

Haptic-enabled teleoperation of single-rod hydraulic manipulators

by

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ABSTRACT

Hydraulically powered machines such as backhoes, excavators, and underwater manipulators are commonly used in industries and require extensive interaction with highly unstructured environments. Proper operation of these machines with manipulator-like implements depends on the ability of human operators who currently use hand controllers to command in response to various circumstances. This approach, known as teleoperation, combines the strength and accuracy of the manipulator with the intelligence of the human operators who normally use visual information, directly or through cameras, to accomplish a task. Providing haptic sensation about the task environment to operators, augments their ability to perform tele-manipulation. The focus of this thesis is thus on enabling a hydraulic manipulator to interact with the environment using the concept of haptic-enabled teleoperation. The application area is towards the utilization of hydraulic manipulators to perform hazardous and/or difficult tasks such as earth-moving, live-line and underwater maintenance activities.

Two main issues in teleoperation control are stability and telepresence. The overall closed-loop control system should be stable irrespective of the behavior of the human operator or the task environment. Although the haptic feedback to the master side enables the human operator to rely on his tactile senses, it can make the overall teleoperation system unstable. Teleoperation control of hydraulic manipulators is generally more challenging than that of their electromechanical counterparts, as hydraulic actuators exhibit significant nonlinear characteristics. In this research, a bilateral control scheme has been developed for haptic teleoperation of single-rod hydraulic actuators considering nonlinear dynamics

of hydraulic actuation, haptic device, and the human operator. Stability of the complete control system is confirmed theoretically by building a proper Lyapunov function. In terms of telepresence, haptic sensation is provided to the operator based on the position error between the haptic device end-point and the hydraulic actuator implement displacements. Proposed control scheme can be used in a wide variety of applications where the interaction force between the hydraulic actuator and the task environment cannot be measured. The proposed control scheme is easy to implement as it only requires the measurements of the actuator line pressures and displacements of the master and slave. This control scheme is further improved to incorporate base disturbances of single-rod hydraulic actuators. Stability of this controller with an estimated upper value for the base disturbance is analytically proven. Simulation studies are conducted, which confirms that the developed controllers can effectively stabilize the system while interacting with a task environment. They are further tested experimentally on a hydraulic test rig to verify their practicality and effectiveness in real applications.

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CHAPTER 1

1 INTRODUCTION

1.1 Statement of problem

Over the years, researchers have adopted various approaches in the operation of hydraulic manipulators with the intention of improving safety and efficiency. At one end, they aim to develop fully autonomous robots, wherein, humans are removed from direct operation of the machine [1]. However, several applications require interaction with unstructured environments, in which, a fully autonomous hydraulic machine is not practical. Examples are backhoes, excavators, forklifts, harvesters and underwater manipulators [2, 3, 4]. Due to extensive interaction with environments, these manipulator-like machines rely on the intelligence of the human operators, who currently use hand controllers to command in response to various circumstances [5]. Similarly, the human operator may not be present in the task environment for a number of other reasons such as when the environment is too dangerous or hazardous. Therefore, separating human

operators from task environment is essential in some circumstances and in other cases it is solely for convenience [6]. This approach, known as teleoperation, thus combines the accuracy, high power to mass ratio, and good performance of a robot manipulator with the intelligence and decision-making abilities of the human operator. Operators typically use visual information, directly or through cameras, to perform a task.

A teleoperated system is composed of a master side, in which an operator utilizes a hand-controller; a slave side, where a manipulator emulates the behavior of the master side; and a communication channel, which connects both slave and master sides and various types of information (position, velocity and/or interaction force) between the master site and the slave site are exchanged. If only information of the master is transmitted to the slave side, the teleoperation system is called unilateral. If the slave manipulator reflects information back to the master, by using feedbacks, the system is called bilateral [7].

In order to ensure quick and proper commands by the operators, in the operation of above-mentioned machines, some sensory feedback from the task environment is required. Touch, or *haptic* sensation is one of the most fundamental ways in which people perceive and effect changes in the world around them and can enhance the ability to perform tele-manipulation [6, 8]. Haptic interaction can greatly increase task quality, productivity and safety in operation of hydraulic machines compared to traditional manual levers [8]. Touch interaction differs fundamentally from all our other sensory modalities in that it is intrinsically bilateral. We exchange energy between ourselves and the physical world as we push on it and it pushes back on us. In this exchange, information and intent are conveyed in a physically direct and cognitively primal way [9]. It has been shown that in some applications, haptic feedback alone can even be more valuable than visual feedback

[10]. With respect to bilateral control, when the assigned task implies a contact with the environment (force tracking), or the operator needs to follow particular trajectory (position tracking), the use of haptic sensation can be helpful [8].

Haptic teleoperation of electrically-actuated manipulators has been extensively investigated by many researchers and several control strategies have been proposed, in which, the stability and performance issues were addressed. However, teleoperation control of hydraulic manipulators, in general, is more challenging than that of their electro-mechanical counterparts. Electric actuators normally behave as force/torque sources, while in hydraulic actuators, the control voltage acts to move the spool valve that controls the flow of hydraulic fluid into and out of the actuator. This flow in turn causes a pressure differential buildup that is proportional to the actuator force. Even if some of the system dynamics is ignored, the control voltage fundamentally controls the derivative of the actuator force and not the force itself [11]. Hydraulic actuators also exhibit significant nonlinear characteristics [11]. These challenges make the stability analysis difficult. Nevertheless, the use of hydraulics offers several advantages such as good stiffness, high power to mass ratio, good performance at low speed, and the ability to work underwater [12]. Furthermore, hydraulic actuators are capable of applying high constant torques for a long period of time, which in case of electrically-actuated actuators causes overheating of motors. These advantages and lack of enough research in the field of haptic-enabled teleoperation of hydraulic manipulators provided the motivation to study and develop stable control strategies for hydraulic manipulators.

This thesis focuses on design of control systems applied to hydraulic manipulators using the Lyapunov stability theorems that incorporate nonlinearities and guarantee the system

stability [6, 13, 14, 15]. The aim is to design and employ stable feedback control systems in order to bilaterally control hydraulic manipulators with the capability of perceiving the interaction force between the hydraulic manipulator end-effector and the task environment when it is not feasible to measure the interaction force. In addition, the controller should be capable of reducing position errors at master and slave sides.

In bilateral teleoperation of hydraulic manipulators, designing feedback control system needs to consider some issues when the slave manipulator interacts (constrained motion), or is not in contact (free motion) with the environment. Additionally, the complete bilateral control system should be stable regardless of the input commands or the task environment. Contact information, in constrained motion, is helpful to the operator in controlling the contact force, and therefore, reducing damage to the manipulator and environment as well as in probing an uncertain environment. With respect to the position tracking in free motion, hydraulic actuators at the slave site could start lagging or stay behind the master motion depending on the configuration of the slave and responsiveness of hydraulic actuation systems. Therefore, the requirement of position tracking is more challenging in hydraulic manipulators than that of the electromechanical counterparts. Furthermore, hydraulic actuators exhibit significant nonlinear characteristics, making the stability analysis difficult. Thus, the main challenges are: (i) achieving proper feedback from slave side to reduce the position error in free motion, (ii) estimating the interaction force between the manipulator end-effector and the environment in constrained motion and (iii) guaranteeing stability of the control system.

Another important research problem that is addressed in this thesis is the control of hydraulic manipulator with base motion. Most industrial manipulators have a base fixed in

an inertial frame. There are, however, manipulators for which this is not the case. An example is manipulators mounted on cranes for live-line maintenance or underwater vehicles. This research is built upon the work done previously in [16, 17], in which various force feedback schemes were developed to control a hydraulic manipulator for performing typical live line maintenance tasks. In [16, 17], the experiments were, however, conducted on a hydraulic manipulator with a stationary base only. In some real applications, the slave is installed on top of a crane bucket, and therefore, in such a configuration, the base will be subjected to unwanted motion due to wind gust, unstable soil, and compliance in the crane actuation. When the manipulator's base is excited, the difficulty in performing tasks naturally increases. Thus, the issue of base disturbance in haptic-enabled teleoperated systems needs more explorations.

1.2 Objectives of this thesis

The objectives of the thesis are:

- (i) To investigate effects of base disturbance on the performance of otherwise well-performing teleoperated hydraulic manipulators. Thoroughly test the previously developed force feedback schemes in presence of base disturbance and evaluate the performance of the overall teleoperated system in performing maintenance tasks.
- (ii) To design a stable control scheme in order to control a single-rod hydraulic actuator bilaterally with the capability of regenerating the interaction force between the hydraulic actuator and the environment using the haptic device to be applied to the operator's hand. Control scheme should also be capable of reducing the position error at both master and slave sides.

- (iii) To design a stable bilateral control system for single-rod hydraulic actuator under base excitation without having any prior knowledge about the base motion.

1.3 Scope of this thesis

In the first part of the research, the available teleoperated hydraulic manipulator [16, 17] is retrofitted to include a Stewart platform for emulating motion of the crane bucket in live-line maintenance. The hydraulic manipulator is controlled bilaterally using a haptic device. The teleoperated hydraulic system includes a PHANTOM Desktop haptic device (master site) and a 4-degrees of freedom (DOF) slave Kodiak hydraulic manipulator placed on a 6-DOF Stewart platform. First, the previously developed force feedback schemes are rigorously tested in the presence of base disturbance. The tested schemes are: (i) virtual fixture scheme, (ii) force augmentation scheme, and (iii) augmented virtual scheme. The effectiveness of each of the three schemes is validated by emulating two live power line maintenance tasks in a laboratory setting. Performance of each scheme is evaluated under three measures: (i) task completion time, (ii) number of failed trials and, (iii) displacement of the manipulator end-effector.

For the second part of the research, two advanced nonlinear controllers are designed and experimentally validated for bilateral control of single-rod hydraulic actuators. Two main issues in bilateral control systems are stability and performance [18]. Foremost, the overall closed-loop system should be stable irrespective of the input commands (provided by the human operator) or the task environment [7]. Furthermore, a feel of performing task at the remote site must be available to the operator. In general, there is a tradeoff between accurate tracking and sufficient stability margins [19]. Many research studies have

addressed this tradeoff for electrically-actuated manipulators (as presented in [7] and the references cited therein). In comparison to electrically-actuated robots, research on application of bilateral control applied to hydraulic manipulators is sparse. There are some researches (see [20] and references cited therein) on applying the existing control schemes, originally designed for electrically-actuated robots, to the hydraulic robots; however, none of them investigated the stability of the entire nonlinear system. There also exists studies on development of controllers for double-rod hydraulic actuators which guarantees the system stability [21]. However, there are no bilateral control schemes developed for haptic control of *single-rod hydraulic manipulators* that includes nonlinear dynamics of hydraulic functions and the human operator's dynamics. The lack of enough studies on design of stable controllers for haptic-enabled single-rod hydraulic actuators motivated the research undertaken in this thesis. The aim is to theoretically prove the system stability, and to experimentally validate the practicality of the proposed control system by applying the concepts to real-world applications, using a specially developed test-rig.

In regards to the controller design and stability analysis, comparing different control schemes showed that those using nonlinear methods realize higher accuracy compared to the controllers directly based on linear models [22]. Therefore, selecting a nonlinear method will result in better performance. Nonlinear stability analysis techniques are generally based on the energy of a system, such as Lyapunov stability and passivity. The passivity approach, however, is quite conservative in many cases degrading the transparency and performance of the system [20], while the Lyapunov stability analysis has shown better results for highly nonlinear systems including electro-hydraulic manipulators [6, 13, 14, 15].

1.4 Application area of interest

The targeted application of this research is live transmission line maintenance operation. However, this research is equally relevant to applications such as backhoes, excavators, forklifts and underwater manipulators, which involves hydraulic tele-manipulation. Due to the laboratory setup, most experiments are conducted for live-line maintenance tasks.

Live-line maintenance is carried out for several reasons such as changing an insulator, replacement of damaged section of a conductor, testing an insulator, or relocating a conductor to a higher pole. In live-line maintenance, it is imperative that the power transmission system must always be available under high voltage [23]. Inspection and maintenance of an overhead power distribution system, however, is a dangerous task to perform, especially in places with acute climatic conditions preventing human exposure over extended periods of time. Therefore, utilities around the world have started to develop and examine the application of robotic systems for the inspection and maintenance of power line distribution networks [24, 25]. The application of teleoperated robotic systems in live power line maintenance possesses some advantages. They increase the operator's safety and allow to combine the accuracy and good performance of the system with the intelligence of the operator [26, 27]. In spite of these advantages, the adoption of robotics technology in live-line maintenance is still new and presents challenges that need investigation [16, 17].

1.5 Thesis outline

In Chapter 2, a detailed literature review along with some definitions is provided. The advantages of using a haptic interface in tele-manipulation; and unilateral and bilateral

control modes are described first. Next, the previously developed force feedback schemes are explained, followed by a literature review on teleoperation of manipulators with mobile base and Lyapunov control technique for hydraulic actuators.

Chapter 3 describes the experimental setup with detailed information about system components. The derivation of dynamic model of the complete system consisting of single-rod hydraulic actuator interacting with an environment, human operator, and the haptic device is also presented.

Chapter 4 serves as a prelude to understand how base disturbance effects the previously developed unilateral and bilateral controllers for hydraulic teleoperation. In this chapter, the force feedback schemes developed previously, namely, virtual fixture scheme, force augmentation scheme, and augmented virtual scheme are tested on a hydraulic manipulator retrofitted with a Stewart platform to create base excitation.

In Chapters 5, a bilateral control scheme is developed, implemented, and experimentally validated for a fixed base single-rod hydraulic actuator. The stability of the complete bilateral control scheme has been theoretically proven using Lyapunov techniques. Stability is proven considering nonlinear hydraulic functions, servo-valve dynamics, haptic device dynamics, human operator dynamics, and dynamics of the task environment. Simulations and experiments are conducted to verify the practicality of the proposed controller.

In Chapter 6, the control scheme developed in Chapter 5 is modified for base excited single-rod hydraulic actuators and the system stability is theoretically proven to be insensitive to the uncertainties of the physical parameters and of the measurement of the

base point motion. Effectiveness of the proposed controller is verified by simulation and experimental studies.

Chapter 7 presents the contributions made by the research as well as some recommendations for future work.

CHAPTER 2

2 RELEVANT BACKGROUND

2.1 Basic concepts and definitions

2.1.1 *Teleoperation*

The prefix tele from Greek origin means at a distance and teleoperation naturally indicates operating at a distance [3, 7]. The term “distance” could refer to a physical distance or could also refer to a change in scale. Teleoperation extends the human capability to manipulating objects remotely by providing the operator with similar conditions as those at the remote location. This is achieved via installing a similar manipulator or joystick, called the master, at the human’s end that provides motion commands to the slave which is performing the actual task. In a general setting, the human

imposes a force on the master manipulator which in turn results in a displacement that is transmitted to the slave that mimics that movement.

Tele-manipulation has shown to have the following major advantages [25]:

- It keeps the operator away from the hazardous areas where the task is to be performed and consequently increases the operator's safety;
- It allows for a more compact design suitable for hard to reach and constrained working spaces;
- It combines the accuracy and strength of the robot with the intelligence of the human operator.

A teleoperation system is composed of [7]: (i) a master site, (ii) a remote site, (iii) a communication channel which connects both sites through a feedback system. Standard teleoperation system architecture is shown in Figure 2.1.

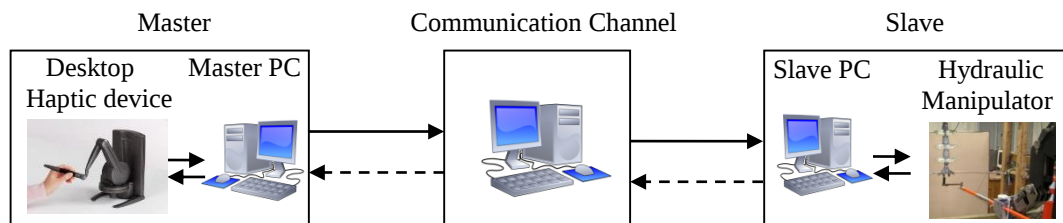


Figure 2.1 Standard teleoperation system.

In Figure 2.1, the arrows can either be from left to right (solid arrows) or in both ways (both solid and dashed arrows) [7]. Figure 2.2a and 2.2b show how information exchanges between master and slave sites in unilateral and bilateral control modes, respectively. In unilateral mode, only information of the master device is sent to the slave side to be used in slave controller, while if the slave manipulator also redirects some information back to the master, by using feedbacks, the system is bilateral.

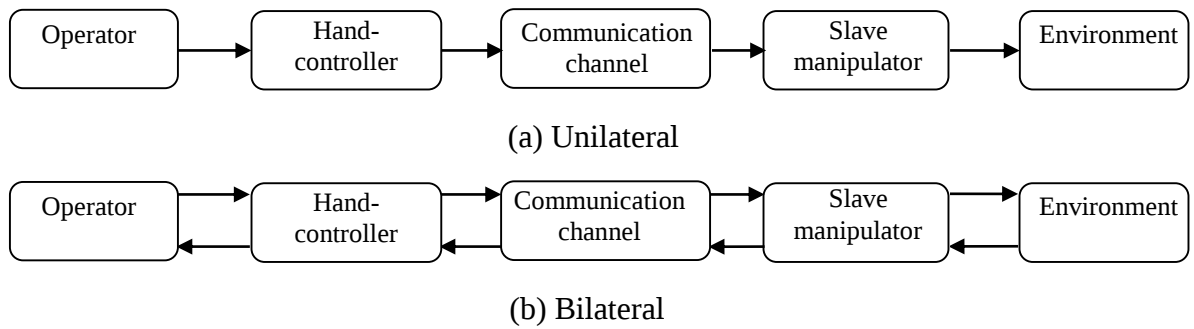


Figure 2.2 Teleoperation system.

2.1.2 Haptics

The definition of haptics is of or relating to the sense of touch. Haptic control implies that the human-machine interface can be programmed to artificially supply the user with arbitrary force sensations. Typically the haptic force is used to relay information about the force acting on a remote or virtual environment. Haptic feedback means that information can be fed back to the operator in the form of force signals. The availability of both haptic and motion information exchange between operator and remote manipulator sites allows a sense of “telepresence” to the operator [9].

Haptic controls have been used in wide variety of applications such as hazardous material handling in nuclear services [28], medical robotics, tele-ultrasound, tele-surgery [29, 30, 31, 32, 33, 34], underwater robotics [35], mobile robots [36, 37, 38, 39], and micro-manipulation and assembly [40]. As far as the haptic control of hydraulic actuators is concerned, research in this area is limited to a few studies. Kontz *et al.* [8] showed that by shaping the impedance, a better feel of telepresence can be achieved for operators in a human-in-the-loop excavator test-bed [8]. The force feedback was generated using a virtual spring which couples the displacement of the haptic device to the displacement of the

bucket, *i.e.*, the force feedback is proportional to the position error between the master and slave manipulators. In a different application, Kontz *et al.* [2] used virtual fixtures (constrained motions) to generate force signals in a forklift truck. Rather than making the system completely transparent, they allowed it to react to virtual forces acting on the end-effector. Haptic control of excavators has also been explored by Parker *et al.* [5] and Lawrence *et al.* [41], who developed a magnetically-levitated joystick with stiffness feedback to control a 4-DOF hydraulic excavator. Zarei-nia *et al.* [20] experimentally compared the performance of a number of teleoperation control schemes, on a double-rod hydraulic actuator. Most of the previous studies did not investigate the stability of the entire control system considering combined operator-haptic-actuator-environment dynamics and nonlinear hydraulic functions specifically for single-rod hydraulic actuator. Research in the area of bilateral control of hydraulic actuators is still new, and there remain many challenges to make haptic control ready for hydraulics in practice [42].

2.2 Prior relevant work

In this section, the previous work done in developing force feedback schemes for hydraulic manipulators is discussed. Literature review related to teleoperation of mobile manipulators and Lyapunov controllers for hydraulic actuators is presented next.

2.2.1 Force feedback schemes

Virtual fixture: The concept of virtual fixtures is usually adopted for unilateral haptic control of a hydraulic manipulator [17]. Virtual fixture constrains large movements of the operator's hand into constrained regions or along desired paths defined in the slave

manipulator working space. The concept of virtual fixture was first introduced by Rosenberg [43]. To understand the concept of virtual fixture, a simple case of a real physical fixture such as a ruler is typically used. A simple task of drawing a straight line on a piece of paper without using any tool is generally difficult. But, by using a simple device like a ruler, the pen can be guided along a straight line, which increases the task easiness and accuracy while decreasing the task completion time [44]. Furthermore, if the ruler is used to guide a cutting tool to cut a work-piece, it works as a barrier to safeguard against dangerous or damaging failures to increase safety.

With respect to live transmission line maintenance application, virtual fixture was used to define a barrier for the slave manipulator in order to prevent it from hitting insulators or other elements of transmission lines that can possibly be hazardous or damaging [17]. The virtual fixture force generated is proportional to the manipulator's penetration into the forbidden region. The force can be generated by a virtual spring which pulls the operator's hand back on track or out of the forbidden region. Concept of virtual fixture is described in Figure 2.3.

With reference to Figure 2.3, when the haptic end-effector is on desired trajectory, the virtual spring remains in rest position ($x_a = x_d, y_a = y_d, z_a = z_d$), and no force is generated ($\vec{F}_{VF} = 0$). However, when the operator moves the end-effector away from desired trajectory (off-track), the virtual spring generates a force ($\vec{F}_{VF} \neq 0$) which is proportional to the amount of penetration into the forbidden region. As seen, the force generated by the haptic device is computed based on the distance between the actual position of master implement, $\vec{P}_A = [x_a \quad y_a \quad z_a]^T$, and the desired position of master implement, $\vec{P}_B = [x_d \quad y_d \quad z_d]^T$.

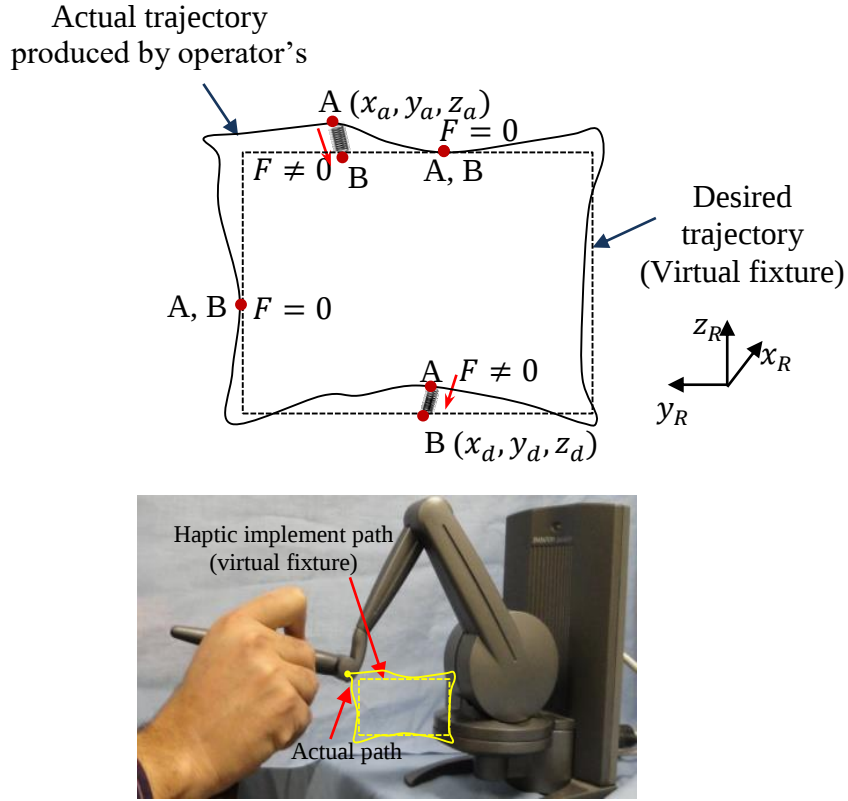


Figure 2.3 Virtual spring pulls the operator's hand toward desired trajectory. Haptic end-effector position can be on-track ($\vec{F}_{VF} = 0$) or off-track ($\vec{F}_{VF} \neq 0$). A and B are haptic end-effector actual position (x_a, y_a, z_a) and target (desired) position (x_d, y_d, z_d) , respectively.

The virtual fixture force generated by the haptic device, $\vec{F}_{VF}$, is given by [17]:

$$\vec{F}_{VF} = -G_{VF}\vec{R} \quad (2.1)$$

where G_{VF} is the impedance of the virtual fixture, or basically the stiffness of the virtual spring pushing/pulling the operator's hand toward the desired trajectory and the haptic end-effector position error (*i.e.* distance between the actual and desired position), $\vec{R}$, is defined as:

$$\vec{R} = \begin{bmatrix} x_a - x_d \\ y_a - y_d \\ z_a - z_d \end{bmatrix} \quad (2.2)$$

Augmentation force: In unilateral control, the concept of virtual fixture was shown to reduce master position errors originating from the operator's unwanted hand motion. While the virtual fixture force keeps the operator's hand on the defined virtual path, desired motion at the slave side cannot be assured. For example, the position tracking of the slave manipulator can simply be violated by the fast motion of the operator's hand at the master side reflecting the mismatch between the master and slave dynamics. This is predominantly evident in hydraulic manipulators, since hydraulic actuators exhibit significant nonlinear characteristics. Therefore, the concept of position referenced force augmentation scheme, in bilateral mode, was developed to compensate for positioning errors in hydraulics for live-line maintenance [16].

The augmentation force (or simply position error force), signals the operator to slow down the haptic implement (hand) motion when position error becomes larger than the accuracy expected from the controller at the slave end-effector. The direction of this force should be opposite to the operator's hand velocity vector at the master implement. This force allows the operator to realize position error at the slave site. When position error at the slave end-effector is apparent, the augmentation force is initiated alerting the operator to slow down the hand motion allowing the slave manipulator to catch up. The augmentation force is defined as [16]:

$$\vec{F}_{AU} = \begin{cases} -|G_{AU}(\vec{R}_e - \vec{R}_t)|\hat{v} & \|\vec{R}_e\| > \|\vec{R}_t\| \\ 0 & \|\vec{R}_e\| \leq \|\vec{R}_t\| \end{cases} \quad (2.3)$$

As shown in Figure 2.4, $\vec{R}_e$ is the vector of position error at the manipulator end-effector. As the controller has some inherent error, the augmentation force is only generated when the position error is greater than a threshold ($\vec{R}_t$). This threshold is defined based on the

steady-state positioning error originating from the manipulator's controller and sensors resolution and helps to prevent repeated activation-deactivation cycles. When $\|\vec{R}_e\| \leq \|\vec{R}_t\|$, the haptic device does not produce any augmentation force. When $\|\vec{R}_e\| > \|\vec{R}_t\|$, $\vec{F}_{AU}$ is proportional to $\vec{R}_e - \vec{R}_t$ in terms of magnitude, and parallel to $\hat{v}$ which is the unit vector of haptic implement instantaneous velocity. The negative sign indicates that the augmentation force acts in the opposite direction of the haptic device instantaneous velocity. G_{AU} is a diagonal matrix representing the impedance of augmented force.

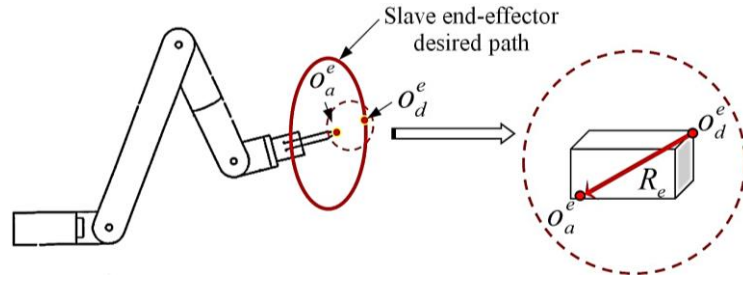


Figure 2.4 Desired (O_d^e) and actual (O_a^e) positions of the slave manipulator end-effector.

Augmented virtual fixture: The augmented virtual fixture is a combination of virtual fixture scheme and force augmentation scheme [16]. Figure 2.5 shows how the augmented virtual fixture force is calculated using the virtual fixture and the augmentation forces. While the virtual fixture is intended to aid the operator in following a predefined path, the proposed augmentation force makes the master dynamics a better match with the dynamics of the hydraulic manipulator. Using this scheme, the combined virtual fixture and augmentation force can reduce position errors at both the master device implement and the slave manipulator end-effector. As shown in Figure 2.5, while the virtual fixture force ($\vec{F}_{VF}$) pulls the operator's hand toward the haptic desired path, the augmentation force ($\vec{F}_{AU}$) slows

down the operator's hand motion. The augmented virtual fixture force ($\vec{F}_{AVF}$) is then calculated as below [16]:

$$\vec{F}_{AVF} = \rho_{VF}\vec{F}_{VF} + \rho_{AU}\vec{F}_{AU} \quad (2.4)$$

where ρ_{VF} and ρ_{AU} are the weighing factors to adjust the relative effect of virtual fixture and augmentation forces, respectively.

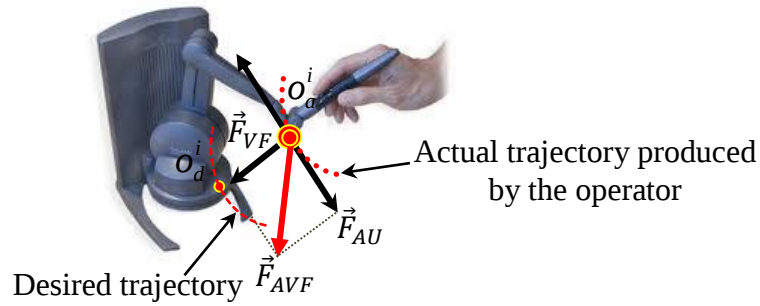


Figure 2.5 Augmented virtual fixture scheme.

2.2.2 Teleoperation of mobile manipulators

A mobile robotic system is composed of a manipulator mounted on top of a movable base, and is normally used in unstructured and unknown environments which makes its operation more challenging than in a case where the environment is known [45, 46]. Teleoperation of manipulators with moving base has many promising applications such as live transmission line maintenance, underwater inspection, flight simulators, planets exploration, navigation in hazardous environments, and inspection of industrial constructions [47]. When the base motion is undesirable, and originates from disturbing forces and torques, the system is called base-excited system [48]. In live-line maintenance, as an example, the slave manipulator must be installed on top of a crane bucket. In such a configuration, the base will be subjected to unwanted motion owing to wind gust, unstable

soil and compliance in the crane actuation. The unwanted motion is highly unpredictable owing to the lack of realistic and accurate models of the working environment. When the manipulator's base is excited, the difficulty in performing tasks naturally increases.

Research activities in the area of teleoperation of manipulators with mobile base, are limited to a few studies, which mostly discusses the design and development of new mechanisms for mobile platforms [23, 49, 50, 51, 24]. None of them, however, dealt with investigating the effect of base excitation of the slave manipulator in performing teleoperation.

2.2.3 *Lyapunov controller for hydraulic actuators*

The performance achievable by classical linear controllers are usually limited due to highly nonlinear behavior of the hydraulic dynamics. Lyapunov-based control technique incorporates all nonlinearities and improves the performance of the system while guaranteeing the stability [6, 13, 14, 15]. Lyapunov stability analysis has been extensively used in controller design for hydraulic actuators. Most researches have been carried out on double-rod hydraulic actuators. For example, Sekhavat *et al.* [52, 53] developed a controller based on Lyapunov stability analysis to regulate the impacts of a hydraulic actuator that comes in contact with a non-moving environment. Niksefat *et al.* [54] proposed a Lyapunov-based control scheme that allows a hydraulic actuator to follow a free space trajectory and then make and maintain contact with the environment to apply a desired force. Halanay *et al.* [55] used Lyapunov stability analysis method for a hydraulic actuator to study the effect of the mounting structure of an airplane on hydraulic-powered control. Becker *et al.* [56] proposed a model-based robust controller for electro-hydraulic

robots in which the stability proof was based on the Lyapunov method. With regards to bilateral teleoperation of double-rod hydraulic actuators, only Zarei-Nia *et al.* [21, 57] and Li and Krishnaswamy [58, 59] have developed a bilateral control scheme till to date. However, both of these studies have certain limitations. The controller designed by Zarei-Nia *et al.* [21] was based on the interaction force readings between the slave manipulator and the task environment. Thus, it is not suitable for applications where mounting a force sensor at the end-effector is not feasible. The approach proposed by Li and Krishnaswamy [58] was based on the passivity, which, although stable, is very conservative. Moreover, the condition of “sufficiently slow manipulation” is another limitation of the passivity approach.

In contrast to double-rod hydraulic actuators, the two chambers of a single-rod hydraulic actuator have different areas. As a result, the dynamics in which the pressure changes in the two chambers cannot be combined into a single load pressure equation. This complicates the controller design since it not only increases the dimension of the system to be dealt with, but also brings in the stability issue of the added internal dynamics. Thus, research in this field is very limited and often restricted due to the many assumptions that need to be made [60]. Guan *et al.* [61], presented a nonlinear adaptive robust control method based on Lyapunov stability method, with adaptation laws to compensate for the uncertain nonlinear parameters due to the varieties of the original control volumes. Furthermore, by combining back-stepping techniques and a simple robust control method, the whole system’s controller and adaptation laws were presented, which can compensate for all unknown parameters and uncertain nonlinearities. Guo *et al.* [62], developed a nonlinear cascade controller based on an extended disturbance observer to track desired

position trajectory for electro-hydraulic single-rod actuators in the presence of both external disturbances and parameter uncertainties. The proposed extended disturbance observer accounted for external perturbations and parameter uncertainties separately. In addition, the outer position tracking loop used sliding mode control to compensate for disturbance estimation error with desired cylinder load pressure as control output; the inner pressure control loop was designed using the back-stepping technique. The stability of the overall closed-loop system was proven based on the Lyapunov theorem. Another approach was proposed by Kim *et al.* [63], in which a nonlinear controller based on an extended second-order disturbance observer was presented to track desired position for an electro-hydraulic single-rod actuator in the presence of both external disturbances and parameter uncertainties. The extended second-order disturbance observer, based on Lyapunov stability method, dealt with not only the external perturbations, but also parameter uncertainties. None of these studies, however, considered dynamics of haptic device and human operator in their design. Thus, according to the literature review, there is a lack of research on bilateral control of single-rod hydraulic actuators.

2.3 Summary

In this chapter, a comprehensive literature review was presented. Haptic teleoperation of hydraulic manipulators in unilateral and bilateral control modes was described first. Next, the previously developed force feedback schemes were explained, where virtual fixture, force augmentation and augmented virtual fixture were used to control a hydraulic manipulator for performing typical live line maintenance tasks. It was found that the issue of base disturbance in haptic-enabled teleoperated systems needed more explorations as all

the previous force schemes were validated on hydraulic manipulator with a stationary base. Later in this chapter, the literature review on teleoperation of manipulators with mobile base and Lyapunov control technique applied to hydraulic systems was presented. It was noted that for hydraulic systems, none of the previous studies included haptic force feedback in their design, nor was the stability of a bilateral system studied for single-rod hydraulic actuators. This thesis fills the void in literature, by designing and experimentally validating two stable advanced nonlinear control schemes based on Lyapunov's stability theory to control single-rod hydraulic actuators bilaterally.

CHAPTER 3

3 EXPERIMENTAL SETUP AND MODELING

In this chapter, the experimental setup is explained first, followed by describing the general dynamics of the bilateral tele-manipulation system which consists of human operator, haptic device, single-rod hydraulic actuator, and task environment.

3.1 Test rig

The setup has been constructed to validate the feedback control schemes designed previously in the presence of base disturbance and has been used to test the proposed advanced nonlinear controllers for single-rod hydraulic actuators.

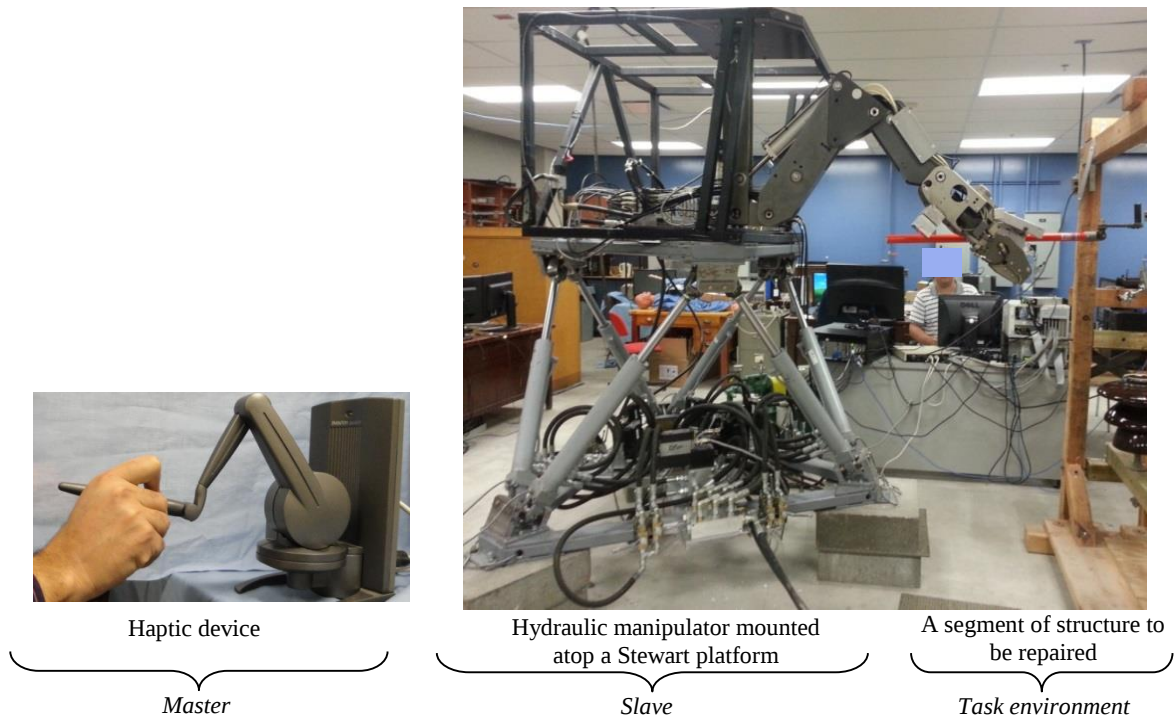


Figure 3.1 Hardware-in-the-loop teleoperated hydraulic manipulator test rig.

The hardware-in-the-loop test rig (see Figure 3.1) comprises of an industrial six degree-of-freedom (DOF) hydraulic manipulator (MAGNUM 7-function hydraulic manipulators manufactured by International Submarine Engineering Ltd., Canada) mounted atop a 6-DOF Stewart platform, a PHANTOM haptic device that allows control of the manipulator by the operator while creating a feel of force, a frame replicating a segment of structure to be repaired, and a tool attached to the hydraulic manipulator. Compared to electrically-actuated manipulators, hydraulic manipulators combine compactness with high power and are more adaptable with regards to insulation and environmental considerations.

The hydraulic manipulator DOFs are: a rotation about vertical axis (arm), three rotations about horizontal axes (shoulder, main elbow, and extended elbow), and two wrist rotations (yaw and roll). In addition, an actuator is provided at the end-effector, which is not considered as a DOF, and is used to open and close the jaw. The movable base is composed

of six single-rod hydraulic actuators; each controlled by a proportional valve. In the middle of each actuator, a linear displacement sensor is installed that measures the actual position of corresponding cylinder. The manipulator is placed on a corner of the top plate. There exist three linear movements (lateral, longitudinal and vertical), and three rotations (pitch, roll and yaw). A computer controls the Stewart platform using a QuaRC interfacing board. The developed test rig with the parallel platform (Stewart Platform) can be used to simulate various motions (see Figure 3.1). The Stewart platform produces all possible degrees of freedom such as surge, sway, heave, roll, pitch and yaw.

The master and slave devices are connected to a PC using parallel port and data acquisition boards, respectively. Data acquisition boards are used to send control signals to the servo-valves and read the manipulator's joint angle encoders. The arrangement described above enables a commercially available hydraulic manipulator to work with a haptic device for performing tasks.

3.2 Kinematic equations

A schematic of the slave hydraulic manipulator and corresponding coordinate frames is shown in Figure 3.2. In designed experiments, the actuators creating the yaw and roll rotations were switched off as they were not required for the application (typical live-line tasks) in this thesis. Only the first four DOFs were employed to run the manipulator; DOFs are: vertical rotation (θ_1^s), shoulder (θ_2^s), main elbow (θ_3^s), and extended elbow (θ_4^s). The tool is attached to the last link and is always kept horizontal. In Figure 3.2, $\{x_e^s y_e^s z_e^s\}$ denotes the coordinate system attached to the manipulator end-effector, and the fixed (global) coordinate system is denoted by $\{x_0^s y_0^s z_0^s\}$. The superscript "s" indicates the parameter that belongs to the slave (hydraulic manipulator). In order to always keep the

tool parallel to the horizontal plane, the joint angle θ_4^s (extended elbow) is expressed in terms of θ_2^s and θ_3^s , as follows:

$$\theta_4^s = -\theta_3^s - \theta_2^s \quad (3.1)$$

Thus, only the first three angular displacements of the manipulator ($\theta_{i=1,2,3}^s$) are required to solve the kinematics.

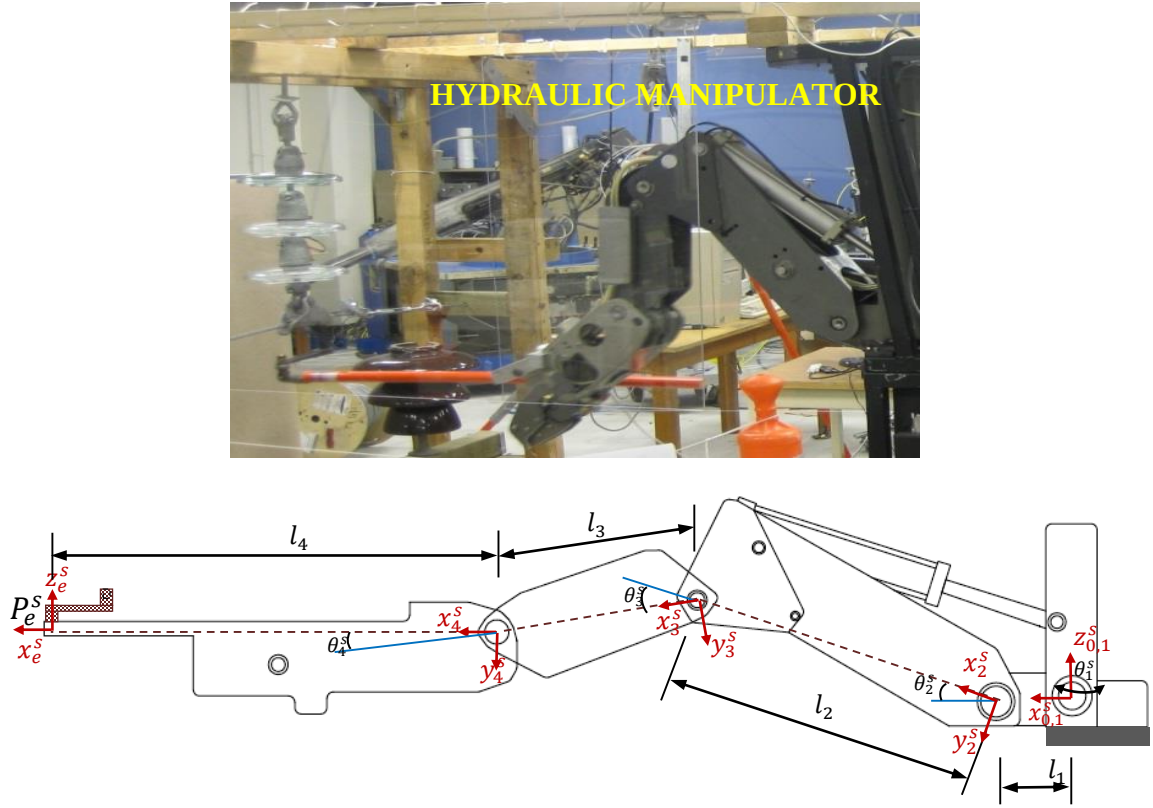


Figure 3.2 Coordinate frames of Kodiak 1000 hydraulic manipulator.

Let $\vec{p}_{e,d}^s = [x_e^s \ y_e^s \ z_e^s]^T$ denote the desired coordinate of hydraulic manipulator end-effector. By solving the inverse kinematics of the manipulator, the desired angular displacements ($\theta_{i=1..4}^s$) are calculated using:

$$\theta_1^s = \tan^{-1} \left(\frac{y_e^s}{x_e^s} \right) \quad (3.2)$$

$$\theta_2^s = \tan^{-1} \left(\frac{k_2}{k_1} \right) + \tan^{-1} \left(\frac{\pm \sqrt{k_1^2 + k_2^2 - k_3^2}}{k_3} \right) \quad (3.3)$$

$$\theta_3^s = \tan^{-1} \left(\frac{2l_2 k_2 c_2 - 2l_2 k_1 s_2}{k_1^2 + k_2^2 - l_2^2 - l_3^2} \right) \quad (3.4)$$

Using Eq. (3.1), we have:

$$\begin{aligned} \theta_4^s = & -\tan^{-1} \left(\frac{k_2}{k_1} \right) - \tan^{-1} \left(\frac{\pm \sqrt{k_1^2 + k_2^2 - k_3^2}}{k_3} \right) \\ & - \tan^{-1} \left(\frac{2l_2 k_2 c_2 - 2l_2 k_1 s_2}{k_1^2 + k_2^2 - l_2^2 - l_3^2} \right) \end{aligned} \quad (3.5)$$

where $c_i = \cos(\theta_i^s)$, $s_i = \sin(\theta_i^s)$, and

$$k_1 = x_e^s c_1 + y_e^s s_1 - l_1 - l_4 \quad (3.6)$$

$$k_2 = -z_e^s \quad (3.7)$$

$$k_3 = \frac{k_1^2 + k_2^2 + l_2^2 - l_3^2}{2l_2} \quad (3.8)$$

A schematic of the Stewart platform is shown in Figure 3.3. As observed, there are six hydraulic actuators that are commanded to generate the desired posture (position and

orientation) of frame $\{X^t Y^t Z^t\}$. The position, D , and orientation, R , of frame $\{X^t Y^t Z^t\}$ with respect to frame $\{X^b Y^b Z^b\}$ are defined as follows:

$$D = [D_x \ D_y \ D_z]^T \quad (3.9)$$

$$R = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} \quad (3.10)$$

With reference to Figure 3.3, and as shown in [64], the length of each actuator, $L_{i=1\dots 6}$, is given by:

$$\begin{aligned} L_i &= |S_i| \\ &= \sqrt{S_{i,x}^2 + S_{i,y}^2 + S_{i,z}^2} \end{aligned} \quad (3.11)$$

where,

$$S_i = [S_{i,x} \ S_{i,y} \ S_{i,z}]^T = D + RT_i - B_i \quad (3.12)$$

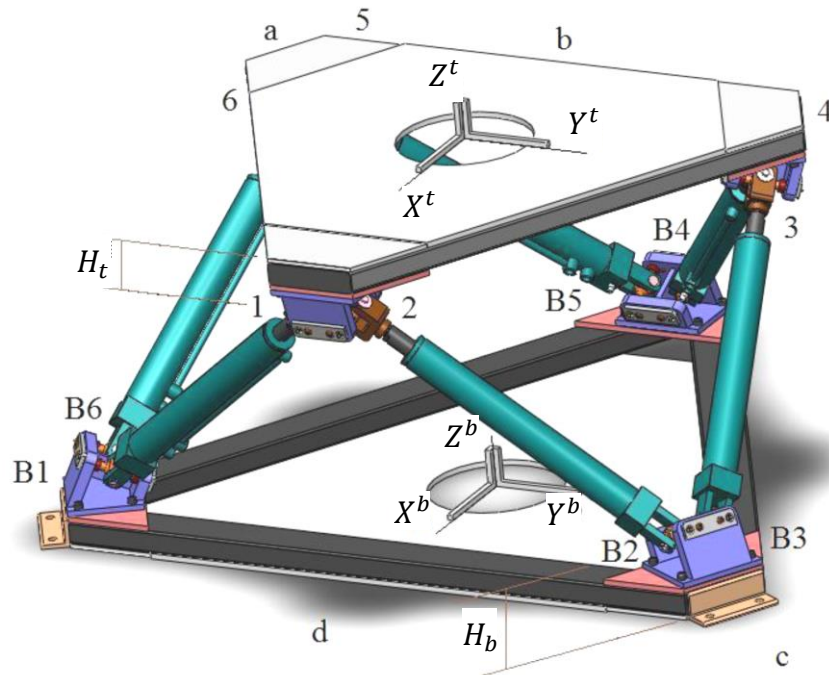


Figure 3.3 Coordinate systems of Stewart platform.

In Eq. (3.12), D and R are given by Eq. (3.9) and Eq. (3.10). B_i and T_i are position vectors of the endpoints of each actuator with respect to frames $\{X^b Y^b Z^b\}$ and $\{X^t Y^t Z^t\}$, respectively. They are defined as below:

$$\left\{ \begin{array}{l} T_1 = \begin{bmatrix} L_t - r_t & \frac{-a}{2} & -H_t \end{bmatrix}^T \\ T_2 = \begin{bmatrix} L_t - r_t & \frac{a}{2} & -H_t \end{bmatrix}^T \\ T_3 = \begin{bmatrix} \frac{a}{2} \sin 60^\circ - (L_t - r_t) \sin 30^\circ & (L_t - r_t) \cos 30^\circ + \frac{a}{2} \sin 30^\circ & -H_t \end{bmatrix}^T \\ T_4 = \begin{bmatrix} -r_t & (L_t - r_t) \cos 30^\circ - \frac{a}{2} \sin 30^\circ & -H_t \end{bmatrix}^T \\ T_5 = \begin{bmatrix} -r_t & -(L_t - r_t) \cos 30^\circ + \frac{a}{2} \sin 30^\circ & -H_t \end{bmatrix}^T \\ T_6 = \begin{bmatrix} \frac{a}{2} \sin 60^\circ - (L_t - r_t) \sin 30^\circ & -(L_t - r_t) \cos 30^\circ - \frac{a}{2} \sin 30^\circ & -H_t \end{bmatrix}^T \end{array} \right. \quad (3.13)$$

and,

$$\left\{ \begin{array}{l} B_1 = \begin{bmatrix} r_b & -(L_b - r_b) \cos 30^\circ + \frac{c}{2} \sin 30^\circ & H_b \end{bmatrix}^T \\ B_2 = \begin{bmatrix} r_b & (L_b - r_b) \cos 30^\circ - \frac{c}{2} \sin 30^\circ & H_b \end{bmatrix}^T \\ B_3 = \begin{bmatrix} (L_b - r_b) \sin 30^\circ - \frac{c}{2} \sin 60^\circ & (L_b - r_b) \cos 30^\circ + \frac{c}{2} \sin 30^\circ & H_b \end{bmatrix}^T \\ B_4 = \begin{bmatrix} -(L_b - r_b) & \frac{c}{2} & H_b \end{bmatrix}^T \\ B_5 = \begin{bmatrix} -(L_b - r_b) & -\frac{c}{2} & H_b \end{bmatrix}^T \\ B_6 = \begin{bmatrix} (L_b - r_b) \sin 30^\circ - \frac{c}{2} \sin 60^\circ & -(L_b - r_b) \cos 30^\circ - \frac{c}{2} \sin 30^\circ & H_b \end{bmatrix}^T \end{array} \right. \quad (3.14)$$

where L_t , L_b , r_t and r_b are calculated using:

$$L_t = (a + b) \sin 60^\circ, \quad L_b = (c + d) \sin 60^\circ \quad (3.15)$$

$$r_t = \frac{\sqrt{3}}{6} (2a + b), \quad r_b = \frac{\sqrt{3}}{6} (2c + d) \quad (3.16)$$

The geometries of the Kodiak manipulator and the Stewart platform are listed in Table 3-1.

Table 3-1 Geometries of Kodiak 1000 hydraulic manipulator and Stewart platform.

Component	Parameter	Length (mm)
Kodiak manipulator	Extension link, l_1	133
	Shoulder, l_2	459
	Elbow, l_3	342
	Extended elbow, l_4	1354
Stewart platform	Top plate, short side, a	101.6
	Top plate, long side, b	1045.5
	Bottom plate, short side, c	101.6
	Bottom plate, long side, d	1368.0
	Joint distance from bottom plate, H_b	140.5
	Joint distance from top plate, H_t	145.8

Figure 3.4 depicts coordinate frames of the PHANToM Desktop haptic device. The actual trajectory of operator's hand at master site (haptic device), forms the desired trajectory of manipulator end-effector.

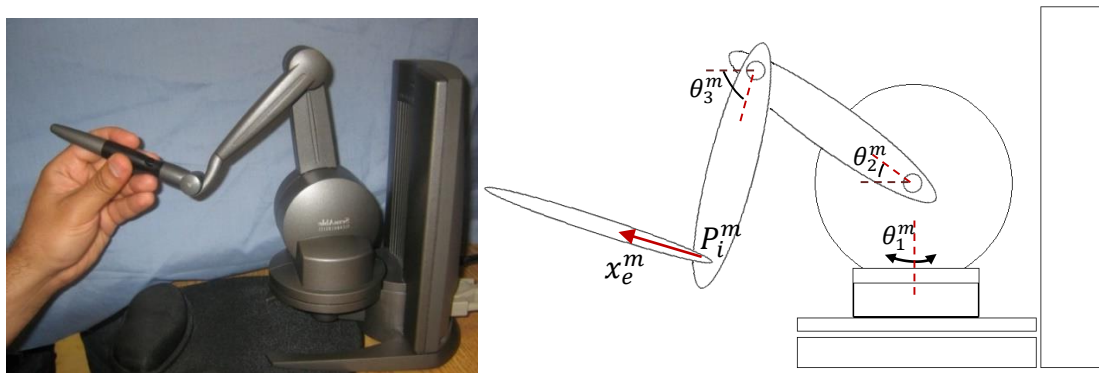


Figure 3.4 Coordinate frames of PHANToM Desktop haptic device.

Figure 3.5 illustrates the schematic of a typical link, which is driven by a single-rod hydraulic actuator. P_s and P_r are the hydraulic pump (supply) and tank pressures (return), respectively. u_i denotes the control signal applied to the hydraulic value. $\theta_{i,a}^s$ is the actual value of the joint angular displacement and is measured by a Hall effect sensor. The actual position of the operator's hand is continuously recorded, $\vec{P}_{i,a}^m = (x_{e,a}^m, y_{e,a}^m, z_{e,a}^m)$, and then multiplied by a mapping factor (α_s) in order to obtain the desired position of manipulator end-effector, $\vec{P}_{e,d}^s = (x_{e,d}^s, y_{e,d}^s, z_{e,d}^s)$. The desired angular displacement vector of manipulator joints ($\vec{\theta}_{i,d}^s$) is then obtained using Eqs. (3.1) to (3.5). The difference between the actual and desired joint displacements (e_i^s) is continuously calculated and sent to the controller in order to generate the appropriate control signal reducing the position error.

A nonlinear PI (NPI) controller was implemented for the first part of the research for each joint of the manipulator to test the previously developed haptic force schemes. The NPI controller (i) has excellent tracking and regulating ability, (ii) responds quickly to variations of the set point, (iii) reverses the directions quickly without overshoot, and (iv) retains above properties for both large and small changes in the set point [16]. The NPI controller was shown to improve tracking accuracy by a factor of five, as compared to the conventional PI controller with fixed gains, without sacrificing regulation accuracy or robustness. The accuracy of the NPI controller is about 0.2 degree [65].

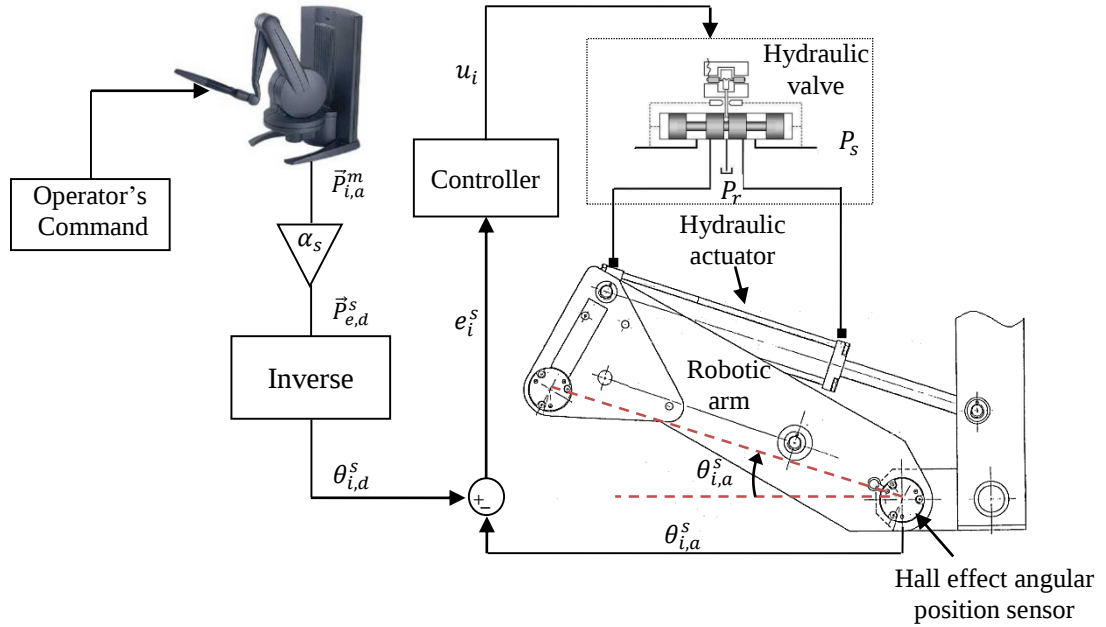


Figure 3.5 Schematic of a manipulator link driven by a single-rod hydraulic actuator.

3.3 Derivation of dynamic models

In this section, the dynamic model of the bilateral tele-manipulation system consisting of human operator, haptic device, single-rod hydraulic actuator, and task environment is presented. The PHANTOM haptic device and human operator's hand is shown in Figure 3.6.

The combined dynamics of the master manipulator and the human arm, in one dimension, is described as [18, 66, 67, 68]:

$$m_m \ddot{x}_m + k_d \dot{x}_m + k_h x_m = F_h + F_m \quad (3.17)$$

where, m_m is the combined inertia of the master manipulator and human arm, k_d is the combined viscous coefficient of the master manipulator and the human arm, and k_h is the combined stiffness of the human arm and the haptic device. F_h is the force generated by

the human operator's hand, x_m , $\dot{x}_m$, $\ddot{x}_m$, is the displacement, velocity and acceleration of the master implement, respectively, and F_m is the master force generated by the master manipulator actuators based on a control law. Note that although the master haptic device is a three degrees-of-freedom (DOF) system, only 1DOF (the horizontal motion of the master implement x_m as seen in Figure 3.6) is employed to control the single-rod slave actuator.

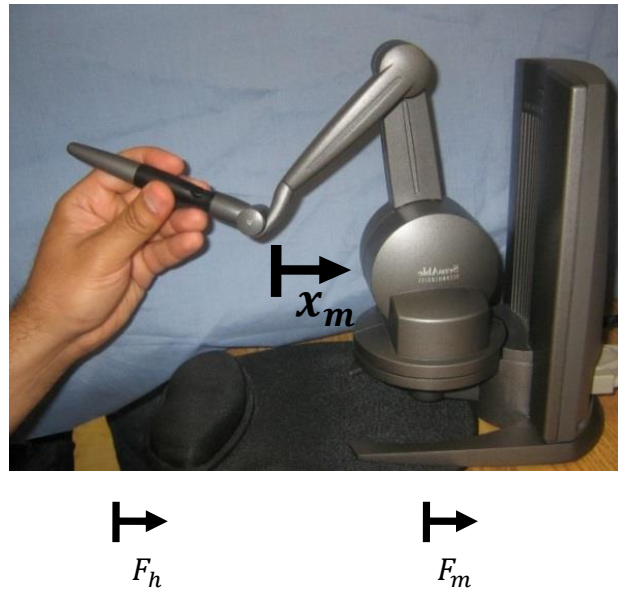


Figure 3.6 Haptic device and human operator's hand.

The schematic of a valve-controlled single-rod hydraulic actuator attached to a spring is shown in Figure 3.7. In the hydraulic actuator, the control signal activates the spool valve to move and control the flow of the hydraulic fluid into and out of the actuator. This flow in turn causes a pressure differential build-up in the actuator and makes it to move. In contrast to the double-rod hydraulic actuators, the two chambers of a single-rod hydraulic actuator have different areas. As a result, the dynamic equations describing the pressure changes in the two chambers cannot be combined into a single load pressure equation. This

complicates the controller design since it not only increases the dimension of the system to be dealt with but also brings in the stability issue of the added internal dynamics.

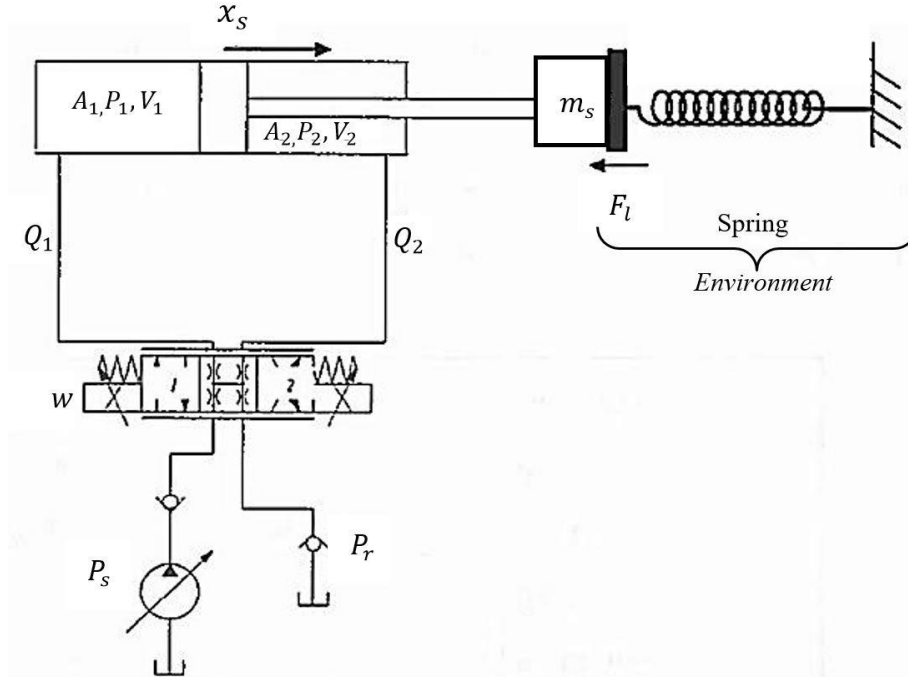


Figure 3.7 Schematic of single-rod hydraulic actuator interacting with an environment

The actuator is considered to be activated by an ideal critical centre servo-valve, with matched and symmetrical orifices. For control flows Q_1 and Q_2 through the valve, the nonlinear governing equations can be written in the following compact form [69]:

$$Q_1 = k_v w_1 x_{sp} \sqrt{\frac{P_s - P_r}{2} + \text{sgn}(x_{sp}) \left(\frac{P_s + P_r}{2} - P_1 \right)} \quad (3.18)$$

$$Q_2 = k_v w_2 x_{sp} \sqrt{\frac{P_s - P_r}{2} + \text{sgn}(x_{sp}) \left(P_2 - \frac{P_s + P_r}{2} \right)} \quad (3.19)$$

where k_v is the flow coefficient, and w_1 and w_2 refer to the width of the rectangular port cut into the valve bushing through which the fluid flows in each side of the chamber. The supply and tank pressures are denoted by P_s and P_r , respectively. Variables P_1 and P_2 refer

to the hydraulic pressures in each of the actuator chambers and x_{sp} is the spool displacement. For simplification it is considered that $P_r = 0$, and assumed valve to be symmetric, $w_1 = w_2 = w$;

$$Q_1 = k_v w x_{sp} \sqrt{\frac{P_s}{2} - \text{sgn}(x_{sp}) \left(\frac{P_s}{2} - P_1 \right)} \quad (3.20)$$

$$Q_2 = k_v w x_{sp} \sqrt{\frac{P_s}{2} + \text{sgn}(x_{sp}) \left(P_2 - \frac{P_s}{2} \right)} \quad (3.21)$$

The function $\text{sgn}(*)$ is the sign function and defined as follows:

$$\text{sgn}(*) = \begin{cases} +1 & * > 0 \\ 0 & * = 0 \\ -1 & * < 0 \end{cases}$$

The continuity equations that describe the pressure changes in each actuator chamber as a function of flow into and out of the actuator, Q_1 and Q_2 , can be written as follows [69]:

$$\dot{P}_1 = \frac{\beta}{V_1 + A_1 x_s} (Q_1 - A_1 \dot{x}_s) \quad (3.22)$$

$$\dot{P}_2 = \frac{\beta}{V_2 - A_2 x_s} (-Q_2 + A_2 \dot{x}_s) \quad (3.23)$$

where A_1 and A_2 is the annulus area of the piston in each of the actuator chambers. The volumes of fluid contained in the connecting lines between the servo-valve and the actuator are given by V_1 and V_2 . The fluid bulk modulus is given by β , $\dot{x}_s$ is the velocity and x_s is the displacement of the end-effector, respectively.

The ratio between effective piston areas, A_1 and A_2 is defined as:

$$\alpha A_1 = A_2 \quad (3.24)$$

This ratio assumption was also made in [70, 71, 72], which helps in combining the dynamic equations describing the pressure changes in the two chambers, into a single load pressure equation.

In a single-rod actuator, assuming the actuator around $x_s = 0$ and $\gamma = \frac{V_2}{V_1} < 1$, the pressure differential equations, from Eqs. (3.22) and (3.23), will be as follows:

$$\dot{P}_1 = \frac{\beta}{V_1} (Q_1 - A_1 \dot{x}_s) \quad (3.25)$$

$$\dot{P}_2 = \frac{\beta}{\gamma V_1} (A_2 \dot{x}_s - Q_2) \quad (3.26)$$

According to the steady-state flow relation in single-rod actuators, following is the relation between fluid flows:

$$Q_2 = \alpha Q_1 \quad (3.27)$$

From Eqs. (3.20) and (3.21),

$$\frac{P_s}{2} + \text{sgn}(x_{sp}) \left(P_2 - \frac{P_s}{2} \right) = \alpha^2 \frac{P_s}{2} + \alpha^2 \text{sgn}(x_{sp}) \left(\frac{P_s}{2} - P_1 \right) \quad (3.28)$$

For $x_{sp} > 0$,

$$P_2 = \alpha^2 P_s - \alpha^2 P_1 \quad (3.29)$$

$$P_s = P_1 + \frac{P_2}{\alpha^2} \quad (3.30)$$

And for $x_{sp} < 0$,

$$P_s - P_2 = \alpha^2 P_1 \quad (3.31)$$

$$P_s = \alpha^2 P_1 + P_2 \quad (3.32)$$

Thus,

$$P_s = \begin{cases} P_1 + \frac{P_2}{\alpha^2} & \text{if } x_{sp} > 0 \\ \alpha^2 P_1 + P_2 & \text{if } x_{sp} < 0 \end{cases} \quad (3.33)$$

Defining ε as:

$$\varepsilon = \begin{cases} \frac{1}{\alpha^2} & \text{if } x_{sp} > 0 \\ 1 & \text{if } x_{sp} < 0 \end{cases} \quad (3.34)$$

Following can be a general formula for P_s :

$$P_s = \varepsilon(\alpha^2 P_1 + P_2) \quad (3.35)$$

By defining $P_L = P_1 - \alpha P_2$ as output pressure and using above equation, the pressure of each side of cylinder can be obtained in terms of supply and differential pressures as follows:

$$P_1 = \frac{\alpha P_s + \varepsilon P_L}{\varepsilon(1 + \alpha^3)} \quad (3.36)$$

$$P_2 = \frac{P_s - \varepsilon \alpha^2 P_L}{\varepsilon(1 + \alpha^3)} \quad (3.37)$$

Thus,

$$P_1 = \begin{cases} \frac{\alpha^3 P_s + P_L}{\alpha^3 + 1} & x_{sp} > 0 \\ \frac{\alpha P_s + P_L}{\alpha^3 + 1} & x_{sp} < 0 \end{cases} \quad (3.38)$$

$$P_2 = \begin{cases} \frac{\alpha^2 (P_s - P_L)}{\alpha^3 + 1} & x_{sp} > 0 \\ \frac{P_s - P_L}{\alpha^3 + 1} & x_{sp} < 0 \end{cases} \quad (3.39)$$

To obtain the differential equation for P_L , using the steady state flow relation, we have:

$$\dot{P}_L = \dot{P}_1 - \alpha \dot{P}_2 \quad (3.40)$$

$$\dot{P}_L = \frac{\beta}{V_1} (Q_1 - A_1 \dot{x}_s) - \alpha \frac{\beta}{\gamma V_1} (-Q_2 + A_2 \dot{x}_s) \quad (3.41)$$

$$\dot{P}_L = \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma} \right) \left(\frac{Q_2}{\alpha} - A_1 \dot{x}_s \right) \quad (3.42)$$

From Eqs. (3.21), (3.34) and (3.37), we have:

$$Q_2 = k_v w x_{sp} \sqrt{\frac{P_s}{2} + \text{sgn}(x_{sp}) \left(\frac{P_s - \varepsilon \alpha^2 P_L}{\varepsilon(1 + \alpha^3)} - \frac{P_s}{2} \right)} \quad (3.43)$$

$$Q_2 = \begin{cases} k_v w x_{sp} \frac{\alpha}{\sqrt{1 + \alpha^3}} \sqrt{P_s - P_L} & x_{sp} > 0 \\ k_v w x_{sp} \frac{\alpha}{\sqrt{1 + \alpha^3}} \sqrt{\alpha P_s + P_L} & x_{sp} < 0 \end{cases} \quad (3.44)$$

Then a general formula for Q_2 can be written as follows:

$$Q_2 = k_v w x_{sp} \frac{\alpha}{\sqrt{1 + \alpha^3}} \sqrt{(1 + \alpha) \frac{P_s}{2} + \text{sgn}(x_{sp}) \left((1 - \alpha) \frac{P_s}{2} - P_L \right)} \quad (3.45)$$

Now, putting the value of Q_2 from Eq. (3.45) in Eq. (3.42). Following equation represents the single-rod hydraulic actuator differential equation for load pressure P_L :

$$\dot{P}_L = \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma} \right) \left(\frac{k_v w x_{sp}}{\sqrt{1 + \alpha^3}} \sqrt{(1 + \alpha) \frac{P_s}{2} + \text{sgn}(x_{sp}) \left((1 - \alpha) \frac{P_s}{2} - P_L \right)} - A_1 \dot{x}_s \right)$$

or,

$$\dot{P}_L = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \dot{x}_s + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_{sp}}{\sqrt{1 + \alpha^3}} (\sqrt{\Delta}) \quad (3.46)$$

where,

$$\Delta = (1 + \alpha) \frac{P_s}{2} + \text{sgn}(x_{sp}) \left((1 - \alpha) \frac{P_s}{2} - P_L \right) \quad (3.47)$$

The equation of motion for a hydraulic actuator is [73]:

$$A_1 P_L = m_s \ddot{x}_s + d \dot{x}_s + F_l + F_{fr} \quad (3.48)$$

where m_s is the inertia of the moving part of the actuator. $\dot{x}_s$ and $\ddot{x}_s$ are the velocity and the acceleration of the end-effector, respectively. Parameter d is the equivalent viscous damping coefficient describing the combined effect of the viscous friction between the piston and the cylinder walls and the damping of the load. F_{fr} is the dry friction and F_l denotes the contact force.

The hydraulic actuator manipulates a stiffness dominant environment. Thus, the contact force, F_l , can be stated as:

$$F_l = k_s x_s \quad (3.49)$$

where k_s is the stiffness of the environment. Note that, when the hydraulic actuator moves in free motion, the stiffness of the environment is zero, *i.e.* $k_s = 0$, therefore $F_l = 0$, and when the hydraulic actuator is manipulating the environment, $k_s \neq 0$.

The dynamics between the valve input voltage, u , and the spool displacement, x_{sp} , are described as a first-order model which is adequate for many industrial applications [73, 74, 75]:

$$\dot{x}_{sp} = \frac{-1}{\tau} x_{sp} + \frac{k_{sp}}{\tau} u \quad (3.50)$$

where, k_{sp} and τ are the valve gain and time constant, respectively.

The hydraulic function parameters are shown in Table 3-2 below, which were obtained directly from manufacturer's specifications sheet or by experimental measurement/verification to resemble the actual test rig on which all the experiments were performed.

Table 3-2 Hydraulic system parameters.

Parameter	Symbol	Value
Supply pressure	P_s	6.895 MPa (1000 psi)
Combined mass of piston and rod	m_s	15.7 kg
Viscous damping coefficient	d	250 Ns/m
Piston area (blind side)	A_1	$3.17 \times 10^{-3} m^2$
Piston area (rod side)	A_2	$2.66 \times 10^{-3} m^2$
Initial fluid volume (blind side)	V_1	$1.58 \times 10^{-4} m^3$
Initial fluid volume (rod side)	V_2	$1.33 \times 10^{-4} m^3$
Hydraulic compliance	C	$2 \times 10^{-13} m^5/N$
Orifice coefficient of discharge	c_d	0.6
Hydraulic fluid density	ρ	847.15 kg/m ³
Orifice area gradient	w	$1.01 \times 10^{-2} m$
Valve gain	k_{sp}	$4.064 \times 10^{-5} m/V$
Valve time constant	τ	0.03 s
Inertia of master (haptic)	m_m	0.045 kg
Viscous coefficient at the master side	k_d	0.2 Ns/m
Stiffness of human arm	k_h	10 N/m

The model described above is now used to build the state space model. Defining the state vector $\vec{x}$ as:

$$\begin{aligned}\vec{x} &= [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \\ &= [x_s \quad \dot{x}_s \quad P_L \quad x_{sp} \quad x_m \quad \dot{x}_m]^T\end{aligned}\tag{3.51}$$

State space equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{A_1}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 \\ \dot{x}_3 = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} x_4 (\sqrt{\Delta}) \\ \dot{x}_4 = \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} u \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{1}{m_m} (F_h + F_m - k_d x_6 - k_h x_5) \end{cases}\tag{3.52}$$

where, $\Delta = (1 + \alpha) \frac{P_s}{2} + \text{sgn}(x_4) \left((1 - \alpha) \frac{P_s}{2} - P_L \right)$

Note that in Eq. (3.52), the friction term F_{fr} is neglected, as depending on the type of actuator, this friction can be small or significant [54]. This assumption was needed to make the Lyapunov stability analysis manageable. Nevertheless, as will be seen later in the experimental results, the controllers developed here performed well for the real actuator with friction. u and F_m are computed by the control laws to be designed in Chapter 5.

3.4 Summary

This chapter described specifications of the experimental test rig employed in this thesis, followed by describing the required formulations. The test rig used in this research

comprises of an industrial hydraulic manipulator mounted atop a Stewart platform, a PHANTOM haptic device that allows control of the manipulator by the operator while creating a feel of force, a frame replicating a segment of structure to be repaired, and a tool attached to the hydraulic manipulator. Furthermore, in this chapter, the dynamic model of the bilateral tele-manipulation system which consists of human operator, haptic device, single-rod hydraulic actuator, and task environment was presented.

CHAPTER 4

4 CONTROL OF HYDRAULIC MANIPULATORS WITH BASE MOTION¹

4.1 Introduction

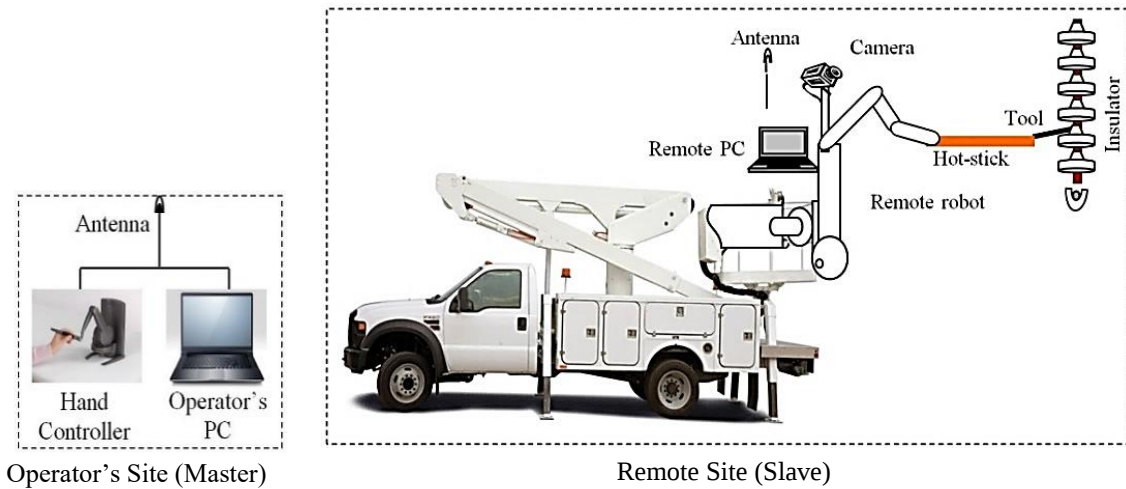
This chapter serves as a prelude to understand how base motion effects the previously developed unilateral and bilateral controllers for hydraulic teleoperation. In this chapter, the force feedback schemes developed in [16, 17] have been rigorously tested in the presence of base disturbance, as those were validated only when the base of hydraulic manipulator was stationary.

¹ A version of this chapter has been published in V. Banthia, Y. Maddahi, K. Zareinia, S. Liao, T. Olson, W. K. Fung, S. Balakrishnan and N. Sepehri, "A prototype telerobotic platform for live transmission line maintenance: Review of design and development," *Transactions of the Institute of Measurement and Control*, vol. 40, no. 11, pp. 3273-3292, 2018 [76]

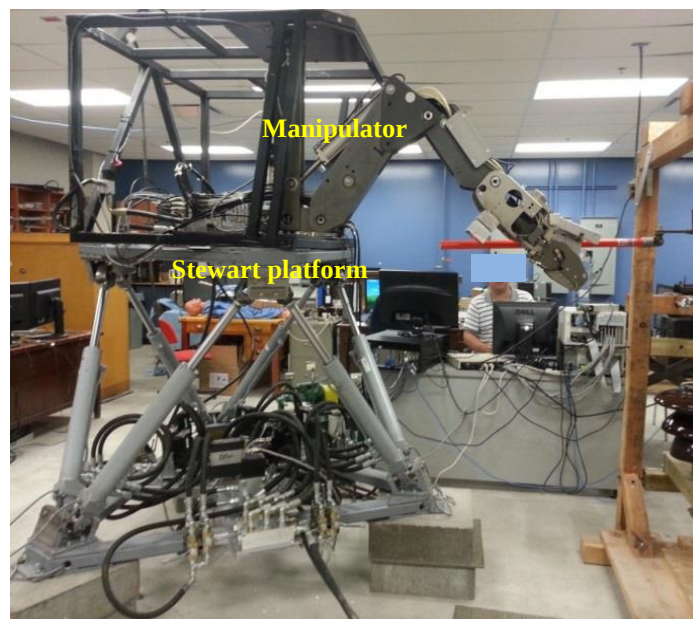
and partially presented at V. Banthia, Y. Maddahi, S. Balakrishnan and N. Sepehri, "Haptic-enabled teleoperation of base-excited hydraulic manipulators applied to live-line maintenance," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Chicago, IL, 2014 [77]

The focus is on conducting live-line maintenance using teleoperation whereby a manipulator is installed on top of a live-line bucket truck to perform tasks using a hot-stick.

The concept is described in Figure 4.1a. A hot-stick (with associated tools) is attached to



(a)



(b)

Figure 4.1 (a) Concept developed to perform live-line maintenance near energized lines, (b) Test rig to simulate base motions of the crane bucket.

the end-effector of a robotic arm. The robotic manipulator is then remotely controlled by a lineman/operator enabling him to perform typical live-line maintenance tasks from a safe distance away from the high voltage power lines. As the slave manipulator is installed on top of a crane bucket, its base is subjected to unwanted motion due to wind gust, unstable soil, and compliance in the crane actuation. Thus, the issue of base disturbance needs proper investigation on otherwise well-performing teleoperated system.

Experimental investigations performed on the test rig in the real field, indicated that the hydraulic manipulator base was disturbed with displacement amplitude around 45 mm and frequency of about 0.12 Hz [76, 77]. Therefore, for experiments, the Stewart platform (see Figure 4.1b) was programmed to generate random top plate displacements (T_x, T_y, T_z) as depicted in Figure 4.2. Note that the radial displacement applied to the top plate of Stewart has maximum amplitude and frequency of 90 mm and 0.24 Hz, respectively, which will confirm that the system can work in presence of larger displacements as well. When the displacement of the top plate is determined, the program solves the inverse kinematics of the Stewart platform given in Eq. (3.14) and finds the displacement of each hydraulic actuator. The displacement of each hydraulic actuator ($B_{i=1..6}$) is illustrated in Figure 4.3. During the operation, the operator could not recognize or predict the amplitude or the frequency of the base motion; therefore, the motion was completely unpredictable. This unpredictability is obvious when the frequency of motion changes every 20 seconds (see arrows in Figure 4.2).

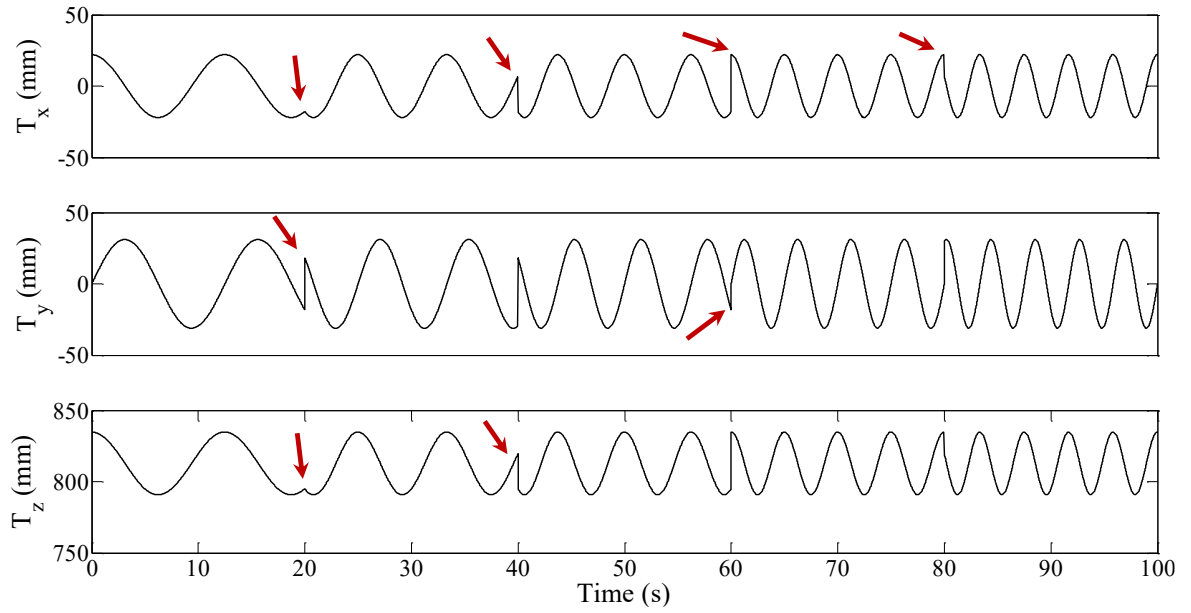


Figure 4.2 Displacement of platform top plate over a 100-second period. The arrows show the unpredictability of the base motion.

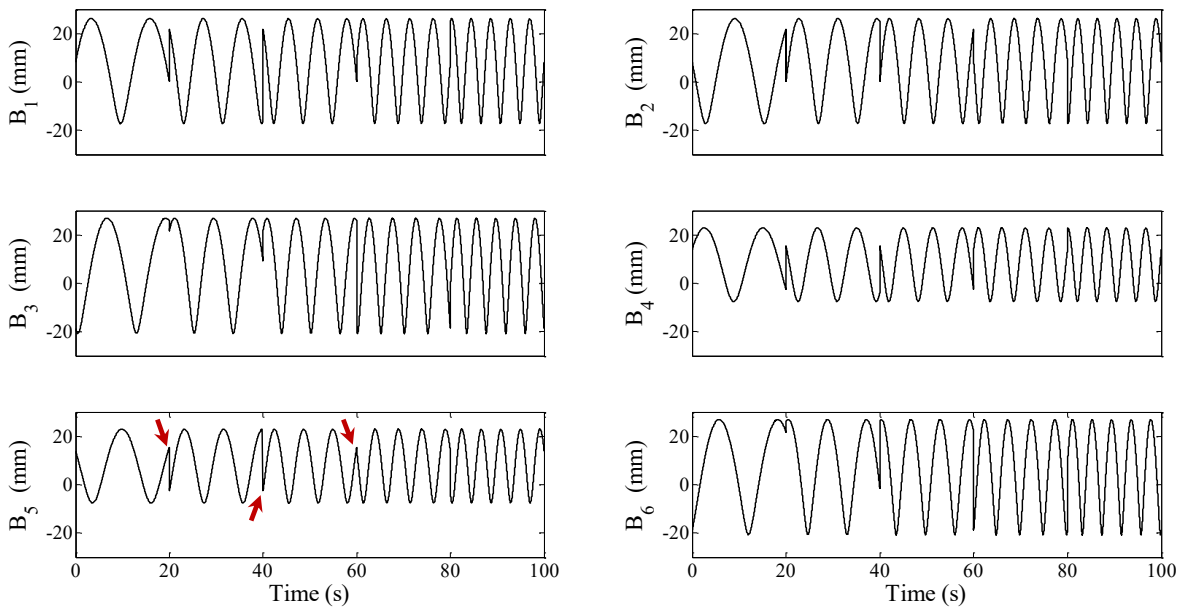


Figure 4.3 Actuator displacements of 6-DOF Stewart platform.

4.2 Experimental evaluation of control schemes

Performances of the three control schemes, explained in Section 2.2.1, were evaluated in the presence of base excitation. The schemes were: virtual fixture scheme, force augmentation scheme, and augmented virtual scheme. The performance of each scheme was then compared with a mode in which no force was generated by the haptic device. The effectiveness of each of the three schemes was validated by emulating two live power line maintenance tasks in a laboratory setting. First, the performance of the hydraulic telemanipulator was evaluated under no force mode (NF), whereby haptic device applied no force to the operator's hand. The same tests were repeated when the operator ran the system under virtual fixture scheme (VF), *i.e.* a trajectory was defined as virtual fixture, and the operator's hand is kept on the virtual fixture path. The experiments were then repeated when no virtual fixture was added to the system, and the haptic device was augmented by the force augmentation scheme (AU). As mentioned in Section 2.2.1, using this scheme, the generated force helps the operator realize how far the slave end-effector moves behind or ahead of the slave implement. Lastly, the operator was asked to repeat the experiments when the augmented virtual fixture (AVF) force was utilized by the haptic device.

In all schemes, a nonlinear PI (NPI) controller was used to control the joint angles of the slave manipulator [65]. This NPI controller was built upon a conventional linear PI controller and three modifications were performed to it. The first modification was to multiply the accumulated error integral by a novel velocity error varying factor at each control step. This modification has shown to effectively prevent integral windup and allows the use of larger integral gains, therefore improving both regulating and tracking abilities. The second modification addressed the problems of hydraulic flow deadband and stiction

at the joints. A nonlinear filter was introduced which reliably detects the occurrence of stalling by calculating a stick-induced velocity error signal. This signal was then used as a switch to boost the control signal as required. Implementing this modification enabled the manipulator to follow changes in set-point without any delay. The third modification allowed the reduction of overshoot in the deceleration response. This was accomplished by boosting the position error by a factor proportional to a deceleration term in the calculation of the integral portion of the controller at certain periods. More details on the controller and the gain parameters are provided in Appendix A.1.

4.3 Test procedure

There were eight graduate students from the University of Manitoba who contributed in this study. All participants were right-handed and reported a normal to corrected-to-normal vision. They were first briefed and trained about the concept of schemes that were employed in this study, operation process, and possible effects of base motion on quality of task. They were then asked to try the experiment in the simulation mode while the robot was switched off and force capability of haptic device was deactivated. The training phase was completed once the operator felt comfortable and confident to use the system. Finally, they practiced with the real system and went through the experiments. Participants were asked to hold the stylus of the haptic device like holding a pen and trace some paths.

There were two typical tasks emulated by the experimental setup shown in Figure 4.4. As illustrated in Figure 4.4a, to twist a tie wire around an electrical cable, a circular curve in $x_0^s z_0^s$ plane is traced (Task A). In another task, shown in Figure 4.4b, the operator pulls the cotter pin out (Task B). As observed, a combination of several lines, located on $y_0^s z_0^s$

plane, forms the defined trajectory for completing this task. In each task, the operator moves the haptic implement along depicted trajectory, from point A to point D. In the case of testing virtual fixture scheme, corresponding virtual fixture for the task of twisting a tire wire was defined as shown in Figure 4.5. Virtual fixture is intended to aid the operator in following a predefined path. As was discussed in Section 2.2.1, when the hand implement is on desired trajectory, the virtual spring remains in rest position ($x_a = x_d, y_a = y_d, z_a = z_d$), and no force is generated ($\vec{F}_{VF} = 0$). However, when the operator moves the implement away from desired trajectory (off-track), the virtual spring generates a force ($\vec{F}_{VF} \neq 0$) which is proportional to the amount of penetration into the forbidden region. From Eq. (2.1), virtual fixture force generated by the haptic device, $\vec{F}_{VF}$, is given by [17]:

$$\vec{F}_{VF} = -G_{VF}\vec{R} \quad (4.1)$$

where G_{VF} is the impedance of the virtual fixture, or basically the stiffness of the virtual spring pushing/pulling the operator's hand toward the desired trajectory and $\vec{R}$ is the haptic end-effector position error, $\vec{R}$. In tests under the VF scheme, the stiffness of the virtual spring (G_{VF}) was set to [16]:

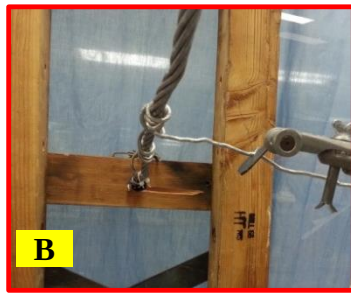
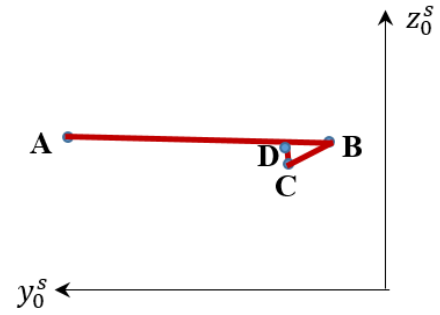
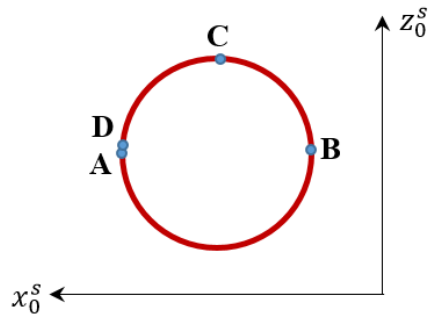
$$G_{VF} = \begin{bmatrix} 1500N/m & 0 & 0 \\ 0 & 1500N/m & 0 \\ 0 & 0 & 1500N/m \end{bmatrix} \quad (4.2)$$

When force augmentation scheme was tested, the haptic force was generated based on the position error $\vec{E}_e^s$ at the slave end-effector as shown in Figure 4.6. From EQ. (2.3), the augmentation force is defined as [16]:

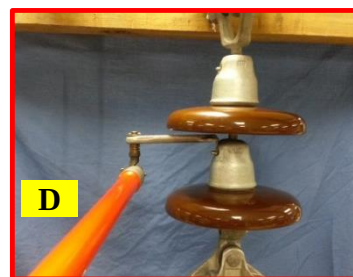
$$\vec{F}_{AU} = \begin{cases} -|G_{AU}(\vec{E}_e^s - \vec{E}_t^s)|\hat{v} & \|\vec{E}_e^s\| > \|\vec{E}_t^s\| \\ 0 & \|\vec{E}_e^s\| \leq \|\vec{E}_t^s\| \end{cases} \quad (4.2)$$

As the NPI controller has some inherent error, the augmentation force is only generated when the position error is greater than a threshold ($\vec{E}_t^s$). This threshold is defined based on the steady-state positioning error originating from the manipulator's NPI controller and sensors resolution and helps to prevent repeated activation-deactivation cycles. When $\|\vec{E}_e^s\| \leq \|\vec{E}_t^s\|$, the haptic device does not produce any augmentation force. NPI controller produced maximum steady-state error reflecting a threshold of 1.54 mm, 1.06 mm and 2.07 mm at the end-effector along x_0^s , y_0^s and z_0^s , respectively [16]. Thus, the threshold is $\vec{E}_t^s = [1.54mm \ 1.06mm \ 2.07mm]^T$.

In the case of augmented virtual scheme, both virtual fixtures and augmentation forces were combined to reduce position errors at both the master device implement and the slave manipulator end-effector.



(a) Twisting a tie wire (Task A)



(b) Pulling out a cotter pin (Task B)

Figure 4.4 Typical live-line maintenance tasks used to validate performance of teleoperated system.

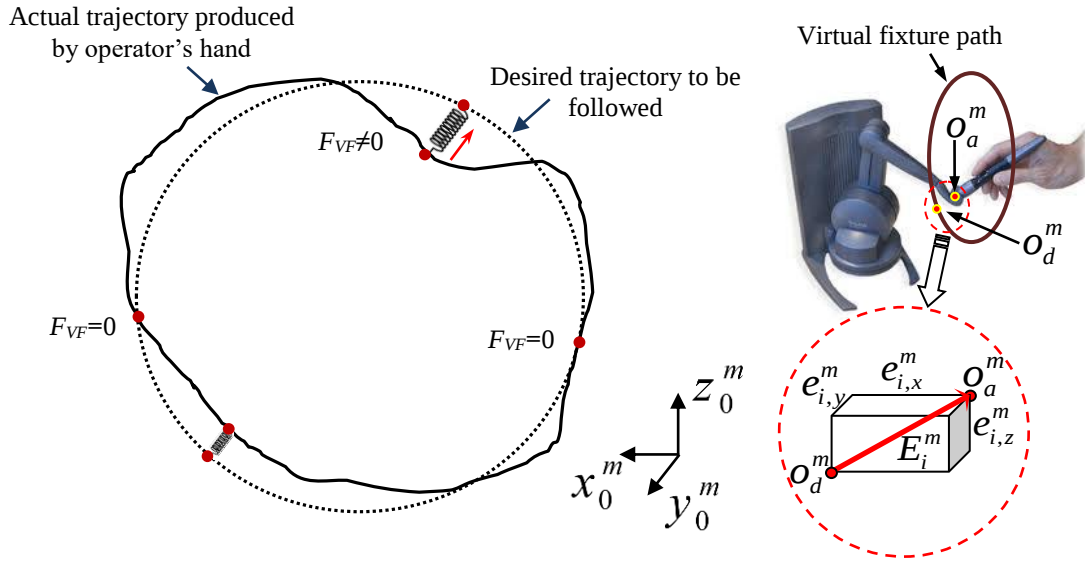


Figure 4.5 Virtual fixture at the master side for the task of twisting a tie wire.

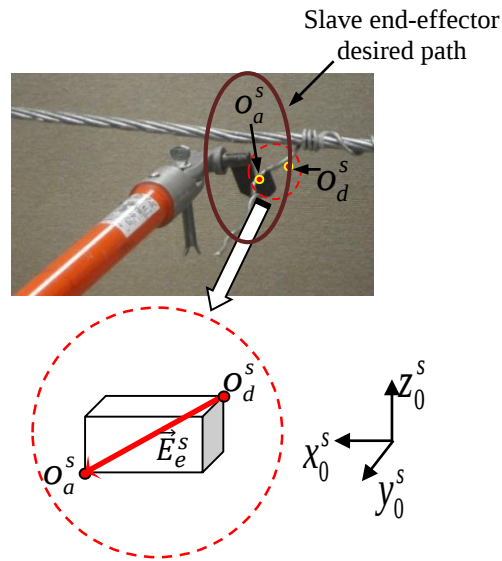


Figure 4.6 Desired (O_d^s) and actual (O_a^s) positions of the slave manipulator end-effector for the task of twisting a tie wire. Position error at the end-effector is expressed by $\vec{E}_e^s$.

4.4 Experimental results

The augmented virtual fixture was observed to adjust the operator's hand movement (force augmentation scheme) and keep the hand on defined trajectory (virtual fixture scheme). As seen in Figure 4.4a, the maintenance tool might collide with the cable because of the manipulator base movement. Therefore, although the virtual fixture part of the scheme intended to keep the operator on the circular trajectory, the operator was compelled to move the haptic implement against the virtual fixture force, and deviate from the defined trajectory, in order to avoid colliding. The operators believed that working under virtual fixture and augmented virtual fixture schemes was tiring because of applying such a resistive force over a long time.

Figure 4.7 and Figure 4.8 depict the task completion times, for both tasks under all four force modes. As shown the NF mode was less time consuming than the other force modes. In contrary, AVF mode showed more value of task completion time as compared to the AU and VF modes. The reason can be found in the nature of AVF that it (i) helps the operators to slow down the hand motion when position error appear at the slave site, and (ii) tries to pull back the operator's hand toward the virtual fixture trajectory, and therefore redirects the hand from its intended motion.

Figure 4.9 and Figure 4.10 show mean value of slave manipulator displacements for tasks A and B, respectively. The slave end-effector displacement, in each test, is summation of displacements between two adjacent sampling. As observed in both tasks, the end-effector in NF mode travels longer distance than the other force modes. Moreover, the augmented virtual fixture exhibited the best in terms of end-effector distance. The displacements, for virtual fixture and force augmentation schemes, were almost close to

each other. However, in task A, the virtual fixture seems more helpful to the operator than task B.

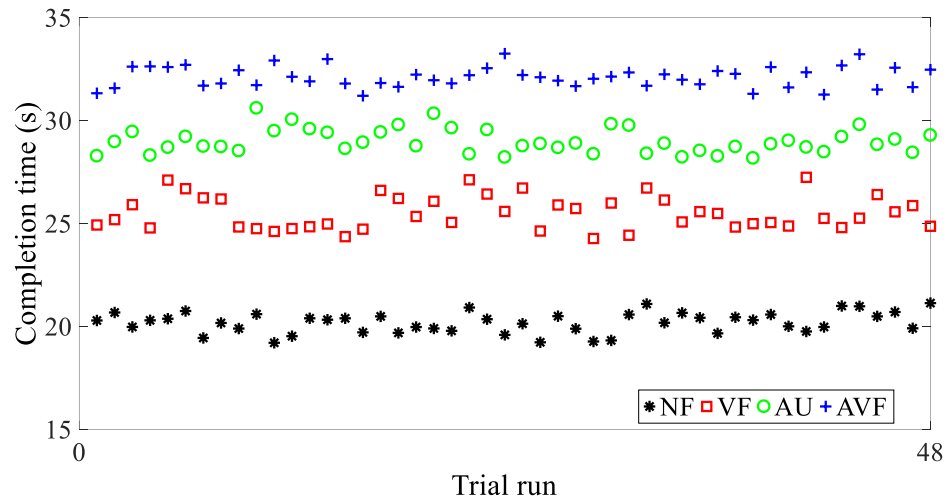


Figure 4.7 Task completion times for tie-wire task.

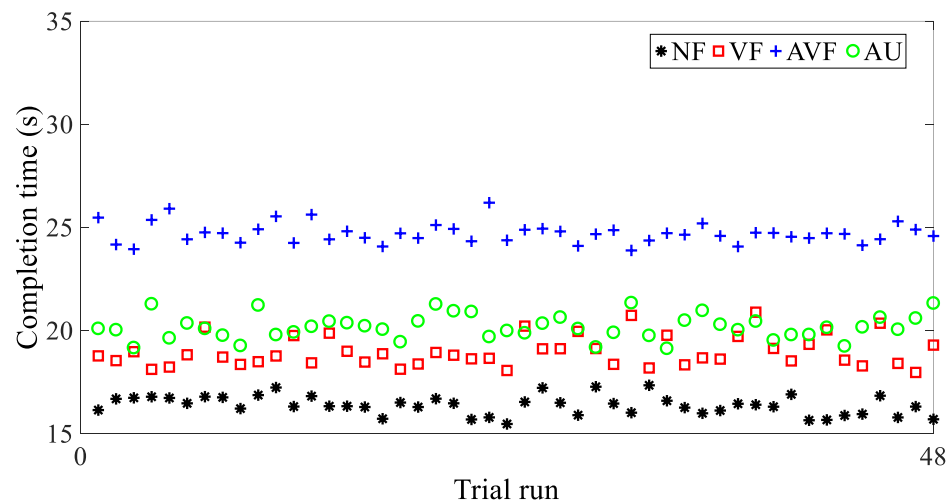


Figure 4.8 Task completion times for cotter pin task.

The overall information about performance measures is given in Table 4-1. As seen in the fourth column, the number of failed trials, in VF, is more than the other schemes. More specifically, the operators failed the operation, under the VF, AVF, NF, and AU schemes, for 22, 17, 13, and 5 times, respectively. The shaded cells, in Table 2, show the worst force

mode in each significant criterion. As observed, the AU is not the worst scheme in any of the three measures. With reference to results obtained from the three performance measures, especially the number of failed trials, the AU mode was found better than the other schemes.

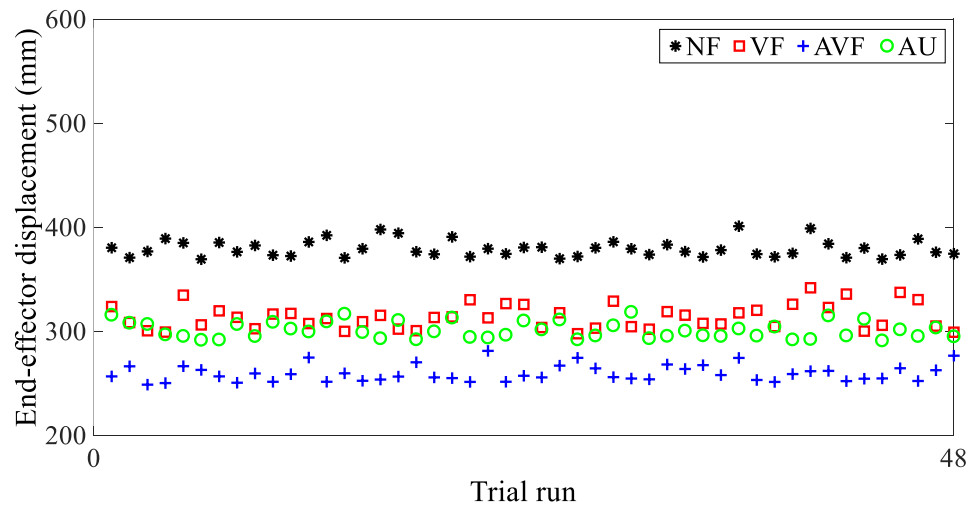


Figure 4.9 Slave end-effector displacements for tie-wire task.

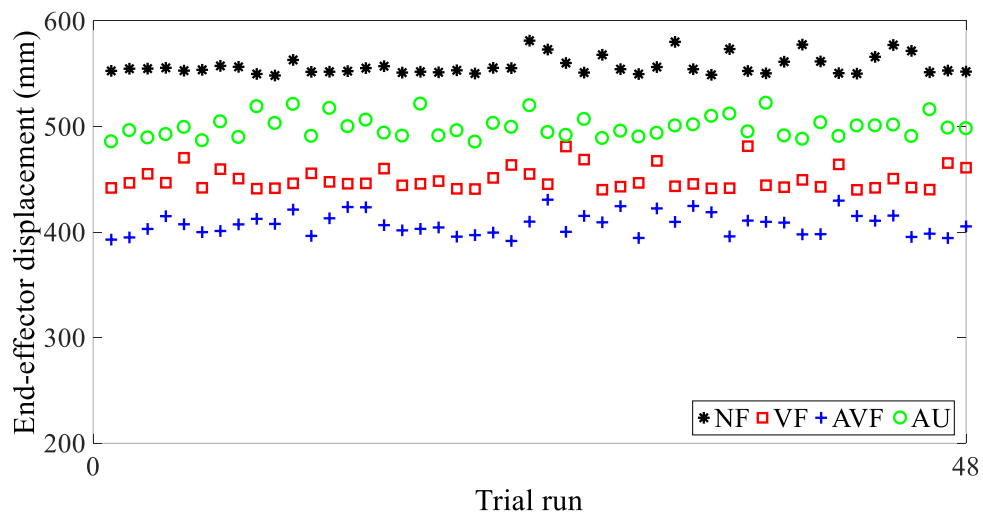


Figure 4.10 Slave end-effector displacements for cotter pin task.

Table 4-1 Performance measure of the teleoperated system under no force (NF), virtual fixture (VF), augmentation (AU), and augmented virtual fixture (AVF).

Task	Force mode	Task time (s)	Number of failed trials	End-effector displacement (mm)
Twisting a tie wire	NF	19.18	8	380.91
	VF	25.45	14	315.60
	AU	29.46	3	256.54
	AVF	32.08	11	290.41
Pulling a cotter pin out	NF	16.56	5	548.32
	VF	18.21	8	452.98
	AU	21.23	2	401.12
	AVF	24.15	6	492.35

Figure 4.11 shows the variations of force for tests presented in Figure 4.4 under force augmentation mode. Once the position error appears at the slave side, the haptic device generates the augmentation force, and alerts the operators to slow down the hand motion. This allows the hydraulic actuators at the slave side to catch up with the haptic motion, and the end-effector moves towards the intended trajectory. Therefore, the position error at the slave end-effector decreases.

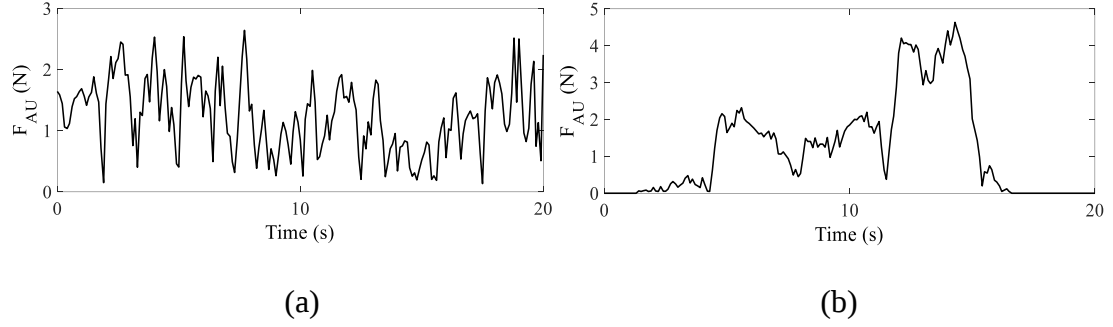


Figure 4.11 Augmentation force for (a) tie-wire task; (b) cotter pin task.

Figure 4.12 to Figure 4.15 illustrates typical trajectories of the slave manipulator end-effector along x_0^s , y_0^s , and z_0^s axes and radial position error of slave end-effector, $|\vec{E}_e^s| = \sqrt{(x_e^s)^2 + (y_e^s)^2 + (z_e^s)^2}$, when the haptic device works under either the force augmentation or no force mode. Dashed (red) and solid lines (black) show slave end-effector actual and desired trajectories, respectively. As seen, there was an unavoidable position error at the slave manipulator end-effector which originates from responses of the slave controller and the actuation system. In addition, in some situations, the maintenance tool was about to collide with the cable due to the unpredictable motion of the base. Therefore, the operator tried to keep the maintenance tool from colliding and moved the haptic device fast. The fast motion resulted in position error appearing at the slave end-effector. However, in force-enabled mode, this problem was then alleviated by implementing the concept of the force augmentation.

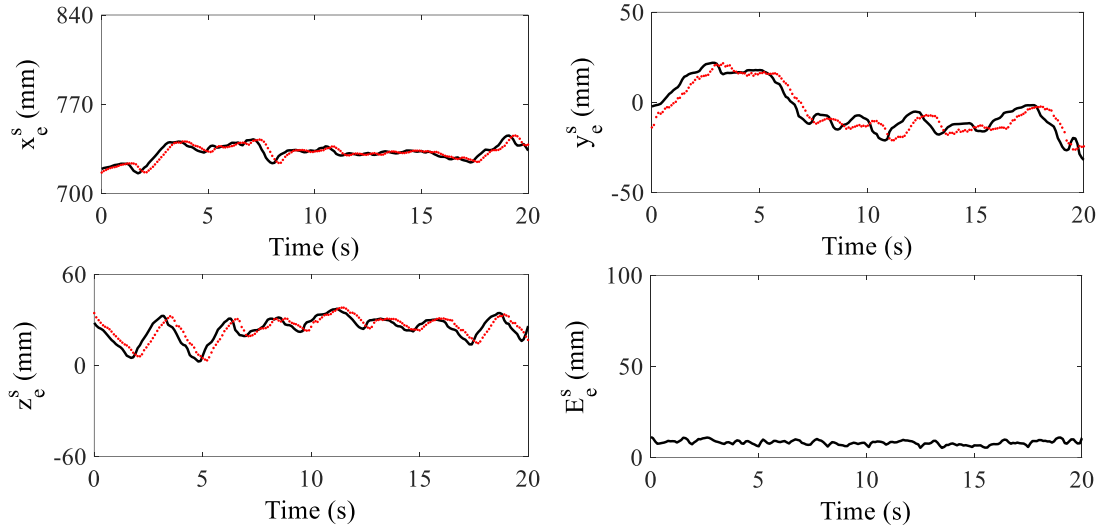


Figure 4.12 Typical test trajectory of hydraulic manipulator end-effector under augmented mode for tie-wire task and radial position error of slave end-effector.

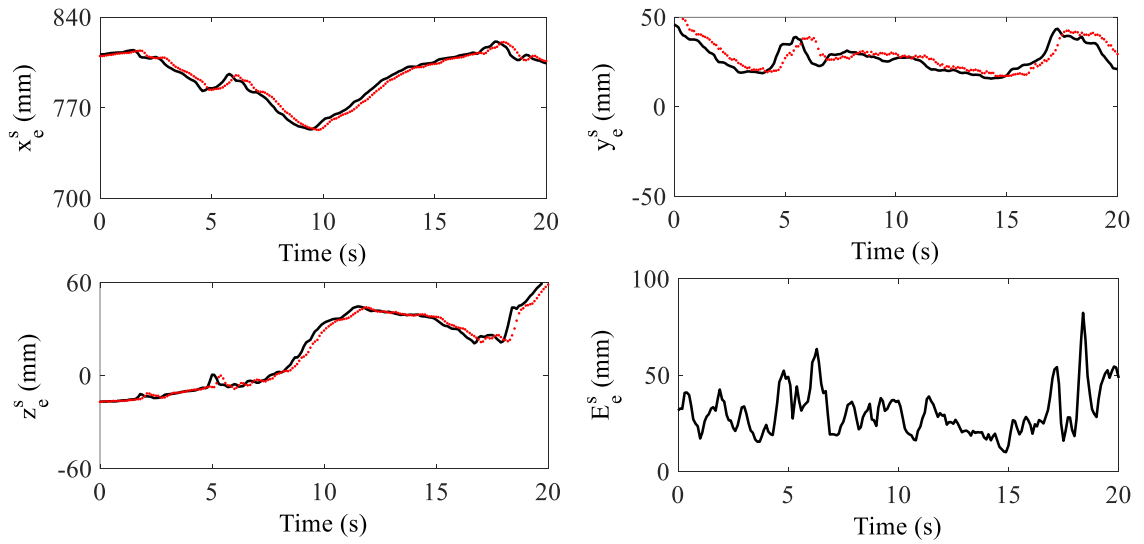


Figure 4.13 Typical test trajectory of hydraulic manipulator end-effector under no force mode for tie-wire task and radial position error.

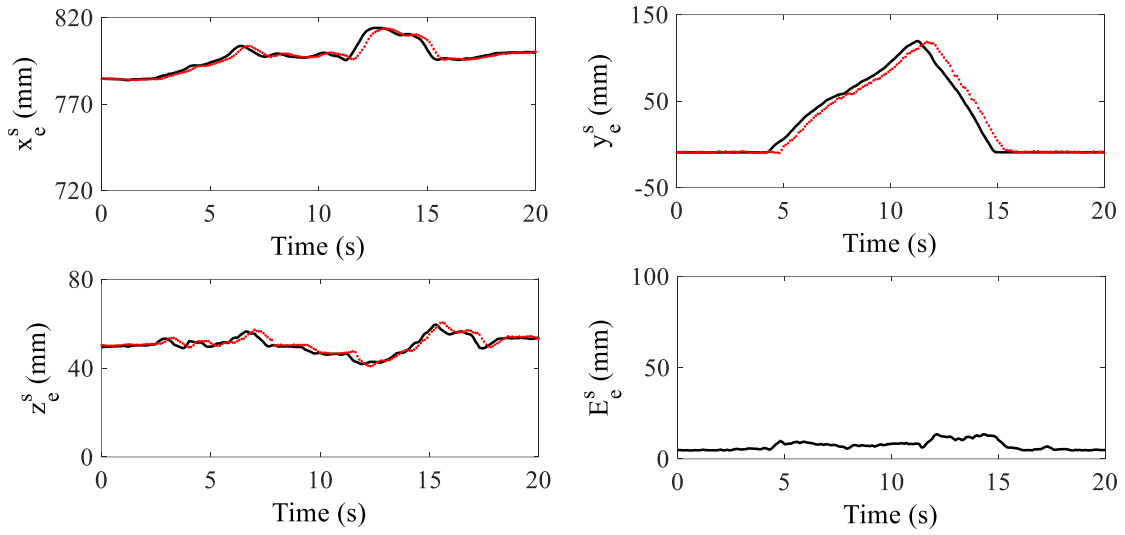


Figure 4.14 Typical test trajectory of hydraulic manipulator end-effector under augmented mode for cotter pin task and radial position error.

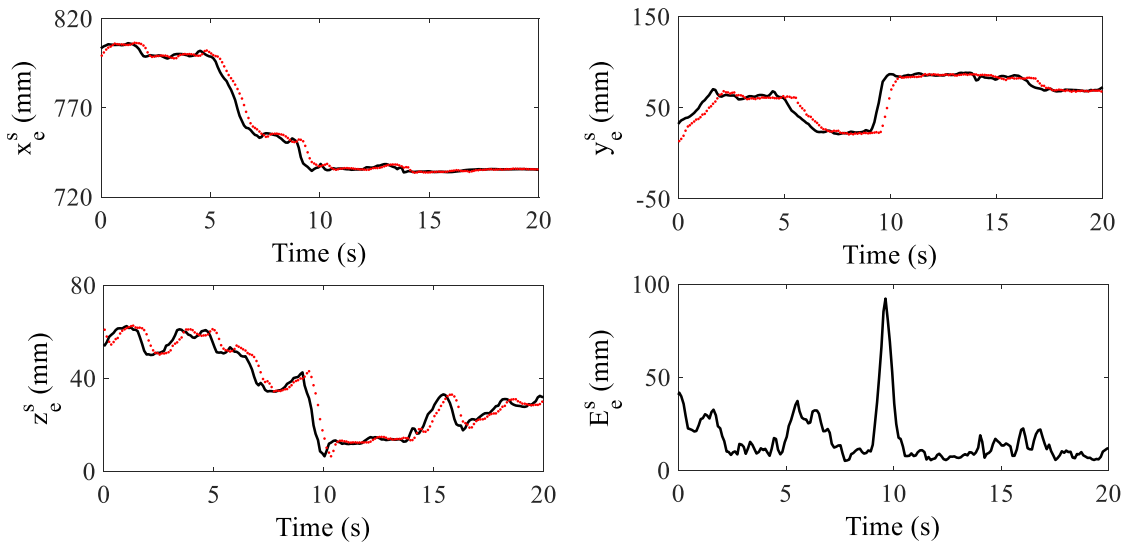


Figure 4.15 Typical test trajectory of hydraulic manipulator end-effector under no force mode for cotter pin task; and radial position error of slave end-effector.

4.5 Performance evaluation

This section evaluates the effectiveness of the developed comprehensive system in performing typical live-line maintenance tasks. The complete system setup includes an operator positioned to view the wireless monitors (see Figure 4.16). The operator ① has direct visual access to the robotic arm and hot-stick tool as well. The haptic device ② is used to interface with the robotic arm to direct the tool. In addition, the augmentation force is added to the haptic device to reduce the position error at the manipulator's end-effector as this force feedback scheme was found to be the most effective as per Section 4.4. Multiple wireless cameras may be positioned on the robot to provide the operator with the best visual feedback ③. In this work, only one camera is used for demonstration purposes. Finally, the hydraulic manipulator ④ completes the setup and performs the tasks, translating the operator's hand movements to the hot-stick tool. There are two hot-sticks attached to the arm to receive various hot-stick tools: the pan tool ⑤ for transferring and manipulating insulator strings, and the multi-tool ⑥ for cotter pin and other hardware manipulation. The tasks were performed on a 3-bell string ⑦ in this laboratory demonstration. As mentioned, the Stewart platform ⑧ simulates the crane bucket movement.

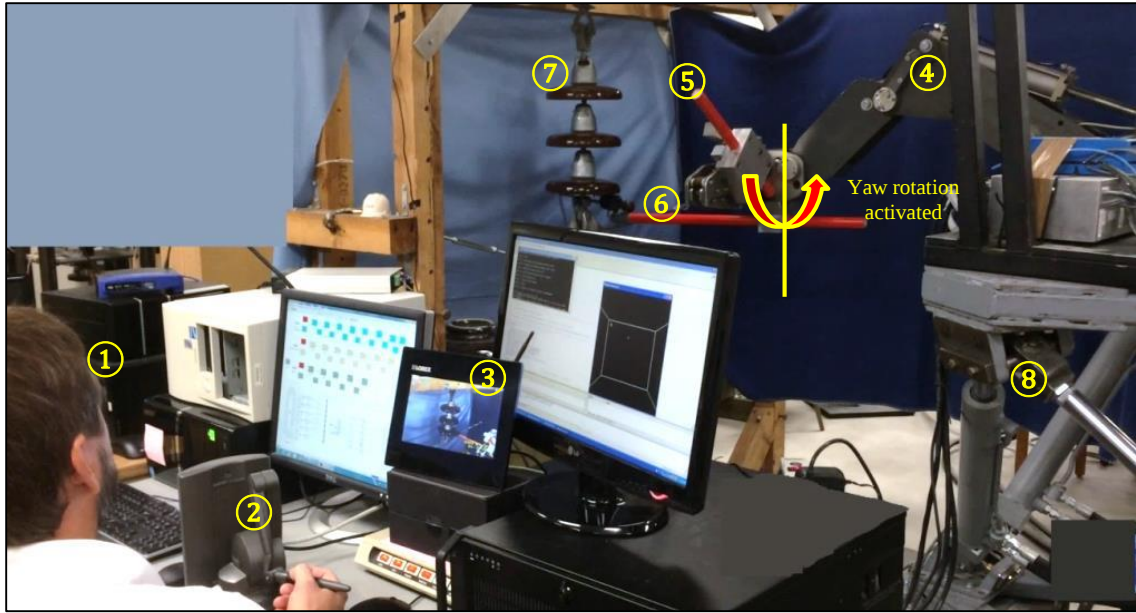
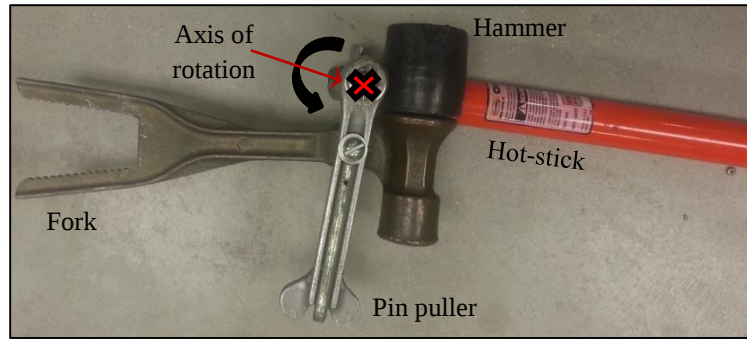


Figure 4.16 Complete system setup; ① operator ② haptic device ③ cameras ④ manipulator ⑤ pan tool attachment ⑥ multi-tool ⑦ insulator ⑧ Stewart platform.

Special tools were combined to provide a single multi-tool (see Figure 4.17) capable of performing several tasks without interrupting or reconfiguring the setup. The cotter pin puller was combined with the hammer head for removal and installation of the cotter pin keeper on the insulator. Lastly, the ball socket adjuster (fork) allows the operator to manipulate the socket for release or capture of the ball on the insulator.



Pin puller

Hammer

Fork

Figure 4.17 Multi-tool used to perform multiple live-line maintenance tasks.

The actuator creating the yaw rotation is switched on this time and the pan tool is attached on that link, so that all the operations required for changing the insulator could be performed without reconfiguring the setup. The pan tool (Figure 4.18) is used to remove the insulator or insulator string from the Y-ball or adjacent insulator as required.

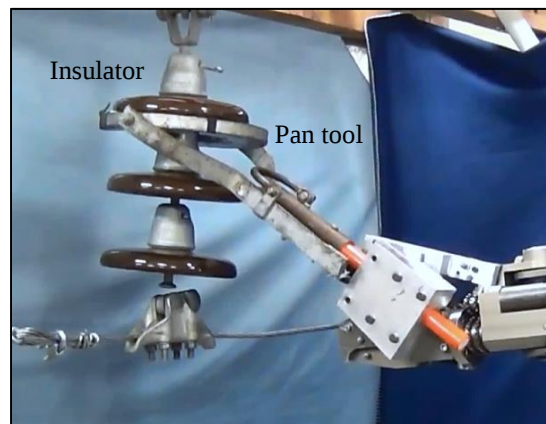
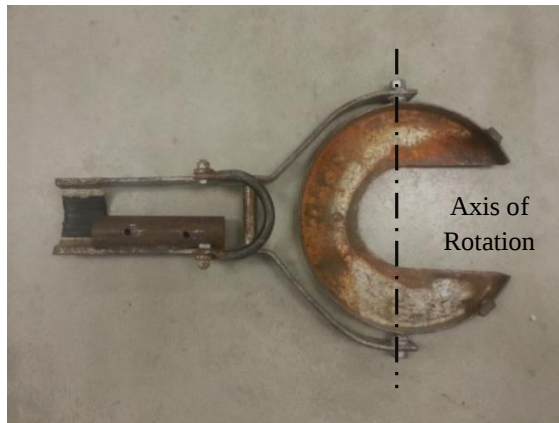


Figure 4.18 Pan tool being used to remove insulator.

It is then used to transfer and install the new insulator string. Corresponding work envelope and end-effector trajectories are also shown for each task, which demonstrates that with minimal movements, all the tasks were effortlessly performed. The operator begins by unpinning the insulator keeper using the multi-tool pin puller (Figure 4.19). Inset shows how the operator's hand movement corresponds to the hydraulic manipulator. End-effector motion and corresponding work envelope is shown in Figure 4.20.

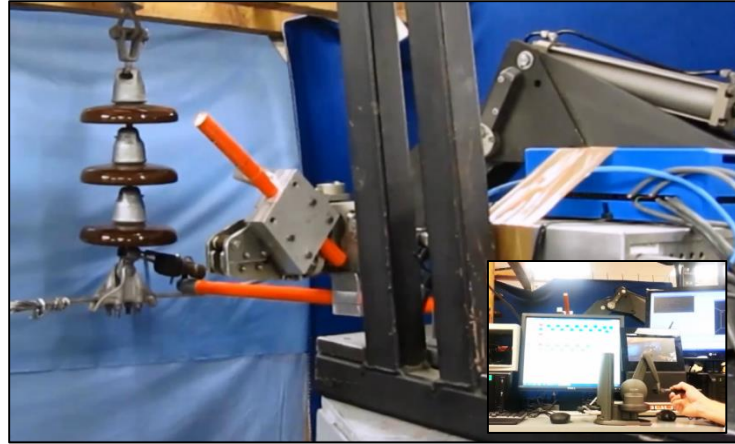
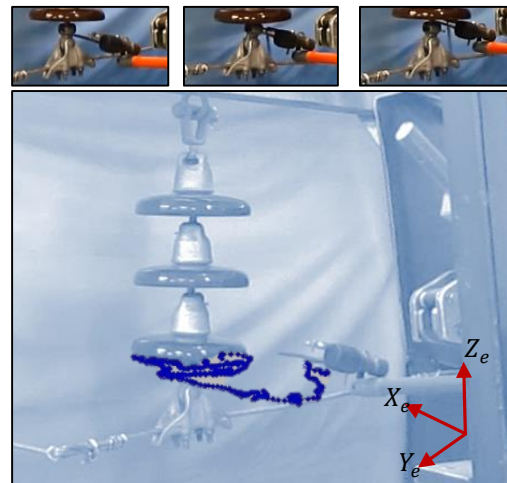
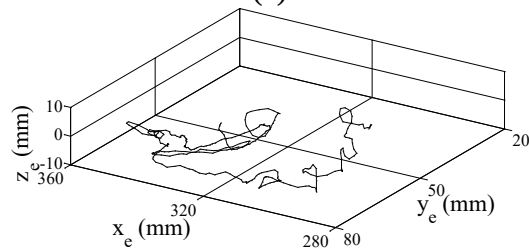


Figure 4.19 Unpinning the insulator keeper.



(a)



(b)

Figure 4.20 (a) Pulling out the pin using pin puller, and (b) corresponding work envelope.

Next, the lower end of the string is released (Figure 4.21) from the conductor shoe using the ball socket adjuster (fork). The haptic device allows the operator to make small positional changes to the hot-stick tool in any direction as though the hot-stick were a pen in hand. Complete three-dimensional motion and control is provided by the hydraulic robotic arm. Normally, the operator's whole body would be involved in manipulating the hot-stick tool from the bucket, ladder or tower. Using the new system, the operator's wrist is most active. This greatly reduces worker's fatigue.

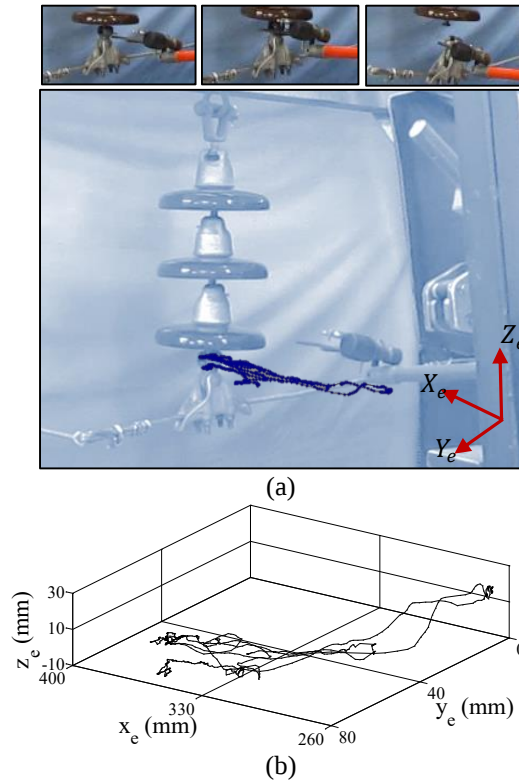


Figure 4.21 (a) Releasing the lower string using the ball socket adjuster (fork), and (b) corresponding work envelope.

Once the lower end of the insulator string is disconnected, the pan tool is attached and maneuvered into place as shown in Figure 4.22. The string is picked off of the Y-ball connector. Upon removal of the defective string, the pan tool is re-charged with a new insulator and the operator now maneuvers the socket, capturing the Y-ball connector (Figure 4.20).

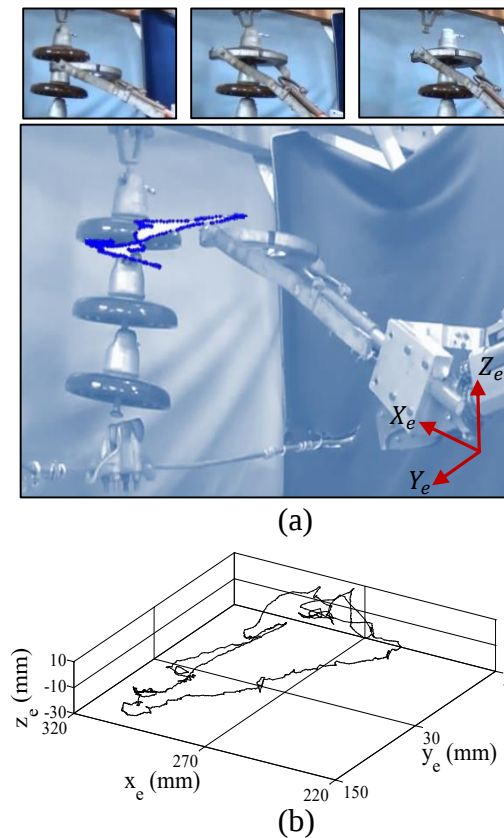


Figure 4.22 (a) Removing the broken insulator using the pan tool, and (b) corresponding work envelope.

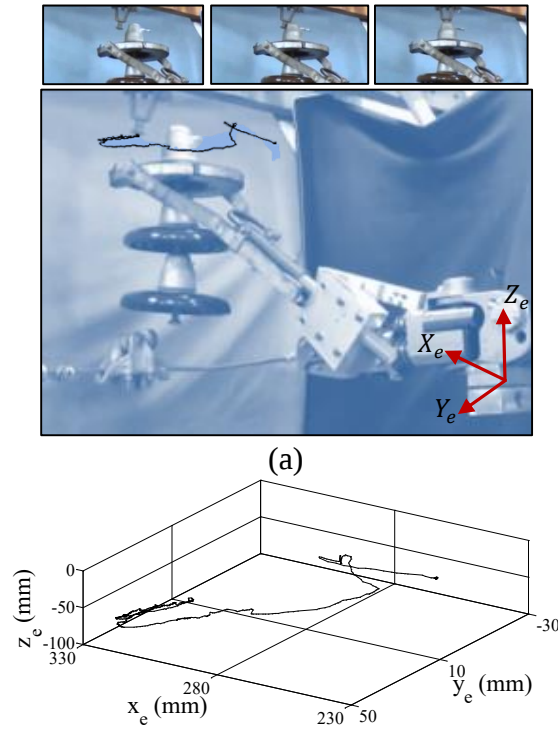
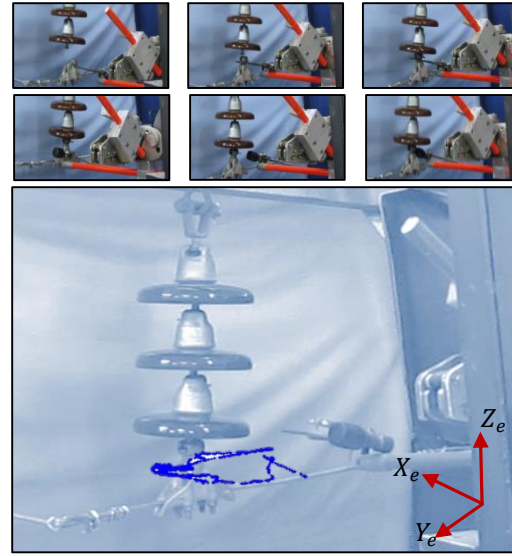
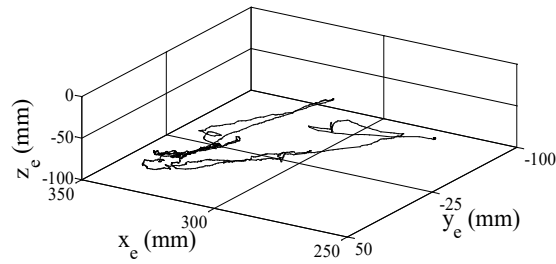


Figure 4.23 (a) Replacing a new insulator using the pan tool, and (b) corresponding work envelope.

Finally, after changing the tool attachment from the pan tool back to the multi-tool, the lower attachment is made using the ball socket adjuster (fork). Once the socket is seated, the hammer completes the task, driving home the cotter pin keeper as shown in Figure 4.24. This successfully completes the task of the changing an insulator string.



(a)



(b)

Figure 4.24 (a) Connecting the lower joint using the ball socket adjuster (fork) followed by hammering, and (b) corresponding work envelope.

4.6 Summary

In this chapter, the performance of a teleoperated hydraulic manipulator in performing two tasks relevant to live-line maintenance was evaluated. Experimental studies were conducted when the manipulator base was excited by the Stewart platform creating random motions. There were three tested force modes: virtual fixture, force augmentation, and augmented virtual fixture schemes. The performance of the teleoperated system, under each force mode, was evaluated and then compared with a mode in which no force was

generated by the haptic device. Performance of each scheme was evaluated under three measures: task completion time, number of failed trials and displacement of the manipulator end-effector. Results showed that the augmentation force (position error force) was the best scheme by which the operators could complete the typical tasks efficiently and comfortably. Since implementing the augmented force is straightforward and effective, for the proposed application, it is recommended as a tool to perform the maintenance tasks. Similar concept has been used in the following chapters where position error between slave and master sides is used to generate the haptic force. In addition, this force is proven to be proportional to the interaction force between the hydraulic actuator and the environment making it the best choice in applications where installing a force sensor on the end-effector of the hydraulic manipulator is unfeasible.

CHAPTER 5

5 BILATERAL CONTROL OF SINGLE-ROD HYDRAULIC ACTUATORS WITH FIXED BASE²

5.1 Introduction

In this chapter, a bilateral control scheme is developed and experimentally validated for a fixed base single-rod hydraulic actuator. The stability of the complete bilateral control scheme has been theoretically proven using Lyapunov techniques. In this scheme, the position error between displacements of the haptic device and the hydraulic actuator is used at both master and slave sides to maintain good position tracking at the actuator side while providing a feel of performing task at the remote site without the need for direct measurement of the interaction force. It allows an operator to use a haptic device to manipulate a hydraulic actuator in either free or constrained motion.

² A version of this chapter has been published in V. Banthia, K. Zareinia, S. Balakrishnan and N. Sepehri, "A Lyapunov Stable Controller for Bilateral Haptic Teleoperation of Single-Rod Hydraulic Actuators," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 139, no. 11, pp. 111001-111017, 2017 [78]

There are various methods for creating and applying haptic force to the operator's hand in teleoperation systems [7]. A simple way to create and apply haptic sensation is to use the contact force between the slave manipulator and the task environment [20]. However, in certain applications as in controlling backhoes and excavators, providing haptic force feedback based on measurement of interaction force, is not feasible and presents many challenges [6]. When interaction force information is not readily available, position information can potentially be used to generate haptic feedback [6]. A simple approach in using position information for providing haptic feedback is to use the positions of master and slave manipulators [6, 8, 79]. Rather than making the system transparent, *i.e.*, having direct force feedback on the operator's hand, the haptic device alerts the operator of the reactions as a result of forces acting on the implement.

The control scheme proposed in this chapter consists of two control laws at the master and slave sides. At the slave side, the hydraulic actuator has a stable position tracking for both unconstrained and constrained motions as well as the transition phase between these two motions. At the master side, the haptic device provides the “feel” of telepresence to the operator by creating a force that has two components. One component uses position error information between the master and the slave. The second component reflects the stiffness of the environment to the operator. At the steady-state, this term is a scaled version of interaction force. The proposed controller can be tuned to give different weights to each of the two haptic sensations that are built based on the position error or the interaction force. The proposed control scheme is also easy to implement, as the only required measurements are hydraulic system pressures (supply pressure and actuator's line pressures), and displacements of the master and slave device.

The control laws are obtained during the non-straightforward process of constructing a Lyapunov function to guarantee the stability of the overall closed-loop system. A description of the Lyapunov stability analysis method along with some other definitions used in this thesis are provided in Appendix A.2. Owing to the discontinuity in the proposed control laws, the control system is non-smooth. With regard to classical solution theories, a solution cannot even be defined; much less discuss its existence, uniqueness, and stability for such systems. Therefore, Filippov's solution concept is first employed to prove the existence, continuation and uniqueness of the Filippov's solution [80, 81]. Following, the extended Lyapunov's stability theory [82, 83, 84, 85] is used for the stability analysis of the resulting bilateral control system. Simulation and experimental studies are presented to validate practicality and effectiveness of the proposed controller.

5.2 Development of controller

The bilateral teleoperation system model (see Figure 5.1) described in Eq. (3.52) is now employed to design a stable bilateral control scheme.

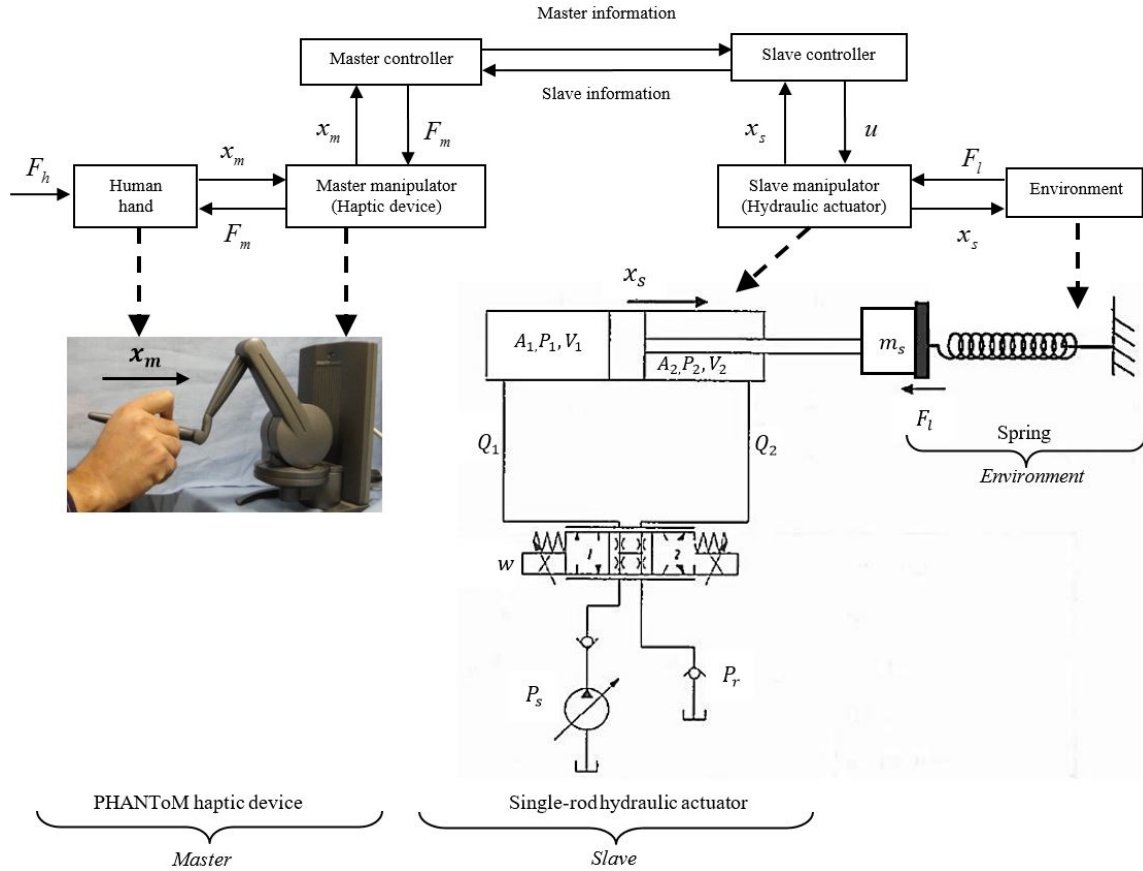


Figure 5.1 Bilateral teleoperation system.

The aim is to develop a Lyapunov stable feedback control scheme that is capable of doing the regulating task, *i.e.* given a constant force by human operator and an arbitrary initial condition, the system should approach to an equilibrium point and remain there. The control laws are constructed during the non-straightforward process of finding the proper Lyapunov function for the system as explained in Appendix A.3. Here, the control laws are proposed first and then the stability using the Lyapunov stability analysis is presented. The following control laws are proposed to determine values for u and F_m :

$$u = [-K_{p1}(x_s - x_m) - K_{p2}P_L]\sqrt{\Delta} \quad (5.1)$$

$$F_m = K_{p3}(x_s - x_m) + K_{p3}\frac{Q}{A_1}P_L \quad (5.2)$$

$$u = [-K_{p1}(x_1 - x_5) - K_{p2}x_3]\sqrt{\Delta} \quad (5.3)$$

$$F_m = K_{p3}(x_1 - x_5) + K_{p3}\frac{Q}{A_1}x_3 \quad (5.4)$$

where,

$$\Delta = (1 + \alpha)\frac{P_s}{2} + \text{sgn}(x_4)\left((1 - \alpha)\frac{P_s}{2} - x_3\right) \quad (5.5)$$

$$Q = \frac{V_1}{\beta\left(1 + \frac{\alpha^2}{\gamma}\right)}$$

In Eqs. (5.1) and (5.2), K_{p1} , K_{p2} , and K_{p3} are real positive gains. The first term in Eq. (5.4) acts as a proportional controller to minimize the displacement error between the master and slave manipulators, which results in a faithful position tracking by the slave. This first term constrains the operator's hand motion when the slave manipulator is behind/ahead in tracking master manipulator's displacement. The second term, which is related to the load pressure, P_L , of the hydraulic actuator is added for the stability requirement. The entire control law helps the operator to feel as if he/she is directly manipulating the hydraulic actuator creating the level of telepresence, or "feel" of the remote site. Similar terms, but scaled by term $\sqrt{(1 + \alpha)\frac{P_s}{2} + \text{sgn}(x_v)\left((1 - \alpha)\frac{P_s}{2} - P_L\right)}$ to ensure stability, are used for calculating the control signal u , which help the hydraulic actuator to have a stable position tracking.

Note that the above controller is discontinuous due to term $sgn(x_4)$ in (5.3) when $x_4 = 0$. Replacing u and F_m in Eq. (3.52) with Eqs. (5.3) and (5.4), results in the following state space equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{A_1}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 \\ \dot{x}_3 = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} x_4 (\sqrt{\Delta}) \\ \dot{x}_4 = \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{\Delta} \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{1}{m_m} \left[F_h + K_{p3}(x_1 - x_5) + K_{p4} \frac{Q}{A_1} x_3 - k_d x_6 - k_h x_5 \right] \end{cases} \quad (5.6)$$

where, $\Delta = (1 + \alpha) \frac{P_s}{2} + sgn(x_4) \left((1 - \alpha) \frac{P_s}{2} - x_3 \right)$

The control system described by Eq. (5.6) is non-smooth due to term $sgn(x_4)$ on the right-hand side of the equation describing $\dot{x}_4$. These types of systems violate the fundamental assumption of conventional solution theories of ordinary differential equations, known as the Lipschitz-continuous requirement. With respect to classical solution theories, one cannot even define a solution, much less discuss its existence, uniqueness and stability. Filippov's solution theory [80, 81] is one of the earliest and most conceptually straightforward approaches developed for analysis of non-smooth systems. Based on the Filippov's solution concept, the conventional Lyapunov stability theory initially developed for smooth systems, has been extended to non-smooth systems [83, 84, 85] and will be used in the next section to study the stability.

5.3 Stability analysis

The first step in the stability analysis is the Filippov's solution analysis that establishes a new definition of solution for differential equations with discontinuous terms. It provides the required theorems to prove the existence, uniqueness, and continuity of non-smooth systems [80, 81]. The second step is to construct a smooth Lyapunov function for the non-smooth dynamic system of Eq. (5.6) to prove the stability of the system.

5.3.1 Existence, uniqueness, and continuation of Filippov's solution

The dynamic system presented by Eq. (5.6) consists of nonlinear differential equations with discontinuous right-hand sides. The discontinuity arises from the term $sgn(x_4)$ originated by the control law Eq. (5.3). Here, Filippov's solution concept is used for the non-smooth system described by Eq. (5.6). The discontinuity surface of the system described in Eq. (5.6) is:

$$S_1 := \{\vec{x}: x_1 \neq 0 \text{ and } x_4 = 0\} \quad (5.7)$$

The discontinuity surface, S_1 , divides the solution region, Ω_{S_1} , into two regions:

$$\Omega_{S_1}^+ := \{\vec{x}: x_1 \neq 0 \text{ and } x_4 > 0\} \quad (5.8)$$

$$\Omega_{S_1}^- := \{\vec{x}: x_1 \neq 0 \text{ and } x_4 < 0\} \quad (5.9)$$

The right-hand sides of Eq. (5.6) are piecewise continuous and defined everywhere in Ω_{S_1} . They are measurable and bounded as well. Therefore, Eq. (5.6) satisfies condition B of Filippov's solution theory and according to theorems 4 and 5 of Filippov [80], the local existence and continuity of a solution is confirmed. Next, the uniqueness of the solution is proven. Since the right-hand sides of Eq. (5.6) are all continuous before and after the discontinuity surface, and the discontinuity surface, S_1 , is smooth and independent

of time, conditions A, B and C of Filippov's solution theory are satisfied [80, 81].

Following the procedure described in [80], the limiting values of the vector function of the right-hand sides of Eq. (5.6), i.e. $f_{S_1}^+$ and $f_{S_1}^-$ when S_1 is approached from $\Omega_{S_1}^+$ and $\Omega_{S_1}^-$, are:

$$f_{S_1}^+ = \begin{cases} x_2 \\ \frac{A_1}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{P_s - x_3}) \\ \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2} x_3] \sqrt{P_s - x_3} \\ x_6 \\ \frac{1}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + \frac{Q K_{p3}}{A_1} x_3 - k_d x_6 - k_h x_5\right) \end{cases} \quad (5.10)$$

$$f_{S_1}^- = \begin{cases} x_2 \\ \frac{A}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{\alpha P_s + x_3}) \\ \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2} x_3] \sqrt{\alpha P_s + x_3} \\ x_6 \\ \frac{1}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + \frac{Q K_{p3}}{A_1} x_3 - k_d x_6 - k_h x_5\right) \end{cases} \quad (5.11)$$

The projections of $f_{S_1}^+$ and $f_{S_1}^-$ along the normal to the discontinuity surface, i.e., $N_{S_1} = \{0,0,0,1,0,0\}$ are denoted by $f_{N_{S_1}}^+$ and $f_{N_{S_1}}^-$:

$$f_{N_{S_1}}^+ = \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2}(x_3)] \sqrt{P_s - x_3} \quad (5.12)$$

$$f_{N_{S_1}}^- = \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2}(x_3)] \sqrt{\alpha P_s + x_3} \quad (5.13)$$

$f_{N_{S_1}}^+$ and $f_{N_{S_1}}^-$ have the same sign (note that $\sqrt{P_s \pm x_3}$ is always non-negative). This satisfies conditions of Lemma 9 of Filippov's solution [80].

The other discontinuity surface:

$$S_2 := \{\vec{x}: x_1 = 0 \text{ and } x_4 \neq 0\} \quad (5.14)$$

Then,

$$\Omega_{S_2}^+ := \{\vec{x}: x_1 > 0 \text{ and } x_4 \neq 0\} \quad (5.15)$$

$$\Omega_{S_2}^- := \{\vec{x}: x_1 < 0 \text{ and } x_4 \neq 0\} \quad (5.16)$$

$$f_{S_2}^+ = \begin{cases} x_2 \\ \frac{A_1}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{\Delta}) \\ \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{\Delta} \\ x_6 \\ \frac{1}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + \frac{Q K_{p3}}{A_1} x_3 - k_d x_6 - k_h x_5\right) \end{cases} \quad (5.17)$$

$$f_{S_2}^- = \begin{cases} x_2 \\ \frac{A}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{\Delta}) \\ \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{\Delta} \\ x_6 \\ \frac{1}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + \frac{Q K_3}{A_1} x_3 - k_d x_6 - k_h x_5\right) \end{cases} \quad (5.18)$$

Projections of $f_{S_2}^+$ and $f_{S_2}^-$ along $N_{S_2} = \{1,0,0,0,0\}$

$$f_{N_{S_2}}^+ = x_2 \quad (5.19)$$

$$f_{N_{S_2}}^- = x_2 \quad (5.20)$$

$f_{N_{S_2}}^+$ and $f_{N_{S_2}}^-$ have the same sign. Thus, Lemma 9 of Filippov satisfies [80].

$$S_3 := \{\vec{x}: x_1 = 0 \text{ and } x_4 = 0\} \quad (5.21)$$

Note that S_3 is the intersection of S_1 and S_2 . Uniqueness analysis of S_3 is also carried out in a similar way.

5.3.2 Stability proof

Extension of Lyapunov's second method for non-smooth dynamic systems [83], based on Filippov's solution theory is used for stability analysis. A smooth Lyapunov function for the non-smooth dynamic system is constructed. By imposing $\dot{x}_i (i=1..6) = 0$, the system described by Eq. (5.6) is shown to have the following equilibrium point:

$$\begin{aligned}\vec{x}_{eq} &= [x_{1eq} \quad x_{2eq} \quad x_{3eq} \quad x_{4eq} \quad x_{5eq} \quad x_{6eq}]^T \\ &= [x_{1ss} \quad x_{2ss} \quad x_{3ss} \quad x_{4ss} \quad x_{5ss} \quad x_{6ss}]^T\end{aligned}\tag{5.22}$$

Defining the following states from Eq. (5.6):

when $\dot{x}_1 = 0$, we get:

$$x_{2ss} = 0\tag{5.23}$$

Similarly when $\dot{x}_2 = 0$, we get:

$$\frac{A_1}{m_s} x_{3ss} - \frac{d}{m_s} x_{2ss} - \frac{k_s}{m_s} x_{1ss} = 0$$

Since $x_{2ss} = 0$ from Eq. (5.23), we get:

$$\frac{A_1}{m_s} x_{3ss} = \frac{k_s}{m_s} x_{1ss}$$

or,

$$x_{3ss} = \frac{k_s}{A_1} x_{1ss}\tag{5.24}$$

Now when $\dot{x}_3 = 0$, we get:

$$-\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_{2ss} + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} x_{4ss} (\sqrt{\Delta}) = 0$$

Since $x_{2ss} = 0$,

$$x_{4ss} = 0 \quad (5.25)$$

Now when $\dot{x}_4 = 0$, we get:

$$\begin{aligned} \frac{-1}{\tau} x_{4ss} + \frac{k_{sp}}{\tau} [-K_{p1}(x_{1ss} - x_{5ss}) - K_{p2}x_{3ss}] \sqrt{\Delta} \\ = 0 \end{aligned}$$

or,

$$x_{5ss} = \left(1 + \frac{K_{p2}k_s}{K_{p1}A_1}\right) x_{1ss} \quad (5.26)$$

When $\dot{x}_5 = 0$, we get:

$$x_{6ss} = 0 \quad (5.27)$$

Finally, when $\dot{x}_6 = 0$, we get:

$$\frac{1}{m_m} \left[F_h + K_{p3}(x_{1ss} - x_{5ss}) + K_{p4} \frac{Q}{A_1} x_{3ss} - k_d x_{6ss} - k_h x_{5ss} \right] = 0 \quad (5.28)$$

Putting the values of x_{3ss} and x_{5ss} from Eqs. (5.24) and (5.26) in above, we get:

$$x_{1ss} = \frac{K_{p1}A_1^2 F_h}{K_{p3}K_{p2}A_1k_s - QK_{p3}K_{p1}k_s + K_{p1}A_1^2k_h + K_{p2}A_1k_hk_s} \quad (5.29)$$

and from Eq. (5.24)

$$x_{3ss} = \frac{K_{p1}A_1k_s F_h}{K_{p3}K_{p2}A_1k_s - QK_{p3}K_{p1}k_s + K_{p1}A_1^2k_h + K_{p2}A_1k_hk_s} \quad (5.30)$$

and from Eq. (5.26)

$$x_{5ss} = \frac{K_{P1}A_1^2F_h + K_{P2}A_1k_sF_h}{K_{P3}K_{P2}A_1k_s - QK_{P3}K_{P1}k_s + K_{P1}A_1^2k_h + K_{P2}A_1k_hk_s} \quad (5.31)$$

Finally we get:

$$x_{1ss} = \frac{K_{P1}A_1^2F_h}{K_{P3}K_{P2}A_1k_s - QK_{P3}K_{P1}k_s + K_{P1}A_1^2k_h + K_{P2}A_1k_hk_s} \quad (5.32)$$

$$x_{3ss} = \frac{K_{P1}A_1k_sF_h}{K_{P3}K_{P2}A_1k_s - QK_{P3}K_{P1}k_s + K_{P1}A_1^2k_h + K_{P2}A_1k_hk_s} \quad (5.33)$$

$$x_{5ss} = \frac{K_{P1}A_1^2F_h + K_{P2}A_1k_sF_h}{K_{P3}K_{P2}A_1k_s - QK_{P3}K_{P1}k_s + K_{P1}A_1^2k_h + K_{P2}A_1k_hk_s} \quad (5.34)$$

Thus,

$$\begin{aligned} \vec{x}_{eq} &= [x_{1eq} \quad x_{2eq} \quad x_{3eq} \quad x_{4eq} \quad x_{5eq} \quad x_{6eq}]^T \\ &= [x_{1ss} \quad 0 \quad x_{3ss} \quad 0 \quad x_{5ss} \quad 0]^T \end{aligned} \quad (5.35)$$

Defining $\vec{e} = \vec{x} - \vec{x}_{eq}$ the following error states are then defined:

$$\begin{aligned} \vec{e} &= [e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5 \quad e_6]^T \\ &= [(x_1 - x_{1ss}) \quad x_2 \quad (x_3 - x_{3ss}) \quad x_4 \quad (x_5 - x_{5ss}) \quad x_6]^T \end{aligned} \quad (5.36)$$

$$\begin{cases} e_1 = x_1 - x_{1ss} \\ e_2 = x_2 - x_{2ss} \\ e_3 = x_3 - x_{3ss} \\ e_4 = x_4 - x_{4ss} \\ e_5 = x_5 - x_{5ss} \\ e_6 = x_6 - x_{6ss} \end{cases} \quad (5.37)$$

Since x_{2ss}, x_{4ss} and x_{6ss} are all zeros, we get:

$$\begin{cases} e_1 = x_1 - x_{1ss} \\ e_2 = x_2 \\ e_3 = x_3 - x_{3ss} \\ e_4 = x_4 \\ e_5 = x_5 - x_{5ss} \\ e_6 = x_6 \end{cases} \quad (5.38)$$

First derivative:

$$\begin{cases} \dot{e}_1 = \dot{x}_1 \\ \dot{e}_2 = \dot{x}_2 \\ \dot{e}_3 = \dot{x}_3 \\ \dot{e}_4 = \dot{x}_4 \\ \dot{e}_5 = \dot{x}_5 \\ \dot{e}_6 = \dot{x}_6 \end{cases} \quad (5.39)$$

Putting the above values in Eq. (5.6), we get:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \frac{A_1}{m_s} (e_3 + x_{3ss}) - \frac{d}{m_s} e_2 - \frac{k_s}{m_s} (e_1 + x_{1ss}) \\ \dot{e}_3 = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) e_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} e_4 (\sqrt{\Delta}) \\ \dot{e}_4 = \frac{-1}{\tau} e_4 + \frac{k_{sp}}{\tau} [-K_{p1}(e_1 + x_{1ss} - e_5 - x_{5ss}) - K_{p2}(e_3 + x_{3ss})] \sqrt{\Delta} \\ \dot{e}_5 = e_6 \\ \dot{e}_6 = \frac{1}{m_m} \left(F_h + K_{p3}(e_1 + x_{1ss} - e_5 - x_{5ss}) + K_{p3} \frac{Q}{A_1} (e_3 + x_{3ss}) - k_d e_6 - k_h (e_5 + x_{5ss}) \right) \end{cases} \quad (5.40)$$

Putting the values of $x_{1ss}, x_{3ss}, x_{5ss}$:

$$\dot{e}_2 = \frac{A_1}{m_s} e_3 - \frac{d}{m_s} e_2 - \frac{k_s}{m_s} e_1 \quad (5.41)$$

$$\dot{e}_4 = \frac{-1}{\tau} e_4 + \frac{k_{sp}}{\tau} [-K_{p1}(e_1 - e_5) - K_{p2} e_3] \sqrt{\Delta} \quad (5.42)$$

$$\dot{e}_6 = \frac{1}{m_m} \left[K_{p3}(e_1 - e_5) + K_{p3} \frac{Q}{A_1} e_3 - k_d e_6 - k_h e_5 \right] \quad (5.43)$$

Eq. (5.40) changes to:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \frac{A_1}{m_s} e_3 - \frac{d}{m_s} e_2 - \frac{k_s}{m_s} e_1 \\ \dot{e}_3 = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) e_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} e_4 (\sqrt{\Delta}) \\ \dot{e}_4 = \frac{-1}{\tau} e_4 + \frac{k_{sp}}{\tau} [-K_{p1}(e_1 - e_5) - K_{p2}e_3] \sqrt{\Delta} \\ \dot{e}_5 = e_6 \\ \dot{e}_6 = \frac{1}{m_m} \left[K_{p3}(e_1 - e_5) + K_{p3} \frac{Q}{A_1} e_3 - k_d e_6 - k_h e_5 \right] \end{cases} \quad (5.44)$$

Remark:

At steady-state, force generated by the haptic device, F_{mss} , is related to the contact force F_l between the hydraulic actuator and the environment, as follows:

From Eq. (5.4),

$$F_{mss} = K_{p3}(x_{1ss} - x_{5ss}) + K_{p3} \frac{Q}{A_1} x_{3ss} \quad (5.45)$$

$$F_{mss} = K_{p3}x_{1ss} - K_{p3}x_{5ss} + K_{p3} \frac{Q}{A_1} x_{3ss} \quad (5.46)$$

Putting the value of x_{5ss} and x_{3ss} in terms of x_{1ss} from Eqs. (5.23) and (5.24),

$$F_{mss} = \left[\frac{K_{p3}K_{p1}Q - K_{p3}K_{p2}A_1}{K_{p1}A_1^2} \right] k_s x_{1ss} \quad (5.47)$$

$$F_{mss} = \left[\frac{K_{p1}Q - K_{p2}A_1}{K_{p1}A_1^2} \right] K_{p3}F_{lss} \quad (5.48)$$

It is observed that even though the contact force is not directly measured, the force generated by the haptic device F_{mss} is proportional to the interaction force F_l . The proportionality can be adjusted via controller gain, K_{p3} .

The right-hand sides of Eq. (5.44) are discontinuous, but measurable and bounded. To construct a smooth Lyapunov function for the non-smooth system of Eq. (5.44), the procedure described by Wu *et al.* [83] is used. The following Lyapunov function is constructed for the system described by Eq. (5.44):

$$\begin{aligned}
V(e_1, e_2, e_3, e_4, e_5, e_6) &= \left(\frac{13K_{p1}A_1^2\beta^2\left(1+\frac{\alpha^2}{\gamma}\right)^2 + 14k_s\beta\left(1+\frac{\alpha^2}{\gamma}\right)\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]}{28V_1\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} \right) e_1^2 \\
&+ \frac{K_{p1}V_1 + K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right)}{4\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} e_3^2 + \frac{K_{p1}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right)}{\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} e_1e_3 + \frac{m_s\beta\left(1+\frac{\alpha^2}{\gamma}\right)}{2V_1} e_2^2 \\
&+ \frac{Ak_vw\tau\beta^2\left(1+\frac{\alpha^2}{\gamma}\right)^2}{2k_{sp}V_1\sqrt{1+\alpha^3}\left(K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right)} e_4^2 \\
&+ \frac{K_{p1}A_1^2\beta^2\left(1+\frac{\alpha^2}{\gamma}\right)^2}{28V_1\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} (e_1 - 14e_5)^2 + \frac{1}{4} \left(e_3 - \frac{2K_{p1}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right)}{\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} e_5 \right)^2 \\
&+ \frac{(-13K_{p1}A_1^2K_{p3} + K_{p1}A_1^2k_h)\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]\beta^2\left(1+\frac{\alpha^2}{\gamma}\right)^2}{2V_1K_{p3}\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]^2} e_5^2 \\
&- \frac{2K_{p1}^2V_1K_{p3}A_1^2\beta^2\left(1+\frac{\alpha^2}{\gamma}\right)^2}{2V_1K_{p3}\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]^2} e_5^2 + \frac{K_{p1}m_mA_1^2\beta^2\left(1+\frac{\alpha^2}{\gamma}\right)^2}{2V_1K_{p3}\left[K_{p2}A_1\beta\left(1+\frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} e_6^2
\end{aligned} \tag{5.49}$$

From Eq. (5.49), function V is continuous. Next, V is proven positive and definite.

Rewriting V as follows:

$$V = V_1 + V_2$$

where:

$$V_1 = \left(\frac{13K_{p1}A_1^2\beta^2\left(1 + \frac{\alpha^2}{\gamma}\right)^2 + 14k_s\beta\left(1 + \frac{\alpha^2}{\gamma}\right)\left[K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]}{28V_1\left[K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} \right) e_1^2$$

$$+ \frac{K_{p1}V_1 + K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right)}{4\left[K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} e_3^2 + \frac{K_{p1}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right)}{\left[K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} e_1 e_3$$

$$V_1 = (term1)e_1^2 + (term2)e_3^2 + (term3)e_1e_3 \quad (5.50)$$

$$\begin{aligned}
V_2 &= \frac{m_s \beta \left(1 + \frac{\alpha^2}{\gamma}\right)}{2V_1} e_2^2 + \frac{Ak_v w \tau \beta^2 \left(1 + \frac{\alpha^2}{\gamma}\right)^2}{2k_{sp} V_1 \sqrt{1 + \alpha^3} (K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1} V_1)} e_4^2 \\
&+ \frac{K_{p1} A_1^2 \beta^2 \left(1 + \frac{\alpha^2}{\gamma}\right)^2}{28V_1 \left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1} V_1\right]} (e_1 - 14e_5)^2 \\
&+ \frac{1}{4} \left(e_3 - \frac{2K_{p1} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right)}{\left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1} V_1\right]} e_5 \right)^2 \\
&+ \frac{\left(-13K_{p1} A_1^2 K_{p3} + K_{p1} A_1^2 k_h\right) \left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1} V_1\right] \beta^2 \left(1 + \frac{\alpha^2}{\gamma}\right)^2}{2V_1 K_{p3} \left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1} V_1\right]^2} e_5^2 \\
&- \frac{2K_{p1}^2 V_1 K_{p3} A_1^2 \beta^2 \left(1 + \frac{\alpha^2}{\gamma}\right)^2}{2V_1 K_{p3} \left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1} V_1\right]^2} e_5^2 + \frac{K_{p1} m_m A_1^2 \beta^2 \left(1 + \frac{\alpha^2}{\gamma}\right)^2}{2V_1 K_{p3} \left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1} V_1\right]} e_6^2
\end{aligned} \tag{5.51}$$

To prove V_1 is positive, it can be re-written in the following form:

$$V_1 = [e_1 \quad e_3] M \begin{bmatrix} e_1 \\ e_3 \end{bmatrix} \tag{5.52}$$

where matrix M is:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} \text{term1} & (\text{term3})/2 \\ (\text{term3})/2 & \text{term2} \end{bmatrix} \tag{5.53}$$

As per Sylvester's theorem [86], for V_1 to be positive definite, a necessary and sufficient condition for the symmetric matrix M is that its principal minors, i.e., M_{11} and $\det(M) = (M_{11}M_{22} - M_{12}M_{21})$ must be strictly positive. M_{11} is shown below:

$$M_{11} = \text{term1} \quad (5.54)$$

$$\begin{aligned} \det(M) &= (M_{11}M_{22} - M_{21}M_{12}) \\ &= (\text{term1}) * (\text{term2}) - (\text{term3})^2/4 \end{aligned} \quad (5.55)$$

$$\begin{aligned} M_{11} &= \text{term1} \\ &= \frac{13K_{p1}A_1^2\beta^2\left(1 + \frac{\alpha^2}{\gamma}\right)^2 + 14k_s\beta\left(1 + \frac{\alpha^2}{\gamma}\right)\left[K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]}{28V_1\left[K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1\right]} \end{aligned} \quad (5.56)$$

M_{11} will be strictly positive subject to the following condition:

$$K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1 > 0 \quad (5.57)$$

Thus,

$$K_{p2} > \frac{K_{p1}V_1}{A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right)} \quad (5.58)$$

Let's define:

$$H = K_{p2}A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right) - K_{p1}V_1 \quad (5.59)$$

and

$$J = \beta \left(1 + \frac{\alpha^2}{\gamma} \right) \quad (5.60)$$

For $\det(M)$ we have:

$$\det(M) = (\text{term1}) * (\text{term2}) - (\text{term3})^2 / 4 \quad (5.61)$$

$$\begin{aligned} \det(M) = & \frac{13K_{p1}A_1^2J^2 + 14k_sJH}{28V_1H} * \frac{K_{p1}V_1 + K_{p1}A_1J}{4H} \\ & - \frac{(K_{p1}A_1J)^2}{4H^2} \end{aligned} \quad (5.62)$$

If we rewrite $\det(M)$ and have terms pertaining to k_s separate, we have:

$$\begin{aligned} \det(M) = & k_s \frac{14JH(K_{p1}V_1 + K_{p1}A_1J)}{112V_1H^2} + \frac{13K_{p1}A_1^2J^2(K_{p1}V_1 + K_{p1}A_1J)}{112V_1H^2} \\ & - \frac{(K_{p1}A_1J)^2}{4H^2} \end{aligned} \quad (5.63)$$

$$\begin{aligned} \det(M) = & k_s \frac{14JH(K_{p1}V_1 + K_{p1}A_1J)}{112V_1H^2} \\ & + \frac{13K_{p1}^2A_1^2J^2V_1 + 13K_{p1}A_1^3J^3K_{p2} - 28V_1K_{p1}^2A_1^2J^2}{112V_1H^2} \end{aligned} \quad (5.64)$$

$$\begin{aligned} \det(M) = & k_s \frac{14JH(K_{p1}V_1 + K_{p1}A_1J)}{112V_1H^2} \\ & + K_{p1}A_1^2J^2 \frac{13A_1JK_{p2} - 15V_1K_{p1}}{112V_1H^2} \end{aligned} \quad (5.65)$$

The first term is positive, for the second term to be positive:

$$13A_1JK_{p2} > 15V_1K_{p1} \quad (5.66)$$

or,

$$K_{p2} > \frac{15V_1K_{p1}}{13A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right)} \quad (5.67)$$

Note that if condition (5.67) satisfies then (5.58) will be satisfied, because:

$$K_{p2} > \frac{15V_1K_{p1}}{13A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right)} > \frac{K_{p1}V_1}{A_1\beta\left(1 + \frac{\alpha^2}{\gamma}\right)} \quad (5.68)$$

Thus, V_1 is proven to be positive. Now for V_2 .

All terms in V_2 are positive subject to condition (5.68), except for the term pertaining to e_5^2 , rewriting that term we have:

$$\frac{(-13K_{p1}A_1^2K_{p3} + K_{p1}A_1^2k_h)HJ^2 - 2K_{p1}^2V_1K_{p3}A_1^2J^2}{2V_1K_{p3}H^2} > 0 \quad (5.69)$$

$$(-13K_{p1}A_1^2K_{p3} + K_{p1}A_1^2k_h)HJ^2 - 2K_{p1}^2V_1K_{p3}A_1^2J^2 > 0 \quad (5.70)$$

$$K_{p3}(-13H - 2K_{p1}V_1) + k_hH > 0 \quad (5.71)$$

$$k_hH > K_{p3}(13H + 2K_{p1}V_1) \quad (5.72)$$

Thus,

$$K_{p3} < \frac{k_hH}{13H + 2K_{p1}V_1} \quad (5.73)$$

Thus, subject to satisfying conditions (5.68) and (5.73), function V is positive definite.

Next, V is proven to be a smooth Lyapunov function and $\dot{V}$ is negative and at least semi-definite. The derivative of V with respect to time is:

$$\begin{aligned}
\dot{V} = & \frac{13K_{p1}A_1^2J^2 + 14k_sJH}{28V_1H} 2e_1\dot{e}_1 + \frac{K_{p1}V_1 + K_{p2}A_1J}{2H} e_3\dot{e}_3 + \frac{K_{p1}A_1J}{H} e_1\dot{e}_3 + \frac{K_{p1}A_1J}{H} e_3\dot{e}_1 \\
& + \frac{m_sJ}{2V_1} e_2\dot{e}_2 + \frac{Ak_vw\tau J^2}{k_{sp}V_1\sqrt{1+\alpha^3}H} e_4\dot{e}_4 + \frac{K_{p1}A_1^2J^2}{14V_1H} (e_1 - 14e_5)(\dot{e}_1 - 14\dot{e}_5) \\
& + \frac{1}{2} \left[e_3 - \frac{2K_{p1}A_1J}{H} e_5 \right] \left[\dot{e}_3 - \frac{2K_{p1}A_1J}{H} \dot{e}_5 \right] \\
& + \frac{(-13K_{p1}A_1^2K_{p3} + K_{p1}A_1^2k_h)HJ^2 - 2K_{p1}^2V_1K_{p3}A_1^2J^2}{2V_1K_{p3}H^2} e_5\dot{e}_5 \\
& + \frac{K_{p1}m_mA_1^2J^2}{2V_1K_{p3}H} e_6\dot{e}_6
\end{aligned} \tag{5.74}$$

Replacing $\dot{e}_{i(i=1..6)}$ with right-hand sides of (5.44):

$$\begin{aligned}
\dot{V} = & \frac{13K_{p1}A_1^2J^2}{14V_1H}e_1e_2 + \frac{k_sJ}{V_1}e_1e_2 + \frac{K_{p1}V_1}{2H}\left[-\frac{A_1J}{V_1}e_2e_3 + \frac{Jk_vw}{V_1\sqrt{1+\alpha^3}}e_3e_4(\sqrt{\Delta})\right] \\
& + \frac{K_{p2}A_1J}{2H}\left[-\frac{A_1J}{V_1}e_2e_3 + \frac{Jk_vw}{V_1\sqrt{1+\alpha^3}}e_3e_4(\sqrt{\Delta})\right] \\
& + \frac{K_{p1}A_1J}{H}\left[-\frac{A_1J}{V_1}e_1e_2 + \frac{Jk_vw}{V_1\sqrt{1+\alpha^3}}e_1e_4(\sqrt{\Delta})\right] + \frac{K_{p1}A_1J}{H}e_3e_2 \\
& + \frac{m_sJ}{2V_1}e_2\left[\frac{A_1}{m_s}e_3 - \frac{d}{m_s}e_2 - \frac{k_s}{m_s}e_1\right] \\
& + \frac{Ak_vw\tau J^2}{k_{sp}V_1\sqrt{1+\alpha^3}H}\left[\frac{-1}{\tau}e_4^2 + \frac{k_{sp}}{\tau}\left[-K_{p1}e_4(e_1-e_5) - K_{p2}e_4e_3\right]\sqrt{\Delta}\right] \\
& + \frac{K_{p1}A_1^2J^2}{14V_1H}(e_1e_2 - 14e_1e_6 - 14e_5e_2 + 14^2e_5e_2) \\
& + \frac{1}{2}e_3\left[-\frac{A_1J}{V_1}e_2 + \frac{Jk_vw}{V_1\sqrt{1+\alpha^3}}e_4(\sqrt{\Delta})\right] - \frac{K_{p1}A_1J}{H}e_3e_6 \\
& - \frac{K_{p1}A_1J}{H}e_5\left[-\frac{A_1J}{V_1}e_2 + \frac{Jk_vw}{V_1\sqrt{1+\alpha^3}}e_4(\sqrt{\Delta})\right] + \frac{2K_{p1}^2A_1^2J^2}{H^2}e_5e_6 \\
& + \frac{(-13K_{p1}A_1^2K_{p3} + K_{p1}A_1^2k_h)HJ^2 - 2K_{p1}^2V_1K_{p3}A_1^2J^2}{V_1K_{p3}H^2}e_5e_6 \\
& + \frac{K_{p1}m_mA_1^2J^2}{m_mV_1K_{p3}H}e_6\left[K_{p3}(e_1-e_5) + \frac{V_1K_{p3}}{JA_1}e_3 - k_de_6 - k_he_5\right]
\end{aligned} \tag{5.75}$$

Now, arranging terms and multiplying brackets, we have:

$$\begin{aligned}
\dot{V} = & e_1 e_2 \left[\frac{13K_{p1}A_1^2J^2}{14V_1H} + \frac{k_sJ}{V_1} - \frac{K_{p1}A_1^2J^2}{V_1H} - \frac{m_sJk_s}{V_1m_s} + \frac{K_{p1}A_1^2J^2}{14V_1H} \right] \\
& + e_2 e_3 \left[-\frac{K_{p1}V_1A_1J}{2HV_1} - \frac{K_{p2}A_1^2J^2}{2V_1H} + \frac{K_{p1}A_1J}{H} + \frac{m_sJA_1}{V_1m_s} - \frac{A_1J}{2V_1} \right] \\
& + e_3 e_4 (\sqrt{\Delta}) \left[\frac{K_{p1}V_1Jk_vw}{2HV_1\sqrt{1+\alpha^3}} + \frac{K_{p2}A_1J^2k_vw}{2HV_1\sqrt{1+\alpha^3}} - \frac{AK_vw\tau J^2k_{sp}}{\tau k_{sp}V_1\sqrt{1+\alpha^3}H} \right. \\
& \left. + \frac{Jk_vw}{2V_1\sqrt{1+\alpha^3}} \right] + e_1 e_4 (\sqrt{\Delta}) \left[\frac{K_{p2}A_1J^2k_vw}{HV_1\sqrt{1+\alpha^3}} - \frac{AK_vw\tau J^2k_{sp}K_{p1}}{\tau k_{sp}V_1\sqrt{1+\alpha^3}H} \right] \\
& + e_2^2 \left[-\frac{m_sJd}{V_1m_s} \right] + e_4^2 \left[-\frac{Ak_vw\tau J^2}{k_{sp}V_1\sqrt{1+\alpha^3}H} \right] \\
& + e_4 e_5 (\sqrt{\Delta}) \left[-\frac{Ak_vw\tau J^2}{k_{sp}V_1\sqrt{1+\alpha^3}H} - \frac{K_{p1}A_1J^2k_vw}{HV_1\sqrt{1+\alpha^3}} \right] \\
& + e_1 e_6 \left[-\frac{14K_{p1}A_1^2J^2}{14V_1H} + \frac{K_{p1}m_mA_1^2J^2K_{p3}}{m_mV_1K_{p3}H} \right] \\
& + e_3 e_6 \left[-\frac{K_{p1}A_1J}{H} + \frac{K_{p1}m_mA_1^2J^2V_1K_{p3}}{m_mV_1K_{p3}HJA_1} \right] \\
& + e_2 e_5 \left[-\frac{K_{p1}A_1^2J^2}{V_1H} + \frac{K_{p1}A_1^2J^2}{V_1H} \right] \\
& + e_5 e_6 \left[\frac{14K_{p1}A_1^2J^2}{V_1H} - \frac{K_{p1}m_mA_1^2J^2K_{p3}}{m_mV_1K_{p3}H} - \frac{K_{p1}m_mA_1^2J^2K_h}{m_mV_1K_{p3}H} + \frac{2K_{p1}^2A_1^2J^2}{H^2} \right. \\
& \left. - \frac{13K_{p1}A_1^2J^2HK_{p3}}{V_1K_{p3}H^2} + \frac{K_{p1}HA_1^2J^2K_h}{V_1K_{p3}H^2} - \frac{2K_{p1}^2A_1^2J^2V_1K_{p3}}{V_1K_{p3}H} \right] \\
& + e_6^2 \left[-\frac{K_{p1}m_mA_1^2J^2k_d}{m_mV_1K_{p3}H} \right]
\end{aligned} \tag{5.76}$$

For bracket pertaining to $e_1 e_2$:

$$\left[\frac{13K_{p1}A_1^2 J^2 + 14Hk_s J - 14K_{p1}A_1^2 J^2 - 14Hk_s J + K_{p1}A_1^2 J^2}{14V_1 H} \right] = 0 \quad (5.77)$$

Similarly, for bracket pertaining to:

$e_2 e_3$; $e_3 e_4(\sqrt{\Delta})$; $e_1 e_4(\sqrt{\Delta})$; $e_4 e_5(\sqrt{\Delta})$; $e_1 e_6$; $e_3 e_6$; $e_2 e_5$; $e_5 e_6$: are all zeros when solving them.

Thus, the remaining terms in $\dot{V}$ are as follows:

$$\begin{aligned} \dot{V} = e_2^2 \left[-\frac{Jd}{V_1} \right] + e_4^2 \left[-\frac{Ak_v w \tau J^2}{k_{sp} V_1 \sqrt{1 + \alpha^3} H} \right] \\ + e_6^2 \left[-\frac{K_{p1} A_1^2 J^2 k_d}{V_1 K_{p3} H} \right] \end{aligned} \quad (5.78)$$

$$\begin{aligned} \dot{V} = -e_2^2 \left[\frac{\beta \left(1 + \frac{\alpha^2}{\gamma} \right) d}{V_1} \right] - e_4^2 \left[\frac{Ak_v w \tau \beta^2 \left(1 + \frac{\alpha^2}{\gamma} \right)^2}{k_{sp} V_1 \sqrt{1 + \alpha^3} \left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma} \right) - K_{p1} V_1 \right]} \right] \\ - e_6^2 \left[\frac{K_{p1} A_1^2 \beta^2 \left(1 + \frac{\alpha^2}{\gamma} \right)^2 k_d}{V_1 K_{p3} \left[K_{p2} A_1 \beta \left(1 + \frac{\alpha^2}{\gamma} \right) - K_{p1} V_1 \right]} \right] \end{aligned} \quad (5.79)$$

As seen from Eq. (5.79), $\dot{V}$ is continuous; thus, V is a smooth Lyapunov function for the non-smooth system described by Eq. (5.44). Moreover, $\dot{V}$ is negative semi-definite. Note that all parameters are positive numbers. Therefore, the control system is stable in the sense

of Lyapunov, according to the theorem defined in [83]. The overall conditions required to prove the stability are summarized below:

$$K_{p2} > \frac{15V_1K_{p1}}{13A_1\beta \left(1 + \frac{\alpha^2}{\gamma}\right)} \quad (5.80)$$

$$K_{p3} < \frac{k_h H}{13H + 2K_{p1}V_1} \quad (5.81)$$

5.4 Results

5.4.1 Simulation results

Simulation studies were conducted to verify the developed system model, prove the stability of the system and confirm the tracking ability of the control laws. System parameters used in simulations are given in Table 3-2 in Chapter 3. Values of the hydraulic function parameters were obtained directly from manufacturer's specifications sheet or by experimental measurement/verification to resemble the actual test rig on which all the experiments were performed. The parameters used at the master side, k_h , k_d and m_m were chosen to be similar to the ones given in reference [66, 87, 88].

Simulations were generated in C++ using the fourth-order Runge-Kutta integration method. Eq. (3.52) was used as the system's equations and Eqs. (5.3) and (5.4) were used for the purpose of control. Gains were appropriately tuned to ensure a good tracking performance of the system. The controller gains were chosen as $K_{p1} = 0.1$, $K_{p2} = 4.6 \times 10^{-12}$ and $K_{p3} = 0.38$. The controller gains were chosen based on the following procedure. First, a value for K_{p1} in Eq. (5.3) is chosen that is high enough to produce a fast response without saturating the control signal or causing unacceptable overshoots. Values

of gains K_{p2} and K_{p3} are governed by inequalities (5.80) and (5.81), respectively; they ensure stability of the system. Next, K_{p2} was chosen to be the smallest value satisfying (5.80). This makes position error between the master and slave smallest, for given K_{p1} . Finally, K_{p3} was chosen to be a large value satisfying (5.81).

In the first simulation study, the human input force F_h , was set to a constant value of $0.4N$, and initial conditions for all states were chosen as zero. The aim of this simulation was to show that the system can reach an equilibrium point given a constant input in free motion *i.e.* $k_s = 0kN/m$. From Eq. (5.35), the equilibrium point of the system was determined to be:

$$\begin{aligned}\vec{x}_{eq} &= [x_{1eq} \quad x_{2eq} \quad x_{3eq} \quad x_{4eq} \quad x_{5eq} \quad x_{6eq}]^T \\ &= [40.0 \text{ mm} \quad 0.0 \text{ m/s} \quad 0.0 \text{ Pa} \quad 0.0 \text{ mm} \quad 40.0 \text{ mm} \quad 0.0 \text{ m/s}]^T\end{aligned}\tag{5.82}$$

where $\vec{x} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T = [x_s \quad \dot{x}_s \quad P_L \quad x_{sp} \quad x_m \quad \dot{x}_m]^T$

Figure 5.2 show results. As is seen, all states reached their equilibrium points.

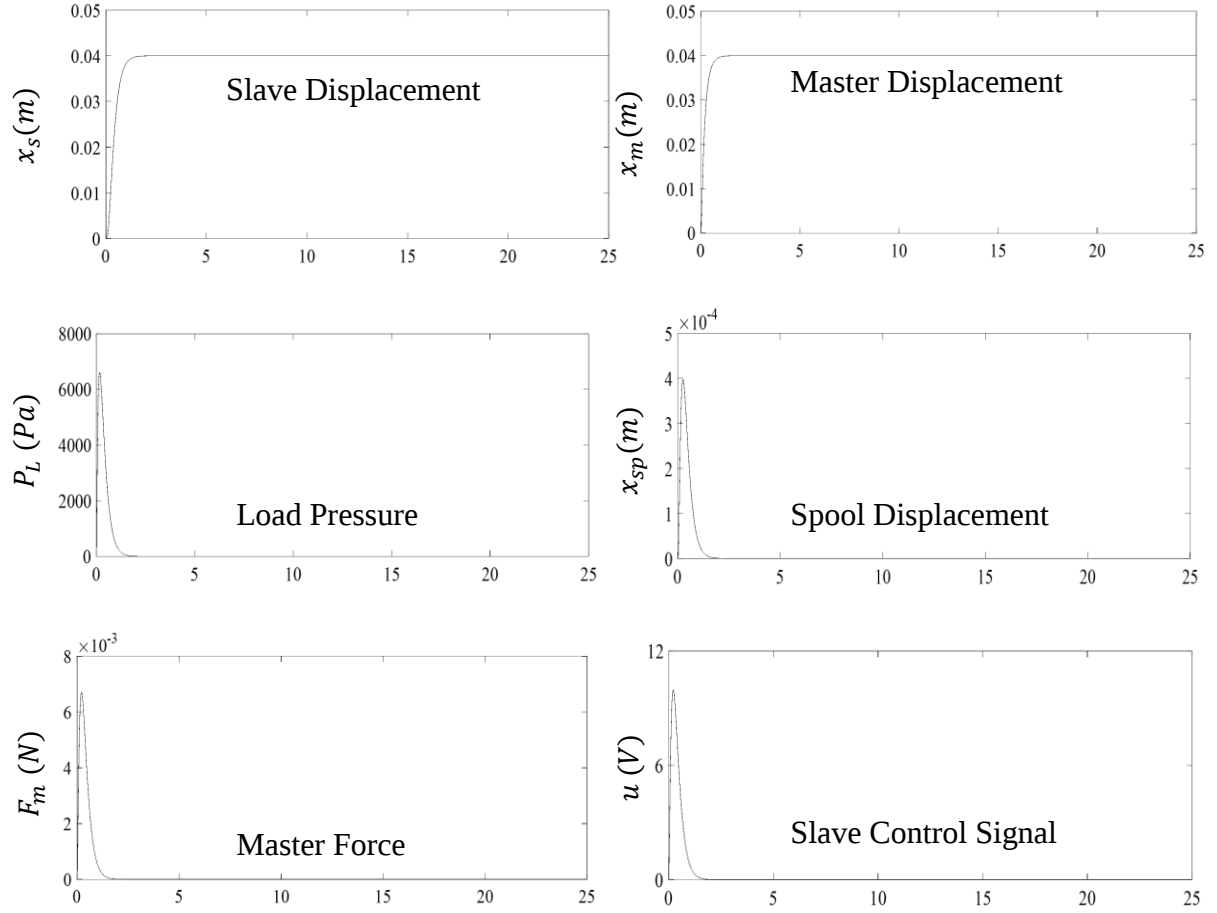


Figure 5.2 Simulation results given a constant human input ($F_h = 0.4N$).

The same simulation test was repeated, but this time, the hydraulic actuator interacted with a spring having stiffness of $k_s = 17kN/m$. The goal was to demonstrate that the proposed system can reach an equilibrium point while interacting with an environment exhibiting stiffness. The equilibrium point of the system was determined to be:

$$\begin{aligned} \vec{x}_{eq} &= [x_{1eq} \quad x_{2eq} \quad x_{3eq} \quad x_{4eq} \quad x_{5eq} \quad x_{6eq}]^T \\ &= [39.9 \text{ mm} \quad 0.0 \text{ m/s} \quad 0.21 \text{ MPa} \quad 0.0 \text{ mm} \quad 40.0 \text{ mm} \quad 0.0 \text{ m/s}]^T \end{aligned} \quad (5.83)$$

Figure 5.3 show results of this simulation. As is seen, all states reached their equilibrium points.

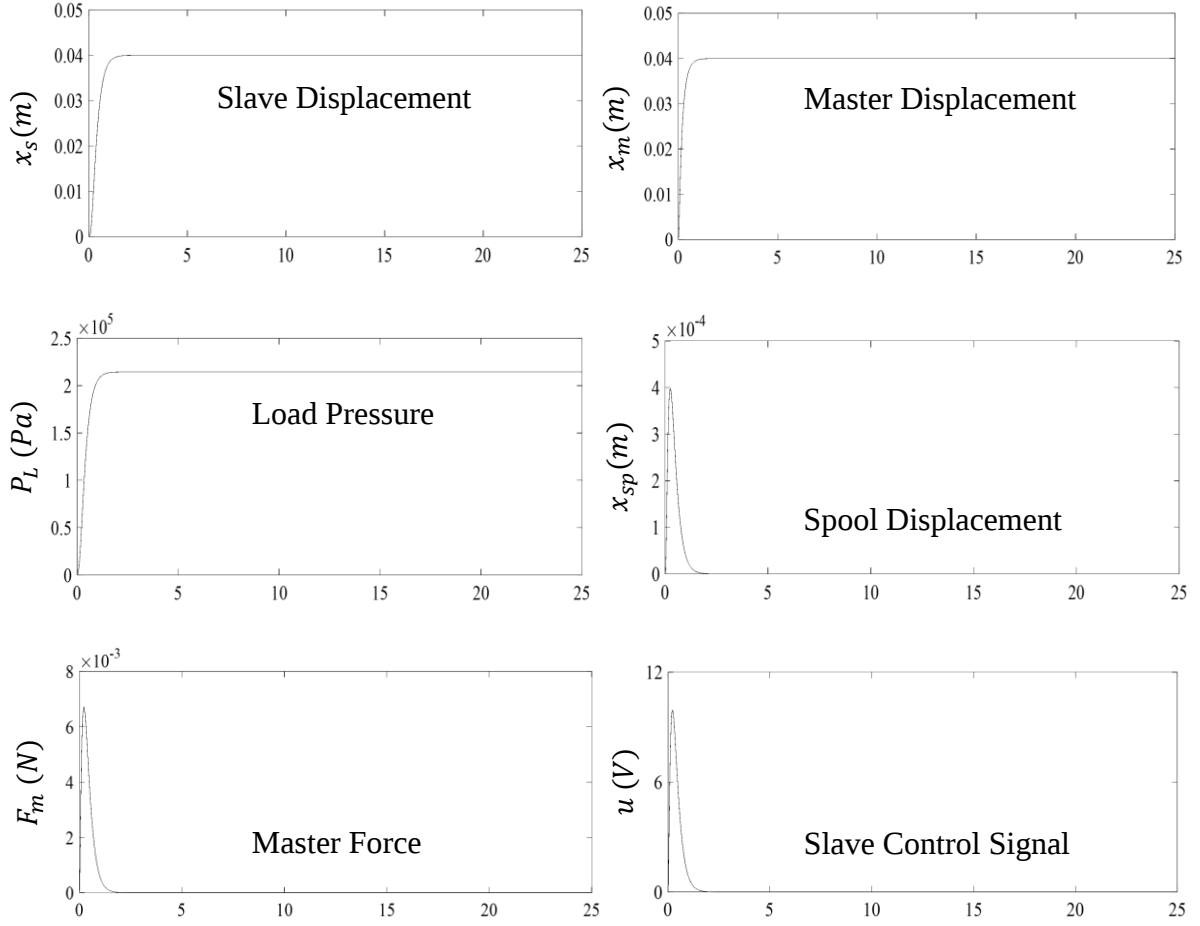


Figure 5.3 Simulation results given a constant human input ($F_h = 0.4N$). The hydraulic actuator is pushing against a spring having stiffness $k_s = 17kN/m$.

The second set of simulation studies was designed to observe the tracking ability of the control laws resembling the actual experiments to be performed later. The human input was set to be a sinusoidal force with an amplitude of $0.4N$ and frequency of $0.13Hz$. The same controller gains as in the previous test were used. With reference to Figure 5.4, the hydraulic actuator started in free motion *i.e.* $k_s = 0kN/m$ and then came in contact with a spring having stiffness of $k_s = 17kN/m$ at $20mm$. As is seen, displacements of the haptic

and the hydraulic actuator are almost the same, which validates the excellent tracking performance of the control system.

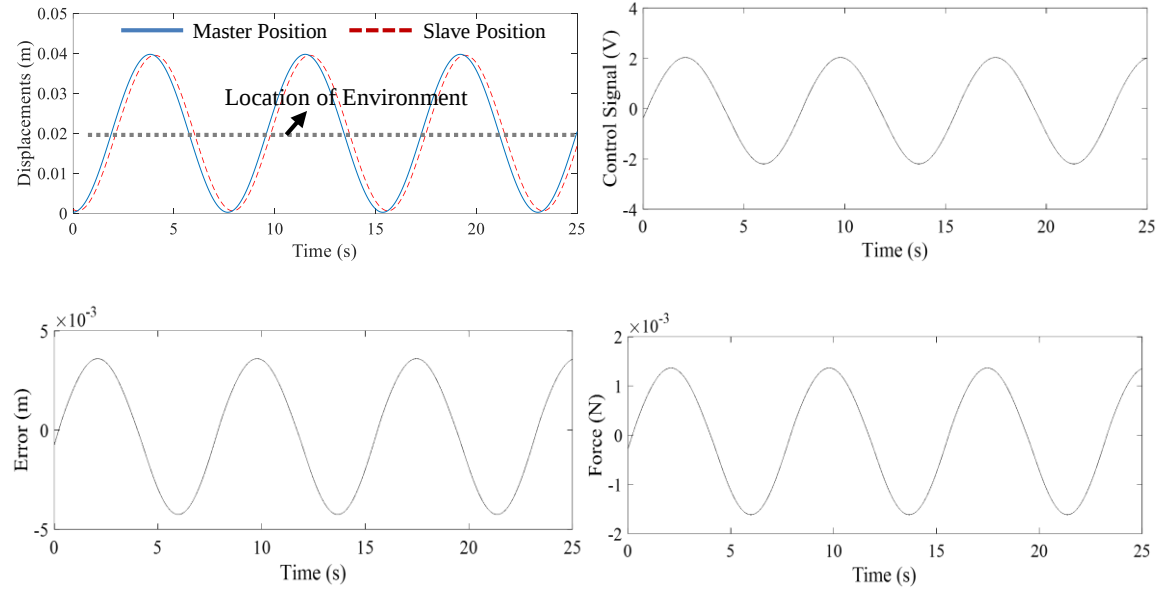


Figure 5.4 Simulation results given a sinusoidal human input. Hydraulic actuator starts in free motion and makes contact with a spring having stiffness of $k_s = 17kN/m$ at $20mm$.

5.4.2 Experimental results

For conducting the experiments, one link (shoulder) of the available teleoperated hydraulic manipulator (see Figure 5.5) was developed to be bilaterally controlled by a haptic device, while all the other actuators were locked in place. As shown in Figure 5.5, experiments were performed on the swivel motion control of the manipulator arm. As there is a nonlinear transformation between the joint swivel angle θ and the cylinder position x_s , the equivalent mass of swivel inertia depends on the swivel angle as well as the cylinder position. However, by limiting the movement of the swivel cylinder in its middle range, this equivalent mass will not vary much and can be treated as constant [60]. The load

pressure, P_L and the displacement of the hydraulic actuator, x_s were sent from the slave side to the master. These were used at the master side to compute the master force, F_m using Eq. (5.4). The displacement of the haptic device, x_m was sent from the master side to the slave to determine the control signal of the hydraulic actuator, u using Eq. (5.3). Other variables needed by the controllers were pump pressure, P_s , and differential pressures, P_L , which were easily obtained via online measurements. The master haptic device was connected to a PC using parallel port, while the slave manipulator was connected to the same PC with data acquisition boards. Data acquisition boards were used to send control signals to the servo-valves and read the manipulator's joint angle encoders. Since the haptic device and the hydraulic manipulator were connected to the same PC, there was no noticeable communication delay. The control loop worked at 500 Hz (sampling time of 2 ms). In all experimental studies, the same controller gains were used as in the simulations.

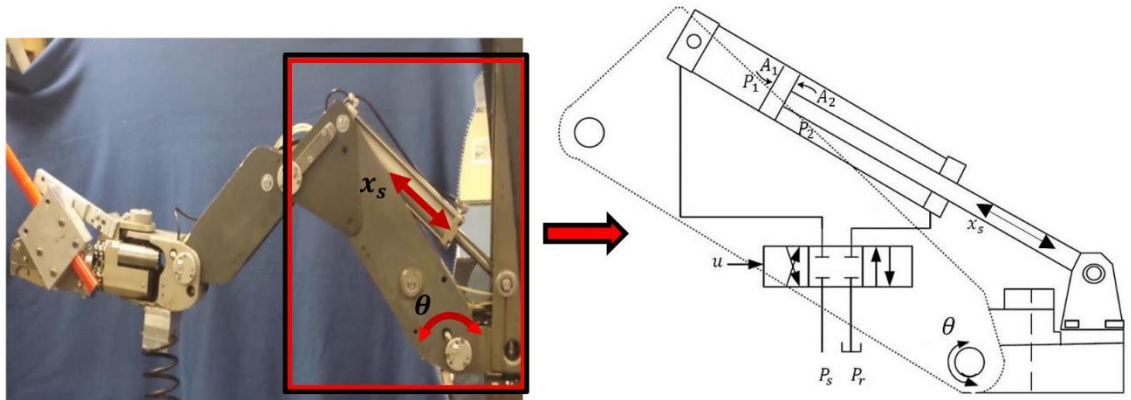


Figure 5.5 Shoulder link of the manipulator used for experiments.

Three different experiments were created to test the designed controller during free motion and when interacting with the environment. Experimental results are shown in Figure 5.6 to Figure 5.10. With reference to Figure 5.6a, hydraulic actuator moved in free

motion and the operator applied a command to the haptic device similar to a step displacement. The displacement tracking error between the haptic device and the hydraulic actuator is around $\pm 5mm$. In the next set of experiments, the operator moved the haptic device back and forth and the hydraulic actuator moved in free motion. The results are shown in Figure 5.6b. The displacement tracking error between the haptic device and the hydraulic actuator in this set of experiment is around $\pm 10mm$. Another point to note is that the use of discontinuous control strategy, creates some chattering as seen in the zoomed-in plot in Figure 5.6a.

The proposed controller makes the master dynamics a better match with the dynamics of the hydraulic actuator. As seen in Figure 5.6, the operator feels additional force on his hands whenever there is a quick change in the direction of motion alerting the operator to slow down the hand motion allowing the slave manipulator to catch up as position error gets apparent. The entire control law essentially assisted the operator to feel as if he is actually moving the hydraulic actuator, providing a sense of telepresence.

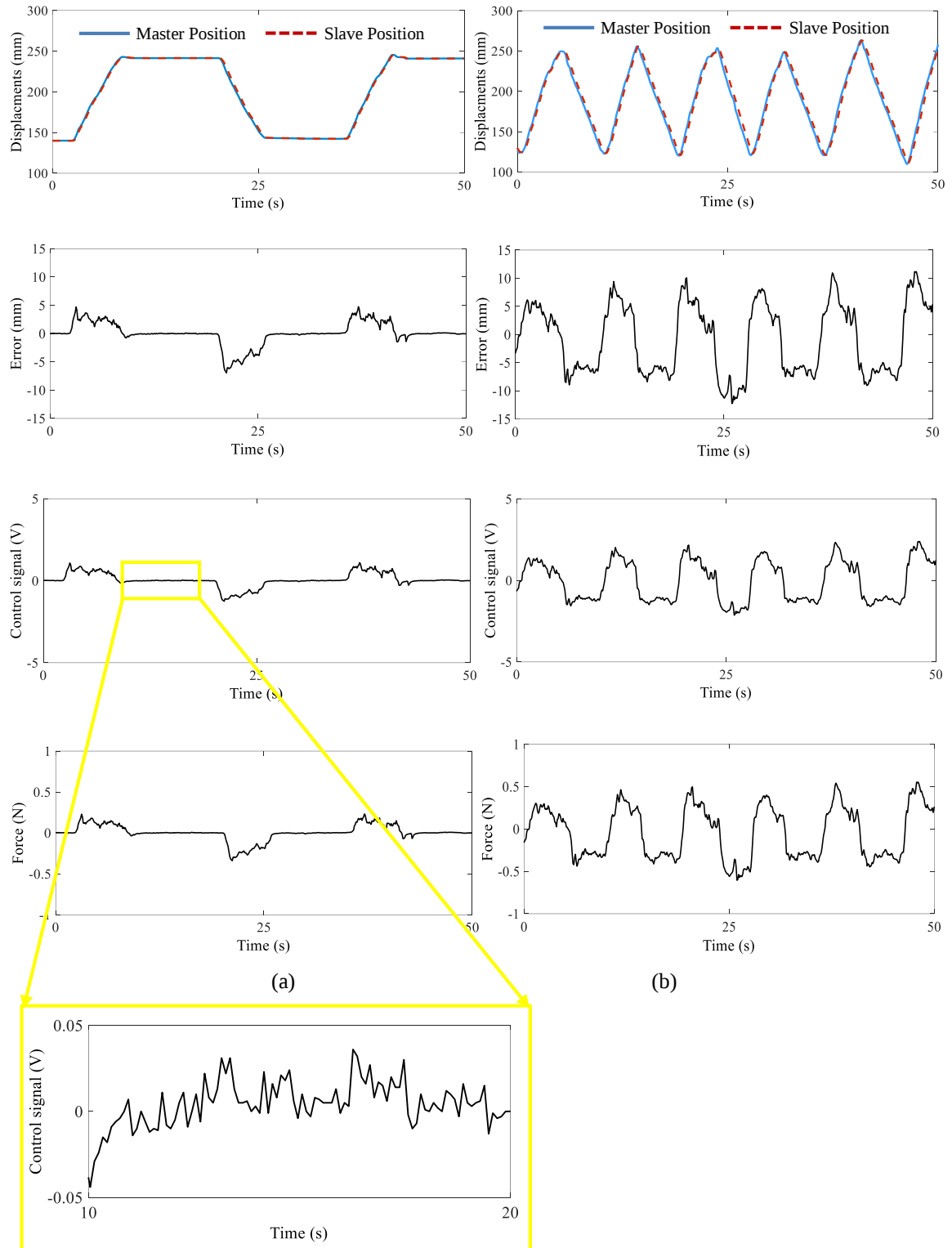


Figure 5.6 Experimental results of hydraulic actuator moving in free motion for (a) step-like master input, (b) sinusoid-like master input.

Figure 5.8 show results of the next experiment in which the hydraulic actuator starts in free motion and comes in contact with a spring having stiffness of $k_s = 17 \text{ kN/m}$, at $x_s \approx 200\text{mm}$. The spring used in the experiments could not travel more than 50mm (see Figure 5.7). Thus, during the experiments, the operator was restricted not to push beyond this limit.

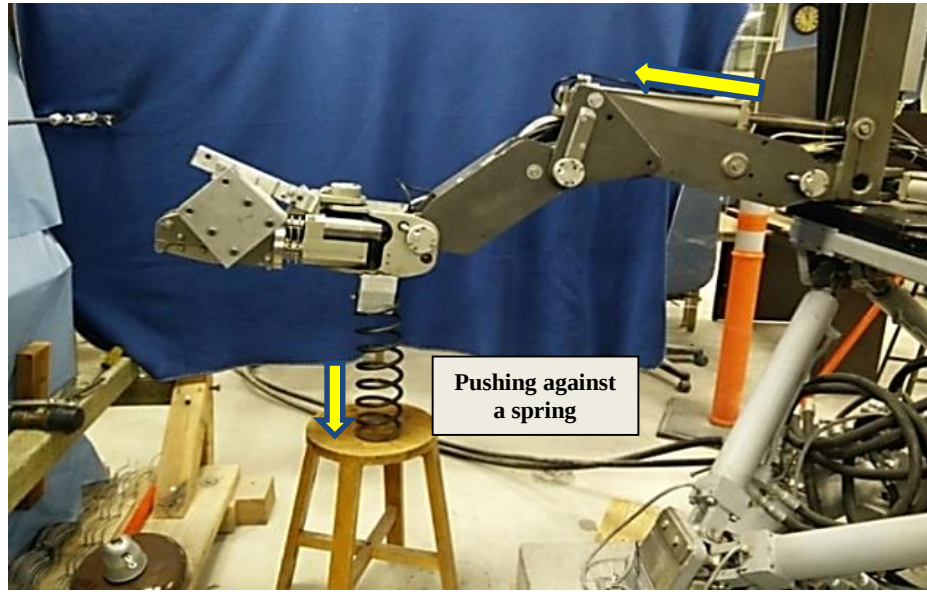


Figure 5.7 Hydraulic actuator starts in free motion and pushes against a spring having stiffness of $k_s = 17 \text{ kN/m}$ at $x_s \approx 200\text{mm}$.

With reference to Figure 5.8a, the operator applied a command to the haptic device similar to a step displacement, while in Figure 5.8b, the operator moves the haptic device back and forth.

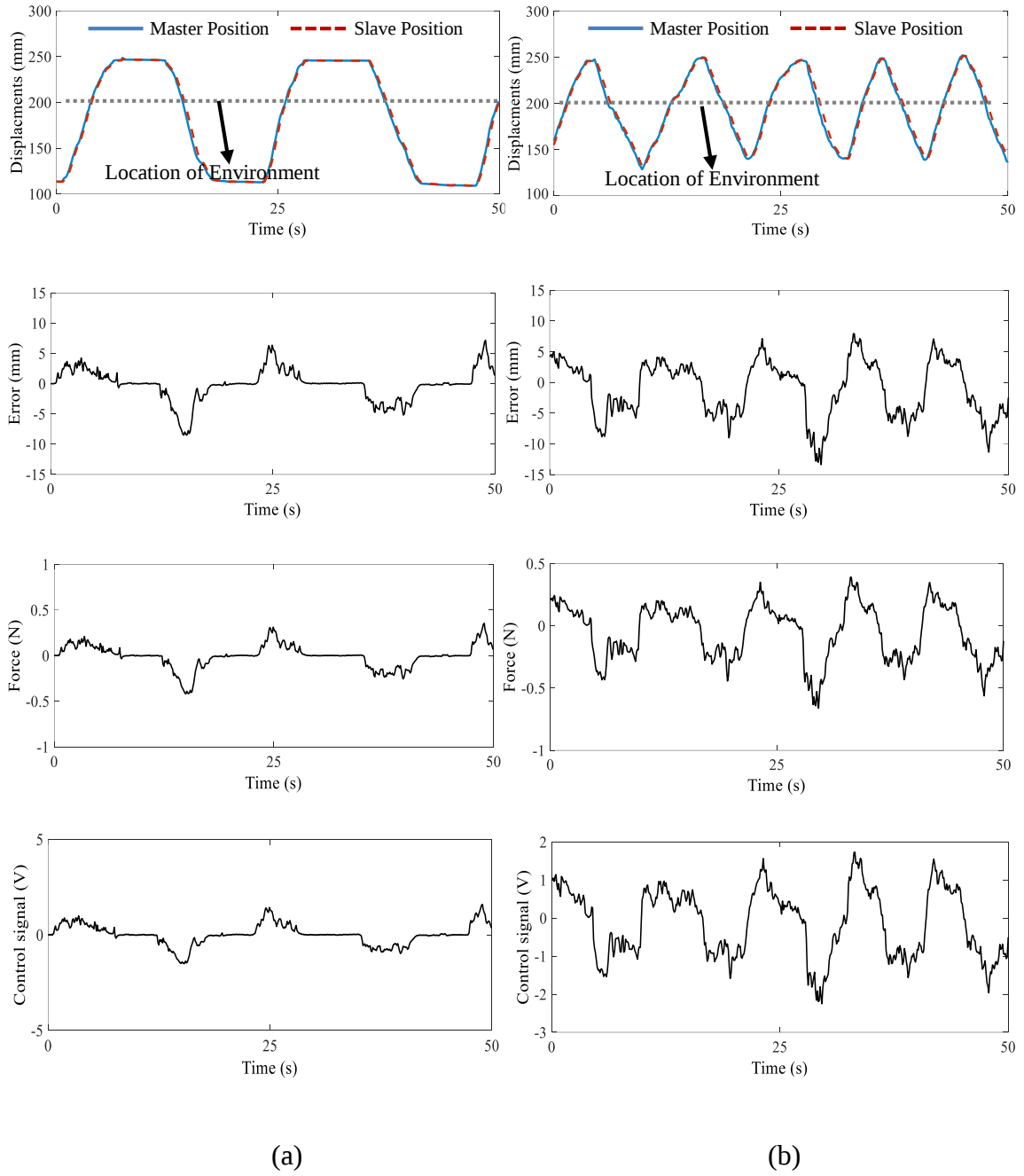


Figure 5.8 Experimental results of hydraulic actuator moving in free motion and making contact with a spring having stiffness of $k_s = 17 \text{ kN/m}$ for (a) step-like master input, (b) sinusoid-like master input.

The next two experiments are conducted to investigate the effect of interacting with the real environment of unknown stiffness. Figure 5.9 shows the test setup whereby the hydraulic actuator is pushing against a live line conductor wire having very high stiffness with a hot-stick. Figure 5.10 show results of experiment in which the hydraulic actuator starts in free motion and comes in contact with a conductor wire, at $x_s \approx 200mm$. With reference to Figure 5.10a, the operator applied a command to the haptic device similar to a step displacement, while in Figure 5.10b, the operator moves the haptic device back and forth.

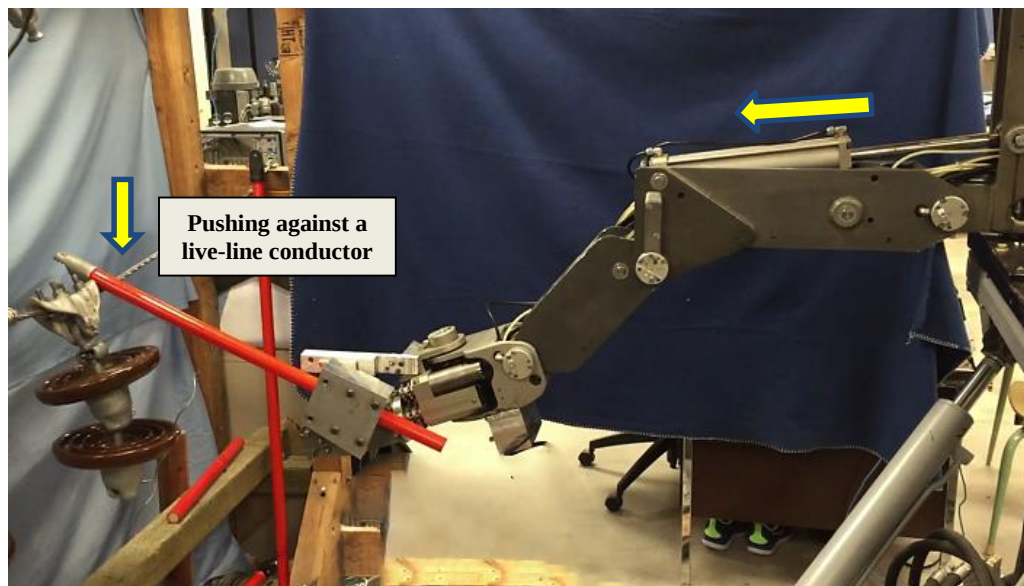


Figure 5.9 Hydraulic actuator starts in free motion and makes contact with a live-line conductor wire at $x_s \approx 200mm$.

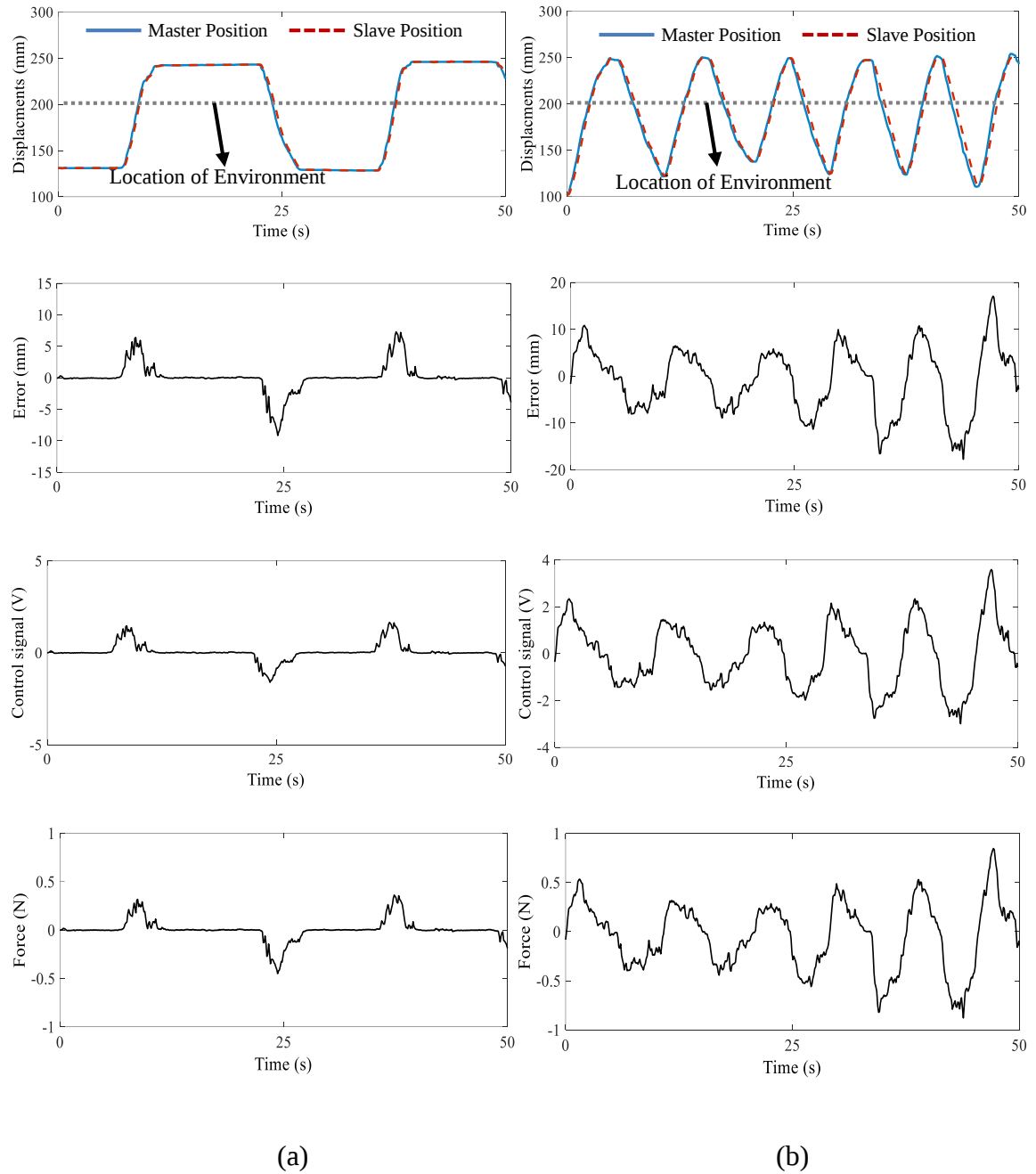


Figure 5.10 Experimental results of hydraulic actuator moving in free motion and making contact with a live-line conductor wire for (a) step-like master input, (b) sinusoid-like master input.

In all the experiments, the control signal is within range and not saturated. As evident through all the results, the system is stable and exhibits good tracking response while providing haptic force feedback to the operator. The experimental results confirm that even though actuator dry friction was not explicitly considered as part of the controllers' design, the experimental system performed well.

5.5 Summary

This chapter presented the design and implementation of a theoretically sound and practically simple to implement bilateral controller that allows teleoperation of single-rod hydraulic actuators. The controller works for both free and constrained motions and is based on Lyapunov's stability technique. Stability of the proposed control scheme considering nonlinear hydraulic functions, and combined dynamics of the operator-haptic, was analytically proven. Owing to the discontinuity in the control laws, the resulting control system was non-smooth. Thus, the existence, continuation and uniqueness of the solution were first proven using Filippov's solution theories. The extended Lyapunov's stability theory was used next for the stability analysis of the subsequent control system. Simulation results were provided, which confirmed that the controller could effectively stabilize the system. As far as the validation is concerned, experimental results confirmed that the controller could effectively stabilize the system while having good position tracking by the hydraulic actuator, and providing a feel of performing tasks at the remote site to the operator.

The controller was based on position information of master and slave devices. This controller is the best choice in applications in which installing a force sensor on the end-

effector of the hydraulic manipulator is not practical. The results of this chapter contribute to enhancing the operator's ability to carry out stable haptic-enabled teleoperation of hydraulic robots in hazardous (nuclear plants or underwater) environments.

In the next chapter, this control scheme has been modified for base excited single-rod hydraulic actuators and the system stability is proven to be insensitive to the uncertainties of the physical parameters and of the measurement of the base point motion.

CHAPTER 6

6 BILATERAL CONTROL OF SINGLE-ROD HYDRAULIC ACTUATORS SUBJECTED TO BASE DISTURBANCE³

6.1 Introduction

Teleoperation of manipulators with moving base has many promising applications such as live transmission line maintenance, underwater inspection, flight simulators, planets exploration, navigation in hazardous environments, and inspection of industrial constructions [47]. When the base motion is undesirable, and originates from disturbing forces and torques, the system is called base-excited system [48]. In these hydraulic machines, the base of the manipulator is attached to a non-inertial coordinate system.

³ A version of this chapter has been published in V. Bantia, A. Maddahi, K. Zareinia, S. Balakrishnan and N. Sepehri, "A Lyapunov Stable Controller for Bilateral Haptic Teleoperation of Single-Rod Hydraulic Actuators Subjected to Base Disturbance," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 141, no. 3, pp. 031013 - 031033, 2019 [89].

This base disturbance would affect the control quality and stability due to un-modelled dynamics. Therefore, the effect of base disturbance should be included in the design of controller.

In this chapter, a control scheme is developed and evaluated for stable bilateral haptic teleoperation of a single-rod hydraulic actuator subjected to base disturbance. The proposed controller is built upon Lyapunov stability technique and is capable of reducing position errors at master and slave sides and provides a feel of the contact force between the actuator and the task environment to the operator without a need for direct measurement. The controller requires only the measurements of the actuator line pressures and displacements of the master and slave. The system stability is insensitive to the uncertainties of the physical parameters and of the measurement of the base point motion. Stability of the proposed controller incorporating hydraulic nonlinearities and operator dynamics with an estimated upper value for the base disturbance is analytically proven. Simulation studies validate that the proposed control system is stable while interacting with a task environment. Experimental results demonstrate the effectiveness of control scheme in maintaining stability, while having good position tracking by the hydraulic actuator as well as providing a haptic feel to the operator without direct measurement of interaction force, while the hydraulic actuator is subjected to base disturbance.

6.2 Dynamic equation of single-rod actuator with base disturbance

In this section, the dynamic model of the bilateral tele-manipulation system (see Figure 6.1) which consists of human operator, master haptic device, slave single-rod hydraulic actuator with base disturbance, and task environment is presented.

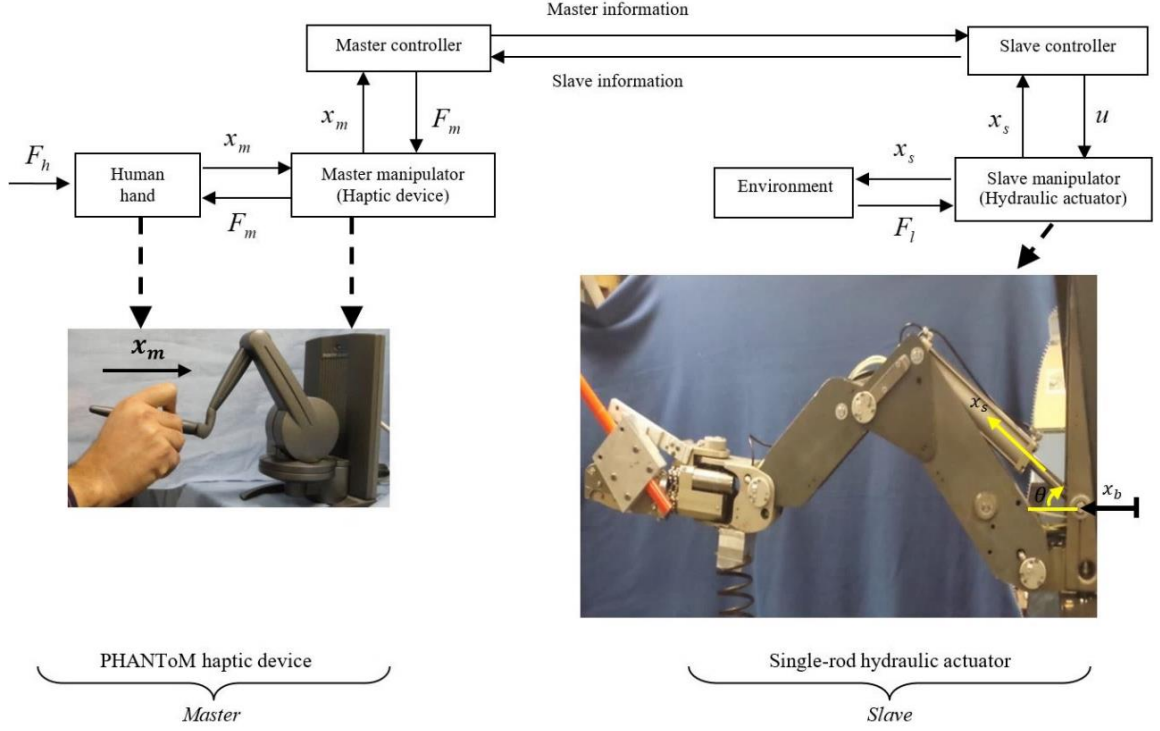


Figure 6.1 Bilateral teleoperation system.

The single-rod hydraulic actuator link studied has one degree of rotational freedom and the base point can move freely in the horizontal plane (see Figure 6.2). Considering the rotational motion of the Kodiak manipulator link, the dynamic equation of actuator with base disturbance is rewritten as follows from Eq. (3.48):

$$m_s \ddot{x}_s = A_1 P_L - d \dot{x}_s - k_s x_s - k_s x_b \cos \theta$$

$$\ddot{x}_s = \frac{A_1}{m_s} P_L - \frac{d}{m_s} \dot{x}_s - \frac{k_s}{m_s} x_s - \underbrace{\frac{k_s}{m_s} x_b \cos \theta}_{D_b} \quad (6.1)$$

where x_s , $\dot{x}_s$ and $\ddot{x}_s$ represents the displacement, velocity and acceleration of the implement, attached to the actuator, respectively; m_s is the inertia of the moving part of

the actuator; d is the equivalent viscous damping coefficient; k_s denotes the stiffness of external load applied along the actuator, while x_b represents the base motion displacement and D_b represents the base disturbance term. Taking $\theta = zx_s$ for further analysis, where z is a constant that converts the displacement of piston into the rotation of link. Note that the term of dry friction has been neglected to make the Lyapunov stability analysis manageable.

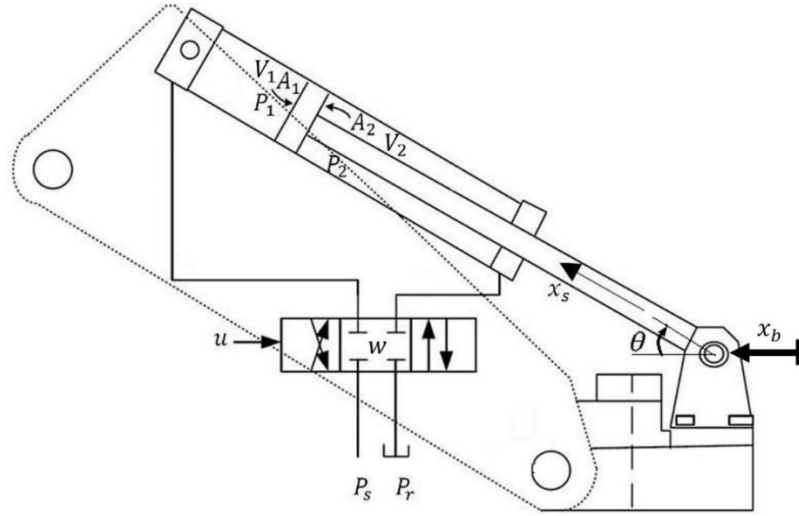


Figure 6.2 Schematic of valve-controlled single-rod hydraulic actuator.

The state space variables equations are as follows:

$$\vec{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [x_s \ \dot{x}_s \ P_L \ x_{sp} \ x_m \ \dot{x}_m]^T$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{A_1}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 - \frac{k_s}{m_s} x_b \cos(\alpha x_1) \\ \dot{x}_3 = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} x_4 (\sqrt{\Delta}) \\ \dot{x}_4 = \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} u \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{1}{m_m} (F_h + F_m - k_d x_6 - k_h x_5) \end{cases} \quad (6.2)$$

Terms u and F_m are the control laws to be designed in Section 6.3.

6.3 Development of controller

In this section, the model described in Eq. (6.2) is employed to design a stable controller. The aim is to design a Lyapunov control scheme for both free and constrained motions of the hydraulic actuator. The control laws are constructed during the non-straightforward process of constructing a proper Lyapunov function for the system. The control laws are first proposed and then the stability is analyzed using the generalized Lyapunov stability theorem. The proposed control laws are:

$$u = [-K_{p1}(x_s - x_m) - K_{p2}P_L]\sqrt{\Delta} + C \quad (6.3)$$

$$F_m = K_{p3}(x_s - x_m) + (K_{p3} \frac{V_1}{JA_1} + B')P_L + Bx_s \quad (6.4)$$

where $\Delta = (1 + \alpha) \frac{P_s}{2} + \text{sgn}(x_{sp}) \left((1 - \alpha) \frac{P_s}{2} - P_L \right)$ and C is the compensation term added to the control signal to overcome the effect of base disturbance and still ensure system stability. In Eq. (6.3), $C = \frac{Q}{c_s} \hat{D}_b$, and $\hat{D}_b$ is referred to as the estimated upper value

of the base disturbance term. The reason for using an estimated upper value for D_b in the control signal has been discussed in Section 6.4.2. Other terms and relevant parameters are defined as follows:

$$\begin{aligned}
C_5 &= \frac{A_1 P J^2}{V_1 H} - \frac{Q P k_{sp} \sqrt{\Delta}}{\tau A_1} + \frac{A_1 Q}{m_s K_{p2} \sqrt{\Delta}}; \quad Q = \frac{-B_Q + \sqrt{(B_Q^2 - 4A_Q C_Q)}}{2A_Q}; \\
J &= \beta \left(1 + \frac{\alpha^2}{\gamma}\right); \quad H = K_{p2} A_1 J - K_{p1} V_1; \quad P = \frac{k_v w}{\sqrt{(1+\alpha^3)}}; \\
A_Q &= (1 - ML); \quad B_Q = \frac{-M J m_s}{V_1} - LS; \quad C_Q = \frac{-J m_s S}{V_1}; \\
M &= \frac{-P \sqrt{\Delta}}{A_1} + \frac{A_1 \tau}{m_s k_{sp} K_{p2} \sqrt{\Delta}}; \quad L = \frac{K_{p1} k_s V_1 \sqrt{\Delta} m_s}{A_1^2 \tau J} - \frac{V_1 K_{p1}}{J P K_{p2} \sqrt{\Delta}}; \quad S = \frac{A_1 P \tau J^2}{k_{sp} V_1 H}; \\
B &= \frac{K_{p1} k_{sp} \sqrt{\Delta} Q}{Y \tau} + \frac{14T}{Y}; \quad B' = \frac{-Z}{2Y}; \quad T = \frac{-Q K_{p1} k_{sp} \sqrt{\Delta}}{14\tau} + \frac{A_1^2 Q K_{p1}}{14 m_s P K_{p2} \sqrt{\Delta}}; \\
Y &= \frac{K_{p1} A_1^2 J^2}{V_1 K_{p3} H}; \quad Z = \frac{2V_1 Q K_{p1} k_{sp} \sqrt{\Delta}}{A_1 \tau J^2} - \frac{2A_1 Q K_{p1} V_1}{m_s J P K_{p2} \sqrt{\Delta}};
\end{aligned} \tag{6.5}$$

Note that the term C , in Eq. (6.3), must be measurable. To meet this condition, we need to prove that C_5 , in Eq. (6.5), is measurable and a positive value. The necessary condition for C_5 to be measurable is that the expression H (in the first term of C_5) is positive and can never be zero.

From Eq. (6.5),

$$C_5 = \frac{A_1 P J^2}{V_1 H} - \frac{Q P k_{sp} \sqrt{\Delta}}{\tau A_1} + \frac{A_1 Q}{m_s K_{p2} \sqrt{\Delta}}$$

and,

$$H = K_{p2}A_1J - K_{p1}V_1$$

Thus,

$$H = K_{p2}A_1J - K_{p1}V_1 > 0$$

or,

$$K_{p2} > \frac{K_{p1}V_1}{A_1J} \quad (6.6)$$

Also,

$$C_5 = \frac{A_1PJ^2}{V_1H} - \frac{QPk_{sp}\sqrt{\Delta}}{\tau A_1} + \frac{A_1Q}{m_s K_{p2}\sqrt{\Delta}} > 0 \quad (6.7)$$

Thus, the chosen value of K_{p2} must ensure that C_5 is always a measurable and positive number.

In Eqs. (6.3) and (6.4), K_{p1} , K_{p2} , and K_{p3} are real positive gains. Note that the control laws are discontinuous due to term $sgn(x_{sp})$. The entire control law helps the operator to feel a level of telepresence. The first term in Eq. (6.4) basically generates a feedback to the human operator proportional to the position error between master and slave manipulators. This force feedback notifies the human operator about the position error between master and slave implements and constrains the operator's hand motions when the slave manipulator is behind/ahead in tracking master manipulator's displacement. The second term relates to the load pressure, P_L , of the hydraulic actuator and is added for stabilizing the overall control system. Similar terms are used for defining the control signal u .

Replacing u and F_m in Eq. (6.2) with Eqs. (6.3) and (6.4) in terms of state space variables, we get:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{A_1}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 - \frac{k_s}{m_s} x_b \cos(zx_1) \\ \dot{x}_3 = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} x_4 (\sqrt{\Delta}) \\ \dot{x}_4 = \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3]\sqrt{\Delta} + C) \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{1}{m_m} \left[F_h + K_{p3}(x_1 - x_5) + (K_{p3} \frac{V_1}{JA_1} + B')x_3 + Bx_1 - k_d x_6 - k_h x_5 \right] \end{cases} \quad (6.8)$$

where, $\Delta = (1 + \alpha) \frac{P_s}{2} + \text{sgn}(x_4) \left((1 - \alpha) \frac{P_s}{2} - x_3 \right)$

The control system described above is non-smooth due to the term $\text{sgn}(x_4)$. As per conventional solution theories, a solution cannot even be defined for such systems. Thus, Filippov's solution theory [80, 81], developed for analysis of non-smooth systems, is used. Based on the Filippov's solution theorems, the generalized Lyapunov stability theory is extended to non-smooth systems [83, 84, 85].

6.4 Stability analysis

Filippov's solution theorems establish definition of solution for differential equations with discontinuous terms. The existence, uniqueness, and continuity of non-smooth systems can be proven based on the Filippov's solution theorems [80, 81]. The next step is to construct a smooth Lyapunov function for the non-smooth dynamic system of Eq. (6.8).

6.4.1 Existence, uniqueness, and continuation of Filippov's solution

According to the Filippov's solution concept, the first discontinuity surface of the non-smooth system described in Eq. (6.8) is defined as:

$$S_1 := \{\vec{x}: x_1 \neq 0 \text{ and } x_4 = 0\} \quad (6.9)$$

The discontinuity surface, S_1 , divides the solution region, Ω_{S_1} , into two regions:

$$\Omega_{S_1}^+ := \{\vec{x}: x_1 \neq 0 \text{ and } x_4 > 0\} \quad (6.10)$$

$$\Omega_{S_1}^- := \{\vec{x}: x_1 \neq 0 \text{ and } x_4 < 0\} \quad (6.11)$$

The right-hand sides (RHS) of Eq. (6.8) are piecewise continuous and defined everywhere in Ω_{S_1} . The RHS of Eq. (6.8) implies the values of state derivatives ($\dot{x}_i$) in terms of the state variables (x_i), system parameters and control inputs. The state variables represent the physical parameters of the system like displacement and velocities of master and slave, load pressure, and spool valve position which are bounded. Moreover, there is no term in RHS of Eq. (6.8) comprising the state variables in the denominator; therefore, they are always measurable and bounded. Thus, Eq. (6.8) satisfies condition B of Filippov's solution theory and as per Theorems 4 and 5 of Filippov [80], the local existence and continuity of a solution are established.

Next, the uniqueness of the solution is proven. Since the right-hand sides of Eq. (6.8) are all continuous before and after the discontinuity surface, and the discontinuity surface, S_1 , is smooth, conditions A, B and C of Filippov's solution are satisfied [80, 81]. Based on the procedure described in [80], the limiting values of the vector function of the right-hand sides of Eq. (6.8), *i.e.* $f_{S_1}^+$ and $f_{S_1}^-$ when S_1 is approached from $\Omega_{S_1}^+$ and $\Omega_{S_1}^-$, are:

$$f_{S_1}^+ = \begin{cases} \frac{x_2}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 - \frac{k_s}{m_s} x_b \cos(zx_1) \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{P_s - x_3}) \\ -\frac{1}{\tau} x_4 + \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{P_s - x_3} + C) \\ \frac{x_6}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + (K_{p3} \frac{V_1}{JA_1} + B')x_3 + Bx_1 - k_d x_6 - k_h x_5\right) \end{cases} \quad (6.12)$$

$$f_{S_1}^- = \begin{cases} \frac{x_2}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 - \frac{k_s}{m_s} x_b \cos(zx_1) \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{\alpha P_s + x_3}) \\ -\frac{1}{\tau} x_4 + \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{\alpha P_s + x_3} + C) \\ \frac{x_6}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + (K_{p3} \frac{V_1}{JA_1} + B')x_3 + Bx_1 - k_d x_6 - k_h x_5\right) \end{cases} \quad (6.13)$$

The projections of $f_{S_1}^+$ and $f_{S_1}^-$ along the normal to the discontinuity surface, *i.e.*, $N_{S_1} = \{0,0,0,1,0,0\}$ are denoted by $f_{N_{S_1}}^+$ and $f_{N_{S_1}}^-$:

$$f_{N_{S_1}}^+ = \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{P_s - x_3} + C) \quad (6.14)$$

$$f_{N_{S_1}}^- = \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{\alpha P_s + x_3} + C) \quad (6.15)$$

The proof that $f_{N_{S_1}}^+$ and $f_{N_{S_1}}^-$ have the same sign has been presented below.

From Eq. (6.8):

$$\dot{x}_4 = \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3] \sqrt{\Delta} + C)$$

If it can be proven that $C = G\sqrt{\Delta}$ and G is a constant, then:

$$\dot{x}_4 = \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3 + G] \sqrt{\Delta}) \quad (6.16)$$

Plotting C vs $\sqrt{\Delta}$:

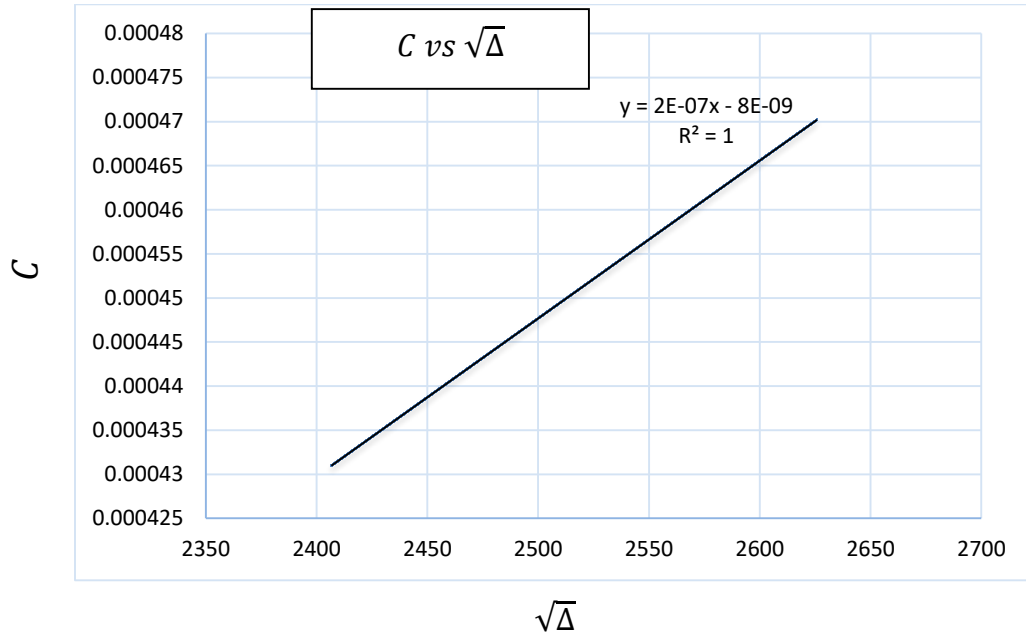


Figure 6.3 Plot of C vs $\sqrt{\Delta}$ to prove G is a constant.

From the plot above, we get $C = 2e^{-7}\sqrt{\Delta} - 8e^{-9} \approx 2e^{-7}\sqrt{\Delta}$, thus G is a constant.

The projections of $f_{S_1}^+$ and $f_{S_1}^-$ along the normal to the discontinuity surface, i.e., $N_{S_1} = \{0,0,0,1,0,0\}$ are denoted by $f_{N_{S_1}}^+$ and $f_{N_{S_1}}^-$:

$$f_{N_{S_1}}^+ = \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3 + G]\sqrt{P_s - x_3}) \quad (6.17)$$

$$f_{N_{S_1}}^- = \frac{k_{sp}}{\tau} ([-K_{p1}(x_1 - x_5) - K_{p2}x_3 + G]\sqrt{\alpha P_s + x_3}) \quad (6.18)$$

The first part of the right hand side of the equations above, i.e., $\frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2}x_3 + G]$, does not change sign while passing through the discontinuity surface S_1 , and since the terms $\sqrt{P_s - x_3}$ and $\sqrt{\alpha P_s + x_3}$ are always non-negative and have the same sign, the projections of $f_{S_1}^+$ and $f_{S_1}^-$ along the normal to the discontinuity surface S_1 , i.e., $f_{N_{S_1}}^+$ and

$f_{N_{S_1}}^-$ have the same sign. This satisfies the conditions of Lemma 9 of Filippov's solution [80].

The other discontinuity surfaces S_2 and S_3 are defined as:

$$S_2 := \{\vec{x}: x_1 = 0 \text{ and } x_4 \neq 0\} \quad (6.19)$$

$$S_3 := \{\vec{x}: x_1 = 0 \text{ and } x_4 = 0\} \quad (6.20)$$

The analysis of the discontinuity surfaces S_2 is presented below:

$$S_2 := \{\vec{x}: x_1 = 0 \text{ and } x_4 \neq 0\} \quad (6.21)$$

which divides the solution region, Ω_{S_2} , into two regions:

$$\Omega_{S_2}^+ := \{\vec{x}: x_1 > 0 \text{ and } x_4 \neq 0\} \quad (6.22)$$

$$\Omega_{S_2}^- := \{\vec{x}: x_1 < 0 \text{ and } x_4 \neq 0\} \quad (6.23)$$

$$f_{S_2}^+ = \begin{cases} x_2 \\ \frac{A_1}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 - D_b \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{\Delta}) \\ \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2} x_3] \sqrt{\Delta} \\ x_6 \\ \frac{1}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + \frac{V_1 K_{p3}}{J A_1} x_3 - k_d x_6 - k_h x_5\right) \end{cases} \quad (6.24)$$

$$f_{S_2}^- = \begin{cases} x_2 \\ \frac{A}{m_s} x_3 - \frac{d}{m_s} x_2 - \frac{k_s}{m_s} x_1 - D_b \\ -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) x_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w x_4}{\sqrt{1 + \alpha^3}} (\sqrt{\Delta}) \\ \frac{-1}{\tau} x_4 + \frac{k_{sp}}{\tau} [-K_{p1}(x_1 - x_5) - K_{p2} x_3] \sqrt{\Delta} \\ x_6 \\ \frac{1}{m_m} \left(F_h + K_{p3}(x_1 - x_5) + \frac{V_1 K_{p3}}{J A_1} x_3 - k_d x_6 - k_h x_5\right) \end{cases} \quad (6.25)$$

Projections of $f_{S_1}^+$ and $f_{S_1}^-$ along $N_{S_2} = \{1,0,0,0,0\}$ are:

$$f_{N_{S_2}}^+ = x_2 \quad (6.26)$$

$$f_{N_{S_2}}^- = x_2 \quad (6.27)$$

$f_{N_{S_2}}^+$ and $f_{N_{S_2}}^-$ have the same sign. Thus, Lemma 9 of Filippov is satisfied [80].

The surface S_3 is the intersection of S_1 and S_2 . Analysis of S_3 is also carried out in a similar way.

6.4.2 Stability proof

Extension of Lyapunov's second method applicable to non-smooth dynamic systems with no equilibrium points [81, 90] is used to prove stability. Since the hydraulic system under base excitation does not have fixed equilibrium point(s), the concept of total stability must be applied for the stability analysis [90]. In total stability, the equilibrium point(s) of undisturbed system are obtained first. If the equilibrium points are asymptotically stable, the disturbed system will be totally stable [90]. In the previous chapter, the stability of the same hydraulic manipulator with no base disturbance (*i.e.*, undisturbed system) was proven. Based on the results, the equilibrium point of the undisturbed system was asymptotically stable; therefore, the disturbed system (Eq. (6.8)) is totally stable.

In order to design the control laws for the disturbed system, a Lyapunov function must be constructed first. Based on the Lyapunov's stability theories, a Lyapunov function is always defined based on the system equations whose equilibrium point is at the origin. Therefore, the set of Eqs. (6.8) must be rewritten in terms of the vector of equilibrium point

$\vec{e}$ obtained below. By allowing $\dot{x}_i (i=1..6) = 0$, the undisturbed version of the system described by Eq. (6.8) is shown to have the following equilibrium point:

$$\vec{x}_{eq} = [x_{1ss} \quad x_{2ss} \quad x_{3ss} \quad x_{4ss} \quad x_{5ss} \quad x_{6ss}]^T \quad (6.28)$$

where

$$\begin{aligned} x_{1ss} &= \frac{K_{P1}A_1^2F_hJ}{K_{P3}K_{P2}A_1k_sJ - V_1K_{P3}K_{P1}k_s + K_{P1}A_1^2k_hJ + K_{P2}A_1k_hk_sJ} \\ x_{2ss} &= 0 \\ x_{3ss} &= \frac{K_{P1}A_1k_sF_hJ}{K_{P3}K_{P2}A_1k_sJ - V_1K_{P3}K_{P1}k_s + K_{P1}A_1^2k_hJ + K_{P2}A_1k_hk_sJ} \\ x_{4ss} &= 0 \\ x_{5ss} &= \frac{K_{P1}A_1^2F_hJ + K_{P2}A_1k_sF_hJ}{K_{P3}K_{P2}A_1k_sJ - V_1K_{P3}K_{P1}k_s + K_{P1}A_1^2k_hJ + K_{P2}A_1k_hk_sJ} \\ x_{6ss} &= 0 \end{aligned} \quad (6.29)$$

Defining $\vec{e} = \vec{x} - \vec{x}_{eq}$, we will have the vector of equilibrium point as follows:

$$\begin{aligned} \vec{e} &= [e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5 \quad e_6]^T \\ &= [(x_1 - x_{1ss}) \quad x_2 \quad (x_3 - x_{3ss}) \quad x_4 \quad (x_5 - x_{5ss}) \quad x_6]^T \end{aligned} \quad (6.30)$$

Rewriting Eq. (6.8) in terms of vector $\vec{e}$:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \frac{A_1}{m_s} e_3 - \frac{d}{m_s} e_2 - \frac{k_s}{m_s} e_1 - D_b \\ \dot{e}_3 = -\frac{\beta A_1}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) e_2 + \frac{\beta}{V_1} \left(1 + \frac{\alpha^2}{\gamma}\right) \frac{k_v w}{\sqrt{1 + \alpha^3}} e_4 (\sqrt{\Delta}) \\ \dot{e}_4 = \frac{-1}{\tau} e_4 + \frac{k_{sp}}{\tau} [(-K_{p1}(e_1 - e_5) - K_{p2} e_3) \sqrt{\Delta} + C] \\ \dot{e}_5 = e_6 \\ \dot{e}_6 = \frac{1}{m_m} \left(F_h + K_{p3}(e_1 - e_5) + (K_{p3} \frac{V_1}{JA_1} + B') e_3 + B e_1 - k_d e_6 - k_h e_5 \right) \end{cases} \quad (6.31)$$

Eq. (6.31) represents the dynamic model of base-disturbed system in terms of vector $\vec{e}$. The right-hand sides of Eq. (6.31) are although discontinuous but are measurable and bounded. Now, the method defined by Wu *et al.* [83, 90] is used to construct a Lyapunov function for the disturbed system Eq. (6.31):

$$\begin{aligned} V = & C_1 e_1^2/2 + C_2 e_3^2/2 + C_3 e_1 e_3 + C_4 e_2^2/2 + C_5 e_4^2/2 + C_6 (e_1 - 14e_5)^2/2 \\ & + \frac{1}{4} [e_3 - C_7 e_5]^2 + C_8 e_5^2/2 + Y e_6^2/2 + Q e_2 e_4 \end{aligned} \quad (6.32)$$

where, $C_{i(i=1,...,8)}$ are the coefficients shown as follows:

$$\begin{aligned} C_1 = & \frac{13K_{p1}A_1^2J^2 + 14k_sJH}{14V_1H} + \frac{13K_{p1}k_{sp}\sqrt{\Delta}Q}{14\tau} + \frac{k_sA_1Q}{P\sqrt{\Delta}m_s} - \frac{K_{p1}k_{sp}Q\sqrt{\Delta}}{\tau} + \frac{A_1^2QK_{p1}}{Pm_sK_{p2}\sqrt{\Delta}} \\ & + \frac{K_{p1}k_{sp}k_sQV_1\sqrt{\Delta}}{A_1^2\tau J} + \frac{V_1QK_{p1}k_s}{JPK_{p2}m_s\sqrt{\Delta}} + \frac{QK_{p1}k_{sp}\sqrt{\Delta}}{14\tau} - \frac{A_1^2QK_{p1}}{14m_sPK_{p2}\sqrt{\Delta}} \\ C_2 = & \frac{K_{p1}V_1 + K_{p2}A_1J}{2H} - \frac{k_{sp}K_{p2}\sqrt{\Delta}V_1Q}{\tau A_1J} + \frac{V_1^2Qk_s}{J^2P\sqrt{\Delta}m_sA_1} \\ C_3 = & \frac{K_{p1}A_1J}{H} + \frac{V_1k_sQ}{JP\sqrt{\Delta}m_s} - \frac{K_{p1}k_{sp}QV_1\sqrt{\Delta}}{A_1\tau J} + \frac{V_1A_1QK_{p1}}{JPm_sK_{p2}\sqrt{\Delta}} \end{aligned}$$

$$\begin{aligned}
C_4 &= \frac{m_s J}{V_1} + \frac{K_{p1} k_{sp} Q V_1 \sqrt{\Delta} m_s}{A_1^2 \tau J} - \frac{V_1 Q K_{p1}}{J P K_{p2} \sqrt{\Delta}} \\
C_5 &= \frac{A_1 P J^2}{V_1 H} - \frac{Q P k_{sp} \sqrt{\Delta}}{\tau A_1} + \frac{A_1 Q}{m_s K_{p2} \sqrt{\Delta}} \\
C_6 &= \frac{K_{p1} A_1^2 J^2}{14 V_1 H} + \frac{K_{p1} k_{sp} \sqrt{\Delta} Q}{14 \tau} - \frac{Q K_{p1} k_{sp} \sqrt{\Delta}}{14 \tau} + \frac{A_1^2 Q K_{p1}}{14 m_s P K_{p2} \sqrt{\Delta}} \\
C_7 &= -\frac{2 K_{p1} A_1 J}{H} + \frac{2 V_1 Q K_{p1} k_{sp} \sqrt{\Delta}}{A_1 \tau J} - \frac{2 A_1 Q K_{p1} V_1}{m_s J P K_{p2} \sqrt{\Delta}} \\
C_8 &= \frac{(-13 K_{p1} A_1^2 k_{p3} + K_{p1} A_1^2 k_h) H J^2 - 2 K_{p1}^2 V_1 k_{p3} A_1^2 J^2}{V_1 k_{p3} H^2} - \frac{14 K_{p1} k_{sp} Q \sqrt{\Delta}}{\tau} - 196 T \\
&\quad - 0.5 Z^2
\end{aligned} \tag{6.33}$$

Positive semi-definiteness of the Lyapunov function candidate is proven according to the Sylvester theorem [86] as shown in the previous chapter. It is found that subject to satisfying the following conditions, the function V is positive definite:

$$K_{p2} > \frac{V_1^2 A_1^2 \tau J Q K_{p1}}{J P \sqrt{\Delta} (m_s J^2 A_1^2 \tau + K_{p1} k_{sp} Q V_1^2 \sqrt{\Delta} m_s)} \tag{6.34}$$

$$K_{p3} <$$

$$\frac{2 K_{p1} k_h A_1^2 J^2 H \tau}{26 K_{p1} A_1^2 J^2 H \tau + 4 K_{p1}^2 A_1^2 J^2 V_1 \tau + 28 K_{p1} k_{sp} Q \sqrt{\Delta} V_1 H^2 + 392 T V_1 H^2 \tau + Z^2 V_1 H^2 \tau} \tag{6.35}$$

Controller gains are appropriately tuned to ensure the positiveness of the Lyapunov function and a good tracking performance of the system. The procedure for choosing the control parameters is as follows: first, the value of the gain K_{p1} is chosen to be high enough to produce a fast response without saturating the control signal or causing unacceptable

overshoots. Then, value of K_{p1} is inserted in the inequality (6.34) and an appropriate value of K_{p2} is obtained. The selected value of K_{p2} is then substituted in inequality (6.7) to ensure term C_5 is positive and bounded. Finally, both K_{p1} and K_{p2} are substituted in inequality (6.35), and the value of K_{p3} is determined. Details on obtaining values of K_{p2} and K_{p3} are explained in Appendix A.4. Note that the value of Δ in inequalities (6.34) and (6.35) is always positive and well defined due to the physical characteristic of the system.

Next, $\dot{V}$ must be proven to be negative and at least semi-definite. The derivative of V with respect to time is:

$$\begin{aligned}\dot{V} = & C_1 e_1 \dot{e}_1 + C_2 e_3 \dot{e}_3 + C_3 e_1 \dot{e}_3 + C_3 e_3 \dot{e}_1 + C_4 e_2 \dot{e}_2 + C_5 e_4 \dot{e}_4 + C_6 (e_1 - 14e_5)(\dot{e}_1 \\ & - 14\dot{e}_5) + \frac{1}{2} [e_3 - C_7 e_5][\dot{e}_3 - C_7 \dot{e}_5] + C_8 e_5 \dot{e}_5 + Y e_6 \dot{e}_6 + Q \dot{e}_2 e_4 \\ & + Q e_2 \dot{e}_4\end{aligned}\tag{6.36}$$

Replacing $\dot{e}_{i(i=1..6)}$ of Eq. (6.31) and rearranging terms in Eq. (6.36), we have:

$$\begin{aligned}\dot{V} = & -e_2^2 \left[\frac{Jd}{V_1} + \dots \right] - e_4^2 \left[\frac{Ak_v w \tau J^2}{k_{sp} V_1 \sqrt{1+\alpha^3} H} + \dots \right] - e_6^2 \left[\frac{K_{p1} A_1^2 J^2 k_d}{V_1 K_{p3} H} + \dots \right] + \\ & e_2 [-C_4 D_b + QC] + e_4 [C_5 C - Q D_b]\end{aligned}\tag{6.37}$$

The squares terms in Eq. (6.37) are always negative, and so, the negativeness of $\dot{V}$ can be guaranteed as long as coefficients of e_2 and e_4 can be made zero; i.e., $-C_4 D_b + QC = 0$ and $C_5 C - Q D_b = 0$, which results in $C = \frac{Q}{C_5} D_b$ and $C = \frac{C_4}{Q} D_b$. Note that $\frac{Q}{C_5} = \frac{C_4}{Q}$; therefore, we can consider either $C = \frac{Q}{C_5} D_b$ or $C = \frac{C_4}{Q} D_b$ for further analyses.

If we can measure the disturbance term D_b accurately, the compensation term C , used in the control signal u , will easily be determined using $C = \frac{Q}{c_5} D_b$, and the coefficients of e_2 and e_4 in $\dot{V}$ will become zero which make the derivative of Lyapunov function negative semi-definite. However, in reality, it is not practically possible to measure the disturbance accurately. Also, a constant term C , which does not depend on the instantaneous measurement of the disturbance, should be used in the control signal for a range of the disturbance variations. Therefore, the coefficients of e_2 and e_4 will not always be equal to zero, and the negativeness of $\dot{V}$ at all times cannot be guaranteed in practice. Such a Lyapunov function does not decrease monotonically which conflicts with the requirements of a Lyapunov function; however, if it is guaranteed that the amount of increase of the Lyapunov function is always lower than the amount of decrease in the adjacent regions, the Lyapunov function will be decreasing overall, and the system will be stable. Such a Lyapunov function is known as quasi-Lyapunov function [90]. To meet the requirements of quasi-Lyapunov assumption, one solution is to consider an estimated upper value for the disturbance term inside the term C of the control signal. By taking the estimated upper value of D_b , the overall trend of the defined quasi-Lyapunov function (Eq. (6.32)) should be decreasing as will be shown in Section 6.5.1. This type of stability is weaker than the stability in the sense of Lyapunov, but, has been found very helpful for engineering problems [91].

6.5 Results

The proposed controller has been evaluated based on both simulation and experimental results.

6.5.1 Simulation results

Simulations were performed to validate the developed system model, prove the overall stability of the system and confirm the tracking ability of the control laws developed in this chapter. Table 3-2 in Chapter 3 shows the system parameters used in these simulations as well.

Eq. (6.2) was used to describe the system, and Eqs. (6.3) and (6.4) were used for the purpose of control as discussed in Section 6.3. The control parameters were properly tuned to ensure a good tracking performance of the system as discussed in Section 6.4.2. The controller gains were chosen as $K_{p1} = 0.07$, $K_{p2} = 4 \times 10^{-11}$ and $K_{p3} = 50$.

As mentioned earlier, the control signal u (Eq. (6.3)) must contain an estimated upper value of the base disturbance term, *i.e.*,

$$u = [-K_{p1}(x_s - x_m) - K_{p2}P_L]\sqrt{\Delta} + \frac{Q}{C_5}\hat{D}_b \quad (6.38)$$

where, $\hat{D}_b$ denotes the estimated upper value of base disturbance, and is defined as:

$$\hat{D}_b = \frac{\hat{k}_s}{m_s}\hat{x}_b\cos(zx_1) \quad (6.39)$$

In Eq. (6.39), $\hat{k}_s$ and $\hat{x}_b$ are the estimated upper values of environment stiffness and the base displacement, respectively. To stress this point, these estimated upper values are used for two purposes: (i) to guarantee the overall stability of the system in presence of a certain range of environmental uncertainties (*i.e.*, various loads and base disturbances) and (ii), to remove the need of measuring environmental parameters (*e.g.*, base displacement and the

external load). Table 6-1 lists the estimated upper and nominal values used in the simulation.

The human input force F_h , was set to a constant step value of 0.4 N , and initial conditions for all states were chosen as zero. The base displacement, x_b , was taken as a sine wave with amplitude $\alpha_b (= 0.15\text{ m})$ and frequency $\omega_b (= 36\text{ rad/s})$. From Eq. (6.1), the actual disturbance term, used in the simulation, is:

$$D_b = \frac{k_s}{m_s} x_b \cos(zx_1) = \frac{k_s}{m_s} [\alpha_b \sin(\omega_b t)] \cos(zx_1) \quad (6.40)$$

Note that the term D_b is used only in Eq. (6.2) to simulate the base disturbance in the hydraulic system and is different from $\hat{D}_b$ which is used in the control signal. It should also be noted that the control signal constructed based on the estimated upper values is applicable to the hydraulic actuator for a range of environment uncertainties as shown in Table 6-1.

Table 6-1 Values of environment parameters used in simulation study.

Parameter	Nominal value used in Eqs. (16) and (42)	Estimated upper values used in control signal Eq. (40)	Range of uncertainties
Base displacement	$x_b = 0.15 \sin(36t)$	$\hat{x}_b = 0.15\text{ m}$	$[-0.15, 0.15]\text{ m}$
Environment stiffness	$k_s = 37.5 + 37.5 \sin t\text{ kN/m}$	$\hat{k}_s = 75\text{ kN/m}$	$[0, 75]\text{ kN/m}$

The system response under sinusoidal base disturbance is plotted in Figure 6.4, which demonstrate the good tracking ability of the controller.

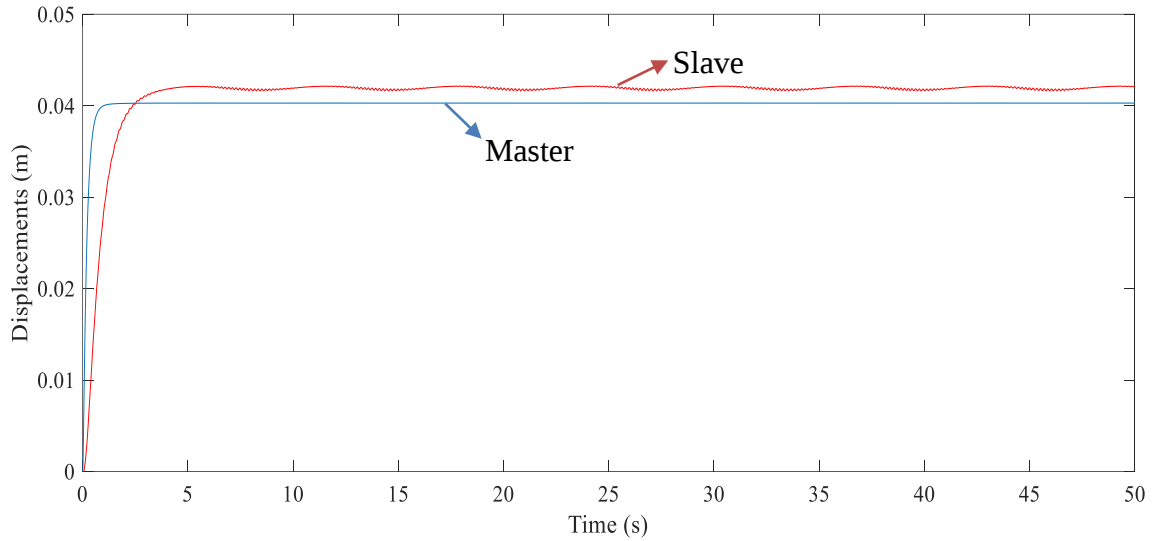


Figure 6.4 System response under sinusoidal base disturbance.

The quasi-Lyapunov function for the estimated upper value of base disturbance is plotted in Figure 6.5. As seen in the inset, the quasi-Lyapunov function increases during a certain time period. The amount of the increase is, however, lower than the decrease in the adjacent regions and the function continues to decrease overall. Through the simulations, it is seen that the numerical value of rate of change of the quasi-Lyapunov function is negative, which proves that the system is totally stable.

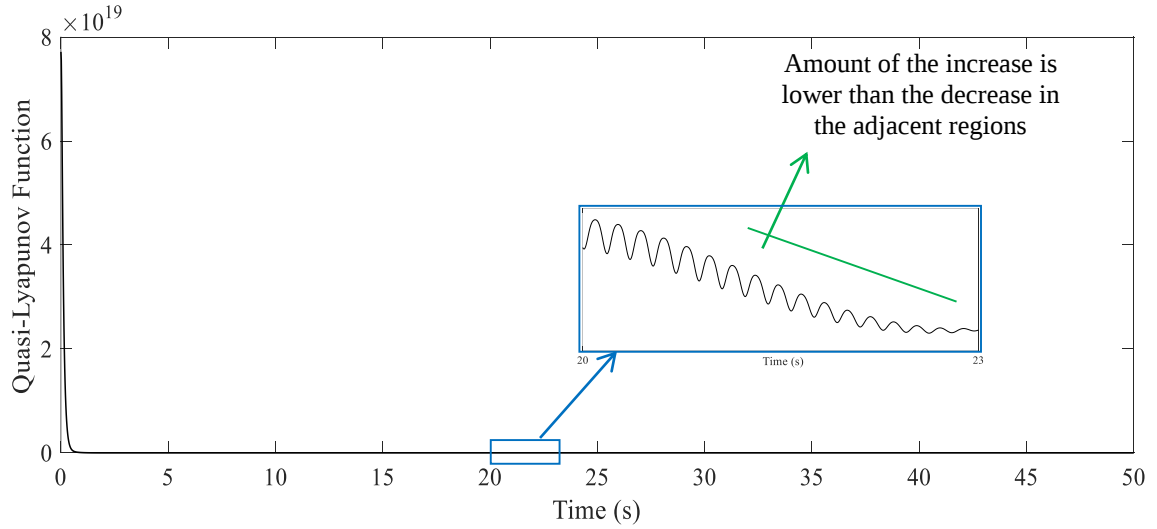


Figure 6.5 Quasi-Lyapunov function. The inset shows that the amount of the increase of function is lower than the decrease in the adjacent regions.

6.5.2 Experimental results

Experiments were conducted on the swivel motion control of the shoulder link of the Kodiak manipulator. The master was a PHANToM haptic device controlled by a PC's parallel port. The slave manipulator was also connected to the same PC with data acquisition boards to send control signals to the servo-valves and receive the manipulator joint angle measured by an encoder. The control loop worked at 500 Hz (sampling time of 2 ms). For the experimental studies, the same controller gains were used as in the simulations.

Various experiments were conducted to examine the proposed controller. The Stewart platform (see Figure 6.6) was programmed to generate low frequency disturbances similar to the ones used in [92]. For the experiments, the platform was programmed to generate horizontal displacement of the platform top plate first with amplitude and frequency of 50

mm and 7.5 rad/s , respectively and then with amplitude and frequency of 100 mm and 15 rad/s , respectively. This was followed by random step horizontal displacements of the platform top plate as shown in Figure 6.7. To determine the overall estimated upper value of disturbance, the estimated upper value of stiffness of the environment k_s was taken to be 75 kN/m , while the amplitude and frequency of base motion was taken as 150 mm and 36 rad/s , respectively.

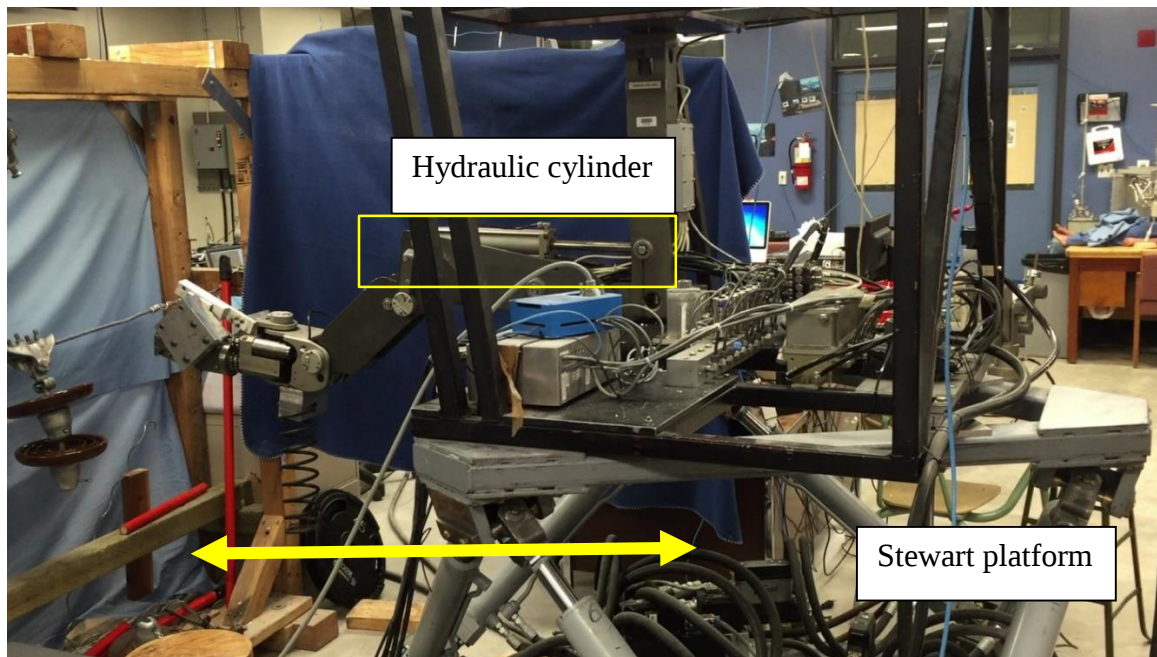


Figure 6.6 Stewart platform programmed to generate random low frequency base disturbances.

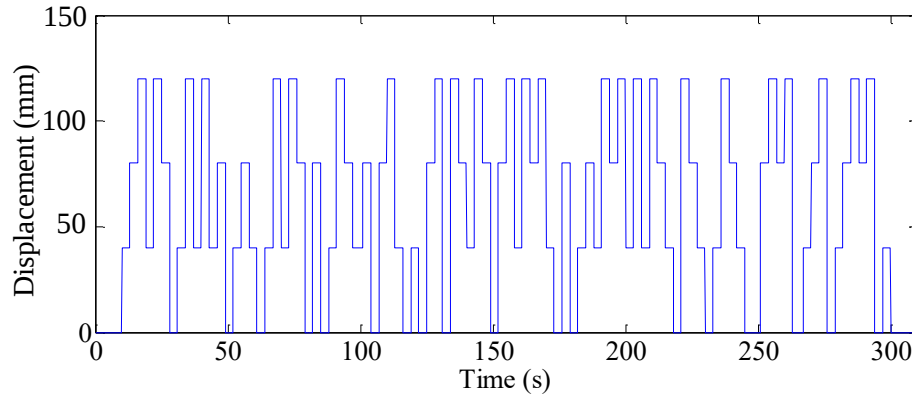


Figure 6.7 Stewart platform top plate displacements.

The first set of experimental results are shown in Figure 6.8 to Figure 6.10. The plots in Figure 6.8a, represent system response when the hydraulic actuator moved in free motion and the operator applied a command to the haptic device similar to a step displacement, while the base of the manipulator was given a sinusoidal motion of amplitude 50 mm and frequency of 7.5 rad/s . In the next experiment, the operator moved the haptic device back and forth and the hydraulic actuator moved in free motion and once again the base of the manipulator was given a sinusoidal motion with amplitude of 50 mm and frequency 7.5 rad/s . The results for this case are shown in Figure 6.8b. Results show that the controller stabilized the system effectively while having good position tracking by the hydraulic actuator and in addition providing a feel of performing task at the remote site to the operator. The proposed controller makes the master dynamics a better match with the dynamics of the hydraulic actuator. As seen in Figure 6.8, the operator feels additional force on his hands whenever there is a quick change in the direction of motion alerting the operator to slow down the hand motion allowing the slave manipulator to catch up as

position error gets apparent. The entire control law essentially assisted the operator to feel as if he is actually moving the hydraulic actuator, providing a sense of telepresence.

The same two sets of experiments were repeated again, wherein the base motion was a sinusoidal with amplitude 100 *mm* and frequency 15 *rad/s*. Figure 6.9 shows results of this experiment. This was followed by the experiments wherein random step motions (as seen in Figure 6.7) were given to the base of manipulator and results of this experiment are shown in Figure 6.10.

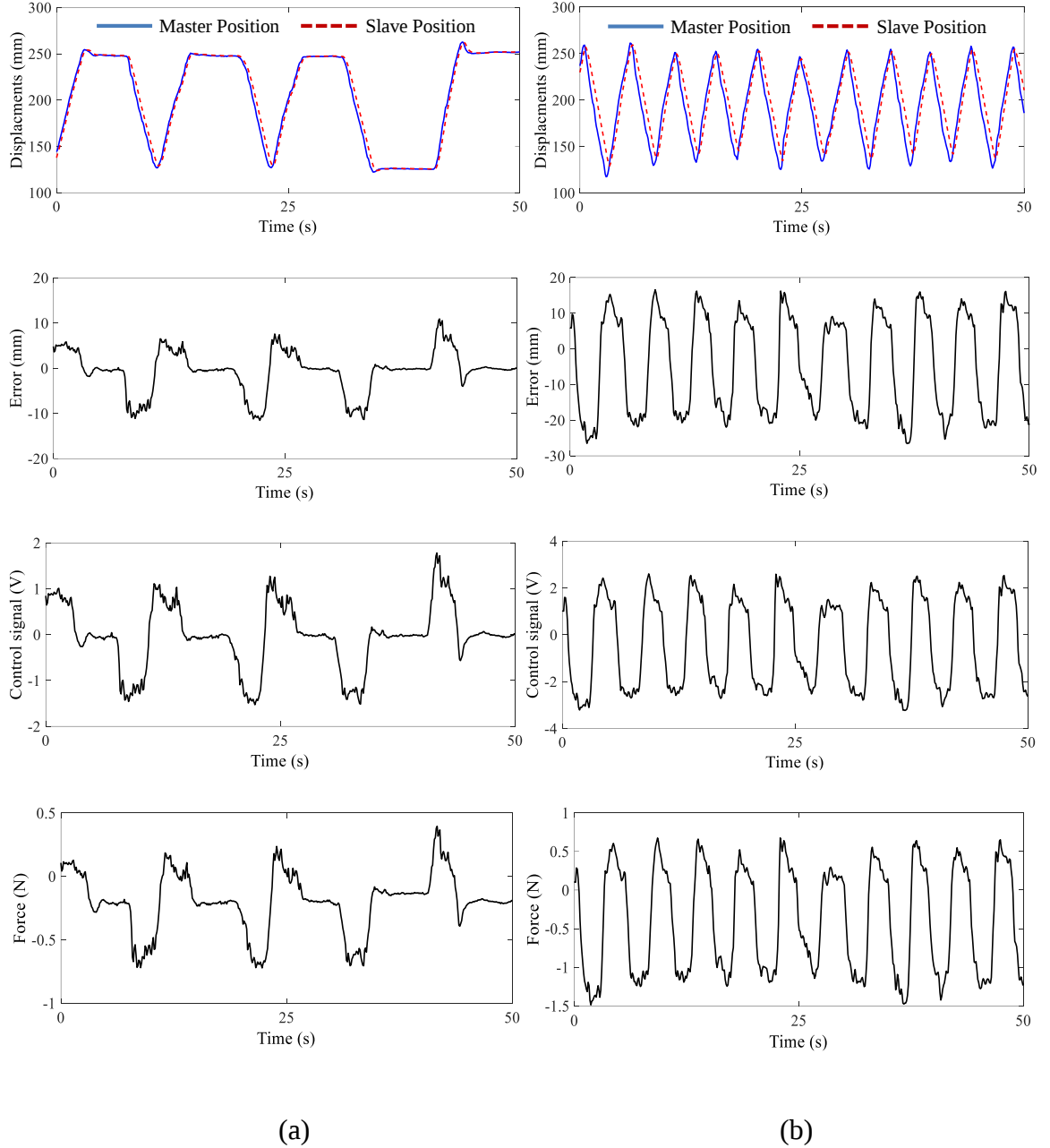


Figure 6.8 Experimental results of hydraulic actuator moving in free motion for (a) step-like master input, (b) sinusoid-like master input. Base of the manipulator is moving with amplitude α_b 50 mm and frequency ω 7.5 rad/s.

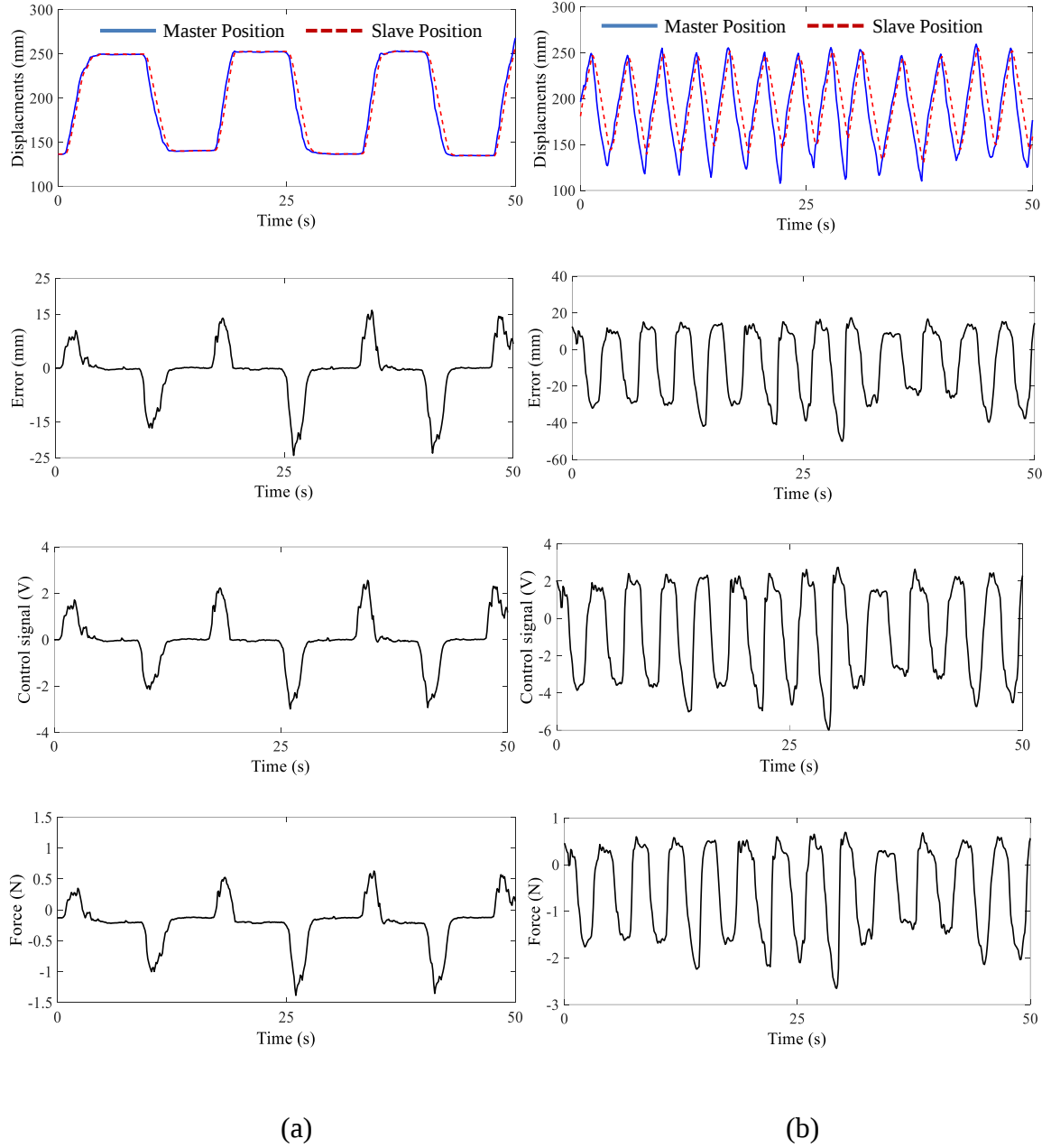


Figure 6.9 Experimental results of hydraulic actuator moving in free motion for (a) step-like master input, (b) sinusoid-like master input. Base of the manipulator is moving with amplitude α_b 100 mm and frequency ω 15 rad/s.

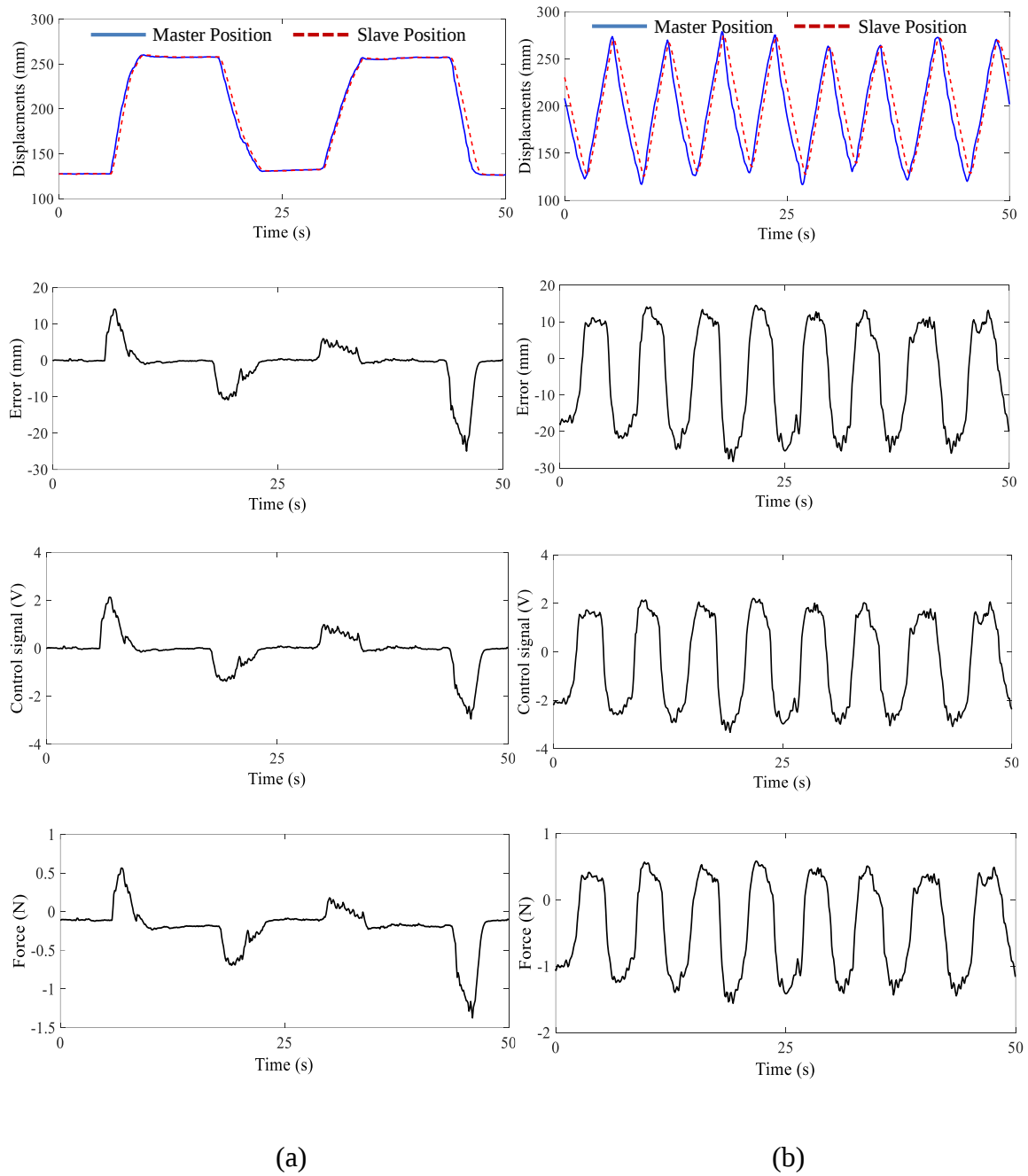


Figure 6.10 Experimental results of hydraulic actuator moving in free motion for (a) step-like master input, (b) for sinusoid-like master input. Base of the manipulator is given random step inputs as in Figure 6.7.

In the next set of experiments, the hydraulic actuator started in free motion and came in contact with a spring (environment) having stiffness of $k_s = 17 \text{ kN/m}$ at $x_s \approx 200 \text{ mm}$. The spring used in the experiments could not be compressed more than 50 mm (see Figure 6.11). Figure 6.12 to Figure 6.14 show the results of these experiments. With reference to Figure 6.12, the base of the manipulator was given a sinusoidal motion of amplitude 50 mm and frequency of 7.5 rad/s . As seen in the Figure 6.12, when the slave actuator makes contact with the environment (see the displacement plot) at 200 mm , the operator feels the force on his hands through the master haptic device. At this point, a large error is seen since the slave is interacting with a stiff environment and cannot follow the master movement, which gets translated into the haptic force based on the proposed control law. The control law thus assisted the operator to feel as if he is actually operating the hydraulic actuator against an environment. In Figure 6.13, base of the manipulator was given a sinusoidal motion of amplitude 100 mm and frequency of 15 rad/s . This was followed by the next set of experiments wherein random step motions were given to the base of manipulator and results of this experiment are shown in Figure 6.14.

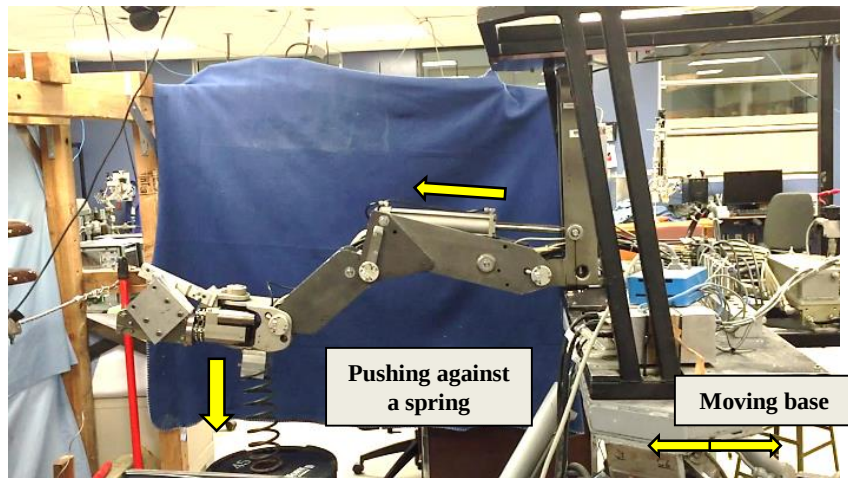


Figure 6.11 Hydraulic actuator starts in free motion and pushes against a spring having stiffness of $k_s = 17 \text{ kN/m}$, while the base of the manipulator is moving.

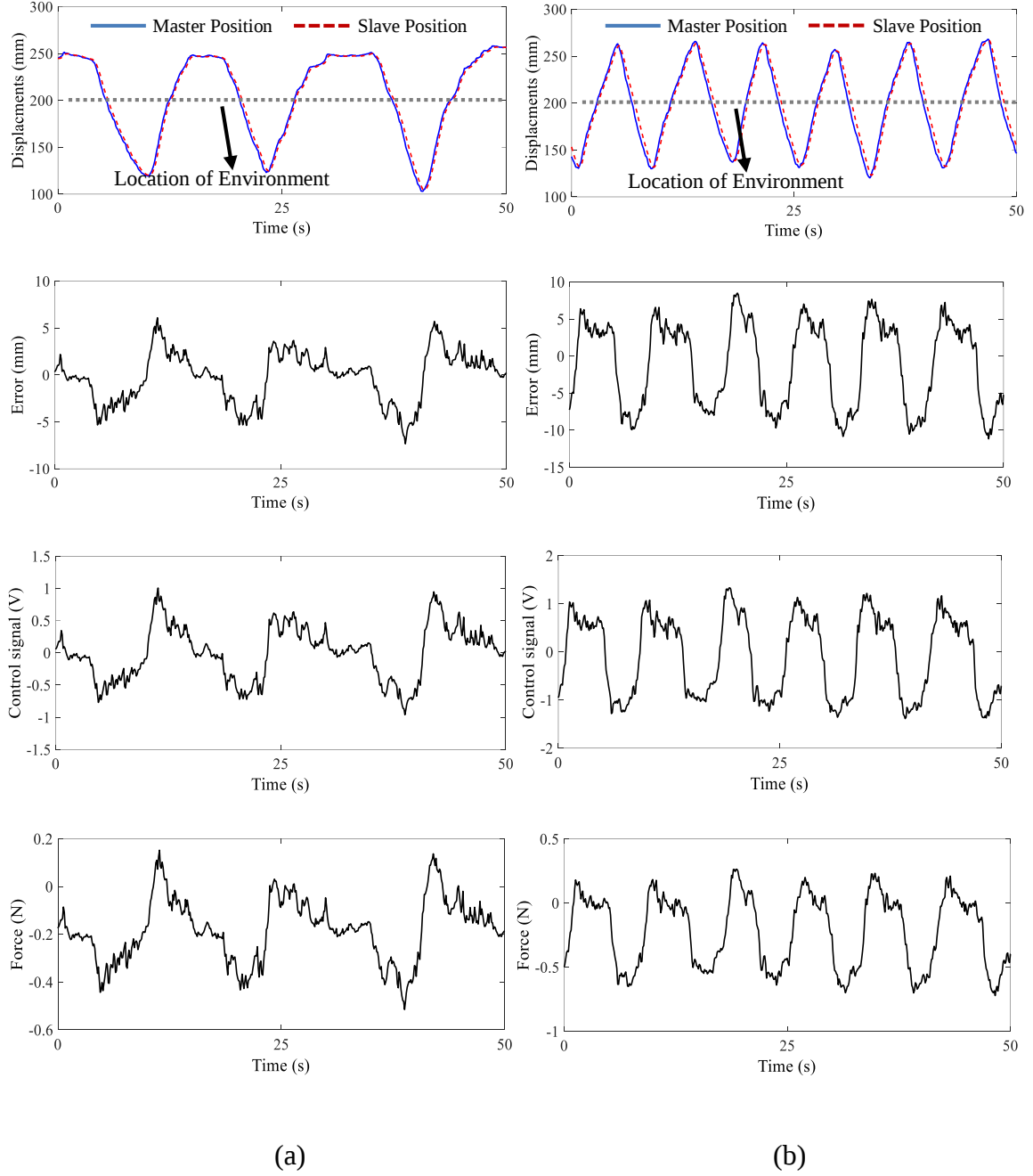


Figure 6.12 Experimental results of hydraulic actuator moving in free motion and making contact with a spring having stiffness of $k_s = 17 \text{ kN/m}$ for (a) step-like master input, (b) sinusoid-like master input. Base of the manipulator is moving with an amplitude α_b 50 mm and frequency ω 7.5 rad/s.

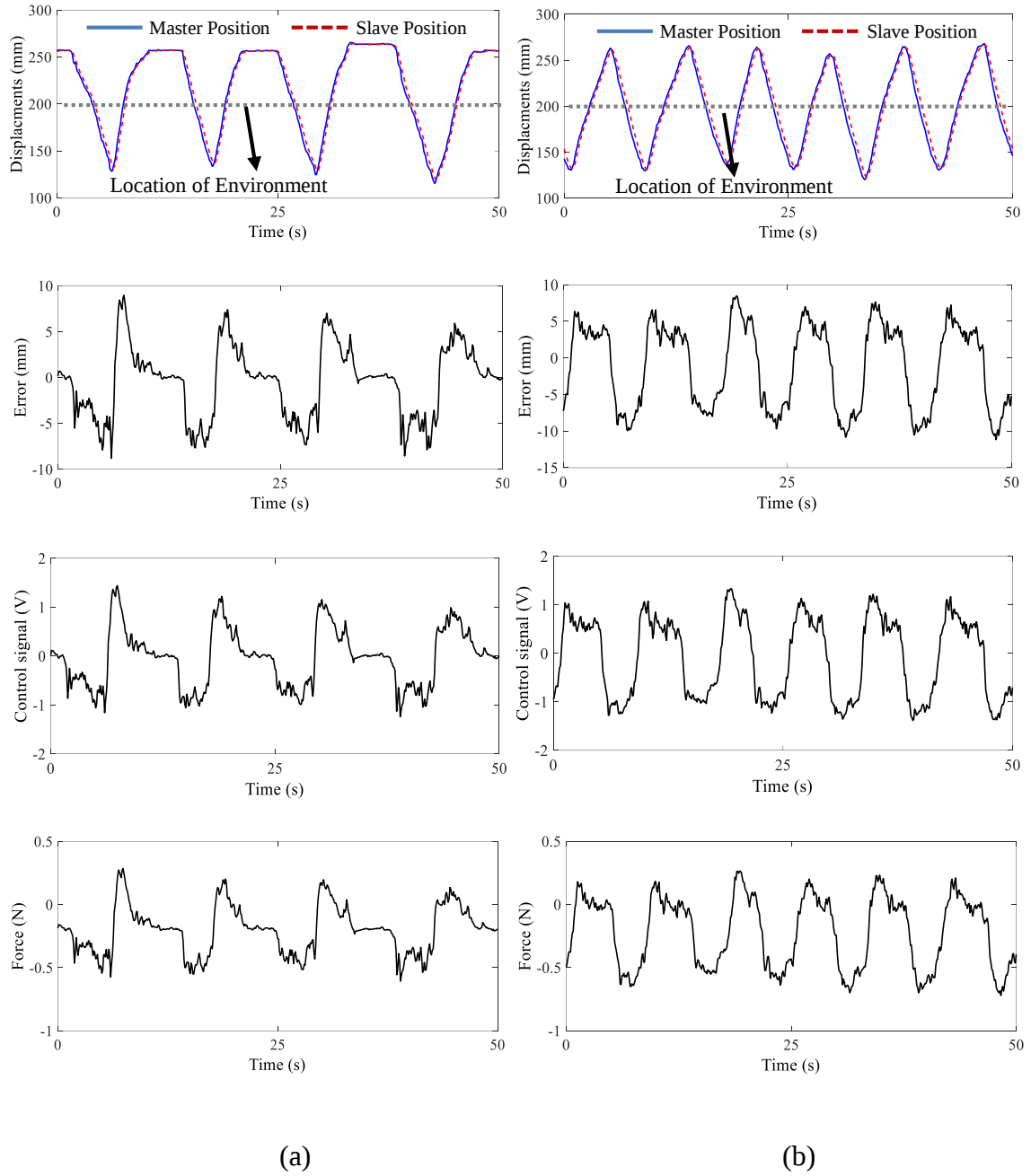


Figure 6.13 Experimental results of hydraulic actuator moving in free motion and making contact with a spring having stiffness of $k_s = 17 \text{ kN/m}$ for (a) step-like master input, (b) sinusoid-like master input. Base of the manipulator is moving with an amplitude α_b 100 mm and frequency ω 15 rad/s.

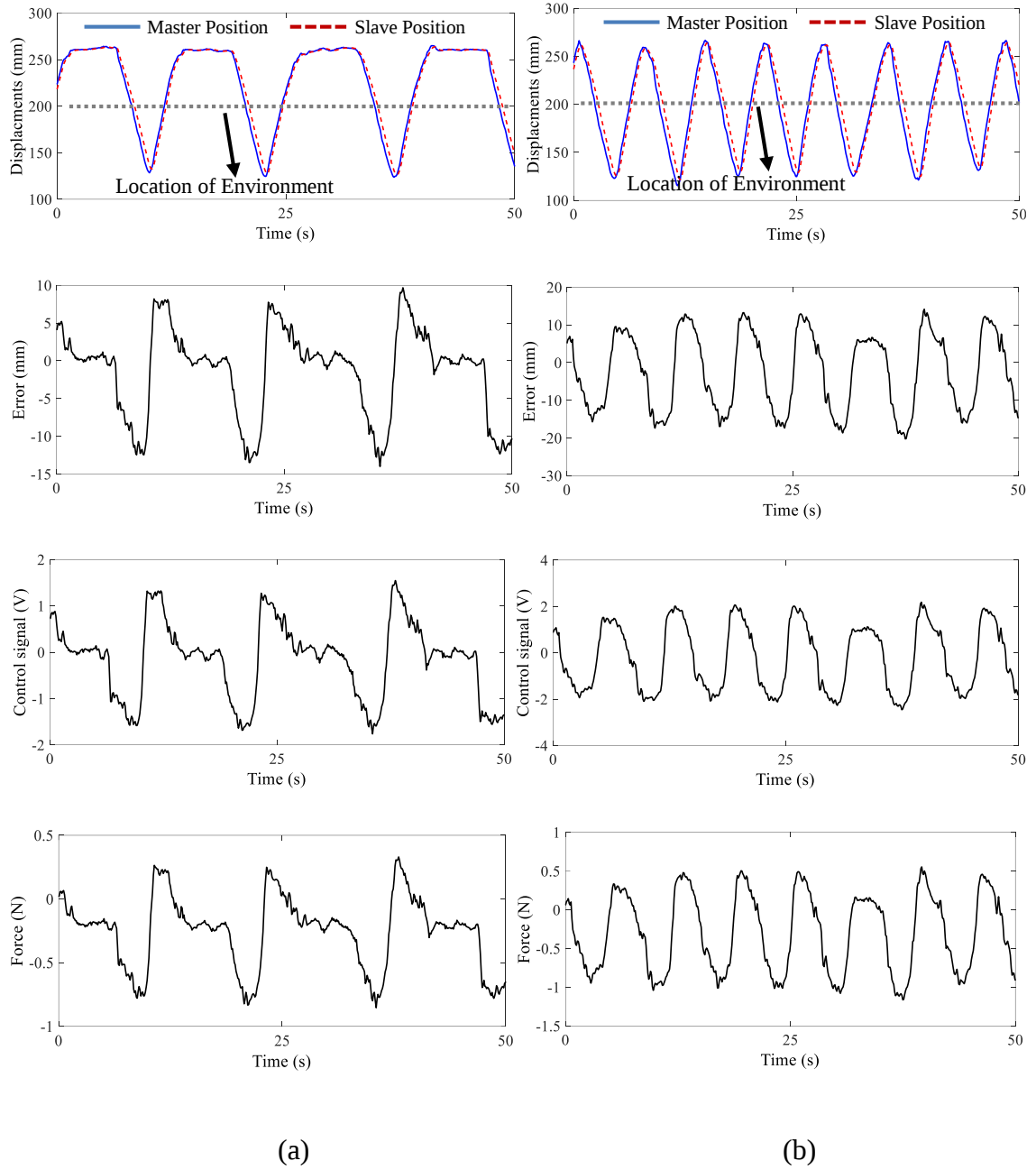


Figure 6.14 Experimental results of hydraulic actuator moving in free motion and making contact with a spring having stiffness of $k_s = 17 \text{ kN/m}$ for (a) step-like master input, (b) sinusoid-like master input. Base of the manipulator is given random step inputs.

The next two sets of experiments were performed to examine the performance of the controller when the hydraulic actuator was pushing against a live-line conductor wire having very high stiffness with a hot-stick (see Figure 6.15). Figure 6.16 to Figure 6.18 show results of experiment in which the hydraulic actuator started in free motion and came in contact with a conductor wire (environment) at $x_s \approx 200 \text{ mm}$. In these experiments, the base of the manipulator was first given a sinusoidal motion of amplitude 50 mm and frequency of 7.5 rad/s , and then a sinusoidal motion of amplitude 100 mm and frequency of 15 rad/s . This was again followed by the experiments wherein random step motions were given to the base of manipulator.

The experimental results confirm that the system was stable and exhibited good tracking response while providing haptic force feedback to the operator.

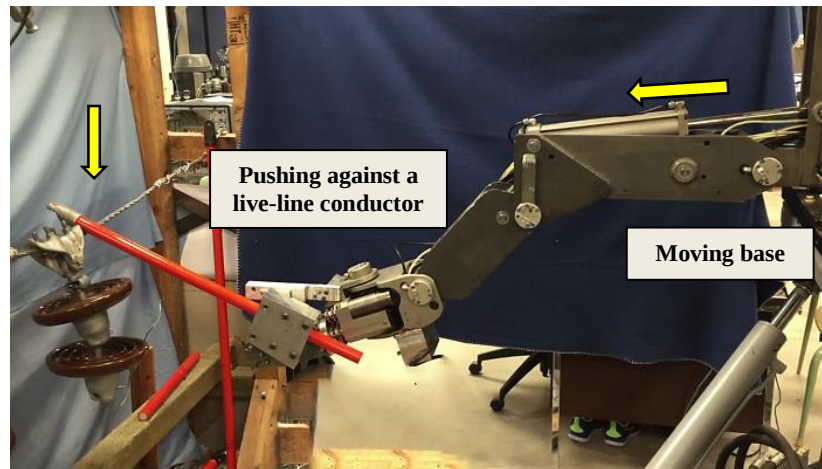


Figure 6.15 Hydraulic actuator starts in free motion and makes contact with a live-line conductor.

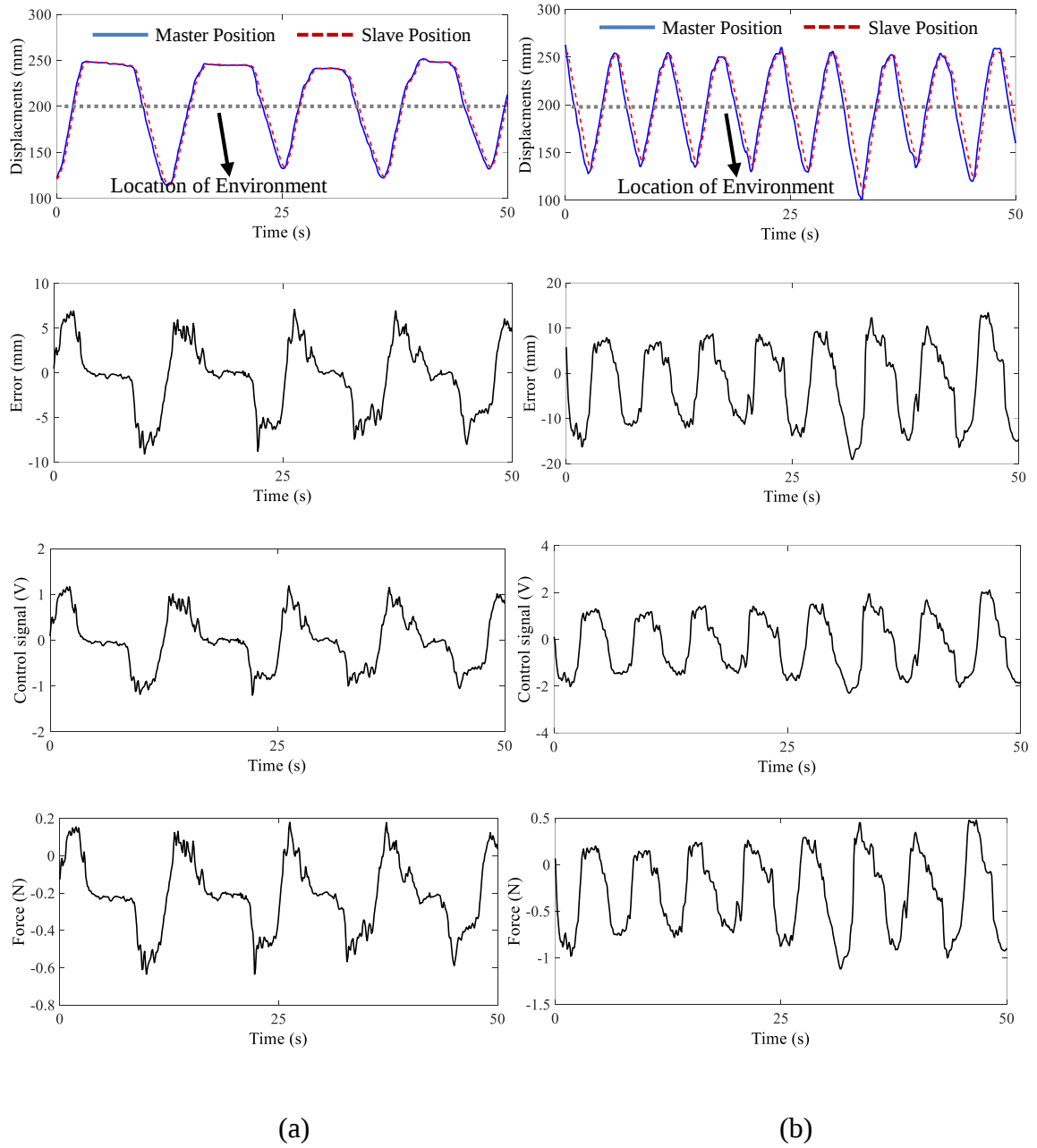


Figure 6.16 Experimental results of hydraulic actuator moving in free motion and making contact with a live-line conductor wire for (a) step-like master input, (b) sinusoid-like master input. Base of the manipulator is moving with an amplitude α_b 50 mm and frequency ω 7.5 rad/s.

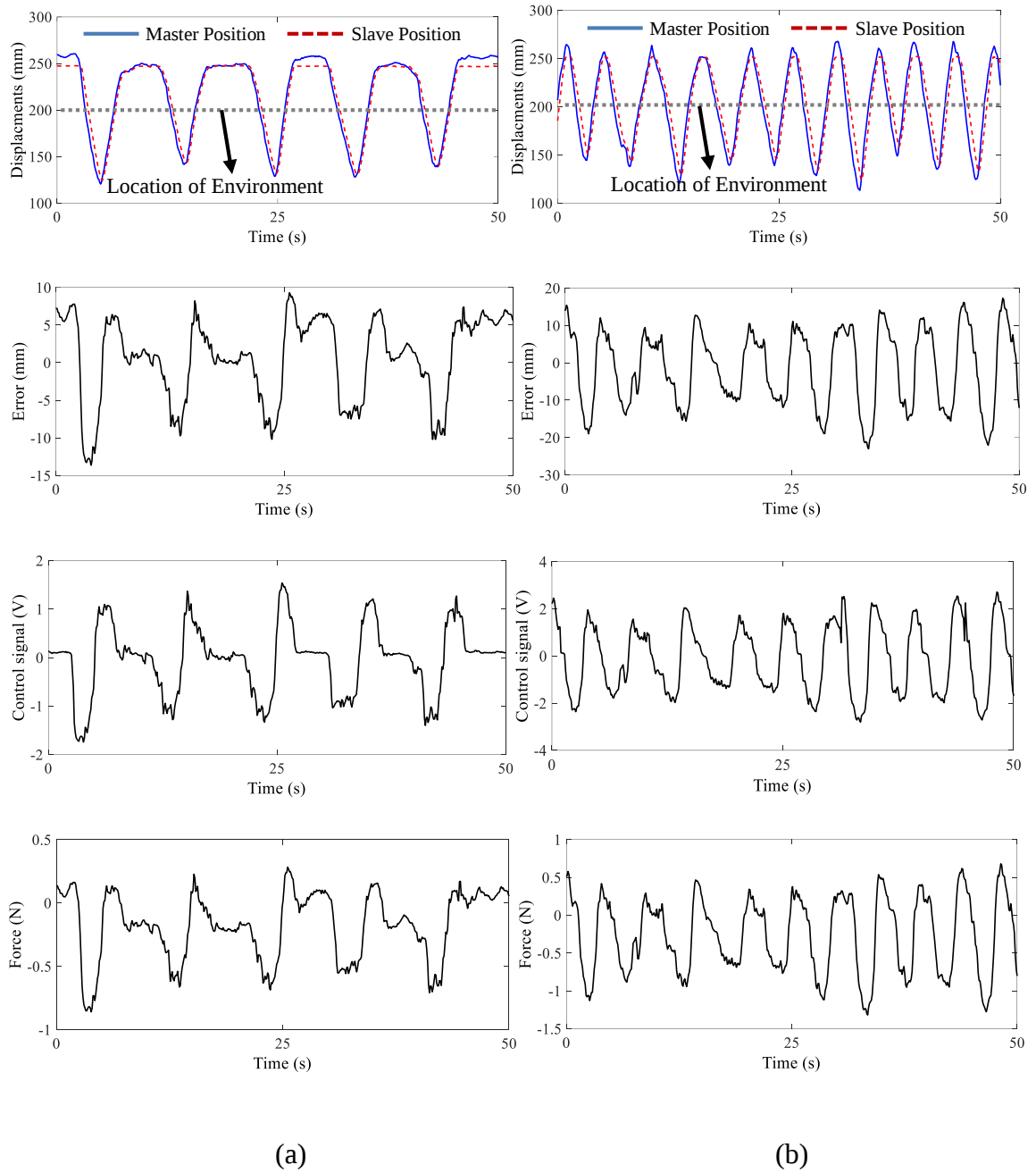


Figure 6.17 Experimental results of hydraulic actuator moving in free motion and making contact with a live-line conductor wire for (a) step-like master input, (b) sinusoid-like master input. Base of the manipulator is moving with an amplitude α_b 100 mm and frequency ω 15 rad/s.

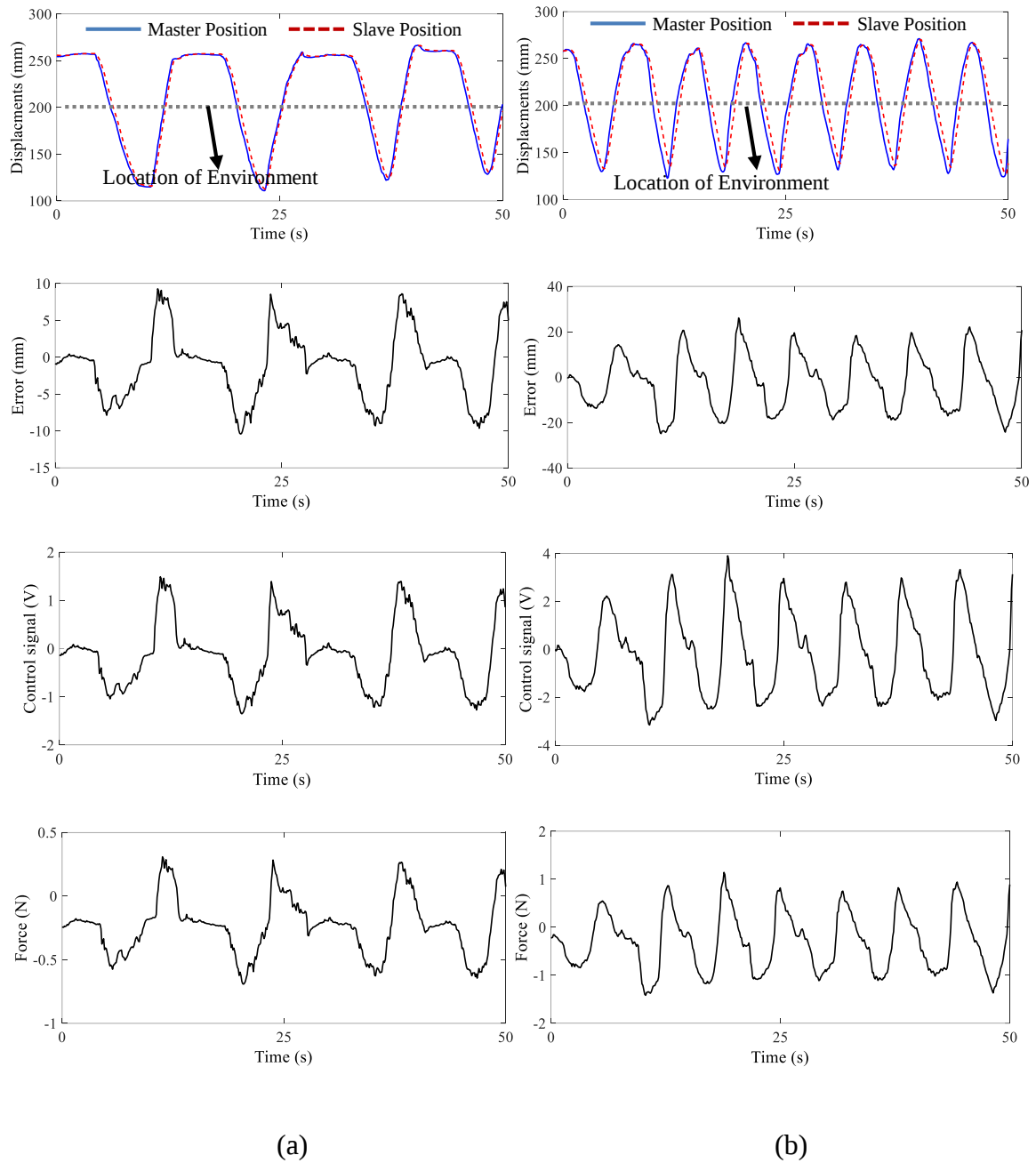


Figure 6.18 Experimental results of hydraulic actuator moving in free motion and making contact with a live-line conductor wire for (a) step-like master input, (b) sinusoidal master input. Base of the manipulator is given random step inputs.

6.6 Summary

This chapter presented the design and implementation of a Lyapunov-based controller for bilateral haptic teleoperation of a single-rod hydraulic actuator subjected to base disturbance. Due to the presence of a sign function in the proposed control law, the resulting control system was non-smooth. Thus, for stability proof, Filippov solution theories were first used to prove the existence, continuation, and uniqueness of the solution of the non-smooth system. The extended Lyapunov's stability theory was used next for carrying out the stability analysis of the resulting control system. Stability of the proposed controller with an estimated upper value for the base disturbance was analytically proven. Simulation studies were conducted, which confirmed that the controller could effectively stabilize the system while interacting with a task environment. Experimental results further demonstrated the effectiveness of the controller in maintaining stability of the system, while having good position tracking by the hydraulic actuator. In addition, the controller provided a feel of the contact force between the actuator and the task environment to the operator, at the master side, without direct measurement of the interaction force. This type of haptic force is suitable in applications whereby mounting a force sensor on the implement is not practical and interaction force information is not readily available, for example excavators and backhoes.

CHAPTER 7

7 CONCLUDING REMARKS

7.1 Contributions of this thesis

Haptic control of base-excited hydraulic manipulators was investigated in this thesis. Both unilateral and bilateral controls of hydraulic manipulators were thoroughly investigated using experimental validations on actual systems in a laboratory setting. Due to the laboratory setup, most experiments were conducted for live-line maintenance tasks. However, this research is equally relevant to applications which involves hydraulic tele-manipulation.

The three objectives outlined in Chapter 1.2 have been successfully achieved. First, the performance of the previously developed force feedback control schemes for teleoperated hydraulic manipulators was evaluated in the presence of base disturbance. This study demonstrated the effects of base disturbance on the performance of otherwise well-

performing teleoperated hydraulic manipulators. Experimental studies were conducted when the manipulator base was excited by a Stewart platform. Two typical live-line maintenance tasks were defined. There were three tested force modes: virtual fixture, force augmentation, and augmented virtual fixture schemes. The performance of the entire system, under each force mode, was evaluated and then compared with a mode in which no force was generated by the haptic device. Performance of each scheme was evaluated under three measures: task completion time, number of failed trials and displacement of the manipulator end-effector. Results showed that the augmentation force (position error force) was the best scheme by which the operators could complete the typical tasks efficiently and comfortably. Since implementing the augmented force is straightforward and effective, for the proposed application, it is recommended as a tool to perform the live-line maintenance tasks.

Based on a similar concept of using position errors between the haptic device and the hydraulic actuator positions to generate the haptic force, a bilateral control scheme was developed for single-rod hydraulic actuators for the first time in literature. The extension of Lyapunov stability theory to non-smooth systems, based on Filippov's solution theory, was employed to develop this control scheme. Theoretical stability of the control scheme was thoroughly investigated considering nonlinear hydraulic functions, servo-valve dynamics, haptic device dynamics, human operator dynamics, and dynamics of the task environment in the analysis. Control scheme was tested experimentally on a hydraulic test rig to verify its practicality and effectiveness in real applications. Proposed control scheme can be used in a wide variety of applications where the interaction force between the

hydraulic actuator and the task environment cannot be measured, such as backhoes, excavators, forklifts, harvesters and underwater manipulators.

This control scheme was further improved to incorporate base disturbances of single-rod hydraulic actuators. Stability of the proposed controller with an estimated upper value for the base disturbance was analytically proven. Simulation studies were conducted, which confirmed that the controller could effectively stabilize the system while interacting with a task environment. Experimental results further demonstrated the effectiveness of the controller in maintaining stability of the system, while having good position tracking by the hydraulic actuator.

In summary, **the major contribution** of this thesis is the thorough examination of the entire bilateral control system for a single-rod hydraulic actuator from theoretical stability to experimental validation, making it a very comprehensive study in the field.

7.2 Future work

This research will lead to the future work, which includes but is not limited to the following:

- Results can be extended to multi-degree-of-freedom single-rod hydraulic manipulators.
- Stable control schemes could be developed using the original hydraulic equations without imposing assumptions on the piston initial position and range of movements.

- As the proposed controller in this research is discontinuous, the control may chatter as the state trajectories reach the discontinuous region. This could be rectified by the replacement of the discontinuous terms from the controller by continuous functions.
- The hydraulic actuator's dry friction can be considered in dynamic equations.

REFERENCES

- [1] S. G. Ponnambalam, J. Parkkinen and K. C. Ramanathan, "Trends in Intelligent Robotics, Automation, and Manufacturing," in *IRAM 2012*, Kuala Lumpur, Malaysia, November 28-30, 2012.
- [2] M. E. Kontz, J. Beckwith and W. J. Book, "Evaluation of a teleoperated haptic forklift," in *Proceedings of IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, Monterey, USA, 2005.
- [3] S. Tafazoli, S. E. Salcudean, K. Hashtrudi-Zaad and P. D. Lawrence, "Impedance control of a teleoperated excavator," *IEEE Transactions on Control System Technology*, vol. 10, no. 3, pp. 355-367, 2002.
- [4] E. Papadopoulos, B. Mu and R. Frenette, "On modeling, identification, and control of a heavy-duty electrohydraulic harvester manipulator," *IEEE/ASME Transactions on Mechatronics*, pp. 178-187, 2003.
- [5] N. R. Parker, S. E. Salcudean and P. D. Lawrence, "Application of force feedback to heavy duty hydraulic machines," in *Proceedings of IEEE International Conference on Robotics and Automation*, Atlanta, USA, 1993.
- [6] M. E. Kontz and W. J. Book, "Position/rate haptic control of a hydraulic forklift," in *Proceedings of ASME International Mechanical Engineering Congress*, 2003.
- [7] P. F. Hokayem and M. W. Spong, "Bilateral teleoperation: An historical survey," *Journal of Automatica*, vol. 42, no. 12, pp. 2035-2057, 2006.
- [8] M. E. Kontz, M. C. Herrera, J. D. Huggins and W. J. Book, "Impedance shaping for improved feel in hydraulic systems," in *Proceedings of ASME International Mechanical Engineering Congress and Exposition*, Seattle, USA, 2008.
- [9] J. K. Salisbury and M. A. Srinivasan, "Phantom-based haptic interaction with virtual objects," *IEEE Computer Graphics and Applications*, pp. 6-10, 1997.
- [10] O. Y. Ming, D. V. Beard and F. P. Brooks Jr., "Force display performs better than visual display in simple 6-D docking task," in *Proceedings of IEEE Conference on Robotics and Automation*, Scottsdale, USA, 1989.
- [11] B. Heinrichs, N. Sepehri and A. Thornton-Trump, "Position-based impedance control of an industrial hydraulic manipulator," *IEEE Control Systems Magazine*, vol. 17, pp. 46-52, 1997.
- [12] H. E. Merritt, *Hydraulic control systems*, NY: John Wiley, 1967.
- [13] M. Sirouspour and S. Salcudean, "On the nonlinear control of hydraulic servo-systems," in *Proceedings of 2000 IEEE International Conference on Robotics and Automation*, San Francisco, CA, 2000.
- [14] A. Alleyne and R. Liu, "A simplified approach to force control for electro-hydraulic systems," *Control Engineering Practice*, vol. 8, no. 12, p. 1347-1356, December 2000.

- [15] M. R. Sirouspour and S. E. Salcudean, "Nonlinear Control of Hydraulic Robots," *IEEE Transactions on Robotics and Automation*, vol. 17, no. 2, pp. 173-182, April 2001.
- [16] Y. Maddahi, "Bilateral Control of Base-Excited Hydraulic Manipulators Operating under a Delayed and Lossy Network," PhD Thesis, University of Manitoba, Winnipeg, 2013.
- [17] K. Zarei-nia, "Haptic-enabled teleoperation of hydraulic manipulators: theory and application," PhD Thesis, University of Manitoba, Winnipeg, 2011.
- [18] D. A. Lawrence, "Stability and transparency in bilateral teleoperation," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 5, pp. 624-637, 1993.
- [19] B. Hannaford, "Stability and performance tradeoffs in bi-lateral telemanipulation," in *Proceedings of IEEE International Conference on Robotics and Automation*, Scottsdale, USA, 1989.
- [20] K. Zarei-nia, A. Y. Goharrizi, N. Sepehri and W. K. Fung, "Experimental evaluation of bilateral control schemes applied to hydraulic actuators: a comparative study," *Transactions of the Canadian Society for Mechanical Engineering*, vol. 33, no. 3, pp. 377-398, 2009.
- [21] K. Zarei-nia, N. Sepehri and Q. Wu, "A Lyapunov controller for stable haptic manipulation of hydraulic actuators," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 3, pp. 241-261, 2012.
- [22] A. Bonchis, P. I. Corke and D. C. Rye, "Experimental evaluation of position control methods for hydraulic systems," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 6, pp. 876 - 882, November 2002.
- [23] K. Toussaint, N. Pouliot and S. Montambault, "Transmission line maintenance robots capable of crossing obstacles: State-of-the-art review and challenges ahead," *Journal of Field Robotics*, vol. 26, no. 5, p. 477-499, 2009.
- [24] K. Takaoka, K. Yokoyama, H. Wakisako, K. Yano, K. Higashitima and S. Murakami, "Development of the fully-automatic live-line maintenance robot- phase III," in *Proceedings of IEEE International Symposium on Assembly and Task Planning Soft Research Park*, Fukuoka, Japan, 2009.
- [25] J. Lessard, J. Robert and P. Rondot, "Evaluation of working techniques using teleoperation for power line maintenance," in *International Conference on Telemanipulator and Telepresence Technologies*, 1994.
- [26] K. Takaoka, K. Yokoyama, H. Wakisako, K. Yano, K. Higashijima and S. Murakami, "Development of the fully-automatic live-line maintenance robot-phase III," in *Proceedings of International Symposium on Assembly and Task Planning*, Fukuoka, Japan, 2001.
- [27] S. Montambault and N. Pouliot, "The HQ LineROVer: Contributing to innovation in transmission line maintenance," in *ESMO Proceedings: the Power is in Your Hands*, 2003.
- [28] G. Clement, "An overview of CAT control in nuclear services," in *Proceedings of IEEE International Conference on Robotics and Automation*, 1985.

- [29] M. Tavakoli, R. Patel and M. Moallem, "A force reflective master-slave system for minimally invasive surgery," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, Las Vegas, USA, 2003.
- [30] A. J. Madhani, "The Black Falcon: a teleoperated surgical instrument for minimally invasive surgery," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, Victoria, Canada, 1998.
- [31] L. Sun, F. Van Meer, Y. Bailly and C. K. Yeung, "Design and development of a da Vinci surgical system simulator," in *Proceedings of IEEE International Conference on Mechatronics and Automatio*, Harbin, China, 2007.
- [32] A. Baheti, "RoSS: virtual reality robotic surgical simulator for the da Vinci surgical system," in *Proceedings of Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems*, Reno, USA, 2008.
- [33] Y. Matsumoto, "Dexterous manipulation in constrained bilateral teleoperation using controlled supporting point," *IEEE Transactions on Industrial Electronics*, vol. 54, no. 2, pp. 1113-1121, 2007.
- [34] F. Najafi and N. Sepehri, "A novel hand-controller for remote ultrasound imaging," *Journal of Mechatronics*, vol. 18, no. 10, pp. 578-590, 2008.
- [35] J. Funda, "A symbolic teleoperator interface for time-delayed underwater robot manipulation," in *Proceedings of Ocean Technologies and Opportunities in the Pacific for the 90's*, 1991.
- [36] N. Diolaiti, "Teleoperation of a mobile robot through haptic feedback," in *IEEE International Workshop on Haptic Virtual Environments and their Applications*, 2002.
- [37] S. G. Hong, "Generating artificial force for feedback control of teleoperated mobile robots," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, Kyongju, Korea, 1999.
- [38] J. Lim, J. Ko and J. Lee, "Internet-based teleoperation of a mobile robot with force-reflection," in *Proceedings of IEEE Conference on Control Applications*, 2003.
- [39] O. J. Rosch, K. Schilling and H. Roth, "Haptic interfaces for the remote control of mobile robots," *Journal of Control Engineering Practice*, vol. 10, pp. 1309-1313, 2002.
- [40] M. Boukhniifer, "Scaled teleoperation controller design for micromanipulation over Internet," in *Proceedings of IEEE International Conference on Robotics and Automation*, New Orleans, USA, 2004.
- [41] P. Lawrence, S. Salcudean, N. Sepehri, D. Chan, S. Bachmann, N. Parker, M. Zhu and R. Frenette, "Coordinated and force-feedback control of hydraulic excavators," in *Proceedings of International Symposium on Experimental Robotics IV*, Berlin, Germany, 1997.
- [42] W. J. Book, "Haptics for and by Hydraulics," in *The 16th IEEE International Symposium on Robot and Human Interactive Communication*, Jeju, South Korea, 26-29 August 2007.

- [43] L. B. Rosenberg, "Virtual fixtures: perceptual tools for telerobotic manipulation," in *Proceedings of IEEE Annual International Symposium on Virtual Reality*, Seattle, USA, 1993.
- [44] N. Turro, O. Khatib and E. Coste-Maniere, "Haptically augmented teleoperation," in *Proceedings of IEEE International Conference on Robotics and Automation*, Seoul, Korea, 2001.
- [45] T. Sheridan, *Telerobotics, Automation and Human Supervisory Control*, MIT Press, 1992.
- [46] C. Sayers, *Remote Control Robotics*, New York: Springer Verlag, 1999, pp. 41-50.
- [47] I. Farkhatdinov and J. A. J. Ryu, "Preliminary experimental study on haptic teleoperation of mobile robot with variable force feedback gain," in *Proceedings of IEEE Haptics Symposium*, Waltham, USA, 2010.
- [48] Q. C. Wu, "Lyapunov stability analysis of a class of base-excited inverted pendulums with application to bipedal locomotion systems," PhD Thesis, University of Manitoba, Winnipeg, 1996.
- [49] Y. Song, H. Wang, Y. Jiang and L. Ling, "AApe-D: a novel power transmission line maintenance robot for broken strand repair," in *Proceedings of International Conference on Applied Robotics for the Power Industry (CARPI)*, Zurich, Switzerland, 2012.
- [50] J. Jian, Z. Guoxian and Z. Tingyu, "Design of a mobile robot for the innovation in power line inspection and maintenance," in *Proceedings of International Conference on Reconfigurable Mechanisms and Robots*, London, UK, 2009.
- [51] J. Park, J. Lee, B. Cho and K. Oh, "Development of inspection robot system for live-line suspension insulator strings in 345kV power transmission lines park," in *Proceedings of European Conference on Power Electronics and Applications*, Barcelona, Spain, 2009.
- [52] P. Sekhavat, N. Sepehri and Q. Wu, "Impact stabilizing controller for hydraulic actuators with friction: Theory and experiments," *Control Engineering Practice*, vol. 14, no. 12, p. 1423–1433, December 2006.
- [53] P. Sekhavat, Q. Wu and N. Sepehri, "Impact control in hydraulic actuators with friction: theory and experiments," in *Proceedings of the 2004 American Control Conference*, Boston, MA, USA, 2004.
- [54] N. Niksefat, C. Q. Wu and N. Sepehri, "Design of a Lyapunov Controller for an Electro-hydraulic Actuator During Contact Tasks," *Journal of Dynamic Systems, Measurement, and Control*, vol. 123, no. 2, pp. 299-307, 2000.
- [55] A. Halanay, C. Safta, F. Ursu and I. Ursu, "Stability analysis for a nonlinear model of a hydraulic servomechanism in a servoelastic framework," *Nonlinear Analysis: Real World Applications*, vol. 10, no. 2, pp. 1197-1209, April 2009.
- [56] O. Becker, I. Pietsch and J. Hesselbach, "Robust task-space control of hydraulic robots," in *Proceedings of 2003 IEEE International Conference on Robotics and Automation*, Taipei, Taiwan, September 2003.

- [57] K. Zareinia and N. Sepehri, "A Hybrid Haptic Sensation for Teleoperation of Hydraulic Manipulators," *Journal of Dynamic Systems, Measurement, and Control*, vol. 137, no. 9, pp. 1-13, 2015.
- [58] P. Y. Li and K. Krishnaswamy, "Passive Bilateral Teleoperation of a Hydraulic Actuator using an Electrohydraulic Passive Valve," *International Journal of Fluid Power*, vol. 5, no. 2, pp. 43-55, 2004.
- [59] K. Krishnaswamy and P. Y. Li, "Passive control of an electrohydraulic actuator using an electrohydraulic passive valve," in *Proceedings of the American Control Conference*, Arlington, VA, USA, 25-27 June 2001.
- [60] B. Yao, F. Bu, J. Reedy and G. Chiu, "Adaptive robust motion control of single-rod hydraulic actuators: theory and experiments," *IEEE/ASME Transactions on Mechatronics*, vol. 5, no. 1, pp. 79-91, March 2000.
- [61] C. Guan and S. Pan, "Nonlinear Adaptive Robust Control of Single-Rod Electro-Hydraulic Actuator With Unknown Nonlinear Parameters," *IEEE Transactions on Control Systems Technology*, vol. 16, no. 3, pp. 434-445, May 2008.
- [62] K. Guo, J. Wei, J. Fang, R. Feng and X. Wang, "Position tracking control of electro-hydraulic single-rod actuator based on an extended disturbance observer," *Mechatronics*, vol. 27, pp. 47-56, April 2015.
- [63] W. Kim, D. Shin, D. Won and C. C. Chung, "Disturbance-Observer-Based Position Tracking Controller in the Presence of Biased Sinusoidal Disturbance for Electrohydraulic Actuators," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 6, pp. 2290 - 2298, November 2013.
- [64] E. Fitcher, "A Stewart platform-based manipulator: General theory and practical construction," *International Journal of Robotics Research*, vol. 5, no. 2, pp. 157-182, 1986.
- [65] N. Sepehri, A. A. Khayyat and B. Heinrichs, "Development of a nonlinear PI controller for accurate positioning of an industrial hydraulic manipulator," *Journal of Mechatronics*, vol. 7, pp. 683-700, 1997.
- [66] Y. Yokokohji and T. Yoshikawa, "Bilateral control of master-slave manipulators for ideal kinesthetic coupling-formulation and experiment," *IEEE Transactions on Robotics and Automation*, vol. 10, no. 5, pp. 605 - 620, October 1994.
- [67] A. Abdossalami and S. Sirouspour, "Adaptive Control for Improved Transparency in Haptic Simulations," *IEEE Transactions on Haptics*, vol. 2, no. 1, pp. 2-14, March 2009.
- [68] G. Christiansson and F. Helm, "The Low-Stiffness Teleoperator Slave — a Trade-off between Stability and Performance," *The International Journal of Robotics Research*, vol. 26, no. 3, pp. 287-299, 2007.
- [69] H. E. Merritt, *Hydraulic Control Systems*, New York, 1967.
- [70] V. Durbha and P. Li, "A nonlinear spring model of hydraulic actuator for passive controller design in bilateral tele-operation," in *American Control Conference (ACC)*, Montreal, Quebec, 27-29 June 2012.

- [71] H. Heyong, H. Qingxue, W. Jun and W. Jing, "The analysis of new hydraulic rolling shear servo system dynamic characteristics," *Journal of Engineering Manufacture*, vol. 227, no. 3, pp. 453-459, March 2013.
- [72] T. Schmidt, "Adaptive Backstepping Control of Asymmetric Electro-Hydraulic System," Aalborg University, Master's Thesis, Denmark, May 2012.
- [73] N. Niksefat, N. Sepehri and Q. Wu, "Design and experimental evaluation of a QFT contact task controller for electro-hydraulic actuators," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 2-3, p. 225–250, January 2007.
- [74] R. Liu and A. Alleyne, "Nonlinear force/pressure tracking of an electrohydraulic actuator," *Journal of Dynamic Systems, Measurement, and Control*, vol. 122, no. 1, pp. 232-236, 1998.
- [75] M. Jerouane, N. Sepehri and F. Lamnabhi-Lagarriguea, "Dynamic analysis of variable structure force control of hydraulic actuators via the reaching law approach," *International Journal of Control*, vol. 77, no. 4, pp. 1260-1268, 2004.
- [76] V. Banthia, Y. Maddahi, K. Zareinia, S. Liao, T. Olson, W. K. Fung, S. Balakrishnan and N. Sepehri, "A prototype telerobotic platform for live transmission line maintenance: Review of design and development," *Transactions of the Institute of Measurement and Control*, vol. 40, no. 11, pp. 3273-3292, 2017.
- [77] V. Banthia, Y. Maddahi, S. Balakrishnan and N. Sepehri, "Haptic-enabled teleoperation of base-excited hydraulic manipulators applied to live-line maintenance," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Chicago, IL, 2014.
- [78] V. Banthia, K. Zareinia, S. Balakrishnan and N. Sepehri, "A Lyapunov Stable Controller for Bilateral Haptic Teleoperation of Single-Rod Hydraulic Actuators," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 139, no. 11, pp. 111001-111017, 2017.
- [79] W. S. Kim, "Developments of new force reflecting control schemes and an application to a teleoperation training simulator," in *Proceedings of 1992 IEEE International Conference on Robotics and Automation*, Nice, France, May 1992.
- [80] A. F. Filippov, "Differential equations with discontinuous right-hand side," *American Mathematical Society Translations*, vol. 42, pp. 199-231, 1964.
- [81] A. F. Filippov, "Differential equations with second members discontinuous on intersecting surfaces," *Differential Equations*, vol. 15, pp. 1292-1299, 1980.
- [82] Q. Wu, N. Sepehri, A. B. Thornton-Trump and S. Onyshko, "An extended integral method to derive Lyapunov functions for nonlinear systems," *International Journal of Control*, vol. 62, no. 3, pp. 717-736, 1995.
- [83] Q. Wu, S. Onyshko, N. Sepehri and A. B. Thornton-Trump, "On construction of smooth Lyapunov functions for non-smooth systems," *International Journal of Control*, vol. 69, no. 3, pp. 443-457, 1998.
- [84] Q. Wu and N. Sepehri, "On Lyapunov's stability analysis of non-smooth systems with applications to control engineering," *International Journal of Non-Linear Mechanics*, vol. 36, no. 7, p. 1153–1161, October 2001.

- [85] D. Shevitz and B. Paden, "Lyapunov stability theory of nonsmooth systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 9, pp. 1910 - 1914, September 1994.
- [86] J. Slotine and W. Li, *Applied Nonlinear Control*, New Jersey: Prentice-Hall, 1990.
- [87] T. Tsuji, P. G. Morasso, K. Goto and K. Ito, "Human hand impedance characteristics during maintained posture," *Biological Cybernetics*, vol. 72, no. 6, pp. 475-485, May 1995.
- [88] M. Pierre and R. Kirsch, "Measuring dynamic characteristics of the human arm in three dimensional space," in *Proceedings of the Second Joint EMBSBMES Conference*, Houston, TX, USA, October 2002.
- [89] V. Banthia, A. Maddahi, K. Zareinia, S. Balakrishnan and N. Sepehri, "A Lyapunov Stable Controller for Bilateral Haptic Teleoperation of Single-Rod Hydraulic Actuators Subjected to Base Disturbance," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 141, no. 3, pp. 031013 - 031033, 2019.
- [90] Q. Wu, A. B. Thornton-Trump and N. Sepehri, "Lyapunov stability control of inverted pendulums with general base point motion," *International Journal of Non-Linear Mechanics*, vol. 33, no. 5, pp. 801-818, 15 September 1998.
- [91] W. Hahn, *Theory and Application of Lyapunov's Direct Method*, Englewood, NJ: Prentice Hall, 1963.
- [92] C. Wronka and M. Dunnigan, "Derivation and analysis of a dynamic model of a robotic manipulator on a moving base," *Robotics and Autonomous Systems*, vol. 59, no. 10, pp. 758-769, October 2011.

APPENDIX

A.1

Nonlinear PI controller:

$$u(t) = K_p e(t) + K_I I(t) \quad (\text{A.1})$$

$$I(t) = (I(t - \Delta t) + e(t)\Delta t) \frac{\alpha}{\alpha + \dot{e}^2(t)} \quad (\text{A.2})$$

where,

$$\alpha(\dot{q}_s) = \alpha_{max} (0.001 + \frac{|\dot{q}_s|}{\dot{q}_{max}}) \quad (\text{A.3})$$

where $u(t)$ is the control signal; $e(t)$ is error between the joint position, $q(t)$, and the joint set-point, $q_s(t)$; K_p is proportional gain; K_I is the integral gain; $I(t)$ is the time integral of $e(t)$; α is an arbitrary constant; $\dot{q}_{max}$ is maximum velocity; velocity error, $\dot{e}(t) = \dot{q}_s(t) - \dot{q}(t)$, where $\dot{q}_s(t)$ denotes the set-point velocity. The values of gains are as follows:

Proportional gains $K_p[5] = \{0.25, 1.0, 0.35, 0.25, 0.08\}$;

Integral gains $K_I[5] = \{1.8, 0.5, 1.6, 1.8, 1.6\}$;

$\alpha_{max}[5] = \{80000.0, 35000.0, 15000.0, 102000.0, 80000.0\}$;

$\dot{q}_{max}[5] = \{100.0, 80.0, 140.0, 220.0, 200.0\}$.

A.2

In this appendix, some terms used in stability are defined first and then Lyapunov's stability is explained [86].

Notations used:

$\forall$: for any; $\exists$: there exist; $\in$: in the set; $\Rightarrow$: implies that; $\Leftrightarrow$: equivalent

Nonlinear systems

A nonlinear dynamic system can usually be presented by the set of nonlinear differential equations in the form:

$$\dot{x} = f(x, t) \quad (\text{A.4})$$

where, $f \in \mathbb{R}^n$: nonlinear vector function

$x \in \mathbb{R}^n$: state vectors

n : order of the system

A special class of nonlinear systems is linear system. The dynamics of linear systems are of the form $\dot{x} = A(t)x$ with $A \in \mathbb{R}^{n \times n}$.

Autonomous and non-autonomous systems

Linear systems are classified as either time-varying or time-invariant. For nonlinear systems, these adjectives are replaced by autonomous and non-autonomous.

The nonlinear system (A.4) is said to be autonomous if f does not depend explicitly on time, *i.e.*, if the system's state equation can be written

$$\dot{x} = f(x) \quad (\text{A.5})$$

Otherwise, the system is called non-autonomous.

Equilibrium point

It is possible for a system trajectory to correspond to only a single point. Such a point is called an equilibrium point. Many stability problems are naturally formulated with respect to equilibrium points.

A state x^* is an equilibrium state (or equilibrium points) of the system if once $x(t)$ is equal to x^* , it remains equal to x^* for all future time.

Mathematically, this means that the constant vector x^* satisfies

$$0 = f(x^*) \quad (\text{A.6})$$

Equilibrium points can be found using the above nonlinear algebraic equation.

A linear time-invariant system,

$$\dot{x} = Ax \quad (\text{A.7})$$

has a single equilibrium point (the origin 0) if A is nonsingular. If A is singular, it has an infinity of equilibrium points, which contained in the null-space of the matrix A , i.e., the subspace defined by $Ax = 0$. A nonlinear system can have several (or infinitely many) isolated equilibrium points. We usually transform the system differential equations in such a way to have the equilibrium point as the origin of the state space.

Stability and instability

The equilibrium state $x = 0$ is said to be stable (in the sense of Lyapunov) if, for any $R > 0$, there exist $r > 0$, such that if $\|x(0)\| \leq r$ then $\|x(t)\| \leq R$ for all $t \geq 0$. Otherwise, the equilibrium point is unstable. In other words, Lyapunov stability of an equilibrium means that solutions starting "close enough" to the equilibrium remain "close enough" forever.

An equilibrium points 0 is asymptotically stable if it is stable (in the sense of Lyapunov), and if in addition there exist some $r > 0$ such that $\|x(0)\| \leq r$ implies that $x(t) \rightarrow 0$ as $t \rightarrow \infty$. Or in other words, asymptotic stability means that solutions that start close enough not only remain close enough but also eventually converge to the equilibrium.

Lyapunov's second method for stability

The basic idea of the Lyapunov's second method is that if the total energy of the electrical or mechanical system is continuously dissipated, then the system should eventually settle down at an equilibrium point. The Lyapunov's second method is commonly employed to prove the stability of nonlinear systems, by using a Lyapunov's function, V as follows. Consider a function $V(x) : \mathbb{R}^n \rightarrow \mathbb{R}$, with continuous first order derivative such that:

- $V(x) > 0$
- $V(x) = 0$ if and only if $x = 0$
- $\dot{V}(x) = \frac{d}{dt}V(x) \leq 0$

The first two conditions imply that the function $V(x)$ is positive definite, and the third condition implies that $V(x)$ with respect to time is negative semi-definite. In other words, if we can find such a scalar function that satisfies the above conditions, then the system is stable in the sense of Lyapunov. Finding a proper Lyapunov function is not a straightforward process for all systems and we may not be able to find one for a particular system. However, if one cannot be found for a system, it does not mean that the system is unstable, *i.e.*, failure of proof is not proof of failure.

A.3

In general, there are 3 approaches for Lyapunov's stability control design:

- 1) Design the controller, conduct stability analysis (construct a Lyapunov function);
- 2) Construct a Lyapunov function candidate, design a controller to validate the candidate;
- 3) Adjust both the controller and the Lyapunov function candidate simultaneously.

In this research, approach #2 was employed. There are few methods described in the literature for finding Lyapunov function candidates such as the variable gradient method, Krasovskii's method and the extended integral method; however, they are applicable only to specific systems. For most systems, the function of total energy is a good candidate for the Lyapunov function. Any Lyapunov function candidate must satisfy the following conditions:

- $V(x) > 0$
- $V(x) = 0$ if and only if $x = 0$
- $\dot{V}(x) = \frac{d}{dt}V(x) \leq 0$

To find a Lyapunov function for Eq. (3.52), once the equilibrium points were transformed to zeros, a quadratic function was considered as follows:

$$V(x_1, x_2, x_3, x_4, x_5, x_6) = \sum_{i=1}^6 x_i^2 \quad (\text{A.8})$$

or

$$V(x_1, x_2, x_3, x_4, x_5, x_6) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \quad (\text{A.9})$$

Next, the derivative of $V(x_1, x_2, x_3, x_4, x_5, x_6)$ is found:

$$\dot{V} = 2x_1\dot{x}_1 + 2x_2\dot{x}_2 + 2x_3\dot{x}_3 + 2x_4\dot{x}_4 + 2x_5\dot{x}_5 + 2x_6\dot{x}_6 \quad (\text{A.10})$$

Values of $x_1, x_2, x_3, x_4, x_5, x_6$ and $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5, \dot{x}_6$ are then substituted in Eq. (A.10). Having done this step, some terms will be remaining on the right-hand side of $\dot{V}$. In order to prove that $\dot{V}$ is negative semi definite, one needs to get rid of as many terms as possible in $\dot{V}$. There is a need for adding and modifying the coefficients of $x_1^2, x_2^2, x_3^2, x_4^2, x_5^2, x_6^2$ in the original function $V(x_1, x_2, x_3, x_4, x_5, x_6)$. This process requires several iterations and

must be repeated until all three conditions, mentioned above, are met. The leftover terms in $\dot{V}$ will form the control laws as presented in Eq. (5.1) and Eq. (5.2).

A.4

This appendix provides details about the steps followed to determine values of controller parameters K_{p2} and K_{p3} . According to Eq. (6.3),

$$\Delta = (1 + \alpha) \frac{P_s}{2} + \text{sgn}(x_{sp}) \left((1 - \alpha) \frac{P_s}{2} - P_L \right)$$

Thus, value of Δ is dependent on the values of $\text{sgn}(x_{sp})$ and P_L ; however, it will only change within a certain range. To find a constant value of gains K_{p2} and K_{p3} , an acceptable range for P_L is considered as $\frac{1}{3}P_s < P_L < \frac{2}{3}P_s$. Value of $\text{sgn}(x_{sp})$ can be either +1 or -1.

Thus, four limit values of Δ will be as follows:

- 1) When $\text{sgn}(x_{sp}) = +1$ and $P_L = \frac{1}{3}P_s$, $\Delta = \frac{2}{3}P_s$
- 2) When $\text{sgn}(x_{sp}) = +1$ and $P_L = \frac{2}{3}P_s$, $\Delta = \frac{1}{3}P_s$
- 3) When $\text{sgn}(x_{sp}) = -1$ and $P_L = \frac{1}{3}P_s$, $\Delta = (\alpha + \frac{1}{3})P_s$
- 4) When $\text{sgn}(x_{sp}) = -1$ and $P_L = \frac{2}{3}P_s$, $\Delta = (\alpha + \frac{2}{3})P_s$

Next, four values of Δ , obtained above, are substituted in Eq. (6.34), one at a time to find corresponding values for the right-hand side term of Eq. (6.34). Finally, one value is chosen for K_{p2} , which meets the inequality (6.34) for all values of Δ .

After K_{p2} is found, value of K_{p3} is obtained by putting value of K_{p1} , K_{p2} and different values of Δ in Eq. (6.35) according to above procedure. Then, one value is chosen for K_{p3} , which meets the inequality (6.35) for all values of Δ .