. As a result, customers' decision to purchase in the first period depends on the following factors: their valuation for the product, "V_i", price of the product, "P_i", and value of the reward, "R_i", $i = A$ or B. Without loss of generalizability, we normalize both firms' costs to zero and market size to 1, respectively. Thus, $V_i \in [0, 1]$, $R_i \in (0, 1]$, and $P_i \in (0, 1]$. We assume prices of the same goods are static across the two periods ($P_A = P_A^1 = P_A^2$) for model tractability. This assumption seems strong at first sight; however, academic precedents and business practices help justify it. From an academic perspective, according to the concept of price-stickiness (Keynes, 2018), firms face menu costs when updating prices. What is more, frequent price updates may irritate loyal customers and undermine the degree of loyalty as well as brand name. Hence, firms in our model are assumed to fix prices and adjust reward values to maximize profit. From reality, the department of operations management usually elects the pricing strategy while the marketing department determines the rewarding tactic, which can take place in a sequential order. Since our model focuses on the rewarding scheme/marketing function of a coalition, pricing decision is beyond the scope of this study. As a result, we choose to pin the product prices. In addition, it is natural to rule that $P_i \geq R_i$ as neither firm would offer a reward for free. Following Gandomi and Zolfaghari (2013) and Hotelling (1929), among others, we assume that consumers are uniformly distributed over the market for each firm to capture heterogeneity. A customer will make a purchase if the surplus of doing so is nonnegative, denoted as "S_i", $i = A$ or B.

---Period One of the Non-Coalition Case:

Customers' surplus of buying a unit from *A* in period one is

$$S_A^1 = V_A - P_A + V_A - (P_A - R_A) = 2V_A - 2P_A + R_A \tag{1}$$

And the purchase would occur if

$$S_A^1 \geq 0 \ \rightarrow V_A \geq P_A - \frac{R_A}{2} \tag{2}$$

The graphic demonstration of product valuations and purchase probabilities is in Figure 1:

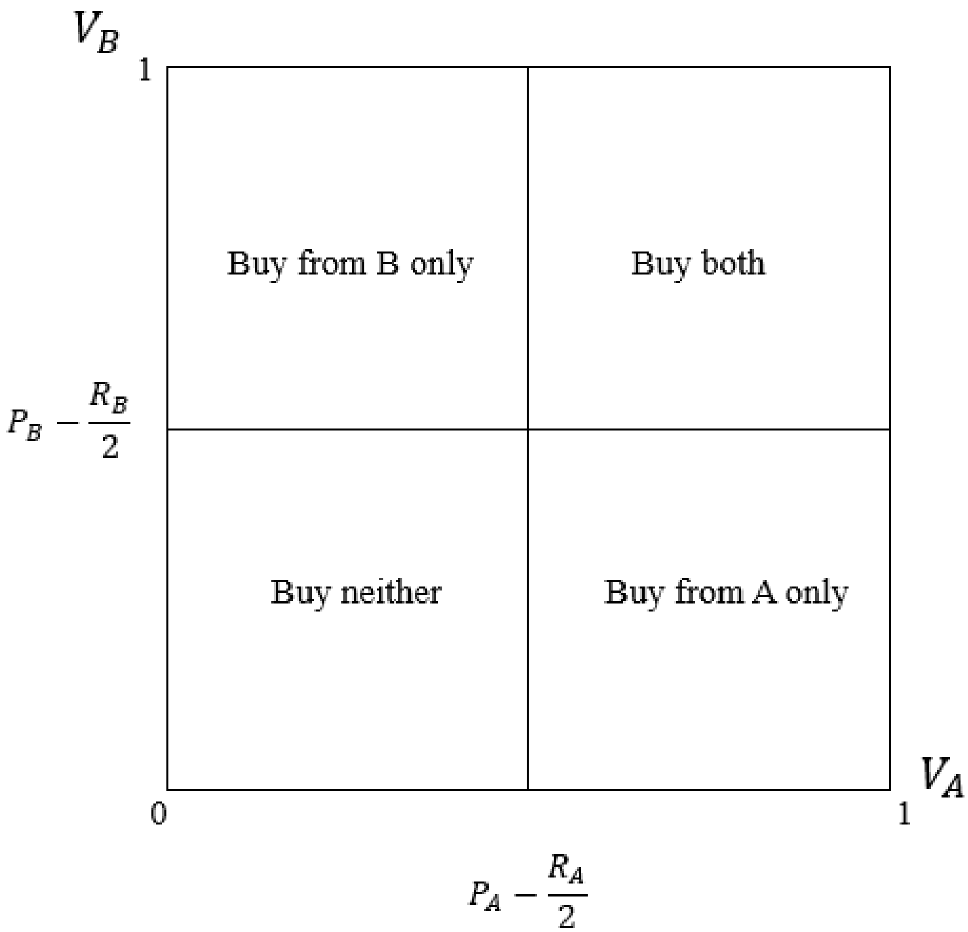


Figure 1. Product Valuations and Purchase Probabilities

Therefore, the probability of making a purchase of A is

$$\Pi_A^1 = 1(1 - V_A) = 1\left(1 - P_A + \frac{R_A}{2}\right) = 1 - P_A + \frac{R_A}{2} \tag{3}$$

, where Π is the probability of purchase. Since firms' cost is zero, total profit (TP thereafter) would then equal to total revenue, so

$$TP_A^1 = P_A \Pi_A^1 = P_A(1 - P_A + \frac{R_A}{2}) \tag{4}$$

---Period Two of the Non-Coalition Case:

In period two, if a customer who bought a unit from *A* in period one and is considering repeating the purchase would have the following valuation

$$S_A^2 = V_A - (P_A - R_A) \tag{5}$$

And a non-negative surplus would lead to the purchase:

$$S_A^2 \geq 0 \;\rightarrow\; V_A \geq P_A - R_A \tag{6}$$

Since

$$P_A - R_A < P_A - \frac{R_A}{2} \tag{7}$$

All consumers who bought from *A* in period one would repeat the purchase. Thus,

$$\Pi_A^2 = \Pi_A^1 = 1 - P_A + \frac{R_A}{2} \tag{8}$$

Consequently, the *TP* for firm *A* in period two is

$$TP_A^2 = (P_A - R_A)(1 - P_A + \frac{R_A}{2}) \tag{9}$$

Then, the sum of *TP* for firm *A* in both periods is

$$TP_A = TP_A^1 + TP_A^2 = (2P_A - R_A)\left(1 - P_A + \frac{R_A}{2}\right) \tag{10}$$

Firm *A* maximizes its total profits over the two periods by setting the optimal reward value.

$$\max_{R_A} TP_A = TP_A^1 + TP_A^2 = (2P_A - R_A)\left(1 - P_A + \frac{R_A}{2}\right) \tag{11}$$

$$s.t. \begin{cases} 0 < P_A \leq 1 \\ 0 < R_A \leq 1 \\ P_A \geq R_A \end{cases}$$

After confirming the concavity of the *TP* function (available in Appendix A), the optimal reward value can be obtained by solving the first-order condition (FOC). And we have

$$R_A^* = 2P_A - 1 \tag{12}$$

Also, it is necessary to determine the range for the parameter (P_A):

$$\begin{cases} 0 < R_A \leq 1 \rightarrow 0 < 2P_A - 1 \leq 1 \\ 0 \leq P_A \leq 1 \end{cases} \text{, so } 0.5 < P_A \leq 1 \tag{13}$$

We can now parameterize TP_A by substituting the optimized reward back, and

$$TP_A^* = 0.5 \quad (14)$$

Interestingly enough, the optimized expression of TP_A is equivalent to that in the case where firm A chooses to optimize its price P_A, without the help of a loyalty program. As a result, we can reach the following proposition.

Proposition 1: ***If a monopoly firm operates without any loyalty program, it will gain flexibility in pricing by introducing an independent loyalty program.***

The demonstration of Proposition 1 is available in the appendix B. It suggests that the monopoly firm can achieve optimal profit by setting a desired reward value, making the pricing decision versatile rather than rigid. This is clear as the comparative statics shows

$$\frac{\partial TP_A^*}{\partial P_A} = 0 \quad (15)$$

The optimal TP_A doesn't change as P_A does. Obviously, having an independent loyalty program benefits the monopoly firm in multiple ways.

First, changing prices in real world can be inconvenient and costly. Nominal prices are "sticky" and incur costs when updated (Keynes, 2018), so firms avoid them by maintaining the same prices, deviating from the profit-maximization pricing decisions. For example, restaurants are reluctant to update their menus as the brochures impose significant costs if reprinted seasonally or monthly and underpricing certain dishes when the market prices for them have increased hurts profit. Thanks to the loyalty program, firms can adjust the reward value to obtain optimal profits and keep using the same price tags and menus. Thus, restaurants give out coupons for customer retention by guaranteeing a certain discount of percentage or dollar amount if consumption reaches a threshold next time. In this way, the restaurants save money by keeping menus and printing inexpensive coupons and more importantly, optimize profits through a loyalty program.

Second, we have normalized both firms' costs to zero throughout non-coalition and coalition cases. However, in reality, costs are omnipresent, from production to after-sale customer service. Therefore, the prices need rearranging to maintain the profit margin for the monopoly, and that's where firms find a loyalty program valuable. While typically utilized by the marketing department for customer retention and acquisition, a loyalty program also balances out costs, an extra tool for the operations department besides pricing strategies. Consequently, optimizing the reward value for profit maximization requires joint effort by both departments, promoting collaboration and synergy.

Last but not least, customers tend to feel more comfortable accepting changes in reward values than in prices as they treat the reward as “extra gain” from the monopoly. Hence, profit maximization through reward optimization alleviates customers’ averseness towards the monopoly. Take the sale of airline tickets as an example, a price drop would displease the consumers who just completed the purchase, harming the goodwill of the company. In addition, rational consumers would anticipate further decrease of the ticket prices and delay purchases, driving the airline’s business into a vicious circle. With the help of a loyalty program, the airline could issue consumers points as a reward that can be redeemed later for numerous benefits, an “extra gain” that can stimulate loyalty behavior and enhance the airline’s goodwill. Furthermore, the company doesn’t have to alter its ticket prices anymore, preserving a good relationship with customers. In short, a loyalty program maintains customer relationship and induces new purchases, leaving the prices intact.

From equation (12), we can derive the relationship between P_A and the optimized reward.

$$\frac{\partial R_A^*}{\partial P_A} = 2 > 0 \tag{16}$$

It is intuitive that as P_A increases, R_A^* rises as well in order to comfort customers and thus, maintain and improve monopoly’s profitability.

For firm *B*, its optimized *TP* is symmetrically 0.5, and a coalition could exist when *TP* for the coalition is greater than or equal to

$$TP_A^* + TP_B^* = 1 \tag{17}$$

Now, we can look at the social welfare: The key is to compare total surpluses under the two cases. For the non-coalition case, the total surplus is (we assume $R_A = R_B$),

$$1\left(1-\ P_A+\ \frac{R_A}{2}\right)+1\left(1-\ P_B+\ \frac{R_B}{2}\right)-\left(1-\ P_A+\ \frac{R_A}{2}\right)*\left(1-\ P_B+\ \frac{R_B}{2}\right)$$
$$=1-P_A*P_B+\frac{R}{2}(P_A+P_B)-\frac{R^2}{4} \quad (18)$$

Chapter 4. Coalition Case: two firms in different industries have a joint LP

Firm A and B have decided to form a coalition for enhanced profitability. The coalition seeks to maximize the joint profit over two periods with one integrated loyalty program, similar to Plenti (Fruend, 2017). Specifically, firm A will be the focal firm and firm B would abandon its own LP. Thus, the rewarding scheme will be unified, $R_A = R_B = R$, and adjusted accordingly: if a customer purchases from firm A or B (or both) in the first period and purchases from firm A in the second period, s/he would receive "R" (or double "R") from the coalition. Otherwise, consumers receive no reward. This setting mimics reality as it has an asymmetric structure, a natural result of the collaboration between firms in disparate industries (Delta Airlines, 2019). Consumers are still forward-looking in making the purchase decisions and would proceed if anticipating a nonnegative surplus. The assumptions of zero marginal cost for the coalition and rewarding rationality ($P_i \geq R$) hold.

There are two possible scenarios within the coalition case. First, we probe the one where $P_A + P_B - R \leq 1$.

Scenario One: $\boldsymbol{P_A + P_B - R \leq 1}$

---Period One:

Based on the model setup and assumptions, customers' surplus of buying a unit from A in period one is:

$$S_A^1 = V_A - P_A + V_A - (P_A - R) = 2V_A - 2P_A + R \quad (19)$$

And customers' surplus of buying a unit from B in period one is:

$$S_B^1 = V_B - P_B + V_A - (P_A - R) \quad (20)$$

Since the two independent LPs are integrated, it is critical to consider the case where a customer buys from both firms in period one, aiming to receive two rewards in period two. Therefore, on the one hand, the surplus for a customer who has already bought from A and considers buying from B is S_{AB}^1 = surplus of buying both minus S_A^1:

$$S_{AB}^1 = V_A - P_A + V_B - P_B + V_A - (P_A - 2R) - S_A^1 = V_B - P_B + R \quad (21)$$

On the other hand, the surplus for a customer who has already bought from B and considers buying from A is S_{BA}^1 = surplus of buying both minus S_B^1:

$$S_{BA}^1 = V_A - \ P_A + V_B - \ P_B + V_A - (P_A - 2R) - S_B^1 = V_A - P_A + R \tag{22}$$

Purchases will occur if the anticipated surplus is nonnegative. Figure Two illustrates the product valuations and purchase probabilities:

According to the figure, we are ready to calculate the probabilities (shaded areas) of purchase in period one.

1. the probability of purchasing from Firm *A* only is the trapezoid in orange:

$$\Pi_A^1 = \frac{1}{2}\left(P_A - P_A + \frac{R}{2} + P_B + P_A - R - P_A + \frac{R}{2}\right)(P_B - R) = \frac{1}{2}P_B(P_B - R) \tag{23}$$

2. the probability of purchasing from Firm *B* only is the trapezoid in green:

$$\Pi_B^1 = \frac{1}{2}(P_A - R)(2 + R - 2P_B - P_A) \tag{24}$$

3. the probability of purchasing from both firms is in blue:

$$\Pi^1 = 1 + R - P_A - \frac{1}{2}P_B^2 + \ \frac{1}{8}\ R^2 \tag{25}$$

The total profit for period one (TP^1) would equal to total revenue:

$$TP^1 = P_A\Pi_A^1 + P_B\Pi_B^1 + (P_A + P_B)\Pi^1 \tag{26}$$

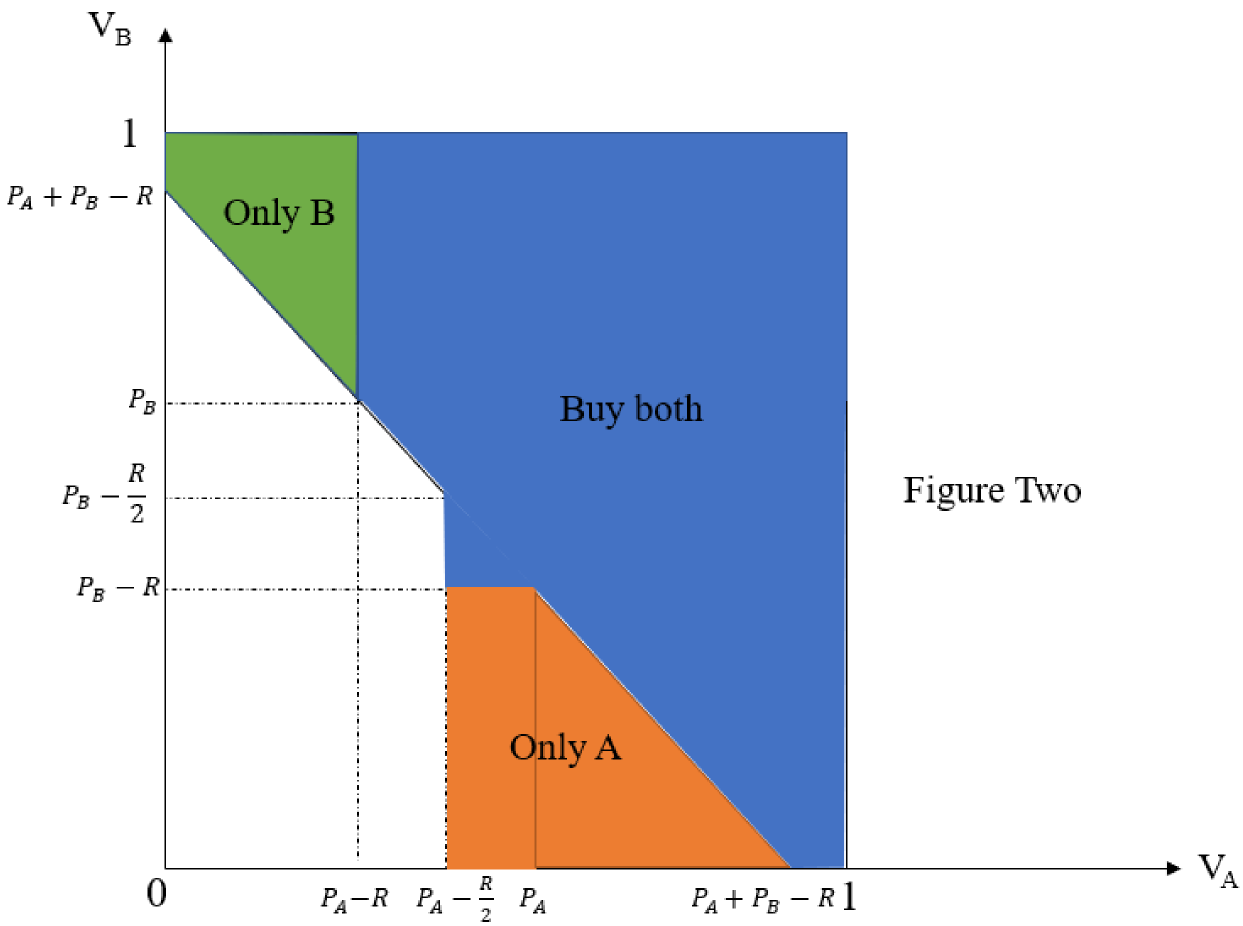


Figure 2. product valuations and purchase probabilities when $P_A + P_B - R \leq 1$

---Period Two:

Consumers who have made a purchase/purchases in period one need to decide whether to purchase from Firm *A* to claim the reward/rewards.

1) For customers who only bought from Firm *A* in period one, their valuation is

$$S_A^2 = V_A - (P_A - R) \tag{27}$$

For the purchase to take place, the surplus must be nonnegative:

$$S_A^2 \geq 0 \rightarrow V_A \geq P_A - R \tag{28}$$

It's easy to observe that $P_A - R < P_A - \frac{R}{2}$, so all the customers who only bought *A*'s product would repeat the purchase:

$$\Pi_A^2 = \Pi_A^1 = \frac{1}{2}P_B(P_B - R) \tag{29}$$

2) For customers who only bought from Firm *B* in period one, their valuation is

$$S_B^2 = V_A - (P_A - R) \tag{30}$$

For the purchase to take place, the surplus must be nonnegative:

$$S_B^2 \geq 0 \rightarrow V_A \geq P_A - R \tag{31}$$

However, we can observe from Figure Two that consumers who only bought *B*'s product have $V_A \leq P_A - R$. Thus,

$$\Pi_B^2 = 0 \tag{32}$$

3) For customers who bought from both firms in period one, their valuation is

$$S_{both}^2 = V_A - (P_A - 2R) \tag{33}$$

For the purchase to take place, the surplus must be nonnegative:

$$S_{both}^2 \geq 0 \rightarrow V_A \geq P_A - 2R \tag{34}$$

It is obvious that $P_A - 2R < P_A - R$, so all the customers who bought from both firms in period one would purchase from Firm *A* in period two. Thus,

$$\Pi^2 = \Pi^1 \tag{35}$$

Besides buying from *A*, customers decide if they would like to purchase from *B* in period two, even though they would not receive any rewards:

$$S_{BB}^2 = V_B - P_B \tag{36}$$

And consumers would make a purchase of *B* if

$$S_{BB}^2 \geq 0 \rightarrow V_B \geq P_B \tag{37}$$

Thus, the probability (area) of purchasing from Firm *B* is

$$\Pi_{BB}^2 = 1(1 - P_B) = 1 - P_B \tag{38}$$

Now we can calculate the *TP* for the coalition in period two:

$$TP^2 = (P_A - R)\,\Pi_A^2 + (P_A - 2R)\Pi^2 + P_B\Pi_{BB}^2 \tag{39}$$

The coalition maximizes its total profits over the two periods by setting the optimal reward value.

$$\max_R TP = TP^1 + TP^2 \tag{40}$$

$$= 2P_A + 2P_B - 2P_A^2 - P_B^2 - \frac{1}{2} P_B^3 - \frac{1}{2} P_A^2 P_B + 4P_A R + \frac{1}{4} P_A R^2 + \frac{3}{2} P_B^2 R - P_A P_B^2 - 2R - 2R^2 - \frac{1}{4} R^3 + \frac{1}{8} P_B R^2$$

$$s.t. \begin{cases} 0 < R \leq 1 \\ 0 < P_A \leq 1 \\ 0 < P_B \leq 1 \\ P_A \geq R \\ P_B \geq R \\ P_A + P_B - R \leq 1 \end{cases}$$

After confirming the concavity of the *TP* function (available in Appendix C), the optimal reward value can be obtained by solving the FOC. And we have

$$R^* = \frac{1}{6}(-16 + 2P_A + P_B) + \frac{1}{6}\sqrt{160 + 128P_A + 4{P_A}^2 - 32P_B + 4P_A P_B + 73{P_B}^2} \quad (41)$$

if 1) $0 < P_B < \frac{2}{3}$ and $\frac{1}{8}(4 - 3{P_B}^2) < P_A < \frac{4 + 8P_B - 2{P_B}^2}{8 + P_B}$;

or 2) $\frac{2}{3} < P_B < 0.7192$ and $\frac{1}{2}(4 - 3P_B) - \frac{1}{2}\sqrt{36 - 68P_B + 25{P_B}^2} < P_A < \frac{4 + 8P_B - 2{P_B}^2}{8 + P_B}$;

or 3) $0.7192 < P_B < 0.72$ and $\frac{1}{2}(4 - 3P_B) - \frac{1}{2}\sqrt{36 - 68P_B + 25{P_B}^2} < P_A < \frac{1}{2}(4 - 3P_B) + \frac{1}{2}\sqrt{36 - 68P_B + 25{P_B}^2}$.

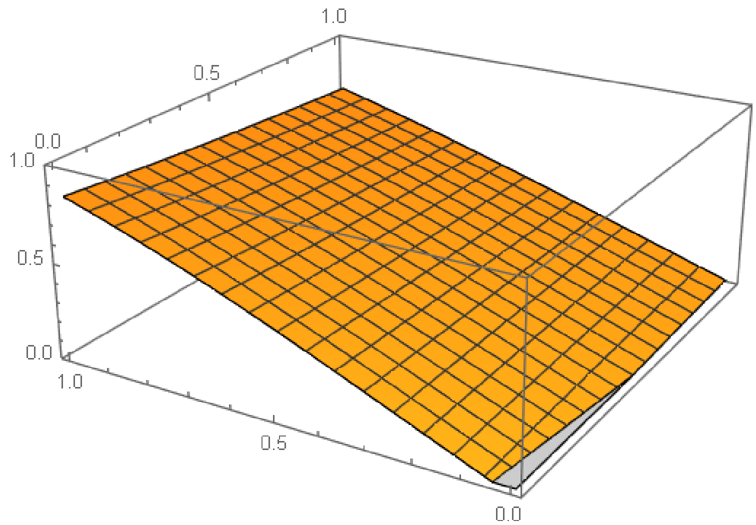
Parameter P_B has three sub-ranges, and P_A has three sub-ranges accordingly. It is pertinent to probe how prices affect the optimal reward and *TP*.

$$\frac{\partial R^*}{\partial P_A} = \frac{1}{3} + \frac{\frac{1}{12}(128 + 8P_A + 4P_B)}{\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_A P_B + 73P_B^2}} \quad (42)$$

$$\frac{\partial R^*}{\partial P_B} = \frac{1}{6} + \frac{\frac{1}{12}(-32 + 4P_A + 146P_B)}{\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_A P_B + 73P_B^2}} \quad (43)$$

Observing equations (42) and (43), we can reach the following proposition.

Proposition 2: ***The optimal reward of a monopolistic coalition will increase as the product prices rise.***



Plot 1: graph of $\frac{\partial R^*}{\partial P_B}$

Holding P_B constant, if P_A increases in its corresponding range, R^* will increase since $\frac{\partial R^*}{\partial P_A} > 0$ for any values of P_A and P_B. Similarly, holding P_A constant, if P_B increases, R^* will increase as $\frac{\partial R^*}{\partial P_B} >$ 0 for most values of P_A and P_B, according to the graphic demonstration of equation (43) (Plot One). Such results are plausible because the increase of P_A gives the coalition an incentive to raise the optimal reward level to attract and retain more customers for a higher profit. On the contrary, raising P_B does not improve the optimized reward if P_A is smaller than 0.5 in our model. In other words, variations of P_B would not affect the optimized reward if P_A is too small due to the fact that firm A is recognized and designed to be the focal point of the coalition.

Given the analytical results, it would not be arduous to predict the course of actions from the department head of marketing within a coalition: if prices of product grow, she/he would boost the level of optimized reward to seize the opportunity of accruing profits. We can take the coalition between United Airlines (UA) and Chase Bank (Chase) as an example. Due to the COVID-19, Heathrow Airport in the UK added a twelve-dollar “regulatory charge” to all tickets (Schlappig, 2021a). Unfortunately, “the airport claims that the airlines have asked this tax to simply be added to ticket prices.” (Schlappig, 2021a). Therefore, prices of tickets for UA have inflated by twelve dollars for the consumers. Fearing of impairing customer loyalty and keen to survival during the pandemic, the coalition launched a promotion on purchased points with a bonus of points up to 100% (Schlappig, 2021b) in order to combat potential profit loss because of the price inflation, a decision forecasted by our analytical model. To be frank, it is likely that the raised regulatory fee is not the only motivation for the decision of point promotion. However, this exemplifies the fact that changes of product prices can be external, elevating the external validity of the economic model.

Next, we look at how changes in prices affect optimized total profit.

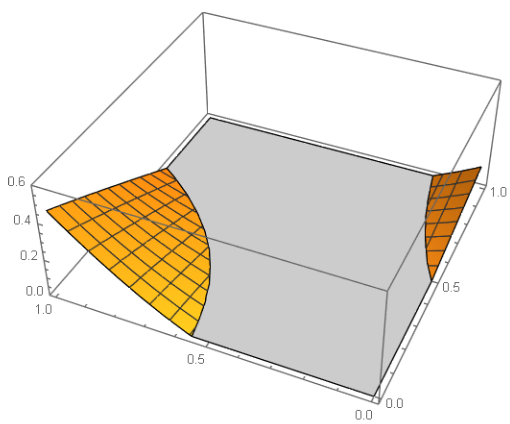
$$\begin{aligned}\frac{\partial TP^*}{\partial P_A} = \frac{1}{\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}}(71.1 + 0.11P_A^3 + 1.0139P_B^3 \\ - 5.78\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2} + P_B^2(32 \\ - 0.486\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}) + P_A^2(5.3 \\ + \frac{1}{6}P_B + 0.056\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}) \\ + P_B(-12 + 0.22\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}) \\ + P_A(61.3 + 2.083P_B^2 \\ - 2.2\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2} + P_B(2.67 \\ - 0.94\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2})))\end{aligned} \tag{44}$$

$$\frac{\partial^2 TP^*}{\partial P_A^2} = \frac{1}{\sqrt{160. + 128.A + 4.A^2 - 32.B + 4.AB + 73.B^2}}(71.1111111111111 + 0.1111111111111109A^3 + 1.0138888888888888B^3 - 5.777777777777777\sqrt{160. + 128.A + 4.A^2 - 32.B + 4.AB + 73.B^2} + B^2(32. - 0.4861111111111111\sqrt{160. + 128.A + 4.A^2 - 32.B + 4.AB + 73.B^2}) + A^2(5.333333333333333 + 0.16666666666666666B + 0.05555555555555555\sqrt{160. + 128.A + 4.A^2 - 32.B + 4.AB + 73.B^2}) + B(-12. + 0.2222222222222222\sqrt{160. + 128.A + 4.A^2 - 32.B + 4.AB + 73.B^2}) + A(61.33333333333333 + 2.0833333333333333B^2 - 2.22222222222222223\sqrt{160. + 128.A + 4.A^2 - 32.B + 4.AB + 73.B^2} + B(2.6666666666666665 - 0.9444444444444444\sqrt{160. + 128.A + 4.A^2 - 32.B + 4.AB + 73.B^2}))) \tag{45}$$

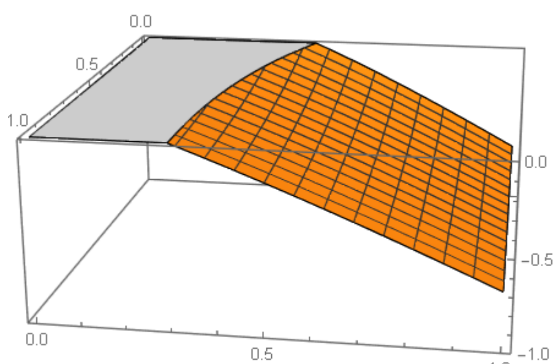
$$\frac{\partial TP^*}{\partial P_B} = \frac{1}{\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}}(-17.78 + 0.056P_A^3 + 37.01P_B^3 + 3.4\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2} + P_B(84.67 - 10.2\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}) + P_B^2(-24.3 - 0.74\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}) + P_A^2(1.33 + 2.083P_B - 0.472\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}) + P_A(-12 + 3.0417P_B^2 + 0.2\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2} + P_B(64 - 0.972\sqrt{160 + 128P_A + 4P_A^2 - 32P_B + 4P_AP_B + 73P_B^2}))) \tag{46}$$

$$
\begin{aligned}
\frac{\partial^2 TP^*}{\partial P_B^2} = {} & 1/Sqrt[4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2-32.B \\
& +160.]\,(A\text{^}2\,(2.08333\;-(0.236111\,(4.A+146.B \\
& -32.))/Sqrt[4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2-32.B+160.]) \\
& +A\,(-((0.486111\,B\,(4.A+146.B-32.))/Sqrt[4.A\text{^}2+4.A\,B \\
& +128.A+73.B\text{^}2-32.B+160.])-0.972222\,Sqrt[4.A\text{^}2 \\
& +4.A\,B+128.A+73.B\text{^}2-32.B+160.]+(0.111111\,(4.A \\
& +146.B-32.))/Sqrt[4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2-32.B \\
& +160.]+6.08333\,B+64.)+2\,B\,(-0.743056\,Sqrt[4.A\text{^}2+4.A\,B \\
& +128.A+73.B\text{^}2-32.B+160.]-24.3333) \\
& -10.2222\,Sqrt[4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2-32.B+160.] \\
& -(0.371528\,B\text{^}2\,(4.A+146.B-32.))/Sqrt[4.A\text{^}2+4.A\,B \\
& +128.A+73.B\text{^}2-32.B+160.]-(5.11111\,B\,(4.A+146.B \\
& -32.))/Sqrt[4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2-32.B+160.] \\
& +(1.72222\,(4.A+146.B-32.))/Sqrt[4.A\text{^}2+4.A\,B+128.A \\
& +73.B\text{^}2-32.B+160.]+111.021\,B\text{^}2+84.6667) \\
& -1/(2\,(4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2-32.B \\
& +160.)\text{^}(3/2))\,(4.A+146.B \\
& -32.)\,(A\,(B\,(64.-0.972222\,Sqrt[4.A\text{^}2+4.A\,B+128.A \\
& +73.B\text{^}2-32.B+160.])+0.222222\,Sqrt[4.A\text{^}2+4.A\,B \\
& +128.A+73.B\text{^}2-32.B+160.]+3.04167\,B\text{^}2-12.) \\
& +B\,(84.6667\;-10.2222\,Sqrt[4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2 \\
& -32.B+160.])+A\text{^}2\,(-0.472222\,Sqrt[4.A\text{^}2+4.A\,B+128.A \\
& +73.B\text{^}2-32.B+160.]+2.08333\,B+1.33333) \\
& +B\text{^}2\,(-0.743056\,Sqrt[4.A\text{^}2+4.A\,B+128.A+73.B\text{^}2-32.B \\
& +160.]-24.3333)+3.44444\,Sqrt[4.A\text{^}2+4.A\,B+128.A \\
& +73.B\text{^}2-32.B+160.]+0.0555556\,A\text{^}3+37.0069\,B\text{^}3 \\
& -17.7778)
\end{aligned}
\tag{47}
$$

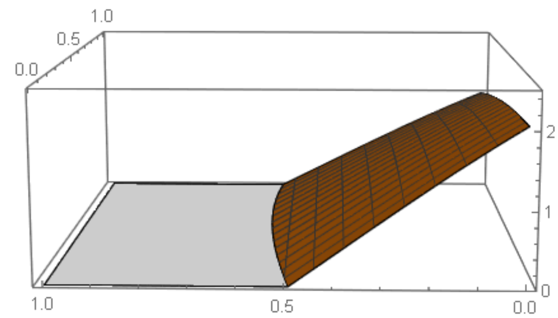
Plot Two describes equation (44): Holding P_B constant and greater than 0.55, the optimized total profit for the coalition will increase since $\frac{\partial TP^*}{\partial P_A} > 0$ for $P_A \in [0, 0.7]$. Meanwhile, if P_B is sufficiently small, optimized TP increases when P_A is bigger than 0.5. Furthermore, the total profit function is concave in P_A when P_B is greater than approximately 0.45 as $\frac{\partial^2 TP^*}{\partial P_A^2}$ (SOC) < 0, according to Plot Three. From equation (45) and the corresponding Plot Four, we can notice that holding P_A constant, the optimized TP will increase if P_B is smaller than 0.5 as $\frac{\partial TP^*}{\partial P_B} > 0$. In addition, the TP function is concave in P_B within its general domain since $\frac{\partial^2 TP^*}{\partial P_B^2}$ (SOC) < 0 from Plot Five.



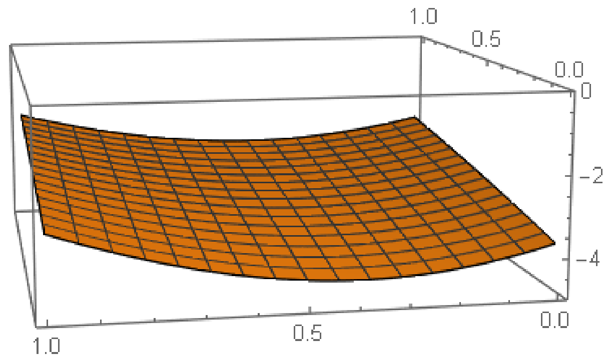
Plot 2: graph of $\frac{\partial TP^*}{\partial P_A}$



Plot 3: graph of $\frac{\partial^2 TP^*}{\partial P_A^2}$



Plot 4: graph of $\frac{\partial TP^*}{\partial P_B}$



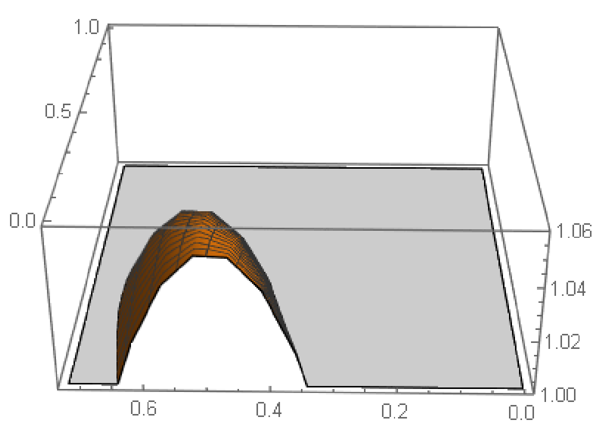
Plot 5: graph of $\frac{\partial^2 TP^*}{\partial P_B^2}$

Based on Plots Two-Five, we can generate the following proposition.

Proposition 3: ***It is the auxiliary product/joining firm that has a more significant effect on reshaping the optimized total profit for the coalition, around the middle of the valuation distribution. Meanwhile, the price change of the main product/focal firm would not be a dominant influencer on the optimized total profit.***

This finding seems surprising and counter-intuitive at first sight. Nevertheless, observation from the Plots provides plausible explanation. It is obvious from Plot Two that P_A and P_B cannot be high concurrently; while one firm has a relatively high price, the other firm has to have a low price to allow a growing optimized profit. Moving on to Plot Three, if product price for firm B is low (less than 0.45), the total profit function would be convex in P_A: the optimized total profit increases with P_A indefinitely in order to make up for the loss of profit due to underpricing of P_B. In other words, if P_B is too small, the Law of Diminishing Returns does not apply to the P_A - TP^* relationship. Thus, P_B has to take on value/values that are moderately high. But what value/values of P_B lead to the coalition's globally optimized profit? Plot Four indicates that values around 0.5 sound proper because exceeding those would result in a TP^* that is not a global maximum. By contrast, values of P_A does not affect the concavity of P_B on TP^* (see Plot Five), meaning that P_A does not confine the choice of P_B, granting a larger degree of freedom and suggesting less influence on shaping the optimized total profit than P_B does. (We can name the optimized total profit for the coalition as profit ceiling as it represents the highest attainable profit given P_A and P_B.) In conclusion, it would be tough for a coalition to achieve a global profit ceiling if P_B (price of the auxiliary product) is too high or low. On the contrary, if P_B lies around the middle of the valuation distribution, a coalition would be able to obtain a global profit ceiling. As firm *A* being the focus of the coalition, a high P_B may discourage customers with low valuation on *B*'s product from purchasing. Also, a low P_B may induce too many purchases and over redemption of the reward, shrinking the profit ceiling.

Proposition 3 explicates the reason for the coalition between Delta Air Lines (Delta) and American Express (Amex) to raise the annual fees for several of its co-branded credit cards (White, 2020). For the consumers, the main goal is to satisfy the purchase requirement for the opening bonus: "spend \$3000 within 90 days of the card opening and get 100,000 points/miles" (White, 2020). Meantime, they have to pay fees associated with the open status and usage of the card, such as Annual Percentage Rate, foreign transaction fee, and annual fee. Therefore, the coalition treats the spending target as the main price to pay (P_A, equivalently) and the associated fees as the auxiliary price (P_B). The decision to raise the annual fees by the operations department aims to enlarge the chance for the coalition to achieve a higher optimized *TP* to combat the severely dropped demand for travel during the pandemic.



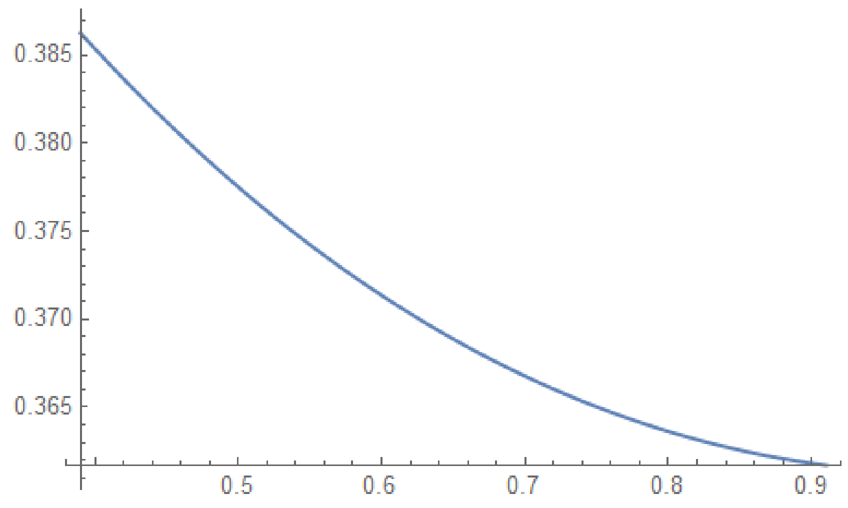
Plot 6: Numerical Experiment of TP^*

Now we can proceed to the numerical experiment of the profit ceilings (TP^*) to map out accurate ranges for P_B and P_A that initiate a superior TP^*. According to Plot Six, P_B ranges from 0 to 0.72 and $P_A \in [0,1]$, and the coalition can generate a TP^* bigger than 1 (non-coalition case) when P_B is between 0.34 and 0.66. However, since the range of P_A depends on the value of P_B, Plot Six should be chopped as P_B and P_A evolve. As a result, the range for TP^* should shrink accordingly. For example, the first TP^* bigger than 1 actually appears at $P_B = 0.42$, but not 0.34. Also, the corresponding range for P_A is [0.43385, 0.832] but not (0, 1]. In addition, the coalition will not produce a superior TP^* if P_B exceeds 0.66. The range [0.42, 0.66] for P_B reconciles with Plots Four and Five that the *TP* function is concave in P_B and centered around P_B = 0.5. Furthermore, the global profit ceiling for the coalition in this $P_A + P_B - R \leq 1$ scenario is about 1.03 when $P_B \in [0.55, 0.56]$, a value lower than that in Plot Six. This result echoes the finding in Proposition 3: A global profit ceiling appears around the middle of the valuation distribution of the auxiliary product price (P_B).

Valuation of P_A depends on P_B due to the design of the coalition, and according to Plot Two, for $P_B \in [0.42, 0.66]$, $\frac{\partial TP^*}{\partial P_A} < 0$ as all the corresponding ranges for P_A fall under the grey area. Thus, holding P_B constant within [0.42, 0.66], the TP^* would decrease as P_A increases, reducing the global profit ceiling and shortening the spectrum of superior TP^*s. As per this result, we can come to the subsequent proposition.

Proposition 4: ***Given an optimal auxiliary product price, the optimized total profit decreases as the main product price increases within its feasible range, regardless of the elevation of the optimized reward.***

This proposition seems counter-intuitive as well, but we can get a hint from Proposition 2: the optimized reward (R^*) increases as P_A does. What is more, from Plot Seven we can see that as P_A increases within its feasible range, [0.39, 0.91], the difference between P_A and R^* decreases; the growth rate for R^* is higher than that of P_A. Still, TP^* decreases as P_A rises, implying that the increased reward level for customer acquisition cannot keep up with the loss of demand due to price inflation. Therefore, it is plausible to conclude that P_A has a more significant effect on deciding the profit ceiling than the optimal reward. When the external main product price fluctuates, the marketing manager of the coalition will most likely rearrange the reward in the same direction as predicted in the Proposition 2 so as to maintain and enhance profitability. Unfortunately, such effort will not outcompete the more magnified changes of P_A.



Plot 7: Difference between P_A and R^* as P_A increases when P_B = 0.55

For the coalition case, when $P_A + P_B - R \leq 1$, total surplus is the total area "1" minus the unfilled trapezoid area.

$$1 - \left(0.5 * (P_B - R/2 + P_A + P_B - R) * (P_A - R/2)\right)$$
$$= 1 - 0.375R^2 + RP_A - 0.5P_A^2 + 0.5RP_B - P_AP_B \quad (48)$$

If we take the difference of equations (46) and (18), then we have,

$$(46) - (18) = -0.125R^2 + 0.5RP_A - 0.5P_A^2 \quad (49)$$

Since $R > 0$ and $P_A \geq R$, so (46) < (18), the social welfare/total surplus shrinks as two firms form a coalition.

Scenario Two: $1 < P_A + P_B - R < 2$

---Period One:

The expressions for consumers' valuations are the same as in Scenario One, and we are here to discuss the other possible range of $P_A + P_B - R$.

$$S_B^{11} = V_B - P_B + V_A - (P_A - R) \tag{50}$$

S_B^{11} is the valuation function for consumers who consider purchasing from Firm B in period one for this scenario. For the purchase to take place, the surplus must be nonnegative. And since V_B and V_A range from 0 to 1, $P_A + P_B - R$ could be greater than 1. Consequently, the graphic illustration, areas as probability, and optimal solutions need adjusting accordingly. We have converted the valuations to Figure Three for this scenario:

By Figure Three, we are ready to calculate the NEW probabilities (shaded areas) of purchase in period one.

1. the probability of purchasing from Firm A only is the combination of a rectangle and a trapezoid in orange:

$$\Pi_A^{11} = (P_B - R)\frac{R}{2} + \frac{1}{2}(1 - P_A)(P_A + P_B - R - 1 + P_B - R) \tag{51}$$

2. the probability of purchasing from Firm B only is the triangle in blue:

$$\Pi_B^{11} = \frac{1}{2}(1 - P_B)(1 - P_B) \tag{52}$$

3. the probability of purchasing from both firms is in green:

$$\Pi^{11} = \frac{1}{2}(1 - P_A - P_B + R + 1)^2 - \Pi_B^{11} + \frac{1}{8} R^2 \tag{53}$$

The total profit (TP) would then equal to total revenue for the coalition:

$$TP^{11} = P_A\Pi_A^{11} + P_B\Pi_B^{11} + (P_A + P_B)\Pi^{11} \tag{54}$$

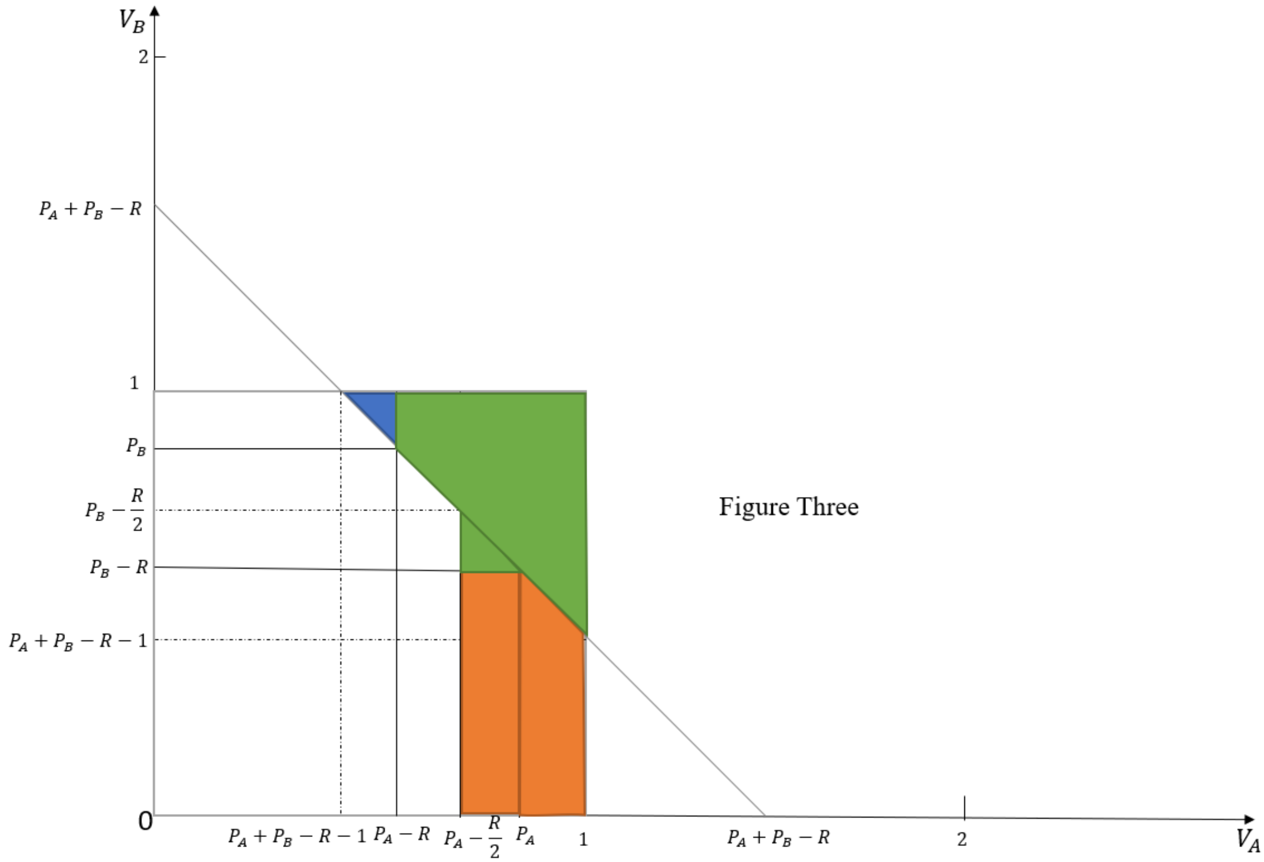


Figure 3. product valuations and purchase probabilities when $1 < P_A + P_B - R < 2$

---Period Two:

Consumers have to decide whether to purchase from Firm *A* to claim the reward/rewards.

1) For the customers who only bought from Firm *A* in period one, their valuation is

$$S_A^{22} = V_A - (P_A - R) \tag{55}$$

For the purchase to take place, the surplus must be nonnegative, and from Figure Three we can observe that $P_A - R < P_A - \frac{R}{2}$, so all the customers who bought *A*'s product would repeat the purchase:

$$\Pi_A^{22} = \Pi_A^{11} \tag{56}$$

2) For the customers who only bought from Firm *B* in period one, their valuation is

$$S_B^{22} = V_A - (P_A - R) \tag{57}$$

For the purchase to take place, the surplus must be nonnegative:

$$S_B^{22} \geq 0 \rightarrow V_A \geq \; P_A - R \tag{58}$$

However, we can observe from Figure Three that consumers who bought *B*'s product have $V_A \leq P_A - R$. Thus,

$$\Pi_B^{22} = 0 \tag{59}$$

3) For the customers who bought from both firms in period one, their valuation is

$$S_{both}^{22} = \; V_A - (P_A - 2R) \tag{60}$$

For the purchase to take place, the surplus must be nonnegative:

$$S_{both}^{22} \geq 0 \rightarrow V_A \geq \; P_A - 2R \tag{61}$$

It is obvious that $P_A - 2R < \; P_A - R$, so all the customers who bought from both firms in period one would purchase from Firm *A* in period two. Thus,

$$\Pi^{22} = \Pi^{11} \tag{62}$$

Besides buying from *A*, customers would decide if they would like to purchase from *B*, even though they would not receive any rewards. And the valuation is

$$S_{BB}^{22} = \; V_B - \; P_B \tag{63}$$

And consumers would make a purchase from *B* if

$$S_{BB}^{22} \geq 0 \rightarrow \; V_B \geq \; P_B \tag{64}$$

Thus, the probability (area) of purchasing from *B* is

$$\Pi_{BB}^{22} = 1(1 - P_B) = 1 - P_B \tag{65}$$

With all the probabilities figured out, we can now calculate the *TP* for the coalition in period two:

$$TP^{22} = (P_A - R)\, \Pi_A^{22} + \; (P_A - 2R)\Pi^{22} + \; P_B \Pi_{BB}^{22} \tag{66}$$

And the combined *TP* from both periods is

$$\max_R TP_2 = \; TP^{11} + TP^{22} = (2P_A - R)\, \Pi_A^{22} + P_B \Pi_B^{11} + (2P_A + P_B - 2R)\Pi^{22} + P_B \Pi_{BB}^{22} \tag{67}$$

$$s.t. \begin{cases} 0 < R \leq 1 \\ 0 < \; P_A \leq 1 \\ 0 < \; P_B \leq 1 \\ P_A \geq R \\ P_B \geq R \\ 1 < P_A + P_B - R < 2 \end{cases}$$

After checking the concavity of the *TP* function (available in Appendix D), it is necessary to add an additional constraint $-6 + 2.5P_A + 4.25P_B - 4.5R < 0$. The optimal reward value can be obtained by solving the FOC. And we have

$$R^* = \frac{1}{18}(-24 + 10P_A + 17P_B) + \frac{1}{18}\sqrt{216 + 240P_A + 28{P_A}^2 - 384P_B - 92P_AP_B + 145{P_B}^2} \quad (68)$$

If 1) $\frac{2}{3} < P_B < 0.7192\ and\ 5 - 3P_B - \sqrt{20 - 24P_B + 7{P_B}^2} < P_A < 0.5(4 - 3P_B) - 0.5\sqrt{36 - 68P_B + 25{P_B}^2}$;

or 2) $0.7192 < P_B < 0.72\ and\ 5 - 3P_B - \sqrt{20 - 24P_B + 7{P_B}^2} < P_A < 0.5(4 - 3P_B) - 0.5\sqrt{36 - 68P_B + 25{P_B}^2}$;

or 3) $0.7192 < P_B < 0.72\ and\ 0.5(4 - 3P_B) + 0.5\sqrt{36 - 68P_B + 25{P_B}^2} < P_A < 1$;

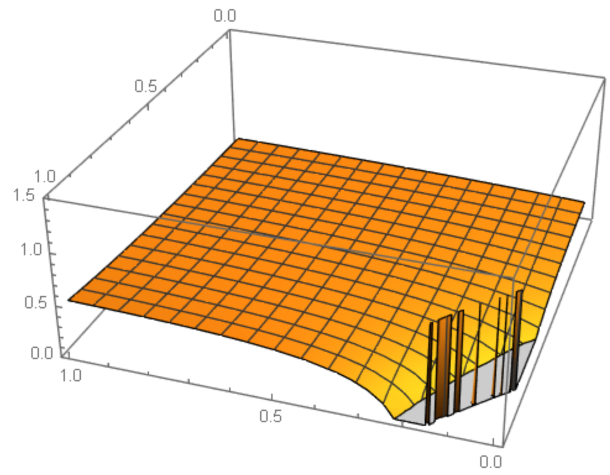
or 4) $0.72 < P_B < 1\ and\ 5 - 3P_B - \sqrt{20 - 24P_B + 7{P_B}^2} < P_A < 1$.

Parameter P_B has four sub-ranges, and P_A has four sub-ranges accordingly. It is pertinent to probe how prices affect R^* and TP_2^*.

$$\frac{\partial R^*}{\partial P_A} = 0.56 + \frac{0.0278(240 + 56P_A - 92P_B)}{\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2}} \quad (69)$$

$$\frac{\partial R^*}{\partial P_B} = 0.94 + \frac{0.0278(-384 - 92P_A + 290P_B)}{\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2}} \quad (70)$$

Observing equations (65) and (66), we draw a conclusion similar to Proposition 2: Holding P_B constant, if P_A increases in its corresponding range, R^* will increase since $\frac{\partial R^*}{\partial P_A} > 0$ for any values of P_A and P_B. Similarly, holding P_A constant, if P_B increases, R^* will increase as $\frac{\partial R^*}{\partial P_B} > 0$ for most values of P_A and P_B, according to the graphic demonstration of equation (66) (Plot Eight).



Plot 8: graph of $\frac{\partial R^*}{\partial P_B}$

Such results are plausible because the increase of P_A gives the coalition an incentive to raise the optimal reward level to attract and retain more customers for a higher profit. On the contrary, raising P_B does not improve the optimized reward if P_A is smaller than 0.25 in our model. Given the analytical results similar to Scenario One's, the department head of marketing within a coalition would be expected to boost the level of optimized reward to seize the opportunity of accruing profits if prices of products grow.

Next, we look at how changes in prices affect optimized total profit.

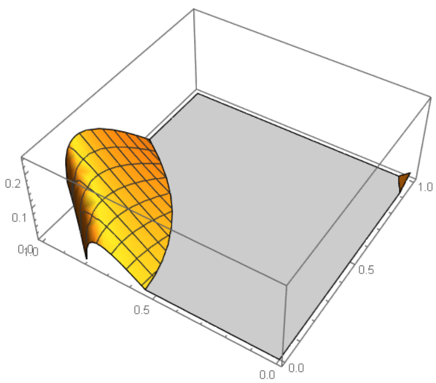
$$
\begin{aligned}
\frac{\partial TP_2^*}{\partial P_A} = {} & \frac{1}{\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2}}(20 + 0.60P_A^3 \\
& - 5.15P_B^3 - 1.6\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2} \\
& + P_B^2(27.056 \\
& - 0.159\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2}) \\
& + P_A^2(7.78 - 2.98P_B \\
& - 0.062\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2}) \\
& + P_B(-43.2 \\
& + 2.09\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2}) \\
& + P_A(26.89 + 6.4P_B^2 \\
& - 0.815\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2} \\
& + P_B(-25.3 \\
& - 0.654\sqrt{216 + 240P_A + 28P_A^2 - 384P_B - 92P_AP_B + 145P_B^2})))
\end{aligned} \tag{71}
$$

$$
\begin{aligned}
\frac{\partial^2 TP_2^*}{\partial P_A^2} = {} & (1/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.])(-((0.0308642\,A\text{^}2\,(56.A - 92.B + 240.))/Sqrt[28.A\text{^}2 \\
& - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B + 216.]) \\
& + A\,(-((0.32716\,B\,(56.A - 92.B + 240.))/Sqrt[28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.]) - (0.407407\,(56.A - 92.B \\
& + 240.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.]) + 2\,A\,(-0.0617284\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A \\
& + 145.B\text{^}2 - 384.B + 216.] - 2.98148\,B + 7.77778) \\
& + B\,(-0.654321\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] - 25.3333) - 0.814815\,Sqrt[28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.] - (0.0794753\,B\text{^}2\,(56.A \\
& - 92.B + 240.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] + (1.0463\,B\,(56.A - 92.B + 240.))/Sqrt[28.A\text{^}2 \\
& - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B + 216.] - (0.805556\,(56.A \\
& - 92.B + 240.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] + 1.81481\,A\text{^}2 + 6.39815\,B\text{^}2 + 26.8889) \\
& - 1/(2\,(28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.)\text{^}(3/2))\,(56.A - 92.B + 240.)\,(B\text{^}2\,(27.0556 \\
& - 0.158951\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.]) + A\text{^}2\,(-0.0617284\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A \\
& + 145.B\text{^}2 - 384.B + 216.] - 2.98148\,B + 7.77778) \\
& + A\,(B\,(-0.654321\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] - 25.3333) - 0.814815\,Sqrt[28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.] + 6.39815\,B\text{^}2 + 26.8889) \\
& - 1.61111\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.] + B\,(2.09259\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] - 43.2222) + 0.604938\,A\text{^}3 - 5.1466\,B\text{^}3 + 20.)
\end{aligned} \tag{72}
$$

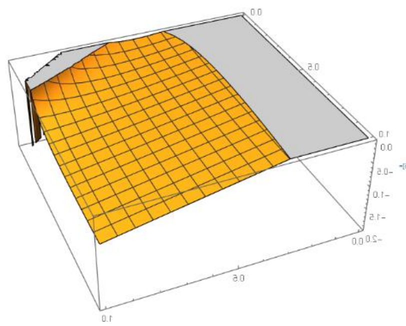
$$
\begin{aligned}
\frac{\partial TP_2^*}{\partial P_B} = {} & \frac{1}{\sqrt{216+240P_A+28P_A^2-384P_B-92P_AP_B+145P_B^2}}(-32-0.9938P_A^3 \\
& +16.22P_B^3 \\
& +4.194\sqrt{216+240P_A+28P_A^2-384P_B-92P_AP_B+145P_B^2} \\
& +P_B(81.056 \\
& -8.37\sqrt{216+240P_A+28P_A^2-384P_B-92P_AP_B+145P_B^2}) \\
& +P_A^2(-12.67+6.398P_B \\
& -0.327\sqrt{216+240P_A+28P_A^2-384P_B-92P_AP_B+145P_B^2}) \\
& +P_B^2(-64.4 \\
& +2.46\sqrt{216+240P_A+28P_A^2-384P_B-92P_AP_B+145P_B^2}) \\
& +P_A(-43.2-15.44P_B^2 \\
& +2.1\sqrt{216+240P_A+28P_A^2-384P_B-92P_AP_B+145P_B^2}+P_B(54.1 \\
& -0.32\sqrt{216+240P_A+28P_A^2-384P_B-92P_AP_B+145P_B^2})))
\end{aligned} \tag{73}
$$

$$
\begin{aligned}
\frac{\partial^2 TP_2^*}{\partial P_B^2} = {} & 1/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.]\,(A\text{^}2\,(6.39815\ - (0.16358\,(-92.A + 290.B \\
& - 384.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.]) + A\,(-((0.158951\,B\,(-92.A + 290.B \\
& - 384.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.]) - 0.317901\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] + (1.0463\,(-92.A + 290.B \\
& - 384.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.] - 30.8796\,B + 54.1111) - 8.37037\,Sqrt[28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.] + 2\,B\,(2.45756\,Sqrt[28.A\text{^}2 \\
& - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B + 216.] - 64.4444) \\
& + (1.22878\,B\text{^}2\,(-92.A + 290.B - 384.))/Sqrt[28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.] - (4.18519\,B\,(-92.A \\
& + 290.B - 384.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] + (2.09722\,(-92.A + 290.B \\
& - 384.))/Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B \\
& + 216.] + 48.669\,B\text{^}2 + 81.0556) - 1/(2\,(28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.)\text{^}(3/2))\,(-92.A + 290.B \\
& - 384.)\,(A\,(B\,(54.1111\ - 0.317901\,Sqrt[28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.]) + 2.09259\,Sqrt[28.A\text{^}2 \\
& - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B + 216.] - 15.4398\,B\text{^}2 \\
& - 43.2222) + B\,(81.0556\ - 8.37037\,Sqrt[28.A\text{^}2 - 92.A\,B \\
& + 240.A + 145.B\text{^}2 - 384.B + 216.]) \\
& + A\text{^}2\,(-0.32716\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] + 6.39815\,B - 12.6667) + 4.19444\,Sqrt[28.A\text{^}2 \\
& - 92.A\,B + 240.A + 145.B\text{^}2 - 384.B + 216.] \\
& + B\text{^}2\,(2.45756\,Sqrt[28.A\text{^}2 - 92.A\,B + 240.A + 145.B\text{^}2 \\
& - 384.B + 216.] - 64.4444) - 0.993827\,A\text{^}3 + 16.223\,B\text{^}3 - 32.)
\end{aligned} \tag{74}
$$

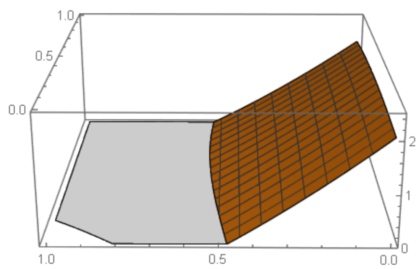
Observing Plots Nine-Twelve, we draw a conclusion similar to Proposition 3: the most prominent difference is in Plot Ten. If product price for firm B is low (less than 0.3 instead of 0.45), the total profit function would be convex in P_A.



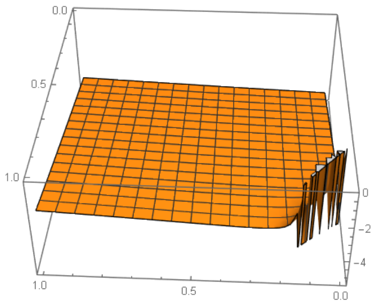
Plot 9: graph of $\frac{\partial TP_2^*}{\partial P_A}$



Plot 10: graph of $\frac{\partial^2 TP_2^*}{\partial P_A^2}$

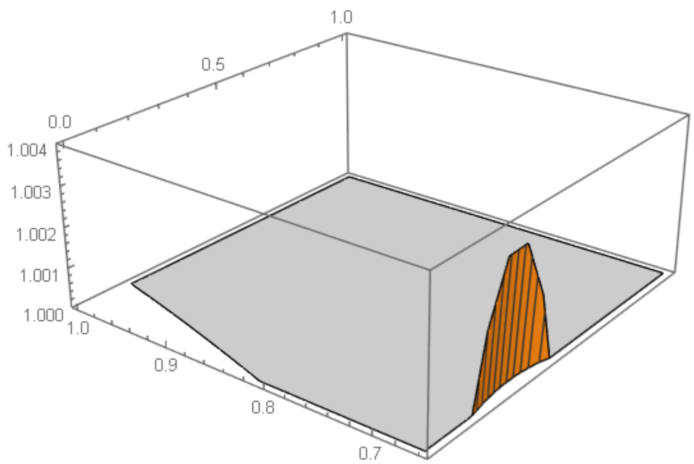


Plot 11: graph of $\frac{\partial TP_2^*}{\partial P_B}$



Plot 12: graph of $\frac{\partial^2 TP_2^*}{\partial P_B^2}$

Scenario Two describes a different relationship between P_A, P_B, and R, and it is necessary to enumerate the values of TP_2^* in various external environments. In Plot Thirteen, the range for P_B is [0.66, 1] and $P_A \in [0,1]$. The optimized *TP* of the coalition ranges up to 1.004, bigger than that in the non-coalition case when P_B is slightly higher than its lower bound. However, if we set P_B = 0.667 for instance, then P_A ranges from 0.33325 to 0.335, and none of the TP_2^* is bigger than 1. This finding holds for $P_B \in [0.66, 0.7]$ by a numerical experiment, eliminating the possibility of having a superior *TP*. Thus, it is intrinsic to state that when $1 < P_A + P_B - R < 2$, forming a coalition will not yield a greater total profit for the member firms. Compare the findings about the optimized *TP* with those in Scenario One, we induce the subsequent proposition.



Plot 13: Numerical Experiment of TP_2^*

Proposition 5: ***If the optimal reward value is far less than the sum of product prices, the coalition would not be able to generate a superior total profit, undermining the survival of the entity.***

This proposition appears to be more rational than the previous ones. On the one hand, as an auxiliary tool for profit improvement, the ability for loyalty programs to guide profits are deemed to be less influential than product prices, without considering the effect of cumulated rewards (Chun et al., 2020). Therefore, maneuvering the optimal reward value exhibits a relatively insignificant impact on profit expansion if exercised alone. On the other hand, if product prices are much higher than the reward value, then the reward is often viewed as inadequate by customers for acquisition or retention purposes, compromising the optimized *TP*. To battle against the adversity of big difference between the reward value and the sum of product prices, the marketing manager of the coalition will have to discuss the pricing strategy with the operations department before deciding the reward value. In other words, the two departments within the coalition need to cooperate to reach a decision for both the product prices and the reward value simultaneously but not sequentially in order to grant flexibility and effectiveness for the optimization process, boosting the chance for producing a better total profit. This proposition may unveil the truth why Plenti, a coalition program introduced by American Express in 2015 (Fruend, 2017), failed and ended in 2018. If we take a closer look at the reasons of a missing central focal point and different shopping patterns among various industries, it is really the immense divergence between reward value and the sum of product prices that hindered the members' sales. "It wasn't familiar", as Pearson stated (2018), customers participating in the program did not realize the value of the program and "had to wait too long for rewards" (Pearson, 2018). Consequently, the program was treated as unvaluable by customers due to insufficient advertising, and the value of the reward was low compared to the product prices for shoppers had to take excessive time to accumulate the rewards for a single redemption. These hassles ultimately devaluated the reward value and damaged the reputation of Plenti, leading to a vicious circle of sales decline.

When $1 < P_A + P_B - R < 2$, social welfare/total surplus is the total area "1" minus the unfilled area.

$$1-(1*\left(P_A-\frac{R}{2}\right)-0.5*\left(P_A-\frac{R}{2}-(P_A+P_B-R-1)\right)\left(1-P_B+\frac{R}{2}\right)) \tag{75}$$
$$=1+\frac{R}{2}-P_A+0.5(1+\frac{R}{2}-P_B)^2$$

If we take the difference of equations (71) and (18), then we have

$$(71)-(18)=0.5+R+0.375R^2-P_A-\frac{RP_A}{2}-P_B-RP_B+P_AP_B+0.5P_B^2 \tag{76}$$

It is hard to tell if the coalition creates a bigger welfare when $1<P_A+P_B-R<2$.

Proposition 6: ***If the optimal reward value is far less than the sum of product prices, it is unclear how the social welfare/total surplus compares to the one in the non-coalition case; on the contrary, the coalition would generate a total smaller surplus.***

Chapter 5. Discussion, Conclusion, and Limitation with Future Research

In this paper, we intend to probe the external and internal conditions that make a coalition more profitable than the sum of two independent LPs if it is entirely cooperative. Therefore, we design a dynamic economic model for both non-coalition and coalition cases to solve their profit maximization problems over two periods, with consumers being strategic when making purchasing decisions. The optimization problem is then solved by taking the first-order condition to further parameterize the total profit (TP) function. Afterwards, we perform comparative statics to check how exogenous variables (product prices) affect the optimized rewarding decision and TP. In the case of non-coalition, we find that a loyalty program would offer a monopoly firm more flexibility in pricing, and the optimized TP is independent of the product prices and optimal rewards. In the coalition case, there are two scenarios for the relationship between reward and the sum of product prices, and a coalition would not be able to generate a superior total profit if the optimal reward value is far less than the sum of product prices. Instead, when $P_A + P_B - R \leq 1$, the coalition would create a superior $TP^* \in [1, 1.301]$ when $P_B \in [0.42, 0.66]$, and the TP function is concave in both prices, coinciding with reality. According to the 3-D plots of comparative statics, we discover that the optimal reward of a monopolistic coalition will increase as the product prices rise. What is more, the auxiliary product would have a more significant effect on shaping the optimized total profit compared with the main product. Surprising but reasonable, given an optimal auxiliary product price, the optimized total profit decreases as the main product price increases within its feasible range.

This research contributes to academia by offering a preliminary study on coalition regarding customer loyalty; it uses a simplified yet illustrative economic model to simulate the interaction between the coalition and its customers, examining the external and internal conditions that would make a collaboration sustainable. Such original work paves the way for future research to advance for more sophisticated study on collaborative business partnerships. Also, this paper guides the practitioners within the coalition identifying break-even points, facilitating the generation of strategic operational and marketing plans, and thus, securing the success and continuance of the coalition in the long run.

As the first analytical attempt to study the coalition business model, this paper has a few limitations, mainly due to the simplifications of our model for tractability, and later research can build upon it by lifting them. First, we set prices as exogenous rather than endogenous, which may be unrealistic since a firm's marketing or OM department often treats both prices and reward values as internal. Upcoming related work could refine the model by incorporating prices as choice variables so that the model mimics reality more closely. Second, we assume customer valuations are static across the two periods, which may lead to inaccurate analytical results because the valuation can be influenced by such factors as prices of the merchandise, the value of the reward, and quality of goods/service received. For remediation, a subsequent analytical investigation can add more impact factors to the valuation function, making it stochastic and a finer reflection of product demand. Last but not least, we suppose firm *B* abandons its LP when forming the coalition, which might be an uncommon operation in reality. Following exploration could expand current work by keeping the non-focal firm's LP to observe how a coalition interacts with all the individual LPs.

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Appendices

Appendix A

We can check the concavity of the *TP* function by taking FOC and second-order condition (SOC)

$$FOC = \frac{\partial\, TP_A}{\partial R_A} = 2P_A - R_A - 1 \tag{A1}$$

$$SOC = \frac{\partial^2\, TP_A}{\partial R_A^2} = -1 < 0 \tag{A2}$$

Since $SOC < 0$, the *TP* function is concave in R_A so we can solve the FOC to obtain the maximizer of the *TP* function.

Appendix B

Demonstration of Proposition 1:

If firm *A* did not have an LP, consumers would not be forward-looking in making the purchase decisions. Thus,

$$S_A^1 = V_A - P_A \tag{B1}$$

And total profit for period one is

$$TP_A^1 = P_A \Pi_A^1 = P_A(1 - P_A) \tag{B2}$$

Since $TP_A^1 = TP_A^2$,

$$TP_A = TP_A^1 + TP_A^2 = 2P_A(1 - P_A) \tag{B3}$$

Price for A can be optimized:

$$P_A^* = 0.5 \tag{B4}$$

Consequently, the optimal total profit is

$$TP_A^* = 0.5 \tag{B5}$$

Appendix C

We can check the concavity of the *TP* function by taking the SOC:

$$FOC = \frac{\partial TP}{\partial R} = 4P_A + 0.5P_A R + 1.5P_B^2 - 2 - 4R - 0.75R^2 + 0.25P_B R \tag{C1}$$

$$SOC = \frac{1}{2}P_A - 4 - \frac{3}{2}R + \frac{1}{4}P_B \tag{C2}$$

It is obvious that the SOC < 0 for all possible combinations of values of P_A, P_B, and *R*. Thus, the *TP* function is concave in *R*. As a result, we can solve the FOC to obtain the maximizer of the *TP* function. Please note, in the Mathematica Notebook, *A* and *B* represent P_A and P_B, respectively.

Appendix D

We can check the concavity of the *TP* function by taking the SOC:

$$SOC = \frac{\partial^2 TP_2}{\partial R^2} = -6 + 2.5P_A + 4.25P_B - 4.5R \tag{D1}$$