DEVELOPMENT OF LONG-RANGE FLOW FORECASTING CAPABILITIES FOR A LARGE MULTI-RESERVOIR SYSTEM

BY

Michael J. Bender

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE
IN
CIVIL ENGINEERING

Department of Civil Engineering
University of Manitoba
Winnipeg, Manitoba

ISBN 0-315-78042-8

DEVELOPMENT OF LONG-RANGE FLOW FORECASTING CAPABILITIES

FOR A LARGE MULTI-RESERVOIR SYSTEM

BY

MICHAEL J. BENDER

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

ABSTRACT

This paper compares presently available methods for long-range water supply forecasting with theoretical statistical time-series models as a means of evaluating forecast performance and reliability of long-range monthly probabilistic stream flow forecasts. Seasonal AutoRegressive Integrated Moving Average (SARIMA) models are used in conjunction with deseasonalized AutoRegressive Moving Average (ARMA) models to produce forecasts for various inflow types under a range of flow scenarios. Evaluation of model performance under these conditions provided possible insight into development of a hybrid technique using engineering knowledge and experience to improve the quality and reliability of the forecasts. Ranking of model choices from analysis of forecast errors within a flow sensitivity analysis allowed the formation of a set of rules to govern the selection of a single model when more than one model is available. These modelling procedures provided a basis for comparison with existing methods of long-range forecasting at large utilities such as Manitoba Hydro. Increased confidence in the optimal forecasted operating and planning policies of a large utility are consequences of improved forecasts.

Manitoba Hydro was chosen as a case study utility from a selection of several other large utilities that were surveyed. Manitoba Hydro is a large utility which operates a multi-reservoir electric power generation system in the province of Manitoba. The needs and priorities of the system demand forecasts of up to a year in advance for planning of budgets and release policies. Manitoba Hydro needs and requirements were used to govern the scenario within forecasts are made, allowing forecasts from Manitoba Hydro to be compared with forecasts produced from other statistical time series modelling techniques.

ACKNOWLEDGEMENTS

There are many people who have made this thesis possible. First of all, I would like to thank the Manitoba Hydro Research and Development committee for supporting this project through a Manitoba Hydro post-graduate scholarship. I also appreciate the time and effort that Dave Cormie has allocated to provide me with test forecasts from Manitoba Hydro, data for the analysis, and hours of consultation time. My program advisor, Dr. Slobodan Simonovic, was always very supportive, understanding, and encouraging. Thank you, Simon, for offering me the chance to work for you.

For my progress, completion of this project, personal growth, and sanity, I would like to express my sincere gratitude to, my buddies, my comrades, my friends in FIDS. Thanks, guys, for always being there for me.

I would like to thank my parents for always encouraging me, and supporting me. I am very fortunate that they are such a positive impact on my life. Most importantly I would like to thank my dearest love, Marilyn, for your thoughts and prayers, for putting up with me, and for standing beside me every step of the way.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF NOTATION

a_t	white noise series
B	backshift operator
d,D ..	degree of differencing (nonseasonal, and seasonal)
E[...]	expected value function
f	forecast value
k	time lag, number of forecasts
L	seasonal lag
m	number of harmonics
n,N ...	number of data points
p,P ...	number of AR terms (nonseasonal, and seasonal)
q,Q ...	number of MA terms (nonseasonal, and seasonal)
r_k	autocorrelation at lag k
t,τ	position in time
w_i	forecast weight
x_t, y_t, z_t	data series
$\hat{x}_i$	forecasted variable
ϕ	AR coefficient
θ	MA coefficient
μ	mean
σ	standard deviation
λ	Box-Cox transformation parameter
λ_j	parametric deseasonalization coefficient
ν_x	historical statistical parameter: μ , or σ
ν_τ	monthly statistical parameter: μ , or σ

ω .... time series periods

1 INTRODUCTION

1.1 Hydrologic forecasting

Hydrologic forecasting plays an ever increasing role in water resource management for improvement of irrigation practices, flood control, and hydro-electric generation optimization. The ability of the engineer to make competent forecasts of natural inflows to reservoirs has improved. Developments in statistical theory, contact with statisticians, and experience in evaluating the interaction of various physical processes enable us to better comprehend the content of our data sets. Techniques for forecasting vary with the system purpose, physical characteristics, and availability of data.

Hydrologic forecasting, water supply forecasting in particular, can be separated into two general types: short-term, or real-time forecasting; and long-range forecasting. Each is suited for different, specific system requirements. They differ in physical nature and require individual mathematical approaches. Short-term forecasting has been thoroughly researched, and well documented, but long range forecasting remains somewhat mysterious to engineers. Yet, it can not be ignored because optimization techniques for reservoir release policies demand an indication of what inflows can be expected in the coming months. This paper will explore time series analysis techniques as they apply to long-range water supply forecasting, and use them as a basis for comparison with forecasting within Manitoba Hydro.

1.2 Project scope

The purpose of this paper is to evaluate the effectiveness of long-term water supply forecasting by several large utilities, Manitoba Hydro in particular, and compare their capabilities with available theoretical statistical tools. A comparison of time series models such as those produced by AR and MA (AutoRegressive and Moving Average) processes

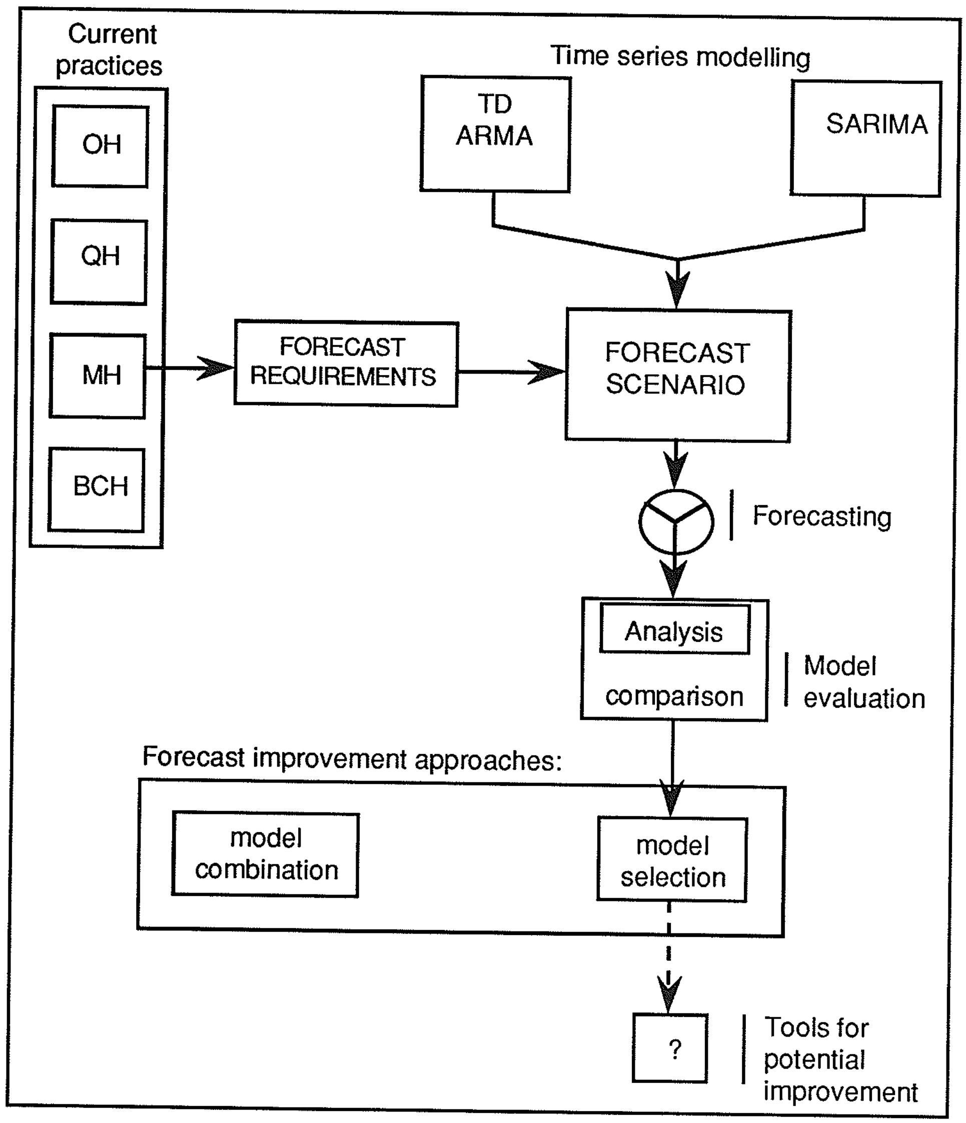


Figure 1. Project Overview

with the current mathematical methods may indicate the relative quality of the forecasts, and suggest improvements in use of the data.

A sensitivity analysis for flow scenarios is used to aid in the selection of a superior model when several seemingly indistinguishable models are available. This type of analysis is used to prepare a ranking of the models, defining a simple set of rules that govern the choice of best forecast model under given specific flow conditions. A flow chart of the project components is outlined in Figure 1.

1.3 The Manitoba Hydro System

Manitoba Hydro (frequently referred to as MH within this paper) operates a multi-reservoir power generation system within the province of Manitoba, Canada. Large reservoirs regulate a considerable portion of the downstream flow. The largest, Lake Winnipeg, is approximately 23,700 square kilometres. The extent of natural or man-made storage capacity provided by these reservoirs reduces the need for extensive management of extreme or sudden flood events by damping the effects of precipitation and flow events. Because of the level of flow regulation, or magnitude of capacity, time lags of natural inflows do not greatly affect the performance of the system. For this reason, short-term forecast improvements are not considered to be highly beneficial. Conversely, long range forecasts up to a year in advance are necessary for the implementation of robust reservoir release policies to maximize system benefits from power production.

Forecasting at Manitoba Hydro is dependent on stream flow data because flow gauges are the only comprehensive source of available data. Other system information regularly associated with real-time conceptual forecasting models such as precipitation data and soil moisture conditions is available only on a regional basis. The information gained from these regional or general data sets is more useful in defining short-term expectations. Forecasts are used as input to a linear programming (LP) package that optimizes system benefits, and develops an efficient operational plan of reservoir releases. At MH, various forecasts are studied so that a robust plan can be developed. The operating plan developed by the LP is optimal given that particular forecast materializes. At best, it is near optimal or optimal given the percieved range of flow possibilities. At worst, it is one of many feasible solutions. The quality of LP output is a function of the quality of forecasted inflows to the system. Improvements in forecast accuracy will improve the quality of operation of the system, although the magnitude of improvement will vary according to the system characteristics (Georgakakos, 1989; Mishalani and Palmer, 1988).

2 WATER SUPPLY FORECASTING

2.1 Short-term forecasting

Short-term forecasts are primarily used to manage flooding events, although they are also valuable for management of power generation and irrigation on a real-time basis. They range from an hour to a number of days, up to a week. Modelling for short-term forecasting, because of the recent nature of the system knowledge, is a matter of calculating the system response to known events or situations.

Much of the work for real-time or short-term forecasting for stream flow approach the problem as a dynamic state-space system to produce a conceptual rainfall-runoff model. Various physical parameters such as precipitation, soil moisture content, and others are used to form the state of the system. The system state is then forecasted, or estimated, with the use of techniques such as Kalman filtering. For an in-depth study of dynamic state-space systems and Kalman filtering, see Abraham and Ledolter (1983). The general formulation of a dynamic linear state-space system is given (2.1a,b).

$$y_{t+1} = Ay_t + Ga_{t-1} \qquad (2.1a)$$

$$z_t = Hy_t + b_t \qquad (2.1b)$$

y_t = forecast, or dependent, variable

z_t = known, independent, variable

A, G, H are coefficient matrices

a_t, b_t are other functions such as error terms

The best known example of short-term forecasting is provided by the National Weather Service River Forecast System (NWSRFS) in the US. Their Extended Streamflow Prediction (ESP) program is frequently used and referenced in the literature (Day, 1985; Georgakakos and Smith, 1990; Hudlow, 1988; Kitanidis, 1980a, 1980b). It consists of a conceptual rainfall-runoff model that is estimated using Kalman filtering. There are numerous examples of experiments in using different formulations of the system ranging from linear to nonlinear, and variations of the Kalman Filter such as the Extended Kalman Filter (EKF) (Georgakakos and Smith, 1990). Other work for real-time forecasting with conceptual rainfall-runoff models experiment with various estimation techniques. Puente and Bras (1987) reviewed the use of EKF along with other, more complicated, filters for nonlinear systems. Another example is the work by Sen (1991) who combined Kalman filtering with Orthogonal Walsh Series.

The practice of formulating rainfall-runoff models in state-space systems for real-time forecasting has been well established. Possible improvements in the system estimation method have been explored but reduction in estimation error is quickly approaching the limits of the data, and the extent of our knowledge about these natural processes. Nonlinear filtering using EKF has been demonstrated to be effective (Puente and Bras, 1987; Georgakakos and Smith, 1990).

2.2 Long-term forecasting

The approach for long-term forecasting is not as clearly defined. These kinds of forecasts range in length from weeks to months, up to a year, using time steps of a day, week, or month. The problem with making forecasts of several months in advance is that Nature is quite unpredictable. The timing, number, and intensity of precipitation events, or changes in the state of the system, are more or less random occurrences that are not strongly correlated. Long-term forecast accuracy is confined or constrained in that engineers are restricted to making general observations about stream flow behaviour from limited information within the data.

Some work has been done in conceptual state-space modelling to apply those techniques to long range forecasting. Day (1985) used the ESP model at the NWS, described in section 2.1, to produce probabilistic forecasts using simulation techniques. However, examples of this are limited and the potential of this approach is uncertain because the focus with conceptual models is based on the day to day calculation of responses of a simulated or historical meteorological record, and general system flow behaviours are merely implied, or ignored.

Instead, long range forecasting practices, and theory, concentrate on using some form of pattern recognition regression, or correlation of time dependence of system behaviour to forecast time series data such as stream flow.

2.2.1 Time series analysis

The theoretical statistical approach for modelling single variate time series data is to correlate previous time lags with the present or forecast lag (Box and Jenkins, 1976). Multivariate modelling is also possible by cross-correlating two or more data sets. Correlation of time lags demands the calculation of 2 functions: the Autocorrelation Function (ACF), and the Partial Autocorrelation Function (PACF). They are quite tedious and complicated to calculate, but statistical packages such as the Statistical Analysis System (SAS) will easily calculate these functions. Both functions are valuable aids in the identification of a stationary series, and the identification of relevant significant time lags to be included in a time series model. The general, biased, ACF is given below in (2.2) for lag k of time series x_t. For a detailed discussion of these functions, see Abraham and Ledolter (1983), or Bowerman and O'Connell (1987).

$$r_k = \frac{\sum_{t=1}^{N-k} (x_t - \overline{x})(x_{t+k} - \overline{x})}{\sum_{t=1}^{N} (x_t - \overline{x})^2} \qquad (2.2)$$

For a model to be considered stationary, it must first satisfy stationarity conditions that state the roots of the equation (2.3)

$$u^p - \phi_1 u^{p-1} - \phi_2 u^{p-2} - \dots - \phi_p = 0 \quad \text{,for AR(p)} \qquad (2.3)$$

lie inside the unit circle. That is,

$$|u_i| < 1 \quad , i = 1, \dots, p \qquad (2.4)$$

Where u_i in (2.4) are the roots of the equation. In general, a stationary model has coefficients ranging from -1 to +1. The practical application of this for identifying a stationary series is to consider whether the two correlation functions, ACF and PACF, become insignificant after a reasonably limited number of lags. This can become difficult for a complicated seasonal data set. Experience in modelling time series data, and familiarity with identifying stationary processes, is a great asset.

By correlating previous time lags, time series models are expressed as a combination of AutoRegressive (AR) and Moving Average (MA) parameters (Box and Jenkins, 1976) shown in (2.6a,b). Model notation commonly uses the backshift operator

(B) to simplify complicated model terms. It is not a variable, but operates on a variable, as in (2.5a,b), to represent a variable lag according to the power in which B is raised.

$$Bx_t = x_{t-1} \tag{2.5a}$$

$$B^2x_t = x_{t-2} \tag{2.5b}$$

$$\text{MA(1):} \quad x_t = (1 - \theta_1 B)a_t \tag{2.6a}$$

$$\text{AR(2):} \quad (1 - \phi_1 B - \phi_2 B^2)x_t = a_t \tag{2.6b}$$

The AR and MA processes are inversely related by the following equations (2.7a,b,c,d):

$$(1 - \phi_1 B\,)x_t = a_t \tag{2.7a}$$

$$x_t = \frac{1}{(1 - \phi_1 B\,)} a_t \tag{2.7b}$$

Then, by Taylor series expansion in (2.7c):

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad , |x| < 1 \tag{2.7c}$$

the AR(1) model can be represented by an infinite series of exponentially decreasing error, or MA, lags as in (2.7d). In the same way, a MA(1) model is the equivalent of an infinite, exponentially decreasing significant series of AR terms.

$$x_t = (1 + \phi_1 B + \phi_2 B^2 + \phi_3 B^3 + \cdots)a_t \qquad , |\phi_i| < 1 \tag{2.7d}$$

The ACF is controlled by MA processes, and the PACF is controlled by AR processes. Examination of these two functions will provide a thorough basis for analysis of the system behaviour under time dependence, and will suggest the appropriate parameters to include in the model. Model parameters can then be estimated using a technique such as Maximum Liklihood estimation.

2.2.2 Time series application

In Civil Engineering, modelling with ARMA (AutoRegressive Moving Average) processes have been available for years. An ARMA modelling approach has been developed that is specific to stream flow. A complete discussion of this type of modelling is available within Salas et. al. (1980).

Considerable data manipulation is usually required before an ARMA model can be fit. The first manipulation stage is transformation of the data. Monthly stream flows typically demand transformation to create a normal or gaussian data distribution. Stream flow data is generally skewed such that there is a long tail on the high flow side and a non-negative condition on the low flow side of the data distribution. A Box-Cox transformation can be used (2.8a,b), but flows are generally lognormally distributed and can be transformed accordingly by (2.8b).

$$z_t = \frac{1}{\lambda}[x_t^{\lambda} - 1] \qquad , \lambda \neq 0 \qquad -2 \leq \lambda \leq 2 \tag{2.8a}$$

$$z_t = \ln[x_t] \qquad , \lambda = 0 \tag{2.8b}$$

Flows are then deseasonalized, or standardized, to account for variations in monthly mean (2.10) and standard deviation (2.11). Most stream flow series exhibit significant seasonality, or changes in flow characteristics from month to month. Spring flows tend to be high and winter flows tend to be very low. The variance also tends to change. The range of possibilities for flow in spring is much greater than in the winter. Nonparametric deseasonalization uses Fourier analysis to estimate the properties of mean and standard deviation with a smoothing effect from the historical statistics. Another technique is parametric deseasonalization. It simply uses the standard normalized variable, with historical characteristics, to handle seasonality. Both parametric and nonparametric deseasonalization employ the standard variable (2.9):

$$y_{p,\tau} = \frac{z_{p,\tau} - \mu_\tau}{\sigma_\tau} \tag{2.9}$$

$$\mu_\tau = \frac{1}{n}\sum_{p=1}^{n} z_{p,\tau} \tag{2.10}$$

$$\sigma_\tau = \left[\frac{1}{n-1}\sum_{p=1}^{n} (z_{p,\tau} - \mu_\tau)^2 \right]^{\frac{1}{2}} \tag{2.11}$$

Knowing that the historical monthly means and standard errors vary throughout the year, Fourier analysis is used when the engineer expects the true monthly characteristics to display a more smooth transition through the year than the historical approximations. This technique is best suited for when the data set is limited. As the set gets larger, the historical characteristics will approach the true values and Fourier analysis becomes unnecessary. Fourier series estimations of the monthly characteristics are made by defining the means, and standard errors, as a series of harmonics that vary about the annual mean of the statistic in question (2.12).

$$\nu_\tau = \nu_x + \sum_{j=1}^{m} (A_j \cos\lambda_j\tau + B_j \sin\lambda_j\tau) \qquad , \lambda_j = \frac{2\pi j}{\omega}$$
$$A_j = \frac{2}{\omega}\sum_{t=1}^{\omega} (\nu_\tau - \nu_x)\cos\frac{2\pi j\tau}{\omega} \tag{2.12}$$
$$B_j = \frac{2}{\omega}\sum_{t=1}^{\omega} (\nu_\tau - \nu_x)\sin\frac{2\pi j\tau}{\omega} \qquad , \omega = 12 \text{ for monthly data}$$

A sensitivity analysis must also be performed to determine the optimal number of harmonics, m, to be used.

After deseasonalization, the working data set is no longer a stream flow time series, but a set of non-dimensional numbers expressed in terms of number of standard errors from a mean value. The Autocorrelation and Partial Autocorrelation functions of this transformed deseasonalized data set are usually stationary with significant AR terms, and possibly an MA term. Typical model results are: AR(1), AR(2), ARMA(1,1).

2.2.3 Statistical time series approach

Current statistical practices for time series analysis consists of differencing the data to achieve stationary correlation structures. Stationary structures of this kind are usually more complicated than those representing the transformed deseasonalized data set. This is because periodic trends or tendencies are not drowned out with differencing as they are with deseasonalization. Seasonal AutoRegressive Integrated Moving Average (SARIMA) models and other modern variations can be used to model stream flow by correlating recent and seasonal time lags to the present or forecast time lag. Combinations of AR, MA parameters, seasonal AR, MA parameters, seasonal, and nonseasonal differencing produce a SARIMA model with general notation of:

$$\text{SARIMA}(p,d,q)x(P,D,Q)_L \tag{2.13}$$

or

$$\phi(B)\Phi(B^L)(1 - B)^d(1 - B^L)^D x_t = \theta(B)\Theta(B^L)a_t$$

p = no. AR terms P = no. seasonal AR terms

d = degree differencing D = degree seasonal differencing

q = no. MA terms Q = no. seasonal MA terms

L = seasonal lag

Initially, the stream flow data set needs to be transformed in the same manner as deseasonalized ARMA models to produce a gaussian distribution. Differencing the data series without transformation may also produce a gaussian distribution, but does not stipulate strict non-negativity of the flows. This is why transformation should be used.

The transformed data set still exhibits all or most of the seasonal properties of the original data set. The correlation structure is usually non-stationary with a general periodicity that appears in the shape of a cosine series. To produce a stationary series, one with a limited or finite number of significant time lags, data values from previous lags are subtracted from the present lag to give a differenced series. Stream flow series usually require differencing of similar months, seasonal differencing, such as April with April. Typical seasonal differencing for annually periodic data series, such as stream flow, are shown in (2.14, 2.15):

$$X(12): \quad (1 - B^{12})X_t = X_t - X_{t-12} \tag{2.14}$$

$$X(1,12): \quad (1 - B)(1 - B^{12})X_t = X_t - X_{t-1} - X_{t-12} + X_{t-13} \tag{2.15}$$

such that, for X(1,12)

$$E[X_t - X_{t-1} - X_{t-12} + X_{t-13}] = E[X_t - X_{t-12}] + E[-X_{t-1} + X_{t-13}] = 0 \tag{2.16}$$

a stationary series is produced (2.16). This seasonal differencing is an implicit measurement of variation in flows. Subtracting the monthly flow from one year ago produced a series with an expected value of zero. In this way, seasonal differencing allows variation of expected value forecasts for each month in accordance with the historical monthly characteristics, with emphasis on recent behaviour.

Selection of relevant parameters to include in the model has been described in section 2.2.1. Seasonal parameters are standard inclusions in the model structure. Seasonal MA parameters represent the monthly mean, biased toward more recent flows in that particular month. Seasonal AR parameters represent annual trends or patterns in the data. Examples of SARIMA models follow in (2.17) and (2.18). Equation (2.17) has AR terms which are nonmultiplicative. Equation (2.18) is a multiplicative seasonal model with respect to the AR process. Most seasonal models tend to be more representative of the system if they are multiplicative.

$\text{SARIMA}(2,0,0)\text{x}(0,1,1)_{12}$ (2.17)

or

$$(1 - B^{12})(1 - \phi_1 B - \phi_2 B^2)x_t = (1 - \theta_{1,12}B^{12})a_t$$

$\text{SARIMA}(1,1,0)\text{x}(1,1,1)_{12}$ (2.18)

or

$$(1 - B)(1 - B^{12})(1 - \phi_1 B)(1 - \phi_{1,12} B^{12}) x_t = (1 - \theta_{1,12} B^{12}) a_t$$

Developed models are evaluated for model adequacy by three tests. A t-test is used to check the significance of individual parameters. A parameter is accepted if the t-ratio exceeds 2.0. The greater the t-ratio, the more significant a parameter. The actual value of the parameter will be proportional to the t-ratio. If a parameter is statistically insignificant by the t-ratio test, it may be removed from the model structure and the remaining parameters should be recalculated.

A Chi-square test is used to evaluate the validity of the model in general. The probability values associated with the residual autocorrelation Chi-square test should be greater than 5%. The greater the probabilities, the better the test. If one or more of the calculated Chi-square probabilities are less than 0.05 (5%), and the model passes the other tests, then the model structure may be completely inadequate and a new differencing scheme should be found that gives a more stationary correlation structure.

Finally, the AutoCorrelation Function (ACF) and Partial AutoCorrelation Function (PACF) of model residuals can be checked for additional significant lags. Review of these functions will give an indication whether all of the appropriate model parameters have been included. If the residual correlation functions indicate a significant correlation at one of the lags, the relevant parameter can be added to the model structure. Addition of another parameter to the model also tends to increase the Chi-square probabilities.

More than one differencing scheme may produce a reasonably stationary series, so more than one acceptable model structure may be found. These models may be immediately compared by observing any differences in Chi-square probabilities. If one model exhibits considerably greater probabilities, it may be chosen above the others.

To make this choice with a greater degree of confidence, forecasts of previous flows, values already used to define the model, can be made and residuals compared. This kind of forecast is called a backcast. It can be used in conjunction with other tests to verify observations about relative model adequacy. There are occasions, though, where one model may not be recognizably superior to other models.

Care must also be taken that model structures from dissimilar modelling techniques are not directly compared. Deseasonalized ARMA models do not easily compare to SARIMA models with these tests. It is like comparing apples to oranges. Other statistics for model adequacy are also unavailable. The Akaike Information Criterion (AIC) is one statistic that is sometimes used to evaluate relative model improvement from the inclusion of different parameters. It's value is irrelevant for comparing models of varied differencing, or SARIMA models with deseasonalized ARMA models.

2.2.4 Comparison of approaches

There are several differences between ARMA modelling by deseasonalization, and differenced SARIMA models. First of all, the reasoning for deseasonalization is the perceived importance to preserve the historical monthly characteristics of mean and variance. Once this priority condition is met, testing for additional information within the data can begin. Unfortunately, in deseasonalizing the flow, the series is no longer in terms of discharge but expressed as a number of standard errors from a mean value. This usually drowns out any seasonal information not strictly related to the mean and variance. In this way, a significant portion of information available within the data may be lost. Models of this type display stationary correlation structures without differencing. They are usually restricted to recent lags such as one, or two time steps to predict the next flows.

Deseasonalized ARMA models produce expected value forecasts, but are not available to supply extreme value forecasts. The historical characteristics of the data series are preserved, so that an extreme forecast for an individual month is historically, statistically correct, but it is not a conditional forecast according to the information already available. If low percentile extreme forecast flow volumes are summed for all of the months in the forecast interval, the total volume will be much more extreme than percentile for the interval history. This is because the probability of extreme values occurring in every single month, over the duration, is very low. For this reason, an extreme forecast may be used for a single month, but has decreasing validity as the forecast interval increases.

The differenced flow models are obtained in the same way as the deseasonalized models, except the transformed data set is differenced instead of being deseasonalized. For stream flows, differencing of the present flow with lag 12 usually represents the annual periodicity, or mean, that states March flows tend to be like other March flows. A number of differencing arrangements can be tested to generate one or more correlation structures which are recognizably stationary. Monthly characteristics of the stream are not explicitly defined, but a stationary, seasonally differenced series will ensure that forecasted flows for March will be similar to historical March flows. The hope in using a SARIMA model is that any possible annual trend or pattern in flows will be recognized in addition to flow dependence on recent flows of one or two lags. The ability to detect annual or seasonal behaviour trends is the advantage SARIMA has over a deseasonalized ARMA model which tends to drown out these effects through extensive data manipulation. These kinds of trends may be due to a number of natural occurrences. Meteorological patterns may cause alternating wet and dry years, a progressive warming trend, or some system flushing effect every few years. Regardless of the actual physical or meteorological processes, the effects on flow can be detected with the correlation structure within the SARIMA "black box".

The weakness in applying SARIMA models to produce water supply forecasts is that probabilistic forecasts are usually required to produce risk-based reservoir operating policies. It is important that basic minimum needs are met. Overestimation of flows within the forecasts are very costly. Underestimation is rewarded with excess water for such things as power production, or irrigation. For this reason, a "Price is Right" mentality emerges where forecasts of the flow attempt to come close to the actual value without going over. Low percentile forecasts are required by utilities like Manitoba Hydro to produce risk-based policies. SARIMA models, in their most basic format, assume constant variance throughout the year. This produces extreme forecasts that are meaningless for most months of the year, and ridiculous for annual system flow volumes for the same reason as deseasonalized ARMA models. In this way extreme forecasts for SARIMA models are irrelevant on a monthly as well as an annual scale, where deseasonalized ARMA models are also invalid, but are still statistically meaningful for an independent study of a single month.

2.2.5 Forecast selection

Based on MSE analysis, and visual observations of forecast series of several available models, models can be ranked for each flow scenario to generate simple guidelines for selecting a specific model given the flow conditions. This is the forecast strategy that will be used in this project to produce forecast improvements over single forecast model development.

Another available approach is weighted model combinations. Research has been conducted to produce various ingenious methods of weighting each model to reduce the overall forecast error (McLeod et. al., 1987; Newbold and Granger, 1974). The general formulation of a weighted forecast, f_c, for k model forecasts is shown in (2.19):

$$f_c = \sum_{i=1}^{k} w_i f_i \tag{2.19}$$

The motivation behind the use of combinations is that models produced from different approaches will focus on different aspects of information available within the data. If more than one model is used to make forecasts, then a more complete understanding of the system processes will be achieved. Ultimately, the more models that are used the closer the forecast should be to the actual value. However, in water supply forecasting, there are a limited number of models available, and they will usually all overestimate the flow, or they will all underestimate the flow as in Figure 2. In this case no combination, under any weighting scheme, will provide a better forecast than the closest single model forecast.

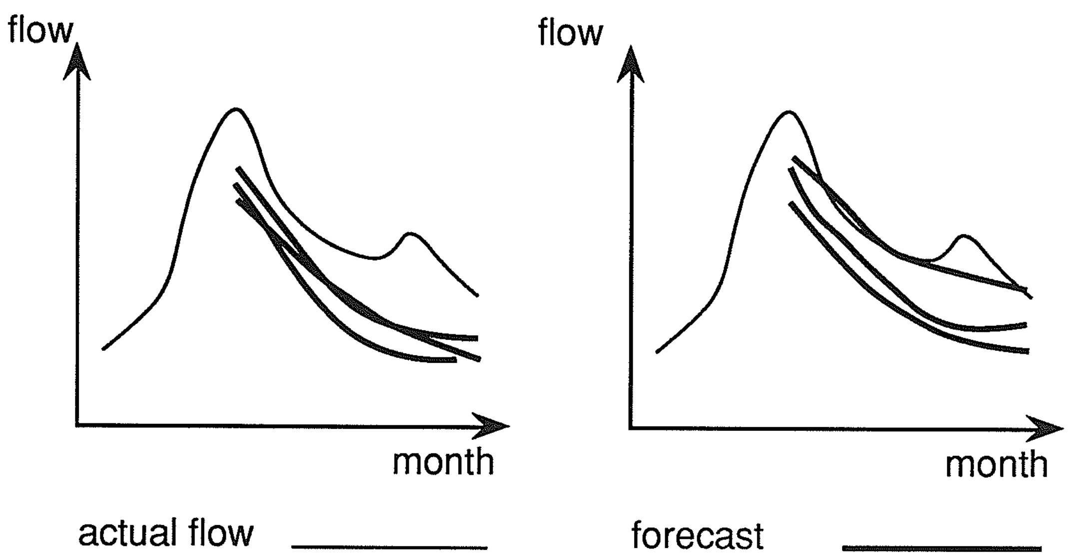


Figure 2. Selection of forecast model

2.2.6 Current forecasting practices

The long-range flow forecasting techniques that large utilities have developed are widely varying. Several Canadian utilities are presently developing statistical time series models for long-range forecasts using SARIMA models, but most methods still tend to be empirical manipulations of traditional engineering tools such as multiple linear regression, stepwise regression, curvilinear regression, simulation, and other less common methods such as principal component analysis, and pattern recognition techniques (Shafer and Huddleston, 1984). These empirical techniques are designed around the engineer's experience with the physical processes at work. Providing forecasts in the long term requires discharge state estimation for several time steps ahead, usually months. The difficulty lies in the fact that forecast accuracy decreases as the number of time lags increases, and the present state becomes less of a factor in determining future states.

Probabilistic forecasts are typically required from long-term forecasting procedures. Presently used methods formulate the modelling tools within a framework such that some determination of distribution is possible. However, the statistical significance and validity of extreme forecasts is difficult to verify for methods or procedures that are not standard. Care must be taken to produce extreme monthly forecasts that also produce annual or seasonal flow volumes that are meaningful to the system. Disagreggation processes have been accepted as the basis for stream flow generation, but this approach may also be a viable method of ensuring long range forecasts are valuable to the engineer.

Our ability to forecast has been improving, but future improvements appear to be limited. Forecast errors generally vary as a function of flow variation (Shafer and Huddleston, 1984). Certainly, there is a limited amount of information that can be extracted from a set of data, regardless of the complexity or quality of the model. These limitations can be extended by combining several data sets or data types. Perhaps we are approaching the limit of usable information from a single data source, but it is only recently that more extensive data sets have emerged. There has not been a clearly defined procedure or technique that will best suit the expanding databases that are quickly becoming available. Forecast accuracy will improve with an increase in available relevant hydrometeorological information. This will also be limited as the errors approach nil because we will always experience an element of randomness from Nature. Until a suitably flexible forecast modelling tool and framework becomes standard, this limitation of total possible system information will not be constraining.

2.2.7 Review of current practices

2.2.7.1 Manitoba Hydro

Long range forecasting is presently done for the Manitoba Hydro system streams using linear regression models as the basic mathematical tool, equating historical water volumes for the forecast interval to the month by month serial flow volume regressions. The Manitoba Hydro method is a disaggregation process by definition, and acts in a sequential manner from month to month as an AR(1) model, defined as the Serial Correlation, that is controlled by the annual historical characteristics, or Period Correlation (Fig. 3).

The Period Correlation, is used to correlate the response of the forecast period historical flow volume with the most recent month on record. Expected value forecasts are made for the months in the forecast interval by setting the sum of forecasted monthly volumes equal to the annual or period volume.

Extreme percentile forecasts can be obtained by moving a number of standard deviations from the expected line on the Period Correlation, forcing the Serial Correlations to forecast flows in compliance with the annual volume. That is, forecasts for each month are dependent on the previous one and extremities are dampened by knowing the range of annual volumes. Both the Period Correlation and the Serial Correlations are functions of the last known month of flow.

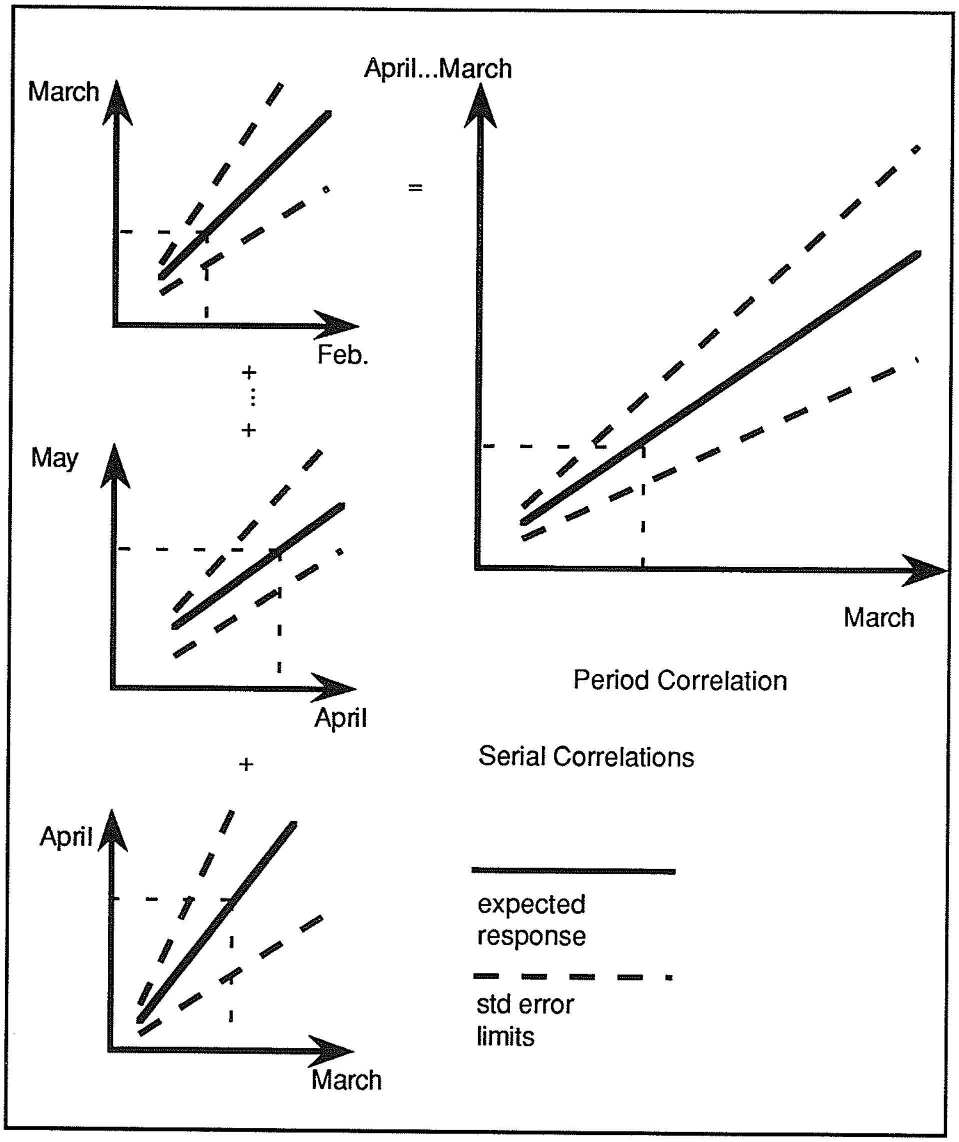


Figure 3. Manitoba Hydro long-range forecast method

In practice, other sources of information are used to enhance the engineer's knowledge about system conditions. They are the Manitoba Water Resources Branch, Saskatchewan Water Corporation, and Alberta River Forecast Center. This additional information is used to manually adjust the model forecasts.

2.2.7.2 BC Hydro

BC Hydro produces real-time forecasts with a conceptual rainfall-runoff model, and long range forecasts of monthly data with a multiple linear regression model called VOLCAST (Fast, 1990). A comprehensive data set is gathered. In addition to natural inflows: snowpack information, precipitation levels, and maximum monthly temperatures are used. This monthly data is organized into a set of five indices that are used as variables to forecast the stream flow at a number of gauging station locations for particular months. In this way, the total number of multiple linear regressions is 12n, where n equals the number of stations and there are 12 months in a year. Figure 4 depicts the general forecast procedure.

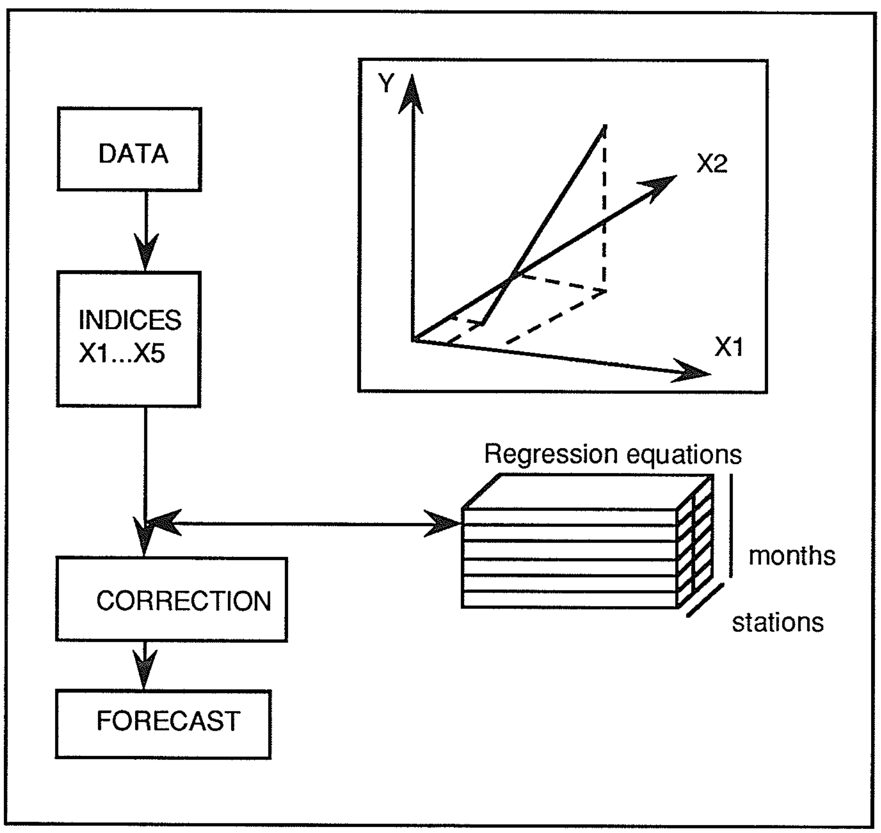


Figure 4. BC Hydro long-range forecast method

Inclusion of snowpack, precipitation, and temperature data appear to significantly improve the forecasting ability of BC Hydro. For snowmelt data in particular, forecast errors tend to decrease as inflow becomes more dependent on snowmelt rather than precipitation.

2.2.7.3 Quebec Hydro

Another technique used to produce a probabilistic forecast is simulation. Quebec Hydro uses a conceptual model with precipitation and temperature data to generate series of natural inflows from historical sequences of meteorological input (see Fig. 5). The generated series are then statistically analyzed to infer a probabilistic forecast. Care is taken to preserve the proper hydrograph shape.

2.2.7.4 Ontario Hydro

Ontario Hydro provides an example of several engineering tools being used in combination. Long range forecasting at Ontario Hydro compares records of flows to choose a flow sequence from the history of record (Tao, 1991). A probabilistic forecast for each day up to 400 days is made based on comparison of volumes or peak flows. Ontario Hydro's forecasting technique ranges from short-term expected forecasts, through medium range heuristic forecasts, to the long-range probabilistic forecasts. Short-term forecasts are made using either a conceptual model or a time series approach for daily forecasts up to 4 days. To bridge the gap between short and long-term forecasts, medium range heuristic forecasts use polynomial regressions to provide a smooth transition from the two extremes.

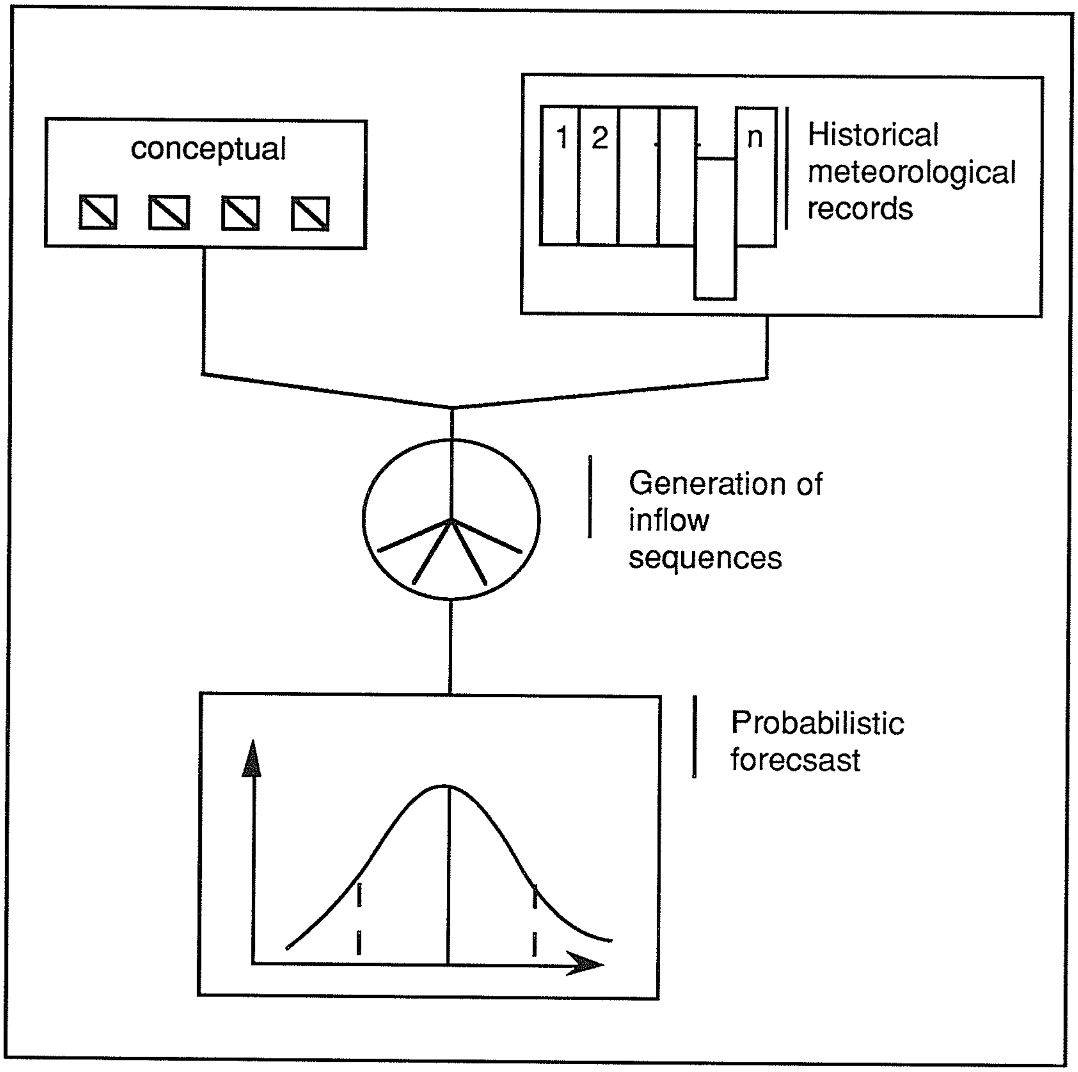


Figure 5. Quebec Hydro long-range forecasting method

3 CASE STUDY

3.1 Requirements

The varying techniques of these large utilities are driven by the engineer's understanding of the system behaviour and forecasting needs. For example, BC Hydro incorporates a glacial melt index. Because of the mountainous terrain in BC and the resulting fast runoff, flooding of valleys is a concern. BC Hydro has tailored their forecasting capabilities to cope with such problems by incorporating other indicators such as precipitation and temperature. Manitoba Hydro differs in that linear regression of historical flows is used to forecast long range flows. Real-time or short-term forecasts have limited applicability for water supply forecasting in general because of the lack of sensitivity to the timing of natural inflows due to the large reservoir storage capacity in relation to the size of the inflows.

Techniques for forecasting will differ due to the varying physical demands of the system. Manitoba Hydro regulates flow over a large basin. Streamflow gauges are supplied at key locations. Precipitation and soil moisture information is also available, but only for general use to provide indications of relative physical system conditions. The addition of more gauges may produce a reliable source of data for modelling and forecasting. However, much of the flow originates from outside the province. Cooperation with adjacent provinces and states is required to secure a reliable source of data to improve the linear regression forecasting model capabilities.

3.2 Inflow sources

There are three identifiable types of natural inflow to the Manitoba Hydro system. They are: streams with considerable long-term upstream storage such as a series of small lakes; streams with relatively little upstream storage dependent on the water supplied by overland flow and spring runoff from snowmelt; and inflow data consisting of various, minor processes.

Three data sets were chosen for analysis, one representing each type of natural inflow. All three of the data sets are modelled and their forecasts are compared with forecasts from Manitoba Hydro. The data is in terms of monthly average flow (kcfs) and each data set is a minimum of 30 years of historical record.

The Grass River basin was selected as an example of a river with considerable upstream storage. Measured at Standing Stone Falls, it is a small river that connects a series of lakes and eventually drains into the Nelson River in northern Manitoba. These lakes act as capacitors in an electrical circuit. Effects from precipitation events are dampened by the lake storage, reducing the monthly variation of flow volumes.

The Red River was chosen to be examined as an example of a river basin with relatively little upstream storage capacity, dependent on precipitation and spring snowmelt runoff. The Red River data set consists of measurements taken at Lockport, a control structure north of the city of Winnipeg. This river drains a large basin immediately north and west of the Mississippi River head waters, entering Manitoba at the North Dakota, USA, border and ending at Lake Winnipeg. Behaviour of the Red River is very seasonal. Half of the annual flow volume occurs in the 2 peak months of April and May. Where standard errors in peak months on the Red River are nearly as great as the mean flow, the Grass River standard errors are only half of the mean.

The third data set examined is the Partial Inflow Available for Outflow to Lake Winnipeg (PIAO) data set. It consists of left over processes and inflow to Lake Winnipeg after all known stream flow sources are subtracted from the total inflow of the lake. The series appears as a random or white noise process with annual mean of zero. The data consists of both positive and negative values because the data set is based upon changes in storage. Negative values usually occur in the summer or fall seasons when depleting processes such as evaporation outweigh the runoff from precipitation events.

3.3 Model development

3.3.1 Grass River

The Grass River data is a typical flow series. The monthly average flow means and variances for each month are smooth in transition with the peak flow and peak variance usually occuring in July. The data set was transformed by taking logarithms to produce a gaussian distribution. Some skewness still remained, but was reasonably low and the non-negativity condition was imposed.

Two general approaches were used to produce a stationary correlation structure, as previously described. Nonparametric deseasonalization is one technique that was used. The transformed deseasonalized data exhibited a stationary correlation structure with nonstationary exponentially decreasing ACF, and a significant lag one PACF. For a review of SAS output, see Appendix E.

The best resulting model structure, given the name GTD (3.1), is an ARMA(1,1). It passed all three statistical tests, including very high Chi-square probabilities: indicating a good fit. The model equation is:

$$(1 - 0.93B)X_t = (1 + 0.24B)a_t \tag{3.1}$$

Removing seasonality with differencing produced two reasonably stationary correlation structures (3.2a,b) with the following differencing schemes:

$$(1 - B^{12})X_t = a_t \tag{3.2a}$$

$$(1 - B)(1 - B^{12})X_t = a_t \tag{3.2b}$$

A SARIMA model was produced for each of these differenced series. The model in (3.3, 3.4) was produced from (3.2a).

$$\text{SARIMA}(2,0,0)\text{x}(0,1,1) \tag{3.3}$$

It passed all 3 tests of t-ratio test, Chi-square model adequacy test, and residual correlation functions. The equation for this model, called GS1, is:

$$(1 - B^{12})(1 - 1.32B + 0.392B^2)X_t = (1 - 0.91B^{12})a_t \tag{3.4}$$

The AR lag 1 parameter is greater than 1.0 but the model appears stable despite this. From (3.2b), the model structure GS2 (3.5) was found:

$$\text{SARIMA}(1,1,1)\text{x}(0,1,1) \tag{3.5}$$

and although it is reasonably adequate, the first Chi-square probability did not meet the 5% requirement. This is a minor violation, and the model is still acceptable (3.6).

$$(1\text{-}B)(1\text{-}B^{12})(1 + 0.175B^3)X_t = (1 + 0.38B)(1 - 0.92B^{12})a_t \tag{3.6}$$

Backcasts were produced for GS1, GS2, but no clearly superior model was chosen. A complete summary of SAS output for the models is provided in Appendix E.

3.3.2 Red River

The Red River data is similar to the Grass River, except that spring runoff produces a dominate peak. Transformation, deseasonalization, and differencing procedures are similar to the Grass River. Nonparametric deseasonalization produced RTD (3.7), given as:

$$\text{AR(2):} \quad (1 - 0.646B - 0.12B^2)X_t = a_t \tag{3.7}$$

It passed all statistical tests of t-ratios, Chi-square probabilities, and residual correlations.

Differencing produced two SARIMA models, RS1 (3.8), and RS2 (3.9) respectively:

$$\text{SARIMA}(1,0,0)\text{x}(1,1,1) \tag{3.8}$$

or

$$(1 - B^{12})(1 - 0.75B)(1 + 0.114B^{12})X_t = (1 - 0.845B^{12})a_t$$

for RS1, and

$$\text{SARIMA}(1,1,1)\text{x}(1,1,1) \tag{3.9}$$

or

$$(1 - B)(1 - B^{12})(1 - 0.64B)(1 + 0.14B^{12})X_t = (1 - 0.92B)(1 - 0.85B^{12})a_t$$

for RS2. Examination of the Chi-square probabilities and backcasts suggested that RS2 is superior to RS1. RS1 was then discarded. The two remaining models are RTD, RS2. A complete summary is available in Appendix E.

3.3.3 PIAO

The PIAO data set is not a typical flow series. Because both negative and positive values exist, and the series appears to be a white noise process, no transformation was used. Examination of seasonality with differencing produced the model WS1 (3.10).

$$\text{SARIMA}(0,0,0)\text{x}(0,1,1) \tag{3.10}$$

or

$$(1 - B^{12})X_t = (1 - 0.87B^{12})a_t - 0.288$$

The constant, 0.288, is necessary to adjust the forecast because the mean value of the differenced series is significantly different from zero. Very little significance was evident for sequential lags of 1 or 2. The only correlation between time lags was seasonal. Since deseasonalization will yield an ARMA model with only recent lags as significant, a TD model was not produced.

3.4 Model comparisons

The modelling techniques previously described generally yield several models for each data set which cannot be differentiated. For example, there are three available models for the Grass River. In the case of the Red River, one of the models was noticeably inferior and was subsequently removed from the list, but there are still two models to choose from. Table 1 summarizes the models available for forecasting.

TABLE 1. Summary of available forecast models

dataset	model	notation	AR lag	MA lag	diff.
Grass	GTD	ARMA(1,1)	1	1	0
	GS1	SARIMA(2,0,0)x(0,1,1)	1,2	12	12
	GS2	SARIMA(1,1,1)x(0,1,1)	3	1,12	1,12
Red	RTD	AR(2)	1,2	0	0
	RS2	SARIMA(1,1,1)x(1,1,1)	1,12	1,12	1,12
PIAO	WS1	SARIMA(0,0,0)x(0,1,1)	0	12	12

3.4.1 Sensitivity analysis

Further examination is necessary to differentiate between possible forecast models. One way in which this can be accomplished is to produce a number of forecasts and then compare the errors (Oron et. al., 1991). This was initially done in the form of backcasts, but was inadequate to show superior or inferior models. A more comprehensive forecast comparison plan is required.

River flows change from year to year. They range from high flow years to years of drought. It may be that the models developed in the previous section will vary in effectiveness for different flow scenarios. A model sensitivity analysis to flow will produce a large number of forecasts to compare, and evaluate the sensitivity of the models to various flow conditions to provide an in-depth understanding of model performance.

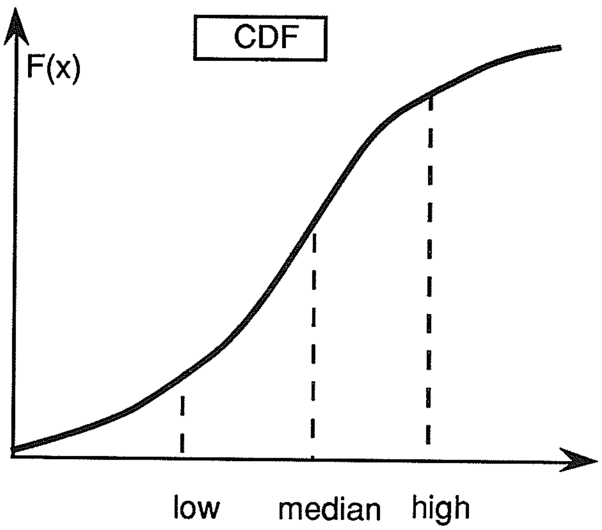


Figure 6. Selection of model sensitivity analysis years

For the Grass and Red rivers, historical annual volumes were plotted. From that, recent years were selected that represent high, median, and low flow years, such as in Figure 6. Care was taken to choose recent years in the record so that when forecasts are made, they are based on a sufficient number of previous flows. Table 2 lists the years chosen to represent the flow conditions.

TABLE 2. Case years for sensitivity analysis

flow	Grass	Red	PIAO
high	1985	1979	
median	1982	1983	
low	1990	1990	1990

3.4.2 Forecast scenario

Once relevant years have been chosen for forecasting, a specific forecast procedure or scenario must be defined. This should be specific to the system needs. It may be that models behave differently when the demands on them vary. A modelling approach represents a certain portion of the system. If a model represented all of the system, there would be no error. However, there is always some error component within the model. Depending on the focus of the system needs, and system characteristics, the model performances may be sensitive to the forecast scenario.

For this project, the data was supplied by Manitoba Hydro and is presently forecasted within their system. Therefore, the project forecast scenario should be consistent with MH. That way, MH forecasts can be compared with these models and any possibility of incompatability or inconsistency is removed. Forecast comparisons, then, can only be made when the forecast scenario, or the demands, are consistent.

The MH forecast scenario consists of a series of updates over a period of a year that begins with March being the last known month of the previous flow year (Fig. 7). That is, forecasts run from April to April and are updated every month as new data is available. For one forecast year to be completed, 12 forecast runs range from 12 lags to 1 lag in length. In all, 78 forecasts are made for one forecast year, and 234 individual forecasts are required to complete the sensitivity analysis for one data set to various flow conditions.

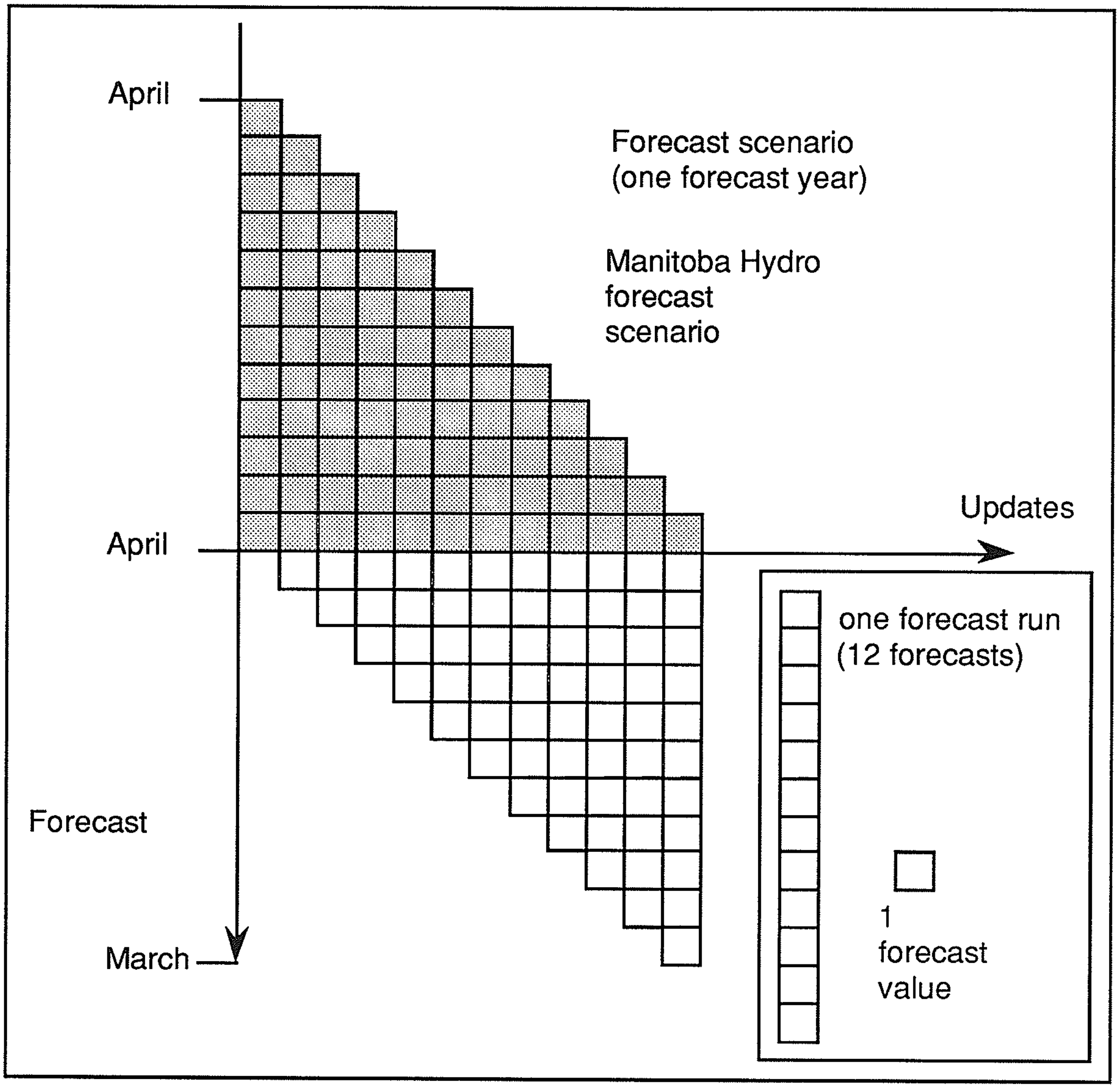


Figure 7. Forecast scenario

3.4.3 Error analysis

After forecasts are completed for the Grass and Red rivers, errors are analyzed to give an efficient review of model performance. Visual examination of actual forecasts is beneficial in making observations on model behaviour concerning the appropriateness of the model curve characteristics, or ability to handle large system fluxes such as spring runoff. Because the model parameters are recalculated for each update within each forecast run, variations in the parameters should also be evaluated by statistical confidence limits to ensure stability and consistency of the parameters. A t-test of each model verified that the model parameters are stable.

The Mean Squared Error (MSE) is a statistical parameter that provides a convenient measure of performance in the same units and magnitude as variance.

$$MSE = \frac{SSE}{n} = \frac{1}{n}\sum_{i=1}^{n} (\hat{x}_i - x_i)^2 \tag{3.11}$$

To ensure a thorough understanding of model performance from the errors, the MSE is calculated in three ways. The first is to calculate MSE for each forecast run (Fig. 8). Twelve MSE values are available for each model, one for each run. MSE is a function of average variance to the end of the forecast year. That is, after the high spring and summer flows the errors should drop because variance in the fall and winter months is low.

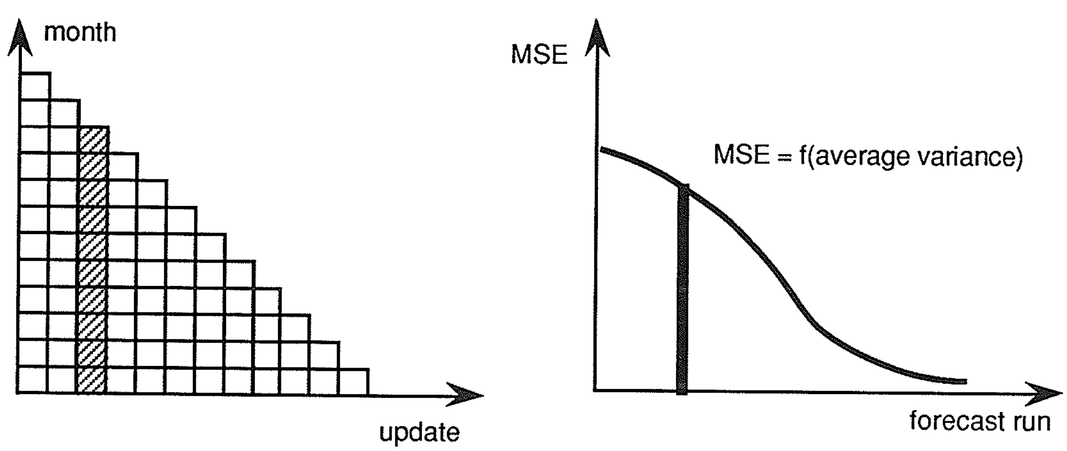


Figure 8. MSE comparison: by forecast run

Another way MSE is evaluated is by month (Fig. 9). A number of forecast errors are available for each month of the year because the same month is included in a number of forecast runs, where the lag depends on which forecast run is relevant. MSE values change with the variance of each particular month.

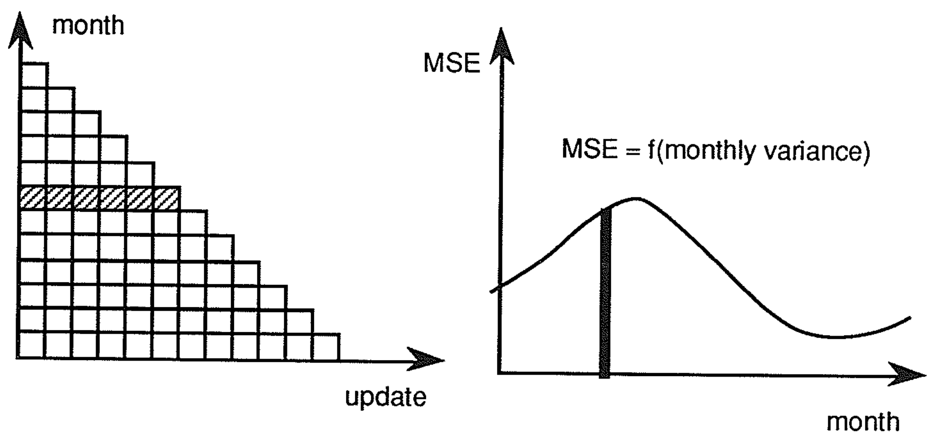


Figure 9. MSE comparison: by month

Lastly, MSE is calculated for specific lags in the forecast runs (Fig. 10), so MSE is available for each lag (1 to 12). In general, MSE should increase with increased lag. This is often offset by the fall and winter lags which have comparatively low errors regardless of the lag.

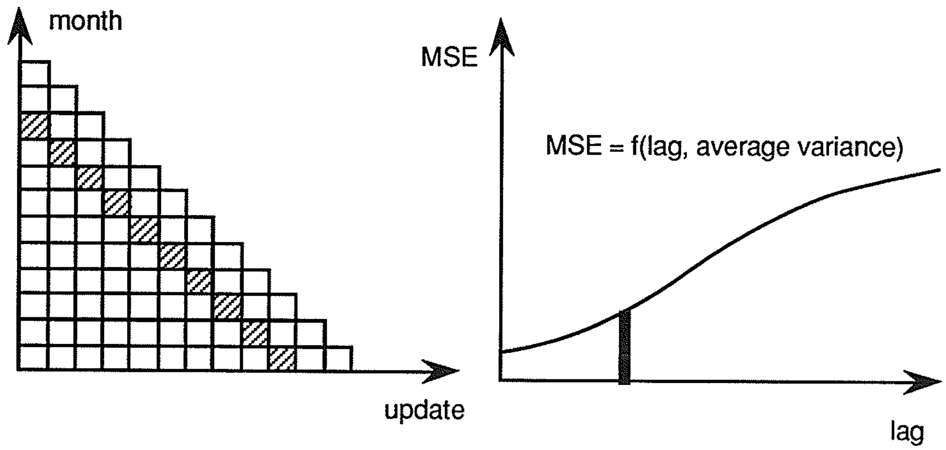


Figure 10. MSE comparison: by forecast lag

Once the MSE analysis is complete, generalizations can be made concerning choosing the appropriate model given the flow conditions. Choices need to be made for the Grass River and the Red River.

3.5 Case study observations

3.5.1 Evaluation criteria

The error analysis was completed for each model within the forecast scenario previously described. Model selection is based on minimum MSE, and observations on general forecast curve characteristics. Special attention needs to be focused upon tendancies to overestimate or underestimate the flow. This is important because of possible shortages resulting from allowing reservoir releases to be too high when forecasts overestimate the flow.

3.5.2 Grass River

There are three time series models developed for the Grass River. One is a transformed deseasonalized (TD) type model, GTD. Two seasonal models were also selected for forecasting: GS1, and GS2. GS1 indicates dependence on the 2 most recent months as well as the seasonal distinction of each month with a differencing of the lag 12 flows, and seasonal MA term. GS2 uses differencing of both lag one and lag 12 to produce a stationary structure. A lag 3 AR term is also included. Its significance is not especially high, but it does suggest the effect of the lakes on the travel time through the basin. Or, it may suggest other natural tendencies within the forecast year that have a period of 3 months.

The two seasonal models had difficulty approximating the flow curve characteristics as flow decreases month by month from the spring peak. In the median and high years, they both tended to overestimate the flow from the peak to the end of the forecast year. This is evident in plots of the forecast run for the high flow case. For these high flows, and also median flows, the GTD model performed much better, fitting the general curve shape, leaving relatively small errors, and only slightly overestimating the flow in places. Error analysis for the Grass River high flow case can be found in Appendix A.2. MSE comparisons, for individual months in particular, shows the dominance of the GTD model over the other time series models in that flow case. Median flow case analysis for the Grass River is in Appendix A.3.

The low year case, 1990, was preceded by another low flow year (Appendix A.4). Together, these 2 years were the lowest combination in history. The GTD model did poorly in this case because recent flows before the forecast years were several standard deviations below the monthly means. No model did well in this case, but Manitoba Hydro forecasts appear to do better than GS2 which was the most successful time series model for that case.

3.5.3 Red River

Two models were selected for forecasting of the Red River: RTD and RS2. The TD type model is an AR(2) which states the two most recent months affect the current month. The SARIMA model for Red River, RS2, is seasonally differenced by both lag one and lag 12, the same differencing as with GS2. The lag one AR term shows dependence on recent flows, and the lag 12 term indicates a possible seasonal trend or dependence. The MA terms of lag one and 12 show an exponentially decreasing dependence on both recent lags and recent years. The MA lag 12 term is usually included in seasonal models as a weighted monthly mean, but the AR lag 12 term suggests the possibility of annual trends otherwise neglected by techniques such as the standard TD civil engineering approach. Forecasts for the 3 flow cases confirmed this observation (Appendix B.2, B.3, B.4). The seasonal model, RS2, consistently outperformed RTD forecasts in all cases, and Manitoba Hydro forecasts for the critical low flow year.

Both theoretical model types, and the MH technique failed to predict the spring runoff. The runoff from snowmelt appears to have no correlation with previous flow. An examination of dependence of the peak flow (Appendix B.1) for plots of peak flow for the Red River) found no month, year, or combination with which a dependence could be associated. This inability to predict the peak flows in the spring, from stream flow data alone, is a major weakness in the models. On the average the spring runoff surge accounts for nearly half of the annual flow volume in the Red River. The two spring months of April and May exhibit the greatest forecast errors, as well as the greatest variability in flow.

In order to improve forecasts of the peak, other physical data such as snowpack levels, and soil moisture indicators are needed. In practice, the MH technique uses the Manitoba Water Resources Branch flow forecasts to establish a volume forecast for the spring snow melt runoff. In this way, Manitoba Hydro is able to incorporate some of the physical dependencies such as snowpack into forecasts. However, the mathematical models alone, time series, regression, or other, are unable to estimate the peak from flow data only.

3.5.4 PIAO

The SARIMA model for PIAO required the estimation of only one parameter (MA lag 12). This suggests that each month is distinct and that recent lags have little or no effect on the outcome of the present state of the system. The seasonal differencing of the model reinforces the annual dependence, and independence of the months.

The lack of significance from recent lags is evident in the forecast runs for the low flow case of 1990 (Appendix C.2). As updates are made for each forecast run, the forecasts for the upcoming months are largely unaffected. The MH regression procedure also shows little dependence on recent lags. For the MH method, the month to month Serial Correlation is not available but simply set to their historical characteristics of mean and standard deviation. Manitoba Hydro does exhibit some adaptability in updates for lag one forecasts. This is due to the inherent AR(1) form of the MH forecasts. SARIMA forecasts were generally comparable to MH forecasts. However, MH forecasts were able to adapt to a low period during the year for the one step ahead forecast while more distant forecasts quickly reverted to the historical mean. SARIMA model forecasts were completely insensitive to new developments throughout the forecast year.

3.6 Model selections

If it is possible to determine which model produces the best forecast under a general array of flow possibilities, then that model should be used. Table 3 summarizes the time series model rankings for the case study, including Manitoba Hydro rankings where they are available. Another way of showing the ranks of the models is in Figure 11 below for the Grass River models. From Table 3, a simple set of rules can be derived to govern model selection.

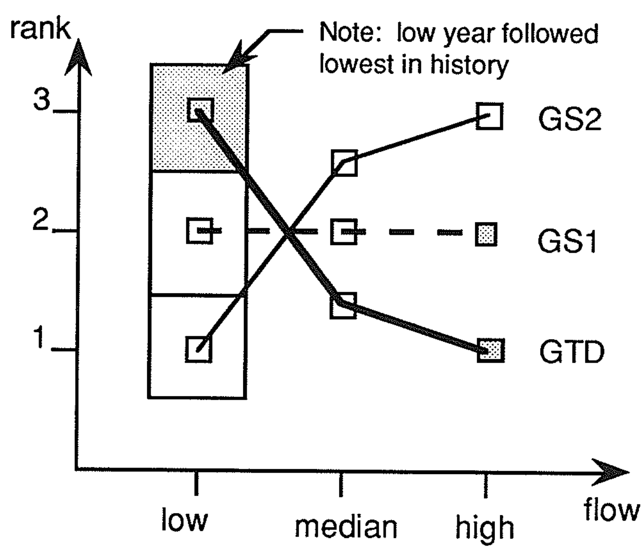


Figure 11. Ranking of the Grass River models

TABLE 3. Ranking of forecasting models

dataset	flow	year	rank						
			GS1	GS2	GTD	RS2	RTD	WS1	MH
Grass	high	1985	2	3	1				
	median	1982	2	3	1				
	low	1990	2	1	3				1
Red	high	1979				1	1		
	median	1983				1	2		
	low	1990				1	2		3
PIAO	low	1990						1	1

Now the fuzzy terms of high, median, and low flow need to be defined so the engineer can differentiate between them. One suggestion is to define median monthly flow as being within 0.5 standard deviations of the historical monthly mean. Then median flow over a period of months would demand the average flow to be within 0.5 standard deviations of the average mean over the relevant period of months. The high and low flows are then above the 0.5 standard deviation threshold on the positive or negative side. This project, for the decision support application in the following section, allows the user to make a qualitative judgement without restrictions of statistical limits .

For PIAO, WS1 is the only available time series model. A TD type model was not attempted because of the limited correlation found in the data. If one had been developed, it would have been an AR(1) that converged to the historical mean after 2 or 3 lags. This is the same behaviour shown by the MH forecasts. PIAO is largely a random series of minor processes.

In all flow cases, RS2 outperformed both RTD and MH forecasts. It should be chosen in all situations, but care should be taken if the spring runoff has not yet occurred.

Additional information would be beneficial for this river because of the dependence of the annual flow volume on the two peak months of April and May. April is the typical peak month, but late springs will shift the peak to May.

The median and high flow cases for the Grass River are best forecasted by GTD. There is some question about the low flow case because the previous year was also very low. The MH method performed well for the low case. Of the time series models, a safe choice may be GS1 in the low flow case if the peak flow is still uncertain.

4 DECISION SUPPORT APPLICATION

4.1 Purpose

As a means of documenting the techniques, procedures, and accumulated knowledge in time series analysis, a prototype system was developed to aid users in understanding and performing time series analysis for water supply forecasting. The model selection process and observations concerning the quality of forecasts suggests the use of a rule-based system incorporating artificial intelligence techniques to represent the knowledge.

4.2 System

A Unix workstation, SUN Sparc Station 1+, was chosen to develop the system. It enabled the execution of multiple programs and displays for a more flexible environment. Networking capabilities of the Unix workstation provided the ability to transfer data or knowledge to and from numerous sources.

4.3 Tools

A user interface and system development tool was then chosen. Nexpert Object is an expert system development tool that provides the means by which rules can be written to: control the analysis process; execute external programs and packages to perform various tasks; and control the display of text and questions for the user. Other tools used to develop the system are SAS (Statistical Analysis System), Xgraph, Unix scripts, and

FORTRAN programs. SAS is the primary tool used to perform statistical analysis of the data. Xgraph is a plotting tool that graphs a specified set of data within the X-window graphical environment. It is useful in plotting forecasts or historical series of data. Unix scripts are analogous to DOS batch files. A script is a text file consisting of a series of operating system commands that can be executed by the operating system. Several of these files are used to control the formation of input files and execution of both SAS and Xgraph applications. Numerous FORTRAN programs control data manipulation such as transformation, deseasonalization, and transfer of data to other formats for input or display. Summaries and printouts of programs can be found in Appendix D.2. Arrangement of these tools is shown in Figure 12. They can be classified into two categories: control, and execution. Nexpert Object uses a knowledge base, stored within the application, to control access and execution of the programming, statistical, and display tools on the right of the figure.

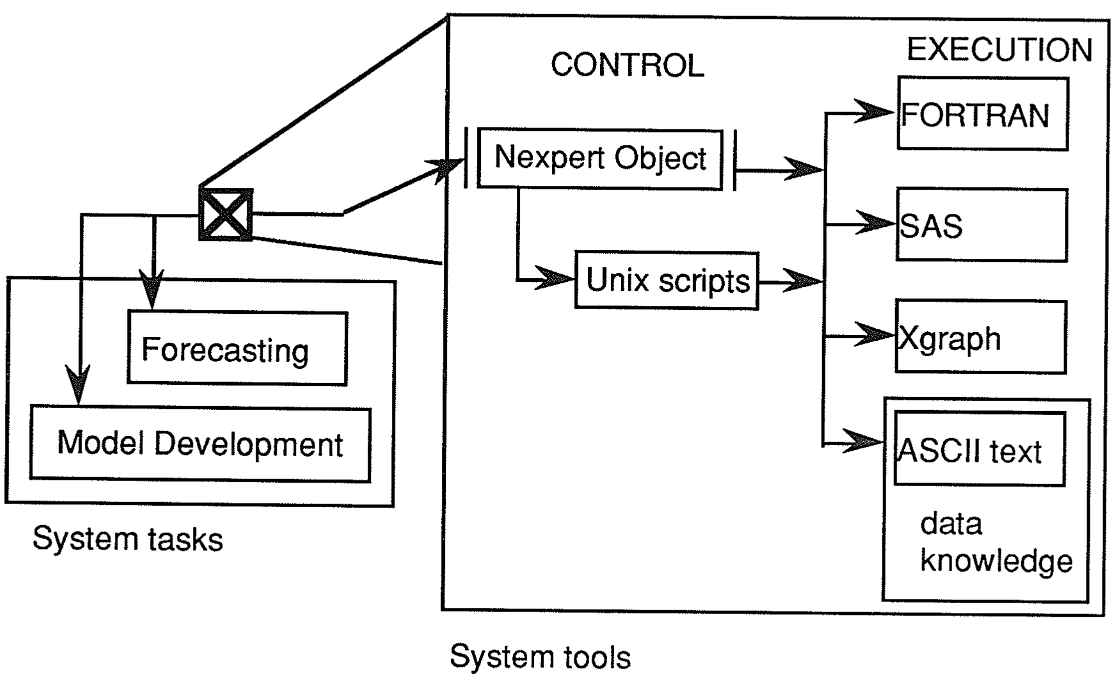


Figure 12. DSA for water supply long-range forecasting tasks and tools

4.4 User modes

Nexpert Object controls the display from the session control window which asks questions, and provides answer options. Text windows, defined within Nexpert, control the display of textual instructions and discussion. The decision support application has two specific modes or tasks (Figure 12). The first, forecasting, queries information from the user for choosing a specific forecast model. Forecasts are then automatically produced for the chosen model. Figure 13 represents the process of producing a forecast within the application.

The second task is designed to aid the user in the development of a new model. This model development mode provides the user with relevant knowledge and instruction to produce an adequate model using one of the time series analysis procedures discussed in this paper (Figure 14). The system allows the user to produce advanced time series models and forecasts of data without having to be an expert in this field of study.

4.5 Knowledge base

A set of If...Then rules control the direction of the application. By organizing the knowledge in this fashion, the developer can restrict actions to knowledge that is relevant. Model rankings for the study cases supplied Nexpert with a straight-forward means by which the rules could be built. There are four different types of rules: to control direction of the application; to assign properties describing the system; to govern the selection of a model; and to check statistical tests for the system or model. A listing of all of the rules within the Water Supply Forecasting DSA can be found in Appendix D.1.

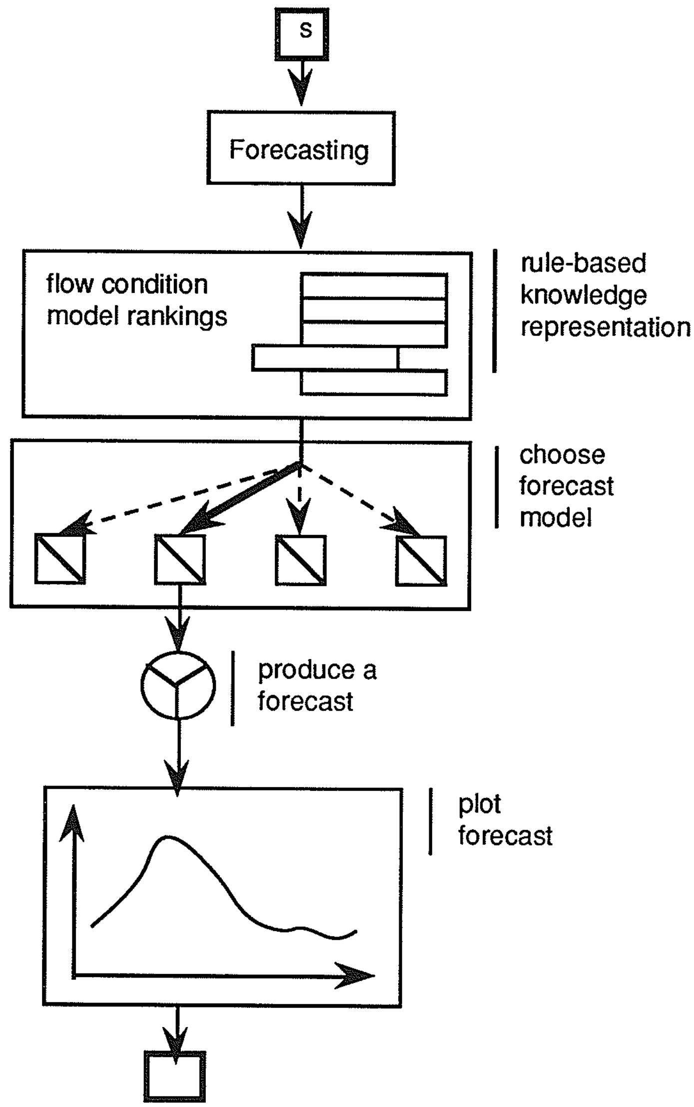


Figure 13. Forecasting user mode

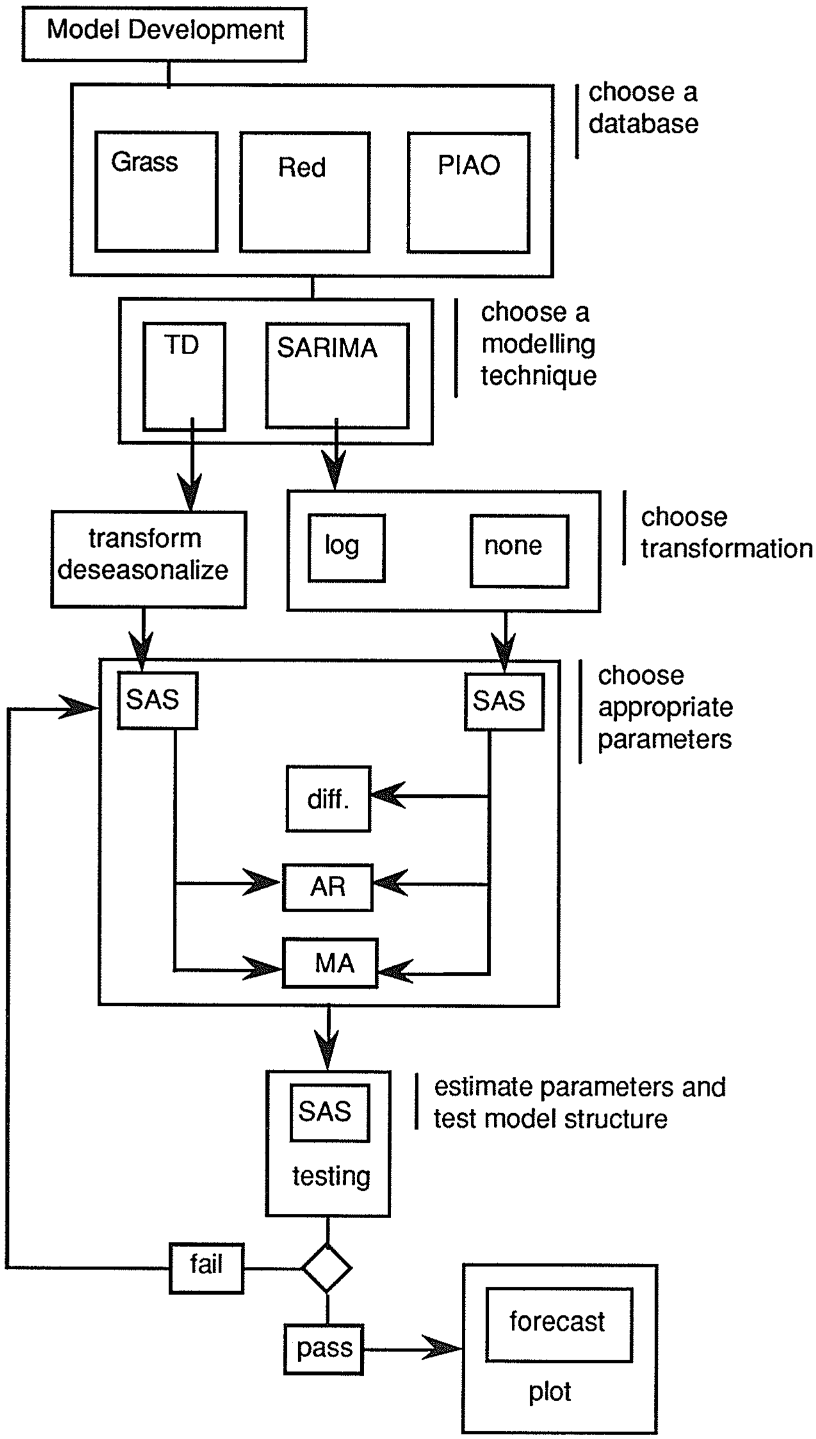


Figure 15. Model development user mode

4.6 Consultations

Two consultations are discussed below to demonstrate the use of the application, one for each task.

4.6.1 Forecasting

The initial application prompt requires the user to choose a task. If the forecasting task is chosen, a text window appears with a list of data sets with available models. The question "Which data set do you wish to use?" is posed to the user. If 'grass' is chosen, a text window halts the program to allow for the user to perform an update of the data set. Next, the user is queried in an attempt to pass rules for choosing the appropriate model as in Figure 13. The user is asked to enter recent flow behaviour in general terms of high, median, or low. Uncertain is also an option if the user is unfamiliar with the data. If this is selected, the data series is plotted with Xgraph and a text window advises the user on how to decide which option to select. For the selection of 'low' flow behaviour, another question asks "What is the relative magnitude of the peak flow?" with options of high, median, low, and uncertain. If 'low' is selected, a text message states that model GS2 has been chosen. SAS is simultaneously executed. The output file for the forecasts is displayed on the screen, and then the forecasts are plotted against historical monthly means. This ends the consultation.

4.6.2 Model Development

When the model development task is selected (Figure 14), a message appears to remind the modeller of data requirements for an adequate model. Next, the relevant data set is chosen. The user is then allowed to update the flows. Information is then given concerning transformation and techniques for handling seasonality including strengths and weaknesses of each approach. The user must then choose the approach to proceed with. Options are 'seasonal' and 'td' (transform and deseasonalize). If 'seasonal' is chosen, the next question asks for the type of transformation with options of 'log' and 'none'. A text window recommends that 'log' be udes, so it is chosen.

SAS is then executed to generate the correlation functions (ACF, PACF) for a number of differencing schemes. The user must then enter the desired differencing scheme and parameters to include with help from text windows displaying typical selections and describing properties of the functions. Once this is complete, SAS is executed to estimate model parameters. The SAS output is shown, including statistical tests and residual correlation functions. Three questions then ask whether the model passed the tests for: t-ratio test, Chi-square test, and residual correlations. If all of the tests are satisfactory, the model is accepted. Otherwise, the user is sent back to select new model parameters, or a new model structure if all tests are passed except the Chi-square test. When the model is accepted after this iterative procedure of adjustment, the user has the option of producing forecasts. If the user chooses to, forecasts for the next year are produced and plotted for the acceptable model. Otherwise, the consultation ends.

5 CONCLUSIONS

5.1 Research goals

This project applies advanced time series models to make better use of information contained in data sets for forecasting monthly water supply. Improvements in the forecasting accuracy were sought as a means of realizing benefits from more confident and accurate optimal operating policies of a reservoir system. SARIMA models were chosen as a likely candidate to improve currently used forecasting models at Manitoba Hydro. Numerous models were developed and applied to three types of data series within a sensitivity analysis study of flow conditions, using the Manitoba Hydro forecast scenario. A method of analyzing the errors took advantage of the simplicity and flexibility of the MSE statistic. Ranking of the models under various flow conditions suggested a simple set of rules to govern the choice of model for forecasting that will produce the best available forecast.

Many of the methods that are used in large utilities to produce long-range forecasts have been developed out of familiarity with the engineer and simplicity. Previous use of time series techniques in Water Resources engineering has been rigidly developed around the engineer's impression of what is proper. Recognizing the possible improvements in accuracy from our expanding quantitative knowledge of natural systems, SARIMA models may form the basic tool around which an efficient and flexible standard forecasting framework can be built.

5.2 Summary of results

Linear regression and simulation are the most widely used techniques in use today for long-range forecasting. Deseasonalized ARMA models are theoretical engineering tools already available for use to improve forecasts, but their potential is restricted compared to more general techniques such as SARIMA models.

These statistical approaches, TD ARMA models and SARIMA models, have been used to provide a basis of comparison to evaluate long range forecasting techniques at Manitoba Hydro. Both expected and low percentile probabilistic forecasts are needed by Manitoba Hydro to produce risk-based release policies. Only the mean forecasts could be evaluated because low and high percentile forecasts for the time series models are statistically correct for a single month, but are meaningless for the system when several time lags are involved. The standard errors typically provided with statistical forecasts do not consider any possible transition period for changes in flow patterns, only simple historical extremes.

Manitoba Hydro forecasts performed well for Grass River, but was inferior to RS2 on the Red River for presented case study forecasts. This may be due to the fact that changes in the Grass River basin are gradual, and sequential correlations are high. The Red River experiences drastic changes from month to month, and is susceptible to physical and meteorological trends or patterns. SARIMA modelling provides a more flexible framework of evaluating sources of correlation such as seasonal patterns so as to make more complete use of the historical data. The Manitoba Hydro method is dependent on the most recent month of known flow.

In general, standard SARIMA modelling performed as well as the Manitoba Hydro linear regression models, and has potential for improvement if an appropriate modelling framework is developed to allow probabilistic forecasts to be made. Engineering concerns about manipulation of data by differencing is unsupported. Historical characteristics of the data are maintained implicitly through the correlation of similar months. In fact, the differencing may be considered as the equivalent to deseasonalization of the data. SARIMA modelling has been shown to be a good application of statistical time series modelling to water supply forecasting. Expected forecasts of system natural inflow for hydroelectric utilities may benefit from the use of this technique.

5.3 Potential time series use

The SARIMA models presented in this paper are standard, basic, applications of the work by Box and Jenkins (1976). There are advanced techniques that can be added to these models to improve their effectiveness.

One possibility is the development of Transfer Function Noise (TFN) models by adding additional data sets to the presently used flow. There may be other rivers in the same proximity that display similar flow characteristics, precipitation data, temperature data, etc. A TFN model has the same form as SARIMA models in terms of AR and MA processes, but simply adds crosscorrelation of data sets to identify any additional information relating one data set to another. This type of modelling is the next logical extension of time series modelling. The procedure and calculations are similar to those presented in this paper.

Another extension to SARIMA modelling is to identify particular events that affect the system, and estimate AR and MA processes within the event in the form of an Intervention model. Intervention analysis can be used to model the effects of mud slides, man-made diversions, or specifically addressing annual spring runoff effects.

In their present form, these SARIMA and deseasonalized ARMA models do not supply probabilistic forecasts that have any meaning to the system. Extreme forecast values for each month are at best representative of the historical monthly extreme and do not consider the system characteristics on an annual level where the sum of the extreme monthly volumes are not indicative of the historical annual volumes. The sum of extremes would be much more extreme for the annual volume history. It is this form of problem that suggests the use of disaggregation processes. Disaggregation models simply approach the data as being governed by a hierarchy of system control with 2 levels: annual, and monthly (or weekly) to produce a modelling framework. The most beneficial possibility of producing probabilistic forecasts that are meaningful to the system may be to formulate the time series model in a disaggregation form. The linear regression method used by Manitoba Hydro has been formulated within a disaggregation framework. For SARIMA models to produce probabilistic forecasts, they may need to be formulated in a similar fashion.

REFERENCES

Abraham, B. and Ledolter, J., Statistical Methods for Forecasting, Wiley and Sons, New York, 1983.

Bowerman, B.L. and O'Connell, R.T., Time Series Forecasting, Duxbury Press, Boston, 1987.

Chen, C.L., et. al., "A flow Forecasting System for Hydro Power", Waterpower, p. 771-780, 1991.

Day, G.N., "Extended Streamflow Forecasting Using NWSRFS", J. Water Resources Planning and Management, ASCE, Vol. 111, NO. 2, p. 157-170, 1985.

Fast, B., "Analysis of Seasonal Inflow Volume Forecasts Produced Using Regression Equations", Internal Report for Operations Control Department, BC Hydro, 1990.

Georgakakos, K.P. and Smith, G.F., "On Improved Hydrologic Forecasting - Results From A WMO Real-Time Forecasting Experiment", J. Hydrology, Vol. 114, p. 17-45, 1990.

Georgakakos, A.P., "The Value of Streamflow Forecasting in Reservoir Operation", Water Resources Bulletin, Vol. 25, NO. 4, p. 789-800, 1989.

Hudlow, M.D., "Technological Developments in Real-Time Operational Hydrologic Forecasting in the United States", J. Hydrology, Vol. 102, p. 69-92, 1988.

Kitanidis, P.K. and Bras, R.L., "Real-Time Forecasting With a Conceptual Hydrologic Model: Analysis of Uncertainty", Water Resources Research, Vol. 16, NO. 6, p. 1025-1033, 1980.

Kitanidis, P.K. and Bras, R.L., "Real-Time Forecasting With a Conceptual Hydrologic Model: Applications and Results", Water Resources Research, Vol. 16, NO. 6, p. 1034-1044, 1980.

Koch, R.W. and Smillie, G.M., "Bias in Hydrologic Prediction using Log-Transformed Regression Models", Water Resources Bulletin, Vol. 22, NO. 5, p. 717-723, 1986.

McKerchar, A.I. and Delleur, J.W., "Application of Seasonal Parametric Linear Stochastic Models to Monthly Flow Data", Water Resources Research, Vol. 10, No. 2, p. 246, 1974.

McLeod, A.I., Noakes, D.J., Hipel, K.W., Thompstone, R.M., "Combining Hydrologic Forecasts", Journal of Water Resources Planning and Management, Vol. 113, No. 1, p. 29-41, 1987.

Mishalani, N.R. and Palmer, R.N., "Forecast Uncertainty in Water Supply Reservoir Operation", Water Resources Bulletin, Vol. 24, NO. 6, p. 1237-1245, 1988.

Nash, J.E. and Barsi, B.I., "A Hybrid Model For Flow Forecasting on Large Catchments", J. Hydrology, Vol. 65, p. 125-137, 1983.

Newbold, P., Granger, C.W.J., "Experience with Forecasting Univariate Time Series and the Combination of Forecasts", J. R. Statistics Society, A, 137, p. 131-165, 1974.

Nexpert Object version 2.0, Introduction Manual, Neuron Data Inc., Palo Alto, California, 1991.

Nexpert Object version 2.0, Reference Manual, Neuron Data Inc., Palo Alto, California, 1991.

Nexpert Object version 2.0, User's Guide, Neuron Data Inc., Palo Alto, California, 1991.

Oron, G., Mehrez, A., Rabinowitz, G., "Forecasting in Optimizing Dual System for Energy generation and Irrigation", J. Wat. Res. Planning and Management, Vol. 117, No. 3, p. 287-300, 1991.

Puente, C.E. and Bras, R.L., "Application of Nonlinear Filtering in the Real Time Forecasting of River Flows", Water Resources Research, Vol. 23, NO. 4, p. 675-682, 1987.

Salas, J.D. et. al., Applied Modelling of Hydrologic Time Series, Water Resources Publications, Littleton, Colorado, 1980.

Sen, Z., "Adaptive Prediction of Hydrologic Series by Walsh-Kalman Model", J. Hydrology, Vol. 122, p. 221-234, 1991.

Shafer, B.A. and Huddleston, J.M., "Analysis of Seasonal Volume Streamflow Forecast Errors in the Western U.S.", American Water Resource Association, June, p. 117-126, 1984.

Smith, J.A., "Long-range Streamflow Forecasting Using Nonparametric regression", Water Resources Bulletin, Vol. 27, No. 1, p. 39-46, 1991.

Smith, J.A., Day, G.N., Kane, M.D., "Nonparametric Framework for Long-Range Streamflow Forecasting", J. Water Resource Planning and Management, Vol. 118, No. 1, p. 82-92, 1992.

Tao, T. and Lai, L.M.S., "Design Specifications of Short Term Inflow Forecaster", Internal Report for Operations Planning Facilities Department, DWP-OP-24, 31 pp, 1990.

APPENDIX A: Grass River

Appendix A.1 Historical characteristics

Figure A.1.1 Grass River data series

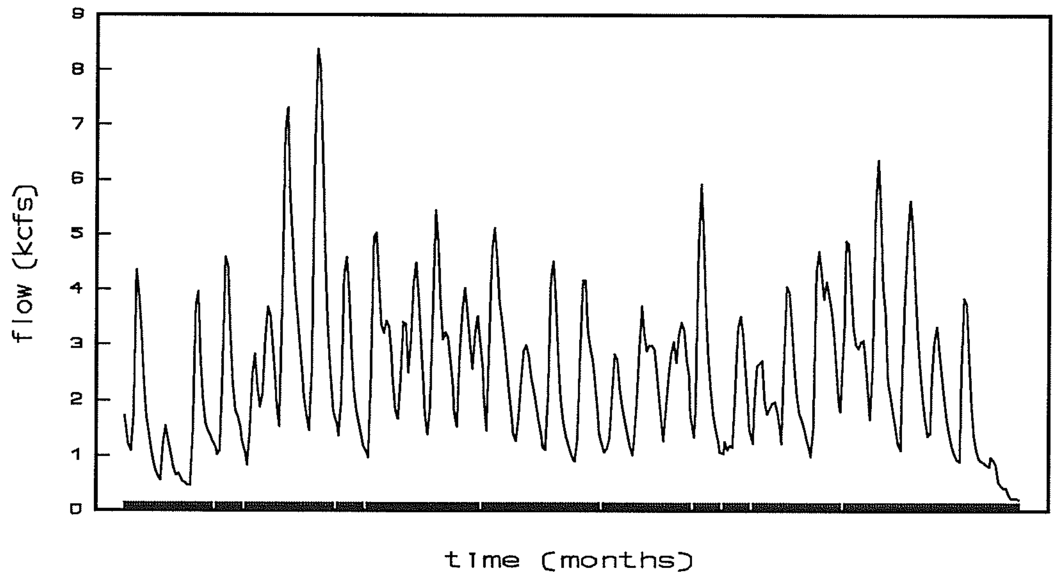


Figure A.1.2 Grass River annual characteristics

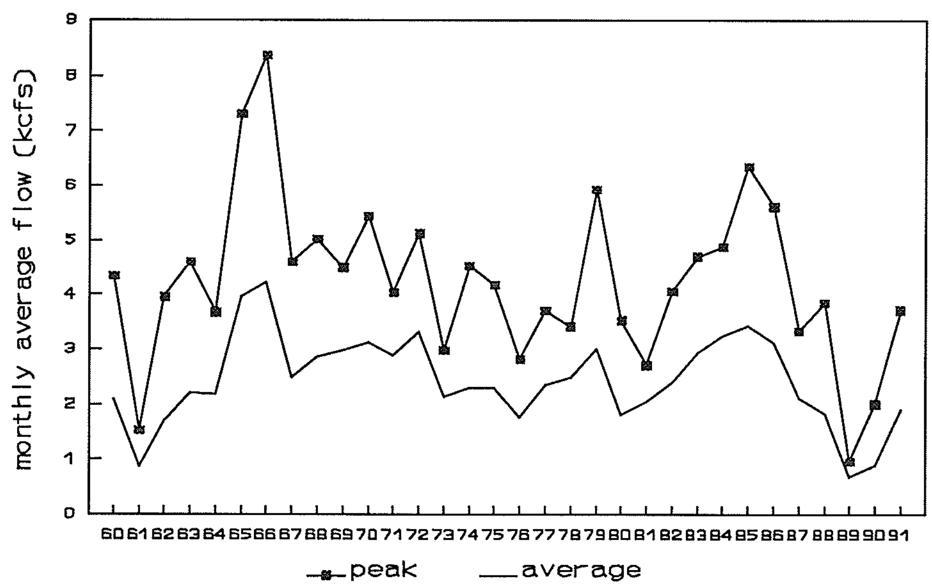
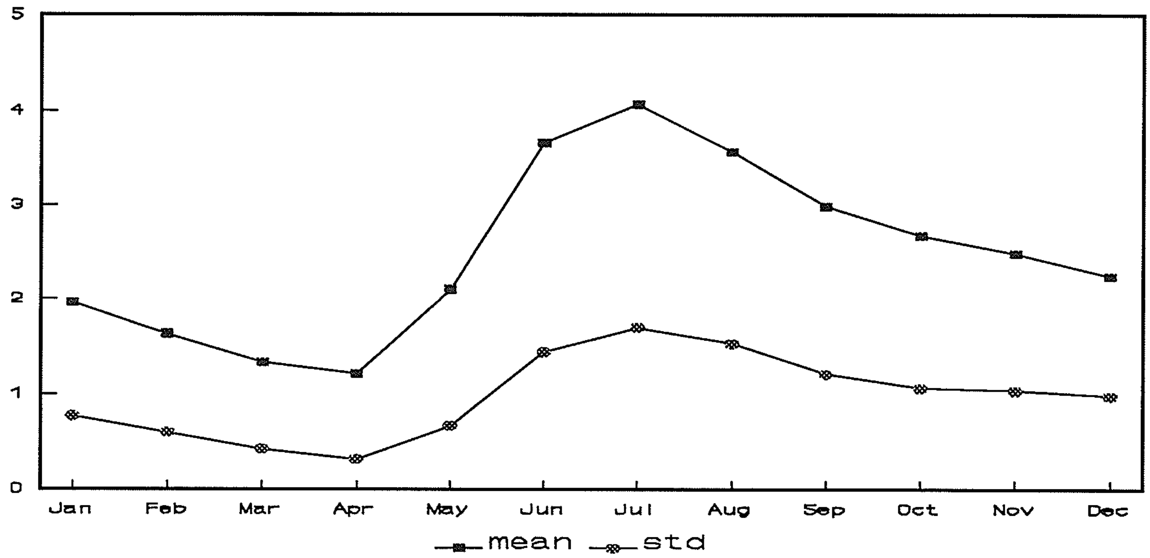


Figure A.1.3 Grass River historical characteristics



Appendix A.2 High case

Figure A.2.1 Grass River high flow case forecast

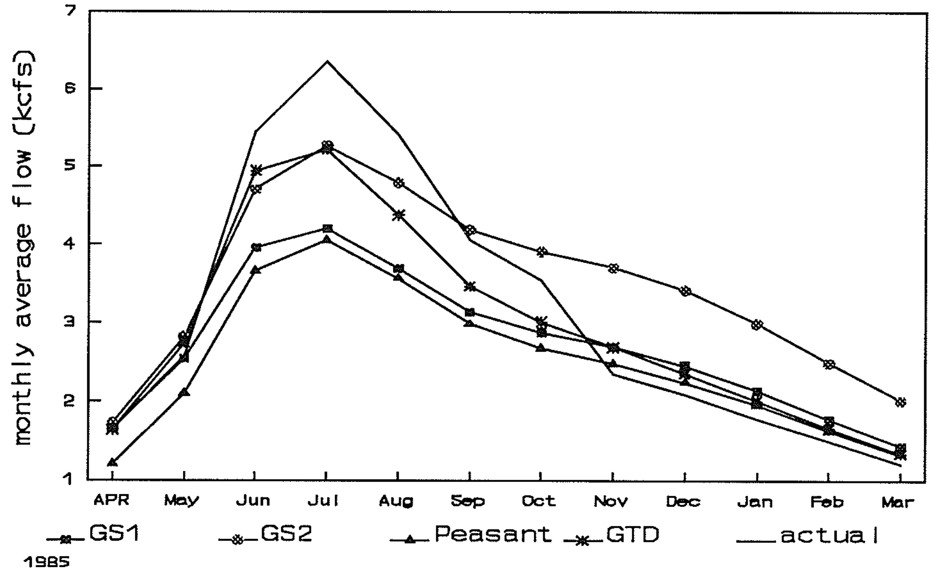


Figure A.2.2 Grass River high flow case forecast

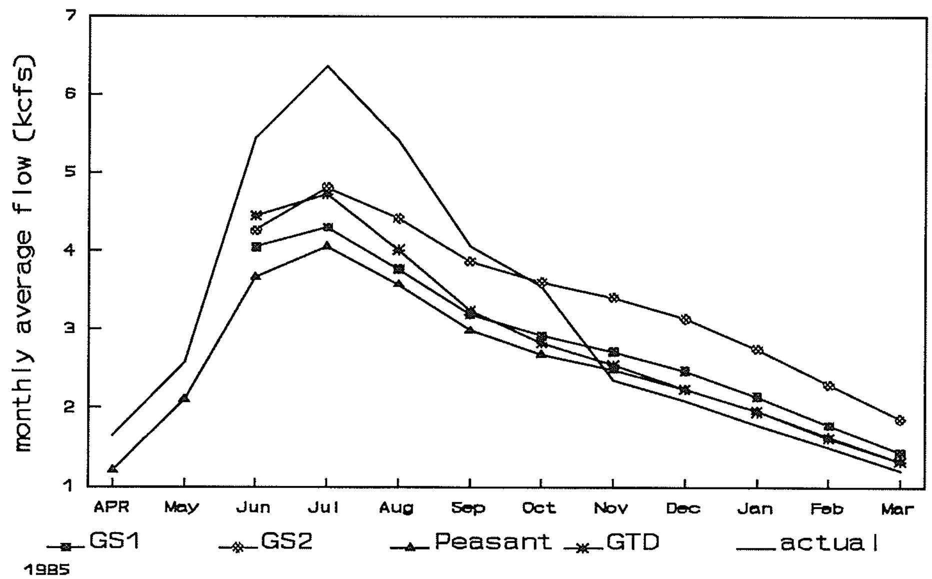


Figure A.2.3 Grass River high flow case forecast

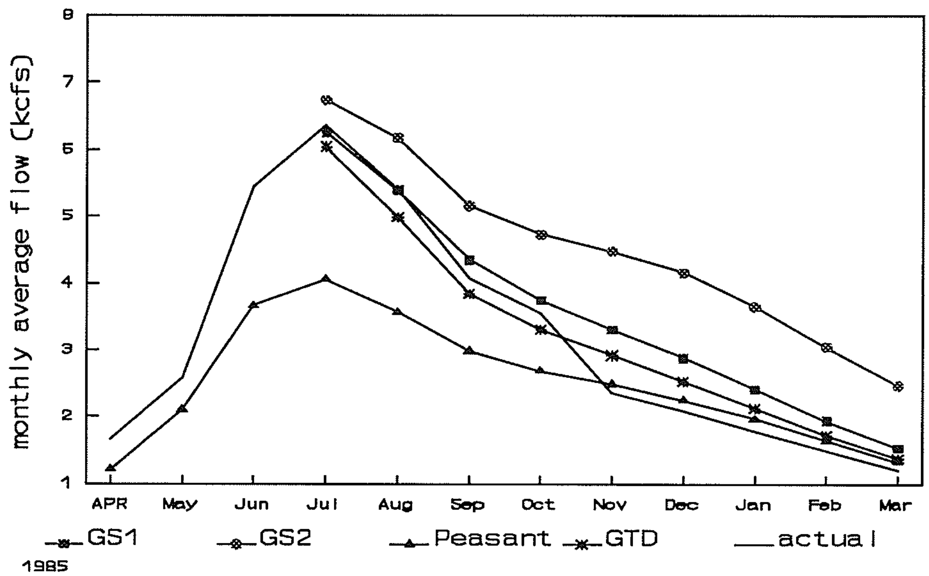


Figure A.2.4 Grass River high flow case forecast

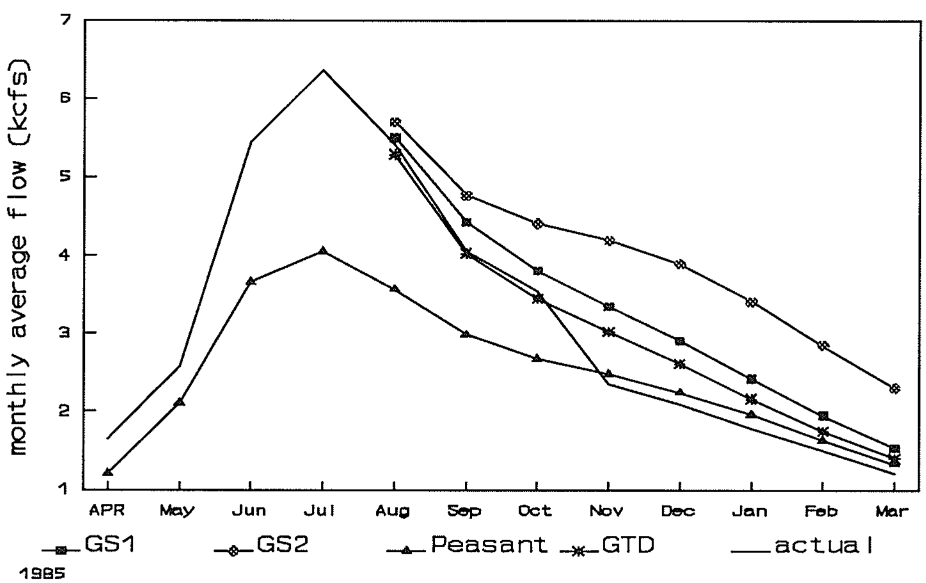


Figure A.2.5 Grass River high flow case MSE by forecast run

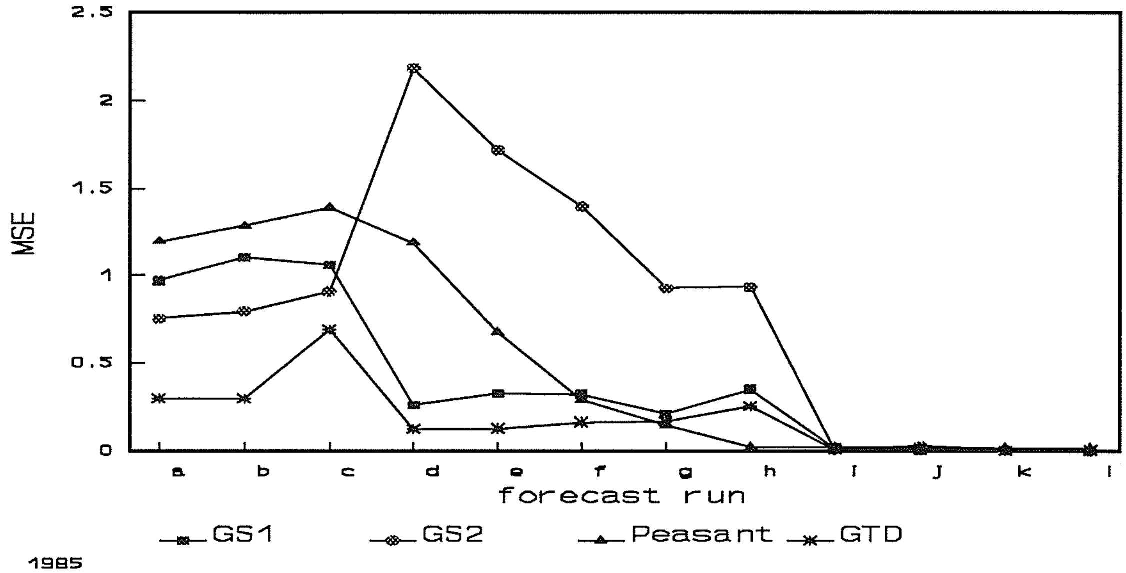


Figure A.2.6 Grass River high flow case MSE by month

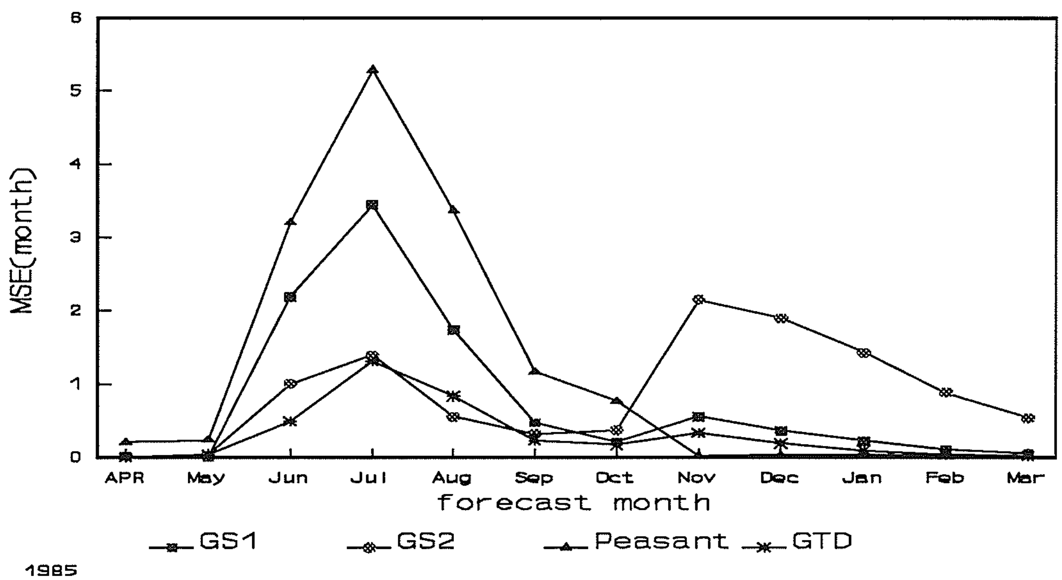
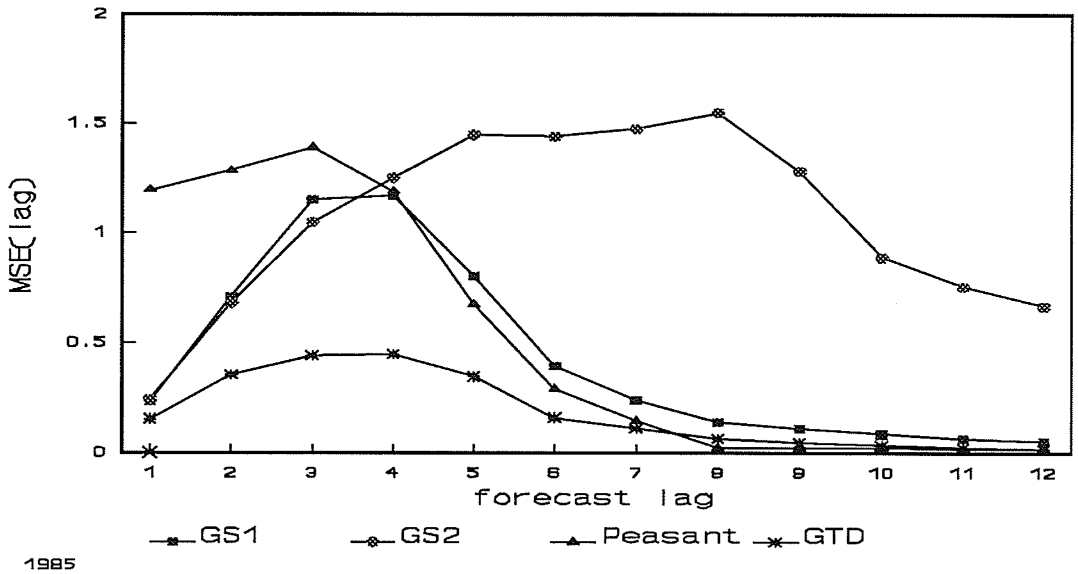


Figure A.2.7 Grass River high flow case MSE by lag



Appendix A.3 Median case

Figure A.3.1 Grass River median flow case forecast

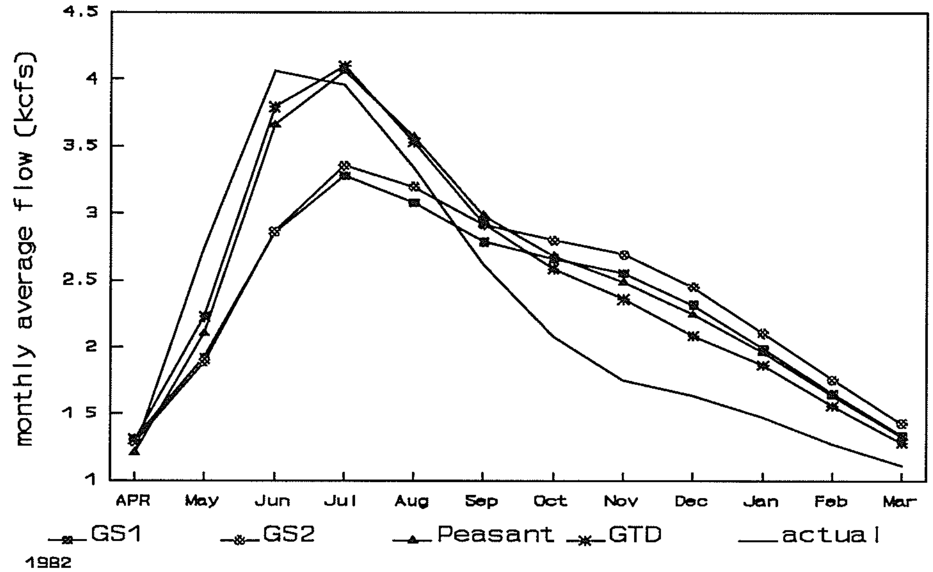


Figure A.3.2 Grass River median flow case forecast

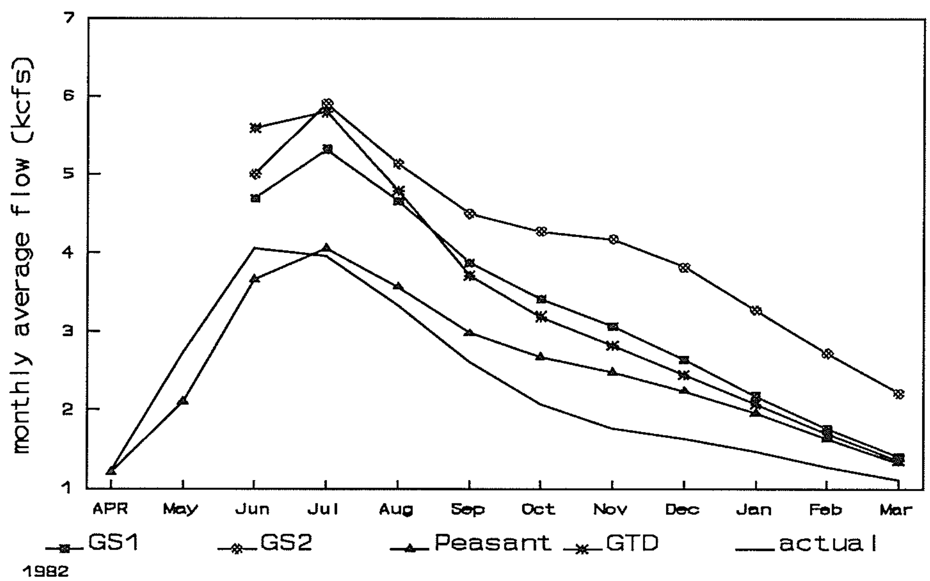


Figure A.3.3 Grass River median flow case forecast

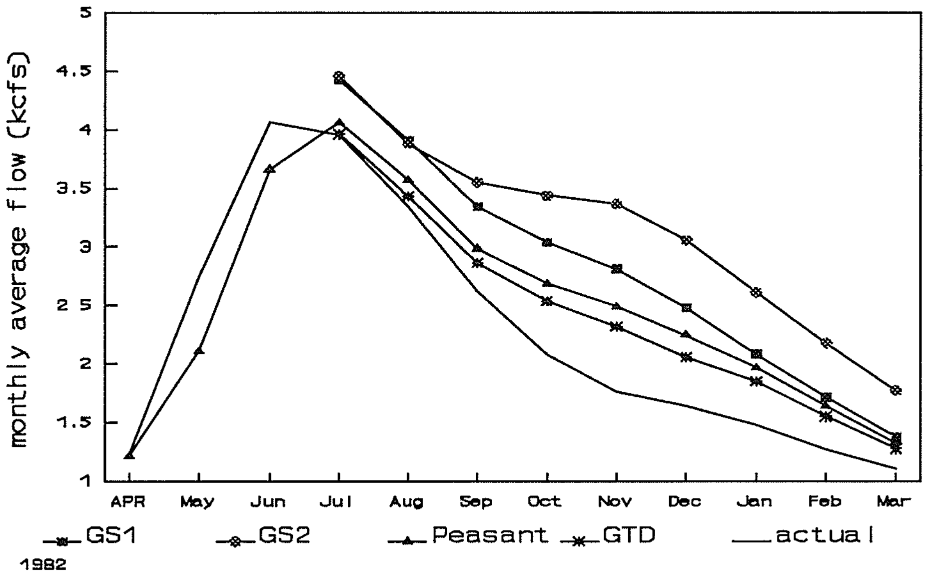


Figure A.3.4 Grass River median flow case forecast

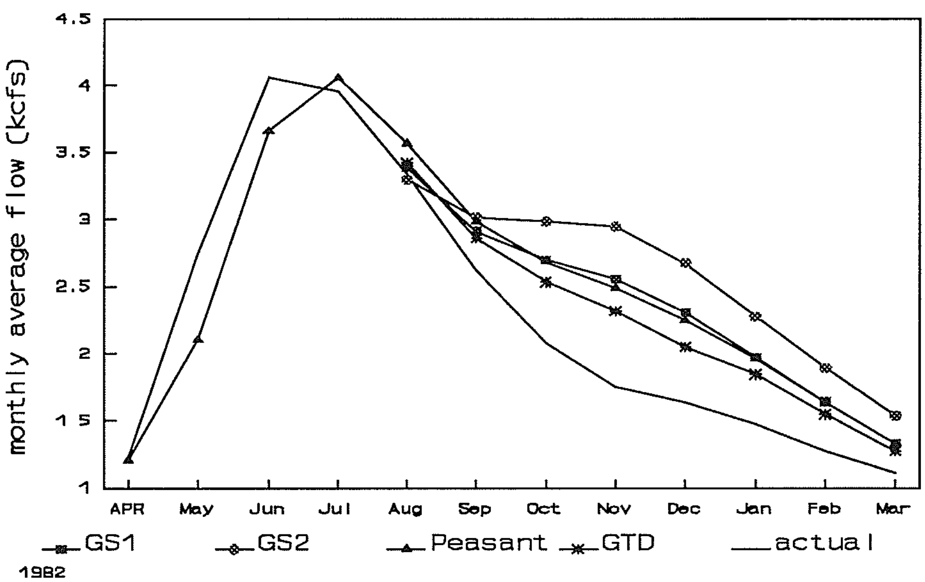


Figure A.3.5 Grass River median flow case MSE by forecast run

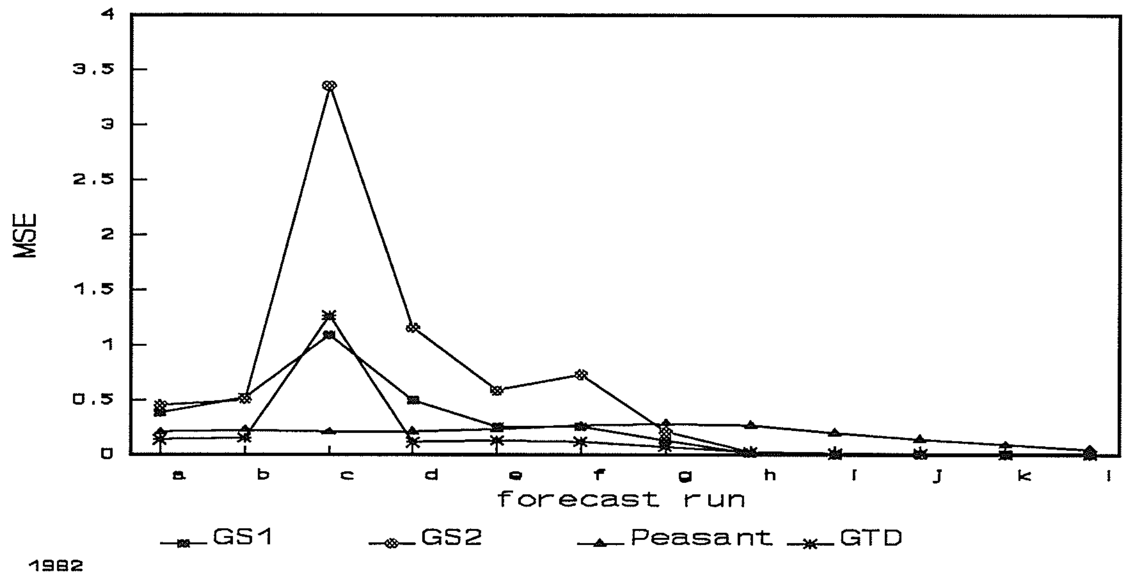


Figure A.3.6 Grass River median flow case MSE by month

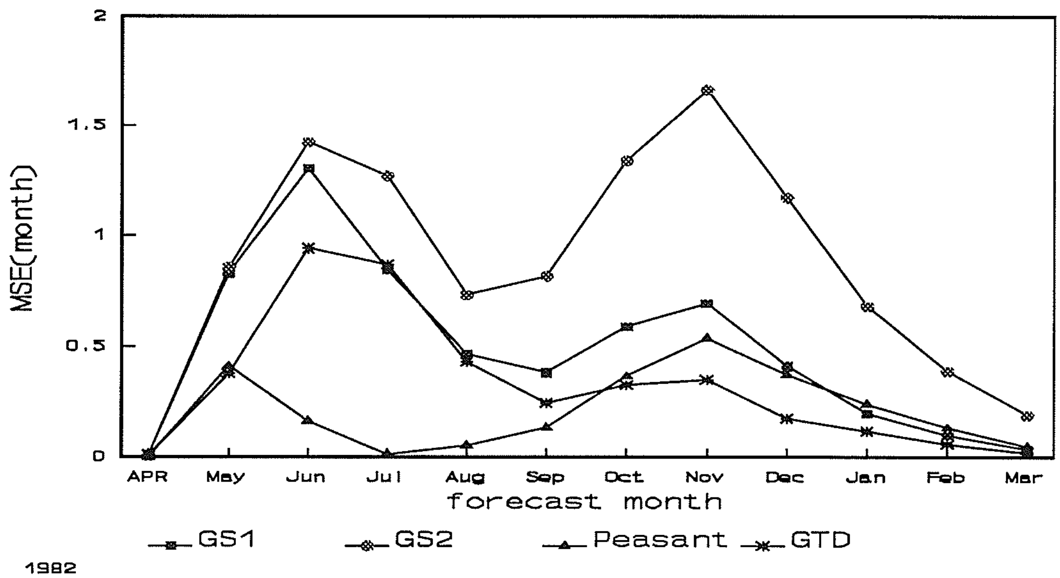
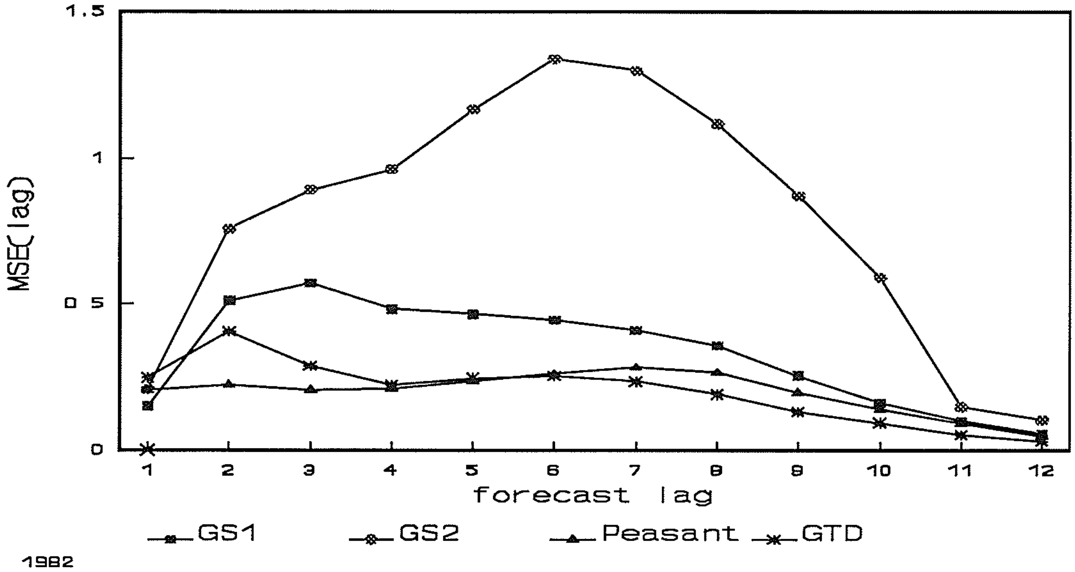


Figure A.3.7 Grass River median flow case MSE by lag



Appendix A.4 Low case

Figure A.4.1 Grass River lo^Xxflow case forecast

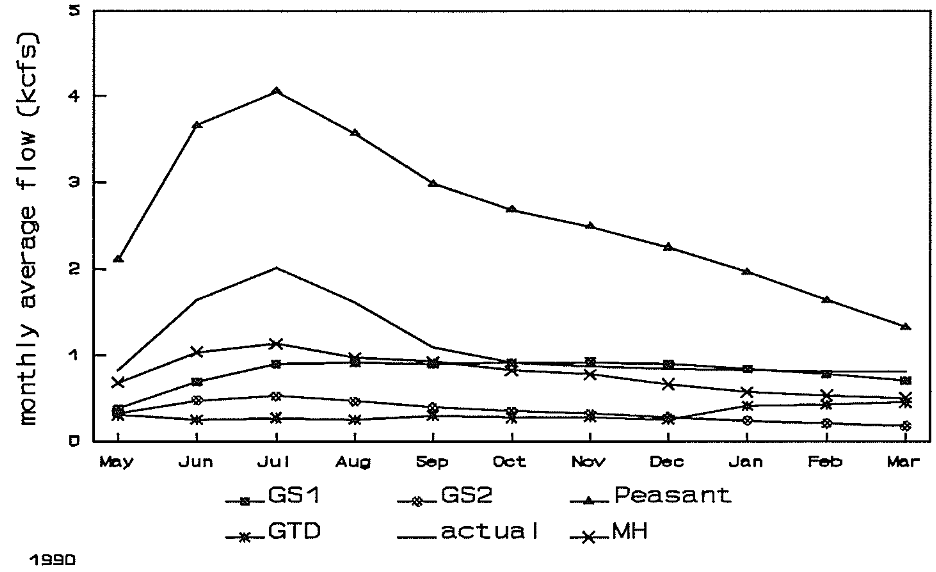


Figure A.4.2 Grass River low flow case forecast

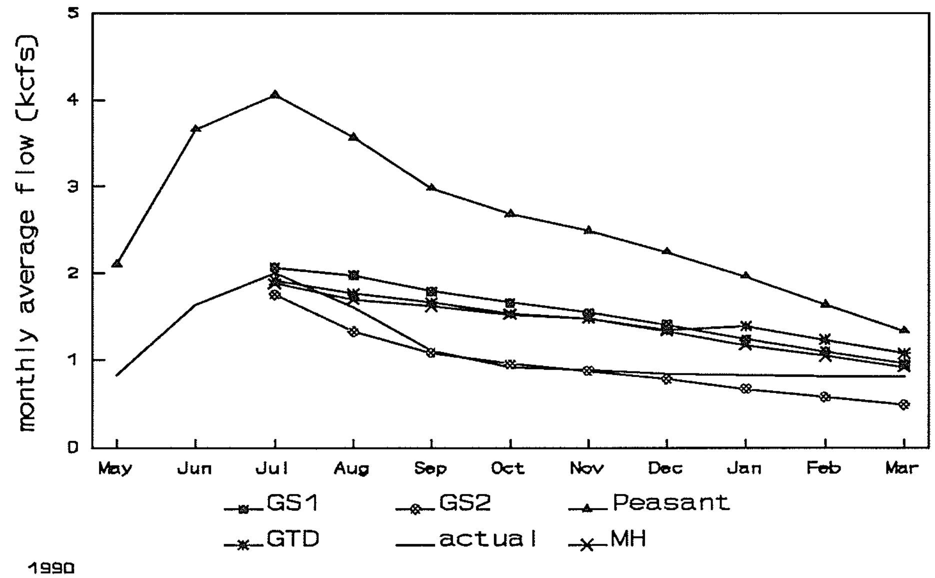


Figure A.4.3 Grass River low flow case forecast

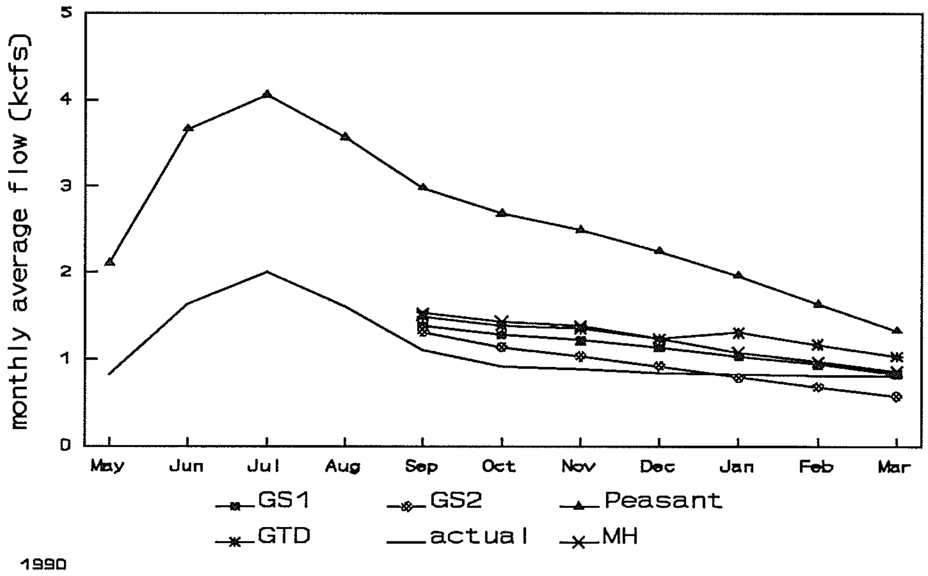


Figure A.4.4 Grass River low flow case MSE by forecast run

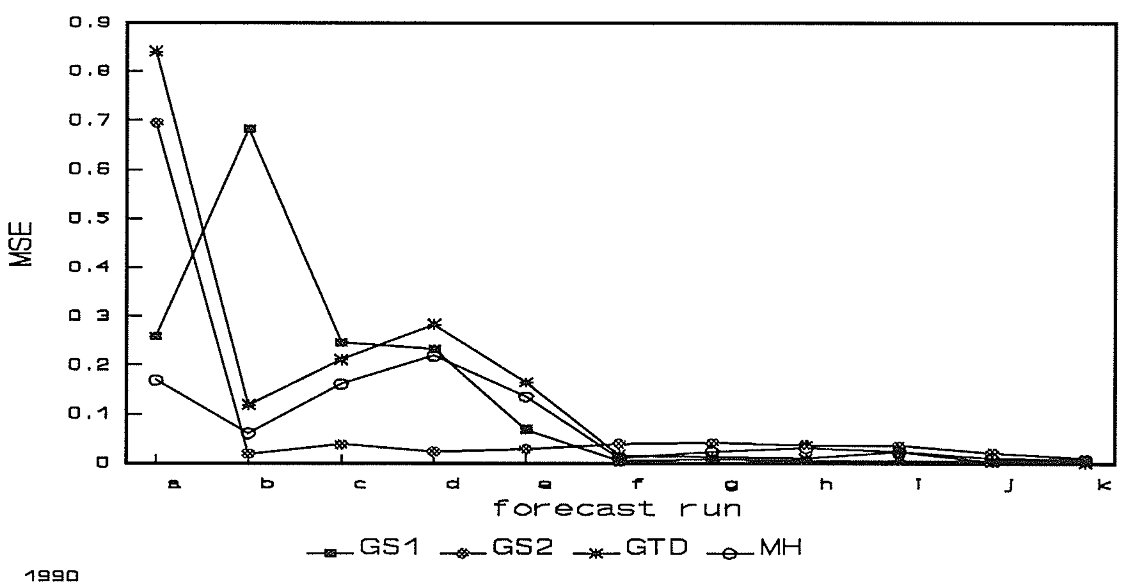


Figure A.4.5 Grass River low flow case MSE by month

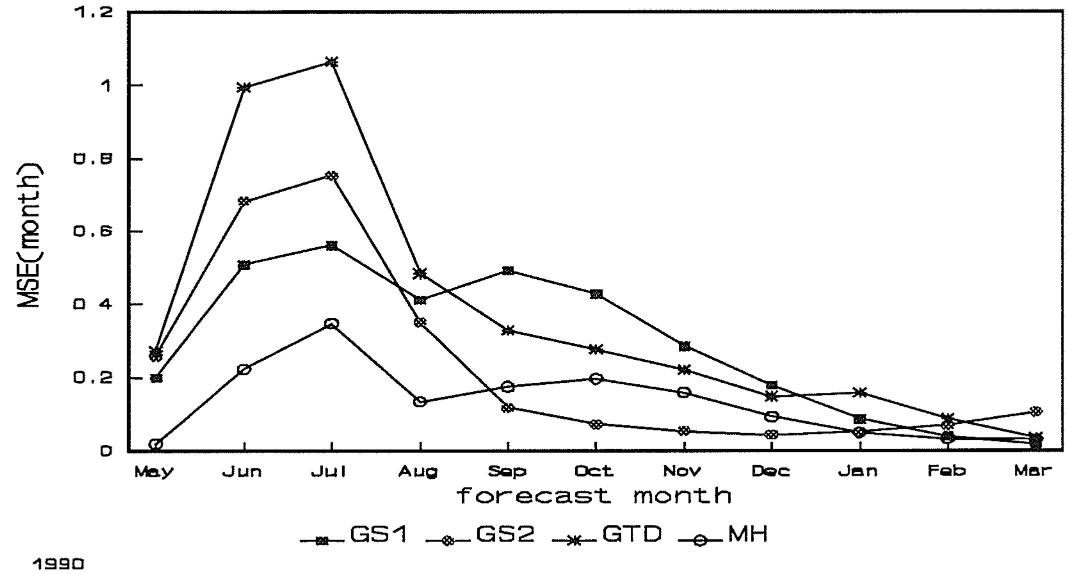
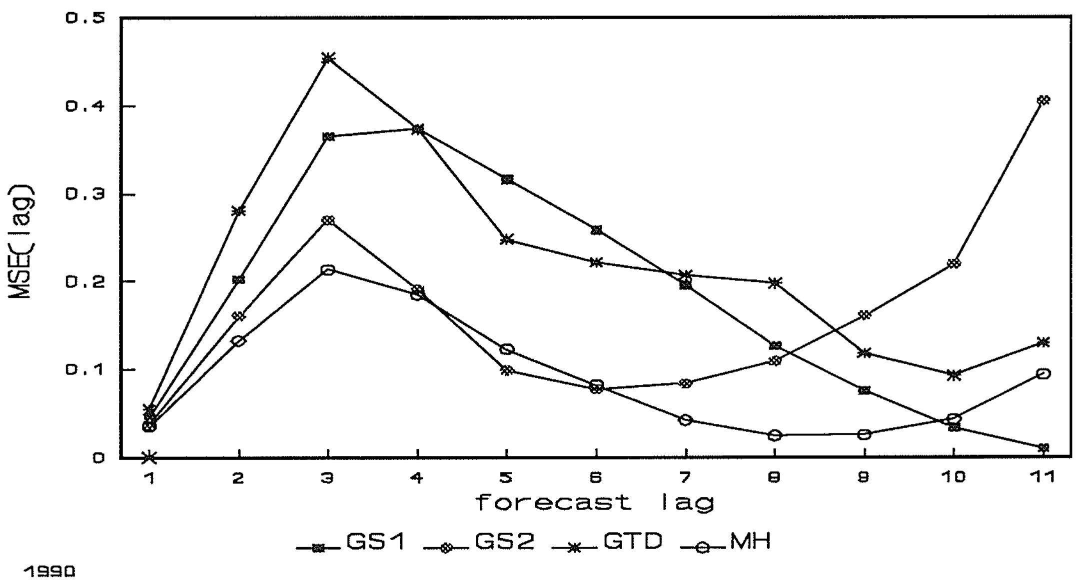


Figure A.4.6 Grass River low flow case MSE by lag



APPENDIX B: Red River

Appendix B.1 Historical characteristics

Figure B.1.1 Red River data series

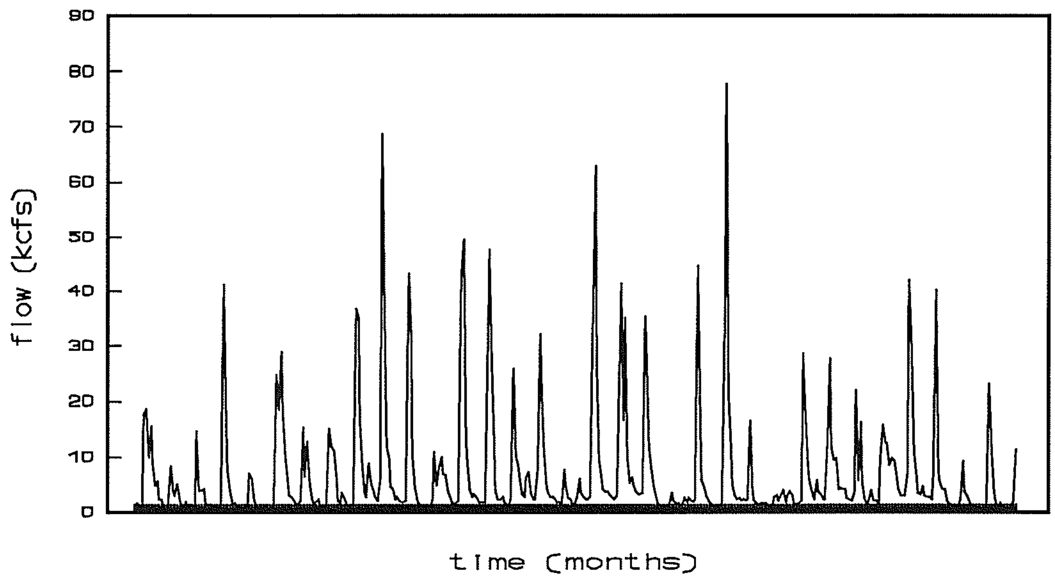


Figure B.1.2 Red River annual characteristics

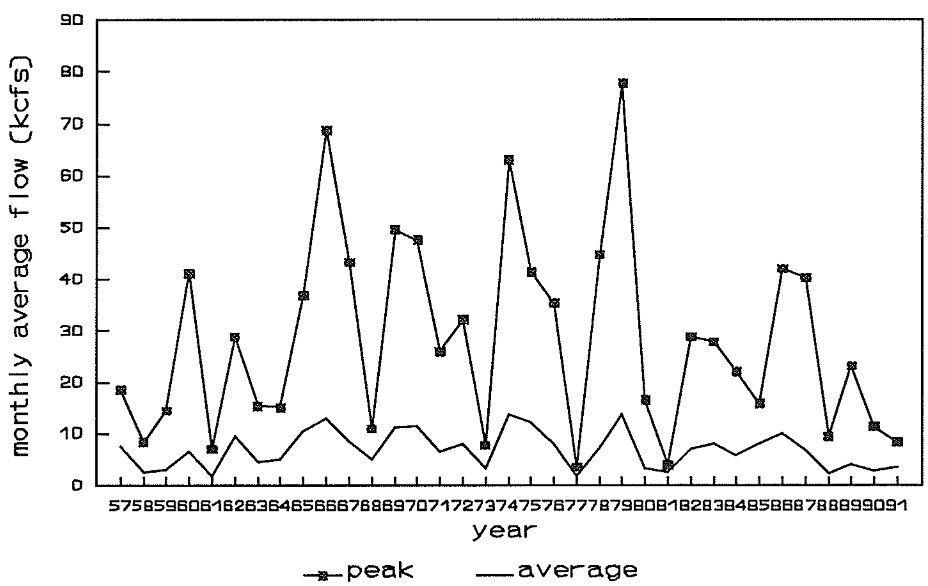


Figure B.1.3 Red River historical characteristics

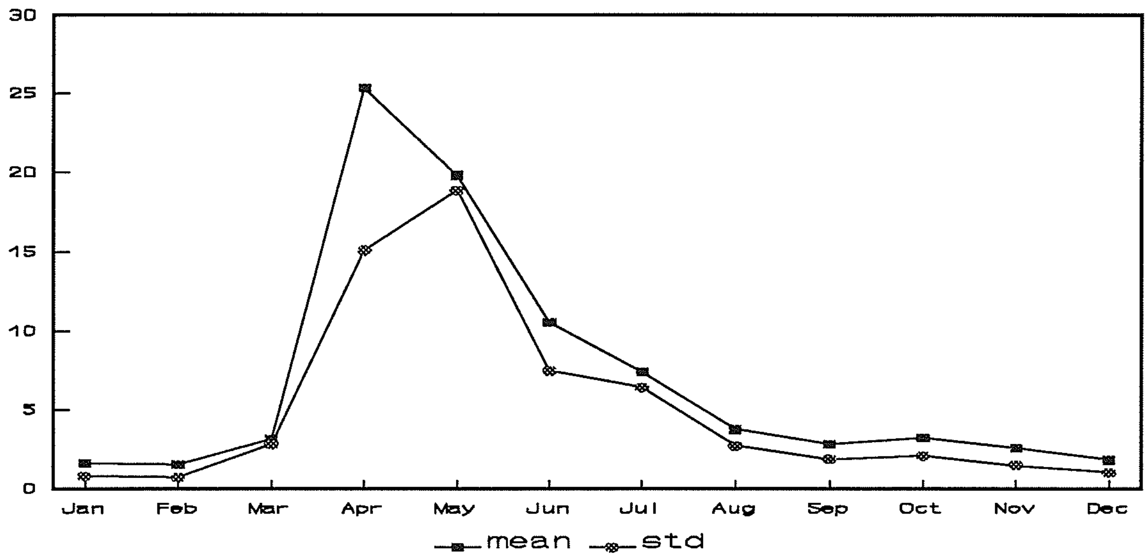


Figure B.1.4 Red River peak correlation: average

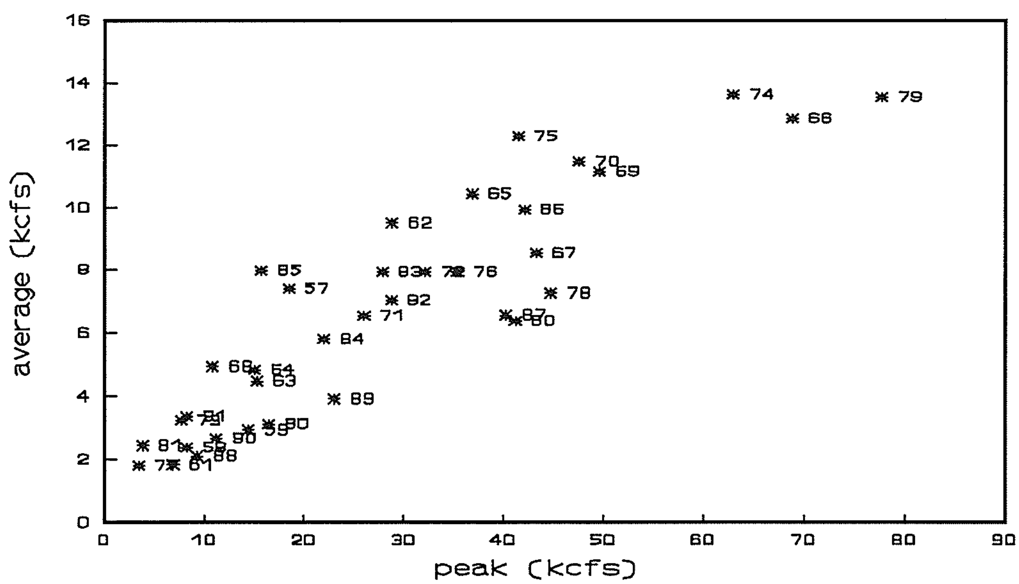


Figure B.1.5 Red River peak correlation: March

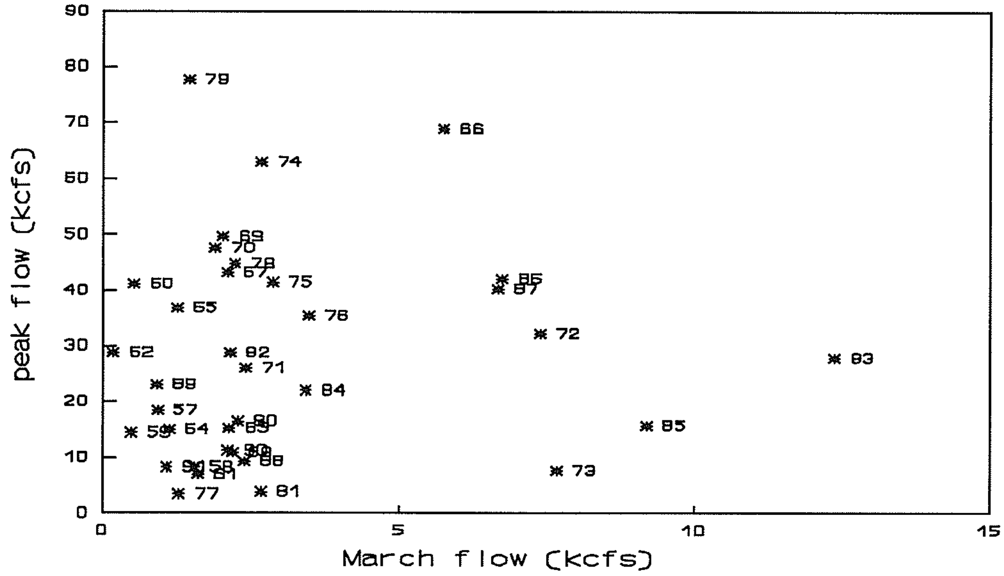


Figure B.1.6 Red River peak correlation: average flow from January through the end of March

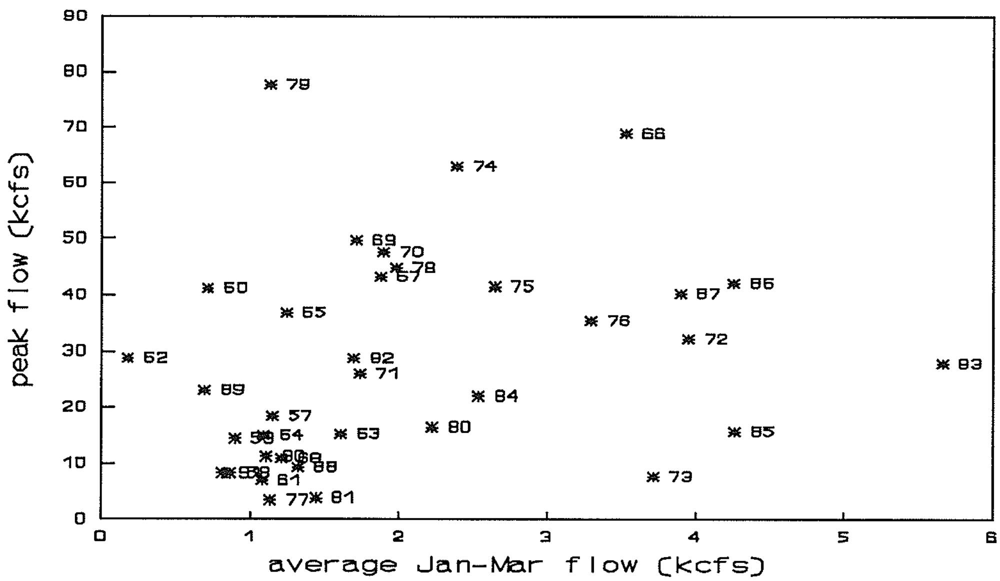
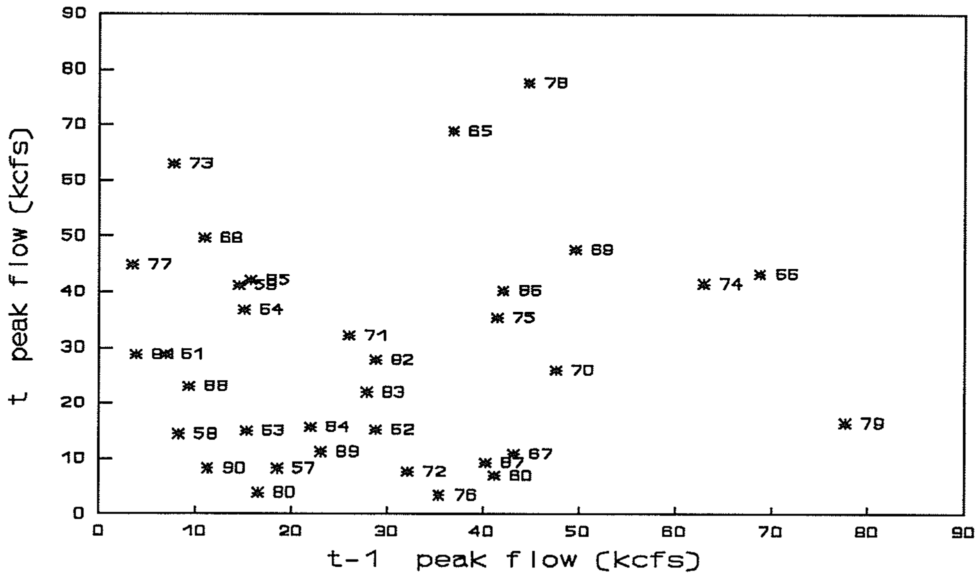


Figure B.1.7 Red River peak correlation with previous peak



Appendix B.2 High case

Figure B.2.1 Red River high flow case forecast

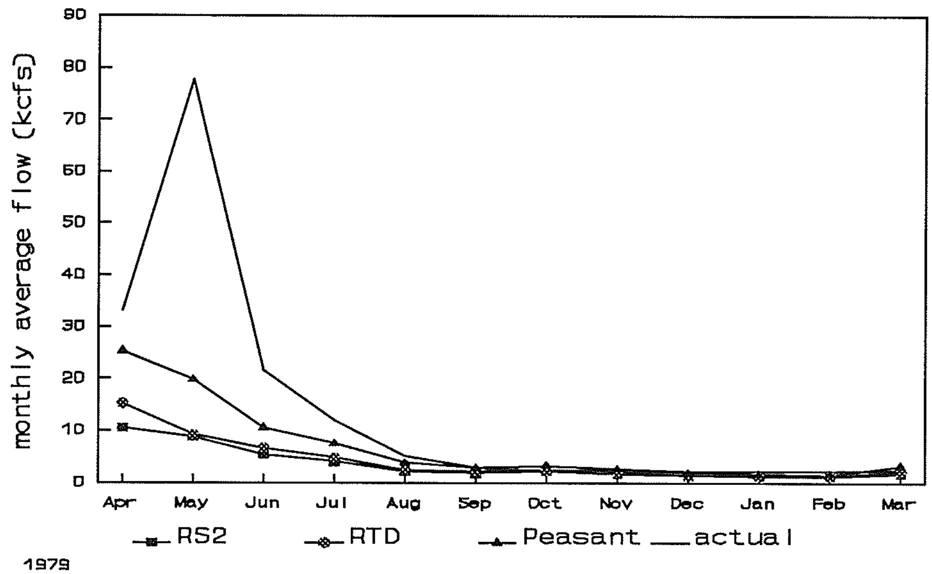


Figure B.2.2 Red River high flow case forecast

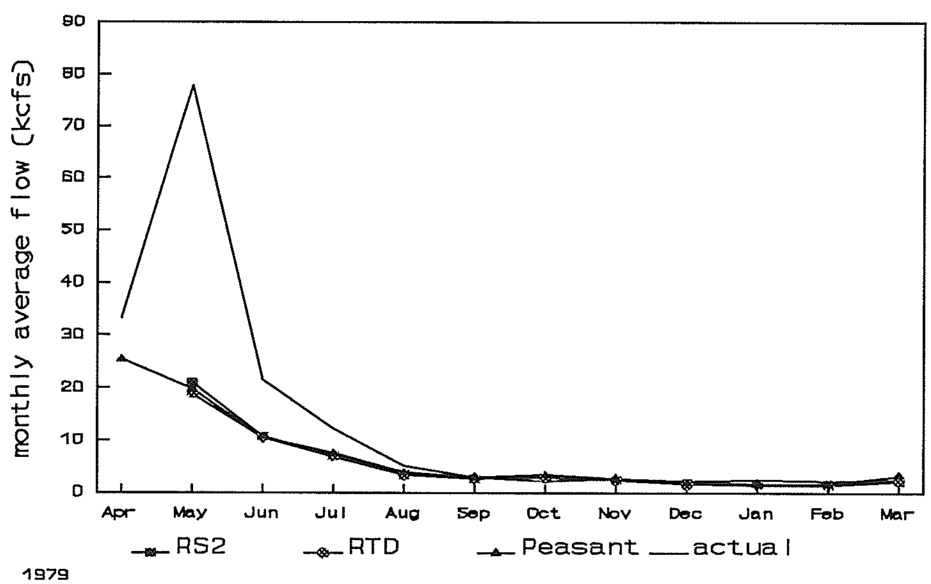


Figure B.2.3 Red River high flow case forecast

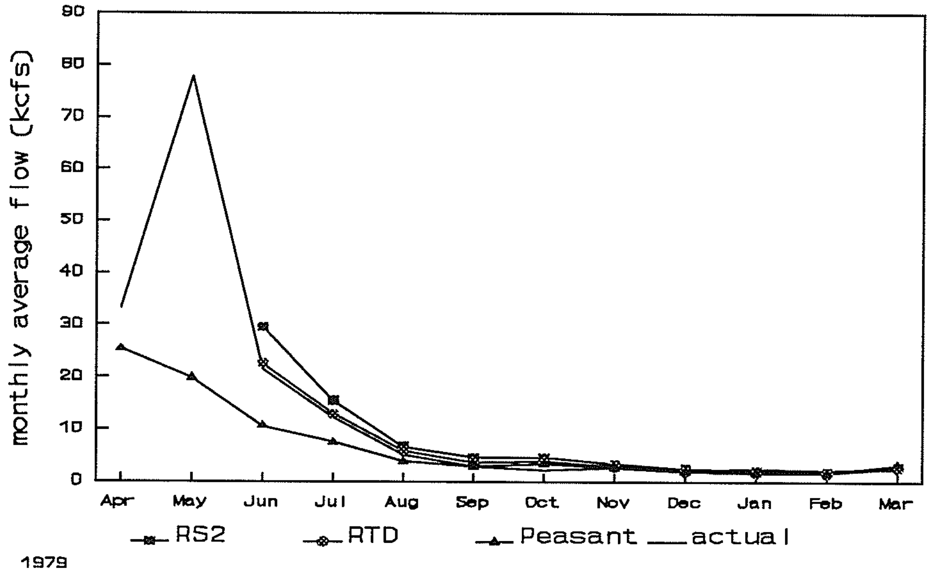


Figure B.2.4 Red River high flow case forecast

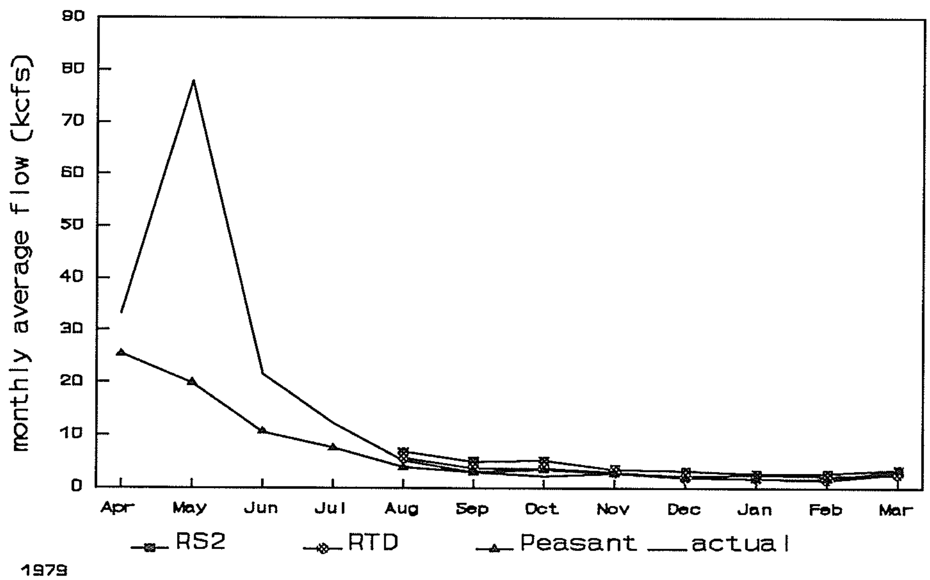


Figure B.2.5 Red River high flow case RMSE by forecast run

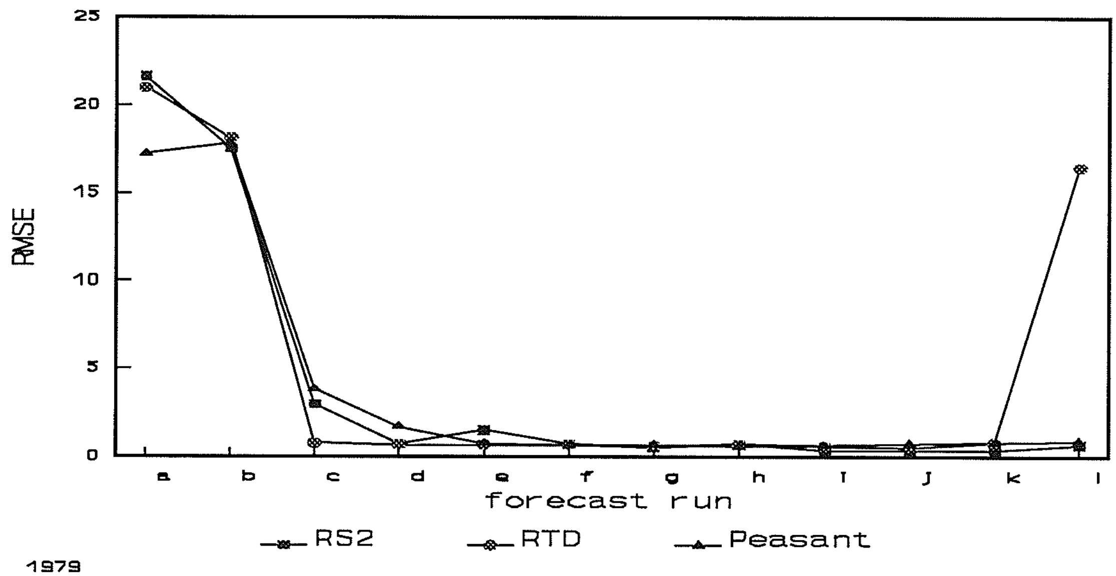


Figure B.2.6 Red River high flow case RMSE by month

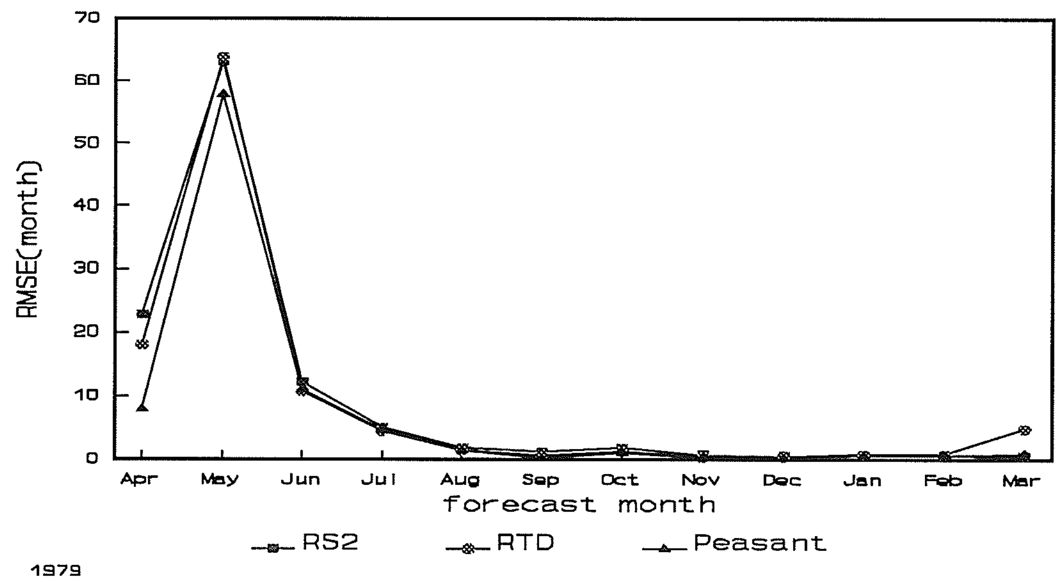
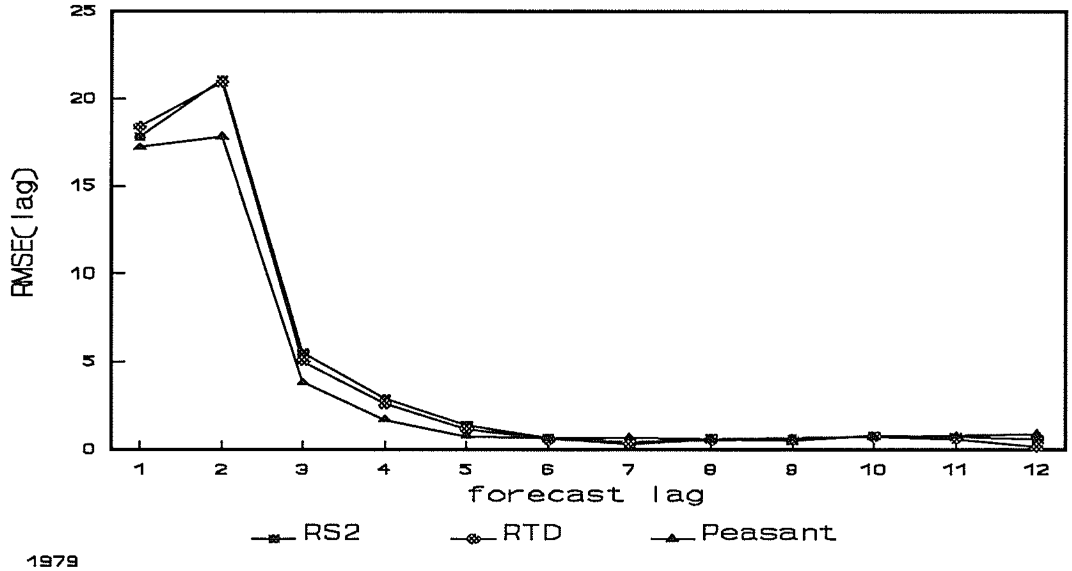


Figure B.2.7 Red River high flow case RMSE by lag



Appendix B.3 Median case

Figure B.3.1 Red River median flow case forecast

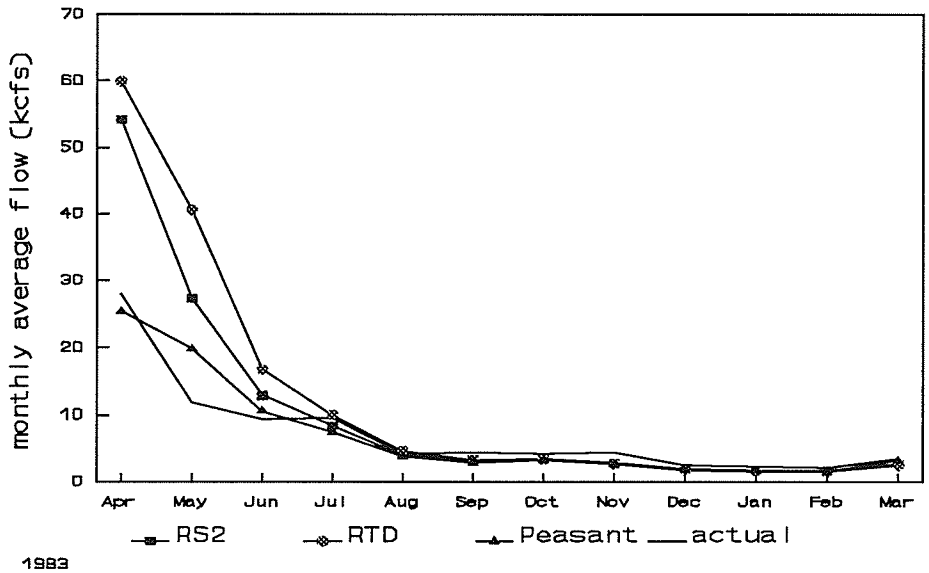


Figure B.3.2 Red River median flow case forecast

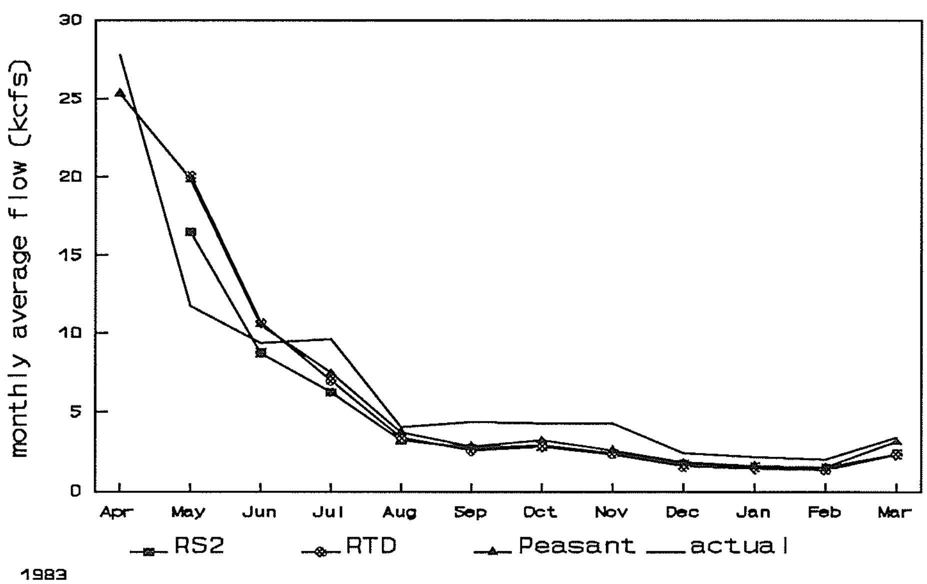


Figure B.3.3 Red River m^Xxian flow case forecast

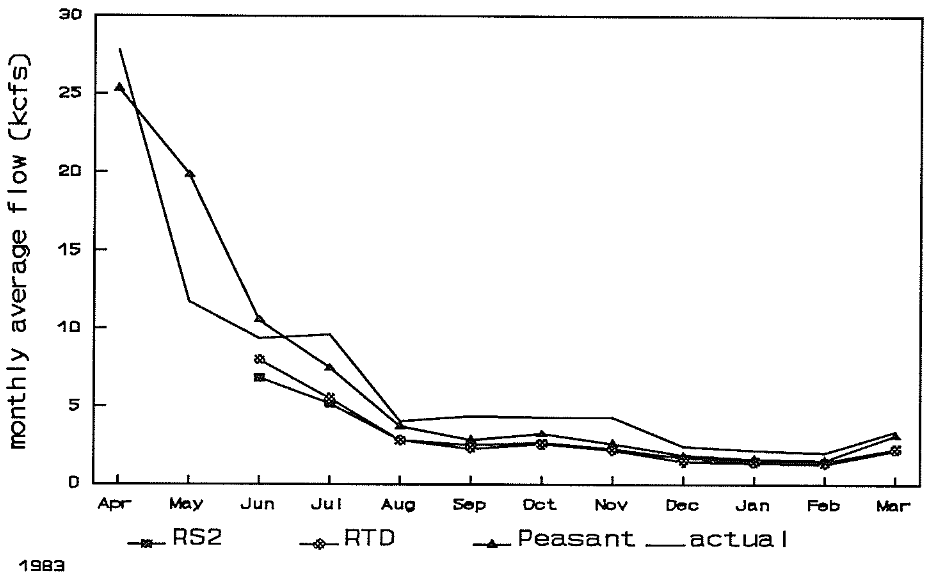


Figure B.3.4 Red River median flow case forecast

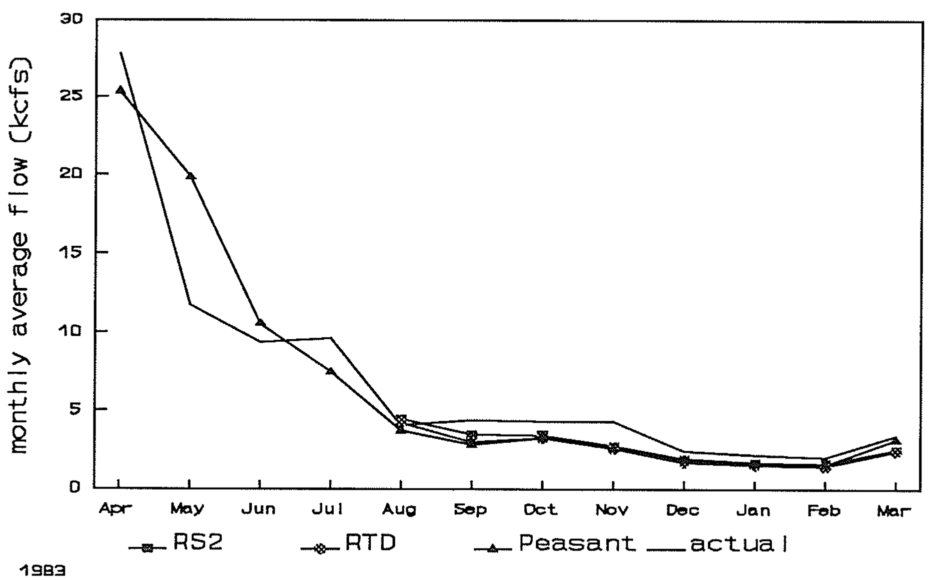


Figure B.3.5 Red River median flow case RMSE by forecast run

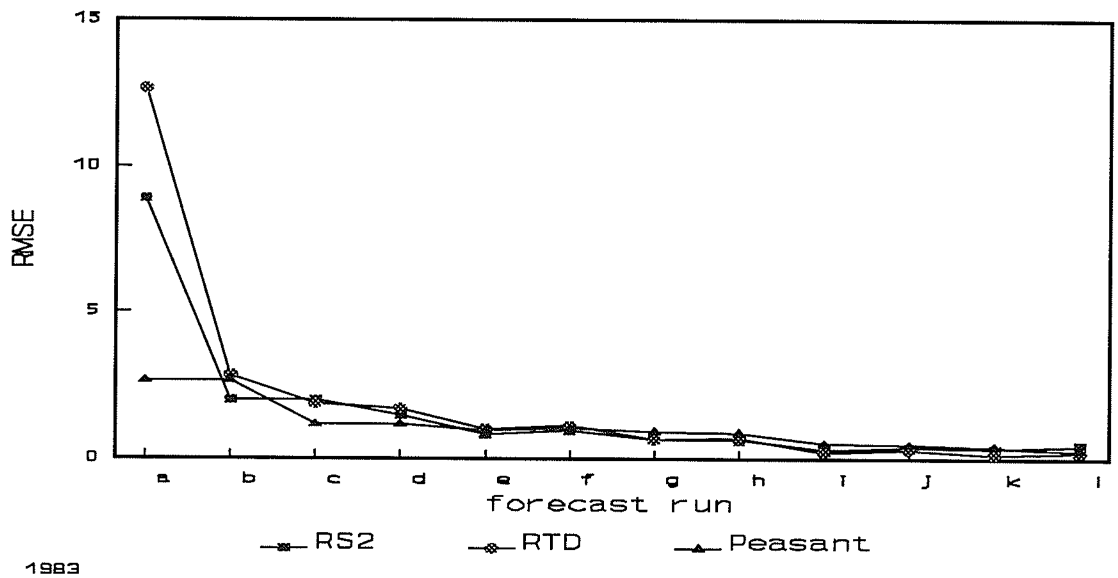


Figure B.3.6 Red River median flow case RMSE by month

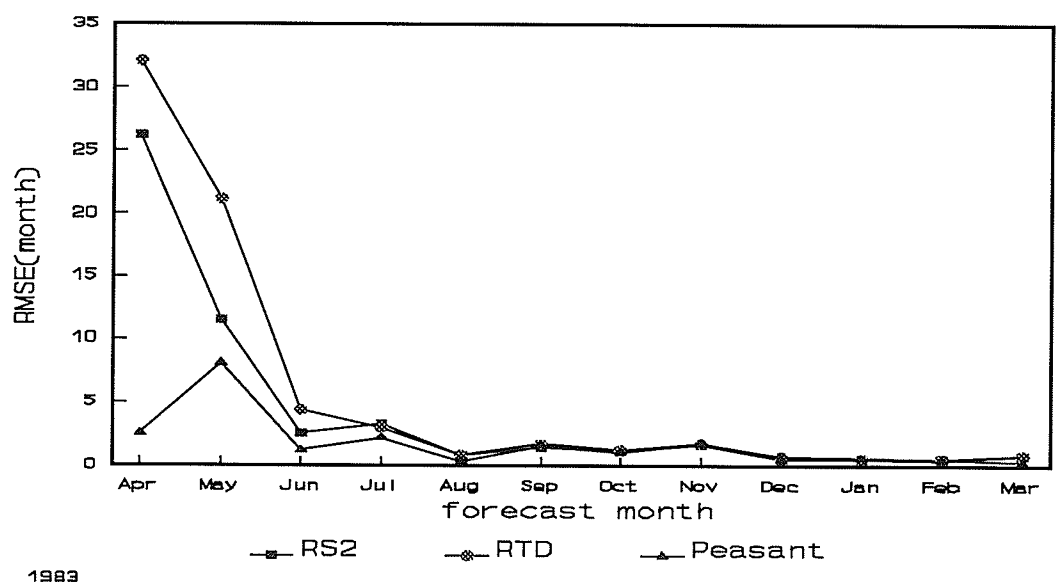
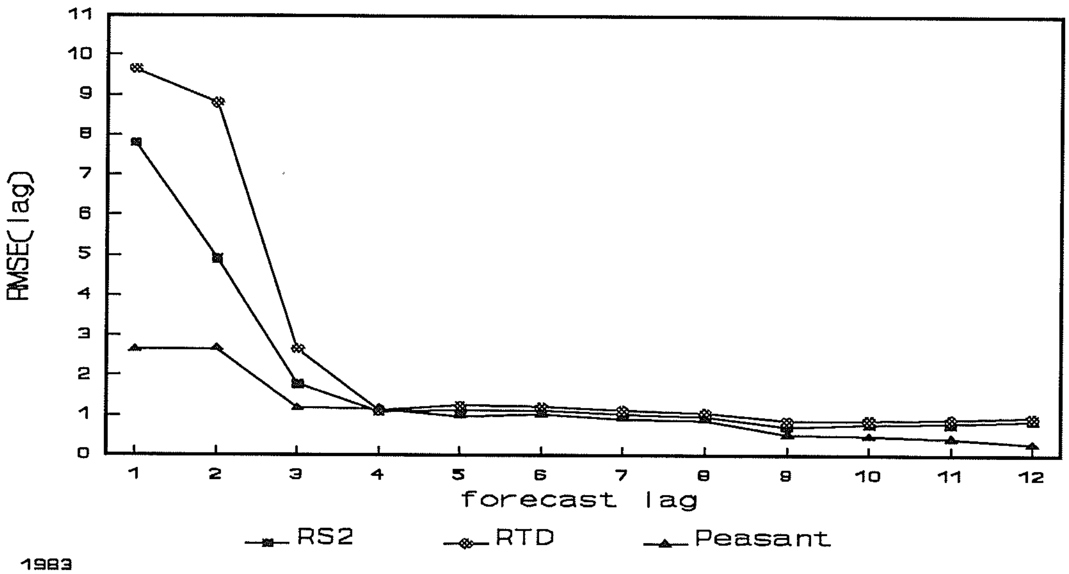


Figure B.3.7 Red River median flow case RMSE by lag



Appendix B.4 Low case

Figure B.4.1 Red River low flow case forecast

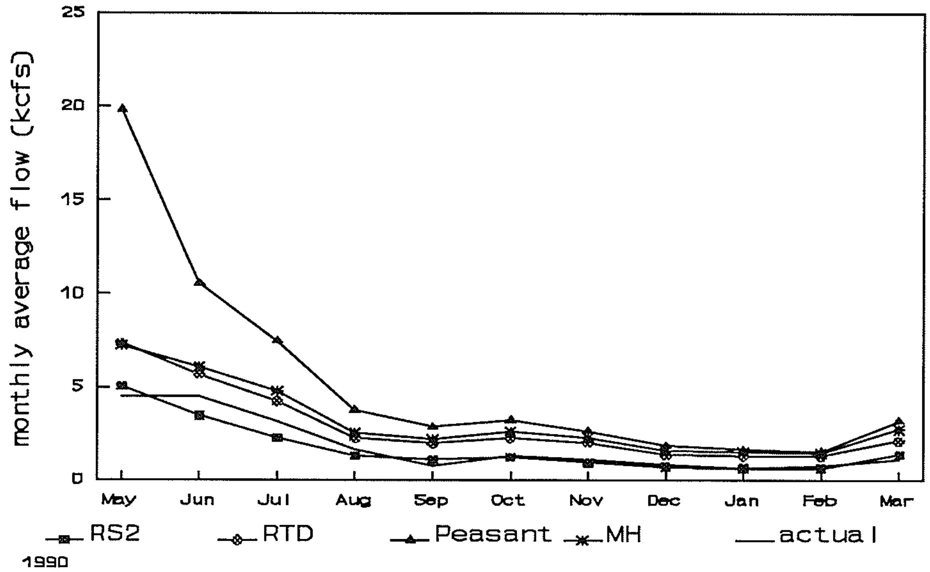


Figure B.4.2 Red River low flow case forecast

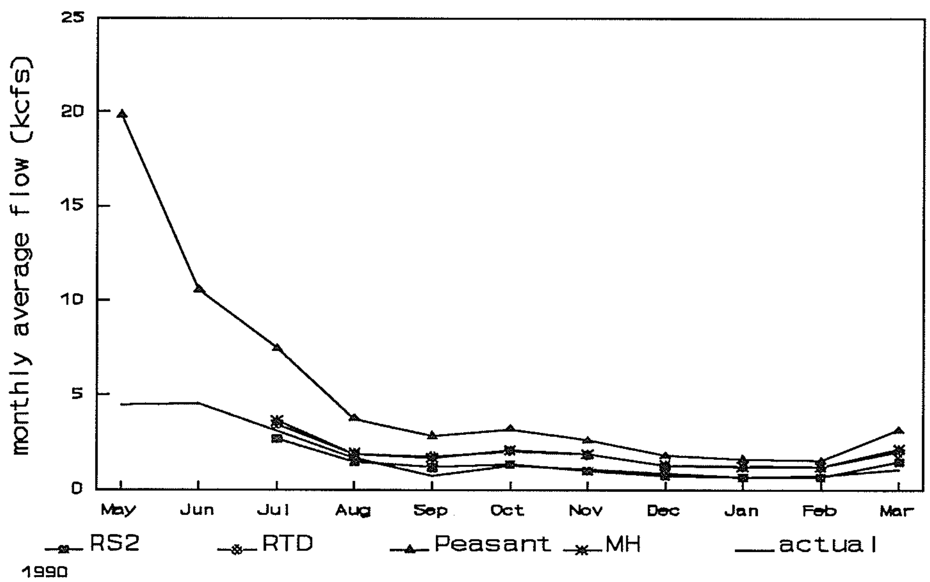


Figure B.4.3 Red River low flow case forecast

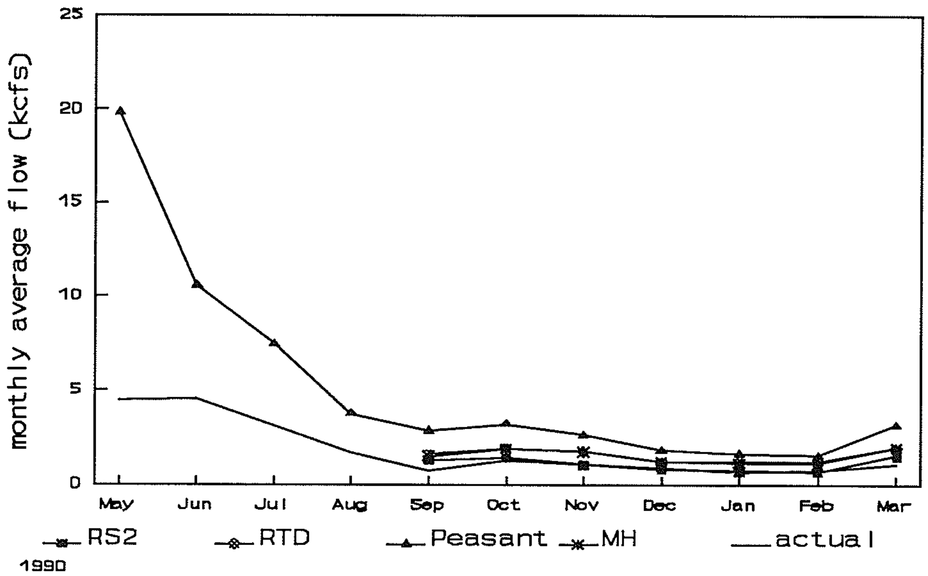


Figure B.4.4 Red River low flow case MSE by forecast lag

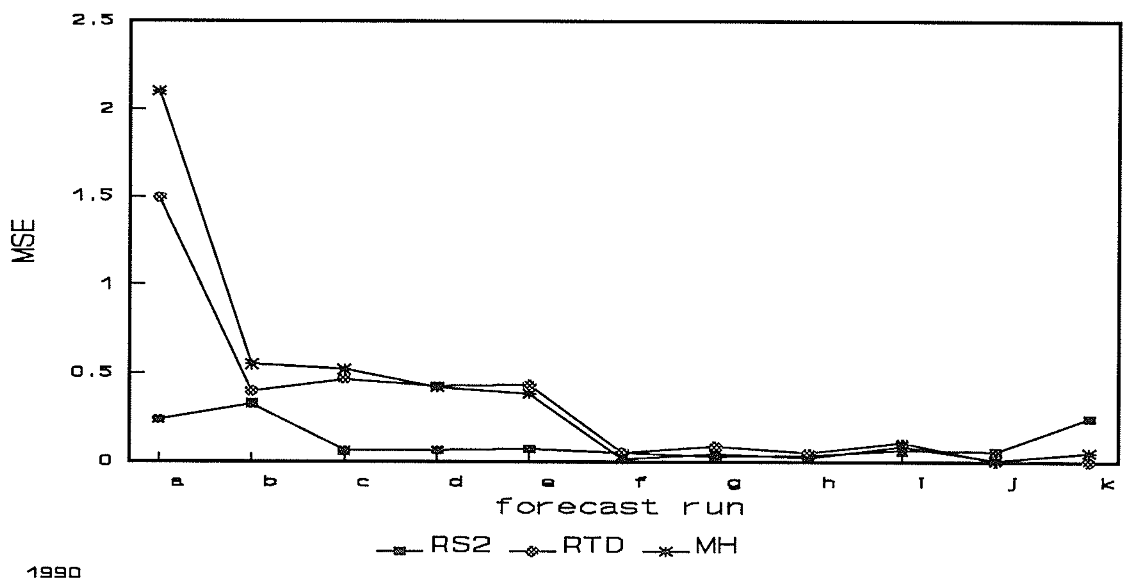


Figure B.4.5 Red River low flow case MSE by month

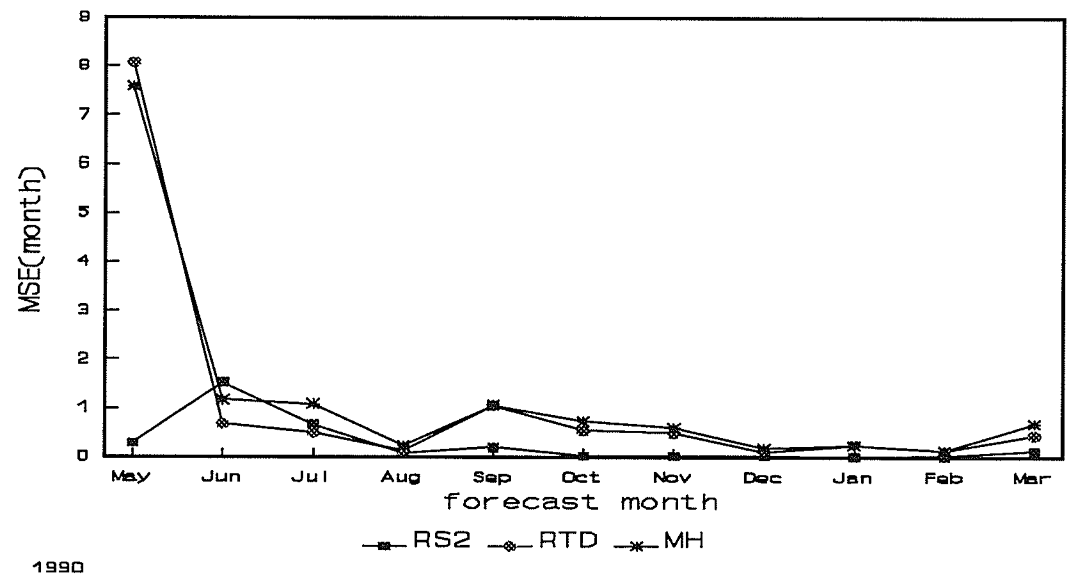
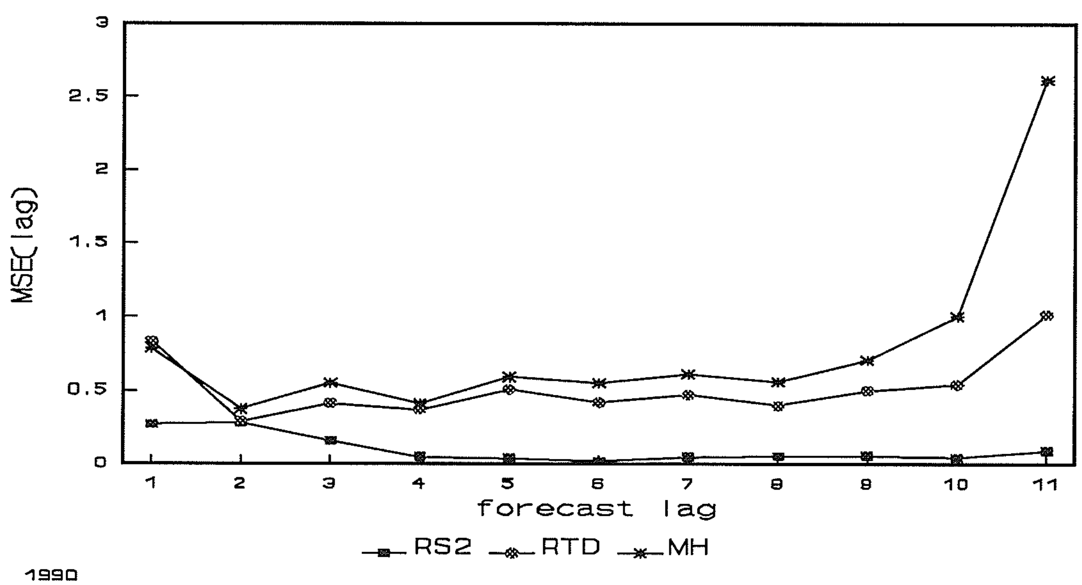


Figure B.4.6 Red River low flow case MSE by lag



APPENDIX C: PIAO

Appendix C.1 Historical characteristics

Figure C.1.1 PIAO data series

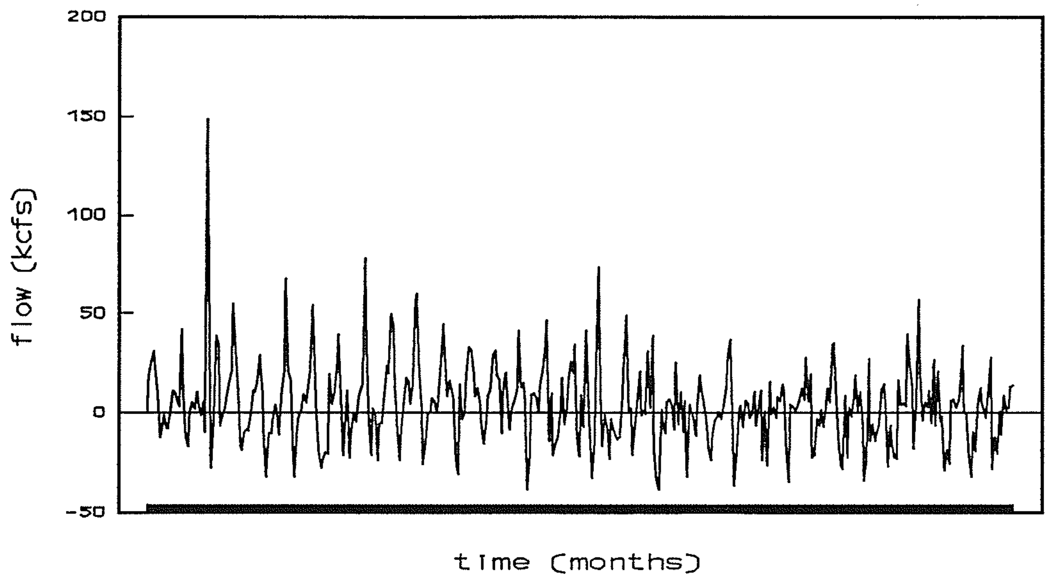
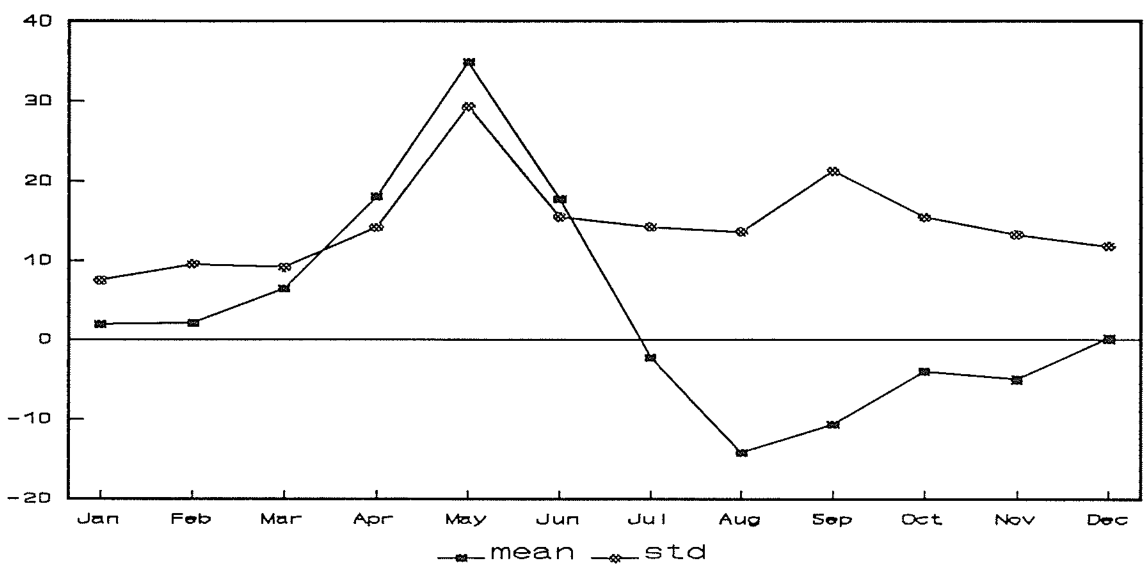


Figure C.1.2 PIAO historical characteristics



Appendix C.2 Low case

Figure C.2.1 PIAO low flow case forecast

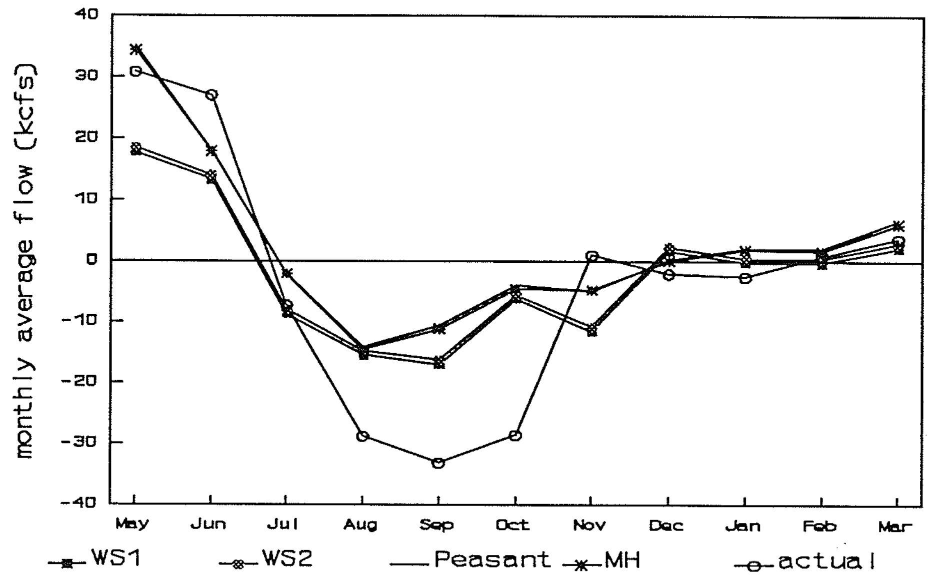


Figure C.2.2 PIAO low flow case forecast

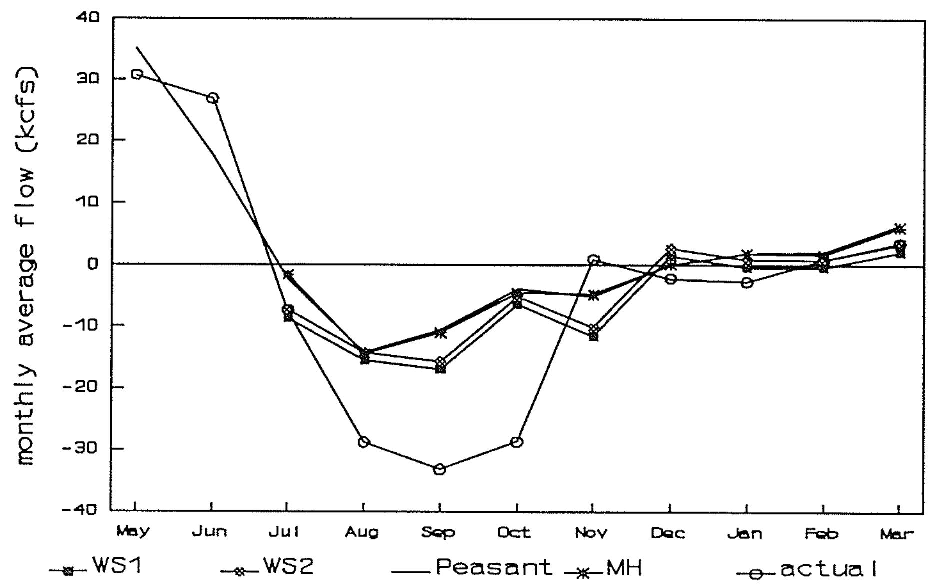


Figure C.2.3 PIAO low flow case forecast

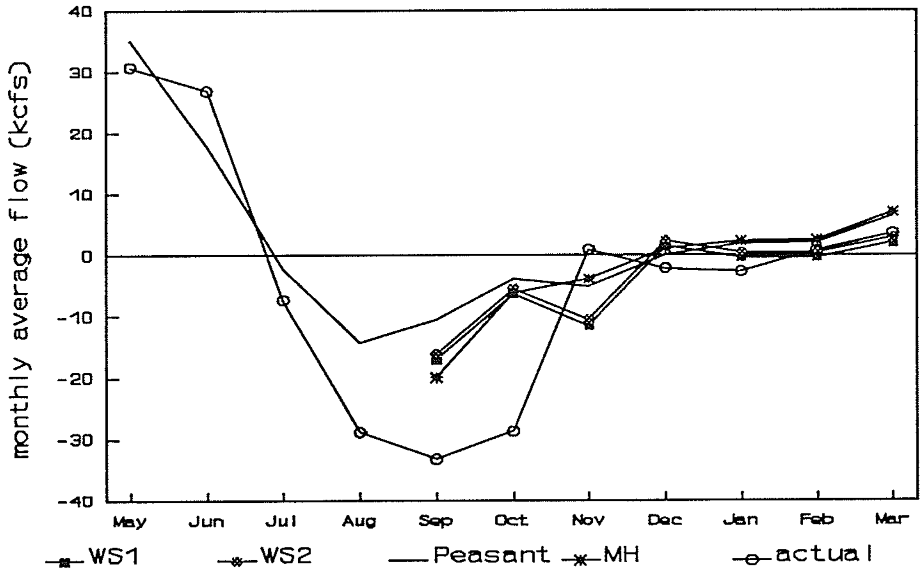


Figure C.2.4 PIAO low flow case MSE by forecast run

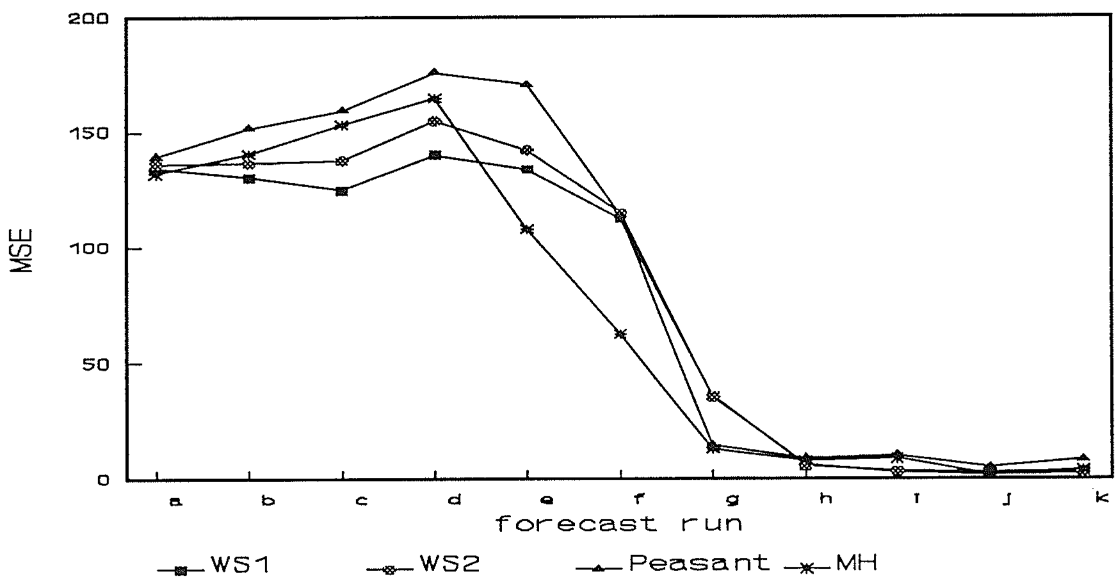


Figure C.2.5 PIAO low flow cast MSE by month

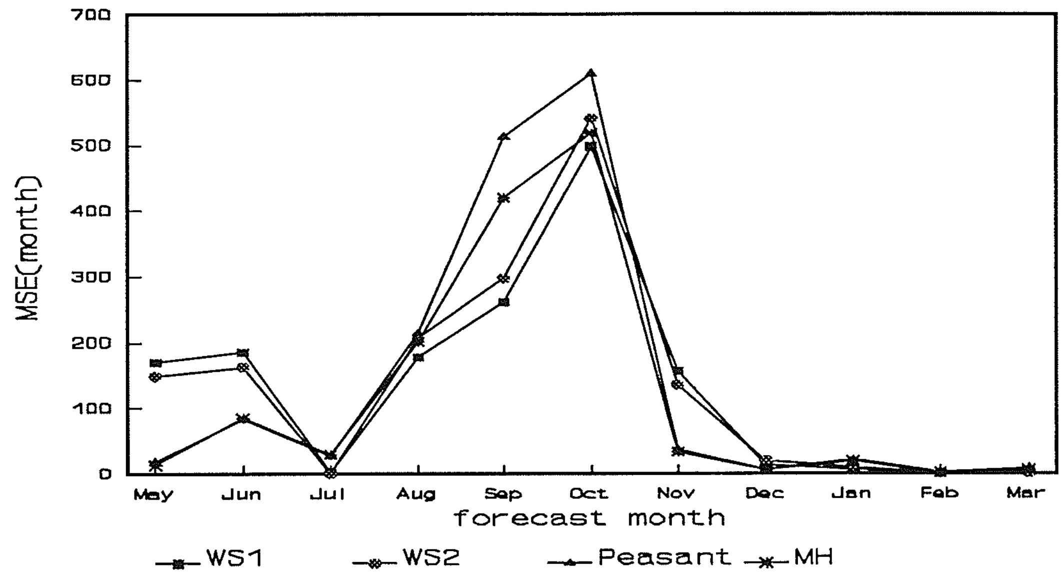
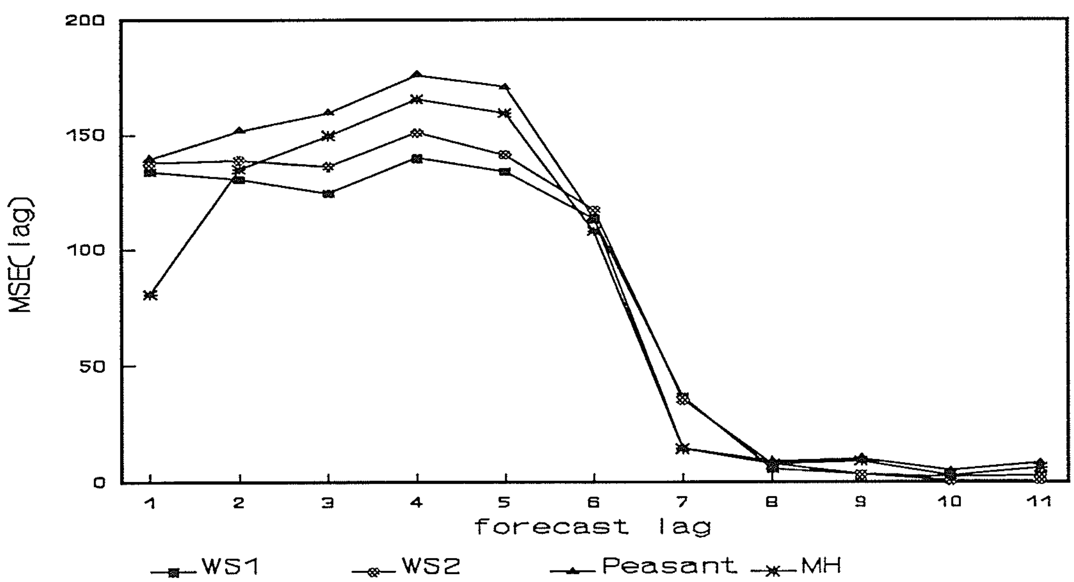


Figure C.2.6 PIAO low flow case MSE by lag



APPENDIX D: Decision Support Application

Appendix D.1 Knowledge base

The following is a listing of the rules that make up the Decision Support Application for water supply long-range forecasting developed within the Nexpert Object expert system development tool on a Unix workstation.

```
RULE :  Rule model_development_steps (#1)
If
        1 is precisely equal to 1
Then dofirst
        is confirmed.
        And Show "tf2.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,200,400,650;
        And identify is assigned to identify
        And Execute "rm -f sasfile"(@TYPE=EXE;)
        And Execute "msas @V(set) @V(transform) identify"(@TYPE=EXE;)
        And Show "sasfile.txt"
 @KEEP=TRUE;@WAIT=FALSE;@RECT=250,250,900,600;

RULE :  Rule model_development_steps (#2)
If
        1 is precisely equal to 1
        And estimate is assigned to estimate
        And var is assigned to var
        And plags is assigned to plags
        And qlags is assigned to qlags
Then dosecond
        is confirmed.
        And Execute "rm -f sasfile"(@TYPE=EXE;)
        And  STRCAT("\"",STRCAT(var,STRCAT("\"  \"",STRCAT(plags,STRCAT("\"
\"",STRCAT(qlags,"\""))))))  is assigned to parms
        And Execute "msas @V(set) @V(transform) estimate @V(parms)"(@TYPE=EXE;)
        And Show "sasfile.txt"
 @KEEP=TRUE;@WAIT=FALSE;@RECT=0,250,1100,600;

RULE :  Rule model_development_forecast (#3)
If
        make_forecast is TRUE
Then dothird
        is confirmed.
```

```
        And Execute "rm -f sasfile"(@TYPE=EXE;)
        And Execute "msas @V(set) @V(transform) forecast @V(parms)"(@TYPE=EXE;)
        And Show "sasfile.txt"
 @KEEP=TRUE;@WAIT=FALSE;@RECT=400,250,700,600;
        And Execute "mfore"(@TYPE=EXE;)
        And Execute "xgraph_for @V(set)
 @V(transform)"(@TYPE=EXE;@WAIT=TRUE;)

RULE : Rule estimate_model_structure (#6)
If
        procedure is "seasonal"
        And transform is "none"
        And task is "model_development"
Then estimate
        is confirmed.
        And Show "sparm.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,200,300,650;
        And Show "s_non_opt.txt"
 @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,300,200;

RULE : Rule estimate_model_structure (#5)
If
        procedure is "seasonal"
        And transform is "log"
        And task is "model_development"
Then estimate
        is confirmed.
        And Show "sparm.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,200,300,650;
        And Show "s_log_opt.txt"
 @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,300,200;

RULE : Rule estimate_model_structure (#4)
If
        procedure is "td"
        And task is "model_development"
Then estimate
        is confirmed.
        And Show "dparm.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,200,300,650;
        And Show "d_opt.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,300,200;
        And "X" is assigned to var

RULE : Rule graph_series (#7)
If
        flow is "uncertain"
Then get_flow
```

```
        is confirmed.
        And Show "flow_help.txt"
 @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,350,300;
        And Execute "xgraph_ser @V(set)"(@TYPE=EXE;)
        And Reset flow
        And Reset get_flow

RULE :  Rule get_flow_3 (#8)
If
        flow is "low"
Then get_flow
        is confirmed.

RULE :  Rule get_flow_2 (#9)
If
        flow is "median"
Then get_flow
        is confirmed.

RULE :  Rule get_flow_1 (#10)
If
        flow is "high"
Then get_flow
        is confirmed.

RULE :  Rule identify_model_structure (#12)
If
        procedure is "td"
Then identify
        is confirmed.
        And "td" is assigned to transform

RULE :  Rule identify_model_structure (#11)
If
        procedure is "seasonal"
Then identify
        is confirmed.
        And set_transform is assigned to set_transform

RULE :  Rule make_model (#13)
If
        there is evidence of dofirst
        And there is evidence of dosecond
        And parameters is "significant"
```

```
        And chi_square is "adequate"
        And residuals is "insignificant"
Then make_model
        is confirmed.
        And Show "make.txt" @KEEP=TRUE;@WAIT=TRUE;@RECT=0,0,400,200;

RULE :  Rule reject_model_structure (#16)
If
        parameters is "insignificant"
Then reject_model
        is confirmed.
        And Show "r_parms.txt" @KEEP=TRUE;@WAIT=TRUE;@RECT=0,0,400,200;
        And Reset parameters
        And Reset chi_square
        And Reset residuals
        And Reset plags
        And Reset qlags
        And Reset reject_model
        And Reset dosecond
        And Reset make_model
        And make_model is assigned to make_model

RULE :  Rule reject_model_structure (#15)
If
        residuals is "significant"
Then reject_model
        is confirmed.
        And Show "r_res.txt" @KEEP=TRUE;@WAIT=TRUE;@RECT=0,0,400,200;
        And Reset dosecond
        And Reset plags
        And Reset qlags
        And Reset parameters
        And Reset chi_square
        And Reset residuals
        And Reset reject_model
        And Reset make_model
        And make_model is assigned to make_model

RULE :  Rule reject_model_structure (#14)
If
        chi_square is "inadequate"
Then reject_model
        is confirmed.
        And Show "r_chi.txt" @KEEP=TRUE;@WAIT=TRUE;@RECT=0,0,400,200;
```

```
        And Reset parameters
        And Reset chi_square
        And Reset residuals
        And Reset qlags
        And Reset plags
        And Reset qlags
        And Reset reject_model
        And Reset dosecond
        And Reset make_model
        And make_model is assigned to make_model

RULE :  Rule choose_model_ws1 (#17)
If
        task is "forecasting"
        And set is "piao"
Then set_model
        is confirmed.
        And "ws1" is assigned to model
        And "none" is assigned to transform
        And Show "piao.txt" @KEEP=FALSE;@WAIT=TRUE;@RECT=0,0,400,200;
        And Show "ws1.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule choose_model_rs2 (#18)
If
        task is "forecasting"
        And set is "red"
Then set_model
        is confirmed.
        And "rs2" is assigned to model
        And "log" is assigned to transform
        And Show "red.txt" @KEEP=FALSE;@WAIT=TRUE;@RECT=0,0,400,200;
        And Show "rs2.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule choose_model_gtd_4 (#19)
If
        task is "forecasting"
        And set is "grass"
        And there is evidence of get_flow
        And flow is "low"
        And peak is "median"
Then set_model
        is confirmed.
        And "gtd" is assigned to model
        And "td" is assigned to transform
```

```
        And Show "gtd.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule choose_model_gtd_3 (#20)
If
        task is "forecasting"
        And set is "grass"
        And there is evidence of get_flow
        And flow is "low"
        And peak is "high"
Then set_model
        is confirmed.
        And "gtd" is assigned to model
        And "td" is assigned to transform
        And Show "gtd.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule choose_model_gtd_2 (#21)
If
        task is "forecasting"
        And set is "grass"
        And there is evidence of get_flow
        And flow is "median"
Then set_model
        is confirmed.
        And "gtd" is assigned to model
        And "td" is assigned to transform
        And Show "gtd.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule choose_model_gtd_1 (#22)
If
        task is "forecasting"
        And set is "grass"
        And there is evidence of get_flow
        And flow is "high"
Then set_model
        is confirmed.
        And "gtd" is assigned to model
        And "td" is assigned to transform
        And Show "gtd.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule choose_model_gs2 (#23)
If
        task is "forecasting"
        And set is "grass"
        And there is evidence of get_flow
```

```
        And flow is "low"
        And peak is "low"
Then set_model
        is confirmed.
        And "gs2" is assigned to model
        And "log" is assigned to transform
        And Show "gs2.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule choose_model_gs1 (#24)
If
        task is "forecasting"
        And set is "grass"
        And there is evidence of get_flow
        And flow is "low"
        And peak is "uncertain"
Then set_model
        is confirmed.
        And "gs1" is assigned to model
        And "log" is assigned to transform
        And Show "gs1.txt" @KEEP=TRUE;@WAIT=FALSE;@RECT=0,0,400,200;

RULE :  Rule get_seasonal_transform (#26)
If
        transform is "none"
Then set_transform
        is confirmed.

RULE :  Rule get_seasonal_transform (#25)
If
        transform is "log"
Then set_transform
        is confirmed.

RULE :  Rule end_session (#27)
If
        task is "quit"
Then what_to_do
        is confirmed.

RULE :  Rule decide_to_forecast (#28)
If
        task is "forecasting"
Then what_to_do
        is confirmed.
```

```
And Show "forecast.txt" @KEEP=TRUE;@WAIT=TRUE;@RECT=0,0,400,200;
And set_model is assigned to set_model
And Execute "fsas @V(set) @V(model)"(@TYPE=EXE;)
And Show "sasfile.txt" @KEEP=T
```

Appendix D.2 Program listings

Program td.f is written in FORTRAN to transform and deseasonalize a set of monthly flow values and return the deseasonalized values to 'td.out' output file to be used by SAS.

```
c        ***********************************
c        Transform deseasonalize program
c        ***********************************
        DIMENSION X(12,50), x1(12), x2(12)
        OPEN (1,FILE='td.in')
         OPEN (2,FILE='td.out')
         J=1
 2       READ (1,*,END=99) (X(I,J), I=1, 12)
         DO 6 I=1,12
           X(I,J)=ALOG(X(I,J))
           x1(i)=x1(i)+X(i,j)
           x2(i)=x2(i)+X(i,j)**2
 6       continue
         j=j+1
         go to 2
 99      close (1)
         I=I-1
         do 7 k=1,I
           X(k,j)=alog(x(k,j))
           x1(k)=x1(k)+X(k,j)
           x2(k)=x2(k)+X(k,j)**2
 7       continue
         do 8 k=1,12
           x2(k)=(x2(k)-x1(k)**2/j)/(j-1)
           x1(k)=x1(k)/j
 8       continue
         do 9 l=1,j-1
           do 11 k=1,12
             X(k,l)=(X(k,l)-x1(k))/x2(k)
 11        continue
           write (2,10) (X(k,l), k=1,12)
 9       continue
         do 12 k=1,i
           X(k,j)=(X(k,j)-x1(k))/x2(k)
 12      continue
         write (2,10) (X(k,j), k=1,i)
         close (2)
```

```
10      format (12f7.2)
        stop
        end
```

Program real0.f is written in FORTRAN to produce a compatible data file for plotting when no transformation or deseasonalization manipulation is used to produce the model forecasts.

```
c        ***********************************
c        de - program
c        ***********************************
        DIMENSION X(12,50), q1(12)
        OPEN (1,FILE='real.in')
         OPEN (2,FILE='xsas.dat')
         OPEN (3,FILE='real.out')
        OPEN (4,FILE='means.out')
         J=1
 2       READ (1,*,END=99) (X(I,J), I=1, 12)
         DO 6 I=1,12
            q1(i)=q1(i)+X(i,j)
 6       continue
         j=j+1
         go to 2
 99      close (1)
         I=I-1
         do 7 k=1,I
            q1(k)=q1(k)+X(k,j)
 7       continue
         do 8 k=1,12
            q1(k)=q1(k)/j
 8       continue
        do 12 k=i+1,12
           READ (2,10,end=14) obs, rnum
           write (3,10) obs, rnum
           write (4,10) obs, q1(k)
 12      continue
        do 14 k=1,i
           READ (2,10,end=14) obs, rnum
           write (3,10) obs, rnum
           write (4,10) obs, q1(k)
 14      continue
        close (2)
        close (3)
        close (4)
 10      format (i3,x,f8.3)
         stop
         end
```

Program real1.f is written in FORTRAN to produce real value forecasts for plotting when the data set has been transformed by the model to generate forecasts.

```
c        ***********************************
c        de - Transform program
c        ***********************************
        DIMENSION X(12,50), q1(12)
        OPEN (1,FILE='real.in')
         OPEN (2,FILE='xsas.dat')
         OPEN (3,FILE='real.out')
        OPEN (4,FILE='means.out')
         J=1
 2       READ (1,*,END=99) (X(I,J), I=1, 12)
         DO 6 I=1,12
           q1(i)=q1(i)+X(i,j)
           X(I,J)=ALOG(X(I,J))
 6       continue
         j=j+1
         go to 2
 99      close (1)
         I=I-1
         do 7 k=1,I
           q1(k)=q1(k)+X(k,j)
           X(k,j)=alog(x(k,j))
 7       continue
         do 8 k=1,12
           q1(k)=q1(k)/j
 8       continue
        do 12 k=i+1,12
          READ (2,10,end=14) obs, tnum
          rnum=exp(tnum)
          write (3,10) obs, rnum
          write (4,10) obs, q1(k)
 12      continue
        do 14 k=1,i
          READ (2,10,end=14) obs, tnum
          rnum=exp(tnum)
          write (3,10) obs, rnum
          write (4,10) obs, q1(k)
 14      continue
        close (2)
        close (3)
        close (4)
```

```
10    format (i3,x,f8.3)
      stop
      end
```

Program real2.f is written in FORTRAN to produce real value forecasts for plotting when the data set has been transformed and deseasonalized by the model to generate forecasts.

```
c       ***********************************
c       de - Transform deseasonalize program
c       ***********************************
        DIMENSION X(12,50), x1(12), x2(12), q1(12), q2(12)
        OPEN (1,FILE='real.in')
        OPEN (2,FILE='xsas.dat')
        OPEN (3,FILE='real.out')
        OPEN (4,FILE='means.out')
        J=1
 2      READ (1,*,END=99) (X(I,J), I=1, 12)
        DO 6 I=1,12
          q1(i)=q1(i)+X(i,j)
          q2(i)=q2(i)+X(i,j)**2
          X(I,J)=ALOG(X(I,J))
          x1(i)=x1(i)+X(i,j)
          x2(i)=x2(i)+X(i,j)**2
 6      continue
        j=j+1
        go to 2
 99     close (1)
        I=I-1
        do 7 k=1,I
          q1(k)=q1(k)+X(k,j)
          q2(k)=q2(k)+X(k,j)**2
          X(k,j)=alog(x(k,j))
          x1(k)=x1(k)+X(k,j)
          x2(k)=x2(k)+X(k,j)**2
 7      continue
        do 8 k=1,12
          q2(k)=(q2(k)-q1(k)**2/j)/(j-1)
          q1(k)=q1(k)/j
          x2(k)=(x2(k)-x1(k)**2/j)/(j-1)
          x1(k)=x1(k)/j
 8      continue
        do 12 k=i+1,12
          READ (2,10,end=14) obs, tdnum
          rnum=exp(tdnum*x2(k)+x1(k))
          write (3,10) obs, rnum
          write (4,10) obs, q1(k)
 12     continue
```

```
      do 14 k=1,i
        READ (2,10,end=14) obs, tdnum
        rnum=exp(tdnum*x2(k)+x1(k))
        write (3,10) obs, rnum
        write (4,10) obs, q1(k)
14    continue
      close (2)
      close (3)
      close (4)
10    format (i3,x,f8.3)
      stop
      end
```

Program mfore.f is written in FORTRAN to translate SAS output for model forecasts into a compatible form for plotting.

```
c -----------------------------------------------
c  MFORE
c
c  reads forecast values from sasfile.txt
c  writes in x,y format to xsas.dat for XGRAPH
c -----------------------------------------------
      character*80 line, sasline
      dimension x(5)
c
      open (1,file='sasfile.txt')
      open (2,file='xsas.dat')
c
      sasline='Obs'
      do while (line .ne. sasline)
      read (1,*,end=99) line
c     write (*,*) line
      end do
      do 10 i=1,12
      read (1,*,end=99,err=99) n, (x(j), j=1,4)
c     write (*,*) (x(j), j=1,4)
      write (2,20) n, x(1)
 10   continue
 99   continue
      close (1)
      close (2)
 20   format (i3,x,f8.3)
      stop
      end
```

Program data.f is written in FORTRAN to translate data files into a compatible form for plotting historical data series.

```
c --------------------------------------
c  DATA
c
c  write out data set (from data.in) in
c  x,y format for XGRAPH
c --------------------------------------
      DIMENSION X(12)
      OPEN (1,FILE='data.in')
      OPEN (2,FILE='data.out')
      n=0
 2    READ (1,*,END=99) (X(I), I=1, 12)
      DO 6 I=1,12
      m=n+i
      write (2,10) m, X(I)
 6    continue
      n=n+12
      go to 2
 99   close (1)
      I=I-1
      do 7 k=1,i
      m=n+k
      write (2,10) m, X(k)
 7    continue
      close (2)
 10   format (i3,x,f7.2)
      stop
      end
```

<u>fsas</u> is a Unix script that controls the formation of a SAS input file to forecast a previously developed model, and executes SAS.

```
echo DATA FLOW\; > sasfile
echo INPUT X@@\; >> sasfile
if [ $2 = ?td ]
then
        echo CARDS\; >> sasfile
         cp $1 td.in
         td
         cat td.out >> sasfile
else
        echo LX=LOG\(X\)\; >> sasfile
        echo CARDS\; >> sasfile
         cat $1 >> sasfile
fi
echo PROC ARIMA\; >> sasfile
cat $2 >> sasfile
echo FORECAST LEAD=12\; >> sasfile
sas sasfile
mv sasfile.lst sasfile.txt
```

msas is a Unix script that controls the formation of a SAS input file for various stages of model development, and executes SAS.

```
echo DATA FLOW\; > sasfile
echo INPUT X@@\; >> sasfile
if [ $2 = td ]
then
        echo CARDS\; >> sasfile
        cp $1 td.in
        td
        cat td.out >> sasfile
fi
if [ $2 = log ]
then
        echo LX=LOG\(X\)\; >> sasfile
        echo CARDS\; >> sasfile
        cat $1 >> sasfile
fi
if [ $2 = none ]
then
        echo CARDS\; >> sasfile
        cat $1 >> sasfile
fi
echo PROC ARIMA\; >> sasfile
if [ $3 = identify ]
then
        if [ $2 = td ]
        then
                echo IDENTIFY VAR=X\; >> sasfile
        fi
        if [ $2 = log ]
        then
                echo IDENTIFY VAR=LX\; >> sasfile
                echo IDENTIFY VAR=LX\(12\)\; >> sasfile
                echo IDENTIFY VAR=LX\(1,12\)\; >> sasfile
        fi
        if [ $2 = none ]
        then
                echo IDENTIFY VAR=X\; >> sasfile
                echo IDENTIFY VAR=X\(12\)\; >> sasfile
                echo IDENTIFY VAR=X\(1,12\)\; >> sasfile
        fi
fi
```

```
if [ $3 = estimate ]
then
        echo IDENTIFY VAR= $4 NOPRINT\; >> sasfile
        echo ESTIMATE P= $5 Q= $6 NOCONSTANT METHOD=ML PLOT\; >>
sasfile
fi
if [ $3 = forecast ]
then
        echo IDENTIFY VAR= $4 NOPRINT\; >> sasfile
        echo ESTIMATE P= $5 Q= $6 NOCONSTANT METHOD=ML NOPRINT\; >>
sasfile
        echo FORECAST LEAD=12\; >> sasfile
fi
sas sasfile
mv sasfile.lst sasfile.txt
```

xgraph_for is a Unix script that controls the execution of FORTRAN programs for data manipulation of model forecasts, and executes the plotting utility Xgraph.

```
cp $1 real.in
if [ $2 = td ]
then
        real2
fi
if [ $2 = log ]
then
        real1
fi
if [ $2 = none ]
then
        real0
fi
echo \"forecast\" > xsas.dat
cat real.out >> xsas.dat
echo " " >> xsas.dat
echo \"monthly means\" >> xsas.dat
cat means.out >> xsas.dat
xgraph -P -t "Forecasts" -x "month" -y "kcfs" xsas.dat
```

xgraph_ser is a Unix script that executes Xgraph to plot historical data series.

```
cp $1 data.in
data
xgraph -t "Data series" -x "month" -y "kcfs" data.out
```

APPENDIX E: Time Series Output

Grass River
correlation structure

```
                      Name of variable = X.
                      Mean of working series = 2.467951
                      Standard deviation     = 1.385322
                      Number of observations =      364
                               Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   1.919117     1.00000  |                    |********************|
  1   1.602615     0.83508  |                  . |*****************   |
  2   0.971014     0.50597  |                 .  |**********          |
  3   0.407375     0.21227  |                .   |****                |
  4   0.065954     0.03437  |                .   |*   .              |
  5  -0.091502    -0.04768  |                .  *|    .              |
  6  -0.132494    -0.06904  |                .  *|    .              |
  7  -0.111554    -0.05813  |                .  *|    .              |
  8  -0.023609    -0.01230  |                .   |    .              |
  9   0.173858     0.09059  |                .   |**  .              |
 10   0.511714     0.26664  |                .   |*****              |
 11   0.896672     0.46723  |                .   |*********          |
 12   1.091623     0.56882  |                .   |***********        |
 13   0.919769     0.47927  |                .   |**********         |
 14   0.509975     0.26573  |               .    |*****              |
 15   0.121003     0.06305  |               .    |*    .             |
 16  -0.126174    -0.06575  |               .   *|     .             |
 17  -0.249198    -0.12985  |               . ***|     .             |
 18  -0.289951    -0.15109  |               . ***|     .             |
 19  -0.262813    -0.13694  |               . ***|     .             |
 20  -0.165799    -0.08639  |               .  **|     .             |
 21   0.034250     0.01785  |               .    |     .             |
 22   0.364616     0.18999  |               .    |****.              |
 23   0.727967     0.37932  |               .    |********           |
 24   0.882241     0.45971  |               .    |*********          |

                           Partial Autocorrelations
           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.83508  |                  . |*****************   |
             2    -0.63239  |        *************| .                 |
             3     0.27886  |                  . |******              |
             4    -0.07759  |                 **| .                  |
             5    -0.03882  |                 .*| .                  |
             6     0.04615  |                  . |*.                  |
             7    -0.01683  |                  . | .                  |
             8     0.14319  |                  . |***                 |
             9     0.20299  |                  . |****                |
            10     0.29859  |                  . |******              |
            11     0.14896  |                  . |***                 |
            12    -0.14921  |                ***| .                  |
            13    -0.21219  |               ****| .                  |
            14     0.08715  |                  . |**                  |
```

```
 15      0.02009 |                  . | .                  |
 16     -0.09146 |                  **| .                  |
 17     -0.03671 |                  .*| .                  |
 18     -0.01354 |                  . | .                  |
 19      0.04249 |                  . |*.                  |
 20      0.01788 |                  . | .                  |
 21      0.09818 |                  . |**                  |
 22      0.13805 |                  . |***                 |
 23      0.00740 |                  . | .                  |
 24     -0.17158 |                 ***| .                  |

                 Name of variable = LX.
                 Mean of working series = 0.731437
                 Standard deviation     = 0.627061
                 Number of observations =      364
                              Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.393205    1.00000  |                    |********************|
  1   0.340793    0.86670  |                  . |*****************   |
  2   0.244326    0.62137  |                 .  |************        |
  3   0.153468    0.39030  |                .   |********            |
  4   0.090531    0.23024  |                .   |*****               |
  5   0.055701    0.14166  |                .   |***.                |
  6   0.039881    0.10143  |                .   |**  .               |
  7   0.034494    0.08772  |                .   |**  .               |
  8   0.044060    0.11205  |                .   |**  .               |
  9   0.074395    0.18920  |                .   |****                |
 10   0.127870    0.32520  |                .   |*******             |
 11   0.185916    0.47282  |                .   |*********           |
 12   0.210478    0.53529  |                .   |***********         |
 13   0.174752    0.44443  |               .    |*********           |
 14   0.103993    0.26447  |               .    |*****               |
 15   0.038506    0.09793  |               .    |**   .              |
 16 -0.0052241   -0.01329  |               .    |     .              |
 17  -0.028682   -0.07294  |               .   *|     .              |
 18  -0.038497   -0.09791  |               .  **|     .              |
 19  -0.036839   -0.09369  |               .  **|     .              |
 20  -0.019877   -0.05055  |               .   *|     .              |
 21   0.018204    0.04630  |               .    |*    .              |
 22   0.077529    0.19717  |               .    |****.               |
 23   0.139719    0.35533  |               .    |*******             |
 24   0.164686    0.41883  |               .    |********            |

                          Partial Autocorrelations
            Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1     0.86670 |                  . |*****************   |
              2    -0.52168 |          **********| .                  |
              3     0.12671 |                  . |***                 |
              4     0.04415 |                  . |*.                  |
              5     0.01146 |                  . | .                  |
              6     0.00999 |                  . | .                  |
              7     0.02126 |                  . | .                  |
```

```
  8    0.16416  |                    .  |***                 |
  9    0.18560  |                    .  |****                |
 10    0.26365  |                    .  |*****               |
 11    0.10437  |                    .  |**                  |
 12   -0.17618  |                ****|  .                   |
 13   -0.31352  |              ******|  .                   |
 14    0.00809  |                    .  |  .                 |
 15    0.07860  |                    .  |**                  |
 16   -0.05770  |                   .*|  .                  |
 17   -0.04869  |                   .*|  .                  |
 18   -0.00320  |                    .  |  .                 |
 19    0.06815  |                    .  |*.                  |
 20    0.06866  |                    .  |*.                  |
 21    0.08373  |                    .  |**                  |
 22    0.10363  |                    .  |**                  |
 23    0.06290  |                    .  |*.                  |
 24   -0.14058  |                 ***|  .                   |

              Name of variable = LX.
              Period(s) of Differencing = 1.
              Mean of working series = -0.00593
              Standard deviation     = 0.299572
              Number of observations =      363
                       Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.089743    1.00000  |                    |********************|
  1   0.044939    0.50075  |                  . |**********          |
  2 -0.0057956   -0.06458  |                 . *|  .                 |
  3  -0.027869   -0.31055  |              ******|  .                 |
  4  -0.026087   -0.29069  |              ******|  .                 |
  5  -0.016865   -0.18793  |                ****|  .                 |
  6  -0.010560   -0.11767  |                 .**|  .                 |
  7  -0.014181   -0.15801  |                 ***|  .                 |
  8  -0.020128   -0.22429  |                ****|  .                 |
  9  -0.020587   -0.22940  |               *****|  .                 |
 10 -0.0039992   -0.04456  |                 . *|  .                 |
 11   0.033647    0.37492  |                 .  |*******             |
 12   0.059236    0.66006  |                 .  |*************       |
 13   0.035273    0.39305  |                .   |********            |
 14 -0.0050393   -0.05615  |                .  *|   .                |
 15  -0.021835   -0.24330  |               *****|   .                |
 16  -0.019528   -0.21759  |                ****|   .                |
 17  -0.012700   -0.14152  |                .***|   .                |
 18  -0.010202   -0.11368  |                . **|   .                |
 19  -0.013133   -0.14634  |                .***|   .                |
 20  -0.018855   -0.21010  |                ****|   .                |
 21  -0.019491   -0.21718  |                ****|   .                |
 22 -0.0026449   -0.02947  |                .  *|   .                |
 23   0.034213    0.38124  |                .   |********            |
 24   0.056820    0.63314  |                .   |*************       |

                    Partial Autocorrelations
```

```
Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.50075  |                  . |**********          |
  2   -0.42087  |            ********| .                  |
  3   -0.08734  |                  **| .                  |
  4   -0.09901  |                  **| .                  |
  5   -0.11304  |                  **| .                  |
  6   -0.12543  |                 ***| .                  |
  7   -0.23951  |               *****| .                  |
  8   -0.24647  |               *****| .                  |
  9   -0.29284  |              ******| .                  |
 10   -0.12538  |                 ***| .                  |
 11    0.25824  |                  . |*****               |
 12    0.29671  |                  . |******              |
 13   -0.11258  |                  **| .                  |
 14   -0.05624  |                  .*| .                  |
 15    0.09569  |                  . |**                  |
 16    0.01218  |                  . | .                  |
 17   -0.02454  |                  . | .                  |
 18   -0.04719  |                  .*| .                  |
 19   -0.02931  |                  .*| .                  |
 20   -0.08959  |                  **| .                  |
 21   -0.10539  |                  **| .                  |
 22   -0.00685  |                  . | .                  |
 23    0.17114  |                  . |***                 |
 24    0.15138  |                  . |***                 |

                    Name of variable = LX.
                    Period(s) of Differencing = 12.
                    Mean of working series = -0.05161
                    Standard deviation     = 0.517147
                    Number of observations =      352
                             Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.267441    1.00000  |                    |********************|
  1   0.235589    0.88090  |                  . |*****************   |
  2   0.185067    0.69199  |                 .  |**************      |
  3   0.137099    0.51263  |                .   |**********          |
  4   0.101686    0.38022  |                .   |********            |
  5   0.079911    0.29880  |                .   |******              |
  6   0.064455    0.24101  |               .    |*****               |
  7   0.048145    0.18002  |               .    |****.               |
  8   0.028472    0.10646  |               .    |**  .               |
  9  0.0030407    0.01137  |               .    |    .               |
 10  -0.026734   -0.09996  |               .  **|    .               |
 11  -0.055472   -0.20742  |               .****|    .               |
 12  -0.068661   -0.25673  |               *****|    .               |
 13  -0.051392   -0.19216  |               .****|    .               |
 14  -0.029540   -0.11045  |               .  **|    .               |
 15  -0.009884   -0.03696  |               .   *|    .               |
 16  0.0032887    0.01230  |               .    |    .               |
 17   0.010501    0.03926  |               .    |*   .               |
 18   0.014650    0.05478  |               .    |*   .               |
```

```
  19    0.019444     0.07270  |                .    |*    .               |
  20    0.022783     0.08519  |                .    |**   .               |
  21    0.023594     0.08822  |                .    |**   .               |
  22    0.022176     0.08292  |                .    |**   .               |
  23    0.016873     0.06309  |                .    |*    .               |
  24    0.010126     0.03786  |                .    |*    .               |

                               Partial Autocorrelations
               Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                 1    0.88090   |                   . |*****************   |
                 2   -0.37496   |             *******| .                   |
                 3    0.02489   |                   . | .                  |
                 4    0.06726   |                   . |*.                  |
                 5    0.04908   |                   . |*.                  |
                 6   -0.04271   |                   .*| .                  |
                 7   -0.07348   |                   .*| .                  |
                 8   -0.06495   |                   .*| .                  |
                 9   -0.13047   |                  ***| .                  |
                10   -0.12683   |                  ***| .                  |
                11   -0.08016   |                   **| .                  |
                12    0.14230   |                   . |***                 |
                13    0.35018   |                   . |*******             |
                14   -0.20306   |                 ****| .                  |
                15    0.07830   |                   . |**                  |
                16    0.05890   |                   . |*.                  |
                17    0.04043   |                   . |*.                  |
                18   -0.03730   |                   .*| .                  |
                19   -0.00030   |                   . | .                  |
                20   -0.05369   |                   .*| .                  |
                21   -0.10363   |                   **| .                  |
                22   -0.08036   |                   **| .                  |
                23   -0.08427   |                   **| .                  |
                24    0.14658   |                   . |***                 |

                          Name of variable = LX.
                          Period(s) of Differencing = 1,12.
                          Mean of working series = -0.00201
                          Standard deviation     =  0.23957
                          Number of observations =      351
                                Autocorrelations
 Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
   0    0.057394    1.00000  |                    |********************|
   1    0.019051    0.33193  |                  . |*******             |
   2  -0.0025383   -0.04423  |                  .*| .                  |
   3   -0.012618   -0.21985  |                ****| .                  |
   4   -0.012188   -0.21235  |                ****| .                  |
   5  -0.0071205   -0.12406  |                 .**|  .                 |
   6  -0.0001181   -0.00206  |                 .  |  .                 |
   7   0.0025455    0.04435  |                 .  |* .                 |
   8   0.0049217    0.08575  |                 .  |**.                 |
   9   0.0051220    0.08924  |                 .  |**.                 |
  10  -0.0002725   -0.00475  |                 .  |  .                 |
```

```
11   -0.013402      -0.23351   |                 *****|  .                   |
12   -0.027125      -0.47262   |             *********|  .                   |
13 -0.0043943       -0.07656   |                   .**|  .                   |
14  0.0021257        0.03704   |                   .  |* .                   |
15  0.0062288        0.10853   |                   .  |**.                   |
16  0.0063093        0.10993   |                   .  |**.                   |
17  0.0041788        0.07281   |                   .  |* .                   |
18 -0.0001194       -0.00208   |                   .  |  .                   |
19  0.0017633        0.03072   |                   .  |* .                   |
20  0.0032659        0.05690   |                   .  |* .                   |
21  0.0027066        0.04716   |                   .  |* .                   |
22  0.0040345        0.07030   |                   .  |* .                   |
23 0.00080489        0.01402   |                   .  |  .                   |
24 -0.0022791       -0.03971   |                   . *|  .                   |

                       Partial Autocorrelations
            Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1     0.33193  |                   . |*******              |
              2    -0.17352  |                  ***| .                   |
              3    -0.16804  |                  ***| .                   |
              4    -0.09793  |                   **| .                   |
              5    -0.06078  |                   .*| .                   |
              6    -0.00126  |                   . | .                   |
              7    -0.02376  |                   . | .                   |
              8     0.03633  |                   . |*.                   |
              9     0.03932  |                   . |*.                   |
             10    -0.04849  |                   .*| .                   |
             11    -0.23025  |                *****| .                   |
             12    -0.38666  |             ********| .                   |
             13     0.20440  |                   . |****                 |
             14    -0.16072  |                  ***| .                   |
             15    -0.07853  |                   **| .                   |
             16    -0.03051  |                   .*| .                   |
             17    -0.01273  |                   . | .                   |
             18    -0.03197  |                   .*| .                   |
             19     0.03457  |                   . |*.                   |
             20     0.10079  |                   . |**                   |
             21     0.05193  |                   . |*.                   |
             22     0.05512  |                   . |*.                   |
             23    -0.18373  |                 ****| .                   |
             24    -0.18567  |                 ****| .                   |


Red River
correlation structure

                  Name of variable = LX.
                  Mean of working series = 1.218286
                  Standard deviation     = 1.150616
                  Number of observations =      400
                           Autocorrelations
```

```
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   1.323917    1.00000  |                    |********************|
  1   0.902389    0.68161  |                  . |**************      |
  2   0.470937    0.35571  |                 .  |*******             |
  3   0.145206    0.10968  |                 .  |**.                 |
  4  -0.103843   -0.07844  |                 .**|  .                 |
  5  -0.137886   -0.10415  |                 .**|  .                 |
  6  -0.122699   -0.09268  |                 .**|  .                 |
  7  -0.162624   -0.12284  |                 .**|  .                 |
  8  -0.175281   -0.13240  |                 ***|  .                 |
  9  0.0022527    0.00170  |                 .  |  .                 |
 10   0.246083    0.18588  |                 .  |****                |
 11   0.575453    0.43466  |                 .  |*********           |
 12   0.801899    0.60570  |                 .  |************        |
 13   0.514288    0.38846  |                .   |********            |
 14   0.179672    0.13571  |                .   |***.                |
 15  -0.087535   -0.06612  |                .  *|   .                |
 16  -0.290877   -0.21971  |               ****|   .                |
 17  -0.300328   -0.22685  |              *****|   .                |
 18  -0.264040   -0.19944  |               ****|   .                |
 19  -0.296605   -0.22404  |               ****|   .                |
 20  -0.295432   -0.22315  |               ****|   .                |
 21  -0.097345   -0.07353  |                .  *|   .                |
 22   0.155377    0.11736  |                .   |**  .               |
 23   0.467163    0.35286  |                .   |*******             |
 24   0.715406    0.54037  |                .   |***********         |
```

```
                          Partial Autocorrelations
           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.68161  |                  . |**************      |
             2    -0.20334  |               ****|  .                 |
             3    -0.08471  |                 **|  .                 |
             4    -0.12000  |                 **|  .                 |
             5     0.10379  |                 . |**                  |
             6    -0.04516  |                 .*|  .                 |
             7    -0.11772  |                 **|  .                 |
             8    -0.02769  |                 .*|  .                 |
             9     0.25836  |                 . |*****               |
            10     0.17867  |                 . |****                |
            11     0.32210  |                 . |******              |
            12     0.20741  |                 . |****                |
            13    -0.40052  |           ********|  .                 |
            14    -0.07358  |                 .*|  .                 |
            15    -0.03678  |                 .*|  .                 |
            16    -0.06381  |                 .*|  .                 |
            17    -0.01061  |                 . |  .                 |
            18    -0.02219  |                 . |  .                 |
            19    -0.05090  |                 .*|  .                 |
            20    -0.02044  |                 . |  .                 |
            21     0.11117  |                 . |**                  |
            22     0.05792  |                 . |*.                  |
            23     0.08163  |                 . |**                  |
```

```
          24      0.21466  |                   .  |****                 |

                    Name of variable = LX.
                    Period(s) of Differencing = 1.
                    Mean of working series = 0.005073
                    Standard deviation     = 0.916423
                    Number of observations =      399
                                 Autocorrelations
 Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
   0   0.839831    1.00000  |                    |********************|
   1   0.014225    0.01694  |                  . |  .                 |
   2  -0.102873   -0.12249  |                 ** |  .                 |
   3  -0.069821   -0.08314  |                 ** |  .                 |
   4  -0.215834   -0.25700  |              ***** |  .                 |
   5  -0.051150   -0.06091  |                 .* |  .                 |
   6   0.054891    0.06536  |                  . |*.                  |
   7  -0.029480   -0.03510  |                 .* |  .                 |
   8  -0.189769   -0.22596  |              ***** |  .                 |
   9  -0.069298   -0.08251  |                 ** |  .                 |
  10  -0.089083   -0.10607  |                 ** |  .                 |
  11   0.099615    0.11861  |                  . |**                  |
  12   0.510733    0.60814  |                  . |************        |
  13   0.055483    0.06606  |                 .  |*  .                |
  14  -0.063687   -0.07583  |                .** |   .                |
  15  -0.059554   -0.07091  |                . * |   .                |
  16  -0.196139   -0.23355  |              ***** |   .                |
  17  -0.046492   -0.05536  |                . * |   .                |
  18   0.068548    0.08162  |                .   |**.                 |
  19  -0.035019   -0.04170  |                . * |   .                |
  20  -0.199562   -0.23762  |              ***** |   .                |
  21  -0.057185   -0.06809  |                . * |   .                |
  22  -0.059055   -0.07032  |                . * |   .                |
  23   0.058877    0.07011  |                .   |*  .                |
  24   0.528477    0.62927  |                .   |*************       |

                            Partial Autocorrelations
            Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1     0.01694  |                  . |  .                 |
              2    -0.12281  |                 ** |  .                 |
              3    -0.07996  |                 ** |  .                 |
              4    -0.27571  |             ****** |  .                 |
              5    -0.09058  |                 ** |  .                 |
              6    -0.02063  |                  . |  .                 |
              7    -0.11251  |                 ** |  .                 |
              8    -0.34862  |            ******* |  .                 |
              9    -0.22777  |              ***** |  .                 |
             10    -0.33715  |            ******* |  .                 |
             11    -0.19177  |               **** |  .                 |
             12     0.39888  |                  . |********            |
             13     0.05013  |                  . |*.                  |
             14    -0.00235  |                  . |  .                 |
             15     0.01407  |                  . |  .                 |
```

```
 16   -0.04061  |                  .*| .                  |
 17   -0.02304  |                  . | .                  |
 18    0.00146  |                  . | .                  |
 19   -0.03154  |                  .*| .                  |
 20   -0.16208  |                 ***| .                  |
 21   -0.08262  |                  **| .                  |
 22   -0.08618  |                  **| .                  |
 23   -0.20564  |                ****| .                  |
 24    0.27742  |                  . |******              |

               Name of variable = LX.
               Period(s) of Differencing = 12.
               Mean of working series = -0.03185
               Standard deviation     = 0.994561
               Number of observations =      388
                          Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.989152    1.00000  |                    |********************|
  1   0.675613    0.68302  |                  . |**************      |
  2   0.500144    0.50563  |                 .  |**********          |
  3   0.373571    0.37767  |                 .  |********            |
  4   0.265750    0.26866  |                 .  |*****               |
  5   0.194367    0.19650  |                 .  |****                |
  6   0.156811    0.15853  |                 .  |***                 |
  7   0.130990    0.13243  |                .   |***.                |
  8   0.070454    0.07123  |                .   |*  .                |
  9  -0.029069   -0.02939  |                .  *|   .                |
 10  -0.132142   -0.13359  |                .***|   .                |
 11  -0.227715   -0.23021  |               *****|   .                |
 12  -0.432635   -0.43738  |           *********|   .                |
 13  -0.297411   -0.30067  |              ******|   .                |
 14  -0.216554   -0.21893  |                ****|   .                |
 15  -0.171401   -0.17328  |                .***|   .                |
 16  -0.162975   -0.16476  |                .***|   .                |
 17  -0.139171   -0.14070  |                .***|   .                |
 18  -0.137908   -0.13942  |                .***|   .                |
 19  -0.130479   -0.13191  |                .***|   .                |
 20  -0.139479   -0.14101  |                .***|   .                |
 21  -0.136049   -0.13754  |                .***|   .                |
 22  -0.103201   -0.10433  |                . **|   .                |
 23  -0.101209   -0.10232  |                . **|   .                |
 24  -0.062898   -0.06359  |                .  *|   .                |

                      Partial Autocorrelations
           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.68302  |                  . |**************      |
             2    0.07331  |                  . |*.                  |
             3    0.01424  |                  . | .                  |
             4   -0.02304  |                  . | .                  |
             5    0.00561  |                  . | .                  |
             6    0.02927  |                  . |*.                  |
             7    0.01707  |                  . | .                  |
```

```
    8   -0.06877   |                  .*| .                  |
    9   -0.13422   |                 ***| .                  |
   10   -0.12806   |                 ***| .                  |
   11   -0.12162   |                  **| .                  |
   12   -0.36840   |             *******| .                  |
   13    0.35881   |                  . |*******             |
   14    0.04345   |                  . |*.                  |
   15    0.00451   |                  . | .                  |
   16   -0.07848   |                  **| .                  |
   17    0.03499   |                  . |*.                  |
   18   -0.01615   |                  . | .                  |
   19    0.03887   |                  . |*.                  |
   20   -0.10883   |                  **| .                  |
   21   -0.12270   |                  **| .                  |
   22   -0.04643   |                  .*| .                  |
   23   -0.07388   |                  .*| .                  |
   24   -0.20463   |                ****| .                  |

                      Name of variable = LX.
                      Period(s) of Differencing = 1,12.
                      Mean of working series = 0.000098
                      Standard deviation     = 0.791276
                      Number of observations =      387
                                Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.626117    1.00000  |                    |********************|
  1  -0.136207   -0.21754  |                ****| .                  |
  2  -0.047542   -0.07593  |                  **| .                  |
  3  -0.021147   -0.03378  |                  .*| .                  |
  4  -0.037942   -0.06060  |                  .*| .                  |
  5  -0.032960   -0.05264  |                  .*| .                  |
  6  -0.011787   -0.01883  |                  . | .                  |
  7   0.035216    0.05625  |                  . |*.                  |
  8   0.036971    0.05905  |                  . |*.                  |
  9  0.0037377    0.00597  |                  . | .                  |
 10 -0.0043012   -0.00687  |                  . | .                  |
 11   0.107585    0.17183  |                  . |***                 |
 12  -0.336994   -0.53823  |         ***********| .                  |
 13   0.053486    0.08542  |                 .  |**.                 |
 14   0.032640    0.05213  |                 .  |* .                 |
 15   0.039259    0.06270  |                 .  |* .                 |
 16  -0.016885   -0.02697  |                 . *|  .                 |
 17   0.023020    0.03677  |                 .  |* .                 |
 18 -0.0066232   -0.01058  |                 .  |  .                 |
 19   0.015626    0.02496  |                 .  |  .                 |
 20  -0.012233   -0.01954  |                 .  |  .                 |
 21  -0.027836   -0.04446  |                 . *|  .                 |
 22   0.029348    0.04687  |                 .  |* .                 |
 23  -0.033933   -0.05420  |                 . *|  .                 |
 24   0.044244    0.07066  |                 .  |* .                 |

                          Partial Autocorrelations
```

```
Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1   -0.21754  |                ****| .                  |
  2   -0.12938  |                 ***| .                  |
  3   -0.08602  |                  **| .                  |
  4   -0.10692  |                  **| .                  |
  5   -0.11561  |                  **| .                  |
  6   -0.09313  |                  **| .                  |
  7   -0.00484  |                  . | .                  |
  8    0.04625  |                  . |*.                  |
  9    0.02549  |                  . |*.                  |
 10    0.01023  |                  . | .                  |
 11    0.20929  |                  . |****                |
 12   -0.48253  |          **********| .                  |
 13   -0.12283  |                  **| .                  |
 14   -0.07611  |                  **| .                  |
 15    0.00850  |                  . | .                  |
 16   -0.11200  |                  **| .                  |
 17   -0.04773  |                  .*| .                  |
 18   -0.09972  |                  **| .                  |
 19    0.04815  |                  . |*.                  |
 20    0.04281  |                  . |*.                  |
 21   -0.03544  |                  .*| .                  |
 22   -0.00525  |                  . | .                  |
 23    0.11276  |                  . |**                  |
 24   -0.27484  |               *****| .                  |
```

Partial Inflow Available for Outflow (PIAO) for Lake Winnipeg
correlation structure

```
                      Name of variable = X.
                      Mean of working series =  4.01445
                      Standard deviation     = 20.02696
                      Number of observations =      400
                                Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    401.079    1.00000  |                    |********************|
  1    146.663    0.36567  |                  . |*******             |
  2  13.941080    0.03476  |                  . |*.                  |
  3 -63.108661   -0.15735  |                 ***| .                  |
  4 -41.001195   -0.10223  |                  **| .                  |
  5 -49.201587   -0.12267  |                  **| .                  |
  6 -58.073120   -0.14479  |                 ***| .                  |
  7 -58.163076   -0.14502  |                 ***| .                  |
  8 -49.281789   -0.12287  |                  **| .                  |
  9 -36.393983   -0.09074  |                  **| .                  |
 10  31.106511    0.07756  |                  . |**                  |
 11    123.507    0.30794  |                  . |******              |
 12    196.241    0.48928  |                 .  |**********          |
 13    136.556    0.34047  |                 .  |*******             |
 14  24.643295    0.06144  |                 .  |* .                 |
```

```
 15 -53.482776   -0.13335 |                 ***|  .                 |
 16 -64.832376   -0.16164 |                 ***|  .                 |
 17 -59.857698   -0.14924 |                 ***|  .                 |
 18 -56.469570   -0.14079 |                 ***|  .                 |
 19 -53.110472   -0.13242 |                 ***|  .                 |
 20 -53.292721   -0.13287 |                 ***|  .                 |
 21 -38.254258   -0.09538 |                 .**|  .                 |
 22   8.529879    0.02127 |                 .  |  .                 |
 23    105.059    0.26194 |                 .  |*****               |
 24    175.918    0.43861 |                 .  |*********           |

                    Partial Autocorrelations
           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.36567 |                  . |*******             |
             2    -0.11423 |                  **| .                  |
             3    -0.15174 |                 ***| .                  |
             4     0.01971 |                  . | .                  |
             5    -0.11047 |                  **| .                  |
             6    -0.11049 |                  **| .                  |
             7    -0.07910 |                  **| .                  |
             8    -0.09346 |                  **| .                  |
             9    -0.08077 |                  **| .                  |
            10     0.10178 |                  . |**                  |
            11     0.23805 |                  . |*****               |
            12     0.33357 |                  . |*******             |
            13     0.14561 |                  . |***                 |
            14     0.00097 |                  . | .                  |
            15    -0.04267 |                  .*| .                  |
            16    -0.04121 |                  .*| .                  |
            17    -0.04399 |                  .*| .                  |
            18    -0.02390 |                  . | .                  |
            19    -0.00403 |                  . | .                  |
            20    -0.04597 |                  .*| .                  |
            21    -0.06315 |                  .*| .                  |
            22    -0.06488 |                  .*| .                  |
            23     0.06779 |                  . |*.                  |
            24     0.15697 |                  . |***                 |

                  Name of variable = X.
                  Period(s) of Differencing = 1.
                  Mean of working series = 0.029617
                  Standard deviation     = 22.58029
                  Number of observations =      399
                            Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    509.870     1.00000 |                    |********************|
  1   -121.888    -0.23906 |               *****| .                  |
  2 -55.528494    -0.10891 |                  **| .                  |
  3    -99.331    -0.19482 |                ****| .                  |
  4  30.175566     0.05918 |                  . |*.                  |
  5   1.050295     0.00206 |                  . | .                  |
  6  -9.214640    -0.01807 |                  . | .                  |
```

```
 7   -8.486393   -0.01664 |                 . | .                 |
 8   -4.160920   -0.00816 |                 . | .                 |
 9  -54.411047   -0.10672 |                 **| .                 |
10  -26.275392   -0.05153 |                 .*| .                 |
11   20.152826    0.03953 |                 . |*.                 |
12     133.136    0.26112 |                 . |*****              |
13   52.267683    0.10251 |                 . |**                 |
14  -33.957597   -0.06660 |                 .*| .                 |
15  -67.157258   -0.13171 |                ***| .                 |
16  -15.957831   -0.03130 |                 .*| .                 |
17    1.970417    0.00386 |                 . | .                 |
18   -0.332363   -0.00065 |                 . | .                 |
19    4.051435    0.00795 |                 . | .                 |
20  -15.391792   -0.03019 |                 .*| .                 |
21  -32.298937   -0.06335 |                 .*| .                 |
22  -49.908632   -0.09789 |                 **| .                 |
23   24.993043    0.04902 |                 . |*.                 |
24     112.845    0.22132 |                 . |****               |

                      Partial Autocorrelations
            Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1   -0.23906  |              *****| .                 |
              2   -0.17612  |               ****| .                 |
              3   -0.29284  |             ******| .                 |
              4   -0.12426  |                 **| .                 |
              5   -0.12014  |                 **| .                 |
              6   -0.14398  |                ***| .                 |
              7   -0.11822  |                 **| .                 |
              8   -0.12357  |                 **| .                 |
              9   -0.26355  |              *****| .                 |
             10   -0.33576  |            *******| .                 |
             11   -0.37743  |           ********| .                 |
             12   -0.16467  |                ***| .                 |
             13   -0.01475  |                 . | .                 |
             14    0.02737  |                 . |*.                 |
             15    0.02116  |                 . | .                 |
             16    0.02071  |                 . | .                 |
             17   -0.00116  |                 . | .                 |
             18   -0.02258  |                 . | .                 |
             19    0.01917  |                 . | .                 |
             20    0.03241  |                 . |*.                 |
             21    0.02801  |                 . |*.                 |
             22   -0.10083  |                 **| .                 |
             23   -0.18022  |               ****| .                 |
             24   -0.08971  |                 **| .                 |

                     Name of variable = X.
                     Period(s) of Differencing = 12.
                     Mean of working series = -0.16992
                     Standard deviation     = 20.20382
                     Number of observations =      388
                              Autocorrelations
```

```
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    408.194    1.00000  |                    |********************|
  1  28.513655    0.06985  |                  . |*.                  |
  2 -31.212934   -0.07647  |                  **| .                  |
  3 -37.287606   -0.09135  |                  **| .                  |
  4  31.326836    0.07674  |                  . |**                  |
  5  21.095786    0.05168  |                  . |*.                  |
  6   0.807854    0.00198  |                  . | .                  |
  7 -12.477816   -0.03057  |                  .*| .                  |
  8  -3.203905   -0.00785  |                  . | .                  |
  9  29.446108    0.07214  |                  . |*.                  |
 10  41.031840    0.10052  |                  . |**                  |
 11  -5.245924   -0.01285  |                  . | .                  |
 12   -184.531   -0.45207  |           *********| .                  |
 13 -10.818843   -0.02650  |                  .*| .                  |
 14  -3.645224   -0.00893  |                  . | .                  |
 15   0.881887    0.00216  |                  . | .                  |
 16 -23.202817   -0.05684  |                  .*| .                  |
 17 -21.245833   -0.05205  |                  .*| .                  |
 18  -4.136355   -0.01013  |                  . | .                  |
 19  -0.701995   -0.00172  |                  . | .                  |
 20  -3.666029   -0.00898  |                  . | .                  |
 21   8.726806    0.02138  |                  . | .                  |
 22 -28.553314   -0.06995  |                  .*| .                  |
 23 -14.208605   -0.03481  |                 . *|  .                 |
 24 -21.172115   -0.05187  |                 . *|  .                 |
```

```
                      Partial Autocorrelations
            Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1    0.06985  |                  . |*.                  |
              2   -0.08174  |                  **| .                  |
              3   -0.08079  |                  **| .                  |
              4    0.08442  |                  . |**                  |
              5    0.02771  |                  . |*.                  |
              6   -0.00006  |                  . | .                  |
              7   -0.01141  |                  . | .                  |
              8   -0.00418  |                  . | .                  |
              9    0.06577  |                  . |*.                  |
             10    0.08714  |                  . |**                  |
             11   -0.01568  |                  . | .                  |
             12   -0.44070  |           *********| .                  |
             13    0.04622  |                  . |*.                  |
             14   -0.08973  |                  **| .                  |
             15   -0.07853  |                  **| .                  |
             16    0.01895  |                  . | .                  |
             17   -0.04033  |                  .*| .                  |
             18   -0.01169  |                  . | .                  |
             19   -0.01675  |                  . | .                  |
             20   -0.00724  |                  . | .                  |
             21    0.08820  |                  . |**                  |
             22   -0.00504  |                  . | .                  |
             23   -0.02381  |                  . | .                  |
```

```
            24   -0.30107  |              ******| .                  |

                      Name of variable = X.
                      Period(s) of Differencing = 1,12.
                      Mean of working series = 0.017615
                      Standard deviation      = 27.57583
                      Number of observations =      387
                               Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    760.427    1.00000  |                    |********************|
  1   -320.183   -0.42106  |            ********| .                  |
  2 -53.005333   -0.06970  |                  .*| .                  |
  3 -74.269242   -0.09767  |                  **| .                  |
  4  77.340248    0.10171  |                  . |**                  |
  5  10.335637    0.01359  |                  . | .                  |
  6  -7.113473   -0.00935  |                  . | .                  |
  7 -22.498589   -0.02959  |                  .*| .                  |
  8 -23.575200   -0.03100  |                  .*| .                  |
  9  22.405083    0.02946  |                  . |*.                  |
 10  55.806903    0.07339  |                  . |*.                  |
 11    135.433    0.17810  |                  . |****                |
 12   -353.899   -0.46540  |           *********| .                  |
 13    166.730    0.21926  |                 .  |****                |
 14   2.141605    0.00282  |                 .  |  .                 |
 15  28.808010    0.03788  |                 .  |* .                 |
 16 -28.714098   -0.03776  |                 . *|  .                 |
 17 -12.972307   -0.01706  |                 .  |  .                 |
 18  14.027537    0.01845  |                 .  |  .                 |
 19   6.256454    0.00823  |                 .  |  .                 |
 20 -15.320286   -0.02015  |                 .  |  .                 |
 21  49.009062    0.06445  |                 .  |* .                 |
 22 -49.829953   -0.06553  |                 . *|  .                 |
 23  18.894520    0.02485  |                 .  |  .                 |
 24 -34.697409   -0.04563  |                 . *|  .                 |

                           Partial Autocorrelations
           Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.42106  |            ********| .                  |
             2   -0.30022  |              ******| .                  |
             3   -0.35032  |             *******| .                  |
             4   -0.21903  |                ****| .                  |
             5   -0.15582  |                 ***| .                  |
             6   -0.12502  |                 ***| .                  |
             7   -0.11811  |                  **| .                  |
             8   -0.16772  |                 ***| .                  |
             9   -0.16298  |                 ***| .                  |
            10   -0.05244  |                  .*| .                  |
            11    0.34669  |                  . |*******             |
            12   -0.18522  |                ****| .                  |
            13   -0.03611  |                  .*| .                  |
            14   -0.05093  |                  .*| .                  |
            15   -0.13539  |                 ***| .                  |
```

```
 16   -0.07354   |                    .*| .                    |
 17   -0.09321   |                    **| .                    |
 18   -0.07973   |                    **| .                    |
 19   -0.08435   |                    **| .                    |
 20   -0.16258   |                   ***| .                    |
 21   -0.05707   |                    .*| .                    |
 22   -0.03664   |                    .*| .                    |
 23    0.22068   |                    . |****                 |
 24   -0.15981   |                   ***| .                    |
```

Grass River
GS1 model structure

```
                 Maximum Likelihood Estimation
        Parameter    Estimate      Std Error   T Ratio  Lag
        MA1,1         0.90913        0.03600     25.26  12
        AR1,1         1.31841        0.04779     27.59   1
        AR1,2        -0.39233        0.04865     -8.06   2

        Variance  Estimate = 0.02654837
        Std Error Estimate = 0.16293671
        AIC                = -252.17185
        SBC                = -240.58096
        Number of Residuals=        352

                 Autocorrelation Check of Residuals
  To    Chi                          Autocorrelations
 Lag  Square DF   Prob
   6    6.44  3  0.092  0.048 -0.053 -0.088 -0.043 -0.022  0.053
  12    9.77  9  0.370  0.017  0.055  0.038  0.055 -0.037  0.002
  18   13.84 15  0.538  0.092 -0.006  0.021  0.036  0.012 -0.026
  24   15.77 21  0.782  0.009  0.044  0.021  0.048  0.020 -0.000
  30   16.80 27  0.936 -0.025  0.016  0.026  0.029 -0.010 -0.015
  36   24.53 33  0.856 -0.010 -0.008  0.007 -0.073  0.118  0.016
  42   25.16 39  0.958 -0.012 -0.012 -0.013 -0.015  0.026  0.014

 Model for variable LX
 Period(s) of Differencing = 12.
 Autoregressive Factors
 Factor 1: 1 - 1.3184 B**(1) + 0.39233 B**(2)
 Moving Average Factors
 Factor 1: 1 - 0.90913 B**(12)

                 Autocorrelation Plot of Residuals
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.026548     1.00000  |                    |********************|
  1  0.0012785     0.04816  |                  . |*.                  |
  2 -0.0014007    -0.05276  |                  .*| .                  |
  3 -0.0023414    -0.08819  |                  **| .                  |
  4 -0.0011351    -0.04275  |                  .*| .                  |
```

```
  5  -0.0005864    -0.02209  |                  . | .                  |
  6   0.0014095     0.05309  |                  . |*.                  |
  7  0.00044591     0.01680  |                  . | .                  |
  8   0.0014703     0.05538  |                  . |*.                  |
  9   0.0010112     0.03809  |                  . |*.                  |
 10   0.0014531     0.05474  |                  . |*.                  |
 11  -0.0009797    -0.03690  |                  .*| .                  |
 12  0.00004156     0.00157  |                  . | .                  |
 13   0.0024405     0.09193  |                  . |**                  |
 14  -0.0001722    -0.00649  |                  . | .                  |
 15  0.00054567     0.02055  |                  . | .                  |
 16  0.00096159     0.03622  |                  . |*.                  |
 17  0.00031619     0.01191  |                  . | .                  |
 18  -0.0006902    -0.02600  |                  .*| .                  |
 19  0.00022598     0.00851  |                  . | .                  |
 20   0.0011715     0.04413  |                  . |*.                  |
 21  0.00054667     0.02059  |                  . | .                  |
 22   0.0012663     0.04770  |                  . |*.                  |
 23  0.00054264     0.02044  |                  . | .                  |
 24   -3.098E-6    -0.00012  |                  . | .                  |

                             Partial Autocorrelations
             Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
               1    0.04816  |                  . |*.                  |
               2   -0.05521  |                  .*| .                  |
               3   -0.08330  |                  **| .                  |
               4   -0.03782  |                  .*| .                  |
               5   -0.02777  |                  .*| .                  |
               6    0.04439  |                  . |*.                  |
               7    0.00326  |                  . | .                  |
               8    0.05477  |                  . |*.                  |
               9    0.04136  |                  . |*.                  |
              10    0.06331  |                  . |*.                  |
              11   -0.02570  |                  .*| .                  |
              12    0.01960  |                  . | .                  |
              13    0.10425  |                  . |**                  |
              14   -0.01834  |                  . | .                  |
              15    0.03037  |                  . |*.                  |
              16    0.04011  |                  . |*.                  |
              17    0.01567  |                  . | .                  |
              18   -0.02472  |                  . | .                  |
              19    0.00838  |                  . | .                  |
              20    0.04587  |                  . |*.                  |
              21    0.00625  |                  . | .                  |
              22    0.04079  |                  . |*.                  |
              23    0.01103  |                  . | .                  |
              24    0.01445  |                  . | .                  |
```

Grass River

GS2 model structure

Maximum Likelihood Estimation

Parameter	Estimate	Std Error	T Ratio	Lag
MA1,1	-0.38200	0.04862	-7.86	1
MA2,1	0.91662	0.03621	25.31	12
AR1,1	-0.17546	0.05198	-3.38	3

```
Variance  Estimate = 0.02668969
Std Error Estimate =  0.1633698
AIC                = -250.58946
SBC                =  -239.0071
Number of Residuals=        351
```

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations					
6	9.91	3	0.019	-0.000	0.005	-0.007	-0.114	-0.116	-0.034
12	12.53	9	0.185	-0.057	-0.004	-0.005	0.037	-0.051	-0.004
18	16.33	15	0.360	0.086	-0.031	0.005	0.028	-0.005	-0.034
24	18.11	21	0.642	0.000	0.042	0.011	0.044	0.030	0.006
30	18.84	27	0.876	-0.021	0.008	0.008	0.009	-0.028	-0.021
36	29.06	33	0.664	-0.034	-0.011	0.019	-0.082	0.132	0.021
42	29.69	39	0.859	-0.006	0.017	0.008	-0.006	0.031	0.012

```
Model for variable LX
Period(s) of Differencing = 1,12.
Autoregressive Factors
Factor 1: 1 + 0.17546 B**(3)
Moving Average Factors
Factor 1: 1 + 0.382 B**(1)
Factor 2: 1 - 0.91662 B**(12)
```

Autocorrelation Plot of Residuals

```
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.026690    1.00000  |                    |********************|
  1 -5.5105E-7   -0.00002  |                  . | .                  |
  2 0.00014492    0.00543  |                  . | .                  |
  3 -0.0001891   -0.00709  |                  . | .                  |
  4 -0.0030330   -0.11364  |                  **| .                  |
  5 -0.0031053   -0.11635  |                  **| .                  |
  6 -0.0009132   -0.03422  |                  .*| .                  |
  7 -0.0015128   -0.05668  |                  .*| .                  |
  8 -0.0001063   -0.00398  |                  . | .                  |
  9 -0.0001436   -0.00538  |                  . | .                  |
 10 0.00097503    0.03653  |                  . |*.                  |
 11 -0.0013662   -0.05119  |                  .*| .                  |
 12 -0.0001179   -0.00442  |                  . | .                  |
 13  0.0022869    0.08568  |                  . |**                  |
 14  -0.000831   -0.03114  |                  .*| .                  |
 15 0.00014185    0.00531  |                  . | .                  |
 16 0.00075248    0.02819  |                  . |*.                  |
```

```
 17 -0.0001348    -0.00505  |                   .  |  .                   |
 18 -0.0009189    -0.03443  |                   . *|  .                   |
 19 6.54339E-6     0.00025  |                   .  |  .                   |
 20  0.0011124     0.04168  |                   .  |* .                   |
 21 0.00030237     0.01133  |                   .  |  .                   |
 22  0.0011752     0.04403  |                   .  |* .                   |
 23 0.00079693     0.02986  |                   .  |* .                   |
 24 0.00014889     0.00558  |                   .  |  .                   |
```

```
                    Partial Autocorrelations

 Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
   1    -0.00002  |                   .  |  .                   |
   2     0.00543  |                   .  |  .                   |
   3    -0.00709  |                   .  |  .                   |
   4    -0.11368  |                   **|  .                   |
   5    -0.11789  |                   **|  .                   |
   6    -0.03566  |                   . *|  .                   |
   7    -0.05976  |                   . *|  .                   |
   8    -0.02116  |                   .  |  .                   |
   9    -0.03445  |                   . *|  .                   |
  10     0.01288  |                   .  |  .                   |
  11    -0.07490  |                   . *|  .                   |
  12    -0.02622  |                   . *|  .                   |
  13     0.07585  |                   .  |**                    |
  14    -0.03474  |                   . *|  .                   |
  15    -0.00767  |                   .  |  .                   |
  16     0.01232  |                   .  |  .                   |
  17     0.00889  |                   .  |  .                   |
  18    -0.03245  |                   . *|  .                   |
  19    -0.00412  |                   .  |  .                   |
  20     0.05410  |                   .  |* .                   |
  21     0.01830  |                   .  |  .                   |
  22     0.03996  |                   .  |* .                   |
  23     0.02346  |                   .  |  .                   |
  24     0.03037  |                   .  |* .                   |
```

Grass River
GTD model structure

```
                        Name of variable = X.
                        Mean of working series = -0.05472
                        Standard deviation     = 1.108327
                        Number of observations =      364

                              Autocorrelations

Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   1.228390     1.00000  |                    |********************|
  1   1.099397     0.89499  |                  . |******************  |
  2   0.956644     0.77878  |                .   |****************    |
```

```
   3    0.829294     0.67511  |               .   |**************      |
   4    0.719714     0.58590  |              .    |************        |
   5    0.653898     0.53232  |              .    |***********         |
   6    0.603831     0.49156  |              .    |**********          |
   7    0.551527     0.44898  |              .    |*********           |
   8    0.502193     0.40882  |              .    |********            |
   9    0.460548     0.37492  |             .     |*******             |
  10    0.424906     0.34590  |             .     |*******             |
  11    0.393351     0.32022  |             .     |******              |
  12    0.364996     0.29713  |             .     |******              |
  13    0.345587     0.28133  |             .     |******              |
  14    0.318034     0.25890  |             .     |*****.              |
  15    0.295253     0.24036  |             .     |*****.              |
  16    0.271140     0.22073  |             .     |**** .              |
  17    0.247505     0.20149  |             .     |**** .              |
  18    0.217003     0.17666  |             .     |**** .              |
  19    0.189289     0.15410  |             .     |***  .              |
  20    0.169803     0.13823  |             .     |***  .              |
  21    0.163856     0.13339  |             .     |***  .              |
  22    0.165815     0.13499  |             .     |***  .              |
  23    0.165240     0.13452  |             .     |***  .              |
  24    0.155850     0.12687  |             .     |***  .              |

                       Partial Autocorrelations
            Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1    0.89499  |                 . |******************  |
              2   -0.11170  |                 **| .                  |
              3    0.00113  |                 . | .                  |
              4    0.00491  |                 . | .                  |
              5    0.12024  |                 . |**                  |
              6    0.01473  |                 . | .                  |
              7   -0.02618  |                 .*| .                  |
              8    0.00346  |                 . | .                  |
              9    0.03091  |                 . |*.                  |
             10    0.01326  |                 . | .                  |
             11   -0.00128  |                 . | .                  |
             12    0.00363  |                 . | .                  |
             13    0.03670  |                 . |*.                  |
             14   -0.03684  |                 .*| .                  |
             15    0.01929  |                 . | .                  |
             16   -0.01462  |                 . | .                  |
             17    0.00473  |                 . | .                  |
             18   -0.04545  |                 .*| .                  |
             19    0.00381  |                 . | .                  |
             20    0.01786  |                 . | .                  |
             21    0.04315  |                 . |*.                  |
             22    0.01578  |                 . | .                  |
             23   -0.00667  |                 . | .                  |
             24   -0.02047  |                 . | .                  |

                    Maximum Likelihood Estimation
                Parameter   Estimate    Std Error   T Ratio  Lag
```

MA1,1	-0.24337	0.05451	-4.46	1
AR1,1	0.93434	0.02513	37.19	1

```
Variance  Estimate = 0.15786028
Std Error Estimate = 0.39731635
AIC                = 365.555783
SBC                =  373.35009
Number of Residuals=        364
```

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations					
6	2.97	4	0.562	0.001	-0.008	-0.027	-0.062	-0.041	0.041
12	4.45	10	0.925	-0.004	0.002	0.034	0.009	0.033	0.040
18	7.87	16	0.953	0.067	-0.044	0.003	0.031	0.038	0.008
24	10.33	22	0.983	0.006	-0.027	-0.013	0.027	0.068	-0.002
30	11.60	28	0.997	-0.031	0.004	0.005	0.036	0.004	-0.029
36	18.01	34	0.989	-0.015	-0.022	-0.032	-0.013	0.117	0.015
42	20.49	40	0.996	-0.064	0.023	0.002	0.023	0.029	0.012

```
                        Autocorrelation Plot of Residuals
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.157860     1.00000 |                    |********************|
  1 0.00022611     0.00143 |                  . | .                  |
  2 -0.0012845    -0.00814 |                  . | .                  |
  3 -0.0042822    -0.02713 |                  .*| .                  |
  4 -0.0097580    -0.06181 |                  .*| .                  |
  5 -0.0065229    -0.04132 |                  .*| .                  |
  6  0.0064947     0.04114 |                  . |*.                  |
  7 -0.0005866    -0.00372 |                  . | .                  |
  8 0.00031778     0.00201 |                  . | .                  |
  9  0.0053270     0.03374 |                  . |*.                  |
 10  0.0014601     0.00925 |                  . | .                  |
 11  0.0051662     0.03273 |                  . |*.                  |
 12  0.0062946     0.03987 |                  . |*.                  |
 13   0.010582     0.06703 |                  . |*.                  |
 14 -0.0069974    -0.04433 |                  .*| .                  |
 15 0.00050561     0.00320 |                  . | .                  |
 16  0.0049395     0.03129 |                  . |*.                  |
 17  0.0060448     0.03829 |                  . |*.                  |
 18  0.0013258     0.00840 |                  . | .                  |
 19 0.00091207     0.00578 |                  . | .                  |
 20 -0.0042733    -0.02707 |                  .*| .                  |
 21 -0.0021087    -0.01336 |                  . | .                  |
 22  0.0043018     0.02725 |                  . |*.                  |
 23   0.010714     0.06787 |                  . |*.                  |
 24 -0.0003076    -0.00195 |                  . | .                  |
```

```
                 Partial Autocorrelations
Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.00143 |                  . | .                  |
  2    -0.00814 |                  . | .                  |
```

```
 3   -0.02711  |                    .*| .                   |
 4   -0.06186  |                    .*| .                   |
 5   -0.04191  |                    .*| .                   |
 6    0.03948  |                    . |*.                   |
 7   -0.00771  |                    . | .                   |
 8   -0.00343  |                    . | .                   |
 9    0.03094  |                    . |*.                   |
10    0.01236  |                    . | .                   |
11    0.03616  |                    . |*.                   |
12    0.04015  |                    . |*.                   |
13    0.07361  |                    . |*.                   |
14   -0.03773  |                    .*| .                   |
15    0.00910  |                    . | .                   |
16    0.04248  |                    . |*.                   |
17    0.04633  |                    . |*.                   |
18    0.00637  |                    . | .                   |
19   -0.00001  |                    . | .                   |
20   -0.01822  |                    . | .                   |
21   -0.00976  |                    . | .                   |
22    0.02222  |                    . | .                   |
23    0.06450  |                    . |*.                   |
24   -0.01102  |                    . | .                   |

                 Model for variable X
                    Autoregressive Factors
                    Factor 1: 1 - 0.93434 B**(1)
                    Moving Average Factors
                    Factor 1: 1 + 0.24337 B**(1)
```

```
Red River
RS2 model structure

                    Maximum Likelihood Estimation
               Parameter    Estimate      Std Error   T Ratio  Lag
               MA1,1         0.92022       0.03132     29.38    1
               MA2,1         0.85106       0.03633     23.43   12
               AR1,1         0.63660       0.05751     11.07    1
               AR2,1        -0.13845       0.05814     -2.38   12

               Variance  Estimate = 0.27856633
               Std Error Estimate = 0.52779384
               AIC                = 627.456813
               SBC                = 643.290512
               Number of Residuals=        387

                     Autocorrelation Check of Residuals
     To   Chi                     Autocorrelations
    Lag  Square DF   Prob
     6     3.66  2  0.160 -0.034  0.013  0.055 -0.026  0.038 -0.053
```

```
12     8.91  8  0.350  0.096  0.011  0.008 -0.049  0.035  0.014
18    13.88 14  0.459 -0.060  0.017  0.034 -0.054 -0.003 -0.065
24    17.18 20  0.642  0.026 -0.049 -0.030 -0.014 -0.052 -0.033
30    19.82 26  0.800 -0.062  0.031 -0.015  0.026 -0.019 -0.017
36    22.06 32  0.906 -0.033 -0.033  0.053  0.008 -0.014 -0.004
42    27.06 38  0.907 -0.026  0.065  0.029  0.020  0.070  0.021
```

```
Model for variable LX
Period(s) of Differencing = 1,12.
Autoregressive Factors
Factor 1: 1 - 0.6366 B**(1)
Factor 2: 1 + 0.13845 B**(12)
Moving Average Factors
Factor 1: 1 - 0.92022 B**(1)
Factor 2: 1 - 0.85106 B**(12)
```

```
                    Autocorrelation Plot of Residuals
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.278566    1.00000  |                    |********************|
  1  -0.009607   -0.03449  |                  .*|  .                 |
  2  0.0035767    0.01284  |                  . |  .                 |
  3   0.015241    0.05471  |                  . |*.                  |
  4 -0.0071437   -0.02564  |                  .*|  .                 |
  5   0.010627    0.03815  |                  . |*.                  |
  6  -0.014887   -0.05344  |                  .*|  .                 |
  7   0.026765    0.09608  |                  . |**                  |
  8  0.0031815    0.01142  |                  . |  .                 |
  9  0.0023078    0.00828  |                  . |  .                 |
 10  -0.013560   -0.04868  |                  .*|  .                 |
 11   0.009675    0.03473  |                  . |*.                  |
 12  0.0039572    0.01421  |                  . |  .                 |
 13  -0.016798   -0.06030  |                  .*|  .                 |
 14  0.0048010    0.01723  |                  . |  .                 |
 15  0.0094919    0.03407  |                  . |*.                  |
 16  -0.015040   -0.05399  |                  .*|  .                 |
 17 -0.0008532   -0.00306  |                  . |  .                 |
 18  -0.018136   -0.06511  |                  .*|  .                 |
 19  0.0071681    0.02573  |                  . |*.                  |
 20  -0.013694   -0.04916  |                  .*|  .                 |
 21 -0.0083699   -0.03005  |                  .*|  .                 |
 22 -0.0040182   -0.01442  |                  . |  .                 |
 23  -0.014525   -0.05214  |                  .*|  .                 |
 24 -0.0092161   -0.03308  |                  .*|  .                 |
```

```
                      Partial Autocorrelations
            Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1   -0.03449  |                  .*|  .                 |
              2    0.01166  |                  . |  .                 |
              3    0.05562  |                  . |*.                  |
              4   -0.02210  |                  . |  .                 |
              5    0.03524  |                  . |*.                  |
              6   -0.05371  |                  .*|  .                 |
```

```
             7     0.09517  |                  . |**                  |
             8     0.01379  |                  . | .                  |
             9     0.01502  |                  . | .                  |
            10    -0.06425  |                  .*| .                  |
            11     0.03955  |                  . |*.                  |
            12     0.00693  |                  . | .                  |
            13    -0.04465  |                  .*| .                  |
            14    -0.00229  |                  . | .                  |
            15     0.04019  |                  . |*.                  |
            16    -0.05764  |                  .*| .                  |
            17     0.00318  |                  . | .                  |
            18    -0.07116  |                  .*| .                  |
            19     0.02524  |                  . |*.                  |
            20    -0.04840  |                  .*| .                  |
            21    -0.01265  |                  . | .                  |
            22    -0.03632  |                  .*| .                  |
            23    -0.03778  |                  .*| .                  |
            24    -0.04124  |                  .*| .                  |


Red River
RTD model structure

                      Name of variable = X.
                      Mean of working series = -0.01841
                      Standard deviation     = 0.964225
                      Number of observations =      400

                               Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.929730    1.00000  |                    |********************|
  1   0.686436    0.73832  |                  . |***************     |
  2   0.559787    0.60210  |                 .  |************        |
  3   0.466477    0.50173  |                 .  |**********          |
  4   0.379770    0.40847  |                .   |********            |
  5   0.342725    0.36863  |                .   |*******             |
  6   0.297936    0.32045  |                .   |******              |
  7   0.282650    0.30401  |                .   |******              |
  8   0.254463    0.27370  |                .   |*****               |
  9   0.209550    0.22539  |                .   |*****               |
 10   0.179784    0.19337  |                .   |****                |
 11   0.182101    0.19586  |                .   |****                |
 12   0.139867    0.15044  |                .   |***.                |
 13   0.124335    0.13373  |                .   |***.                |
 14   0.115164    0.12387  |                .   |** .                |
 15   0.103701    0.11154  |                .   |** .                |
 16   0.098628    0.10608  |                .   |** .                |
 17   0.080267    0.08633  |                .   |** .                |
 18   0.056834    0.06113  |                .   |*  .                |
 19   0.055165    0.05933  |                .   |*  .                |
```

```
20   0.023621    0.02541 |                . |*  .                |
21  0.0052139    0.00561 |                . |   .                |
22   0.022715    0.02443 |                . |   .                |
23   0.026625    0.02864 |                . |*  .                |
24   0.048733    0.05242 |                . |*  .                |

                          Partial Autocorrelations
          Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.73832 |                  . |***************     |
            2     0.12527 |                  . |***                 |
            3     0.04555 |                  . |*.                  |
            4    -0.00904 |                  . | .                  |
            5     0.07660 |                  . |**                  |
            6     0.00578 |                  . | .                  |
            7     0.06252 |                  . |*.                  |
            8    -0.00212 |                  . | .                  |
            9    -0.03753 |                  .*| .                  |
           10    -0.00209 |                  . | .                  |
           11     0.07397 |                  . |*.                  |
           12    -0.07188 |                  .*| .                  |
           13     0.01384 |                  . | .                  |
           14     0.01364 |                  . | .                  |
           15     0.00881 |                  . | .                  |
           16     0.00446 |                  . | .                  |
           17    -0.01295 |                  . | .                  |
           18    -0.04035 |                  .*| .                  |
           19     0.02878 |                  . |*.                  |
           20    -0.05303 |                  .*| .                  |
           21    -0.01556 |                  . | .                  |
           22     0.05406 |                  . |*.                  |
           23     0.01948 |                  . | .                  |
           24     0.04173 |                  . |*.                  |

                       Maximum Likelihood Estimation
              Parameter   Estimate     Std Error   T Ratio  Lag
              AR1,1        0.64610      0.04976     12.98    1
              AR1,2        0.12355      0.04976      2.48    2

              Variance  Estimate = 0.41780608
              Std Error Estimate = 0.64637921
              AIC                = 788.865442
              SBC                = 796.848371
              Number of Residuals=        400

                     Autocorrelation Check of Residuals
       To   Chi                         Autocorrelations
      Lag  Square DF   Prob
        6    2.03  4  0.731 -0.004 -0.032  0.021 -0.046  0.035 -0.011
       12    9.51 10  0.484  0.060  0.063 -0.006 -0.029  0.096 -0.022
       18   10.64 16  0.831 -0.006  0.015  0.005  0.040  0.016 -0.023
       24   17.30 22  0.747  0.057 -0.023 -0.068  0.015 -0.013  0.083
       30   18.35 28  0.917 -0.020  0.011 -0.013  0.010 -0.036  0.019
```

```
36    21.04 34  0.960  0.004 -0.010  0.013 -0.020  0.061 -0.041
42    24.33 40  0.976 -0.019  0.057 -0.023  0.015  0.054  0.013

                       Autocorrelation Plot of Residuals
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0   0.417806    1.00000   |                    |********************|
 1 -0.0017655   -0.00423   |                  . | .                  |
 2  -0.013433   -0.03215   |                  .*| .                  |
 3  0.0087647    0.02098   |                  . | .                  |
 4  -0.019421   -0.04648   |                  .*| .                  |
 5   0.014554    0.03484   |                  . |*.                  |
 6 -0.0047474   -0.01136   |                  . | .                  |
 7   0.024981    0.05979   |                  . |*.                  |
 8   0.026436    0.06327   |                  . |*.                  |
 9 -0.0024212   -0.00580   |                  . | .                  |
10  -0.012055   -0.02885   |                  .*| .                  |
11   0.040179    0.09617   |                  . |**                  |
12 -0.0091266   -0.02184   |                  . | .                  |
13 -0.0025685   -0.00615   |                  . | .                  |
14  0.0061810    0.01479   |                  . | .                  |
15  0.0021996    0.00526   |                  . | .                  |
16   0.016864    0.04036   |                  . |*.                  |
17  0.0068258    0.01634   |                  . | .                  |
18 -0.0094156   -0.02254   |                  . | .                  |
19   0.023638    0.05658   |                  . |*.                  |
20  -0.009594   -0.02296   |                  . | .                  |
21  -0.028378   -0.06792   |                  .*| .                  |
22  0.0062004    0.01484   |                  . | .                  |
23 -0.0055417   -0.01326   |                  . | .                  |
24   0.034789    0.08327   |                  . |**                  |

                                 Partial Autocorrelations
                Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                  1   -0.00423   |                  . | .                  |
                  2   -0.03217   |                  .*| .                  |
                  3    0.02072   |                  . | .                  |
                  4   -0.04742   |                  .*| .                  |
                  5    0.03601   |                  . |*.                  |
                  6   -0.01482   |                  . | .                  |
                  7    0.06447   |                  . |*.                  |
                  8    0.05920   |                  . |*.                  |
                  9    0.00280   |                  . | .                  |
                 10   -0.03015   |                  .*| .                  |
                 11    0.10118   |                  . |**                  |
                 12   -0.02296   |                  . | .                  |
                 13   -0.00091   |                  . | .                  |
                 14    0.00404   |                  . | .                  |
                 15    0.01033   |                  . | .                  |
                 16    0.02784   |                  . |*.                  |
                 17    0.02505   |                  . |*.                  |
                 18   -0.02995   |                  .*| .                  |
                 19    0.04935   |                  . |*.                  |
```

```
   20   -0.02100  |                      . | .                    |
   21   -0.06019  |                      .*| .                    |
   22   -0.00458  |                      . | .                    |
   23   -0.01065  |                      . | .                    |
   24    0.07547  |                      . |**                    |
```

```
Model for variable X
Autoregressive Factors
Factor 1: 1 - 0.6461 B**(1) - 0.12355 B**(2)
```

```
PIAO
WS1 model structure
```

```
                   Maximum Likelihood Estimation
          Parameter   Estimate     Std Error   T Ratio  Lag
          MU          -0.28839       0.12735     -2.26    0
          MA1,1        0.87153       0.03145     27.71   12
          Constant Estimate   = -0.2883877
          Variance  Estimate = 234.669861
          Std Error Estimate =  15.318938
          AIC                = 3237.96752
          SBC                = 3245.88953
          Number of Residuals=        388
```

```
                 Autocorrelation Check of Residuals
  To     Chi                       Autocorrelations
 Lag   Square DF   Prob
   6    11.73  5  0.039  0.107 -0.060 -0.086  0.077  0.038  0.001
  12    14.05 11  0.230 -0.012  0.031  0.048  0.041  0.019  0.021
  18    16.79 17  0.469  0.062 -0.018 -0.043 -0.026 -0.010 -0.001
  24    20.04 23  0.640  0.000  0.001  0.036 -0.053 -0.050 -0.035
  30    23.29 29  0.763  0.058  0.046 -0.012 -0.011  0.042  0.016
  36    24.61 35  0.905  0.011 -0.015 -0.015 -0.016 -0.047 -0.006
  42    32.59 41  0.823  0.035  0.090  0.081  0.034 -0.030  0.021
```

```
Model for variable X
Estimated Mean = -0.2883877
Period(s) of Differencing = 12.
Moving Average Factors
Factor 1: 1 - 0.87153 B**(12)
```

```
                  Autocorrelation Plot of Residuals
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0     234.670    1.00000  |                    |********************|
 1   25.181145    0.10730  |                  . |**                  |
 2  -14.150375   -0.06030  |                  .*| .                  |
 3  -20.103261   -0.08567  |                  **| .                  |
 4   18.164470    0.07740  |                  . |**                  |
 5    8.804776    0.03752  |                  . |*.                  |
```

```
  6   0.291505    0.00124  |                  . | .                  |
  7  -2.701569   -0.01151  |                  . | .                  |
  8   7.254238    0.03091  |                  . |*.                  |
  9  11.211778    0.04778  |                  . |*.                  |
 10   9.592490    0.04088  |                  . |*.                  |
 11   4.360850    0.01858  |                  . | .                  |
 12   4.849970    0.02067  |                  . | .                  |
 13  14.543168    0.06197  |                  . |*.                  |
 14  -4.141506   -0.01765  |                  . | .                  |
 15 -10.120450   -0.04313  |                  .*| .                  |
 16  -6.040165   -0.02574  |                  .*| .                  |
 17  -2.249870   -0.00959  |                  . | .                  |
 18  -0.143186   -0.00061  |                  . | .                  |
 19   0.095656    0.00041  |                  . | .                  |
 20   0.306964    0.00131  |                  . | .                  |
 21   8.448963    0.03600  |                  . |*.                  |
 22 -12.381445   -0.05276  |                  .*| .                  |
 23 -11.732227   -0.04999  |                  .*| .                  |
 24  -8.329319   -0.03549  |                  .*| .                  |

                          Partial Autocorrelations
           Lag Correlation  -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.10730 |                  . |**                  |
             2    -0.07265 |                  .*| .                  |
             3    -0.07214 |                  .*| .                  |
             4     0.09270 |                  . |**                  |
             5     0.00885 |                  . | .                  |
             6    -0.00110 |                  . | .                  |
             7     0.00552 |                  . | .                  |
             8     0.02927 |                  . |*.                  |
             9     0.03754 |                  . |*.                  |
            10     0.03454 |                  . |*.                  |
            11     0.02111 |                  . | .                  |
            12     0.02337 |                  . | .                  |
            13     0.05945 |                  . |*.                  |
            14    -0.03437 |                  .*| .                  |
            15    -0.03305 |                  .*| .                  |
            16    -0.01492 |                  . | .                  |
            17    -0.02776 |                  .*| .                  |
            18    -0.00833 |                  . | .                  |
            19    -0.00106 |                  . | .                  |
            20    -0.00146 |                  . | .                  |
            21     0.03351 |                  . |*.                  |
            22    -0.06665 |                  .*| .                  |
            23    -0.03599 |                  .*| .                  |
            24    -0.02407 |                  . | .                  |
```