
A Physical and Mathematical Study of the Supercooling Process and Frazil Evolution

by Shiqiang Ye

A thesis submitted to the Faculty of Graduate Studies in partial
fulfillment of the requirements for the degree of

Doctor of Philosophy

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**A PHYSICAL AND MATHEMATICAL STUDY OF THE SUPERCOOLING
PROCESS AND FRAZIL EVOLUTION**

BY

SHIQIANG YE

**A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University
of Manitoba in partial fulfillment of the requirements of the degree
of
Doctor of Philosophy**

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**I define LOVE thus: The will to extend one's self for
the purpose of nurturing one's own or another's spiritual growth.**

**---Dr. M.S. Peck
"Road-less Traveled --A new psychology of love,
traditional values and spiritual growth"**

For the great LOVE from

my grandma,

and

my parents

who have been giving their love during my life.

Abstract

The supercooling of water in nature and the associated production of frazil can have a profound impact on the management of water resources infrastructure in cold regions. Herein, a series of experiments were carried out using the unique counter-rotating flume that is housed in a computer controlled cold room. A Digital Image Process System (DIPS) was used to observe frazil ice processes. In particular, the effects of air temperature, flow velocity, and bottom roughness on the supercooling and frazil ice processes were examined.

Based on these experimental findings and previous research, a new approach for the supercooling process and frazil evolution is proposed. The principal supercooling process is defined as a process whereby frazil formation reaches the entrained frazil generating capacity of the flow. The residual supercooling stage represents, therefore, a two-layer frazil ice stage, involving both entrained and surface frazil ice. The model describing the frazil size and its distribution in flows is developed based on characteristics of the flow turbulence. Furthermore, an equation estimating the entrained frazil generating capacity of a flow is proposed, which is governed by air temperature and flow turbulence.

A numerical model for the supercooling process and frazil ice evolution is developed. The model avoids the need to simulate seeding, secondary nucleation, flocculation/break-up, and gravitational removal. Only the overall heat balance is considered; consequently, less assumptions and calibrated parameters are needed. The model successfully simulates the characteristics of frazil ice evolution and the supercooling process of water, and as well as the vertical distribution of frazil ice. Sensitivity analysis shows that the cooling rate, the initial ice concentration assumed, and the bed friction factor are much more sensitive than other hydraulic parameters.

This new approach provides an opportunity to estimate the frazil ice characteristics and supercooling period under varying thermo-hydraulic conditions in practical engineering. Besides, the approach and its concepts can be considered fundamentals to various river ice problems, and can be extended to develop models for anchor ice growth during the supercooling process.

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Nomenclature

Symbol	Dimensions	Definition
A	m^2	surface area of open channel
A_f, A_e	m^2	areas of surface and edge of a frazil ice disk
a		constant
a_{floc}		constant in size distribution
B	m	width of channel
b		constant
C		constant
$C, \bar{C}$	%	concentration of frazil ice, and its depth-averaged value
C, C'		Chezy coefficient and gain-related Chezy coefficient
C_d		drag coefficient
C_p	$Jkg^{-1} ^\circ C$	specific heat of water, 4.18×10^3
D	m	diameter of frazil surface
D_e	m	frazil thickness
D_{min}	m	minimum frazil surface diameter
D_{50}	m	mean frazil surface diameter
D_{max}	m	surface diameter of maximum frazil
D_{imax}	m	instantaneous biggest frazil size
D_i	m	instantaneous frazil surface diameter
D_n	m	nominal diameter of frazil ice particles
E	J	turbulence energy
F	N	fall velocity coefficient
F_b	N	buoyancy force of frazil particle
F_t	N	vertical mixing force due to flow turbulence
f		fraction factor
G	J	turbulence energy generation
g	m/s^2	acceleration due to gravity
g'	m/s^2	reduced acceleration due to gravity

H	m	water depth
H_{wi}	$W/m^{2\circ}C$	heat exchange coefficient between water and ice
H_{wi}^{max}	$W/m^{2\circ}C$	heat exchange coefficient at saturated state
h_{wi}	$W/m^{2\circ}C$	heat exchange coefficient between water and single frazil particle
h	$W/m^{2\circ}C$	heat transfer coefficient
h_{wif}	$W/m^{2\circ}C$	heat transfer at frazil face
h_{wie}	$W/m^{2\circ}C$	heat transfer at frazil edge
i		i^{th} time increment
K_v		frazil volumetric factor, 0.785
k		coefficient of carrying capacity
k_s	m	roughness of bed
k_w	$W/m^{2\circ}C$	thermal conductivity of water, 0.5659
$k, \bar{k}$	m^2/s^2	kinetic energy of low turbulence, its depth-averaged value
L_i	J/kg	latent heat of fusion, 3.34×10^5
l_o		Prandtl's mixing length
l_l		characteristic length for the vertical frazil movement
M	%	concentration of frazil ice in volume
M_i	%	instantaneous frazil ice concentration in volume
M_*	%	entrained frazil ice carrying capacity at air temperature of $-10^\circ C$
M_{*T}	%	frazil ice carrying capacity at an air temperature T_a
M_v	%	frazil ice concentration by heat balance
m^*		variable $m^* = \bar{r}/\eta$
N		sample number
N_i		number of frazil ice particles
N_u		Nusselt number
N_{uf}		Nusselt number of frazil surface
N_{ue}		Nusselt number of frazil edge
n		Manning coefficient
$\bar{n}_{max}$		maximum frazil number
P_r		Prandtl number
P		probability of frazil size
$P(D_i)$		probability of frazil less than D_i

Q	m^3/s	flow discharge
Q_{aw}	W	heat exchange rate between air and water
Q_{wi}	W	heat exchange rate between water and ice
Q_i	W	heat due to ice formation
R	m	hydraulic radius
R^2		coefficient of determination
R_e		Reynolds number
r	m	radius of frazil face
$\bar{r}$	m	characteristic length of frazil
T	$^{\circ}C$	water temperature
T_f	$^{\circ}C$	freezing temperature of water
T_e	$^{\circ}C$	equilibrium temperature
T_N	$^{\circ}C$	nucleation temperature
T_c	$^{\circ}C$	characteristic supercooling
T_a	$^{\circ}C$	air temperature
T_b	$^{\circ}C$	bed chamber temperature of the flume
T_{in}	$^{\circ}C$	inner chamber temperature of the flume
T_{out}	$^{\circ}C$	outer chamber temperature of the flume
T_i	$^{\circ}C$	instantaneous water temperature
$T_{min}, \Delta T_{min}$	$^{\circ}C$	maximum supercooling of water
$T_r, \Delta T_r$	$^{\circ}C$	residual water temperature
T_{of}	$^{\circ}C$	offset of the measured water temperature
t	s	length of a time series
t_{sp}	<i>minutes</i>	length of principal supercooling
t_n	s	time of initial appearance of ice particle
t_c	s	period of initial ice formation
U	m/s	water velocity
u_*	m/s	bed shear velocity
u'	m/s	horizontal fluctuating velocity
v'	m/s	vertical fluctuating velocity
W	kg	self-weight of ice particle
y	m	vertical distance measured from the bed

Z		variable in size distribution, $= \sqrt{\beta_1 D_i / \sigma_v}$
z		suspension index
α_T		turbulence intensity
α_H		slope of linear function between heat transfer and frazil ice
β_1	m/s^2	frazil ice coefficient
β_3		factor of channel bed
Δt	s	time step
Δd	km	length of open channel in which generate frazil ice
$\varepsilon, \bar{\varepsilon}$	m^2/s^2	dissipative rate and its depth-averaged
ε_s		diffusion coefficient
η	m	dissipation length
ϕ	$W/m^2 \text{ } ^\circ C$	heat transfer coefficient
$\Phi(Z)$		standard normal probability distribution of Z
μ		constant
ν	m^2/s	kinematic viscosity
ρ	kg/m^3	water density
ρ_i	kg/m^3	ice density
ρ_s	kg/m^3	sediment density
$\sigma_{v'}$	m/s	root mean square of the vertical velocities
$\sigma_{u'}$	m/s	root mean square of the longitudinal velocities
τ	N/m^2	bed shear stress
ω	m/s	rise velocity of frazil particles , or fall velocity of sediment
ξ		relative vertical location $\xi = y / H$
ζ		variable in size distribution, $= \sqrt{\beta_1 D / \sigma_v}$

Note: Watt = Joule per second

Ice has an effect on a significant portion of the populated world. In northern countries, ice represents an environmental hazard that can endanger human lives and influence human activities such as industrial production, transport of goods, housing, buildings and winter sports. These facts have triggered a specialized field of engineering: Ice engineering. The main goal of ice engineering is to protect human life and property against the harmful effects of ice (Ashton, 1986).

1.1 Ice Engineering: A Long History

The presence of ice in rivers is an important phenomenon to be considered in the development of water resources in cold regions. Ice formation can affect the design, operation, and maintenance of hydraulic engineering facilities (Billfalk, 1987; Horjen, 1994; Shen, 1996; Anderean, 1997; Anderean et al 1998). Major engineering concerns related to river ice are flooding induced by ice jams (Beltaos, 1995), hydropower operation, inland navigation, water transfers (Shen, 1996; Lia, 1998), environmental, ecological (Prowse and Gridley, 1993; Prowse, 2001a, b), and morphological effects (Ettema and Muste, 2000).

River ice engineering can be traced back to the 19th century. The first engineering studies on ice were connected with public works construction. Arentein's study (Ashton, 1986) on the ice conditions in the winters of 1847-48 and 1848-49 on the Hungarian Danube river was associated with the construction of the first permanent bridge between Buda and Pest, which had been heavily debated

since the two cities were flooded in 1838 as a consequence of ice jams. Early studies of ice formation, ice jamming and thermal ice pressures were published in the second half of the 19th century. Comprehensive studies were started on lake ice as well; Von Chohnoky's (1909) study of Lake Balaton is an early example.

Scientific studies on river ice formation began early in the 20th century. In 1914, the complete clogging of the water supply system for the city of Leningrad precipitated the upsurge in Russian ice research. The measurements of supercooling in the Neva River and other Russian rivers enabled Altberg (1936) to explain the formation of anchor ice as a result of heat convection. In the 1930s enough information had been collected to characterize the ice regime of many rivers, such as the Danube (Laszloffy, 1934). Ice became a subject in engineering manuals (Barnes, 1928; Schocklitsch, 1937), and detailed studies were described in journal papers (Devik, 1932; Korzhavin, 1962).

Ice problems originate from the generation of frazil ice in rivers. Since the well-known experiment conducted by Michel (1963) on supercooling and ice formation in turbulent waters, more useful information about the time histories of water temperature and ice formation has been collected in laboratory experiments (Carstens, 1966; Hanley and Michel 1975, 1977; Muller, 1978; Tsang and Hanley, 1985; Tsang and Trapp, 1990; Tsang and Cui, 1994) and field observations (Osterkamp et al., 1983; Tsang, 1986). Osterkamp (1977) discussed different nucleation theories. Reviews on supercooling and frazil evolution were written by Michel (1971), Osterkamp (1978), Martin (1981), and Tsang (1982), Ashton (1986). Comprehensive books on river engineering were written by Starosolszky (1970) and Michel (1971). Michel (1978) published the first book on ice mechanics.

In the 1980s-1990s, much more attention was given to ice engineering in rivers and oceans, especially in cold regions. Several reviews on river ice processes and past research have been published (Ashton, 1986; Gerard, 1990; Prowse and Gridley 1993; Beltaos 1995; Carson and Groeneveld, 1997; Healy et al., 1997; and Tatinclaux, 1998). Andres (1982), Ettema et al.(1984), Hanley and Tsang (1984), Daly and Colbeck (1986) and Tsang and Cui (1994) conducted a series of experiments on frazil formation in supercooled water. A Primer on Hydraulics of Ice Covered River (Davar et al., 1996) summarized several aspects on ice cover and ice jams. Daly (1984, 1994) wrote very complete and state-of-the-art reviews on frazil ice.

A unique aspect of Canadian hydrology is the influence of winter on streamflow behavior. Virtually all of the rivers in Canada experience some ice effects each year, and in many cases river ice has

produced the most extreme and dangerous flood events on record. River ice causes other problems of particular concern. For example, on rivers regulated for hydro-power production, river ice can severely limit the power production potential of such facilities (Raban, 1995).

In very turbulent reaches, supercooled surface water can be transported to a considerable depth within the flow, thereby permitting the build-up of “anchor” ice on the riverbed (Tsang, 1982; Parkinson, 1984; Petryk and Marcotte, 1984, 1986; Marcotte and Robert, 1986; Kerr et al., 1997, 1998). Given the right set of hydrometeorological conditions, anchor ice can grow and thicken to form extensive blankets of ice on the riverbed. As an example, Manitoba Hydro continues to experience significant losses in generation revenue due to ice growth at Sundance Rapids downstream of the Limestone Generating Station. At these rapids, four kilometers downstream from the generating station, anchor ice is forming on a shallow, granite rock shelf. This initiates the formation of an ice bridge that covers 80-90% of the channel’s width resulting in up to 1.5 meters of staging in the tailrace of the station (Girling and Groeneveld, 1999). It was proposed to excavate this rock shelf, but there are concerns that modifying the natural channel will only provoke anchor ice growth further upstream or in other areas that would result in a greater headloss. Unfortunately, the technology in anchor ice prediction is not sufficiently advanced to determine with any certainty what the governing conditions for ice growth is (Acres, 2000). A decision cannot be made until further research is conducted to get a better understanding of the process of anchor ice formation.

1.2 Literature Review

This section briefly describes a history of observation and studies on the supercooling process and frazil formation occurring in streams and rivers during the winter regime as well as some of the important problem situations that can result. Certain important points will be emphasized since they are fundamentals to the present study.

1.2.1 Supercooling Process

Tsang (1988b) idealized the process of supercooling as shown in Figure 1.1. It is seen from Figure 1.1 that as the water is supercooled below the freezing temperature T_f , normally 0°C , at a certain nucleation temperature T_N , frazil begins to form. Under natural conditions, T_N has been found to be a few hundredths of a degree below T_f . At first, the latent heat of fusion released by the formation of

frazil ice is not sufficient to compensate for the heat loss from the water due to the small number of ice particles formed and their associated slow rate of frazil production. Therefore, the water temperature continues to decrease but at a slower rate. At the point where the latent heat of fusion released is equal to the rate of heat loss to the air, the water no longer has a net heat loss and its temperature reaches the lowest point $T_{\min}$. Thereafter, increased latent heat liberation (probably by reason of accelerated secondary frazil nucleation) leads to a slow increase of the water temperature until an equilibrium temperature (T_e) is reached, at which point the rate of heat loss again is equal to the rate of latent heat liberation. It should be noted that the equilibrium temperature is a variable depending on the hydrometeorological conditions under which frazil is formed. The equilibrium temperature T_e is reached where the rate of latent heat liberation and the rate of heat loss are once again equal. Since there will be many frazil crystals to contribute to the heat loss at this time, T_e does not have to be much less than T_f to remove the small amount of heat that is contributed by the individual ice crystals. In fact, T_e is close to T_f and the system approaches the equilibrium state. However, the determination of the point where T_e is first reached is difficult. To overcome this hurdle, Hanley and Tsang (1984) chose the point where 90 percent of the maximum temperature depression ($T_f - T_{\min}$) is recovered as the characteristic point to mark the end of the initial period of frazil production. The temperature of the water at this point, T_c , is equal to $T_f + 0.1(T_{\min} - T_f)$, noting that $T_{\min}$ is negative. The time from the instant when T_N is reached to the instant when T_c is reached, t_c , serves as the characteristic time of initial frazil production period. By conducting systemic experiments, Tsang and Hanley (1985) were able to establish relationships relating T_N , $T_{\min}$, and t_c , and the corresponding ice concentration. Mercier (1984) determined an analytical expression for the time to maximum supercooling that depends on the four basic system parameters: the rates of heat loss, seeding, turbulent dissipation, and secondary nucleation. Daly and Axelson (1989) further developed a method to estimate the time to maximum supercooling.

1.2.2 Frazil Ice

According to the US Army Engineer Cold Regions Research & Engineering Laboratory (CRREL) (1997), frazil ice is defined as “fine, small, needle-like structures or thin, flat, circular plates of ice suspended in water”. It is formed when the water temperature drops below the freezing point, which leads to the supercooling of the water. Frazil ice is the origin of almost all the other forms of river ice including anchor ice. Frazil will grow, multiply and agglomerate into flocs and float to the surface to form frazil slush, floes and eventually a stationary ice cover. However, descriptions of the size and shape of frazil ice, and the correlation of frazil ice geometry with water temperature have not been well

delineated. In addition, rather varied terminology is used in published literature to describe frazil ice (Ettema et al., 1984).

1.2.2.1. Frazil Ice Evolution

According to Daly (1991, 1994), three rather general phases of evolution of frazil ice are identified (Figure 1.2). The first phase is formation, where the frazil ice dynamics take place. It is characterized by supercooled water, turbulent flow, rapid growth of disk-shaped crystals and the creation of new crystals by secondary nucleation. The length scales of the ice associated in this stage (Figure 1.3) range from several microns to perhaps a few millimeters (Daly, 1994). This stage usually lasts for a relatively short time during very cold periods when the heat transfer from the open water surface is large.

The second phase is transformation and transport. It is characterized by water at more or less the equilibrium temperature, and frazil in the form of flocs, anchor ice, and floes. The length scales of the ice associated with this stage (Figure 1.3) range from several millimeters to several meters. The frazil is largely moving under the influence of the river or stream, generally at the surface. This ice may travel long distances and move for many days. After cold nights, it is typical to see slush, formed of frazil flocs, moving along at the water surface of northern rivers and streams. This slush may eventually form large moving floes.

The third phase is the stationary ice cover. It is characterized by stationary, floating ice covers that may be quite large and last for the entire winter season. These ice covers are formed by a variety of mechanisms, depending on the form of the frazil ice when it arrives at the stationary ice cover and the hydraulic conditions at the cover's leading edge. These floating covers may raise stages and cause flooding, or cause increases in the channel head losses that can disrupt power production and interfere with navigation.

1.2.2.2 Frazil Ice Morphology

In this thesis, the development of the first stage, frazil formation, will be studied. The term frazil ice refers mainly to the frazil disks in Daly's classification (Figure 1.3). However, a description of morphology of frazil ice is not simple (Daly, 1984, 1991). The various shapes of ice crystals appear to result from a complex interaction of the imposed heat transfer conditions and the intrinsic crystallography of ice.

Virtually all field observations of frazil ice crystals in the shape of six-pointed stars, hexagonal plates or spheres and small pieces of dendritic ice eventually evolve in natural water bodies into the disk shape (Tsang, 1982; Daly, 1984, 1991, 1994). The observations of Margolis (1969) indicate that the thickness of a frazil disk is $0.68r \pm 16.7\%$, where r is the major radius of the frazil disk. Laboratory observations by Arakawa (1954) showed that disk-shaped crystals have a diameter of 0.5 to 3 mm and a diameter-to-thickness ratio of 5:1 to 100:1. The disk shape taken by ice at low supercooling levels is apparently the result of the anisotropic growth kinetics of ice (Daly, 1991).

Daly and Colbeck (1986) conducted a series of experiments on frazil ice evolutions in a laboratory flume. The aspect ratios of frazil particles are shown in Figure 1.4, and sizes of ice particles have been measured in the range of about 35 μm to 0.5 mm. The mean of the size distributions was generally about 0.1 mm. An interesting result is the degree to which the observed size distribution could be approximated by log normal size distribution (Daly and Colbeck, 1986).

Although it is difficult to measure frazil size or concentration (Daly, 1994), a few attempts and estimates have been made. Estimates of the frazil crystals concentration in natural rivers range from about 10,000 to 1 million crystals/ m^3 (Schaefer, 1950; Osterkamp et al, 1983). Measurements in a laboratory flume (Daly and Colbeck, 1986) indicated concentrations ranging from 1.80×10^5 to 9.82×10^5 crystals/ m^3 . Field data by Tsang (1986) indicated the upper limit for the concentration of frazil ice that can be formed in supercooled water of streams, rivers, and lakes is in the order of 0.5% by weight.

1.2.2.3 Rise Velocity

There have been a few attempts to estimate the rise velocity of frazil (Ashton, 1983; Daly, 1984; Gosink and Osterkamp, 1983; and Andreasson et al., 1998). Daly (1984) reviewed the rise velocity of frazil ice discs. His findings are summarized in Figure 1.5, where a simplified equation is shown for the intermediate range by the solid thick line. This equation (Svensson and Omstedt, 1994) is given by

$$\omega = 30r^{1.2}, \quad (1.1)$$

where ω is the rise velocity, and r is radius of a frazil ice surface particle. The units of r and ω are cm and cm/s, respectively. Within the specific ranges, the following equations were developed (Daly, 1984):

Stokes range ($r < 0.03$ cm)

$$\omega = 0.08(g' \nu^{-1} r^2); \quad (1.2)$$

Intermediate range ($0.03 < r < 0.14$ cm)

$$\omega = 0.16(g'^{0.715} \nu^{-0.428} r^{1.14}); \quad (1.3)$$

and fully turbulent range ($r > 0.14$ cm)

$$\omega = 0.707(g' r)^{1/2}; \quad (1.4)$$

where g' , the reduced gravity is 129.6 cm/s^2 (Daly, 1984) for a kinematic viscosity, ν , of $1.79 \times 10^{-6} \text{ m}^2/\text{s}$ at 0°C .

Gosink And Osterkamp (1983) reported their observations on the rise velocity of frazil ice in an Alaska river (Figure 1.6). Their observation shows the rise velocity is much less than the theoretical prediction of Daly (1984), specifically for larger frazil particles over 0.2 cm. The findings of Gosink and Osterkamp (1983) could be regressed as

$$\omega = 9D^{1.7}, \quad (1.5)$$

where the units of D and ω are cm and cm/s, respectively.

1.2.2.4 Heat Transfer from a Disk

The heat transfer from a disk in a stagnant fluid was determined by Wadia (1974). Two trends can be observed from his findings (Figure 1.7). At a low value of $\bar{r}/\eta$, the ratio of h_e/h_f increases rapidly

as the aspect ratio (thickness/face diameter) departs from unity. The characteristic length, $\bar{r}$, is equal to surface radius (r) or thickness (D_e) for the face and edge of a disk particle, respectively, and η is the dissipation length. h_e and h_f are the heat transfer at the edge and face of the disk, respectively. Wadia's (1974) explanation of this result was the local shear of the fluid was higher near the edge of the disk than on the face. Second, for a fixed aspect ratio, as $\bar{r}$ increases the ratio of h_e/h_f decreases, approaching a value of 1 asymptotically. Wadia (1974) speculated that since the face dimension is larger than the edge dimension, the face of the disk can respond to a larger wave-number range of the small eddies, which can enhance surface renewal of the disk boundary layer. It is at the larger values of $\bar{r}$ that turbulent intensity plays a major role in influencing transport. Therefore, the turbulent intensity would enhance h_f more than it would h_e . At larger values of $\bar{r}$, the effects of higher local shear at the edges increases h_e , to a comparable magnitude as the turbulent intensity enhancement of h_f , causing the ratio of h_e/h_f to approach unity (Figure 1.7).

In order to determine the relative value of the actual heat transfer coefficient, the turbulent Nusselt number N_u is used. Its definition is (Daly, 1984)

$$N_u = \frac{h\eta}{k_w}, \quad (1.6)$$

where h is a heat transfer coefficient (i.e., h_e or h_f) and k_w is the thermal conductivity of water.

Defining $m^* = \bar{r}/\eta$, where $\bar{r}$ may be radius of a face or thickness of a disk, Daly (1984) summarized the following relationships (Figure 1.8), which were developed by Batchelor (1980) and Wadia (1974), between N_u , m^* , and P_r .

$$N_u = \left(\frac{1}{m^*} \right) + 0.17 P_r^{1/2} \quad \text{if } m^* < \frac{1}{P_r^{1/2}} \quad (1.7)$$

and

$$N_u = \left[\left(\frac{1}{m^*} \right) + 0.55 \left(\frac{P_r}{m^*} \right)^{1/3} \right] \quad \text{if } \frac{1}{P_r^{1/2}} < m^* < 10 \quad (1.8)$$

in which P_r is the Prandtl number. For large particles, i.e., $m^* > 1$

$$N_u = 1.1 \left[\left(\frac{1}{m^*} \right) + 0.8 \alpha_T^{0.035} \left(\frac{P_r}{m^*} \right)^{1/3} \right] \quad \text{if } \alpha_T m^{*4/3} < 1000 \quad (1.9)$$

and

$$N_u = 1.1 \left[\left(\frac{1}{m^*} \right) + 0.8 \alpha_T^{0.24} (P_r)^{1/3} \right] \quad \text{if } \alpha_T m^{*4/3} \geq 1000 \quad (1.10)$$

where $\alpha_T = \sqrt{(2k/U)}$ is the turbulence intensity (Hammer and Shen, 1995), U is the mean flow velocity, and k is the kinetic energy. It is of interest to note that when N_u decreases, the m^* term increases, provided that $m^* > 1$. Therefore, the thermal growth rate of frazil particles decreases rapidly with the increase in particle size.

1.2.3 Lab and Field Observations

Many experimental observations of frazil ice have been made over the last 30 years. These observations include the water temperature response to frazil particle growth (Michel, 1963), the effects of fluid turbulence on the water temperature response (Carsten, 1966; Muller, 1978; Ettema et al., 1984), the influence of salinity (Tsang and Hanely, 1985) and the size distribution of frazil ice particles (Bukina, 1967; Daly and Colbeck, 1986). Some of these observations are as follows.

1.2.3.1 Experiments of Michel's (1963)

Michel (1963) carried out his experiments in a frazil ice flume set outside on the Campus of Laval University. He reported a typical sequence of water temperatures and frazil formation (Figure 1.9). At the start of a sequence the water in the flume cooled at a constant rate. A time history of the water temperature followed a straight line with a constant negative slope during this initial period. The cooling rate remained constant, even though the water temperature passed through 0°C, until it

reached a few hundredths of a degree below the freezing point. As the water temperature approached its maximum supercooling, the cooling rate decreased rapidly until it became zero. At this time, tiny ice particles formed that were too small to be readily seen by the naked eye, but could be detected if viewed with a strong light.

After the water reached its minimum temperature, it returned to 0°C. This warming period was characterized by two stages, an initial steep constant rise in temperature followed by an asymptotical approach to 0°C. The water temperature measured by Michel during the course of an experiment reflected a balance between the heat loss at the water's surface and the latent heat released by the growing frazil crystals. When the rate of temperature change became zero, the latent heat released by the growing crystals exactly balanced the heat loss at the water surface.

1.2.3.2. Carstens' Experiments (1966)

Carstens (1966) conducted a series of experiments in a test flume where he investigated the extent to which water could be supercooled. He varied the rate of heat through the water surface and varied the turbulence of the flow. His experiments produced temperature-time history curves very similar to Michel (1963). However, the temperature of the water did not always return to the freezing point; it would often reach a constant value of supercooling, which he termed the residual supercooling. When the rate of heat loss from the water surface was increased, the maximum supercooling of the water increased, the residual supercooling also increased and the rise in temperature from the maximum supercooling to residual supercooling required less time. Carstens (1966) also felt that the number of ice crystals increased. He saw large qualitative differences in the temperature-time history curves when the turbulence of the flow was varied. "Strong" turbulence produced relatively small maximum supercooling levels, and the temperature would return relatively quickly to the freezing point with little or no residual supercooling; conversely, "weak" turbulence produced a relatively large maximum supercooling level that would persist for relatively long periods.

Carstens (1966) concluded that the level of supercooling reached in a particular experiment reflected the balance of the heat loss at the surface and the heat transferred from the growing frazil crystals, which was controlled by turbulence of the flow. He suggested that the heat transfer from the particles to the water could be related by an expression such as $N_u = CR_e^m$ where N_u is the Nusselt number (nondimensional heat transfer), C and m are physical constants in his nomenclature, and R_e is the

Reynolds number based on the characteristic velocity of the flow and the characteristic linear dimension of a frazil crystal.

1.2.3.3 Osterkamp's Observation (1971)

Field measurements of water temperatures in a turbulent stream are hard to achieve and therefore quite rare. An exception is the Goldstream Creek study conducted in interior Alaska during frazil ice production in 1972. Osterkamp et al. (1983) reported the time-temperature history of the Creek (Figure 1.10), and Gosink and Osterkamp (1983) presented the thermal and hydraulics parameters. It is worth noting that the field supercooling process they observed is similar to laboratory observations (Michel, 1963; Carstens, 1966), except the supercooling period in the field lasted for hours while the laboratory observations were measured in minutes. Osterkamp's data is also in disagreement with Daly and Axelson (1989) whose estimation of supercooling in the field is also on the order of minutes.

1.2.3.4 Turbulence Jar Experiments

Hanley and Michel (1975, 1977) studied ice formation at several air temperatures and water speeds in a cold room at Laval University. The ice formation under controlled conditions was studied in the cold room using a cylindrical steel tank 120 cm in diameter and 76 cm deep. Paddles turned by a variable speed motor moved the water about the axis of the tank. Twenty-one thermometers were used to record temperatures above and below the surface of the tank. During experiments conducted, with the paddle rotating, ice grew on the tank wall that was called border ice. Hanley and Michel (1977) concluded that in flowing water the rate of growth of border ice was nearly independent of water speed, but was related to the temperature of the ambient air by a simple power function. A heat-balance equation for the growth of frazil ice was developed to the point where it could be related to the experimental data. Values of frazil ice mass obtained by applying the equation were compared with direct observations.

Muller (1978) first conducted several experiments with supercooling temperatures between -0.05°C and -0.30°C in a refrigerated turbulence tank ($17.2 \times 12 \times 20$ cm), in which turbulence was generated by an oscillating grid. He investigated the rate of heat transfer from the frazil crystals to the supercooled water and the rate at which the number of frazil particles increased. He observed no frazil at a supercooling level down to -1°C unless the water was seeded with ice particles. He also observed

that the number of ice particles increased during the course of an experiment. Muller (1978) concluded that some type of multiplication process was at work, that "intensified" with an increase in supercooling or an increase in turbulence. He also found that the apparent heat transfer from the frazil particles could be normalized with the supercooling level and the size of the particles. This normalized heat transfer was a constant for all the experiments with an average representative Nusselt number of 10.

Andres (1982) conducted an experiment to investigate the nucleation and frazil production during the supercooling period for various air temperatures and turbulence intensities. He performed his experiments using an octagonal tank. A propeller was mounted at the base of the tank in order to generate a turbulent flow in which the frazil ice formed. From his data, Andres (1982) observed that the amount of frazil produced during the supercooling period varied considerably but was primarily a function of the air temperature, while the duration of supercooling was dependent on the turbulence intensity. The average rate of production during the entire supercooling period was less than the rate following supercooling due to the existence of a residual temperature.

Ettema et al.(1984) reported his experiments on an investigation of the influence of initial water temperature on the nature and rate of frazil-ice formation in a turbulence tank. He concluded that the most important factor in frazil evolution was the initial supercooling temperature. He also observed a constant frazil size throughout the process, and the rate of frazil formation increased with increasing intensity of turbulence.

1.2.4 Numerical modeling

There are a few models of frazil ice dynamics in a river. Daly (1984) presented a dynamic frazil ice model that can be quantitatively discussed through the use of a continuity equation for the number of frazil ice particles and a heat balance equation. However, these equations are dimensionally incompatible and strongly nonlinear (Daly and Axelson 1989). Omstedt and Svensson (1984) developed a frazil ice model for a turbulent Ekman layer in oceans, which was applied to rivers by Nyberg (1985). The mathematical complexities and uncertainty in Omstedt's (1984) computer model, involving 15 differential equations and a number of assumptions and numerous coefficients, make it difficult to implement in a practical engineering problems (Tsang and Cui, 1994).

Mercier (1984) formulated a kinetic model of frazil growth using a Monte Carlo technique to simulate frazil formation in channels. Svensson and Omstedt (1994) presented a mathematical description of frazil ice dynamics in a river. Hammer and Shen (1995) used a two-dimensional turbulence model to examine the evolution of frazil ice in channels by considering thermal growth, secondary nucleation and flocculation processes. These three models were verified by these model developers using Carstens' (1966) and Michel's (1963) experiments.

1.2.4.1 Mercier (1984)

Mercier (1984) extended the work of Daly (1984) and developed a general comprehensive model of the reactive transport of suspended frazil in turbulent flow. The analytical expression assumes a homogeneous, isotropic water body. Initial frazil production is assumed to be caused by seed particles, whose size is near the critical radius, introduced into the water at a constant seeding rate. The value of the critical radius is calculated using thermodynamic principles, and is estimated to be $0.4\ \mu\text{m}$ for the range of supercooling found in rivers and stream. Additional crystals are produced by secondary nucleation. It is assumed that secondary nucleation is dominated by newly created particles through the collision of large crystals by the action of turbulent shear.

Mercier's (1984) analytical expression is governed by four system parameters: rate of heat loss, the particle seeding rate per unit volume, turbulent energy dissipation rate, and the number of crystals per unit collision energy. Two of the parameters, the rate of heat loss and turbulent energy dissipation rate, can be measured and are controlled in every frazil experiment. The seeding rate is much more difficult to control and measure. The number of particles produced per unit energy is a material property that has been estimated for other materials, but can only be estimated by numerical experiment for frazil ice (Daly and Axelson, 1989). Mercier (1984) found that a constant number of particles produced per unit energy (4×10^{10} nuclei/erg) and an optimized seeding rate ranging 10^{-1} to 10^{-6} particles/cm³s appear to be reasonable based on the experimental conditions (Daly and Axelson, 1989).

The thermal energy balance is determined in the following manner. The water is supercooled by a constant heat loss rate per unit volume. Above the equilibrium temperature, any seed particles introduced will melt. Below this temperature, seed particles can grow and produce new particles by secondary nucleation. It is assumed that as the ice particles grow, the latent heat of fusion released is negligible before the point of maximum supercooling is reached. At the point of maximum

supercooling, the rate of the latent heat of fusion released equals the heat loss rate. After this point the water temperature rises as frazil ice growth continues. The point of the maximum increase in temperature corresponds to the maximum rate of frazil growth.

Mercier also determined the particle size distributions and profiles of mass concentration at various times and depths. He described the frazil size distribution as (Mercier, 1984)

$$g(l) = Al^{-b}, \quad (1.11)$$

where $g(l)$ is the number density distribution (number of frazil crystals per unit fluid volume per unit particle length), l the particle length scale (e.g., diameter) and A and b are constants. Mercier (1984) found that setting $b = 4$ gave the correct slope of the distribution.

1.2.4.2 Svensson and Omstedt (1994)

Svensson and Omstedt (1994) also assumed a number of initial frazil ice particles, which are classified into N discrete radius intervals, where all particles in them are assumed to have equal radii. The number of particles in each group is a function of initial seeding, particle growth, secondary nucleation, gravitational removal, and growth due to cooling of the water volume and flocculation/break up.

Particle growth due to supercooling is determined by the heat transfer rate between the water and the particles, for which a constant Nusselt number, $N_u = 1.0$, is assumed for all groups of ice particles.

Secondary nucleation is related to the likelihood for collisions. However, flocculation is assumed to take place by the sintering mechanism. Large flocs can also be expected to break up under the action of shear. It is assumed that the net effect of the two processes is the formation of larger aggregates. If a steady state situation is to be considered, it is necessary to consider the gravitational removal, since the particles are big enough to rise to the surface.

The water temperature (T) in a well-mixed box can be calculated from the overall heat balance.

$$\frac{d}{dt}(\rho C_p T) = -Q_{aw} + \sum_{i=1}^{N-1} Q_i, \quad (1.11)$$

where ρ is the density of water, C_p is the specific heat for water, Q_{aw} is the net heat loss per unit volume at the surface, and Q_i release of heat due to freezing for radius interval i .

Svensson and Omstedt's mathematical model contains two more parameters than Mercier's (1984) model, which have to be calibrated; they are: $\bar{n}_{max}$ the maximum frazil number and a_{floc} a number sensitive to the size distribution.

1.2.4.3 Hammer and Shen (1995)

Their model uses the same expressions of flow turbulence as Mercier (1984), and three differential equations expressing the mean flow, frazil concentration, and water temperature. The frazil particles are assumed to be thin circular disks with a constant ratio of 1:10 between the thickness and diameter. The frazil ice size distribution is divided into eight logarithmically spaced size groups. The size of groups range from 4 μm to 1.435 mm. Seed crystals and secondary nuclei are assumed to be in the lowest size group. The formulation for frazil evolution includes thermal growth, secondary nucleation, flocculation, and the treatment of source/sink terms were discussed. Turbulent shear can cause flocculated particles to break up. However, since particle breakup is limited to large, weakly bounded aggregated particles under high shear (Mercier, 1984), particle breakup is not considered.

In their simulation, frazil formation is initiated by the mass exchange of seeding particles at the free surface. Their model of frazil crystal growth is based on the rate of heat transfer between particles and the ambient turbulent flow; Nusselt number relationships (Wadia, 1974; Batchelor, 1980; Daly, 1984) are employed. Secondary nucleation and flocculation are simulated using binary collisions of frazil particles. The model simulated the size distribution of frazil particles size in a turbulent channel flow. The size distribution was a result of crystal seeding, thermal growth and flocculation of frazil particles.

1.2.4.4 Numerical Modeling Comments

In these modeling efforts, frazil ice dynamics have generally been considered. The time-temperature evolution has been the main concern. The former models (Hammer and Shen, 1995; Svensson and Omstedt, 1994) of frazil evolution assume the geometry of frazil ice and divide them into various groups of frazil size. Furthermore, the modelers consider frazil breakup and flocculation, about which very little is known at this time (Daly, 1991, 1994).

Although great advancements on frazil dynamics have been made in these modelling efforts, some limitations still exist, and a number of assumptions have to be made and calibrated. Frazil size and its distribution, frazil and flow turbulence, and furthering the relationship between the heat exchange and frazil growth are not quite understood. Much more work is needed to elucidate frazil ice dynamics.

1.3 Objective of this Study

The main objectives of this research are to investigate several frazil processes. In particular, they are:

- to investigate flow turbulence characteristics and their effect on frazil ice size and evolution during the supercooling process;
- to develop a new approach for the supercooling process and frazil ice formation;
- to develop a model of the vertical distribution of frazil;
- to develop a numerical model based of a new approach to simulate the supercooling process; and
- to undertake an extensive series of laboratory experiments to investigate the hydraulic parameters and thermal conditions that influence frazil evolution, and verify the approaches and models presented and developed herein.

Overall, this study will provide valuable data that will increase our knowledge of the parameters and thermal conditions influencing frazil. In addition, the approach and its concepts in this study can be considered fundamentals to various river ice problems, and can be extended to develop models for anchor ice growth during the supercooling process. This will help to develop better management strategies to mitigate ice related complications associated with drinking and cooling water intakes, hydroelectric generation, frazil ice induced flooding, and operation of irrigation canals and hydraulic structures in cold regions. Since these problems are of considerable economic and social importance,

and an advancement of our understanding of these processes would be of considerable benefit to Canada and to industry worldwide.

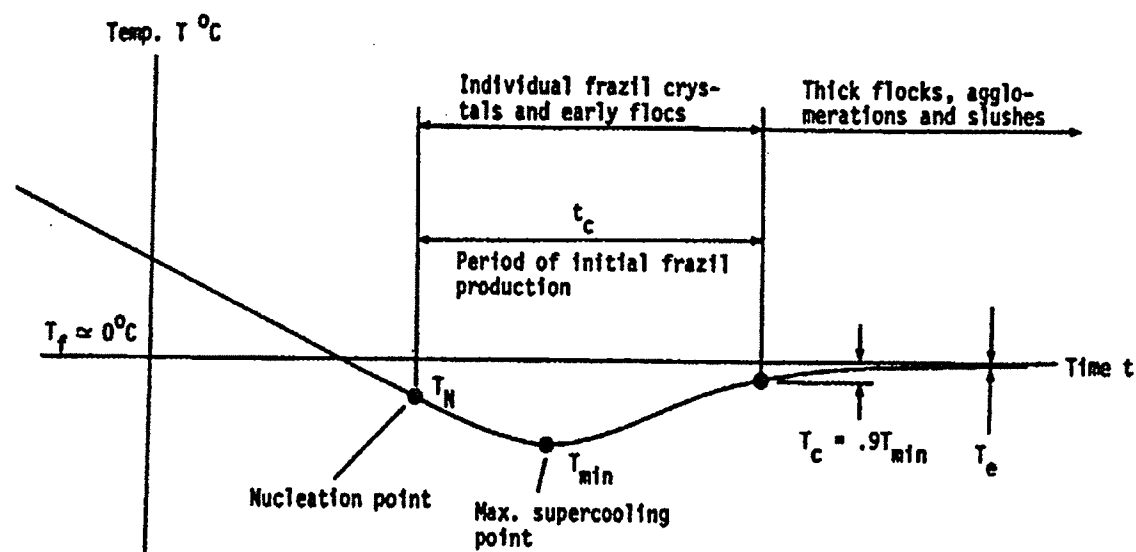


Figure 1.1. Idealized supercooling process (Tsang, 1988).

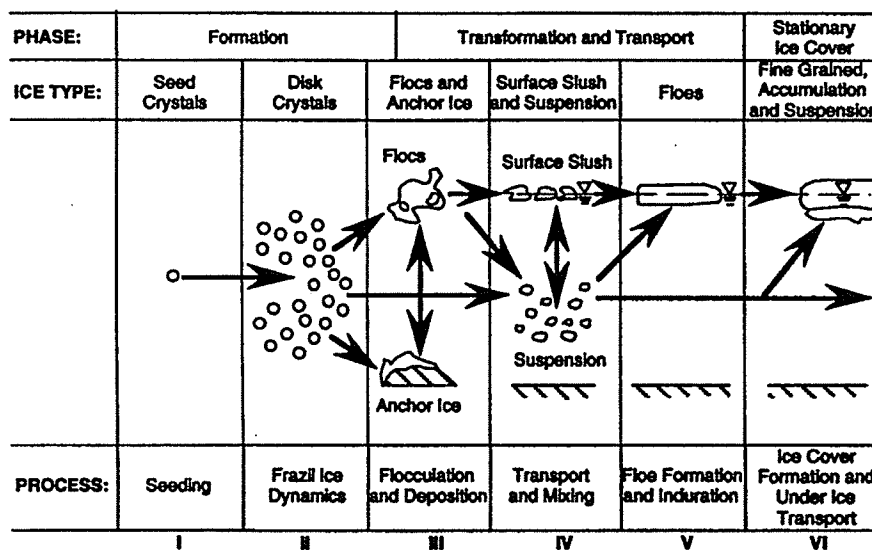


Figure 1.2. Evolution of frazil ice in natural water bodies (Daly, 1994).

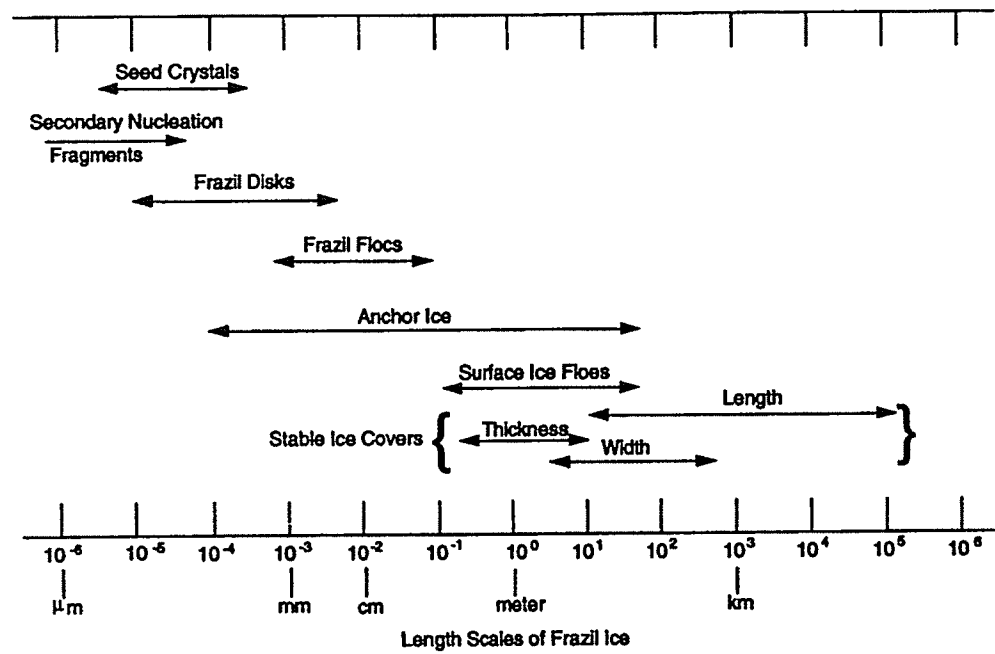


Figure 1.3. Length scales of frazil ice (Daly, 1984).

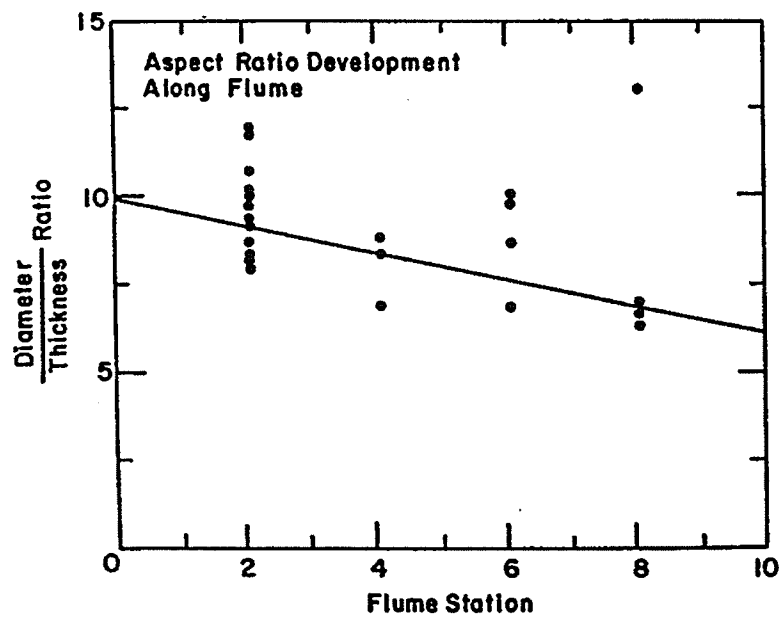


Figure 1.4. Aspect ratio of frazil crystals in a laboratory flume (Daly and Colbeck, 1986).

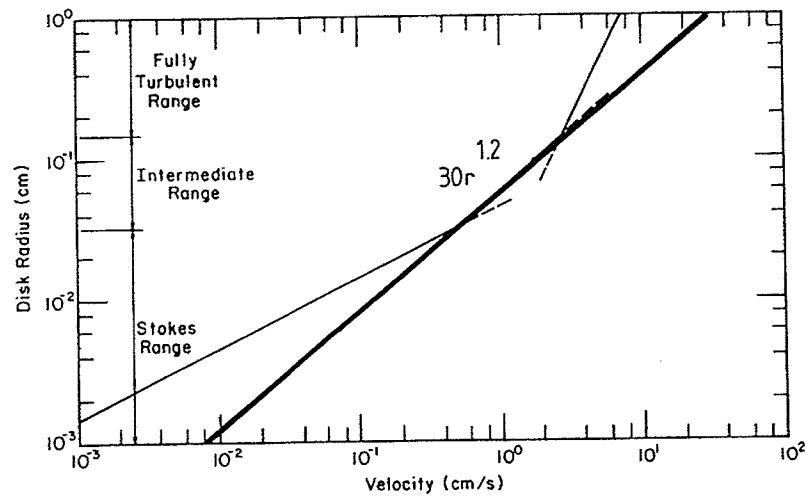


Figure 1.5. Terminal rise velocity of a frazil disk (Svensson and Omstedt, 1994 after Daly, 1984).

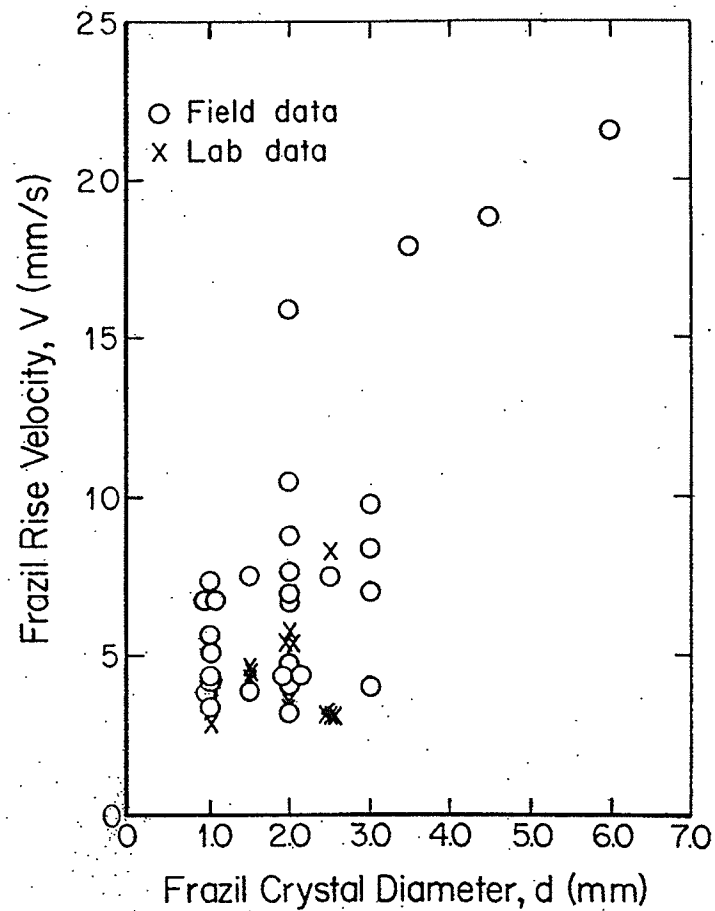


Figure 1.6. Measured frazil ice rise velocity vs. diameter (Gosink and Osterkamp, 1983).

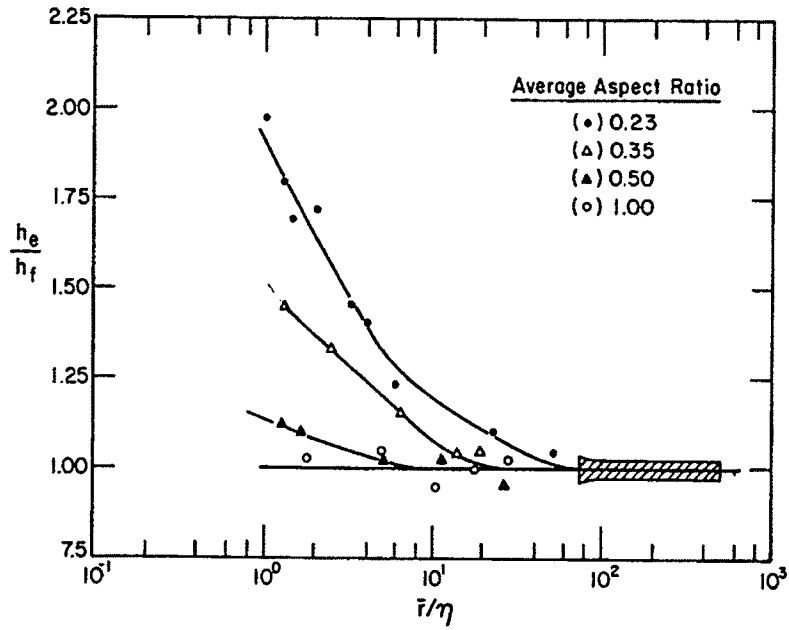


Figure 1.7. Edge vs. face heat transfer (Daly, 1984 after Wadia, 1974).

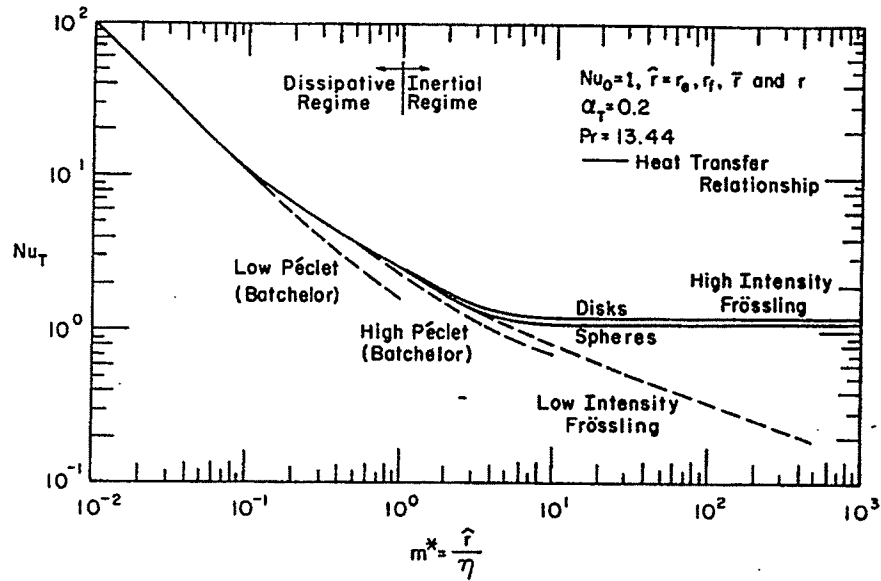


Figure 1.8. Nondimensional turbulent Nusselt number (Daly, 1984).

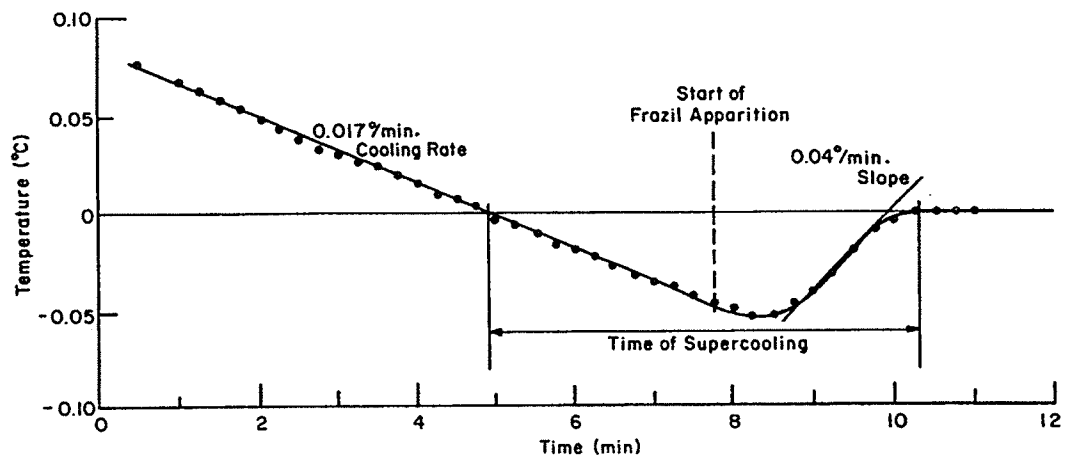


Figure 1.9. Typical supercooling process (Daly, 1984 after Michel, 1963).

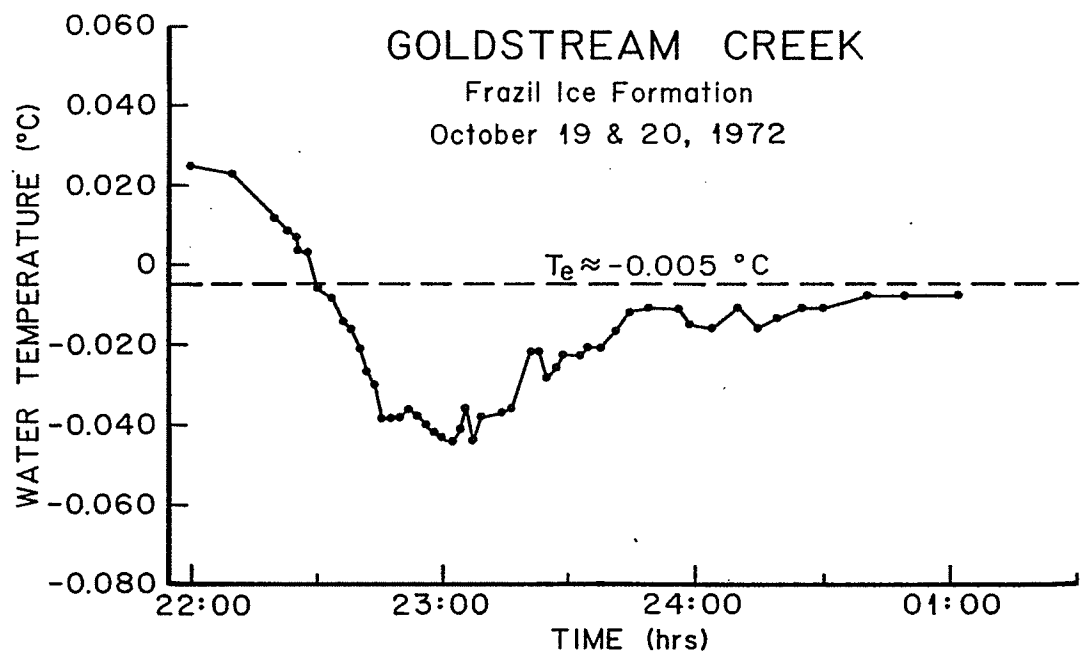


Figure 1.10. Water temperature in Goldstream Creek (Osterkamp et al., 1983).

Frazil and anchor ice can be observed from two fundamental frames of reference. One possible frame is the Eulerian frame of reference where observations of ice are made from a stationary position. An example of this would be watching ice form on a rock while you are standing on the riverbank. Another way of observing ice growth is through a Lagrangian frame of reference where a parcel of ice and water is observed as it floats flows down the river. These observations would be made from a frame of reference that drifts with the particle. This is arguably the more practical method for observing frazil ice process in a laboratory. A long flume could theoretically simulate the drifting parcel in the laboratory, but would not be practical for such research because the flume would be uneconomically long due to the nature of the formation and evolution of frazil and anchor ice. To overcome this problem a circular flume can be used.

2.1 Counter-Rotating Flume

Figure 2.1 shows the counter-rotating flume used for this research (Tsang and Cui, 1994; Doering et al., 2001; Morris, 2002; Clark and Doering, 2002), which is located in the Hydraulics Research and Testing Facility (HRTF) at the University of Manitoba. It consists of an inner drum placed inside of an outer cylinder, both of which are supported underneath by a turntable (Figure 2.2). The bottom lining of the flume has separate supports, allowing it to rotate independently. The width of the flume,

from the inside of the outer cylinder to the outside of the inner drum, is 0.20 m. The centerline diameter of the flume channel is 1.20 m and has a height of 0.35 m. The bottom of the flume is lined with six plastic plates, each embedded with a uniform gravel layer. The roughness of the flume bottom can be changed by exchanging the bed plates with ones that have a different size of embedded gravel (Figure 2.3).

The simulation of straight channel flow with the use of a circular flume produces some undesirable effects. The centripetal force associated with a rotating column of water causes an uneven lateral distribution of ice flocs, changing the frazil size and concentration and producing an unrealistic water surface profile. To overcome these effects or at least reduce them to an acceptable level, the bottom of the flume rotates clockwise while the walls rotate counterclockwise. The rotation rates are adjusted until the shear forces acting on the water from the walls are in equilibrium with the shear force from the bottom. The result is the water in the flume is stationary to an observer on the ground and therefore eliminates any centripetal forces induced by the rotation of the flume. Although the water in the flume is stationary to an absolute frame of reference, it has a velocity with respect to the bed commensurate with open channel flow (Figure 2.4).

There are two water velocity profiles in the counter-rotating flume (Figure 2.5). One is the absolute velocity with respect to the ground (2.5b), and another is the relative velocity with respect to the bed (2.5a). Both profiles have a maximum velocity equal to the bed speed, and a minimum velocity equal to zero. The velocity profile generated in the flume is realistic in open channel when viewed from a frame of reference relative to the bed. The maximum velocity occurs at the surface while the velocity decreases as depth increases until the velocity reaches zero at the boundary layer over the bed (Figure 2.5 (a)). This velocity profile shape would be similar to that in a natural stream. However, in an absolute frame of reference, the velocity profile produced by the flume is different. The velocity at the surface is zero while the maximum velocity occurs at the bed (Figure 2.5 (b)).

It is the relative velocity that controls the turbulence characteristics of the main flow and hence the evolution and redistribution of frazil. It is also the relative velocity that simulates the prototype velocity in a river (Tsang and Cui, 1994).

The velocity profiles are governed by both the bed and wall speeds as well as the bed roughness. For a bed roughness diameter of 5 mm, the calibration procedure was carried out with a flow meter placed at 0.6 of water depth (Figure 2.6). For a given bed speed, the wall speed was adjusted to match the

angular velocity of the probe. In such a situation, the water would be stationary with respect to an observer in the cold room. The relationships between the wall and bed speeds (for the bed roughness of 5 mm) are as follows:

$$S_w = 0.9798S_b, \quad (H = 10 \text{ cm}) \quad (2.1a)$$

$$S_w = 0.8451S_b, \quad (H = 15 \text{ cm}) \quad (2.1b)$$

$$S_w = 0.7185S_b, \quad (H = 20 \text{ cm}) \quad (2.1c)$$

where H is the water depth in the flume. S_w and S_b are the speeds of the wall and bed [rpm], respectively

The mean relative water flow velocity, which is referred to as water velocity hereafter, is calculated by converting the rotational rate of the bed or wall, measured in rpm (revolutions per minute), to its angular velocity. The mean velocity could be wall speed or the speed converted based on (2.1). However, in practice, both values are rarely identical such that the average of these two values is used. This conversion can be made using the following equations

$$\begin{aligned} U &= (\pi d) \times (S_w + \alpha_1 S_b) / 2 \\ &= 3.14(S_w + \alpha_1 S_b) \end{aligned} \quad (2.2)$$

where U is the mean water velocity [cm/s], α_1 = ratio of wall speed to bed speed (0.9798, 0.8451 and 0.7185 for water depths of 10, 15, and 20 cm, respectively), and d is the centerline diameter of the flume (1.2 m).

In the counter-rotating flume, this “reversal” of the velocity profile may alter the natural interaction at the water/air interface. Since the cooling force of the water is driven by the heat exchange at this interface, changing the characteristics of the water/air interface will have a direct effect on the cooling rate of the water. Surface turbulence produced by a natural velocity profile increases the surface area and enhances mixing, and thereby influencing the rate of heat exchange. The quiescent nature of the surface produced in the flume may alter these characteristics and change the cooling rate for a given air temperature. Therefore, the experiments are described using their cooling rates as well as the air temperature. This allows for comparison with other experiments conducted by other authors.

The flume is located inside a custom designed cold room. The cold room (Figure 2.7) is a 4.3 m wide, 4.3 m deep and 2.7 m high chamber constructed of 4" thick Norbec insulated panels with an insulation-value of R30. It is equipped with two Blanchard Ness outdoor air cooled condensing units, each with a capacity of 48,000 BTU's. Each condensing unit is coupled with a Blanchard Ness low silhouette evaporator coil via a Sporlan CDS-8 step motor evaporator control valve. The temperature of the cold room is regulated with an Omron E5GN temperature controller mounted outside the cold room (Figure 2.8). The temperature in the cold room can be adjusted between room temperature and -35°C and is held within $\pm 0.1^\circ\text{C}$.

An air jacket below the base plates circulates warm air to simulate a geothermal heat flux from the bed. The air pocket is equipped with a heater and fan (Figure 2.9) connected to a controller to maintain a preset constant temperature. The air temperature in this pocket is separated from the water by a 5 cm thick layer of UHMW (Ultra High Molecular Weight) plastic, which acts as an insulator.

To prevent the sidewalls of the flume from icing, warm air is circulated in ducts surrounding the flume walls. A heater lamp connected to a controller (Figure 2.10) is activated whenever the air temperature in the ducts falls below a preset temperature. The warm air is circulated through the ducts with a fan (Figure 2.10). The temperature in all three ducts (inner, outer and bottom) is monitored, controlled, and recorded using an array of RTD's.

2.2 Data acquisition system

2.2.1 Temperature Recording Equipment

Water temperature is measured with a Guildline 9540 thermometer which uses a platinum RTD (Figure 2.11). The microprocessor in the unit converts the resistance in the probe to voltage and then to degrees Celsius. The temperature is displayed on the 9540 (Figure 2.12), while the corresponding voltage is sent to the data acquisition board in the computer outside of the cold room. The probe is inserted into the flow from the side to prevent any interaction between the water/air interface that would occur if the probe was submerged from the surface. Figure 2.11 shows the RTD submerged in the water from the side of the flume, 6 cm above the bed, and 6 cm outside of the wall drum. The

temperature voltage signal from the microprocessor ranges from 0/10 Volts for a temperature ranging from -1 to $+2^{\circ}\text{C}$.

The thermometer was calibrated by the manufacturer on August 28, 2001 at an ambient air temperature of $23 \pm 2^{\circ}\text{C}$, and a relative humidity of $45 \pm 15\%$. The probe was immersed into stirred liquid baths. The results of the calibration are in the Table 2.1.

Table 2.1. Calibration of the thermometer by manufacturer on Aug. 28, 2001.

Standard Temperature ($^{\circ}\text{C}$)	Measured Temperature ($^{\circ}\text{C}$)	Total Uncertainty ($\pm mK$)
40.000	39.999	3
1.000	0.998	3
0.000	-0.003	3
-1.000	-0.999	3
-40.000	-40.003	3

The zero of the thermometer was verified by Roy Hartle upon receipt by placing the RTD in a distilled ice bath. It was noticed in March 2002 that the offset of the thermometer had shifted. The thermometer was validated in situ, at an air temperature of 1°C . The relationship between voltage and temperature is shown in Figure 2.12. The thermometer calibration was checked routinely thereafter with no significant change in offset noted.

A Keithley Metrabyte DAS-1600 data acquisition board was used to record the water temperature, the temperatures of the outer, inner and bottom cavities, as well as the rotation rates for the bed and walls. The data were sampled at 1 Hz using LABTECH NOTEBOOK, version 10 running on a 400 MHz Pentium II with 64 MB of RAM (Figure 2.13).

2.2.2 Image Recording Equipment

The flume is equipped with two cameras located in a cavity on the side of the flume (Figure 2.14a). Figure 2.1 shows a technician adjusting the cameras in the camera box. Each camera transmits its signal to a monitor located just outside the cold room. The first camera is a Panasonic color CCTV

that records a wide-angle view of the water column. The signal from this camera is not used in the analysis process; rather it is used to monitor the progress of an experiment. The second camera (Figure 2.14b) is a Sentech STC-1100a monochrome progressive scan CCD camera with a 12 mm lense yielding a field of view of 20 cm (horizontal) by 15 cm (vertical). The analog signal from this camera is sent to a DIPIX FPG-44 Power Grabber board where the signal is converted to a digital image consisting of 640x480 pixels with 256 shades of grey. This digital image is then recorded directly to the hard drive of the computer. The image recording system can currently capture and record up to 15 frames per sec (Morris, 2002).

The image recordings are obtained using cross-polarized light conditions as illustrated in Figure 2.15. Located inside the inner drum, opposite the cameras, is a light source (two 27W compact fluorescent bulbs) and a polarizing sheet (polarizer 1). Each camera lens is also fitted with a polarizer (polarizer 2) that is positioned so that the optic sills are perpendicular to the optic sills on the first polarizing sheet. This creates a cross-polarizing condition. The first polarizing sheet filters light oscillations perpendicular to its optic sills, while the polarizing sheet on the lenses filters the light in the other principle direction, preventing any light from directly entering the camera. The only way light can enter the camera is after it has passed through a "particle" of ice, regenerating oscillations that were filtered out by the first polarizing sheet. This allows the normally transparent ice crystals to be seen distinctly by the camera.

The camera position was set so that the surface of the water was a few millimeters above the camera's field of view. Due to the cross-polarized lighting conditions, the interface of the water surface and the air regenerated some of the light oscillations that were just filtered out by the polarizing sheet (polarizer 1). These redeveloped oscillations caused the water surface to appear very bright. Since ice particles also appear very bright, the system used to analyze the images might falsely identify the surface as ice. Therefore by "moving" the surface out of view of the camera, this problem was avoided.

The location of the camera in its rotation around the flume is an important parameter necessary in the image processing procedures. The rotational location is recorded using an LED (Light Emitting Diode) (Figure 2.14a) and switch. Mounted on the underside of the rotating flume, is a switch (Figure 2.16) that is triggered once every rotation to turn on an LED for 1 sec. The LED is positioned inside the camera box such that it can be seen on the bottom left corner of each image. By observing when the LED is turned on, the rotational position of the cameras can be determined. Figure 2.17

shows two images, one with the LED on and the other with the LED off. This is used to obtain a position specific reference image for image processing.

In order to extract information about the evolution of frazil ice particles within the counter-rotating ice flume, a series of images are recorded. After all the images are collected, they are analyzed to compute the size and location of each individual ice particle within each image. This information is then used to study the change in size and spatial distribution over the course of an experiment.

2.2.3 Image Processing

The digital images collected, using the equipment described above, are gray scale TIFF (Tag(ged) Image File Format) images with a resolution of 640 by 480 pixels. Each image is actually a 640x480 matrix with an entry value ranging from 0 to 255, where 0 represents black, 255 represents white, and all numbers in between are shades of gray (higher numbers are brighter). Since each image is a matrix of numbers, it can easily (and digitally) be manipulated. All image manipulation/analysis algorithms were developed using Matlab[®] Release 12 equipped with the image and signal processing toolboxes.

The first step in the processing system is the removal of the background “noise” from each image. Inherent in each image are small scratches and non-uniform lighting conditions. It is necessary to remove these features before the processing system begins the particle recognition process, since they could be misinterpreted as a frazil ice particles. The removal process begins by comparing an ice image to a reference image. The term “ice image” refers to an image taken during an experiment and includes particles of frazil ice, while “reference image” refers to an “average” image captured prior to any ice formation. By comparing these two images, individual ice particles can be recognized. Any difference between the images can be attributed to frazil ice.

The subtraction process is not perfect. It does not result in a precise solid shape representing a frazil particle. Rather it produces frazil ice particles represented by a blotchy shape filled with holes that can sometimes be surrounded by small random spots. Figure 2.18 shows a reference image, an ice image, and the subtracted image. The subtracted image is composed of zeros, negative, and positive numbers, with frazil ice particles maintaining positive values. Since frazil ice particles show as brighter pixels, a darker pixel in the ice image (relative to the reference image) is assumed to be noise. Therefore, all negative pixel values are reassigned to zero.

Low values in the subtracted matrix are also attributed to noise. Histograms (not shown) of the values in subtracted images indicate that values greater than a certain criteria may be safely considered to be frazil ice particles, while lesser values (including negative values) can be safely considered to be noise. Therefore, all values less than the criteria are changed to zero while values greater than the criteria are changed to 1, creating a binary image (composed of either 0 or 1). The criteria's value depends on light intensity, lenses, and location of the camera, but is typically about 5 shades of grey.

The processing of the binary image starts by cleaning the image. All pixels that are surrounded by 0's are removed from the image. As the name suggests this process cleans up the image by removing the stand alone pixels that are not associated with any ice particles. As a result of the cleaning process, the effective resolution of the system is reduced to two pixels instead of just one. A sequence of erosion and dilatation operations are used to process a binary image. These operations reduce the amount of noise and create coherent particles of frazil ice that are easily counted. Figures 2.19a and 2.19b show the original subtracted (grayscale) image and the final binary image after applying the morphological procedures described above. A comparison of Figures 2.19a and 2.19b indicates that a particle in Figure 2.19b tends to look larger (to the unaided eye) than the corresponding particle in figure 2.19a. However, an inspection of the pixel values in figure 2.19a indicates that the edges of the particles are as shown in figure 2.19b. The ability of the digital image processing system to discern the true edge of these particles is, of course, one of the advantages of digital manipulation. It should be noted that the processing methodology has undergone extensive manual verification to ensure it correctly detects the edge of a particle and that there is no appreciable distortion of a particle's shape.

The image grabbing and processing technologies were developed by Morris (2002). The end result of the image processing system is two files. The first file contains the vertical distribution of ice particles for each cycle while the second file contains information about the individual frazil particles. The second file is created in such a way that the size and location of every particle can be recalled from any image. For example, the number, size and location of every particle on the third image of the tenth cycle can be examined. This storage method allows flexibility in the examination of the results.



Figure 2.1 Counter-rotating flume.

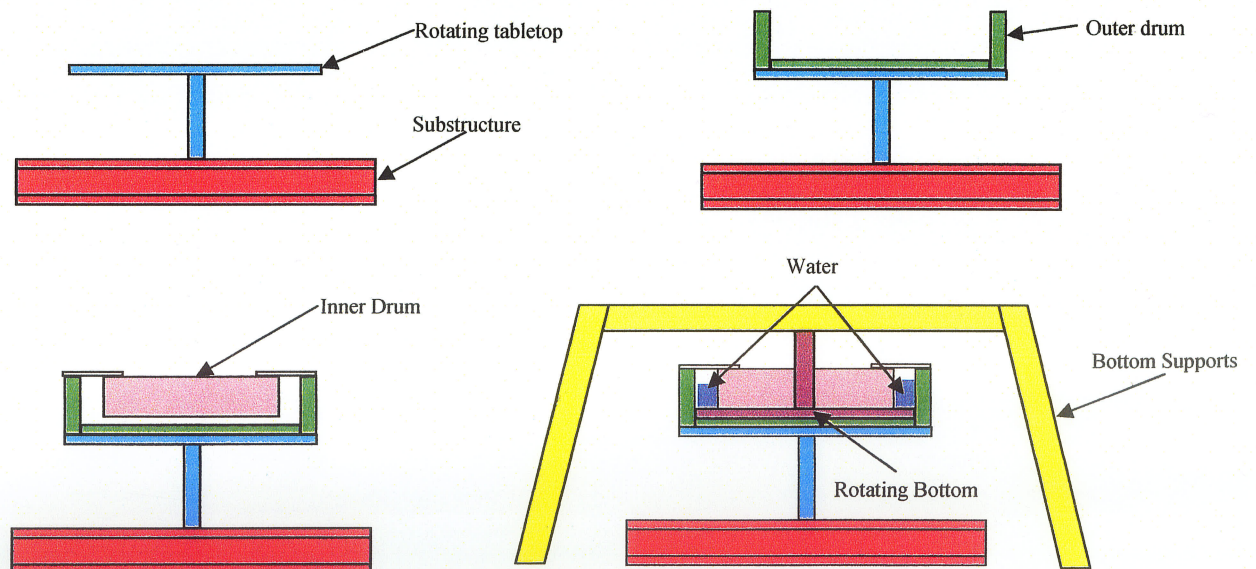


Figure 2.2. Schematic of structural components of the counter-rotating flume (Morris, 2002).

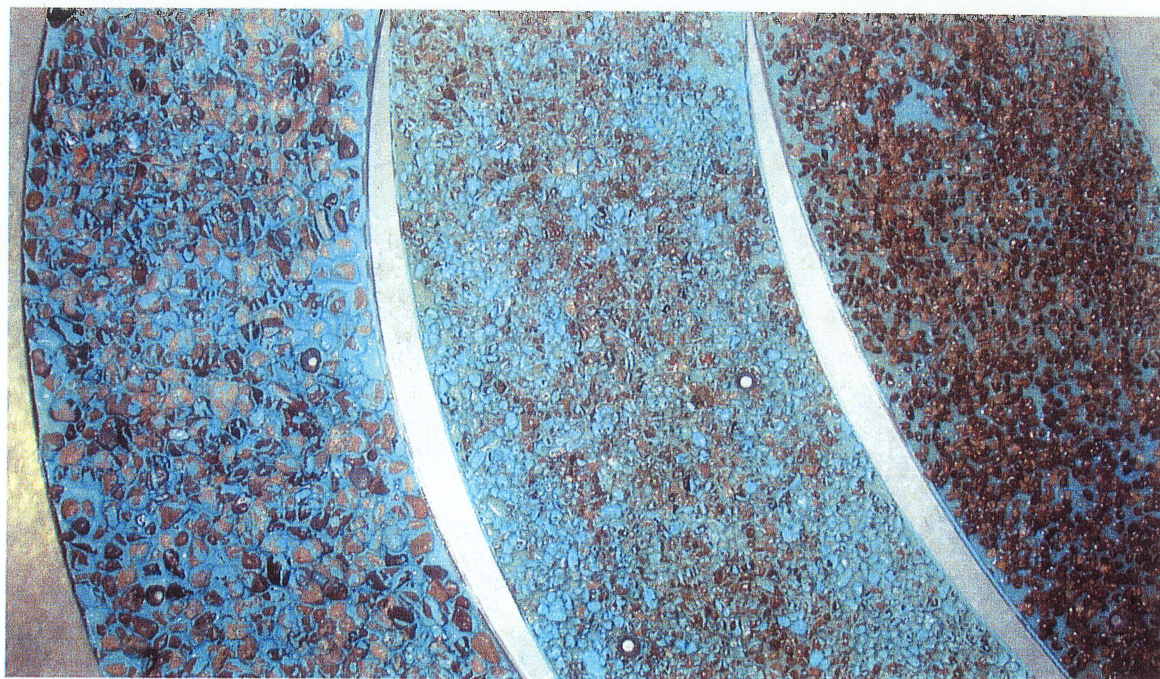


Figure 2.3. Three bedplates with different sizes of embedded gravel.

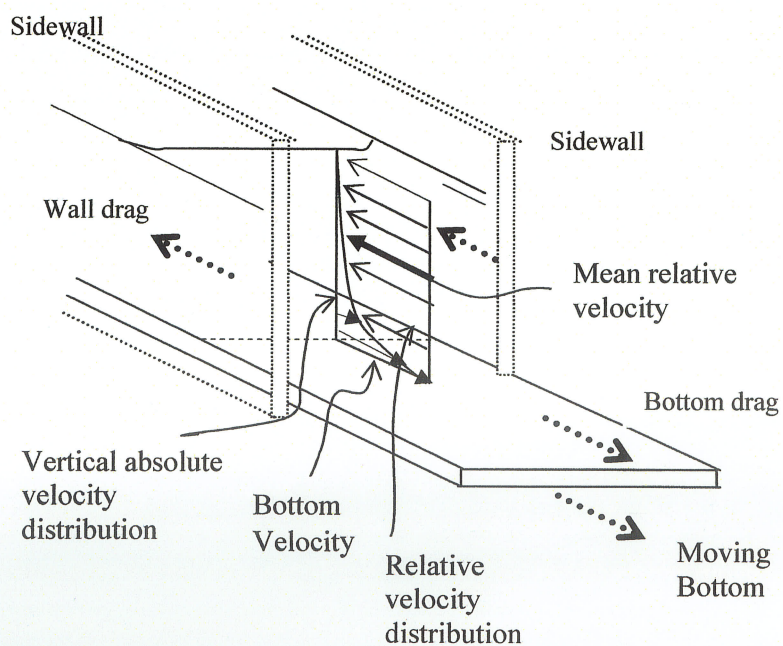


Figure 2.4. Generation of flow in the counter-rotating flume.

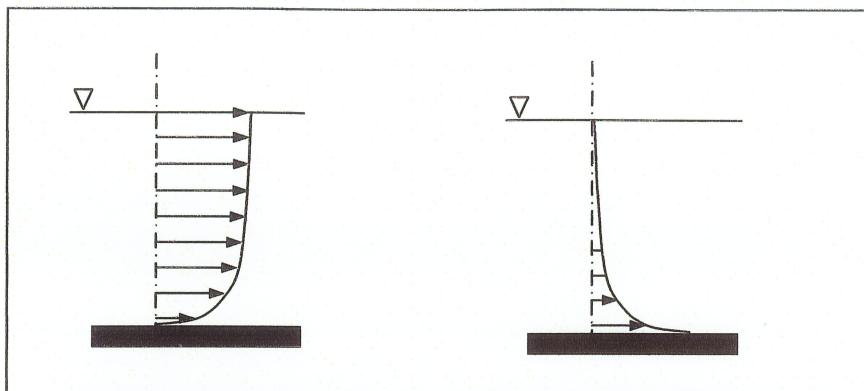


Figure 2.5. Velocity profile generated in (a) open channel flow and (b) in the counter-rotating flume.

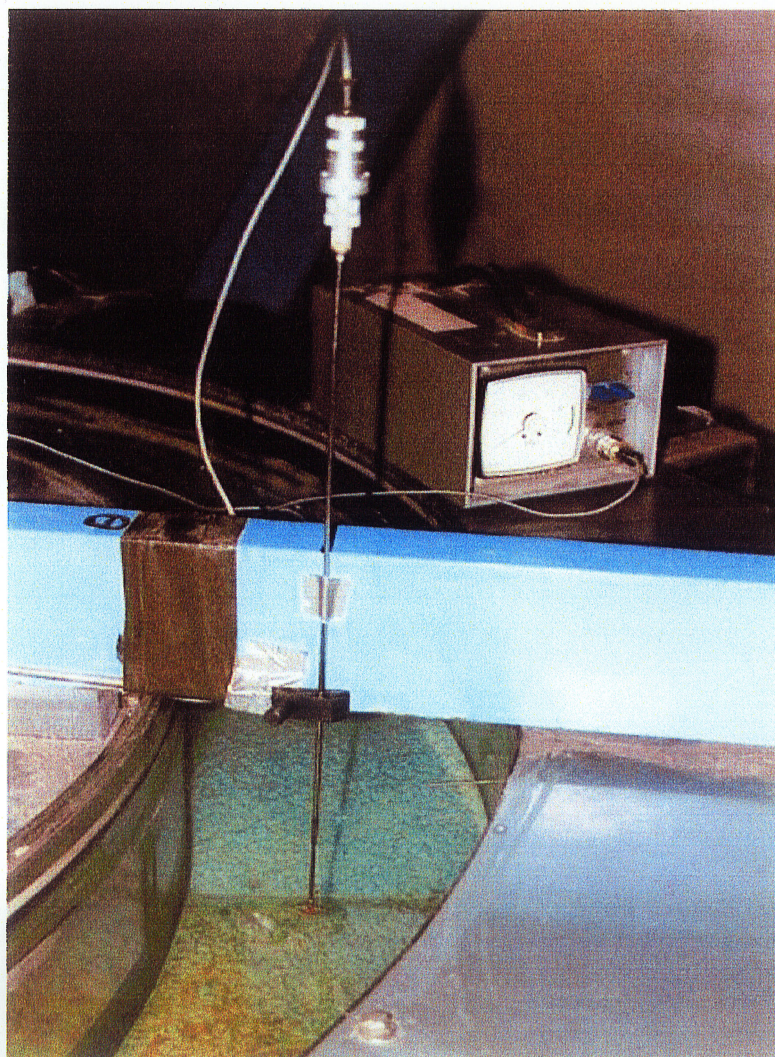


Figure 2.6. Flowmeter used in counter-rotating rates calibration to balance shear stresses.

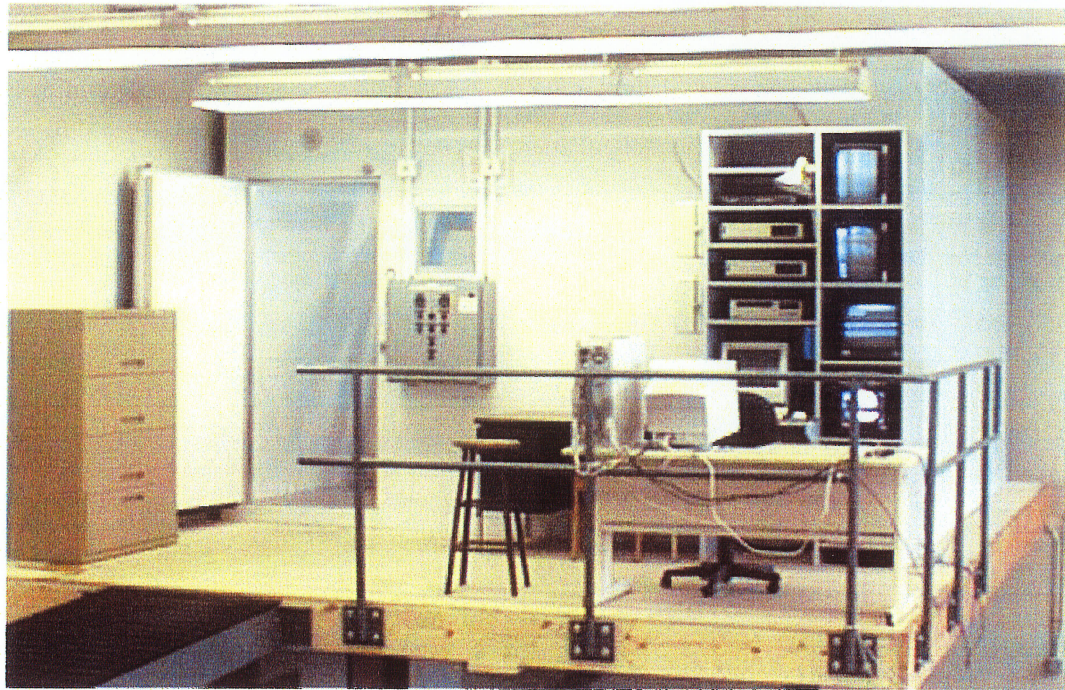


Figure 2.7. HRTF's cold room.

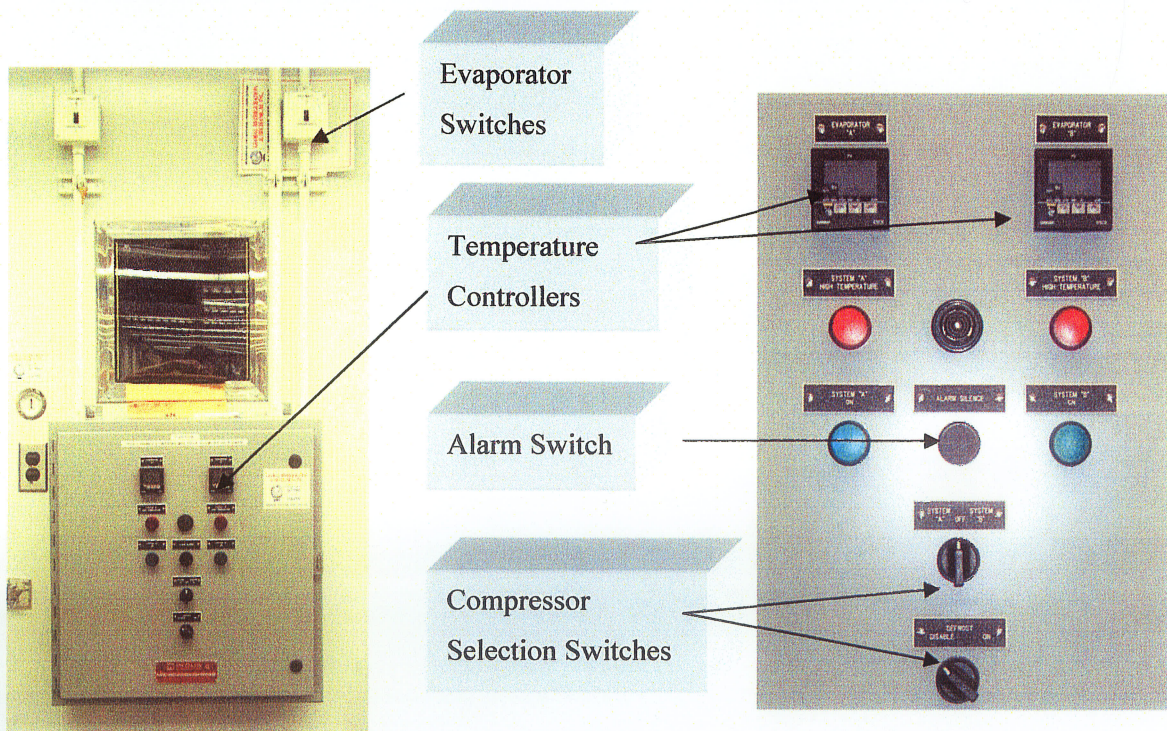


Figure 2.8. Air temperature control board.

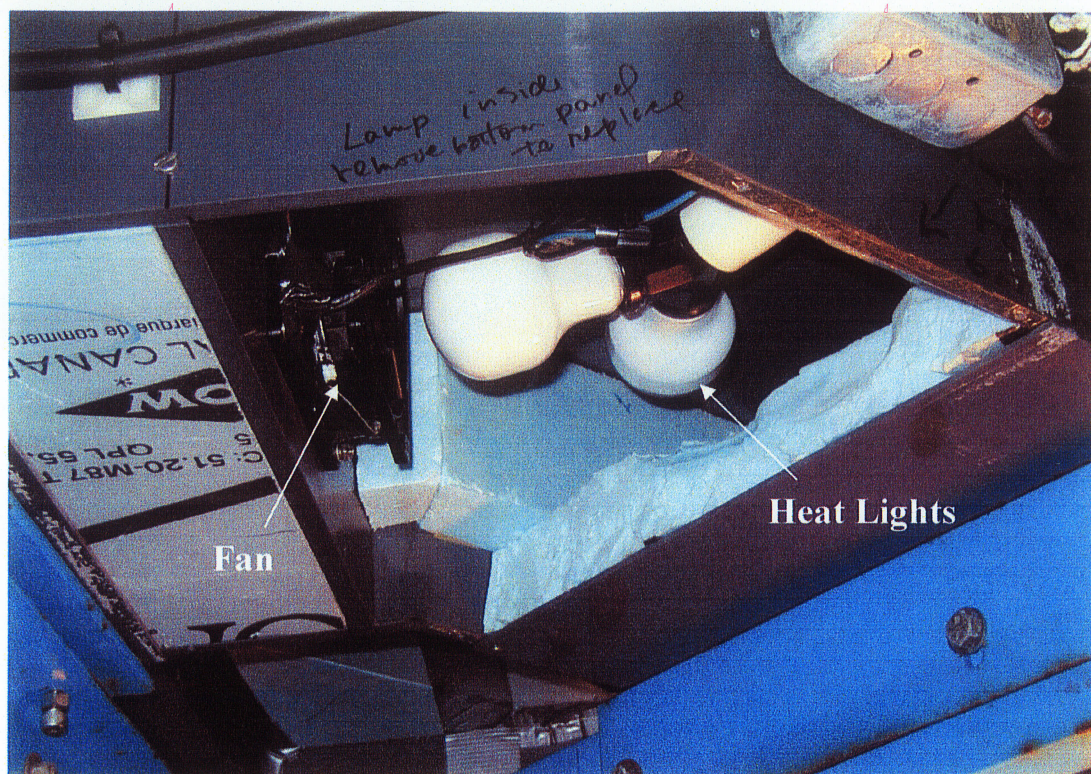


Figure 2.9. Heater and fan of air jacket below the flume bed.

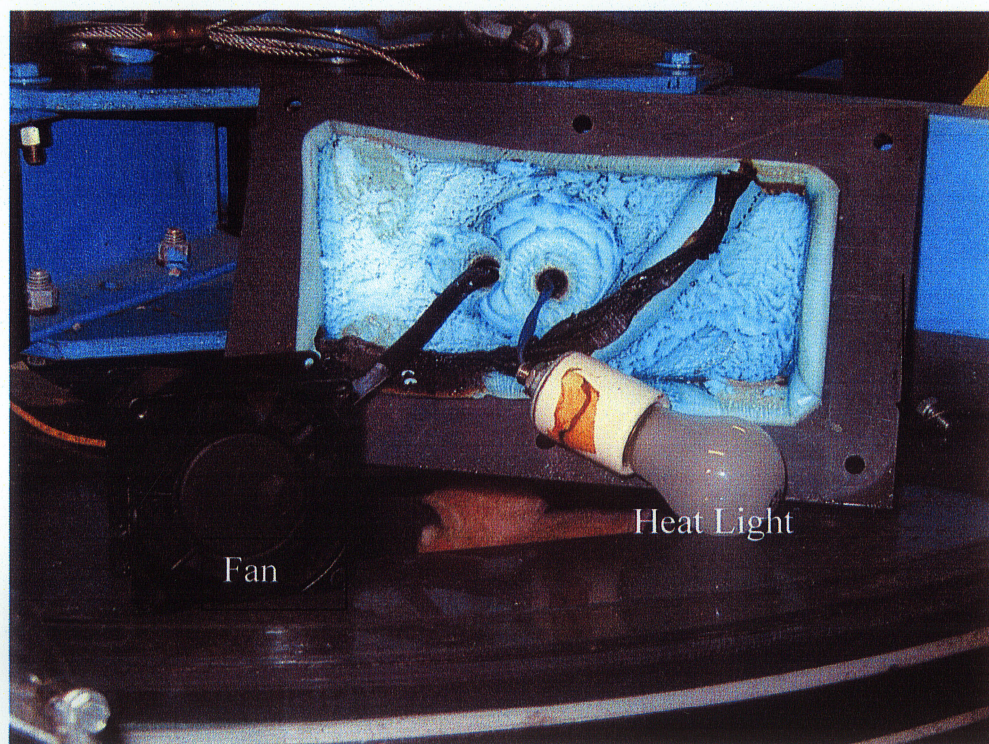


Figure 2.10. Heater lamp and fan in the inner duct.

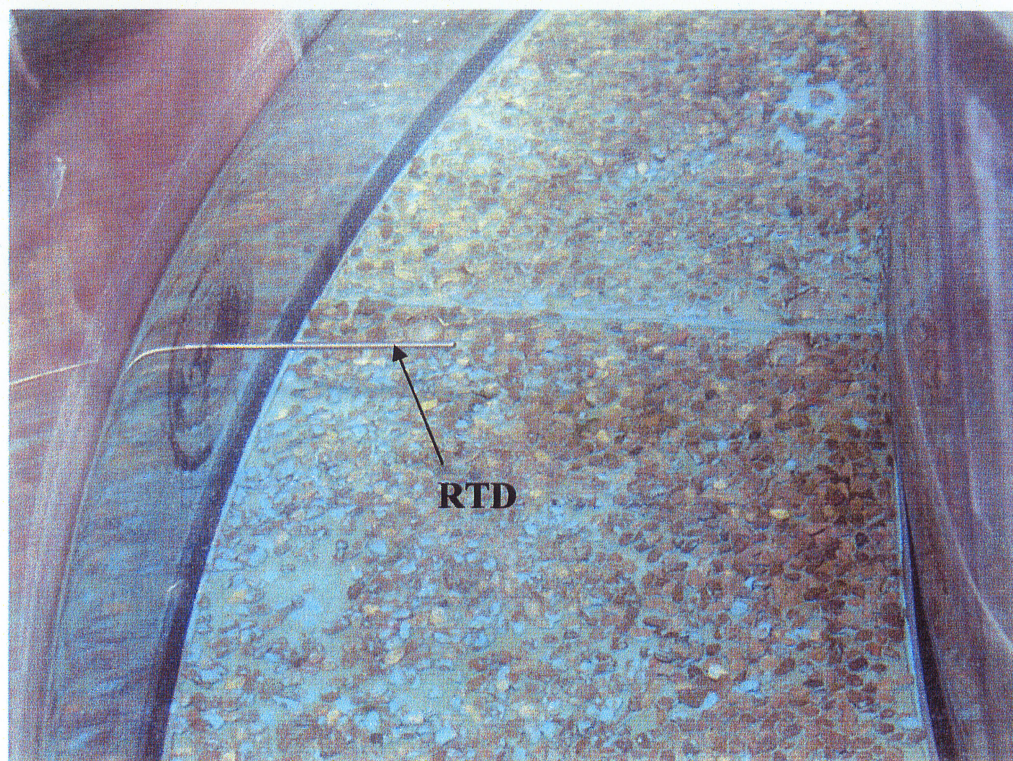


Figure 2.11. The RTD submerged in the water from the side of the flume.

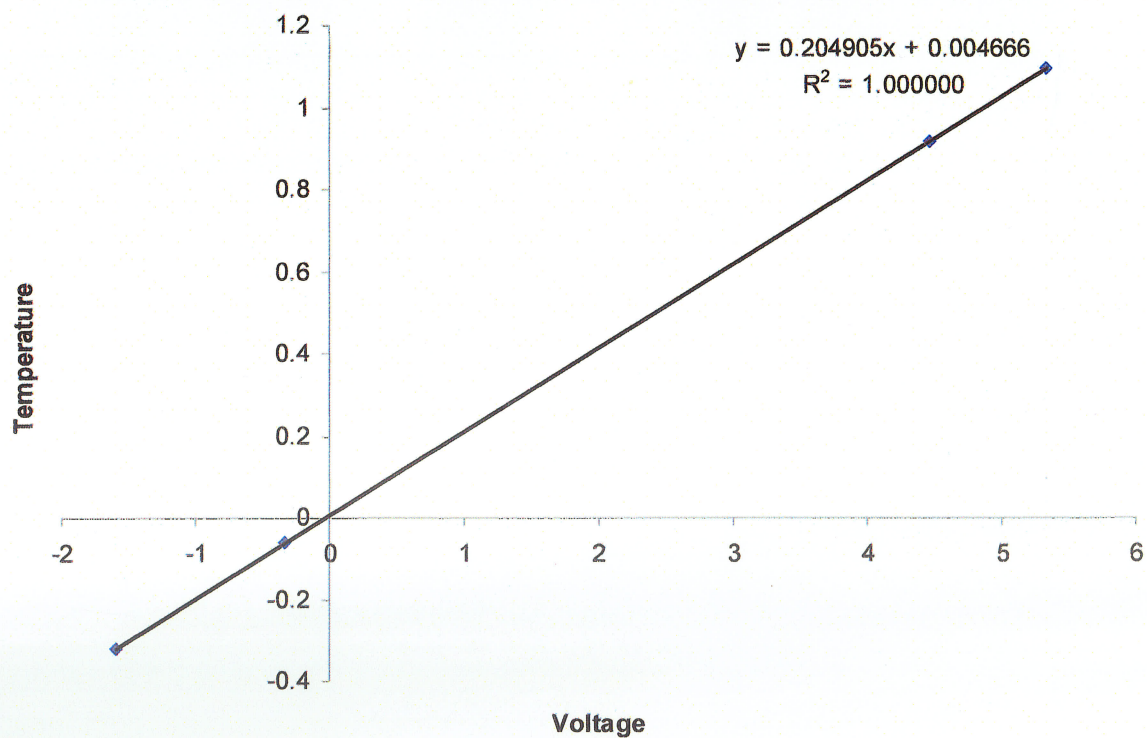


Figure 2.12. Calibration of thermometer.

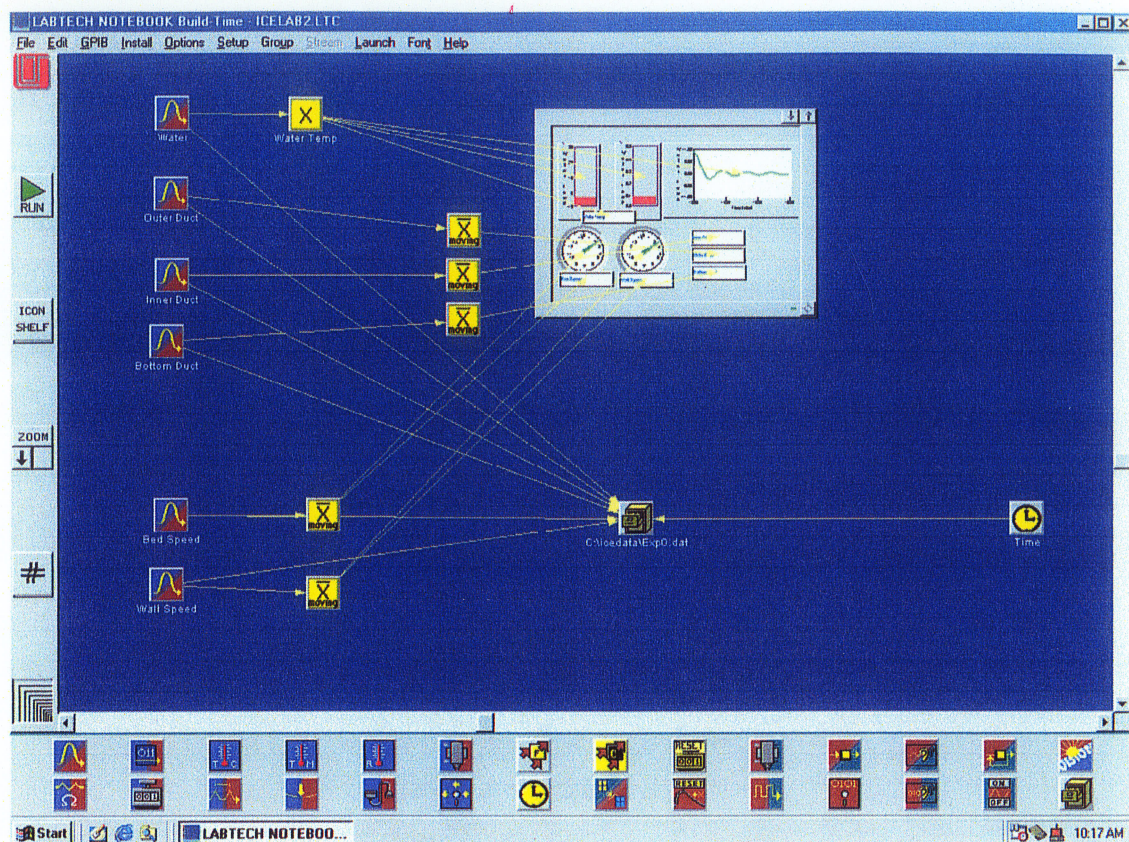
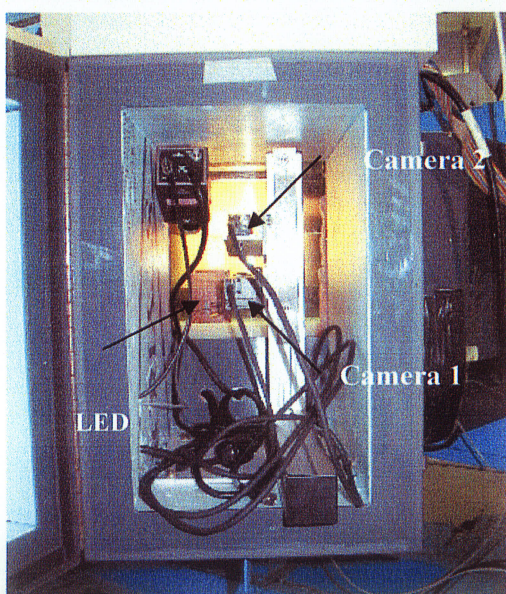
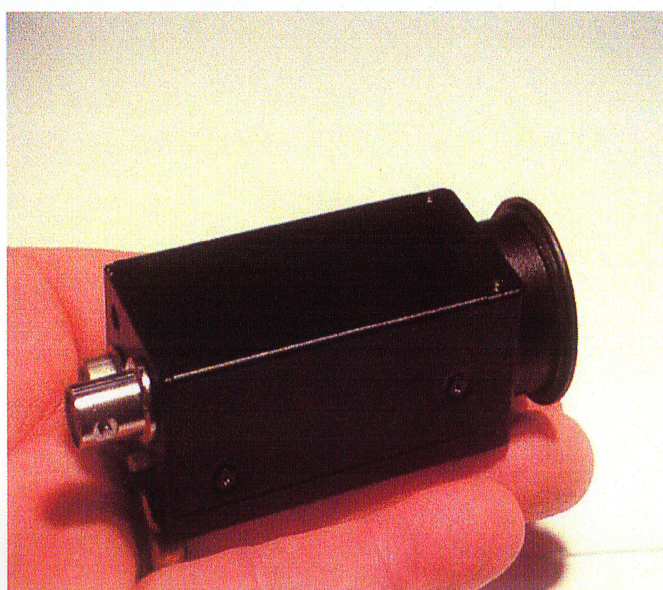


Figure 2.13. Interface of icelab2.itc.



(a)



(b)

Figure 2.14. (a) Cameras and LED housed in the camera box. (b) Camera 2, a Sentech STC-1100a monochrome progressive scan camera (no lense is shown).

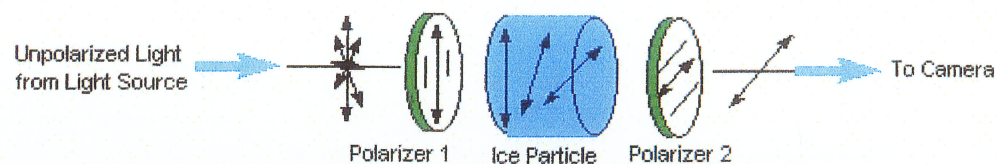


Figure 2.15. Cross polarized light scheme (Morris, 2002).

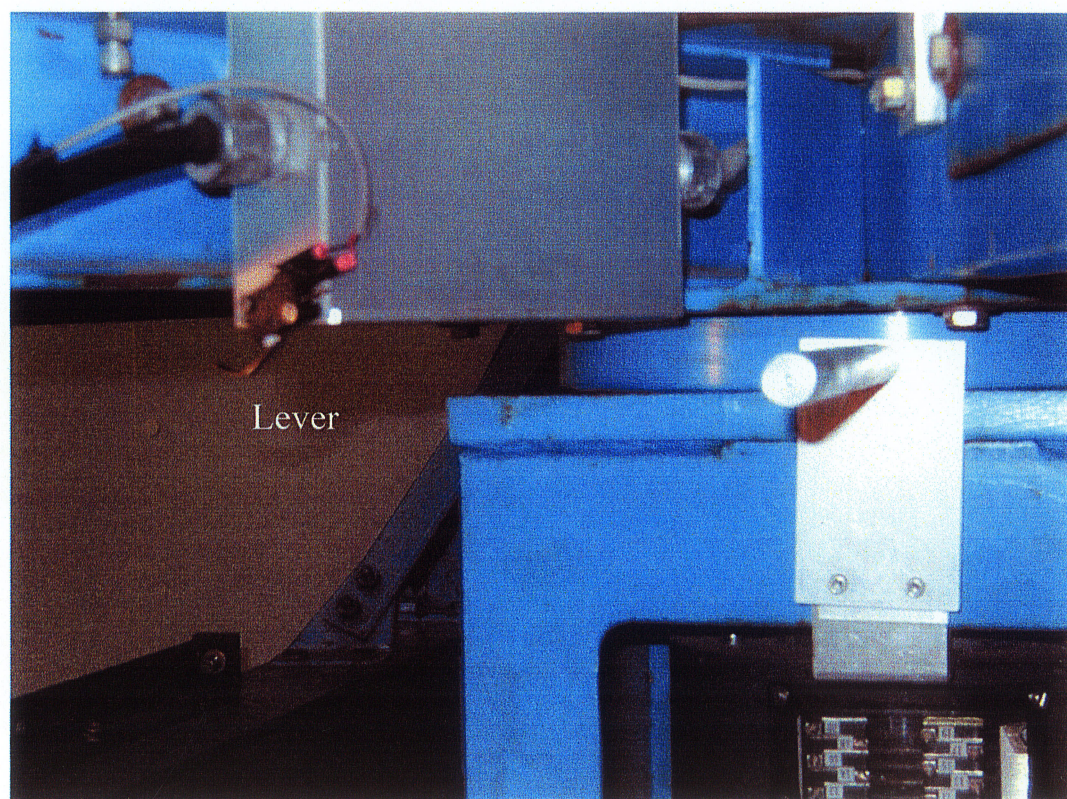


Figure 2.16. LED switch lever under the flume.

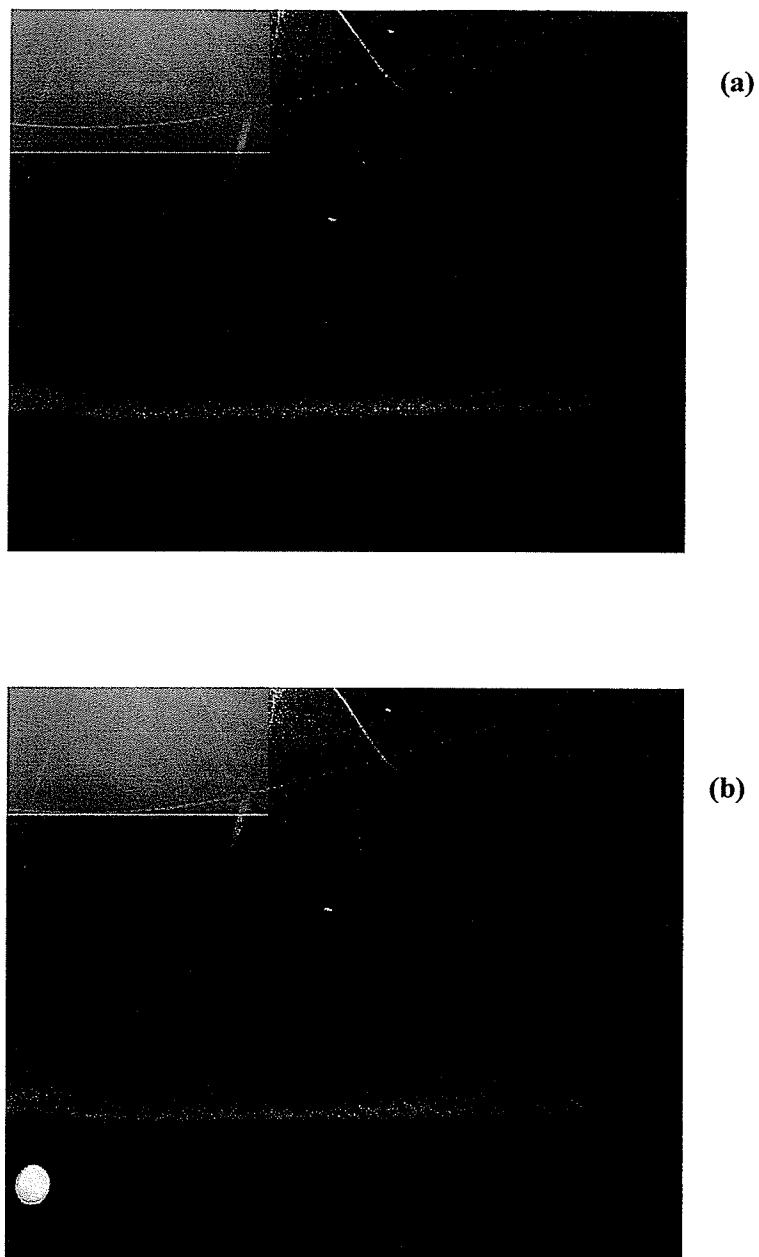


Figure 2.17. Image with (a) the LED off and (b) the LED on.

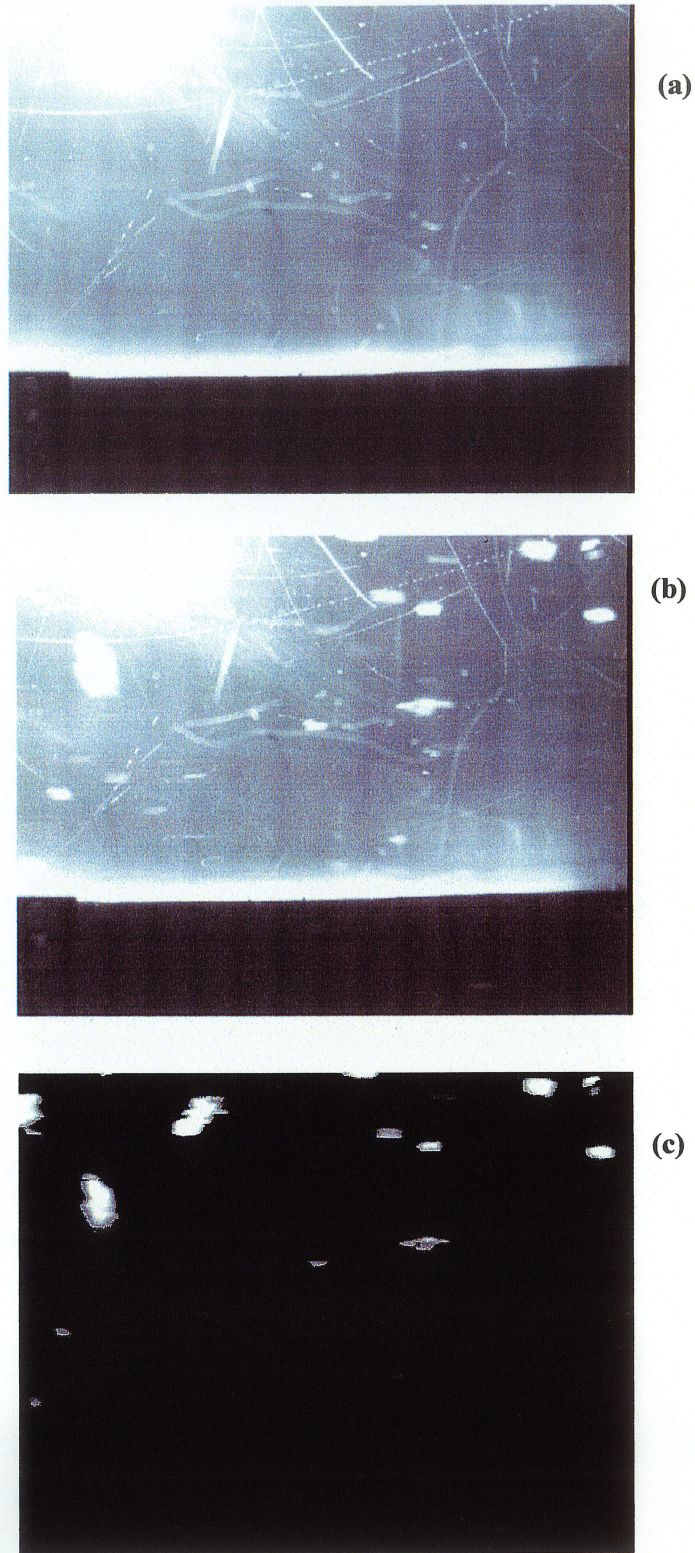


Figure 2.18. (a) Reference image, (b) frazil ice image, and
(c) subtracted image showing frazil ice particles.



Figure 2.19. (a) Grayscale (subtracted) image of frazil ice particles.

(b) Processed (binary) image of frazil ice particles.

A series of experiments were conducted using the counter-rotating ice flume. A total of 42 experiments were collected using water velocity, water depth, and air temperature as variables. The water temperature histories were recorded and analyzed. Using the image processing system, the experiments were analyzed and information about the supercooling process and the evolution of frazil ice were retrieved. By comparing the results, a qualitative description of the effects of thermohydraulic parameters on these processes was achieved.

3.1 Experimental Parameters

A total of 42 experiments (Table 3.1) were collected using the counter-rotating ice flume. The experiments were divided into three groups with three water depths: 10, 15, and 20 cm. In these experiments, the temperatures of the outer, bottom, and bed chambers, as well as bed and wall speeds were recorded. The mean values of these variables recorded during the last 60 minutes of these experiments are listed in Table 3.1. The water velocity was computed using equation (2.2).

In general, an experiment was conducted as follows. The cold room was kept at a constant air temperature of 1°C overnight. Systems were left power on, but the bed and wall were not in moving. On the morning of an experiment, the water temperature would be about 1.0°C, sometimes with a thin ice cover. The ice cover was removed, and new water was added (if needed) to the flume until the

desired water level was obtained. The bed and walls were then set to their designated speeds and the air temperature was reduced to the desired value.

Table 3.1a. Experiment run summary (Group 1, $H = 15$ cm).

Exp. No.	Air Temp. [°C]	Inner Temp. [°C]	Outer Temp. [°C]	Bed Temp. [°C]	Bed Speed [rpm]	Wall Speed [rpm]	Water Velocity [m/s]
56	-10	5.84	2.59	1.68	8.28	6.82	0.434
57	-10	2.63	2.62	1.65	8.31	6.81	0.435
58	-10	3.02	2.65	1.65	9.61	8.33	0.517
59	-10	2.52	3.27	1.67	11.47	9.56	0.604
60	-10	2.87	2.68	1.67	8.12	6.66	0.424
61	-10	2.76	2.68	1.68	8.20	6.49	0.421
62	-10	2.77	2.66	1.67	10.66	9.58	0.584
63	-10	2.66	2.65	1.65	11.43	9.83	0.612
64	-10	3.17	2.66	1.67	7.42	5.88	0.382
65	-10	-3.98	2.65	1.65	7.46	6.13	0.391
66	-10	3.82	2.62	1.68	11.63	10.24	0.630
67	-10	3.54	2.64	1.65	7.71	5.99	0.393
68	-10	3.74	2.66	1.69	7.78	6.28	0.404
69	-10	3.24	2.65	1.65	11.66	10.61	0.642
70	-10	3.85	2.61	1.67	9.65	8.42	0.521
71	-10	3.67	2.66	1.67	8.42	6.57	0.430
72	-10	3.73	2.64	1.68	7.95	6.76	0.423
90	-15	2.87	3.35	1.64	8.40	6.60	0.430
91	-15	1.63	3.50	1.59	9.46	8.41	0.515
92	-15	1.52	3.48	1.61	9.05	8.28	0.500
93	-15	1.38	3.42	1.59	11.67	10.26	0.632
94	-6.5	4.25	3.06	1.66	8.58	6.39	0.428
95	-6.5	4.19	3.06	1.63	8.44	6.51	0.428
102	-10	2.71	3.32	1.70	8.62	6.36	0.428

When the water temperature dropped to 0.10°C , 400 reference images were taken. When the water temperature reached 0.05°C images were again acquired according to a prescribed “duty cycle” until the water temperature stabilized following the end of the supercooling process. After each experiment, the temperature and image data were processed.

Table 3.1b. Experiment run summary (Group 2, H = 10 cm).

Exp. No.	Air Temp. [°C]	Inner Temp. [°C]	Outer Temp. [°C]	Bed Temp. [°C]	Bed Speed [rpm]	Wall Speed [rpm]	Water Velocity [m/s]
73	-10	2.96	3.26	1.66	9.18	6.63	0.491
74	-10	3.33	3.21	1.69	5.7	5.56	0.350
75	-10	2.99	3.24	1.69	8.09	7.88	0.496
76	-10	3.35	2.69	1.66	10.91	11.45	0.695
77	-10	3.94	2.68	1.68	7.56	7.75	0.476
97	-15	1.62	3.48	1.63	7.87	8.05	0.495
99	-12	3.31	3.20	1.64	7.69	7.80	0.482
101	-10	3.60	3.15	1.64	8.01	7.83	0.492

Table 3.1c. Experiment run summary (Group 3, H = 20 cm).

Exp. No.	Air Temp. [°C]	Inner Temp. [°C]	Outer Temp. [°C]	Bed Temp. [°C]	Bed Speed [rpm]	Wall Speed [rpm]	Water Velocity [m/s]
78	-10	4.14	2.72	1.64	11.12	7.43	0.484
79	-10	4.19	2.69	1.68	11.04	7.55	0.486
80	-10	2.53	2.74	1.66	7.34	6.06	0.356
81	-10	2.44	2.72	1.66	8.39	5.65	0.367
82	-10	2.48	2.70	1.66	8.02	5.60	0.357
83	-10	1.77	2.75	1.77	8.35	5.96	0.376
84	-10	2.21	2.78	1.64	10.24	8.13	0.486
85	-10	2.37	2.73	1.64	11.21	8.18	0.510
86	-10	2.51	2.74	1.66	10.61	7.75	0.483
87	-10	2.61	2.73	1.66	10.78	7.75	0.485

3.2 Water Temperature Measurements

3.2.1 Data Processing

The high sampling rate of the water sensor RTD results in data that shows turbulent fluctuations of the water temperature. The 1 Hz water temperature data was preprocessed to remove ‘spikes’ and

ensure data quality. One minute block average was used to identify outliers, which were replaced with the average of the one minute block. The data were subsequently reduced to 0.033 Hz data (a point every 30 s) by averaging over 30 points (Figure 3.1). This averaged data was used for all subsequent analysis. The last 5 minutes of data were averaged to obtain an offset. The offset was then added to the averaged data. The processed temperature data (dash line in Figure 3.1) were used to obtain the characteristics of the supercooling process. The point of maximum supercooling ($T_{\min}$) was noted, and a straight line was fitted through all data points 5 minutes before and 5 minutes after the supercooling phase started in order to calculate the cooling rate of the water. This method of measuring the cooling rate of the water is consistent with previous investigations (e.g., Michel, 1963). The time from the instant when the freezing temperature 0°C is reached to the instant when 0°C is recovered, is defined as t_{sp} the characteristic time of the principal supercooling process and the initial period of frazil production. By carefully controlling the environmental variables, such as the bed, inner, and outer cavity temperatures, as well as the air temperature, an experiment is remarkably repeatable as shown in Figure 3.2; here all three experiments with similar hydraulic and environmental variables produce a very repeatable supercooling curve response.

The measured water temperatures of the experiments were analyzed and presented in Table 3.2. The offset values are not consistent. Some are even positive. This anomaly could be caused by a few of the following considerations.

At first this anomaly was attributed to accumulation of ice on the thermometer. This explanation was verified by observations. In Exp 97 and 99, ice was found on the thermometer, the offsets for these experiments are 0.0026 and 0.0053 $^{\circ}\text{C}$, respectively. After the ice was taken off manually, these offsets changed to -0.0085 and -0.0081 $^{\circ}\text{C}$, respectively. It can be concluded that the ice on thermometer acts as an insulator; its impact depends on the amount and density of ice mass on the thermometer. In some cases, for instance Exp. 96, the accumulation and release of ice from the thermometer produces a fluctuating water temperature (Figure 3.3).

The second factor contributing to this abnormality in the offset is the air temperature. As mentioned in Chapter 2, the thermometer was calibrated at air temperatures of 23°C (manufacturer) and 1°C (HRTF). Since the air temperature surrounding the thermometer during the experiments was below the suggested operating temperature for the thermometer, some variability in signal quality is expected. A warmer ambient operating condition could improve the stability of this thermometer.

The third factor potential causing this anomaly is possibly the quality of the Winnipeg tap water used for these experiments. The equilibrium temperature and freezing point of water varies with the air temperature and water quality (Ashton, 1986, Osterkamp, 1977). An equilibrium temperature of -0.005°C has been observed in rivers (Osterkamp and Gosink 1982, Tsang 1982) and calculations (Osterkamp, 1977).

Table 3.2a. Experiment temperature characteristics (Group 1, $H=15$ cm).

Exp. No.	Air Temp. [$^{\circ}\text{C}$]	Water Velocity [m/s]	Cooling Rate [$^{\circ}\text{C}/\text{min}$]	Period of Super-cooling [Minutes]	Maximum Super-cooling [$^{\circ}\text{C}$]	Offset [$^{\circ}\text{C}$]	Frazil Ice formed (M_v) [%]
56	-10	0.434	-0.0038	23.0	-0.037	0.0010	0.118
57	-10	0.435	-0.0042	24.5	-0.037	-0.0172	0.142
58	-10	0.517	-0.0044	19.5	-0.039	-0.0145	0.116
59	-10	0.604	-0.0032	26	-0.042	0.0116	0.114
60	-10	0.424	-0.0040	23.5	-0.045	0.0067	0.129
61	-10	0.421	-0.0040	21	-0.040	-0.0171	0.116
62	-10	0.584	-0.0044	21	-0.038	-0.0072	0.127
63	-10	0.612	-0.0038	20	-0.042	-0.0114	0.105
64	-10	0.382	-0.0036	28	-0.030	-0.0225	0.136
65	-10	0.391	-0.0038	25	-0.047	-0.0167	0.130
66	-10	0.630	-0.0041	20.5	-0.043	-0.0113	0.114
67	-10	0.393	-0.0035	27.5	-0.033	-0.0155	0.131
68	-10	0.404	-0.0037	26	-0.031	-0.0227	0.132
69	-10	0.642	-0.0038	19	-0.041	-0.0007	0.098
70	-10	0.521	-0.0038	23	-0.031	-0.0178	0.120
71	-10	0.430	-0.0037	26	-0.044	-0.0136	0.133
72	-10	0.423	-0.0038	22.5	-0.041	-0.0241	0.118
90	-15	0.432	-0.0071	17.5	-0.042	0.0243	0.169
91	-15	0.515	-0.0060	18.0	-0.032	-0.0014	0.149
92	-15	0.500	-0.0060	18.5	-0.041	0.0052	0.152
93	-15	0.632	-0.0065	15.5	-0.052	0.0077	0.138
94	-6.5	0.428	-0.0021	41.5	-0.030	0.0092	0.117
95	-6.5	0.428	-0.0020	39	-0.035	0.0087	0.107
102	-10	0.428	-0.0035	28.5	-0.048	-0.0054	0.138

The calculation of the percentage of "frazil ice formed" given in Table 3.2a to 3.2c is explained in the following section. Water temperature measurements of these experiments are shown in Appendix A.

Table 3.2b. Experiment temperature characteristics (Group 2, H = 10 cm).

Exp. No.	Air Temp. [°C]	Water Velocity [m/s]	Cooling Rate [°C/min]	Period of Super-cooling [Minutes]	Maximum Super-cooling [°C]	Offset [°C]	Frazil Ice formed (M_v) [%]
73	-10	0.491	-0.0045	22	-0.044	0.0059	0.134
74	-10	0.350	-0.0043	23.5	-0.044	0.0005	0.138
75	-10	0.496	-0.0042	26	-0.044	-0.0007	0.151
76	-10	0.695	-0.0056	16	-0.043	0.0112	0.122
77	-10	0.476	-0.0047	23.5	-0.044	0.0044	0.152
97	-15	0.495	-0.0083	17	-0.052	0.0026	0.194
99	-12	0.482	-0.0046	24.5	-0.045	-0.0053	0.155
101	-10	0.492	-0.0035	29	-0.039	-0.0032	0.138

Table 3.2c. Experiment temperature characteristics (Group 3, H = 20 cm).

Exp. No.	Air Temp. [°C]	Water Velocity [m/s]	Cooling Rate [°C/min]	Period of Super-cooling [Minutes]	Maximum Super-cooling [°C]	Offset [°C]	Frazil Ice formed (M_v) [%]
78	-10	0.484	-0.0029	31.5	-0.037	-0.0014	0.125
79	-10	0.486	-0.0028	28	-0.043	0.0019	0.107
80	-10	0.356	-0.0030	27	-0.028	-0.0083	0.110
81	-10	0.367	-0.0037	28.5	-0.039	-0.0108	0.143
82	-10	0.357	-0.0034	30	-0.028	-0.0074	0.140
83	-10	0.376	-0.0049	21.5	-0.038	-0.0137	0.143
84	-10	0.486	-0.0037	22.5	-0.040	-0.0101	0.113
85	-10	0.510	-0.0028	25.5	-0.036	-0.0059	0.099
86	-10	0.483	-0.0029	24.5	-0.040	-0.0113	0.099
87	-10	0.485	-0.0029	30	-0.038	-0.0052	0.119

3.2.2 Thermodynamic Estimate of Frazil Volume

An estimate of the expected concentration of frazil ice in a supercooled environment can be derived from thermodynamic considerations. Consider a fully mixed (Figure 3.4), thermally insulated

container of water without a lid. For a unit volume (i.e., 1 m³) of water the heat balance can be conceptualized as

$$Q_{aw} = Q_i \pm Q_w, \quad (3.1)$$

where Q_{aw} is the exchange at the air - water interface, Q_i is the formation of ice, and Q_w represents any cooling (Q_w will be positive) or warming (Q_w will be negative) of the water. At the end of the principal period of supercooling, t_{sp} , when an offset (T_{off}) exists (Figure 3.1), the heat balance is given by

$$Q_{aw}(t_{sp} + \Delta t) = T_{off}(1 - M_v)C_p\rho + M_vL_i\rho_i, \quad (3.2)$$

where M_v is the volume of ice formed over time $t_{sp} + \Delta t$, Δt is the time length of freezing point variation due to the offset, C_p is the specific heat of water, and L_i is the latent heat of fusion of ice. The left hand side of (3.2) represents the heat exchange across the air-water interface, which is a release of heat from the water surface. The term $T_{off}(1 - M_v)C_p\rho$ denotes the heat released from the water due to the offset of the water temperature. The remaining term ($M_vL_i\rho_i$) is the heat released due to the formation of frazil ice. For known conditions (i.e., Q_{aw} , t_{sp} , and T_{off}) equation (3.2) can be rearranged to estimate the concentration of frazil ice (in the volume),

$$M_v = \frac{Q_{aw}t_{sp} + Q_{aw}\Delta t - T_{off}C_p\rho}{L_i\rho_i - T_{off}C_p\rho}. \quad (3.3)$$

In general, the heat exchange between the water surface and the overlying air (Q_{aw}) is governed by (Hanley and Michel, 1975, 1977)

$$Q_{aw} = \phi(T_a - T), \quad (3.4)$$

where ϕ is the heat transfer coefficient, T_a is the air temperature, and the T is the water temperature. The parameter ϕ , which is expressed in $\text{W/m}^2/\text{°C}$, is quite sensitive to wind speed but is constant for a given air temperature and wind speed. The heat exchange between the water surface and the air (Q_{aw}) can also be calculated from the cooling rate of the water, i.e.,

$$Q_{aw} = \rho C_p \frac{dT}{dt}. \quad (3.5)$$

Replacing Q_{aw} in (3.3) with (3.5), then,

$$M_v = \frac{C_p \rho \left(t_{sp} \frac{dT}{dt} + (\Delta t \frac{dT}{dt} - T_{off}) \right)}{L_i \rho_i - T_{off} C_p \rho}. \quad (3.6)$$

As shown in Figure 3.1, the following approximation can be used

$$\Delta t \frac{dT}{dt} \cong T_{off}. \quad (3.7)$$

T_{off} is typically on the order of one thousandth of a degree. For typical values of L_i , C_p , ρ , and ρ_i , $L_i \rho_i$ is much larger than the term $T_{off} C_p \rho$ by 10^4 or 10^5 . Therefore, neglecting the very small terms in (3.6), the frazil ice concentration can be approximated by

$$M_v = \frac{C_p \rho}{L_i \rho_i} t_{sp} \frac{dT}{dt}, \quad (3.8)$$

where dT/dt is the cooling rate, which is given by the slope of the time (t) versus temperature (T) curve 5 minutes before and after freezing point is reached.

This simple thermodynamic model was used to predict the percentage of frazil ice formed given in Table 3.2. However, this estimate relies on the characteristics of the supercooling processes as measured by the Guildline thermometer and is therefore subject to the shortcomings of this thermometer.

3.2.3 Characteristics of Supercooling Process

The temperature characteristics of each experiment can be summarized with three statistics: cooling rate, period of supercooling, and degree of supercooling; these predict the amount (percentage) of frazil ice formed. The cooling rate was calculated by determining the slope of the temperature plot during initial supercooling period. The second parameter, period of supercooling, measures the time each experiment was in a state of supercooling or below 0°C . The third parameter, degree of supercooling (or maximum supercooling), measures the minimum temperature reached during the experiment. The analyses are mainly made using Group 1 experiments, with water depths of 0.15 m, because there are a relatively larger number of experiments in this group.

Figure 3.5 shows the cooling rates plotted against the air temperature for the same or similar water velocities. It appears that the cooling rate is strongly affected by the air temperature. The data shows that the cooler air temperatures yield higher cooling rates, and shorter durations of supercooling process for each water velocity. From the relative shapes of these supercooling curves it can be inferred that colder air temperature generate more frazil ice. This result is consistent with expectation since a cooler air temperature provides a larger "heat sink" which drives the cooling of the water surface. This result is similar to the results of Hanley and Michel (1977) who also concluded that the cooling rate is strongly affected by the surrounding air temperature.

If the air temperature is kept constant, the effects of water velocity on the process are presented in Figure 3.6. From Figure 3.6, the effect of the water velocity on the cooling rate is unclear. However, Figure 3.7 indicates that the rate of cooling increases slightly for increasing velocity, which is not inconsistent with expectation. Hanley and Michel (1977) concluded that the water velocity has a very

weak influence on the cooling rate. The Lagrangian frame of reference in the counter-rotating flume could further weaken any influence of velocity. As shown in Figure 3.8, the relationship of maximum supercooling and water velocity is not well defined, but the weakly observed trend of increased supercooling for increased velocity is also not inconsistent with expectation.

However, the length of supercooling was longer for slower velocities (Figure 3.9). As a result, smaller velocities yield more frazil ice formation (Figure 3.10). Similar influences of air temperature and water velocity on the supercooling process can be found for Group 2 and 3 experiments. The similar tendency of ice formation with water velocity is also found for Group 2 and 3 experiments (Figure 3.11) which show a different linear relationship from Group 1.

Figure 3.12 shows the effect of water depth on the supercooling process. The deeper flow produces a smaller cooling rate and maximum supercooling. This is not inconsistent with expectation because the deeper water needs more time to cool down.

Figure 3.13 shows the Reynolds number and frazil ice formed for the three groups, which better shows the turbulence influence on the ice formed. The stronger the turbulence, the less ice that is formed.

3.3 Frazil Ice Image Observations

3.3.1 Data Processing

In the early stages of this research, the camera position was set and fixed such that the water surface would not appear in the images, but would be just out of site of the camera. This was done because the water surface might be misinterpreted as ice particles in the ice particle recognition process since the surface can appear as bright as a frazil particle. At the start of each experiment, the water level in the flume would be adjusted until the water surface was H cm above the bedplates, where H was the desired water depth for an experiment. Theoretically, this adjustment would bring the water surface out of view of the camera. Unfortunately, in some experiments the water depth was not sufficiently precise so the water surface did appear in the top few millimeters of the images. As expected, the results of the particle recognition process were misleading.

Some experiments also encountered a growth of border ice on the Plexi-Glass, especially Group 3 experiments. The ice initiated at the air/water/Plexi-Glass interface and progressed downward a few centimeters, even though the cavities surrounding the flume walls were heated. This caused a bright patch to appear on the top of each image, which slowly progressed downward. Since this ice was very bright, it would be mistaken by the particle recognition process as a large ice particle floating on the surface. Figure 3.14 shows an image illustrating the border ice growth phenomenon. Note the bright ice and the dim ice. The brighter ice is located on the back of the flume wall while the dull ice is attached to the front of the flume wall.

Table 3.3a. Experiment frazil image characteristics (Group 1, $H = 15$ cm).

Exp. No.	Water Velocity [m/s]	Air Temp. [°C]	Frazil ice formed [%]	Peak frazil number	Gap with supercooling [min]	Peak frazil Area [%]	Gap with supercooling [min]
56	0.434	-10	0.118	48.84	2.25	4.71	-0.75
57	0.435	-10	0.142	42.90	6.25	5.16	3.25
58	0.517	-10	0.116	93.76	2.75	7.02	1.75
60	0.424	-10	0.129	56.86	3.25	4.71	2.25
61	0.421	-10	0.116	38.94	1.75	4.49	2.75
62	0.584	-10	0.127	75.04	1.75	5.52	-0.25
63	0.612	-10	0.105	76.32	0.75	5.40	-1.25
64	0.382	-10	0.136	24.90	16.25	3.37	0.25
65	0.391	-10	0.130	29.58	4.75	4.32	0.75
66	0.630	-10	0.114	87.6	0.75	7.20	-1.25
67	0.393	-10	0.131	25.08	1.25	2.04	-2.75
68	0.404	-10	0.132	22.54	2.75	3.62	-5.25
69	0.641	-10	0.098	82.04	2.25	6.93	1.25
71	0.430	-10	0.133	47.24	4.75	4.69	3.75
72	0.423	-10	0.118	44.24	1.25	3.92	-7.25

To overcome the misinterpretation of ice from the water surface and the migration of ice on the flume walls, a portion of the image was not used. Obviously if ice growing on the flume walls can be seen in the first 50 lines of an image, then those lines can not be used in the image analysis. A visual inspection of several images determined the number of lines that cannot be used in an experiment. Before an experiment was analyzed, several images in the latter portion of the run were inspected to establish how far the ice had crept down the wall. It was important to examine images near the end of

the experiment to determine the farthest reach of the migrating wall ice since this ice is continually expanding. The outcome of this inspection yielded the number of lines that the processing system omitted. In fact, the number of lines omitted depended on the hydrothermal variables. As mentioned, the Group 3 experiments were particularly susceptible to this phenomenon. In order to compare frazil images between experiment groups, a maximum value of 100 lines, was removed in the processing of all of these experiments.

During the experimental runs, the camera and LED was adjusted twice in order to view the full depth of water, consequently a total of 23 experimental images were processed and presented in two groups in Table 3.3. It is worth noting that frazil images can only be compared for the same group with identical camera, and LED locations.

3.3.2 Number and Area of Frazil Particles

The field of view of the camera is approximately 20 cm by 15 cm and is converted to a digital image having 640 by 480 square pixels. This results in a single pixel being approximately 250 μm square. Although the length of one pixel is about 250 μm , the smallest particles that can be recognized as frazil ice are around 4 pixels, or about 1.0 mm if all four pixels are in a line. The frazil particles, having different sizes, were grouped into various bins by size in the interval (5 pixels), and the number of frazil particles in these bins were counted. Therefore, the smallest frazil diameter can be considered with certainty is from the second bin (5-10 pixels); the relevant frazil diameter is 2 mm. Figure 3.15a shows a typical disk shaped frazil particle. Frazil particles with larger sizes tend to have both non-uniform thicknesses and shape. It is common to see a small circular area of different light intensity in the middle of the larger frazil particles. The diameter to thickness ratio is difficult to measure since the frazil particles rarely travel exactly perpendicular to the flow with respect to either of their axes. It is more common to see the particles as oval shaped (Figure 3.15a) because of their skewed orientation in the water.

As the amount of frazil increases, the particles flocculate and sinter together. It is common to see larger, irregular shaped particles sintered together than small perfectly rounded particles. Both the size and the density of these flocs increases as an experiment progresses. It is common at the end of the principal supercooling to have a few very large flocs with a low concentration of individual frazil particles. Particle sintering and flocculation is shown in Figure 3.15b.

The frazil particles' information obtained from the Digital Image Processing System (DIPS) is stored in the column vectors described in section 2.2.3. This information is used to visualize the evolving vertical distribution of ice particles. Two important features of frazil ice evolution are the change in number and change in volume of frazil ice particles over the length of an experiment. The change in the number of ice particles is easily measured by simply counting the number of particles in each image. By plotting the matrix containing the number of particles for a given size versus time, a 3-D plot is obtained (Figure 3.16). The data used to generate this figure was taken from Exp.63, which lasted 35 minutes and collected images with a cycle length of 1 minute. The y-axis represents the frazil size, measured in pixels. The z-axis corresponds to the number of frazil particles that were present at each size bin for each time cycle.

Table 3.3b. Experiment frazil image characteristics (Group 3, H =20 cm).

Exp. No.	Water Velocity [m/s]	Air Temp. [°C]	Frazil ice formed [%]	Peak frazil number	Gap with supercooling [min]	Peak frazil area [%]	Gap with supercooling [min]
80	0.356	-10	0.110	45.96	8.75	12.36	-0.25
81	0.367	-10	0.143	72	11.25	8.98	5.25
82	0.357	-10	0.140	109.32	6.75	8.63	5.75
83	0.376	-10	0.143	95.58	5.25	10.76	-24.75
84	0.486	-10	0.113	136.4	3.25	10.67	1.25
85	0.510	-10	0.099	142.66	4.25	9.73	2.25
86	0.483	-10	0.099	140.88	3.75	10.98	0.75
87	0.486	-10	0.119	121.54	7.25	8.96	5.25

However, the change in the volume of frazil ice is not so straightforward and simple. The image processing system can measure the area of ice particles observed on each image simply by counting the number of pixels excited. To convert the area of ice particles seen on a 2-D image into a volume of ice requires some assumptions about the shape of the frazil ice particles. Because the shape and density of a frazil particle changes as it grows and differs depending on the flow and atmospheric conditions (Daly, 1984 and Ashton, 1986), the task of converting an area seen on the image to a volume of ice becomes quite difficult. Due to these difficulties, no conclusive conjecture will be made. Therefore instead of reporting the evolution of the volume of ice, the area analysis of these data herein will be limited to a presentation of the area of ice. Figure 3.17 shows the total number

and area variation during the same experiment as Figure 3.16, i.e., Exp. 63. It is worth noting that there is a time gap, of about 2 minutes, between the peak values of frazil number and area.

Figure 3.18 compares the supercooling process and frazil area evolution during this process. Frazil ice grows at a slow but increasing rate as the water gets colder. Frazil formation occurs rapidly during the warming stage of the water. About the end of the principal supercooling (t_r), the frazil ice sticks together and aggregates to form big floes, which float to the surface, and form surface ice. This finding can be considered a conclusion from all the experiments shown in Appendix B.

Table 3.3 presents the image characteristics, such as the peak frazil number and peak frazil area as well as their time gaps with the end of the principal supercooling. Figures 3.19 and 3.20 show the peak number of frazil particles versus water velocity. Figures 3.21 and 3.22 show the peak frazil area versus water velocity. In these figures, the larger squares indicate the averaged values surrounding this point. Obviously, the higher water velocity produces a higher number of frazil particles and a greater area (volume) of frazil. This tendency is contrary to the estimate of frazil ice formed based on a heat budget approach. This controversy will be explained later in the Chapter 7.

The image observations indicate that turbulence has a strong influence on the number (Figure 3.23) and size of frazil particles, its distribution, and the total volume of frazil formed during the supercooling process. It is worth noting the image grabbing and processing system is unable to detect a frazil particle smaller than a certain size (i.e., 2 mm). The image information is two dimensional and represents an across flume integration.

The frazil particle information can be displayed in different ways, such as vertical distribution, and temporal and spatial variations over the supercooling process. These features of frazil evolution will be discussed in a subsequent chapter in this study.

3.4 Summary of Experimental Findings

In summary, preliminary findings of the water temperatures are as follows:

- i) The cooling rate of the water depends strongly on the air temperature. Higher cooling rates occur for colder air temperatures.

- ii) The cooling rate is very weakly dependent on water speed; higher velocities cause a slightly larger rate of cooling.
- iii) The length of the principal supercooling is strongly related to air temperature; colder air causes a shorter supercooling.
- iv) The length of supercooling is weakly dependent on water speed; the length of supercooling decreases slightly for increased velocity.
- v) The maximum amount of supercooling depends mainly on air temperature; colder air temperatures increase the maximum supercooling.
- vi) The maximum amount of supercooling is weakly related to water velocity; supercooling increases for decreased velocity
- vii) Ice formed in the principal supercooling based on a heat budget approach is proportional to cooler air temperature, but inversely related to water speed.

A digital image processing system has developed (Morris, 2002) in parallel with this research to characterize the temporal evolution of frazil ice size, density, and spatial distribution. An analysis of these data show:

- i) Frazil ice grows at a slow but increasing rate as water supercools. Frazil formation occurs rapidly during the warming stage of the water. At the end of the principal supercooling, the frazil ice sticks together and aggregates to form big floes, which float to the surface.
- ii) Frazil particle growth seemingly follows a certain rule which may be governed by flow turbulence. The flow turbulence has stronger influence on frazil size than the amount of frazil formed.
- iii) The influences of flow turbulence on supercooling process and frazil evolution have been identified, and a further model needs to be developed to explore its role in the processes.

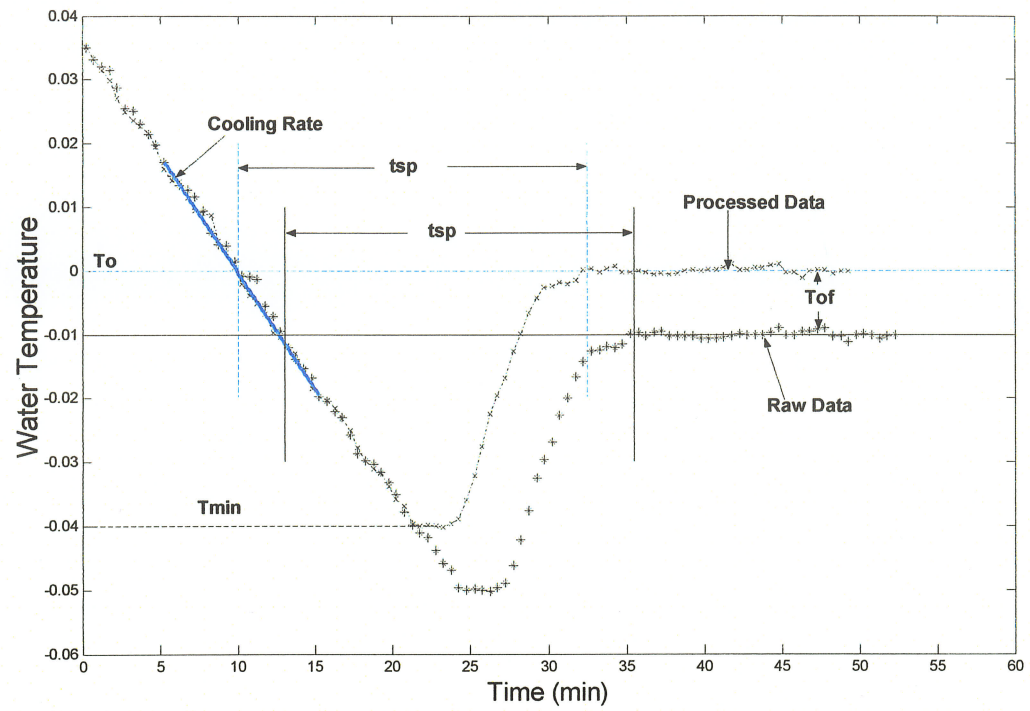


Figure 3.1. Supercooling process analysis.

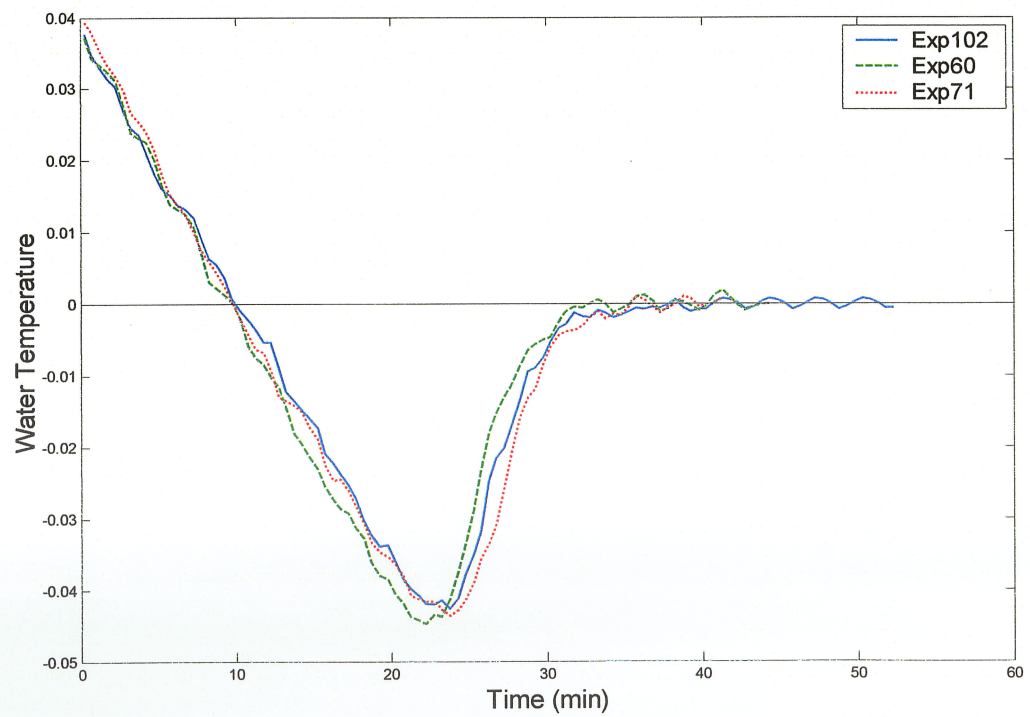


Figure 3.2. Repeatability of experiments.

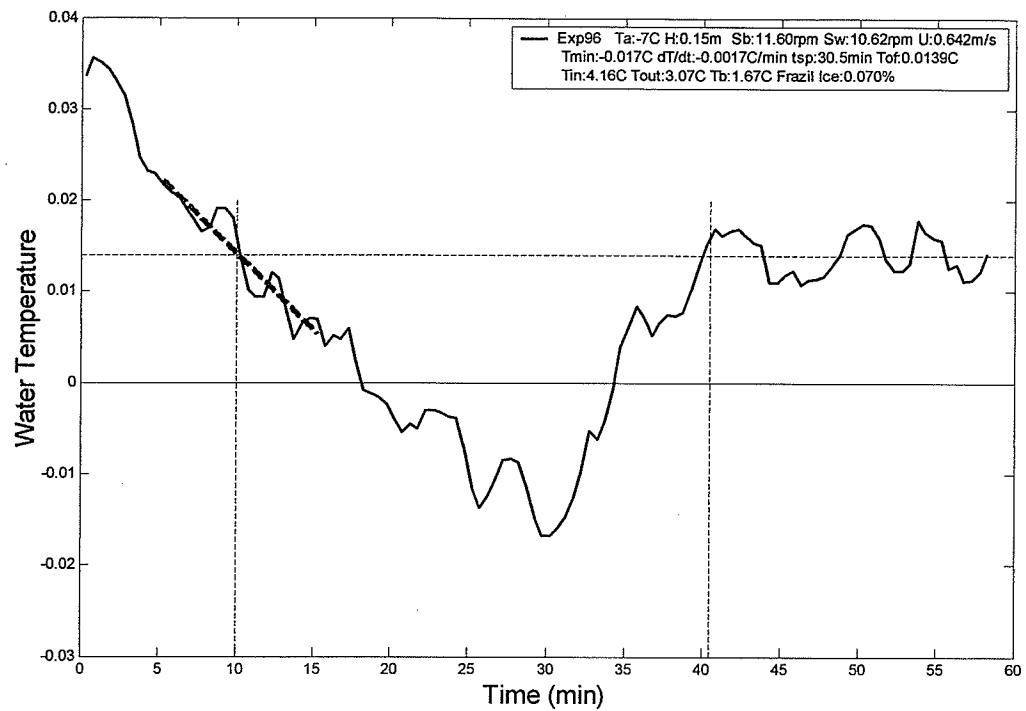


Figure 3.3. Impact of ice build-up on the thermometer to the supercooling process.

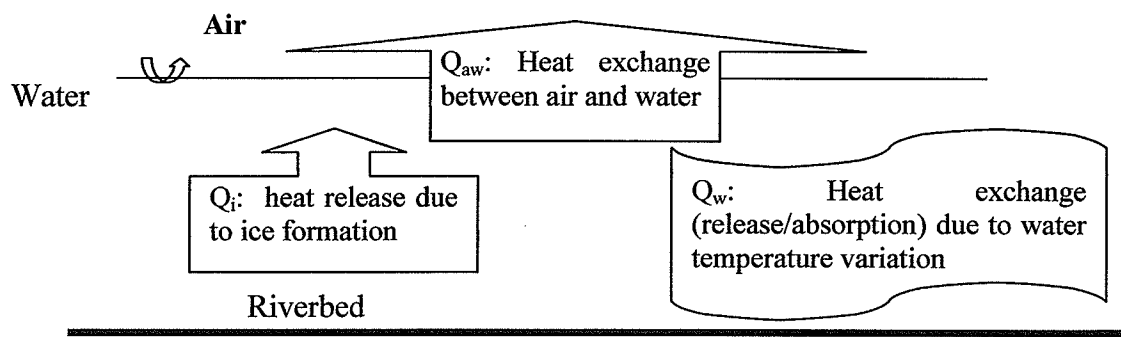


Figure 3.4. Schematic heat balance in a reach of river.

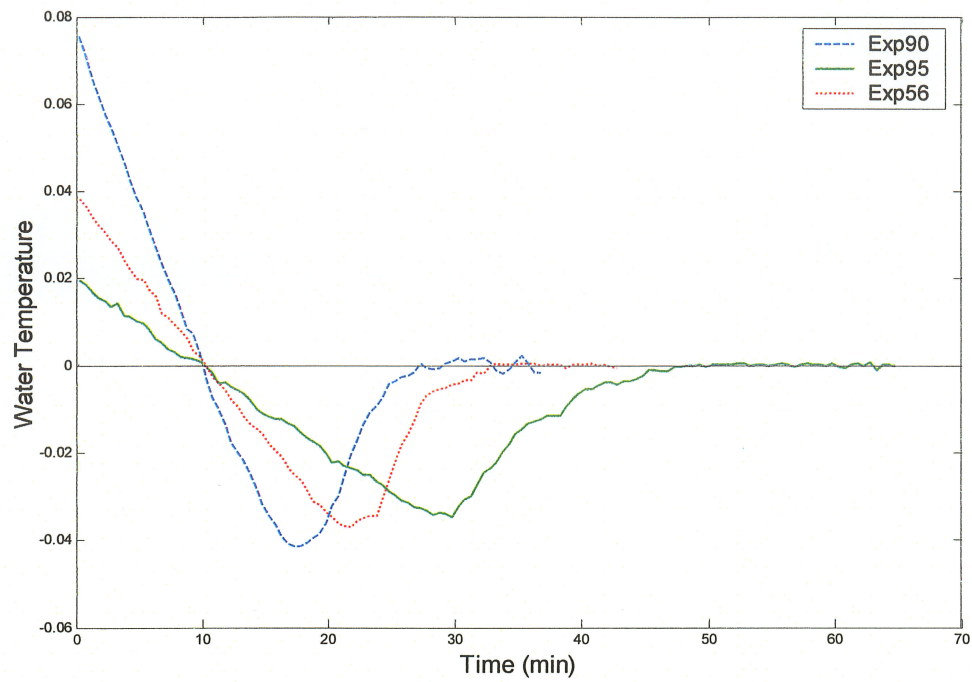


Figure 3.5. Effects of air temperature on the supercooling process.

Note: Exp. 90, $T_a = -15^\circ\text{C}$; Exp. 56, $T_a = -10^\circ\text{C}$; Exp. 95, $T_a = -6.5^\circ\text{C}$.

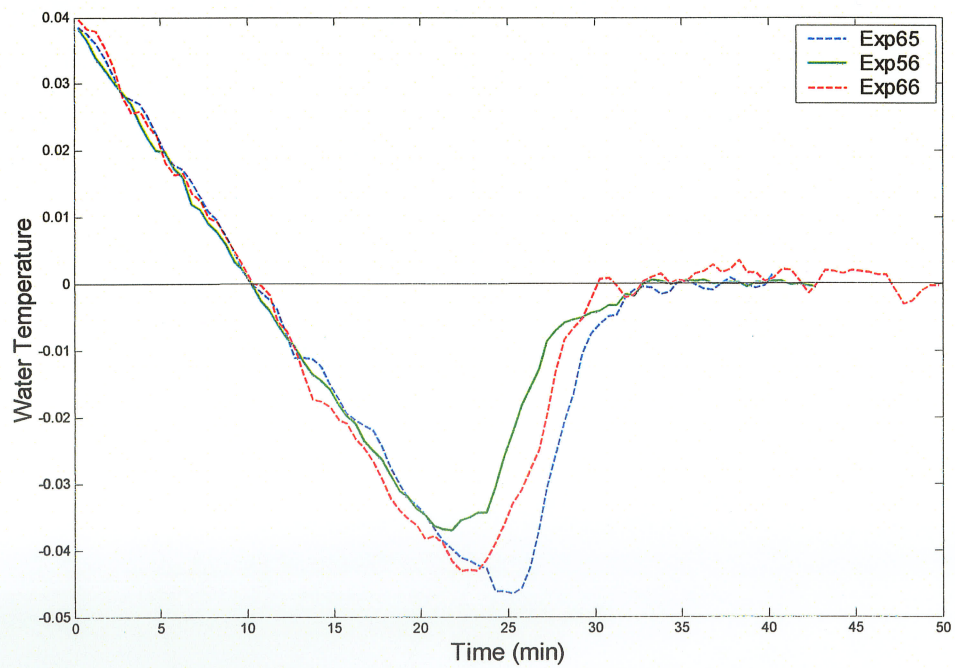


Figure 3.6. Influence of water velocity on the supercooling process.

Note, water velocities for Exps. 65, 56, and 66 are 0.391, 0.434 and 0.630, respectively.

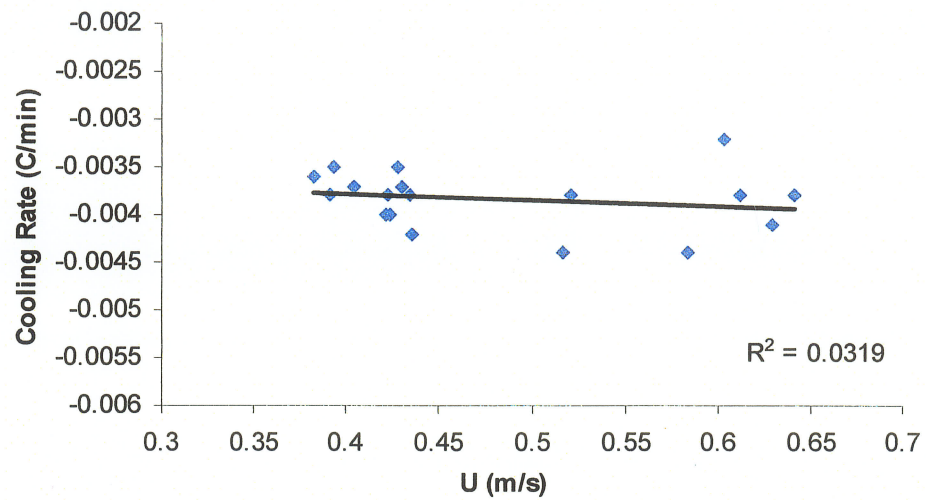


Figure 3.7. Influence of water velocity on cooling rate ($T_a = -10$ °C, $H=15$ cm).

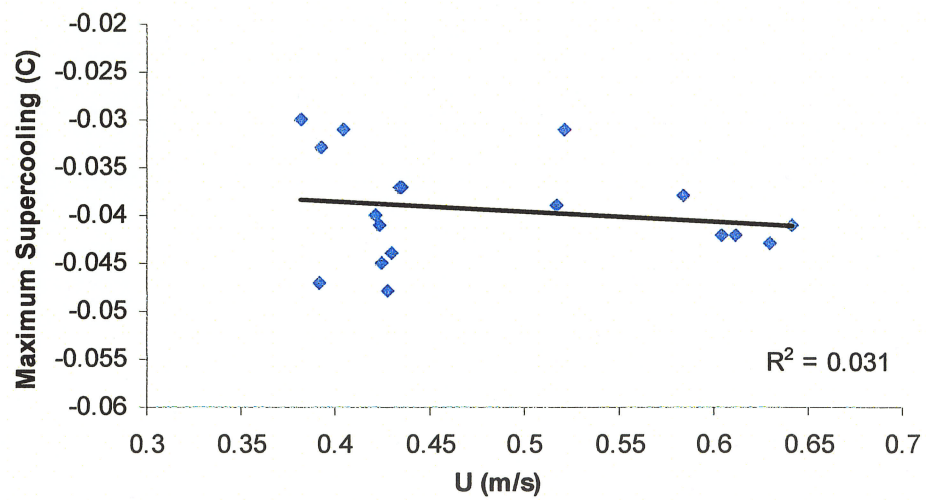


Figure 3.8. Influence of water velocity on maximum supercooling ($T_a = -10$ °C, $H=15$ cm).

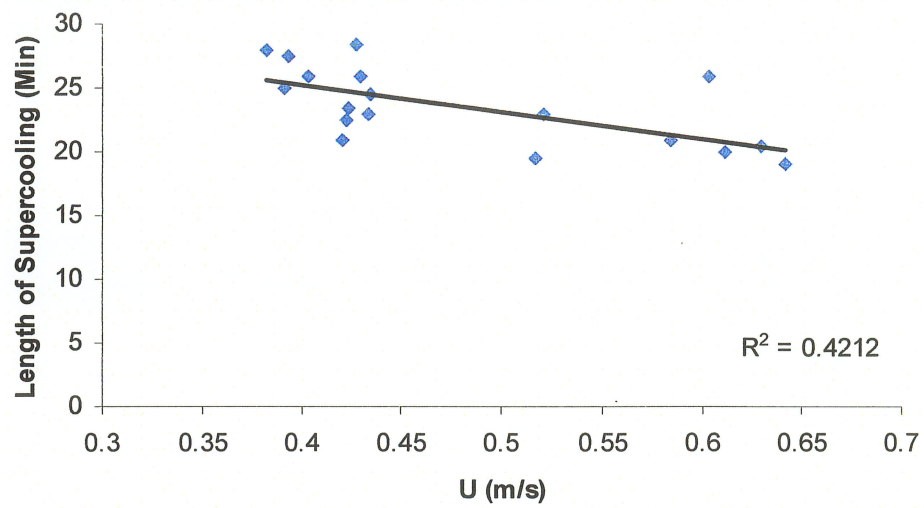


Figure 3.9. Influence of water velocity on length of supercooling ($T_a = -10$ °C, $H=15$ cm).

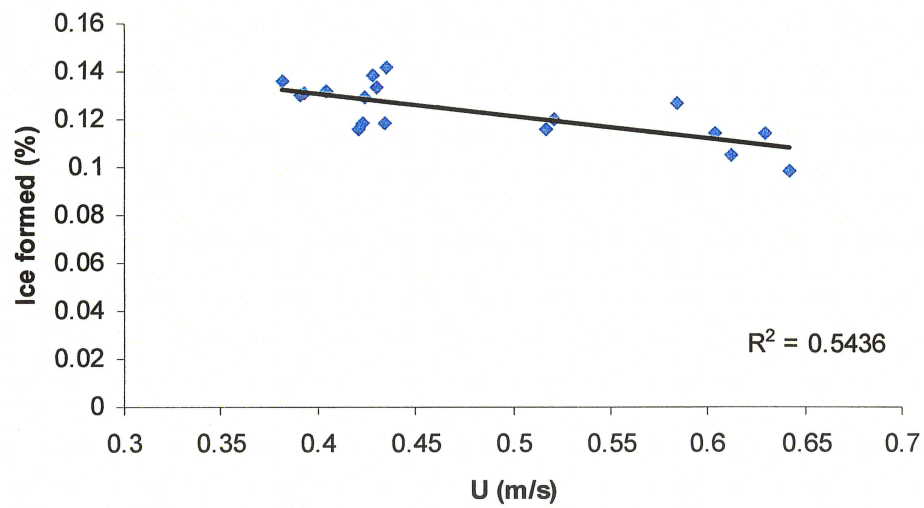


Figure 3.10. Percentage of frazil ice formed vs. water velocity for Group 1 experiments.

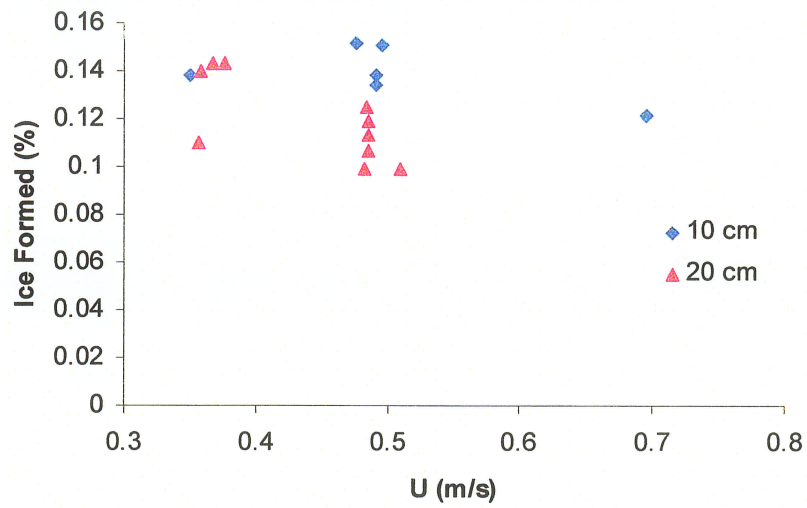


Figure 3.11. Percentage of frazil ice formed vs. water velocity for Groups 2 and 3 experiments.

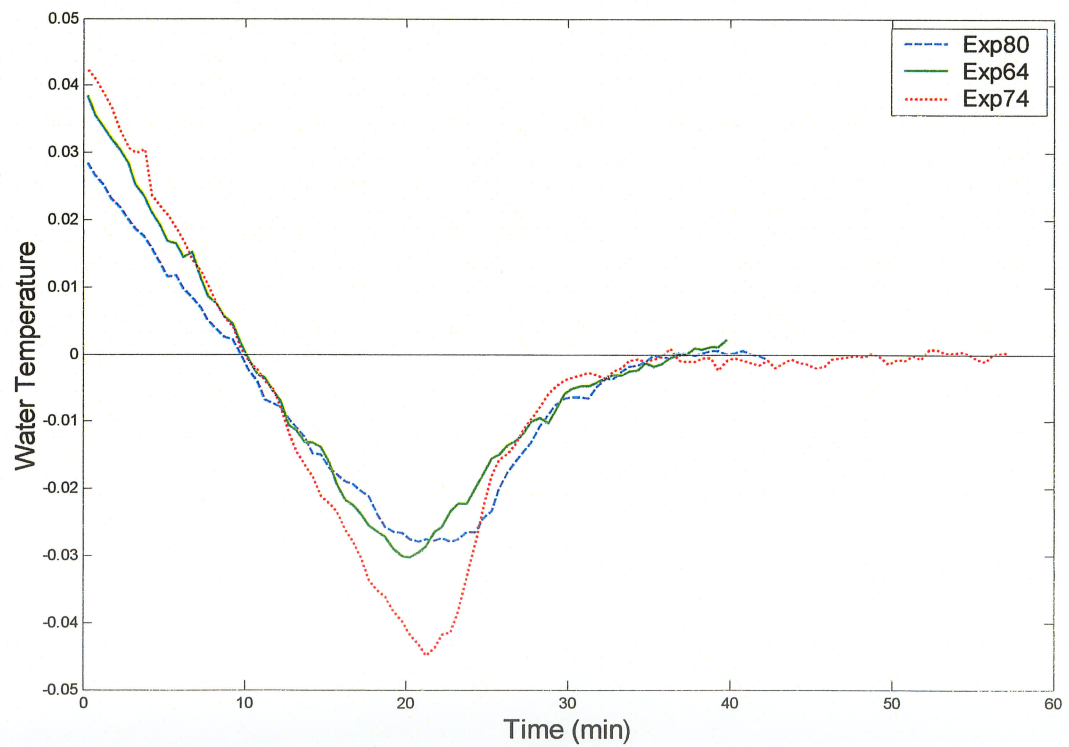


Figure 3.12. Influence of water depth on the supercooling process.

Note, the water depths for Exp 80, 64, 74 are 20,15,10 cm, respectively.

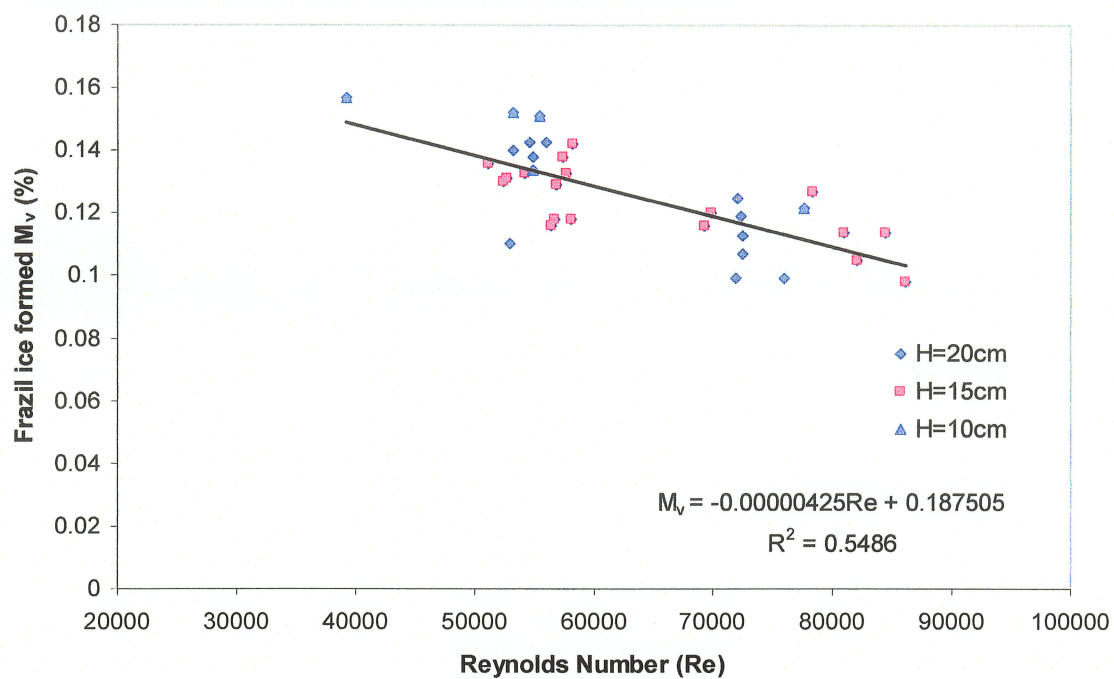


Figure 3.13. Percentage of frazil ice formed vs. Reynolds number for all experiments.

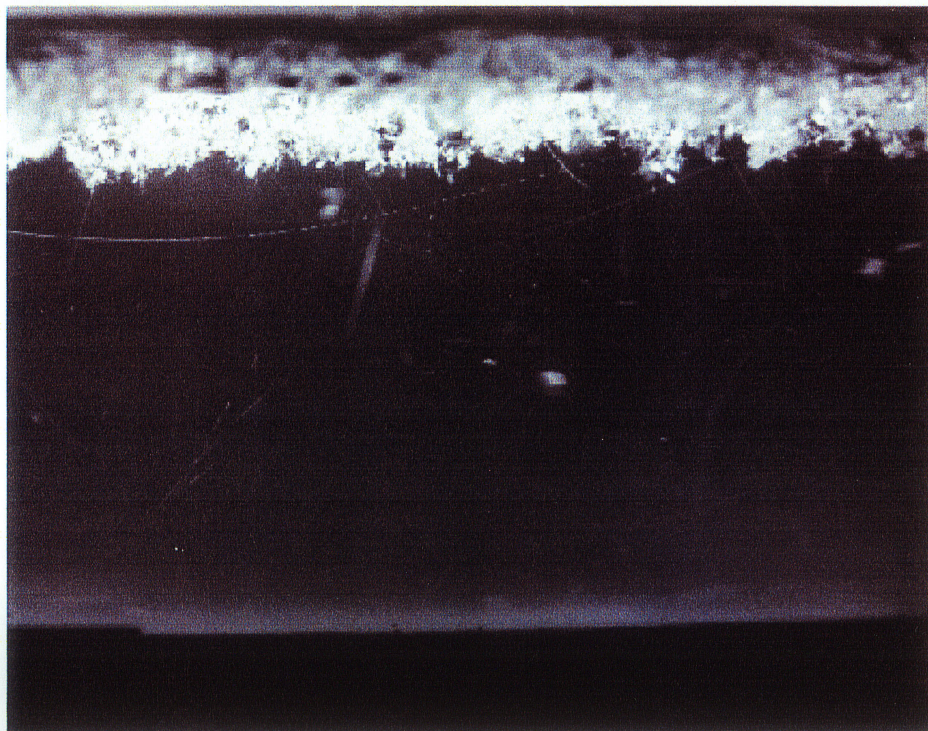
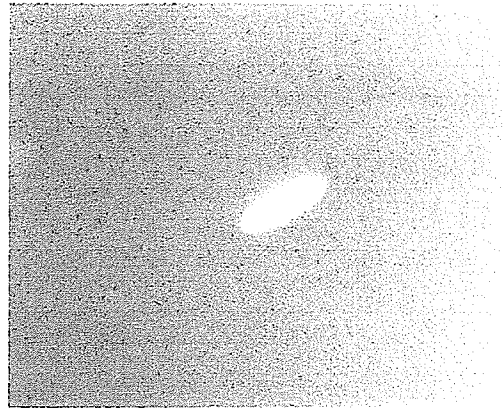
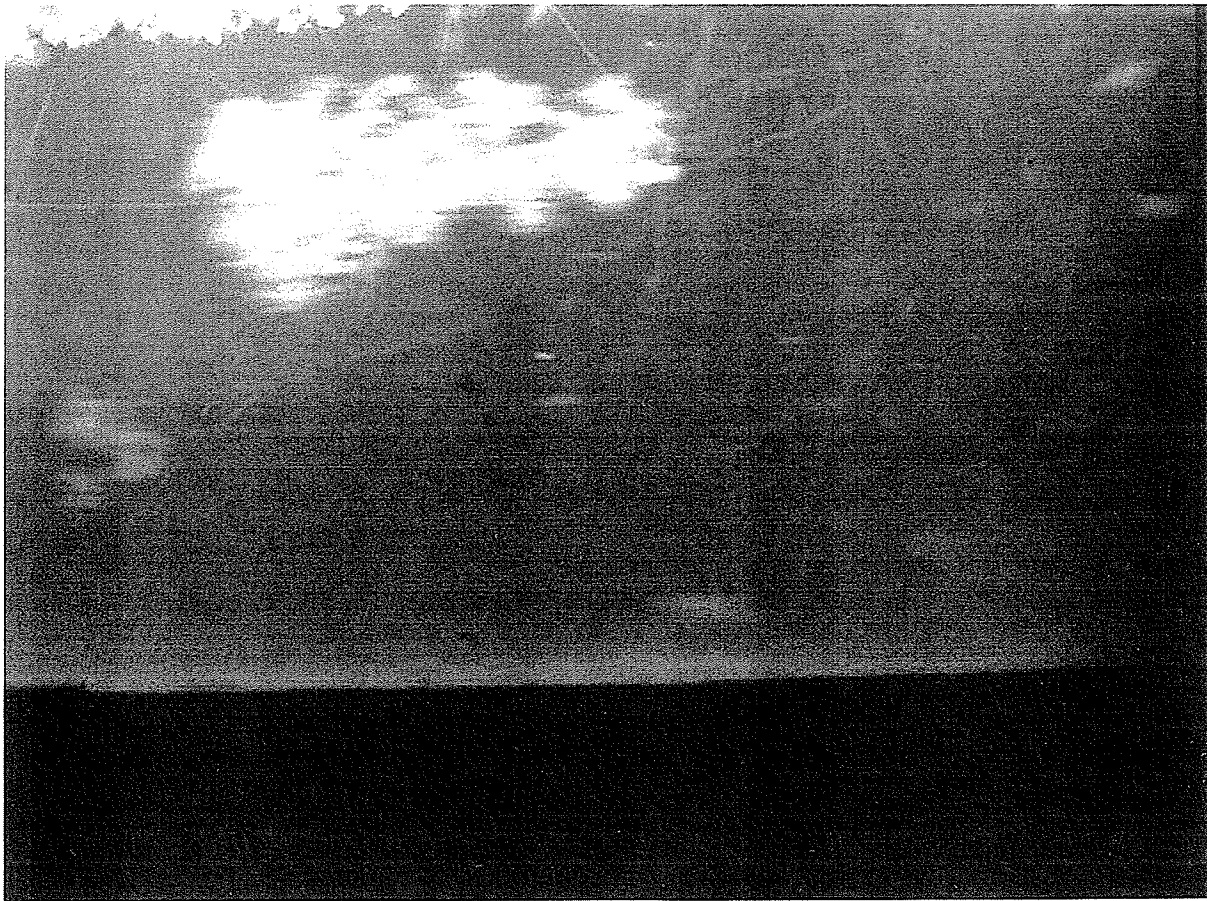


Figure 3.14. Ice attached to the wall near the surface (Exp. 14).



a)



b)

Figure 3.15. a) Typical frazil disk, 4.15 mm in diameter.
b) Sintering and flocculation of particles (Exp. 63).

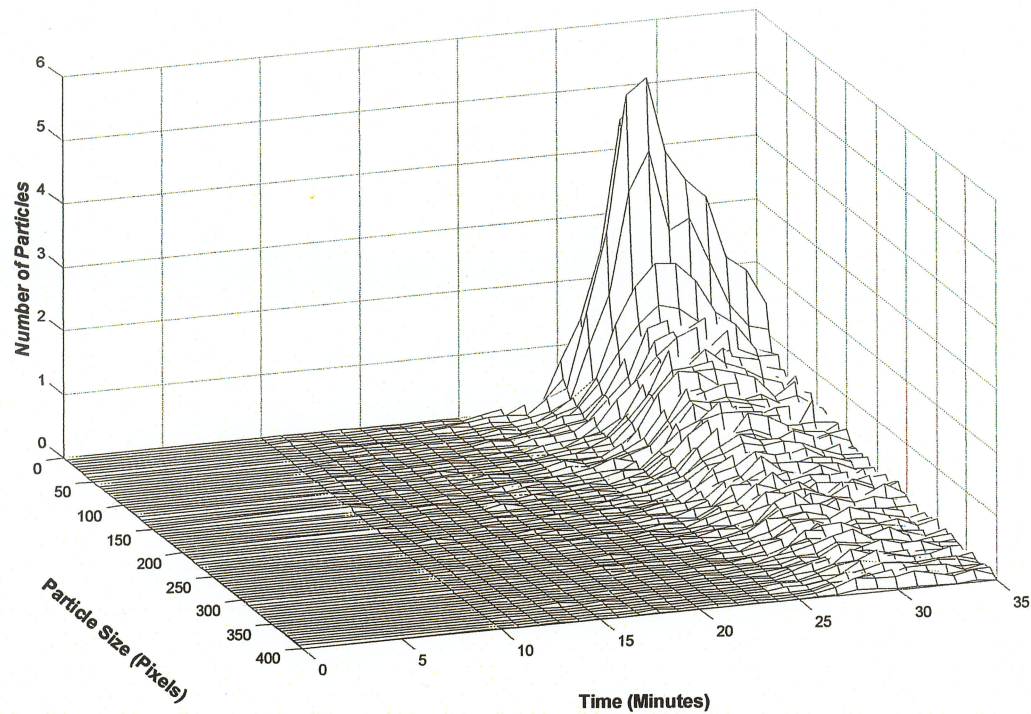


Figure 3.16. Temporal variation of the size distribution of frazil ice particles for Exp. 63.

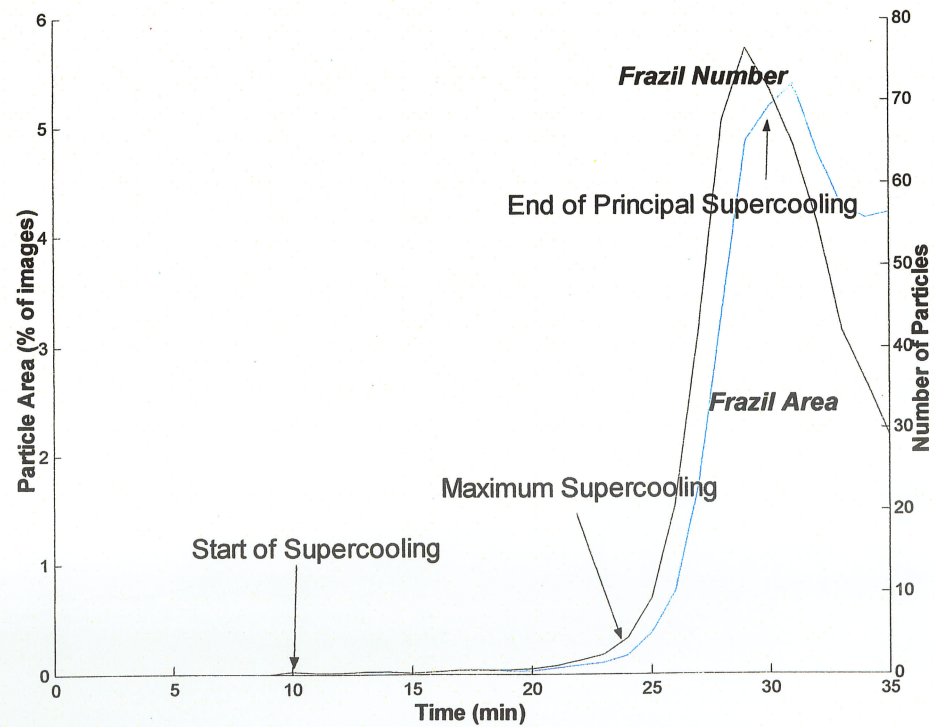


Figure 3.17. Variation of frazil number and area for Exp. 63 during its evolution.

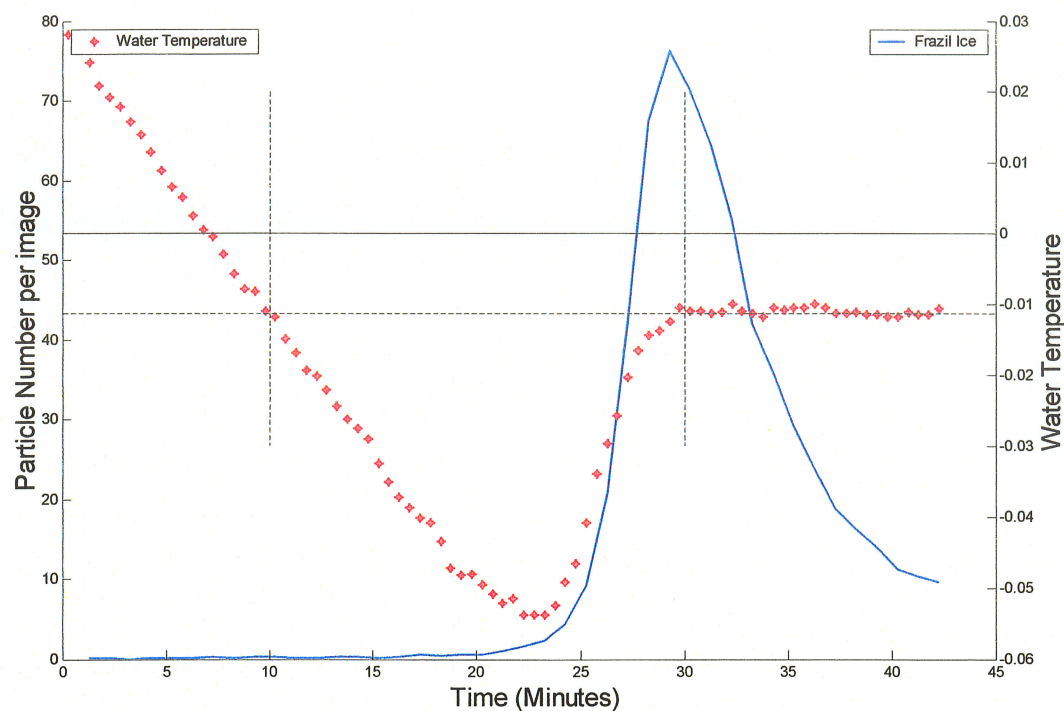


Figure 3.18. Frazil evolution during supercooling process (Exp 63).

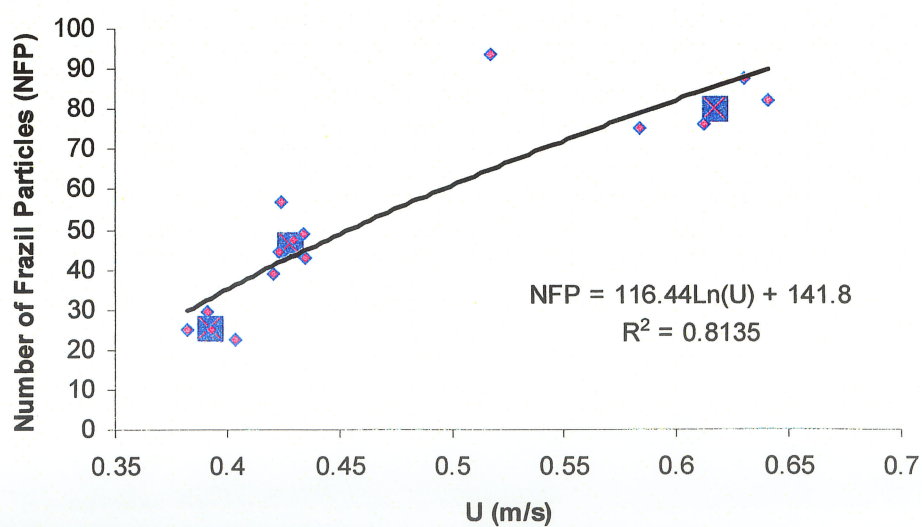


Figure 3.19. Frazil number vs. water velocity for data in Table 3.3a (Larger squares are averages).

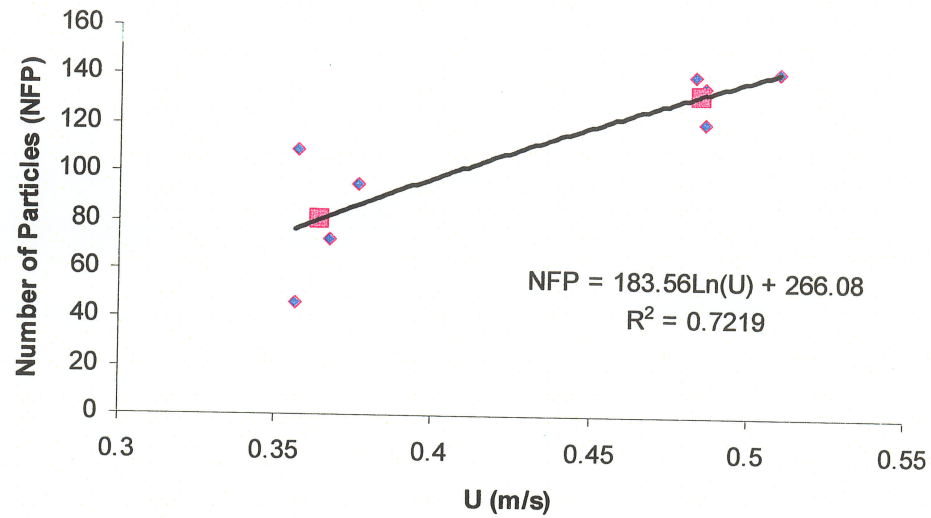


Figure 3.20. Frazil number vs. water velocity for data in Table 3.3b (Squares are the averages).

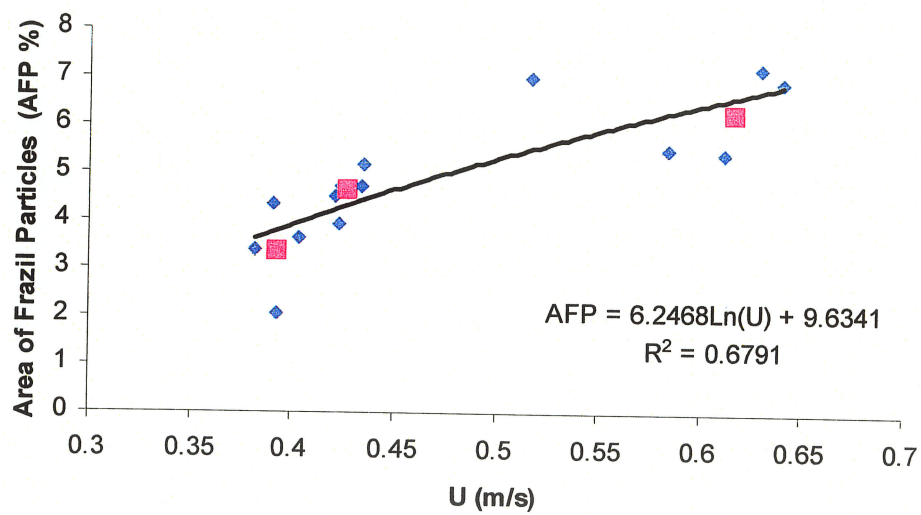


Figure 3.21. Frazil area vs. water velocity for the data in Table 3.3a (Squares are averages).

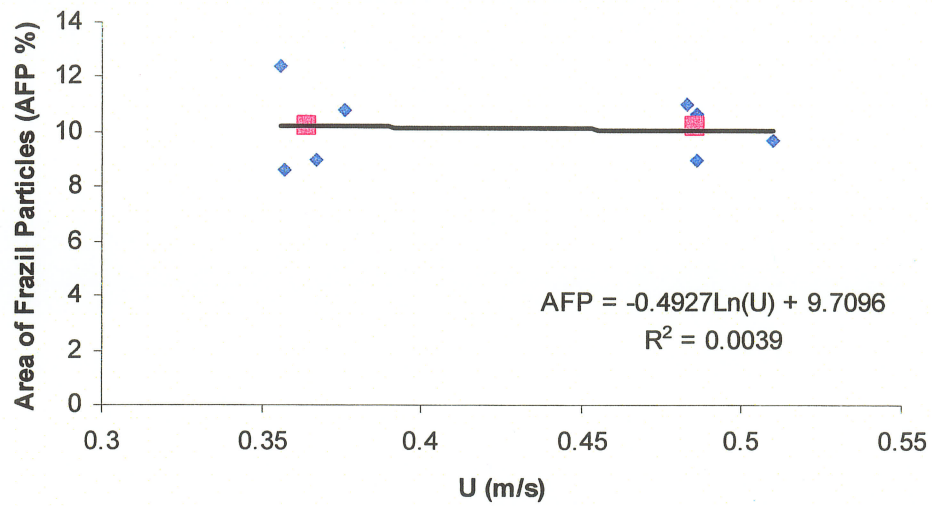


Figure 3.22. Frazil area vs. water velocity for the data in Table 3.3b (Larger squares are averages)

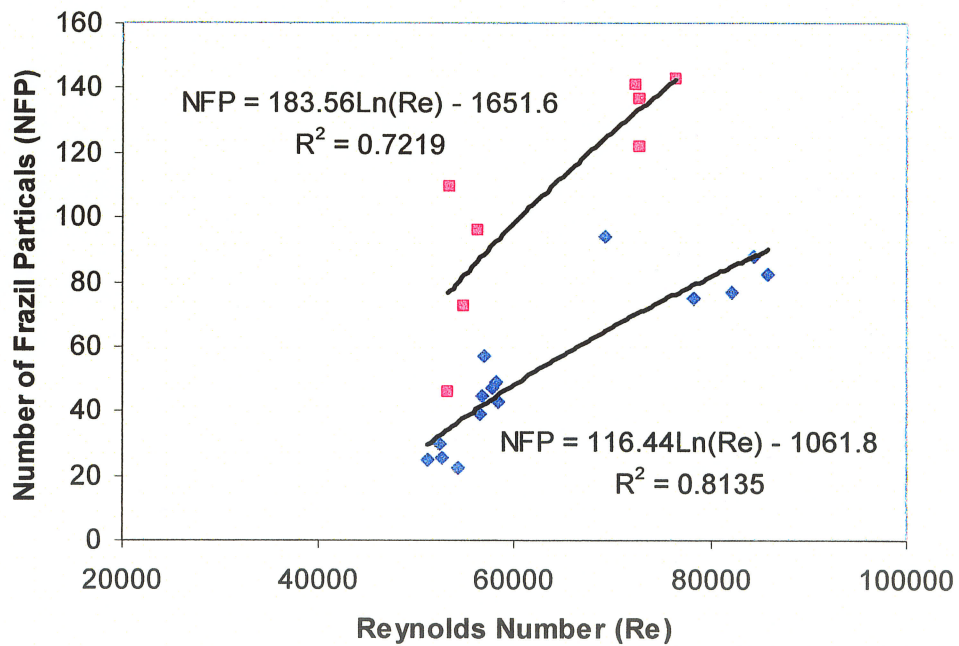


Figure 3.23. Frazil number vs. Reynolds number.

Note, upper indicates experiments of $H = 20$ cm, lower indicates experiments of $H = 15$ cm.

The formation and evolution of frazil ice in turbulent water that undergoes a small degree of supercooling is a problem that involves several fundamental questions (Daly, 1994). One of these important questions is “how is the size of frazil ice particles related to the hydraulics and thermodynamics of the flow?” In this chapter, a model is proposed to elucidate this topic. The hypothesis proposed is the turbulent eddy size is related to the frazil size, and therefore, the vertical fluctuation plays a central role in frazil evolution. Based on frazil buoyancy and flow turbulence, the dominant frazil size and its distribution is derived.

4.1 Turbulence in an Open-channel

Nezu and Nakagawa (1993) provide a good summary of the characteristics of turbulence in open-channel flow. The three well-known spectral subregions of velocity fluctuations are productive, inertial, and viscous subranges (Figure 4.1). Turbulent energy is extracted from the mean flow in the production subrange. It is transferred to small-scale eddies in the inertial subrange, and then dissipated into heat in the viscous subrange.

The energy transfer in wall-turbulence fields is analogous to this cascade process through the spectral subrange. From this analogy, the turbulent structure of open-channel flows can be divided into the following three subregions (Nezu and Nakagawa, 1993), as shown in Figure 4.2.

- 1) Wall region [$y/H < 0.15$, H is the water depth, and y is a vertical coordinate]. This region corresponds to the 'inner layer' of classical boundary-layer treatments; length and velocity scales are ν/u_* and u_* , respectively, where ν is the kinematic viscosity of water and u_* is the bed shear velocity. The turbulent energy generation (G) consequently exceeds the dissipation rate (ε), in this region, i.e., $G > \varepsilon$.
- 2) Intermediate region [$0.15 \leq y/H \leq 0.6$]. This region is not strongly influenced by either the wall properties or the free surface. Instead it corresponds loosely to the inertial subrange of the spectral distribution. The length and velocity scales are y and $\sqrt{\tau/\rho}$, respectively. This region maintains a near-equilibrium turbulent energy budget, $G \approx \varepsilon$. The intermediate and free-surface regions, together, are called the 'outer region' or 'outer layer', a region in which viscous effect are negligible.
- 3) Free-surface region [$0.6 < y/H < 1.0$]. In this region, the turbulent structure is controlled by the outer variables, with length and velocity scales specified as the flow depth H and the maximum mainstream velocity $U_{\max}$, respectively. The turbulent dissipation rate ε becomes larger than the generation rate G . Consequently, turbulent energy must be supplied from the wall region to the free-surface region by turbulent diffusion.

According to Zhang et al. (1990) and Ni et al. (1991, 2000), fluctuating velocities in the longitudinal and vertical direction u' and v' can be approximately described by a normal distribution, i.e.,

$$f_1(u') = \frac{1}{\sqrt{2\pi}\sigma_{u'}} \exp\left(-\frac{u'^2}{2\sigma_{u'}^2}\right), \quad (4.1)$$

and

$$f_2(v') = \frac{1}{\sqrt{2\pi}\sigma_{v'}} \exp\left(-\frac{v'^2}{2\sigma_{v'}^2}\right), \quad (4.2)$$

where $\sigma_{u'}$ and $\sigma_{v'}$ are the root mean squares of the longitudinal and vertical fluctuating velocities, respectively, or the fluctuating (turbulent) intensities in the longitudinal and vertical directions. From (4.1) and (4.2), the following relations for $|u'|$ and $|v'|$ can be derived

$$f_3(|u'|) = \frac{2}{\sqrt{2\pi}\sigma_{u'}} \exp\left(-\frac{u'^2}{2\sigma_{u'}^2}\right), \quad (4.3)$$

$$f_4(|v'|) = \frac{2}{\sqrt{2\pi}\sigma_{v'}} \exp\left(-\frac{v'^2}{2\sigma_{v'}^2}\right). \quad (4.4)$$

Observations of the fluctuation of vertical velocity (Zhang et al., 1990; Ni et al., 1991) indicate that

$$\sigma_{v'} = (0.45 \text{ to } 1.05) u_* \quad (4.5)$$

Hui et al. (1997) demonstrated a vertical uniform distribution of $\sigma_{v'}$ in a wide 2-D uniform flow, i.e.,

$$\sigma_{v'} = \alpha u_* \quad (4.6)$$

where α is a constant, and according to their measurements $\alpha \geq 1$. Herein α is set equal to 1.05, thus, $\sigma_{v'} = 1.05u_*$.

4.2 Governing Principles

Hou (1982) suggested that a local isotropic eddy could suspend a corresponding sand grain size in the flow, that is, the eddy size is equal to the size of the sand particle that the flow can deliver. But Ni et al. (1991) questioned this theory asserting that the eddy size should be much larger than the sand because sand is much heavier than water (their density ratio is 2.65:1). Herein, Hou's (1982) theory,

is adapted for frazil ice, because the density of frazil ice is much closer to that of water than sand. The density difference between frazil ice and water is approximately 0.92:1. However, a modification for the shape of the frazil ice particles is required. Andreasson et al. (1998) noticed the importance of vertical turbulent velocity and believed that eddies smaller than a certain diameter (D) of frazil crystal will have a negligible influence on the mixing of ice particles of size (D). Based on this, Andreasson et al. (1998) estimated the size of the minimum ice particle present in the surface layer of a flow, the size of a minimum particle floating on the surface, and the maximum particle size at and near the surface.

This chapter proposes the following hypothesis regarding frazil evolution in a flow with supercooling conditions: that the vertical fluctuation of the flow plays a central role in determining the frazil size distribution and that the supercooling process progresses in a manner whereby frazil formation reaches a specific entrained frazil capacity given by the characteristics of the flow. The frazil size distribution in a turbulent flow is derived according to the buoyant force and vertical mixing force.

The description of frazil ice morphology is not simple (Daly, 1984). The observed shape of a frazil ice crystal appears to result from a complex interaction between the imposed heat transfer conditions and the intrinsic crystallography of ice. Disk shaped frazil is typically observed in laboratory and field research. In this study, the diameter to thickness ratio is difficult to measure since the frazil particles rarely travel exactly perpendicular to the flow with respect to either of their axes. It is more common to see the particles as oval shaped (Figure 3.15a) because of their skewed orientation in the water. These crystals grow edgewise forming plates. The radius of the face (r) and the edge thickness (D_e) are commonly used as the characteristic length for the face and edge (Figure 4.3), respectively. Laboratory flume experiments (Daly and Colbeck, 1986) indicate that the radius-to-thickness ratio is 4:1. Following Daly (1984), this ratio can be used to define a volumetric shape factor, K_v , i.e.,

$$K_v r^3 = \pi r^2 D_e = \frac{\pi}{4} r^3. \quad (4.7)$$

Solving for K_v yields,

$$K_v = \frac{\pi}{4} = 0.785. \quad (4.8)$$

4.3 Determination of Characteristic Frazil Sizes

Theoretically, the smallest frazil particle size might be assumed to be very small (i.e., comparable to the size of a water molecule). However, the size of a frazil ice particle is restricted by turbulent eddies. According to the characteristics of turbulent energy transfer described in 4.1, if the eddy size is smaller than the inner length, the turbulent energy exists mainly in the form of heat, which simply dissipates. Therefore, it is assumed that frazil ice that is less the inner length can not exist because of heat dissipation. In this study, an inner length (Nezu and Nakagawa, 1993) is assumed to determine the minimum particle size ($D_{\min}$), i.e.,

$$D_{\min} = \frac{\nu}{u_*}. \quad (4.9)$$

The dissipation length scale is given by Kolmogorov (η), which characterizes the inertial subrange and viscous subrange. The dissipation length scale is defined as

$$\eta = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}, \quad (4.10)$$

where the dissipative rate (ε) can be determined according to the following (Mercier, 1984, and Hammer and Shen, 1995)

$$\varepsilon(y) = \frac{u_*^3}{\kappa y} \left(1 - \frac{y}{H} \right). \quad (4.11)$$

The depth-averaged value of $\varepsilon(y)$ can be determined from (Mercier, 1984)

$$\bar{\varepsilon} = \frac{1}{H - \nu/u_*} \int_0^H \varepsilon(y) dy \cong \frac{u_*^3}{\kappa H} \left(\ln \frac{u_* H}{\nu} - 1 \right). \quad (4.12)$$

The turbulent kinetic energy can be approximated by (Mercier, 1984)

$$k(y) = \frac{u_*^2}{0.3} \left(1 - \frac{y}{H} \right), \quad (4.13)$$

where the depth-averaged value of $k(y)$ is determined in a way similar to that of $\varepsilon(y)$. It is approximated as

$$\bar{k} = \frac{1}{H} \int_0^H k(y) dy = 1.67 u_*^2. \quad (4.14)$$

4.4. Frazil Size Distribution

4.4.1 Force Balance

It is assumed that frazil particles are neutral buoyant in a flow when the summation of the vertical forces is zero; the vertical forces include the self-weight of an ice particle (W), the buoyancy force (F_b), and the drag force (F_d). Suspension is maintained provided

$$F_b = F_d + W, \quad (4.15)$$

where

$$F_b = \rho g K_v \left(\frac{D}{2} \right)^3, \quad (4.16)$$

$$W = \rho_i g K_v \left(\frac{D}{2} \right)^3, \quad (4.17)$$

and

$$F_d = \frac{1}{2} \pi \left(\frac{D}{2} \right)^2 C_d \rho v'^2. \quad (4.18)$$

ρ and ρ_i are the densities of water and ice, respectively, C_d is the drag coefficient, and g is the acceleration due to gravity. The drag coefficient of a disk whose major radius is perpendicular to the flow is a well-known function of the Reynolds number and can be readily found in many fluid mechanics textbooks. However, it should be recognized that real frazil particles rise in an unstable fashion that is not always normal to the direction of flow. Wuebben (1984) suggested that the oscillations of a rising frazil disc are related to the alternating shedding of vortices. These oscillations significantly increase the effective drag coefficient. Wuebben (1984) conducted a series of experiments using artificial discs and found the effective drag coefficient C_d is around 1.75 for a broad range of disc sizes.

Substituting (4.16), (4.17), and (4.18), into (4.15) yields

$$(\rho - \rho_i) g K_v \left(\frac{D}{2} \right)^3 = \frac{1}{2} \pi \left(\frac{D}{2} \right)^2 C_d \rho v'^2, \quad (4.19)$$

which can be expressed in the form

$$v'^2 = \beta_1 D \quad (4.20)$$

provided

$$\begin{aligned} \beta_1 &= \frac{\rho - \rho_i}{\rho} g \frac{K_v}{\pi C_d} \\ &= \frac{g'}{2\pi C_d}. \end{aligned} \quad (4.21)$$

Here g' (a modified form of reduced gravity) has a value of 1.294 m/s^2 , $C_d = 1.75$ and therefore $\beta_1 = 0.118 \text{ m/s}^2$.

4.4.2 Frazil Size Distribution

Combining (4.20) with (4.4) yields the frazil ice size density distribution

$$\phi_1(D) = \frac{\sqrt{\beta_1}}{\sigma_v \sqrt{2\pi} \sqrt{D}} \exp\left(-\frac{\beta_1 D}{2\sigma_v^2}\right). \quad (4.22)$$

If the frazil ice size density distribution follows (4.22), then the cumulative density distribution is given by

$$P(D_i) = \int_0^{D_i} \phi_1(D_i) dD. \quad (4.23)$$

Note, for uniform frazil ice this method would yield approximately the same results as that obtained by a weight-percentage method. Substituting (4.22) into (4.23), gives

$$P(D_i) = \int_0^{D_i} \frac{\sqrt{\beta_1}}{\sqrt{2\pi} \sigma_v} D^{\frac{1}{2}} \exp\left(-\frac{\beta_1 D}{2\sigma_v^2}\right) dD. \quad (4.24)$$

Letting $\zeta = \frac{\sqrt{\beta_1 D}}{\sigma_v}$ and rearranging gives $D = \frac{\sigma_v^2}{\beta_1} \zeta^2$ therefore, $dD = \frac{2\sigma_v^2 \zeta}{\beta_1} d\zeta$, so

$$P(D_i) = \frac{2}{\sqrt{2\pi}} \int_0^Z \exp\left(-\frac{\zeta^2}{2}\right) d\zeta = 2\Phi(Z) - 1 \quad (4.25)$$

where

$$Z = \frac{\sqrt{\beta_1 D_i}}{\sigma_v} \quad (4.26)$$

and

$$\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^Z \exp\left(-\frac{\zeta^2}{2}\right) d\zeta. \quad (4.27)$$

Here, $\Phi(Z)$ is the standard normal probability distribution. The mean frazil size (D_{50}) can be determined using (4.25) since $P(D_{50}) = 0.5$.

4.5. Frazil Size in Experiments

The evolution of frazil ice size and concentration are difficult to measure and monitor in nature (laboratory and field experiment). Carstens (1966) did not describe frazil size and concentration in his well-known experiments. Bukina (1967) represented his results of experimentally determined size distribution function of frazil crystal which were formed and grew in turbulent flow. Daly and Colbeck (1986) conducted a series of experiments designed to measure the size distribution of frazil ice crystals.

4.5.1 Observations of Frazil Evolution

To test this developed frazil size distribution model is difficult. To date no complete data set is available for such purpose. The frazil size information obtained from the DIPS is not complete (small particles are not detected), however, this system can still provide valuable information on the evolution of larger frazil ice particles.

A typical experiment, Exp. 63, is chosen to clarify the frazil evolution and supercooling process. Figure 4.4 shows evolution of frazil particles of different sizes in this experiment. The number of frazil particles starts to increase when supercooling begins at 10 minutes into this experiment, but their numbers grow slowly. The number of particles increases rapidly when the maximum supercooling is reached around 25 minutes. The frazil population explodes mainly in the warming stage of the supercooling process, i.e., between 25 -30 minutes. After the principal supercooling ends at 30 minutes, the number of frazil particles drops rapidly. This kind of frazil evolution is found in almost all of the experiments (refer to Appendix B).

Figure 4.5 depicts the frazil evolution in another manner: the distribution of frazil size during the period of principal supercooling. In this figure, these three distribution curves look parallel to each other. This might lead to the conclusion that there is a frazil size distribution curve to be always obeyed during the supercooling process. In other words, the distribution of frazil size is governed by flow turbulence and remains consistent as long as the effect of frazil formation on the flow turbulence can be ignored. Therefore, the distribution of frazil size and its D_{50} are consistent. This is a significant finding for further studies on the distribution of frazil size.

After the principal period of supercooling is the residual supercooling. It is reasonable to assume some flow turbulence variation due to the formation of a surface layer of frazil and large flocs. Consequently, the frazil size distribution changes. Figure 4.6 presents a comparison of the frazil distributions near the end of principal supercooling, and 3 and 5 minute after the end. The curves no longer look parallel to each other, especially in the region, larger than 80 pixels. Appendix B shows that the distribution curves during the residual supercooling are quite different than during the principal supercooling.

4.5.2 Channel Bottom Friction

During the principal supercooling, it is assumed that the frazil formation does not change the flow turbulent characteristics. In order to apply the developed frazil size distribution model, the flow turbulence has to be estimated in advance. The flow turbulence is expressed in the form of shear velocity, which is determined by (Carter et al., 1963)

$$u_* = U \sqrt{\frac{f}{8}}, \quad (4.28)$$

where f is a friction factor. Assuming smooth boundaries, the friction factor f can be determined from

$$\frac{1}{\sqrt{f}} = 2 \log(R_e \sqrt{f}) - 0.8 \quad (4.29)$$

provided $R_e > 10^5$. If $R_e < 10^5$, the Blasius formula

$$f = \frac{0.3164}{R_e^{0.25}} \quad (4.30)$$

can be used. R_e is the Reynolds number given by $4UR/\nu$, where U is the depth averaged flow velocity and R is the hydraulic radius.

For turbulent flow f can be estimated using the Colebrook-White equation (Colebrook, 1939)

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{k_s}{3.71(4R)} + \frac{2.51}{R_e \sqrt{f}} \right), \quad (4.31)$$

where k_s is a representative roughness height. f may be less accurately approximated using Altshul's explicit formulation for f , which is given by (Idelchik, 1986)

$$f = 0.1 \left(1.46 \frac{k_s}{4R} + \frac{100}{R_e} \right)^{0.25}. \quad (4.32)$$

The present experiments were conducted in the counter-rotating flume at the Hydraulics Research and Testing Facility (HRTF), the University of Manitoba using a bed consisting of a layer of gravel ($d_{50} = 5$ mm), which was glued to the underlying bedplate. The flume is 0.20 m wide, 0.35 m deep, and has a centerline diameter of 1.2 m.

The flow turbulence can be estimated using fundamental hydraulic variables, such as the channel width, water depth, bottom roughness, and flow velocity. The process of predicting frazil size is done using Matlab algorithms.

Table 4.1 presents the applications of the frazil size distribution model to 7 selected HRTF tests. The prediction of these characteristic frazil particle parameters, viz., $D_{\min}$ and D_{50} , and the associated distribution are an important milestone in the determination of the entrained frazil capacity. Table 4.1 also summarizes the Reynolds number (R_e), friction factor (f), and shear velocity (u_*) computed for the experiments. Figure 4.7 shows the full size distribution for a number of these experiments.

Table 4.1. Characteristic frazil diameters of typical observations.

Data set		U [m/s]	B [m]	H [m]	R [m]	R_e	f	u_* [m/s]	$D_{\min}$ [m]	D_{50} [m]
HRTF (2002)	Exp 60	0.424	0.2	0.15	0.06	56849	0.042	0.031	0.000058	0.004
	Exp 63	0.613	0.2	0.15	0.06	82054	0.042	0.044	0.000040	0.0084
	Exp 69	0.642	0.2	0.15	0.06	86078	0.042	0.047	0.000038	0.0092
	Exp 102	0.428	0.2	0.15	0.06	57385	0.042	0.031	0.000057	0.0041
	Exp 76	0.695	0.2	0.10	0.05	77654	0.044	0.052	0.000035	0.0113
	Exp 101	0.492	0.2	0.10	0.05	54972	0.044	0.037	0.000049	0.0057
	Exp 84	0.486	0.2	0.20	0.067	72402	0.041	0.035	0.000051	0.0051
Carstens (1966)	Case I	0.5	0.2	0.2	0.067	74488	0.019	0.024	0.000070	0.0024
	Case II	0.33	0.2	0.2	0.067	49162	0.021	0.017	0.000110	0.0012

Two cases of Carstens (1966) are also shown in Table 4.1. The first (Case I in Table 4.1) corresponds to Case 'A' given in figure 6 of Carstens (1966) whereas the second (Case II) corresponds to his figure 7. The size distributions predicted by the model for these two cases are shown in Figure 4.8.

These two cases have been used by previous researchers (Mercier, 1984; Svensson and Ostmedt, 1994; and Hammer and Shen, 1995) to simulate the supercooling process.

Table 4.1 and Figures 4.7 and 4.8 show that the frazil size gets bigger when the flow turbulence gets stronger because Exp. 69 has a higher turbulent intensity than Exp 101. The HRTF experiments are stronger than that in Carstens' tests. Based on the model, the frazil particle size and number are related to flow velocity. Higher velocities produce bigger and more frazil ice particles than smaller velocities, which is consistent with the experimental observations (see Chapter 3) using a digital image processing system.

4.5.3 Discussion

The frazil size distribution curves predicted by the model herein look like typical sediment size distributions (Figures 4.7 and 4.8). In order to compare the distributions of the size of frazil particles, a relative frequency of frazil particles is used in Figure 4.9. In the figure, the frazil sizes are converted to frazil diameter from pixels, in which the minimum frazil diameter that the DIPS can distinguish is about 2 mm. As shown in this figure, the three curves of observations and the modeled distribution look similar. This supports the assertion that there is a frazil size distribution curve to always be obeyed during the principal supercooling process, and the developed model fits the curves reasonably well.

Shen and Wang (1995) reported mean frazil diameters of 7 mm and 11 mm at the Hequ reach of the Yellow River, China during the winters of 1986-87 and 1987-88, respectively. Gosink and Osterkamp (1983) found that the frazil size in Goldstream Creek ranged from 1 mm to 6 mm. The mean frazil size predicted by this model is similar to the measured typical size of frazil ice in the field.

Like the measurements of Daly and Colbeck (1986), minimum frazil size predictions (Table 4.1) range from 35 μm to 110 μm . This is bigger than the size predicted by the model of Mercier (1984) who estimated the critical radius of frazil ice should be about 0.4 μm for the range of supercooling found in rivers and stream using thermodynamic principles. The reason for this difference is the philosophy behind the estimations. The present model is developed based on the principles of turbulent, open-channel flow, which yields the frazil size distribution. The present model also

provides a physical mechanism to estimate both the minimum and mean size of frazil in a turbulent flow. Such measurements would otherwise be difficult to obtain.

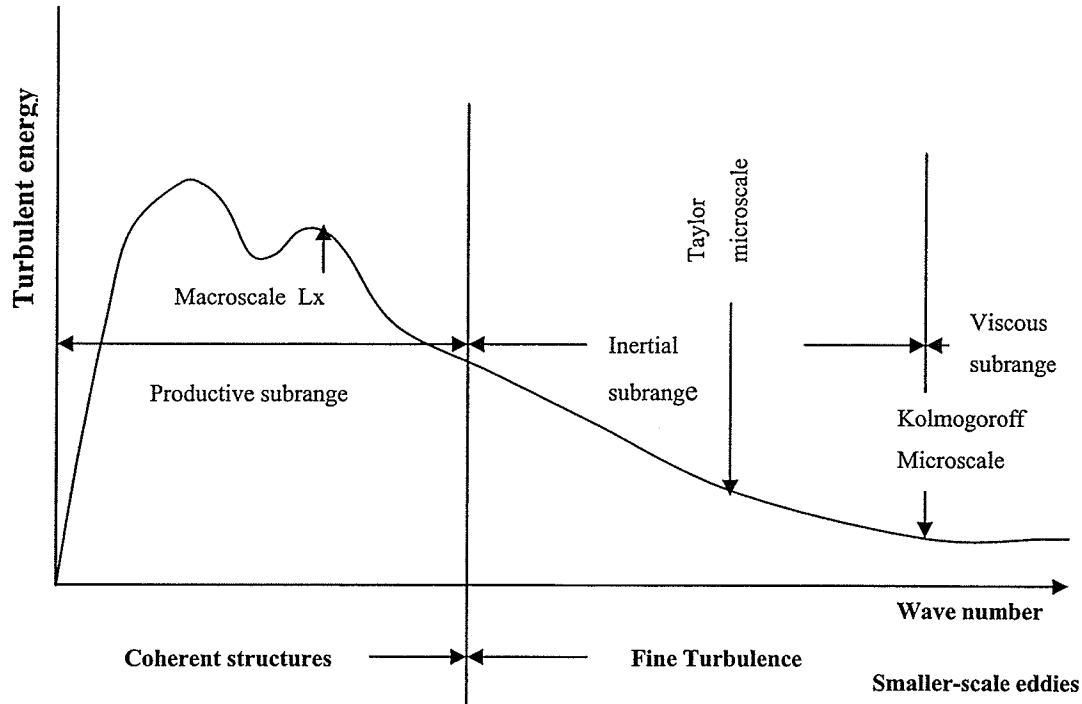


Figure 4.1. Turbulent energy spectrum (after Nezu and Nakagawa, 1993)

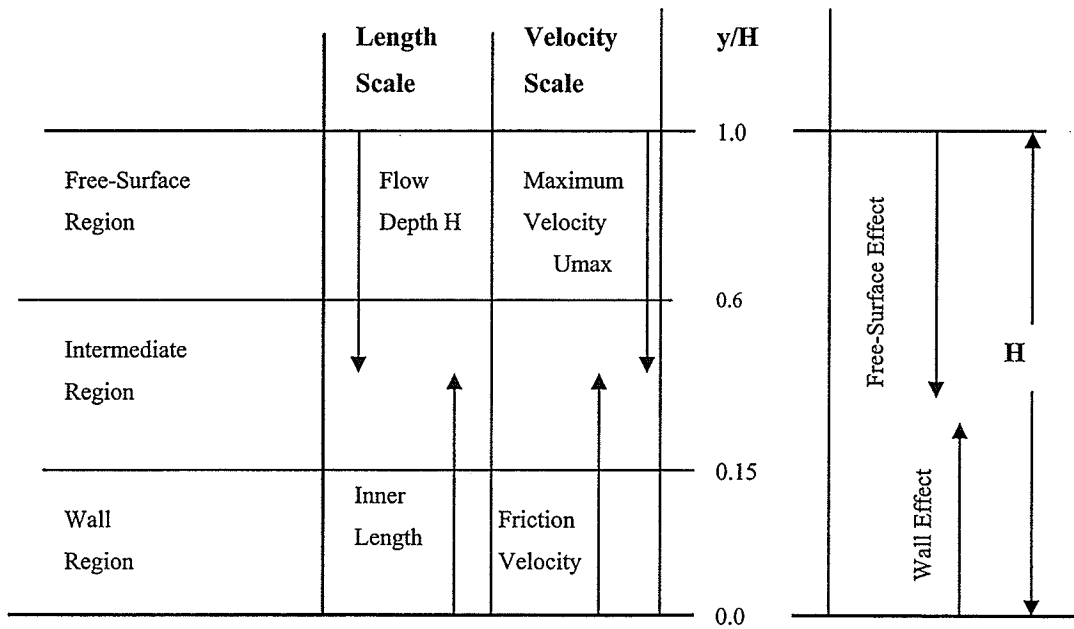


Figure 4.2. Sub-division of the open-channel flow field (after Nezu and Nakagawa, 1993).

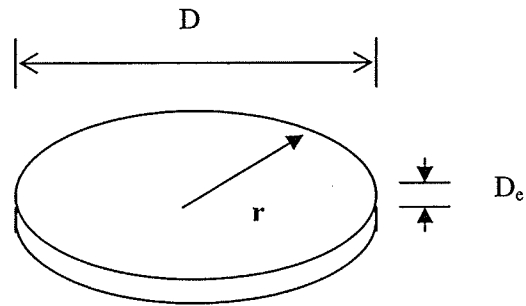


Figure 4.3. Definition of frazil disk morphology.

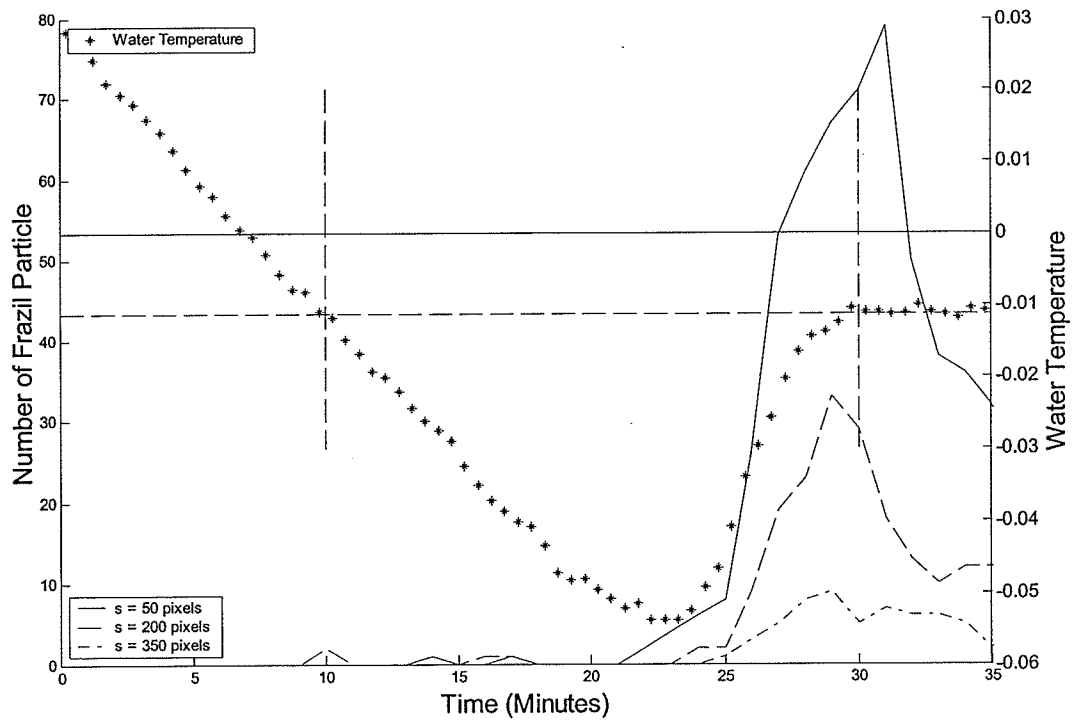


Figure 4.4. Evolution of three frazil particle sizes during the principal period of supercooling.

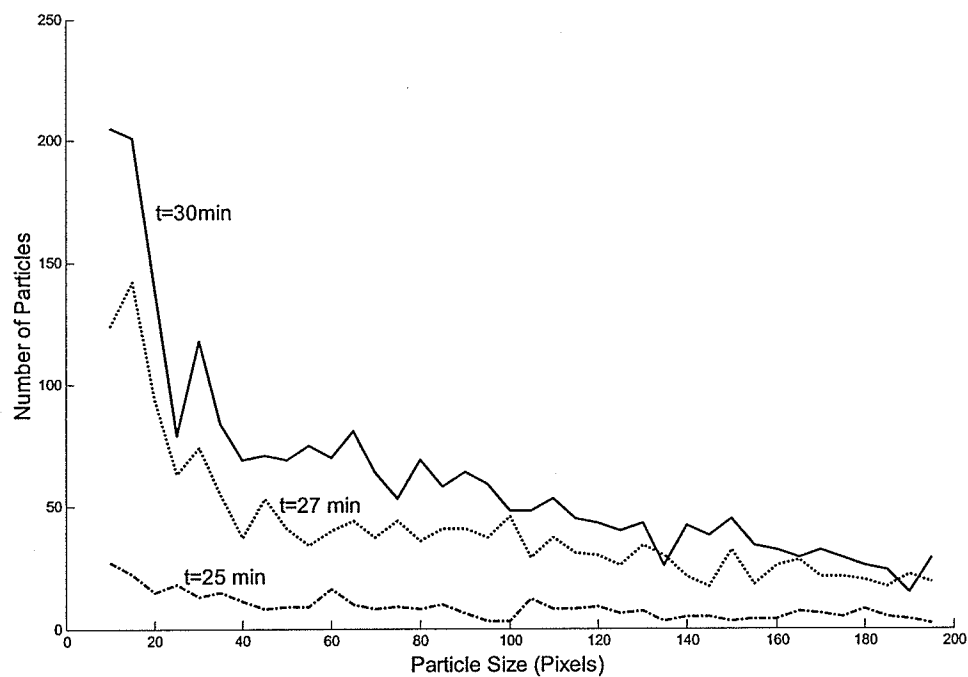


Figure 4.5. Evolution of frazil particle distributions at various times during the period of principal supercooling.

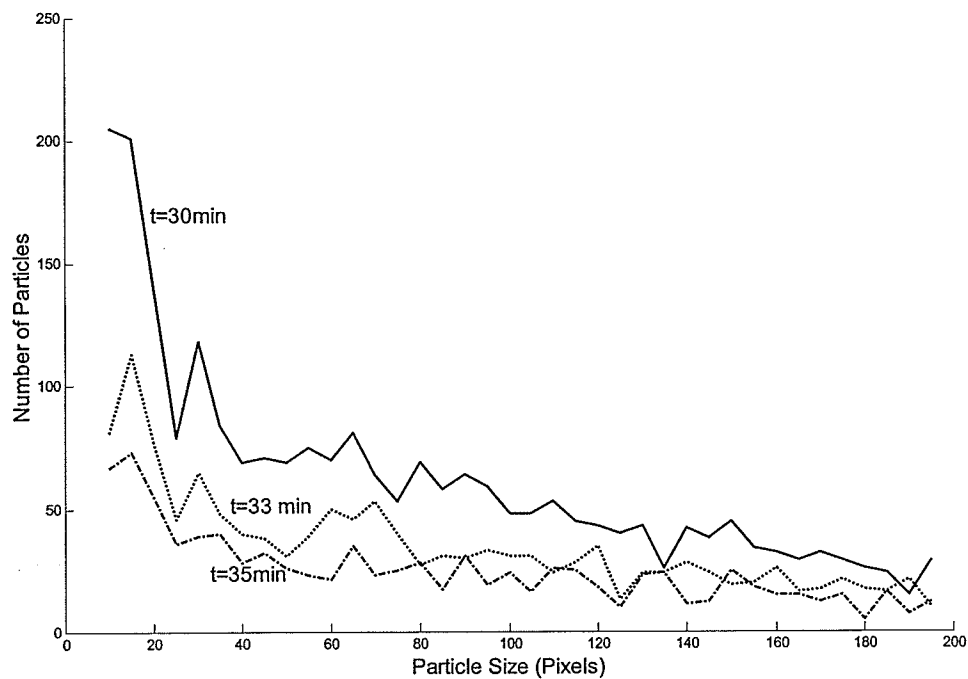


Figure 4.6. Evolution of frazil particle distributions at various times during the period of residual supercooling.

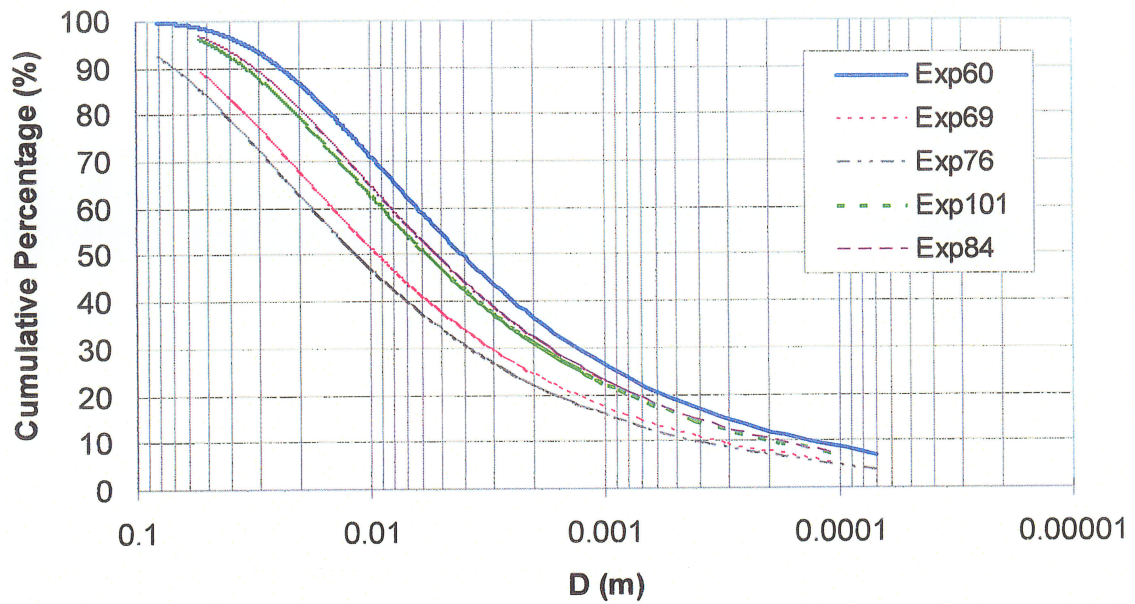


Figure 4.7. Cumulative frazil size distributions for selected HRTF tests.

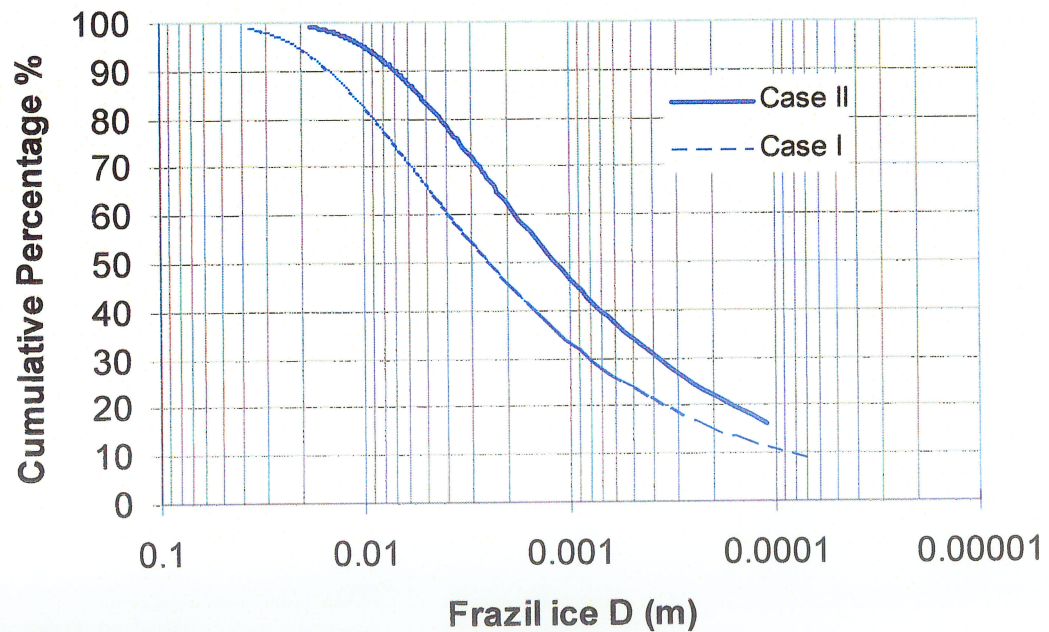


Figure 4.8. Cumulative Frazil size distributions for two of Carstens' tests.

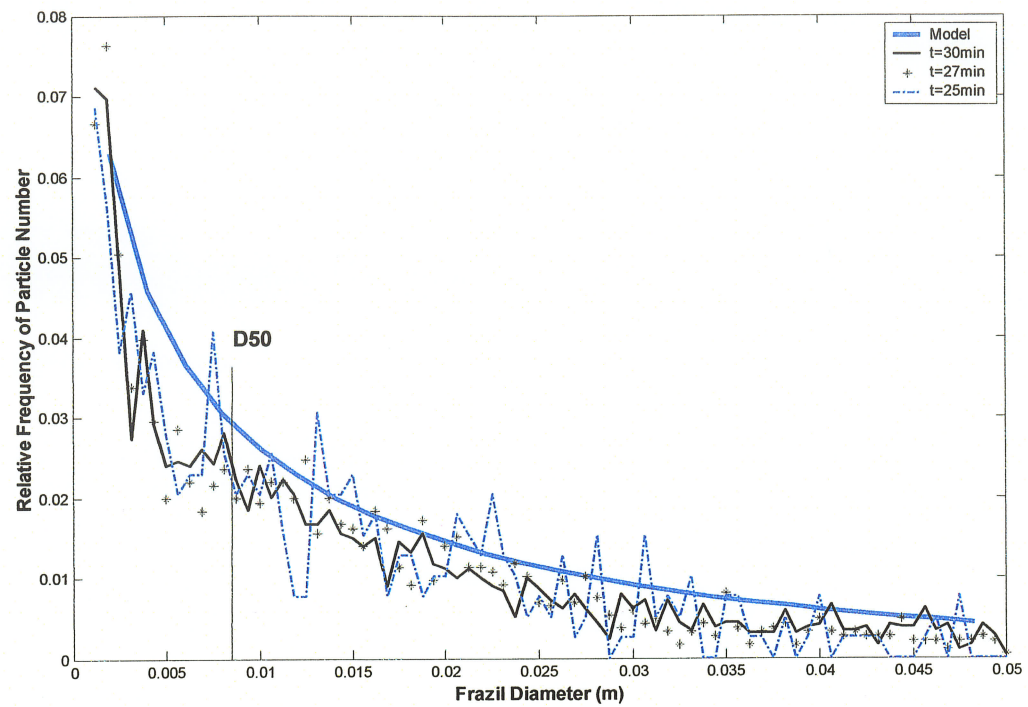


Figure 4.9. Comparison of size distributions of frazil ice particles.

Frazil Generating Capacity of a Flow

In this chapter, a new model describing the principal supercooling process is proposed. It is asserted that frazil formation reaches the entrained-frazil generating capacity of a flow, i.e., the maximum amount of entrained frazil ice particles that the flow can generate. The frazil capacity of a flow is related to the flow turbulence intensity and to the frazil size/volume. A relationship is developed to calculate the maximum well-mixed frazil in a flow.

5.1 Frazil Ice Generation and Transport in a River

Figure 5.1 shows the typical ice conditions downstream of a hydro-electric development. The river can be divided into three segments: cooling, frazil generation, and ice jam, cover formation and breakup. It is in the generation segment that frazil ice is continuously generated, and provides the frazil ice to transport downstream.

In the frazil generation segment, the water temperature experiences the entire period of the principal supercooling process: the water is cooled to a maximum supercooling, then rises up to a residual level of supercooling. This means that the generation segment is responsible for the principal supercooling period whereas the ice jam and cover segment is responsible for the residual supercooling.

Once the frazil ice is generated, there are two kinds of ice transportation in rivers, suspended or entrained frazil ice and surface (frazil) ice. Arden and Wigle (1972) observed frazil ice in suspension on the Niagara River, where frazil ice was largely in the form of disks.

Gosink and Osterkamp (1983) compared the buoyant and the mixing time scales to determine "well mixed vs. layered" flow. If the buoyant time scale (defined by the entire depth) was less than the time required for a frazil disk to diffuse downward through the entire depth, then they expected a distinct surface layer to form. Interestingly, the ratio of the time scales is independent of the flow depth, and depends only on the mean flow velocity, the channel roughness, and the rise velocity of the frazil. They assumed the rise velocity to be 0.01 m/s, which is appropriate, based on their analysis, for disks 2.5 mm in diameter with a thickness-to-diameter ratio of 1:10.

Matousek (1984) determined the flow conditions under which a distinct surface layer could be expected. Matousek (1984) compared the buoyant rise velocity of the frazil disks and the mean magnitude of the vertical turbulent velocity fluctuation. If the rise velocity exceeded this mean magnitude, then a surface layer should form. Empirical correlations were used to determine both the rise velocity and the vertical velocity fluctuations. Tsang (1988b) compared the kinetic energy available for mixing and the increase in potential energy resulting from the suspension of the frazil ice in the flow. From this, he derived a "distribution equation," which, when solved numerically, provided the vertical concentration distribution of the frazil ice, including a distinct surface layer, if present.

Liou and Ferrick (1992) developed a model in which the net upward migration of frazil disks attributable to buoyancy was opposed by intermittent mixing induced by large energy-containing eddies. A surface renewal model was used to describe the effects of large eddy mixing. They supposed that a critical surface layer thickness was necessary for the surface to develop structure. In general, they found that the heat loss rate from the water surface, the surface renewal frequency, and the critical surface layer thickness determined whether the frazil evolved towards a well-mixed equilibrium state or a layered state.

The two-layer (surface frazil and suspended frazil) ice transport concept was first introduced by Lal (1989), Lal and Shen (1991) in the model RICE. Wang (1993) and Shen et al. (1995) further modified this concept in the RICEN model. In their models, the total discharge was first calculated. The ice discharge in each reach was then separated into surface ice and suspended ice discharges based on

distribution coefficients determined from the two-layer analysis. Andres (1995) assumed that the distribution coefficient is related in some sense to the rise velocity of the frazil particles; the rate at which the suspended frazil is transported into the surface layer is a function of the distribution coefficient and the volumetric concentration of suspended frazil.

In the previous research efforts, frazil ice properties, such as its size and rise velocity, couldn't be determined. These parameters were assumed empirically, or considered as calibration parameters (Shen, 2002).

5.2 A New Model of Frazil Ice Formation

In Chapter 3, experiments showed that the frazil formed during the principal period of supercooling is related to flow turbulence, not only the amount of frazil, but also the frazil size and quantity. Chapter 4 attempted to develop a model relating the distribution of frazil size to the flow turbulence characteristics. These efforts provide an opportunity to relate flow turbulence to the characteristics of frazil ice generated in the flow. Herein, a new model of frazil ice formation is proposed.

In this new model, it is assumed that the water temperature during the supercooling process drops below the freezing point (0°C , see Figure 5.2a) to a minimum value ($T_{\min}$) at time $t_{\min}$. Beyond this point, the water temperature rises up to a residual temperature (T_r), which occurs at time t_r . During the supercooling process (until t_r), the instantaneous frazil particle size distribution is governed by flow turbulence as outlined in the previous chapter. The distributions are the same as the final distribution assuming frazil formation has a negligible impact on the turbulence of the flow. Nevertheless, the total number of frazil particles increases gradually as shown in Figure 5.2b. The increase is slow during the cooling stage, but faster during the warming stage. When the residual temperature is reached, the number of frazil particles reaches its peak value, then decreases due to flocculation forming a layer of frazil (surface) ice.

However, the total frazil ice mass (M_i) increases from zero to the maximum entrained frazil generating capacity (M_v) of the flow is reached at time t_r (Figure 5.2c), which is simply referred to as the frazil capacity or entrained frazil capacity hereinafter. Beyond the time t_r , M_i increases but M_v decreases resulting in a two-layer process: entrained frazil ice and surface frazil, which are governed

by the residual supercooling and entrained frazil ice processes. These aforementioned characteristics of the supercooling process and frazil evolution were observed and confirmed in the current experiments described in Chapter 3.

The experimental observations in the counter-rotating flume are consistent with an entrained frazil capacity existing in the flow. Figure 5.3 shows three experiments with similar velocities, and similar experimental environment, the supercooling processes are remarkably repeatable. The frazil formed in these three experiments is almost identical with a discrepancy of about 5%. Figure 5.4 and 5.5 show another three experiments with similar velocities, but with small fluctuation in the bed and wall cavity temperatures resulting in a small variation of these supercooling processes; nonetheless, the frazil formed in these experiments is within 10%.

The concept of an entrained frazil capacity during the supercooling process can also be found in the tank experiments conducted by Carstens (1966). His experiments varied the cooling rate of a small tank with insulated walls and bottom. Constant circulation was maintained in the tank by means of a propeller, which can be considered a constant turbulent intensity. According to the model proposed above, the entrained frazil ice generating capacity should be constant for a constant turbulent intensity and independent of the cooling rate. Figure 5.6 shows $t - T$ curves for six experiments with different rates of cooling. The curves are plotted so that the times at which the water temperature is exactly 0°C coincide, in order to make comparison easier. An inspection of figure 5.6 suggests the influence on the water temperature caused by an increase in the rate of heat loss through the free water surface produces the following:

- a) an increase in the maximum amount of supercooling,
- b) a decrease in the time to the rise from maximum supercooling to the residual supercooling,
- c) an increase in the residual magnitude of supercooling, and
- d) the triangular areas bounded by the $t - T$ curves of supercooling seem equal.

5.3 Frazil Generating Capacity of a Flow

The entrained frazil generating capacity of a flow is defined as the maximum amount of entrained frazil ice particles that a flow can generate. Observations show this capacity can be considered as the maximum amount of frazil ice that can be entrained and suspended in the flow without the formation

of a surface frazil ice layer. Hence, the capacity is assumed to be analogous to the suspended sediment carrying capacity of a flow, with the important distinction that sediment is negatively buoyant while frazil ice is positively buoyant. Following this idea, the entrained frazil capacity should be related to the turbulent intensity, and frazil size.

In order to represents the condition of flow and frazil ice composition, the dimensionless parameter $U^3 / gR\omega$, is proposed. The frazil generating capacity is assumed to have the form

$$M_* = f\left(\frac{U^3}{gR\omega}\right), \quad (5.1)$$

where, U is the mean flow velocity, R is the hydraulic radius, and g is the gravity acceleration. ω is the rise velocity of a mean frazil particle size with D_{50} . Svensson and Omstedt (1994) simplified Daly's equations (1984) describing the rise velocity of frazil ice to

$$\omega = 32.8D_{50}^{1.2}. \quad (5.2)$$

The equation was developed by assuming that a disk rises steadily with its axis perpendicular to the vertical. This may be an overestimation of the rise velocity (Daly, 1984) particularly for frazil diameters larger than 0.28 cm. In this study, Equation (5.2) is used for open channel flow because of smaller frazil sizes, such as Carstens experiments (1966). For the experiments in the counter-rotating flume, the mean frazil diameters are in general bigger than 0.28 cm. A regression analysis of the field and laboratory data from Gosink and Osterkamp (1983) yields

$$\omega = 226D_{50}^{1.7}. \quad (5.3)$$

Note, in (5.2) and (5.3) the rise velocity and diameter of frazil particles are in units of m/s and m, respectively. The parameters used to compute estimates of the dimensionless parameter $U^3 / gR\omega$ are summarized in Table 5.1 to 5.4.

Figure 5.7 plots the dimensionless parameter $U^3 / gR\omega$ versus the frazil formed by heat budget for all of the experiments which have varying air temperatures. It looks very scattered, but shows a

tendency for the frazil formed to increase with increasing $U^3 / gR\omega$. The relationship is regressed in a linear function as:

$$M_v = 0.0227 \left(\frac{U^3}{gR\omega} \right) - 0.0199. \quad (5.4)$$

Figure 5.8 plots the parameter and frazil formed for the experiments at air temperature -10°C , the relationship is regressed as

$$M_* = 0.0231 \left(\frac{U^3}{gR\omega} \right) - 0.0253. \quad (5.5)$$

Table 5.1. Summary of frazil characteristics (Group 1, $H = 15$ cm).

	T_a [°C]	U [m/s]	R [m]	D_{50} [cm]	ω [cm/s]	$U^3/gR\omega$	t_{sp} [min]	dT/dt [°C/min]	M_v [%]
56	-10	0.434	0.06	0.42	2.06	6.74	23	-0.0038	0.118
57	-10	0.435	0.06	0.42	2.06	6.79	24.5	-0.0042	0.142
58	-10	0.517	0.06	0.6	3.78	6.22	19.5	-0.0044	0.116
59	-10	0.604	0.06	0.82	6.42	5.83	26	-0.0032	0.114
60	-10	0.424	0.06	0.4	1.9	6.83	23.5	-0.004	0.129
61	-10	0.421	0.06	0.4	1.9	6.69	21	-0.004	0.116
62	-10	0.584	0.06	0.76	5.64	6	21	-0.0044	0.127
63	-10	0.612	0.06	0.84	6.69	5.82	20	-0.0038	0.105
64	-10	0.382	0.06	0.32	1.3	7.3	28	-0.0036	0.136
65	-10	0.391	0.06	0.32	1.11	7.07	25	-0.0038	0.13
66	-10	0.63	0.06	0.89	7.36	5.78	20.5	-0.0041	0.114
67	-10	0.393	0.06	0.34	1.44	7.17	27.5	-0.0035	0.131
68	-10	0.404	0.06	0.37	1.66	6.75	26	-0.0037	0.133
69	-10	0.642	0.06	0.92	7.81	5.76	19	-0.0038	0.098
70	-10	0.521	0.06	0.61	3.88	6.19	23	-0.0038	0.12
71	-10	0.43	0.06	0.41	1.98	6.83	26	-0.0037	0.133
72	-10	0.423	0.06	0.4	1.9	6.78	22.5	-0.0038	0.118
90	-15	0.432	0.06	0.42	2.06	6.65	17.5	-0.0071	0.169

91	-15	0.515	0.06	0.59	3.67	6.32	18	-0.006	0.149
92	-15	0.5	0.06	0.56	3.36	6.32	18.5	-0.006	0.152
93	-15	0.632	0.06	0.89	7.38	5.81	15.5	-0.0065	0.138
94	-6.5	0.427	0.06	0.41	1.98	6.69	41.5	-0.0021	0.117
95	-6.5	0.428	0.06	0.41	1.98	6.74	39	-0.002	0.107
102	-10	0.428	0.06	0.41	1.98	6.74	28.5	-0.0035	0.138

Table 5.2. Summary of frazil characteristics (Group 2, $H = 10$ cm).

	T_a [°C]	U [m/s]	R [m]	D_{50} [cm]	ω [cm/s]	$U^3/gR\omega$	t_{sp} [min]	dT/dt [°C/min]	M_v [%]
73	-10	0.491	0.05	0.56	3.36	7.19	22	-0.0045	0.134
74	-10	0.351	0.05	0.29	1.1	8.03	24.5	-0.0042	0.157
75	-10	0.496	0.05	0.58	3.54	7.03	28	-0.0036	0.151
76	-10	0.695	0.05	1.13	11.08	6.18	25	-0.0038	0.122
77	-10	0.476	0.05	0.53	3.06	7.19	23.5	-0.0047	0.152
97	-15	0.495	0.05	0.57	3.46	7.14	17	-0.0083	0.194
99	-12	0.482	0.05	0.54	3.16	7.23	26	-0.0037	0.155
101	-10	0.492	0.05	0.57	3.46	7.01	29	-0.0035	0.138

It is suggested that (5.5) can be used to determine the entrained frazil generating capacity at an air temperature of -10°C . If the air temperature is not -10°C , the following air conversion, which is based on the data of Hanley and Michel (1977) and HRTF data (Figure 5.9), is proposed.

$$\frac{M_{*T}}{M_*} = 0.6868e^{\{0.4149(T_a/10)\}} \quad (5.6)$$

M_{*T} is the flow's entrained frazil generating capacity at an air temperature of T_a ; the absolute value of the temperature is used in equation (5.6). As a result, if the air temperature and flow hydraulic parameters are known, it is possible to estimate the frazil size formed in the flow using (4.25), the capacity of frazil formed (5.5) and the volume of entrained frazil ice formed (5.6) for this hydrothermal condition. Figure 5.10 shows good agreement between the frazil ice concentration and the air-temperature adjusted capacity of the flow.

Table 5.3. Summary of frazil characteristics (Group 3, H = 20 cm).

	T_a [°C]	U [m/s]	R [m]	D_{50} [cm]	ω [cm/s]	$U^3/gR\omega$	t_{sp} [min]	dT/dt [°C/min]	M_v [%]
78	-10	0.484	0.067	0.51	2.86	6.05	31.5	-0.0029	0.125
79	-10	0.486	0.067	0.51	2.86	6.13	28	-0.0028	0.107
80	-10	0.356	0.067	0.27	0.97	7.1	27	-0.003	0.11
81	-10	0.367	0.067	0.29	1.1	6.89	28.5	-0.0037	0.143
82	-10	0.357	0.067	0.28	1.03	6.73	30	-0.0034	0.14
83	-10	0.376	0.067	0.3	1.16	6.99	21.5	-0.0049	0.143
84	-10	0.486	0.067	0.52	2.96	5.93	22.5	-0.0037	0.113
85	-10	0.51	0.067	0.57	3.46	5.86	25.5	-0.0028	0.099
86	-10	0.483	0.067	0.51	2.86	6.01	24.5	-0.0029	0.099
87	-10	0.485	0.067	0.51	2.86	6.09	30	-0.0029	0.119

Figure 5.4 Summary of frazil characteristics of Carstens' experiments (1966)

	T_a [°C]	U [m/s]	R [m]	D_{50} [cm]	ω [cm/s]	$U^3/gR\omega$	t_{sp} [min]	dT/dt [°C/min]	M_v [%]
Case I	-10	0.5	0.067	0.24	0.25	7.77	6	-0.02	0.16
Case II	-10	0.33	0.067	0.12	0.98	5.60	10	-0.0078	0.099

5.4 Model Application to Nelson River

Traditionally, the volume of ice in an open channel is estimated by the following equation.

$$M = \frac{AC_0(T - T_a)t}{\rho_i L} \quad (5.7)$$

where; M is volume of ice generated in time t [m^3], A is exposed surface area of open water [m^2], C_0 is a coefficient from about 15 to 25 $W/m^2°C$, (20 for Nelson River), T is water temperature (near 0 usually) [$°C$], T_a is air temperature [$°C$], t is time in seconds, ρ_i is density of ice (920 kg/m^3), and L_i is latent heat of fusion ($333,000 \text{ Joules/kg}$).

This equation is used to estimate the total ice (both frazil ice and other forms (Beltaos, 1985)). The new approach attempts to predict the frazil size based on the hydraulics of the flow, together with thermal data. It can predict the amount of frazil ice that can be generated during the period of principal supercooling. In river reaches after a hydropower station such as that shown in Figure 5.1, the concept of frazil generating capacity and its equations have the following potential practical applications:

- to predict the characteristics of frazil and its distribution in a river based on the hydraulics of the flow;
- to predict the limit of frazil that can be entrained in a flow based on hydraulic and thermal condition;
- to predict the length of the principal period supercooling for a known cooling rate; and
- to predict the length of an open reach in a cold regions that generates frazil ice.

As an example of these applications, the following parameters of the Nelson River are assumed:

- River width is 1000 m,
- Discharge is 3000 m³/s,
- Slope = 1/1000,
- Manning coefficient is 0.03.
- Air temperature $T_a = -13\text{ }^{\circ}\text{C}$,
- $C_0 = 20\text{ W/m}^2\text{ }^{\circ}\text{C}$

Hydraulic calculations yield:

- Water velocity $U = 1.61\text{ m/s}$,
- Water depth $H = 1.92\text{ m}$, and
- Bed shear velocity $u_* = 0.04\text{ m/s}$.

Bringing these hydro-thermal parameters into the model, produces a frazil generating capacity M_{*T} of 0.16 %, and mean frazil diameter of 0.68 cm.

The cooling rate is determined by a heat balance of a water column in Figure 5.11, the equation is

$$\begin{aligned}\frac{dT}{dt} &= \frac{C_0(T - T_a)}{\rho C_p H} \\ &= 3.26 \times 10^{-5} \text{ C/sec} \\ &= 0.002 \text{ C/min}\end{aligned}\tag{5.8}$$

To reach the generating capacity of 0.16% in the river, the flow has to move a certain distance from the 0°C isotherm.

According to the heat balance (3.8), the length of the principal period of supercooling process would be

$$\begin{aligned}t_{sp} &= \frac{M_v L_i \rho_i}{C_p \rho \frac{dT}{dt}} \\ &= 58.8 \text{ min} \\ &\cong 1.0 \text{ hour}\end{aligned}\tag{5.9}$$

The length of the open water reach during which the supercooling occurs can be estimated by

$$\begin{aligned}\Delta d &= U \times t_{sp} \\ &= 1.61 \times 3600 \\ &= 5796 \text{ m} \\ &\cong 5.8 \text{ km}\end{aligned}\tag{5.10}$$

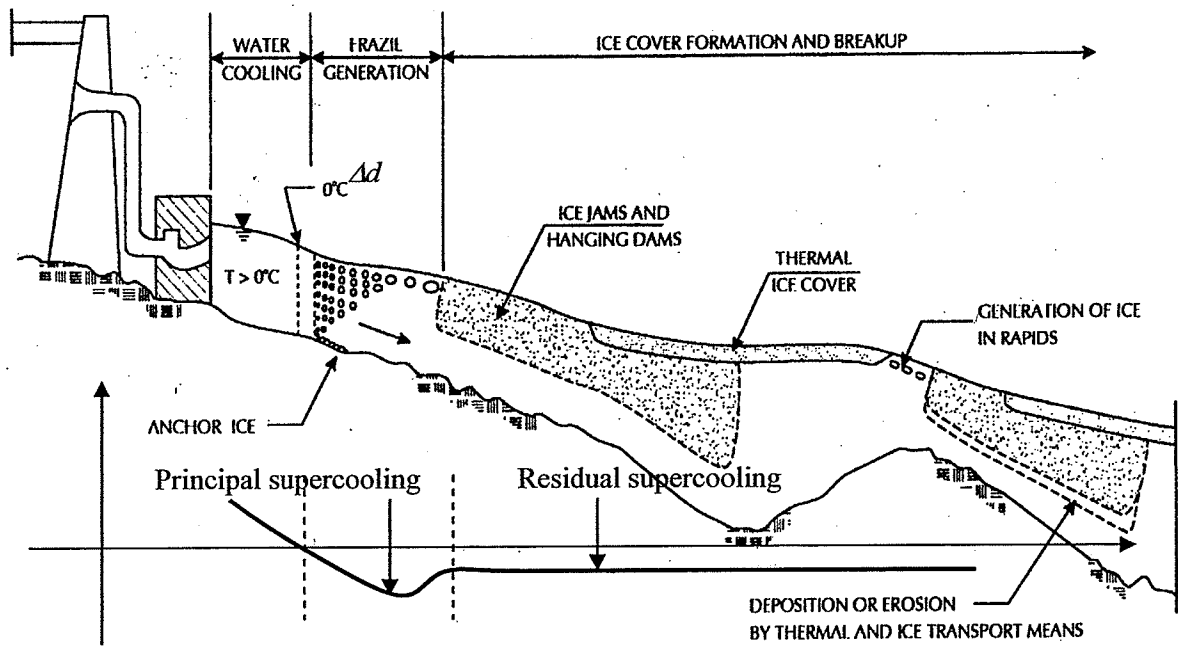


Figure 5.1. Ice conditions and water temperature in a river (modified from Beltaos, 1985)

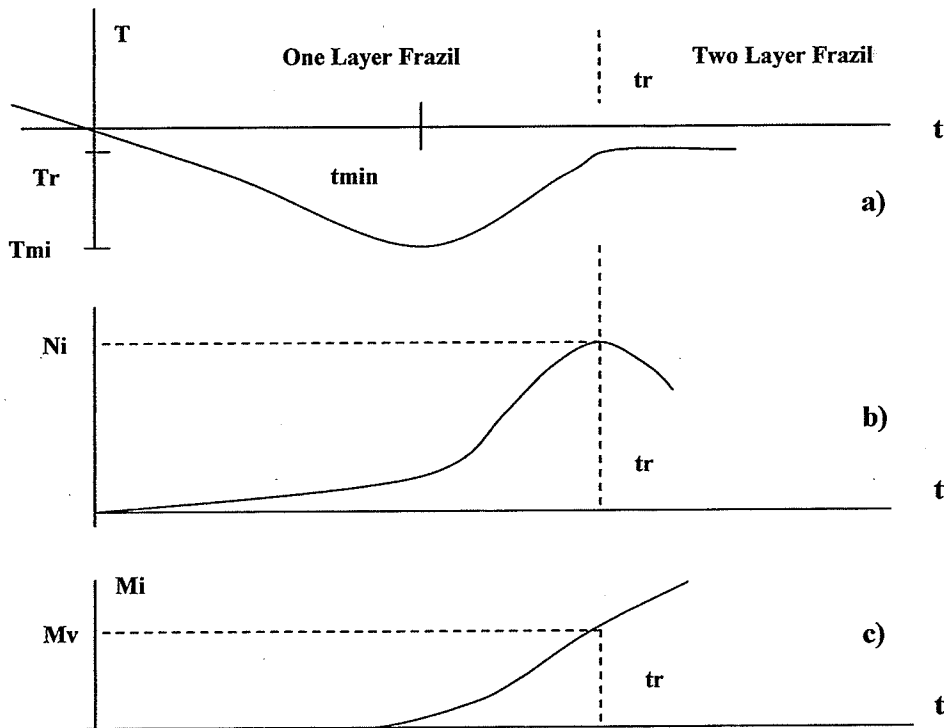


Figure 5.2. Schematic of frazil formation during the supercooling process in a box of water.

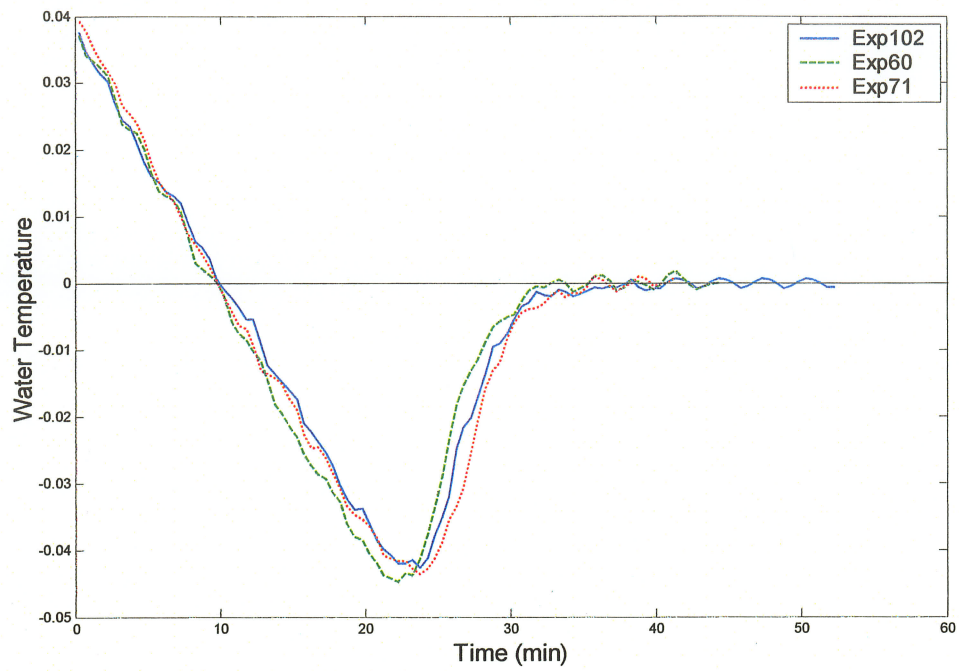


Figure 5.3. Water temperature vs. time for Exp. 102, 60 and 71 ($H=15$ cm, $T_a = -10^\circ\text{C}$).

Note, the area bounded by the temperature trends and $^\circ\text{C}$ is related to frazil generation capacity.

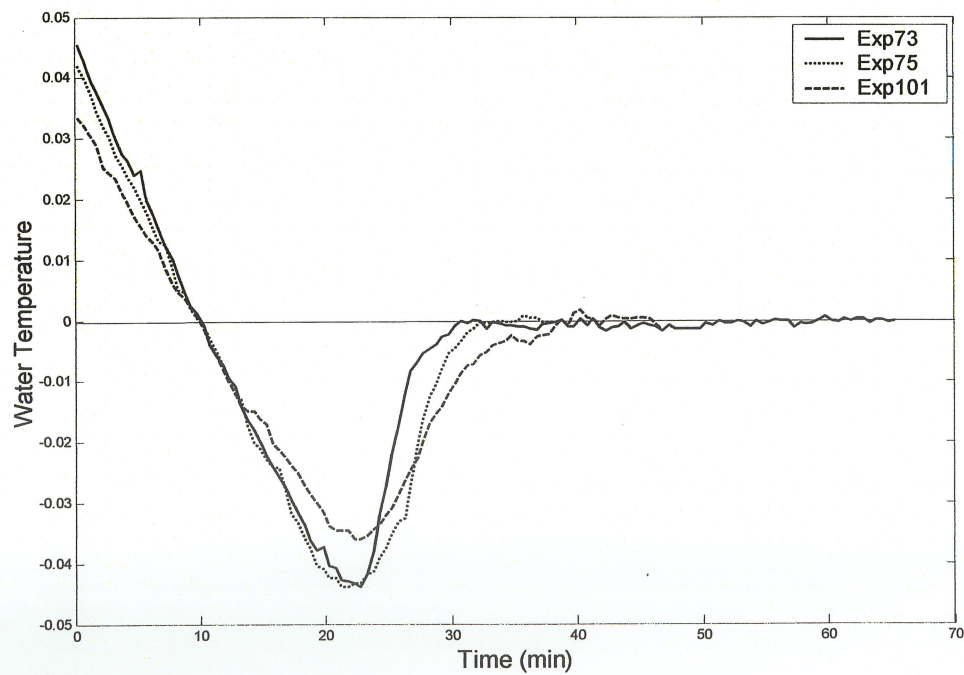


Figure 5.4. Water temperature vs. time for Exp. 73, 75 and 101 ($H=10$ cm, $T_a = -10^\circ\text{C}$).

Note, the area bounded by the temperature trends and $^\circ\text{C}$ is related to frazil generation capacity.

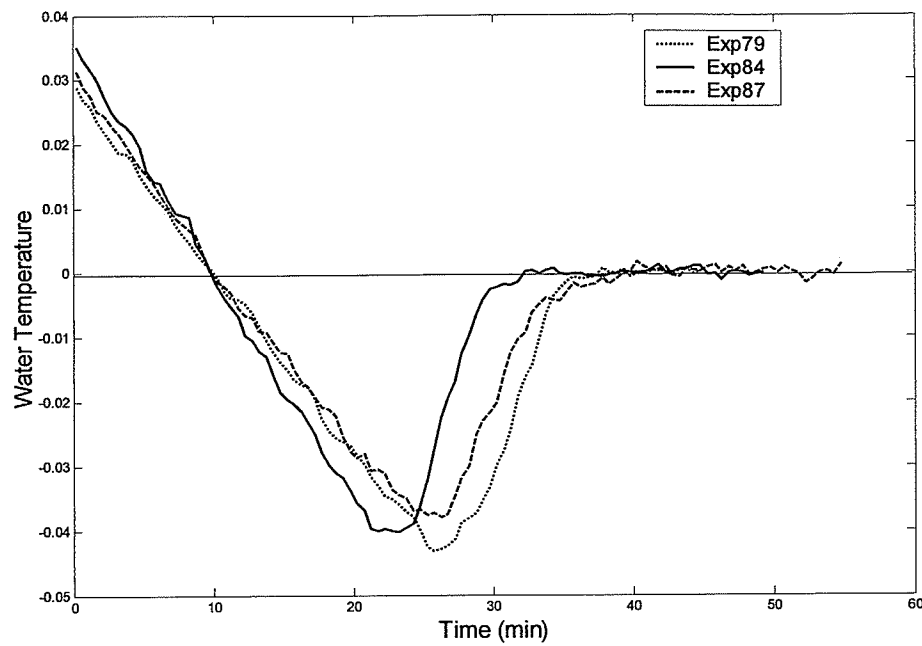


Figure 5.5. Water temperature vs. time for Exp. 79, 84 and 89 ($H=20$ cm, $T_a = -10^\circ\text{C}$).

Note, the area bounded by the temperature trends and $^\circ\text{C}$ is related to frazil generation capacity.

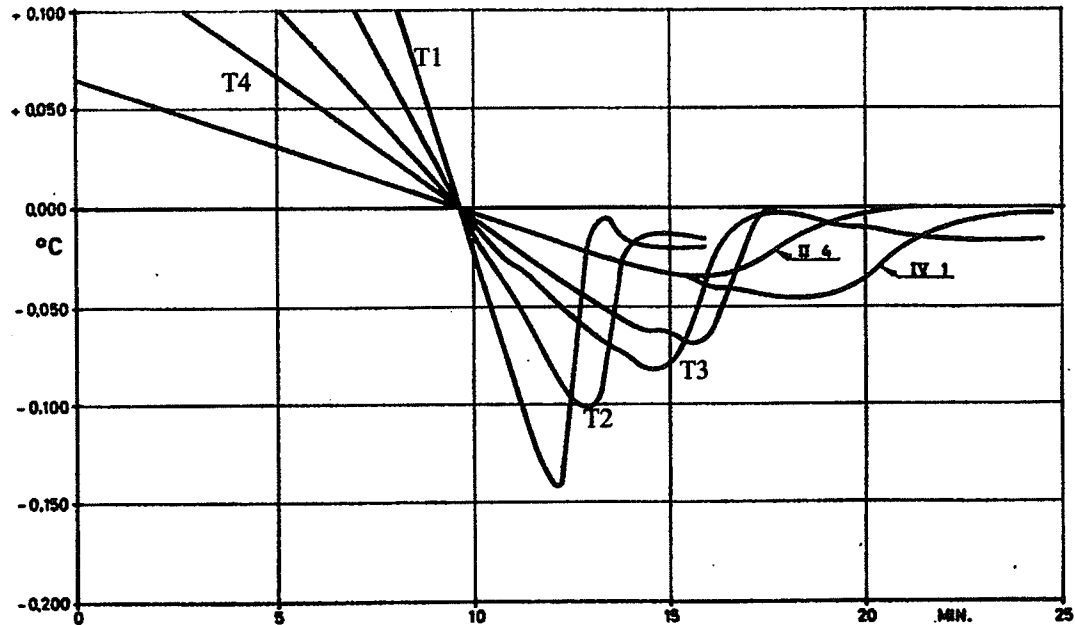


Figure 5.6. Effect of cooling rate on supercooling in a tank (Carstens, 1966).

Note, the area bounded by the temperature trends and $^\circ\text{C}$ is related to frazil generation capacity.

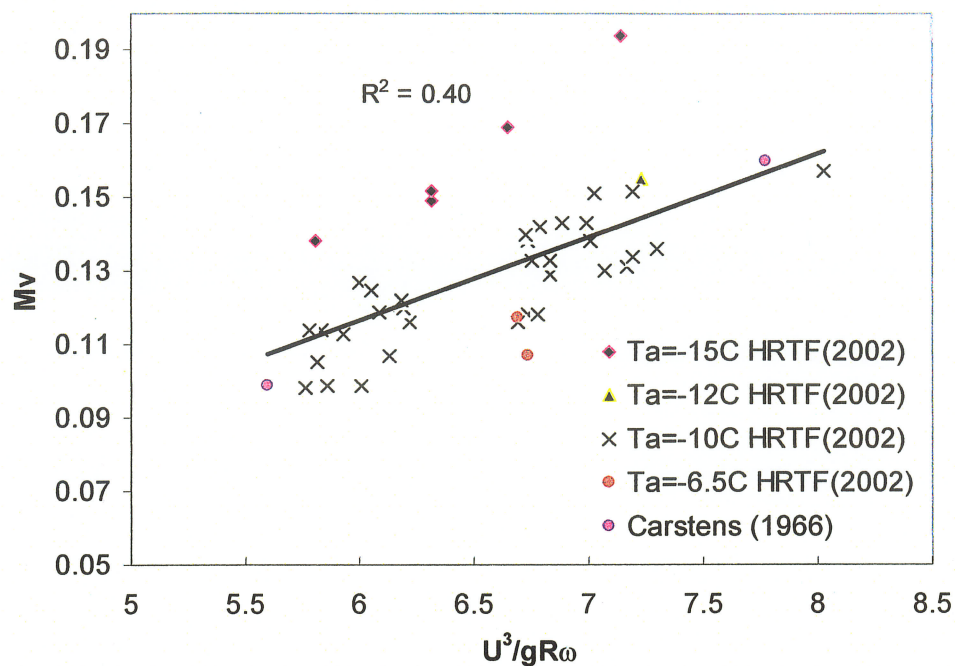


Figure 5.7. Comparison of frazil concentrations without air temperature adjustment.

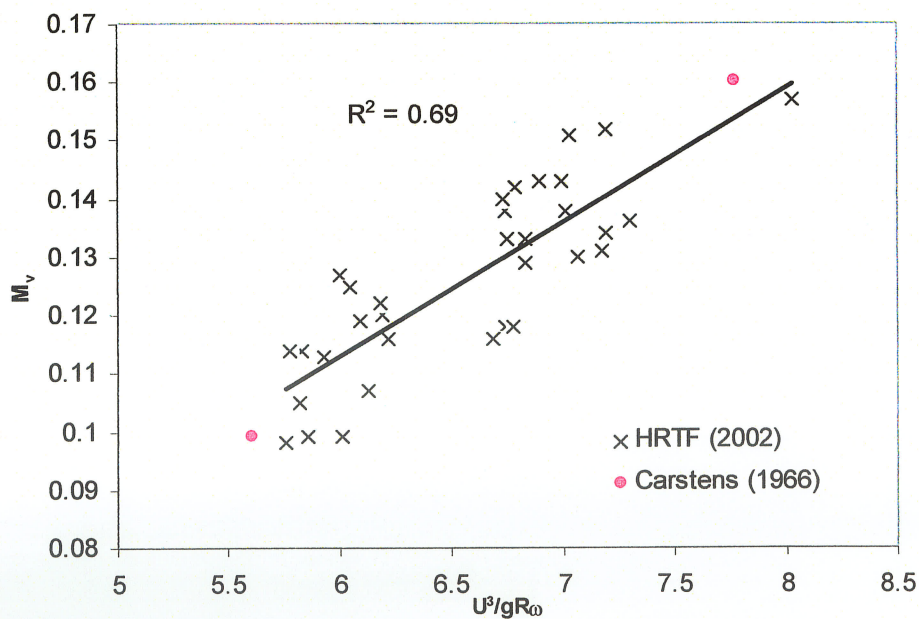


Figure 5.8. Comparison of frazil concentrations for experiments at $T_a = -10^\circ\text{C}$

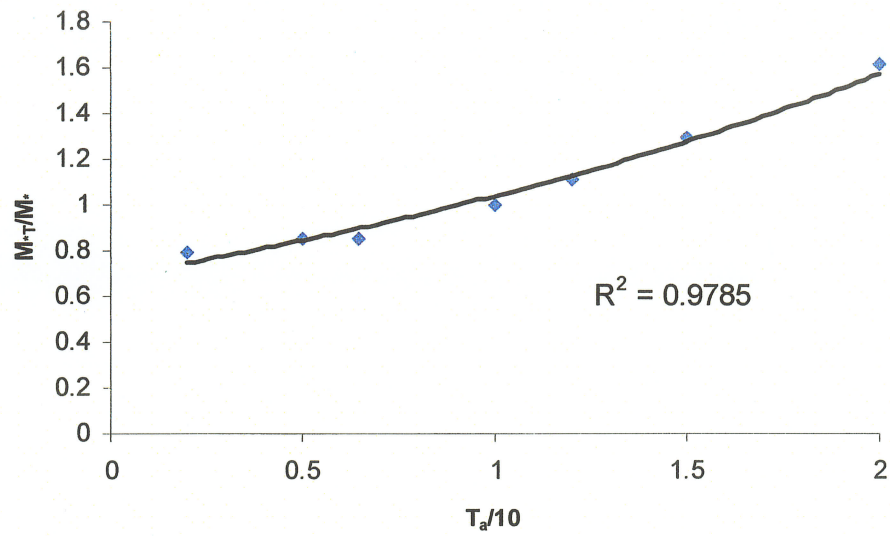


Figure 5.9. Air conversion factor for entrained frazil capacity.

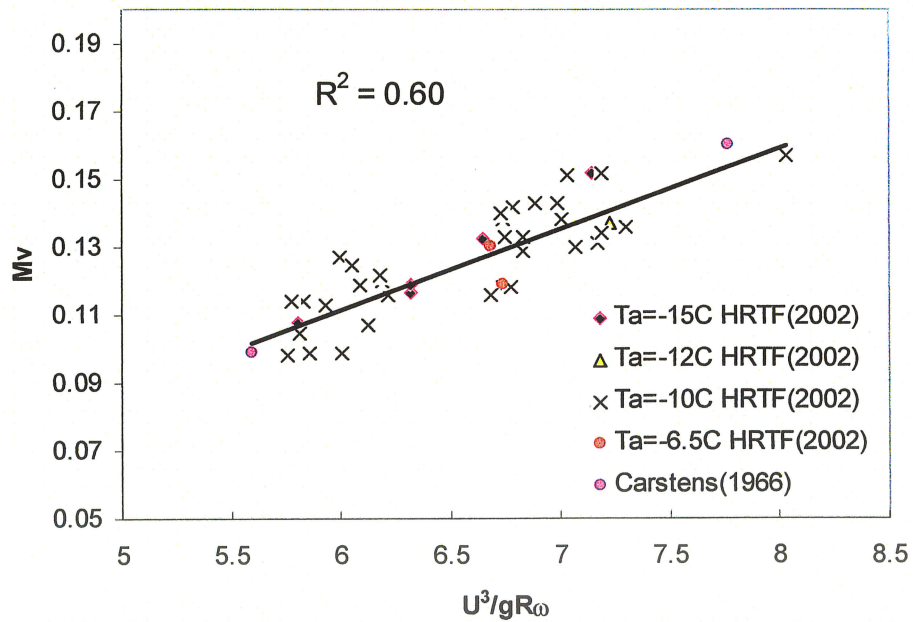


Figure 5.10. Comparison of frazil concentrations with air temperature adjustment.

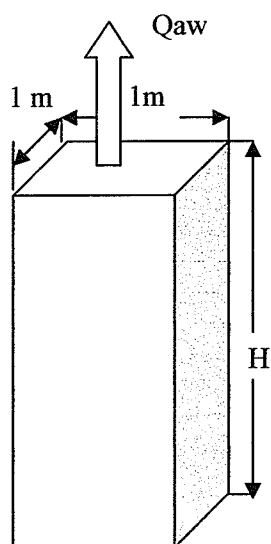


Figure 5.11 Schematic heat exchanges in a water column

Frazil can grow in size, agglomerate, and float to the surface to form surface ice, or be brought to the bottom and adhere to the bed to form anchor ice. The vertical distribution of frazil ice is governed by its buoyancy and turbulent mixing. Our current knowledge of the vertical distribution of frazil is quite limited. Only a single vertical concentration profile obtained from Lachine Rapids on the St. Lawrence River has been reported in the literature (Tsang 1986). Previously herein, a model of frazil size distribution based on the flow turbulence characteristics in a river was proposed. In this chapter, a model for the vertical distribution of frazil ice is derived based on similar principles.

6.1 Background

Tsang (1988b) proposed a theoretical model for the vertical distribution of frazil. In his model the increase in potential energy of the frazil-water system due to frazil entrainment to the lower layers is assumed to be derived from the kinetic energy of the flow turbulence, which itself can be related to the shear stress of the frazil-laden flow. Such considerations lead to an integral equation whose general analytical solution could not be obtained. A particular numerical solution, however, was obtained, which was qualitatively supported by data from the Lachine Rapids (Tsang, 1988b).

A computer model involving 15 differential equations and a number of assumptions was proposed by Omstedt (1985a, b) for the production and distribution of frazil in a turbulent flow of saline water. Among the many assumptions in the model was that discrete frazil crystals are suspended in the flow.

Omstedt's model is difficult to use for practical engineering because of its complexity, large number of assumptions, and numerous coefficients (Tsang and Cui, 1994).

A model proposed by Liou and Ferrick (1992) assumed probable pulldown of surface frazil by large eddies at a certain frequency and subsequent dispersion and free rise of frazil towards the surface. At later stages, the surface pulldown will cease and frazil will accumulate at the surface as a layer of constant concentration.

In view of the mathematical difficulty in Tsang's theoretical model (1988b) and the complexity and uncertainty in Omstedt's computer model (1985), Tsang and Cui (1994) attempted to develop a simple empirical equation for engineering use based on the data obtained from their experiments. They found that the distribution of frazil in a flow is determined by three factors: total heat loss, flow turbulence, and evolutionary state of the frazil.

Priset and Hausser (1961) proposed that frazil ice might be considered as a sediment and that one might use sediment transport equations to describe frazil ice transport in rivers. However, the validity of this analogy is not clear yet (Daly, 1994). The greatest difficulty with this analogy is with respect to frazil size, a description of its variation with time, and heat transfer properties (Tsang, 1988a, b). Shen and Wang (1995) attempted to consider frazil ice transport under ice-cover as a specific type of sediment transport. Hammer et al. (1995, 1996) conducted theoretical studies on the formation and growth of anchor ice which discussed the vertical mixing, frazil size, as well as the distribution of bed gravel.

6.2 Governing Principles

In chapter 4, a model for the size distribution of frazil ice was developed based on the probability distribution of v' . Herein, a model for the vertical distribution of frazil ice is derived based on similar principles. It is worth noting that a frazil ice particle is treated as a "negative" sediment, i.e., its rise velocity, ω , has a negative value. For any given point on a vertical line in two-dimensional flow, the diffusion equation has the form

$$\frac{dC}{dy} = \frac{\omega}{\varepsilon_s} C, \quad (6.1)$$

where ε_s is the diffusion coefficient, C is the point volumetric concentration at y , and y is the vertical distance measured from the bed. For sediment transport, Ni and Wang (1991) proposed

$$\varepsilon_s = \frac{1}{2} l_1 \overline{|v'|}, \quad (6.2)$$

where v' is the vertical fluctuating velocity. The characteristic length for the vertical frazil movement is l_1 , or the vertical component of the Lagrangian trajectory of the particle, which is different from the so-called mixing length l_0 (Ni and Wang, 1991). To account for the differences between frazil ice shape and density in comparison to sediment, a factor is proposed and incorporated into (6.2) for frazil ice-laden flow, i. e.,

$$\varepsilon_s = \frac{1}{2} \frac{\rho_s}{K_v \rho_i} l_1 \overline{|v'|}, \quad (6.3)$$

in which ρ_s is the density of sediment, typically 2650 kg/m³. Since v' is described by a normal distribution, i.e.,

$$f(v') = \frac{1}{\sqrt{2\pi}\sigma_{v'}} \exp\left[-\frac{v'^2}{2\sigma_{v'}^2}\right] \quad (6.4)$$

where

$$\sigma_{v'} = \sqrt{\overline{v'^2}} \quad (6.5)$$

then

$$\overline{|v'|} = \int_0^{+\infty} f(|v'|) v' dv' = \sqrt{\frac{2}{\pi}} \sqrt{\overline{v'^2}} \quad (6.6)$$

By combining (6.1), (6.3), and (6.6), a diffusion equation is obtained

$$\frac{dC}{dy} = \frac{K_v \frac{\rho_i}{\rho_s} \sqrt{2\pi} \frac{\omega}{u_*} \cdot C}{l_1 \frac{\sqrt{v'^2}}{u_*}} \quad (6.7)$$

Since $\sqrt{v'^2}/u_* = F(y/h) = \sigma_v/u_* = \alpha$, a value of 1.05 is used in this study. This approximation is acceptable in most of the flow region according to several measurements (Zhang et al., 1990). Then

$$l = l_1 \frac{\sqrt{v'^2}}{u_*} = l_1 \alpha. \quad (6.8)$$

The following form of the characteristic length

$$l_1 = \frac{\rho_s l}{\rho_i \alpha} = \frac{\rho_s y}{\rho_i \alpha} \left(1 - \frac{y}{H}\right), \quad (6.9)$$

is chosen, simplified from Ni and Wang (1991) with consideration for the density of ice.

Substituting (6.9) into (6.7), the following solution is obtained by integration

$$\int_{C_a}^C \frac{dC}{C} = \int_a^H \frac{\alpha K_v \frac{\rho_i^2}{\rho_s^2} \sqrt{2\pi} \frac{\omega}{u_*}}{y \left(1 - \frac{y}{H}\right)} dy \quad (6.10)$$

Equation (6.10) can be written in the form

$$\frac{C}{C_a} = \left(\frac{a}{y} \cdot \frac{1 - \frac{y}{H}}{1 - \frac{a}{H}} \right)^{-\alpha K_v \left(\frac{\rho_i}{\rho_s} \right)^2 \sqrt{2\pi} \frac{\omega}{u_*}} = \left[\frac{a}{y} \cdot \frac{1 - \frac{y}{H}}{1 - \frac{a}{H}} \right]^{-z}, \quad (6.11a)$$

where

$$z = \alpha K_v \left(\frac{\rho_i}{\rho_s} \right)^2 \sqrt{2\pi} \frac{\omega}{u_*} \quad (6.11b)$$

is called the frazil suspended index, and C_a is the frazil ice concentration by volume at a distance $y = a$ from the bed. Using the stable characteristic of $\sqrt{v'^2}$, a very interesting result is obtained.

With the exception of the negative power in (6.11) and a factor of $\alpha K_v \left(\frac{\rho_i}{\rho_s} \right)^2$, which accounts for the difference between frazil and sediment, the suspension index $\sqrt{2\pi} (\omega / u_*)$ has exactly the same form as the well-known Rouse equation. This occurs because the equality $\sqrt{2\pi} = 1/\kappa \approx 2.5$ is always true for the commonly used $\kappa = 0.4$. Thus (6.11a) could be called the Rouse equation of frazil ice distribution.

6.3 A Model of Vertical Frazil Distribution

The well-known Rouse vertical distribution has been proven to be qualitatively true for suspended sediment. However, its quantitative examination is difficult because it requires the fall velocity of the sediment and the concentration of the sediment at one point and they are difficult to obtain. The application of the Rouse equation to frazil distribution, of course, will be further handicapped because of the varied rise velocity of frazil ice particles (Tsang, 1988). However, based on our proposed model of the supercooling process and frazil evolution, at any time during the principal supercooling, the frazil ice and frazil distribution in channels could be satisfied with a size distribution, and the corresponding frazil ice concentration determined by flow turbulence and heat balance. At the end of the principal supercooling, the mean frazil diameter (D_{50}) and the entrained frazil generating capacity of a flow (M_v) is known, the vertical frazil ice distribution and the capacity have a relationship as follows

$$M_v = \int_0^H u C dy. \quad (6.12)$$

The von Karman-Prandtl logarithmic law is

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y}{H} + \text{const} \quad (6.13)$$

or

$$\frac{u}{u_*} = \left(\frac{U}{u_*} + \frac{1}{\kappa} \right) + \frac{1}{\kappa} \ln \frac{y}{H}. \quad (6.14)$$

Using (6.12) and (6.14), letting $\xi = y/H$, following the procedure of Guo and Wood (1995), and considering the negative buoyant velocity of frazil ice, the frazil ice capacity equation (6.12) can be written as follows

$$M_v = H \int_0^1 u C d\xi \quad (6.14a)$$

$$= H u_* C_a \left(\frac{\xi_a}{1-\xi_a} \right)^{-z} \int_0^1 \left[\left(\frac{U}{u_*} + \frac{1}{\kappa} \right) + \frac{1}{\kappa} \ln \xi \right] \left(\frac{1-\xi}{\xi} \right)^{-z} d\xi \quad (6.14b)$$

$$= H u_* C_a \left(\frac{\xi_a}{1-\xi_a} \right)^{-z} \left[\left(\frac{U}{u_*} + \frac{1}{\kappa} \right) \int_0^1 \left(\frac{1-\xi}{\xi} \right)^{-z} d\xi + \frac{1}{\kappa} \int_0^1 \left(\frac{1-\xi}{\xi} \right)^{-z} \ln \xi d\xi \right] \quad (6.14c)$$

Defining

$$J_1 = \int_0^1 \left(\frac{1-\xi}{\xi} \right)^{-z} d\xi = \int_0^1 \left(\frac{\xi}{1-\xi} \right)^z d\xi \quad (6.15)$$

and

$$J_2 = \int_0^1 \left(\frac{1-\xi}{\xi} \right)^{-z} \ln \xi d\xi = \int_0^1 \left(\frac{\xi}{1-\xi} \right)^z \ln \xi d\xi \quad (6.16)$$

then (6.14) is rewritten as

$$\begin{aligned}
 M_v &= Hu_* C_a \left(\frac{\xi_a}{1-\xi_a} \right)^{-z} \left[\left(\frac{U}{u_*} + \frac{1}{\kappa} \right) J_1 + \frac{1}{\kappa} J_2 \right] \\
 &= Hu_* C_a \left(\frac{1-\xi_a}{\xi_a} \right)^z \left[\left(\frac{U}{u_*} + \frac{1}{\kappa} \right) J_1 + \frac{1}{\kappa} J_2 \right]
 \end{aligned} \tag{6.17}$$

It now becomes necessary to solve J_1 and J_2 .

J_1 is a beta function if $z < 1$,

$$J_1 = B(1-z, 1+z) = B(1+z, 1-z) = \Gamma(1+z)\Gamma(1-z) = \frac{z\pi}{\sin z\pi} \tag{6.18}$$

in which B is a beta function, and Γ is a gamma function.

As derived in Appendix C, J_2 has a solution of the form

$$J_2 = \frac{z\pi}{\sin z\pi} [f(z) - 1], \tag{6.19}$$

where

$$f(z) = z \sum_{n=0}^{500} [(n+1)(z+n+1)]^{-1}. \tag{6.20}$$

Then

$$M_v = Hu_* C_a \left(\frac{1-a}{a} \right)^z \left[\frac{U}{u_*} + \frac{f(z)}{\kappa} \right] \frac{z\pi}{\sin z\pi}. \tag{6.21}$$

Alternatively, integrating (6.11) can give the mean concentration $\bar{C}$ as

$$\frac{\bar{C}}{C_a} = \left(\frac{1 - \xi_a}{\xi_a} \right)^z \frac{z\pi}{\sin z\pi}. \quad (6.22)$$

The corresponding position $\bar{y}$ or $\bar{\xi}$ for $\bar{C}$ is

$$\bar{y} = H\bar{\xi} = H \left[1 + \left(\frac{z\pi}{\sin z\pi} \right)^{-1/z} \right]^{-1}. \quad (6.23)$$

If (6.23) is substituted in (6.21), the capacity of the flow is

$$M_v = Hu_* \bar{C} \left[\frac{U}{u_*} + \frac{f(z)}{\kappa} \right]. \quad (6.24)$$

If the mean concentration and its location are chosen as the reference point, then (6.11) becomes

$$\frac{C}{\bar{C}} = \left[\frac{\bar{\xi}}{1 - \bar{\xi}} \cdot \frac{1 - \xi}{\xi} \right]^{-z}. \quad (6.25)$$

6.4 Model Verification and Discussion

The only published field frazil ice measurement is from Lachine Rapids (Tsang, 1986). However, no detailed hydrothermal data accompany these measurements to verify this model. Frazil ice distributions from the counter-rotating flume data were published, but no information was collected on the supercooling process (Tsang and Cui, 1994).

The vertical distribution model can be verified by frazil observations obtained herein. These observations contain information about the vertical distribution of frazil, but are subject to limitations of the DIPS, namely grabbing and recognizing a frazil ice particle smaller than a certain size. However, the system can still provide important quantitative data regarding the vertical distribution of frazil ice.

In order to compare the modeled vertical distributions of frazil ice, the capacity of entrained frazil ice (M_v) is replaced by an instantaneous mass of frazil ice, which is computed from the sum of all frazil ice particles in an image; the mean frazil ice diameter is determined by hydraulic variables. Therefore, the modeled vertical distribution is a relative profile, showing only the tendency/shape of this profile; it is not an absolute value. An absolute profile could be obtained if all the frazil particles/mass existing in the flow could be measured. However, the DIPS has not yet been developed to provide this information.

To consider the impact of flow velocities, 5 tests are selected to verify the model (Table 6.1). All of the experiments selected belong to Group 1 with a water depth of 15 cm and an air temperature of -10 °C. the other test groups produce unreasonable profiles due to several factors. For example, the Group 2 experiments developed a significant amount of border ice.

Table 6.1. Experiments for examining the vertical distribution of frazil ice.

Exp.	U [m/s]	H [m]	M* [%]	u* [m/s]	D ₅₀ [cm]	W [cm/s]	z
60	0.424	0.15	0.133	0.0309	0.4	1.9	0.152
102	0.429	0.15	0.133	0.0312	0.41	1.96	0.156
63	0.612	0.15	0.106	0.0444	0.84	6.66	0.373
66	0.630	0.15	0.104	0.0457	0.89	7.36	0.400
69	0.641	0.15	0.104	0.0466	0.92	7.83	0.417

Figures 6.1 to 6.5 show the vertical distribution of frazil at the end of the principal supercooling process, that is, at the observed peak frazil area during the supercooling period for these experiments. All the figures show that the observed and modeled distributions are quite comparable. These distributions also show that higher velocities yield a less uniform profile due to larger frazil particles and the associated higher rise velocities. Consequently, this yields a higher suspension index, z . However, the higher velocities yield a higher absolute profile, which is opposite to that predicted by the model. This apparent inconsistency can be explained by the limitations of the current image grabbing/recognizing system, and are further confirmed by a numerical model of the supercooling process and frazil evolution presented in the next chapter.

Figure 6.6 shows the vertical frazil distributions at other times during the supercooling process for Exp. 63. The three distributions are similar except for the absolute values. All of the distributions can be described using the same mean diameter of frazil size and the same rise velocity as at the saturated state of the entrained frazil ice formation, i.e., at the end of the principal supercooling process. This finding further supports the approach that the frazil size distribution is governed by the flow turbulence. It can be considered the same during the principal supercooling process, unless the hydraulic conditions change due to the formation of a surface frazil ice layer.

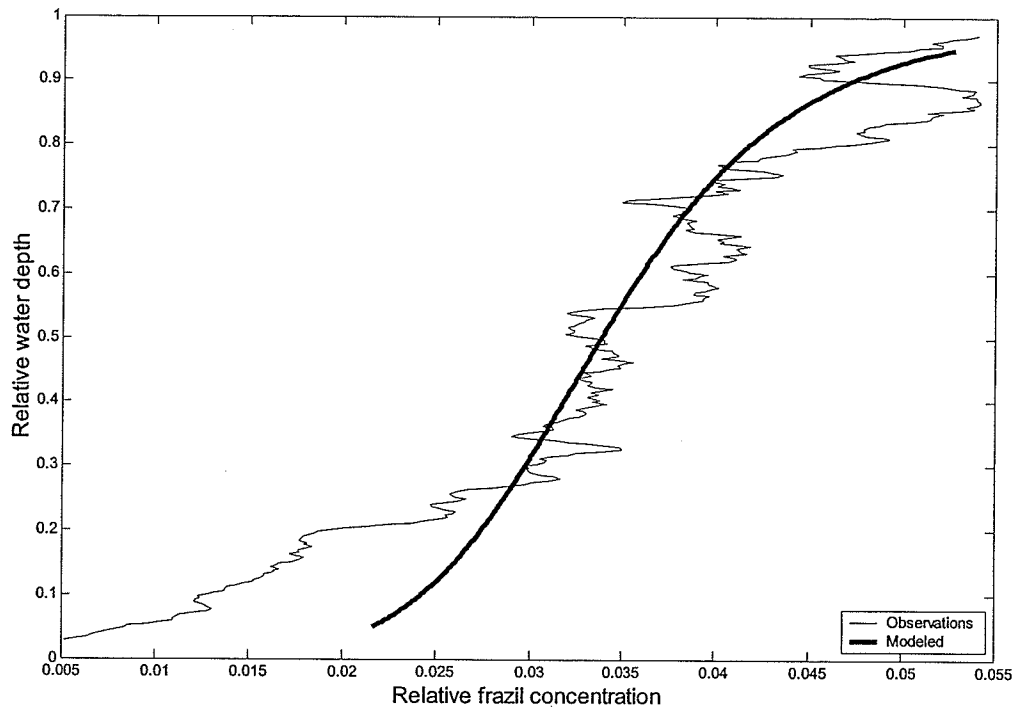


Figure 6.1. Vertical profile of frazil concentration for Exp. 60 at $t = 27$ minutes, $U = 0.424$ m/s.

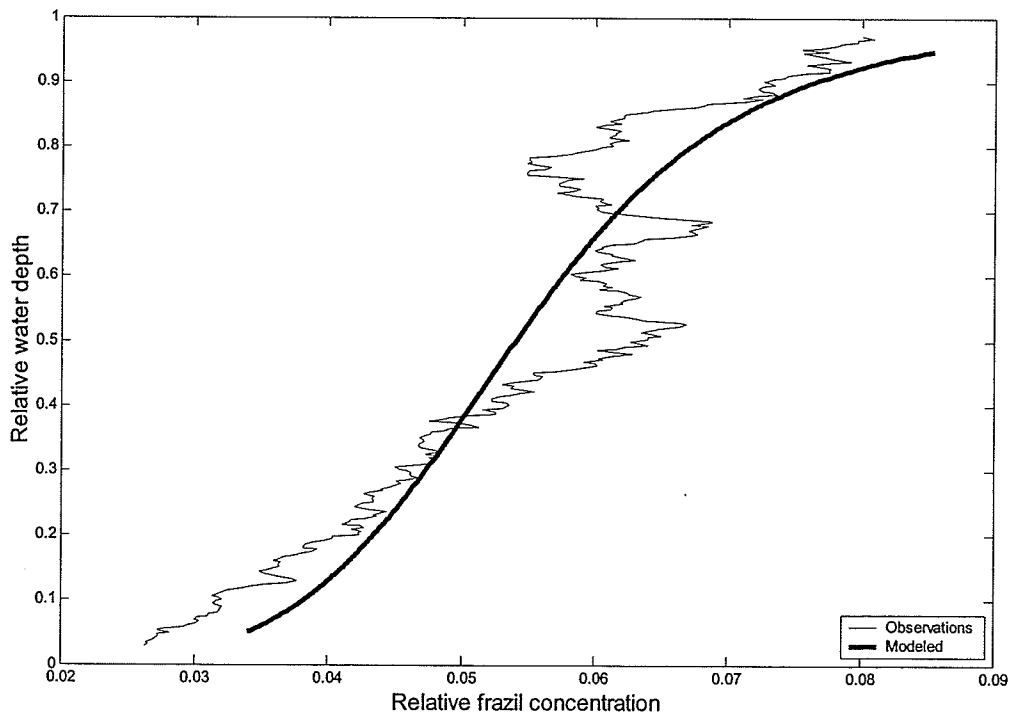


Figure 6.2. Vertical profile of frazil concentration for Exp. 102 at $t = 38$ minutes, $U = 0.429$ m/s.

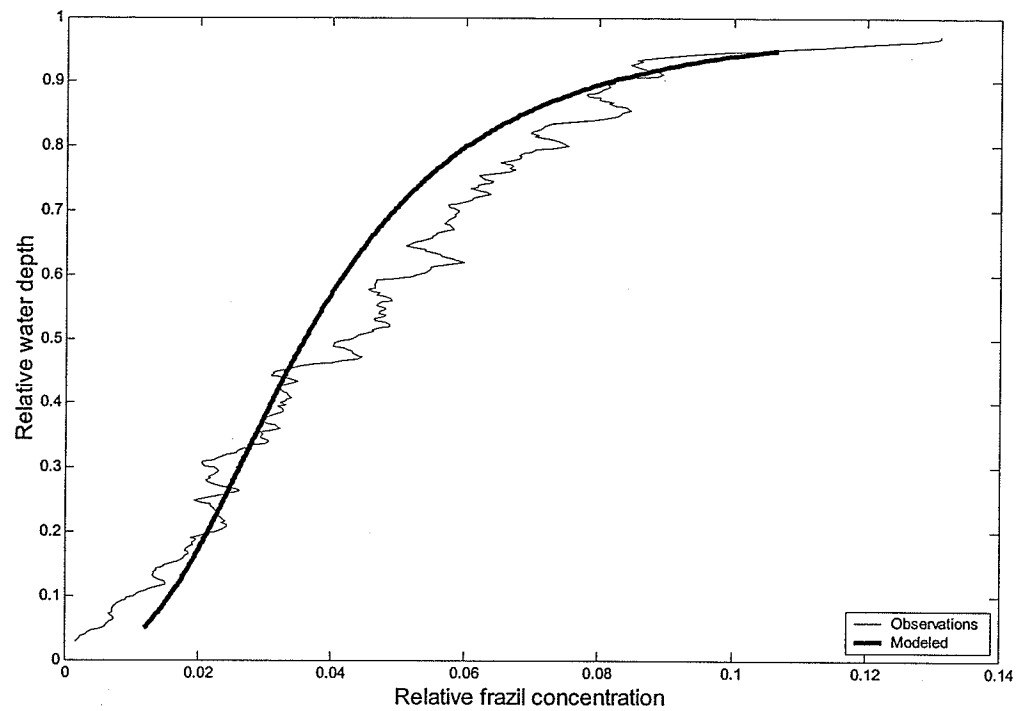


Figure 6.3. Vertical profile of frazil concentration for Exp. 63 at 30 minutes, $U = 0.612$ m/s.

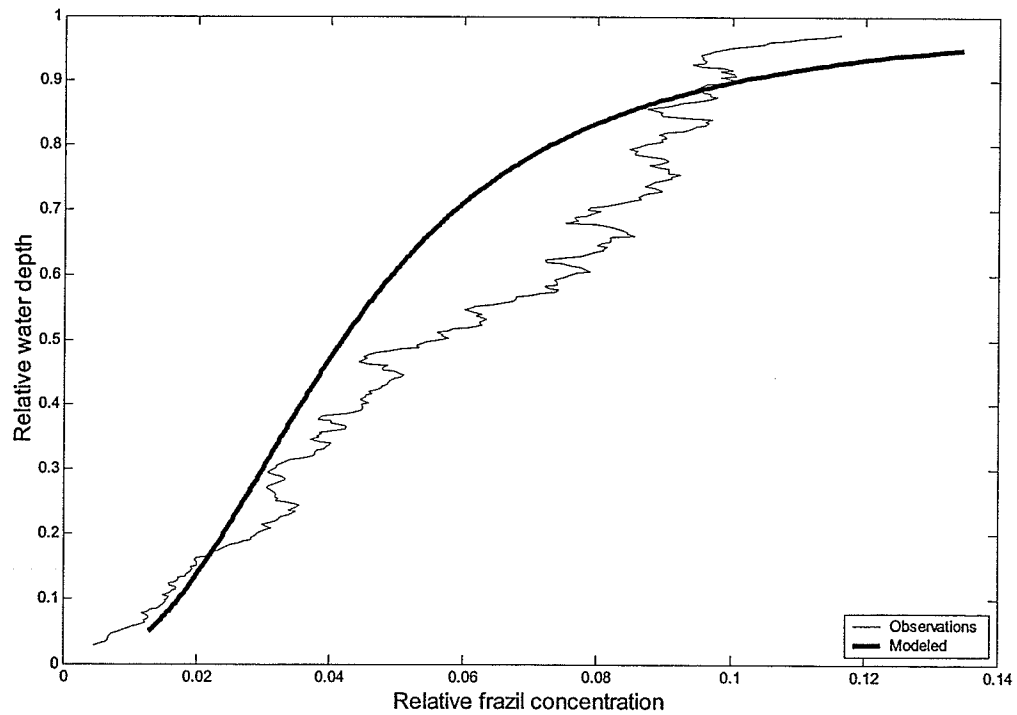


Figure 6.4. Vertical profile of frazil concentration for Exp. 66 at 25 minutes, $U = 0.630$ m/s.

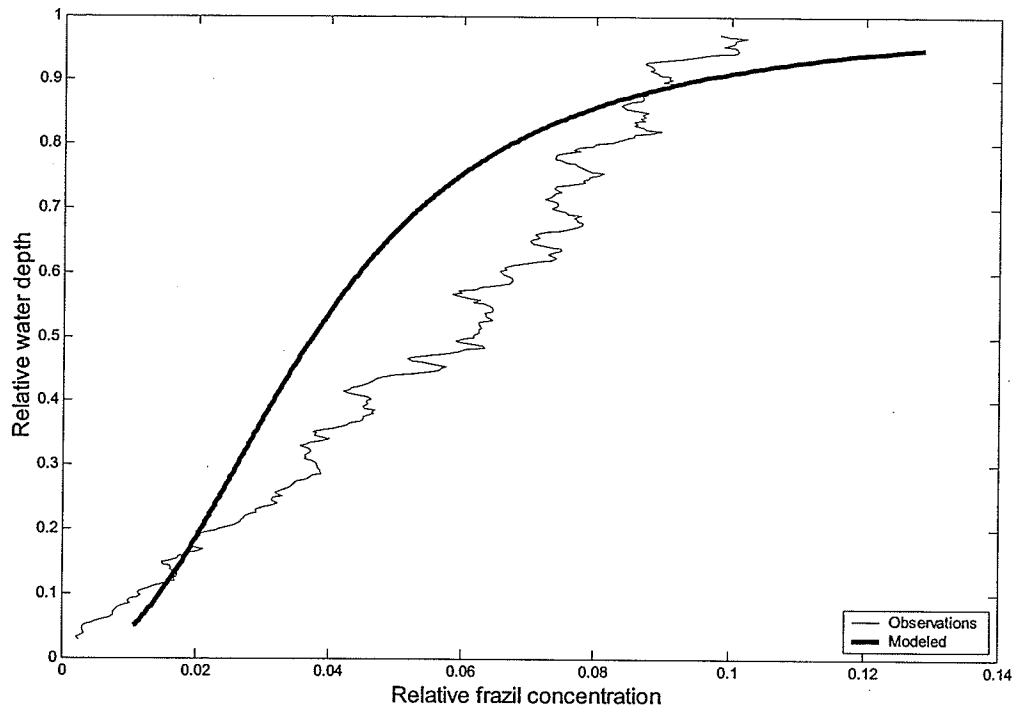


Figure 6.5. Vertical profile of frazil concentration for Exp. 69 at 26 minutes, $U = 0.641$ m/s.

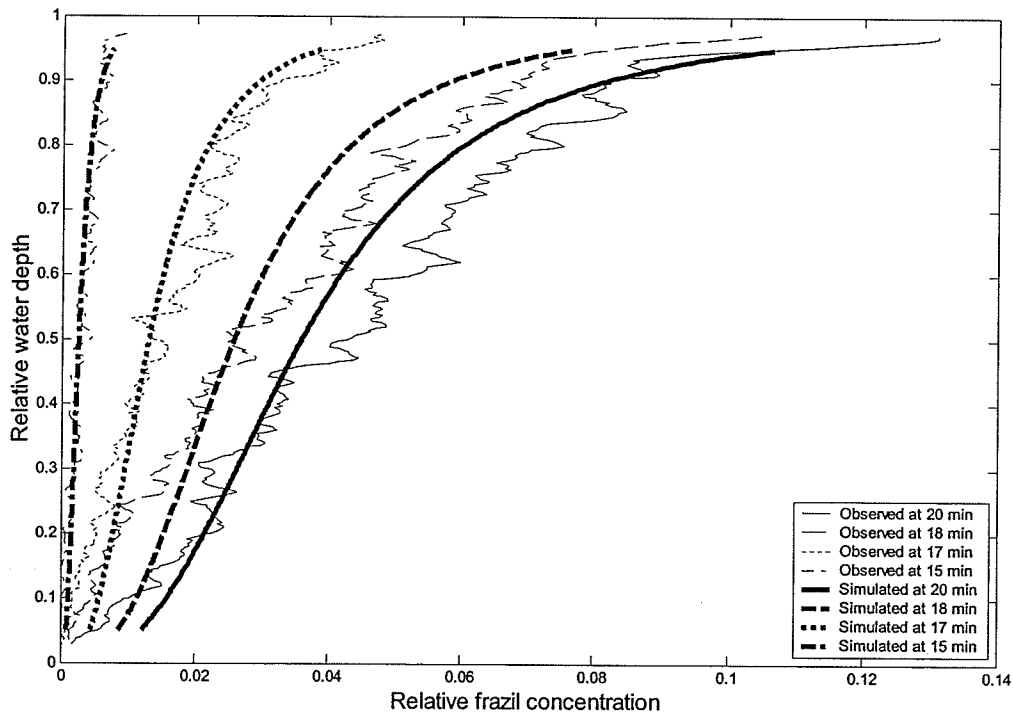


Figure 6.6. Variation of the vertical profile of frazil at various times during supercooling for Exp. 63.

In previous modeling efforts, frazil ice dynamics was typically treated in a way where primarily the temperature response due to frazil ice formation was considered. In these efforts, the time-temperature evolution is the main concern. Furthermore, in order to simulate the supercooling process, a model of evolution had to be assumed a priori for the frazil morphology, as well as a size/distribution of the frazil ice. This chapter describes and tests a new model of the supercooling process based on the work described in the previous chapters.

7.1 Model Formulation

The components of the model consist of flow turbulence, frazil size distribution, rise velocity, entrained frazil generating capacity, frazil growth, heat transfer, and water temperature variation. Experiment 63 is chosen as an example for the description of this model.

7.1.1 Flow Turbulence

Like Mercier (1984) and Hammer and Shen (1995), in a well-mixed flow, the depth-averaged turbulence is estimated herein from the observed flow data. The dissipative rate, $\varepsilon(y)$, can be determined using (4.11). Correspondingly, the depth-averaged value of $\bar{\varepsilon}$ can be determined by

(4.12). The turbulent kinetic energy can be approximated by (4.13) and the depth-averaged value of $\bar{k}$ is approximated by (4.14).

In these calculations, the shear velocity is determined by (4.28). Assuming smooth boundaries, the friction factor f can be determined from Karman-Nikuradse's formula (4.29 or 4.30). The bed roughness coefficient (f) of the counter-rotating flume is estimated using Altshul's formula (4.32).

7.1.2 Distribution of Frazil Size

In Chapter 4, a model to describe the distribution of frazil size was developed based on turbulence in open channel flow. The probability of a given size of frazil disk is given by (4.22 to 4.24). The minimum frazil particle size ($D_{\min}$) is determined using the inner length (4.9). Therefore, based on the hydraulic characteristics of a flow, i.e., width (B), water depth (h), and flow velocity (U), and the bed shear velocity (u_*), the percentage of $D_{\min}$ expected in a frazil sample may be calculated using (4.25). In the same way, the mean frazil size (D_{50}) can be determined using (4.25) when $P(D_{50}) = 0.5$.

7.1.3 Entrained Frazil Capacity of a Flow

In Chapter 5, the concept of the entrained frazil generating capacity of a flow was proposed. It was shown that the entrained frazil capacity of the flow can be related to the flow turbulence intensity and frazil size or volume. A semi-theoretical and semi-empirical relationship (5.5) was proposed to predict the entrained frazil capacity (M_*) of a flow by volume at an air temperature of -10°C . In the determination of this capacity, the rise velocity of a characteristic frazil ice particle (D_{50}) was obtained by (5.2) or (5.3). If the air temperature is not -10°C , (5.6) is used to adjust the capacity.

7.1.4 Frazil Growth during the Supercooling Process

Frazil growth is governed by flow turbulence and heat exchange. It has been determined from the

analysis of frazil images that the distribution of frazil ice size is apparently satisfied during the principal supercooling process. This leads to a mechanism of frazil growth in which D_{50} is constant during the entire period of principal supercooling. The finding that the frazil distribution is apparently satisfied makes it possible to neglect the complicated process of frazil growth. The assumption of a constant D_{50} is not valid when particle sintering and agglomeration become significant.

In this model, the inner length, $D_{\min}(\nu/u_*)$ was adopted for the minimum frazil size. During the entire principal supercooling process, the frazil size distribution is assumed to always follow (4.25) with constant mean frazil diameter (D_{50}) but the number and amount of the various size of frazil particles vary with time. Although the absolute number and volume of a given frazil size change, the relative number and volume of a given size of frazil is invariant. The corresponding instantaneous amount (M_i) of frazil ice at any time can be determined by heat balance.

At any time, the amount of frazil ice is known (M_i). Therefore, based on the known distribution of frazil ice size (4.25), the number of a particular size of frazil ice can be obtained at any time. In this model, the distribution of frazil ice size is computed in multiples of the minimum size $D_{\min}$, i.e.,

$$D_i = iD_{\min}, \quad (7.1)$$

where i is the i -th increment of frazil size. Thus, the probability interval of this size $\Delta P(D_i)$ is calculated using (4.25)

$$\Delta P(D_i) = P(D_i) - P(D_{i-1}) = 2(\Phi(Z_i) - \Phi(Z_{i-1})) \quad (7.2)$$

where $\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^Z \exp\left(-\frac{\zeta^2}{2}\right) d\zeta$, is the standard normal cumulative probability distribution.

In this expression, Z is expressed as

$$Z_i = \frac{\sqrt{\beta_1 D_i}}{\sigma_v} \quad (7.3)$$

and

$$Z_{i-1} = \frac{\sqrt{\beta_1 D_{i-1}}}{\sigma_v}. \quad (7.4)$$

The volume (ΔM_i) of this frazil size (D_i) in a unit volume (1 m^3) is obtained by

$$\Delta M_i(D_i) = M_i \Delta P(D_i). \quad (7.5)$$

This volume of frazil size D_i divided by the volume of single frazil particle, i.e., $K_v \left(\frac{D_i}{2} \right)^3$, yields the number ($N(D_i)$) of frazil with a diameter D_i

$$N(D_i) = \frac{\Delta M_i(D_i)}{V(D_i)} = \frac{M_i \Delta P(D_i)}{K_v \left(\frac{D_i}{2} \right)^3}. \quad (7.6)$$

Consequently, a quantitative distribution of frazil size during the supercooling process can be set up (Figure 7.1), yielding the total number of frazil particles at any time (Figure 7.2).

7.1.5 Frazil Concentration and Heat Transfer Coefficient

Previous research regarding the formation of new frazil ice particles considered thermal growth and secondary nucleation/breakup; two processes that are not well understood (Daly, 1994). As a result, a number of assumptions were made, and several parameters were calibrated. One of the merits of the current model is only the heat balance in a fully mixed volume of water needs to be considered. No matter how the secondary nucleation and breakup processes work, the heat balance must always be satisfied. In other words, the heat released from the water surface must be balanced by the heat

release of water cooling/warming and the latent fusion of the ice. However, the new ice formation is governed by the heat transfer between the existing ice and the supercooled water. Thus, the relationship between the heat transfer coefficient (H_{wi}) and the frazil ice concentration (M_i) has to be setup in advance. Herein, it is assumed that the disc-shaped crystals grow at the edge and the surface and always at thickness to diameter ratio of 1:8. The heat transfer coefficient for each ice-crystal can be estimated by (Daly, 1984)

$$h_{wi} = \frac{k_w N_u A_e}{\beta_3 \eta}, \quad (7.8)$$

where A_e is the area of a frazil disk edge, and k_w is the thermal conductivity of water. N_u is the Nusselt number for the frazil ice particle, which is governed by the frazil geometry (characteristic length, $\bar{r}$), and by the flow turbulent intensities (η and $\bar{k}$, etc.), and can be determined using (1.7) to (1.10) (Daly, 1984). β_3 is a factor related to the bed roughness as suggested in this study. For a smooth bed $\beta_3=1$. For the counter-rotating flume $\beta_3=3$. The characteristic length ($\bar{r}$) is calculated based on the surface and edge areas of the frazil particle, $((A_s + A_e)/4\pi)^{1/2}$ in this study.

The Kolmogorov (η) dissipation length scale is defined as

$$\eta = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}. \quad (7.9)$$

Figure 7.3 shows the relationship between a single frazil particle and its heat transfer coefficient. As shown, h_{wi} gets larger as a frazil particle gets larger. However, the number of small frazil particles exceeds the number of larger particles shown in Figure 7.1 and 7.2. Consequently, smaller frazil ice particles play a more important role in the heat transfer process than larger particles (Figure 7.4).

To simulate the total heat transfer coefficient (H_{wi}) at an instantaneous frazil ice concentration (M_i), the following expression is proposed

$$H_{wi} = \sum_{D_i=D_{\min}}^{D_{i\max}} h_{wi}(D_i)N(D_i). \quad (7.10)$$

Here, $N(D_i)$ is the number of frazil-crystals with a surface diameter D_i , $D_{i\max}$ is the largest instantaneous frazil size. From the above, the curve of $M_i - H_{wi}$ is built up. Due to the assumed frazil growth mechanism, it must be a linear function (Figure 7.5) expressed as

$$H_{wi} = \alpha_H M_i, \quad (7.11)$$

where α_H is a constant that represents the slope of the linear function.

7.1.6 Water Temperature

When the water temperature in a well-mixed 1 m^3 box is simulated from the overall heat balance, the three components of the heat budget, viz., air to water, water itself, and water to ice, must be balanced, i.e.,

$$\frac{d}{dt}(\rho C_p (1 - M_i)T) = -Q_{aw} - Q_{wi}, \quad (7.12)$$

where ρ is the density of water, C_p is the specific heat for water, T is the water temperature, and M_i is the ice concentration. The left hand side of the equation is the heat release/absorption due to water cooling/warming. The first term of the right hand side, Q_{aw} , is the net heat loss per unit

volume at the surface and the second term, Q_{wi} , is the heat transfer to ice from water, which is given by

$$Q_{wi} = H_{wi}(T_i - T) = -a_H M_i T. \quad (7.13)$$

T_i is the ice temperature (0°C). Q_{wi} also determines the formation of new frazil ice particles, i.e.,

$$\frac{dM_i}{dt} = \frac{H_{wi}(T_i - T)}{\rho_i L_i} = \frac{Q_{wi}}{\rho_i L_i}. \quad (7.14)$$

ρ_i is the density of ice and L_i is the latent of fusion of ice. In general, the heat exchange between the water surface and air (Q_{aw}) is governed by (Hanley and Michel, 1975, 1977)

$$Q_{aw} = \phi(T_a - T), \quad (7.15)$$

where ϕ is the heat transfer coefficient expressed in $\text{W/m}^2\text{C}$, which is very sensitive to wind speed, but is a constant for a given air temperature and wind speed. However, Q_{aw} can also be calculated based on the cooling rate of water

$$Q_{aw} = \rho C_p \frac{dT}{dt}. \quad (7.16)$$

dT/dt is the cooling rate measured from the $t-T$ curve. dT/dt and Q_{aw} can be considered constants during the supercooling process. Therefore, the water temperature and ice formation are governed by equations (7.12) and (7.14), respectively.

7.1.7 Computational Scheme

The set of equations (7.12, 7.13, and 7.14) can be solved by a simple explicit numerical method, such as Euler or a second-order Runge-Kutta technique. Both methods yield similar simulations when a time step of 1 second is used. However, for larger time step, such as 5 seconds, a second-order Runge-Kutta method is much more stable and accurate than Euler. As a result, Euler's method with a 1 second time step is used in the numerical model.

The initial ice concentration is determined by an optimization scheme designed to match the lowest water temperature. An initial ice concentration is assumed in advance, and then the lowest simulated water temperature is compared with the measured maximum supercooling. By trial and error, when the discrepancy between these water temperatures is less than 0.0001 °C, the corresponding value of initial ice concentration is considered to be optimized.

7.1.8 Vertical Frazil Ice Distribution

In the simulation, a frazil growth mechanism is assumed based on experimental findings and theoretical development. D_{50} is assumed to be constant during the principal supercooling process. This finding makes it possible not only to neglect the complexities of the frazil growth process, but also to develop a model for the vertical distribution of frazil ice.

In Chapter 6, a vertical distribution model of frazil was developed. At any time during the principal supercooling process, when the frazil ice concentration (M_i) is known, the position of the corresponding mean concentration, $\bar{C}$, at a point, $\bar{\xi} = \bar{y}/H$, are determined by (6.24) and (6.23), respectively. Therefore, the vertical frazil ice distribution C could be described using (6.25). an example is shown in Figure 7.6

7.2 Descriptions of Tests Simulated

The numerical model was included in the data post-processing procedure. Using this procedure, the cooling rate and hydraulic parameters are automatically obtained from the experimental variables.

The initial ice concentration is calibrated using an optimization scheme in order to best match the water temperature curves. The model is readily able to simulate the characteristics of the supercooling process and frazil ice evolution. The simulation takes about 10 minutes on a PIII-1.7 GHz computer.

In order to demonstrate the applicability of this model, five HRTF experiments are selected to verify the numerical model. The input data and parameters are presented in Table 7.1. All the experiments will be verified by the water temperature history. Exp. 60 and 71 will be used to compare frazil ice number and area concentrations obtained from image analysis. Exp. 73, 84 and 92 are examples of experiments with a different water depth and/or a different air temperature.

Table 7.1. Input parameters of tests used for simulations.

		Ta [°C]	U [m/s]	B [m]	H [m]	ks [m]	dT/dt [°C/min]
HRTF (2002)	Exp. 63	-10	0.612	0.2	0.15	0.005	-0.0038
	Exp. 71	-10	0.41	0.2	0.15	0.005	-0.0037
	Exp. 73	-10	0.491	0.2	0.10	0.005	-0.0045
	Exp. 84	-10	0.486	0.2	0.20	0.005	-0.0037
	Exp. 92	-15	0.500	0.2	0.15	0.005	-0.0060
Carstens (1966)	Case I	-10	0.5	0.2	0.2	0	-0.02
	Case II	-10	0.33	0.2	0.2	0	-0.0078

Table 7.2. Characteristics of flow turbulence and frazil size.

		Re	f	u* [cm/s]	$\bar{k}$ [m ² /s ²]	$\bar{\varepsilon}$ [m ² /s ³]	D _{min} [cm]	D ₅₀ [cm]
HRTF (2002)	Exp. 63	82054	0.042	4.44	0.0033	0.0106	0.0040	0.84
	Exp. 71	57621	0.042	3.31	0.0016	0.0035	0.0057	0.41
	Exp. 73	54811	0.044	3.65	0.0022	0.0080	0.0049	0.56
	Exp. 84	72440	0.041	3.49	0.0020	0.0039	0.0051	0.84
	Exp. 92	67079	0.042	3.64	0.0022	0.0056	0.0049	0.56
Carstens (1966)	Case I	74488	0.019	2.45	0.0010	0.0013	0.0073	0.25
	Case II	49162	0.021	1.70	0.0005	0.0004	0.0105	0.12

Like previous modelling efforts, e.g., Svensson and Omstedt (1994) and Hammer and Shen (1995), the new model is also validated using experimental data from Carstens (1966) in order to compare the principles behind these models. The two well-known Carstens' experiments are selected in the present simulations. The first case (denoted Case I herein) corresponds to Case A in figure 6 (Carstens, 1966), while the second (Case II) corresponds to figure 7 (Carstens, 1966). His frazil ice experiments were conducted in a recirculating flume, 20 cm wide by 30 cm deep by 600 cm long, located in a cold room with an air temperature of -10°C . The water was 20 cm deep and was circulated by a variable speed propeller. The bottom and sides of the acrylic flume were insulated, and cooling of the water was aided by a fan blowing along the straight portion of the flume. Water temperature was measured at a point located 0.15 to 0.10 m below the water surface. Due to the mixing effect of the propeller, the vertical temperature gradient was found to be negligible. Consequently, the well-mixed condition will be imposed in the numerical model for comparison with the experimental data.

Table 7.3. Frazil size, number and entrained capacity of the flows.

		R [m]	D_{50} [cm]	ω [cm/s]	$\frac{U^3}{gR\omega}$	M_v [%]	Frazil [$10^8/\text{m}^3$]	M_{i0} [%]
HRTF (2002)	Exp. 63	0.06	0.84	6.66	5.84	0.106	71.12	0.5×10^{-4}
	Exp. 71	0.06	0.41	1.99	6.76	0.131	51.76	1.6×10^{-4}
	Exp. 73	0.05	0.56	3.40	7.08	0.139	69.65	1.9×10^{-4}
	Exp. 84	0.067	0.84	2.89	6.08	0.112	52.32	1.8×10^{-4}
	Exp. 92	0.06	0.56	3.37	6.31	0.152	72.35	4.5×10^{-4}
Carstens (1966)	Case I	0.067	0.25	2.46	7.77	0.16	5.78	3.5×10^{-4}
	Case II	0.067	0.12	0.098	5.60	0.099	2.04	3.5×10^{-4}

The turbulence parameters (depth-averaged kinetic energy $\bar{k}$ and dissipation rate $\bar{\varepsilon}$ shown in Table 7.2) are calculated from the measured flow data (U , B , and h), and the hydraulic parameters (R_e , f , and u_*). In Table 7.2, the characteristic frazil particle diameters ($D_{\min}$ and D_{50}) are also presented.

In Table 7.3, the initial ice concentrations (M_{i0}) are determined by the optimization scheme. These values shown are very small and are considered as impurities in the water. The water used in the

experiments is Winnipeg tap water, which does not pass through a water treatment plant. It is therefore normal to have small particulate matter suspended in the water.

Figure 7.7 shows the relationship between the Reynolds number and the optimized value of the initial ice concentration. This line regressed indicates that the value of initial ice concentration is inversely related to the flow turbulence. The high value associated with of Exp. 92 indicates that the initial ice concentration may be related to air temperature as well.

The difference between the approach herein and previous modelling efforts is the initiation of frazil ice formation. Here frazil starts to form at the freezing point (0°C) as a result of the heat exchange between the supercooling water and the existing initial ice particles. One of the previous researchers assumed a seeding process where tiny ice particles dropped into the water even though the water was not supercooled (Osterkamp and Gosink, 1983); a mechanism resulting from the water vapor that forms ice particles in cold air. This mechanism was applied in the modelling of Mercier (1984) and Hammer and Shen (1995). Another mechanism initiates frazil a short period after the water is supercooled by manually seeding with a piece of ice (Ettema et al., 1984; Daly and Cobeck, 1986; Tsang, 1982).

Another explanation of the initial ice is nucleation at the water surface. The temperature at which ice nucleates in the water can be greater than 0°C (Osterkamp, 1978). In our experiments spicules of surface ice have been seen when the water temperature is 0.07°C , and small frazil disks have been seen at 0.05°C (Clark and Doering, 2002). These observations were made during experiments with air temperatures of -10°C or -15°C . It is important to note that the thermometer is located 6 cm from the bottom of the flume, while typical water depths range from 10 to 20 cm. Since the heat loss is from the surface, the water temperature must be slightly cooler near the surface even though the water is typically well mixed by the counter-rotating motion.

There is generally also intermittent border ice along the flume walls that develops during an experiment. It is not clear whether the border ice acts as a seeding material or initial ice for the frazil formation, but this seems likely. There is no deliberate seeding of the water. It is not clear whether mass exchange processes originating in the water, such as splashing, bubble bursting etc. causes the frazil ice nucleation, although since particles can be seen first at the surface this is also possible.

7.3 Verification of Simulations

The temperature variations (Figures 7.8 to 7.12) for validated experiments are simulated using the model described above. The comparisons proved the applicability of the model in engineering practice.

An interesting finding in these simulations is the frazil growth rate is constant after the principal supercooling process, i.e., the residual supercooling. This can be explained by (7.14). At the end of the supercooling process the frazil reaches its capacity of the flow while the heat transfer coefficient (H_{wi}) reaches its maximum constant value ($H_{wi}^{\max}$). The residual supercooling (T_r) maintains a constant value if the isolation effect of surface ice is ignored, thus,

$$\frac{dM_i}{dt} = \frac{T_r H_{wi}^{\max}}{\rho_i L_i} = \frac{Q_{aw}}{\rho_i L_i}, \quad (7.17)$$

and the residual water temperature may be approximated as

$$T_r = \frac{Q_{aw}}{H_{wi}^{\max}}. \quad (7.18)$$

Table 7.4. Residual temperatures compared to offsets.

		U [m/s]	M_v [%]	$H_{wi}^{\max}$ [$\times 10^4 \text{ W m}^{-2} \text{ } ^\circ\text{C}$]	dM_i/dt [$\times 10^{-7}/\text{s}$]	T_{off} [$^\circ\text{C}$]	T_r [$^\circ\text{C}$]
HRTF (2002)	Exp. 63	0.612	0.106	6.80	8.75	-0.0114	-0.0039
	Exp. 71	0.430	0.131	7.15	8.53	-0.0136	-0.0036
	Exp. 73	0.491	0.139	8.31	10.10	0.0059	-0.0037
	Exp. 84	0.486	0.112	6.34	8.39	-0.0101	-0.0040
	Exp. 92	0.500	0.152	8.82	13.6	0.0052	-0.0047
Carstens (1966)	Case I	0.50	0.16	22.8	45.6	-0.0052	-0.0062
	Case II	0.33	0.099	11.76	27.5	-0.0085	-0.0080

The residual temperatures of the experiments are shown in Table 7.4. The simulations yield comparable values to Carstens' experiments, but different from the counter-rotating flume values that possess unstable offsets due to difficulties described in Chapter 3, therefore, the simulations of the supercooling processes for these experiments used the simulated residual temperature instead of the measured offsets.

Figures 7.13 shows that the number of frazil ice particles increase with time. The simulations are comparable to observations. Note the present model can only simulate the principal supercooling stage. One interesting finding is the concentration of frazil ice increases very slowly at the beginning of the supercooling process until the maximum supercooling (lowest water temperature) occurs. After the maximum supercooling, the growth rate of frazil ice accelerates. The peak rate of growth is between $t_{\min}$ and t_r . This finding is also confirmed by other laboratory observations (Michel 1963; Carstens, 1966) and field measurements (Osterkamp et al., 1983).

The digital image processing system is under development. The minimum frazil size that the system can distinguish depends on many factors that are hard to be quantitatively determined. The minimum frazil particle the DIPS can see is reasonably considered to be about 2 mm. Based on this, the simulated and observed number of frazil ice particles are quite comparable (Figure 7.14 and 7.15).

Table 7.5 shows a comparison of the number of frazil particles and the amount of frazil for the three experiments. All three experiments were conducted at an air temperature of -10°C and water depth of 0.15 m. The simulation shows the tendency for the number of frazil particles to increase for higher water velocities, as well as the size of the frazil particles (measured by D_{50}) to get larger, while the capacity gets smaller.

Table 7.5. Comparison of frazil ice particles number and capacity

	U	observed	Particles (>2 mm)	Total	M_v	M_*	D_{50}
	m/s	Particles/image	$10^3/\text{m}^3$	$10^8/\text{m}^3$	%	%	cm
Exp. 67	0.393	25	28	48	0.131	0.138	0.34
Exp. 71	0.43	47	44	52	0.133	0.131	0.41
Exp. 63	0.612	76	55	71	0.105	0.106	0.84

7.4 Sensitivity Analysis and Discussion

To examine the sensitivity of the process to various model parameters, additional simulations were performed for Exp.63. Figures 7.17, 7.18 and 7.19 show the effects of different parameters. A larger initial ice concentration results in a faster water temperature recovery (Figure 7.17). The initial values can vary within a reasonable range during the experiment.

As expected, a larger heat loss rate (cooling rate) leads to a higher degree of supercooling and a faster temperature recovery (Figure 7.18).

In this model, turbulence intensity is related to the bottom shear velocity, which determines the frazil size distribution and capacity of the flow as well as the heat transfer coefficient between the frazil ice particles and the water. Table 7.6 shows that the turbulence greatly affects the frazil size distribution and flow capacity, and the residual temperature, but has only a slight effect on the water temperature history (Figure 7.19).

Table 7.6. Effect of turbulence intensity to frazil size and capacity.

Run	u^* [cm/s]	D_{min} [cm]	D_{50} [cm]	M_v [%]	T_r [°C]
1	4.44	0.0040	0.84	0.106	-0.0039
2	4.00	0.0045	0.68	0.18	-0.0025
3	4.89	0.0037	1.01	0.066	-0.0061

The mathematical modeling of frazil ice dynamics includes many intricate physical processes, many of which are not well understood (Svensson and Omstedt, 1994). Compared with other simulations (Mercier, 1984; Svensson and Omstedt, 1994; Hammer and Shen, 1995) this mathematical formulation, based on a new approach, is fairly simple. However, it is the first attempt to relate the flow hydraulics, turbulence, and frazil size/distribution, thereby giving a clear physical meaning to the supercooling process. A two-layer frazil model provides a new way to consider the frazil transportation in river and accretions on the bed and the vertical frazil profile and evolution.

This new numerical model avoids having to simulate secondary nucleation and flocculation processes, as well as the associated calibration required for some coefficients. The model reproduces some important experimental findings, in particular, the time/temperature history during supercooling and ice formation. This modeling approach is promising, and together with more experimental data on crystal size distribution can easily be further developed. It is suggested that the present model is considered as a synthesis of experiments. It is then expected that improved understanding of frazil ice dynamics can be achieved.

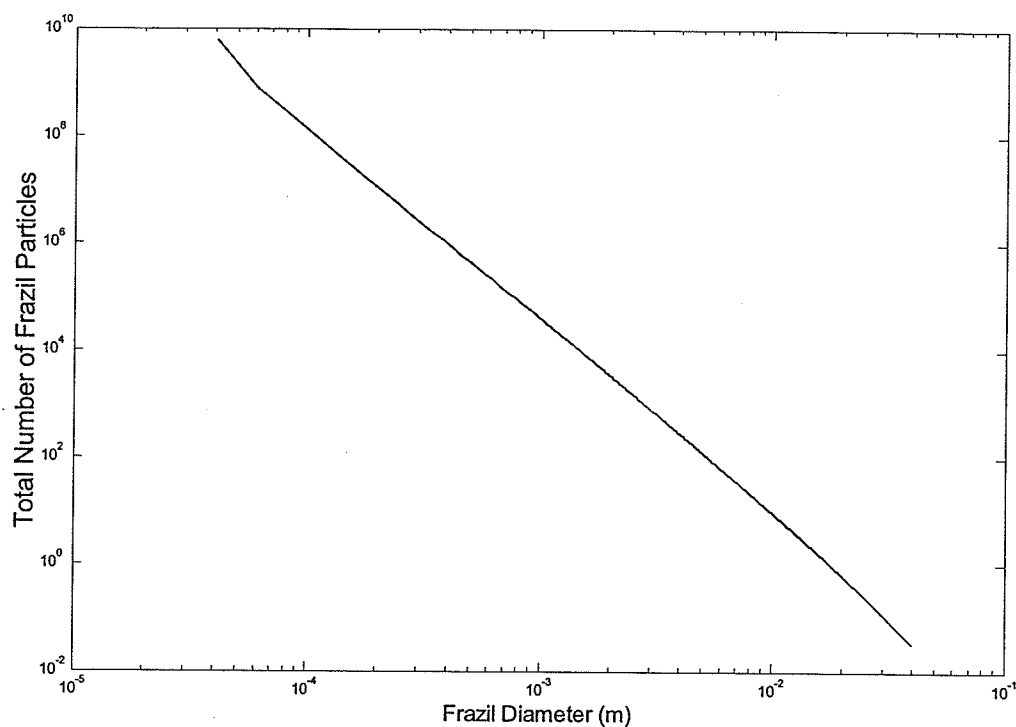


Figure 7.1. Frazil size and number at the saturated state (Exp. 63).

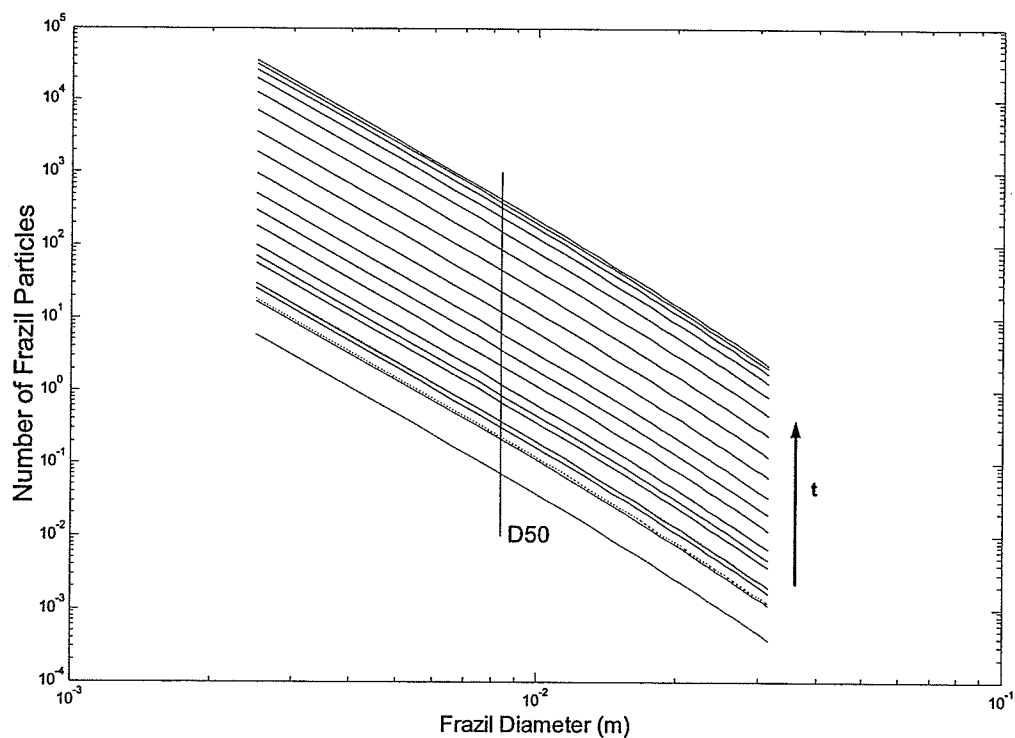


Figure 7.2. Frazil growth in Exp. 63.

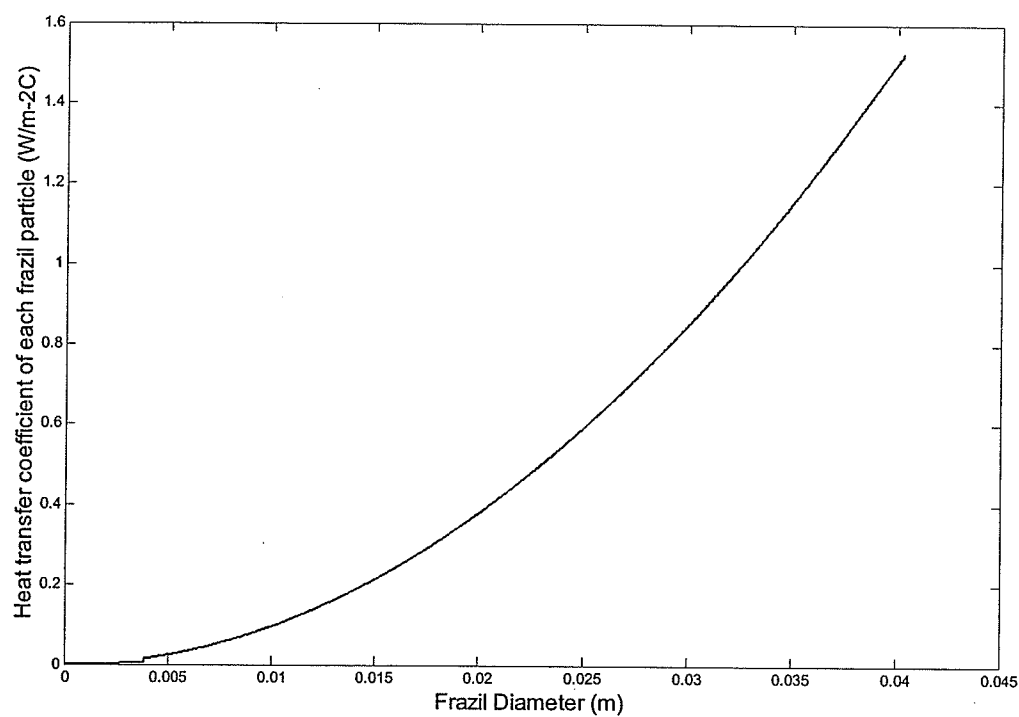


Figure 7.3. Heat transfer coefficient of each frazil particle.

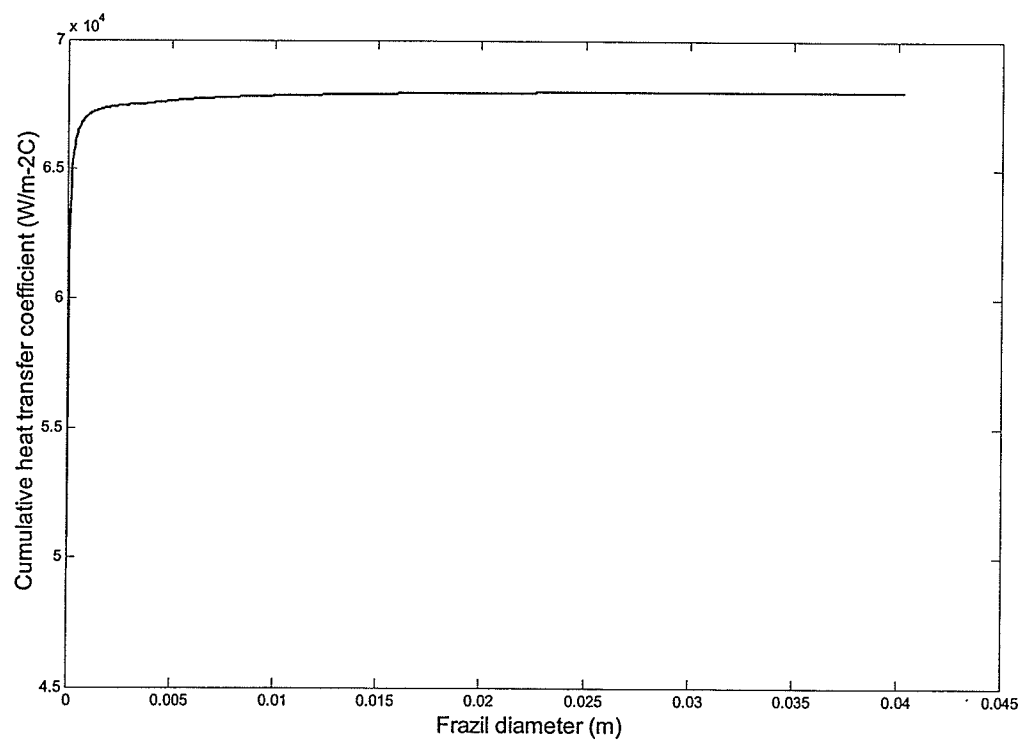


Figure 7.4. Cumulative heat transfer coefficient of frazil particle.

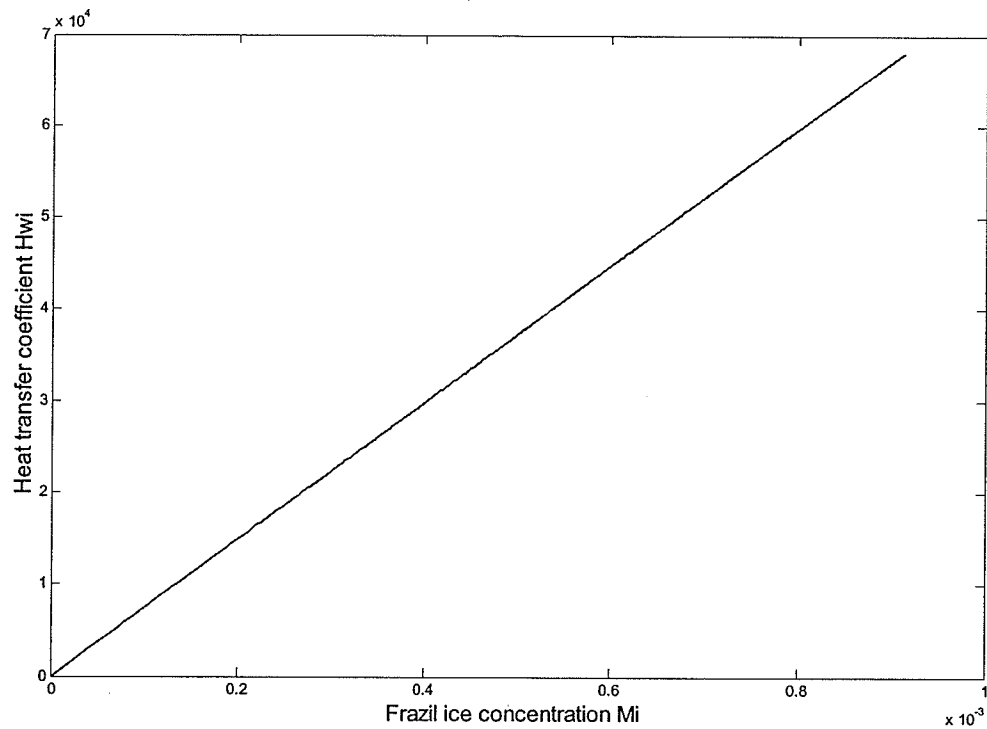


Figure 7.5. Linear M_i vs. H_{wi} relationship for Exp. 63

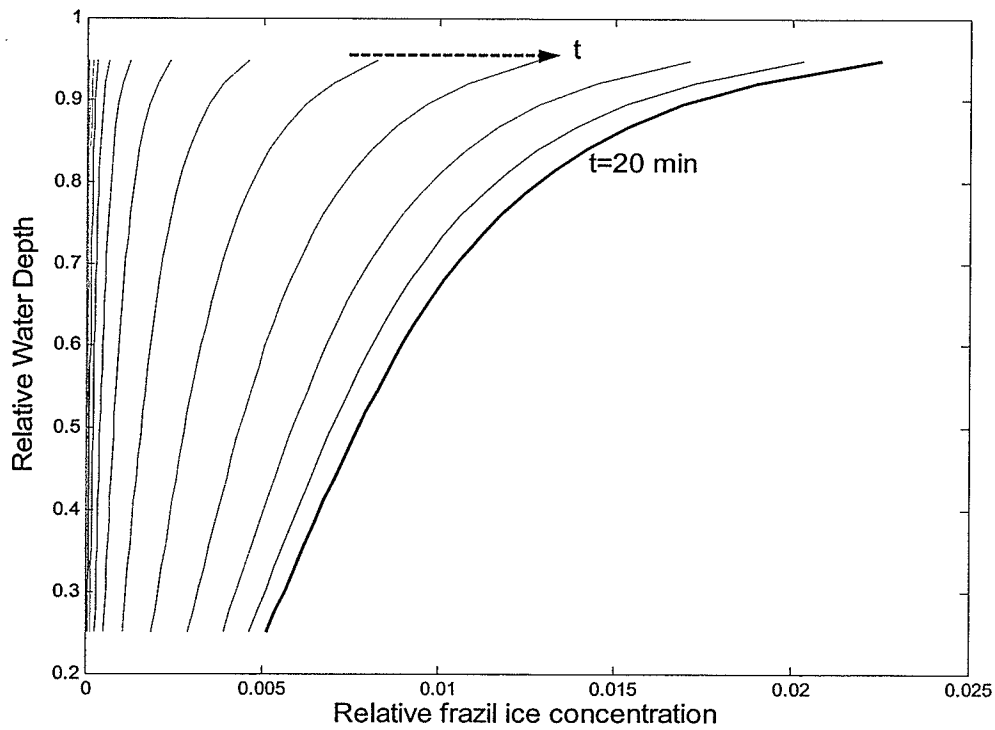


Figure 7.16 Simulation of vertical profiles of frazil concentration for Exp. 63

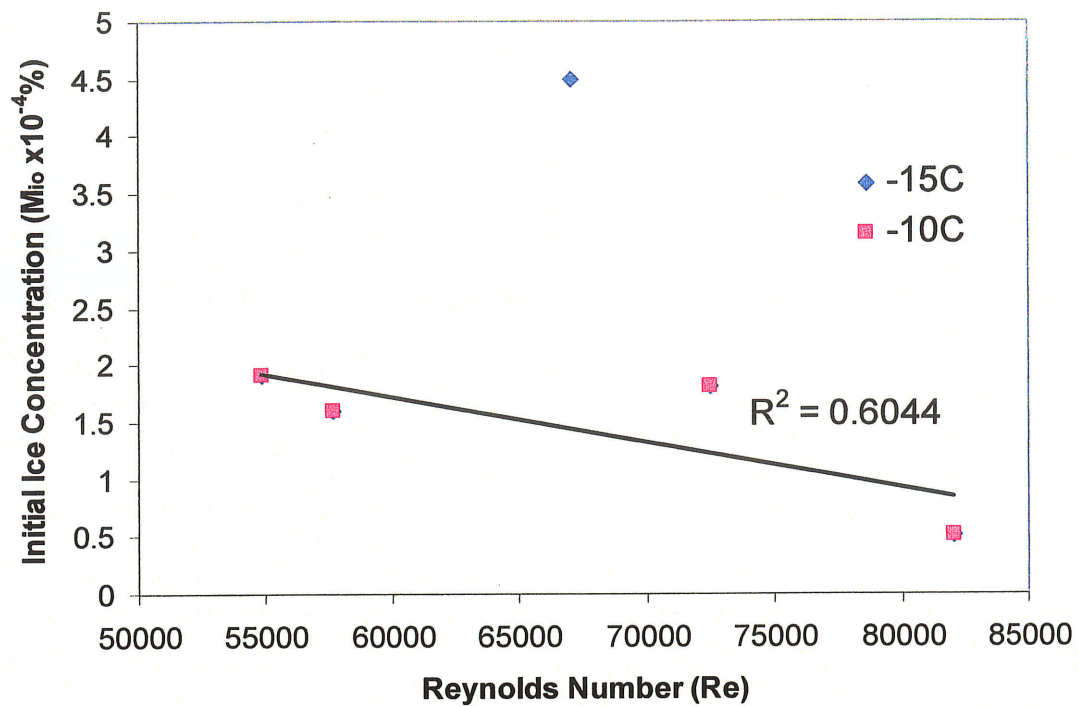


Figure 7.7. Reynolds number vs. initial ice concentration

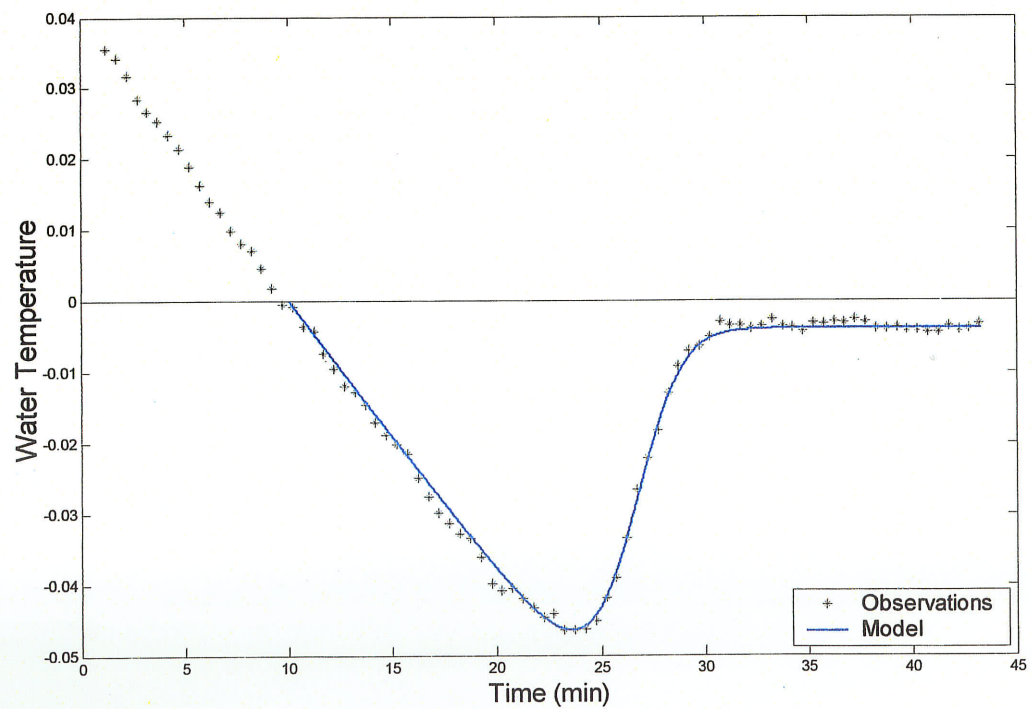


Figure 7.8. Simulation of Exp. 63.

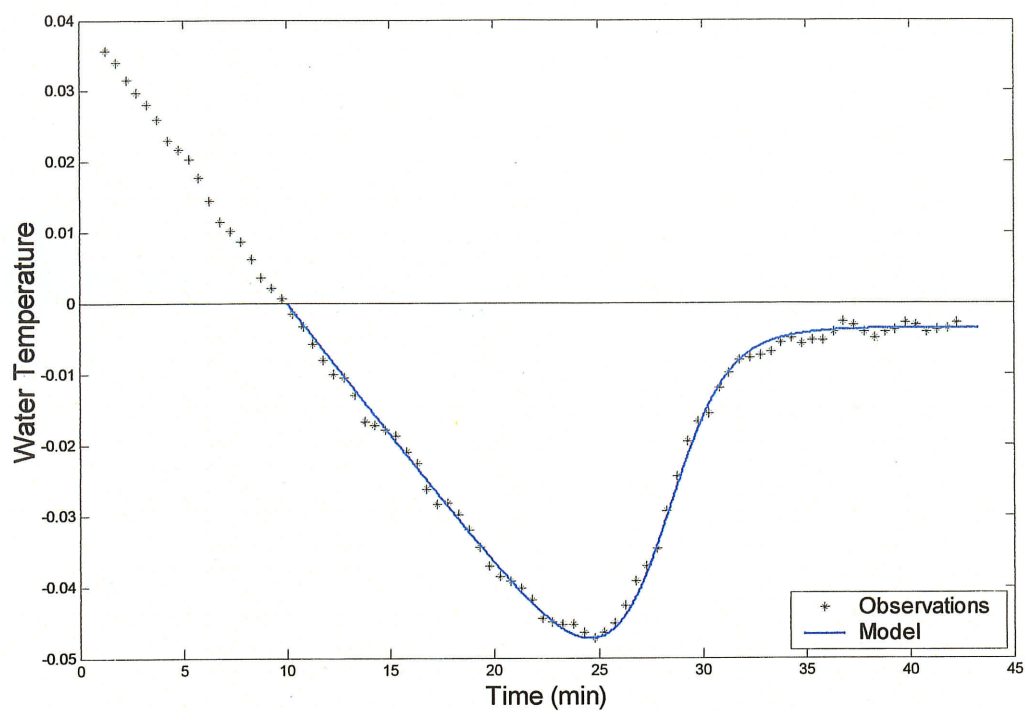


Figure 7.9. Simulation of Exp. 71.

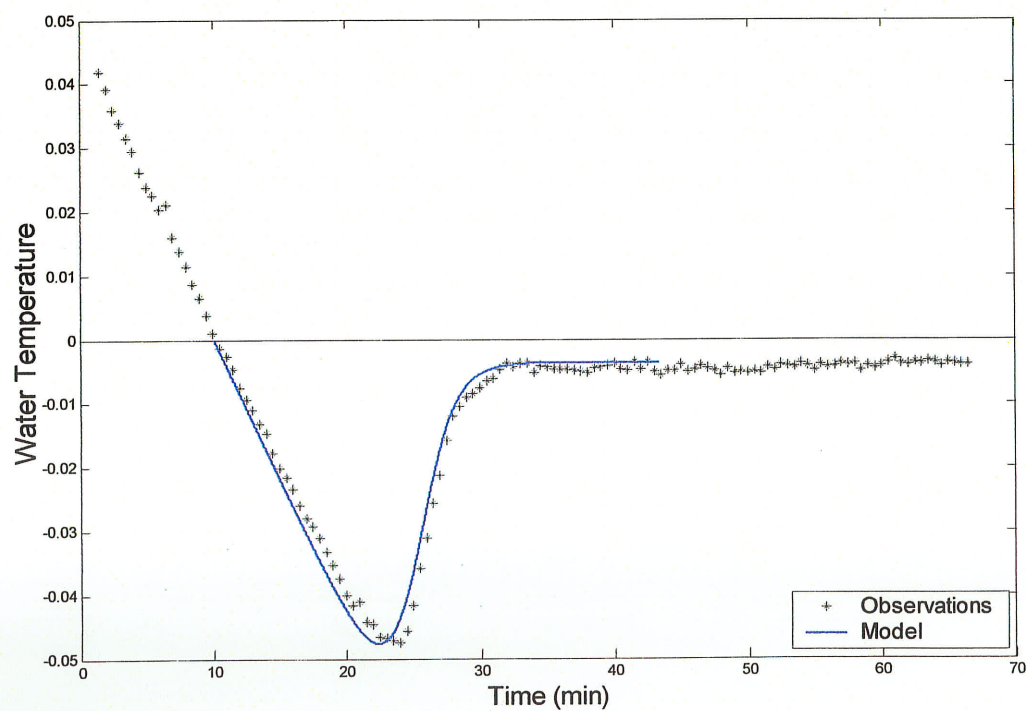


Figure 7.10. Simulation of Exp. 73.

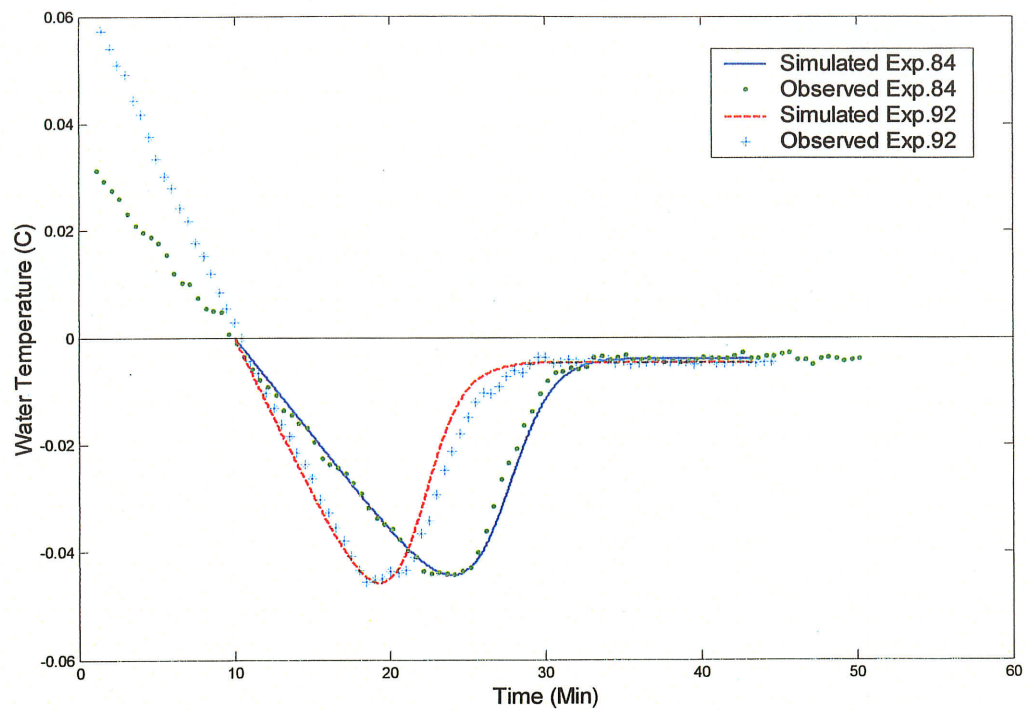


Figure 7.11. Simulation of Exp. 84 & 92.

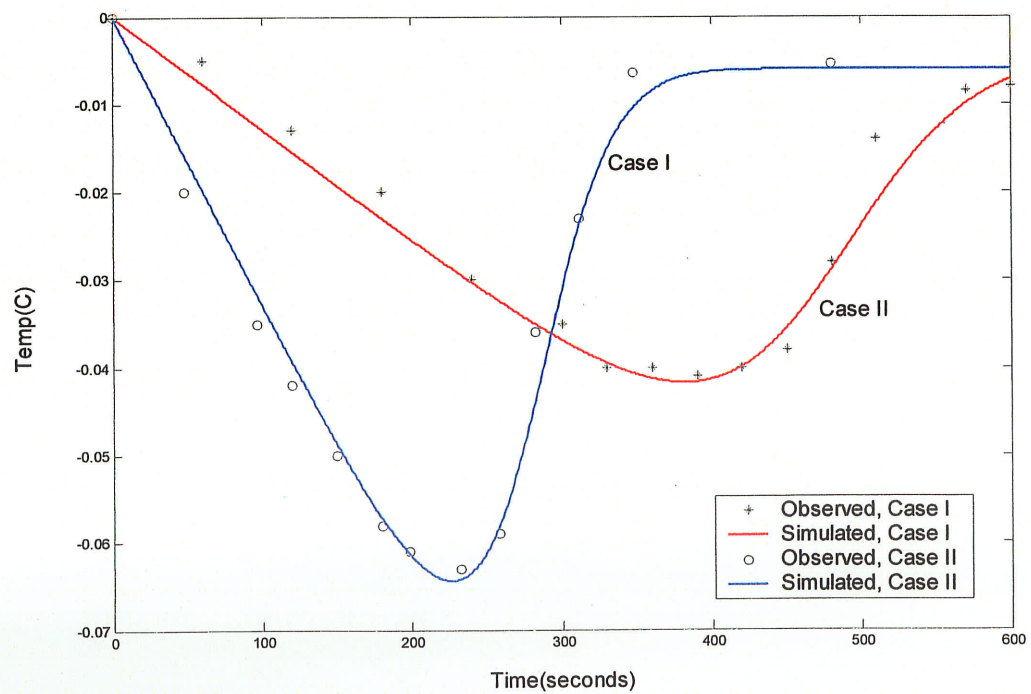


Figure 7.12. Simulations of two Carstens' experiments.

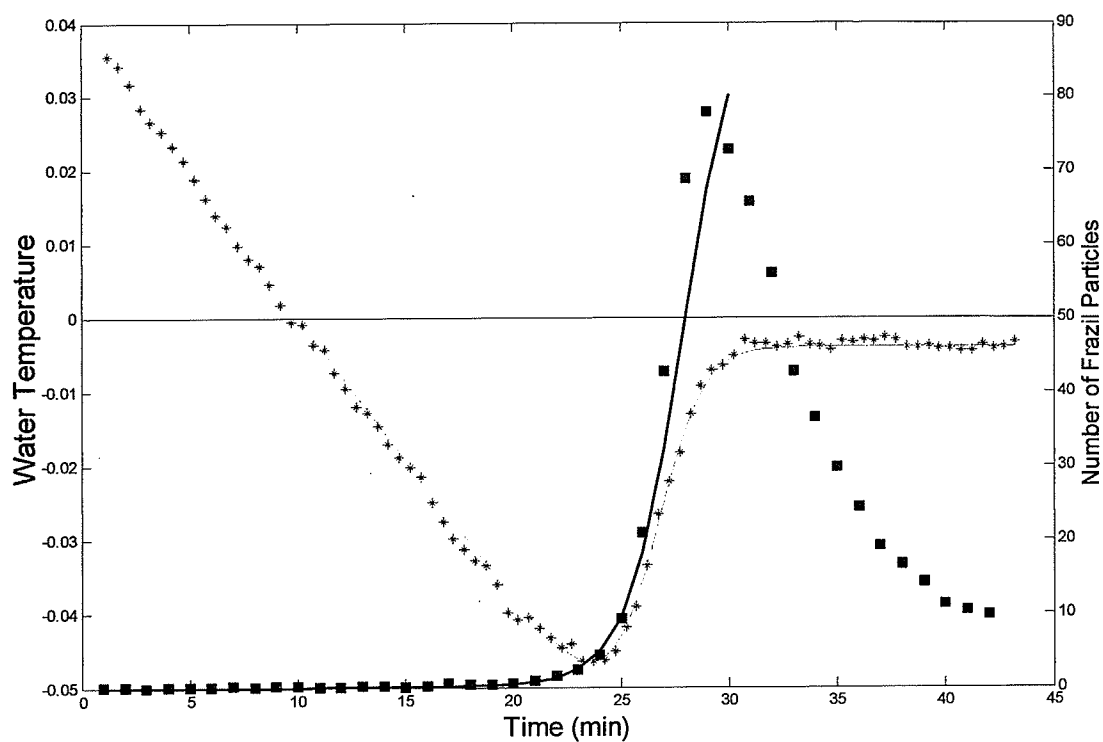


Figure 7.13. Frazil growth observation in Exp. 63.

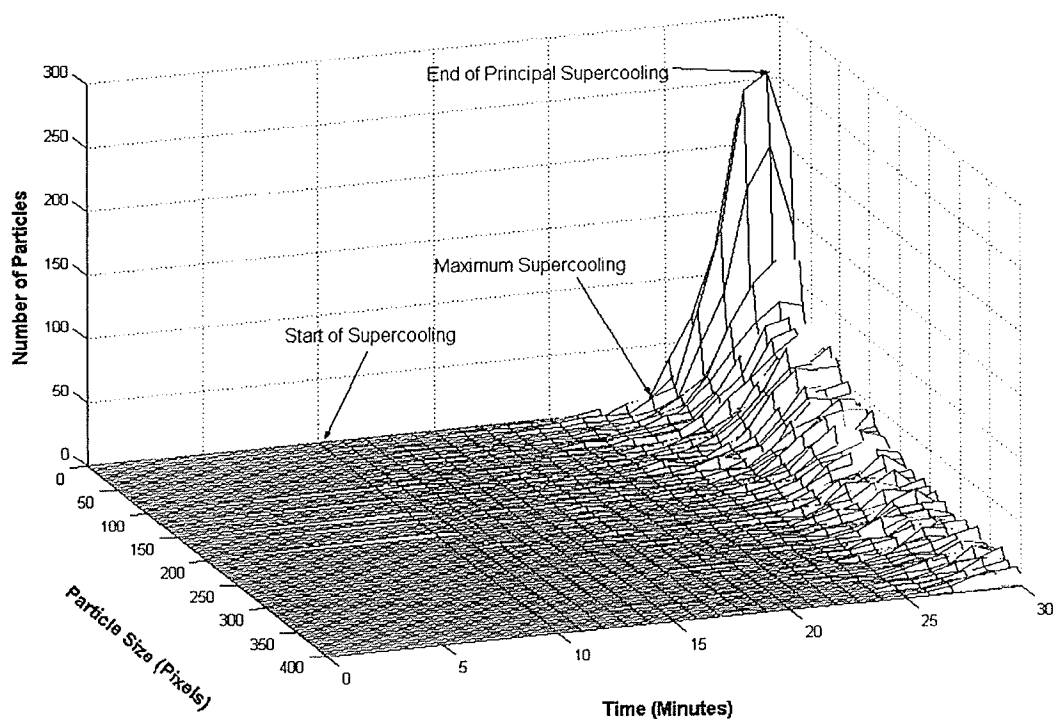


Figure 7.14. Observed 3-D frazil distribution for Exp 63.

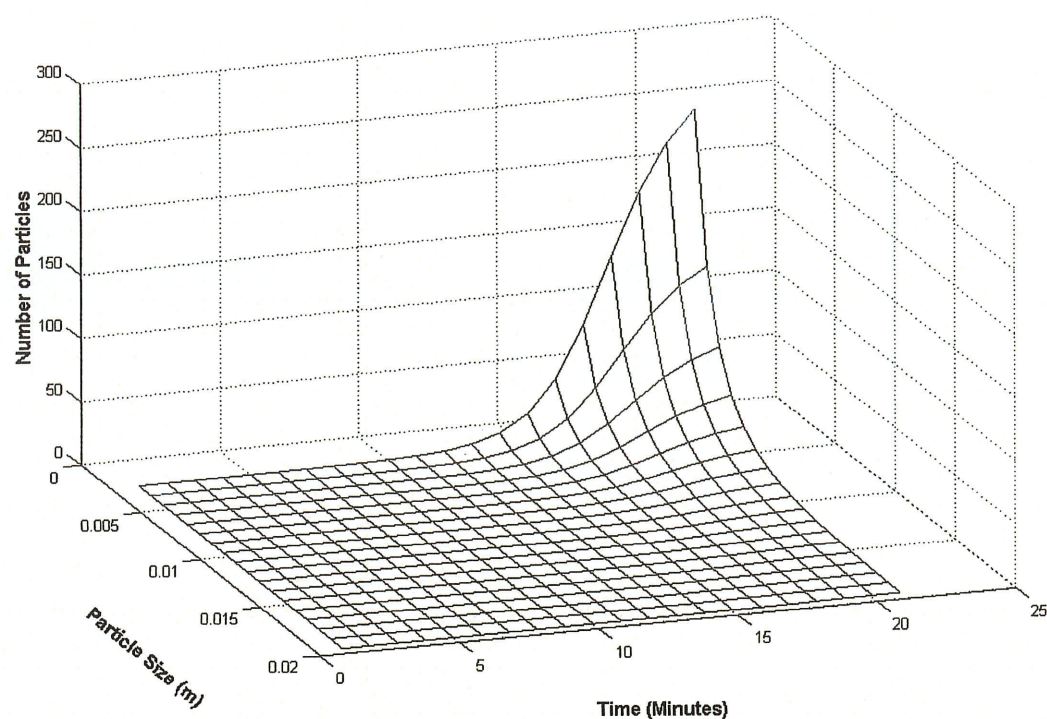


Figure 7.15. Simulated 3-D frazil distribution for Exp 63.

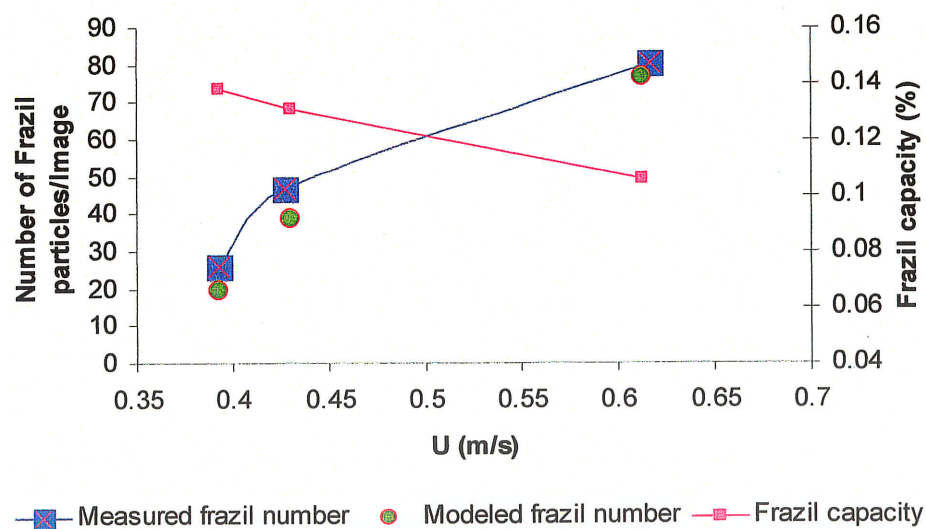


Figure 7.16. Comparison of frazil particle numbers and frazil capacities.

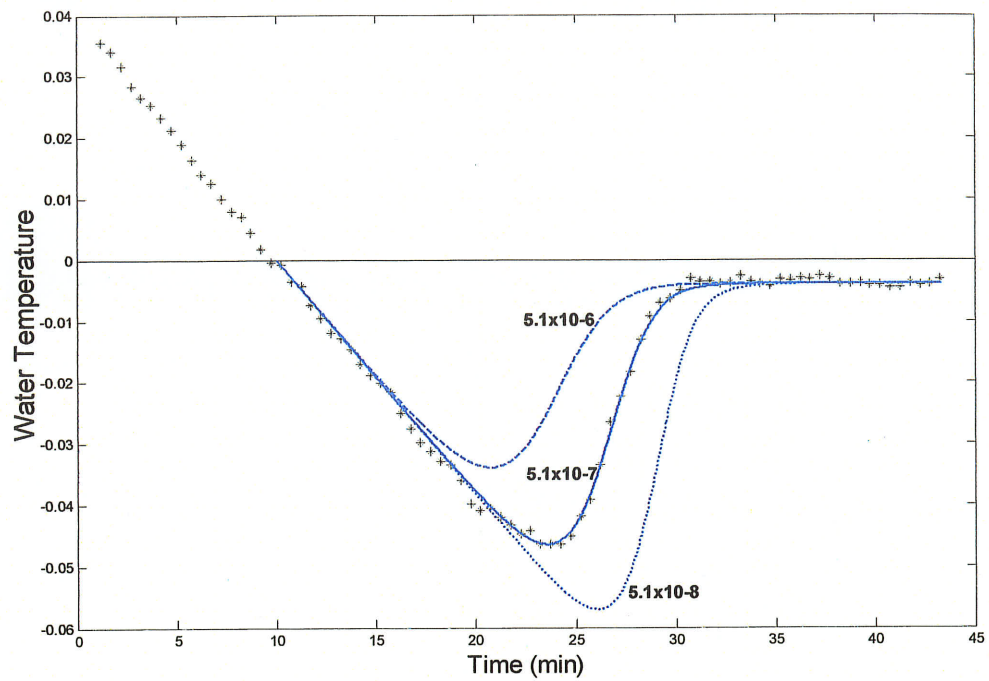


Figure 7.17. Effects of initial ice concentration.

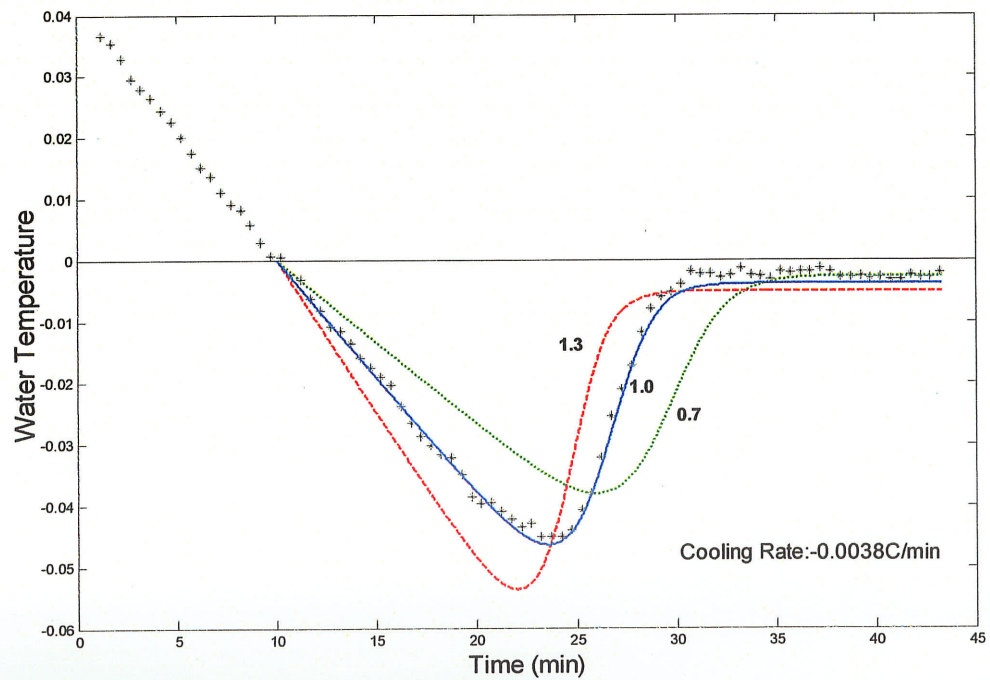


Figure 7.18. Effects of cooling rate.

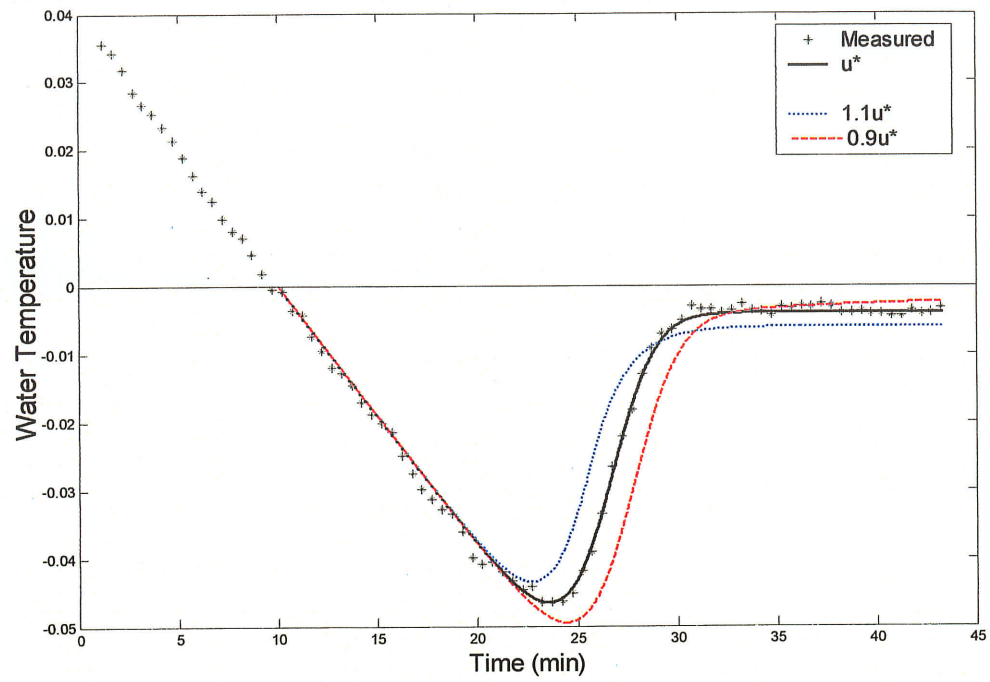


Figure 7.19. Effects of flow turbulence.

In this thesis, based on a series of experiments conducted in the counter-rotating flume, a new approach for the supercooling process and frazil evolution is proposed. It relates hydraulic conditions with frazil size and the entrained frazil generating capacity of a flow. This yields a simple but clear physical link between the supercooling process and frazil growth. The numerical model based on this new approach, which has developed herein, provides predictions in good agreement with observations.

8.1 Summary

8.1.1 Findings of Experiments

Frazil particles

The observed shape of a frazil ice crystal results from a complex interaction between the imposed heat transfer conditions and the intrinsic crystallography of ice. Disk shaped frazil was typically observed. The diameter to thickness ratio was difficult to measure since the frazil particles rarely travel exactly perpendicular to the flow with respect to either of their axes. As the amount of frazil increases, the particles flocculate and sinter together. It was common at the end of an experiment to have a few very large flocs with a low concentration of individual frazil particles.

Supercooling process and frazil evolution

The DIPS program counted the number of particles, the size of the particles, as well as their vertical distribution in the flume. At the beginning of supercooling both the number and size of frazil particles was quite small. The water became supercooled after frazil ice particles were detected. Secondary nucleation of these particles then leads to a rapid proliferation of frazil ice that was associated with a commensurate increase in the water temperature as a result of the release of the latent heat of fusion. When equilibrium was attained (i.e., a residual temperature at the end of the principal supercooling process), the number of frazil ice particles decreased rapidly as a result of flocculation processes. Observations are consistent with the assumption that a frazil size distribution with a fixed D_{50} was maintained and governed by the flow turbulence during the principal supercooling process.

Factors affecting supercooling process and frazil evolution

For a given flume bed roughness and other environmental variables, the air temperature was the primary factor governing the supercooling process and frazil evolution. As expected, a colder air temperature caused a shorter supercooling process and a larger maximum supercooling value, and led to more frazil ice production. The experiments with similar velocities but different cooling rates yielded a similar amount of frazil ice formed. This confirmed the concept of a predetermined frazil generating capacity of a flow. The concept proposed states that for given air temperature and flow turbulence, there is a maximum value of entrained frazil ice formed in a flow. When the air temperature was kept constant, the length of supercooling was longer for lower velocities. As a result, this yielded more frazil ice formation. However, the size of frazil got larger when the flow turbulence got stronger.

8.1.2 Theoretical development

Based on these experimental findings and previous research (Carstens, 1966; Daly and Colbeck, 1986), a new approach for the supercooling process and frazil evolution was proposed. The approach proposes that the principal supercooling process is a process whereby frazil formation reaches the entrained frazil generating capacity of the flow, and the residual supercooling represents a two-layer frazil ice stage with entrained and floating surface frazil ice. Two models were developed to describe

the frazil size distribution and the vertical distribution of frazil ice in flows, respectively, based on characteristics of the flow turbulence. Furthermore, in analogy to suspended sediment transport, an equation estimating the entrained frazil generating capacity of a flow was proposed, which is governed by air temperature and flow turbulence.

A numerical model for the supercooling process and frazil ice evolution in open channels was developed based on the proposed approach to these issues. This model avoids the need to simulate seeding, secondary nucleation, flocculation/break-up, and gravitational removal, which are not well understood. Only the overall heat balance was considered during the entire process. Therefore, fewer assumptions and calibrated parameters are needed. The model successfully simulated the characteristics of frazil ice evolution and the supercooling process. Sensitivity analysis showed that the cooling rate, the assumed initial ice concentration, and the bed friction factor are much more sensitive than other hydraulic parameters. The cooling rate and hydraulic parameters were automatically obtained from the experimental variables. The initial ice concentration was calibrated using an optimization scheme in order to best match the water temperature curves. The initial ice concentration was found to be reasonably well predicted by the Reynolds number.

8.2 Conclusions

The following conclusions can be drawn from the above summary of this research.

- i) Disk shaped frazil ice is generally observed during the supercooling experiments conducted in the counter-rotating flume. Larger flocs are the result of flocculation and sintering together of several or many frazil disks.
- ii) Frazil formation initiates at or near the onset of supercooling. The bulk of frazil ice formations occur during the warming stage of the principal supercooling process. The residual supercooling represents two-layer stage of frazil ice: entrained frazil and surface frazil.
- iii) Air temperature is the primary factor affecting the supercooling process. Colder air increases the cooling rate, the maximum supercooling, and yields a shorter period of supercooling. Warmer air decreases the cooling rate, the maximum supercooling, and yields a longer period of supercooling. Colder air generates more frazil ice.

- iv) The principal period of supercooling is a period of entrained frazil ice formation. There exists an entrained frazil generating capacity of a flow for a given thermo-hydraulic conditions.
- v) The flow turbulence influences the period of principal supercooling and the entrained frazil generating capacity; higher turbulence produces more and larger frazil particles but with a lesser amount of frazil particles per unit volume.
- vi) In the principal supercooling stage, the relative frazil size distribution is constant and governed primarily by flow turbulence.
- vii) A model describing the frazil size distribution was developed based on the characteristics of the flow.
- viii) A model for the vertical distribution of frazil was developed, and verified by experimental observations.
- ix) Equations were developed to estimate the entrained frazil generating capacity of a flow.
- x) A numerical model was developed that successfully stimulated the time-history of the supercooling process, as well as the distribution and evolution of frazil ice. The only "free" parameter in the model is the initial ice concentration.

8.3 Future works

While high-resolution images of various frazil ice formations have been acquired, there are still particles that cannot be detected by the current system. Daly and Colbeck (1986) were able to view particles as small as $30\text{ }\mu\text{m}$. The ability to view particles as they just begin to form would help to accurately determine the temperature at which nucleation occurs. This small field of view would nevertheless limit the ability to have an overall view of what is happening in the flume. One possible solution would be to have two or more cameras acquiring images simultaneously. This would allow one camera to be acquiring a very small field of view while the second should acquire images covering the entire height of the flume. The ability to use two identical cameras with two identical frame grabber boards acquiring simultaneous images has sparked the question as to whether stereoscopic images of the frazil particles can be obtained. This would help to resolve the third dimension (width into the flume) and perhaps help solve the problem of overlapping particles appearing as a single particle in two-dimensional images. It would also help to characterize the diameter to thickness ratio of a frazil disk as well as any change during frazil evolution.

An ADV (Acoustic Doppler Velocimeter) and hot film have been installed in the counter-rotating flume. In later research, this system will be able to measure 3-D velocity fluctuations, from which the flow turbulent characteristics can be obtained. These measurements are expected to confirm two assumptions invoked in the new approach proposed: that the vertical velocity fluctuation plays a central role in the frazil size and its distribution; and the turbulence intensity follows a normal distribution, which is expressed by a function of the bottom shear velocity. This system would focus on measurements of the flow velocity profile, the vertical turbulent velocity distribution, and bottom shear velocity. It can also provide information on the variation of the bottom roughness during experiments. Hence, it would also indirectly monitor the roughness variation due to anchor ice growth and the vertical mixing process of the frazil and flow.

Advanced DIPS and turbulence measurement techniques would allow the possibility to explore the complicated interaction between flow turbulence and frazil ice formation. The frazil geometry and distribution could vary over the supercooling process. More measurement and observation of frazil ice could provide some modifications of the new approach in order for a better understanding of this complicated process.

The current research is strongly related to the management of hydraulic structures in cold climates. A closely related problem is the anchor ice growth, which can be divided into: 1) frazil ice formation; 2) frazil ice distribution (specifically the concentration near the riverbed); and 3) frazil adhesion to the bottom and anchor ice growth. The approach proposed in this thesis could be extended to explore the mechanisms of anchor ice growth with the assistance of additional experimental research.

Bibliography

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- Acres Manitoba Limited, (2000) Report on Limestone Ice Mitigation Study, Winnipeg.
- Altberg, V. I. (1936) Twenty years of work in the domain of underwater ice formation. International Association for Scientific Hydrology, Bulletin, Vol. 23, 373-407.
- Andersson A. (1997) Frazil ice at water intakes, Doctoral thesis, Lulea University of Technology.
- Andreasson, P., Hammer, L. and Shen, H. T. (1998) The influence of surface turbulence on the formation of ice pans. 14th IAHR Symposium. On Ice, Potsdam, USA., 69-76.
- Andres, D. D. (1982) Nucleation and frazil ice production during the supercooling period. Workshop on Hydraulics of ice-covered rivers, June, Edmonton, Alberta, 168-175.
- Andres, D. D. (1995) Frazil generation and ice floe formation on a regulated river, Proceedings of the 8th workshop on the hydraulics of ice covered rivers, Kamloops, BC.59-75.
- Arakawa, K. (1954) Studies on the freezing of water. International Association of Scientific Hydrology, Publication No. 39, 474-477.
- Arden, R. S. and Wigle, T. E. (1972) Dynamics of ice formation in the Upper Niagara River. In Proceedings, international Symposium on the Role of Snow and Ice in hydrology, Baff, Alberta, Vol. 2. UNESCO-WMO-IHAS.
- Ashton, G. D. ed. (1986) River and Lake Ice Engineering. Water Resources Publications, Littleton, Colorado, U.S.A.
- Barnes, H. T. (1928) Ice Engineering. Renouf Publishing Co., Montreal.
- Batchelor, G. K. (1980) Mass transfer from small particles suspended in turbulent fluid. J. of Fluid Mechanics. 98(3): 609-623.
- Beltaos, S. ed. (1995) River Ice Jams. Water Resources Publications, LLC.

- Billfalk, L., (1987) Ice problems in hydropower plant. Report U(L) 1987/15, Swedish State Power Board, Sweden.
- Bukina, L.A. (1967) Size distribution of frazil ice crystals in turbulent flows. *Izv., Atmospheric and Oceanic Physics*, 3(1): 58-68.
- Carson, R. W., Groeneveld, J. L. (1997) Evolution of the ICESIM model. 9th workshop on River Ice, Fredericton. 289-302.
- Carstens, T. (1966) Experiments with supercooling and ice formation in flowing water. *Geofysiske Publikasjoner*, 26(9): 1-18.
- Carter, R.W., Einstein, H.A., Hinds, Julian, Powell, R.W. and Silberman, E. (1963) Friction factors in open channels. *J. Hydraulics Division, ASCE*, 89HY2, 97-143.
- Chapra, S.C. and Canale, R. P. (1988) Numerical methods for engineers. McGraw-Hill, Inc.
- Clark, S and Doering, J. C. (2002) Laboratory observation of frazil ice. *IAHR Ice Symposium, New Zealand*, Vol. 1 127-133.
- Colebrook, C. F. (1939) Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws. *Journal of the Institute of Civil Engineers*, 1938-1939, No. 4, 133-156.
- Daly, S. F. (1984) Frazil ice dynamics, CRREL monograph 84-1
- Daly, S. F. (1987) The evolution of frazil ice in river and streams: research and control. *Int. Sym. On cold regions heat transfer*. Edmonton, 11-16.
- Daly, S. F. (1991) Frazil ice. In K.C. Cheng and N. Seki (editors), *Freezing and melting Heat Transfer and Engineering*. Hemisphere Washington, D.C. Ch. 16, 523-544.
- Daly, S. F. (1994) Report on Frazil ice, USA Cold Regions research and Engineering Laboratory, Special Report 94-23.
- Daly, S. F. and Axelson, K. D. (1989) Estimation of time to maximum supercooling during dynamic frazil ice formation. USA Cold Regions research and Engineering Laboratory, Special Report 89-26.
- Daly, S. F. and Colbeck, S. C. (1986) Frazil ice measurements in CRREL's flume facility, *IAHR Ice Symposium, Iowa city, U.S.A.* 427-436
- Davar et al. ed. (1996) A Primer on Hydraulics of Ice Covered River. The Canadian Committee on River Ice Processes and the Environment.
- Devik, O. (1932) Thermal and dynamic conditions of ice in streams, as applied to Norway. (Thermische und Dynamische Bedingungen der Eisbildung in Wasserlaufen auf Norwegische Verhältnisse angewandt) (In German). *Geofysiske Publikasjoner*, Vol. 9, 5-100. U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, N.H., Draft Translation 513(1976).

- Doering, J. C. Bekeris, L. E. Morris, M. P. Dow, K. E. and Girling, W. C. (2001) Laboratory study of anchor ice growth. *J. of Cold Regions Engineering*, 15 (1), 60-66.
- Doering, J.C. and Morris, M.P. (2002) A digital image processing system to characterize frazil ice. *Canadian Journal of Civil Engineering*; Special Issue, (to appear).
- Ettema, R. and Muste, M. (2000) Method for estimating sediment transport in ice-covered channels, *J. of Cold Regions Engineering*, 14(3) 130-144.
- Ettema, R., Karim, M. F. and Kennedy, J. F. (1984) Frazil ice formation. US Cold Regions Research and Engineering Laboratory (CRREL), Report 84-18.
- Fang, H. W. and Wang, G. Q. (2000) Three-dimensional mathematical model of suspended-sediment transport, *J. Hydraulic Engineering*, ASCE. 126 578-592.
- Gerard, R. (1990) Hydrology of floating ice. *Northern Hydrology: Canadian perspectives*, Environment Canada, Notional Hydrology Research Institute, Saskatoon, Canada.
- Girling, W. C. and Groeneveld, J. (1999) Anchor ice formation below Limestone Generating Station, 10th workshop on ice, Winnipeg, Manitoba. 160-173.
- Gosink J. P. and Osterkamp T.E. (1983) Measurements and analyses of velocity profiles and frazil crystal rise velocities during period of frazil ice formation in rivers. *Annals of Glaciology* 4: 79-84.
- Guo, J. K. and Wood, W. L. (1995) Fine suspended sediment transport rates, *J. Hydr. Engrg.* ASCE. Vol. 121 No. 12, 919-922.
- Hammer, L., and Shen, H. T. (1994) Anchor ice growth and frazil accretion, *IAHR ice Sympo.* Trondheim, 1059-1067.
- Hammer, L., and Shen, H. T. (1995) Frazil evolution in channels. *J. Hydraulic Res.* 33(3), 291-306.
- Hammer, L., Kerr, D. J., Shen, H. T. and Liu, L. (1996) Anchor ice growth in gravel-bedded channels, 13th *IAHR Sympo. on Ice*, Beijing, 843-850.
- Hammer, L. and Shen, H. T. (1995) Anchor ice growth in channels, 8th workshop on river ice, Kamloops, BC. 77-92.
- Han, Q., and He, M. (1987) Mathematical model of reservoir sedimentation and fluvial process. *Journal of sediment research*, Vol3 14-29 (in Chinese).
- Hanley, T. O'D. and Michel, B. (1975). Temperature patterns during the formation of border ice and frazil in a laboratory tank. *Proc. 3rd int. Symp. On Ice Problems*. Edited by G. E. Frankenstein. Cold Regions Res. Eng. Lab. 211-220.
- Hanley, T. O'D and Michel, B. (1977) Laboratory formation of border ice and frazil slush. *Canadian Journal of Civil Engineering*, 4: 153-160.

- Hanley, T. O'D. and Tsang, G. (1984) Formation and properties of frazil ice in saline water. *Cold Regions Sciences and Technology*, 8(3): 209-221.
- Healy, D., Hicks, F. Beltaos, S. (1997) A comparison of the ICEJAM and RIVJAM profile models. 9th Workshop on River Ice, Fredericton, N. B. 269-288.
- Horjen, I. (1994) Frazil and Glaze ice accretion on trash racks in slightly supercooled flows, 12th IAHR Sympo. On Ice, Trondheim, Norway, 571-580.
- Hou, H. C. (1982) Basic topics of river dynamics, Water resources press, Beijing (In Chinese).
- Hui, Y. J., Li, Y. T. and Hu C.H. (1997) Studies on turbulence and sediment movement of hyperconcentration of sediment. Project report for Natural Science Foundation of China, No.59339170. Beijing
- Idelchik, I. E. (1986) Handbook of Hydraulic Resistance. 2nd revived and augment edn. (Hemisphere: New York, USA).
- Kerr, D. J., Shen, H. T., and Daly, S. F. (1998) Evolution and hydraulic resistance of anchor ice on gravel beds. IAHR Ice Symposium. Potsdam, NY
- Kerr, D., J., Shen, H. T., Daly, S. (1997) Anchor ice formation and growth on gravel channel bed, 9th workshop on River ice, Fredricton, N. B. 153-172.
- Korzhavin, K. N. (1962) Action of ice on engineering structure (in Russian). Publishing House of Siberian Branch of U. S. S. R. Academy of Sciences. U.S. CRREL, Hanover, N. H., Draft Translation 260 (1971).
- Lal, A. M. W. (1989) A mathematical model for river ice processes, thesis presented to Clarkson University, at Potsdam, N. Y., in partial fulfillment of the requirements for the degree of doctor of philosophy.
- Lal, A. M. W. and Shen, H. T. (1991) Mathematical model for river ice processes. *J. Hydraulic Engineering*, ASCE, 117(7), 851-867.
- Laszloffy, W. (1934) Folyoink Jegviszonyai Kulonos Tekintettel a Dunara. (Ice Regime of Rivers with Special regard to the Danube). Vizugyi Kozlemenyek, Budapest.
- Lia, L. (1998) Snow and ice blocking of tunnels. IAHR Symposium. on Ice, Potsdam, Y.N.
- Liou, C. P. and Ferrick, M. G. (1992) A model for vertical frazil distribution, *Water Resources Research*, Vol. 28, No. 5, 1329-1337.
- Marcotte, N. (1984) Anchor ice in Lachine Rapids, results of observations and analysis. IAHR Symp.on ice, Hamburg, Germany, Vol. 1, 151-159.
- Marcotte, N. (1986) A simple mathematical modeling of moving sheet ice, *Proceeding IAHR Symposium on Ice*, Iowa City, August 18-22, 89-100.

- Marcotte, N. and Robert, S. (1986) Elementary mathematical modeling of anchor ice, Proceeding IAHR Symposium on Ice, Iowa City, August 18-22, 493-506.
- Margolis, G. (1969) The nucleation and growth rates of ice in a well-stirred crystallizer. Ph.D. Dissertation. Department of Chemical Engineering. Cambridge: Massachusetts Institute of Technology.
- Martin, S. (1981) Frazil ice in rivers and oceans. *Annual Review of Fluid Mechanics*, 13:379-397.
- Matousek, V. (1984) Types of ice runs and conditions for their formation, Vol.1, 1984 IAHR ice Symposium, Hamburg, 316-327.
- Mercier, S. (1984) The reactive transport of suspended particles: Mechanics and Modelling. Ph.D. dissertation, Joint Program in Ocean Engineering, Massachusetts Institute of Technology, Cambridge, MA.
- Michel, B. (1963) Theory of formation and deposit of frazil ice. In Eastern Snow Conference, Proc. Annual Meeting, Quebec.
- Michel, B. (1971) Winter regime of rivers and lakes. U. S. Army Cold Regions research and Engineering laboratory, Hanover, N.H., Monograph III-Bla.
- Michel, B. (1978) Ice Mechanics. Les Presses de l'Universite Laval, Quebec.
- Morris, M. P. (2002) A digital image processing system for the characterization of frazil ice. M. Sc. Thesis, University of Manitoba.
- Muller, A. (1978) Frazil ice formation in turbulent flow. The University of Iowa, Iowa Institute of Hydraulic Research, Report No. 214.
- Nezu, I and Nakagawa, H. (1993) Turbulence in open-channel flows, IAHR Monograph, A. A. Balkema, Rotterdam.
- Ni, J. R. and Wang, G.Q. (1991) Vertical sediment distribution, *J. Hydr. Engrg., ASCE*, Vol. 117, No. 9, 1185-1194.
- Ni, J. R. Wang, G.Q. and Zhang, H.W. (1991) Basic theories of solid-liquid two-phase flow and their recent application (in Chinese), Science Press, Beijing, China.
- Nyberg, L. (1985) Ice formation in rivers, In numerical Simulation of Fluid Flow and heat/Mass Transfer Processes, C.A. Brebbia and S. A. Orsag (editors), Springer Verlag.
- Omstedt, A. (1985a) On supercooling and ice formation in turbulent sea water. *Journal of Glaciology*, 31(109): 263-271.
- Omstedt, A. (1985b) Modelling frazil ice and grease ice in the upper layers of the ocean. *Cold Regions Science and Technology*, 11: 87-98.
- Omstedt, A. and Svensson, U. (1984) Modelling supercooling and ice formation in turbulent Ekman layer. *Journal of Geophysical Research*, 89(C1): 735-744.

- Osterkamp, T. E. (1977) Frazil nucleation by mass-exchange processes at the air-water interface, *J. of Glaciology*, 19(8), 619-625.
- Osterkamp, T. E. (1978) Frazil ice formation: a review, *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, 104(hy9), 1239-1255.
- Osterkamp, T. E., Gilfilian, R. E., Gosink, J. P. and Bensen, C. S. (1983) Water temperature measurements in turbulent streams during periods of frazil-ice formation, *Annals of Glaciology, international Glaciological Society, Proceedings of the second Symposium on Applied Glaciology*, 4, 209-215.
- Osterkamp, T.E. and Gosink, J. P. (1982) Selected aspects of frazil ice formation and ice cover in Turbulent streams, *Proceedings of the workshop on Hydraulics of ice-covered Rivers*, June, Edmonton, AB, 131-147, National Research Council of Canada.
- Pariset, E. and Hausser, R. (1961) Formation and evolution of the ice covers on rivers. *Trans., Engineering Institute of Canada, Ottawa*, 5(1) 41-49.
- Parkinson F. E. (1984) Anchor ice effect on water levels. *Workshop on the hydraulics of river ice, Fredrickton*, 345-370.
- Petryk, S. Raban, R. Marcotte, N. (1995) Research on River Ice Processes Related to Hydro-electric Installations, *Proceedings of the 8th workshop on the hydraulics of ice covered rivers*, August 16-18, Kamloops, British Columbia, 523-539.
- Prowse, T. D. and Gridley, N. C., ed. (1993) Environmental aspects of river ice. *NHRI Science Report No. 5*, National Hydrology Research Institute, Saskatoon, 155p.
- Prowse, T. D. (2001a) River-ice ecology. I: Hydrologic geomorphic, and water-quality aspects. *J. Cold Reg. Engrg., ASCE*. 15(1), 1-16.
- Prowse, T. D. (2001b).. River-ice ecology. II: Biological aspects. *J. Cold Reg. Engrg., ASCE*. 15(1), 17-33.
- Raban R. (1995), An Ice Engineering Overview, 8th Workshop of River Ice, Kamloop, BC, Canada, 225-231.
- Schaefer, V. J. (1950) The formation of frazil and anchor ice in cold water. *Transactions of the American Geophysical Union*, 31(6) 885-893.
- Schocklitsch, A. (1937) *Hydraulic Structures*. Vol. I. American Society of Mechanical Engineers. New York.
- Shen, H. T. (1996) River ice process-State of research, 13th IAHR Symposium. on Ice, Beijing, 1-7.
- Shen, H. T., Wang, D. S. (1995) Under cover transport and accumulation of frazil granules. *J. Hydraulic Engineering*. 121(2). 184-195.

- Shen, H. T., Wang, D. S. and Lal, A. M. W. (1995) Numerical simulation of river ice process. *J. Cold Regions Engineering*, 9(3). 107-118.
- Shen, H. T. (2002) Development of a comprehensive river ice simulation system, 16th international symposium on ice, Dunedin, New Zealand, Vol. 1, 142-148.
- Spiegel, M. R. (1968) *Mathematical Handbook of formulas and tables*. McGraw-Hill Book Company.
- Strarosolszky, O. (1970) Ice in hydraulic engineering. Institute for Vassbygging, University of Trondheim, Norway, Report No. 70-1.
- Svensson, U. & Omstedt, A. (1994) Simulation of supercooling and size distribution in frazil ice dynamics, *Cold Regions Science and Technology*. 22: 221-233.
- Svensson, U., Billfalk, L. and Hammer, L. (1989) A mathematical model for model-ice formation in rivers, *cold regions Science and Technology*, 16 (2), 179-189.
- Tatinclaux, J. C. (1998) Recent progress in river ice engineering research at CRREL, *Journal of Cold Regions Engineering*, 12(3), 114-135.
- Tsang, G. (1982) Frazil and anchor ice: A monograph. National Research Council of Canada, Subcommittee on Hydraulics of Ice-Covered Rivers, Ottawa, Canada, 90 pages.
- Tsang, G. (1986) Preliminary report on field study at Lachine Rapids on cooling of river and river and formation of frazil and anchor ice . *Proceedings of the 4th Workshop on Hydraulics of River Ice*, Montreal, F5.1-5.51.
- Tsang, G. (1988a) Development of mathematical modeling for prediction of river cooling and frazil and anchor ice formation, *Proceeding of 5th workshop on the hydraulics of river ice*, June 1988, Winnipeg, Man., 393-416.
- Tsang, G. (1988b) A theory for frazil distribution in turbulent flow, 9th IAHR Symposium. on Ice, Sapporo, Japan 152-169 .
- Tsang, G. and Trapp, B. (1990) Preliminary findings from experiments on frazil and anchor ice in a circular flume, *Proceedings of Northern Hydrology Symposium*, Saskatoon, Editor Prowse & Ommanney 405-418.
- Tsang, G. (1993) Size distribution of frazil flocs. *Proceedings of 7th workshop on hydraulics of ice covered rivers-environmental aspect of river ice*, Saskatoon.
- Tsang, G. and Cui, W. (1994) Laboratory study of frazil distribution in a flow, *Canadian J. of Civil Engineering*, 21, 696-709.
- Tsang, G. and Hanley, T. O'D. (1985) Frazil formation in water of different salinities and supercooling. *J. of Glaciology*, Vol. 31, No. 108, 74-85.
- USACRREL, (1997) *Ice engineering*, U.S. Army Cold Regions research and Engineering Laboratory, Hanover, New Hampshire.

- Von Chelnoky, E. (1909) The ice of Lake Balaton (Das Eis des Balatonsees) (In German). Geographischen Gesellschaft, Vol. 1, pt. 5.
- Wadia, P. H. (1974) Mass transfer from spheres and discs in turbulent agitated vessels. Ph.D. dissertation. Department of Chemical Engineering. Cambridge: Massachusetts Institute of Technology.
- Wang, D. S. (1993) A study on frazil and anchor ice processes and development of a river ice model. Doctoral dissertation, Clarkson University
- Wigle, T. E. (1970) Investigation into frazil, bottom ice and surface ice formation in the Niagara River. In IAHR 1970.
- Wilson, K. C. (1987) Analysis of bed-load transport. Proc. IAHR Congress. Lausanne, Switzerland.
- Wuebben, J. L. (1984) The rise pattern and velocity of frazil ice. Workshop on hydraulics of river ice, University of New Brunswick, June 20-21, 297-316.
- Yalin, M. S. (1977) Mechanics of sediment transport. Pergamon Press. Oxford, England.
- Zhang, H. W. Jiang, N. H. (1994) Law of similarity for models of flood flows of the Lower Reach of the Yellow River (in Chinese), Henan scientific press, China
- Zhang, H. W., Liu, Y. L. and Wang, G. D. (1990) Size distributions of suspended particles in turbulent flow field, Proceedings of the fourth International Symposium on refined flow modeling and turbulence measurements, Sept. 20-23, Wuhan, China.

Water Temperature Measurements

The data presented in this appendix shows the water temperature history for each of the experiments carried out in the counter-rotating flume. A total of 42 experiments were divided into three groups with three water depths: 10, 15, and 20 cm. In these experiments, the temperatures of the outer, bottom and bed chambers, as well as bed and wall speeds were recorded, respectively. Mean values of these variables during the last 60 minutes of these experiments are listed in the figures.

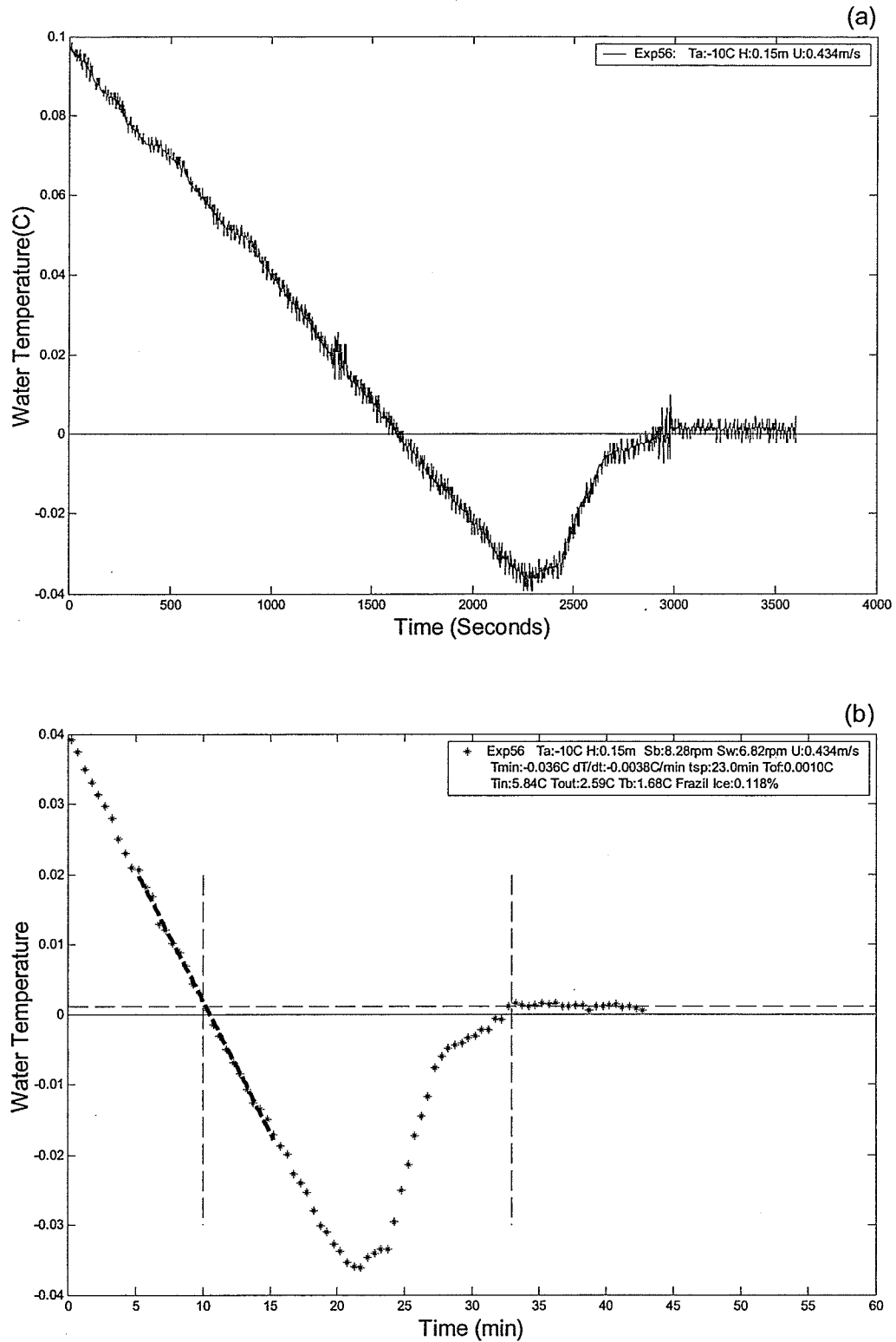


Figure A1. Group 1. Exp. 56. $T_a = -10$ °C, $H = 0.15$ m, $U = 0.434$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

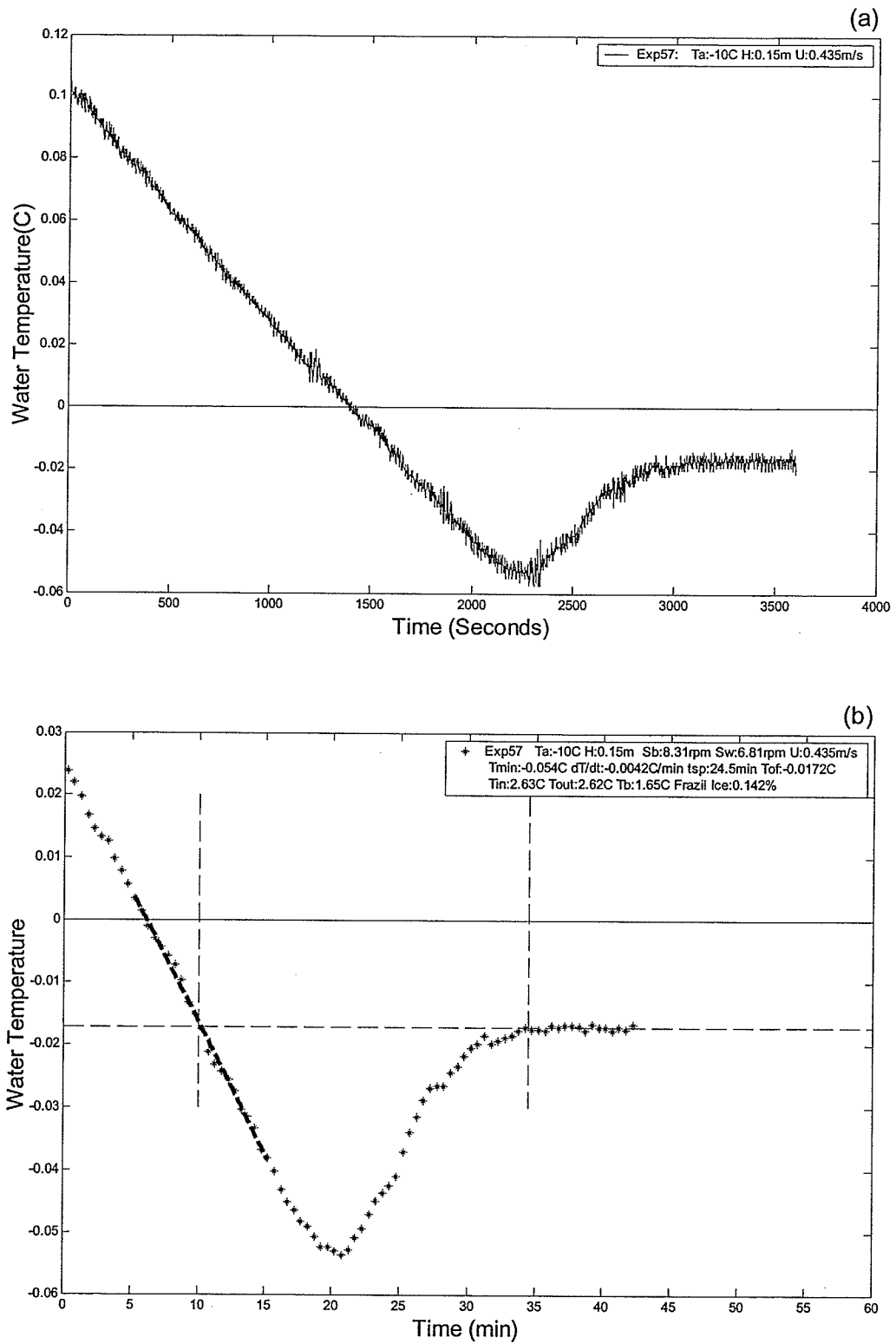


Figure A1. Group 1. Exp.57. $T_a = -10^\circ\text{C}$, $H = 0.15\text{m}$, $U = 0.435\text{m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

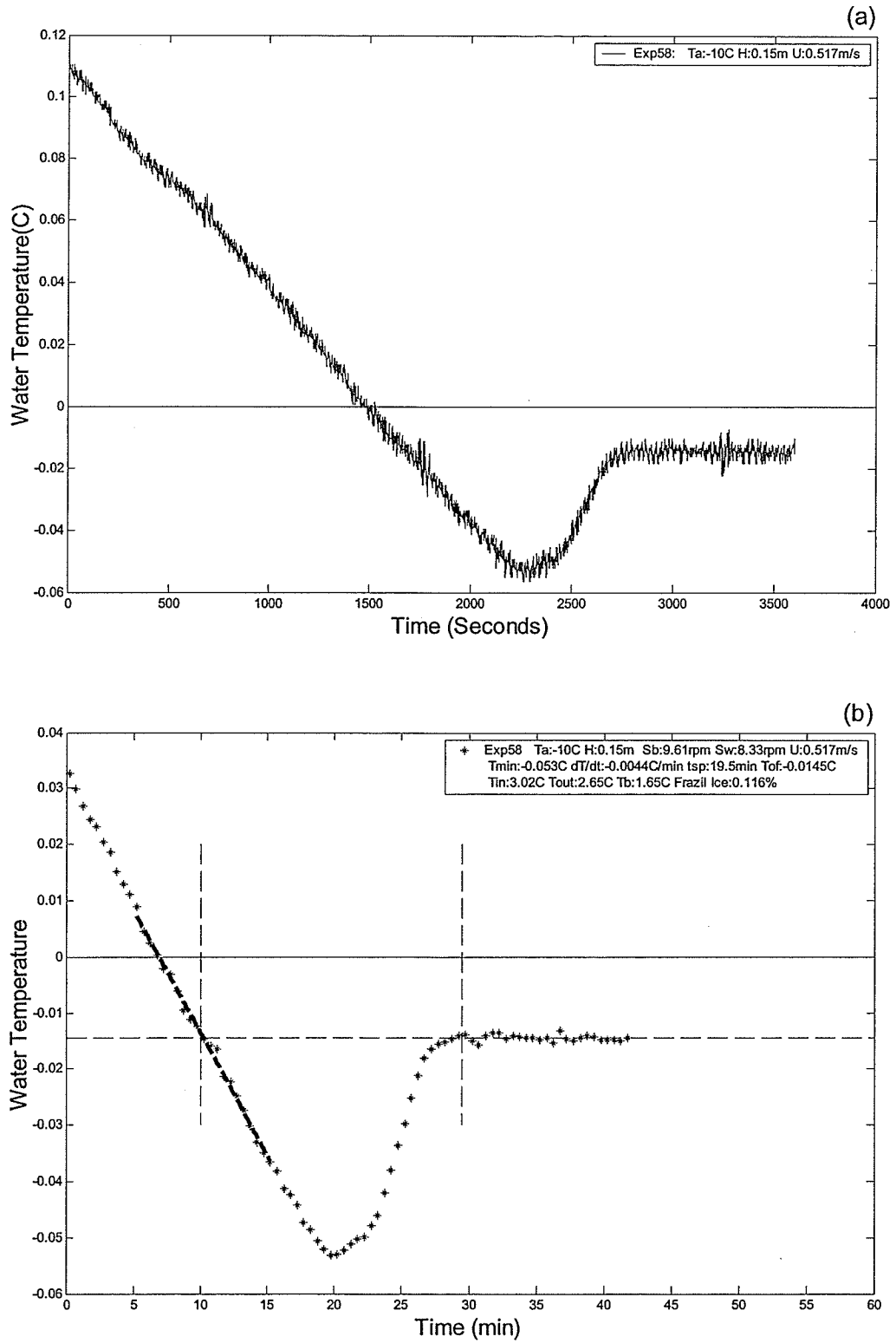


Figure A1. Group 1. Exp. 58. $T_a = -10$ °C, $H = 0.15$ m, $U = 0.517$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

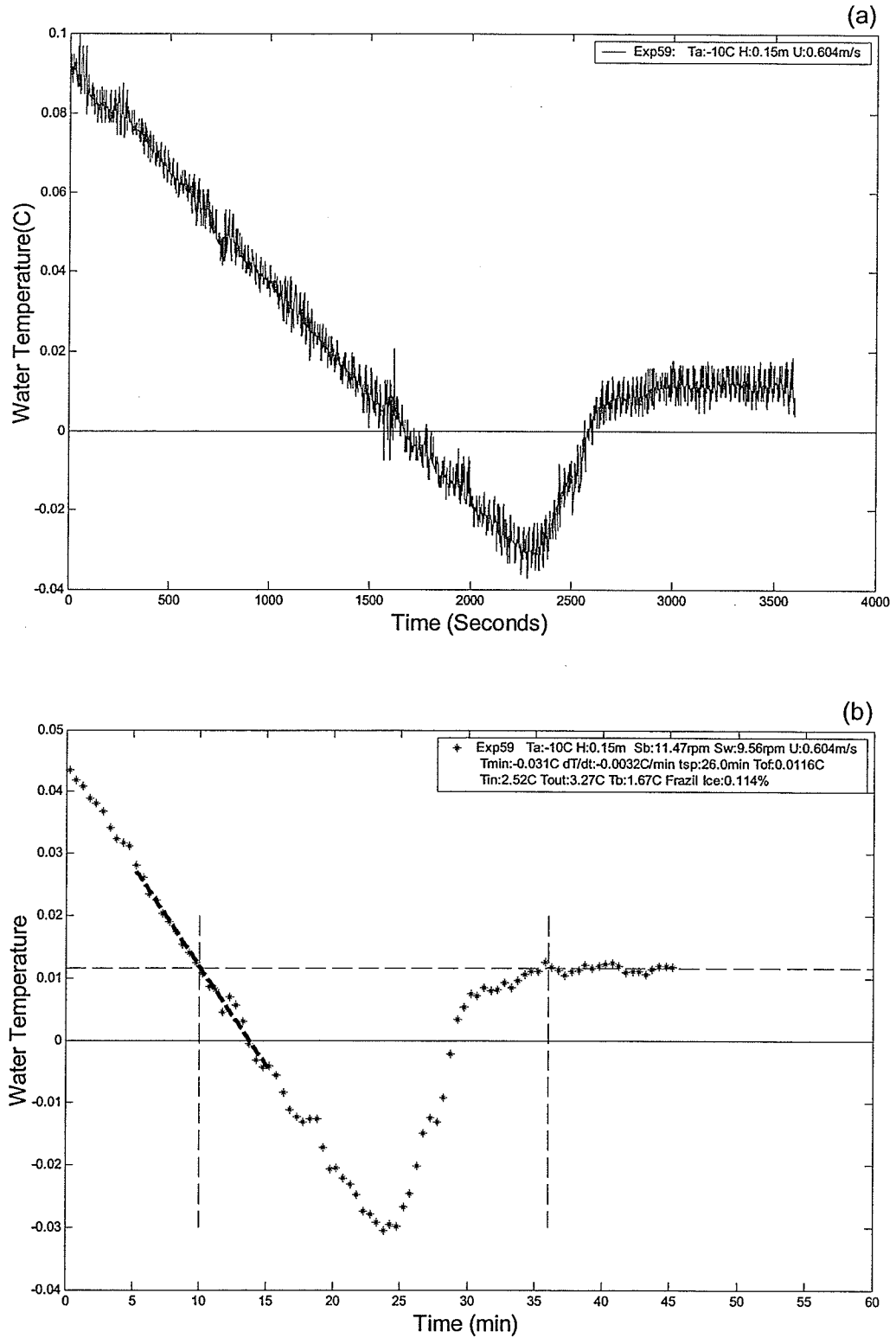


Figure A1. Group 1. Exp. 59. $T_a = -10^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.604\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

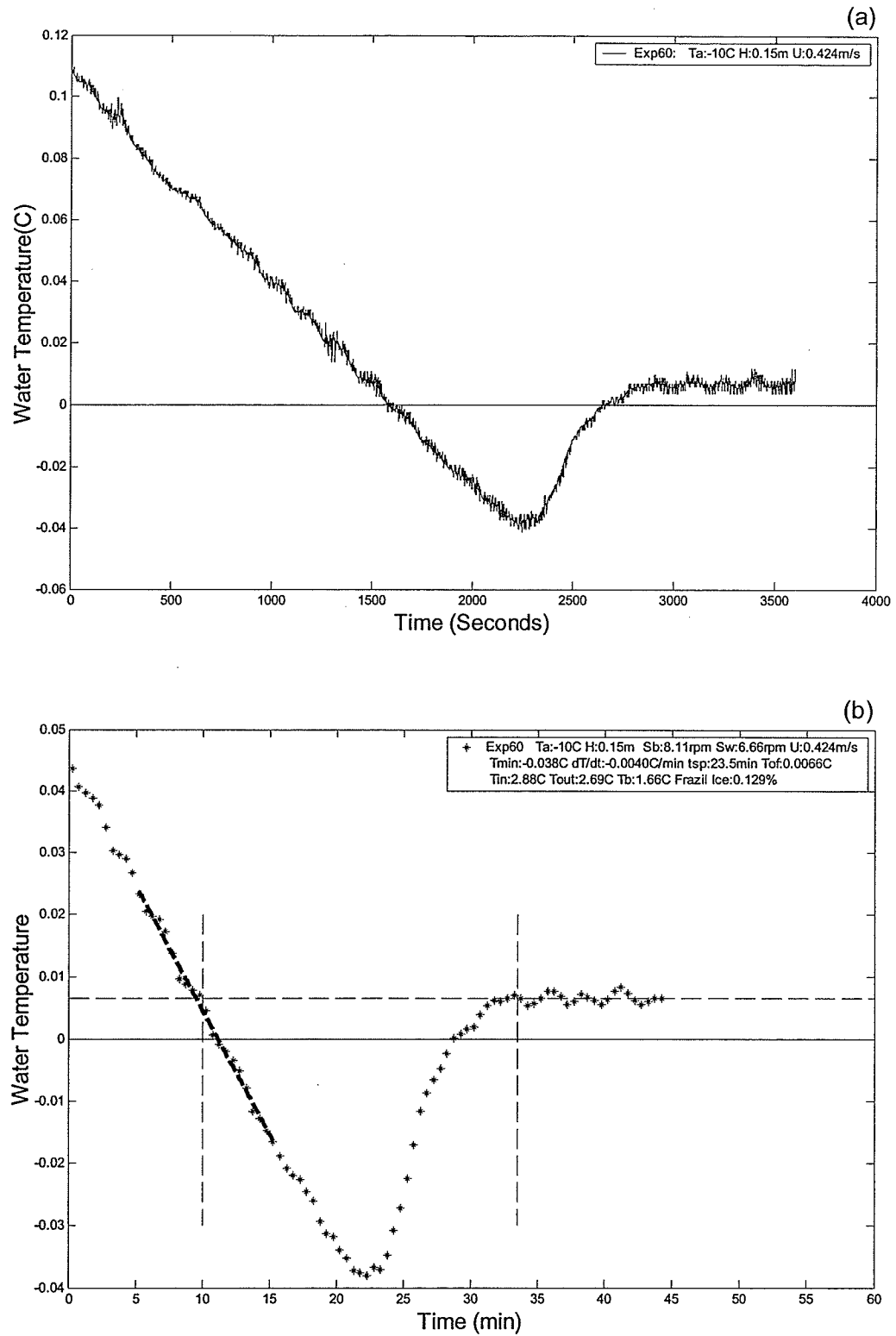


Figure A1. Group 1. Exp. 60. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.424\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

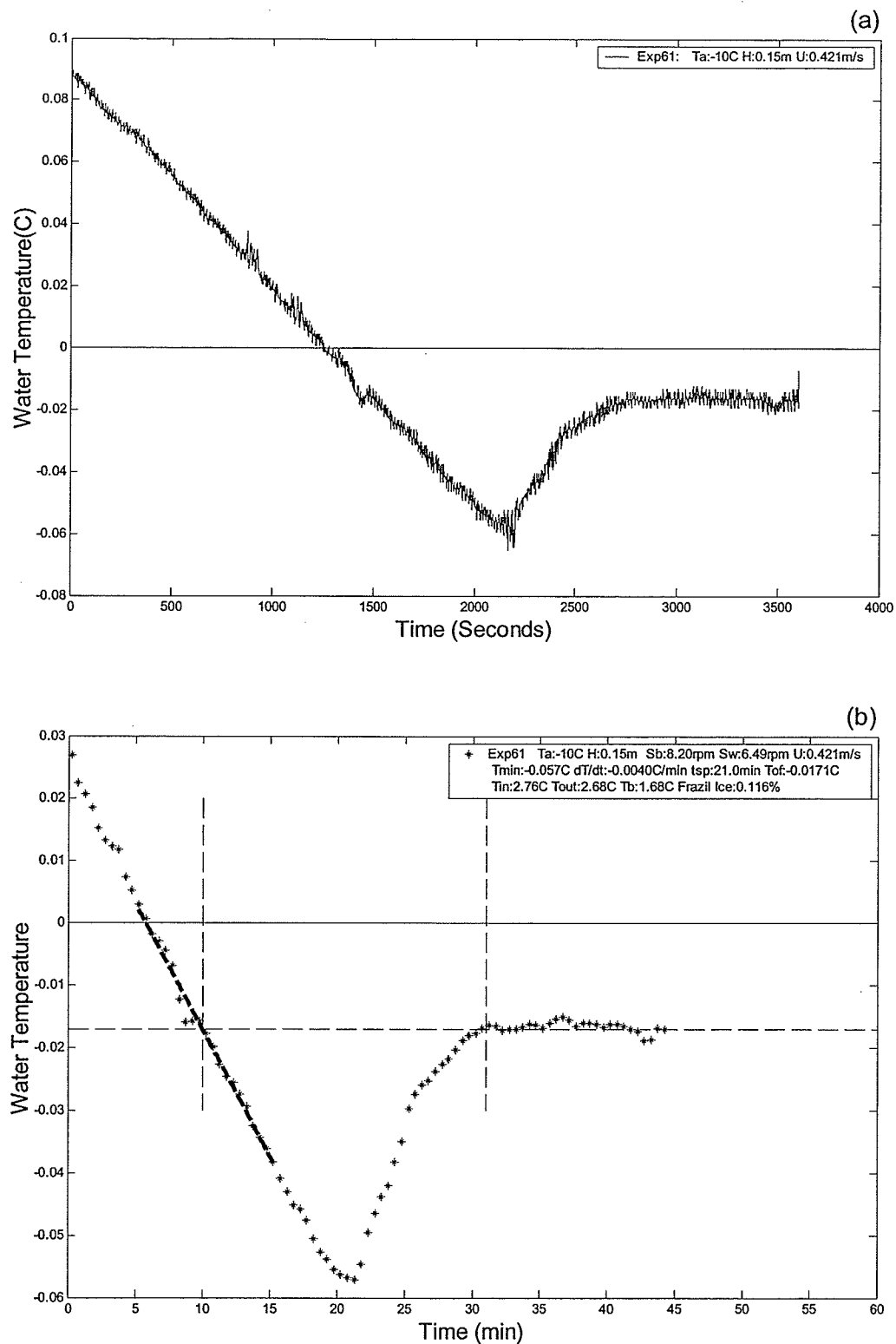


Figure A1. Group 1. Exp. 61. $T_a = -10^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.421\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

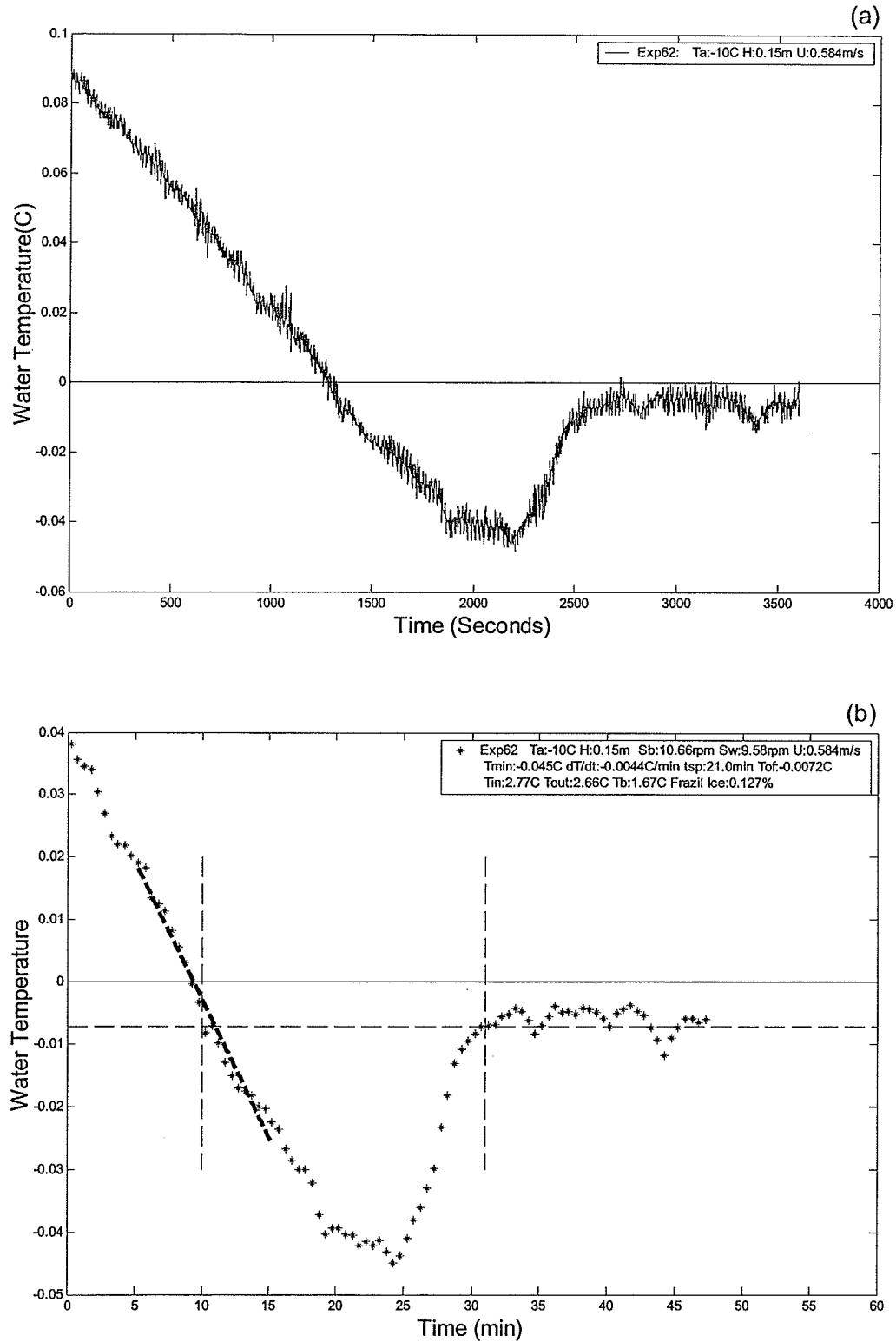


Figure A1. Group 1. Exp. 62. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.584\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

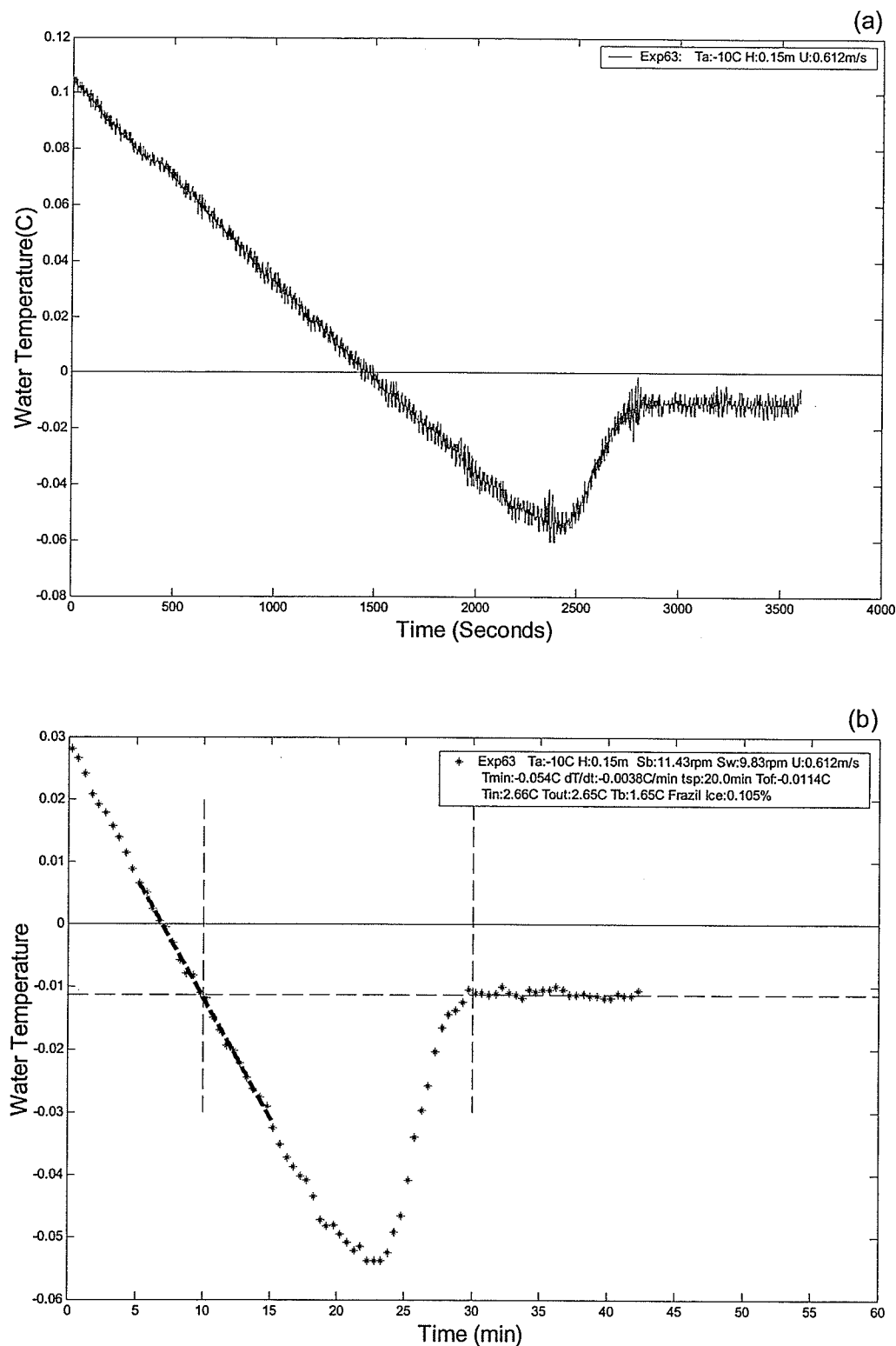


Figure A1. Group 1. Exp. 63. $T_a = -10$ °C, $H = 0.15$ m, $U = 0.612$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

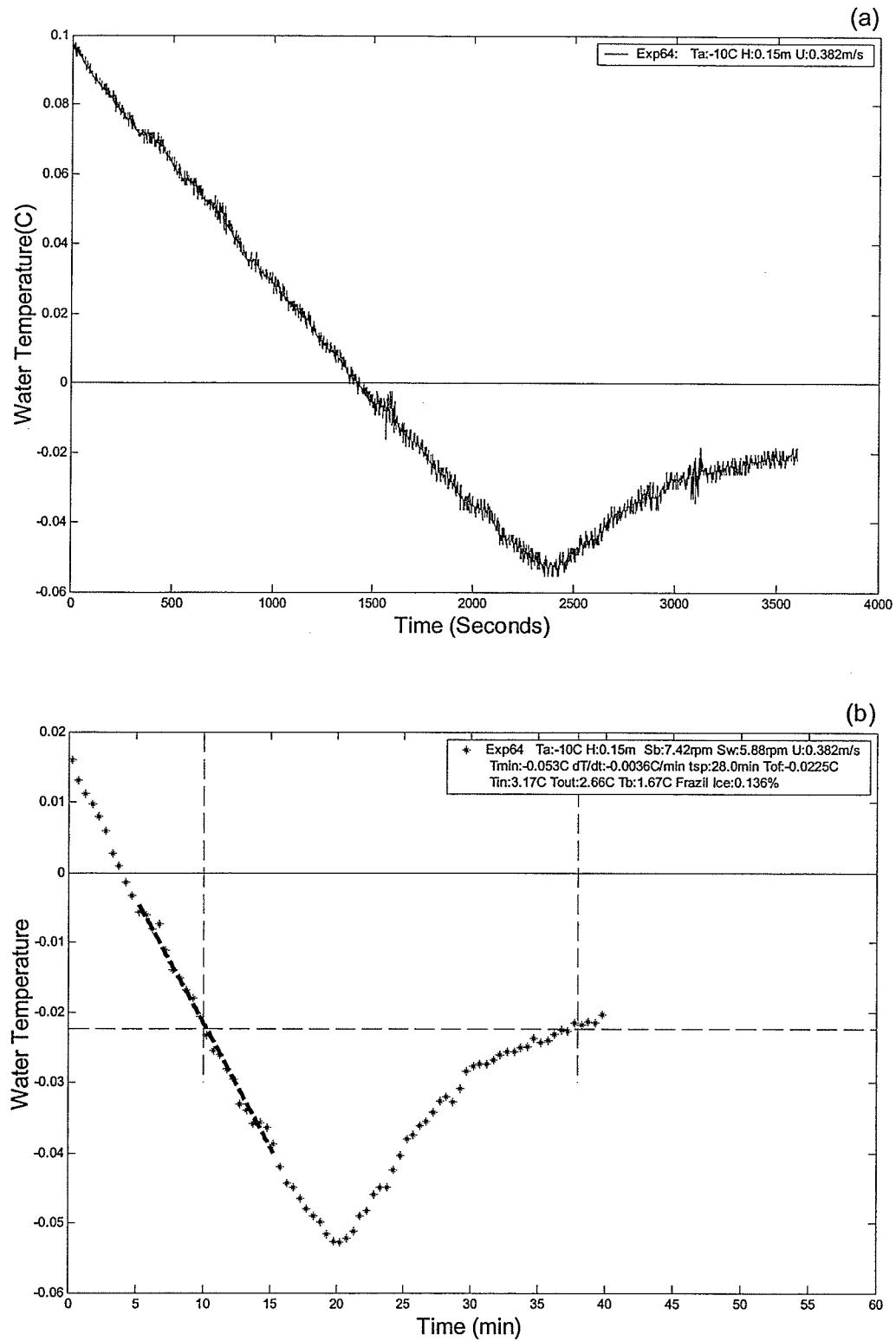


Figure A1. Group 1. Exp. 64. $T_a = -10^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.382\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

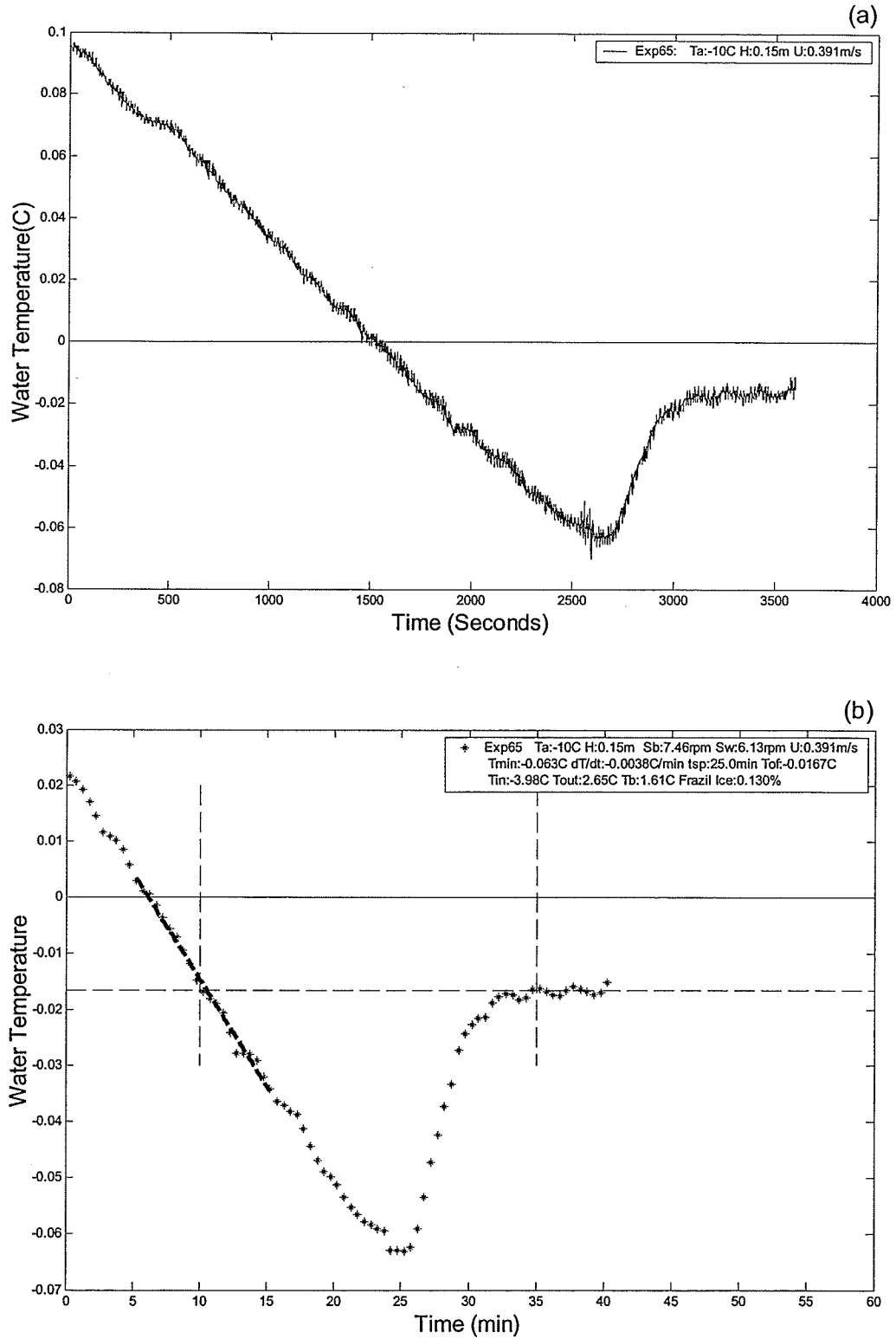


Figure A1. Group 1. Exp. 65. $T_a = -10^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.391\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

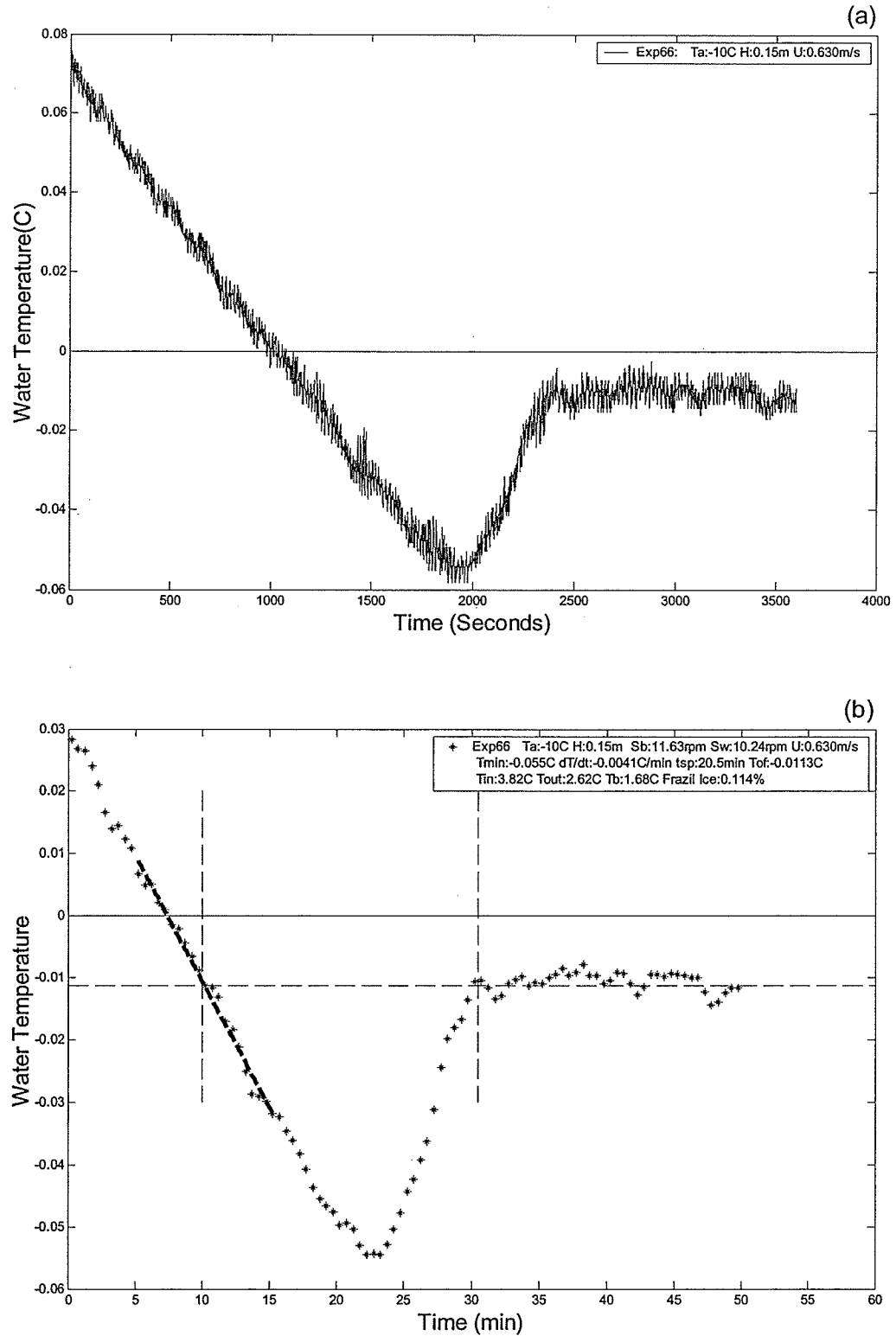


Figure A1. Group 1. Exp. 66. $T_a = -10$ °C, $H = 0.15$ m, $U = 0.630$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

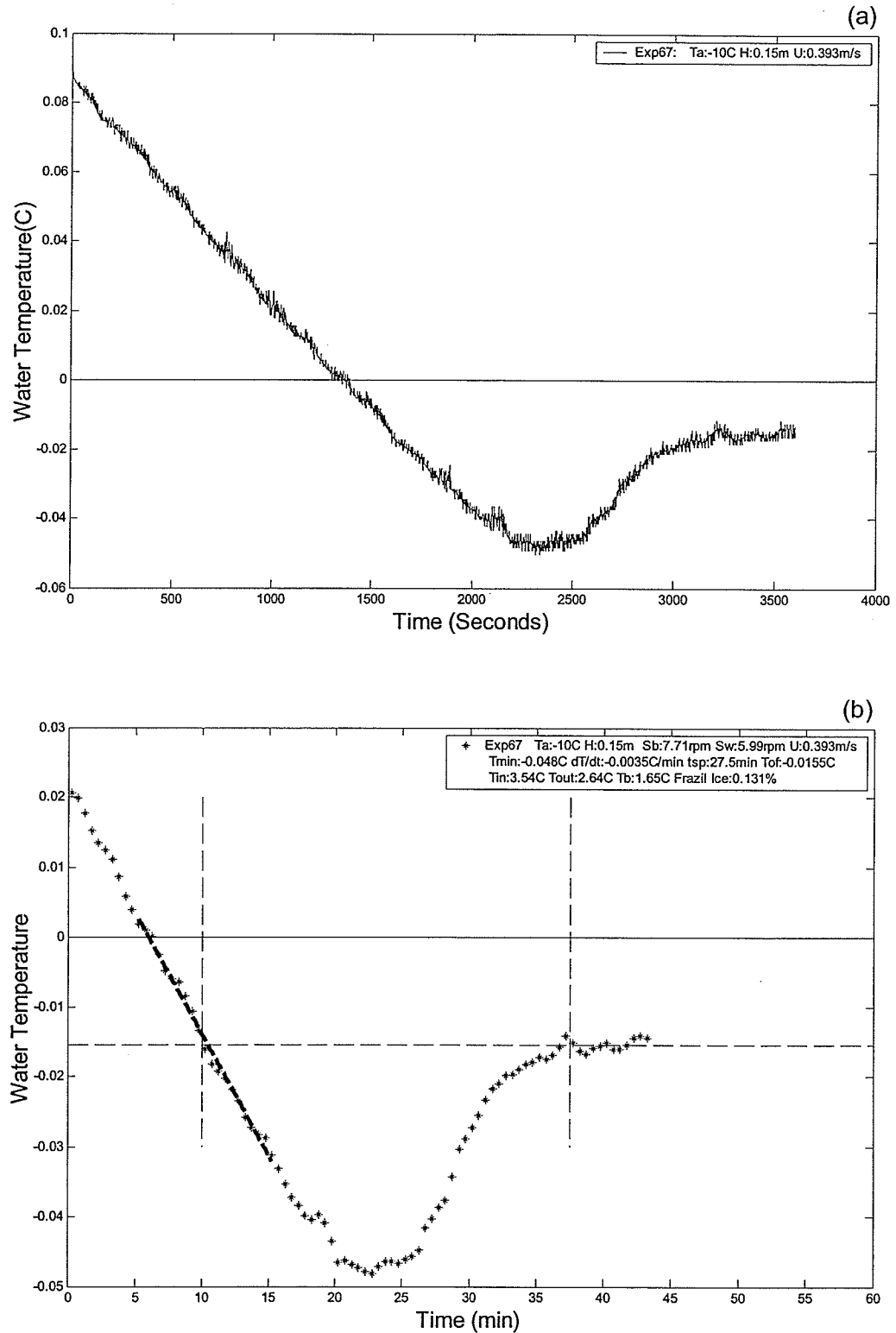


Figure A1. Group 1. Exp. 67. $T_a = -10^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.393\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

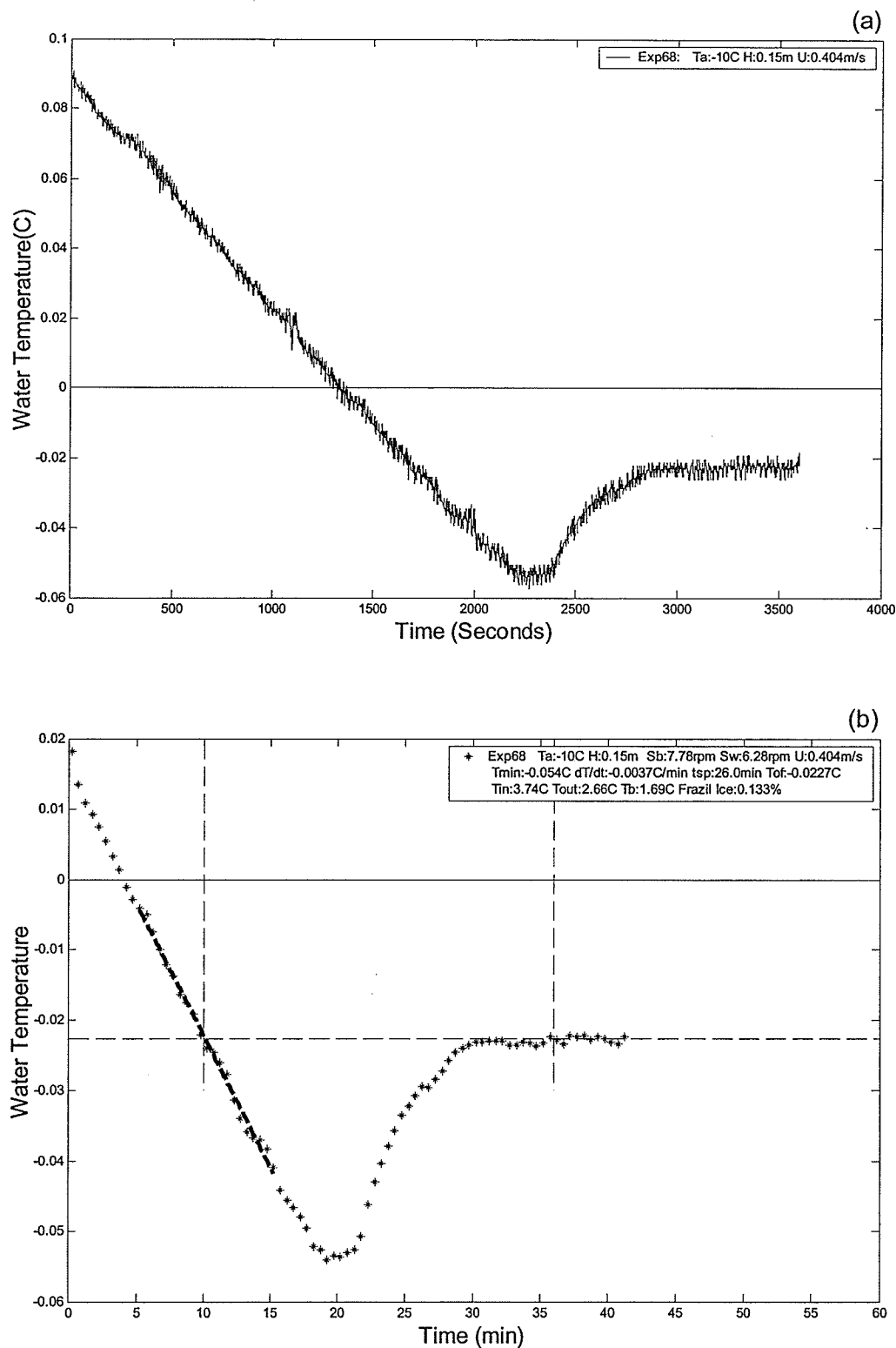


Figure A1. Group 1. Exp. 68. $T_a = -10^\circ\text{C}$, $H = 0.15\text{m}$, $U = 0.404\text{m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

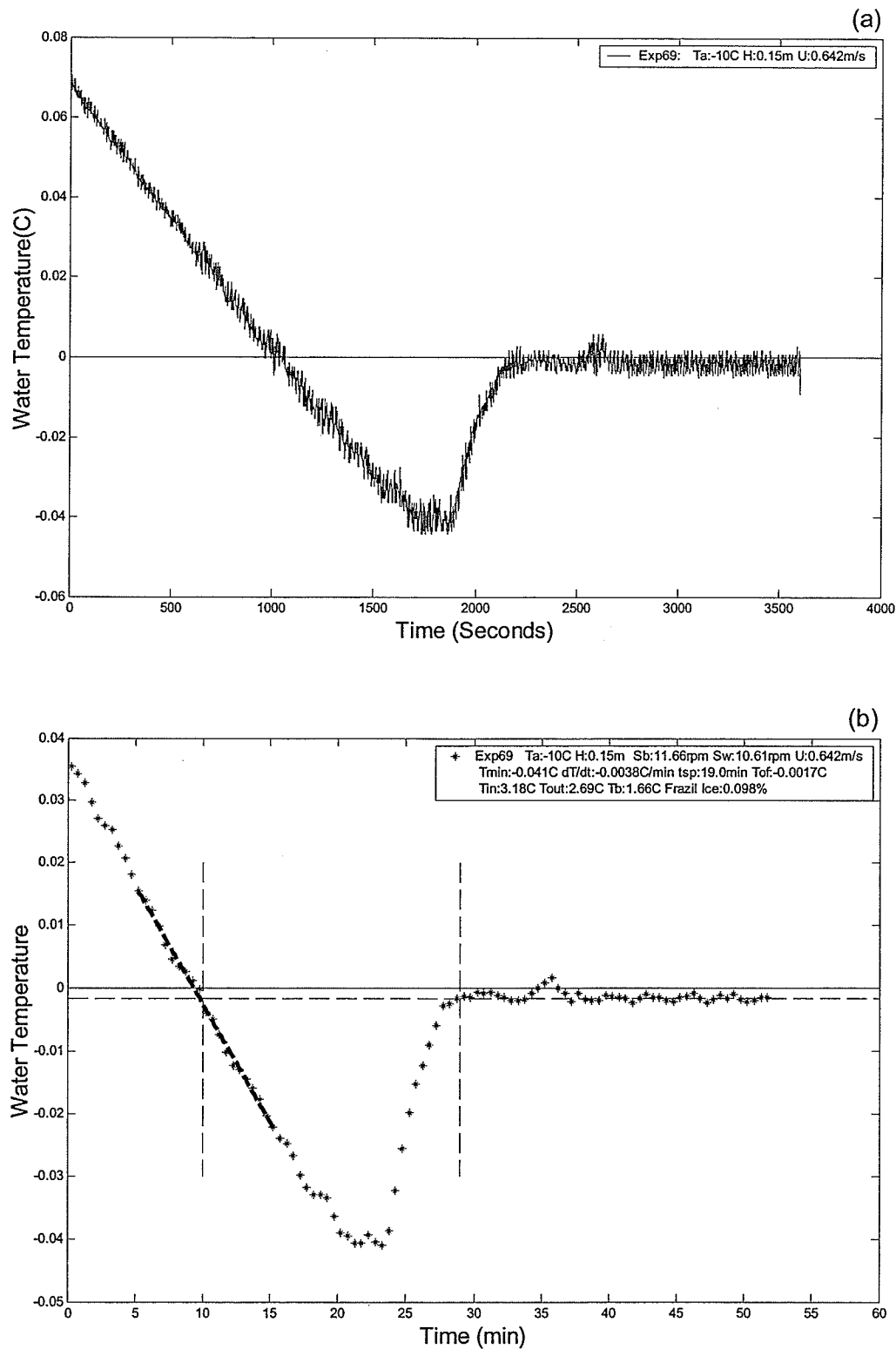


Figure A1. Group 1. Exp. 69. $T_a = -10$ °C, $H = 0.15$ m, $U = 0.642$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

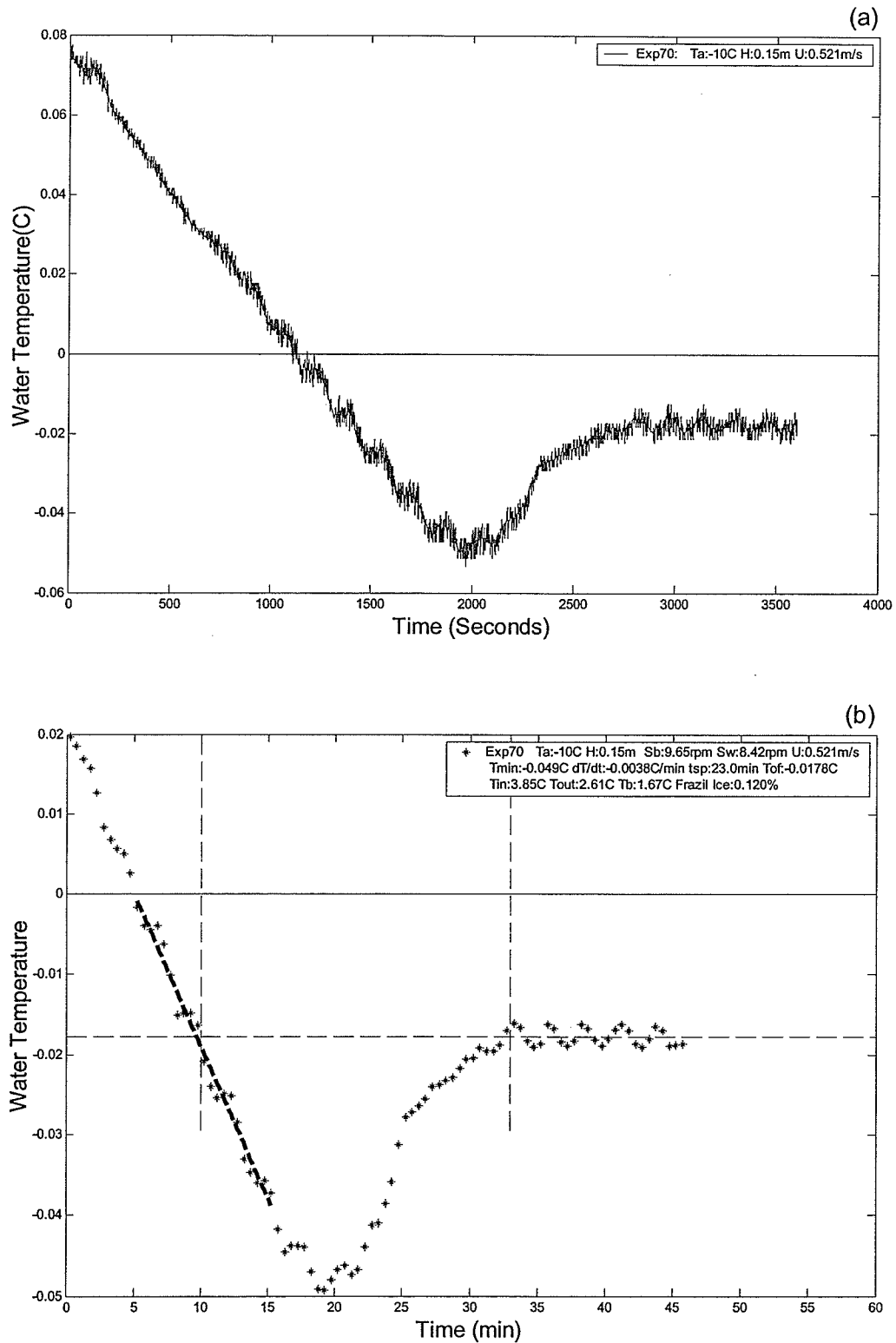


Figure A1. Group 1. Exp. 70. $T_a = -10$ °C, $H = 0.15$ m, $U = 0.521$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

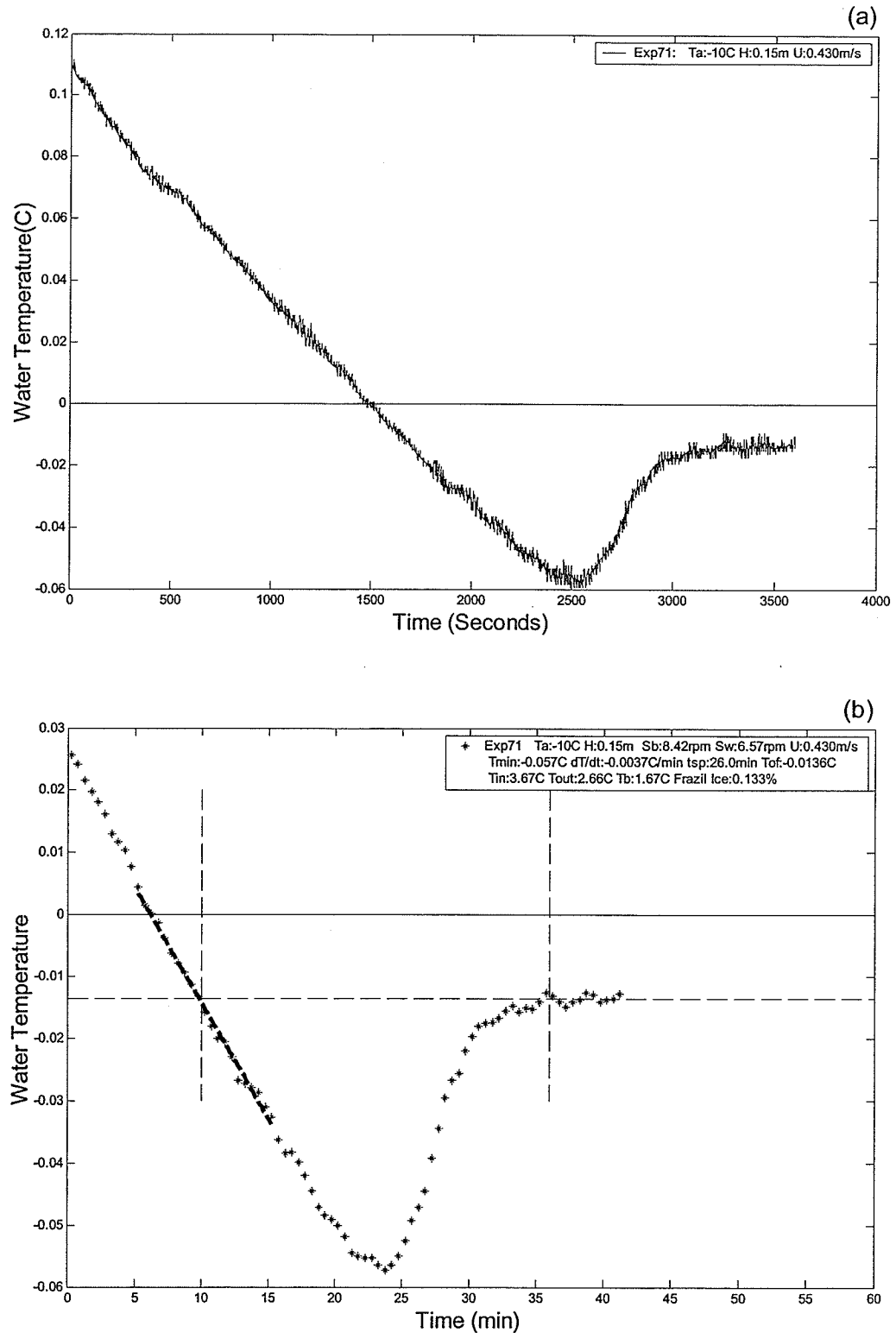


Figure A1, Group 1. Exp71: $T_a = -10^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.430\text{ m/s}$

(a) Despiked water temperature vs. time, **(b)** Characteristics of supercooling process

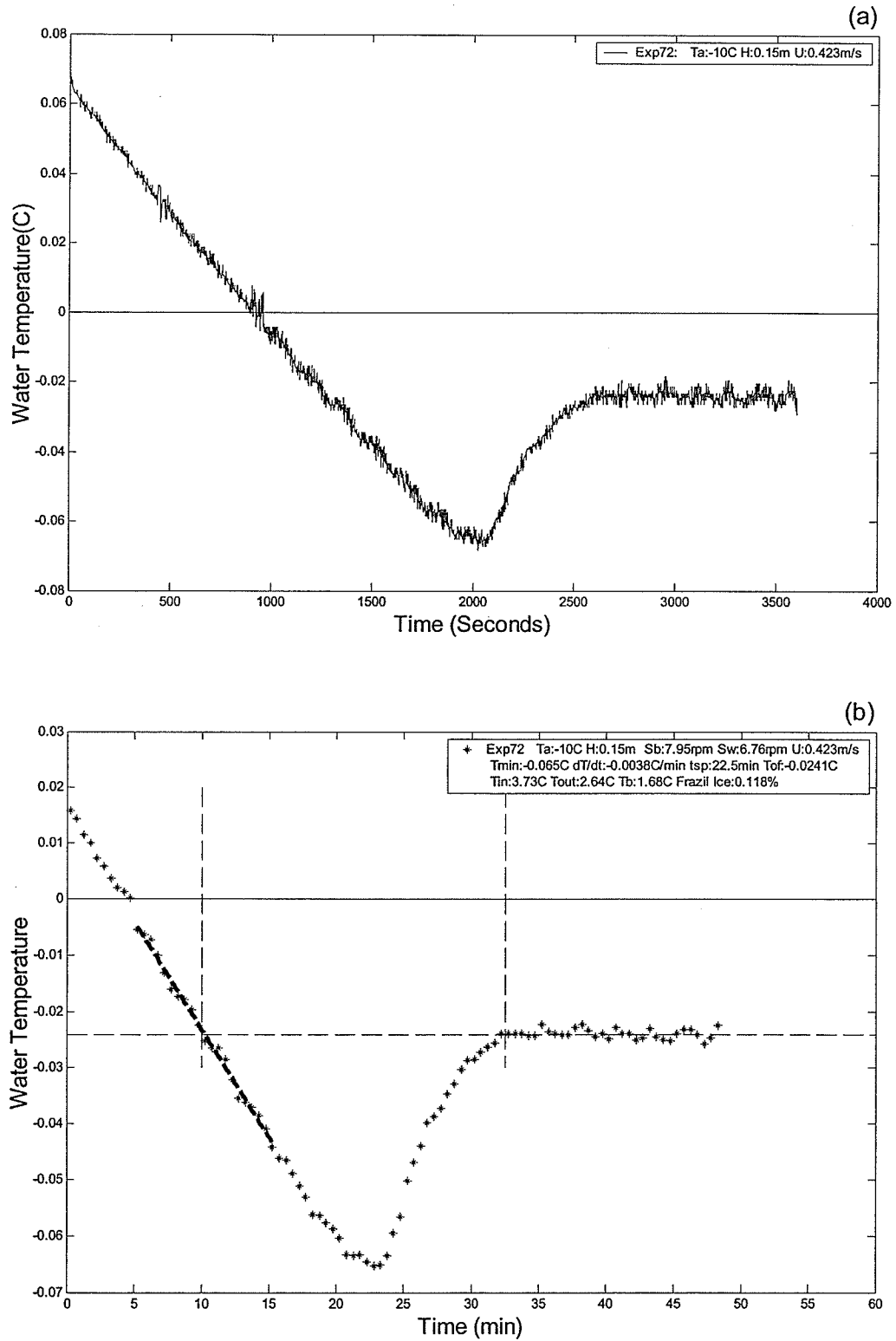


Figure A1, Group 1. Exp72: $T_a = -10$ °C, $H = 0.15$ m, $U = 0.423$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

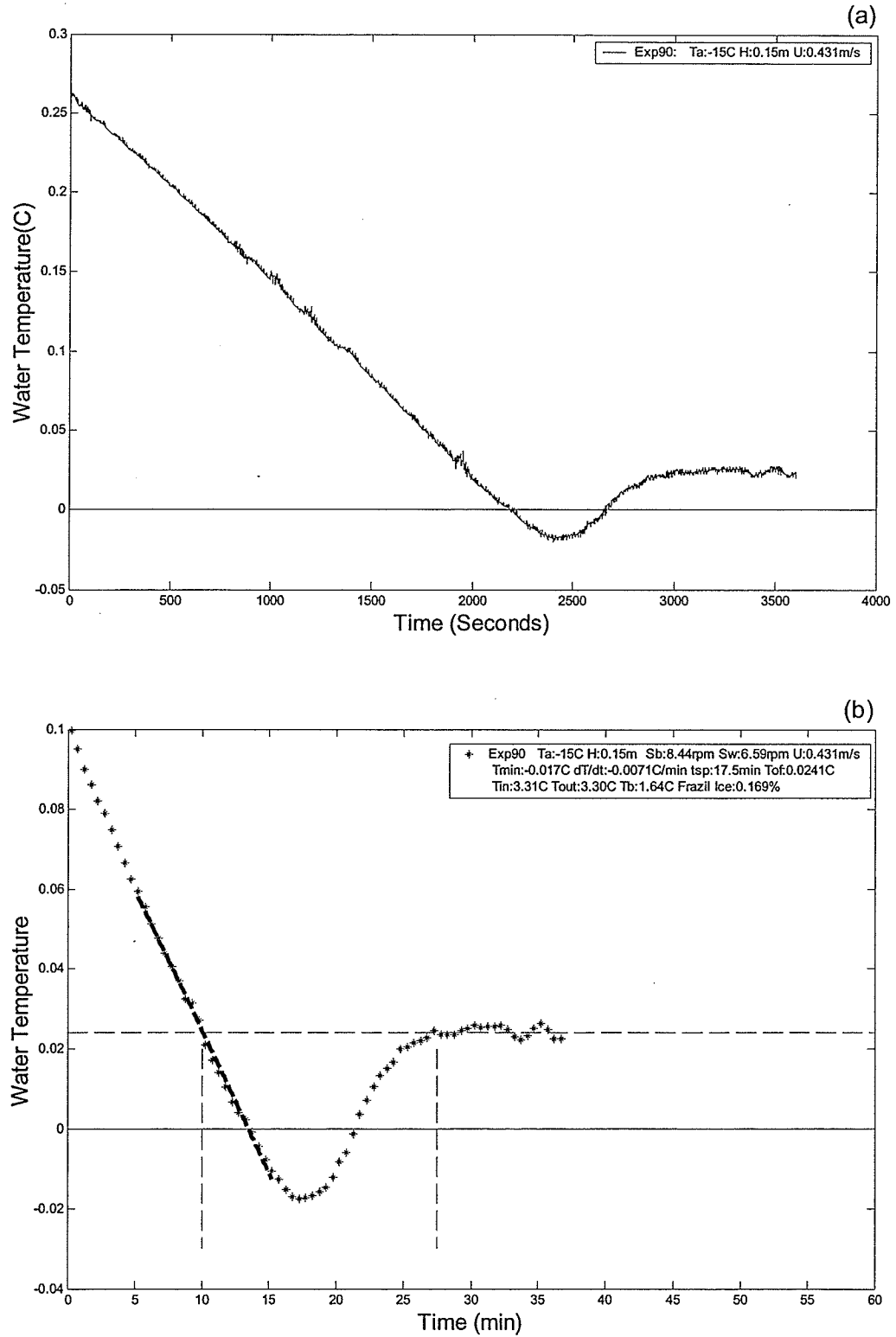


Figure A1, Group 1. Exp90: $T_a = -15^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.431\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

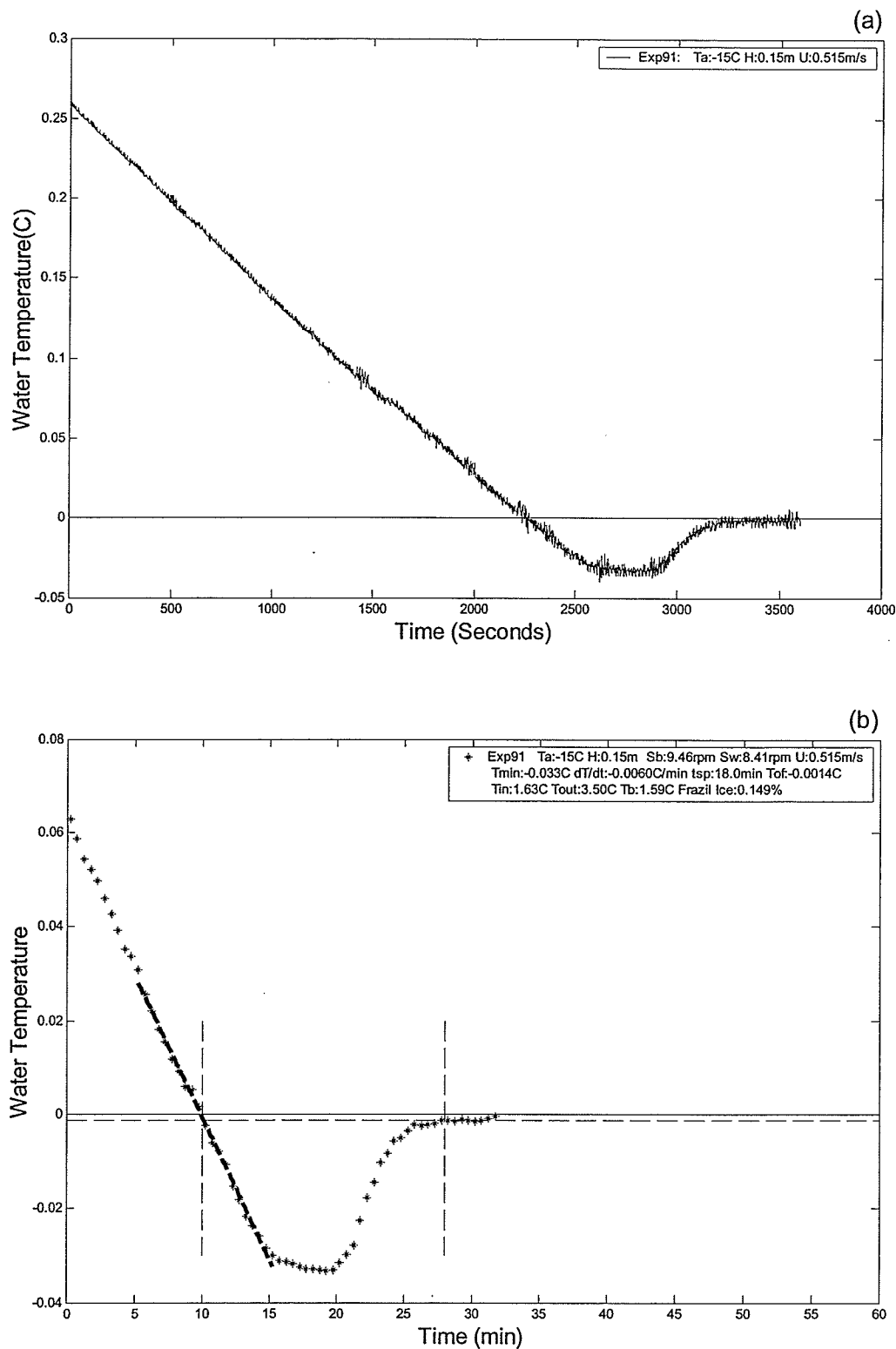


Figure A1, Group 1. Exp91: T_a = -15 °C, H = 0.15 m, U = 0.515 m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

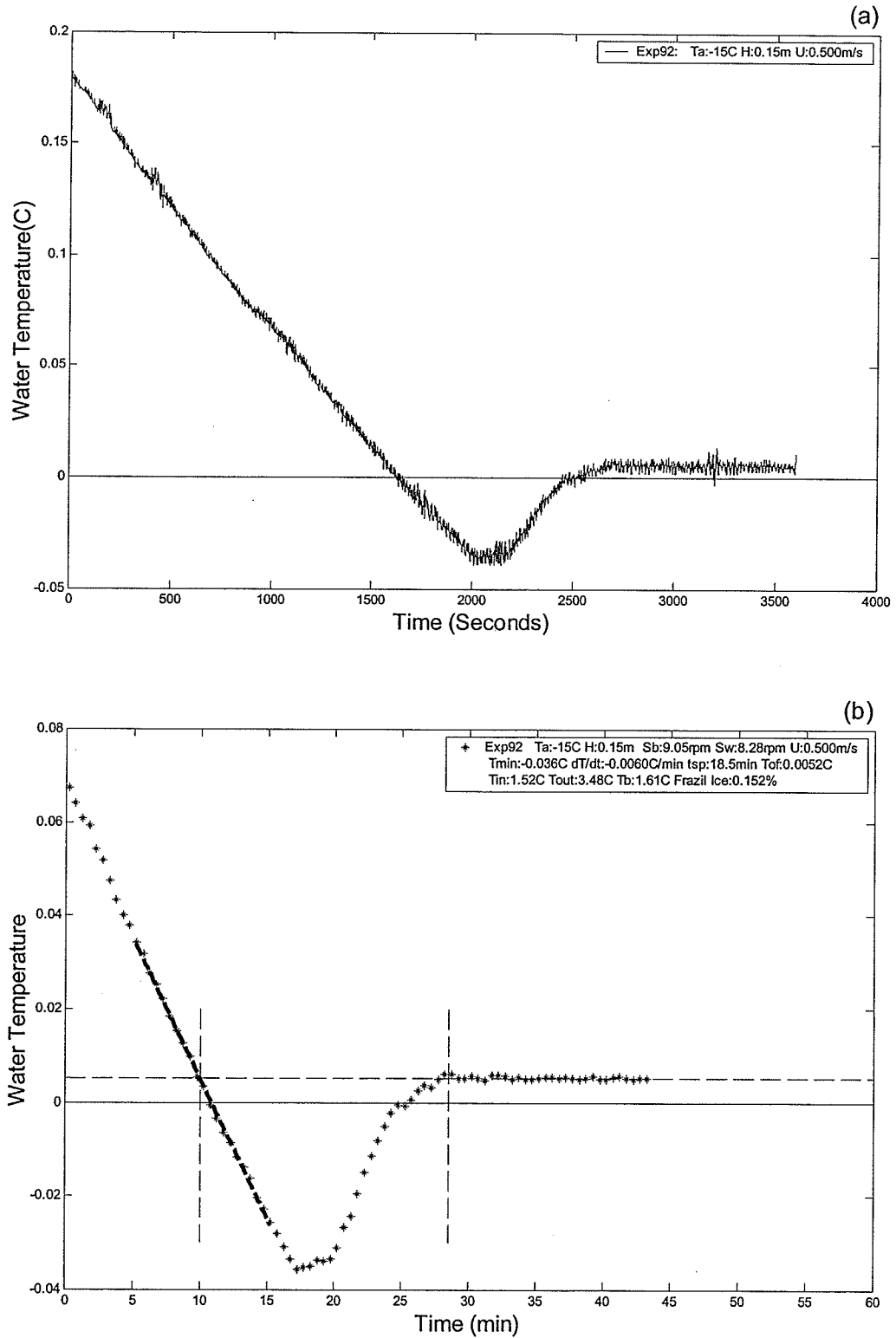


Figure A1, Group 1. Exp92: $T_a = -15^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.500\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

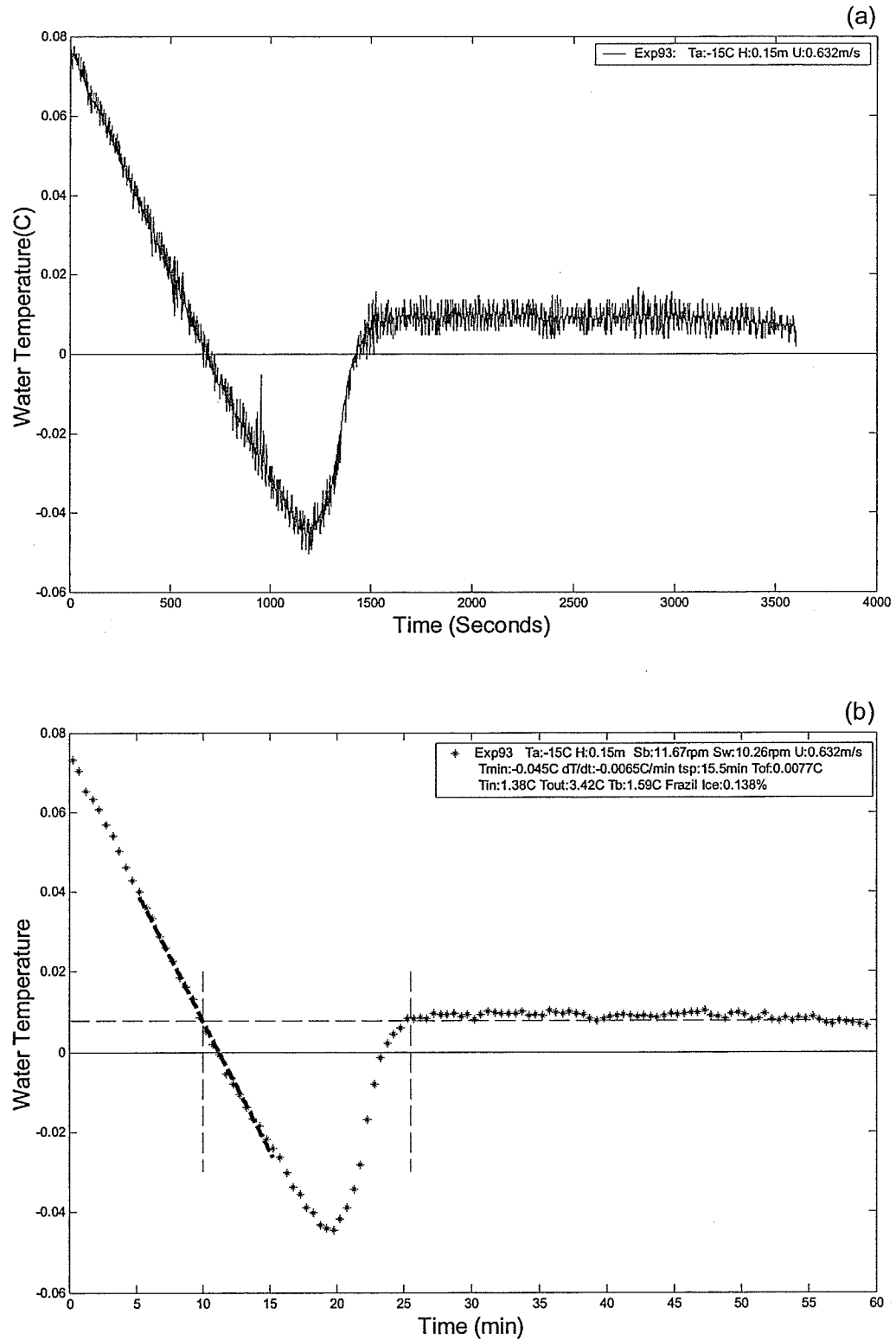


Figure A1, Group 1. Exp93: $T_a = -15^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.632\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

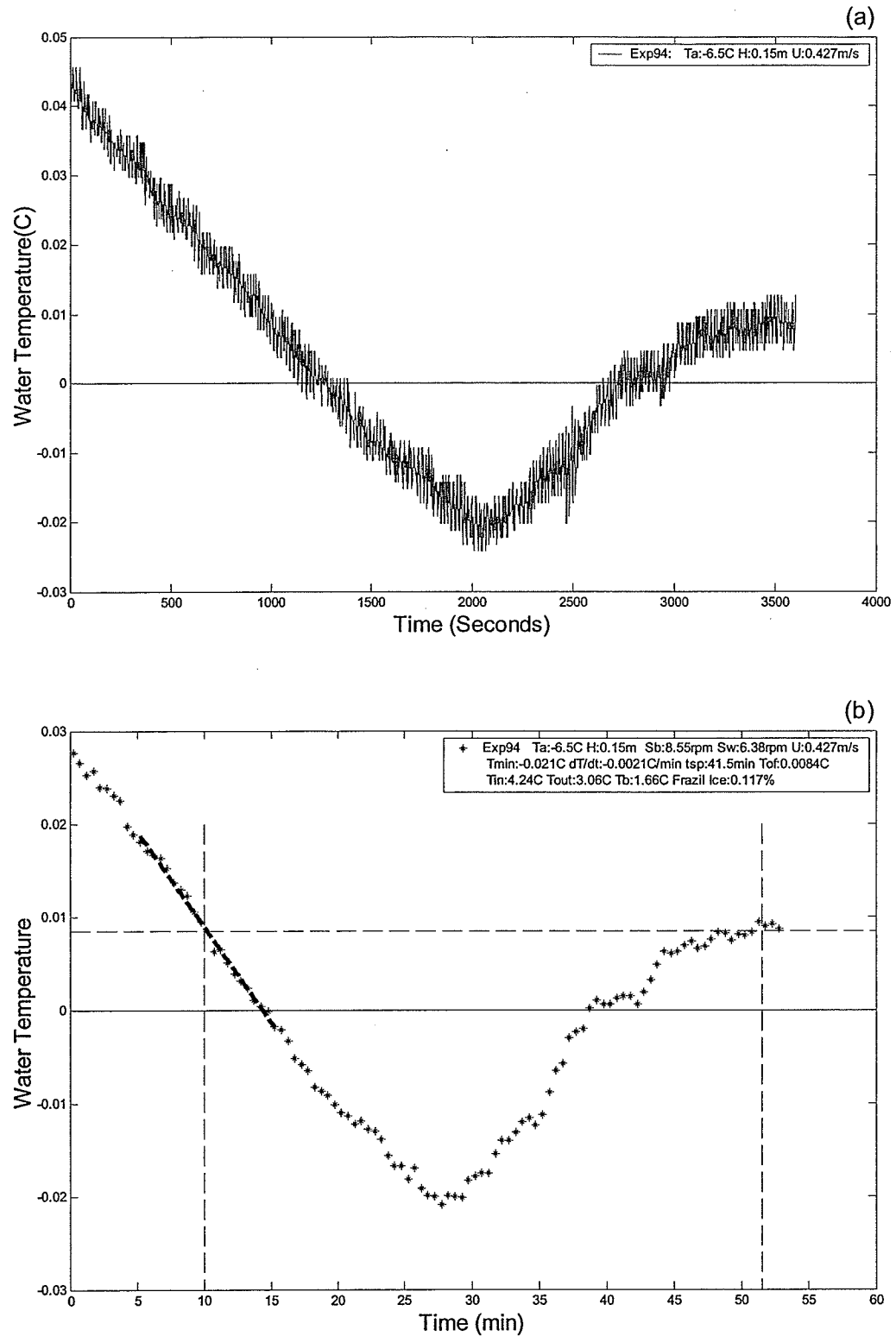


Figure A1. Group 1. Exp. 94. $T_a = -6.5^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.427\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

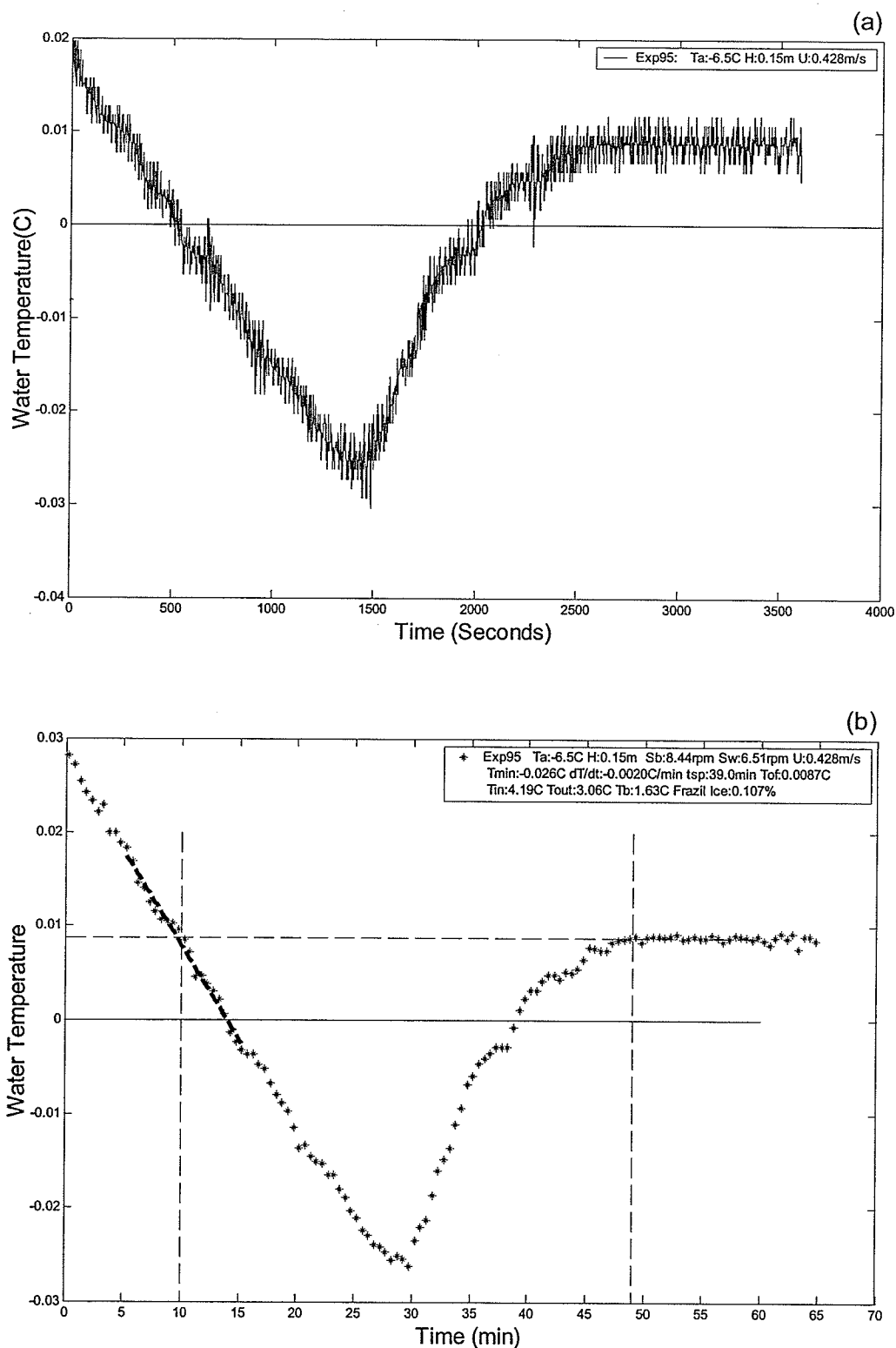


Figure A1. Group 1. Exp. 95. $T_a = -6.5^\circ\text{C}$, $H = 0.15\text{ m}$, $U = 0.428\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

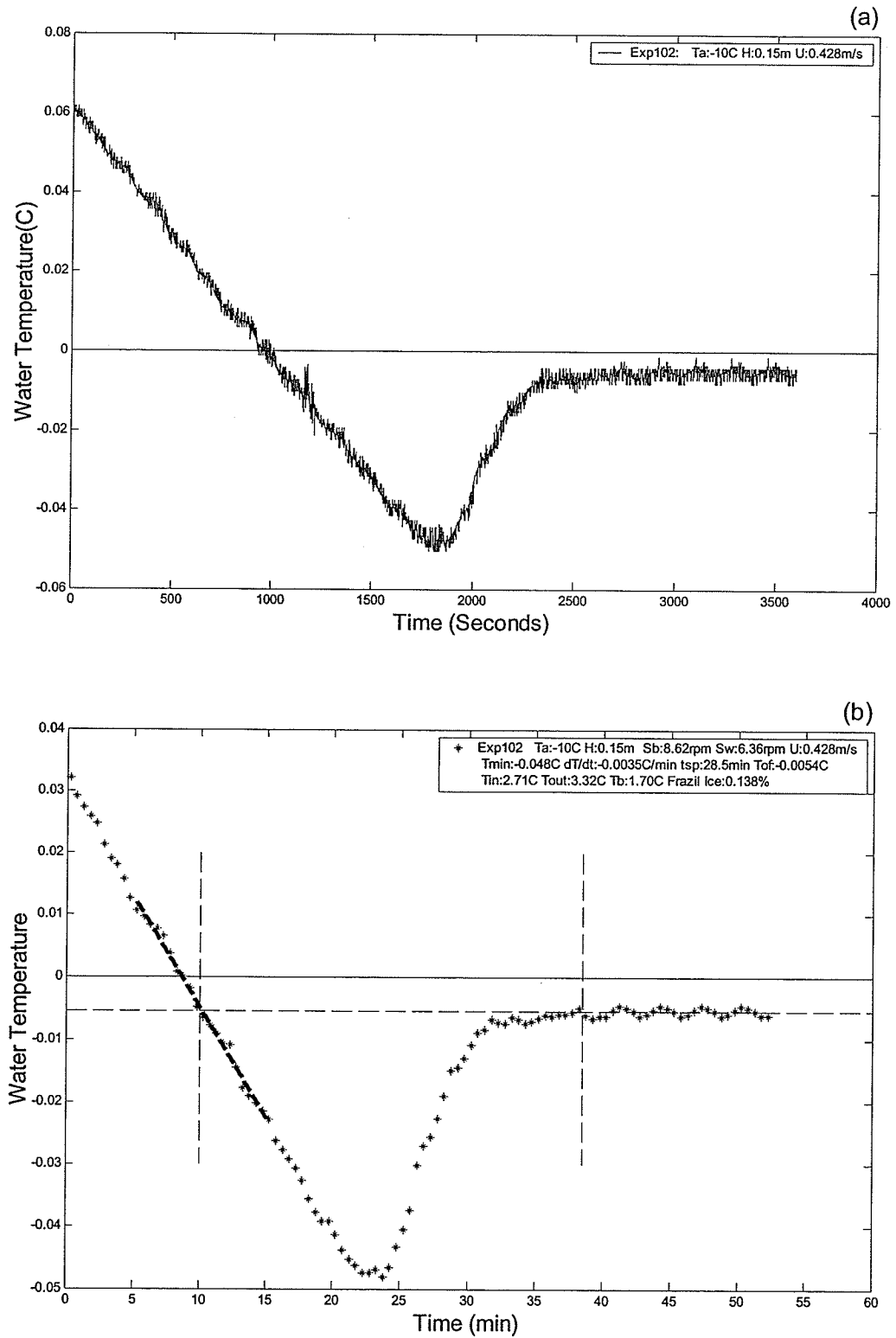


Figure A1. Group 1. Exp. 102. $T_a = -10$ °C, $H = 0.15$ m, $U = 0.428$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

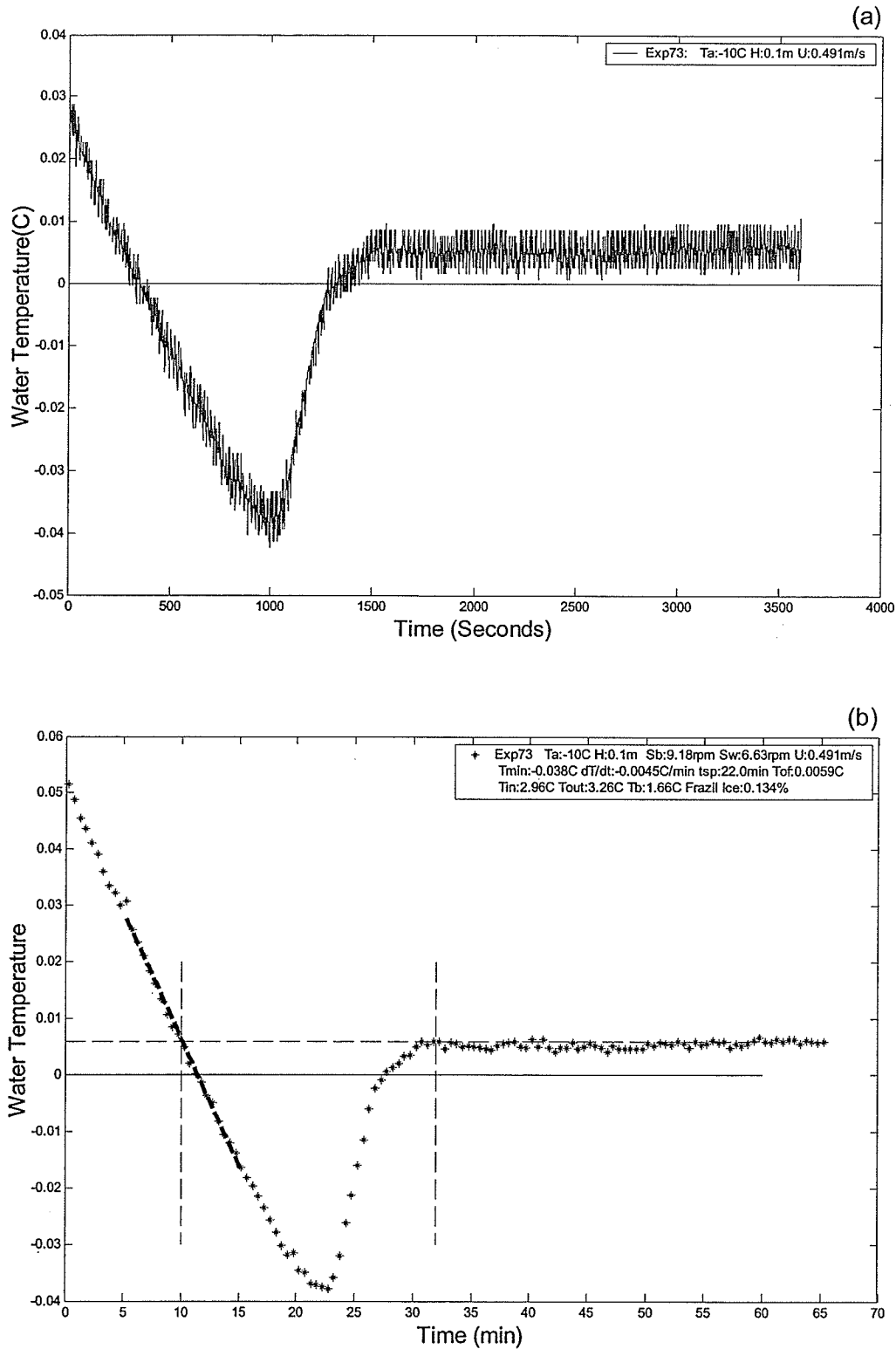


Figure A2. Group 2. Exp. 73. $T_a = -10$ °C, $H = 0.10$ m, $U = 0.491$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

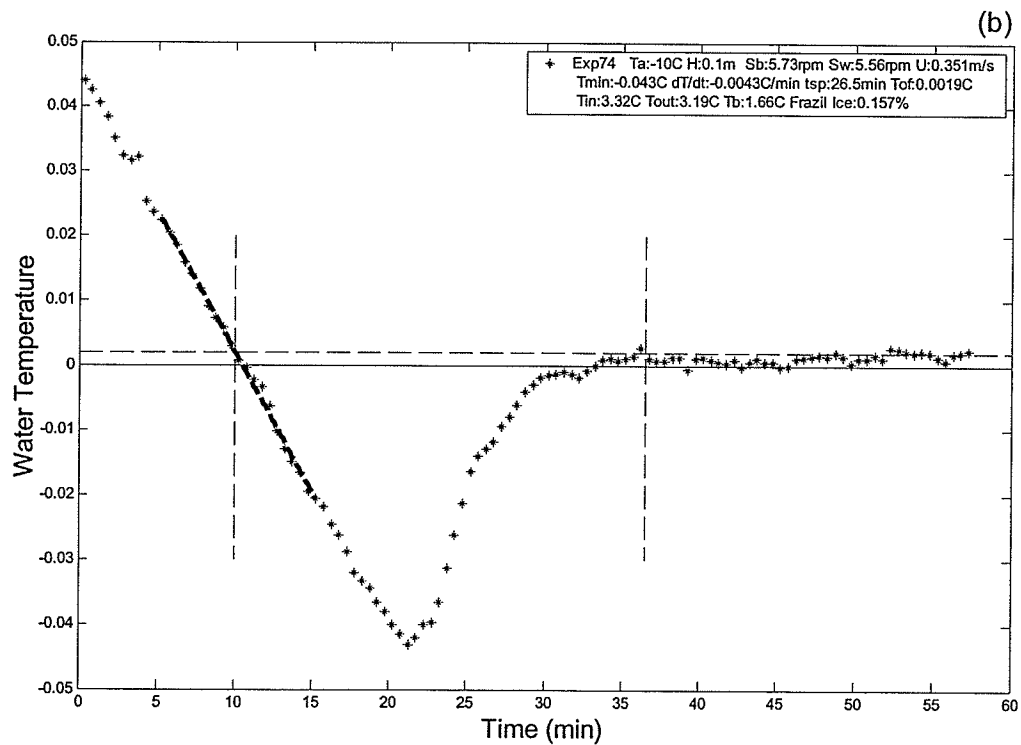
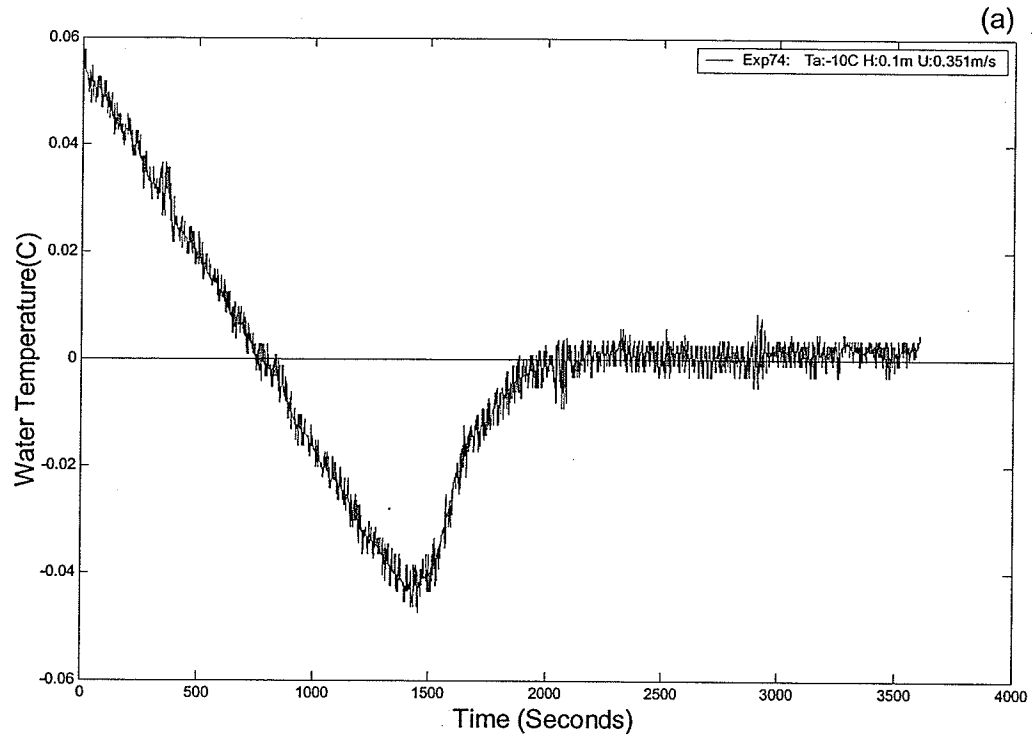


Figure A2. Group 2. Exp. 74. $T_a = -10^\circ\text{C}$, $H = 0.10\text{ m}$, $U = 0.351\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

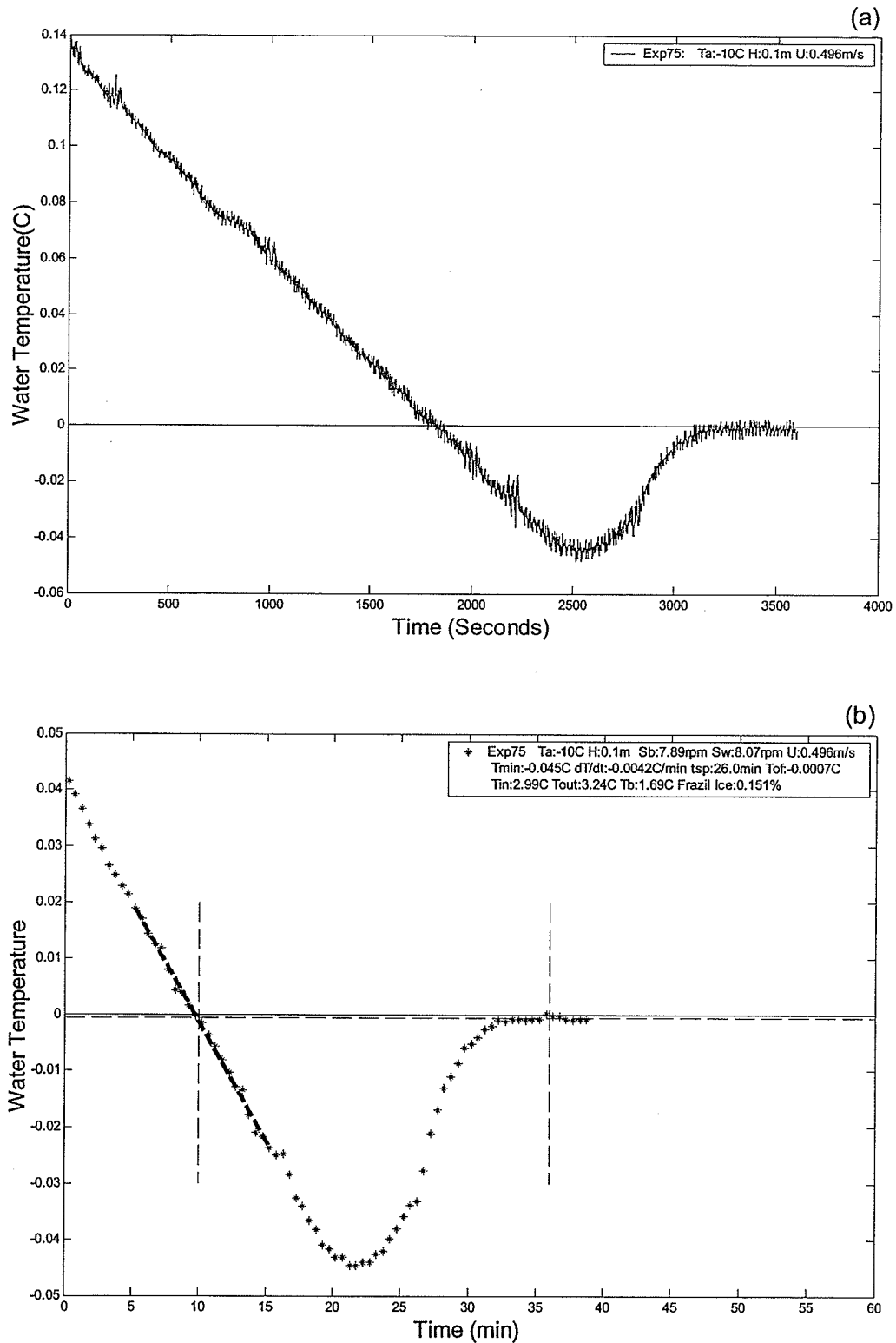


Figure A2. Group 2. Exp.75. $T_a = -10^{\circ}\text{C}$, $H = 0.10\text{ m}$, $U = 0.496\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

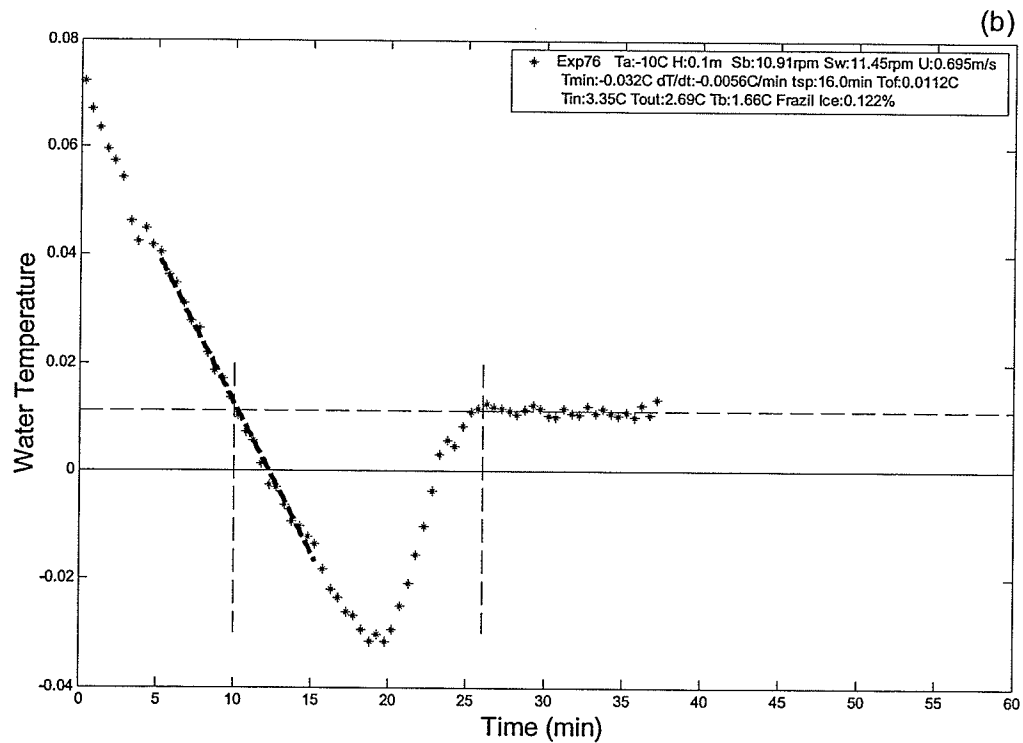
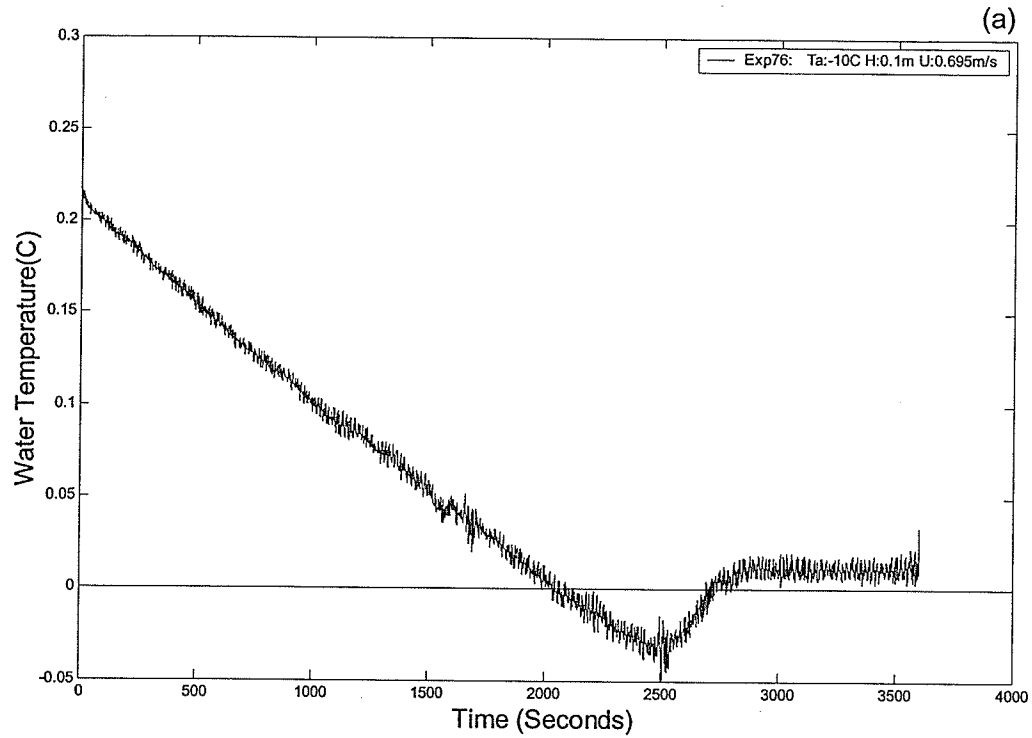


Figure A2. Group 2. Exp.76. $T_a = -10^\circ\text{C}$, $H = 0.10\text{ m}$, $U = 0.695\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

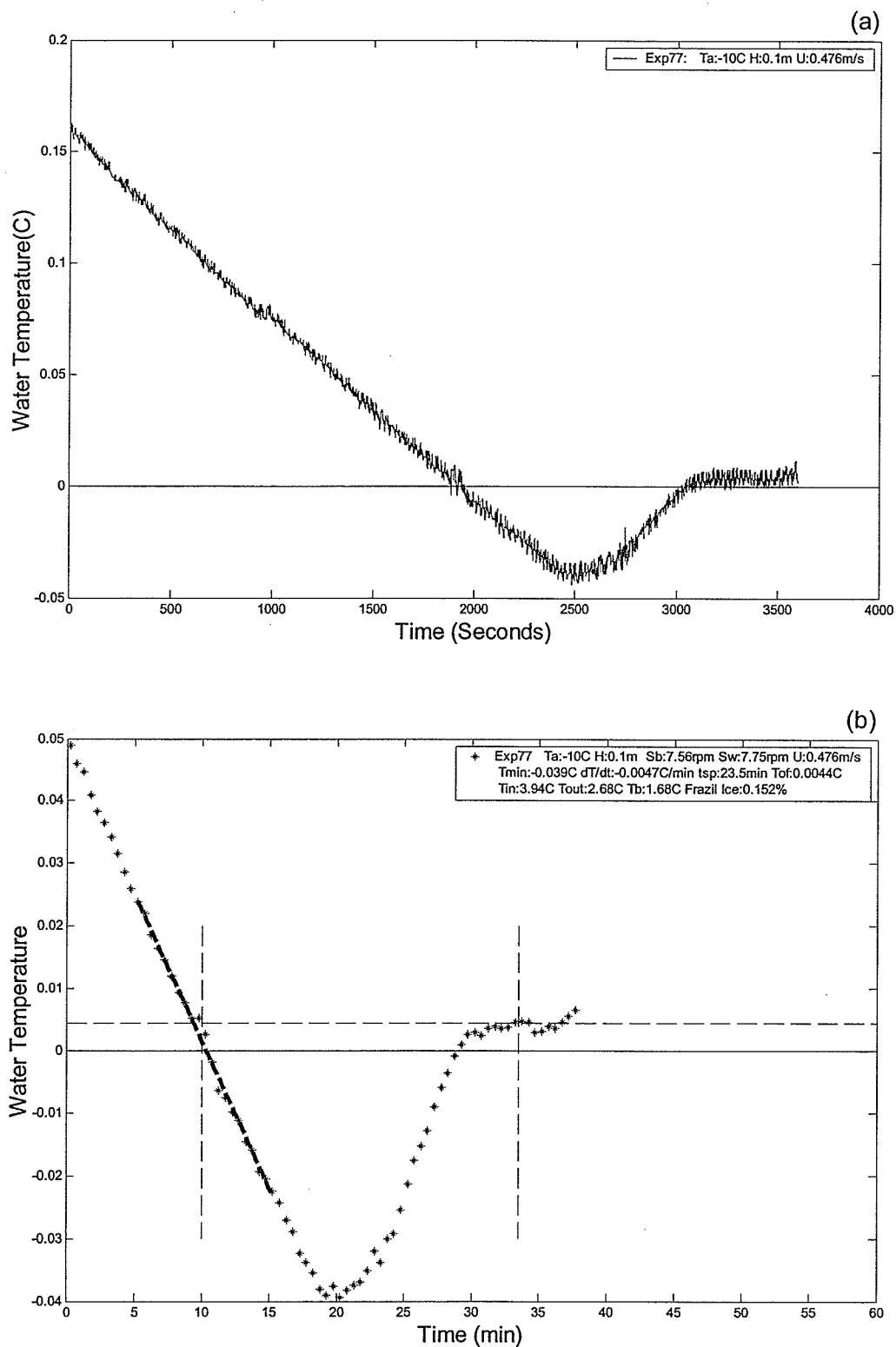


Figure A2. Group 2. Exp.77. $T_a = -10^\circ\text{C}$, $H = 0.10\text{ m}$, $U = 0.476\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

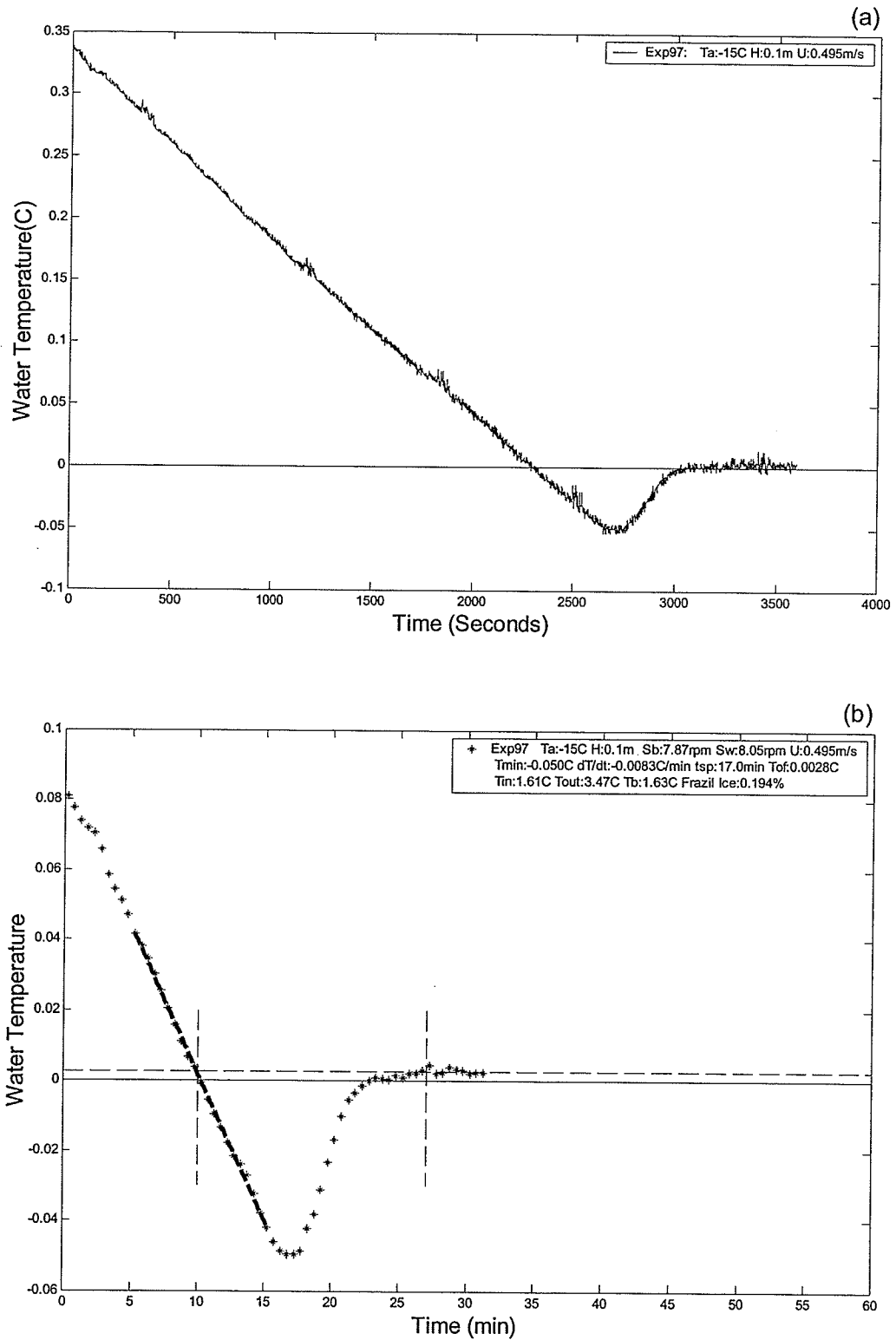


Figure A2. Group 2. Exp. 97. $T_a = -15$ °C, $H = 0.10$ m, $U = 0.495$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

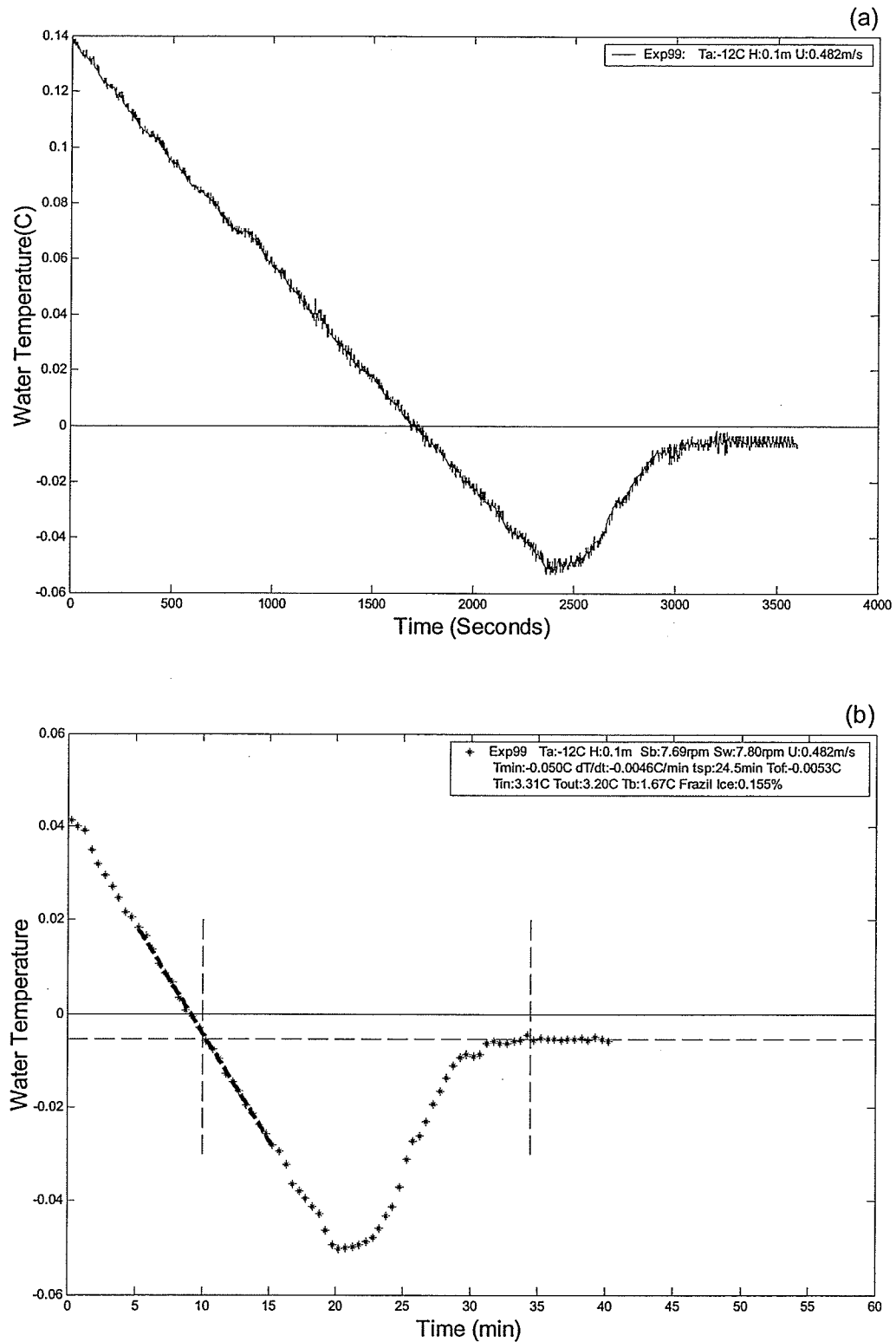


Figure A2. Group 2. Exp.99. $T_a = -11^\circ\text{C}$, $H = 0.10\text{ m}$, $U = 0.482\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

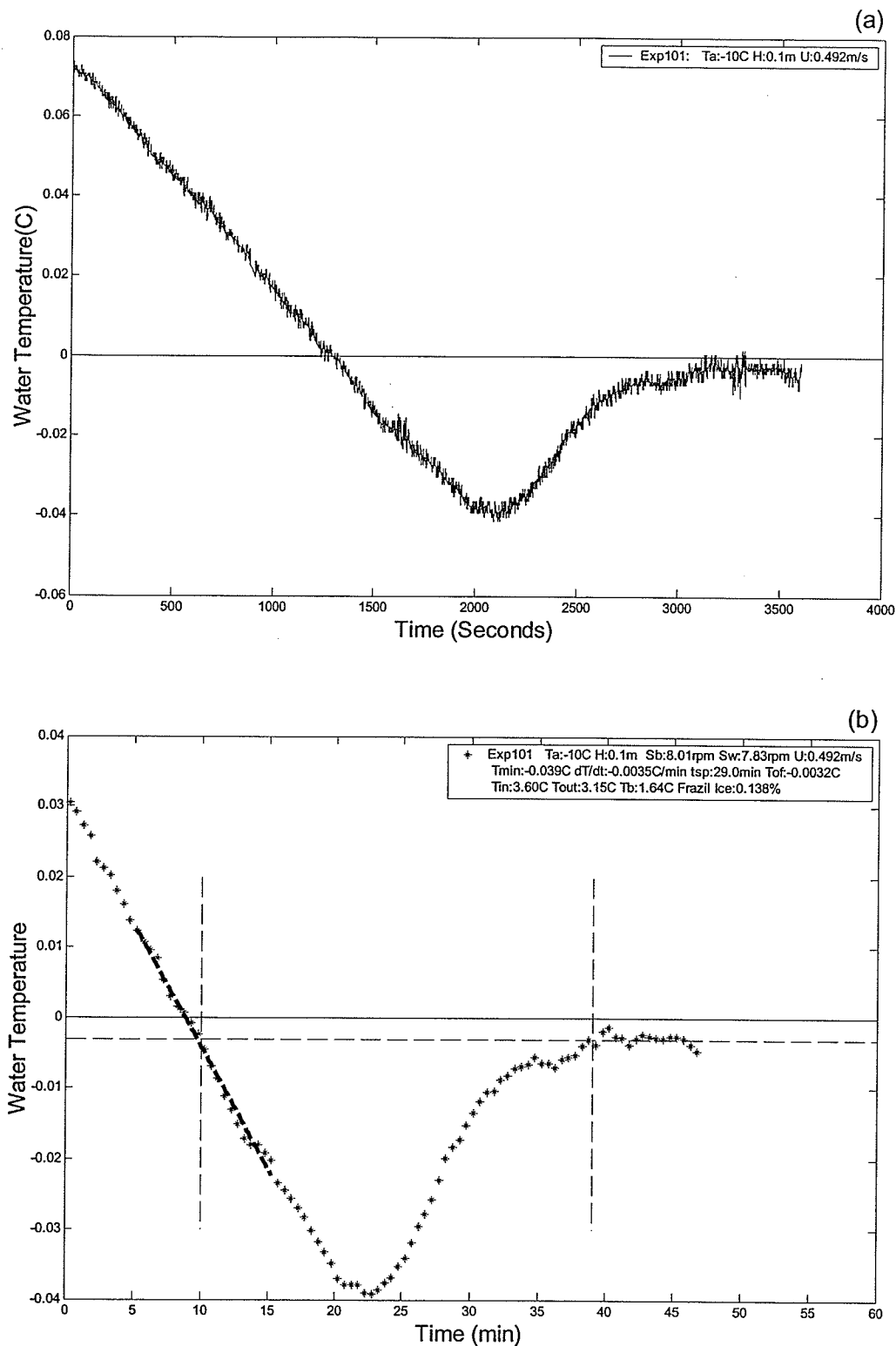


Figure A2. Group 2. Exp. 101. $T_a = -7^\circ\text{C}$, $H = 0.10\text{ m}$, $U = 0.492\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

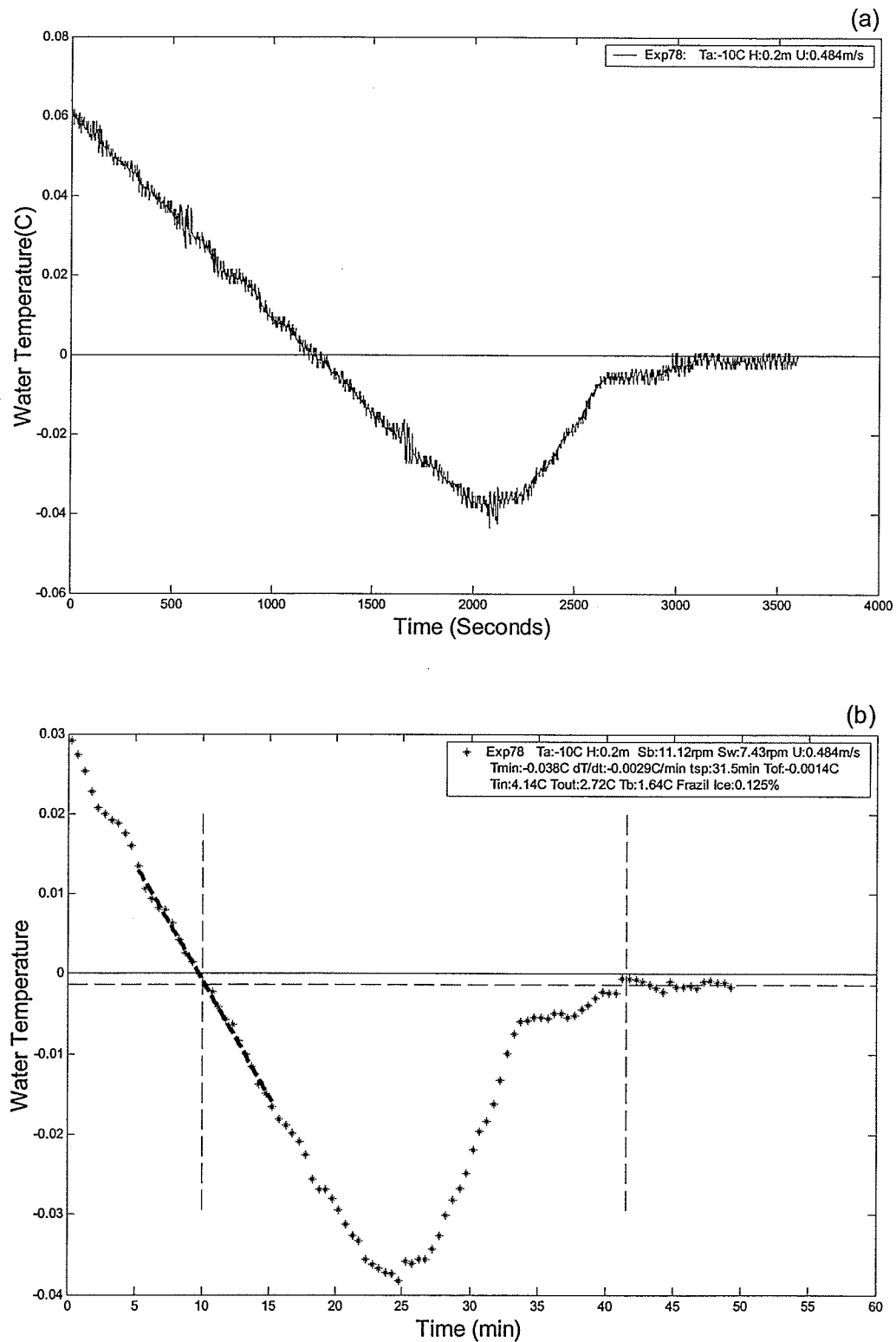


Figure A3. Group 3. Exp. 78. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.484\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

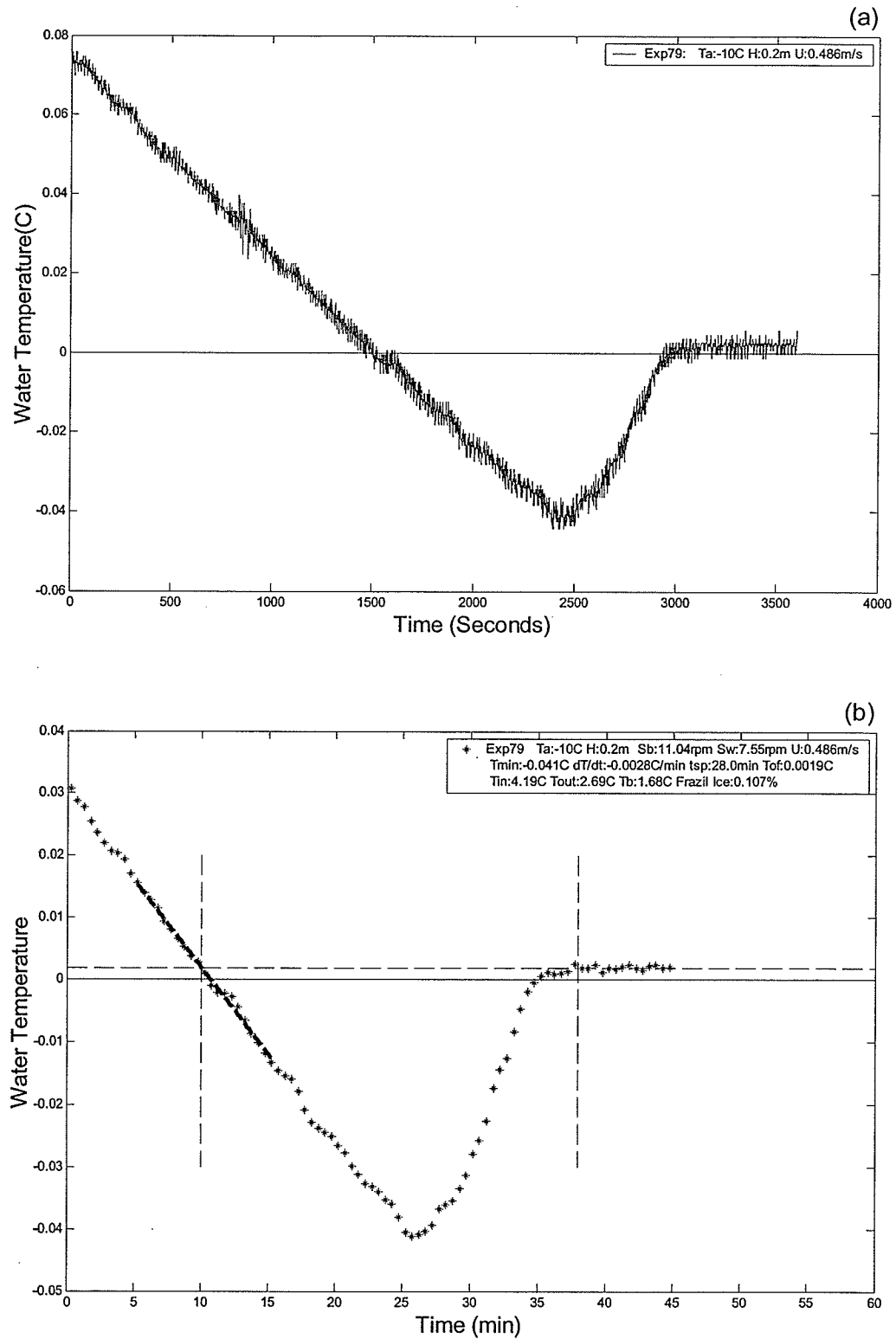


Figure A3. Group 3. Exp. 79. $T_a = -10$ °C, $H = 0.20$ m, $U = 0.486$ m/s

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

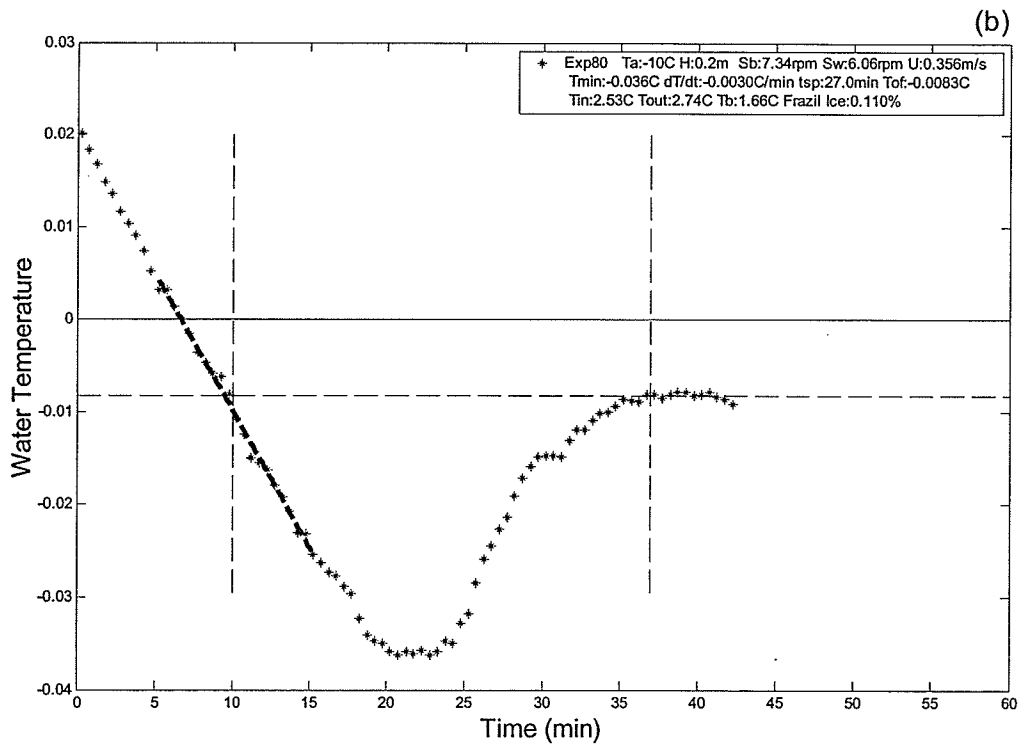
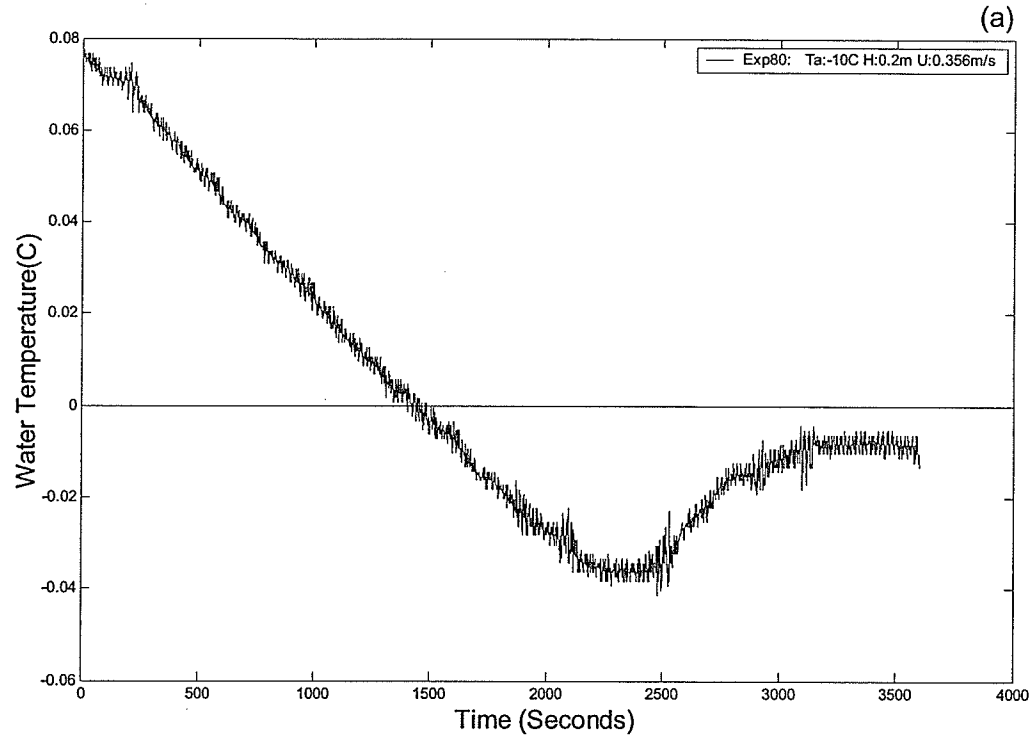


Figure A3. Group 3. Exp. 80. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.356\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

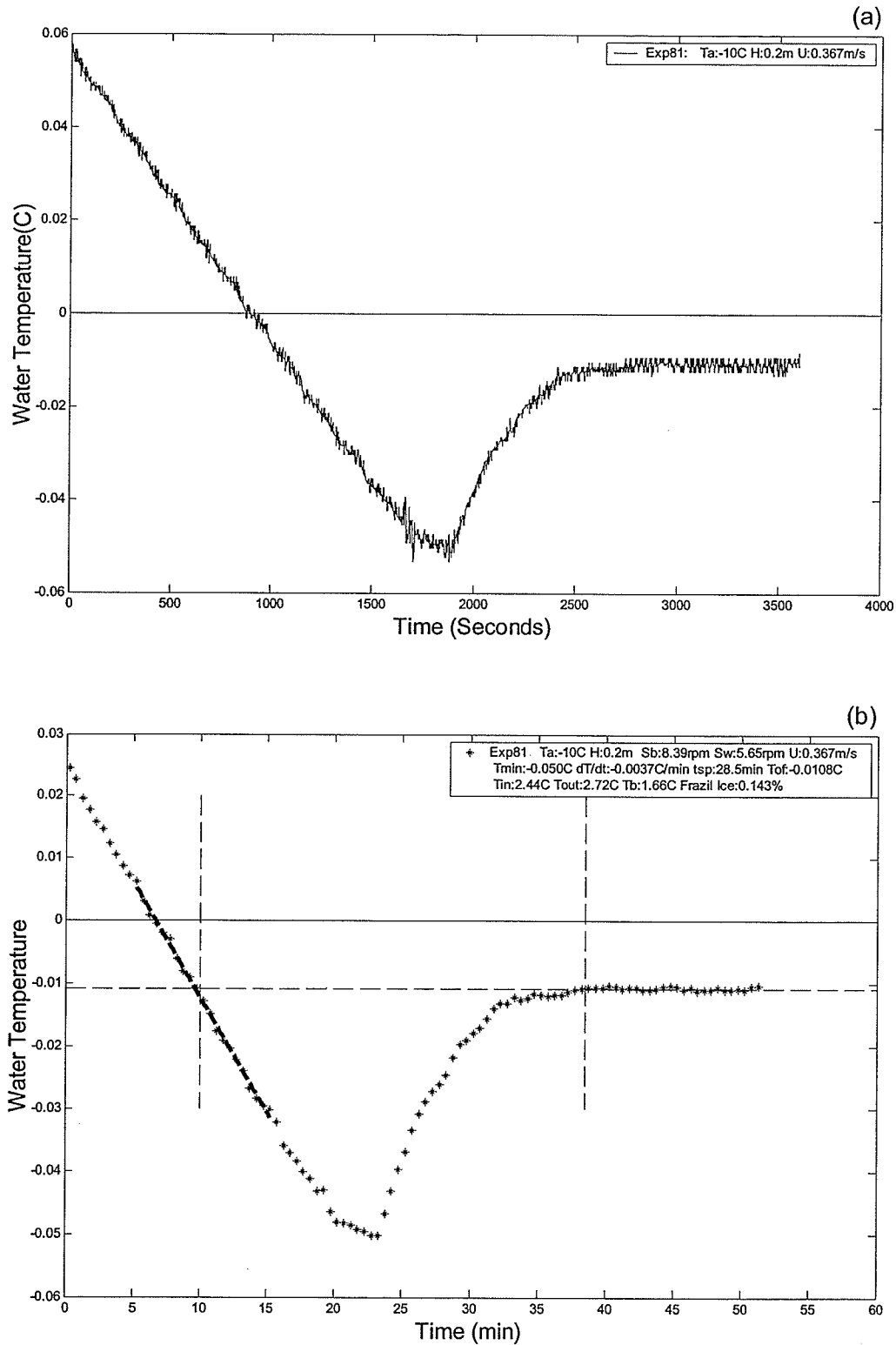


Figure A3. Group 3. Exp. 81. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.367\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

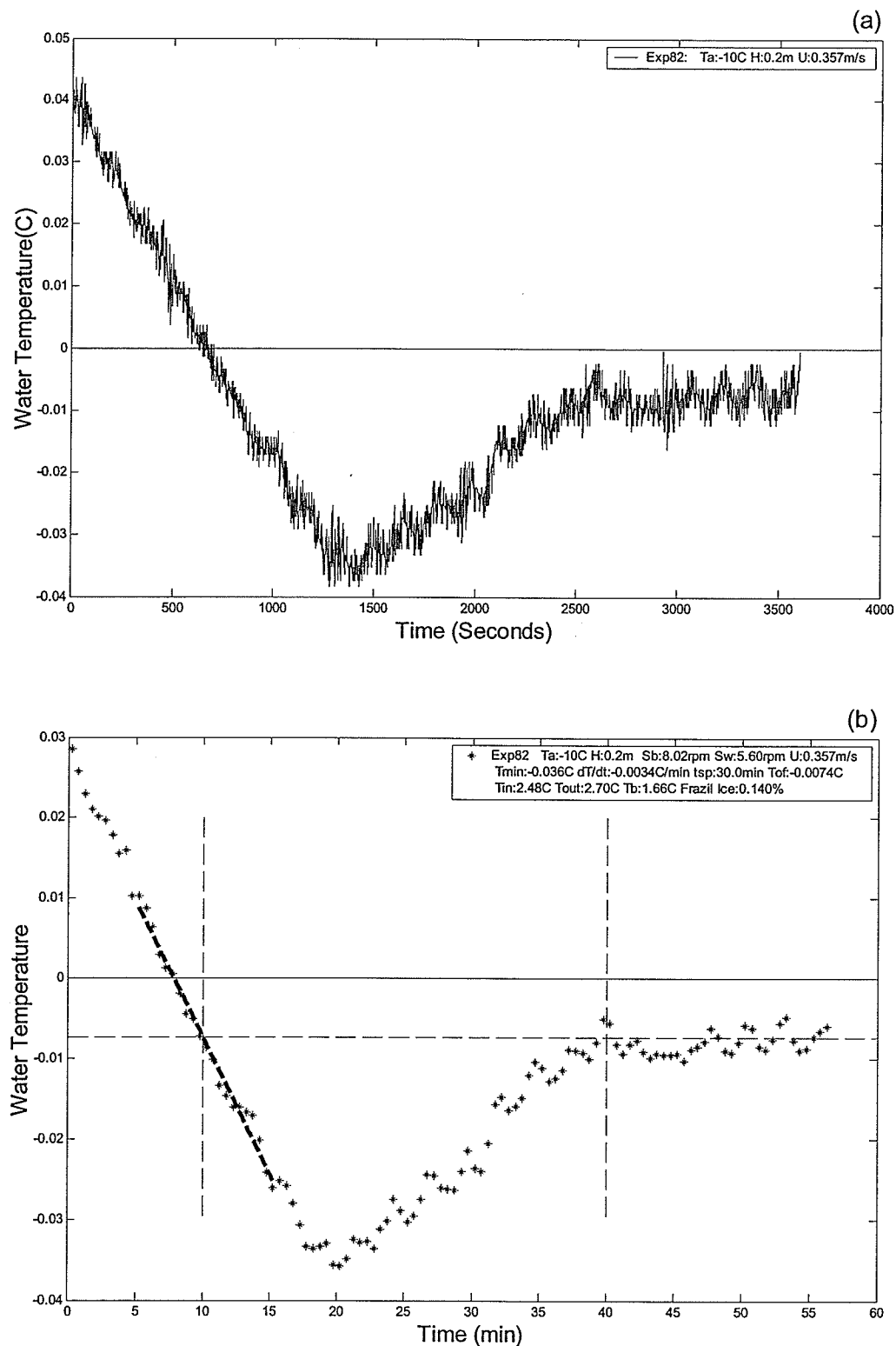


Figure A3. Group 3. Exp. 82. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.357\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

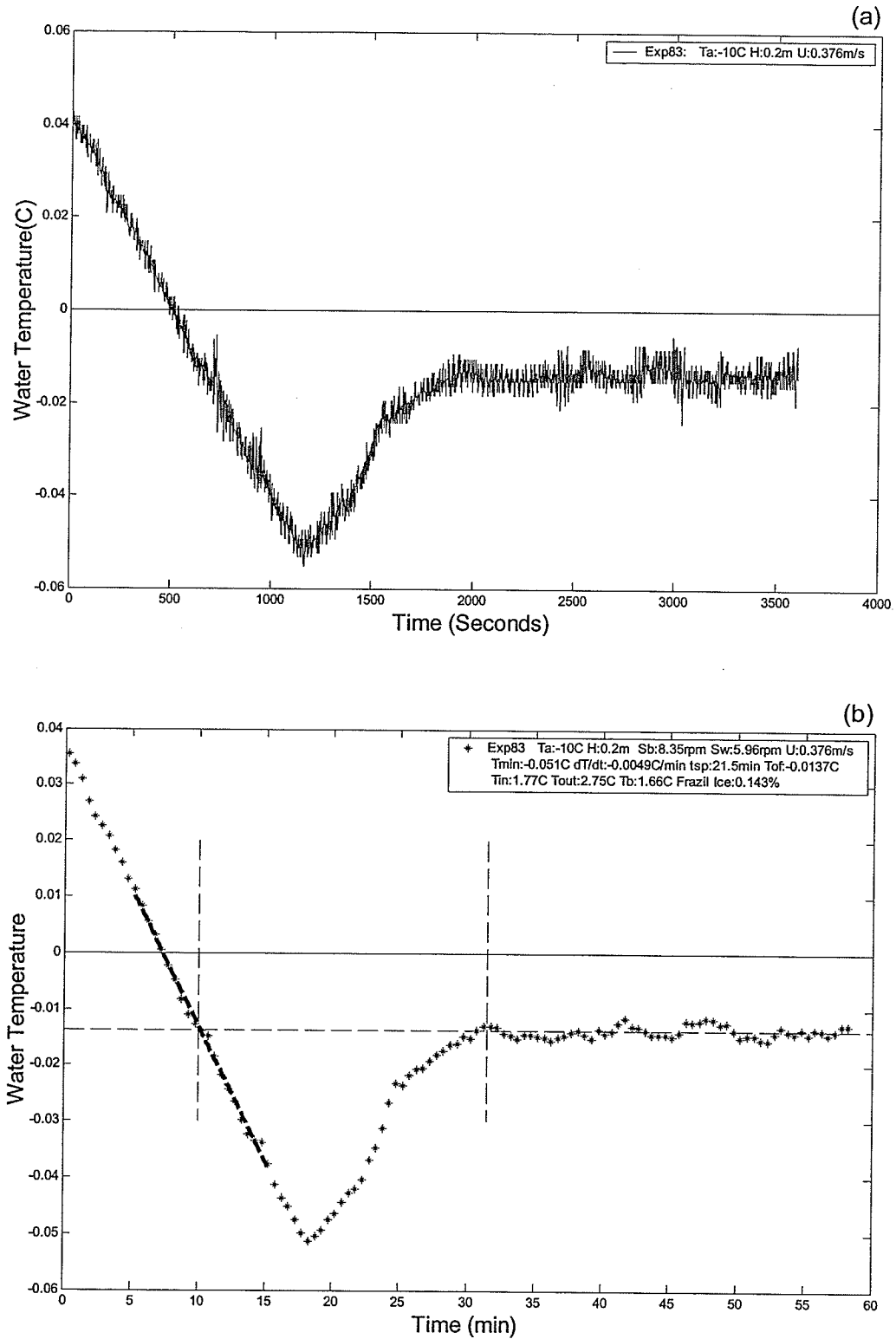


Figure A3. Group 3. Exp. 83. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.376\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

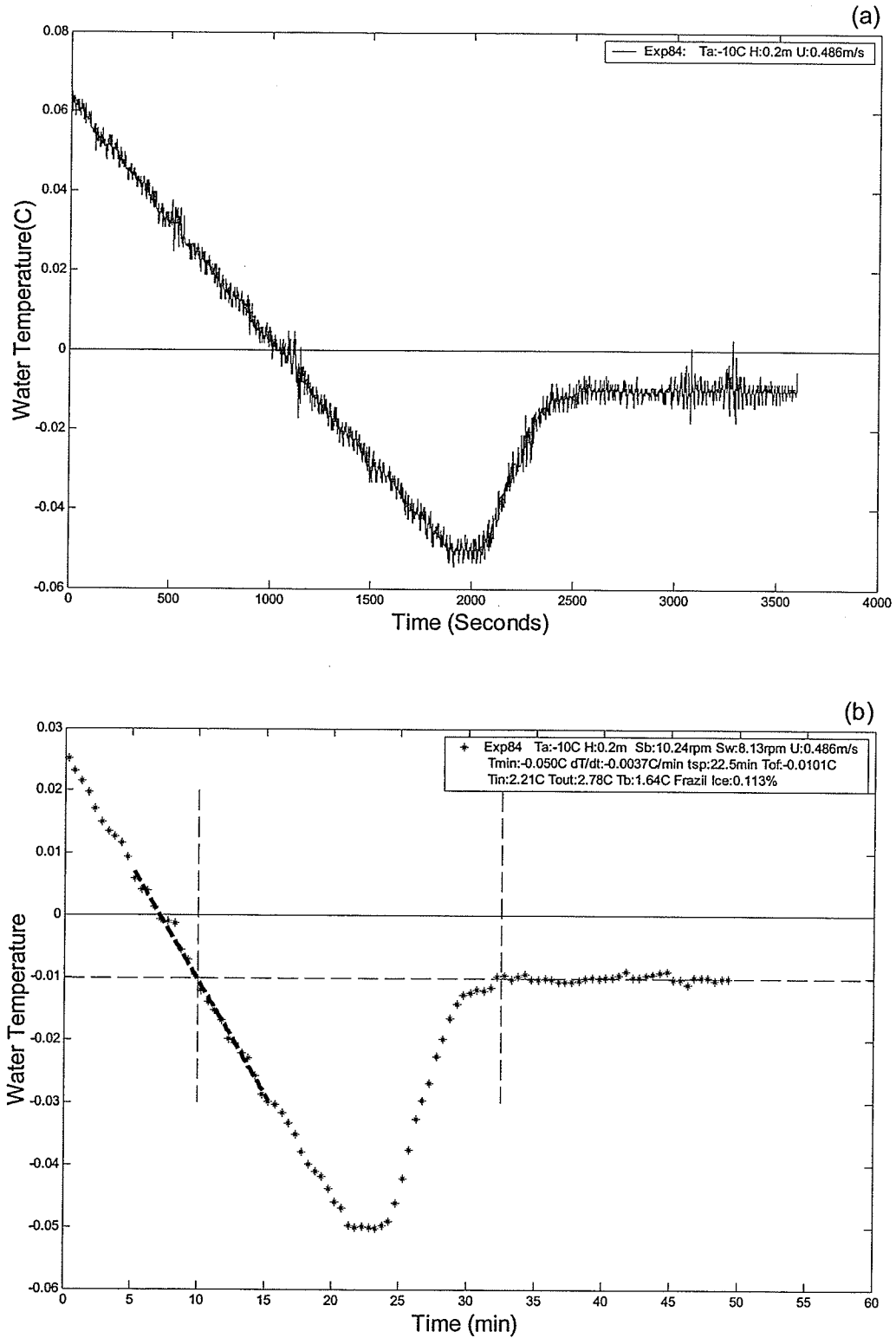


Figure A3. Group 3. Exp. 84. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.486\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

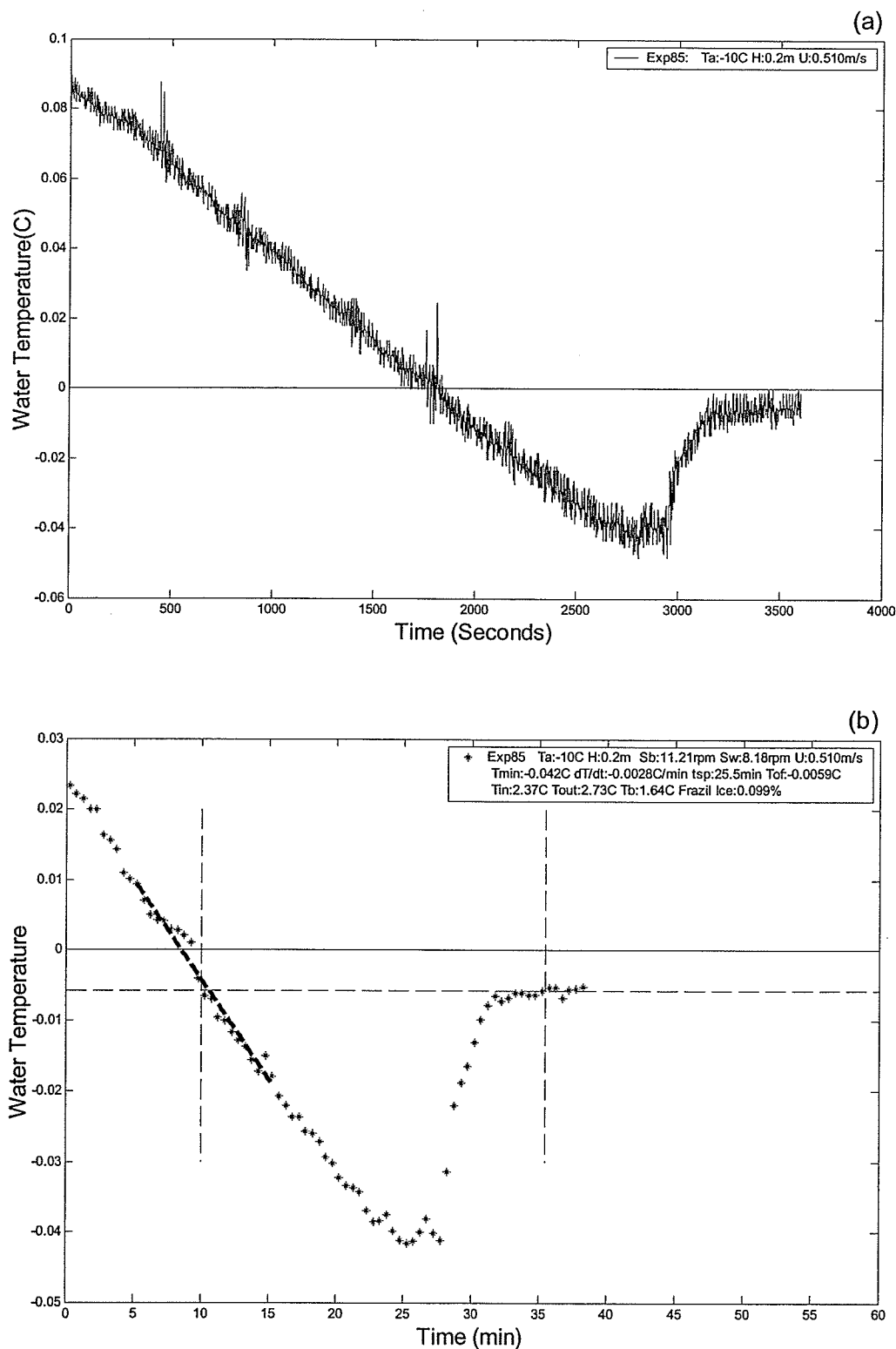


Figure A3. Group 3. Exp. 85. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.510\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

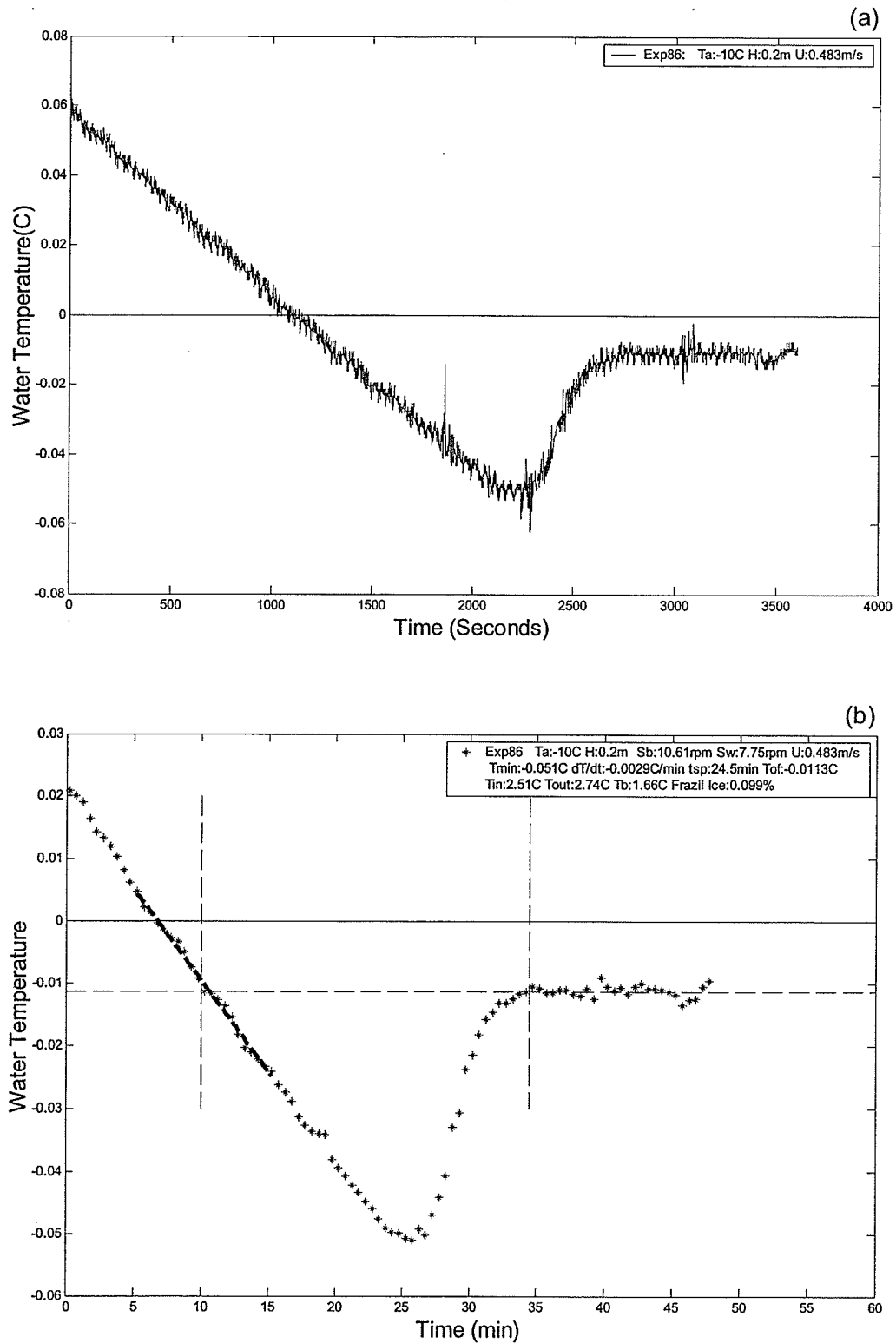


Figure A3. Group 3. Exp. 86. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.483\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

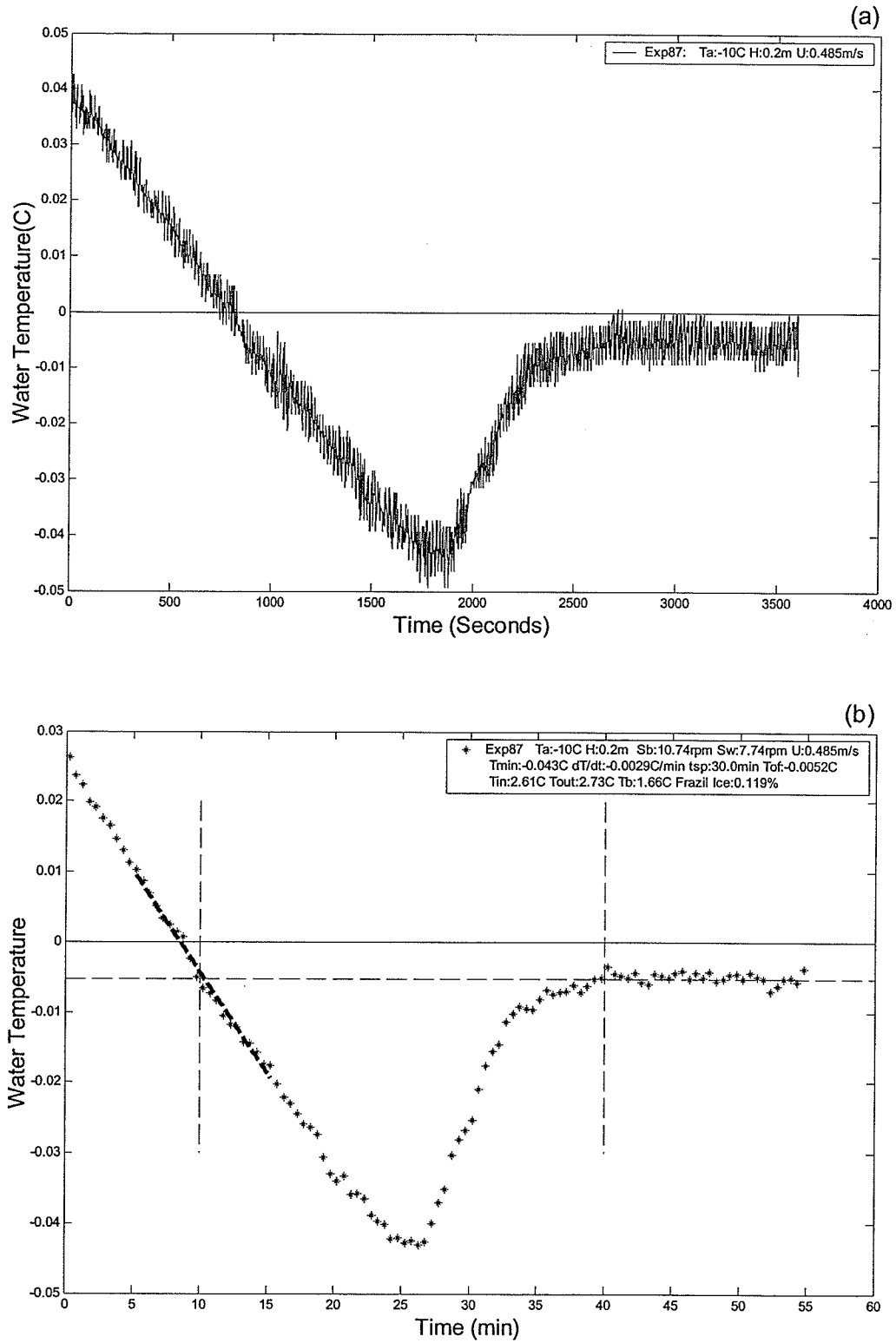


Figure A3. Group 3. Exp. 87. $T_a = -10^\circ\text{C}$, $H = 0.20\text{ m}$, $U = 0.485\text{ m/s}$

(a) Despiked water temperature vs. time, (b) Characteristics of supercooling process

Frazil Image Observations

The data presented in this appendix shows the results of the digital image processing (DIPS) of the experiments carried out in the counter-rotating flume. Due to uncertainties and technical problems with the DIPS, only some of the experiments in Groups 1 and 3 were processed and presented here. Group 1 has 15 tests and Group 3 has 8 tests.

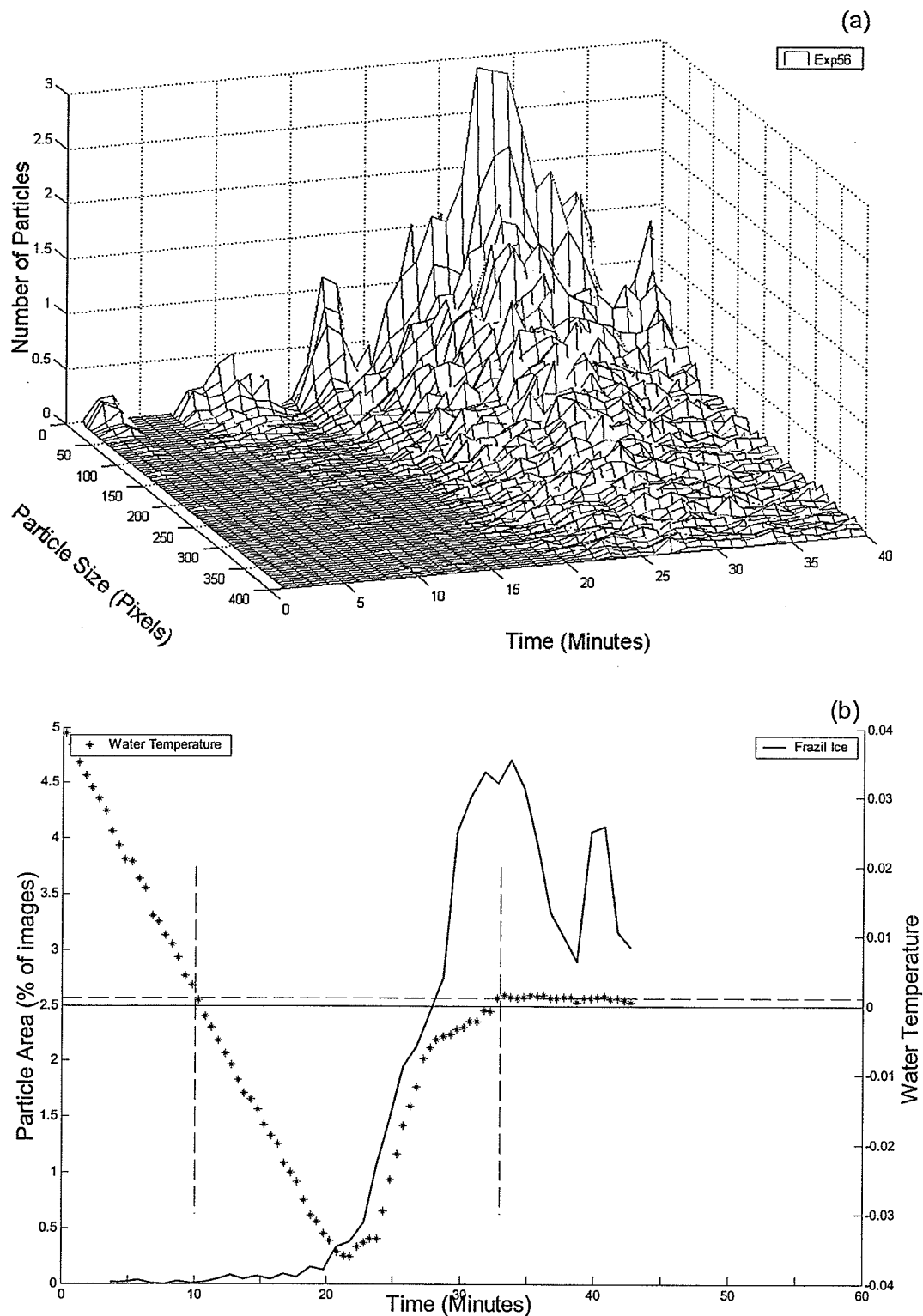


Figure B1. Group1. Exp. 56. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.434\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

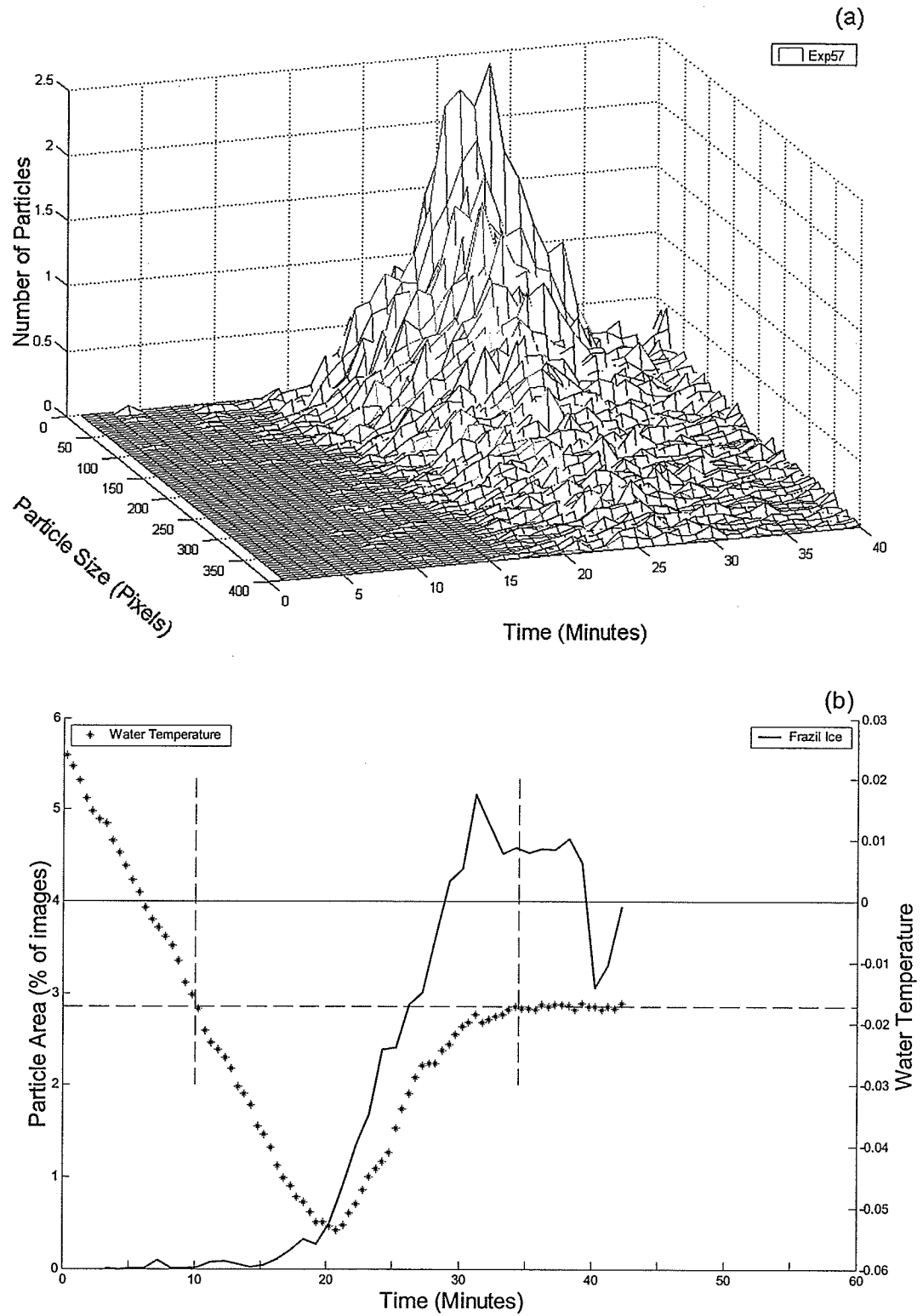


Figure B1. Group1. Exp. 57. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.435\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

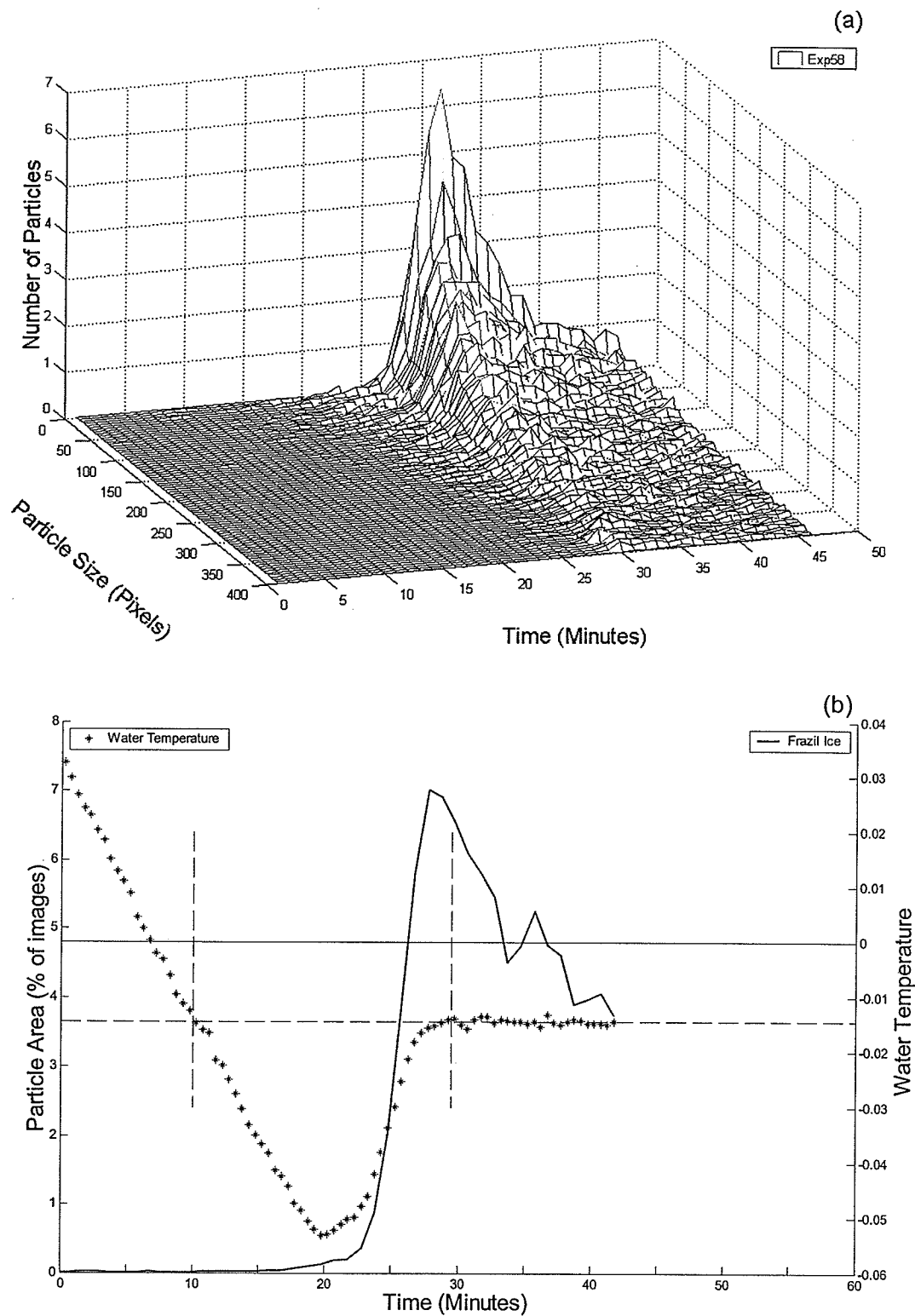


Figure B1. Group 1. Exp. 58. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.517\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

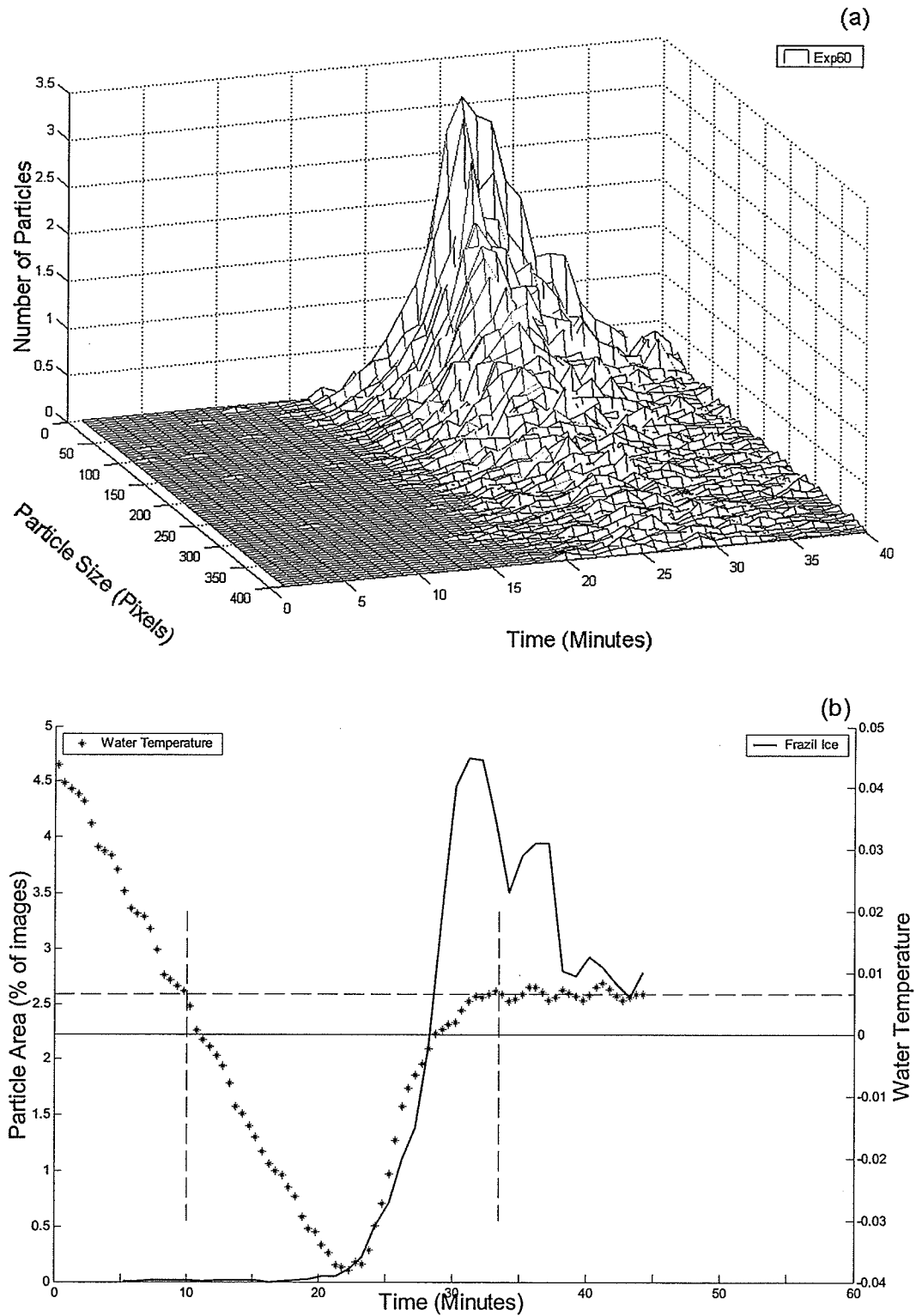


Figure B1. Group1. Exp. 60. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.424\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

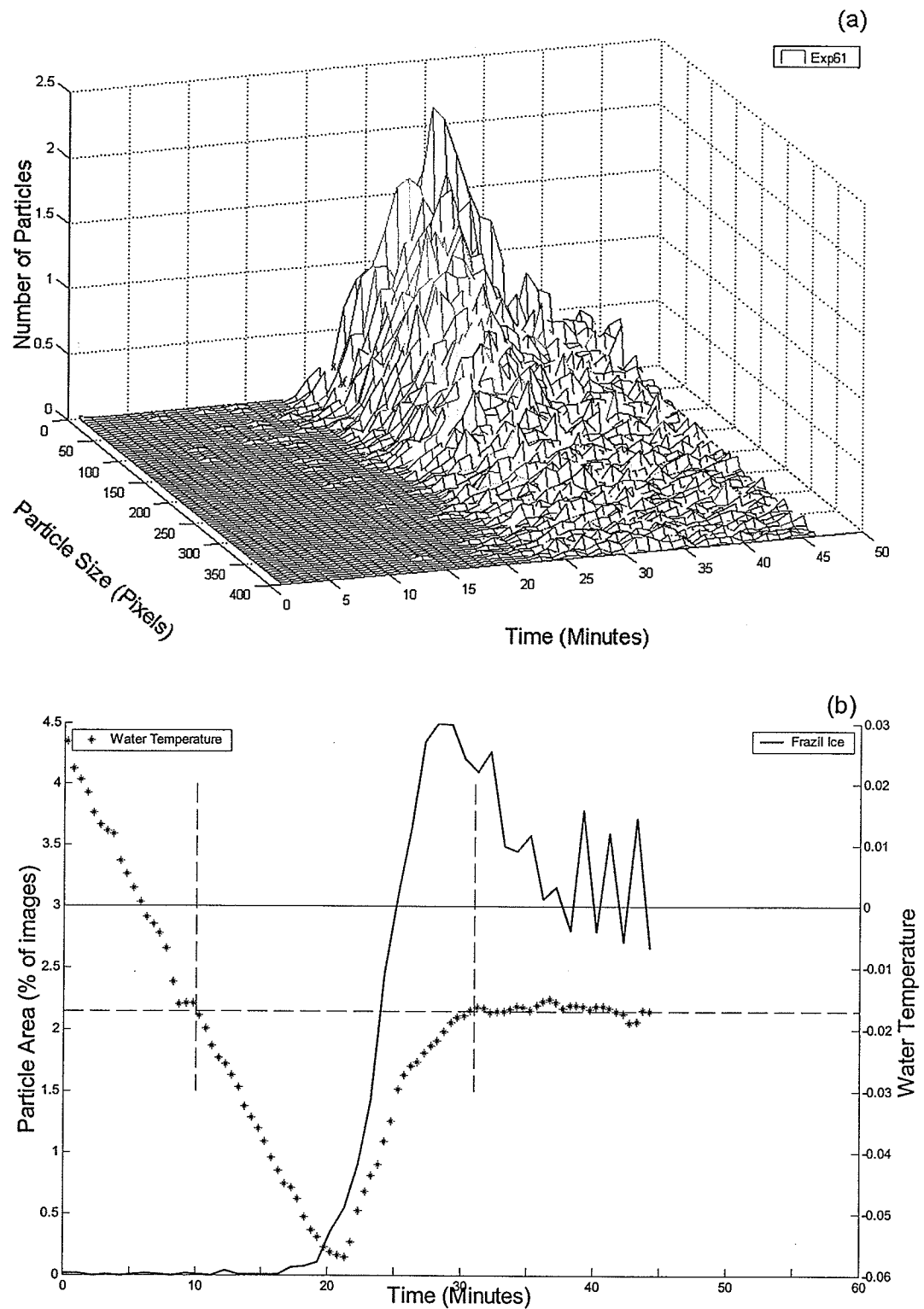


Figure B1. Group 1. Exp. 61. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.421\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

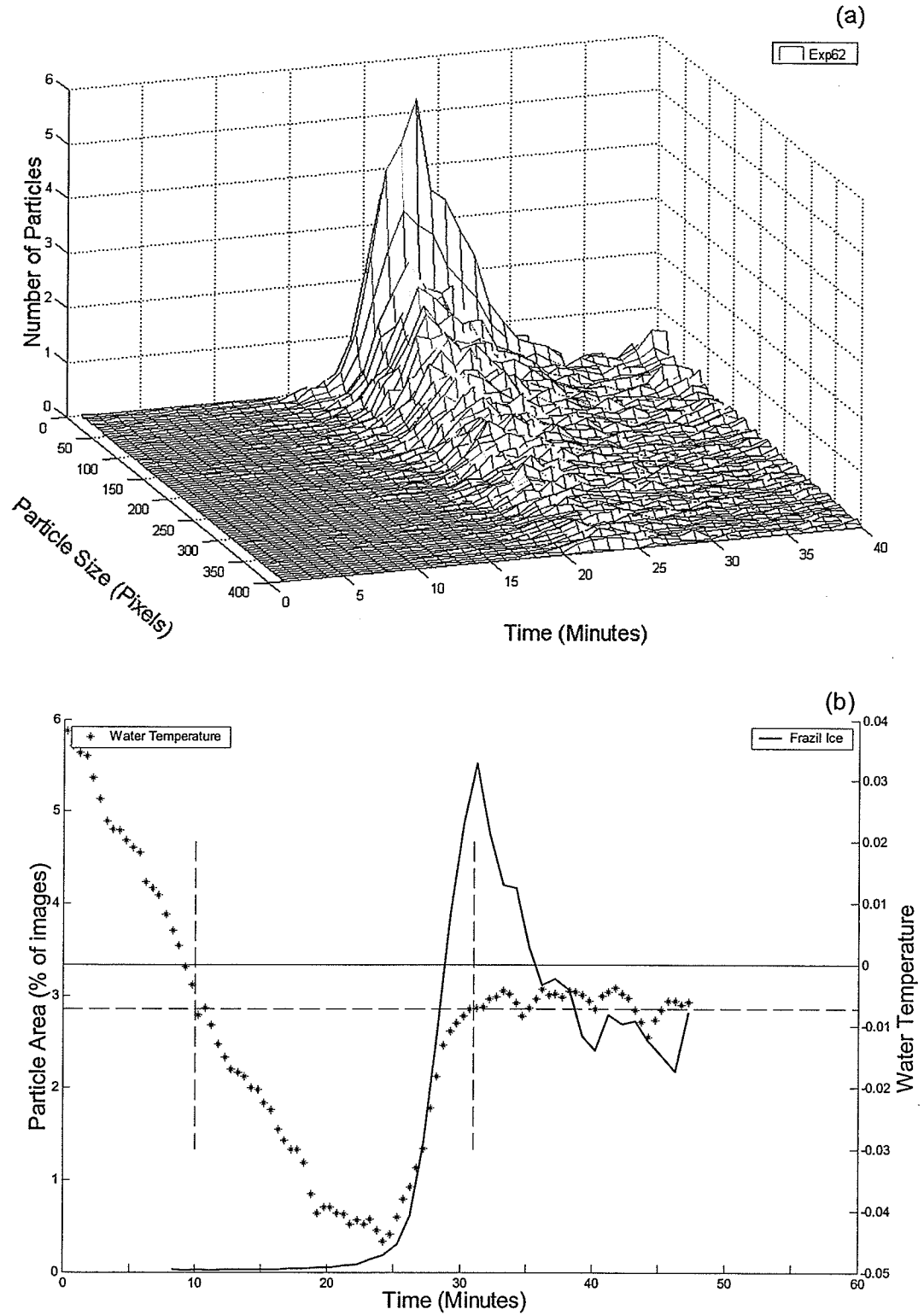


Figure B1. Group 1. Exp. 62. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.584\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

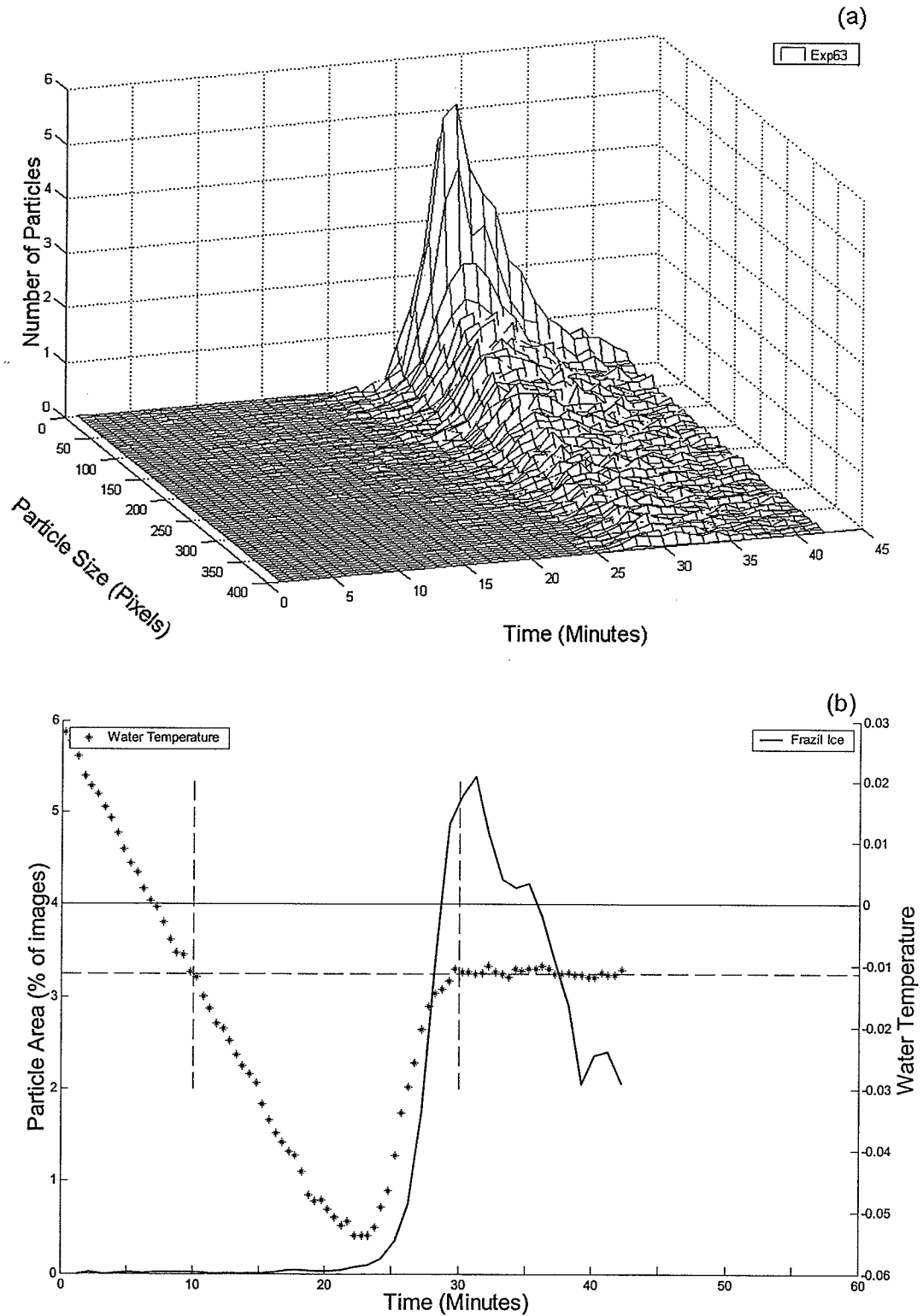


Figure B1. Group 1. Exp. 63. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.612\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process.

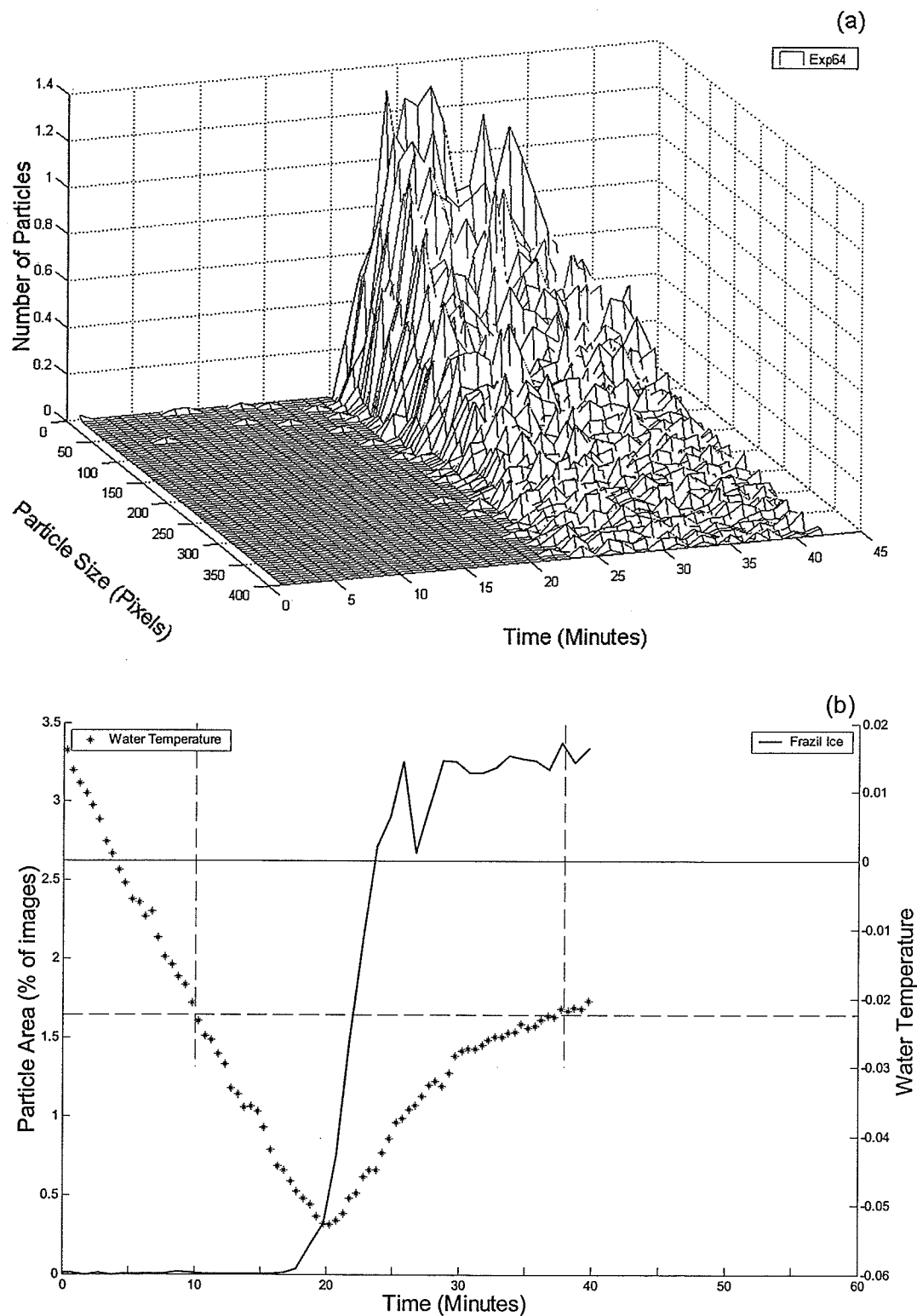


Figure B1, Group 1. Exp. 64. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.382\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

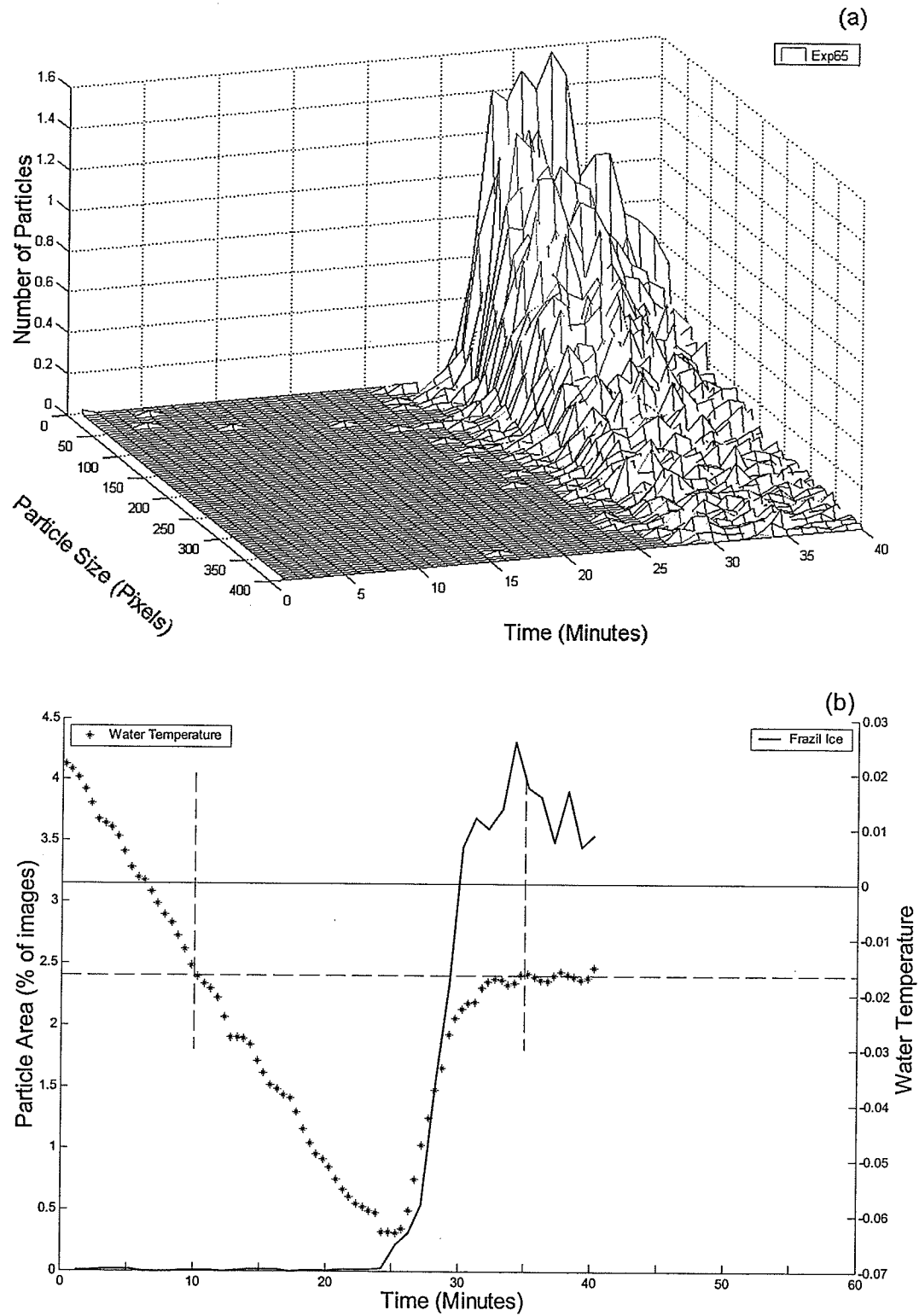


Figure B1. Group 1. Exp. 65. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.391\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

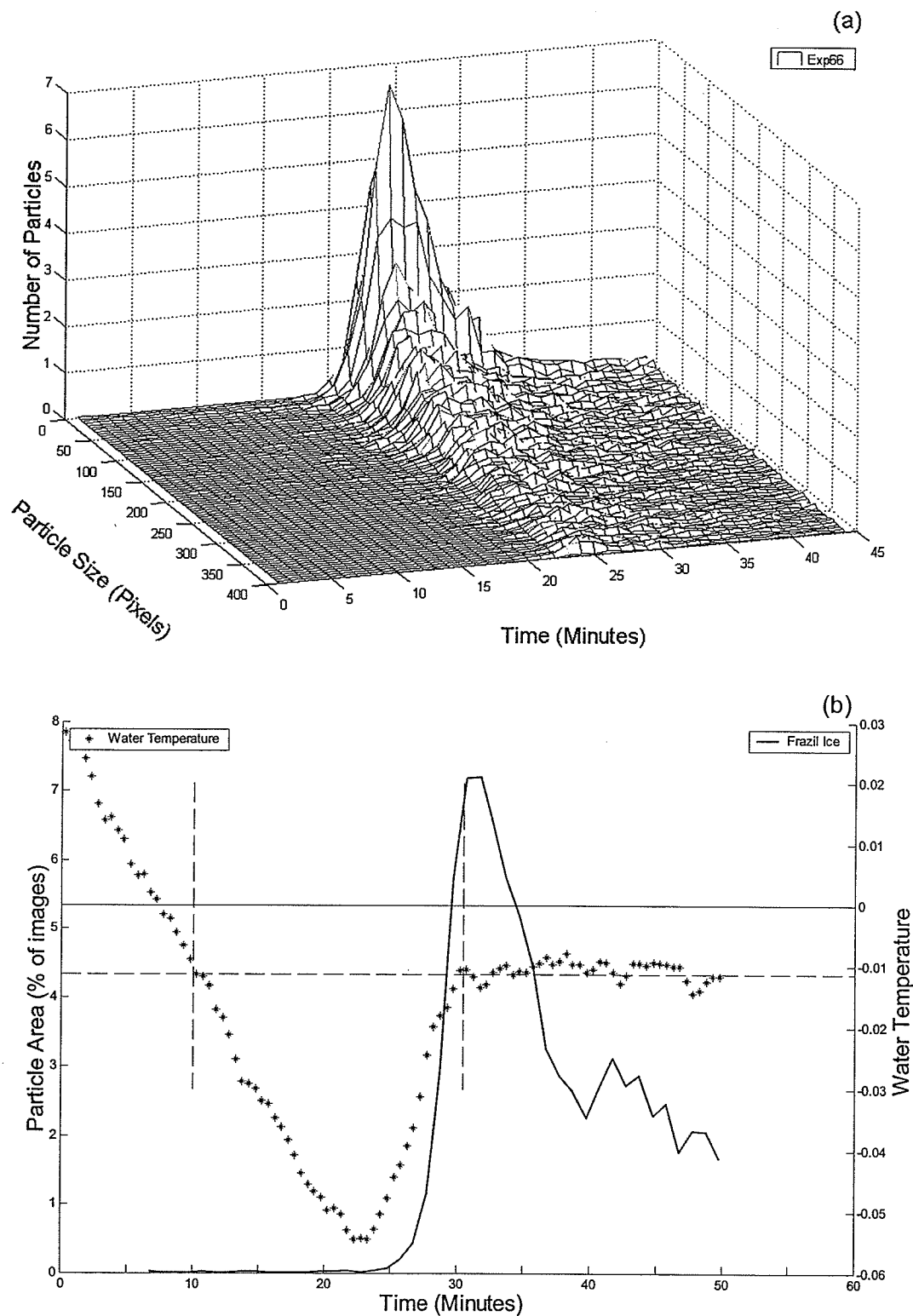


Figure B1. Group 1. Exp. 66. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.630\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

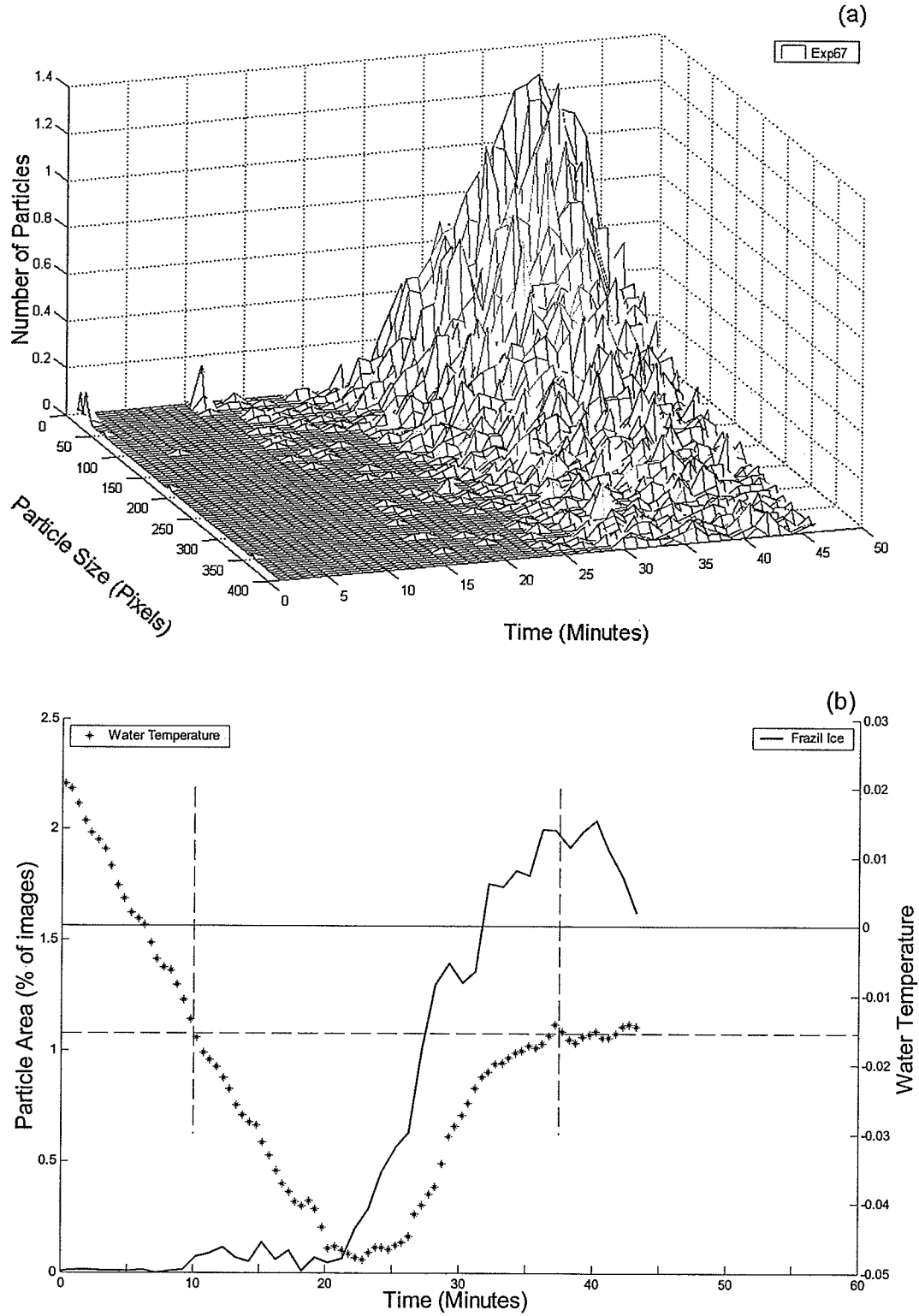


Figure B1. Group 1. Exp. 67. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.393\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

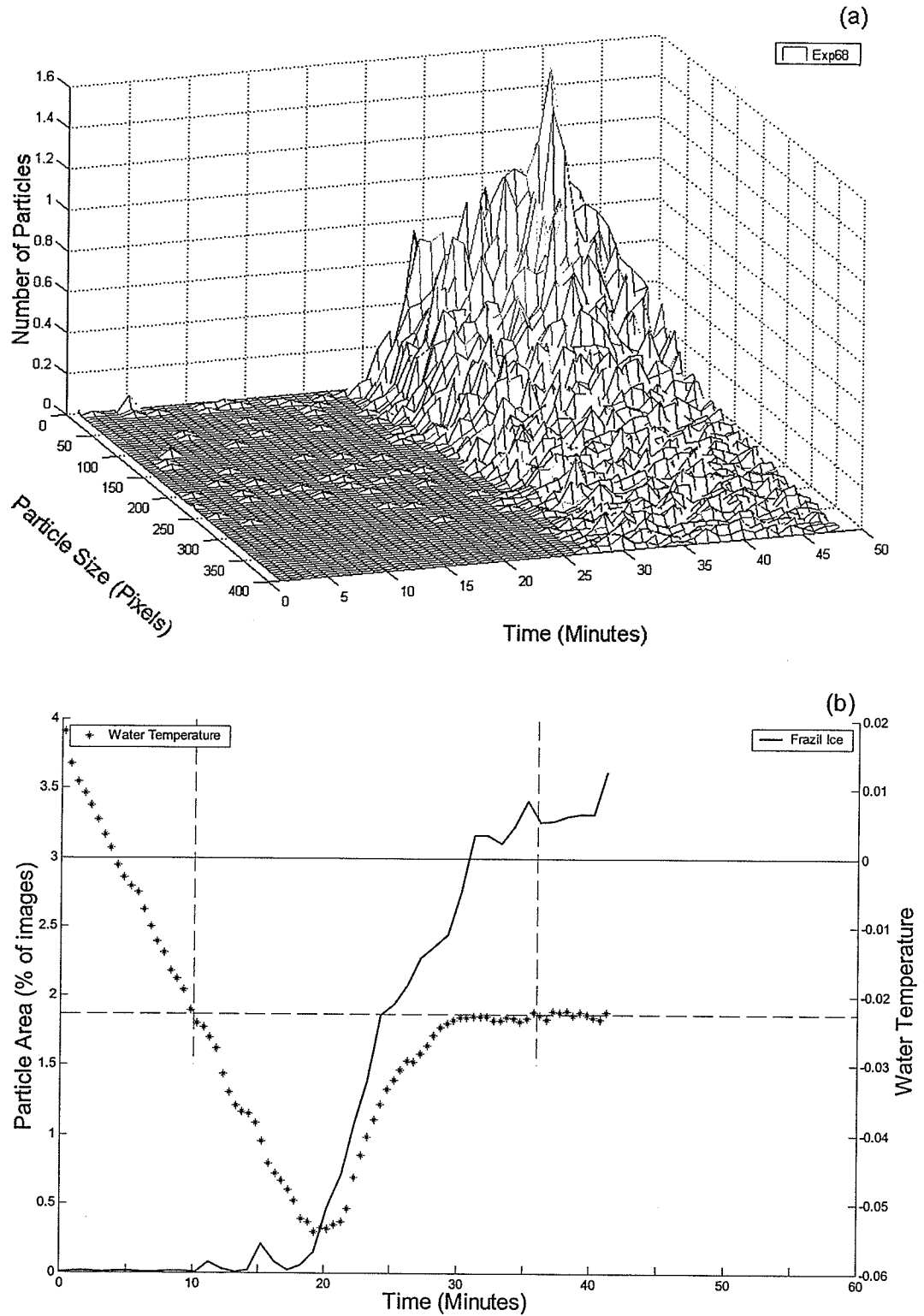


Figure B1. Group 1. Exp. 68. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.404\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

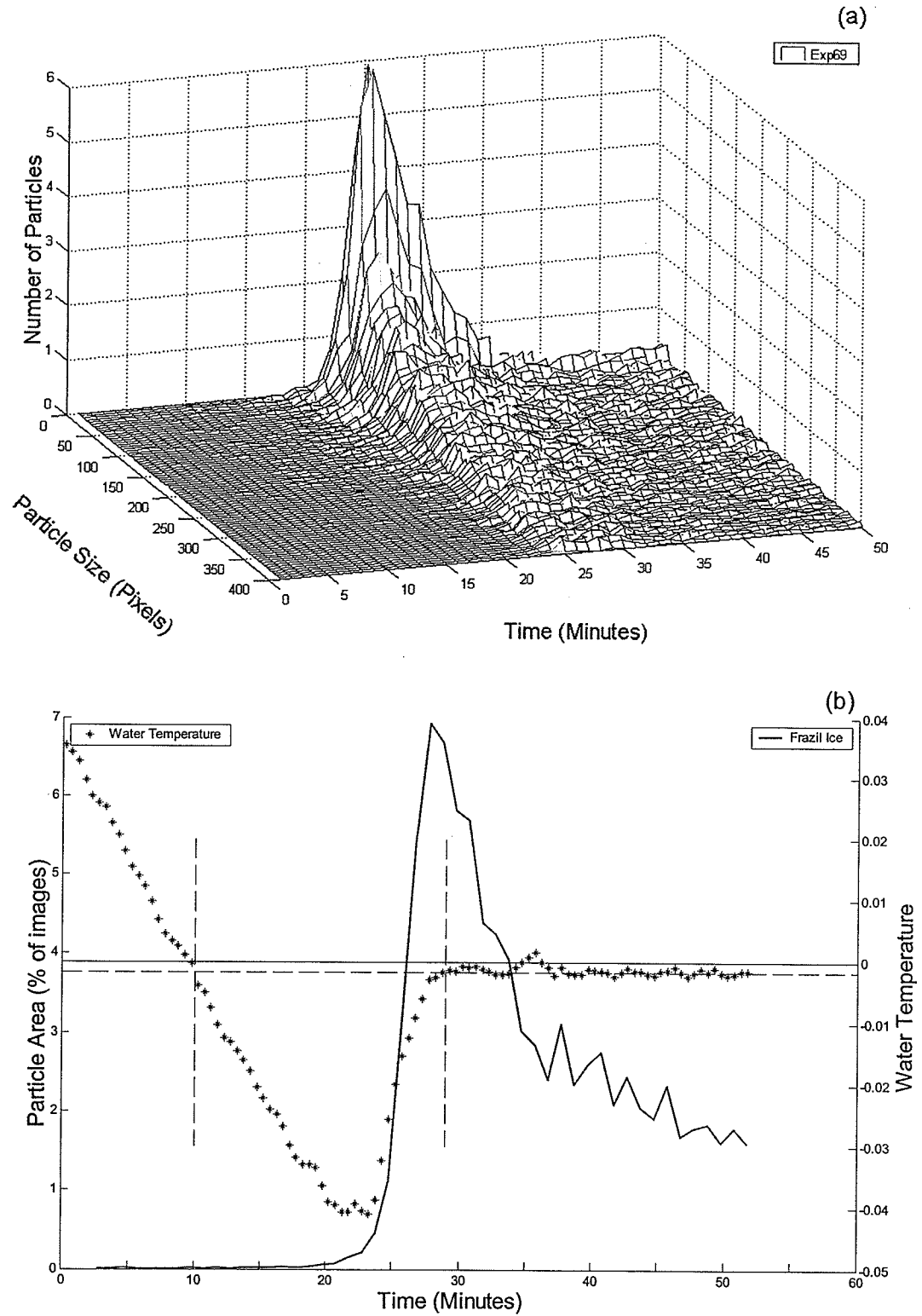


Figure B1. Group 1. Exp. 69. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.642\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

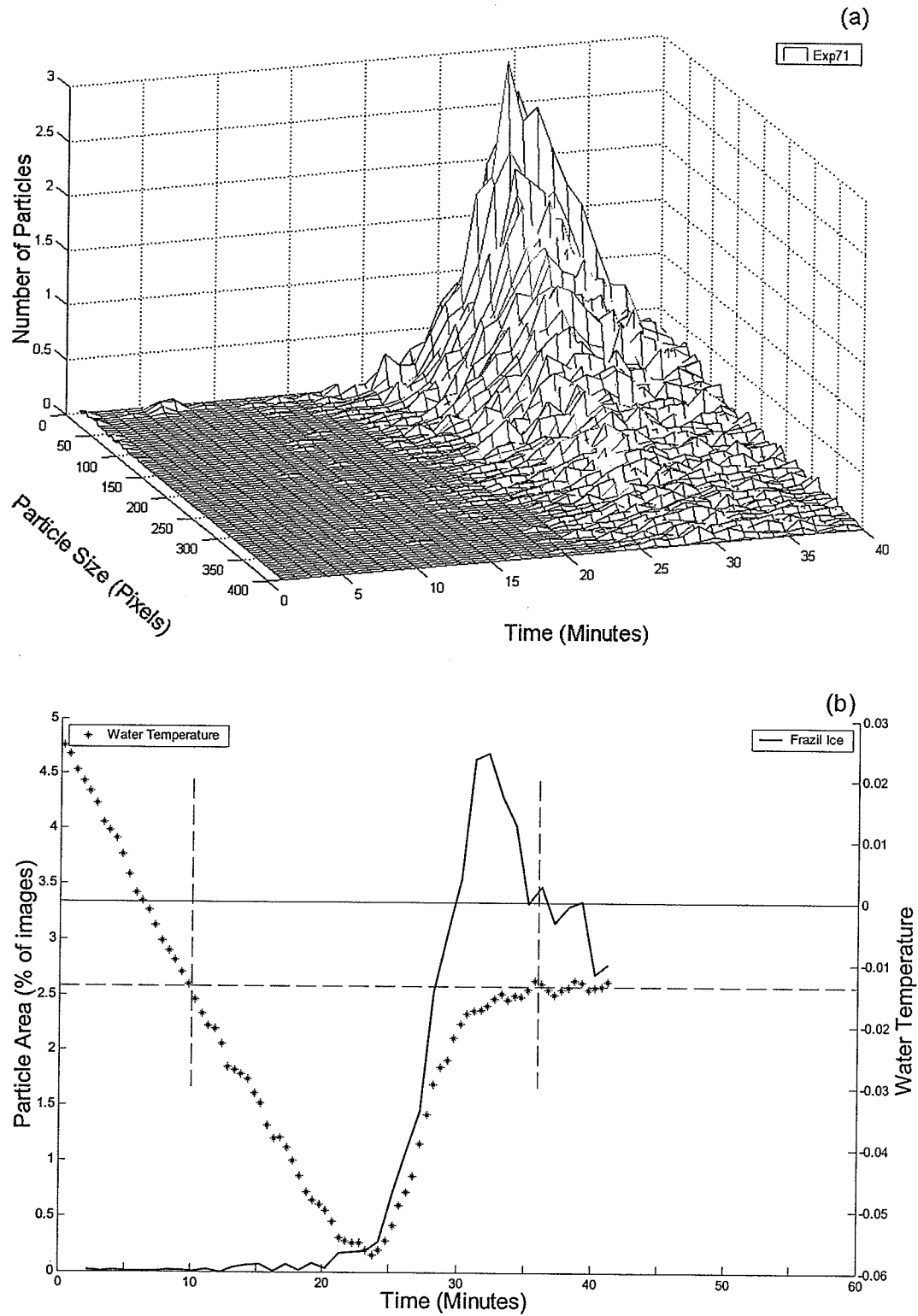


Figure B1. Group 1. Exp. 71. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.430\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

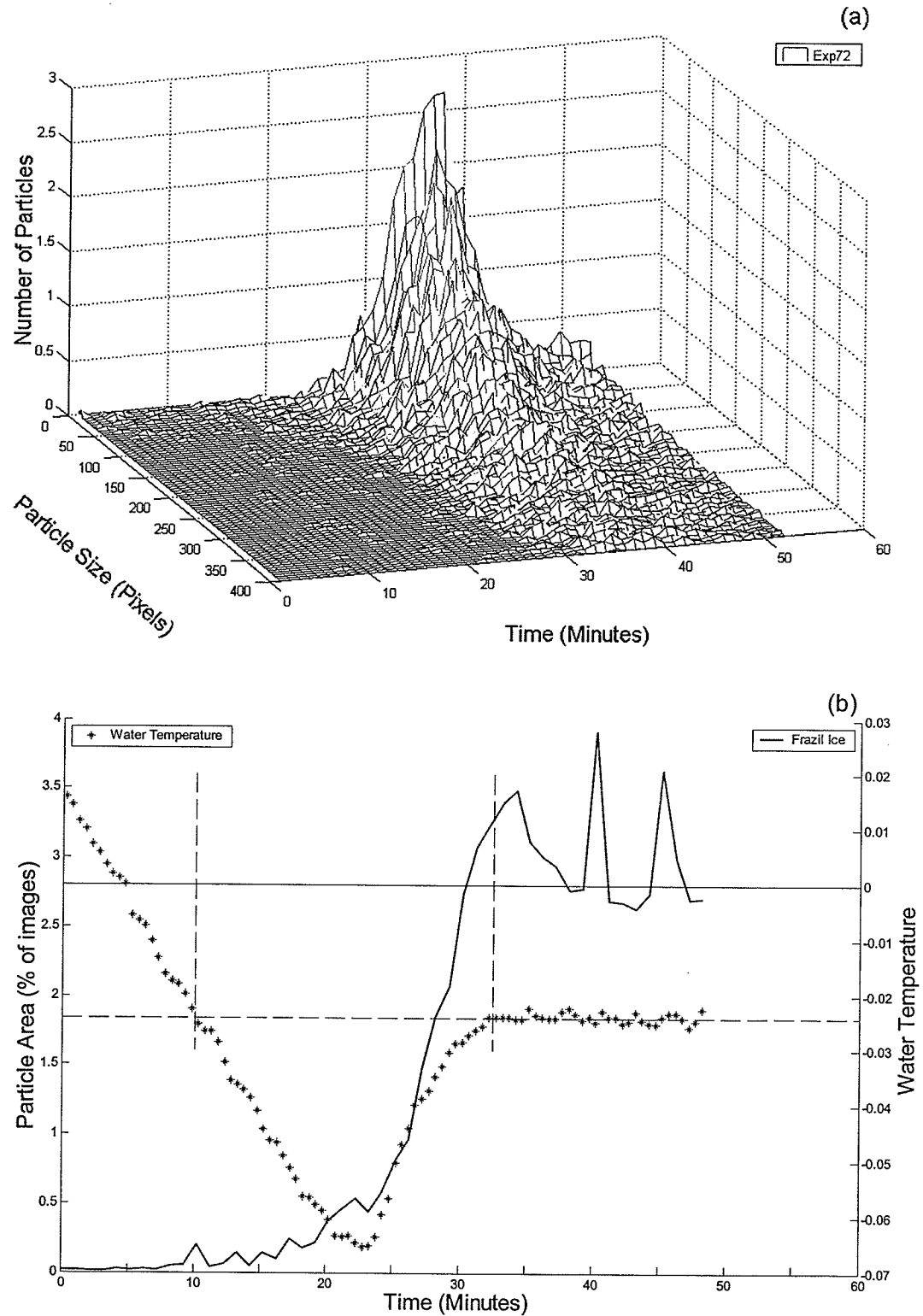


Figure B1. Group 1. Exp. 72. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.15\text{ m}$, $U = 0.430\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

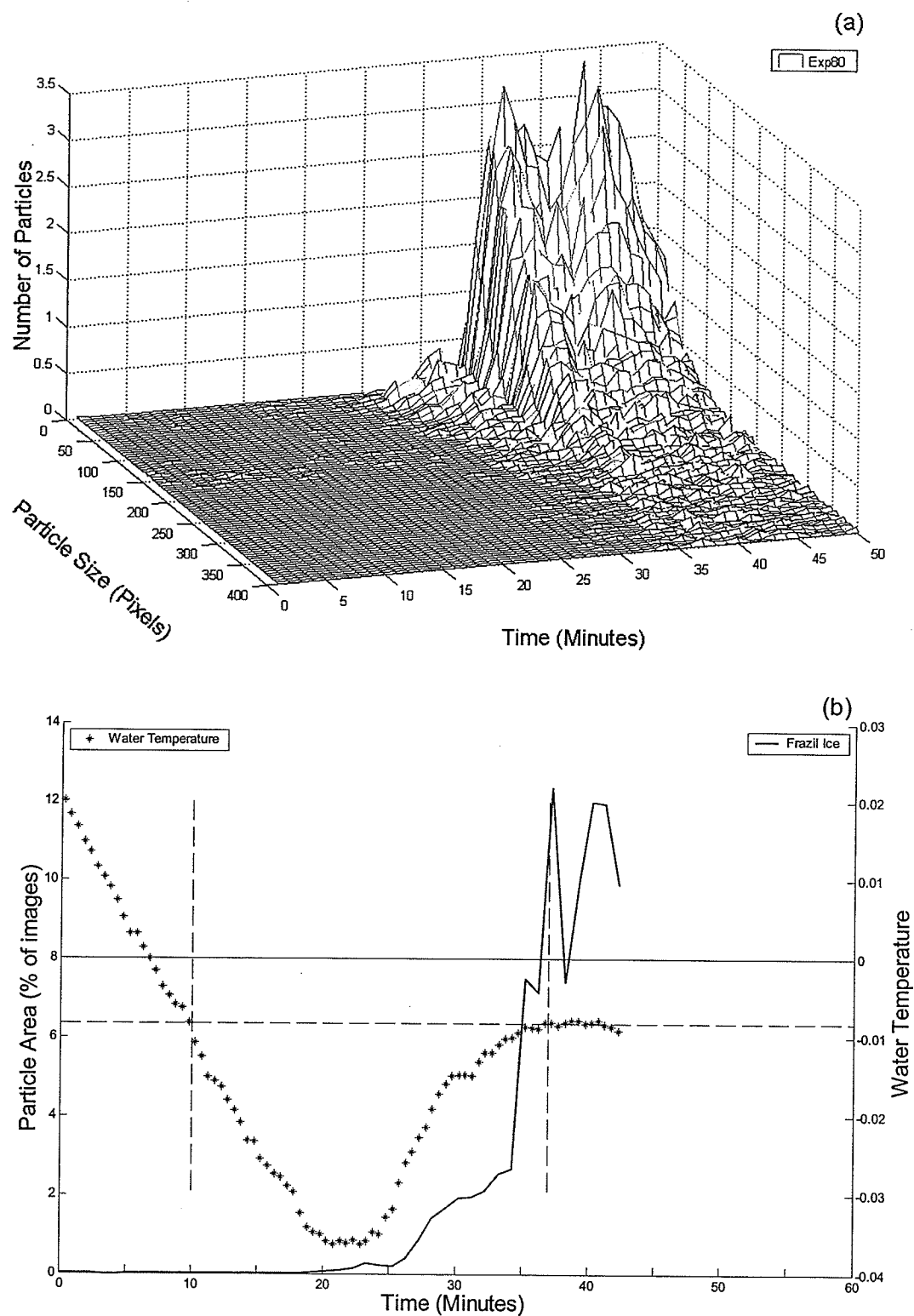


Figure B2. Group 3. Exp. 80. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.356\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

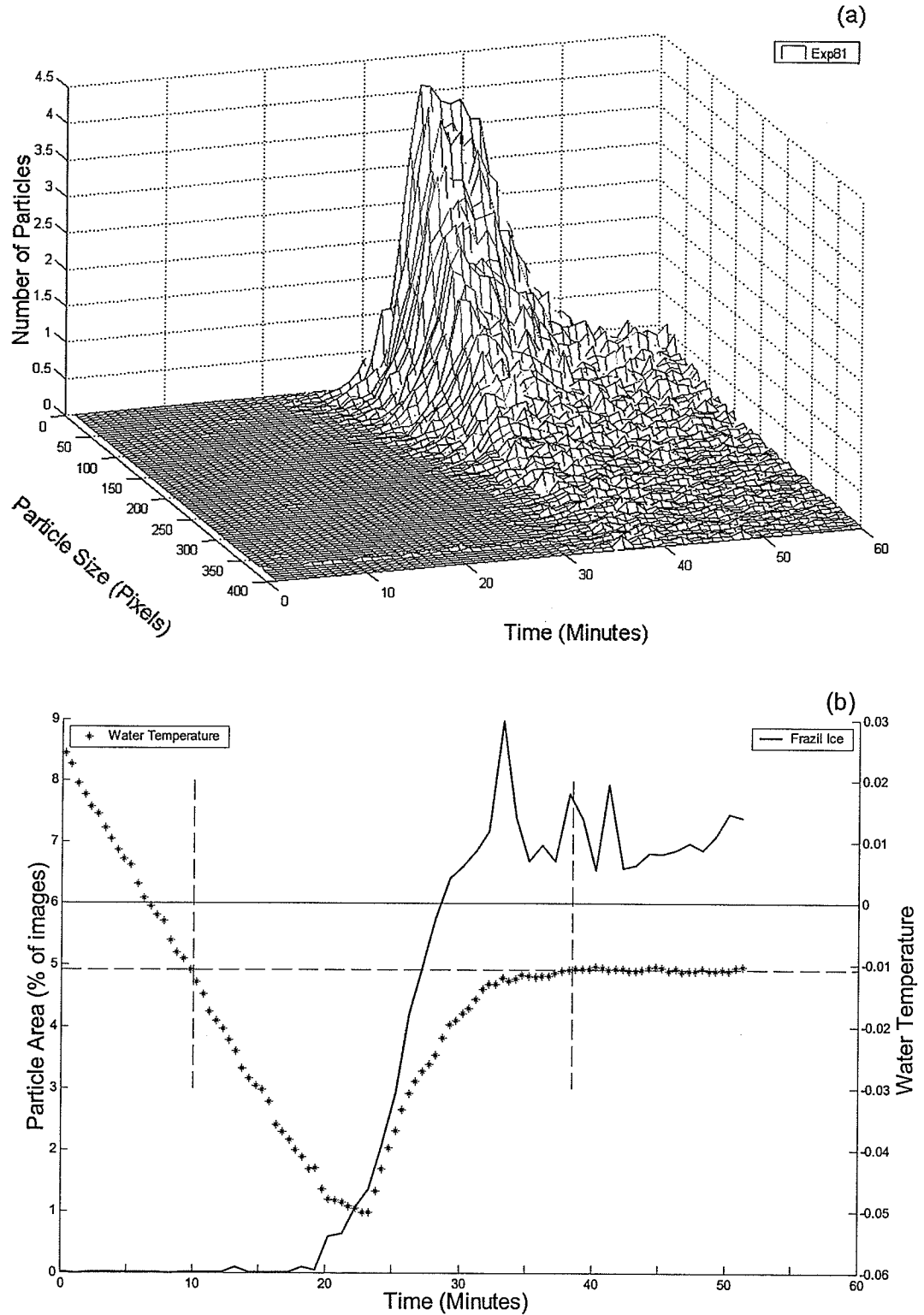


Figure B2. Group 3. Exp. 81. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.367\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

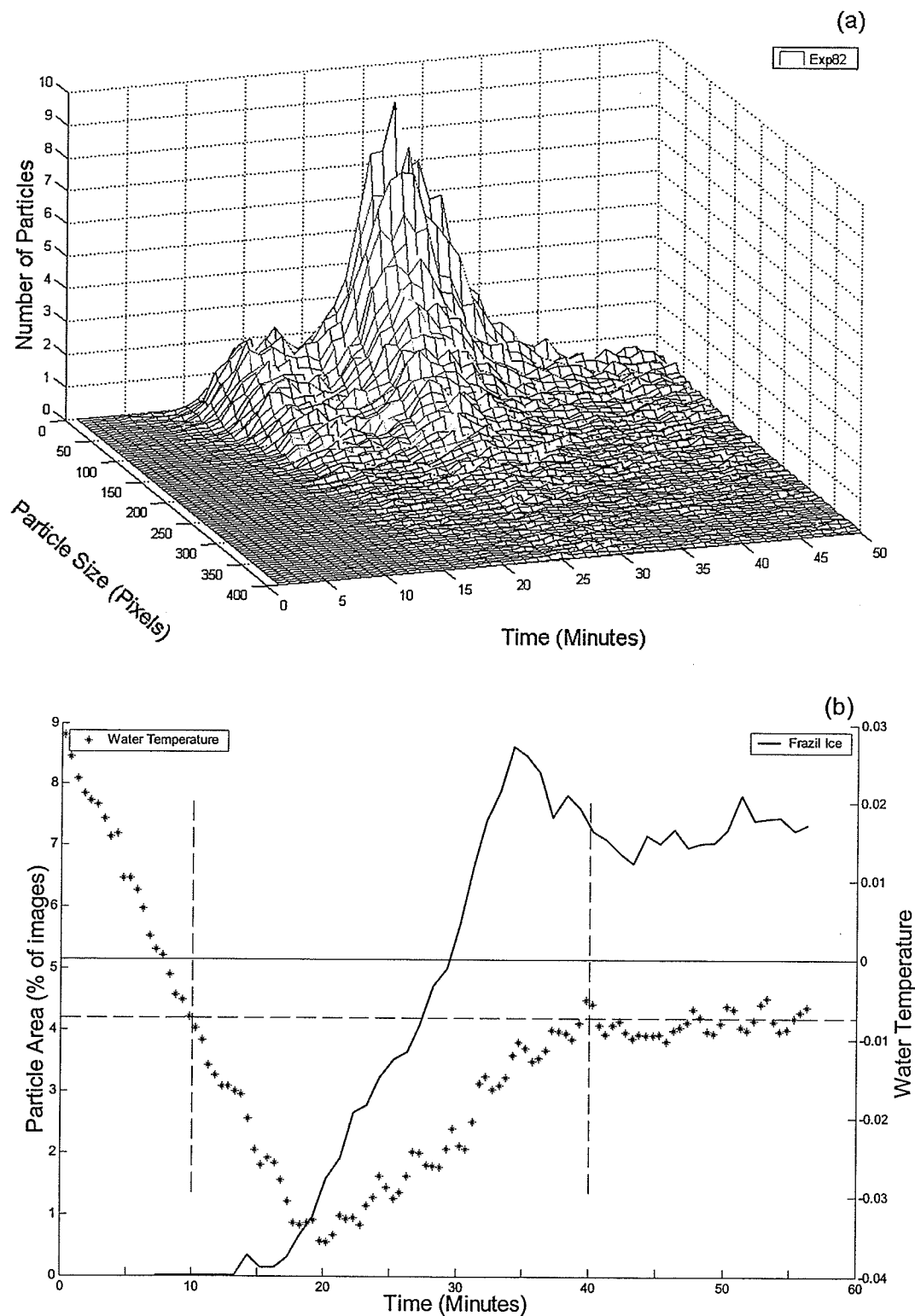


Figure B2. Group 3. Exp. 82. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.357\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

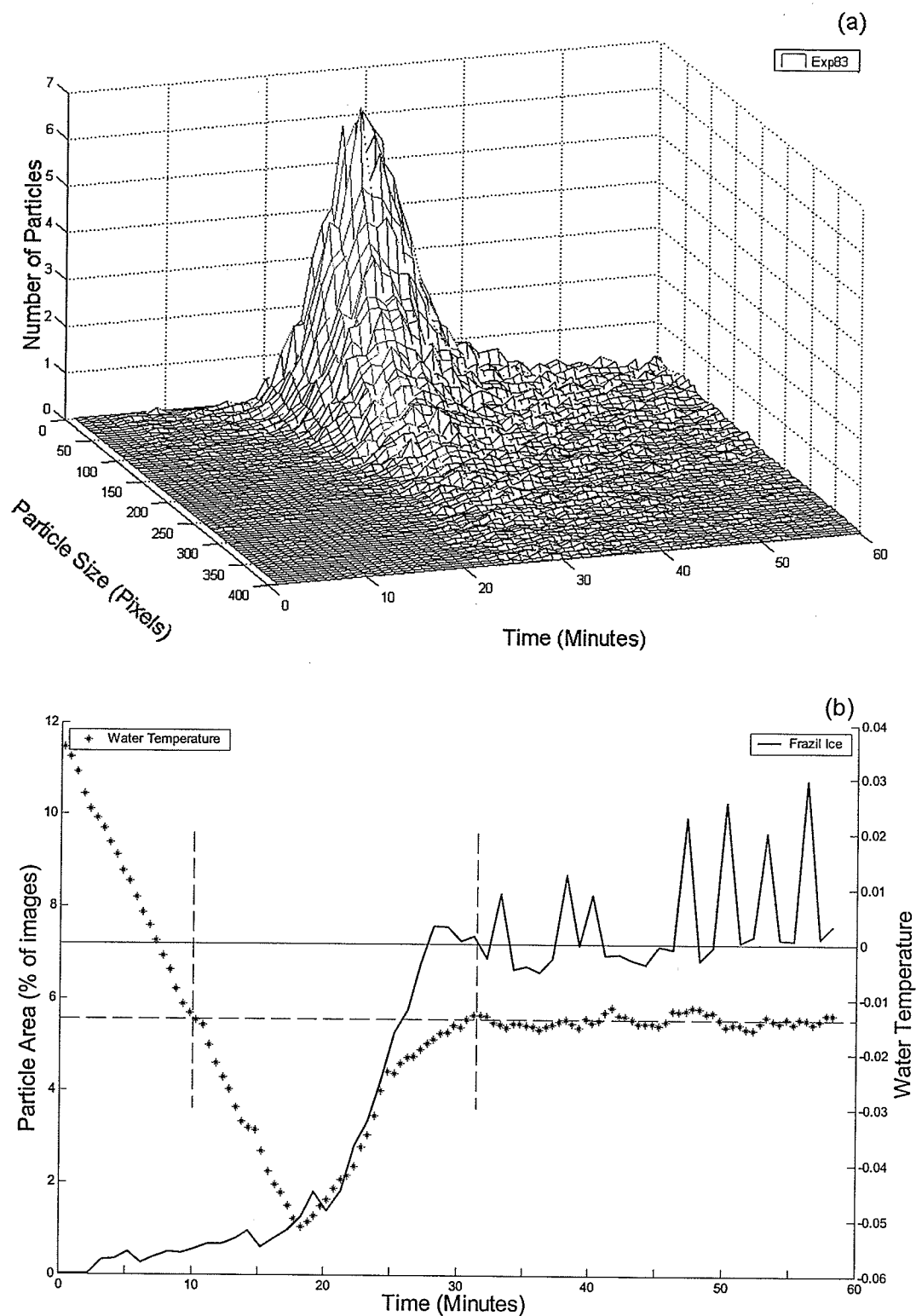


Figure B2. Group 3. Exp. 83. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.376\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

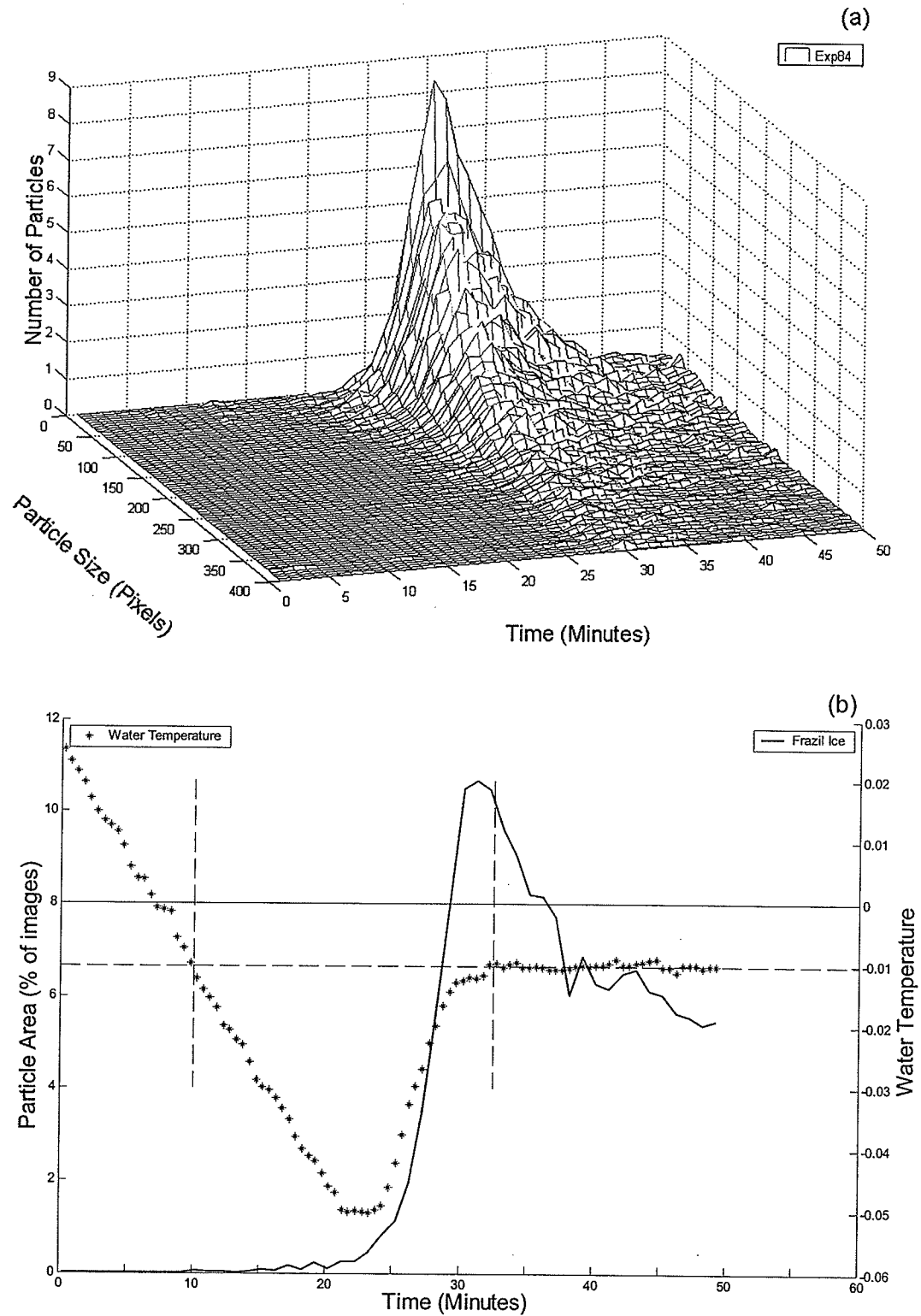


Figure B2. Group 3. Exp. 84: $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.486\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

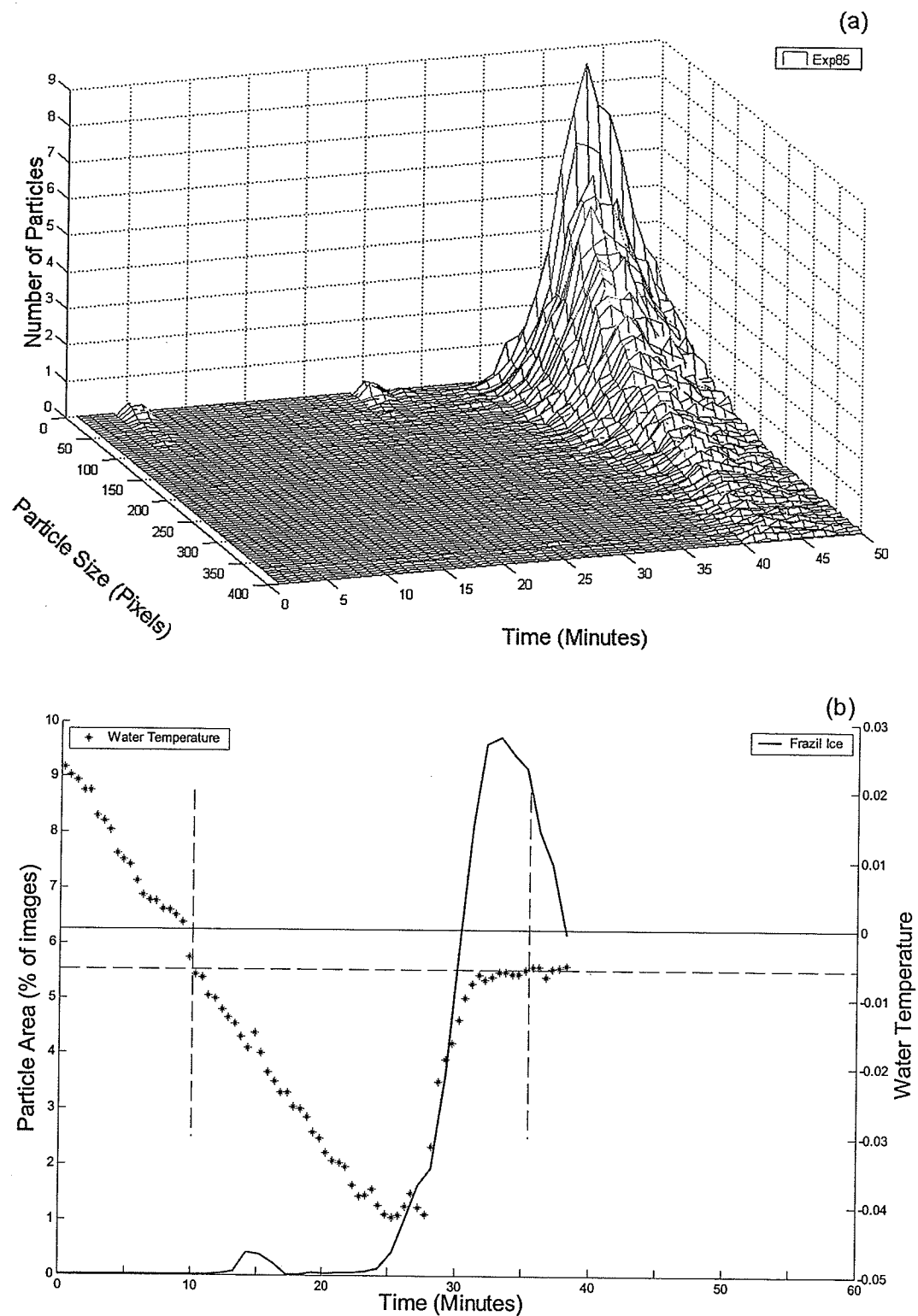


Figure B2. Group 3. Exp. 85. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.510\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

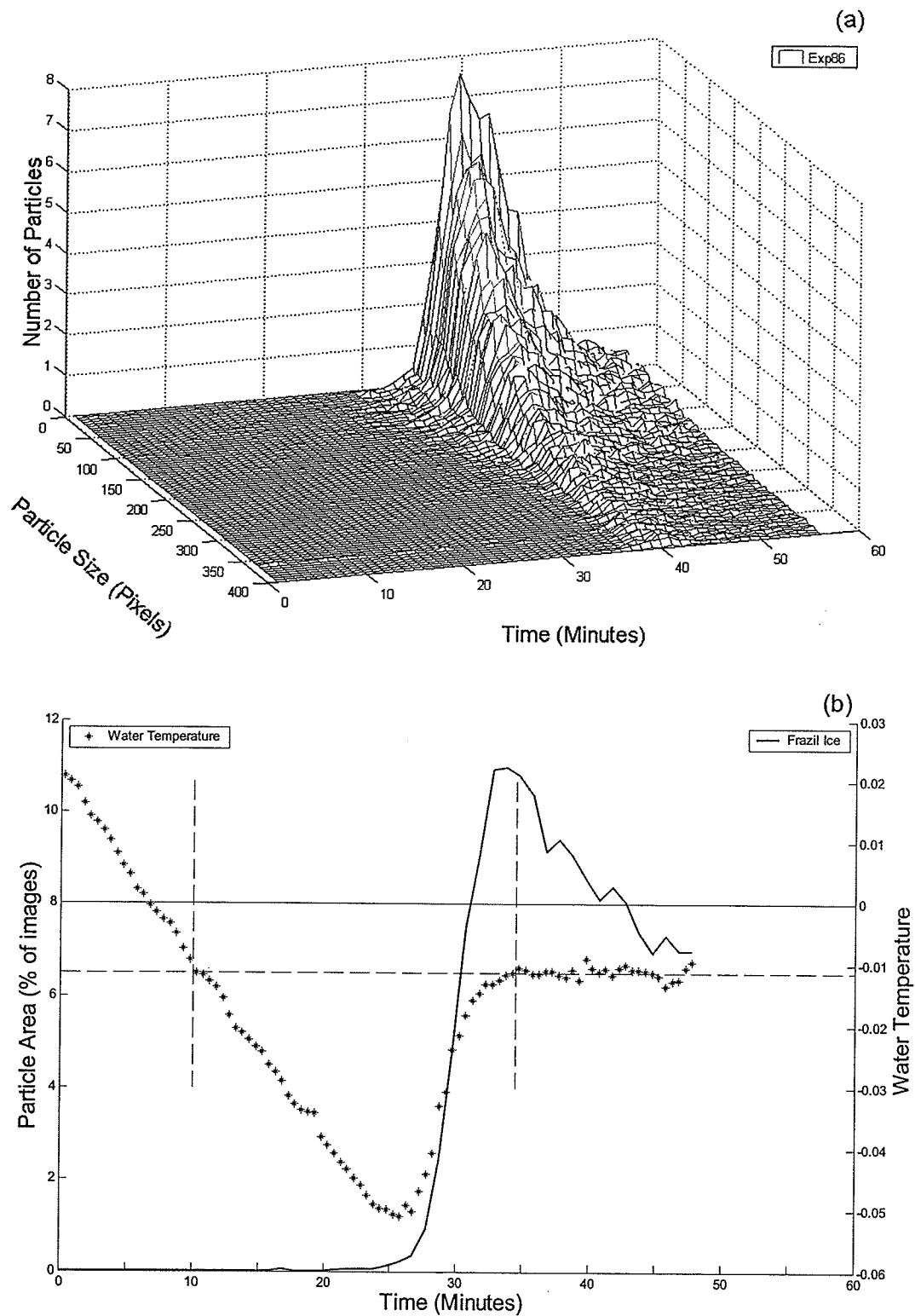


Figure B2. Group 3. Exp. 86. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.483\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

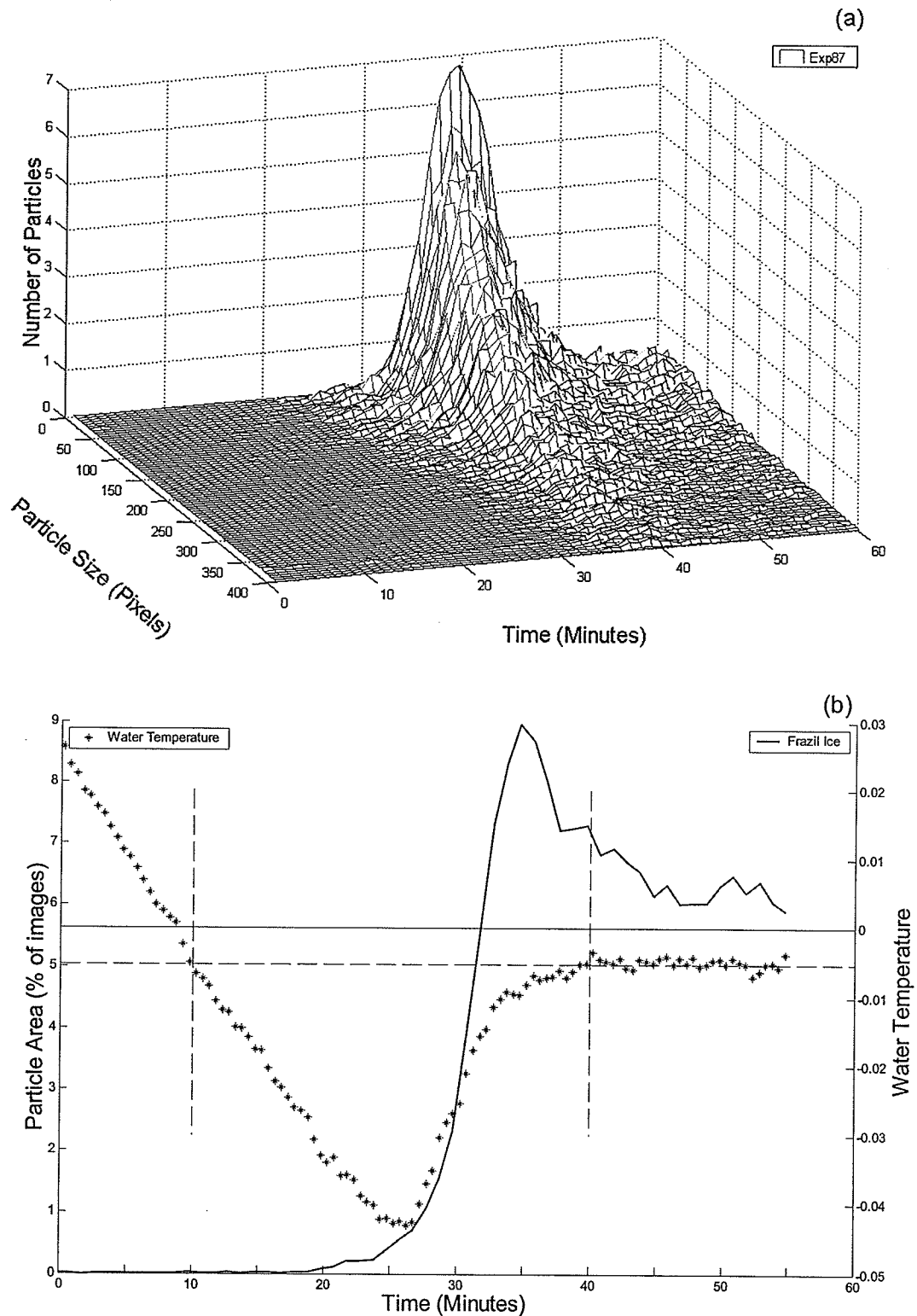


Figure B2. Group 3. Exp. 87. $T_a = -10\text{ }^{\circ}\text{C}$, $H = 0.20\text{ m}$, $U = 0.485\text{ m/s}$

(a) 3D frazil distribution, (b) Frazil evolution during supercooling process

Integrals J_1 and J_2

As defined in Chapter 6

$$J_1 = \int_0^1 \left(\frac{1-\xi}{\xi} \right)^{-z} d\xi = \int_0^1 \left(\frac{\xi}{1-\xi} \right)^z d\xi \quad (6.15)$$

and

$$J_2 = \int_0^1 \left(\frac{1-\xi}{\xi} \right)^{-z} \ln \xi d\xi = \int_0^1 \left(\frac{\xi}{1-\xi} \right)^z \ln \xi d\xi. \quad (6.16)$$

J_1 is a beta function if $z < 1$,

$$J_1 = B(1-z, 1+z) = B(1+z, 1-z) = \Gamma(1+z)\Gamma(1-z) = \frac{z\pi}{\sin z\pi}, \quad (C1)$$

in which B is a beta function, and Γ is a gamma function.

To solve for J_2 the following transformation is introduced

$$\ln x = \lim_{\beta \rightarrow 0} \frac{1}{\beta} (x^\beta - 1) \quad (C2)$$

Substituting (C2) into (6.16) and exchanging the integral and limit results in

$$J_2 = \lim_{\beta \rightarrow 0} \frac{1}{\beta} \int_0^1 \left(\frac{\xi}{1-\xi} \right)^z (\xi^\beta - 1) d\xi, \quad (C3a)$$

$$J_2 = \lim_{\beta \rightarrow 0} \frac{1}{\beta} \int_0^1 \frac{\xi^{\beta+z}}{(1-\xi)^z} d\xi - \int_0^1 \left(\frac{\xi}{1-\xi} \right)^z d\xi, \quad (C3b)$$

$$J_2 = \lim_{\beta \rightarrow 0} \frac{1}{\beta} [B(1-z, 1+z+\beta) - B(1-z, 1+z)], \quad (C3c)$$

$$J_2 = \lim_{\beta \rightarrow 0} \frac{1}{\beta} \left[\frac{\Gamma(1-z)\Gamma(1+z+\beta)}{\Gamma(2+\beta)} - \frac{\Gamma(1-z)\Gamma(1+z)}{\Gamma(2)} \right], \text{ and} \quad (C3d)$$

$$J_2 = \Gamma(1-z) \lim_{\beta \rightarrow 0} \frac{1}{\beta} \left[\frac{\Gamma(1+z+\beta)}{\Gamma(2+\beta)} - \frac{\Gamma(1+z)}{\Gamma(2)} \right]. \quad (C3e)$$

By the derivative definition

$$J_2 = \Gamma(1-z) \frac{d}{dx} \left[\frac{\Gamma(1+z+x)}{\Gamma(2+x)} \right]_{x=0}, \quad (C3f)$$

$$J_2 = \Gamma(1-z) \cdot \left[\frac{\Gamma'(1+z+x)\Gamma(2+x) - \Gamma(1+z+x)\Gamma'(2+x)}{\Gamma^2(2+x)} \right]_{x=0}, \text{ and} \quad (C3g)$$

$$J_2 = \Gamma(1-z) \frac{\Gamma'(1+z)\Gamma(2) - \Gamma(1+z)\Gamma'(2)}{\Gamma^2(2)}. \quad (C3h)$$

Since $\Gamma(2) = 1$, then

$$J_2 = \Gamma(1-z)\Gamma(1+z) \left[\frac{\Gamma'(1+z)}{\Gamma(1+z)} - \Gamma'(2) \right]. \quad (\text{C3i})$$

since $\Gamma'(2) = 1 - \gamma$, where γ is Euler constant (0.577215669) then

$$J_2 = \frac{z\pi}{\sin z\pi} \left[\frac{\Gamma'(x)}{\Gamma(x)} \Big|_{x=1+z} - (1-\gamma) \right]. \quad (\text{C3j})$$

However, the derivative of the Gamma function could be approximated as (Spiegel, 1968)

$$\begin{aligned} \frac{\Gamma'(x)}{\Gamma(x)} \Big|_{x=1+z} &= -\gamma + \left(\frac{1}{1} - \frac{1}{x} \right) + \left(\frac{1}{2} - \frac{1}{x+1} \right) + \cdots + \left(\frac{1}{n} - \frac{1}{x+n-1} \right) + \cdots \\ &= -\gamma + (x-1) \sum_{n=0}^{\infty} \left[\frac{1}{(n+1)(x+n)} \right]^{-1} \end{aligned} \quad (\text{C3k})$$

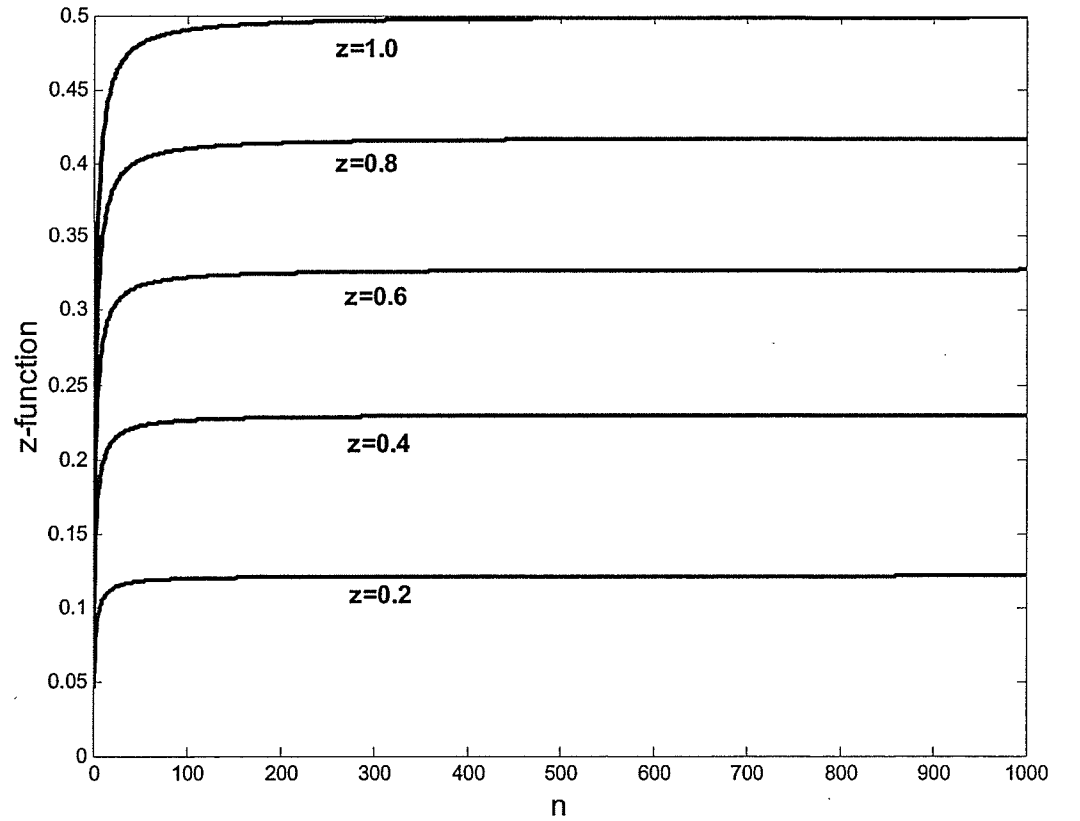


Figure C1: Numerical experiments of function of $f(z)$.

Replacing (C3j) with (C3k), then

$$J_2 = \frac{z\pi}{\sin z\pi} \left[z \sum_{n=0}^{\infty} [(n+1)(z+n+1)]^{-1} - 1 \right]. \quad (\text{C4})$$

Numerical experiments show that the first term in the square bracket tends to a constant when n is chosen to be larger than 500 (Figure C1). An approximation is obtained by using $n = 500$.

Defining

$$f(z) = z \sum_{n=0}^{500} [(n+1)(z+n+1)]^{-1} \quad (\text{C5})$$

then

$$J_2 = \frac{z\pi}{\sin z\pi} [f(z) - 1]. \quad (\text{C6})$$