CAPACITY PLANNING UNDER FUZZY ENVIRONMENT

BY

MANOJ KUMAR VERMA

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Department of Mechanical and Industrial Engineering
University of Manitoba
Winnipeg, Manitoba

0-612-57589-6

THE UNIVERSITY OF MANITOBA

FACULTY OF GRADUATE STUDIES

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submitted by

Manoj Kumar Verma

in partial fulfillment of the requirements for the degree of

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The Thesis/~~Practicum~~ Examining Committee certifies that the thesis/~~practicum~~ (and oral examination ~~if required~~) is:

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☒ **Thesis**

☐ **~~Practicum~~**

Advisor: C. R. Bector
Dr. C.R. Bector

Balakrishnan
Dr. S. Balakrishnan

D. R. Strong
Dr. D.R. Strong

Date: April 18, 2001

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A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirements of the degree

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TO MY GRANDPARENTS

ABSTRACT

In recent years, in manufacturing industry, their has been a great deal of interest in capacity planning because the focus is shifting on techniques that determine flexibility of the amount and timing of work center capacity to satisfy the master production schedule. There are several techniques available for preparing work center capacity plans under crisp environment, but there is a scarcity of technique available for finding the required capacity in terms of labor hours under fuzzy environment.

In the present thesis, we analyze the Bill of Labor (BOL), Resource Profile (RP) and Capacity Requirements Planning (CRP) approaches under fuzzy environment with a variety of assumptions. Chapter 1 provides an introduction to the concepts of capacity planning problems considered in the thesis, followed by the literature survey in Chapter 2. The capacity analysis under fuzzy environment using BOL approach for rough cut capacity planning (RCCP) is considered in Chapter 3. Assuming that all the components of an item are manufactured in the same time period as the end item, i.e. lead-time offsets are considered to zero. Chapter 4 deals with the capacity analysis under fuzzy environment using RP approach for RCCP by including the lead-time dimension in it. Chapter 5 deals with the capacity requirements planning under fuzzy environment. Finally, conclusion, contribution and recommendations for further research on the afore-mentioned problems are presented in Chapter 6.

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CHAPTER 1

INTRODUCTION

The architecture of any in-house production system, built up from several production cells, may be implemented in different fashions (flow lines or work centers for instance). This macro-structure further refines as each production cell provides the capability to perform a group of operations. Raw materials and component parts float concurrently through the complex system in order to be processed and assembled, until a final product comes out ready for delivery.

Capacity analysis and planning is one of the most challenging subjects for the management. It appears to be a hierarchical process ranging from long to medium to short term decisions. Validating the master schedule (MS) with respect to capacity is one of the very important step in material resource planning (MRP), because an overstated MS can cause a variety of problems in an organization (for example, because of an overstated MS an organization can end up with increased raw materials and work–in–process inventories due to the reason that more materials are purchased and released to the shop than are completed and shipped out). Furthermore, to use the capital-intensive equipment of modern manufacturing optimally, the accurate capacity planning decisions at both strategic and tactical levels are of utmost importance (Gunasekaran et. al, 1998).

According to Fogarty et. al (1991), the Bill of Labor (BOL), Resource Profile (RP) and Capacity Requirements Planning (CRP) approaches use data on the time standards (a time standard is the time an average worker takes working at a normal place to produce one unit of an item) for each product at the key resources. The time standard for any part has built into it for a worker an allowance for rest to overcome fatigue, and an allowance for unavoidable delays etc. This leads to imprecise estimates of the time standards because of which many companies are reluctant to use time standards in performing the capacity analysis for the purpose of capacity management. Furthermore, fast changes in technology, continual changes in the production processes, and heavy dependence of many manufacturing processes on technology, use of crisp estimates of time standards is less than satisfactory and much less reliable. Also, usually the figures in the MS are forecast by the marketing and finance departments and thus are not precise. Therefore, it is reasonable to assume that the everyday problems in many organizations are becoming more and more complex and the relevant information available is becoming more and more imprecise, vague, and sometimes incomplete. Hence, it is natural to deal with such problems through fuzzy systems. The fuzzy numbers, for example can be obtained from experts who, instead of one but possibly forecasted estimate, provide three imprecise estimates in the case of triangle fuzzy numbers (four imprecise estimates in the case of trapezoidal fuzzy numbers) of the parameters involved. Such an approach is useful because, most of the data available in capacity planning problems in the industry is in the form of fuzzy estimates. Under such circumstances, using the fuzzy approach yields a relatively "more satisfactory and flexible solution". The fuzzy approach used in the present thesis is a small step in this direction.

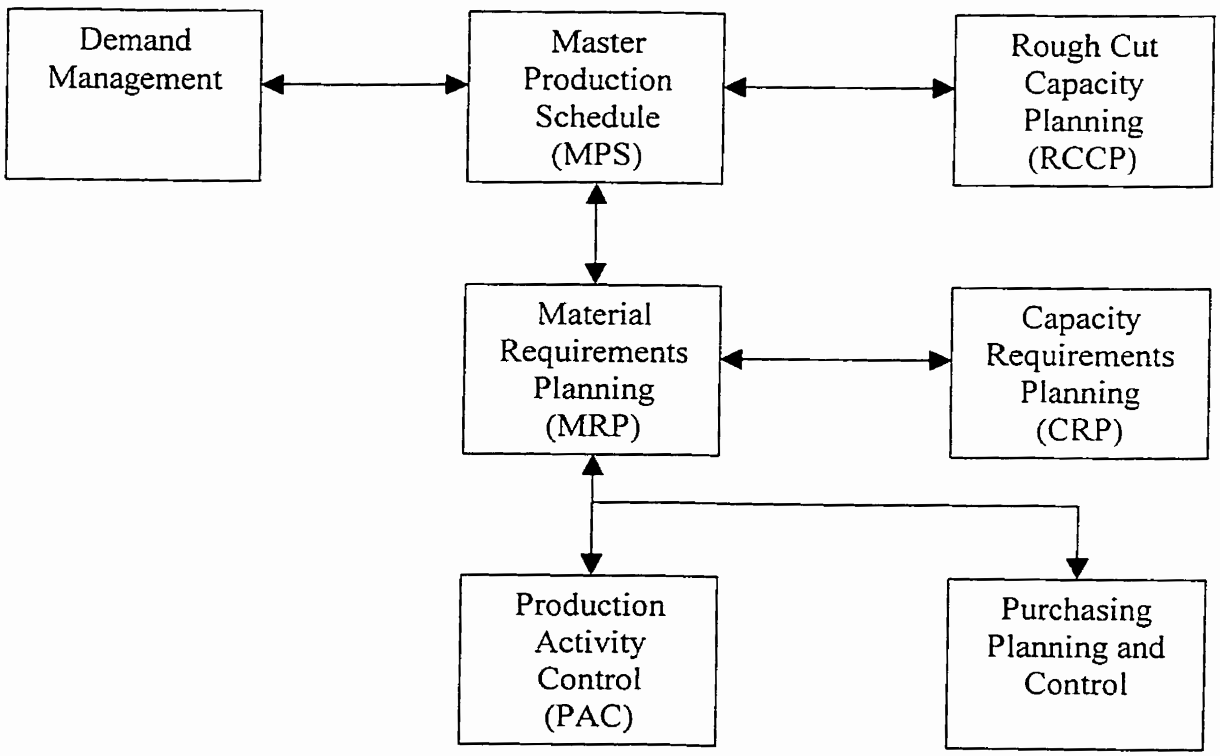
The present thesis seeks to provide an alternate, easy to understand and hopefully improved means of obtaining satisfactory capacities in terms of labor hours. We consider bill of labor, resource profile and capacity requirements planning approaches, under fuzzy environments, with a finite planning horizon.

1.1 Basic Concepts of Capacity Planning

Capacity Requirements Planning

A Capacity Requirements Planning (CRP) is the process of determining how much labor and machine resources are required to accomplish the task of production. Open shop orders, and planned orders in the material requirements planning (MRP) system, are input to CRP, which 'translates' these orders into hours of work by work center by time period. Capacity requirements planning is a detailed comparison of the capacity required by the material requirements planning and orders currently in process verses available capacity. According to Smunt (1996), the area of capacity planning is receiving increased emphasis in the management of operations due to the financial benefits of efficiently utilizing capacity and to the importance of accurate capacity plans for use with MRP and other information oriented planning systems.

Figure 1.1 Flow Diagram of Planning Activities



Benefits of Capacity Requirements Planning

- Provision of capacity planning data highlights overloads and under-loads.
- Early warning of jobs, which will be late due to insufficient availability of resources.
- Gives the information to plan shift changes or sub-contracting and to review due dates if necessary.
- Advance overtime planning improves labor relations.
- Highlights bottlenecks in production.
- Ability to plan the availability and improve the utilization of resources (labor and machinery).
- Warning of jobs approaching to each work center allows time for tools required to be requisition and in place when required.
- Jobs automatically prioritized at each work center.

Taal and Wortmann (1997) address the capacity problems by improving capacity planning at the material requirements planning level through integration of MRP and infinite capacity planning. The human planner is still a critical factor in capacity planning because the MRP-II ignores capacity constraints and leaves the capacity problems to the planner. Also the MRP-II does not give clear insight into the cause of capacity problems. Therefore, the human factor cannot be ignored when designing planning methods. This view is reflected in the basic concepts discussed below:

Supporting, not Replacing the Planner

It is not possible to model every aspect of a production system within an information system. This means that the human planner still has (and will continue to have) an important function in capacity planning. The human evaluates and controls the planning process. The main function of a planning system is to support the planner in the planning process, i.e. give a clear insight into capacity problems and suggest solutions.

Mathematical versus Human Perspective

Many planning methods are designed from a mathematical perspective and focus on reaching a mathematical optimal value. From a planner's point of view, a planning process over which the planner has control reaches Optimality. A planner's optimum often corresponds to a feasible and reasonable plan that reflects the planning methods of the planner.

Robustness of Capacity Planning Methods

Planning methods are called robust if they are relatively insensitive to the dynamic behavior of a production environment. Robust planning methods react smoothly on changes in the production environment, i.e. they avoid nervousness. This is especially important in production system where the situation changes constantly, such as machines go down, people get sick and customer demand is unknown etc. A primary characteristic of a robust planning method is that it has memory. This memory function result of the previous planning run is used to compute a new plan. Using robust planning methods results in a less nervous and more stable planning process over which the planner has better control.

Aggregated versus Detail Planning

When designing planning methods it is important to explicitly decide on the aggregation level of the planning. The approach of the Bake and Hellberg (1993) is to plan in as much detail as possible to prevent the loss of information caused by aggregation. However, using detail information can result in a very nervous planning process. This nervousness depends on the reliability of the information and the robustness of the planning method. Unreliable information frequently changes and therefore results in a nervous planning process.

1.2 Time Horizon for Capacity Planning

Long Range Planning

The long-range-planning horizon should exceed the time required to acquire new facilities and equipment. This may require 10 years or longer for organizations involved in the extraction process, where new mines must be developed. It may be as short as 18 months for the machine shop, where the facilities and equipment are catalog items.

Medium Range Planning

The medium range planning usually covers a period beginning 1 to 2 months in the future and ending 12 to 18 months in the future. In medium range planning, capacity may be varied by such alternatives as hiring, layoffs, new tools, minor equipment purchases, and subcontracting.

Short Range Planning

There is no precise definition for the length of the short term planning horizon. This is tied into the daily or weekly scheduling process and involves making adjustments to eliminate the variance between planned output and actual output. This includes alternatives such as overtime, personnel transfers and alternative production routing.

1.3 Rough Cut Capacity Planning (RCCP)

According to Fogarty et. al (1991), before management approves the production plan or the MPS, it must verify the organization's ability to carry out the plan. Rough cut capacity planning includes the following:

- Determining that sufficient working capital will be available to meet the cash flow requirements.
- Determining that key vendors have the required capacity and obtaining commitment of that capacity.

Advantages of Rough Cut Capacity Planning (RCCP)

- A routing for every part number is not necessary.
- It does not require as much detail as capacity requirements planning (CRP).
- It quickly defines potential capacity problems with few critical work centers.
- If capacity imbalances exist, it can aid in the rescheduling of the MPS until capacity problems can be solved.
- It can be run prior to MRP to validate production and purchasing plans.
- It can be run more frequently because it does not require computer capacity.

1.4 The Role of RCCP in the Production Planning and Control System

Production and inventory planning is the process of dealing with flexibility to meet the desires of the customer, the need for stability in manufacturing and the resultant inventory levels to compensate for the mismatch. The process involves performing two functions effectively:

- Developing an achievable Master Production Schedule.
- Planning and controlling capacities.

Master Schedule (MS)

It is a plan to manufacture specific items or provide specific service (s) within a given time period. The Master Scheduling is a key link in the manufacturing planning and control chain. The MS interfaces with marketing, distribution planning, production planning and capacity planning. Master Scheduling calculates the quantity required to meet demand requirements from all resources.

Capacity Planning

Capacity planning is the task of determining how much output is needed from the facilities and from the supplier.

Capacity Control

Capacity control is the comparison between planned levels and actual output achieved and the identification of significant variances above or below plan. Corrective action must be initiated promptly if control is to be maintained, that usually means adjusting capacity or preferable in most cases to the alternative of changing the master schedule.

1.5 Adjusting Capacity Available and/or Capacity Required

According to Fogarty et. al (1991), when capacity available is less than the capacity required then four basic options are available to increase the available capacity by using overtime, subcontracting, alternate route, or adding personnel. If no combination of the four options can provide sufficient capacity, then the master production schedule will have to be reduced.

Overtime

Overtime is probably the popular solution to insufficient capacity because few advance arrangements must be made. Increasing labor capacity by scheduling overtime avoids the cost of hiring and training and also does not increase the total fringe benefit cost for holidays, vacations, and insurance. However, direct costs usually increase due to both premium wages and decreasing productivity rates. This decrease in productivity is especially true when the weekly overtime becomes excessive or lasts for more than a month or so. Over-time/under-time should be the first choice in case the due dates are rigid.

Subcontracting

A second way to obtain additional capacity is through subcontracting. Using other firms on a regular basis to perform manufacturing and professional services, such as engineering, marketing research, and software development, can be an effective method of balancing supply and demand. Subcontracting can be especially valuable when treated as an important link in the production chain. Arrangements for subcontracting must begin well in advance to permit time to find a vendor capable of performing quality work. Subcontracting usually is more expensive than building an item in house on regular time. However, subcontracting may be cheaper than building the part in house on overtime.

Disadvantages of subcontracting:

- Lead-time usually increases.
- Transportation cost may increase.
- It is more difficult to guarantee a quality product.

Alternate Routing

If only a few work centers have excess work, the remaining work centers will tend to have too little work during a given period. It is, therefore, possible to consider a temporary change in the routing of specific parts so that work usually performed in work center A temporary is performed in work center B. If work center B cannot achieve the needed quality, then alternate routing should not be consider. If work center B presently is not used because of time, then alternate routing should be considered.

Adding Personnel

There are three ways to add personnel: add a shift, add new hires to an existing shift, or move existing personnel from an underused work center. The time to consider adding a shift is when the master schedule is formulated, and when the demand chase versus level production versus mixed strategy choice is made. Adding new hires to an existing shift is likely to be an option only when the budget for the next fiscal year is being approved. Thus, the only short-term way to obtain additional personnel is to shift people from an underused work center to one that is overload.

Revising the Master Production Schedule

Most companies consider a revision to the MPS to be a solution of last resort in the event of insufficient capacity, only when all other options are exhausted. The MPS revision actually should be the first thing a company considers because if insufficient capacity exists, it is impossible to complete all orders on time. Then the management has to decide whose order is going to be late based on the impact of the whole enterprise or to have a worker on the shop floor to make the choice based on the convenience of one department. When capacity overloads exist we may prefer to have jobs completed out of strict due date sequence. Rescheduling of MPS should be the first choice in case the due dates are flexible and the overtime working expensive.

According to Pandy and Hasin (1997), alternative, adding personnel (i.e. hiring man power and purchasing machines) is a long-term strategy, which cannot be considered under short-term capacity planning. Two other planning alternatives, e.g. (a) allocation of overtime/under-time to existing workers and machines, and (b) rescheduling of the MPS should be exercised together for the purpose of short-term capacity adjustments.

1.6 Material Requirements Planning (MRP)

Material Requirements Planning (MRP) is a technique aimed to fulfill the replenishment function in industry. It "creates schedules, identifying the specific parts and materials required to produce an end item, the exact numbers needed, and the dates when orders for these materials should be released and be received or completed within the production cycle".

The principle of MRP is a logical consequence of attempting to meet a dependent demand (i.e., the demand for materials, parts, and components depends on the demand for an end product). Ultimately, this approach will govern the ordering of the diverse requirements and consequently their inventory behavior. The main purpose of a basic MRP system are to control inventory levels, assign operating priorities for items, and plan capacity to load the production system. These may be briefly expanded as follows:

Inventory

- Order the right part.
- Order in the right quantity.
- Order at the right time.

Priorities

- Order with the right due date.
- Keep the due date valid.

Capacity

- Plan for a complete load.
- Plan an accurate load.
- Plan for an adequate time to view future load.

The theme of MRP is “getting the right materials to the right place at the right time”. It also removes the guesswork from forward procurement plans, eliminates tedious calculations and triggers procurement activity at the precise time.

Benefits of Material Requirements Planning

- MRP quickly converts the MPS to net material and component requirements.
- Quickly determines shortage within lead-time threshold and avoids unnecessarily premature commitment to purchase or manufacture.
- Reduced costs by lowering stock and work in progress levels.
- Improved productivity by ensuring that materials are available when required.
- Improved customer service levels through deliveries consistent with promises.
- Ability to negotiate better material prices with the knowledge of scheduled requirements.
- Potential shortages revealed at planning stage, prior to kitting, and insufficient time to allow for purchasing action or component manufacture.
- Greater management control over materials and work in progress.
- Greater flexibility and changes in plan can be quickly put into effect.

1.7 Bill of Material (BOM)

This is a very common document issued by the design department. To be suitable for an MRP analysis, it is not enough that this bill unambiguously identify materials, components or sub-assemblies (product specification), but also that it provides other features such as the support for the planning via the identification of relationships among components. The bill of materials will show the structure of the actual products, from raw materials, components, and parts subassemblies up to the end item. We may be able to say that the language in a master production schedule is stated in the same terms as it is for the bill of materials. Basically there are two types of BOM as follows:

Single Level Bill of Material

The simplest format is a single level BOM. It consists of a list of all components needed to make the end item, including for each component (1) a unique part number, (2) a short verbal description, (3) the quantity needed for each single end item, and (4) the part's unit of measure.

Multilevel Bill of Material

The single level BOM is sufficient when a product is assembled at one time from a set of purchased parts and raw materials, it does not adequately describe a product that has subassemblies. In multilevel BOM the numbers for the components of each subassembly are intended under the respective subassembly numbers. When a component is used in more than one subassembly a common parts bill may be produced for use by inventory planning. In this type of bill there is only one occurrence of the item along with its total quantity per final assembly.

1.8 Other Definitions (Fogarty et. al (1991))

Order Release

Order release initiates the execution phase of production. It authorizes production and/or purchasing. The planned order becomes a released (open) order. Placement of a purchase order or the initiation of manufacturing follows shortly. Order release planning may take place until the movement of the order release. Authorization of order release is based first on the planned orders in the MRP output, the current priority, the availability of materials and tooling, and the loads specified by input/output planning. Release of an order triggers the release of the following:

- Requisitions for material and components required by the order. If some of these items are not required immediately and have not been allocated previously, they are allocated now.
- Production order documentation to the plant. This documentation may include a set of both engineering drawing and manufacturing specifications and a manufacturing routing sheet.
- Requisitions for tools required in the first week or so of production. Tooling, including tapes for numerically controlled machines, required in later operations is reserved for the appropriate period. Tooling can be included in the master production schedule and the bill of material.

Dispatching

Dispatching informs first line supervisor of the released orders and their priority, that is, the sequence in which orders should be run. This information can be transmitted via a hard copy (handwritten, typed, or computer printout). Dispatching list identifies the date, the plant, and the work center, it includes the work center capacity, and it lists the orders, their quantity, their capacity requirements, and their priority. If orders take a day or less to process, dispatch lists usually are prepared daily.

Lead Times

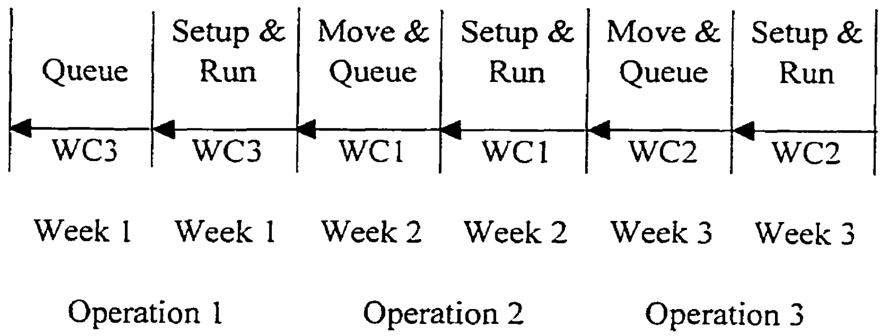
The time intervals necessary to either manufacture or purchase the components are referred to as the lead-times. The lead times are used to compute lead-time offsets for each component.

Scheduled Receipts

Scheduled receipts come from orders already released either to manufacturing (production, manufacturing, or shop orders) or to suppliers (purchase orders). When an order is released it becomes an open order and has schedule receipt.

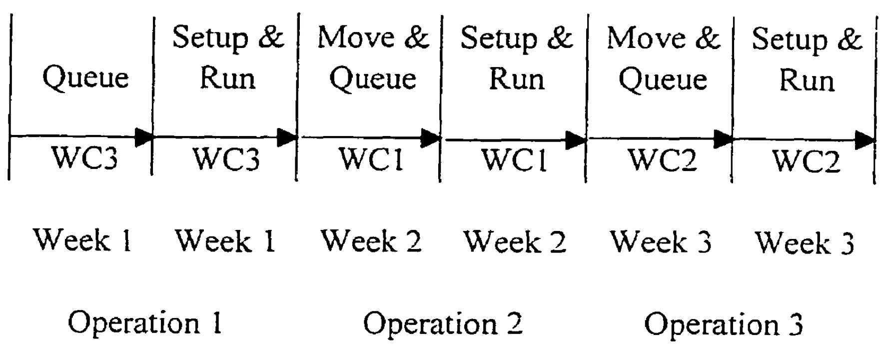
Backward Scheduling

In Backward Scheduling, activities start at the planned receipt date (due date) and move backward in time.



Forward Scheduling

In Forward Scheduling, activities start at the planned release date and move forward in time. In Chapter 5 of this thesis, we used a forward scheduling. For example, Part 3 goes to WC3, WC1 and WC2.



Chase Strategy

The chase strategy is designed to allow for sufficient capacity and flexibility to enable production output to match the demand. The rationale of the chase strategy is to avoid high inventory carrying cost when demand varies substantially by varying employment levels, using overtime, subcontracting, and/or assigning production employees to maintenance or training activities during low demand periods. However the chase strategy is not necessary or economically practical in many situations. Example includes situation in which employees have a guaranteed annual wage and those in which equipment capacity is well below the maximum demand rate.

Level Production Strategy

The level production strategy is designed to allow for the same production rate throughout the year and to have inventory or backorders absorb variations in demand. This makes sense when the demand is relatively stable, but following this approach in some situations, such as the manufacture or artificial Christmas trees, will result in excessive inventory carrying costs.

Standard Time

The observer will observe one or more operators continuously and record the time taken. This time is called observe time. Then this observed time is adjusted (given a rating) to obtain the time that a typical experienced operator would take.

$$\text{Normal Time} = (\text{Observed time}) \times (\text{Rating})$$

However, normal time is not representative of the time an experienced worker would take working all day. Additional time must be given for personal requirements, fatigue, and delay allowances. The resulting time is called standard time.

$$\text{Standard Time} = (\text{Normal time}) \times (1 + \text{work time allowances}).$$

1.9 Fuzzy Set Theory

In this section we introduce some of the basic concepts and terminology of fuzzy set theory. Theory of fuzzy sets is basically a theory of graded concepts (Zimmerman, 1991). A central concept of fuzzy set theory is that it is permissible for an element to belong partly to a fuzzy set.

1.9.1 Fuzzy Set

Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. The membership in a crisp subset of X is viewed as a characteristic function μ_A from X to [0, 1] such that:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \notin A \\ 1 & \text{if } x \in A \end{cases}$$

where [0, 1] is called a valuation set (Lai and Hwang, (1992)).

If the valuation set is allowed to be the real interval [0, 1], A is called a fuzzy set proposed by Zadeh (1996). $\mu_A(x)$ is the degree of membership of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. Therefore, A is completely characterized by the set of ordered pairs:

$$A = \{(x, \mu_A(x)) / x \in X\}$$

where $\mu_A(x)$ maps X to the membership space [0, 1]. Elements with zero degree of membership are usually not listed. If Sup $\mu(x) = 1$, $\forall x \in R$, then the fuzzy set A is called a normal fuzzy set in R. A fuzzy set that is not normal is called subnormal fuzzy set.

1.9.2 α – Level Set or α – Cut

One of the most important concepts of fuzzy sets is the concept of an α-cut or α-level set. An α-cut denoted by A_α is the crisp set of elements x in R whose degree of belongings to the fuzzy set A is at least $\alpha \in [0, 1]$. This means

$$A_\alpha = \{x \in R \mid \mu_A(x) \geq \alpha, \alpha \in [0, 1]\}$$

that is, the α-cut or α-level set of a fuzzy set is the crisp set A_α that contains all elements of the universal set $X \in R$ whose membership grades in A are greater than or equal to the specified value of α, $\alpha \in [0, 1]$.

1.9.3 Support of a Fuzzy Set

The support of a fuzzy set A is a set S(A) such that $x \in S(A) \Leftrightarrow \mu_A(x) > 0$. If $\mu_A(x)$ is constant over S(A), then A is non-fuzzy.

1.9.4 Intersection of Fuzzy Sets

Intersection of two fuzzy sets A and B is a fuzzy set C denoted by $C = A \cap B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \min\, [\mu_A(x), \mu_B(x)], \quad \forall\, x \in X$$

1.9.5 Algebraic Operations on Fuzzy Sets

In addition to the set theoretic operations, we can also define a number of other ways of forming combinations of fuzzy sets and relating them to one another. Here we present some more important operations among those:

1. Algebraic product of two fuzzy sets A and B, is A (·) B, whose membership function is

$$\mu_{A(\cdot)B}(x) = \mu_A(x)\ (\cdot)\ \mu_B(x), \quad \forall\, x \in X$$

2. The algebraic sum of A and B is A + B whose membership function is defined as

$$\mu_{(A+B)}(x) = \mu_A(x)\ (+)\ \mu_B(x), \quad \forall\, x \in X$$

provided $\mu_A(x)\ (+)\ \mu_B(x) \leq 1, \quad \forall\, x \in X$

3. The absolute difference $|A - B|$, of A and B is given by

$$\mu_{A-B}(x) = |\mu_A(x)\ (-)\ \mu_B(x)| \quad x \in X$$

1.9.6 Convexity of Fuzzy Set

The notion of convexity can be extended to fuzzy sets in such a way as to preserve many of the properties that it has in case of crisp sets. In what follows, we assume that the set X is the n-dimensional space R^n. We now have the following two equivalent definitions of convexity of a fuzzy set.

A fuzzy set A is convex if and only if every set $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ for all $\alpha \in [0, 1]$ is a convex set.

The second definition of convexity of a fuzzy set is as follows:

A fuzzy set A is said to be a convex set if

$$\mu\,(\lambda x_1 + (1-\lambda)x_2) \geq \min\,(\mu(x_1), \mu(x_2)), \quad x_1, x_2 \in X,\ \lambda \in [0, 1].$$

1.10 Fuzzy Arithmetic

The first definition of a fuzzy set allows us to extend various properties of crisp sets and operations on crisp sets to their fuzzy counterparts.

An ordinary number 'a' can be characterized by using the notation of membership function as,

$$\mu_A(x) = \begin{cases} 1 & \text{if } x = a \\ 0 & \text{if } x \neq a \end{cases}$$

A fuzzy number A is a fuzzy set on the real line R, which possesses the following properties:

- A is a normal, convex fuzzy set on R
- The α-level set A_α must be a closed interval for every $\alpha \in [0, 1]$
- The support of A, $S(A) = \{x \mid \mu_A(x) > 0\}$, must be bounded.

Fuzzy arithmetic is based on two properties of fuzzy numbers:

1. Each fuzzy set and thus, each fuzzy number can be fully and uniquely represented by its α-level sets.
2. α-level sets of each fuzzy numbers are closed intervals of real numbers for all $\alpha \in [0, 1]$

These properties enable us to define an arithmetic operation on fuzzy numbers in terms of arithmetic operations on their α-level sets (i.e. arithmetic operations on closed intervals).

1.10.1 Fuzzy Arithmetic Based On Operations On Closed Intervals

A fuzzy number can be characterized by an interval of confidence at level α,

$$A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$$

which has the property

$$\alpha \leq \alpha' \Rightarrow A_{\alpha'} \subset A_\alpha$$

According to Kaufmann and Gupta (1985, 1988), let A = [a, b] $\in$ R and B = [c, d] $\in$ R be two fuzzy numbers, then the arithmetic operations on them as follows:

Addition	$A + B = [a + c, b + d]$
Subtraction	$A - B = [a - d, b - c]$
Multiplication	$AB = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$
Division	$A/B = [\min(a/c, a/d, b/c, b/d), \max(a/c, a/d, b/c, b/d)]$
Minimum ($\wedge$)	$A \wedge B = [a \wedge c, b \wedge d]$
Maximum ($\vee$)	$A \vee B = [a \vee c, b \vee d]$

Let A and B be two fuzzy numbers. $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ be the α-level set of A, and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$ be the α-level set of B.

Let * denote any of the arithmetic operations $+, -, \cdot, /, \wedge$ and $\vee$ on fuzzy numbers. Then, we define a fuzzy set A * B in R, by defining its α-level sets $(A * B)_\alpha$ as

$$(A * B)_\alpha = A_\alpha * B_\alpha \text{ for any } \alpha \in [0, 1]$$

Since $(A * B)_\alpha$ is a closed interval for each $\alpha \in [0, 1]$ and A and B are fuzzy numbers, A * B is also a fuzzy number.

The multiplication of fuzzy number $A \subset R$ by an ordinary number $k \in R^+$ can also be defined as

$$k\,(\cdot)\,A_\alpha = [ka_1^{(\alpha)}, ka_2^{(\alpha)}]$$

or equivalently, $\mu_{k.A}(x) = \mu_A(x/k) \quad \forall\ x \in R$.

1.10.2 Triangular Fuzzy Number

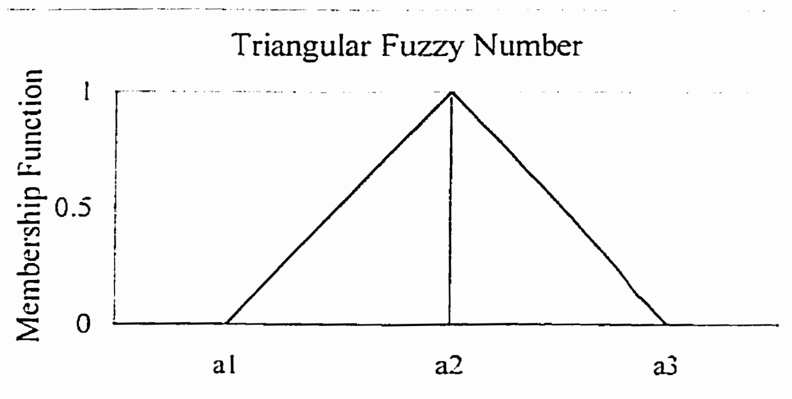
A triangular fuzzy number (T.F.N.), A, is denoted by the triplet (a_1, a_2, a_3) and its membership function is written as

$$\mu_A(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & x \ge a_3 \end{cases}$$

The α-level set of a triangular fuzzy number is

$$A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3] \qquad \forall \alpha \in [0, 1]$$

Figure 1.2 Graphic representation of a Triangular Fuzzy Number



Algebraic Operations on T.F.N.

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two T.F.Ns then,

- Addition $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- Subtraction $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

For the following two operations, we assume that a_i and b_i, $i = 1, 2, 3$ are positive.

- Multiplication $A (.) B = (a_1 b_1, a_2 b_2, a_3 b_3)$
- Division $A (:) B = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right)$

1.10.3 Trapezoidal Fuzzy Number

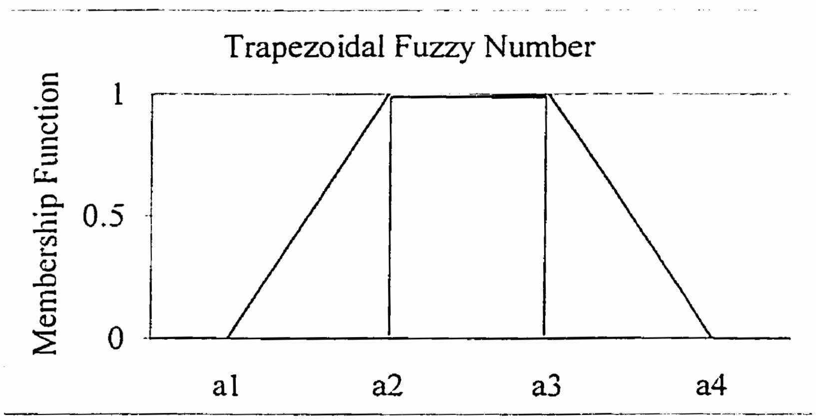
A trapezoidal fuzzy number (Tr.F.N.), A, is denoted by a quadruplet (a_1, a_2, a_3, a_4) whose membership function is written as

$$\mu_A(x) = \begin{cases} 0 & x \le a_1 \\ \frac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3} & a_3 \le x \le a_4 \\ 0 & x \ge a_4 \end{cases}$$

The α-level set of a trapezoidal fuzzy number A is

$$A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] = [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\alpha + a_4] \qquad \forall\, \alpha \in [0, 1]$$

Figure 1.3 Graphic representation of a Trapezoidal Fuzzy Number



Algebraic Operations on Tr.F.N.

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be two Tr.F.Ns then,

- Addition $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- Subtraction $A - B = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

For the following two operations, we assume that a_i and b_i, $i = 1, 2, 3$ are positive.

- Multiplication $A\ (.)\ B = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
- Division $A\ (:)\ B = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$

1.11 Organization of the Thesis

In the present thesis, an important problem in the field of Industrial Engineering i.e. capacity planning problem (addressed by Forgarty et al, 1991) has been considered. The bill of labor for RCCP, resource profile for RCCP and capacity requirements planning approaches has been modeled under fuzzy environments.

Chapter 1 provides an introduction to the concepts of capacity planning problems considered in the thesis. Chapter 2 deals with the literature review of the related work done by other researchers. The capacity analysis under fuzzy environment, using bill of labor approach for rough cut capacity planning (RCCP) is considered in Chapter 3. Chapter 4 deals with the capacity analysis under fuzzy environment using the resource profile approach for RCCP. Chapter 5 deals with the capacity requirements planning under fuzzy environment. Finally, the conclusion and the discussion on the contributions made by the thesis, along with some recommendations for further research, are given in Chapter 6.

CHAPTER 2

LITERATURE SURVEY

This chapter provides a survey of the literature dealing with capacity analysis and planning problems, and other concepts considered in this thesis. The purpose of this chapter is to review the developments, and to identify the status of existing literature in these areas.

2.1 Review of Literature on Capacity Analysis and Planning Problems

Recently, several papers dealing with various aspects of capacity analysis and planning have appeared in the literature. Some major sources of this literature include the APICS publications (such as Production and Inventory Management Journal, Journal of Operation Management), International Journal of Production Research (IJPR), European Journal of Operations Research (EJOR) etc. According to William et. al (1982), in many firms the execution of MPS is handicapped because adequate capacity has not been planned at individual departments or work centers. Also, in the resource profile approach, the planning horizon must be sufficiently long, so that the few periods do not exhibit an end of horizon effect. The first few periods of a planning horizon are ones that must be accurate to provide MPS verification. Jones and Williams (1992) discuss a comparison between rough cut capacity planning and capacity requirements planning. Many manufacturing companies have moved from intermittent to flow production in order to reduce lead-time, work in process, order quantity, and transaction processing. These improvements have not reduced the need for capacity planning but will influence the decision regarding the use of the appropriate tools. Smith and Pickard (1993) formulate an integer-programming model for a capacity planning problem and illustrate their method using a real example. Bretthauer (1996) and Bretthauer and Cote (1997) address the problem of capacity planning in manufacturing and computer networks and use the branch and bound technique to solve the mathematical models of those problems. Zijm and Buitenhek (1996) discuss a framework for capacity planning and lead time management in manufacturing companies with an emphasis on the machine shop. Glasserman (1996) consider the problem of allocating production capacity among multiple items, assuming that a fixed proportion of overall capacity can be dedicated exclusively to the production of each item. Pandey and Hasin (1997a) address the problems of, considering the actual processing sequence used in the processing of a product as described by shop routing, and accounting for the effect of rescheduling of the MS on the inventory holding cost, using rough cut capacity planning (RCCP).

Crepeau and Eugene (1995) aims to help people understand and apply the fundamentals of capacity planning and control, in both the traditional job shop and flow manufacturing environments and also help people to recognize and avoid common blunders, such as blindly trying to maximize utilization (often a mistake in process industries as well as the job shop), loading in hot order and other such as paths to career termination. Aghezzaf and Houssaine (2000) developed an algorithm that produces an efficient mixed strategies-based staffing plan. The labor capacities resulting from this algorithm are then translated into available regular time and overtime unit production capacities. Pandy and Hasin (1997b) implement a scheme for the integration of Rough-

Cut-Capacity-Planning with MRPII, with more constraints and elements than considered presently by considering shop capacity during MPS generation (i.e, making the MPS more feasible/realistic). According to Pandy and Hasin (1997c), two important factors (the actual processing sequence used in the processing of a product and the effect of rescheduling of the MPS on inventory holding cost) must be considered during RCCP and rescheduling of MPS.

Akkan and Can (1997) develop a heuristic to minimize the present value of the cost of rejecting orders and inventory holding cost due to early completion. Tall et. al (1997) considers the capacity problems by improving capacity planning at the material requirements planning (MRP) level through integration of MRP and finite capacity planning. Also the planning method is based on a new and more accurate primary process model, giving the planning algorithm more flexibility in solving capacity problems. Guide et. al (1997) develops a new capacity planning techniques and implemented it along with some standard capacity planning techniques in a remanufacturing environment. The results show that the new techniques are significantly better than the standard techniques.

Smunt (1996) discusses an overview of learning curve analysis for rough cut capacity planning and illustrates the effective use of learning curves for capacity planning through a comparison of traditional approaches with ones that incorporate the learning curve concept. To solve capacity planning problems, Bretthauer and Kurt (1996) present a branch and bound algorithm that globally minimizes a concave cost function over a single convex nonlinear performance constraint and lower and upper bounds on the discrete capacity variables. Chung and Shu (1995) designed capacity requirements planning (CRP) system for flexible manufacturing systems (FMS) to determines the requested capacities by carrying out part type selection, batch size determination and process plan selection, and by estimating tool slot request. If there is insufficient capacity, then the CRP system calculates the overtime hours required to complete the production of demand quantity for a specified part type.

According to Toyce and Charles (1990), the CRP logic must be expanded and refined to give manufactures more efficient tools for planning and controlling both material and capacity resources. So that, to provide realistic delivery promise dates. improve on time delivery performance and reduce work in process inventory through lead-time reduction and queue control. Bemelmans (1985) developed a capacity oriented system of production and inventory control and then compared its performance with the performance of a product-oriented system. Matsuura and Haruki (1993) deals with the problem of determining planned lead times in a periodic loading system, such as material requirements planning (MRP). The planned lead-time should be determined under a management policy, taking a trade off between work in process quantity and capacity requirements variations into consideration.

2.2 Effect of Over-time/Under-time

Pandy and Hasin (1997a) analyze and formulate the impact of capacity planning functions on inventory holding cost time period. Operating with over-time and under-time can have a significant impact on inventory holding time and cost because when production of a batch continues over a period of time, the finished products, at a certain time, or day are stored temporarily until the batch is complete, specially when the total throughput/cycle time for the batch extends beyond a day. The time for which a partially processed batch waits in the stockroom until complete processing of the entire batch, is termed as the inventory holding time period during production (IHTPDP), where as the total inventory holding time period (IHTP) can be defined as the summation of IHTP during production and the period from the completion of the batch up-to the due date or shipping/dispatching date. By using overtime, IHTPDP is shortened, thereby decreasing inventory-holding costs.

2.3 Lead Times for Capacity Planning

Pandy and Hasin (1996b), formulate a strategy for the measurement of manufacturing lead time based on shop routing, and accessing the impact of overtime capacity planning on manufacturing lead time (MLT). According to Pandy and Hasin (1996b), the manufacturing lead-time for a batch of parts is the time period that the batch spends in the production shop. It is the difference between the time when the raw materials enter the production, and the instant the completed batch exits the shop as a finished product. MLT is one of the important elements used in the computations of the total lead-time for use in manufacturing resource-planning systems. The total lead-time comprises the sum of purchasing, ordering, transportation, inspection, inventory, manufacturing, delivery lead-time etc.

2.4 Production Capacity Planning and Control

To support capacity planning at the production function level, Gunasekaran et. al (1998) develop a mathematical model for determining the optimum lot sizes for a set of products and the capacity required to produce them in a multistage production system. The purpose of the modeling is to determine the capacity required and batch sizes at each stage by minimizing the total system cost per unit time, which consists of (i) set up cost. (ii) cost due to the quenching of batches, and (iii) hiring cost of the machines. Accurate capacity planning decisions at strategic and tactical levels are essential for the optimal use of the capital-intensive equipments of modern manufacturing. The level of demand and lot size determine the capacity required at each machine stage and forms the basis of the production model and associated costs to be integrated into the corporate model. The optimization of batch sizes and capacity reduce the hiring cost of machine and other costs such as set-up cost and inventory cost due to queuing of matches.

2.5 Motivation and Objectives of the Research

The present research was largely motivated by the benefits of capacity planning in terms of labor hours. The selection of research problems and the objectives in this thesis depends on the following factors discussed below:

- Throughout industrial establishments worldwide, it is commonly experienced that the demand for the products manufactured by them varies periodically. According to Fogarty et. al (1991), the BOL, RP and CRP approaches use data on the time standards for each product at the key resources. The time standard units for any part has built into it for a worker an allowance for rest to overcome fatigue, and an allowance for unavoidable delays etc. This leads to imprecise estimates of the time standards because of which many companies are reluctant to use time standards in performing the capacity analysis for the purpose of capacity management.

- The fast changes in technology, continual changes in the production processes, and heavy dependence of many manufacturing processes on technology, use of crisp estimates of time standards is less than satisfactory and much less reliable. Also, the figures in the MS are forecast by the marketing and finance departments and thus are not precise. Therefore, it is reasonable to assume that everyday problems in many organizations are becoming more and more complex and relevant information available is becoming more and more imprecise, vague, and some-times incomplete. Hence, it is natural to deal with such problems through fuzzy system.
- Validating the master schedule (MS) with respect to capacity is one of the very important step in material resource planning (MRP), because an overstated MS can cause a variety of problems in an organization (for example, because of an overstated MS, an organization can end up with increased raw materials and work–in–process inventories due to the reason that more materials are purchased and released to the shop than are completed and shipped out).

Most of the data available in capacity planning problems in industry is in the form of fuzzy estimates. Under such circumstances, using the fuzzy system approach is more realistic and yields a relatively "more satisfactory" solution.

CHAPTER 3

CAPACITY ANALYSIS UNDER FUZZY ENVIRONMENT USING BILL OF LABOR APPROACH

In this chapter, we consider a problem in which the number of master schedule items, the number of products in the bill of labor, and the number of work centers are finite. We extend the crisp Bill of Labor (BOL) approach to find the capacity required in Work Center i for Period j, assuming that the BOL amount for a given Product k in a given Work Center i is represented, in terms of time units, not by a crisp number but by a triangular fuzzy number, and similarly, the MS amount for the given Product k in a given Period j is represented, in terms of product units, by another triangular fuzzy number. The approach can be extended further when the Bill of Labor amount for a given Product k in a given Work Center i is represented, in terms of time units, by a trapezoidal fuzzy number, and the MS amount for the given Product k in a given Period j is represented, in terms of product units, by another trapezoidal fuzzy number.

3.1 Introduction

According to Fogarty et. al (1991), the bill of labor (BOL) (also known as the bill of capacity or bill of resources) is one of the various important techniques used in the process of RCCP. Rough cut capacity planning is a process that, using various techniques, determines whether the organization has sufficient capacity to carry out a given production plan. The bill of labor approach assumes that all components are built in the same time period as the end item, i.e. lead-time offsets are not considered. BOL is designed to convert the MS from units of end items to be produced into the amount of time required for certain key resources. Using RCCP, Fogarty et. al (1991) deals with the capacity-planning problem in which all of the number of MS items, the number of products in the BOL and the number of work centers are finite. It may be emphasized here that in all of the above references (Chapter 1 and 2), the data and the values of the parameters are assumed to be crisp.

3.2 Bill of Labor Approach for RCCP under Crisp Environment

We assume that k represents the number of products in an MS, j represents the number of periods, and i represents the number of work centers, where $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$.

3.2.1 Assumptions: We assume that

1. each bill of labor is time phased, and
2. all components of an item are manufactured in the same time period as the end item, i.e. lead-time offsets are not considered.

3.2.2 Notations

Let, a_{ik} = BOL amount in Work Center i for Product k (measured in time units),

b_{kj} = MS amount Product k in Period j (measured in product units),

c_{ij} = Capacity required in Work Center i for Period j (measured in time units),

P_k = Product k, $k = 1, 2, \ldots, n$,

WC_i = Work Center i, $i = 1, 2, \ldots, p$,

M_j = Period j, $j = 1, 2, \ldots, m$.

In matrix form we can represent a_{ik}'s, b_{kj}'s and c_{ij}'s as follows:

BOL in time units

	P_1	P_2	...	P_n
WC_1	a_{11}	a_{12}	...	a_{1n}
WC_2	a_{21}	a_{22}	...	a_{2n}
...	...	...	...	...
WC_p	a_{p1}	a_{p2}	...	a_{pn}

MS in product units

	M_1	M_2	...	M_m
P_1	b_{11}	b_{12}	...	b_{1m}
P_2	b_{21}	b_{22}	...	b_{2m}
...	...	...	...	...
P_n	b_{n1}	b_{n2}	...	b_{nm}

3.2.3 General Formulation

Then the formula to compute c_{ij}'s is given as follows:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} (\cdot) b_{kj}, \quad i = 1, 2, \ldots, p \text{ ; and } j = 1, 2, \ldots, m. \tag{1}$$

In (1), if for all values of i, j, and k, a_{ik}'s, b_{kj}'s be crisp numbers, then their multiplication is also a crisp number, and as a result each c_{ij}, the sum of those individual multiplications, is also a crisp number. Thus, in this case, it is easy to compute each capacity value c_{ij}. In view of (1), we obtain the following capacity matrix

Capacity in time units

	M_1	M_2	...	M_m
WC_1	c_{11}	c_{12}	...	c_{1m}
WC_2	c_{21}	c_{22}	...	c_{2m}
...	...	...	...	...
WC_p	c_{p1}	c_{p2}	...	c_{pm}

3.3 Bill of Labor Approach for RCCP under Fuzzy Environment

When at least one of a_{ik}'s, b_{kj}'s is a fuzzy number, then their product is also a fuzzy number, and as a result the corresponding c_{ij} the sum of the individual products, is also a fuzzy number. Thus, in this case computing c_{ij}'s is relatively more involved than the crisp case.

We now assume that in (1), each of a_{ik} and b_{kj} for $k = 1, 2, \dots, n$; $i = 1, 2, \dots, p$; and $j = 1, 2, \dots, m$; is a T.F.N. of the type

$$a_{ik} = (a_{ik1}, a_{ik2}, a_{ik3}) \text{ and } b_{kj} = (b_{kj1}, b_{kj2}, b_{kj3}) \tag{2}$$

The values of $a_{ik} = (a_{ik1}, a_{ik2}, a_{ik3})$ and $b_{kj} = (b_{kj1}, b_{kj2}, b_{kj3})$ can be obtained by using the experts who share the same information but different opinions on certain problems.

In (1) if we set $a_{ik} = (a_{ik1}, a_{ik2}, a_{ik3})$ and $b_{kj} = (b_{kj1}, b_{kj2}, b_{kj3})$, then

(i) each term in the right hand side expression is obtained by multiplying two T.F.N.'s,

(ii) each term obtained in (i) is itself a fuzzy number but not necessarily a T.F.N., and

(iii) c_{ij} is obtained by adding the results of each individual multiplication obtained in (i).

Thus, we have

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \, (.) \, b_{kj} \, , \qquad i = 1, 2, \dots, p \, ; \text{and } j = 1, 2, \dots, m \, . \tag{3}$$

In view of (ii), it is important to point out here that in (3) though each of the values for c_{ij} is a fuzzy number yet it is not necessarily a T.F.N. (Kaufmann and Gupta (1985, 1988)). Thus, c_{ij} is a fuzzy number given by

$$c_{ij} = (c_{ij1}, c_{ij2}, c_{ij3}) \tag{4}$$

for, $i = 1, 2, \dots, p$; and $j = 1, 2, \dots, m$. Each fuzzy number c_{ij} in (4) and its membership function are determined on the lines of Kaufmann and Gupta (1985, 1988) by using the interval of confidence for a_{ik} and b_{kj} respectively, at α–level (see Definition 1.9.2).

$$[a_{11}^{\alpha}, a_{12}^{\alpha}] = [(a_{ik2} - a_{ik1})\alpha + a_{ik1}, -(a_{ik3} - a_{ik2})\alpha + a_{ik3}] \quad \forall \alpha \in [0, 1]. \tag{5}$$

$$[b_{11}^{\alpha}, b_{12}^{\alpha}] = [(b_{kj2} - b_{kj1})\alpha + b_{kj1}, -(b_{kj3} - b_{kj2})\alpha + b_{kj3}] \quad \forall \alpha \in [0, 1]. \tag{6}$$

for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$. We multiply the two intervals in (5) and (6) and add the individual multiplications as suggested in Kaufmann and Gupta (1985, 1988).

This yields the following interval of confidence with quadratic functions in α .

Thus, for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$., we get

$$[\sum_{k=1}^{n} [(a_{ik2} - a_{ik1})(b_{kj2} - b_{kj1})]\alpha^2 + \sum_{k=1}^{n} [a_{ik1}(b_{kj2} - b_{kj1}) + (a_{ik2} - a_{ik1}) b_{kj1}]\alpha + \sum_{k=1}^{n} a_{ik1} b_{kj1},$$

$$\sum_{k=1}^{n} [(a_{ik3} - a_{ik2})(b_{kj3} - b_{kj2})]\alpha^2 - \sum_{k=1}^{n} [a_{ik3}(b_{kj3} - b_{kj2}) + (a_{ik3} - a_{ik2}) b_{kj3}]\alpha + \sum_{k=1}^{n} a_{ik3} b_{kj3}].$$

In this interval of confidence

1. Setting $\alpha = 0$, we get the end points c_{ij1} and c_{ij3}

$$c_{ij1} = \sum_{k=1}^{n} a_{ik1} b_{kj1} \text{ and } c_{ij3} = \sum_{k=1}^{n} a_{ik3} b_{kj3}$$

   of the fuzzy number c_{ij} .

2. Setting $\alpha = 1$ gives the following middle point c_{ij2} of c_{ij}

$$c_{ij2} = \sum_{k=1}^{n} [(a_{ik2} - a_{ik1})(b_{kj2} - b_{kj1})] + \sum_{k=1}^{n} [a_{ik1}(b_{kj2} - b_{kj1}) + (a_{ik2} - a_{ik1})\, b_{kj1}]$$

$$+ \sum_{k=1}^{n} a_{ik1}\, b_{kj1}$$

Also,

$$c_{ij2} = \sum_{k=1}^{n} [(a_{ik3} - a_{ik2})(b_{kj3} - b_{kj2})] - \sum_{k=1}^{n} [a_{ik3}(b_{kj3} - b_{kj2}) + (a_{ik3} - a_{ik2})\, b_{kj3}]$$

$$+ \sum_{k=1}^{n} a_{ik3}\, b_{kj3}$$

The membership function is obtained from the interval of confidence by setting separately, each of the quadratic function, equal to x and solving each of those two quadratic equations for α. Thus,

$$\sum_{k=1}^{n} [(a_{ik2} - a_{ik1})(b_{kj2} - b_{kj1})]\alpha^2 + \sum_{k=1}^{n} [a_{ik1}(b_{kj2} - b_{kj1}) + (a_{ik2} - a_{ik1})\, b_{kj1}]\alpha$$

$$+ \sum_{k=1}^{n} a_{ik1}\, b_{kj1} = x \qquad (7)$$

Solving (7) for positive root α gives membership function between c_{ij1} and c_{ij2} satisfying $0 \leq \alpha \leq 1$.

Next, we set

$$\sum_{k=1}^{n} [(a_{ik3} - a_{ik2})(b_{kj3} - b_{kj2})]\alpha^2 - \sum_{k=1}^{n} [a_{ik3}(b_{kj3} - b_{kj2}) + (a_{ik3} - a_{ik2})\, b_{kj3}]\alpha$$

$$+ \sum_{k=1}^{n} a_{ik3}\, b_{kj3} = x \qquad (8)$$

Solving (8) for positive root α gives membership function between c_{ij2} and c_{ij3} satisfying $0 \leq \alpha \leq 1$.

3.3.1 Numerical Example under Fuzzy Environments

In this section we consider a numerical example in which we have five products P_1, P_2, P_3, P_4 and P_5, four work centers WC_1, WC_2, WC_3 and WC_4, and six time periods M_1, M_2, M_3, M_4, M_5 and M_6. It is assumed that the bill of labor (BOL) is measured in terms of hours, master schedule (MS) is measured in terms of number of units of products, and the T.F.N.'s. for the BOL and the MS are as follows:

Table 3.1 Bill of Labor in time units

	P_1	P_2	P_3	P_4	P_5
WC_1	(.25, .30, .31)	(.12, .14, .15)	(.25, .26, .28)	(.24, .28, .30)	(.12, .14, .16)
WC_2	(.24, .27, .29)	(.18, .20, .23)	(.36, .40, .41)	(.25, .26, .30)	(.30, .32, .35)
WC_3	(.15, .17, .19)	(.30, .32, .33)	(.40, .43, .44)	(.50, .52, .55)	(.36, .40, .41)
WC_4	(.12, .16, .18)	(.35, .37, .40)	(.15, .17, .20)	(.50, .55, .60)	(.40, .43, .46)

Table 3.2 Master Schedule in product units (Month 1 to Month 3)

	M_1	M_2	M_3
P_1	(50, 60, 65)	(42, 46, 55)	(40, 45, 55)
P_2	(60, 72, 75)	(75, 78, 80)	(90, 92, 95)
P_3	(75, 80, 90)	(80, 90, 100)	(115, 120, 128)
P_4	(90, 101, 105)	(100, 105, 108)	(107, 109, 110)
P_5	(70, 80, 85)	(80, 90, 95)	(60, 70, 80)

Table 3.3 Master Schedule in product units (Month 4 to Month 6)

	M_4	M_5	M_6
P_1	(100, 105, 110)	(70, 75, 78)	(80, 90, 95)
P_2	(32, 40, 42)	(80, 90, 92)	(65, 75, 80)
P_3	(75, 78, 80)	(110, 114, 120)	(80, 95, 100)

P_4	(50, 52, 64)	(72, 78, 84)	(70, 75, 85)
P_5	(100, 105, 110)	(55, 65, 70)	(90, 95, 110)

Using (5), (4) and (3), we obtain the capacities, measured in terms of hours, as follows:

Table 3.4 Fuzzy Capacities in hours (Month 1 to Month 3)

	M_1	M_2	M_3
WC_1	(68.45, 88.36,101.7)	(73.10, 90.12, 104.65)	(82.43, 97.90, 112.94)
WC_2	(93.3, 114.46, 134.25)	(101.38, 120.12, 141)	(111.95, 129.29, 151.28)
WC_3	(125.7, 152.16, 169.3)	(139.6, 162.08, 179.2)	(154.1, 173.37, 191.42)
WC_4	(111.25, 139.79, 161.8)	(125.29, 147.97, 170.4)	(131.05, 151.69, 176.3)

Table 3.5 Fuzzy Capacities in hours (Month 4 to Month 6)

	M_4	M_5	M_6
WC_1	(71.59, 86.64, 99.6)	(78.48, 95.68, 107.98)	(75.4, 96.5, 112.55)
WC_2	(99.26, 114.67, 132.06)	(105.3, 124.93, 142.68)	(104.2, 127.2, 150.95)
WC_3	(115.6, 133.23, 150.26)	(134.3, 157.13, 172.88)	(130.9, 157.15, 180.3)
WC_4	(99.45, 118.61, 141.6)	(110.9, 135.53, 157.44)	(115.35, 140.4, 170.7)

These capacities computed above are compared with the available capacities to permit planning for expansion of these resources in a timely fashion by revealing those resources that may be short in capacity.

We now demonstrate the calculation for c_{11} using the approach suggested above and in Kaufmann and Gupta (1985, 1988). The rest of the c_{ij}'s are calculated on similar lines. However, for all c_{ij}'s, we provide the numerical results in the form of the intervals of confidence for various values of α in Appendix 1, and the bar graph of the average fuzzy capacities and the graphs of the membership functions in Appendix 2.

3.3.2 Intervals of Confidence

Using Definition (1.9.6) and (1.10.2), we have the intervals of confidence as follows:

3.3.2.1 Intervals of Confidence of Bill of Labor

$a_{11}^{\alpha} = [.05\alpha + .25, .31 - .01\alpha]$ $\quad a_{12}^{\alpha} = [.02\alpha + .12, .15 - .01\alpha]$

$a_{13}^{\alpha} = [.01\alpha + .25, .28 - .02\alpha]$ $\quad a_{14}^{\alpha} = [.04\alpha + .24, .30 - .02\alpha]$

$a_{15}^{\alpha} = [.02\alpha + .12, .16 - .02\alpha]$ $\quad a_{21}^{\alpha} = [.03\alpha + .24, .29 - .02\alpha]$

$a_{22}^{\alpha} = [.02\alpha + .18, .23 - .03\alpha]$ $\quad a_{23}^{\alpha} = [.04\alpha + .36, .41 - .01\alpha]$

$a_{24}^{\alpha} = [.01\alpha + .25, .30 - .04\alpha]$ $\quad a_{25}^{\alpha} = [.02\alpha + .30, .35 - .03\alpha]$

$a_{31}^{\alpha} = [.02\alpha + .15, .19 - .02\alpha]$ $\quad a_{32}^{\alpha} = [.02\alpha + .30, .33 - .01\alpha]$

$a_{33}^{\alpha} = [.03\alpha + .40, .44 - .01\alpha]$ $\quad a_{34}^{\alpha} = [.02\alpha + .50, .55 - .03\alpha]$

$a_{35}^{\alpha} = [.04\alpha + .36, .41 - .01\alpha]$ $\quad a_{41}^{\alpha} = [.04\alpha + .12, .18 - .02\alpha]$

$a_{42}^{\alpha} = [.02\alpha + .35, .40 - .03\alpha]$ $\quad a_{43}^{\alpha} = [.02\alpha + .15, .20 - .03\alpha]$

$a_{44}^{\alpha} = [.05\alpha + .50, .60 - .05\alpha]$ $\quad a_{45}^{\alpha} = [.03\alpha + .40, .46 - .03\alpha]$

3.3.2.2 Intervals of Confidence of Master Schedule

$b_{11}^{\alpha} = [10\alpha + 50, 65 - 5\alpha]$ $\quad b_{21}^{\alpha} = [12\alpha + 60, 75 - 3\alpha]$

$b_{31}^{\alpha} = [5\alpha + 75, 90 - 10\alpha]$ $\quad b_{41}^{\alpha} = [11\alpha + 90, 105 - 4\alpha]$

$b_{51}^{\alpha} = [10\alpha + 70, 85 - 5\alpha]$ $\quad b_{12}^{\alpha} = [4\alpha + 42, 55 - 9\alpha]$

$b_{22}^{\alpha} = [3\alpha + 75, 80 - 2\alpha]$ $\quad b_{32}^{\alpha} = [10\alpha + 80, 100 - 10\alpha]$

$b_{42}^{\alpha} = [5\alpha + 100, 108 - 3\alpha]$ $\quad b_{52}^{\alpha} = [10\alpha + 80, 95 - 5\alpha]$

$b_{13}^{\alpha} = [5\alpha + 40, 55 - 10\alpha]$ $\quad b_{23}^{\alpha} = [2\alpha + 90, 95 - 3\alpha]$

$b_{33}^{\alpha} = [5\alpha + 115, 128 - 8\alpha]$ $\quad b_{43}^{\alpha} = [2\alpha + 107, 110 - 1\alpha]$

$b_{53}^{\alpha} = [10\alpha + 60, 80 - 10\alpha]$ $\quad b_{14}^{\alpha} = [5\alpha + 100, 110 - 5\alpha]$

$b_{24}^{\alpha} = [8\alpha + 32, 42 - 2\alpha]$ $b_{34}^{\alpha} = [3\alpha + 75, 80 - 2\alpha]$

$b_{44}^{\alpha} = [2\alpha + 50, 64 - 12\alpha]$ $b_{54}^{\alpha} = [5\alpha + 100, 110 - 5\alpha]$

$b_{15}^{\alpha} = [5\alpha + 70, 78 - 3\alpha]$ $b_{25}^{\alpha} = [10\alpha + 80, 92 - 2\alpha]$

$b_{35}^{\alpha} = [4\alpha + 110, 120 - 6\alpha]$ $b_{45}^{\alpha} = [6\alpha + 72, 84 - 6\alpha]$

$b_{55}^{\alpha} = [10\alpha + 55, 70 - 5\alpha]$ $b_{16}^{\alpha} = [10\alpha + 80, 95 - 5\alpha]$

$b_{26}^{\alpha} = [10\alpha + 65, 80 - 5\alpha]$ $b_{36}^{\alpha} = [15\alpha + 80, 100 - 5\alpha]$

$b_{46}^{\alpha} = [5\alpha + 70, 85 - 10\alpha]$ $b_{56}^{\alpha} = [5\alpha + 90, 110 - 15\alpha]$

3.3.3 Calculating the Required Capacity in Work Center 1 for Period 1 (c_{11})

Using (3), we have

$$c_{11}^{(\alpha)} = [\sum_{k=1}^{5} a_{1k}^{(\alpha)} (.)\, b_{k1}^{(\alpha)}]$$

$$= [a_{11}^{\alpha} (.)\, b_{11}^{\alpha} + a_{12}^{\alpha} (.)\, b_{21}^{\alpha} + a_{13}^{\alpha} (.)\, b_{31}^{\alpha} + a_{14}^{\alpha} (.)\, b_{41}^{\alpha} + a_{15}^{\alpha} (.)\, b_{51}^{\alpha}]$$

where (.) is multiplication of two intervals of confidence.

$$c_{11}^{(\alpha)} = [(.05\alpha + .25, .31 - .01\alpha) (.) (10\alpha + 50, 65 - 5\alpha)$$

$$+ (.02\alpha + .12, .15 - .01\alpha) (.) (12\alpha + 60, 75 - 3\alpha)$$

$$+ (.01\alpha + .25, .28 - .02\alpha) (.) (5\alpha + 75, 90 - 10\alpha)$$

$$+ (.04\alpha + .24, .30 - .02\alpha) (.) (11\alpha + 90, 105 - 4\alpha)$$

$$+ (.02\alpha + .12, .16 - .02\alpha) (.) (10\alpha + 70, 85 - 5\alpha)]$$

$$= [(1.43\alpha^2 + 18.48\alpha + 68.45,\ .46\alpha^2 - 13.8\alpha + 101.7)]$$

We now set

$$1.43\alpha^2 + 18.48\alpha + 68.45 = x \quad \text{and} \quad .46\alpha^2 - 13.8\alpha + 101.7 = x$$

This yields,

$$1.43\alpha^2 + 18.48\alpha + (68.45 - x) = 0 \tag{9}$$

and $$.46\alpha^2 - 13.8\alpha + (101.7 - x) = 0 \tag{10}$$

In (9) setting $\alpha = 0$ we get $x = 68.45$

In (10) setting $\alpha = 0$ we get $x = 101.7$

Setting $\alpha = 1$ in either $1.43\alpha^2 + 18.48\alpha + (68.45 - x) = 0$

or $.46\alpha^2 - 13.8\alpha + (101.7 - x) = 0$

yields $x = 88.36$, therefore $c_{11} = (68.45, 88.36, 101.7)$

Now, the membership function is obtained follows.

Solving the quadratic equation $1.43\alpha^2 + 18.48\alpha + (68.45 - x) = 0$

for α we obtain

$$\alpha = \frac{-18.48 + \sqrt{-50.02 + 5.72\,x}}{2.86} \qquad \text{for } 68.45 \le x \le 88.36$$

and solving the quadratic equation $.46\alpha^2 - 13.8\alpha + (101.7 - x) = 0$

For α we obtain

$$\alpha = \frac{13.8 - \sqrt{3.31 + 1.84\,x}}{.92} \qquad \text{for } 88.36 \le x \le 101.7$$

Thus, the membership function for $c_{11} = (68.45, 88.36, 101.7)$ is

$$\mu_{c_{11}}(x) = \begin{cases} 0 & x \le 68.45 \\ \dfrac{-18.48 + \sqrt{-50.02 + 5.72x}}{2.86} & 68.45 \le x \le 88.36 \\ \dfrac{13.8 - \sqrt{3.31 + 1.84x}}{.92} & 88.36 \le x \le 101.7 \\ 0 & x \ge 101.7 \end{cases}$$

Similarly, we obtain the fuzzy capacities for rest of c_{ij}'s.

3.3.4 Results

Table 3.6 Fuzzy Capacities (in terms of labor hours) in Work Center i for Period j

(68.45, 88.36, 101.7) c_{11}	(73.10, 90.12, 104.65) c_{12}	(82.43, 97.90, 112.94) c_{13}
(71.59, 86.64, 99.6) c_{14}	(78.48, 95.68, 107.98) c_{15}	(75.4, 96.5, 112.55) c_{16}
(93.3, 114.46, 134.25) c_{21}	(101.38, 120.12, 141) c_{22}	(111.95, 129.29, 151.28) c_{23}
(99.26, 114.67, 132.06) c_{24}	(105.3, 124.93, 142.68) c_{25}	(104.2, 127.2, 150.95) c_{26}
(125.7, 152.16, 169.3) c_{31}	(139.6, 162.08, 179.2) c_{32}	(154.1, 173.37, 191.42) c_{33}
(115.6, 133.23, 150.26) c_{34}	(134.3, 157.13, 172.88) c_{35}	(130.9, 157.15, 180.3) c_{36}
(111.25, 139.79, 161.8) c_{41}	(125.29, 147.97, 170.4) c_{42}	(131.05, 151.69, 176.3) c_{43}
(99.45, 118.61, 141.6) c_{44}	(110.9, 135.53, 157.44) c_{45}	(115.35, 140.4, 170.7) c_{46}

with the membership functions given below:

Membership function for $c_{12} = (73.10, 90.12, 104.65)$,

$$\mu_{c_{12}}(x) = \begin{cases} 0 & x \le 73.10 \\ \dfrac{-16.26 + \sqrt{42.16 + 3.04x}}{1.52} & 73.10 \le x \le 90.12 \\ \dfrac{15 - \sqrt{28.25 + 1.88x}}{.94} & 90.12 \le x \le 104.65 \\ 0 & x \ge 104.65 \end{cases}$$

Membership function for $c_{13} = (82.43, 97.90, 112.94)$,

$$\mu_{c13}(x) = \begin{cases} 0 & x \le 82.43 \\ \dfrac{-14.85 + \sqrt{16.09 + 2.48x}}{1.24} & 82.43 \le x \le 97.90 \\ \dfrac{15.55 - \sqrt{11.40 + 2.04x}}{1.02} & 97.90 \le x \le 112.94 \\ 0 & x \ge 112.94 \end{cases}$$

Membership function for $c_{14} = (71.59, 86.64, 99.6)$,

$$\mu_{c14}(x) = \begin{cases} 0 & x \le 71.59 \\ \dfrac{-14.43 + \sqrt{30.68 + 2.48x}}{1.24} & 71.59 \le x \le 86.64 \\ \dfrac{13.41 - \sqrt{.54 + 1.8x}}{.9} & 86.64 \le x \le 99.6 \\ 0 & x \ge 99.6 \end{cases}$$

Membership function for $c_{15} = (78.48, 95.68, 107.98)$,

$$\mu_{c15}(x) = \begin{cases} 0 & x \le 78.48 \\ \dfrac{-16.27 + \sqrt{-27.23 + 3.72x}}{1.86} & 78.48 \le x \le 95.68 \\ \dfrac{12.69 - \sqrt{-7.41 + 1.56x}}{.78} & 95.68 \le x \le 107.98 \\ 0 & x \ge 107.98 \end{cases}$$

Membership function for $c_{16} = (75.4, 96.5, 112.55)$,

$$\mu_{c16}(x) = \begin{cases} 0 & x \le 75.4 \\ \dfrac{-19.95 + \sqrt{51.16 + 4.6x}}{2.30} & 75.4 \le x \le 96.5 \\ \dfrac{16.75 - \sqrt{-34.58 + 2.8x}}{1.4} & 96.5 \le x \le 112.55 \\ 0 & x \ge 112.55 \end{cases}$$

Membership function for $c_{21} = (93.3, 114.46, 134.25)$,

$$\mu_{c_{21}}(x) = \begin{cases} 0 & x \le 93.3 \\ \dfrac{-20.11 + \sqrt{12.55 + 4.2x}}{2.10} & 93.3 \le x \le 114.46 \\ \dfrac{20.39 - \sqrt{93.55 + 2.4x}}{1.2} & 114.46 \le x \le 134.25 \\ 0 & x \ge 134.25 \end{cases}$$

Membership function for $c_{22} = (101.38, 120.12, 141)$,

$$\mu_{c_{22}}(x) = \begin{cases} 0 & x \le 101.38 \\ \dfrac{-17.91 + \sqrt{-15.82 + 3.32x}}{2.86} & 101.38 \le x \le 120.12 \\ \dfrac{21.49 - \sqrt{117.78 + 2.44x}}{1.22} & 120.12 \le x \le 141 \\ 0 & x \ge 141 \end{cases}$$

Membership function for $c_{23} = (111.95, 129.29, 151.28)$,

$$\mu_{c_{23}}(x) = \begin{cases} 0 & x \le 111.95 \\ \dfrac{-16.73 + \sqrt{6.73 + 2.44x}}{1.22} & 111.95 \le x \le 129.29 \\ \dfrac{22.7 - \sqrt{85.65 + 2.84x}}{1.42} & 129.29 \le x \le 151.28 \\ 0 & x \ge 151.28 \end{cases}$$

Membership function for $c_{24} = (99.26, 114.67, 132.06)$,

$$\mu_{c_{24}}(x) = \begin{cases} 0 & x \le 99.26 \\ \dfrac{-14.86 + \sqrt{2.44 + 2.2x}}{2.30} & 99.26 \le x \le 114.67 \\ \dfrac{18.2 - \sqrt{-96.63 + 3.24x}}{1.62} & 114.67 \le x \le 132.06 \\ 0 & x \ge 132.06 \end{cases}$$

Membership function for $c_{25} = (105.3, 124.93, 142.68)$,

$$\mu_{c_{25}}(x) = \begin{cases} 0 & x \le 105.3 \\ \dfrac{-18.86 + \sqrt{31.37 + 3.08x}}{1.54} & 105.3 \le x \le 124.93 \\ \dfrac{18.32 - \sqrt{10.31 + 2.28x}}{1.14} & 124.93 \le x \le 142.68 \\ 0 & x \ge 142.68 \end{cases}$$

Membership function for $c_{26} = (104.2, 127.2, 150.95)$,

$$\mu_{c_{26}}(x) = \begin{cases} 0 & x \le 104.2 \\ \dfrac{-21.75 + \sqrt{-47.94 + 5x}}{2.5} & 104.2 \le x \le 127.2 \\ \dfrac{24.9 - \sqrt{-74.36 + 4.6x}}{2.3} & 127.2 \le x \le 150.95 \\ 0 & x \ge 150.95 \end{cases}$$

Membership function for $c_{31} = (125.7, 152.16, 169.3)$,

$$\mu_{c_{31}}(x) = \begin{cases} 0 & x \le 125.7 \\ \dfrac{-25.25 + \sqrt{29.17 + 4.84x}}{2.42} & 125.7 \le x \le 152.16 \\ \dfrac{17.54 - \sqrt{36.77 + 1.6x}}{.8} & 152.16 \le x \le 169.3 \\ 0 & x \ge 169.3 \end{cases}$$

Membership function for $c_{32} = (139.6, 162.08, 179.2)$,

$$\mu_{c_{32}}(x) = \begin{cases} 0 & x \le 139.6 \\ \dfrac{-21.54 + \sqrt{-60.93 + 3.76x}}{1.88} & 139.6 \le x \le 162.08 \\ \dfrac{17.56 - \sqrt{-7.03 + 1.76x}}{.88} & 162.08 \le x \le 179.2 \\ 0 & x \ge 179.2 \end{cases}$$

Membership function for $c_{33} = (154.1, 173.37, 191.42)$,

$$\mu_{c_{33}}(x) = \begin{cases} 0 & x \le 154.1 \\ \dfrac{-18.54 + \sqrt{-106.24 + 2.92x}}{1.46} & 154.1 \le x \le 173.37 \\ \dfrac{18.49 - \sqrt{4.98 + 1.76x}}{.88} & 173.37 \le x \le 191.42 \\ 0 & x \ge 191.42 \end{cases}$$

Membership function for $c_{34} = (115.6, 133.23, 150.26)$,

$$\mu_{c_{34}}(x) = \begin{cases} 0 & x \le 115.6 \\ \dfrac{-17.04 + \sqrt{17.54 + 2.36x}}{1.18} & 115.6 \le x \le 133.23 \\ \dfrac{17.58 - \sqrt{-21.52 + 2.2x}}{1.1} & 133.23 \le x \le 150.26 \\ 0 & x \ge 150.26 \end{cases}$$

Membership function for $c_{35} = (134.3, 157.13, 172.88)$,

$$\mu_{c_{35}}(x) = \begin{cases} 0 & x \le 134.3 \\ \dfrac{-21.89 + \sqrt{-25.80 + 3.76x}}{1.88} & 134.3 \le x \le 157.13 \\ \dfrac{16.12 - \sqrt{3.99 + 1.48x}}{.74} & 157.13 \le x \le 172.88 \\ 0 & x \ge 172.88 \end{cases}$$

Membership function for $c_{36} = (130.9, 157.15, 180.3)$,

$$\mu_{c_{36}}(x) = \begin{cases} 0 & x \le 130.9 \\ \dfrac{-25.1 + \sqrt{27.87 + 4.6x}}{2.30} & 130.9 \le x \le 157.15 \\ \dfrac{23.8 - \sqrt{97.66 + 2.6x}}{1.3} & 157.15 \le x \le 180.3 \\ 0 & x \ge 180.3 \end{cases}$$

Membership function for $c_{41} = (111.25, 139.79, 161.8)$,

$$\mu_{c_{41}}(x) = \begin{cases} 0 & x \le 111.25 \\ \dfrac{-26.95 + \sqrt{18.75 + 6.36x}}{3.18} & 111.25 \le x \le 139.79 \\ \dfrac{22.85 - \sqrt{-21.52 + 3.36x}}{1.68} & 139.79 \le x \le 161.8 \\ 0 & x \ge 161.8 \end{cases}$$

Membership function for $c_{42} = (125.29, 147.97, 170.4)$,

$$\mu_{c_{42}}(x) = \begin{cases} 0 & x \le 125.29 \\ \dfrac{-21.71 + \sqrt{-14.80 + 3.88x}}{1.94} & 125.29 \le x \le 147.97 \\ \dfrac{23.27 - \sqrt{-31.05 + 3.36x}}{1.68} & 147.97 \le x \le 170.4 \\ 0 & x \ge 170.04 \end{cases}$$

Membership function for $c_{43} = (131.05, 151.69, 176.3)$,

$$\mu_{c_{43}}(x) = \begin{cases} 0 & x \le 131.05 \\ \dfrac{-19.9 + \sqrt{8.10 + 2.96x}}{1.48} & 131.05 \le x \le 151.69 \\ \dfrac{25.49 - \sqrt{29.16 + 3.52x}}{1.76} & 151.69 \le x \le 176.3 \\ 0 & x \ge 176.3 \end{cases}$$

Membership function for $c_{44} = (99.45, 118.61, 141.6)$,

$$\mu_{c_{44}}(x) = \begin{cases} 0 & x \le 99.45 \\ \dfrac{-18.49 + \sqrt{75.35 + 2.68x}}{1.34} & 99.45 \le x \le 118.61 \\ \dfrac{23.96 - \sqrt{24.67 + 3.88x}}{1.94} & 118.61 \le x \le 141.6 \\ 0 & x \ge 141.6 \end{cases}$$

Membership function for $c_{45} = (110.9, 135.53, 157.44)$,

$$\mu_{c_{45}}(x) = \begin{cases} 0 & x \le 110.9 \\ \dfrac{-23.55 + \sqrt{75.51 + 4.32x}}{2.16} & 110.9 \le x \le 135.53 \\ \dfrac{22.66 - \sqrt{41.15 + 3x}}{1.5} & 135.53 \le x \le 157.44 \\ 0 & x \ge 157.44 \end{cases}$$

Membership function for $c_{46} = (115.35, 140.4, 170.7)$.

$$\mu_{c_{46}}(x) = \begin{cases} 0 & x \le 115.35 \\ \dfrac{-23.75 + \sqrt{-35.75 + 5.2x}}{2.6} & 115.35 \le x \le 140.4 \\ \dfrac{31.65 - \sqrt{79.94 + 5.4x}}{2.7} & 140.4 \le x \le 170.7 \\ 0 & x \ge 170.7 \end{cases}$$

3.3.5 Interpretation of the Results

From Table 3.6, we calculate the average fuzzy required capacity in terms of hours for each Work Center for the six Periods. The average required capacity in Work Center 1, Work Center 2, Work Center 3 and Work Center 4 are (75, 93, 107), (103, 122, 142), (133, 156, 174), and (116, 139, 163) respectively for 6 Periods, which are required to satisfy the requirements of the master production schedule (MPS). The bill of labor approach is superior to capacity planning because it better predicts the actual change in hours required from week to week.

In Appendix 1 and 2, we calculate the various values of x, when α lies between 0 and 1 and plot the membership function graphs for different values of x. For example, the required capacity in Work Center 1 for Period 1 is (68.45, 88.36, 101.70). As we can see in graph c_{11}, when $68.45 \leq x \leq 88.36$ the membership function increases monotonically to the left and goes to its maximum value of 1 at an interior point $x = 88.36$, and when $88.36 \leq x \leq 101.7$ the membership function decreases monotonically to the right and goes to 0 at a right end point x = 101.70, starting from 1 at x = 88.36. Similarly we can see the membership function graphs for rest of c_{ij}'s along with the values of x. Most of the data available in capacity planning problems in the industry is in the form of fuzzy estimates. Fuzzy set theory permits the partial belonging of an element to a fuzzy set characterized by a membership function that takes values in the interval [0, 1]. Thus, fuzzy approach yields a relatively "more satisfactory and flexible solution" within a pre-specified intervals, whereas a conventional crisp set theory only permits an element either to belong (membership grade 1) or not to belong (membership grade 0) to the set. Another advantage of the present approach under fuzzy environments is that, it also provided us a range of labor hours showing the lower and upper bounds of the possible solution.

According to Cox (1995), fuzzy models are much more compact, encode a higher degree of knowledge, usually execute faster, are less prone to error, work well with missing decision points, and can be maintained much more easily. This means less experienced knowledge engineers can build more complex models.

CHAPTER 4

CAPACITY ANALYSIS UNDER FUZZY ENVIRONMENT USING RESOURCE PROFILE APPROACH

In this chapter, we consider a problem in which the number of master schedule items, the number of products in the resource profile, and the number of work centers are finite. We extend the crisp resource profile approach to find the capacity required in Work Center i for Period j, assuming that the resource profile amount for each Product k in a given Work Center i due in Period $t - j$ is represented, in terms of time units, not by a crisp number but by a triangular fuzzy number, and similarly, the master schedule amount for each Product k in a given Period t is represented, in terms of product units, by another triangular fuzzy number. The approach can be extended further when the resource profile amount for each Product k in a given Work Center i due in Period $t - j$ is represented, in terms of time units, by a trapezoidal fuzzy number, and the master schedule amount for each Product k in a given Period t is represented, in terms of product units, by another trapezoidal fuzzy number.

4.1 Introduction

Neither the bill of labor approach nor the capacity using overall factors considers lead-time offset. According to Fogarty et. al (1991), it is not uncommon to find products, whose manufacturing lead-time runs over several months. For parts having lengthy lead-times, the resource profile approach might be very useful because the bill of labor approach assumes that all components are built in the same time period as the end item. The resource profile approach relaxes the Assumptions 3.2.1 by including the lead-time dimension in it. A resource profile is different from the bill of labor approach in the sense that the time at each department or work center is now associated with a specific time period, reflecting lead-time for the corresponding part. To create a resource profile approach, the lead-time must be converted to periods prior to the period in which the order is promised.

4.2 Resource Profile Approach for RCCP under Crisp Environment

We assume that k represents the number of products in an MS, j represents the number of periods, t represents the time to due date, and i represents the number of work centers, where $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, m$; and $t = j, j + 1, \ldots, m$.

4.2.1 Notations

Let,

$a_{ik(t-j)}$ = Resource profile amount in Work Center i for Product k due in Period $t - j$ (measured in time units), $j = 1, 2, \ldots, m$; and $t = j, j + 1, \ldots, m$.

b_{kt} = MS amount Product k in Period t (measured in product units),

c_{ij} = Capacity required in Work Center i for Period j (measured in time units),

P_k = Product k, $k = 1, 2, \ldots, n$,

WC_i = Work Center i, $i = 1, 2, \ldots, p$,

M_j = Period j, $j = 1, 2, \ldots, m$.

In matrix form we can represent $a_{ik(t-j)}$'s, b_{kt}'s and c_{ij}'s as follows:

RP in Time Units

	m − 1	. . .	1	0
P_1	$a_{i1(m-1)}$	. . .	a_{i11}	a_{i10}
P_2	$a_{i2(m-1)}$	. . .	a_{i21}	a_{i20}
. . .	. . .	. . .	. . .	. . .
P_n	$a_{in(m-1)}$	. . .	a_{in1}	a_{in0}

MS in Product Units

	M_1	M_2	. . .	M_m
P_1	b_{11}	b_{12}	. . .	b_{1m}
P_2	b_{21}	b_{22}	. . .	b_{2m}
. . .	. . .	. . .	. . .	. . .
P_n	b_{n1}	b_{n2}	. . .	b_{nm}

4.2.2 General Formulation

Then the formula to compute c_{ij}'s is given as follows:

$$c_{ij} = \sum_{k=1}^{n} \sum_{t=j}^{m} a_{ik(t-j)} \, (.) \, b_{kt} \,, \qquad i = 1, 2, \ldots, p \,; \text{and } j = 1, 2, \ldots, m \,. \tag{1}$$

In (1), if for all values of i, j, t, and k, $a_{ik(t-j)}$'s, b_{kt}'s be crisp numbers, then their multiplication is also a crisp number, and as a result each c_{ij}, the sum of those individual multiplications, is also a crisp number. Thus, in this case, it is easy to compute each capacity value c_{ij}.

In view of (1), we obtain the following capacity matrix

Capacity in Time Units

	M_1	M_2	. . .	M_m
WC_1	c_{11}	c_{12}	. . .	c_{1m}
WC_2	c_{21}	c_{22}	. . .	c_{2m}
. . .	. . .	. . .	. . .	. . .
WC_p	c_{p1}	c_{p2}	. . .	c_{pm}

4.3 Resource Profile Approach for RCCP under Fuzzy Environment

When at least one of $a_{ik(t-j)}$'s, b_{kt}'s is a fuzzy number, then their product is also a fuzzy number, and as a result the corresponding c_{ij} the sum of the individual products, is also a fuzzy number. Thus, in this case computing c_{ij}'s is relatively more involved than the crisp case.

We now assume that in (1), each of $a_{ik(t-j)}$ and b_{kt} for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, m$; and $t = j, j+1, \ldots, m$; is a T.F.N. of the type

$$a_{ik(t-j)} = (a^{(1)}_{ik(t-j)}, a^{(2)}_{ik(t-j)}, a^{(3)}_{ik(t-j)}) \text{ and } b_{kt} = (b_{kt1}, b_{kt2}, b_{kt3}) \quad (2)$$

As already suggested in Chapter 3, the values of $a_{ik(t-j)} = (a^{(1)}_{ik(t-j)}, a^{(2)}_{ik(t-j)}, a^{(3)}_{ik(t-j)})$ and $b_{kt} = (b_{kt1}, b_{kt2}, b_{kt3})$ also can be obtained by using the experts who share the same information but different opinions on certain problems.

In (1) if we set $a_{ik(t-j)} = (a^{(1)}_{ik(t-j)}, a^{(2)}_{ik(t-j)}, a^{(3)}_{ik(t-j)})$ and $b_{kt} = (b_{kt1}, b_{kt2}, b_{kt3})$, then

(i) each term in the right hand side expression is obtained by multiplying two T.F.N.'s,

(ii) each term obtained in (i) is itself a fuzzy number but not necessarily a T.F.N., and

(iii) c_{ij} is obtained by adding the results of each individual multiplication obtained in (i).

Thus, we have

$$c_{ij} = \sum_{k=1}^{n} \sum_{t=j}^{m} a_{ik(t-j)} (.) b_{kt}, \quad i = 1, 2, \ldots, p. \quad (3)$$

In view of (ii), it is important to point out here that in (3) though each of the values for c_{ij} is a fuzzy number yet it is not necessarily a T.F.N. (Kaufmann and Gupta (1985, 1988)). Thus, c_{ij} is a fuzzy number given by

$$c_{ij} = (c_{ij1}, c_{ij2}, c_{ij3}) \quad (4)$$

for $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$. Each fuzzy number c_{ij} in (4) and its membership function are determined on the lines of Kaufmann and Gupta (1985, 1988) by using the interval of confidence for $a_{ik(t-j)}$ and b_{kt} respectively, at α–level (see Def. 1.9.2).

$$[a_{11}^{\alpha}, a_{12}^{\alpha}] = [(a^{(2)}_{ik(t-j)} - a^{(1)}_{ik(t-j)})\alpha + a^{(1)}_{ik(t-j)}, -(a^{(3)}_{ik(t-j)} - a^{(2)}_{ik(t-j)})\alpha + a^{(3)}_{ik(t-j)}] \quad \forall \alpha \in [0, 1]. \tag{5}$$

$$[b_{11}^{\alpha}, b_{12}^{\alpha}] = [(b_{kt2} - b_{kt1})\alpha + b_{kt1}, -(b_{kt3} - b_{kt2})\alpha + b_{kt3}] \quad \forall \alpha \in [0, 1]. \tag{6}$$

for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, m$; and $t = j, j+1, \ldots, m$. We multiply the two intervals in (5) and (6) and add the individual multiplications as suggested in Kaufmann and Gupta (1985, 1988).

This yields the following interval of confidence with quadratic functions in α .

Thus, for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, m$; and $t = j, j+1, \ldots, m$., we get

$$[\sum_{k=1}^{n}\sum_{t=j}^{m} [(a^{(2)}_{ik(t-j)} - a^{(1)}_{ik(t-j)})(b_{kt2} - b_{kt1})]\alpha^2 + \sum_{k=1}^{n}\sum_{t=j}^{m} [a^{(1)}_{ik(t-j)}(b_{kt2} - b_{kt1})$$

$$+ (a^{(2)}_{ik(t-j)} - a^{(1)}_{ik(t-j)})\, b_{kt1}]\alpha + \sum_{k=1}^{n}\sum_{t=j}^{m} a^{(1)}_{ik(t-j)}\, b_{kt1},$$

$$\sum_{k=1}^{n}\sum_{t=j}^{m} [(a^{(3)}_{ik(t-j)} - a^{(2)}_{ik(t-j)})(b_{kt3} - b_{kt2})]\alpha^2 - \sum_{k=1}^{n}\sum_{t=j}^{m} [a^{(3)}_{ik(t-j)}(b_{kt3} - b_{kt2})$$

$$+ (a^{(3)}_{ik(t-j)} - a^{(2)}_{ik(t-j)})\, b_{kt3}]\alpha + \sum_{k=1}^{n}\sum_{t=j}^{m} a^{(3)}_{ik(t-j)}\, b_{kt3}].$$

In this interval of confidence

1. Setting $\alpha = 0$, we get the end points c_{ij1} and c_{ij3}

$$c_{ij1} = \sum_{k=1}^{n}\sum_{t=j}^{m} a^{(1)}_{ik(t-j)}\, b_{kt1} \quad \text{and} \quad c_{ij3} = \sum_{k=1}^{n}\sum_{t=j}^{m} a^{(3)}_{ik(t-j)}\, b_{kt3}$$

of the fuzzy number c_{ij} .

2. Setting $\alpha = 1$ gives the following middle point c_{ij2} of c_{ij}

$$c_{ij2} = \sum_{k=1}^{n} \sum_{t=j}^{m} [(a^{(2)}_{ik(t-j)} - a^{(1)}_{ik(t-j)}) (b_{kt2} - b_{kt1})] + \sum_{k=1}^{n} \sum_{t=j}^{m} [a^{(1)}_{ik(t-j)} (b_{kt2} - b_{kt1})$$

$$+ (a^{(2)}_{ik(t-j)} - a^{(1)}_{ik(t-j)}) b_{kt1}] + \sum_{k=1}^{n} \sum_{t=j}^{m} a^{(1)}_{ik(t-j)} b_{kt1}$$

Also,

$$c_{ij2} = \sum_{k=1}^{n} \sum_{t=j}^{m} [(a^{(3)}_{ik(t-j)} - a^{(2)}_{ik(t-j)}) (b_{kt3} - b_{kt2})] - \sum_{k=1}^{n} \sum_{t=j}^{m} [a^{(3)}_{ik(t-j)} (b_{kt3} - b_{kt2})$$

$$+ (a^{(3)}_{ik(t-j)} - a^{(2)}_{ik(t-j)}) b_{kt3}] + \sum_{k=1}^{n} \sum_{t=j}^{m} a^{(3)}_{ik(t-j)} b_{kt3}$$

The membership function is obtained from the interval of confidence by setting separately, each of the quadratic function, equal to x and solving each of those two quadratic equations for α. Thus,

$$\sum_{k=1}^{n} \sum_{t=j}^{m} [(a^{(2)}_{ik(t-j)} - a^{(1)}_{ik(t-j)}) (b_{kt2} - b_{kt1})]\alpha^2 + \sum_{k=1}^{n} \sum_{t=j}^{m} [a^{(1)}_{ik(t-j)} (b_{kt2} - b_{kt1})$$

$$+ (a^{(2)}_{ik(t-j)} - a^{(1)}_{ik(t-j)}) b_{kt1}]\alpha + \sum_{k=1}^{n} \sum_{t=j}^{m} a^{(1)}_{ik(t-j)} b_{kt1} = x \qquad (7)$$

Solving (7) for positive root α gives membership function between c_{ij1} and c_{ij2} satisfying $0 \le \alpha \le 1$.

Next, we set

$$\sum_{k=1}^{n} \sum_{t=j}^{m} [(a^{(3)}_{ik(t-j)} - a^{(2)}_{ik(t-j)}) (b_{kt3} - b_{kt2})]\alpha^2 - \sum_{k=1}^{n} \sum_{t=j}^{m} [a^{(3)}_{ik(t-j)} (b_{kt3} - b_{kt2})$$

$$+ (a^{(3)}_{ik(t-j)} - a^{(2)}_{ik(t-j)}) b_{kt3}]\alpha + \sum_{k=1}^{n} \sum_{t=j}^{m} a^{(3)}_{ik(t-j)} b_{kt3} = x \qquad (8)$$

Solving (8) for positive root α gives membership function between c_{ij2} and c_{ij3} satisfying $0 \leq \alpha \leq 1$.

4.3.1 Numerical Example under Fuzzy Environments

In this section we consider a numerical example in which we have five products P_1, P_2, P_3, P_4 and P_5, four work centers WC_1, WC_2, WC_3 and WC_4, six time periods M_1, M_2, M_3, M_4, M_5 and M_6 and six months lead time (time to due date). It is assumed that the resource profile (RP) is measured in terms of hours, MS is measured in terms of number of units of products, and the T.F.N.'s. for the resource profile and the MS are as follows:

Table 4.1 Resource Profile in time units for Work Center 1

	Time to due date					
	5	4	3	2	1	0
P_1	(.2, .3, .7)	(.3, .4, .8)	(.3, .7, .8)	(.4, .6, .8)	(.2, .3, .7)	(.35, .4, .7)
P_2	(.6, .7, .9)	(.4, .5, .9)	(.2, .6, .9)	(.1, .5, .6)	(.15, .2, .5)	(.6, .75, .8)
P_3	(.65, .7, .9)	(.6, .8, .9)	(.1, .5, .7)	(.4, .8, .9)	(.2, .8, .9)	(.3, .4, .5)
P_4	(.6, .8, .9)	(.12, .15, .2)	(.15, .25, .4)	(.4, .9, .95)	(.3, .6, .8)	(.4, .6, .9)
P_5	(.6, .7, .9)	(.4, .5, .9)	(.2, .6, .9)	(.1, .5, .6)	(.15, .2, .5)	(.6, .75, .8)

Table 4.2 Resource Profile in time units for Work Center 2

P_1	(.25, .3, .31)	(.15, .2, .25)	(.4, .5, .8)	(.2, .3, .7)	(.4, .6, .8)	(.4, .5, .9)
P_2	(.1, .2, .4)	(.2, .4, .9)	(.1, .8, .9)	(.7, .8, .9)	(.6, .9, .95)	(.3, .8, .9)
P_3	(.2, .4, .9)	(.8, .85, .95)	(.1, .3, .7)	(.3, .8, .9)	(.1, .5, .7)	(.2, .7, .9)
P_4	(.1, .6, .9)	(.3, .7, .9)	(.7, .8, .9)	(.6, .9, .95)	(.5, .6, .9)	(.1, .4, .7)
P_5	(.2, .3, .7)	(.3, .4, .8)	(.3, .7, .8)	(.4, .6, .8)	(.2, .3, .7)	(.35, .4, .7)

Table 4.3 Resource Profile in time units for Work Center 3

P_1	(.1, .2, .4)	(.2, .4, .7)	(.1, .5, .7)	(.7, .8, .9)	(.6, .8, .9)	(.3, .8, .9)
P_2	(.4, .5, .7)	(.7, .8, .9)	(.4, .5, .7)	(.1, .8, .9)	(.2, .7, .8)	(.7, .8, .9)
P_3	(.1, .6, .9)	(.3, .7, .9)	(.2, .4, .6)	(.3, .5, .8)	(.1, .2, .3)	(.3, .5, .6)
P_4	(.2, .3, .7)	(.3, .4, .8)	(.4, .6, .9)	(.4, .6, .8)	(.2, .3, .6)	(.3, .7, .8)
P_5	(.5, .7, .8)	(.3, .4, .7)	(.6, .8, .9)	(.15, .2, .3)	(.6, .7, .9)	(.1, .3, .5)

Table 4.4 Resource Profile in time units for Work Center 4

P_1	(.2, .3, .7)	(.3, .5, .7)	(.3, .7, .8)	(.4, .6, .8)	(.2, .4, .6)	(.1, .2, .5)
P_2	(.2, .4, .9)	(.8, 85, 9)	(.1, .3, .7)	(.1, .5, .7)	(.3, .5, .8)	(.15, .2, .3)
P_3	(.1, .8, .9)	(.1, .2, .8)	(.3, .6, .9)	(.4, .6, .8)	(.7, .8, .9)	(.3, .7, .9)
P_4	(.2, .3, .7)	(.2, .7, .9)	(.6, .7, .9)	(.5, .6, .8)	(.5, .7, .8)	(.3, .4, .7)
P_5	(.6, .8, .9)	(.2, .3, .7)	(.6, .7, .8)	(.4, .7, .8)	(.2, .7, .9)	(.1, .4, .6)

Table 4.5 Master Schedule in product units (Month1 to Month 3)

	M_1	M_2	M_3
P_1	(200, 230, 270)	(200, 205, 250)	(170, 240, 250)
P_2	(100, 175, 210)	(230, 240, 260)	(95, 100, 150)
P_3	(295, 299,309)	(300, 310, 390)	(200, 210, 260)
P_4	(300, 305, 390)	(200, 295, 350)	(210, 220, 280)
P_5	(100, 175, 210)	(230, 240, 260)	(95, 100, 150)

Table 4.6 Master Schedule in product units (Month 4 to Month 6)

	M_4	M_5	M_6
P_1	(300, 325, 375)	(100, 150, 160)	(100, 175, 200)
P_2	(105, 190, 195)	(100, 140, 200)	(200, 250, 270)
P_3	(250, 255, 280)	(125, 175, 200)	(100, 170, 175)

P_4	(110, 120, 200)	(100, 130, 200)	(210, 230, 290)
P_5	(105, 190, 195)	(100, 140, 200)	(200, 250, 270)

Using (5), (4) and (3), we obtain the capacities, measured in terms of hours, as follows:

Table 4.7 Fuzzy Capacities in hours (Month1 to Month 3)

	M_1	M_2	M_3
WC_1	(1727, 3527.6, 5656)	(1375.2, 2735.5, 4429.5)	(793.5, 1963.5, 3289)
WC_2	(1582.5, 3362.55, 5812.10)	(1324.5, 2884, 4878.75)	(979.25, 2293, 3717.25)
WC_3	(1655.75, 3411, 5405.4)	(1504.25, 2865, 4478)	(1047.5, 2218, 3300)
WC_4	(1733, 3402.8, 5738.1)	(1388, 2717, 4555.5)	(1022.25, 2065.5, 3390)

Table 4.8 Fuzzy Capacities in hours (Month 4 to Month 6)

	M_4	M_5	M_6
WC_1	(659, 1606, 2463.5)	(395.5, 844.5, 1511.5)	(389, 651, 920.5)
WC_2	(1827.75, 1786, 2851.5)	(455, 1045.5, 1773)	(211, 598.5, 972.5)
WC_3	(688.5, 1683.5, 2478.5)	(449.5, 1045.5, 1569.5)	(283, 661, 895)
WC_4	(656.75, 1509.5, 2468)	(397.5, 955.5, 1548.5)	(153, 396.5, 703.5)

These capacities computed above are compared with the available capacities to permit planning for expansion of these resources in a timely fashion by revealing those resources that may be short in capacity.

We now demonstrate the calculation for c_{11} using the approach suggested above and in Kaufmann and Gupta (1985, 1988). The rest of the c_{ij}'s are calculated on similar lines. However, for all c_{ij}'s, we provide the numerical results in the form of the intervals of confidence for various values of α in Appendix 3, and the bar graph of the average fuzzy capacities and the graphs of the membership functions in Appendix 4.

4.3.2 Intervals of Confidence

Using Definition (1.9.6) and (1.10.2), we have the intervals of confidence as follows:

4.3.2.1 Intervals of Confidence of Resource Profile for Work Center 1

$a_{110}^{\alpha} = [.05\alpha + .35, .7 - .3\alpha]$ $\quad$ $a_{111}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$

$a_{112}^{\alpha} = [.2\alpha + .4, .8 - .2\alpha]$ $\quad$ $a_{113}^{\alpha} = [.4\alpha + .3, .8 - .1\alpha]$

$a_{114}^{\alpha} = [.1\alpha + .3, .8 - .4\alpha]$ $\quad$ $a_{115}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$

$a_{120}^{\alpha} = [.15\alpha + .6, .8 - .05\alpha]$ $\quad$ $a_{121}^{\alpha} = [.05\alpha + .15, .5 - .3\alpha]$

$a_{122}^{\alpha} = [.4\alpha + .1, .6 - .1\alpha]$ $\quad$ $a_{123}^{\alpha} = [.4\alpha + .2, .9 - .3\alpha]$

$a_{124}^{\alpha} = [.1\alpha + .4, .9 - .4\alpha]$ $\quad$ $a_{125}^{\alpha} = [.1\alpha + .6, .9 - .2\alpha]$

$a_{130}^{\alpha} = [.1\alpha + .3, .5 - .1\alpha]$ $\quad$ $a_{131}^{\alpha} = [.6\alpha + .2, .9 - .1\alpha]$

$a_{132}^{\alpha} = [.4\alpha + .4, .9 - .1\alpha]$ $\quad$ $a_{133}^{\alpha} = [.4\alpha + .1, .7 - .2\alpha]$

$a_{134}^{\alpha} = [.2\alpha + .6, .9 - .1\alpha]$ $\quad$ $a_{135}^{\alpha} = [.05\alpha + .65, .9 - .2\alpha]$

$a_{140}^{\alpha} = [.2\alpha + .4, .9 - .3\alpha]$ $\quad$ $a_{141}^{\alpha} = [.3\alpha + .3, .8 - .2\alpha]$

$a_{142}^{\alpha} = [.5\alpha + .4, .95 - .05\alpha]$ $\quad$ $a_{143}^{\alpha} = [.1\alpha + .15, .4 - .15\alpha]$

$a_{144}^{\alpha} = [.03\alpha + .12, .2 - .05\alpha]$ $\quad$ $a_{145}^{\alpha} = [.2\alpha + .6, .9 - .1\alpha]$

$a_{150}^{\alpha} = [.15\alpha + .6, .8 - .05\alpha]$ $\quad$ $a_{151}^{\alpha} = [.05\alpha + .15, .5 - .3\alpha]$

$a_{152}^{\alpha} = [.4\alpha + .1, .6 - .1\alpha]$ $\quad$ $a_{153}^{\alpha} = [.4\alpha + .2, .9 - .3\alpha]$

$a_{154}^{\alpha} = [.1\alpha + .4, .9 - .4\alpha]$ $\quad$ $a_{155}^{\alpha} = [.1\alpha + .6, .9 - .2\alpha]$

4.3.2.2 Intervals of Confidence of Resource Profile for Work Center 2

$a_{210}^{\alpha} = [.1\alpha + .4, .9 - .4\alpha]$ $\quad$ $a_{211}^{\alpha} = [.2\alpha + .4, .8 - .2\alpha]$

$a_{212}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$ $\quad$ $a_{213}^{\alpha} = [.1\alpha + .4, .8 - .3\alpha]$

$a_{214}^{\alpha} = [.05\alpha + .15, .25 - .05\alpha]$ $\quad$ $a_{215}^{\alpha} = [.05\alpha + .25, .31 - .01\alpha]$

$a_{220}^{\alpha} = [.5\alpha + .3, .9 - .1\alpha]$ $a_{221}^{\alpha} = [.3\alpha + .6, .95 - .05\alpha]$

$a_{222}^{\alpha} = [.1\alpha + .7, .9 - .1\alpha]$ $a_{223}^{\alpha} = [.7\alpha + .1, .9 - .1\alpha]$

$a_{224}^{\alpha} = [.2\alpha + .2, .9 - .5\alpha]$ $a_{225}^{\alpha} = [.05\alpha + .25, .31 - .01\alpha]$

$a_{230}^{\alpha} = [.5\alpha + .2, .9 - .2\alpha]$ $a_{231}^{\alpha} = [.4\alpha + .1, .7 - .2\alpha]$

$a_{232}^{\alpha} = [.5\alpha + .3, .9 - .1\alpha]$ $a_{233}^{\alpha} = [.2\alpha + .1, .7 - .4\alpha]$

$a_{234}^{\alpha} = [.05\alpha + .8, .95 - .1\alpha]$ $a_{235}^{\alpha} = [.2\alpha + .2, .9 - .5\alpha]$

$a_{240}^{\alpha} = [.3\alpha + .1, .7 - .3\alpha]$ $a_{241}^{\alpha} = [.1\alpha + .5, .9 - .3\alpha]$

$a_{242}^{\alpha} = [.3\alpha + .6, .95 - .05\alpha]$ $a_{243}^{\alpha} = [.1\alpha + .7, .9 - .1\alpha]$

$a_{244}^{\alpha} = [.4\alpha + .3, .9 - .2\alpha]$ $a_{245}^{\alpha} = [.5\alpha + .1, .9 - .3\alpha]$

$a_{250}^{\alpha} = [.05\alpha + .35, .7 - .3\alpha]$ $a_{251}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$

$a_{252}^{\alpha} = [.2\alpha + .4, .8 - .2\alpha]$ $a_{253}^{\alpha} = [.4\alpha + .3, .8 - .1\alpha]$

$a_{254}^{\alpha} = [.1\alpha + .3, .8 - .4\alpha]$ $a_{255}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$

4.3.2.3 Intervals of Confidence of Resource Profile for Work Center 3

$a_{310}^{\alpha} = [.5\alpha + .3, .9 - .1\alpha]$ $a_{311}^{\alpha} = [.2\alpha + .6, .9 - .1\alpha]$

$a_{312}^{\alpha} = [.1\alpha + .7, .9 - .1\alpha]$ $a_{313}^{\alpha} = [.4\alpha + .1, .7 - .2\alpha]$

$a_{314}^{\alpha} = [.2\alpha + .2, .7 - .3\alpha]$ $a_{315}^{\alpha} = [.1\alpha + .1, .4 - .2\alpha]$

$a_{320}^{\alpha} = [.1\alpha + .7, .9 - .1\alpha]$ $a_{321}^{\alpha} = [.5\alpha + .2, .8 - .1\alpha]$

$a_{322}^{\alpha} = [.7\alpha + .1, .9 - .1\alpha]$ $a_{323}^{\alpha} = [.1\alpha + .4, .7 - .2\alpha]$

$a_{324}^{\alpha} = [.1\alpha + .7, .9 - .1\alpha]$ $a_{325}^{\alpha} = [.1\alpha + .4, .7 - .2\alpha]$

$a_{330}^{\alpha} = [.2\alpha + .3, .6 - .1\alpha]$ $a_{331}^{\alpha} = [.1\alpha + .1, .3 - .1\alpha]$

$a_{332}^{\alpha} = [.2\alpha + .3, .8 - .3\alpha]$ $a_{333}^{\alpha} = [.2\alpha + .2, .6 - .2\alpha]$

$a_{334}^{\alpha} = [.4\alpha + .3, .9 - .2\alpha]$ $a_{335}^{\alpha} = [.5\alpha + .1, .9 - .3\alpha]$

$a_{340}^{\alpha} = [.4\alpha + .3, .8 - .1\alpha]$ $a_{341}^{\alpha} = [.1\alpha + .2, .6 - .3\alpha]$

$a_{342}^{\alpha} = [.2\alpha + .4, .8 - .2\alpha]$ $a_{343}^{\alpha} = [.2\alpha + .4, .9 - .3\alpha]$

$a_{344}^{\alpha} = [.1\alpha + .3, .8 - .4\alpha]$ $a_{345}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$

$a_{350}^{\alpha} = [.2\alpha + .1, .5 - .2\alpha]$ $a_{351}^{\alpha} = [.1\alpha + .6, .9 - .2\alpha]$

$a_{352}^{\alpha} = [.05\alpha + .15, .3 - .1\alpha]$ $a_{353}^{\alpha} = [.2\alpha + .6, .9 - .1\alpha]$

$a_{354}^{\alpha} = [.1\alpha + .3, .7 - .3\alpha]$ $a_{355}^{\alpha} = [.2\alpha + .5, .8 - .1\alpha]$

4.3.2.4 Intervals of Confidence of Resource Profile for Work Center 4

$a_{410}^{\alpha} = [.1\alpha + .1, .5 - .3\alpha]$ $a_{411}^{\alpha} = [.2\alpha + .2, .6 - .2\alpha]$

$a_{412}^{\alpha} = [.2\alpha + .4, .8 - .2\alpha]$ $a_{413}^{\alpha} = [.4\alpha + .3, .8 - .1\alpha]$

$a_{414}^{\alpha} = [.2\alpha + .3, .7 - .2\alpha]$ $a_{415}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$

$a_{420}^{\alpha} = [.05\alpha + .15, .3 - .1\alpha]$ $a_{421}^{\alpha} = [.2\alpha + .3, .8 - .3\alpha]$

$a_{422}^{\alpha} = [.4\alpha + .1, .7 - .2\alpha]$ $a_{423}^{\alpha} = [.2\alpha + .1, .7 - .4\alpha]$

$a_{424}^{\alpha} = [.05\alpha + .8, .9 - .05\alpha]$ $a_{425}^{\alpha} = [.2\alpha + .2, .9 - .5\alpha]$

$a_{430}^{\alpha} = [.4\alpha + .3, .9 - .2\alpha]$ $a_{431}^{\alpha} = [.1\alpha + .7, .9 - .1\alpha]$

$a_{432}^{\alpha} = [.2\alpha + .4, .8 - .2\alpha]$ $a_{433}^{\alpha} = [.3\alpha + .3, .9 - .3\alpha]$

$a_{434}^{\alpha} = [.1\alpha + .1, .8 - .6\alpha]$ $a_{435}^{\alpha} = [.7\alpha + .1, .9 - .1\alpha]$

$a_{440}^{\alpha} = [.1\alpha + .3, .7 - .3\alpha]$ $a_{441}^{\alpha} = [.2\alpha + .5, .8 - .1\alpha]$

$a_{442}^{\alpha} = [.1\alpha + .5, .8 - .2\alpha]$ $a_{443}^{\alpha} = [.1\alpha + .6, .9 - .2\alpha]$

$a_{444}^{\alpha} = [.5\alpha + .2, .9 - .2\alpha]$ $a_{445}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$

$a_{450}^{\alpha} = [.3\alpha + .1, .6 - .2\alpha]$ $a_{451}^{\alpha} = [.5\alpha + .2, .9 - .2\alpha]$

$a_{452}^{\alpha} = [.3\alpha + .4, .8 - .1\alpha]$ $a_{453}^{\alpha} = [.1\alpha + .6, .8 - .1\alpha]$

$a_{454}^{\alpha} = [.1\alpha + .2, .7 - .4\alpha]$ $a_{455}^{\alpha} = [.2\alpha + .6, .9 - .1\alpha]$

4.3.2.5 Intervals of Confidence of Master Schedule for Month 1 to Month 6

$b_{11}^{\alpha} = [30\alpha + 200, 270 - 40\alpha]$ $b_{12}^{\alpha} = [5\alpha + 200, 250 - 45\alpha]$

$b_{13}^{\alpha} = [70\alpha + 170, 250 - 10\alpha]$ $b_{14}^{\alpha} = [25\alpha + 300, 375 - 50\alpha]$

$b_{15}^{\alpha} = [50\alpha + 100, 160 - 10\alpha]$ $b_{16}^{\alpha} = [75\alpha + 100, 200 - 25\alpha]$

$b_{21}^{\alpha} = [75\alpha + 100, 210 - 35\alpha]$ $b_{22}^{\alpha} = [10\alpha + 230, 260 - 20\alpha]$

$b_{23}^{\alpha} = [5\alpha + 95, 150 - 50\alpha]$ $b_{14}^{\alpha} = [85\alpha + 105, 195 - 5\alpha]$

$b_{25}^{\alpha} = [40\alpha + 100, 200 - 60\alpha]$ $b_{26}^{\alpha} = [50\alpha + 200, 270 - 20\alpha]$

$b_{31}^{\alpha} = [4\alpha + 295, 309 - 10\alpha]$ $b_{32}^{\alpha} = [10\alpha + 300, 390 - 80\alpha]$

$b_{33}^{\alpha} = [10\alpha + 200, 260 - 50\alpha]$ $b_{34}^{\alpha} = [5\alpha + 250, 280 - 25\alpha]$

$b_{35}^{\alpha} = [50\alpha + 125, 200 - 25\alpha]$ $b_{36}^{\alpha} = [70\alpha + 100, 175 - 5\alpha]$

$b_{41}^{\alpha} = [5\alpha + 300, 390 - 85\alpha]$ $b_{42}^{\alpha} = [5\alpha + 290, 350 - 55\alpha]$

$b_{43}^{\alpha} = [10\alpha + 210, 280 - 60\alpha]$ $b_{44}^{\alpha} = [10\alpha + 110, 200 - 80\alpha]$

$b_{45}^{\alpha} = [30\alpha + 100, 200 - 70\alpha]$ $b_{46}^{\alpha} = [20\alpha + 210, 290 - 60\alpha]$

$b_{51}^{\alpha} = [75\alpha + 100, 210 - 35\alpha]$ $b_{52}^{\alpha} = [10\alpha + 230, 260 - 20\alpha]$

$b_{53}^{\alpha} = [5\alpha + 95, 150 - 50\alpha]$ $b_{54}^{\alpha} = [85\alpha + 105, 195 - 5\alpha]$

$b_{55}^{\alpha} = [40\alpha + 100, 200 - 60\alpha]$ $b_{56}^{\alpha} = [50\alpha + 200, 270 - 20\alpha]$

4.3.3 Calculating the Required Capacity in Work Center 1 for Period 1 (c_{11})

Using (3), we have

$$c_{11}^{(\alpha)} = [\sum_{k=1}^{5} \sum_{t=1}^{6} a_{1k(t-1)}^{(\alpha)} (.)\, b_{kt}^{(\alpha)}]$$

$$c_{11}^{(\alpha)} = [(a_{110}^{\alpha}\,(.)\,b_{11}^{\alpha} + a_{111}^{\alpha}\,(.)\,b_{12}^{\alpha} + a_{112}^{\alpha}\,(.)\,b_{13}^{\alpha} + a_{113}^{\alpha}\,(.)\,b_{14}^{\alpha} + a_{114}^{\alpha}\,(.)\,b_{15}^{\alpha}$$

$$+ a_{115}^{\alpha}\,(.)\,b_{16}^{\alpha} + a_{120}^{\alpha}\,(.)\,b_{21}^{\alpha} + a_{121}^{\alpha}\,(.)\,b_{22}^{\alpha} + a_{122}^{\alpha}\,(.)\,b_{23}^{\alpha} + a_{123}^{\alpha}\,(.)\,b_{24}^{\alpha}$$

$$+ a_{124}^{\alpha}\,(.)\,b_{25}^{\alpha} + a_{125}^{\alpha}\,(.)\,b_{26}^{\alpha} + a_{130}^{\alpha}\,(.)\,b_{31}^{\alpha} + a_{131}^{\alpha}\,(.)\,b_{32}^{\alpha} + a_{132}^{\alpha}\,(.)\,b_{33}^{\alpha}$$

$$+ a_{133}^{\alpha}\,(.)\,b_{34}^{\alpha} + a_{134}^{\alpha}\,(.)\,b_{35}^{\alpha} + a_{135}^{\alpha}\,(.)\,b_{36}^{\alpha} + a_{140}^{\alpha}\,(.)\,b_{41}^{\alpha} + a_{141}^{\alpha}\,(.)\,b_{42}^{\alpha}$$

$$+ a_{142}^{\alpha}\,(.)\,b_{43}^{\alpha} + a_{143}^{\alpha}\,(.)\,b_{44}^{\alpha} + a_{144}^{\alpha}\,(.)\,b_{45}^{\alpha} + a_{145}^{\alpha}\,(.)\,b_{46}^{\alpha} + a_{150}^{\alpha}\,(.)\,b_{51}^{\alpha}$$

$$+ a_{151}^{\alpha}\,(.)\,b_{52}^{\alpha} + a_{152}^{\alpha}\,(.)\,b_{53}^{\alpha} + a_{153}^{\alpha}\,(.)\,b_{54}^{\alpha} + a_{154}^{\alpha}\,(.)\,b_{55}^{\alpha} + a_{155}^{\alpha}\,(.)\,b_{56}^{\alpha})].$$

where (.) is multiplication of two intervals of confidence.

$$c_{11}^{(\alpha)} = [(.05\alpha + .35,\ .7 - .3\alpha)\,(.)\,(30\alpha + 200,\ 270 - 40\alpha)$$

$$+ (.1\alpha + .2,\ .7 - .4\alpha)\,(.)\,(5\alpha + 200,\ 250 - 45\alpha)$$

$$+ (.2\alpha + .4,\ .8 - .2\alpha)\,(.)\,(70\alpha + 170,\ 250 - 10\alpha)$$

$$+ (.4\alpha + .3,\ .8 - .1\alpha)\,(.)\,(25\alpha + 300,\ 375 - 50\alpha)$$

$$+ (.1\alpha + .3,\ .8 - .4\alpha)\,(.)\,(50\alpha + 100,\ 160 - 10\alpha)$$

$$+ (.1\alpha + .2,\ .7 - .4\alpha)\,(.)\,(75\alpha + 100,\ 200 - 25\alpha)$$

$$+ (.15\alpha + .6,\ .8 - .05\alpha)\,(.)\,(75\alpha + 100,\ 210 - 35\alpha)$$

$$+ (.05\alpha + .15,\ .5 - .3\alpha)\,(.)\,(10\alpha + 230,\ 260 - 20\alpha)$$

$$+ (.4\alpha + .1,\ .6 - .1\alpha)\,(.)\,(5\alpha + 95,\ 150 - 50\alpha)$$

$$+ (.4\alpha + .2,\ .9 - .3\alpha)\,(.)\,(85\alpha + 105,\ 195 - 5\alpha)$$

$$+ (.1\alpha + .4,\ .9 - .4\alpha)\,(.)\,(40\alpha + 100,\ 200 - 60\alpha)$$

$$+ (.1\alpha + .6,\ .9 - .2\alpha)\,(.)\,(50\alpha + 200,\ 270 - 20\alpha)$$

$$+ (.1\alpha + .3,\ .5 - .1\alpha)\,(.)\,(4\alpha + 295,\ 309 - 10\alpha)$$

$$+ (.6\alpha + .2,\ .9 - .1\alpha)\,(.)\,(10\alpha + 300,\ 390 - 80\alpha)$$

$$+ (.4\alpha + .4,\ .9 - .1\alpha)\,(.)\,(10\alpha + 200,\ 260 - 50\alpha)$$

$$+ (.4\alpha + .1, .7 - .2\alpha)\,(.)\,(5\alpha + 250, 280 - 25\alpha)$$

$$+ (.2\alpha + .6, .9 - .1\alpha)\,(.)\,(50\alpha + 125, 200 - 25\alpha)$$

$$+ (.05\alpha + .65, .9 - .2\alpha)\,(.)\,(70\alpha + 100, 175 - 5\alpha)$$

$$+ (.2\alpha + .4, .9 - .3\alpha)\,(.)\,(5\alpha + 300, 390 - 85\alpha)$$

$$+ (.3\alpha + .3, .8 - .2\alpha)\,(.)\,(5\alpha + 290, 350 - 55\alpha)$$

$$+ (.5\alpha + .4, .95 - .05\alpha)\,(.)\,(10\alpha + 210, 280 - 60\alpha)$$

$$+ (.1\alpha + .15, .4 - .15\alpha)\,(.)\,(10\alpha + 110, 200 - 80\alpha)$$

$$+ (.03\alpha + .12, .2 - .05\alpha)\,(.)\,(30\alpha + 100, 200 - 70\alpha)$$

$$+ (.2\alpha + .6, .9 - .1\alpha)\,(.)\,(20\alpha + 210, 290 - 60\alpha)$$

$$+ (.15\alpha + .6, .8 - .05\alpha)\,(.)\,(75\alpha + 100, 210 - 35\alpha)$$

$$+ (.05\alpha + .15, .5 - .3\alpha)\,(.)\,(10\alpha + 230, 260 - 20\alpha)$$

$$+ (.4\alpha + .1, .6 - .1\alpha)\,(.)\,(5\alpha + 95, 150 - 50\alpha)$$

$$+ (.4\alpha + .2, .9 - .3\alpha)\,(.)(85\alpha + 105, 195 - 5\alpha)$$

$$+ (.1\alpha + .4, .9 - .4\alpha)\,(.)\,(40\alpha + 100, 200 - 60\alpha)$$

$$+ (.1\alpha + .6, .9 - .2\alpha)\,(.)\,(50\alpha + 200, 270 - 20\alpha)].$$

$$= [(191.3\alpha^2 + 1609.3\alpha + 1727, 219\alpha^2 - 2347.4\alpha + 5656)]$$

We now set

$$191.3\alpha^2 + 1609.3\alpha + 1727 = x \quad \text{and} \quad 219\alpha^2 - 2347.4\alpha + 5656 = x$$

This yields,

$$191.3\alpha^2 + 1609.3\alpha + (1727 - x) = 0 \tag{9}$$

and $$219\alpha^2 - 2347.4\alpha + (5656 - x) = 0 \tag{10}$$

In (9) setting $\alpha = 0$ we get $x = 1727$

In (10) setting $\alpha = 0$ we get $x = 5656$

Setting $\alpha = 1$ in either $191.3\alpha^2 + 1609.3\alpha + (1727 - x) = 0$

or $219\alpha^2 - 2347.4\alpha + (5656 - x) = 0$

yields $x = 3527.6$, therefore $c_{11} = (1727, 3527.6, 5656)$

Now, the membership function is obtained as follows.

Solving the quadratic equation $191.3\alpha^2 + 1609.3\alpha + (1727 - x) = 0$

for α we obtain

$$\alpha = \frac{-1609.3 + \sqrt{1268346.09 + 765.2\,x}}{382.6} \qquad \text{for } 1727 \leq x \leq 3527.6$$

and solving the quadratic equation $219\alpha^2 - 2347.4\alpha + (5656 - x) = 0$

for α we obtain

$$\alpha = \frac{2347.4 - \sqrt{555630.76 + 876\,x}}{438} \qquad \text{for } 3527.6 \leq x \leq 5656$$

Thus, the membership function for $c_{11} = (1727, 3527.6, 5656)$ is

$$\mu_{c_{11}}(x) = \begin{cases} 0 & x \leq 1727 \\ \dfrac{-1609.3 + \sqrt{1268346.09 + 765.2x}}{382.6} & 1727 \leq x \leq 3527.6 \\ \dfrac{2347.4 - \sqrt{555630.76 + 876x}}{438} & 3527.6 \leq x \leq 5656 \\ 0 & x \geq 5656 \end{cases}$$

Similarly, we obtain the fuzzy capacities for rest of c_{ij}'s .

4.3.4 Results

Table 4.9 Fuzzy Capacities (in terms of labor hours) in a Work Center i for Period j

(1727, 3527.6, 5656) c_{11}	(1375.2, 2735.5, 4429.5) c_{12}	(793.5, 1963.5, 3289) c_{13}
(659, 1606, 2463.5) c_{14}	(395.5, 844.5, 1511.5) c_{15}	(389, 651, 920.5) c_{16}
(1582.5, 3362.55, 5812.1) c_{21}	(1324.5, 2884, 4878.75) c_{22}	(979.25, 2293, 3717.25) c_{23}
(827.75, 1786, 2851.5) c_{24}	(455, 1045.5, 1773) c_{25}	(211, 598.5, 972.5) c_{26}
(1655.75, 3411, 5405.4) c_{31}	(1504.25, 2865, 4478) c_{32}	(1047.5, 2218, 3300) c_{33}
(688.5, 1683.5, 2478.5) c_{34}	(449.5, 1045.5, 1569.5) c_{35}	(283, 661, 895) c_{36}
(1733, 3402.8, 5738.1) c_{41}	(1388, 2717, 4555.5) c_{42}	(1022.25, 2065.5, 3390) c_{43}
(656.75, 1509.5, 2468) c_{44}	(397.5, 955.5, 1548.5) c_{45}	(153, 396.5, 703.5) c_{46}

with the membership functions given below:

Membership function for $c_{12} = (1375.2, 2735.5, 4429.5)$,

$$\mu_{c_{12}}(x) = \begin{cases} 0 & x \le 1375.2 \\ \dfrac{-1151.45 + \sqrt{176995.02 + 835.4x}}{417.7} & 1375.2 \le x \le 2735.5 \\ \dfrac{1884.5 - \sqrt{176061.25 + 762x}}{381} & 2735.5 \le x \le 4429.5 \\ 0 & x \ge 4429.5 \end{cases}$$

Membership function for $c_{13} = (793.5, 1963.5, 3289)$,

$$\mu_{c_{13}}(x) = \begin{cases} 0 & x \le 793.5 \\ \dfrac{-968 + \sqrt{295876 + 808x}}{404} & 793.5 \le x \le 1963.5 \\ \dfrac{1442.5 - \sqrt{541554.25 + 468x}}{234} & 1963.5 \le x \le 3289 \\ 0 & x \ge 3289 \end{cases}$$

Membership function for $c_{14} = (659, 1606, 2463.5)$,

$$\mu_{c_{14}}(x) = \begin{cases} 0 & x \le 659 \\ \dfrac{-776.75 + \sqrt{154561.56 + 681x}}{340} & 659 \le x \le 1606 \\ \dfrac{968.5 - \sqrt{-155801.75 + 444x}}{222} & 1606 \le x \le 2463.5 \\ 0 & x \ge 2463.5 \end{cases}$$

Membership function for $c_{15} = (395.5, 844.5, 1511.5)$,

$$\mu_{c_{15}}(x) = \begin{cases} 0 & x \le 395.5 \\ \dfrac{-363 + \sqrt{-4283 + 344x}}{172} & 395.5 \le x \le 844.5 \\ \dfrac{734 - \sqrt{133674 + 268x}}{134} & 844.5 \le x \le 1511.5 \\ 0 & x \ge 1511.5 \end{cases}$$

Membership function for $c_{16} = (389, 651, 920.5)$,

$$\mu_{c_{16}}(x) = \begin{cases} 0 & x \le 389 \\ \dfrac{-232.25 + \sqrt{764906 + 119x}}{59.5} & 389 \le x \le 651 \\ \dfrac{297.5 - \sqrt{-14589.75 + 112x}}{56} & 651 \le x \le 920.5 \\ 0 & x \ge 920.5 \end{cases}$$

Membership function for $c_{21} = (1582.5, 3362.55, 5812.1)$,

$$\mu_{c_{21}}(x) = \begin{cases} 0 & x \leq 1582.5 \\ \dfrac{-1541.55 + \sqrt{866671.40 + 954x}}{477} & 1582.5 \leq x \leq 3362.55 \\ \dfrac{2722.3 - \sqrt{1069916.19 + 1091x}}{545.5} & 3362.55 \leq x \leq 5812.10 \\ 0 & x \geq 5812.10 \end{cases}$$

Membership function for $c_{22} = (1324.5, 2884, 4878.75)$,

$$\mu_{c_{22}}(x) = \begin{cases} 0 & x \leq 1324.5 \\ \dfrac{-1400.25 + \sqrt{1116993.56 + 637x}}{318.5} & 1324.5 \leq x \leq 2884 \\ \dfrac{2197.5 - \sqrt{872340 + 811x}}{405.5} & 2884 \leq x \leq 4878.75 \\ 0 & x \geq 4878.75 \end{cases}$$

Membership function for $c_{23} = (979.25, 2293, 3717.25)$,

$$\mu_{c_{23}}(x) = \begin{cases} 0 & x \leq 979.25 \\ \dfrac{-1124.5 + \sqrt{523208 + 757x}}{378.5} & 979.25 \leq x \leq 2293 \\ \dfrac{1565 - \sqrt{356413 + 563x}}{281.5} & 2293 \leq x \leq 3717.25 \\ 0 & x \geq 3717.25 \end{cases}$$

Membership function for $c_{24} = (827.75, 1786, 2851.5)$,

$$\mu_{c_{24}}(x) = \begin{cases} 0 & x \leq 827.75 \\ \dfrac{-791 + \sqrt{71916.25 + 669x}}{334.5} & 827.75 \leq x \leq 1786 \\ \dfrac{1191 - \sqrt{-12972 + 502x}}{251} & 1786 \leq x \leq 2851.5 \\ 0 & x \geq 2815.5 \end{cases}$$

Membership function for $c_{25} = (455, 1045.5, 1773)$,

$$\mu_{c_{25}}(x) = \begin{cases} 0 & x \le 455 \\ \dfrac{-464.5 + \sqrt{-13559.75 + 504x}}{252} & 455 \le x \le 1045.5 \\ \dfrac{814.5 - \sqrt{46406.25 + 348x}}{174} & 1045.5 \le x \le 1773 \\ 0 & x \ge 1773 \end{cases}$$

Membership function for $c_{26} = (211, 598.5, 972.5)$,

$$\mu_{c_{26}}(x) = \begin{cases} 0 & x \le 211 \\ \dfrac{-311.5 + \sqrt{32888.25 + 304x}}{152} & 211 \le x \le 598.5 \\ \dfrac{411 - \sqrt{24991 + 148x}}{74} & 598.5 \le x \le 972.5 \\ 0 & x \ge 972.5 \end{cases}$$

Membership function for $c_{31} = (1655.75, 3411, 5405.4)$,

$$\mu_{c_{31}}(x) = \begin{cases} 0 & x \le 1655.75 \\ \dfrac{-1552.7 + \sqrt{1069388.64 + 810.2x}}{405.1} & 1655.75 \le x \le 3411 \\ \dfrac{2228.4 - \sqrt{-93687.84 + 936x}}{468} & 3411 \le x \le 5405.4 \\ 0 & x \ge 5405.4 \end{cases}$$

Membership function for $c_{32} = (1504.25, 2865, 4478)$,

$$\mu_{c_{32}}(x) = \begin{cases} 0 & x \le 1504.25 \\ \dfrac{-1160 + \sqrt{137687.25 + 803x}}{401.5} & 1504.25 \le x \le 2865 \\ \dfrac{1792 - \sqrt{5016 + 716x}}{358} & 2865 \le x \le 4478 \\ 0 & x \ge 4478 \end{cases}$$

Membership function for $c_{33} = (1047.5, 2218, 3300)$,

$$\mu_{c_{33}}(x) = \begin{cases} 0 & x \le 1047.5 \\ \dfrac{-956.5 + \sqrt{18232.25 + 856x}}{428} & 1047.5 \le x \le 2218 \\ \dfrac{1236.5 - \sqrt{-114467.75 + 498x}}{249} & 2218 \le x \le 3300 \\ 0 & x \ge 3300 \end{cases}$$

Membership function for $c_{34} = (688.5, 1683.5, 2478.5)$,

$$\mu_{c_{34}}(x) = \begin{cases} 0 & x \le 688.5 \\ \dfrac{-847 + \sqrt{309817 + 592x}}{296} & 688.5 \le x \le 1683.5 \\ \dfrac{874.5 - \sqrt{-23412.75 + 318x}}{159} & 1683.5 \le x \le 2478.5 \\ 0 & x \ge 2478.5 \end{cases}$$

Membership function for $c_{35} = (449.5, 1045.5, 1569.5)$,

$$\mu_{c_{35}}(x) = \begin{cases} 0 & x \le 449.5 \\ \dfrac{-483 + \sqrt{30115 + 452x}}{226} & 449.5 \le x \le 1045.5 \\ \dfrac{579.5 - \sqrt{-12608.75 + 222x}}{111} & 1045.5 \le x \le 1569.5 \\ 0 & x \ge 1569.5 \end{cases}$$

Membership function for $c_{36} = (283, 661, 895)$,

$$\mu_{c_{36}}(x) = \begin{cases} 0 & x \le 283 \\ \dfrac{-303.5 + \sqrt{7778.25 + 298x}}{149} & 283 \le x \le 661 \\ \dfrac{249 - \sqrt{8301 + 60x}}{30} & 661 \le x \le 895 \\ 0 & x \ge 895 \end{cases}$$

Membership function for $c_{41} = (1733, 3402.8, 5738.1)$,

$$\mu_{c_{41}}(x) = \begin{cases} 0 & x \le 1733 \\ \dfrac{-1455.45 + \sqrt{632460.50 + 857.4x}}{428.7} & 1733 \le x \le 3402.8 \\ \dfrac{2592.3 - \sqrt{821252.49 + 1028x}}{514} & 3402.8 \le x \le 5738.1 \\ 0 & x \ge 5738.1 \end{cases}$$

Membership function for $c_{42} = (1388, 2717, 4555.5)$,

$$\mu_{c_{42}}(x) = \begin{cases} 0 & x \le 1388 \\ \dfrac{-1144 + \sqrt{281616 + 740x}}{370} & 1388 \le x \le 2717 \\ \dfrac{2042.5 - \sqrt{454518.25 + 816x}}{408} & 2717 \le x \le 4555.5 \\ 0 & x \ge 4555.5 \end{cases}$$

Membership function for $c_{43} = (1022.25, 2065.5, 3390)$,

$$\mu_{c_{43}}(x) = \begin{cases} 0 & x \le 1022.25 \\ \dfrac{-843.5 + \sqrt{-105285.5 + 799x}}{399.5} & 1022.25 \le x \le 2065.5 \\ \dfrac{1458.5 - \sqrt{310182.25 + 536x}}{268} & 2065.5 \le x \le 3390 \\ 0 & x \ge 3390 \end{cases}$$

Membership function for $c_{44} = (656.75, 1509.5, 2468)$,

$$\mu_{c_{44}}(x) = \begin{cases} 0 & x \le 656.75 \\ \dfrac{-702.5 + \sqrt{98799.5 + 601x}}{300.5} & 656.75 \le x \le 1509.5 \\ \dfrac{1069.5 - \sqrt{48038.25 + 444x}}{222} & 1509.5 \le x \le 2468 \\ 0 & x \ge 2468 \end{cases}$$

Membership function for $c_{45} = (397.5, 955.5, 1548.5)$,

$$\mu_{c_{45}}(x) = \begin{cases} 0 & x \le 110.9 \\ \dfrac{-455 + \sqrt{43255 + 412x}}{206} & 110.9 \le x \le 135.53 \\ \dfrac{661.5 - \sqrt{13293.25 + 274x}}{137} & 135.53 \le x \le 157.44 \\ 0 & x \ge 157.44 \end{cases}$$

Membership function for $c_{46} = (153, 396.5, 703.5)$

$$\mu_{c_{46}}(x) = \begin{cases} 0 & x \le 153 \\ \dfrac{-188 + \sqrt{1684 + 220x}}{110} & 153 \le x \le 396 \\ \dfrac{340 - \sqrt{2414 + 130x}}{65} & 396 \le x \le 703.5 \\ 0 & x \ge 703.5 \end{cases}$$

4.3.5 Interpretation of the Results

From Table 4.9, we calculated the average fuzzy required capacity in terms of hours for each Work Center for the six Periods. The average required capacity in Work Center 1, Work Center 2, Work Center 3, and Work Center 4 are (890, 1888, 3045), (897, 1995, 3334), (938, 1981, 3021), and (892, 1841, 3067) respectively for 6 Periods, which are required to satisfy the requirements of the master production schedule (MPS). The advantage of RP approach over BOL approach is that, the resource profile approach relaxes the Assumptions 3.2.1 by including the lead-time dimension in it. The potential benefit of the resource profile is that it accommodates both product mix variations and production lead times in the preparation of capacity plans.

In Appendix 3 and 4, we calculate the various values of x, when α lies between 0 and 1 and plot the membership function graphs for different values of x. For example, the required capacity in Work Center 1 for Period 1 is (1727, 3527.6, 5656). As we can see in the membership function graph c_{11}, when $1727 \leq x \leq 3527.6$ the membership function increases monotonically to the left and goes to its maximum value of 1 (level of one's believe about the belongingness of x to A, or level of truth of x belonging to A, or degree of compatibility of x to A) at an interior point x = 3527.6, and when $3527.6 \leq x \leq 5656$ the membership function decreases monotonically to the right and goes to 0 at a right end point x = 5656, starting from 1 at x = 3527.6. Similarly we can see the membership function graph for rest of c_{ij}'s along with the values of x. Most of the data available in capacity planning problems in the industry is in the form of fuzzy estimates. Fuzzy set theory permits the partial belonging of an element to a fuzzy set characterized by a membership function that takes values in the interval [0, 1]. Thus, fuzzy approach yields a relatively "more satisfactory and flexible solution" within a pre-specified intervals, whereas a conventional crisp set theory only permits an element either to belong (membership grade 1) or not to belong (membership grade 0) to the set. Another major advantage of the present approach under fuzzy environments that it also provided us a range of labor hours showing the lower and upper bounds of the possible solution.

CHAPTER 5

CAPACITY REQUIREMENTS PLANNING UNDER FUZZY ENVIRONMENT

In this chapter, we consider a problem in which, number of the setup times per lot and run times per piece for planned order releases and for the released orders, the number of lot size of each parts, and the number of work centers, are finite. We extend the crisp capacity requirements planning (CRP) approach to find the capacity required in Work Center i for Period j, assuming that the setup time per lot and the run time per piece for the planned order releases and for the released order amount for each Part k in a given Work Center i for a given Period j is represented, in terms of time units, not by a crisp number but by a triangular fuzzy number, and similarly, the lot size amount for each Part k in a given Period j is represented, in terms of product units, by another triangular fuzzy number.

5.1 Introduction

Implementing and operating an MRP system is a major challenge for many companies. Success of MRP requires accurate data, timely data processing, a realistic master production schedule, methods of controlling as well as planning priority, and a balance approach to processing changes (the handling of unplanned events). Material Requirements Planning (MRP) calculates the exact quantity, need date, and planned order release date for each of the subassemblies, components, and materials required to manufacture the products listed on the master production schedule. Open shop orders, and planned orders in the MRP system are the input to CRP, which translate these orders into hours of work-by-work center by time period. Planned order releases are taken from the MRP system and used to perform a deterministic simulation that uses lead-time offsets to determine the time each order passes through each workstation and continues by including jobs already released to the shop floor. From this simulation, a machine load report is produced. The machine load report for each workstation is compared to capacity available at that station. Thus, implementing CRP requires both a far more detailed Industrial Engineering database, e.g., work standards and routing files, but also formal system for handling transaction on the shop floor and in the store room.

5.2 Capacity Requirements Planning under Crisp Environment

We assume that k represents the number of parts in setup and run time matrices, j represents the number of periods, and i represents the number of work centers, where $k = 1, 2, \dots, n$; $i = 1, 2, \dots, p$; and $j = 1, 2, \dots, m$.

5.2.1 Assumptions: We assume that

1. Machines do not break down, and are available throughout the planning horizon.
2. Machine utilization up to the extent of 100% is possible.

5.2.2 Notations

Let,

a_{ikj} = Setup time per lot in Work Center i for Part k in Period j.

b_{ikj} = Run time per piece in Work Center i for Part k in Period j.

L_{kj} = Lot Size amount Part k in Period j (measured in product units).

c_{ij} = Capacity required in Work Center i for Period j (measured in time units),

P_k = Part k, $k = 1, 2, \ldots, n$,

M_j = Period j, $j = 1, 2, \ldots, m$,

WC_i = Work Center i, $i = 1, 2, \ldots, p$.

In matrix form we can represent a_{ikj}'s, b_{ikj}'s, L_{kj}'s and c_{ij}'s as follows:

Setup time per lot in time units

	M_1	M_2	. . .	M_m
P_1	a_{i11}	a_{i12}	. . .	a_{i1m}
P_2	a_{i21}	a_{i22}	. . .	a_{i2m}
. . .	. . .	. . .	. . .	. . .
P_n	a_{in1}	a_{in2}	. . .	a_{inm}

Run time per piece (.) Lot Size

	M_1	M_2	. . .	M_m
P_1	$b_{i11}L_{11}$	$b_{i12}L_{12}$	. . .	$b_{i1m}L_{1m}$
P_2	$b_{i21}L_{21}$	$b_{i22}L_{22}$	. . .	$b_{i2m}L_{22}$
. . .	. . .	. . .	. . .	. . .
P_n	$b_{in1}L_{n1}$	$b_{in2}L_{n2}$	. . .	$b_{inm}L_{nm}$

5.2.3 General Formulation

Then the formula to compute c_{ij}'s is given as follows:

$$c_{ij} = \sum_{k=1}^{n} \left(a_{ikj} + b_{ikj}\,(.)\,L_{kj}\right), \quad i = 1, 2, \ldots, p \text{ ; and } j = 1, 2, \ldots, m. \tag{1}$$

In (1), if for all values of i, j, and k, a_{ikj}'s, b_{ikj}'s, and L_{kj}'s be crisp numbers, then c_{ij} in (1) is also a crisp number. Thus, in this case, it is easy to compute each capacity value c_{ij}.

In view of (1), we obtain the following capacity matrix

Capacity in Time Units

	M_1	M_2	...	M_m
WC_1	c_{11}	c_{12}	...	c_{1m}
WC_2	c_{21}	c_{22}	...	c_{2m}
...	...	...	...	...
WC_p	c_{p1}	c_{p2}	...	c_{pm}

5.3 Capacity Requirements Planning under Fuzzy Environment

When at least one of a_{ikj}'s, b_{ikj}'s, and L_{kj}'s is a fuzzy number, then as a result the corresponding c_{ij}, the sum of the individual multiplications with additions, is also a fuzzy number. Thus, in this case computing c_{ij}'s is relatively more involved than the crisp case.

We now assume that in (1), each of a_{ikj}, b_{ikj} and L_{kj}, for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$; is a T.F.N. of the type

$a_{ikj} = (a^{(1)}_{ikj}, a^{(2)}_{ikj}, a^{(3)}_{ikj})$, $b_{ikj} = (b^{(1)}_{ikj}, b^{(2)}_{ikj}, b^{(3)}_{ikj})$ and $L_{kj} = (L^{(1)}_{kj}, L^{(2)}_{kj}, L^{(3)}_{kj})$ (2)

As already suggested in Chapter 3 and 4, the values of $a_{ikj} = (a^{(1)}_{ikj}, a^{(2)}_{ikj}, a^{(3)}_{ikj})$, $b_{ikj} = (b^{(1)}_{ikj}, b^{(2)}_{ikj}, b^{(3)}_{ikj})$ and $L_{kj} = (L^{(1)}_{kj}, L^{(2)}_{kj}, L^{(3)}_{kj})$, also can be obtained by using the experts who share the same information but different opinions on certain problems.

In (1) if we set $a_{ikj} = (a^{(1)}_{ikj}, a^{(2)}_{ikj}, a^{(3)}_{ik})$, $b_{ikj} = (b^{(1)}_{ikj}, b^{(2)}_{ikj}, b^{(3)}_{ikj})$ and $L_{kj} = (L^{(1)}_{kj}, L^{(2)}_{kj}, L^{(3)}_{kj})$, then

(i) each term in the right hand side expression is obtained by $(a_{ikj} + b_{ikj}\ (.)\ L_{kj})$ T.F.N's, which may not be a T.F.N.

(ii) each term obtained in (i) is itself a fuzzy number but not necessarily a T.F.N., and

(iii) c_{ij} is obtained by adding the results of each individual term $(a_{ikj} + b_{ikj}\ (.)\ L_{kj})$ obtained in (i).

Thus, we have

$$c_{ij} = \sum_{k=1}^{n} \left(a_{ikj} + b_{ikj}\ (.)\ L_{kj}\right), \quad i = 1, 2, \ldots, p\ ; \text{and } j = 1, 2, \ldots, m\ . \tag{3}$$

In view of (ii), it is important to point out here that in (3) though each of the values for c_{ij} is a fuzzy number yet it is not necessarily a T.F.N. (Kaufmann and Gupta (1985, 1988)). Thus, c_{ij} is a fuzzy number given by

$$c_{ij} = (c_{ij1},\ c_{ij2},\ c_{ij3}) \tag{4}$$

for, $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$. Each fuzzy number c_{ij} in (4) and its membership function is determined on the lines of Kaufmann and Gupta (1985, 1988) by using the interval of confidence for a_{ikj}, b_{ikj}, and L_{kj} respectively, at α–level (see Definition 1.9.2).

$$[a^{(\alpha)}_{11}, a^{(\alpha)}_{12}] = [(a^{(2)}_{ikj} - a^{(1)}_{ikj})\alpha + a^{(1)}_{ikj},\ -(a^{(3)}_{ikj} - a^{(2)}_{ikj})\alpha + a^{(3)}_{ikj}] \quad \forall\ \alpha \in [0, 1]. \tag{5}$$

$$[b^{(\alpha)}_{11}, b^{(\alpha)}_{12}] = [(b^{(2)}_{ikj} - b^{(1)}_{ikj})\alpha + b^{(1)}_{ikj},\ -(b^{(3)}_{ikj} - b^{(2)}_{ikj})\alpha + b^{(3)}_{ikj}] \quad \forall\ \alpha \in [0, 1]. \tag{6}$$

$$[L^{(\alpha)}_{11}, L^{(\alpha)}_{12}] = [(L^{(2)}_{kj} - L^{(1)}_{kj})\alpha + L^{(1)}_{kj},\ -(L^{(3)}_{kj} - L^{(2)}_{kj})\alpha + L^{(3)}_{kj}] \quad \forall\ \alpha \in [0, 1]. \tag{7}$$

for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$. We multiply the two intervals in (6) and (7) and then add with interval (5) as suggested in Kaufmann and Gupta (1985, 1988).

This yields the following interval of confidence with quadratic functions in α .

Thus, for $k = 1, 2, \ldots, n$; $i = 1, 2, \ldots, p$; and $j = 1, 2, \ldots, m$, we get

$$[\sum_{k=1}^{n} [(b^{(2)}_{ikj} - b^{(1)}_{ikj})(L^{(2)}_{kj} - L^{(1)}_{kj})]\alpha^2 + \sum_{k=1}^{n} [(b^{(1)}_{ikj}\ (L^{(2)}_{kj} - L^{(1)}_{kj}) + (b^{(2)}_{ikj} - b^{(1)}_{ikj})\ L^{(1)}_{kj})$$

$$+ (a^{(2)}{}_{ikj} - a^{(1)}{}_{ikj})]\alpha + \sum_{k=1}^{n} a^{(1)}{}_{ikj} + (b^{(1)}{}_{ikj} L^{(1)}{}_{kj}),$$

$$\sum_{k=1}^{n} [(b^{(3)}{}_{ikj} - b^{(2)}{}_{ikj})(L^{(3)}{}_{kj} - L^{(2)}{}_{kj})]\alpha^2 - \sum_{k=1}^{n} [(b^{(3)}{}_{ikj} (L^{(3)}{}_{kj} - L^{(2)}{}_{kj}) + (b^{(3)}{}_{ikj} - b^{(2)}{}_{ikj}) L^{(3)}{}_{kj})$$

$$+ (a^{(3)}{}_{ikj} - a^{(2)}{}_{ikj})]\alpha + \sum_{k=1}^{n} a^{(3)}{}_{ikj} + (b^{(3)}{}_{ikj} L^{(3)}{}_{kj})].$$

In this interval of confidence

1. Setting $\alpha = 0$, we get the end points c_{ij1} and c_{ij3}

$$c_{ij1} = \sum_{k=1}^{n} a^{(1)}{}_{ikj} + (b^{(1)}{}_{ikj} (.) L^{(1)}{}_{kj}) \text{ and } c_{ij3} = \sum_{k=1}^{n} a^{(3)}{}_{ikj} + (b^{(3)}{}_{ikj} (.) L^{(3)}{}_{kj})$$

   of the fuzzy number c_{ij}.

2. Setting $\alpha = 1$ gives the following middle point c_{ij2} of c_{ij}

$$c_{ij2} = \sum_{k=1}^{n} [(b^{(2)}{}_{ikj} - b^{(1)}{}_{ikj})(L^{(2)}{}_{kj} - L^{(1)}{}_{kj})] + \sum_{k=1}^{n} [(b^{(1)}{}_{ikj} (L^{(2)}{}_{kj} - L^{(1)}{}_{kj}) + (b^{(2)}{}_{ikj} - b^{(1)}{}_{ikj}) L^{(1)}{}_{kj})$$

$$+ (a^{(2)}{}_{ikj} - a^{(1)}{}_{ikj})] + \sum_{k=1}^{n} a^{(1)}{}_{ikj} + (b^{(1)}{}_{ikj} L^{(1)}{}_{kj})$$

Also,

$$c_{ij2} = \sum_{k=1}^{n} [(b^{(3)}{}_{ikj} - b^{(2)}{}_{ikj})(L^{(3)}{}_{kj} - L^{(2)}{}_{kj})] - \sum_{k=1}^{n} [(b^{(3)}{}_{ikj} (L^{(3)}{}_{kj} - L^{(2)}{}_{kj}) + (b^{(3)}{}_{ikj} - b^{(2)}{}_{ikj}) L^{(3)}{}_{kj})$$

$$+ (a^{(3)}{}_{ikj} - a^{(2)}{}_{ikj})] + \sum_{k=1}^{n} a^{(3)}{}_{ikj} + (b^{(3)}{}_{ikj} L^{(3)}{}_{kj})$$

The membership function is obtained from the interval of confidence by setting separately, each of the quadratic function, equal to x and solving each of those two quadratic equations for α.

Thus,

$$\sum_{k=1}^{n} [(b^{(2)}_{ikj} - b^{(1)}_{ikj})(L^{(2)}_{kj} - L^{(1)}_{kj})]\alpha^2 + \sum_{k=1}^{n} [(b^{(1)}_{ikj} (L^{(2)}_{kj} - L^{(1)}_{kj}) + (b^{(2)}_{ikj} - b^{(1)}_{ikj}) L^{(1)}_{kj})$$

$$+ (a^{(2)}_{ikj} - a^{(1)}_{ikj})]\alpha + \sum_{k=1}^{n} a^{(1)}_{ikj} + (b^{(1)}_{ikj} L^{(1)}_{kj}) = x \qquad (8)$$

Solving (8) for positive root α gives membership function between c_{ij1} and c_{ij2} satisfying $0 \leq \alpha \leq 1$.

Next, we set

$$\sum_{k=1}^{n} [(b^{(3)}_{ikj} - b^{(2)}_{ikj})(L^{(3)}_{kj} - L^{(2)}_{kj})]\alpha^2 - \sum_{k=1}^{n} [(b^{(3)}_{ikj} (L^{(3)}_{kj} - L^{(2)}_{kj}) + (b^{(3)}_{ikj} - b^{(2)}_{ikj}) L^{(3)}_{kj})$$

$$+ (a^{(3)}_{ikj} - a^{(2)}_{ikj})]\alpha + \sum_{k=1}^{n} a^{(3)}_{ikj} + (b^{(3)}_{ikj} L^{(3)}_{kj}) = x \qquad (9)$$

Solving (9) for positive root α gives membership function between c_{ij2} and c_{ij3} satisfying $0 \leq \alpha \leq 1$.

5.3.1 Numerical Example under Fuzzy Environments

In this section we consider a numerical example in which we have four parts P_1, P_2, P_3, and P_4, three work centers WC_1, WC_2, and WC_3, and six time periods M_1, M_2, M_3, M_4, M_5 and M_6. It is assumed that the setup time per lot and run time per piece for planned order releases and for the released orders are measured in terms of time units, lot size is measured in terms of number of units of products, and the T.F.N.'s for the order quantity, on hand, on order, setup time and run time are as follows:

Table 5.1 Item Master Record Files for (XYZ)

Part	Order Quantity	On Hand	On Order	Due Date	Ld-Time
P_1	(190, 200, 205)	(90, 100, 105)	(190, 200, 205)	First wk	1 week
P_2	(395, 400, 410)	(395, 400, 410)	(395, 400, 410)	Second wk	2 weeks
P_3	(2395, 2400, 2410)	(1490, 1500, 1505)	(2395, 2400, 2410)	Second wk	3 weeks
P_4	(5995, 6000, 6010)	(2490, 2500, 2505)	(5995, 6000, 6010)	Second wk	4 weeks

Table 5.2 Routing Files for (XYZ)

Part	Work Center	Setup Time per Lot (minutes)	Run Time per Piece (minutes)
P_1	1	(25, 30, 35)	(2, 2.5, 2.7)
P_2	2	(8, 10, 25)	(.6, .75, .8)
P_2	1	(5, 15, 35)	(.4, .5, .7)
P_3	3	(10, 15, 30)	(.2, .3, .35)
P_3	1	(10, 25, 30)	(.2, .25, .40)
P_3	2	(12, 15, 25)	(.15, .25, .3)
P_4	2	(20, 25, 45)	(.7, .75, .85)
P_4	3	(10, 30, 40)	(.1, .15, .3)
P_4	1	(50, 75, 90)	(.3, .5, .6)
P_4	3	(20, 30, 35)	(.3, .35, .45)

Table 5.3 Planned Order Releases of (XYZ) for Month 1 to Month 3

Part	1	2	3
P_1	(190, 200, 205)	(190, 200, 205)	(190, 200, 205)
P_2	(395, 400, 410)	(395, 400, 410)	(395, 400, 410)
P_3	(2395, 2400, 2410)	—	(2395, 2400, 2410)
P_4	(5995, 6000, 6010)	—	—

Table 5.4 Planned Order Releases of (XYZ) for Month 4 to Month 6

Part	4	5	6
P_1	(190, 200, 205)	(190, 200, 205)	(190, 200, 205)
P_2	(395, 400, 410)	(395, 400, 410)	(395, 400, 410)
P_3	—	(2395, 2400, 2410)	(2395, 2400, 2410)
P_4	(5995, 6000, 6010)	—	—

The setup time per lot matrix in time units is created directly from the planned order releases of the MRP system.

Table 5.5 Setup time per lot computation for planned order releases in time units

		Week					
	Part	1	2	3	4	5	6
WC_1	P_1	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)
	P_2	—	(5, 15, 35)	(5, 15, 35)	(5, 15, 35)	(5, 15, 35)	(5, 15, 35)
	P_3	—	(10, 25, 30)	—	(10, 25, 30)	—	(10, 25, 30)
	P_4	—	—	(50, 75, 90)	—	—	(50, 75, 90)
WC_2	P_1	—	—	—	—	—	—
	P_2	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)
	P_3	—	—	(12, 15, 25)	—	(12, 15, 25)	—
	P_4	(20, 25, 45)	—	—	(20, 25, 45)	—	—
WC_3	P_1	—	—	—	—	—	—
	P_2	—	—	—	—	—	—
	P_3	(10, 15, 30)	—	(10, 15, 30)	—	(10, 15, 30)	(10, 15, 30)
	P_4	—	(10, 30, 40)	—	(20, 30, 35)	(10, 30, 40)	—

Table 5.6 Run time per piece computation for planned order releases in time units

		Week					
	Part	1	2	3	4	5	6
WC_1	P_1	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)
	P_2	—	(.4, .5, .7)	(.4, .5, .7)	(.4, .5, .7)	(.4, .5, .7)	(.4, .5, .7)
	P_3	—	(.2, .25, .4)	—	(.2, .25, .4)	—	(.2, .25, .4)
	P_4	—	—	(.3, .5, .6)	—	—	(.3, .5, .6)
WC_2	P_1	—	—	—	—	—	—
	P_2	(.6, .75, .8)	(.6, 75, .8)	(.6, .75, .8)	(.6, 75, .8)	(.6, .75, .8)	(.6, 75, .8)
	P_3	—	—	(.15, .25, .3)	—	(.15, .25, .3)	—
	P_4	(.7, .75, .85)	—	—	(.7, .75, .85)	—	—
WC_3	P_1	—	—	—	—	—	—
	P_2	—	—	—	—	—	—
	P_3	(.2, .3, .35)	—	(.2, .3, .35)	—	(.2, .3, .35)	(.2, .3, .35)
	P_4	—	(.1, .15, .3)	—	(.3, .35, .45)	(.1, .15, .3)	—

The setup time per lot added to the (run time per piece) (.) (lot size at each work center for a given time period) to produce required capacity by the planned order releases from the MRP system.

Table 5.7 Fuzzy Capacity Requirements in Work Center i for Period j of Planned Order Releases

(405, 530, 598.5) c_{11}	(1057, 1370, 1914.5) c_{12}	(2416.5, 3820, 4616.5) c_{13}
(1057, 1370, 1914.5) c_{14}	(568, 745, 920.5) c_{15}	(2905.5, 4445, 5610.5) c_{16}

(4461.5, 4835, 5506.5) c_{21}	(245, 310, 353) c_{22}	(616.25, 925, 1101) c_{23}
(4461.5, 4835, 5506.5) c_{24}	(616.25, 925, 1101) c_{25}	(245, 310, 353) c_{26}
(489, 735, 873.5) c_{31}	(609.5, 930, 1843) c_{32}	(489, 735, 873.5) c_{33}
(1818.5, 2130, 2739.5) c_{34}	(1098.5, 1665, 2716.5) c_{35}	(489, 735, 873.5) c_{36}

According to Table 4.1, four orders (190, 200, 205), (395, 400, 410), (2395, 2400, 2410), and (5995, 6000, 6010) have been released to the shop floor. These four orders are all on schedule, that is, the number of operations remaining to be completed is equal to the number of weeks until due date. Part P_1 has one operation to be completed, the other three orders have two-operation (the final two) to be completed.

In matrix form we can represent a_{ikj}'s, b_{ikj}'s, L_{kj}'s, and c_{ij}'s as follows:

Setup time per lot in time units

	M_1	M_2	. . .	M_m
P_1	a_{i11}	a_{i12}	. . .	a_{i1m}
P_2	a_{i21}	a_{i22}	. . .	a_{i2m}
. . .	. . .	. . .	. . .	. . .
P_n	a_{in1}	a_{in2}	. . .	a_{inm}

Run time per piece (.) Lot Size

	M_1	M_2	. . .	M_m
P_1	$b_{i11}L_{11}$	$b_{i12}L_{12}$	. . .	$b_{i1m}L_{1m}$
P_2	$b_{i21}L_{21}$	$b_{i22}L_{22}$	. . .	$b_{i2m}L_{22}$
. . .	. . .	. . .	. . .	. . .
P_n	$b_{in1}L_{n1}$	$b_{in2}L_{n2}$	. . .	$b_{inm}L_{nm}$

5.3.2 General Formulation

Then the formula to compute c_{ij}'s is given as follows.

$$c_{ij} = \sum_{k=1}^{n} \left(a_{ikj} + b_{ikj}\,(.)\,L_{kj}\right), \quad i = 1, 2, \ldots, p\,; \text{ and } j = 1, 2, \ldots, m\,. \qquad \text{(i)}$$

When at least one of a_{ikj}'s, b_{ikj}'s, and L_{kj}'s is a fuzzy number, then as a result the corresponding c_{ij}, the sum of the individual multiplications with additions, is also a fuzzy number. Thus, in this case computing c_{ij}'s is relatively more involved than the crisp case. In view of (i), we obtain the fo llowing capacity matrix

Capacity in Time Units

	M_1	M_2	...	M_m
WC_1	c_{11}	c_{12}	...	c_{1m}
WC_2	c_{21}	c_{22}	...	c_{2m}
...	...	...	...	...
WC_p	c_{p1}	c_{p2}	...	c_{pm}

Table 5.8 Setup time per lot and Run time (.) Lot size for Released Orders

Part	WC	Week	Setup time	Run time (.) Lot size	Total time
P_1	1	1	(25, 30, 45)	(380, 500, 553.5)	(405, 530, 598.5)
P_2	2	1	(8, 10, 25)	(237, 300, 328)	(245, 310, 353)
P_2	1	2	(5, 15, 35)	(158, 200, 287)	(163, 215, 322)
P_3	1	1	(10, 25, 30)	(479, 600, 964)	(489, 625, 994)
P_3	2	2	(12, 15, 25)	(359.25, 600, 723)	(371.25, 615, 748)
P_4	1	1	(50, 75, 90)	(1798.5, 3000, 3606)	(1848.5, 3075, 3696)
P_4	3	2	(20, 30, 35)	(1798.5, 2100, 2704.5)	(1818.5, 2130, 2739.5)

Table 5.9 Fuzzy Capacity Required by the Released Orders for c_{ij}'s

Work Center	Week 1	Week 2
WC_1	(2742.5, 4230, 5288.5)	(163, 215, 3220
WC_2	(245, 310, 353)	(371.25, 615, 748)
WC_3	—	(1818.5, 2130, 2739.5)

The capacity required by planned order releases (Table 5.7) is now added to the capacity required by the released orders (Table 5.9) to the shop floor to produce the total fuzzy capacity requirements plan in Table 5.10 and 5.11.

Using (5), (4) and (3), we obtain the capacities, measured in terms of minutes, as follows.

Table 5.10 Fuzzy Capacities in minutes for Month 1 to Month 3

	M_1	M_2	M_3
WC_1	(3147.5, 4760, 5887)	(1220, 1585, 2236.5)	(2416.5, 3820, 4616.5)
WC_2	(4706.5, 5145, 5859.5)	(616.25, 925, 1101)	(616.25, 925, 11010
WC_3	(489, 735, 873.5)	(2428, 3060, 4582.5)	(489, 735, 873.5)

Table 5.11 Fuzzy Capacities in minutes for Month 4 to Month 6

	M_4	M_5	M_6
WC_1	(1057, 1370, 1914.5)	(568, 745, 920.5)	(2905.3, 4445, 5610.5)
WC_2	(4461.5, 4835, 5506.5)	(616.25, 925, 1101)	(245, 310, 353)
WC_3	(1818.5, 2130, 2739.5)	(1098.5, 1665, 2716.5)	(489, 735, 873.5)

These capacities computed above are compared with the available capacities to permit planning for expansion of these resources in a timely fashion by revealing those resources that may be short in capacity.

We now demonstrate the calculation for c_{11} using the approach suggested above and in Kaufmann and Gupta (1985, 1988). The rest of the c_{ij}'s are calculated on similar lines. However, for all c_{ij}'s, we provide the numerical results in the form of the intervals of confidence for various values of α in Appendix 5, and the bar graph of the average fuzzy capacities and the graphs of the membership functions in Appendix 6.

5.3.3 Intervals of Confidence of Planned Order Releases

Using Definition (1.9.6) and (1.10.2), we have the intervals of confidence as follows:

5.3.3.1 Intervals of Confidence of Setup Time per lot for Planned Order Releases (from Page 85).

Each of a_{111}^{α}, a_{112}^{α}, a_{113}^{α}, a_{114}^{α}, a_{115}^{α}, $a_{116}^{\alpha} = [5\alpha + 25, 45 - 15\alpha]$,

Each of a_{122}^{α}, a_{123}^{α}, a_{124}^{α}, a_{125}^{α}, $a_{126}^{\alpha} = [10\alpha + 5, 35 - 20\alpha]$,

Each of a_{132}^{α}, a_{134}^{α}, $a_{136}^{\alpha} = [15\alpha + 10, 30 - 5\alpha]$,

Each of a_{143}^{α}, $a_{146}^{\alpha} = [25\alpha + 50, 90 - 15\alpha]$,

Each of a_{121}^{α}, a_{131}^{α}, a_{133}^{α}, a_{135}^{α}, a_{141}^{α}, a_{142}^{α}, a_{144}^{α}, $a_{145}^{\alpha} = [0, 0, 0]$,

Each of a_{221}^{α}, a_{222}^{α}, a_{223}^{α}, a_{224}^{α}, a_{225}^{α}, $a_{226}^{\alpha} = [2\alpha + 8, 25 - 15\alpha]$,

Each of a_{233}^{α}, $a_{235}^{\alpha} = [3\alpha + 12, 25 - 10\alpha]$,

Each of a_{241}^{α}, $a_{244}^{\alpha} = [5\alpha + 20, 45 - 20\alpha]$,

Each of a_{211}^{α}, a_{212}^{α}, a_{213}^{α}, a_{214}^{α}, a_{215}^{α}, $a_{216}^{\alpha} = [0, 0, 0]$,

Each of a_{231}^{α}, a_{232}^{α}, a_{234}^{α}, a_{236}^{α}, a_{242}^{α}, a_{243}^{α}, a_{245}^{α}, $a_{246}^{\alpha} = [0, 0, 0]$,

Each of a_{331}^{α}, a_{333}^{α}, a_{335}^{α}, $a_{336}^{\alpha} = [5\alpha + 10, 30 - 15\alpha]$,

Each of a_{342}^{α}, $a_{345}^{\alpha} = [20\alpha + 10, 40 - 10\alpha]$,

$a_{344}^{\alpha} = [10\alpha + 20, 35 - 5\alpha]$,

Each of a_{311}^{α}, a_{312}^{α}, a_{313}^{α}, a_{314}^{α}, a_{315}^{α}, $a_{316}^{\alpha} = [0, 0, 0]$,

Each of a_{321}^{α}, a_{322}^{α}, a_{323}^{α}, a_{324}^{α}, a_{325}^{α}, a_{326}^{α}, a_{332}^{α}, a_{334}^{α}, a_{341}^{α}, a_{343}^{α}, $a_{346}^{\alpha} = [0, 0, 0]$.

5.3.3.2 Intervals of Confidence of Run Time per piece for Planned Order Releases (from Page 86).

Each of b_{111}^{α}, b_{112}^{α}, b_{113}^{α}, b_{114}^{α}, b_{115}^{α}, $b_{116}^{\alpha} = [.5\alpha + 2, 2.7 - .2\alpha]$, Each of b_{122}^{α}, b_{123}^{α}, b_{124}^{α}, b_{125}^{α}, $b_{126}^{\alpha} = [.1\alpha + .4, .7 - .2\alpha]$,

Each of b_{132}^{α}, b_{134}^{α}, $b_{136}^{\alpha} = [.05\alpha + .2, .4 - .15\alpha]$,

Each of b_{121}^{α}, b_{131}^{α}, b_{133}^{α}, b_{135}^{α}, b_{141}^{α}, b_{142}^{α}, b_{144}^{α}, $b_{145}^{\alpha} = [0, 0, 0]$,

Each of b_{143}^{α}, $b_{146}^{\alpha} = [.2\alpha + .3, .6 - .1\alpha]$,

Each of b_{221}^{α}, b_{222}^{α}, b_{223}^{α}, b_{224}^{α}, b_{225}^{α}, $b_{226}^{\alpha} = [.15\alpha + .6, .8 - .05\alpha]$,

Each of b_{233}^{α}, $b_{235}^{\alpha} = [.1\alpha + .15, .3 - .05\alpha]$,

Each of b_{241}^{α}, $b_{244}^{\alpha} = [.05\alpha + .7, .85 - .1\alpha]$,

Each of b_{211}^{α}, b_{212}^{α}, b_{213}^{α}, b_{214}^{α}, b_{215}^{α}, $b_{216}^{\alpha} = [0, 0, 0]$,

Each of b_{231}^{α}, b_{232}^{α}, b_{234}^{α}, b_{236}^{α}, b_{242}^{α}, b_{243}^{α}, b_{245}^{α}, $b_{246}^{\alpha} = [0, 0, 0]$,

Each of b_{331}^{α}, b_{333}^{α}, b_{335}^{α}, $b_{336}^{\alpha} = [.1\alpha + .2, .35 - .05\alpha]$,

Each of b_{342}^{α}, $b_{345}^{\alpha} = [.05\alpha + .1, .3 - .15\alpha]$,

$$b_{344}^{\alpha} = [.05\alpha + .3, .45 - .1\alpha],$$

Each of b_{311}^{α}, b_{312}^{α}, b_{313}^{α}, b_{314}^{α}, b_{315}^{α}, $b_{316}^{\alpha} = [0, 0, 0]$,

Each of b_{321}^{α}, b_{322}^{α}, b_{323}^{α}, b_{324}^{α}, b_{325}^{α}, b_{326}^{α}, b_{332}^{α}, b_{334}^{α}, b_{341}^{α}, b_{343}^{α}, $b_{346}^{\alpha} = [0, 0, 0]$.

5.3.3.3 Intervals of Confidence of Lot Sizes for Part k in Period j (from Page 84).

$L_{1j} = [10\alpha + 190, 205 - 5\alpha]$, $\quad L_{2j} = [5\alpha + 395, 410 - 10\alpha]$,

$L_{3j} = [5\alpha + 2395, 2410 - 10\alpha]$, $\quad L_{4j} = [5\alpha + 5995, 6010 - 10\alpha]$.

5.3.4 Intervals of Confidence of Released Orders

Using Definition (1.9.6) and (1.10.2), we have the intervals of confidence as follows:

5.3.4.1 Intervals of Confidence of Setup Time per lot for Released Orders (from Page 88).

$a_{111}^{\alpha} = [5\alpha + 25, 45 - 15\alpha]$, $\quad a_{131}^{\alpha} = [15\alpha + 10, 30 - 5\alpha]$,

$a_{141}^{\alpha} = [25\alpha + 50, 90 - 15\alpha]$, $a_{122}^{\alpha} = [10\alpha + 5, 35 - 20\alpha]$,

$a_{221}^{\alpha} = [2\alpha + 8, 25 - 15\alpha]$, $a_{232}^{\alpha} = [3\alpha + 12, 25 - 10\alpha]$,

$a_{342}^{\alpha} = [10\alpha + 20, 35 - 5\alpha]$, and

Rest of a_{ikj}'s = [0, 0, 0].

5.3.4.2 Intervals of Confidence of Run Time per piece for Released Orders (from Page 88).

$b_{111}^{\alpha} = [.5\alpha + 2, 2.7 - .2\alpha]$, $b_{131}^{\alpha} = [.05\alpha + .2, .4 - .15\alpha]$,

$b_{141}^{\alpha} = [.2\alpha + .3, .6 - .1\alpha]$, $b_{122}^{\alpha} = [.1\alpha + .4, .7 - .2\alpha]$,

$b_{221}^{\alpha} = [.15\alpha + .6, .8 - .05\alpha]$, $b_{232}^{\alpha} = [.1\alpha + .15, .3 - .05\alpha]$,

$b_{342}^{\alpha} = [.05\alpha + .3, .45 - .1\alpha]$, and

Rest of b_{ikj}'s = [0, 0, 0].

5.3.5 Calculating Required Capacity for Planned Order Releases in Work Center 1 for Period 1 (c_{11})

Using (3), we have

$$c_{11}^{(\alpha)}(P) = \sum_{k=1}^{4}\left(a_{1k1}^{(\alpha)} + b_{1k1}^{(\alpha)} (.)\, L_{k1}^{(\alpha)}\right)_P$$

$$c_{11}^{(\alpha)}(P) = [(a_{111}^{\alpha} + b_{111}^{\alpha} (.)\, L_{11}^{\alpha}) + (a_{121}^{\alpha} + b_{121}^{\alpha} (.)\, L_{21}^{\alpha}) + (a_{131}^{\alpha} + b_{131}^{\alpha} (.)\, L_{31}^{\alpha}) + (a_{141}^{\alpha} + b_{141}^{\alpha} (.)\, L_{41}^{\alpha})]_P$$

where $c_{11}^{(\alpha)}(P)$ stands for required capacity of planned order releases, and

(.) is multiplication of two intervals of confidence.

$$c_{11}^{(\alpha)}(P) = [(5\alpha + 25, 45 - 15\alpha) + (.5\alpha + 2, 2.7 - .2\alpha) (.) (10\alpha + 190, 205 - 5\alpha) + 0 + 0 + 0]$$

$$= [(5\alpha^2 + 120\alpha + 405, 1\alpha^2 - 69.5\alpha + 598.5)] \qquad (10)$$

5.3.6 Calculating Required Capacity for Released Orders in Work Center 1 for Period 1 (c_{11})

Using (3), we have

$$c_{11}^{(\alpha)}(R) = \sum_{k=1}^{4}\left(a_{1k1}^{(\alpha)} + b_{1k1}^{(\alpha)}\,(.)\,L_{k1}^{(\alpha)}\right)_R$$

$$c_{11}^{(\alpha)}(R) = [(a_{111}^{\alpha} + b_{111}^{\alpha}\,(.)\,L_{11}^{\alpha}) + (a_{121}^{\alpha} + b_{121}^{\alpha}\,(.)\,L_{21}^{\alpha})$$
$$+ (a_{131}^{\alpha} + b_{131}^{\alpha}\,(.)\,L_{31}^{\alpha}) + (a_{141}^{\alpha} + b_{141}^{\alpha}\,(.)\,L_{41}^{\alpha})]_R$$

where $c_{11}^{(\alpha)}(R)$ stands for required capacity of released orders.

$$c_{11}^{(\alpha)}(R) = [(5\alpha + 25, 45 - 15\alpha) + (.5\alpha + 2, 2.7 - .2\alpha)\,(.)\,(10\alpha + 190, 205 - 5\alpha)$$
$$+ 0 + (15\alpha + 10, 30 - 5\alpha) + (.05\alpha + .2, .4 - .15\alpha)\,(.)\,(5\alpha + 2395, 2410 - 10\alpha)$$
$$+ (25\alpha + 50, 90 - 15\alpha) + (.2\alpha + .3, .6 - .1\alpha)\,(.)\,(5\alpha + 5995, 6010 - 10\alpha)]$$
$$= [(6.25\alpha^2 + 1481.25\alpha + 2742.5, 3.5\alpha^2 - 1062\alpha + 5288.5)] \qquad (11)$$

By adding (10) and (11), we get the total required capacity for c_{11}. Therefore,

$$c_{11}^{(\alpha)} = c_{11}^{(\alpha)}(P) + c_{11}^{(\alpha)}(R)$$

$$c_{11}^{(\alpha)} = (11.25\alpha^2 + 1601.25\alpha + 3147.5, 4.5\alpha^2 - 1131.5\alpha + 5887)$$

We now set

$$11.25\alpha^2 + 1601.25\alpha + 3147.5 = x \quad \text{and} \quad 4.5\alpha^2 - 1131.5\alpha + 5887 = x$$

This yields,

$$11.25\alpha^2 + 1601.25\alpha + (3147.5 - x) = 0 \qquad (12)$$

$$\text{and} \qquad 4.5\alpha^2 - 1131.5\alpha + (5887 - x) = 0 \qquad (13)$$

In (12) setting $\alpha = 0$ we get $x = 3147.5$

In (13) setting $\alpha = 0$ we get $x = 5887$

Setting $\alpha = 1$ in either $11.25\alpha^2 + 1601.25\alpha + (3147.5 - x) = 0$

or $4.5\alpha^2 - 1131.5\alpha + (5887 - x) = 0$

yields $x = 4760$, therefore $c_{11} = (3147.5, 4760, 5887)$

Now, the membership function is obtained follows.

Solving the quadratic equation $11.25\alpha^2 + 1601.25\alpha + (3147.5 - x) = 0$

for α we obtain

$$\alpha = \frac{-1601.25 + \sqrt{2422364.06 + 45x}}{22.50} \qquad \text{for } 3147.5 \le x \le 4760$$

and solving the quadratic equation $4.5\alpha^2 - 1131.5\alpha + (5887 - x) = 0$

For α we obtain

$$\alpha = \frac{1131.5 - \sqrt{1174326.25 + 18x}}{9} \qquad \text{for } 4760 \le x \le 5887$$

Thus, the membership function for $c_{11} = (3147.5, 4760, 5887)$ is

$$\mu_{c_{11}}(x) = \begin{cases} 0 & x \le 3147.5 \\ \dfrac{-1601.25 + \sqrt{2422364.06 + 45x}}{22.50} & 3147.5 \le x \le 4760 \\ \dfrac{1131.5 - \sqrt{1174326.25 + 18x}}{9} & 4760 \le x \le 5887 \\ 0 & x \ge 5887 \end{cases}$$

Similarly, we obtain the fuzzy capacities for rest of c_{ij}'s.

5.3.7 Results

Table 5.12 Fuzzy Capacities (in terms of labor hours) in Work Center i for Period j

(3147.5, 4760, 5887) c_{11}	(1220, 1585, 2236.5) c_{12}	(2416.5, 3820, 4616.5) c_{13}
(1057, 1370, 1914.5) c_{14}	(568, 745, 920.5) c_{15}	(2905.5, 4445, 5610.5) c_{16}
(4706.5, 5145, 5859.5) c_{21}	(616.25, 925, 1101) c_{22}	(616.25, 925, 1101) c_{23}
(4461.5, 4835, 5506.5) c_{24}	(616.25, 925, 1101) c_{25}	(245, 310, 353) c_{26}
(489, 735, 873.5) c_{31}	(2428, 3060, 4582.5) c_{32}	(489, 735, 873.5) c_{33}
(1818.5, 2130, 2739.5) c_{34}	(1098.5, 1665, 2716.5) c_{35}	(489, 735, 873.5) c_{36}

with the membership functions given below:

Membership function for $c_{12} = (1220, 1585, 2236.5)$,

$$\mu_{c_{12}}(x) = \begin{cases} 0 & x \le 1220 \\ \dfrac{-358.75 + \sqrt{98201.56 + 25x}}{12.50} & 1220 \le x \le 1585 \\ \dfrac{658 - \sqrt{374815 + 26x}}{13} & 1585 \le x \le 2236.5 \\ 0 & x \ge 2236.5 \end{cases}$$

Membership function for $c_{13} = (2416.5, 3820, 4616.5)$,

$$\mu_{c_{13}}(x) = \begin{cases} 0 & x \le 2416.5 \\ \dfrac{-1397 + \sqrt{1888780 + 26x}}{13} & 2416.5 \le x \le 3820 \\ \dfrac{800.5 - \sqrt{566936.25 + 16x}}{8} & 3820 \le x \le 4616.5 \\ 0 & x \ge 4616.5 \end{cases}$$

Membership function for $c_{14} = (1057, 1370, 1914.5)$,

$$\mu_{c_{14}}(x) = \begin{cases} 0 & x \le 659 \\ \dfrac{-307.25 + \sqrt{70091.56 + 23x}}{11.50} & 659 \le x \le 1606 \\ \dfrac{549 - \sqrt{266940 + 18x}}{9} & 1606 \le x \le 2463.5 \\ 0 & x \ge 2463.5 \end{cases}$$

Membership function for $c_{15} = (568, 745, 920.5)$,

$$\mu_{c_{15}}(x) = \begin{cases} 0 & x \le 568 \\ \dfrac{-171.5 + \sqrt{16916.25 + 22x}}{11} & 568 \le x \le 745 \\ \dfrac{178.5 - \sqrt{20816.25 + 12x}}{6} & 745 \le x \le 920.5 \\ 0 & x \ge 920.5 \end{cases}$$

Membership function for $c_{16} = (2905.5, 4445, 5610.5)$,

$$\mu_{c_{16}}(x) = \begin{cases} 0 & x \le 2905.5 \\ \dfrac{-1532.75 + \sqrt{2270874.06 + 27x}}{13.5} & 2905.5 \le x \le 4445 \\ \dfrac{1171 - \sqrt{1247810 + 22x}}{11} & 4445 \le x \le 5610.5 \\ 0 & x \ge 5610.5 \end{cases}$$

Membership function for $c_{21} = (4706.5, 5145, 5859.5)$,

$$\mu_{c_{21}}(x) = \begin{cases} 0 & x \le 4706.5 \\ \dfrac{-436.75 + \sqrt{157805.06 + 7x}}{3.5} & 4706.5 \le x \le 5145 \\ \dfrac{716.5 - \sqrt{466496.25 + 8x}}{4} & 5145 \le x \le 5859.5 \\ 0 & x \ge 5859.5 \end{cases}$$

Membership function for $c_{22} = (616.25, 925, 1101)$,

$$\mu_{c_{22}}(x) = \begin{cases} 0 & x \le 616.25 \\ \dfrac{-307.5 + \sqrt{91475 + 5x}}{2.5} & 616.25 \le x \le 925 \\ \dfrac{177 - \sqrt{26925 + 4x}}{2} & 925 \le x \le 1101 \\ 0 & x \ge 1101 \end{cases}$$

Membership function for $c_{23} = (616.25, 925, 1101)$,

$$\mu_{c_{23}}(x) = \begin{cases} 0 & x \le 616.25 \\ \dfrac{-307.5 + \sqrt{91475 + 5x}}{2.5} & 616.25 \le x \le 925 \\ \dfrac{177 - \sqrt{26925 + 4x}}{2} & 925 \le x \le 1101 \\ 0 & x \ge 1101 \end{cases}$$

Membership function for $c_{24} = (4461.5, 4835, 5506.5)$,

$$\mu_{c_{24}}(x) = \begin{cases} 0 & x \le 4461.5 \\ \dfrac{-372.5 + \sqrt{120910.25 + 4x}}{2} & 4461.5 \le x \le 4835 \\ \dfrac{673 - \sqrt{419890 + 6x}}{3} & 4835 \le x \le 5506.5 \\ 0 & x \ge 5506.5 \end{cases}$$

Membership function for $c_{25} = (616.25, 925, 1101)$,

$$\mu_{c_{25}}(x) = \begin{cases} 0 & x \le 616.25 \\ \dfrac{-307.5 + \sqrt{91475 + 5x}}{2.5} & 616.25 \le x \le 925 \\ \dfrac{177 - \sqrt{26925 + 4x}}{2} & 925 \le x \le 1101 \\ 0 & x \ge 1101 \end{cases}$$

Membership function for $c_{26} = (245, 310, 353)$,

$$\mu_{c_{26}}(x) = \begin{cases} 0 & x \le 245 \\ \dfrac{-64.25 + \sqrt{3393.06 + 3x}}{1.5} & 245 \le x \le 310 \\ \dfrac{43.5 - \sqrt{1186.25 + 2x}}{1} & 310 \le x \le 353 \\ 0 & x \ge 353 \end{cases}$$

Membership function for $c_{31} = (489, 735, 873.5)$,

$$\mu_{c_{31}}(x) = \begin{cases} 0 & x \le 489 \\ \dfrac{-245.5 + \sqrt{59292.25 + 2x}}{1} & 489 \le x \le 735 \\ \dfrac{139 - \sqrt{17574 + 2x}}{1} & 735 \le x \le 873.5 \\ 0 & x \ge 873.5 \end{cases}$$

Membership function for $c_{32} = (2428, 3060, 4582.5)$,

$$\mu_{c_{32}}(x) = \begin{cases} 0 & x \le 2428 \\ \dfrac{-631.5 + \sqrt{393936.25 + 2x}}{1} & 2428 \le x \le 3060 \\ \dfrac{1525 - \sqrt{2279800 + 10x}}{5} & 3060 \le x \le 4582.5 \\ 0 & x \ge 4582.5 \end{cases}$$

Membership function for $c_{33} = (489, 735, 873.5)$,

$$\mu_{c_{33}}(x) = \begin{cases} 0 & x \le 489 \\ \dfrac{-245.5 + \sqrt{59292.25 + 2x}}{1} & 489 \le x \le 735 \\ \dfrac{139 - \sqrt{17574 + 2x}}{1} & 735 \le x \le 873.5 \\ 0 & x \ge 873.5 \end{cases}$$

Membership function for $c_{34} = (1818.5, 2130, 2739.5)$,

$$\mu_{c_{34}}(x) = \begin{cases} 0 & x \le 1818.5 \\ \dfrac{-311.25 + \sqrt{95058.06 + 1x}}{.5} & 1818.5 \le x \le 2130 \\ \dfrac{610.5 - \sqrt{361752.25 + 4x}}{2} & 2130 \le x \le 2739.5 \\ 0 & x \ge 2739.5 \end{cases}$$

Membership function for $c_{35} = (1098.5, 1665, 2716.5)$,

$$\mu_{c_{35}}(x) = \begin{cases} 0 & x \le 1098.5 \\ \dfrac{-565.75 + \sqrt{316777.56 + 3x}}{1.5} & 1098.5 \le x \le 1665 \\ \dfrac{1053.5 - \sqrt{1088130.25 + 8x}}{4} & 1665 \le x \le 2716.5 \\ 0 & x \ge 2716.5 \end{cases}$$

Membership function for $c_{36} = (489, 735, 873.5)$.

$$\mu_{c_{36}}(x) = \begin{cases} 0 & x \le 489 \\ \dfrac{-245.5 + \sqrt{59292.25 + 2x}}{1} & 489 \le x \le 735 \\ \dfrac{139 - \sqrt{17574 + 2x}}{1} & 735 \le x \le 873.5 \\ 0 & x \ge 873.5 \end{cases}$$

5.3.8 Interpretation of the Results

The Capacity requirements planning (CRP) approach is better than rough cut capacity planning (RCCP) approaches (bill of labor and resource profile) because the RCCP processes often plan by month or by week, whereas CRP processes plan by week, by day or even hourly. Also, the RCCP does not consider the on hand inventory or work in process, whereas the CRP does. Capacity requirements planning (CRP) is a detailed comparison of the capacity required by the material requirements planning and the orders currently in process versus available capacity. CRP examines cumulative capacity while RCCP is generally interpreted using average capacity.

As shown in Table 5.10 and 5.11, we calculate the total required capacity plan in Work Center i for Period j in time units. In Appendix 5 and 6, we calculate the various values of x, when α lies between 0 and 1 and plot the membership function graphs for different values of x. For example, the required capacity in Work Center 1 for Period 1 is (3147.5, 4760, 5887). As we can see in the membership function graph c_{11}, when $3147.5 \leq x \leq 4760$ the membership function increases monotonically to the left and goes to its maximum value of 1 at an interior point x = 4760, and when $4760 \leq x \leq 5887$ the membership function decreases monotonically to the right and goes to 0 at a right end point x = 5887, starting from 1 at x = 4760. Similarly we can see the membership function graph for rest of c_{ij}'s along with the values of x.

CHAPTER 6

CONCLUSION, CONTRIBUTION AND RECOMMENDATIONS

In the present chapter, we state the contributions and conclusions of this thesis. Finally, we give some recommendations for further research on the problems considered in this thesis.

6.1 Conclusion and Contribution

The major contribution of the present thesis is that the results obtained here, using the theory of fuzzy sets and fuzzy logic, provide more satisfactory solution and flexibility in the form of range of estimates/values in the area of Capacity Planning and Capacity Resource Planning. In Chapter 3, we discuss the problem of finding the required capacity of various workstations for different time periods under fuzzy environment. Assuming that the data for both bill of labor (BOL) (measured in time units) and master schedule (MS) (measured in product units) is known in the form of triangular fuzzy numbers (T.F.N.'s), we find the required capacity (measured in time units) in the form of fuzzy numbers, which are not necessarily T.F.N's.

Literature is full of definitions of resource profile but there is no general formulation available for the resource profile approach under fuzzy environments. In Chapter 4, we develop a general formula to calculate the required capacity for a specific work center and a particular period (For example: the i^{th} work center for j^{th} period) under fuzzy environment by including the lead-time dimension in it. Furthermore, we discuss the problem of finding the required capacity of various workstations for different time periods. Assuming that the data for both resource profile (RP) (measured in time units) and master schedule (measured in product units) is known in the form of T.F.N.'s, we find the required capacity (measured in time units) in the form of fuzzy numbers, which are not necessarily T.F.N's.

In Chapter 5, we develop a general formula to calculate the required capacity for a specific work center and a particular period under fuzzy environment. Furthermore, we discuss the problem of finding the required capacity of various workstations for different time periods. Assuming that the data for setup time per lot (measured in time units), run time per piece (measured in time units) and lot size (measured in product units) of planned order releases and for released orders is known in the form of T.F.N.'s, we find the required capacity (measured in time units) in the form of fuzzy numbers, which are not necessarily T.F.N's. The results obtained in this chapter consider both on hand inventory or work in process.

It is suggested that the methods presented in this thesis are computationally effective and useful for determining the satisfactory solution to capacity planning problems.

6.2 Recommendations for Future Research

It is believed that a number of extensions are possible to the capacity planning problems. The results (3)–(6) of Chapter 3 can be extended to the case when the estimates for BOL and the MS are provided in the form of trapezoidal fuzzy numbers (Tr.F.N.'s).

Similarly, the results (3)–(6) of Chapter 4 can be extended to the case when the estimates for RP and the MS are provided in the form of Tr.F.N.'s. Also, the results (3)–(7) of Chapter 5 can be extended to the case when the estimates of setup time per lot and run time per piece for planned order releases and for the released orders. and the lot sizes are provided in the form of Tr.F.N.'s.

The same concepts can be used:

- To solve the aggregate planning problems, when the production rate and demand of the products are imprecise/uncertain.
- In economic order quantity model, when the period requirements, preparation cost, carrying cost and the cost of one unit are uncertain and can be represented in the form of triangular fuzzy numbers or trapezoidal fuzzy numbers.
- In operation overlapping problem when the management is unable to provide the exact information about the lot size, processing time per unit for different operations, and set up time.
- In Material Requirements Planning (MRP) to calculate the least total cost (the minimum sum of ordering and carrying costs) when the gross requirements. schedule receipt, projected on hand inventory and planned order releases are fuzzy.

Also, the same concepts can be utilized in the field of financial management for determining the master investment schedule under the condition when resources in the form of available fund are imprecise.

In brief, in any type of industry, the basic goal remains the same: to identify the most cost effective or profitable way of getting the right product to the right place at the right time, given a host of working and business constraints and parameters. Therefore, for an organization with multiple locations, production processes, products, models and customers, where instinct and experience are not able to cope up with the size and complexity of the operation, use of fuzzy logic and fuzzy sets is an aid to be examined.

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APPENDIX 1

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Interval of Confidence for $0 \le \alpha \le 1$

Corresponding to C_{11}

α	$x = 1.43\,\alpha^2 + 18.48\,\alpha + 68.45$	α	$x = .46\,\alpha^2 - 13.8\,\alpha + 101.70$
0	68.45	1	88.36
0.1	70.31	0.9	89.65
0.2	72.20	0.8	90.95
0.3	74.12	0.7	92.27
0.4	76.07	0.6	93.59
0.5	78.05	0.5	94.92
0.6	80.05	0.4	96.25
0.7	82.09	0.3	97.60
0.8	84.15	0.2	98.96
0.9	86.24	0.1	100.32
1	88.36	0	101.70

Corresponding to C_{12}

α	$x = .76\,\alpha^2 + 16.26\,\alpha + 73.10$	α	$x = .47\,\alpha^2 - 15\,\alpha + 104.65$
0	73.10	1	90.12
0.1	74.73	0.9	91.53
0.2	76.38	0.8	92.95
0.3	78.05	0.7	94.38
0.4	79.73	0.6	95.82
0.5	81.42	0.5	97.27
0.6	83.13	0.4	98.73
0.7	84.85	0.3	100.19
0.8	86.59	0.2	101.67
0.9	88.35	0.1	103.15
1	90.12	0	104.65

Corresponding to C_{13}

α	$x = .62\,\alpha^2 + 14.85\,\alpha + 82.43$	α	$x = .51\,\alpha^2 - 15.55\,\alpha + 112.94$
0	82.43	1	97.90
0.1	83.92	0.9	99.36
0.2	85.42	0.8	100.83
0.3	86.94	0.7	102.30
0.4	88.47	0.6	103.79
0.5	90.01	0.5	105.29
0.6	91.56	0.4	106.80
0.7	93.13	0.3	108.32
0.8	94.71	0.2	109.85
0.9	96.30	0.1	111.39
1	97.90	0	112.94

Corresponding to C_{14}			
α	$x = .62\ \alpha^2 + 14.43\ \alpha + 71.59$	α	$x = .45\ \alpha^2 - 13.41\ \alpha + 99.6$
0	71.59	1	86.64
0.1	73.04	0.9	87.90
0.2	74.50	0.8	89.16
0.3	75.97	0.7	90.43
0.4	77.46	0.6	91.72
0.5	78.96	0.5	93.01
0.6	80.47	0.4	94.31
0.7	81.99	0.3	95.62
0.8	83.53	0.2	96.94
0.9	85.08	0.1	98.26
1	86.64	0	99.60

Corresponding to C_{15}			
α	$x = .93\ \alpha^2 + 16.27\ \alpha + 78.48$	α	$x = .39\ \alpha^2 - 12.69\ \alpha + 107.98$
0	78.48	1	95.68
0.1	80.12	0.9	96.87
0.2	81.77	0.8	98.08
0.3	83.44	0.7	99.29
0.4	85.14	0.6	100.51
0.5	86.85	0.5	101.73
0.6	88.58	0.4	102.97
0.7	90.32	0.3	104.21
0.8	92.09	0.2	105.46
0.9	93.88	0.1	106.71
1	95.68	0	107.98

Corresponding to C_{16}			
α	$x = 1.15\ \alpha^2 + 19.95\ \alpha + 75.40$	α	$x = .7\ \alpha^2 - 16.75\ \alpha + 112.55$
0	75.40	1	96.50
0.1	77.41	0.9	98.04
0.2	79.44	0.8	99.60
0.3	81.49	0.7	101.17
0.4	83.56	0.6	102.75
0.5	85.66	0.5	104.35
0.6	87.78	0.4	105.96
0.7	89.93	0.3	107.59
0.8	92.10	0.2	109.23
0.9	94.29	0.1	110.88
1	96.50	0	112.55

Corresponding to C_{21}			
α	$x = 1.05\,\alpha^2 + 20.11\,\alpha + 93.30$	α	$x = .6\,\alpha^2 - 20.39\,\alpha + 134.25$
0	93.30	1	114.46
0.1	95.32	0.9	116.39
0.2	97.36	0.8	118.32
0.3	99.43	0.7	120.27
0.4	101.51	0.6	122.23
0.5	103.62	0.5	124.21
0.6	105.74	0.4	126.19
0.7	107.89	0.3	128.19
0.8	110.06	0.2	130.20
0.9	112.25	0.1	132.22
1	114.46	0	134.25

Corresponding to C_{22}			
α	$x = .83\,\alpha^2 + 17.91\,\alpha + 101.38$	α	$x = .61\,\alpha^2 - 21.49\,\alpha + 141$
0	101.38	1	120.12
0.1	103.18	0.9	122.15
0.2	105.00	0.8	124.20
0.3	106.83	0.7	126.26
0.4	108.68	0.6	128.33
0.5	110.54	0.5	130.41
0.6	112.42	0.4	132.50
0.7	114.32	0.3	134.61
0.8	116.24	0.2	136.73
0.9	118.17	0.1	138.86
1	120.12	0	141.00

Corresponding to C_{23}			
α	$x = .61\,\alpha^2 + 16.73\,\alpha + 111.95$	α	$x = .71\,\alpha^2 - 22.7\,\alpha + 151.28$
0	111.95	1	129.29
0.1	113.63	0.9	131.43
0.2	115.32	0.8	133.57
0.3	117.02	0.7	135.74
0.4	118.74	0.6	137.92
0.5	120.47	0.5	140.11
0.6	122.21	0.4	142.31
0.7	123.96	0.3	144.53
0.8	125.72	0.2	146.77
0.9	127.50	0.1	149.02
1	129.29	0	151.28

Corresponding to C_{24}

α	$x = .55\ \alpha^2 + 14.86\ \alpha + 99.26$	α	$x = .81\ \alpha^2 - 18.2\ \alpha + 132.06$
0	99.26	1	114.67
0.1	100.75	0.9	116.34
0.2	102.25	0.8	118.02
0.3	103.77	0.7	119.72
0.4	105.29	0.6	121.43
0.5	106.83	0.5	123.16
0.6	108.37	0.4	124.91
0.7	109.93	0.3	126.67
0.8	111.50	0.2	128.45
0.9	113.08	0.1	130.25
1	114.67	0	132.06

Corresponding to C_{25}

α	$x = .77\ \alpha^2 + 18.86\ \alpha + 105.3$	α	$x = .57\ \alpha^2 - 18.32\ \alpha + 142.68$
0	105.30	1	124.93
0.1	107.19	0.9	126.65
0.2	109.10	0.8	128.39
0.3	111.03	0.7	130.14
0.4	112.97	0.6	131.89
0.5	114.92	0.5	133.66
0.6	116.89	0.4	135.44
0.7	118.88	0.3	137.24
0.8	120.88	0.2	139.04
0.9	122.90	0.1	140.85
1	124.93	0	142.68

Corresponding to C_{26}

α	$x = 1.25\ \alpha^2 + 21.75\ \alpha + 104.2$	α	$x = 1.15\ \alpha^2 - 24.9\ \alpha + 150.95$
0	104.20	1	127.20
0.1	106.39	0.9	129.47
0.2	108.60	0.8	131.77
0.3	110.84	0.7	134.08
0.4	113.10	0.6	136.42
0.5	115.39	0.5	138.79
0.6	117.70	0.4	141.17
0.7	120.04	0.3	143.58
0.8	122.40	0.2	146.02
0.9	124.79	0.1	148.47
1	127.20	0	150.95

Corresponding to C_{31}			
α	$x = 1.21\,\alpha^2 + 25.25\,\alpha + 125.7$	α	$x = .4\,\alpha^2 - 17.54\,\alpha + 169.30$
0	125.70	1	152.16
0.1	128.24	0.9	153.84
0.2	130.80	0.8	155.52
0.3	133.38	0.7	157.22
0.4	135.99	0.6	158.92
0.5	138.63	0.5	160.63
0.6	141.29	0.4	162.35
0.7	143.97	0.3	164.07
0.8	146.67	0.2	165.81
0.9	149.41	0.1	167.55
1	152.16	0	169.30

Corresponding to C_{32}			
α	$x = .94\,\alpha^2 + 21.54\,\alpha + 139.60$	α	$x = .44\,\alpha^2 - 17.56\,\alpha + 179.20$
0	139.60	1	162.08
0.1	141.76	0.9	163.75
0.2	143.95	0.8	165.43
0.3	146.15	0.7	167.12
0.4	148.37	0.6	168.82
0.5	150.61	0.5	170.53
0.6	152.86	0.4	172.25
0.7	155.14	0.3	173.97
0.8	157.43	0.2	175.71
0.9	159.75	0.1	177.45
1	162.08	0	179.20

Corresponding to C_{33}			
α	$x = .73\,\alpha^2 + 18.54\,\alpha + 154.10$	α	$x = .44\,\alpha^2 - 18.49\,\alpha + 191.42$
0	154.10	1	173.37
0.1	155.96	0.9	175.14
0.2	157.84	0.8	176.91
0.3	159.73	0.7	178.69
0.4	161.63	0.6	180.48
0.5	163.55	0.5	182.29
0.6	165.49	0.4	184.09
0.7	167.44	0.3	185.91
0.8	169.40	0.2	187.74
0.9	171.38	0.1	189.58
1	173.37	0	191.42

Corresponding to C_{34}			
α	$x = .59\,\alpha^2 + 17.04\,\alpha + 115.60$	α	$x = .55\,\alpha^2 - 17.58\,\alpha + 150.26$
0	115.60	1	133.23
0.1	117.31	0.9	134.88
0.2	119.03	0.8	136.55
0.3	120.77	0.7	138.22
0.4	122.51	0.6	139.91
0.5	124.27	0.5	141.61
0.6	126.04	0.4	143.32
0.7	127.82	0.3	145.04
0.8	129.61	0.2	146.77
0.9	131.41	0.1	148.51
1	133.23	0	150.26

Corresponding to C_{35}			
α	$x = .94\,\alpha^2 + 21.89\,\alpha + 134.30$	α	$x = .37\,\alpha^2 - 16.12\,\alpha + 172.88$
0	134.30	1	157.13
0.1	136.50	0.9	158.67
0.2	138.72	0.8	160.22
0.3	140.95	0.7	161.78
0.4	143.21	0.6	163.34
0.5	145.48	0.5	164.91
0.6	147.77	0.4	166.49
0.7	150.08	0.3	168.08
0.8	152.41	0.2	169.67
0.9	154.76	0.1	171.27
1	157.13	0	172.88

Corresponding to C_{36}			
α	$x = 1.15\,\alpha^2 + 25.1\,\alpha + 130.90$	α	$x = .65\,\alpha^2 - 23.8\,\alpha + 180.30$
0	130.90	1	157.15
0.1	133.42	0.9	159.41
0.2	135.97	0.8	161.68
0.3	138.53	0.7	163.96
0.4	141.12	0.6	166.25
0.5	143.74	0.5	168.56
0.6	146.37	0.4	170.88
0.7	149.03	0.3	173.22
0.8	151.72	0.2	175.57
0.9	154.42	0.1	177.93
1	157.15	0	180.30

Corresponding to C_{41}			
α	$x = 1.59\,\alpha^2 + 26.95\,\alpha + 111.25$	α	$x = .84\,\alpha^2 - 22.85\,\alpha + 161.80$
0	111.25	1	139.79
0.1	113.96	0.9	141.92
0.2	116.70	0.8	144.06
0.3	119.48	0.7	146.22
0.4	122.28	0.6	148.39
0.5	125.12	0.5	150.59
0.6	127.99	0.4	152.79
0.7	130.89	0.3	155.02
0.8	133.83	0.2	157.26
0.9	136.79	0.1	159.52
1	139.79	0	161.80

Corresponding to C_{42}			
α	$x = .97\,\alpha^2 + 21.71\,\alpha + 125.29$	α	$x = .84\,\alpha^2 - 23.27\,\alpha + 170.40$
0	125.29	1	147.97
0.1	127.47	0.9	150.14
0.2	129.67	0.8	152.32
0.3	131.89	0.7	154.52
0.4	134.13	0.6	156.74
0.5	136.39	0.5	158.98
0.6	138.67	0.4	161.23
0.7	140.96	0.3	163.49
0.8	143.28	0.2	165.78
0.9	145.61	0.1	168.08
1	147.97	0	170.40

Corresponding to C_{43}			
α	$x = .74\,\alpha^2 + 19.9\,\alpha + 131.05$	α	$x = .88\,\alpha^2 - 25.49\,\alpha + 176.30$
0	131.05	1	151.69
0.1	133.05	0.9	154.07
0.2	135.06	0.8	156.47
0.3	137.09	0.7	158.89
0.4	139.13	0.6	161.32
0.5	141.19	0.5	163.78
0.6	143.26	0.4	166.24
0.7	145.34	0.3	168.73
0.8	147.44	0.2	171.24
0.9	149.56	0.1	173.76
1	151.69	0	176.30

Corresponding to C_{44}

α	$x = .67\,\alpha^2 + 18.49\,\alpha + 99.45$	α	$x = .97\,\alpha^2 - 23.96\,\alpha + 141.60$
0	99.45	1	118.61
0.1	101.31	0.9	120.82
0.2	103.17	0.8	123.05
0.3	105.06	0.7	125.30
0.4	106.95	0.6	127.57
0.5	108.86	0.5	129.86
0.6	110.79	0.4	132.17
0.7	112.72	0.3	134.50
0.8	114.67	0.2	136.85
0.9	116.63	0.1	139.21
1	118.61	0	141.60

Corresponding to C_{45}

α	$x = 1.08\,\alpha^2 + 23.55\,\alpha + 110.90$	α	$x = .75\,\alpha^2 - 22.66\,\alpha + 157.44$
0	110.90	1	135.53
0.1	113.27	0.9	137.65
0.2	115.65	0.8	139.79
0.3	118.06	0.7	141.95
0.4	120.49	0.6	144.11
0.5	122.95	0.5	146.30
0.6	125.42	0.4	148.50
0.7	127.91	0.3	150.71
0.8	130.43	0.2	152.94
0.9	132.97	0.1	155.18
1	135.53	0	157.44

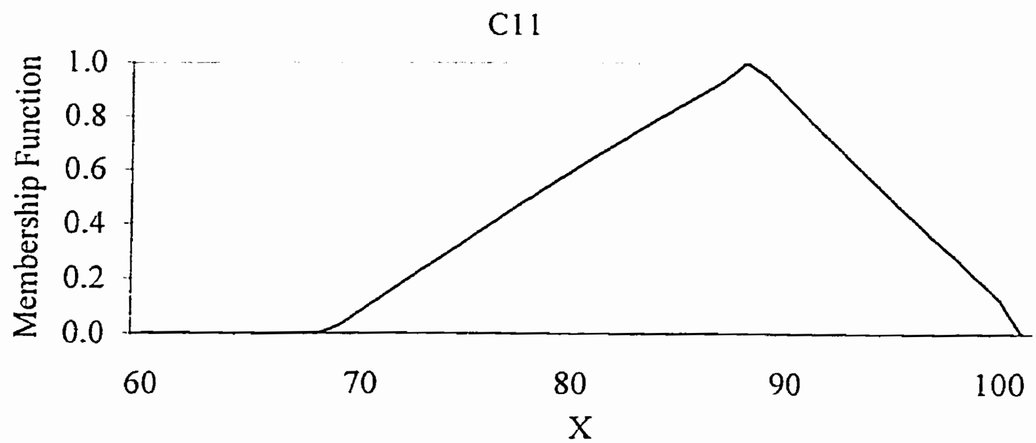
Corresponding to C_{46}

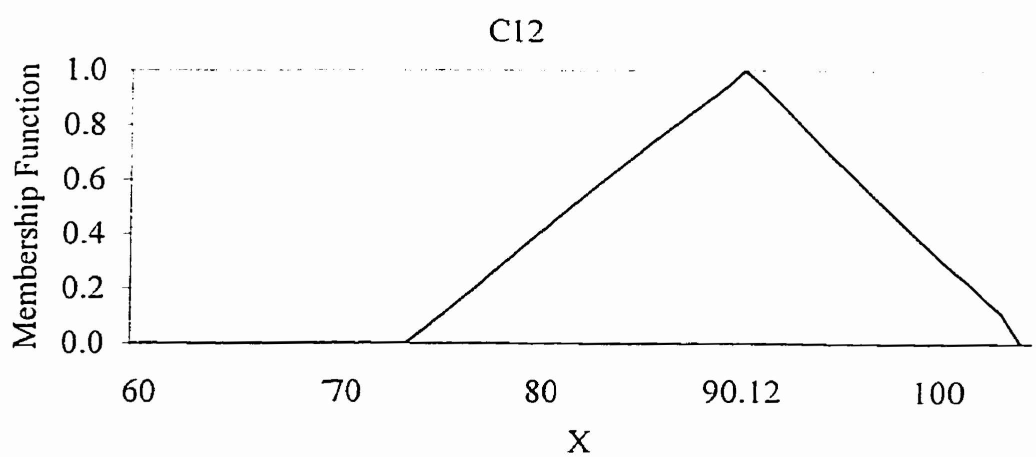
α	$x = 1.3\,\alpha^2 + 23.75\,\alpha + 115.35$	α	$x = 1.35\,\alpha^2 - 31.65\,\alpha + 170.7$
0	115.35	1	140.40
0.1	117.74	0.9	143.31
0.2	120.15	0.8	146.24
0.3	122.59	0.7	149.21
0.4	125.06	0.6	152.20
0.5	127.55	0.5	155.21
0.6	130.07	0.4	158.26
0.7	132.61	0.3	161.33
0.8	135.18	0.2	164.42
0.9	137.78	0.1	167.55
1	140.40	0	170.70

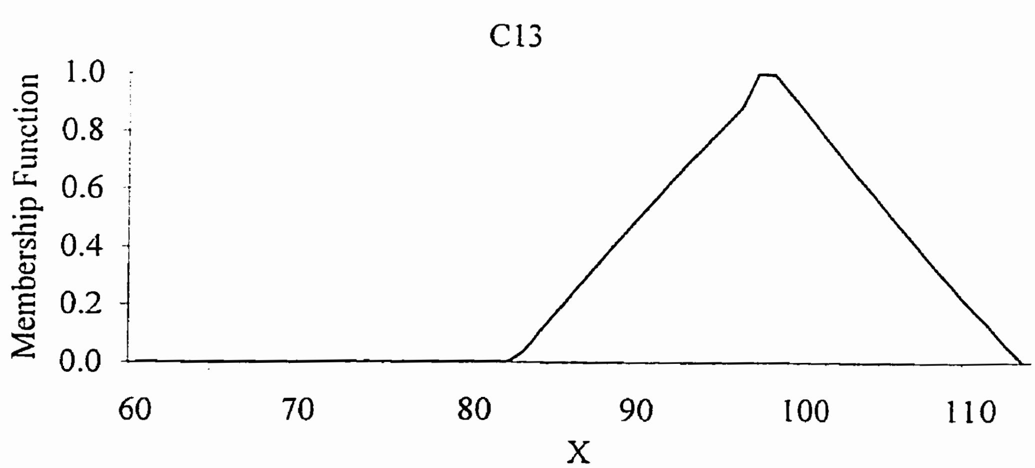
APPENDIX 2

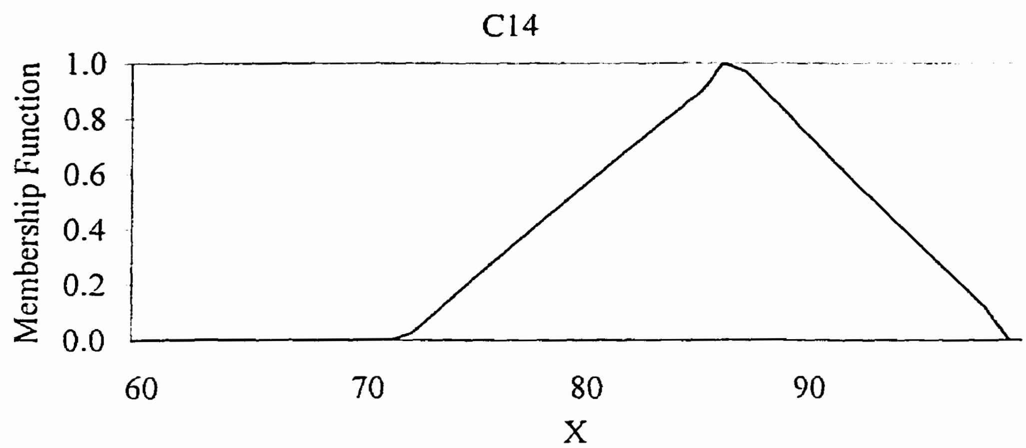
(PAGE 119 – 127)

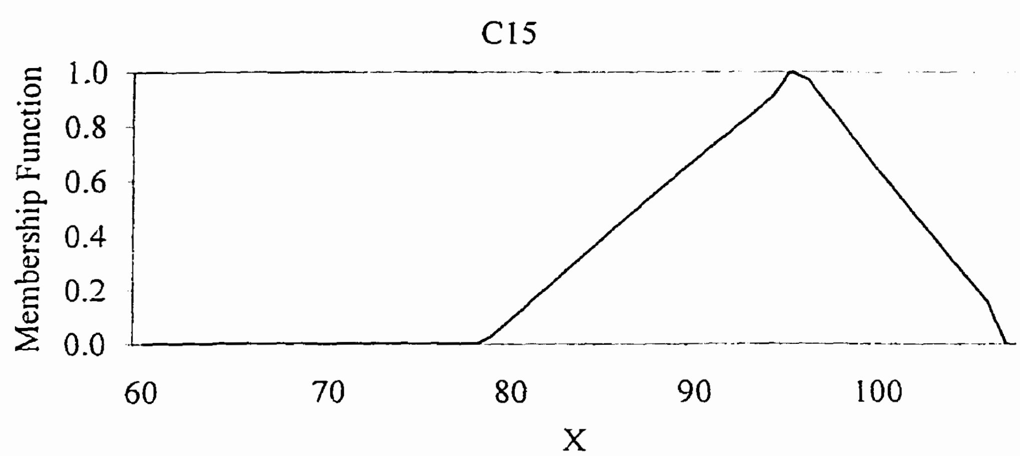
Graph of Membership Functions for cij 's








C14
Membership Function
1.0
0.8
0.6
0.4
0.2
0.0
60
70
80
90
X


C15
Membership Function
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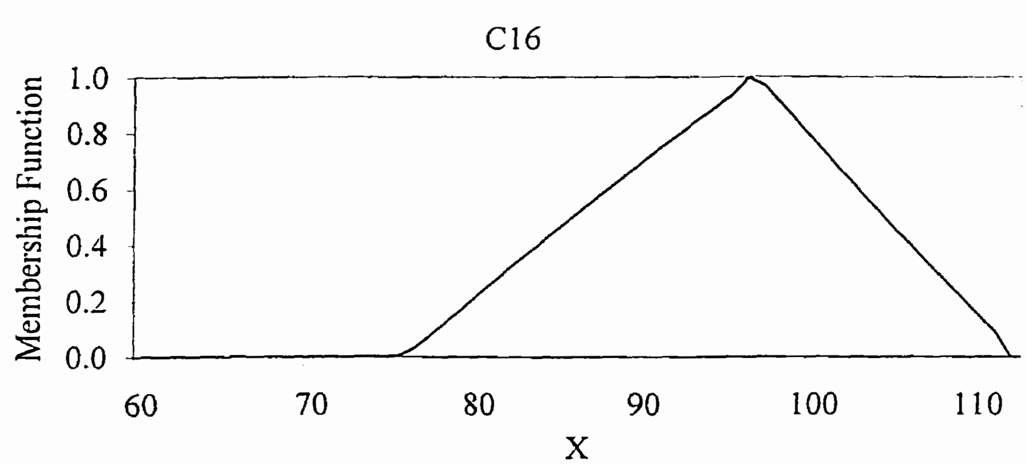
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
X


C16
Membership Function
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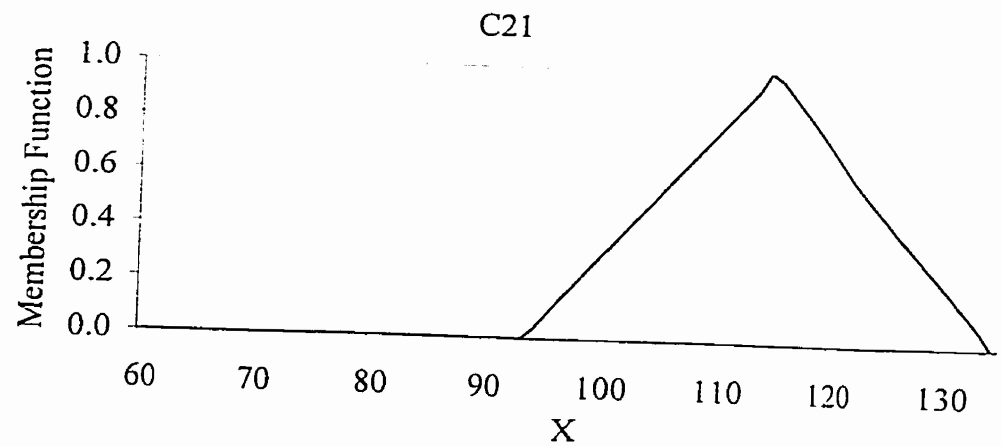
0.6
0.4
0.2
0.0
60
70
80
90
100
110
X


C21
Membership Function
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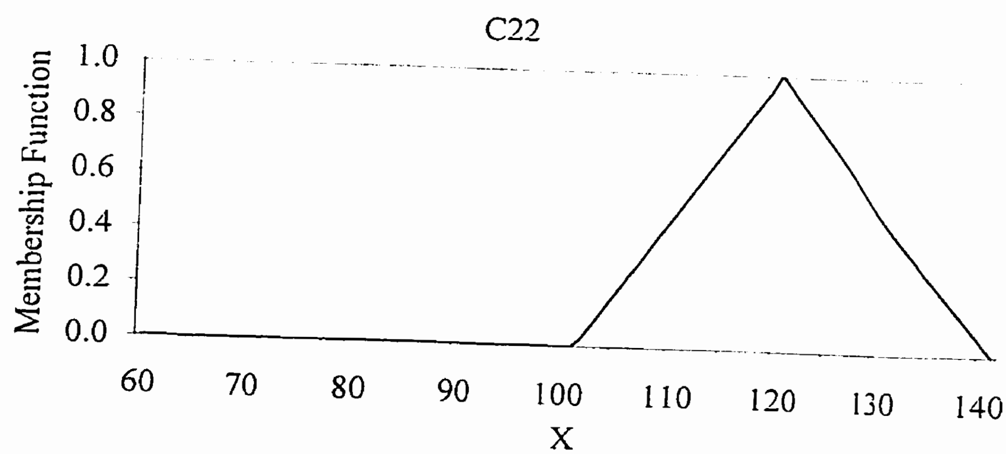
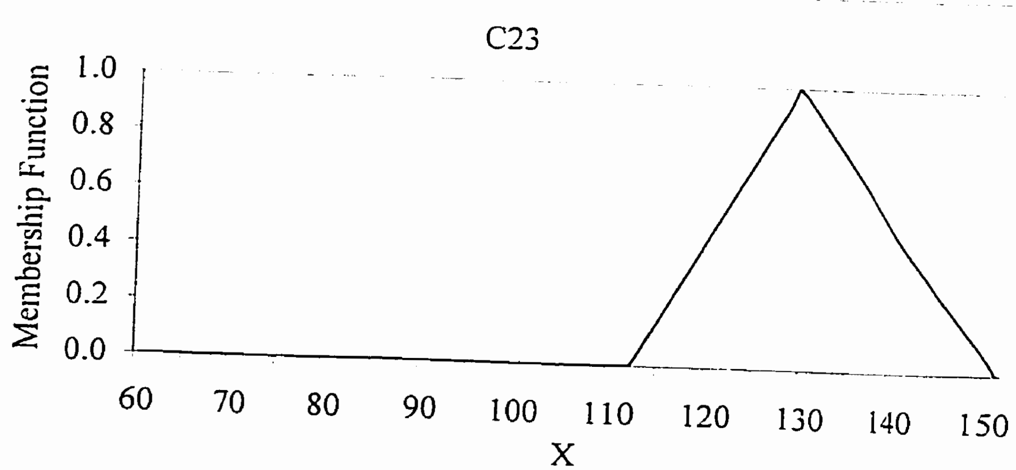
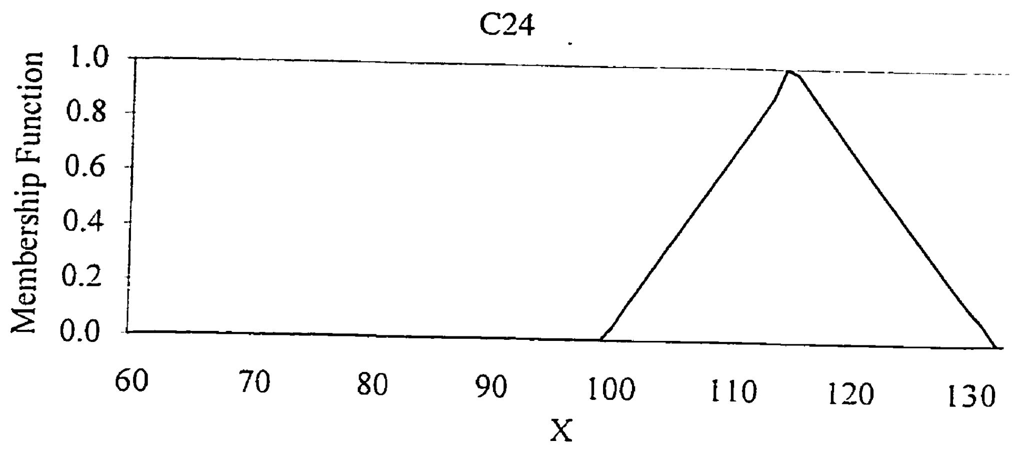
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
X


C22
Membership Function
1.0
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
X


C23
Membership Function
1.0
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
150
X


C24
Membership Function
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0.6
0.4
0.2
0.0
60
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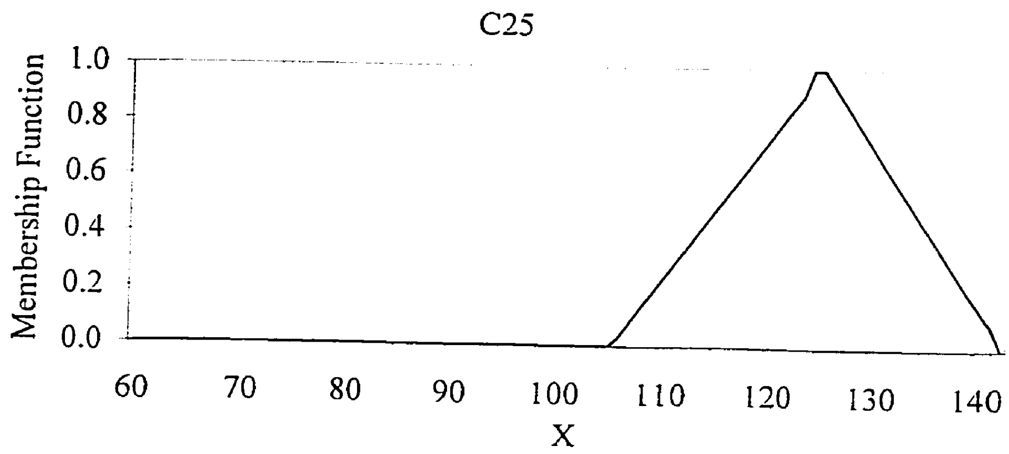
80
90
100
110
120
130
X


C25
Membership Function
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0.2
0.0
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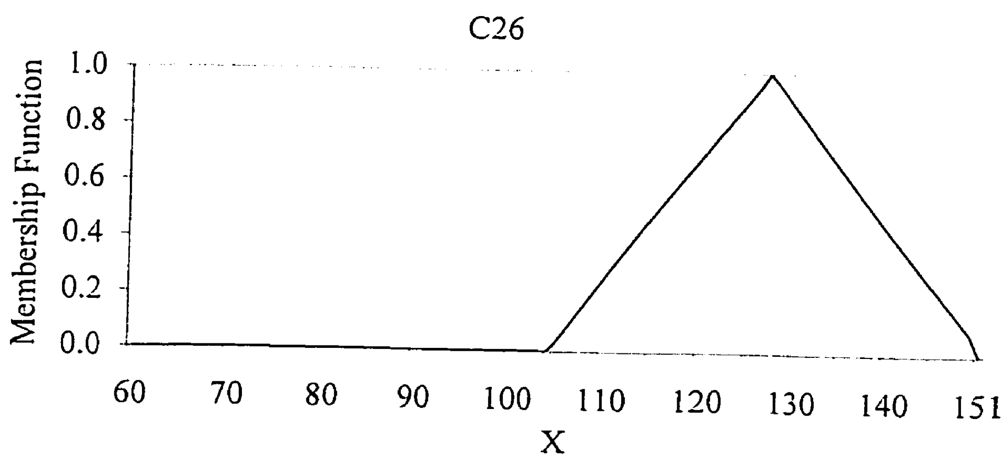
80
90
100
110
120
130
140
X


C26
Membership Function
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0.4
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0.0
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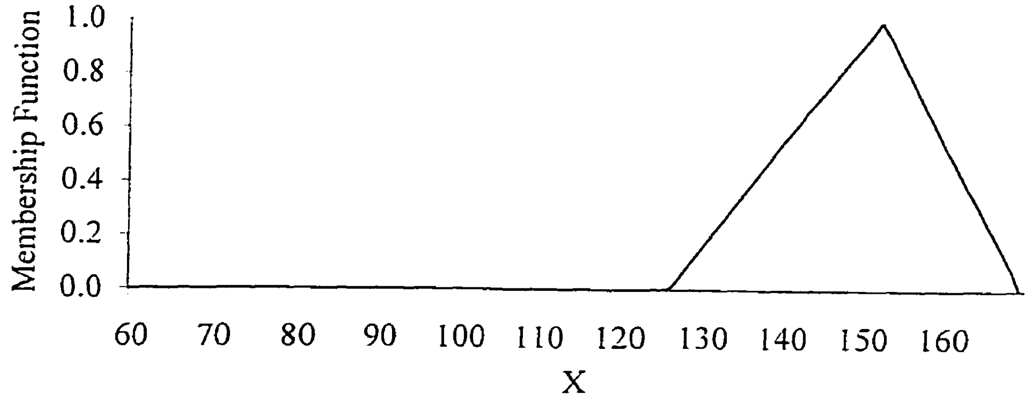
80
90
100
110
120
130
140
151
X

C31


Membership Function
1.0
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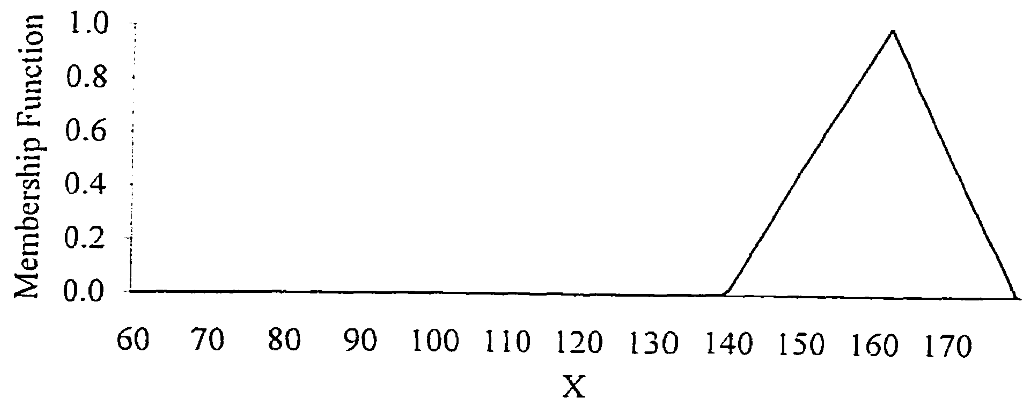
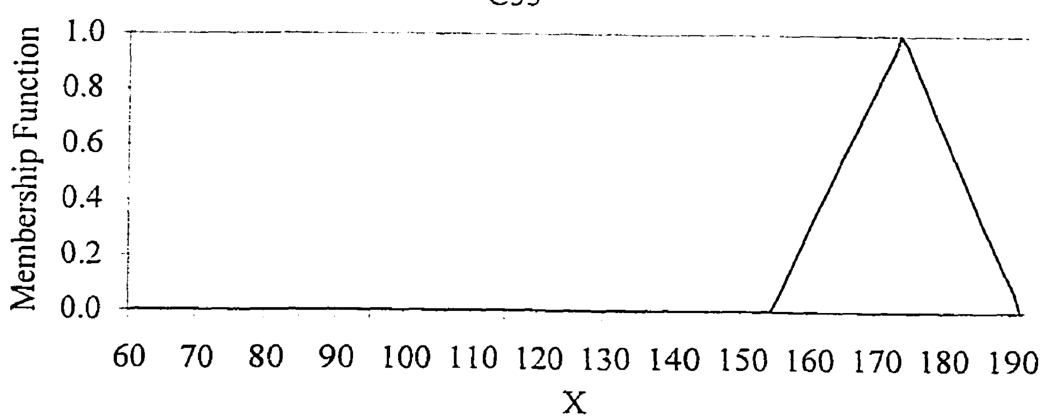
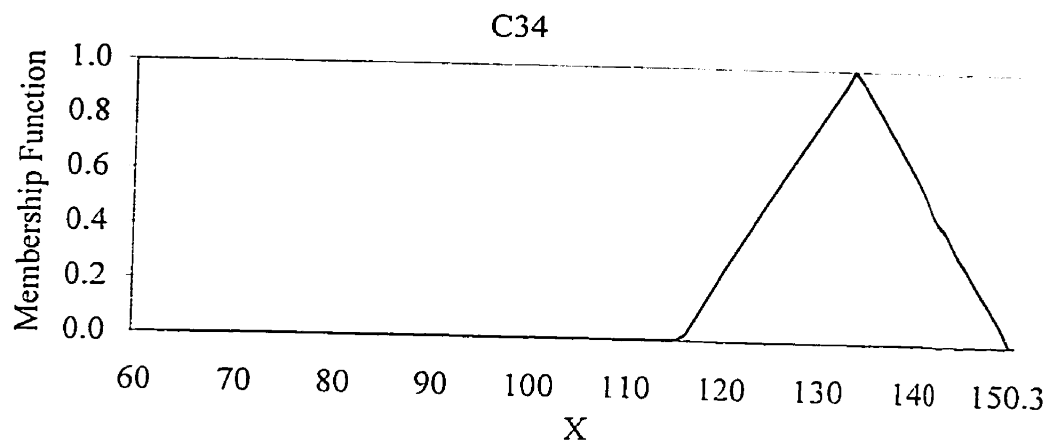
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
150
160
X

C32


Membership Function
1.0
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
150
160
170
X

C33


Membership Function
1.0
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
150
160
170
180
190
X


C34
Membership Function
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0.4
0.2
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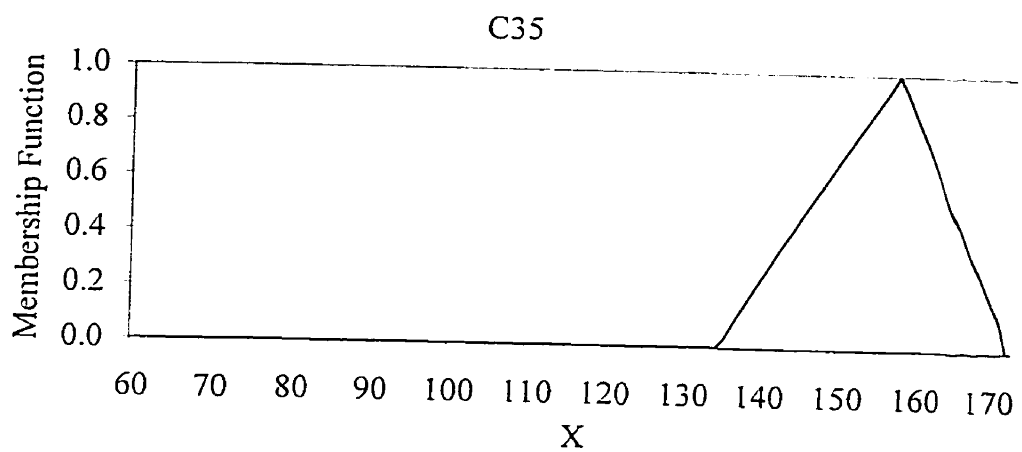
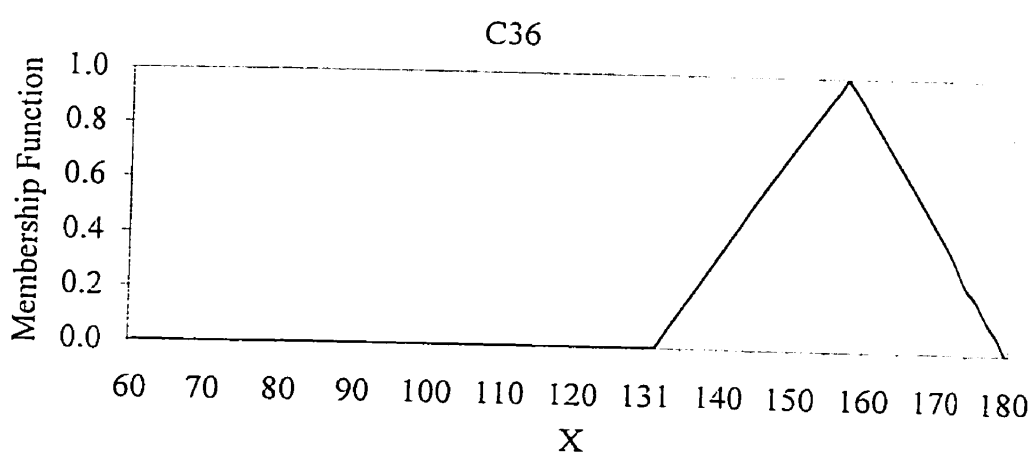
70
80
90
100
110
120
130
140
150.3
X


C35
Membership Function
1.0
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
150
160
170
X


C36
Membership Function
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0.6
0.4
0.2
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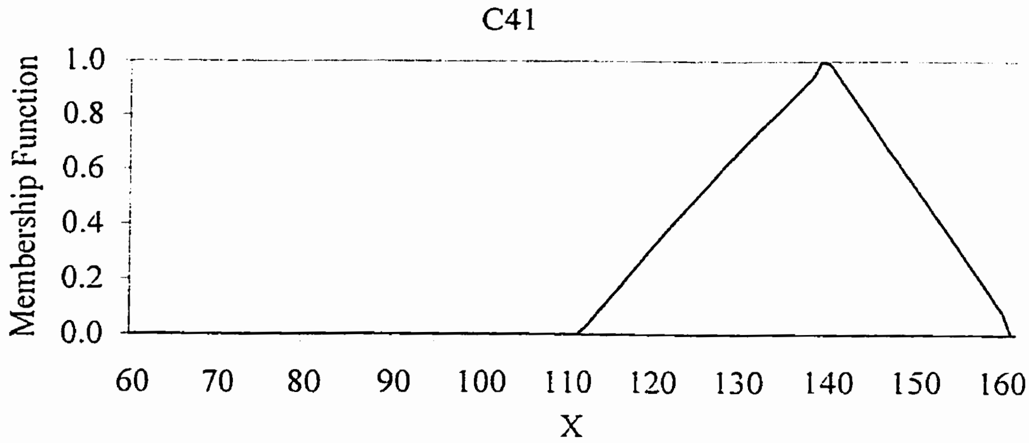
70
80
90
100
110
120
131
140
150
160
170
180
X


C41
Membership Function
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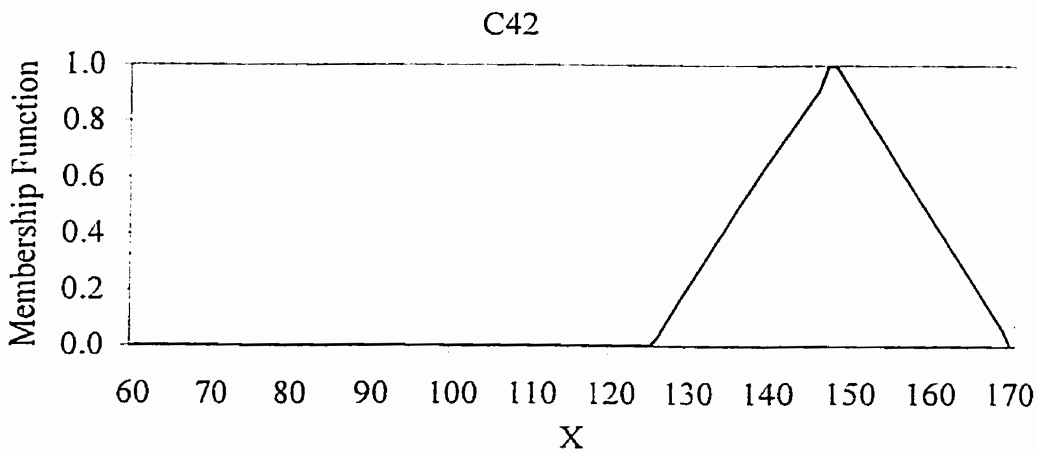
0.6
0.4
0.2
0.0
60 70 80 90 100 110 120 130 140 150 160
X


C42
Membership Function
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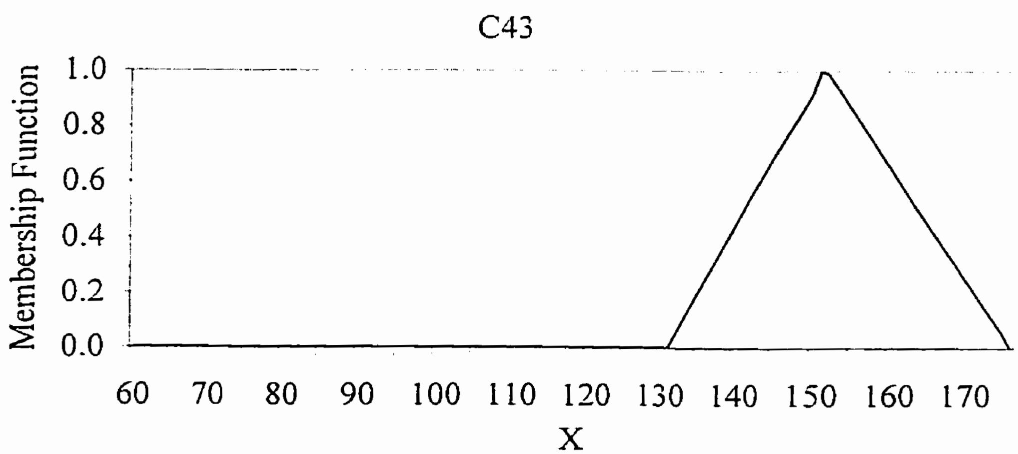
0.8
0.6
0.4
0.2
0.0
60 70 80 90 100 110 120 130 140 150 160 170
X


C43
Membership Function
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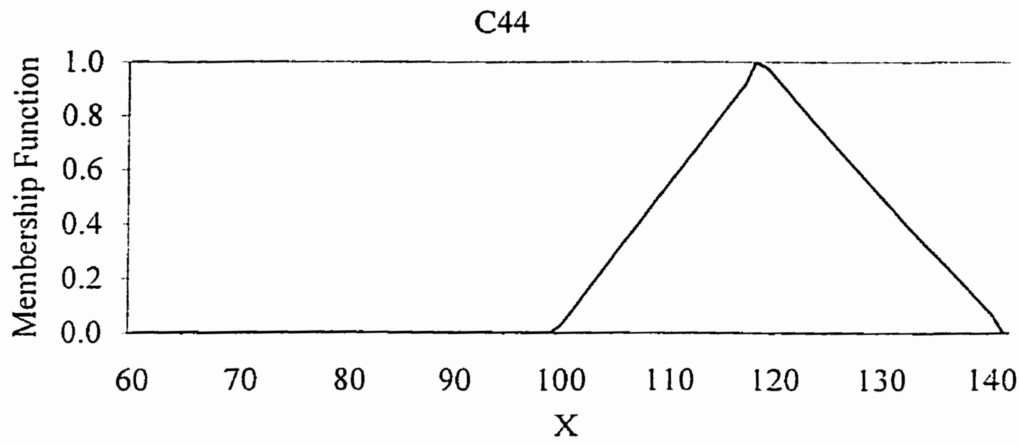
0.8
0.6
0.4
0.2
0.0
60 70 80 90 100 110 120 130 140 150 160 170
X


C44
Membership Function
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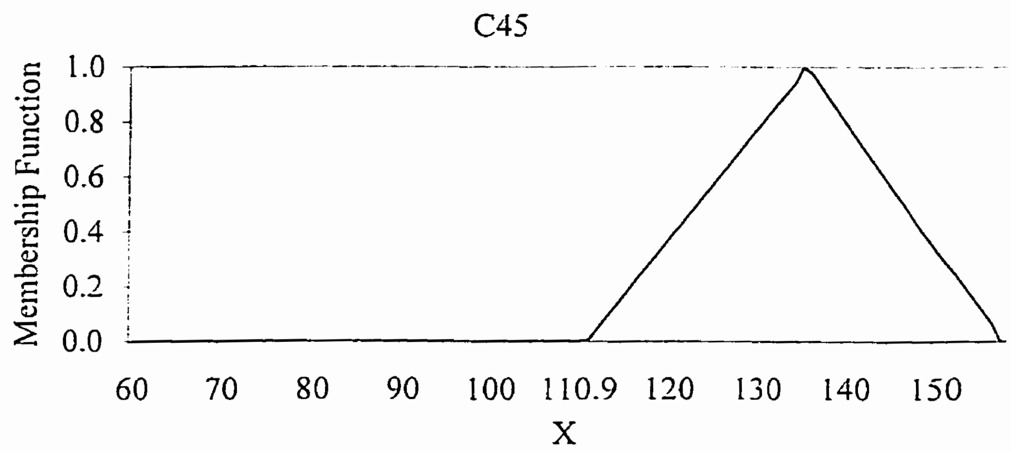
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
X


C45
Membership Function
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0.4
0.2
0.0
60
70
80
90
100
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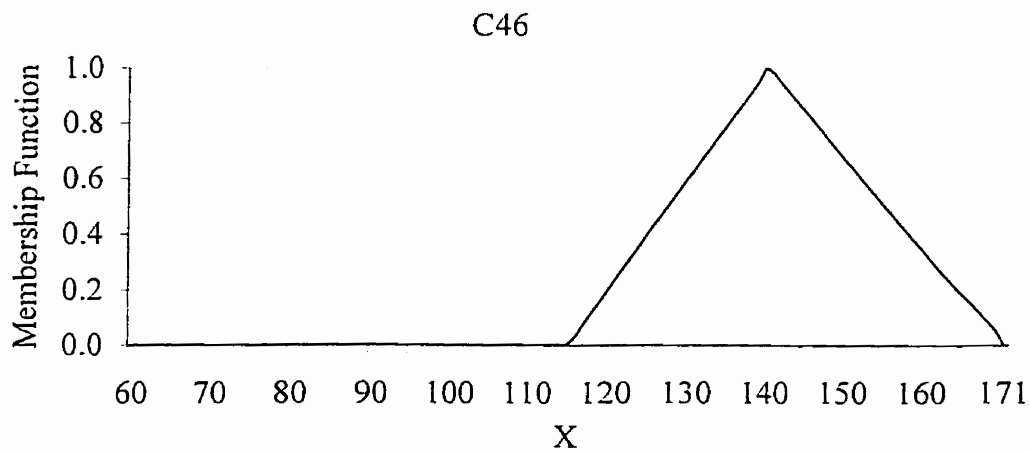
120
130
140
150
X


C46
Membership Function
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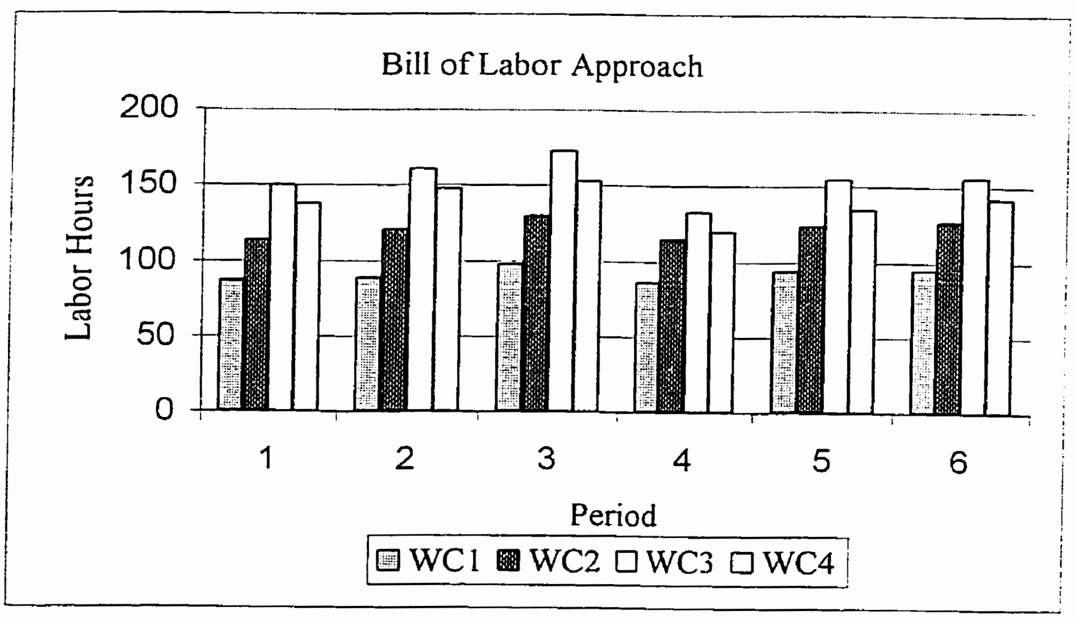
0.8
0.6
0.4
0.2
0.0
60
70
80
90
100
110
120
130
140
150
160
171
X

From Table 3.6, we calculate the average fuzzy capacities in Work Center i for Period j.

Period	1	2	3	4	5	6
WC1	87	89	98	86	94	95
WC2	114	121	130	115	124	127
WC3	150	161	173	133	155	156
WC4	138	148	153	120	135	142



APPENDIX 3

(PAGE 128 – 135)

Interval of Confidence for $0 \le \alpha \le 1$

Corresponding to C_{11}

α	$x = 191.3\,\alpha^2 + 1609.3\,\alpha + 1727$	α	$x = 219\,\alpha^2 - 2347.4\,\alpha + 5656$
0	1727.00	1	3527.60
0.1	1889.84	0.9	3720.73
0.2	2056.51	0.8	3918.24
0.3	2227.01	0.7	4120.13
0.4	2401.33	0.6	4326.40
0.5	2579.48	0.5	4537.05
0.6	2761.45	0.4	4752.08
0.7	2947.25	0.3	4971.49
0.8	3136.87	0.2	5195.28
0.9	3330.32	0.1	5423.45
1	3527.60	0	5656.00

Corresponding to C_{12}

α	$x = 208\,\alpha^2 + 1151.45\,\alpha + 1375.2$	α	$x = 190.5\alpha^2 - 1884.5\,\alpha + 4429.5$
0	1375.20	1	2735.50
0.1	1492.43	0.9	2887.76
0.2	1613.84	0.8	3043.82
0.3	1739.43	0.7	3203.70
0.4	1869.20	0.6	3367.38
0.5	2003.14	0.5	3534.88
0.6	2141.26	0.4	3706.18
0.7	2283.55	0.3	3881.30
0.8	2430.02	0.2	4060.22
0.9	2580.67	0.1	4242.96
1	2735.50	0	4429.50

Corresponding to C_{13}

α	$x = 202\,\alpha^2 + 968\,\alpha + 793.5$	α	$x = 117\,\alpha^2 - 1442.5\,\alpha + 3289$
0	793.50	1	1963.50
0.1	892.32	0.9	2085.52
0.2	995.18	0.8	2209.88
0.3	1102.08	0.7	2336.58
0.4	1213.02	0.6	2465.62
0.5	1328.00	0.5	2597.00
0.6	1447.02	0.4	2730.72
0.7	1570.08	0.3	2866.78
0.8	1697.18	0.2	3005.18
0.9	1828.32	0.1	3145.92
1	1963.50	0	3289.00

Corresponding to C_{14}			
α	$x = 170.25\,\alpha^2 + 776.75\,\alpha + 659$	α	$x = 111\,\alpha^2 - 968.5\,\alpha + 2463.5$
0	659.00	1	1606.00
0.1	738.38	0.9	1681.76
0.2	821.16	0.8	1759.74
0.3	907.35	0.7	1839.94
0.4	996.94	0.6	1922.36
0.5	1089.94	0.5	2007.00
0.6	1186.34	0.4	2093.86
0.7	1286.15	0.3	2182.94
0.8	1389.36	0.2	2274.24
0.9	1495.98	0.1	2367.76
1	1606.00	0	2463.50

Corresponding to C_{15}			
α	$x = 86\,\alpha^2 + 363\,\alpha + 395.5$	α	$x = 67\,\alpha^2 - 734\,\alpha + 1511.5$
0	395.50	1	844.50
0.1	432.66	0.9	905.17
0.2	471.54	0.8	967.18
0.3	512.14	0.7	1030.53
0.4	554.46	0.6	1095.22
0.5	598.50	0.5	1161.25
0.6	644.26	0.4	1228.62
0.7	691.74	0.3	1297.33
0.8	740.94	0.2	1367.38
0.9	791.86	0.1	1438.77
1	844.50	0	1511.50

Corresponding to C_{16}			
α	$x = 29.75\,\alpha^2 + 232.25\,\alpha + 389$	α	$x = 28\,\alpha^2 - 297.5\,\alpha + 920.5$
0	389.00	1	651.00
0.1	412.52	0.9	675.43
0.2	436.64	0.8	700.42
0.3	461.35	0.7	725.97
0.4	486.66	0.6	752.08
0.5	512.56	0.5	778.75
0.6	539.06	0.4	805.98
0.7	566.15	0.3	833.77
0.8	593.84	0.2	862.12
0.9	622.12	0.1	891.03
1	651.00	0	920.50

Corresponding to C_{21}			
α	$x = 238.5\ \alpha^2 + 1541.55\ \alpha + 1582.5$	α	$x = 272.75\ \alpha^2 - 2722.3\ \alpha + 5812.1$
0	1582.50	1	3362.55
0.1	1739.04	0.9	3582.96
0.2	1900.35	0.8	3808.82
0.3	2066.43	0.7	4040.14
0.4	2237.28	0.6	4276.91
0.5	2412.90	0.5	4519.14
0.6	2593.29	0.4	4766.82
0.7	2778.45	0.3	5019.96
0.8	2968.38	0.2	5278.55
0.9	3163.08	0.1	5542.60
1	3362.55	0	5812.10

Corresponding to C_{22}			
α	$x = 159.25\ \alpha^2 + 1400.25\ \alpha + 1324.5$	α	$x = 202.75\ \alpha^2 - 2197.5\ \alpha + 4878.75$
0	1324.50	1	2884.00
0.1	1466.12	0.9	3065.23
0.2	1610.92	0.8	3250.51
0.3	1758.91	0.7	3439.85
0.4	1910.08	0.6	3633.24
0.5	2064.44	0.5	3830.69
0.6	2221.98	0.4	4032.19
0.7	2382.71	0.3	4237.75
0.8	2546.62	0.2	4447.36
0.9	2713.72	0.1	4661.03
1	2884.00	0	4878.75

Corresponding to C_{23}			
α	$x = 189.25\ \alpha^2 + 1124.5\ \alpha + 979.25$	α	$x = 140.75\ \alpha^2 - 1565\ \alpha + 3717.25$
0	979.25	1	2293.00
0.1	1093.59	0.9	2422.76
0.2	1211.72	0.8	2555.33
0.3	1333.63	0.7	2690.72
0.4	1459.33	0.6	2828.92
0.5	1588.81	0.5	2969.94
0.6	1722.08	0.4	3113.77
0.7	1859.13	0.3	3260.42
0.8	1999.97	0.2	3409.88
0.9	2144.59	0.1	3562.16
1	2293.00	0	3717.25

Corresponding to C_{24}

α	$x = 167.25\,\alpha^2 + 791\,\alpha + 827.75$	α	$x = 125.5\,\alpha^2 - 1191\,\alpha + 2851.5$
0	827.75	1	1786.00
0.1	908.52	0.9	1881.26
0.2	992.64	0.8	1979.02
0.3	1080.10	0.7	2079.30
0.4	1170.91	0.6	2182.08
0.5	1265.06	0.5	2287.38
0.6	1362.56	0.4	2395.18
0.7	1463.40	0.3	2505.50
0.8	1567.59	0.2	2618.32
0.9	1675.12	0.1	2733.66
1	1786.00	0	2851.50

Corresponding to C_{25}

α	$x = 126\,\alpha^2 + 464.5\,\alpha + 455$	α	$x = 87\,\alpha^2 - 814.5\alpha + 1773$
0	455.00	1	1045.50
0.1	502.71	0.9	1110.42
0.2	552.94	0.8	1177.08
0.3	605.69	0.7	1245.48
0.4	660.96	0.6	1315.62
0.5	718.75	0.5	1387.50
0.6	779.06	0.4	1461.12
0.7	841.89	0.3	1536.48
0.8	907.24	0.2	1613.58
0.9	975.11	0.1	1692.42
1	1045.50	0	1773.00

Corresponding to C_{26}

α	$x = 76\,\alpha^2 + 311.5\,\alpha + 211$	α	$x = 37\,\alpha^2 - 411\,\alpha + 972.5$
0	211.00	1	598.50
0.1	242.91	0.9	632.57
0.2	276.34	0.8	667.38
0.3	311.29	0.7	702.93
0.4	347.76	0.6	739.22
0.5	385.75	0.5	776.25
0.6	425.26	0.4	814.02
0.7	466.29	0.3	852.53
0.8	508.84	0.2	891.78
0.9	552.91	0.1	931.77
1	598.50	0	972.50

Corresponding to C_{31}			
α	$x = 202.55\,\alpha^2 + 1552.7\,\alpha + 1655.7$	α	$x = 234\,\alpha^2 - 2228.4\,\alpha + 5405.4$
0	1655.75	1	3411.00
0.1	1813.05	0.9	3589.38
0.2	1974.39	0.8	3772.44
0.3	2139.79	0.7	3960.18
0.4	2309.24	0.6	4152.60
0.5	2482.74	0.5	4349.70
0.6	2660.29	0.4	4551.48
0.7	2841.89	0.3	4757.94
0.8	3027.54	0.2	4969.08
0.9	3217.25	0.1	5184.90
1	3411.00	0	5405.40

Corresponding to C_{32}			
α	$x = 200.75\,\alpha^2 + 1160\,\alpha + 1504.25$	α	$x = 179\,\alpha^2 - 1792\,\alpha + 4478$
0	1504.25	1	2865.00
0.1	1622.26	0.9	3010.19
0.2	1744.28	0.8	3158.96
0.3	1870.32	0.7	3311.31
0.4	2000.37	0.6	3467.24
0.5	2134.44	0.5	3626.75
0.6	2272.52	0.4	3789.84
0.7	2414.62	0.3	3956.51
0.8	2560.73	0.2	4126.76
0.9	2710.86	0.1	4300.59
1	2865.00	0	4478.00

Corresponding to C_{33}			
α	$x = 214\,\alpha^2 + 956.5\,\alpha + 1047.5$	α	$x = 124.5\,\alpha^2 + 1236.5\,\alpha + 3330$
0	1047.50	1	2218.00
0.1	1145.29	0.9	2318.00
0.2	1247.36	0.8	2420.48
0.3	1353.71	0.7	2525.46
0.4	1464.34	0.6	2632.92
0.5	1579.25	0.5	2742.88
0.6	1698.44	0.4	2855.32
0.7	1821.91	0.3	2970.26
0.8	1949.66	0.2	3087.68
0.9	2081.69	0.1	3207.60
1	2218.00	0	3330.00

Corresponding to C_{34}			
α	$x = 148\ \alpha^2 + 847\ \alpha + 688.5$	α	$x = 79.5\ \alpha^2 + 874.5\ \alpha + 2478.5$
0	688.50	1	1683.50
0.1	774.68	0.9	1755.85
0.2	863.82	0.8	1829.78
0.3	955.92	0.7	1905.31
0.4	1050.98	0.6	1982.42
0.5	1149.00	0.5	2061.13
0.6	1249.98	0.4	2141.42
0.7	1353.92	0.3	2223.31
0.8	1460.82	0.2	2306.78
0.9	1570.68	0.1	2391.85
1	1683.50	0	2478.50

Corresponding to C_{35}			
α	$x = 113\ \alpha^2 + 483\ \alpha + 449.5$	α	$x = 55.5\ \alpha^2 + 579.5\ \alpha + 1569.5$
0	449.50	1	1045.50
0.1	498.93	0.9	1092.91
0.2	550.62	0.8	1141.42
0.3	604.57	0.7	1191.05
0.4	660.78	0.6	1241.78
0.5	719.25	0.5	1293.63
0.6	779.98	0.4	1346.58
0.7	842.97	0.3	1400.65
0.8	908.22	0.2	1455.82
0.9	975.73	0.1	1512.11
1	1045.50	0	1569.50

Corresponding to C_{36}			
α	$x = 74.5\ \alpha^2 + 303.5\ \alpha + 283$	α	$x = 15\ \alpha^2 + 249\ \alpha + 895$
0	283.00	1	661.00
0.1	314.10	0.9	683.05
0.2	346.68	0.8	705.40
0.3	380.76	0.7	728.05
0.4	416.32	0.6	751.00
0.5	453.38	0.5	774.25
0.6	491.92	0.4	797.80
0.7	531.96	0.3	821.65
0.8	573.48	0.2	845.80
0.9	616.50	0.1	870.25
1	661.00	0	895.00

Corresponding to C_{41}			
α	$x = 214.35\ \alpha^2 + 1455.45\ \alpha + 1733$	α	$x = 257\ \alpha^2 + 2592.3\ \alpha + 5738.1$
0	1733.00	1	3402.80
0.1	1880.69	0.9	3613.20
0.2	2032.66	0.8	3828.74
0.3	2188.93	0.7	4049.42
0.4	2349.48	0.6	4275.24
0.5	2514.31	0.5	4506.20
0.6	2683.44	0.4	4742.30
0.7	2856.85	0.3	4983.54
0.8	3034.54	0.2	5229.92
0.9	3216.53	0.1	5481.44
1	3402.80	0	5738.10

Corresponding to C_{42}			
α	$x = 185\ \alpha^2 + 1144\ \alpha + 1388$	α	$x = 204\ \alpha^2 + 2042.5\ \alpha + 4555.5$
0	1388.00	1	2717.00
0.1	1504.25	0.9	2882.49
0.2	1624.20	0.8	3052.06
0.3	1747.85	0.7	3225.71
0.4	1875.20	0.6	3403.44
0.5	2006.25	0.5	3585.25
0.6	2141.00	0.4	3771.14
0.7	2279.45	0.3	3961.11
0.8	2421.60	0.2	4155.16
0.9	2567.45	0.1	4353.29
1	2717.00	0	4555.50

Corresponding to C_{43}			
α	$x = 199.75\ \alpha^2 + 843.5\ \alpha + 1022.25$	α	$x = 134\ \alpha^2 + 1458.5\ \alpha + 3390$
0	1022.25	1	2065.50
0.1	1108.60	0.9	2185.89
0.2	1198.94	0.8	2308.96
0.3	1293.28	0.7	2434.71
0.4	1391.61	0.6	2563.14
0.5	1493.94	0.5	2694.25
0.6	1600.26	0.4	2828.04
0.7	1710.58	0.3	2964.51
0.8	1824.89	0.2	3103.66
0.9	1943.20	0.1	3245.49
1	2065.50	0	3390.00

Corresponding to C_{44}			
α	$x = 150.25\ \alpha^2 + 702.5\ \alpha + 656.75$	α	$x = 111\ \alpha^2 + 1069.5\ \alpha + 2468$
0	656.75	1	1509.50
0.1	728.50	0.9	1595.36
0.2	803.26	0.8	1683.44
0.3	881.02	0.7	1773.74
0.4	961.79	0.6	1866.26
0.5	1045.56	0.5	1961.00
0.6	1132.34	0.4	2057.96
0.7	1222.12	0.3	2157.14
0.8	1314.91	0.2	2258.54
0.9	1410.70	0.1	2362.16
1	1509.50	0	2468.00

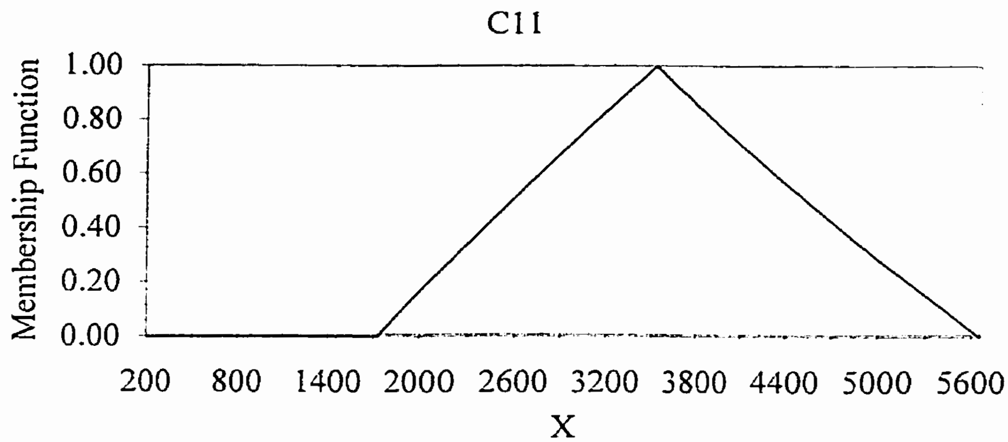
Corresponding to C_{45}			
α	$x = 103\ \alpha^2 + 455\ \alpha + 397.5$	α	$x = 68.5\ \alpha^2 + 661.5\ \alpha + 1548.5$
0	397.50	1	955.50
0.1	444.03	0.9	1008.64
0.2	492.62	0.8	1063.14
0.3	543.27	0.7	1119.02
0.4	595.98	0.6	1176.26
0.5	650.75	0.5	1234.88
0.6	707.58	0.4	1294.86
0.7	766.47	0.3	1356.22
0.8	827.42	0.2	1418.94
0.9	890.43	0.1	1483.04
1	955.50	0	1548.50

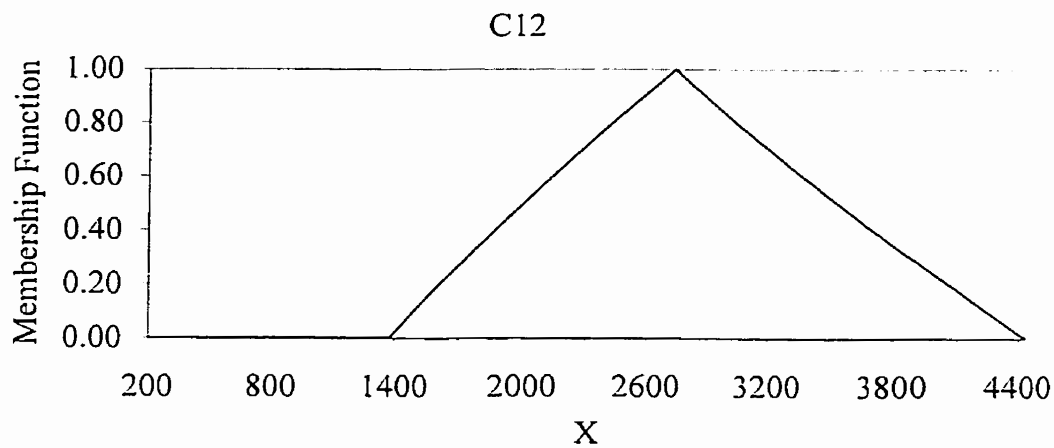
Corresponding to C_{46}			
α	$x = 55\ \alpha^2 + 188\ \alpha + 153$	α	$x = 32.5\ \alpha^2 + 340\ \alpha + 703.5$
0	153.00	1	396.00
0.1	172.35	0.9	423.83
0.2	192.80	0.8	452.30
0.3	214.35	0.7	481.43
0.4	237.00	0.6	511.20
0.5	260.75	0.5	541.63
0.6	285.60	0.4	572.70
0.7	311.55	0.3	604.43
0.8	338.60	0.2	636.80
0.9	366.75	0.1	669.83
1	396.00	0	703.50

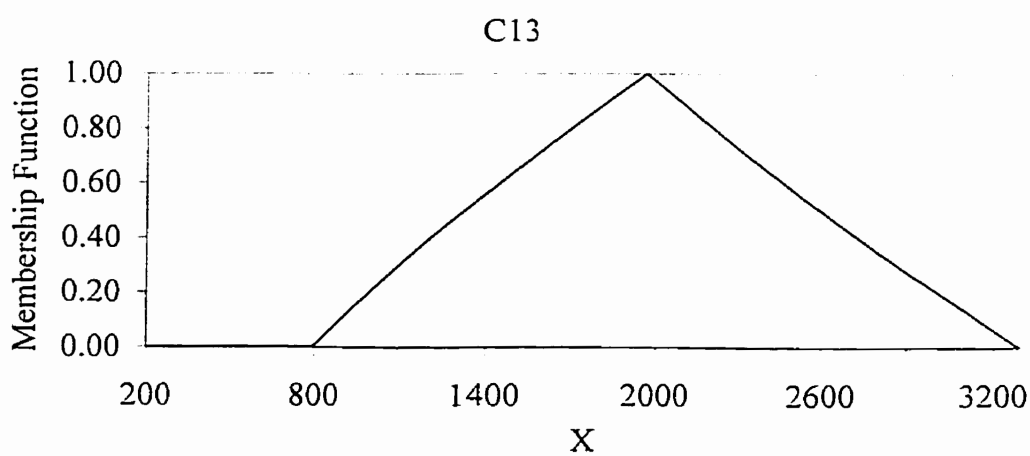
APPENDIX 4

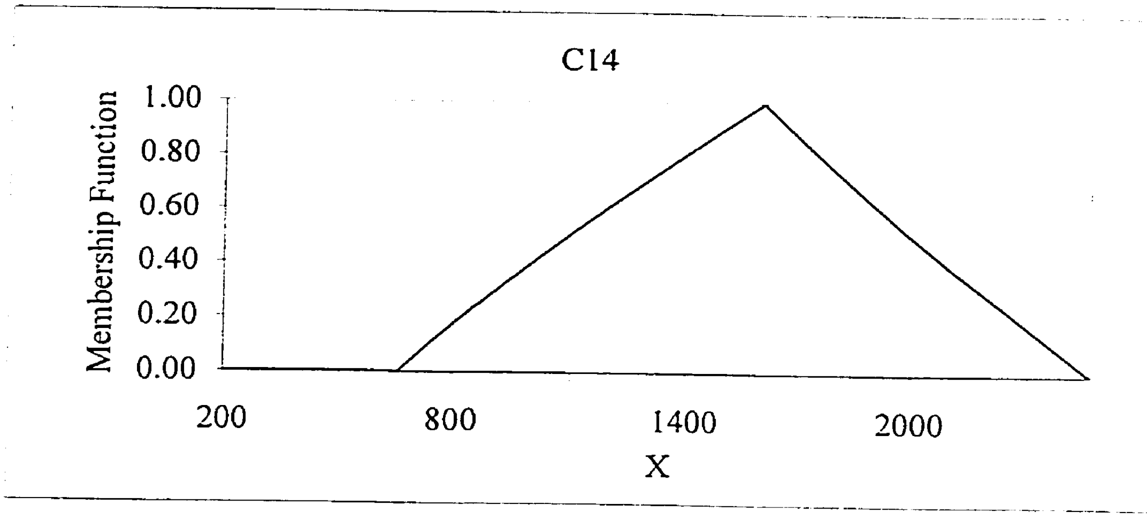
(PAGE 136 – 144)

Graph of Membership Functions for cij 's





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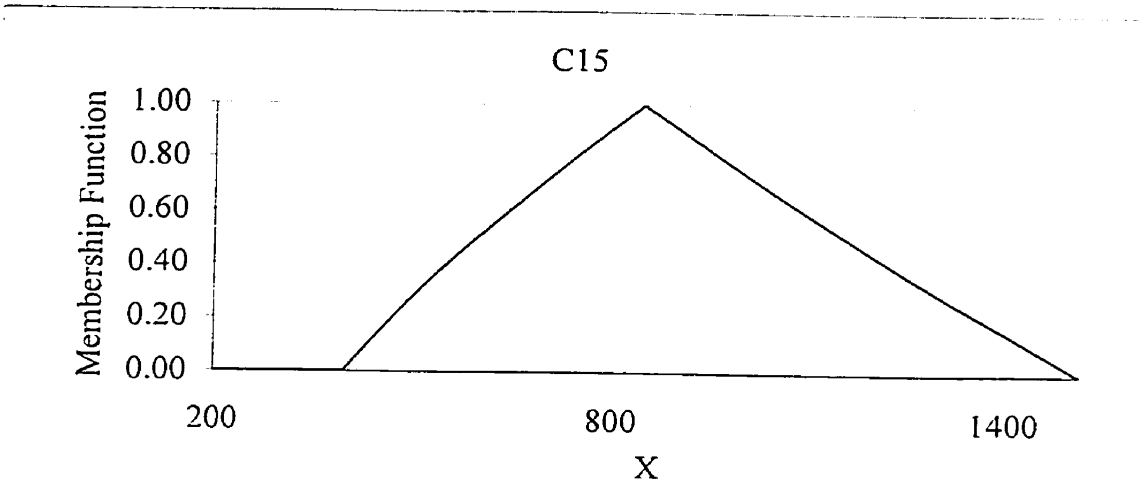
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
X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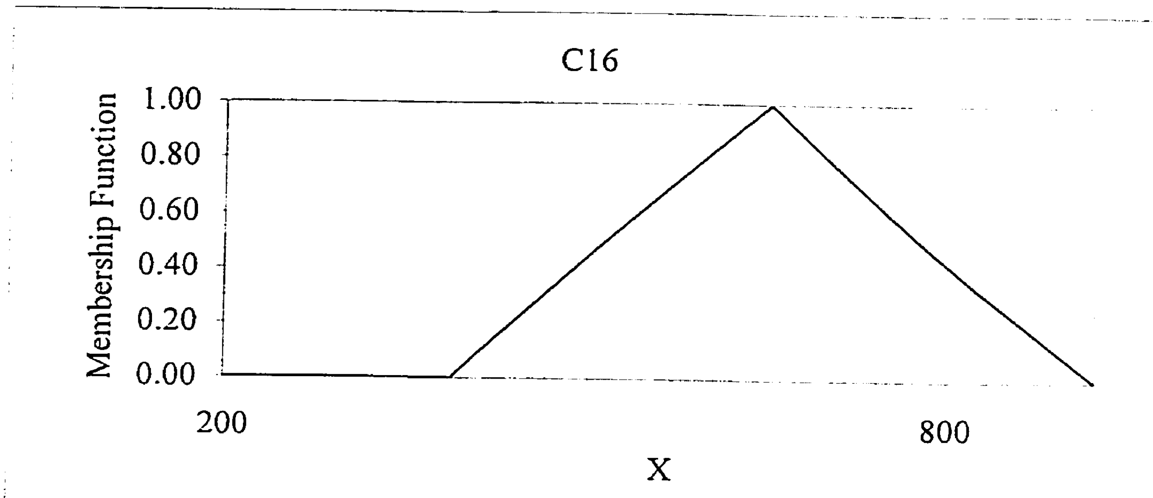
200
800
1400
X


C16
Membership Function
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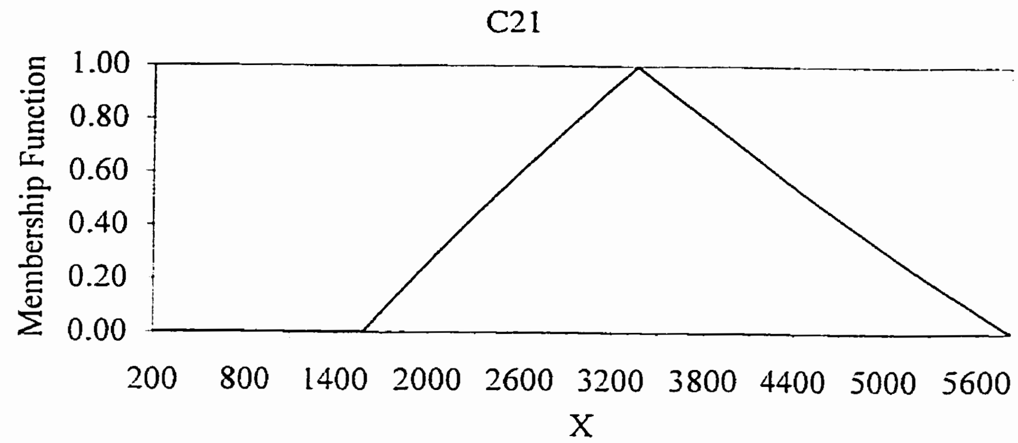
0.80
0.60
0.40
0.20
0.00
200
800
X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Membership Function
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0.80
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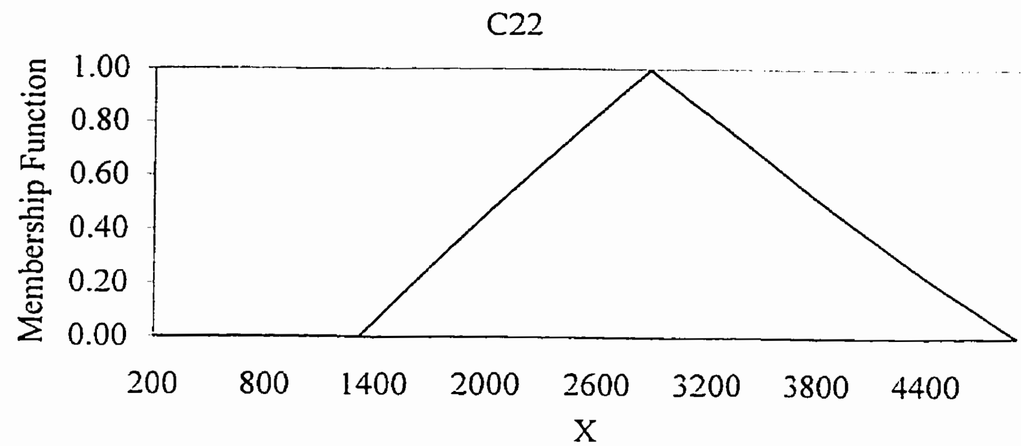
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
5000
5600
X


C22
Membership Function
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0.80
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0.20
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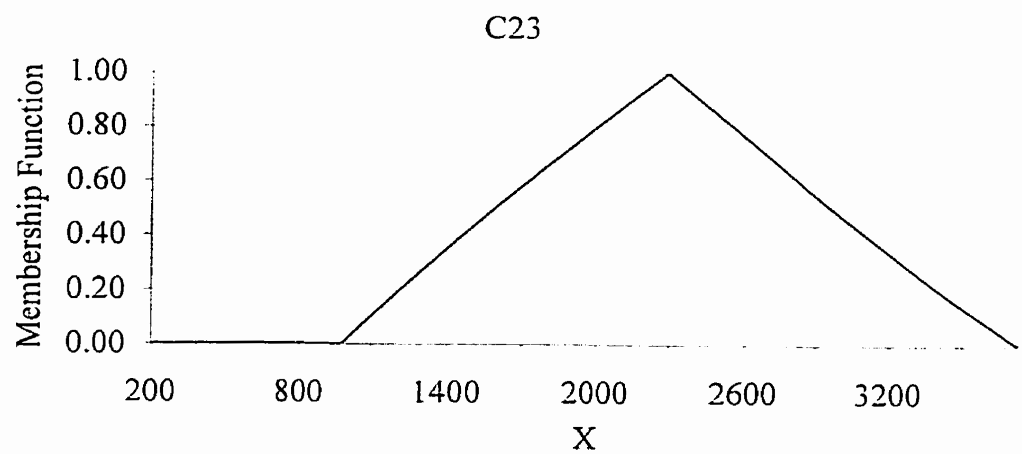
200
800
1400
2000
2600
3200
3800
4400
X


C23
Membership Function
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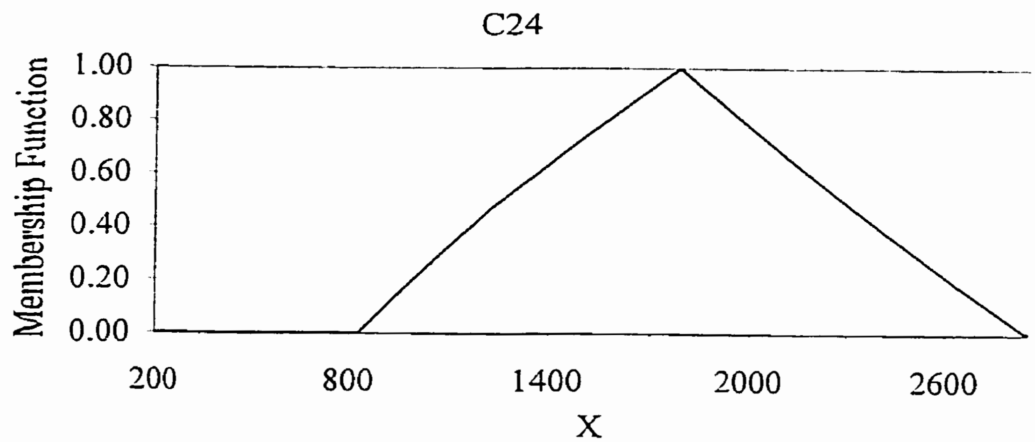
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
X


C24
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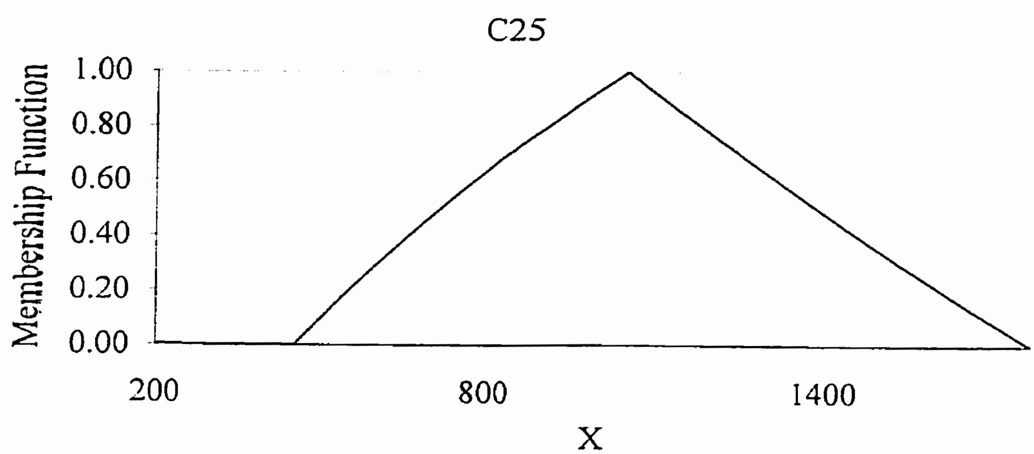
0.40
0.20
0.00
200
800
1400
2000
2600
X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Membership Function
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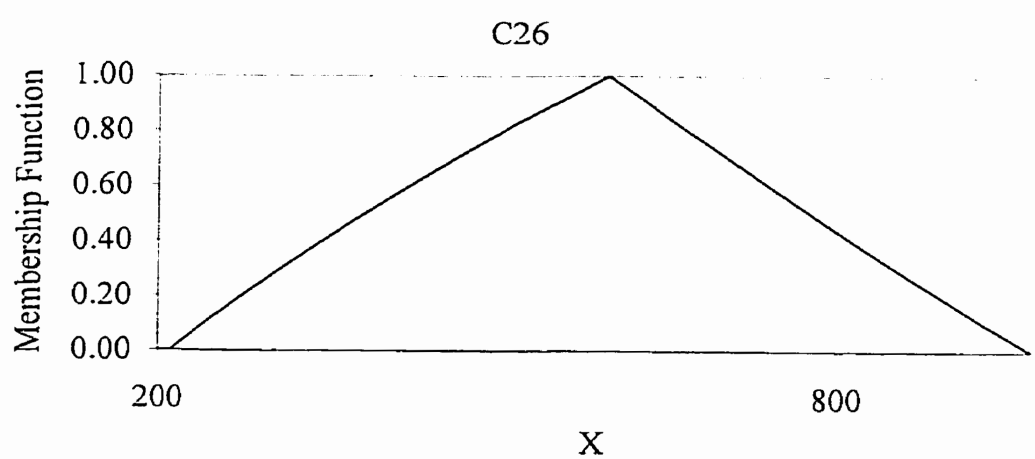
0.00
200
800
1400
X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Membership Function
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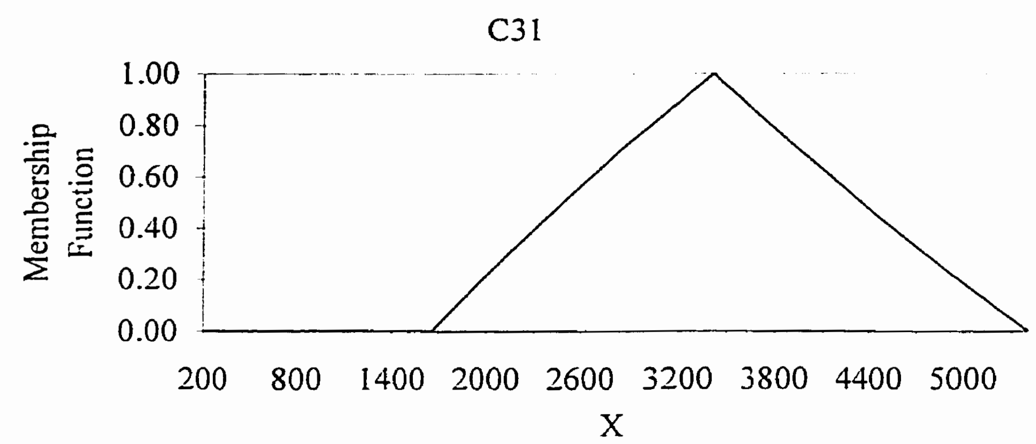
0.80
0.60
0.40
0.20
0.00
200
800
X



C31
Membership Function
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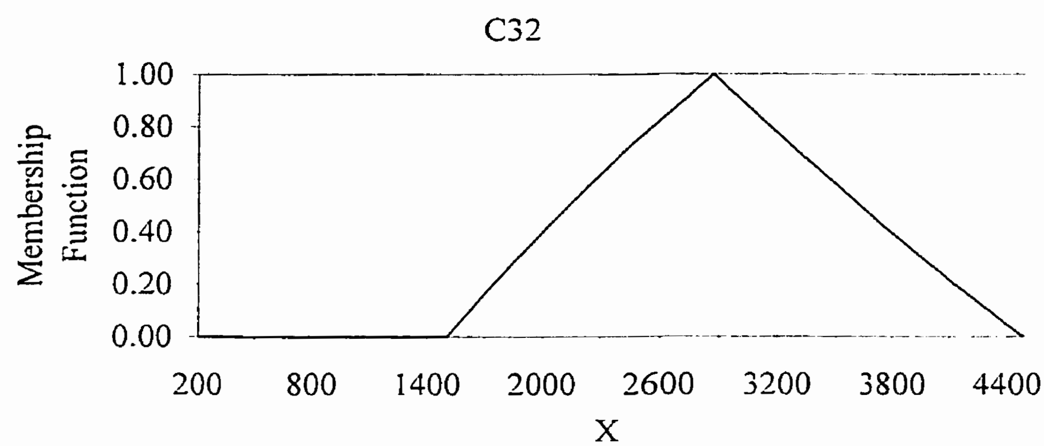
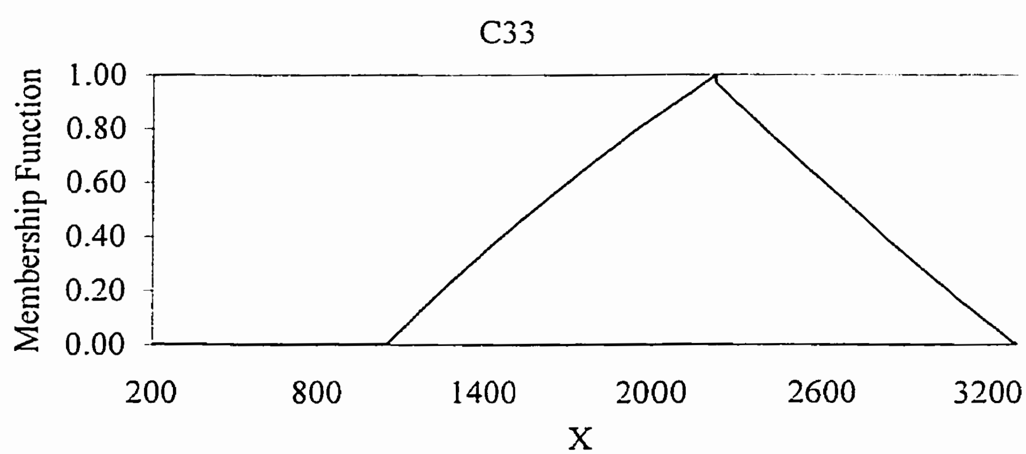
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
5000
X



C32
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
X



C33
Membership Function
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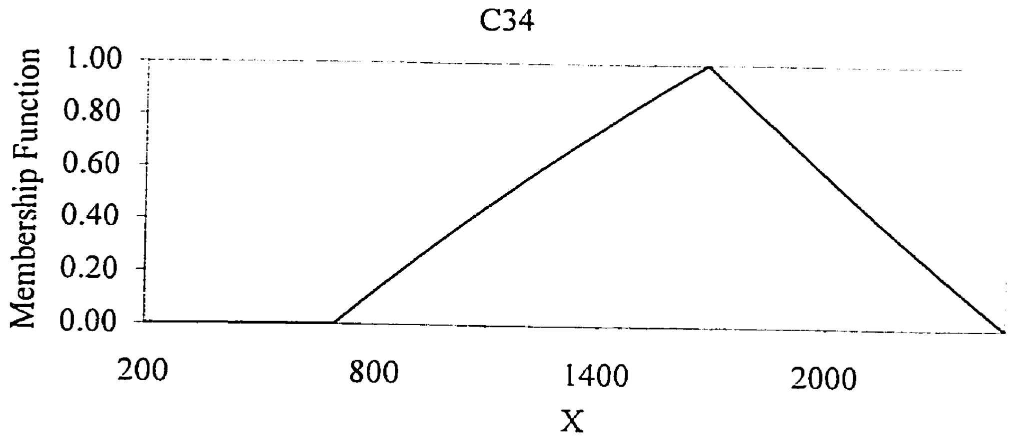
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
X


C34
Membership Function
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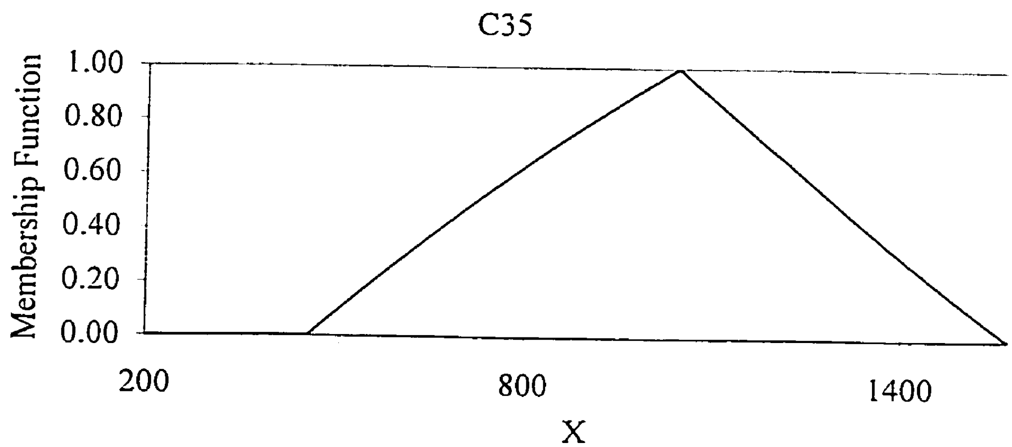
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
X


C35
Membership Function
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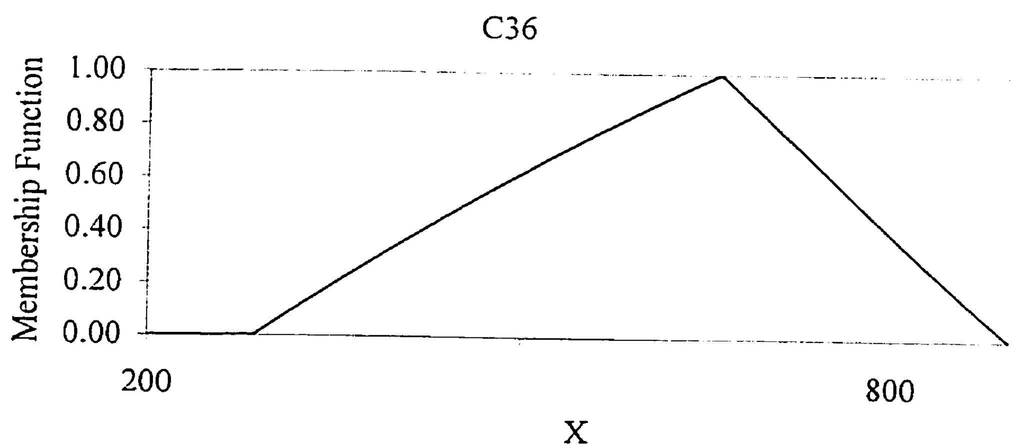
0.80
0.60
0.40
0.20
0.00
200
800
1400
X


C36
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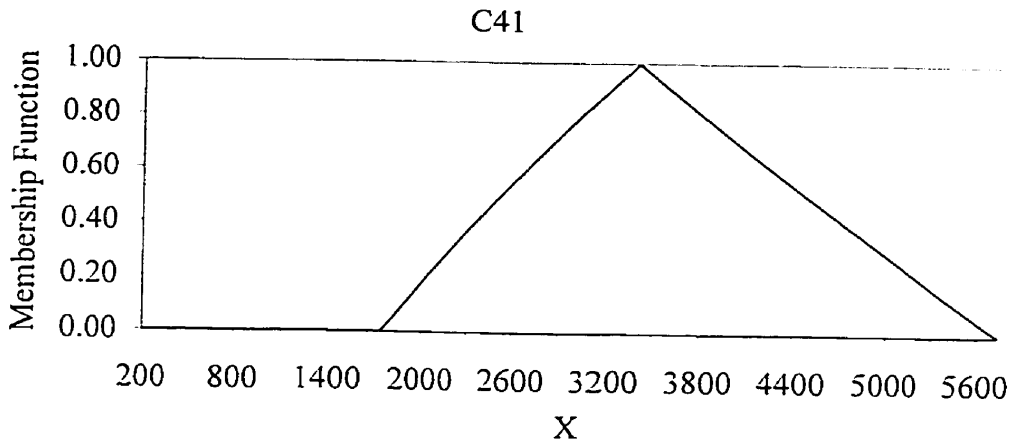
0.40
0.20
0.00
200
800
X


C41
Membership Function
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0.60
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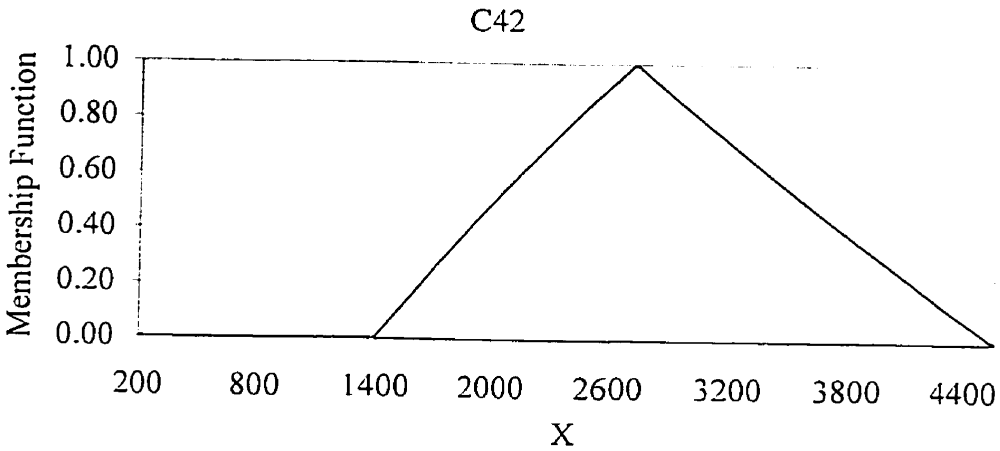
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
5000
5600
X


C42
Membership Function
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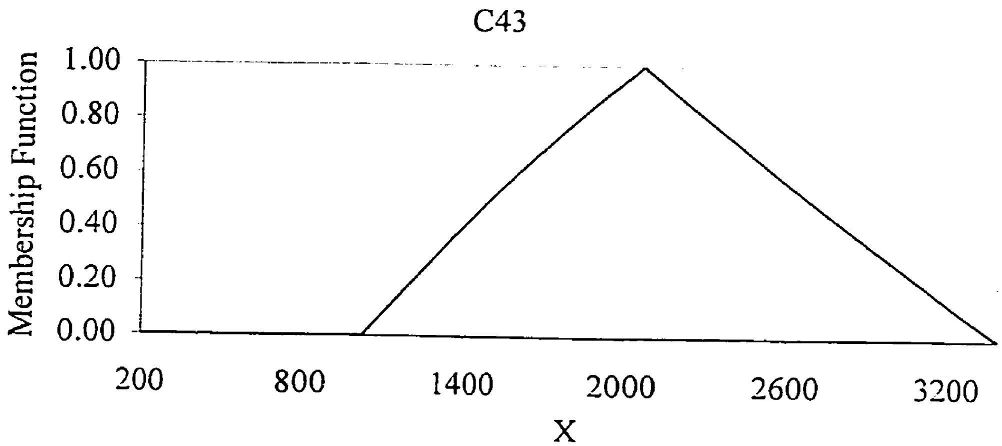
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
X


C43
Membership Function
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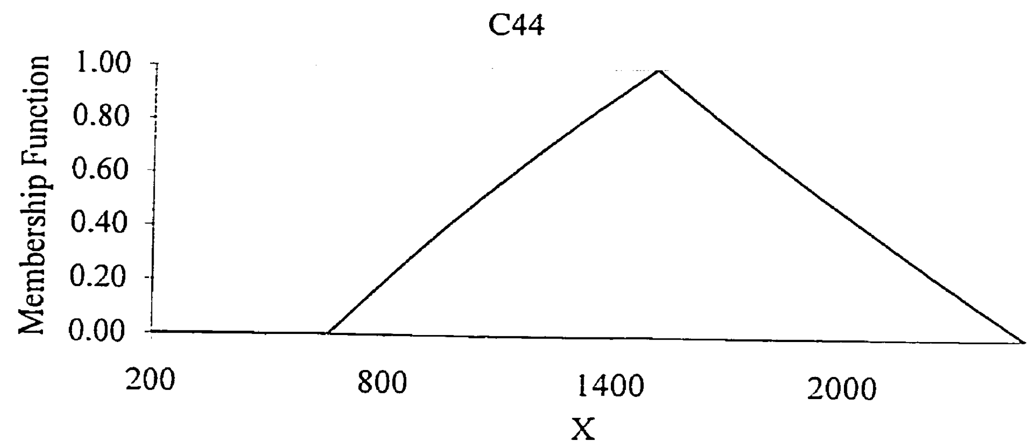
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
X


C44
Membership Function
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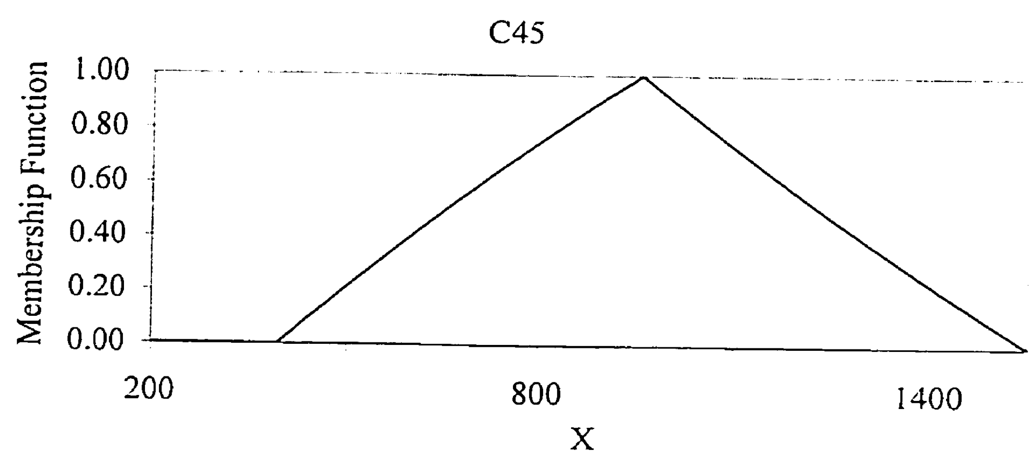
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
X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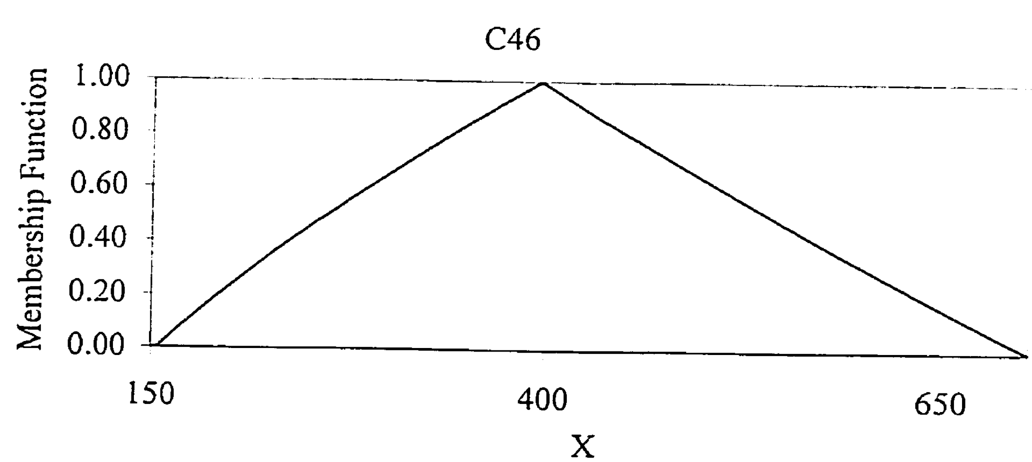
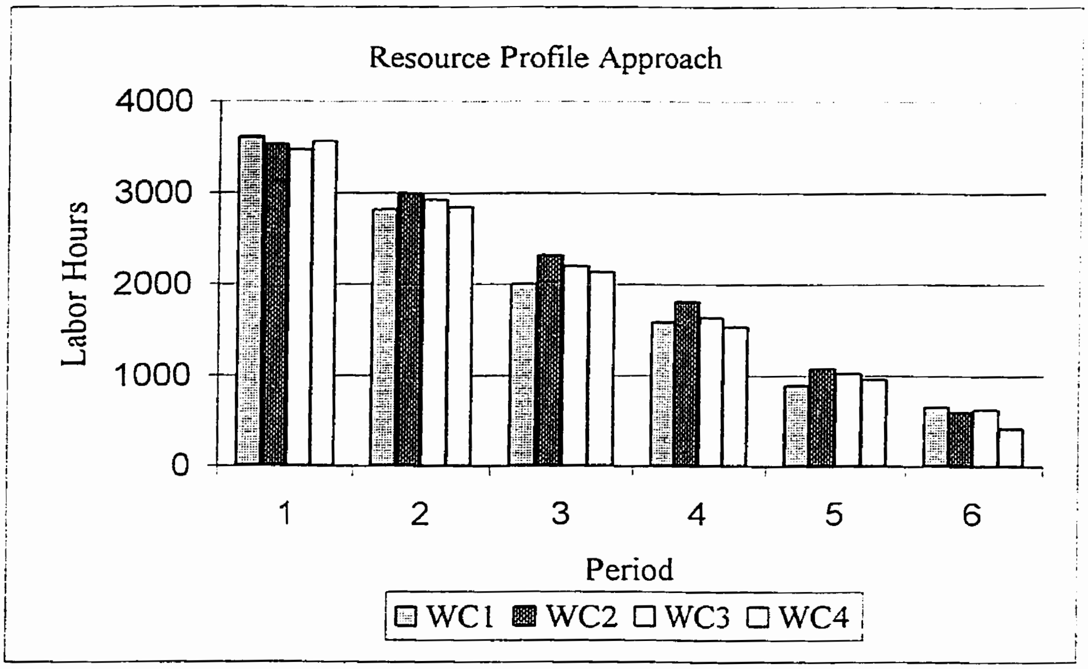
0.40
0.20
0.00
200
800
1400
X


C46
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
150
400
650
X

Resource Profile Approach

From Table 4.9, we calculate the average fuzzy capacities in Work Center i for Period j.

Period	1	2	3	4	5	6
WC1	3610	2819	2002	1584	899	652.9
WC2	3530	2993	2321	1813	1080	595.1
WC3	3471	2928	2196	1634	1028	625
WC4	3569	2844	2136	1536	964	412



APPENDIX 5

(PAGE 145 – 150)

Interval of Confidence for $0 \leq \alpha \leq 1$

Corresponding to C_{11}

α	$x = 11.25\,\alpha^2 + 1601.25\,\alpha + 3147.5$	α	$x = 4.5\,\alpha^2 - 1131.5\,\alpha + 5887$
0	3147.50	1	4760.00
0.1	3307.74	0.9	4872.30
0.2	3468.20	0.8	4984.68
0.3	3628.89	0.7	5097.16
0.4	3789.80	0.6	5209.72
0.5	3950.94	0.5	5322.38
0.6	4112.30	0.4	5435.12
0.7	4273.89	0.3	5547.96
0.8	4435.70	0.2	5660.88
0.9	4597.74	0.1	5773.90
1	4760.00	0	5887.00

Corresponding to C_{12}

α	$x = 6.25\,\alpha^2 + 1131.5\,\alpha + 1220$	α	$x = 6.5\alpha^2 - 658\,\alpha + 2236.5$
0	1220.00	1	1585.00
0.1	1255.94	0.9	1649.57
0.2	1292.00	0.8	1714.26
0.3	1328.19	0.7	1779.09
0.4	1364.50	0.6	1844.04
0.5	1400.94	0.5	1909.13
0.6	1437.50	0.4	1974.34
0.7	1474.19	0.3	2039.69
0.8	1511.00	0.2	2105.16
0.9	1547.94	0.1	2170.77
1	1585.00	0	2236.50

Corresponding to C_{13}

α	$x = 6.5\,\alpha^2 + 1397\,\alpha + 2416.5$	α	$x = 4\,\alpha^2 - 800.5\,\alpha + 4616.5$
0	2416.50	1	3820.00
0.1	2556.27	0.9	3899.29
0.2	2696.16	0.8	3978.66
0.3	2836.19	0.7	4058.11
0.4	2976.34	0.6	4137.64
0.5	3116.63	0.5	4217.25
0.6	3257.04	0.4	4296.94
0.7	3397.59	0.3	4376.71
0.8	3538.26	0.2	4456.56
0.9	3679.07	0.1	4536.49
1	3820.00	0	4616.50

Corresponding to C_{14}			
α	$x = 5.75\alpha^2 + 307.25\alpha + 1057$	α	$x = 4.5\alpha^2 - 549\alpha + 1914.5$
0	1057.00	1	1370.00
0.1	1087.78	0.9	1424.05
0.2	1118.68	0.8	1478.18
0.3	1149.69	0.7	1532.41
0.4	1180.82	0.6	1586.72
0.5	1212.06	0.5	1641.13
0.6	1243.42	0.4	1695.62
0.7	1274.89	0.3	1750.21
0.8	1306.48	0.2	1804.88
0.9	1338.18	0.1	1859.65
1	1370.00	0	1914.50

Corresponding to C_{15}			
α	$x = 5.5\alpha^2 + 171.5\alpha + 568$	α	$x = 3\alpha^2 - 178.5\alpha + 920.5$
0	568.00	1	745.00
0.1	585.21	0.9	762.28
0.2	602.52	0.8	779.62
0.3	619.95	0.7	797.02
0.4	637.48	0.6	814.48
0.5	655.13	0.5	832.00
0.6	672.88	0.4	849.58
0.7	690.75	0.3	867.22
0.8	708.72	0.2	884.92
0.9	726.81	0.1	902.68
1	745.00	0	920.50

Corresponding to C_{16}			
α	$x = 6.75\alpha^2 + 1532.75\alpha + 2905.5$	α	$x = 5.5\alpha^2 - 1171\alpha + 5610.5$
0	2905.50	1	4445.00
0.1	3058.84	0.9	4561.06
0.2	3212.32	0.8	4677.22
0.3	3365.93	0.7	4793.50
0.4	3519.68	0.6	4909.88
0.5	3673.56	0.5	5026.38
0.6	3827.58	0.4	5142.98
0.7	3981.73	0.3	5259.70
0.8	4136.02	0.2	5376.52
0.9	4290.44	0.1	5493.46
1	4445.00	0	5610.50

Corresponding to C_{21}			
α	$x = 1.75\,\alpha^2 + 436.75\,\alpha + 4706.5$	α	$x = 2\,\alpha^2 - 716.5\,\alpha + 5859.5$
0	4706.50	1	5145.00
0.1	4750.19	0.9	5216.27
0.2	4793.92	0.8	5287.58
0.3	4837.68	0.7	5358.93
0.4	4881.48	0.6	5430.32
0.5	4925.31	0.5	5501.75
0.6	4969.18	0.4	5573.22
0.7	5013.08	0.3	5644.73
0.8	5057.02	0.2	5716.28
0.9	5100.99	0.1	5787.87
1	5145.00	0	5859.50

Corresponding to C_{22}			
α	$x = 1.25\,\alpha^2 + 307.5\,\alpha + 616.25$	α	$x = 1\,\alpha^2 - 177\,\alpha + 1101$
0	616.25	1	925.00
0.1	647.01	0.9	942.51
0.2	677.80	0.8	960.04
0.3	708.61	0.7	977.59
0.4	739.45	0.6	995.16
0.5	770.31	0.5	1012.75
0.6	801.20	0.4	1030.36
0.7	832.11	0.3	1047.99
0.8	863.05	0.2	1065.64
0.9	894.01	0.1	1083.31
1	925.00	0	1101.00

Corresponding to C_{23}			
α	$x = 1.25\,\alpha^2 + 307.5\,\alpha + 616.25$	α	$x = 1\,\alpha^2 - 177\,\alpha + 1101$
0	616.25	1	925.00
0.1	647.01	0.9	942.51
0.2	677.80	0.8	960.04
0.3	708.61	0.7	977.59
0.4	739.45	0.6	995.16
0.5	770.31	0.5	1012.75
0.6	801.20	0.4	1030.36
0.7	832.11	0.3	1047.99
0.8	863.05	0.2	1065.64
0.9	894.01	0.1	1083.31
1	925.00	0	1101.00

Corresponding to C_{24}

α	$x = 1\ \alpha^2 + 372.5\ \alpha + 4461.5$	α	$x = 1.5\ \alpha^2 - 673\ \alpha + 5506.5$
0	4461.50	1	4835.00
0.1	4498.76	0.9	4902.02
0.2	4536.04	0.8	4969.06
0.3	4573.34	0.7	5036.14
0.4	4610.66	0.6	5103.24
0.5	4648.00	0.5	5170.38
0.6	4685.36	0.4	5237.54
0.7	4722.74	0.3	5304.74
0.8	4760.14	0.2	5371.96
0.9	4797.56	0.1	5439.22
1	4835.00	0	5506.50

Corresponding to C_{25}

α	$x = 1.25\ \alpha^2 + 307.5\ \alpha + 616.25$	α	$x = 1\ \alpha^2 - 177\ \alpha + 1101$
0	616.25	1	925.00
0.1	647.01	0.9	942.51
0.2	677.80	0.8	960.04
0.3	708.61	0.7	977.59
0.4	739.45	0.6	995.16
0.5	770.31	0.5	1012.75
0.6	801.20	0.4	1030.36
0.7	832.11	0.3	1047.99
0.8	863.05	0.2	1065.64
0.9	894.01	0.1	1083.31
1	925.00	0	1101.00

Corresponding to C_{26}

α	$x = .75\ \alpha^2 + 64.25\ \alpha + 245$	α	$x = .5\ \alpha^2 - 43.5\ \alpha + 353$
0	245.00	1	310.00
0.1	251.43	0.9	314.26
0.2	257.88	0.8	318.52
0.3	264.34	0.7	322.80
0.4	270.82	0.6	327.08
0.5	277.31	0.5	331.38
0.6	283.82	0.4	335.68
0.7	290.34	0.3	340.00
0.8	296.88	0.2	344.32
0.9	303.43	0.1	348.66
1	310.00	0	353.00

Corresponding to C_{31}

α	$x = .5\,\alpha^2 + 245.5\,\alpha + 489$	α	$x = .5\,\alpha^2 - 139\,\alpha + 873.5$
0	489.00	1	735.00
0.1	513.56	0.9	748.81
0.2	538.12	0.8	762.62
0.3	562.70	0.7	776.45
0.4	587.28	0.6	790.28
0.5	611.88	0.5	804.13
0.6	636.48	0.4	817.98
0.7	661.10	0.3	831.85
0.8	685.72	0.2	845.72
0.9	710.36	0.1	859.61
1	735.00	0	873.50

Corresponding to C_{32}

α	$x = .5\,\alpha^2 + 631.5\,\alpha + 2428$	α	$x = 2.5\,\alpha^2 - 1525\,\alpha + 4582.5$
0	2428.00	1	3060.00
0.1	2491.16	0.9	3212.03
0.2	2554.32	0.8	3364.10
0.3	2617.50	0.7	3516.23
0.4	2680.68	0.6	3668.40
0.5	2743.88	0.5	3820.63
0.6	2807.08	0.4	3972.90
0.7	2870.30	0.3	4125.23
0.8	2933.52	0.2	4277.60
0.9	2996.76	0.1	4430.03
1	3060.00	0	4582.50

Corresponding to C_{33}

α	$x = .5\,\alpha^2 + 245.5\,\alpha + 489$	α	$x = .5\,\alpha^2 - 139\,\alpha + 873.5$
0	489.00	1	735.00
0.1	513.56	0.9	748.81
0.2	538.12	0.8	762.62
0.3	562.70	0.7	776.45
0.4	587.28	0.6	790.28
0.5	611.88	0.5	804.13
0.6	636.48	0.4	817.98
0.7	661.10	0.3	831.85
0.8	685.72	0.2	845.72
0.9	710.36	0.1	859.61
1	735.00	0	873.50

Corresponding to C_{34}			
α	$x = .25\,\alpha^2 + 311.25\,\alpha + 1818.5$	α	$x = 1\,\alpha^2 + 610.5\,\alpha + 2739.5$
0	1818.50	1	2130.00
0.1	1849.63	0.9	2190.86
0.2	1880.76	0.8	2251.74
0.3	1911.90	0.7	2312.64
0.4	1943.04	0.6	2373.56
0.5	1974.19	0.5	2434.50
0.6	2005.34	0.4	2495.46
0.7	2036.50	0.3	2556.44
0.8	2067.66	0.2	2617.44
0.9	2098.83	0.1	2678.46
1	2130.00	0	2739.50

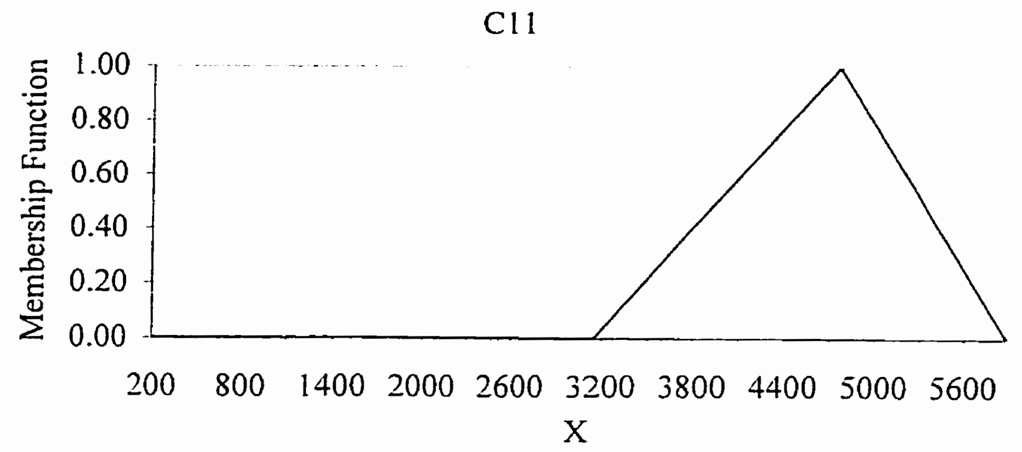
Corresponding to C_{35}			
α	$x = .75\,\alpha^2 + 565.75\,\alpha + 1098.5$	α	$x = 2\,\alpha^2 + 1053.5\,\alpha + 2716.5$
0	1098.50	1	1665.00
0.1	1155.08	0.9	1769.97
0.2	1211.68	0.8	1874.98
0.3	1268.29	0.7	1980.03
0.4	1324.92	0.6	2085.12
0.5	1381.56	0.5	2190.25
0.6	1438.22	0.4	2295.42
0.7	1494.89	0.3	2400.63
0.8	1551.58	0.2	2505.88
0.9	1608.28	0.1	2611.17
1	1665.00	0	2716.50

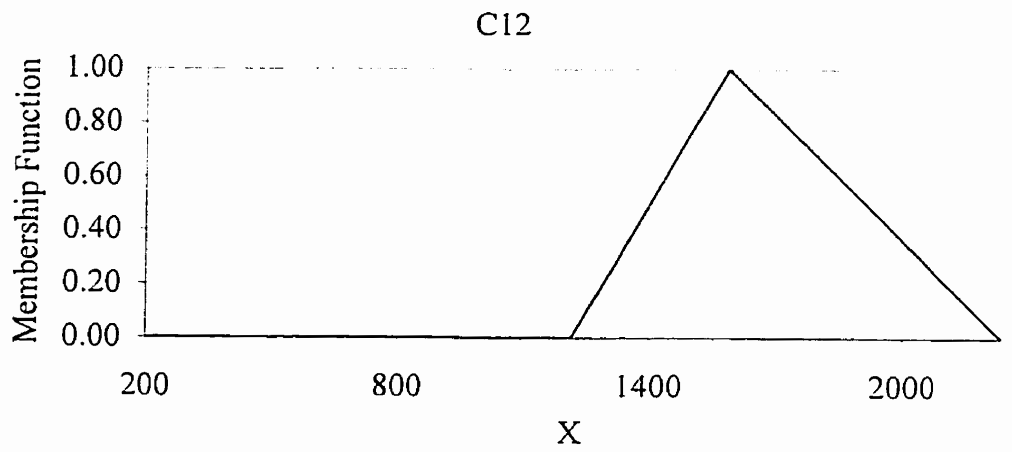
Corresponding to C_{36}			
α	$x = .5\,\alpha^2 + 245.5\,\alpha + 489$	α	$x = .5\,\alpha^2 - 139\,\alpha + 873.5$
0	489.00	1	735.00
0.1	513.56	0.9	748.81
0.2	538.12	0.8	762.62
0.3	562.70	0.7	776.45
0.4	587.28	0.6	790.28
0.5	611.88	0.5	804.13
0.6	636.48	0.4	817.98
0.7	661.10	0.3	831.85
0.8	685.72	0.2	845.72
0.9	710.36	0.1	859.61
1	735.00	0	873.50

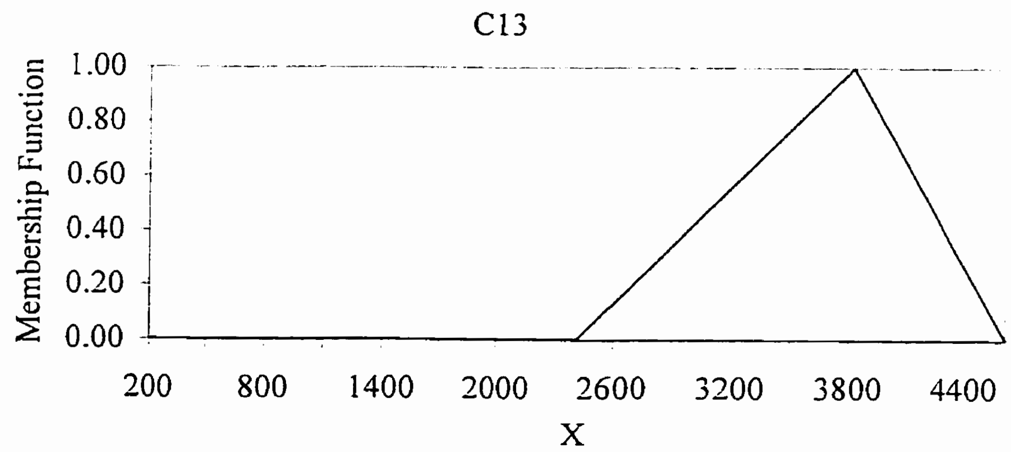
APPENDIX 6

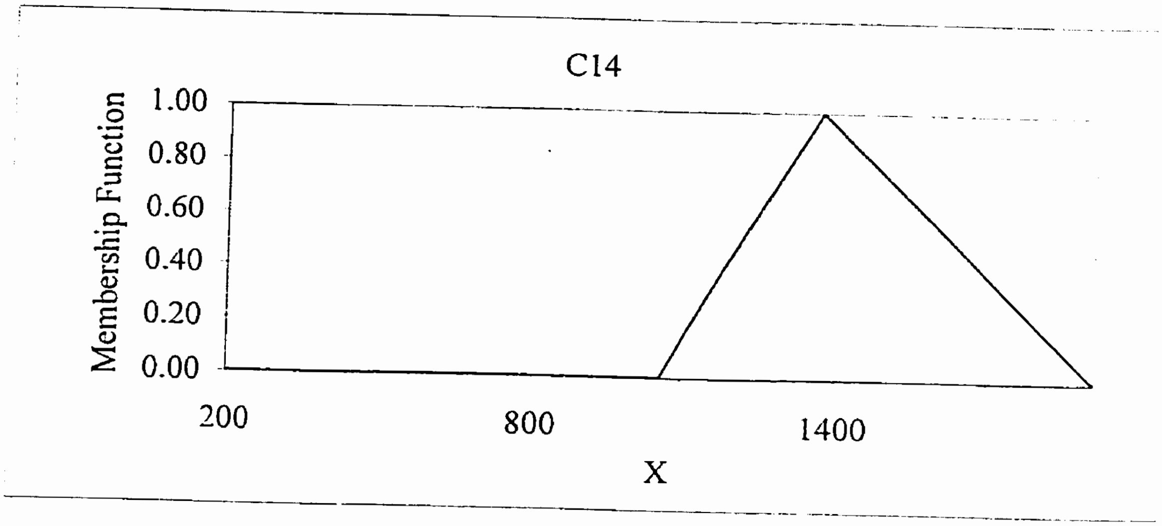
(PAGE 151 – 157)

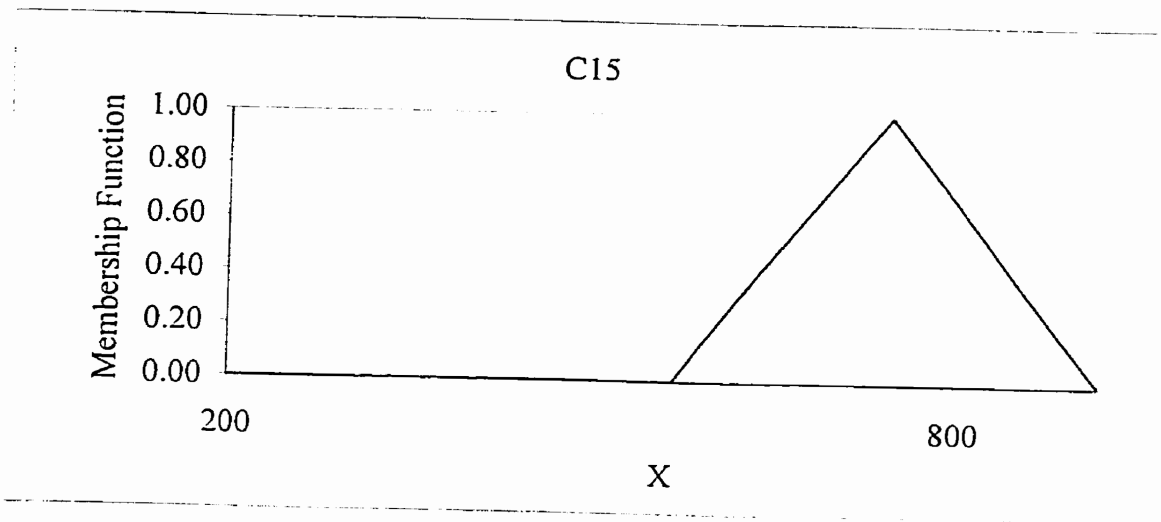
Graph of Membership Functions for cij 's








C14
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
X


C15
Membership Function
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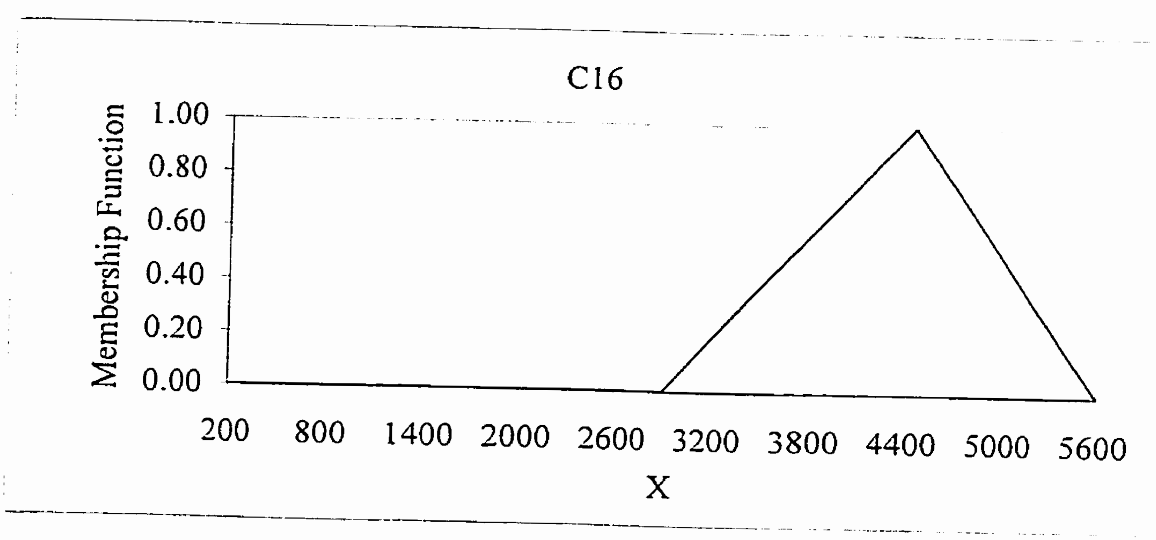
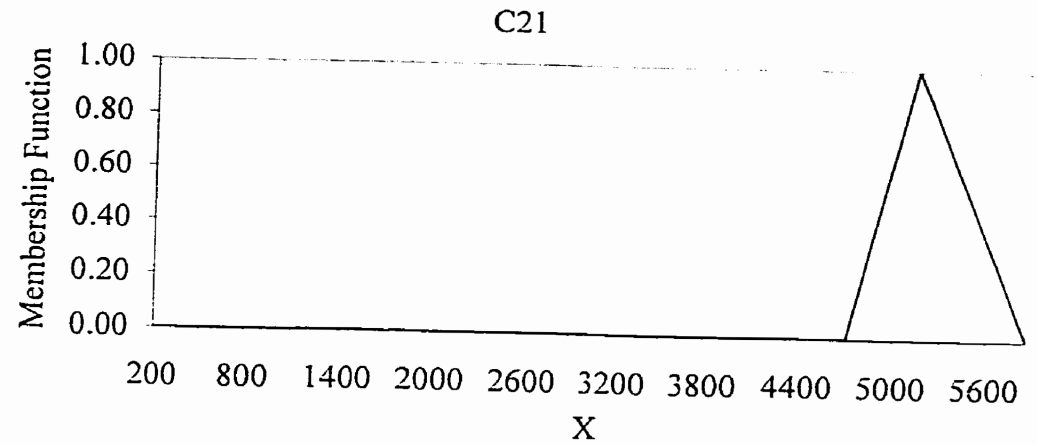
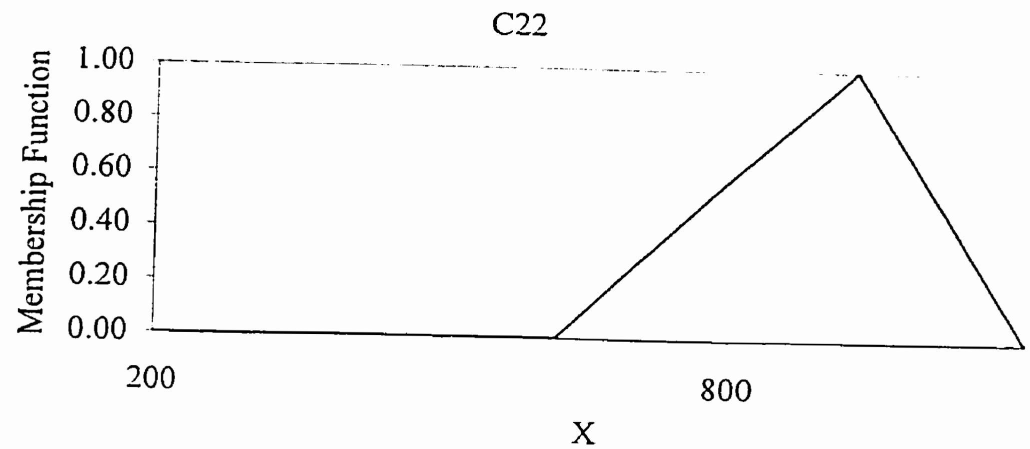
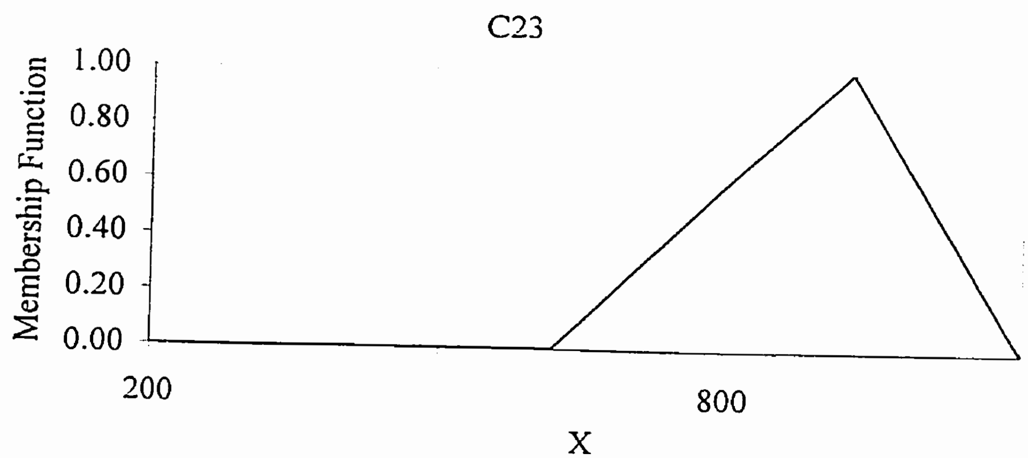
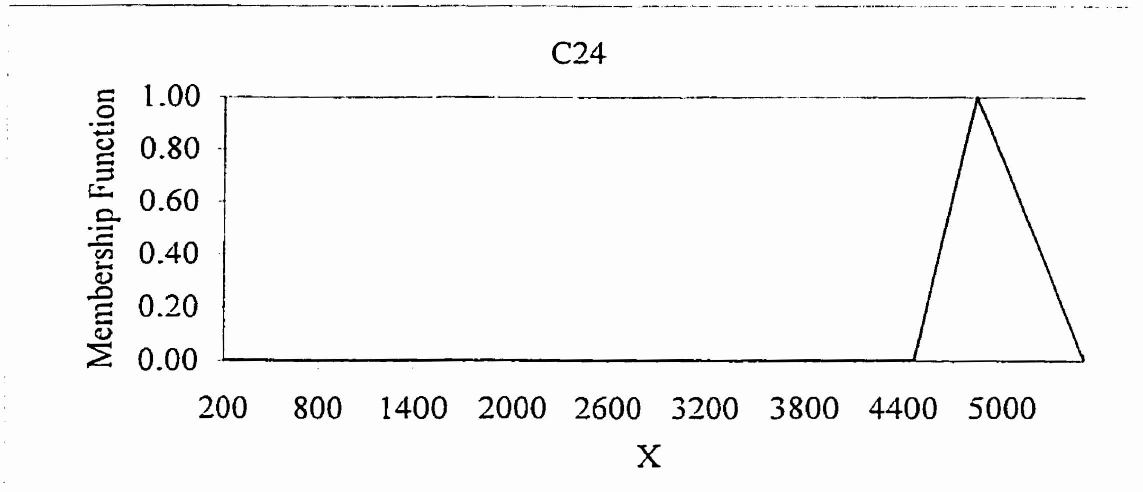
0.80
0.60
0.40
0.20
0.00
200
800
X

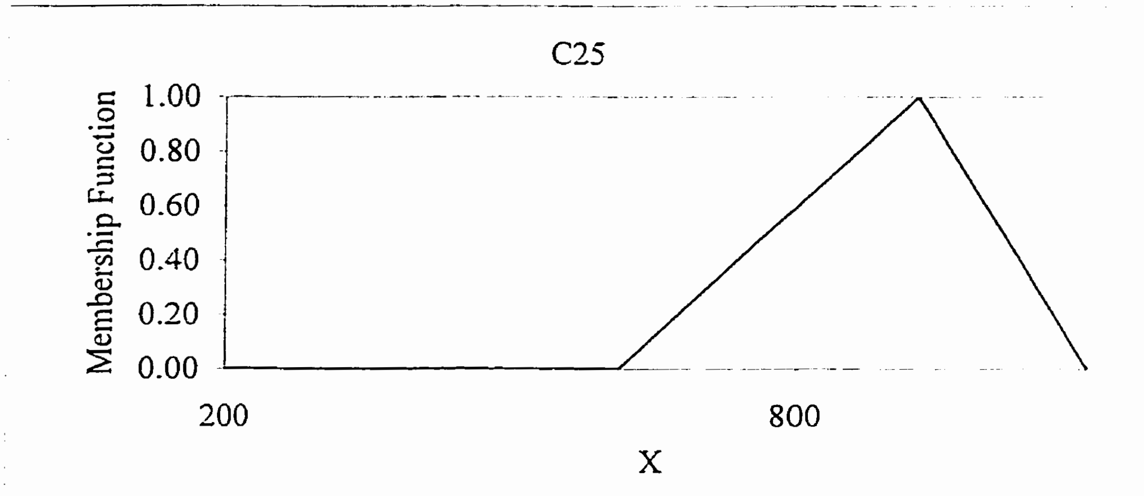

C16
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
5000
5600
X

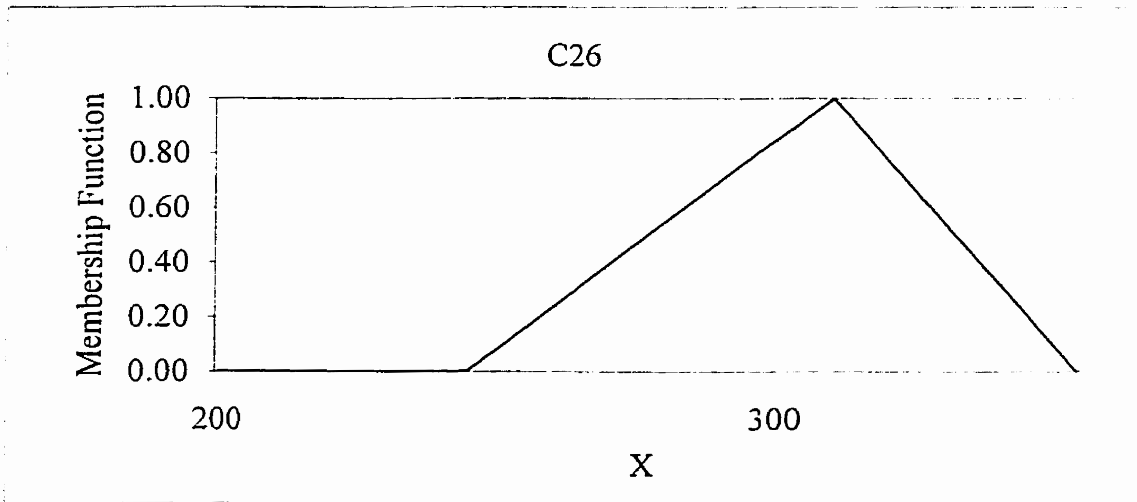

C21
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
5000
5600
X

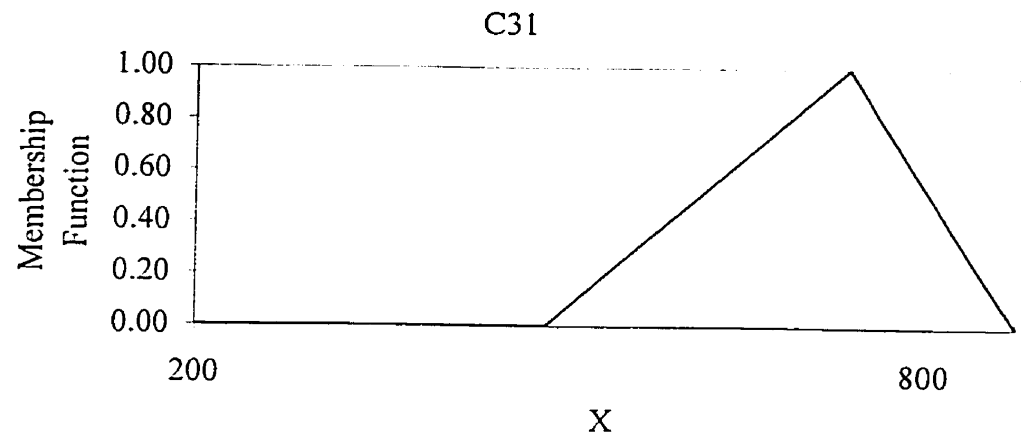

C22
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
X

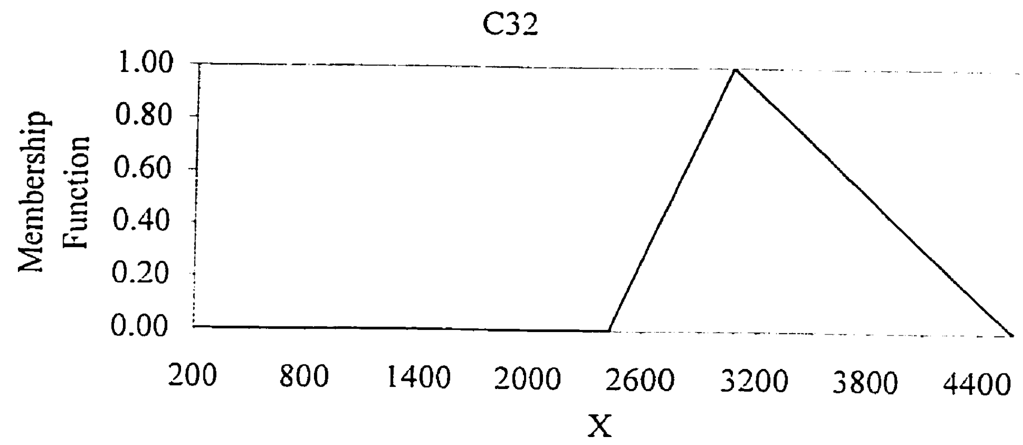

C23
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
X

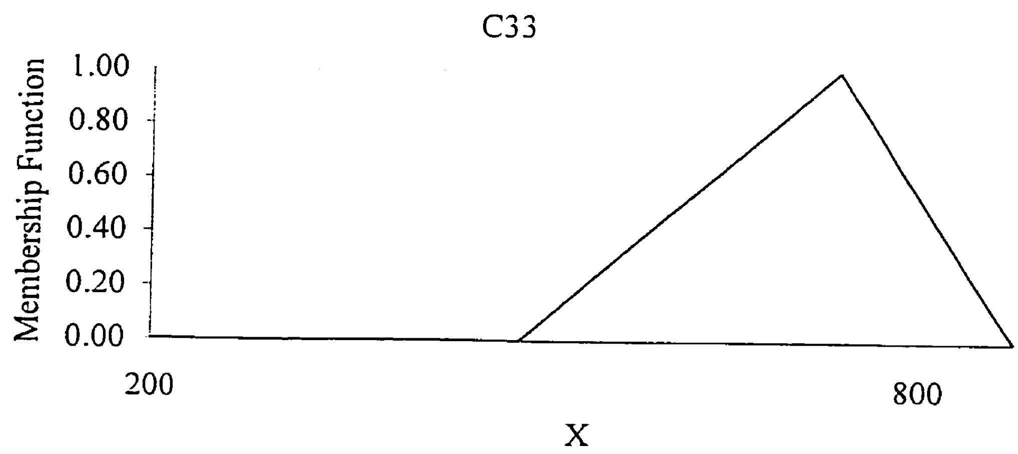

C24
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
3200
3800
4400
5000
X

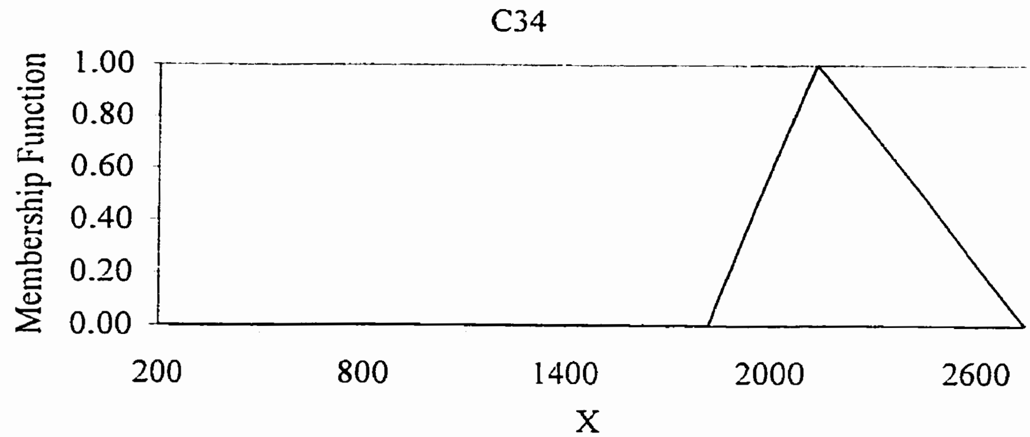

C25
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
X

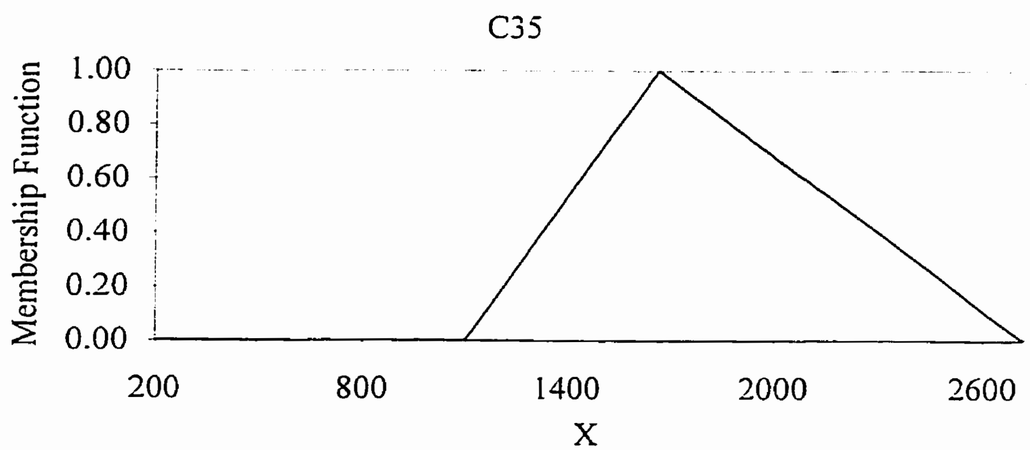

C26
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
300
X

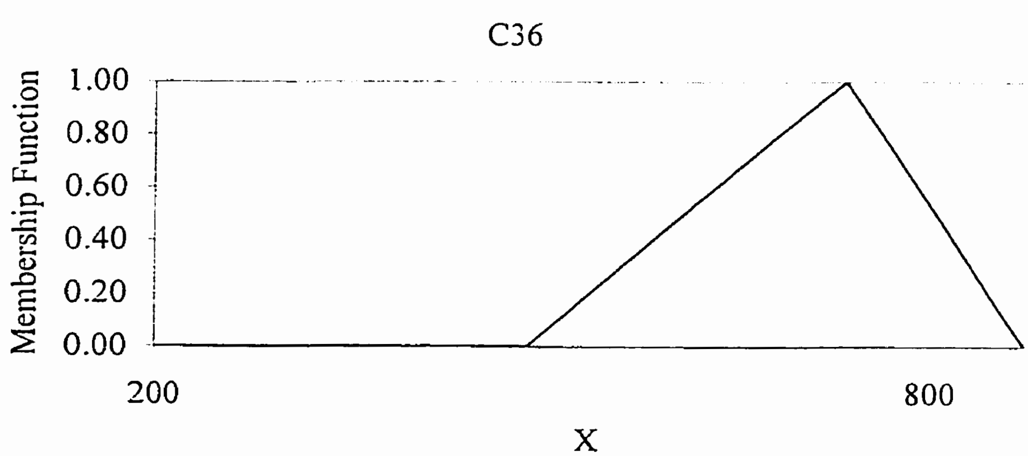

C31
1.00
0.80
0.60
0.40
0.20
0.00
Membership Function
200
800
X


C32
1.00
0.80
0.60
0.40
0.20
0.00
Membership Function
200
800
1400
2000
2600
3200
3800
4400
X


C33
1.00
0.80
0.60
0.40
0.20
0.00
Membership Function
200
800
X


C34
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
X


C35
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
1400
2000
2600
X


C36
Membership Function
1.00
0.80
0.60
0.40
0.20
0.00
200
800
X

From Table 5.12, we calculate the average fuzzy capacities in Work Center i for Period j.

Period	1	2	3	4	5	6
WC1	4639	1657	3668	1428	745	4352
WC2	5214	892	892	4910	892	305
WC3	708	3283	708	2205	1786	708