ACTIVE REALIZATION OF ALL-PASS TRANSFER FUNCTIONS IN UNBALANCED STRUCTURE

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ABSTRACT

An active RLC synthesis is presented for transfer functions containing both real and complex zeros in the right half s-plane. The poles of the function are restricted to the open left half s-plane.

Particular attention is paid to the all-pass portion of the transfer function. An unbalanced network representation, which requires a negative gyrator, is obtained for an arbitrary all-pass function. This network is then put in cascade with another unbalanced network which realizes the minimum-phase portion of the transfer function. Thus any transfer function is realizable in the form of two cascade networks which are unbalanced.

ACKNOWLEDGEMENT

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1. INTRODUCTION

A two-port is named "constant-resistance" if its driving-point impedance is equal to a constant R when the two-port is terminated in a resistance R. The symmetrical lattice shown in FIGURE 1.1 will be a constant-resistance two-port if $Z_a Z_b = R^2$.

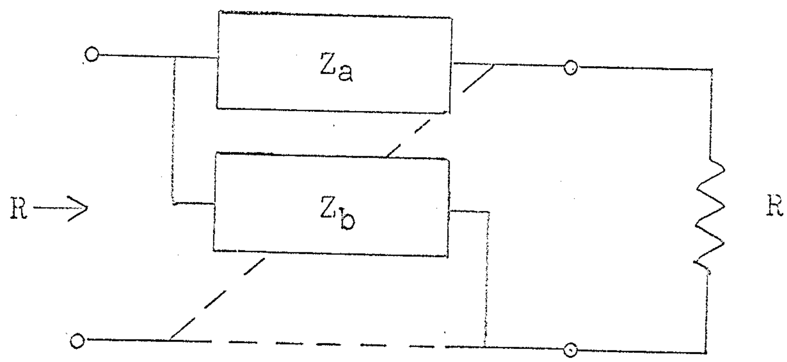

FIGURE 1.1 Constant-Resistance Symmetrical Lattice with $Z_a Z_b = R^2$.

Because of this constant-resistance property, all transfer functions of such a two-port differ only by a multiplicative constant. In this thesis T_{12} will be used to represent any of those transfer functions. Under the constant-resistance condition the voltage transfer function of the lattice in FIGURE 1.1 is

$$T_{12} = R(R-Z_a)/(R+Z_a). \qquad (1)$$

Since R may take on any practical value, normalize equation (1) with respect to R to obtain

$$T_{12} = (1-Z_a)/(1+Z_a) \qquad (2)$$

or,

$$Z_a = (1-T_{12})/(1+T_{12}). \qquad (3)$$

To realize the transfer function as a constant-resistance lattice requires that Z_a be a positive real function (1).*

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The conditions imposed upon T_{12} such that Z_a is positive real are:

1. $T_{12}(s)$ has no poles on the jw-axis.
2. $|T_{12}(jw)| \leq 1$.

The above conditions can always be satisfied. Firstly, we must assume that the given transfer function has no poles on the jw-axis. Secondly, we need only reduce the gain constant to satisfy the second condition. It is, therefore, always possible to realize a transfer function in a symmetrical lattice.

This form of realization however, has several disadvantages. The most obvious disadvantages are the need for an excessive number of elements and the low gain constant. Another undesirable feature of a symmetrical lattice is its balanced nature. It may be possible to obtain an unbalanced equivalent of the lattice in certain cases. This, however, is not always possible in the general case when only passive elements are used. It maybe that active elements are required to unbalance the general symmetrical lattice. If such an unbalanced equivalent exists, it would be preferable to establish realization procedures for obtaining, directly, such an unbalanced structure.

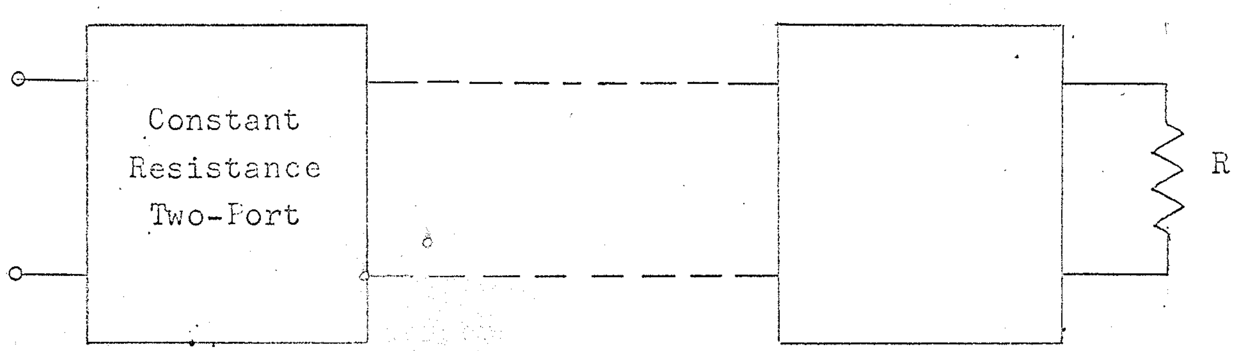

FIGURE 1.2 Cascade of Constant-Resistance Two-Ports.

The impedances that form the lattice will become relatively complicated for higher order transfer functions. This disadvantage can be overcome. Consider a cascade connection of constant-resistance two-ports shown in FIGURE 1.2. Each two-port is seen to be terminated properly to yield a constant input impedance. The result is that the over all network will be constant-resistance. Such a network structure proves to be very useful.

Let us consider the decomposition of a given transfer function in the form

$$T_{12} = T_{12a}T_{12b}\cdots T_{12n}. \tag{4}$$

It is shown by Balabanian (1)* that each component T_{12} can be realized as a constant-resistance lattice. Thus the cascade connection of these two-ports will realize the given function to within a multiplicative constant.

It has been shown (1)** that a nonminimum-phase function can be written as the product of a minimum-phase function and an all-pass function. Thus, we can write

$$T_{12}(s) = T_{12m}(s)T_{12o}(s) \tag{5}$$

where $T_{12m}(s)$ is a minimum-phase function and $T_{12o}(s)$ is an all-pass function.

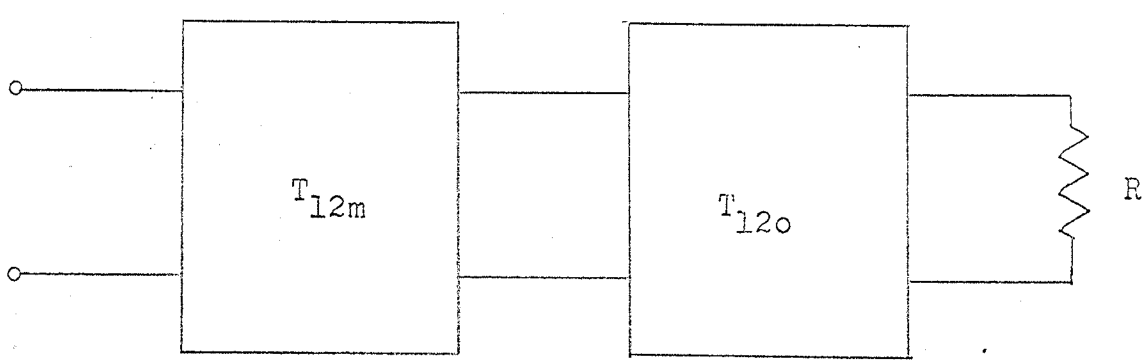

FIGURE 1.3. Realization of Nonminimum-Phase Transfer Function.

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The network realizing this function will take the form of a cascade connection of a minimum-phase two-port and an all-pass two-port. This network is shown in FIGURE 1.3.

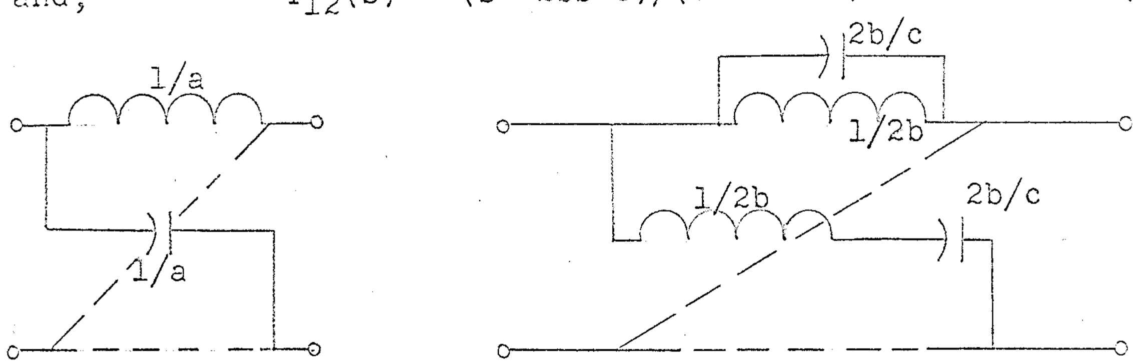
Let us now restrict ourselves to the minimum-phase function. In this class we shall include those which have zeros on the jw-axis as well as in the interior of the left-half plane. Balabanian (1)* proves that any realizable minimum-phase transfer function having no poles on the jw-axis can be realized as a constant-resistance ladder network. The minimum-phase component of the general transfer function can therefore be realized in an unbalanced structure.

It remains now to consider the all-pass portion of the general transfer function. A general all-pass function can be written as the product of simple first-order and second-order all-pass functions. The first- and second-order all-pass functions can be written, respectively, as (a,b,c>o)

$$T_{12}(s) = (a-s)/(a+s) \tag{6}$$

and,

$$T_{12}(s) = (s^2-2bs+c)/(s^2+2bs+c). \tag{7}$$



(a) (b)

FIGURE 1.4 Realization of First-Order All-Pass Function (a), and Second-Order All-Pass Function (b)[1].

[1] All elements are represented as impedances (henrys, ohms, and farads) unless otherwise stated.

*Pages 331-338.

The constant-resistance lattice realizations of these two functions can easily be found to be the networks of FIGURE 1.4. There is still the possibility that unbalanced equivalents of some or all of the all-pass lattices may be found. The first-order lattice has a positive-real transmission zero; and, as a result, there is obviously no unbalanced equivalent of such a lattice when only passive elements are used. The second-order lattice can be unbalanced (with only passive elements) provided that the element values satisfy certain restrictions (1).* These are as follows:

1. $1/2b \geqq 2b/c$. (8)

2. $c/2b^2 \geqq 1$. (9)

It is the purpose of this thesis to obtain an unbalanced structure for both the general first- and second-order all-pass lattice. This may then be extended to the realization of the general nth order all-pass lattice. It is shown that any nth order all-pass lattice may be unbalanced with the aid of a single negative resistor plus conventional passive elements excluding transformers. We can then claim to be able to unbalance any transfer function that has only poles in the open left-half s-plane. Furthermore, the proposed network, which is of constant-resistance π-structure, can be applied to the realization of maximally flat delay networks or phase correction networks (2).

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2. REALIZATION OF Nth ORDER ALL-PASS NETWORK IN UNBALANCED FORM

Turn now to the problem of realizing the general nth order all-pass function in an unbalanced structure. The following procedure will employ negative resistors to obtain such an unbalanced equivalent.

The zeros of an all-pass function are the negatives of its poles. Such a function can, therefore, be written as

$$T_{12o} = (m-n)/(m+n) = (1-n/m)/(1+n/m) \tag{10}$$

where m and n are, respectively, the even and odd parts of a Hurwitz polynomial. It is seen that the above transfer function can be realized as a constant-resistance lattice. The lattice arm impedance has the form $Z_a = n/m$. The impedance Z_a is therefore lossless. The general nth order all-pass function is, therefore, realized as a lossless constant-resistance lattice. We now seek an unbalanced equivalent of such a structure.

The unbalanced equivalent of the above lattice will be of the constant-resistance π-structure. Consider the y-parameters of a lattice network.

$$y_{11} = \tfrac{1}{2}(Y_b+Y_a) \tag{11}$$

$$y_{12} = \tfrac{1}{2}(Y_b-Y_a). \tag{12}$$

The admittances, Y_b and Y_a, are defined in FIGURE 1.1. Let us now rearrange the terms on the right-hand side of equations (11) and (12).

$$\begin{matrix} y_{11} \\ y_{12} \end{matrix} = \begin{bmatrix} Y_b \\ o \end{bmatrix} + \begin{bmatrix} -\tfrac{1}{2}Y_b \\ \tfrac{1}{2}Y_b \end{bmatrix} + \begin{bmatrix} \tfrac{1}{2}Y_a \\ -\tfrac{1}{2}Y_a \end{bmatrix} \qquad \begin{matrix} (13) \\ (14) \end{matrix}$$

The brackets merely serve to group the terms. Each set of brackets represents a component network. The composite network will be the parallel connection of the three sub-networks.

Our present purpose is to show that each of these subnetworks is of an unbalanced nature and that each is physically realizable.

Thus the net effect has been to obtain three simplier unbalanced networks. The three sets of y-parameters are seen to be:

$$y_{11a} = Y_b = n/m \quad (15)$$

$$y_{12a} = 0 \quad (16)$$

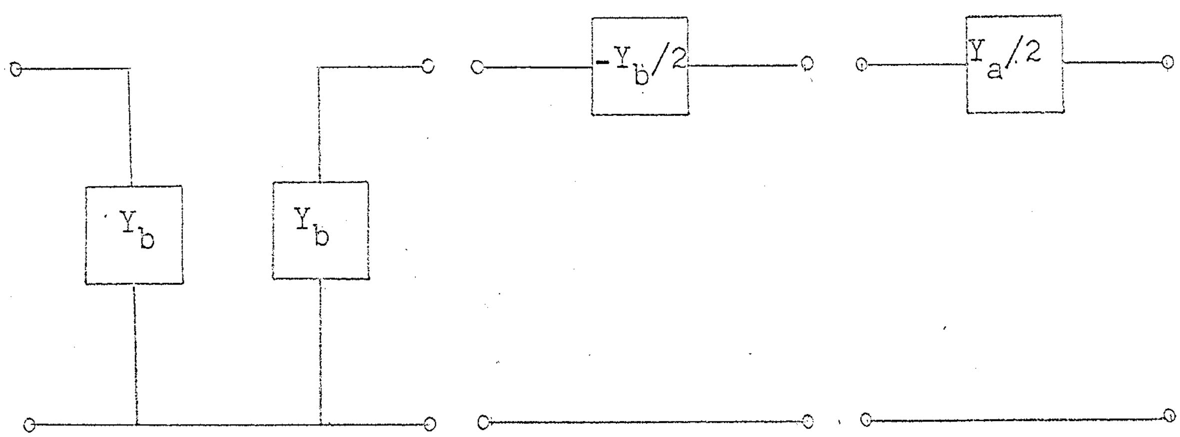
$$y_{11b} = -\tfrac{1}{2}Y_b = -n/2m \quad (17)$$

$$y_{12b} = \tfrac{1}{2}Y_b = n/2m \quad (18)$$

$$y_{11c} = \tfrac{1}{2}Y_a = m/2n \quad (19)$$

$$y_{12c} = -\tfrac{1}{2}Y_a = -m/2n \quad (20)$$

The subnetworks represented by equations (15), (16), (17), (18), (19), and (20) are shown in FIGURE 2.1.



(a) (b) (c)

FIGURE 2.1 Subnetworks for the Realization of the Nth Order All-Pass Function.

Note that in FIGURE 2.1 the admittances are defined as $Y_a = m/n$ and $Y_b = n/m$.

In FIGURE 2.1 it is seen that the realization of the arms in the π-network is reduced to the realization of lossless driving-point functions. The realization of networks (a) and (c) is straight forward. Network (b) is shown to contain the negative of a lossless driving-point function. The realization of network (b) is nevertheless accomplished with the aid of a negative gyrator (see Appendix A). Network (b) is shown in its final form in FIGURE 2.2.

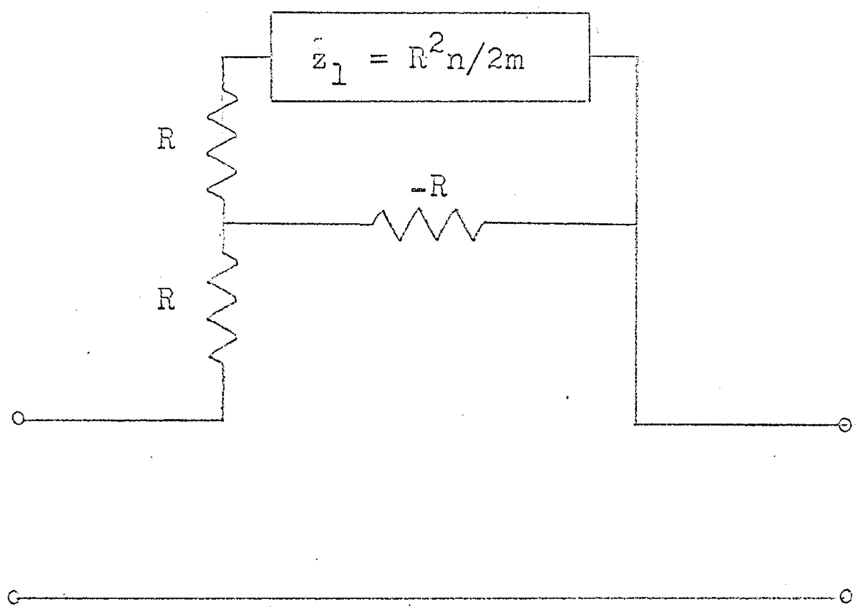


FIGURE 2.2 Realization of $-\frac{1}{2}Y_b$.

The complete realization of the nth order all-pass function is shown in FIGURE 2.3.

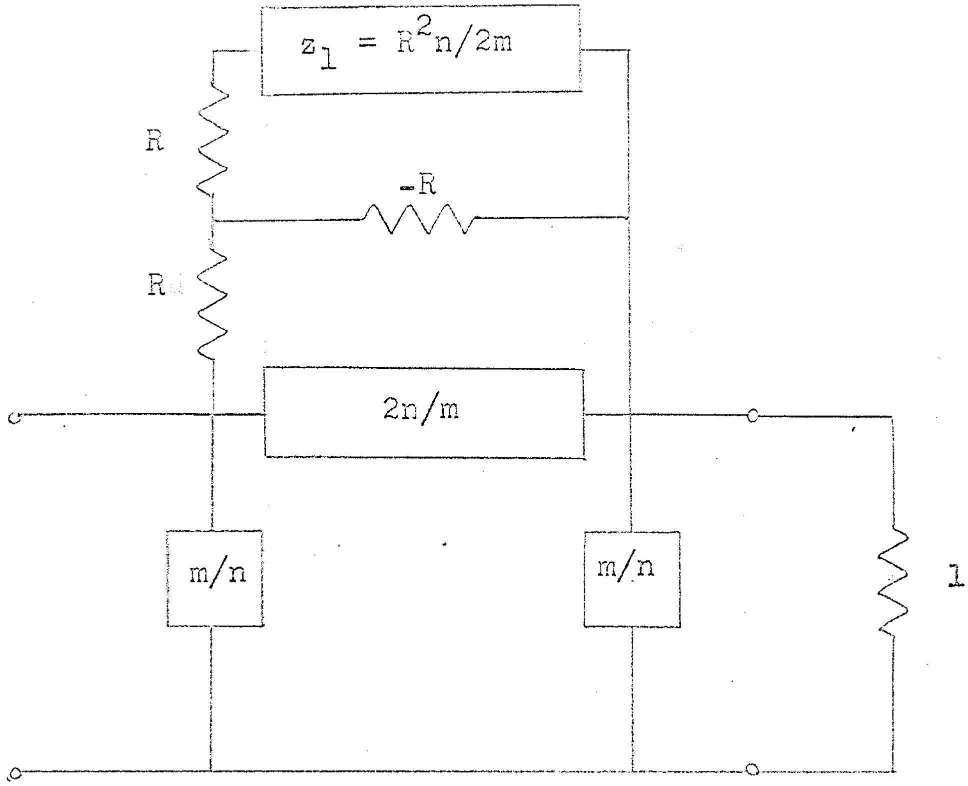


FIGURE 2.3 Realization of the Nth Order All-Pass Function in Unbalanced Structure.

Note that the elements represented by blocks in FIGURE 2.3 are all physically realizable as lossless driving-point impedances. Therefore, this procdure is straight forward regardless of the complexity of the given all-pass function. Another interesting observation is that the general all-pass function can be realized in an unbalanced structure with the aid of one negative resistor. The value of this negative resistor may of course take on any practical value.

3. SPECIFIC EXAMPLES OF ALL-PASS NETWORKS

It was previously stated that the realization of the general all-pass function in a constant-resistance $\top\!\top$-network reduces to the realization of a driving-point impedance. The complexity of this driving-point impedance increases as the order of the all-pass function is increased. To overcome this drawback factor the general all-pass function into first- and second-order all-pass functions. This may be accomplished by arranging the poles and zeros in an obvious way. The realization of the general all-pass network will then be a cascade connection of constant-resistance $\top\!\top$-networks which are in turn the realizations of simple first- and second-order all-pass functions.

3.1 First-Order Case

Consider the typical first-order all-pass transfer function

$$T_{12} = (a-s)/(a+s) = (1-s/a)/(1+s/a). \tag{21}$$

The branch impedance arm is found by comparing equation (21) to equation (2). The result is

$$Y_a = a/s \tag{22}$$

and,

$$Y_b = s/a. \tag{23}$$

We refer now to equations (11) and (12) to obtain the y-parameters of this first-order all-pass network. The results are

$$\begin{matrix} y_{11} = \tfrac{1}{2}(s/a+a/s) = \\ y_{12} = \tfrac{1}{2}(s/a-a/s) = \end{matrix} \begin{bmatrix} s/a \\ o \end{bmatrix} + \begin{bmatrix} -a/2s \\ s/2a \end{bmatrix} + \begin{bmatrix} a/2s \\ -a/2s \end{bmatrix}. \qquad \begin{matrix} (24) \\ (25) \end{matrix}$$

The appropriate decomposition of the y-parameters has been made. The subnetworks represented by equations (24) and (25) are shown in FIGURE 3.1.

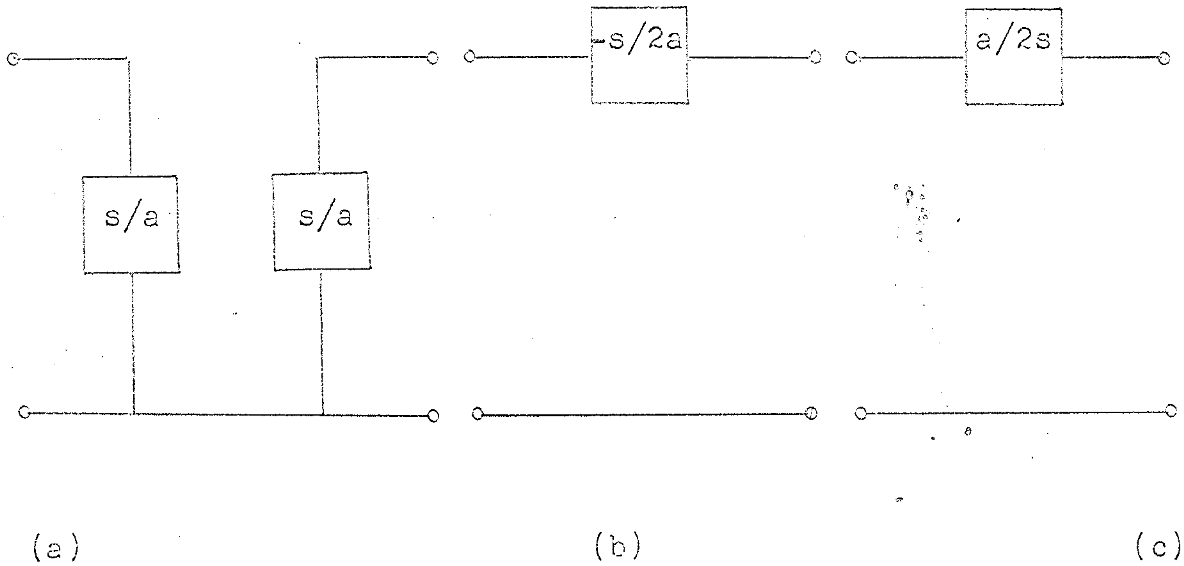


FIGURE 3.1 Subnetworks for the Realization of the First-Order All-Pass Function.

The negative gyrator must now be employed to realize the network given in FIGURE 3.1 (b). The subnetworks are redrawn in FIGURE 3.2 using actual circuit elements.

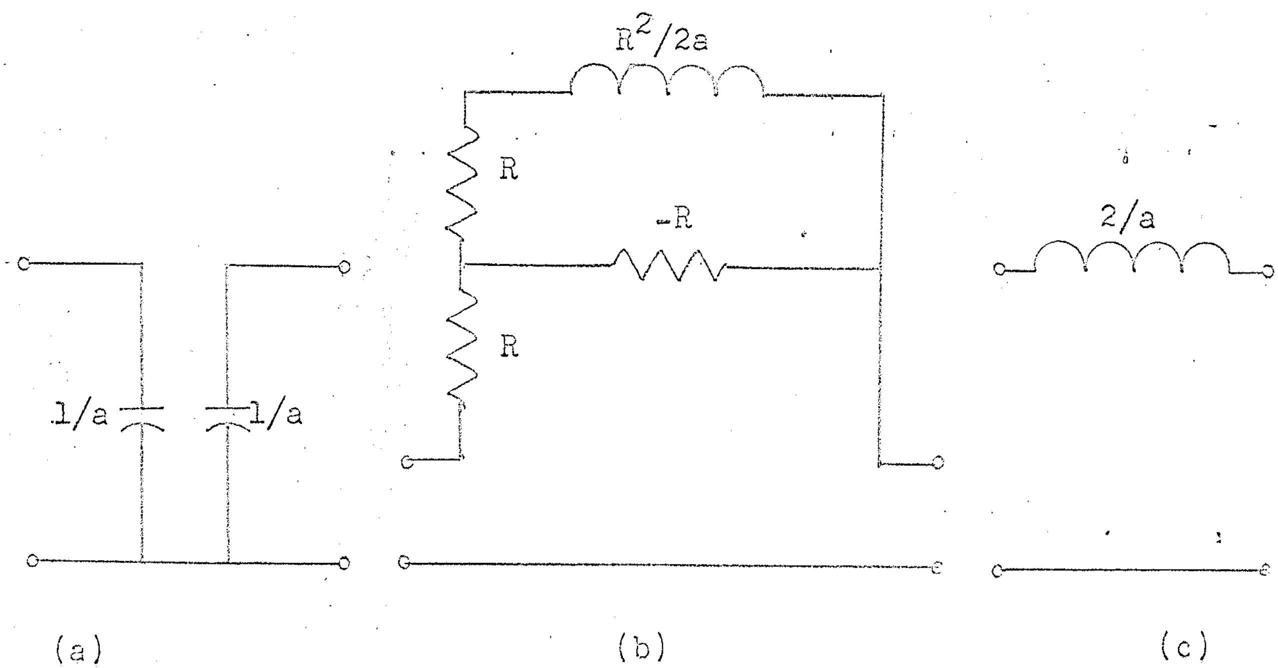


FIGURE 3.2 Realization of Component Networks of FIGURE 3.1.

The end result is the circuit in FIGURE 3.3. This network may be placed in cascade with any other constant-resistance structure used to realize the minimum phase part of a general transfer function.

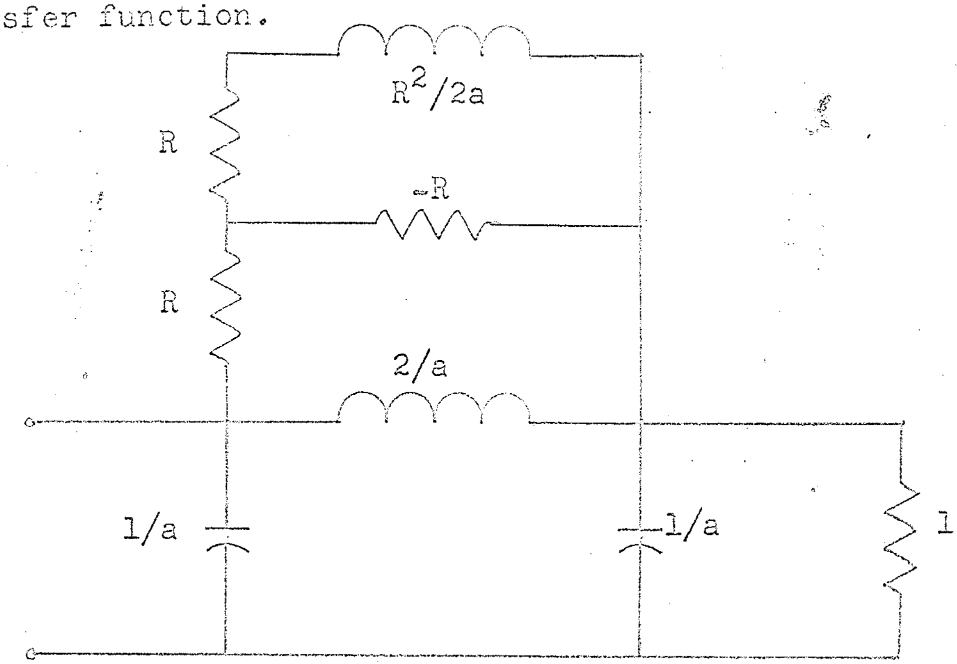


FIGURE 3.3 Realization of the First-Order All-Pass Function in Constant-Resistance π-Structure.

3.2 Second-Order Case

In a similar manner, one may obtain a constant-resistance structure for the second-order all-pass function. The procedure employed in section 3.1 may now be applied to the second-order case. The results are as stated below.

$$T_{12} = (s^2-2bs+c)/(s^2+2bs+c) \tag{26}$$

$$T_{12} = \frac{1-2bs/(s^2+c)}{1+2bs/(s^2+c)} = \frac{1-Z_a}{1+Z_a} \tag{27}$$

$$Y_b = 1/Y_a \tag{28}$$

$$y_{11} = bs/(s^2+c) + (s^2+c)/4bs \tag{29}$$

$$y_{12} = bs/(s^2+c) - (s^2+c)/4bs \tag{30}$$

$$\begin{array}{l} y_{11} = \\ y_{12} = \end{array} \begin{bmatrix} 2bs/(s^2+c) \\ 0 \end{bmatrix} + \begin{bmatrix} -bs/(s^2+c) \\ bs/(s^2+c) \end{bmatrix} + \begin{bmatrix} (s^2+c)/4bs \\ -(s^2+c)/4bs \end{bmatrix} \begin{array}{l} (31) \\ (32) \end{array}$$

The subnetworks represented by equations (31) and (32) are shown in FIGURE 3.4.

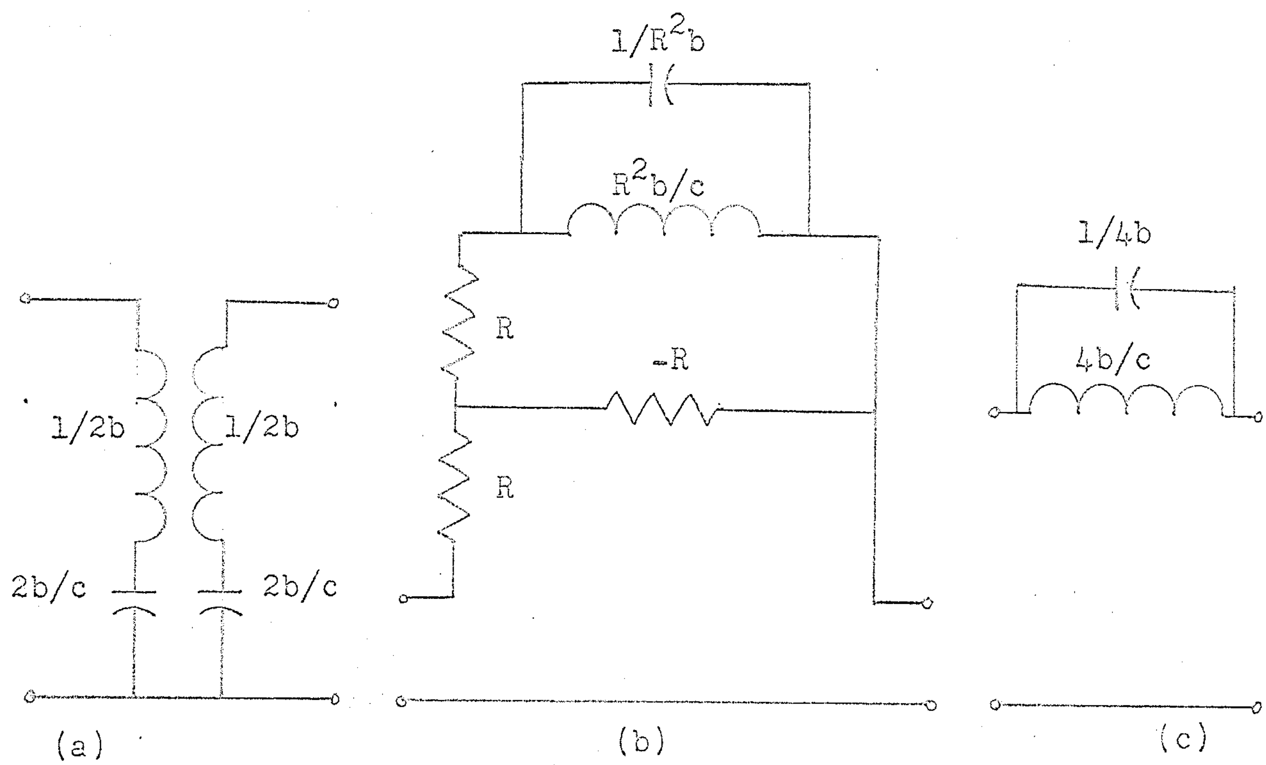


FIGURE 3.4 Subnetworks for the Realization of the Second-Order All-Pass Function.

The end result is shown in FIGURE 3.5. Note that the negative resistor may take on any practical value.

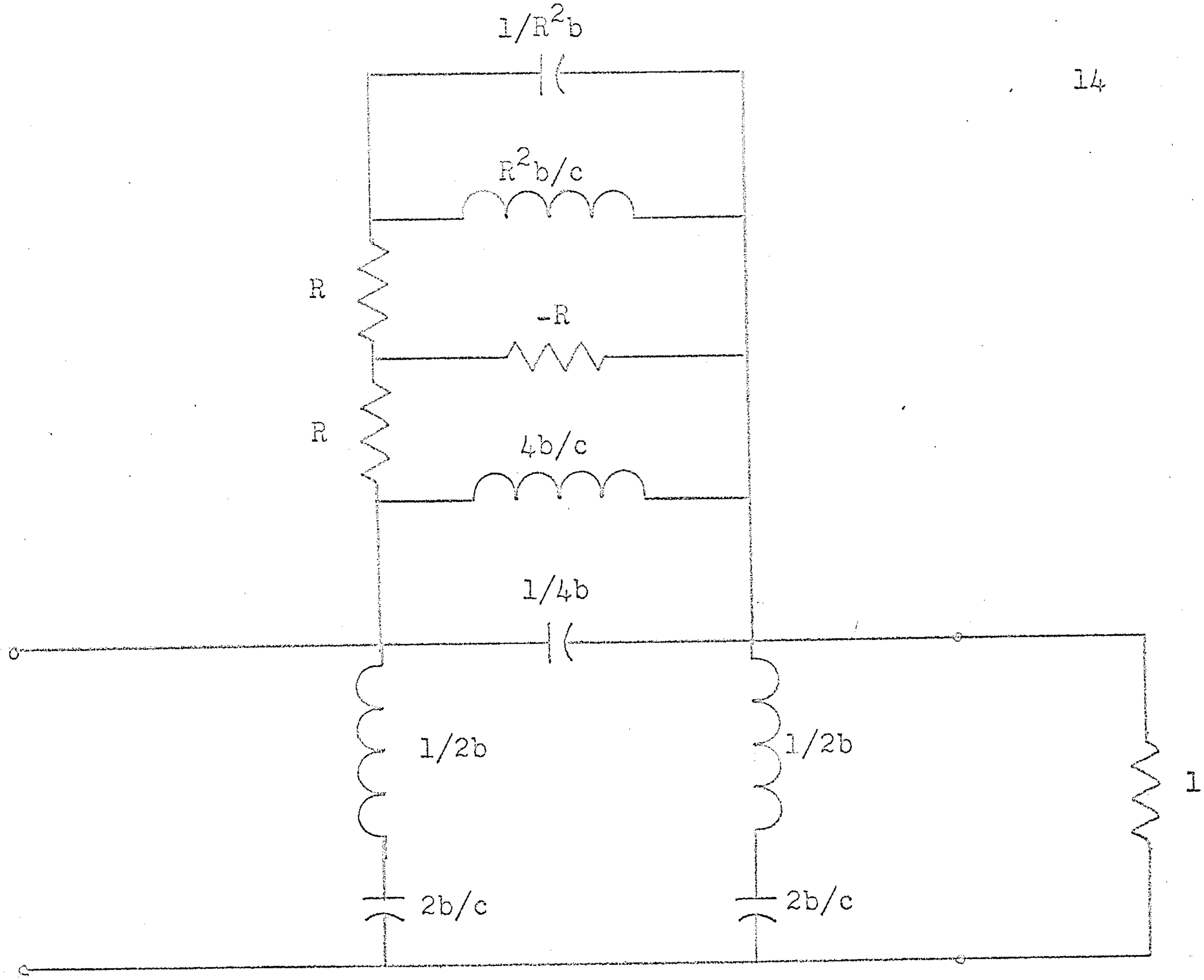


FIGURE 3.5 Realization of the Second-Order All-Pass Function in Constant-Resistance π-Structure.

4. CONCLUDING REMARKS

Of immediate interest is the fact that we can realize any order transfer function which may have zeros anywhere in the right-half s-plane and poles only in the open left-half s-plane. This thesis, therefore, proposes a new method of realizing transfer functions in an unbalanced structure. No transformers are used and the only active element used is a single negative resistance. The negative resistor can be chosen to be any practical value. Since the proposed network is constant-resistance, we can realize several transmission zeros by inserting constant-resistance two-ports in cascade. This does not disturb the rest of the network.

The procedure set forth in this thesis is reduced to a realization of lossless driving-point impedances of the types m/n and n/m where (m + n) is a Hurwitz polynomial. The complete problem as stated here has thus far been investigated in the case of rather specialized classes of networks. Allemandou {4} presents a realization procedure in which a ratio Z_1/Z_2 is found. Z_1/Z_2 is seen to be a ratio of two polynomials. There is, however, a restriction on the roots of these polynomials. The realization of Z_1 and Z_2 as RC driving point functions is only possible if these polynomials have negative real roots. This procedure is immediately applicable to the first-order all-pass function. However, in the general case we must assume that the two polynomials defining Z_1/Z_2 have complex roots. The general RLC synthesis procedures must then be employed with the possibility that ideal transformers may be necessary in the realization of Z_1 and Z_2. The technique described in this thesis places no such conditions on the driving point functions Y_a and Y_b. We, therefore, submit that the proposed technique is more general. The network proposed in this thesis can also be applied to the realization of maximally flat time delay networks and phase correction networks in unbalanced form {3}.

5. APPENDIX

This Appendix deals with the negative gyrator {3}*
previously mentioned in Chapter 2.

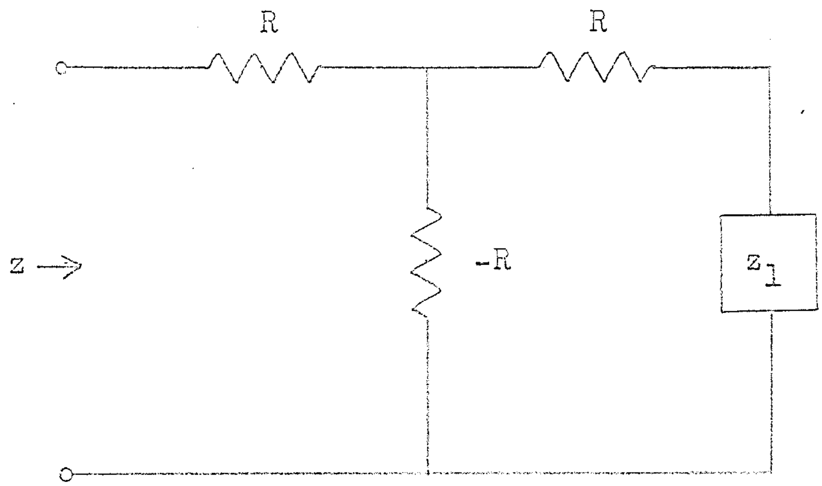


FIGURE 5.1 Negative Gyrator Network.

The driving-point impedance for the circuit in FIGURE 5.1 is

$$z = R + \frac{1}{-1/R + 1/(R+z_1)} \tag{33}$$

$$z = -R^2/z_1 \tag{34}$$

or,

$$z_1 = -R^2/z. \tag{35}$$

Refer now to the general nth order all-pass function given in equation (10). The following relations are seen to hold:

$$z = -2/Y_b \tag{36}$$

$$Y_b = n/m, \tag{37}$$

therefore,

$$z = -2m/n \tag{38}$$

and,

$$z_1 = -R^2/z = R^2n/2m. \tag{39}$$

It is noted that the impedance function z_1 is a lossless driving-point function. Hence, z_1 is realizable in terms of passive lossless elements. We have thus illustrated the realization procedure of a negative real lossless driving-point function.

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The value of the negative resistor shown in FIGURE 5.1 is of considerable practical importance. The realization of a practical negative resistance is obtained with the aid of a tunnel diode. Ghausi {5}* outlines the negative-resistance characteristic of a tunnel diode.

*Pages 168-171.

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