

CRITERIA FOR MODELING ONE-DIMENSIONAL UNSTEADY
DRAINAGE OF LAYERED SOILS

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ABSTRACT

A review of some of the existing theories for physical modeling of unsteady drainage from unsaturated porous media indicated the choice of Brooks and Corey's theory and it was applied for one-dimensional unsteady drainage from layered soils. Another necessary criterion was developed, in addition to those offered by Brooks and Corey, for layered soils. Some experiments were done and the results showed that the developed criterion must be satisfied to enable prediction of the prototype behavior from that of the model.

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LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
C	Hydraulic conductivity.....	LT^{-1}
$^{\circ}C$	Degrees centigrade.....	none
d	Differential operator.....	none
d	Microscopic characteristic length.....	L
e	Subscript meaning "effective".....	none
f	Body force per unit volume.....	FL^{-3}
g	Acceleration due to gravity.....	LT^{-2}
H	Hydraulic head.....	L
K	Maximum permeability to a particular fluid phase when only one phase occupies the medium.....	L^2
K_e	Effective permeability, the permeability when the medium is occupied by more than one fluid phase.....	L^2
K_r	Relative permeability, K_e/K	none
L	Length.....	L
m	General subscript.....	none
n	General subscript.....	none
nw	Subscript meaning "nonwetting phase".....	none
P	Pressure.....	FL^{-2}
P_b	Bubbling pressure, approximately the maximum capillary pressure on the drainage cycle at which a continuous nonwetting phase exists in a porous medium.....	FL^{-2}
P_c	Capillary pressure, the pressure difference ($P_{nw}-P_w$).....	FL^{-2}
Q	Volume discharged per unit area.....	L
q	Liquid volume flux.....	LT^{-1}
r	Position vector.....	L

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
S	Saturation, the ratio of the volume of the wetting fluid to the volume of the voids.....	none
S_e	Effective saturation $(S-S_r)/(1-S_r)$	none
S_r	Residual saturation, the saturation at which K_{ew} approaches zero.....	none
t	Time.....	T
w	Subscript meaning "wetting phase".....	none
z	Vertical coordinate direction.....	L
α	Contact angle of wetting-nonwetting fluid interface with	
$f(\alpha)$	Dimensionless representation for contact angle.....	none
γ	Specific weight of fluid, weight per unit volume of fluid.....	FL^{-3}
ϵ	The exponent in the equation $K_{rw} = S_e^\epsilon$	none
η	The exponent in the equation $K_{rw} = (P_b/P_c)^\eta$ called the pore-size distribution index....	none
λ	The exponent in the equation $S_e = (P_b/P_c)^\lambda$ called the pore-size distribution index....	none
μ	Dynamic viscosity.....	$FL^{-2}T$
μ^2	Square microns - a unit of permeability....	L^2
ρ	Fluid density.....	$FL^{-4}T^2$
ρ_b	Bulk density of the porous medium.....	$FL^{-4}T^2$
ρ_s	Particle density of the medium.....	$FL^{-4}T^2$
σ	Interfacial tension.....	FL^{-1}
ϕ	Porosity, the volume of pore space expressed as a fraction of bulk volume of the medium.....	none
ϕ_e	Effective porosity, $(1-S_r)\phi$	none
Δ	Denotes a difference.....	

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
∇	Gradient operator.....	L^{-1}
div	The divergence operator.....	L^{-1}

A dotted subscript denotes the scaled variable.

A primed quantity refers to the model.

CHAPTER I

INTRODUCTION

The relevant laws of nature and the geometrical relations of the physical situation sometimes cannot be formulated mathematically because of the complexity of the problem. There are many cases which can be represented as functional equations, usually as differential equations. However, it is not always possible to solve these equations for different conditions because the mathematical difficulties are too great.

This need not discourage investigators from finding solutions for such problems. In both cases these problems can be understood well by means of direct experimentation. Many phenomena cannot be investigated directly because of the great amount of time and money involved. Through proper model design, systems of comparatively large size can be reproduced in the laboratory in a smaller scale. The laboratory results can faithfully duplicate the behavior in the prototype, but on a miniature dimensional scale and in a comparatively short span of time.

There are two methods of defining scaling criteria. One is the well-known dimensional analysis which is based on Buckingham's π theorem. The other called "inspectional

analysis" by Rurak (17) consists of transforming the equations of a problem differential or otherwise, so that all the variables are dimensionless. Simple inspection then shows how these dimensionless variables are related.

Many investigators, both petroleum and drainage engineers, have proposed scaling laws. Brooks and Corey (2) in 1964 developed a theory of similitude for nonsteady drainage of unsaturated porous media by use of Richard's equation. Their theory was tested for one-dimensional and two-dimensional drainage for a single layer of soil by Corey et al. (6) and Hedstrom et. al. (10) respectively in 1965 and 1971 with good results.

The purpose of this study is to apply Brooks and Corey's theory of similitude to one-dimensional drainage for two or more layers of soil and to find out the other necessary similitude requirements if any. This study is also a basis for the study of the two-dimensional case which is being completed by I.G. Henderson, a graduate student at the University of Manitoba, as an M.Sc. thesis.

CHAPTER II

BACKGROUND AND THEORY

Studies of the problems of flow through porous media have been done by many investigators in the petroleum industry and water resource area. Petroleum engineers are interested in two-phase liquid flow or the displacement of oil by water or gas since most often raw petroleum occurs in the porous mantle of the earth. Water resource engineers in irrigation and drainage problems are faced with the liquid phase in a water-air system in which water is the only phase flowing.

Some of the modeling studies which have been done are merely qualitative and there has not been any effort to apply the results to the prototype. However, in many quantitative studies, in which the main purpose has been to find out the prototype behavior by use of models, failure occurred because of not considering the microscopic geometry as well as the macroscopic geometry.

Some of the investigators in the petroleum industry who have developed scaling criteria are Leverett (13), Rapoport (15), Croes and Schwarz (8), Craig et. al (7), Scheidegger (18) and Richardson (16).

Several other investigators have proposed scaling

theories for flow of one liquid in unsaturated porous materials. Stallman (19), Miller and Miller (14) and Brooks and Corey (2) are in this group.

Because Richardson (16) has suggested additional criteria for modeling layered soils and has done a few qualitative experiments a few pages are devoted here to explain the principles of his theory as well as the theory of Miller and Miller (14) and the similarity of it with that of Brooks and Corey (2).

The present author has taken advantage of using the converted scaling criteria to similar units which has been done by Corey et. al. (6) for ease of comparison.

Richardson (16) has scaled the differential equation describing the displacement of oil from a porous medium by water or gas. He has pointed out that the solution of this equation satisfies both model and prototype if the following are true:

- (1) The model and prototype are geometrically similar
- (2) The model and prototype have similar initial and boundary conditions
- (3) The model and prototype have the same values for

the parameters $\frac{\sigma f(\alpha)}{q \mu_o L} \sqrt{K\phi}$, $\frac{K \Delta \rho g}{q \mu_o}$, $\frac{\mu_w}{\mu_o}$

with times given by $qt/\phi L$ where $f(\alpha)$ is merely a dimensionless representation for the contact angle and q is the volume flux. Other symbols are σ for interfacial tension, μ for viscosity, K for absolute permeability, ϕ for porosity and the subscripts w and o refer to water and oil respectively.

- (4) The model and prototype have the same relative permeability and dimensionless capillary pressure-saturation relations.

For stratified sands he proposed one more criterion in addition to those mentioned above; model must have the same relative distribution of permeabilities as prototype. In other words

$$\left(\frac{K_b}{K_a}\right)_{\text{model}} = \left(\frac{K_b}{K_a}\right)_{\text{prototype}} \dots \dots \left(\frac{K_m}{K_a}\right)_{\text{model}} = \left(\frac{K_m}{K_a}\right)_{\text{prototype}}$$

where subscripts $a, b, \dots m$ refer to different layers.

He performed some qualitative tests using this relation between layers and in his experiments he made gravitational forces negligible by selecting a very small depth of model and by allowing only flow parallel to the layers.

Miller and Miller (14) were the first to consider the scaling laws which would be valid for any flow system whether it be drainage or imbibition. They used inspectional

analysis and gave attention to microscopic as well as macroscopic scaling. They call two media "similar" when their solid geometries differ only by a constant magnifying factor. When the interface geometries are also "similar" they describe the two media as being in "similar" states. To satisfy this, the values $\frac{P_{cd}}{\sigma}$, α , $\frac{K_e}{d^2}$ should be the same in the two media where P_c is the capillary pressure, K_e the effective permeability and d is a characteristic length of the media.

They developed the following scaled or "reduced" variables (a dot subscript is used to denote each scaled variable).

Scaled Microscopic Interface Geometry

$$\frac{1}{r_{m.}} \equiv \frac{d}{r_m}$$

$$\frac{1}{r_m} = \text{mean film curvature}$$

$$P_{c.} \equiv \frac{P_{cd}}{\sigma}$$

$$\alpha_{.} \equiv \alpha$$

$$\alpha = \text{contact angle (radians)}$$

Scaled Flow Properties of Medium

$$S_{.} \equiv S$$

$$S = \text{degree of saturation}$$

$$K_{e.} \equiv \frac{K_e}{d^2}$$

Scaled Macroscopic Variables

(L = macroscopic characteristic length)

$$r. \equiv \frac{r}{L}$$

r = position vector

$$f. \equiv \frac{Ld}{\sigma} f$$

f = body force per unit volume

For gravity $f = \rho g$

$$q. \equiv \frac{\mu_L}{\sigma d} q$$

$$t. \equiv \frac{\sigma d}{\mu L^2} t$$

t = time

Klute et. al. (11), Elrick et. al. (9) and Wilkinson et. al. (20) tested the validity of Miller and Miller's theory in different ways.

Klute et. al. (11) prepared a series of "similar" media consisting of different fractions of sand. They obtained capillary pressure-saturation curves for both drainage and imbibition cycles. They also obtained capillary pressure-permeability curves for each fraction. The "reduced" capillary pressure-saturation curves, as well as "reduced" capillary pressure-permeability curves, coalesced into one curve within the limits of experimental error.

Elrick et. al. (9) ran four different tests to examine Miller and Miller's theory. They ran hysteresis loop experiments and unsaturated conductivity tests for a unique medium but using different liquids. For the hysteresis loop

experiments the agreement with theory was good for clean sand but deteriorated as the clay content increased.

They also ran infiltration and drainage tests for systems constructed from various "similar" media; only one liquid was used. The results were in excellent agreement with the theory.

Wilkinson et. al. (20) drained the initially saturated columns of different fractions of sand which were "similar". The "reduced" outflow was plotted against "reduced" time and the agreement was reasonable within experimental error except for the finest fraction. When the tests were done for infiltration into dry columns the curve for coarsest fraction showed some deviation from other curves.

Another scaling theory was proposed by Brooks and Corey (2) in 1964 and this is the theory which has been used by the author for vertical drainage from two horizontal layers of sand. Before explaining their theory of similitude it is better first to mention some of the properties of porous media from Brooks and Corey (2).

In the two-fluid system discussed herein, the fluids are assumed to be immiscible. They are referred to as non-wetting and wetting phases. Even though the discussion is confined to a gas-liquid system, the theory applies to any two-fluid system in which the fluids are immiscible.

It is assumed that Darcy's equation applies to flow of both the gas and liquid phases occupying a porous medium. For this assumption to be valid, both phases must form continuous networks within the medium and any isolated bubbles of gas must be regarded as part of the porous matrix.

When two immiscible fluids occupy the pores of a porous matrix, the permeability to one fluid phase is called "effective permeability", K_e . Usually the effective permeability will be smaller than K , the permeability when only one fluid occupies the medium. The ratio K_e/K is called "relative permeability" and varies from 0 to 1.0.

The fraction of the total pore space occupied by the wetting phase is defined as saturation S . It is considered here as a quantity which can vary from point to point within the medium.

In general, the functional relationship, $K_e = f(S)$ is not single-valued but is affected by hysteresis. Hysteresis affects both the liquid and the gas permeabilities.

Another assumption is that the liquid and gaseous fluid phases are separated by curved interfaces, concave with respect to the liquid. This assumption is valid at relatively high liquid saturations in a medium containing at least some pores large enough that the liquid permeability is substantial. It is not always valid for saturation less

than field capacity or for porous media having only extremely small pores. The pressure differences across the interface is called capillary pressure P_c .

$$P_c = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = P_{\text{gas}} - P_{\text{liquid}} \quad (1)$$

where σ is the interfacial tension, r_1 and r_2 are major and minor radii of curvature at a point on the interface. The liquid is at a pressure less than the gas and P_c is therefore, positive. Saturation is a function of P_c and K_e is also a function of P_c , both being affected by hysteresis.

According to the analysis of Burdine (3), the relative permeability K_e/K_w is approximated by the relation

$$K_{rw} \approx \left(\frac{S - S_r}{1 - S_r} \right)^2 \frac{\int_0^S \frac{dS}{P_c^2}}{\int_0^1 \frac{dS}{P_c^2}} \quad (2)$$

where S_r is the residual saturation. The permeability is assumed to approach zero at this finite saturation. Corey called the quantity $(S - S_r)/(1 - S_r)$ the effective saturation, S_e . By changing S to S_e in the above equation it becomes

$$K_{rw} = (S_e)^2 \frac{\int_0^{S_e} \frac{dS_e}{P_c^2}}{\int_0^1 \frac{dS_e}{P_c^2}} \quad (3)$$

and for the non-wetting phase

$$K_{rnw} = (1-S_e)^2 \frac{\int_{S_e}^1 \frac{dS_e}{P_c^2}}{\int_0^1 \frac{dS_e}{P_c^2}} \quad (4)$$

Corey (4) approximated the ratio of integrals in equations 3 and 4 by S_e^2 and $(1-S_e^2)$ respectively. Then the two above equations become

$$K_{rw} \approx S_e^4 \quad (5)$$

$$K_{rnw} \approx (1-S_e)^2 (1-S_e^2) \quad (6)$$

The equations given by Corey imply that effective saturation, S_e , is a linear function of $\frac{1}{P_c^2}$, i.e.

$$S_e = \left(\frac{c}{P_c} \right)^2 \text{ for } P_c \geq c \quad (7)$$

where c is some constant.

If equation 7 is substituted into equation 5 the result is

$$K_{rw} = \left(\frac{c}{P_c} \right)^8 \quad (8)$$

but for media with relatively uniform pores the exponent in equation 8 is much larger than 8 and theoretically could increase without bound.

The theory developed by Brooks and Corey for properties of porous media is based on observations from a large number of experimental data which indicate that

$$S_e = \left(\frac{P_b}{P_c} \right)^\lambda \quad \text{for } P_c \geq P_b \quad (9)$$

where λ and P_b are characteristic constants of the medium.

This equation was discovered by plotting S_e as a function of $\log P_c/\gamma$ on the drainage cycle. The result is a straight line with a negative slope λ for $P_c/\gamma \geq P_b/\gamma$. The parameter P_b/γ is defined by the intercept where the straight line meets the ordinate representing $S_e = 1.0$ and it is called the bubbling pressure of the medium. The bubbling pressure, P_b is a measure of the maximum pore size forming a continuous network of flow channels within the medium.

To describe S_e it is necessary to have capillary pressure-desaturation data. Such curves can be seen in Appendix A which have been completed for different fractions of the sand used for the drainage test.

The negative of the slope of the curve of S_e as a function of P_c/γ is designated as λ , and it is called the pore-size distribution index of the medium.

Substituting equation 9 into equation 3 and performing the integration gives

$$K_{rw} = (S_e)^{\frac{2+3\lambda}{\lambda}} = (S_e)^\varepsilon \quad (10)$$

or

$$K_{rw} = \left(\frac{P_b}{P_c} \right)^\eta \quad \text{for } P_c \geq P_b \quad (11)$$

where $\eta = 2+3\lambda$ and $\varepsilon = (2+3\lambda)/\lambda$

Similarly for the non-wetting phase

$$K_{rnw} = (1-S_e)^2 \left(1-S_e^{\frac{2+\lambda}{\lambda}} \right) \quad (12)$$

or

$$K_{rnw} = \left(1 - \left(\frac{P_b}{P_c} \right)^\lambda \right)^2 \left(1 - \left(\frac{P_b}{P_c} \right)^{2+\lambda} \right) \quad (13)$$

for $P_c \geq P_b$.

Equations 9 to 13 present the theory of Brooks and Corey for the properties of porous media.

Theoretically, λ could have any positive value greater than zero, being small for media having a wide range of pore sizes and large for media with a relatively uniform pore size. The theoretical lower limit of η is 2. Values of η for different fractions of sand used for drainage test can be seen from the capillary pressure-relative permeability data in Appendix A.

Having become familiar with Brooks and Corey's theory for properties of porous media, it is more convenient to discuss their theory for similitude requirements.

The assumption of Miller and Miller (14), that is, isotropic porous systems containing two immiscible fluid phases (air and water) in which the air pressure is considered to be constant but both air and water are continuous phases, were accepted by Brooks and Corey (2).

Their analysis is in many respects the same as Miller and Miller's, but it is restricted to the drainage cycle in order to avoid the complication of hysteresis. In addition, the theoretical functional relationships among P_c , S_e , K_{rw} and K_{rnw} are used to simplify the resulting expressions for the criteria of similitude.

They begin from the Richards equation which is obtained from combining Darcy's equation and the continuity equation. Darcy's equation is

$$q = \frac{\gamma K}{\mu} \nabla \left(\frac{P}{\gamma} + z \right) \quad (14)$$

and the continuity equation

$$\text{div } q = -\phi \frac{\partial S}{\partial t} \quad (15)$$

They substitute ϕ_e for ϕ , the "effective porosity", which is given by

$$\phi_e = (1 - S_r) \phi \quad (16)$$

and S_e for S and it gives

$$\text{div} \left[\frac{\gamma K_e}{\mu} \nabla \left(\frac{P}{\gamma} + z \right) \right] = \phi_e \frac{\partial S_e}{\partial t} \quad (17)$$

This equation is then written in a dimensionless form by scaling the variables in the equation. They scale the variables by choosing standard units of permeability, length, pressure and time which are known to be significant characteristics of the system. The standard units are designated as K_0 , L_0 , P_0 and t_0 respectively. the standard unit K_0 is the fully-saturated (maximum) permeability K of the medium.

Using the standard units as scaling factors, equation 17 becomes

$$\frac{t_0 K_0 \gamma}{L_0 \mu \phi_e} \operatorname{div.} \left[K_r \nabla \cdot \left(\frac{P}{P_0} + \frac{z \gamma}{P_0} \right) \right] = \frac{\partial S_e}{\partial (t/t_0)} \quad (18)$$

where the dots after the divergence and gradient operator symbols indicate that the operations are performed with respect to the scaled coordinate length.

As standard units they chose bubbling pressure, P_b for P_0 . In this case, however, it is necessary to use P_b/γ as the standard unit of length. Similarly the appropriate standard unit of time is $\phi_e P_b \mu / K \gamma^2$. Scaled variables are obtained by dividing the unscaled variables by their respective standard units. Then, equation 18 becomes

$$\operatorname{div.} \left[K_r \nabla \cdot (P. + z.) \right] = \frac{\partial S_e}{\partial \tau} \quad (19)$$

where the dots designate scaled variables or operators with respect to scaled variables.

Equation 19 will yield identical particular solutions in terms of scaled variables provided that

(1) geometric similitude exists

(2) the boundary and initial conditions in terms of scaled variables are identical

(3) the functional relationships among K_r , S_e and P_c/P_b are identical for the both systems.

Scaled variables resulting from Brooks and Corey's theory (from Corey et. al.(6)) are:

Microscopic geometry

$$P. \equiv \frac{1}{P_b} P \quad \text{Pressure of wetting phase}$$

$$\eta. \equiv \eta \quad \text{Pore size distribution index}$$

Macroscopic variables

$$L. \equiv \frac{\rho g}{P_b} L \quad \text{Length}$$

$$t. \equiv \frac{(\rho g)^2 K}{P_b \mu \phi_e} t \quad \text{Time}$$

$$q. \equiv \frac{\mu}{\rho g K} q \quad \text{Flow volume flux}$$

$$Q. \equiv q.t. = \frac{\rho g}{P_b \phi_e} q t = \frac{\rho g}{P_b \phi_e} Q \quad \text{Flow volume}$$

The properties of the medium

$$K. \equiv K_r \equiv \frac{K_e}{K} \quad \text{Permeability}$$

$$S. \equiv S_e \equiv \frac{S-S_r}{1-S_r} \quad \text{Degree of saturation}$$

$$\phi. \equiv \phi_e \equiv (1-S_r)\phi \quad \text{Porosity}$$

In the approach of Brooks and Corey (2) the microscopic variables are taken care of implicitly by using a pore-size distribution index (λ or η) and bubbling pressure, P_b , while Miller and Miller (14) have explicitly scaled microscopic variables.

Laliberte et. al. (12) who have examined some of the problems associated with the theory of Brooks and Corey indicate that the pore-size distribution index is changed only slightly over a wide range of porosities but permeability and bubbling pressure may be changed several fold over the same range.

Corey et. al. (6) tested the validity of Brooks and Corey's scaling criteria for one-dimensional unsteady flow through a single layer of sand and obtained exceptional results. They also found that the value of the parameter η does significantly affect the drainage process. Differences in η , however, are more pronounced with low values than with high values of η . They comment that prototype materials possessing high η -values (above 15) could undoubtedly be modeled with materials having η -values within ± 20 percent of the prototype media. However, media with low values (3-8) should be modeled with media having η -values within ± 5 percent of the true values.

Corey (5) offers the following criteria for similitude for layered media:

Consider m layers of media with the flow being perpendicular to the layers. Individual layers between

model and prototype are scaled if

$$\left(\frac{L \rho g}{P_b}\right)'_n = \left(\frac{L \rho g}{P_b}\right)_n \quad n = 1, 2, 3 \dots m \quad (20)$$

$$\left(\frac{t K (\rho g)^2}{P_b \mu \phi_e}\right)'_n = \left(\frac{t K (\rho g)^2}{P_b \mu \phi_e}\right)_n \quad n = 1, 2, 3 \dots m \quad (21)$$

$$\eta'_n = \eta_n \quad n = 1, 2, 3 \dots m \quad (22)$$

where the primed quantities refer to the model and the unprimed to the prototype.

It is highly unlikely that layers of media in a prototype would exhibit the same length and time scales. In other words, in general

$$\left(\frac{L}{P_b}\right)_1 \neq \left(\frac{L}{P_b}\right)_2 \neq \dots \neq \left(\frac{L}{P_b}\right)_m \quad (23)$$

$$\left(\frac{tK}{P_b \phi_e}\right)_1 \neq \left(\frac{tK}{P_b \phi_e}\right)_2 \neq \dots \neq \left(\frac{tK}{P_b \phi_e}\right)_m \quad (24)$$

The fluid properties have been omitted since each layer is assumed to contain the same fluid. Then, there are unique length and time scales for each prototype layer. These scales are linearly related or

$$L_{.1} = a_2 L_{.2} = a_3 L_{.3} = \dots a_m L_{.m} \quad (25)$$

$$t_{.1} = b_2 t_{.2} = b_3 t_{.3} = \dots b_m t_{.m} \quad (26)$$

where the a 's and b 's are constants dependent upon the properties of the individual layers.

In order to model such layers, the same ratio of length and time scales must exist among the model layers, or

$$L!_1 = a_2 L!_2 = a_3 L!_3 = \dots a_m L!_m \quad (27)$$

$$t!_1 = b_2 t!_2 = b_3 t!_3 = \dots b_m t!_m \quad (28)$$

Should one be able to secure materials to effectively meet these criteria, the different length and time scales would still exist among the various model layers.

Equations 20 and 28 according to Corey (5) are the criteria necessary to model layered soils. There is the further requirement that each layer must be treated separately since each has unique length and time scales.

Further it is suggested that because of the difficulty in meeting the above criteria it is possible to use prototype materials for the model and to effect the modeling by using a different liquid in the model. The larger the scale reduction desired the lower the surface tension of liquid necessitated in the model.

In the previous pages some of the different criteria for similitude were mentioned. As could be seen there are very few discussions concerning layered media. Here, the author would like to comment on the theory for layered soils suggested by Corey (5) and to mention another criterion which has not been pointed out for this case.

It is apparent if Brooks and Corey's scaling laws are followed equations 25 and 27 proposed by Corey (5) will

be satisfied. Considering scaled times, equations 21 and 26 and 28 theoretically are correct but in practice it is almost impossible to apply different time scales to different layers. The author in doing experiments for non-steady drainage of two layers of sand has noticed that when the top layer is still draining the second layer begins to drain even when the coarser layer is over the finer layer. In other words, at a period of time, which is not necessarily short, both layers contribute simultaneously to the flow. The reason for such behavior, probably, is that the largest pores (which drain first) in the finer layer are larger than the smallest pores in the coarser layer. It would be possible to distinguish the time in which each layer has contributed to the flow if the top layer is much coarser than the bottom layer (or layers) which is a very special case.

A very important characteristic in modeling is that the distribution of scaled pressure is identical for model and prototype. For the single-layer case Brooks and Corey's criteria fully satisfy this requirement. For layered soils another necessary condition must be satisfied to achieve this as the following discussion shows.

Consider a prototype, in a one-dimensional case, in which there are two layers of different materials with length L_1 and L_2 and bubbling pressures P_{b_1} and P_{b_2} . Also consider a model with two layers of material different from those of prototype with lengths L'_1 and L'_2 and

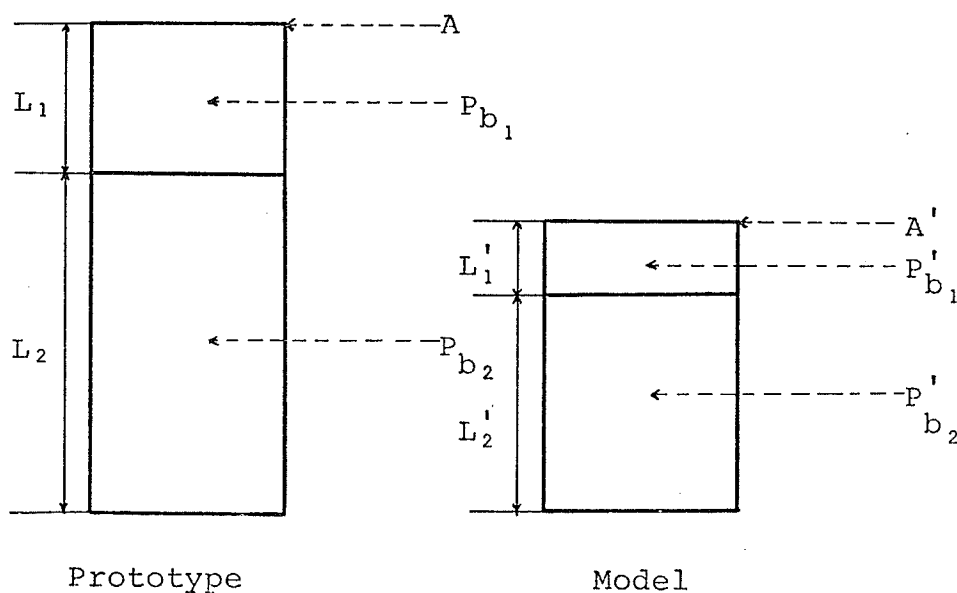
bubbling pressures P_{b_1}' and P_{b_2}' . It must be pointed out that Brooks and Corey's scaling laws have been applied to them. In other words, the following relations exist:

$$\frac{L_1}{P_{b_1}/\gamma} = \frac{L_1'}{P_{b_1}'/\gamma} \quad \text{or} \quad L_1' = \frac{L_1 \times P_{b_1}'/\gamma}{P_{b_1}/\gamma} \quad (29)$$

$$\frac{L_2}{P_{b_2}/\gamma} = \frac{L_2'}{P_{b_2}'/\gamma} \quad \text{or} \quad L_2' = \frac{L_2 \times P_{b_2}'/\gamma}{P_{b_2}/\gamma} \quad (30)$$

where the primed quantities refer to the model and the unprimed to the prototype.

Consider a section in the top layer of the prototype and the similar section in the model. It can be any section but for the sake of specificity sections A and A', the top of the prototype and model, are chosen (refer to diagram below).



Scaled capillary pressure head at A = $\frac{L_1 + L_2}{P_{b_1} / \gamma}$ as time approaches ∞ (31)

Scaled capillary pressure head at A' = $\frac{L_1' + L_2'}{P_{b_1}' / \gamma}$ as time approaches ∞ (32)

These two quantities must be equal, or

$$\frac{L_1 + L_2}{P_{b_1}} = \frac{L_1' + L_2'}{P_{b_1}'} \quad (33)$$

$\gamma = \rho g$ have been omitted because for both model and prototype it is the same provided the same liquid is used in both model and prototype.

Substituting L_1' and L_2' in equation 33 by their respective equations 29 and 30 gives

$$\frac{L_1 + L_2}{P_{b_1}} = \frac{\frac{L_1 \times P_{b_1}'}{P_{b_1}} + \frac{L_2 \times P_{b_2}'}{P_{b_2}}}{P_{b_1}'} \quad (34)$$

or

$$\frac{L_1 + L_2}{P_{b_1}} = \frac{L_1 \times P_{b_1}' \times P_{b_2} + L_2 \times P_{b_2}' \times P_{b_1}}{P_{b_1}' \times P_{b_1} \times P_{b_2}} \quad (35)$$

Simplifying equation 35 gives

$$L_1 + L_2 = L_1 + \frac{L_2 \times P_{b_1} \times P_{b_2}'}{P_{b_2} \times P_{b_1}'} \quad (36)$$

or

$$1 = \frac{P_{b_1} \times P_{b_2}'}{P_{b_2} \times P_{b_1}'} \quad (37)$$

$$\frac{P_{b_1}}{P_{b_2}} = \frac{P'_{b_1}}{P'_{b_2}} \quad \text{or} \quad \frac{P_{b_1}}{P'_{b_1}} = \frac{P_{b_2}}{P'_{b_2}} \quad (38)$$

Equation 38 is a very important condition which must be satisfied if the modeling for layered soils is done by using Brooks and Corey's theory. It will be shown later if this condition does not exist among the layers the model behavior is completely different from that of the prototype.

Using the same reasoning for any other section will give the above result. However, it should be pointed out when the analysis is done for the bottom layer there will not be any such condition derived because the analysis is the same as drainage from a single layer for which the newly derived condition is superfluous. However, for two or more layers the general condition must be satisfied, that is,

$$\frac{P_{b_1}}{P'_{b_1}} = \frac{P_{b_2}}{P'_{b_2}} = \frac{P_{b_3}}{P'_{b_3}} \dots\dots\dots = \frac{P_{b_n}}{P'_{b_n}} \quad (39)$$

where $n = 1, 2, 3 \dots m$ layers.

CHAPTER III

EXPERIMENTAL PROCEDURE

The experiments were done for one-dimensional nonsteady flow through two layers of material following Brooks and Corey's (2) scaling laws.

All the tests, model and prototype, were done with columns of sand packed into plastic tubes. The cross-sectional area of columns were 11.40 cm^2 and underneath the sand there was constructed a coarse filter to retain the sand and leading to an outflow tube. The technique was to use a filter coarse enough to reduce the head loss and fine enough to hold the finest grains of the sand as well. These tests were different from those by Corey et. al. (6) since air was allowed to enter only from the top of the tube. Otherwise, beside the fact that layered soils were used in this study, the procedures were essentially the same.

Media and Liquid

Different fractions of Selkirk silica sand, a product of the Winnipeg Supply and Fuel Company, Winnipeg, Manitoba, were used as media. The sieve analysis of this sand is shown in Table 1B, Appendix B. These data were supplied to the author by Dr. K.M.Adam.

The liquid used was a hydrocarbon oil, which is a

core test fluid referred to as soltrol "C". It has the advantage over water that the physical structure of the media remains much more stable in its presence. Also it has a low surface tension (22.9 dynes/cm) and facilitates experimental work involving a mixture of liquid and air. Moreover, the model size can be reduced appreciably if this oil is used instead of water.

The dynamic viscosity, the density and the ratio of dynamic viscosity to specific weight are tabulated in Table 2B, Appendix B as reported by Laliberte et. al. (12).

Determining Media Properties

The properties of different fractions of sand were determined before any decision for the modeling study was being made. This was due especially to the fact that η must be almost the same for corresponding layers in the model and the prototype. The required data were mostly obtained from the capillary pressure-relative permeability and capillary pressure-desaturation data. These relations for six fractions of sand used in the drainage tests as well as a brief discussion about how they were obtained are in Appendix A. The capillary pressure-relative permeability data as well as particle densities, ρ_s , were supplied to the author by Dr. K.M. Adam.

Having chosen the combinations for prototypes and models, the materials were packed in the tubes and were vacuum-saturated. Before running the sample for the non-steady drainage test the saturated permeability of the sample, that is, the permeability of both layers, was measured by the falling head permeability method. This was done because, as mentioned before, assigning different time factors for each layer is not possible and for the whole sample there must be one time factor as well as one discharge factor, that is, a unique value of $\frac{(\rho g)^2 K}{\phi_e P_b \mu}$ and $\frac{\rho g}{P_b \phi_e}$, respectively.

The effective porosity for each layer was calculated using the equation

$$\phi_e = (1 - S_r) \phi$$

where $\phi = 1 - \frac{\rho_b}{\rho_s}$ and ρ_s is 2.65 since Selkirk silica sand is 99.9 percent pure silica.

Testing the Theory - Details of Tests

Six different fractions of Selkirk silica sand were chosen for nonsteady drainage tests. In Tables 1 and 2 their values of P_b/γ and η and also the ratio of their bubbling pressures has been shown.

The proper fractions for prototypes and models were chosen and the length of each layer in the prototype was selected arbitrarily. The length of the corresponding layers in the model was then calculated using the relation

Table 1
Pore-size Distribution Index and Bubbling
Pressure for Different Fractions of
Selkirk Silica Sand

Fraction*	η	P_b/γ cm
150-200	19.71	37.55
100-150	18.03	28.41
65-100	18.20	18.51
48-65	20.30	14.67
35-48	20.00	10.16
28-35	18.43	7.68

*Tyler standard.

Table 2
Ratio of Bubbling Pressures of Different Fractions of Selkirk Silica Sand

$\frac{P_b \text{ 150-200}}{P_b \text{ 100-150}} = 1.32$					
$\frac{P_b \text{ 150-200}}{P_b \text{ 65-100}} = 2.03$	$\frac{P_b \text{ 100-150}}{P_b \text{ 65-100}} = 1.53$				
$\frac{P_b \text{ 150-200}}{P_b \text{ 48-65}} = 2.56$	$\frac{P_b \text{ 100-150}}{P_b \text{ 48-65}} = 1.94$	$\frac{P_b \text{ 65-100}}{P_b \text{ 48-65}} = 1.26$			
$\frac{P_b \text{ 150-200}}{P_b \text{ 35-48}} = 3.70$	$\frac{P_b \text{ 100-150}}{P_b \text{ 35-48}} = 2.98$	$\frac{P_b \text{ 65-100}}{P_b \text{ 35-48}} = 1.82$	$\frac{P_b \text{ 48-65}}{P_b \text{ 35-48}} = 1.44$		
$\frac{P_b \text{ 150-200}}{P_b \text{ 28-35}} = 4.89$	$\frac{P_b \text{ 100-150}}{P_b \text{ 28-35}} = 3.70$	$\frac{P_b \text{ 65-100}}{P_b \text{ 28-35}} = 2.41$	$\frac{P_b \text{ 48-65}}{P_b \text{ 28-35}} = 1.91$	$\frac{P_b \text{ 35-48}}{P_b \text{ 28-35}} = 1.32$	

$$\frac{L}{P_b/\gamma} = \frac{L'}{P_b'/\gamma}$$

where the primed quantities refer to the model and unprimed to the prototype. The samples were vacuum-saturated and then were allowed to drain. The instant that the sample began to drain, time measurement was started and outflow measurement versus time was observed. The measurements were continued until the flow rate was very, very small. The data were then properly scaled and the scaled discharge, $Q_s = \frac{\rho g}{P_b \phi_e} Q$ was plotted versus scaled time, $t_s = \frac{K(\rho g)^2}{P_b \mu \phi_e} t$.

For the prototype the two fractions of Selkirk silica sand chosen were 150-200 and 100-150. The ratio of the bubbling pressure of the finer layer (150-200) to the coarser layer (100-150) is $37.55/28.41 = 1.32$. Two prototypes were run, one with the 150-200 layer on the top and the 100-150 at the bottom and the other, the 100-150 layer on the top and the 150-200 layer at the bottom. With each prototype three models were run, one with the ratio of bubbling pressure of the finer layer to that of the coarser layer exactly the same as that of the prototype (35-48, 28-35). The second one with the ratio slightly different and the third one with the ratio appreciably different to prove the criterion that

$$\frac{P_{b_1}}{P_{b_2}} = \frac{P_{b_1}'}{P_{b_2}'}$$

The details of different arrangements and results are shown in the next section.

Here, it should be pointed out that for the second series of tests, that is, with the finer layer at the bottom, suction equal to 0.6 times the bubbling pressure head of the bottom layer was applied at the bottom of the samples, for both model and prototype, and because this suction is scaled it does not change the results. That is to say $P_c \equiv \frac{P_c}{P_b}$

To calculate Q and t , the following parameters were used in the equations $Q = \frac{\rho g}{P_b \phi_e} Q$ and $t = \frac{(\rho g)^2 K}{P_b \phi_e} t$.

For P_b the value obtained from the capillary pressure-relative permeability was used. For ϕ_e , the effective porosity for each layer was weighted according to the lengths of two layers and the average value was used. K was the saturated permeability for the whole sample as was measured by the falling head permeability method. Furthermore, temperatures were taken during the tests and the values of $\mu/\rho g$ were read from Table 2B, Appendix B.

Because of the different ratios of bubbling pressures when the criterion $\frac{P_{b1}}{P'_{b1}} = \frac{P_{b2}}{P'_{b2}}$ is not satisfied, for each test two scaled discharges and scaled times, that is, $Q = \frac{\rho g}{P_b \phi_e} Q$ and $t = \frac{(\rho g)^2 K}{P_b \phi_e \mu} t$ were calculated, one using the bubbling pressure of the finer layer and the other the bubbling pressure of the coarser layer.

The nonsteady drainage data for the prototypes and models are shown in Appendix C.

CHAPTER IV

DETAILS OF TESTS AND RESULTS

SERIES NO. 1

Prototype No. 1

Fraction	L cm	η	$P_b/\rho g$ cm	S_r	ϕ	ϕ_e
150-200	30	19.71	37.55	0.127	0.378	0.330
100-150	60	18.03	28.41	0.101	0.357	0.318
$\phi_{e \text{ ave}} = \frac{30 \times 0.330 + 60 \times 0.318}{30 + 60} = 0.322$				$\frac{P_b \text{ 150-200}}{P_b \text{ 100-150}} = 1.32$		
$K = 5.40 \mu^2 = 5.40 \times 10^{-8} \text{ cm}^2$				Suction = 0		

Considering P_b of the finer layer

$$Q_s = 0.08270 Q$$

$$t_s = 0.4466 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

Considering P_b of the coarser layer

$$Q_s = 0.1093 Q$$

$$t_s = 0.5903 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

Model No. 1A

Fraction	L cm	η	$P_b/\rho g$ cm	S_r	ϕ	ϕ_e
35-48	8.12	20.00	10.16	0.101	0.303	0.272
28-35	16.22	18.43	7.68	0.100	0.353	0.318
$\phi_e \text{ ave} = \frac{8.12 \times 0.272 + 16.22 \times 0.318}{8.12 + 16.22} = 0.303 \quad \frac{P_b \text{ 35-48}}{P_b \text{ 28-35}} = 1.32$						
$K = 73.09 \mu^2 = 73.09 \times 10^{-8} \text{ cm}^2 \quad \text{Suction} = 0$						

Considering P_b of the finer layer

$$Q_e = 0.3249 Q$$

$$t_e = 11.31 \times 10^{-3} t \text{ (temperature the same during the test)}$$

Considering P_b of the coarser layer

$$Q_e = 0.4297 Q$$

$$t_e = 14.96 \times 10^{-3} t$$

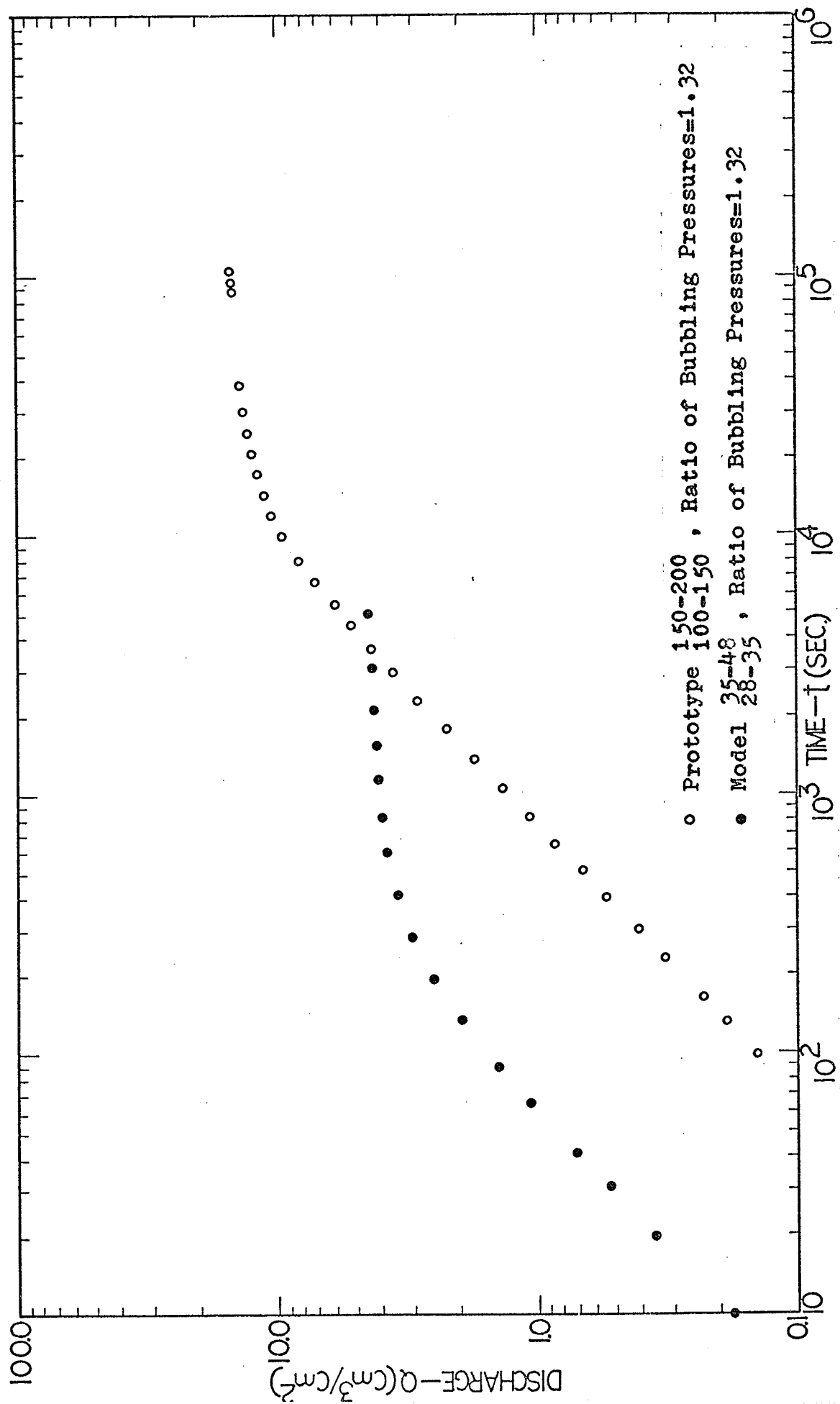


Figure 1. Discharge-time relationship of unsteady drainage from prototype No.1 and model No.1A.

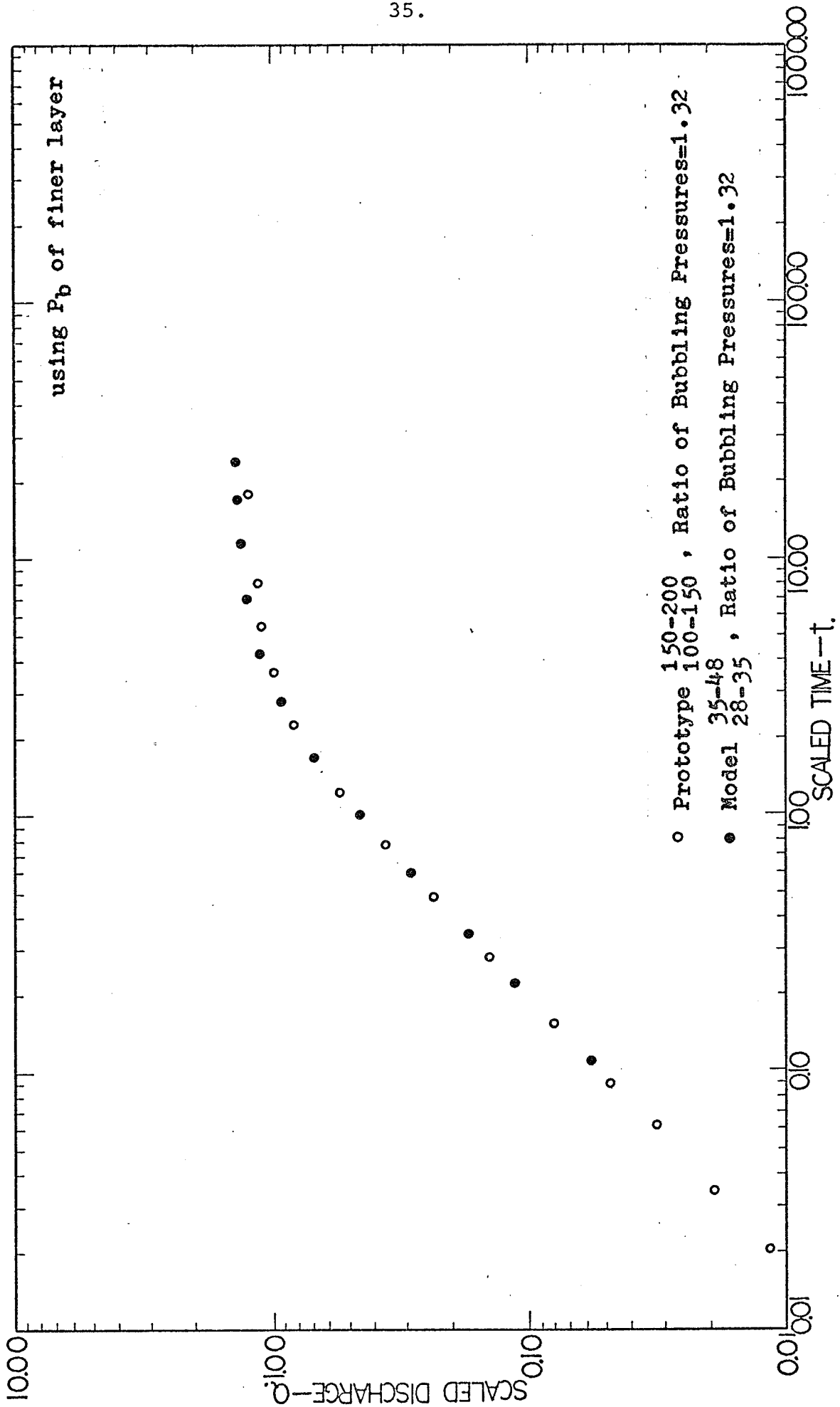


Figure 2. Scaled discharge-time relationship of unsteady drainage from prototype No.1 and model No.1A.

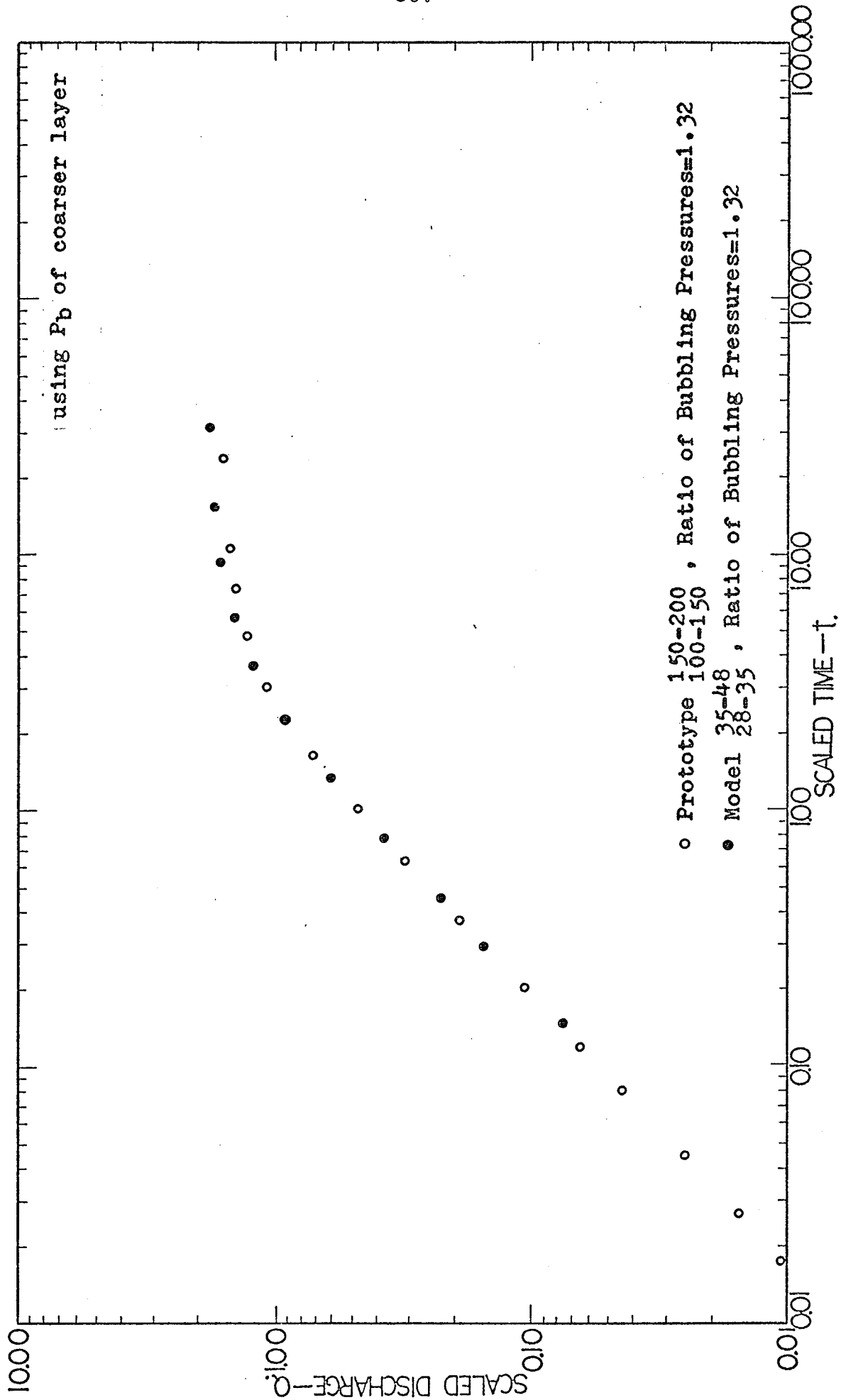


Figure 3. Scaled discharge-time relationship of unsteady drainage from prototype No.1 and model No.1A.

Model No. 1B

Fraction	L cm	η	$P_b/\rho g$ cm	S_r	ϕ	ϕ_e
65-100	14.79	18.20	18.51	0.112	0.332	0.295
48-65	30.98	20.30	14.67	0.100	0.334	0.301
$\phi_e \text{ ave} = \frac{14.79 \times 0.295 + 30.98 \times 0.301}{14.79 + 30.98} = 0.299 \quad \frac{P_b \text{ 65-100}}{P_b \text{ 48-65}} = 1.26$						
$K = 19.59 \mu^2 = 19.59 \times 10^{-8} \text{ cm}^2 \quad \text{Suction} = 0$						

Considering P_b of the finer layer

$$Q_e = 0.1807 Q$$

$$t_e = 3.540 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

Considering P_b of the coarser layer

$$Q_e = 0.2280 Q$$

$$t_e = 4.467 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

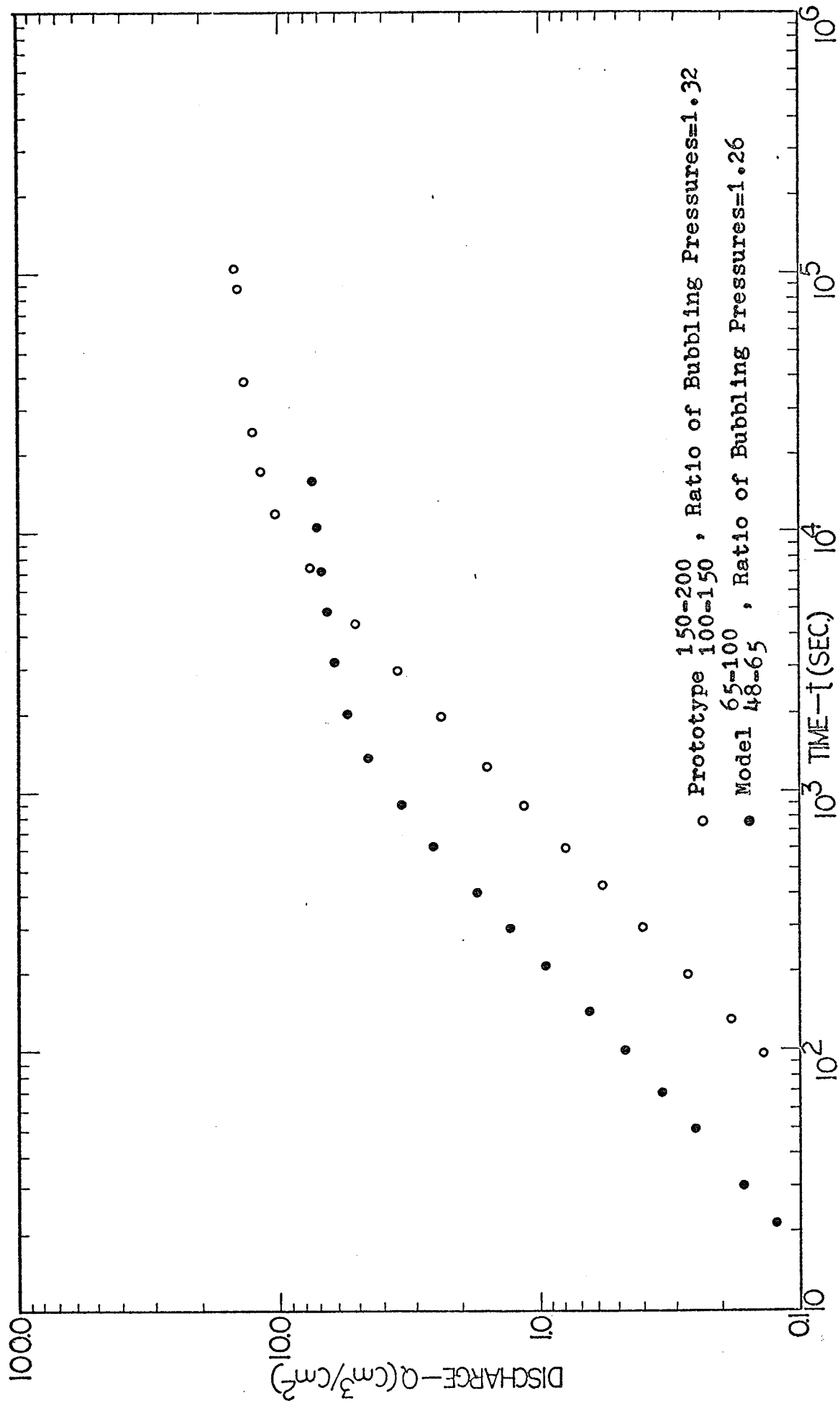


Figure 4. Discharge-time relationship of unsteady drainage from prototype No.1 and model No.1B.

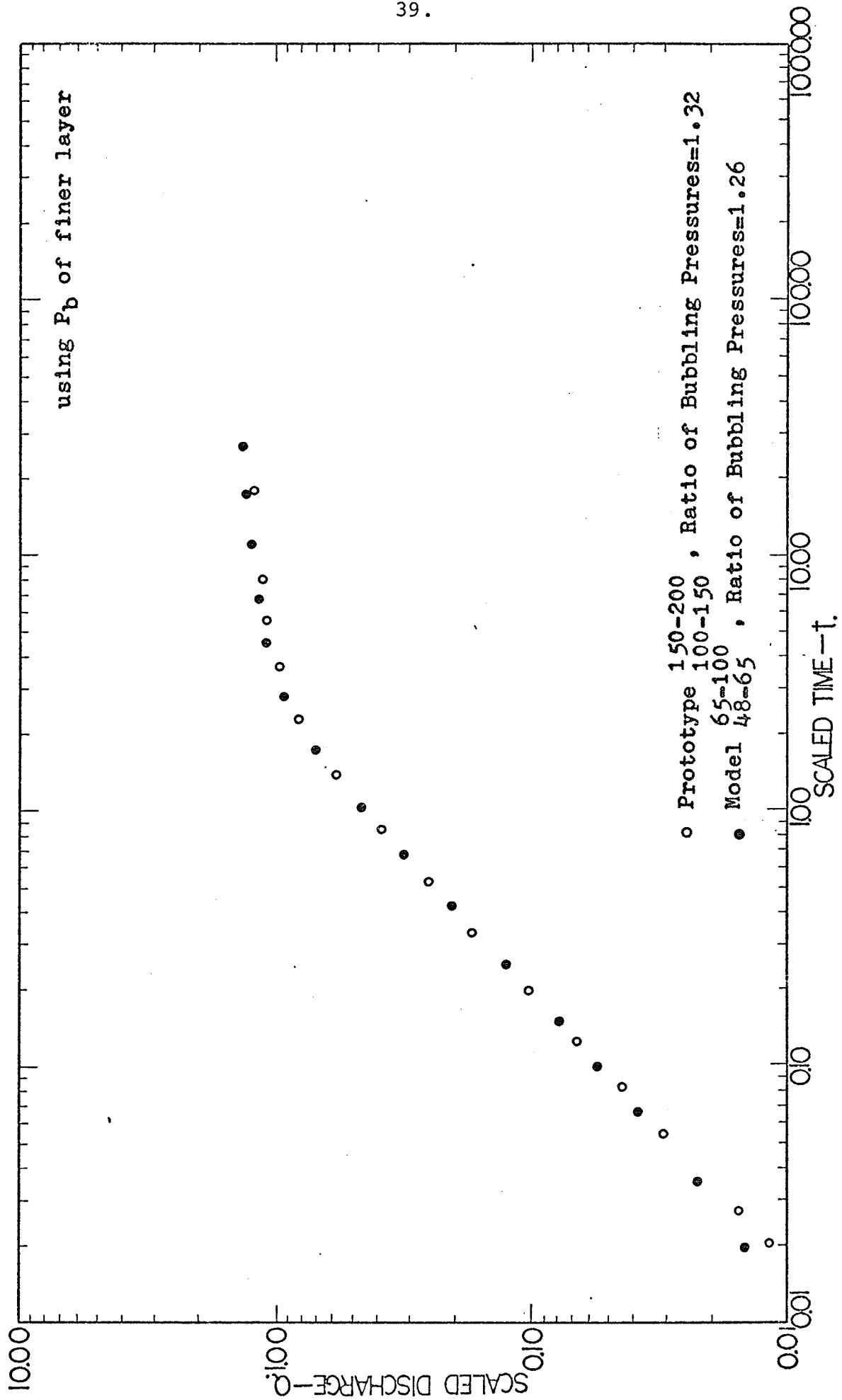


Figure 5. Scaled discharge-time relationship of unsteady drainage from prototype No.1 and model No.1B.

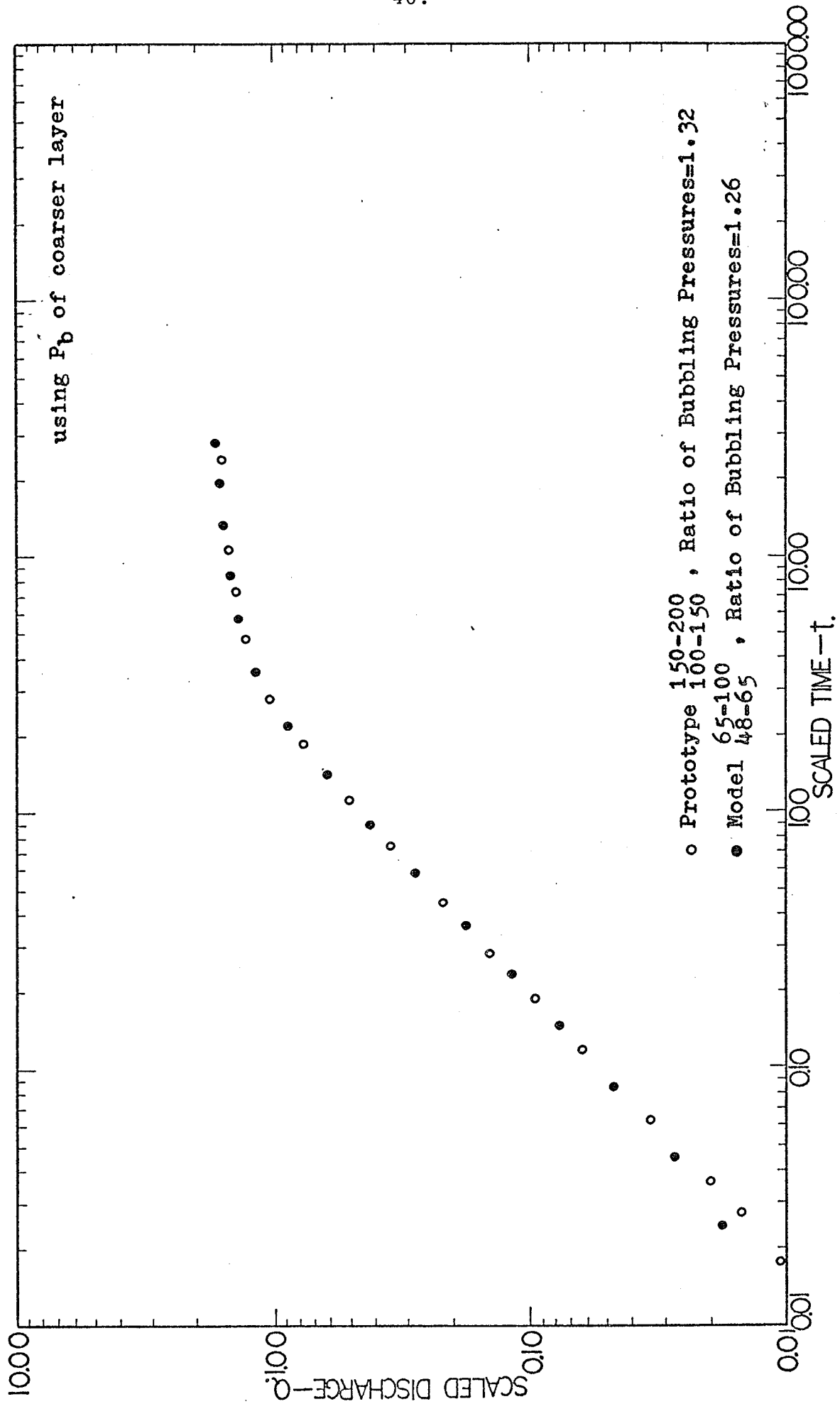


Figure 6. Scaled discharge-time relationship of unsteady drainage from prototype No.1 and model No.1B.

Model No. 1C

Fraction	L cm	η	$P_b/\rho g$ cm	S_r	ϕ	ϕ_e
65-100	14.79	18.20	18.51	0.112	0.304	0.270
35-48	21.50	20.00	10.16	0.101	0.325	0.292
$\phi_e \text{ ave} = \frac{14.79 \times 0.270 + 21.50 \times 0.292}{14.79 + 21.50} = 0.283$ $\frac{P_b \text{ 65-100}}{P_b \text{ 35-48}} = 1.82$						
$K = 21.06 \mu^2 = 21.06 \times 10^{-8} \text{ cm}^2$				Suction = 0		

Considering P_b of the finer layer

$$Q_e = 0.1909 Q$$

$$t_e = 4.021 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

Considering P_b of the coarser layer

$$Q_e = 0.3478 Q$$

$$t_e = 7.325 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

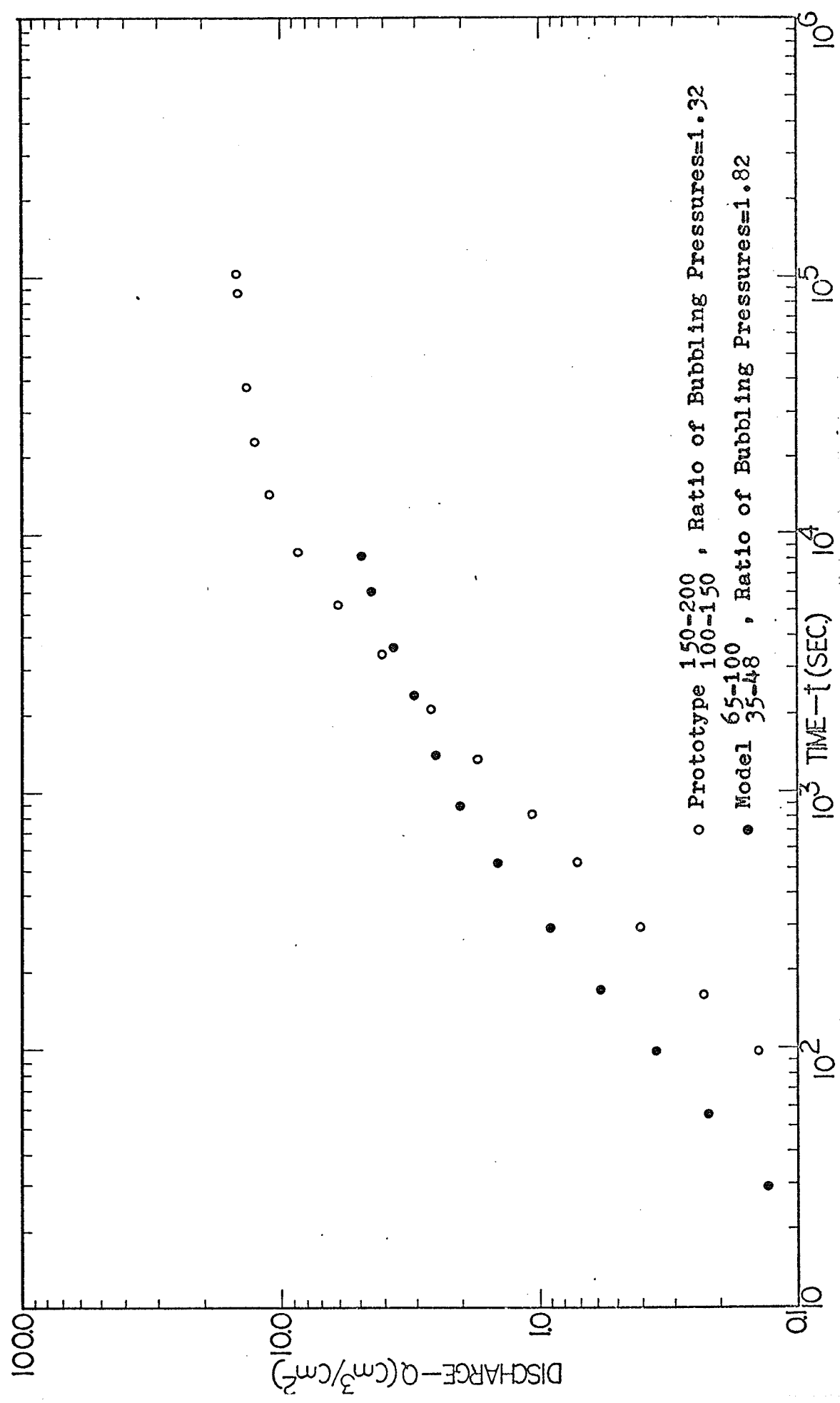


Figure 7. Discharge-time relationship of unsteady drainage from prototype No.1 and model No.1C.

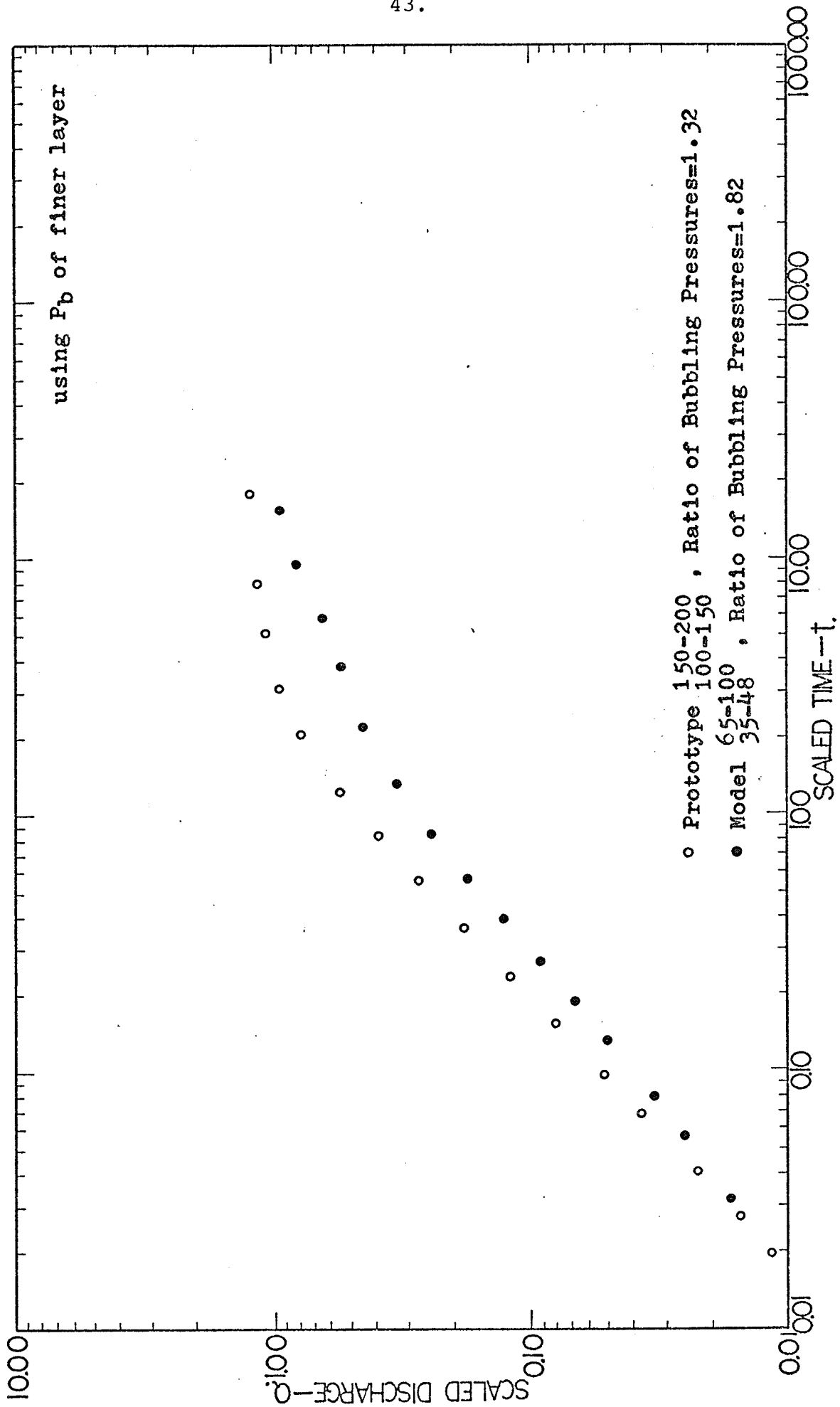


Figure 8. Scaled discharge-time relationship of unsteady drainage from prototype No.1 and model No.1C.

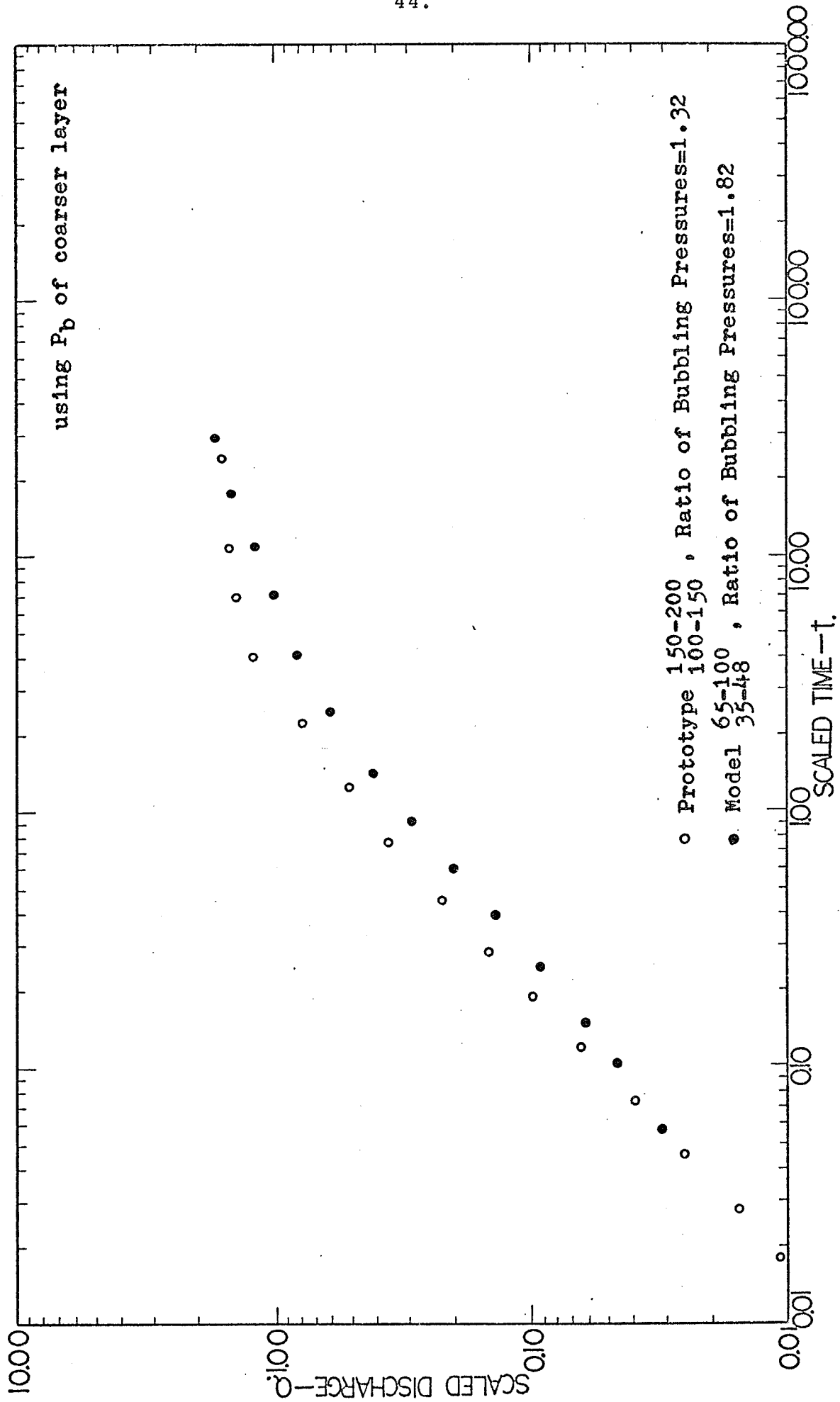


Figure 9. Scaled discharge-time relationship of unsteady drainage from prototype No.1 and model No.1C.

SERIES NO. 2

Prototype No. 2

Fraction	L cm	η	$P_b/\rho g$ cm	S_r	ϕ	ϕ_e
100-150	30	18.03	28.41	0.110	0.369	0.328
150-200	45	19.71	37.55	0.127	0.389	0.340

$$\phi_e \text{ ave} = \frac{30 \times 0.328 + 45 \times 0.340}{30 + 45} = 0.335 \quad \frac{P_b \text{ 150-200}}{P_b \text{ 100-150}} = 1.32$$

$$K = 5.72 \mu^2 = 5.72 \times 10^{-8} \text{ cm}^2 \quad \text{Suction} = .6 P_b \text{ 150-200} = 22.53 \text{ cm}$$

Considering P_b of the finer layer

$$Q_s = 0.07950 Q$$

$$t_s = 0.4547 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

Considering P_b of the coarser layer

$$Q_s = 0.1051 Q$$

$$t_s = 0.6010 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

Model No. 2A

Fraction	L cm	η	$P_b/\rho g$ cm	S_r	ϕ	ϕ_e
28-35	8.11	18.43	7.68	0.100	0.332	0.299
35-48	12.18	20.00	10.16	0.101	0.324	0.291
$\phi_e \text{ ave} = \frac{8.11 \times 0.299 + 12.18 \times 0.291}{8.11 + 12.18} = 0.294$ $\frac{P_b \text{ 35-48}}{P_b \text{ 28-35}} = 1.32$						
$K = 58.89 \mu^2 = 58.89 \times 10^{-8} \text{ cm}^2 \quad \text{Suction} = .6 P_b \text{ 35-48} = 6.10 \text{ cm}$						

Considering P_b of the finer layer

$$Q_s = 0.3348 Q$$

$$t_s = 9.287 \times 10^{-3} t \quad (\text{temperature the same during test})$$

Considering P_b of the coarser layer

$$Q_s = 0.4429 Q$$

$$t_s = 12.28 \times 10^{-3} t$$

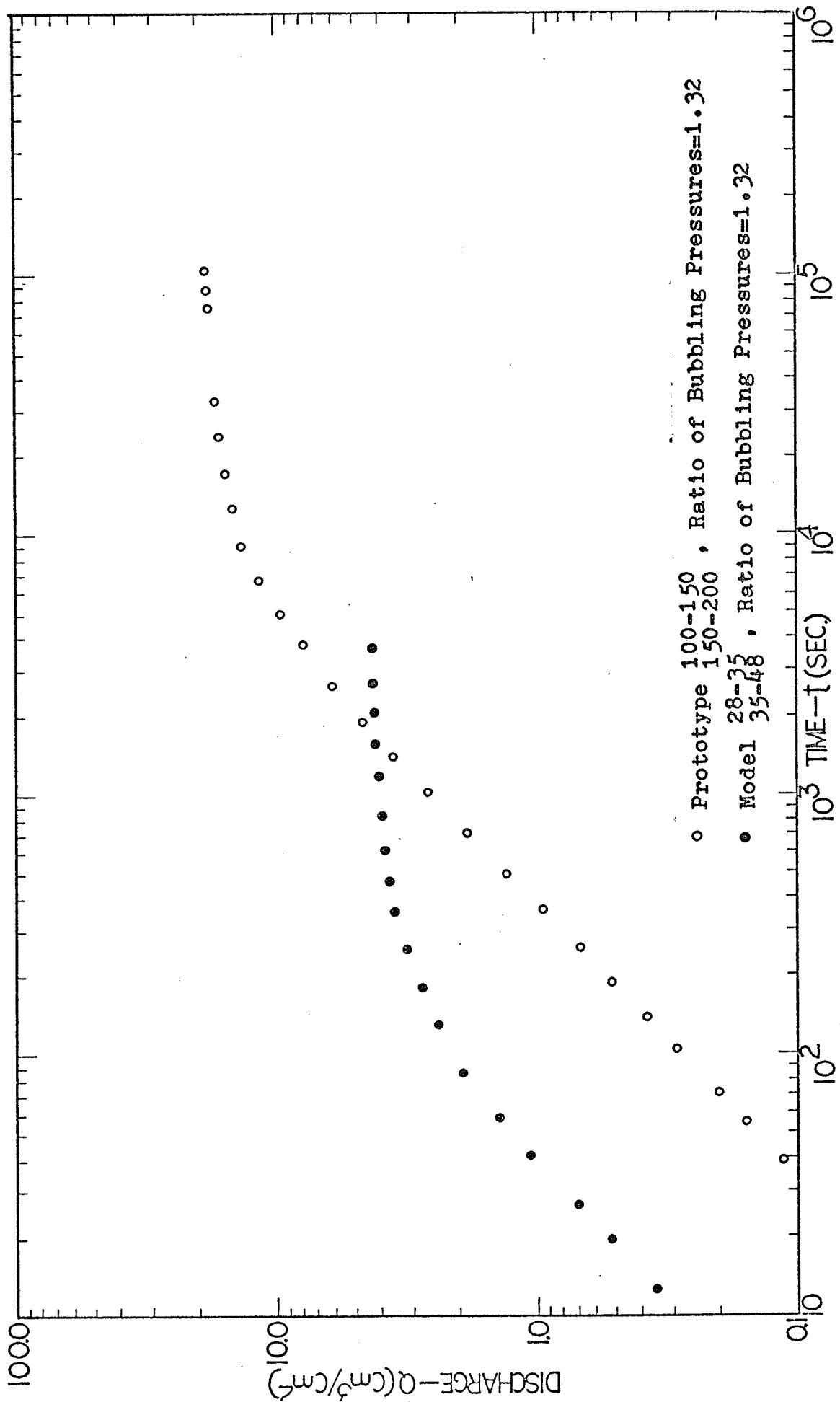


Figure 10. Discharge-time relationship of unsteady drainage from prototype No. 2 and model No. 2A.

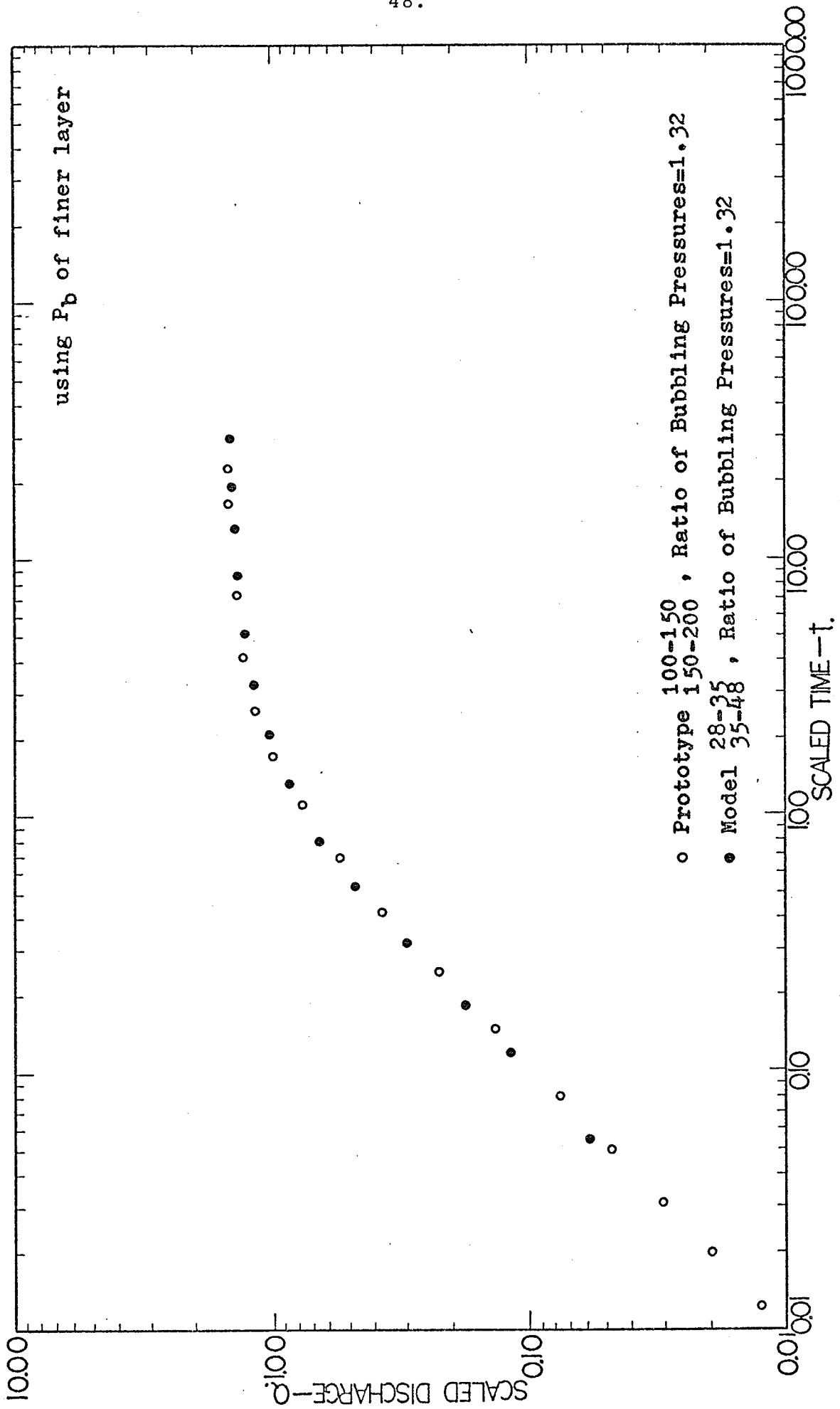


Figure 11. Scaled discharge-time relationship of unsteady drainage from prototype No.2 and model No.2A.

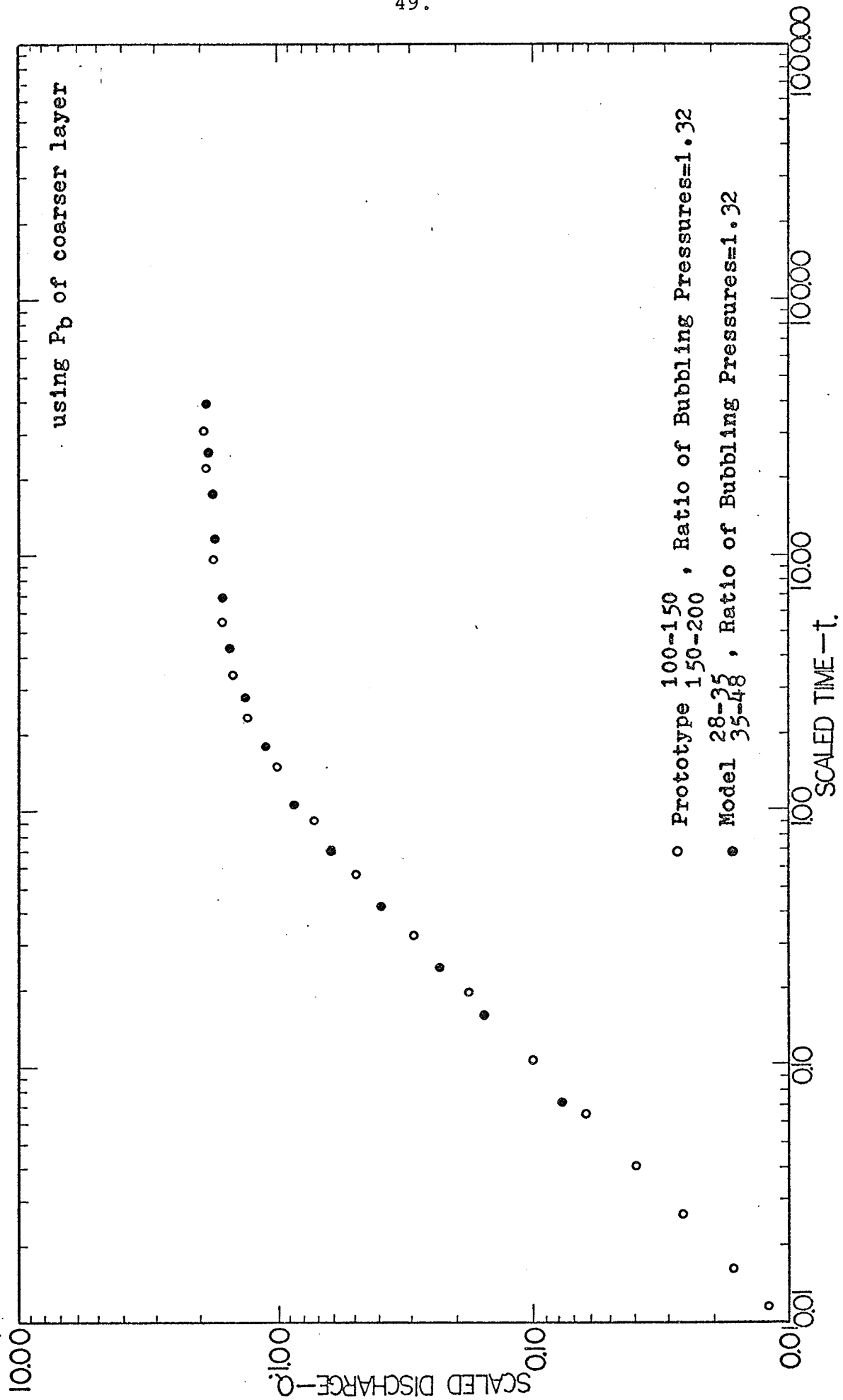


Figure 12. Scaled discharge-time relationship of unsteady drainage from prototype No.2 and model No.2A.

Model No. 2B

Fraction	L cm	η	$\bar{P}_b/\rho g$ cm	S_r	ϕ	ϕ_e
35-48	10.73	20.00	10.16	0.101	0.349	0.314
48-65	17.58	20.30	14.67	0.100	0.351	0.316

$$\phi_e \text{ ave} = \frac{10.73 \times 0.314 + 17.58 \times 0.316}{10.73 + 17.58} = 0.315 \quad \frac{P_b \text{ 48-65}}{P_b \text{ 35-48}} = 1.44$$

$$K = 30.40 \mu^2 = 30.40 \times 10^{-8} \text{ cm}^2 \quad \text{Suction} = .6 P_b \text{ 48-65} = 8.80 \text{ cm}$$

Considering P_b of the finer layer

$$Q_s = 0.2164 Q$$

$$t_s = 6.579 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

Considering P_b of the coarser layer

$$Q_s = 0.3125 Q$$

$$t_s = 9.500 \times \frac{1}{\mu/\rho g \times 10^5} \times 10^{-3} t$$

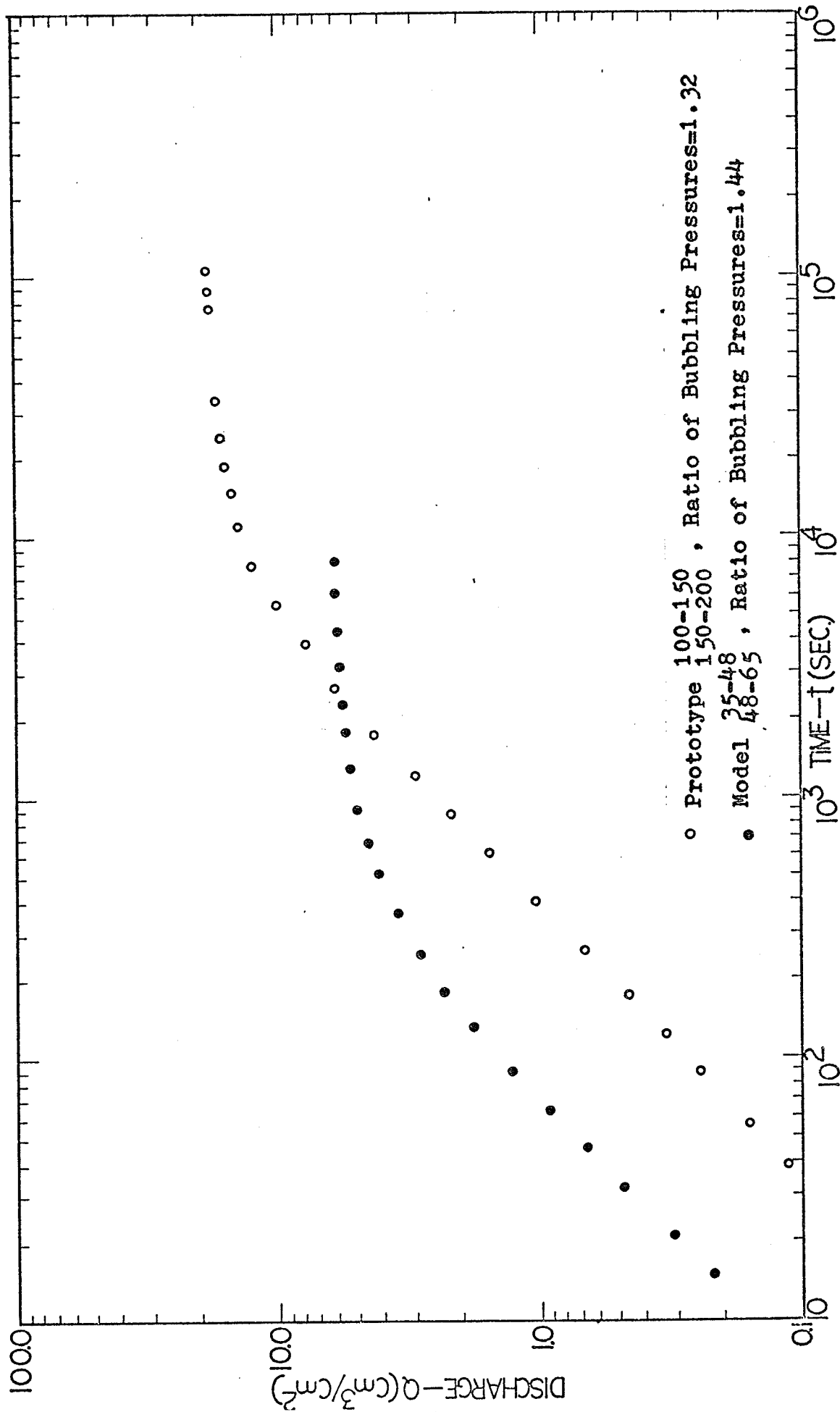


Figure 13. Discharge-time relationship of unsteady drainage from prototype No.2 and model No.2B.

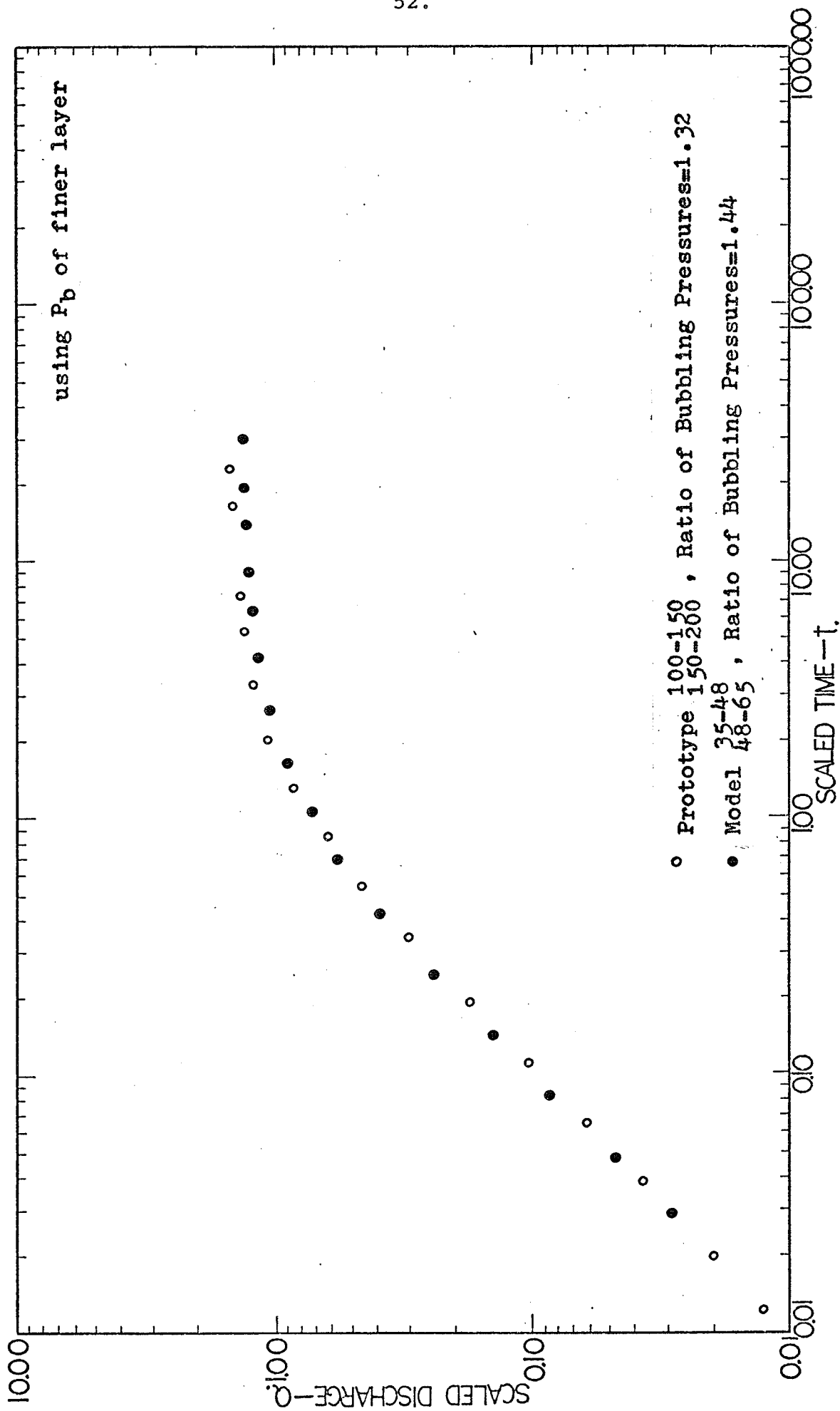


Figure 14. Scaled discharge-time relationship of unsteady drainage from prototype No.2 and model No.2B.

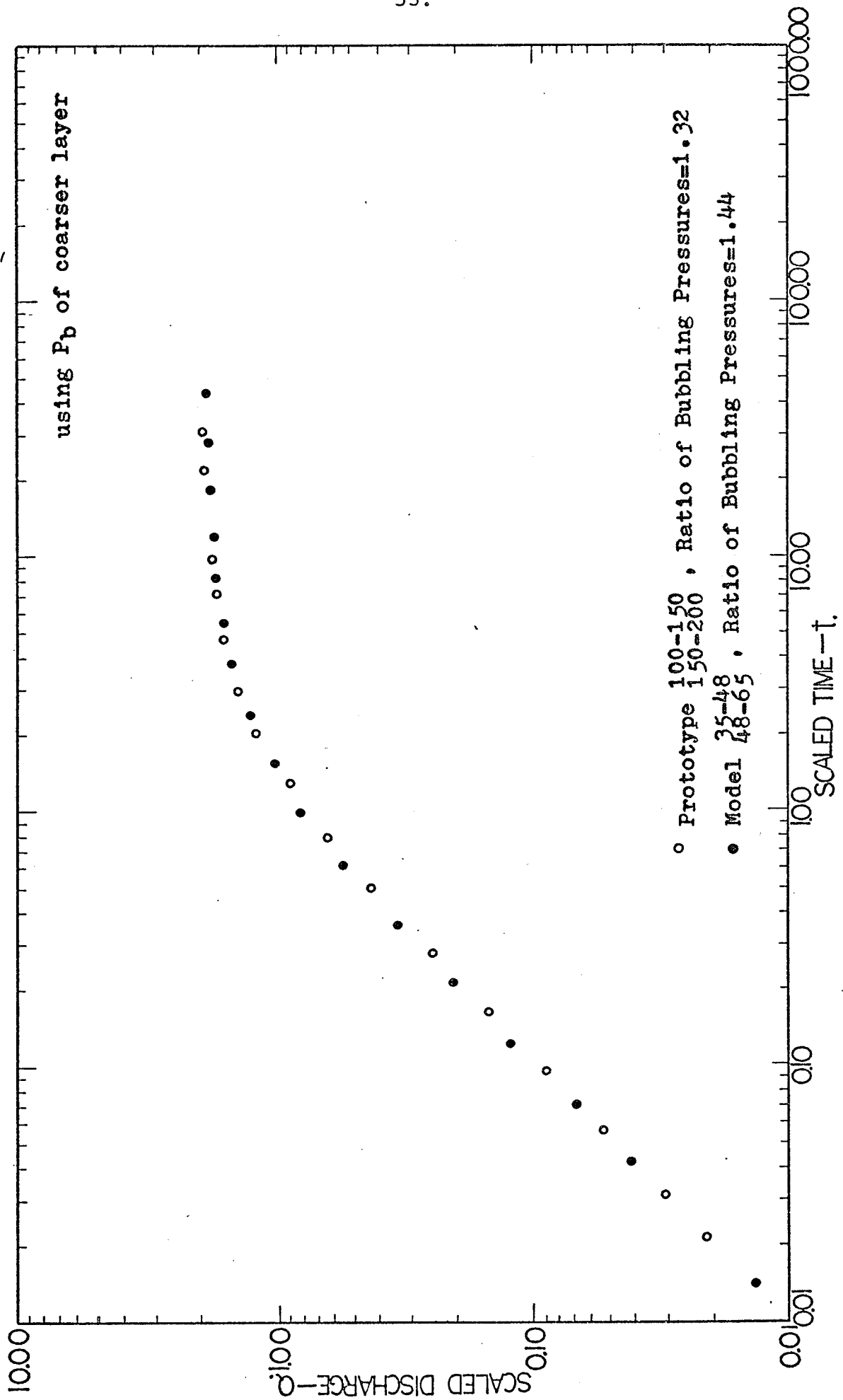


Figure 15. Scaled discharge-time relationship of unsteady drainage from prototype No.2 and model No.2B.

Model No. 2C

Fraction	L_{cm}	η	$P_c/\rho g$	S_r	ϕ	ϕ_e
28-35	8.11	18.43	7.68	0.100	0.329	0.296
48-65	17.58	20.30	14.67	0.100	0.375	0.338

$$\phi_e \text{ ave} = \frac{8.11 \times 0.296 + 17.58 \times 0.338}{8.11 + 17.58} = 0.325 \quad \frac{P_b \text{ 48-65}}{P_b \text{ 28-35}} = 1.91$$

$$K = 39.03 \mu^2 = 39.03 \times 10^{-8} \text{ cm}^2 \quad \text{Suction} = .6 P_b \text{ 48-65} = 8.80 \text{ cm}^+$$

Considering P_b of the finer layer

$$Q_s = 0.2097 Q$$

$$t_s = 4.064 t \text{ (temperature the same during test)}$$

Considering P_b of the coarser layer

$$Q_s = 0.4006 Q$$

$$t_s = 7.764 t$$

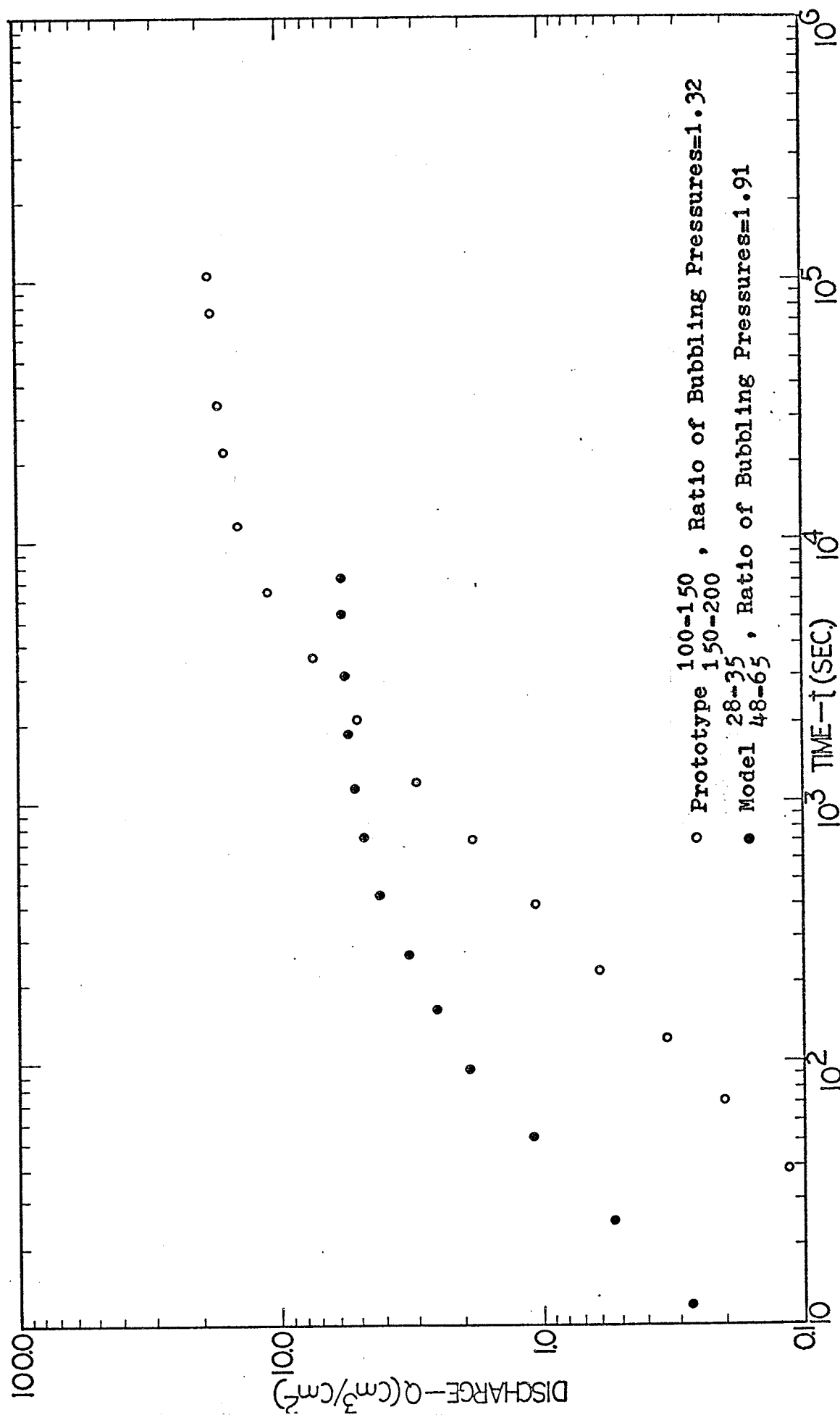


Figure 16. Discharge-time relationship of unsteady drainage from prototype No.2 and model No.2C.

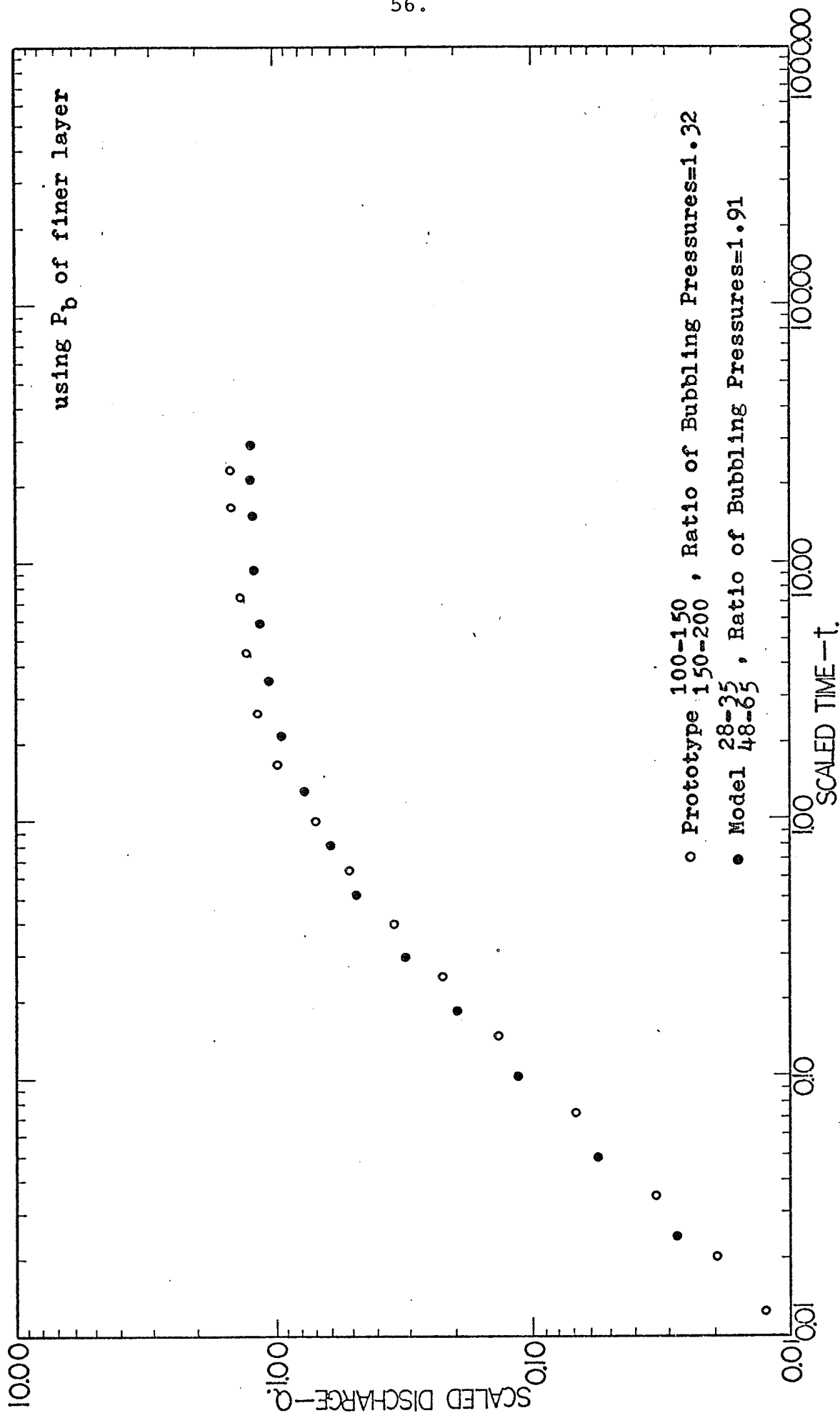


Figure 17. Scaled discharge-time relationship of unsteady drainage from prototype No.2 and model No.2C.

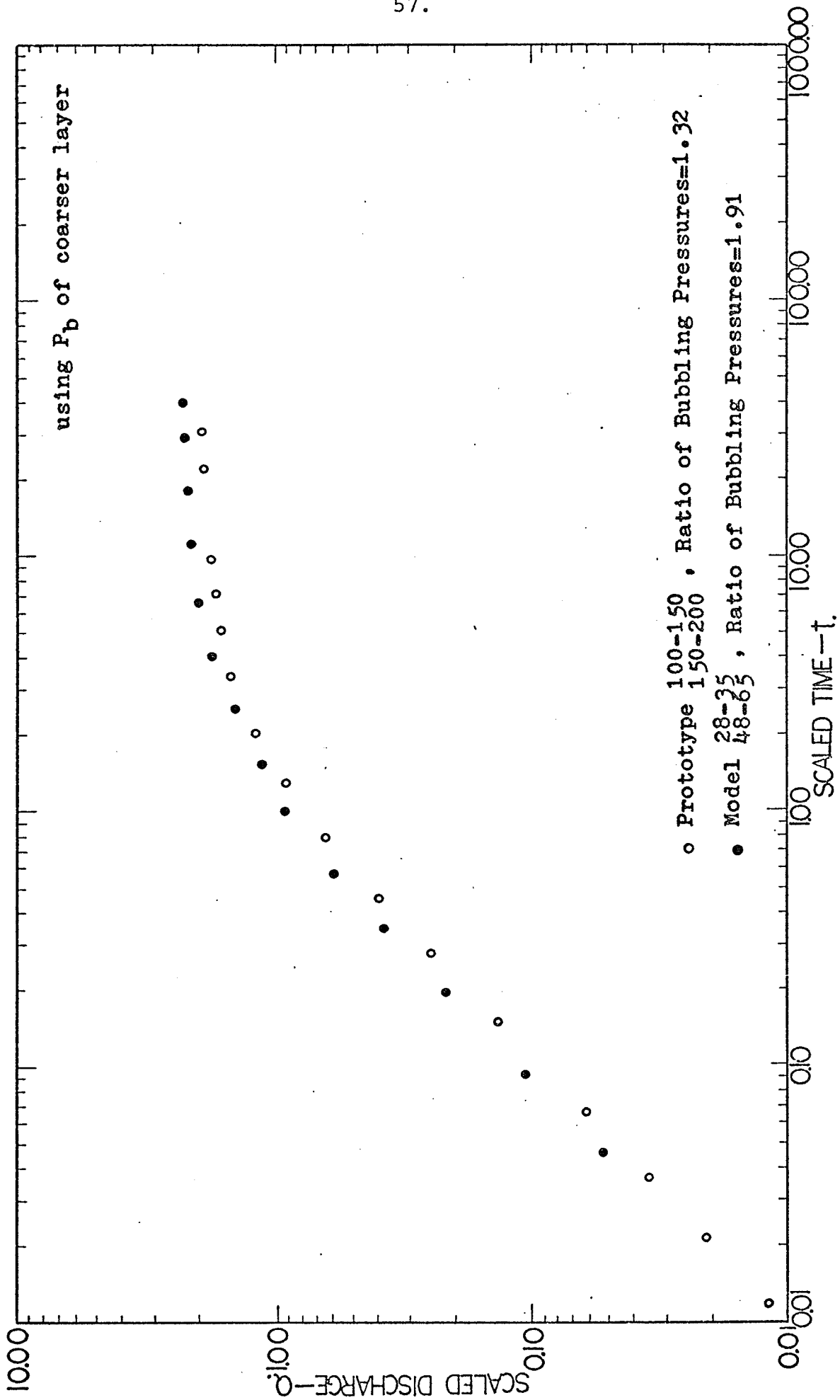


Figure 18. Scaled discharge-time relationship of unsteady drainage from prototype No.2 and model No.2C.

CHAPTER V

DISCUSSION OF THE RESULTS

The specifications of each prototype and model were shown in the previous section with the results of the tests in the corresponding figures. The following is a brief discussion of the results.

In prototype No. 1 the 150-200 fraction was on the top and the 100-150 fraction at the bottom with the ratio of the bubbling pressure of the finer layer to the coarser layer equal to 1.32. With this prototype three models were run; model No. 1A with the ratio of the bubbling pressure of the layers the same as that of the prototype, that is 1.32; model No. 1B with the ratio of 1.26, slightly different from the prototype and model No. 1C with the ratio of 1.82, much different from the prototype. In all the cases the length of each layer in the model was calculated according to Brooks and Corey's theory, that is:

$$\frac{L}{P_b/\gamma} = \frac{L'}{P_b'/\gamma}$$

or

$$L' = \frac{L \times P_b'/\gamma}{P_b/\gamma}$$

where the primed quantities refer to the model and unprimed to the prototype.

From the results of the first series of the tests which are shown in Figures 1 to 9 it can be seen easily that the agreement between prototype No. 1 and model No. 1A and model No. 1B is very good, at least within experimental error, while there is no agreement between prototype No. 1 and model No. 1C. Remembering that the ratio of the bubbling pressures among the layers of model No. 1A and model No. 1B are the same and very near to that of the prototype No. 1, and that the ratio for model No. 1C, for which there is no agreement between the model and the prototype behavior, is far different from that of the prototype No. 1, it is evident that the criterion which was developed in this study is substantiated. That is,

$$\frac{P_{b_1}}{P'_{b_1}} = \frac{P_{b_2}}{P'_{b_2}} = \dots\dots\dots = \frac{P_{b_m}}{P'_{b_m}}$$

where subscripts 1, 2, ... m refer to the layers in the prototype and model.

It should be noted that for all the models the η requirement was satisfied and the largest difference between the η 's was for the model No. 1B which was less than 13% and smaller than 20% recommended by Corey et. al. (6). Therefore, this is not likely to be a major factor in the difference in the behaviors of the model No. 1C and the prototype No. 1 and the reason is, in fact, that the bubbling

pressure ratio criterion was not satisfied.

Before discussing the second series of the tests it should be pointed out, here, that for model No. 1A, with the ratio of bubbling pressures of the layers exactly the same as the prototype, at large scaled times there is a small difference between the prototype and model behavior while for model No. 1B, with the ratio of the bubbling pressures of the layers very slightly different from the prototype, the larger scaled time data are in better agreement with the prototype. This can be attributed to the experimental error because there are ways in which error may occur. For instance, calculating the bubbling pressure of the medium from the data obtained in capillary pressure-relative permeability test may involve a small percentage error. Likewise error may be introduced in determining the effective porosity of the medium which is calculated by using the residual saturation.

Also, during the unsteady drainage test the adjustment of the outflow tube for the zero suction (or for any suction) may involve a small error in the sense that the outflow tube may be fixed a few millimeters too high or too low. This involves two effects, one on drainage ratio and the other on total drainage since an error in suction may cause the sample to drain a few millimeters more or less than it should. Furthermore, when the column containing two layers is under

vacuum for saturation at the junction of the two layers there is a small zone where the two layers slightly mix with each other. Although each layer is "modeled", one would not expect this mixing process to be identical nor the resulting mixed depth to be modeled. For this reason, mixing of layers would be expected to account for some deviation between the scaled data of the model and prototype. Further, where the fine material is placed on top (model No. 1A) one would expect the fines to enter the voids of the coarse more easily when aided by gravity. Also, where the fine material was on the bottom (model No. 2A) it would be less probable that for the coarse grains would enter the voids of the fine material by gravity, and mixing would be at a minimum. Comparison of the scaled data of Figure 2 and 11 shows that model No. 2A did in fact show less discrepancy than model No. 1A. It is impossible, of course, to verify if in fact the above argument does account for the lesser discrepancy, however, it is nevertheless a noteworthy point. Another point which must be considered is that the prototype drains for a longer time than the model and evaporation from the outflow tube for the prototype is more than the model and this causes that the scaled discharge-time relationship for the prototype, at large values of scaled time, be lower than the model.

In the second series of the tests, for which the

results are shown in Figures 10 to 18, the 100-150 fraction (coarser layer) was placed at the top and the 150-200 fraction at the bottom for the prototype No. 2. In these series three models were run with prototype No. 2, namely, model No. 2A with the ratio of the bubbling pressures of the layers equal to 1.32, exactly the same as that of the prototype; model No. 2B with the ratio of 1.44, slightly different from the prototype, and model No. 2C with the ratio of 1.91, which is much different from that of the prototype. The materials in model No. 2A were the same as those in model No. 1A but like prototype No. 2 the coarser layer was on the top and the finer one at the bottom. The materials for model No. 2B were not the same as those for model No. 1B and so the ratio of the bubbling pressures of the layers was different than for model No. 1B. The ratio of the bubbling pressures for model No. 2B was 1.44, compared to 1.26 for model No. 1B. The difference of the prototype and model ratios was slightly greater for series 2 than for series 1 and the agreement is slightly worse, as one should expect. The behavior of model No. 2C does not agree with that of prototype No. 2 because the ratio of the bubbling pressures of the model layers, 1.92, is much different from that of the prototype where the ratio was 1.32.

It should be pointed out that in the second series of the tests the largest difference between η 's was for model

No. 2B which was less than 11 percent and it satisfies the recommendations by Corey et. al. (6). Therefore, the entire difference in the behaviors of the model No. 2C and prototype No. 2 cannot be attributed to the η -value and it would be attributed to the difference in the ratios of the bubbling pressures.

CHAPTER VI

CONCLUSION

When the ratio of the bubbling pressures among the model layers is the same as or very near to that of the prototype the scaled discharge-time relationships for the model and the prototype are within the limits of experimental error. This indicates that the prototype behavior can be predicted from the model, provided that all modeling criteria are fulfilled. The larger the difference between the ratio of the bubbling pressures of the layers, the larger will be the difference in behavior of the model and the prototype. This statement exemplifies the importance of the additional modeling criterion required, as found in this study.

For practical purposes when the proper materials for the model are not available to satisfy the pore-size distribution index requirement and also the ratios of bubbling pressures of the prototype layers, then, materials with the ratio of the bubbling pressures a small percentage different from that of the prototype may be used.

Because of boundary effects between layers, it is recommended that some caution be exercised in choosing the materials for the model such that extremely large model, prototype scale ratios do not result especially where relatively thin prototype layers must be modeled. The reason

is that if the thickness of the model layers are too small the experimental error will be reflected to such an extent that the results may be seriously affected.

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APPENDIX A

CAPILLARY PRESSURE-DESATURATION MEASUREMENT AND DATA
AND
CAPILLARY PRESSURE-RELATIVE PERMEABILITY MEASUREMENT AND DATA

CAPILLARY PRESSURE-SATURATION MEASUREMENT

This measurement was done by using the pressure controller. The pressure controller consisted of an acrylic tube, open at the top, approximately 5-cm inside diameter and 2-cm height with a capillary barrier fixed inside (Figure 1A). The capillary barrier was made of special kind of ceramic which could hold a few feet of negative pressure without bubbling. Sand was packed inside the pressure controller and was covered with clear plastic wrap and was vacuum-saturated. Then, it was connected to a reservoir by a flexible tube. There were some tiny holes in the wrap to allow air to pass in and out of the sample freely. The wrap remained around the controller during the measurement to minimize the occurrence of hydraulic gradients in response to evaporation.

Capillary pressures were applied to the sample in increments by changing the level of the reservoir. At each capillary pressure, after equilibrium between the sand and the measurement system had been achieved the weight of the sample was determined and saturation was calculated.

To calculate the saturation and some other necessary data the following weights must be known: 1) the weight of the saturated controller; 2) the weight of the saturated controller plus dry sample; 3) the weight of the saturated

controller plus dry sample plus plastic wrap, and 4) the weight of the whole controller after saturation.

From these four weights it is possible to find out the bulk density, ρ_b , which is used to calculate porosity, ϕ , (assuming particle density, ρ_s , for the sample is known). Also the weight of the oil in the sample is given by item 4 minus 3 which is used to determine the saturation at each capillary pressure.

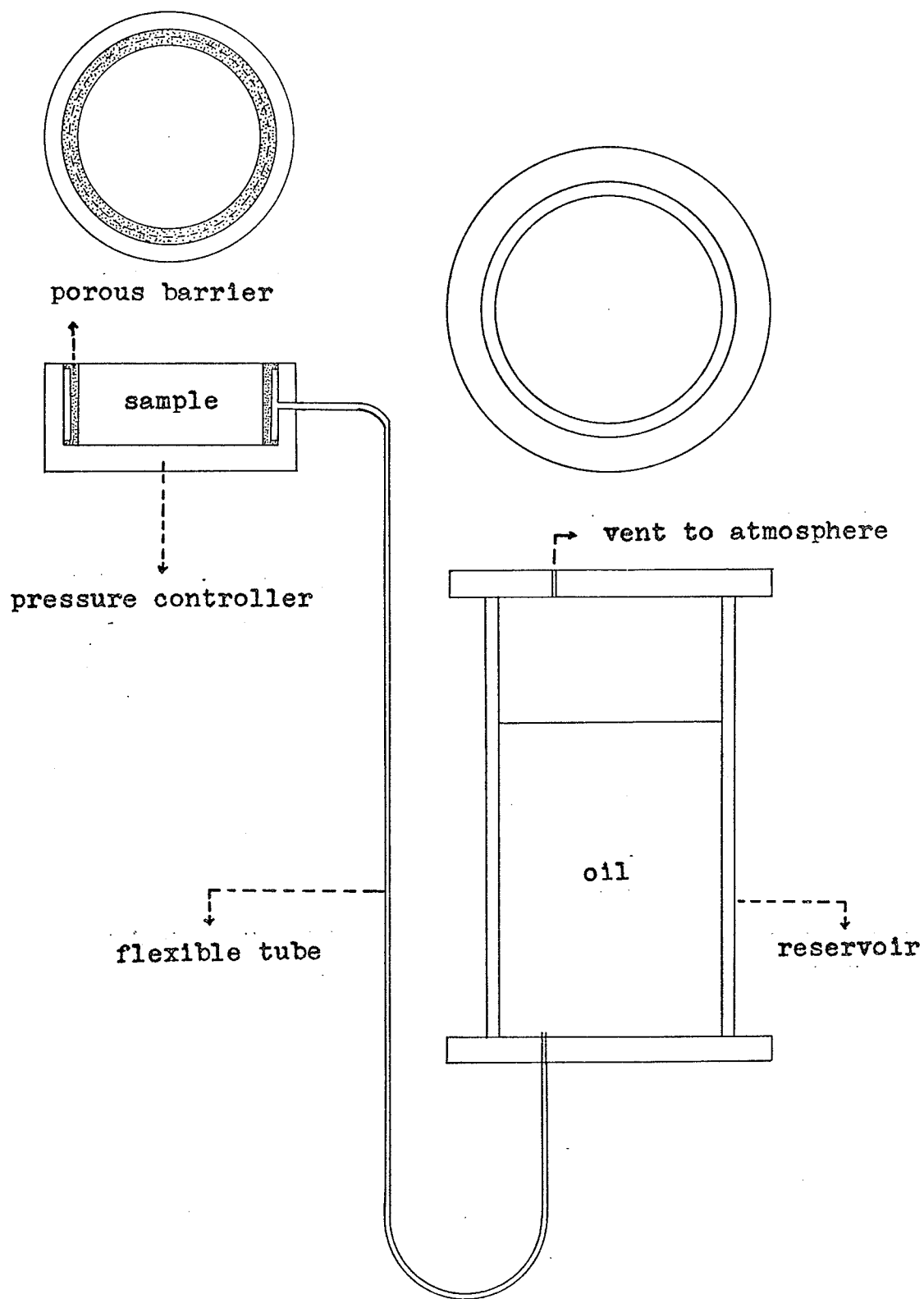


Figure 1A. Sketch of apparatus used to determine capillary pressure-desaturation relationships.

CAPILLARY PRESSURE-PERMEABILITY MEASUREMENT

This measurement was done by using apparatus shown in Figure 2A. This apparatus consisted of an acrylic tube approximately 15-cm in length and 5-cm I.D. which was inserted between two pressure controllers. The sample was packed in the tube and pressure controllers and tensiometers were attached to the tube. The inflow pressure controller was connected to a supply bottle and the tensiometers to two manometer tubes made of glass to measure capillary pressure and hydraulic gradients.

Capillary pressures were applied to the sample in increments by changing the elevations of the supply bottle, the sample or outlet tube in order to create a unit hydraulic gradient for which the saturation is uniform. After equilibrium the rate of flow through the sample was measured at each capillary pressure and the corresponding permeability was calculated.

For further discussion about these two measurements the reader is referred to papers by Anat et. al. (1), Corey et. al. (6) and Laliberte et. al. (12).

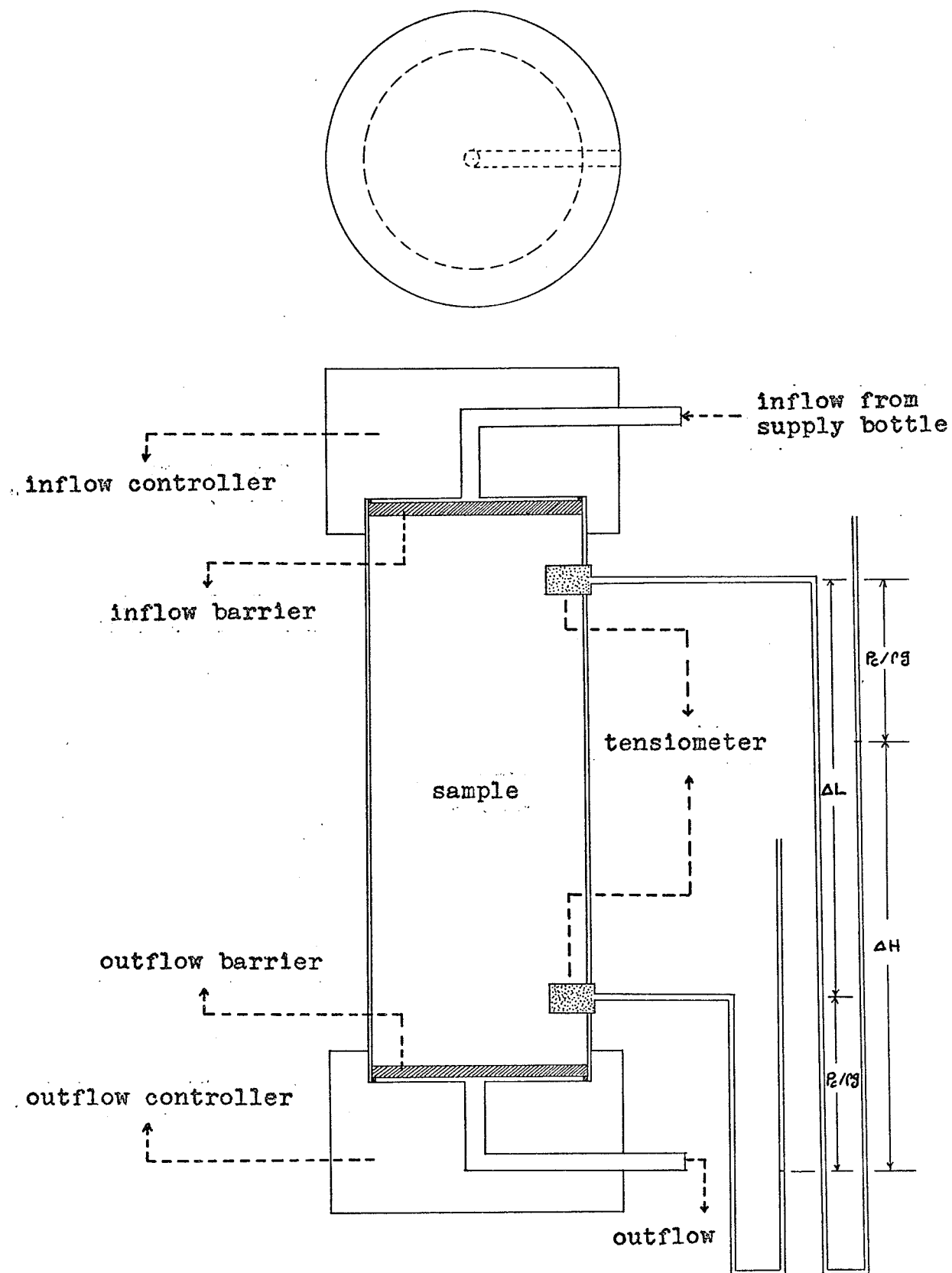


Figure 2A. Sketch of apparatus used to determine capillary pressure-permeability relationships.

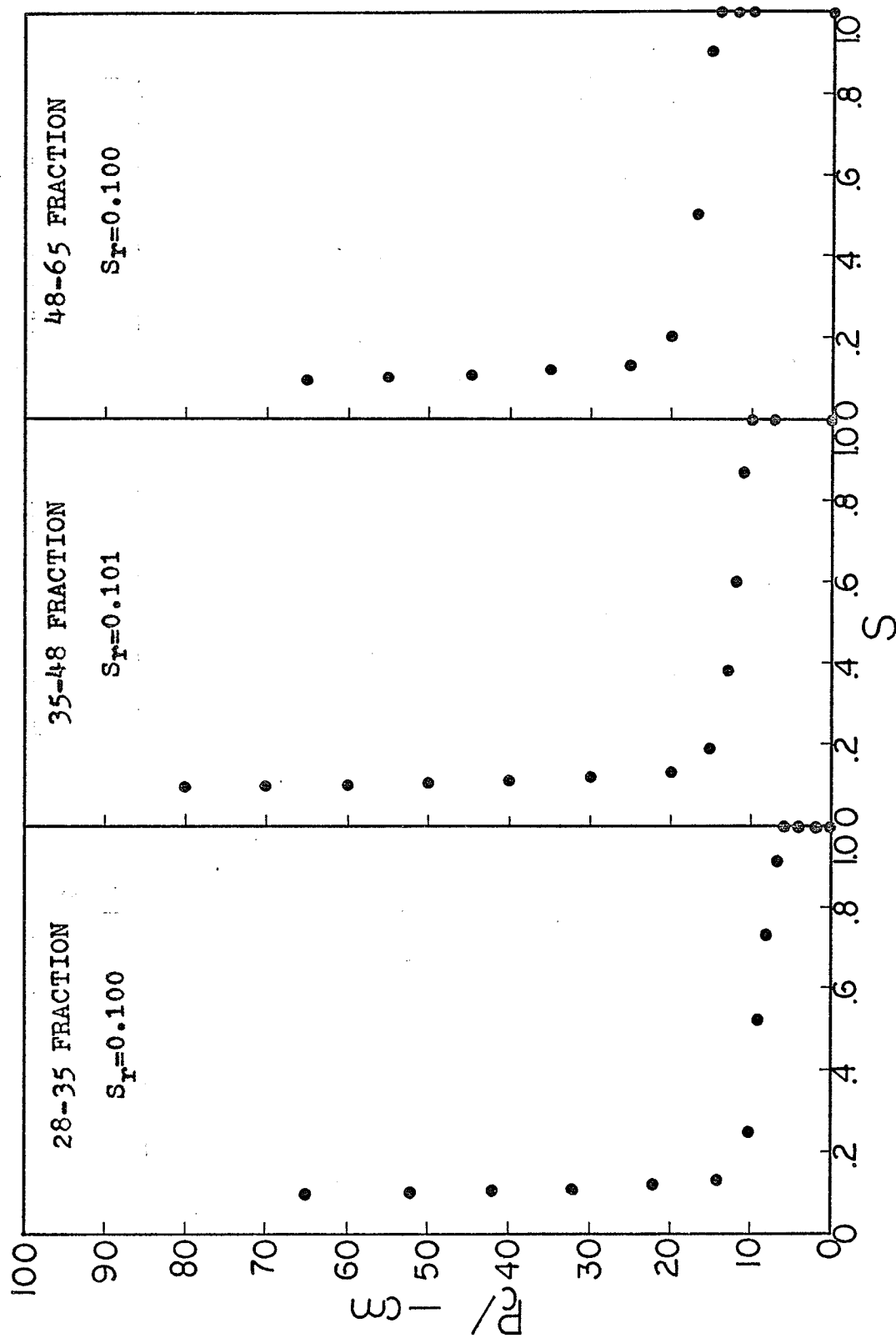


Figure 3A. Capillary pressure-desaturation relationships for different fractions of Selkirk Silica Sand.

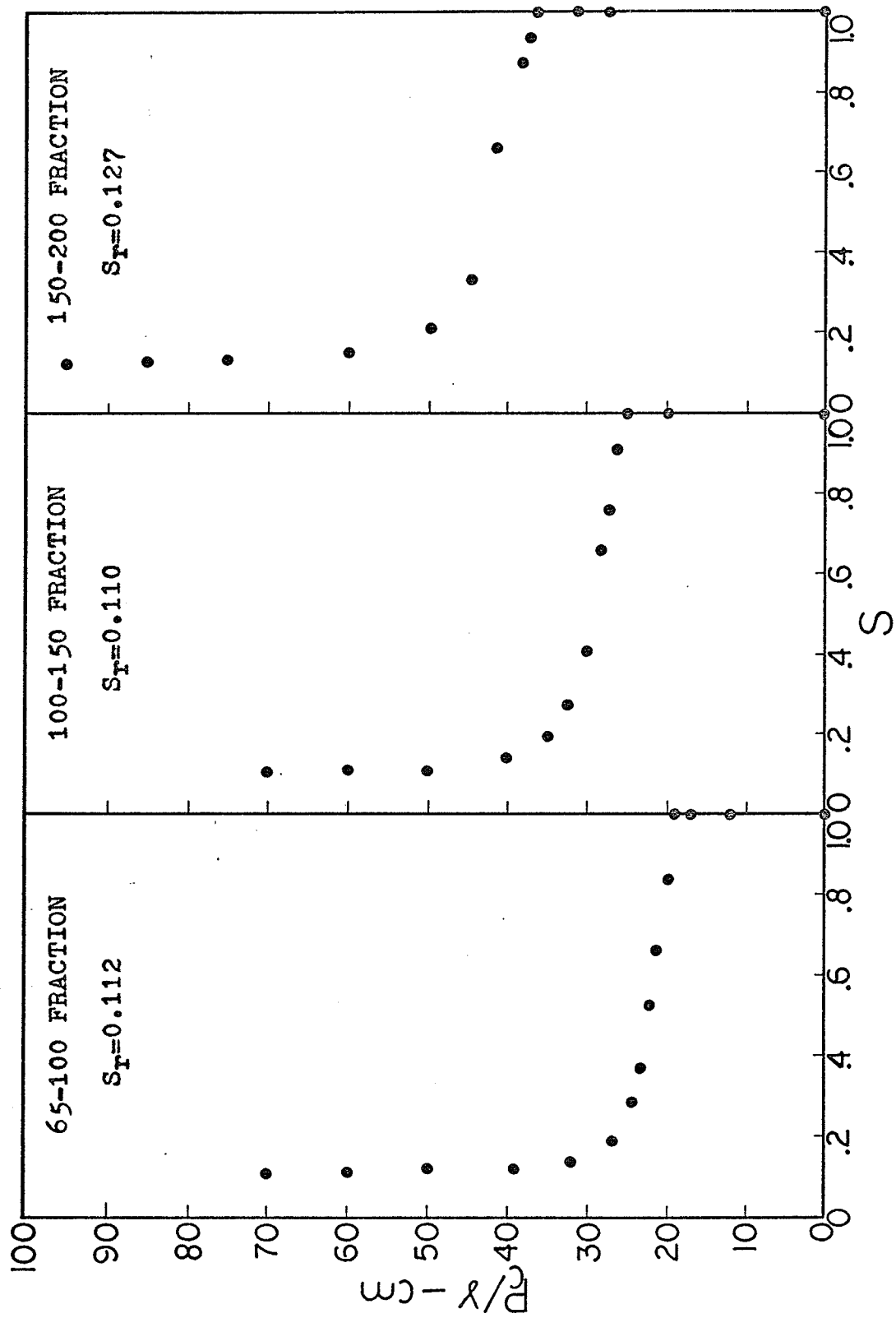


Figure 4A. Capillary pressure-desaturation relationships for different fractions of Selkirk Silica Sand.

Table 1A
Capillary Pressure-Desaturation
Data for Selkirk Silica Sand (28-35 Fraction)

$p_c/\rho g$ cm	S
6.0	1.000
7.0	0.916
8.0	0.733
9.0	0.520
10.0	0.254
11.0	0.159
14.0	0.136
22.0	6.124
32.0	0.113
42.0	0.107
52.0	0.103
65.0	0.100

$$\phi = 0.352$$

Table 2A
Capillary Pressure-Desaturation
Data for Selkirk Silica Sand (35-48 Fraction)

$p_c/\rho g$ cm	S
10.0	1.000
11.0	0.868
12.0	0.595
13.0	0.381
15.0	0.192
20.0	0.135
30.0	0.123
40.0	0.114
50.0	0.108
60.0	0.105
70.0	0.101
80.0	0.101

$$\phi = 0.360$$

Table 3A
 Capillary Pressure-Desaturation
 Data for Selkirk Silica Sand (48-65 Fraction)

$p_c/\rho g$ cm	S
14.0	1.000
15.0	0.909
17.0	0.503
20.0	0.209
25.0	0.135
35.0	0.123
45.0	0.110
55.0	0.106
65.0	0.100

$$\phi = 0.373$$

Table 4A
 Capillary Pressure-Desaturation
 Data for Selkirk Silica Sand (65-100 Fraction)

$p_c/\rho g$ cm	S
19.0	1.000
20.0	0.844
21.0	0.664
22.0	0.529
23.0	0.375
24.0	0.289
27.0	0.192
32.0	0.139
39.0	0.123
50.0	0.118
60.0	0.114
70.0	0.112

$$\phi = 0.386$$

Table 5A

Capillary Pressure-Desaturation
Data for Selkirk Silica Sand (100-150 Fraction)

$p_c/\rho g$ cm	S
25.0	1.000
26.0	0.914
27.0	0.764
28.0	0.631
30.0	0.407
32.0	0.273
35.0	0.192
40.0	0.145
50.0	0.111
60.0	0.110
70.0	0.110

$$\phi = 0.379$$

Table 6A

Capillary Pressure-Desaturation
Data for Selkirk Silica Sand (150-200 Fraction)

$p_c/\rho g$ cm	S
36.0	1.000
37.0	0.945
38.0	0.898
41.0	0.663
45.0	0.335
50.0	0.217
60.0	0.153
75.0	0.137
85.0	0.132
95.0	0.127

$$\phi = 0.403$$

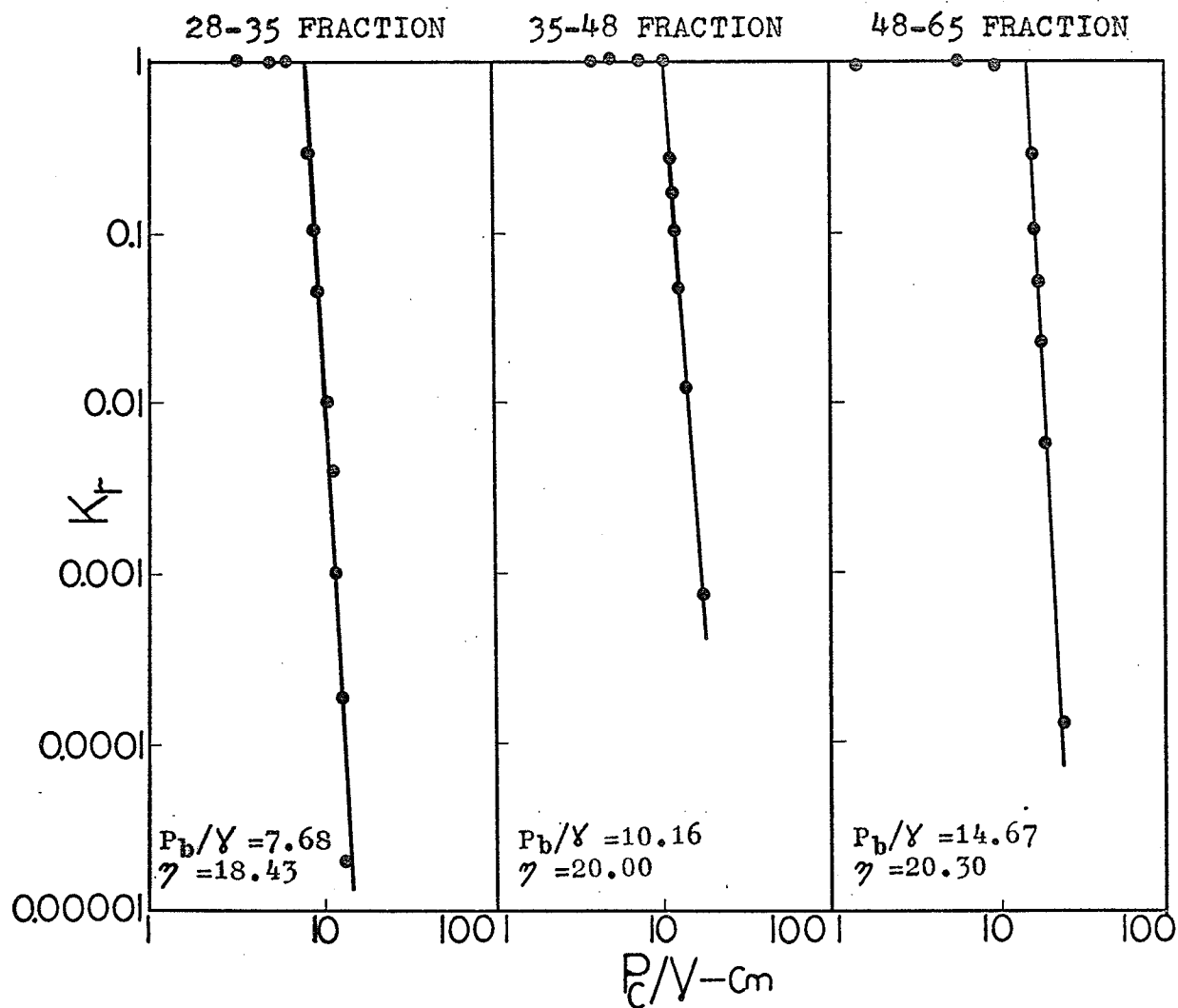


Figure 5A. Capillary pressure-relative permeability relationships for different fractions of Selkirk Silica Sand.

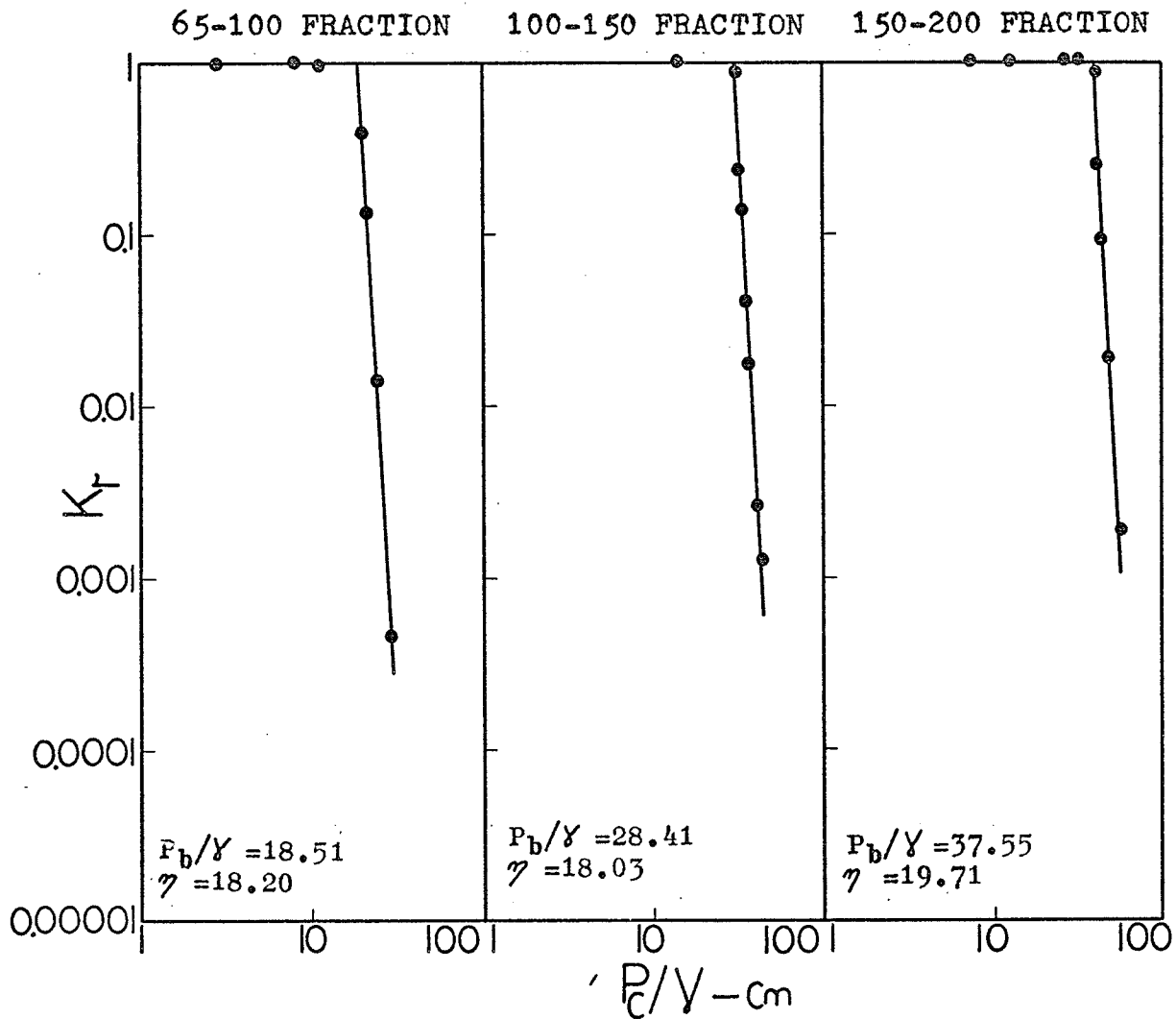


Figure 6A. Capillary pressure-relative permeability relationships for different fractions of Selkirk Silica Sand.

Table 7A
Capillary Pressure-Permeability Data for Selkirk Silica Sand (28-35 Fraction)

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$\gamma \times 10^{-3}$ cm/sec	K μ^2	K_r	$P_c/\rho g$ cm
21.0	2.089	0.461	30.77	170.4	0.984	3.22
21.8	2.055	0.087	7.24	169.9	0.978	5.08
21.7	2.055	0.085	7.187	173.7	1.000	5.89
22.0	2.051	0.349	8.192	52.42	0.301	8.05
23.0	2.014	1.003	8.95	17.91	0.103	8.65
22.3	2.036	0.950	6.90	14.75	0.0848	8.74
21.0	2.089	0.953	3.64	7.96	0.0458	9.02
20.5	2.108	0.962	0.801	1.75	0.0101	4.96
21.0	2.089	0.938	0.318	0.705	0.00406	10.86
20.0	2.127	1.053	0.0875	0.176	0.00101	11.43
19.5	2.136	1.050	0.0161	0.0327	0.000188	12.32
19.0	2.146	1.053	0.00166	0.00338	0.0000195	13.12

$$\frac{P_b}{\rho g} = 7.68 \text{ cm} \quad \eta = 18.43$$

Table 8A
Capillary Pressure-Permeability Data for Selkirk Silica Sand (35-48 Fraction)

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$\gamma \times 10^{-3}$ cm/sec	K μ^2	K_r	$p_c/\rho g$ cm
23.8	1.988	0.822	24.87	60.23	0.999	3.90
23.2	2.005	0.467	15.11	64.63	1.070	5.02
21.5	2.070	0.938	27.61	62.73	1.040	7.29
21.5	2.070	0.937	27.37	60.27	1.000	9.89
21.0	2.089	0.972	8.38	17.95	0.298	10.93
21.0	2.089	0.991	5.06	10.63	0.176	11.35
18.0	2.203	0.974	2.974	6.57	0.109	11.79
17.0	2.279	0.968	1.25	2.93	0.0486	12.42
22.0	2.051	0.923	0.352	0.778	0.0129	13.35
23.0	2.014	1.090	0.0161	0.0296	0.000730	16.72

$\frac{p_b}{\rho g} = 10.16 \text{ cm}$ $\eta = 20.00$

Table 9A
Capillary Pressure-Permeability Data for Selkirk Silica Sand (48-65 Fraction)

Temp. °C	$\mu/\rho \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$\gamma \times 10^{-3}$ cm/sec	K_2 μ	K_r	$P_c/\rho g$ cm
18.0	2.203	0.735	11.78	35.19	0.937	1.45
19.0	2.165	0.697	12.13	37.56	1.000	6.58
20.0	2.127	0.682	11.71	36.42	0.970	9.57
23.5	1.996	1.109	6.27	11.26	0.300	15.70
23.8	1.988	1.064	4.84	9.01	0.240	15.92
23.5	1.996	1.082	3.90	7.18	0.191	16.04
21.0	2.089	1.063	2.02	3.95	0.105	16.42
21.5	2.070	1.059	1.57	3.05	0.0813	16.70
21.0	2.089	0.997	0.929	1.94	0.0517	16.75
19.0	2.165	1.057	0.404	0.825	0.0220	17.64
20.5	2.108	1.084	0.114	0.222	0.00590	18.70
20.5	2.108	1.014	00.00233	0.00483	0.000129	23.00

$$\frac{P_b}{\rho g} = 14.67 \text{ cm} \quad \eta = 20.30$$

Table 10A

Capillary Pressure-Permeability Data for Selkirk Silica Sand (65-100 Fraction)

Temp °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$\gamma \times 10^{-3}$ cm/sec	$K_{\frac{2}{\mu}}$	K_r	$p_c/\rho g$ cm
19.0	2.165	1.012	13.79	29.42	0.985	2.79
19.0	2.165	0.991	13.71	29.88	1.000	7.83
19.0	2.165	1.017	13.79	29.27	0.980	11.14
19.2	2.159	0.998	5.61	12.34	0.413	19.33
20.0	2.127	0.885	1.71	4.10	0.137	20.65
19.0	2.165	0.834	0.157	0.406	0.0136	23.72
19.0	2.165	0.780	0.00482	0.0133	0.000446	28.10

$$\frac{p_b}{\rho g} = 18.51 \quad \eta = 18.20$$

Table 11A
Capillary Pressure-Permeability Data for Selkirk Silica Sand (100-150 Fraction)

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$\gamma \times 10^{-3}$ cm/sec	K_2 μ	K_r	$P_c/\rho g$ cm
21.9	2.055	0.885	3.35	7.78	1.000	13.41
21.9	2.055	0.799	2.66	6.82	0.876	29.03
20.5	2.108	0.806	0.735	1.91	0.246	30.91
20.9	2.093	0.825	0.404	1.024	0.132	31.75
18.8	2.173	0.860	0.122	0.308	0.0396	33.39
20.4	2.112	0.930	0.0591	0.134	0.0172	34.43
18.8	2.173	1.585	0.0153	0.0210	0.00270	39.80
21.5	2.070	1.445	0.00655	0.00938	0.00121	41.55

$$\frac{P_b}{\rho g} = 28.41 \text{ cm} \quad \eta = 18.03$$

Table 12A
Capillary Pressure-Permeability Data for Selkirk Silica Sand (150-200 Fraction)

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$\gamma \times 10^{-3}$ cm/sec	K μ^2	K_r	$p_c/\rho g$ cm
19.0	2.165	0.923	2.37	5.55	1.000	7.45
19.0	2.165	0.971	2.50	5.55	1.000	12.20
20.0	2.127	1.135	3.00	5.59	1.007	25.82
20.5	2.108	1.098	2.95	5.65	1.018	32.27
20.0	2.127	0.729	1.63	4.73	0.852	39.62
19.0	2.165	1.164	0.755	1.40	0.252	40.38
19.0	2.165	1.186	0.284	0.517	0.0932	42.34
19.0	2.165	1.179	0.0574	0.105	0.0189	45.70
19.0	2.165	0.578	0.00277	0.0103	0.00186	51.79

$$\frac{p_b}{\rho g} = 37.55 \text{ cm} \quad \eta = 19.71$$

APPENDIX B

SIEVE ANALYSIS OF SELKIRK SILICA SAND
AND
PROPERTIES OF SOLTROL "C"

Table 1B
Sieve Analysis of Selkirk Silica Sand

Mesh*	Opening mm	Retained on %
10	1.651	.6
20	0.833	1.2
28	0.589	4
35	0.417	10
48	0.295	16.6
65	0.208	27
100	0.147	29
150	0.104	9
200	0.074	1.8
PAN		.2

*Tyler standard.

Table 2B
Dynamic Viscosity and Density of Soltrol "C"
Oil Used in Experiments

Temp. °C	Viscosity, μ Centipoises	Density, ρ grams/ml	$\mu/\rho g$ cm/seconds
20.0	1.589	0.7582	2.127
20.5	1.571	0.7579	2.108
21.0	1.555	0.7576	2.089
21.5	1.539	0.7573	2.070
22.0	1.524	0.7569	2.051
22.5	1.509	0.7566	2.032
23.0	1.494	0.7562	2.014
23.5	1.481	0.7559	1.996
24.0	1.468	0.7556	1.979
24.5	1.454	0.7553	1.962
25.0	1.440	0.7549	1.945
25.5	1.427	0.7546	1.927
26.0	1.414	0.7542	1.910
26.5	1.401	0.7539	1.893
27.0	1.388	0.7536	1.877
27.5	1.375	0.7533	1.861
28.0	1.362	0.7529	1.845
28.5	1.348	0.7526	1.829
29.0	1.337	0.7522	1.814
29.5	1.326	0.7519	1.799
30.0	1.315	0.7515	1.783

APPENDIX C
UNSTEADY DRAINAGE DATA

Table 1C
Unsteady Drainage Data from Prototype No. 1

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	t second	Q cm^3/cm^2	t. (p_b of finer layer)	Q. (p_b of finer layer)	t. (p_b of coarser layer)	Q. (p_b of coarser layer)
19.8	2.135	66	0.0965	0.0965	0.0138	0.0182	0.0105
19.8	2.135	100	0.140	0.0209	0.0116	0.0277	0.0153
19.8	2.135	133	0.184	0.0278	0.0152	0.0368	0.0201
19.8	2.135	166	0.228	0.347	0.0189	0.0459	0.0249
19.8	2.135	199	0.272	0.0416	0.0225	0.0550	0.0297
19.8	2.135	233	0.316	0.0487	0.0261	0.0644	0.0345
19.8	2.135	267	0.360	0.0559	0.0298	0.0738	0.0393
19.8	2.135	301	0.404	0.0630	0.0334	0.0832	0.0442
19.8	2.135	335	0.447	0.0701	0.0370	0.0926	0.0489
19.8	2.135	403	0.535	0.0843	0.0442	0.111	0.0585
19.8	2.135	438	0.579	0.0916	0.0479	0.121	0.0633
19.8	2.135	473	0.623	0.0990	0.0515	0.131	0.0681
19.8	2.135	508	0.667	0.106	0.0552	0.140	0.0729
19.8	2.135	542	0.711	0.113	0.0588	0.150	0.0777
19.8	2.135	610	0.798	0.128	0.0660	0.169	0.0872
19.8	2.135	645	0.842	0.135	0.0696	0.178	0.0920
19.8	2.135	680	0.886	0.142	0.0733	0.188	0.0968
19.8	2.135	750	0.974	0.157	0.0805	0.207	0.106
19.8	2.135	820	1.061	0.172	0.0877	0.227	0.116
19.8	2.135	891	1.149	0.186	0.0950	0.246	0.126
19.8	2.135	962	1.237	0.201	0.102	0.266	0.135
19.8	2.135	1034	1.325	0.216	0.110	0.286	0.145
19.8	2.135	1143	1.456	0.239	0.120	0.316	0.159
19.8	2.135	1252	1.588	0.262	0.131	0.346	0.174

Table 1C continued

19.8	2.135	1363	1.719	0.285	0.142	0.377	0.188
19.8	2.135	1624	2.026	0.340	0.168	0.449	0.221
19.8	2.135	1775	2.202	0.371	0.182	0.491	0.241
19.8	2.135	1928	2.377	0.403	0.197	0.533	0.260
19.8	2.135	2122	2.596	0.444	0.215	0.587	0.284
20.1	2.123	2316	2.816	0.487	0.233	0.644	0.308
20.1	2.123	2514	3.035	0.529	0.251	0.699	0.332
20.1	2.123	2713	3.254	0.571	0.269	0.754	0.356
20.1	2.123	2956	3.518	0.622	0.291	0.822	0.385
20.1	2.123	2450	4.044	0.726	0.334	0.959	0.442
20.1	2.123	3706	4.316	0.780	0.357	1.030	0.472
20.1	2.123	4048	4.667	0.852	0.386	1.125	0.510
20.2	2.119	4476	5.105	0.942	0.422	1.247	0.558
20.2	2.119	5370	5.982	1.130	0.495	1.496	0.654
20.2	2.119	5964	6.509	1.255	0.538	1.662	0.711
20.2	2.119	6601	7.035	1.389	0.582	1.839	0.769
20.2	2.119	7276	7.561	1.531	0.625	2.027	0.826
20.2	2.119	7997	8.088	1.683	0.669	2.228	0.884
20.2	2.119	8633	8.526	1.816	0.705	2.405	0.932
20.2	2.119	10019	9.345	2.108	0.777	2.791	1.027
20.5	2.108	10846	9.833	2.298	0.813	3.037	1.075
20.5	2.108	11889	10.272	2.519	0.849	3.329	1.123
20.7	2.100	14149	10.974	3.009	0.908	3.977	1.199
20.8	2.097	15601	11.351	3.323	0.939	4.392	1.241
20.8	2.097	17184	11.702	3.660	0.968	4.837	1.279
20.8	2.097	20784	12.325	4.427	1.019	5.851	1.347
21.0	2.089	22973	12.605	4.912	1.042	6.492	1.378
21.0	2.089	24461	12.763	5.230	1.056	6.913	1.395
20.8	2.097	26085	12.912	5.556	1.068	7.343	1.411

Table 1C continued

20.8	2.097	30108	13.211	6.413	1.093	8.475	1.444
20.6	2.104	38200	13.588	8.110	1.124	10.719	1.485
19.8	2.135	86670	14.614	18.131	1.209	23.964	1.597
19.8	2.135	94730	14.746	19.818	1.219	26.193	1.612
20.3	2.116	103384	14.868	21.824	1.230	28.844	1.625

Table 2C
Unsteady Drainage Data from Model No. 1A

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	t second	$\frac{Q}{cm^3/cm^2}$	t. (p_b of finer layer)	Q. (p_b of finer layer)	t. (p_b of coarser layer)	Q. (p_b of coarser layer)
20.7	2.100	10	0.175	0.113	0.0569	0.150	0.0752
20.7	2.100	20	0.351	0.226	0.114	0.299	0.151
20.7	2.100	31	0.526	0.350	0.171	0.464	0.226
20.7	2.100	42	0.702	0.475	0.228	0.628	0.302
20.7	2.100	53	0.877	0.599	0.285	0.793	0.377
20.7	2.100	65	1.063	0.735	0.345	0.972	0.457
20.7	2.100	90	1.404	1.017	0.456	1.346	0.603
20.7	2.100	135	1.930	1.526	0.627	2.019	0.829
20.7	2.100	152	2.105	1.718	0.684	2.273	0.905
20.7	2.100	195	2.456	8.205	0.798	2.916	1.055
20.7	2.100	249	2.807	2.815	0.912	3.724	1.206
20.7	2.100	283	2.982	3.199	0.969	4.233	1.281
20.7	2.100	381	3.333	4.307	1.083	5.698	1.432
20.7	2.100	414	3.421	4.680	1.111	6.192	1.470
20.7	2.100	589	3.728	6.659	1.211	8.809	1.602
20.7	2.100	630	3.772	7.122	1.226	9.422	1.621
20.7	2.100	816	3.904	9.225	1.268	12.204	1.678
20.7	2.100	1038	3.991	11.735	1.397	15.524	1.715
20.7	2.100	1130	4.018	12.775	1.305	16.900	1.727
20.7	2.100	1520	4.096	17.184	1.331	22.733	1.760
20.7	2.100	2117	4.175	23.934	1.356	31.662	1.794
20.7	2.100	3083	4.254	34.855	1.382	46.109	1.828
20.7	2.100	5026	4.351	56.821	1.411	75.169	1.870

Table 3C
Unsteady Drainage Data from Model No. 1B

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	t second	Q cm^3/cm^2	t. (p_b of finer layer)	Q. (p_b of finer layer)	t. (p_b of coarser layer)	Q. (p_b of coarser layer)
20.0	2.127	12	0.0789	0.0200	0.0143	0.0252	0.0180
20.0	2.127	22	0.123	0.0366	0.0222	0.0462	0.0280
20.0	2.127	31	0.167	0.0516	0.0302	0.0651	0.0381
20.0	2.127	41	0.211	0.0682	0.0881	0.0861	0.0481
20.0	2.127	51	0.254	0.0849	0.0459	0.107	0.0579
20.0	2.127	61	0.298	0.102	0.0538	0.128	0.0679
20.0	2.127	71	0.342	0.118	0.0618	0.149	0.0780
20.0	2.127	92	0.430	0.153	0.0777	0.193	0.0980
20.0	2.127	102	0.474	0.170	0.0857	0.214	0.108
20.0	2.127	112	0.518	0.186	0.0936	0.235	0.118
20.0	2.127	143	0.649	0.238	0.117	0.300	0.148
20.0	2.127	153	0.693	0.255	0.125	0.321	0.158
20.0	2.127	174	0.781	0.290	0.141	0.365	0.178
20.0	2.127	215	0.956	0.358	0.173	0.451	0.218
20.0	2.127	258	1.132	0.429	0.205	0.542	0.258
20.0	2.127	279	1.219	0.464	0.220	0.586	0.278
20.0	2.127	300	1.307	0.499	0.236	0.630	0.298
20.0	2.127	411	1.746	0.684	0.316	0.863	0.398
20.0	2.127	434	1.833	0.722	0.331	0.911	0.418
20.0	2.127	626	2.535	1.042	0.458	1.315	0.578
20.0	2.127	677	2.711	1.127	0.490	1.422	0.618
20.0	2.127	895	3.412	1.489	0.617	1.879	0.778
20.0	2.127	1051	3.851	1.749	0.696	2.207	0.878
20.0	2.127	1353	4.553	2.252	0.823	2.841	1.038

Table 3C continued

20.0	2.127	1700	5.123	2.829	0.926	3.570	1.168
20.0	2.127	2000	5.474	3.328	0.989	4.200	1.248
20.0	2.127	2770	6.000	4.610	1.084	5.817	1.368
20.0	2.127	3172	6.175	5.279	1.116	6.661	1.408
20.0	2.127	4064	6.439	6.763	1.164	8.534	1.468
20.0	2.127	5039	6.614	8.386	1.195	10.581	1.508
20.0	2.127	6328	6.789	10.531	1.227	13.288	1.548
20.0	2.127	6712	6.833	11.170	1.235	14.095	1.558
20.0	2.127	7117	6.877	11.844	1.243	14.945	1.568
20.2	2.119	9238	7.096	15.432	1.282	19.472	1.618
20.2	2.119	10517	7.192	17.569	1.300	22.168	1.640
20.2	2.119	13251	7.360	22.136	1.330	27.930	1.678
20.2	2.119	15862	7.482	26.497	1.352	33.434	1.706

Table 4C
Unsteady Drainage Data from Model No. 1C

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	t second	$\frac{Q}{cm^3/cm^2}$	$\frac{t}{(p_b \text{ of finer layer})}$	$\frac{Q}{(p_b \text{ of finer layer})}$	$\frac{t}{(p_b \text{ of coarser layer})}$	$\frac{Q}{(p_b \text{ of coarser layer})}$
20.5	2.108	17	0.0877	0.0324	0.0167	0.0591	0.0305
20.5	2.108	30	0.132	0.0572	0.0252	0.104	0.0459
20.5	2.108	43	0.175	0.0820	0.0334	0.149	0.0609
20.5	2.108	57	0.219	0.109	0.0418	0.198	0.0762
20.5	2.108	71	0.263	0.135	0.0502	0.247	0.0915
20.5	2.108	100	0.351	0.191	0.0670	0.347	0.122
20.5	2.108	114	0.395	0.217	0.0754	0.396	0.137
20.5	2.108	143	0.482	0.273	0.0920	0.497	0.168
20.5	2.108	173	0.570	0.330	0.109	0.601	0.198
20.5	2.108	203	0.658	0.387	0.126	0.705	0.229
20.5	2.108	266	0.833	0.507	0.159	0.924	0.290
20.5	2.108	301	0.921	0.574	0.176	1.046	0.320
20.5	2.108	409	1.184	0.780	0.226	1.421	0.412
20.5	2.108	448	1.272	0.854	0.243	1.557	0.442
20.5	2.108	532	1.447	1.015	0.276	1.849	0.503
20.5	2.108	705	1.754	1.345	0.335	2.450	0.610
20.5	2.108	885	2.018	1.688	0.385	3.075	0.702
20.5	2.108	1169	2.345	2.230	0.444	4.062	0.809
20.5	2.108	1401	2.500	2.648	0.477	4.825	0.870
20.0	2.127	2032	2.851	3.841	0.544	6.998	0.992
20.0	2.127	2427	3.026	4.588	0.578	8.358	1.052
20.0	2.127	3156	3.377	5.965	0.645	10.869	1.175
20.0	2.127	3695	3.640	6.984	0.695	12.725	1.266

Table 4C continued

20.0	2.127	5125	4.202	9.687	0.802	17.650	1.461
20.0	2.127	6077	4.465	11.487	0.852	20.929	1.553
20.0	2.127	8377	4.877	15.834	0.931	28.850	1.696

Table 5C
Unsteady Drainage Data from Prototype No. 2

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	t second	$\frac{Q}{cm^3/cm^2}$	t. (p _b of finer layer)	Q. (p _b of finer layer)	t. (p _b of coarser layer)	Q. (p _b of coarser layer)
22.9	2.018	40	0.114	0.00901	0.00906	0.0119	0.0120
22.9	2.018	56	0.158	0.0126	0.0126	0.0167	0.0166
22.9	2.018	73	0.202	0.0164	0.0161	0.0217	0.0212
22.9	2.018	90	0.246	0.0203	0.0196	0.0268	0.0259
22.9	2.018	107	0.289	0.0241	0.0230	0.0319	0.0304
22.9	2.018	124	0.333	0.0279	0.0265	0.0369	0.0350
22.9	2.018	141	0.377	0.0318	0.0300	0.0420	0.0396
22.9	2.018	158	0.421	0.0356	0.0335	0.0471	0.0442
22.9	2.018	175	0.465	0.0394	0.0370	0.0521	0.0489
22.9	2.018	192	0.509	0.0433	0.0405	0.0572	0.0535
22.9	2.018	226	0.596	0.0509	0.0474	0.0673	0.0626
22.9	2.018	260	0.684	0.0586	0.0544	0.0774	0.0719
22.9	2.018	295	0.772	0.0665	0.0614	0.0879	0.0811
22.9	2.018	329	0.860	0.0741	0.0684	0.0980	0.0904
22.9	2.018	364	0.947	0.0820	0.0753	0.108	0.0995
22.9	2.018	398	1.035	0.0897	0.0823	0.119	0.109
22.9	2.018	503	1.298	0.113	0.103	0.150	0.136
22.9	2.018	556	1.430	0.125	0.114	0.166	0.150
22.9	2.018	609	1.561	0.137	0.124	0.181	0.164
22.9	2.018	662	1.693	0.149	0.135	0.197	0.178
22.9	2.018	715	1.825	0.161	0.145	0.213	0.192
22.9	2.018	860	2.175	0.194	0.173	0.256	0.229
22.9	2.018	933	2.351	0.210	0.187	0.278	0.247
22.9	2.018	1025	2.570	0.231	0.204	0.305	0.270

Table 5C continued

22.9	2.018	1116	2.789	0.251	0.222	0.332	0.293
22.9	2.018	1209	3.009	0.272	0.239	0.360	0.316
22.9	2.018	1414	3.491	0.319	0.278	0.421	0.367
22.9	2.018	1528	3.754	0.344	0.298	0.455	0.395
22.9	2.018	1679	4.105	0.378	0.326	0.500	0.431
22.9	2.018	1760	4.298	0.397	0.342	0.524	0.452
22.9	2.018	1914	4.649	0.431	0.370	0.570	0.489
22.9	2.018	2073	5.000	0.467	0.398	0.617	0.526
22.9	2.018	2442	5.702	0.550	0.453	0.727	0.599
22.9	2.018	2653	6.053	0.598	0.451	0.790	0.636
22.9	2.018	2866	6.404	0.646	0.509	0.853	0.673
22.9	2.018	3121	6.798	0.703	0.540	0.929	0.714
22.8	2.021	3576	7.456	0.805	0.593	1.064	0.784
22.8	2.021	3819	7.808	0.859	0.621	1.136	0.821
22.8	2.021	4350	8.553	0.979	0.680	1.294	0.899
22.8	2.021	4447	8.693	1.001	0.691	1.323	0.914
22.8	2.021	5033	9.482	1.132	0.754	1.497	0.997
22.8	2.021	5458	10.009	1.228	0.796	1.623	1.052
22.8	2.021	5878	10.491	1.323	0.834	1.748	1.103
22.8	2.021	6334	10.974	1.425	0.872	1.884	1.153
22.8	2.021	6827	11.456	1.536	0.911	2.030	1.204
22.8	2.021	7322	11.895	1.647	0.946	2.178	1.250
22.8	2.021	7822	12.289	1.760	0.977	2.326	1.292
22.8	2.021	9106	13.175	2.049	1.047	2.708	1.385
22.8	2.021	9987	13.623	2.247	1.084	2.970	1.433
22.8	2.021	10886	14.009	2.449	1.114	3.237	1.472
22.8	2.021	11515	14.237	2.591	1.132	3.425	1.496
22.8	2.021	12782	14.623	2.876	1.163	3.801	1.537
22.8	2.021	14.854	15.105	3.342	1.201	4.418	1.588

Table 5C continued

22.8	2.021	16070	15.333	3.616	1.219	4.779	1.611
22.8	2.021	17386	15.544	3.912	1.236	5.171	1.634
22.8	2.021	18635	15.719	4.193	1.250	5.542	1.652
22.8	2.021	20045	15.895	4.510	1.264	5.961	1.671
22.8	2.021	22070	16.114	4.966	1.281	6.564	1.694
22.7	2.024	23987	16.289	5.390	1.295	7.122	1.712
22.7	2.024	28045	16.596	6.302	1.319	8.327	1.744
22.7	2.024	33032	16.895	7.422	1.343	9.807	1.776
22.7	2.024	74492	18.035	16.738	1.434	22.117	1.895
22.6	2.028	87367	18.219	19.588	1.448	25.887	1.915
22.5	2.032	103866	18.386	23.245	1.462	30.724	1.932

Table 6C
Unsteady Drainage Data from Model No. 2A

Temp. C°	$\mu/\rho g \times 10^5$ cm-sec	t second	Q cm ³ /cm ²	t. (p _b of finer layer)	Q.	t. (p _b of coarser layer)	Q.
20.1	2.123	6	0.175	0.0557	0.0586	0.0737	0.0775
20.1	2.123	13	0.351	0.121	0.118	0.160	0.155
20.1	2.123	20	0.526	0.186	0.176	0.246	0.233
20.1	2.123	27	0.702	0.251	0.235	0.332	0.311
20.1	2.123	35	0.877	0.325	0.294	0.430	0.388
20.1	2.123	42	1.053	0.390	0.353	0.516	0.466
20.1	2.123	50	1.228	0.464	0.411	0.614	0.544
20.1	2.123	58	1.404	0.539	0.470	0.712	0.622
20.1	2.123	67	1.579	0.622	0.529	0.823	0.699
20.1	2.123	87	1.930	0.808	0.646	1.069	0.855
20.1	2.123	132	2.368	1.226	0.793	1.622	1.049
20.1	2.123	147	2.500	1.365	0.837	1.806	1.107
20.1	2.123	183	2.763	1.700	0.925	2.248	1.224
20.1	2.123	229	3.026	2.127	1.013	2.813	1.340
20.1	2.123	258	3.158	2.396	1.057	3.169	1.399
20.1	2.123	359	3.465	3.334	1.160	4.410	1.535
20.1	2.123	471	3.640	4.374	1.219	5.786	1.612
20.1	2.123	560	3.728	5.201	1.249	6.879	1.651
20.1	2.123	615	3.772	5.712	1.263	7.555	1.671
20.1	2.123	843	3.904	7.829	1.307	10.355	1.729
20.1	2.123	951	3.947	8.832	1.321	11.682	1.748
20.1	2.123	1182	4.018	10.977	1.345	14.520	1.780
20.1	2.123	1429	4.070	13.271	1.363	17.554	1.803
20.1	2.123	1575	4.096	14.627	1.371	19.347	1.814
20.1	2.123	2081	4.167	19.327	1.395	25.563	1.846
20.1	2.123	2704	4.219	25.113	1.413	33.216	1.869
20.1	2.123	3204	4.246	29.756	1.422	39.358	1.881
20.1	2.123	3674	4.263	34.121	1.427	45.132	1.888

Table 7C
Unsteady Drainage Data from Model No. 2B

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	t second	Q cm^3/cm^2	t. (p_b of finer layer)	Q. (p_b of finer layer)	t. (p_b of coarser layer)	Q.
23.4	2.000	3	0.0439	0.00987	0.00950	0.0143	0.0137
23.4	2.000	9	0.132	0.0296	0.0286	0.0428	0.0413
23.4	2.000	15	0.219	0.0493	0.0474	0.0713	0.0684
23.4	2.000	21	0.307	0.0691	0.0664	0.0998	0.0959
23.4	2.000	26	0.395	0.0855	0.0855	0.124	0.123
23.4	2.000	32	0.482	0.105	0.104	0.152	0.151
23.4	2.000	45	0.658	0.148	0.142	0.214	0.206
23.4	2.000	63	0.921	0.207	0.199	0.299	0.288
23.4	2.000	76	1.096	0.250	0.237	0.361	0.343
23.4	2.000	89	1.272	0.293	0.275	0.423	0.398
23.4	2.000	131	1.798	0.431	0.389	0.622	0.562
23.4	2.000	178	2.325	0.568	0.503	0.846	0.727
23.4	2.000	212	2.588	0.697	0.560	1.007	0.809
23.4	2.000	253	2.851	0.832	0.617	1.202	0.891
23.4	2.000	328	3.289	1.079	0.712	1.558	1.028
23.4	2.000	262	3.465	1.191	0.750	1.720	1.083
23.4	2.000	485	3.991	1.595	0.864	2.304	1.247
23.4	2.000	509	4.079	1.674	0.883	2.418	1.275
23.4	2.000	633	4.535	2.181	0.981	3.149	1.417
23.4	2.000	803	4.798	2.641	1.038	3.814	1.499
23.4	2.000	896	4.930	2.947	1.067	4.256	1.541
23.4	2.000	1161	5.193	3.819	1.124	5.515	1.623
23.4	2.000	1294	5.281	4.256	1.143	6.147	1.650
23.4	2.000	1769	5.491	5.819	1.188	8.403	1.716
23.4	2.000	1987	5.561	6.536	1.203	9.438	1.738
23.4	2.000	2247	5.623	7.391	1.217	10.673	1.757
23.4	2.000	2530	5.675	8.322	1.228	12.018	1.773
23.4	2.000	2812	5.719	9.250	1.238	13.357	1.787

Table 7C continued

23.4	2.000	3140	5.763	10.322	1.247	14.915	1.801
22.6	2.028	3877	5.833	12.577	1.262	18.161	1.823
22.6	2.028	4326	5.868	14.033	1.270	20.265	1.834
22.6	2.028	6031	5.956	19.564	1.289	28.252	1.861
22.6	2.028	8028	6.035	26.042	1.306	37.606	1.886
22.6	2.028	9361	6.061	30.366	1.312	43.851	1.894

Table 8C
Unsteady Drainage Data from Model No. 2C

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	t second	$\frac{Q}{cm^3/cm^2}$	$\frac{t}{(P_b \text{ of finer layer})}$	$\frac{Q}{(P_b \text{ of finer layer})}$	$\frac{t}{(P_b \text{ of coarser layer})}$	Q.
23.0	2.014	6	0.132	0.0244	0.0277	0.0466	0.0529
23.0	2.014	12	0.263	0.0488	0.0552	0.0932	0.105
23.0	2.014	25	0.526	0.102	0.110	0.194	0.211
23.0	2.014	45	0.921	0.183	0.193	0.349	0.369
23.0	2.014	52	1.053	0.211	0.221	0.404	0.422
23.0	2.014	73	1.447	0.297	0.303	0.567	0.580
23.0	2.014	95	1.886	0.386	0.395	0.738	0.755
23.0	2.014	128	2.280	0.520	0.478	0.994	0.913
23.0	2.014	161	2.544	0.654	0.533	1.250	1.019
23.0	2.014	197	2.807	0.801	0.589	1.530	1.124
23.0	2.014	257	3.201	1.045	0.671	1.995	1.282
23.0	2.014	278	3.333	1.130	0.699	2.158	1.335
23.0	2.014	324	3.597	1.317	0.754	2.516	1.441
23.0	2.014	440	4.123	1.789	0.865	3.416	1.652
23.0	2.014	523	4.386	2.126	0.920	4.061	1.757
23.0	2.014	732	4.781	2.975	1.003	5.683	1.915
23.0	2.014	850	4.956	3.455	1.039	6.600	1.985
23.0	2.014	1109	5.132	4.508	1.076	8.610	2.056
23.0	2.014	1443	5.263	5.866	1.104	11.204	2.108
23.0	2.014	1831	5.368	7.443	1.126	14.216	2.150
23.0	2.014	2343	5.465	9.524	1.146	18.192	2.189
23.0	2.014	2995	5.544	12.174	1.163	23.254	2.221
23.0	2.014	3758	5.605	15.276	1.175	29.178	2.245
23.0	2.014	5203	5.684	21.149	1.192	40.397	2.277
23.0	2.014	7150	5.737	29.063	1.203	55.514	2.298