by

## MOOSA CHEUNG

## A Thesis

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The Faculty of Graduate Studies
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# In Partial Fulfillment of the Requirements for the Degree of Master of Science 

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BUBBLE DYNAMICS IN CONSTANT-FLOW BARBOTAGE.
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by
Moosa Cheung

A thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

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## ABSTRACT

In order to improve our understanding of complex $=$ boiling heat transfer, there has recently been an increased interest in simulating boiling by barbotage systems (bubbling of a gas into a liquid), as both phenomena are known to promote heat transfer between a heat-transfer surface and a liquid through the stirring of the boundary layer. The present work, in yet further detail, examines and compares bubble hydrodynamics in pool barbotage and saturated pool boiling. The hydrodynamics referred to here are bubble growth ( $R$ vs. $t$, where $R$ is the bubble radius, and $t$ is the growth time), bubble growth rate (dR/dt vs. $t$ ), and bubble departure radius.

The present study concentrates exclusively on constantflow barbotage and nucleate pool boiling as it has been found that certain aspects of bubble growth have more similarities between constant-flow barbotage and boiling bubbles rather than the other limiting case of constantpressure barbotage bubbles. The barbotage experiments presented in this thesis were devised in which it was possible to compare the hydrodynamics of constant-flow barbotage bubbles with those of nucleate-boiling bubbles

- reported by Cole and Shulman for the same liquids and the same liquid temperatures. The flow rates for barbotage were chosen to give growth curves ( R vs. t ) falling, for a substantial portion of the growth time, between the
uppermost and the lowest growth curves for boiling bubbles. (For any one set of fixed boiling conditions, Cole and Shulman show a number of bubble growth curves.). Acetone, methanol, carbon tetrachloride, distilled water, and toluene (the same liquids employed by Cole and Shulman) were used as the experimental liquids and air as the injected gas. Bubble growth and departure radii were determined by highspeed cine photography for all tested liquids and conditions (air flow rates, liquid temperatures and orifice diameters). The experimental results were then quantitively compared with the constant-flow barbotage theories and nucleate boiling.

The comparisons between the experimental results and the barbotage theories showed:
(i) The experimental growth results were in excellent agreement with the constant-flow bubble growth equation. Thus, it could be concluded that the present apparatus genuinely generated constant-flow barbotage bubbles.
(ii) Bubble departure radii were found to be in good agreement with predictions from the literature with a maximum deviation of $9 \frac{1}{2} \%$ in toluene.

For barbotage and boiling, the comparisons indicated:
(i) The bubble growth curves (R vs. t) of the present - barbotage experiments had shapes similar to (but not identical to) the boiling results for the same bulk liquid temperatures.
(ii) The bubble growth rates (dR/dt vs. t) of
barbotage bubbles had similar shapes and magnitudes to those of lower boiling bubbles.
(iii) The bubble departure radii of barbotage bubbles fell within the range of those for the boiling bubbles. $=$.
$=\quad$ The author wishes to express his deep gratitude to Dr. G.E. Sims, the author's thesis advisor, for his invaluable guidance and encouragement he received during the course of this work. Special thanks are due to Mr. L.W. Wilkins of the Mechanical Engineering Machine Shop and to Mr. F. Kapitoler, a technician in the Mechanical Engineering Department for their construction of the experimental apparatus. Finally, financial support for the project by the Mechanical Engineering Department in the form of a teaching assistantship and by the National Research Council of Canada are gratefully acknowledged.

## TABLE OF CONTENTS

Page
ABSTRACT ..... iii
ACKNOWLEDGEMENTS ..... vi
TABLE OF CONTENTS ..... vii
LIST OF FIGURES ..... ix
LIST OF TABLES ..... xi
NOMENCLATURE ..... $x i v$
CHAPTER 1 INTRODUCTION ..... 1
1.1 Background ..... 1
1.2 Purpose and Scope ..... 4
1.3 Layout of Thesis ..... 5
CHAPTER 2 LITERATURE REVIEW ..... 7
2.1 Barbotage Bubble Dynamics ..... 7
2.1.1 Introductory remarks ..... 7
2.1.2 Bubble growth rate ..... 8
2.1.3 Bubble departure size ..... 11
2.2 Nucleate Boiling Dynamics ..... 16
2.2.1 Introductory remarks ..... 16
2.2.2 Bubble growth rate ..... 17
2.3 Barbotage as an Analog of Boiling ..... 19
CHAPTER 3 APPARATUS ..... 20
3.1 Introductory Remarks ..... 20
3.2 Test Section ..... 20
3.3 Air Supply System ..... 23
3.4 Photographic Equipment ..... 28
CHAPTER 4 PROCEDURE ..... 32
4.1 Introductory Remarks ..... 32
4.2 High-Speed Cine Photography ..... 32
4.3 Calculation of Instantaneous Volume of A Bubble ..... 34
4.4 Experimental Conditions ..... 35
CHAPTER 5 RESULTS AND DISCUSSIONS ..... 38
5.1 Bubble Behavior ..... 38
5.2 Bubble Growth and Departure Size ..... 42
5.2.1 Presentation of results ..... 42
5.2.2 Comparison of experimental and theoretical bubble growth ..... 55
5.2.3 Comparison of experimental and theoretical bubble departure size ..... 65
5.3 Comparison of Present Barbotage and Nucleate Boiling Bubble Growth Rates ..... 65
CHAPTER 6 SUMMARY AND CONCLUSIONS ..... 82
REFERENCES ..... 85
APPENDIX A - Repeatability Tests ..... 92
APPENDIX B - Calibration of the Rotameter ..... 98
APPENDIX C - Physical Properties of Experimental Liquids ..... 106
APPENDIX D - Calculation of Bubble Volume ..... 108
APPENDIX E - Calculation of Actual Flow Rate at Orifice - ..... 111
APPENDIX F - Bubble Identification and Tabulated Data ..... 116
Figure Page
2.1 Idealized Sequence of Bubble Formation ..... 14
3.1 Orifice Plate Arrangements ..... 21
3.2 Test Section Arrangement ..... 24
3.3 Schematic of Air Supply and Bubbling Unit ..... 26
3.4 Arrangement of Photographic Equipment ..... 30
5.1 Typical Behavior of Double Bubbles ..... 39
5.2 Typical Behavior of Three-bubble Series ..... 41
5.3 Bubble Growth Data in Acetone (Actual Flow Rate at Orifice, $15.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 45
5.4 Bubble Growth Data in Acetone (Actual Flow Rate at Orifice, $34.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 46
5.5 Bubble Growth Data in Methanol
(Actual Flow Rate at Orifice, $42.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 47
5.6 Bubble Growth Data in Methanol
(Actual Flow Rate at Orifice, $56.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 48
5.7 Bubble Growth Data in Carbon Tetrachloride
(Actual Flow Rate at Orifice, $21.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 49
5.8 Bubble Growth Data in Carbon Tetrachloride (Actual Flow Rate at Orifice, $37.9 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 50
5.9 Bubble Growth Data in Distilled Water
(Actual Flow Rate at Orifice, $113.5 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 51
5.10 Bubble Growth Data in Distilled Water
(Actual Flow Rate at Orifice, $180.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 52
5.11 Bubble Growth Data in Toluene
(Actual Flow Rate at Orifice, $130.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 53
5.12 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Acetone - Actual Flow Rate, $15.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 56
5.13 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Acetone - Actual Flow Rate, $34.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 57
5.14 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Methanol - Actual Flow Rate, $42.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 58
5.15 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Methanol - Actual Flow Rate, $56.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 59
5.16 Comparison of Experimental Results and Bubble Growth Egn. 2.4 for Carbon Tetrachloride - Actual Flow Rate, $21.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 60
5.17 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Carbon Tetrachloride - Actual Flow Rate, $37.9 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 61
5.18 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Distilled Water - Actual Flow Rate, $113.5 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 62
5.19 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Distilled Water - Actual Flow Rate, $180.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 63
5.20 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Toluene - Actual Flow Rate, $130.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ..... 64
5.21 Comparison of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Acetone ..... 68
5.22 Comparsion of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Methanol ..... 69
5.23 Comparison of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Carbon Tetrachloride ..... 70
5.24 Comparison of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Distilled Water ..... 71
5.25 Comparison of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Toluene ..... 72
5.26 Comparison of Bubble Growth Rate Between Present Barbotage and Boiling Bubbles for Acetone ..... 73
5.27 Comparison of Bubble Growth Rate Between Present Barbotage and Boiling Bubbles for Mathanol ..... 74
5.28 Comparison of Bubble Growth Rate Between Present Barbotage and Boiling Bubbles for Carbon Tetrachloride ..... 75
5. 29 Comparison of Bubble Growth Rate Between Present Barbotage and Boiling Bubbles for Distilled Water ..... 76
5.30 Comparison of Bubble Growth Rate Between Present Barbotage and Boiling Bubbles for Toluene ..... 77
A. 1 Comparison of Bubble Growth Curves between the Repeatability Test and the Original Experimental Results for Distilled Water ..... 94
A. 2 Comparison of Bubble Growth Curve between the Repeatability Test and the Original Experimental Results for Methanol ..... 95
B. 1 Calibration Apparatus Arrangement for Rotameter ..... 99
B. 2 Comparison of Calibration Results with Manufacturer's Curve for Rotameter ..... 104
D.1. Enlarged Bubble Outline for Volume Calculation ..... 109
Table Page
4.1 Experimental Conditions for the Bubble Growth ..... 36
Experiments
5.1 Summary of Bubble Type Observed in Different Liquids and Conditions ..... 43
5.2 Bubble Departure Times and Radii ..... 54
5.3 Comparison of Experimental Bubble Departure Radii with the Predictions of Kumar and Kuloor ..... 66
5.4 Comparsion of Bubble Departure Radii between Present Barbotage and Boiling Bubbles ..... 79
A. 1 Repeatability Test:Bubble Growth Data for Distilled Water (Actual Flow Rate, $\left.180.0 \mathrm{~cm}^{3} / \mathrm{sec}\right)$. ..... 96
A. 2 Repeatability Test:Bubble Growth Data for Methanol (Actual Flow Rate, $42.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ). ..... 97
B. 1 Operating Conditions and Calibration Results for Rotameter ..... 103
C. 1 Physical Properties of the Experimental Liquids ..... 107
E. 1 Calculation Results of the Actual Flow Rate for All Tested Liquids and Corresponding Experimental Conditions ..... 114
F. 1 Bubble Growth Data for Acetone (Actual Flow Rate, $15.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 117
F. 2 Bubble Growth Data for Acetone (Actual Flow Rate, $34.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 118
F. 3 Bubble Growth Data for Methanol (Actual Flow Rate, $42.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 120
F. 4 Bubble Growth Data for Methanol (Actual Flow Rate, $56.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 122
F. 5 Bubble Growth Data for Carbon Tetrachloride (Actual Flow Rate, $21.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 124
F. 6 Bubble Growth Data for Carbon Tetrachloride (Actual Flow Rate, $37.9 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 126
F. 7 Bubble Growth Data for Distilled Water (Actual Flow Rate, $113.5 \mathrm{~cm}^{3} / \mathrm{sec}$ ) ..... 127
F. 8 Bubble Growth Data for Distilled Water
(Actual Flow Rate, $180.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ) 130
F. 9 Bubble Growth Data for Toluene (Actual Flow Rate, $130.1 \mathrm{~cm}^{3} / \mathrm{sec}$ )132

| C | Bubble growth constant (Eqn. 2.11) |  |
| :---: | :---: | :---: |
| $\mathrm{C}_{\mathrm{p}}$ | Specific heat of liquid at constant pressure | $\mathrm{J} / \mathrm{kg} \cdot \mathrm{K}$ |
| D | Diameter of bubble | $m$ or cm |
| f | Frequency | $\sec ^{-1}$ |
| 9 | Acceleration due to gravity | $\mathrm{m} / \mathrm{sec}^{2}$ or $\mathrm{cm} / \mathrm{sec}^{2}$ |
| $\mathrm{H}_{\mathrm{fg}}$ | Latent heat of vaporization | $\mathrm{J} / \mathrm{kg} \cdot \mathrm{K}$ |
| Ja | Jacob number (Eqn. 2.11) | - |
| K | Orifice constant | - |
| M | Virtual mass | kg or gm |
| P | Pressure | $\mathrm{N} / \mathrm{m}^{2}$ or cm Hg |
| P | Pressure in the plenum chamber | $\mathrm{N} / \mathrm{m}^{2}$ |
| P | Pressure at a large distance | $\mathrm{N} / \mathrm{m}^{2}$ |
| $Q$ | Volumetric air flow rate | $\mathrm{ft}^{3} / \mathrm{hr}$ or $\mathrm{cm}^{3} / \mathrm{sec}$ |
| R | Radius of bubble | $m$ or cm |
| T | Temperature | ${ }^{\circ} \mathrm{C}$ or K |
| t | Time | sec |
| V | Volume of bubble | $\mathrm{m}^{3}$ or $\mathrm{cm}^{3}$ |

## Greek Symbols

| $\rho$ | Density | $\mathrm{kg} / \mathrm{m}^{3}$ or $\mathrm{g} / \mathrm{cm}^{3}$ |
| :--- | :--- | :---: |
| $\sigma$ | Surface tension | $\mathrm{N} / \mathrm{m}$ or dynes $/ \mathrm{cm}$ |
| $\theta$ | Contact angle | degree |
| $\alpha$ | Thermal diffusivity of liquid | $\mathrm{m}^{2} / \mathrm{sec}$ |

Subscript
act Actual flow rate at orifice
b Bubbling conditions (or conditions at orifice)
d Departure
eq Equivalent
e Expansion stage
F Final
fb Force balance at the expansion stage
9 gas
mix Mixture of air and liquid vapor
I Indicated air flow rate
1 Liquid
m Metering conditions

- Orifice
sat Saturation temperature of liquid
v,vap Vapor
w Wall (or surface)


## CHAPTER 1

## INTRODUCTION

## $=1.1$ Background

Pool barbotage is the bubbling of a gas through an immersed orifice or porous surface into a pool of initially stationary liquid. Pool boiling is boiling on a heater surface submerged in a pool of initially stationary liquid. One commom type of pool boiling is nucleate boiling which is characterized by the growth of bubbles in a liquid at specfic points on an immersed hot solid surface with the temperature sufficiently greater than the saturation temperature of the liquid. The relation between boiling and barbotage is of great interest as both phenomena are known to promote heat transfer between a heat-transfer surface and a liquid through a bubble-stirred boundary layer.

In general, there are two limiting types of flow in barbotage systems. One case, commonly referred to as the "constant-flow" case, pertains to bubble formation in which the rate of gas flow into a bubble is essentially constant. The other limiting case, referred to as the "constantpressure" case, pertains to bubble formation at an orifice which is supplied with gas from an ante-chamber at constant pressure. A more detailed discussion of these two limiting cases will be given in Chapter 2, Literature Review.
bubble-stirred boundary layers because, in contrast with boiling, the generation rate of a barbotage system is independent of the rate of heat transfer and can be accurately controlled and measured. Further, in boiling, there are heat transfer mechanisms involving both latent heat and agitation effects while in barbotage only agitation effects are present. Agitation effects in both barbotage and boiling are related to bubble hydrodynamics: generally speaking, the phenomena associated with bubble growth and departure. Since only agitation effects are present in barbotage systems, this can be an advantageous simplication; thus barbotage studies should help to improve the understanding of heat transfer across bubble-stirred boundary layers and hopefully shed additional light on the mechanisms in boiling.

When simulating nucleate boiling by barbotage, the aspects considered may be purely hydrodynamic, or may include heat transfer. As regards hydrodynamics, the similarities in appearence of flow regimes in barbotage and saturated nucleate boiling have been noted[43,44,49]; the similarities of bubble formation with time have also been pointed out [37,43]. Some investigators[2,35,44] have used barbotage to simulate the critical heat flux. With respect to heat transfer, the average heat-transfer coefficients in both boiling and barbotage have been compared[2,50,51]. Barakat and Sims[4,5] have compared the instantaneous heattransfer coefficients and liquid flow patterns in pool boiling and pool barbotage, commenting on the similarities
and dissimilarities. They have also shown, through the examination of bubble growth behavior that there is more similarity between boiling bubbles and barbotage bubbles under constant-flow conditions rather than constant-pressure conditions. It should be noted that the constant-flow barbotage bubbles generated by Barakat and Sims[5] for the heat transfer comparison were at very small gas flow rates ( $<1.0 \mathrm{~cm}^{3} / \mathrm{sec}$ ); the departure sizes of the barbotage bubbles were considerably smaller than the specific boiling bubbles in the comparison and the barbotage bubble growth times were longer. The liquid under consideration was toluene.

The present work attempted to answer the question: "How well can the hydrodynamics of boiling bubbles be simulated by barbotage bubbles using a simple system, namely constant-flow barbotage?" The hydrodynamic phenomena studied here were bubble growth ( $R$ vs. $t$, where $R$ is the bubble radius, and $t$ is the bubble growth time), bubble growth rate (dR/dt vs. t), and bubble departure radius. These are no doubt important in determining agitation effects. The approach was to find boiling results in saturated nucleate pool boiling which would allow for the operation of the barbotage system at atmosphere pressure. (It was desired to run at the same liquid temperatures as those for boiling in order to obtain similar liquid properties). An excellent set of boiling results-was available in the work of Cole and Shulman[6] who showed a number of bubbles for nominally the same conditions in
different liquids; for each bubble they also showed a complete growth curve ( R vs. t ) from bubble initiation to bubble break-off radius. Their results were generally at sub-atmospheric pressures, allowing in a number of cases, by virture of the low saturation temperatures, the corresponding barbotage systems to be run at room temperature or some degrees above room temperature at atmospheric pressure.

For the longer term, if the hydrodynamics of boiling bubbles can be more closely simulated by barbotage systems, heat transfer experiments similar to those of Barakat and Sims[5] could be repeated.

### 1.2 Purpose and Scope

The present study concentrates exclusively on constant-flow barbotage and nucleate boiling. The objectives of the study were:
(i) to conduct experiments in different liquids to determine the bubble growth and bubble departure radii of constant-flow barbotage under conditions such as to compare with certain of the boiling results reported by Cole and Shulman[6],
(ii) to compare the above barbotage experimental results with the bubble growth equation and theoretical predictions of bubble departure radii based on the model of Kumar and Kuloor[22].

For comparison of the hydrodynamics between boiling
and the present barbotage results, gas flow rates, based on the barbotage bubble growth equation, were chosen to give growth curves (R vs. $t$ ) falling, for most of the growth time, roughly mid-way between the uppermost and the lowest bubble growth curves for the boiling results of Cole and Shulman[6]. (It should be kept in mind that Cole and Shulman indicate a large variation in bubble growth, $R$ vs. $t$ for a fixed set of conditions.) The liquids used were acetone, methanol, carbon tetrachloride, distilled water, and toluene. The temperatures of the liquids used in the present investigation were essentially the same as the saturation temperatures corresponding to the pressures stated by Cole and Shulman, thereby obtaining similar liquid properties for both barbotage and boiling. Air was used as the bubbling gas. Orifice diameters of $0.3 \mathrm{~cm}, 0.4 \mathrm{~cm}$, and 0.6 cm were used.

### 1.3 Layout of Thesis

A literature review on bubble hydrodynamics in pool barbotage and nucleate pool boiling is presented in Chapter 2. Chapter 3 gives details of the experimental apparatus, and photographic equipment while Chapter 4 covers the experimental procedures and conditions. Chapter 5 includes the results and discussion and is divided into three main parts as follows:
(i) Bubble behavior,
(ii) Bubble growth and departure size,
(iii) Comparison of present barbotage and nucleate-
boiling bubble hydrodynamics. The summary and conclusions are presented in Chapter 6. Repeatability tests, calibration of the rotameter, physical properties of experimental liquids, calculation of bubble volume, calculation of actual flow rate at the orifice, and bubble identification and tabulated data are presented in appendices.

The general behavior of barbotage bubbles is shown in Figs. 5.1 and 5.2; the bubble type observed is summarized in Table 5.1. The bubble growth results in different liquids and conditions are given in Figs. 5.3 through 5.11 while the comparison of experimental results and bubble growth equation is presented in Figs. 5.12 to 5.20. Table 5.3 summarizes the comparison of experimental bubble departure radii with theoretical predictions from the literature. Figures 5.21 to 5.30 and Table 5.4 give comparisons of the hydrodynamics between the present barbotage and boiling bubbles in terms of the bubble growth ( $R$ vs.t), bubble growth rate (dR/dt vs. $t$ ), and bubble departure radius.

## CHAPTER 2

## REVIEW OF THE LITERATURE

### 2.1 Barbotage Dynamics

### 2.1.1 Introductory remarks

As a barbotage bubble is formed by the flow of a gas through an upward-facing orifice above a gas chamber (sometimes also called an ante-chamber), the pressure within the bubble changes with time. If the pressure beneath the orifice is constant, the gas flow rate will vary with time. If there is a high pressure drop restriction, such as a very long thin capillary, between the orifice and gas chamber, the pressure fluctuations due to the forming bubbles are much smaller than the pressure drop between the orifice and the gas chamber. In this case, the gas flow rate can be treated as a constant, a limiting case. If the volume of the gas chamber upstream of the orifice is very large by comparison with the volume of bubble being formed and if the pressure drop across the orifice is small, the situation corresponds to the other limiting case of bubble formation under constant-pressure-supply conditions. For conditions intermediate between the limits of constant flow rate and constant pressure, the chamber volume must be taken into account. The phenomenon of bubble formation under these various conditions was first reported by Hughes et al.[19]
and later thoroughly reviewed by Park[26], and by Kumar and Kuloor[22]. The following review keeps the emphasis more on barbotage bubble formation under constant-flow-rate conditions than on barbotage under constant-pressure-supply and intermediate conditions. The review is made under two separate headings, namely, 'bubble growth rate' and 'bubble departure size'.

### 2.1.2 Bubble growth rate

Bubble formation from a submerged orifice under constant-flow-rate conditions can be achieved by passing the gas through a long thin capillary in line with the orifice or through a porous plate attached to the underside of the orifice plate. The pressure drop across the capillary or porous plate is very large so that the pressure fluctuations within the bubbles due to changes in the radius of curvature have a negligible effect on the gas flow rate. Since the gas flow in this case is constant, the bubble growth is, therefore, simply expressed as

$$
d v / d t=Q,
$$

where $V=$ volume of the bubble,
$t=t$ ime,
Q=gas flow rate.
If the bubble volume throughout the period of growth is assumed to be of a spherical shape, then Eqn. 2.1 can be written as

$$
\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{4}{3} \pi R^{3}\right)=\mathrm{Q}
$$

where $R$ is the radius of the bubble. Rearranging Eqn. 2.2 gives

$$
\frac{d R}{d t}=\frac{Q}{4 \pi R^{2}}
$$

The integration of Eqn. 2.3 with $R=0$ at $t=0$ as intial condition would yield

$$
R=\left(\frac{3 Q}{4 \pi}\right)^{1 / 3} t^{1 / 3}
$$

Substitution of $R$ from this equation in Eqn. 2.3 gives

$$
\frac{d R}{d t}=0.2068 Q^{\frac{1}{3}} t^{-\frac{2}{3}}
$$

Equations 2.4 and 2.5 are to be used to compare with the present investigation of constant-flow barbotage in the five different liquids mentioned earlier in Chapter 1.

For the other limiting case of bubble formation under constant-pressure conditions, bubble growth is more complicated as compared with bubble growth under constantflow conditions. In the constant-pressure case, the small pressure drop across the orifice, coupled with the changing pressure in the bubble and the constant plenum chamber pressure, results in a changing gas flow rate during the entire period of bubble growth.

Subash and Sims[37] formulated a bubble growth equation under constant-pressure conditions by combining the equation of motion for an expanding bubble (the Rayleigh equation [29]) with the orifice equation. The result of the
formulation is given in the following dimensionless relation:

$$
\mathrm{R}^{\star} \ddot{\mathrm{R}}+1.5 \dot{R}^{\star}+8 \mathrm{ER} \star^{4} \dot{R}^{\star^{2}}+\frac{2}{\mathrm{R}^{\star}}=2 \Delta \mathrm{P} *
$$

$$
\text { Where } \begin{aligned}
R^{*} & \equiv \frac{R}{R_{O}}, \\
\dot{R}^{*} & \equiv \frac{\partial R^{*}}{\partial t^{*}}, \\
\ddot{R}^{*} & \equiv \frac{\partial}{\partial t^{*}}\left(\frac{\partial R^{*}}{\partial t^{*}}\right), \\
t^{*} & \equiv t\left[\frac{\sigma}{\rho_{L} R_{O}^{3}}\right]^{\frac{1}{2}}, \\
\Delta P^{*} & \equiv \frac{\Delta \mathrm{P}}{\Delta P_{C r i t}} \equiv\left[\frac{p_{1}-p_{\infty}}{\frac{2 \sigma}{R_{\odot}}}\right], \\
E & =\frac{1}{(K)^{2}} \frac{\rho_{g}}{\rho_{L}},
\end{aligned}
$$

R=radius of the bubble,

$$
\mathrm{R}_{0}=\text { radius of the orifice, }
$$

$$
k=\text { orifice constant, }
$$

$$
P_{1}=\text { Pressure in the plenum chamber, }
$$

$$
P_{\infty}=p r e s s u r e \text { at a large distance from the bubble at the }
$$

level of the orifice,
t=time,
ब=surface tension,

$$
\rho_{g}=g a s \text { density, }
$$

$$
\rho_{1}=1 \text { quid density. }
$$

The solution of En. 2.6 can be carried out by numerical integration, using the fourth order Runge-Kutta formula with conditions at time $t=0$,

$$
R^{*}(0)=1
$$

$$
\dot{R}^{*}(0)=0
$$

The bubble growth rates predicted by Eqn. 2.6 have been found to be in good agreement with the experimental data for water, acetone, and hexane. The complete derivation of Eqn. 2.6 is contained in Subash and Sims[37].

A survey of barbotage literature indicated that, so far, the analytical solution to bubble growth under intermediate conditions has not yet been available.

### 2.1.3 Bubble departure size

The determination of bubble departure volume (also called the 'terminal bubble volume') is strongly dependent on the gas flow rate in the constant-flow-rate case. In addition, orifice size, surface tension, liquid density, and liquid viscosity have been found to be the most important factors influencing the bubble departure size. A good review of the literature on the above-mentioned factors which influence the bubble departure size was presented by Kumar and Kuloor[22]. The theoretical predictions of bubble departure size under constant-flow-rate conditions are briefly discussed below.

Since the liquids involved in the present investigation have small viscosities ( $\leqslant 1.0 \mathrm{cp}$ ), emphasis will be placed on the theoretical predictions of bubble departure-size for inviscid liquids while for viscous liquids, the reader is referred to Refs.[10,22].

At very small flow rates ( $\leqslant 1.0 \mathrm{~cm}^{3} / \mathrm{sec}$ ) $[22,23]$, the bubble departure volume can be determined when the upward buoyant force is balanced by the downward force due to surface tension. The balance of these two static forces, therefore, directly gives the departure bubble volume as

$$
v_{d}=\frac{\pi D_{o} \sigma \cos \theta}{g\left(\rho_{1}-\rho_{g}\right)}
$$

where $V_{d}=$ bubble departure volume,
$D_{0}=$ orifice diameter,
$\theta=$ contact angle,
$\mathbf{g}=$ gravitational acceleration
The contact angle here is defined as the angle between the vertical and the stretched interface at the base of the bubble.

As the flow rate is increased, the liquid inertial force which is associated with the gas flow rate becomes significant. For small orifice sizes, Davidson and Schüler[11], assuming the bubble to be forming at a point source where the gas is supplied, have developed a simple theoretical equation considering only buoyant and liquid inertial forces. This equation is given by

$$
v_{d}=1.378 \frac{Q^{6 / 5}}{g^{6} / 5}
$$

where $V_{d}$ is the bubble departure volume and $Q$ is the gas flow rate. Eqn. 2.8 has been used to verify bubble departure volumes obtained for flow rates up to $3.0 \mathrm{~cm}^{3} / \mathrm{sec}$ and orifice radii from 0.0143 to 0.0334 cm . Davidson and

Schüler[11] find that theory and experiment agree excellently only in the flow range of 1.5 to $3.0 \mathrm{~cm}^{3} / \mathrm{sec}$. For larger orifices, Davidson and Schuler have also developed another equation which takes into consideration the residual bubble that forms the nucleus of the succeeding bubble. However, this equation has been compared with their experimental values obtained for larger orifices (0.15 to 0.25 cm in radii) and found to have considerable deviation for flow rates beyond $20.0 \mathrm{~cm}^{3} / \mathrm{sec}$.

Kumar and Kuloor[22] assume bubble formation takes place in two stages, namely, the expansion stage and detachment stage. The bubble is assumed to stay at the orifice in the first stage, whereas in the second stage it is assumed to travel away from the orifice until it detaches itself. An idealized sequence of bubble formation according to the theoretical prediction of Kumar and Kuloor[22] is shown in Fig. 2.1. At the end of the expansion stage, they consider that the bubble volume is determined by the balance between the upward buoyancy and the downward surface tension and inertial forces, for this situation, the bubble volume is expressed as

$$
v_{f b}-\frac{11 Q^{2} V_{f b}^{2 / 3}}{192 \pi(3 / 4 \pi)^{2 / 3} g}=\frac{\pi D_{0} \sigma \cos \theta}{\rho_{1} g}
$$

where $V_{f b}$ wolume of bubble at the end of the expansion stage, and the other symbols are as introduced earlier. During the detachment stage, the bubble moves upward while

growing and attaches to the orifice by a neck as shown in Fig. 2.1; the bubble detaches when its base has moved a distance equal to the radius $\left(R_{f b}\right)$ of the bubble volume at the end of the expansion stage. From the Newton's second law of motion, they obtain the equation of the final bubble volume at the end of the detachment stage. The equation is giveñ as

$$
\begin{align*}
R_{f b}= & \frac{P}{4 Q}\left(V_{F}^{2}-V_{f b}^{2}\right)-\frac{9 N}{Q}\left(V_{F}^{1 / 3}-v_{f b}^{1 / 3}\right)-\frac{J}{Q}\left(V_{F}-V_{f b}\right) \\
& +\frac{1}{Q}\left[\left(J V_{f b}+3 N V_{f b}^{1 / 3}-\frac{P}{2} v_{f b}^{2}\right)\right]\left(1 n V_{F}-1 n V_{f b}\right)
\end{align*}
$$

where $P=\equiv \frac{16 \mathrm{~g}}{11 Q}$,

$$
\begin{aligned}
& J=\equiv \frac{16 \pi D_{0} \sigma \cos \theta}{11 Q \rho_{1}}, \\
& N=\equiv \frac{Q}{12 \pi(3 / 4 \pi)^{2 / 3}},
\end{aligned}
$$

$V_{F}=f i n a l$ bubble volume,
$R_{f b}=r a d i u s$ of bubble at the end of the expanison
stage.

It should be noted that the gas density in both Eqns. 2.9 and 2.10 does not appear since it is assumed to be negligible in comparison with the liquid density (i.e. From Eqn. 2.10, the final bubble volume, $V_{F}$ can be calculated by iteration. The value of $R_{f b}$ to be used in Eqn. 2.10 is evaluated from the expansion stage, i.e. from Eqn. 2.9 with $\cos \theta$ taken as unity.

The above equations have been verified by Kumar and

Kuloor[22] with their own experiments and with experimental data including those experiments of Davidson and Schüler[11] and Datta et al.[8]; the theoretical values of bubble departure diameter are in good agreement with the experimental data. Equation 2.10, with $R_{f b}$ from Eqn. 2.9 will be used for comparison with the experimental results in the present investigation.

### 2.2 Nucleate Boiling Dynamics

### 2.2.1 Introductory remarks

Nucleate pool boiling occurs when a heater is submerged in a pool of initially stationary liquid. When the surface temperature of the heater sufficiently exceeds the saturation temperature of the liquid, vapor bubbles grow rapidly in the superheated liquid layer next to the surface until they depart and move out into the bulk liquid while rising as the result of buoyancy. If the bulk liquid is subcooled the bubbles collapse. Thus in nucleate boiling a complex fluid motion around the heater is initiated and maintained by the nucleation, growth , departure and collapse of bubbles as well to some degree by natural convection.

Good reviews of boiling bubble dynamics can be found in Refs.[18,30,40,42]. The bubble growth rate of nucleate pool boiling is briefly discussed below.

### 2.2.2 Bubble growth rate

The bubble growth period can be divided into three regions, namely the dynamically controlled region which occurs in the very early stage of bubble growth, the thermally controlled region in the later stage of bubble growth and the transition region where both dynamic and thermal effects can be important. The motion in the dynamically controlled region can be described by the Rayleigh solution[29]. The classical analyses, e.g.[12,13,27], have as a result for the thermally controlled region in a uniformly superheated liquid of infinite extent, that the bubble radius increases with the square root of time. The equation is of the form

$$
R=2 C J a \sqrt{\alpha t}
$$

where $C=$ growth constant,

$$
\alpha=\text { thermal diffusivity of the liquid, }
$$

$$
J a \equiv \frac{C_{p} \rho_{1}\left(T_{w}-T_{s a t}\right)}{H_{f g} \rho_{v}},
$$

$C_{p}=$ specific heat at constant pressure, $T_{w}=$ wall temperature at superheated conditions, $T_{\text {sat }}=$ saturation temperature of the liquid, $H_{f g}=$ latent heat of vaporization of the liquid.

The values of the growth constant, $C$ given by Fritz and Ende[13], Forster and Zuber[12], and Plesset and Zwick[27] are $\sqrt{1 / \pi}, \sqrt{\pi / 2}$, and $\sqrt{3 / \pi}$ respectively. Of special interest in the current investigation, Cole and Shulman[6] correlated
their experimental data for water and organic liquids at a heat transfer surface in saturated nucleate pool boiling by the expression

$$
R=2.5 \mathrm{Ja}^{3} / \sqrt[4]{\alpha t}
$$

In most practical situations, by far the major portion of bubble life is in the thermally controlled region.

As regards experimental data, especially for bubble growth at a heating surface, besides the radius increasing with a time exponent of $1 / 2$, one sees other values of this exponent as well. Westwater et al.[46], fitting their bubble growth results in the form $R \backsim t^{n}$ for pentane and ether, found that the mean time exponent, $n$, varied from 0.190 to 0.525 . Akiyama et al.[1] studied the effect of system pressure on the growth characteristics of an isolated vapor bubble using water, ethanol and carbon tetrachloride. They found that in the large pressure region $(1.0$ to 30.0 atm.), the time exponent decreased roughly from 0.5 to nearly 0.1 for water, while in the low pressure region (0.4 to 1.0 atm.$)$, the time exponents for the three liquids ranged from 0.4 to 1.0. Subash and Sims[37] used a leastsquares analysis to obtain the time exponents for some experimental bubble growth data published by various investigators[15,16,21, 36,41$]$ for saturated nucleate boiling of water at atmospheric pressure. They found that the time exponents ranged from 0.33 to 0.64 .

> It is understood that there are some situations,
especially in the boiling of liquid metals, where the transition region (both dynamic and thermal effects need to be considered) is important. Work such as that of Mikic et al.[25] and Theofanous et al.[38] pay special attention to this region.

### 2.3 Barbotage as an Analog of Boiling

In this section the existing literature comparing the bubble growth rates in barbotage and boiling will be briefly discussed. Zuber[49] considered Davidson and Amick's[9] description of the appearance of barbotage bubbles forming at an orifice under constant flow rate conditions and noted that the description fitted well the formation in Yamagata and Nishikawa's[48] experiments in nucleate boiling. Wallis[43] compared the bubble growth rate of Siemes and Kauffmann[33] for the constant-flow-rate case in barbotage to Stanizewski's[36] boiling growth rate results for water at atmospheric pressure; he concluded that the (volumetric) growth process in both cases was linear in nature during the major part of a bubble's history. Subash and Sims[37] indicated that the experimental bubble growth rate results in boiling appeared to fall between the growth rate results determined for the two limiting cases of barbotage, viz, the constant-flow case and constant-pressure case. Recently, Barakat[3] has compared barbotage with the boiling data of Cooper and Lloyd[7] for toluene and illustrated the similarity of the bubble growth behavior between boiling bubbles and constant-flow barbotage bubbles.

## CHAPTER 3

## APPARATUS

### 3.1 Introductory Remarks

The experimental apparatus used in the present work was designed to measure the growth of barbotage bubbles formed at submerged orifices of different sizes under constant flow conditions for different air-liquid systems. The apparatus can be divided into three major components, namely, the test section, air-supply system, and photographic equipment.

### 3.2 Test Section

The test section essentially consisted of an orifice plate, capillary tube and bubbling tank. The orifice plate arrangements are shown in Fig. 3.1. Three identical circular plates were made of stainless steel, each with a diameter of 16.8 cm and a thickness of 1.0 cm . An orifice was drilled in the centre of each plate. The orifice diameters of the three plates were $0.3 \mathrm{~cm}, 0.4 \mathrm{~cm}$, and 0.6 cm . For orifice diameters of 0.3 cm and 0.4 cm with lower gas flow rates, the capillary was divided into two sections. The upper section of the capillary that was directly glued to the plate in line with the orifice and had the same internal diameter as the orifice, while the lower section of the capillary had an internal diameter smaller than the orifice to ensure a large pressure drop for obtaining constant-flow-rate conditions. The arrangement for the

(A) 0.3 cm Orifice plate

(B) 0.4 cm Orifice plate

Fig. 3.1 Orifice Plate Arrangements.


Fig. 3.1- (Cont.) Orifice Plate Arrangements.
larger orifice diameter of 0.6 cm with higher gas flow rates was basically the same as those for the smaller orifices except the upper section of the capillary was packed with glass powder to make certain that a large pressure drop was achieved. The complete test section is shown in Fig. 3.2. The bubbling tank was made by bonding four glass plates of identical dimensions to the four sides of a 0.48 cm thick brass plate which formed the bottom of the tank The inside dimensions of the tank were $25 \mathrm{~cm} \times 25 \mathrm{~cm} \times 20 \mathrm{~cm}$. The hollow stainless steel cylinder served as a support for the bubbling tank and accommodation for the capillary tubes. For experimental runs at liquid temperatures higher than room temperature, the whole bubbling unit was immersed in a water bath tank which was a glass tank, 40.6 cm x 38.1 cm x 30.5 cm with a brass base. The left and right sides of the water bath were insulated to minimize the heat loss from the water bath to the surroundings while the rear and front were left uninsulated for photographic purposes.

### 3.3 Air Supply System

Figure 3.3 shows a schematic of the air supply system which consisted of a pressure regulator, filter, dryer, flow meter (rotameter), pressure gauge, saturators and a heater (used only for those experiments conducted at temperatures higher than room temperature).

Air was supplied from the building air compressor operating between $80 \mathrm{psig}(552 \mathrm{kPa})$ and $100 \mathrm{psig}(690 \mathrm{kPa})$ and reduced in pressure to the experimental operating

(A) For Room-temperature Experiments

Fig. 3.2 Test Section Arrangement.

(B) For Higher-temperature Experiments

Fig. 3.2-(Conť.): Test Section Arrangement.

(A) Arrangement for Room-temperature Experiments

Fig. 3.3 Schematic of Air Supply and Bubbling Unit.

(B) Arragement for Higher-temperature Experiments

Fig, 3,3 (Cont.) Schematic of Air Supply and Bubbling Unit.
condition in passing through the pressure regulator valve. As the air passed through the filter and the gas dryer, oil particles, dust and moisture present in the air were removed. Next, the air passed through the flow meter (Brooks Instrument Canada Ltd., Model No. 1560) which had a built-in needle valve to control accurately the flow range up to $125 \mathrm{~cm}^{3} / \mathrm{sec}$. The air was saturated with the vapor of the liquid in passing through two saturators containing that liquid. Finally, the air passed through the capillary and orifice into the bubbling tank containing the experimental liquid. The line pressure between the flow meter and the saturators was measured by a pressure gauge for calculation of the air flow rate at the flow meter to be corrected subsequently for conditions at the orifice. For experimental runs with liquid temperatures higher than room temperature, a 500 -watt (at 115 volts) General Electric Hot Point immersion heater was used, at variable voltage in the water bath surrounding the bubbling unit (as shown in Fig. 3.3b), thus raising the air and the liquid pool temperature to the required value.

The flow meter was calibrated before running the experiments. The calibration method and procedure are presented in Appendix $B$.

### 3.4 Photographic Equipment

A Hycam Model 41-0004 16 mm high-speed motion picture camera with a speed range of $10-11000$ frames per second was used. The air bubbles were photographed using as the
objective lens a Cosmicar 4 in. focal length, $f / 2.5$ with 30 mm 'C' mount extension. The camera was equipped with two built-in timing lights. One light generating at 100 pulses/sec was used to obtain the frame speed. The film stock was Kodak 16 mm , Ektachrome, VNX 430.7250, colour film in 100-ft rolls. The camera was located at a distance from the orifice plane, such that the field-of-view was slightly larger than the size of the terminal bubbles forming at the orifice. Before the filming began, the camera was focussed on a wire placed in the centre of the orifice. Two 650 watt (at 115 volts) lamps were used in conjunction with a ground glass screen to provide illumination for the photography. The light arrangement resulting in sharply defined bubble profiles is shown in Fig. 3.4, and was based on the experience of Subash and Sims[37] and Barakat[3]. To choose the correct combination of the film speed (ASA 400), f-stop, and camera speed, a Pentax Spotmeter III was used.

In order to determine the true volume of a bubble from a magnified image, a stainless steel rod whose actual diameter was known was suspended above the orifice vertically, and its image recorded at 500 frames/sec. The actual diameter of the rod was 0.955 cm .

The developed photographic films were projected frame by frame on a PCD Model 2AE-3A Viewer equipped with a digital $X-Y$ reader which enabled the accurate measurement of bubble volumes and bubble formation times. A Goodkin projector with an objective lens having a focal length of


High Speed
Camera

## Elevation



Plan View

Fig. 3.4 Arrangement of Photographic Equipment.

150 mm was employed to enlarge the figures of boiling data of Cole and Shulman[6] for the accurate measurement of their bubble growth results which were to be used later to compare with those of the present barbotage bubbles.

## CHAPTER 4

## PROCEDURE

### 4.1 Introductory Remarks

Before running the experiments, some air flow rates based on Eqn. 2.4 were chosen to generate growth curves (R vs. t) which fell within the band of boiling results, for fixed conditions, of Cole and Shulman[6] while the others were selected arbitrarily. High-speed cine photography was used to record the detail of bubble formation. While a systematic study of orifice size was not performed, in general, the idea was to keep the orifice size small consistent with obtaining a sufficient number of analyzable bubbles (typically 6 to 19 in approximately 3000 frames); the orifice sizes used were $0.3,0.4$, and 0.6 cm in diameters.

### 4.2 High-speed Cine Photography

Prior to each experimental run, the laboratory ventilation system was turned on; the bubbling tank was checked for leaks; and the inside compartment and objective lens of the camera were cleaned to ensure they were free of dust and foreign objects. The procedure before and during the photographing of bubbles growing at the orifice was then as follows:
(i) The bubbling tank was washed with ordinary soap and rinsed thoroughly with water. It was then dried with a
piece of clean cloth.
(ii) The two saturators were cleaned and filled to the two-third level with the experimental liquid.
(iii) The air supply system was connected to the bubbling unit and then turned on so that there was steady flow through the orifice. This precaution was necessary to prevent liquid leaking through the orifice while the tank was being filled.
(iv) The bubbling tank was filled with the experimental liquid to a depth of 12 cm above the orifice.
(v) The air flow rate was adjusted by means of the flow meter needle valve to obtain the desired value.
(vi) The camera was then focussed on the orifice plane with a piece of stainless wire placed in the centre of the orifice.
(vii) Two 650 watt lamps were turned on and the illumination was checked with the light meter for the correct combination of frame speed and aperture. The lamps were turned on only during the periods of the illumination check and photographing to prevent damage to the groundglass screen from overheating.
(viii) The camera was loaded with film. The timing light was set at 100 pulses/sec and the aperture was adjusted according to the exposure-meter reading. A rod with known diameter was put in the plane of the orifice and photographed at 500 frames/sec for 2 sec .
(ix) With the rod removed, the air flow rate was rechecked to ensure the desired value and steady state
conditions. Values of indicated flow rate, line pressure downstream of the flow meter, and pool temperature were then recorded.
(x) The frame speed was set at 2000 frames/sec, the aperture readjusted, and the bubble images photographed.

For the experimental runs with liquid temperatures higher than room temperature, the liquid was preheated to approximatly two degree Celsius above the desired temperature. The air was turned on. The heated liquid was then slowly poured into the bubbling tank surrounded by the water bath the temperature of which was being maintained by the immersion heater. During these experiments the saturators were in the water bath as well; the air to the orifice would then be at the same temperature as, and saturated with the vapor of, the pool liquid. Once equilibrium was established, the procedure in steps (v) to (x) was then repeated.

### 4.3 Calculation of Instantaneous Volume of A Bubble

The developed photographic films recording the sequence of the formation of each bubble were projected frame by frame on a screen to obtain a two-dimensional picture of the bubble. The screen, equipped with an $X-Y$ digital reader, provided an easy way to measure bubble dimensions with an accuracy of approximately 0.01 cm in the actual bubble. The procedure for measuring and computing bubble volumes using the method of L'Ecuyer and Murthy[23] is illustrated in Appendix D.

By assuming the measured volume equal to a sphere of equal volume, the equivalent radius of the bubble, $R_{e q}$ was obtained from the following equation:

$$
R_{e q}=\left[\frac{3}{4 \pi} \times v_{\text {measured }}\right]^{1 / 3}
$$

The subscript "eq" in $R_{\text {eq }}$ is to be omitted later in the thesis while the implication of equivalent radius remains.

### 4.4 Experimental Conditions

The conditions under which the experiments were run are summarized in Table 4.1. Since the calibration results of the rotameter were in very good agreement with the manufacturer's calibration curve (see Appendix B), the manufacturer's curve was, therefore, used to obtain the indicated air flow rates corresponding to the rotameter scale readings recorded during the experiments. The indicated air flow rates were corrected to actual metering conditions according to the manufacturer's instructions; these flow rates in turn were then used to calculate the actual flow rate (air saturated with vapour at the pool temperature) at the orifice. The relevant calculations are presented in Appendix $E$. The values of the actual flow rates at the orifice were the ones used to determine the theoretical growth rate in a later section.

The liquids used in the present investigation were acetone, methanol, carbon tetrachloride, distilled-water, and toluene. The first four liquids were supplied by Fisher Chemical Company and had a purity of 99.9 percent while the

| Liquid | Orifice <br> Dia. <br> cm | Room <br> Temp. | Liquid <br> Temp. | Indicated <br> Air Flow <br> Rate | Actual <br> Flow Rate <br> at <br> Orifice <br> ${ }^{\circ} \mathrm{C}$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Acetone | 0.3 | 24.4 | 24.4 | 11.0 |  <br> $\mathrm{cm}^{3} / \mathrm{sec}$ |
| 0.3 | 24.4 | 24.4 | 24.0 | 34.8 |  |
| Methanol | 0.3 | 23.3 | 23.3 | 35.0 | 42.3 |
| Carbon |  |  |  |  |  |
| Tetra- <br> Chloride | 0.4 | 25.6 | 25.6 | 44.4 | 56.3 |
|  | 0.3 | 23.3 | 31.1 | 29.3 | 37.9 |
| Distilled <br> Water | 0.6 | 25.5 | 25.5 | 88.7 | 113.5 |
| Toluene | 0.6 | 23.2 | 50.0 | 111.5 | 180.1 |

Table 4.1 Experimental Conditions For the Bubble Growth Experiments.
distilled water was readily available from the laboratory still. The liquid properties of the above liquids are presented in Appendix $C$.

The liquid temperatures were controlled within approximately one degree Celsius of the saturation temperatures corresponding to the pressures used by Cole and Shulman[6].

## CHAPTER 5

## RESULTS AND DICUSSIONS

5.1 Bubble Behavior

There were three types of bubble formation observed in the films and they could be classified as follows:
(a) Double bubbles,
(b) Series of three bubbles,
(c) Series of more than three bubbles.

The typical behavior of double-bubble formation is shown in Fig. 5.1. The first bubble (sometimes referred as the "leading bubble") started with a small meniscus at the orifice. This small meniscus increased its size due to the mass flow into the bubble and generally became somewhat pear-shaped for the smaller orifices but more spherical for the largest orifice. As the size of the first bubble increased further, a stage was reached when the upward forces acting upon the bubble had increased sufficiently to start lifting the bubble off the orifice. A very short stem started forming and connected the first bubble to the orifice. As the first bubble travelled up further, the stem either was broken clear or ended in a sharp neck. The lower part of the stem at the orifice formed the second bubble (sometimes referred as the "secondary bubble") which always caught up to the first bubble staight away. As the second


Fig. 5.1 Typical Behavior of Double Bubbles.
bubble continued to grow, it protruded into the first bubble, subsequently causing deformation of the top of the first bubble. The size of the second bubble was smaller than that of the first bubble and ended in a stem and clean break. After the second bubble had detached from the orifice, it combined with the first bubble and travelled up in the liquid column as a single unit.

Figure 5.2 shows the typical behavior of a threebubble series in which the first and second bubbles of the series behaved the same as those of the double bubbles described above. The third bubble in the series started completely free after the second bubble broke free at the orifice. As the third bubble grew at the orifice, it caught up to the combination of the first and second bubbles. During the later part of growth period, the third bubble fed the preceding bubble (combination of first and second bubbles) causing it to enlarge in size. The third bubble usually ended in a stem and clean break or sharp neck giving finally for the three bubbles a shape like that of a mushroom. In a few cases, mainly in toluene, the first bubble, for a small portion of its growth, overlapped the bottom of the previous bubble, but with its outline clearly visible such that its volume could be calculated; in this case such a bubble was still considered as a first bubble.

For series of more than three bubbles, the behavior of the first and second bubbles in the series was very similar to that of double bubbles. In fact, the bubble series of


Fifg. 5.2 Typical Behavior of Three-bubble Series.
this type could be broken down into: a number of double bubbles, a number of three-bubble series, or some combination of double bubbles and three-bubble series. The bubble connecting these double bubbles and/or three-bubble series together was a "first-like" bubble which generally acted like a first bubble, except this "first-like" bubble, while at the orifice, contacted the previous bubble for part of its growth period during which its volume could not be measured. A "first-like" bubble was followed by a typical second bubble.

Table 5.1 gives the summary of the bubble types for different liquids and conditions. It was always first bubbles which were analyzed in terms of bubble volume against time.

### 5.2 Bubble Growth and Departure Size

### 5.2.1 Presentation of results

Figures 5.3 through 5.11 show plots of bubble equivalent radius against time for all liquids (the subscript "eq" in Req is dropped in the figures). The tabulated bubble growth data and bubble identification are given in Appendix $F$. The bubble departure time, $t_{d}$, and bubble departure radius, $\mathrm{R}_{\mathrm{d}}$, are presented in Table 5.2.

As mentioned earlier, the bubble growth results reported here are for "first bubbles", as described in the previous section. The zero time for bubbles was taken

| Film No. | $\begin{gathered} \text { Liquid } \\ \text { and } \\ \text { Conditions } \end{gathered}$ | Frame Nos. Analyzed | Bubble Type Observed |
| :---: | :---: | :---: | :---: |
| 3A11 | Acetone; <br> Orifice Dia., $0.3 \mathrm{~cm} ;$ Actual Flow Rate, $15.8 \mathrm{~cm}^{3} / \mathrm{sec}$; Liquid Temp., $24.4^{\circ} \mathrm{C}$. | 0-3000 | 15 Double Bubbles; 4 5-Bubble Series. |
| 3A24 | Acetone; <br> Orifice Dia., $0.3 \mathrm{~cm} ;$ <br> Actual Flow Rate, $34.8 \mathrm{~cm}^{3} / \mathrm{sec} ;$ <br> Liquid Temp., $24.4^{\circ} \mathrm{C}$. | 29-3072 | 5 Double Bubbles; <br> 1 4-Bubble Series; <br> 3 5-Bubble Series; <br> 2 6-Bubble Series; <br> 1 12-Bubble Series. |
| 3M35 | Methanol; <br> Orifice Dia., 0.3 cm ; Actual Flow Rate, $42.3 \mathrm{~cm}^{3} / \mathrm{sec}$; Liquid Temp., $23.3^{\circ} \mathrm{C}$. | 84-2854 | 7 Double Bubbles; <br> 1 4-Bubble Series; <br> 1 6-Bubble Series; <br> 1 11-Bubble Series; <br> 1 12-Bubble Series. |
| 4M44.4 | Methanol; <br> Orifice Dia., $0.4 \mathrm{~cm} ;$ <br> Actual Flow Rate, $56.3 \mathrm{~cm}^{3} / \mathrm{sec}$; <br> Liquid Temp., $25.6^{\circ} \mathrm{C}$. | 16-3540 | 1 Double Bubble; 1 4-Bubble Series; 1 7-Bubble Series; 1 8-Bubble Series; 1 18-Bubble Series; 1 14-Bubble Series. |
| 3C16.5 | Carbon Tetrachloride; Orifice Dia., 0.3 cm ; Actual Flow Rate, $21.1 \mathrm{~cm}^{3} / \mathrm{sec}$; Liquid Temp., $31.1^{\circ} \mathrm{C}$. | 16-2834 | $\begin{array}{r} 13 \text { Double Bubbles; } \\ 1 \text { 3-Bubble Series; } \\ 1 \text { 5-Bubble Series; } \\ 1 \text { 6-Bubble Series; } \\ 1 \text { 9-Bubble Series. } \end{array}$ |

Continued on next page.

Table 5.1 Summary of Bubble Type Observed in Different Liquids and Conditions.

| Film No. | $\begin{aligned} & \text { Liquid } \\ & \text { and } \\ & \text { Conditions } \end{aligned}$ | Frame Nos. Analyzed | Bubble Type Observed |
| :---: | :---: | :---: | :---: |
| 3C29.3 | Carbon <br> Tetrachloride; <br> Orifice Dia., $0.3 \mathrm{~cm} ;$ <br> Actual Flow Rate, <br> $37.9 \mathrm{~cm}^{3} / \mathrm{sec}$; <br> Liquid Temp., $31.1^{\circ} \mathrm{C}$. | 12-3011 | 8 Double Bubbles; <br> 1 5-Bubble Series; <br> 1 6-Bubble Series; <br> 2 7-Bubble Series; |
| 6W88. 7 | Distilled Water; Orifice Dia., 0.6 cm ; Actual Flow Rate, $113.5 \mathrm{~cm}^{3} / \mathrm{sec}$; Liquid Temp., $25.5^{\circ} \mathrm{C}$. | 2-2995 | ```1 Double Bubble; 5 3-Bubble Series; 2 4-Bubble Series; 2 5-Bubble Series.``` |
| 6W111.5 | Distilled Water; Orifice Dia., 0.6 cm ; Actual Flow Rate, $180.1 \mathrm{~cm}^{3} / \mathrm{sec} ;$ Liquid Temp., $50.0^{\circ} \mathrm{C}$. | 49-2918 | 6 Double Bubbles; <br> 5 3-Bubble Series; <br> 1 4-Bubble Series; <br> 1 5-Bubble Series. |
| 6T91.9 | Toluene; <br> Orifice Dia., 0.6 cm ; Actual Flow Rate, $130.1 \mathrm{~cm}^{3} / \mathrm{sec} ;$ Liquid Temp., $35.6^{\circ} \mathrm{C}$. | 56-3166 | 6 3-Bubble Series; <br> 1 4-Bubble Series; <br> 3 6-Bubble Series. |

Table 5.1 - (Cont.) Summary of Bubble Type Observed in Different Liquids and Conditions.

Acetone


Fig. 5.3 Bubble Growth Data in Acetone. (Actual Flow Rate at Orifice, $15,8 \mathrm{~cm}^{3} / \mathrm{sec}$ ).


Fig. 5.4 Bubble Growth Data in Acetone. (Actual Flow Rate at Orifice, $34.8 \mathrm{~cm}^{3} / \mathrm{sec}$.)

## Methanol

O Bubble 3M35-800
$\triangle$ Bubble 3M35-1749
$\square$ Bubble 3M35-2640
Liquid Temp $=23.3^{\circ} \mathrm{C}$
Orifiće Dia $=0.3 \mathrm{~cm}$


Fig. 5.5 Bubble Growth Data in Methanol. (Actual Flow Rate at Orifice, $42.3 \mathrm{~cm}^{3} / \mathrm{sec}$.)

## Methanol

O Bubble 4M44.4-452
$\triangle$ Bubble 4M44.4-1004
$\square$ Bubble 4M44.4-2652
Liquid Temp $=25.6^{\circ} \mathrm{C}$
Orifice Dia= 0.4 cm


Fig. 5.6 Bubble Growth Data in Methanol. (Actual Flow Rate at Orifice, $56.3 \mathrm{~cm}^{3} / \mathrm{sec}$. )

## Carbon Tetrachloride



Fig. 5.7 Bubble Growth Data in Carbon Tetrachloride. (Actual Flow Rate at Orifice, $21.1 \mathrm{~cm}^{3} / \mathrm{sec}$.)

## Carbon Tetrachloride



Fig. 5.8 Bubble Growth Data in Carbon Tetrachloride. (Actual Flow Rate at Orifice, $37.9 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

## Distilled Water



Fig. 5.9 Bubble Growth Data in Distilled Water. (Actual Flow Rate at Orifice, $113.5 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

## Distilled Water



Fig. 5.10 Bubble Growth Data in Distilled Water. (Actual Flow Rate at Orifice, $180.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Toluene


Fig. 5.11 Bubble Growth Data in Toluene.
(Actual Flow Rate at Orifice, $130,1 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

| Liquid | Bubble <br> Identification | Orifice Dia. cm | Actual <br> Flow rate $\mathrm{cm}^{3} / \mathrm{sec}$ | ```Bubble Departure Time, td ms``` | ```Bubble Departure Radius, Rd cm``` |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Acetone | $\begin{aligned} & 3 A 11-597 \\ & 3 A 11-1237 \\ & 3 A 11-1382 \\ & 3 A 24-481 \\ & 3 A 24-1111 \\ & 3 A 24-1228 \end{aligned}$ | $0.3$ $0.3$ | $15.8$ $34.8$ | 34.2 <br> 39.1 <br> 35.0 <br> 37.5 <br> 37.4 <br> 36.8 | 0.507 0.510 0.503 (Ave.:0.507) 0.689 0.684 0.680 (Ave.: 0.684 ) |
| Methanol | $\begin{aligned} & 3 M 35-800 \\ & 3 M 35-1749 \\ & \text { 3M35-2640 } \end{aligned}$ <br> 4M44.4-452 <br> 4M44.4-1004 <br> 4M44.4-2652 | $0.3$ $0.4$ | $42.3$ $56.3$ | $\begin{aligned} & 37.9 \\ & 38.2 \\ & 41.4 \\ & 42.9 \\ & 40.0 \\ & 41.4 \end{aligned}$ | 0.759 0.760 0.747 (Ave.:0.755) 0.807 0.788 0.810 (Ave.:0.802) |
| Carbon Tetrachloride | 3C16.5-1829 $3 \mathrm{C} 16.5-1931$ $3 \mathrm{C} 16.5-2033$ $3 \mathrm{C} 29.3-1945$ 3C29.3-2558 3C29.3-2839 | $0.3$ $0.3$ | $21.1$ $37.9$ | $\begin{aligned} & 37.0 \\ & 37.9 \\ & 37.3 \\ & 38.5 \\ & 33.3 \\ & 37.1 \end{aligned}$ | 0.610 0.600 0.590 (Ave.:0.600) 0.687 0.670 0.657 (Ave.:0.671) |
| Distilled <br> Water | 6W88.7-1404 <br> 6W88.7-1729 <br> 6W88.7-2611 <br> 6W111.5-1833 <br> 6W111.5-1994 <br> 6W111.5-2300 | $0.6$ $0.6$ | $\begin{aligned} & 113.5 \\ & 180.1 \end{aligned}$ | 55.5 <br> 56.0 <br> 51.9 <br> 57.0 <br> 51.2 <br> 48.1 | 1.130 1.188 1.174 (Ave.:1.164) 1.351 1.352 1.283 (Ave.:1.329) |
| Toluene | $\begin{aligned} & 6 \mathrm{~T} 91.9-293 \\ & 6 \mathrm{~T} 91.9-2461 \\ & 6 \mathrm{~T} 91.9-2694 \end{aligned}$ | 0.6 | 130.1 | $\begin{aligned} & 54.9 \\ & 54.4 \\ & 55.8 \end{aligned}$ | $\begin{gathered} 1.248 \\ 1.266 \\ 1.265 \\ \text { (Ave.:1.260) } \end{gathered}$ |

Table 5.2 Bubble Departure Times and Radii
corresponding to the frame when the first bubble appeared as a meniscus which was left behind at the orifice by the preceding bubble and had a finite radius. For the three bubbles reported for toluene, each bubble was in 'contact with the preceding bubble for typically four frames (of a bubble life of approximately 130 frames) during which time it was still possible to calculate the bubble volume as the outline was still clear in the small overlap area. The bubble departure time, $t_{d}$, and departure radius, $R_{d}$, were taken corresponding to the frame when it showed a clean break or sharp neck between the first and the second bubbles.

For each experimental condition (liquid temperature, orifice diameter, and air flow rate), three sets of bubble growth data were analyzed in order to check if they were similar. It is seen from Figs. 5.3 to 5.11 that the bubble growth results for the three bubbles analysed in each case fall close to one another for the same conditions.

Repeatability tests were performed on methanol and distilled water and the results are presented in Appendix A. The bubble growth results of the repeatability tests for both liquids are in good agreement with those of the original experiments (see Appendix A for quantitative discussion).
5.2.2 Comparison of experimental and theoretical bubble growth

Figures 5.12 through 5.20 show experimental growth


Fig. 5,12 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Acetone - Actual Flow Rate, $15.8 \mathrm{~cm}^{3} / \mathrm{sec}$.


Fig. 5.13 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Acetone - Actual Flow Rate, $34.8 \mathrm{~cm}^{3} / \mathrm{sec}$.

## Methanol

|  | Bubble $3 \mathrm{M} 35-800$ <br> $\triangle$ <br> Bubble $3 \mathrm{M} 35-1749$ |
| ---: | :--- |
| $\square$ | Bubble $3 \mathrm{M} 35-2640$ |
| $\square$ | Eqn. 2.4 |
|  | Liquia Temp $=23.3^{\circ} \mathrm{C}$ |
|  | Orifice Dia $=0.3 \mathrm{~cm}$ |



Fig. 5.14 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Methanol - Actual Flow Rate, $42.3 \mathrm{~cm}^{3} / \mathrm{sec}$.

## Methanol

O Bubble 4m44.4-452
$\triangle$ Bubble 4M44.4-1004
$\square$ Bubble 4M44.4-2652
——Eqn. 2.4
Liquid Temp $=25.6^{\circ} \mathrm{C}$
Orifice Dia $=0.4 \mathrm{~cm}$


Fig. 5.15 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Methanol - Actual Flow Rate, $56.3 \mathrm{~cm}^{3} / \mathrm{sec}$.

Carbon Tetrachloride


Fig. 5.16 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Carbon Tetrachloride Actual Flow Rate, $21.1 \mathrm{~cm}^{3} / \mathrm{sec}$.

Carbon Tetrachloride


Fig. 5.17 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Carbon Tetrachloride Actual Flow Rate, $37.9 \mathrm{~cm}^{3} / \mathrm{sec}$.

## Distilled Water



Fig. 5.18 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Distilled Water - Actual Flow Rate, $113.5 \mathrm{~cm}^{3} / \mathrm{sec}$.

## Distilled Water



Fig. 5.19 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Distilled Water - Actual Flow Rate, $180.1 \mathrm{~cm}^{3} / \mathrm{sec}$.

Toluene


Fig. 5.20 Comparison of Experimental Results and Bubble Growth Eqn. 2.4 for Toluene - Actual Flow Rate, $130.1 \mathrm{~cm}^{3} / \mathrm{sec}$.
results obtained in the present work, along with the corresponding theoretical curves from Eqn. 2.4, i.e. $R=(3 Q / 4 \pi)^{1 / 3} t^{1 / 3}$. It can be seen that the experimental results are in excellent agreement with the theoretical curves for the entire period of growth in all liquids except at zero time when the experimental results have a finite radius. However, it can be concluded that the present apparatus genuinely operated under constant-flow conditions.

### 5.2.3 Comparison of experimental and theoretical bubble departure size

Table 5.3 gives the present experimental departure radii for all test liquids as well as the departure radii predicted by the theory of Kumar and Kuloor[22]. The experimental value for each air flow rate is the average of the three bubbles analyzed for the same conditions (see Table 5.2). It was pointed out in Sec. 2.4 that Kumar and Kuloor have proposed a model by assuming the bubble formation to take place in two stages, namely, the expansion stage and the detachment stage. During the first stage, the bubble expands while its base remains attached to the orifice whereas in the detachment stage the bubble base moves away from the orifice and remains in contact with the orifice through a stem. This idealized model of bubble formation is quite similar to the behavior of forming a present first bubble. It is seen that the results presented in Table 5.3 show good agreement between the theoretical predictions and experimental values with a maximum deviation

| Liquid | Orifice <br> Dia. <br> cm | Actual Flow Rate at Orifice $\mathrm{cm}^{3} / \mathrm{sec}$ | Bubble Departure Radius, $\mathrm{R}_{\mathrm{d}}, \mathrm{cm}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | *Present Experiment | Theory | $\frac{\text { Experiment }}{\text { Theory }}$ |
| Acetone | 0.3 | 15.8 | 0.507 | 0.513 | 0.99 |
|  | 0.3 | 34.8 | 0.684 | 0.666 | 1.03 |
| Methanol | 0.3 | 42.3 | 0.755 | 0.724 | 1.04 |
|  | 0.4 | 56.3 | 0.802 | 0.829 | 0.98 |
| Carbon Tetrachoride | 0.3 | 21.1 | 0.600 | 0.556 | 1.08 |
|  | 0.3 | 37.9 | 0.671 | 0.704 | 0.95 |
| Distilled Water | 0.6 | 113.5 | 1.164 | 1.100 | 1.06 |
|  | 0.6 | 180.1 | 1.329 | 1.320 | 1.01 |
| Toluene | 0.6 | 130.1 | 1.260 | 1.150 | 1.10 |

${ }^{*} R_{d}$ shown is the average of the three values presented in Table 5.2 for each actual flow rate.

Table 5.3 Comparison of Experimental Bubble Departure Radii with Predictions of Kumar and Kuloor[19]
of $9 \frac{1}{2} \%$ in toluene.
5.3 Comparison of Present Barbotage and Nucleate Boiling

The quantitative comparison of bubble growth curves ( $R$ vs. $t$ ) between the present constant flow barbotage and nucleate boiling bubbles of Cole and Shulman[6] are presented in Figs. 5.21 to 5.25 for essentially the same bulk liquid temperatures. The saturation temperatures quoted in the figures for various liquids at the pressures stated by Cole and Shulman were obtained from Refs. $[14,20,39]$ and are given to an accuracy of approximately half a degree Celsius to accommodate the discrepancy among the sources for the same conditions. Cole and Shulman indicated that bubble growth results varied in a statistical fashion for apparently the same conditions. In the figures, only the two boiling bubbles represented by the two extreme bubble growth curves (the uppermost and the lowest curves) are replotted for comparison with the present barbotage bubbles. Here it could be mentioned again that barbotage flow rates were chosen to give growth curves falling, for most of the growth time, roughly mid-way between the extremes of the growth curves for the boiling bubbles. The barbotage bubbles shown in the figures belong, in each case, to the bubble whose growth result lies the closest to the bubble growth Eqn. 2.4 among the three bubbles analyzed for the same conditions. Figures 5.26 to 5.30 give the bub̄ble growth rates (dR/dt vs. $t$, or $\dot{R}$ vs. $t$ ) of the boiling and barbotage bubbles. The growth rates were obtained for both

## Acetone



Fig. 5.21 Comparison of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Acetone.

Methanol


Fig. 5.22 Comparison of Bubble Growth Curves Between Present Barbotage and Bołling Bubbles for Methanol.

## Carbon Tetrachloride



Fig. 5.23 Comparison of Bubble Growth Curves between Present Barbotage and Boiling Bubbles for Carbon Tetrachloride.


Fig. 5.24 Comparison of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Distilled Water.


Fig. 5.25 Comparison of Bubble Growth Curves Between Present Barbotage and Boiling Bubbles for Toluene.

## Acetone



Fig. 5.26 Comparison of Bubble Growth Rates Between Present Barbotage and Boiling Bubbles for Acetone.

## Methanol



Fig. 5.27 Comparison of Bubble Growth Rates Between Present Barbotage and Boiling Bubbles for Methanol.

## Carbon Tetrachloride



Fig. 5.28 Comparison of Bubble Growth Rates Between Present Barbotage and Boiling Bubbles for Carbon Tetrachloride.


Fig. 5.29. Comparison of Bubble Growth Rates Between Present Barbotage and Boiling Bubbles for Distilled Water.

Toluene

boiling and barbotage by measuring the slope, $\Delta R / \Delta t$, between two consecutive $R$ vs. $t$ data points and plotting this at the mean of the time of the two data points. Along with the barbotage bubbles, Eqn. 2.5 for constant-flow barbotage bubble growth rates, is shown. The following observations can be made regarding these figures:
(i) The growth curves ( $R$ vs. $t$ ) of barbotage bubbles are slightly higher than those of the two boiling bubbles during the early part of the growth period and have shapes similar to those of the boiling bubbles for most of the growth time. Cole and Shulman[5] indicated that the bubble growth shapes ( $R$ vs. $t$ ) of their boiling results were best described by the square root of time variation whereas the shapes of the present barbotage bubbles growth vary with the cube root of time as indicated in Eqn. 2.4 and the previous Sec. 5.2.2. Further, in boiling, the bubble growth depends on the fluid properties and superheat conditions while in constant-flow barbotage, the bubble growth is simply controlled by the gas flow rate.
(ii) The plots of the bubble growth rates ( $\dot{R}$ vs. $t$ ) show that the growth rates of barbotage bubbles appear to have shapes quite similar to those of the two boiling bubbles with the magnitudes approximately the same as the lower boiling bubble except in toluene where the barbotage bubble shows rather lower growth rates than the boiling bubbles.
(iii) Table 5.4 gives a comparison of bubble departure radii between the present barbotage and boiling bubbles

| Liquid | Boiling <br> (Cole and Shulman) <br> Range of Bubble Departure <br> Radius, Rd, cm | Barbotage <br> (Present) <br> Bubble Departure <br> Radius, Rd, cm |
| :--- | :---: | :---: |
| Acetone | $0.48-0.95$ | 0.69 |
| Methanol <br> Carbon <br> Tetrachloride <br> Distilled <br> Water <br> Toluene$0.80-1.09$ | 0.80 |  |

* Average for the three bubbles analyzed for any set of conditions (see Table 5.2).

Table 5.4 Comparison of Bubble Departure Radii between the Present Barbotage and Boiling Bubbles
presented in Figs. 5.21 to 5.25. The bubble departure radii for barbotage bubbles are the average of the three bubbles analyzed for each set of conditions (see Table 5.2). It is seen that the departure radii of barbotage bubbles fall within the range of those for the boiling bubbles at nominally the same conditions.

The above observations of bubble growth curves ( $R$ vs. $t$ ) and bubble growth rates ( $\mathrm{dR} / \mathrm{dt}$ vs.t) reveal that there are some similarities between the present barbotage and boiling bubbles regarding both the shape and magnitude of these curves. The present barbotage bubbles also give a reasonably good "combination" of $R$ vs. $t$ and $\dot{R}$ vs. $t$ in comparison with boiling bubbles. For the same liquid properties in boiling and barbotage, the combination referred to means: a bubble growth curve (R vs. t) for barbotage roughly mid-way between the extremes of the boiling data for a substantial portion of the growth time; and barbotage growth rates ( $\dot{R}$ vs. $t$ ) which fall close to the lower boiling data.

As regards constant-flow barbotage as a simulation of nucleate pool boiling, it should be noted that there is a limitation on the exponent in $R \backsim t^{n}$. As mentioned earlier, Cole and Shulman used

$$
R=2.5 \mathrm{Ja}^{3 / 4} \sqrt{\alpha t}
$$

to correlate their boiling data, or $R \backsim t^{1 / 2}$. The simple
barbotage system used here results in $R \sim t^{1 / 3}$ in which the time exponent is different from that in boiling. (It should be noted that, however, as pointed out in Sec. 2.2.2, a wide range of $n$ in $R \backsim t^{n}$ have been reported for boiling.) Further, if it is desired to use barbotage systems conveniently for the simulation of particular boiling conditions (i.e., the liquid temperature to be the same as the saturation temperature in boiling), the boiling saturation temperature must be low enough to allow for the operation of barbotage systems at atmospheric pressure (the most convenient pressure for operation); this often implies low-pressure boiling conditions.

The departure radii in boiling and barbotage as obtained here are similar (see Table 5.4), i.e. the barbotage results fall within the range of the boiling results.

## CHAPTER 6

## SUMMARY AND CONCLUSIONS

(1) Barbotage experiments under constant-flow conditions were devised in which it was possible to compare the hydrodynamics of the present barbotage bubbles with nucleate boiling results of Cole and Shulman[6] for essentially the same bulk liquid properties. The hydrodynamics referred to here were bubble growth (R vs. t), bubble growth rate (dR/dt vs. t), and bubble departure radius. A barbotage flow rate, in each condition, was chosen to give the growth curve ( $R$ vs.t) falling within the two extremes of the growth curves (the uppermost and the lowest curves) of boiling bubbles for most of the growth time. The experiments were performed in acetone, methanol, carbon tetrachloride, distilled water, and toluene (the same liquids as used by Cole and Shulman). Air was used as the injected gas. High-speed photography was employed to record the sequence of bubble formation. The volume of a bubble at any instant during formation was determined from the frames of the high speed motion picture in the manner described by L'Ecuyer and Murthy[23].
(2) Three types of bubble formation were distinguished in the present study. They were the formation of double bubbles, three-bubble series, and series of more than three bubbles. In general, the bubble formation of the last type, long bubble series, could be broken down into: a number of double bubbles, three-bubble series, or combinations of some
double bubbles and three-bubble series. The first bubbles of the three types were chosen to be analyzed for measuring the bubble volumes against time.
(3) The bubble growth Eqn. 2.4 for constant-flow barbotage is presented in Chapter 2, Literature Review. The comparison of barbotage $R$ vs. $t$ results with Eqn. 2.4 indicated that the apparatus indeed generated constant-flow bubbles.
(4) The measured bubble departure radii of the present work were compared with the theoretical predictions based on the model of Kumar and Kuloor[22]. The theoretically predicted bubble departure radii were found to be in good agreement with the experimental results with a maximum deviation of $9 \frac{1}{2} \%$ in toluene.
(5) Quantitative comparisons of the hydrodynamics between the present barbotage bubbles and boiling bubbles of Cole and Shulman are presented in Figs. 5.22 to 5.31 and Table 5.4. The conclusions drawn are as follows:
(i) The growth curves (R vs. t) of barbotage bubbles are slightly higher than those of the two boiling bubbles during the early part of the growth period, but then lie roughly mid-way between the extremes of the boiling results for the major portion of the growth time.
(ii) Considering the scatter in the actual $\dot{R}$ vs. $t$ results (Figs. 26 to 30 ), the barbotage and the lower boiling results lie very close together except for toluene where the barbotage results are distinguishably lower
than the boiling results.
(iii) The departure radii of barbotage bubbles fall within the range of those for boiling at nominally the same conditions.

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## APPENDIX A

## REPEATABILITY TESTS

Repeatability tests were performed on methanol and distilled water with essentially the same flow rates, orifice diameters and liquid bulk temperatures as those employed in the experiments with the same liquids reported in the body of this thesis. Figures A. 1 and A. 2 show the comparison of bubble growth results ( R vs.t) between the repeatability tests and the original experimental work. It is seen that the bubble growth results of the repeatability tests for both liquids are in good agreement with those of original experiments. A quantitative measure of the repeatability is given below where the experimental $R$ vs. $t$ data are compared with Eqn. 2.4 for the original
experimental conditions; the algebraic mean deviation, $\bar{d}$ and root mean square deviation, $\overline{\text { d }}$ rms are given in each case:

Distilled water

Original experiment:

Bubble 6W111.5-1833 $\bar{d}=5.5 \%, \bar{d}_{r m s}=8.7 \%$
Bubble 6W111.5-1994 $\bar{d}=5.3 \%, \bar{d}_{r m s}=7.0 \%$
Bubble $6 \mathrm{~W} 111.5-2300 \quad \bar{d}=-0.8 \%, \bar{d}_{r m s}=5.0 \%$

Repeatability
test:
Bubble 6W111.5R-1276
$\bar{d}=1.0 \%, \bar{d}_{r m s}=5.2 \%$
Bubble $6 \mathrm{~W} 111.5 \mathrm{R}-2038 \quad \bar{d}=0.7 \%, \bar{d}_{r m s}=1.5 \%$
Methanol
Original
experiment:

| Repeatability | Bubble $3 M 35 R-2814$ | $\bar{d}=2.0 \%, \bar{d} m s=6.4 \%$ |
| :--- | :--- | :--- |
| test: | Bubble $3 M 35 R-1784$ | $\bar{d}=0.7 \%, \bar{d} r m s=2.4 \%$ |

It can be seen that all the original and repeatability data fall with an algebraic mean deviation in the range of -1.8 to $5.5 \%$ and a root meam square deviation in the range of 1.5 to $8.7 \%$.

Table A. 1 and A. 2 gives the tabulated results of the repeatability tests for both distilled water and methanol respectively. The letter " $R$ " right after the indicated air flow rate in the bubble identification numbers (see Appendix F for explanation) shown in both tables and figures indicates a repeatability test. These repeatability tests were run after the main experimental program, i.e., after obtaining the data reported in the body of the thesis.

Distilled Water


Fig. A. 1 Comparison of the Bubble Growth Curves between the Repeatability Test and the Original Experimental Results for Distilled Water.

Methanol


Fig. A. 2 Comparison of the Bubble Growth Curves between the Repeatability Test and the Original Experimental Results for Methanol

Table A. 1 Repeatability Test:Bubble Growth Data for Distilled Water (Actual Flow Rate, $180.0 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter: Liquid temperature: Room temperature:
Liquid height above the orifice: Indicated air flow rate:
0.6 cm
$50.0^{\circ} \mathrm{C}$
$23.0^{\circ} \mathrm{C}$
12.0 cm
$111.5 \mathrm{~cm}^{3} / \mathrm{sec}$

Bubble Identification No. 6W111.5R-1276 (Time/frame, $4.167 \times 10^{-4} \mathrm{sec} /$ frame)

| Frame No. | Time ms | $\begin{aligned} & \text { Bubble } \\ & \text { Volume } \\ & \mathrm{cm}^{3} \times 10^{-3} \end{aligned}$ | Bubble Equ Radius cII |
| :---: | :---: | :---: | :---: |
| 1276 | 0.0 | 18.15 | 0.163 |
| 1280 | 1.7 | 491.60 | 0.490 |
| 1287 | 4.6 | 890.05 | 0.597 |
| 1296 | 8.3 | 1592.84 | 0.724 |
| 1306 | 12.5 | 2240.85 | 0.812 |
| 1316 | 16.7 | 2995.00 | 0.894 |
| 1330 | 22.5 | 4067.53 | 0.990 |
| 1345 | 28.8 | 5014.50 | 1.062 |
| 1360 | 35.0 | 6038.28 | 1.129 |
| 1375 | 41.3 | 7251.93 | 1.201 |
| 1390 | 47.5 | 8134.60 | 1.248 |
| 1405 | 53.8 | 9080.16 | 1.294 |
| 1410 | 55.8 | 9397.43 | 1.309 |

Bubble Identification No. 6W111.5R-2038 (Time/frame, $4.762 \times 10^{-4} \mathrm{sec} /$ frame)

| 2038 | 0.0 | 12.76 | 0.145 |
| ---: | ---: | ---: | ---: |
| 2042 | 1.9 | 384.21 | 0.451 |
| 2050 | 5.7 | 1005.04 | 0.632 |
| 2060 | 10.5 | 1961.22 | 0.777 |
| 2075 | 17.6 | 3152.10 | 0.910 |
| 2085 | 22.4 | 3860.54 | 0.973 |
| 2100 | 29.5 | 5333.57 | 1.084 |
| 2115 | 36.7 | 6711.12 | 1.170 |
| 2130 | 43.8 | 8139.51 | 1.248 |
| 2145 | 51.0 | 9408.04 | 1.310 |
| 2156 | 56.2 | 10204.91 | 1.340 |

Table A. 2 Repeatability Test: Bubble Growth Data for Methanol (Actual Flow Rate, $42.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ).
Orifice diameter:
Liquid temperature:
Room temperature:
Liquid height above the orifice:
0.3 cm
22.8 O
22.8
${ }^{\circ} \mathrm{C}$
12.0 cm
$35.0 \mathrm{~cm} 3 / \mathrm{sec}$

Bubble Identification No. 3M35R-1784 (Time/frame, $4.545 \times 10^{-4} \mathrm{sec} /$ frame)

Frame No.
Time
Bubble
volume
ms
$\mathrm{cm}^{3} \times 10^{-3}$
Bubble Equiv. Radius cm

1784
1787 1790 1796 1802 1810 1820 1830 1840 1850 1860 1868
0.0
1.4
2.7
5.5
8.2
11.8
16.4
20.9
25.5
30.0
34.5
38.2
3.85
52.19
121.70
248.97
377.38
534.18
741.48
848.93
1132.86
1325.38
1478.55
1631.65
0.097
0.232
0.307
0.390
0.448
0.503
0.561
0.587
0.647
0.681
0.707
0.730

Bubble Identification No. 3M35R-2814 (Time/frame, $4.348 \times 10^{-4}$ sec/frame)

2814
2816
2820
2825
2834
2844
2854
2864
2874
2884
2894
2904
2909
0.0
0.9
2.6
4.8
8.7
13.0
17.4
21.7
26.1
31.4
34.5
39.1
41.3
2.45
0.084
0.245
0.327
146.87
0.385
239.45
0.421
13.0
312.08
0.515
572.09
0.558
21.7
727.86
0.593
26.1
874.03
0.639
31.4
1090.76
0.670
34.5
1260.45
0.707
41.3
1603.57
0.726
1641.75
0.732

## APPENDIX B

## CALIBRATION OF THE ROTAMETER

The rotameter (Brook Instrument Co., Model. 1560, Tube No. 2-2-25C) was calibrated using a gas meter manufactured by Precison Scientific Co., USA. The arrangement of the calibration apparatus is given in Fig. B.1. The wet gas meter was filled with distilled water to the required level before the calibration. The compressed air passing through the pressure regulator valve reduced in pressure to the operating condition. The air then passed through a filter, gas dryer and rotameter. Finally, the air entered the wet gas meter where the volumetric flow rate of the air which was saturated with water vapor was measured. The line pressure between the rotameter and the wet gas meter was recorded; room temperature, wet gas meter temperature, and barometric pressure were also recorded before and after the calibration process. The measured saturated air flow rate was corrected for vapor content, then corrected to standard conditions ( 14.7 psia and $70^{\circ} \mathrm{F}$ ) as described below for comparison with the manufacturer's calibration curve for the rotameter. Calculations, etc., in this appendix are in Imperial units since the manufacturer's materials were in these units.

Determination of the dry air flow rate at the wet gas meter

Since the air passing through the wet gas meter was saturated with water vapor, it was necessary to correct the


Fig B.l Calibration Apparatus Arrangement for Rotameter.
measured flow rate to dry air conditions. It is well known from thermodynamics that in a mixture of ideal gases, the volume fraction of one component is equal to the ratio of the partial pressure of that component at the temperature of the mixture to the total pressure of the mixture. That is

$$
\begin{equation*}
\frac{Q_{\text {vap }}}{Q_{\text {mix }}}=\frac{Q_{\text {vap }}}{Q_{\text {vap }}+Q_{\text {air }}}=\frac{P_{\text {vap }}}{P_{\text {tot }}} \tag{B. 1}
\end{equation*}
$$

where $Q_{\text {air }}, Q_{\text {vap }}$, and $Q_{\text {mix }}$ are the volumetric flow rates of air, water vapor and mixture (vapor +air), respectively. $P_{\text {vap }}$ is the partial pressure of the vapor and $P_{\text {tot }}$ is the total pressure of the mixture. Thus, the dry air flow rate, Qair can be obtained from Eqn. B.l with known values of the mixture flow rate, temperature and pressures ( $p_{\text {vap }}$ and $\left.P_{\text {tot }}\right)$.

Correction of the air flow rate at the wet meter to air flow rate at rotameter operating conditions

The air flow rate at the wet meter $Q_{a i r}$ is related to the air flow rate at rotameter conditions through

$$
\begin{equation*}
Q_{m}=Q_{\text {air }} \times \frac{P_{\text {wet meter }} \mathrm{x}_{\mathrm{m}}}{\mathrm{P}_{\mathrm{m}} \times \mathrm{T}_{\text {wet meter }}} \tag{B. 2}
\end{equation*}
$$

where $Q_{m}=A i r$ flow rate at rotameter conditions,
$T_{m}=$ Temperature at rotameter,
$P_{m}=$ Pressure at rotameter,
$Q_{\text {air }}=$ Air flow rate at wet gas meter conditions,
$T_{\text {wet meter }}=$ Temperature at wet gas meter,
$P_{\text {wet meter }}=$ Pressure at wet gas meter

In the present case, the pressures at the rotameter and in the wet gas meter are essentially the same, as are the temperatures, i.e. $P_{m} \simeq P_{\text {wet meter }}$ and $T_{m} \simeq T_{\text {wet meter }}$. We therefore have

$$
Q_{m} \simeq Q_{a i r}
$$

Correction of air flow rate at rotameter operating conditions to standard conditions

For comparison with the manufacturer's calibration curve a further correction is necessary [B.1, B. 2] through

$$
\begin{equation*}
Q_{I}^{\prime}=Q_{m} \sqrt{\frac{S . G . \times(70.0+459.7) \times P_{m}}{1.0 \times T_{m} \times 14.7}} \tag{B. 3}
\end{equation*}
$$

where $Q_{I}^{\prime}$ is the volumetric air flow rate at standard conditions ( 14.7 psia and 70 F ) for the same float height. In this equation $P_{m}$ would be in psia and $T_{m}$ in ${ }^{\circ} R$.

## Example

$$
\begin{array}{ll}
\text { Indicated rotameter air flow rate (scale) } & =3 \mathrm{~cm} \\
\text { Measured flow rate at wet meter, } Q_{\text {mix }} & =1.84 \mathrm{ft}^{3} / \mathrm{hr} \\
\mathrm{P}_{\text {tot }}\left(\mathrm{P}_{\text {wet meter }}+\mathrm{P}_{\text {atm }}\right) \\
\text { Temperature at wet gas meter, } \mathrm{T}_{\text {wet meter }} & =14.19 \mathrm{psia} \\
\text { Water vapor at } 65 \mathrm{~F}, \mathrm{P}_{\text {vap }} & =65.0^{\circ} \mathrm{F} \\
\text { Temperature at rotameter (room temp.), } \mathrm{T}_{\mathrm{m}} & =6.31 \mathrm{psia} \\
\text { Specific gravity of air, S.G. } & =1.0
\end{array}
$$

Substituing the values of $Q_{\text {mix }}, P_{\text {tot }}$, and $P_{\text {vap }}$ in Eqn. B.l,

[^0]we have
$$
\frac{Q_{\text {vap }}}{Q_{\operatorname{mix}}}=\frac{Q_{\text {vap }}}{Q_{\text {vap }}+Q_{\text {air }}}=\frac{0.31}{14.19}=0.0218
$$

Therefore,

$$
\begin{aligned}
Q_{\text {vap }} & =0.0218 \times Q_{\operatorname{mix}}=0.0218 \times 1.84 \\
& =0.04 \mathrm{ft}^{3} / \mathrm{hr}
\end{aligned}
$$

Thus, the air flow rate is

$$
\begin{aligned}
Q_{\text {air }} & =Q_{\text {mix }}-Q_{\text {vap }}=1.84-0.04 \\
& =1.80 \mathrm{ft}^{3} / \mathrm{hr}
\end{aligned}
$$

As indicated earlier, in the present case essentially $Q_{m}=Q_{\text {Qir }}$. Substituting $Q_{m}$ into Eqn. B. 3 , we obtain

$$
\begin{aligned}
Q_{I}^{\prime} & =1.80 \sqrt{\frac{1.0 \times 14.19 \times(70.0+459.7)}{1.0 \times 14.7 \times(65.2+459.7)}} \\
& =1.78 \mathrm{ft}^{3} / \mathrm{hr}
\end{aligned}
$$

Table B. 1 summarizes the operating conditions and calibration results for ten values of air flow rate. The corrected values of air flow rate at standard conditions are plotted on the manufacturer's calibration curve and are shown in Fig. B.2. It is seen that the calibration results are in excellent agreement with the manufacturer's curve. Therefore, the manufacturer's curve was used to obtain the indicated air flow rates for the the present barbotage experiments.

| Rotameter calibrated |  | Brook Instrument, Model Tube No. R-2-25-C |  |
| :---: | :---: | :---: | :---: |
| Wet gas meter |  | Precision Scientific Co |  |
| Barometric ${ }^{\text {c pressure }}$ | : | Start 14.18 psia; End | 14.18 psia |
| Room temperature |  | Start 65.2 ${ }^{\circ} \mathrm{F}$; End | $65.2{ }^{\circ} \mathrm{F}$ |
| Standard conditions | : | $70.0{ }^{\circ} \mathrm{F}$ and 14.7 psia | , |


| Rotameter | Wet gas meter |  |  |
| :---: | :---: | :---: | :---: |
| Air flow rate - <br> cm scale | $\Delta P_{\text {wet meter }}$ <br> in. of liquid <br> $(S . G .=0.827)$ | $Q_{\text {mix }}$ <br> Vapi +air <br> $\left(\mathrm{ft}^{3} / \mathrm{hr}\right)$ | $Q_{1}^{\prime}$ <br> at <br> standard <br> conditions |
| 3 | 0.20 | 1.84 | 1.78 |
| 4 | 0.25 | 2.52 | 2.44 |
| 5 | 0.25 | 3.18 | 3.07 |
| 6 | 0.30 | 3.85 | 3.72 |
| 12 | 0.35 | 5.84 | 5.64 |
| 15 | 0.40 | 7.86 | 7.59 |
| 19 | 0.45 | 9.95 | 9.60 |
| 22 | 0.60 | 12.68 | 12.24 |
| 25 | 0.70 | 14.69 | 14.18 |
|  | 0.80 | 16.52 | 15.96 |

Table B.l Operating Conditions and Calibration Results.


## References for Appendix B

B. 1 Brooks Instruments General Catalog, P.50, (Identification on back cover 1171-B-5M-B)
B. 2 VARIABLE AREA FLOWMETER HANDBOOK:

Vol. 1, BASIC ROTAMETER PRINCIPLES, Fisher and
Porter Catalog 10A 1021 (1970); Vol. 2, ROTAMETER CALCULATIONS, Fisher and Porter Catolog 10A 1022 (1969).

## APPENDIX C

PHYSICAL PROPERTIES OF THE EXPERIMENTAL LIQUIDS

The properties of the liquids used in the present experiments are listed in Table C.l. The properties for acetone, methanol, carbon tetrachloride and toluene were taken from Ref.[C.l] while those for distilled water from Ref.[C.2] The liquid properties listed correspond to the present experimental conditions.

## References for Appendix C

C.l Gallant, R.W., PHYSICAL PROPERTIES OF HYDROCARBONS, Vol. 1 and Vol. 2, Gulf Publishing Co. (1974).
C. 2 HANDBOOK OF CHEMISTRY AND PHYSICAL, 62nd Edition, CRC Press (1981-1982).

| Liquid | Temp. <br> ${ }^{\circ} \mathrm{C}$ | Density <br> gm/cc | Surface <br> Tension <br> dynes/cm | Boiling Point <br> at Atmospheric <br> Pressure <br> ${ }^{\circ} \mathrm{C}$ |
| :--- | :---: | :---: | :---: | :---: |
| Acetone | 24.4 | 0.78 | 22.8 | 56.0 |
| Methanol | 23.3 | 0.79 | 22.3 | 65.0 |
| Carbon <br> Tetrachloride | 31.3 | 1.58 | 25.4 | 77.0 |
| Distilled <br> Water | 25.0 | 1.00 | 72.0 | 100.0 |
| Toluene | 50.0 | 0.99 | 67.9 |  |

Table C.l Physical Properties of the Experimental Liquids.

## APPENDIX D

## CALCULATION OF BUBBLE VOLUME

The volume of a bubble (at any instant during formation) was determined from the frames of high-speed motion pictures and was based on the calculation method of L'Ecuyer and Murthy[D.1]. A description of the procedure is as follows:

1. The picture frame with the rod image whose actual diameter was known was projected on a screen equipped with a digital $x-Y$ reader.
2. The scale of the digital $\mathbb{X}-\mathrm{Y}$ reader was adjusted to the actual diameter of the rod corresponding to the diameter of the projected image. This would give the scale factor with a l:l ratio.
3. A picture frame of the same film from which the bubble volume was to be calculated was then projected on the screen.
4. The enlarged outline of the bubble image was divided into a series of truncated cones by means of horizontal lines. Figure D. 1 illustrates one such outline. The volume of a bubble was then computed from the following equation:

$$
\begin{aligned}
V= & \frac{\pi}{3} H_{1}^{2}\left(3 R_{1}-H_{1}\right)+\sum_{i=1}^{n-1} \frac{\pi}{12} H_{2}\left(d_{i}^{2}+d_{i+1}^{2}+d_{i} d_{i+1}\right) \\
& +\frac{\pi}{12} H_{3}\left(d_{n}^{2}+d_{n+1}^{2}+d_{n \cdot n+1}\right)
\end{aligned}
$$



Fig. D. 1 Enlarged Bubble Outline for Volume Calculation.
5. The true bubble volume was directly calculated with the measured values from Eqn. D. 1 using a programmable calculator.

Reference for Appendix D
D. 1 L'Ecuyer, M.R. and S.N.B. Murthy, Energy transfer from
a liquid to gas bubbles forming at a submerged orifice, NASA Report TND-2547 (1965).

## APPENDIX E

## CALCULATION OF ACTUAL FLOW RATE AT ORIFICE

Before attempting to determine the actual flow rate (liquid vapor + air) at the orifice, two corrections for air flow rates have to be made, i.e., the indicated air flow rate obtained from the calibration curve is first converted from standard conditions ( 14.7 psia and $70^{\circ} \mathrm{F}$, or 76 cm Hg and $21.1^{\circ} \mathrm{C}$ ) to metering conditions; next, the air flow rate at metering conditions is converted to the bubbling conditions immediately downstream of the orifice.

The indicated air flow rate can be corrected using Eqn. B. 3 given in Appendix B. Equation B.3, rearranged and expressed in S.I. unit, is

$$
\begin{equation*}
Q_{\mathrm{m}}=Q_{I} \sqrt{\frac{1.0 \times 76.0 \times \mathrm{T}_{\mathrm{m}}}{\text { S.G. } \times \mathrm{P}_{\mathrm{m}} \times(21.1+273.2)}} \tag{E. 1}
\end{equation*}
$$

where $Q_{\mathrm{m}}=$ Air flow rate at metering conditions

$$
\begin{aligned}
Q_{I}= & \text { Indicated air flow rate at standard conditions, } \\
& (76 \mathrm{~cm} \mathrm{Hg} \text { and }(21.1+273.2) \mathrm{K}),
\end{aligned}
$$

$$
T_{m}=\text { Temperature at metering conditions, }
$$

$$
P_{m}=\text { Pressure at metering conditions. }
$$

Assuming air as an ideal gas, the air flow rate at metering conditions is then corrected to the bubbling conditions using the ideal gas law which is given by

$$
\begin{equation*}
\frac{Q_{m} P_{m}}{T_{m}}=\frac{Q_{b} P_{b}}{T_{b}} \tag{E. 2}
\end{equation*}
$$

where $Q_{b}$ is the air flow rate at the orifice, $T_{b}$ is the
temperature of the pool, and $g_{b}$ is the pressure in the plane of the orifice. Finally, the actual flow rate at the orifice was determined by Eqn. B.l presented in Appendix B assuming both air and liquid vapor behave as ideal gases. With some changes in symbols, Eqn. B.l is rewritten as follows:

$$
\begin{equation*}
\frac{Q_{\text {vap }}}{Q_{\text {mix }}}=\frac{Q_{\text {vap }}}{Q_{b}+Q_{\text {vap }}}=\frac{P_{\text {vap }}}{P_{\text {tot }}} \tag{E.}
\end{equation*}
$$

where $Q_{\text {vap }}, Q_{b}$, and $Q_{\text {mix }}$ are the volumetric flow rates of liquid vapor, air and mixture respectively. $P_{\text {vap }}$ is the partial pressure of the liquid vapor and $P_{\text {tot }}$ is the total pressure of the mixture. $P_{\text {tot }}$ is also equal to the pressure of $P_{b}$ which is the sum of liquid head above the orifice and the atmospheric pressure.

## Example (acetone)

Indicated air flow rate, $Q_{I} \quad=11.0 \mathrm{~cm}^{3} / \mathrm{sec}$
Temperature at metering conditions, $T_{m} \quad=24.4{ }^{\circ} \mathrm{C}$
Pressure at metering conditions, $P_{m}$ (gauge) $=3.6 \mathrm{~cm} \mathrm{Hg}$ Temperature of pool liquid, $T_{b} \quad=24.4^{\circ} \mathrm{C}$ $P_{\text {tot }}=P_{b}$ at orifice, $\frac{(12.0 \times 0.79)}{13.6}+76.0=76.7 \mathrm{~cm} \mathrm{Hg}$ $P_{\text {vap }}$ liquid vapor pressure at $24.4 \mathrm{C} \quad=22.2 \mathrm{~cm} \mathrm{Hg}$ S.G. of air

$$
=1.0
$$

Substituting the values of $Q_{I}, T_{m}$, and $P_{m}$ in Eqn. E.I, we have

$$
\begin{aligned}
Q_{\mathrm{m}} & =11.0 \sqrt{\frac{1.0 \times 76.0 \times(24.4+273.2)}{1.0 \times(3.6+76.0) \times(21.1+273.2)}} \\
& =10.8 \mathrm{~cm}^{3} / \mathrm{sec}
\end{aligned}
$$

Then the air flow rate at bubbling conditions ( or orifice conditions) is calculated from Egn. E.2. Thus;

$$
\begin{aligned}
Q_{b} & =10.8 \times \frac{(3.6+76.0) \times(24.4+273.2)}{76.7 \times(24.4+273.2)} \\
& =11.2 \mathrm{~cm}^{3} / \mathrm{sec}
\end{aligned}
$$

Finally, the values of $Q_{b}, P_{\text {vap }}$, and $P_{\text {tot }}$ are substituted in Eqn. E. 3 to solve for the actual flow rate (liquid vapor + air) at the orifice:

$$
\frac{Q_{\mathrm{vap}}}{Q_{\operatorname{mix}}}=\frac{Q_{\mathrm{vap}}}{11.2+Q_{\text {vap }}}=\frac{22.2}{76.7}=0.289
$$

Therefore,

$$
\begin{aligned}
Q_{\text {vap }} & =0.289 \times Q_{v a p}+11.2 \times 0.289 \\
& =4.6 \mathrm{~cm}^{3} / \mathrm{sec}
\end{aligned}
$$

Thus, the actual flow rate at orifice is

$$
\begin{aligned}
Q_{\text {act }} & =Q_{\text {mix }}=Q_{\text {vap }}+Q_{b} \\
& =4.6+11.2 \\
& =15.8 \mathrm{~cm}^{3} / \mathrm{sec}
\end{aligned}
$$

Table E.l summarizes the calculation results of the actual flow rates for all liquids and the corresponding experimental conditions. The liquid vapor pressures were obtained from Refs.[E.1,E.2].

| Liquid | Orifice <br> Dia. <br> cm | $\begin{aligned} & \mathrm{T}_{\mathrm{m}} \\ & { }^{\circ} \mathrm{C} \end{aligned}$ | $\begin{aligned} & \mathrm{T}_{\mathrm{b}} \\ & { }^{0} \mathrm{C} \end{aligned}$ | $\begin{gathered} \mathrm{P}_{\mathrm{m}} \\ \mathrm{~cm} \mathrm{Hg} \end{gathered}$ | $\begin{gathered} Q_{I} \\ \mathrm{~cm}^{3} / \mathrm{sec} \end{gathered}$ | $\begin{gathered} \mathrm{Q} \\ \mathrm{~cm}^{3} / \mathrm{sec} \end{gathered}$ | $Q_{b}$ $\mathrm{cm}^{3} / \mathrm{sec}$ | $\begin{gathered} Q_{\text {act }} \\ (\text { vap }+ \text { air }) \\ \mathrm{cm}^{3} / \mathrm{sec} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Acetone | $\begin{aligned} & 0.3 \\ & 0.3 \end{aligned}$ | $\begin{aligned} & 24.4 \\ & 24.4 \end{aligned}$ | $\begin{aligned} & 24.4 \\ & 24.4 \end{aligned}$ | $\begin{aligned} & 3.6 \\ & 5.2 \end{aligned}$ | $\begin{aligned} & 11.0 \\ & 24.0 \end{aligned}$ | $\begin{aligned} & 10.8 \\ & 23.4 \end{aligned}$ | $\begin{aligned} & 11.2 \\ & 24.7 \end{aligned}$ | $\begin{aligned} & 15.8 \\ & 34.8 \end{aligned}$ |
| Methanol | $\begin{aligned} & 0.3 \\ & 0.4 \end{aligned}$ | $\begin{aligned} & 23.3 \\ & 25.6 \end{aligned}$ | $\begin{aligned} & 23.3 \\ & 25.6 \end{aligned}$ | $\begin{array}{r} 6.5 \\ 11.6 \end{array}$ | $\begin{aligned} & 35.0 \\ & 44.4 \end{aligned}$ | $\begin{aligned} & 33.7 \\ & 41.7 \end{aligned}$ | $\begin{aligned} & 36.3 \\ & 47.6 \end{aligned}$ | $\begin{aligned} & 42.3 \\ & 56.3 \end{aligned}$ |
| Carbon Tetrachloride | $\begin{aligned} & 0.3 \\ & 0.3 \end{aligned}$ | $\begin{aligned} & 23.3 \\ & 23.3 \end{aligned}$ | $\begin{aligned} & 31.1 \\ & 31.1 \end{aligned}$ | $\begin{array}{r} 7.8 \\ 10.3 \end{array}$ | $\begin{aligned} & 16.5 \\ & 29.3 \end{aligned}$ | $\begin{aligned} & 16.0 \\ & 28.0 \end{aligned}$ | $\begin{aligned} & 17.3 \\ & 31.2 \end{aligned}$ | $\begin{aligned} & 21.1 \\ & 37.9 \end{aligned}$ |
| Distilled Water | $\begin{aligned} & 0.6 \\ & 0.6 \end{aligned}$ | $\begin{aligned} & 25.5 \\ & 23.2 \end{aligned}$ | $\begin{gathered} 25.5 \\ 50.0 \end{gathered}$ | $\begin{aligned} & 41.4 \\ & 56.9 \end{aligned}$ | $\begin{array}{r} 88.7 \\ 111.5 \end{array}$ | $\begin{aligned} & 71.9 \\ & 84.6 \end{aligned}$ | $\begin{aligned} & 109.8 \\ & 159.5 \end{aligned}$ | $\begin{aligned} & 113.5 \\ & 180.1 \end{aligned}$ |
| Toluene | 0.6 | 23.3 | 35.6 | 49.1 | 91.9 | 71.9 | 122.0 | 130.1 |

Table E. 1 Calculation Results of the Actual Flow Rates for All Tested Liquids and Corresponding Experimental Conditions.

References for Appendix E
E. 1 Gallant, R.W., PHYSICAL PROPERTIES OF HYDROCARBONS, Vol. 1 and 2, Gulf Publishing Co. (1974).
E. 2 Keenan, J.H., F.G. Keyes, P.G. Hill and J.G. Moore, STEAM TABLES, Wiley (1969).

## APPENDIX F

## BUBBLE INDENTIFICATION AND TABULATED DATA

Each bubble analyzed is identified by a code in which the first number refers to the orifice diameter in millimetres. The second letter in the code refers to the name of the liquid tested. The third number is the indicated air flow rate which is given as cubic centimetre per second. Following the indicated air flow rate is a dash and then the picture frame number at which the bubble taken to be analyzed started to form at the orifice. The frame number for each film was set to zero when a completely blank frame was seen on the screen just before the bubble images appeared following the filming of the rod. Should this be required for later review, the frame number in the code serves as a means of locating the analyzed bubble in the film. The following example illustrates the application of the code. Let 3 A11-597 be the identification of a bubble analyzed. The first number(3) refers to the orifice diameter of 3 mm ; the letter(A) refers to the liquid as acetone; the second number(11) refers to the indicated air flow rate of $11.0 \mathrm{~cm}^{3} / \mathrm{sec}$ while the last number(597) is the frame number which gives the location of the start of the bubble formation.

The tabulated data for the bubble growth obtained during the experimental investigation are presented in Tables F. 1 to F.9. Three "first bubbles" (see Sec. 5.1) were analyzed, generally at random, for each set of conditions.

Table F. 1 Bubble Growth Data for Acetone ( Actual Flow Rate, $15.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter:
Liquid temperature:
Room temperature:
Liquid height above the orifice:
Indicated air flow rate:
0.3 cm
$24.4{ }^{\circ} \mathrm{C}$
$24.4{ }^{\circ} \mathrm{C}$
12.0 cm
$11.0 \mathrm{~cm} 3 / \mathrm{sec}$

Bubble Identification No. 3A11-597
(Time/frame, $4.167 \times 10^{-4} \mathrm{sec} /$ frame)

| Frame No. | Time, <br> t <br> ms | Bubble <br> Volume <br> $\mathrm{cm}^{3} \times 10^{-3}$ | Bubble Equi <br> Radius <br> cm |
| :---: | :---: | :---: | :---: |
| 597 | 0.0 | 2.15 | 0.080 |
| 601 | 1.7 | 24.40 | 0.080 |
| 610 | 5.4 | 80.63 | 0.268 |
| 620 | 9.6 | 153.35 | 0.332 |
| 630 | 13.8 | 226.24 | 0.378 |
| 640 | 17.9 | 278.26 | 0.405 |
| 650 | 20.9 | 328.41 | 0.428 |
| 670 | 30.4 | 463.25 | 0.480 |
| 679 | 34.2 | 545.70 | 0.507 |

Bubble Identification No. 3A11-1237
(Time/frame, $4.348 \times 10^{-4} \mathrm{sec} /$ frame)

1237
1238
1241
1244
1250
1259
1269
1279
1289
1299
1317

1382
1385
1389
1397
1407
1419
1434
1444
1459

| 0.0 | 1.75 | 0.075 |
| ---: | ---: | ---: |
| 0.4 | 5.38 | 0.109 |
| 1.3 | 13.68 | 0.148 |
| 2.6 | 37.80 | 0.208 |
| 5.2 | 79.34 | 0.267 |
| 9.1 | 142.48 | 0.324 |
| 13.5 | 196.92 | 0.361 |
| 17.8 | 261.75 | 0.397 |
| 22.2 | 313.92 | 0.422 |
| 26.5 | 385.06 | 0.451 |
| 34.3 | 520.46 | 0.499 |
| 39.1 | 554.17 | 0.510 |

Bubble Identification No. 3A11-1382
(Time/frame, $4.545 \times 10^{-4} \mathrm{sec} /$ frame)

| 0.0 | 1.15 | 0.065 |
| ---: | ---: | ---: |
| 4.4 | 8.52 | 0.120 |
| 3.2 | 37.62 | 0.238 |
| 6.8 | 88.86 | 0.277 |
| 13.6 | 168.43 | 0.343 |
| 16.8 | 248.43 | 0.390 |
| 23.6 | 347.04 | 0.436 |
| 28.2 | 434.90 | 0.470 |
| 35.0 | 532.05 | 0.503 |

Table F. 2 Bubble Growth Data for Acetone ( Actual Flow Rate, $34.8 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter:
Liquid temperature:
Room temperature:
Liquid height above the orifice:
Indicated air flow rate:
0.3 cm
$24.4{ }^{\circ} \mathrm{C}$
$24.4{ }^{\circ} \mathrm{C}$
12.0 cm
$24.0 \mathrm{~cm} / \mathrm{sec}$

Bubble Identification No. 3A24-481
(Time/frame, $4.167 \times 10^{-4} \mathrm{sec} /$ frame)

| Frame No. | Time, <br> t <br> ms | Bubble <br> Volume <br> $\mathrm{cm}^{3} \times 10^{-3}$ | Bubble Equi <br> Radius <br> cm |
| :---: | ---: | :---: | :---: |
|  |  |  |  |
| 481 | 0.0 | 5.58 | 0.110 |
| 483 | 0.8 | 32.57 | 0.198 |
| 486 | 2.1 | 94.35 | 0.282 |
| 492 | 4.6 | 197.82 | 0.361 |
| 499 | 7.5 | 305.82 | 0.418 |
| 507 | 11.0 | 410.51 | 0.461 |
| 516 | 14.6 | 549.14 | 0.508 |
| 526 | 18.8 | 670.42 | 0.543 |
| 536 | 22.9 | 799.73 | 0.576 |
| 546 | 27.1 | 950.29 | 0.610 |
| 556 | 31.3 | 1095.61 | 0.640 |
| 566 | 35.4 | 1247.66 | 0.668 |
| 571 | 37.5 | 1363.09 | 0.689 |

Bubble Identification No. 3A24-1111 (Time/frame, $4.348 \times 10^{-4} \mathrm{sec} /$ frame)

| 0.0 | 3.71 | 0.096 |
| ---: | ---: | ---: |
| 0.9 | 21.90 | 0.174 |
| 2.2 | 80.97 | 0.268 |
| 3.9 | 172.73 | 0.345 |
| 6.1 | 236.23 | 0.833 |
| 8.7 | 356.67 | 0.4440 |
| 11.7 | 449.85 | 0.475 |
| 15.7 | 571.08 | 0.515 |
| 20.0 | 702.83 | 0.552 |
| 24.3 | 838.83 | 0.585 |
| 28.7 | 1005.98 | 0.622 |
| 33.0 | 1174.83 | 0.655 |
| 37.4 | 1342.34 |  |

Bubble Identification No. 3A24-1228 (Time/frame, $4.545 \times 10^{-4} \mathrm{sec} /$ frame)

1228
1230
1233
1236
1239
1243
0.0
0.9
2.3
3.6
5.0
6.8
6.37
29.69
103.91
184.07
0.115
0.192
0.292
0.353
204.28
0.365
0.405

Table F. 2 - Cont'd

| 1248 | 9.1 | 363.47 | 0.443 |
| :--- | ---: | ---: | ---: |
| 1254 | 11.8 | 449.81 | 0.475 |
| 1261 | 15.0 | 557.21 | 0.511 |
| 1268 | 18.2 | 656.33 | 0.539 |
| 1277 | 22.3 | 821.22 | 0.581 |
| 1287 | 26.8 | 978.73 | 0.616 |
| 1297 | 31.4 | 1105.50 | 0.641 |
| 1309 | 36.8 | 1315.49 | 0.680 |

Table F. 3 Bubble Growth Data for Methanol( Actual Flow Rate, $42.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter: Liquid temperature: Room temperature: Liquid height above the orifice: Indicated air flow rate:

Bubble Identification No. 3M35-800
(Time/frame, $4.167 \times 10^{-4} \mathrm{sec} /$ frame)
0.3 cm
$23.3^{\circ} \mathrm{C}$
$23.3^{\circ} \mathrm{C}$
12.0 cm
$35.0 \mathrm{~cm}^{3} / \mathrm{sec}$

| Frame No. | Time, <br> t | Bubble <br> Volume <br> $\mathrm{cm}^{3} \times 10^{-3}$ | Bubble Equi <br> Radius <br> cm |
| :---: | :---: | :---: | :---: |
|  | ms |  |  |
| 800 | 0.0 | 2.06 | 0.079 |
| 802 | 0.8 | 25.74 | 0.183 |
| 804 | 1.7 | 69.60 | 0.255 |
| 806 | 2.5 | 125.14 | 0.310 |
| 808 | 3.3 | 168.30 | 0.343 |
| 810 | 4.2 | 200.24 | 0.363 |
| 812 | 5.0 | 216.50 | 0.372 |
| 820 | 8.3 | 384.52 | 0.451 |
| 830 | 12.5 | 557.94 | 0.511 |
| 840 | 16.7 | 752.61 | 0.556 |
| 850 | 20.8 | 936.90 | 0.607 |
| 860 | 25.0 | 1129.50 | 0.625 |
| 870 | 29.1 | 1302.80 | 0.678 |
| 880 | 33.3 | 1528.30 | 0.715 |
| 891 | 37.9 | 1829.70 | 0.759 |

Bubble Identification No. 3m35-1749
(Time/frame, $4.545 \times 10^{-4} \mathrm{sec} /$ frame)

1749
1754 1757 1760 1763 1766 1769 1779 1789 1799 1809 1819 1833
0.0
2.3
3.6
5.0
6.3
7.7
9.1
13.6
18.2
22.7
27.3
31.8
38.2

| 3.59 | 0.095 |
| ---: | ---: |
| 90.17 | 0.278 |
| 186.43 | 0.354 |
| 226.16 | 0.378 |
| 314.85 | 0.422 |
| 345.18 | 0.435 |
| 401.38 | 0.458 |
| 627.44 | 0.531 |
| 802.42 | 0.578 |
| 1067.16 | 0.634 |
| 1210.00 | 0.662 |
| 1305.80 | 0.678 |
| 1837.24 | 0.760 |

Bubble Identification No. 3M35-2640 (Time/frame, $4.545 \times 10^{-4} \mathrm{sec} /$ frame)

| 2640 | 0.0 | 1.56 | 0.072 |
| ---: | ---: | ---: | ---: |
| 2642 | 0.9 | 16.94 | 0.159 |
| 2644 | 1.8 | 66.21 | 0.251 |
| 2646 | 2.7 | 114.19 | 0.301 |

Table F. 3 - Cont'd

| 2648 | 3.6 | 137.08 | 0.320 |
| ---: | ---: | ---: | ---: |
| 2650 | 4.5 | 165.63 | 0.341 |
| 2660 | 9.1 | 375.44 | 0.447 |
| 2670 | 13.6 | 627.66 | 0.531 |
| 2680 | 18.2 | 833.55 | 0.584 |
| 2690 | 22.7 | 1051.77 | 0.631. |
| 2700 | 27.3 | 1237.42 | 0.667 |
| 2710 | 31.8 | 1406.96 | 0.695 |
| 2720 | 36.4 | 1624.63 | 0.729 |
| 2731 | 41.4 | 1747.86 | 0.747 |

Table F. 4 Bubble Growth Data for Methanol (Actual Flow Rate, $56.3 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter:
Liquid temperature:
Room temperature:
Liquid height above the orifice:

> 0.4 cm
> $25.6{ }^{\circ} \mathrm{C}$
> $25 . \mathrm{C}^{\circ} \mathrm{C}$
> 12.0 cm
> $44.4 \mathrm{~cm} 3 / \mathrm{sec}$

Bubble Identification No. 4M44.4-452
(Time/frame, $4.167 \times 10^{-4} \mathrm{sec} /$ frame)

| Frame No. | Time, <br> t | Bubble <br> Volume <br> $\mathrm{cm}^{3} \times 10^{-3}$ | Bubble Equ <br> Radius <br> cm |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 452 | 0.0 | 7.24 | 0.120 |
| 454 | 0.8 | 38.30 | 0.209 |
| 457 | 2.1 | 138.64 | 0.315 |
| 460 | 3.3 | 218.78 | 0.374 |
| 464 | 5.0 | 270.59 | 0.401 |
| 470 | 7.5 | 414.78 | 0.463 |
| 476 | 10.0 | 562.84 | 0.512 |
| 486 | 14.2 | 800.21 | 0.576 |
| 496 | 18.3 | 969.14 | 0.614 |
| 506 | 22.5 | 1158.20 | 0.652 |
| 516 | 26.7 | 1420.87 | 0.697 |
| 526 | 30.8 | 1593.95 | 0.725 |
| 536 | 35.0 | 1772.59 | 0.751 |
| 546 | 39.2 | 2063.02 | 0.790 |
| 555 | 42.9 | 2203.81 | 0.807 |

Bubble Identification No. 4M44.4-1004
(Time/frame, $4.167 \times 10^{-4} \mathrm{sec} /$ frame)
1004
1006
1009
1014
1020
1027
1035
1044
1054
1064
1074
1084
1094
1100
0.0
0.8
2.1
4.2
6.7
9.6
12.9
16.7
20.8
25.0
29.2
33.3
37.5
40.0
7.24
0.121

## 

0.8
32.18
0.197

2.
129.13
0.314

7
256.87
0.394

6
423.38
0.466
12.
16.7
555.11
0.510
20.8
668.07
0.542
25.0
887.42
0.594
29.2
1026.64
0.626
33.3
1265.63
0.671
37.5
1464.59
0.705
40.0
1958.82
0.738

Bubble Identification No. 4M44.4-2652
(Time/frame, $4.545 \times 10^{-4} \mathrm{sec} /$ frame)

| 0.0 | 5.56 | 0.110 |
| :--- | ---: | ---: |
| 0.9 | 42.39 | 0.216 |
| 2.3 | 164.92 | 0.340 |

Table F. 4 - Cont'd

| 2661 | 4.1 | 229.73 | 0.380 |
| :--- | ---: | ---: | ---: |
| 2666 | 6.4 | 389.18 | 0.453 |
| 2672 | 9.1 | 562.89 | 0.512 |
| 2679 | 12.3 | 744.83 | 0.562 |
| 2877 | 15.9 | 942.00 | 0.608 |
| 2706 | 20.0 | 1242.25 | 0.667 |
| 2716 | 24.5 | 1457.90 | 0.703 |
| 2726 | 33.6 | 1940.80 | 0.751 |
| 2736 | 38.2 | 2152.90 | 0.801 |
| 2743 | 41.4 | 2228.90 | 0.810 |

Table F. 5 Bubble Growth Data for Carbon Tetrachloride ( Actual Flow Rate, $21.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter:
Liquid temperature:
Room temperature:
Liquid height above the orifice:
Indicated air flow rate:

> 0.3 cm
> $31.1 \mathrm{Co}^{\circ} \mathrm{C}$
> 23.3 C
> 12.0 cm
> $16.5 \mathrm{~cm}^{3} / \mathrm{sec}$

Bubble Identification No. 3C16.5-1829
(Time/frame, $5.000 \times 10^{-4} \mathrm{sec} / \mathrm{frame}$ )

| Frame No. | Time, | Bubble | Bubble Equiv. |
| :---: | :---: | :---: | :---: |
|  | t | Volume | Radius |
| ms | $\mathrm{cm}^{3} \times 10^{-3}$ | cm |  |

1829
1831
1834
1838
1843
1849
1856
1864
1873
1883
1893
1903
0.0
$1.0 \quad 17.94$
0.760
0.160
$\begin{array}{lll}2.5 & 60.89 & 0.240\end{array}$
$4.5 \quad 99.95$
183.91
$\begin{array}{ll}10.0 & 257.41 \\ 13.5 & 331.84\end{array}$
0.290
$7.0 \quad 183.91 \quad 0.350$
0.400
0.430
$17.5 \quad 419.76$
0.460
$22.0 \quad 492.80$
0.490
$27.0 \quad 622.53$
0.530
$32.0 \quad 801.21$
0.580
$37.0 \quad 930.34$
0.610

Bubble Identification No. 3C16.5-1931 (Time/frame, $5.263 \times 10^{-4} \mathrm{sec} /$ frame)

| 0.0 | 3.60 | 0.095 |
| ---: | ---: | ---: |
| 1.1 | 24.60 | 0.180 |
| 3.2 | 93.64 | 0.280 |
| 5.8 | 145.41 | 0.330 |
| 9.0 | 217.32 | 0.370 |
| 13.2 | 321.47 | 0.430 |
| 17.9 | 402.88 | 0.460 |
| 23.2 | 513.92 | 0.500 |
| 28.4 | 676.16 | 0.540 |
| 33.7 | 820.02 | 0.580 |
| 37.9 | 927.11 | 0.600 |

Bubble Indentification No. 3C16.5-2033 (Time/frame, $4.878 \times 10^{-4} \mathrm{sec} /$ frame)

2033
2036
2039
2043
2048
2054
2061
2069
2078
0.0
1.5
2.9
4.9
7.3
10.2
13.7
17.6
22.0
5.58
0.110
0.200
0.250
0.290
0.350
0.390
0.420
0.450
0.480

Table F. 5 - Cont'd

| 2088 | 26.8 | 576.02 | 0.520 |
| :--- | :--- | :--- | :--- |
| 2098 | 31.7 | 714.41 | 0.560 |
| 2110 | 37.3 | 841.65 | 0.590 |

Table F. 6 Bubble Growth Data for Carbon Tetrachloride (Actual Flow Rate, $37.9 \mathrm{~cm}^{3} / \mathrm{sec}$ ).
Orifice diameter:
Liquid temperature:
Room temperature:
Iiquid height above the orifice:

$$
\begin{aligned}
& 0.3 \mathrm{~cm} \\
& 23.3 \mathrm{~cm}^{\circ} \mathrm{C} \\
& 23.3{ }^{\circ} \mathrm{C} \\
& 12.0 \mathrm{~cm} \\
& 29.3 \mathrm{~cm} 3 / \mathrm{sec}
\end{aligned}
$$

Indicated air flow rate:
Bubble Identification No. 3C29.3-1945 (Time/frame, $5.000 \times 10^{-4} \mathrm{sec} /$ frame)
Frame No.

| Time, | Bubble |
| :---: | :---: |
| t | Volume |
| ms | $\mathrm{cm}^{3} \times 10^{-3}$ |

1945
1947
1950
1953
1957
1962
1969
1977
1987
1997
2007
2017
2022
0.0
1.0
6.89
$2.5 \quad 35.55$
$\begin{array}{ll}2.5 & 163.08\end{array}$
6.0
8.5
12.0
16.0
21.0
237.69
341.76
474.10

Bubble Equiv. Radius
cm
0.118
0.204
0.299
0.339
0.384
0.434
0.484
0.528
26.0
31.0
36.0
617.82
0.568
0.606
0.635
0.673
0.687

Bubble Identification No. 3 3 $29.3-2558$
(Time/frame, $4.762 \times 10^{-4}$ sec/frame)
2558
2560
2564
2568
2574
2581
2589
2598
2608
2618
2628

| 0.0 | 2.23 |  |
| ---: | ---: | ---: |
| 1.0 | 17.63 | 0.081 |
| 2.9 | 86.26 | 0.162 |
| 4.8 | 149.02 | 0.274 |
| 7.6 | 294.44 | 0.329 |
| 11.0 | 385.18 | 0.413 |
| 14.8 | 533.83 | 0.451 |
| 19.0 | 665.88 | 0.503 |
| 23.8 | 8566.78 | 0.542 |
| 28.6 | 985.05 | 0.589 |
| 33.3 | 1257.71 | 0.617 |
|  |  | 0.670 |

Bubble Identification No. 3C29.3-2841
(Time/frame, $4.762 \times 10^{-4} \mathrm{sec} /$ frame)

2839
2843
2846
2851
2856
2862
2870
2879

| 0.0 | 6.37 |  |
| ---: | ---: | ---: |
| 1.0 | 39.53 | 0.115 |
| 2.4 | 86.04 | 0.211 |
| 4.8 | 177.72 | 0.274 |
| 7.1 | 259.45 | 0.349 |
| 10.0 | 351.72 | 0.396 |
| 13.8 | 479.46 | 0.438 |
| 18.1 | 608.92 | 0.486 |
|  |  | 0.526 |

Table F. 6 - Cont'd 2889 2900 2910 2919
22.9
746.57
898.75
1052.51
1190.61
0.563
0.599
0.631
0.657

Table F. 7 Bubble Growth Data for Distilled Water (Actual Flow Rate, $113.5 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter:
Liquid temperature:
Room temperature:
Liquid height above the orifice:
0.6 cm

Indicated air flow rate:
$25.5^{\circ} \mathrm{C}$
$25.5^{\circ} \mathrm{C}$
12.0 cm
$88.7 \mathrm{~cm} 3 / \mathrm{sec}$

Bubble Identification No. 6W88.7-1404
(Time/frame, $4.545 \times 10^{-4} \mathrm{sec} /$ frame)
Frame No.

1404
1406
1408
1411
1415
1420
1427
1436
1446
1456
1466
1476
1487
1497
1513
1526

1729
1731
1735
1739
1745 1753 1763 1773 1783 1793 1803

0.0
1.0
1.8
3.2
5.0
7.3
10.5
14.5
19.1
23.6
28.2
32.7
37.7
42.3
49.5
55.5
Bubble
Volume $\mathrm{cm}^{3} \times 10^{-3}$
7.24
177.83
370.32
434.88
632.52
909.09
1275.05
1687.80
2218.67
2620.43
3131.87
3558.42
4092.22
4614.34
5400.61
6069.07
5400.61
6069.07

Bubble Identification No. 6w88.7-1729 (time/frame, $5.000 \times 10^{-4} \mathrm{sec} / \mathrm{frame}$ )

| 0.0 | 10.31 | 0.135 |
| ---: | ---: | ---: |
| 1.0 | 122.35 | 0.308 |
| 3.0 | 444.56 | 0.475 |
| 5.0 | 620.85 | 0.529 |
| 8.0 | 1008.56 | 0.622 |
| 12.0 | 1488.56 | 0.708 |
| 17.0 | 2041.62 | 0.787 |
| 22.0 | 2653.14 | 0.859 |
| 27.0 | 3304.70 | 0.924 |
| 32.0 | 3900.81 | 0.977 |
| 37.0 | 4613.29 | 1.033 |
| 42.0 | 5381.48 | 1.087 |
| 47.0 | 6170.04 | 1.138 |
| 56.0 | 7028.56 | 1.188 |

10.31
0.135
0.308
0.475
0.529
0.622
0.708
0.787
0.859
0.924
0.977
1.033
1.087
1.188

Bubble Identification No. 6W88.7-2611 (Time/frame, $4.762 \times 10^{-4}$ sec/frame)
0.0
1.9
5.89
0.112
254.25
0.393

| Table F.7 -Cont'd |  |  |  |
| ---: | ---: | ---: | ---: |
| 2620 | 4.3 | 877.91 | 0.594 |
| 2638 | 12.9 | 1499.21 | 0.710 |
| 2648 | 17.6 | 2177.00 | 0.804 |
| 2660 | 23.3 | 2806.16 | 0.875 |
| 2675 | 30.5 | 3764.18 | 0.965 |
| 2690 | 37.6 | 4484.52 | 1.023 |
| 2710 | 47.1 | 5605.75 | 1.102 |
| 2720 | 51.9 | 6777.85 | 1.174 |

Table F. 8 Bubble Growth Data for Distilled Water (Actual Flow Rate, $180.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter:
0.6 cm
$50.0{ }^{\circ} \mathrm{C}$
$23.2^{\circ} \mathrm{C}$
12.0 cm
$111.5 \mathrm{~cm} 3 / \mathrm{sec}$

Bubble Identification No. 6W111.5-1833 (Time/frame, $5.000 \times 10^{-4} \mathrm{sec} / \mathrm{frame}$ )

| Frame No. | Time, <br> $t$ <br> ms | Bubble <br> Volume <br> $\mathrm{cm}^{3} \times 10^{-3}$ | Bubble Equiv. <br> Radius <br> cm |
| :---: | ---: | :---: | :---: |
|  |  |  |  |
| 1833 | 0.0 | 13.06 | 0.146 |
| 1835 | 1.0 | 294.52 | 0.413 |
| 1837 | 2.0 | 638.35 | 0.534 |
| 1841 | 4.0 | 973.98 | 0.615 |
| 1848 | 7.5 | 1730.76 | 0.745 |
| 1858 | 12.5 | 2628.73 | 0.856 |
| 1868 | 17.5 | 3436.11 | 0.936 |
| 1878 | 22.5 | 4251.94 | 1.005 |
| 1888 | 27.5 | 5191.85 | 1.074 |
| 1898 | 32.5 | 6124.98 | 1.135 |
| 1908 | 37.5 | 6857.08 | 1.179 |
| 1918 | 42.5 | 7721.95 | 1.226 |
| 1928 | 47.5 | 8330.53 | 1.256 |
| 1947 | 57.0 | 10331.15 | 1.351 |

Bubble Identification No. 6W111.5-1994
(Time/frame, $4.651 \times 10^{-4} \mathrm{sec} / \mathrm{frame}$ )

1994
1996
2000
2006
2014
2024
2034
2044
2059
2074
2086
15.91
0.156
0.9
260.25
0.396
2.8
690.81
0.548
5.6
9.3
14.0
18.6
1252.20
0.669
0.765
23.3
1873.35
0.873
23.3
2790.46
0.954
30.2
37.2
42.8
51.2
3640.83
1.021
1.111
1.196
1.263
1.352

Bubble Identification No. 6W111.5-2300
(Time/frame, $4.762 \times 10^{-4} \mathrm{sec} / \mathrm{frame}$ )

2300
2302
2304
2308
2315
2323
0.0
11.74
0.141
$1.0 \quad 104.80$
0.292
1.9
$3.8 \quad 713.21$
314.81
0.422
$\begin{array}{ll}7.1 & 1297.29\end{array}$
0.554
$11.0 \quad 2059.53$
0.677
0.789

Table F. 8 - Cont'd

| 2333 | 15.7 | 2833.20 | 0.878 |
| :--- | :--- | :--- | :--- |
| 2343 | 20.0 | 3640.06 | 0.954 |
| 2353 | 25.2 | 4690.07 | 1.038 |
| 2363 | 30.0 | 5668.39 | 1.106 |
| 2378 | 37.1 | 7158.07 | 1.196 |
| 2396 | 45.7 | 8346.51 | 1.258 |
| 2401 | 48.1 | 8863.49 | 1.283 |

Table F. 9 Bubble Growth Data for Toluene (Actual Flow Rate, $130.1 \mathrm{~cm}^{3} / \mathrm{sec}$ ).

Orifice diameter: Liquid temperature:
Room temperature:
Liquid height above the orifice:
Indicated air flow rate:
0.6 cm $35.6^{\circ} \mathrm{C}$ $23.3^{\circ} \mathrm{C}$ 12.0 cm $91.9 \mathrm{~cm}^{3} / \mathrm{sec}$

Bubble Identification No. 6T91.9-293 (Time/frame, $4.255 \times 10^{-4} \mathrm{sec} /$ frame)
Frame No.


Bubble Equiv. Radius cm

293 296 300 305 314 324 334 344 354 364 374 389 404 422
0.0
8.18 $1.3 \quad 177.84$ 493.41 776.75
1356.55
1961.48
2541.19
2886.72
3844.16
4412.02
5103.46
6084.20
7105.02
8147.77
0.125
0.348
0.490
0.570
0.687
0.777
0.847
0.883
0.972
1.017
1.068
1.132
1.193
1.248

Bubble Identification No. 6T91.9-2461
(Time/frame, $4.651 \times 10^{-4} \mathrm{sec} /$ frame)

| 0.0 | 6.37 | 0.115 |
| ---: | ---: | ---: |
| 0.9 | 137.26 | 0.320 |
| 1.9 | 346.30 | 0.436 |
| 4.2 | 733.09 | 0.559 |
| 7.9 | 1230.62 | 0.665 |
| 12.6 | 1922.92 | 0.771 |
| 17.2 | 2741.09 | 0.868 |
| 24.2 | 3323.77 | 0.926 |
| 32.1 | 4478.63 | 1.023 |
| 39.1 | 5580.15 | 1.103 |
| 46.1 | 6629.09 | 1.165 |
| 54.4 | 8508.17 | 1.266 |

Bubble Identification No. 6T91.9-2694
(Time/frame, $4.651 \times 10^{-4} \mathrm{sec} /$ frame)

| 2694 | 0.0 | 9.20 | 0.130 |
| ---: | ---: | ---: | ---: |
| 2700 | 2.8 | 365.84 | 0.444 |
| 2705 | 5.1 | 704.06 | 0.552 |
| 2708 | 6.5 | 927.45 | 0.605 |
| 2719 | 11.6 | 1544.77 | 0.717 |
| 2729 | 16.3 | 2181.71 | 0.805 |

Table F. 9 - Cont'd

| 2739 | 20.9 | 2701.65 | 0.864 |
| :--- | :--- | :--- | :--- |
| 2749 | 25.6 | 3371.89 | 0.930 |
| 2759 | 30.2 | 4026.07 | 0.987 |
| 2774 | 37.2 | 5137.21 | 1.070 |
| 2790 | 44.7 | 6104.53 | 1.134 |
| 2804 | 51.2 | 7290.47 | 1.203 |
| 2814 | 55.8 | 8482.92 | 1.265 |


[^0]:    *Equation quoted requires additional development in order to obtain Eqn. B. 3

