IMPROVING THE ACCURACY OF NORMAL FORM ANALYSIS

By

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Niraj Kshatriya

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CHAPTER 1 Introduction

1.1 Power system stability

A Power system normally remains in a state of equilibrium under normal operating conditions and regains an acceptable state of equilibrium after being subjected to a disturbance. Small signal stability is the ability of a power system to maintain synchronism when subject to a small disturbance. The disturbances are considered sufficiently small if the model used for small signal analysis is a true representation of the nonlinear system. During the disturbances, power systems exhibit oscillations of various parameters viz. voltage, active and reactive power flow, frequency etc., and control systems are called upon to damp out those oscillations. Also during normal operations, control systems' objective is to operate as efficiently as possible by maintaining voltage and frequency close to nominal values.

The equations governing the dynamics of power systems are nonlinear, from which it is difficult to understand its behavior and to identify the inputs to control them. Hence, it is necessary to transform and/or make few approximations about the system before analyzing it. In small signal analysis, the traditional way of analyzing power systems and designing controllers, the differential equations governing the dynamics of power systems are expanded about a Stable Equilibrium Point (SEP) using Taylor series and the system is linearized using the first term of the series. By doing so it is not possible to model the nonlinearity associated with the dynamics. But now a days power demands and market structure are causing the system to operate more and more close to the point of steady state instability, which causes the system to be more and more nonlinear. So, nonlinear analysis is emerging as an area of increasing importance in the study of stressed power systems.

1.2 Nonlinear analysis

Recently Normal Form (NF) technique has been used for nonlinear analysis of power systems. The idea behind NF technique is to model the system up to certain degree of nonlinearities and to apply a sequence of nonlinear transformations that removes the nonlinear terms, and then the closed form solution of the original system can be obtained. It has been shown that NF analysis can be used to study nonlinear interaction among the fundamental modes [3][7][8][11]. Applications of NF analysis include control systems design [6][9][15], and to predict inter area separation in power systems [7][8]. More recent work has been reported as 'Modal Series', where a new method for nonlinear analysis has been suggested and is claimed it to be more robust than NF analysis [12][14].

1.3 Motivation and objective

Nonlinear analysis of power systems is emerging as a new area and motivation behind this project was to find benefits and limitations of using NF analysis in the study of power systems. There are inherent assumptions in the theory of NF analysis, the implications of which were never explored in the practical use of NF analysis.

The objective of the research work was to explore the implications of these assumptions on accuracy, and to investigate the methods of improving the accuracy of NF analysis.

In this thesis it has been demonstrated that when the inherent assumption of NF technique are violated, it does not provide reliable information about the power system. Three indices have been proposed to predict violation of these assumptions so that NF analysis can be applied accurately.

1.4 Outline of thesis

In **Chapter 2** the basic theory of linear analysis and NF analysis are presented. Linear participation factors using linear analysis, which are being used in traditional small signal analysis are discussed. Nonlinear participation factors and NF indices that have been suggested to quantify the nonlinearity using NF analysis are also discussed here.

Chapter 3 discusses the two assumptions underlying the NF technique. Three indices are proposed here, which can predict violation of one of these basic assumptions.

Effect of two assumption underlying the NF technique were investigated using two test systems: *1.* a set of four differential equations with *2^nd^* order nonlinearity and *2.* Two-Area, Four-Generator system. These are described in detail in **Chapter 4**.

In **Chapter 5**, using test systems, it is demonstrated that when the assumptions are violated NF analysis becomes inaccurate and when those are complied with, reliable results can be produced. It is also shown that the suggested indices can quickly identify the violation of assumptions of NF analysis.

Chapter 6 concludes the thesis with a discussion on accuracy of indices and future direction for the research.

CHAPTER 2 Linear analysis and nonlinear analysis

A power system is inherently nonlinear as generators introduce nonlinearity because of the swing equation, excitation limiting and magnetic saturation; and loads introduce nonlinearity due to variation in characteristics with voltage and frequency. For small signal analysis purposes it is assumed that the generator excitation limit is not reached and that the magnetic material of generator does not saturate, and the load is modeled as either of or combination of constant impedance, constant current and constant power. With above background, the power system is analyzed in small signal stability as discussed in following sections.

2.1 Representation of power system for stability analysis

2.1.1 State space representation

A power system can be described by a set of n first order nonlinear ordinary differential equations as

$$\dot{x}_i = f_i(x_1, x_2, \ldots x_n) \qquad i = 1, 2, \ldots, n \tag{2.1}$$

by considering that the inputs to the system are derived from the state variables. In a vector-matrix notation form Equation 2.1 can be written as

$$\dot{X} = F(X) \tag{2.2}$$

where

$$X = [x_1, x_2, \ldots, x_n]^T \qquad\qquad F = [f_1, f_2, \ldots, f_n]^T$$

X is referred to as *state vector*, and its entries as *state variables*. For the system, *equilibrium points* are those points where $\dot{X} = 0$, and at those points the system is at rest and all the variables are constant and unvarying with time. These are also called *fixed, steady-state, or singular points* of the system. The system is said to be *asymptotically stable* if, when subjected to a small perturbation, it returns to the original state. Henceforth we will refer to the equilibrium points, where the system is asymptotically stable, as *stable equilibrium points (SEP)*. The equilibrium points by definition satisfy following equation

$$F(X_{sep}) = 0 \tag{2.3}$$

where X_{sep} is the numerically evaluated state vector X at the SEP. The equilibrium points are truly characteristic of the dynamical system and therefore we can draw conclusions about the stability from their nature. It is difficult to estimate the true nature of the power system from the set of nonlinear differential equations given by Equation 2.2 and for that purpose it is common practice to simplify the power system using Taylor series expansion as discussed in following subsection.

2.1.2 Taylor series expansion

The general form of Taylor series of a real function in n variables, $f(x_1, \ldots, x_n)$, about $x = [x_1', \ldots, x_n']^T$ for a small deviation $\Delta x_1, \ldots, \Delta x_n$ is given by

$$f(x_1' + \Delta x_1, \ldots, x_n' + \Delta x_n) = \sum_{j=0}^{\infty} \left\{ \frac{1}{j!} \left[\sum_{k=1}^{n} \Delta x_k \frac{\partial}{\partial x_k} \right]^j f(x_1, \ldots, x_n) \right\}_{x_1 = x_1', \ldots, x_n = x_n'} \tag{2.4}$$

Let's see how this can be applied to the problem of the power system. Let X_{sep} be the initial state vector corresponding to the SEP, about which the power system is being analyzed. From Equation 2.1 we have

$$\dot{x}_{isep} = f_i(X_{sep}) = 0 \tag{2.5}$$

If the state variables are perturbed by a small deviation ΔX, then the new value of the state variables is $x_i = x_{isep} + \Delta x_i$. The new state must satisfy Equation 2.2. Hence

$$\dot{x}_i = \dot{x}_{isep} + \Delta\dot{x}_i = f_i(X_{sep} + \Delta X) \tag{2.6}$$

As perturbations are small, the nonlinear function in Equation 2.6 can be expressed in terms of Taylor's series expansion using Equation 2.4, with explicit terms for $j = 0, \ldots, 2$ and $O(|\Delta X|^3)$ for $j \geq 3$, as

$$f_i(X_{sep} + \Delta X) = f_i(X_{sep}) + \left.\frac{\partial f_i(X)}{\partial x_1}\right|_{X_{sep}} \Delta x_1 + \ldots + \left.\frac{\partial f_i(X)}{\partial x_n}\right|_{X_{sep}} \Delta x_n \qquad \textbf{(2.7)}$$
$$+ \left.\frac{1}{2}\frac{\partial^2 f_i(X)}{\partial x_1 \partial x_1}\right|_{X_{sep}} \Delta x_1 \Delta x_1 + \left.\frac{1}{2}\frac{\partial^2 f_i(X)}{\partial x_1 \partial x_2}\right|_{X_{sep}} \Delta x_1 \Delta x_2$$
$$+ \ldots + \left.\frac{1}{2}\frac{\partial^2 f_i(X)}{\partial x_n \partial x_{n-1}}\right|_{X_{sep}} \Delta x_n \Delta x_{n-1} + \left.\frac{1}{2}\frac{\partial^2 f_i(X)}{\partial x_n \partial x_n}\right|_{X_{sep}} \Delta x_n \Delta x_n$$
$$+ O(|\Delta X|^3)$$

where

$\left.\frac{\partial f_i(X)}{\partial x_k}\right|_{X_{sep}}$ is first partial derivative of function $f_i(X)$ with respect to state variable x_k evaluated at SEP.

$\left.\frac{\partial^2 f_i(X)}{\partial x_{k1} \partial x_{k2}}\right|_{X_{sep}}$ is double partial derivative of function $f_i(X)$ with respect to state variables x_{k1} and x_{k2} evaluated at SEP.

$O(|\Delta X|^3)$ is homogeneous polynomial of degree three and higher order of state variables ΔX, for $j = 3, \ldots, \infty$ in Equation 2.4.

$\Delta \dot{x}_i$ can be obtained from Equation 2.5 and 2.6 and 2.7 and in compact form is given by

$$\Delta \dot{x}_i = A_i \Delta X + \frac{1}{2}\Delta X^T H^i \Delta X + O(|\Delta X|^3) \qquad \textbf{(2.8)}$$

where A_i is i^{th} row of $n \times n$ sized *Jacobian matrix A,* also called *state or plant matrix*, evaluated at SEP and is given by

$$A = \left.\begin{bmatrix} \frac{\partial f_1(X)}{\partial x_1} & \cdots & \frac{\partial f_1(X)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(X)}{\partial x_1} & \cdots & \frac{\partial f_n(X)}{\partial x_n} \end{bmatrix}\right|_{X_{sep}} \qquad \textbf{(2.9)}$$

and H^i is i^{th} $n \times n$ sized symmetric *Hessian matrix* evaluated at SEP and is given by

$$H^i = \begin{bmatrix} \frac{\partial^2 f_i(X)}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 f_i(X)}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f_i(X)}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f_i(X)}{\partial x_n \partial x_n} \end{bmatrix}_{X_{sep}} \quad \textbf{(2.10)}$$

As per customary notation replacing ΔX with X in Equation 2.8, we get

$$\dot{x}_i = A_i X + \frac{1}{2} X^T H^i X + O(|X|^3) \quad \textbf{(2.11)}$$

2.1.3 Neighborhood of SEP

A neighborhood of an SEP in n-dimensional real space $\mathbb{R}^n$ is the set of points inside an *n-sphere* with center at the SEP and radius $\varepsilon > 0$. In linear and NF analysis the system is simplified by assuming that it is in a neighborhood of SEP having certain properties as discussed in Section 2.3 and Section 2.4 respectively. Before that eigen analysis of a matrix is discussed in the following section, which forms the basis of linear analysis and NF analysis.

2.2 Eigen properties of a matrix

Let A be an $n \times n$ square matrix, then there are special set of scalars associated with it known as eigenvalues and each eigenvalue is paired with a corresponding right and left eigenvectors. This concept is further discussed in following section.

2.2.1 Eigenvalue of a matrix

For a given $n \times n$ square matrix A, if there is a $n \times 1$ vector $U \in \mathbb{R}^n \neq 0$ such that

$$AU = \lambda U \tag{2.12}$$

for some scalar λ, then λ is called eigenvalue of A with corresponding right eigenvector U. Equation 2.12 can be written as

$$(A - \lambda I)U = 0 \tag{2.13}$$

where I is identity matrix. Above equation has nontrivial solution if and only if

$$det(A - \lambda I) = 0 \tag{2.14}$$

This equation is known as the characteristic equation of A, and the left-hand side known as the *characteristic polynomial*. The n solutions of $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_n$ are called *eigenvalues* of A. If A is real (which in fact is the case with the power system), eigenvalues may be real and/or complex conjugate pairs.

2.2.2 Eigenvectors associated with eigenvalues

For any eigenvalue λ_i, the n-column vector U_i which satisfies Equation 2.12 is called the *right eigenvector* of A associated with eigenvalue λ_i. Therefor we have

$$AU_i = \lambda_i U_i \qquad i = 1, 2, \ldots, n \tag{2.15}$$

Similarly, $1 \times n$ vector $V_i \in \mathbb{R}^n \neq 0$ which satisfies

$$V_i A = \lambda_i V_i \qquad i = 1, 2, \ldots, n \tag{2.16}$$

is called *left eigenvector* of A associated with eigenvalue λ_i. The left and right eigenvectors corresponding to different eigenvalues are orthogonal

$$V_i U_j = 0 \tag{2.17}$$

and the product of eigenvectors corresponding to the same eigenvalues are constant

$$V_i U_i = C_i \tag{2.18}$$

where C_i is a non-zero constant. It is common practice to normalize these vectors so that

$$V_i U_i = 1 \tag{2.19}$$

2.2.3 The modal matrices

From above definitions of eigenvalues and associated right eigenvectors and left eigenvectors, modal matrices are defined as

$$U = \begin{bmatrix} U_1 & U_2 & \dots & U_n \end{bmatrix} \tag{2.20}$$

$$V = \begin{bmatrix} V_1^T & V_2^T & \dots & V_n^T \end{bmatrix} \tag{2.21}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} \tag{2.22}$$

Each of above matrices are of size $n \times n$. In terms of these matrices, Equation 2.15, 2.16 and 2.19 can be expanded as

$$AU = U\Lambda \tag{2.23}$$

$$V^T A = V^T \Lambda \tag{2.24}$$

$$V^T U = I \qquad \textit{and hence} \qquad V^T = U^{-1} \tag{2.25}$$

and from above it follows that

$$U^{-1}AU = \Lambda \qquad and \qquad V^{T}AV^{T^{-1}} = \Lambda \tag{2.26}$$

2.3 Linear analysis

2.3.1 Basic theory

The Taylor series expansion of the system, given by Equation 2.11, is still nonlinear and difficult to analyze, hence in small signal stability analysis it is assumed that the system is in a certain neighborhood of SEP where $O(|X|^2)$ is negligible and Equation 2.11 can be approximated to be

$$\dot{X} = AX \tag{2.27}$$

The set of differential equations of above form gives rate of change of each state variable, which is a linear combination of all the state variables. Due to the assumption that the system is linear it is not possible to obtain the nonlinear characteristic of the power system, which in some case may not be small enough to ignore.

Let U and V^T be the right and left eigenvector matrix associated with Λ, the diagonal matrix of n distinct eigenvalues λ_i, $i = 1, 2, \ldots, n$ for state matrix A. Then using transformation

$$X = UY \tag{2.28}$$

Equation 2.27 is transforms into

$$\dot{Y} = \Lambda Y \tag{2.29}$$

where Y is a new state vector. Equation 2.29 represents n uncoupled first order differential equations

$$y_i = \lambda_i y_i \quad \textbf{(2.30)}$$

The time domain solution of above in terms of initial condition of state vector Y, (Y_0), is given by

$$y_i(t) = y_{i0}e^{\lambda_i t} \quad \textbf{(2.31)}$$

From Equation 2.28, the time domain solution of the original state vector X is given by

$$x_i(t) = \sum_{j=1}^{n} U_{ij}y_{j0}e^{\lambda_j t} \quad \textbf{(2.32)}$$

Thus, the response of the system is given by a linear combination of n-dynamic modes corresponding to the n-eigenvalues of the state matrix. The scalar y_{j0} represents the magnitude of the excitation of the i^{th} mode resulting from the initial condition.

2.3.2 Participation factor

If only k^{th} state variable is excited by magnitude of unity, $X_0 = [0, \ldots, 0, 1, 0, \ldots 0]$, then from Equation 2.25 and 2.28, $y_{i0} = V^T_{ik}$, and time domain solution of state variable from Equation 2.32 is given by

$$x_i(t) = \sum_{j=1}^{n} U_{ij}V^T_{jk}e^{\lambda_j t} \quad \textbf{(2.33)}$$

The elements $U_{ki}V_{ik}^T$ are termed as *participation factors* p_{ki} and are measures of the relative participation of the k^{th} state variable in the i^{th} mode and vice versa [13]. In terms of participation factors Equation 2.33 can be written as

$$x_i(t) = \sum_{j=1}^{n} p_{ki}e^{\lambda_j t} \tag{2.34}$$

Participation matrix is given by

$$P = \begin{bmatrix} U_{11}V^T{}_{11} & \dots & U_{1n}V^T{}_{n1} \\ \vdots & \ddots & \vdots \\ U_{n1}V^T{}_{1n} & \dots & U_{nn}V^T{}_{nn} \end{bmatrix} \tag{2.35}$$

2.3.3 Initial condition in linear analysis

Though linear analysis is initial condition independent (Section 2.5), to verify the plant matrix A, of the power system, closed form time domain solution of the state variables given by Equation 2.32 is compared with that of nonlinear model obtained by numerical integration method. For that purpose it may be necessary to obtained the initial conditions of the modal variables, which can be derived using first four steps, described in Subsection 2.4.4.

2.4 Normal Form (NF) analysis

2.4.1 Basic theory

To understand the physical phenomena of a nonlinear system it is necessary to model it with nonlinearity. In NF analysis the system is modeled with nonlinearities up to a certain degrees, then using a sequence of transformations these nonlinear terms are removed successively and finally reducing the system to the linear one. Here we will restrict discussion to models with 2^{nd} order nonlinearity and the transformations of up to 2^{nd} order.

By assuming that the system is in a certain neighborhood of SEP where $O(|X|^3)$ is negligible, Equation 2.11 can be approximated to be

$$\dot{x}_i = A_i X + \frac{1}{2} X^T H^i X \tag{2.36}$$

This is a set of differential equations with homogeneous polynomial of the state variables of degree 1 and 2, and gives rate of change of the state variables as a quadratic function of the state variables. Using different notation Equation 2.36 can be written as

$$\dot{x}_i = \sum_{j=1}^{n} A_{ij} x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} H^i_{jk} x_j x_k \tag{2.37}$$

By using the transformation given by Equation 2.28, the same transformation that was used for linear analysis, Equation 2.36 in terms of new state vector Y becomes

$$\dot{y}_i = \lambda_i y_i + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} C^i_{jk} y_j y_k \tag{2.38}$$

where

$$C^i_{jk} = \frac{1}{2} \sum_{l=1}^{n} \sum_{m=1}^{n} \sum_{o=1}^{n} V^T_{il} H^l_{mo} U_{mj} U_{ok} \tag{2.39}$$

In Equation 2.38 the first term is linearly uncoupled, however the second term is still nonlinearly coupled. According to Poincaré's theorem a power series given by Equation 2.38 can be reduced to its linear form at an SEP by applying series of nonlinear transformation if the eigenvalues are nonresonant ([2], §22 and 23). However, here we will not apply series of transformation, instead we will limit to one nonlinear transformation. In the absence of second order resonance, i.e. $\lambda_i \neq \lambda_j + \lambda_k$ for all n-tuple of eigenvalues, the transformation given by

$$y_j = z_j + \sum_{k=1}^{n} \sum_{l=1}^{n} h2^j_{kl} z_k z_l \qquad \textbf{(2.40)}$$

where

$$h2^j_{kl} = \frac{C^i_{jk}}{\lambda_i - \lambda_j + \lambda_k} \qquad \textbf{(2.41)}$$

eliminates the *2*nd order term in Equation 2.38 and in terms of new variable Z it becomes

$$\dot{z}_j = \lambda_j z_j + O(|Z|^3) \qquad \textbf{(2.42)}$$

Please note that due to nonlinear transformation, the *2*nd term has been eliminated, however the *3*rd and higher order terms have been introduced, which were originally absent in differential equations in state space and modal space. By assuming that the system is in the neighborhood of SEP of Z space where $O(|Z|^3)$ is negligible then the system is linearly uncoupled and is given by

$$\dot{z}_j = \lambda_j z_j \qquad \textbf{(2.43)}$$

The time domain solution of Z variable ($Z(t)$) is given by

$$z_j = z_{j0} e^{\lambda_j t} \qquad \textbf{(2.44)}$$

Time domain solution for Y variables, ($Y(t)$), and state variables, ($X(t)$), can be constructed from above and from the transformation definitions, and are given by

$$y_j(t) = z_{j0}e^{\lambda_j t} + \sum_{k=1}^{n}\sum_{l=1}^{n} h2^j_{kl} z_{k0} z_{l0} e^{(\lambda_k + \lambda_l)t} \tag{2.45}$$

$$x_i(t) = \sum_{j=1}^{n} U_{ij} z_{j0} e^{\lambda_j t} + \sum_{j=1}^{n} U_{ij}\left[\sum_{k=1}^{n}\sum_{l=1}^{n} h2^j_{kl} z_{k0} z_{l0} e^{(\lambda_k + \lambda_l)t}\right] \tag{2.46}$$

2.4.2 Nonlinearity indices

Several measures of the nonlinearity in *Y* coordinates were proposed in [7] based on the *h2* coefficients and the initial condition of a transient.

An interaction coefficient is defined as

$$h2^i_{jk} z_{j0} z_{k0} \tag{2.47}$$

to quantify the effects of second order terms on the transient solution.

The nonlinear interaction index *I1* for mode *j* is defined as

$$I1(j) = \left| \, y_{j0} - z_{j0} + \max_{k,l}(h2^j_{kl} z_{k0} z_{l0}) \, \right| \tag{2.48}$$

It gives the measure of the effect of the nonlinear terms in the solution by comparing the linear solution to the second order solution of modal *Y* coordinates.

Nonlinear interaction index *I2* for mode *j* is defined as

$$I2(j) = \frac{\max_{k,l}(h2^j_{kl} z_{k0} z_{l0})}{z_{j0}} \tag{2.49}$$

It determines whether the nonlinear effects arises from the second order terms indicating a strong modal interaction, or whether the second order terms affect the initial solution in the *Z* coordinates indicating a dominant fundamental mode [16].

Large value of interaction coefficient or indices *I1* and *I2* correctly reflect the high nonlinearity of the system in modal coordinates, but do not necessarily imply that the system has significant nonlinearity in the original *X* coordinates [10].

2.4.3 Nonlinear participation factor

Nonlinear participation factor is defined by extending the idea of linear participation factor [8]. If only k^{th} state variable is excited by magnitude of 1, $X_0 = [0, \ldots, 0, 1, 0, \ldots 0]$, then from Equation 2.25 and 2.28, $y_{i0} = V_{ik}^T = v_{ik}$.

Since $Y = O(|Z|)$, the inverse of Equation 2.40 takes the form ([1], 2.3)

$$z_j = y_j - h2(Y) + O(|Y|^3) \tag{2.50}$$

By neglecting $O(|Y|^3)$ approximate solution of Z variables is given by

$$z_{j0} \approx v_{jk} - \sum_{p=1}^{n} \sum_{q=1}^{n} h2_{pq}^{j} v_{pk} v_{qk} \tag{2.51}$$

and from Equation 2.46, the time domain solution of state variables is given by

$$x_k(t) \approx \sum_{i=1}^{n} p2_{ki} e^{\lambda_i t} + \sum_{p=1}^{n} \sum_{q=1}^{n} p2_{kpq} e^{(\lambda_p + \lambda_q)t} \tag{2.52}$$

where,

$$p2_{ki} = u_{ki}\left(v_{ik} - \sum_{p=1}^{n} \sum_{q=1}^{n} h2_{pq}^{i} v_{pk} v_{qk}\right) \tag{2.53}$$

There are two types of second-order participation factors. The first participation factor $p2_{ki}$ represents the second order participation of k^{th} state in the i^{th} mode. It can be seen that the linear participation factor $p_{ki} = u_{ki} v_{ik}$ is the first term in above and the second term can be thought of as providing second order corrections to it. The second type of second order participation factor $p2_{kpq}$ represents the participation of the k^{th} state in a new mode $\lambda_p + \lambda_q$, formed by combination of the original mode p and q.

Please note that above definition of nonlinear participation factor is approximate due to the fact that solution of Z variable, the inverse of nonlinear transformation, is approximate.

2.4.4 Initial conditions in NF analysis

The closed form time domain solution of modal variables Y, and state variables X in NF analysis can be obtained as shown above. In order to obtain these solutions, the initial condition in Z space is required, which can be obtained as follow [7].

1. Values of state variables immediately after clearing a disturbance or from the trajectories of state variables after clearing a disturbance is obtained as X_{cl}
2. Values of state variables at post disturbance SEP is obtained as X_{sep}.
3. The initial condition for state variables with respect to SEP is obtained as $X_0 = X_{cl} - X_{sep}$.
4. Using Equation 2.28, the initial condition of modal variable can be obtained as $Y_0 = U^{-1}X_0$.
5. Then simultaneous solution of following n-nonlinear equations, using known value of Y_0, gives initial condition Z_0.

$$z_{j0} + h2(Z) - y_{j0} = 0 \tag{2.54}$$

No explicit solution exists for above equations and one has to resort to a numerical technique to solve it, which needs an initial guess value, $Z0guess$. There exist multiple solutions to Equation 2.54 and the converged value depends on $Z0guess$. However, the most sought after Z_0 is the value which gives minimum $O(|Z|^3)$ in Equation 2.42, hence selection of initial guess is one of the key steps in NF analysis. Any random value can be used as a guess value, however it is unlikely that it can produce converged value of Z_0 which can be useful in NF analysis. Following are suggested $Z0guess$ values that can lead to a correct solution.

1. From Equation 2.50 it can be said that best guess value can be

$$z0guess_j = y_{j0} - h2(Y_0) \quad \textbf{(2.55)}$$

2. Second suggested guess value is

   $z0guess_j = y_{j0}$

3. Third suggested guess value is

   $z0guess_j = 0$

4. Fourth suggested guess value is the converged value of Z_0 in a different case or any random guess.

2.5 Initial condition independent linear analysis

The Taylor expansion of the nonlinear system around an SEP is again nonlinear. In linear analysis, for analysis purpose, it is assumed that the system is in the neighborhood of SEP where $O(|X|^2)$ is negligible and the system is linear. If the system is free and not at SEP then the system will move from its initial condition, defined as state of the system with respect to a fixed point, towards an SEP with time. If the initial condition of the system is not in the neighborhood of the SEP, where the system can be assumed to be linear, then while moving towards the SEP at a particular instant it will enter that neighborhood and will remain inside of it and settle to an SEP. As the system is assumed to be linear, the properties of the system predicted by linear analysis are unique, irrespective of the state of the system when it enters the neighborhood. Hence linear analysis is initial condition independent, when analyzing the system in the neighborhood of SEP where $O(|X|^2)$ is negligible. Because of its initial condition independent nature, linear analysis is a widely used tool to study small signal stability analysis of power systems, which can be used to identify local area modes, inter area modes and to select location and input of controller and design controller to improve the system performance.

2.6 Initial condition dependent NF analysis

In NF analysis it is assumed that the system is in the neighborhood of SEP where $O(|X|^3)$ is negligible and the system can be assumed to be quadratic. As described in Section 2.5 if the system is not in that neighborhood of SEP then, while moving from an initial condition, it will enter that neighborhood at a particular instant and will remain inside of that neighborhood and settle to an SEP. However, the system is assumed to be quadratic in that neighborhood hence the properties of the system predicted by NF analysis will depend on the state of the system when it enter into that neighborhood. Hence, NF analysis an initial condition dependent analysis, when analyzing the system in the neighborhood of SEP where $O(|X|^3)$ is negligible. In the reported studies to date, the NF analysis has been applied to the power system by considering a disturbance and using the state of the system immediately after clearing a disturbance to calculate initial condition.

Next we will investigate NF analysis by using initial condition obtained by applying a disturbance and using the value of state variables from their trajectories, in addition to that obtained using the values of the state variables immediately after clearing a disturbance.

CHAPTER 3 Validating initial conditions in NF analysis

As mentioned in Subsection 2.4.1 there are two basic assumptions underlying the NF analysis and as discussed in Section 2.6 NF analysis is initial condition dependent. It is necessary for initial condition to be used for NF analysis to comply with the assumptions, otherwise NF analysis will not be accurate. The assumptions are discussed in detail here and three indices have been proposed that can identify failure of one of the assumptions.

3.1 Assumption A

3.1.1 In state space $O(|X|^3)$ is negligible

Consider a power system in which a disturbance occurs, and upon clearing the disturbance, the system settles to an SEP. Let the value of state variables at SEP be X_{sep}, which is used to derive A and H matrices for second order small signal model given by Equation 2.36. Then let X be the state of the system at a given instant on the post disturbance trajectories of the state variables. Then initial condition for the model is given by

$X_0 = X - X_{sep}$. One of the possible choice for X is the state of the system immediately after clearing a disturbance, X_{cl}, which has been used in reported studies to date.

In NF analysis Equation 2.36 is assumed to be an approximation of Equation 2.11. In other words the system should be in the neighborhood of SEP where $O(|X|^3)$ is negligible. From Equation 2.4 and 2.7 it can be seen that two parameters decide the size of $O(|X|^3)$. The first parameter is k^{th} partial derivative of $F(X)$ evaluated at X_{sep}, which is system dependent, and second parameter is its corresponding multiplier, $(\Delta x_k)^j$. For a given operating point only choice left to user is to select initial condition X_0 (= ΔX) appropriately, i.e. sufficiently close to the SEP, such that $O(|X|^3)$ becomes negligible. Hence it is necessary to validate X_0 for *Assumption A*

3.1.2 Strategy to validate *Assumption A*

The time domain solution of Equation 2.2, which represents the true nonlinear dynamics of the states, and Equation 2.36, which is an approximate representation of the nonlinear system for NF analysis, can be compared. The time domain solution of these equations can be obtained using numerical integration technique and initial condition X_0. Equation 2.2 has SEP at X=X_{sep} while Equation 2.36 has SEP at X=*[0]*, hence care needs to be exercised while comparing the time domain solution. If the time domain solutions of the state variables obtained by the above two methods are not in good agreement then it can be said that the chosen initial condition is not sufficiently close to the SEP so that $O(|X|^3)$

becomes negligible. Or in other words, the system is not in the neighborhood of SEP where $O(|X|^3)$ is negligible.

As a theoretical approach *Assumption A* shall be validated using the time responses of all the state variables, which can be a formidable task in the practice. Hence, from the practical application point of view we suggest to compare the time responses of few most non-linear state variables, e.g. speed or angle variation of generators in the system.

The time domain solution obtained for above two equations will be compared using prony analysis method. Practical issues in use of prony analysis, and the strategy to be used are discussed in Subsection 3.1.3.

In the reported results so far, the values of the state variables immediately after clearing a disturbance have been selected to calculate initial condition X_0. If X_0 derived at the instant of clearing a disturbance does not meet the *Assumption A*, then we propose to select the initial condition from the trajectories of the state variables at a subsequent instant that can meet these assumptions. Because, as discussed in Section 2.6, even if initial condition is not in a certain neighborhood of SEP, with time the system will move closer to an SEP and at a particular instant it will enter that neighborhood.

3.1.3 Using Prony analysis to compare time domain responses

Prony analysis is a methodology that extends Fourier analysis by estimating frequency, damping, strength and relative phase of that component present in a given signal. And here prony analysis will be used to compare the two time responses as mentioned in earlier subsection. However, the following important practical issues need to be considered for the sensible use of the method [17]:

1. Sampling time selection, i.e. time interval between two consecutive sample of the signal
2. Data window selection, i.e. the time duration of the signal
3. Data de-trending, i.e. removal of steady state values from the signal
4. Model order selection, i.e. estimating of number of modes present in the signal

Prony analysis will produce most accurate result when sampling time is in the order of 0.01 s (issue 1) and that is what we will use in our analysis. Results using prony analysis will be inaccurate for very small or very large sample data length (issue 2). And also the result is inconsistent for two different data length of minor difference, e.g. prony of time length 20.0s and 21.0s will be different. To perform prony analysis, the signal will be generated using numerical integration technique, and data will not have any white noise present in the signal. Hence, issue 3 does not pose any problem in the application considered. In the commercially available software that is used here to perform prony analysis, the users do not have the choice of model order selection. Hence, we do not have control over issue 4.

Though issue 1 has been taken care of, and issue 3 does not pose a problem, because of other limitations prony analysis of a given signal for two different time window of minor difference will generate inconsistent results and care needs to be exercised in interpreting those results. However, prony analysis predicts the low frequency and low damped modes with good accuracy. Hence, we recommend to perform prony analysis of the signals for a given time length and compare the lightly damped low frequencies.

3.2 Assumption B

3.2.1 In *Z* space $O(|Z|^3)$ is negligible

From state space, the system is transformed to the modal space using linear transformation and from modal space, it is transformed to *Z* space using a nonlinear transformation. After these two successive transformations the system takes the form of

$$\dot{z}_j = \lambda_j z_j + O(|Z|^3) \tag{3.1}$$

where the 2^{nd} order terms are absent and the 3^{rd} and higher order terms appear. To obtain time domain solution of *Z* variables it is assumed that $O(|Z|^3)$ is negligible and Equation 3.1 can be approximated to be

$$\dot{z}_j = \lambda_j z_j \tag{3.2}$$

Or, in other words, it is assumed that in *Z* space the system is in the neighborhood of SEP where $O(|Z|^3)$ is negligible. For this to be true Z_0 should be sufficiently small or should be sufficiently closer to an SEP of *Z* space. And for that reason it is necessary to validate converged value of Z_0 for *Assumption B*.

3.2.2 Criteria to validate *Assumption B*

Following subsections suggest indices and corresponding criteria that can be used to validate Z_0 for *Assumption B*.

3.2.2.1 Criterion 1: Relative value of $O(|Z|^3)$ should be small

Numerical value of $O(|Z|^3)$ at $t=t_0$ (at the instant when X_0 is being calculated) in Equation 3.1 can be calculated by computing (a) the exact value of time derivative of Z variables at $t=t_0$, $\dot{Z}_{actual}$, which takes into account both the terms in Equation 3.1, and (b) the approximate value of time derivative of Z variables at $t=t_0$, $\dot{Z}_{appx}$, which takes into account only the first term.

The numerical value of $\dot{Z}_{actual}$, can be calculated using the exact value of time derivative of X variables at $t=t_0$, $\dot{X}_{actual}$, computed using the known values of X_0, and using X to Y and Y to Z variables transformation definition and using converged value of Z_0 as follows. Substituting value of X_0 in Equation 2.37 gives $\dot{X}_{actual}$ as

$$\dot{x}_{actual_i} = \sum_{j=1}^{n} A_{ij}x_j + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} H^i_{jk}\, x_j x_k \tag{3.3}$$

Using definition of X to Y transformation $\dot{Y}_{actual}$ can be calculated as

$$\dot{Y}_{actual} = V^T \dot{X}_{actual} \tag{3.4}$$

Taking time derivative of Y to Z transformation defined by

$$y_j = z_j + \sum_{k=1}^{n}\sum_{l=1}^{n} h2^j_{kl} z_k z_l \tag{3.5}$$

and substituting converged value of Z_0 gives following relation between $\dot{Y}_{actual}$ and $\dot{Z}_{actual}$ as

$$\dot{y}_{actual_j} = \dot{z}_{actual_j} + \sum_{k=1}^{n}\sum_{l=1}^{n} h2^j_{kl}\dot{z}_{actual_k} z_{l0} + \sum_{k=1}^{n}\sum_{l=1}^{n} h2^j_{kl} z_{k0}\dot{z}_{actual_l} \tag{3.6}$$

Rearranging above yields

$$\dot{y}_{actual_j} = \dot{z}_{actual_j} + \sum_{k=1}^{n}\sum_{l=1}^{n} h2^j_{lk}\dot{z}_{actual_l} z_{k0} + \sum_{k=1}^{n}\sum_{l=1}^{n} h2^j_{kl} z_{k0}\dot{z}_{actual_l} \tag{3.7}$$

$$\dot{y}_{actual_j} = \dot{z}_{actual_j} + \sum_{l=1}^{n}\left(\sum_{k=1}^{n}(h2^j_{lk} + h2^j_{kl}) z_{k0}\right)\dot{z}_{actual_l} \tag{3.8}$$

By using following definition of M matrix elements, m_{jl}, which is first partial derivative of $h2(Z)$ in Equation 3.5, evaluated at $Z=Z_0$, $Dh2(Z_0)$,

$$m_{jl} = \sum_{k=1}^{n}(h2^j_{lk} + h2^j_{kl}) z_{k0} \tag{3.9}$$

Equation 3.7 can be written as

$$\dot{y}_{actual_j} = \dot{z}_{actual_j} + \sum_{l=1}^{n} m_{jl}\dot{z}_{actual_l} \tag{3.10}$$

Above can be written in the matrix form as

$$\begin{bmatrix} \dot{y}_{actual_1} \\ \vdots \\ \dot{y}_{actual_n} \end{bmatrix} = \begin{bmatrix} \dot{z}_{actual_1} \\ \vdots \\ \dot{z}_{actual_n} \end{bmatrix} + \begin{bmatrix} \sum_{l=1}^{n} m_{1l}\dot{z}_{actual_l} \\ \vdots \\ \sum_{l=1}^{n} m_{nl}\dot{z}_{actual_l} \end{bmatrix} \tag{3.11}$$

Upon rearranging

$$\begin{bmatrix} \dot{y}_{actual_1} \\ \vdots \\ \dot{y}_{actual_n} \end{bmatrix} = \begin{bmatrix} \dot{z}_{actual_1} \\ \vdots \\ \dot{z}_{actual_n} \end{bmatrix} + \begin{bmatrix} m_{11} & \cdots & m_{1n} \\ \vdots & \ddots & \vdots \\ m_{n1} & \cdots & m_{nn} \end{bmatrix} \begin{bmatrix} \dot{z}_{actual_1} \\ \vdots \\ \dot{z}_{actual_n} \end{bmatrix} \tag{3.12}$$

In more compact form above can be written as

$$\dot{Y}_{actual} = [I + M]\dot{Z}_{actual} \tag{3.13}$$

Using Equation 3.4 and 3.13, exact numerical value of time derivative of Z variables, $\dot{Z}_{actual}$, can be calculated as follows

$$\dot{Z}_{actual} = [I + M]^{-1}\dot{Y}_{actual} \tag{3.14}$$

The values calculated using above formulae gives true numerical value of time derivative of all Z variables at $t=t_0$ and takes into account both the terms in Equation 3.1.

The numerical value of approximated time derivative value of Z variables, $\dot{Z}_{appx}$, can be computed by substituting the value of Z_0 in Equation 3.2 as

$$\dot{z}_{appx_j} = \lambda_j z_{j0} \tag{3.15}$$

Therefore, the relative value of $O(|Z|^3)$ in Equation 3.1 at $t=t_0$, as a % of $\dot{Z}_{actual}$ can be calculated as

$$OZ3 = \frac{|\dot{Z}_{actual} - \dot{Z}_{appx}|}{|\dot{Z}_{actual}|} 100\ \% \tag{3.16}$$

The above value gives the size of $O(|Z|^3)$ that is being neglected and gives an estimate on how farther the system is in the Z space from the neighborhood of SEP where $O(|Z|^3)$ is negligible. Larger the value farther the system is from this neighborhood and *Assumption B* is more likely to be inaccurate. Hence, we propose index *EI1* as

$$EI1 = max(OZ3) \tag{3.17}$$

which is the maximum value in *n*-row vector *OZ3*.

While validating Z_0 for *Assumption B*, this criterion is said to be met if $EI1 < \varepsilon 1$, where $\varepsilon 1$ is a system dependent pre-specified value, which can be in the range of 500-1500 from the many simulations performed on the small test system used in this thesis.

3.2.2.2 Criterion 2: The effect of neglected $O(|Z|^3)$ should be small in state space

By neglecting $O(|Z|^3)$ the time response of *Z* variables can be obtained and using the definition of the transformations, the time response of state variables in terms of *Z* variables can be obtained. The approximate value of $\dot{X}$ at $t=t_0$, $\dot{X}_{appx}$, by neglecting $O(|Z|^3)$ can be calculated as follows

$$\dot{Y}_{appx} = [I+M]\dot{Z}_{appx} \tag{3.18}$$

$$\dot{X}_{appx} = U\dot{Y}_{appx} \tag{3.19}$$

Relative error introduced in time derivative of states in state space is given by

$$OX3 = \frac{|\dot{X}_{actual} - \dot{X}_{appx}|}{|\dot{X}_{actual}|} 100\ \% \tag{3.20}$$

The above value gives the size of the error introduced in state space by neglecting $O(|Z|^3)$ in *Z* space. Large value is an indicator of significant error introduced in state space by making an approximation in *Z* space. Hence, we propose index *EI2* as

$$EI2 = max(OX3) \tag{3.21}$$

which is the maximum value in n-row vector $OX3$. While validating Z_0 for *Assumption B*, this criterion is said to be met if $EI2 < \varepsilon 2$, where $\varepsilon 2$ is a system dependent pre-specified value, which can be in the range of 500-1500 from the many simulations performed.

3.2.2.3 Criterion 3: Eigenvalue of $Dh2(Z_0)$ should be sufficiently small

Differential equation of modal variables is given by Equation 2.38 and in general form it can be written as

$$\dot{Y} = \Lambda Y + g2(Y) \tag{3.22}$$

where $g2(Y)$ is second order function of modal variables. Then by using transformation

$$Y = Z + h2(Z) \tag{3.23}$$

Equation 3.22 can be transformed to

$$\dot{Z} = \Lambda Z + O(|Z|^3) \tag{3.24}$$

where 2^{nd} order terms are absent. Following describes how using a suitable choice of $h2(Z)$ above can be achieved (section 2.3 of [1]).

Taking time derivative of Equation 3.23 and expressing in terms of $\dot{Z}$ yields

$$\dot{Z} = (I + Dh2(Z))^{-1}\dot{Y} \tag{3.25}$$

where $Dh2(Z)$ is first partial derivative of $h2(Z)$ with respect to all the Z variables, which is a first order function. In above, substituting expression of $\dot{Y}$ from Equation 3.22, and expression of Y from Equation 3.23, yields

$$\dot{Z} = (I + Dh2(Z))^{-1}(\Lambda(Z + h2(Z)) + g2(Z + h2(Z))) \tag{3.26}$$

Using Binomial series if we can assume

$$(I + Dh2(Z))^{-1} = (I - Dh2(Z) + O(|Z|^2)) \tag{3.27}$$

then Equation 3.26 can be written as

$$\dot{Z} = (I - Dh2(Z) + O(|Z|^2))(\Lambda(Z + h2(Z)) + g2(Z + h2(Z))) \tag{3.28}$$

Expansion of above yields

$$\dot{Z} = (I - Dh2(Z) + O(|Z|^2))(\Lambda Z + \Lambda h2(Z) + g2(Z) + O(|Z|^3)) \tag{3.29}$$

$$\dot{Z} = \Lambda Z - (Dh2(Z)\Lambda Z - \Lambda h2(Z)) + g2(Z) + O(|Z|^3) \tag{3.30}$$

The transformation matrix *h2(Z)* is chosen such that

$$Dh2(Z)\Lambda Z - \Lambda h2(Z) = g2(Z) \tag{3.31}$$

This is known as the homological equation associated with linear vector field ΛZ ([1], 2.3). When Equation 3.31 is satisfied, Equation 3.30 essentially becomes Equation 3.24.

So, in the NF theory it is assumed that $(I + Dh2(Z))^{-1} = I - Dh2(Z) + O(|Z|^2)$ using Binomial series. The $O(|Z|^2)$ is minimum when maximum of absolute eigenvalues of *Dh2(Z)* at $t=t_0$, $Dh2(Z_0)$ (=M, given by Equation 3.9), very close to zero. Hence we propose index *EI3* as

$$EI3 = max(abs(eigenvalue(M))) \tag{3.32}$$

which is the maximum absolute value in *n*-eigenvalues of matrix *M*. While validating Z_0 for *Assumption B*, this criterion is said to be met if $EI3 < \varepsilon 3$, where $\varepsilon 3$ is a system dependent pre-specified value, which can be in the range of 0.6 to 0.85 from the many simulations performed.

3.2.3 Strategy to validating *Assumption B*

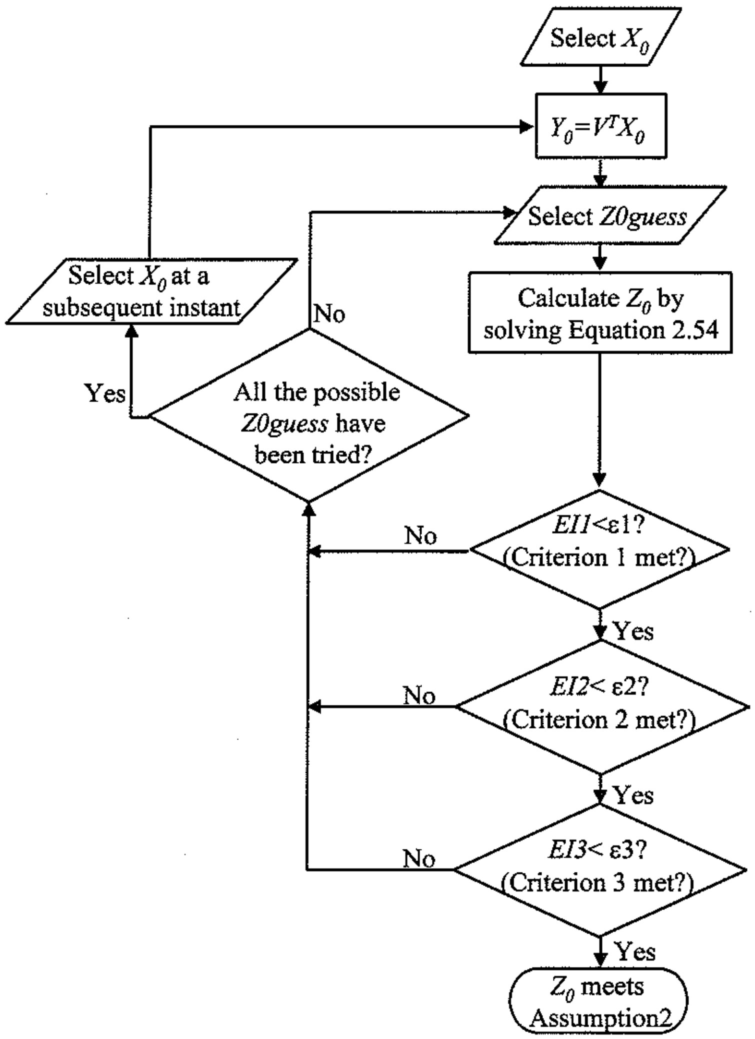


Figure 3.1 Flowchart to validate *Assumption B*

The value of Z_0 can be computed as described in Subsection 2.4.4, which has multiple solution for a given value of Y_0. Once the X_0 is validated for *Assumption A*, the converged value of Z_0 can be validated for *Assumption B* using above three criteria as shown in following flow chart.

3.3 Validating initial conditions

As discussed in above sections, there are two assumptions, one in state space and another in Z space, and it is necessary for initial condition in the respective space to comply with the assumptions.

To completely validate initial conditions for NF analysis, first state space initial condition, X_0, should be validated for *Assumption A* as described in Subsection 3.1.2. If X_0 complies with this assumption at a given instant, then X_0 derived at a subsequent instant will also comply with the *Assumption A*, and without validating new X_0 it can be assumed that X_0 derived after that instant is an appropriate initial condition in state space. This is because of the fact that once the system enters into a certain neighborhood of SEP, then the system will remain in that neighborhood forever if the system is free and asymptotically stable (Section 2.5).

Then Z space initial condition, Z_0, should be validated for *Assumption B* as described in Subsection 3.2.3. As nonlinear transformation is used to map the state space into Z space, the system being in the neighborhood of the SEP of state space where $O(|X|^3)$ is negligible (or *Assumption A* complied with) does not necessarily mean that the system is also in the neighborhood of Z space SEP where $O(|Z|^3)$ is negligible (or *Assumption B* also complied with). Hence, this is a necessary step in the process of complete validation of the initial conditions. If Z_0 complies with this assumption at a given instant, then Z_0 derived using X_0 of a subsequent instant will *not* necessarily comply with the *Assumption B* and it is necessary to follow all the steps described in Subsection 3.2.3. This is because of the fact that there exists multiple solution for Z_0 for a given Y_0. And depending on *Z0guess*, new converged value of Z_0 may not fall on the trajectories of Z variables, given by the solution of the set of differential equations using the previous appropriate Z_0.

CHAPTER 4 Test Systems

The application of NF analysis will be investigated using two test systems. The criteria developed so far will be tested on the set of four nonlinear differential equations having four state variables, which will be easier to analyze and understand. Then, developed criteria will be further tested using 2-Area, 4-Generator system. These systems are described in the following sections.

4.1 Test System 1: Nonlinear differential equations

Following is the set of four nonlinear differential equations, which has 2^{nd} order nonlinearity [10].

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ \mu & -1 & 0 & 1 \\ -1 & 0 & -1 & 1 \\ 0 & -1 & \mu & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2}\varepsilon x_1^2 \\ 0 \\ 0 \end{bmatrix} \tag{4.1}$$

where μ and ε are small real constants. ε decides nonlinearity in the system and the non-linearity increases with increasing value of $|\varepsilon|$. For $\varepsilon = 0$ the system is linear. The A matrix of the system has two pairs of complex conjugate pair of eigenvalues

$$\lambda_1, \lambda_3 = -1 + \sqrt{\mu} \pm i \qquad \lambda_2, \lambda_4 = -1 - \sqrt{\mu} \pm i \tag{4.2}$$

Hence, μ decides proximity to first order resonance, defined as $\lambda_i = \lambda_j, i \neq j$. When $|\mu| \to 0$, the two pairs of eigenvalues moves closer and when $\mu = 0$, the two pairs of eigenvalues coincide with strong resonance at $-1 \pm i$. In this study, we will select the case with $\mu \neq 0$. For the system, right and left eigenvector matrices, Hessian matrix and non-linear transformation matrix $h2(Z)$ are

$$U = \frac{1}{\sqrt{2(1+\mu)}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ \sqrt{\mu} & -\sqrt{\mu} & \sqrt{\mu} & -\sqrt{\mu} \\ i & i & -i & -i \\ i\sqrt{\mu} & -i\sqrt{\mu} & -i\sqrt{\mu} & i\sqrt{\mu} \end{bmatrix} \tag{4.3}$$

$$V^T = \frac{\sqrt{1+\mu}}{2\sqrt{2}}\begin{bmatrix} 1 & \frac{1}{\sqrt{\mu}} & -i & \frac{-i}{\sqrt{\mu}} \\ 1 & \frac{-1}{\sqrt{\mu}} & -i & \frac{i}{\sqrt{\mu}} \\ 1 & \frac{1}{\sqrt{\mu}} & i & \frac{i}{\sqrt{\mu}} \\ 1 & \frac{-1}{\sqrt{\mu}} & i & \frac{-i}{\sqrt{\mu}} \end{bmatrix} \tag{4.4}$$

$$\begin{aligned} H^i_{jk} &= \varepsilon \qquad ; i = 2 \qquad j = k = 1 \\ &= 0 \qquad ; \textit{Otherwise} \end{aligned} \tag{4.5}$$

$$C^i_{jk} = \frac{\varepsilon(-1)^{i+1}}{8\sqrt{2\mu(\mu+1)}} \tag{4.6}$$

$$h2_{jk}^{i} = \frac{\varepsilon(-1)^{i+1}}{8(\lambda_j + \lambda_k - \lambda_i)\sqrt{2\mu(\mu+1)}} \qquad (4.7)$$

4.2 Test system 2: Power system

2-Area, 4-Generator power system, shown in following figure, is used for analysis purposes.

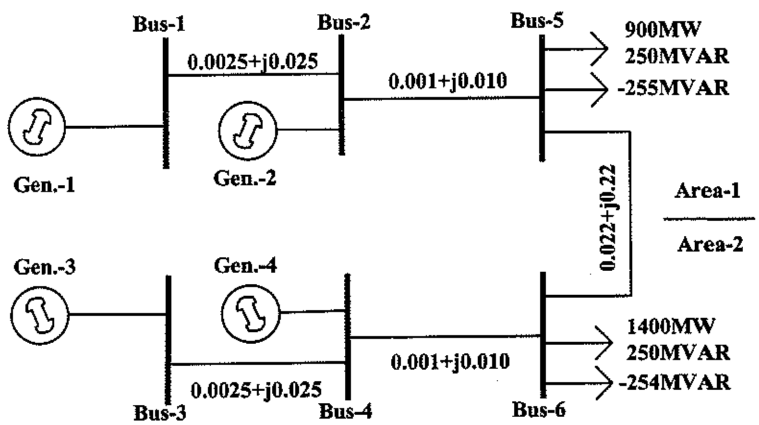


Figure 4.1 Single line diagram of 2-Area, 4-Generator power system

Area-1 is sending area and Area-2 is receiving area. Bus-1 is slack bus for the Area-1, and Bus-3 is slack bus for the system and for the Area-2 as well. The two areas are connected through a weak transmission line having high impedance, because of which the system has low damped inter-area mode. Here generators are represented by a two-axis model with exciter and load is modeled as constant impedance. More details about the power system can be found in Appendix A. NF analysis will be studied for the power system at various operating points and with different contingency. It is further discussed in Chapter 5.

4.3 Performing NF analysis

4.3.1 Software and executable files

To perform NF analysis following software packages are being used:

1. Power World Simulator (PWS):

   To generate converged load flow data in raw data and IEEE format.

2. Dynamic Security Assessment (DSA) software:

   Transient Security Assessment Tool (TSAT) and Small Signal Analysis Tool (SSAT) are part of DSA software. TSAT is used to simulate fault on the system, to generate time response of various variables, and to perform prony analysis of time response of a variable. SSAT is used to calculate the eigenvalues of the system, which can be compared with that obtained during NF analysis.

3. MATLAB:

   To plot time response of state variables, calculate eigenvalue of M matrix.

4. NF analysis software:

   This is set of executable programs written in Fortran, originally developed at Iowa State University, and we greatly acknowledge the support provided by Dr. V.V. Vittal by supplying Fortran code of this software.

   Parts of this software are executable files qybus.exe and qybus.exe to generate Y-bus data of the system, and main.exe to perform NF analysis on the system and generate all the necessary data for analysis purpose.

5. Auxiliary software:

   Executable program makegstate.exe, developed in Fortran, is used to generate initial condition data.

4.3.2 Performing NF analysis for power system

Following describes the set of sequences and input data file need for complete NF analysis of the power system described above.

1. Solve load flow case in PWS.

   User Input: Generator bus voltages, load and inter-area power flow

   Output files: Saved cases of the load flow in raw data and IEEE format.

2. Run TSAT case to simulate a disturbance. Disturbance will be applied such that after removal of the disturbance the system will settle to the same operating point that before disturbance.

   Input files: Case file, load flow data file generated during Step 1, dynamic data file, contingency file, and output specification file.

   Output files: Time response of generator variables voltages, active power, reactive power, and state variables speed deviation, rotor angle, and exciter state variables xe1, state 2, and Efd.

3. In MATLAB simulate exciter models using simulink toolbox.

   Input files: Time response of exciters state xe1 generated in Step 2.

   Output files: Time response of exciters state xe2.

4. Run Gstate.exe to generate the system data at a given time.

   Input files: Generators variables created in Step 2 and Step 3.

   User input: Time *t* at which to produce the system data.

   Output files: Data file of state of the system at *t*.

5. Run qybus.exe.

   Input file: Load flow data from Step 1, and data file indicating internal reactance of the machine and list of buses that are to be retained.

   Output file: Temporary Y-bus

6. Run mqy.exe.

   Input files: Temporary Y-bus from Step 5, and inter reactance of the machine but with -ve sign for each bus.

   Output file: Y-bus of the reduced system.

7. Run main.exe

   Input files: Load flow data from Step 1, dynamic data, Y-bus data from Step 6, state of the system at SEP and at $t=t_0$ from Step 4.

   User input: Initial guess for Z_0, state variable number, time step and time length to generate time response of state variable using numerical integration and other closed form solution expressions.

   Output files: X_0, Y_0, Z_0, interaction indices *I1* and *I2*, time response of state variable, data of M matrix elements and other auxiliary data.

8. Execute MATLAB program files.

   Input files: Time response of state variable from Step 2 and Step 7, data file containing *M* matrix data.

   Output files: Graph of state variable time response given by various methods and expressions, eigenvalue of *M* matrix.

To perform a particular analysis it may not be necessary to use complete procedure mentioned above, however it describes general flow of various data files.

CHAPTER 5 Test results

So far two assumptions underlying the NF analysis have been identified. One test strategy has been proposed to validate X_0 for *Assumption A*, and three test criteria have been proposed to validate the converged value of Z_0 for *Assumption B*. Test systems described in Chapter 4 will be used to verify that the violation of these assumptions can produce inaccurate information about the system and test strategy and criteria proposed can easily and quickly identify this failure.

5.1 Case with nonlinear equations

The test system described in Section 4.1 is quadratic by itself and does not have any higher order nonlinearity, hence only *Assumption B* applies to this system. Let $\mu = 0.65$, $\varepsilon = 2.5$ and $X_0 = [0.9, 0.9, 0.9, 0.9]^T$ for this system.

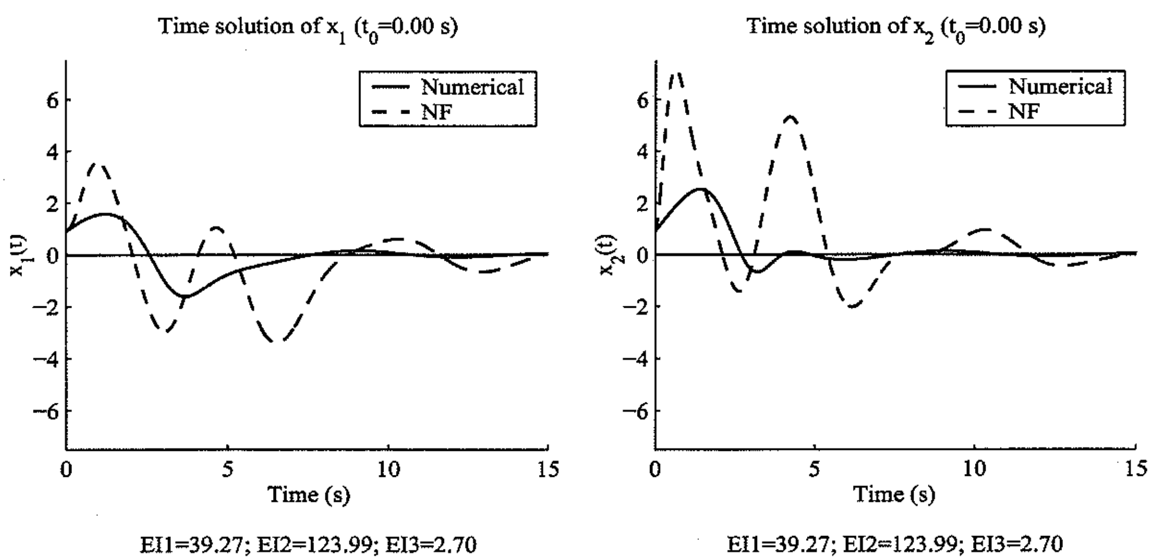
5.1.1 NF analysis may fail if *Assumption B* is violated

Using initial guess $Z0guess = Y_0 - h2(Y_0)$, converged value of Z_0, and values of indices are

$$Z_0 = [-4.82 + 3.77i, 6.23 - 31.57i, -4.82 - 3.77i, 6.23 + 31.57i]^T$$

$$EI1 = 39.27 \qquad EI2 = 123.99 \qquad EI3 = 2.70$$

Figure 5.1 shows time domain solution of all the state variables obtained using numerical integration method fourth-order Runge-Kutta (RK4), shown as Numerical, and that given by NF closed form expression given by Equation 2.46, shown as NF. It can be seen that time domain solution given by two method are not in agreement at all for all the states. Hence, it can be said that NF analysis failed and cannot produce accurate results with this initial condition of Z_0. That is because of the fact that in Z space the system is *not* in the neighborhood of SEP where $O(|Z|^3)$ is negligible or, in other words, converged value of Z_0 failed to meet *Assumption B*.



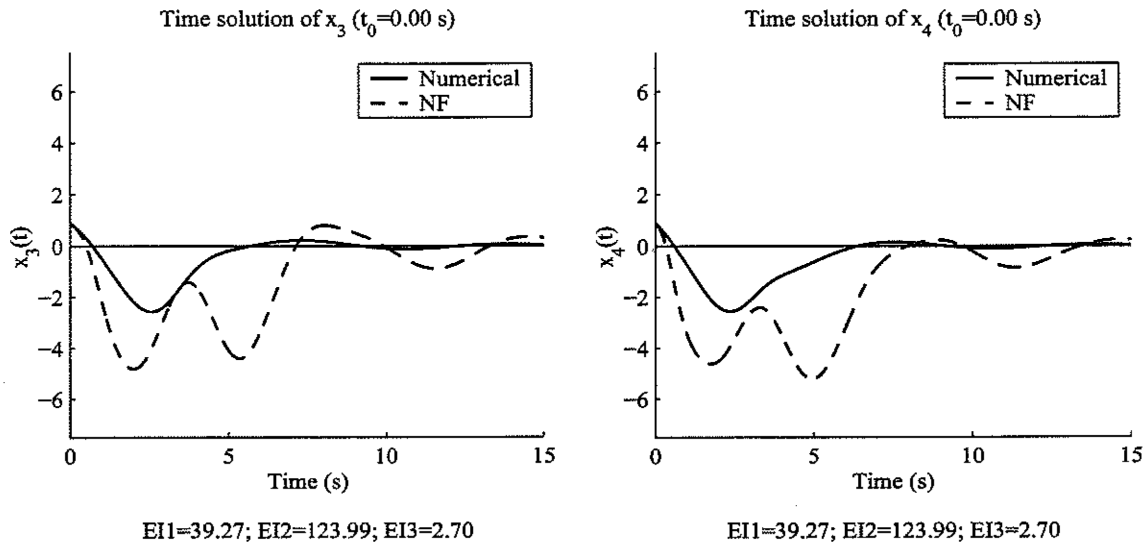
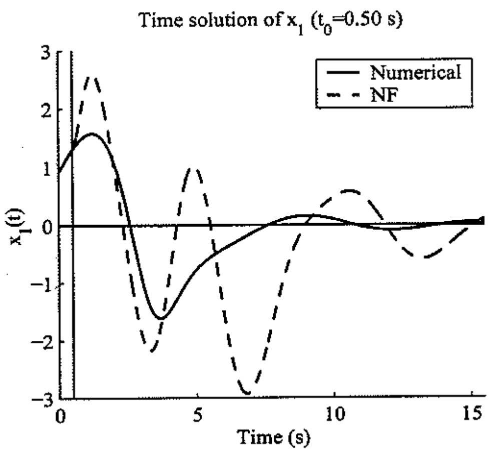


Figure 5.1 Case 1: Numerical and NF response with X_0 at t_0=*0.0 s.*

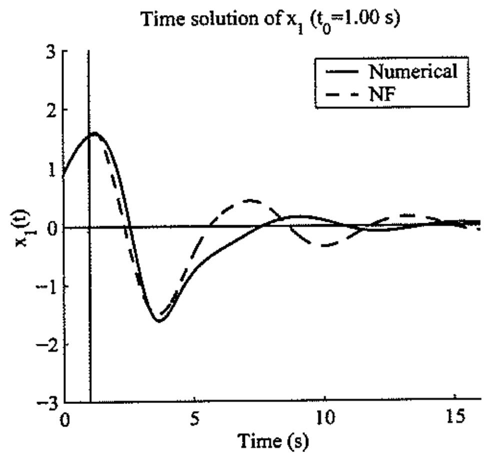
5.1.2 Selecting initial condition at a subsequent instant

Figure 5.2 shows time domain solution of state x_1 with initial condition selected from the trajectories of the state variables at a different time instant. Using initial condition at time 0.5s till 4.5s (Figure 5.2, *a* to *i*), NF failed to produce correct time domain solution of the states (only shown are time response of state x_1, however this is the case with the other states as well). While using the initial condition from the state of the system at $t=5.0$ the NF analysis can produce correct time domain response (Figure 5.2, *j*). So, if the initial condition at any given instant is not proper, then initial condition at a subsequent instant can be used for NF analysis if it is found to be appropriate. This is because of the fact that with time the system moves towards an SEP (if the system is free and asymptotically stable) and if at a given instant initial condition is not in a certain neighborhood of SEP then the system will also moves towards that neighborhood and eventually enters into it.



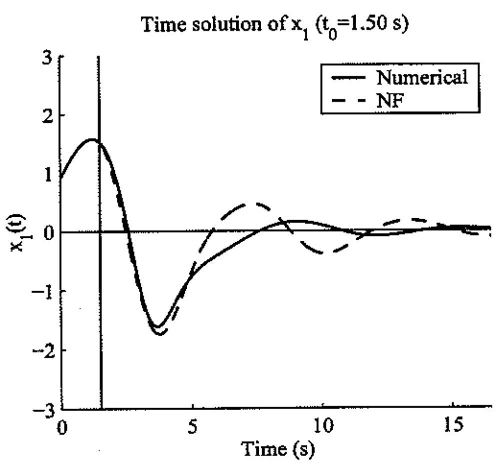
EI1=108.97; EI2=868.51; EI3=3.21

a.



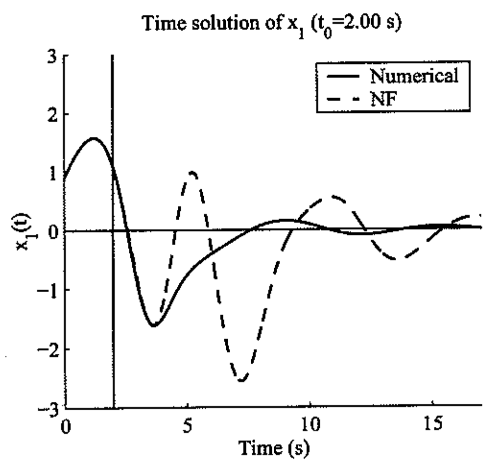
EI1=43.27; EI2=491.00; EI3=1.59

b.



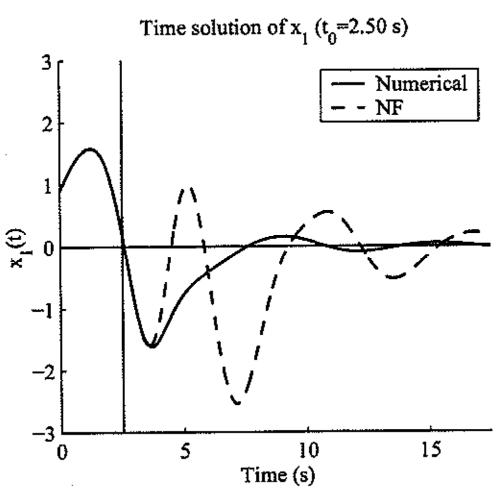
EI1=116.80; EI2=800.04; EI3=1.37

c.



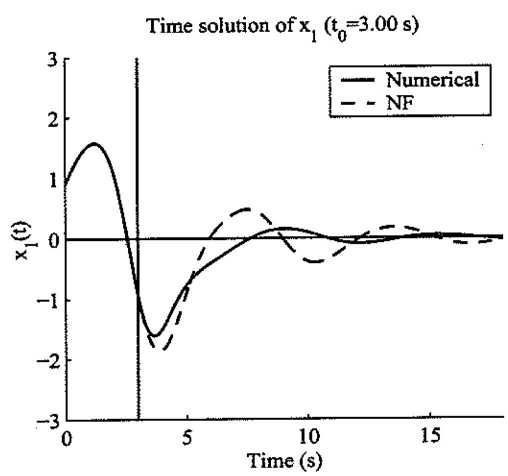
EI1=4.34; EI2=8.65; EI3=3.46

d.



EI1=5.88; EI2=10.13; EI3=3.52

e.



EI1=140.29; EI2=69.16; EI3=1.54

f.

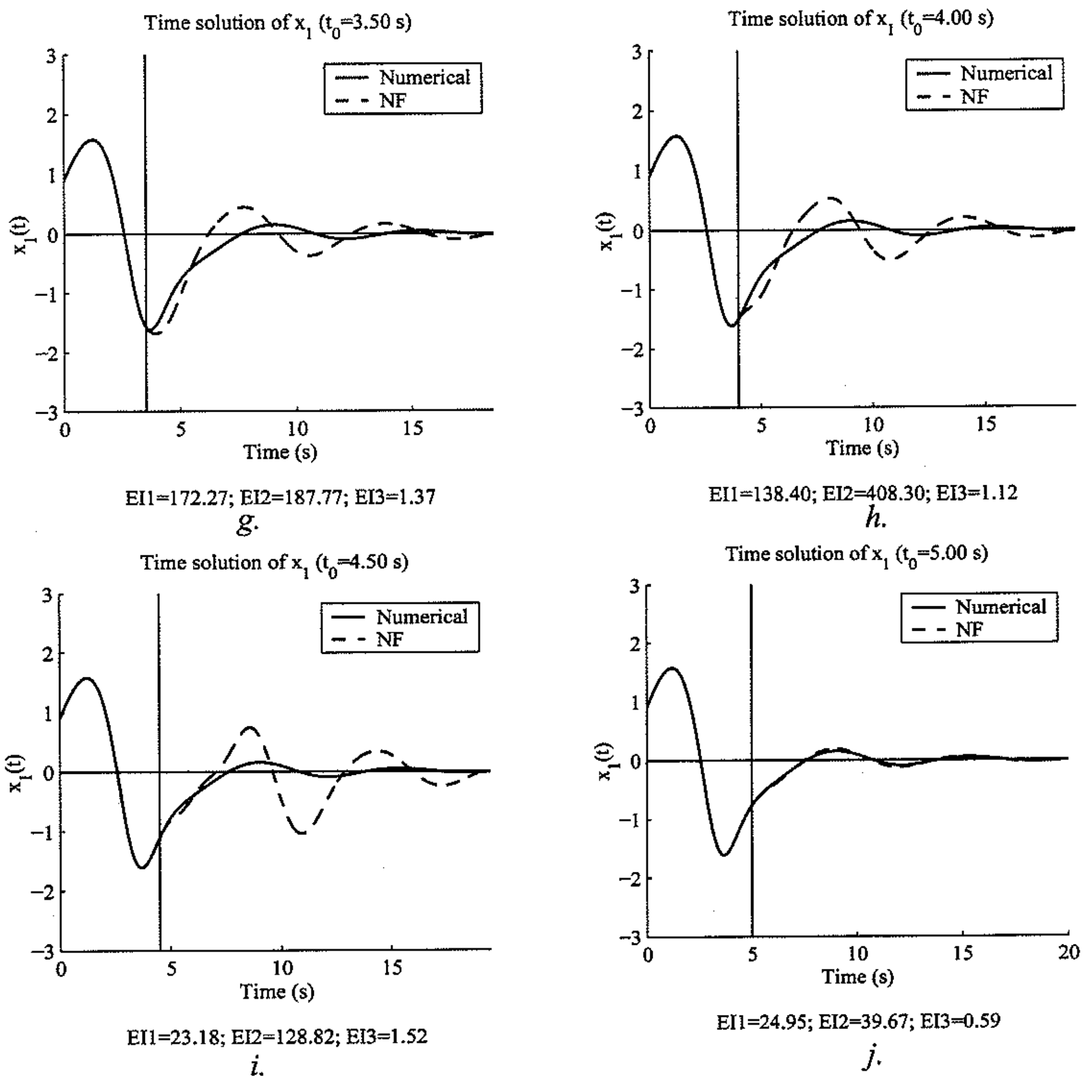


Figure 5.2 Case 1: Numerical and NF response with X_0 at t_0=*0.0, 0.5,...,5.0 s*

5.1.3 Using suggested criteria to identify failure of *Assumption B*

Three criteria are suggested in Subsection 3.2.2.3 to identify failure of initial condition Z_0 to meet *Assumption B*. Based on many simulations performed we select $\varepsilon 1 = 500.0$, $\varepsilon 2 = 500.0$ and $\varepsilon 3 = 0.85$.

5.1.3.1 Case 1

For above case, Table 5.1 shows the values of *EI1, EI2* and *EI3*, whether these criteria are met or not, and whether time domain solution is a match or not. It can be seen that when all the three of the criteria are met, the time domain solution is a close match. Hence, suggested criteria can successfully predict whether *Assumption B* has been complied with or not.

Table 5.1 Case 1: Z_0 Validation result

t_0	*EI1*	*EI2*	*EI3*	$EI1 \le \varepsilon 1$	$EI2 \le \varepsilon 2$	$EI3 \le \varepsilon 3$	**Time response match?**
0.5	108.96	868.5	3.20	Yes	No	No	No
1.0	43.27	491.00	1.59	Yes	Yes	No	No
1.5	116.79	800.04	1.37	Yes	No	No	No
2.0	4.34	8.65	3.45	Yes	Yes	No	No
2.5	5.87	10.13	3.52	Yes	Yes	No	No
3.0	140.28	69.16	1.53	Yes	Yes	No	No
3.5	172.27	187.77	1.37	Yes	Yes	No	No
4.0	138.40	408.30	1.12	Yes	Yes	No	No
4.5	23.166	128.82	1.51	Yes	Yes	No	No
5.0	24.95	39.67	0.59	Yes	Yes	Yes	Yes

5.1.3.2 Case 2

In previous cases while using initial condition at t_0=*5.0s*, converged value of Z_0 found to be appropriate using initial guess

$$\begin{aligned} Z0guess &= Y_0 - h2(Y_0) \\ &= [-0.180 - 0.269i, -0.364 - 0.2903i, -0.180 + 0.269i, -0.364 + 0.2903i]^T \end{aligned}$$

Using two other different initial guess values

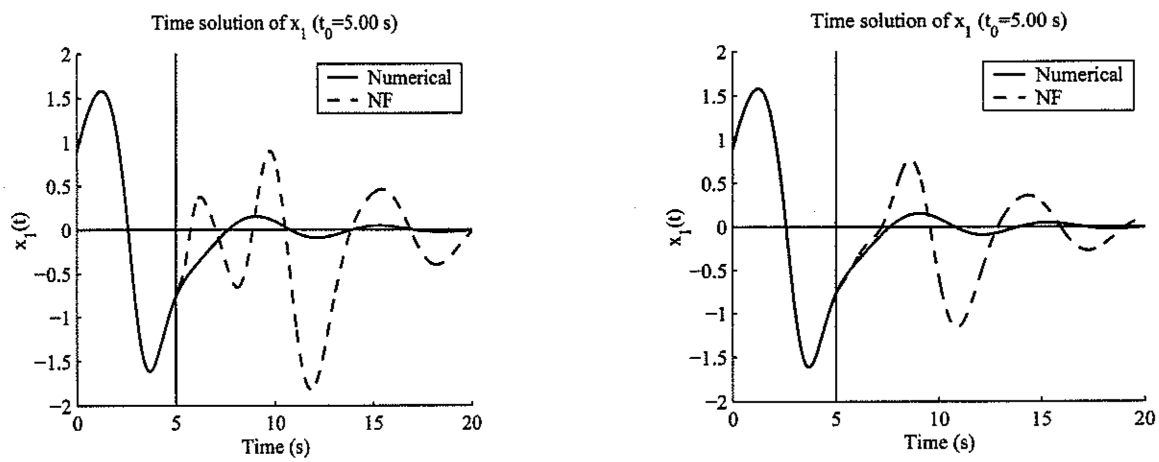
$$Z0guess1 = [0.612 + 0.776i, 0.651 + 0.043i, 0.612 - 0.776i, 0.651 - 0.043i]^T$$

and

$$Z0guess2 = [0.305 + 0.369i, 1.068 + 0.883i, 0.3053 - 0.369i, 1.068 - 0.883i]^T$$

solution converged to different values of Z_0. Figure 5.3 shows time response of *X1* with these guess values. Again using proposed indices, it is possible to predict failure of initial condition Z_0 to comply with *Assumption B.*

This shows using suggested criteria proper Z_0 can be selected from multiple solutions. This is a useful property of the criteria, as while applying NF analysis to the problem of power systems, initial guess may not converge to a proper Z_0, though there exist a proper Z_0, and these criteria can identify and eliminate those values.



EI1=876.65; EI2=3039.84; EI3=1.82

a. with Z0guess1

EI1=38.51; EI2=225.20; EI3=1.83

b. with Z0guess2

Figure 5.3 Case 1: Numerical and NF response with X_0 at t_0=*5.0 s* and various *Z0guess*

5.1.4 One set of indices is sufficient for different initial condition

Let's select initial condition $X_0 = [0.4, 0.25, 0.25, 0.25]^T$, different from that used in previous subsection. Figure 5.4 shows time response of state x_1 with initial condition at t_0=0.0 s. In this case Z_0 is valid initial condition and that can be seen from time response and indices *EI1 (<ε1)*, *EI2 (<ε2)* and *EI3 (<ε3)*.

From above, it can be said that the above criteria set, $[\varepsilon 1, \varepsilon 2, \varepsilon 3]$, for a given system can be used to validate initial condition Z_0 for various state space initial conditions, X_0.

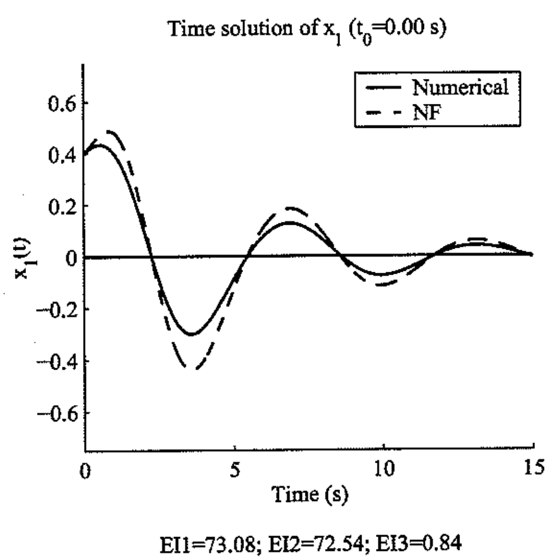


Figure 5.4 Case 2: Numerical and NF response with X_0 at t_0=0.0 s

5.2 Case with Power system

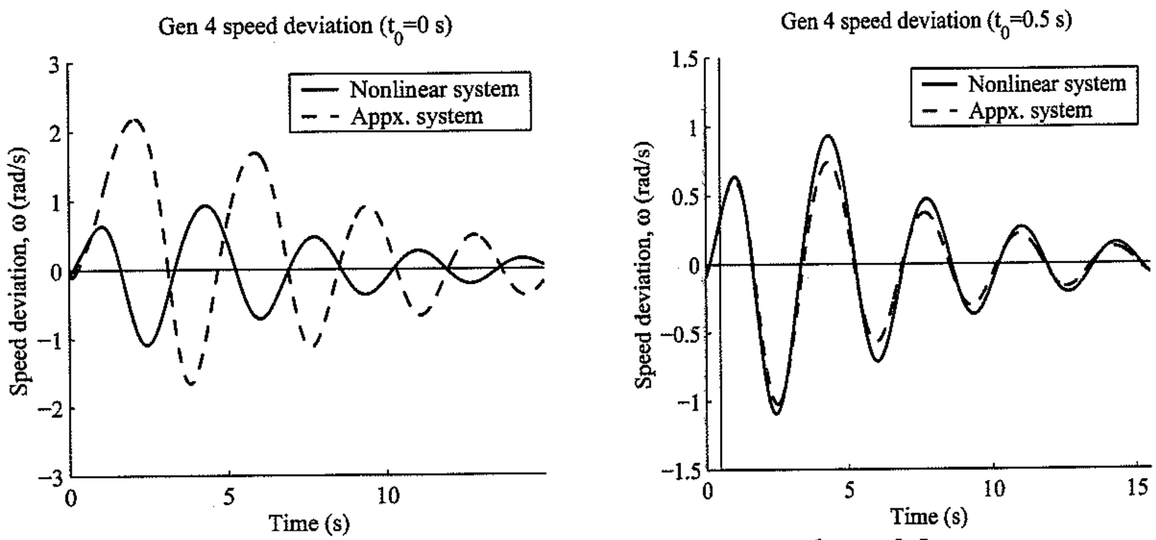
The power system are described in Section 4.2, while considering the cases with the scenario 2 described in Table 5.5. Here both the assumptions are applied and are investigated in following subsections.

5.2.1 Validating *Assumption A*

Assumption A can be validated by comparing original nonlinear differential equations $\dot{X} = F(X)$ and its approximate system given by first two terms of the Taylor series as $\dot{x}_i = A_i X + \frac{1}{2} X^T H^i X$ shown as 'Nonlinear system' and 'Approx. system' respectively.

Here time domain solution is obtained using numerical integration method RK4. Figure 5.5 *a.* and *b.* shows the time domain solution of Gen-4 speed deviation given by above two methods with initial condition at *a.* $t_0=0.0\ s$ (i.e. immediately after clearing a disturbance) and *b.* $t_0=0.5\ s$ respectively. It can be seen that the two solutions are not exact match.

Table 5.2 and Table 5.3 shows low frequency and low damped modes present in above two time responses given by prony analysis for case *a.* and case *b.* respectively.



a. $t_0=0.0\ s$ *b.* $t_0=0.5\ s$

Figure 5.5 Nonlinear and approximate response of ω_4 with X_0 at $t_0=0.5$ and 5.0 s.

Table 5.2 Prony analysis of Gen-4 speed for t_0=*0.0 s* (case *a.)*

Nonlinear system

No.	Mag	Freq Hz	Damping %
1	1.85	0.304	9.11
2	0.20	0.606	9.07
3	0.02	0.894	10.53

Approximate system

No.	Mag	Freq Hz	Damping %
1	4.34	0.306	8.97
2	0.48	0.607	7.58
3	0.30	0.823	11.47

Table 5.3 Prony analysis of Gen-4 speed for t_0=*0.5 s* (case *b.)*

Nonlinear system

No.	Mag	Freq Hz	Damping %
1	1.7	0.304	9.12
2	0.17	0.606	9.10
3	0.015	0.891	10.77

Approximate system

No.	Mag	Freq Hz	Damping %
1	1.41	0.304	9.241
2	0.11	0.608	9.20
3	0.0063	0.910	9.054

With initial condition at t_0=*0.0 s* (Figure 5.5 *a.* and Table 5.2) it can be seen that the magnitude of similar damped frequencies differ significantly. For initial condition at t_0=*0.5 s* (Figure 5.5 *b.* and Table 5.3) the magnitudes of similar damped frequencies are still different (magnitude difference of 17.64 % for damped frequency 1)

For *0.5 s* from the instant of removal of the disturbance, the original nonlinear system cannot be represented by first two terms of the Taylor series. That is because of the fact that even after *0.5 s* the system has not entered into the neighborhood of SEP where $O(|X|^3)$ is negligible. It can be said that initial condition, X_0, selected from the trajectories of the state even within *0.5 s* from the instant of removal of disturbance does *not* comply with *Assumption A*.

Now, let's select initial condition at an instant later at $t_0=0.75\ s$. Figure 5.6 shows the time responses of Gen-4 speed deviation and Table 5.4 shows prony results in this case. The time response given by two systems are in good agreement and prony analysis shows presence of very similar damped frequencies and their magnitude (magnitude difference of 7.10% for damped frequency 1). Hence we can say that now the system is in the neighborhood of SEP where $O(|X|^3)$ is negligible and X_0 complies with *Assumption A*.

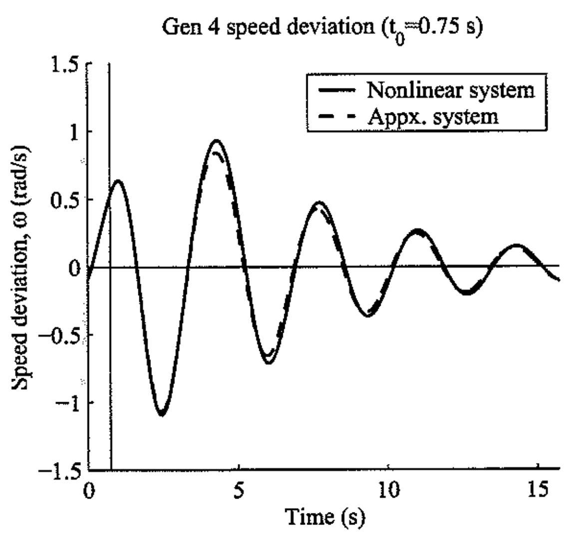


Figure 5.6 Approximate and NF response of ω_4 with X_0 at $t_0=0.75\ s$.

Table 5.4 Prony analysis of Gen-4 speed for $t_0=0.75\ s$

Nonlinear system

No.	Mag	Freq Hz	Damping %
1	1.633	0.304	9.136
2	0.1511	0.606	9.073
3	0.0121	0.892	10.492

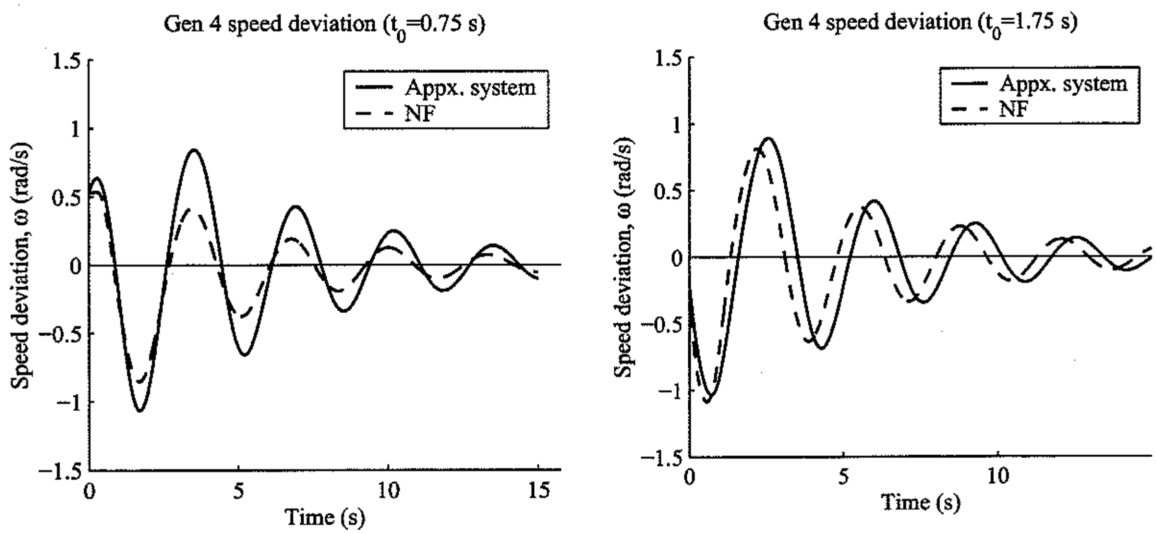
Approximate system

No.	Mag	Freq Hz	Damping %
1	1.517	0.305	9.201
2	0.1262	0.608	9.117
3	0.0083	0.910	9.101

Once it is assured that the approximate system is a good representation of the original nonlinear system then we can carry forward the NF analysis by solving for Z_0 and validating it for *Assumption B*.

5.2.2 Validating *Assumption B*

Figure 5.7 shows time domain response of Gen-4 speed deviation given by RK4 numerical integration of the approximate system, shown as 'Appx. system', and time response given by close form solution of NF analysis, shown as 'NF'. For case *a.* it can be seen that both time responses are not in agreement with each other. Hence, in Z space the system is *not* in the neighborhood of SEP where $O(|Z|^3)$ is negligible. For this system we select $\varepsilon 1 = 1100.0$, $\varepsilon 2 = 1100.0$ and $\varepsilon 3 = 0.85$ based on many simulation performed. Then failure of *Assumption B* can be appropriately predicted using indices *EI1*, *EI2* and *EI3*.



EI1=5509.08; EI2=15088.43;EI3=1.40

EI1=1285.17; EI2=89878.68;EI3=1.00

a. t_0=*0.75 s* *b.* t_0=*1.75s*

Figure 5.7 Approximate and NF response of ω_4 with X_0 at t_0=*0.75 s* and *1.75 s*.

As the system is not in the neighborhood of SEP of Z space, we have to select X_0 at a subsequent instant to perform NF analysis accurately. So, let's select X_0 at an instant of t_0=*1.75* s. Figure 5.7 *b.* shows the time response in this case and it can be seen that both time responses are not in agreement with each other here as well. Failure of Z_0 to meet *Assumption B* can again be predicted using all the three indices. So, let's select initial condition X_0 at a still further instant of $t_0=2.25\ s$ and Figure 5.8 *a.* shows time response in this case. Close time response given by closed form solution of NF analysis and the approximate system, and $EI1 < 1100.0$, $EI2 < 1100.0$ and $EI3 < 0.85$ suggests, in Z space the system is in the neighborhood of SEP where $O(|Z|^3)$ can be neglected. NF analysis performed using these values of X_0 and Z_0 will be most accurate. Figure 5.8 *b.* shows time response given by the nonlinear system, the approximate system and the closed form solution given by NF analysis.

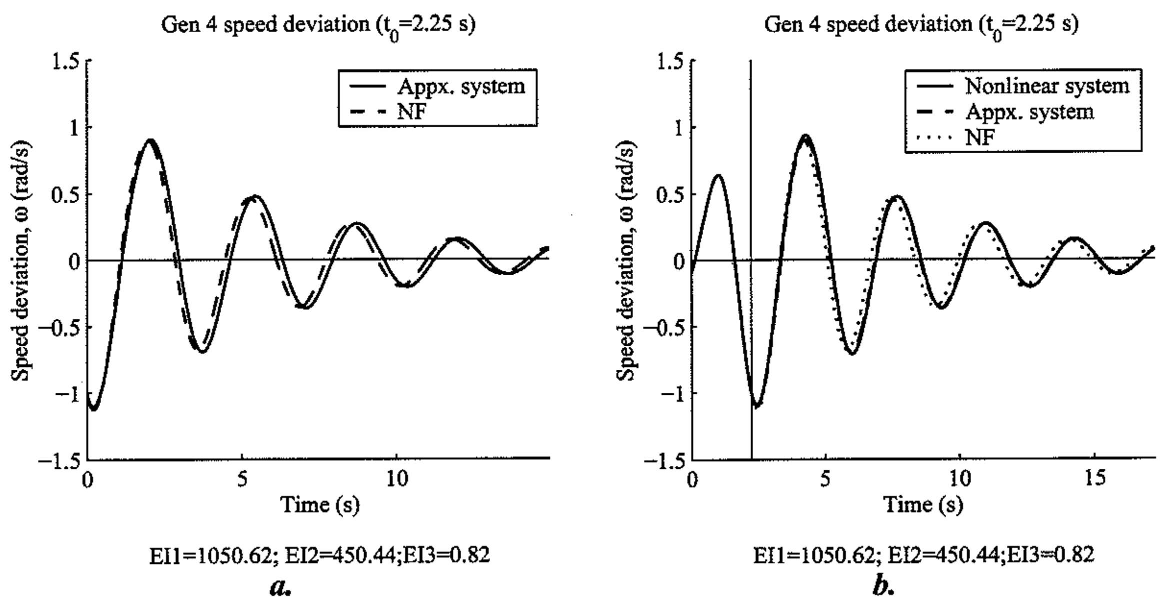


Figure 5.8 Nonlinear, approximate and NF response of ω_4 with X_0 at $t_0=2.25\ s$.

5.3 Comprehensive test using the power system

Table 5.5 shows details of various *Scenarios* using which NF analysis and suggested criteria were investigated. In Table 5.6 to 5.11, the results with initial conditions at various instant, t_0, after clearing a disturbance for each scenario are tabulated. In the 'Case#', the first column of the tables, the first digit refers to scenario and last two digits refers to the case with that scenario, values in second column are the time, t_0, at which initial condition was selected from the instant of removal of disturbance. Column 3 lists whether *Assumption A* is valid or not by comparing the time response of nonlinear system and its approximate system. Column 5-7 lists whether three suggested criteria are met or not. Using these criteria whether *Assumption B* is valid or not is predicted in column 8, while column 9 lists whether that assumption is valid or not by comparing the time responses of approximate system and that given by NF analysis.

Table 5.5 Power system scenario details

Scenario*	Inter-area power (MW)	Bus2 Volt (pu)	Bus4 Volt (pu)	Gen1 generation (MW)	Gen2 generation (MW)	Gen3 generation (MW)	Bus5 Load (MW)	Contingency	$\varepsilon 1$	$\varepsilon 2$	$\varepsilon 3$
1	400.0	1.01	1.01	665.0	664.0	566.8	900.0	Bus-2, 3-ph fault, 10ms.	1100.0	1100.0	0.85
2	400.0	1.01	1.01	665.0	664.0	566.8	900.0	Bus-2, 3-ph fault, 20ms.	1100.0	1100.0	0.85
3	400.0	1.01	1.01	665.0	664.0	566.8	900.0	Bus-6, 3-ph fault, 20ms.	1100.0	1100.0	0.85
4	400.0	1.01	1.01	665.0	664.0	566.8	900.0	Bus-5, 3-ph fault, 20ms.	1100.0	1100.0	0.85
5	415.0	1.01	1.01	681.3	664.0	557.8	900.0	Bus-4, 3-ph fault, 10ms.	1100.0	1100.0	0.82
6	415.0	1.02	1.02	680.7	664.0	554.6	900.0	Bus-5, 3-ph fault, 40ms.	1500.0	1500.0	0.85

*For all the scenarios Bus1 Volt=1.02 pu, Bus3 Volt=1.02 pu, Gen4 generation=500.0 MW, Bus6 Load=1400.0 MW. (refer Figure 4.1 on page 38 for single line diagram)

Table 5.6 Cases with scenario 1

Case#	t0	Assumption A	Assumption B				
		Nonlinear and Appx time response match?	*EI1≤ε1* C1	*EI2≤ε2* C2	*EI3≤ε3* C3	C1.&. C2.&. C3	NF and Appx. time response match?
1	2	3	4	5	6	7	8
101	0.00	No	Yes	No	No	No	No
102	0.25	No	Yes	Yes	Yes	Yes	Yes
103	0.50	No	Yes	Yes	Yes	Yes	Yes
104	0.75	Yes	Yes	Yes	Yes	Yes	Yes
105	1.00	Yes	Yes	Yes	Yes	Yes	Yes
106	1.25	Yes	Yes	Yes	Yes	Yes	Yes

Table 5.7 Cases with scenario 2

Case#	t0	Assumption A	Assumption B				
		Nonlinear and Appx time response match?	*EI1≤ε1* C1	*EI2≤ε2* C2	*EI3≤ε3* C3	C1.&. C2.&. C3	NF and Appx. time response match?
1	2	3	4	5	6	7	8
201	0.00	No	Yes	No	No	No	No
202	0.25	Yes	Yes	Yes	No	No	No
203	0.50	Yes	Yes	No	No	No	No
204	0.75	Yes	No	No	No	No	No
205	1.00	Yes	No	No	No	No	No
206	1.25	Yes	No	No	No	No	No
207	1.50	Yes	No	No	Yes	No	No
208	1.75	Yes	No	No	No	No	No
209	2.00	Yes	No	No	No	No	No
210	2.25	Yes	Yes	Yes	Yes	Yes	Yes
211	2.50	Yes	No	Yes	Yes	No	Yes
212	2.75	Yes	Yes	Yes	Yes	Yes	Yes
213	3.00	Yes	No	Yes	Yes	No	Yes
214	3.25	Yes	Yes	Yes	Yes	Yes	Yes
215	3.50	Yes	Yes	Yes	Yes	Yes	Yes
216	3.75	Yes	No	No	Yes	No	Yes

Table 5.8 Cases with scenario 3

Case#	t0	*Assumption A* Nonlinear and Appx time response match?	*Assumption B* $EI1 \le \varepsilon 1$ C1	$EI2 \le \varepsilon 2$ C2	$EI3 \le \varepsilon 3$ C3	C1.&. C2.&. C3	NF and Appx. time response match?
1	2	3	4	5	6	7	8
301	0.00	No	No	Yes	No	No	No
302	0.25	No	Yes	Yes	Yes	Yes	Yes
303	0.50	Yes	Yes	Yes	Yes	Yes	Yes
304	0.75	Yes	Yes	Yes	Yes	Yes	Yes

Table 5.9 Cases with scenario 4

Case#	t0	*Assumption A* Nonlinear and Appx time response match?	*Assumption B* $EI1 \le \varepsilon 1$ C1	$EI2 \le \varepsilon 2$ C2	$EI3 \le \varepsilon 3$ C3	C1.&. C2.&. C3	NF and Appx. time response match?
1	2	3	4	5	6	7	8
401	0.25	No	Yes	Yes	No	No	No
402	0.50	No	Yes	No	Yes	No	No
403	0.75	Yes	Yes	Yes	No	No	No
404	1.00	Yes	Yes	Yes	No	No	No
405	1.25	Yes	No	No	Yes	No	No
406	1.50	Yes	Yes	No	Yes	No	No
407	1.75	Yes	Yes	No	Yes	No	Yes
408	2.00	Yes	Yes	Yes	Yes	Yes	Yes
409	2.25	Yes	Yes	Yes	Yes	Yes	Yes
410	2.50	Yes	No	Yes	Yes	No	Yes
411	2.75	Yes	Yes	Yes	Yes	Yes	Yes
412	3.00	Yes	Yes	Yes	Yes	Yes	Yes
413	3.25	Yes	No	Yes	Yes	No	Yes
414	3.50	Yes	Yes	Yes	Yes	Yes	Yes
415	3.75	Yes	Yes	No	Yes	No	Yes

Table 5.10 Cases with scenario 5

Case#	t0	*Assumption A*	*Assumption B*				
		Nonlinear and Appx time response match?	*EI1≤ε1* C1	*EI2≤ε2* C2	*EI3≤ε3* C3	C1.&. C2.&. C3	NF and Appx. time response match?
1	2	3	4	5	6	7	8
501	0.00	No	No	No	No	No	No
502	0.25	No	Yes	Yes	Yes	Yes	Yes
503	0.50	No	Yes	Yes	Yes	Yes	Yes
504	0.75	Yes	Yes	Yes	Yes	Yes	Yes
505	1.00	Yes	Yes	Yes	Yes	Yes	Yes

Table 5.11 Cases with scenario 6

Case#	t0	Assumption A: Nonlinear and Appx time response match?	Assumption B: $EI1 \leq \varepsilon 1$ C1	$EI2 \leq \varepsilon 2$ C2	$EI3 \leq \varepsilon 3$ C3	C1.&. C2.&. C3	NF and Appx. time response match?
1	2	3	4	5	6	7	8
601	0.00	No	Yes	No	No	No	No
602	1.50	No	No	Yes	Yes	No	No
603	2.00	No	Yes	Yes	No	No	No
604	3.00	No	Yes	Yes	No	No	No
605	3.50	No	Yes	Yes	Yes	Yes	No
606	4.00	No	Yes	No	No	No	No
607	4.50	No	Yes	Yes	No	No	No
608	5.00	No	No	Yes	No	No	No
609	5.50	No	No	Yes	No	No	No
610	7.00	Yes	Yes	Yes	No	No	No
611	7.50	Yes	Yes	No	Yes	No	No
612	8.00	Yes	Yes	No	No	No	No
613	8.50	Yes	No	Yes	No	No	No
614	9.00	Yes	No	Yes	Yes	No	No
615	9.50	Yes	No	Yes	Yes	No	No
616	10.00	Yes	No	Yes	No	No	No
617	10.50	Yes	Yes	Yes	No	No	No
618	11.00	Yes	Yes	No	No	No	No
619	11.50	Yes	Yes	Yes	Yes	Yes	Yes
620	12.00	Yes	Yes	No	Yes	No	Yes
621	12.50	Yes	Yes	No	No	No	Yes
622	13.00	Yes	No	Yes	No	No	Yes
623	13.50	Yes	Yes	Yes	Yes	Yes	Yes
624	14.00	Yes	Yes	Yes	No	No	Yes
625	14.50	Yes	Yes	Yes	No	No	Yes
626	15.00	Yes	No	Yes	No	No	Yes
627	15.50	Yes	Yes	Yes	Yes	Yes	Yes
628	16.00	Yes	Yes	Yes	Yes	Yes	Yes
629	16.50	Yes	Yes	Yes	No	No	Yes

5.4 Discussion

5.4.1 Validating *Assumption A*

From Subsection 5.2.1 it can be seen that if the system is not in the neighborhood of SEP where $O(|X|^3)$ is negligible, or in other words initial condition X_0 at a given instant (e.g. at $t_0=0.0\ s$ and $t_0=0.5\ s$ in this particular case) violates *Assumption A*, then the original non-linear system cannot be approximated using the first two terms of the Taylor series, which is an approximate representation of the system for NF analysis. Hence, NF analysis cannot provide accurate information about the system using an inaccurate approximate system. With time the system will move towards an SEP and hence we can select the initial condition at a subsequent instant when the system is in the neighborhood of SEP where $O(|X|^3)$ is negligible, or in other words initial condition X_0 at a given instant complies with *Assumption A* (e.g. at $t_0=0.75\ s$ in this particular case). In that case the approximate system is an accurate representation of the system, using which NF analysis can provide accurate information about structural property of the system. Prony analysis has been successfully used to validate this assumption.

While using NF analysis for the power system, it has been observed that initial condition selected immediately after clearing a disturbance usually does not meet this assumption (case 101,...,601 in Section 5.3).

5.4.2 Validating *Assumption B*

For NF analysis to produce accurate information about the system an another approximation has to be validated, that in Z space the system is in the neighborhood of SEP where $O(|Z|^3)$ is negligible or in other words Z_0 should be sufficiently small so that $O(|Z|^3)$ can be neglected. Using cases with the nonlinear system (Subsection 5.1.1, 5.1.2 (case *a.* to *i.)* and 5.1.4 (case *a.*)) and the power system (Subsection 5.2.2) it has been shown that if this assumption is violated, or in other words if Z_0 does not comply with *Assumption B*, then time response of states given by NF analysis closed form solution cannot produce close match to that given by the approximate system. Again using the same argument that was used to validate *Assumption A*, the initial condition X_0 at a subsequent instant shall be used such that in Z space the system is in the neighborhood of SEP where $O(|Z|^3)$ becomes negligible, or in other words Z_0 complies with *Assumption B* (Subsection 5.1.2 (case *j.*) and 5.1.4 using the nonlinear equations and Subsection 5.2.2 using the power system)

While using initial condition immediately after clearing a disturbance, if we neglect *Assumption A* and assume that 2nd order approximation is a valid assumption, then converged value of Z_0 usually fails to comply with *Assumption B* (case 101,...,601 in Section 5.3).

5.4.3 Indices and their accuracy

In Subsection 3.2.2 three indices, *EI1*, *EI2* and *EI3,* have been proposed to validate *Assumption B*. It has been shown that indices can be used to validate *Assumption B* (Subsection 5.1.3 and 5.2.2), and were further tested using all together 75 cases with different load flow and contingencies (Section 5.3). Following can be verified from the test results

1. In the cases with power system, when *Assumption B* is invalid (36 cases), indices correctly predicted failure except once (case 605).
2. In the cases with power system, for 39 cases the *Assumption B* is valid, i.e. the time response given by NF analysis is close match to that obtained by numerical integration of approximate system. Out of 39 cases, for 25 cases the proposed indices could correctly predicted that *Assumption B* is valid. While for 14 cases it incorrectly predicted that *Assumption B* is invalid.
3. One set of criteria is sufficient for a given operating point e.g. in Scenario 1-4 with power system (Table 5.5) and in Subsection 5.1.4 with the set of nonlinear quadratic equations only one set of criteria are used.

From the above it can be said that indices can predict failure of *Assumption B* accurately. However, sometime it may predict failure of *Assumption B* while it is not. Following explains the limitations of the indices in validating *Assumption B*.

Index *EI1* is based on relative numerical value of $O(|Z|^3)$ in $\dot{z}_j = \lambda_j z_j + O(|Z|^3)$. It is possible that minor error may appear large for smaller value of $\lambda_j z_j$. The appreciable error in highly damped modes may not be reflected in time response of states as those may disappear and / or may reduce quickly with time.

Similarly for index *EI2*, minor error in small magnitude of $\dot{x}_i$ may appear very large. Also large error in $\dot{x}_i$ may be due to highly damped modes which may disappear and / or reduce quickly with time and may not appear as appreciable error in time response of state.

Index *EI3* decides whether $(I - Dh2(Z_0) + O(|Z|^3))$ is an appropriate approximation of $(I + Dh2(Z_0))^{-1}$ or not based on eigenvalue of $Dh2(Z_0)$. Even if largest eigenvalue is greater than unity may not appear as error in time response of state variables, as it may have introduced an error into particular time derivative of z_{i0}, which may be of smaller in magnitude and/or corresponds to highly damped mode.

Because of the these characteristic, indices may predict that *Assumption B* is invalid while it may not be the case.

CHAPTER 6 Conclusions and future work

6.1 Conclusions

Recently NF analysis has been reported as a tool for nonlinear analysis of the power system. As analysis is nonlinear, an initial condition of the system has to be used for analysis purpose, and the reported studies to date it has been derived using the state of the system immediately after clearing a disturbance.

In this thesis two basic assumptions underlying NF analysis have been identified: *a.* in state space $O(|X|^3)$ is negligible and *b.* in Z space $O(|Z|^3)$ is negligible. It has been shown that initial conditions, X_0 and Z_0, to be used for NF analysis do not meet these assumptions then NF analysis becomes inaccurate. Usually this is the case when initial condition X_0 is derived using state of the system immediately after clearing the disturbance. If we let go the asymptotically stable system then it will move closer to an SEP with time and eventually enters the neighborhood of SEP where these assumptions are valid. Hence, we can derive the initial condition using the state of the system from the trajectories of the states at a subsequent instant so that these assumptions are met, using which NF analysis can produce accurate results.

To validate the first assumption, that the system is in the neighborhood of SEP where $O(|X|^3)$ is negligible, the time response of nonlinear system and its quadratic approximation can be compared using any frequency domain analysis. Prony analysis has been successfully used here.

To validate the second assumption, that the system is in the neighborhood of SEP where $O(|Z|^3)$ is negligible, three indices have been proposed. *EI1* gives relative measure of the $O(|Z|^3)$ that is being neglected in *Z* space. *EI2* gives relative measure of the effect in state space when $O(|Z|^3)$ is neglected in *Z* space. *EI3* gives measure how accurately the equation $(I + Dh2(Z_0))^{-1}$ can be represented using binomial series, by calculating largest eigen value of $Dh2(Z_0)$. It has been shown that these indices can be used to predict failure of assumption that $O(|Z|^3)$ is negligible in *Z* space.

6.2 Future work

It has been shown in Subsection 5.4.3 that the proposed indices may occasionally fail to predict that *Assumption B* is valid. As a future work, accuracy of indices *EI1* and *EI2* can be improved by considering the magnitude of variables *Z*, and frequency and damping of the modes associated with it. Similarly accuracy of index *EI3* can be improved by considering the magnitude of variable *Z* and frequency and damping associated with it, while estimating the error introduced when largest eigen value of $Dh2(Z_0)$ is greater than pre-specified value.

When NF analysis predicts the time response of the state variable same as that given by approximate system, the effect of neglected $O(|Z|^3)$ on magnitude of different modes that predicted by NF analysis can further be investigated. Because different components of a given set of frequencies is not uniquely defined to produce given time responses. Various characteristic of nonlinear transformation can further be analyzed for this purpose.

6.3 Contributions

6.3.1 Programs developed

In the research work, the programs developed at Iowa State University were used to perform NF analysis of the power system. In addition to that following programs were developed as a further research work

1. To compute time response of state variable by numerical integration of quadratic equations, an approximate representation of the nonlinear system.
2. To compute time response of state variable given by NF analysis.
3. To compute the magnitude of the modes present in a state variable predicted by NF analysis.
4. To simulate the exciter to produce time response of exciter states.
5. To produce numerical values of the variables of the system to be used as initial condition in NF analysis.
6. To compute indices *EI1*, *EI2* and *EI3*.

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Appendix A Power System Data

Following describes completely describes the 2-area and 4-generator system that is used for analysis purpose here.

A.1 Generator Data

In the power system used for analysis, generators are modelled with two-axis model. Various generator constants are:

Table A.1: Generator Data for MVAbase=100

Parameter	Value
Ra	0.000277
Xd	0.2000
Xq	0.1888
Xd'	0.0333
Xq'	0.06111
Tdo'	8.0000
Tqo'	0.4000
H	58.500
D1,...,D4	81,9,99,10.8

A.2 Exciter data

Each generator is modeled with exciter as shown in following figure.

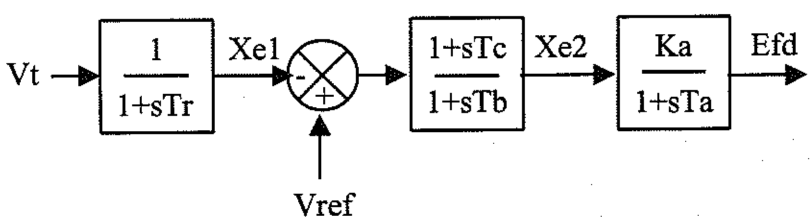

Fig. A. 1 Exciter model

Following table lists various exciter constants, which are same for all the exciters.

Table A.2: Exciter Data

Parameter	Value
Ka	100
Ta	0.01
Tb	10.0
Tc	1.0
Tr	0.01
Vrmax	5.0
Vrmin	-5.0

A.3 State variables for power system

Following lists the states used to describe the power system and their location in state vector using above models of generators and exciters.

$$X^T = [Eq_1', \ldots, Eq_4', Ed_1', \ldots, Ed_4', \omega_1, \ldots, \omega_4, \delta_{21}, \ldots, \delta_{41},$$
$$Xe1_1, \ldots, Xe1_4, Xe2_1, \ldots, Xe2_4, Efd_1, \ldots, Efd_4]$$

where:

Eq_i' = q-axis component of voltage behind the transient impedence, Ei', for i^{th} generator (States 1 to 4).

Ed_i' = d-axis component of Ei' (States 5 to 8).

ω_i = Speed deviation for i^{th} generator (States 9 to 12).

δ_{i1} = Relative rotor angle of i^{th} generator w.r.t. Generator 1, $\delta_i - \delta_1$ (States 13 to 15).

Efd_i = Field voltage, an exciter output, for i^{th} generator (States 16 to 19).

$Xe1_i$ = Output of *Tr* transfer function of exciter for i^{th} generator (States 20 to 23).

$Xe2_i$ = Output of *lead-lag* transfer function of exciter for i^{th} generator (States 24 to 27).

A.4 Load flow data

Power system load flow is simulated using Power World Simulator (PWS). One of the load flow data in raw data format, generated using PWS, is listed below. For transient simulation using TSAT requires load flow in raw data format while NF analysis program accepts load flow data in IEEE format, which again can be generated using PWS.

```
0  100.00
  1,'Area1G1 ', 230.0000,2,   0.000,   0.000,  1,  1,1.02000, 68.8723,  0
  2,'Area1G2 ', 230.0000,2,   0.000,   0.000,  1,  1,1.02000, 59.4416,  0
  3,'Area2G1 ', 230.0000,3,   0.000,   0.000,  2,  2,1.02000,  0.0000,  0
  4,'Area2G2 ', 230.0000,2,   0.000,   0.000,  2,  2,1.02000, -7.6842,  0
  5,'LoadBus1', 230.0000,1,   0.000,   0.000,  1,  1,0.97667, 51.9766,  0
  6,'LoadBus2', 230.0000,1,   0.000,   0.000,  2,  2,0.97496, -13.4941,  0
0 / END OF BUS DATA, BEGIN LOAD DATA
  5,'1 ',1,  1,  1,  900.000,  250.000,   0.000,   0.000,   0.000,  -0.000,  0
  5,'2 ',1,  1,  1,    0.000, -255.100,   0.000,   0.000,   0.000,  -0.000,  0
  6,'1 ',1,  2,  2, 1400.000,  250.000,   0.000,   0.000,   0.000,  -0.000,  0
  6,'2 ',1,  2,  2,    0.000, -254.300,   0.000,   0.000,   0.000,  -0.000,  0
0 / END OF LOAD DATA, BEGIN GENERATOR DATA
  1,'1 ',  680.715,  -11.825, 9900.000,-9900.000,1.02000,  0,  900.000,  0.00000,  9.00000,  0.00000,  0.00000,1.00000,1,  100.0,
10000.000,   0.000,  0,1.0000
  2,'1 ',  664.000,  516.283, 9900.000,-9900.000,1.02000,  0,  900.000,  0.00000,  9.00000,  0.00000,  0.00000,1.00000,1,  100.0,
10000.000,   0.000,  0,1.0000
  3,'1 ',  554.652,  -18.094, 9900.000,-9900.000,1.02000,  0,  900.000,  0.00000,  9.00000,  0.00000,  0.00000,1.00000,1,  100.0,
10000.000,   0.000,  0,1.0000
  4,'1 ',  500.000,  497.901, 9900.000,-9900.000,1.02000,  0,  900.000,  0.00000,  9.00000,  0.00000,  0.00000,1.00000,1,  100.0,
10000.000,   0.000,  0,1.0000
0 / END OF GENERATOR DATA, BEGIN BRANCH DATA
  1,    2,'1 ', 0.00250, 0.02500,0.00000,10000.00,  0.00,  0.00,,, 0.00000, 0.00000, 0.00000, 0.00000,1,  0.0,  0,1.0000
  2,    5,'1 ', 0.00100, 0.01000,0.00000,10000.00,  0.00,  0.00,,, 0.00000, 0.00000, 0.00000, 0.00000,1,  0.0,  0,1.0000
  3,    4,'1 ', 0.00250, 0.02500,0.00000,10000.00,  0.00,  0.00,,, 0.00000, 0.00000, 0.00000, 0.00000,1,  0.0,  0,1.0000
  4,    6,'1 ', 0.00100, 0.01000,0.00000,10000.00,  0.00,  0.00,,, 0.00000, 0.00000, 0.00000, 0.00000,1,  0.0,  0,1.0000
  5,    6,'1 ', 0.02200, 0.22000,0.00000,10000.00,  0.00,  0.00,,, 0.00000, 0.00000, 0.00000, 0.00000,1,  0.0,  0,1.0000
0 / END OF BRANCH DATA, BEGIN TRANSFORMER ADJUSTMENT DATA
0 / END OF TRANSFORMER ADJUSTMENT DATA, BEGIN AREA DATA
  1,  1,  415.000,   0.010,'1         '
  2,  3, -415.000,   0.100,'2         '
0 / END OF AREA DATA, BEGIN TWO-TERMINAL DC DATA
0 / END OF TWO-TERMINAL DC DATA, BEGIN SWITCHED SHUNT DATA
0 / END OF SWITCHED SHUNT DATA, BEGIN IMPEDANCE CORRECTION DATA
0 / END OF IMPEDANCE CORRECTION DATA, BEGIN MULTI-TERMINAL DC DATA
0 / END OF MULTI-TERMINAL DC DATA, BEGIN MULTI-SECTION LINE DATA
0 / END OF MULTI-SECTION LINE DATA, BEGIN ZONE DATA
```

```
  1,'1     '
  2,'2     '
0 / END OF ZONE DATA, BEGIN INTER-AREA TRANSFER DATA
  1,  2,'1',  415.00
0 / END OF INTER-AREA TRANSFER DATA, BEGIN OWNER DATA
0 / END OF OWNER DATA, BEGIN FACTS CONTROL DEVICE DATA
0 / END OF FACTS CONTROL DEVICE DATA
```

Appendix B Power system test results

Table B.1 Cases with scenario 1

Case#	t0	*EI1*	*EI2*	*EI3*
1	2	3	4	5
101	0.00	468.26	890.37	24.75
102	0.25	125.45	31.92	0.52
103	0.50	48.70	481.39	0.45
104	0.75	103.14	183.26	0.42
105	1.00	83.74	133.79	0.46
106	1.25	240.41	310.00	0.43

Table B.2 Cases with scenario 2

Case#	t0	*EI1*	*EI2*	*EI3*
1	2	3	4	5
201	0	168.08	3290.44	52.43
202	0.25	230.07	121.69	1.55
203	0.5	251.57	2905.37	1.44
204	0.75	5509.08	15088.43	1.40
205	1	4181.91	2912.57	1.29
206	1.25	1921.59	1701.40	1.01
207	1.5	1840.42	14227.11	0.71
208	1.75	1285.17	89878.68	1.00
209	2	6776.37	2575.17	1.06
210	2.25	1050.62	450.44	0.82
211	2.5	1611.67	177.14	0.49
212	2.75	166.93	97.50	0.48
213	3	10264.30	198.49	0.50
214	3.25	667.90	232.11	0.62
215	3.5	1044.41	814.54	0.79
216	3.75	14645.99	5974.21	0.77

Table B.3 Cases with scenario 3

Case#	t0	*EI1*	*EI2*	*EI3*
1	2	3	4	5
301	0.00	3096.44	129.45	6.92
302	0.25	32.92	57.29	0.41
303	0.50	155.98	57.81	0.34
304	0.75	119.85	24.69	0.24

Table B.4 Cases with scenario 4

Case#	t0	*EI1*	*EI2*	*EI3*
1	2	3	4	5
401	0.25	361.86	112.16	0.90
402	0.50	227.22	2645.09	0.72
403	0.75	397.05	885.48	0.83
404	1.00	570.71	904.80	0.86
405	1.25	1411.98	2060.80	0.78
406	1.50	507.56	4361.51	0.57
407	1.75	359.45	7553.89	0.77
408	2.00	338.61	572.27	0.82
409	2.25	834.39	669.88	0.71
410	2.50	4612.72	597.40	0.48
411	2.75	210.33	94.88	0.35
412	3.00	457.49	98.81	0.42
413	3.25	3220.20	743.88	0.44
414	3.50	305.51	208.31	0.60
415	3.75	205.71	6740.42	0.61

Table B.5 Cases with scenario 5

Case#	t0	*EI1*	*EI2*	*EI3*
1	2	3	4	5
501	0.00	1690.71	103578.88	10.69
502	0.25	82.53	8.82	0.21
503	0.50	52.96	342.04	0.44
504	0.75	27.57	285.37	0.45
505	1.00	98.73	11.67	0.34

Table B.6 Cases with scenario 6

Case#	t0	*EI1*	*EI2*	*EI3*
1	2	3	4	5
601	0.00	158.16	7922.97	36.60
602	1.50	3450.24	141.13	0.72
603	2.00	373.66	1133.92	1.44
604	3.00	292.69	208.90	1.15
605	3.50	694.65	143.69	0.60
606	4.00	1290.09	1654.86	1.31
607	4.50	589.17	242.84	2.01
608	5.00	5755.89	336.77	2.31
609	5.50	2582.90	960.09	2.32
610	7.00	683.84	550.45	0.89
611	7.50	1091.50	5875.24	0.78
612	8.00	772.40	10686.29	1.96
613	8.50	6775.17	558.91	1.92
614	9.00	3456.42	154.32	0.49
615	9.50	7497.59	126.44	0.73
616	10.00	2133.83	275.03	1.57
617	10.50	1314.04	377.34	1.93
618	11.00	584.76	3812.24	1.62
619	11.50	206.09	206.16	0.59
620	12.00	806.34	3370.92	0.76
621	12.50	412.62	4390.10	1.55
622	13.00	1878.54	289.03	1.27
623	13.50	891.09	67.29	0.48
624	14.00	520.58	126.30	0.87
625	14.50	655.72	394.69	1.35
626	15.00	20850.68	426.85	1.14
627	15.50	208.92	76.22	0.45
628	16.00	485.09	726.52	0.73
629	16.50	100.36	914.25	1.13