Information Content of Time Durations of Limit Order Book Updates

by

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Abstract

The limit order book (LOB) of an exchange contains detailed information about the trading process. Research performed using LOB data has focused on bid-ask spreads, quotes and depths at different LOB levels. However, other studies argue that the timing of trades also conveys information to market participants. Informed traders submit orders only when new information enters the market, so long durations between trades suggest that there is no new information and short durations indicate the presence of "informed" orders. Most research related to durations is based on transaction data, but less is known about the information carried by the time duration of events in the LOB. The objective of this research is to assess the effect of the duration of LOB updates on the price discovery process in agricultural futures markets. We estimate a measure of information share to examine the contribution of quotes, depth, and time durations on transaction prices. Our findings suggest that LOB durations do convey information. It is found that, on average, depth has a larger contribution to prices relative to durations, and that more information is concentrated in the first two levels of the LOB.

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1. Introduction

Agricultural commodity futures contracts were traditionally traded in the open outcry system, however, the Chicago Mercantile Exchange (CME) Group, the largest exchange for agricultural commodities, transitioned most of the futures pits to an electronic trading platform in 2015 (Gousgounis and Onur 2017). In the electronic platform, the limit order book (LOB) contains detailed information of the trading process, including the ask and bid prices of the limit orders submitted to the exchange, the number of contracts associated with each price (i.e., quantities or market depth), and the time at which each order entered the market. Limit orders submitted by informed traders contain their private information, a reason for which the LOB is believed to hold information that is relevant in the price discovery process. However, there is no clear evidence in the literature on the proportion of information attributable to each component of the LOB. Some studies find that trades (Hasbrouck 1991), and in particular trade size (Easley and O'Hara 1997), convey information, while other studies argue that the time between trades contains information as well (Engle and Russell 1998). Some other studies point at the quotes and quantities present at the top of the LOB, whereas others suggest that information is stored in the orders placed beyond the top of the LOB (Cao et al. 2009, Arzandeh and Frank 2019). To our knowledge no research has examined the relative contribution of information of each of these components of the LOB, jointly, to prices. In particular, even though the price discovery process has been studied extensively in agricultural commodity markets, research in this area at the microstructure level and for the entire LOB is scarce. Easley and O'Hara (1992) argue that the absence of trade is correlated with volume and it reflects no new information, thus volume can be regarded as a signal of existence of information. Later, Knez and Ready (1996) point out that the market depth significantly affects trader's trading strategies, and Hasbrouck and Seppi (2001) prove that low depth influences the relationship between sizes and returns. It has also been found that the speed of order arrival impacts traders' beliefs and order submission decisions (Lo and Sapp 2008). Informed traders submit orders only when new information enters the market, so long durations suggest that there is no new information and short durations indicate the presence of "informed" orders. Further, previous studies suggest that if time can be correlated with influential factors related to the price adjustment mechanism, then both the presence or absence of trade can convey information to market participants (Allen, Peiris and Yang 2005, Easley and O'Hara 1992). Many studies indicate that trade duration is not exogenous to the price discovery process (Hausman et al. 1992; Easley et al. 1997 and Dufour and Engle 2000). This is because informed traders prefer to trade a large number of contracts in a hurry to make a profit out of their private information (Easley and O'Hara 1992), submitting either a large number of small orders or a small number of large orders in a certain period of time. A consequence of this is that the trade duration should be correlated to trade volume and hence both the trade duration and trade volume may affect price adjustments of an asset (Manganelli 2005). Lo and Hall (2015) endogenize duration between order events in their specification and point out that the duration between order events is positively related to existing depth in the LOB, particularly at the best price.

Most previous studies related to time duration are based on transaction data (Russel and Engle 1994; Allen et al. 1998; Manganelli 2005; Chen et al. 2008). Less is known about the information carried by the duration between order book events, and none of the studies focus on time duration of LOB updates in agricultural markets. Few studies made for agricultural commodities have found that quotes and depth beyond the best bid and ask also contain information and contribute to price discovery (Arzandeh and Frank 2019), however incorporating LOB event durations may help better explain the price discovery process by presenting a more complete picture of the LOB dynamics. Therefore, this paper aims at filling this gap in the literature by extending the studies of the information content of the LOB in agricultural futures markets. More specifically, we apply the vector error correction (VEC) model and a measure of information share to study the informational role of both depth and time duration in the LOB in the price discovery process. The products are five main agricultural commodities in the CME Group, including lean hogs, live cattle, corn, soybean, and wheat. This study will help market participants better understand the information contained in the LOB. The empirical results can be used to improve their trading strategies and thus reduce their price risk.

2. Literature Review

Price discovery

Price discovery is the incorporation of information to prices of an underlying asset (O'Hara 2003). Informed and uninformed traders are motivated by information and liquidity needs, respectively. With relevant private information about the real value of the asset, informed traders will act strategically to make a profit. Then, private information will be gradually incorporated into market trading. Thus, uninformed traders can learn the true value of the asset by observing trading activities, such as price, volume, or time between trades (Manganelli 2005). Hasbrouck's (1991) analysis reveals that the change in prices depends on characteristics of trades (trade direction and size) and the market environment as measured by bid-ask spread, in addition to the current and past levels of prices. Further, Hautsch and Huang (2012) show that price adjustment depends on orders' aggressiveness, their sizes, and the state of theLOB.

Information Contained in the LOB

Several studies have been conducted on the information contained in the LOB, however most of them have focused on the type of order (market vs. limit) and their associated depth, and the results appear to be mixed. Some studies contend that informed traders prefer to submit market orders to make a profit out of the private information, suggesting that the LOB contains little information (Seppi 1997). In contrast, other studies show that informed traders submit more limit orders over market orders (Bloomfield, O'Hara and Saar 2005; Kaniel and Liu 2006), and therefore their private information is incorporated into the LOB (Cao et al. 2008; Cao et al. 2009). Compared to market orders, limit orders lead to lower trading costs and don't reveal trader's information too rapidly (Cao et al. 2008). Even if the limit orders are not being executed, passive order placement through limit orders incurs significant market impact (Parlour and Seppi 2008; Boulatov and George 2008). For agricultural commodities, Arzandeh and Frank (2019) examine the role of market depth on the price discovery of futures contracts and find that information is embedded on orders beyond the best bid and best ask prices.

Order events update the LOB at irregular time intervals, including incoming new limit or market orders and amendments or cancellations of existing limit orders. It is standard practice in the literature to aggregate observations into regular time intervals (Hasbrouck 1995; Cao et al 2009), where the appropriate interval length varies significantly among markets because of different level of limit order activity. For example, Cao et al (2009) use a one-second interval for high frequently traded products such as the E-mini S&P 500 while Arzandeh and Frank (2019) choose a longer snapshot duration based on average transaction price change for five agricultural commodities. Lo and Hall (2015) indicate that important information would be likely missed if all order events aggregate into a regular time interval. In order to examine the information contained in the time duration of order events, our data is irregularly time spaced.

The study of time duration between trades goes back to Diamond and Verrecchia (1987) who first accord time a prominent informational role. In theoretical models, informed traders only trade when an informational event occurs. Thus, an informational event and the ensuing trading is positive to trading frequency (Easley and O'Hara 1992, Manganelli 2005). Trading frequency, trading volume and speed of price movements will increase when new unexpected information hits the markets. Thus, volume and time between trades are expected to be correlated with information about the asset's true value and they will influence its price. In other words, the presence and absence of information can be reflected by the presence and absence of trade. Further, Easley et al. (1997) and Dufour and Engle (2000) examine the most frequently traded stocks on the New York Stock Exchange (NYSE) and confirm the effect of durations on transaction prices and mid-quote revisions, finding that longer durations lead to lower price volatility, whereas shorter durations lead to higher price volatility. Ruan and Ma (2017) examine the intraday informational and liquidity effects of unexpected durations between two consecutive trades on bid-ask spreads and depths. They show that an unexpected short duration has a permanent effect of increasing the quoted spread and has a positive correlation with the adverse selection component of the effective spread. Engle and Patton (2004) also prove that short duration and medium volume trades have the largest impacts on quoted prices and a long duration likely represents a liquidity trade rather than one based on valuable information.

While the informational role of time between trades is well established in the extant literature, less is known about the information carried by the duration between order book events, such as new orders, order modifications, and cancellations. Lo and Hall (2007) study the liquidity replenishment process of the LOB in the Australian Securities Exchange (ASX), and their results suggest that the effect of durations between LOB events on existing depth and the best price is positive but not significant. Another study by Lo and Sapp (2008) investigates the influence of the duration between two new orders and characteristics of the LOB on the arrival time of limit or market order submissions, and find that the expected time between the arrival of successive orders in the foreign exchange market depends on the previous type of order submitted and the slope on both sides of the order book.

The information content in agricultural futures markets may deviate from other markets due to differences in trading volume, trading frequency, commodity availability, etc. Manganelli (2005) and Russell and Tsay (2001) argue that in more frequently traded markets, volume is more consistent, and price is quicker to return to its equilibrium. Arzandeh and Frank (2019) compared five major agricultural commodities with the E-mini S&P 500, which has relatively higher trading frequency, finding that the E-mini S&P 500 has the higher information share contained in the LOB beyond the best bid and best ask spread (BAS).

Modified Information Share Measure

The contribution of related series to price discovery has commonly been assessed in previous studies by using Hasbrouck's information share (IS) and Gonzalo and Granger's (1995) permanent-transitory (PT) measures. However, weaknesses have been identified in both IS and PT. Ballie et al. (2000) and Hasbrouck (2000) argue that PT is a unique price discovery measure but ignores innovation variance (e.g., the correlations between different price series). On the other hand, IS incorporates both the system dynamics and the innovation variances but depends on the ordering of the price series used in the estimation, as a result of using a Choleski decomposition of the innovations.

Lien and Shrestha (2009) developed a modified information share (MIS), which is unique and independent of the ordering of the price series in the model. Unlike IS and PT measure, MIS is not established under the assumption that the common factor shared by interrelated market have the same long-run equilibrium price. In other words, a one-to-one cointegration relation between pairs is not necessary for the MIS and it is realistic only for almost identical assets such as cross-listed stocks (Chau et al. 2017). The MIS only requires that all I(1) time series share one and only one common trend. Thus, we apply Lien and Shrestha's (2009) MIS, which is unique and independent of the ordering of the price series.

3. Methods

Empirical model

Hasbrouck (1991) proposes the vector autoregression (VAR) framework to capture the relationship between price and trading, indicating that unexpected trade has a positive, concave and persistent impact on prices. Dufour and Engle (2000) extend Hasbrouck's (1991) research by incorporating the duration between trades into the bivariate model, finding that price durations play an important informational role in the dynamics between trades and quotes. Cao et al. (2009) find that order book information behind the market is moderately informative of price discovery based on Hasbrouck's (1995) information share measure. A trader's intention may execute a large order that is unable to be filled by depth at the best prices. The volume of standing limit orders behind the best prices will influence execution strategy as it determines the cost of immediate execution.

In order to examine the informational role of the duration between order events on price revisions, we extend Arzandeh and Frank's (2019) study by adding duration between order events in their model of LOB prices and quantities. Thus, the variables in our model are transaction price ($Price_t$), quantity-weighted price (QWP_t^{i-j}) and duration-weighted price (DWP_t^{i-j}).

The quantity-weighted price reflects the price and quantity aspects of the LOB at any given point in time and it is defined as:

$$QWP_t^{i-j} = \frac{\Sigma_{n=i}^{j}(Q_{n,t}^{b}P_{n,t}^{b}+Q_{n,t}^{a}P_{n,t}^{a})}{\Sigma_{n=i}^{j}(Q_{n,t}^{b}+Q_{n,t}^{a})} \qquad (1)$$

where QWP_t^{i-j} is the quantity-weighted price from step i to step j, P_n^b (P_n^a) is the bid (ask) price at step n, and Q_n^b (Q_n^a) is the associated quantity.

The duration-weighted price reflects the price and duration aspects of the LOB at any given point in time and it is defined as:

$$DWP_t^{i-j} = \frac{\Sigma_{n=i}^{j}(W_{n,t}^{b}P_{n,t}^{b}+W_{n,t}^{a}P_{n,t}^{a})}{\Sigma_{n=i}^{j}(W_{n,t}^{b}+ W_{n,t}^{a})} \qquad (2)$$

where DWP_t^{i-j} is the duration-weighted price from step i to step j, and $W_{n,t}^{b}(W_{n,t}^{a})$ represents the reciprocal duration of the bid (ask) price or quantity change at step n, $W_{n,t}^{b}(W_{n,t}^{a}) = 1/[t - (t-1)]$. We define the weight for the duration as the reciprocal of the time elapsed between two order events to make it consistent with the quantity-weighted measure. Shorter durations, as well as larger quantities, are indicative of the presence of new information.

The order events may only occur in certain step *n*, which may not update all steps in the LOB. For example, a new order may enter the LOB in step 3 on the ask side, which means that there is no change from step 1 to 2. Thus, the weight $W_{n,t}^{b}(W_{n,t}^{a})$ is not defined if there is no update on step *n*. We define two alternative weights to overcome this problem. The first one is to replace the current weight with the previous weight, $W_{n,t}^{b}(W_{n,t}^{a}) = W_{n,t-1}^{b}(W_{n,t-1}^{a})$. That is, the weight changes only if there is new information coming into the LOB. When there is no update on step *n* we assume there is no new information and therefore the weight is the same as the previous one. The second one is to define the weight as zero, $W_{n,t}^{b}(W_{n,t}^{a}) = 0$, so that we only consider the cases with new information on step *n*. The second alternative weight still has a problem when we aggregate duration-weighted price from step *i* to step *j*, since DWP_{t}^{i-j} will still be not defined if all weights $W_{n,t}^{b}(W_{n,t}^{a})$ from step *i* to *j* equal to zero. Without any new information, duration-weighted price will not change, $DWP_{t}^{i-j} = DWP_{t-1}^{i-j}$. In this research, we will apply the first alternative weights and use the second alternative weights as robustness check.

The price series (i.e., $Price_t$, $QWP_t^{i\text{-}j}$, and $DWP_t^{i\text{-}j}$) for each commodity cointegrate naturally and tend to move together, thus we use a cointegrated VAR model. The vector error correction (VEC) model is,

$$\Delta X_t = \mu + \alpha\beta' X_{t-1} + \sum_{l=1}^{p-1} \Gamma_l \Delta X_{t-l} + \varepsilon_t \tag{3}$$

where $X_t = [Price_t\ QWP_t^{i\text{-}j}\ DWP_t^{i\text{-}j}]'$ is the vector of endogenous variables; μ is a constant; α represents the loading matrices, reflecting how quickly price series adjust to their long-run equilibrium; β denotes the cointegrating matrix (the coefficients of the cointegration process); and

Γ_l is a parameter matrix with *l*=1…*p*-1, representing the short-term adjustment process. The covariance matrix of the error term ε_t is denoted by $\Omega = E[\varepsilon_t, \varepsilon_t^T]$.

The VEC model can be characterized by the following vector moving average (VMA) representation (Lütkepohl 2005), where X_t depends on current and past innovations.

$$\Delta X_t = \Psi(L)\varepsilon_t, \text{ where } \Psi(L) = \Psi_0\varepsilon_t + \Psi_1\varepsilon_{t-1} + \Psi_2\varepsilon_{t-2} + \cdots \qquad (4)$$

Or, alternatively,

$$X_t = X_0 + \Psi(1)\sum_{l=1}^{t}\varepsilon_l + \Psi^*(L)\varepsilon_t \qquad (5)$$

where $\Psi(1)\varepsilon_t$ represents the long-run impact of innovations on prices. Because all price series are cointegrated, the long-run impact on all prices is the same, thus $\psi = (\psi_1, \psi_2, \cdots, \psi_k)$ is the common row in $\Psi(1)$.

Modified Information Share (MIS)

For a general case where the innovations are not independent (i.e., the matrix Ω is not diagonal), we consider the following factor structure for innovations:

$$\varepsilon_t = Fz_t, E[z_t] = 0, E[z_t z_t'] = I \qquad (6)$$

where $F = \left[G\Lambda^{-1/2}G^TV^{-1}\right]^{-1}$ is based on the correlation matrix and $\Omega = FF'$. Letting Φ represent the innovation correlation matrix, Λ is a diagonal matrix with diagonal elements being the eigenvalues of the correlation matrix Φ, where the corresponding eigenvectors are given by the columns of matrix G. V is a diagonal matrix containing the innovation standard deviations on the diagonal; i.e., $V = diag(\sqrt{\Omega_{11}}, \sqrt{\Omega_{22}}, \dots . \sqrt{\Omega_{kk}})$. Under this factor structure, the contribution of the kth price series to the efficient price is measured in the MIS by means of variance decomposition as,

$$MIS_j = \frac{[\psi F]_k^2}{[\psi F]\Omega[\psi F]'}. \quad (7)$$

It is important to note that under this factor structure, the resulting MIS is independent of the ordering of the price series and leads to a unique measure of price discovery.

4. Description of Data

We use the *Market Depth* data files from the CME Group, which include all the market data messages required to recreate the LOB and trade data, for five agricultural commodities (lean hogs, live cattle, corn, soybean and wheat). The dataset is for the period from November 23, 2015 to February 14, 2017, with millisecond granularity. We reconstruct five-depth book levels for lean hogs and live cattle and ten-depth book levels for corn, soybean, and wheat. Our dataset is similar to that used in Arzandeh and Frank (2019), except that we use the complete LOB instead of taking snapshots at regular time intervals (to compute LOB events time durations), and we use a longer period. We keep the LOB events' time durations associated with transaction price changes.

Lean hogs and live cattle futures contracts trade in one morning session (8:30 –13:05 CT). Futures contracts for corn, soybeans and wheat trade in a morning session (8:30 – 13:20 CT) and an evening session (19:00 – 7:45 CT) on Sundays and weekdays. Following Arzandeh and Frank's (2019) procedure, we remove the data with low traded volume, usually in the evening session and on Sundays and only keep the most active trading days (weekdays) and trading hours (morning session). Also, we eliminate days with diverse opening and closing times which is mostly the case on holidays as well as the business days prior to or after a holiday. We construct the dataset by rolling forward futures contracts when the aggregate volume of the second nearest contract is higher than that of the current contract for two consecutive days, as suggested by Arzandeh and Frank (2019) for these markets.

Descriptive Statistics

Table 1 shows the average interval between order events with transaction price change by weekday. Market traders submit more orders with transaction price change on Tuesdays, Wednesdays, and Thursdays than Mondays and Fridays. Out of the five commodities under study, order events update more frequently in grains. Every 355.89, 212.97 and 267.44 milliseconds after the previous order, on average, there is an order event with transaction price change in the corn, soybean, and wheat markets, respectively.

Table 1: Average duration of order events with transaction price change in the LOB by weekday (milliseconds)

		Monday	Tuesday	Wednesday	Thursday	Friday	Average
Lean hogs	Obs	96496	108500	109727	107742	102240	104941
	Mean	611.21	596.68	610.16	598.39	634.84	610.00
	Median	36.00	39.00	40.00	37.00	37.00	38.00
Live cattle	Obs	138719	159748	151436	161421	150708	152406
	Mean	513.83	542.63	559.40	530.25	543.79	537.98
	Median	36.00	47.00	53.00	49.00	51.00	47.20
Corn	Obs	123053	155299	148212	142253	133518	140467
	Mean	381.64	357.32	352.04	330.31	358.13	355.89
	Median	108.00	98.00	96.00	80.00	92.00	94.80
Soybean	Obs	183598	224501	211011	215128	212854	183598
	Mean	212.97	192.45	193.32	179.81	193.08	212.97
	Median	39.00	37.00	38.00	30.00	33.00	39.00
Wheat	Obs	96218	109088	108628	112710	102603	105849
	Mean	304.32	134.90	309.75	287.49	300.74	267.44
	Median	36.00	34.00	37.00	34.00	33.00	34.80

The average time durations between order events every 5 minutes are shown in Figure 1 and Figure 2 for meats and grains, respectively. During lunch time (around 12:00 pm to 12:30 pm), the time duration for these five commodities is the longest, which means less trading during lunchtime.

This finding is consistent with Eaves and Jeffrey (2010) and McInish and Wood (1990). Also, for lean hogs and live cattle there are two big drops of time duration between 9:00 am to 9:05 am and around 10 minutes before the market closing. Traders may submit many orders before closing of the market to close their position and avoid "overnight" volatility (Lo and Sapp 2005), which results in the second drop in duration for all commodities.

Figure 1. Average duration of order events with transaction price change for meats in the LOB every five minutes (milliseconds)

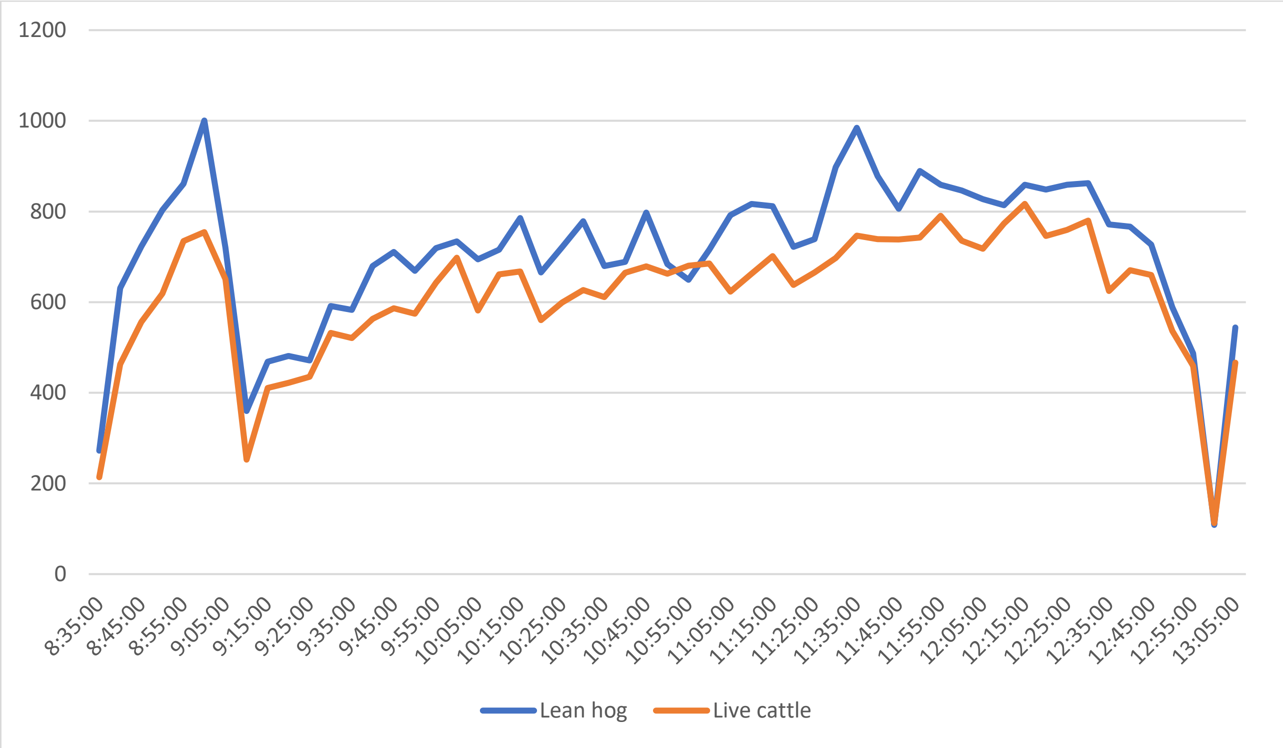


Figure 2. Average duration of order events with transaction price change for grains in the LOB every five minutes (milliseconds)

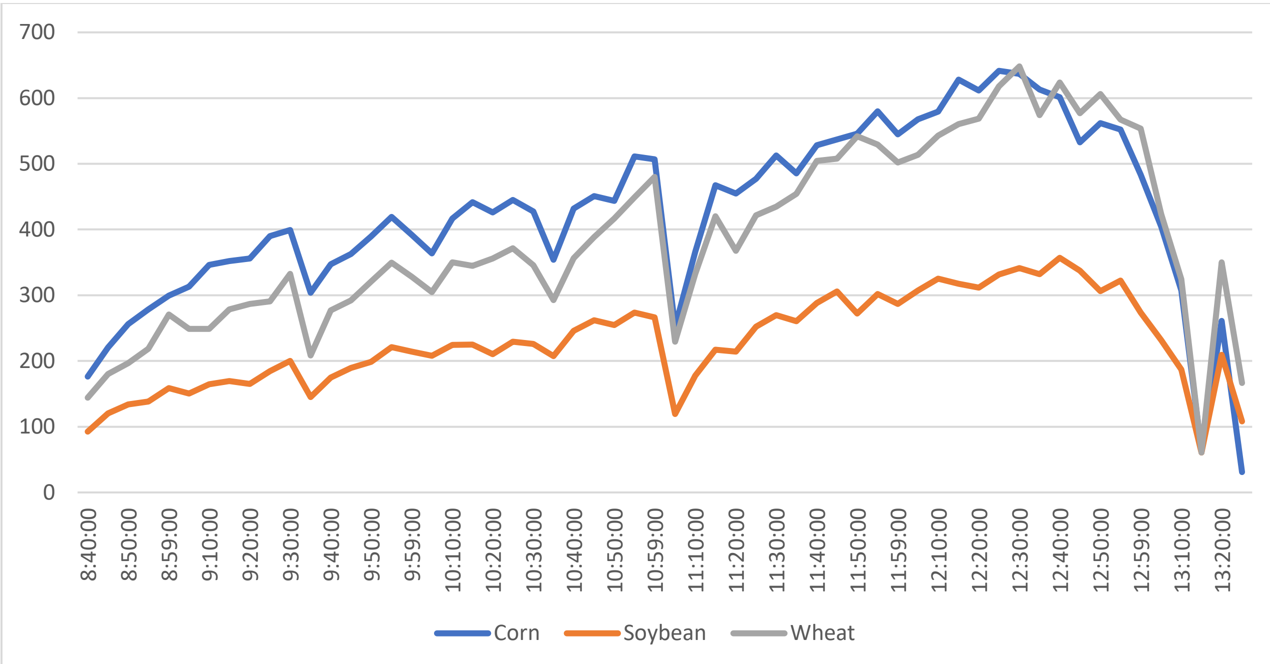


Figure 3 shows the average quantity of new order events every five minutes. The quantities change more for grains than meats. The average quantity changes for lean hogs and live cattle are 9.71 and 9.65, respectively, while the average quantity change for corn is 343.94, for soybean is 47.55 and for wheat is 59.47.

Figure 3. Average quantity of new order events with transaction price change for meats in the LOB every five minutes

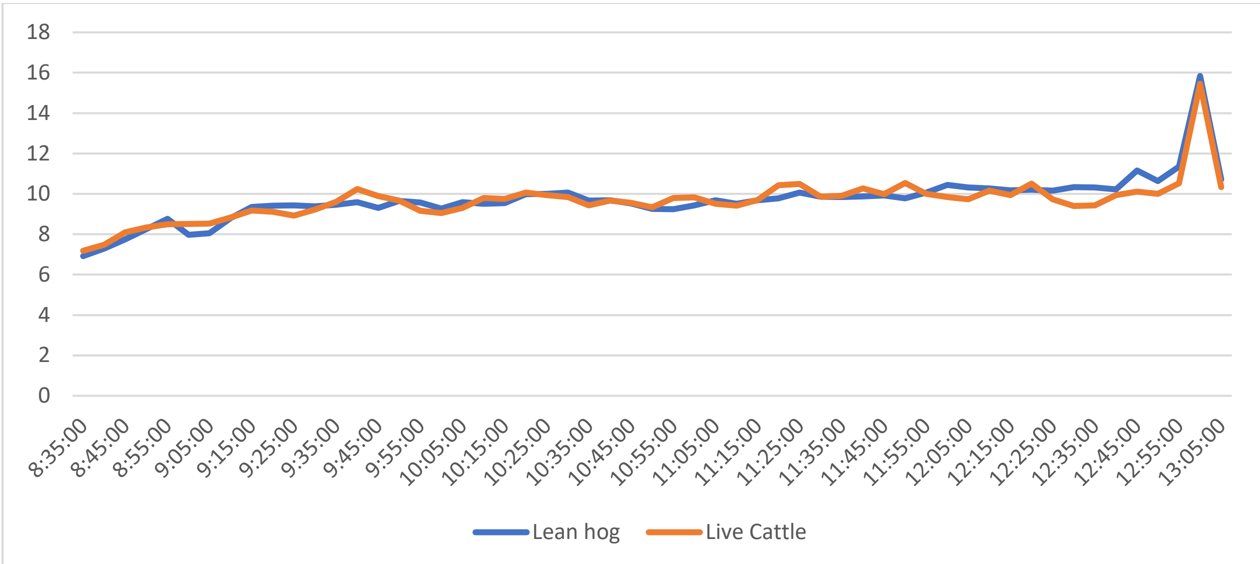
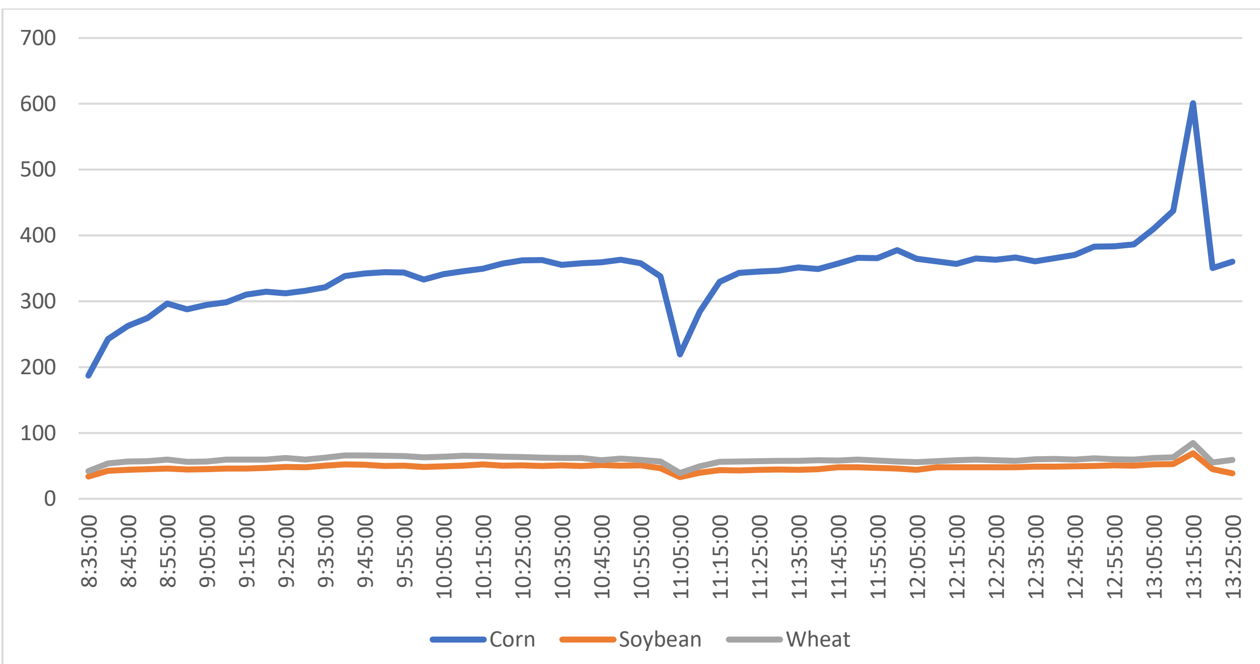


Figure 4. Average quantity of new order events with transaction price change for grains in the LOB every five minutes



Stationarity and cointegration tests

We use the Augmented Dickey-Fuller (ADF) test to assess stationarity of the time series used in our model. To determine the optimal number of lags in the ADF test, we use those that minimize the Akaike's information criterion (AIC). The results show all price series are not stationary in levels but are stationary after taking first differences on each day. Table 2 shows the average daily results of the ADF test. We also use the Kwiatkowski, Philips, Schmidt and Shin (KPSS) test for stationary. The results of KPSS are consistent with the results of the ADF test and are shown in table 3.

Table 2: Augmented Dickey-Fuller test for unit root

		CT	CNT	NCNT		CT	CNT	NCNT
Lean hogs	Price_t (7)	-2.327	-1.452	-0.088	$\Delta\ \text{Price}_t$ (7)	-15.142	-15.117	-15.083
	QWP_t^{1-2} (7)	-2.333	-1.459	-0.085	$\Delta\ QWP_t^{1-2}$ (7)	-15.200	-15.175	-15.141
	QWP_t^{3-5} (12)	-2.388	-1.483	-0.103	ΔQWP_t^{3-5} (12)	-11.879	-11.849	-11.811
	DWP_t^{1-2} (6)	-2.315	-1.444	-0.094	ΔDWP_t^{1-2} (6)	-16.931	-16.907	-16.876
	DWP_t^{3-5} (15)	-2.332	-1.456	-0.086	ΔDWP_t^{3-5} (15)	-15.142	-15.117	-15.083
Live cattle	Price_t (6)	-2.146	-1.287	-0.086	$\Delta\ \text{Price}_t$ (6)	-19.716	-19.701	-19.675
	QWP_t^{1-2} (7)	-2.156	-1.293	-0.082	ΔQWP_t^{1-2} (7)	-18.508	-18.492	-18.464
	QWP_t^{3-5} (8)	-2.243	-1.355	-0.083	ΔQWP_t^{3-5} (8)	-17.336	-17.321	-17.297
	DWP_t^{1-2} (7)	-2.124	-1.266	-0.083	ΔDWP_t^{1-2} (7)	-19.212	-19.196	-19.168
	DWP_t^{3-5} (15)	-2.169	-1.275	-0.080	ΔDWP_t^{3-5} (15)	-15.430	-15.410	-15.377

Corn	Price_t (14)	-2.288	-1.627	-0.055	$\Delta\ \text{Price}_t$ (14)	-12.102	-12.059	-12.010
	QWP_t^{1-2} (20)	-2.276	-1.595	-0.054	ΔQWP_t^{1-2} (20)	-10.038	-9.986	-9.933
	QWP_t^{3-10} (23)	-2.262	-1.551	-0.042	ΔQWP_t^{3-10} (23)	-9.026	-8.960	-8.887
	DWP_t^{1-2} (16)	-2.306	-1.628	-0.059	ΔDWP_t^{1-2} (16)	-11.785	-11.742	-11.694
	DWP_t^{3-10} (25)	-2.839	-1.948	-0.052	ΔDWP_t^{3-10} (25)	-12.016	-11.992	-11.963
Soybean	Price_t (8)	-2.103	-1.417	0.049	$\Delta\ \text{Price}_t$ (8)	-20.532	-20.513	-20.487
	QWP_t^{1-2} (10)	-2.121	-1.423	0.050	ΔQWP_t^{1-2} (10)	-18.432	-18.410	-18.383
	QWP_t^{3-10} (18)	-2.186	-1.464	0.048	ΔQWP_t^{3-10} (18)	-13.621	-13.593	-13.559
	DWP_t^{1-2} (10)	-2.097	-1.414	0.053	ΔDWP_t^{1-2} (10)	-18.655	-18.632	-18.604
	DWP_t^{3-10} (26)	-2.160	-1.439	0.041	ΔDWP_t^{3-10} (26)	-13.911	-13.883	-13.848
Wheat	Price_t (11)	-2.238	-1.597	-0.129	$\Delta\ \text{Price}_t$ (11)	-12.213	-12.179	-12.143
	QWP_t^{1-2} (10)	-2.263	-1.615	-0.130	ΔQWP_t^{1-2} (10)	-12.767	-12.735	-12.702
	QWP_t^{3-10} (16)	-2.299	-1.577	-0.134	ΔQWP_t^{3-10} (16)	-10.002	-9.961	-9.917
	DWP_t^{1-2} (10)	-2.228	-1.607	-0.125	ΔDWP_t^{1-2} (10)	-13.340	-13.308	-13.276
	DWP_t^{3-10} (23)	-2.548	-1.777	-0.116	ΔDWP_t^{3-10} (23)	-11.295	-11.270	-11.244

CT: constant and trend; CNT: constant and no trend; NCNT: no constant and no trend. Numbers between parenthesis indicate the number of lags used in the ADF test. 5% critical value: CT: -3.410; CNT: -2.860; NCNT: -1.950.

Table 3: KPSS test for stationarity

		T	L		T	L
Lean hogs	Price_t	0.801	4.020	$\Delta\ \text{Price}_t$	0.063	0.146
	QWP_t^{1-2}	0.777	3.732	ΔQWP_t^{1-2}	0.063	0.149
	QWP_t^{3-5}	0.764	3.699	ΔQWP_t^{3-5}	0.061	0.139
	DWP_t^{1-2}	0.781	3.737	ΔDWP_t^{1-2}	0.061	0.145
	DWP_t^{3-5}	0.778	3.725	ΔDWP_t^{3-5}	0.044	0.099
Live cattle	Price_t	1.178	5.796	$\Delta\ \text{Price}_t$	0.069	0.140
	QWP_t^{1-2}	1.027	5.030	ΔQWP_t^{1-2}	0.070	0.142
	QWP_t^{3-5}	1.013	4.984	ΔQWP_t^{3-5}	0.066	0.130
	DWP_t^{1-2}	1.029	5.044	ΔDWP_t^{1-2}	0.070	0.140
	DWP_t^{3-5}	1.030	5.052	ΔDWP_t^{3-5}	0.050	0.100
Corn	Price_t	0.814	4.180	$\Delta\ \text{Price}_t$	0.054	0.139
	QWP_t^{1-2}	0.813	4.171	ΔQWP_t^{1-2}	0.059	0.155
	QWP_t^{3-10}	0.815	4.107	ΔQWP_t^{3-10}	0.069	0.185
	DWP_t^{1-2}	0.743	4.040	ΔDWP_t^{1-2}	0.053	0.136
	DWP_t^{3-10}	0.813	4.171	ΔDWP_t^{3-10}	0.018	0.035
Soybean	Price_t	1.421	7.025	$\Delta\ \text{Price}_t$	0.069	0.154
	QWP_t^{1-2}	1.420	7.023	ΔQWP_t^{1-2}	0.070	0.156
	QWP_t^{3-10}	1.385	6.910	ΔQWP_t^{3-10}	0.066	0.147
	DWP_t^{1-2}	1.422	7.023	ΔDWP_t^{1-2}	0.069	0.156
	DWP_t^{3-10}	1.409	7.008	ΔDWP_t^{3-10}	0.031	0.066

	Price_t	0.261	1.203	$\Delta\ \text{Price}_t$	0.064	0.148
	QWP_t^{1-2}	0.261	1.203	ΔQWP_t^{1-2}	0.074	0.164
Wheat	QWP_t^{3-10}	0.255	1.193	ΔQWP_t^{3-10}	0.034	0.161
	DWP_t^{1-2}	0.262	1.203	ΔDWP_t^{1-2}	0.074	0.164
	DWP_t^{3-10}	0.261	1.199	ΔDWP_t^{3-10}	0.058	0.110

T: trend stationary; L: level stationary

We adopt Johansen's trace test to assess cointegration of all five agricultural commodities. The results show that about 29.81% of the days in the period under study do not show a cointegration relationship (36.69% for lean hogs, 31.05 for live cattle, 28.08% for corn, 32.75 for soybean and 20.48 for wheat). Hu et Al. (2020) indicate that intraday prices may not be cointegrated because of inverse carrying charges, market inefficiency and delivery related price change. We only keep the days with cointegration relationships for the analysis. Table 4 shows the daily average of the cointegration test results, indicating that we can reject the null hypothesis of no cointegration and establish the presence of four cointegrating equations (CE) for each commodity based on the trace value statistics.

Table 4. Johansen Cointegration Test

No. of CE		Eigenvalue	Trace statistics	5% Critical value
	None	.	131.076	68.52
	At most 1	0.031	80.967	47.21
Lean hogs	At most 2	0.022	44.811	29.68
	At most 3	0.016	18.745	15.41
	At most 4	0.010	**1.869**	3.76
	None	.	173.064	68.52
	At most 1	0.026	109.316	47.21
Live cattle	At most 2	0.020	60.745	29.68
	At most 3	0.014	24.533	15.41
	At most 4	0.009	**1.781**	3.76
	None	.	114.785	68.52
	At most 1	0.022	68.034	47.21
Corn	At most 2	.018	35.048	29.68
	At most 3	0.012	12.972	15.41
	At most 4	0.006	**2.306**	3.76
	None	.	205.868	68.52
	At most 1	0.026	128.745	47.21
Soybean	At most 2	0.017	68.591	29.68
	At most 3	0.013	23.318	15.41
	At most 4	0.006	**2.119**	3.76

	None	.	116.960	68.52
	At most 1	0.030	69.518	47.21
Wheat	At most 2	0.021	35.689	29.68
	At most 3	0.014	13.391	15.41
	At most 4	0.007	**2.205**	3.76

5. Results

In our model, the vector of endogenous variables for the meats is given by $X_t = [Price_t\ , QWP_t^{1-2}, QWP_t^{3-5}, DWP_t^{1-2}, DWP_t^{3-5}]'$ and for the grains is given by $X_t = [Price_t\ , QWP_t^{1-2}, QWP_t^{3-10}, DWP_t^{1-2}, DWP_t^{3-10}]'$. In the VEC model we add 5 lags for lean hogs, 6 lags for live cattle, 7 lags for corn and wheat, and 8 lags for soybean, based on the AIC of the model and the ACF of the model residuals.

Table 5 reports summary statistics of the MIS for all five commodities. Transaction price, quantity-weighted price and duration-weighted price at different steps contribute different information shares to price discovery. For all commodities, transaction prices ($Price_t$) have the highest contribution to its own price, accounting for 56.61% in lean hogs, 60.32% in live cattle, 41.61% in corn, 52.60% in soybean, and 45.89% in wheat. For meats, the information contained in the first two steps of the LOB is higher than the information contained in the rest of the book, for both quantities and durations. For grains, except soybean, quantities beyond the first two steps of the LOB appear to contain more information than that contained in the first two steps, whereas for durations the first two steps have a higher MIS relative to further away steps. These results are consistent with Arzandeh and Frank (2019), showing that there is information beyond the best bid and ask contributing to price discovery and that the contribution may be larger relative to the top of the LOB. Also, the results suggest that the information share of duration-weighted price is smaller than that of quantity-weighted price, indicating that quantity conveys more information than duration. For durations, the top two steps contribute, on average, more information than the rest of the steps in the LOB for all five commodities. Moreover, durations contribute more information, on average, for meats than grains. The average contribution of steps 1 and 2 for meats is 12.64% and for grains is about 10.54% whereas the contribution beyond steps 1 and 2 for meats is 3.78% and for grains is about 1.21%.

Table 5. Summary statistics of the average MIS (%)

		$Price_t$	QWP_t^{1-2}	QWP_t^{3-5}	DWP_t^{1-2}	DWP_t^{3-5}
Lean hogs	**Mean**	**56.61%**	**15.71%**	**8.86%**	**14.62%**	**4.19%**
	Median	57.48%	15.25%	8.33%	14.40%	3.84%
	Min	38.65%	8.00%	3.18%	4.64%	0.41%
	Max	70.95%	32.52%	21.04%	34.59%	13.62%
	Sd	0.070	0.040	0.033	0.053	0.023
		$Price_t$	QWP_t^{1-2}	QWP_t^{3-5}	DWP_t^{1-2}	DWP_t^{3-5}
Live Cattle	**Mean**	**60.32%**	**16.85%**	**8.82%**	**10.66%**	**3.36%**
	Median	61.17%	15.79%	8.47%	10.23%	3.05%
	Min	36.57%	8.30%	2.35%	1.33%	0.23%
	Max	74.21%	50.11%	19.52%	20.53%	11.50%
	Sd	0.067	0.054	0.029	0.037	0.021

		$Price_t$	QWP_t^{1-2}	QWP_t^{3-10}	DWP_t^{1-2}	DWP_t^{3-10}
Corn	**Mean**	**41.61%**	**25.56%**	**27.42%**	**4.56%**	**0.86%**
	Median	41.46%	23.67%	24.25%	3.98%	0.43%
	Min	8.59%	7.91%	4.15%	0.67%	0.00%
	Max	83.42%	51.79%	72.86%	19.88%	8.48%
	Sd	0.133	0.082	0.154	0.029	0.012
		$Price_t$	QWP_t^{1-2}	QWP_t^{3-10}	DWP_t^{1-2}	DWP_t^{3-10}
Soybean	**Mean**	**52.60%**	**22.18%**	**19.31%**	**5.33%**	**0.58%**
	Median	53.64%	21.21%	16.17%	3.49%	0.30%
	Min	12.59%	11.94%	5.66%	0.76%	0.00%
	Max	79.09%	36.02%	62.83%	23.35%	6.55%
	Sd	0.116	0.054	0.107	0.046	0.008
		$Price_t$	QWP_t^{1-2}	QWP_t^{3-10}	DWP_t^{1-2}	DWP_t^{3-10}
Wheat	**Mean**	**45.89%**	**18.20%**	**22.53%**	**11.19%**	**2.20%**
	Median	47.47%	17.71%	18.60%	11.01%	1.78%
	Min	10.30%	7.78%	4.52%	2.18%	0.14%
	Max	66.06%	33.43%	70.25%	24.23%	8.82%
	Sd	0.117	0.042	0.136	0.045	0.018

Figures 5 to 9 report the daily MIS results of the top two steps for all five commodities. These figures show that in 33.61% of the days for meats and 4.89% of days for grains in our sample period, duration does contribute more than quantity to price discovery (49.20% for lean hogs, 18.01% for live cattle, 0.47% for corn, 4.17% for soybean, and 10.10% for wheat). These results indicate that time durations are more important to price discovery for meats than grains, especially for lean hogs.

For the days in which time durations appear to contribute more to price discovery relative to quantity, we looked for informational events that may help explain this finding. It has been shown that USDA reports contain new information (Karali et al. 2019), and that price reactions to the new information occur soon after the reports are publicly released (Lehecka et al. 2014). However, we find no significant correlation between the main USDA reports[1] containing information about lean hogs, live cattle, corn, soybean, and wheat and the days in which the durations show a higher share of information.

1 The most important USDA reports for lean hogs are Livestock Slaughter (LS), Cold Storage (CS), and Hog and Pigs (HP) reports; for live cattle are Cattle on Feed (CoF), Livestock Slaughter (LS), and Cold Storage (CS); for grains (corn, soybean and wheat) are WASDE, Prospective planting (PP), Grain Stock (GS), and Crop Production (CP). There are a total of 28 report days for lean hogs, 32 report days for live cattle and 18 report days for grains. We find that only 11 report days for lean hog, 9 report days for live cattle, 1 report day for corn and soybean, and 2 report days for wheat with more information in duration.

Figure 5. Daily MIS of the top two steps of the LOB for lean hogs.

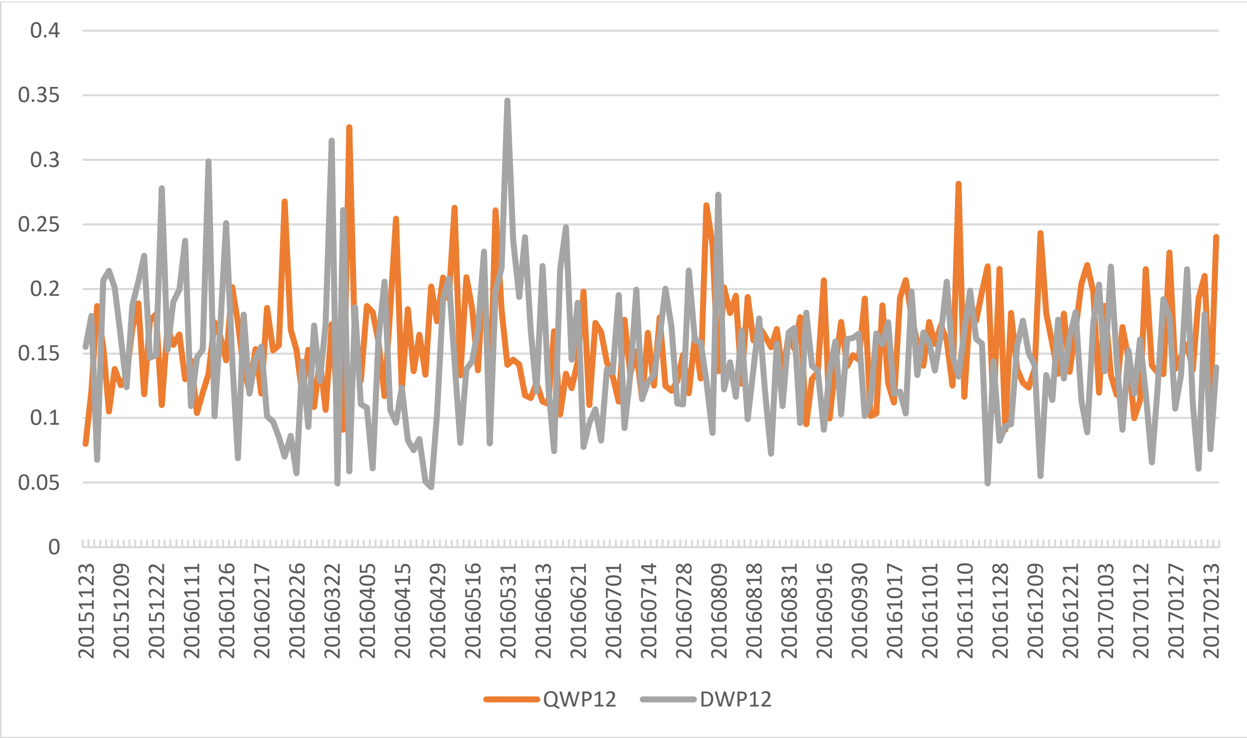


Figure 6. Daily MIS of the top two steps of the LOB for live cattle.

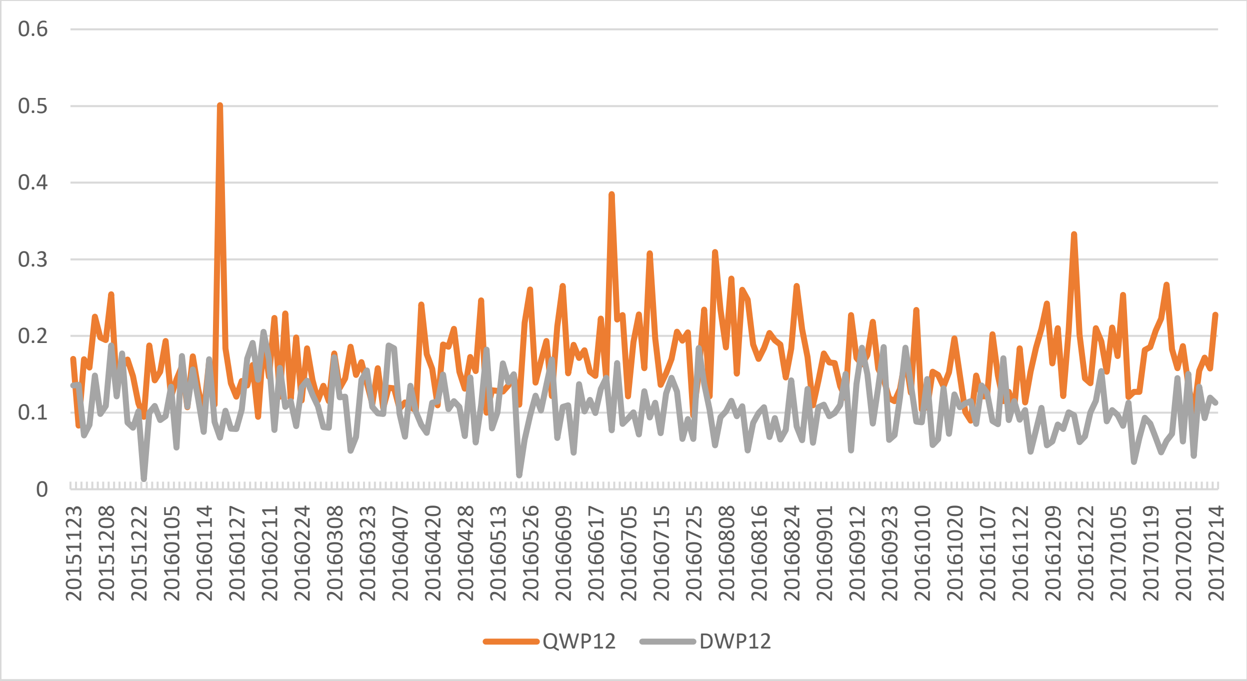


Figure 7. Daily MIS of the top two steps of the LOB for corn.

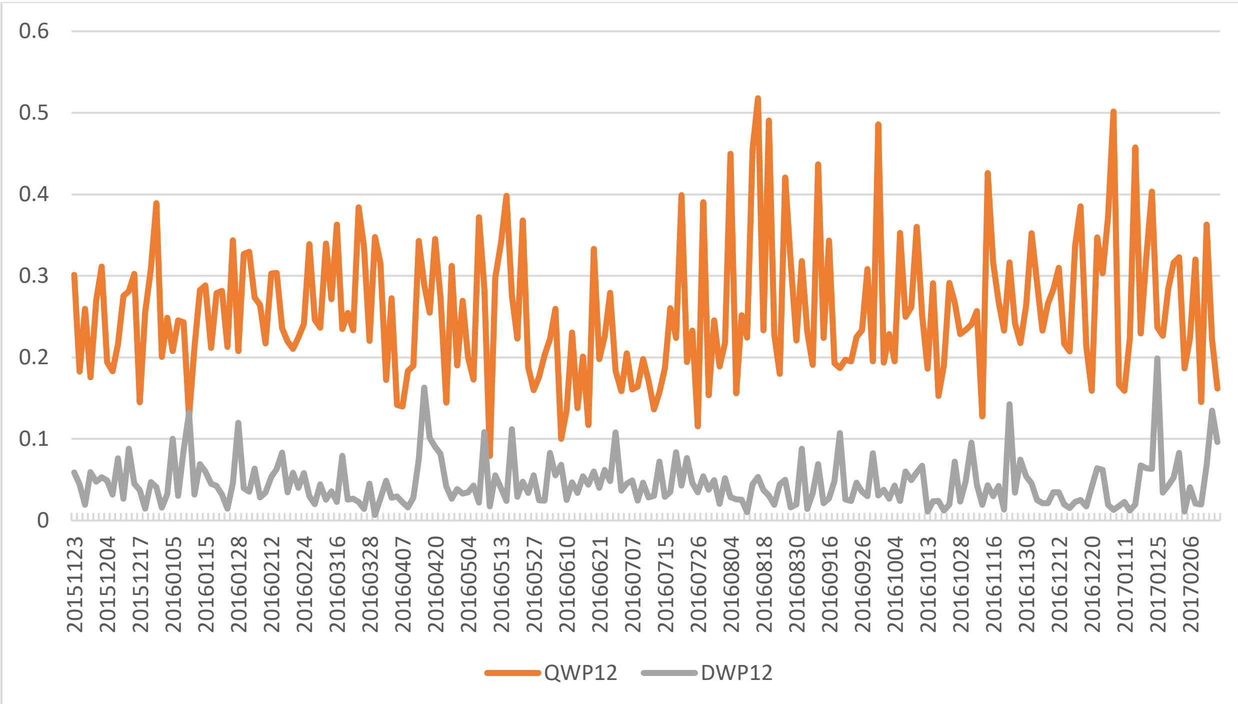


Figure 8. Daily MIS of the top two steps of the LOB for soybean.

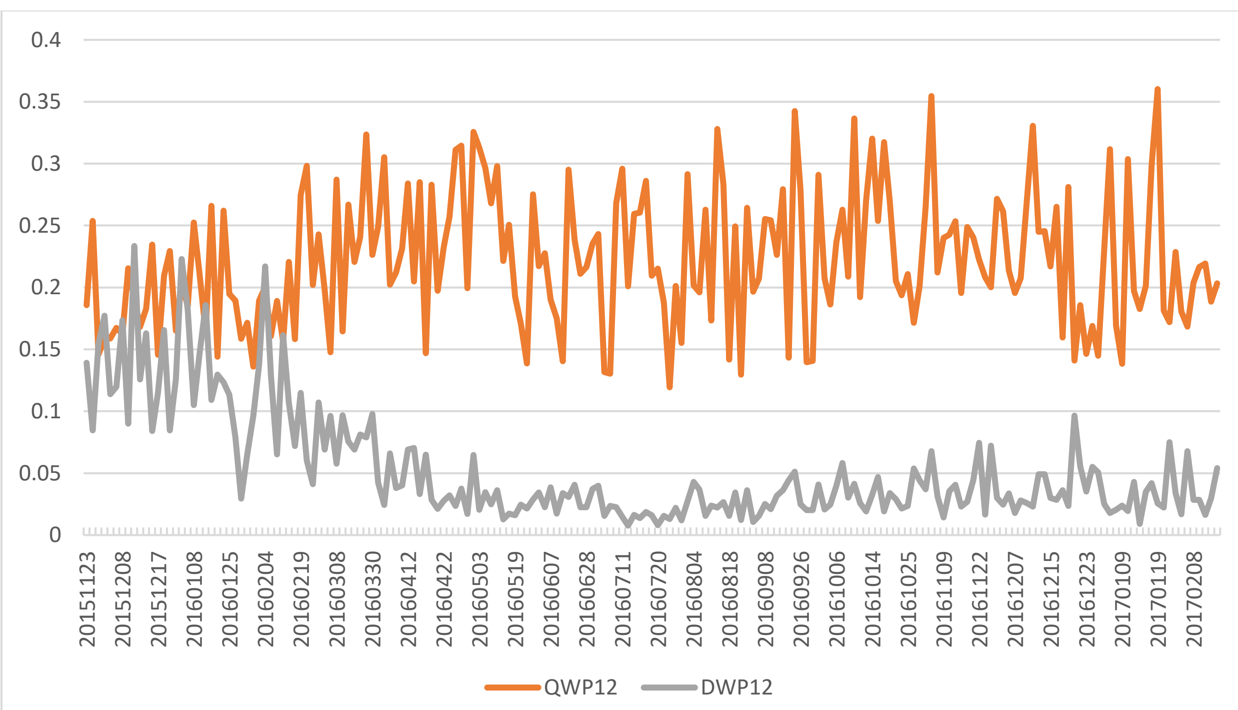
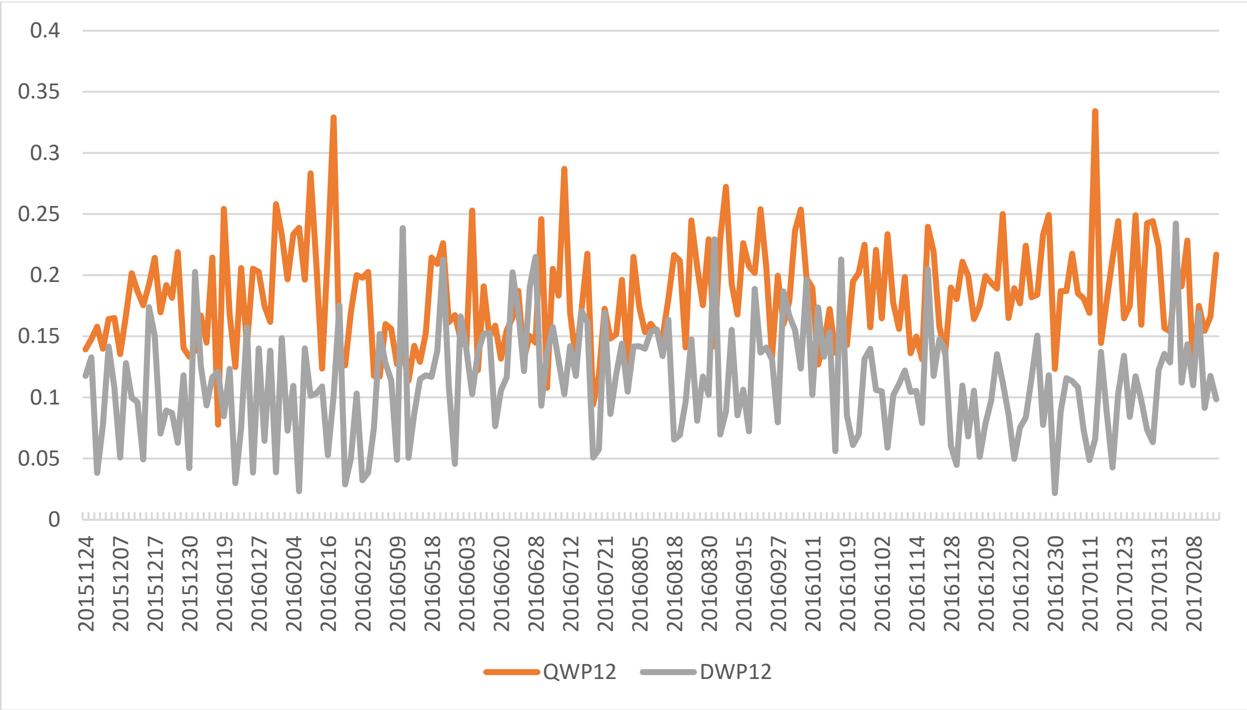


Figure 9. Daily MIS of the top two steps of the LOB for wheat.



Robustness of Results

The LOB updates continuously in an irregular timely manner, but not every step will change when we construct the duration-weighted price based on transaction price changes. If there is no update on step *n*, we can 1) use the previous duration as weight or 2) use zero duration as weight. The results of the first option have been shown in table 5. Table 6 shows the results for the second option. We find that these results are consistent with the results from table 5.

Table 6. Summary statistics of the average MIS (%)

		$Price_t$	QWP_t^{1-2}	QWP_t^{3-5}	DWP_t^{1-2}	DWP_t^{3-5}
Lean hogs	**Mean**	**57.01%**	**16.14%**	**8.83%**	**14.16%**	**3.86%**
	Median	57.77%	15.72%	8.33%	13.74%	3.52%
	Min	35.74%	8.55%	3.43%	5.53%	0.29%
	Max	71.69%	32.43%	21.11%	35.01%	14.38%
	Sd	0.070	0.042	0.033	0.051	0.023
		$Price_t$	QWP_t^{1-2}	QWP_t^{3-5}	DWP_t^{1-2}	DWP_t^{3-5}
Live cattle	**Mean**	**60.15%**	**17.54%**	**8.87%**	**10.22%**	**3.23%**
	Median	60.60%	16.62%	8.54%	10.21%	2.98%
	Min	36.52%	9.28%	2.70%	3.41%	0.37%
	Max	73.50%	50.95%	20.89%	19.67%	12.20%
	Sd	0.066	0.056	0.029	0.035	0.019
		$Price_t$	QWP_t^{1-2}	QWP_t^{3-10}	DWP_t^{1-2}	DWP_t^{3-10}
Corn	**Mean**	**42.63%**	**26.66%**	**24.93%**	**4.62%**	**1.16%**
	Median	43.47%	26.83%	23.51%	4.09%	0.73%
	Min	12.60%	11.05%	8.21%	0.88%	0.00%
	Max	63.98%	41.50%	67.41%	12.87%	8.31%
	Sd	0.106	0.062	0.125	0.026	0.015
		$Price_t$	QWP_t^{1-2}	QWP_t^{3-10}	DWP_t^{1-2}	DWP_t^{3-10}
Soybean	**Mean**	**48.25%**	**19.73%**	**19.38%**	**11.28%**	**1.36%**
	Median	48.70%	18.53%	16.22%	10.52%	1.00%

	Min	17.27%	12.92%	6.92%	4.41%	0.21%
	Max	67.61%	30.42%	57.16%	22.80%	3.54%
	Sd	0.117	0.045	0.108	0.039	0.009
		$Price_t$	QWP_t^{1-2}	QWP_t^{3-10}	DWP_t^{1-2}	DWP_t^{3-10}
Wheat	**Mean**	**44.93%**	**17.04%**	**25.79%**	**10.38%**	**1.87%**
	Median	46.00%	16.94%	22.17%	9.57%	1.43%
	Min	13.30%	7.87%	7.19%	2.71%	0.04%
	Max	67.67%	25.20%	71.15%	19.18%	7.69%
	Sd	0.133	0.035	0.159	0.045	0.017

6. Conclusions

The effect of duration on transaction prices and mid-quote revisions has been shown by many studies (Easley et al. 1997, Dufour and Engle 2000, Ruan and Ma 2017). Most studies on time duration are based on transaction data. Research focusing on the time duration of LOB updates seems to be lacking. In particular, the information content of time durations in agricultural commodity markets has not been studied. We examine the LOB for lean hogs, live cattle, corn, soybean, and wheat for the period of November 2015 to February 2017 and study how new information is conveyed by traders and incorporated into transaction prices. We assess the informational content of the observed depth and time durations of book updates at different levels of the LOB. We find that both depth and durations do convey information as they reflect informed traders' order submission strategies when new information enters the market. On average, market depth conveys more information than durations for all five commodities. Nonetheless, in about

26.13% of the sampled days the information contained in the durations is higher than that contained in the depth. For the meats, market depth at the top of the book appears to have more information that is incorporated in transaction prices relative to steps further away in the LOB. For corn and wheat, depth beyond the first two steps of the book contain more information relative to the top of the book. For corn, the information contained in the depth seems to be distributed more evenly across all steps of the book. Durations at the top of the book contain more information than durations further away for all five commodities. Moreover, durations contribute more to price discovery in meats than in grains. Our results are in line with those in Arzandeh and Frank (2019) in that the LOB beyond the best bids and asks contain information that is incorporated into prices. We further show that in addition to the number of contracts associated with each price level, the time duration of book updates also contains information that contributes to the price discovery of agricultural commodities.

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Appendix

Summary of daily optimal lags for the VEC model based on AIC for five agricultural commodities

	Mean	Min	Max	P50	P75	P90
Lean hogs	3.65	2	10	3	4	5
Live cattle	4.42	1	10	4	5	6
Corn	5.15	2	10	5	6	7
Soybean	5.60	2	10	5	6	8
Wheat	4.62	2	10	4	5	7

P50 (the median), P75, and P90 are the 50th, 75th and 90th percentiles, respectively.