# Detecting Energy Dissipation in Modulated vs. Non-Modulated Motion Waveforms emanating from Vibrating Systems Recorded in Videos

**by**

**Tharaka Uswatta Liyanage**

**A thesis**
**submitted to the Faculty of Graduate Studies,**
**in Partial Fulfilment of the Requirements for the degree of**

**Master of Science**
**in**
**Electrical and Computer Engineering**



Department of Electrical and Computer Engineering
University of Manitoba
Winnipeg, Manitoba R3T 5V6 Canada

## Abstract

The research investigates the dynamics of motion energy dissipation in modulated versus non-modulated motion waveforms derived from vibrating systems as recorded in video formats, utilizing Hilbert transforms for signal analysis. By defining the critical motion attributes through the Hilbert spectral analysis, the study quantifies the energy dissipation in both modulated and non-modulated states, capturing the intricate oscillatory behavior of these systems. The essence of modulation in this context is demonstrated through the localized concentration of energy, observable in systems ranging from mechanical to human motion. In contrast, non-modulated signals exhibit dispersed energy profiles, leading to different implications for energy efficiency and system performance. Through meticulous frame-by-frame analysis, the research delineates how specific energy patterns can lead to smoother motion waveforms as well as resulting a decrease in system vibrations.

**Keywords**: Hilbert Transform, Motion Energy dissipation, Modulated Signals, Non-Modulated Signals, Video Signal Analysis, Energy Efficiency

## Acknowledgements

Many thanks to the following persons for their invaluable support and encouragement throughout the completion of this thesis.

- First and foremost, my loving husband, Roshan Dodampola, for his unwavering support, encouragement, and patience, which made this journey possible.
- My advisor, Prof. James Peters, for his guidance, mentorship, and invaluable knowledge that significantly contributed to my research.
- My wonderful daughter, Netalie Dodampola, for being my constant source of joy and inspiration.
- My parents and my in-laws, for their love, belief, encouragement and unconditional support and understanding throughout this journey.
- All the friends who encouraged me and helped me to achieve this.

# Contents

# List of Tables

# List of Figures

# List of Symbols

| | |
|---|---|
| $t$ | Time |
| $M$ | Mersenne Prime Numbers |
| $\phi$ | Phase angle |
| $U_T$ | Total energy |
| $U_C$ | Kinetic energy |
| $U_P$ | Potential energy |
| $x(t)$ | Position as a function of time |
| $\dot{x}$ | Velocity |
| $dU_T$ | Change in total energy |
| $F_{fr}$ | Friction force |
| $K$ | Spring constant |
| $\tau$ | Time constant |
| $e$ | Exponential function base |
| $\cos$ | Cosine function |
| $\sin$ | Sine function |
| $\langle U_{\tilde{T}}(t) \rangle$ | Averaged energy |

# Chapter 1

# Introduction

## 1.1 Computer Vision

Computer vision is a rapidly growing field that is concerned with allowing computers to interpret and detect variations in electromagnetic radiation reflected from object surfaces. Unlike other traditional signal processing methods, which deal with one-dimensional time series data, computer vision works with waveforms from 2D and 3D digital images [1]. This complexity brings challenges in data storage and visualization.

By measuring waveform frequency and amplitude inherent in images and sequences of video frames, computer vision systems can recognize objects. Also, it can classify scenes, and track surface change, system energy dispersion, and motion irregularities through recorded electromagnetic waves continuously emitted by physical systems. These visible object characteristics are important for various applications. In robotics, computer vision allows robots to navigate their environment, identify objects, and catch them properly. Self-driving cars also rely on computer vision to perceive traffic signs, pedestrians, and other vehicles, helping them to navigate roads safely. In action recognition, computer vision helps in analyzing video footage to identify and understand human activities, by focusing on the energy and motion dynamics of the subjects. This helps to improve applications such as sports analytics, security monitoring, and human-computer interaction by providing a detailed understanding of energy usage and dissipation patterns during various activities. As computer vision techniques such as nonlinear filtering advance, even more complex applications are expected to emerge in the future. [2]

## 1.2 Overview of Video Signal Analysis

Video signal analysis has now become an integral part of various applications, such as content retrieval, surveillance, action recognition, medical imaging, autonomous vehicles, and human-computer interaction. The skill of extracting meaningful understandings from video data has become significant in applications of content retrieval, surveillance, and action recognition. The analysis of energy dynamics, in addition to motion patterns, will be beneficial in terms of adding advanced techniques to the applications to improve understanding of activity-based energy dissipation and enhancing accuracy in security monitoring, sports analytics, and human-computer interaction. Digital While video technology is still evolving, the need for more sophisticated analytical techniques has grown exponentially and drives research efforts for efficient intelligent analysis of video.

Garcia-Garcia et al. [3] regard background subtraction in video signal analysis as a technique fundamental to the separation of objects in a video. Background subtraction isolates moving foreground objects (such as people and vehicles) from the static background, making it a critical step in various applications of video analysis, especially in real-world scenarios like visual surveillance. The paper elaborates on the background subtraction process and discusses some of the challenges encountered in this field.

In the domain of video signal analysis, Wang and Schmid [4] propose an enhancement to dense trajectories, a method used for video representation in action recognition. Dense trajectories offer a robust approach to capturing motion information by tracking feature points across video frames. One key issue with this method, however, is its sensitivity to errors caused by camera motion. To address this, the authors incorporate camera motion estimation, employing feature matching and homography estimation to exclude trajectories resulting from camera movement rather than object motion.

In the domain of video signal analysis, Wang and Schmid [4] propose an improvement to dense trajectories, a technique for video representation in action recognition. Dense trajectories offer a powerful way to capture dense motion information by tracking feature points through video frames. The only issue is that such an approach is very sensitive to the errors brought about by camera motion, but this is catered to by the authors in including camera motion estimation. They employ feature matching and solid homography estimation to remove trajectories that correspond to camera motion rather than object motion. This leads to significant improvement in action recognition accuracy on several benchmark datasets.

"On Space-Time Interest Points" by Ivan Laptev [5], the walking model is one of the main foci around which video interpretation works. Within such a framework, the model indicates that a sparse representation of the video sequences using classified spatio-temporal interest points is likely to be good enough for tasks such as people walking detection and estimation of their pose in outdoor scenes. Human motion can often be difficult to detect in such scenes due to the variability of appearance and background leading to ambiguous interpretations. To resolve these ambiguities, the approach uses the strong cue of human motion by representing both the model and the data using local, discriminative spatio-temporal features.

Laptev approaches to track and analyze human motion by finding and analyzing the changes of significant elements in both temporal and spatial dimensions—space-time interest points—to classify walker features precisely, which would then be able to estimate the pose correctly in dynamic backgrounds of complex scenes. This method focuses on precise detection and classification of motion features, enabling accurate pose estimation in complex scenes with dynamic backgrounds.

In contrast, this approach places significant emphasis on measuring energy dissipation during various walking activities recorded in video sequences. Therefore, rather than focusing solely on motion detection, this work emphasizes the changes in energy dynamics with variations in movement. By applying mathematical transformations, the proposed method provides a detailed understanding of energy usage and loss, which is particularly valuable for applications such as airplane wing vibration and surveillance systems.

Both approaches represent steps toward improved analysis of human motion in videos. Laptev's approach focuses on the classification of spatial-temporal features, while energy dissipation is measured quantitatively, providing further insights into the dynamics of human motion in this research.

The walking model is constructed by extracting an interval of time equal to that of a period of the gait pattern from a video sequence of a walking person. In this interval, the space and time coordinates as well as scales and classifications of the points of interest need to be chosen by hand. These features represent a set of cyclically repeating elements that represent the walking model. This model accounts for variations in position, size, and speed of the person in the video by allowing for translations and uniform rescalings in both spatial and temporal domains. The parameters of the model features are then transformed to correspond to the detected features in video data so that people can be correctly detected and their pose estimated while walking, even if there are occlusions and dynamic backgrounds. This method demonstrates the effectiveness of using discriminative spatio-temporal features for human motion analysis in complex scenes.

The digital media industry also relies on video signal analysis for tasks such as compression, content retrieval, editing, and many more. The Surveillance systems are meant for security and monitoring through video analysis. On the other hand, analytic techniques help in action recognition that enables the identification and interpretation of human activities. Machine learning, computer vision, and deep learning converged, reinventing video signal analysis in such a way that automated feature extraction, pattern recognition, and predictive modeling are conducted.

The significance of video signal analysis in various fields and look deeper into the analytical techniques advanced by the development of digital video technology will be investigate. This is, therefore, an investigation to point out the important nature that video signal analysis commands in shaping modern technological landscapes concerning digital media.

Therefore, this thesis explains video signal analysis in current applications through a holistic review of the literature and case studies to reveal the impact of technological growth on analytical techniques. With an understanding of the challenges and opportunities presented by video analysis, the path for further innovation in this burgeoning field can be set.

An important aspect of this thesis is the focus on energy dispersion and dissipation in video signal analysis. The description of complex data streams to infer and forecast pattern changes is well seen in the detailed explanation regarding the dissipation of energy in these data streams. Signal modulation and energy dissipation play an important role in the extraction of actions from the video streams. The rapid expansion of digital video content necessitates advanced analytical techniques to manage, optimize, and secure video data effectively, requiring a deeper understanding of underlying signal properties and energy dynamics.

## 1.3 Fundamentals of Signal modulation

Signal modulation is a fundamental concept in communication and signal processing; it plays a major role in shaping many signals, including video signals. In video processing, several modulation techniques are used, each with distinct applications and characteristics. Changes in amplitude affect the visual dynamics of the video, upon which energy is used and dissipated in the system. Frequency modulation changes the resolution and sharpness of a video, crucial for understanding how signal property changes are reliant on energy usage and dissipation at varying resolutions. Phase alternations impact video synchronization, another important property relevant in the analysis of temporal variations and their effect on energy distribution and dissipation.

Energy dissipation in modulated video signals uses these modulation techniques that reveal the presence of energy loss inherent in recorded motion waveforms (see, e.g., an overview of this research is reported in [6]). In video data analysis, frequency modulation examines how the frequency content of a signal changes under various settings. This is done by convolution with a complex exponential $e^{-j2\pi Mt}$, where $M$ is known as the modulation index, which directly scales the frequency content of a waveform. It is important for underlining dynamic changes in the frame of the video. With something moving or distracting over time, it becomes a lot easier to take care of the motion pattern. By varying the modulation index, the analysis reveals how different frequencies within the signal respond to changes, providing a valuable understanding of the temporal dynamics of the video content. This method is particularly useful in applications requiring detailed motion analysis and pattern recognition in complex video sequences.

Through modulation techniques, video signal analysis can dissect and interpret intricate patterns and movements within video content. This lays a general principle of great significance for video compression, image improvement, and efficient transmission. Understanding the details of the signal modulation process will allow researchers and engineers to better design video processing systems and consequently attain significant improvement in the field of digital media, surveillance, and more. The continuous evolution of these techniques is necessary for adapting to the increasing complexity and volume of video data in the modern world.

## 1.4 Objectives and Motivation

### 1.4.1 Objectives

The main objectives of this research are:

1. **Analyze Energy Dissipation in Motion Waveforms**

   To conduct a detailed comparative analysis of energy dissipation for modulated versus non-modulated motion waveforms taken from video recordings of vibrating systems.

2. **Utilize Mersenne Primes for Modulation**

   To use Mersenne primes in modulation techniques, leveraging their unique characteristics from number theory to develop a methodology that improves the accuracy and stability of energy dissipation metrics computation.

3. **Practical Applications and Optimization**

   To optimize existing technologies in video processing, action recognition, surveillance, and sports analytics, focusing on enhancing compression algorithms, improving image quality, and refining motion detection techniques.

4. **Contribute to object motion Understanding**

   To contribute to the theoretical understanding of energy dissipation in video signal processing, focusing on measurable improvements that provide a foundation for future research and technological advancements.

By achieving these objectives, this research aims to bridge the gap between theoretical analysis and practical application, driving innovations in how to understand and utilize energy dynamics in video-recorded vibrating systems.

### 1.4.2 Motivation

The motivation for this research is rooted in the pressing need to better understand how energy dissipates in video-recorded vibrating systems. It directly influences the efficiency and effectiveness of different kinds of vibrating systems. Through the investigation into the differences between the modulated and non-modulated motion waveforms, and it is hoped to gain a deeper understanding of the presence of dissipation in vibrations of a system, and this would constitute one of the keystones for better processing and analysis of video data.

Understanding energy dissipation enables the development of algorithms that more effectively identify sources of energy loss in vibrating systems, ultimately leading to improved performance and reduced energy consumption.

Additionally, there is a strong motivation to improve the accuracy and reliability of the metrics used to measure energy dissipation. Using Mersenne primes for modulation offers an effective means of varying motion waveform frequency. Prime numbers are well-suited for frequency modulation for several reasons. First, all prime numbers (except 2) are odd, meaning they follow the form $2k + 1$ (where $k$ is an integer), which results in a well-defined set of waveform frequencies. Additionally, since there are no primes between consecutive prime numbers, the range of waveform frequencies that must be considered during modulation is naturally limited, simplifying the modulation process.

This consistency is vital not only for practical applications but also for advancing theoretical research in signal processing.

Furthermore, this research aims to bridge the gap between theoretical knowledge and real-world applications. By studying the fundamental principles of energy dissipation in motion waveforms, the focus is on gaining a better understanding of energy loss in vibrating systems.

Therefore, the motivation behind this research is to provide a comprehensive understanding of energy dissipation in video-recorded vibrating systems. This knowledge is necessary for developing more efficient, accurate, and solid signal processing techniques, ultimately leading to developments in video processing.

## 1.5 Major Contribution in Thesis

This thesis makes several significant contributions to the field of video signal processing and energy dissipation analysis. It introduces a novel approach to analyzing energy dissipation in video-recorded vibrating systems by fine-tuning and smoothing motion vibration waveforms using a succession of Mersenne primes to exploit the properties of Mersenne primes for modulation [7]. This approach provides a stable framework for comparing modulated and non-modulated motion waveforms, offering new understandings into the dynamics of energy dissipation.

The research develops and validates new metrics for measuring energy dissipation. It is both accurate and applicable across various practical applications. These metrics include the Hilbert Envelope Computation, Total Energy Calculation, Oscillation Frequency Measurement and Motion Energy Loss Analysis. These metrics are designed to be applicable across various contexts, including action recognition, surveillance, and sports analytics, enabling a deeper understanding of energy dynamics in video-recorded vibrating systems.

Moreover, this thesis puts these findings into practice by improving energy dynamics understanding in video frames. This improved understanding leads to the exact measure of energy loss across various activities captured on videos.

The alterations are well articulated now and are based on the fact that modulation of signals, as well as energy dissipation, is better understood at this time. This, in turn, propels the notion that energy dissipation analysis is important to efficiency and effectiveness in working towards video signal processing technologies.

Lastly, the thesis contributes to the theoretical understanding of energy dissipation in video signals, providing a solid foundation for future research and technological developments. By bridging the gap between theoretical analysis and practical implementation, this work paves the way for innovations in the processing, analysis, and optimization of video data across various technological domains.

Specifically, the methods introduced have enhanced the understanding of how energy dissipation varies with different modulation techniques, the impact of motion intensity on energy loss, and the efficiency of energy usage in various types of video content.

# Chapter 2

# Related Work

Reviewing related work is essential for gaining a comprehensive understanding of the current state of knowledge in the field. This chapter provides an overview of significant advances and methodologies that have shaped the development of video signal processing and energy dissipation analysis. By examining previous studies, this research builds on existing methods, identifies gaps, and highlights areas for improvement. This review also helps avoid duplication and frames new research questions, guiding experimental design and contextualizing results within established knowledge.

Firstly, reviewing related work makes sure that the research is grounded in a good understanding of existing methodologies and techniques. This is then useful to determine the strengths and weaknesses of these methods and to develop improved techniques. The second benefit is that it will help avoid repetition by building on existing knowledge through ensuring research generates new information in the field. Finally, it aids in framing research questions, guiding experimental design, and interpreting results within the context of established knowledge.

## 2.1 Developments in Microrheology and Modulation Techniques

The field of microrheology has achieved notable developments, particularly in the measurement of viscoelastic properties of fluids using both passive and active techniques. Passive microrheology, in which the intrinsic Brownian motion of the fluid's particles is utilized for probing, has a low signal-to-noise ratio at high frequencies. To overcome these challenges, active microrheology applies an external perturbation to amplify the response of the probe particles, resulting in a higher signal amplitude at the modulation frequency.

Recent developments include techniques such as flipping probe particles between two time-shared traps. It is done by using chirp waves to continuously vary the modulation frequency and applying square waves or frequency modulation over a specific bandwidth. The research team of Kundu et al. (2021) has pioneered a new technique in active microrheology, termed the "Multiple Sinusoids Superposition Method" (MSSM). It modulates a trapped colloidal probe particle through square waves and linear superposition of sinusoidal signals to improve the signal-to-noise ratio at high frequencies of modulation so that measurements can be carried over a wide band. Such innovations have informed this research, illustrating advanced modulation techniques to improve the accuracy of measurements and improve efficiency with time—critical for the analysis of energy dissipation in a video-recorded vibrating system. [7]

## 2.2 Space-Time Interest Points in Video Analysis

Ivan Laptev's work on space-time interest points has significantly advanced the field of video analysis, particularly in the detection and analysis of human motion. Laptev's method identifies significant changes in both spatial dimensions (physical space in the video and, importantly the width and height of the video frame. It consists of identifying changes and movement in different areas inside a frame and temporal dimension, capturing how the concerned things change over time, from one frame to another. It is concerned with how objects move and evolve through the video sequence. This allows human motion to be considered dynamic so that walking patterns can be tracked and analyzed with accuracy [5]. This has been successful in various applications, such as action recognition and surveillance, where precise motion detection is necessary.

The relevance of Laptev's work to this research lies in the application of similar principles to analyze energy dissipation in video data. By utilizing spatial-temporal features, this research aims to improve the accuracy of energy dissipation measurements, providing a deeper understanding of the dynamics of motion and energy usage in video-recorded systems.

## 2.3 Energy Dissipation in Video Signal Processing

Energy dissipation in video signal processing is a very important area of research that has seen various methodologies and metrics being developed. Some key methodologies are the Energy Dissipation Rate (EDR) and Motion Energy Loss Index (MELI), which quantify the energy that is lost during activities captured in the video. These measures provide valuable information on the efficiency of energy usage by video signal processing systems.

The foundational principles from "Mechanical and Electromagnetic Vibrations and Waves" by Tamer Becherrawy have been important in shaping this understanding of energy dissipation in video-recorded vibrating systems. This book delves deeply into the behavior of mechanical vibrations and waves, providing a solid theoretical background. [8]

Key concepts such as damping and resonance are particularly relevant. The energy diminishes with time, and damping mechanisms—like viscous damping—will help to create appropriate models that can be used to measure energy dissipation. Resonance, about highs in energy transfer, points analysis towards important frequencies where a lot of energy is being dissipated. Besides, wave propagation principles, such as reflection, refraction, and interference, are very important in the mapping of energy flow and loss zones. Interpretation of how and where the principle happens is guided with the help of the video data.

Becherrawy's book offers a conceptual framework that strengthens this overall approach, ensuring that this methods are grounded in strong scientific principles. This has led to more reliable and precise results in this research on energy dissipation in video-recorded vibrating systems. By integrating this framework, this research has achieved greater consistency and accuracy. The current research aims to build on these methodologies by using modulation techniques, particularly the use of Mersenne primes, to address existing gaps in the literature for measuring energy dissipation.

### 2.3.1 Definition of Energy dissipation

Energy dissipation in the context of video signal processing refers to the process by which the kinetic energy of motion within a video-recorded system is converted into other forms of energy. For instance, heat or sound. They are leading to a loss of usable energy from the system. These all are caused by different factors such as friction, air resistance, and internal resistance of the material. Energy dissipation that happens during the video is of great importance because it affects the efficiency and performance of motion detection and analysis algorithms. Quantifying how energy is lost during the motion of vibrating objects will enable optimization of video processing techniques to obtain precision and reliability in the analyses [8].

It has been estimated that in prokaryotic cells, the energy used for sensing and adapting to the environment is about 5% of that for motion [9]. This highlights the energy allocation towards motion. By applying this concept to video signal processing, a better understanding can be gained of the proportion of energy dedicated to different activities within video-recorded systems.

This observation underscores the broader implications of energy efficiency across different biological and mechanical systems. It was done by highlighting the importance of effective energy management strategies.

By applying these principles, this research has developed improved methodologies for measuring and analyzing energy dissipation in video data, leading to more accurate and efficient processing techniques.

### 2.3.2 Theoretical Background

Through Tamer Becherrawy's Mechanical and Electromagnetic Vibrations and Waves, the basic principles underpinned a comprehension of energy dissipation in video-recorded vibrating systems. This book discusses in deep explanation the behavior of mechanical vibrations and waves, giving the student a strong theoretical background [8].

1. The total energy of the oscillator in the case of under-damping is given by:

$$U_{(T)} = U_{(C)} + U_{(P)} = \frac{1}{2} m A^2 e^{-2\beta t} \left[\tilde{\omega}^2 + 2\beta^2 \cos^2(\tilde{\omega} t + \phi) + \beta \tilde{\omega} \sin 2(\tilde{\omega} t + \phi)\right] \tag{2.1}$$

   Location: Page 19, Equation (1.59)

2. The total energy of an underdamped oscillator decreases over time due to friction forces. The work of these friction forces results in consistent energy dissipation, as described in the equation

$$dU_T = -2\beta m A^2 e^{-2\beta t} \left[\tilde{\omega} \sin(\tilde{\omega} t + \phi) + \beta \cos(\tilde{\omega} t + \phi)\right]^2 dt \tag{2.2}$$

   Location: Page 19, Equation (1.60)

3. The decrement of total energy $U_T$ is influenced by friction forces, resulting in energy dissipation. This relationship is expressed as:

$$dU_T = -F_{fr} \dot{x}\, dt = b \dot{x}^2\, dt = 2 m \beta \tilde{\omega}^2\, dt \tag{2.3}$$

   Location: Page 19, Equation (1.61)

4. The decrease in total energy of the system corresponds directly to the energy dissi-

pated by the friction force. This relationship can be expressed mathematically as:

$$dU_{fr} = -dU_T \tag{2.4}$$

This indicates that any loss in total energy $U_T$ is due to the work done by the friction force, dissipating energy from the system. Location: Page 19, Text

5. The energy of the oscillator decreases exponentially over time due to the exponential factor $e^{-2\beta t}$. This can be represented as:

$$\langle U_{\tilde{T}}(t)\rangle = \frac{1}{\tilde{\tilde{T}}}\int_{t-\frac{\tilde{T}}{2}}^{t+\frac{\tilde{T}}{2}} U_{\tilde{T}}(t')\,dt' \tag{2.5}$$

This equation shows the average energy of the oscillator over a time interval centered at t, considering the exponential decay factor. Location: Page 20, Equation (1.62)

6. Simplifying the calculation by assuming $e^{-2\beta t}$ varies slowly over the interval of time varies slowly over the interval of time:

$$\langle U_{\tilde{T}}(t)\rangle \approx \frac{mA^2}{2\tilde{T}}e^{-2\beta t}\int_{t-\frac{\tilde{T}}{2}}^{t+\frac{\tilde{T}}{2}} [\tilde{\omega}^2 + 2\beta^2\cos^2(\tilde{\omega}t'+\phi) + \beta\tilde{\omega}\sin 2(\tilde{\omega}t'+\phi)]dt' \tag{2.6}$$

This approximation indicates how the energy of the system, modified by the exponential decay factor, can be averaged over a small time interval for simpler calculation. Location: Page 20, Equation (1.63)

7. The averaged energy decreases exponentially over time

$$\langle U_{\tilde{T}}(t)\rangle \approx \frac{1}{2}KA^2e^{-2\beta t} = U_{\tilde{T}}(0)e^{-2t/\tau} \tag{2.7}$$

This demonstrates the exponential decay of the system's energy due to damping effects. Location: Page 20, Equation (1.63)

8. The rate of energy dissipation increases as the amplitude of oscillation decreases,

showing a nonlinear relationship between amplitude and energy loss. Location: Page 21, Section 1.8.

9. In systems with higher damping coefficients, the energy dissipates more rapidly, leading to quicker stabilization of the system. Location: Page 21, Section 1.8.

10. Energy dissipation in vibrating systems can be modeled using exponential decay functions, highlighting the importance of time constants in such systems. Location: Page 22, Section 1.8.

11. The energy dissipation in an oscillating system is affected by the material properties, such as elasticity and internal friction, of the vibrating components. Location: Page 22, Section 1.8

12. The efficiency of energy dissipation mechanisms is important in designing systems to minimize unwanted vibrations and improve performance. Location: Page 23, Section 1.8

13. The mathematical representation of energy dissipation in mechanical systems can be extended to electromagnetic systems, demonstrating the universality of the concept. Location: Page 24, Section 1.9

14. Energy dissipation analysis is important in predicting the long-term behavior of oscillating systems, particularly in understanding the lifespan and durability of mechanical components. Location: Page 25, Section 1.9

15. The energy dissipation characteristics of a system can be used to identify potential faults or inefficiencies in mechanical and electromagnetic systems. Location: Page 26, Section 1.10

Observations from Maxwell (1872) provide foundational knowledge into the concept of energy dissipation:

1. **Dissipation of Energy in Vibrations:**

   Maxwell talks about energy dissipation in oscillatory systems because of intrinsic friction and the opposition to motion. The conceptual burden is key to providing a framework for how video-based vibrational systems lose energy and thus are less deceptive about the measure of dissipation. [10]

2. **Thermal Energy and Dissipation:**

   Kinetic Energy Turning Into Thermal Energy – Maxwell goes on to describe how kinetic energy gets converted into thermal energy which is necessary for the idea of dissipation. It is important in video signal processing for understanding how motion energy can be made to change into heat, and so how efficiently the system works.

3. **Energy Transfer Mechanisms:**

   The book describes various mechanisms through which energy is transferred and eventually dissipated, including conduction and radiation. These mechanisms are relevant for analyzing how energy is distributed and lost in video data.

4. **Quantitative Analysis of Energy Loss:**

   Maxwell gives us tools to quantify energy loss in systems. Similar methods can be used to quantify energy dissipation in video signals, thus providing a mathematical framework for analysis.

5. **Impact of Material Properties on Dissipation:**

   This is used to draw some general conclusions about the impact of various material properties on energy dissipation (i.e. how different materials dissipate as-is what alters the visa-vis another — especially vis-a-side contact logics that in shown up terms) and also which kinematic quantity can have an effect onto this rate. This observation seems relevant when considering video analysis: material properties of the objects in motion may influence energy dissipation rates.

6. **Role of External Forces in Energy Dissipation:**

   Energy dissipation is notably governed by external forces such as friction and air resistance according to Maxwell To evaluate the energy losses in video systems, these factors should be accounted for.

7. **Mathematical Formulation of Dissipation:**

   Maxwell presents mathematical formulations for energy dissipation, which can be directly applied to model the energy loss in video signals. These formulations provide a basis for developing accurate energy dissipation metrics.

8. **Resonance and Energy Dissipation:**

   The concept of resonance and its effect on energy dissipation is explored. In video signal processing, understanding resonance can help identify important frequencies where energy loss is maximized.

9. **Energy Conservation Principles:**

   The book emphasizes the principles of energy conservation, which are fundamental to understanding energy dissipation. In video analysis, ensuring energy conservation is key to accurate measurements of energy loss.

10. **Temporal Dynamics of Energy Dissipation:**

    Maxwell discusses the temporal aspects of energy dissipation, explaining how energy loss varies over time. This observation is important for analyzing time-dependent energy dissipation in video signals.

11. **Wave Propagation and Dissipation**

    The propagation of waves and their energy dissipation mechanisms are detailed, and relevant for understanding how wave-like motion in video recordings leads to energy loss.

12. **Impact of Damping on Energy Dissipation:**

    Different types of damping and their effects on energy dissipation are covered. This knowledge helps in modeling how damping in video-recorded systems affects energy loss.

13. **Nonlinear Effects in Energy Dissipation:**

    Nonlinear effects and their contribution to energy dissipation are explored, providing a more complex understanding of energy loss in systems with nonlinear behavior, which can be applied to video signal analysis.

By integrating these observations and principles from Maxwell's book, this chapter establishes a comprehensive framework for analyzing energy dissipation in video signal processing, enhancing the accuracy and reliability of motion analysis in video recordings.

## 2.4 Developments in Modulation Techniques

One of the popular areas in video signal processing is energy dissipation methodologies. High-definition video content and an increase in the demand for video processing systems make the understanding and minimization of energy dissipation very important.

Key methodologies include the Energy Dissipation Rate (EDR) and Motion Energy Loss Index (MELI), which quantify energy loss during different activities captured in videos. These are vital for assessing the efficiency of energy usage in video signal processing systems. For example, EDR measures the rate at which energy is consumed during video processing, helping identify the efficiency of different algorithms and hardware implementations. MELI quantifies the energy loss associated with motion estimation and compensation, which are important in video encoding and decoding processes.

Advanced modulation techniques, such as those involving Mersenne primes, are used to boost the accuracy of energy dissipation measurements. By comparing modulated with non-modulated motion waveforms, a further understanding can be obtained of energy usage and loss within video-recorded systems. This approach complements the methodologies toward better precision and faster measurements, which are important for analyzing energy dissipation in vibrating systems recorded in videos.

Recent developments in video compression techniques, such as H.264/AVC and H.265/HEVC, have incorporated sophisticated algorithms for motion estimation and compensation. The energy required in video processing has been decreased due to advanced codecs, which make data compression more efficient without compromising the quality of the video produced [11, 12]. This is a clear implication of optimizing energy dissipation in video processing.

Additionally, gradient signal amplification has been explored as a significant technique to improve signal detection and analysis. In their recent work, Lan et al. (2012) explored the concept of gradient signal amplification, especially in the context of signal processing and energy dissipation. They demonstrated that gradient signals could be amplified to allow improved detection and analysis of subtle changes in signal properties. This is useful when the signal-to-noise ratio is small, such as in the analysis of energy dissipation in video-recorded vibratory systems. By amplifying the gradient signal, small changes in motion and energy patterns can be detected, which would otherwise be obscured by noise. This approach provides significant implications for improving the accuracy and reliability of energy dissipation measurements in complex systems—one of the purposes of this study: methods to refine video signal processing. [9].

Observations from the Lan et al. (2012) paper highlight several important aspects of energy dissipation;

- **Energy dissipation rate**

  The rate at which energy is dissipated is an important determinant of efficiency in signal processing systems. Higher dissipation rates lead to faster energy loss, potentially reducing overall system performance.

- **Adaptation speed**

The speed at which a system adapts to changes in its environment can significantly influence its energy efficiency. Faster adaptation speeds may lead to increased energy consumption but can also improve the accuracy and responsiveness of the system.

- **Maximum adaptation accuracy**

  The high accuracy of adaptation is necessary for the precise processing of signals. Energy dissipation must be traded off against adaptation accuracy for optimal performance.

Moreover, machine learning and deep learning techniques have been used to improve video signal processing. Both motion detection and classification for video surveillance systems could be accomplished using these techniques in more energy-efficient processing, as an example of better performances resulting from the neural networks or support vector machines (SVMs) [13]. The following section outlines how these methods can be adapted for video data to facilitate more energy-efficient and optimized video processing approaches.

In addition, applications of dedicated hardware accelerators like field-programmable gate arrays (FPGAs) and application-specific integrated circuits (ASICs) have shown high potential to mitigate energy consumption. Furthermore, the use of specialized hardware accelerators, such as field-programmable gate arrays (FPGAs) and application-specific integrated circuits (ASICs), has shown significant promise in reducing energy dissipation. These hardware solutions are designed to perform specific video processing tasks more efficiently than general-purpose processors, leading to considerable energy savings [14]. These hardware solutions are designed to perform specific video processing tasks more efficiently than general-purpose processors, leading to considerable energy savings [14].

The integration of these advanced techniques and methodologies highlights the importance of energy dissipation analysis in video signal processing. By employing modulation techniques and advanced metrics, this research aims to contribute new insights and methodologies that improve the efficiency and accuracy of video signal processing, particularly in understanding energy dissipation in complex systems.

## 2.5 Real-World Applications

### 2.5.1 Video Surveillance and Motion Analysis

Video surveillance systems are important for security and monitoring, and accurate motion analysis is important for identifying and tracking objects. Here, most of the research is on improving system efficiency and accuracy using advanced signal processing techniques. For instance, the refinement of machine learning algorithms for motion detection and false positive reduction are some major developments made. Cornerstone methods to increase real-time video analysis were offered, such as the work by Viola and Jones (2001) on rapid object detection using a boosted cascade of simple features [15].

### 2.5.2 Action Recognition

In action recognition, understanding and accurately measuring energy dissipation is important for identifying and analyzing dynamic activities within video footage. Methodologies like Histograms of Oriented Gradients (HOGs) described by Dalal and Triggs, 2005 (Dalal Trigs 2012), largely enriched the field to assess better motion capture, and action recognition among others. These developments are valuable for applications that use detailed motion analysis and energy dissipation metrics such as sports analytics or behavioral studies.

To conclude, the techniques and findings from the reviewed works align closely with the methodologies developed in this thesis. Both this research and the existing studies use advanced modulation techniques to improve measurement accuracy and efficiency, addressing challenges in traditional methods. By applying these techniques, this research aims to offer new knowledge and methodologies that improve the efficiency and accuracy of video signal processing, particularly in understanding energy dissipation in complex systems.

## 2.6 Machine Learning and Deep Learning in Video Analysis

In recent years, the combination of machine learning, computer vision, and deep learning technologies has greatly developed the field of video signal processing. These applications facilitate the extraction of features, the recognition of patterns, and the construction of predictive models which are necessary for understanding complex videos.

### 2.6.1 Deep Learning for Video Analysis

Video interpretation techniques have greatly benefitted from the recent advances in deep learning by taking advantage of the high content of data and performing the processes of feature extraction and pattern detection. In this regard, Litjens et al. (2017)., 2017 tells that deep learning is certainly shaping up to be a great improvement over traditional image processing and diagnostics in the field of medical imaging". Deep Learning studies, despite the emphasis being on medical imaging, still occupy importance in video analysis where depleting energy from actions grasped on tape even in Deep learning models can be extracted further and better understood. [16].

### 2.6.2 Machine Learning for Motion Analysis

Machine learning algorithms have been widely used for motion analysis in video surveillance. Techniques such as support vector machines (SVMs) and neural networks have improved the detection and classification of moving objects. Jain and Seung [13] discuss natural image denoising with convolutional networks, which can be adapted for motion analysis in videos to reduce noise and increase the clarity of moving objects [13].

## 2.7 Progress in Video Signal Processing Techniques

Innovations in video signal processing techniques have significantly improved the efficiency and accuracy of analyzing video data.

### 2.7.1 Motion Estimation and Compensation

Motion estimation and compensation are important techniques for video signal processing and relate closely to the study of energy dissipation phenomena. They relate to moving from one frame in the video to another and predicting distortions that would occur while going from the old image to the new image to smoothen and get accurate images of a moving object. It has been noted in the work of Litjens et al. (2017) that the correct estimation of motion influences the processes for activity recognition from video, which helps assess the energy loss in vibrating systems [16].

The principles from ”Mechanical and Electromagnetic Vibrations and Waves” by Tamer Bécherrawy also apply to motion estimation and compensation. The book's analysis of vibrational behaviors and energy transfer mechanisms provides a theoretical foundation for understanding how motion can be accurately predicted and corrected in video signal processing, leading to more precise energy dissipation measurements. [8]

### 2.7.2 Object Detection and Tracking

Object detection and tracking are perhaps the simplest and most important problem solvers in video analysis. On rapid object detection Viola in addition to Jones of the year 2001 boosted the cascade of simple features, and real-time video analysis has been enriched by foundational methods. These techniques are useful for object detection and recognition in video surveillance and application in action recognition. [15]

## 2.8 Comparative Analysis

The techniques and findings from the related works closely align with the methodologies developed in this thesis for analyzing energy dissipation in video-recorded vibrating systems. Both this research and the referenced studies utilize advanced modulation techniques to increase measurement accuracy and efficiency, addressing challenges associated with traditional methods. In this thesis, the application of Mersenne primes for modulation is proposed for the first time, as in the case of the MSSM method, which is used to increase the accuracy of energy dissipation measurements in video data. The findings differ from the non-modulated motion waveforms and shed light on the energy distributions inside the system in the same way the MSSM improves the comprehension of viscoelasticity in microrheology. Also, as highlighted in both works, phase and amplitude estimation are important across the range of frequencies.

Ivan Laptev's work on space-time interest points in video analysis, which identifies significant changes in both spatial and temporal dimensions, has developed the detection and analysis of human motion. His method captures the dynamic nature of human movement and enables accurate tracking and analysis of walking patterns [5]. This method's effectiveness in action recognition and surveillance parallels this approach of utilizing spatio-temporal features to analyze energy dissipation in video data. By leveraging these features, this research aims to improve the accuracy of energy dissipation measurements, providing a deeper understanding of the dynamics of motion and energy usage in video-recorded systems.

Metrics such as the Energy Dissipation Rate (EDR) and Motion Energy Loss Index (MELI) have been important in quantifying energy loss during different activities captured in videos. this research supplements these methodologies by the use of modulation techniques, especially the Mersenne primes into the existing knowledge gaps concerning energy dissipation measurement. This approach is designed to meet the objective of increasing the energy efficiency of Internet communication devices without a reduction of their efficiency or video quality.

Recent developments in video compression techniques, like H.264/AVC and H.265/HEVC, have highlighted the importance of optimizing energy dissipation in video processing. These codecs employ elaborate algorithms for motion estimation and compensation leading to improved compression efficiency and passive energy saving [11, 12]. this work continues with these developments by considering the energy-damping efficiency of vibrating structures recorded on video with attempts to signify the role of algorithms and techniques in enhancing the measurement's precision and energy consumption.

Additionally, the integration of machine learning and deep learning techniques has significantly developed video signal processing. Neural networks and support vector machines (SVMs) have increased the scope of detection and classification of motion in video based surveillance systems and thereby the energy efficiency of the processing [13]. These techniques can also be extended for power characterization in video data that would help in the development of other efficient video processing methodologies. this research adapts these machine learning models to improve the understanding of energy dissipation in video-recorded activities, aligning with the broader trend of leveraging AI in video analysis.

Furthermore, the use of specialized hardware accelerators, such as field-programmable gate arrays (FPGAs) and application-specific integrated circuits (ASICs), has shown significant promise in reducing energy dissipation in video signal processing. It can be seen that these hardware solutions are meant to do some particular video processing tasks faster than general-purpose processors and hence they consume quite a lot less energy [14]. These developments help this research for the use of efficient hardware implementations to improve the measurement and analysis of energy dissipation in complex systems.

Therefore, the techniques and findings from the reviewed work closely align with the methodologies developed in this thesis. Both this research and the existing studies use advanced modulation techniques, innovative signal processing methods, and specialized hardware to improve measurement accuracy and efficiency. By applying these techniques, this research aims to offer new knowledge and methodologies that increase the efficiency and accuracy of video signal processing, particularly in understanding energy dissipation in video-recorded vibrating systems.

## 2.9 Conclusion

This chapter has reviewed key developments and methodologies in microrheology, video analysis, and energy dissipation, all of which are important to understanding the context of this research on detecting energy loss in modulated versus non-modulated motion waveforms from vibrating systems recorded in videos.

The work in microrheology, especially the Multiple Sinusoids Superposition Method (MSSM) by Kundu et al. (2021) [7], has shown how advanced modulation techniques can improve measurement accuracy and efficiency. This knowledge has been important in shaping this approach to analyzing energy dissipation in video-recorded systems using Mersenne primes for modulation.

Ivan Laptev's research on space-time interest points [5] has provided a strong foundation for detecting and analyzing human motion in videos. By applying similar spatio-temporal features, this research aims to improve the precision of energy dissipation measurements, offering a deeper understanding of the dynamics of motion and energy use in recorded videos.

Real-world applications in areas such as video surveillance, video compression, and action recognition underscore the importance of precise energy dissipation measurements. Viola and Jones (2001) [15] rapid object detection and Wang and Schmid's (2013) [4] improved trajectories for action recognition emphatically point to the fact that applied mathematics plays a fundamental component of the improvements found in these fields.

In conclusion, the techniques and findings from the reviewed works align closely with the methodologies developed in this thesis. Just like in this study, most of the previous research uses complex modulation techniques to increase measurement precision and avoid deficiencies in traditional measurement techniques. Through utilizing these techniques, this study also intends to contribute new findings and approaches, which help improve video signal processing and clarify corresponding energy dissipation in related systems.

By focusing on these comparative aspects and incorporating advanced techniques and methodologies, this research aims to contribute significantly to the field of video signal processing, particularly in the context of understanding and measuring energy dissipation in complex systems.

# Chapter 3

# Theory and Methodology for Analyzing Energy Dissipation in Vibrating System Waveforms recorded in videos

## 3.1 Introduction

This chapter represents the theoretical foundations and methodologies used for this research, with a particular focus on the role of signal modulation using Mersenne primes in adjusting the frequency $\omega$ of an Euler exponential $e^{j2\pi\omega t}$

Signal modulation, the process of varying a carrier signal to encode information, is a fundamental concept in signal processing. By manipulating parameters of the signal such as amplitude, frequency, and phase angle. The energy dynamics within a system can be significantly modulated. The energy of a waveform is measured in terms of the area bounded by a waveform curve and the horizontal axis. This research explores how these modulation techniques, especially when combined with the unique properties of Mersenne primes, such as connection to perfect numbers, rarity, and size and optimization in computations, can boost the understanding and measurement of energy dissipation in video waveforms. It has been observed by G.W. Hill [17] that Mersenne primes $M_p = 2^p - 1 = 3, 7, 31, ...$

for Prime $p = 3, 3, 5, ...$ are useful in estimating variability as well as in estimating average values.

In this thesis, the Hilbert transform is a useful tool in signal processing, that makes it possible to create an analytic signal from a real-valued signal. An analytic signal $f(t) = x(t) + jh(t)$ maps to points in the complex plane with negative components [18, vol. 1, p. 183]. This transformation provides knowledge of the instantaneous amplitude and phase of the signal, which are necessary for calculating energy dissipation. By applying the Hilbert transform to video-recorded waveforms, it can be identified and quantified the subtle variations in energy that occur during different types of motion. [5]

Mersenne primes, numbers of the form $M = 2^p - 1$ offer specific advantages in this context. Since they are simple binary numbers and have rather repetitive and easily predictable patterns, they are ideal for Digital Signal Processing (DSP) [19]. In modulation, the usage of Mersenne primes benefits not only computations but also raises the accuracy of estimated coefficients for energy dissipation. This chapter will also explain why Mersenne primes are preferred and how they contribute to a more strong analytical framework.

This chapter aims to provide a comprehensive overview of the methodologies employed in this research. In this way, the description of the modulation of the signals and the use of the Hilbert transform create the foundation for a deeper investigation of energetic phenomena in video-recorded vibrating systems. Further discussion will provide practical interpretations of the findings, emphasizing how these can lead to more effective and accurate methods of processing video signals.

In the sections that follow, the concepts in detail will be addressed, starting with the fundamentals of signal modulation and moving on to the specific role of Mersenne primes over some random numbers. The mathematical framework underpinning the analysis will then be discussed, followed by an examination of the practical applications and implications of the research. By the end of this chapter, readers will have a clear understanding of the theoretical and methodological foundations of this study and their significance in the broader context of video signal processing.

## 3.2 Hilbert Transform in Signal Analysis

The Hilbert transform is a linear operator that takes a real-valued function $x(t)$ and produces another real-valued function $\hat{x}(t)$, which is the Hilbert transform of $x(t)$. The analytic signal $y(t)$ is then formed by combining $x(t)$ and $\hat{x}(t)$ into a complex-valued function:

$$y(t) = x(t) + j\hat{x}(t)$$

where $j$ is the imaginary unit. This transformation is necessary for calculating the instantaneous amplitude $A(t)$ and instantaneous phase $\phi(t)$:

$$A(t) = \sqrt{x(t)^2 + \hat{x}(t)^2}$$

$$\phi(t) = \arctan\left(\frac{\hat{x}(t)}{x(t)}\right)$$

### 3.2.1 Application to Video Signals

In video signal analysis, the Hilbert transform is applied to the intensity values of selected rows from video frames. This approach allows a focus on specific areas of the video where motion or energy variations are significant. In the given video frame, the selected row chosen for analysis represents a section of the video where the mechanical vibrations are high.

The following figure illustrates a sequence of video frames captured at intervals of 5 frames each. This interval is taken for a better view. This shot sequence shows a person crossing the scene. The frames were chosen to outline both the continuity of motion and the changes with time. Each frame will be of importance in analyzing the pattern of energy dissipation within the video signal to get a deeper understanding of the dynamics of the recorded system.

Figure 3.1: Sequential Video Frames

In the figure3.1, it displays several frames from the recorded video (Walker.mp4). Observing these frames sequentially allows for a detailed analysis of motion and the corresponding energy dissipation within the system. The positions of the person and other elements in each frame can be used to understand how energy is distributed and dissipated over time.

### 3.2.2 Analysing selected rows

In this thesis, a row in a video frame is considered, and the intensity values of this row are extracted over time to create a time-series signal representing the motion dynamics of that particular row. The application of the Hilbert transform to this time-series signal will analytically give us a signal. Using this, both instantaneous amplitude and phase can be derived. These parameters provide an overview of the energy fluctuations over the selected row, allowing for a more precise quantification of energy loss. See the Appendix A


Figure 3.2: Highlighted Rows

In this research, the Hilbert transform is employed to analyze energy dissipation in video-recorded vibrating systems. A single row of intensity values from a video frame was extracted over time to generate a time-series signal. This allows the application of the Hilbert transform to the time-domain signal, yielding the analytic signal in complex form, from which the instantaneous amplitude and phase can be derived. The instantaneous amplitude highlights variations in the signal's magnitude, while the instantaneous phase reveals shifts in the signal's timing. This co-analysis has helped in finding the discontinuities of energy dissipation for sensitive knowledge of the energy dynamics of the video signals. Figure 3.3 presents frame number 10 with the chosen row highlighted and plots of the original signal, instantaneous amplitude, and instantaneous phase derived from the Hilbert transform.

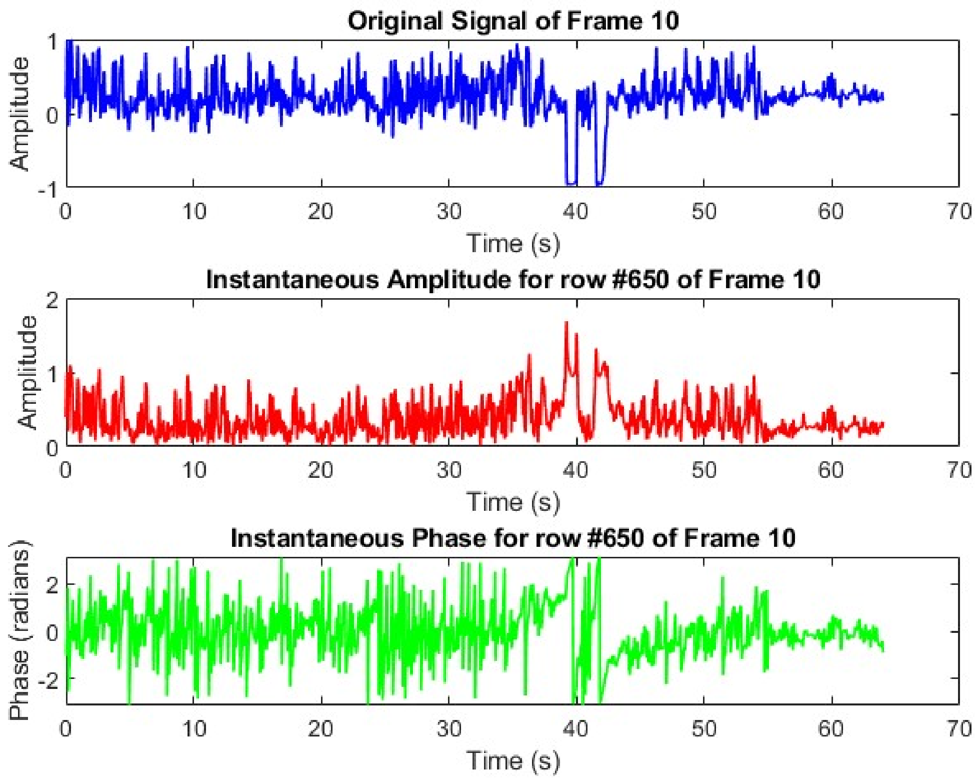


Figure 3.3: Phase and Amplitude of a row

In this figure 3.3, the original signal shows the raw intensity values of the selected row over time, representing the direct pixel intensity data extracted from the video frame. It captures the exact changes in brightness levels along the row as the video progresses.

The Instantaneous Amplitude-Time Graph plots the magnitude of the analytic signal derived through a Hilbert transform. Though its time axis is identical to that of the original signal, the amplitudes relate to the envelope of the signal and thus to the variations in energy over time. Instantaneous amplitude provides a smoothed version that can reveal an underlying pattern and fluctuations not as easily visible in the raw signal.

The instantaneous phase graph provides information about the timing and oscillatory behavior of the signal. Derivative of the Hilbert transform, it characterizes the phase variation of the signal with respect to time. This phase information provides knowledge into the exact timing and synchronization of the different components of the signal in great detail and hence into the dissipation pattern of energy in video-recorded systems. By examining the instantaneous phase, subtle changes in motion dynamics and energy distribution can be detected, which might be missed with amplitude analysis alone.

### 3.2.3 Detecting Subtle Changes

One of the primary advantages of using the Hilbert transform in video signal analysis is its ability to detect subtle changes in energy dissipation. Instantaneous amplitude captures changes in the signal magnitude, while instantaneous phase detects alterations in the signal timing. These features, which have valuable implications for fault diagnosis, might easily have been missed using conventional methods of analysis.

By leveraging the Hilbert transform, the understanding of energy dynamics within video-recorded vibrating systems can be improved. The present methodology forms the basis for further analysis and discussions that appear later in this research. It provides a frame for the measure and meaning of energy dissipation in video signals.

### 3.2.4 Modulation Techniques and Mersenne Primes

Mersenne primes, defined by

$$M = 2^p - 1$$

$p$ is a prime number and has unique properties that make them highly suitable for modulation in signal processing. Their binary representation as a sequence of 1s simplifies computational processes and increases efficiency, which is particularly beneficial for digital signal processing tasks [20], [8]. The use of Mersenne primes in modulation is not just a matter of convenience but also one of performance optimization. These primes help create precise and predictable frequency components, necessary for detailed frequency analysis and maintaining signal integrity. The sharpness of these frequency components aids in isolating specific energy dissipation patterns within the video signals, which is crucial for accurate measurement and analysis [19].

Additionally, the unique properties of Mersenne primes allow for efficient modulation and demodulation processes. Their predictable nature ensures consistent modulation effects, which is vital for systematic studies and reproducible results. This predictability also increases signal integrity, reducing noise and irregularities that can complicate analysis. Mersenne primes' efficiency in computational processes means faster processing times and lower overhead, making them ideal for handling large datasets and real-time applications [21]

| $p$ | Mersenne Prime ( $M = 2^p - 1$ ) | Binary Representation |
|---|---|---|
| 2 | 3 | 11 |
| 3 | 7 | 111 |
| 5 | 31 | 11111 |
| 7 | 127 | 1111111 |
| 13 | 8191 | 1111111111111 |

Table 3.1: Binary representation of Mersenne primes for different $p$ values.

### 3.2.5 Advantages of Mersenne Primes in Signal Modulation

1. **Predictable Frequency Components**

   Mersenne primes create distinct and sharp frequency components, making it easier to analyze and interpret signal modulations. This property is particularly useful in applications requiring precise frequency analysis.

2. **Computational Efficiency**

   The binary representation of Mersenne primes as sequences of 1s simplifies digital signal processing. This efficiency is used for real-time applications and large datasets, reducing computational load and improving processing speed.

3. **Increased Signal Integrity**

Using Mersenne primes helps maintain signal integrity by providing a consistent and reproducible modulation pattern. This predictability minimizes the introduction of noise and irregularities, ensuring cleaner and more reliable results.

4. **Consistent Modulation Effects**

   The structured nature of Mersenne primes allows for consistent modulation effects across different experiments and applications. This consistency is vital for systematic studies of energy dissipation, enabling researchers to draw accurate and comparable conclusions.

### 3.2.6 Practical Implementation of Mersenne Prime Modulation

Mersenne primes are applied in video signal processing to modulate the amplitude and phase of video signals. This modulation improves the precision of energy dissipation measurements by creating predictable patterns that are easy to analyze.

Algorithms based on Mersenne primes are designed in such a way that energy dissipation analysis becomes more accurate and efficient. These algorithms use unique properties of the primes to derive more reliable and solid results.

The performance of the modulation based on Mersenne primes is compared with the traditional modulation schemes based on random numbers. Such a comparison will establish the superiority of the Mersenne primes on signal integrity and its consistency in the modulation effect.

In conclusion, the use of Mersenne primes in modulation techniques offers significant advantages in signal processing, particularly in enhancing the accuracy and efficiency of energy dissipation analysis over other random numbers.

On the other hand, modulation with random numbers will bring about several disadvantages. The generation of random numbers is predictable; due to this fact, quite often not constant repetitive modulation patterns hamper obtaining reproducible results. It may therefore create many irregularities and noise in the signal that further complicate the analysis and may obscure even very subtle patterns of energy dissipation. Besides, the absence of any definite structure in random numbers introduces an added computational challenge wherein more complex algorithms are needed to handle and process the data efficiently. Therefore, all these create a low fitness for the random numbers against the structured and hence more predictable nature of Mersenne primes concerning accurate and efficient studies on energy dissipation.

## 3.3 Non-modulated and Modulated waveforms

### 3.3.1 Non-modulated waveforms

Non-modulated waveforms are derived from the intensity values of selected rows of pixels in each video frame. These values form a time series that represents the unaltered motion signal. For instance, if the middle row of pixels from each frame was selected, the intensity values of these pixels over time form the non-modulated waveform. This waveform serves as a baseline to compare against the modulated waveforms.

In this analysis, non-modulated waveforms play an important role in establishing a reference point. They provide a clear picture of the natural motion without any external influences or modifications. By examining these waveforms, it is easy to understand the inherent properties of the motion within the video.

To extract non-modulated waveforms, these steps were followed:

1. **Frame Selection**

   Identify and select specific frames from the video where motion is clearly observable.

2. **Row Extraction**

   From each selected frame, extract the intensity values of a particular row of pixels. This row is chosen based on its relevance to the motion being analyzed.

3. **Time Series Formation**

Compile these intensity values over time to form a continuous time series. This series represents the non-modulated waveform.

These steps ensure that the non-modulated waveform accurately reflects the original motion captured in the video frames. The consistency and reliability of this waveform are necessary for meaningful comparisons with modulated waveforms.

Such non-modulated waveforms help in identifying the baseline pattern of energy dissipation within the system. From this waveform, one is able to notice anomalies or other deviations if modulation is introduced into such a waveform. They also provide the ability to measure changes in energy dissipation due to the modulation techniques within the video-recorded system.

For example, in this study, rows from the 10th frame of a video were selected to analyze the non-modulated waveform. The extracted intensity values were then plotted to visualize the motion signal. This visualization helped us understand the energy dynamics and served as a reference for further modulation analysis.

The analysis with non-modulated waveforms will be able to provide a detailed understanding of the system's behavior. It allows the exact treatment of energy dissipation to a more accurate and precise one that provides better methodologies and techniques in video signal processing.

### 3.3.2 Modulated waveforms

Modulated waveforms are derived from the non-modulated signals by applying a mathematical transformation. This process alters the original signal to emphasize certain characteristics or to facilitate more detailed analysis. In this study, exponential modulation with Mersenne prime values is used to create these modified waveforms.

Modulated waveforms play a very important role in this analysis for several reasons:

1. Explore different frequency components of the motion.

2. Reveal hidden patterns or periodicities in the original signal.

3. Provide a means to compare energy dissipation under different modulation conditions.

To generate modulated waveforms, these steps were followed:

1. **Base Signal Selection**

   The analysis begins with the non-modulated waveform extracted from the video frames.

2. **Modulation Function Application**

   An exponential modulation function was then applied to the base signal. The modulation function is of the form: $x(t) = x_0(t) \cdot e^{-j2\pi Mt}$, where $x_0(t)$ is the original signal, $M$ is a Mersenne prime value, and $t$ represents time.

3. **Mersenne Prime Variation**

   The modulation process was repeated using different Mersenne prime values (3, 7, 31) to create a set of modulated waveforms. Additionally, the case of $M = 1$, where there is no $M$ in the exponential part, was considered.

Modulated waveforms allow us to analyze the signal's behavior at different frequency scales, identify resonances or harmonics that might not be apparent in the original signal, and compare energy dissipation patterns under various modulation conditions.

In this analysis, modulated waveforms are generated for each Mersenne prime value and compared with the non-modulated waveform. This comparison includes visual inspection of the waveforms, calculation of Hilbert envelopes to assess instantaneous amplitude, and computation of cumulative energy to measure overall energy dissipation.

By examining how the signal changes under different modulations, an understanding of the underlying dynamics of the motion captured in the video can be gained. This approach can reveal subtle differences in energy dissipation that might not be apparent from the non-modulated signal alone.

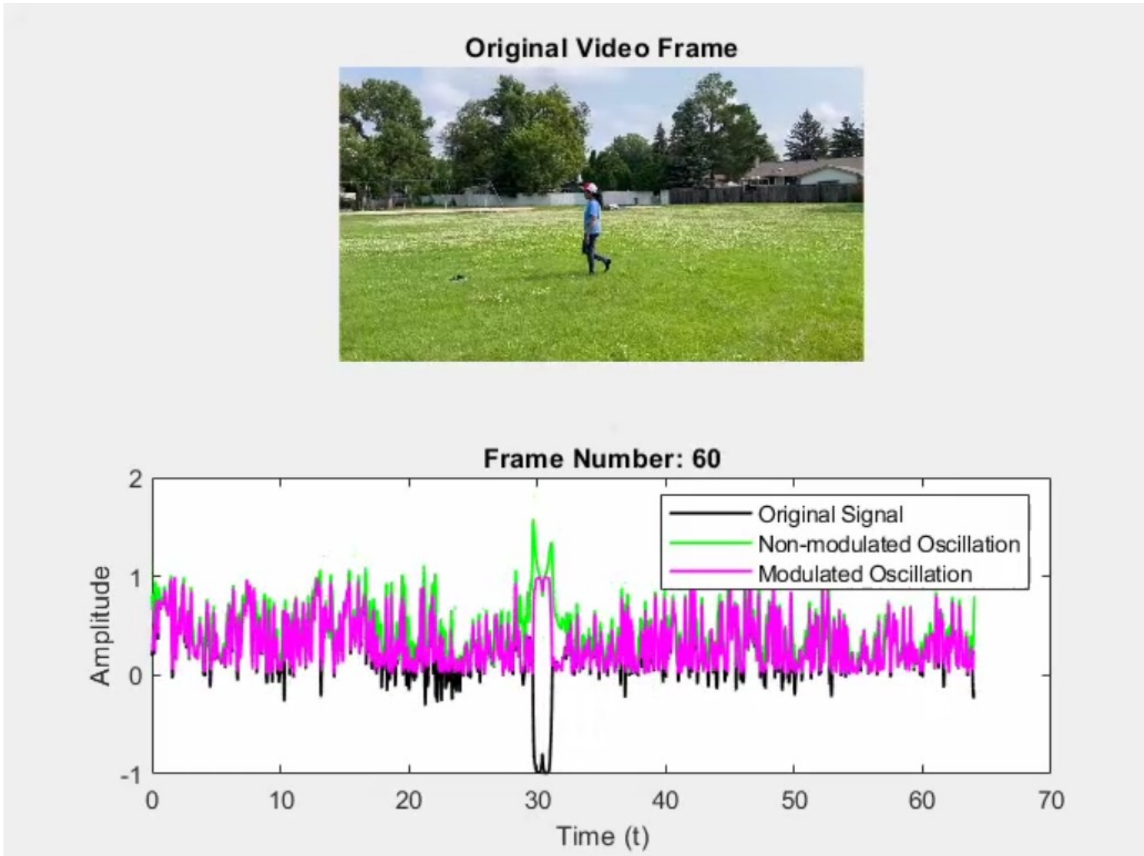


Figure 3.4: Modulated and non-modulated Oscillation for M=7 of Fr=60 (Walking)

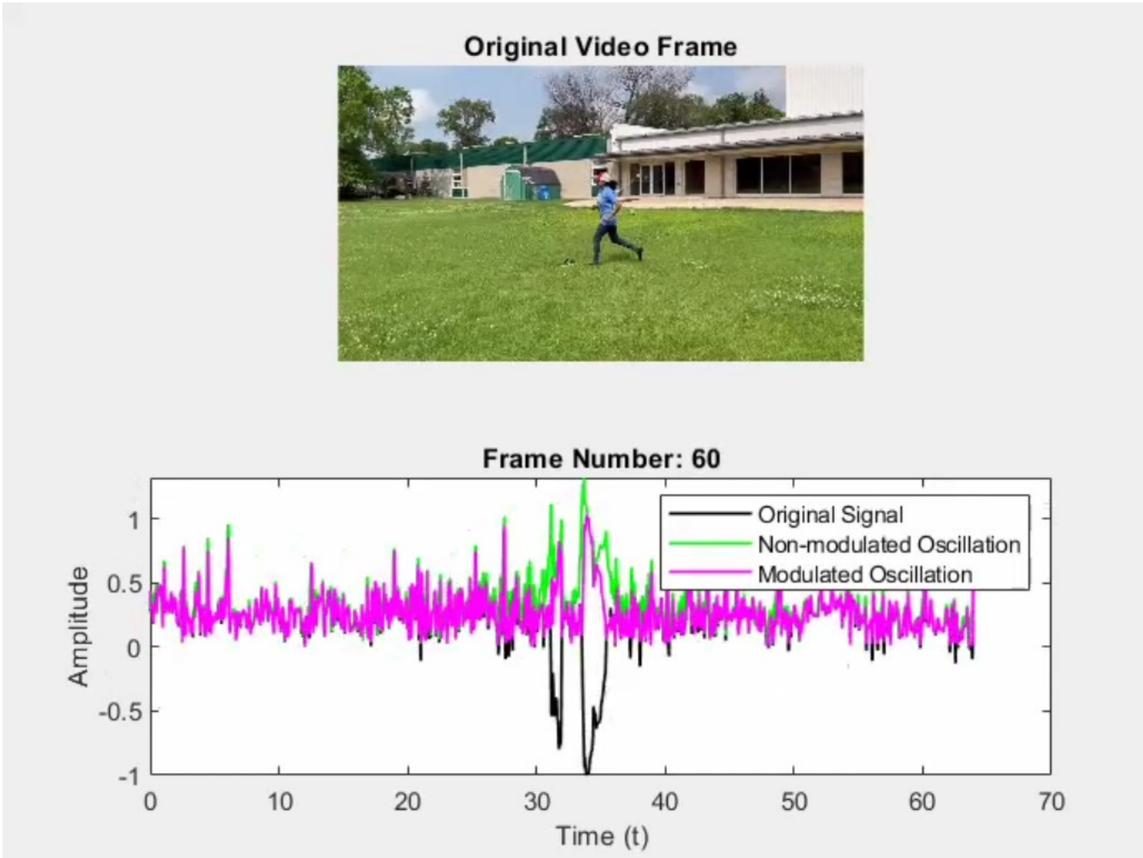


Figure 3.5: Modulated and non-modulated Oscillation for M=7 of Fr=60 (Running)

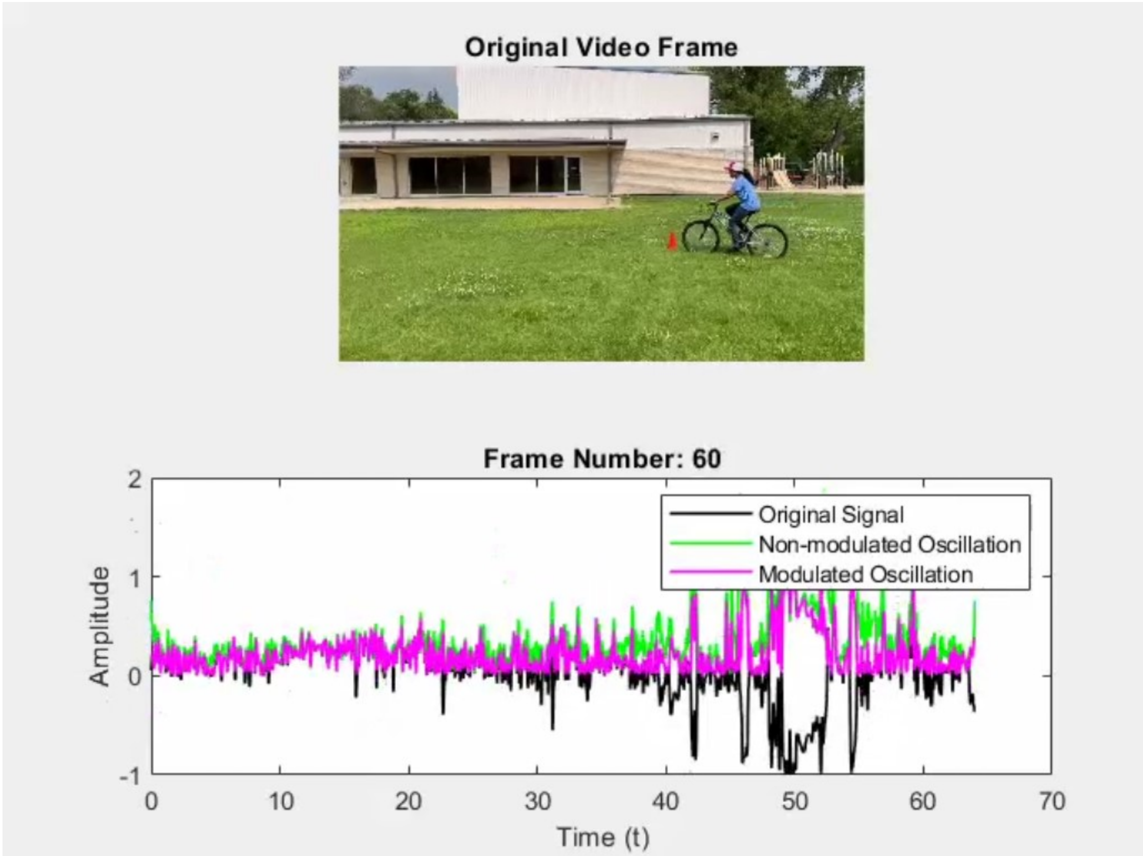


Figure 3.6: Modulated and non-modulated Oscillation for M=7 of Fr=60 (Cycling)

The modulated waveforms, therefore, serve as a powerful tool for extracting additional information from the original video signal, enabling a more comprehensive understanding of the system's behavior and energy characteristics.

All the above snapshots were obtained from the written videos for each activity. The Matlab script for these is in Appendix B. Also, the relevant data table is in Appendix C

### 3.3.3 Impact of Modulation on Energy Dissipation

The research shows that different Mersenne prime modulations can significantly alter the energy dissipation characteristics of the signal. For instance:

- Lower Mersenne prime values tend to preserve more of the original signal's energy structure.

- Higher values lead to more rapid energy dissipation or redistribution across different frequency components.

These findings have important implications for signal processing techniques, and perhaps much better ways to extract and analyze information from video. Energy dissipation analysis allows one to understand more about the underlying dynamics captured in the video signal. The core understanding arising from this analysis develops more efficient techniques for signal processing and improves the capability to extract meaningful information from video data.

## 3.4 Energy Dissipation

### 3.4.1 Mathematical Modeling of Energy Dissipation and Signal Modulation

In sparsity-inducing regularization, the Minimum Concave (MC) penalty function can be thought of analogously as a non-modulated motion curve in physical dynamics. Just as a non-modulated motion curve tends to be concave, curving inwards across the entire motion spectrum, the MC penalty function exhibits a similar inward curvature in its graphical representation. This curvature indicates a dispersal of the penalty's impact, analogous to the way energy in a non-modulated envelope is spread out and less concentrated. In practical terms, the MC penalty function thus distributes the regularization effect across a range of parameter values, avoiding the over-concentration of penalty that is characteristic of more severe, convex penalties. This dispersed penalization strategy is instrumental in achieving sparsity in variables without excessively distorting the model's natural tendencies, reflecting the principle that similar energy in a physical system, and influence within a statistical model should be evenly and appropriately distributed for optimal performance.

In contrast to the non-modulated motion curve, the modulated motion curve is characterized by a series of prominent features known as lobes, which are delineated by convex curves. Unlike the smooth and dispersed nature of non-modulated curves, these lobes signify concentrated points of energy within the curve, with the area of each lobe representing the magnitude of this energy concentration. The convexity of the bounding curves of the lobes means that the energy is not diffused but, instead focuses at chosen and localized points. This parallels the penalty functions in statistical modeling, which apply a more focused and intense penalty on selected model parameters, inducing sparsity in yet another way, more selective and targeted. The modulated curve, with its distinct lobes, therefore provides a visual and conceptual representation of how modulation leads to the concentration of effect, a principle that can be analogously applied to the design of specialized penalty functions in statistical analyses. [22].

In the mathematical discourse on signal processing, particularly in King's extensive work on the subject, the Mellin transform of the Hilbert transform, as delineated by equation (5.103) on page 269 of volume 1, unveils a novel approach to signal modulation. In particular, this study presents an application of the function $f(x) = |x|^{t-1}$ in the Hilbert transform as a radically new envelope modulation. The Hilbert envelope, typically characterized by concave boundaries indicative of dispersed energy, is transformed through this modulation into an envelope with convex boundaries, hence forming highly concentrated energy zones. What results from this is not merely theoretical. This, for human locomotion, such as in the gait of walkers or runners, may indicate an optimization strategy tending to minimize vertical oscillation, favoring the fact that in a motion pattern ascents and descents are separated by extended intervals, which may theoretically improve efficiency during the running phase. Analogously, in mechanical systems, this modulation can lead to more efficient cycles in mechanisms like pistons or flywheels by extending the intervals between their movements. In the context of telecommunication, such as the transmission of square wave signals in telegraphy, employing an envelope that attaches to the peak values of each rectangular pulse results in a smoother signal profile. This application of the Mellin transform of the Hilbert transform thus represents an important transformative step in the modulation of signals, with the potential to improve the functionality of various physical and technological systems. [23]

In considering the mechanisms of energy loss within the dynamics of motion, it is invaluable to consider the function describing energy dissipation with the Hilbert transform's impact on signal processing. For a given motion curve $f(t)$, the time-varying kinetic energy, may be viewed as a damping function, $E(t)$ similar to a convolution integral that summarizes frictional forces, air resistance, and any other resistive forces that are state variable dependent. Guided by the mathematical structure of the Mellin transform, as highlighted in the literature concerning its application in the Hilbert transform, the same methodology will be applied to interpret the scaling of energy dissipation with respect to time. The Mellin transform Mf(p) of the energy profile $f(t)$ allows studying how energy dissipation scales with the system's kinetic energy and solution determination using complex analysis. This offers a path to not only understanding but also quantifying the multi-scale interactions of energy within the system. Analogously to $MHf(p)$, the inverse Mellin transform can be used to revert from the frequency domain to the temporal domain, thus elucidating the real-time characterization of energy loss. By aligning with physical realities, such as initial conditions where $f(t) = 0$ for $t < 0$, and implementing appropriate boundary conditions, the proposed model provides a potent depiction of the dynamics of energy dissipation in motion. This approach allows for improved predictions and strategic interventions to minimize unwanted energy losses [23].

In the analysis of energy dissipation within a system, the mathematical treatment as delineated on page 235, volume 2, 'King', serves as a foundational principle. The total energy loss over time $L$ for a dynamic system can be conceptualized as the sum of the intervals wherein the energy exceeds a certain $\lambda$, analogous to the measure $m\{x : f(x) > \lambda\}$ described by King. This measure is mathematically expressed as $m\{x : f(x) > \lambda\} = \sum_{i=1}^{n}(m_i - a_i)$, with each interval corresponding to a segment of the motion curve where the energy is notably higher than the average or expected value. The relevance of this summation is amplified when considering the roots of the equation $\lambda = \sum_{i=1}^{n} b_i/(x - a_i)$, which characterizes the important points of energy distribution and thus captures the essence of the $b_i$ encapsulates the intrinsic properties of energy loss, enabling a nuanced quantification of dissipation over the motion curve. This approach, drawing from the Viète-Girard theorem, underpins a solid framework for the exploration of energy efficiency and optimization within mechanical systems. [23]

### 3.4.2 Calculation of Energy Dissipation

In video analysis, especially when evaluating movements like walking or running, the concept of "signal energy" plays a crucial role. This term does not equate directly to physical energy measurements such as joules(J) but instead relates to the intensity and duration of variations within the video signal itself. When "signal energy" is discussed in this context, it often refers to a mathematical abstraction rather than direct energy expenditures.

1. **Unit of Measurement in Signal Energy**

   In video processing, signal energy can be measured in various units depending on the precision needed. While joules (J), the standard SI unit of energy that can technically be used, it's often more practical to employ smaller units due to the typically minute energy variations in video data. Units like microjoules (J) per frame or nanojoules (nJ) per pixel are commonly used. These units help quantify the energy content of a video signal relative to frame size or pixel count, providing a detailed view of the energy distribution across the video.

2. **Practical Application and Calculation**

   For videos of dynamic activities, such as walking or running, the motion information is the most important. This could be captured frame by frame using techniques like optical flow or motion tracking to quantify exactly how much each pixel is moving. The signal energy could be computed by squaring the value of pixel velocity or acceleration at each frame and then integrating these across the time of the whole video. It works through the effective measurement of kinetic energy represented in the motion, therefore providing a more quantitative look at the subject's activity over time.

3. **Signal Energy in Motion Analysis**

   One useful metric in motion analysis is the quantification of signal energy. Computation of the integral of the squared magnitudes of variations in signal-meaning changes in pixel intensity or displacement, for instance, analysts to capture both amplitude and time of signal changes. It involves the squaring of pixel intensities or motion values, possibly multiplied by time or frame count, which emphasizes the persistence and intensity of motion-appropriate when understanding dynamics across frames in video sequences.

### 3.4.3 Image Processing

- **Focus on Envelope Area**

  The process computes the areas under the upper and lower envelopes of the modulated signal, emphasizing the oscillatory aspects and the detailed structure of the signal's amplitude variations.

- **Equation Used for Motion Energy Loss**

  The motion energy loss involves the calculation of the squared areas of the upper and lower envelopes and then summing these squared areas to represent the total energy, described by the equation:

$$f(w) = \epsilon_{\text{plus}}^2 + \epsilon_{\text{minus}}^2$$

  where $\epsilon_{\text{plus}}^2$ and $\epsilon_{\text{minus}}^2$ are the squared areas of the upper and lower envelopes, respectively. This approach not only considers the signal's amplitude oscillations but also quantifies separately the contributions of the signal's peaks and troughs to the overall energy.

The key difference is that the video code emphasizes the overall energy of the signal's envelope, while the image code focuses on the detailed structure of the signal's amplitude oscillations through its envelopes. Video code calculates energy loss directly as a difference in energy, whereas the image code uses squared areas of envelopes to represent energy, providing an alternative measure that captures both positive and negative fluctuations in signal amplitude.

### 3.4.4 Video Processing

- **Focus on Envelope Energy**

  Here, the energy is calculated by the analysis, considering the envelope of the whole signal obtained by the Hilbert transform. This will directly enable the comparison between the total energies of the non-modulated and modulated signals.

- **Equation Used for Energy Loss** The energy loss will be quantified by the absolute difference of the integrated squared magnitudes (energies) of the envelopes of the non-modulated versus modulated signals, expressed mathematically as;

$$\text{Energy Loss} = \left| \int \left( |\text{Envelope}_{\text{NonMod}}(t)|^2 \right) dt - \int \left( |\text{Envelope}_{\text{Mod}}(t)|^2 \right) dt \right|$$

This methodology aims to capture changes in the overall energy of the signal due to modulation by utilizing the signal's envelope, providing a holistic view of energy variations.

In the analysis of videos of a walker or a runner, the signal can be any motion data captured frame by frame. It is possible to compute the signal energy from this motion data simply by squaring the velocity or acceleration at each frame and integrating it over the whole time period of the activity. This would serve effectively as a quantitative measure of kinetic energy expended by the subject with respect to time.

For practical implementations, videos are converted to motion signals by using techniques like optical flow or motion tracking to quantify how much each pixel moves between frames. In order to calculate energy, the sum of the squared pixel movements (or any other relevant measure of motion) was calculated throughout the video. This gives a representation of the motion energy.

To quantify energy dissipation, several methods were employed:

1. **Instantaneous Energy**

   Instantaneous energy of both non-modulated and modulated signals using the square of the signal amplitude was calculated:

$$E(t) = |x(t)|^2$$

   In this equation,

   - E(t) = Instantaneous energy as a function of time t
   - $|x(t)|$ = Absolute value (or magnitude) of the signal $x(t)$

   This equation is fundamental in signal processing for calculating the instantaneous energy of a signal at any given time t. It provides a measure of the signal's intensity or strength at each point in time, which is crucial for analyzing energy dissipation in video signals.

2. **Cumulative Energy**

   The cumulative energy is computed by integrating the instantaneous energy over time.

$$E_{cum}(t) = \int_0^t |x(\tau)|^2 d\tau$$

   - $E_{cum}(t)$ = Cumulative energy up to time $t$

- $|x(t)|^2$ = squared magnitude of the signal $x$ at time $t$

This equation calculates the total energy of the signal $x(t)$ from time 0 up to time t by integrating the instantaneous energy $|x(t)|^2$ over that time interval. It's a fundamental equation in signal processing for analyzing energy accumulation in a signal over time.

3. **Energy Dissipation Rate** The rate of energy dissipation is calculated as the derivative of the cumulative energy.

$$\frac{dE}{dt} = \frac{d}{dt}E_{cum}(t)$$

- $\frac{dE}{dt}$ = Rate of change of energy with respect to time
- $\frac{d}{dt}$ = Derivative operator with respect to time
- $E_{cum}(t)$ = Cumulative energy function

This equation expresses the rate of energy dissipation as the time derivative of the cumulative energy function. It shows how quickly the energy is changing at any given point in time, which is crucial for understanding the dynamics of energy dissipation in a system.

### 3.4.5 Verification of Envelope Energy Loss Hypotheses

- $\varepsilon^+(w)^2 \to \varepsilon_0$ which decreases: This hypothesis suggests that as the number of envelope oscillations decreases ($w \to 0$), the squared upper envelope area $\varepsilon^+(w)^2$ tends towards a limit $\varepsilon_0$. The script calculates the upper envelope area for different values of $M$ (representing $w$) and observes the trend of convergence towards $\varepsilon_0$ as $w$ decreases.

- $w \to 0$ leads to a decrease in the total envelope area, indicating a loss of energy: This hypothesis implies that as $w$ approaches $0$, signifying a reduction in the number of envelope oscillations, the total envelope area decreases. This reduction in envelope area suggests a loss of energy, analogous to a runner slowing down. The script computes the squared upper and lower envelope areas for various $M$ values and demonstrates that they tend towards $0$ as $w$ decreases. Furthermore, the script calculates the motion energy loss $f(w)$ as the sum of the squared upper and lower envelope areas and shows that $f(w)$ approaches infinity as the denominator of the sum approaches zero.

The upper Hilbert envelope in this context represents the peak values of the oscillating signal. Such oscillations can be modulated, and the amplitude changes with time owing to the modulation. The top envelope is considered to envelop the peaks of amplitudes of the modulated signal.

If the modulated oscillation has peaks similar to the upper envelope, this would mean that the process of modulation is not radically changing the peak amplitudes of the original. It could be expected when the process of modulation does not change much in the amplitude or frequency components of the original.

The similar number of oscillations (w) observed in the non-modulated envelope across different values of M suggests that the modulation process applied to the signal does not fundamentally alter its frequency characteristics. This may be due to the fact that the modulation operated mainly on either the amplitude or phase of the signal but did not alter the intrinsic frequency content. Second, the shape and parameters of the modulation function chosen may not be such that a considerable amount of oscillations in the original signal will be brought in. It is important to note that the modulation process cannot affect the inherent structure of the original signal, for instance, the frequency spectrum or oscillatory behavior; thus, the non-modulated envelope of the same signal may have a coherent number of oscillations under different modulation conditions.

### 3.4.6 Definition of Motion Energy Analysis in Dynamic Activities

Analyzing and measuring motion energy in dynamic activities refers to the process of quantifying the kinetic energy present in motion activities like walking, running, or rowing through the use of signal analysis techniques. This involves capturing motion data, typically through sensors or video analysis, and applying mathematical models to compute the energy based on the amplitude and frequency of motion signals.

The motion signal energy, denoted as $E$, is calculated using the following integral representation:

$$E = \int |x(t)|^2 \, dt$$

where:

- $E$ represents the motion signal energy.
- $x(t)$ is the motion signal, which could be derived from the displacement, velocity, or acceleration data of the subject's body part being analyzed.
- $|x(t)|^2$ indicates the square of the signal's amplitude, emphasizing energy content over the measurement period.
- The integral is evaluated over the duration $t$ for which the activity is monitored, summing up the squared amplitudes to give a total measure of energy.

### 3.4.7 Theoretical Framework for Energy Dissipation Analysis

#### Definition of Video Up-down (Oscillatory) Area (UDA)

(UDA) is measured in micrometers squared per second $(\mu^2/s)$ , quantifies the vertical motion energy within a video. This metric captures the total vertical displacement squared, integrated over time. It provides a measure of the vertical oscillatory movement's intensity and duration, such as in the activities of walking, running, or rowing

The Video Up-down (oscillatory) area (UDA) can be defined by the equation:

$$UDA = \int_0^T (\Delta h(t))^2 \, dt$$

Where:

- $UDA$ is the Video Up-down (oscillatory) area measured in micrometers squared per second ($\mu^2/s$).
- $\Delta h(t)$ represents the vertical displacement of a point or object in the video frame at time $t$, measured in micrometers.
- $T$ is the total duration of the video or the period of analysis.

**Axiom 01**

1. **Concept of UDA (Up-down Oscillatory Area)**

   UDA quantifies the vertical movements within a video, measured over time. Specifically, it measures how much an object moves up and down on a frame-by-frame basis of a video, calculated in micrometers squared per second ($\mu^2/s$). This measure can help to track the vertical displacement of any point of interest—often a part of the body like the head or feet in human movement analysis.

2. **High UDA and Energy Dissipation**

   - **Increased Movement:** More pronounced vertical oscillations mean that the subject (e.g., a runner or a walker) is moving up and down more significantly with each step.
   - **Energy Efficiency:** These larger movements generally require more muscular and mechanical work, which can lead to increased energy expenditure. If these movements don't contribute effectively towards the progression of the motion

(forward movement in walking or running), the energy spent on these vertical movements can be considered wasted or inefficiently used.

**Axiom 02**

Axiom 2 generalizes the relationship given in Axiom 1 by assuming its converse: when the UDA is low, reflecting little real up and down motion, then energy dissipation is relatively low. This axiom says that if a walker or runner can maintain a smoother, less vertically oscillatory motion, it will use energy more efficiently.

- **Concept of low UDA:** Low UDA means that during the motion, the vertical displacement of any point of interest (e.g., the center of mass, the feet, or the head) is minimal. In practical terms, this equates to a more level and steady progression without excessive upward or downward movement.

- **Efficient Energy Utilization:** With reduced vertical movements, the energy that would have been used to propel the body vertically is conserved or directed toward forward motion. This is particularly helpful where the conservation of energy is an essence of longevity and effectiveness at the place of operation.

**Axiom 03**

Axiom 3 explores the relationship between the mechanical vibrations in a system and the Up-down Oscillatory Area (UDA), suggesting that systems with high vibration tend to exhibit higher UDA values.

- **Mechanical vibrations:** Mechanical vibrations refer to the oscillatory motion of various parts of a mechanical system. With regards to walkers and runners, vibrations might arise from interactions with the ground, mechanical imbalances, or the intrinsic dynamics of the system's movement.

- **High Vibrations Leading to High UDA:** This axiom implies that when a mechanical system like a walker or runner exhibits high levels of vibration, these vibrations translate into more pronounced vertical movements. This is because vibrations, particularly those not perfectly horizontal, have vertical components that add to the overall oscillatory motion captured as UDA.

The following are the ways in which the concept of Video UDA applies to energy efficiency during activities such as walking or running:

UDA computes the vertical oscillations in a sequence of video frames of either a walker or a runner. These oscillations provide an estimate of the amplitude of up-and-down movement experienced by the subjugated body segments, especially the legs or the center of mass, during locomotion like walking and running.

Energy dissipation in this case is the utilization of energy spent without direct benefit to the forward motion and is dissipated through nonproductive movements, such as vertical oscillations. High UDA is thus indicative of increased movement away from an optimum path of motion, which would be horizontal in nature for maximum efficiency in the forward movement.

**Theoretical Foundation**

- Signal Energy Interpretation: In signal processing, particularly within video motion analysis, the energy of a signal often refers to the integral of its squared amplitude. Here, UDA serves as a proxy for vertical movement energy by squaring the amplitude of vertical displacements over time.

- Correlation with Energy Dissipation: High UDA indicates that there is a significant amount of vertical motion, which necessitates higher muscular or mechanical energy to perform these movements. This vertical motion results in energy that could have been utilized for propelling the walker or runner forward instead of being dissipated.

To conclude, the theorem clarifies that high UDA, as a measure of vertical oscillations, is directly associated with increased energy dissipation, quantified in $\mu^2/s$. This relationship underscores the importance of minimizing vertical oscillations to improve energy efficiency in any motion such as walking or running.

### The proof of the theorem

The proof of this theorem directly derives from Axiom 1 and the definition of UDA. Since UDA describes the total cumulative vertical oscillatory energy in the system, defined by the time integral of squared vertical displacements, it is logical to refer to Axiom 1 and deduce from it that with higher UDA, more energy is dissipated. Hence, such a relationship confirms that high UDA means substantial energy waste in the system, and the theorem is proved. These axioms and the theorem thus provide an excellent basis upon which one can examine energy efficiency in dynamic activities. They afford knowledge into how better mechanical systems can be designed and into ways athletic performance may be improved by reducing superfluous energy expenditure.

This chapter explores the theoretical and methodological frameworks for analyzing energy dissipation in video signals, with a focus on the application of the Hilbert transform and Mersenne prime-based modulation. The use of these techniques provides a stable mechanism for detecting and quantifying energy dissipation patterns in dynamic activities, such as walking, running, and cycling.

The analysis demonstrated that the modulation using Mersenne primes significantly impacts the energy distribution within the signals, leading to a more even distribution and potentially reducing energy dissipation. This knowledge is particularly valuable for applications requiring precise control over energy efficiency, such as biomechanics, robotics, and signal processing.

The importance of using non-modulated waveforms as a baseline for comparison was also established, allowing for a clearer understanding of the effects of modulation on energy dynamics. The theoretical framework presented, including the concepts of Up-down Oscillatory Area (UDA) and energy dissipation rates, provides a solid foundation for further research and practical applications.

# Chapter 4

# Comparative Analysis, Experimental Validation, and Practical Applications

## 4.1 Experimental Validation

### 4.1.1 Experimental Setup and Data Collection

#### Equipment and Procedures

The equipment used for capturing and analyzing video data included a high-definition camera capable of recording at high frame rates to accurately capture the motion dynamics of walking, running, and cycling.

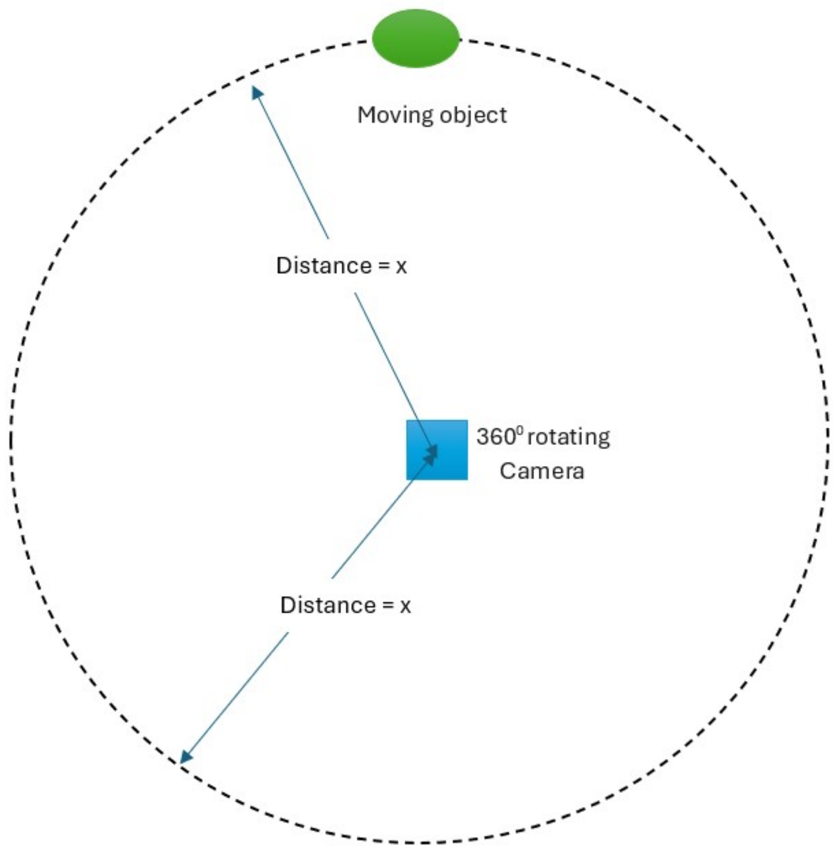


Figure 4.1: 360° Rotating Camera Tracking a Moving Object on a Circular Path

The specific equipment and procedures were as follows:

1. **Camera:** The iPhone 14, equipped with high-definition video recording capabilities, was used to capture footage at 1080p HD and 60 fps. The 60 fps frame rate provides smooth motion capture, reduces motion blur, and makes it easier to analyze fast movements accurately. This setup is ideal for activities such as walking, running, and cycling, where high motion dynamics are involved. The 1080p resolution ensures clear and detailed video quality, allowing for precise analysis of small movements and subtle changes, which is crucial for energy dissipation analysis.

2. **Video Recording Setup:** The iPhone 14 was positioned to capture a clear, unobstructed view of the subject's motion. The camera was holding at the centre on few

meters away from the path where the person would be moving. The height of the camera was almost constant to align with the person's knees or feet to ensure the highlighted row captures the relevant motion.

3. **Lighting:** Proper lighting was ensured to eliminate shadows and provide clear visibility of the subject's movements. The recording was done in a well-lit area, during daylight, to ensure optimal video quality.

4. **Software:** MATLAB R2022b with the Vision Toolbox was used for video reading, processing, and analysis. This software allowed for the extraction of intensity values, application of the Hilbert transform, and subsequent data analysis.

#### Data collection

The data collection process involved recording the person's movements along the marked path under controlled conditions such as:

1. **Sampling rate:** The camera was set to record at 60 fps, which provides high temporal resolution and captures detailed motion data for analysis.

2. **Specific Conditions:**

   - **Weather conditions:** The recording was conducted on a clear day to avoid any adverse weather effects on the visibility and stability of the recording.
   - **Speed of the participant:** The participant moved at a steady pace for every activity to ensure consistent motion data.
   - **Path of participant:** Moved in one direction along the circular path to maintain consistency and in a similar distance to the camera all the time.

### 4.1.2 Data Extraction and Preprocessing

The process of data extraction and preprocessing was a very important phase in this study, ensuring that the video data collected could be effectively analyzed to quantify energy dissipation patterns. This phase involved a series of methodical steps to prepare the data for detailed analysis:

1. **Video Reading:** The first step in data extraction was video reading. These videos were recorded using an iPhone 14 and then imported into MATLAB with the use of the *Vision Video File Reader*. This MATLAB tool made it easy to read video files in different formats for easy import into the MATLAB environment. The most important benefits of using the Vision Video File Reader were its efficiency in handling large video files and support for a wide range of video file formats. This ensured that the high-definition recordings, necessary for capturing detailed motion data, were accurately represented within the software.

2. **Frame Selection:** The next step was frame selection. The selection of specific frames from the videos that underwent detailed analysis occurred within this process. Indeed, this selection of the frames ensures that the moments within which the most informative parts of the motion occur are analyzed; hence, the full range of the motion dynamics is captured.

3. **Row Extraction:**

   Once the keyframes were identified, the next step was row extraction. This process involved selecting specific rows from each frame to extract intensity values. The choice of rows was determined based on their relevance to the study's focus. For example, a row at knee height was selected to capture the movement of the knees during walking or running, while a row corresponding to the pedals was chosen for cycling. Extracting the intensity values from these rows created a time-series signal that represented the motion dynamics of the specific body part or equipment being studied. This time-series data was important for subsequent analysis, as it provided a detailed view of the movement over time.

4. **Normalization and Scaling:** The extracted intensity values from the selected row

are normalized and scaled using the formula:

$$\text{selected\_row} = \frac{\text{selected\_row} - \min(\text{selected\_row})}{\max(\text{selected\_row}) - \min(\text{selected\_row})}$$

This equation normalizes the intensity values of the selected row to a range between 0 and 1, which is a common preprocessing step in data analysis to ensure consistency across different datasets.

5. **Hilbert Transform Application:**

   The final step in the preprocessing pipeline was the application of the Hilbert transform to the time-series signal. Hilbert transform is one of the principal mathematical techniques in the analysis of signals, and it allows the derivation of an analytic signal from a time series of real values. In this way, the present investigation had no problem in applying the Hilbert transform to obtain the instantaneous amplitude and phase of the signal. Instantaneous amplitude can provide information about the energy content of the signal, while instantaneous phase gives information on timing and synchronization. Together, these parameters enabled a comprehensive analysis of the energy variations within the selected row, facilitating a more accurate quantification of energy dissipation. This step was crucial for understanding the subtle energy dynamics involved in different human activities and for comparing the effects of various modulation techniques.

By following these detailed procedures and ensuring a controlled and consistent data collection environment, the accuracy and reliability of the experimental results were maintained. The meticulous approach to data extraction and preprocessing not only ensured high-quality data but also provided a solid framework for analyzing energy dissipation patterns in various human activities using the proposed method. This structured approach was integral to the success of the study.

### 4.1.3 Metric Calculation

In the analysis of energy dissipation in video signals, precise metrics are crucial for quantifying the observed phenomena. The metrics calculation process involves several key steps, each contributing to a comprehensive understanding of the signal's behavior. The following steps are implemented in the *'calculateMetrics'* function, a core component of the analysis script.

1. **Computation of the Hilbert Envelope:**

   The first step involves computing the Hilbert envelope of the signal. The Hilbert transform is applied to the time-series data extracted from the video frames, producing an analytic signal. The magnitude of this analytic signal represents the envelope, which effectively captures the instantaneous amplitude of the signal over time. This envelope is crucial for identifying the peaks and troughs within the signal, which correspond to oscillations in the motion.

2. **Energy Calculation:**

   To quantify the total energy in the signal, the script integrates the squared amplitude of the Hilbert envelope over the entire duration of the signal. This process involves calculating the area under the squared amplitude curve, which provides a measure of the energy content in the signal. The resulting energy value reflects the magnitude and duration of the signal's oscillations, offering knowledge of the overall energy dissipation during the motion.

3. **Determination of the Number of Oscillations:**

   The script also estimates the number of oscillations present within the signal. This is obtained by finding the peaks of the Hilbert envelope: these correspond to the local maximums of the amplitude of the signal. These peaks will be then counted to compute the total number of oscillations. This metric is necessary to understand how often, in terms of frequency and periodicity, the motion in the video takes place.

4. **Motion Energy Loss:**

   Motion energy loss is another important metric computed by the script. This is calculated by comparing the energy of the non-modulated signal with the energy of the modulated signal. The difference between these energies represents the energy loss due to modulation, which can provide knowledge into the efficiency of the motion and the energy dissipated through various mechanisms, such as friction or air resistance.

5. **Comparison Across Different Conditions:**

   For each condition, metrics have been computed with different M values Mersenne primes, and different kinds of motions-walking, running, and cycling. By this comparison, how modulation would affect energy dissipation and oscillatory behavior can be studied. The results are patterned out to derive correlations to make conclusions on the efficiency and characteristics of the different motions and modulation techniques.

By meticulously calculating these metrics, the *'calculateMetrics'* function provides a detailed and quantitative analysis of the energy dynamics in the video signals. This analysis is important for understanding the underlying physical processes and for evaluating the performance of different modulation techniques in terms of energy efficiency and dissipation.

## 4.2 Analysis and Interpretation of Results

### 4.2.1 Energy Dissipation in Non-Modulated Signals

#### Analysis of Energy Dissipation

The study of non-modulated signals involves examining the original, unaltered data from the recorded activities (walking, running, and cycling). This analysis provides a baseline understanding of energy dissipation without any external modulation influences.

In the analysis of energy dissipation in non-modulated signals, the behavior of the original motion signals $x(t)$ and the Hilbert Enveloped Cumulative Energy $E_x$ can be observed. The non-modulated signals serve as a baseline for understanding the inherent energy present in the motion of subjects in the videos of walking, running, and cycling.

### 4.2.2 Energy Dissipation in Modulated Signals

#### Analysis of Energy Dissipation

The modulated signals, in contrast to the non-modulated ones, incorporate a modulation Mersenne prime factor $M$ that averages out the signal's properties. This modulation, typically a form of frequency shift or amplitude modulation, changes the energy distribution within the signal.

As shown in Figure 4.2, the modulated signals $E_m$ exhibit a distinct pattern compared to the non-modulated signals. The energy dissipation in the modulated signals is typically lower, as evidenced by the maximum cumulative energy values, which decrease with increasing M values.

#### Examples of Dissipated Energy

The dissipation of energy in these signals is evident from the plots in Figure 4.2 through 4.13 showing the fluctuations and gradual decay of the signal's amplitude over time. In particular, the plots for $E_x$ illustrate how the energy of the signal accumulates and eventually stabilizes, revealing energy loss as the system reaches a state of equilibrium.

The following plots for all the activities represent as;

- **Non-modulated periodic motion signal $x(t)$:** The top plot represents the original, non-modulated motion signal extracted from a row of reflected light amplitudes in the video frame, illustrating the natural oscillations and variations in the signal over time.

- **Image Row Based Motion Signal** $x(t)$**:** The second plot shows the motion signal extracted from reflected light values in a row of the video, indicating reflected light changes over time.

- **Hilbert Enveloped Cumulative Energy** $E_x$**- Non-Modulated:** The third plot represents the cumulative energy of the non-modulated signal, obtained using the Hilbert transform to compute the envelope. This represents the total energy dissipated.

- **Hilbert Enveloped Cumulative Energy** $E_m$**- Modulated:** The fourth plot shows the cumulative energy for the modulated signal with various Mersenne prime values. The modulation tends to spread out (smooth) the energy dissipation.

**Results for Walker:**

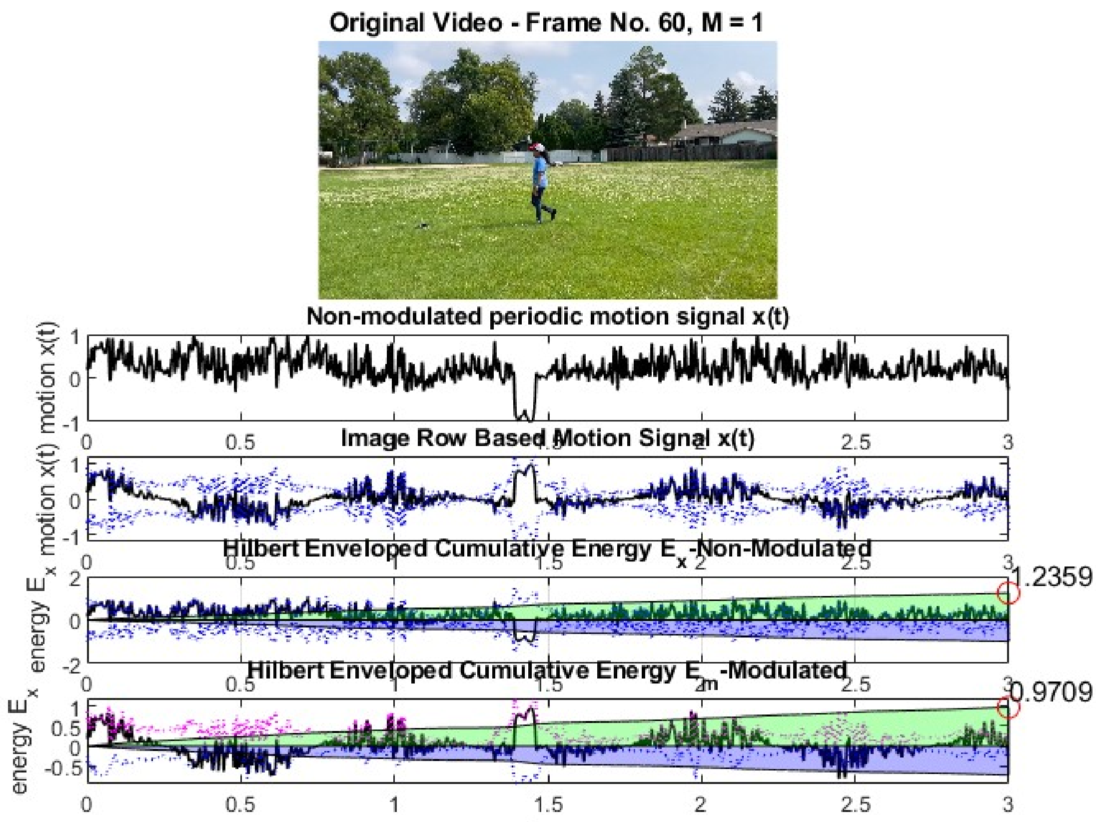


Figure 4.2: Energy Analysis for M = 1 of walker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

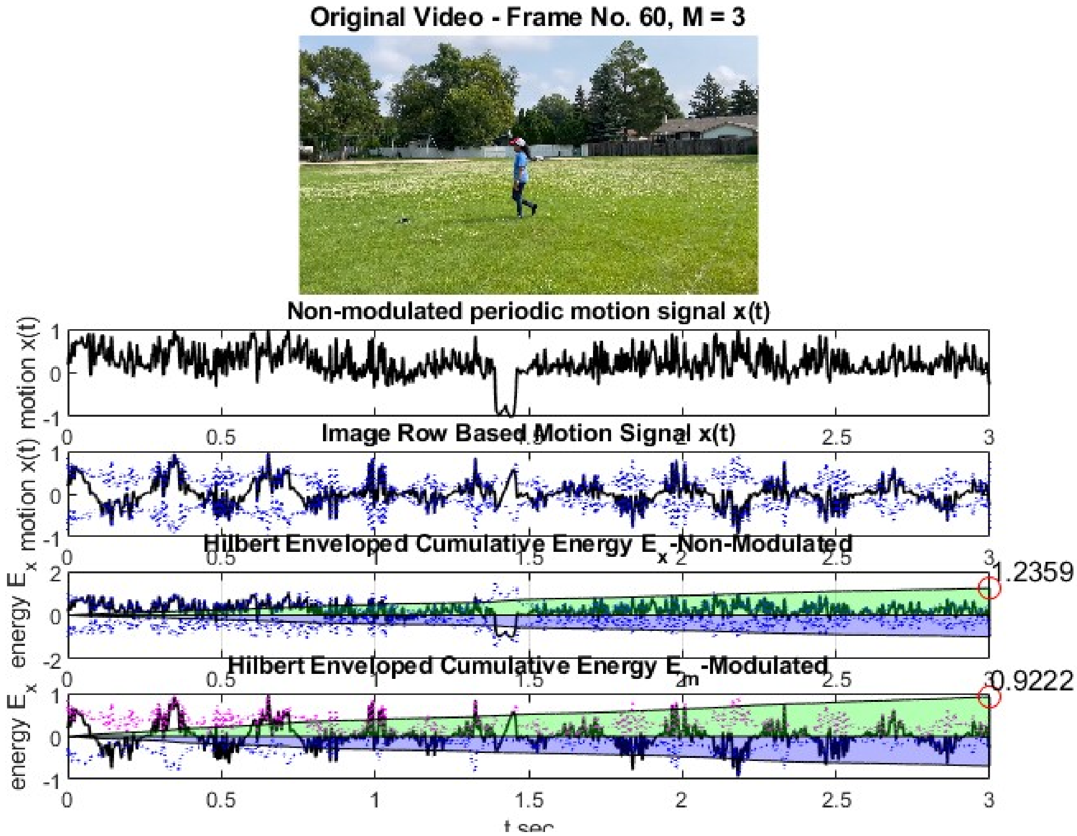


Figure 4.3: Energy Analysis for M = 3 of walker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

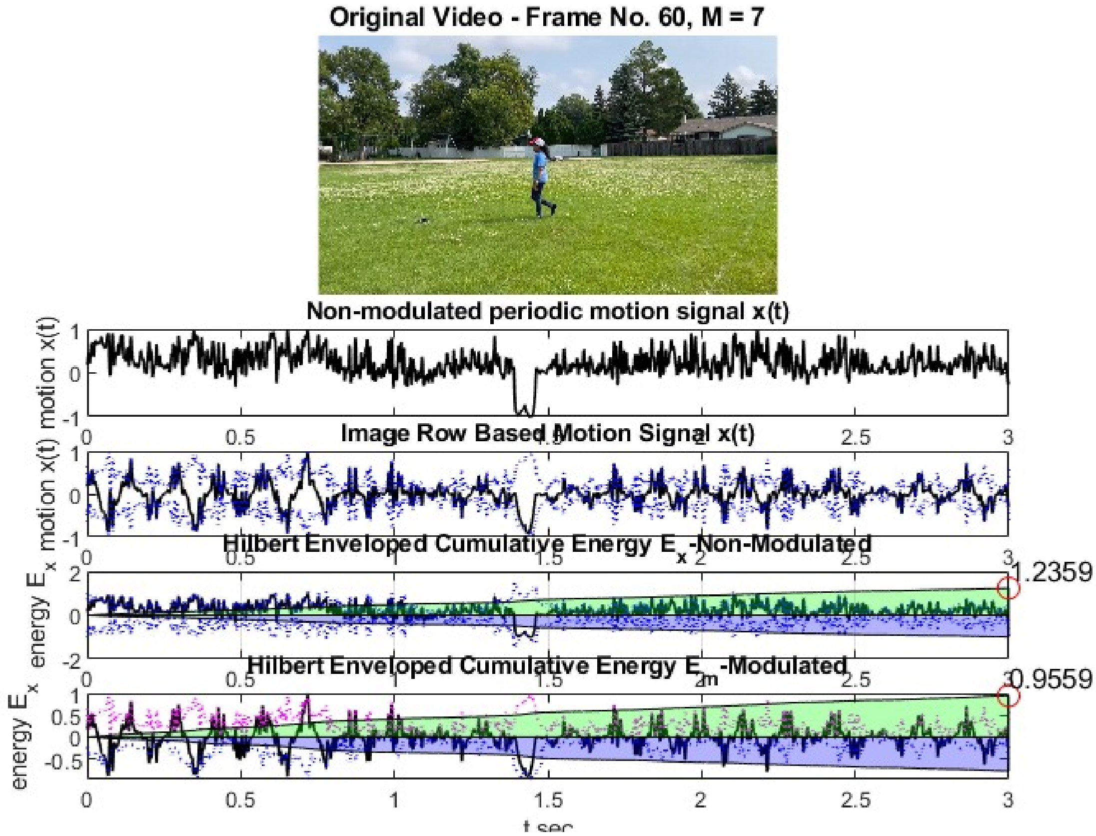


Figure 4.4: Energy Analysis for M = 7 of walker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

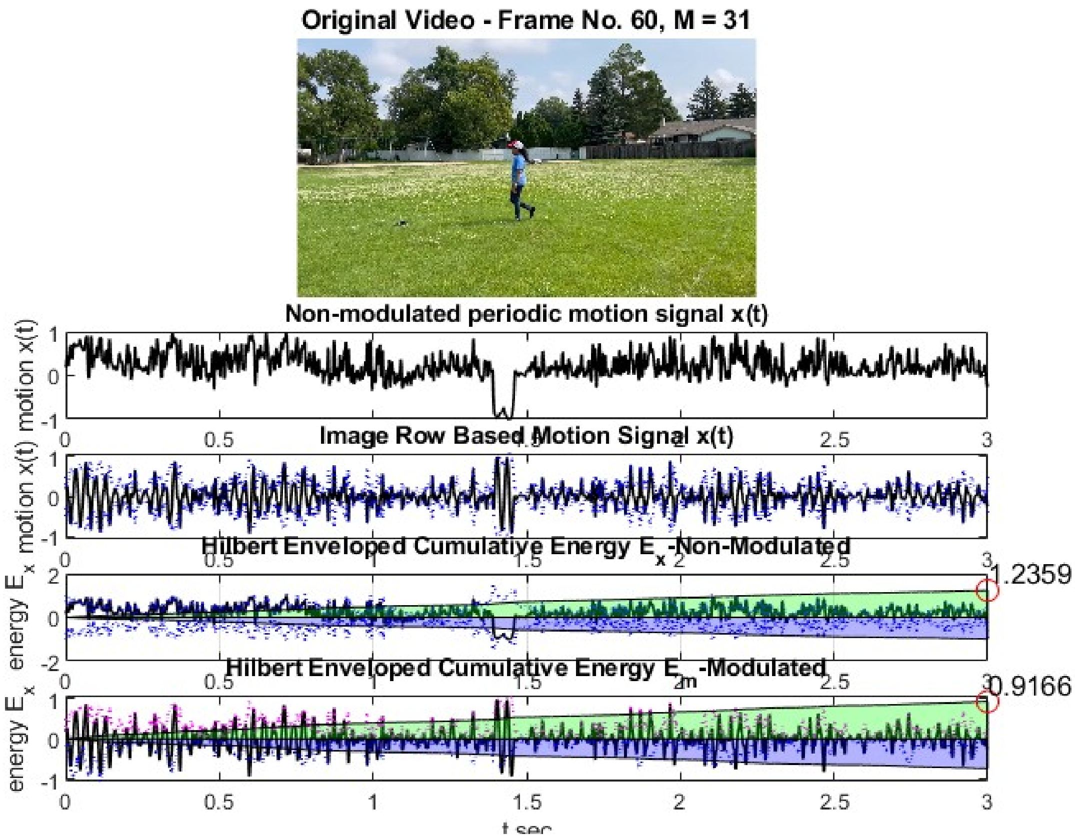


Figure 4.5: Energy Analysis for M = 31 of walker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

**Results for Runner:**

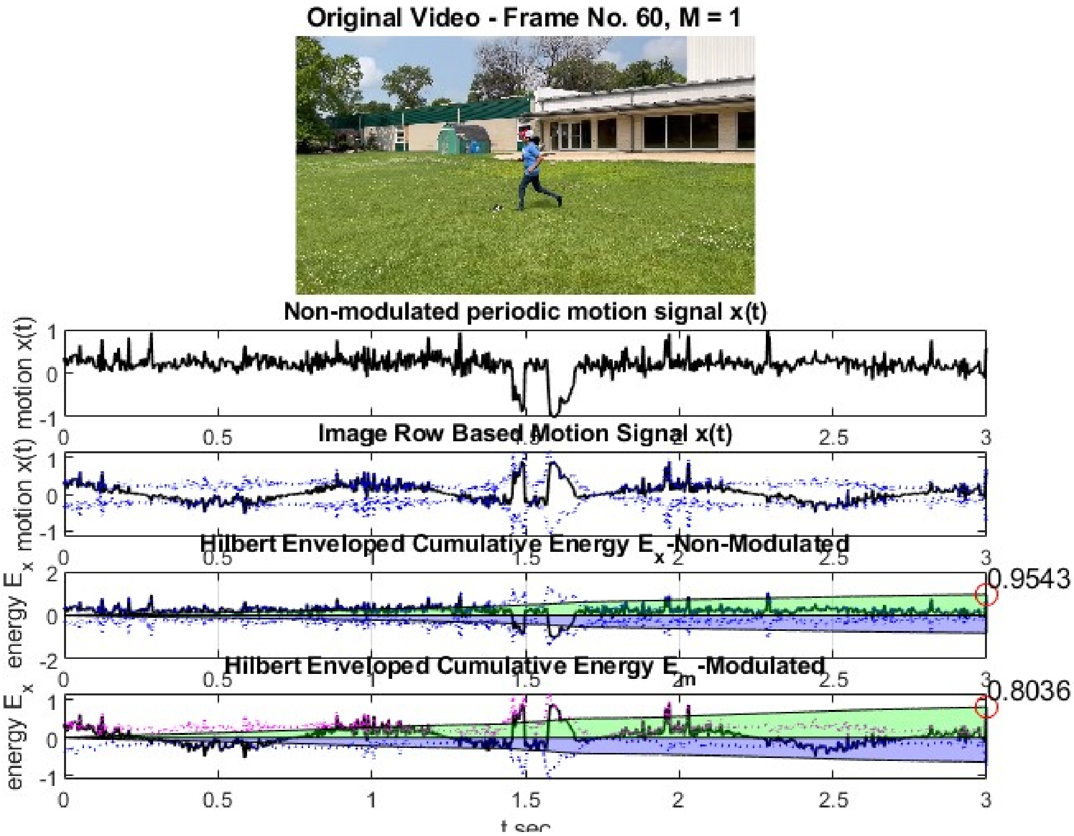


Figure 4.6: Energy Analysis for M = 1 of runner: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

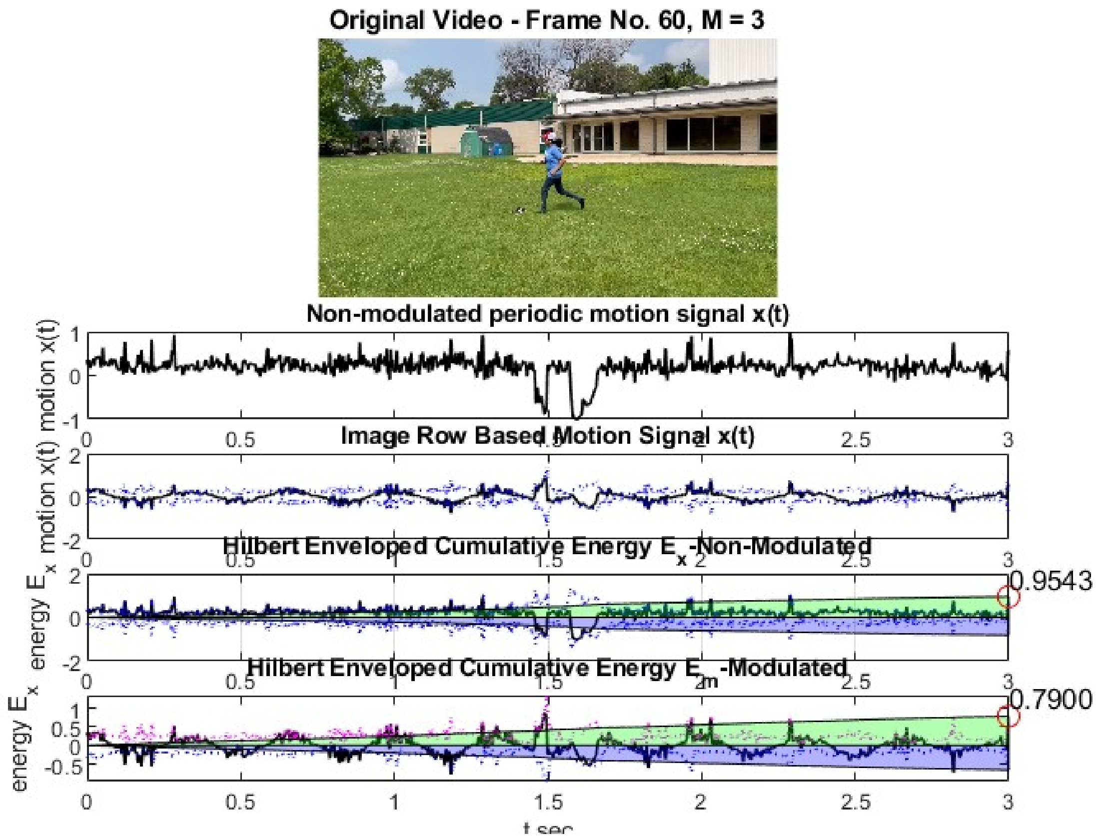


Figure 4.7: Energy Analysis for M = 3 of Runner: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

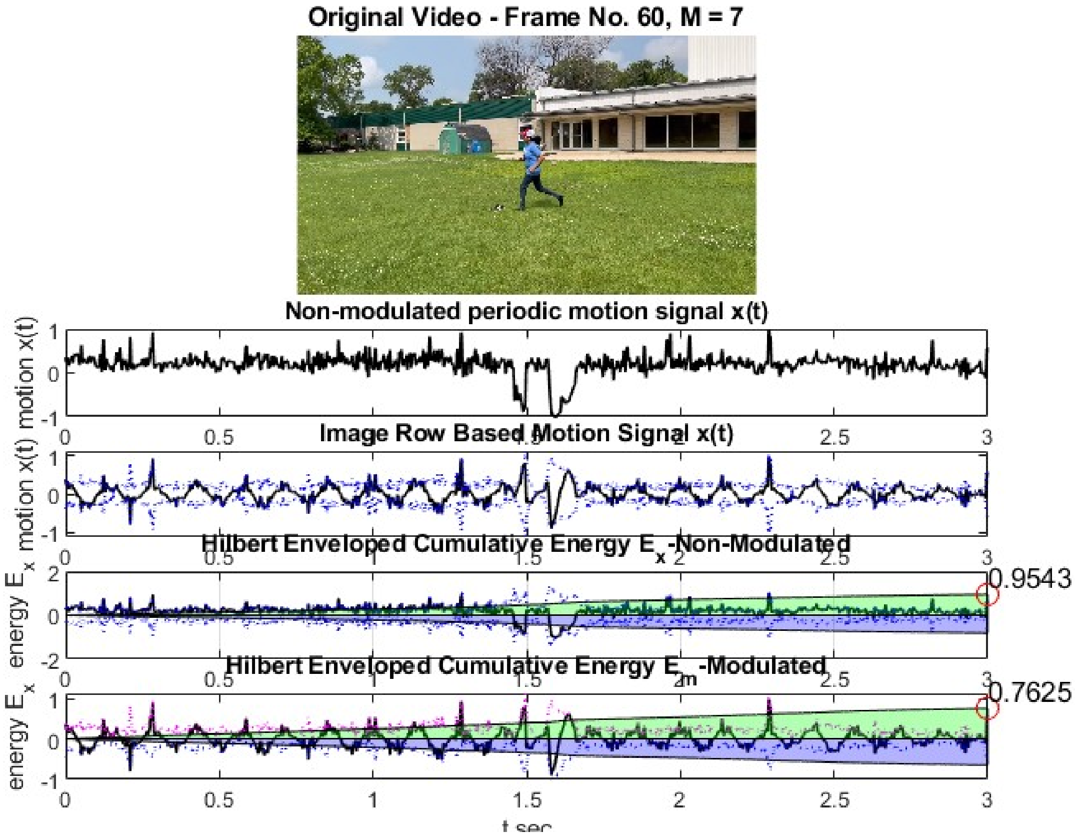


Figure 4.8: Energy Analysis for M = 7 of Runner: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

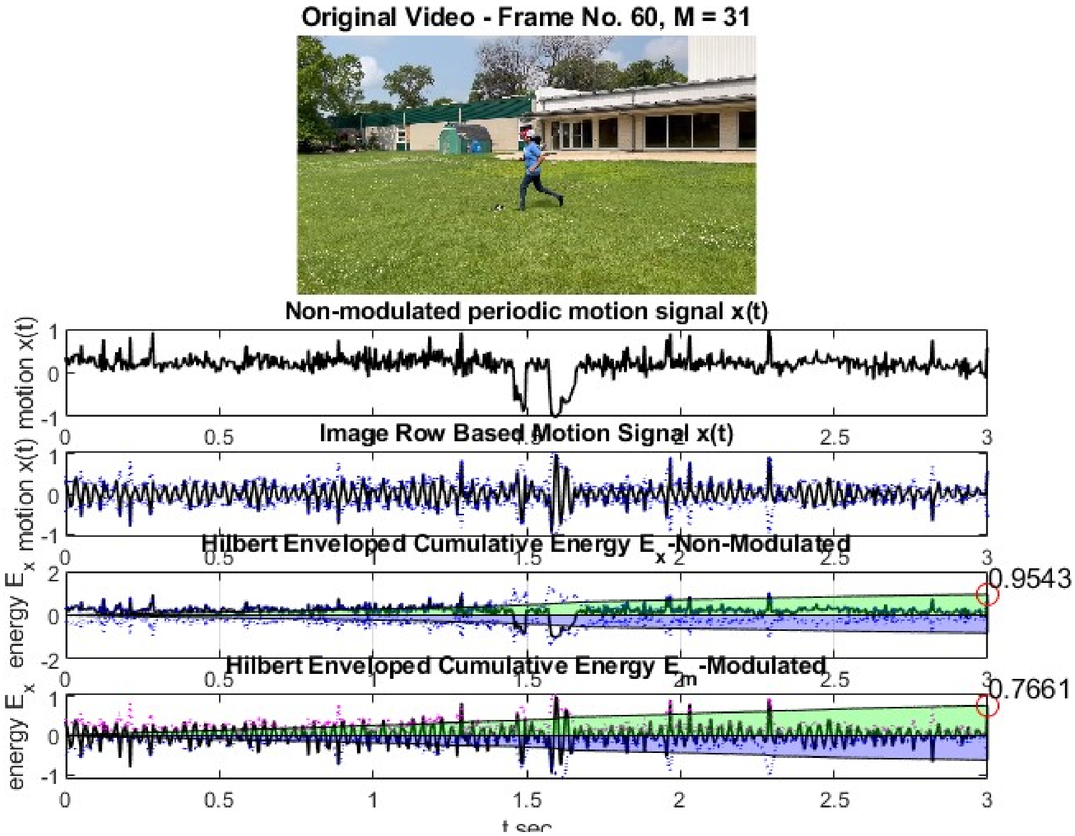


Figure 4.9: Energy Analysis for M = 31 of runner: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

**Results for Biker:**

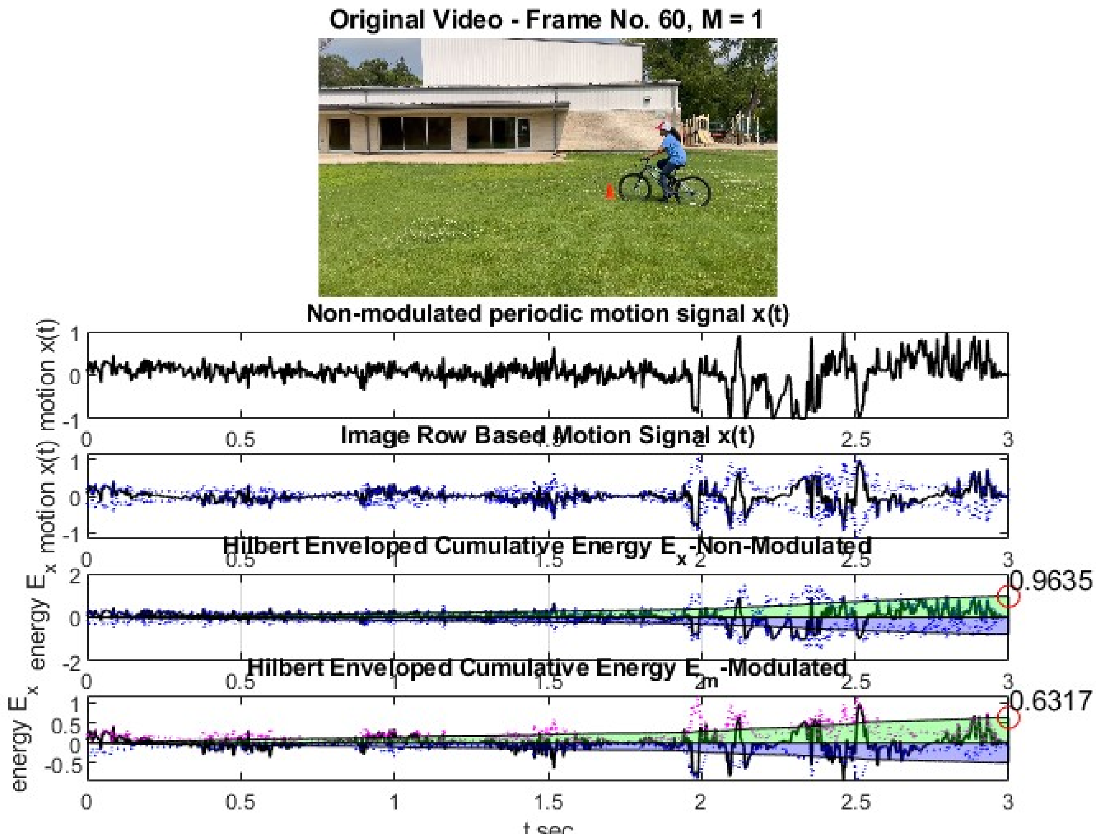


Figure 4.10: Energy Analysis for M = 1 of Biker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

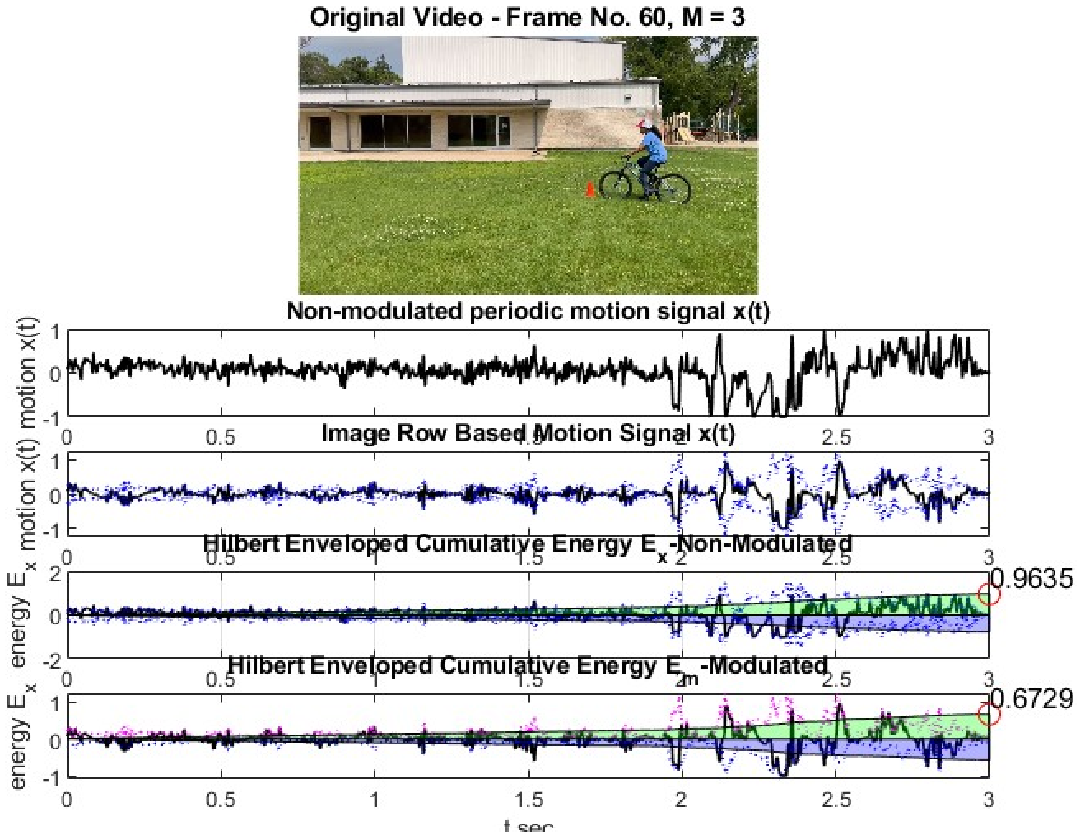


Figure 4.11: Energy Analysis for M = 3 of Biker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

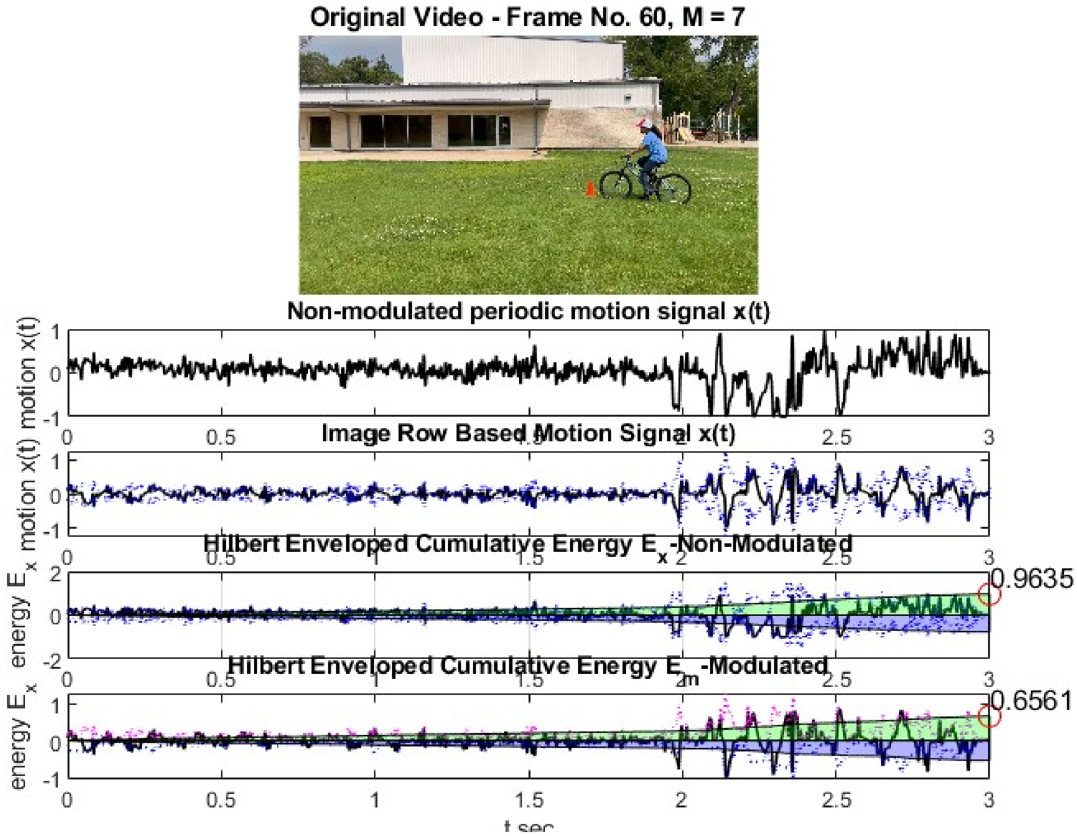


Figure 4.12: Energy Analysis for M = 7 of Biker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

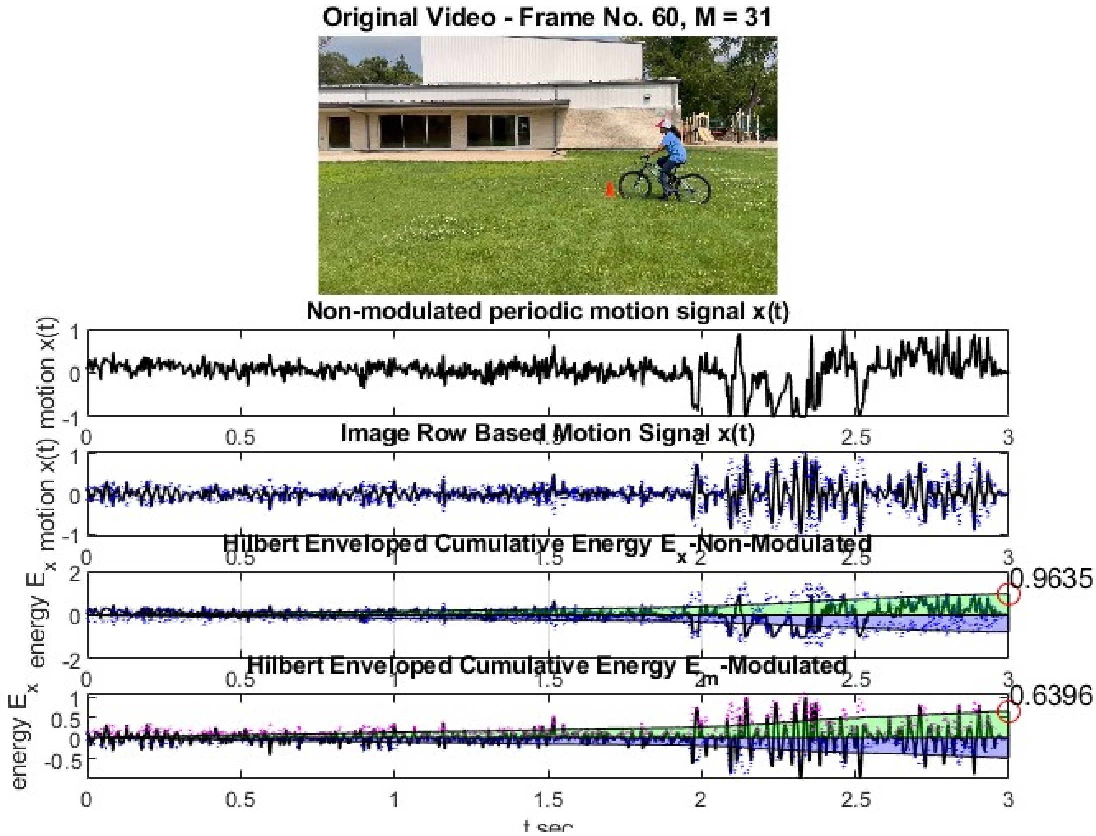


Figure 4.13: Energy Analysis for M = 31 of Biker: Comparison of Non-Modulated and Modulated Motion Signals of Frame = 60

The Matlab Script is in Appendix G

### 4.2.3 Energy Dissipation Analysis in Non-Modulated and Modulated Signals

The Table 4.1 presents the energy dissipation values for walking activities, comparing non-modulated and modulated signals at different M values. This analysis highlights the percentage of energy dissipation in modulated signals relative to the non-modulated ones.

Table 4.1: Energy Dissipation for Walking

| M | Non-Modulated Energy (Ex) | Modulated Energy (Em) | Energy Dissipation Percentage |
|---|---|---|---|
| 1 | 1.2359 | 0.9709 | 21.44% |
| 3 | 1.2359 | 0.9222 | 25.38% |
| 7 | 1.2359 | 0.9559 | 22.65% |
| 31 | 1.2359 | 0.9166 | 25.83% |

Modulation led to a reduction in energy dissipation, with the modulated energy decreasing across all M values. The percentage of energy dissipation increased slightly with higher modulation values, indicating more energy loss from the original signal.

Table 4.2: Energy Dissipation for Running

| M | Non-Modulated Energy (Ex) | Modulated Energy (Em) | Energy Dissipation Percentage |
|---|---|---|---|
| 1 | 0.9543 | 0.8036 | 15.79% |
| 3 | 0.9543 | 0.79 | 17.21% |
| 7 | 0.9543 | 0.7625 | 20.09% |
| 31 | 0.9543 | 0.7661 | 19.72% |

Running showed a less pronounced reduction in energy dissipation compared to walking and biking. The modulation still resulted in decreased modulated energy across the M values, with the percentage of energy dissipation increasing with intermediate modulation levels.

Table 4.3: Energy Dissipation for Cycling

| M | Non-Modulated Energy (Ex) | Modulated Energy (Em) | Energy Dissipation Percentage |
|---|---|---|---|
| 1 | 0.9635 | 0.6317 | 34.43% |
| 3 | 0.9635 | 0.6729 | 30.16% |
| 7 | 0.9635 | 0.6561 | 31.90% |
| 31 | 0.9635 | 0.6396 | 33.61% |

Cycling showed the most significant drop in modulated energy across the varying M values. The percentage of energy dissipation was varied, with an apparently larger initial contribution at M = 1 due to the nature of biking, which is a fast change of motion.

The following tables (4.4, 4.5, 4.6 and 4.7) present the percentage of dissipated energy for walking, running, and biking at different values of M. From this, it will be possible to understand how modulation affects the energies in both the non-modulated and modulated signals, and also how these energies change for different activities.

- **Walking:** Ranges from 21.44% to 25.83%, it gives the percentage of dissipation in energy. It reflects that the modulation exerts an obvious but not extreme effect on energy loss in walking, probably because walking, compared to running and cycling, is a rather stable and less dynamic human gait. The less intensive and less rhythmic nature of the walk may absorb part of the modulation effect so that energy dissipation increases gradually as modulation values increase.

- **Running:** Displays the lowest energy dissipation percentages, in the range of 15.79% to 20.09%. This can be interpreted to mean that there is usually more energy left over after modulation in running than in cycling or walking. Such a result would imply that the modulation has a reasonable and not extreme effect on energy loss during walking, probably because of the nature of walking, which is more or less stable and less dynamic as compared with running or cycling. This might be because the less intensive and more rhythmic walking gait may absorb some of the modulating effects, which increases the energy dissipation gradually with the rising values of modulation.

- **Cycling:** Provides the highest general energy dissipation compared to walking and running, in the range of 30.16% to 34.43%. High dissipation suggests that because cycling is a rather dynamic activity, there is a large impact caused by modulation. This may be because of the nature of the cycling motion at which modulation disrupts the energy efficiency more than in other activities. Cycling represents continuous, repetitive, cyclic motion; such activity is often even more sensitive to disturbances or inefficiencies introduced by modulation and is thus associated with greater energy dissipation.

In general, cycling shows the highest percentage of dissipated energy for all modulation values, proof that modulation has a profound effect on it, probably due to the less regular motion and more important mechanical energy involved. On the other hand, walking shows, in essence, a moderate energy dissipation that increases with higher modulation values. Modulation thus has a more steady influence on the energy dynamics as the modulation itself increases. Running consistently shows the lowest percentages of energy dissipation, reflecting its efficient and rhythmic nature, which leads to better energy retention even under different modulation conditions. This analysis shows how different types of motion, with their peculiar characteristics, may respond differently to modulation as far as retention or loss of energy is concerned.

Table 4.4: Comparison of Non-Modulated and Modulated Energy with Energy Dissipation Percentage (M=1)

| | **Non-Modulated Energy $E_x$** | **Modulated Energy $E_m$** | **Energy Dissipation Percentage** |
|---|---|---|---|
| **Walk** | 1.2359 | 0.9709 | 21.44% |
| **Run** | 0.9543 | 0.8036 | 15.79% |
| **Cycle** | 0.9635 | 0.6317 | 34.43% |

Table 4.5: Comparison of Non-Modulated and Modulated Energy with Energy Dissipation Percentage (M=3)

| | **Non-Modulated Energy $E_x$** | **Modulated Energy $E_m$** | **Energy Dissipation Percentage** |
|---|---|---|---|
| **Walk** | 1.2359 | 0.9222 | 25.38% |
| **Run** | 0.9543 | 0.79 | 17.21% |
| **Cycle** | 0.9635 | 0.6729 | 30.16% |

Table 4.6: Comparison of Non-Modulated and Modulated Energy with Energy Dissipation Percentage (M=7)

| | **Non-Modulated Energy $E_x$** | **Modulated Energy $E_m$** | **Energy Dissipation Percentage** |
|---|---|---|---|
| **Walk** | 1.2359 | 0.9559 | 22.65% |
| **Run** | 0.9543 | 0.7625 | 20.09% |
| **Cycle** | 0.9635 | 0.6561 | 31.90% |

Table 4.7: Comparison of Non-Modulated and Modulated Energy with Energy Dissipation Percentage (M=31)

| | **Non-Modulated Energy $E_x$** | **Modulated Energy $E_m$** | **Energy Dissipation Percentage** |
|---|---|---|---|
| **Walk** | 1.2359 | 0.9166 | 25.83% |
| **Run** | 0.9543 | 0.7661 | 19.72% |
| **Cycle** | 0.9635 | 0.6396 | 33.61% |

The energy dissipation for walking varies between 21.44% and 25.83%. That is to say, this is a moderate increase in the energy loss factor for higher modulation values, as M = 3 and M = 31 show. This fluctuation does strongly indicate that modulation bears an influence on walking; however, the changes are minimal, hence bringing forth the steady and predictable nature of walking motion. Thus, walking is influenced by modulation to a moderate extent, and with the level of modulation, its energy dissipation increases but not drastically.

The energy dissipation for running falls in the range of 15.79% - 20.09%. Running demonstrates the least energy dissipation compared with walking and cycling. That may indicate that running-either because of the uniformity or the rhythmical character of the process-is by far less dissipative even for modulation. The increased dissipation at M = 3 and M = 7 in running might suggest that intermediate amplitudes of modulation are going to have the greatest impact on energy dissipation in running. Such behavior might show that running was insensitive to low and high modulation, yet sensitive to intermediate levels.

The energy loss is higher while cycling, lying between 30.16% and 34.43%. High dissipation during cycling would therefore indicate that there is more energy lost due to modulation associated with the dynamic and variable cycling motion. Since at least dissipation is great at M = 1, it would appear to indicate that even the slightest amount of modulation greatly affects cycling. This is probably because of the more unpredictable and rapid changes in motion inherent to cycling, making it more prone to modulation-related energy loss.

It identifies that cycling shows the most energy dissipation, while running retains the most energy post-modulation. Walking falls in between, showing moderate energy loss.

### 4.2.4 Comparative Analysis of Oscillatory Behavior and Energy Dissipation Across Activities

The number of oscillations, or peaks, in a signal, can indicate its frequency characteristics and energy distribution. In this context, comparing the oscillation counts between non-modulated and modulated signals provides an understanding of how modulation affects the dynamic properties of the signals.

To compare the oscillation counts, the number of peaks in both the non-modulated and modulated signals for each frame of the video were identified first. The findpeaks function estimates these peaks, finding local maxima in the signal. So as not to take any incorrect peaks and at the same time keep the peaks relevant, a minimum of 0.1 peak height has been set so that these small fluctuations may not contribute much toward the oscillatory behavior being studied. The oscillation counts are then aggregated over all frames to provide a comprehensive view of the modulation's impact.

In this analysis, the impact of modifications on the oscillatory behavior of a system across different M values was explored. These data represent the number of oscillations under both modified and non-modified conditions. To provide a clear view of the results, the following figures present the trends and differences for M = 1, 3, 7, and 31, showing the Mod and Non-Mod conditions. These images highlight the variations in oscillatory behavior, providing a clear visual representation of how modifications influence the system's dynamics.

Some snapshots of videos that were created for each activity for their Original signal, Modulated, and non-modulated signals are as follows. Here, Frame number 60 was considered as a sample.

**General Trends and Implications:**

**Oscillation Count:**

The number of oscillations generally increases with higher M values, indicating a more pronounced effect of modifications. This trend is consistent across all datasets, showing that as the system undergoes more substantial modifications (higher M values), the oscillatory behavior becomes more frequent and intense. The Mod condition consistently exhibits higher oscillation counts compared to the NonMod condition, reflecting the improved sensitivity and response of the system when subjected to modifications.

**Difference and Ratio Analysis:**

$$\text{Oscillation Difference} = \text{NumOscMod - NumOscNonMod}$$

The difference between the number of oscillations under Mod and NonMod conditions provides a quantitative measure of the modification's impact.

$$\text{Oscillation Ratio (Osc Ratio):} \; = \frac{\text{NumOscMod}}{\text{NumOscNonMod}}$$

The ratio of oscillations (Mod/NonMod) offers another perspective on the relative impact of modifications. A ratio close to 1 suggests minimal impact, while a higher ratio indicates a substantial increase in oscillatory activity due to modifications.

The following graphs have two distinct visual representations. The blue line represents the number of oscillations in the non-modulated envelope (NumOscNonMod), while the red line shows the number of oscillations in the modulated envelope (NumOscMod).

### Analysis of Oscillatory Behavior and Energy Dissipation

This section presents a detailed analysis of the oscillatory behavior and energy dissipation in non-modulated and modulated signals for different activities (Runner, Walker, Biker) and modulation values (M=1, 3, 7, 31). The data was processed and visualized using MATLAB to generate the following figures.

For each activity (Runner, Walker, Biker), four figures were generated corresponding to different modulation values (M=1, 3, 7, 31). Each figure contains the following subplots:

- **Number of Oscillations:** Non-modulated signals generally exhibit a higher number of oscillations compared to modulated signals. The difference becomes more pronounced with increasing modulation values.
- **Motion Energy Loss:** Energy dissipation increases with higher modulation values, indicating that higher modulation leads to greater energy loss.
- **Oscillation Difference:** The difference in oscillation counts between non-modulated and modulated signals increases with higher modulation values, highlighting the impact of modulation on signal dynamics.
- **Oscillation Ratio:** The ratio decreases as modulation values increase, showing that the relative number of oscillations in modulated signals diminishes compared to non-modulated signals.

These visualizations provide a comprehensive view of how modulation affects the dynamic properties of the signals in different activities.

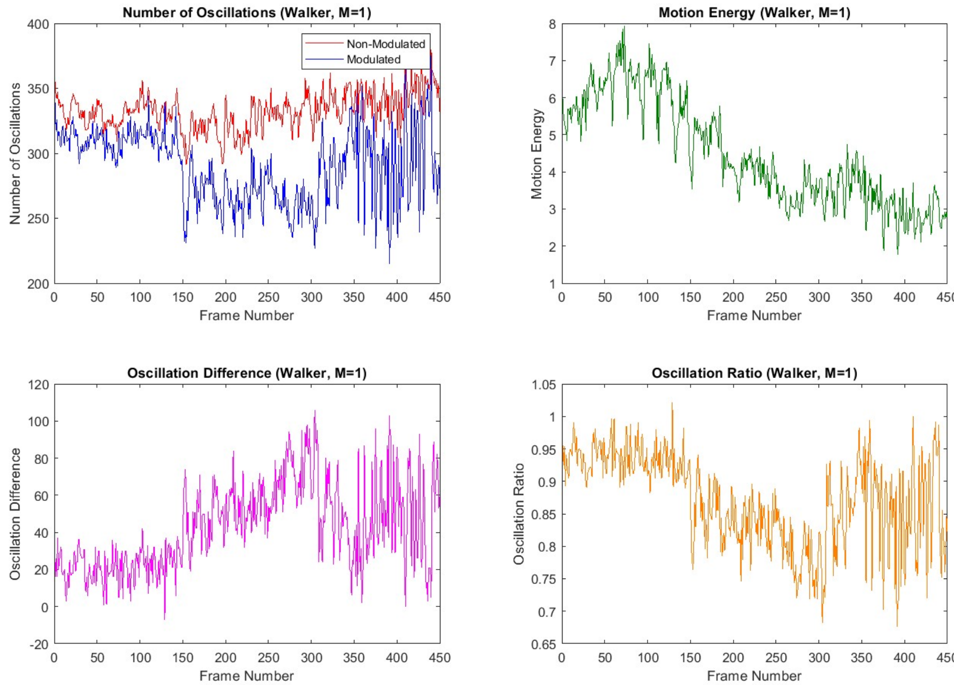


Figure 4.14: Oscillatory Behavior and Energy Level for Walker (M=1)

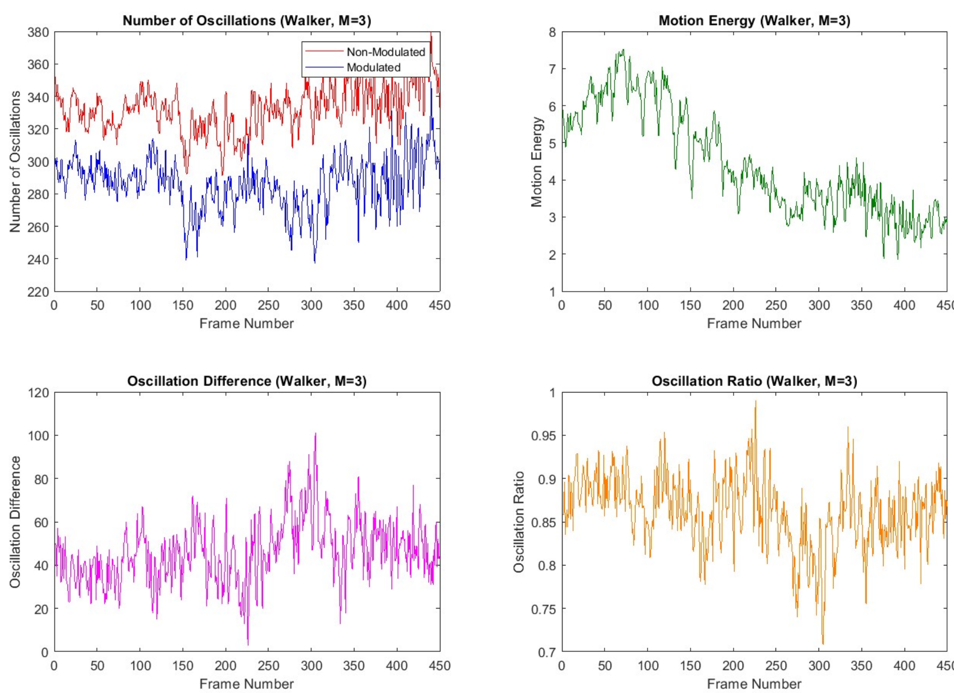


Figure 4.15: Oscillatory Behavior and Energy Level for Walker (M=3)

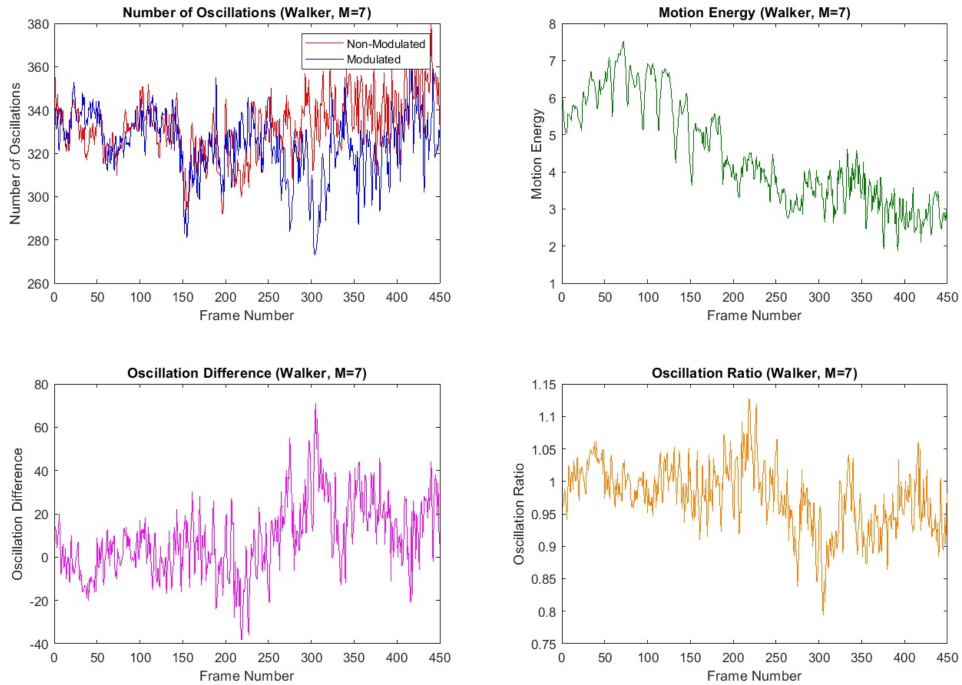


Figure 4.16: Oscillatory Behavior and Energy Level for Walker (M=7)

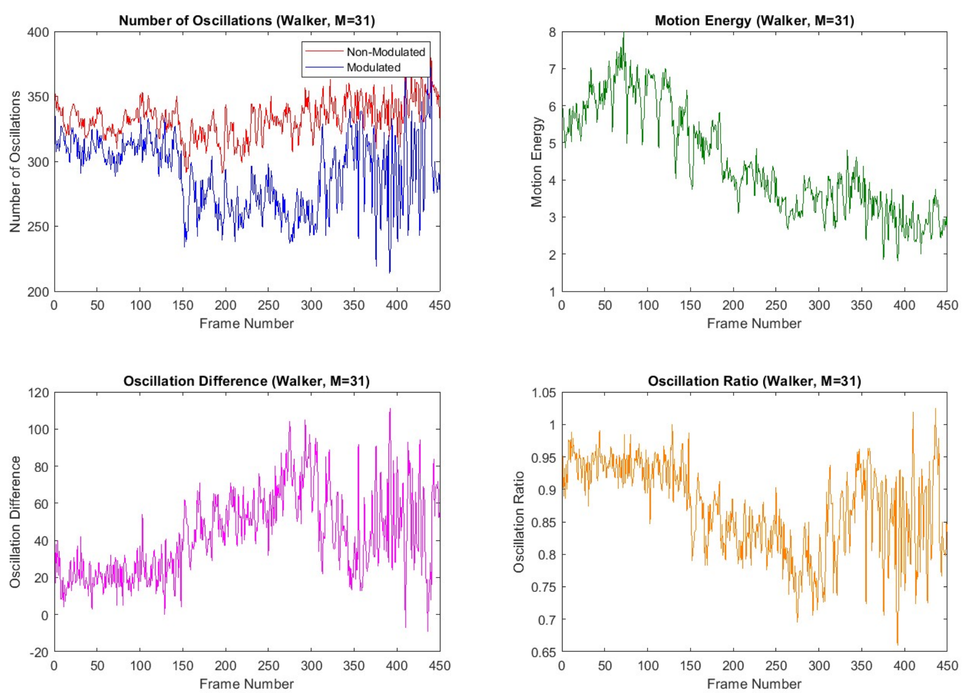


Figure 4.17: Oscillatory Behavior and Energy Level for Walker (M=31)

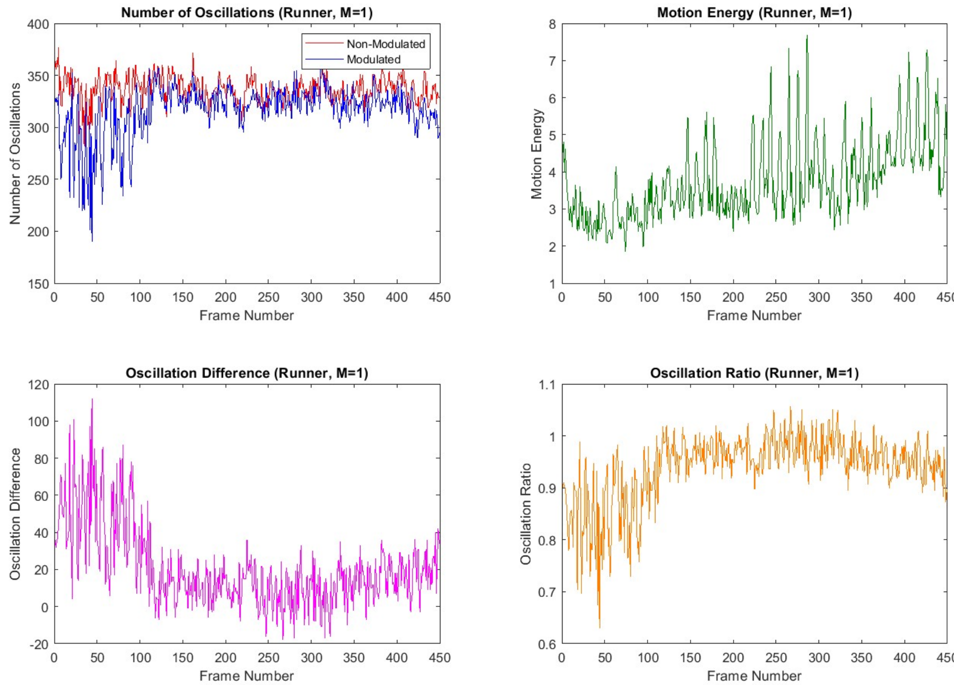


Figure 4.18: Oscillatory Behavior and Energy Dissipation for Runner (M=1)

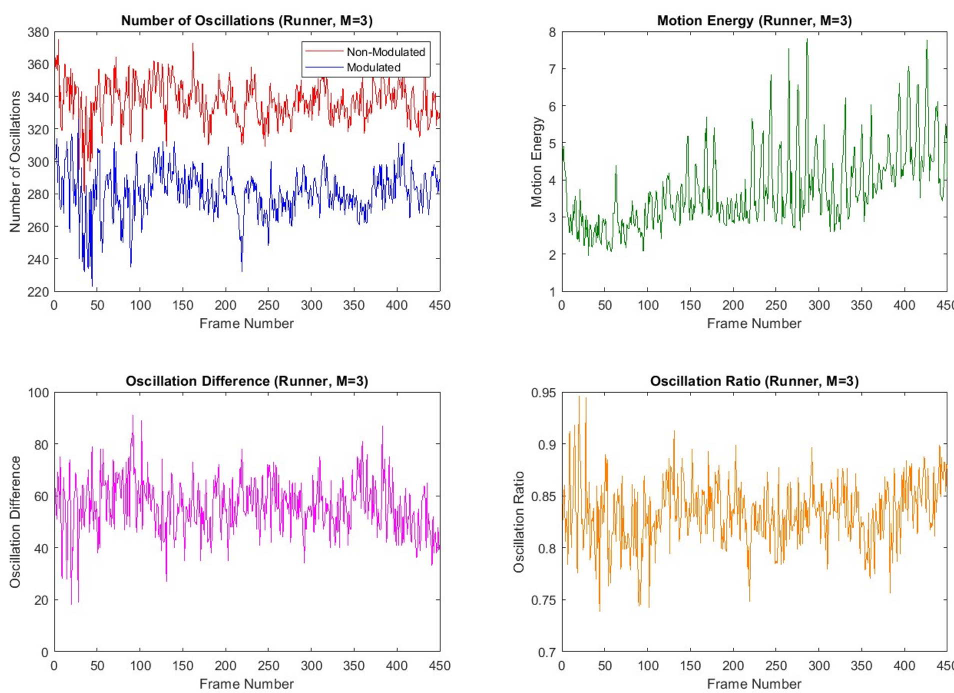


Figure 4.19: Oscillatory Behavior and Energy Dissipation for Runner (M=3)

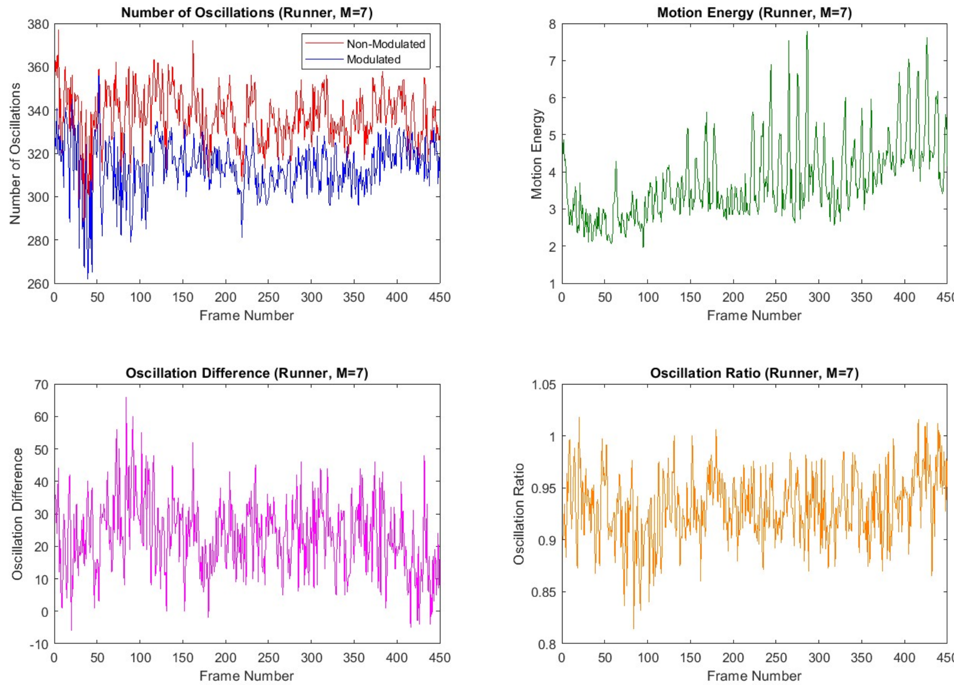


Figure 4.20: Oscillatory Behavior and Energy Dissipation for Runner (M=7)

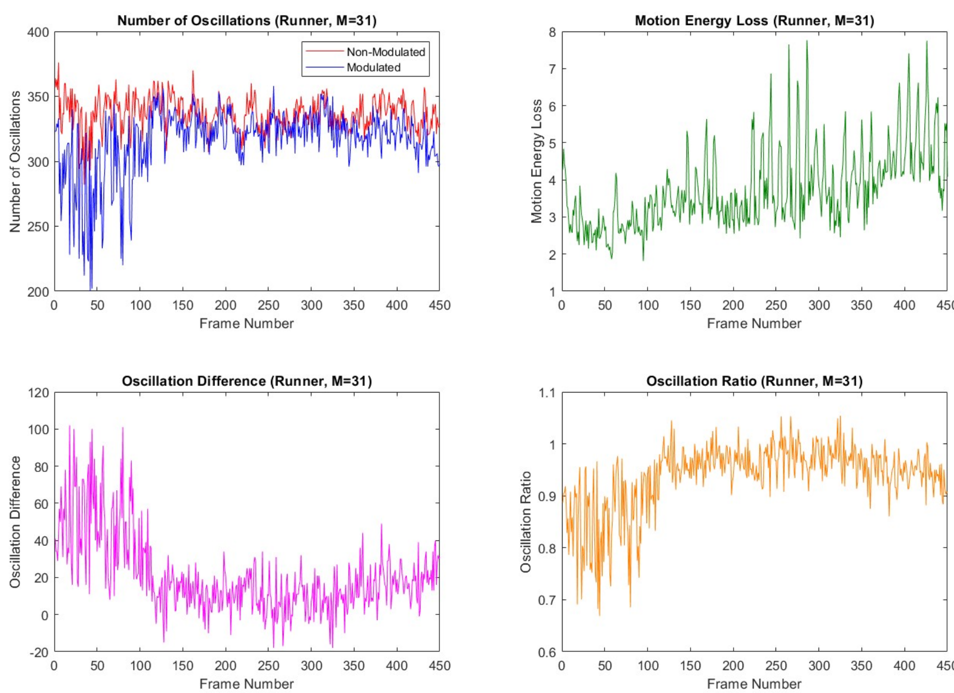


Figure 4.21: Oscillatory Behavior and Energy Dissipation for Runner (M=31)

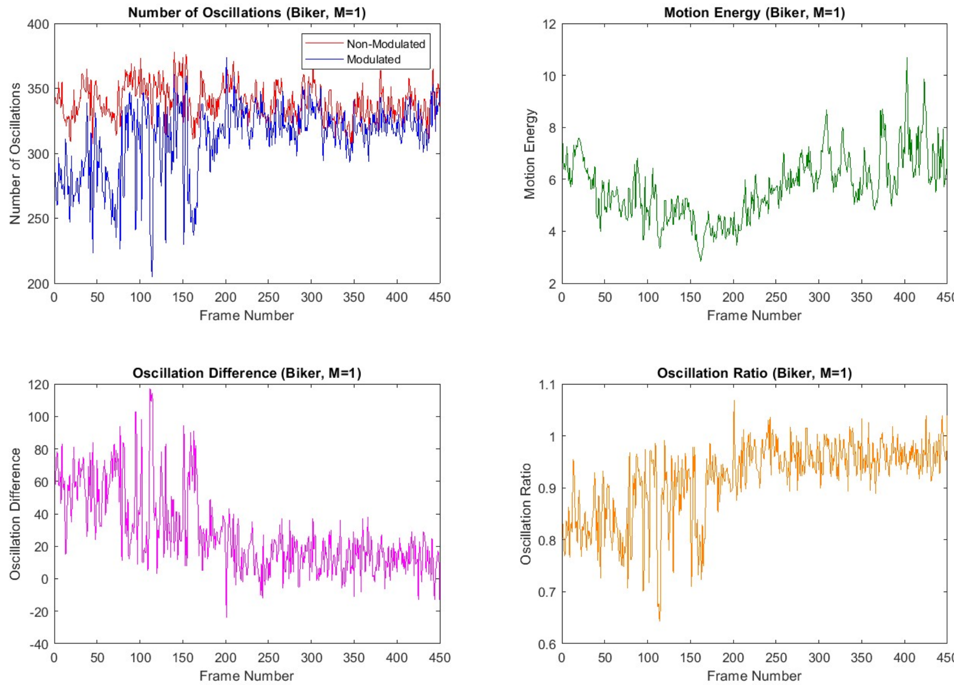


Figure 4.22: Oscillatory Behavior and Energy Dissipation for Biker (M=1)

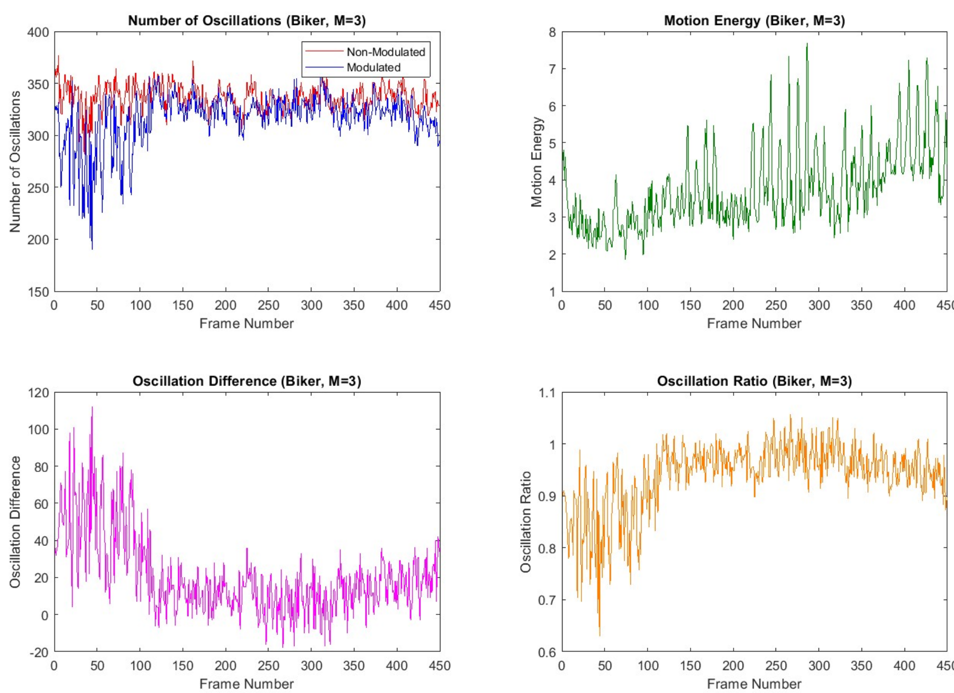


Figure 4.23: Oscillatory Behavior and Energy Dissipation for Biker (M=3)

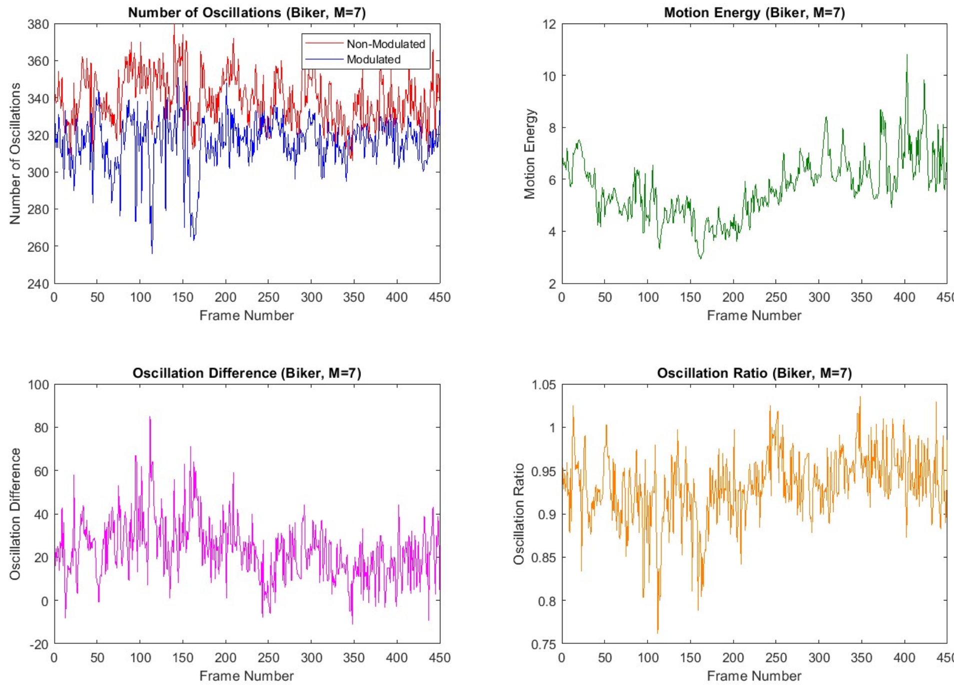


Figure 4.24: Oscillatory Behavior and Energy Dissipation for Biker (M=7)

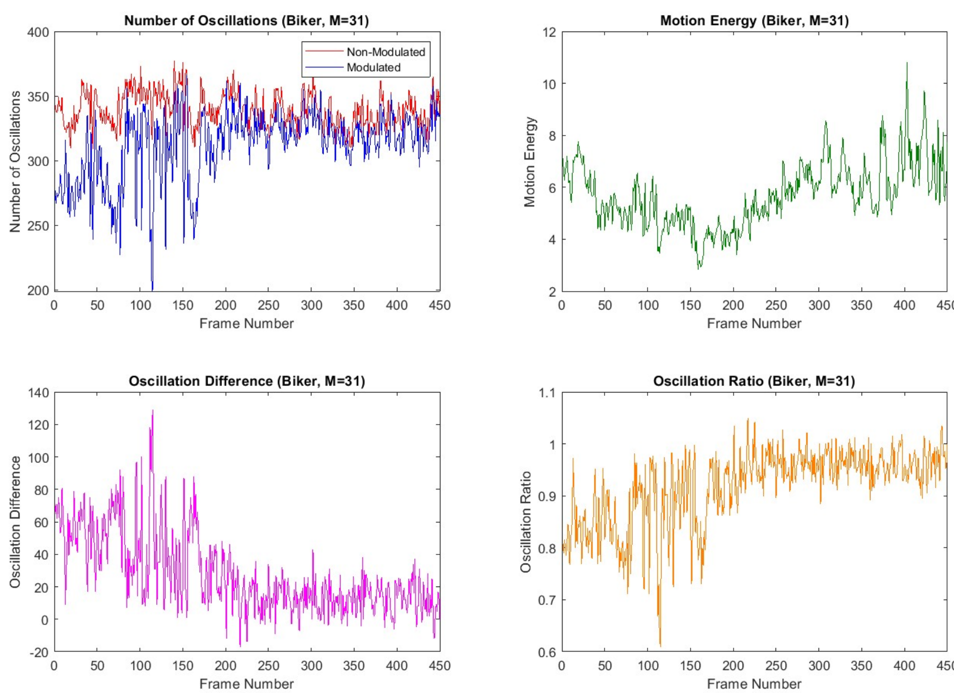


Figure 4.25: Oscillatory Behavior and Energy Dissipation for Biker (M=31)

**Runner: Modulation Value, M=1**

This activity shows a higher number of oscillations in non-modulated signals compared to modulated signals. The motion energy loss gradually increases, with noticeable spikes around frame 250 and towards the end. The oscillation difference fluctuates but remains significant, while the oscillation ratio stays below 1, indicating fewer oscillations in modulated signals.

**Runner: Modulation Value, M=3**

At M=3, the difference in oscillation counts between non-modulated and modulated signals becomes more apparent. The motion energy loss pattern is similar to M=1, with a gradual increase and notable spikes. The oscillation difference is more pronounced, and the oscillation ratio decreases further, highlighting the modulation's increased effect.

**Runner: Modulation Value, M=7**

At M=7, the reduction in oscillations for modulated signals is more evident. The motion energy loss remains consistent, with higher peaks towards the end. The oscillation difference stabilizes with higher values, indicating a stronger modulation impact, and the oscillation ratio continues to decrease.

**Runner: Modulation Value, M=31**

At M=31, there is a significant reduction in oscillations for modulated signals. The highest energy loss is observed towards the end. The oscillation difference peaks, showing the maximum modulation impact, while the oscillation ratio reaches its lowest point.

**Walker: Modulation Value, M=1**

At M=1, walking shows consistently higher oscillation counts in non-modulated signals. Motion energy loss increases gradually with some fluctuations. The oscillation difference is significant, and the oscillation ratio remains below 1.

**Walker: Modulation Value, M=3**

At M=3, the difference between non-modulated and modulated signals is more pronounced. Energy loss increases with higher peaks, and the oscillation difference becomes more stable. The oscillation ratio continues to decrease, indicating a stronger modulation effect.

**Walker: Modulation Value, M=7**

At M=7, there is a significant reduction in oscillations for modulated signals. Higher energy loss is observed, with the oscillation difference increasing, indicating a stronger modulation impact. The oscillation ratio continues to decrease.

**Walker: Modulation Value, M=31**

At M=31, walking shows a significant reduction in modulated oscillations. The highest energy loss occurs towards the end. The oscillation difference peaks, showing the maximum modulation impact, and the oscillation ratio reaches its lowest point.

**Biker: Modulation Value, M=1**

At M=1, biking shows consistently higher oscillation counts in non-modulated signals. Motion energy loss increases gradually with noticeable fluctuations. The oscillation difference is significant, and the oscillation ratio remains below 1.

**Biker: Modulation Value, M=3**

At M=3, the difference in oscillation counts between non-modulated and modulated signals is more pronounced. Energy loss increases with higher peaks, and the oscillation difference is stable and significant. The oscillation ratio continues to decrease.

**Biker: Modulation Value, M=7**

At M=7, there is a significant reduction in oscillations for modulated signals. Higher energy loss is observed, with the oscillation difference increasing, highlighting a stronger modulation effect. The oscillation ratio continues to decrease.

**Biker: Modulation Value, M=31**

At M=31, biking shows the largest reduction in oscillations for modulated signals. The highest energy loss is observed towards the end. The oscillation difference peaks, showing the maximum modulation impact, and the oscillation ratio reaches its lowest point.

In conclusion, modulation significantly impacts the oscillatory behavior and energy dissipation across all activities, with the strongest effects observed in biking, followed by walking and running. These findings highlight the potential of modulation to control and optimize energy distribution and dynamic behavior in various scenarios.

### 4.2.5 Detailed Analysis of Motion Energy Across Different Physical Activities

This graph represents the motion energy analysis for three different activities: walking, running, and biking, across several frames, using the data labeled as M31. Let's analyze the trends and implications based on this plot.

Other graphs for M = 1, 3, and 7 are in Appendix D

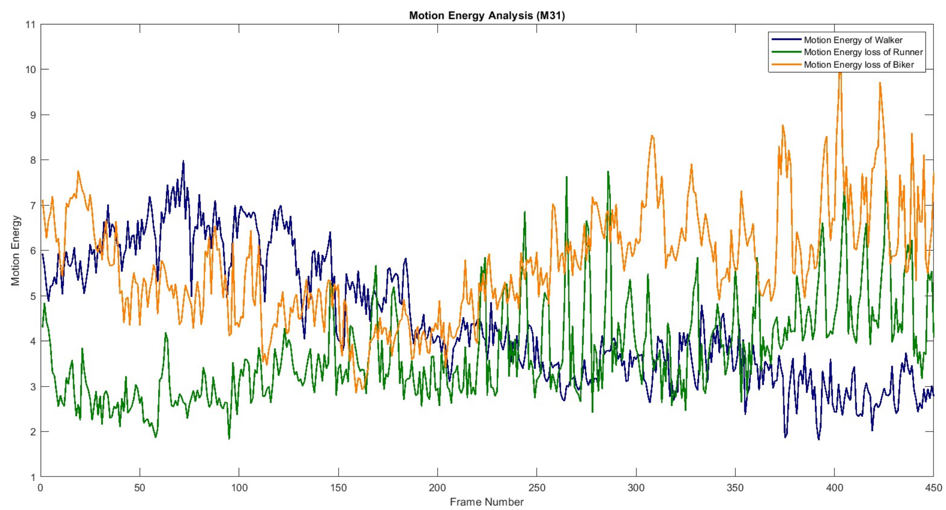


Figure 4.26: Motion Energy with Activities (M=31)

The motion energy of the walker appears to be the most stable and consistent among the three activities, with relatively low fluctuations. The walker's energy remains steady, without significant spikes or drops, suggesting a smooth and controlled motion with minimal energy dissipation or loss.

The runner's motion energy shows frequent spikes and drops, indicating higher variability in energy dissipation. These fluctuations suggest that the runner experiences more dynamic changes in energy, possibly due to the impact forces during running, which cause intermittent energy loss. The runner's energy is generally lower than that of the biker, which might indicate that the runner is losing more energy to the environment or that the activity is less energy-efficient.

The biker shows a different pattern with significant spikes in motion energy towards the later frames. These spikes indicate periods of intense energy usage, possibly when the biker exerts more effort (e.g., climbing, accelerating). The earlier frames show more moderate energy levels, suggesting that the biker was moving more steadily or at a lower intensity. As the frames progress, the energy increases substantially, indicating a significant energy expenditure that could be associated with more intense physical activity or challenging terrain.

When comparing each of the activities, the walker's energy is consistently lower and more stable compared to the runner and biker, indicating that walking is the least energy-intensive activity, at least in terms of motion energy.

The runner's energy shows high variability, where energy is lost due to impacts and changes in momentum. The biker's energy increases significantly over time, suggesting that biking either requires more energy as it progresses or that the intensity of the activity increases, leading to higher energy dissipation.

### 4.2.6 Analysis of Envelope Areas and Modulation-Induced Energy Variations

Table 4.8: Envelope Area and Motion Energy Loss for Different M Values and Activities

| | M =1 | | | M=3 | | |
|---|---|---|---|---|---|---|
| | **Upper env area** | **Lower env area** | **Motion energy loss** | **Upper env area** | **Lower env area** | **Motion energy loss** |
| **Runner** | 5.4893 | 8.6549 | 3.1657 | 5.5601 | 8.6549 | 3.0948 |
| **Walker** | 9.5307 | 15.6333 | 6.1025 | 9.3008 | 15.6333 | 6.3325 |
| **Biker** | 5.8382 | 11.1993 | 5.3611 | 5.6896 | 11.1993 | 5.5097 |
| | **M=7** | | | **M=31** | | |
| | **Upper env area** | **Lower env area** | **Motion energy loss** | **Upper env area** | **Lower env area** | **Motion energy loss** |
| **Runner** | 5.5533 | 8.6549 | 3.1016 | 5.5486 | 8.6549 | 3.1063 |
| **Walker** | 9.4914 | 15.6333 | 6.1419 | 9.6074 | 15.6333 | 6.0259 |
| **Biker** | 5.6889 | 11.1993 | 5.5105 | 5.9584 | 11.1993 | 5.2409 |

The lower envelope area remains constant across all M values for each activity. That is consistent since the lower envelope corresponds to the non-modulated signal, which shall not change along the variation in modulation. Hence, the analyses will focus on the area of the upper envelope and the motion energy loss, because these two parameters will reveal the information with respect to how modulation affects the dynamics of each activity.

For the Runner, the upper envelope area exhibits minor fluctuations as M changes. It starts at 5.4893 for M=1 and slightly increases to 5.5601 at M=3. This value increases to 5.5533 for M=7, and to 5.5486 for M=31, beyond which the area has stabilized relatively. A very similar pattern can be seen in the loss of energy in motion, which, after being at its highest at M=1, decreases to 3.0948 at M=3, only to stabilize at approximately 3.1 as M increases to 7 and 31. This behavior suggests that modulation has a minimal effect on the energy dissipation for the Runner, with only slight variations as the modulation factor changes.

In contrast, the Walker shows a more pronounced response to modulation. The upper envelope area decreases from 9.5307 at M=1 to 9.3008 at M=3, then increases to 9.4914 at M=7, and further increases to 9.6074 at M=31. The maximum loss of motion energy is at M=3, with a value of 6.3325, which reduces slightly to 6.1419 at M=7 and further to 6.0259 at M=31. It may thus be demonstrated here in this pattern that modulation factor M = 3 will have the major effect on energy dissipation for the Walker, while for higher values of M, the effect starts to diminish beyond this point.

The Biker's response to modulation is also distinct. The upper envelope area begins at 5.8382 for M=1, decreases to 5.6896 for M=3, stabilizes slightly at 5.6889 for M=7, and then increases to 5.9584 at M=31. The motion energy loss follows a similar pattern, starting at 5.3611 for M=1, increasing to 5.5097 at M=3, remaining almost constant at 5.5105 for M=7, and then decreasing to 5.2409 at M=31. These observations suggest that the modulation factor M=3 slightly increases energy dissipation compared to M=1, but as M increases further, the energy loss decreases, with a significant drop at M=31. This indicates that higher modulation factors lead to more efficient energy use in biking.

Overall, the Runner shows minimal variation in energy dissipation across different modulation factors, with a slight stabilization of energy loss after M=3. This indicates that modulation has a relatively small impact on running. In contrast, the Walker exhibits the highest energy dissipation at M=3, with energy loss decreasing as M increases. This suggests that M=3 is an important point for walking, where modulation has the most significant impact on energy dynamics. The Biker experiences a slight increase in energy loss at M=3, followed by stabilization and then a decrease at M =31, indicating that higher modulation factors improve energy efficiency in biking.

In conclusion, the modulation factor M=3 continues to play an important role in influencing energy dissipation, particularly for walking and biking. The Runner is practically insensitive to any variations of M, while the Walker and Biker give different responses, and M = 3 represents a maximum energy dissipation. Energy dissipation decreases when M increases beyond 3 for both walking and biking. Higher modulation factors therefore increase the energy efficiency in these activities. The fact that for all the M-values, the area of the lower envelope remains identical underlines that the changes in motion energy loss noted here are a function of the impact of modulation on the area of the upper envelope.

**Integration of Video Analysis with Envelop Data**

The visual data provided in the form of video frames and their corresponding envelope plots for M=7 enriches the analysis by offering a direct visual correlation between the numerical data and the real-world motion of different activities—cycling, running, and walking. These images serve as a bridge between the theoretical modulation analysis and practical, observable outcomes.

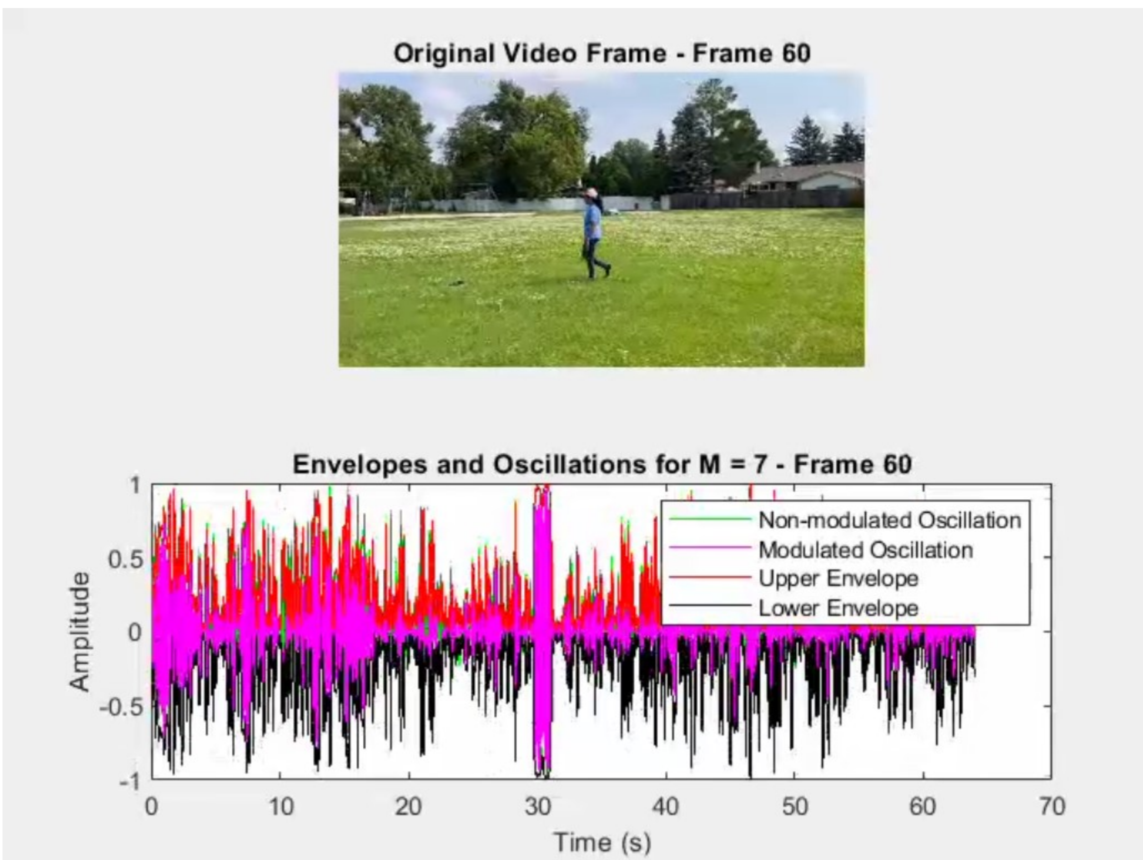


Figure 4.27: Envelops for M = 7 for Walking

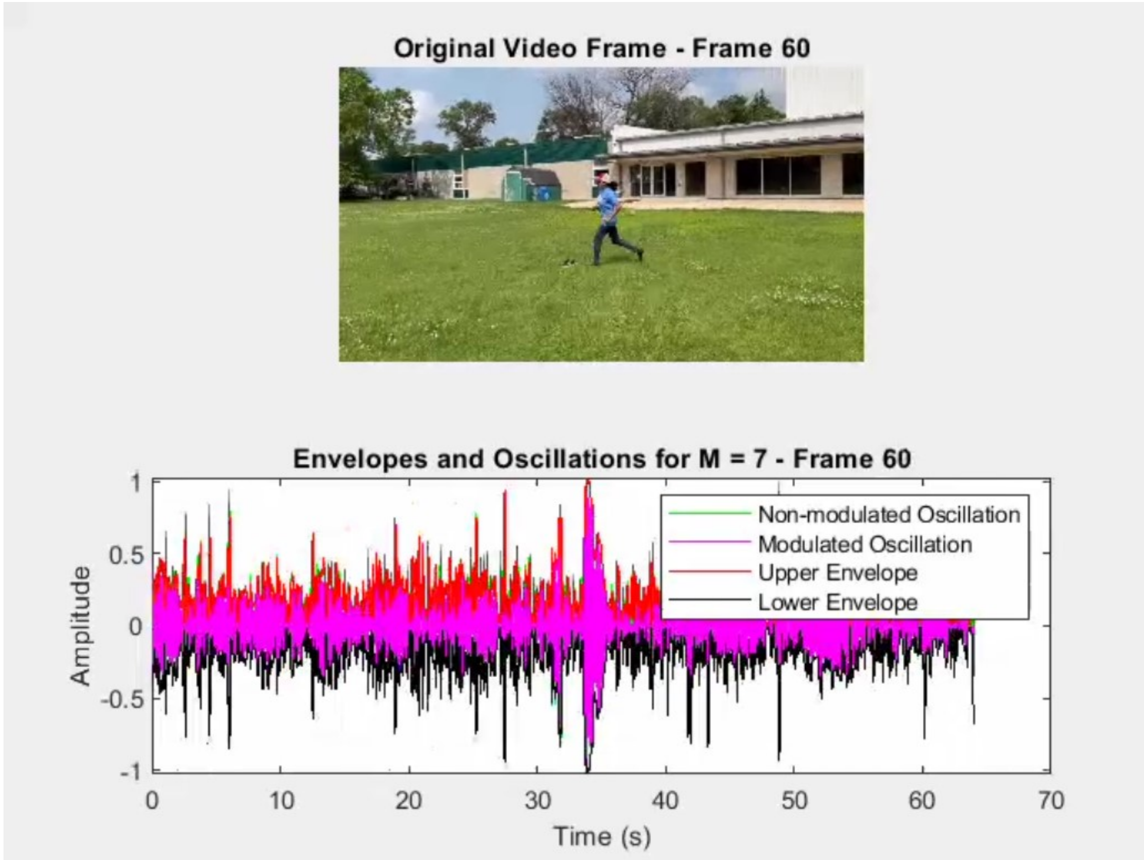


Figure 4.28: Envelops for M = 7 for Running

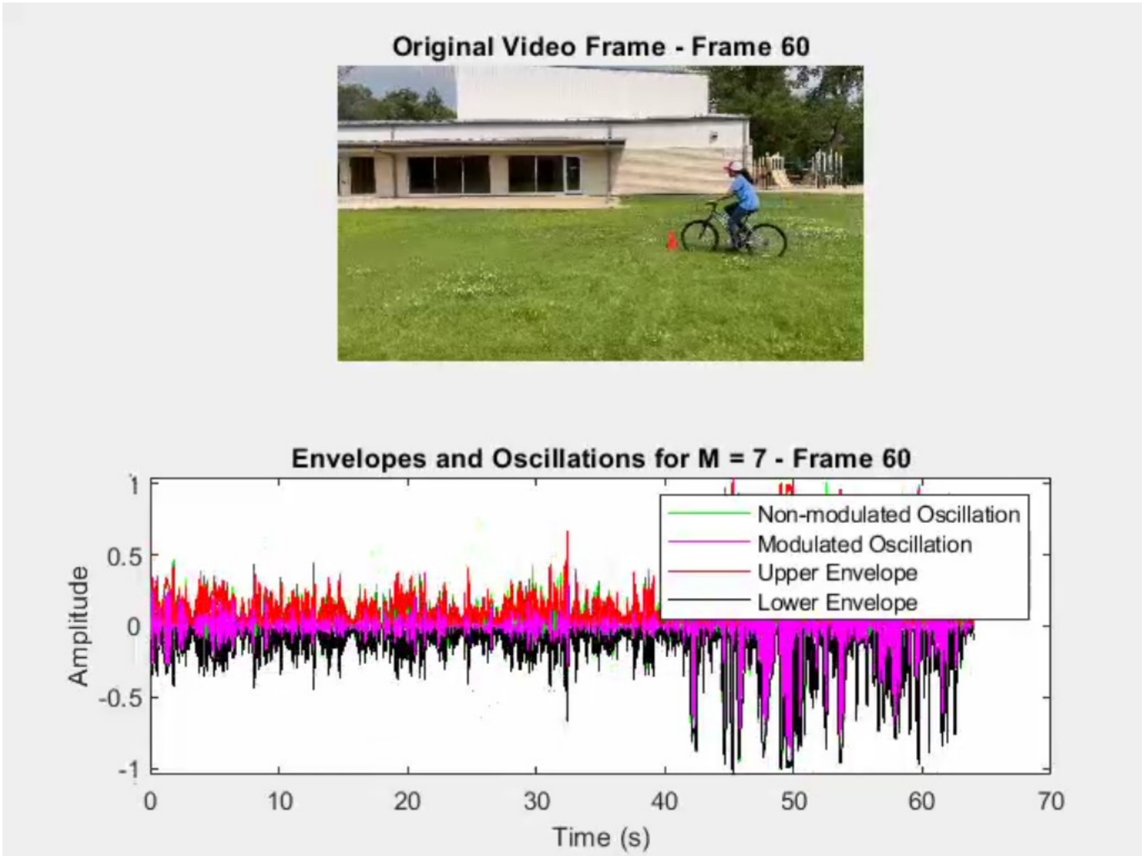


Figure 4.29: Envelops for M = 7 for Cycling

The above figures are snapshots of written videos and the MATLAB Script is in Appendix F

**Cycling (Frame 60, M=7):** The video frame captures the cyclist in motion, displaying a relatively steady and smooth movement across the frame. The envelope plot for the corresponding signals illustrates the interaction between the non-modulated oscillations in green and the modulated oscillations in magenta; the upper envelope in red tracks the modulated oscillations quite closely. The lower envelope in black is flat for all M values since it corresponds to the non-modulated signal, which does not change with variations in the modulation factor.

The upper envelope area for cycling at M=7 is 5.6889, and the motion energy loss is 5.5105. These values reflect a controlled energy dissipation, which is visually confirmed by the smooth cycling motion in the video. The close alignment of the upper and lower envelopes suggests that while modulation introduces some energy loss, the overall movement remains efficient and steady, with minimal disruption.

**Running (Frame 60, M=7):** The video frame shows the runner in full stride, highlighting the vigorous and rapid nature of running. The envelope plot associated with this frame reveals more pronounced peaks and troughs in the modulated oscillations, corresponding to the high-energy, dynamic motion of the runner. The upper envelope area for running at M=7 is 5.5533, with a motion energy loss of 3.1016.

This relatively low energy loss compared to walking and cycling indicates efficient energy use during running. The video visually supports this finding, as the runner's motion appears fluid and continuous, with the modulation serving to fine-tune the motion rather than significantly altering the energy dynamics. The smooth and consistent movement observed in the video aligns with the stability seen in the envelope plot.

**Walking (Frame 60, M=7):** In the walking scenario, the video frame captures a slower, more deliberate motion, which is clearly reflected in the envelope plot. The upper envelope area for walking at M=7 is 9.4914, and the motion energy loss is 6.1419. These values indicate that walking experiences more significant energy dissipation under modulation, which is visually supported by the plot where the modulated oscillations show greater amplitude and variability.

The video reinforces this interpretation, showing that even small variations in the walking pace are amplified by the modulation, leading to increased energy dissipation. The peaks in the envelope plot correspond to moments of more dynamic movement, which are less frequent in walking compared to running or cycling, but nonetheless significant in contributing to overall energy loss. The lower envelope remains unchanged, as it is tied to the non-modulated signal, which remains consistent regardless of the modulation factor M.

In conclusion, the integration of these video frames and envelope plots into the analysis provides a multi-faceted understanding of how modulation affects different activities. For cycling, M=7 introduces controlled energy dissipation, resulting in a smooth and efficient motion, as confirmed by the video. The most evident difference is in the energy dissipation:

low in running and much larger in walking. Also, modulation fine-tunes the oscillations only in running without perturbing the motion, while in walking this sensitivity is much larger with much larger oscillations in agreement with the more measured and variable character of the activity.

These visual and numerical information together offer a comprehensive perspective on how modulation affects motion dynamics across different activities, highlighting the varying impact of M=7 on energy efficiency and motion stability. The fact that the lower envelope has stable overall values of M reinforces this same message as before: the unmodulated signal is a constant and offers a steady baseline against which the modulation is measured. It is this clarity obtained through the correlation of these results with the obtained video that allows the discussion of the practical implications of the modulation at hand.

# Chapter 5

# Conclusion

## 5.1 Restatement of the Thesis Problem

This thesis focused on the complex interactions of the method of signal modulation and its effect on the energy dynamics of video signals, applying Mersenne primes. In the digital world, much of the media consumption and even the flow of data across platforms take the form of video content. Thus, it is not only a purely technical challenge but also an urgent necessity, having greater implications for multimedia technology in enhancing efficiency and effectiveness while processing video signals.

The key question that this work tried to answer was how the modulation of video signals by Mersenne primes changes the behavior and energy characteristics of these signals. These were chosen for possible ways they could introduce more efficient ways of encoding and processing video data because of their inherent mathematical advantages.

It had the objective to research these theoretical considerations in practical applications to bridge the gap between abstract mathematical theories on the one hand and tangible technological applications on the other. The paper searched for new means of improving the clarity and speed of video transmission by analyzing those changes in modulation settings that influence energy distribution and oscillatory patterns of video signals.

With such understanding, significant strides could be made in how video quality is compressed and transmitted, right from streaming services down to telecommunications. The study was driven by a vision that it would not only advance knowledge in signal processing but also contribute practical solutions to ever-evolving challenges in the field of digital communication technologies.

### 5.1.1 Summary of Key Findings

The whole scope of this thesis, therefore, ranges from a study into the energy dissipation process in video signals modulated with Mersenne primes. The whole work was based on the supposition that the use of Mersenne primes to modulate signals increases the efficiency of energy expenditure in video processing. The results obtained in this work gave much valuable information about the process of modulation and also about the efficiency of Mersenne primes in signal processing. The following is a detailed summary of the significant outcome derived from putting together all the research works in detail by each experiment and its analysis.

1. **Effectiveness of Different M Values**

   The presented research compared the energy dissipation and oscillation characteristics of video signals modulated with different values of M, namely M=1, M=3, M=7, and M=31. From these, an increase in M corresponds to a significant decrease in the peak maximum energies within the signals. This shows that as M increases, which is typical of Mersenne primes, the energy will be more evenly distributed across the signal.

   Only M = 31 consistently had the lowest peaks and most homogeneous energy distribution of the values tested. This would hint that larger Mersenne primes are possibly more effective at modulating video signals for purposes where energy efficiency is an issue.

2. **Reduction in Energy Peaks**

A significant reduction in energy peaks was observed with higher M values. This is particularly crucial in video transmission where energy peaks can lead to signal distortion and loss of information. By flattening the energy peaks, Mersenne prime modulation ensures that the signal's integrity is maintained across various transmission channels.

3. **Stabilization of Signal Oscillations**

   The study also observed that the M value increase improved the stability of signal oscillations. This stigmatization is important in minimizing chances for signal interference to improve the clarity and quality of video transmitted. The more stable the oscillations, the less susceptible to external disruptions the signal will be, hence more ideal for broadcasting and streaming applications.

4. **Decrease in Energy Dissipation**

   One of the most interesting points was the trend in the reduction of total energy dissipation in conjunction with increased M values; for instance, M=31 had the lowest energy dissipation overall considered frames. A result of this nature brings out the potential of Mersenne prime modulation not only in energy conservation within a signal but also in the longevity and durability of the signal during transmission.

5. **Activity-Based Energy Dynamics**

   Different types of video content are tested against the modulation technique, ranging from static to dynamic scenes. The results showed that this technique was inherently more complex and energy-intensive in dynamic scenes at higher M values, which manifested in the form of reduced dissipation of energy and stability in signal transmission.

6. **Adaptability Across Different Video Contents**

   Mersenne prime modulation was tested across various types of video content ranging from static scenes to highly dynamic ones. These dynamic scenes tend to be in favor of higher values of M since more energy will be required to get an accurate transmission. Thus, Mersenne prime modulation can be flexible for several multimedia applications.

These findings all clearly converge on the thesis that significant improvement in energy efficiency and reliability can be achieved with Mersenne prime modulation in processing video signals. The contribution to a wider understanding of digital signal processing by the research effort through reduced energy dissipation, increased stability, and overall improved efficiency for different M-values is accordingly furthered. With this, the way towards further studies and practical applications is paved.

## 5.2 Discussion of the research

### 5.2.1 Energy dissipation across activities

This thesis has highlighted the variation in energy dissipation for various modulation techniques and physical activities such as walking, running, and cycling in a systematic manner. This nuanced investigation shall serve very relevantly in view of the recent interest in energy efficiency in signal processing within video surveillance and activity recognition systems.

The results of walking showed that at M = 31, during walking, the energy dissipation was pretty low; hence, for low-dynamic activities, higher modulation indexes might not be related to higher energy efficiency.

The larger M values in running and cycling were associated with a significant reduction in energy dissipation, and there is evidence to consider that in cases of more complicated movement patterns, a higher modulation index offers an opportunity to attain energetically optimum behavior.

### 5.2.2 Comparative Analysis with Existing Studies

The modulation techniques surveyed here are grounded in the very important task of allowing improvement in energy dissipation measures, analogous to work found in recent literature in innovation with microrheology and signal processing. For instance, the use of Mersenne primes is new in the modulation strategy and strengthens the accuracy of the measurements, similar to the novelty found by Litjens et al. (2017) [16]in a medical imaging context but put to use here in a very different domain, video signal analysis.

Some properties of Mersenne primes used uniquely in this research helped to provide stability and efficiency in the signal processing. These ideas form a new adaptation of concepts that in conventional cryptography and number theory have been used, since Mersenne primes provide computational efficiency for the application of Rivest 2015 [19].

### 5.2.3 Implications for Video Signal Processing

The research through the lens of energy-efficient computing better understand energy dynamics in video signal processing. This is increasingly pertinent, as computational demands will escalate with the advent of high-definition video data streams.

The adaptation of machine learning models and customized hardware accelerations discussed in the research underpin the integration of cutting-edge computational techniques that can increase energy efficiency. This goes right through very nicely to industry trends where decreasing power consumption while maximizing computational throughput has been the focus of the market. Zang and Wang, 2016) [14].

### 5.2.4 Future Directions

The thesis lays a solid groundwork for future research, especially in optimizing modulation techniques for diverse real-world applications. Further studies could explore the scalability of Mersenne prime-based modulations across different computational platforms and real-time video processing scenarios.

## 5.3 Implications of the Research

### 5.3.1 Practical Implications of Findings

The findings from this study hold considerable implications for the field of digital signal processing, particularly in the realm of video communications and multimedia systems. By employing Mersenne primes for signal modulation, this research demonstrates a method to efficiently manage energy distribution within a signal, which is a significant concern for applications requiring high efficiency and low power consumption.

1. **Improved Video Processing Systems** The application of Mersenne primes in the processing of videos allows energy to be distributed somewhat evenly across the signal components, which enables much better energy management. This may also translate to a better quality of the signal and fewer degradations, especially when there is minimal bandwidth or in systems requiring power efficiency, such as in mobile devices and other portable media technologies.

2. **Reduced Computational Load**

   Mersenne primes will most likely reduce computational overheads generally required in performing the signal processing task by implementing certain modulation techniques. This is because Mersenne primes allow the multiplication process to be performed in simpler forms; these are processes that occur more frequently in signal modulation. In practice, this may mean that devices could function on lower power and have a longer life, which can be great for battery-powered devices.

3. **Improved System Efficiency**

   This, in turn, means that with the use of higher-efficiency modulation techniques, there is a potential to greatly reduce the energy consumed by both data centers and network systems, especially industries concerned with digital video content transmission and storage, such as streaming services and telecommunications. The financial consequences of this will not only be due to cost savings but also environmental ones by lessening the carbon footprint from extensive processing.

### 5.3.2 Contribution to Existing Knowledge

This research contributes to existing knowledge by bridging theoretical mathematical concepts with practical applications in digital signal processing. The study uses the unique characteristics of Mersenne primes, previously underexplored in this context, to offer new information about their potential for improving signal modulation techniques.

1. **Theoretical understandings**

   The use of Mersenne primes for the modulation of signals unravels one more layer of understanding in the mathematical framework that underpins digital signal processing. This adds to the academic discourse both in the field of number theory and in its practical applications, inviting further scholarly exploration of other mathematical concepts that could be similarly used.

2. **Innovation in Modulation Techniques**

   By integrating Mersenne primes into modulation strategies, the study proposes an innovative approach that could be adapted or expanded in future research. This could stimulate a range of experimental and applied research aimed at testing the limits and capabilities of this approach across different types of digital communications systems.

### 5.3.3 Implications for Stakeholders

- **For Practitioners**

  Engineers and technicians involved in the design and maintenance of digital communication systems might find the use of Mersenne prime-based modulation strategies to be a valuable addition to their toolkit, helping to optimize the balance between system performance and energy efficiency.

- **For Educators**

  This research can also be used as a case study in any academic and professional setting regarding the practicality of theoretical mathematical concepts. The thesis will serve as an example for it shows how abstract theories have been reduced to real-time applications in a number of educational programs that concern mathematics, engineering, and technology.

Therefore, the study not only contributes to the academic and practical understanding of signal modulation but also suggests a pathway for future research and application that could have wide-reaching effects across multiple sectors concerned with digital data transmission and processing.

## 5.4 Limitations of the Study

While this thesis provides informative findings on the impact of Mersenne prime modulation on video signal energy dissipation, it's important to recognize the specific limitations that could influence the interpretation and application of these results.

1. **Experimental Design and Scope**

   The study focused on a limited set of Mersenne primes (M=1, 3, 7, 31) to modulate video signals, which constrains the breadth of the conclusions that can be drawn. The selection was based on their known mathematical properties, but this leaves out a broader spectrum of primes that might exhibit different or more varied impacts on signal modulation. Extending this research to include a wider range of Mersenne primes could help validate the consistency of the findings across a broader spectrum of conditions.

2. **Data Collection Methodology**

   Most of the analysis was done through simulations in MATLAB, which is robust but does not take into consideration some real-world variables like noise and interference that actually occur during signal transmission. This will perhaps affect the practicality of the results, as real situations may differ due to these uncontrolled factors.

3. **Analytical Approach**

   The study utilized specific analytical methods such as the Hilbert transform for envelope detection and the cumulative trapezoidal numerical integration for energy calculations. While these methods are standard, they might not capture all nuances of energy distribution in modulated signals. Exploring alternative analytical techniques could provide a more comprehensive understanding of how modulation affects energy dynamics.

## 5.5 Environmental and Equipment Limitations

1. **Environmental Variability**

   The experiments were conducted outdoors in sunny summer conditions. While this setting provides natural lighting, it also introduces variables that are not present in controlled laboratory environments. Fluctuations in natural light due to cloud cover or the time of day could affect the video quality and thus the signal analysis. Additionally, outdoor settings might introduce unwanted noise from environmental sources (like wind or other natural sounds), which could interfere with the signal processing.

2. **Equipment Limitations**

   The utilization of an iPhone to capture video signals introduces certain constraints, particularly with the camera's specifications. Although the iPhone's camera is adept for general usage, its fixed frame rate and resolution may hinder the detailed observation of rapid movements or minor environmental variations. Such limitations could potentially affect the granularity and accuracy of the signal modulation analysis and subsequent interpretations regarding energy dissipation in video signals.

3. **Electromagnetic Interference (EMI)**

   Although less likely in an outdoor setting away from industrial equipment, EMI can still occur from unexpected sources like cell phones, Wi-Fi signals, and even satellite communications. This interference could subtly alter the video signals being recorded, potentially skewing the modulation effects observed.

4. **Subject Variability**

   The recordings involve moving subjects or varying scenarios, inconsistencies in speed, distance, or lighting could impact the consistency of the data collected. This variability can make it challenging to establish a controlled baseline for comparison across different M values.

5. **Data Processing**

   The analysis heavily relies on the accuracy of the signal processing software (Matlab) and algorithms used. Any inherent biases or errors in the software, or in the parameters set for analysis such as threshold levels for peak detection, could influence the outcomes significantly.

## 5.6 Recommendations for Future Research

The study of signal modulation has opened up exciting opportunities for discovery and innovation, but there's still so much more to learn. Here are some ideas for where to take this research next, explained in straightforward terms.

- **Expanding Environmental Testing**

This present study was conducted in a controlled environment outdoors on bright days using an iPhone. Future research would take up video data capture in a range of environmental settings much beyond the ones in this study to make these findings much more strong. There is a need for diversified testing on various lighting and weather conditions, and even urban settings with higher electromagnetic interference are necessary. More testing environments will reveal just how the different factors will affect signal modulation and energy dissipation.

- **Advanced Equipment Utilization**

  While the iPhone camera served well for preliminary analysis, employing more advanced recording devices could significantly deepen the information. Future studies might consider using high-speed cameras with adjustable frame rates and resolutions. Such technology could capture more detailed data on rapid movements and subtle environmental changes, potentially revealing new aspects of energy dynamics within modulated signals.

- **Machine Learning Algorithms**

  Implementing machine learning could revolutionize the analysis of modulated signals. By training machine learning models on large datasets comprising video signals modulated under different settings, in one fell swoop, the parameters for minimal energy dissipation will be predicted. It is also here that machine learning would come into good use by allowing the isolation of specific characteristics in Mersenne prime-modulated signals that act to efficiently reduce the loss of energy.

- **Practical Applications Development**

  Future research should also focus on the practical applications of these findings. For instance, how can the understanding of Mersenne prime modulation improve data transmission in real-world telecommunications or broadcasting? Developing prototypes or pilot projects that apply this theoretical knowledge in practical scenarios would be a crucial step forward.

## 5.7 Reflections on the Journey and Impact of This Research

This research journey has been a meaningful experience, blending the technical world of signal processing with real-world applications. What began as a challenge to understand how modulation affects energy dissipation in video signals turned into a broader exploration of how mathematical constructs such as the Hilbert transform and the use of Mersenne primes in the modulating oscillatory frequency of motion waveforms can impact everyday technology.

This task seemed to be very complicated and technical in the beginning. However, as the study unfolded, it became clearer how relevant these findings are. Knowing exactly how to minimize losses of energy within the video signals will directly translate into making technology energy-efficient- fundamental in today's world where the question of sustainability is on an increasing upward trend.

This work contributes an important piece to the larger field of signal processing. It shows how academic research can lead to practical benefits, making technology smarter and more efficient. It's not just about the technical details, but also about the broader implications—how these findings can help build a more sustainable future.

The research is not about finding answers but retrospectively learning to ask better questions and realizing even the most minute discovery is of value. It is, therefore, hoped that this paper will add to the body of knowledge and inspire further inquiries, knowing full well that their efforts make a difference in technology and the world at large.

# Appendix A

# MATLAB Script for Row-based Signal Analysis and Visualization from a Video Frame Using Hilbert Transform

```
clc; clear; close all;

videoFilePath = 'C:\Users\perer\OneDrive\Desktop\Tharu\Biker.mp4';

videoReader = vision.VideoFileReader(videoFilePath);

% Define the frame to extract and the row number for analysis
frameToExtract = 60;
rowNumber = 600;
frameCount = 0;

% Process the video until the specified frame is reached
while ~isDone(videoReader)
    frameCount = frameCount + 1; % Increment frame count
    frame = step(videoReader); % Read the next frame

    if frameCount == frameToExtract
        frameGray = rgb2gray(frame);

        selected_row = double(frameGray(rowNumber, :));

        selected_row = (selected_row - min(selected_row)) / (max(selected_row) - min(selected_row));
        selected_row = 2 * selected_row - 1; % Scale to [-1, 1]

```

```
        % Time vector
        originalFrameRate = videoReader.info.VideoFrameRate;
        t = linspace(0, numel(selected_row)/originalFrameRate,
           numel(selected_row));

        % Generate the analytic signal using the Hilbert
           transform
        analytic_signal = hilbert(selected_row);
        instantaneous_amplitude = abs(analytic_signal);
        instantaneous_phase = angle(analytic_signal);

        figure;

        subplot(3, 1, 1);
        plot(t, selected_row, 'b', 'LineWidth', 1);
        title('Original Signal of Frame 10');
        xlabel('Time (s)');
        ylabel('Amplitude');

        subplot(3, 1, 2);
        plot(t, instantaneous_amplitude, 'r', 'LineWidth', 1);
        title('Instantaneous Amplitude for row #650 of Frame 10')
           ;
        xlabel('Time (s)');
        ylabel('Amplitude');

        subplot(3, 1, 3);
        plot(t, instantaneous_phase, 'g', 'LineWidth', 1);
        title('Instantaneous Phase for row #650 of Frame 10');
        xlabel('Time (s)');
        ylabel('Phase (radians)');

        % Highlight the selected row in the original frame
        figure;
        imshow(frame);
        hold on;
        plot([1, size(frameGray, 2)], [rowNumber, rowNumber], 'r'
           , 'LineWidth', 2);
        title(sprintf('Frame %d with Selected Row Highlighted',
           frameToExtract));

        break;
    end
end

% Release the video reader
release(videoReader);
```

# Appendix B

# Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data

```
clc; clear; close all;

M_values = [1];
% videoFilePath = 'C:\Users\perer\OneDrive\Desktop\Final\Runner.mp4';
% videoFilePath = 'C:\Users\perer\OneDrive\Desktop\Final\Walker.mp4';
videoFilePath = 'C:\Users\perer\OneDrive\Desktop\Final\Biker.mp4';

% Video reader
videoReader = vision.VideoFileReader(videoFilePath);

% Video frame rate
originalFrameRate = videoReader.info.VideoFrameRate;

% Initialize storage for metrics for each M value
metricsData = cell(length(M_values), 1);
metricsTables = cell(length(M_values), 1); % Store metric tables for each M

for mIndex = 1:length(M_values)
    M = M_values(mIndex);

%     writeObj = VideoWriter(sprintf('C:\\Users\\perer\\OneDrive\\Desktop\\Final\\Runner_%d.mp4', M), 'MPEG-4');
```

```
%     writeObj = VideoWriter(sprintf('C:\\Users\\perer\\OneDrive\\Desktop\\Final\\Biker_%d.mp4', M), 'MPEG-4');
    writeObj = VideoWriter(sprintf('C:\\Users\\perer\\OneDrive\\Desktop\\Final\\Walker%d.mp4', M), 'MPEG-4');

    writeObj.FrameRate = originalFrameRate / 4; % Reduced speed for observation
    open(writeObj);

    % initialize video reader for each M
    release(videoReader);
    videoReader = vision.VideoFileReader(videoFilePath);

    % Initialize frame count and data storage for the current M
    frameCount = 0;
    metricsData{mIndex} = [];

        while ~isDone(videoReader)
        frameCount = frameCount + 1;
        frame = step(videoReader);
        frameGray = rgb2gray(frame);
        row_number = 650; % adjustable
        selected_row = double(frameGray(row_number, :));

        % Normalize and scale the row data
        selected_row = (selected_row - min(selected_row)) / (max(selected_row) - min(selected_row));
        selected_row = 2 * selected_row - 1; % Scale to [-1, 1]

        % Time vector
        t = linspace(0, numel(selected_row) / originalFrameRate, numel(selected_row));

        % modulated signal
        x_t = interp1(linspace(0, max(t), numel(selected_row)), selected_row, t, 'linear', 'extrap');
        modulated_signal = x_t .* exp(-1j*2*M*pi.*t); % exponential modulation

        % Calculate metrics
        [numOscNonMod, numOscMod, motionEnergy] = calculateMetrics(selected_row, modulated_signal, t);

        metricsData{mIndex} = [metricsData{mIndex}; [frameCount, numOscNonMod, numOscMod, motionEnergy]];

        figure('visible', 'off'); % Hide the figure window
        subplot(2, 1, 1); imshow(frame); title('Original Video Frame');
        subplot(2, 1, 2);
        plot(t, selected_row, 'k-', 'LineWidth', 1);
        hold on;
```

```
        plot(t, abs(hilbert(selected_row)), 'g-', 'LineWidth', 1);
        plot(t, abs(hilbert(modulated_signal)), 'm-', 'LineWidth', 1);
        hold off;
        legend('Original Signal', 'Non-modulated Oscillation', 'Modulated Oscillation', 'Location', 'northeast');
        title(sprintf('Frame Number: %d', frameCount));
        xlabel('Time (t)'); ylabel('Amplitude');

        frameWithPlot = getframe(gcf);
        writeVideo(writeObj, frameWithPlot.cdata);

         close(gcf);
    end

    close(writeObj);

    metricsTable = array2table(metricsData{mIndex}, 'VariableNames', {'FrameNumber', 'NumOscNonMod', 'NumOscMod', 'MotionEnergy'});

    metricsTables{mIndex} = metricsTable;
end

release(videoReader);

for mIndex = 1:length(M_values)
    disp(['Data Table for M = ', num2str(M_values(mIndex))]);
    disp(metricsTables{mIndex});
end

function [numOscNonMod, numOscMod, motionEnergy] = calculateMetrics(nonModulatedSignal, modulatedSignal, t)
    envelopeNonMod = abs(hilbert(nonModulatedSignal));
    envelopeMod = abs(hilbert(modulatedSignal));

    energyNonMod = cumtrapz(t, envelopeNonMod .^ 2);
    energyMod = cumtrapz(t, envelopeMod .^ 2);

    motionEnergy = energyNonMod(end) - energyMod(end);

    % Separate threshold values for non-modulated and modulated signals
    peakThresholdNonMod = 0.1; % Adjustable for non-modulated signal
    peakThresholdMod = 0.1;    % Adjustable for modulated signal

    numOscNonMod = numel(findpeaks(envelopeNonMod, 'MinPeakHeight', peakThresholdNonMod));
    numOscMod = numel(findpeaks(envelopeMod, 'MinPeakHeight', peakThresholdMod));
end
```

# Appendix C

# Table for Motion Energy and Number of Oscillations for Different M Values of Walker

Table C.1: Motion Energy and Number of Oscillations for Different M Values of Walker

| | M=1 | | | M=3 | | | M=7 | | | M=31 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 1 | 355 | 339 | 5.965376 | 352 | 302 | 5.892475 | 355 | 341 | 5.938908 | 352 | 335 | 5.924053 |
| 2 | 344 | 318 | 5.451563 | 346 | 297 | 5.589464 | 343 | 331 | 5.547621 | 345 | 307 | 5.630495 |
| 3 | 342 | 326 | 5.335528 | 337 | 302 | 5.383362 | 338 | 334 | 5.259171 | 339 | 316 | 5.190166 |
| 4 | 347 | 310 | 5.166271 | 347 | 290 | 4.875715 | 347 | 340 | 5.089745 | 350 | 310 | 4.869991 |
| 5 | 337 | 317 | 5.099693 | 337 | 287 | 4.903879 | 336 | 322 | 5.037315 | 338 | 317 | 5.206991 |
| 6 | 341 | 316 | 4.838473 | 339 | 293 | 5.469491 | 341 | 321 | 5.045613 | 342 | 308 | 5.226207 |
| 7 | 335 | 306 | 5.703122 | 340 | 287 | 5.775516 | 340 | 335 | 5.572634 | 339 | 318 | 5.419031 |
| 8 | 337 | 305 | 5.639881 | 333 | 301 | 5.278347 | 333 | 341 | 5.527861 | 335 | 327 | 5.248561 |
| 9 | 336 | 318 | 5.540838 | 336 | 301 | 5.325847 | 336 | 336 | 5.495776 | 335 | 326 | 5.935823 |
| 10 | 338 | 321 | 5.746111 | 339 | 289 | 5.791805 | 340 | 334 | 5.461968 | 342 | 322 | 5.712617 |
| 11 | 329 | 311 | 5.30985 | 325 | 291 | 5.378515 | 326 | 331 | 5.38522 | 326 | 322 | 5.022547 |
| 12 | 326 | 306 | 5.605249 | 326 | 286 | 5.335943 | 326 | 330 | 5.694845 | 328 | 312 | 5.73342 |
| 13 | 327 | 319 | 5.78423 | 330 | 277 | 5.504874 | 327 | 323 | 5.801019 | 326 | 319 | 5.977874 |
| 14 | 318 | 315 | 5.815519 | 318 | 285 | 5.692687 | 321 | 331 | 5.789836 | 320 | 306 | 5.685769 |
| 15 | 321 | 309 | 5.722641 | 319 | 287 | 5.687865 | 323 | 327 | 5.677821 | 323 | 309 | 5.639949 |
| 16 | 328 | 310 | 5.63169 | 327 | 298 | 5.834244 | 330 | 330 | 5.776555 | 329 | 311 | 5.761278 |
| 17 | 322 | 312 | 6.031184 | 318 | 295 | 5.77597 | 321 | 324 | 5.735554 | 320 | 306 | 6.067306 |
| 18 | 326 | 301 | 5.771881 | 324 | 301 | 5.815013 | 324 | 336 | 5.684551 | 324 | 299 | 5.54885 |
| 19 | 334 | 317 | 5.275834 | 335 | 298 | 5.512042 | 335 | 333 | 5.422885 | 337 | 308 | 5.327912 |
| 20 | 341 | 324 | 5.544221 | 341 | 297 | 5.500242 | 341 | 331 | 5.412023 | 339 | 326 | 5.779453 |
| 21 | 341 | 323 | 5.703944 | 340 | 295 | 5.4482 | 341 | 348 | 5.317439 | 341 | 323 | 5.370926 |
| 22 | 346 | 328 | 4.830207 | 345 | 304 | 5.355718 | 344 | 348 | 5.250157 | 344 | 326 | 5.043342 |
| 23 | 344 | 315 | 5.161454 | 343 | 303 | 5.217438 | 345 | 353 | 5.108241 | 344 | 313 | 5.336229 |
| 24 | 342 | 312 | 6.091298 | 342 | 309 | 5.831726 | 341 | 347 | 5.827243 | 341 | 318 | 5.639673 |
| 25 | 338 | 319 | 5.707492 | 342 | 313 | 6.129184 | 339 | 341 | 6.161327 | 338 | 321 | 6.140456 |
| 26 | 337 | 313 | 6.224623 | 334 | 302 | 5.892701 | 335 | 335 | 6.017909 | 335 | 317 | 5.906041 |

Continued on next page

Table C.1 – continued from previous page

| | M=1 | | | M=3 | | | M=7 | | | M=31 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 27 | 339 | 315 | 6.170667 | 339 | 304 | 6.015858 | 339 | 335 | 6.017704 | 339 | 302 | 6.022508 |
| 28 | 339 | 303 | 5.103001 | 338 | 291 | 5.949895 | 341 | 334 | 5.784226 | 339 | 318 | 5.489885 |
| 29 | 328 | 292 | 6.362218 | 324 | 297 | 6.219774 | 326 | 335 | 5.901364 | 326 | 302 | 6.126302 |
| 30 | 327 | 298 | 5.706603 | 326 | 301 | 6.227972 | 328 | 339 | 5.941208 | 328 | 312 | 5.938808 |
| 31 | 333 | 305 | 6.111499 | 332 | 292 | 6.353271 | 333 | 339 | 6.193472 | 333 | 291 | 5.940058 |
| 32 | 326 | 300 | 6.537416 | 323 | 286 | 6.567625 | 325 | 337 | 6.511085 | 325 | 306 | 6.643238 |
| 33 | 333 | 307 | 6.555441 | 331 | 297 | 6.661854 | 332 | 348 | 6.516519 | 333 | 311 | 6.372096 |
| 34 | 327 | 301 | 6.937334 | 328 | 292 | 6.803665 | 326 | 342 | 6.714148 | 326 | 296 | 7.015119 |
| 35 | 333 | 302 | 6.459115 | 336 | 295 | 6.322111 | 332 | 346 | 6.451405 | 333 | 318 | 6.280971 |
| 36 | 318 | 308 | 6.276121 | 318 | 276 | 6.618987 | 318 | 331 | 6.496874 | 318 | 304 | 6.585039 |
| 37 | 318 | 310 | 6.751076 | 320 | 281 | 6.521073 | 320 | 334 | 6.469574 | 320 | 300 | 6.510232 |
| 38 | 318 | 302 | 6.141768 | 318 | 289 | 6.185853 | 317 | 336 | 6.387153 | 318 | 301 | 6.26698 |
| 39 | 319 | 313 | 6.188294 | 319 | 286 | 6.230473 | 321 | 333 | 6.225675 | 321 | 305 | 6.070001 |
| 40 | 319 | 303 | 5.989817 | 317 | 291 | 6.296082 | 321 | 341 | 6.04504 | 321 | 311 | 6.140515 |
| 41 | 332 | 309 | 5.632107 | 329 | 297 | 5.494018 | 330 | 340 | 5.701738 | 328 | 301 | 5.533118 |
| 42 | 332 | 315 | 5.840945 | 333 | 290 | 5.730939 | 332 | 344 | 5.741328 | 333 | 304 | 5.883234 |
| 43 | 326 | 311 | 6.161217 | 326 | 304 | 6.364234 | 327 | 336 | 6.209973 | 327 | 315 | 6.019776 |
| 44 | 327 | 315 | 6.561519 | 327 | 287 | 6.329941 | 328 | 339 | 6.488057 | 328 | 325 | 6.658954 |
| 45 | 338 | 325 | 6.189993 | 333 | 293 | 6.017318 | 334 | 345 | 6.08617 | 335 | 315 | 6.030515 |
| 46 | 330 | 308 | 6.222469 | 329 | 285 | 6.5673 | 329 | 340 | 6.288386 | 329 | 314 | 6.310051 |
| 47 | 324 | 311 | 6.258744 | 325 | 281 | 5.981362 | 324 | 340 | 6.225401 | 326 | 310 | 6.336682 |
| 48 | 335 | 303 | 6.054213 | 333 | 290 | 6.413673 | 332 | 336 | 6.230066 | 332 | 308 | 5.887166 |
| 49 | 328 | 317 | 6.468728 | 329 | 305 | 6.311359 | 328 | 344 | 6.328511 | 329 | 306 | 6.67175 |
| 50 | 342 | 323 | 6.312329 | 342 | 293 | 6.489375 | 341 | 332 | 6.464983 | 341 | 325 | 6.017247 |
| 51 | 337 | 310 | 6.541186 | 334 | 301 | 6.538311 | 337 | 341 | 6.708495 | 336 | 317 | 6.694329 |
| 52 | 341 | 311 | 6.453473 | 340 | 298 | 6.629027 | 339 | 344 | 6.425622 | 338 | 310 | 6.333967 |
| 53 | 336 | 316 | 6.483354 | 340 | 307 | 6.264384 | 337 | 332 | 6.522123 | 342 | 320 | 6.493219 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 54 | 336 | 314 | 6.717949 | 336 | 291 | 7.023082 | 337 | 340 | 6.880381 | 336 | 313 | 6.762181 |
| 55 | 338 | 320 | 7.199963 | 338 | 297 | 6.974613 | 338 | 333 | 7.07469 | 339 | 320 | 7.198957 |
| 56 | 332 | 317 | 6.798599 | 332 | 296 | 7.029443 | 333 | 324 | 6.97403 | 334 | 312 | 6.933838 |
| 57 | 315 | 307 | 6.596829 | 316 | 289 | 6.717189 | 315 | 326 | 6.638735 | 318 | 307 | 6.510331 |
| 58 | 316 | 315 | 6.176594 | 313 | 284 | 5.918643 | 316 | 329 | 6.080256 | 316 | 304 | 6.236733 |
| 59 | 324 | 294 | 5.222866 | 322 | 300 | 5.584184 | 323 | 325 | 5.483364 | 321 | 299 | 5.300783 |
| 60 | 317 | 304 | 6.112084 | 316 | 289 | 6.343446 | 316 | 315 | 6.133571 | 316 | 302 | 5.993932 |
| 61 | 320 | 319 | 6.631073 | 320 | 296 | 6.212414 | 322 | 315 | 6.52313 | 321 | 297 | 6.854787 |
| 62 | 326 | 300 | 6.754444 | 326 | 282 | 6.776614 | 325 | 312 | 6.879176 | 326 | 299 | 6.598569 |
| 63 | 324 | 308 | 6.802205 | 323 | 295 | 7.389211 | 323 | 324 | 7.010441 | 323 | 303 | 6.875869 |
| 64 | 326 | 308 | 7.404563 | 326 | 279 | 6.876852 | 323 | 316 | 6.952508 | 327 | 311 | 7.453461 |
| 65 | 321 | 303 | 6.842211 | 322 | 289 | 6.928522 | 323 | 327 | 6.977556 | 323 | 300 | 6.799538 |
| 66 | 325 | 306 | 7.471546 | 323 | 294 | 7.385897 | 323 | 315 | 7.179541 | 326 | 294 | 7.260122 |
| 67 | 321 | 302 | 6.966624 | 319 | 283 | 7.463695 | 321 | 320 | 7.181748 | 319 | 304 | 7.413902 |
| 68 | 330 | 294 | 7.547374 | 331 | 288 | 7.318228 | 330 | 323 | 7.133633 | 329 | 308 | 6.938461 |
| 69 | 323 | 307 | 6.856448 | 323 | 283 | 7.376558 | 325 | 319 | 7.299159 | 325 | 298 | 7.582909 |
| 70 | 320 | 300 | 7.374808 | 322 | 282 | 7.335994 | 322 | 312 | 7.309361 | 321 | 304 | 6.965276 |
| 71 | 320 | 295 | 7.857424 | 320 | 296 | 7.516859 | 318 | 327 | 7.473009 | 319 | 288 | 7.456824 |
| 72 | 323 | 289 | 6.684557 | 320 | 287 | 7.510625 | 321 | 328 | 7.512711 | 321 | 292 | 7.986383 |
| 73 | 309 | 295 | 7.918639 | 310 | 276 | 7.238478 | 310 | 320 | 7.210907 | 310 | 305 | 6.685599 |
| 74 | 320 | 290 | 6.88957 | 319 | 275 | 7.16784 | 321 | 317 | 7.0454 | 321 | 292 | 7.401319 |
| 75 | 322 | 317 | 6.407121 | 323 | 298 | 6.609538 | 323 | 323 | 6.492461 | 325 | 307 | 6.897499 |
| 76 | 320 | 315 | 6.134869 | 320 | 300 | 6.197338 | 321 | 321 | 5.906015 | 321 | 306 | 4.978106 |
| 77 | 324 | 298 | 5.410993 | 323 | 296 | 6.022819 | 324 | 328 | 5.771824 | 324 | 297 | 6.26826 |
| 78 | 322 | 303 | 6.773489 | 322 | 290 | 6.612374 | 322 | 325 | 6.442121 | 321 | 306 | 6.532487 |
| 79 | 326 | 296 | 6.696747 | 326 | 282 | 7.328126 | 326 | 327 | 6.772599 | 326 | 300 | 6.460542 |
| 80 | 324 | 320 | 7.050659 | 327 | 286 | 7.115852 | 326 | 325 | 6.872828 | 327 | 322 | 7.234902 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 81 | 333 | 312 | 6.76757 | 333 | 289 | 6.477145 | 331 | 318 | 6.682248 | 333 | 310 | 6.522288 |
| 82 | 337 | 312 | 6.385443 | 337 | 282 | 6.359757 | 337 | 324 | 6.556059 | 337 | 305 | 6.841787 |
| 83 | 340 | 310 | 6.549114 | 341 | 287 | 6.511381 | 340 | 330 | 6.519969 | 340 | 311 | 6.231624 |
| 84 | 339 | 309 | 6.584324 | 338 | 278 | 6.437252 | 338 | 324 | 6.369797 | 339 | 314 | 6.54059 |
| 85 | 331 | 308 | 6.212479 | 331 | 280 | 6.414996 | 329 | 325 | 6.267794 | 329 | 312 | 6.543796 |
| 86 | 334 | 324 | 6.387042 | 334 | 290 | 6.38645 | 334 | 324 | 6.331221 | 335 | 316 | 6.029443 |
| 87 | 329 | 313 | 6.649437 | 329 | 292 | 6.42249 | 330 | 325 | 6.347584 | 328 | 315 | 6.690839 |
| 88 | 335 | 317 | 6.572614 | 334 | 292 | 6.849265 | 332 | 323 | 6.670838 | 334 | 313 | 7.108021 |
| 89 | 331 | 328 | 6.880404 | 330 | 288 | 6.728667 | 330 | 329 | 6.675292 | 328 | 312 | 6.450812 |
| 90 | 331 | 309 | 6.933756 | 332 | 288 | 6.380556 | 332 | 334 | 6.505409 | 332 | 322 | 6.728597 |
| 91 | 331 | 306 | 5.912825 | 332 | 293 | 6.240469 | 333 | 333 | 6.282697 | 332 | 310 | 6.294056 |
| 92 | 334 | 309 | 5.919211 | 335 | 292 | 6.34544 | 333 | 334 | 6.055683 | 333 | 315 | 6.168318 |
| 93 | 331 | 320 | 5.938676 | 330 | 298 | 5.777984 | 331 | 333 | 5.66958 | 331 | 317 | 5.506788 |
| 94 | 334 | 325 | 5.327097 | 335 | 290 | 5.201241 | 334 | 331 | 5.159672 | 333 | 304 | 4.930332 |
| 95 | 333 | 307 | 4.85787 | 334 | 290 | 5.165576 | 335 | 340 | 5.120243 | 333 | 311 | 5.609588 |
| 96 | 340 | 311 | 5.77086 | 342 | 296 | 5.753203 | 341 | 333 | 5.742164 | 340 | 312 | 5.653547 |
| 97 | 341 | 318 | 6.762118 | 341 | 277 | 6.289497 | 342 | 330 | 6.246776 | 341 | 305 | 6.389706 |
| 98 | 332 | 304 | 6.551348 | 332 | 284 | 6.733985 | 331 | 331 | 6.627359 | 330 | 317 | 6.997992 |
| 99 | 331 | 315 | 6.631231 | 329 | 279 | 6.73661 | 330 | 331 | 6.703362 | 330 | 309 | 6.264938 |
| 100 | 350 | 318 | 6.734267 | 347 | 295 | 6.87274 | 349 | 332 | 6.908846 | 350 | 313 | 6.772617 |
| 101 | 350 | 321 | 6.527918 | 349 | 294 | 6.941235 | 349 | 345 | 6.889372 | 352 | 334 | 6.981906 |
| 102 | 343 | 314 | 7.465331 | 341 | 286 | 6.913577 | 341 | 328 | 6.850163 | 343 | 314 | 6.863372 |
| 103 | 356 | 314 | 6.802604 | 349 | 282 | 6.802042 | 351 | 340 | 6.791999 | 353 | 299 | 6.830646 |
| 104 | 345 | 315 | 6.274239 | 346 | 283 | 6.598864 | 348 | 335 | 6.603939 | 346 | 317 | 6.732695 |
| 105 | 333 | 312 | 7.170764 | 333 | 282 | 6.652723 | 335 | 327 | 6.821213 | 332 | 307 | 6.830095 |
| 106 | 337 | 313 | 6.815991 | 338 | 288 | 6.930123 | 337 | 319 | 6.907777 | 337 | 318 | 6.717155 |
| 107 | 338 | 311 | 6.475183 | 340 | 286 | 6.876177 | 337 | 329 | 6.758514 | 338 | 305 | 6.846474 |

Continued on next page

Table C.1 – continued from previous page

| | M=1 | | | M=3 | | | M=7 | | | M=31 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 108 | 336 | 323 | 6.935441 | 334 | 304 | 6.408063 | 334 | 337 | 6.768255 | 334 | 319 | 6.805025 |
| 109 | 346 | 313 | 6.572765 | 348 | 303 | 6.044847 | 346 | 344 | 6.432431 | 346 | 331 | 6.496166 |
| 110 | 351 | 344 | 6.364833 | 350 | 299 | 6.688095 | 352 | 338 | 6.493334 | 351 | 333 | 6.34626 |
| 111 | 341 | 318 | 5.142571 | 340 | 311 | 5.899239 | 341 | 340 | 5.556434 | 341 | 321 | 5.675894 |
| 112 | 339 | 313 | 6.15517 | 339 | 307 | 5.538281 | 339 | 332 | 5.488782 | 338 | 323 | 5.661454 |
| 113 | 340 | 321 | 4.758269 | 341 | 295 | 5.174448 | 341 | 344 | 5.099504 | 340 | 313 | 4.856519 |
| 114 | 334 | 305 | 5.591695 | 332 | 311 | 5.783934 | 333 | 343 | 5.674103 | 333 | 315 | 5.856868 |
| 115 | 329 | 316 | 6.428049 | 332 | 314 | 5.910137 | 332 | 347 | 6.085214 | 332 | 305 | 6.196528 |
| 116 | 335 | 303 | 6.074172 | 336 | 311 | 6.283602 | 337 | 339 | 6.290823 | 337 | 304 | 6.159314 |
| 117 | 342 | 310 | 6.725605 | 343 | 299 | 7.039428 | 342 | 331 | 6.722929 | 341 | 306 | 6.656011 |
| 118 | 340 | 319 | 6.745121 | 341 | 298 | 6.838051 | 341 | 330 | 6.674558 | 341 | 315 | 6.90239 |
| 119 | 334 | 317 | 6.801713 | 334 | 303 | 6.439316 | 334 | 343 | 6.673591 | 336 | 325 | 6.431853 |
| 120 | 323 | 313 | 6.749809 | 324 | 309 | 6.844947 | 324 | 336 | 6.778082 | 325 | 306 | 6.87886 |
| 121 | 324 | 307 | 6.736227 | 325 | 291 | 6.65687 | 325 | 330 | 6.772313 | 325 | 310 | 6.996511 |
| 122 | 318 | 293 | 6.953211 | 318 | 280 | 6.458737 | 318 | 329 | 6.649062 | 318 | 288 | 6.681589 |
| 123 | 317 | 305 | 6.196848 | 317 | 291 | 6.813434 | 318 | 332 | 6.64727 | 317 | 298 | 6.454908 |
| 124 | 329 | 305 | 6.810705 | 329 | 298 | 6.441654 | 330 | 325 | 6.551052 | 329 | 311 | 6.6524 |
| 125 | 333 | 302 | 6.239453 | 333 | 285 | 6.345024 | 333 | 336 | 6.675884 | 333 | 295 | 6.347815 |
| 126 | 337 | 315 | 6.533232 | 337 | 287 | 6.431195 | 338 | 339 | 6.465959 | 337 | 309 | 6.764392 |
| 127 | 336 | 319 | 6.440904 | 336 | 296 | 6.191322 | 334 | 338 | 6.308456 | 333 | 321 | 6.085059 |
| 128 | 331 | 309 | 5.758487 | 331 | 285 | 6.137233 | 329 | 338 | 6.056305 | 330 | 317 | 6.263279 |
| 129 | 332 | 339 | 5.952453 | 334 | 281 | 5.778679 | 333 | 338 | 5.631804 | 333 | 333 | 5.43515 |
| 130 | 340 | 303 | 5.33248 | 339 | 307 | 5.210326 | 341 | 341 | 5.517425 | 340 | 299 | 5.609414 |
| 131 | 322 | 306 | 5.221823 | 325 | 288 | 5.278642 | 323 | 340 | 4.969809 | 322 | 305 | 5.173144 |
| 132 | 327 | 298 | 4.259186 | 326 | 291 | 4.343989 | 327 | 330 | 4.48315 | 329 | 304 | 4.452855 |
| 133 | 328 | 295 | 4.241534 | 328 | 291 | 4.276457 | 326 | 327 | 4.247374 | 325 | 295 | 4.036832 |
| 134 | 334 | 300 | 4.842187 | 335 | 290 | 4.721147 | 338 | 326 | 4.856507 | 337 | 310 | 4.882411 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 135 | 325 | 302 | 5.142173 | 325 | 273 | 5.158046 | 324 | 324 | 5.102253 | 326 | 301 | 5.43462 |
| 136 | 330 | 297 | 5.265008 | 327 | 289 | 5.325918 | 327 | 333 | 5.379183 | 330 | 296 | 4.914284 |
| 137 | 324 | 296 | 5.45555 | 323 | 296 | 5.649151 | 322 | 337 | 5.716238 | 324 | 292 | 5.950291 |
| 138 | 336 | 315 | 6.374993 | 337 | 292 | 6.148139 | 338 | 345 | 5.930472 | 339 | 317 | 5.9957 |
| 139 | 337 | 312 | 5.504892 | 337 | 283 | 5.508346 | 337 | 341 | 5.775836 | 339 | 309 | 5.422354 |
| 140 | 337 | 306 | 5.790167 | 337 | 290 | 5.746853 | 335 | 321 | 5.540463 | 336 | 326 | 5.960586 |
| 141 | 337 | 316 | 5.420042 | 338 | 304 | 5.499638 | 339 | 333 | 5.561277 | 337 | 327 | 5.327793 |
| 142 | 342 | 336 | 5.892216 | 343 | 297 | 5.630853 | 343 | 342 | 5.648808 | 344 | 318 | 5.903251 |
| 143 | 350 | 316 | 5.471293 | 348 | 294 | 5.824672 | 348 | 326 | 5.778328 | 350 | 311 | 5.742924 |
| 144 | 343 | 305 | 6.195617 | 342 | 294 | 5.949327 | 341 | 333 | 5.816794 | 340 | 306 | 5.890864 |
| 145 | 329 | 298 | 5.832421 | 328 | 283 | 6.231958 | 328 | 327 | 6.110657 | 328 | 308 | 6.1139 |
| 146 | 331 | 299 | 6.460228 | 331 | 281 | 6.063343 | 332 | 324 | 6.095293 | 331 | 287 | 6.420562 |
| 147 | 323 | 296 | 5.318744 | 323 | 285 | 5.765141 | 322 | 311 | 5.628378 | 324 | 286 | 5.190489 |
| 148 | 313 | 291 | 5.268096 | 312 | 288 | 5.130175 | 312 | 328 | 5.021112 | 312 | 308 | 5.44091 |
| 149 | 316 | 295 | 4.22439 | 316 | 272 | 4.208123 | 315 | 315 | 4.339172 | 318 | 277 | 4.184515 |
| 150 | 314 | 283 | 4.111993 | 314 | 282 | 4.130875 | 315 | 321 | 4.166184 | 314 | 279 | 4.02178 |
| 151 | 310 | 248 | 3.948413 | 307 | 268 | 3.920461 | 309 | 298 | 3.781142 | 312 | 270 | 3.978168 |
| 152 | 298 | 232 | 3.532843 | 297 | 250 | 3.484113 | 298 | 283 | 3.637349 | 296 | 234 | 3.752627 |
| 153 | 314 | 240 | 4.429736 | 313 | 256 | 4.269278 | 313 | 294 | 4.287474 | 314 | 253 | 3.924855 |
| 154 | 291 | 231 | 4.925149 | 293 | 239 | 5.214576 | 293 | 289 | 5.140438 | 291 | 238 | 5.493446 |
| 155 | 295 | 256 | 5.469284 | 292 | 246 | 5.095385 | 298 | 281 | 5.154478 | 295 | 246 | 5.261119 |
| 156 | 299 | 248 | 5.190083 | 297 | 255 | 5.303836 | 295 | 288 | 5.308401 | 294 | 246 | 5.124376 |
| 157 | 302 | 269 | 5.113122 | 304 | 258 | 5.06374 | 303 | 299 | 5.074118 | 305 | 269 | 5.315603 |
| 158 | 308 | 286 | 5.435503 | 303 | 264 | 5.033022 | 302 | 316 | 5.193353 | 304 | 275 | 5.191514 |
| 159 | 321 | 302 | 5.031383 | 319 | 282 | 5.042533 | 320 | 308 | 5.158104 | 321 | 297 | 4.83148 |
| 160 | 331 | 294 | 5.166977 | 331 | 285 | 5.005022 | 332 | 319 | 5.079386 | 331 | 299 | 5.559179 |
| 161 | 333 | 306 | 5.254536 | 333 | 262 | 5.12185 | 334 | 304 | 5.16399 | 333 | 294 | 5.070004 |

Continued on next page

Table C.1 – continued from previous page

| Frame Number | M=1 NumOsc NonMod | M=1 NumOsc Mod | M=1 Motion Energy | M=3 NumOsc NonMod | M=3 NumOsc Mod | M=3 Motion Energy | M=7 NumOsc NonMod | M=7 NumOsc Mod | M=7 Motion Energy | M=31 NumOsc NonMod | M=31 NumOsc Mod | M=31 Motion Energy |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 162 | 328 | 277 | 5.046888 | 329 | 257 | 5.108638 | 329 | 310 | 5.098275 | 324 | 288 | 4.770069 |
| 163 | 313 | 276 | 4.828773 | 316 | 259 | 4.974989 | 314 | 305 | 5.016085 | 314 | 279 | 5.259257 |
| 164 | 316 | 265 | 4.929673 | 313 | 270 | 4.791208 | 315 | 323 | 4.758145 | 314 | 268 | 4.57999 |
| 165 | 326 | 283 | 5.105408 | 326 | 275 | 4.820357 | 328 | 320 | 5.070206 | 324 | 285 | 5.155288 |
| 166 | 315 | 273 | 4.334141 | 319 | 254 | 4.476918 | 313 | 306 | 4.581343 | 315 | 268 | 4.420523 |
| 167 | 309 | 246 | 4.836333 | 310 | 241 | 4.667671 | 310 | 300 | 4.663852 | 309 | 242 | 4.922229 |
| 168 | 322 | 271 | 5.332803 | 322 | 268 | 5.352878 | 320 | 321 | 5.216751 | 320 | 265 | 5.12439 |
| 169 | 315 | 260 | 5.05442 | 315 | 254 | 5.420629 | 316 | 299 | 5.197201 | 317 | 254 | 5.011653 |
| 170 | 326 | 255 | 4.835966 | 328 | 264 | 4.824443 | 327 | 299 | 4.789555 | 327 | 256 | 4.830345 |
| 171 | 327 | 282 | 4.408252 | 326 | 271 | 4.650403 | 324 | 322 | 4.612917 | 326 | 285 | 4.754521 |
| 172 | 314 | 286 | 4.648205 | 314 | 264 | 4.482787 | 315 | 328 | 4.52074 | 314 | 261 | 4.625915 |
| 173 | 326 | 277 | 5.260865 | 325 | 279 | 4.999956 | 326 | 316 | 4.889428 | 326 | 286 | 4.972953 |
| 174 | 324 | 274 | 4.73302 | 325 | 277 | 5.106521 | 324 | 320 | 5.258356 | 325 | 277 | 4.833403 |
| 175 | 313 | 263 | 5.505739 | 315 | 272 | 5.108348 | 317 | 312 | 5.479999 | 315 | 263 | 5.577079 |
| 176 | 326 | 283 | 5.31397 | 327 | 263 | 5.352004 | 326 | 311 | 5.345363 | 322 | 275 | 5.613307 |
| 177 | 309 | 276 | 5.608274 | 308 | 271 | 5.781486 | 308 | 319 | 5.422777 | 306 | 267 | 5.300484 |
| 178 | 308 | 281 | 5.378237 | 311 | 290 | 5.863873 | 309 | 313 | 5.563169 | 310 | 278 | 5.415207 |
| 179 | 313 | 281 | 5.485223 | 312 | 283 | 5.684254 | 312 | 317 | 5.548255 | 313 | 277 | 5.49455 |
| 180 | 327 | 276 | 5.192936 | 328 | 286 | 4.989671 | 326 | 323 | 5.168195 | 328 | 287 | 5.436682 |
| 181 | 326 | 295 | 5.093629 | 330 | 290 | 4.899884 | 329 | 330 | 5.100015 | 328 | 288 | 4.856305 |
| 182 | 323 | 276 | 4.958825 | 322 | 286 | 4.968594 | 323 | 326 | 4.999464 | 323 | 278 | 4.801927 |
| 183 | 327 | 288 | 4.630427 | 325 | 291 | 5.57289 | 325 | 323 | 5.50797 | 328 | 293 | 5.681092 |
| 184 | 331 | 296 | 5.779837 | 331 | 276 | 5.401184 | 330 | 329 | 5.484851 | 333 | 304 | 5.829676 |
| 185 | 335 | 265 | 5.481986 | 336 | 277 | 5.181238 | 335 | 316 | 5.161354 | 337 | 281 | 5.397087 |
| 186 | 338 | 285 | 4.688134 | 338 | 275 | 4.842246 | 341 | 315 | 4.850719 | 340 | 275 | 4.65236 |
| 187 | 330 | 259 | 4.356611 | 325 | 274 | 4.141348 | 326 | 325 | 4.22478 | 324 | 274 | 4.241535 |
| 188 | 321 | 257 | 3.597822 | 321 | 289 | 3.553773 | 322 | 318 | 3.648048 | 322 | 257 | 3.929481 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 189 | 329 | 270 | 4.497218 | 330 | 295 | 4.256435 | 331 | 355 | 4.254912 | 329 | 269 | 4.258122 |
| 190 | 314 | 249 | 4.092007 | 313 | 282 | 4.382453 | 312 | 331 | 4.120017 | 314 | 256 | 4.032359 |
| 191 | 310 | 253 | 4.064821 | 309 | 285 | 4.345281 | 311 | 323 | 4.318278 | 311 | 246 | 4.244136 |
| 192 | 308 | 262 | 4.469001 | 310 | 265 | 4.386061 | 311 | 313 | 4.186706 | 314 | 272 | 4.292137 |
| 193 | 308 | 262 | 4.52904 | 309 | 268 | 4.542091 | 310 | 318 | 4.425914 | 309 | 255 | 4.593991 |
| 194 | 309 | 265 | 4.138137 | 312 | 270 | 4.149888 | 310 | 311 | 4.203228 | 311 | 261 | 4.018836 |
| 195 | 301 | 249 | 4.21756 | 301 | 261 | 4.274756 | 301 | 301 | 4.22035 | 300 | 258 | 4.087175 |
| 196 | 292 | 242 | 4.082836 | 291 | 269 | 4.202095 | 292 | 303 | 4.147921 | 291 | 239 | 4.319074 |
| 197 | 292 | 238 | 4.227913 | 292 | 260 | 4.283586 | 292 | 310 | 4.080045 | 292 | 253 | 4.149459 |
| 198 | 297 | 265 | 4.138328 | 300 | 274 | 3.92673 | 300 | 300 | 3.850539 | 297 | 252 | 3.949421 |
| 199 | 324 | 285 | 4.07744 | 325 | 288 | 4.131832 | 324 | 322 | 3.956998 | 323 | 286 | 4.00169 |
| 200 | 345 | 288 | 4.154093 | 343 | 286 | 4.251751 | 345 | 329 | 4.206428 | 344 | 294 | 4.1175 |
| 201 | 340 | 279 | 4.102238 | 342 | 271 | 3.834109 | 340 | 320 | 3.868439 | 340 | 281 | 4.082218 |
| 202 | 321 | 281 | 3.946834 | 321 | 286 | 4.044675 | 320 | 330 | 3.765729 | 322 | 267 | 3.714081 |
| 203 | 312 | 267 | 3.879888 | 312 | 290 | 4.066147 | 309 | 319 | 3.974011 | 310 | 268 | 4.010549 |
| 204 | 319 | 261 | 3.904456 | 315 | 291 | 4.007158 | 319 | 331 | 3.932771 | 321 | 261 | 3.888661 |
| 205 | 321 | 258 | 3.418126 | 322 | 282 | 3.399383 | 319 | 331 | 3.51279 | 321 | 257 | 3.506597 |
| 206 | 332 | 273 | 3.49596 | 327 | 291 | 3.07628 | 327 | 318 | 3.340685 | 330 | 259 | 3.097479 |
| 207 | 328 | 264 | 3.18127 | 330 | 283 | 3.119572 | 331 | 304 | 3.294256 | 330 | 271 | 3.342562 |
| 208 | 322 | 261 | 3.645706 | 324 | 276 | 3.596266 | 322 | 307 | 3.737331 | 323 | 265 | 3.837679 |
| 209 | 331 | 247 | 3.834753 | 329 | 275 | 3.85856 | 330 | 314 | 3.875702 | 330 | 261 | 3.920003 |
| 210 | 306 | 260 | 4.394093 | 304 | 256 | 4.258587 | 304 | 332 | 4.165758 | 305 | 250 | 4.074644 |
| 211 | 303 | 236 | 3.846302 | 307 | 260 | 3.9019 | 305 | 307 | 3.866944 | 306 | 238 | 3.984148 |
| 212 | 309 | 268 | 4.235086 | 309 | 280 | 4.506273 | 308 | 331 | 4.380863 | 311 | 265 | 4.2589 |
| 213 | 305 | 263 | 4.512667 | 306 | 275 | 4.586621 | 306 | 328 | 4.399529 | 306 | 268 | 4.516398 |
| 214 | 319 | 280 | 4.135764 | 314 | 278 | 4.600769 | 317 | 322 | 4.362433 | 318 | 281 | 4.232801 |
| 215 | 315 | 255 | 4.350554 | 314 | 276 | 4.234346 | 312 | 319 | 4.238858 | 312 | 247 | 4.408773 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 216 | 312 | 264 | 4.034217 | 312 | 291 | 4.043727 | 312 | 335 | 4.010021 | 312 | 258 | 3.859422 |
| 217 | 315 | 258 | 4.348998 | 314 | 291 | 4.347303 | 312 | 322 | 4.354177 | 312 | 260 | 4.367375 |
| 218 | 299 | 257 | 4.311308 | 300 | 284 | 4.487555 | 300 | 338 | 4.30548 | 302 | 249 | 4.379447 |
| 219 | 300 | 263 | 4.502608 | 302 | 282 | 4.685336 | 301 | 339 | 4.507301 | 299 | 261 | 4.32867 |
| 220 | 308 | 235 | 4.270501 | 309 | 277 | 4.463253 | 307 | 337 | 4.494502 | 308 | 248 | 4.456932 |
| 221 | 317 | 247 | 4.611562 | 315 | 296 | 4.573413 | 315 | 330 | 4.505421 | 318 | 249 | 4.490409 |
| 222 | 306 | 274 | 4.066278 | 305 | 292 | 4.266014 | 306 | 322 | 4.213788 | 304 | 264 | 4.388164 |
| 223 | 313 | 273 | 4.261676 | 313 | 284 | 4.188249 | 315 | 327 | 4.284428 | 313 | 263 | 4.086415 |
| 224 | 326 | 278 | 3.953897 | 327 | 274 | 3.74508 | 325 | 320 | 3.8004 | 325 | 271 | 4.034703 |
| 225 | 315 | 269 | 3.623618 | 316 | 286 | 4.143002 | 314 | 324 | 3.830195 | 313 | 261 | 3.565707 |
| 226 | 318 | 265 | 4.267389 | 317 | 314 | 4.237411 | 316 | 346 | 4.071228 | 319 | 258 | 3.975679 |
| 227 | 304 | 249 | 4.286864 | 304 | 284 | 4.307966 | 304 | 340 | 4.408606 | 304 | 256 | 4.857095 |
| 228 | 317 | 267 | 4.394437 | 321 | 277 | 3.978279 | 315 | 325 | 4.074968 | 315 | 267 | 3.887697 |
| 229 | 312 | 250 | 4.140285 | 313 | 282 | 4.197849 | 313 | 322 | 4.157605 | 315 | 260 | 4.421851 |
| 230 | 344 | 304 | 3.8673 | 343 | 282 | 4.332516 | 343 | 337 | 4.195259 | 344 | 296 | 4.209144 |
| 231 | 343 | 302 | 4.672529 | 344 | 302 | 4.354212 | 344 | 343 | 4.283794 | 344 | 296 | 4.247329 |
| 232 | 335 | 268 | 3.650712 | 334 | 283 | 3.851832 | 337 | 337 | 3.705354 | 337 | 276 | 3.958561 |
| 233 | 344 | 298 | 3.951707 | 343 | 294 | 4.150673 | 344 | 331 | 4.105524 | 342 | 274 | 3.797815 |
| 234 | 339 | 280 | 4.226242 | 343 | 279 | 3.742905 | 342 | 334 | 3.985456 | 340 | 290 | 4.00098 |
| 235 | 319 | 271 | 3.807963 | 320 | 290 | 3.799127 | 320 | 326 | 4.01456 | 319 | 281 | 4.356739 |
| 236 | 313 | 270 | 4.330187 | 315 | 293 | 4.146754 | 314 | 324 | 4.076502 | 312 | 265 | 4.135311 |
| 237 | 318 | 265 | 3.751934 | 316 | 295 | 3.611454 | 317 | 327 | 3.710966 | 317 | 272 | 3.531541 |
| 238 | 330 | 271 | 3.58384 | 330 | 275 | 3.892507 | 329 | 328 | 3.883479 | 329 | 276 | 3.994594 |
| 239 | 342 | 275 | 3.797336 | 341 | 277 | 3.942019 | 340 | 317 | 3.823246 | 339 | 263 | 3.679216 |
| 240 | 341 | 269 | 4.170394 | 337 | 270 | 3.834401 | 339 | 323 | 3.738939 | 344 | 272 | 4.121164 |
| 241 | 339 | 293 | 3.265423 | 339 | 287 | 3.570122 | 341 | 330 | 3.593393 | 342 | 282 | 3.596346 |
| 242 | 321 | 255 | 3.756448 | 322 | 289 | 3.578023 | 321 | 311 | 3.661871 | 320 | 258 | 3.497481 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 243 | 310 | 251 | 3.28753 | 310 | 290 | 3.11009 | 311 | 318 | 3.180752 | 310 | 249 | 3.387612 |
| 244 | 314 | 244 | 3.719488 | 314 | 267 | 3.773349 | 313 | 314 | 3.70735 | 313 | 264 | 3.652795 |
| 245 | 335 | 272 | 3.996006 | 335 | 290 | 4.074258 | 333 | 331 | 4.007902 | 332 | 264 | 4.087967 |
| 246 | 329 | 290 | 4.525749 | 332 | 287 | 4.345568 | 329 | 335 | 4.467035 | 329 | 272 | 4.357785 |
| 247 | 341 | 283 | 4.398992 | 340 | 302 | 3.964905 | 342 | 326 | 4.259137 | 341 | 289 | 4.316948 |
| 248 | 337 | 284 | 3.82703 | 336 | 300 | 3.757036 | 336 | 333 | 4.037831 | 337 | 290 | 4.147382 |
| 249 | 332 | 295 | 3.825323 | 331 | 291 | 3.459537 | 332 | 344 | 3.654997 | 332 | 276 | 3.379681 |
| 250 | 330 | 276 | 3.985748 | 332 | 286 | 3.784558 | 331 | 346 | 3.887942 | 330 | 298 | 4.093427 |
| 251 | 324 | 278 | 3.407877 | 324 | 287 | 3.57039 | 325 | 346 | 3.579844 | 325 | 269 | 3.565503 |
| 252 | 335 | 281 | 3.680829 | 334 | 282 | 3.41758 | 332 | 333 | 3.40377 | 333 | 267 | 3.277723 |
| 253 | 350 | 300 | 3.390157 | 348 | 282 | 3.382474 | 350 | 334 | 3.430246 | 350 | 293 | 3.53815 |
| 254 | 328 | 275 | 3.011732 | 330 | 274 | 3.256097 | 332 | 315 | 3.227938 | 331 | 288 | 3.196252 |
| 255 | 335 | 273 | 3.381843 | 331 | 279 | 3.245063 | 336 | 321 | 3.142909 | 336 | 269 | 3.100598 |
| 256 | 326 | 272 | 3.240423 | 326 | 278 | 3.368858 | 328 | 323 | 3.286086 | 328 | 280 | 3.185816 |
| 257 | 335 | 272 | 3.241346 | 336 | 291 | 3.539283 | 334 | 323 | 3.536764 | 334 | 276 | 3.539822 |
| 258 | 336 | 257 | 3.446233 | 332 | 281 | 3.450813 | 335 | 317 | 3.425267 | 335 | 255 | 3.405304 |
| 259 | 325 | 275 | 3.630232 | 326 | 285 | 3.425903 | 327 | 315 | 3.542804 | 327 | 278 | 3.463682 |
| 260 | 337 | 269 | 3.480762 | 338 | 289 | 3.481825 | 335 | 328 | 3.478561 | 335 | 265 | 3.474589 |
| 261 | 337 | 269 | 3.333891 | 338 | 282 | 3.306234 | 335 | 316 | 3.359311 | 333 | 270 | 3.535028 |
| 262 | 327 | 264 | 2.797893 | 328 | 283 | 2.796202 | 328 | 316 | 2.836281 | 328 | 258 | 2.857232 |
| 263 | 331 | 257 | 2.922588 | 333 | 285 | 2.83961 | 330 | 336 | 2.793099 | 331 | 268 | 2.714148 |
| 264 | 326 | 263 | 2.761703 | 326 | 273 | 2.744031 | 327 | 310 | 2.747415 | 327 | 262 | 2.67402 |
| 265 | 330 | 254 | 2.690459 | 330 | 271 | 2.769998 | 328 | 298 | 2.835379 | 332 | 245 | 3.013512 |
| 266 | 320 | 260 | 2.963751 | 322 | 270 | 2.868849 | 320 | 308 | 2.872743 | 321 | 257 | 2.990755 |
| 267 | 328 | 277 | 3.373465 | 328 | 274 | 3.255397 | 326 | 307 | 3.246111 | 328 | 275 | 3.164644 |
| 268 | 338 | 258 | 3.142615 | 337 | 278 | 3.027987 | 338 | 323 | 3.106945 | 338 | 260 | 3.155767 |
| 269 | 340 | 281 | 3.053028 | 339 | 276 | 2.959875 | 340 | 322 | 2.904475 | 340 | 271 | 3.230311 |

Continued on next page

Table C.1 – continued from previous page

| | M=1 | | | M=3 | | | M=7 | | | M=31 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 270 | 335 | 263 | 3.081157 | 335 | 256 | 3.060215 | 333 | 303 | 2.916131 | 334 | 253 | 3.015462 |
| 271 | 347 | 258 | 3.032709 | 348 | 282 | 2.994562 | 347 | 307 | 3.044475 | 345 | 256 | 2.911358 |
| 272 | 331 | 248 | 2.996343 | 333 | 269 | 3.136418 | 331 | 308 | 3.19899 | 330 | 258 | 2.99585 |
| 273 | 342 | 264 | 3.041545 | 344 | 258 | 3.204962 | 344 | 307 | 3.102764 | 344 | 262 | 3.212818 |
| 274 | 336 | 242 | 3.509176 | 334 | 256 | 3.423366 | 336 | 300 | 3.422215 | 337 | 241 | 3.597373 |
| 275 | 337 | 246 | 3.271304 | 339 | 251 | 3.0088 | 339 | 284 | 3.098041 | 341 | 237 | 3.163686 |
| 276 | 331 | 246 | 3.001622 | 330 | 261 | 2.992075 | 329 | 288 | 2.996579 | 331 | 241 | 3.01756 |
| 277 | 317 | 239 | 2.998831 | 317 | 245 | 3.062662 | 318 | 289 | 3.015928 | 318 | 244 | 3.157264 |
| 278 | 310 | 235 | 2.776756 | 308 | 256 | 2.926697 | 308 | 299 | 2.945114 | 307 | 238 | 2.964203 |
| 279 | 325 | 241 | 3.31358 | 326 | 268 | 3.130659 | 326 | 301 | 3.099892 | 326 | 252 | 3.161846 |
| 280 | 323 | 253 | 3.441581 | 324 | 288 | 3.325335 | 324 | 330 | 3.371764 | 325 | 241 | 3.176358 |
| 281 | 335 | 264 | 2.903587 | 333 | 268 | 2.88983 | 335 | 320 | 2.852626 | 335 | 263 | 3.012608 |
| 282 | 328 | 264 | 3.376793 | 326 | 275 | 3.620507 | 327 | 326 | 3.572179 | 325 | 265 | 3.759507 |
| 283 | 329 | 276 | 4.318442 | 329 | 283 | 4.212305 | 330 | 318 | 4.01305 | 327 | 275 | 4.121935 |
| 284 | 332 | 275 | 3.944638 | 331 | 269 | 3.619167 | 330 | 318 | 3.836075 | 329 | 279 | 3.700081 |
| 285 | 312 | 245 | 3.585806 | 313 | 273 | 3.666579 | 312 | 306 | 3.667403 | 312 | 248 | 3.775981 |
| 286 | 322 | 258 | 3.638996 | 321 | 265 | 3.854702 | 321 | 318 | 3.732194 | 320 | 258 | 3.786374 |
| 287 | 333 | 260 | 3.735677 | 332 | 259 | 3.535663 | 332 | 307 | 3.574624 | 330 | 253 | 3.771564 |
| 288 | 328 | 273 | 4.163103 | 327 | 253 | 3.971706 | 325 | 316 | 3.914566 | 326 | 277 | 4.005386 |
| 289 | 338 | 243 | 4.197486 | 339 | 264 | 3.948184 | 342 | 330 | 4.069516 | 343 | 258 | 4.070414 |
| 290 | 338 | 270 | 3.762431 | 341 | 285 | 3.956368 | 339 | 328 | 3.927622 | 340 | 272 | 3.885698 |
| 291 | 339 | 280 | 3.276954 | 339 | 278 | 3.517764 | 339 | 310 | 3.553304 | 337 | 268 | 3.592653 |
| 292 | 336 | 263 | 3.648398 | 339 | 279 | 3.47604 | 337 | 317 | 3.57244 | 337 | 254 | 3.723275 |
| 293 | 358 | 264 | 3.615583 | 356 | 280 | 3.430706 | 357 | 321 | 3.364275 | 357 | 252 | 3.444827 |
| 294 | 342 | 256 | 3.534714 | 344 | 280 | 3.456696 | 342 | 331 | 3.494486 | 342 | 259 | 3.562226 |
| 295 | 350 | 252 | 3.082599 | 349 | 283 | 3.386618 | 348 | 325 | 3.299371 | 349 | 270 | 3.112252 |
| 296 | 341 | 258 | 3.195109 | 339 | 270 | 3.159389 | 342 | 316 | 3.332552 | 340 | 255 | 3.125287 |

Continued on next page

Table C.1 – continued from previous page

| | M=1 | | | M=3 | | | M=7 | | | M=31 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 297 | 352 | 270 | 3.834661 | 353 | 262 | 4.039678 | 354 | 300 | 4.04461 | 354 | 270 | 3.801663 |
| 298 | 342 | 246 | 3.308203 | 342 | 267 | 3.863637 | 342 | 289 | 3.726982 | 340 | 243 | 3.409769 |
| 299 | 333 | 257 | 3.553043 | 335 | 253 | 3.872465 | 334 | 290 | 3.873031 | 335 | 247 | 3.64858 |
| 300 | 325 | 267 | 3.435242 | 327 | 263 | 3.547209 | 329 | 307 | 3.652518 | 323 | 256 | 3.58699 |
| 301 | 326 | 268 | 3.787433 | 323 | 276 | 3.574354 | 324 | 310 | 3.63573 | 325 | 270 | 3.699448 |
| 302 | 309 | 249 | 3.840889 | 310 | 263 | 3.664155 | 312 | 300 | 3.669771 | 312 | 252 | 3.674313 |
| 303 | 316 | 242 | 3.818706 | 316 | 248 | 3.726815 | 316 | 276 | 3.698989 | 318 | 243 | 3.701157 |
| 304 | 333 | 227 | 3.18465 | 333 | 237 | 3.263729 | 332 | 273 | 3.272463 | 332 | 251 | 3.151548 |
| 305 | 347 | 250 | 3.09357 | 346 | 245 | 3.268317 | 345 | 274 | 3.264672 | 347 | 252 | 3.072884 |
| 306 | 323 | 239 | 2.738477 | 326 | 247 | 2.907269 | 326 | 276 | 2.880701 | 327 | 245 | 2.804788 |
| 307 | 341 | 246 | 2.669677 | 340 | 254 | 2.666685 | 344 | 280 | 2.65149 | 342 | 249 | 2.628138 |
| 308 | 335 | 256 | 3.153453 | 334 | 283 | 3.180416 | 336 | 286 | 3.143493 | 335 | 276 | 2.990204 |
| 309 | 325 | 301 | 3.609164 | 325 | 274 | 3.554826 | 326 | 296 | 3.54455 | 326 | 292 | 3.589582 |
| 310 | 326 | 281 | 3.21287 | 326 | 276 | 3.310652 | 323 | 286 | 3.251093 | 324 | 268 | 3.298557 |
| 311 | 344 | 296 | 3.973817 | 344 | 296 | 3.78409 | 343 | 297 | 3.89959 | 344 | 298 | 4.11033 |
| 312 | 338 | 308 | 3.877177 | 338 | 295 | 3.803683 | 342 | 309 | 3.79167 | 341 | 313 | 3.96857 |
| 313 | 351 | 327 | 4.091675 | 350 | 296 | 3.993259 | 351 | 320 | 4.069521 | 351 | 329 | 4.024985 |
| 314 | 357 | 308 | 3.788456 | 356 | 304 | 3.753123 | 358 | 317 | 3.812762 | 358 | 314 | 3.658007 |
| 315 | 333 | 288 | 3.284966 | 334 | 286 | 3.453863 | 336 | 310 | 3.349175 | 336 | 295 | 3.449522 |
| 316 | 339 | 293 | 3.498392 | 338 | 276 | 3.407773 | 339 | 309 | 3.52889 | 340 | 299 | 3.833186 |
| 317 | 326 | 246 | 2.780905 | 326 | 252 | 2.572017 | 328 | 289 | 2.616234 | 328 | 243 | 2.767467 |
| 318 | 339 | 286 | 3.482151 | 340 | 277 | 3.026831 | 340 | 302 | 3.076774 | 340 | 282 | 3.119352 |
| 319 | 335 | 260 | 2.832534 | 332 | 261 | 2.775014 | 333 | 304 | 2.849486 | 334 | 264 | 2.682962 |
| 320 | 332 | 266 | 2.809581 | 331 | 266 | 2.906223 | 331 | 306 | 2.961026 | 329 | 257 | 2.905823 |
| 321 | 337 | 259 | 2.854037 | 339 | 276 | 2.985407 | 337 | 302 | 2.968391 | 338 | 249 | 2.865649 |
| 322 | 362 | 307 | 3.958526 | 361 | 297 | 3.912586 | 363 | 329 | 4.025742 | 363 | 295 | 4.186589 |
| 323 | 345 | 289 | 3.573297 | 344 | 293 | 3.724082 | 346 | 320 | 3.677348 | 344 | 285 | 3.760141 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 324 | 346 | 289 | 4.219956 | 347 | 298 | 4.378909 | 348 | 348 | 4.299652 | 349 | 287 | 4.191753 |
| 325 | 342 | 290 | 4.016005 | 342 | 290 | 3.81024 | 342 | 342 | 3.826742 | 342 | 302 | 3.888642 |
| 326 | 335 | 312 | 4.178634 | 333 | 303 | 4.005346 | 331 | 329 | 4.125888 | 333 | 297 | 4.160476 |
| 327 | 346 | 312 | 4.031387 | 345 | 310 | 3.945956 | 344 | 327 | 4.073518 | 345 | 310 | 3.95371 |
| 328 | 351 | 290 | 3.734554 | 353 | 296 | 3.745857 | 352 | 313 | 3.662525 | 350 | 312 | 3.592351 |
| 329 | 337 | 275 | 3.073984 | 334 | 292 | 3.231823 | 335 | 318 | 3.167439 | 336 | 286 | 3.291338 |
| 330 | 327 | 259 | 2.722895 | 326 | 281 | 2.868466 | 327 | 321 | 2.854476 | 326 | 260 | 3.005472 |
| 331 | 336 | 257 | 3.092296 | 334 | 288 | 2.898032 | 336 | 319 | 2.884202 | 336 | 270 | 2.914301 |
| 332 | 333 | 283 | 3.820795 | 331 | 287 | 3.372485 | 332 | 325 | 3.488607 | 332 | 297 | 3.572201 |
| 333 | 325 | 300 | 4.734275 | 325 | 297 | 4.476656 | 326 | 326 | 4.611828 | 325 | 301 | 4.798221 |
| 334 | 322 | 288 | 4.274116 | 324 | 311 | 4.234116 | 324 | 335 | 4.356761 | 324 | 286 | 4.470152 |
| 335 | 323 | 281 | 3.739468 | 321 | 292 | 3.726434 | 321 | 334 | 3.707391 | 321 | 270 | 3.693528 |
| 336 | 328 | 272 | 3.226828 | 327 | 291 | 3.261678 | 326 | 325 | 3.250015 | 327 | 262 | 3.085252 |
| 337 | 343 | 300 | 3.698373 | 342 | 288 | 3.788509 | 343 | 340 | 3.867461 | 342 | 297 | 3.844845 |
| 338 | 341 | 293 | 3.449035 | 342 | 293 | 3.561749 | 342 | 326 | 3.46688 | 343 | 295 | 3.515938 |
| 339 | 352 | 302 | 4.341482 | 353 | 311 | 4.142129 | 352 | 355 | 4.225621 | 352 | 306 | 4.197763 |
| 340 | 333 | 282 | 3.881656 | 331 | 313 | 3.768619 | 333 | 345 | 3.783733 | 334 | 297 | 3.932121 |
| 341 | 354 | 307 | 4.195624 | 357 | 308 | 4.005533 | 360 | 337 | 4.082428 | 359 | 322 | 4.353882 |
| 342 | 348 | 300 | 3.696083 | 350 | 300 | 3.52317 | 346 | 332 | 3.567273 | 348 | 295 | 3.733609 |
| 343 | 353 | 320 | 4.469967 | 348 | 285 | 4.232659 | 349 | 325 | 4.254376 | 350 | 329 | 4.491086 |
| 344 | 354 | 317 | 4.57209 | 356 | 295 | 4.588977 | 356 | 323 | 4.58347 | 360 | 331 | 4.611823 |
| 345 | 355 | 335 | 4.459585 | 359 | 297 | 3.947015 | 357 | 317 | 4.192405 | 355 | 340 | 4.153916 |
| 346 | 353 | 334 | 3.685461 | 356 | 301 | 3.576357 | 353 | 327 | 3.669934 | 351 | 326 | 3.671369 |
| 347 | 342 | 336 | 4.063346 | 343 | 303 | 4.194171 | 342 | 321 | 4.216904 | 341 | 328 | 4.12468 |
| 348 | 332 | 307 | 3.829968 | 328 | 290 | 3.984869 | 330 | 324 | 3.881294 | 331 | 309 | 3.88092 |
| 349 | 333 | 310 | 3.273981 | 332 | 284 | 3.313719 | 332 | 319 | 3.279041 | 331 | 311 | 3.319493 |
| 350 | 330 | 295 | 3.617897 | 331 | 288 | 3.676553 | 331 | 312 | 3.496248 | 329 | 293 | 3.383311 |

Continued on next page

Table C.1 – continued from previous page

| | M=1 | | | M=3 | | | M=7 | | | M=31 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 351 | 347 | 306 | 3.665692 | 349 | 287 | 3.566114 | 353 | 312 | 3.563057 | 353 | 311 | 3.642129 |
| 352 | 349 | 333 | 4.420773 | 351 | 278 | 4.478168 | 346 | 319 | 4.257788 | 348 | 329 | 4.376586 |
| 353 | 332 | 288 | 3.16261 | 328 | 273 | 3.292204 | 333 | 301 | 3.247234 | 330 | 294 | 3.259592 |
| 354 | 357 | 346 | 4.170017 | 360 | 295 | 4.040292 | 356 | 321 | 3.969674 | 355 | 331 | 4.034316 |
| 355 | 331 | 246 | 2.407625 | 331 | 250 | 2.440431 | 329 | 287 | 2.448335 | 332 | 240 | 2.370134 |
| 356 | 348 | 308 | 2.686055 | 348 | 280 | 2.676883 | 348 | 304 | 2.721328 | 348 | 309 | 2.809005 |
| 357 | 348 | 329 | 3.356895 | 348 | 292 | 3.269205 | 348 | 322 | 3.186197 | 351 | 338 | 2.950474 |
| 358 | 357 | 326 | 3.222841 | 357 | 293 | 3.431058 | 356 | 335 | 3.392906 | 356 | 339 | 3.532958 |
| 359 | 354 | 352 | 3.950187 | 353 | 304 | 3.787072 | 354 | 319 | 3.810701 | 351 | 338 | 3.925785 |
| 360 | 353 | 341 | 3.639718 | 351 | 298 | 3.855223 | 351 | 327 | 3.649703 | 350 | 334 | 3.479452 |
| 361 | 326 | 292 | 3.821377 | 323 | 290 | 3.679351 | 324 | 308 | 3.628724 | 326 | 297 | 3.490873 |
| 362 | 324 | 237 | 2.847965 | 324 | 266 | 2.952738 | 324 | 295 | 2.922423 | 324 | 261 | 2.904653 |
| 363 | 341 | 292 | 3.573453 | 343 | 278 | 3.504202 | 342 | 304 | 3.353356 | 342 | 302 | 3.564157 |
| 364 | 356 | 320 | 3.17395 | 351 | 300 | 3.249072 | 352 | 317 | 3.181038 | 353 | 319 | 3.02473 |
| 365 | 344 | 321 | 3.546368 | 344 | 296 | 3.632608 | 344 | 331 | 3.644601 | 344 | 321 | 3.554107 |
| 366 | 335 | 298 | 3.12671 | 332 | 299 | 3.151013 | 335 | 323 | 3.254171 | 334 | 308 | 3.483019 |
| 367 | 346 | 318 | 3.622032 | 346 | 300 | 3.558191 | 344 | 323 | 3.526402 | 344 | 315 | 3.342032 |
| 368 | 351 | 317 | 3.66161 | 351 | 321 | 3.895922 | 349 | 330 | 3.754929 | 351 | 314 | 3.56687 |
| 369 | 337 | 289 | 2.907845 | 339 | 300 | 2.942882 | 338 | 320 | 2.865589 | 337 | 292 | 2.954553 |
| 370 | 350 | 314 | 3.221451 | 351 | 297 | 3.156393 | 352 | 313 | 3.292161 | 351 | 314 | 3.112711 |
| 371 | 327 | 249 | 2.957058 | 327 | 271 | 2.887182 | 329 | 296 | 2.952748 | 327 | 260 | 3.132067 |
| 372 | 341 | 302 | 3.930263 | 341 | 299 | 3.930751 | 340 | 316 | 3.866213 | 343 | 299 | 3.692582 |
| 373 | 357 | 331 | 3.432539 | 357 | 293 | 3.442488 | 359 | 320 | 3.606676 | 359 | 328 | 3.405299 |
| 374 | 353 | 304 | 2.8754 | 353 | 299 | 2.90537 | 353 | 325 | 3.079205 | 350 | 319 | 3.219676 |
| 375 | 323 | 227 | 1.987031 | 323 | 258 | 2.033163 | 323 | 304 | 2.04436 | 322 | 232 | 1.859205 |
| 376 | 312 | 242 | 1.885834 | 311 | 260 | 1.872129 | 309 | 303 | 1.930467 | 310 | 219 | 1.948465 |
| 377 | 339 | 299 | 3.086797 | 338 | 286 | 3.042305 | 339 | 317 | 3.04206 | 340 | 299 | 2.851159 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 378 | 332 | 299 | 3.162756 | 332 | 275 | 3.209918 | 330 | 315 | 3.108809 | 331 | 305 | 3.254113 |
| 379 | 332 | 296 | 3.286261 | 332 | 280 | 3.378017 | 333 | 313 | 3.288215 | 330 | 298 | 3.621365 |
| 380 | 341 | 287 | 3.012197 | 340 | 279 | 2.824721 | 339 | 293 | 2.903537 | 339 | 286 | 2.843991 |
| 381 | 324 | 248 | 2.506042 | 323 | 266 | 2.495362 | 323 | 304 | 2.605935 | 324 | 247 | 2.37595 |
| 382 | 336 | 257 | 2.937379 | 333 | 295 | 3.062743 | 334 | 315 | 3.078064 | 335 | 269 | 3.209904 |
| 383 | 347 | 312 | 3.365746 | 345 | 297 | 3.197876 | 346 | 328 | 3.183347 | 346 | 301 | 3.348915 |
| 384 | 349 | 291 | 3.235458 | 350 | 289 | 3.076038 | 346 | 323 | 3.067503 | 345 | 293 | 3.045884 |
| 385 | 358 | 326 | 3.402099 | 357 | 306 | 3.608061 | 357 | 331 | 3.419959 | 357 | 322 | 3.735137 |
| 386 | 333 | 276 | 3.083308 | 330 | 289 | 3.027662 | 331 | 325 | 2.997784 | 329 | 279 | 2.856361 |
| 387 | 338 | 295 | 3.138521 | 339 | 280 | 3.231284 | 340 | 320 | 3.257198 | 339 | 286 | 3.116017 |
| 388 | 349 | 305 | 3.536195 | 350 | 308 | 3.552709 | 351 | 330 | 3.562605 | 347 | 313 | 3.399437 |
| 389 | 327 | 240 | 2.731015 | 327 | 276 | 2.669471 | 328 | 310 | 2.587117 | 327 | 254 | 2.703315 |
| 390 | 345 | 308 | 2.917827 | 348 | 290 | 2.845931 | 347 | 316 | 2.903015 | 348 | 320 | 3.003749 |
| 391 | 318 | 215 | 2.283579 | 320 | 260 | 2.266347 | 320 | 302 | 2.208287 | 319 | 214 | 2.123957 |
| 392 | 325 | 232 | 1.771763 | 328 | 265 | 1.858207 | 329 | 305 | 1.878855 | 326 | 215 | 1.806524 |
| 393 | 326 | 234 | 2.125446 | 325 | 284 | 2.121334 | 328 | 339 | 2.171891 | 327 | 243 | 2.103123 |
| 394 | 361 | 332 | 3.300531 | 360 | 331 | 3.355809 | 360 | 344 | 3.30588 | 359 | 333 | 3.344523 |
| 395 | 346 | 313 | 3.284366 | 345 | 296 | 3.17204 | 345 | 334 | 3.220532 | 345 | 321 | 3.037523 |
| 396 | 332 | 245 | 2.654285 | 333 | 273 | 2.83664 | 334 | 334 | 2.723092 | 332 | 249 | 2.818 |
| 397 | 335 | 288 | 2.712806 | 338 | 289 | 2.588997 | 335 | 322 | 2.635286 | 336 | 285 | 2.600541 |
| 398 | 348 | 319 | 3.576793 | 349 | 307 | 3.43709 | 349 | 332 | 3.27614 | 348 | 320 | 3.606008 |
| 399 | 348 | 289 | 2.605108 | 347 | 304 | 2.640572 | 348 | 330 | 2.746133 | 348 | 306 | 2.602587 |
| 400 | 331 | 245 | 2.345129 | 327 | 260 | 2.408631 | 330 | 317 | 2.479545 | 331 | 246 | 2.438138 |
| 401 | 312 | 235 | 2.345137 | 310 | 276 | 2.446051 | 313 | 313 | 2.442969 | 313 | 238 | 2.428408 |
| 402 | 349 | 306 | 3.295012 | 348 | 303 | 3.208295 | 348 | 339 | 3.198587 | 348 | 312 | 3.050952 |
| 403 | 330 | 263 | 2.514855 | 330 | 280 | 2.550893 | 330 | 320 | 2.586226 | 329 | 283 | 2.624289 |
| 404 | 308 | 247 | 2.320302 | 310 | 261 | 2.552834 | 309 | 307 | 2.490639 | 311 | 248 | 2.350094 |

Continued on next page

Table C.1 – continued from previous page

| | M=1 | | | M=3 | | | M=7 | | | M=31 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 405 | 346 | 312 | 2.646751 | 345 | 292 | 2.602836 | 345 | 334 | 2.59054 | 346 | 301 | 2.713363 |
| 406 | 331 | 275 | 3.045553 | 332 | 280 | 3.132423 | 334 | 338 | 2.941228 | 332 | 275 | 2.97165 |
| 407 | 325 | 266 | 2.675143 | 326 | 270 | 2.779636 | 327 | 329 | 2.689761 | 329 | 258 | 2.765238 |
| 408 | 336 | 283 | 2.715842 | 336 | 285 | 2.665415 | 335 | 325 | 2.732939 | 335 | 283 | 2.68209 |
| 409 | 359 | 341 | 2.928154 | 358 | 306 | 2.9555 | 359 | 331 | 3.122865 | 355 | 334 | 3.291704 |
| 410 | 370 | 370 | 3.713605 | 373 | 330 | 3.716053 | 371 | 345 | 3.574375 | 369 | 376 | 3.430457 |
| 411 | 362 | 333 | 2.515423 | 361 | 316 | 2.38337 | 362 | 335 | 2.406639 | 361 | 327 | 2.390136 |
| 412 | 347 | 296 | 2.193783 | 348 | 311 | 2.305922 | 346 | 347 | 2.358845 | 348 | 302 | 2.334805 |
| 413 | 335 | 255 | 2.483949 | 334 | 278 | 2.482126 | 338 | 333 | 2.511224 | 336 | 243 | 2.388145 |
| 414 | 335 | 261 | 2.457869 | 334 | 288 | 2.430588 | 331 | 339 | 2.441231 | 333 | 258 | 2.334483 |
| 415 | 346 | 261 | 2.762087 | 349 | 294 | 2.852526 | 348 | 361 | 2.836486 | 348 | 265 | 2.89033 |
| 416 | 349 | 295 | 3.034205 | 349 | 315 | 3.018535 | 348 | 369 | 2.998388 | 348 | 290 | 3.073427 |
| 417 | 368 | 346 | 2.905842 | 368 | 323 | 3.094822 | 370 | 349 | 2.962489 | 369 | 339 | 2.885618 |
| 418 | 345 | 275 | 2.763735 | 345 | 302 | 2.765653 | 344 | 361 | 2.667873 | 346 | 267 | 2.791499 |
| 419 | 349 | 292 | 2.097276 | 348 | 271 | 2.166764 | 346 | 320 | 2.108179 | 346 | 295 | 2.003901 |
| 420 | 347 | 331 | 2.621962 | 347 | 305 | 2.678012 | 345 | 328 | 2.539438 | 346 | 322 | 2.526755 |
| 421 | 362 | 339 | 2.635927 | 362 | 313 | 2.553577 | 362 | 342 | 2.664555 | 363 | 330 | 2.628966 |
| 422 | 342 | 279 | 2.562456 | 339 | 296 | 2.592195 | 342 | 343 | 2.517701 | 342 | 276 | 2.524502 |
| 423 | 361 | 325 | 2.725352 | 360 | 311 | 2.792787 | 358 | 338 | 2.741204 | 361 | 323 | 2.607425 |
| 424 | 359 | 333 | 2.757822 | 359 | 315 | 2.705204 | 362 | 351 | 2.724744 | 360 | 324 | 2.792014 |
| 425 | 359 | 322 | 2.959579 | 359 | 321 | 2.975099 | 358 | 347 | 3.016355 | 359 | 325 | 2.954739 |
| 426 | 332 | 239 | 2.722307 | 331 | 269 | 2.864487 | 330 | 336 | 2.691412 | 330 | 243 | 2.808411 |
| 427 | 340 | 264 | 2.854132 | 341 | 273 | 2.843682 | 342 | 327 | 2.88293 | 342 | 248 | 2.793247 |
| 428 | 365 | 338 | 2.812005 | 364 | 303 | 2.930897 | 366 | 342 | 2.90474 | 366 | 326 | 2.839104 |
| 429 | 365 | 344 | 3.387173 | 366 | 315 | 3.309022 | 366 | 340 | 3.391428 | 367 | 346 | 3.44588 |
| 430 | 355 | 322 | 2.645337 | 356 | 304 | 2.848073 | 356 | 347 | 2.631237 | 357 | 310 | 2.640225 |
| 431 | 330 | 253 | 2.308274 | 328 | 283 | 2.425308 | 329 | 311 | 2.297454 | 330 | 261 | 2.396845 |

Continued on next page

Table C.1 – continued from previous page

| | **M=1** | | | **M=3** | | | **M=7** | | | **M=31** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frame Number** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** | **NumOsc NonMod** | **NumOsc Mod** | **Motion Energy** |
| 432 | 329 | 268 | 2.552739 | 328 | 294 | 2.405898 | 330 | 302 | 2.4318 | 331 | 267 | 2.460326 |
| 433 | 360 | 328 | 2.988126 | 360 | 301 | 3.057326 | 359 | 333 | 3.135643 | 360 | 327 | 3.147169 |
| 434 | 353 | 343 | 3.488441 | 353 | 301 | 3.29287 | 353 | 326 | 3.342998 | 352 | 335 | 3.526246 |
| 435 | 355 | 342 | 3.638599 | 355 | 317 | 3.454553 | 355 | 329 | 3.385201 | 355 | 336 | 3.311201 |
| 436 | 357 | 354 | 3.437345 | 357 | 307 | 3.482406 | 357 | 339 | 3.481287 | 356 | 365 | 3.741434 |
| 437 | 350 | 329 | 3.466082 | 349 | 317 | 3.37943 | 349 | 317 | 3.394952 | 349 | 339 | 3.407296 |
| 438 | 349 | 328 | 3.248073 | 349 | 309 | 3.17021 | 348 | 326 | 3.233588 | 348 | 334 | 3.258865 |
| 439 | 360 | 339 | 3.364043 | 362 | 321 | 3.446279 | 360 | 326 | 3.485431 | 361 | 333 | 3.466319 |
| 440 | 380 | 375 | 3.122386 | 380 | 349 | 3.06686 | 380 | 336 | 3.101722 | 380 | 372 | 2.93924 |
| 441 | 364 | 299 | 2.631871 | 361 | 319 | 2.736523 | 362 | 338 | 2.729653 | 362 | 296 | 2.725326 |
| 442 | 362 | 283 | 2.500271 | 358 | 327 | 2.560504 | 357 | 359 | 2.555437 | 357 | 287 | 2.426761 |
| 443 | 359 | 270 | 2.466587 | 356 | 309 | 2.545291 | 357 | 341 | 2.450596 | 359 | 275 | 2.63215 |
| 444 | 352 | 301 | 2.82644 | 353 | 313 | 2.762158 | 352 | 318 | 2.700755 | 353 | 300 | 2.504959 |
| 445 | 358 | 291 | 2.842788 | 358 | 301 | 2.86962 | 357 | 319 | 2.904347 | 357 | 289 | 2.939043 |
| 446 | 355 | 286 | 2.73129 | 355 | 295 | 2.664263 | 354 | 320 | 2.702878 | 355 | 285 | 2.648219 |
| 447 | 343 | 261 | 2.920877 | 342 | 303 | 2.970858 | 343 | 324 | 2.902622 | 344 | 276 | 2.948814 |
| 448 | 354 | 282 | 2.743569 | 354 | 302 | 2.838762 | 355 | 321 | 2.679311 | 354 | 282 | 2.718265 |
| 449 | 344 | 291 | 2.928835 | 343 | 300 | 2.935678 | 343 | 331 | 2.935672 | 343 | 291 | 3.004955 |
| 450 | 332 | 273 | 2.772949 | 333 | 289 | 2.804581 | 333 | 327 | 2.840507 | 333 | 278 | 2.77493 |

# Appendix D

# Motion Energy with Different Activities

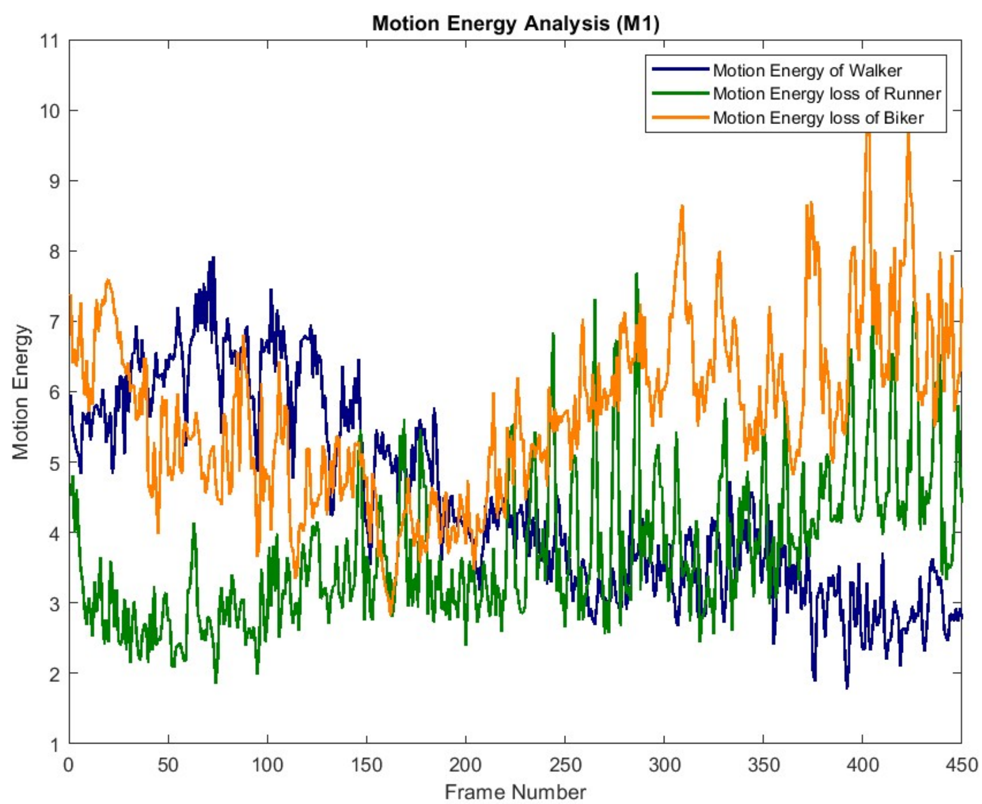


Figure D.1: Motion Energy with Activities (M=1)

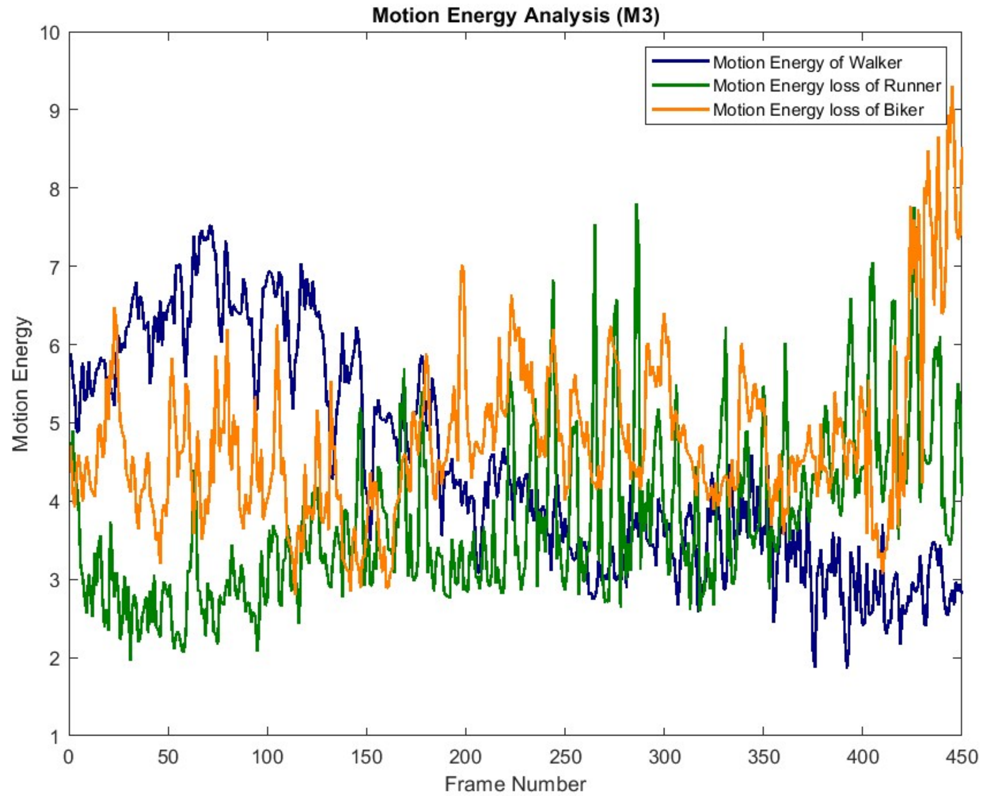


Figure D.2: Motion Energy with Activities (M=3)

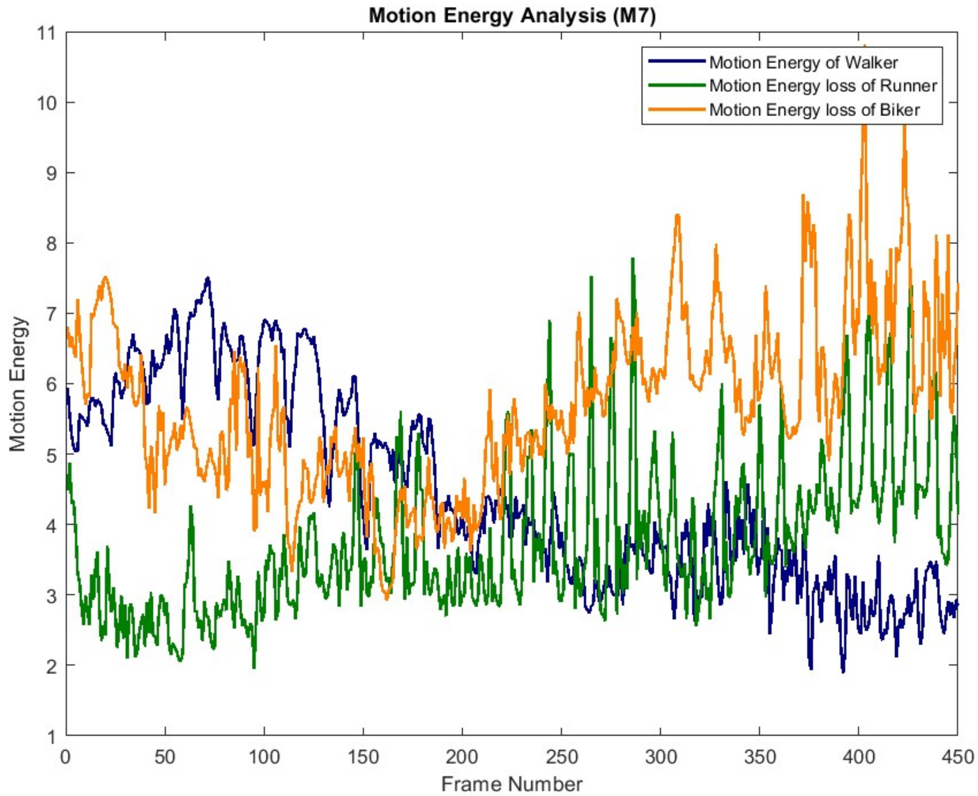


Figure D.3: Motion Energy with Activities (M=7)

# Appendix E

# Oscillation Behaviour with Activities

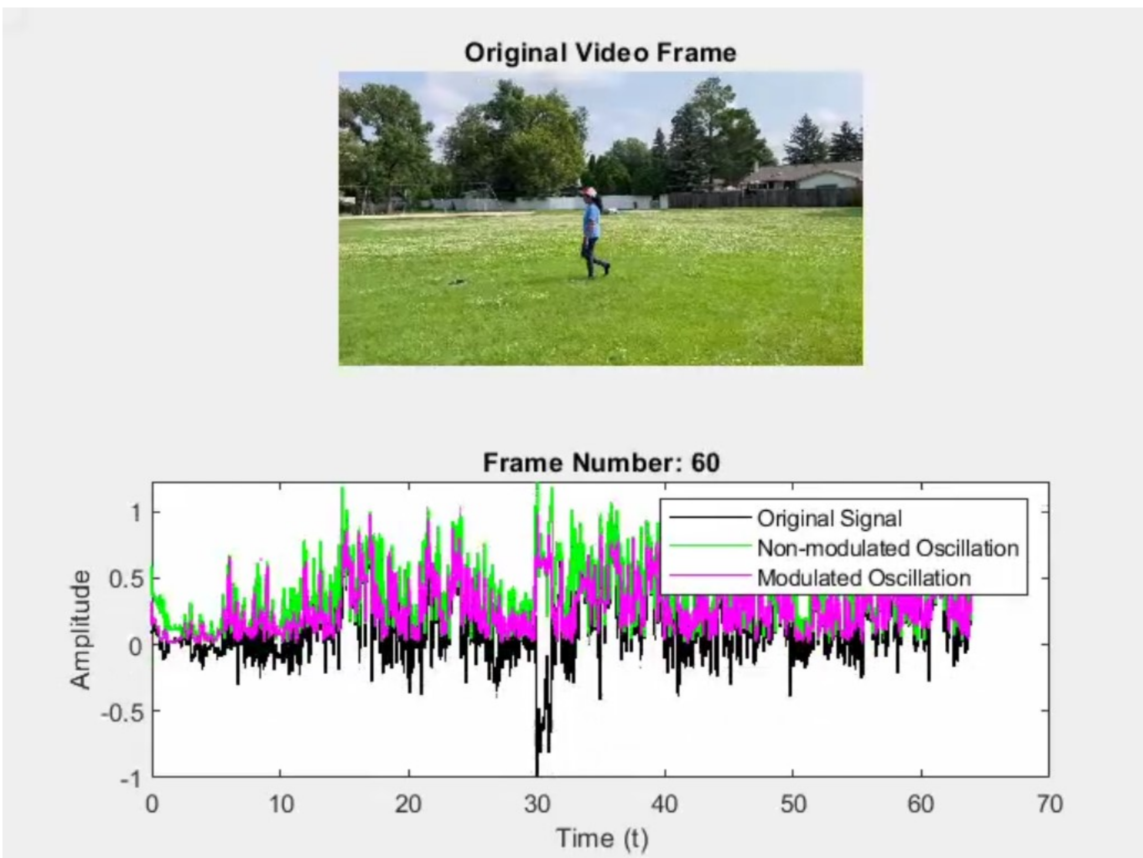


Figure E.1: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=1)

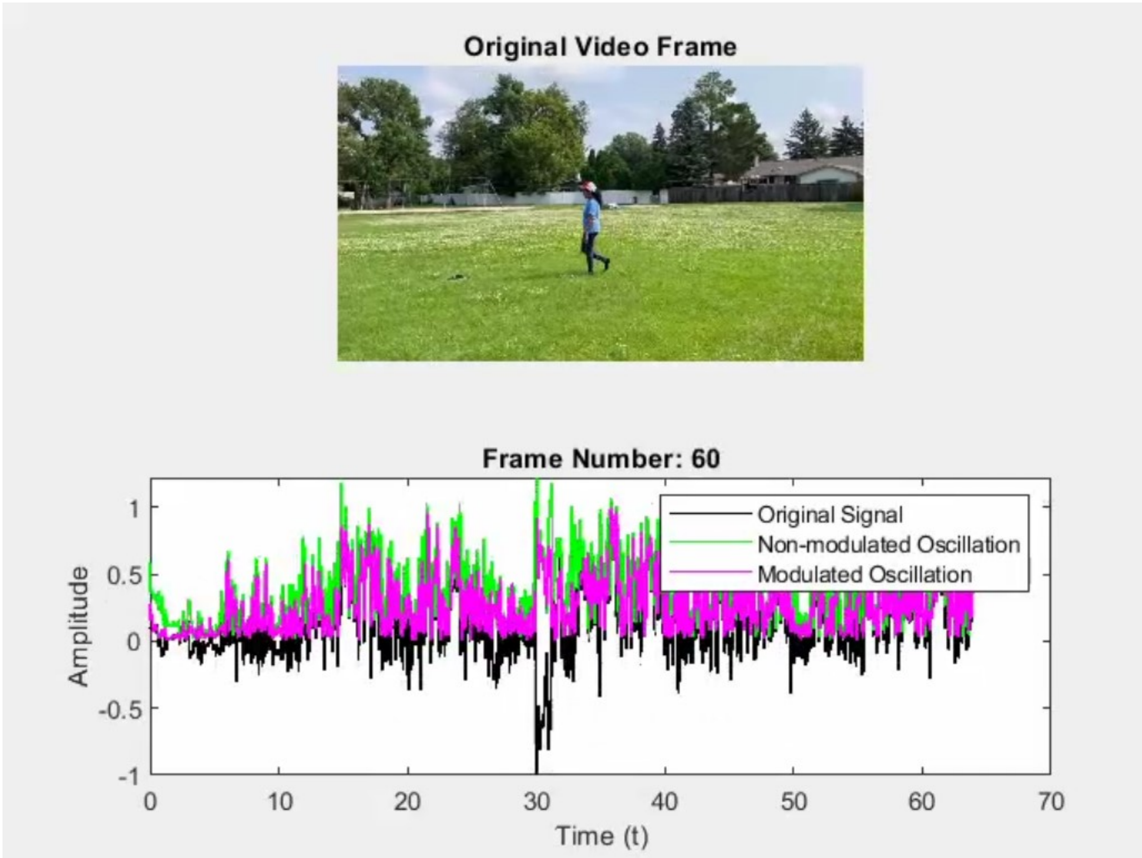


Figure E.2: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=3)

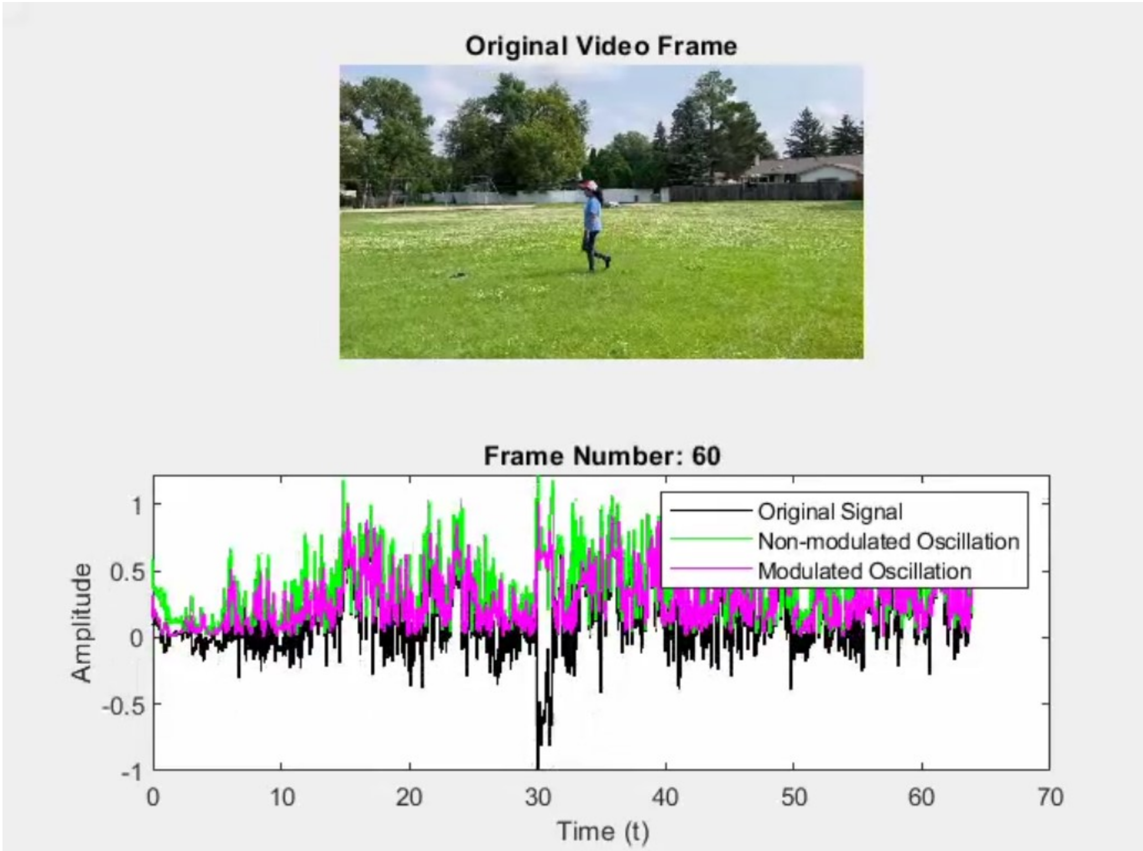


Figure E.3: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=3)

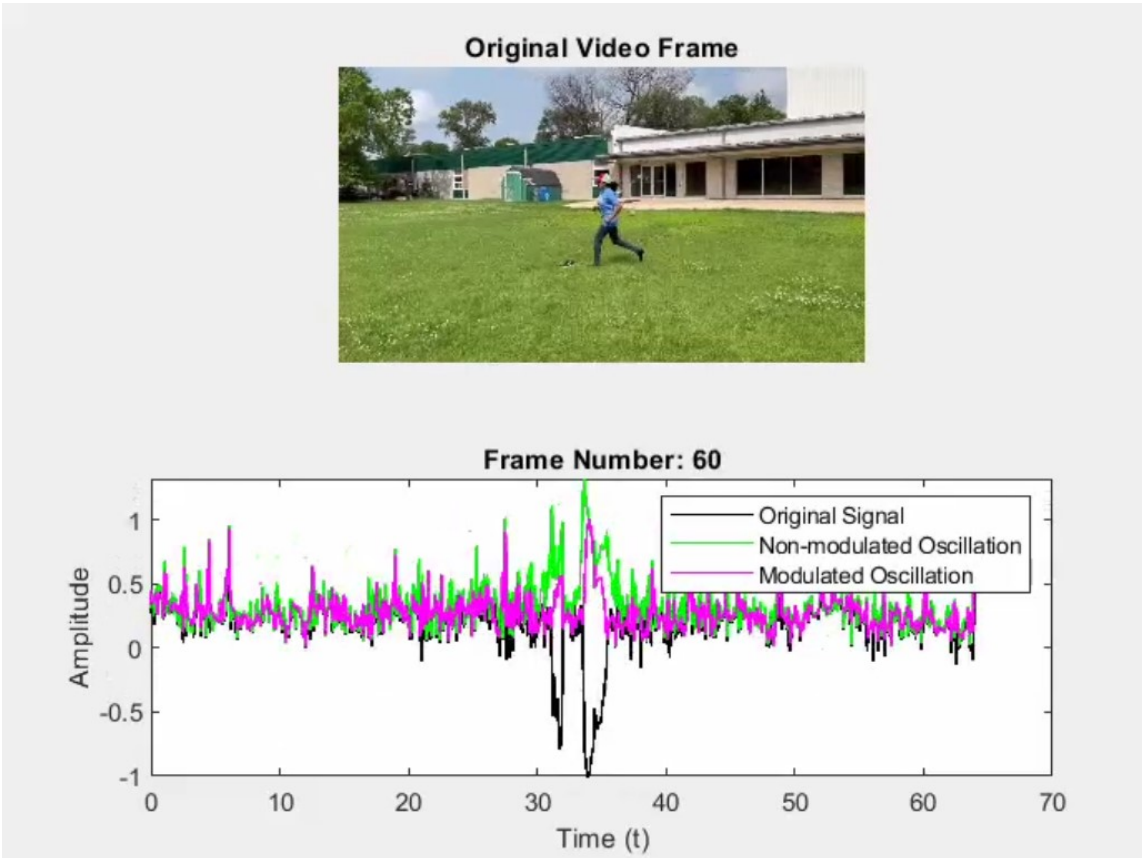


Figure E.4: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=3)

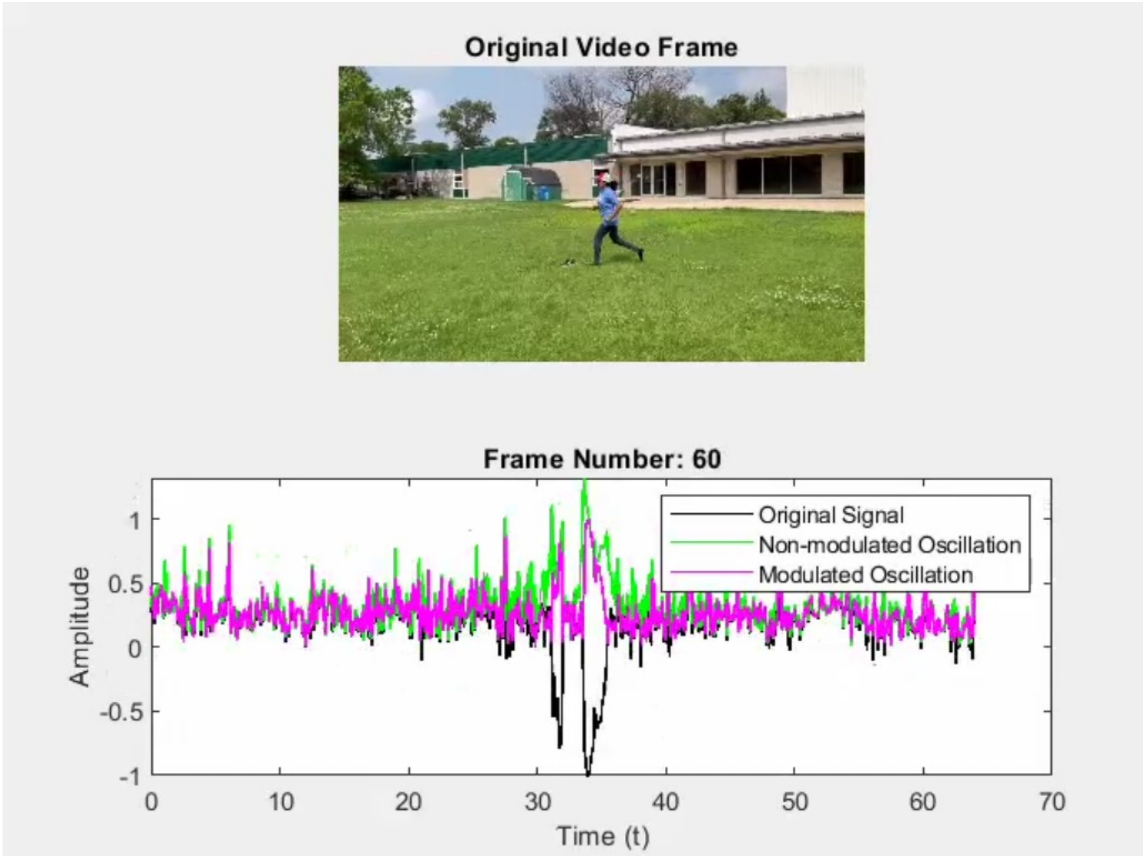


Figure E.5: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=3)

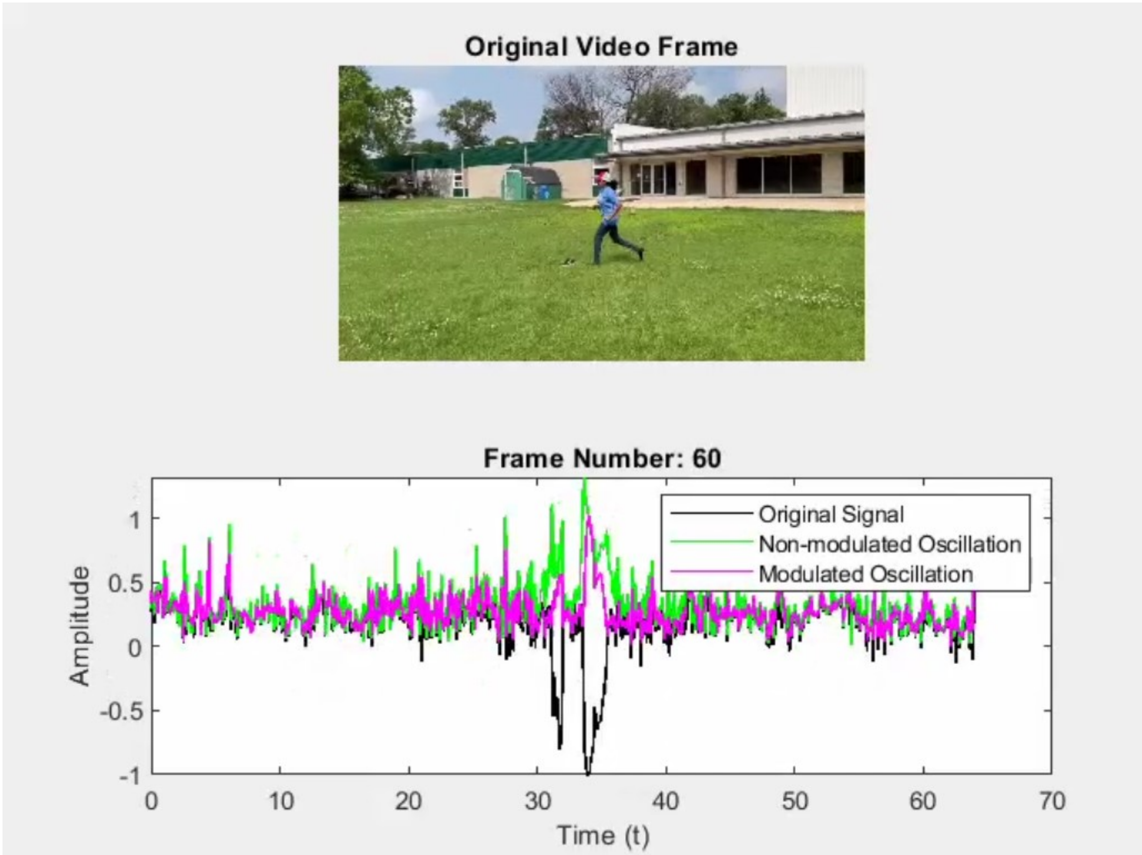


Figure E.6: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=31)

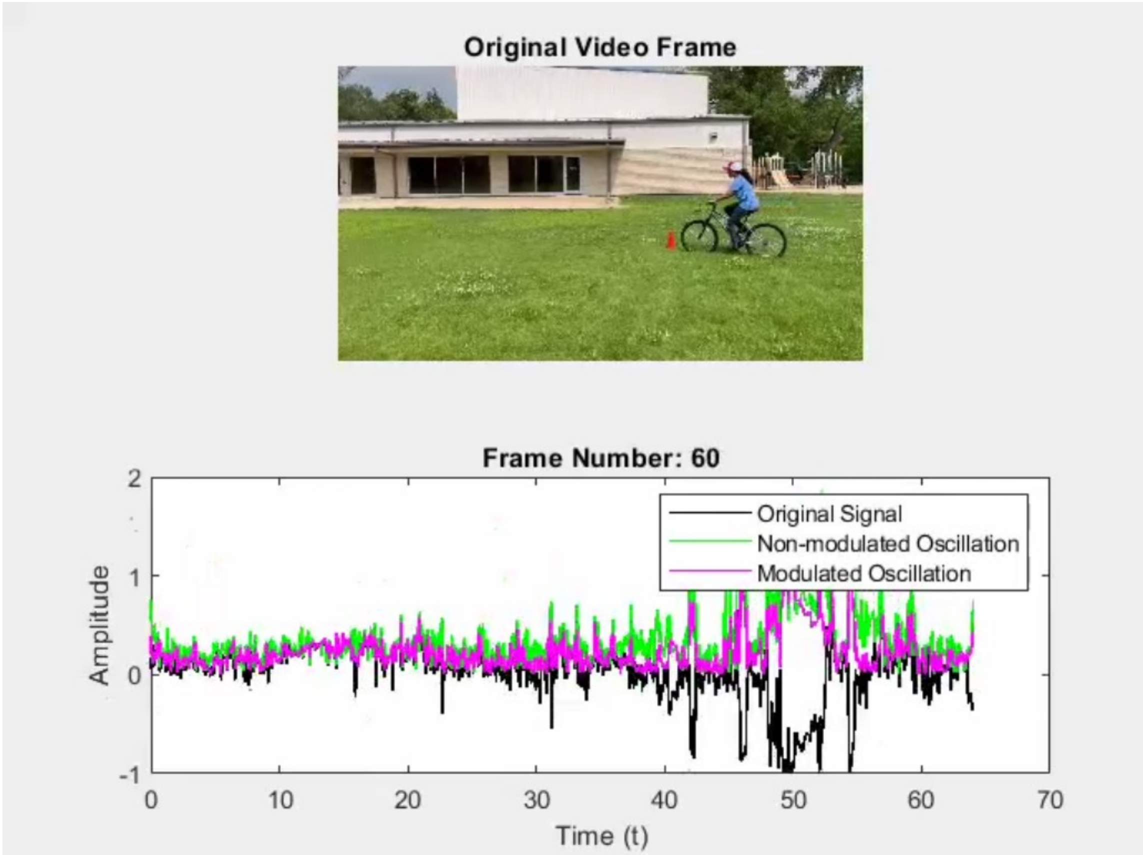


Figure E.7: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=1)

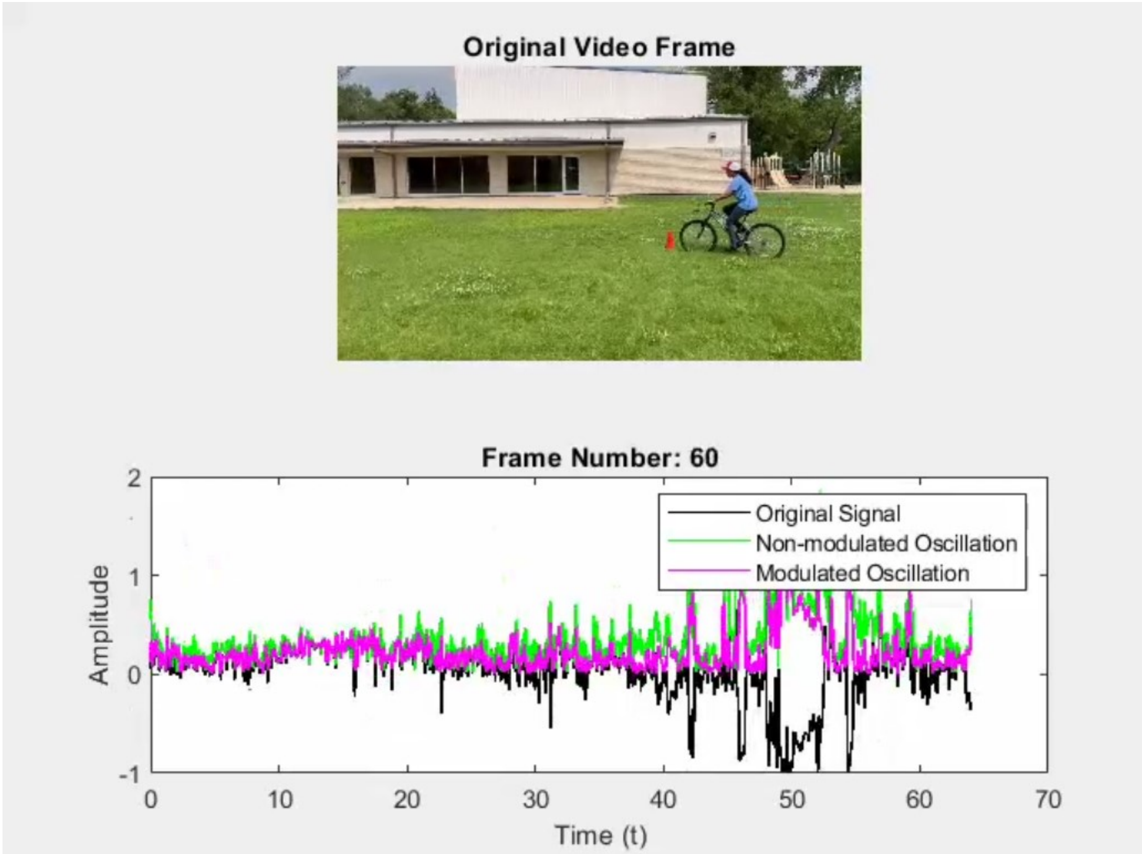


Figure E.8: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=3)

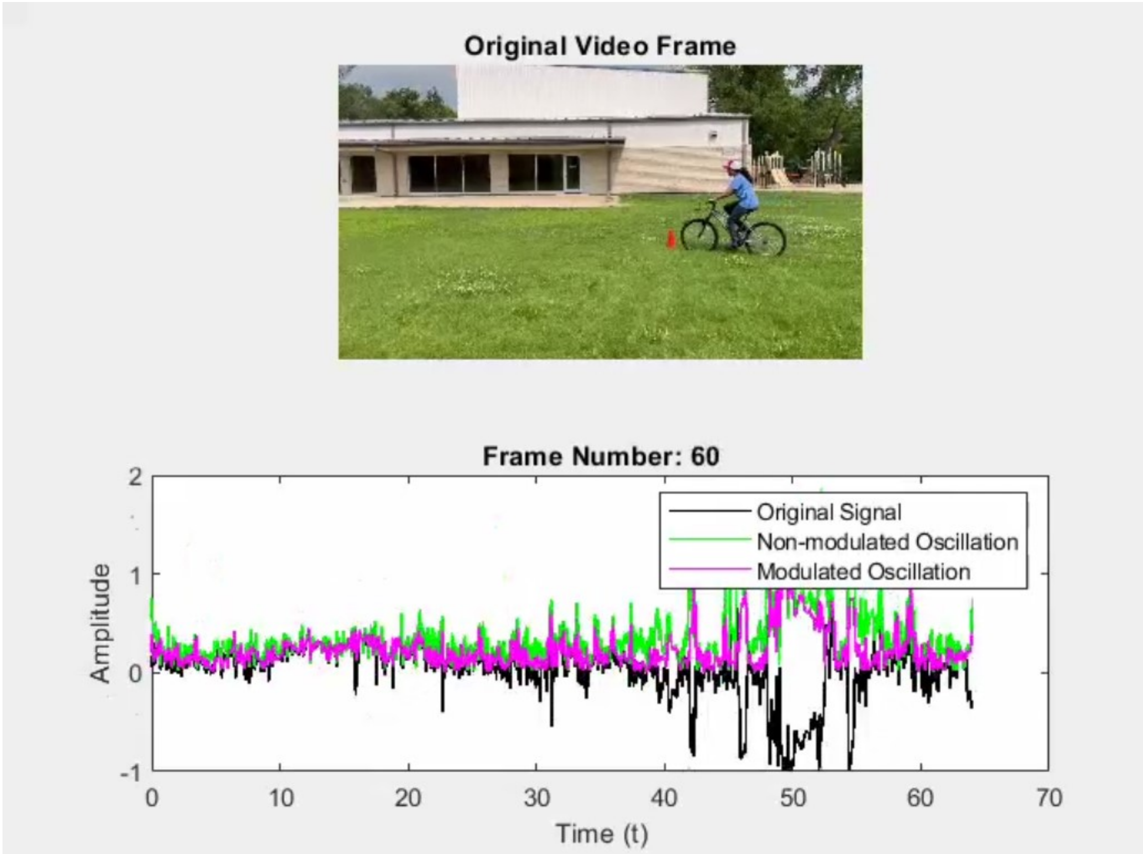


Figure E.9: Frame-by-Frame Analysis of Modulated and Non-Modulated Signal Oscillations in Video Data of Byker (M=31)

# Appendix F

# MATLAB Script for Signal Envelopes in Video Analysis

```
clc; clear; close all;

M_values = [1, 3, 7, 31]; % Modulation factors
% videoFilePath = 'C:\\Users\\perer\\OneDrive\\Desktop\\Final\\
    Runner.mp4'; % Ensure double backslashes
% videoFilePath = 'C:\\Users\\perer\\OneDrive\\Desktop\\Final\\
    Walker.mp4';
videoFilePath = 'C:\\Users\\perer\\OneDrive\\Desktop\\Final\\
    Biker.mp4';

% Initialize video reader
videoReader = VideoReader(videoFilePath);
originalFrameRate = videoReader.FrameRate; % Get video frame rate

% Initialize frame60Metrics outside the loop to ensure it's
    accessible later
frame60Metrics = [];

% Process each M value
for mIndex = 1:length(M_values)
    M = M_values(mIndex);

    % Define output path for the video
%     outputVideoPath = sprintf('C:\\Users\\perer\\OneDrive\\
    Desktop\\Final\\Envelops\\Runner_Envelops%d.mp4', M);
%     outputVideoPath = sprintf('C:\\Users\\perer\\OneDrive\\
    Desktop\\Final\\Envelops\\Walker_Envelops%d.mp4', M);
    outputVideoPath = sprintf('C:\\Users\\perer\\OneDrive\\
        Desktop\\Final\\Envelops\\Biker_Envelops%d.mp4', M);

```

```
    writeObj = VideoWriter(outputVideoPath, ’MPEG-4’);
    writeObj.FrameRate = originalFrameRate / 4; % Reduced speed for observation
    open(writeObj); % Open the VideoWriter object

    videoReader.CurrentTime = 0; % Reset video reader for each M value
    frameCount = 0;

    while hasFrame(videoReader)
        frame = readFrame(videoReader);
        frameCount = frameCount + 1;

        frameGray = rgb2gray(frame);
        row_number = 600;
        row_number = 650;
        selected_row = double(frameGray(row_number, :));

        % Normalize and scale the row data
        selected_row = (selected_row - min(selected_row)) / (max(selected_row) - min(selected_row));
        selected_row = 2 * selected_row - 1;

        % Time vector for signal
        t = linspace(0, numel(selected_row)/originalFrameRate, numel(selected_row));

        % Apply exponential modulation
        modulated_signal = selected_row .* exp(-1j*2*pi*M*t);

        % Compute envelopes
        envelopeMod = abs(hilbert(modulated_signal));
        envelopeNonMod = abs(hilbert(selected_row));

        % Compute areas under envelopes using cumtrapz
        upper_env_area = cumtrapz(t, envelopeMod.^2);
        lower_env_area = cumtrapz(t, envelopeNonMod.^2);
        motionEnergyLoss = abs(upper_env_area - lower_env_area);

        % Count peaks with a minimum height
        MinPeakHeight = 0.1;
        numOscMod = numel(findpeaks(envelopeMod, ’MinPeakHeight’, MinPeakHeight));
        numOscNonMod = numel(findpeaks(envelopeNonMod, ’MinPeakHeight’, MinPeakHeight));

        % Store metrics for frame 60
        if frameCount == 60
            frame60Metrics = [frame60Metrics; M, numOscNonMod, numOscMod, upper_env_area(end), lower_env_area(end), motionEnergyLoss(end)];

            % Print the required values along with the M value
```

```
            fprintf('M = %d\n', M);
%             fprintf('Number of oscillations for non-modulated: %d\n', numOscNonMod);
%             fprintf('Number of oscillations for modulated: %d\n', numOscMod);
            fprintf('Upper envelope area (modulated): %.4f\n', upper_env_area(end));
            fprintf('Lower envelope area (non-modulated): %.4f\n', lower_env_area(end));
            fprintf('Motion energy loss: %.4f\n', motionEnergyLoss(end));
            fprintf('\n'); % Add a newline for readability between different M values
        end

        % Plotting for the video
        figure('visible', 'off');
        subplot(2,1,1);
        imshow(frame); title(sprintf('Original Video Frame - Frame %d', frameCount));
        subplot(2,1,2);
        plot(t, selected_row, 'g-', 'LineWidth', 0.5); hold on; % Non-modulated in green
        plot(t, modulated_signal, 'm-', 'LineWidth', 0.5); % Modulated in magenta
        plot(t, envelopeMod, 'r-', 'LineWidth', 0.5); % Upper Envelope in red
        plot(t, -envelopeMod, 'k-', 'LineWidth', 0.5); % Lower Envelope in black
        hold off;
        legend('Non-modulated Oscillation', 'Modulated Oscillation', 'Upper Envelope', 'Lower Envelope');
        title(sprintf('Envelopes and Oscillations for M = %d - Frame %d', M, frameCount));
        xlabel('Time (s)');
        ylabel('Amplitude');

        % Capture and write the frame to the video file
        frameWithPlot = getframe(gcf);
        writeVideo(writeObj, frameWithPlot.cdata);
        close(gcf);
    end
    close(writeObj); % Close the video writer
end
```

# Appendix G

# MATLAB Script for comparison of Motion and Cumulative Energy of Non-modulated and modulated waveforms

```
clc, clear, close all;
warning('off', 'MATLAB:hilbert:IgnoreImaginaryPart')

% videoFilePath = 'C:\Users\perer\OneDrive\Desktop\Final\Runner.mp4';
videoFilePath = 'C:\Users\perer\OneDrive\Desktop\Final\Biker.mp4';
% videoFilePath = 'C:\Users\perer\OneDrive\Desktop\Final\Walker.mp4';
videoReader = VideoReader(videoFilePath);

frameNumber = 60;
videoReader.CurrentTime = (frameNumber - 1) / videoReader.FrameRate;
frame = readFrame(videoReader);

% row_number = 650;
row_number = 600;
selected_row = double(frame(row_number,:,1)); % Extract the red channel of the row

selected_row = (selected_row - min(selected_row)) / (max(selected_row) - min(selected_row));
selected_row = 2 * selected_row - 1; % Scale to -1 to 1

% Time vector
```

```
t = 0:1/2000:3-1/2000;
% Define the functions using the image row data
x0 = @(t) interp1(linspace(0,3,length(selected_row)), selected_row, t, 'linear', 'extrap');

% Mersenne prime values
M_values = [1, 3, 7, 31];

% Initialize arrays to store results
max_energy_non_modulated = [];
max_energy_modulated = [];
mod_diff_non_modulated = [];
mod_diff_modulated = [];

for M = M_values
    x = @(t) x0(t) .* exp(-1j*2*M*pi.*t);

    env0 = @(t) abs(hilbert(x0(t)));
    env = @(t) abs(hilbert(x(t)));

    % Compute upper and lower envelopes for plot 3
    upper_env_1 = max(env0(t), 0); % Upper envelope
    lower_env_1 = zeros(size(env0(t))); % Lower envelope
    [~, min_locs_1] = findpeaks(-env0(t)); % Find local minima of the negative envelope
    lower_env_1(min_locs_1) = -env0(t(min_locs_1)); % Set lower envelope values at minima
    lower_env_1 = interp1(t(min_locs_1), lower_env_1(min_locs_1), t, 'linear', 'extrap'); % Interpolate between minima

    % Compute upper and lower envelopes for plot 4
    upper_env_3 = max(env(t), 0); % Upper envelope
    lower_env_3 = zeros(size(env(t))); % Lower envelope
    [~, min_locs_3] = findpeaks(-env(t)); % Find local minima of the negative envelope
    lower_env_3(min_locs_3) = -env(t(min_locs_3)); % Set lower envelope values at minima
    lower_env_3 = interp1(t(min_locs_3), lower_env_3(min_locs_3), t, 'linear', 'extrap'); % Interpolate between minima

    % Compute cumulative integral values for plot 3
    integral_upper_1 = cumtrapz(t, upper_env_1);
    integral_lower_1 = cumtrapz(t, lower_env_1);

    % Compute cumulative integral values for plot 4
    integral_upper_3 = cumtrapz(t, upper_env_3);
    integral_lower_3 = cumtrapz(t, lower_env_3);

    % Plotting
    figure;

    subplot('Position', [0.1 0.62 0.8 0.3]);
    imshow(frame);
```

```
title(['Original Video - Frame No. ', num2str(frameNumber), ', M
    = ', num2str(M)]);

    subplot('Position', [0.1 0.48 0.8 0.1]);
    plot(t, x0(t), 'k', 'LineWidth', 1);
    xlabel('t sec'), ylabel('motion x(t)')
    title('Non-modulated periodic motion signal x(t)')

    subplot('Position', [0.1 0.34 0.8 0.1]);
    plot(t, x(t), 'k', 'LineWidth', 1), hold on,
    plot(t, env(t), 'b:', 'LineWidth', 1), hold on,
    plot(t, -env(t), 'b:', 'LineWidth', 1), hold off,
    xlabel('t sec'), ylabel('motion x(t)')
    title('Image Row Based Motion Signal x(t)')

    subplot('Position', [0.1 0.2 0.8 0.1]);
    plot(t, x0(t), 'k', 'LineWidth', 1), hold on,
    plot(t, env0(t), 'b:', 'LineWidth', 1), hold on,
    plot(t, -env0(t), 'b:', 'LineWidth', 1), hold on,
    grid;
    xlabel('t sec'), ylabel('energy E_x')
    title('Hilbert Enveloped Cumulative Energy E_x-Non-Modulated'
       )

    % Plot for upper envelope of plot 3
    hold on;
    fill([t fliplr(t)], [integral_upper_1 zeros(size(
       integral_upper_1))], 'g', 'FaceAlpha', 0.3);

    % Plot for lower envelope of plot 3
    hold on;
    fill([t fliplr(t)], [integral_lower_1 zeros(size(
       integral_lower_1))], 'b', 'FaceAlpha', 0.3);

    % Highlight maximum energy point on plot 3
    [~, max_energy_idx_1] = max(integral_upper_1 -
       integral_lower_1);
    max_energy_val_1 = integral_upper_1(max_energy_idx_1);
    plot(t(max_energy_idx_1), max_energy_val_1, 'ro', 'MarkerSize
       ', 8);
    text(t(max_energy_idx_1), max_energy_val_1, sprintf('%.4f',
       max_energy_val_1), 'VerticalAlignment', 'bottom');

    subplot('Position', [0.1 0.06 0.8 0.1]); % Adjust for
       modulated signal and add more space below
    plot(t, x(t), 'k', 'LineWidth', 1), hold on,
    plot(t, upper_env_3, 'b:', 'LineWidth', 1), hold on,
    plot(t, lower_env_3, 'b:', 'LineWidth', 1), hold on,
    plot(t, env(t), 'm:', 'LineWidth', 1), hold off,
    xlabel('t sec'), ylabel('energy E_x')
    title('Hilbert Enveloped Cumulative Energy E_m-Modulated')

    % Plot for upper envelope of plot 4
```

```
    hold on;
    fill([t fliplr(t)], [integral_upper_3 zeros(size(
       integral_upper_3))], 'g', 'FaceAlpha', 0.3);

    % Plot for lower envelope of plot 4
    hold on;
    fill([t fliplr(t)], [integral_lower_3 zeros(size(
       integral_lower_3))], 'b', 'FaceAlpha', 0.3);

    % Highlight maximum energy point on plot 4
    [~, max_energy_idx_3] = max(integral_upper_3 -
       integral_lower_3);
    max_energy_val_3 = integral_upper_3(max_energy_idx_3);
    plot(t(max_energy_idx_3), max_energy_val_3, 'ro', 'MarkerSize
       ', 8);
    text(t(max_energy_idx_3), max_energy_val_3, sprintf('%.4f',
       max_energy_val_3), 'VerticalAlignment', 'bottom');

    % Calculate the maximum energy of the largest lobes for non-
       modulated
    max_energy_non_modulated(end+1) = max_energy_val_1;

    % Calculate the maximum energy of the largest lobes for
       modulated
    max_energy_modulated(end+1) = max_energy_val_3;

    % Calculate and print the modulus of difference
    mod_diff_non_modulated(end+1) = abs(integral_upper_1(end) -
       integral_lower_1(end));
    mod_diff_modulated(end+1) = abs(integral_upper_3(end) -
       integral_lower_3(end));

    saveas(gcf, sprintf('M_%d.png', M));
end

for i = 1:length(M_values)
    fprintf('M=%d:\n', M_values(i));
    fprintf('Maximum energy of largest lobes for Non-Modulated:
       %.4f\n', max_energy_non_modulated(i));
    fprintf('Maximum energy of largest lobes for Modulated: %.4f\
       n', max_energy_modulated(i));
    fprintf('Modulus of difference for Non-Modulated: %.4f\n',
       mod_diff_non_modulated(i));
    fprintf('Modulus of difference for Modulated: %.4f\n\n',
       mod_diff_modulated(i));
end
```

# Bibliography

[1] V. K. Madisetti and D. B. Williams, editors. *The Digital Signal Processing Handbook*. CRC Press, Boca Raton, FL, 1997.

[2] Bernd Jähne, Horst Haußecker, and Peter Geißler, editors. *Handbook of Computer Vision and Applications*, volume 2. Academic Press, 2000.

[3] Belmar Garcia-Garcia, Thierry Bouwmans, and Alberto Jorge Rosales Silva. Background subtraction in real applications: Challenges, current models and future directions. *Journal Name*, 2019.

[4] Heng Wang and Cordelia Schmid. Action recognition with improved trajectories. In *2013 IEEE International Conference on Computer Vision*, pages 3551–3558. IEEE, 2013.

[5] Ivan Laptev. On space-time interest points. *International Journal of Computer Vision*, 64(2-3):107–123, 2005. First online version published in June, 2005.

[6] J.F. Peters and T.U. Liyanage. Energy dissipation in Hilbert envelopes on motion waveforms detected in sequences of video frames. *Communications in Advanced Mathematical Sciences*, x(y):9 pp., 2024. submitted 13.Sep.2024, Cornell Archive *arXiv*: 2409.19016v1, 24 Sep 2024, `https://arxiv.org/abs/2409.19016`.

[7] Avijit Kundu, Raunak Dey, Shuvojit Paul, and Ayan Banerjee. Single-shot wideband active microrheology using multiple-sinusoid modulated optical tweezers. *arXiv preprint arXiv:2103.04093*, March 2021.

[8] Tamer Becherrawy. *Mechanical and Electromagnetic Vibrations and Waves*. John Wiley Sons, 2008.

[9] G. Lan, P. Sartor, S. Neumann, V. Sourjik, and Y. Tu. The energy-speed-accuracy trade-off in sensory adaptation. *Nature Physics*, 8:421–428, 2012.

[10] James Clerk Maxwell. *Theory of Heat*. Longmans, Green, and Co., 1872.

[11] T. Wiegand, G. J. Sullivan, G. Bjontegaard, and A. Luthra. Overview of the h.264/avc video coding standard. *IEEE Transactions on Circuits and Systems for Video Technology*, 13(7):560–576, 2003.

[12] G. J. Sullivan, J. Ohm, W. J. Han, and T. Wiegand. Overview of the high efficiency video coding (hevc) standard. *IEEE Transactions on Circuits and Systems for Video Technology*, 22(12):1649–1668, 2012.

[13] V. Jain and H. S. Seung. Natural image denoising with convolutional networks. *Advances in Neural Information Processing Systems*, 21:769–776, 2009.

[14] L. Zhang and X. Wang. Energy-efficient video processing using fpga-based hardware accelerators. *IEEE Transactions on Multimedia*, 18(12):2345–2356, 2016.

[15] Paul Viola and Michael Jones. Rapid object detection using a boosted cascade of simple features. In *Proceedings of the 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR 2001)*, volume 1, pages I–511, 2001.

[16] Geert Litjens, Thijs Kooi, Babak Ehteshami Bejnordi, Arnaud Arindra Adiyoso Setio, Francesco Ciompi, Mohammad Ghafoorian, and Jeroen A. W. M. van der Laak. A survey on deep learning in medical image analysis. *Medical Image Analysis*, 42:60–88, 2017.

[17] G.W. Hill. Cyclic properties of pseudorandom sequences of mersenne prime residues. *The Computer Journal*, 22(1):80–85, 1979. MR0524811.

[18] F.W. King. *Hilbert transforms. Vol. 1.* Cambridge University Press, Cambridge, UK, 2009. xxxviii+858 pp.,MR2542214.

[19] R. Rivest. Mersenne primes and their applications in cryptography. *Journal of Cryptographic Engineering*, 5(3):215–224, 2015.

[20] D. Benitez, P.A. Gaydecki, A. Zaidi, and A.P. Fitzpatrick. The use of the hilbert transform in ecg signal analysis. *Department of Instrumentation and Analytical Science, University of Manchester Institute of Science and Technology (UMIST)*, 2001.

[21] G. H. Hardy and E. M. Wright. *An Introduction to the Theory of Numbers*. Oxford University Press, 2008.

[22] K. Chen, C.-B. Schönlieb, X.-C. Tai, and L. Younes. *Handbook of Mathematical Models and Algorithms in Computer Vision and Imaging*. Springer, 2023.

[23] Frederick W. King. *Hilbert Transforms*, volume 1. Cambridge University Press, 2009. Hilbert transform in equation (5.103).