A COST MODEL FOR DEFERRED STATE LIFE TEST PLANS WITH REPLACEMENT

BY

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A thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of

MASTER OF SCIENCE

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ABSTRACT

This research is concerned with the cost model of the deferred state life test plan, which is one of the lot-by-lot acceptance sampling plans by attributes. In most of the sampling plans, the decision to accept or reject a submitted lot depends only on the sampling test results of the lot concerned. Other information will not be considered. Deferred state life test plan is a sampling plan which uses subsequent lots information for making decisions to accept or reject the current lot. The advantages of the deferred state life test plan are in reducing both the sampling test time and the resulting sampling cost. Also, it provides an indicator of quality degradation.

The objective of this research is to evaluate the total test cost of using the deferred state life test plan to see whether the use of this life test plan can reduce the total test cost of the sampling test. In order to calculate the expected total test cost of the deferred state life test plan, a cost model is developed for the deferred state life plan with replacement, i.e. the failed items during the life test will be replaced by new ones drawn from the remainder of the same lot.

A cost comparison is made between a deferred state life test plan and a military standard sampling plan such that both plans can provide the same producer's and consumer's protection. Based on some assumed input values, such as the cost of testing an item, the cost of conducting the life test per unit time, etc., the expected total test cost of conducting the deferred state life test plan appears to be less than the expected test cost of conducting the military standard sampling plan. As a result, it is believed that the use of the deferred state life test plan can reduce the total test cost under some situations, such as the example discussed in this dissertation.

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CHAPTER I

INTRODUCTION

Generally, inspection plays an important role in appraising the quality of incoming and outgoing items of a company. The results of the inspection may be reported to the related departments so that corrective actions may be taken whenever it is necessary. A company may form an independent inspection department or the inspection can be done by one or more existing departments inside the company. In general, there are three alternatives to sentence a submitted lot:

(1) inspect all items in the lot and return the defective items to the producer;

(2) use acceptance sampling to make the decision of acceptance or rejection of the entire lot; and

(3) accept the lot without inspection.

Inspecting all items is usually not economical since it is likely to be expensive and time consuming. Also, the inspection costs may be higher than the cost of accepting defective items. Unless the cost of passing defective items is very high, 100% inspection is not desirable. On the other hand, accepting the lot without inspection is risky because a large amount of defective items may be accepted. Unless it is certain that the submitted lots have high quality or that the cost of accepting defective items is very low, inspection is recommended. So acceptance sampling is the most common alternative to sentence lots and the most economical way to obtain the information about the quality of a product in a reasonable time.

Acceptance Sampling

In some situations inspection is possible only after a process has been completed.

For example, a company receives a shipment of raw materials which will be converted into finished products. In this situation acceptance sampling is often used under one of the following conditions:

(1) When it is very expensive to inspect all items, a sampling test may reduce the inspection cost.

(2) When the items will be destroyed after inspection such as the life testing of light bulbs, sampling tests are needed. If all the items are inspected, they will all be destroyed and no items can be used.

(3) When inspections are done manually, a high percentage of defective items may be passed by 100% inspection due to fatigue or boredom.

(4) When it is impossible to inspect all items due to limited time and inspection resources, sampling tests should be used.

Lot-by-lot acceptance sampling by attributes is one of the common types of acceptance sampling. In this dissertation, all sampling plans discussed will be lot-by-lot acceptance sampling by attributes. When using this type of sampling, a predetermined number of items is taken from each lot and inspected by attributes. Attributes are quality characteristic which are shown on a " go-not-go " basis and each item in the lot can be classified by attributes such as conforming or nonconforming, good or defective, pass or fail, etc. Based on the information on the inspected sample, a decision of acceptance or rejection of the entire lot would be made. This decision is referred to as lot sentencing. For example, if the number of defective items dose not exceed a predetermined number, the lot is accepted; otherwise, it is rejected. Each lot is sampled and either accepted or rejected. Accepted lots are placed into the production system of the company and rejected lots are returned to the supplier or subjected to some other remedial action.

Acceptance sampling is one of the major areas of statistical quality control. With an acceptance sampling plan, a random sample is taken from the submitted lot. Quality evaluation of such a sampling plan requires the use of probability theory and statistical methods. Several types of probability distributions used in this dissertation are presented in Appendix A.

Some Properties of Acceptance Sampling

(1) All items in a lot should come from the same source; otherwise, the effectiveness of the sampling plan will be affected (Besterfield, 1986).

(2) Large lot size is preferable. When the lot size increases, the sample size does not increase as rapidly as the lot size in most sampling plans. As a result, inspection cost is reduced (Besterfield, 1986).

(3) Random sampling must be used to avoid biases when selecting items from the entire lot. Under random sampling, all items in the lot should have the same chance to be chosen in the sample (Montogomery, 1985).

(4) The purpose of acceptance sampling is lot sentencing, not quality estimation, i.e. it is not used to control or improve the quality of products (Montogomery, 1985).

Advantages and Disadvantages of Sampling

The advantages of sampling tests when compared with 100% inspection are the following:

(1) The inspection cost is lower in sampling since fewer items are inspected.

(2) The costs of training inspectors and keeping records are lower in sampling since fewer inspectors and fewer records are needed.

(3) When the items are destroyed after inspection, cost will be reduced by using sampling.

(4) Since fewer items are handled by using sampling, the damage cost of handling will

decrease.

(5) The aggregate inspection error may be reduced by using sampling since fewer items are inspected.

The disadvantages of sampling tests when compared with 100% inspection are the following:

(1) Since not all the items are inspected, there are risks of rejecting good lots and of accepting defective lots.

(2) The cost of planning is higher in sampling testing than 100% inspection.

(3) Not all information about the lot for quality evaluation can be obtained by using sampling.

General Assumptions

Lot-by-lot acceptance sampling by attributes is one of the common types of acceptance sampling. In this dissertation, all sampling plans discussed will be lot-by-lot acceptance sampling by attributes. When a company receives a shipment of goods, each item is packed inside a lot. With lot-by-lot acceptance sampling by attributes, a sample of a predetermined number of items is taken from each lot. Each item in the sample is inspected by attributes. That is, a quality characteristic of the items in the sample is inspected. After inspection, each item can be classified either as a good item when it conforms to the required standard or as a defective item when it does not. Based on the information of this inspected sample, a decision of acceptance or rejection of the entire lot will be made.

The operating characteristic (OC) curve is a means to evaluate a sampling plan. The OC curves are discussed in Appendix B. There are two types of OC curve: type A OC curve and type B OC curve. If the lot is an isolated lot with finite size, a type A OC curve is used. But if the lots are taken from a steady flow of items which are produced by a single source, a type B OC curve should be used. With lot-by-lot acceptance sampling by attributes, a type B OC curve may also be used.

A sampled item may also be tested using a life test plan. For example, a sample of a predetermined number of items is taken from a submitted lot. Each item, for example a light bulb, is allowed to operate until it fails or a predetermined total test time is reached. Then the number of failed items is counted. Based on this information, a decision regarding disposition of the submitted lot could be made. The failure distribution for each item is assumed to be an exponential distribution and each failing item fails during its useful life period, i.e. it dose not fail due to wearout effects. The probability that an item will fail within the total test time can be evaluated. Then the probability of acceptance of a submitted lot can also be evaluated.

Producer's and Consumer's Risk

Since not all items in a submitted lot are inspected when using acceptance sampling, there exists the risk of rejecting good lots and of accepting defective lots. That is, a lot may be classified as non-acceptable when, in fact, it meets the quality criterion and a lot may be accepted when, in fact, it does not meet the quality criterion. If all items are inspected and there is no error in the inspection, the ideal OC curve looks like the curve shown in Figure 1.1. The OC curve is an evaluation technique to show the discriminatory power of a sample plan and is discussed in Appendix B. For example, in the situation of Figure 1.1, the submitted lot will be accepted if there are 2% or less defective items in the lot and the lot will be rejected if there are more than 2% defective items in the lot. Therefore, there is no risk of accepting defective lots and rejecting good lots.

When sampling is used, two types of risks are encountered. The first is the producer's risk (α), which is the probability of rejecting a good lot. The second is the consumer's risk (β), which is the probability of accepting a defective lot. There are two fraction defective values related to these two risks. Denote the probability of accepting a lot as P_a. The first fraction defective value is denoted by p_{AQL} - if the fraction defective of a submitted lot is p_{AQL}, the probability of acceptance of the lot will be $P_a = 1 - \alpha$. When we set the producer's risk to be α, the corresponding fraction defective is p_{AQL} and is called the acceptable quality level (AQL). The AQL shows the poorest level or highest fraction defective that is acceptable as a process average. The second value is denoted by p_{LQL} - if the fraction defective of a submitted lot is p_{LQL}, the probability of acceptance of the lot will be $P_a = \beta$. When we set the consumer's risk to be β, the corresponding fraction defective is p_{LQL} and is called the limiting quality level (LQL). The LQL shows the poorest level or highest fraction defective that is acceptable as a lot average.

An example is given in Figure 1.2, the probability of acceptance of a lot with x fraction defective is 0.95. Although the lot is good, it may be rejected with a probability of 0.05. If we set the producer's risk to be 0.05, the fraction defective at x is the AQL. On the other hand, if the fraction defective of a submitted lot is y, the lot will be classified as a defective lot. Although the lot is defective, it may be accepted with a probability of 0.1. If we set the consumer's risk to be 0.1, the fraction defective at y is the LQL.

Objectives

In most of the sampling plans, the decision to accept or reject a submitted lot depends only on the sampling test results of the lot concerned. Other information will not be considered. For example, a sample of n items is taken from a submitted lot of size N and placed on test. If the number of defective items in the sample is less than the predetermined maximum allowable number of defective items, the entire lot is accepted; otherwise, it is rejected. Using this procedure, it is possible that a lot will be rejected when, for example, the last ten lots all have been accepted. For this observation, there are two possibilities. The first is that the results of the sampling test indicate the degradation of quality in the manufacturing process. The second is that the results of the sampling test are not representative of the entire lot since the sample items are taken randomly and an inordinate number of defective items has been selected. Similarly, a submitted lot may be accepted when, for example, the last ten lots have been rejected. The results of the sampling test may indicate an upgrading of quality in the manufacturing process or that the sample does not represent the entire lot.

On the other hand, some acceptance sampling plans will consider not only the sampling results of the current lot but also information about other lots for making decision to accept or reject the current lot. They may use the information on the past lots, future lots, or both past and future lots. A deferred state attribute acceptance plan is a sampling plan which uses information about subsequent lots for making the decision to accept or reject the current lot. The advantages of this sampling plan are in reduction of the total sampling test time and of the resulting sampling cost. In this dissertation, cost evaluation of the deferred state sampling plan will be discussed. The results can be used to select a lot-by-lot attribute acceptance sampling plan which will reduce the overall cost of sampling tests.

Figure 1.1. An ideal OC curve.

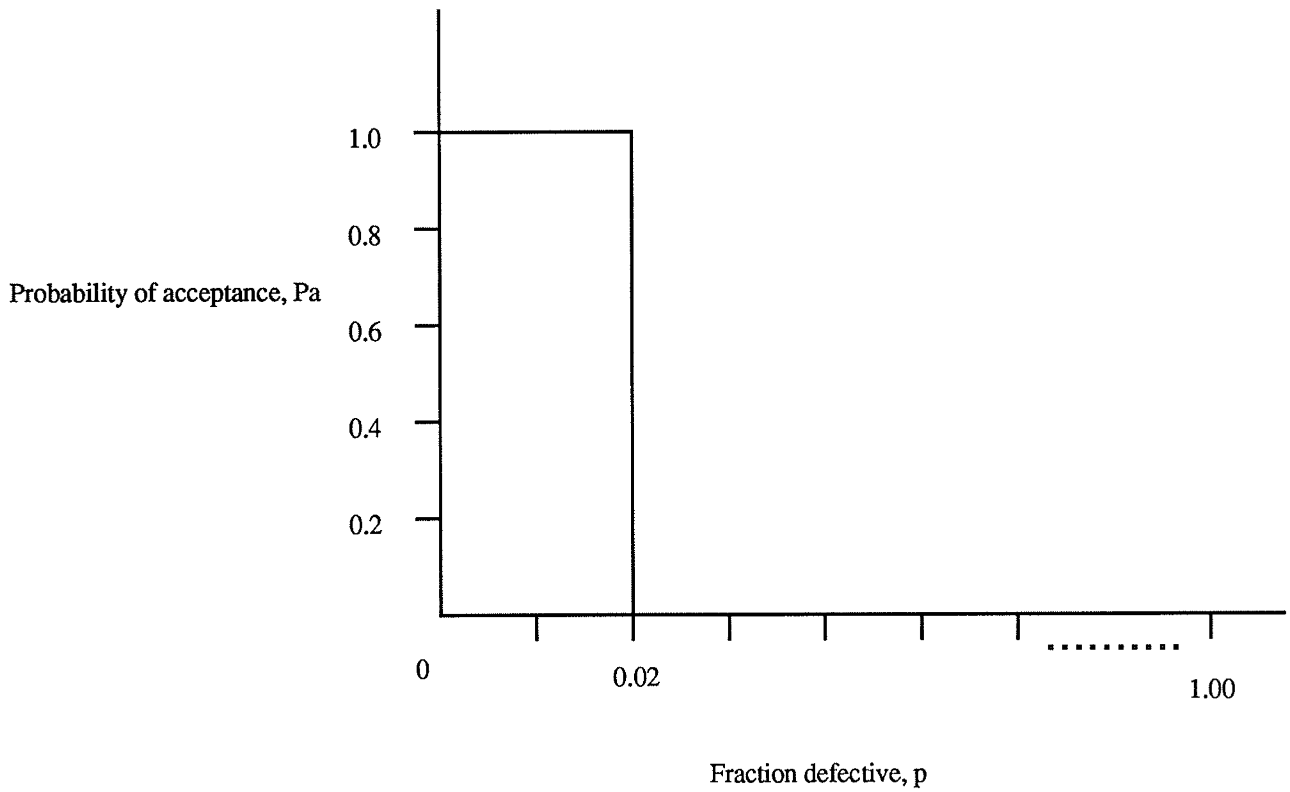
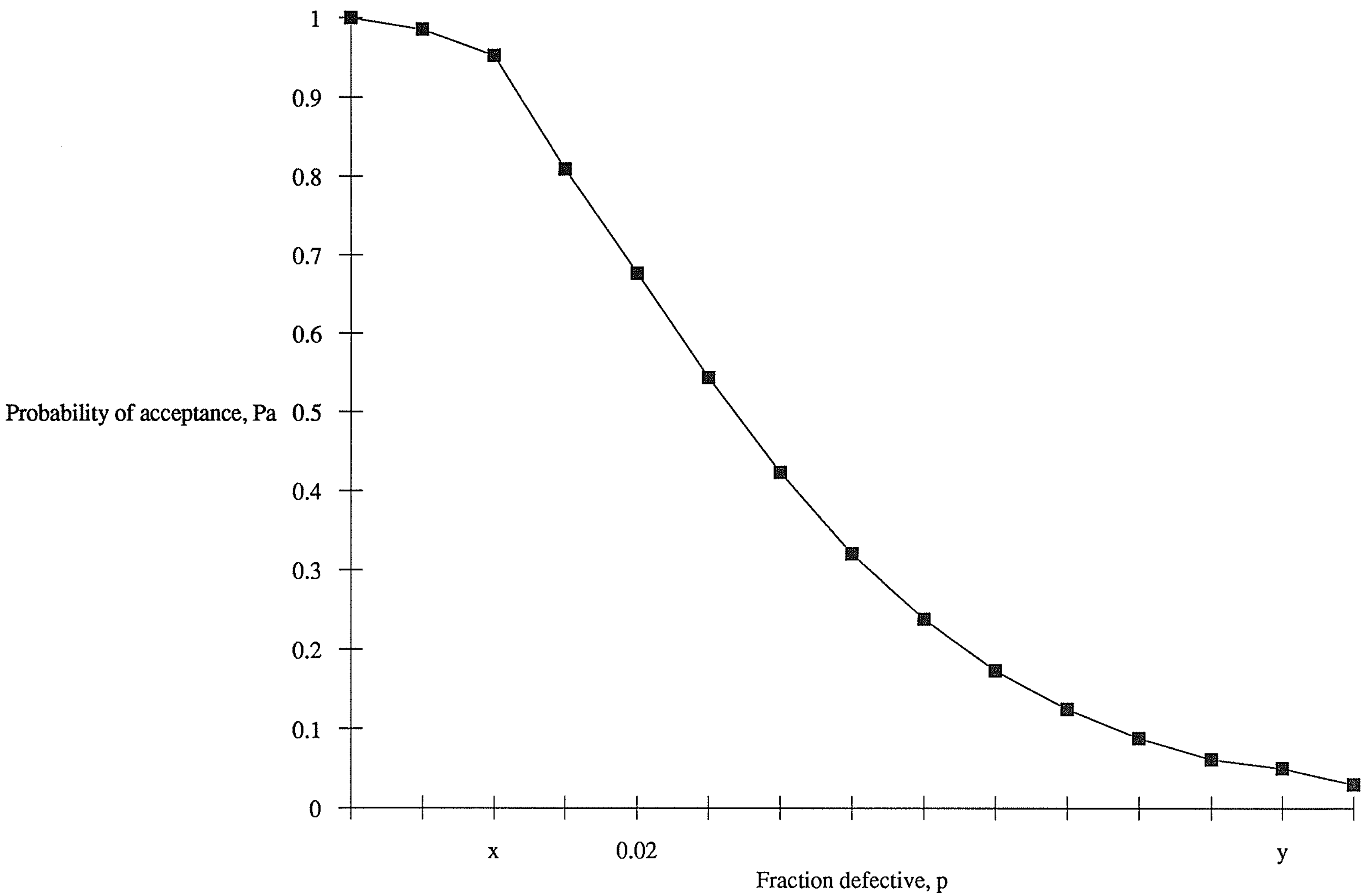


Figure 1.2. A typical OC curve.



CHAPTER II

LITERATURE REVIEW

Lot-by-lot acceptance sampling by attributes is one of the common types of acceptance sampling. With such a sampling plan, a sample of a predetermined number of items is taken from each lot and a quality characteristic of the items in the sample is inspected. Based on the information from this sample, the lot may be accepted if it conforms to the required standard or rejected if it does not do so. There are three basic types of sampling plans: single sampling, double sampling, and multiple sampling. We will first discuss these sampling plans in detail and then other common types of acceptance sampling by attributes. In order to understand the characteristics of each plan, some evaluation techniques for acceptance sampling are presented in Appendix B.

Single Sampling Plan

In the single sampling plan, the predetermined numbers are

N = the lot size,

n = the sample size, and

c = the acceptance number.

When using the single sampling plan by attributes, one sample of size n is taken from the lot of size N and inspected. If there are c or less defective items in the sample, the lot is accepted. If there are more than c defective items in the sample, the lot is rejected. In other words, the acceptance or rejection of the lot depends on the inspection results of a single sample.

For example, consider a steady flow of items which are produced by a single source. Lots of size $N = 5000$ are taken from the flow and samples of size n are inspected.

Since the lots are selected from a steady flow, an infinite population is assumed. For this situation, the probability of accepting the lot should be calculated from the binomial probability distribution. However, it is much simpler to use the Poisson probability distribution, which is a good approximation to the binomial probability distribution (for $n \geq 20$, $p \leq 0.1$ and $np \leq 5$). Therefore the Poisson probability distribution is used to calculate the probability of acceptance of a lot for this sampling plan.

In our example, a sample of size $n = 100$ is selected from each lot of size $N = 5000$ and the acceptance number, c, is equal to two; that is, the lot will be accepted if there are at most two defective items in the sample. So the probability of acceptance of the lot is equal to the probability of at most two defective items in the sample, denoted $P_a = P_{2 \text{ or less}}$. When plotting the OC curve, different fraction defective p values are assumed and then the np values are calculated. Finally, the P_a values can be found from the Poisson table by using the appropriate np and c values. In our example, we assume p is equal to 0.05, then

$$np = 100 \times 0.05 = 5.0$$

and

$$P_a = P_{2 \text{ or less}} = 0.125.$$

Some P_a values are calculated in Table 2.1. The OC curve for this single sampling plan is plotted in Figure 2.1. When the OC curve is obtained, we can use the curve to find the probability of acceptance of a lot in which the fraction defective, p, is known. For example, if the fraction defective of a submitted lot is 0.035, the probability of acceptance will be approximately 0.32 from reading Figure 2.1. Some publications which discuss various versions of the single sampling plan are listed in Table 2.5.

Double Sampling Plan

The double sampling plan is more complicated than the single sampling plan because a second sample may be required. Generally, the sample sizes of double sampling plan are smaller and the total number of inspections may be reduced. As a result, the total inspection cost is reduced. The predetermined numbers are

N = lot size,

n_1 = sample size for the first sample,

c_1 = acceptance number for the first sample,

r_1 = rejection number for the first sample,

n_2 = sample size for the second sample,

c_2 = acceptance number for both samples, and

r_2 = rejection number for both samples.

When using a double sampling plan by attributes, a first sample of size n_1 is taken from the lot of size N and inspected. One of the following three decisions is made after inspection:

(1) If c_1 or fewer defective items are found in the first sample, accept the lot;

(2) If r_1 or more defective items are found in the first sample, reject the lot; and

(3) If more than c_1 and fewer than r_1 defective items are found in the first sample, a second sample of size n_2 is required.

If a second sample is required, n_2 items are taken from the same lot which has $N - n_1$ items remaining. One of the following two decisions is made after inspection:

(1) If c_2 or fewer defective items are found in both samples, accept the lot; and

(2) If r_2 or more defective items are found in both samples, reject the lot.

In other words, the decision of acceptance or rejection of the lot is based on the inspection results from both samples when a second sample is required.

In the double sampling plan, two curves are required for the OC curves. The first one is for the probability of acceptance of a lot after inspecting the first sample if the second sample is not required. If a second sample is needed, the second curve is for the probability of acceptance of that lot after inspecting the second sample. The formation of the OC curves may be illustrated by the following example. In a lot of size N = 5000, the first sample of size $n_1 = 80$ is taken and inspected. If there is 1 defective or none in the sample, the lot is accepted, i.e. $c_1 = 1$. If there are 3 or more defective items in the sample, the lot is rejected, i.e. $r_1 = 3$. When there are 2 defective items in the sample, a second sample of size $n_2 = 100$ is required and taken from the same lot which has $N - n_1$ items remaining. If there are 3 or fewer defective items in both samples, the lot is accepted; otherwise, it is rejected, i.e. $c_2 = 3$ and $r_2 = 4$. In determining the first curve, the probability of acceptance of the lot after inspecting the first sample, $(P_a)_{n_1}$, is equal to the probability of having 1 or less defective in the first sample, $(P_{1 \text{ or less}})_{n_1}$, i.e.

$$(P_a)_{n_1} = (P_{1 \text{ or less}})_{n_1}. \tag{2.1}$$

Some $(P_a)_{n_1}$ values are calculated in Table 2.2. When determining the second curve, the probability of acceptance of the lot after inspecting the second sample, $(P_a)_{n_1+n_2}$, is equal to the sum of the probability of acceptance of the lot on the first sample, $(P_a)_{n_1}$, and the probability of acceptance of the lot on the second sample, $(P_a)_{n_2}$, i.e. $(P_a)_{n_1+n_2} = (P_a)_{n_1} + (P_a)_{n_2}$. By using formula (2.1), we get

$$(P_a)_{n_1+n_2} = (P_{1 \text{ or less}})_{n_1} + (P_a)_{n_2}. \tag{2.2}$$

But it is obvious that the lot is accepted in the second sample if there are 2 defective items in the first sample and 1 defective or none in the second sample, therefore

$$(P_a)_{n_2} = (P_2)_{n_1}(P_{1 \text{ or less}})_{n_2}. \tag{2.3}$$

Substituting formula (2.3) into formula (2.2), we have

$$(P_a)_{n_1+n_2} = (P_{1 \text{ or less}})_{n_1} + (P_2)_{n_1}(P_{1 \text{ or less}})_{n_2}. \tag{2.4}$$

Some values of $(P_a)_{n_1+n_2}$ are calculated in Table 2.2. The OC curves for this double sampling plan are plotted in Figure 2.2. When the OC curves are obtained, we can use the curves to find the probability of acceptance of a submitted lot in which the fraction defective is known. For example, if the fraction defective of a submitted lot is 0.035, we read from Figure 2.2 that the probability of acceptance of the lot after inspecting the first sample is 0.23, and the probability of acceptance of the lot after inspecting the second sample is 0.26. Some publications which discuss various versions of the double sampling plan are listed in Table 2.5.

Multiple Sampling Plan

Multiple sampling plans are extensions of the double sampling plan. Instead of requiring two samples in a double sampling plan, a multiple sampling plan may require three or more samples with smaller sample sizes. The technique is similar to that used in the double sampling plan. The following illustration is a multiple sampling plan which requires at most three samples. The predetermined numbers for this plan are

N = lot size,

n_1 = sample size for the first sample,

c_1 = acceptance number for the first sample,

r_1 = rejection number for the first sample,

n_2 = sample size for the second sample,

c_2 = acceptance number for both first and second samples,

r_2 = rejection number for both first and second samples,

n_3 = sample size for the third sample,

c_3 = acceptance number for all three samples, and

r_3 = rejection number for all three samples.

When using multiple sampling plan by attributes, a first sample of size n_1 is taken from the lot of size N and inspected. One of the following three decisions is made after inspection:

(1) If c_1 or fewer defective items are found in the first sample, accept the lot;

(2) If r_1 or more defective items are found in the first sample, reject the lot; and

(3) If more than c_1 and fewer than r_1 defective items are found in the first sample, a second sample of size n_2 is required.

If a second sample is required, n_2 items are taken from the same lot which has $N - n_1$ items remaining. One of the following three decisions is made after inspection:

(1) If c_2 or fewer defective items are found in both first and second samples, accept the lot;

(2) If r_2 or more defective items are found in both first and second samples, reject the lot; and

(3) If more than c_2 and fewer than r_2 defective items are found in both first and second samples, a third sample of size n_3 is required.

If a third sample is required, n_3 items are taken from the same lot which has $N - n_1 - n_2$ items remaining. One of the following two decisions is made after inspection:

(1) If c_3 or fewer defective items are found in all three samples, accept the lot; and

(2) If r_3 or more defective items are found in all three samples, reject the lot.

In other words, the decision of acceptance or rejection of the lot is based on the inspection results from all three samples when three samples are required.

As a matter of fact, it is possible to make the probability of acceptance of a specific lot under a single sampling plan equal to the probability of acceptance of that lot under an appropriate double or multiple sampling plan. In other words, the selection of a sampling plan does not depend on it effectiveness since all three types of sampling plans can result in the same effectiveness when appropriate predetermined numbers are selected. The effectiveness is the ability to reduce both the producer's and consumer's risk. So, when selecting the type of a sampling plan, one should consider other factors such as cost of sampling, psychological effect and so on.

Generally, for the same degree of effectiveness, the total number of inspections in a single sampling plan is more than that in a double sampling plan since the decision can sometimes be made in the first sample when using a double sampling plan; therefore, no second sample is needed and the total number of inspections is reduced. Similarly, in a multiple sampling plan, the total number of inspections is usually less than that in a double sampling plan since the decision can be made in the first few samples. As the total number of inspections decreases, the inspection cost decreases. So, on the average, a multiple sampling plan has a lower inspection cost than the double or single sampling plans. On the other hand, a multiple sampling plan is more complicated than the others so that other costs such as training people and recording results are higher. Since the sampling cost is the sum of all these costs, we should consider the overall sampling cost when selecting the type of a sampling plan.

The other factor is the psychological effect, a feeling of having a second chance in the double sampling plan instead of having only one chance in single sampling plan, which we should also consider when selecting the type of a sampling plan. In a single sampling plan, only one sample is taken from a lot and the decision to accept or reject the lot is based on only one sample inspection result. But in a double sampling plan, if the results of the first sample are marginal between the acceptance and rejection decisions, a second chance is given by allowing a second sample. In a multiple sampling plan, multiple chances are given thus improving the psychological effect over that of a double sampling plan.

The formation of OC curves for a multiple sampling plan is an extension of the formation of OC curves for a double sampling plan. Instead of requiring two curves in a double sampling plan, multiple sampling plans require three or more curves to construct OC curves. For example, if there is a multiple sampling plan which involves m samples, $m \geq 3$, the probability of acceptance of the lot after the k^{th} sample has been inspected is

$$(P_a)_{n_1+n_2+\ldots+n_k} = (P_a)_{n_1} + (P_a)_{n_2} + \ldots + (P_a)_{n_k}; \tag{2.5}$$

here $(P_a)_{n_1+n_2+\ldots+n_k}$ = probability of acceptance of a lot after the k^{th} sample is inspected, i.e. after $n_1 + n_2 + \ldots + n_k$ items have been inspected,

$(P_a)_{n_j}$ = probability of acceptance of a lot on the j^{th} sample of size n_j

$$
= \sum_{\substack{c_1<d_1<r_1 \\ c_2<d_1+d_2<r_2 \\ \cdot \\ \cdot \\ \cdot \\ c_{k-1}<\sum_{i=1}^{k-1} d_i<r_{k-1} \\ \sum_{i=1}^{k} d_i \le c_k}} \prod_{i=1}^{j} (P_{d_i})_{n_i} \quad , 1 \le k \le m,
$$

and d_i ($i \le j$) is the number of defective items found in i^{th} sample. By substituting all values of k in formula (2.5), m curves are obtained. We can use these curves to find the probability of acceptance of a specific lot by using a technique similar to the one we use in a double sampling plan. Some publications which discuss various versions of the multiple sampling plan are listed in Table 2.5.

Sequential Sampling Plan

Sequential sampling plan are acceptance sampling plans by attributes for destructive or costly inspection. They were developed by Wald (1947). In this sampling plan, only one item at a time is taken from the lot and inspected. After inspection, we compare the cumulative number of defective items to the acceptance number and rejection number. For this sampling plan, the acceptance number and rejection number are not constant. They are given by the following formulas (Wald, 1973):

$$a_m = \frac{\log\left(\frac{\beta}{1-\alpha}\right)}{\log\left(\frac{p_1}{p_0}\right) - \log\left(\frac{1-p_1}{1-p_0}\right)} + m\left[\frac{\log\left(\frac{1-p_0}{1-p_1}\right)}{\log\left(\frac{p_1}{p_0}\right) - \log\left(\frac{1-p_1}{1-p_0}\right)}\right], \text{ and} \tag{2.6}$$

$$r_m = \frac{\log\left(\frac{1-\beta}{\alpha}\right)}{\log\left(\frac{p_1}{p_0}\right) - \log\left(\frac{1-p_1}{1-p_0}\right)} + m\left[\frac{\log\left(\frac{1-p_0}{1-p_1}\right)}{\log\left(\frac{p_1}{p_0}\right) - \log\left(\frac{1-p_1}{1-p_0}\right)}\right]; \tag{2.7}$$

here m = number of items inspected,

a_m = acceptance number when m items are inspected,

r_m = rejection number when m items are inspected,

α = producer's risk,

β = consumer's risk,

p_0 = fraction defective at the acceptable quality level, AQL, and

p_1 = fraction defective at the limiting quality level, LQL.

If the cumulative number of defective items is less than the acceptance number, accept the lot. If the number of cumulative defective items is greater than the rejection number, reject the lot. Otherwise, continue the inspection until a decision of accepting or rejecting the lot is made. Figure 2.3 illustrates this sampling plan. When using a graphical method, we plot the cumulated defective items curve after each inspection. If the cumulated defective items curve is within the "continue sampling" region, one continues the inspection by taking another item until the curve goes outside this region. The lot is accepted if the cumulated defective items curve intersects with the acceptance number line; and the lot is rejected if the cumulated defective items curve intersects the rejection number line. Theoretically, the sequential sampling plan can continue until all the items in the lot are inspected but, in practice, this sampling plan is truncated when the number of inspected items is equal to three times the sample size of the corresponding single sampling plan. Generally, this sampling plan reduces the number of items inspected, so the inspection cost will decrease for destructive or costly inspection. Detailed information can be found in Wald (1973); some other publications which discuss various versions of the sequential sampling plan are listed in Table 2.5.

Military Standard 105 (MIL-STD-105)

In 1949, the Statistical Research Group of Columbia University proposed an acceptance sampling plan for lot-by-lot inspection by attributes called JAN-STD-105 (1949). After revisions, MIL-STD-105A (1950), MIL-STD-105B (1958), MIL-STD-105C (1961), and MIL-STD-105D (1963) were published. In 1989, the latest version was published and called MIL-STD-105E (1989). Some publications discussing various versions of the military standard are listed in Table 2.5. Generally, this standard is used when the lots are taken from a steady flow of items which are produced by a source, but, after some adjustments it can also be used for isolated lots. It is the most common type of lot-by-lot acceptance sampling plan for attribute inspection and it is extensively used in industry for acceptance sampling. This standard is applicable to inspection of incoming materials, products in process, end products, and so on. The aim of this standard is to maintain a satisfactory level of average outgoing quality.

Three types of sampling plans are included in this standard. They are the single, double, and multiple sampling plans. For each type of sampling plan, it provides three types of inspection: normal, tightened, and reduced. Normal inspection is used to inspect the lots in the beginning of inspection. After a certain number of inspections, if the quality is not satisfactory, the tightened inspection is used. On the other hand, if the quality is good, the reduced inspection is used. The use of the different types of inspection may be switched from one to another following the criteria stated in the standard. Also, the procedures for using this sampling plan are given in the standard.

An example of OC curves for normal, tightened and reduced inspection is given in Figure 2.4. If a lot is submitted for inspection, the probability of acceptance of the lot under reduced inspection is the highest; and the probability of acceptance of the lot under tightened inspection is the lowest among the three types of inspection. In other words, the risk of accepting a defective lot will be the highest when using reduced inspection. Besides, reduced inspection has the smallest sample size and the tightened inspection has the largest sample size so that the inspection cost will be reduced when using reduced inspection.

Truncated Life Test Plan

Epstein (1954) discussed some life test plans which he called truncated life tests. Before these sampling plans start, the sample size, n, the rejection number, r, and the truncated test time, T, beyond which the test will not be run, are determined. Then n items are selected from a submitted lot and simultaneously subjected to life test. If we let x_r denoted a random variable of the time at which the r^{th} failure occurs and T the predetermined truncated test time, the sampling test will be terminated at $\min(x_r, T)$. If the test is terminated at time T, i.e. T less than x_r, the submitted lot is accepted; otherwise, it is rejected. Although truncated life test plans can be used for any life distribution, Epstein considered the case that the life distribution of the tested items has the exponential form. Furthermore, the failed items during the test may or may not be replaced. In the replacement case, less time is required to obtain a given number of failures but more items are needed in the test. In the non-replacement case, more time is required to obtain a given number of failure but less items are needed. First, consider the non-replacement case. The probability distribution of exactly d failures for truncated time T is given by the binomial distribution:

$$P_d = \frac{n!}{d!\,(n-d)!} p^d q^{n-d}, \qquad d = 0, 1, 2, \dots, r - 1; \qquad (2.8)$$

here n = sample size,

d = number of failures,

p = probability of an item fails during the interval (0, T) and it is equal to $1 - e^{-\lambda T}$,

q = 1 - p, and

λ = failure rate.

The probability of acceptance of the submitted lot is

$$P_a = \sum_{d=0}^{r-1} \frac{n!}{d!\,(n-d)!} p^d q^{n-d}. \qquad (2.9)$$

Epstein showed that the expected waiting time to obtain the r^{th} failure for the non-replacement life test plan is

$$E(t) = \frac{1}{\lambda} \sum_{d=1}^{r} \frac{1}{n-d+1}. \qquad (2.10)$$

Now consider the replacement case. The probability distribution of exactly d failures for truncated time T is

$$P_d = \frac{(n\lambda T)^d e^{-n\lambda T}}{d!}, \qquad d = 0, 1, 2, \ldots, r-1. \tag{2.11}$$

The probability of acceptance of the submitted lot is

$$P_a = \sum_{d=0}^{r-1} \frac{(n\lambda T)^d e^{-n\lambda T}}{d!}. \tag{2.12}$$

Epstein also showed that the expected waiting time to obtain the r^{th} failure for the replacement life test plan is

$$E(t) = \frac{r}{\lambda n}. \tag{2.13}$$

Some publications which discuss various versions of the truncated life test plan are listed in Table 2.5.

Chain Sampling Inspection

Dodge (1955a) developed the Chain Sampling Inspection Plan to reduce inspection costs for destructive or costly inspection. When a destructive or costly inspection is encountered, the sample size should be small in order to reduce the inspection cost. If the sample size is small, the acceptance number is usually small and sometimes it is zero. As a matter of fact, the sampling plans have the poor shape of the OC curves when acceptance number is zero because the OC curves will be convex throughout (Montogomery, 1985). Also, the probability of acceptance will decrease rapidly as the fraction defective increases. A better shape for these OC curves can be obtained by using the chain inspection plan.

This plan uses the results of several previous inspections, so it is assumed that the lots should have the same quality and that they come from a steady flow of items which are produced by a single source. Now consider an example in which a sample of size n is taken from a submitted lot of size N. The lot is accepted if there is no defective item in the sample; and the lot is rejected if there are two or more defective items in the sample. When there is only one defective item in the sample, the lot is accepted only if all i preceding lots of same size were accepted; otherwise, reject the lot. The values of i and n depend on how effective an OC curve is required. The formula for OC curves for this plan is

$$P_a = P_0 + P_1(P_0)^i \tag{2.14}$$

with P_a = probability of acceptance of the lot, and

P_d = probability of having d defective items in the sample.

Some publications discussing various versions of the chain sampling inspection are listed in Table 2.5.

Skip-Lot-Sampling Plan

Dodge (1955b) also developed the Skip-Lot-Sampling Plan to minimize inspection costs by reducing inspection after the submitted lots have good quality history. When lots are taken from a steady flow of items of the same quality, this plan may be used. The procedure is as follows:

(1) Each lot is inspected by a specific sampling plan.

(2) When i consecutive lots are accepted, stop inspecting every lot. Then only a sample of a fixed number, f, of subsequent lots are selected randomly and inspected using the same sampling plan.

(3) Whenever a lot is rejected, go to procedure (1).

The values of i and f are related to what AOQL value is required and a table for this sampling plan can be found in Dodge (1955b). Some publications which discuss various versions of the skip-lot-sampling plan are listed in Table 2.5.

Dodge-Romig Tables

Dodge and Romig (1959) developed a set of sampling inspection tables in order to minimise the average total number of inspections. There are two types of sampling inspection tables. The first type is based on Lot Tolerance Percent Defective (LTPD), i.e. limiting quality level (LQL), and the second type is based on average outgoing quality limit (AOQL). For each type of the tables, single and double sampling plans are available. The tables based on LTPD are used when the submitted lots are homogeneous or when the objective of sampling is to assure an average outgoing quality level. The tables based on AOQL are used when the submitted lots are nonhomogeneous or when the objective of sampling is to assure quality no worse than a given target. Whenever the value of LTPD or AOQL is decided and the fraction defective of incoming lots of size N is known, the sample size n may be read directly from the tables of a single or double sampling plan. Some publications discussing various versions of the Dodge-Romig tables are listed in Table 2.5.

Dependent Stage Attribute Acceptance Sampling Plan

In the late 1960's, Mogg (1969) developed a type of sampling plan, called dependent stage attribute acceptance sampling plan, which uses information from prior lots to decide whether to accept or reject the current lot. The advantages of this sampling plan are that it reduces the sample size. The notation used in the dependent stage sampling plan are defined as follows:

n = sample size,

r = the maximum number of allowable defective items from the current sample for unconditional acceptance of the lot,

b = the maximum number of additional defective items for which the decision of acceptance or rejection of the current lot will depend on the acceptance or rejection of prior lots,

$P_{a;k}$ = the probability of accepting lot number k, and

$P_{x,n}$ = the probability that there are exactly x defective items in a sample size of n items.

Mogg designated the dependent stage sampling plan by DSSP-r,b with the operating procedure outlined by the following steps;

Step 1 - At the outset, select a random sample of n items from the first lot submitted and accept the lot if the sample contains r or less defective items.

Step 2 - For each lot number, record the disposition as to whether it was accepted or rejected.

Step 3 - Repeat steps 1 and 2 on subsequent lots for the first b lots.

Step 4 - For lot b+1, select a random sample of n items and accept the lot if the sample contains r or less defective items. For more than r defective items, the decision to accept or reject the current lot will depend on the historical data, and the following courses of action will dictate the decision;

r+1 defective items - Accept the current lot if lot number 1 was accepted.

r+2 defective items - Accept the current lot if lot number 2 was accepted.

... ...

r+b defective items - Accept the current lot if lot number b was accepted.

r+i defective items - Reject the current lot, ($i > b$).

Step 5 - Repeat step 4 for each subsequent lot. That is, check the disposition of lot m-b if r+1 defective items are observed in the m^{th} lot. Check the disposition of lot m-b+1 if r+2 defective items are observed in the m^{th} lot and so on. Reject the lot if more than r+b defective items are observed, or if the lot checked on the review was rejected. Otherwise, accept the lot.

The properties of dependent stage sampling plans could be described by OC curves. The OC curve for such a sampling plan was developed by evaluating the proportion of lots that will be accepted for a product from a process. Mogg considered some elementary dependent stage sampling plans first and then developed the expression for the general OC curve by induction. He showed that the general expression for the OC curve for the dependent stage sampling plan, DSSP-r,b, is

$$P_{a;k} = \frac{\sum_{i=0}^{r} P_{i,n}}{\left(1 - \sum_{j=r+1}^{r+b} P_{j,n}\right)}, \qquad r \geq 0,\ b \geq 1 \qquad (2.15)$$

with $P_{i,n} = \frac{n!}{i!\,(n-i)!} p^i (1-p)^{n-i}$,

$P_{a;k}$ = the probability of acceptance of the submitted lot number k,

n = the sample size, and

p = the fraction defective of the submitted lots.

An example of $P_{a;k}$ values for DSSP-0,1 sampling plan with 15 sample items is shown in Table 2.3. The OC curve for this sampling plan is plotted in Figure 2.5. If this OC curve is used, the probability of acceptance of a submitted lot can be determined when the fraction defective is known.

Mogg also compared some selected dependent stage sampling plans with a variety of single sampling plans and double sampling plans from Freeman (1948). One of the comparisons is among the DSSP-0,1 sampling plan with 15 sample items, the single sampling plan with $n = 20$, $c = 1$, and the double sampling plan with $n_1 = 13$, $c_1 = 0$, $r_1 = 3$, $n_2 = 2$, and $r_2 = 3$. The OC curves of all the above sampling plans are almost identical. It means that all these plans would provide same consumer and producer protection. The average sample number (ASN) of DSSP-0,1 is 15 units and the ASN of the single sampling plan is 21 units. Thus, the ASN of DSSP-0,1 is six units less than the ASN of the single sampling plan. For the double sampling plan, the ASN is a variable from 14 units to 28 units. In other words, the ASN of DSSP-0,1 is as much as 13 units less than the ASN of the double sampling plan. These comparisons showed the advantage of dependent stage sampling plans in saving the average sample number.

Mogg pointed out the limitations of the dependent stage sampling plan as follows:

(1) Production is steady so that results on current and preceding lots are indicative of a continuing process.

(2) Lots are submitted substantially in the order of their production.

(3) A fixed sample size, n, from each lot is assumed.

(4) Inspection by attributes is assumed with quality measured by fraction defective p for a binomial distribution.

He also mentioned that sometimes the dependent stage sampling plan may not be desirable since the OC curve changes from lot to lot in the early stages of this plan and does not settle down to a fixed curve until approximately ten lots have been inspected. The reason is that the plan acts as single sampling plan until b+1 lots are inspected and historical results are considered from that point.

Deferred State Attribute Acceptance Sampling Plan

In the early 1970's, Baker (1971) developed a type of sampling plan which is called the deferred state attribute acceptance sampling plan. This sampling plan uses subsequent lots' information for making a decision to accept or reject the current lot. The operating procedure of this type of sampling plan is similar to the dependent stage sampling plan except that the conditional decisions depend on the disposition of future lots instead of past lots. Thus, the formations of OC curves of dependent stage sampling plans and deferred state sampling plans are similar. The deferred state sampling plan provides an indicator for quality degradation. If a large number of defective items are observed in a sample, the probability that the process quality degraded beyond an acceptable level is high. The indicator concept is based on the assumption that the number of defective items from a sample may truly represent the process quality. The notation used in deferred state sampling plans is as follows:

n = sample size,

r = the maximum number of allowable defective items from the current sample for unconditional acceptance of the lot,

b = the maximum number of additional defective items for which the decision of acceptance or rejection of the current lot will depend on the acceptance or rejection of subsequent lots,

$P_{a;k}$ = the probability of accepting lot number k, and

$P_{x,n}$ = the probability that there are exactly x defective items in a sample size of n items.

Baker designated the deferred state sampling plan by DS(r,b) sampling plan with the operating procedure outlined by the following steps;

Step 1 - For lot number k, select a random sample of n items from the submitted lot and determine the number of defective items.

Step 2 - Accept the lot if the sample contains r or fewer defective items. For more than r defective items, the decision to accept or reject the current lot is dictated by the following courses of action;

r+1 defective items	- Defer the decision until the disposition of lot number k+b is obtained. If lot number k+b is accepted, then accept lot k, otherwise reject lot number k.
r+2 defective items	- Defer the decision until the disposition of lot number k+b-1 is obtained. If lot number k+b-1 is accepted, then accept lot k, otherwise reject lot number k.
. . .	. . .
r+b defective items	- Defer the decision until the disposition of lot number k+1 is obtained. If lot number k+1 is accepted, then accept lot k, otherwise reject lot number k.
r+i defective items (i > b)	- Reject lot number k.

Step 3 - Increment k by 1 and return to step 1.

The properties of deferred state sampling plan could be described by the OC curves. The OC curve for such a sampling plan was developed by evaluating the proportion of lots that will be accepted for a product from a process. Baker considered some elementary deferred state sampling plans first and then developed the expression for the general OC curve by induction. He showed that the general expression for the OC curve for deferred state sampling plan, DS(r,b), is

$$P_{a;k} = \frac{\sum_{i=0}^{r} P_{i,n}}{\left(1-\sum_{j=1}^{b} P_{r+j,n}\right)} \qquad r \geq 0,\ b \geq 1, \qquad (2.16)$$

with $P_{i,n} = \frac{n!}{i!\,(n-i)!}p^{i}(1-p)^{n-i}$, and p = the fraction defective of the submitted lots. An example of $P_{a;k}$ values for a DS(0,1) sampling plan with n = 15 is shown in Table 2.4. The OC curve for this sampling plan is plotted in Figure 2.6. From this OC curve, the probability of acceptance of a submitted lot can be determined when the fraction defective is known.

Baker compared the DS(0,1) sampling plan with n = 15 to the single sampling plan with n = 20 and c = 1, and the double sampling plan with $n_1 = 13$, $c_1 = 0$, $r_1 = 3$, $n_2 = 26$, $c_2 = 2$ and $r_2 = 3$. The OC curves of all three plans are almost identical, but the ASN curves showed a difference of approximately 5 to 13 units between the DS(0,1) sampling plan and the double sampling plan, and a difference of 5 units between the DS(0,1) sampling plan and the single sampling plan. This made the advantage of the deferred stage attribute acceptance sampling in reducing the average number of samples evident.

Baker also discussed the limitations of the deferred state sampling plan. One of the limitations is that a waiting line may be formed when the lots are in a deferred state, so the carrying cost of deferred lots should be considered before a deferred state sampling plan is selected instead of any other sampling plan. In developing the distribution of waiting times, Baker used the following notation:

$P_{x,n}$ = the probability that there are exactly x defective items in a sample of n items,

W = the parameter which denotes the number of lots that a lot must wait before disposition,

$P(W = i)$ = the probability that a lot waits for i lots before disposition, and

$E(W)$ = the expected wait which is the expected value of W.

Baker developed the distribution of waiting times for the DS(0,1) sampling plan. He showed that the probability that a deferred lot waits for i lots before disposition is

$$P(W = i) = P_{1,n}{}^{i} (1 - P_{1,n}) \tag{2.17}$$

and the expected wait, E(W), is

$$E(W) = \sum_{i=0}^{\infty} i\, P(W = i). \tag{2.18}$$

Substituting (2.17) in (2.18) gives

$$E(W) = \frac{P_{1,n}}{1 - P_{1,n}}. \tag{2.19}$$

The general equation for the expected waiting time can be obtained by induction.

The deferred state sampling plan has a problem similar to that of the dependent stage sampling plan. In the early stages of a dependent stage sampling plan, several lots have to be sampled under a single sampling plan before the dependent stage concept can be used. The OC curve of the dependent stage sampling plan changes from lot to lot in the early stages of the plan and does not settle down until approximately ten lots have been inspected. With a deferred state sampling plan, the problem is how to make the disposition decision of the final lots when the lots are waiting for disposition of future lots which will not be produced. One solution is to use a single sampling plan for the last b lots. That is, if the number of defective items from the sample is less than r, accept the entire lot; otherwise, reject the lot. So no submitted lot will wait for future lots which will not be produced. Thus, the OC curve of the deferred state sampling plan changes from lot to lot in the final stages of the plan; it is not a fixed curve in approximately the last ten lots.

After the deferred state sampling plan was developed, Dean (1971) used this concept in truncated life test plans. The difference between the truncated life test plans and the quality control sampling plans is that time is considered as a parameter in truncated life test plans. Dean developed the deferred state life test plans with the same notation as in deferred state sampling plans, except that

T = the total accumulated test time,

λ = the failure rate of items tested,

$\theta = \frac{1}{\lambda}$ = the mean-time-between-failures (MTBF) of items tested, and

$P_x = P_{x,\lambda,T}$ = the probability that there are x failed items during time T, given the failure rate is λ and that the life distribution of the tested items is exponential.

Dean designated the deferred state life test plan by DS(r,b) life test plan with an operating procedure outlined by the following steps;

Step 1 - For lot number k, select a random sample of n items from the lot and test the sample for a total accumulated test time T. Then determine the number of failed items. The failed items during the test may or may not be replaced.

Step 2 - Accept the submitted lot if r or less failed items are observed. If more than r failed items are observed, the decision to accept or reject the current lot is dictated by the following courses of action;

r+1 failed items - Defer the decision until the disposition of lot number k+b is obtained. If lot number k+b is accepted, accept lot number k, otherwise reject lot number k.

r+2 failed items - Defer the decision until the disposition of lot number k+b-1 is obtained. If lot number k+b-1 is accepted, accept lot number k, otherwise reject lot number k.

... ...

r+b failed items - Defer the decision until the disposition of lot number k+1 is obtained. If lot number k+1 is accepted, accept lot number k, otherwise reject lot number k.

r+i failed items (i > b) - Reject lot number k.

Step 3 - Increment k by 1 and return to step 1.

The properties of deferred state life test plans could also be described by evaluating the proportion of lots that will be accepted for a product from a process. Dean considered the probabilities of accepting the submitted lot number k, $P_{a;k}$, for DS(0,1) and DS(2,4) life test plans first and then developed the general expression for the $P_{a;k}$ by induction. He showed that the general expression of the $P_{a;k}$ for the deferred state life test plan, DS(r,b) life test plan, is

$$P_{a;k} = \frac{\sum_{x=0}^{r} P_x}{\left(1 - \sum_{x=1}^{b} P_{r+x}\right)}, \qquad r \geq 0,\ b \geq 1. \qquad (2.20)$$

Dean also assumed the failure distributions to be exponential. Thus, the probability of a certain number of failed items can be described by Poisson distribution. When the total accumulated test time is T and the failure rate is λ, the probability of exactly x failed items is

$$P_x = P_{x,\lambda,T} = \frac{(\lambda T)^x e^{-\lambda T}}{x!}, \qquad x = 0, 1, 2, \dots, \text{ and } \lambda, T > 0. \tag{2.21}$$

By substituting (2.21) into (2.20), the general expression of the $P_{a;k}$ for deferred state life test plans in terms of r, b, λ and T becomes

$$P_{a;k} = \frac{\sum_{x=0}^{r} \frac{(\lambda T)^x e^{-\lambda T}}{x!}}{\left(1 - \sum_{x=1}^{b} \frac{(\lambda T)^{r+x} e^{-\lambda T}}{(r+x)!}\right)}, \qquad r \geq 0, b \geq 1, \text{ and } \lambda, T > 0. \tag{2.22}$$

Since λ is equal to $\frac{1}{\theta}$, this can be written as

$$P_{a;k} = \frac{\sum_{x=0}^{r} \frac{\left(\frac{T}{\theta}\right)^x e^{\frac{-T}{\theta}}}{x!}}{\left(1 - \sum_{x=1}^{b} \frac{\left(\frac{T}{\theta}\right)^{r+x} e^{\frac{-T}{\theta}}}{(r+x)!}\right)}, \qquad r \geq 0, b \geq 1, \text{ and } \theta, T > 0. \tag{2.23}$$

For any DS(r,b) life test plan, the $P_{a;k}$ values can be found by substituting specific values of θ and T into (2.23).

Dean also considered the general expression for the expected wait, E(W), in the deferred state life test plan. The notation used in this analysis is the same as in the deferred state sampling plan. To derive a general expression for the expected wait in the DS(r,b) life test plan, we note that the probability of unconditional disposition of the current lot, i.e. the probability that the expected wait of the current lot is zero, is

$$P(W = 0) = \sum_{x=0}^{r} P_x + \sum_{x=r+b+1}^{\infty} P_x, \tag{2.24}$$

with $P_x = \dfrac{\left(\frac{T}{\theta}\right)^x e^{\frac{-T}{\theta}}}{x!}$.

The first term in formula (2.24) is the sum of the probabilities of observing at most r failed items and the second term is the sum of the probabilities of observing at least r+b+1 failed items. Furthermore, the probability that the current lot waits for one additional lot before disposition is equal to the probability of having exactly r+b failed items in the current lot multiplied with the probability of unconditional disposition of the next lot, i.e.

$$P(W = 1) = P_{r+b}\, P(W = 0). \tag{2.25}$$

The probabilities that the current lot waits for two or three additional lots before disposition are, respectively,

$$P(W = 2) = P_{r+b}\, P(W = 1) + P_{r+b-1}\, P(W = 0) \tag{2.26}$$

and

$$P(W = 3) = P_{r+b}\, P(W = 2) + P_{r+b-1}\, P(W = 1) + P_{r+b-2}\, P(W = 0). \tag{2.27}$$

Continuing the derivation in this manner, we can write the general expression for the probability that the current lot waits for b+j lots as

$$P(W = b+j) = P_{r+b}\, P(W = b+j-1) + P_{r+b-1}\, P(W = b+j-2) + \ldots$$
$$+ P_{r+2}\, P(W = j+1) + P_{r+1}\, P(W = j) \qquad j = 0, 1, 2, \ldots, \qquad (2.28)$$

and the expected wait, E(W), for DS(r,b) life test plan as

$$E(W) = 1\, P(W = 1) + 2\, P(W = 2) + \ldots + k\, P(W = k) + \ldots . \qquad (2.29)$$

The detailed explanations and examples can be found in Dean (1971).

Table 2.1. Probabilities of acceptance for the single sampling plan:

$N = 5000$, $n = 100$, and $c = 2$.

p	np	P_a
0.00	0	1.000
0.01	1	0.920
0.02	2	0.677
0.03	3	0.423
0.04	4	0.238
0.05	5	0.125
0.06	6	0.062
0.07	7	0.029

Table 2.2. Probabilities of acceptance for the double sampling plan:

$N = 5000$, $n_1 = 80$, $c_1 = 1$, $r_1 = 3$, $n_2 = 100$, $c_2 = 3$, and $r_2 = 4$.

p	n_1p $= 80p$	n_2p $= 100p$	$(P_a)_{n_1}$ $= (P_{1 \text{ or less}})_{n_1}$	$(P_a)_{n_2}$ $= (P_2)_{n_1}(P_{1 \text{ or less}})_{n_2}$	$(P_a)_{n_1+n_2}$ $= (P_a)_{n_1} + (P_a)_{n_2}$
0.00	0.0	0	1.000	(0.000) (1.000) = 0.000	1.000
0.01	0.8	1	0.808	(0.144) (0.736) = 0.106	0.914
0.02	1.6	2	0.525	(0.258) (0.406) = 0.105	0.630
0.03	2.4	3	0.309	(0.261) (0.199) = 0.052	0.361
0.04	3.2	4	0.171	(0.209) (0.091) = 0.019	0.190
0.05	4.0	5	0.091	(0.147) (0.041) = 0.006	0.097
0.06	4.8	6	0.047	(0.095) (0.017) = 0.002	0.049
0.07	5.6	7	0.024	(0.065) (0.007) = 0.001	0.025

Table 2.3. Probabilities of acceptance for the DSSP-0,1 sampling plan with 15 sample items.

p	$P_{a;k}$
0.00	1.0000
0.02	0.9543
0.04	0.8198
0.06	0.6360
0.08	0.4571
0.10	0.3135
0.12	0.2102
0.14	0.1396
0.16	0.0925
0.18	0.0618
0.20	0.0405

Table 2.4. Probabilities of acceptance for the DS(0,1) sampling plan with 15 sample items.

p	$P_{a;k}$
0.00	1.0000
0.02	0.9543
0.04	0.8198
0.06	0.6360
0.08	0.4571
0.10	0.3135
0.12	0.2102
0.14	0.1396
0.16	0.0925
0.18	0.0618
0.20	0.0405

Table 2.5. A list of publications of lot-by-lot acceptance sampling plans by attributes.

Single Sampling Plan	Peach and Littauer (1946) Grubbs (1949) Cameron (1952) Golub (1953) U.S. Army Chemical Corps. Eng. Agency (1953) Wise (1955) Horsnell (1957) Hamaker (1958) Guthrie and Johns (1959) Prairie, Zimmer, and Brookhouse (1962) Hald (1965) Hald (1967a) Hald (1967b) Dodge (1969a) Ayoub, Lambert, and Walvekar (1970) Wortham and Mogg (1970a) Guenther (1971a) Minton (1972) Collins, Case, and Bennett (1973) Bennett, Case, and Schmidt (1974) Hald (1977) Schilling, Sheesley, and Nelson (1978) Stephens (1978) Beaing (1981) Guenther (1984) Case and Chen (1985) Jaraiedi and Herrin (1985) Baker (1988) Ohta and Ichihashi (1988) Ohta and Kanagawa (1988) Brooks (1989) Nachlas and Kim (1989) Soundararajan and Arumainayagam (1989) Govindaraju (1990) Soundararajan and Vijayaraghavan (1990) Nelson(1991)
Double Sampling Plan	U.S. Army Chemical Corps. Eng. Agency (1953)

	Hamaker and Van Strik (1955) Horsnell (1957) Guenther (1971b) Chow, Dickinson, and Hughes (1972) Hald (1977) Baker and Brobst (1978) Schilling, Sheesley, and Nelson (1978) Beaing and Case (1981) Chen (1981) Olorunniwo and Salas (1982) Guenther (1983) Case and Chen (1985) Maghsoodloo and Bush (1985) Srivenkataramana and Harishchandra (1985) Govindaraju (1990)
Multiple Sampling Plan	Bartky (1943) U.S. Army Chemical Corps. Eng. Agency (1953) Hald (1975) Schilling, Sheesley, and Nelson (1978) Bryant and Schmee (1979) Flowers and Cole (1985) Baker (1987) Maghsoodloo (1987)
Sequential Sampling Plan	Wald (1945) Anscombe (1946) Barnard (1946) Wald (1947) Hamaker (1953) Epstein and Sobel (1955) Hoel (1955) Kiefer and Weiss (1957) Anderson (1960) Jackson (1960) Johnson (1962) Eagle (1964) Chernoff and Ray (1965) Tallis and Vagholkar (1965) Aroian and Robison (1966) Aroian (1968) Schafer and Takenaga (1972) Wald (1973) Aroian (1976)

	Garrison and Hichey (1984) Kremers (1987) Tantaratana (1988)
Military Standard 105	JAN-STD-105 (1949) MIL-STD-105A (1950) MIL-STD-105B (1958) MIL-STD-105C (1961) Keefe (1963) MIL-STD-105D (1963) Pabst (1963a) Pabst (1963b) Cocca (1964) Stephens and Larson (1967) Dodge (1969b) Kaplan and MacDonald (1969) Koyama (1969) Ohmae and Suga (1969) Yokoh (1969) Koyama, Ohmae, Suga, and Yamamoto (1970) Brown and Rutemiller (1973) Hill (1973) Brown and Rutemiller (1975) Hahn and Schilling (1975) Sheesley (1977) Schilling and Sheesley (1978a) Schilling and Sheesley (1978b) Liebesman (1979) Duncan et al (1980) Schilling and Johnson (1980) Brush, Cautin, and Lewin (1981) Liebesman (1981a) Liebesman (1981b) Liebesman (1982) Schilling (1982) Buswell and Hoadley (1983) Cocca (1983) Schilling (1983) Enell (1984) Keats and Case (1984) Liebesman and Hawley (1984) Bee, Teck, and Keng (1985) Randhawa (1985)

	Nelson, Wall, and Caporal (1986) Baker (1987) Chakraborty and Bapaye (1989) Glenn (1989) MIL-STD-105E (1989) Flott (1990)
Truncated Life Test Plan	Epstein (1954) Burr (1957) Woodal and Kurkjian (1962) Aroian (1963) Aroian (1964) Craig (1968) Guenther (1971) Angus, Schafer, Van Den Berg, and Rutemiller (1985) Mason (1986)
Chain Sampling Inspection	Dodge (1955a) Frishman (1960) Dodge and Stephens (1964) Stephens and Dodge (1965) Dodge and Stephens (1966) Stephens and Dodge (1967) Soundararajan (1978a) Soundararajan (1978b) Soundararajan and Govindaraju (1983) Soundararajan and Doraiswamy (1984) Soundararajan and Arumainayagam (1989) Raju (1990) Soundararajan and Vijayaraghavan (1990)
Skip-Lot-Sampling Plan	Dodge (1955b) Perry (1970) Perry (1973a) Perry (1973b) Hsu (1980) Carr (1982) Liebesman and Saperstein (1983) Flowers and Cole (1985) Jaraiedi and Bem (1989) Kowalewski and Tye (1990) Perry (1990)

Dodge-Romig Tables	Dodge and Romig (1959) Keats and Case (1984) Flott (1990)
Dependent Stage Sampling Plan	Mogg (1969) Wortham and Mogg (1970b)
Deferred State Sampling Plan	Baker (1971) Dean (1971) Wortham and Baker (1971) Wortham and Baker (1976)

Figure 2.1. The OC curve for the single sampling plan: N = 5000, n = 100, and c = 2.

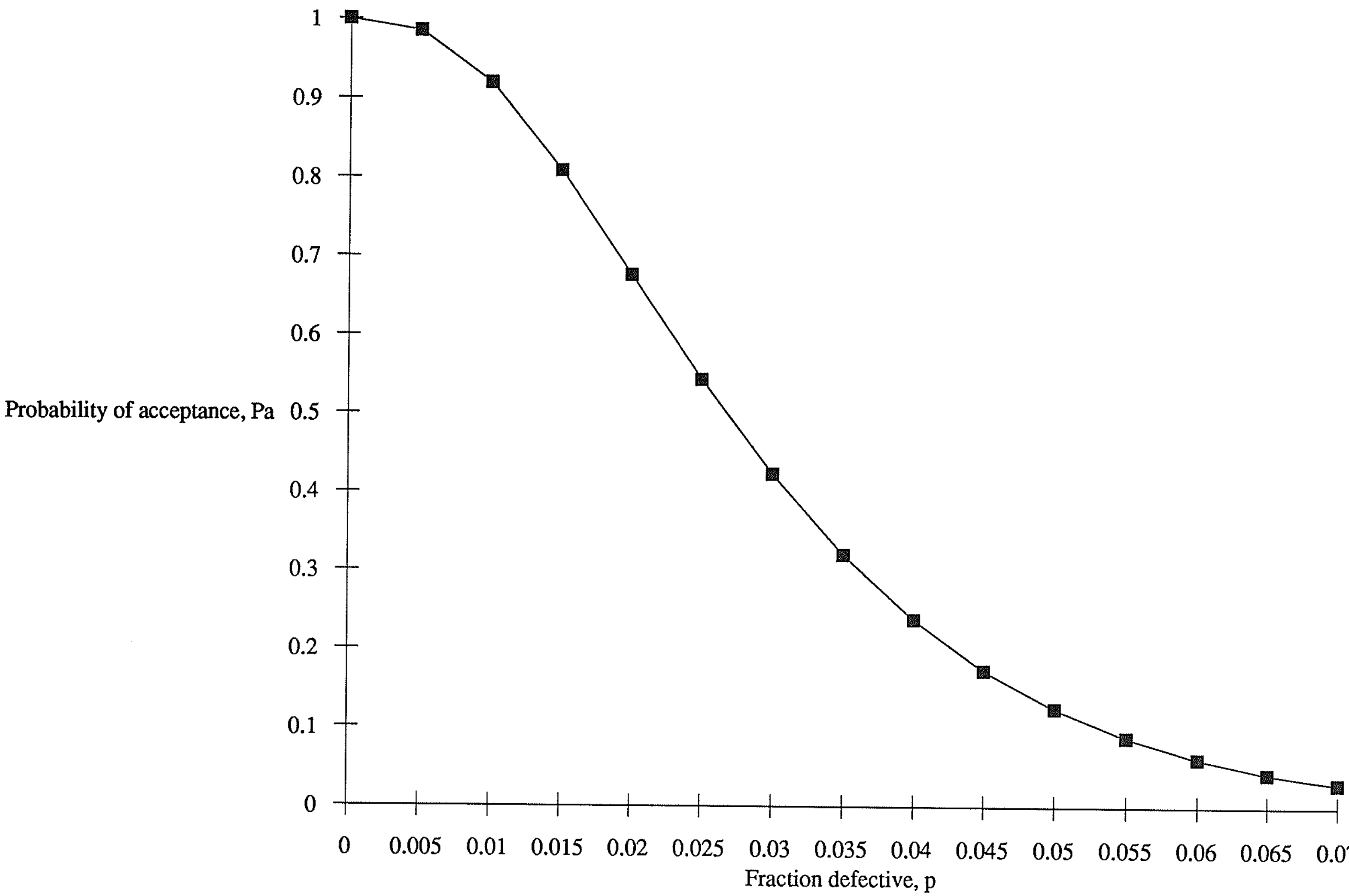


Figure 2.2. The OC curve for the double sampling plan : N = 5000, n1 = 80, c1 = 1, r1 = 3, n2 = 100, c2 = 3 and r2 = 4.

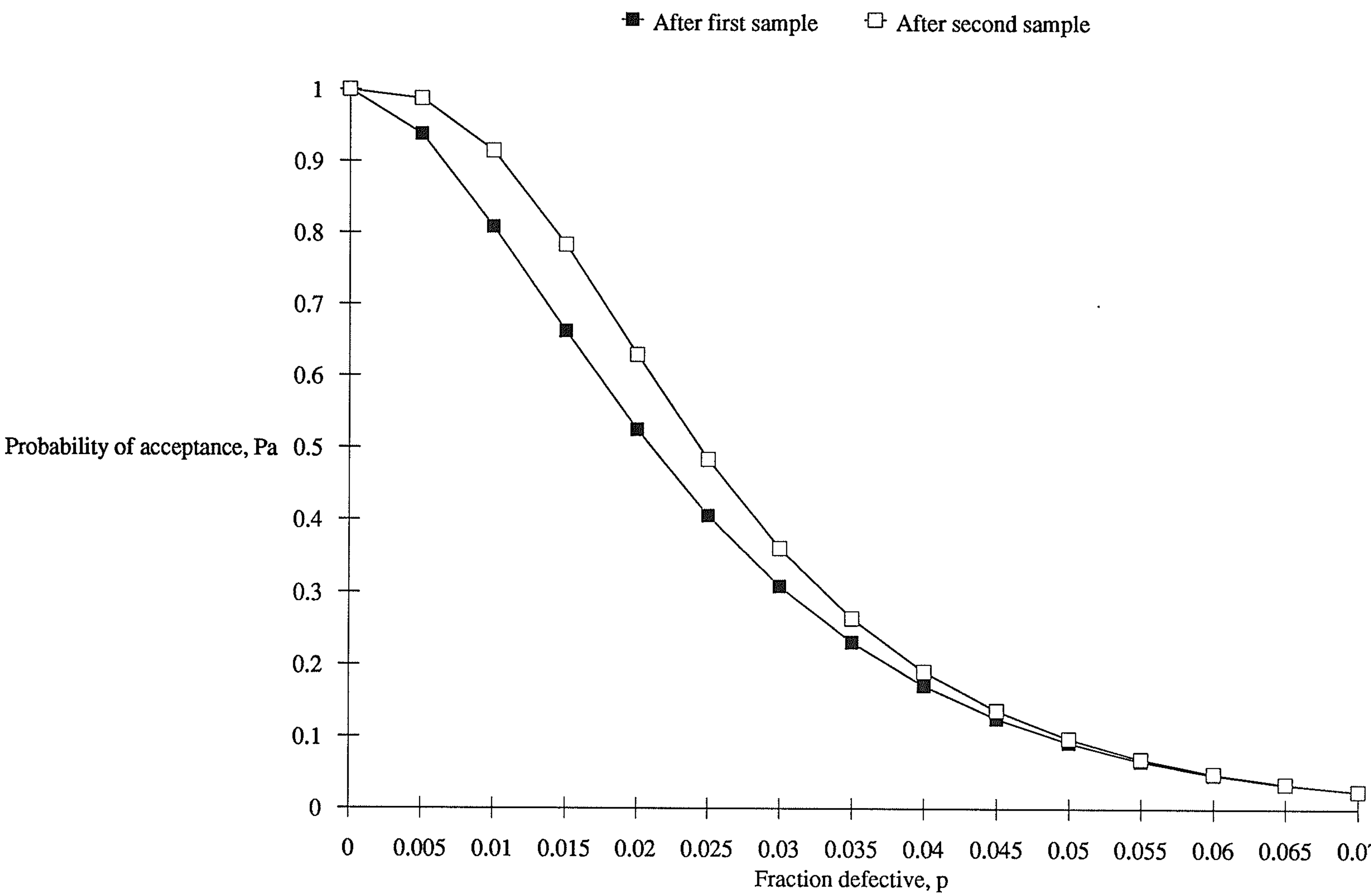


Figure 2.3. An example of a sequential sampling plan.

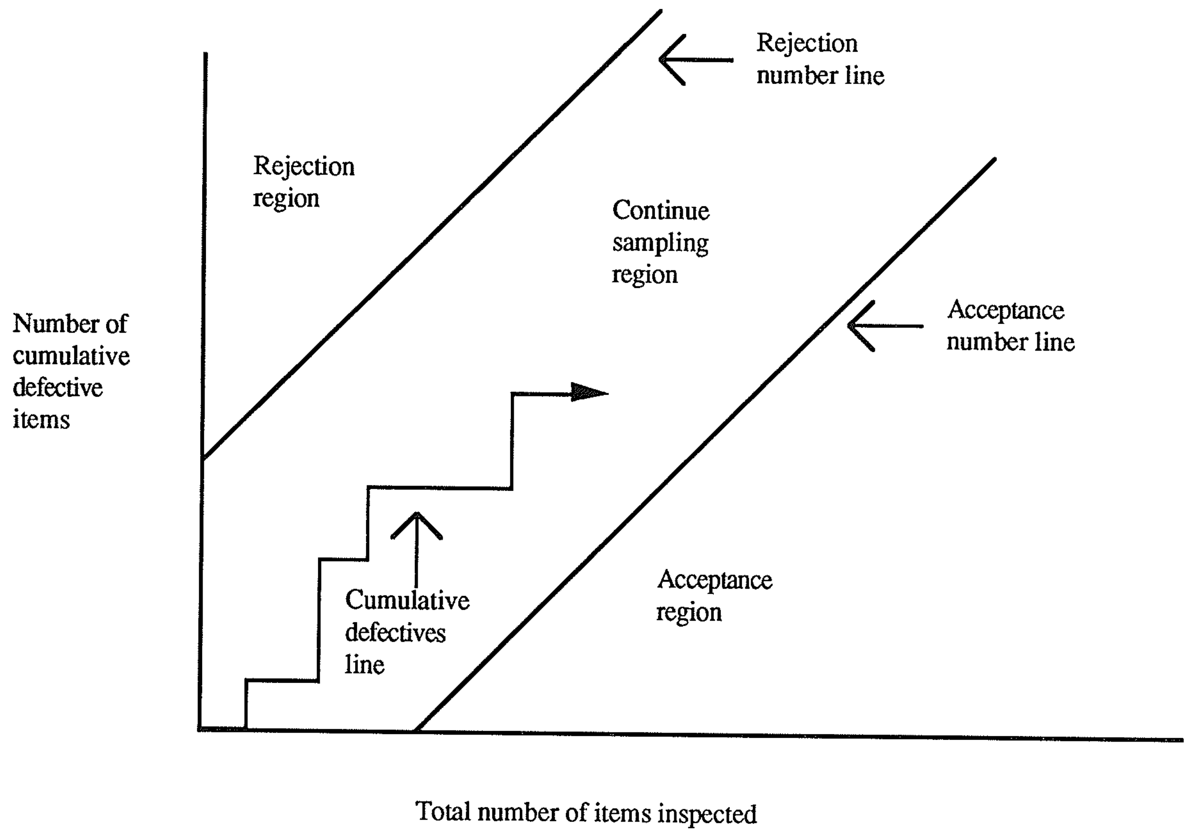


Figure 2.4. An example of OC curve for normal, tightened, and reduced inspection.

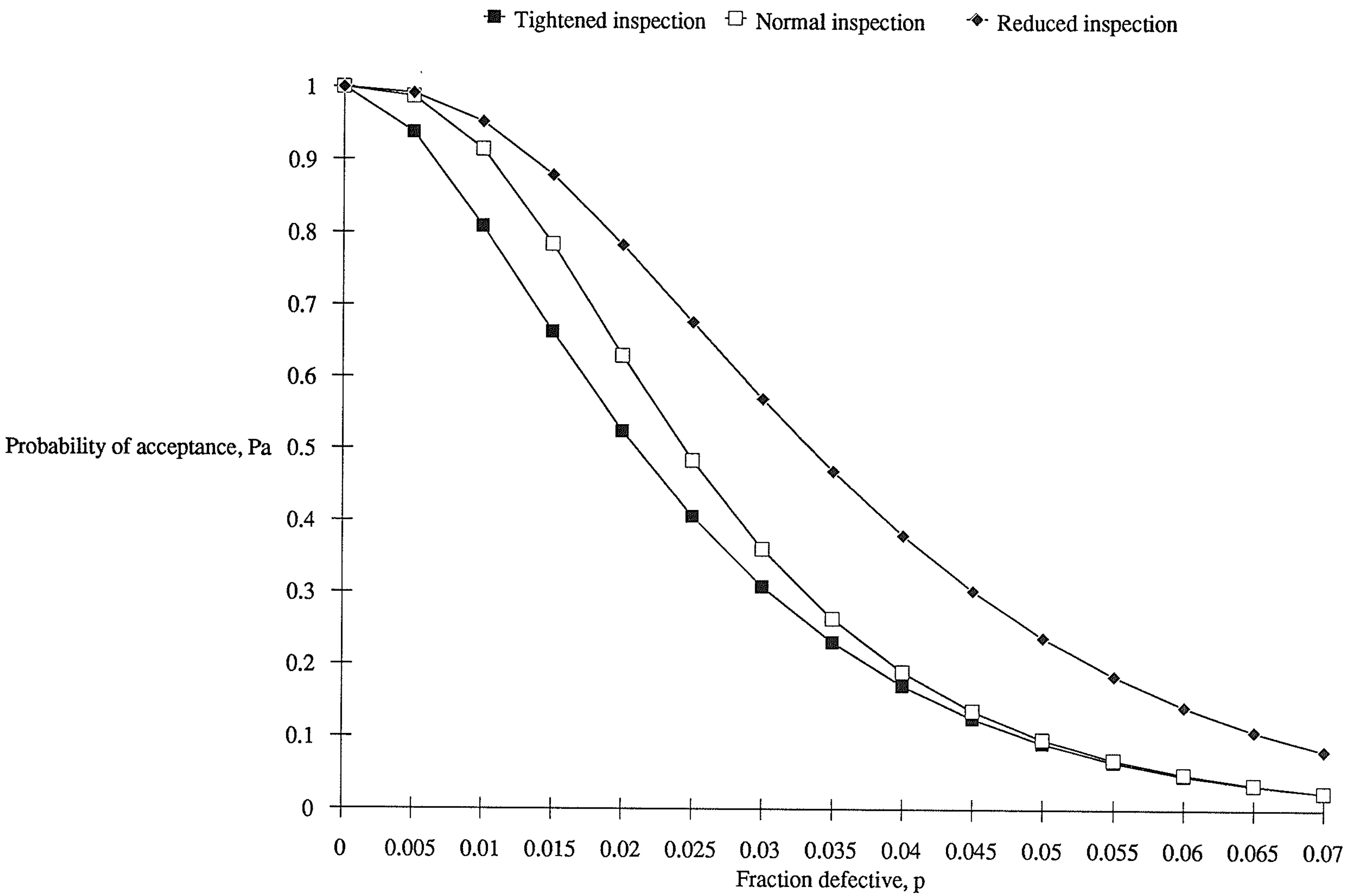


Figure 2.5. The OC curve for the DSSP-0,1 sampling plan with 15 sample items.

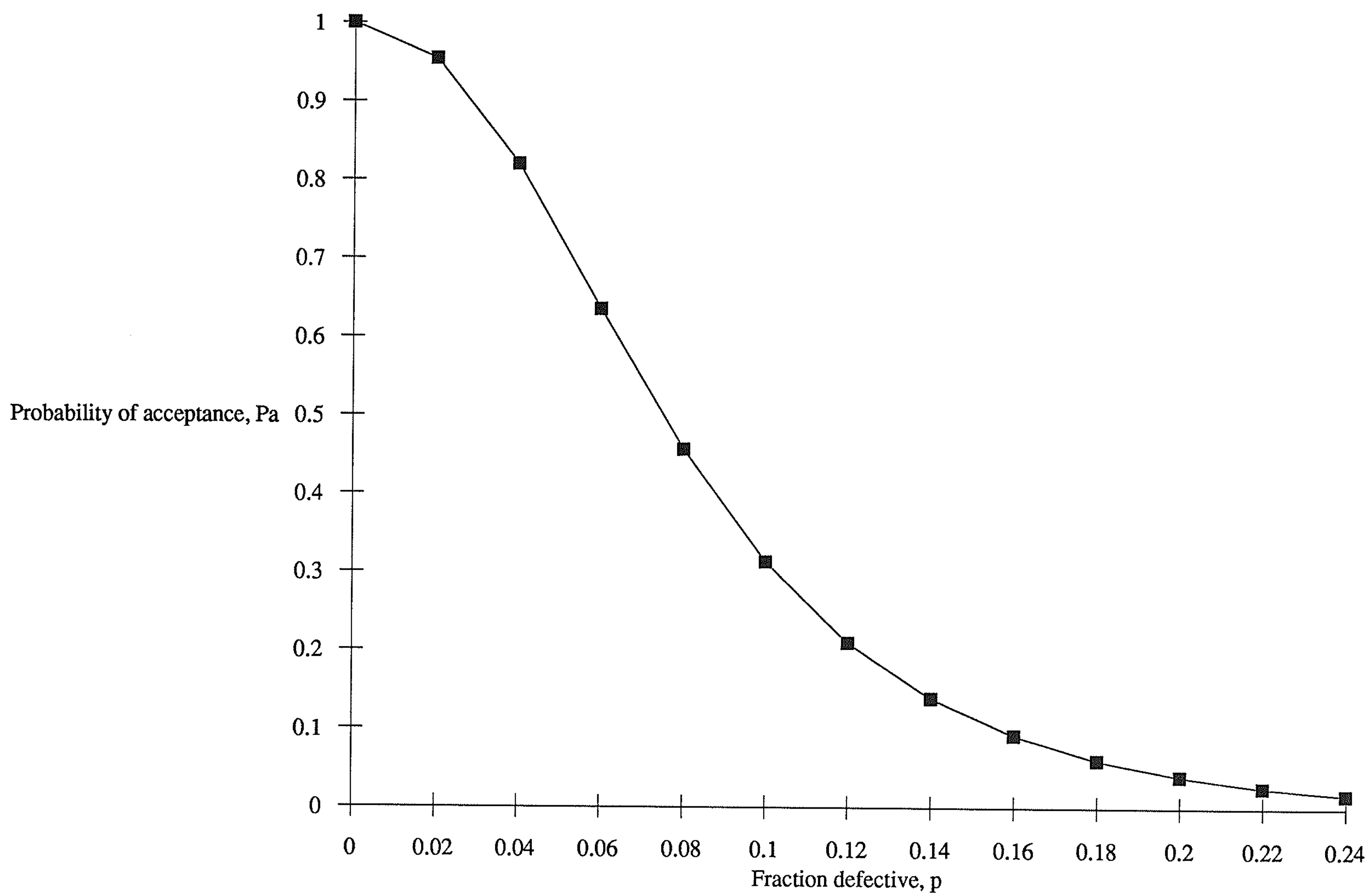
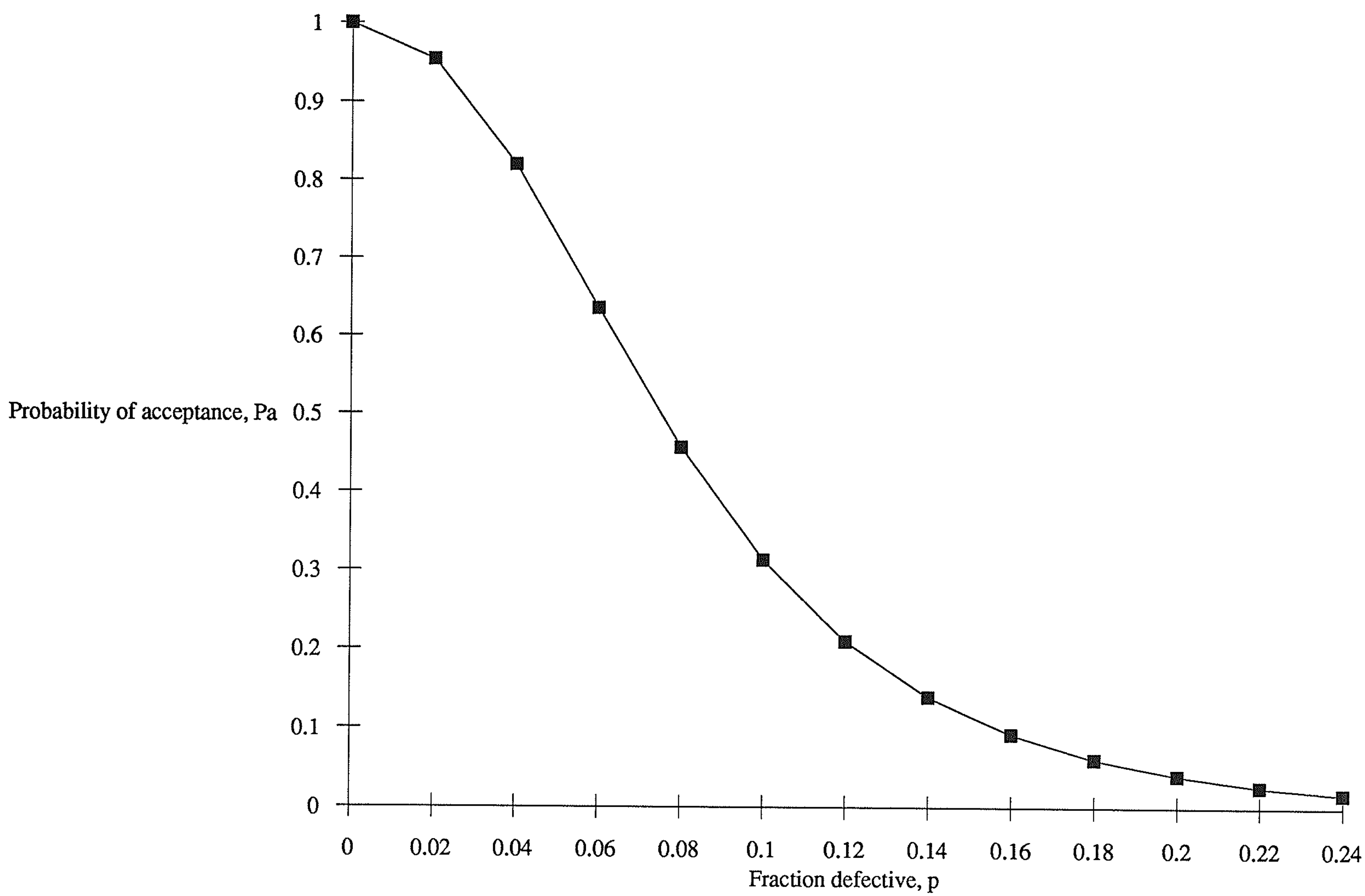


Figure 2.6. The OC curve for the DS(0,1) sampling plan with 15 sample items.



CHAPTER III

DETERMINATION OF TOTAL TEST TIME AND SAMPLE SIZE FOR DEFERRED STATE LIFE TEST PLAN

Before a deferred state life test plan is selected to test the submitted lots, the user must consider many factors, for example, the time until due date, the available human resources, the production facilities, the inventory facilities, etc. Also, the overall testing cost is one of the major factors, causing concern to the user. It includes the total cost of testing time per sample and the total cost of testing each sample. So before a test plan is conducted, the user must determine not only the appropriate test plan but also the total test time and the sample size. The total test time and the sample size for deferred state life test plan were considered by Dean (1971) and we will discuss them here also.

Total Test Time

As shown in formula (2.23), the general expression of the probability of accepting a submitted lot number k, $P_{a;k}$, for deferred state life test plans can be expressed in terms of r, b, θ and T. For any DS(r,b) plan, the values of r and b are known, so that the $P_{a;k}$ can be written as a function of $\frac{T}{\theta}$. Thus, each DS(r,b) plan can be represented by one OC curve with $P_{a;k}$ on the y-axis and $\frac{T}{\theta}$ on the x-axis. The OC curve can easily be obtained by substituting specific values of $\frac{T}{\theta}$ into formula (2.23).

An example of $P_{a;k}$ values for DS(4,3) in terms of $\frac{T}{\theta}$ is shown in Table 3.1. The OC curve for this DS(4,3) plan is plotted in Figure 3.1. Once the OC curve is obtained, we can use it to find the probability of acceptance of a submitted lot in which the value of $\frac{T}{\theta}$ is known. The values of producer's risk (α) and the consumer's risk (β) can also be determined when the discrimination ratio , $\delta > 1$, is given. The discrimination ratio can be calculated from θ_{AQL}, the mean-time-between-failure corresponding to the acceptable quality level and θ_{LQL}, the mean-time-between-failure corresponding to the limiting quality level, by the formula

$$\theta_{AQL} = \delta\,\theta_{LQL}. \tag{3.1}$$

On the other hand, if specific values for α, β and δ are given, the user is responsible for finding a life test plan which satisfies the given requirements. One method to do this is to check the set of OC curves of DS(r,b) plans until an appropriate OC curve is found. For example, suppose a deferred state life test plan is needed satisfying the required conditions: $\alpha = 0.1$, $\beta = 0.1$, and $\delta = 2$. Then a set of OC curves of DS(r,b) plans is checked. When the DS(4,3) plan is considered, it is found that the probability of acceptance ($P_{a;k}$) is 0.9 ($1 - \alpha$) when the value of $\frac{T}{\theta}$ is 4.2, and the probability of acceptance ($P_{a;k}$) is 0.1 (β) when the value of $\frac{T}{\theta}$ is 8.6. It is obvious that

$$\frac{T}{\theta_{AQL}} = 4.2 \tag{3.2}$$

and

$$\frac{T}{\theta_{LQL}} = 8.6. \tag{3.3}$$

That is,

$$T = 4.2\,\theta_{AQL} \tag{3.4}$$

and

$$T = 8.6\,\theta_{LQL}, \tag{3.5}$$

respectively. Dividing formula (3.4) by formula (3.5), we have

$$\frac{\theta_{AQL}}{\theta_{LQL}} = 2, \tag{3.6}$$

and from (3.1), we get $\delta = 2$. Thus, the DS(4,3) plan will be chosen because it satisfies the required conditions.

After a specific life test plan is selected, the total test time can be calculated by (3.4) or (3.5). For example, if the value of θ_{AQL} is 1000 hours for the test items, formula (3.4) shows that the total test time should be 4200 hours ($T = 4.2 \times 1000 = 4200$).

Dean compared the above DS(4,3) plan with the plans in MIL-STD-781B (1967). In MIL-STD-781B, the Test Plan XVIII has exactly the same characteristics as those in DS(4,3), i.e. $\alpha = 0.1$, $\beta = 0.1$, and $\delta = 2$. But the total test time of the Test Plan XVIII is 9.4 θ_{AQL}, while the total test time of the DS(4,3) plan is only $4.2\theta_{AQL}$. The advantage of the deferred state life test plan can be seen, not only through the above comparison, but also through other additional comparisons. As a result, the total cost of testing time per sample can be reduced.

Sample Size

As it was mentioned before, the failed items during a life test may or may not be replaced by a new item drawn from the remainder of the same lot. For the replacement case, the sample size has no restriction since the test can always be terminated at the predetermined total test time T. For the non-replacement case, there exists a possibility that all sample items fail before the total test time is reached, so that the user should determine an appropriate sample size such that the probability of reaching the total test time T is high.

Assuming that all n items of a sample are simultaneously placed on a life test, let t_i

be the time at which the i^{th} item fails. Since the t_i's are independently and exponentially distributed random variables, the sum of all t_i's,

$$y = t_1 + t_2 + \dots + t_n, \tag{3.7}$$

will be a random variable with a gamma distribution (Appendix A). If the mean-time-between-failure is θ, this gamma distribution will have the parameters n and θ. By using formula (A.12), the probability that the sum of all t_i's is greater than the total test time can be written as

$$P(y \geq T) = \sum_{i=0}^{n-1} \frac{\left(\frac{T}{\theta}\right)^i e^{\frac{-T}{\theta}}}{i!}. \tag{3.8}$$

If it is required that the probability of reaching the total test time T must be greater than 0.90, the sample size can be calculated by

$$\sum_{i=0}^{n-1} \frac{\left(\frac{T}{\theta}\right)^i e^{\frac{-T}{\theta}}}{i!} > 0.90. \tag{3.9}$$

The sample size will be that value of n which satisfies formula (3.9); and it must be at least r+b+1 in order to provide the reject decision in all DS(r,b). For example, if we consider the DS(4,3) plan discussed in the last section, the value of $\frac{T}{\theta}$ will be 4.2 with T = 4200 and θ = 1000. The appropriate sample size can then be determined from (3.9), which becomes

$$\sum_{i=0}^{n-1} \frac{(4.2)^i e^{-4.2}}{i!} > 0.90. \qquad (3.10)$$

The above inequality can be solved by an algorithm or a table of Poisson distributions (cumulative probability).

Table 3.2 shows the probability of reaching the total test time of 4200 hours in terms of the sample size n. From Table 3.2, it is obvious that the sample size must be at least eight items to ensure that the probability of reaching the total test time of 4200 hours is greater than 0.90. Dean (1971) also compared the sample sizes between the above DS(4,3) plan and the Test Plan XVIII of MIL-STD-781B (1967). He found that the sample size for Test Plan XVIII is 14 items under the same requirements, thus showing the advantage of the deferred state life test plan in saving on sample size can be shown. As a result, the total cost of testing each sample item can be reduced.

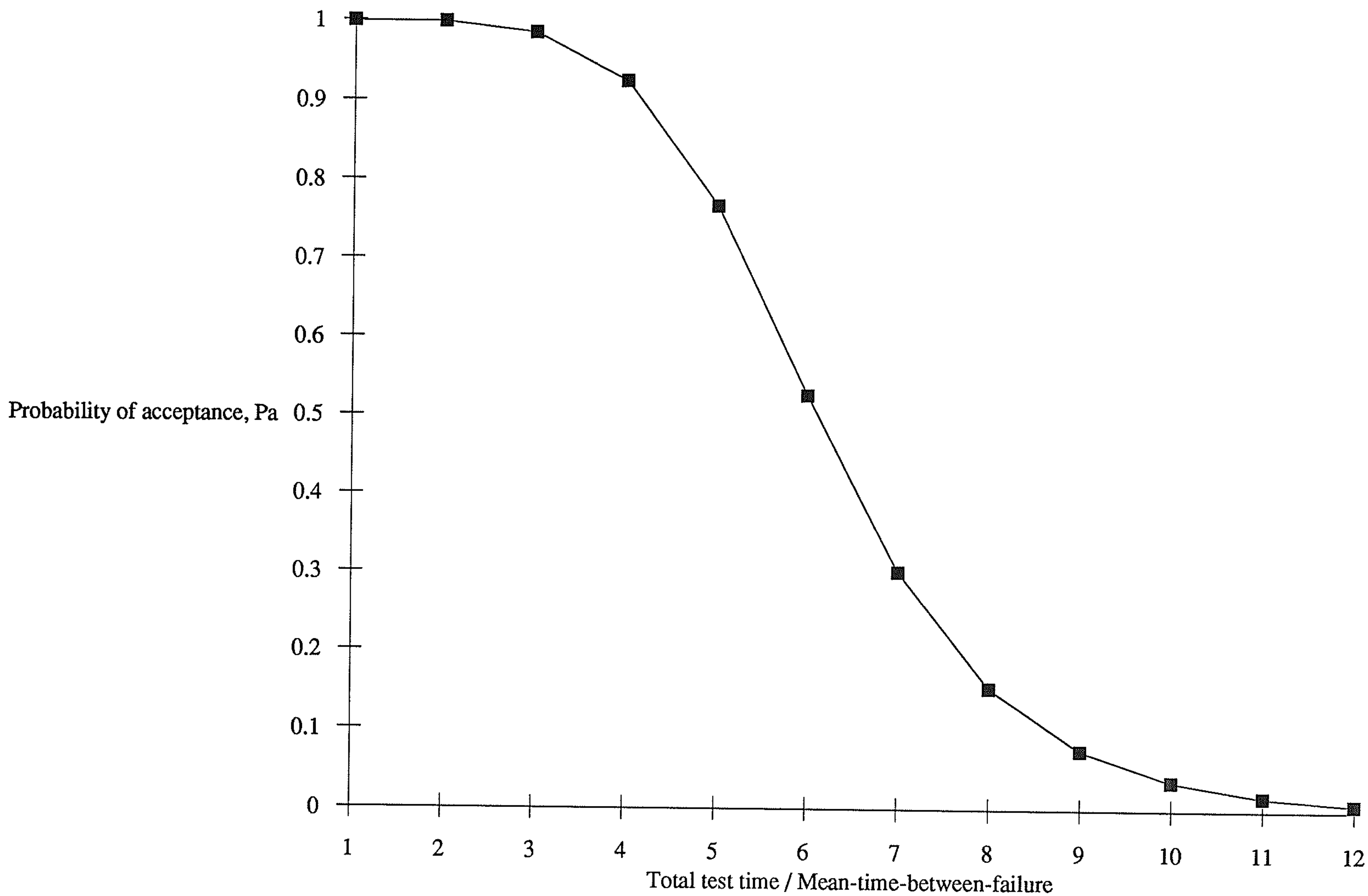
Table 3.1. Probabilities of acceptance a submitted lot for DS(4,3) in terms of $\frac{T}{\theta}$.

$\frac{T}{\theta}$	$P_{a;k}$
1	0.9999
2	0.9988
3	0.9856
4	0.9248
5	0.7676
6	0.5268
7	0.3012
8	0.1541
9	0.0751
10	0.0362
11	0.0173
12	0.0083

Table 3.2. Probabilities of reaching 4200 hours for the DS(4,3) plan with sample size n.

Sample size, n	Probability of reaching 4200 hours
1	0.0150
2	0.0780
3	0.2102
4	0.3954
5	0.5898
6	0.7531
7	0.8675
8	0.9361
9	0.9721
10	0.9889
11	0.9959
12	0.9989
13	0.9996
14	0.9999
15	1.0000

Figure 3.1. The OC curve for the DS(4,3) sampling plan.



CHAPTER IV

THE EXPECTED COST OF THE DEFERRED STATE LIFE TEST PLAN WITH REPLACEMENT

A detailed discussion of the deferred state life test plan was presented in the last two chapters. When a deferred state life test plan is required to test the submitted lots, the user must know how to select an appropriate life test plan when the required conditions are given. Also, the appropriate sample size and total test time must be determined. Another consideration is the total cost of conducting a life test. Dean (1971) proposed a cost model for deferred state life test plans and then evaluated the expected cost for these life test plans. He found that the expected cost of using a deferred state life test plan would be less than the expected cost of using any other life test plans in some situations, but all deferred state test plans can provide the same producer and consumer protection. In order to make the comparisons, a similar cost model will be used in this dissertation, but a different evaluation technique will be presented to see whether the deferred state life test plans may reduce the overall test cost. As mentioned before, the failed items during a life test may or may not be replaced by a new item drawn from the remainder of the same lot. In this chapter we will discuss the replacement case only. The cost model for the deferred state life test plan with replacement is defined as

$$K = \frac{I\,C\,N\,T_{R(r,b)}(W)}{n} + C_s + C_1\,n + C_2\,y, \tag{4.1}$$

with the notation

K = the total cost of conducting the life test,

I = the carrying cost index which is defined as proportional to the average inventory observed during the same year and its range is in the neighbourhood of 0.2 or 0.3 for a specified time interval (This carrying cost includes opportunity costs, space rental cost, labour cost, etc.),

C = the item cost,

N = the total number of items in the lot,

$T_{R(r,b)}(W)$ = the total accumulated waiting time of the current lot in inventory before disposition when using DS(r,b) plans with replacement,

n = the sample size,

C_s = the set-up cost,

C_1 = the cost of testing each items,

C_2 = the cost of testing each items per unit time, and

y = the total accumulated test time of a sample.

The first term in formula (4.1) is the total carrying cost of a submitted lot before disposition. When the total accumulated waiting time, $T_{R(r,b)}(W)$, is divided by the sample size, n, it becomes the calendar waiting time that the lot is hold in inventory. When we multiply the item cost, C, with the total number of items in the lot, N, it becomes the total cost of the submitted lot. Finally, the product of the carrying cost index, I, the total cost of the submitted lot, C N, and the calendar waiting time that the lot is hold in inventory, $\frac{T_{R(r,b)}(W)}{n}$, will give the total carrying cost of a submitted lot before disposition. The second term in formula (4.1) is the set-up cost, C_s. This is a fixed cost, no matter how many items are in the sample or how long the test conducts. The third term is the total cost of testing the sample of n items and the last term is the total cost of testing time per sample.

In the replacement case, a sample of n items is placed on a deferred state life test for the total test time T and each failed item during the test will be replaced by a new item drawn from the remainder of the same lot. In order to develop a general expression of expected cost for DS(r,b) plans with replacement, the DS(0,1) plan will be considered first. Let t_1 and t_2 be the failure times of the first and the second items. Since t_1 and t_2 are independently and exponentially distributed random variables, their density functions would be, in terms of the failure time t and the mean-time-between-failure θ,

$$f(t) = \frac{1}{\theta} e^{\frac{-t}{\theta}} \qquad 0 < t < T. \tag{4.2}$$

Then the sum of t_1 and t_2, i.e. the total accumulated test time of a sample, will be a random variable with a gamma distribution (Appendix A). This sum,

$$y = t_1 + t_2, \tag{4.3}$$

will have a gamma distribution which is truncated at T, and its density function is

$$g(y) = \frac{y e^{\frac{-y}{\theta}}}{\theta^2} \qquad 0 \le y \le T. \tag{4.4}$$

The above formula can easily be obtained by substituting 2 for m, y for t, and $\frac{1}{\theta}$ for λ into formula (A.9). From formula (A.12), we see that the cumulative function of y is

$$G(y) = 1 - \sum_{i=0}^{1} \frac{\left(\frac{y}{\theta}\right)^i e^{\frac{-y}{\theta}}}{i!}$$

$$= 1 - e^{\frac{-y}{\theta}} - \frac{ye^{\frac{-y}{\theta}}}{\theta}. \tag{4.5}$$

Moreover, the expected total accumulated test time of a sample for DS(0,1) plan with replacement, $E_{R(0,1)}(y)$, would be written as

$$E_{R(0,1)}(y) = \int_0^{\infty} yg(y)\, dy \qquad 0 \le y \le T$$

$$= \int_0^{T} yg(y)\, dy + \int_T^{\infty} yg(y)\, dy. \tag{4.6}$$

but, since y is truncated at T, formula (4.6) becomes

$$E_{R(0,1)}(y) = \int_0^{T} yg(y)\, dy + T\int_T^{\infty} g(y)\, dy$$

$$= \int_0^{T} yg(y)\, dy + T\,[1 - G(T)]. \tag{4.7}$$

By using formulas (4.4) and (4.5), formula (4.7) becomes

$$E_{R(0,1)}(y) = \int_0^{T} \frac{y^2 e^{\frac{-y}{\theta}}}{\theta^2} dy + T\left[e^{\frac{-T}{\theta}} + \frac{Te^{\frac{-T}{\theta}}}{\theta} \right]$$

$$= \frac{1}{\theta^2}\int_0^{T} y^2 e^{\frac{-y}{\theta}}\, dy + Te^{\frac{-T}{\theta}} + \frac{T^2 e^{\frac{-T}{\theta}}}{\theta}$$

$$= \frac{1}{\theta^2}\int_0^T y^2(-\theta)de^{\frac{-y}{\theta}} + Te^{\frac{-T}{\theta}} + \frac{T^2e^{\frac{-T}{\theta}}}{\theta}$$

$$= \frac{-1}{\theta}\left[y^2 e^{\frac{-y}{\theta}}\Big|_0^T - \int_0^T e^{\frac{-y}{\theta}} dy^2 \right] + Te^{\frac{-T}{\theta}} + \frac{T^2e^{\frac{-T}{\theta}}}{\theta}$$

$$= \frac{-1}{\theta}\left[T^2e^{\frac{-T}{\theta}} - \int_0^T 2ye^{\frac{-y}{\theta}} dy \right] + Te^{\frac{-T}{\theta}} + \frac{T^2e^{\frac{-T}{\theta}}}{\theta}$$

$$= \frac{-T^2e^{\frac{-T}{\theta}}}{\theta} - 2\int_0^T yde^{\frac{-y}{\theta}} + Te^{\frac{-T}{\theta}} + \frac{T^2e^{\frac{-T}{\theta}}}{\theta}$$

$$= -2\left[y\, e^{\frac{-y}{\theta}}\Big|_0^T - \int_0^T e^{\frac{-y}{\theta}} dy \right] + Te^{\frac{-T}{\theta}}$$

$$= -2Te^{\frac{-T}{\theta}} + 2(-\theta)e^{\frac{-y}{\theta}}\Big|_0^T + Te^{\frac{-T}{\theta}}$$

$$= -2Te^{\frac{-T}{\theta}} - 2\theta e^{\frac{-T}{\theta}} + 2\theta + Te^{\frac{-T}{\theta}}, \text{ or}$$

$$E_{R(0,1)}(y) = 2\theta\left[1 - e^{\frac{-T}{\theta}}\right] - Te^{\frac{-T}{\theta}}. \tag{4.8}$$

Furthermore, let $ET_{R(0,1)}(W = i)$ be the expected total accumulated waiting time of the current lot in inventory when this lot waits for i additional lots before disposition for the DS(0,1) plan with replacement. Let $P_{R(0,1)}(W = i)$ be the probability that the current lot has to wait for i additional lots before disposition for the DS(0,1) plan with replacement. First, consider the expected total accumulated waiting time of the current lot in inventory when a disposition decision can be made on the current sampling test, i.e. the current lot does not have to wait for additional lots before making disposition decision, then

$$ET_{R(0,1)}(W = 0) = E_{R(0,1)}(y). \tag{4.9}$$

Writing $P_i = \dfrac{\left(\dfrac{T}{\theta}\right)^i e^{\frac{-T}{\theta}}}{i!}$, we have

$$P_{R(0,1)}(W = 0) = P_0 + P_2 \tag{4.10}$$

for the probability that the current lot does not have to wait for additional lots before disposition. Second, consider the expected total accumulated waiting time of the current lot which has to wait for one additional lot before disposition, i.e.

$$ET_{R(0,1)}(W = 1) = T + E_{R(0,1)}(y) \tag{4.11}$$

and we obtain

$$P_{R(0,1)}(W = 1) = P_1\ [P_0 + P_2]. \tag{4.12}$$

for the probability that the current lot has to wait for one additional lot before disposition. Similarly, the expected total accumulated waiting time of the current lot which waits for i additional lots before disposition can be written as

$$ET_{R(0,1)}(W = i) = i\,T + E_{R(0,1)}(y) \tag{4.13}$$

with the associated probability that the current lot has to wait for i additional lots before disposition

$$P_{R(0,1)}(W = i) = P_1{}^i\,[P_0 + P_2]. \tag{4.14}$$

The expected total accumulated waiting time of the current lot before disposition for the DS(0,1) plan with replacement, $ET_{R(0,1)}(W)$, is

$$ET_{R(0,1)}(W) = \sum_{i=0}^{\infty} ET_{R(0,1)}(W = i)\, P_{R(0,1)}(W = i). \tag{4.15}$$

Substituting formula's (4.9) to (4.14) into (4.15), we get

$$\begin{aligned} ET_{R(0,1)}(W) &= E_{R(0,1)}(y)\,[P_0 + P_2] + [T + E_{R(0,1)}(y)]\,[P_1\,(P_0 + P_2)] + \ldots \\ &\quad + [i\,T + E_{R(0,1)}(y)]\,[P_1{}^i\,(P_0 + P_2)] + \ldots \\ &= [P_0+P_2]\,[E_{R(0,1)}(y)\,(1 + P_1 + \ldots + P_1{}^i + \ldots) + \\ &\quad T\,P_1\,(1 + 2\,P_1 + 3\,P_1{}^2 + \ldots + i\,P_1{}^{i-1} + \ldots)] \end{aligned}$$

$$= [P_0+P_2] \left[\frac{E_{R(0,1)}(y)}{1-P_1} + \frac{T\,P_1}{(1-P_1)^2} \right]. \tag{4.16}$$

Since P_0, P_1, and P_2 are mutually exclusive and exhaustive, the sum of P_0 and P_2 will be equal to 1 - P_1 and (4.16) becomes

$$ET_{R(0,1)}(W) = E_{R(0,1)}(y) + \frac{T\,P_1}{1-P_1}. \tag{4.17}$$

As a result, the expected total sampling test cost for the DS(0,1) plan with replacement, $EK_{R(0,1)}$, is

$$\begin{aligned} EK_{R(0,1)} &= \frac{ICN}{n} ET_{R(0,1)}(W) + C_s + C_1\, n + C_2\, E_{R(0,1)}(y) \\ &= \frac{ICN}{n} \left[E_{R(0,1)}(y) + \frac{T\,P}{1-P_1} \right] + C_s + C_1\, n + C_2\, E_{R(0,1)}(y). \end{aligned} \tag{4.18}$$

We will now develop a general expression for the expected total test cost of the DS(r,b) plan with replacement. Let t_i be the failure time of the i^{th} item. Since the t_i's are independently and exponentially distributed, and $1 \leq i \leq r+b+1$, their density distributions would be given by formula (4.2). Also, the sum of all t_i's will be a random variable with a gamma distribution. Let y be the sum of all t_i's, i.e. the total accumulated test time of a sample:

$$y = t_1 + t_2 + t_3 + \ldots + t_{r+b+1}. \tag{4.19}$$

It follows that y has a gamma distribution truncated at T, with density function

$$g(y) = \frac{y^{r+b} e^{\frac{-y}{\theta}}}{\theta^{r+b+1}(r+b)!} \qquad 0 \le y \le T. \tag{4.20}$$

The above formula can easily be obtained by substituting r+b+1, y and $\frac{1}{\theta}$ for m, t and λ respectively in formula (A.9). And using formula (A.12), we find the cumulative distribution function

$$G(y) = 1 - \sum_{i=0}^{r+b} \frac{\left(\frac{y}{\theta}\right)^i e^{\frac{-y}{\theta}}}{i!}. \tag{4.21}$$

Moreover, the expected total accumulated test time of a sample for DS(r,b) plans with replacement, $E_{R(r,b)}(y)$, can be written as

$$\begin{aligned} E_{R(r,b)}(y) &= \int_0^{\infty} y g(y)\, dy \qquad 0 \le y \le T \\ &= \int_0^{T} y g(y)\, dy + \int_T^{\infty} y g(y)\, dy. \end{aligned} \tag{4.22}$$

Since y is truncated at T, formula (4.22) becomes

$$\begin{aligned} E_{R(r,b)}(y) &= \int_0^{T} y g(y)\, dy + T\int_T^{\infty} g(y)\, dy \\ &= \int_0^{T} y g(y)\, dy + T\,[1 - G(T)]. \end{aligned} \tag{4.23}$$

By using (4.20) and (4.21), (4.23) becomes

$$E_{R(r,b)}(y) = \int_0^T \frac{y^{r+b+1} e^{\frac{-y}{\theta}}}{\theta^{r+b+1}(r+b)!} dy + T\left[\sum_{i=0}^{r+b} \frac{\left(\frac{T}{\theta}\right)^i e^{\frac{-T}{\theta}}}{i!}\right]. \tag{4.24}$$

Furthermore, let $ET_{R(r,b)}(W=i)$ be the expected total accumulated waiting time of the current lot in inventory when this lot waits for i additional lots before disposition for DS(r,b) plans with replacement. Let $P_{R(r,b)}(W=i)$ be the probability that the current lot has to wait for i additional lots before disposition for DS(r,b) plans with replacement. First, consider the expected total accumulated waiting time of the current lot in inventory if a disposition decision can be made on the current sampling test, i.e. if the current lot does not have to wait for additional lots before making the decision, then

$$ET_{R(r,b)}(W=0) = E_{R(r,b)}(y) \tag{4.25}$$

and the probability that the current lot does not have to wait for additional lots before disposition is

$$P_{R(r,b)}(W=0) = \sum_{i=0}^{r} P_i + \sum_{i=r+b+1}^{\infty} P_i \tag{4.26}$$

Second, consider the expected total accumulated waiting time of the current lot which has to wait for one additional lot before disposition:

$$ET_{R(r,b)}(W = 1) = T + E_{R(r,b)}(y) \tag{4.27}$$

with the probability that the current lot has to wait for one additional lot before disposition

$$P_{R(r,b)}(W = 1) = P_{r+b}\, P_{R(r,b)}(W = 0). \tag{4.28}$$

Third, the approximate expected total accumulated waiting time of the current lot which has to wait for two additional lots before disposition can be written as

$$ET_{R(r,b)}(W = 2) = 2\,T + E_{R(r,b)}(y) \tag{4.29}$$

with the probability that the current lot has to wait for two additional lots before disposition

$$P_{R(r,b)}(W = 2) = P_{r+b}\, P_{R(r,b)}(W = 1) + P_{r+b-1}\, P_{R(r,b)}(W = 0). \tag{4.30}$$

Similarly, the approximate expected waiting time of the current lot which waits for i additional lots before disposition can be written as

$$ET_{R(r,b)}(W = i) = i\,T + E_{R(r,b)}(y) \qquad 1 \le i \le b, \tag{4.31}$$

with the probability that the current lot has to wait for i additional lots before disposition

$$\begin{aligned} P_{R(r,b)}(W = i) &= P_{r+b}\, P_{R(r,b)}(W = i\text{-}1) + P_{r+b-1}\, P_{R(r,b)}(W = i\text{-}2) + \ldots \\ &\quad + P_{r+b-(i-1)}\, P_{R(r,b)}(W = 0). \end{aligned} \tag{4.32}$$

Proceeding in the same fashion we can approximate the expected waiting time of the current lot which waits for b+j additional lots before disposition as

$$ET_{R(r,b)}(W = b+j) = (b+j)\,T + E_{R(r,b)}(y) \qquad j = 0, 1, 2, \ldots, \tag{4.33}$$

with the probability that the current lot has to wait for b+j additional lots before disposition

$$P_{R(r,b)}(W = b+j) = P_{r+b}\, P_{R(r,b)}(W = b+j-1) + P_{r+b-1}\, P_{R(r,b)}(W = b+j-2) + \ldots + P_{r+1}\, P_{R(r,b)}(W = j). \tag{4.34}$$

The expected total accumulated waiting time of the current lot before disposition for DS(r,b) plans with replacement, $ET_{R(r,b)}(W)$, is

$$ET_{R(r,b)}(W) = \sum_{i=0}^{\infty} ET_{R(r,b)}(W = i)\, P_{R(r,b)}(W = i). \tag{4.35}$$

Substituting (4.25) to (4.34) in formula (4.35), we get

$$\begin{aligned} ET_{R(r,b)}(W) &= \sum_{i=0}^{\infty} \left[i\,T + E_{R(r,b)}(y) \right] P_{R(r,b)}(W = i) \\ &= \sum_{i=0}^{\infty} \left[E_{R(r,b)}(y) P_{R(r,b)}(W = i) + i\,T\, P_{R(r,b)}(W = i) \right] \\ &= E_{R(r,b)}(y) \sum_{i=0}^{\infty} P_{R(r,b)}(W = i) + T \sum_{i=0}^{\infty} i\, P_{R(r,b)}(W = i) \end{aligned}$$

$$ET_{R(r,b)}(W) = E_{R(r,b)}(y) + T\, E_{R(r,b)}(W). \tag{4.36}$$

Note that $E_{R(r,b)}(W)$ is the expected wait for the number of additional lots which must be sampled, on the average, before disposition of the current lot, as discussed in Chapter II. As a result, the general expression of the expected total sampling test cost for DS(r,b) plans with replacement, $EK_{R(r,b)}$, is

$$\begin{aligned} EK_{R(r,b)} &= \frac{ICN}{n} ET_{R(r,b)}(W) + C_s + C_1\, n + C_2\, E_{R(r,b)}(y) \\ &= \frac{ICN}{n}\left[E_{R(r,b)}(y) + T\, E_{R(r,b)}(W)\right] + C_s + C_1\, n + C_2\, E_{R(r,b)}(y). \end{aligned} \tag{4.37}$$

In this chapter, the expected total test cost of conducting the deferred state life test plan with replacement was discussed. When a deferred state life test plan is selected to test the submitted lot, not only the appropriate test time can be determined, but also the expected total test cost of performing the deferred state life test with replacement can be calculated. After the expected total test cost is obtained, it may be compared with the expected total test costs of using other type of test plans in order to see whether the use of the deferred state life test plans will reduce the overall test cost.

CHAPTER V

COST COMPARISONS

We have shown that using a deferred state life test plan could reduce the total accumulated test time, but that a waiting line may be formed when there are any lots in a deferred state. So the carrying cost of deferred lots should be considered before a deferred state life test plan is selected to replace any other type of life test plans. In other words, the total cost of testing time per sample are reduced, but the increased total carrying cost for deferred lots may nullity this. In order to demonstrate that the deferred state life test plan can reduce the overall life testing cost, the expected total test cost for the DS(4,3) plan with replacement will be calculated for an example. Some assumed values are as follows: $I = 0.2$, $C = 50$, $N = 100$, $C_s = 1000$, $C_1 = 50$, and $C_2 = 10$. The successive values of the mean-time-between-failures (MTBF) are 1000, 750, 500, 250, and 100; and the sample sizes are 20, 40, 60, 80, and 100.

The cost model and the evaluation method for the deferred state life test plan with replacement was introduced in the last chapter. By substituting the above values in formula (4.37), the expected total sampling test cost for the DS(4,3) plan with replacement can be found. A computer program to do this, written in FORTRAN, is given in Appendix C. This program will read in the required data and calculate the expected total sampling test cost for the required DS(r,b) plan with replacement by using formula (4.37). The results obtained with the above assumed values can be found in Appendix C.

Expected Cost of the Test Plan XVIII With Replacement

As mentioned before, the DS(4,3) plan and the Test Plan XVIII of MIL-STD-781B have almost identical OC curves, i.e. they can provide the same producer and consumer protection. For the Test Plan XVIII of MIL-STD-781B, a sample of n items is selected from a submitted lot of size N. All n items will be placed on life test simultaneously until the predetermined total accumulated test time T is reached or the x+1th failure is found. The total accumulated test time T was discussed in Chapter III and equals 9.4 θ_{AQL}. The x value is the maximum number of allowable failures, 13 in this case. The expected total test cost of Test Plan XVIII with replacement will now be calculated to compare it with the expected total test cost of the DS(4,3) plan with replacement.

The same cost model, given in formula (4.1), will be used for the Test Plan XVIII. The general expression for expected total test cost of Test Plan XVIII with replacement will now be developed. Let t_i be the failure time of the ith item. Since the t_i's are independently and exponential distributed where $1 \leq i \leq x+1$, their density function, in terms of the failure time t and mean-time-between-failure θ, is

$$f(t) = \frac{1}{\theta} e^{\frac{-t}{\theta}} \qquad 0 < t < T. \qquad (5.1)$$

The sum of all t_i's will be a random variable with a gamma distribution. Let y be the sum of all t_i's, i.e. the total accumulated test time of a sample,

$$y = t_1 + t_2 + t_3 + \ldots + t_{x+1}. \qquad (5.2)$$

Then y has a gamma distribution truncated at T with density function

$$g(y) = \frac{y^x e^{\frac{-y}{\theta}}}{\theta^{x+1} x!} \qquad 0 \leq y \leq T \qquad (5.3)$$

and cumulative distribution function

$$G(y) = 1 - \sum_{i=0}^{x} \frac{\left(\frac{y}{\theta}\right)^{i} e^{\frac{-y}{\theta}}}{i!} \tag{5.4}$$

(See formula's A.9 and A.12).

The expected total accumulated test time of a sample for Test Plan XVIII with replacement, $E_{R(XVIII)}(y)$, can be written as

$$E_{R(XVIII)}(y) = \int_0^{\infty} y g(y)\, dy \qquad 0 \leq y \leq T$$

$$= \int_0^{T} y g(y)\, dy + \int_T^{\infty} y g(y)\, dy, \tag{5.5}$$

which, since y is truncated at T, becomes

$$E_{R(XVIII)}(y) = \int_0^{T} y g(y)\, dy + T \int_T^{\infty} g(y)\, dy$$

$$= \int_0^{T} y g(y)\, dy + T\,[1 - G(T)], \tag{5.6}$$

which, by substitution of (5.3) and (5.4), becomes

$$E_{R(XVIII)}(y) = \int_0^T \frac{y^{x+1} e^{\frac{-y}{\theta}}}{\theta^{x+1} x!} dy + T \left[\sum_{i=0}^{x} \frac{\left(\frac{T}{\theta}\right)^i e^{\frac{-T}{\theta}}}{i!} \right]. \tag{5.7}$$

Since there are no deferred lots in the Test Plan XVIII, the disposition decision for the current lot can be made after the current sampling test is done. As a result, the general expression of the expected total sampling test cost for the Test Plan XVIII with replacement, $EK_{R(XVIII)}$, is

$$EK_{R(XVIII)} = \frac{ICN}{n} E_{R(XVIII)}(y) + C_s + C_1 n + C_2 E_{R(XVIII)}(y) \tag{5.8}$$

Cost Comparisons

In order to compare the Test Plan XVIII with the DS(4,3) plan, the previously assumed values for the DS(4,3) plan will also be used in this Test Plan XVIII with replacement. By substituting the assumed values into (5.8), the expected total sampling test cost for the Test Plan XVIII with replacement can be found. A computer program, written in FORTRAN, is given in Appendix D. It will read in the required data and calculate the expected total sampling test cost for the Test Plan XVIII with replacement by using formula (5.8). The results obtained can be found in Appendix D.

For the purpose of comparison, the results from Appendix C and Appendix D are shown in Table 5.1. The expected total test cost is expressed as a function of MTBF and the sample size. It is observed that when the MTBF is kept constant, the expected total test cost will decrease as the sample size increases. When the sample size increases, the total cost of testing each sample will increase. But the calendar waiting time for the lot will decrease when the sample size increases, so that the total carrying cost per lot will decrease. With the assumed values, the increased cost is less than the decreased cost and the expected total test cost will decrease. On the other hand, when the sample size is kept constant, the expected total test cost will decrease as the MTBF decreases. Since the total accumulated test time decreases if the MTBF decreases, the total cost of testing time per sample and the total carrying cost per lot will decrease. As a result, the expected total cost will decrease.

When both MTBF and sample size are fixed, the cost of the DS(4,3) plan and of the Test Plan XVIII in the replacement case can be compared. It is obvious that the expected total test cost of the DS(4,3) plan is less than the expected total test cost of the Test Plan XVIII for each pair of MTBF and sample size. It means that the expected total test cost will be reduced when we use the DS(4,3) plan instead of Test Plan XVIII for all values of MTBF and sample sizes. The percentage saving in costs are shown in Table 5.1. It can be seen that the maximum savings, 27.12%, are obtained when the MTBF is 1000 hours and the sample size is 100 items; the minimum savings, 9.50%, are obtained when the MTBF is 100 hours and the sample size is 20 items, with the previously assumed input values.

Dean (1971) also used the above assumed values to calculate the expected total test costs of both the DS(4,3) plan and the Test Plan XVIII of MIL-STD-781B in the replacement case with his evaluation method. The results are shown in Table 5.2. Again the expected total test cost is expressed as a function of MTBF and the sample size. Also, when the MTBF is kept constant, the expected total test cost will decrease as the sample size increases; and when the sample size is kept constant, the expected total test cost will decrease as the sample size increases. The reasons are the same as before (Table 5.1). It is found again that the expected total test cost of the DS(4,3) plan is less than the expected total test cost of the Test Plan XVIII, for each pair of MTBFs and sample size. This means that using the DS(4,3) plan instead of the Test Plan XVIII will reduce the expected total test cost for all values of MTBF and sample size. In order to compare Dean's results with those presented in this dissertation, the last column in Table 5.2 was added, to show the percentages saved when using the DS(4,3) plan instead of using the Test Plan XVIII in the replacement case. It can be seen that the maximum saved is 30.90%, and occurs when the MTBF is 1000 hours and the sample size is 20 items; and the minimum saved is 21.82%, occurring when the MTBF is 100 hours and the sample size is 100 items again with the previously assumed input values.

In this chapter cost comparisons between the DS(4,3) plan and the Test Plan XVIII of MIL-STD-781B were presented, not only to illustrate the evaluation method introduced in this dissertation but also to discuss the results obtained by Dean. In both examples our results show that the use of the deferred state life test plan can reduce the overall sampling test cost. In some cases the savings are very substantial.

It is important to realise that the deferred state life test plans are not designed to replace all other types of life test plans. The development of the deferred state life test plans provides the user with one more choice when selecting a life test plan to test the submitted lots. The expected total test cost is one of the major factors that concern most of the users. In some situations, the deferred state life test plans can reduce the expected total test cost such as in the example discussed in this chapter.

Table 5.1. Cost comparison between Test Plan XVIII with replacement of MIL-STD-781B and the DS(4,3) plan with replacement.

MTBF (in hour)	Sample size (in unit)	Expected total test cost (in dollar)		Amount saved with DS(4,3) plan (in percentage)
		DS(4,3) plan	Test Plan XVIII	
1000	20	504140	558918	9.80%
	40	274844	327869	16.17%
	60	199079	251519	20.85%
	80	161697	213844	24.39%
	100	139667	191639	27.12%
750	20	378605	419689	9.79%
	40	206883	246652	16.12%
	60	150309	189640	20.74%
	80	122523	161634	24.20%
	100	106250	145230	26.84%
500	20	253070	280460	9.77%
	40	138922	165435	16.03%
	60	101540	127760	20.52%
	80	83348	109423	23.83%
	100	72834	98820	26.30%
250	20	127535	141230	9.70%
	40	70961	84218	15.74%
	60	52770	65880	19.90%
	80	44174	57211	22.79%
	100	39417	52410	24.79%
100	20	52214	57692	9.50%
	40	30184	35487	14.94%
	60	23508	28752	18.24%
	80	20670	25885	20.15%
	100	19367	24564	21.16%

Table 5.2. Cost comparison between Test Plan XVIII with replacement of MIL-STD-781B and the DS(4,3) plan with replacement when using Dean's results.

MTBF (in hour)	Sample size (in unit)	Expected total test cost (in dollar) DS(4,3) plan	Test Plan XVIII	Amount saved with DS(4,3) plan (in percentage)
1000	20	374865	542489	30.90%
	40	235760	336796	30.00%
	60	179641	253487	29.13%
	80	153165	213920	28.40%
	100	139766	193720	27.85%
750	20	281661	407379	30.86%
	40	177557	253334	29.91%
	60	135731	191115	28.98%
	80	116136	161702	28.18%
	100	106312	146777	27.57%
500	20	188458	272269	30.78%
	40	119355	169873	29.74%
	60	91820	128743	28.68%
	80	79108	109485	27.75%
	100	72858	99835	27.02%
250	20	95253	137159	30.55%
	40	61152	86412	29.23%
	60	47910	66372	27.82%
	80	42079	57268	26.52%
	100	39404	52892	25.50%
100	20	39332	56094	29.88%
	40	26231	36335	27.81%
	60	21564	28949	25.51%
	80	19862	25937	23.42%
	100	19332	24727	21.82%

CHAPTER VI
THE MINIMUM EXPECTED TEST COST OF THE DEFERRED STATE LIFE TEST PLAN WITH REPLACEMENT

The selection of an appropriate deferred state life test plan and the determination of the total test time of the selected deferred state life test plan were discussed in Chapter III. In this chapter, the determination of the optimal sample size for a deferred state life test plan will be presented, so that the minimum expected total test cost can be found.

Consider the DS(4,3) plan discussed in the last chapter. The expected total test costs of DS(4,3) plan in terms of the mean-time-between-failures and the sample size were listed in Table 5.1. It is observed that the expected total test cost will decrease as the sample size increases when the mean-time-between-failure is fixed. Obviously, as the sample size increases, the total cost of testing each sample will increase. On the other hand, as the sample size increases, the calendar waiting time of the lot will decrease so that the total carrying cost per lot will decrease. Based on the previously assumed input values, the increase in cost is less than the decrease in cost so that the expected total test cost will decrease as the sample size increases. It is interesting to know, for given mean-time-between-failures, whether the expected total test cost will always be decreasing or whether it will decrease to a certain minimum point and then increase again. Table 6.1 shows the expected total test costs of the DS(4,3) plan with replacement with the previously assumed values except the lot size of 2000 and sample sizes from 100 to 2000, with increments of 100.

Figure 6.1 shows the expected total test cost curves of the mean-time-between-failures of 1000, 750, 500, 250 and 100 hours with various sample sizes. All the curves are concave down with only one minimum point for each curve, and we see that, for the above values of mean-time-between-failure the optimal expected total test costs are

$ 234515.20, $ 198442.40, $ 157573.10, $ 107446.20 and $ 65860.94 for sample sizes of 1900, 1700, 1400, 1000 and 600 respectively.

This leads us to three observations regarding the above example. The first observation is that, for increasing values of mean-time-between-failure, the expected total test cost decreases to a certain minimum and then increases again as the sample size increases. When the sample size increases, the total cost of testing each sample will increase and the total carrying cost per lot will decrease at the same time. When the sample size increases, the increase in cost is less than the decrease in cost before the minimum expected total test cost is reached, so that the expected total test cost is decreasing. When the sample size increases, the increase in cost is greater than the decrease in cost after the minimum expected total test cost is reached, so that the expected total test cost is increasing.

The second observation is that the minimum expected total test cost will decrease when the mean-time-between-failure decreases. It is obvious that the total accumulated test time of the life test will decrease when the mean-time-between-failure decreases since the total accumulated test time is 4.2 times the mean-time-between-failure of the test items. As the total accumulated test time of the life test decreases, the total cost of testing time per sample and the total carrying cost per lot will decrease. Consequently, the expected total test cost will decrease when the mean-time-between-failure of the test items decreases.

The third observation is that the minimum expected total test costs for different values of the mean-time-between-failure do not occur in the same sample size. Although the total cost of testing each sample will increase as sample size increases, the increasing rates are different for different values of the mean-time-between-failure. Similarly, the decreasing rates of the total cost of testing each sample are also different for different values of the mean-time-between-failure. As a result, the minimum expected total test costs occur for different values of sample size for different values of the mean-time-between-failure.

Finally, the exact value of the optimal sample size for minimising the expected total test cost for the DS(4,3) plan with replacement can be easily obtained. For example, consider the test items with mean-time-between-failure of 1000 hours. From Table 6.1, we see that the minimum expected total test cost is \$ 234515.20 for sample size 1900. Since the increments of the sample size in Table 6.1 are 100, the exact minimum expected total test cost might not occur in the sample size of 1900. Hence we constructed Table 6.2 showing the expected total test cost of the DS(4,3) plan with replacement for sample sizes between 1900 and 1930 with increments of one; we find that the exact minimum expected total test cost is \$ 234505.10, for sample sizes 1919, 1920 and 1921. In addition, Figure 6.2 shows the expected total test cost of the DS(4,3) plan in detail, for sample sizes from 1911 to 1928 with increments of one; clearly the minimal expected total test cost will be obtained for sample sizes of 1919, 1920 and 1921.

Table 6.1. The expected total test costs of the DS(4,3) plan with replacement for various mean-time-between-failure values.

MTBF (in hour)	Sample size (in unit)	Expected total test cost (in dollar)
1000	100	1889911.00
	200	973730.50
	300	671669.80
	400	523139.60
	500	436021.30
	600	379609.10
	700	340743.40
	800	312844.00
	900	292255.70
	1000	276785.00
	1100	265036.20
	1200	256078.90
	1300	249269.00
	1400	244146.10
	1500	240372.90
	1600	237696.40
	1700	235923.00
	1800	234902.20
	1900	234515.20
	2000	234666.80
750	100	1418935.00
	200	733048.80
	300	507753.00

	400	397605.20
	500	333516.50
	600	292457.30
	700	264557.80
	800	244883.30
	900	230692.00
	1000	220339.00
	1100	212777.40
	1200	207309.40
	1300	203451.90
	1400	200859.80
	1500	199279.80
	1600	198522.50
	1700	198442.40
	1800	198926.80
	1900	199886.50
	2000	201250.30
500	100	947956.90
	200	492365.60
	300	343835.10
	400	272070.00
	500	231010.80
	600	205304.70
	700	188371.80
	800	176922.10
	900	169127.90
	1000	163892.50
	1100	160518.10
	1200	158539.50
	1300	157634.50
	1400	157573.10
	1500	158186.50
	1600	159348.20

	1700	160961.50
	1800	162951.10
	1900	165257.60
	2000	167833.50
250	100	476978.40
	200	251682.80
	300	179917.50
	400	146535.00
	500	128505.30
	600	118152.30
	700	112185.80
	800	108961.00
	900	107563.90
	1000	107446.20
	1100	108259.00
	1200	109769.70
	1300	111817.20
	1400	114286.50
	1500	117093.20
	1600	120174.10
	1700	123480.70
	1800	126975.50
	1900	130628.80
	2000	134416.70
100	100	194391.20
	200	107273.00
	300	81567.00
	400	71213.94
	500	67002.13
	600	65860.94
	700	66474.31
	800	68184.38

900	70625.56
1000	73578.50
1100	76903.63
1200	80507.88
1300	84326.88
1400	88314.56
1500	92437.25
1600	96669.63
1700	100992.20
1800	105390.10
1900	109851.50
2000	114366.60

Table 6.2. The expected total test costs of the DS(4,3) plan with replacement when the mean-time-between-failure of the test items is 1000 hours.

Sample size (in unit)	Expected total test cost (in dollar)
1900	234515.20
1901	234514.10
1902	234513.20
1903	234512.30
1904	234511.50
1905	234510.60
1906	234510.00
1907	234509.30
1908	234508.60
1909	234508.00
1910	234507.50
1911	234507.00
1912	234506.60
1913	234506.20
1914	234505.90
1915	234505.60
1916	234505.50
1917	234505.30
1918	234505.20
1919	234505.10
1920	234505.10
1921	234505.10
1922	234505.30
1923	234505.40
1924	234505.60
1925	234505.90

1926	234506.20
1927	234506.50
1928	234507.00
1929	234507.40
1930	234508.00

Figure 6.1. The expected total test costs of the DS(4,3) plan with replacement for various mean-time-between-failure values.

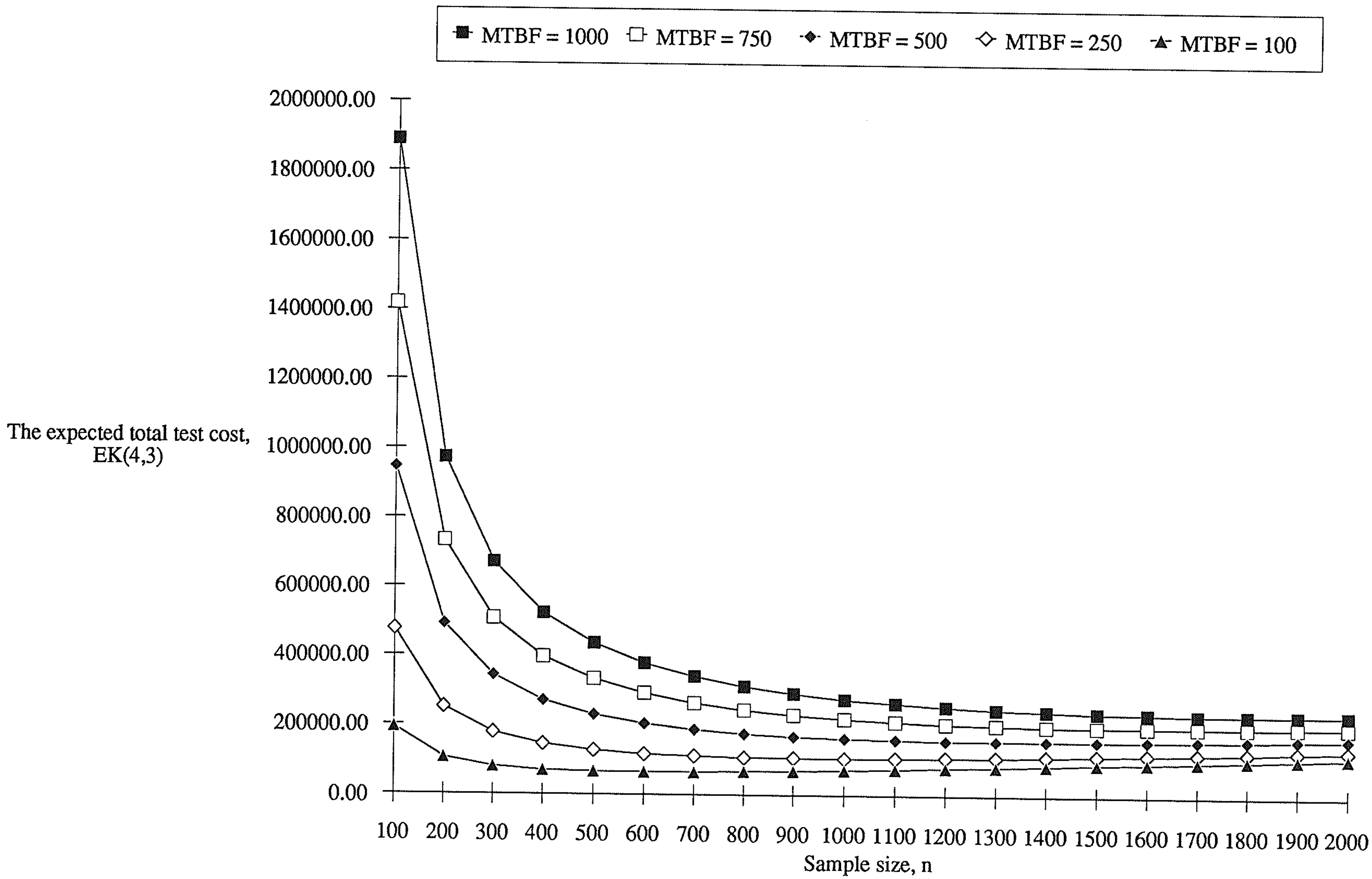
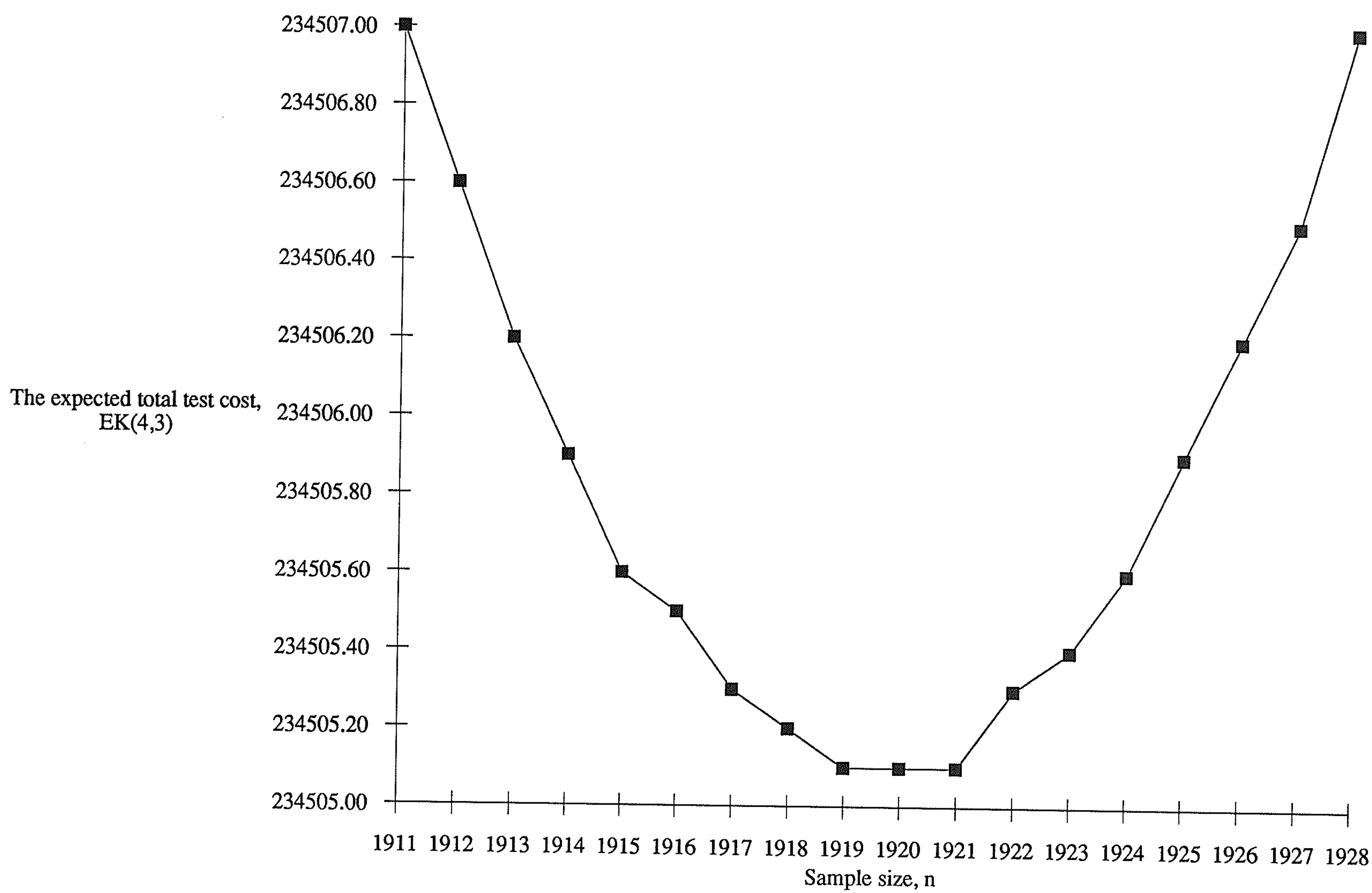


Figure 6.2. The expected total test costs of the DS(4,3) plan with replacement when the mean-time-between-failure of the test items is 1000 hours.



CHAPTER VII
SUMMARY AND CONCLUSIONS

In this dissertation, a review of the current lot-by-lot acceptance sampling plans by attributes was presented in Chapter II. In most current sampling plans, the decision to accept or reject a submitted lot depends only on the sampling test results of the current lot. Information from other lots will not be considered when making this decision. Usually, a submitted lot will be accepted if and only if the number of defective items in the test sample is less than the predetermined maximum allowable defective number; otherwise, the submitted lot will be rejected, without any further considerations. Baker (1971) developed the deferred state attribute acceptance sampling plan in the field of quality control. Those sampling plans use subsequent lots information for the decision to accept or reject the current lot; they are identified as DS(r,b) plans. Instead of making a simple acceptance or rejection decision, one of the following three decisions is made;

1) Accept the entire lot if there are r or fewer defective items in the test sample,
2) Defer the decision if there are r+1 to r+b defective items in the test sample, and
3) Reject the entire lot if there are more than r+b defective items in the test sample.

Dean (1971) used the concept of deferred state attribute acceptance sampling plans in truncated life test plans to develop the deferred state life test plans. The selection of the deferred state life test plans was discussed in Chapter III, as well as calculation of the total accumulated test time T after an appropriate DS(r,b) is chosen. When using a deferred state life test plan, a sample of n items is selected and placed on life test. The life test will be terminated until the predetermined total accumulated test time T is reached or the $r+b+1^{th}$ failed item is found. One of the following three decisions is made after the life test:

1) Accept the entire lot if there are r or fewer failed items in the test sample during the total

accumulated test time T,

2) Defer the disposition decision of the current lot if there are r+1 to r+b failed items in the test sample during the total accumulated test time T, and

3) Reject the entire lot if there are more than r+b failed items in the test sample during the total accumulated test time T.

The failed items during a life test can be replaced by new ones drawn from the remainder of the same lot. Dean developed a cost model for the deferred state life test plan with replacement and he also evaluated the expected total test cost for the DS(4,3) plan with replacement. Then he compared the results with the expected total test cost of the Test Plan XVIII of MIL-STD-781B in the replacement case since these two plans have almost identical operating characteristic curves, i.e. they provide the same producer and consumer protection.

This dissertation focussed on the deferred state life test plans. Our major concern was with the expected total test cost of the deferred state life test plan with replacement, in order to see whether the use of the deferred state life test plan can reduce the overall total test cost. For this comparison, the expected total test costs of the DS(4,3) plan with replacement and the Test Plan XVIII with replacement of MIL-STD-781B were discussed in detail. For the DS(4,3) plan, the total accumulated test time will be 4.2 times θ, the mean-time-between-failure of each test item, as compared to a total accumulated test time of 9.4θ for the Test Plan XVIII of MIL-STD-781B, so that the total accumulated test time would be reduced by using the DS(4,3) plan instead of the Test Plan XVIII of MIL-STD-781B; consequently, the total cost of testing time per sample could be reduced. But one of the limitations of the deferred state life test plans is that a waiting line may be formed by lots placed in a deferred state. Therefore the waiting time of the submitted lots before disposition will increase when the DS(4,3) plan is substituted for the Test Plan XVIII of

MIL-STD-781B. As a result, the carrying cost of the submitted lots will increase with the DS(4,3) plan.

In order to evaluate the expected total cost of the DS(4,3) plan, we introduced in Chapter IV a cost model for the DS(r,b) plan with replacement, based on the assumption that failed items during the life test will be replaced by new ones drawn from the remainder of the same lot. The expected total test cost for the DS(r,b) plan with replacement was then obtained by substituting the required data into the formulas in Chapter IV. In addition, in Chapter V a cost model was developed for the Test Plan XVIII with replacement of MIL-STD-781B, on the same replacement assumption, so that the cost comparisons could be made. The expected total test cost for the Test Plan XVIII with replacement of MIL-STD-781B was then obtained by substituting the required date into the formulas in Chapter V.

Two computer programs written in FORTRAN language are presented in Appendices C and D. The first one, used to calculate the expected total cost of the DS(r,b) plans with replacement by means of the formulas developed in this dissertation, can be found in Appendix C. The other computer program, used to calculate the expected total test cost of the Test Plan XVIII with replacement of MIL-STD-781B by means of the formulas also developed in this dissertation, can be found in Appendix D. On the basis of assumed input values the expected total test costs of the DS(4,3) plan and the Test Plan XVIII of MIL-STD-781B were obtained and the results appear in Appendices C and D. With these results the cost comparisons were made and discussed in Chapter V.

With these assumed input values, the expected total test cost of the DS(4,3) plan appeared to be less than the expected total test cost of the Test Plan XVIII of the MIL-STD-781B for each pair of mean-time-between-failure and the sample size, thus indicating that the expected total test cost will be reduced when the DS(4,3) plan is used instead of the Test Plan XVIII of MIL-STD-781B for all values of mean-time-between-failures and associated sample sizes. The maximum amount saved was around 27% in this example. Furthermore, the results of the cost comparisons in this dissertation are similar to those in Dean (1971) when the same assumed input values are used for the evaluations. In addition, the minimum expected total test cost of the deferred state life test plan with replacement was discussed in Chapter VI.

Finally, it is important to realize that the development of the deferred state life test plan is to provide one more choice in the selection of a life test plan rather than to replace all other types of life test plans. It is believed that the deferred state life test plan can reduce the overall test cost under some situations, such as in the example discussed in this dissertation.

CHAPTER VIII

RECOMMENDATIONS FOR FUTURE RESEARCH

In this dissertation, the deferred state sampling plans discussed are the fundamental models of the deferred state sampling plan, also called fixed deferred state sampling plans. Some other deferred state sampling plans could be formed after making some major or minor modifications in the fixed deferred state sampling plan. In this dissertation, a cost model was developed for the fixed deferred state attribute acceptance sampling plans with replacement. With this cost model, one can evaluate the expected total test cost before a deferred state sampling plan is used. Cost comparisons were also made between the fixed deferred state sampling plan, DS(4,3), and the Test Plan XVIII of MIL-STD-781B in the replacement case, since both plans can provide the same producer and consumer protection. The results of the cost comparisons showed that the fixed deferred state sampling plan can reduce the overall test cost. Further research could concern the development of cost models for modified deferred state sampling plans in order to see whether the overall test cost can be reduced with these modified sampling plans instead of other types of sampling plans which can provide the same producer and consumer protection. Some modified deferred state sampling plans will be suggested in this chapter.

Multiple deferred state sampling plans are an extension of the fixed deferred state sampling plans developed by Baker (1971). The feature of these multiple deferred state sampling plans is that the conditional decisions depend on the disposition of a multiple group of future lots. Early detection of quality degradation is emphasized. Baker categorized these multiple deferred state sampling plans by MDS(r,b,m), here r denotes the maximum number of allowable defective items for unconditional acceptance, b denotes the maximum number of additional defective items for conditional acceptance, and m denotes the number of future lots in which conditional acceptance is based. The operating procedure for the multiple deferred state sampling plans is outlined by the following steps;

Step 1 - For lot number k, select a random sample of n items from the submitted lot and determine the number of defective items.

Step 2 - Accept the lot if there are r or less defective items in the sample. For more than r defective items, the decision to accept or reject the current lot is dictated by the following courses of action;

r+1 to r+b defective items - Defer the decision until the disposition of the next m lots are obtained. If the next m lots are all accepted then accept the current lot number k. If any of the next m lots are rejected then reject the current lot number k.

r+i defective items (i > b) - Reject lot number k.

Step 3 - Increment k by 1 and return to step 1.

Plan evaluation, plan comparisons, and plan limitations of the multiple deferred state sampling plans could be found in Baker (1971).

Baker also developed the dependent-deferred state sampling plans. The feature of these sampling plans is that the decision to accept or reject a submitted lot will use the information of both past lots and future lots. Baker designated the dependent-deferred state sampling plans be DD(r,b,m), here r denotes the maximum number of allowable defective items for unconditional acceptance, b denotes the number of additional defective items for dependent decision, and m denotes the next number of allowable defective items for a deferred decision. The operating procedure for the dependent-deferred state sampling plans is outlined in the following steps;

Step 1 - For lot number k, (k > b), select a random sample of n items from the submitted lot and determine the number of defective items.

Step 2 - Accept the lot if there are r or less defective items in the sample. For more than r defective items, the decision to accept or reject the current lot is dictated by the following courses of actions;

r+1 defective items - Observe the disposition of the b^{th} historical lot which has already been accepted or rejected ignoring those lots in a deferred state, and accept lot number k if the b^{th} historical lot was accepted. Otherwise, reject lot number k.

. . . . . .

r+b defective items - Observe the disposition of the most recent lot that has already been accepted or rejected, and accept lot number k if this most recent lot was accepted. Otherwise, reject lot number k.

r+b+1 defective items - Defer the decision until the disposition of lot number k+m is obtained. If lot number k+m is accepted, then accept lot number k. Otherwise, reject lot number k.

. . . . . .

r+b+m defective items - Defer the decision until the disposition of lot number k+1 is obtained. If lot number k+1 is accepted, then accept lot number k. Otherwise, reject lot number k.

r+b+i defective items ($i > m$) - Reject lot number k.

Step 3 - Increment k by 1 and return to step 1.

Plan evaluation, plan comparisons, and plan limitations of the dependent-deferred state sampling plans can be found in Baker (1971).

Dean (1971) developed another deferred state life test plan which was called group/system deferred state life test plan. The basic assumption for these deferred state life test plans is that a single failure of a system's component parts may not result in failure of the system itself so that the consumer may accept a system which experiences one or more failures. Dean designated the group/system deferred state life test plan by GDS(r,b,m). Here r denotes the maximum number of failures in order for the system to be unconditionally accepted, b denotes the maximum number of additional groups each comprised of m failures which will qualify the system for deferred sentencing, and m denotes the number of system failures comprising one group. The operating procedure for the group/system deferred state life test plan is outlined in the following steps;

Step 1 - Place the submitted system number k to the life test and determine the number of failed parts.

Step 2 - Accept the system if there are r or less failures in the system. For more than r failures, the decision to accept or reject the current system is dictated by the following courses of action;

r+1 to r+m failures	- Defer decision to accept or reject the current system until the disposition of system number k+b has been determined. If system k+b is accepted, accept system number k. If system number k+b is rejected, reject system number k.
r+m+1 to r+2m failures	- Defer decision to accept or reject the current system until the disposition of system number k+b-1 has been determined. If system k+b-1 is

accepted, accept system number k. If system number k+b-1 is rejected, reject system number k.

r+(b-1)m+1 to r+bm failures - Defer decision to accept or reject the current system until the disposition of system number k+1 has been determined. If system k+1 is accepted, accept system number k. If system number k+1 is rejected, reject system number k.

r+i failures (i > bm) - Reject the system number k.

Step 3 - Increment k by 1 and return to step 1.

Detailed information for the group/system deferred state life test plan can be found in Dean (1971).

In addition, some other modified deferred state sampling plans could be developed. For instance, the concepts of the multiple deferred state sampling plans and the dependent-deferred state sampling plans could be combined together to develop a multiple dependent-deferred state sampling plans. Also, the concepts of the group/system deferred state sampling plans and the dependent-deferred state sampling plans could be combined together to develop a group/system dependent-deferred state sampling plans. After a new deferred state sampling plan is introduced, the cost models could be developed in the similar way which was discussed in this dissertation.

In conclusion, different deferred state sampling plans can be introduced in order to satisfy the requirements of different circumstances. Development of cost models could help the sampling user to select an appropriate sampling test plan which dose not only provide the adequate producer and consumer protection but also reduces the overall sampling test cost.

APPENDICES

APPENDIX A

STATISTICAL ASPECTS

The Binomial Probability Distribution

The binomial probability distribution is one of the discrete probability distributions. It represents successive terms in the binomial expansion of $(p + q)^n$ for an integer n, with $p + q = 1$

$$(p+q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{2}p^{n-2}q^2 + \ldots + q^n. \tag{A.1}$$

Sometimes, we are interested only in one or two terms of the binomial expansion. For example, if a sample of items is selected from a population of an infinite number of items or from a steady flow of items produced by a unique source, and each item can be classified by attributes such as good or defective, pass or fail, etc. with probabilities p and q respectively, the binomial distribution will be used, provided that the assumption of independent trials is satisfied, i.e. the probability of occurrence of a defective item in one trial does not affect the probability of occurrence of a defective item in another trial. With the notation

P_x = probability of x defective items in the sample,

n = number of trials or sample size,

x = number of defective items in the sample,

p = fraction defective in the population, and

q = 1 - p = fraction good in the population,

the probability that x items in the sample are defective is

$$P_x = \frac{n!}{x!\,(n-x)!} p^x q^{n-x}. \qquad (A.2)$$

If p is equal to q, the distribution is symmetrical; otherwise, it is asymmetrical. The symmetry property is not affected by the sample size, n. However, the distribution will tend to become symmetrical when the sample size gets large enough regardless of the degree of difference between p and q. The shape of the distribution depends on the sample size, n, and the fraction defective, p. Changing either n or p will result in a different distribution. Values of the binomial probability distribution can be obtained from formula (A.2) or from binomial distribution tables. Since three variables (n, p, and x) are involved in this distribution, these tables require a large amount of space. The mean and the variance of this distribution are np and np(1 - p) respectively.

The Poisson Probability Distribution

The Poisson probability distribution is also a discrete probability distribution. This distribution is used for problems involving the number of occurrences of some event per unit of time or per sample, and these occurrences are random and independent of each other. The probability of x occurrences per unit for a Poisson distribution, in terms of the mean number of occurrences per unit λ, is

$$P_x = \frac{\lambda^x e^{-\lambda}}{x!}. \qquad (A.3)$$

Poisson probabilities can be easily determined from (A.3) or from Poisson distribution tables. Since only two variables (x and λ) are involved in the distribution, such tables do not require a large amount of space.

Generally, the Poisson distribution is skewed; however, it will become symmetric when λ gets larger. The variance of this distribution equals the mean λ. The binomial distribution distribution tends to the Poisson distribution for large n as the mean value, np, gets larger while p (or q) becomes smaller; the simpler Poisson probability distribution is a good approximation to the binomial probability distribution if $n \geq 20$, $p \leq 0.1$, and $np \leq 5$ (Bowker, 1972; Duncan, 1974). Replacing n by λ makes it simple to obtain approximate binomial values from (A.3) or from Poisson distribution tables.

The Hypergeometric Probability Distribution

The hypergeometric probability distribution is also a discrete probability distribution. It is applicable to a population with a finite number of items from which random samples are taken without replacement. In terms of

P_x = probability of x defective items in the sample,

N = lot size,

n = sample size,

X = number of defective items in the lot, and

x = number of defective items in the sample,

the probability of x occurrences is

$$P_x = \frac{\binom{X}{x}\binom{N-n}{X-x}}{\binom{N}{n}}. \qquad \text{(A.4)}$$

The binomial probability distribution is a good approximation to the more complicated hypergeometric probability distribution if N is at least ten times n. When p is replaced by

$\frac{X}{N}$, P_X can be calculated by (A.2) or from binomial distribution tables. Furthermore, if $n \geq 20$, $p \leq 0.1$, and $np \leq 5$, the even simpler Poisson probability can be used to approximate the hypergeometric probability distribution by putting $\lambda = \frac{nX}{N}$. The mean and the variance of a hypergeometric distribution are $\frac{nX}{N}$ and $\left(\frac{nX}{N}\right)\left(1-\frac{X}{N}\right)\left(\frac{N-n}{N-1}\right)$ respectively.

The Exponential Distribution

The exponential distribution is a continuous distribution. It is applicable to problems involving occurrences such as the time to failure of an item or system, if such occurrences are independent of whatever happened before. The exponential density function in terms of rate of occurrence λ and of elapsed time t before occurrence of event often has the form

$$f(t) = \lambda e^{-\lambda t}, \qquad t \geq 0 \tag{A.5}$$

and cumulative distribution function

$$F(t) = \int_0^t f(x)\,dx \qquad t \geq 0$$

$$= \int_0^t \lambda e^{-\lambda x}\,dx, \text{ or}$$

$$F(t) = 1 - e^{-\lambda t}. \tag{A.6}$$

This distribution has the property of "having no memory", i.e. at any time the probability that the next occurrence will take place after a time interval h does not depend on how long ago any previous events may or may not have occurred. This can be shown as follows:

$$P(T \geq t) = \int_t^{\infty} \lambda e^{-\lambda s} ds$$

$$= e^{-\lambda t}. \tag{A.7}$$

Hence, for $t > 0$ and any positive h,

$$P(T \geq t{+}h \mid T \geq t) = \frac{P(T \geq t+h)}{P(T \geq t)}$$

$$= \frac{e^{-\lambda(t+h)}}{e^{-\lambda t}}$$

$$= e^{-\lambda h}$$

$$= P(T > h). \tag{A.8}$$

For example, if the distribution of life of an electric bulb is exponential, the probability that a light bulb will survive to 100 hours given that it has already survived to 50 hours is the same as the probability that a new bulb will survive to 50 hours. But the failure rate, i.e. rate of occurrence, is constant only within the useful life period. If the useful life of this bulb is 1000 hours, the failure rate will remain constant until the operating time approaches 1000 hours, by which time the failure rate will increase.

There exists a relationship between the exponential and Poisson distributions. If the distribution of occurrences of an event is a Poisson distribution, then the distribution of the time between occurrences is an exponential distribution. The mean of an exponential distribution is $\frac{1}{\lambda}$ and the variance of this distribution is $\frac{1}{\lambda^2}$.

The Gamma Distribution

The gamma distribution is a continuous distribution with density function is

$$g(t) = \frac{\lambda^m t^{m-1}}{\Gamma(m)} e^{-\lambda t} \qquad t \geq 0,\ \lambda > 0,\ \text{and } m > 0. \tag{A.9}$$

Usually, m is called the shape parameter and λ is called the scale parameter. With different values chosen for m and λ, the gamma distribution can exhibit many different shapes. $\Gamma(m)$ is called gamma function and it is given by

$$\Gamma(m) = \int_0^{\infty} x^{m-1} e^{-x} dx, \tag{A.10}$$

when m is an integer, $\Gamma(m) = (m - 1)!$. For general m, the cumulative gamma distribution function is

$$G(t) = \int_0^t g(x)\, dx. \tag{A.11}$$

When m is a positive integer, formula (A.11) can be evaluated by integration by parts and may be written as

$$G(t) = 1 - \sum_{i=0}^{m-1} \frac{(\lambda t)^i}{i!} e^{-\lambda t} \qquad t \geq 0. \qquad \text{(A.12)}$$

In formula (A.12), it may be seen that the cumulative gamma distribution can be evaluated as the sum of m Poisson terms with parameter λt.

When $0 < m < 1$, the gamma distribution function, G(t), is an increasing failure rate distribution. If $m = 1$, G(t) is equal to $1 - e^{-\lambda t}$ which is an exponential distribution. When $m > 1$, G(t) is an decreasing failure rate distribution. Furthermore, if the parameter m is an integer, the gamma distribution function is the distribution of the sum of m independently exponential random variables which have the same failure rate, λ. In other words, if $x_1, x_2, \ldots, x_m$ are independently and identically distributed exponential distribution with the same failure rate, λ, then $x_1+x_2+ \ldots +x_m$ is a gamma distribution with parameter m and λ. The mean of a gamma distribution is $\frac{m}{\lambda}$ and the variance of this distribution is $\frac{m}{\lambda^2}$.

APPENDIX B

SOME EVALUATION TECHNIQUES FOR ACCEPTANCE SAMPLING

OC Curve

The operating characteristic (OC) curve is one of the common evaluation techniques and the most important characteristic of sampling plans. It shows the discriminatory power of a sampling plan. The greater the slope of the OC curve, the greater the discriminatory power (Montgomery, 1985). The OC curve gives the probability of acceptance, P_a, of a submitted lot as a function of the fraction defective of that lot is known.

As an example, the OC curve for the single sampling plan: N = 5000, n = 100, and c = 2, is shown in Figure B.1. The fraction defective, p, is on the x-axis and the probability of acceptance of a lot, P_a, is on the y axis. From Figure B.1, the probability of acceptance of a specific lot can be read off for a given fraction defective of the lot. For example, if the fraction defective of a submitted lot is 0.035, the probability of acceptance of the lot is 0.32.

There are two types of OC curve: the type A OC curve and the type B OC curve. If the lot is an isolated lot with finite size, a type A OC curve is used. For this situation, the probability of acceptance of the lot should be calculated from the hypergeometric probability distribution (Appendix A). On the other hand, if the lots are taken from a steady flow of items which are produced by a single source, a type B OC curve is used, and the probability of acceptance of the lots should be calculated from the binomial probability distribution (Appendix A). In this dissertation, all OC curves discussed will be type B OC curves.

Average Outgoing Quality

The average outgoing quality (AOQ) is the basis for an evaluation technique used in acceptance sampling (Besterfield, 1986). Consider lots taken from a steady flow of items and inspected according to a specific sampling plan. Some lots are accepted and shipped to the consumer. But some lots are rejected and returned to the producer. When the producer receives a rejected lot, all items in the lot are inspected and, after all defective items found are replaced by good items the lot with 0% defectives will be sent to the consumer. Such an acceptance sampling plan is called a rectifying inspection plan. In the long run, the average outgoing lot quality (AOQ), i.e. the average fraction defective, for all lots received by the consumer, is determined. For large lot sizes and relatively small sample sizes, AOQ can be approximately determined by multiplying the fraction defective of a submitted lot with the probability of acceptance of the lot (Burr, 1979; Montgomery, 1985), i.e.

$$AOQ \approx p \times P_a. \tag{B.1}$$

For example, consider the single sampling plan with $N = 5000$, $n = 80$ and $c = 1$. First of all, P_a values are calculated. Then the AOQ values follow from formula (B.1). The AOQ values for some assumed fraction defectives are calculated in Table B.1.

The AOQ curve for this single sampling plan is plotted in Figure B.2. From Figure B.2, the AOQ value can be read off for a given fraction defective. For example, if the fraction defective of a submitted lot is 0.035, the average outgoing quality will have a fraction defective of 0.0081. The maximum value in the AOQ curve is called the average outgoing quality limit (AOQL). For our example, the AOQL value is approximately 0.011 fraction defective. Therefore, the average outgoing quality values could not be higher than 0.011 for the single sampling plan of our example, regardless of the fraction defective of the incoming lots. Similarly, the AOQ formulas for a double sampling plan and a multiple sampling plan can be approximately determined by

$$AOQ \approx p \times (P_a)_{n_1+n_2} \quad \text{(B.2)}$$

and

$$AOQ \approx p \times (P_a)_{n_1+ \ldots +n_k} \quad \text{(B.3)}$$

respectively. The average outgoing quality of double and multiple sampling plans can be easily obtained from formulas (B.2) and (B.3).

Average Total Inspection

The average total inspection (ATI) is other an evaluation technique used in rectifying inspection plans. ATI, which is based on the average number of items inspected per lot, includes the number of items inspected in a sample as well as the remaining items inspected in the lot when the lot is rejected. When choosing a sampling plan, it is not sufficient to consider only the AOQ values. Both AOQ and ATI values are required to specify an unique sampling plan. For a single sampling plan, ATI can be determined in terms of lot size N, sample size n and probability of acceptance of the lot P_a from

$$ATI = n\,P_a + N\,(1 - P_a). \quad \text{(B.4)}$$

For example, consider single sample plan N = 5000, n = 80, and c = 1. The ATI values are calculated from (B.4) and are shown in Table B.2. The corresponding ATI curve is plotted in Figure B.3. For a fraction defective of a submitted lot 0, the ATI is equal to 80, i.e. the sample size. Since a submitted lot without defectives must be accepted, no more inspection is needed after inspecting the sample so the ATI must equal the sample size. When the fraction defective of a submitted lot is 1, the ATI is equal to 5000, i.e. the lot size. Since the submitted lot must be rejected, 100% inspection is required, so the ATI must equal the number of items in the lot. For all other p values, the ATI values will lie between the sample size, 80, and the lot size, 5000.

In calculating the ATI for a double sampling plan, three cases should be considered:

(1) The lot is accepted on the first sample with n_1 items inspected;

(2) The lot is accepted on the second sample with $n_1 + n_2$ items inspected; and

(3) The lot is rejected and all items are inspected.

The probability of the first case is $(P_a)_{n_1}$; the probability of the second case is $(P_a)_{n_2}$; and the probability of the third case is $1 - (P_a)_{n_1+n_2}$. Therefore, the ATI formula for a double sampling plan is

$$ATI = n_1(P_a)_{n_1} + (n_1+n_2)\,(P_a)_{n_2} + N\left[1-(P_a)_{n_1+n_2}\right]. \tag{B.5}$$

The ATI formula for a multiple sampling plan can be constructed in a similar way:

$$ATI = n_1(P_a)_{n_1} + \dots + (n_1 + \dots + n_k)\,(P_a)_{n_k} + N\left[1-(P_a)_{n_1+\dots+n_k}\right]. \tag{B.6}$$

ATI values for double and multiple sampling plans can be easily obtained from formulas (B.5) and (B.6).

Average Sample Number

The average sample number (ASN) is the average number of items inspected in an acceptance sampling plan. For a single sampling plan, the ASN is equal to the sample size,

n. For a double sampling plan, the ASN is given in terms of

n_k = the size of the k^{th} sample, and

P_{n_1} = probability of making a decision either accept or reject the lot on the first sample

as

$$ASN = n_1 + n_2 (1 - P_{n_1}) \tag{B.7}$$

For example, if a double sampling plan has $n_1 = 80$, $c_1 = 1$, $r_1 = 3$, $n_2 = 100$, $c_2 = 3$, and $r_2 = 4$, the probability of making a decision on the first sample is equal to the sum of the probability of accepting the lot and the probability of rejecting the lot, i.e.

$$P_{n_1} = (P_{1 \text{ or less}})_{n_1} + (P_{3 \text{ or more}})_{n_1}. \tag{B.8}$$

By substituting formula (B.8) into formula (B.7), we obtain

$$ASN = n_1 + n_2 \left[1 - (P_{1 \text{ or less}})_{n_1} - (P_{3 \text{ or more}})_{n_1}\right] \tag{B.9}$$

for calculating the ASN for this sampling plan.

For a multiple sampling plan, the formula can be derived in a similar way as in double sampling plan; in terms of

n_k = the size of the k^{th} sample, and

P_{n_k} = the probability of making the decision on the k^{th} sample,

it becomes

$$ASN = n_1 P_{n_1} + (n_1 + n_2) P_{n_2} + \ldots + (n_1 + n_2 + \ldots + n_k) P_{n_k}. \qquad \text{(B.10)}$$

Generally, ASN gives useful information for selecting an appropriate sampling plan to minimize the inspection cost.

Up to now, we have discussed four curves: OC, AOQ, ATI, and ASN curves. The OC curve and AOQ curve are protection curves; and ATI curve and ASN curve are cost curves. In practice, these four curves will be unimportant if the fraction defective increases to a value in which the probability of acceptance is 0.5 or less. In this situation, sampling inspection should be discontinued and 100% inspection used since there will be too many rejected lots when using sampling inspection. The producer should take action to improve product quality. Not until a satisfactory quality level is reached, may sampling inspection be used again.

Table B.1. Average outgoing quality for the single sampling plan:

$N = 5000$, $n = 80$, and $c = 1$.

p	np	P_a	$AOQ = p \times P_a$
0.00	0.0	1.000	0.00000
0.01	0.8	0.808	0.00808
0.02	1.6	0.525	0.01050
0.03	2.4	0.309	0.00927
0.04	3.2	0.171	0.00684
0.05	4.0	0.091	0.00455
0.06	4.8	0.047	0.00282
0.07	5.6	0.029	0.00203

Table B.2. Average total inspection for the single sampling plan:

$N = 5000$, $n = 80$, and $c = 1$.

p	np $80p$	P_a	ATI $80 + (1 - P_a)(5000 - 80)$
0	0	1	80
0.01	0.8	0.808	1021
0.02	1.6	0.525	2417
0.03	2.4	0.309	3482
0.04	3.2	0.171	4158
0.05	4.0	0.091	4549
0.06	4.8	0.047	4765
0.07	5.6	0.029	4880

Figure B.1. A typical OC curve for the single sampling plan: N = 5000, n = 100, and c = 2.

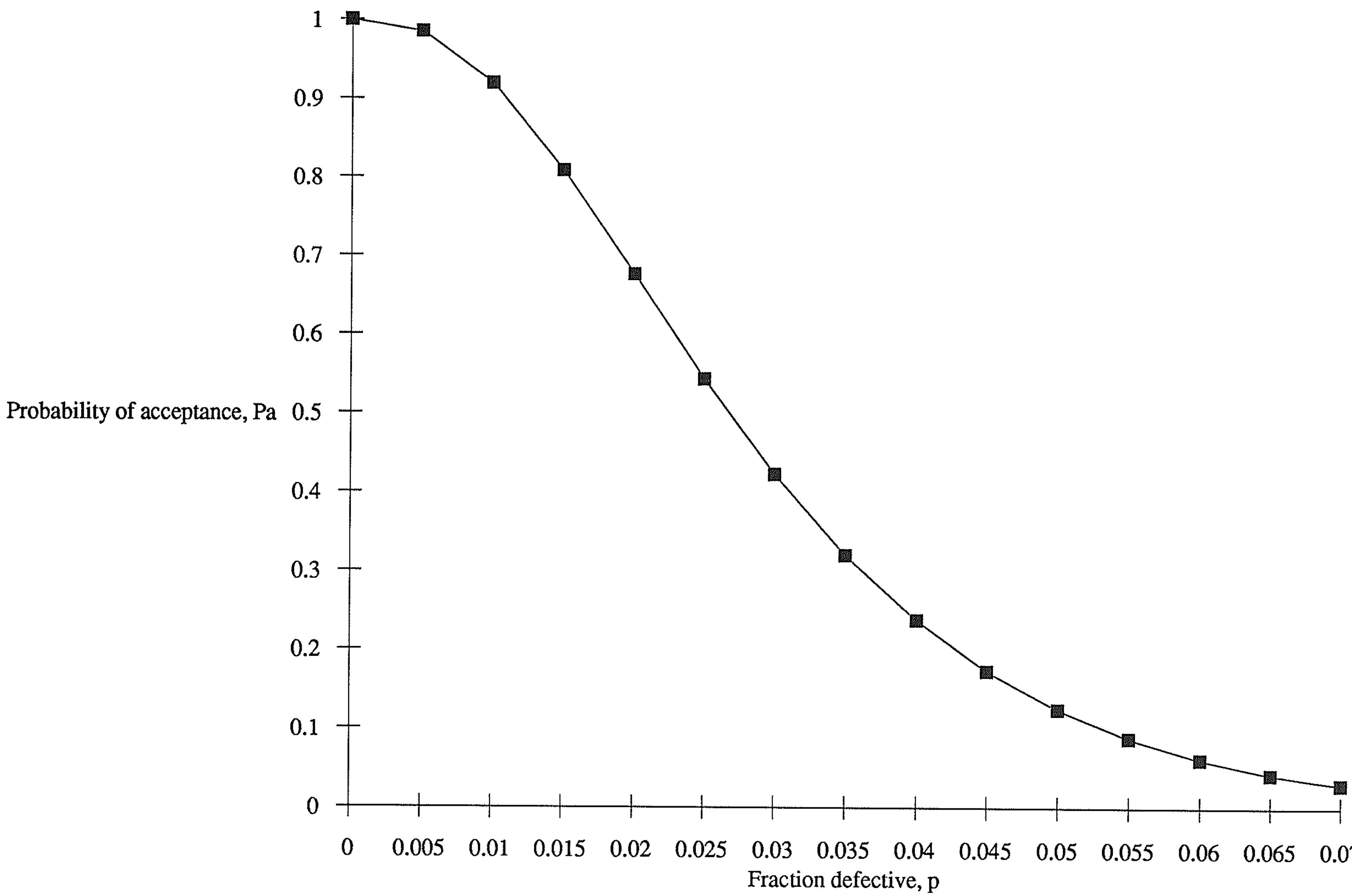


Figure B.2. The AOQ curve for the single sampling plan: N = 5000, n = 80, and c = 1.

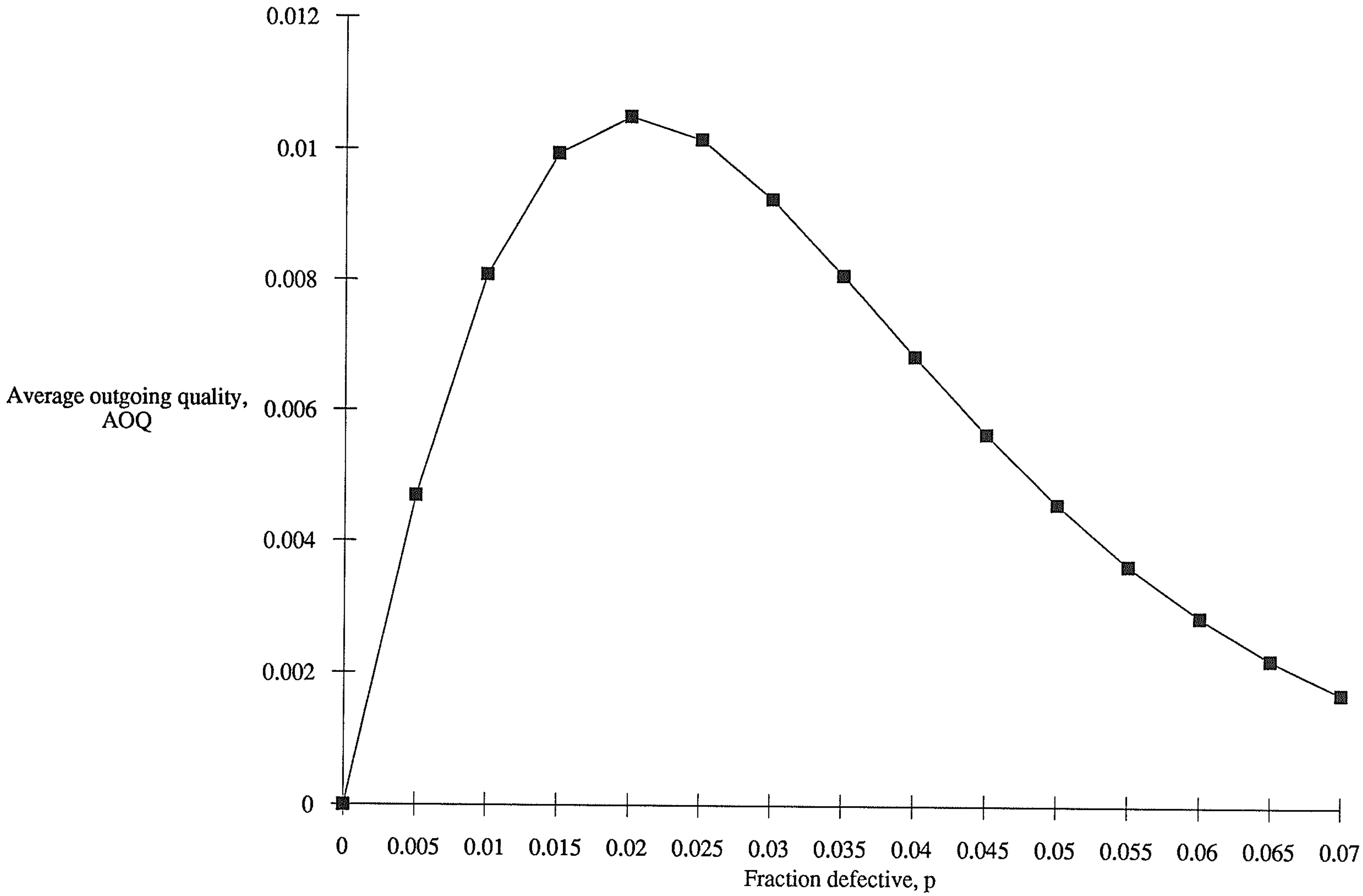
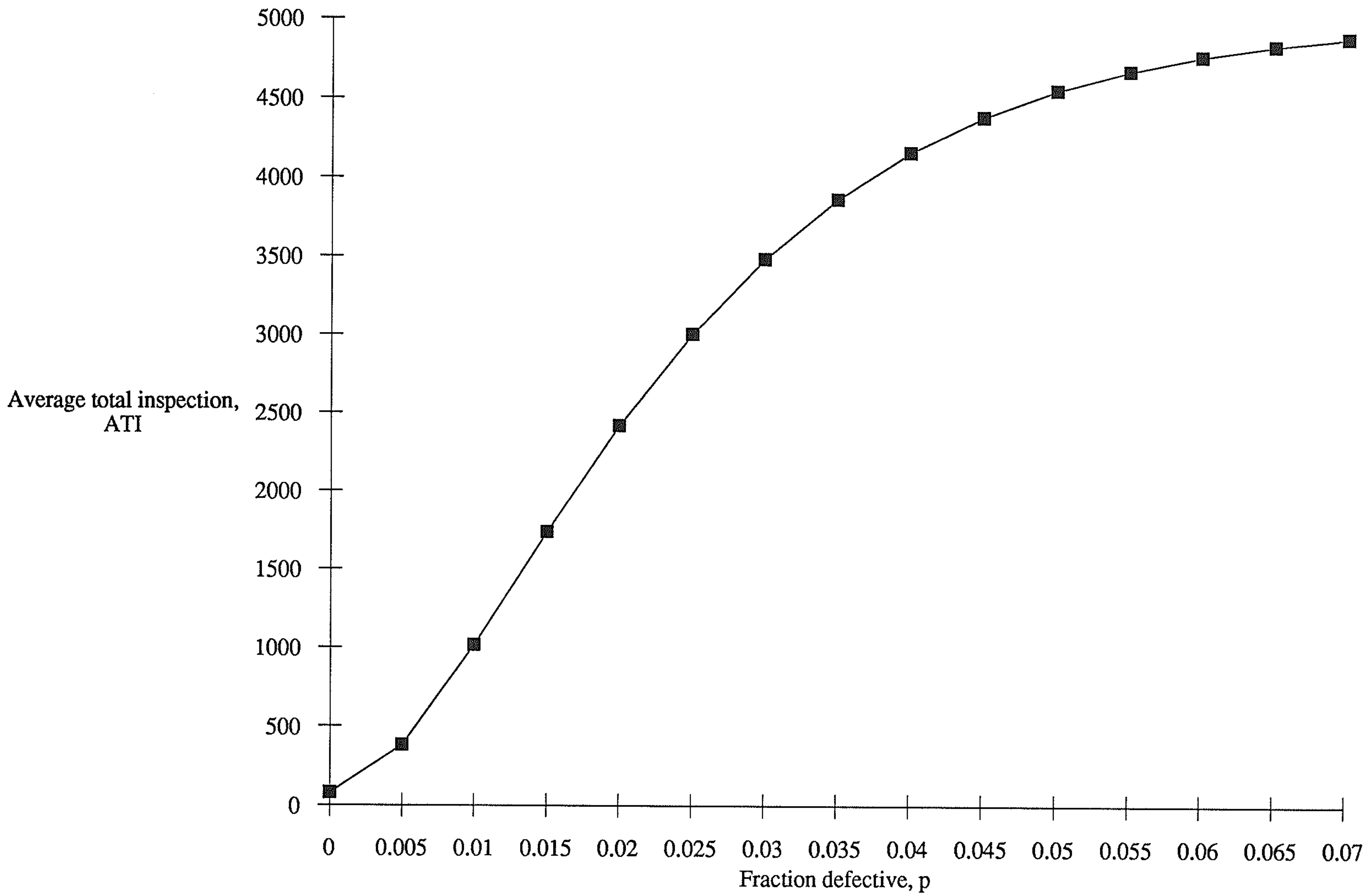


Figure B.3. The ATI curve for the single sampling plan: N = 5000, n = 80, and c = 1.



APPENDIX C

COMPUTATION OF THE EXPECTED TOTAL TEST COST FOR THE DEFERRED STATE LIFE TEST PLAN WITH REPLACEMENT

Computer Program

```
$JOB     WATFIV  DAVE,NOEXT
         INTEGER  R,B,N,ENDATA,LOTSIZ
         REAL   CS,C1,C2,I,C,T,MTBF,EXP1,EXP2,EKRRB
         READ,R,B,I,C,LOTSIZ,CS,C1,C2
C
C  where    R          = the value of r,
C           B          = the value of b,
C           I          = the carrying cost index,
C           C          = the item cost,
C           LOTSIZ     = the total number of items in the lot,
C           CS         = the set-up cost,
C           C1         = the cost of testing each item, and
C           C2         = the cost of testing each items per unit time.
C
C        This program will read in the above values and it will also read in the values of
C  MTBF and the sample size later, then calculate the expected total cost of the DS(r,b) plan
C  with replacement.  The results will be printed out in a table form.
C
         PRINT  100,R,B
```

```
      PRINT  200,I,C,LOTSIZ
      PRINT  300,CS,C1,C2
      PRINT  400
      PRINT  500
      PRINT  600
      ENDATA = 0
      EXCUTE RDDATA
      WHILE ( ENDATA .EQ. 0 ) DO
         CALL  EXPYVA(R,B,T,MTBF,EXP1)
         CALL  EXPWAT(R,B,T,MTBF,EXP2)
         EKRRB = I*C*LOTSIZ / N*(EXP1+T*EXP2) + CS + C1*N + C2*EXP1
         PRINT  700,MTBF,N,EKRRB
         EXECUTE RDDATA
      END WHILE
      PRINT  400
      STOP
C
100   FORMAT('1','THE EXPECTED COST OF DS(',I1,',','I1,') PLAN WITH
     *REPLACEMENT WHEN')
200   FORMAT('0','I = ',F4.2,',',' C = ',F5.2,',',' LOT SIZE = ',I3,',')
300   FORMAT(' ','CS = ',F7.2,',',' C1 = ',F5.2,', AND',' C2 = ',F5.2,',')
400   FORMAT('0','--------------------------------------------------------------------------
     *-------')
500   FORMAT('0',' MTBF ',5X,'SAMPLE SIZE',5X,'EXPECTED TOTAL TEST
     *COST')
```

```
600    FORMAT('0','---------',5X,'-------------------',5X,'----------------------------------
       *-------')
700    FORMAT('0',F7.2,9X,I3,14X,F10.2)
C
C      The following remote block RDDATA will read the values of MTBF and the
C sample size until no data is found, then the calculation of the expected total cost will be
C terminated. It also calculates the value of T.
C
       REMOTE BLOCK RDDATA
       READ,MTBF,N
       AT END DO
          ENDATA = 1
       END AT END
       T = 4.2*MTBF
       END BLOCK
       END
C
C      The following subprogram will calculate the value of the expected test time of a
C lot for the DS(r,b) plan with replacement, i.e. the numerical value of formula (4.24).
C
       SUBROUTINE EXPYVA(R,B,T,MTBF,EXPY)
       INTEGER  R,B,COUNT,NUM1
       REAL  T,MTBF,Y,Y1,Y2,FY1,FY2,SUM,TERM,WIDTH,SUM1,SUM2
      *        ,LAST,RSUM,LSUM,EXPY
C
```

```
      COUNT = 0
      SUM = 0
      TERM = 1
      WHILE ( COUNT .LE. (R+B) ) DO
         SUM = SUM + TERM
         COUNT = COUNT + 1
         TERM = TERM*( T / MTBF ) / COUNT
      END WHILE
      RSUM = T*EXP( -T / MTBF )*SUM
C
C           The following section will use the "Simpson's method" to estimate the numerical
C  results of the integration part in formula (4.24).
C
      WIDTH = T / 100
      SUM1 = SUM2 = 0
      Y = WIDTH
      LAST = T - 3.0*WIDTH
      WHILE ( Y .LE. LAST ) DO
         Y1 = Y
         CALL  F(R,B,MTBF,Y1,FY1)
         SUM2 = SUM2 + FY1
         Y1 = Y + WIDTH
         CALL  F(R,B,MTBF,Y1,FY1)
         SUM1 = SUM1 + FY1
         Y = Y + 2.0*WIDTH
```

```
      END WHILE
      Y1 = T - WIDTH
      CALL  F(R,B,MTBF,Y1,FY1)
      SUM2 = 4.0*( SUM2 + FY1 )
      SUM1 = 2.0*SUM1
      Y1 = T
      CALL  F(R,B,MTBF,Y1,FY1)
      LSUM = ( SUM2 + SUM1 + FY1 )*WIDTH / 3.0
C
      EXPY = LSUM + RSUM
      RETURN
      END
C
C     The following subprogram will evaluate the numerical value of the function inside
C the integration part in formula (4.24).
C
      SUBROUTINE F(R,B,MTBF,Y2,FY2)
      INTEGER  R,B,COUNT
      REAL  MTBF,Y2,FY2
      FY2 = Y2**( R+B+1 )*EXP( -Y2 / MTBF ) / ( MTBF**(R+B+1) )
      COUNT = 1
      WHILE ( COUNT .LE. (R+B) ) DO
         FY2 = FY2 / COUNT
         COUNT = COUNT + 1
      END WHILE
      RETURN
```

```
      END
C
C     The following subprogram will calculate the value of the expected wait for the
C number of additional lots which must be sampled, on the average, before disposition of
C the current lot.  The value will be used for formula (4.37).
C
      SUBROUTINE EXPWAT(R,B,T,MTBF,EXPW)
      INTEGER   R,B,NUM,COUNT,K,U,J,V,W,E,H,C
      REAL  T,MTBF,TOTAL,PROBW(1000),PROBW0,EXPW,LIMIT,VALUE
     *       ,VALUE1
C
C     The following section will calculate the numerical value of P(W = 0) for the
C DS(r,b) plan with replacement.
C
      TOTAL = 0
      COUNT = R + 1
      WHILE ( COUNT .LE. (R+B) ) DO
        CALL  PROB(T,MTBF,COUNT,VALUE)
        TOTAL = TOTAL + VALUE
        COUNT = COUNT + 1
      END WHILE
      PROBW0 = 1 - TOTAL
C
C     The following section will initiate the value of P(W = i) to zero for the DS(r,b)
C plan with replacement.  The range of i is between 1 and 1000 and it is believed that this
```

```
C range is large enough for this calculation purpose.
C
      K = 1
      WHILE ( K .LE. 1000 ) DO
         PROBW(K) = 0
         K = K +1
      END WHILE
C
C        The following section will calculate the numerical value of P(W = 1) for the
C DS(r,b) plan with replacement.
C
      NUM = R + B
      CALL PROB(T,MTBF,NUM,VALUE)
      PROBW(1) = VALUE*PROBW0
C
C        The following section will calculate the numerical values of all P(W = i)'s for the
C DS(r,b) plan with replacement when the range of i is between 2 and b.
C
      K = 2
      WHILE ( K .LE. B ) DO
         TOTAL = 0
         U = 0
         WHILE ( U .LE. (K-2) ) DO
            E = R + B - U
            CALL  PROB(T,MTBF,E,VALUE)
```

```
            TOTAL = TOTAL + VALUE*PROBW(K-1-U)
            U = U + 1
         END WHILE
         CALL  PROB(T,MTBF,E-1,VALUE)
         PROBW(K) = TOTAL + VALUE*PROBW0
         K = K + 1
      END WHILE
C
C     The following section will calculate the numerical values of all P(W = i)'s for the
C  DS(r,b) plan with replacement until the value of P(W = i) is less than 0.0000001.
C
      J = 1
      LIMIT = 1.0
      WHILE ( LIMIT .GE. 1.0E-07 ) DO
         TOTAL = 0
         V = 0
         WHILE ( V .LE. (B-1) ) DO
            H = R + B -V
            CALL  PROB(T,MTBF,H,VALUE1)
            TOTAL = TOTAL + VALUE1*PROBW(B+J-1-V)
            V = V + 1
         END WHILE
         PROBW(B+J) = TOTAL
         LIMIT = PROBW(B+J)
         J = J + 1
```

```
      END WHILE
C
C     The following section will calculate the numerical value of the expected wait for
C  the DS(r,b) plan with replacement.
C
      W = J - 1
      EXPW = 0
      COUNT = 1
      WHILE ( COUNT .LE. W ) DO
         EXPW = EXPW + COUNT*PROBW(COUNT)
         COUNT = COUNT + 1
      END WHILE
      RETURN
      END
C
C     The following subprogram will calculate the probability of have DATA failed
C  items when the total test time is T and the mean-time-between-failure is MTBF.
C
      SUBROUTINE PROB(T,MTBF,DATA,ANS)
      INTEGER  COUNT,D,DATA,ANS1
      REAL  T,MTBF,ANS
      D = DATA
      CALL  FACT(D,ANS1)
      ANS = ( ( ( T / MTBF )**DATA )*EXP( -T / MTBF) ) / ANS1
      RETURN
```

```
      END
C
C     The following subprogram will calculate the numerical value of F factorial.
C
      SUBROUTINE FACT(F,ANS2)
      INTEGER F,ANS2
      ANS2 = 1
      WHILE ( F .GT. 1 ) DO
         ANS2 =ANS2*F
         F = F - 1
      END WHILE
      RETURN
      END
$ENTRY
4  3  0.2  50.0  100  1000.0  50.0  10.0
1000    20
1000    40
1000    60
1000    80
1000    100
750     20
750     40
750     60
750     80
500     100
500     20
```

500	40
500	60
500	80
500	100
250	20
250	40
250	60
250	80
250	100
100	20
100	40
100	60
100	80
100	100

Computer Output

THE EXPECTED COST OF DS(4,3) PLAN WITH REPLACEMENT WHEN

I = 0.20, C = 50.00. LOT SIZE = 100,
CS = 1000.00, C1 = 50.00, AND C2 = 10.00.

MTBF	SAMPLE SIZE	EXPECTED TOTAL TEST COST
1000.00	20	504139.60
1000.00	40	274844.00
1000.00	60	199078.90
1000.00	80	161696.40
1000.00	100	139666.80
750.00	20	378605.20
750.00	40	206883.30
750.00	60	150309.40
750.00	80	122522.50
750.00	100	106250.30
500.00	20	253070.00
500.00	40	138922.10
500.00	60	101539.50
500.00	80	83348.31
500.00	100	72833.50
250.00	20	127535.00
250.00	40	70961.06

250.00	60	52769.79
250.00	80	44174.15
250.00	100	39416.76
100.00	20	52213.98
100.00	40	30184.43
100.00	60	23507.91
100.00	80	20669.66
100.00	100	19366.70

APPENDIX D

COMPUTATION OF THE EXPECTED TOTAL TEST COST FOR THE TEST PLAN XVIII WITH REPLACEMENT OF MIL-STD-781B

Computer Program

```
$JOB     WATFIV  DAVE,NOEXT
         INTEGER  X,N,ENDATA,LOTSIZ
         REAL  CS,C1,C2,I,C,T,MTBF,EXP1,EXVIII
         READ,X,I,C,LOTSIZ,CS,C1,C2
C
C  where     X          = the value of x,
C            I          = the carrying cost index,
C            C          = the item cost,
C            LOTSIZ     = the total number of items in the lot,
C            CS         = the set-up cost,
C            C1         = the cost of testing each item, and
C            C2         = the cost of testing each items per unit time.
C
C        This program will read in the above values and it will also read in the values of
C  MTBF and the sample size later, then calculate the expected total cost for using the Test
C  Plan XVIII with replacement of MIL-STD-781B.  The results will be printed
C  out in a table form.
         PRINT  100
         PRINT  200,I,C,LOTSIZ
```

```
      PRINT  300,CS,C1,C2
      PRINT  400
      PRINT  500
      PRINT  600
      ENDATA = 0
      EXCUTE RDDATA
      WHILE ( ENDATA .EQ. 0 ) DO
         CALL EXPYVA(X,T,MTBF,EXP1)
         EXVIII = I*C*LOTSIZ / N*EXP1 + CS + C1*N + C2*EXP1
         PRINT  700,MTBF,N,EXVIII
         EXECUTE RDDATA
      END WHILE
      PRINT  400
      STOP
C
100   FORMAT('1','THE EXPECTED COST OF THE TEST PLAN XVIII WITH
     *REPLACEMENT OF MIL-STD-781B WHEN')
200   FORMAT('0','I = ',F4.2,',',' C = ',F5.2,',',' LOT SIZE = ',I3,',')
300   FORMAT(' ','CS = ',F7.2,',',' C1 = ',F5.2,', AND',' C2 = ',F5.2,',')
400   FORMAT('0','------------------------------------------------------------------------
     *-------')
500   FORMAT('0', MTBF ',5X,'SAMPLE SIZE',5X,'EXPECTED TOTAL TEST
     *COST')
600   FORMAT('0','---------',5X,'------------------',5X,'--------------------------------
     *-------')
```

```
700     FORMAT('0',F7.2,9X,I3,14X,F10.2)
C
C       The following remote block RDDATA will read the values of MTBF and the
C  sample size until no data is found, then the calculation of the expected total cost will be
C  terminated. It also calculates the value of T.
C
        REMOTE BLOCK RDDATA
        READ,MTBF,N
        AT END DO
           ENDATA = 1
        END AT END
        T = 9.4*MTBF
        END BLOCK
        END
C
C       The following subprogram will calculate the value of the expected test time of a
C  lot for the Test Plan XVIII with replacement of MIL-STD-781B, i.e. the numerical
C  value of formula (5.7).
C
        SUBROUTINE EXPYVA(X,T,MTBF,EXPY)
        INTEGER  X,COUNT,NUM1
        REAL  T,MTBF,Y,Y1,Y2,FY1,FY2,SUM,TERM,WIDTH,SUM1,SUM2
       *        ,LAST,RSUM,LSUM,EXPY
C
        COUNT = 0
```

```
      SUM = 0
      TERM = 1
      WHILE ( COUNT .LE. X ) DO
         SUM = SUM + TERM
         COUNT = COUNT + 1
         TERM = TERM*( T / MTBF ) / COUNT
      END WHILE
      RSUM = T*EXP( -T / MTBF )*SUM
C
C        The following section will use the "Simpson's method" to estimate the numerical
C  result of the integration part in formula (5.7).
C
      WIDTH = T / 100
      SUM1 = SUM2 = 0
      Y = WIDTH
      LAST = T - 3.0*WIDTH
      WHILE ( Y .LE. LAST ) DO
         Y1 = Y
         CALL  F(X,MTBF,Y1,FY1)
         SUM2 = SUM2 + FY1
         Y1 = Y + WIDTH
         CALL  F(X,MTBF,Y1,FY1)
         SUM1 = SUM1 + FY1
         Y = Y + 2.0*WIDTH
      END WHILE
```

```
      Y1 = T - WIDTH
      CALL  F(X,MTBF,Y1,FY1)
      SUM2 = 4.0*( SUM2 + FY1 )
      SUM1 = 2.0*SUM1
      Y1 = T
      CALL  F(X,MTBF,Y1,FY1)
      LSUM = ( SUM2 + SUM1 + FY1 )*WIDTH / 3.0
C
      EXPY = LSUM + RSUM
      RETURN
      END
C
C     The following subprogram will evaluate the numerical value of the function inside
C  the integration part in formula (5.7).
C
      SUBROUTINE F(X,MTBF,Y2,FY2)
      INTEGER  X,COUNT
      REAL  MTBF,Y2,FY2
      FY2 = Y2**( X+1 )*EXP( -Y2 / MTBF ) / ( MTBF**(X+1) )
      COUNT = 1
      WHILE ( COUNT .LE. X ) DO
         FY2 = FY2 / COUNT
         COUNT = COUNT + 1
      END WHILE
      RETURN
      END
```

```
$ENTRY
13   0.2   50.0   100   1000.0   50.0   10.0
1000      20
1000      40
1000      60
1000      80
1000      100
750       20
750       40
750       60
750       80
750       100
500       20
500       40
500       60
500       80
500       100
250       20
250       40
250       60
250       80
250       100
100       20
100       40
100       60
```

100	80
100	100

Computer Output

THE EXPECTED COST OF TEST PLAN XVIII WITH REPLACEMENT OF MIL-STD-781B WHEN

I = 0.20, C = 50.00, LOT SIZE = 100,

CS = 1000.00, C1 = 50.00, AND C2 = 10.00.

MTBF	SAMPLE SIZE	EXPECTED TOTAL TEST COST
1000.00	20	558918.30
1000.00	40	327869.00
1000.00	60	251519.20
1000.00	80	213844.30
1000.00	100	191639.40
750.00	20	419689.30
750.00	40	246652.00
750.00	60	189639.60
750.00	80	161633.50
750.00	100	145229.70
500.00	20	280460.00
500.00	40	165435.00
500.00	60	127760.00
500.00	80	109422.50

500.00	100	98820.06
250.00	20	141230.10
250.00	40	84217.56
250.00	60	65880.06
250.00	80	57211.33
250.00	100	52410.07
100.00	20	57691.96
100.00	40	35486.97
100.00	60	28751.98
100.00	80	25884.49
100.00	100	24563.99

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