THE UNIVERSITY OF MANITOBA

THE EFFECTS OF INSTRUCTION USING RATIONAL SETS OF EXAMPLES AND NONEXAMPLES ON GEOMETRIC CONCEPT AQUISITION AMONG SIXTH-GRADE STUDENTS

BY

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A Thesis
Submitted to the Faculty of Graduate Studies
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ABSTRACT

This study investigated the relative effectiveness of rational sets and randomly arranged sets of examples and nonexamples of the geometric concept parallelogram on sixth-grade students' attainment of the concept at the classifcatory and formal levels. Subjects were exposed to introductory activities (prematerial instruction) which provided opportunity for exploratory work and discussion of the concept. A secondary/exploratory purpose of the study was to investigate whether there was evidence of an interaction between the method of presenting concept instances and the mathematical ability of subjects.

The subjects were 105 grade 6 students from two schools, one from each of two school divisions in Manitoba. Subjects were assigned to groups by stratified random assignment on the basis of classroom, student's sex, and math ability. Teacher ratings were used as a measure of students' general mathematical ability.

All instructional and test materials were researcher-developed. Kuder-Richardson formula 21 reliability coefficients for the subtests at the classificatory and formal levels were .84 and .77 respectively.

The basic design, the posttest - only control group design, was extended into two other nonorthogonal designs: the treatment X school and the treatment X math ability designs. Analysis of variance based on the standard parametric (STP) interpretation was performed separately on each design, testing at the .05 level of significance.

The school factor did not affect results at either level of attainment. No significant treatment effect was found at the classificatory level. A significant treatment effect was found at the formal level

($F = 6.22$, $p < .01$) and ($F = 7.56$, $p < .01$) in favor of rational sets, when school and math ability respectively were used as blocking factors. A significant effect was found for math ability at both the classificatory level ($F = 8.77$, $p < .01$) and the formal level ($F = 6.19$, $p < .01$).

The interaction between the treatments and math ability was not significant at either level of attainment. An estimate of the strength of association between the treatment manipulation and performance at the formal level (partial omega squared) was found to be .06; showing that 6 percent of the variance in scores could be accounted for by the treatment manipulation independently of math ability.

It was concluded that rational sets facilitated geometric concept acquisition at the formal level across all levels of math ability. The strength of the effect was rated as "medium". Implications and recommendations stressing the importance of rational sets were advanced for teaching geometric concepts to grade 6 students of all levels of math ability, and for the design and selection of instructional materials to teach geometric concepts.

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THE PROBLEM AND SIGNIFICANCE OF THE STUDY

Concepts are the basis of formal education; principles - generalizations and rules of procedures - are formulated by means of concepts. Concepts are necessary for improving skills in classification, discrimination, and generalization. Thus, it is important that techniques be investigated which would assist teachers in improving children's learning of mathematical concepts. Nonexamples of concepts can play a useful role in facilitating higher levels of attainment of mathematics concepts. This study investigated the relative effectiveness on concept attainment behavior of two strategies for presenting examples and nonexamples of a geometric concept to sixth-grade students.

Introduction

The results of the Manitoba Mathematics Assessment, as presented in the *Manitoba Mathematics Assessment Program (M.M.A.P.) 1981 Final Report*, revealed that understanding of geometric terminology was weak across all grade levels. Of particular interest was the fact that geometry concepts at grade 6 appeared to be poorly understood. At this level, only 36 percent of the students were able to identify an obtuse angle, and 49 percent correctly identified the diameter of a circle (*M.M.A.P. 1981 Final Report*, p. 30).

An examination of some of the geometry items on the tests showed that the students were required to identify examples of concepts from among diagrams showing examples and nonexamples of the concepts. Such items reflect skills at the classificatory level of concept attainment, as defined by Klausmeier (1976a). Hence, one can say that the Mathematics

Assessment showed that grade 6 students experienced problems at the classificatory level of attainment of geometric concepts.

Preliminary work conducted with grade 5 students in June, 1983, as part of this research, found similar difficulties for the concepts equilateral triangle, quadrilateral, and parallelogram. Students were required to respond to items testing concept attainment at the concrete, identity, classificatory, and formal levels, defined according to Klausmeier, Ghatala, and Frayer (1974) and Diluzio, Katzenmeyer, and Klausmeier (1975). Students were found to perform satisfactorily at the concrete and identity levels, but very poorly at the classificatory and formal levels (See Appendix A). For example, in one set of test items on the concept parallelogram, given 17 instances to classify, 64 percent of the students correctly classified less than 10 instances, while 96 percent of the students scored less than 4 out of 10 on items at the formal level.

Background to the Problem

Research involving the role of nonexamples in concept learning has not always indicated that nonexamples are useful. In fact, some researchers (Ausubel, Novak, & Hanesian, 1978; Bruner, Goodnow, & Austin, 1953; Bruner & Anglin, 1973; Chernick, 1975; Malo, 1975; Sheel, 1981) have pointed to detrimental effects due to the use of nonexamples. However, other studies (Bourne & Guy, 1968; Cohen & Carpenter, 1980; Cook, 1981; Dienes, 1964; Gage, 1977; Shumway, 1971, 1974; Shumway & Lester, 1974; Tennyson, 1973) have stressed the usefulness of nonexamples in concept learning.

A number of studies (Feldman, 1972, 1975; Klausmeier & Feldman, 1973, 1975; McMurray, 1975; McMurray, Bernard, Klausmeier, Schilling, & Vorwerk, 1977; Tennyson, 1973; Tennyson, Woolley, & Merrill, 1972; Tennyson, Steve, & Boutwell, 1975; Tennyson, Chao, & Youngers, 1981) have found that when nonexamples are presented in rational sets, concept acquisition is enhanced. These studies suggest that nonexamples of a concept may facilitate learning only when they are matched with examples in such a manner as to focus attention on the relevant attributes of the concept. This is the basis of rational sets of examples and nonexamples. These studies have, however, not clearly shown how the ability of the students is related to the effectiveness of rational sets of examples and non-examples.

Despite the implication from the research that rational sets better facilitate concept acquisition, only one research (Tennyson, Steve, & Boutwell, 1975) was located in which the effects of rational sets were directly investigated relative to randomly arranged sets of examples and nonexamples. However, this research did not involve mathematics concepts. A related study by Cohen & Carpenter (1980) seemed to refute the findings of Tennyson et al. (1975), by concluding that the method of presenting examples and nonexamples for a geometric concept made no difference at the classificatory level.

Many studies in which the effects of nonexamples and different ways of presenting rational sets have been investigated, have tended to make the prematerial instructional phase (introductory activities) somewhat superficial. In some cases (Shumway, 1971, 1974; Shumway & Lester, 1974; Tennyson et al., 1975) this phase merely involved defining the concept or providing a brief introduction at a computer terminal; Cohen and Carpenter (1980) showed a videotaped lesson which students watched and/or listened; while other studies (Klausmeier & Feldman, 1973, 1975;

Tennyson, Chao, & Youngers, 1981) merely involved the researchers' reading the definition of the concept and the terms to be used while the children listened. After which students were presented with examples and nonexamples of the concept.

A number of mathematics educators have questioned such a superficial instructional phase. According to Geeslin (1974), merely parading examples and nonexamples of a concept in front of students is not a paradigm for learning; some discussion of the concept should be conducted before (or after) the presentation. Cooney (1976) has questioned the applicability of research involving self-instructional booklets to the actual classroom situation. Cohen and Carpenter (1980) have recommended that the scope and sequence of the instructional phase should be a reflection of actual classroom behavior (p. 163). However, Stout and Shumway (1981) have suggested that this depends on whether the research is trying to develop theory or validate theory.

Polya (1977) has stressed the importance of providing students with opportunity for exploratory work prior to being presented with the concept definition. Students should have some opportunity to actively engage in searching for some of the critical attributes of a concept, prior to being exposed to printed instructional material involving examples and nonexamples of the concept. Bruner, Goodnow, and Austin (1956) and Klausmeier (1976b) have emphasized the importance of first establishing an intention on the part of students to learn concepts.

Statement of the Problem

Research was needed to determine the relative effectiveness of instruction using rational sets of examples and nonexamples and instruction using randomly arranged sets, in facilitating higher levels of geometric concept attainment among sixth-grade students, when students have been given opportunity to do exploratory work and discuss the concept prior to studying instructional materials. Research was also needed to determine whether there is an interaction between the methods of presenting concept instances and the mathematical ability of students, in influencing concept attainment behavior.

Specifically, the study sought to answer the following questions:

1) If prematerial instructional activities which reflect actual classroom experiences are provided for students prior to their being presented with printed instructional materials, would sixth-grade students who study rational sets of examples and nonexamples of the geometric concept parallelogram, perform better at the (a) classificatory and (b) formal levels of concept attainment than students who study randomly arranged sets?

Secondary/exploratory research questions were:

2) Is there evidence of an interaction between the method of presenting concept instances and the mathematical ability of the students?

3) What proportion of the variance in concept attainment scores can be attributed to the treatment manipulation, independently of the effects of math ability?

Hypotheses

The following hypothesis was advanced regarding research question 1): Students who study rational sets of examples and nonexamples of the concept will perform at a higher level than students who are presented with randomly arranged sets, for both the (a) classificatory and (b) formal levels of concept attainment.

The rationale underlying the above hypothesis is based on the findings of previous research. The facilitative effects of rational sets of examples and nonexamples on students' concept attainment behavior, have been demonstrated by previous research. These studies involved concepts in different subject areas and across different age groups: Feldman (1972), symmetry; Klausmeier and Feldman (1973, 1975), equilateral triangle; Tennyson (1973), adverbs; Tennyson, Woolley, and Merrill (1972), trochaic meter; Tennyson, Steve, and Boutwell (1975), RX_2 Crystals and adverbs; Tennyson, Chao, and Youngers (1981), equilateral triangle; McMurray, Bernard, Klausmeier, Schilling, and Vorwerk (1977), equilateral triangle and the concept of tree; Suebsonthi (1981), regular polygons. Tennyson et al. (1981) have found that when instructional materials involving rational sets are presented in an expository-interrogatory format, concept attainment is enhanced.

Hence, on the evidence of the above research, one would expect that students who study rational sets (presented in an expository-interrogatory form), would better develop the generalization and discrimination skills necessary for concept attainment at the classificatory and formal levels than subjects receiving randomly arranged sets of examples and nonexamples. One would further expect that the results should not depend on the nature of the introductory activities to which students are exposed prior to studying the instructional materials.

Null hypotheses. The following null hypotheses served to guide interpretation of the results of the research:

1. There is no significant difference in mean performance at the classificatory level between subjects who study rational sets of examples and nonexamples and subjects who study randomly arranged sets.

2. There is no significant difference in mean performance at the formal level between subjects who study rational sets of examples and nonexamples and subjects who study randomly arranged sets.

3. There is no significant interaction between the method of presenting concept instances and the math ability of subjects for performance at the classificatory level.

4. There is no significant interaction between the method of presenting concept instances and the math ability of subjects for performance at the formal level.

Significance of the Problem

According to Tennyson et al. (1981):

> Concept learning is directly related to manipulation of the presentation form, and this manipulation should be included in the design of concept-learning lessons. (P. 333)

However, designing instructional material involving rational sets of examples and nonexamples of a concept, is much more time-consuming and technical than designing randomly arranged sets. Hence, it is important that the relative effectiveness of these two methods of presenting concept instances be compared, in order to determine whether the extra effort and time involved in designing rational sets would be a worthwhile investment on the part of teachers and curriculum developers.

This study would extend research involving the presentation of examples and nonexamples of concepts by ensuring that the prematerial instructional phase reflects actual classroom behavior. According to Klausmeier (1976b):

> At the outset...the learner is engaged actively in searching behaviors directed towards learning the particular concept or concepts at a higher level of attainment. (P. 207)

Such instruction may engage the learners in an active search for the attributes which distinguish examples from nonexamples. However, such instruction and activities are required regardless of the kind of practice instructional material provided for students. During mathematics classroom instruction, it is quite possible that teachers would provide such activities for children without necessarily following them up with instructional material involving rational sets. Hence, the prematerial instructional phase constitutes an important aspect of this research.

A secondary/exploratory purpose of this research examines the role of math ability. According to Skemp (1971, p. 29), the ability to learn concepts among conditions of great "noise" (such as would occur when using nonexamples) is a feature of high intelligence. Klausmeier et al. (1974) have noted that one reason research has shown superior concept mastery by high achieving students is that they are more likely to have discriminated and named attributes than low achieving children of the same age (p. 187).

It seems likely, therefore, that the effectiveness of any method of presenting instances of geometric concepts, may interact with the ability of students in influencing concept attainment behavior, since ability influences the extent to which students can discriminate between critical and variable attributes of a concept. If there is evidence of an interaction effect, then hypotheses for future research can be advanced to further investigate such interaction.

The third (exploratory) research question addresses the issue of practical versus statistical significance. Research has reported statistically significant results regarding the facilitative power of rational sets on concept learning; however, the research has given no indication as to what proportion of the variation in concept attainment behavior can actually be attributed to rational sets independently of other factors. Keppel (1982) and Maxwell, Camp, and Arvey (1981) have emphasized the importance of estimating some measure of strength of association to assist in interpreting results, since statistical significance is greatly influenced by sample size. Hence, a trivial difference can be found to be statistically significant if the sample size is large enough.

The present study has emerged as a result of the findings of the Manitoba Mathematics Assessment, which has highlighted the need for improving the teaching/learning of geometric concepts. This research may provide insight into possible strategies for improving students' acquisition of geometric concepts; as well as, providing insight into criteria for the design and selection of instructional materials and textbooks to be used in the teaching of geometric concepts.

Definition of Terms

The following definitions describe the major terms as used in the problem statement and throughout this study.

Concept

A concept is a set of specific objects, symbols, or events which are grouped together on the basis of shared characteristics and which can be referenced by a particular name or symbol (Merrill & Tennyson, 1977).

Concept Definition

A concept definition is a statement identifying each of the critical attributes and indicating how these attributes are combined.

Attributes

Attributes are defined according to Merrill and Tennyson (1977).

Attribute is a special name used to refer to the characteristics that determine whether a particular symbol or object is a member of a particular class.

A Critical Attribute is a characteristic necessary for determining class membership.

A Variable Attribute is a characteristic shared by some but not all members of the class. It is not necessary for determining class membership.

Instances

Instances are defined according to Merrill and Tennyson (1977).

Instance is a general term used to refer to both members and nonmembers of a concept class.

An Example is a member of the concept class under consideration.

A Nonexample is any instance which is not a member of the concept class under consideration.

Rational Sets of Examples and Nonexamples

A rational set consists of at least two divergent examples each matched to a nonexample (Tennyson et al., 1981). The set of examples should cover all possibilities (Markle & Tiemann, 1970); that is, the rational set must include enough examples of the irrelevant attributes for them to be varied and thus not be accepted as being critical attributes (Sowder, 1980).

Examples are divergent when their variable (irrelevant) attributes are as different as possible. Examples should differ from each other as much as possible while still belonging to the concept class.

An example and nonexample are matched when their variable (irrelevant) attributes are as similar as possible. A nonexample should resemble an example as closely as possible while still being clearly outside the concept class.

Randomly Arranged Sets of Examples and Nonexamples

A selection of varied examples and nonexamples arranged in random order.

Presentation Forms

Presentation forms are defined according to Tennyson et al. (1981).

Expository presentation form. Examples and nonexamples are presented in rational sets with explanations as to why particular geometric figures are not examples of the concept.

Interrogatory presentation form. The student is required to determine whether or not a given instance is an example of the concept; beside each instance is a set of questions which focuses the student's attention on the critical attributes. Following each set of responses the student receives the correct answers.

Expository-interrogatory presentation form. A combination of the above presentation forms such that half of the material reflects an expository format, while half reflects an interrogatory format.

Levels of Concept Attainment

The two dependent variables, the (1) classificatory and (2) formal levels of concept attainment, are defined according to Klausmeier (1976a), and are measured in this study by scores obtained on research-developed subtests at each level. Objectives and item types have been adapted from Klausmeier et al. (1974) and Diluzio et al. (1975).

Classificatory level. Attainment of a concept at the classificatory level is inferred when the individual treats at least two different examples of the same class as equivalent, even though he may not be able to describe the basis for his response. At a higher level the individual can correctly classify a large number of instances as examples and nonexamples, but cannot accurately describe the basis for the grouping in terms of defining attributes.

Specific objectives for the concept parallelogram employed in this research are as follows. The student should be able to:

1. Select the name of the concept, given an example of it.
2. Select an example of the concept, given the concept name.
3. Select a nonexample of the concept, given the concept name.
4. Identify the examples and nonexamples of the concept, given a large number of instances.

Formal level. Attainment of a concept at the formal level is inferred when the individual can give the name of the concept, can accurately designate instances as belonging or not belonging to the concept class, and can state the basis for their inclusion or exclusion in terms of the defining attributes.

In this study, measurement of concept attainment at the formal level includes the following skills/objectives:

1. Discrimination and naming attributes: an assessment of the student's ability to label and discriminate the societally-accepted attributes of the concept parallelogram. Students should be able to: a) select an instance of the attribute, given its name; b) select the name of the attribute, given an instance of it; and c) given two groups of figure drawings, the student should be able to identify the attribute which

fits all the drawings in one group but does not fit all the drawings in the second group.

2. Inferring of relevant and irrelevant attributes. The student should be able to: a) select the name of the relevant attribute of the concept, given the name of the concept; and b) select the name of the irrelevant attribute of the concept, given the concept name.

3. Evaluating and defining: an assessment of the student's ability to define the concept parallelogram and to evaluate instances as belonging or not belonging to the set "parallelogram" based on the presence or absence of the defining attributes. The student should be able to: a) select the name of the concept, given its definition; b) select the definition of the concept, given the concept name; and c) given a drawing of a parallelogram on one side of the page, and three other figure drawings on the opposite side of the page, the student should be able to determine in what way the first drawing (parallelogram) differs from the other drawings.

Prematerial Instruction

Prematerial instruction refers to instruction and activities which are provided for students by a teacher before they are given printed instructional material to study. The instruction and activities relate directly to the content on the printed material.

Prematerial instruction which reflects actual classroom experiences refers to introductory activities which provide some opportunity for students to engage in exploratory work, and to verify (and verbalize) the critical attributes of the concept.

Mathematical Ability

A measure of a student's general ability to perform at mathematical tasks. Mathematical ability has been determined in this study by teachers' (intuitive) ratings of students' general math ability, as being high, average, or low.

Methodology

Following is an overview of the methodological considerations underlying the planning and execution of this research. Such considerations relate to the research design, instrumentation, and analysis of data.

On the basis of the generally weak performance shown by students at the end of grade 5, for attainment of the concept parallelogram at the classificatory and formal levels, no pretest was necessary in this research. The posttest - only control group design was appropriate as a basic design for answering research question one.

To acquire a sufficiently large number of grade 6 students, subjects were taken from two schools; these schools had to be taken from different school divisions. Stratified random assignment to treatment groups in each classroom, and according to student's sex and mathematical ability, along with random assignment of treatments to groups, served to neutralize the effects of any differences within groups and between groups.

To answer research questions two and three, it was necessary to extend the basic design into a treatment X math ability factorial design. Since the schools involved in the research were from different school divisions, it was also informative to extend the basic design into a treatment X school design in order to determine whether the school factor influenced the results. This is in accordance with the suggestions of Keppel (1982), who has stressed the importance of analyzing the effects of control factors used in an experiment; unanalyzed control factors may contribute to error variance.

The school factor did not constitute a crucial aspect of this research. Hence, these two designs (the treatment X math ability and the treatment X school) were analyzed separately for each dependent variable in order to avoid meaningless interactions.

Instructional materials reflecting the different treatment conditions had to be designed. Merrill and Tennyson (1977) and Tennyson et al. (1981) have provided guidelines for concept lesson design based on rational sets, which had to be adapted for the concept parallelogram; material involving randomly arranged sets were then designed, controlling for extraneous factors. A pilot study served as a trial for instructional materials, and appropriate modifications were then made to instruments, and procedures for administration.

A posttest, with subtests measuring skills at the classificatory and formal levels of concept attainment, also had to be designed, since such a (standardized) test was not available for the concept parallelogram. Objectives and item types for designing such tests have been advanced be Klausmeier et al. (1974) for quadrilaterals, and Diluzio et al. (1975) for the equilateral triangle. Appropriate objectives and item types had to be selected and adapted for the design of the posttest in this research. Kuder-Richardson formula 21 (K-R 21) internal consistency reliability coefficient was then estimated using data from the pilot study.

Analysis of Variance (ANOVA)

Choice of an ANOVA strategy for analyzing nonorthogonal designs, that is, designs with unequal cell sizes, must be given careful consideration. For such designs different interpretations could be attached to the usual null hypotheses tested in orthogonal designs.

Given a typical two-factor ANOVA design, the usual null hypotheses of interest are:

H_r: no difference in row main effects,
H_c: no difference in column main effects,
H_i: no interaction. (Herr & Gaebelein, 1978, p. 208)

In analyzing the data from a nonorthogonal design, one is concerned with some particular interpretation of H_r, H_c, and H_i. The sum of squares (SS) that will be used in testing each hypothesis will be determined by the choice of interpretations (Herr & Gaebelein, 1978, p. 210). Herr & Gaebelein have identified five choices of interpretations: a) the standard parametric (STP); b) each main effect adjusted for the other (EAD); c) hierarchical with rows first and columns adjusted for rows (HRC); d) hierarchical with columns first and rows adjusted for columns (HCR); and e) weighted means (WTM) (p. 210).

The STP was employed in this research because the hypotheses related to this strategy are the same as those tested in a design with equal cell sizes; that is, they do not involve weighted means (Herr & Gaebelein 1978, p. 212). Interpretation of the above hypotheses is therefore unqualified. The STP strategy tests the equality of unweighted means, and hence, was most appropriate for this research.

Computer program. The computer package associated with the Statistical Analysis System (SAS) includes four types of least squares sum of squares (Type 1, Type II, Type III, Type IV) for the analysis of nonorthogonal designs using the General Linear Models Procedure (Proc GLM) (SAS USER'S GUIDE, 1982). Type III sum of squares (SS) are based on the STP strategy, and hence, were appropriate for this research.

The Type III tests possess the following properties:

> 1. The hypothesis for an effect does not involve parameters of other effects except for containing effects (which it must involve to be estimable).
> 2. The hypotheses to be tested are invariant to the ordering of effects in the model.
> 3. The hypotheses are the same hypotheses that are tested if there are no missing cells. They are not functions of cell counts.
> 4. The SS do not normally add up to the model SS. (SAS USER'S GUIDE, 1982, p. 165)

All statistical tests were conducted at the .05 level of significance.

A Measure of Strength of Association

According to Maxwell, Camp, and Arvey (1981):

> All other things being equal, the measure of strength obtained for an effect decreases as other powerful effects are included in the design. (P. 530)

To answer research question three, it was necessary to obtain a measure of the strength of an effect relative to error variance rather than total variance. Maxwell et al. (1981) have recommended the use of partial omega squared; partial measures are not dependent on the strengths of other effects in the design.

Hence, use of partial omega squared with the data from the ANOVA summary table for the treatment X math ability design, provided an estimate of the strength of the effect of the treatment manipulation, independently of the effects of math ability. Keppel (1982) has suggested that estimates of the strength of association between treatment effects and a dependent variable may be meaningful even when results are nonsignificant. However, some statisticians are not convinced of the usefulness of such estimates when treatment effects are not statistically significant (Maxwell et al., 1981). Partial omega squared was to be computed only in the event of a significant treatment main effect.

Assumptions

The folloiwng assumptions were associated with the procedures employed in this research:

1. The generally low performance at the classificatory and formal levels of attainment of the concept parallelogram, established during the preliminary pilot work with students at the end of grade 5, was a true reflection of the weaknesses in the population/experimental sample. This was a plausible assumption since the research was conducted early in the grade 6 year, and little or no work in geometry had yet been started in the experimental classes. The parallelogram had not been taught.
2. The average level of mastery of the concept in treatment groups was the same following the prematerial instructional phase. Stratified random assignment of subjects to treatment groups, and random assignment of treatments to groups, served to neutralize the effects of the prematerial instruction. Hence, groups could be assumed to be balanced.
3. Subjects were not exposed to other work involving the concept parallelogram throughout the duration of the experiment.
4. Three subjects were lost during the course of the research. Since the loss of these subjects was not related to the treatments, such loss could be assumed to be random loss. Hence, the randomized nature of the treatment groups was not affected.

The following assumptions are associated with the use of analysis of variance in the statistical analysis of data (Keppel, 1982, pp. 85-87):

5. The individual treatment populations are normally distributed.
6. The variances of the different treatment populations are homogeneous.
7. The error components are independent both within groups and between groups; that is, each observation is in no way related to any other observation in the experiment.

Independence is usually obtained by random assignment to treatment groups (Keppel, 1982, p. 87). Random assignment was a feature of the research design employed in this investigation.

Limitations

Certain limitations should be noted.

Firstly, the study involved sixth-grade students from five mathematics groups in two schools, one from each of two school divisions in Manitoba. Since these schools were not randomly selected, caution would have to be exercised in generalizing conclusions to all sixth-grade students in the school divisions.

Secondly, the instructional and test materials were designed based on one geometric concept, the parallelogram, from the elementary school sixth-grade curriculum. Hence, there is no guarantee that a similar result would be obtained for other geometric concepts.

Thirdly, test items on the classificatory level and the formal level subtests only required students to select instances and attributes respectively. There were no items requiring subjects to produce (that is, draw) instances and attributes.

Fourthly, mathematical ability was measured by teacher ratings of subjects' general ability to perform at mathematical tasks. Since teacher ratings may not be as reliable as ratings based on standardized test results, conclusions which relate to math ability must be regarded as suggestive.

Finally, interpretation of partial omega squared as a measure of the strength of association between the treatments and the dependent variables, can only be made within the context of the research. Thus, for example, the value of partial omega squared may not be generalizable to research involving more than two methods of presenting concept instances, and for which the prematerial instructional phase was not a reflection of classroom experiences.

In a posttest only control group design, mortality or loss of subjects, is a potential threat to internal validity (Gay, 1981). However, because of the short duration of the research in any one school, random loss of subjects could easily be monitored. Such random loss was corrected for by the analysis of variance.

Organization of the Remainder of the Thesis

The remainder of the thesis is organized as follows:

In Chapter 2, the importance of concepts in the development of intellectual skills is discussed, with particular emphasis on the Conceptual Learning and Development (CLD) model. Strategies for instructional design are discussed. The CLD model is related to other general theories of concept teaching/learning.

Chapter 3 reviews related literature on the effects of nonexamples in concept teaching; particularly, as related to the effectiveness of rational sets of teaching examples and nonexamples. In Chapter 4, details of the population and sample, research design, the design of instruments, and data collection procedures are provided.

In Chapter 5, the data are described and analyzed; the results of the analysis are summarized in terms of the (null) hypotheses and research questions of this research. Chapter 6 provides a statement of the conclusions that have been drawn from the research, and the conclusions are related to the theory surrounding the effects of rational sets. Implications for the teaching of geometric concepts are advanced, and recommendations are made regarding the teaching of geometry and for further research.

Chapter 2

THE IMPORTANCE OF CONCEPT LEARNING IN THE DEVELOPMENT OF INTELLECTUAL SKILLS

This chapter presents a detailed description of the Conceptual Learning and Development (CLD) model and its related methodology as advanced by Herbert Klausmeier (Klausmeier, Sipple, & Frayer, 1973; Klausmeier, Ghatala, & Frayer, 1974; Klausmeier, 1976a, 1976b; Klausmeier, Allen, Sipple, & White, 1976). The CLD model is then compared and contrasted with selected aspects of theories of concept learning and/or intellectual development as perceived by other learning theorists: Ausubel, Gagné, Bruner, and Piaget. The nature and importance of concepts are examined. The purpose of the comparisons is to place the CLD model in perspective with other general theories of concept learning. The chapter also attempts to show that the general methodology underlying the CLD model is reflected in the theories propounded by mathematics educators regarding the teaching/learning of mathematical concepts.

The Conceptual Learning and Development (CLD) Model

Klausmeier, Ghatala, and Frayer (1974) define a concept as, "...ordered information about the properties of one or more things - objects, events, or processes - that enables any particular thing or class of things to be differentiated from and also related to other things or classes of things" (p. 4). Concepts are mental constructs which are learned by an individual in a manner which is influenced by previous learning experiences and his/her maturational pattern. These concepts assist the individual in relating to his/her environment. The learning of concepts is affected by the nature of the concepts themselves. Concepts that possess a large number of relevant attributes and a high degree of abstractness provide greater difficulty to learn. The difficulty in learning concepts is also a function of the defining rule (p. 214).

Concepts enable the individual to classify newly encountered objects or events. A concept that has been mastered can be generalized to all possible noninstances including those which closely resemble the members of the concept class. Not only does the acquisition of concepts enable the individual to identify examples and nonexamples of each particular concept, but according to Kalusmeier et al. (1974), it permits the individual to relate concepts to one another in terms of supraordinate - subordinate relationships.

Building on the results of longitudinal studies on concept attainment (Klausmeier et al., 1974; Klausmeier, Sipple, & White, 1973; Klausmeier, Allen, Sipple, & White, 1976), Klausmeier (1976a) has presented a model of conceptual learning and development which incorporates four levels of concept attainment: concrete, identity, classificatory, and formal. Figure 1 represents a diagrammatic representation of the CLD model.

According to Klausmeier (1976a), the CLD model provides a theoretical framework for research on conceptual development and learning as children progress through school. The model enables researchers to describe concept attainment ability in terms of the prerequisite mental operations required at different levels of attainment, and the external conditions and/or instructional considerations that facilitate learning at each level.

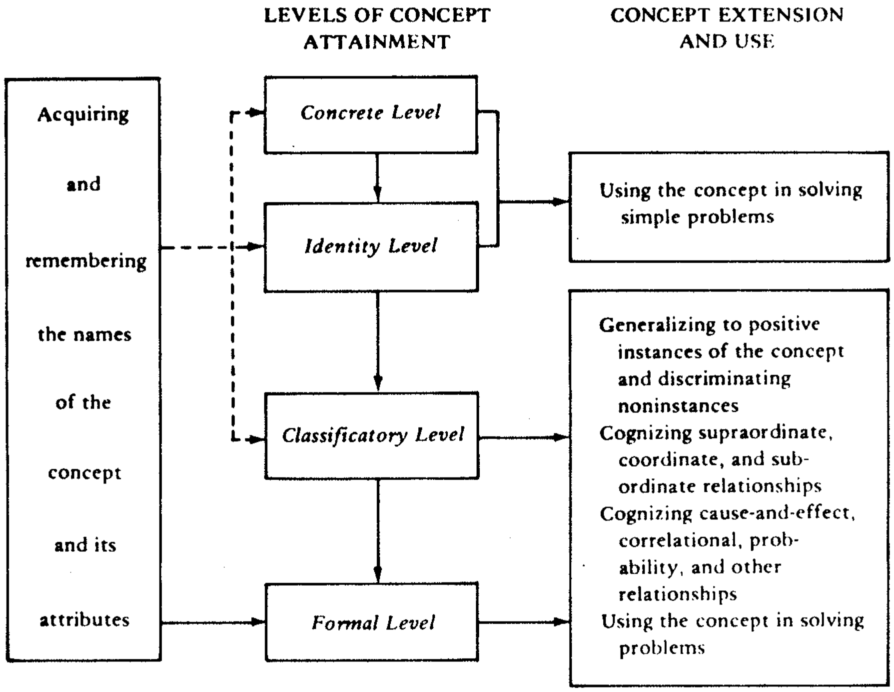

Figure 1. Levels of concept attainment, extension, and utilization. The solid lines and arrows indicate a prerequisite relationship. The broken lines indicate a possible but not prerequisite relationship. (From Klausmeier, 1976a, p. 6).

Concept Attainment

The four levels of concept attainment form an hierarchical system; that is, as the individual masters the prerequisite mental operations at a particular level and has attained the concept at a prior level, he/she progresses from one level to the next. The mental operations at a particular level are a function of maturation and learning. An individual's learning of concepts at a particular level is facilitated by instruction which specifically caters to the individual's level of conceptual development. Further, the extent to which an individual can use a concept is dependent on the level of concept attainment.

Concrete level. When an individual recognizes an object that has been previously encountered, one may infer that the concept has been attained at the concrete level. Attainment of a concept at the concrete level involves the following mental operations: attending to the features of the object, discriminating the object from other objects, and recognizing the object as the same one previously encountered (Klausmeier, 1976a, p. 8).

Identity level. Concept attainment at the identity level may be inferred when a person recognizes an object as the same one encountered on a prior occasion when the object is observed from a different perspective or sensed in a different modality. At the identity level the individual is also involved in attending, discriminating, and remembering as at the concrete level. However, apart from merely discriminating different forms of the same object, concept attainment at the identity level requires that the learner generalize the forms of the particular object as equivalent (Klausmeier, 1976a, p. 9).

Classificatory level. At the classificatory level the individual should be able to generalize that two or more things are alike in some way. At the lowest level of mastery within the classificatory level, at least two different instances of the same concept class are treated as equivalent; however, the individual cannot give the basis for their classifications. At a higher level of mastery the individual can classify a large number of instances as examples or nonexamples of a concept even though he/she still may not be able to define the concept or explain the basis for their classifications in terms of the critical attributes of the concept class (Klausmeier, 1976a, pp. 10-11).

Formal level. One may infer that a concept has been attained at the formal level when an individual can name the concept, name its critical attributes, accurately classify instances as being examples or nonexamples of a concept class, and evaluate instances in terms of the presence or absence of the defining attributes. Concept attainment at the formal level typically involves the following kinds of mental operations and strategies (Klausmeier, 1976a, pp. 11-13): acquiring and remembering the concept name, acquiring and remembering the names of the attributes, acquiring the prior mental operations at the classificatory level, and discriminating the attributes of the concept.

Both inductive and deductive operations are involved. Inductive operations include hypothesizing the relevant attributes and/or rules, cognizing the common attributes and/or rules of positive instances, remembering hypotheses, and evaluating hypotheses using positive and negative instances. Deductive operations include assimilating the concept name and definition, as well as verbal descriptions of examples and nonexamples, remembering the verbal material, and evaluating actual or verbal examples and nonexamples in terms of the presence or absence of the critical attributes.

Principles of conceptual development. Research conducted by Klausmeier and his associates involving the concepts equilateral triangle, noun, tree, and cutting tool have led to the following principles underlying conceptual development (Klausmeier, 1976a, pp. 17-22):

1. Concepts are attained at four successively higher levels in an invariant sequence. Five general patterns have emerged from the research: a) The individual may fail the test at all four levels of concept attainment; b) pass at the concrete level and fail at the other levels; c) pass at the concrete and identity levels but fail at the classificatory and formal levels; d) pass the test at the first three levels and failed at the formal level; and, e) pass at the four levels.

2. The attainment level of any given concept varies among children of the same age.

3. The same children attain various concept at different rates.

4. Use of concepts in understanding supraordinate-subordinate relationships, understanding principles, and solving problems improves as concepts are learned at successively higher levels.

5. Concept attainment at the various levels and the three uses listed in number four above, are facilitated when the individual has the name of the concept and its attributes.

Research conducted by Rampaul (1976) has found that scores on the CLD batteries for the concepts noun, tree, and equilateral triangle correlate highly with scores on other standardized tests involving concept acquisition. Rampaul found a high positive relationship between the CLD measures and measures of school achievement. As noted by Klausmeier (1976a), the knowledge about conceptual learning and instruction which has emerged from the research is sufficiently complete to provide useful guidelines for the design of instructional materials to effectively teach concepts.

Instructional Design

Based on a review of related research, Klausmeier (1976b) has described a general design of instruction associated with the CLD model. This design elaborates on suggestions advanced by Klausmeier et al. (1974). Eight basic steps are involved in conducting a concept analysis for instruction (Klausmeier, 1976b, pp. 192-203):

1. Outline the taxonomy or the hierarchy which incorporates the target concept. Figure 2 illustrates a concept taxonomy for the concept parallelogram.

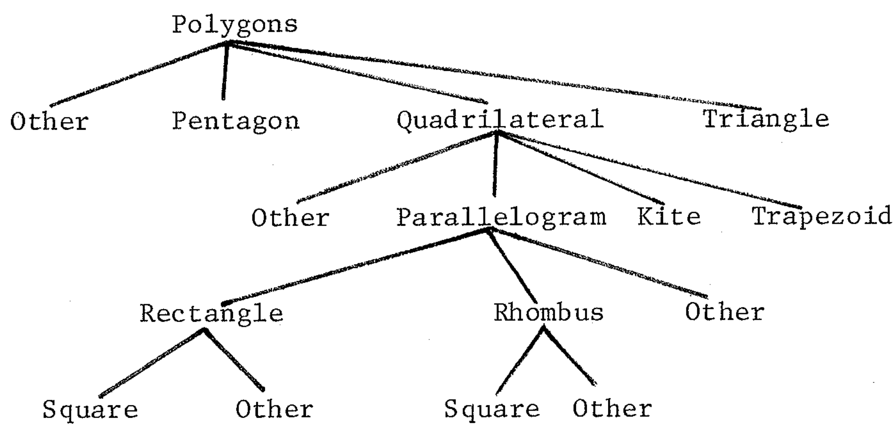

Figure 2. Concept taxonomy for a parallelogram. The superordinate concept is quadrilateral. Coordinate concepts are parallelogram, kite, and trapezoid. Subordinate concepts are rectangle, rhombus, and square.

According to Klausmeier (1976a), a taxonomy differs from an hierarchy in that a taxonomy specifies inclusive-exclusive relationships among classes of things, whereas an hierarchy specifies an ordered relationship among things based on such principles as importance, dependency, or priority. In an hierarchy the order in which an entity occurs in the hierarchy determines when it should be taught; the assumption is that the learning of new material depends on mastery of previous material in the hierarchy. Klausmeier (1976b) and Merrill and Tennyson (1977) suggest that a taxonomy does not necessarily imply prerequisite relationships. A concept or a coordinate set of concepts in a taxonomic relationship can be taught without teaching the subordinate concepts or more than the immediate superordinate concept. The other related concepts in the taxonomy, however, serve to identify defining and variable attributes which can be built into the design of instructional materials.

2. Define the concept.

3. Specify the defining attributes. For example, the defining attributes of a quadrilateral are: four sides, plane, closed, and simple.

4. Specify the variable attributes. Variable attributes are possessed by only some members of the concept class and hence, are irrelevant for defining the concept. Variable attributes of a quadrilateral include: all sides equal, contains four right angles, opposite sides parallel and equal, size of figure, and orientation on page.

5. Select illustrative examples and nonexamples. Both examples and nonexamples are necessary in concept teaching. Klausmeier suggests that at least one rational set of examples and nonexamples is required to prevent errors of overgeneralization, where some nonexamples of the concept are identified as examples; undergeneralization, where examples of the concept are identified as nonexamples; and misconception, in which some examples are identified as nonexamples and some nonexamples are identified as examples (Klausmeier et al., 1974, p. 193).

6. Identify illustrative principles. An example of an illustrative principle involving the concepts rectangle, area, length, and width is: The area of a rectangle is the product of the length and the width.

7. Formulate illustrative problem-solving exercises.

8. Develop a vocabularly list. The vocabularly list would include the name of the concept and the key terms of the critical attributes. How the vocabularly list is used during instruction is, of course, dependent on the level of concept attainment desired.

Higher level mental operations at the classificatory and formal levels may be developed through printed instructional material. Klausmeier (1976b, pp. 207-210) has suggested the following guidelines which should be incorporated into printed instructional materials to facilitate concept learning. These guidelines are based on the trends emerging from research on concept teaching:

1. Establish an intention to learn concept.

2. Elicit student verbalization of the concept name and the defining attributes based on a predetermined vocabulary list.

3. Present a definition of the concept in terms of defining attributes stated in vocabulary appropriate to the target population.

4. Present at least one rational set of properly matched examples and nonexamples of varying difficulty level.

5. Emphasize the defining attributes of the concept by drawing the student's attention to them.

6. Provide a strategy for differentiating examples and nonexamples.

7. Provide for feedback concerning the correctness and incorrectness of the responses.

According to Klausmeier, the research evidence relating to the above guidelines/procedures appears sufficiently strong to support the position that these procedures can be applied to concept teaching in various curricular areas.

Comparing the CLD Model with other Theories of Concept Learning

The major question of interest here is, to what extent does the CLD model and its related strategies correspond to other theories of concept learning? In this section, the CLD model is compared and contrasted with the theories of Piaget, Bruner, Ausubel, and Gagné. In each case, selected aspects of the respective theory would be first discussed, and aspects of each theory would then be related to the CLD model.

Piaget's Stage Theory

To Piaget, the basis of all learning is the child's own activity as he interacts with his physical and social environment (Good & Brophy, 1980, p. 43; Labinowicz, 1980, p. 34). The child does not absorb knowledge passively from the environment, neither is knowledge being formed in the child's mind, ready to emerge as the child matures. Instead, the child constructs knowledge through the interactions between his mental structures and his environment.

Concepts are related to each other and organized into structures called "schemas" or patterns of behavior (Good & Brophy, 1980, p. 45). Concept formation involves the processes of adaptation, organization, assimilation, and accomodation. The process of concept learning begins with the learner possessing some specific structure or pattern of thinking. Some external disturbance or intrusion creates conflict and causes disequilibrium. The individual's intellectual activity operates to compensate for this disturbance and solve the conflict. The result is a new way of thinking and structuring things; that is, a new concept has been formed. A state of new equilibrium has been attained. This process is referred to as equilibration (Labinowicz, 1980).

Piaget has advanced four stages of mental growth through which the child passes. These four stages may be described briefly as follows (Good & Brophy, 1980): 1) The sensorimotor stage (0-2) years, where schema development is totally dependent on learning through the senses as the child interacts with the environment; 2) the pre-operational stage (2-7 years), in which the child begins to internalize his sensorimotor schemas (behaviors) in the form of cognitive schemas, that is, thought; 3) the concrete-operational stage (7-11 years), the child has to depend on concrete experiences to facilitate his thinking and reasoning as he learns concepts; and 4) the formal-operational stage (11 years +), the child becomes capable of thinking logically and rationally about concepts and can use concepts and related principles to solve abstract problems. The child is now capable of organizing information and reasoning scientifically by formulating hypotheses and testing them.

Thus, a child's mental structures undergo quantitative as well as qualitative changes. In seeking to explore and understand the world around him, the perpetual functioning process of the child will generate mental structures and these structures will develop and change the child's growth. Ways of acting and thinking are changed continually as new mental structures emerge from old ones through the process of accommodation. It is the type of mental structure that a child possesses which characterizes the stages of development (Adler, 1977).

Piaget's stage theory may be closely related to the CLD model. Piaget's stages focus on the development of general abilities with qualitative differences in thought processes, which are very much dependent on maturation and social transmission. Klausmeier, on account of his interest in developing practical quidelines for teachers and curriculum developers, has specified cognitive skills at each of the four stages; and these specifications could be used to develop instructional and/or curriculum materials, and assessment tools (Klausmeier et al., 1974; Klausmeier, 1976a, 1976b).

The theories are similar in that both are hierarchical, cumulative, sequential, and invariant. The movement from one level of concept attainment to the other, concrete-formal, results in a qualitative rather than merely quantitative change in the maturing individual's repertoire of classification skills, concepts, and principles. Klausmeier (1976a, p. 10) has noted that concept attainment at the classificatory level as described in the CLD model includes the following sequence of skills, as suggested by Inhelder and Piaget (1964): consistent sorting, exhaustive sorting, conservation of classes, knowledge of multiple-class membership, and horizontal classification.

Instrumental Conceptualism

To Bruner, a concept is an abstraction that represents objects or events having similar properties (Lefrancois, 1982, p. 98). Bruner sees a relationship between thinking and conceptualization; to conceptualize is to form or be aware of concepts, whereas to think is to relate and alter concepts. Conceptualization and categorization are synonymous terms. To categorize is to form concepts. Hence, a category is a concept.

A category may be viewed in two senses. Firstly, it is a representation of objects or events that have similar properties. For example, "quadrilateral" is a category. It is a category that represent polygons with four sides. Secondly, a category may be regarded as a rule. When objects are placed in the same category, one may infer that they are in some way equal (Lefrancois, 1982, p. 98). Categories can be described in terms of attributes and values. An attribute is a property of the concept class. It can vary from one object to another. Values are the variations that are possible for an attribute. For example, the attribute "solid" may refer to cuboids, cylinders, spheres, among others.

There are three basic category types: conjunctive, disjunctive, and relational (Bruner, Goodnow, & Austin, 1956, pp. 41-43). A conjunctive category is defined in terms of the joint presence of the appropriate value of several attributes. A disjunctive category is one which possesses any constituent (singly, or in combination) of the attributes of a particular class. The relational concept is one defined by a specifiable relationship between the defining attributes.

The process of categorization is very essential for human functioning (Bruner et al., 1956; Lefrancois, 1982): 1) Categorization reduces the complexity of human functioning. Categorization allows the individual to respond to different objects as though they are the same, thus simplifying the array of isolated objects or events that one may encounter. 2) Categorization permits the recognition of and classification of objects and events. An object or event is recognized only if it fits into some other previously learned categories. 3) Categorization reduces the necessity for constant learning. The individual can recognize new objects without any actual new learning taking place and can go beyond the information given (Bruner, 1957a, 1957b). That is, following the recognition of an object, the individual can then make inferences about the object based on the characteristics of the concept class.

Hence, categorization provides direction for instrumental activity. In recognizing an object, Bruner suggests that the learner first determines what behaviors (instrumental acts) are appropriate for it (Lefrancois, 1982, p. 101). Bruner's learning theory relating to concept learning may thus be described as "instrumental conceptualism" (Bruner, Olver, & Greenfield et al., 1966; Klausmeier, 1976a, p. 23).

Categories are usually at different levels of generality. Some categories are highly specific, while others are more generic and inclusive. For example, the category "polygons" is generic, whereas "square" is highly specific. A system which illustrates the relationships between generic categories and specific categories is called a coding system (Bruner, 1957a). "A coding system may be defined as a set of contingently related, nonspecific categories. It is the person's manner of grouping and relating information about his world, and it is constantly subject to change and reorganization" (p. 46). To remember some given specific concept, it is often sufficient to remember the coding system into which it fits. For example, knowing that a rectangle is a parallelogram enables the learner to remember that opposite pairs of sides are parallel and equal, since that is one of the characteristics of this coding system. Bruner suggests that generic codes are essential since they facilitate discovery, learning, retention, and transfer (Lefrancois, 1982, p. 101).

Bruner, Olver, and Greenfield et al. (1966) have identified three levels of representation associated with concept learning among children: enactive, iconic, and symbolic. These three levels of representation emerge sequentially as the child develops. The three levels of representation are also reflected in Bruner's methodology regarding concept teaching/learning (Bruner, 1960, 1966; Bruner, Olver, & Greenfield et al., 1966; Shulman, 1977).

At the enactive level, the child directly manipulates objects which illustrate the concept. At the iconic level, the child does not manipulate objects directly but works with mental images (pictorial representations) of the concept. Finally, at the symbolic level, the child works with symbols only. Bruner has emphasized that the three levels or systems of representation are parallel and each is unique, but all are also capable of partial translations one into the other (Bruner, Olver, & Greenfield et al., 1966, p. 11). Bruner suggests that this has implications for cognitive growth. Disequilibrium occurs when two systems of representation do not correspond; for example, what one sees with how one says it, or how an individual is required to act and how he/she perceives the world around him/her. It is when such contradictions occur between systems of representation that the child makes revisions in his/her approach to solving problems involving the concepts under consideration.

Bruner's three levels of representation may be broadly related to the CLD model as follows: concrete/identity - enactive, classificatory - iconic, and formal - symbolic. The three levels of representation - enactive, iconic, symbolic - emerge sequentially and are associated with the age and maturity of individuals. Moreover, classification on the basis of perpetual attributes does not disappear totally, though it is used more by younger children (Bruner, Olver, & Greenfield et al., 1966). Individuals at all age groups classify to some extent on the defining attributes of concepts. This is consistent with the CLD model in that some children at grades 3 and 4 have been able to attain the concept equilateral triangle at the formal level.

However, Klausmeier (1976a) points out that the cognitive operations identified in the CLD model appear to emerge with maturation and learning across a longer time interval than for Bruner's three levels of representation (p. 27). For some concepts, children have been observed to operate at the concrete level before the age of 2; while for other concepts, such as "noun", hypothesizing and evaluating (the formal level) is not found in most students until the age of 16.

Nonexamples play a crucial role in the methodology associated with the CLD model. Bruner, however, questions the usefulness of nonexamples in concept learning. Bruner et al. (1956, p. 62) and Bruner and Anglin (1973, p. 142) have suggested that negative instances place a strain on inference capacity, depending on whether the instance confirms or denies an hypothesis in force. A negative instance confirms the hypothesis in force when it is predicted to be negative; otherwise, it infirms the hypothesis. If the negative instance is denying an hypothesis in force, it necessitates that the hypothesis be changed. A considerable strain on memory results. Hence, according to Bruner, too many encounters with negative instances result in the learner adopting solutions procedures predominantly designed to reducing memory strain. Thus, Bruner and Klausmeier differ somewhat regarding the role of negative instances in concept teaching/learning.

Meaningful Learning

To Ausubel, concepts facilitate the acqusition of new concepts and the categorization of new sensory experiences (Ausubel, Novak, & Hanesian, 1978, p. 95). Existing concepts can be used cognitively to identify less self-evident representatives of a known generic class. Concepts can also be used cognitively in meaningful discovery learning, as exemplified by an individual's ability to: 1) solve the simpler kinds of problem-solving operations which merely require that the learner relate the problem to an already acquired general concept or proposition; and 2) solve more complex problems which require the learner to extend, elaborate, qualify, or reorganize existing concepts and propositions in order to discover means-end relationships.

The central concept underlying Ausubel's learning theory is the idea of meaningful learning, which occurs when new knowledge is consciously incorporated into previously acquired concepts and ideas. What the learner already knows is regarded as the most important single factor influencing learning. Once this is ascertained the child should then be taught accordingly (Novak, 1981).

According to Lefrancois (1982, p. 105), only occasionally does Ausubel use the terms "concept" or "idea". Instead, Ausubel uses the term "subsumer". Basically, a subsumer is a concept. However, the term also implies that one concept incorporates or subsumes other concepts. Cognitive structure is defined by subsumers arranged in hierarchical fashion. The process by which subsumers are developed is referred to as subsumption. Subsumption is facilitated by the use of advance organizers. These refer to statements which are presented to students prior to the introduction of new material or concepts. Advance organizers specify what is to be learned and how it relates to previous learning.

Ausubel uses the term "criterial attributes" to describe the properties common to each member of the concept class, and which distinguish the members of that class from those of other classes (Ausubel et al., 1978, p. 89). Once the individual knows the criterial attributes, he/she would be able to judge whether an instance encountered is an exemplar or nonexemplar of the concept class.

Ausubel and Robinson (1969) and Ausubel et al. (1978) distinguish between concept formation and concept assimilation. Concept formation is the characteristic of a young child's inductive and spontaneous acquisition of generic ideas (Ausubel et al., 1978, p. 93). The criterial attributes are discovered inductively through the child's concrete experiences. For example, a young child who has experienced many varied encounters with cubes which vary in size, color, and texture, would discover inductively some of the criterial attributes of the cube. Older children, adolescents, and adults acquire new concepts through a process of concept assimilation. Concepts are learned when the criterial attributes are presented to the learner by definition, or when the attributes are encountered in context. New conceptual meanings are subsumed with other concepts already in the cognitive structure (Ausubel et al., 1978, p. 94).

This notion of concept assimilation is closely related to the higher levels of concept attainment in the CLD model. When a concept is learned deductively at the formal level, that is, when the individual is given all the essential information for concept learning, this is in effect concept assimilation. Before the new meaning is mastered, the learner is mentally involved in the processes of abstracting, differentiating, generating and testing hypotheses, and making generalization (Ausubel et al., 1978).

One instructional strategy that has emerged from Ausubel's learning theory, and which corresponds to the notion of a concept taxonomy as advanced by Klausmeier (1976b), is the strategy of concept mapping (Novak, 1981, p. 7). Concept maps require students to organize concepts such that the most general, most inclusive (superordinate) concepts are placed at the top, and more specific (subordinate) concepts arranged in two or more levels below the superordinate concepts.

Ausubel et al. (1978) have stressed that the relevant dimensions of a concept should be made quite salient; this facilitates concept acquisition. A similar rationale justifies the emphasis on rational sets of teaching examples and nonexamples emphasized in the CLD model (Klausmeier, 1976b). According to Ausubel et al., if irrelevant information is made too salient, it may have detrimental effect on concept acquisition. Concept acquisition is made more complicated because the task of identifying relevant criterial attributes is increased.

Ausubel is not convinced of the usefulness of nonexamples in concept learning. Ausubel et al. (1978) cite research evidence to support the contention that a sequence of varied positive instances is more effective in enhancing concept acquisition than when negative instances are included. Positive instances convey more explicit information and place a smaller burden on memory than negative instances. According to Ausubel et al., most learners show a disinclination to abstract or process the information presented in negative instances. This position is different from that of Klausmeier (1976b) who proposes extensive use of nonexamples in the design of instructional materials to teach concepts.

Learning Hierarchies

According to Klausmeier (1976a), concept attainment at the concrete and identity levels corresponds to what Gagné (1970) calls multiple discriminations; that is, discrimination learning. In Gagné's learning theory, concept learning and/or rule learning form part of an hierarchical structure involving eight varieties of learning (Gagné, 1970): signal learning, stimulus-response learning, chaining, verbal association, discrimination learning, concept learning, rule learning, and problem solving. The skills at each level are prerequisites for the subsequent level, with problem solving being the highest level of learning. Gagné (1977) has explained this cumulative learning as follows:

> The most prominent implication...is that acquisition of new knowledge depends upon the recall of old knowledge...Learning of any particular capability

> requires the retention of other particular items of subordinate knowledge...the design of an instructional sequence is basically a matter of designing a sequence of topics. (P. 168)

To Gagné (1970, 1977), the hierarchy specifies the order of instruction. Klausmeier (1976b) recognizes the usefulness of establishing an hierarchy in conducting a concept analysis; however, Klausmeier prefers to speak in terms of a concept taxonomy. It is assumed that as long as the related instructional procedures are adhered to, any concept within the taxonomy can be taught without first teaching the subordinate concepts, or after first teaching only the immediate superordinate concept (Klausmeier, 1976b; Merril & Tennyson, 1977).

According to Klausmeier (1976b, p. 193), Gagné's treatment of defined or relational concepts (rule learning) corresponds closely to the formal level of concept attainment in the CLD model. Gagné (1970, pp. 172-211) has distinguished between "concrete" concepts and "defined" or "relational" concepts. Concrete concepts can usually be defined merely by pointing to them; for example, concepts implied by names such as "color", "size", and "shape", or by objects such as "chair", and "lamp". Defined or relational concepts are abstract because they relate to one or more simpler concepts. For example, the concept "perpendicular" is a defined concept. The statement "Two lines are perpendicular when they intersect at right angles", represents a relation between two concepts, "line" and "right angle".

A relational concept may thus be regarded as a type of rule; the process of learning defined concepts is referred to as rule learning in Gagné's hierarchical system. The rule enables the learner to identify objects or events which are embodied by the relation. Hence, it involves skills and abilities similar to those advanced by Klausmeier (1976a) to describe the formal level of concept attainment. Klausmeier (1976b) has therefore provided instructional guidelines to assist children in learning rules, and tools for assessing when children have mastered the rules.

The CLD Model/Methodology Related to Theories of Learning Math Concepts

Many aspects of the theories of teaching/learning mathematics concepts propounded by mathematics educators reflect the general instructional methodology associated with the CLD model.

Dienes (1967) identified four principles underlying the teaching/learning of mathematical concepts: the dynamic principle, the constructively principle, the mathematical variability principle, and the perceptual variability principle. The mathematical and perceptual variability principles are particularly important here in that they emphasize the need for mathematics concepts to be presented in a variety of forms which embody the same conceptual structure. This corresponds to saying that examples should be as varied as possible. Dienes (1964) has also emphasized the need for students to be provided with a variety of situations to facilitate recognition of relevant and irrelevant attributes and discriminating exemplars and nonexemplars of the concept class.

Cooney, Davis, and Henderson (1975) have identified a system of verbal strategies for concept teaching which utilizes nonexamples. These strategies have been described as characterization moves, which mention characteristics or properties of the concept that enable students to find examples and nonexamples; and exemplification moves, in which the instructor names either objects denoted by the concept or objects not denoted by the concept (p. 107). Skemp (1971) also emphasized the need for a variety of examples and nonexamples, and suggests an ordering of concepts or a conceptual hierarchy: primary concepts, concepts which are derived from the child's sensory and motor experiences in the outside world; and secondary concepts which are abstracted from other concepts (p. 25). Skemp also mentioned the role of definitions, suggesting that they add precision to the boundaries of a concept once formed, and explicitly show its relation to other concepts (p. 26).

Other mathematics educators (Charles, 1980; Feinstein, 1979; Nasca, 1978) reflect aspects of the CLD model in their writings. Feinstein has exmphasized the need for instruction to enable students to see a pattern in what they are learning to aid discrimination, and for many and varied examples to be provided for students to encourage generalization, and the need for nonexamples to be included in students' experiences to aid discrimination and generalization. Nasca noted that, among other things, the concept-oriented classroom for young children must provide a range of opportunities for children to engage in discrimination tasks relevant to each concept, and encouraged the use of formal concept names, labels, and definitions by modifying the child's own personal description of classes of objects (p. 49).

Charles (1980) has identified four basic guidelines for selecting and subsequently using examples and nonexamples in teaching geometry concepts: 1) Identify the relevant and most frequently occurring irrelevant characteristics of the concept to be taught; 2) select a variety of examples; 3) select a variety of nonexamples; and 4) draw the students' attention to the relevant and irrelevant characteristics of the concept (pp. 19-20). These general guidelines suggested by Charles support the emphasis on using negative instances in the design of instructional material associated with the CLD model.

Conclusion

The theoretical framework and instructional considerations associated with the CLD model appear to closely correspond to general concept learning theories of other educators, as regards the nature and importance of concepts and the factors influencing concept acquisition. However, some differences have been identified, particularly as related to the usefulness of negative instances in instruction. Ausubel and Bruner question the usefulness of negative instances, which play a crucial role in the instructional design associated with the CLD model. Notwithstanding, the CLD model provides a basis for understanding children's conceptual development and for operationally defining and measuring conceptual development. The model explains the mental operations associated with each level of concept attainment, and specifies the instructional conditions and considerations which tend to facilitate concept attainment.

The instructional procedures associated with the CLD model correspond quite closely to theories relating specifically to the learning of mathematical concepts; in particular as related to the potential usefulness of nonexamples in aiding discrimination and generalization. The instructional considerations underlying the model do not make the study of mathematics/geometry inappropriately formal. Lessons can be developed which potentially provide students with complete understanding of concepts. As noted by Charles (1980), careful thought in the selection and use of examples and nonexamples would enable instructional designers to provide sound instructional lessons not only on geometry concepts, but on any concept.

Chapter 3

RELATED LITERATURE

This chapter presents a review of the research related to the problem being investigated in this study. The purpose of the review is to identify the major trends emerging from the research regarding the presentation of examples and nonexamples of concepts and the factors which improve the effectiveness of instructional materials involving rational sets of concept instances. The chapter is arranged in three broad sections. The first section briefly reviews research involving positive and negative instances but which does not involve rational sets. The second section presents a review of research specifically involving rational sets of teaching examples and nonexamples. A summary of the major trends emerging from the research is provided at the end of the chapter, and the problem and hypotheses of this investigation are re-emphasized in the context of that research.

Research not involving Rational Sets

Research has often sought to determine whether concept learning is better facilitated by presenting students with a set of carefully selected positive instances only, or by providing students with a selection of both positive and negative instances. The results have not always supported the use of negative instances.

Chernick (1975) investigated the effects of the proportion of positive-negative instances and the number of instances presented in the learning of mathematical concepts from the fields of algebra, number theory, and topology. Three treatment conditions were used. One group was exposed to all positive instances, the second to all negative instances, and the third to a mixture of half positive and half negative instances. Subjects in each group were presented with one frame, two frames, or three frames, each frame consisting of four instances.

Chernick hypothesized that performance on achievement, recognition, and transfer tasks related to the mathematical concepts would be increased if the number of positive instances were increased. On analysis of the results, none of the differences between the means of the treatment groups were found to be significant at the five percent level.

Malo (1975) compared the performance in learning disjunctive concepts of students taught how to use information provided by examples and non-examples with a group receiving no such instruction. The study also investigated five exemplification strategies for teaching these concepts. The subjects were 192 college freshmen. Ninety received instruction via programmed booklets in the use of examples and nonexamples in learning disjunctive concepts. The control group consisted of 102 subjects who received no instruction involving examples and nonexamples.

The posttest measured 1) subjects' competencies in concept terminology, translation, and recognition; 2) subjects' abilities to classify and select examples and nonexamples of concepts; and 3) subjects' abilities of analysis, synthesis, and evaluation. No significant differences were found between the experimental and control groups. Malo suggested that one explanation for this result may have been due to the fact that subjects already knew how to use information provided by examples and nonexamples. The results further showed that for items requiring subjects to classify and select examples and nonexamples of the concept, the exemplification strategy which contained all positive instances appeared more effective than strategies which incorporated nonexamples.

Sheel (1981) found no significant differences between an experimental group receiving both positive and negative instances and the control group receiving positive instances only, for initial achievement and retention on "real-world" classroom concepts selected from an introductory calculus course. The subjects were 62 university students. There was also no significant interaction between the treatments and field-independence/field-dependence.

Other studies (Callentine & Warren, 1955; Davidson, 1969; Donaldson, 1959; Freiberg & Tulving, 1961; Hovland & Weiss, 1953; Smoke, 1933a, 1933b; Tagatz, Meinke, & Leinke, 1968) have also found no evidence to support the contention that a selection of positive and negative instances is more facilitative to concept learning than a selection of positive instances only.

However, there is much research evidence to support the usefulness of nonexamples in concept teaching/learning. Bourne and Guy (1968) investigated the effect of negative instances on 216 volunteer undergraduate college students. Concepts which involved a variety of rules were studied. Three treatment conditions were employed. Subjects were exposed to only positive, only negative, or a mixture of positive and negative instances. The effect of the type of conceptual problem, attribute identification versus rule learning, was also explored. The conceptual rule was either conjunctive, disjunctive, or conditional. Bourne and Guy found that the kind of conceptual problem affected performance in different treatment groups. However, subjects performed best on all rules when they were presented with the greatest variety of instances; that is, a mixture of positive and negative instances.

Shumway (1974) investigated the relative effectiveness of an instructional sequence of all positive instances and a sequence of positive and negative instances on ninth-grade students' acquisition of the concepts commutativity and associativity. The study also investigated whether the effects of one concept would transfer to the acquisition of another concept. Treatment conditions required subjects to classify a series of instances as positive or negative. There were four such treatment conditions; each series either contained all positive instances, or some combination of positive and negative instances of commutativity and associativity.

No significant differences were found between treatments for commutativity, and negative instances appeared to increase response time. There was a transfer effect due to the presence of negative instances which led to a significant difference in performance on the posttest for associativity. Shumway concluded that negative instances were essential in concept teaching/learning, and students became more skeptical when they were exposed to negative instances. Classroom teachers were encouraged to use negative instances during instruction.

The generalizability of the reported transfer effect due to negative instances is questionable since commutativity and associativity are not independent concepts. Further, this transfer effect was not demonstrated in follow-up research (Shumway & Lester, 1974). The fact that there was no discussion of the concept before (or after) presentation of the instances limits the applicability of these results to the teaching/learning situation in the classroom.

Not only have subjects receiving both positive and negative instances been found to perform significantly better than those receiving positive instances only, but Cook (1981) also found that subjects receiving both positive and negative instances had a significant improvement on attitudes toward mathematics, while the subjects receiving positive instances only did not. The potential usefulness of a selection of positive and negative instances in facilitating concept learning has been demonstrated by other researchers (Cohen & Carpenter, 1980; Frayer, 1970; Gage, 1977; Huttenlocher, 1962; Logan, 1976; Shumway, 1971, 1972; Shumway & Lester, 1974; Tennyson, 1973). The cumulative effect of such research is to emphasize that negative instances are essential to the learning of advanced concepts, particularly mathematics concepts.

Research on concept teaching has sought to identify techniques for improving the facilitative power of negative instances. One such line of research has advanced the notion of a "rational set" of teaching examples and nonexamples.

Research involving Rational Sets

A behavioral analysis of concepts by Markle and Tiemann (1969, 1970) has led them to propose a "rational set" of examples and nonexamples as essential for producing complete understanding of concepts. Markle and Tiemann (1969) noted that errors in the classification of concepts could be described as: 1) overgeneralization, when the subject classifies a nonexample as an example; 2) undergeneralization, which occurs when an example is classified as a nonexample; and 3) misconception, when items are incorrectly classified on the basis of an irrelevant attribute. A rational set of teaching examples and nonexamples was proposed as a technique for reducing such errors.

Woolley and Tennyson (1972) advanced a theoretical proposition which extended the initial analysis by Markle and Tiemann. Woolley and Tennyson identified three independent variables and four dependent variables. The dependent variables were correct classification, overgeneralization, undergeneralization, and misconception. The independent variables were: 1) probability levels of exemplars/nonexemplars, determined by the ease with which a pre-instructional population recognized the instances; 2) matching, which suggests that an exemplar and nonexemplar are matched when they share most of the same irrelevant attributes, and differ only in some relevant attributes; and 3) pairing, which could be divergent or convergent. Two exemplars in a sequence are said to be divergent when the exemplars differ as much as possible in terms of the irrelevant attributes. Two exemplars are convergent when they differ only slightly in the irrelevant attributes.

This theoretical proposition was investigated by Tennyson, Woolley, and Merrill (1972) using a sample of 76 undergraduate educational psychology university students. A poetry concept class, trochaic meter, was taught. The three independent variables were combined to predict the four dependent variables, which were measured using novel examples and nonexamples.

Tennyson et al. hypothesized that: a) If instances were arranged from high to low probability, divergent, and matched, then correct classification would occur; b) if instances had low probability, and no matching, there would be overgeneralization; c) if instances had high probability, convergent, and no matching, then undergeneralization would occur; and d) if instances were arranged from high to low probability, convergent, and no matching, then there would be misconception. On the basis of these hypotheses, four programs were developed. Subjects were given general directions and a definition of the concept. Self-instructional booklets were used to administer the learning programs.

Four error scores were obtained for each subject according to the predicted responses on the dependent variables. Each subject's grade point average was used as a covariate for each dependent variable. Significant results were found to support each of the hypotheses. Tennyson et al. (1972) suggested that these results showed that instruction does produce certain types of dependent variables which could be controlled by empirically based procedures. Probability rating of exemplars, matching of exemplars and nonexemplars, and pairing of exemplars to ensure divergency appeared to be crucial procedures in instructional design for concept teaching.

Tennyson (1973) replicated the Tennyson et al. (1972) study with seventh-grade students using an adverb learning task. Students responded according to the hypothesized outcomes. In a follow-up experiment, Tennyson (1973) removed the negative instances from the experimental material, and found that subjects did no better than the control group. Concept acquisition even with divergent exemplars arranged according to probability rating appeared to be incomplete without negative instances. These results supported the proposition that concept acquisition is promoted by appropriately manipulating positive and negative instances.

The effectiveness of rational sets of examples and nonexamples designed according to the above theoretical proposition was demonstrated on other concepts by other researchers; for example, Feldman (1972, 1975), Tennyson, Markle and Tiemann, and Swanson (cited in Klausmeier, Ghatala, & Frayer, 1974).

For coordinate concept learning, the nonexamples of one concept are examples of other concepts. Thus, according to Tennyson and Park (1980), a rational set for coordinate concept learning is comprised of examples representing all the coordinate concepts (one example from each concept). The examples within the rational set should be matched according to the similarity of variable attributes.

Very few studies have involved coordinate concepts. However, building on an adaptive design for concept teaching which utilizes computer programs advanced by Tennyson (1975), Tennyson, Tennyson, and Rothen (1980) have found that senior high school students who were taught coordinate concepts simultaneously performed significantly better than those who received the concepts successively or in clusters. Further, Park and Tennyson (1980) have shown that performance was above the criterion level when the presentation order of examples within the rational sets reflected a response - sensitive strategy. This strategy determined the presentation order according to the student's response pattern to the given example.

Designing Instructional Materials

Strategies for the design of printed instructional materials involving rational sets of concept instances have emerged from the literature. Variables which have been manipulated include verbal emphasis of relevant attributes, the role of definitions, the number of rational sets, and presentation form.

Nelson (1973) investigated the effect of the analytic-global cognitive style on the acquisition of geometry concepts presented through written lessons which did or did not contain verbal emphasis of the relevant attributes. Nelson also studied the effect of the reflective-impulsive cognitive style on the acqusition of the geometry concepts through written lessons presented in an expository or discovery format. The dependent variables were two operations at the formal level of concept attainment: discrimating attributes and inferring the concept.

The findings showed that analytic subjects performed better than global subjects and subjects studying the emphasis lessons performed better than subjects studying no emphasis lessons. These results were evidenced on questions which assessed the discrimination of attributes. Emphasis lessons did not benefit global subjects more than analytic subjects. Subjects studying expository lessons performed better than those studying discovery lessons, especially for questions which assessed inference of the concept. Expository lessons did not benefit impulsive subjects more than reflective subjects. These results support the proposition that printed instructional material for concept teaching should provide written cues to assist subjects in discriminating the relevant and irrelevant attributes of the concept class.

Research has sought to determine an optimal number of rational sets necessary for concept mastery. Klausmeier and Feldman (1973, 1975) found that fourth-grade students studying the concept equilateral triangle learned almost equally well from either a definition or one rational set, slightly more from a definition and one rational set, and significantly more from a definition combined with three rational sets. Lessons were administered in self-instructional booklets and were not preceeded or followed-up by any discussion of the concept; one cannot therefore determine the extent to which the findings have applications in classroom situations.

McMurray (1975) compared the effects of four instructional strategies on the learning of a geometric concept using 64 elementary and middle school mentally retarded subjects. The four strategies were: 1) wide variety and paired, four different rational sets of examples and nonexamples presented as matched pairs; 2) wide variety and single, four different rational sets with each instance presented singly; 3) narrow variety and paired, one rational set presented as matched pairs; and 4) narrow variety and single, one rational set of concept instances with each instance presented singly.

Subjects who received a wide variety of concept instances were found to perform significantly better on the acquisition test than those who received a narrow variety regardless of whether instances were encountered singly or as matched pairs. However, significantly fewer overgeneralization errors were made when instances were presented as matched pairs. No significant difference in overgeneralization errors were found between subjects who received a narrow variety of instances however they were presented. These results demonstrate the need for instructional materials designed to teach concepts to include a wide variety of concept instances presented in rational sets as matched pairs.

Steps that should be followed in the design and presentation of instructional lessons to teach concepts have been the focus of some researchers. McMurray, Bernard, and Klausmeier (1974) demonstrated how printed lessons can be designed to teach the concept equilateral triangle to facilitate the formal level of concept attainment. These lessons provided opportunity for students to read descriptions of the relevant attributes, study rational sets of examples and nonexamples, complete practice exercises, and later combine the concepts (attributes) into a definition of the concept equilateral triangle.

Based on an analysis of concept teaching advanced by Klausmeier et al. (1974), McMurray, Bernard, Klausmeier, Schilling, and Vorwerk (1977)

proposed that school-age children can attain subject-matter concepts at higher levels than are normally attained through the use of written instructional lessons which incorporate three basic steps.
1) Analyze the concept to be taught and provide the following elements; concept definition, relevant and irrelevant attributes, and examples and nonexamples; 2) specify the level at which the concept is to be attained and of the cognitive operations underlying the learning of a concept at a particular level; and 3) identify instructional strategies that facilitate attainment of the concept at a specifiable level of mastery.

McMurray et al. (1977) found that experimental lessons based on the above steps and designed to teach elementary students the concepts equilateral triangle and tree, enabled all the experimental subjects to attain the classificatory level. Further, the combined influence of the different strategies facilitated the acquisition of the cognitive operations underlying the formal level. Retention data also indicated that subjects exposed to such lessons remembered most of what they learned two months later.

In a study designed to determine whether presentation form affected concept acquisition, Tennyson, Chao, and Youngers (1981) compared, among other things, three methods of presenting instructional material:
a) expository, where examples and nonexamples were presented in rational sets with labels and statements clearly identifying examples and non-examples; b) interrogatory, with a list of questions based on the relevant attributes of the concept requiring subjects to identify examples and nonexamples; and c) expository-interrogatory, a combination of the first two. The dependent variables were the four levels of concept attainment: concrete, identity, classificatory, and formal.

The sample comprised 120 fourth-grade students and the concept taught was the equilateral triangle. The learning programs for the different treatment conditions followed the same format: general directions and information on subordinate concepts which were read aloud by the experimenter while subjects read silently, the learning task administered by self-instructional booklets, the posttest, and the retention test.

A multivariate analysis of variance showed a significant main effect for presentation form. The findings showed no significant differences in performance at the concrete, identity, and classificatory levels. However, the number of items at the classificatory level (3) may have limited statistical power. Significant differences were found at the formal level. The a posteriori test showed that subjects who were exposed to the expository-interrogatory material were better able to attain the concept at the formal level. Main effects due to differences in presentation form were also found on the formal level subtest in the retention test.

Tennyson et al. (1981) suggested that the expository-only form, in which subjects were exposed to rational sets, enabled subjects to develop a prototype of the concept; and the interrogatory-only form enabled subjects to match their prototype with newly encountered instances, thus developing the necessary generalization skills. Neither one of these forms in isolation greatly facilitated concept attainment at the formal level. The use of a combination of these presentation forms, however, enabled students to acquire the prototype in conjunction with skill development.

On the basis of the findings of their research and the results of previous research, Tennyson et al. (1981) proposed five steps in concept lesson design:

> 1. The taxonomical structure of the concept should be determined...The three levels of the concept's structure - superordinate, coordinate, and subordinate - should be analyzed for critical and variable attributes...
> 2. A definition of the concept should be prepared in terms of critical attributes, and a pool of examples should be prepared on the basis of critical and variable attributes...
> 3. The examples should be arranged in rational sets by appropriate manipulation of the attributes...
> 4. The presentation order of the examples within rational sets should be decided according to student response pattern...
> 5. The presentation form of the rational sets should include expository sets followed by interrogatory sets. (PP. 333-334)

The facilitative power of the expository-interrogatory presentation form has not been supported by other research (Suebsonthi, 1981). Suebsonthi found that third-grade students studying the concept regular polygon, showed no significant difference in performance when material was presented in an expository-inquisitory presentation form or an inquisitory-only presentation form. Suebsonthi also found that concept acquisition and retention were more greatly facilitated for subjects by a presentation of the best examples (examples varied in terms of critical and variable attributes) than by a presentation of a list of concept attributes. Subjects who received a list of attributes were able to recall the list of attributes, but were found incapable of inferring all the attributes.

Rational Sets versus Randomly Arranged Sets of Instances

The above related line of research tends to support the proposition that instructional material should include rational sets of examples and nonexamples in order to enhance higher levels of concept attainment. However, this writer has only been able to locate one research report in which the relative effectiveness on concept acquisition of rational sets and randomly arranged sets of examples and nonexamples were specifically investigated.

In a study designed to test the assumption that concept acquisition is facilitated by instructional design strategies that focus on the critical attributes of concepts, Tennyson, Steve, and Boutwell (1975) investigated two design strategies: a) sequence, organized versus random; and b) analytic explanation, a verbal statement presented with each instance which analyzed the presence or absence of critical attributes. Tennyson et al. hypothesized that the organized relationship would lead to significantly better performance than a random presentation of the same instances. Tennyson et al. also hypothesized that the addition of analytic information to the examples would further improve performance since it focuses attention on the critical attributes by relating the specific characteristic of the given example to the general rule properties of the concept.

The subjects were 87 undergraduate psychology university students. Subjects were randomly assigned to treatment conditions. The instructional objective required that the student would be able to classify a poetry selection as an example or nonexample of trochaic meter. Treatments were administered by computer teletypes. Subjects were given a booklet to read containing general directions for using the computer teletypes, a definition of the concept and some examples. After studying the booklet each student started his/her respective program.

Dependent variables were error scores on the posttest and time to complete the various materials. Error scores on the pretest were used as covariates. On the sequence main effect a significant difference was found between the organized and random groups' error scores. The students in the group receiving the analysis information made significantly fewer errors than those not receiving such statements.

For the time measure, the sequence variable was nonsignificant, but the groups receiving analysis statements spent significantly less time studying the material than those not receiving such statements even though there was more reading involved. The organized sequence group which received analytic material spent significantly less time than for the random sequence group which also received analytic material.

Tennyson et al. (1975) replicated the experiment using the concept RX_2 crystals. Results again showed that the organized sequence group made significantly less classificatory errors than the random sequence group. Tennyson et al. concluded that "the sequencing of instances by similarity or dissimilarity of stimuli can affect performance" (p. 826).

The findings of a somewhat related study by Cohen and Carpenter (1980) seem to contradict these results. Cohen and Carpenter investigated the relative effects of an instructional sequence of examples and nonexamples and a sequence of examples only on the acquisition of the concept semi-regular polyhedra. The study also investigated whether the order in which specific instances are presented has an effect on concept acquisition. The subjects were 54 high-ability geometry students. Subjects were randomly assigned to one of three treatment groups: 1) Control, received eight examples of the concept; 2) Experimental (E_1), received four examples followed by four nonexamples; and 3) Experimental (E_2), received four examples each matched with a closely related nonexample.

The posttest comprised 20 polyhedra which was to be classified as examples or nonexamples of semi-regular polyhedra. Mean posttest scores showed no significant differences between the treatment conditions. Cohen and Carpenter felt that some of the subjects did not attend sufficiently during the video-taped lesson used as the introduction; it is not clear whether the researchers recorded this fact before or after the non-significant findings. However, Cohen and Carpenter reanalyzed the data using 27 of the subjects; that is, 9 subjects per treatment. The results showed that for the reduced sample both experimental groups performed significantly better than the control group; but there was no significant difference between the two experimental groups, demonstrating that the order of presentation of the instances made no difference.

These results are somewhat contrary to what would be expected based on the results of the Tennyson et al. (1975) study. Even though Cohen and Carpenter did not compare rational sets with randomly arranged sets of examples and nonexamples, the experimental condition in which examples and nonexamples were matched according to similarity of irrelevant attributes did in effect involve rational sets. Hence, one would expect a difference in classification behavior between this group and the group receiving four examples followed by four nonexamples, even though the sequence of presentation of examples and nonexamples was not random.

Summary

A review of literature has shown that research findings concerning the usefulness of negative instances in concept teaching/learning have been mixed. Some studies have either found a sequence of positive instances only to be more effective than a sequence of positive and negative instances, or there was no significant difference between the two treatments. However, there is much research evidence demonstrating the potential usefulness of negative instances in concept teaching/learning; and such results appear to be particularly strong for mathematical concepts. The use of rational sets of teaching examples and nonexamples has been proposed as essential for exploiting the usefulness of non-examples in concept learning.

Investigations into the effectiveness of rational sets of teaching examples and nonexamples have demonstrated their effectiveness in facilitating higher levels of concept learning in a variety of curricular areas. Of the 20 studies cited which involved rational sets, 18 reported a significant effect attesting to the facilitative power of instructional material involving rational sets in developing higher levels of concept attainment.

The effects appear to be strongest when the rational sets are supported by an adequate concept definition emphasizing the relevant attributes of the concept, and when the instances within rational sets are divergent, matched and arranged according to difficulty levels. Materials which include a definition, about three or four rational sets, and verbal cues to assist in focussing attention on relevant attributes appear to facilitate higher levels of concept learning. There is evidence that the effectiveness of rational sets is enhanced when the material incorporates an expository-interrogatory presentation form. Such a format appears to provide subjects with a clear prototype of the concept and enables them to test this prototype on novel instances.

Despite the findings of the related line of research attesting to the facilitative power of rational sets, only one research report (Tennyson et al., 1975) has been located in which the effect of rational sets was directly compared to that of a sequence of varied but randomly arranged sets of examples and nonexamples. This research demonstrated the superiority of an organized sequence (rational sets) in reducing errors at the classificatory level of concept attainment, but did not assess concept attainment at the formal level, as defined by Klausmeier (1976a).

The Tennyson et al. (1975) study did not involve mathematical concepts. A related study (Cohen & Carpenter, 1980) has reported somewhat contradictory results. This study found that the order of presentation of negative instances of a slected geometric concept made no difference in concept attainment at the classificatory level.

The research has not clearly shown how/whether the effects attributed to rational sets interact with the mathematical ability of subjects. None of the studies reviewed directly investigated such interaction. However, rational sets have been found to improve learning of a poetry concept at the classificatory level when grade point average was used as a covariate.

Research investigating the effectiveness of negative instances, rational sets or otherwise, has tended to involve very little meaningful instruction or discussion of the concept before or after presentation of the instances. In many of the studies reviewed, subjects were merely presented with a definition of the concept and a description of the attributes, often read aloud by the researcher while the subjects read silently. The extent to which the findings of such research can be generalized to classroom situations may be questioned, since in a typical classroom situation teachers and students discuss concepts before (or after) using printed instructional materials.

The review of literature has therefore not clearly demonstrated the effectiveness of rational sets, as compared with varied but randomly arranged sets of examples and nonexamples, in improving mathematical concept attainment at both the classificatory and formal levels, when subjects have been provided with some opportunity to engage in actively searching out some of the relevant attributes for themselves. The review has also not provided any evidence as to how/whether the effects of rational sets may be related to the mathematical ability of subjects.

The present study was designed to fill these gaps in the research. On the basis of the general research evidence attesting to the effectiveness of rational sets, it has been hypothesized that material involving rational sets would be more effective in facilitating higher levels of concept attainment.

Chapter 4

DESIGN AND PROCEDURES

In this chapter, the research design, the development of instruments, and data collection procedures are described under the following headings:

A. Population and sample

B. Research design

C. The major stages of the research

D. Preliminary pilot work

E. The pilot study

F. The prematerial instruction

G. The treatments

H. The posttest.

Population and Sample

The target population consisted of sixth-grade students in two school divisions in Manitoba. The sample comprised all the sixth-grade students from two elementary schools, one from each school division: school Y and school Z.

These schools were selected because of the willingness of the respective principals and teachers to permit students to participate, and because the classroom teachers themselves also showed enthusiasm for the research. The total number of sixth-grade students enrolled in these schools was 108. School Y contained 71 subjects, arranged into three mathematics classes grouped homogeneously according to ability - high, average, and low. Class sizes were 27, 26, and 18 respectively. Each class was taught by a different teacher. School Z contained 37 sixth-grade students, arranged into two heterogeneous mathematics classes of 19 and 18 subjects respectively. The same teacher taught both mathematics classes.

The total sample consisted of 57 males and 51 females. The experiment was conducted during the first two weeks of November, 1983; hence, the majority of the students had only been in grade six for about two months.

Research Design

The basic design employed in this research was the posttest-only control group design, initially described by Campbell and Stanley (1966) and further explicated by Gay (1981) as shown in Figure 3. The representation in Figure 3 indicates that both groups were exposed to the independent variable.

$$\begin{array}{ccc} R & X_1 & 0 \\ R & X_2 & 0 \end{array}$$

Figure 3. The posttest-only control group design.
R represents random assignment to groups, X_1 and X_2 represent experimental and control treatments respectively, and 0 represents the posttest.
Note. From Gay (1981, p. 230)

In this study, the independent variable was the method of presenting concept instances, with the experimental treatment being rational sets of examples and nonexamples, and the control treatment being randomly arranged sets of examples and nonexamples. The dependent variables were the 1) classificatory and 2) formal levels of concept attainment, as defined by Klausmeier (1976a). The geometric concept taught was the parallelogram.

In each math class, following the completion of the introductory activities on day one, subjects were randomly assigned to treatments using a table of random numbers. The strata were mathematics class, student's sex, and mathematics ability. Experimental and control subjects in each class were then summed. The experimental group consisted of 54 subjects, 29 males and 25 females. The control group consisted of 54 subjects, 28 males and 26 females. Groups were randomly assigned to treatments.

Two subjects, one of each sex and from different groups, did not attend school on the day the treatments were administered; while one female subject from the experimental group did not turn up for the post-test. These three subjects were therefore dropped from the final sample. Since such loss of subjects was not due to the treatments, it could be regarded as random loss; hence, it did not affect the randomized nature of the experimental and control groups. The final sample, therefore, consisted of 105 subjects. The experimental group consisted of 52 subjects, 29 males and 23 females; while 53 subjects, 27 males and 26 females, comprised the control group.

Schools were reluctant to release confidential information on subjects regarding scores on mathematics tests; further, the schools used different standardized instruments. Hence, teacher ratings of mathematics ability had to be used.

Mathematics ability had to be included when assigning subjects to treatment groups in order to maintain consistency in the sampling procedure. In school Y, the three mathematics classes were homogeneous (high, average, and low) according to the teachers; random assignment in each of these classes therefore implied that math ability was being considered. Hence, in school Z where, according to the teacher, the two classes were heterogeneous, the teacher was asked to rate each student's mathematics ability using the general guide: high (75-100), average (50-75), low (below 50). The teacher's ratings were therefore intuitive. This method of rating was satisfactory since the same teacher taught both groups. It is recognized, however, that teacher ratings of mathematics ability may not be as reliable as ratings based on standardized test results.

The basic design was extended into two more extensive designs: 1) a nonorthogonal treatment X school factorial design, with school as a fixed blocking factor, and 2) a nonorthogonal treatment X math ability factorial design, with math ability as a fixed blocking factor. In both designs the treatment factor was fixed. The first design was employed to take into consideration the school variable which may have affected the results. Table 1 shows the cell sizes for the treatment X school design.

Table 1

Cell Sizes for the Treatment X School Design

Schools	Treatments		Total
	Experimental	Control	
Y	34	35	69
Z	18	18	36
Total	52	53	105

To accommodate the exploratory purpose of this research, that is, to determine whether there was evidence of an interaction between the treatments and math ability, mathematics ability was used as a blocking variable irrespective of school. Table 2 shows the cell sizes for the treatment X math ability design.

Table 2

Cells Sizes for the Treatment X Math Ability Design

Levels of Math Ability	Treatments		Total
	Experimental	Control	
High	19	20	39
Average	21	20	41
Low	12	13	25
Total	52	53	105

The cell sizes in Table 2 were obtained by taking the subjects from the respective classes in school Y and adding them to subjects from school Z according to the teacher's rating of each subject's math ability. For example, all the experimental subjects in the high ability class at school Y were added to the experimental subjects rated high ability by the teacher at school Z to obtain the total number of high ability subjects for the experimental group. The same procedure was used for the control group and for the other levels of math ability. Since the purpose of this aspect of the research was exploratory, the above procedure seemed adequate.

The Major Stages of the Research

Prior to commencement of the study, approval to conduct the research was obtained from the school divisions and the school principals. The study was approved by the Ethics Review Committee, University of Manitoba.

Preliminary pilot work was conducted with grade 5 students in school X, a metropolitan Winnipeg school, on June 10th, 1983. The purpose of this preliminary work was to determine students' level of understanding of three selected geometric concepts: equilateral triangle, quadrilateral, and parallelogram, on completion of grade five and to pilot some test material. Analysis of the findings of this phase led to a selection of the concept parallelogram as the focus of this research. Further preliminary work was conducted on July 13th, 1983, using a group of students age 10 to 14 years enrolled in a summer course in microcomputers at the University of Manitoba. The purpose of this exercise was to pilot a variety of instances of the concept parallelogram and to determine the levels of difficulty of the instances for use in designing the rational sets of examples and nonexamples.

The formal pilot study was conducted during the period October 8th-10th, 1983, at school X. This represented a trial run of the experiment. On completion of the pilot study, modifications were made to instruments and procedures.

Prior to the experimental period (September/October, 1983), discussions were held with class teachers regarding the purpose of the study, the rationale behind the design of instructional and test materials, the procedures to be employed in conducting the research, and the role the teachers would play at each stage. These discussions were conducted in order to reduce the effect of the teacher variable by standardizing the instructions and procedures to be employed; the teacher variable was further controlled by randomly assigning subjects to treatments in each classroom. A copy of the detailed written instructions and guidelines that were given to teachers regarding the sequence of activities may be found in Appendix B.

The experiment was conducted in two three-day intervals over a two-week period in November, 1983. No pretest was administered in this research. The results of the preliminary pilot work (June, 1983) indicated little substantial variation in the pattern of performance at the classificatory and formal levels at the end of grade five; the general pattern was one of very low performance at these levels. At the time of the experiment, no work on the concept parallelogram had yet been done at grade 6 in either of the schools involved in the research; in fact, little or no work in geometry had yet been covered by the grade 6 teachers in either school. Hence, assuming that the pattern of weaknesses established during the preliminary pilot work was a fair reflection of weaknesses in the population, stratified random assignment to groups could be assumed to neutralize the effects of whatever individual differences existed, thus balancing the groups. It was felt that administering a pretest would have introduced a confounding factor (pretest-treatment interaction) unnecessarily in the design.

During the period November 7th-9th, the experiment was conducted at school Z; during the following week, November 14th-16th, the experiment was conducted at school Y. In each school, the sequence of activities was as follows: day one, the prematerial instruction, which involved the students in introductory activities and discussion regarding the concept and its characteristics; day two, administering instructional materials, where students were randomly assigned to study instructional booklets designed according to the experimental and control treatments; and day three, administering the posttest. All instruments were researcher-developed.

In the following sections, detailed descriptions of each of the major stages of the research have been provided: preliminary pilot work, the pilot study, the prematerial instruction, the treatments, and the posttest.

Preliminary Pilot Work

Preliminary pilot work was conducted in two phases.

Phase One

This phase occurred in June, 1983. The purpose was to pilot some test material with fifty grade 5 students at school X, in order to gain some insight on the level of students' understanding of certain concepts at the end of grade 5. The results of this phase assisted the researcher in selecting an appropriate concept to be used with grade 6 students.

After examining the grade 6 math curriculum as outlined in the K-6 Mathematics Curriculum (1978) of the Manitoba Department of Education, the researcher selected three concepts for this preliminary investigation: the equilateral triangle, quadrilateral, and the parallelogram. Items were written at the four levels of concept attainment (Klausmeier et al., 1974). Item types were based on objectives advanced by Klausmeier et al. (1974) and Diluzio, Katzenmeyer, and Klausmeier (1975).

Items at the concrete level required subjects to examine an example of a concept on one page of the test booklet, and then on the following page to locate the drawing that looked closest to the first drawing among at least five other polygons. The correct choice had the same spatial orientation as on the preceeding page. Items at the identity level employed similar task and stimulus material as for the concrete level. However, the correct choice on the second page of each item was rotated so that it had a different spatial orientation than the drawing on the preceeding page. A total of 9 items explored skills at the concrete and identity levels for the three concepts.

Items at the classificatory level required subjects to identify a variety of instances of each concept as examples and nonexamples of the respective concepts. Items at the formal level explored students' ability to define the concepts, discriminate the relevant attributes of the concepts, and identify relevant and irrelevant attributes of the concepts. To accommodate a large number of items at the classificatory and formal levels, the items were arranged into two test forms and randomly distributed to the students. Each test form contained the 9 items at the concrete and identity levels; hence, all 50 students responded to these items.

The results of the tests have been tabulated in Appendix A. The general trend showed that the majority of students performed satisfactorily at the concrete and identity levels, but most students experienced considerable difficulty on items at the classificatory and formal levels for all three concepts. Hence, the researcher decided to focus this research on the classificatory and formal levels of concept attainment. The parallelogram was selected as the geometric concept to be taught because many studies involving rational sets had investigated the equilateral triangle (Klausmeier & Feldman, 1973, 1975; Klausmeier, Ghatala, & Frayer, 1974; McMurray, Bernard, & Klausmeier, 1974; Tennyson, Chao, & Youngers, 1981), and the quadrilateral had been used to illustrate item types by Klausmeier et al. (1974). The parallelogram had received much less research attention regarding rational sets.

Phase Two

The second phase of the preliminary pilot work was conducted in July, 1983. This entailed establishing difficulty levels for the instance pool of the concept parallelogram before arranging the rational sets. This procedure was found to be important by previous research (Tennyson, Woolley, & Merrill, 1972) and stressed in concept lesson design (Merrill & Tennyson, 1977).

A pool of instances of the concept parallelogram were developed by the researcher and administered to a group of students age 10 to 14 years for classification. The students were enrolled in a summer course in computers at the Faculty of Education, University of Manitoba. Examples in the pool were matched with nonexamples to form 16 divergent matched pairs. There were 12 other varied but unmatched instances. All the instances were then randomly arranged into a classification test. Subjects first read a definition of the concept parallelogram and then were required to classify each instance as an example or nonexample of a parallelogram.

Merrill and Tennyson (1977) suggest the following percentage ranges for determining difficulty levels: difficult (0-30), medium (30-70), and easy (70-100). This researcher did not find such a range in the classification scores; none of the instances had a difficulty level below 50 percent. Some of the children were older than the target population (grade 6 students) for this research; hence, the researcher retained the entire instance pool. The following levels were arbitrarily adopted for this research: difficult (below 65), medium (65-85), easy (85-100). These difficulty levels were taken into consideration when arranging pairs of instances within rational sets. An attempt was made to arrange pairs of items of similar difficulty level together.

The Pilot Study

The pilot study was conducted during the period October 8th-10th, 1983, at school X. There were 48 grade 6 students involved. The purposes of the pilot study were to: 1) determine the clarity and quality of the introductory activities/student worksheets, and guidelines for teachers covering the prematerial instructional phase; 2) evaluate the quality and clarity of the instructional materials, test, and general instructions to students; 3) obtain estimates of the time required to administer the materials; 4) obtain data to calculate internal consistency reliability coefficients for the classificatory and formal levels subtests; and 5) obtain estimates of power associated with the research.

On completion of the pilot study, modifications were made to instruments and procedures to be employed in the experiment. A discussion of the reliability of the posttest based on the data from the pilot study is presented below with the discussion of the posttest.

Power Analysis

Power refers to the sensitivity of an experiment to identify treatment effects. Statistical power is the apriori probability of correctly rejecting the null hypothesis (Cohen, 1973, p. 226). Although power is improved by other means, such as reducing random variation in treatment conditions, analyzing the effect of control factors used in an experiment, and reducing subject variability, Keppel (1982) has noted that the choice of sample size is the primary means by which statistical power is controlled in an experiment (p. 70).

In this research, only two schools were available to the researcher for the experiment; hence, the sample size was fixed according to the number of grade 6 students in the two schools. A consideration of power was therefore informative. Keppel points out that if the size of a treatment effect is not known to the researcher, a practical approach to estimating statistical power is to use the information provided by a pilot study. Hence, the data from the pilot study in this research was used to estimate statistical power.

Using procedures suggested by Keppel (1982), the power estimates associated with the subtests at the classificatory and formal levels were approximately .78 and .50 respectively. Further information on the power analysis may be found in Appendix E.

The Prematerial Instruction

On day one of the research in each school, subjects were involved in introductory activities and discussion involving the parallelogram and its characteristics. This phase of the research was referred to as the prematerial instruction (Klausmeier, Ghatala, & Frayer, 1974).

The purpose of this phase was to establish an intention on the part of the students to learn the concept parallelogram (Bruner, Goodnow, & Austin, 1956; Klausmeier, 1976b) and to make the instructional phase of the research a better reflection of actual classroom behavior; this was a limitation of much of the research in the literature involving rational sets (Cohen & Carpenter, 1980; Geeslin, 1974). The above objective was achieved by: 1) providing some opportunity for children to do exploratory work and actively engage in searching for some of the critical and variable attributes of the concept (Klausmeier, 1976b; Polya, 1977); and 2) ensuring that children could read the terms and had some understanding of the meaning of the terms which would be encountered when studying the instructional materials (Klausmeier et al. 1974).

The class teachers conducted this phase using lesson objectives and procedures supplied by the researcher. In this way, the activities in each classroom were standardized. Each class was taught as an entire group; thus, both experimental and control subjects were exposed to the same introductory activities. The teachers were told to focus their entire discussion on the examples provided on the worksheet, and not to discuss nonexamples of the relevant attributes of the parallelogram as this was a major objective of the instructional material. A copy of the lesson objectives and procedures for the prematerial instruction has been provided in Appendix B.

The Treatments

On the second day of the research in each school, subjects studied experimental and control materials. Prior to day two, the researcher secured a list of the names of the students in each class, and randomly assigned subjects to treatment conditions using a process of stratified random sampling with class, students' sex, and teacher rating of math ability as strata. Random assignment to treatments following the prematerial instruction served to neutralize the effects of the prematerial instruction across experimental and control groups; hence, it could be assumed that groups were equivalent at the commencement of the treatment conditions. Stratified random assignment also served to ensure that groups were equivalent on other important variables. Finally, random assignment to groups also served to neutralize the effects of the teacher variable.

The instructions to the students prior to their commencing studying the material, encouraged them to take their time when reading/working through the materials and to follow the sequence of numbers accompanying each diagram in the instructional materials. This was in an effort to ensure that subjects studying the rational sets would be more likely to identify similarities and differences between matched pairs of examples and nonexamples.

In studying the instructional materials, subjects were allowed enough time in which to study the material at their own pace. However, as a result of observations made during the pilot study, a minimum time of fifteen minutes was specified because the researcher felt that the material could not be adequately studied in less than fifteen minutes. Details of the instructions given to teachers (students) when administering (studying) the instructional materials may be found in Appendix B.

The Experimental Treatment

Subjects in the experimental group were required to study researcher-developed instructional material involving rational sets of examples and nonexamples of the concept parallelogram. The experimental material consisted of a definition of the concept, a list of the relevant attributes, four rational sets of examples and nonexamples presented in an expository format, and three practice/interrogatory sets each with four novel instances.

Each rational set consisted of four matched pairs of examples and nonexamples of the concept parallelogram. Examples were divergent (varied) in terms of the irrelevant (variable) attributes of the concept. The following were considered to be variable attributes: length of sides, size of angles, size of figure, orientation on the page, and thickness of sides (Sowder, 1980). Each nonexample differed from the corresponding example in at least one of the critical or defining attributes of the concept. The four rational sets were arranged in ascending order of difficulty; difficulty levels for pairs of instances were determined from the results of preliminary pilot work conducted in July, 1983.

The procedures used in designing the experimental material were adapted from procedures advanced by Merrill and Tennyson (1977) and Tennyson, Chao, and Youngers (1981). The steps involved in designing the material were as follows:

1. The taxonomical structure of the concept was first determined. The three levels of the concept's structure - superordinate, coordinate, and subordinate - were analyzed for critical and variable attributes.

2. A definition of the concept was prepared in terms of the critical attributes, and a pool of examples and nonexamples was prepared on the basis of critical and variable attributes.

3. The examples and nonexamples were arranged in rational sets by appropriate manipulation of the attributes.

4. The rational sets were presented in an expository format; that is, each instance was accompanied by a statement specifying whether it was an example or nonexample of a parallelogram, and nonexamples were explained in terms of the critical attributes of the concept. The material also contained interrogatory sets of instances (not necessarily rational sets) which presented subjects with a list of questions based on the relevant attributes of the concept, and required subjects to use these questions in classifying novel instances.

The above steps represented four out of five steps suggested by Tennyson et al. (1981), as required for concept lesson design. The omitted step suggested that the presentation order of examples and nonexamples within the rational sets should depend on the student's response pattern. This step was considered to be inapplicable to this research because it emerged from research involving the simultaneous teaching of coordinate concepts and requires an algorithmic/

adaptive design which involves use of a computer. This step has not been employed in printed instructional material. A copy of the experimental material has been provided in Appendix C.

The Control Treatment

Subjects in the control group were required to study researcher-developed instructional material involving randomly arranged sets of examples and nonexamples of the concept parallelogram. The control material consisted of a definition of the concept, a list of the relevant attributes, four randomly arranged sets of examples and non-examples presented in an expository format, and three practice/interrogatory sets each with four novel instances for subjects to practise their prototype of the concept. A table of random numbers was used to determine the sequence of presentation of instances in the randomly arranged sets.

Establishing control within the material. Keppel (1982) has emphasized that the only difference between conditions in an experimental study should be the independent variable. In this research, the independent variable was the method of presenting concept instances: rational sets of examples and nonexamples versus randomly arranged sets. Hence, every effort was made to ensure that the only difference in the instructional materials was the independent variable.

The following measures were taken to achieve this control:

1. The control method contained the same definition and list of attributes as the experimental material. A summary of research on concept teaching/learning (Tennyson & Park, 1980) has shown that the type of definition given affected concept learning.

2. The total number of instances in the control material was the same as in the experimental material (44); comprised of 20 examples and 24 nonexamples.

3. Presentation form was controlled. There were 32 instances in the rational sets; the same/similar instances were presented in the randomly arranged sets. The expository explanations accompanying specific instances in the control material were the same as the corresponding explanations in the experimental material. This was considered to be very important because there is research evidence (Frayer, 1970; Nelson, 1973; Tennyson, Steve, & Boutwell, 1975) to suggest that the type of verbal cues/analytic information used could affect concept learning. The three practice/interrogatory sets were the same for both groups.

4. All other features of the materials were the same; for example, the size of the instructional booklets (11 pages), and the colour of the paper.

These procedures for establishing control within the materials ensured that any observed differences could be more confidently attributed to the treatment manipulation. A copy of the control material may be found in Appendix C.

The Posttest

On day three of the research in each school, a posttest was administered. The posttest measured performance on two dependent variables: the 1) classificatory and 2) formal levels of concept attainment, as defined by Klausmeier et al. (1974) and Klausmeier (1976a). Subtests measuring these two levels of concept attainment were built into an instrument designed by the researcher. The objectives and item types for the subtests were adapted from objectives and item types advanced by Klausmeier et al. (1974) and Diluzio et al. (1975).

Table 3 shows the specifications for the posttest. A copy of the test may be found in Appendix D.

Table 3

Specifications for Posttest

Level of Attainment	No. of Items	Specific Items
Classificatory	6	1-6
Formal		
Discrimination & Naming	12	7-14, 19-22
Inferring relevant and irrelevant attributes	6	23-25, 27-29
Evaluating and Defining	6	15-18, 26, 30
Total	30	

Validity and Reliability

To determine content validity of the testing instrument, the specific objectives for each level of attainment were given to four mathematics educators at the Faculty of Education, along with incomplete tables of specifications. The judges were requested to identify the items in the test which in their opinion reflected the given objectives for each subtest. This exercise was done individually and the judges did not see the test specifications. Responses were received from only two of the judges; the classification of item types for each of these judges showed 100 percent agreement with the table of specifications.

An objective procedure was used in scoring the posttest. The classificatory level subtest contained 6 items. Items 1-3 were of a typical multiple choice format and were awarded one mark for a correct answer and no marks for an incorrect answer. Items 4-6 required subjects to classify a total of 26 instances as examples or nonexamples of a parallelogram; each instance was awarded one mark if correct. The subject's classification score was the number of correct classifications. Hence, the maximum possible score for the classificatory level was 29.

The formal level subtest consisted of 24 items of a typical multiple choice format and each item was awarded one mark if correct and no marks if incorrect. Hence, the maximum possible score for the formal level subtest was 24. The answer key for the posttest has been provided in Appendix D.

To estimate the reliability of the testing instrument, an internal consistency reliability coefficient was calculated using the scores obtained from the pilot study. Such a coefficient provided an index of the homogeneity of the test items for each subtest (Popham, 1981, p. 142). The Kuder-Richardson method, K-R21, was used to estimate internal consistency for each subtest: 1) classificatory and 2) formal levels of concept attainment.

The reliability coefficient was estimated over the entire set of test scores for the pilot sample (N=48). Table 4 shows the relevant statistics used in estimating reliability coefficients.

Table 4

Maximum Score (K), Means (M), and Standard Deviations (S) for Pilot Study Subtests

Subtests	K	M	S
Classificatory	29	26.02	3.72
Formal	24	13.83	4.71

Using these statistics, the K-R21 reliability coefficients for the classificatory and formal levels subtests were computed to be .84 and .77 respectively. A reliability coefficient for scores on the entire test was not calculated since overall scores were of no importance to this research.

Chapter 5

PRESENTATION AND ANALYSIS OF DATA

This chapter presents and analyzes the data obtained in the research. Descriptive statistics (means and standard deviations) are first presented for each dependent variable (the (1) classificatory and (2) formal levels of concept attainment), to describe the overall performance of the experimental and control groups. Further presentation of descriptive statistics, and statistical analyses, have been organized around the treatment X school and the treatment X math ability factorial designs respectively. An estimate of partial omega squared, as a measure of the strength of association between the treatments and the dependent variables, has been calculated using the data from the treatment X math ability analysis; this estimate has only been computed for the dependent variable which shows a significant treatment main effect. The chapter concludes with a summary of the results of the analysis.

Interpretation of the results of the analyses of variance performed on the data is based on the standard parametric (STP) strategy as described by Herr and Gaebelein (1978). The related computer program employed in performing the ANOVA for the treatment X school and the treatment X math ability factorial designs was SAS 82 Proc GLM with Type III sum of squares (SAS USER'S GUIDE, 1982).

The Overall Performance of Treatment Groups

Table 5 shows the means and standard deviations of the experimental and control groups for each dependent variable. The descriptive statistics are based on the total number of subjects in each group.

Table 5

Group Sizes (N), Means (M), and Standard Deviations (SD) for each Treatment Group and for each Dependent Variable

Treatment	Dependent Variable	
	Classificatory Level	Formal Level
Experimental		
N	52	52
M	27.33	16.44
SD	2.00	3.96
Control		
N	53	53
M	26.57	14.30
SD	2.38	4.84

Note. Maximum possible scores for the classificatory and formal levels subtests were 29 and 24 respectively.

Examination of Table 5 reveals that there was a relatively small difference in means (.76) between experimental and control groups at the classificatory level (27.33 versus 26.57); while the difference in means at the formal level (2.14) was much more substantial (16.44 versus 14.30). In each group, the standard deviation of scores on the formal level subtest was much larger than for the classificatory level, indicating wider variability in performance at the formal level. This variability appeared to be more pronounced in the control group.

The Treatment X School Design

To determine whether the school factor had any effect on the results, school was used as a blocking factor. The school factor was assumed to be fixed (rather than random) in order to improve the power associated with the statistical test (the F test). Further, the number of schools clearly did not reflect a random sample.

Statistical tests of significance were conducted at the .05 level of significance, based on the STP interpretation. The null hypothesis (H_o) and alternate hypothesis (H_1) tested for each effect, and for each dependent variable, may be stated as follows:

School main effect.	H_o:	There is no significant difference in school means.
	H_1:	School means are not equal.
Treatment main effect.	H_o:	There is no significant difference in treatment means.
	H_1:	Treatment means are not equal.
Interaction.	H_o:	There is no interaction.
	H_1:	There is an interaction effect.

Classificatory Level

Descriptive statistics relevant to performance on the classificatory level subtest are shown in Table 6. A comparison of the means in this

Table 6

Group Sizes (N), Means (M), and Standard Deviations (SD) for each School and for Treatments in each School on the Classificatory Level Subtest

	School	Treatment	
		Experimental	Control
	Y		
N	69	34	35
M	27.00	27.32	26.69
SD	2.43	2.21	2.58
	Z		
N	36	18	18
M	26.83	27.33	26.33
SD	1.80	1.53	1.91

table shows that the difference in means for both the treatment effect and the school effect was quite small on the classificatory level subtest. Analysis of variance was used to determine whether there was any significant difference in means. Table 7 shows the ANOVA summary table.

Table 7

ANOVA Summary Table for the Treatment (A) X School (B) Design for Scores on the Classificatory Level Subtest

Source	SS	df	MS	F
A	15.86	1	15.86	3.16
B	.69	1	.69	< 1
A X B	.78	1	.78	< 1
S/AB (error)	506.98	101	5.02	
Total	524.31	104		

None of the F ratios in Table 7 were significant at the .05 level of significance. Hence, the null hypotheses for the treatment and school main effects, and for the interaction effect, cannot be rejected for scores on the classificatory subtest.

Formal Level

Table 8 shows the descriptive statistics for scores on the formal level subtest. The difference in school means for scores on the formal level subtest was very small (15.51 versus 15.08). However, the differences in treatment means in each school were much larger (16.41 versus 14.63, a difference of 1.78; 16.50 versus 13.67, a difference of 2.83). The ANOVA summary table is presented in Table 9.

Table 8

Group Sizes (N), Means (M), and Standard Deviations (SD) for each School and for Treatments in each School on the Formal Level Subtest

	School	Treatment	
		Experimental	Control
	Y		
N	69	34	35
M	15.51	16.41	14.63
SD	4.49	4.03	4.73
	Z		
N	36	18	18
M	15.08	16.50	13.67
SD	4.66	3.82	4.99

Table 9

ANOVA Summary Table for the Treatment (A) X School (B) Design for Scores on the Formal Level Subtest

Source	SS	df	MS	F
A	126.04	1	126.04	6.22*
B	4.51	1	4.51	< 1
A X B	6.52	1	6.52	< 1
S/AB	2044.91	101	20.25	
Total	2181.98	104		

*$p < .01$

The school main effect, and the interaction, were again nonsignificant at the .05 level of significance. However, the F ratio for the treatment main effect was significant ($F=6.22$, $p < .01$). Hence, the null hypothesis for the treatment main effect must be rejected for scores at the formal level, in favor of the alternate hypothesis that treatment means were not equal.

Since the school main effect and the interaction were not significant at either the classificatory or formal level, the school factor can be omitted in further analysis of the data.

The Treatment X Math Ability Design

To facilitate the exploratory purposes of this research math ability was used as a blocking factor, with levels: high, average and low. F tests were conducted at the .05 level of significance, based on the STP interpretation. The null hypothesis (H_o) and alternate hypothesis (H_1) tested for each dependent variable were as follows:

Math ability main effect.	H_o:	There is no significant difference in means.
	H_1:	Math ability means are not equal.
Treatment main effect.	H_o:	There is no significant difference in treatment means.
	H_1:	Treatment means are not equal.
Interaction.	H_o:	There is no interaction.
	H_1:	There is an interaction effect.

Classificatory Level

Descriptive statistics for the classificatory level subtest are shown in Table 10. This table shows that the relative ranking of the experimental and control groups was constant at each level of math ability; however, the difference in treatment means at each level of math ability was less than 1.00. As would be expected, however, the means for high ability and average ability subjects were well above the mean for low ability subjects, suggesting that there might have been a main effect due to math ability.

Table 10

Group Sizes (N), Means (M), and Standard Deviations (SD) for each Level of Math Ability and for Treatments at each Level on the Classificatory Level Subtest

	Level of Math Ability	Treatment	
		Experimental	Control
	High		
N	39	19	20
M	27.85	28.16	27.55
SD	1.46	0.74	1.86
	Average		
N	41	21	20
M	26.90	27.33	26.45
SD	2.09	1.64	2.40
	Low		
N	25	12	13
M	25.60	26.00	25.23
SD	2.71	2.97	2.39

Table 11 provides a summary of the results of the analysis of variance performed on the data. The treatnment main effect was not significant, nor was the interaction effect. Hence, the null hypotheses

Table 11

ANOVA Summary Table for the Treatment (A) X Math Ability (M) Design for Scores on the Classificatory Level Subtest

Source	SS	df	MS	F
A	14.17	1	14.17	3.25
M	76.44	2	38.22	8.77*
A X M	.38	2	.19	< 1
S/AM	431.40	99	4.36	
Total	522.39	104		

*$p < .01$

for the treatment effect and the interaction cannot be rejected. The F ratio for the math ability main effect was significant ($F=8.77$, $p < .01$). Hence, the null hypothesis must be rejected in favor of the alternate hypothesis, that math ability means were not equal.

Formal Level

Descriptive statistics associated with scores on the formal level subtest are shown in Table 12. The differences in means for each level

Table 12

Group Sizes (N), Means (M), and Standard Deviations (SD) for each Level of Math Ability and for Treatments at each Level on the Formal Level Subtest

	Level of Math Ability	Treatment: Experimental	Treatment: Control
	High		
N	39	19	20
M	16.87	18.26	15.55
SD	4.36	3.35	4.77
	Average		
N	41	21	20
M	15.37	15.67	15.05
SD	4.14	3.73	4.50
	Low		
N	25	12	13
M	13.00	14.92	11.23
SD	4.50	4.13	4.08

of math ability were more evident at the formal level. Further, the ranking of the experimental group relative to the control group was the same at all ability levels. The differences in means seemed to be more striking for high ability subjects (18.26 versus 15.55, a difference of 2.71), and for low ability subjects (14.92 versus 11.23, a difference of

3.69); the difference in means for average ability subjects was much smaller (15.67 versus 15.05, a difference of .62). These differences suggest that there might have been an ordinal interaction (Keppel, 1982).

The summary table for the analysis of variance performed on scores on the formal level subtest is shown in Table 13. The treatment main effect ($F=7.56$, $p < .01$) and the math ability main effect ($F=6.19$, $p < .01$) were both significant. Hence, the null hypotheses for the

Table 13

ANOVA Summary Table for the Treatment (A) X Math Ability (M) Design for Scores on the Formal Level Subtest

Source	SS	df	MS	F
A	136.53	1	136.53	7.56*
M	223.60	2	111.80	6.19*
A X M	41.88	2	20.94	1.16
S/AM	1787.48	99	18.06	
Total	2189.49	104		

*p < .01

treatment and math ability main effects respectively must be rejected, in favor of the alternate hypotheses that treatment and math ability means were not equal. The interaction effect was not significant. Hence, the null hypothesis of no interaction cannot be rejected.

Strength of association. Since the treatment effect was significant at the formal level, an estimate of partial omega squared was computed using the data from Table 13. Partial omega squared provides an estimate of the strength of association between the treatment manipulation and scores at the formal level independently of the effect of math ability (Maxwell, Camp, & Arvey, 1981). Partial omega squared was found to be .06. Hence, the treatment manipulation accounted for 6% of the variance in test scores. A further explanation of partial omega squared is given in Appendix E.

Summary of Results

The results of the statistical analysis of data are summarized below. Each null hypothesis listed in chapter one is first stated, and is then followed by a statement concerning its rejection or non-rejection.

Hypothesis One

There is no significant difference in mean performance at the classificatory level between subjects who study rational sets of examples and nonexamples and subjects who study randomly arranged sets.

No significant F ratio was obtained for the treatment effect when school or math ability was used as a blocking factor. Hence, hypothesis one cannot be rejected.

Hypothesis Two

There is no significant difference in mean performance at the formal level between subjects who study rational sets of examples and nonexamples and subjects who study randomly arranged sets.

The F ratios for the treatment effect were significant both when school was used as a blocking factor ($F = 6.22$, $p < .01$), and when math ability was used as a blocking factor ($F = 7.56$, $p < .01$). Hence, hypothesis two must be rejected in favor of subjects who studied rational sets of examples and nonexamples.

Hypothesis Three

There is no significant interaction between the treatments and math ability for performance at the classificatory level.

No significant interaction effect was found for scores at the classificatory level. Hence, hypothesis three cannot be rejected. A significant main effect was found for math ability ($F=8.77$, $p < .01$).

Hypothesis Four

There is no significant interaction between the treatments and math ability for performance at the formal level.

No significant interaction effect was found for scores at the formal level. Hence, hypothesis four cannot be rejected. Since the treatment main effect was significant at the formal level, the nonsignificant interaction suggests that subjects who studied rational sets performed better than subjects who studied randomly arranged sets across all levels of math ability. A significant main effect was also found for math ability ($F=6.19$, $p < .01$).

Strength of Association

No estimate of partial omega squared was computed for performance at the classificatory level when math ability was used as a blocking factor, since the treatment F ratio was not significant. Partial omega squared computed using data from the formal level ANOVA summary table, was estimated to be .06; that is, about 6% of the variance in test scores at the formal level could be accounted for by the treatment manipulation, independently of the effects of math ability.

The School Factor

No significant main effect due to the school factor was found at either the classificatory level or the formal level. There was also no significant interaction between the treatments and the school variable on performance at either level of concept attainment. Hence, the school variable had no effect on student performance.

Chapter 6

CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

This chapter provides a statement of the conclusions drawn from the results of this research, and relates these conclusions to the theory surrounding the effectiveness of rational sets of teaching examples and nonexamples in facilitating higher levels of concept attainment. Conclusions apply only to sixth-grade students from the two Manitoba school divisions which provided the population for this research. Possible implications for the teaching of geometric concepts to grade 6 students are advanced, and recommendations are made concerning the teaching of geometry and for further research.

Conclusions

Based on the results of the analysis of data, the following conclusions are advanced:

1. There is no evidence from this study to support the contention that subjects who study rational sets of examples and nonexamples of a geometric concept perform better than subjects who study randomly arranged sets, at the classificatory level of concept attainment.
2. Rational sets of examples and nonexamples tend to better facilitate geometric concept acquisition at the formal level of concept attainment, when compared to randomly arranged sets.
3. There is no evidence of an interaction between the method of presenting concept instances and the mathematical ability of subjects, for performance at either the classificatory level or the formal level of concept attainment. Rational sets tend to facilitate concept learning at the formal level across all levels of mathematical ability (high, average, and low).

4. The treatment manipulation accounted for about 6 percent of the variance in scores at the formal level, independently of the effects of mathematical ability.

Discussion

The results support only certain aspects of the theory surrounding the effects of rational sets of teaching examples and nonexamples.

Firstly, the results do not support the conclusions of Tennyson et al. (1975) attesting to the superiority of rational sets over randomly arranged sets for the classificatory level of concept attainment. Instead, the results lend some support to the conclusion of Cohen and Carpenter (1980) that for a geometry concept the method of presenting examples and nonexamples makes no difference at the classificatory level.

Secondly, the significant result at the formal level of attainment of the concept parallelogram supports conclusions advanced by McMurray et al. (1977) and Tennyson et al. (1981) for the geometric concept equilateral triangle. These researchers also found that material involving rational sets of examples and nonexamples better facilitated concept acquisition at the formal level, than material which did not incorporate rational sets.

Finally, Tennyson et al. (1981) reported that for the concept equilateral triangle, the expository-interrogatory presentation form made no significant difference at the classificatory level, but generated a significant difference at the formal level. Since the general format underlying the experimental material in this research was the expository-interrogatory presentation form, then the results of this research tend to support the findings of the Tennyson et al. (1981) study.

One possible explanation for the nonsignificant result at the classificatory level may lie in the power estimate calculated on the basis of the results of the pilot study. The estimate of power obtained for the classificatory level subtest was .78; this was slightly below the .80 suggested as acceptable by Cohen (cited in Keppel, 1982). Random loss of subjects during the research also served to reduce power. Hence, a larger sample size may have produced a significant treatment effect at the classificatory level.

It must be noted, however, that because of the conditional interpretation associated with power estimates, power analysis recedes into the background in the presence of significant results (Cohen, 1973, p. 227). Therefore, the apparent low power for the formal level obtained during the pilot phase of this research, in no way detracts from the significant treatment effect found at the formal level.

Ausubel, Novak, and Hanesian (1978) have emphasized that the defining attributes of a concept should be made salient during instruction. In this research, this was the strength of the experimental material; the rational sets made the critical attributes of the concept more salient. One must conclude that the clarity and organized sequence of the instances in the rational sets better facilitated development of the discrimination skills so vital for concept learning at the formal level.

Examination of the estimate of partial omega squared provides a measure of how much practical importance can be attached to the effectiveness of rational sets at the formal level of concept attainment. Cohen (cited in Keppel, 1982, p. 92) has suggested a rough scale for interpreting such estimates: a value of .01 can be regarded as a "small" effect, a "medium" effect is .06, and a "large" effect is .15 or greater.

Partial omega squared in this research was found to be .06; that is, about 6 percent of the variance in test scores at the formal level could be attributed to the treatment manipulation, independently of mathematical ability. One may conclude, therefore, that under the conditions of the research, rational sets of examples and nonexamples produced a "medium" effect at the formal level, relative to randomly arranged sets. Further, since the interaction with math ability was not significant, one must also conclude that this "medium" effect operated across all levels of mathematical ability.

Implications

The results of this study hold important implications for the teaching of geometric concepts to sixth-grade students.

Firstly, material involving rational sets of examples and nonexamples can be effectively and meaningfully employed in the teaching of geometry concepts to grade 6 students. The prematerial instructional phase of this research ensured that the sequence in which the instructional materials were introduced into the unit of work reflected typical mathematics classroom situation; in effect, the prematerial instructional phase served to validate the results for mathematics classroom instruction. The "medium" effect of rational sets in promoting concept learning at the formal level, should make the time spent in designing such sets worthwhile to mathematics teachers and developers of geometric instructional materials.

Secondly, sixth-grade students of all levels of mathematical ability can acquire high levels of attainment of geometric concepts with proper instruction. Some students who studied rational sets of examples and nonexamples, and who were rated "low" in mathematical ability, were able to score very highly on the formal level subtest. Very often teachers tend to expose students of supposedly "low" ability to inferior content, claiming that such students cannot cope with high level tasks. The results of this research serve to question such practice.

In teaching a geometric concept to sixth-grade students, teachers need to spend some time preparing rational sets of examples and nonexamples of the concept, and discuss the instances in terms of the critical and variable attributes of the concept. With such instruction, students of all levels of mathematical ability can develop the generalization and discrimination skills necessary for higher levels of concept attainment.

Thirdly, to the extent that geometric concept instances in some textbooks are arranged in some random order, then too much dependence on such textbooks may be detrimental to the development of higher levels of concept attainment. The results of this research, and results of other studies (McMurray et al. 1977; Tennyson et al., 1975; Tennyson et al., 1981), suggest that such random arrangements may be unproductive in facilitating concept acquisition.

If the teaching/learning of geometric concept is to be improved, then in cases where textbooks do not appropriately present instances of geometric concepts, teachers need to become willing to supplement such textbook materials with other instructional materials which reflect ideas surrounding the design of rational sets of examples and nonexamples. Such instructional materials could then be incorporated into classroom discussions when teaching geometric concepts.

Recommendations

Recommendations are advanced below regarding the teaching of geometric concepts to sixth-grade students, and for further research.

1. In preparing to teach a geometric concept, the concept's structure should be examined for critical and variable attributes. A definition of the concept should then be prepared in terms of the critical attributes. A pool of examples and nonexamples should then be prepared and arranged in rational sets; to avoid extensive use of paper, charts and transparencies for overhead projectors may be employed in arranging rational sets.

2. In teaching a geometric concept, students should first be allowed to do exploratory work regarding the concept. Subsequent discussions among teacher and pupils should focus on the critical and variable attributes of the concept. Rational sets of examples and nonexamples should be presented to the students and discussed in terms of the critical attributes of the concept.

3. Textbook writers, and designers of curriculum/instructional materials, should employ as many of the ideas surrounding the design of rational sets as possible, when presenting geometric concepts.

4. In selecting textbooks, and other printed instructional materials, to be used in teaching geometric concepts, preference should be given to those textbooks/materials which more closely reflect the ideas surrounding the design of rational sets.

The following recommendations relate to further research.

5. This study should be replicated using other geometric concepts, in order to check the nonsignificant result at the classificatory level.

6. In this research, efforts were made to control the effects of the

premateria1 instruction. Hence, the same/similar activities were provided for both groups. It is conceivable, however, that the nature of the premateria1 instruction could interact with the method of presenting concept instances in promoting concept attainment behavior. Hence, instead of controlling the premateria1 instruction, a study should be conducted in which the nature of the premateria1 instruction serves as an independent variable to be manipulated along with the method of presenting concept instances.

7. The interaction between mathematical ability and the method of presenting concept instances on concept attainment behavior, should be further examined using a more reliable measure of mathematical ability; such as, performance on a standardized test. On the basis of this research, it is hypothesized that there would be no significant interaction; rational sets of examples and nonexamples would be more effective at all levels of mathematical ability.

8. Though the interaction was not significant, an interesting result was observed on the formal level subtest. The mean differences in performance between experimental and control groups for subjects of high and low ability, were substantially higher than the mean difference for average ability subjects. Further research involving the interaction between the method of presenting concept instances and the mathematical ability of subjects should investigate whether there is any evidence of this result.

9. Research comparing the effects of rational sets versus randomly arranged sets of teaching examples and nonexamples should also focus on other mathematical concepts besides geometric concepts. The notion of rational sets may be applied to any mathematical concept for which examples and nonexamples can be specified.

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APPENDIX A

Summary of the Results of the Preliminary Pilot Study: Phase One

Table A-1

Results on Items at the Concrete and Identity Levels of Concept Attainment

Number of Items Correct	Number of Students	Percent of Sample
9	27	54
8	9	18
7	8	16
6 or less	6	12

Note. 1) Maximum number of items = 9. 2) Total number of grade 5 students = 50.

Table A-2

Results on Items at the Classificatory and Formal Levels of Concept Attainment: Test Form A

Concept	Results for Selected Scores	
	Classificatory Level	Formal Level
Equilateral Triangle	No. of items/instances=3 24% (of students) scored 0 12% scored 1 64% scored at least 2	No. of items=3 28% scored 0 28% scored 1 44% scored at least 2
Quadrilateral	No. of items/instances=13 40% scored less than 7 60% scored at least 8	No. of items=4 68% scored less than 2.
Parallelogram	No. of items/instances=10 24% scored 0 48% scored below 6 52% scored at least 6	No. of items=10 24% scored 0 32% scored 1 88% scored 3 or less

Note. Number of grade 5 students taking Test Form A = 25.

Table A-3

Results on Items at the Classificatory and Formal Levels of Concept Attainment: Test Form B

	Results for Selected Scores	
Concept	Classificatory Level	Formal Level
Equilateral Triangle	No. of items/instances=3 48% scored 1 or less 52% scored at least 2	No. of items=2 60% scored 0 36% scored 1 4% scored 2
Quadrilateral	No items at this level.	No. of items=7 No students scored above 4 72% scored below 2
Parallelogram	No. of items/instances=17 40% scored below 4 64% scored below 10 36% scored at least 10	No. of items=10 No student scored above 6 64% scored below 2 96% scored below 4

Note. Number of students taking Test Form B = 25.

APPENDIX B

Instructions/Guidelines given to Class Teachers for the Prematerial Instruction and for Administering Instruments

SEQUENCE OF ACTIVITIES

Day 1: Prematerial Instruction

Day 2: Administer Instructional Materials

Day 3: Administer Posttest

DAY 1: PREMATERIAL INSTRUCTION

The following introductory activities and discussion would be organized/directed by the class teacher.

Objectives

The students will be able to identify the following characteristics of a parallelogram:

1. Parallelograms are 4-sided figures.
2. Each pair of opposite sides of any parallelogram is parallel and of equal length.
3. Parallelograms are plane figures.
4. Parallelograms are simple closed figures.

Procedure

1. Divide students into groups of 3. Give each group the worksheet with 4 examples of parallelograms. Ask the students to write down: (a) how they are alike, and (b) how they are different. Children may use any geometrical instruments they wish. Give students about 20 minutes.

Suggestions for assisting students. If students indicate that they do not know how to proceed, the teacher may instruct them to:

(a) Measure the sides of each figure to see which sides are of equal length; look to see which sides are parallel.

(b) Examine the angles to see which are equal and how the angles differ.

2. Conduct a large group discussion in which the teacher serves as a facilitator. Let students talk about their findings; that is, how the shapes are alike and different.

Suggestions for assisting students.

(a) The teacher should refer ONLY to the EXAMPLES ON THE WORKSHEET to reinforce or clarify the characteristics of the parallelogram as outlined in the objectives.

(b) The examples on the worksheet should also be used to draw students attention to irrelevant attributes - for example, the kinds/sizes of angles, the length of the sides, and orientation on the page.

NOTE. The teacher should NOT show nonexamples of any characteristic (the purpose of this study is to determine which type of instructional materials better clarifies the terms and assists children in discriminating nonexamples of the concept).

This section should last about 15 minutes.

3. At the end of the discussion, the major characteristics (see objectives) are to be listed on the chalkboard. Have students write down the characteristics on their worksheets.

4. All worksheets should be collected at the end of the session.

DAY 2: ADMINISTERING INSTRUCTIONAL MATERIALS

1. The name of each student would be written on an instructional booklet by the researcher prior to the start of the study. Students would be required to study their respective booklets INDIVIDUALLY. Teachers should only assist students in reading difficult words.

2. The maximum time allowed for working through the booklet would be 35 minutes. In order to ensure that each student spends a reasonable amount of time on his/her booklet, the MINIMUM time allowed would be 15 minutes. Students who claim to finish in less than 15 minutes should be encouraged to read through their booklets again.

3. At the end of the session, ALL the instructional booklets should be collected.

Instructions to Students

To be read to the students by the class teacher before they commence studying the instructional materials:

1. Each student should work individually.
2. Begin by reading very carefully the definition and directions on the cover page.
3. Do NOT hurry through your booklet. Study each page very carefully. Read each page at least twice. (A note to the teacher: at this point the teacher should refer the students to the first page of their booklets.)

   Follow the sequence of numbers beside the diagrams on each page. That is, after studying figure 1, study figure 2, then figures 3 and 4, and so on.

   You may turn back and forth through your booklet as often as you wish.
4. When you have finished studying the booklet, please raise your hand. You will be allowed to read something while the other students continue working. However, you should take at least 15 minutes studying the material before you ask to be allowed to read something else.

DAY 3: ADMINISTERING THE POSTTEST

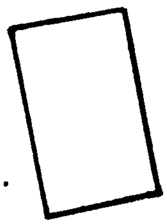
After distributing the test booklets, the teacher should:

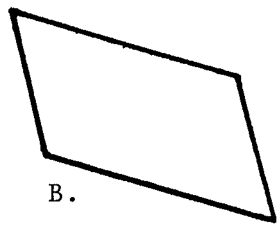
1. Ensure that students write their full names on the cover page.
2. Read the letter/instructions on the cover page to the students.
3. Students should do the test individually. The teacher should only assist students in reading difficult words.
4. The maximum time allowed for the test would be 30 minutes. Students who finish the test before the above time should be provided with reading material while the other students continue working.

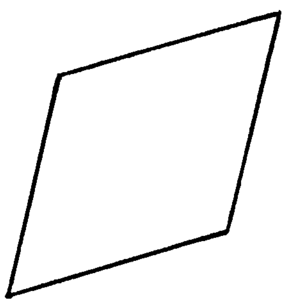
WORKSHEET

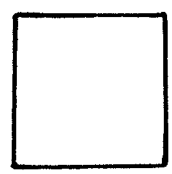
Here are some shapes. They are all called PARALLELOGRAMS.

Examine them closely.

A. 

B. 

C. 

D. 

Please answer these questions. Write your answers in the space below.

HOW ARE THE SHAPES ALIKE?

HOW ARE THEY DIFFERENT?

APPENDIX C

The Treatment Materials
1. The Experimental Treatment
2. The Control Treatment

1. The Experimental Treatment

GEOMETRY

In this lesson you will learn about a special kind of figure. It is called a parallelogram and is defined as follows:

A parallelogram is a four-sided figure with each pair of opposite sides parallel and of equal length.

You will learn six important things about parallelograms.

1. Parallelograms have four straight sides.
2. Each pair of opposite sides is parallel.
3. Each pair of opposite sides is of equal length.

Parallelograms are:

4. plane
5. closed
6. simple

You will need to learn and remember these six important things about parallelograms.

Turn the page.

STUDY THESE FIGURES CAREFULLY

1.

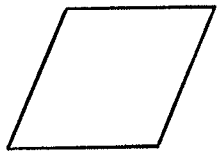


Figure 1 is a parellogram.

2.

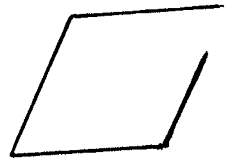


Figure 2 is NOT a parallelogram because it is open. A parallelogram is a closed figure.

3.

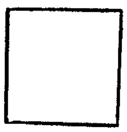


Figure 3 is a parallellogram.

4.

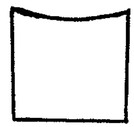


Figure 4 is NOT a parallelogram because it does not have four straight sides.

5.

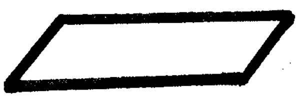


Figure 5 is a parallelogram.

6.

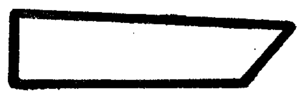


Figure 6 is NOT a parallelogram because each pair of opposite sides is not parallel and of equal length.

7.

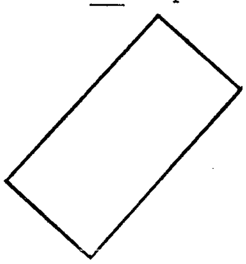


Figure 7 is a parallelogram.

8.

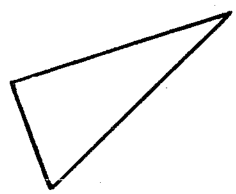


Figure 8 is NOT a parallelogram because it has only three sides. A parallelogram has four (4) sides.

STUDY THESE FIGURES CAREFULLY

9.

Figure 9 is a parallelogram.

10.

Figure 10 is NOT a parallelogram because each pair of opposite sides is not parallel and of equal length.

11.

Figure 11 is a parallelogram.

12.

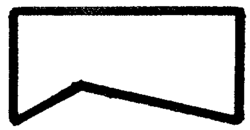


Figure 12 is NOT a parallelogram because it has five sides. A parallelogram has four (4) sides.

13.

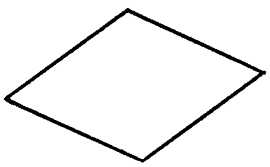


Figure 13 is a parallelogram.

14.

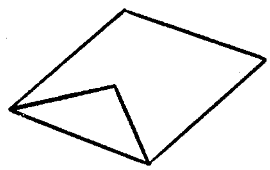


Figure 14 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

15.

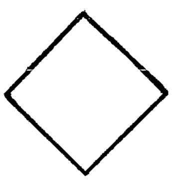


Figure 15 is a parallelogram.

16.

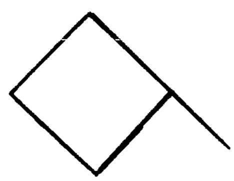


Figure 16 is NOT a parallelogram because it is open. A parallelogram has closed sides.

STUDY THESE FIGURES CAREFULLY

17.

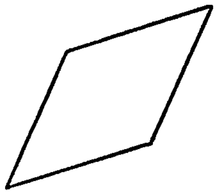


Figure 17 is a parallelogram.

18.

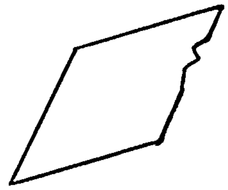


Figure 18 is NOT a parallelogram because it does not have four straight sides.

19.

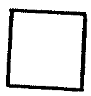


Figure 19 is a parallelogram.

20.

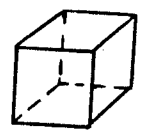


Figure 20 is NOT a parallelogram because it represents a solid. A parallelogram is a plane figure.

21.

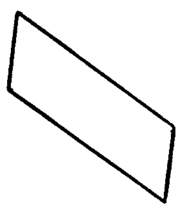


Figure 21 is a parallelogram.

22.

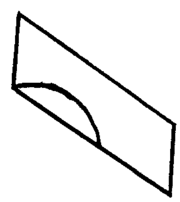


Figure 22 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

23.

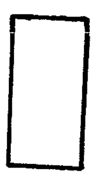


Figure 23 is a parallelogram.

24.

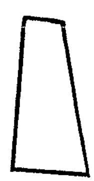


Figure 24 is NOT a parallelogram because each pair of opposite sides is not parallel and of equal length.

STUDY THESE FIGURES CAREFULLY

25.

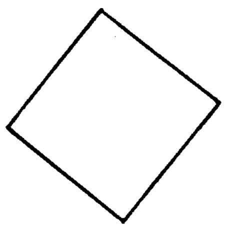


Figure 25 is a parallelogram.

26.

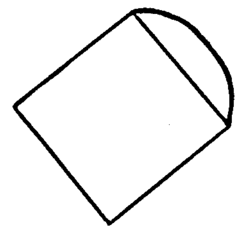


Figure 26 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

27.

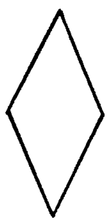


Figure 27 is a parallelogram.

28.

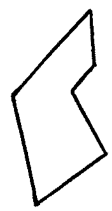


Figure 28 is NOT a parallelogram because it has six sides. A parallelogram has four (4) sides.

29.

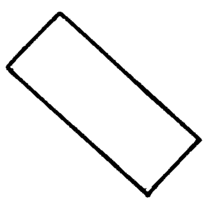


Figure 29 is a parallelogram.

30.

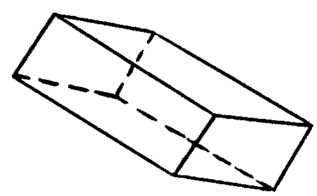


Figure 30 is NOT a parallelogram because it represents a solid. A parallelogram is a plane figure.

31.

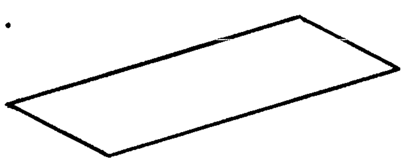


Figure 31 is a parallelogram.

32.

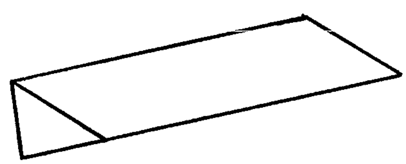
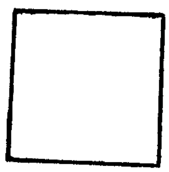


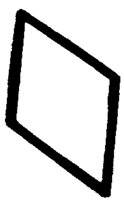
Figure 32 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

You will now see some figures. Your task will be to tell if each figure is a parallelogram. Each time you see a figure, ask yourself the following six questions. If you can answer YES to all six questions, it is a parallelogram. If you answer NO to any of the six questions, it is not.



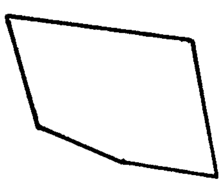
1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO
4. Does it have four straight sides? YES NO
5. Is each pair of opposite sides parallel? YES NO
6. Is each pair of opposite sides of equal length? YES NO

Is it a parallelogram? YES NO



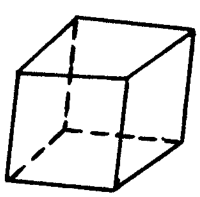
1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO
4. Does it have four straight sides? YES NO
5. Is each pair of opposite sides parallel? YES NO
6. Is each pair of opposite sides of equal length? YES NO

Is it a parallelogram? YES NO



1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO
4. Does it have four straight sides? YES NO
5. Is each pair of opposite sides parallel? YES NO
6. Is each pair of opposite sides of equal length? YES NO

Is it a parallelogram? YES NO

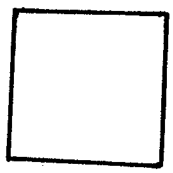


1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO

Is it a parallelogram? YES NO

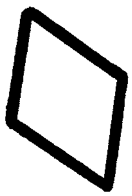
NOW TURN TO THE NEXT PAGE TO SEE IF YOU ARE RIGHT.

Below you will find the answers to the questions. Study the questions which you got wrong very carefully.



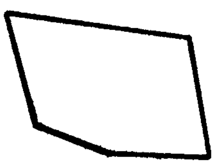
1. Is it a closed figure? **YES** NO
2. Is it a simple figure? **YES** NO
3. Is it a plane figure? **YES** NO
4. Does it have four straight sides? **YES** NO
5. Is each pair of opposite sides parallel? **YES** NO
6. Is each pair of opposite sides of equal length? **YES** NO

Is it a parallelogram? **YES** NO



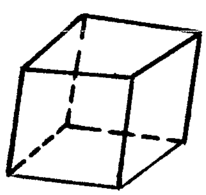
1. Is it a closed figure? **YES** NO
2. Is it a simple figure? **YES** NO
3. Is it a plane figure? **YES** NO
4. Does it have four straight sides? **YES** NO
5. Is each pair of opposite sides parallel? **YES** NO
6. Is each pair of opposite sides of equal length? **YES** NO

Is it a parallelogram? **YES** NO



1. Is it a closed figure? **YES** NO
2. Is it a simple figure? **YES** NO
3. Is it a plane figure? **YES** NO
4. Does it have four straight sides? YES **NO**
5. Is each pair of opposite sides parallel? YES **NO**
6. Is each pair of opposite sides of equal length? YES **NO**

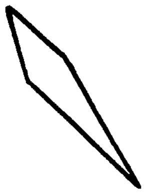
Is it a parallelogram? YES **NO**



1. Is it a closed figure? **YES** NO
2. Is it a simple figure? YES **NO**
3. Is it a plane figure? YES **NO**

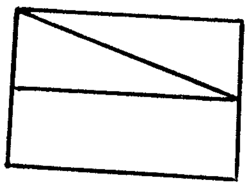
Is it a parallelogram? YES **NO**

You will now see some figures. Your task will be to tell if each figure is a parallelogram. Each time you see a figure, ask yourself the following six questions. If you can answer YES to all six questions, it is a parallelogram. If you answer NO to any of the six questions, it is not.



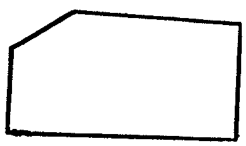
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



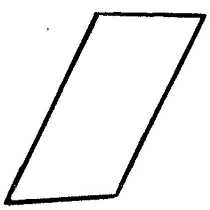
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

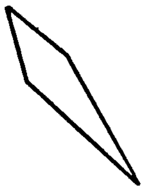


1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

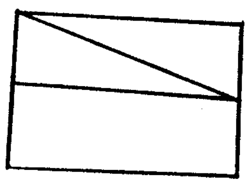
NOW TURN TO THE NEXT PAGE TO SEE IF YOU ARE RIGHT.

Below you will find the answers to the questions. Study the questions which you got wrong very carefully.



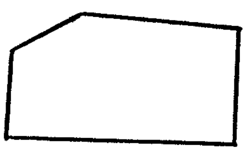
1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? YES **NO**
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? YES **NO**



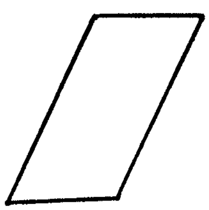
1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? YES **NO**

Is it a parallelogram? YES **NO**



1. Does it have four straight sides? YES **NO**
2. Is each pair of opposite sides parallel? YES **NO**
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

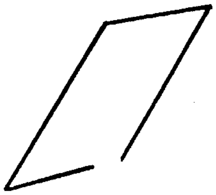
Is it a parallelogram? YES **NO**



1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

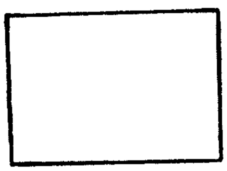
Is it a parallelogram? **YES** NO

You will now see some figures. Your task will be to tell if each figure is a parallelogram. Each time you see a figure, ask yourself the following six questions. If you can answer YES to all six questions, it is a parallelogram. If you answer NO to any of the six questions, it is not.



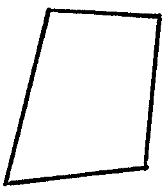
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



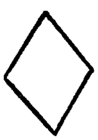
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

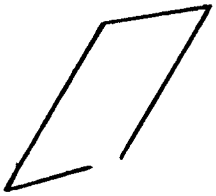


1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

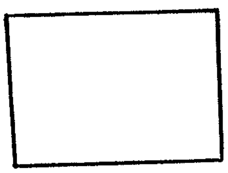
NOW TURN TO THE NEXT PAGE TO SEE IF YOU ARE RIGHT.

Below you will find the answers to the questions. Study the questions which you got wrong very carefully.



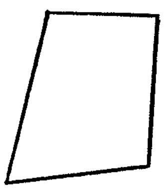
1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? YES **NO**
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? YES **NO**



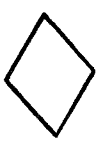
1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? **YES** NO



1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? YES **NO**
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? YES **NO**



1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? **YES** NO

2. The Control Treatment

GEOMETRY

In this lesson you will learn about a special kind of figure. It is called a parallelogram and is defined as follows:

A parallelogram is a four-sided figure with each pair of opposite sides parallel and of equal length.

You will learn six important things about parallelograms.

1. Parallelograms have four straight sides.
2. Each pair of opposite sides is parallel.
3. Each pair of opposite sides is of equal length.

Parallelograms are:

4. plane
5. closed
6. simple

You will need to learn and remember these six important things about parallelograms.

Turn the page.

STUDY THESE FIGURES CAREFULLY

1.

Figure 1 is NOT a parallelogram because each pair of opposite sides is not parallel and of equal length.

2.

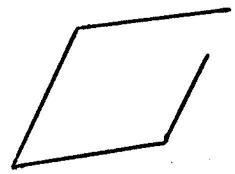


Figure 2 is NOT a parallelogram because it is open. A parallelogram is a closed figure.

3.

Figure 3 is a parallelogram.

4.

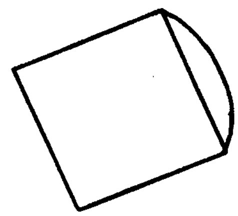


Figure 4 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

5.

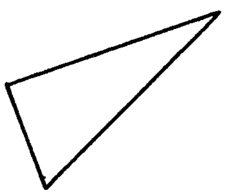


Figure 5 is NOT a parallelogram because it has only three sides. A parallelogram has four (4) sides.

6.

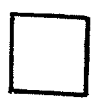


Figure 6 is a parallelogram.

7.

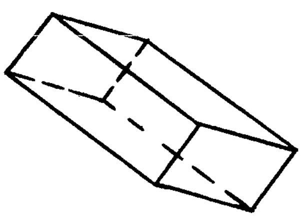


Figure 7 is NOT a parallelogram because it represents a solid. A parallelogram is a plane figure.

8.

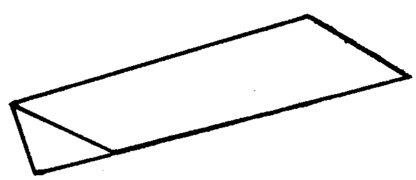


Figure 8 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

STUDY THESE FIGURES CAREFULLY

9.

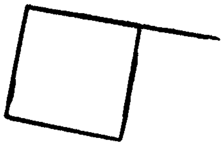


Figure 9 is NOT a parallelogram because it is open. A parallelogram has closed sides.

10.

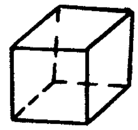


Figure 10 is NOT a parallelogram because it represents a solid. A parallelogram is a plane figure.

11.

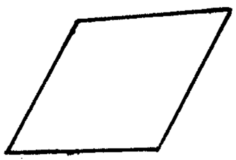


Figure 11 is a parallelogram.

12.

Figure 12 is NOT a parallelogram because each pair of opposite sides is not parallel and of equal length.

13.

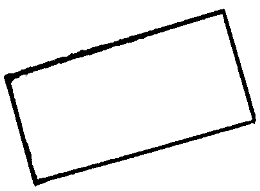


Figure 13 is a parallelogram.

14.

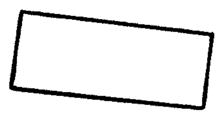


Figure 14 is a parallelogram.

15.

Figure 15 is a parallelogram.

16.

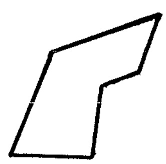


Figure 16 is NOT a parallelogram because it has six sides. A parallelogram has only four (4) sides.

STUDY THESE FIGURES CAREFULLY

17.

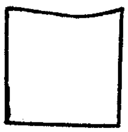


Figure 17 is NOT a parallelogram because it does not have four straight sides.

18.

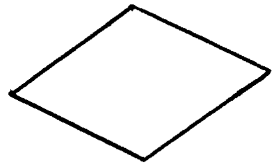


Figure 18 is a parallelogram.

19.

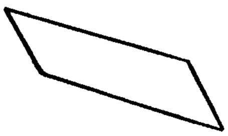


Figure 19 is a parallelogram.

20.

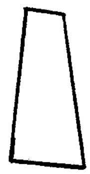


Figure 20 is NOT a parallelogram because each pair of opposite sides is not parallel and of equal length.

21.

Figure 21 is a parallelogram.

22.

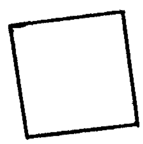


Figure 22 is a parallelogram.

23.

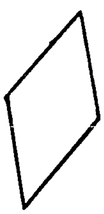


Figure 23 is a parallelogram.

24.

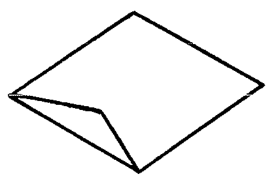


Figure 24 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

STUDY THESE FIGURES CAREFULLY

25.

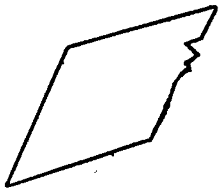


Figure 25 is NOT a parallelogram because it does not have four straight sides.

26.

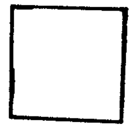


Figure 26 is a parallelogram.

27.

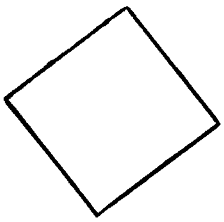


Figure 27 is a parallelogram.

28.

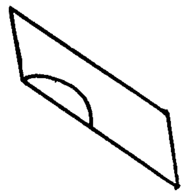


Figure 28 is NOT a parallelogram because it is a complex figure. A parallelogram is a simple figure.

29.

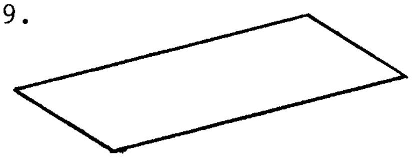


Figure 29 is a parallelogram.

30.

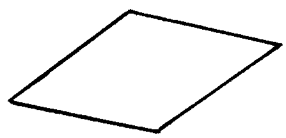


Figure 30 is a parallelogram.

31.

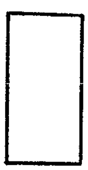


Figure 31 is a parallelogram.

32.

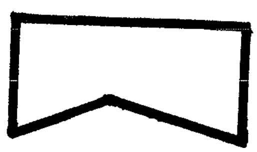
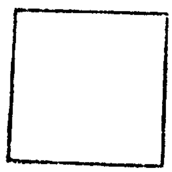


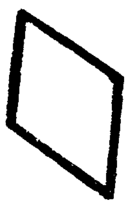
Figure 32 is NOT a parallelogram because it has five sides. A parallelogram has four (4) sides.

You will now see some figures. Your task will be to tell if each figure is a parallelogram. Each time you see a figure, ask yourself the following six questions. If you can answer YES to all six questions, it is a parallelogram. If you answer NO to any of the six questions, it is not.



1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO
4. Does it have four straight sides? YES NO
5. Is each pair of opposite sides parallel? YES NO
6. Is each pair of opposite sides of equal length? YES NO

Is it a parallelogram? YES NO

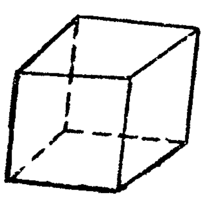


1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO
4. Does it have four straight sides? YES NO
5. Is each pair of opposite sides parallel? YES NO
6. Is each pair of opposite sides of equal length? YES NO

Is it a parallelogram? YES NO

1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO
4. Does it have four straight sides? YES NO
5. Is each pair of opposite sides parallel? YES NO
6. Is each pair of opposite sides of equal length? YES NO

Is it a parallelogram? YES NO

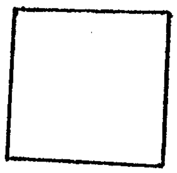


1. Is it a closed figure? YES NO
2. Is it a simple figure? YES NO
3. Is it a plane figure? YES NO

Is it a parallelogram? YES NO

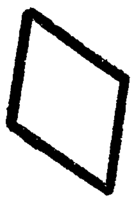
NOW TURN TO THE NEXT PAGE TO SEE IF YOU ARE RIGHT.

Below you will find the answers to the questions. Study the questions which you got wrong very carefully.



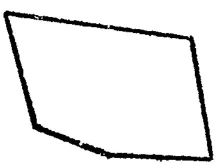
1. Is it a closed figure? **YES** NO
2. Is it a simple figure? **YES** NO
3. Is it a plane figure? **YES** NO
4. Does it have four straight sides? **YES** NO
5. Is each pair of opposite sides parallel? **YES** NO
6. Is each pair of opposite sides of equal length? **YES** NO

Is it a parallelogram? **YES** NO



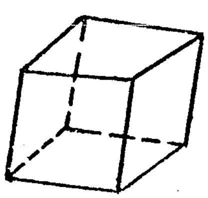
1. Is it a closed figure? **YES** NO
2. Is it a simple figure? **YES** NO
3. Is it a plane figure? **YES** NO
4. Does it have four straight sides? **YES** NO
5. Is each pair of opposite sides parallel? **YES** NO
6. Is each pair of opposite sides of equal length? **YES** NO

Is it a parallelogram? **YES** NO



1. Is it a closed figure? **YES** NO
2. Is it a simple figure? **YES** NO
3. Is it a plane figure? **YES** NO
4. Does it have four straight sides? YES **NO**
5. Is each pair of opposite sides parallel? YES **NO**
6. Is each pair of opposite sides of equal length? YES **NO**

Is it a parallelogram? YES **NO**



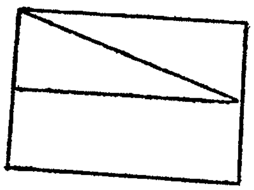
1. Is it a closed figure? **YES** NO
2. Is it a simple figure? YES **NO**
3. Is it a plane figure? YES **NO**

Is it a parallelogram? YES **NO**

You will now see some figures. Your task will be to tell if each figure is a parallelogram. Each time you see a figure, ask yourself the following six questions. If you can answer YES to all six questions, it is a parallelogram. If you answer NO to any of the six questions, it is not.

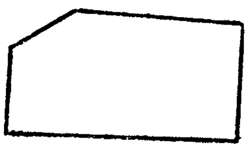
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



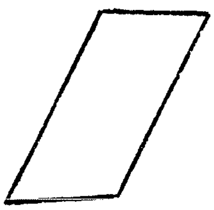
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

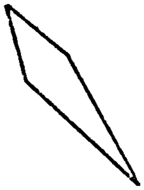


1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

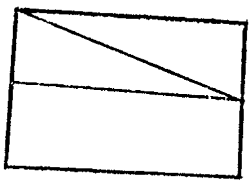
NOW TURN TO THE NEXT PAGE TO SEE IF YOU ARE RIGHT.

Below you will find the answers to the questions. Study the questions which you got wrong very carefully.



1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? YES **NO**
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? YES **NO**

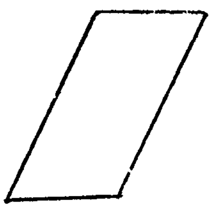


1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? YES **NO**

Is it a parallelogram? YES **NO**

1. Does it have four straight sides? YES **NO**
2. Is each pair of opposite sides parallel? YES **NO**
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

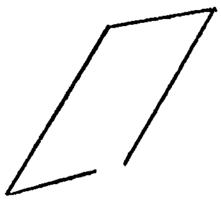
Is it a parallelogram? YES **NO**



1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

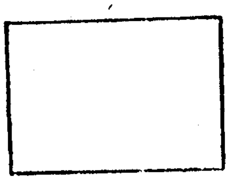
Is it a parallelogram? **YES** NO

You will now see some figures. Your task will be to tell if each figure is a parallelogram. Each time you see a figure, ask yourself the following six questions. If you can answer YES to all six questions, it is a parallelogram. If you answer NO to any of the six questions, it is not.



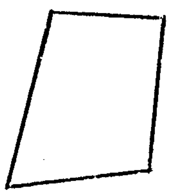
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



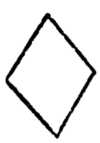
1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO



1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

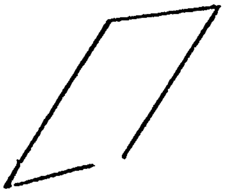


1. Does it have four straight sides? YES NO
2. Is each pair of opposite sides parallel? YES NO
3. Is each pair of opposite sides of equal length? YES NO
4. Is it a closed figure? YES NO
5. Is it a plane figure? YES NO
6. Is it a simple figure? YES NO

Is it a parallelogram? YES NO

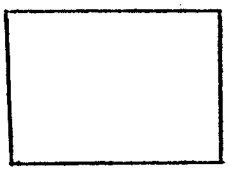
NOW TURN TO THE NEXT PAGE TO SEE IF YOU ARE RIGHT.

Below you will find the answers to the questions. Study the questions which you got wrong very carefully.



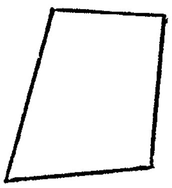
1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? YES **NO**
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? YES **NO**



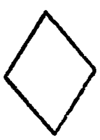
1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? **YES** NO



1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? YES **NO**
3. Is each pair of opposite sides of equal length? YES **NO**
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? YES **NO**



1. Does it have four straight sides? **YES** NO
2. Is each pair of opposite sides parallel? **YES** NO
3. Is each pair of opposite sides of equal length? **YES** NO
4. Is it a closed figure? **YES** NO
5. Is it a plane figure? **YES** NO
6. Is it a simple figure? **YES** NO

Is it a parallelogram? **YES** NO

APPENDIX D

The Posttest and Answer Key

1. The Posttest
2. The Answer Key

1. The Posttest

GEOMETRY TEST

Name: ______________________

School: ______________________

Grade: ______________________

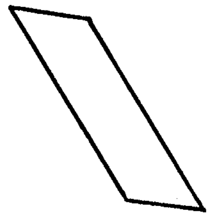
Dear Student:

The purpose of this test is to find out how much you have learned about the geometry topics which you have been studying. Please try your best on the test and attempt all questions.

Read the instructions for each question very carefully. When answering the questions, circle ○ the letter beside the correct answer. You have 30 minutes to do the test.

Thank you.

Osmond Petty

1. This figure  is a

   A. trapezoid.

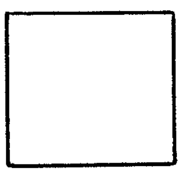
   B. parallelogram.

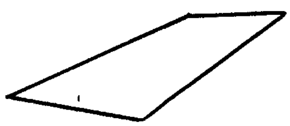
   C. square.

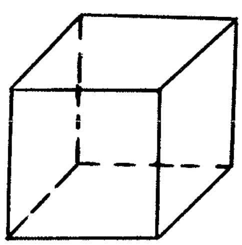
   D. rectangle.

   E. rhombus.

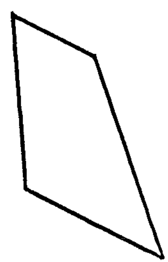
2. Which figure is a parallelogram?

   A. 

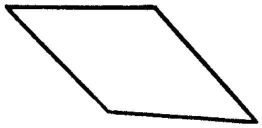
   B. 

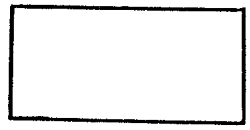
   C. 

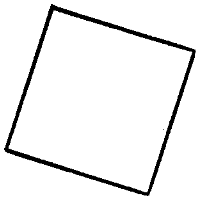
   D. 

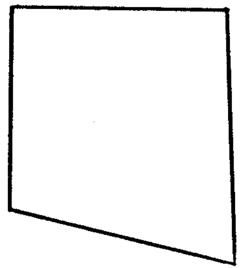
   E. 

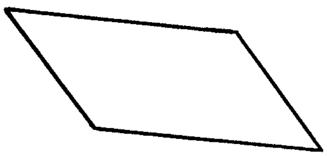
3. Which one of these is <u>not</u> a parallelogram?

A. 

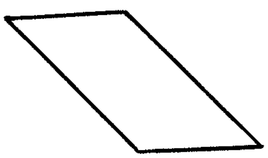
D. 

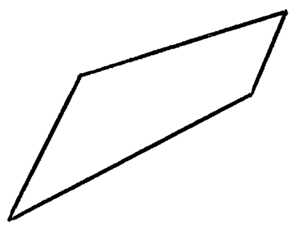
B. 

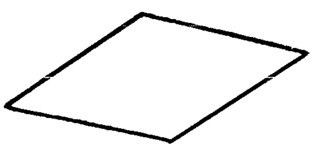
E. 

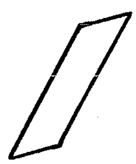
C. 

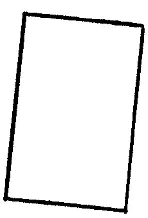
4. Identify ALL the parallelograms in the diagrams below by circling the letter beside the diagram.

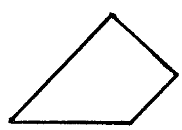
A. 

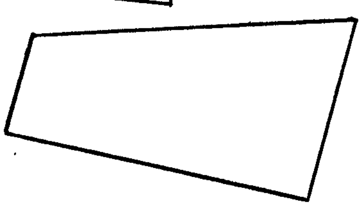
E. 

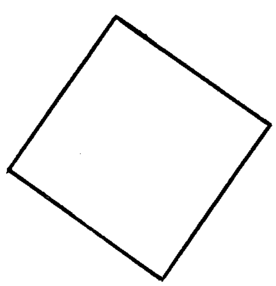
B. 

F. 

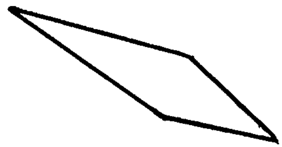
C. 

G. 

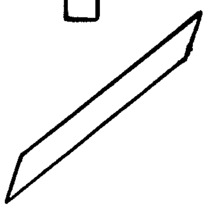
D. 

H. 

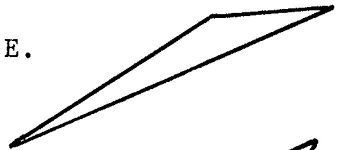
5. Identify ALL the diagrams below which are NOT parallelograms by circling the letter beside the diagram.

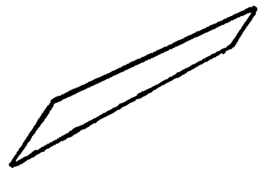
A. 

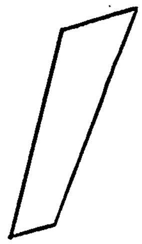
B. 

C. 

D. 

E. 

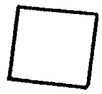
F. 

G. 

H. 

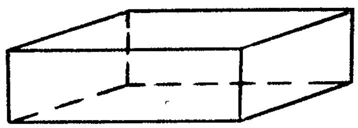
6. Decide whether each figure is a parallelogram. If you think it is a parallelogram, circle YES. If you think it is not a parallelogram, circle NO.

A.



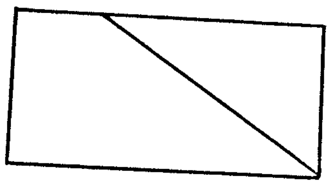
YES NO

B.



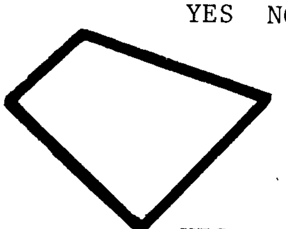
YES NO

C.



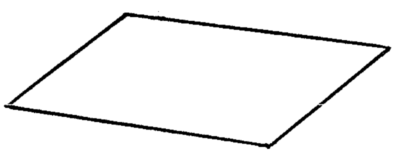
YES NO

D.



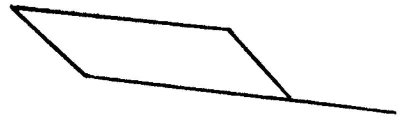
YES NO

E.



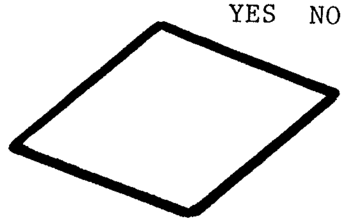
YES NO

F.



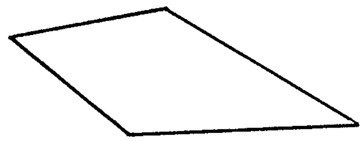
YES NO

G.



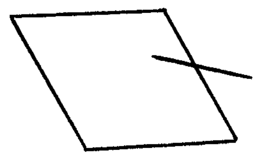
YES NO

H.



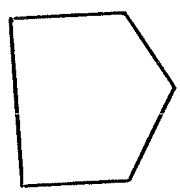
YES NO

I.



YES NO

J.



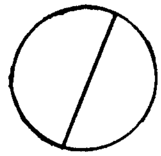
YES NO

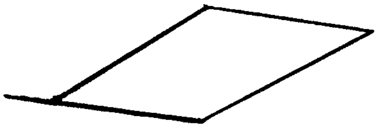
7. Which drawing is a closed figure?

A. 

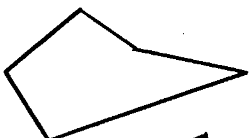
D. 

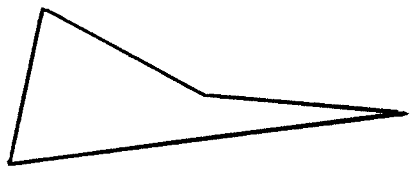
B. 

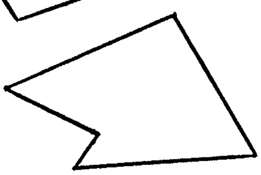
E. 

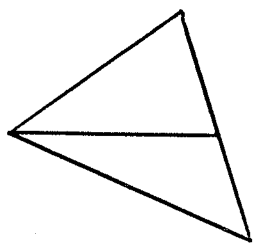
C. 

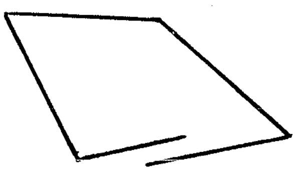
8. Which drawing has four straight sides?

A. 

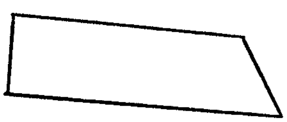
D. 

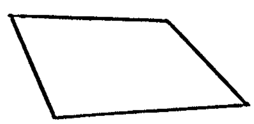
B. 

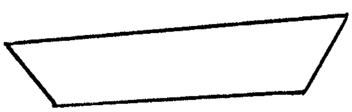
E. 

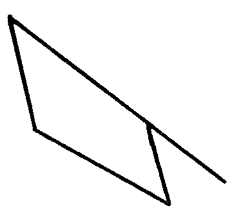
C. 

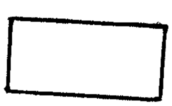
9. Which drawing has two pairs of parallel sides?

A. 

D. 

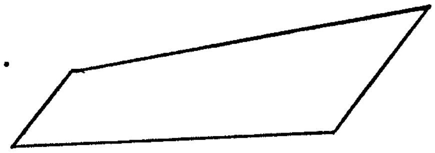
B. 

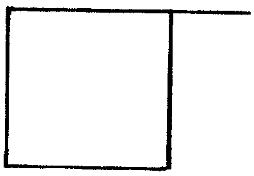
E. 

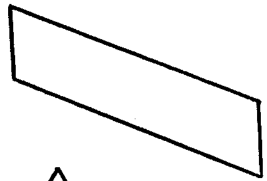
C. 

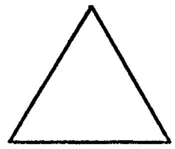
10. Which drawing has each pair of opposite sides equal in length?

A. 

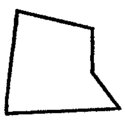
B. 

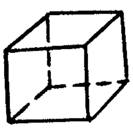
C. 

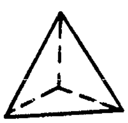
D. 

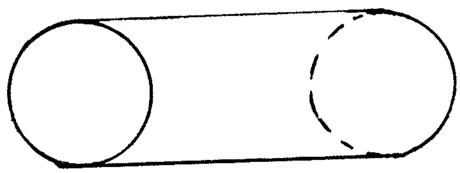
E. 

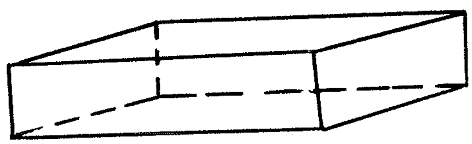
11. Which drawing represents a plane figure?

A. 

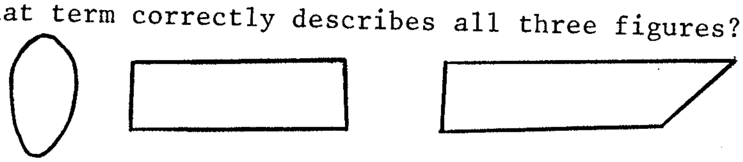
B. 

C. 

D. 

E. 

12. What term correctly describes all three figures?



All three figures are:

A. horizontal.

B. closed.

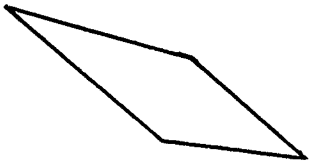
C. solid.

D. quadrilaterals.

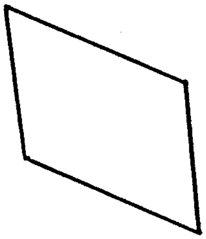
E. right-angled.

13. Which figure has each pair of opposite sides equal and parallel?

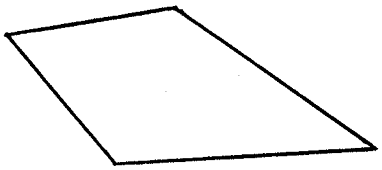
A.



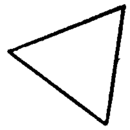
B.



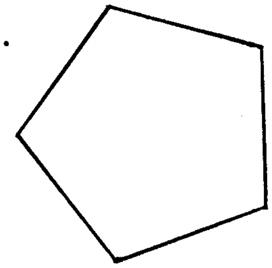
C.



D.

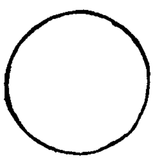


E.

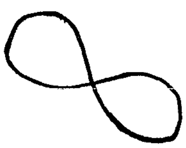


14. Which drawing is a simple figure?

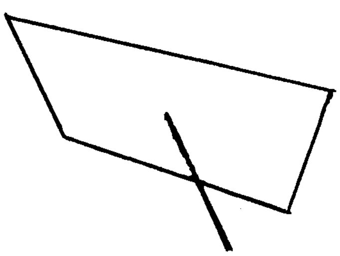
A.



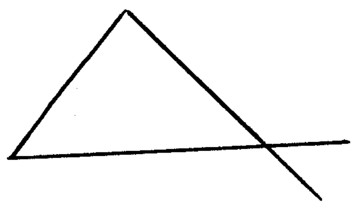
B.



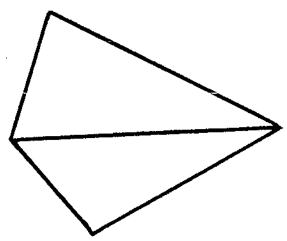
C.



D.



E.



X

15. The drawing X has a property that is not present in any of the other drawings. Which one of the following statements describes that property?

A. X is a plane figure.

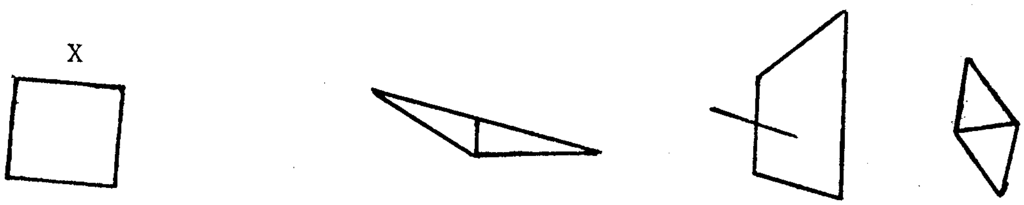
B. X is a complex figure.

C. X has no curve.

D. X is a closed figure.

E. X has no right angles.

X



16. The drawing X has a property that is not present in any of the other drawings. Which one of the following statements describes that property?

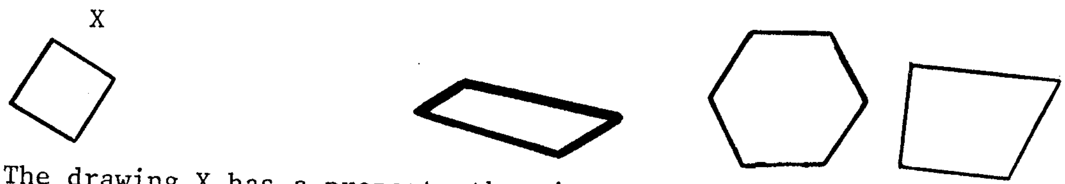
A. X is a plane figure.

B. X is a four-sided figure.

C. X is a small figure.

D. X has straight lines.

E. X is a simple figure.



17. The drawing X has a property that is not present in any of the other drawings. Which one of the following statements describes that property?

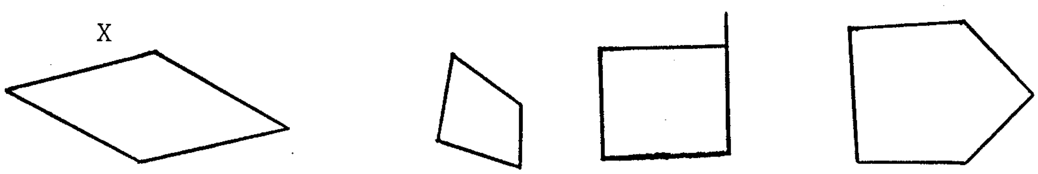
A. X has four thin sides.

B. X has right angles.

C. X has all sides equal in length.

D. X is an open figure.

E. X has each pair of opposite sides parallel.



18. The drawing X has a property that is not present in any of the other drawings. Which one of the statements below describes that property?

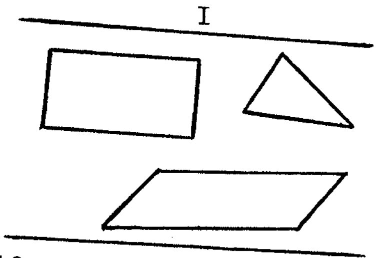
A. X has opposite sides parallel and equal.

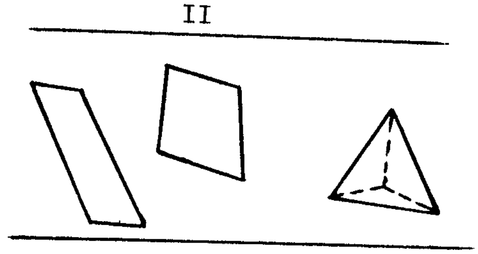
B. X has no right angles.

C. X has each pair of opposite sides parallel.

D. X is a four-sided figure.

E. X is horizontal.





19. ALL of the drawings in group I have a property that is NOT present in ALL of the drawings in group II. Which ONE of the following terms describes that property?

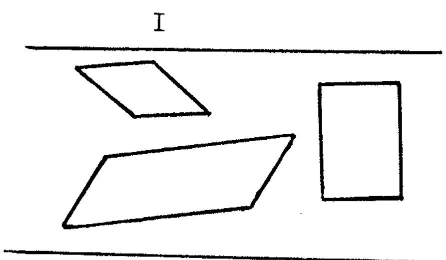
    A. Closed

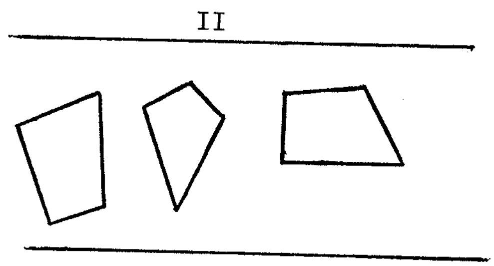
    B. Plane

    C. Complex

    D. Open

    E. Solid





20. ALL the drawings in group I have a property that is NOT present in ALL of the drawings in group II. Which ONE of the following statements describes that property?

    A. Contain four right angles.

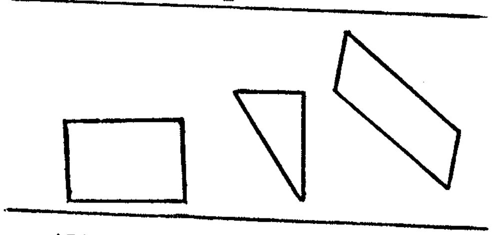
    B. Each pair of opposite sides is straight.

    C. Each pair of opposite sides is perpendicular.

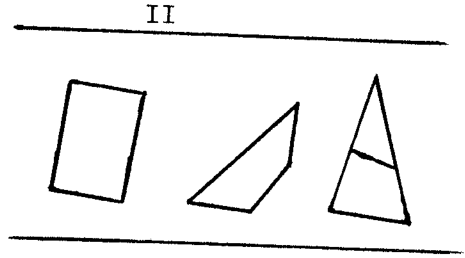
    D. Each pair of opposite sides is parallel and equal.

    E. All the drawings are vertical.

I



II



21. ALL of the drawings in group I have a property that is NOT present in ALL of the drawings in group II. Which ONE of the following terms describes that property?

A. Open

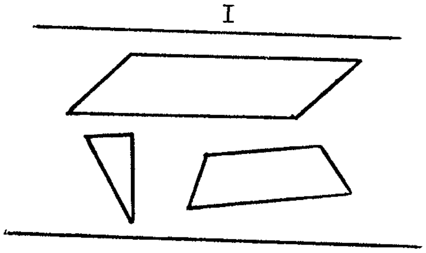
B. Vertical

C. Right-angled

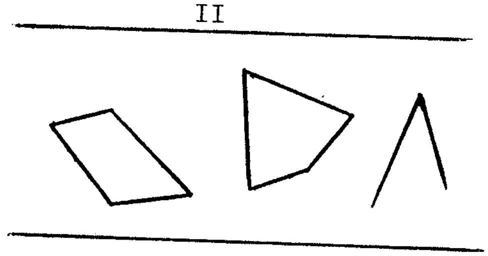
D. Slanting

E. Simple

I



II



22. ALL of the drawings in group I have a property that is NOT present in ALL of the drawings in group II. Which ONE of the following terms describes that property?

A. Right-angled

B. Closed

C. Simple

D. Plane

E. Small

For each of the following items, select the best answer to complete the statement.

23. All parallelograms

 A. are of the same size.

 B. are of medium size.

 C. are plane figures.

 D. have four equal sides.

 E. are small figures.

24. All parallelograms

 A. have four equal angles.

 B. have four equal sides.

 C. have a slanting shape.

 D. are right-angled.

 E. are closed figures.

25. NOT all parallelograms are

 A. four-sided figures.

 B. simple figures.

 C. closed figures.

 D. thin-sided figures.

 E. plane figures.

26. All parallelograms are

 A. plane closed four-sided figures with four right angles.

 B. plane closed four-sided figures with four equal angles.

 C. plane open four-sided figures with each pair of opposite sides equal.

 D. plane closed four-sided figures with each pair of opposite sides equal and parallel.

 E. plane closed four-sided figures with four equal sides.

27. All parallelograms have

    A. four right angles.

    B. four sides of equal length.

    C. opposite sides equal.

    D. thin sides.

    E. vertical sides.

28. All parallelograms do NOT have

    A. four sides.

    B. a slanting shape.

    C. closed sides.

    D. a plane shape.

    E. four angles.

29. NOT all parallelograms have

    A. opposite sides equal.

    B. opposite sides parallel.

    C. a plane shape.

    D. four right angles.

    E. a simple closed shape.

30. All plane closed 4-sided figures with opposite sides equal and parallel are called

    A. rectangles.

    B. squares.

    C. parallelograms.

    D. rhombuses.

    E. trapezoids.

2. The Answer Key

1. B
2. A
3. E
4. Examples: A,B,C,F,H

   Nonexamples: D,E,G
5. Examples: B,C,H

   Nonexamples: A,D,E,F,G
6. Yes: A,E,G

   No: B,C,D,F,H,I,J
7. E
8. D
9. C
10. D
11. A
12. B
13. B
14. A
15. D
16. E
17. E
18. A
19. B
20. D
21. E
22. B
23. C
24. E
25. D
26. D
27. C
28. B
29. D
30. C

APPENDIX E

Statistics and Formulas used in Estimating Power and Partial Omega Squared

1. Estimating Power
2. Estimating Partial Omega Squared

1. Estimating Power

The estimate of effect size used was ϕ_A, defined according to Keppel (1982) as follows:

$$\phi_A^2 = \frac{s' \left[\Sigma(\mu_i - \mu)^2 \right] / a}{\sigma^2_{S/A}}$$

where s' is the sample size, a is the number of treatments, μ_i is the treatment means, μ is the average of the μ_i's, and $\sigma^2_{S/A}$ is the population variance estimated by the within-groups variance. Table F shows the relevant data for the power analysis using the results of the pilot study.

Table A-4

Treatment Means and Within-groups
Variance for Scores on Subtests Based on Pilot Study

Levels of Concept Attainment	Means: Experimental	Means: Control	Within-groups Variance
Classificatory	26.92	25.12	13.58
Formal	14.63	13.04	22.49

Note. N=24 for each group.

The expected total number of subjects in both schools (Y and Z) was 108; thus, it was expected that each treatment group would contain 54 subjects. Hence, with s=54 and .05 level of significance, ϕ_A for the subtests at the classificatory and formal levels was 1.85 and 1.26 respectively. The power functions of the F-distribution (Keppel, 1982) showed that the estimates of statistical power for the classificatory and formal levels were approximately .78 and .50 respectively.

Because of the a priori nature of power analysis, the above power estimates must be subjected to "conditional interpretation" (Cohen, 1973) as follows: For a test with .05 level of significance, using n=54 subjects per treatment group, if the effect size for the classificatory and formal levels subtests be given by 1.85 and 1.26 respectively, then the statistical power would be .78 and .50 respectively. Further, as noted by Keppel (1982), the power estimates are based on the assumption that the treatment effects in the population and the population variance can be closely estimated by the treatment means and within-groups variance from the pilot study. This may or may not be a valid assumption because of the exploratory nature of pilot work.

2. Estimating Partial Omega Squared

Maxwell et al. (1981, p. 530) define partial omega squared as follows:

$$\hat{\omega}^2_{partial} = \frac{df_{effect}\ (F_{effect} - 1)}{df_{effect}\ (F_{effect} - 1) + N},$$

where df = degrees of freedom, F = computed F ratio, and N = total sample size.

Interpretation of the estimate of partial omega squared assumes equal group sizes. Since treatment groups were unequal (n=52 for the experimental group, and n=53 for the control group), N was taken to be 104; that is, 52 subjects per group. The resulting estimate of partial omega squared was therefore slightly conservative.

From Table 13, $df_{effect} = 1$, and $F_{effect} = 7.56$. Substituting these values in the formula above gives a value of .06.