THE STRENGTH AND STATIC FATIGUE
OF A
GRANITE, AN ANORTHOSITE AND A LIMESTONE

by

Rudy H. Schmidtke

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A thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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ABSTRACT

The static fatigue of intact rock strength (long-term strength under static uniaxial compressive loading) has been studied using Lac du Bonnet granite, Beebe anorthosite and Tyndall limestone as the test specimens.

A total of 265 tests were carried out at various constant loads. The experimental data were analyzed using a probabilistic approach based on the Weibull distribution. The static fatigue curve, describing the relationship between strength and time, was constructed by relating the dry short-term strength distribution to the distribution of the time to failure data at constant load.

The static fatigue data have also been analyzed without reference to the Weibull distribution. An exponential function where the asymptote should indicate the static fatigue limit was fitted to the static fatigue data. Static fatigue limits of both granite and anorthosite approximate 56% of their mean compressive strength, whereas the limestone indicates a limit of 43%.

An attempt was made to construct a "universal fatigue curve" using the data from all the tests on the three rock types. This curve suggests a static fatigue limit at 54% of the strength.

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Chapter I

INTRODUCTION

Static fatigue is defined as the delayed failure of a material under sustained loading. French champagne makers never used a bottle twice, owing to observed glass failure attributed to static fatigue (Wiederhorn, 1966). A great deal of attention has been given to the study of static fatigue in ceramic components; many of these are used in satellite technology. In the past, static fatigue failure of rock was not taken into consideration in the design and use of underground openings even though the fact that rocks have a limited "stand-up time" was a recognized problem. Only recently has the static fatigue of rocks been studied. The incentive for this has come from the nuclear industry which intends to bury its waste in deep geological formations.

The strength of a rock mass will depend on the intact rock strength and frictional resistance along discontinuities. The long-term stability will therefore depend on the time-dependency of strength and friction. Amadei and Curran (1980) and Dietrich (1972), however, reported an increase in friction with time for discontinuities in rocks. Lajtai (1985) has also shown that frictional resistance on a discontinuity in Lac du Bonnet granite increases with time. Therefore, the intact rock strength has a role in the long-term stability of a rock mass.

The long-term stability (static fatigue limit) is investigated here for two crystalline rocks; Lac du Bonnet granite and Beebe anorthosite; and a sedimentary carbonate rock; limestone (Tyndallstone).

Chapter II

THEORY OF STATIC FATIGUE

If a material is stressed above a certain level, but below its instantaneous strength, failure by rupture may occur with time. This type of failure is known as failure through static fatigue. The time involved is known as time to failure or lifetime of the material. The lifetime of a material depends on several factors, mainly stress, but also on such environmental factors as temperature and humidity. Materials do not have a specific failure time associated with them. Brittle materials, including rocks, have a wide variation of failure times often through five to six orders of magnitude. Instantaneous strength measurements (compressive strength for example) are similarly distributed although the range of strength values is narrower. In general, neither the strength nor the failure time is a unique material constant, both are statistical quantities.

2.1 Static Fatigue And The Weibull Distribution

The most common theoretical distribution that is used to model both the strength and failure time measurements is the Weibull distribution (Weibull, 1951).

The Weibull distribution is an extreme value distribution which has been developed to predict the occurrence of floods and droughts and other extreme events. Weibull showed that his distribution function adequately described experimental data on the tensile strength and lifetime of steel and other materials. Its usefulness lies in its versatility through a judicious selection of parameters. The distribution may take many shapes and can approximate the widely different normal and exponential distributions.

The Weibull distribution in its general two-parameter form (Snowden, 1977) can be expressed as:

$$P_{(\sigma,v,t)} = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_o}\right)^m \left(\frac{v}{v_o}\right) \left(\frac{t}{t_o}\right)^w\right] \qquad [1]$$

where P is the cumulative probability of failure, v is the volume of the specimen, t is the load duration and σ, v, t, m and w are the Weibull constants. In addition to strength and failure time, this distribution has been shown to account for the well known "size effect" in rock strength as well. However, size effect was eliminated, since all the rock specimens used in the testing program were the same size (constant volume).

The compressive strength distribution represents the strength data (σ) at a constant volume and constant load duration by the equation:

$$P_{(\sigma)} = 1 - \exp[-Z\sigma^m] \qquad [2]$$

where Z is a constant combining the other constants. Time to failure data is also modelled by the same equation given a constant load (σ_{SF}). The probability equation then becomes:

$$P_{(t)} = 1 - \exp[-Yt^{w}] \qquad [3]$$

where Y is a constant combining all other constants. The constants Y and Z are usually referred to as kV and the constants m and w are denoted as m. Therefore, the given probability function from equation [1] is:

$$P_{(\sigma,t)} = 1 - \exp[-kV(\sigma,t)^{m}] \qquad [4]$$

The constants kV and m for each distribution are determined by plotting $\ell n \ell n[1/(1-P_{(\sigma,t)})]$ against $\ell n(\sigma,t)$ and assigning the probability as:

$$P_{(\sigma,t)} = \frac{i}{N + 1} \qquad [5]$$

where i is the rank when the measurements are ordered from the weakest (shortest time to failure) to the strongest (longest time to failure) and N is the number of tests. This transformation puts the Weibull distribution in linear form:

$$\ell n \ell n[\frac{1}{1 - P_{(\sigma,t)}}] = \ell nkV + m\ \ell n(\sigma,t) \qquad [6]$$

where m is the slope of the best straight line fit and ℓn kV is the intercept.

When results from N strength and N time to failure tests are ranked in ascending order, rank 1 corresponds to the weakest strength specimen and the shortest time to failure and rank N corresponds to the strongest strength specimen and the longest time to failure in the test series. One may now make the assumption that the failure times listed at rank i belong to the specimen whose instantaneous strength is listed at the same i-th rank (Burke et.al.,1971). This is based on the expectation that the weaker specimen will fail before the stronger. The same constant load applied to a weak and a strong specimen does not result in the same stressing level, i.e., the weak one is stressed higher than the strong one. This is equivalent to expressing:

$$P_{(\sigma)} = P_{(t)} \qquad [7]$$

which defines the following relationship between stress and time to failure:

$$(kV)_{\sigma}\ \sigma^{m_{\sigma}} = (kV)_{t}\ t^{m_{t}} \qquad [8]$$

From equation [8] the relationship between strength and time to failure at a constant fatigue load can be obtained by solving for σ and t:

$$\sigma = \left(\frac{(kV)_{t}}{(kV)_{\sigma}}\right)^{1/m_{\sigma}} \cdot t^{m_{t}/m_{\sigma}} \qquad [9]$$

or

$$t = \left(\frac{(kV)_{\sigma}}{(kV)_{t}}\right)^{1/m_{t}} \cdot \sigma^{m_{\sigma}/m_{t}} \qquad [10]$$

The strength may be normalized with respect to the static fatigue load applied to the specimen. Therefore, normalizing the strength (stress level) in percent form:

$$\text{stress level}(\sigma_L) = \frac{100 \times \text{applied static fatigue load } (\sigma_{SF})}{\text{strength } (\sigma)} \qquad [11]$$

and substituting equation [11] into [9] and [10]

$$\sigma_L = \left(\frac{kV_t}{kV_\sigma}\right)^{1/m_\sigma} \cdot \sigma_{SF} \cdot t^{-m_t/m_\sigma} \qquad [12]$$

$$t = \left(\frac{kV_t}{kV_\sigma}\right)^{1/m_t} \cdot \left(\frac{\sigma_{SF}}{\sigma_L}\right)^{m_\sigma/m_t} \qquad [13]$$

and linearizing both equations [12] and [13] produces the static fatigue curve:

$$\ell n\sigma_L = \frac{1}{m_\sigma} \ell n\left(\frac{kV_t}{kV_\sigma}\right) + \ell n\, \sigma_{SF} - \frac{m_t}{m_\sigma} \ell nt \qquad [14]$$

or

$$\ell nt = \frac{1}{m_t} \ell n\left(\frac{kV_t}{kV_\sigma}\right) + \frac{m_\sigma}{m_t} \ell n\left(\frac{\sigma_{SF}}{\sigma_L}\right) \qquad [15]$$

It can be shown that the static fatigue curve can also be generated without the use of the Weibull distribution. This approach requires the test specimen population to be the same in both the strength and time to failure tests. The weakest specimen in the strength distribution is paired with the shortest failure time in the time to failure distribution until rank N. A plot of stress level (σ_L) from equation [11] versus time to failure defines the relation between σ and t. The stress level and time to failure are linearized taking the form of a straight line $(y = mx + b)$:

$$\ell n\sigma_L = m\, \ell nt + b \qquad [16]$$

where m = slope and $b = \sigma_L$ intercept.

As discussed above, the static fatigue curve can be generated with Weibull theory or by simply pairing the strength and time to failure data. The static fatigue data of Lac du Bonnet granite subjected to a load (stress) of 160 MPa may be used to demonstrate the use of the Weibull theory and the simple pairing process.

Initially fourteen strength and fourteen time to failure tests (160 MPa) were performed on the granite (Figs. 1, 2 and 3). The data are shown in Tables 1 and 4. (Later strength tests increased the specimen population from $N = 14$ to $N = 70$. However, the difference between the means of the $N = 14$ and $N = 70$ groups is not statistically significant at the 99% confidence level). The test data of Tables 1 and 4 were paired and the resulting stress levels plotted against the time to failure in natural log form. A straight line least square fit to the static fatigue data is shown in Figure 4. The slope of $m_t/m_\sigma = -0.0151$ in equation [14] from Weibull theory compares reasonably well with the slope of $m = -0.0172$ when pairing rank 1 to rank N data for the granite 160 MPa series.

Chapter III

EXPERIMENTAL PROGRAM

Three rock types were used in the experimental program. Test specimens were prepared from blocks of granite purchased from the Cold Spring Quarry located near Lac du Bonnet, Manitoba. The anorthosite came from Beebe, Quebec, and was supplied by the Civil Engineering Department, University of Manitoba. The limestone, commonly referred to as Tyndall stone in Manitoba, comes from a quarry near Garson, Manitoba.

One hundred and thirty instantaneous uniaxial compression tests and 268 time to failure tests were conducted. The number of tests performed on Lac du Bonnet granite, Beebe anorthosite and Tyndall limestone were, 208, 99 and 91, respectively. Strength tests were conducted on dry specimens at room temperature and humidity. Time to failure tests were conducted in a water bath at room temperature (25°C). The saturated specimen was tested in the most appropriate environment that exists in rocks at depth, i.e., 100% humidity.

3.1 Sample Preparation

The blocks of rock were cored with diamond set drill bits on a modified drill press in the laboratory. Blocks of Lac du Bonnet granite were oriented and cored vertically. Orientation of the anorthosite and limestone blocks were not known. A constant supply of water was applied through the drill stem to cool and lubricate the bit. The speed and applied pressure on the bit were determined by the operator.

Several pieces of core, with a diameter of 31.7mm, were drilled from the blocks with their coring axes kept parallel. These pieces of core were then cut with a diamond set saw blade to a length of approximately 68mm. The cut ends were then ground perpendicular to the main core axis to a length of 64mm. This sample dimension satisfied the minimum acceptable length to diameter ratio of two (Hawkes and Mellor, 1970).

3.2 Uniaxial Compressive Strength Test

Compressive strength tests were carried out on a Baldwin hydraulically controlled testing machine with a capacity of 266 kN. The rock specimens were loaded through hardened steel plattens machined to a diameter of 31.7mm. A spherical head was placed at one end to ensure even loading. The loading rate was kept at approximately 2.5 MPa per second. The strength test results of 70 dry specimens of granite, 32 dry specimens of anorthosite, and 28 dry specimens of limestone are listed in Tables 1, 2 and 3. The normal procedure for instantaneous strength determination is to isolate the test specimen from environmental effects in a bath of liquid nitrogen. Lajtai et.al. (in preparation) states that environmental effects at fast loading rates (instantaneous strength test) have little significance on the test results.

3.3 Time To Failure Test

Time to failure tests were carried out on a Structural Behaviour CT-50 loading frame connected to an AP-1000 air on oil pump. This equipment was able to apply a reasonably constant uniaxial compressive load (±5%) to a specimen until failure. Approximately two seconds after opening a valve, a constant load was applied to the specimen.

The environmental conditions for the specimens in the time to failure test differ from those for the strength tests. A coated steel pot was filled with water and the rock specimen was then lowered into it. Specimens for this test were allowed to soak in water under vacuum for at least 48 hours before testing. The specimens remained under water until failure and their failure times were recorded.

One hundred and fifty-two granite specimens were divided into nine groups. Eight groups consisted of fourteen specimens and the ninth group had forty. The latter was tested in static fatigue at 166 MPa and the remaining eight series of fourteen specimens were tested at constant loads of 155, 160, 170, 177, 188, 199, 207 and 215

Three series of fourteen specimens at constant loads of 105, 111 and 115 MPa and one series of twenty-five specimens at a load of 95 MPa comprised the sixty-seven anorthosite time to failure tests.

Time to failure tests on the limestone consisted of forty-nine specimens subjected to a load of 50 MPa (Bures, 1985) and fourteen specimens at a constant load of 57 MPa

(Bell, 1984) totalling sixty-three tests.

The applied static fatigue loads for the various rock types correspond to loading expressed as a percentage of the mean compressive strength, between 69 and 95% for granite, 66 to 80% for anorthosite and 66 to 75% for the limestone. Several specimens failed before the designed load had been reached. One granite specimen subjected to a load of 155 MPa did not fail for forty-five days. These specimens are identified in Tables 5, 6 and 7.

Chapter IV

RESULTS AND DISCUSSION

A comparison and analysis of the strength and time to failure data for the three rock types are presented below.

4.1 Instantaneous Strength Results

Seventy granite, thirty-two anorthosite and twenty-eight limestone specimens were failed and their instantaneous strengths recorded (Tables 1, 2 and 3). The strength test was performed on each rock type to determine its characteristic distribution. Table 7 summarizes the mean strength (C_o) and standard deviation of the granite, anorthosite and limestone.

Values of standard deviation between 3.5% and 10% are considered attainable by Obert and Duvall, (1967) for uniaxial compression tests. The anorthosite has 2/3 the strength of granite and the limestone 1/3 that of the granite.

The shape of the strength distribution for each rock type is not a simple normal distribution. All distributions are skewed to the left. Figure 5 shows the strength values for LDB granite fitted according to the normal distribution. The histogram shows a skewed distribution to the left and the peak of the bell curve over a trough in the distribution. Indeed, the distribution may even be bimodal.

The Weibull distribution has been fitted to the instantaneous compressive strength data and is shown on Figures 6, 7 and 8. The correlation coefficient (r^2) would suggest that Weibull theory fits the strength data reasonably well. A more rigorous analysis involving a statistical test such as the "run" test, however, indicates that the fit is only an approximate one (Mack, 1966).

4.2 Time To Failure Results

The results of each series was fitted to the Weibull distribution. Figure 9 shows that the time to failure distribution for granite at 166 MPa is not a normal distribution. A reasonably good fit ($r^2 = 0.97$) using the Weibull distribution for the same case is shown on Figure 10. As with the strength tests, the Weibull distribution gives the best fit, but again the more rigorous "run" test analysis indicates the fit is only approximate.

4.3 Mean Time To Failure

Calculations of the mean time to failure for each series is perhaps the simplest statistical calculation. A failure time of one second was given to specimens which did not reach the design load and 45 days to the specimen which did not fail in the 155 MPa granite series. Table 8 lists the mean failure times (x) and standard deviation (s) for the three rock types. The wide scatter of data in a time to failure test is further exemplified by the standard deviation value. Deviations about the mean reach 260% which illustrate that the mean value is a poor representation of the data.

Mean failure times for LDB granite have been plotted against the static fatigue load in log-log space shown in Figure 11. A linear and exponential function has been fitted for the ℓn (stress level) versus ℓn (mean time to failure) plot. The equations are shown on Figure 11. The fitted curve has a negative slope, indicating an increase in the mean time to failure with lower applied stresses. Extrapolating the curve to ℓn (stress level) = 0 corresponds to a failure time of 9.2×10^{70} years. One would, however, prefer a failure time equal to infinity at zero load. An exponential function (ℓn stress = $0.86 \times \exp[-0.05 \times \ell n(TTF)] + 4.60$) can also be fitted to the data. Assuming a failure time of infinity, the exponential term in the function becomes zero and the constant becomes the static fatigue limit. According to this exponential function, no failure would occur when the granite is subjected to a compressive stress less than 50 MPa. This value is lower than the mean crack initiation stress of 70.5 MPa (Lajtai et.al. 1982). Crack initiation stresses less than 50 MPa have been measured, but these are individual tests in the distribution.

This analysis illustrates that two very different functions can fit the data with the same degree of confidence. The variability of time to failure data and the physical characteristics of LDB granite suggest the results of the analysis are inconclusive. Mean plotting of all the static fatigue data is also a very poor representation of many tests. Analysis of the strength and time to failure distributions are necessary to make better use of the data and provide more meaningful conclusions. Therefore, the probabilistic approach based on the Weibull distribution and is perhaps the more reasonable approach.

4.4 The Static Fatigue Curve

One may fit a straight line (in log-log space) to the results of each test series as shown previously in Figure 4. The slope of the obtained static fatigue curve for each static fatigue load is listed in Table 9.

The granite, anorthosite and limestone show a general decrease in the static fatigue curve slope when subjected to lower stresses. Equation [14] of the Weibull analysis cannot account for this; it predicts the same slope irrespective of the static fatigue load. The experimental points should follow a linear trend at a constant slope according to equation [14]. In the next step, reliance on the Weibull distribution was abandoned while retaining the "stress level vs. time to failure" pairing aspect of the analysis. All the pairs were grouped regardless of the static fatigue load and analyzed together.

4.5 Static Fatigue Limit

All linear functions attempting to model the stress versus time to failure relationship intersect the time to failure axis at zero stress. The exponential function fitted to stress versus mean time to failure in Figure 11 does not interesect the time axis. A stress level asymptote of 50 MPa occurs when time is infinite. The presence of a static fatigue limit is also shown on Figures 12, 13 and 14. Plotting stress level versus time to failure in log-log space shows a curvature in the family of data points. An exponential function of the general form:

$$Y = A \cdot \exp(BX + C) \qquad [17]$$

was used to model the relationship between $Y = \ln$ (stress level, %) and $X = \ln$ (time to failure, sec). The following fitted equations model the data points between stress levels of 63 to 110% for granite, 60 to 109% for anorthosite, and 67 to 85% for the limestone.

Granite $\ln(\sigma_L) = 0.574 \cdot \exp[-0.0874 \cdot \ln(TTF)] + 4.039$ [18]

Anorthosite $\ln(\sigma_L) = 0.558 \cdot \exp[-0.141 \cdot \ln(TTF)] + 4.022$ [19]

Limestone $\ln(\sigma_L) = 0.708 \cdot \exp[-0.050 \cdot \ln(TTF)] + 3.756$ [20]

For very long failure times ($TTF = \infty$) the exponential term becomes zero and parameter C is the limit. Equation [18] reduces to $\ln(SL) = 4.039$, equation [19] to $\ln(SL) = 4.022$ and equation [20] to $\ln(SL) = 3.756$. The static fatigue limits are expressed more meaningfully as a percentage of the strength, i.e., 57% or 128 MPa for granite; 56% or 80 MPa for anorthosite; and 43% or 33 MPa for the limestone. The above static fatigue limits are within the 95% confidence range at 121 to 136 MPa, 73 to 87 MPa and 30 to 35 MPa for granite, anorthosite and limestone, respectively.

Mould and Southwick, (1959) introduced the concept of the universal fatigue curve. This universal curve was constructed by normalizing both the long-term strength and time to failure data so that test results for different materials could be plotted together. In particular, strength is normalized with the instantaneous strength and time to failure with the measured time to failure when the static fatigue load is one half of the instantaneous strength.

A similiar method has been adopted to construct the universal static fatigue curve for the tested rock types. Strength values have already been normalized to percent form with respect to the average uniaxial compressive strength. The time to failure data was normalized by dividing ℓn (time to failure) by the ℓn (time to failure) at the convenient stress level of 75% for each rock type. Time to failure at a stress level of 50% is difficult to measure, as the static fatigue limit itself may be over this. The normalized time to failure data (standard time to failure) constants at SL 75% are tabulated below.

Granite	ℓn (TTF 75%)	=	8.18
Anorthosite	ℓn (TTF 75%)	=	4.41
Limestone	ℓn (TTF 75%)	=	4.53

The universal static fatigue curve plot of ℓn (stress level, %) versus ℓn (75% std time to failure, sec) is shown on Figure 15. An exponential function in the same form as equation [17] has been fitted to 265 data points producing the following universal static fatigue curve.

$$\ln(SL) = 0.580 \cdot \exp[-0.541 \cdot \ln(75\%\ \mathrm{STD\ TTF})] + 3.990 \quad [21]$$

The universal static fatigue limit according to this function, with 95% confidence boundaries ranges from 51 to 58% of the mean compressive strength. Therefore, according to equation [21] at TTF = ∞, a geologic material exhibits a static fatigue limit at approximately 50% of its mean strength. The static fatigue data of the limestone may however indicate a lower limit (Figure 14), as the limestone data points at the tail-end of the curve seem to have their own trend. Obviously, extrapolation of the results using the concept of the "universal static fatigue curve" must be undertaken with caution. The static fatigue limit obtained this way must be regarded as a crude approximation.

Chapter V

ENGINEERING DESIGN

Defining a function which models the performance of a material for exceptionally long periods of time is very difficult. An exponential function has been fitted to the static fatigue data of granite, anorthosite and limestone in an attempt to model their failure times at stresses ranging from zero to the compressive strength. Experimental data at short failure times (<400 sec) has a wide vertical scatter which reduces with longer failure times. Several experimental points plotting in the short time to failure range at lower stresses may be premature failures just as those tests which failed before the design load had been reached. The possibility of more than one failure mechanism over the length of the experimental failure times should not be excluded. However, the ruptured specimens provided no evidence to support the existence of more than one failure mechanism. An exponential function can also be fitted to experimental points in the lower stress range where data scatter is usually less and the experimental points no longer illustrate a curvature. Either a linear or an exponential curve can easily be fitted to the last 80 data points of the granite curve shown in Figure 16. These functions describe the static fatigue reasonably well for failure times up to two weeks. Estimated stress levels for the linear and exponential fit differ only by 7 MPa for a service life of 100 years. Therefore, for a service life span of 100 years, both functions may serve equally well. For longer service life the extension of these curves would be highly speculative.

Interestingly, the static fatigue limit expressed as a percentage of the mean compressive strength for the granite and anorthosite from equations [18] and [19] is the same. The static fatigue limit of limestone is only 43% of its mean compressive strength suggesting a much lower limit for carbonates. The normalization involved in the universal static fatigue did not remove the 14% difference.

5.1 Short And Long-Term Stability

The available static fatigue data has been modelled by an exponential function. Failure limits can be interpreted for the rock types tested in this study. The model predicts a static fatigue limit for the long-term stability of the rock. The following stresses are those at which no failure will occur.

Granite	128 MPa
Anorthosite	80 MPa
Limestone	33 MPa

If one ignored the probable size effect it is possible to use equations [18], [19], and [20] and measured stresses in the Canadian Shield (Herget, 1980) to predict the service life of underground openings excavated in the test rocks.

Herget, (1980) concluded that the minimum principal compressive stress is oriented vertically which is simply the overburden weight. Maximum principal stresses are oriented sub-horizontally and have been modelled by two linear functions. The first function models the horizontal stress at a depth of 0 - 900m.

$$\sigma = 9.86 + 0.371 \text{ MPa/m} \qquad [22]$$

The second linear function models horizontal stresses from 900 to 2200m.

$$\sigma = 33.41 + 0.011 \text{ MPa/m} \qquad [23]$$

The excavation of underground openings concentrates stresses around an opening. Elastic theory predicts stress concentration factors between zero and three depending on opening geometry and the distance of intersecting and adjacent openings. Obert and Duvall, (1967) report that Hast measured stresses in Swedish mines and discovered that the stress concentration factor was approximately two.

Long-term stability $(t = \infty)$ at depth in granite, anorthosite and limestone can be predicted by combining equations of in-situ stress and time dependent failure.

$$\sigma = \text{static fatigue limit / stress concentration factor} \qquad [24]$$

For example, a critical depth of an underground opening in limestone would be:

$$\sigma = 33/SC = 9.86 + 0.0371/d \qquad [25]$$

$$d_c = 179 \text{ metres.}$$

where SC is the stress concentration assumed to be two and d_c = critical depth. Similarly, d_c can be calculated for the granite and anorthosite.

Granite d_c = 2781 metres

Anorthosite d_c = 812 metres

Similar calculations can be made with finite time simply by equating σ to a form equivalent to the mean compressive strength and substituting equations [18], [19] and [20] into equations [22] and [23].

Failure of underground openings has been experienced by many mines excavated at depth in the Canadian Shield. For example, rock bursts have occurred at a depth of 460 to 600m in the porphyry and tuff at Kirkland Lake, (Herget, 1980) and at a depth of approximately 600m in the andesite at the Campbell Mine in Balmertown, Northwestern Ontario. Herget, 1980 also stated that Hedley reported the occurrence of rock bursts at a depth of 120m at a mine in Newfoundland.

The exponential function fitted to the static fatigue data from uniaxial compression tests is an attempt to model the time-dependent failure of several rock types displaying different compressive strengths. Uniaxial compression is the most critical stress state that occurs in underground openings. The existence of a confining pressure increases the fatigue failure times (Kranz, 1980).

Chapter VI

CONCLUSION

A relationship between strength and failure time data has been proposed for granite, anorthosite and limestone. An attempt to model this relationship utilizing Weibull's probabilistic theory and the exponential curve has been made.

The characteristic strength distribution of each rock type tested in this study was determined through the uniaxial compressive test. The average instantaneous compressive strength (C_o) of each rock was:

Granite	226 MPa
Anorthosite	143 MPa
Limestone	76 MPa

Test specimens were subjected to constant loads below their average compressive strengths and allowed to fail and the time to failure was measured. Higher stresses resulted in shorter times to failure. Data values at the same probability in the strength and time to failure distributions were paired and plotted to create the static fatigue curve. Experimental points, when grouped as a family, indicated a "flattening" at lower stress. An exponential function, relating stress level and time to failure, was fitted to model this static fatigue data. The fitted functions are as follows:

Granite $\ell n(\sigma_L) = 0.574 \cdot \exp[-0.0874 \cdot \ell n(TTF)] + 4.039$

Anorthosite $\ell n(\sigma_L) = 0.558 \cdot \exp[-0.141 \cdot \ell n(TTF)] + 4.022$

Limestone $\ell n(\sigma_L) = 0.708 \cdot \exp[-0.050 \cdot \ell n(TTF)] + 3.756$

The asymptote of the function defines the static fatigue limit and can be expressed as a percentage of the compressive strength. The compressive static fatigue limits of the tested rocks are listed below.

Granite	57% (C_o)	or	128 MPa
Anorthosite	56% (C_o)	or	80 MPa
Limestone	43% (C_o)	or	33 MPa

All experimental tests were normalized for both stress and time to failure and plotted on a universal static fatigue curve. The experimental data points behaved in the same non-linear manner. An exponential function was fitted to the 265 experimental points and produced the following universal static fatigue curve.

$$\ell n(\sigma_L) = 0.580 \cdot \exp[-0.541 \cdot \ell n(75\%\ \text{STD TTF})] + 3.990$$

The universal limit according to the above exponential function is approximately half of the mean compressive strength of geologic materials.

Whether or not the exponential curve models the static fatigue data accurately is very important. The function fits the data reasonably well up to a failure time of two weeks. However, the question is whether or not this function is valid for failure time predictions beyond two weeks. Schmidtke and Lajtai, (in press) show that an exponential or linear function predict stress levels equally well. The two fitted functions begin to diverge significantly beyond failure times of about 100 years. The use of the fitted curves beyond this is highly speculative.

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APPENDIX A

FIGURES

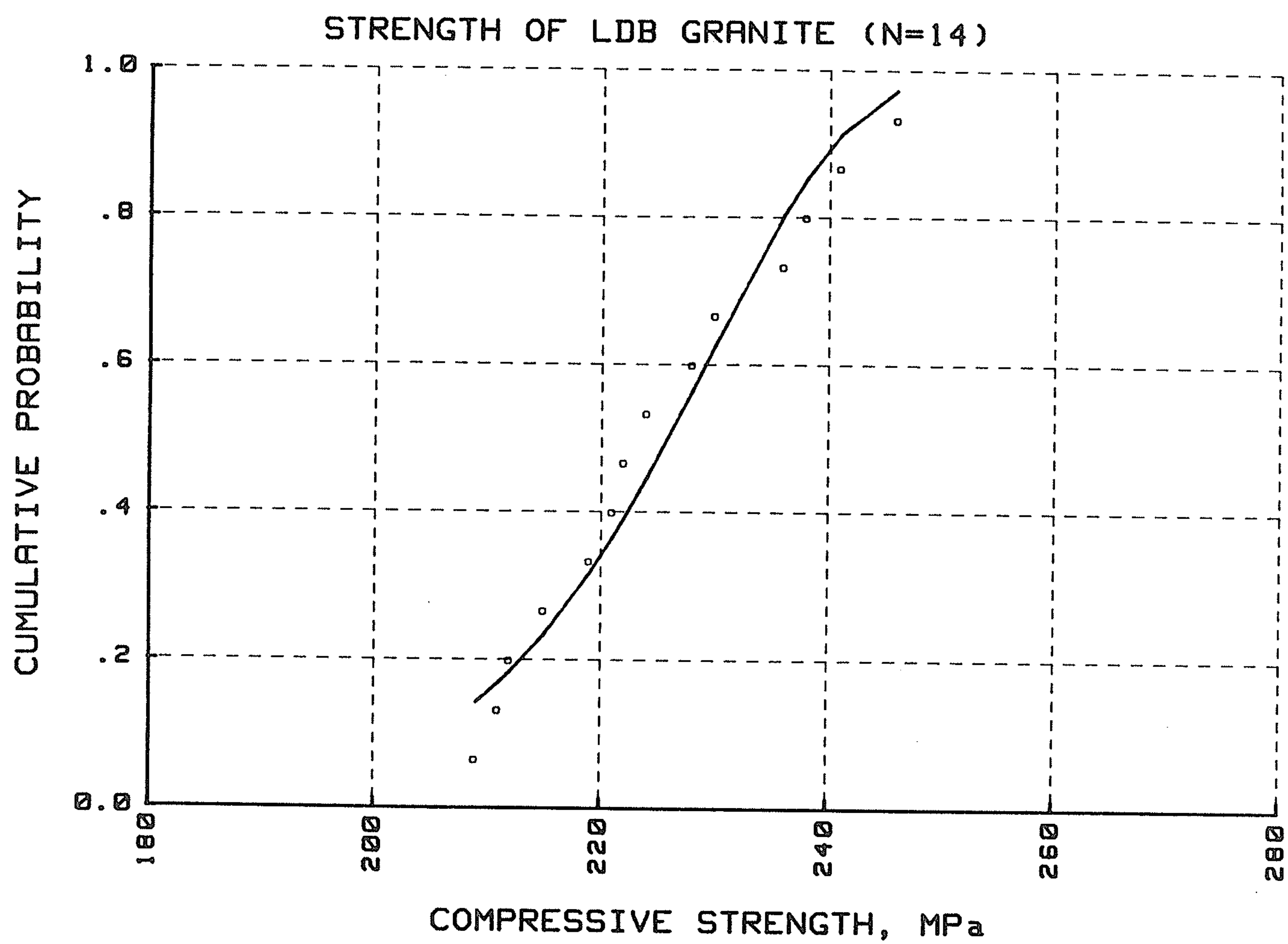

Figure 1: Weibull plot - Uniaxial compressive strength distribution of Lac du Bonnet granite (N = 14).

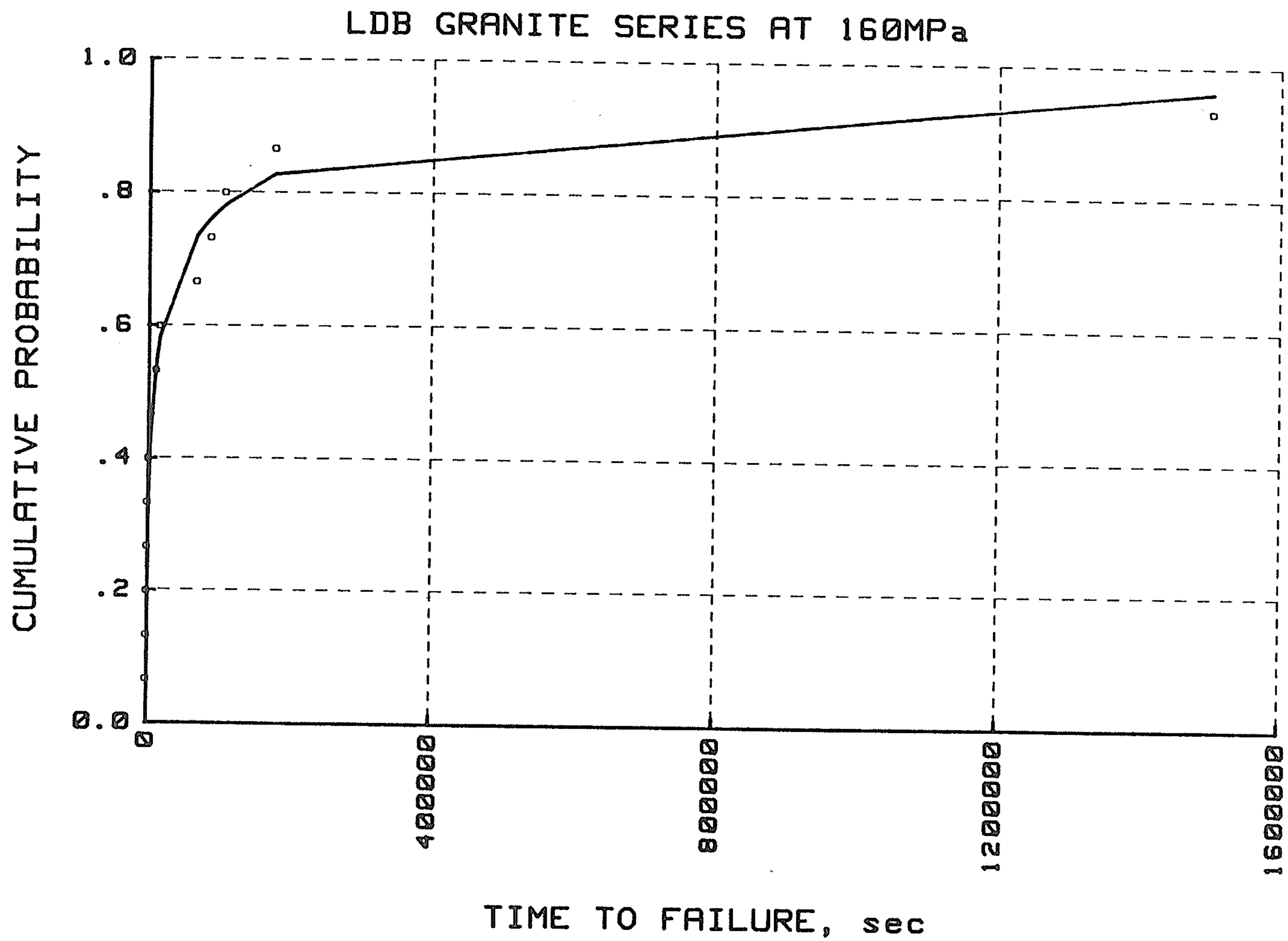

Figure 2: Weibull plot - Time to failure distribution of Lac du Bonnet granite at a constant load of 160 MPa (N = 14). The fitted curve was computer generated at the measured failure times.

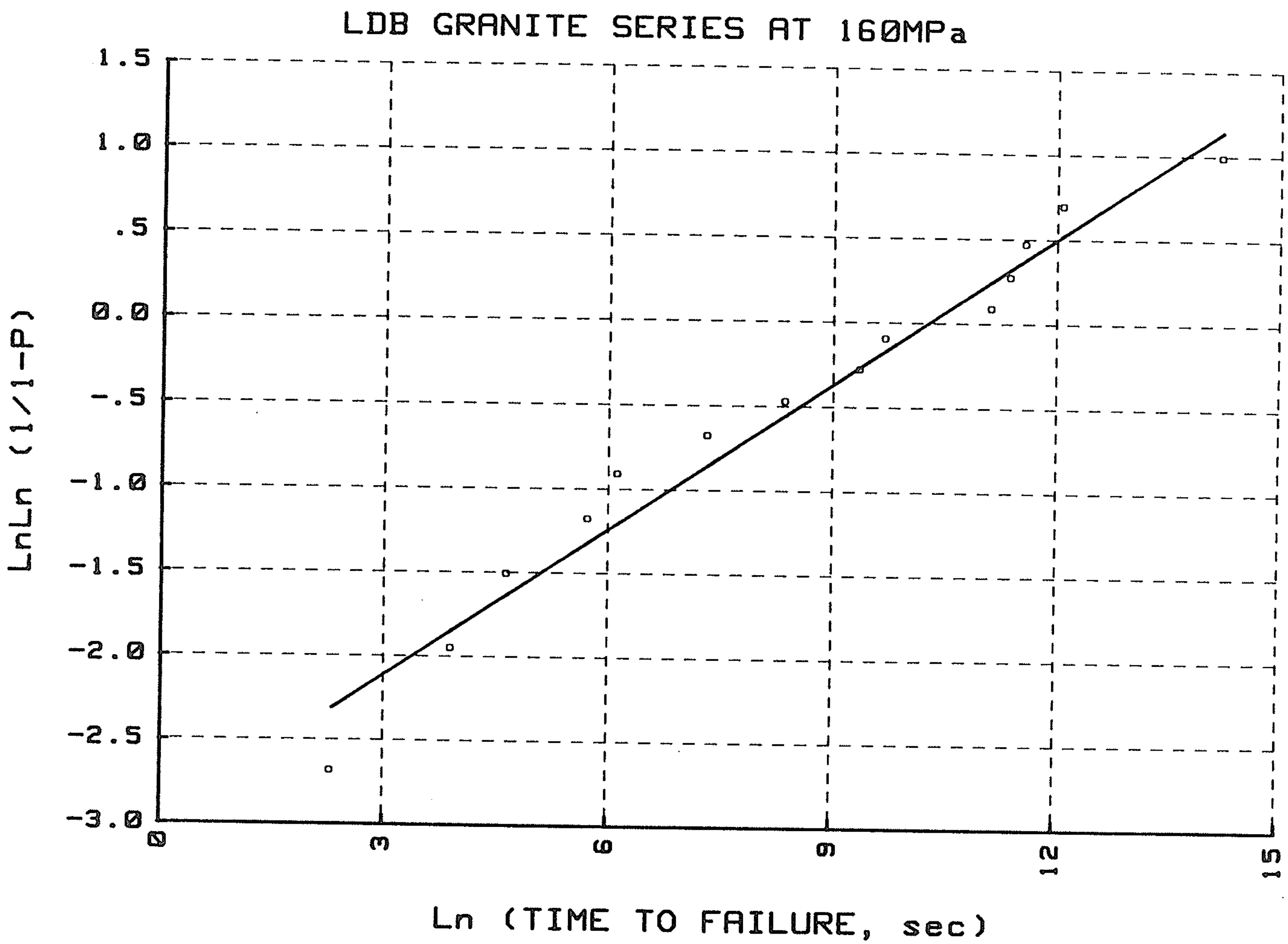

Figure 3: Linear regression line of time to failure data for Lac du Bonnet granite at a constant load of 160 MPa (N = 14).

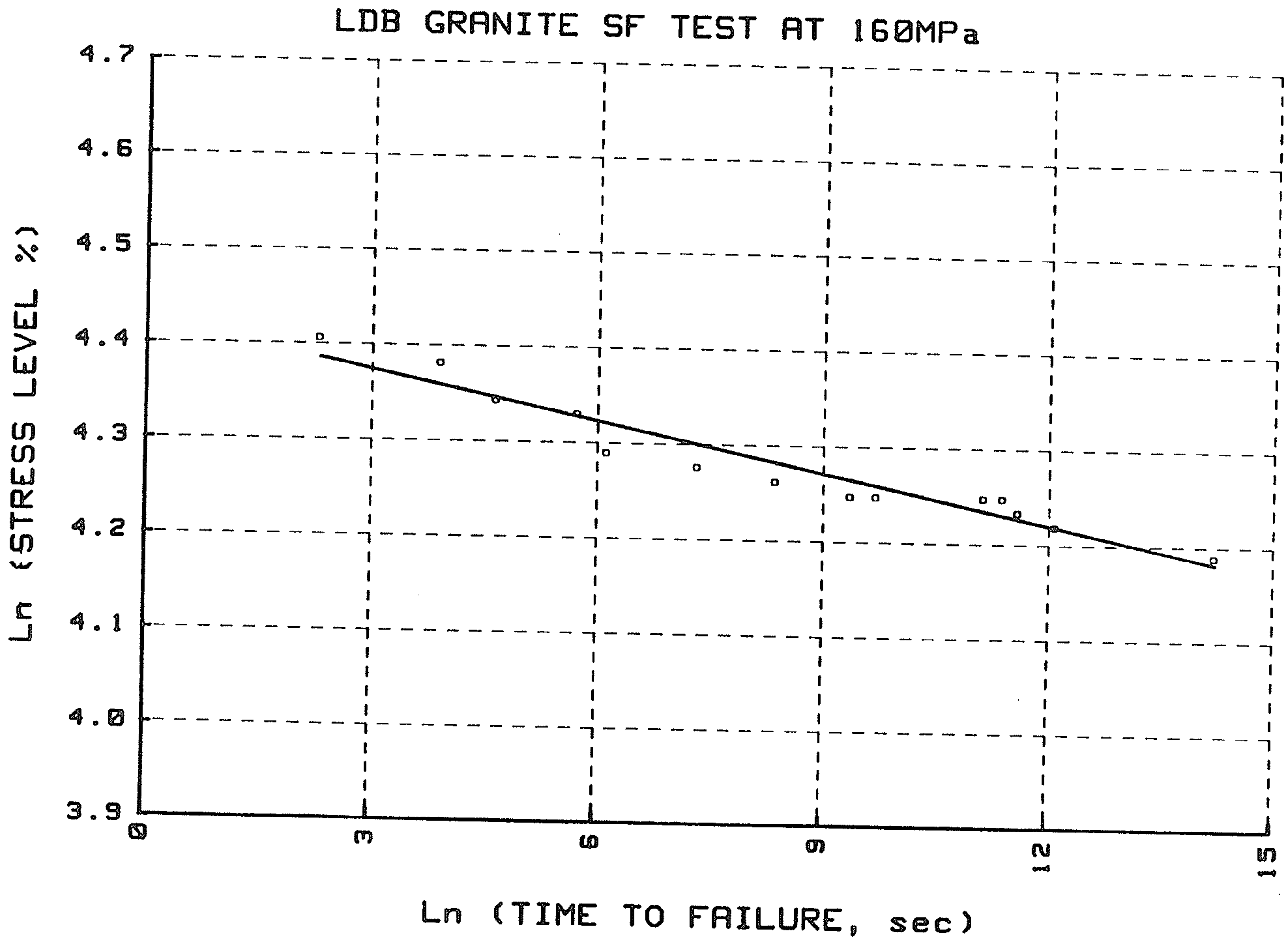

Figure 4: Static fatigue curve of Lac du Bonnet granite at a constant load of 160 MPa (N = 14).

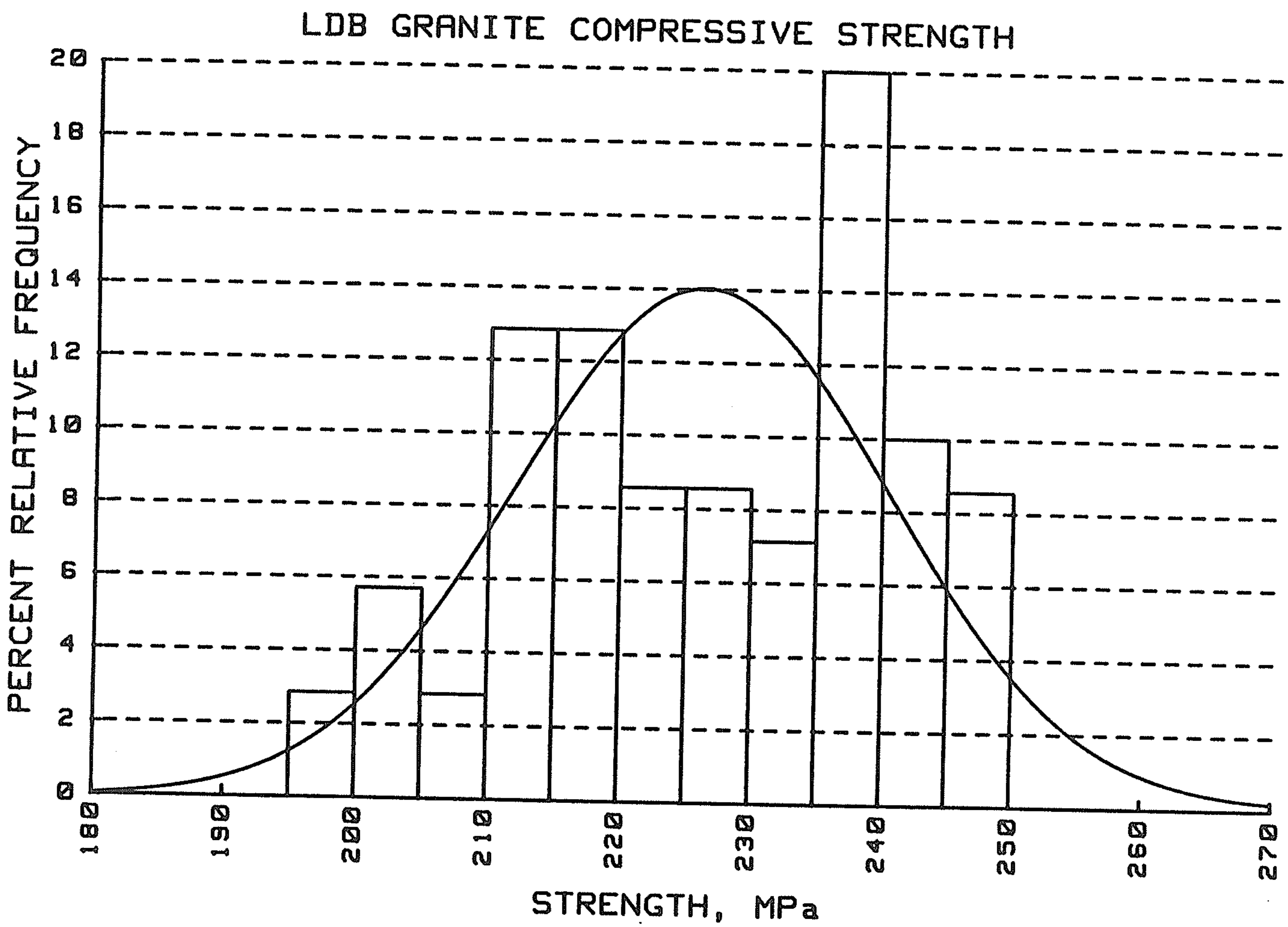

Figure 5: Histogram of Lac du Bonnet granite uniaxial instantaneous compressive strength distribution. Normal distribution fitted to the strength data (N = 70).

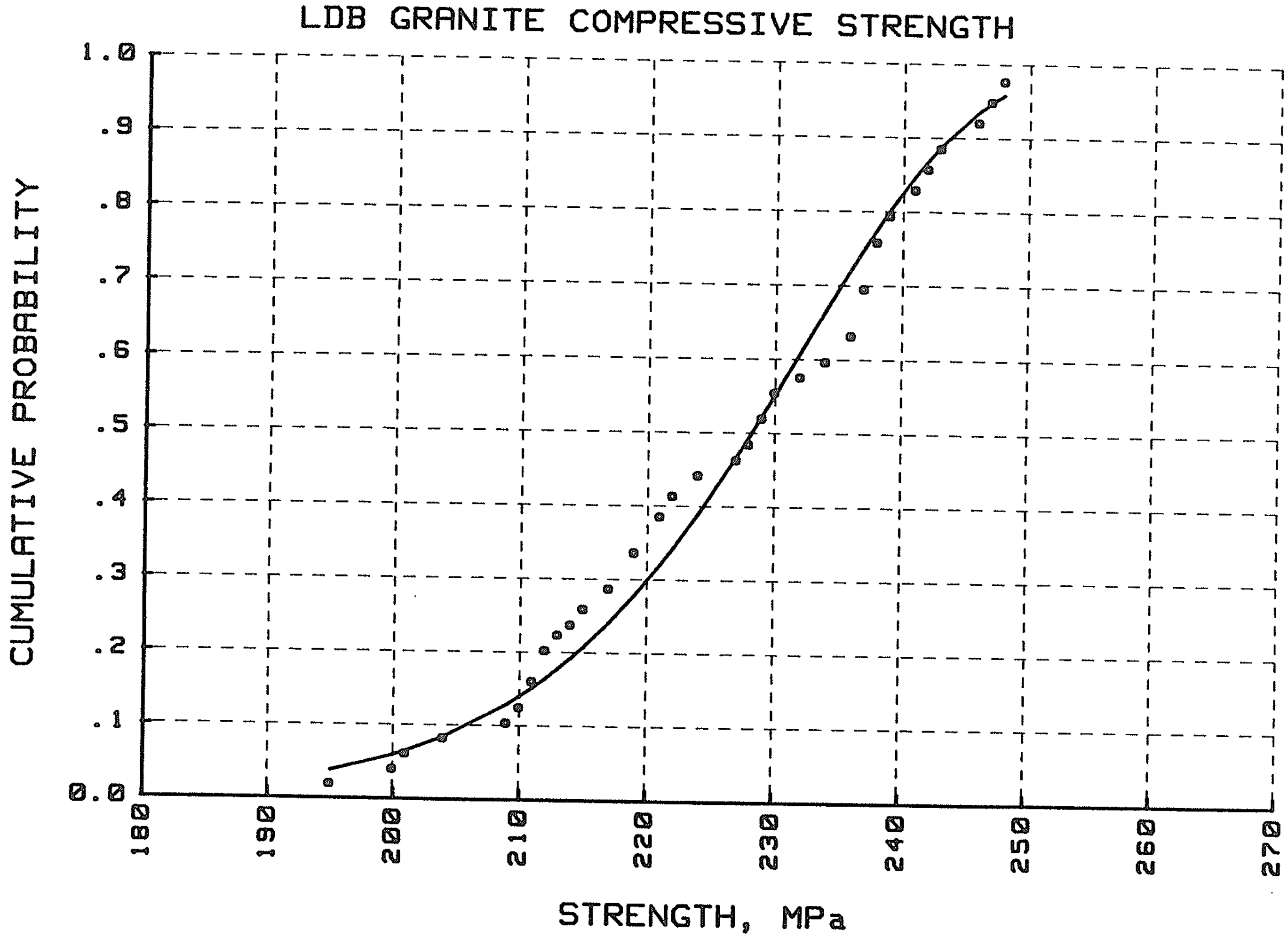

Figure 6: Weibull plot - Uniaxial instantaneous compressive strength distribution of Lac du Bonnet granite (N = 70).

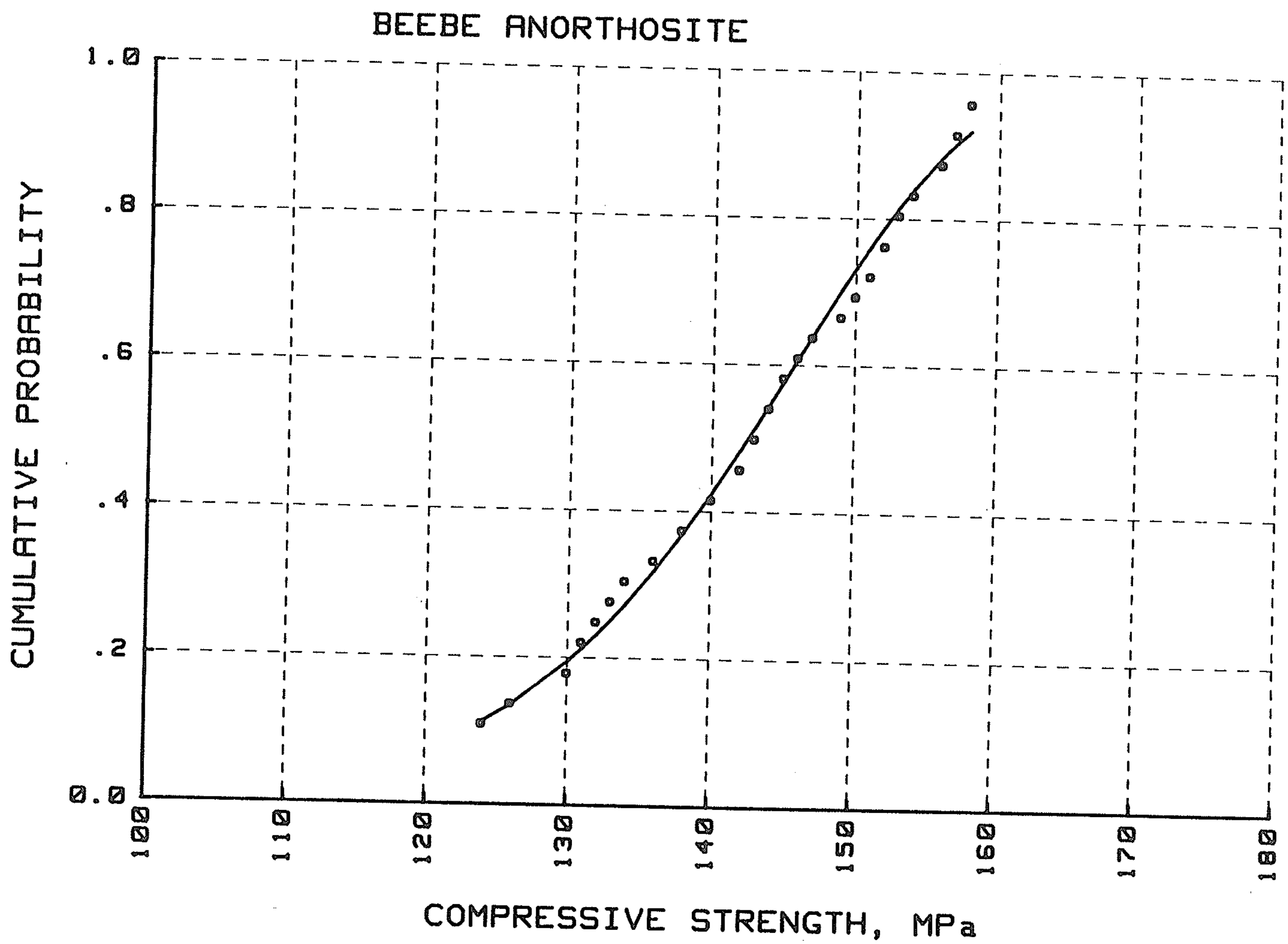

Figure 7: Weibull plot - Uniaxial instantaneous compressive strength distribution of Beebe anorthosite (N = 32).

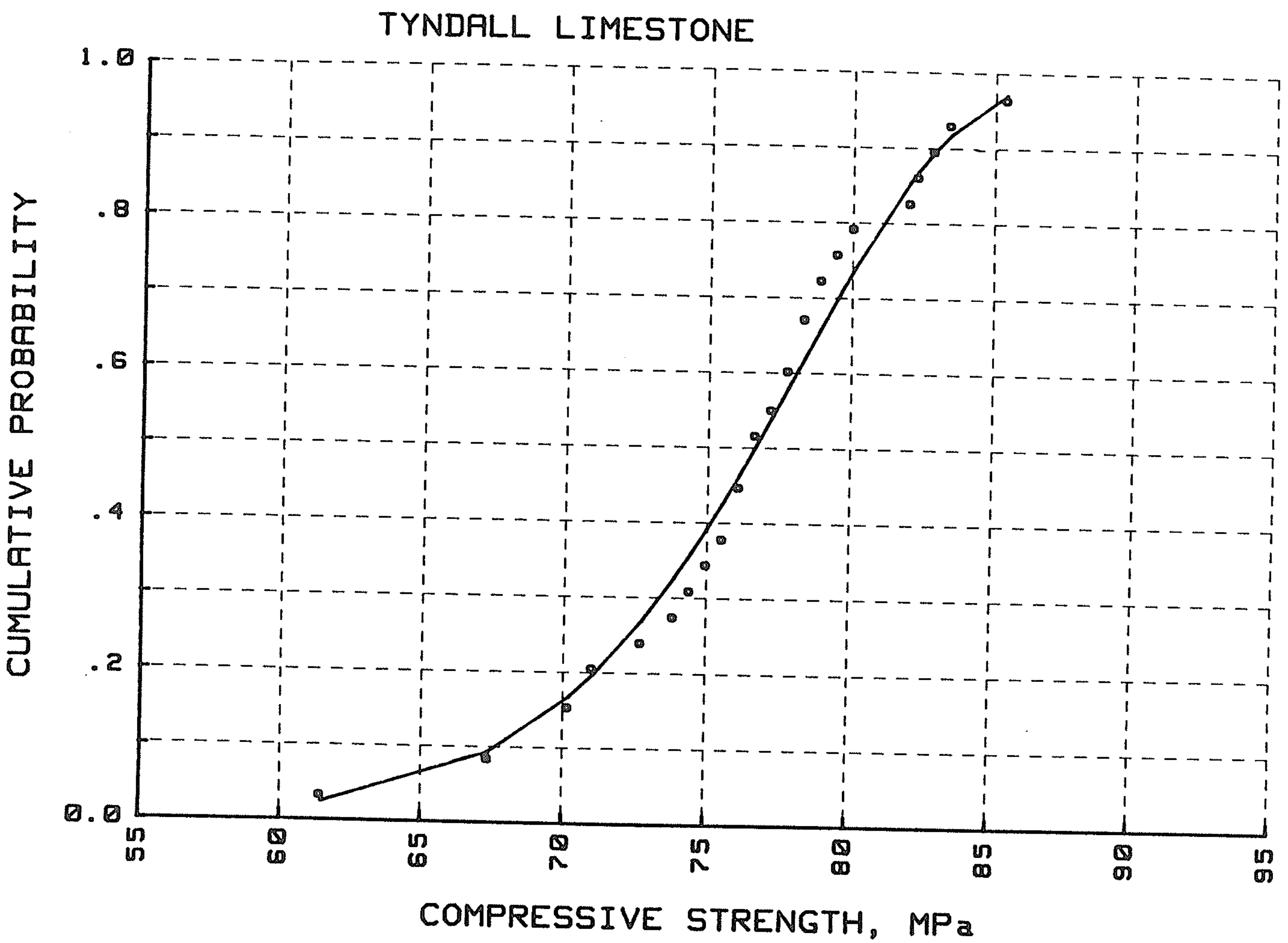

Figure 8: Weibull plot - Uniaxial instantaneous compressive strength distribution of Tyndall limestone (N = 28).

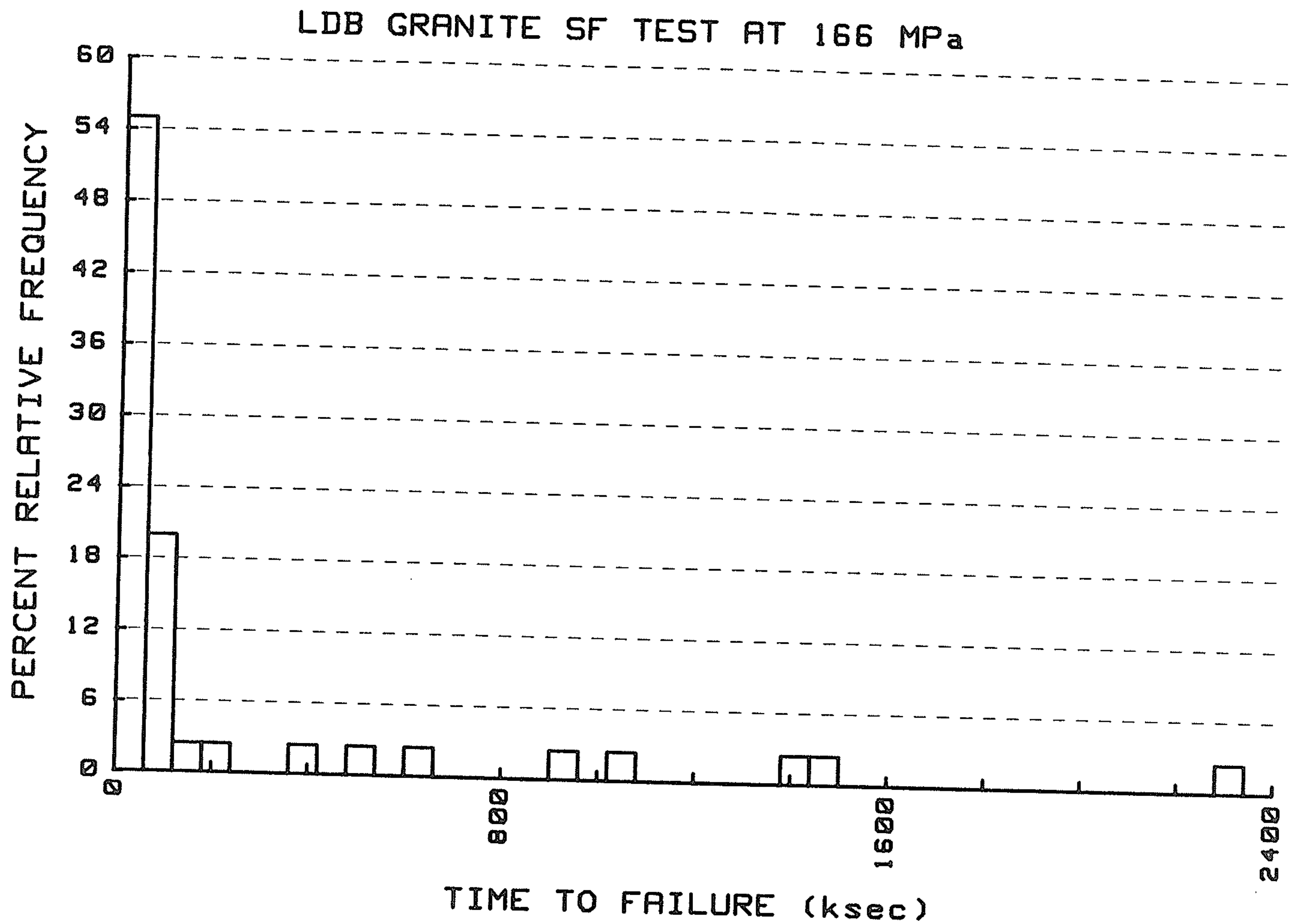

Figure 9: Histogram of time to failure distribution of Lac du Bonnet granite at a constant load of 166 MPa (N = 40).

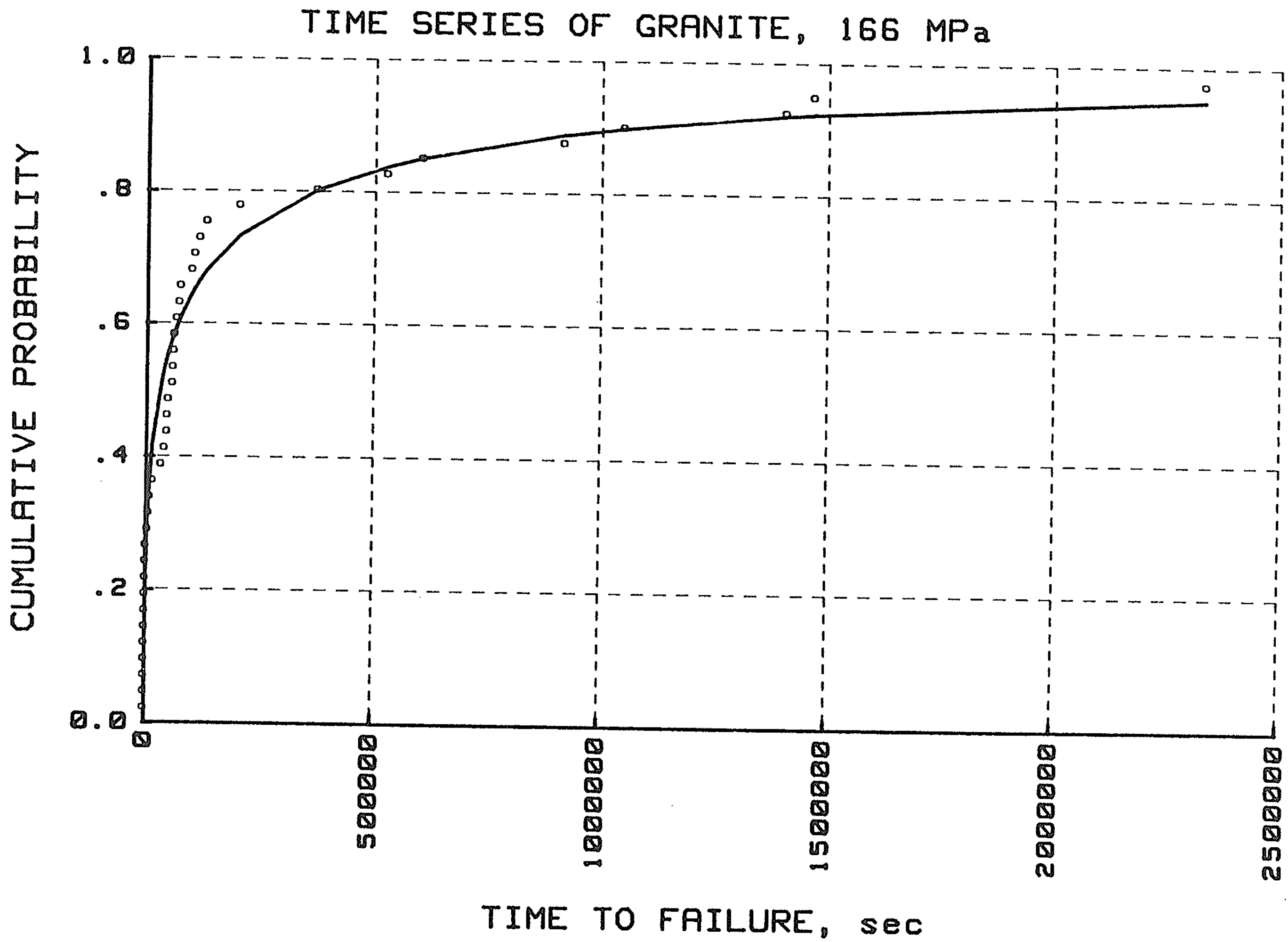

Figure 10: Weibull plot - Time to failure distribution of Lac du Bonnet granite at a constant load of 166 MPa (N = 40). The fitted curve was computer generated at the measured failure times.

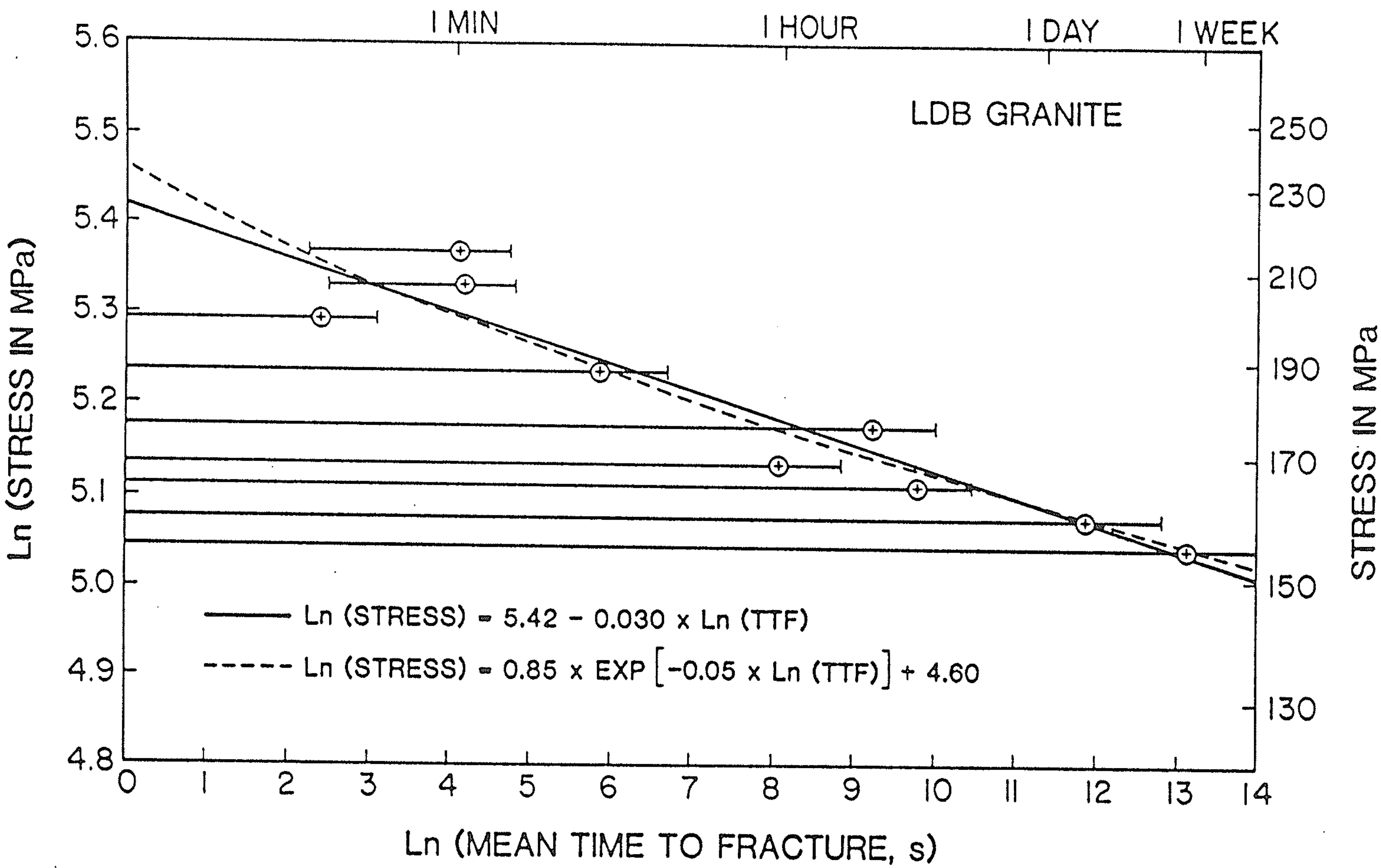

Figure 11: Mean failure time plot - Each point represents the arithmetic mean of nine time to failure test series. The error bars indicate the 95% confidence limits.

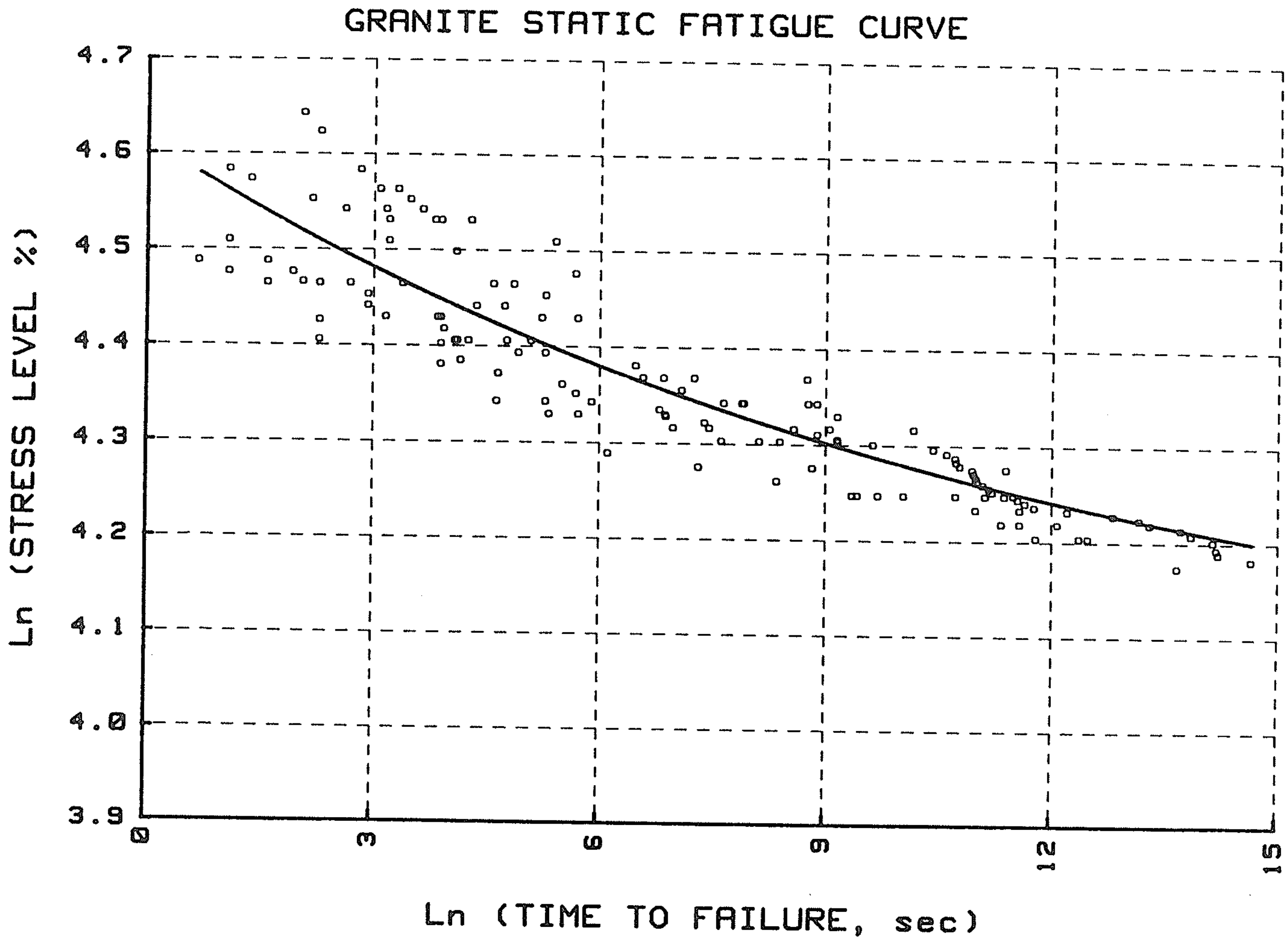

Figure 12: Static fatigue curve of Lac du Bonnet granite fitted to an exponential function (N = 138).

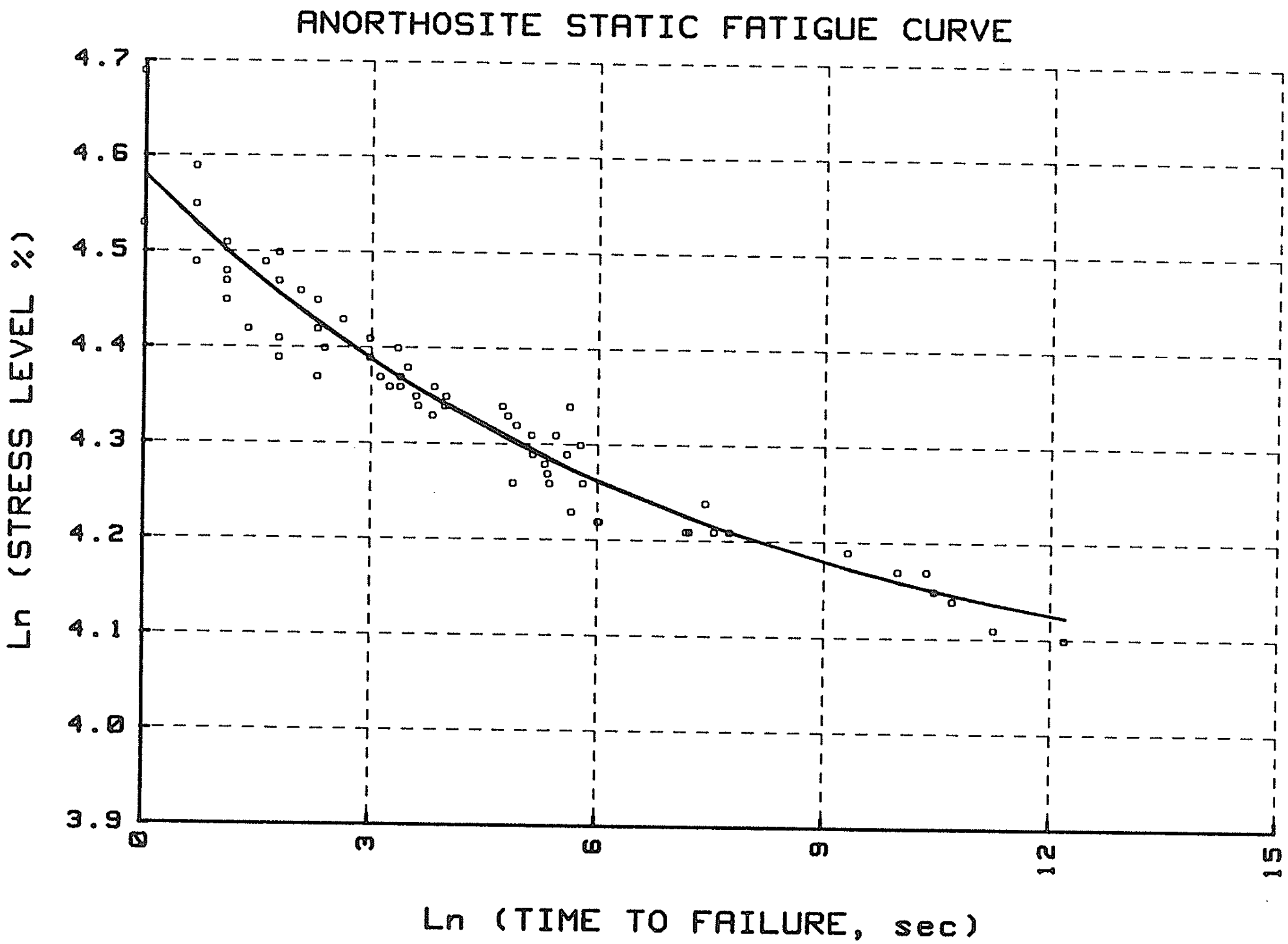

Figure 13: Static fatigue curve of Beebe anorthosite fitted to an exponential function (N = 64).

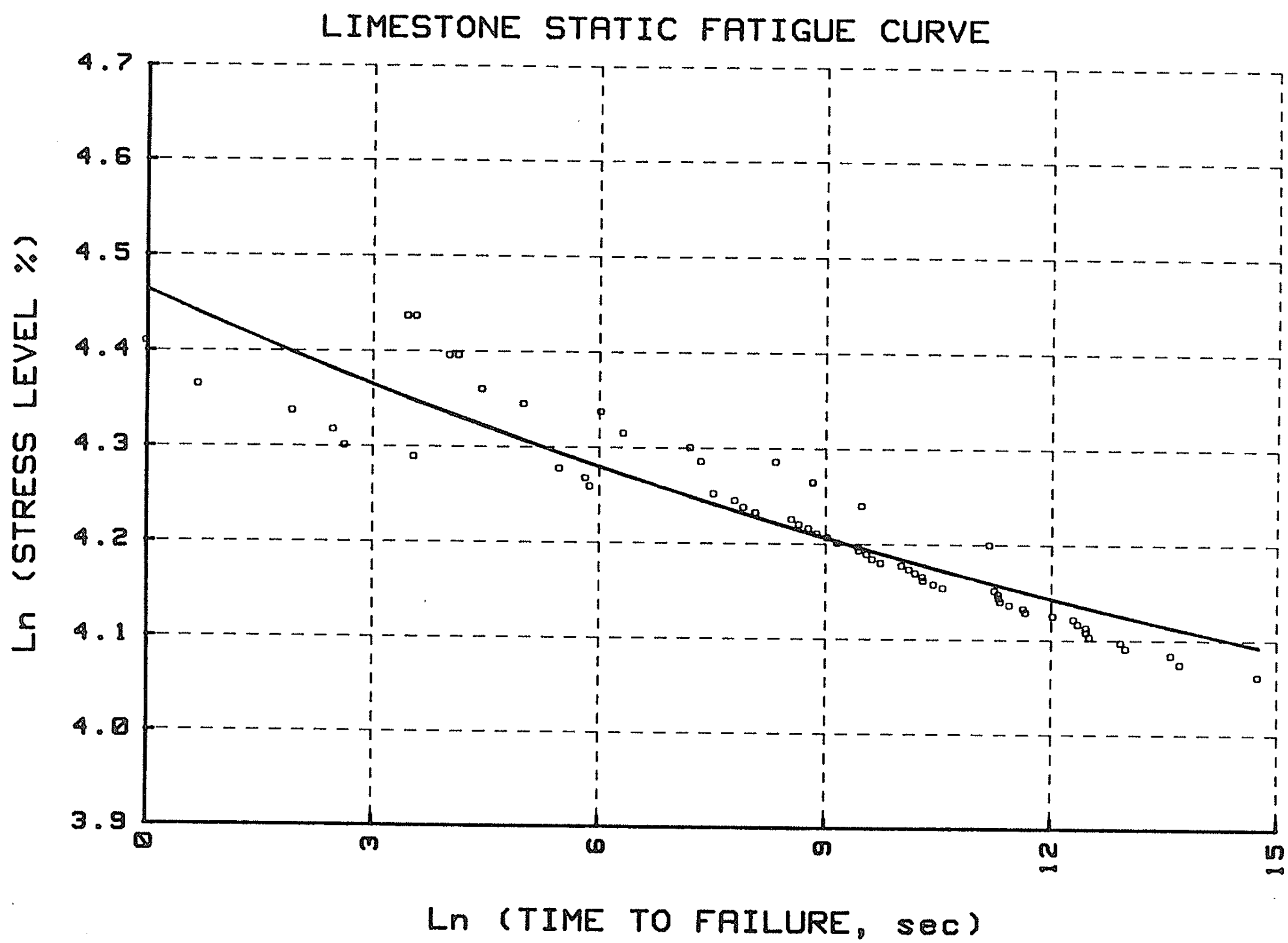

Figure 14: Static fatigue curve of Tyndall limestone fitted to an exponential function (N = 63).

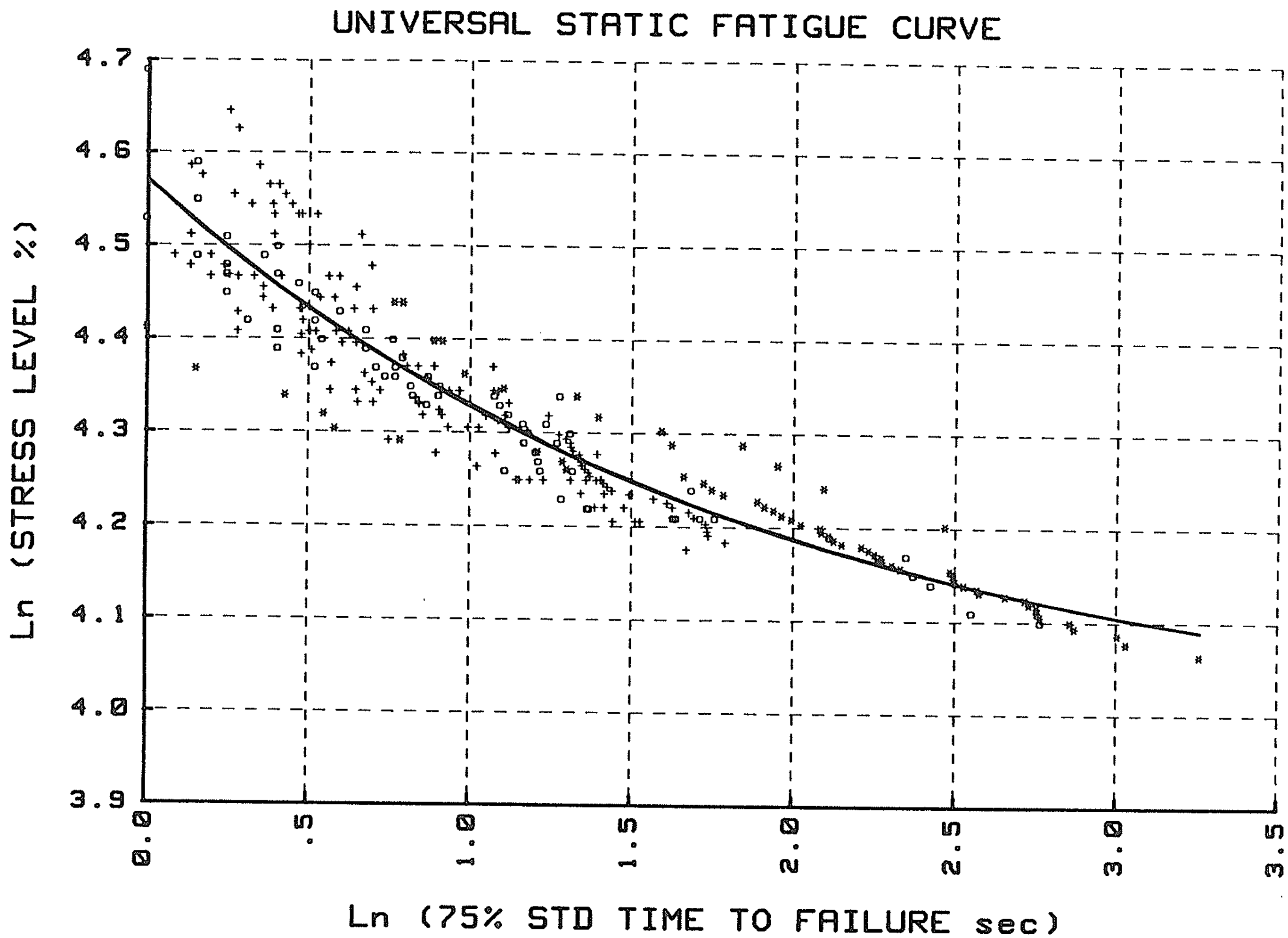

Figure 15: Universal static fatigue curve of all data (N = 265):
+ Lac du Bonnet granite
o Beebe anorthosite
* Tyndall limestone

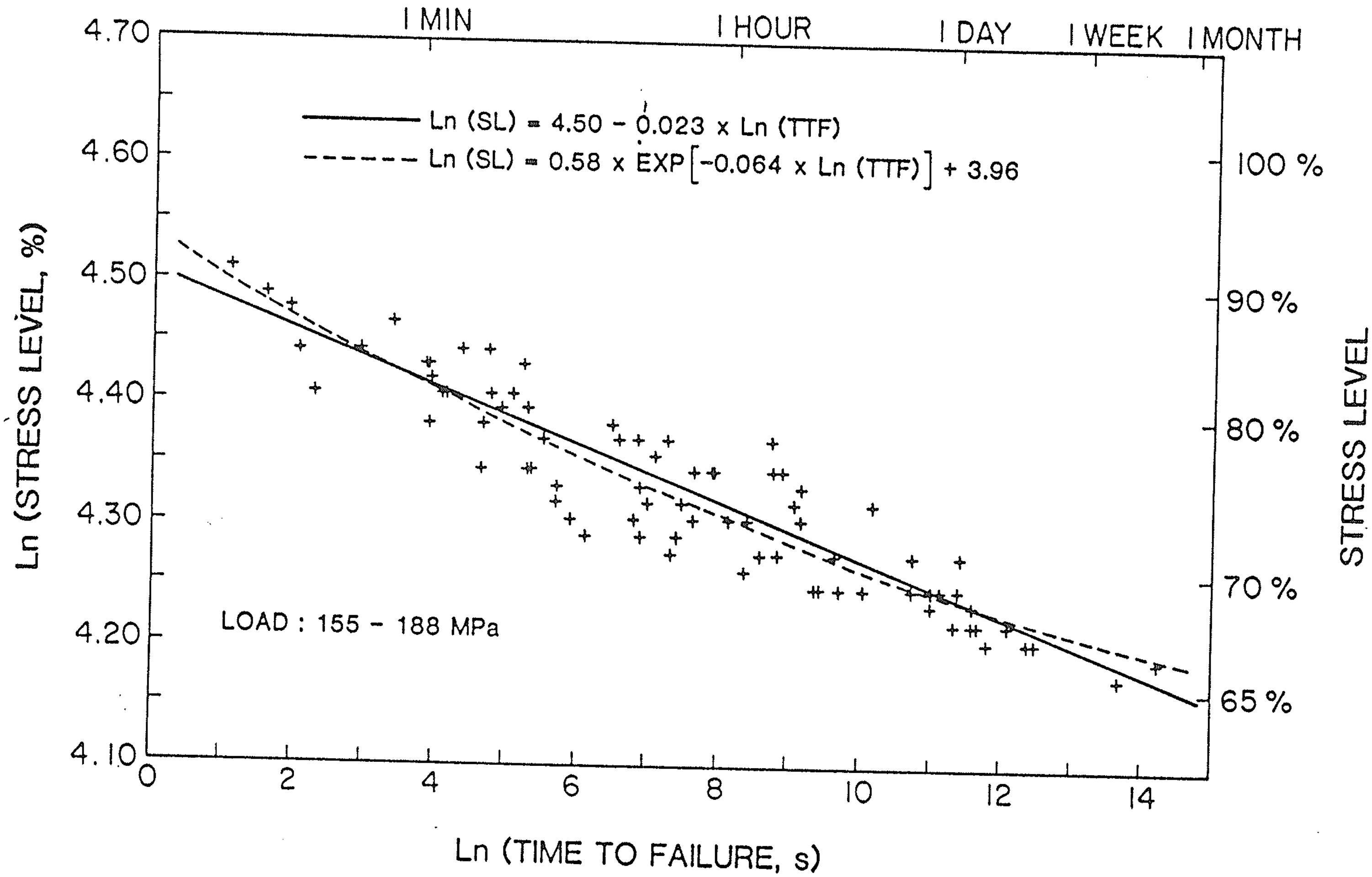

Figure 16: Static fatigue curves (linear and exponential) fitted to the lower stress range of Lac du Bonnet granite (N = 80).

APPENDIX B

TABLES

Table 1: UNIAXIAL COMPRESSIVE STRENGTH OF LAC DU BONNET GRANITE

RANK	STRENGTH (MPa)	RANK	STRENGTH (MPa)	RANK	STRENGTH (MPa)
1	195	26	219	51	237
2	195	27	221*	52	237
3	200	28	221	53	238*
4	201	29	222*	54	238
5	201	30	222	55	238
6	204	31	224*	56	239
7	209*	32	224	57	239
8	210	33	227	58	241*
9	211*	34	228*	59	241
10	211	35	228	60	241
11	211	36	229	61	242
12	211	37	229	62	243
13	211	38	229	63	243
14	212*	39	230*	64	243
15	212	40	230	65	246*
16	213	41	232	66	246
17	214	42	234	67	247
18	215*	43	234	68	247
19	215	44	236*	69	248
20	217	45	236	70	248
21	217	46	236		
22	219*	47	237		
23	219	48	237		
24	219	49	237		
25	219	50	237		

* Test results for N = 14 strength distribution.

N = 70

$\bar{x}$ = 226 MPa

s = 14.23 MPa (6.29%)

WEIBULL CONSTANTS

m = 18.33

ℓnkV = -99.89

N = 14

$\bar{x}$ = 225 MPa

s = 11.78 MPa (5.24%)

WEIBULL CONSTANTS

m = 19.38

ℓnkV = -105.44

Table 2: UNIAXIAL COMPRESSIVE STRENGTH OF BEEBE ANORTHOSITE

RANK	STRENGTH (MPa)
1	124
2	126
3	130
4	130
5	131
6	132
7	133
8	134
9	136
10	138
11	138
12	140
13	142
14	142
15	143
16	144
17	144
18	145
19	146
20	147
21	149
22	150
23	151
24	152
25	152
26	153
27	154
28	156
29	156
30	157
31	158
32	158

$N = 32$

$\bar{x} = 143$ MPa

$s = 9.97$ MPa (6.95%)

WEIBULL CONSTANTS

$m = 12.60$

$\ln kV = -62.88$

Table 3: UNIAXIAL COMPRESSIVE STRENGTH OF TYNDALL LIMESTONE

RANK	**STRENGTH (MPa)**
1	61
2	67
3	67
4	70
5	70
6	71
7	73
8	74
9	74
10	75
11	76
12	76
13	76
14	76
15	77
16	77
17	78
18	78
19	78
20	78
21	79
22	79
23	80
24	82
25	82
26	83
27	83
28	85

$N = 28$

$\bar{x} = 76$ MPa

$s = 5.46$ MPa (7.19%)

WEIBULL CONSTANTS

$m = 15.11$

$\ell nkV = -65.93$

Table 4: LAC DU BONNET GRANITE (TIME TO FAILURE)

215 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	x	110
2	x	107
3	8	104
4	10	102
5	17	98
6	22	96
7	28	96
8	33	95
9	39	94
10	46	93
11	50	93
12	74	93
13	227	91
14	296	88

x - premature failure

207 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	x	99
2	x	98
3	8	98
4	10	97
5	17	95
6	22	94
7	28	94
8	33	93
9	39	91
10	46	90
11	50	87
12	74	87
13	227	86
14	296	84

x - premature failure

(continued)

Table 4: LAC DU BONNET GRANITE (TIME TO FAILURE) (continued)

199 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	x	102
2	x	99
3	x	96
4	x	95
5	x	90
6	x	89
7	2	89
8	3	88
9	5	87
10	10	87
11	15	87
12	19	86
13	24	84
14	72	82

x - premature failure

188 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	x	90
2	x	89
3	3	88
4	5	87
5	19	86
6	48	85
7	50	85
8	52	84
9	60	82
10	62	81
11	165	79
12	200	79
13	1440	78
14	2720	77

x - premature failure

(continued)

Table 4: LAC DU BONNET GRANITE (TIME TO FAILURE) (continued)

177 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	x	85
2	7	84
3	116	83
4	190	82
5	660	81
6	730	80
7	960	80
8	1220	78
9	2130	77
10	2780	77
11	7340	77
12	9620	76
13	26420	75
14	90000	72

x - premature failure

166 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	8	87
2	10	84
3	50	82
4	65	80
5	107	79
6	250	78
7	300	78
8	370	77
9	910	76
10	990	76
11	1650	75
12	5400	75
13	7380	75
14	9600	74
15	15400	74
16	34320	74

(continued)

Table 4: LAC DU BONNET GRANITE (TIME TO FAILURE) (continued)

166 MPa (continued)

RANK	TTF (SEC)	STRESS LEVEL (%)
17	40980	73
18	45900	73
19	46560	73
20	48720	72
21	57600	72
22	58720	72
23	60300	71
24	61380	71
25	66360	71
26	71880	71
27	74880	70
28	99250	70
29	105360	70
30	116000	69
31	130980	69
32	203400	69
33	374220	69
34	528120	68
35	605760	68
36	917280	68
37	1050060	67
38	1406640	67
39	1468800	66
40	2332800	65

Table 4: LAC DU BONNET GRANITE (TIME TO FAILURE) (continued)

160 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	10	82
2	50	80
3	105	77
4	310	76
5	460	73
6	1530	72
7	4320	71
8	11700	70
9	16500	70
10	68400	70
11	88200	70
12	108000	69
13	178200	68
14	1504000	66

155 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	6420	79
2	6540	77
3	8700	75
4	9660	74
5	12540	70
6	46080	70
7	60240	69
8	84780	68
9	108000	68
10	134100	67
11	238140	67
12	266760	67
13	871200	65
14	*	63

* did not fail in 45 days

Table 5: BEEBE ANORTHOSITE (TIME TO FAILURE)

115 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	1	109
2	2	98
3	2	95
4	3	91
5	5	89
6	6	87
7	10	86
8	14	84
9	20	82
10	29	81
11	33	80
12	37	77
13	282	77
14	326	74

111 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	x	108
2	x	98
3	2	89
4	3	87
5	3	86
6	4	83
7	6	81
8	10	79
9	26	78
10	47	78
11	55	77
12	116	77
13	125	76
14	134	70

x - premature failure

(continued)

Table 5: BEEBE ANORTHOSITE (TIME TO FAILURE) (continued)

105 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	x	102
2	6	90
3	8	86
4	10	83
5	11	81
6	23	79
7	30	78
8	38	77
9	46	76
10	235	74
11	273	73
12	337	71
13	1707	69
14	1920	67

x - premature failure

95 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	1	93
2	3	88
3	6	82
4	20	81
5	30	79
6	54	77
7	140	75
8	171	74
9	174	73
10	204	72
11	211	72
12	217	71
13	290	69
14	412	68
15	420	68

(continued)

Table 5: BEEBE ANORTHOSITE (TIME TO FAILURE) (continued)

95 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
16	1326	67
17	1380	67
18	2345	67
19	11271	66
20	21807	65
21	31888	65
22	35242	63
23	44802	63
24	77653	61
25	198790	60

Table 6: TYNDALL LIMESTONE (TIME TO FAILURE)

57 (MPa)

RANK	TTF (SEC)	STRESS LEVEL (%)
1	32	85
2	36	85
3	56	81
4	63	81
5	86	78
6	149	77
7	415	77
8	560	75
9	1360	74
10	1570	73
11	4230	73
12	6920	71
13	13210	69
14	72030	67

50 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
1	1	82
2	2	79
3	7	77
4	12	75
5	14	74
6	35	73
7	240	72
8	339	71
9	360	71
10	1860	70
11	2460	70
12	2761	69
13	3242	69
14	5222	68
15	5760	68

(continued)

Table 6: TYNDALL LIMESTONE (TIME TO FAILURE) (continued)

50 MPa

RANK	TTF (SEC)	STRESS LEVEL (%)
16	6540	68
17	7320	67
18	8400	67
19	9600	67
20	12600	67
21	12780	66
22	14161	66
23	15241	66
24	17041	65
25	22500	65
26	24840	65
27	27000	65
28	29640	65
29	29940	64
30	34382	64
31	38940	64
32	77401	64
33	81180	64
34	81780	63
35	83401	63
36	94260	63
37	113401	63
38	117000	62
39	168240	62
40	222001	62
41	235200	61
42	261960	61
43	262500	61
44	273780	61
45	416520	60
46	444300	60
47	813600	59
48	916500	59
49	2577600	58

Table 7: MEAN INSTANTANEOUS COMPRESSIVE STRENGTH

Rock Type	N	Mean (C_o) MPa	Standard Deviation
LDB granite	70	226	14.23 MPa (6.29%)
Anorthosite	32	143	9.97 MPa (6.95%)
Limestone	28	76	5.46 MPa (7.19%)

Table 8: MEAN FAILURE TIMES

Granite

Load	x (sec)	s
215	60.9	88.46
207	64.6	91.26
199	11.1	19.12
188	312.7	676.48
177	10,160	24,045.33
166	251,219	504,229.69
160	141,610	396,132.29
155	410,083	1,026,272.22

Anorthosite

Load	x (sec)	s
115	55.0	106.53
111	38.1	50.25
105	331.8	638.97
95	17,154	42,362.12

Limestone

Load	x (sec)	s
57	7,194.1	19,032.12
50	154,568.7	402,101.73

Table 9: STATIC FATIGUE CURVE CONSTANTS

GRANITE

LOAD (MPa)	SLOPE (m)	y-INTERCEPT	r^2
215	-0.0382	4.69	0.88
207	-0.0319	4.62	0.97
199	-0.0300	4.51	0.96
188	-0.0217	4.51	0.92
177	-0.0189	4.50	0.92
170	-0.0264	4.50	0.95
166	-0.0191	4.47	0.97
160	-0.0171	4.39	0.95
155	-0.0357	4.63	0.87

ANORTHOSITE

LOAD (MPa)	SLOPE (m)	y-INTERCEPT	r^2
115	-0.0564	4.60	0.87
111	-0.0532	4.54	0.82
105	-0.0461	4.54	0.91
95	-0.0323	4.47	0.92

LIMESTONE

LOAD (MPa)	SLOPE (m)	y-INTERCEPT	r^2
57	-0.0288	4.51	0.95
50	-0.0220	4.39	0.97