THE NEGATIVE BINOMIAL DISTRIBUTION: ITS VALIDITY AS A MODEL IN WHITEFISH SAMPLING

A Thesis

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CHAPTER I

INTRODUCTION

Sampling and inspection of shipments of Whitefish drawn from Canadian Lakes, prior to shipping to the United States market, is necessary in order to maintain quality in the exported fish, as well as avoid rejection of shipments that reach the United States, with the concomitant wastage and loss to the Canadian Fisherman.

Federal inspection services are made available to Fisheries in Canada for the purpose of detecting shipments with an unusually high cyst, Triaenophorus crassus, content and preventing shipment of these parcels to the United States.

An important part of the sampling inspection operation is the statistical procedure that is followed in arriving at a satisfactory sampling plan: For example, the size of the sample, the equating as nearly as possible of the probability of rejecting a satisfactory parcel with the probability of accepting an unsatisfactory parcel and the underlying theoretical distribution and its related assumptions must be considered in the implementation of a sampling plan.

Recent research on the sampling aspect of this problem was based on the assumption that the distribution of fish with cysts followed a Negative Binomial Distribution (N.B.D.) (Paul-1961; Goldsmith-1963).

The object of the present investigation is:

(1) to examine the suitability of the N.B.D. as a model in the Fisheries sampling work, and

(2) to calculate the size of sample necessary to ensure an acceptable probability (producers' and consumers') when Whitefish shipments are sampled to determine the cyst content.

HISTORY

Miller (1952) said, "*Triaenophorus crassus* is a tapeworm which, in one of its immature stages, is very common in the flesh of Whitefish in Canadian lakes. Here it appears as a yellowish cyst about one-half inch long, filled with viscous yellow fluid and a long, coiled, thin worm. These cysts, while harmless to man and animal, are objectional in appearance, and when numerous, render the fish unmarketable."

Since infested fish are unmarketable, the Canadian Government had to pass, under the Fish Inspection Act, a set of Whitefish Inspection Regulations (1954) and its amendments (1958) to control the infestation rate in exported fish.

Kennedy (1948) recommended to the Fisheries Department that a sampling plan, consisting of samples which were sufficiently large enough to give valid computations, should be used to determine whether or not a shipment of Whitefish was acceptable.

Oakland (1950) suggested a sequential sampling plan which had the advantage of requiring a smaller sample size on the average, than did the non-sequential plans. However, this plan proved impractical because more than one sample was sometimes required from a shipment and inspecting officers, as well as Fishermen, objected to this procedure.

PARAMETRIC PROCEDURE

The procedure used in calculating the parameters p and k in the Negative Binomial Distribution is the one set forth by Williamson & Bretherton (1963) in the introduction to their Negative Binomial Tables. These tables are used in the fitting of the Whitefish data as well as in the construction of the tables in determining sample size.

INTERPOLATION

Linear interpolation is used because of its simplicity, and because the difference between it and non-linear interpolation is negligible.

This negligible difference is due to the fact that the numbers involved are quite small (<1) and are negligibly affected by the two means of interpolation.

CHAPTER II

THE NEGATIVE BINOMIAL DISTRIBUTION

The N.B.D., as described by Wilks (1962), is a distribution of a discrete variable x where the variable (the number of cysts) runs from zero to infinity. Since previous workers have described this distribution in detail, only a brief discussion of it is presented here.

From the work of Goldsmith (1963) the N.B.D. has the form $(q - p)^{-k}$ where $p > 0$, $q = 1 + p$ and $k > 0$ is not necessarily an integer. From this expression the general term of the N.B.D. is

$$P(x) = \frac{(k+x-1)!}{x!(k-1)!} \frac{p^x}{q^{k+x}}$$

If one lets x equal the number of cysts ($x = 0, 1, 2, \ldots . n$) then the general term of the N.B.D. yields the expected frequencies for x cysts.

FITTING k and p FROM POPULATION MOMENTS

The population variance σ^2, of the N.B.D. is given by $\sigma^2 = kpq$, and the population mean μ by $\mu = kp$.

Thus, it follows that

$$\sigma^2 = \mu + \mu p$$

Therefore

$$\mu p = \sigma^2 - \mu$$

and hence

$$p = \frac{\sigma^2 - \mu}{\mu}$$

Using this form of p, it follows that

$$k = \frac{\mu^2}{\sigma^2 - \mu}$$

Similarly from sample moments, on replacing σ^2 by s^2 and μ by $\bar{x}$

$$p = \frac{s^2 - \bar{x}}{\bar{x}} \qquad k = \frac{\bar{x}^2}{s^2 - \bar{x}}$$

EFFICIENCY OF MOMENT METHOD

The reciprocal of the efficiency (Fisher 1941) is given by

$$E^{-1} = 1+\frac{4}{3}\frac{p}{q(k+2)} + \frac{3p^2}{q^2(k+2)(k+3)} + \ldots\ldots$$

If $p < 1/9$ for any k, or $k > 18$ for any p, then high efficiency is assured. Also the moment method has a high efficiency for small values of μ when $k/\mu > 6$, for large values of μ when $k > 13$, and for μ in the intermediate zone when $\frac{(k+\mu)(k+2)}{\mu} \geq 15$ (Bliss & Fisher 1953)

Williamson & Bretherton (1963) wrote the N.B.D. in the form $p^k(1-q)^{-k}$, with the general term

$$P(x) = \frac{(k+x-1)!}{x!(k-1)!} p^k q^x$$

where $0 \leq p \leq 1$, $k > 0$, k not necessarily an integer and $q = 1-p$.

Upon letting x equal the number of cysts ($x = 0, 1, 2, \ldots\ldots n$), the general term of the N.B.D. then yields the expected frequencies for x cysts.

METHOD OF MOMENTS AND CUMULANTS

From the general term of the N.B.D. of Williamson & Bretherton, the moment generating function is

$$M(t) = \sum_{x=0}^{\infty} e^{tx} P(x)$$

$$= \sum_{x=0}^{\infty} \frac{e^{tx}(k+x-1)!p^k q^x}{x!(k-1)!}$$

$$= \sum_{x=0}^{\infty} \frac{(k+x-1)!p^k}{x!(k-1)!} (qe^t)^x$$

$$= p^k(1-qe^t)^{-k}$$

The moments of the distribution, however, are more easily obtained from the cumulant generating function, K(t), which is the logarithm of the moment generating function; i.e.,

$$K(t) = \log M(t)$$

$$= \log \left(p^k(1-qe^t)^{-k} \right)$$

$$= -k \log \left(\frac{1-qe^t}{p} \right)$$

$$= -k \log \left(\frac{p+q-qe^t}{p} \right)$$

$$= -k \log \left(1-\frac{q}{p}(e^t-1) \right)$$

$$= -k\left(-\frac{p}{q}(e^t-1) - \frac{p^2}{q^2}(e^t-1)^2 - \ldots\ldots\right)$$

Since $e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \ldots\ldots$

$$K(t) = k\left[\frac{q}{p}\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\right) + \frac{q^2}{2p^2}\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\right)^2 + \frac{q^3}{3p^3}\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\right)^3 + \ldots\ldots\right]$$

The general term of the moment generating function is

$$\sum_{x=0}^{\infty} \mu_r^1 \frac{t^r}{r!}$$

and consequently on equating the coefficients of $\frac{t^r}{r!}$ we obtain

$$\mu_1^{\,1} = \frac{kq}{p} \text{ , the mean } \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(0)$$

$$\mu_2 = \sigma^2 = \frac{kq}{p} + \frac{k2q^2}{2p^2} = \frac{kqp + kq^2}{p^2} = \frac{kq}{p^2}, \text{ the variance.}$$

If the first two moments are estimated from sample moments, then p is estimated by $\frac{\overline{x}}{s^2}$

Setting $q = 1-p$ in (0), k is found to be estimated by $\frac{\overline{x}p}{1-p}$

APPROXIMATION OF p and k BY THE METHOD OF MAXIMUM LIKELIHOOD - HALDANE (1941)

The maximum likelihood method of estimating the parameters p and k has been discussed by many writers, e.g. Haldane (1941), Anscombe (1950), and Sichel (1951). The following approximation makes use of the method described by Haldane (1941) using the (x+1)th term of the N.B.D. which is given by

$$P(x) = \frac{(k+x-1)!\, p^k q^x}{x!(k-1)!} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Let n_x be the observed frequency of x events, R be the maximum value of x observed, the total number of observations $N = \sum_{x=0}^{R} n_x$, and the mean of x, $\mu = \frac{1}{N} \sum_{x=0}^{R} x n_x$.

In this paper, the likelihood is defined by

$$\prod_{x=0}^{R} \left[P(x) \right]^{n_x}$$

and hence the logarithm of the likelihood, L, is given by

$$L = \sum_{x=0}^{R} n_x \log P(x)$$

$$= \sum_{x=0}^{R} n_x \left(k \log p + x \log q + \sum_{s=0}^{x-1} \log(k+s) - \log x! \right)$$

To obtain the value of p and k which gives maximum likelihood, the first partial derivatives of the logarithm of the likelihood with respect to p and k are taken and set equal to zero; i.e.,

$$\text{Maximum } L \text{ when } \frac{\partial L}{\partial p} = 0$$

$$\text{and } \frac{\partial L}{\partial k} = 0$$

$$\text{Since } \frac{\partial L}{\partial p} = \sum_{x=0}^{R} n_x \left(\frac{k}{p} - \frac{x}{1-p} \right) = \sum_{x=0}^{R} n_x \left(\frac{k(1-p)-xp}{p(1-p)} \right)$$

Then L is maximum when

$$\frac{1}{pq} \sum_{x=0}^{R} n_x \left(kq-xp \right) = 0$$

i.e., when

$$\sum_{x=0}^{R} n_x kq = \sum_{x=0}^{R} n_x xp$$

and upon substituting N for $\sum_{x=0}^{R} n_x$ and μN for $\sum_{x=0}^{R} xn_x$ we obtain

$$Nkq = \mu Np$$

from whence

$$p = \frac{kq}{\mu} \quad \text{.......................................(2)}$$

Therefore the best estimate of p makes use of the arithmetic mean.

Similarly, L is maximum when

$$\frac{\partial L}{\partial k} = \sum_{x=0}^{R} n_x \left(\log p + \sum_{s=0}^{x-1} \frac{1}{k+s} \right)$$

$$= N \log p + \sum_{x=0}^{R} \frac{1}{k+x} \sum_{s=x+1}^{R} n_s = 0$$

i.e., when

$$\sum_{x=0}^{R} \frac{1}{k+x} \sum_{s=x+1}^{R} n_s = -N \log p$$

$$= N \log \frac{1}{p} \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

From (2)

$$\mu = \frac{k(1-p)}{p}$$

Thus $$p = \frac{k}{\mu+k}$$

Therefore equation (3) becomes

$$N \log \left(1+\frac{\mu}{k} \right) - \frac{n_1+n_2+\dots+n_R}{k} - \frac{n_2+n_3+\dots+n_R}{(k+1)} - \dots - \frac{n_R}{(k+R-1)} = 0 \dots (4)$$

The solution of this equation, other than $k = \infty$, provides an estimate of k. Using the iterative process of Newton-Raphson (Whittaker and Robinson - 1944), the best estimate of k is obtained.

CHAPTER III

FITTING THE NEGATIVE BINOMIAL DISTRIBUTION TO WHITEFISH DATA

In the following investigation the N.B.D. is fitted to Whitefish data from five Lakes in Manitoba. In each case the goodness of fit is measured by a Chi-Squared test.

Chi-Squared is given by

$$_{0.05}\chi^2_{n-3} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where

O_i is the observed frequency in the ith class,

E_i is the corresponding expected frequency for the ith class,

n-3 is the degree of freedom, and

0.05 is the level of significance of the test (Probability of rejecting the Null Hypothesis when it is true.)

The Null (H_0) and Alternate (H_1) hypotheses

H_0: the N.B.D. is a good model for Whitefish cyst distribution,

H_1: the N.B.D. is not a good model for Whitefish cyst distribution,

must be formulated in order to complete the test.

A significant chi-squared value designates the rejection of the H_0 and the acceptance of H_1, while for a non-significant value the reverse is true.

Expected frequencies less than one are grouped with those immediately following until the expected frequency becomes one or greater. The corresponding observed frequencies are also grouped.

TABLE 3-1. Medium Whitefish Cysts From Knee Lake (1960)

Cysts	Observed Frequency	Expected Probability	Expected Frequency
0	61	.44373	52.80
1	21	.31115	37.03
2	22	.14969	17.81
3	14	.06102	7.26
4^+	1	.03436	4.09
Total	119	.99995	118.99

Mean = 0.93 Variance = 1.25

p = 0.74 k = 2.70

The calculated chi-squared goodness of fit, with two degrees of freedom, is equal to 17.752. This signifies a poor fit of the N.B.D. to the data (since the critical value is 5.991).

TABLE 3-2 Medium Whitefish Cysts From Kisseynew Lake (1961)

Cysts	Observed Frequency	Expected Probability	Expected Frequency
0	29	.54452	43.02
1	19	.14440	11.41
2	16	.08140	6.43
3	6	.05394	4.26
4	4	.03841	3.03
5	1	.02849	2.25
6^+	4	.10875	8.59
Total	79	.99991	78.99

Mean = 1.87 Variance = 13.32

p = 0.14 k = 0.31

The calculated chi-squared goodness of fit, with four degrees of freedom, is equal to 28.120, thus signifying a poor fit (since the critical value is 9.488).

TABLE 3-3 Medium Whitefish Cysts From Dore Lake (1962)

Cysts	Observed Frequency	Expected Probability	Expected Frequency
0	490	.53121	510.49
1	219	.21024	202.04
2	121	.10905	104.80
3	59	.06099	58.61
4	28	.03535	33.97
5	14	.02092	20.10
6	9	.01255	12.06
7	8	.00760	7.30
8	3	.00464	4.46
9	6	.00284	2.73
10^+	4	.00452	4.34
Total	961	.99991	960.90

Mean = 1.10 Variance = 3.04

p = 0.36 k = 0.62

The calculated chi-squared goodness of fit, with eight degrees of freedom, is equal to 12.918, this signifying a good fit (since the critical value is 15.507).

TABLE 3-4 Medium Whitefish Cysts From South Indian Lake (1963)

Cysts	Observed Frequency	Expected Probability	Expected Frequency
0	994	.64650	1013.07
1	338	.20318	318.38
2	134	.08289	129.89
3	62	.03638	57.01
4	15	.01652	25.89
5	10	.00766	12.00
6	6	.00360	5.64
7^+	8	.00321	5.03
Total	1567	.99994	1566.91

Mean = 0.64 Variance = 1.29

p = 0.50 k = 0.63

The calculated chi-squared goodness of fit, with five degrees of freedom, is equal to 8.826, thus signifying a good fit (since the critical value is 11.070).

TABLE 3-5 Medium Whitefish Cysts From Cormorant Lake (1963)

Cysts	Observed Frequency	Expected Probability	Expected Frequency
0	31	.20255	38.28
1	30	.17207	32.52
2	31	.13856	26.19
3	26	.10952	20.70
4	18	.08574	16.20
5	11	.06673	12.61
6	13	.05174	9.78
7	4	.04000	7.56
8	6	.03087	5.83
9	5	.02378	4.49
10	4	.01829	3.46
11	4	.01405	2.66
12	2	.01079	2.04
13^{+}	4	.03523	6.66
Total	189	.99992	188.98

Mean = 3.61 Variance = 15.20

p = 0.24 k = 1.12

The calculated chi-squared goodness of fit, with eleven degrees of freedom, is equal to 8.869, thus signifying a good fit (since the critical value is 19.675).

Thus in three out of the five lakes chosen, the N.B.D. is a good model for the Whitefish data, and the following table provides a summary of the statistics calculated for each of the lakes sampled, including those of the five lakes previously presented in detail.

TABLE 3-6.

A Summary of Statistics Calculated for each of The Lakes Sampled.

Lake	Year	Number of fish	Mean	Var.	p	k	χ^2
Molson	1961	457	2.18	29.51	0.07	0.17	152.158**
"	1962	388	0.98	6.69	0.15	0.17	107.158**
"	1963	793	0.82	5.78	0.14	0.14	128.009**
Knee	1960	119	0.93	1.25	0.74	2.70	17.752*
"	1961	90	0.96	1.48	0.65	1.77	1.992
"	1962	357	1.59	8.27	0.19	0.38	32.357*
"	1963	392	0.98	1.99	0.49	0.95	3.993
"	1964	206	1.19	2.36	0.50	1.21	3.036

TABLE 3-6 CONTINUED

Lake	Year	Number of fish	Mean	Var.	p	k	χ^2
Kisseynew	1961	79	1.87	13.32	0.14	0.31	28.120*
"	1962	94	0.35	0.40	0.88	2.45	0.091
"	1963	97	0.84	4.39	0.19	0.20	20.872*
Attawapiskat	1963	222	0.44	0.72	0.61	0.69	6.764
"	1964	126	0.38	0.53	0.72	0.98	0.840
Thompson	1961	68	0.34	0.62	0.55	0.69	0.605
"	1962	215	0.36	0.89	0.40	0.24	4.540
"	1963	114	0.33	0.77	0.43	0.25	2.935
"	1964	79	0.37	0.83	0.45	0.30	9.808*
Dore	1961	1493	1.11	2.86	0.39	0.70	16.953
"	1962	961	1.10	3.04	0.36	0.62	12.918
"	1963	608	0.86	1.65	0.52	0.94	3.639
Reed	1963	263	9.16	174.06	0.05	0.51	57.894*
" Large	1963	31	5.77	75.71	0.08	0.48	12.584

TABLE 3-6 CONTINUED

Lake	Year	Number of fish	Mean	Var.	p	k	χ^2
South Indian	1961	1594	0.76	3.84	0.20	0.19	500.288**
"	1962	1528	0.64	2.28	0.28	0.25	202.316**
"	1963	1567	0.64	1.29	0.50	0.63	8.826
"	1964	812	0.59	0.96	0.61	0.94	10.620*
" Large	1961	527	0.86	4.96	0.17	0.18	233.451**
" "	1962	81	0.86	1.62	0.53	0.97	0.915
" "	1963	246	0.57	2.85	0.20	0.14	632.499**
" Jumbo	1961	353	0.50	1.65	0.77	1.67	9.305*
" "	1962	35	0.49	0.73	0.67	1.00	4.254*
" "	1963	156	0.58	16.36	0.04	0.02	-------
Cormorant	1963	189	3.61	15.20	0.24	1.12	8.869
Paint	1963	137	3.49	17.06	0.20	0.90	9.478

From table 3-6, one can see that the N.B.D. is a suitable model for Whitefish data in just over one-half of the lakes sampled. This fit or lack of fit in the various sets of data may be due to many factors:

1) A favourable sample may have been drawn even from a highly infested lake.

2) An unfavourable sample may have been drawn from a previously favourable lake.

3) The grouping of the data in the table may have caused a few fits to be unfavourable.

The data for Molson Lake, for example, does not fit the N.B.D. The values of the parameters p and k for this lake are quite compatible with each other, while the mean and variance are larger than average. The same is basically true for South Indian Lake.

The data for Dore Lake, on the other hand, does fit the N.B.D., with the values of the parameters p and k being reasonably well spaced.

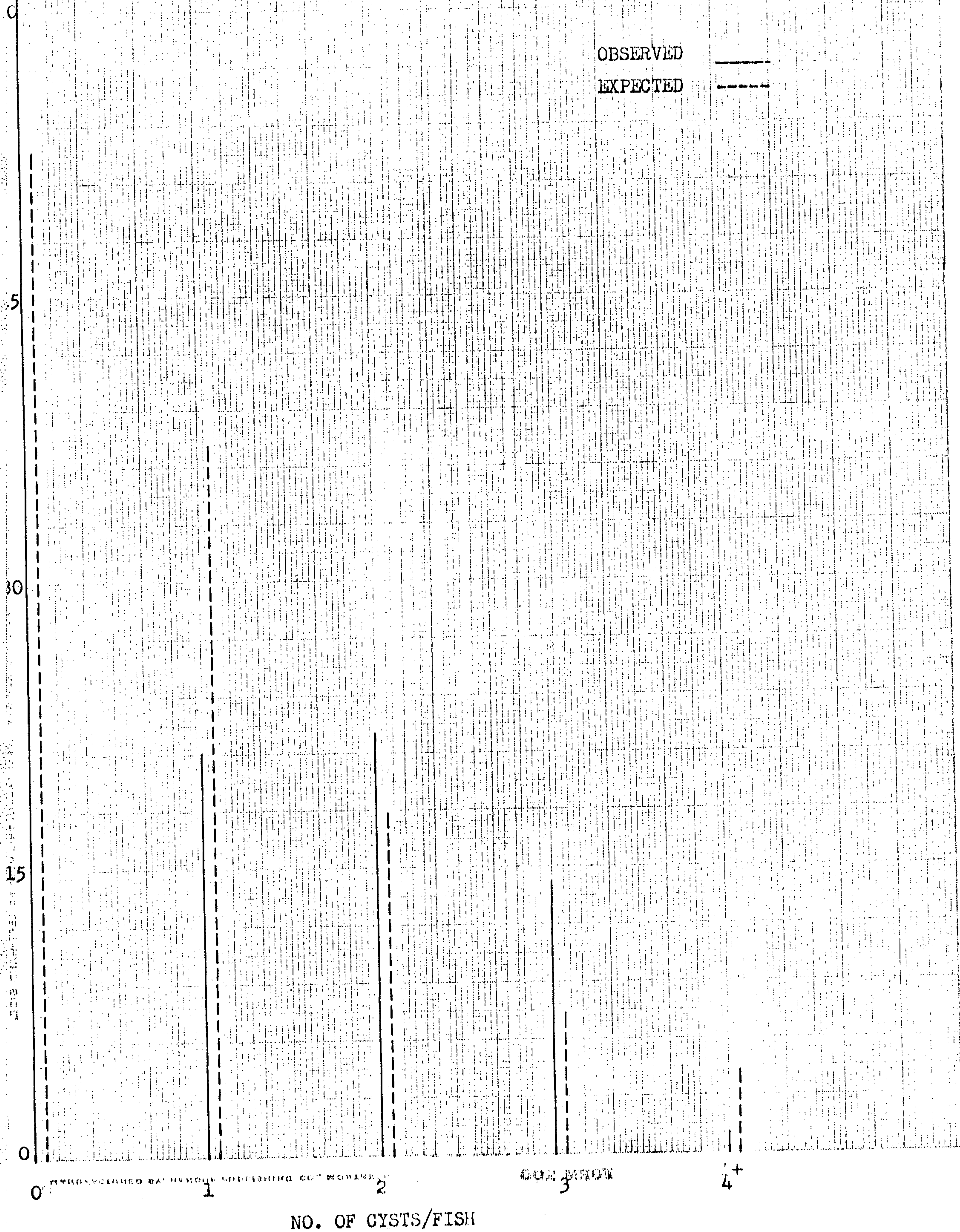
Thus, on the whole, the parameters p and k are quite important in determining goodness of fit of Whitefish data.

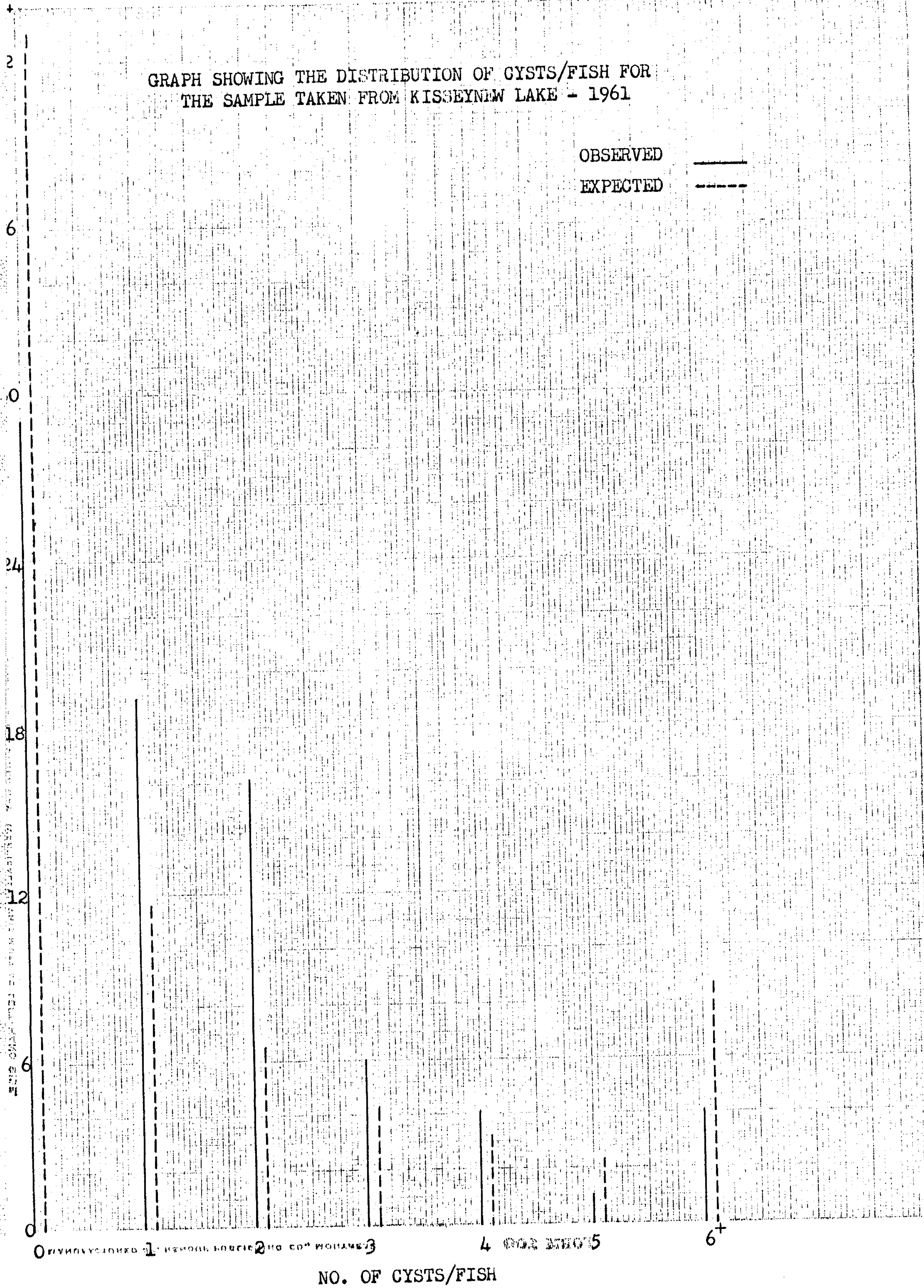
Even though the data from the lakes sampled fits the N.B.D. in only about fifty percent of the cases, and some variable factors enter into the fitting, the N.B.D. will be considered as a suitable model for Whitefish cyst data.

GRAPHICAL REPRESENTATION

The following is a graphical representation of the number of cysts per fish (observed and expected) for the five lakes which were previously dealt with in detail.

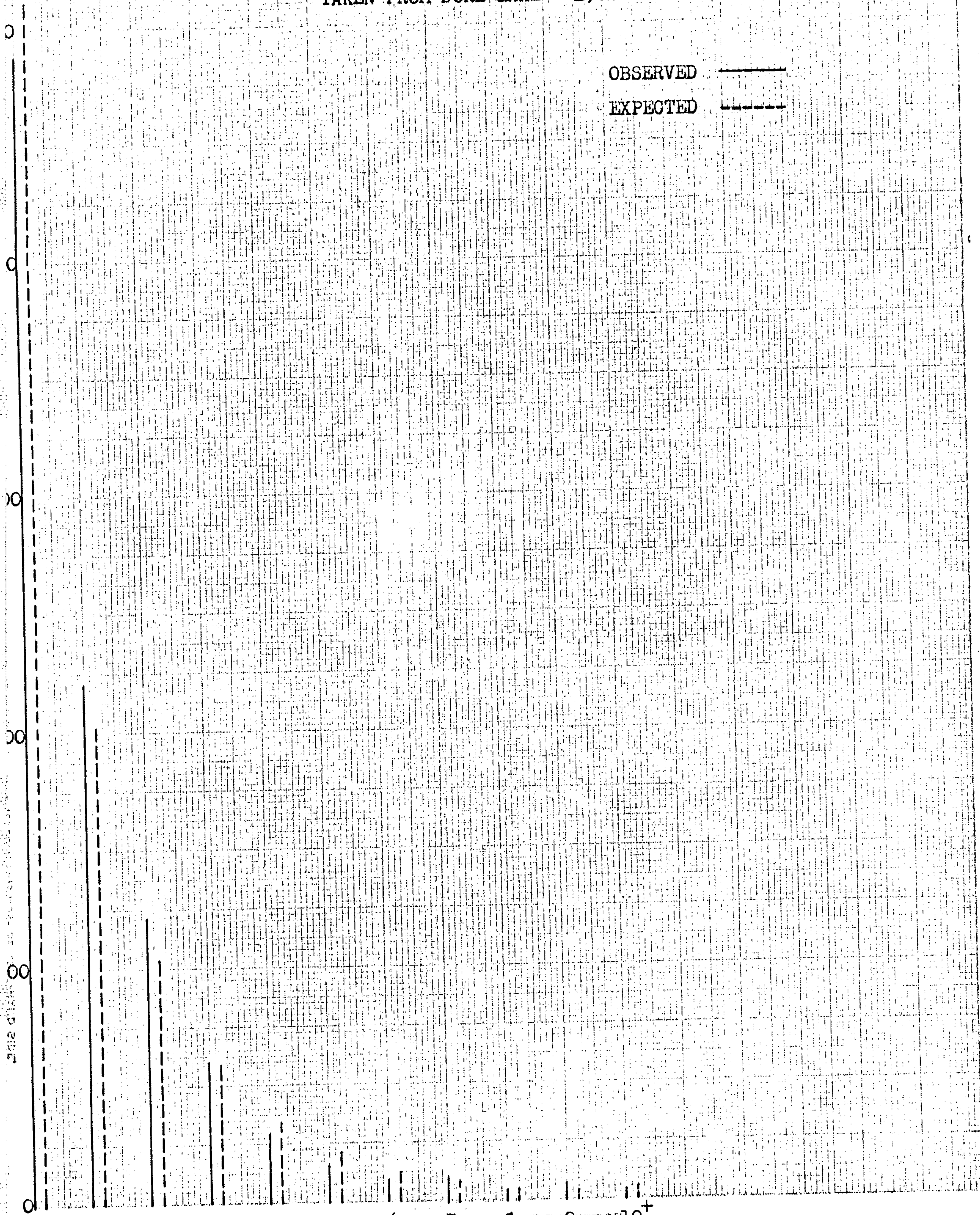
GRAPH SHOWING THE DISTRIBUTION OF CYSTS/FISH FOR THE SAMPLE TAKEN FROM KNEE LAKE - 1960





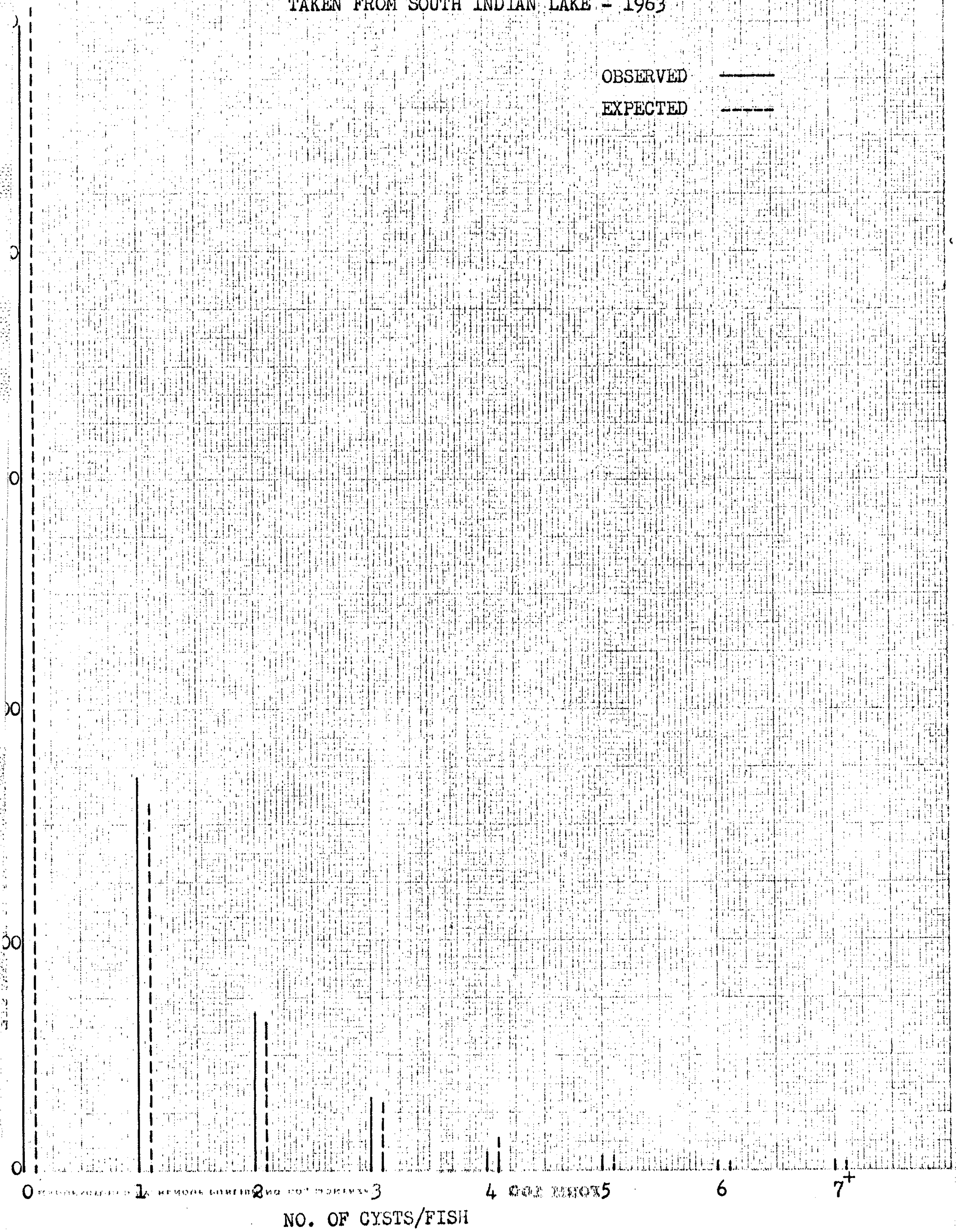
GRAPH SHOWING THE DISTRIBUTION OF CYSTS/FISH FOR THE SAMPLE TAKEN FROM KISSEYNEW LAKE - 1961

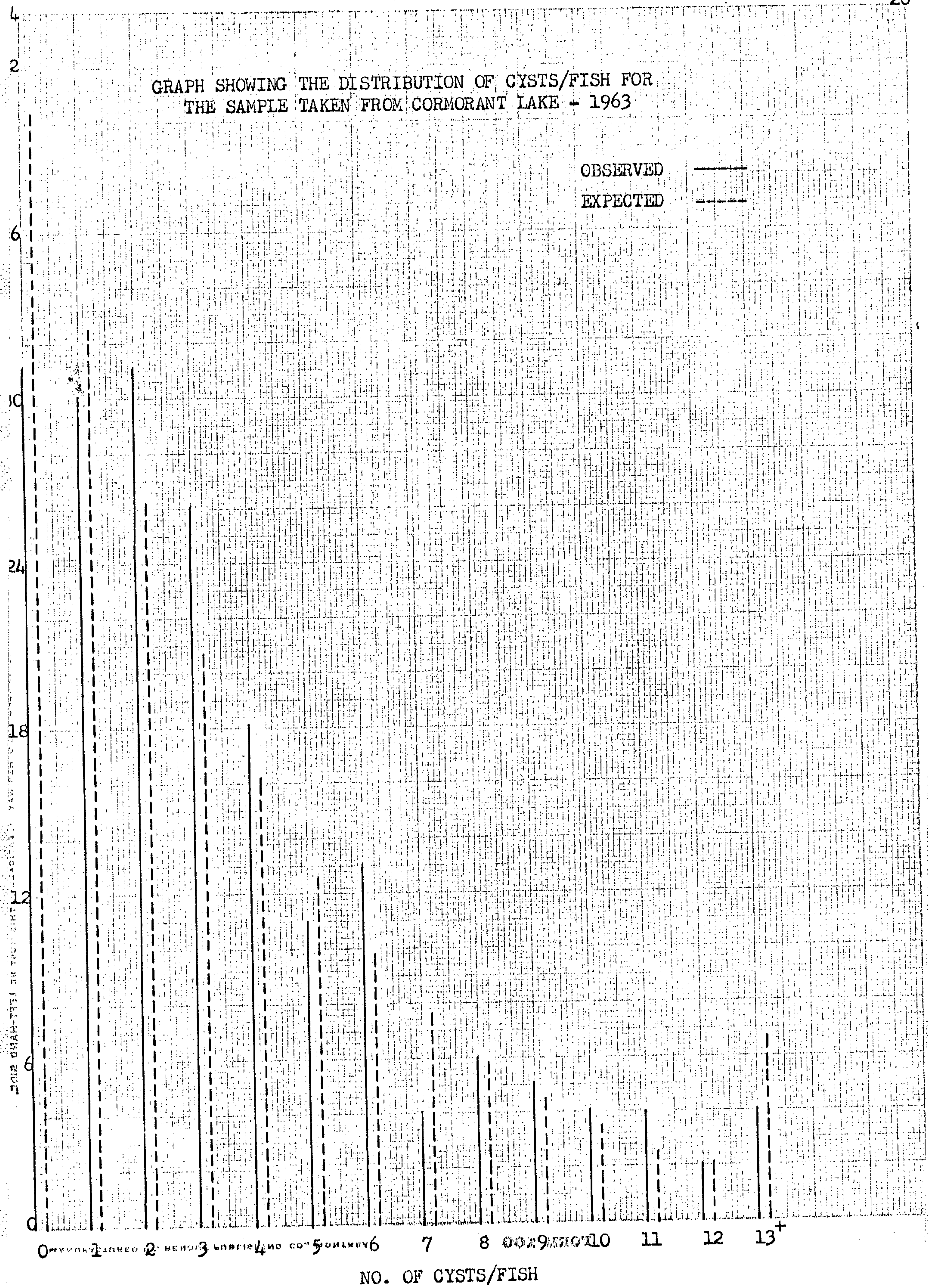
GRAPH SHOWING THE DISTRIBUTION OF CYSTS/FISH FOR THE SAMPLE TAKEN FROM DORE LAKE - 1962



NO. OF CYSTS/FISH

GRAPH SHOWING THE DISTRIBUTION OF CYSTS/FISH FOR THE SAMPLE TAKEN FROM SOUTH INDIAN LAKE - 1963




GRAPH SHOWING THE DISTRIBUTION OF CYSTS/FISH FOR
THE SAMPLE TAKEN FROM CORMORANT LAKE - 1963
OBSERVED
EXPECTED
0
6
12
18
24
0
1
2
3
4
5
6
7
8
9
10
11
12
13+
NO. OF CYSTS/FISH

CHAPTER IV

SAMPLE SIZE AND SENSITIVITY

The calculation of sample size and sensitivity is made using Williamson & Bretherton's 'Tables of the Negative Binomial Probability Distribution'.

Oakland (1950) used the assumption that k remained constant under both the Null and Alternative hypotheses. This assumption, as well as the one stating that the weight of the fish remains constant under both hypotheses, is also used here.

The values in the tables are calculated as follows:

$P = p, p_1, p_1^1$, (none of which are shown in the table) corresponding to $\alpha(10)$ and $\alpha(20)$, $1-\beta(35)$ and $1-\beta(50)$ respectively, were calculated according to the formula

$$P = \frac{1}{n\ w(I.R.) + 1}$$

where

n = sample size

w = weight/fish

I.R. = infestation rate in cysts per hundred pounds of fish.

(e.g. H_0:I.R. = 10 cysts/100 lbs. fish)

$\alpha(10)$ corresponds to a 10% level of significance,

$\alpha(20)$ corresponds to a 20% level of significance,

$1-\beta(35)$ corresponds to

H_1:I.R. = 35 cysts/100 lbs. of fish,

$1-\beta(50)$ corresponds to

H_1:I.R. = 50 cysts/100 lbs. of fish,

$$\alpha = \sum_{x=r+1}^{\infty} \frac{(k+x-1)!}{x!(k-1)!} \frac{p^x}{q^{k+x}} \qquad \text{.........(A)}$$

$$\beta = \sum_{x=0}^{r} \frac{(k+x-1)!}{x!(k-1)!} \frac{p_1^x}{q_1^{k+x}} \qquad \text{.........(B)}$$

$$\text{sensitivity} = 1-\beta = \sum_{x=r+1}^{\infty} \frac{(k+x-1)!}{x!(k-1)!} \frac{p_1^x}{q_1^{k+x}} \qquad \text{.........(C)}$$

where

r, (not shown in the table) is the critical cyst level,

p is the probability under the Null hypothesis, and

p_1 is the probability under the Alternate hypothesis.

I.R. takes the values 0.10, 0.20, 0.30 under H_0 and 0.35 and 0.50 under H_1

w takes the values 1, 2, 3, 4,

k takes the values 0.10, 0.30, 0.60, 0.90, 1.20, 1.50, 1.80, 2.40, 3.00.

After the P values (i.e. p, p_1, p_1^1) have been calculated, it is necessary to refer to the appropriate probabilities in Williamson & Bretherton's 'Tables of the Negative Binomial Probability Distribution'. Under each value of k, the probabilities which immediately bound 0.10 and 0.20 are obtained along with the corresponding sensitivity [1-B(35) and 1-B(50)]

Certain values in the tables are missing because they are beyond the limitations of Williamson & Bretherton's tables, and cannot be calculated.

Relationship of Sample Size to Sensitivity
1-B(35), 1-B(50) as k, w, and I.R. vary.

I.R. = 0.10   w = 1

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
			k = 0.10			
1	.009	.030	.040	.009	.030	.040
	.000	.005	.008	.000	.005	.008
2	.018	.051	.067	.018	.051	.067
	.002	.012	.021	.002	.012	.021
3	.026	.069	.088	.026	.069	.088
	.004	.022	.033	.004	.022	.033
4	.033	.083	.100	.033	.083	.100
	.005	.030	.041	.005	.030	.041
5	.040	.097	.116	.040	.097	.116
	.008	.039	.053	.008	.039	.053
6	.046	.108	.129	.046	.108	.129
	.010	.047	.064	.010	.047	.064

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
7	.051	.116	.141	.051	.116	.141
	.012	.053	.074	.012	.053	.074
8	.056	.126	.149	.056	.126	.149
	.015	.061	.081	.015	.061	.081
9	.061	.133	.158	.061	.133	.158
	.017	.067	.089	.017	.067	.089
10	.067	.141	.165	.067	.141	.165
	.021	.074	.096	.021	.074	.096
11	.071	.145	.172	.071	.145	.172
	.023	.077	.102	.023	.077	.102
12	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
13	.081	.158	.184	.081	.158	.184
	.029	.089	.113	.029	.089	.113
14	.083	.162	.184	.083	.162	.184
	.030	.092	.113	.030	.092	.113

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
15	.088	.167	.191	.088	.167	.191
	.033	.097	.120	.033	.097	.120
16	.092	.172	.198	.092	.172	.198
	.036	.102	.127	.036	.102	.127
17	.095	.178	.198	.095	.178	.198
	.038	.107	.127	.038	.107	.127
18	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135
19	.100	.184	.206	.100	.184	.206
	.041	.113	.135	.041	.113	.135
20	.100	.187	.217	.100	.187	.217
	.041	.116	.146	.041	.116	.146
21	.108	.191	.217	.108	.191	.217
	.047	.120	.146	.047	.120	.146
22	.111	.198	.227	.111	.198	.227
	.050	.127	.156	.050	.127	.156

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
23	.113	.198	.227	.113	.198	.227
	.051	.127	.156	.051	.127	.156
24	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156
			k = 0.30			
1	.028	.085	.112	.028	.085	.112
	.001	.014	.025	.001	.014	.025
2	.053	.146	.188	.053	.146	.188
	.006	.041	.066	.006	.041	.066
3	.076	.193	.240	.076	.193	.240
	.013	.070	.102	.013	.070	.102
4	.097	.229	.283	.097	.229	.283
	.017	.095	.139	.017	.095	.139
5	.112	.264	.313	.112	.264	.313
	.025	.123	.166	.025	.123	.166
6	.129	.290	.340	.129	.290	.340
	.033	.145	.191	.033	.145	.191

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
7	.146	.310	.365	.146	.310	.365
	.041	.163	.216	.041	.163	.216
8	.164	.332	.383	.164	.332	.383
	.051	.184	.235	.051	.184	.235
9	.173	.348	.402	.173	.348	.402
	.057	.199	.255	.057	.199	.255
10	.188	.365	.413	.188	.365	.413
	.066	.216	.266	.066	.216	.266
11	.198	.374	.434	.198	.374	.434
	.073	.226	.290	.073	.226	.290
12	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
13	.223	.402	.458	.223	.402	.458
	.090	.255	.316	.090	.255	.316
14	.229	.412	.465	.229	.412	.465
	.095	.266	.324	.095	.266	.324

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
15	.102	.278	.331	.240	.423	.471
	.049	.199	.251	.102	.278	.331
16	.113	.290	.347	.252	.434	.485
	.057	.210	.268	.113	.290	.347
17	.118	.303	.347	.258	.446	.485
	.060	.223	.268	.118	.303	.347
18	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364
19	.133	.316	.364	.276	.458	.499
	.072	.236	.285	.133	.316	.364
20	.139	.323	.387	.283	.464	.518
	.076	.243	.309	.139	.323	.387
21	.145	.331	.387	.290	.471	.518
	.081	.251	.309	.145	.331	.387
22	.150	.347	.410	.296	.485	.537
	.084	.267	.334	.150	.347	.410

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
23	.157	.347	.410	.303	.485	.537
	.090	.267	.334	.157	.347	.410
24	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.056	.165	.214	.056	.165	.214
	.004	.035	.058	.004	.035	.058
2	.105	.271	.340	.105	.271	.340
	.015	.092	.142	.015	.092	.142
3	.145	.348	.423	.145	.348	.423
	.027	.149	.215	.027	.149	.215
4	.186	.406	.486	.186	.406	.486
	.044	.199	.279	.044	.199	.279
5	.214	.458	.524	.214	.458	.524
	.058	.250	.321	.058	.250	.321
6	.245	.495	.564	.245	.495	.564
	.078	.289	.368	.078	.289	.368

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
7	.271	.526	.597	.271	.526	.597
	.093	.323	.408	.093	.323	.408
8	.288	.554	.619	.288	.554	.619
	.099	.356	.436	.099	.356	.436
9	.124	.381	.467	.317	.575	.643
	.051	.263	.352	.124	.381	.467
10	.142	.408	.483	.340	.597	.655
	.063	.290	.369	.142	.408	.483
11	.155	.422	.516	.356	.608	.680
	.071	.305	.405	.155	.422	.516
12	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
13	.191	.467	.552	.397	.643	.706
	.097	.352	.445	.191	.467	.552
14	.103	.369	.457	.406	.655	.713
	.055	.287	.378	.199	.483	.562

Sample Size	$\alpha(10)$	1-$\beta(35)$	1-$\beta(50)$	$\alpha(20)$	1-$\beta(50)$	1-$\beta(50)$
15	.115	.386	.468	.215	.499	.572
	.063	.304	.389	.115	.386	.468
16	.129	.405	.492	.232	.516	.593
	.074	.323	.414	.129	.405	.492
17	.136	.425	.492	.241	.534	.593
	.079	.344	.414	.136	.425	.492
18	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515
19	.161	.445	.515	.270	.552	.613
	.098	.365	.439	.161	.445	.515
20	.104	.378		.279	.562	
	.066	.315		.169	.457	
21	.110	.389		.288	.572	
	.070	.326		.176	.468	
22	.119	.413		.299	.592	
	.077	.351		.186	.491	

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
23	.127	.413		.310	.592	
	.083	.351		.196	.491	
24	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.082	.237	.302	.082	.237	.302
	.007	.059	.096	.007	.059	.096
2	.144	.378	.464	.144	.378	.464
	.025	.148	.223	.025	.148	.223
3	.209	.473	.562	.209	.473	.562
	.046	.232	.325	.046	.232	.325
4	.265	.542	.631	.265	.542	.631
	.074	.303	.409	.074	.303	.409
5	.301	.601	.672	.301	.601	.672
	.094	.371	.462	.094	.371	.462
6	.125	.422	.519	.344	.641	.713
	.047	.280	.381	.125	.422	.519

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
7	.148	.462	.564	.378	.672	.744
	.059	.321	.431	.148	.462	.564
8	.177	.505	.596	.414	.703	.765
	.075	.366	.467	.177	.505	.596
9	.196	.534	.628	.435	.723	.786
	.091	.397	.505	.196	.534	.628
10	.108	.431	.525	.223	.564	.645
	.053	.331	.429	.108	.431	.525
11	.122	.448	.568	.241	.579	.680
	.062	.348	.476	.122	.448	.568
12	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
13	.162	.505	.613	.292	.628	.716
	.090	.408	.526	.162	.505	.613
14	.171	.526	.625	.303	.646	.726
	.097	.431	.540	.171	.526	.625

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
15	.113	.453	.553	.325	.663	.735
	.067	.376	.481	.191	.547	.637
16	.127	.477	.580	.209	.569	.660
	.077	.401	.511	.127	.477	.580
17	.136	.500	.580	.220	.590	.660
	.084	.425	.511	.136	.500	.580
18	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609
19	.107	.452	.542	.255	.613	.685
	.070	.390	.483	.165	.526	.609
20	.116	.467		.268	.625	
	.077	.405		.176	.540	
21	.125	.481		.280	.637	
	.084	.419		.187	.553	
22	.136	.511		.294	.660	
	.093	.450		.199	.580	

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
23	.101	.450		.211	.580	
	.070	.398		.146	.511	
24	.110	.483		.224	.609	
	.077	.431		.157	.542	
			k = 1.20			
1	.108	.303	.381	.108	.303	.381
	.011	.086	.136	.011	.086	.136
2	.213	.469	.565	.213	.469	.565
	.051	.208	.304	.051	.208	.304
3	.269	.576	.667	.269	.576	.667
	.068	.257	.427	.068	.257	.427
4	.106	.401	.523	.337	.647	.736
	.032	.244	.366	.106	.401	.523
5	.136	.482	.580	.381	.707	.773
	.047	.323	.429	.136	.482	.580
6	.175	.537	.640	.431	.775	.811
	.070	.381	.499	.175	.537	.640

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
7	.208	.580	.685	.208	.580	.685
	.090	.429	.554	.090	.429	.554
8	.115	.481	.594	.244	.625	.716
	.053	.368	.490	.115	.481	.594
9	.134	.517	.693	.270	.655	.746
	.066	.405	.534	.134	.517	.633
10	.160	.554	.653	.304	.685	.761
	.083	.445	.557	.160	.554	.653
11	.179	.573	.695	.327	.700	.793
	.097	.466	.606	.179	.573	.695
12	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
13	.138	.534	.656	.233	.633	.736
	.081	.449	.583	.138	.534	.656
14	.147	.557	.670	.244	.653	.747
	.088	.473	.598	.147	.557	.670

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
15	.104	.499	.613	.269	.674	.758
	.064	.428	.549	.168	.581	.683
16	.122	.527		.295	.695	
	.078	.457		.190	.606	
17	.131	.555		.202	.631	
	.085	.487		.131	.555	
18	.142	.555		.215	.631	
	.094	.487		.142	.555	
19	.110	.517		.240	.656	
	.074	.457		.163	.583	
20	.120	.532		.254	.670	
	.082	.474		.175	.598	
21	.130	.549		.268	.683	
	.090	.491		.187	.613	
22	.100			.201		
	.070			.142		

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
23	.110			.215		
	.078			.154		
24	.121			.230		
	.087			.167		
			k = 1.50			
1	.133	.363	.450	.133	.363	.450
	.017	.115	.284	.017	.115	.284
2	.243	.546	.646	.243	.546	.646
	.050	.267	.381	.050	.267	.381
3	.324	.657	.747	.324	.657	.747
	.091	.395	.519	.091	.395	.519
4	.141	.491	.620	.401	.728	.811
	.046	.319	.451	.141	.491	.620
5	.184	.577	.677	.450	.784	.844
	.071	.411	.529	.184	.577	.677
6	.228	.634	.734	.228	.634	.734
	.098	.477	.603	.098	.477	.603

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
7	.125	.529	.658	.268	.677	.776
	.056	.407	.551	.125	.529	.658
8	.168	.584	.697	.321	.720	.804
	.088	.466	.597	.168	.584	.697
9	.182	.621	.734	.342	.748	.830
	.094	.508	.642	.182	.621	.734
10	.118	.551	.665	.215	.658	.752
	.064	.457	.583	.118	.551	.665
11	.137	.580	.711	.239	.677	.789
	.077	.494	.636	.137	.580	.711
12	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
13	.118	.557	.690	.305	.734	.824
	.072	.480	.627	.191	.642	.756
14	.127	.583	.705	.203	.665	.769
	.079	.509	.644	.127	.583	.705

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
15	.147	.609	.720	.228	.687	.781
	.094	.537	.661	.147	.609	.720
16	.114	.567		.257	.711	
	.075	.503		.172	.636	
17	.124	.596		.272	.734	
	.082	.534		.185	.663	
18	.135	.596		.287	.734	
	.091	.534		.198	.663	
19	.111	.567		.226	.690	
	.077	.512		.159	.627	
20	.123	.586		.242	.705	
	.087	.527		.173	.644	
21	.134	.604		.257	.720	
	.096	.541		.186	.661	
22	.105			.201		
	.075			.146		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
23	.117			.217		
	.085			.160		
24	.129			.233		
	.095			.174		
			k = 1.80			
1	.158	.418	.514	.158	.418	.514
	.020	.146	.225	.020	.146	.225
2	.284	.613	.712	.284	.613	.712
	.066	.328	.454	.066	.328	.454
3	.116	.459	.600	.375	.723	.808
	.032	.287	.426	.116	.459	.600
4	.179	.571	.701	.460	.790	.864
	.065	.393	.548	.179	.571	.701
5	.225	.658	.754	.225	.658	.754
	.091	.494	.617	.091	.494	.617
6	.124	.564	.689	.275	.714	.806
	.054	.435	.579	.124	.564	.689

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
7	.164	.617	.742	.328	.754	.842
	.079	.494	.643	.164	.617	.742
8	.113	.557	.687	.201	.671	.777
	.051	.455	.600	.113	.557	.687
9	.128	.600	.730	.234	.707	.810
	.068	.502	.651	.128	.600	.730
10	.158	.643	.751	.273	.742	.826
	.089	.550	.676	.158	.643	.751
11	.106	.573	.727	.301	.758	.857
	.061	.491	.662	.181	.666	.792
12	.137	.625	.752	.220	.708	.813
	.084	.547	.691	.137	.625	.752
13	.101	.576	.720	.248	.730	.833
	.065	.506	.665	.160	.651	.777
14	.110	.604	.735	.262	.741	.843
	.070	.537	.682	.171	.676	.789

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
15	.132	.632	.749	.293	.771	.852
	.087	.567	.698	.198	.701	.801
16	.106	.600		.227	.727	
	.071	.541		.156	.662	
17	.117	.632		.243	.752	
	.080	.575		.170	.691	
18	.128	.632		.259	.752	
	.089	.575		.183	.691	
19	.110	.612		.214	.720	
	.078	.560		.154	.665	
20	.122	.630		.231	.735	
	.088	.580		.169	.682	
21	.134	.648		.247	.749	
	.098	.599		.183	.698	
22	.109			.265		
	.080			.199		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
23	.122			.216		
	.091			.163		
24	.103			.234		
	.076			.179		
			k = 2.40			
1	.202	.515	.617	.202	.515	.617
	.046	.212	.314	.046	.212	.314
2	.100	.441	.584	.360	.718	.811
	.026	.248	.391	.100	.441	.584
3	.171	.637	.729	.465	.841	.889
	.056	.451	.566	.171	.637	.729
4	.103	.530	.690	.254	.701	.818
	.039	.384	.565	.103	.530	.690
5	.144	.638	.755	.314	.782	.861
	.062	.503	.645	.144	.638	.755
6	.199	.707	.817	.385	.829	.900
	.099	.585	.727	.199	.707	.817

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
7	.132	.645	.784	.248	.755	.859
	.068	.540	.705	.132	.645	.784
8	.170	.707	.820	.301	.803	.884
	.073	.611	.750	.170	.707	.820
9	.120	.658	.793	.206	.746	.853
	.068	.573	.730	.120	.658	.793
10	.153	.705	.814	.249	.784	.869
	.092	.626	.756	.153	.705	.814
11	.110	.650	.806	.279	.800	.898
	.067	.577	.756	.178	.725	.854
12	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
13	.115	.667	.860	.255	.793	.890
	.075	.605	.767	.173	.730	.852
14	.124	.697	.848	.270	.814	.898
	.082	.638	.782	.185	.756	.862

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
15	.104	.671	.797	.218	.782	.873
	.071	.616	.757	.152	.727	.836
16	.128	.704		.252	.806	
	.090	.652		.181	.756	
17	.101	.688		.271	.830	
	.071	.640		.198	.784	
18	.113	.688		.215	.784	
	.081	.640		.157	.736	
19	.104	.678		.253	.810	
	.076	.634		.190	.767	
20	.118	.697		.208	.783	
	.087	.655		.157	.740	
21	.131	.716		.226	.797	
	.098	.675		.173	.757	
22	.112			.247		
	.085			.192		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
23	.126			.210		
	.097			.163		
24	.109			.230		
	.084			.181		
			k = 3.00			
1	.245	.595	.704	.245	.595	.704
	.072	.279	.406	.072	.279	.406
2	.137	.541	.687	.428	.794	.875
	.038	.334	.499	.137	.541	.687
3	.229	.702	.821	.229	.702	.821
	.084	.518	.683	.084	.518	.683
4	.150	.647	.794	.331	.797	.891
	.062	.502	.686	.150	.647	.794
5	.102	.626	.761	.210	.748	.849
	.048	.509	.668	.102	.626	.761
6	.143	.706	.835	.267	.809	.901
	.075	.601	.761	.143	.706	.835

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
7	.105	.668	.815	.334	.849	.925
	.055	.576	.750	.192	.761	.874
8	.140	.745		.240	.824	
	.083	.663		.140	.745	
9	.109	.702		.290	.845	
	.064	.627		.181	.776	
10	.144	.750		.226	.815	
	.089	.683		.144	.750	
11	.111			.260		
	.070			.172		
12	.148			.219		
	.098			.148		
13	.122			.255		
	.082			.178		
14	.136			.274		
	.093			.195		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
15	.120			.232		
	.085			.168		
16	.105			.200		
	.075			.146		
17	.120			.220		
	.087			.164		
18	.135			.240		
	.099			.181		
19	.128			.220		
	.096			.169		
20	.108			.240		
	.081			.186		
21	.125			.208		
	.096			.162		
22	.110			.230		
	.085			.182		

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
23	.117			.243		
	.090			.193		
24	.108			.219		
	.084			.175		

I.R. = 0.10   w = 2

k = 0.10

1	.018	.051	.067	.018	.051	.067
	.002	.012	.021	.002	.012	.021
2	.033	.083	.100	.033	.083	.100
	.005	.030	.041	.005	.030	.041
3	.046	.108	.129	.046	.108	.129
	.010	.047	.064	.010	.047	.064
4	.056	.126	.149	.056	.126	.149
	.015	.061	.081	.015	.061	.081
5	.067	.141	.165	.067	.141	.165
	.021	.074	.096	.021	.074	.096

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
6	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
7	.083	.162	.184	.083	.162	.184
	.030	.092	.113	.030	.092	.113
8	.092	.172	.198	.092	.172	.198
	.036	.102	.127	.036	.102	.127
9	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135
10	.100	.187	.217	.100	.187	.217
	.041	.116	.146	.041	.116	.146
11	.111	.198	.227	.111	.198	.227
	.050	.127	.156	.050	.127	.156
12	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156
			k = 0.30			
1	.053	.146	.188	.053	.146	.188
	.006	.041	.066	.006	.041	.066

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
2	.097	.229	.283	.097	.229	.283
	.017	.095	.139	.017	.095	.139
3	.129	.290	.340	.129	.290	.340
	.033	.145	.191	.033	.145	.191
4	.164	.332	.383	.164	.332	.383
	.051	.184	.235	.051	.184	.235
5	.188	.365	.413	.188	.365	.413
	.066	.216	.266	.066	.216	.266
6	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
7	.229	.412	.465	.229	.412	.465
	.095	.266	.324	.095	.266	.324
8	.113	.290	.347	.252	.434	.485
	.057	.210	.268	.113	.290	.347
9	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
10	.139	.323	.387	.283	.464	.518
	.076	.243	.309	.139	.323	.387
11	.150	.347	.410	.296	.485	.537
	.084	.267	.334	.150	.347	.410
12	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.105	.271	.340	.105	.271	.340
	.015	.092	.142	.015	.092	.142
2	.186	.406	.486	.186	.406	.486
	.044	.199	.279	.044	.199	.279
3	.245	.495	.564	.245	.495	.564
	.078	.289	.368	.078	.289	.368
4	.288	.554	.619	.288	.554	.619
	.099	.356	.436	.099	.356	.436
5	.142	.408	.483	.340	.597	.655
	.063	.290	.369	.142	.408	.483

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
6	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
7	.103	.369	.457	.406	.655	.713
	.055	.287	.378	.199	.483	.562
8	.129	.405	.492	.232	.516	.593
	.074	.323	.414	.129	.405	.492
9	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515
10	.104	.378		.279	.562	
	.066	.315		.169	.457	
11	.119	.413		.299	.592	
	.077	.351		.186	.491	
12	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.144	.378	.464	.144	.378	.464
	.025	.148	.223	.025	.148	.223

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
2	.265	.542	.631	.265	.542	.631
	.074	.303	.409	.074	.303	.409
3	.125	.422	.519	.344	.641	.713
	.047	.280	.381	.125	.422	.519
4	.177	.505	.596	.414	.703	.765
	.075	.366	.467	.177	.505	.596
5	.108	.431	.525	.223	.564	.645
	.053	.331	.429	.108	.431	.525
6	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
7	.171	.526	.625	.303	.646	.726
	.097	.431	.540	.171	.526	.625
8	.127	.477	.580	.209	.569	.660
	.077	.401	.511	.127	.477	.580
9	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
10	.116	.467		.268	.625	
	.077	.405		.176	.540	
11	.136	.511		.294	.660	
	.093	.450		.199	.580	
12	.110	.483		.224	.609	
	.077	.431		.157	.542	
			<u>k = 1.20</u>			
1	.213	.469	.565	.213	.469	.565
	.051	.208	.304	.051	.208	.304
2	.106	.401	.523	.337	.647	.736
	.032	.244	.366	.106	.401	.523
3	.175	.537	.640	.431	.775	.811
	.070	.381	.499	.175	.537	.640
4	.115	.481	.594	.244	.625	.716
	.053	.368	.490	.115	.481	.594
5	.160	.554	.653	.304	.685	.761
	.083	.445	.557	.160	.554	.653

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
6	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
7	.147	.557	.670	.244	.653	.747
	.088	.473	.598	.147	.557	.670
8	.122	.527		.295	.695	
	.078	.457		.190	.606	
9	.142	.555		.215	.631	
	.094	.487		.142	.555	
10	.120	.532		.254	.670	
	.082	.474		.175	.598	
11	.100			.201		
	.070			.142		
12	.121			.230		
	.087			.167		
			<u>k = 1.50</u>			
1	.243	.546	.646	.243	.546	.646
	.050	.267	.381	.050	.267	.381

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
2	.141	.491	.620	.401	.728	.811
	.046	.319	.451	.141	.491	.620
3	.228	.634	.734	.228	.634	.734
	.098	.477	.603	.098	.477	.603
4	.168	.584	.697	.321	.720	.804
	.088	.466	.597	.168	.584	.697
5	.118	.551	.665	.215	.658	.752
	.064	.457	.583	.118	.551	.665
6	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
7	.127	.583	.705	.203	.665	.769
	.079	.509	.644	.127	.583	.705
8	.114	.567		.257	.711	
	.075	.503		.172	.636	
9	.135	.596		.287	.734	
	.091	.534		.198	.663	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
10	.123	.586		.242	.705	
	.087	.527		.173	.644	
11	.105			.201		
	.075			.146		
12	.129			.233		
	.095			.174		
			<u>k = 1.80</u>			
1	.284	.613	.712	.284	.613	.712
	.066	.328	.454	.066	.328	.454
2	.179	.571	.701	.460	.790	.864
	.065	.393	.548	.179	.571	.701
3	.124	.564	.689	.275	.714	.806
	.054	.435	.579	.124	.564	.689
4	.113	.557	.687	.201	.671	.777
	.051	.455	.600	.113	.557	.687
5	.158	.643	.751	.273	.742	.826
	.089	.550	.676	.158	.643	.751

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
6	.137	.625	.752	.220	.708	.813
	.084	.547	.691	.137	.625	.752
7	.110	.604	.735	.262	.741	.843
	.070	.537	.682	.171	.676	.789
8	.106	.600		.227	.727	
	.071	.541		.156	.662	
9	.128	.632		.259	.752	
	.089	.575		.183	.691	
10	.122	.630		.231	.735	
	.088	.580		.169	.682	
11	.109			.265		
	.080			.199		
12	.103			.234		
	.076			.179		
			$k = 2.40$			
1	.100	.441	.584	.360	.718	.811
	.026	.248	.391	.100	.441	.584

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
2	.103	.530	.690	.254	.701	.818
	.039	.384	.565	.103	.530	.690
3	.199	.707	.817	.385	.829	.900
	.099	.585	.727	.199	.707	.817
4	.170	.707	.820	.301	.803	.884
	.073	.611	.750	.170	.707	.820
5	.153	.705	.814	.249	.784	.869
	.092	.626	.756	.153	.705	.814
6	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
7	.124	.697	.848	.270	.814	.898
	.082	.638	.782	.185	.756	.862
8	.128	.704		.252	.806	
	.090	.652		.181	.756	
9	.113	.688		.215	.784	
	.081	.640		.157	.736	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
10	.118	.697		.208	.783	
	.087	.655		.157	.740	
11	.112			.247		
	.085			.192		
12	.109			.230		
	.084			.181		
			k = 3.00			
1	.137	.541	.687	.428	.794	.875
	.038	.334	.499	.137	.541	.687
2	.150	.647	.794	.331	.797	.891
	.062	.502	.686	.150	.647	.794
3	.143	.706	.835	.267	.809	.901
	.075	.601	.761	.143	.706	.835
4	.140	.745		.240	.824	
	.083	.663		.140	.745	
5	.144	.750		.226	.815	
	.089	.683		.144	.750	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
6	.148			.219		
	.098			.148		
7	.136			.274		
	.093			.195		
8	.105			.200		
	.075			.146		
9	.135			.240		
	.099			.181		
10	.108			.240		
	.081			.186		
11	.110			.230		
	.085			.182		
12	.108			.219		
	.084			.175		

I.R. = 0.10   w = 3

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 0.10			
1	.026	.069	.088	.026	.069	.088
	.004	.022	.033	.004	.022	.033
2	.046	.108	.129	.046	.108	.129
	.010	.047	.064	.010	.047	.064
3	.061	.133	.158	.061	.133	.158
	.017	.067	.059	.017	.067	.059
4	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
5	.088	.167	.191	.088	.167	.191
	.043	.097	.120	.043	.097	.120
6	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135
7	.108	.191	.217	.108	.191	.217
	.047	.120	.146	.047	.120	.146

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
8	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156
			k = 0.30			
1	.076	.193	.240	.076	.193	.240
	.013	.070	.102	.013	.070	.102
2	.129	.290	.340	.129	.290	.340
	.033	.145	.191	.033	.145	.191
3	.173	.348	.402	.173	.348	.402
	.057	.199	.255	.057	.199	.255
4	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
5	.102	.278	.331	.240	.423	.471
	.049	.199	.251	.102	.278	.331
6	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364
7	.145	.331	.387	.290	.471	.518
	.081	.251	.309	.145	.331	.387

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
8	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			<u>k = 0.60</u>			
1	.145	.348	.423	.145	.348	.423
	.027	.149	.215	.027	.149	.215
2	.245	.495	.564	.245	.495	.564
	.078	.289	.368	.078	.289	.368
3	.124	.381	.467	.317	.575	.643
	.051	.263	.352	.124	.381	.467
4	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
5	.115	.386	.468	.215	.499	.572
	.063	.304	.389	.115	.386	.468
6	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515
7	.110	.389		.288	.572	
	.070	.326		.176	.468	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
8	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.209	.473	.562	.209	.473	.562
	.046	.232	.325	.046	.232	.325
2	.125	.422	.519	.344	.641	.713
	.047	.280	.381	.125	.422	.519
3	.196	.534	.628	.435	.723	.786
	.091	.397	.505	.196	.534	.628
4	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
5	.113	.453	.553	.325	.663	.735
	.067	.376	.481	.191	.547	.637
6	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609
7	.125	.483		.280	.637	
	.084	.419		.187	.553	

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
8	.110	.483		.224	.609	
	.077	.431		.157	.542	
			k = 1.20			
1	.269	.576	.667	.269	.576	.667
	.068	.257	.427	.068	.257	.427
2	.175	.537	.640	.431	.775	.811
	.070	.381	.499	.175	.537	.640
3	.134	.517	.633	.270	.655	.746
	.066	.405	.534	.134	.517	.633
4	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
5	.104	.499	.613	.269	.674	.758
	.064	.428	.549	.168	.581	.683
6	.142	.555		.215	.631	
	.094	.487		.142	.555	
7	.130	.549		.268	.683	
	.090	.491		.187	.613	

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
8	.121			.230		
	.087			.167		
			k = 1.50			
1	.324	.657	.747	.324	.657	.747
	.091	.395	.519	.091	.395	.519
2	.288	.634	.734	.228	.634	.734
	.098	.477	.603	.098	.477	.603
3	.182	.621	.734	.342	.748	.830
	.094	.508	.642	.182	.621	.734
4	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
5	.147	.609	.720	.228	.687	.781
	.094	.537	.661	.147	.609	.720
6	.135	.596		.287	.734	
	.091	.534		.198	.663	
7	.134	.604		.257	.720	
	.096	.541		.186	.661	

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
8	.129			.233		
	.095			.174		
			k = 1.80			
1	.116	.459	.600	.375	.723	.808
	.032	.287	.426	.116	.459	.600
2	.124	.564	.689	.275	.714	.806
	.054	.435	.579	.124	.564	.689
3	.128	.600	.730	.234	.707	.810
	.068	.502	.651	.128	.600	.730
4	.137	.625	.752	.230	.708	.813
	.084	.547	.691	.137	.625	.752
5	.132	.632	.749	.293	.771	.852
	.087	.567	.698	.198	.701	.801
6	.128	.632		.259	.752	
	.089	.575		.183	.691	
7	.134	.648		.247	.749	
	.098	.599		.183	.698	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
8	.103			.234		
	.076			.179		
			k = 2.40			
1	.171	.637	.729	.465	.841	.889
	.056	.451	.566	.171	.637	.729
2	.199	.707	.817	.385	.829	.900
	.099	.585	.727	.199	.707	.817
3	.120	.658	.793	.206	.746	.853
	.068	.573	.730	.120	.658	.793
4	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
5	.104	.671	.797	.218	.782	.873
	.071	.616	.757	.152	.727	.836
6	.113	.688		.215	.784	
	.081	.640		.157	.736	
7	.131	.716		.226	.797	
	.098	.675		.173	.757	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
8	.109			.230		
	.084			.181		
			k = 3.00			
1	.229	.702	.821	.229	.702	.821
	.084	.518	.683	.084	.518	.683
2	.143	.706	.835	.267	.809	.901
	.075	.601	.761	.143	.706	.835
3	.109	.702		.290	.845	
	.064	.627		.181	.776	
4	.148			.219		
	.098			.148		
5	.120			.232		
	.085			.168		
6	.135			.240		
	.099			.181		
7	.125			.208		
	.096			.162		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
8	.108			.219		
	.084			.175		

I.R. = 0.10 w = 4

k = 0.10

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
1	.033	.083	.100	.033	.083	.100
	.005	.030	.041	.005	.030	.041
2	.056	.126	.149	.056	.126	.149
	.015	.061	.081	.015	.061	.081
3	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
4	.092	.172	.198	.092	.172	.198
	.036	.102	.127	.036	.102	.127
5	.100	.187	.217	.100	.187	.217
	.041	.116	.146	.041	.116	.146
6	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 0.30			
1	.097	.229	.283	.097	.229	.283
	.017	.095	.139	.017	.095	.139
2	.164	.332	.383	.164	.332	.383
	.051	.184	.235	.051	.184	.235
3	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
4	.113	.290	.347	.252	.434	.485
	.057	.210	.268	.113	.290	.347
5	.139	.323	.387	.283	.464	.518
	.076	.243	.309	.139	.323	.387
6	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.186	.406	.486	.186	.406	.486
	.044	.199	.279	.044	.199	.279

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
2	.288	.554	.619	.288	.554	.619
	.099	.356	.436	.099	.356	.436
3	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
4	.129	.405	.492	.232	.516	.593
	.074	.323	.414	.129	.405	.492
5	.104	.378		.279	.562	
	.066	.315		.169	.457	
6	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.265	.542	.531	.265	.542	.631
	.074	.303	.409	.074	.303	.409
2	.177	.505	.596	.414	.703	.765
	.075	.366	.467	.177	.505	.596
3	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
4	.127	.477	.580	.209	.569	.660
	.077	.401	.511	.127	.477	.580
5	.116	.467		.268	.625	
	.077	.405		.176	.540	
6	.110	.483		.224	.609	
	.077	.431		.157	.542	
			k = 1.20			
1	.106	.401	.523	.337	.647	.736
	.032	.244	.366	.106	.401	.523
2	.115	.481	.594	.244	.625	.716
	.053	.368	.490	.115	.481	.594
3	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
4	.122	.527		.295	.695	
	.078	.457		.190	.606	
5	.120	.532		.254	.670	
	.082	.474		.175	.598	

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
6	.121			.230		
	.087			.167		
			k = 1.50			
1	.141	.491	.620	.401	.728	.811
	.046	.319	.451	.141	.491	.620
2	.168	.584	.697	.321	.720	.804
	.088	.466	.597	.168	.584	.697
3	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
4	.114	.567		.257	.711	
	.075	.503		.172	.636	
5	.123	.586		.242	.705	
	.087	.527		.173	.644	
6	.129			.233		
	.095			.174		

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
			k = 1.80			
1	.179	.571	.701	.460	.790	.864
	.065	.393	.548	.179	.571	.701
2	.113	.557	.687	.201	.671	.777
	.051	.455	.600	.113	.557	.687
3	.137	.625	.752	.220	.708	.813
	.084	.547	.691	.131	.625	.752
4	.106	.600		.227	.727	
	.071	.541		.156	.662	
5	.122	.630		.231	.735	
	.088	.580		.169	.682	
6	.103			.234		
	.076			.179		
			k = 2.40			
1	.103	.530	.690	.254	.701	.818
	.039	.384	.565	.103	.530	.690

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
2	.170	.707	.820	.301	.803	.884
	.073	.611	.750	.170	.707	.820
3	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
4	.128	.704		.252	.806	
	.090	.652		.181	.756	
5	.118	.697		.208	.783	
	.087	.655		.157	.740	
6	.109			.230		
	.084			.181		
			<u>k = 3.00</u>			
1	.150	.647	.794	.331	.797	.891
	.062	.502	.686	.150	.647	.794
2	.140	.745		.240	.824	
	.083	.663		.140	.745	
3	.148			.219		
	.098			.148		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
4	.105			.200		
	.075			.146		
5	.108			.240		
	.081			.186		
6	.108			.219		
	.084			.175		

I.R. = 0.20 w = 1

k = 0.10

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
1	.018	.051	.067	.018	.051	.067
	.002	.012	.021	.002	.012	.021
2	.033	.083	.100	.033	.083	.100
	.005	.030	.041	.005	.030	.041
3	.046	.108	.129	.046	.108	.129
	.010	.047	.064	.010	.047	.064
4	.056	.126	.149	.056	.126	.149
	.015	.061	.081	.015	.061	.081

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
5	.067	.141	.165	.067	.141	.165
	.021	.074	.096	.021	.074	.096
6	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
7	.083	.162	.184	.083	.162	.184
	.030	.092	.113	.030	.092	.113
8	.092	.172	.198	.092	.172	.198
	.036	.102	.127	.036	.102	.127
9	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135
10	.100	.187	.217	.100	.187	.217
	.041	.116	.146	.041	.116	.146
11	.111	.198	.227	.111	.198	.227
	.050	.127	.156	.050	.127	.156
12	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
			k = 0.30			
1	.053	.146	.188	.053	.146	.188
	.006	.041	.066	.006	.041	.066
2	.097	.229	.283	.097	.229	.283
	.017	.095	.139	.017	.095	.139
3	.129	.290	.340	.129	.290	.340
	.033	.145	.191	.033	.145	.191
4	.164	.332	.383	.164	.332	.383
	.051	.184	.235	.051	.184	.235
5	.188	.365	.413	.188	.365	.413
	.066	.216	.266	.066	.216	.266
6	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
7	.229	.412	.465	.229	.412	.465
	.095	.266	.324	.095	.266	.324
8	.113	.290	.347	.252	.434	.485
	.057	.210	.268	.113	.290	.347

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
9	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364
10	.139	.323	.387	.283	.464	.518
	.076	.243	.309	.139	.323	.387
11	.150	.347	.410	.296	.485	.537
	.084	.267	.334	.150	.347	.410
12	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			$k = 0.60$			
1	.105	.271	.340	.105	.271	.340
	.015	.092	.142	.015	.092	.142
2	.186	.406	.486	.186	.406	.486
	.044	.199	.279	.044	.199	.279
3	.245	.495	.564	.245	.495	.564
	.078	.289	.368	.078	.289	.368
4	.288	.554	.619	.288	.554	.619
	.099	.356	.436	.099	.356	.436

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
5	.142	.408	.483	.340	.597	.655
	.063	.290	.369	.142	.408	.483
6	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
7	.103	.369	.457	.406	.655	.713
	.055	.287	.378	.199	.483	.562
8	.129	.405	.492	.232	.516	.593
	.074	.323	.414	.129	.405	.492
9	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515
10	.104	.378		.279	.562	
	.066	.315		.169	.457	
11	.119	.413		.299	.592	
	.077	.351		.186	.491	
12	.135	.439		.206	.515	
	.090	.377		.135	.439	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			<u>k = 0.90</u>			
1	.144	.378	.464	.144	.378	.464
	.025	.148	.223	.025	.148	.223
2	.265	.542	.631	.265	.542	.631
	.074	.303	.409	.074	.303	.409
3	.125	.422	.519	.344	.641	.713
	.047	.280	.381	.125	.422	.519
4	.177	.505	.596	.414	.703	.765
	.075	.366	.467	.177	.505	.596
5	.108	.431	.525	.223	.564	.645
	.053	.331	.429	.108	.431	.525
6	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
7	.171	.526	.625	.303	.646	.726
	.097	.431	.540	.171	.526	.625
8	.127	.477	.580	.209	.569	.660
	.077	.401	.511	.127	.477	.580

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
9	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609
10	.116	.467		.268	.625	
	.077	.405		.176	.540	
11	.136	.511		.294	.660	
	.093	.450		.199	.580	
12	.110	.483		.224	.609	
	.077	.431		.157	.542	
			k = 1.20			
1	.213	.469	.565	.213	.469	.565
	.051	.208	.304	.051	.208	.304
2	.106	.401	.523	.337	.647	.736
	.032	.244	.366	.106	.401	.523
3	.175	.537	.640	.431	.775	.811
	.070	.381	.499	.175	.537	.640
4	.115	.481	.594	.244	.625	.716
	.053	.368	.490	.115	.481	.594

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
5	.160	.554	.653	.304	.685	.761
	.083	.445	.557	.160	.554	.653
6	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
7	.147	.557	.670	.244	.653	.747
	.088	.473	.598	.147	.557	.670
8	.122	.527		.295	.695	
	.078	.457		.190	.606	
9	.142	.555		.215	.631	
	.094	.487		.142	.555	
10	.120	.532		.254	.670	
	.082	.474		.175	.598	
11	.100			.201		
	.070			.142		
12	.121			.230		
	.087			.167		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 1.50			
1	.243	.546	.646	.243	.546	.646
	.050	.267	.381	.050	.267	.381
2	.141	.491	.620	.401	.728	.811
	.046	.319	.451	.141	.491	.620
3	.228	.634	.734	.228	.634	.734
	.098	.477	.603	.098	.477	.603
4	.168	.584	.697	.321	.720	.804
	.088	.466	.597	.168	.584	.697
5	.118	.551	.665	.215	.658	.752
	.064	.457	.583	.118	.551	.665
6	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
7	.127	.583	.705	.203	.665	.769
	.079	.509	.644	.127	.583	.705
8	.114	.567		.257	.711	
	.075	.503		.172	.636	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
9	.135	.596		.287	.734	
	.091	.534		.198	.663	
10	.123	.586		.242	.705	
	.087	.527		.173	.644	
11	.105			.201		
	.075			.146		
12	.129			.233		
	.095			.174		
			k = 1.80			
1	.284	.613	.712	.284	.613	.712
	.066	.328	.454	.066	.328	.454
2	.179	.571	.701	.460	.790	.864
	.065	.393	.548	.179	.571	.701
3	.124	.564	.689	.275	.714	.806
	.054	.435	.579	.124	.564	.689
4	.113	.557	.687	.201	.671	.777
	.051	.455	.600	.113	.557	.687

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
5	.158	.643	.751	.273	.742	.826
	.089	.550	.676	.158	.643	.751
6	.137	.625	.752	.220	.708	.813
	.084	.547	.691	.137	.625	.752
7	.110	.604	.735	.262	.741	.843
	.070	.537	.682	.171	.676	.789
8	.106	.600		.227	.727	
	.071	.541		.156	.662	
9	.128	.632		.259	.752	
	.089	.575		.183	.691	
10	.122	.630		.231	.735	
	.088	.580		.169	.682	
11	.109			.265		
	.080			.199		
12	.103			.234		
	.076			.179		

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
			k = 2.40			
1	.100	.441	.584	.360	.718	.811
	.026	.248	.391	.100	.441	.584
2	.103	.530	.690	.254	.701	.818
	.039	.384	.565	.103	.530	.690
3	.199	.707	.817	.385	.829	.900
	.099	.585	.727	.199	.707	.817
4	.170	.707	.820	.301	.803	.884
	.073	.611	.750	.170	.707	.820
5	.153	.705	.814	.249	.784	.869
	.092	.626	.756	.153	.705	.814
6	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
7	.124	.697	.848	.270	.814	.898
	.082	.638	.782	.185	.756	.862
8	.128	.704		.252	.806	
	.090	.652		.181	.756	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
9	.113	.688		.215	.784	
	.081	.640		.157	.736	
10	.118	.697		.208	.783	
	.087	.655		.157	.740	
11	.112			.247		
	.085			.192		
12	.109			.230		
	.084			.181		
			k = 3.00			
1	.137	.541	.687	.428	.794	.875
	.038	.334	.499	.137	.541	.687
2	.150	.647	.794	.331	.797	.891
	.062	.502	.686	.150	.647	.794
3	.143	.706	.835	.267	.809	.901
	.075	.601	.761	.143	.706	.835
4	.140	.745		.240	.824	
	.083	.663		.140	.745	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
5	.144	.750		.226	.815	
	.089	.683		.144	.750	
6	.148			.219		
	.098			.148		
7	.136			.274		
	.093			.195		
8	.105			.200		
	.075			.146		
9	.135			.240		
	.099			.181		
10	.108			.240		
	.081			.186		
11	.110			.230		
	.085			.182		
12	.108			.219		
	.084			.175		

I.R. = 0.20 w = 2

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
			k = 0.10			
1	.033	.083	.100	.033	.083	.100
	.005	.030	.041	.005	.030	.041
2	.056	.126	.149	.056	.126	.149
	.015	.061	.081	.015	.061	.081
3	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
4	.092	.172	.198	.092	.172	.198
	.036	.102	.127	.036	.102	.127
5	.100	.187	.217	.100	.187	.217
	.041	.116	.146	.041	.116	.146
6	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156
			k = 0.30			
1	.097	.229	.283	.097	.229	.283
	.017	.095	.139	.017	.095	.139

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
2	.164	.332	.383	.164	.332	.383
	.051	.184	.235	.051	.184	.235
3	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
4	.113	.290	.347	.252	.434	.485
	.057	.210	.268	.113	.290	.347
5	.139	.323	.387	.283	.464	.518
	.076	.243	.309	.139	.323	.387
6	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.186	.406	.486	.186	.406	.486
	.044	.199	.279	.044	.199	.279
2	.288	.554	.619	.288	.554	.619
	.099	.356	.436	.099	.356	.436
3	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
4	.129	.405	.492	.232	.516	.593
	.074	.323	.414	.129	.405	.492
5	.104	.378		.279	.562	
	.066	.315		.169	.457	
6	.135	.439		.206	.515	
	.090	.377		.135	.439	
k = 0.90						
1	.265	.542	.531	.265	.542	.631
	.074	.303	.409	.074	.303	.409
2	.177	.505	.596	.414	.703	.765
	.075	.366	.467	.177	.505	.596
3	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
4	.127	.477	.580	.209	.569	.660
	.077	.401	.511	.127	.477	.580
5	.116	.467		.268	.625	
	.077	.405		.176	.540	

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
6	.110	.483		.224	.609	
	.077	.431		.157	.542	
			<u>k = 1.20</u>			
1	.106	.401	.523	.337	.647	.736
	.032	.244	.366	.106	.401	.523
2	.115	.481	.594	.244	.625	.716
	.053	.368	.490	.115	.481	.594
3	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
4	.122	.527		.295	.695	
	.078	.457		.190	.606	
5	.120	.532		.254	.670	
	.082	.474		.175	.598	
6	.121			.230		
	.087			.167		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 1.50			
1	.141	.491	.620	.401	.728	.811
	.046	.319	.451	.141	.491	.620
2	.168	.584	.697	.321	.720	.804
	.088	.466	.597	.168	.584	.697
3	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
4	.114	.567		.257	.711	
	.075	.503		.172	.636	
5	.123	.586		.242	.705	
	.087	.527		.173	.644	
6	.129			.233		
	.095			.174		
			k = 1.80			
1	.179	.571	.701	.460	.790	.864
	.065	.393	.548	.179	.571	.701
2	.113	.557	.687	.201	.671	.777
	.051	.455	.600	.113	.557	.687

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
3	.137	.625	.752	.220	.708	.813
	.084	.547	.691	.131	.625	.752
4	.106	.600		.227	.727	
	.071	.541		.156	.662	
5	.122	.630		.231	.735	
	.088	.580		.169	.682	
6	.103			.234		
	.076			.179		
			$k = 2.40$			
1	.103	.530	.690	.254	.701	.818
	.039	.384	.565	.103	.530	.690
2	.170	.707	.820	.301	.803	.884
	.073	.611	.750	.170	.707	.820
3	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
4	.128	.704		.252	.806	
	.090	.652		.181	.756	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
5	.118	.697		.208	.783	
	.087	.655		.157	.740	
6	.109			.230		
	.084			.181		
			k = 3.00			
1	.150	.647	.794	.331	.797	.891
	.062	.502	.686	.150	.647	.794
2	.140	.745		.240	.824	
	.083	.663		.140	.745	
3	.148			.219		
	.098			.148		
4	.105			.200		
	.075			.146		
5	.108			.240		
	.081			.186		
6	.108			.219		
	.084			.175		

I.R. = 0.20   w = 3

Sample Size	$\alpha(10)$	1-$\beta(35)$	1-$\beta(50)$	$\alpha(20)$	1-$\beta(35)$	1-$\beta(50)$
			k = 0.10			
1	.046	.108	.129	.046	.108	.129
	.010	.047	.064	.010	.047	.064
2	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
3	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135
4	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156
			k = 0.30			
1	.129	.290	.340	.129	.290	.340
	.033	.145	.191	.033	.145	.191
2	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
3	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364
4	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.245	.495	.564	.245	.495	.564
	.078	.289	.368	.078	.289	.368
2	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
3	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515
4	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.125	.422	.519	.344	.641	.713
	.047	.280	.381	.125	.422	.519

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
2	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
3	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609
4	.110	.483		.224	.609	
	.077	.431		.157	.542	
			k = 1.20			
1	.175	.537	.640	.431	.775	.811
	.070	.381	.499	.175	.537	.640
2	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
3	.142	.555		.215	.631	
	.094	.487		.142	.555	
4	.121			.230		
	.087			.167		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 1.50			
1	.228	.634	.734	.228	.634	.734
	.098	.477	.603	.098	.477	.603
2	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
3	.135	.596		.287	.734	
	.091	.534		.198	.663	
4	.129			.233		
	.095			.174		
			k = 1.80			
1	.124	.564	.689	.275	.714	.806
	.054	.435	.579	.124	.564	.689
2	.137	.625	.752	.230	.708	.813
	.084	.547	.691	.137	.625	.752
3	.128	.632		.259	.752	
	.089	.575		.183	.691	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
4	.103			.234		
	.076			.179		
			k = 2.40			
1	.199	.707	.817	.385	.829	.900
	.099	.585	.727	.199	.707	.817
2	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
3	.113	.688		.215	.784	
	.081	.640		.157	.736	
4	.109			.230		
	.084			.181		
			k = 3.00			
1	.143	.706	.835	.267	.809	.901
	.075	.601	.761	.143	.706	.835
2	.148			.219		
	.098			.148		

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
3	.135			.240		
	.099			.181		
4	.108			.219		
	.084			.175		

I.R. = 0.20 w = 4

k = 0.10

1	.056	.126	.149	.056	.126	.149
	.015	.061	.081	.015	.061	.081
2	.092	.172	.198	.092	.172	.198
	.036	.102	.127	.036	.102	.127
3	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156

k = 0.30

1	.164	.332	.383	.164	.332	.383
	.051	.184	.235	.051	.184	.235
2	.113	.290	.347	.252	.434	.485
	.057	.210	.268	.113	.290	.347

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
3	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.288	.554	.619	.288	.554	.619
	.099	.356	.436	.099	.356	.436
2	.129	.405	.492	.232	.516	.593
	.074	.323	.414	.129	.405	.492
3	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.177	.505	.596	.417	.703	.765
	.075	.366	.467	.177	.505	.596
2	.127	.477	.580	.209	.569	.660
	.077	.401	.511	.127	.477	.580
3	.110	.483		.224	.609	
	.077	.431		.157	.542	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 1.20			
1	.115	.481	.594	.244	.625	.716
	.053	.368	.490	.115	.481	.594
2	.122	.527		.295	.695	
	.078	.457		.190	.606	
3	.121			.230		
	.087			.167		
			k = 1.50			
1	.168	.584	.697	.321	.720	.804
	.088	.466	.597	.168	.584	.697
2	.114	.567		.257	.711	
	.075	.503		.172	.636	
3	.129			.233		
	.095			.174		
			k = 1.80			
1	.113	.557	.687	.201	.671	.777
	.051	.455	.600	.113	.557	.687

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
2	.106	.600		.227	.727	
	.071	.541		.156	.662	
3	.103			.234		
	.076			.179		
			k = 2.40			
1	.170	.707	.820	.301	.803	.884
	.073	.611	.750	.170	.707	.820
2	.128	.704		.252	.806	
	.090	.652		.181	.756	
3	.109			.230		
	.084			.181		
			k = 3.00			
1	.140	.745		.240	.824	
	.083	.663		.140	.745	
2	.105			.200		
	.075			.146		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
3	.108			.219		
	.084			.175		

I.R. = 0.30 w = 1

k = 0.10

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
1	.026	.069	.088	.026	.069	.088
	.004	.022	.033	.004	.022	.033
2	.046	.108	.129	.046	.108	.129
	.010	.047	.064	.010	.047	.064
3	.061	.133	.158	.061	.133	.158
	.017	.067	.059	.017	.067	.059
4	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
5	.088	.167	.191	.088	.167	.191
	.043	.097	.120	.043	.097	.120
6	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
7	.108	.191	.217	.108	.191	.217
	.047	.120	.146	.047	.120	.146
8	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156
			<u>k = 0.30</u>			
1	.076	.193	.240	.076	.193	.240
	.013	.070	.102	.013	.070	.102
2	.129	.290	.340	.129	.290	.340
	.033	.145	.191	.033	.145	.191
3	.173	.348	.402	.173	.348	.402
	.057	.199	.255	.057	.199	.255
4	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
5	.102	.278	.331	.240	.423	.471
	.049	.199	.251	.102	.278	.331
6	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
7	.145	.331	.387	.290	.471	.518
	.081	.251	.309	.145	.331	.387
8	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.145	.348	.423	.145	.348	.423
	.027	.149	.215	.027	.149	.215
2	.245	.495	.564	.245	.495	.564
	.078	.289	.368	.078	.289	.368
3	.124	.381	.467	.317	.575	.643
	.051	.263	.352	.124	.381	.467
4	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
5	.115	.386	.468	.215	.499	.572
	.063	.304	.389	.115	.386	.468

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
6	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515
7	.110	.389		.288	.572	
	.070	.326		.176	.468	
8	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.209	.473	.562	.209	.473	.562
	.046	.232	.325	.046	.232	.325
2	.125	.422	.519	.344	.641	.713
	.047	.280	.381	.125	.422	.519
3	.196	.534	.628	.435	.723	.786
	.091	.397	.505	.196	.534	.628
4	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
5	.113	.453	.553	.325	.663	.735
	.067	.376	.481	.191	.547	.637

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
6	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609
7	.125	.481		.280	.637	
	.084	.419		.187	.553	
8	.110	.483		.224	.609	
	.077	.431		.157	.542	
			k = 1.20			
1	.269	.576	.667	.269	.576	.667
	.068	.257	.427	.068	.257	.427
2	.175	.537	.640	.431	.775	.811
	.070	.381	.499	.175	.537	.640
3	.134	.517	.633	.270	.655	.746
	.066	.405	.534	.134	.517	.633
4	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
5	.104	.499	.613	.269	.674	.758
	.064	.428	.549	.168	.581	.683

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
6	.142	.555		.215	.631	
	.094	.487		.142	.555	
7	.130	.549		.268	.683	
	.090	.491		.187	.613	
8	.121			.230		
	.087			.167		
			k = 1.50			
1	.324	.657	.747	.324	.657	.747
	.091	.395	.519	.091	.395	.519
2	.228	.634	.734	.228	.634	.734
	.098	.477	.603	.098	.477	.603
3	.182	.621	.734	.342	.748	.830
	.094	.508	.642	.182	.621	.734
4	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
5	.147	.609	.720	.228	.687	.781
	.094	.537	.661	.147	.609	.720

Sample Size	$\alpha(10)$	1-β(35)	1-β(50)	$\alpha(20)$	1-β(35)	1-β(50)
6	.135	.596		.287	.734	
	.091	.534		.198	.663	
7	.134	.604		.257	.720	
	.096	.541		.186	.661	
8	.129			.233		
	.095			.174		
			k = 1.80			
1	.116	.459	.600	.375	.723	.808
	.032	.287	.426	.116	.459	.600
2	.124	.564	.689	.275	.714	.806
	.054	.435	.579	.124	.564	.689
3	.128	.600	.730	.234	.707	.810
	.068	.502	.651	.128	.600	.730
4	.137	.625	.752	.230	.708	.813
	.084	.547	.691	.137	.625	.752
5	.132	.632	.749	.293	.771	.852
	.087	.567	.698	.198	.701	.801

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
6	.128	.632		.259	.752	
	.089	.575		.183	.691	
7	.134	.648		.247	.749	
	.098	.599		.183	.698	
8	.103			.234		
	.076			.179		
			k = 2.40			
1	.171	.637	.729	.465	.841	.889
	.056	.451	.566	.171	.637	.729
2	.199	.707	.817	.385	.829	.900
	.099	.585	.727	.199	.707	.817
3	.120	.658	.793	.206	.746	.853
	.068	.573	.730	.120	.658	.793
4	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
5	.104	.671	.797	.218	.782	.873
	.071	.616	.757	.152	.727	.836

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
6	.113	.688		.215	.784	
	.081	.640		.157	.736	
7	.131	.716		.226	.797	
	.098	.675		.173	.757	
8	.109			.230		
	.084			.181		
			k = 3.00			
1	.229	.702	.821	.229	.702	.821
	.084	.518	.683	.084	.518	.683
2	.143	.706	.835	.267	.809	.901
	.075	.601	.761	.143	.706	.835
3	.109	.702		.290	.845	
	.064	.627		.181	.776	
4	.148			.219		
	.098			.148		
5	.120			.232		
	.085			.168		

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
6	.135			.240		
	.099			.181		
7	.125			.208		
	.096			.162		
8	.108			.219		
	.084			.175		

I.R. = 0.30 w = 2

k = 0.10

1	.046	.108	.129	.046	.108	.129
	.010	.047	.064	.010	.047	.064
2	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
3	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135
4	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 0.30			
1	.129	.290	.340	.129	.290	.340
	.033	.145	.191	.033	.145	.191
2	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
3	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364
4	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.245	.495	.564	.245	.495	.564
	.078	.289	.368	.078	.289	.368
2	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
3	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
4	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.125	.422	.519	.344	.641	.713
	.047	.280	.381	.125	.422	.519
2	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
3	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609
4	.110	.483		.224	.609	
	.077	.431		.157	.542	
			k = 1.20			
1	.175	.537	.640	.431	.775	.811
	.070	.381	.499	.175	.537	.640
2	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631

Sample Size	α(10)	1-β(35)	1-β(50)	α(20)	1-β(35)	1-β(50)
3	.142	.555		.215	.631	
	.094	.487		.142	.555	
4	.121			.230		
	.087			.167		
			k = 1.50			
1	.228	.634	.734	.228	.634	.734
	.098	.477	.603	.098	.477	.603
2	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
3	.135	.596		.287	.734	
	.091	.534		.198	.663	
4	.129			.233		
	.095			.174		
			k = 1.80			
1	.124	.564	.689	.275	.714	.806
	.054	.435	.579	.124	.564	.689

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
2	.137	.625	.752	.230	.708	.813
	.084	.547	.691	.137	.625	.752
3	.128	.632		.259	.752	
	.089	.575		.183	.691	
4	.103			.234		
	.076			.179		
			k = 2.40			
1	.199	.707	.817	.385	.829	.900
	.099	.585	.727	.199	.707	.817
2	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
3	.113	.688		.215	.784	
	.081	.640		.157	.736	
4	.109			.230		
	.084			.181		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 3.00			
1	.143	.706	.835	.267	.809	.901
	.075	.601	.761	.143	.706	.835
2	.148			.219		
	.098			.148		
3	.135			.240		
	.099			.181		
4	.108			.219		
	.084			.175		
			I.R. = 0.30 w = 3			
			k = 0.10			
1	.061	.133	.158	.061	.133	.158
	.017	.067	.059	.017	.067	.059
2	.097	.178	.206	.097	.178	.206
	.039	.107	.135	.039	.107	.135

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 0.30			
1	.173	.348	.402	.173	.348	.402
	.057	.199	.255	.057	.199	.255
2	.123	.303	.364	.264	.446	.499
	.064	.223	.285	.123	.303	.364
			k = 0.60			
1	.124	.381	.467	.317	.575	.643
	.051	.263	.352	.124	.381	.467
2	.143	.425	.515	.250	.534	.613
	.084	.344	.439	.143	.425	.515
			k = 0.90			
1	.196	.534	.628	.435	.723	.786
	.091	.397	.505	.196	.534	.628
2	.145	.500	.609	.231	.590	.685
	.091	.425	.542	.145	.500	.609

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 1.20			
1	.134	.517	.633	.270	.655	.746
	.066	.405	.534	.134	.517	.633
2	.142	.555		.215	.631	
	.094	.487		.142	.555	
			k = 1.50			
1	.182	.621	.734	.342	.748	.830
	.094	.508	.642	.182	.621	.734
2	.135	.596		.287	.734	
	.091	.534		.198	.663	
			k = 1.80			
1	.128	.600	.730	.234	.707	.810
	.068	.502	.651	.128	.600	.730
2	.128	.632		.259	.752	
	.089	.575		.183	.691	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 2.40			
1	.120	.658	.793	.206	.746	.853
	.068	.573	.730	.120	.658	.793
2	.113	.688		.215	.784	
	.081	.640		.157	.736	
			k = 3.00			
1	.109	.702		.290	.845	
	.064	.627		.181	.776	
2	.135			.240		
	.099			.181		
			I.R. = 0.30 w = 4			
			k = 0.10			
1	.077	.153	.178	.077	.153	.178
	.026	.085	.107	.026	.085	.107
2	.116	.206	.227	.116	.206	.227
	.053	.135	.156	.053	.135	.156

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 0.30			
1	.213	.392	.446	.213	.392	.446
	.083	.244	.303	.083	.244	.303
2	.166	.364	.410	.313	.499	.537
	.098	.285	.334	.166	.364	.410
			k = 0.60			
1	.176	.461	.534	.380	.641	.693
	.086	.345	.425	.176	.461	.534
2	.135	.439		.206	.515	
	.090	.377		.135	.439	
			k = 0.90			
1	.145	.485	.590	.271	.611	.698
	.078	.387	.500	.145	.485	.590
2	.110	.483		.224	.609	
	.077	.431		.157	.542	

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 1.20			
1	.120	.510	.631	.210	.612	.716
	.068	.423	.555	.120	.510	.631
2	.121			.230		
	.087			.167		
			k = 1.50			
1	.100	.530	.663	.278	.714	.807
	.058	.452	.596	.168	.618	.734
2	.129			.233		
	.095			.174		
			k = 1.80			
1	.137	.625	.752	.230	.708	.813
	.084	.547	.691	.137	.625	.752
2	.103			.234		
	.076			.179		

Sample Size	$\alpha(10)$	$1-\beta(35)$	$1-\beta(50)$	$\alpha(20)$	$1-\beta(35)$	$1-\beta(50)$
			k = 2.40			
1	.144	.705	.830	.221	.772	.873
	.092	.638	.784	.144	.705	.830
2	.109			.230		
	.084			.181		
			k = 3.00			
1	.148			.219		
	.098			.148		
2	.108			.219		
	.084			.175		

CHAPTER V

SUMMARY

CONCLUSIONS

The mathematical aspect of the N.B.D. has been shown in terms of the work of Goldsmith (1963) and Williamson & Bretherton (1963).

The fitting procedure was shown and five randomly chosen lakes were used as examples of this procedure. A summary of the statistics calculated for each of the lakes sampled was presented along with the chi-squared values for each lake. This summary showed that in just over half of the lakes sampled the N.B.D. was a good model, but that the goodness of fit was influenced by factors subject to variation. On the whole, the N.B.D. was considered to be a suitable model for Whitefish data.

Tables have been constructed which show the relation of sample size to sensitivity ($1-\beta$) at different levels of significance (α) as k, w, and I.R. vary. From these tables, the sample size which gives maximum sensitivity for specific values of k, w, and I.R. may be determined.

RECOMMENDATIONS

(1) An extension of the set of tables showing the relation of sample size to sensitivity to include further values of the parameter k, such as 0.20, 0.40, 0.50.

(2) A more extensive study of the effect of the parameters p and k on the χ^2 criterion for goodness of fit of the N.B.D. to Whitefish data.

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