DESIGN OF A PFM DEVICE
FOR CONTROL APPLICATIONS

A Thesis
Submitted to
The Faculty of Graduate Studies
of
The University of Manitoba

In Partial Fulfilment
of the Requirements
for the Degree

Master of Science

Department of Electrical Engineering

Gordon Robert TRUSCOTT

April 1977

"DESIGN OF A PFM DEVICE
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by

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Abstract

The object of this thesis was the design of a pulse frequency modulator for control system applications. Closed loop systems with type 0 plants were considered. A mathematical analysis of the plant response to a pulse train of arbitrary fixed frequency was presented to facilitate the design. The resulting design was tested on both regulator and non regulator systems involving plants of first to third order. Results indicate the modulator provides a well regulated response relatively insensitive to plant parameter variations.

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ACKNOWLEDGMENTS

The author wishes to express his deep gratitude to Dr. S. Onyshko for suggesting the thesis topic as well as for his valuable guidance and encouragement during the course of this work. The author also wishes to acknowledge the National Research Council for financially supporting this work under Grant No. A8008.

Chapter I

INTRODUCTION

There has been an increased interest in the use of pulse control in recent years. This interest has been the result of a number of reasons, among them being: improved signal to noise ratios over analogue controllers, ease of physical implementation of the control element, more efficient modulation of large power sources, and the ease of encoding and decoding a pulse train into a digital signal for remote control applications.

The types of modulation schemes used in these pulse controllers have been in general either pulse-amplitude (PAM), pulse-width(PWM), pulse-position (PPM), pulse-frequency (PFM), or combinations of these four. Of the four basic schemes, pulse-frequency modulation is the most difficult to analyze because the sampling frequency is completely signal dependent and the modulating process is in general highly nonlinear. However, PFM does possess the advantages of uniform pulse size, and the control is not limited by a preset sampling or reference frequency.

1.1 Background

A great deal of interest in PFM was triggered by the discovery (5) in physiological systems of the existance of pulse trains along nerve fibers that possessed both logarithmic and direct relationships between frequency and stimulus intensity. It was thought that because the physiological system represents a high degree of perfection of feedback systems, PFM, being an important element in such a system, would offer advantages also to technical feedback systems. Consequently, various methods of modulating the pulse frequency were developed.

There are basically two types of PFM schemes in existance to date, commonly known as PFM of the first and second kind. Systems employing PFM of the first kind, are systems in which the value of the modulated parameter T_n (time to next pulse, sampling interval) at the n^{th} instant $t = t_n$, depends only on the value of the modulating function $\sigma(t)$ at that same instant: $T_n = f(\sigma(t_n))$ (see Figure 1.1). Clark (2) was one of the first to investigate this type of modulation and proposed a number of possible functions (f) to be applied to the sampled input $(\sigma(t_n))$. The development of state transition equations for systems whose plants had both real and imaginary roots was carried out.

A modified version of Liapunov's second method was used to analyze stability and show the existence of a limit annulus for steady state operation of certain systems.

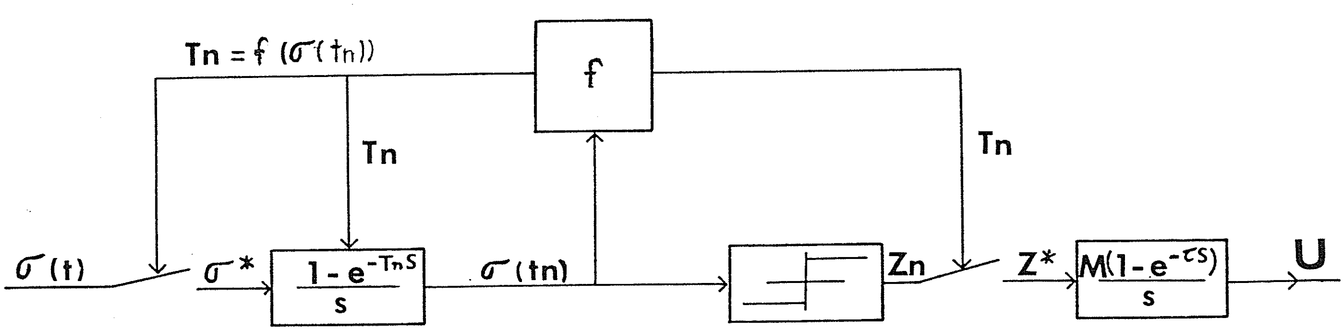

Figure 1.1 PFM of the first kind

Pulse frequency modulators employing modulation of the second kind, are schemes in which the modulated parameter p is determined by a certain function on the modulator input $\sigma(t)$; $p(t) = f(\sigma(t), Z^*)$ (Figure 1.2).

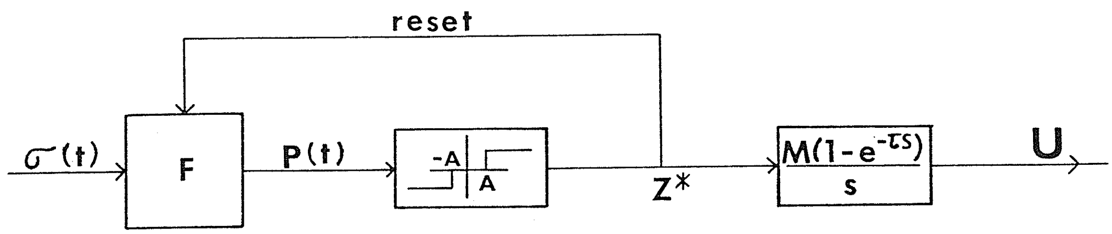

Figure 1.2 PFM of the second kind

Modulators of this type emit a pulse whenever the modulated parameter p reaches a preset threshold ($\pm A$) and then a reset operation is performed on p. The generalized functional law for PFM of the second kind is described by the following equations:

$$\frac{dp}{dt} + g(p) = \sigma(t) - A\ \mathrm{sgn}(p)\ \delta\ (|p| - A) \tag{1.1}$$

$$Z^* = \mathrm{sgn}(p)\ \delta\ (|p| - A) \tag{1.2}$$

Referring to Equation (1.1), the case when g(p) is a continuous odd nondecreasing function of p has been defined (15) as sigma-pulse frequency modulation (ΣPFM). This setting of g(p) corresponds to the passing of the modulating signal $\sigma(t)$ through a first order low pass filter and emitting a pulse whenever the output reaches a preset threshold. Pavlidis and Jury (5) have studied the use of this type of modulator in control systems and its suitability as a model for physiological systems. The study in control systems included the dynamic response analysis of the modulator and a Liapunov stability study of a feedback system employing a sigma-pulse frequency modulator. Their main concern, however, was the investigation of sustained oscillations using a specially developed quasi describing function for the modulator.

ΣPFM systems with g(p) (Equation (1.1)) set equal to zero are known as integral pulse frequency modulation (IPFM) systems. Both Li (10) and Meyer (11) have studied the application of IPFM to control systems, investigating such topics as transient response, stability, and the effects of noise. Farrenkoph, Sabroff and Wheeler (3) applied an IPFM system to space vehicle attitude control, and Murphy and West (2) have studied its use in an outer loop around an existing system to obtain an adaptive autopilot for military aircraft.

Limited work has been done in the area of optimal control systems. A study by Pavlidis (14) concerned the minimum time and fuel problem for a PFM system, the derivation of the optimal control being achieved by a heuristic argument. Onyshko and Noges (13) attacked the problem of finding the optimal PFM control function for a system by means of Pontryagin's Maximum Principle and Dynamic Programming. However, the actual design of an optimal pulse frequency modulator was not attempted, as the design would depend on the specific system under consideration.

Various Russian authors (6), (7), (8), (9), (16), have made significant contributions in the area of stability of PFM control systems. The majority of these studies involved PFM of the second kind and employed Liapunov stability analysis.

1.2 Motivation

PFM schemes of the first and second kind, although interesting from an analytical point of view, are not in general practically applied to the modulation of control system error. When employed in this context, it has been shown (2), (10), (11), (15), that the steady-state performance of the system is highly dependant on the modulator and plant parameters, and usually takes the form of irregular oscillations. It is possible to decrease the amplitude of these limit oscillations by appropriate settings of the modulator parameters, but this results in poor transient performance. These irregular steady-state oscillations are primarily the result of the modulator over driving the plant in one direction, and then compensating for this overcontrol by driving the plant with pulses of the opposite polarity. This type of performance, besides producing large steady state system error, degrades the inherent high power efficiency of pulse modulation through overcontrol of the plant.

The typical performance of these PFM devices when used as control system error modulators is not surprising when it is realized that they were not developed with this object in mind. IPFM was originally proposed because it lent itself to mathematical analysis (11) and ΣPFM was introduced as a mathematical generalization of IPFM (15). PFM of the first kind was developed because the author was interested in a pulse frequency modulator which sampled the input signal (2).

In view of the existing pulse frequency modulator's limitation when used for control system error modulation, it was felt that there was a need for a PFM element designed specifically for this task. The resulting design would hopefully not display the short-comings of existing pulse frequency modulators, but rather: complement the natural efficiency of pulse modulation in controlling large power sources by conserving the control effort, minimize some performance index based on the steady state error (independent of plant parameters), and maintain acceptable transient performance.

1.3 Outline of Analysis

The objective of this investigation is the design of a PFM control system error modulator which is efficient, minimizes a steady-state error performance index, and maintains acceptable transient performance. The only assumption made about the modulator (prior to the design) is that it emits identically shaped rectangular pulses of either polarity. Because the control object, which the modulator is to precede, is assumed to be a single-input-single-output linear time invariant plant, it was possible to use a heuristic approach to the design of the modulator. For the sake of clarity and brevity only the fundamental design techniques and decisions are presented.

Prior to the actual design of the modulator, it is necessary to specify the characteristics of the system it is to be associated with. This is accomplished in Chapter 2. The discussion of the exact nature of the performance index to be minimized, the development of the open loop system response leading to the modulator design and the design of the prototype PFM modulator is presented in Chapter 3.

Physical realization of the prototype modulator, final design modifications, and system simulation are presented in Chapter 4. The conclusions follow in Chapter 5.

Chapter 2

SYSTEM CONFIGURATION

2.1 Preliminary Remarks

The purpose of this study has been stated as being the design of a PFM device to be used to modulate control system error, Figure 2.1.1. It is well to realize in the early design stages of any system controller that it is not possible to create a device which has universal applicability, that controls every type of plant for every type of reference input. This is especially true for nonlinear controllers, to which class PFM belongs. Therefore, it was felt advisable to precede the design of the modulator with a specification of the system it is to be associated with. In particular, specifications will be placed on the nature of the reference input r, the form of the control u, and the plant G. These specifications will be derived on the basis of physical limitations of pulse modulation, stability arguments, and the design objectives outlined in Section 1.3. They are not meant to be binding restrictions on the modulation's use, but rather are intended as a logical framework for the design to evolve from.

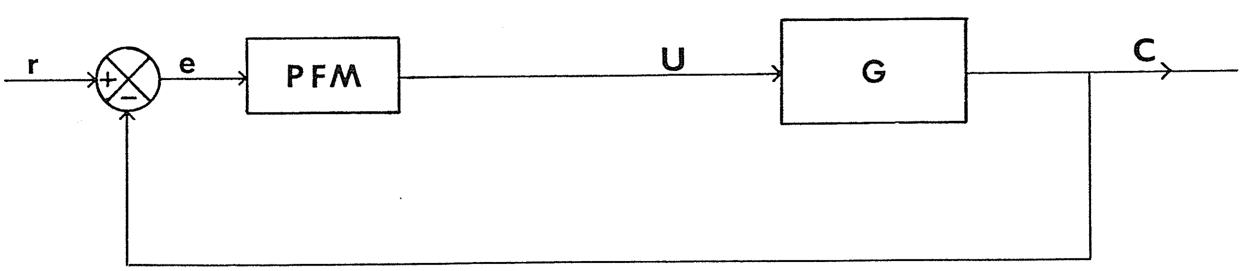

Figure 2.1.1. PFM control system

2.2 System Input

In Chapter 1 it was stated that the modulator emits identically shaped rectangular pulses of either polarity. An integral part of the modulator will therefore be some device which generates a pulse when triggered to do so. A common property of any physical element that produces pulses is the refractory period or dead time following the emission of a pulse during which it is impossible for the pulse element to generate another pulse. This refractory period in general increases as the amount of energy being carried by the pulse increases. Consequently, there are intervals of time following the initiation of each pulse during which the control function is not "controllable." That is, once a pulse is initiated the control cannot be altered for a period of time equal to the pulse width plus the dead time.

In view of this, it would appear that a PFM controller would not function well with systems for which the reference input was subject to sudden changes. Therefore, the system input r will be restricted to be of low frequency, in particular, the modulator will be designed for regulator systems, simplifying the design procedure somewhat.

2.3 Control Signal

One of the design objectives has been stated as being the conservation of the control effort. In order to realize this goal it will be necessary to make some preliminary assumptions about the control object or plant and determine the type of pulse control signal required by these plants for a regulated response.

The control object, a single-input-single-output linear time invariant plant, is assumed to exhibit a stable (i.e. bounded) response to the unit pulse control signal. This will limit the plant to be either of type 0 or type 1 having all poles situated in the left hand complex plane. The type designation refers to the number of poles located at the origin of this plane. As the plant response is being fed back and compared to the reference input, the plant is also required to give a net positive response to a positive pulse input.

A type 1 plant, due to the pole at the origin, will exhibit a non zero equilibrium state for a single pulse input. If excited with n pulses, the steady state response will be a constant equal to n times the single pulse steady-state response. Due to this multiplicity of non zero equilibrium states, the type 1 plant is well suited to pulse controlled regulation. These type 1 plants are quite adequately controlled by PFM of the second kind.

It has been shown [1], [15], [17] that by appropriate setting of the IPFM/ΣPFM parameters with respect to type 1 plant parameters, the system will exhibit zero steady-state error (i.e. perfect regulation) It was therefore felt to be not sufficiently rewarding to attempt to improve on this performance. Consequently, type 0 plants will be considered as the control objects in this study.

Type 0 plants exhibit only one equilibrium state, the origin. Upon termination of excitation to these systems the response will voluntarily relax to zero. Because of this fact, when these types of plants are employed in pulsed regulator application, they require a constant pulse frequency input to maintain the response oscillating about the reference input. The presence of both positive and negative pulses in the control signal, as is the case in PFM of the first and second kind, will quite likely only make the steady-state oscillations larger in magnitude. It would appear that the natural stability of these plants could be used to advantage in conserving the control effort. The pulse modulator need only drive the plant with pulses of the same polarity as the reference input to maintain the response at a non zero value. The presence of pulses of the opposite polarity is not required as the plant will relax in this direction naturally. Consequently in the interests of conserving the control effort and reducing the magnitude of the steady-state error, the control signal u will be restricted to be pulses of the same polarity as the reference input.

2.4 Control Object

The type 0 plants applicable to this study can be further divided into three categories as to whether the transfer function contains distinct, multiple or complex poles. For the case where the transfer function contains complex poles, the unit pulse response of the plant will be oscillatory if these complex poles are dominant and lightly damped. To maintain simplicity in the design of the modulator it was decided to concentrate on the control of "strictly stable" plants (non oscillatory unit pulse response). Therefore, type 0 plants with; i) distinct poles, ii) multiple poles, and iii) non dominant complex poles, or dominant heavily damped complex poles were considered when designing the PFM element. As the latter class (iii) can in general be successfully approximated by transfer functions involving only distinct or multiple poles, it will not be included in the open loop response analysis of the next chapter.

2.5 Summary

The system for which the PFM control element is to be designed has been defined to be of the regulator class involving type 0 plants. Furthermore, a pulse sign control law relating to the sign of the reference input (r) has also been determined.

Chapter 3

CONTROL STRATEGY

As yet, the design objectives have only been stated in general terms: to minimize a performance index of the steady-state error, to maintain acceptable transient performance, and to conserve the control effort. The decision to make the signs of the emitted pulses correspond to the sign of the reference input has somewhat ensured the latter. In order to realize the other objectives, it will be necessary to closely define what these objectives are, and also determine on what properties of the system error to operate.

3.1 Performance Specifications

System performance specfications normally include references to both the transient and steady-state system response to a specific reference input signal, usually a unit step. In general, it is found impossible to satisfy stringent specifications of both the transient and steady-state response simultaneously. Consequently, one set of specifications is usually given priority, and the setting of this priority is of course dependent on the intended use of the system. The system for which the pulse frequency modulator is to be designed is a regulator system and therefore the steady-state performance specifications are of primary importance.

Steady-state performance specifications are normally a function of the magnitude of the steady-state system error. A performance index for the PFM system based on this quantity would be indicative of the "goodness" of the system, however, the minimum value of this index would fluctuate with changes in plant or pulse parameters. What is desired, is a steady-state performance index the minimum value of which is independent of these parameters. A modulator which is designed to minimize such an index would then function equally well with all plants of the allowed class.

The index which was chosen is one which is a measure of the difference between the minimum and maximum values of the steady-state error oscillations.

$$P = [|\, e_{max} | - |\, e_{min} |] \text{ as } t \rightarrow \infty \tag{3.1.1}$$

The minimum of this "minimax" index (P=0) would indicate a mean value of zero for the maximum and minimum of the steady-state error oscillations (i.e. symmetric excursions of the plant response from the reference input).

Upon examination of the standard transient response specifications it is found that the majority of them are inapplicable to the system under consideration here. For instance, the commonly used transient response error performance indexes (integral square-error (ISE), integral-of time-multiplied square-error (ITSE), etc.) all increase without bound if the steady-state system error is non zero. This is the case for the PFM type 0 plant regulator system as it will exhibit steady-state oscillations. The transient solution time specifications (delay time, maximum overshoot, settling time, etc.) are equally hard to apply to the nonlinear system. The one solution time specification which is readily apparent in any system is the rise time. This is defined as the time required for the system error to achieve its first zero. As the steady-state performance is of primary importance, the PFM system will be required to exhibit as short a rise time as the satisfaction of the steady-state specifications will allow.

3.2 Open Loop Response

In order to achieve a modulator design which satisfies the design objectives it is necessary to know the nature of the information available to the modulator. Since the modulator is to be designed for a regulator system, the modulator input, which is the system error, is directly related to the system response. Obtaining the plant response to a train of positive rectangular pulses of arbitrary fixed frequency will therefore give an indication of the modulator input signal. In addition, an attempt is made to find the pulse frequency or period T which will result in a "minimax" response. This T can later be used as a comparison against the modulator action which is still to be designed. The type 0 plants applicable to this system have been limited to two cases, distinct and multiple poles. In this study, the pulse width, although finite, is assumed to be small in comparison to the smallest plant time constant. Consequently, for second and higher order plants, the pulses will be approximated by impulses, easing the analysis. First order plants, however, will be dealt with in a straight forward manner.

3.2.1 G(s) Containing Distinct Poles

In this section first order, and second and higher order plants are dealt with separately.

First Order Plant

A first order plant $G(s) = \frac{g}{s - \lambda}$ (3.2.1)

is to be excited by a control function of the form

$$U(t) = M \qquad nT \leq t \leq nT + \tau$$
$$= 0 \qquad nT + \tau < t < (n+1)\, T \tag{3.2.2}$$
$$n = 0, 1, 2, \ldots$$

That is, $U(t)$ consists of a train of rectangular pulses of magnitude M and width τ, which occur every T seconds.

The general open loop system and excitation is illustrated in Figure 3.2.1.

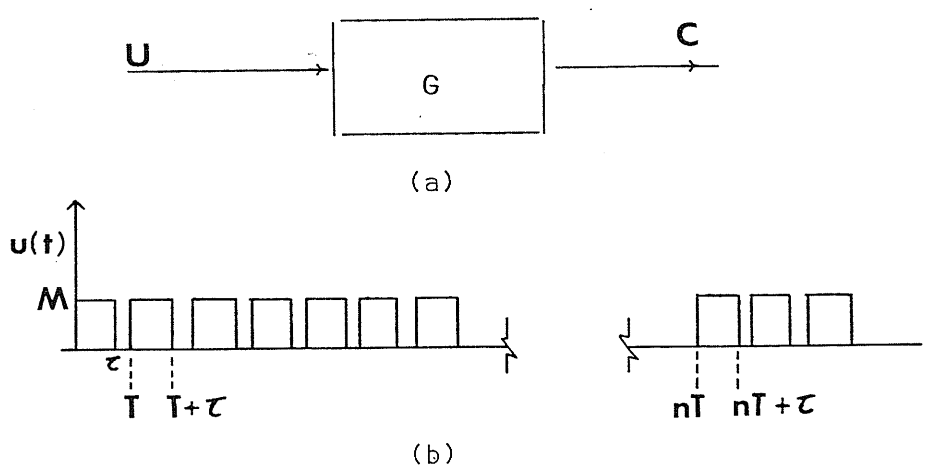

Figure 3.2.1 (a) Open loop system
(b) Control function u

The plant response resulting from an excitation of this form will be:

for $nT \leq t \leq nT + \tau$

$$C(t) = C(nT)e^{\lambda(t-nT)} + \frac{M}{\lambda} g \left(e^{\lambda(t-nT)} - 1\right) \tag{3.2.3}$$

and for $nT + \tau \leq t \leq (n + 1)T$

$$C(t) = C(nT)e^{\lambda(t-nT)} + \frac{M}{\lambda} g \; e^{\lambda(t-nT)}(1-e^{-\lambda\tau}) \tag{3.2.4}$$

where $C(nT)$ is the magnitude of the plant response at the time of the n^{th} pulse initiation.

The transition equation giving the value of the response at the beginning of one pulse period in terms of the response value at the beginning of the previous period is obtained by substituting $t = (n+1)T$ into Equation (3.2.4).

$$C((n+1)T) = C(nT)\; e^{\lambda T} + \frac{M}{\lambda} g \; e^{\lambda T}(1-e^{-\lambda\tau}) \tag{3.2.5}$$

Displaying the total response in graphical form, it would appear as in Figure 3.2.2.

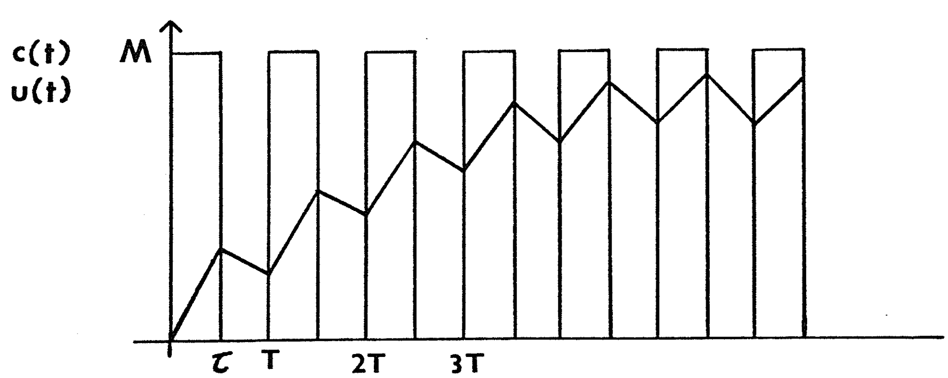

Figure 3.2.2 Open loop response, first order plant

As can be seen from this figure the maximum values of the response occur at the times $t = nT + \tau$ and are

$$C(nT + \tau) = C(nT)\, e^{\lambda\tau} + \frac{M}{\lambda}\, g\, (e^{\lambda\tau} - 1) \tag{3.2.6}$$

The minimum values of the response occur at the times $t = nT$ and have the values $C(nT)$.

In the limit as time approaches infinity (i.e. steady-state), these maxima and minima tend to constant values. For the minima, in the steady state,

$$C((n+1)T) = C(nT) \tag{3.2.7}$$

Using this equality in Equation 3.2.5. results in an expression for the minimum value of the plant response in terms of the plant and pulse parameters.

$$\underset{n \to \infty}{C_{min}(nT)} = \frac{M}{\lambda}\, g\, e^{\lambda T}\, \frac{(1 - e^{-\lambda\tau})}{(1 - e^{\lambda\tau})} \tag{3.2.8}$$

Substituting this value for $C_{min}(nT)$ into Equation 3.2.6. results in an expression for the steady-state maximum in terms of these same parameters

$$\underset{n \to \infty}{C_{max}(nT+\tau)} = \frac{M}{\lambda}\, \frac{(e^{\lambda\tau} - 1)}{(1 - e^{\lambda\tau})} \tag{3.2.9}$$

In order to zero the regulator system performance index, Equation 3.1.1., the system response must satisfy the relation,

$$\frac{C_{max} + C_{min}}{2.0} = r(t) \tag{3.2.10}$$

For a unit step reference input $r(t) = 1.0$ and by substituting Equations 3.2.9 and 3.2.8 into Equation 3.2.10, an expression for the pulse period required to "minimax" the system error is obtained

$$T = \frac{1}{\lambda} \text{ Ln } \left[\frac{2\lambda + M\, g\, (1 - e^{\lambda\tau})}{2\lambda + M\, g\, (1 - e^{-\lambda\tau})}\right] \tag{3.2.11}$$

For example, for a plant of the form

$$G(s) = \frac{1}{S+2} \tag{3.2.12}$$

subject to a pulse train with the parameters

$$\begin{aligned} M &= 5.0 \text{ units} \\ \tau &= 0.025 \text{ seconds} \end{aligned} \tag{3.2.13}$$

the pulse period required for a minimax response is found to be, $T =$ 0.062509 seconds.

n^{th} Order Plant

For an n^{th} order plant containing distinct poles the plant transfer function can be expressed in the partial fraction form.

$$G(s) = \frac{N(s)}{D(s)} = \sum_{i=1}^{m} \frac{g_i}{s-\lambda_i} \tag{3.2.14}$$

where

$$g_i = \left. \frac{N(s)}{\frac{d}{ds} D(s)} \right|_{s = \lambda_i} \tag{3.2.15}$$

The general open loop system representation of Figure 3.2.1.(a) can then be replaced by that of Figure 3.2.3.

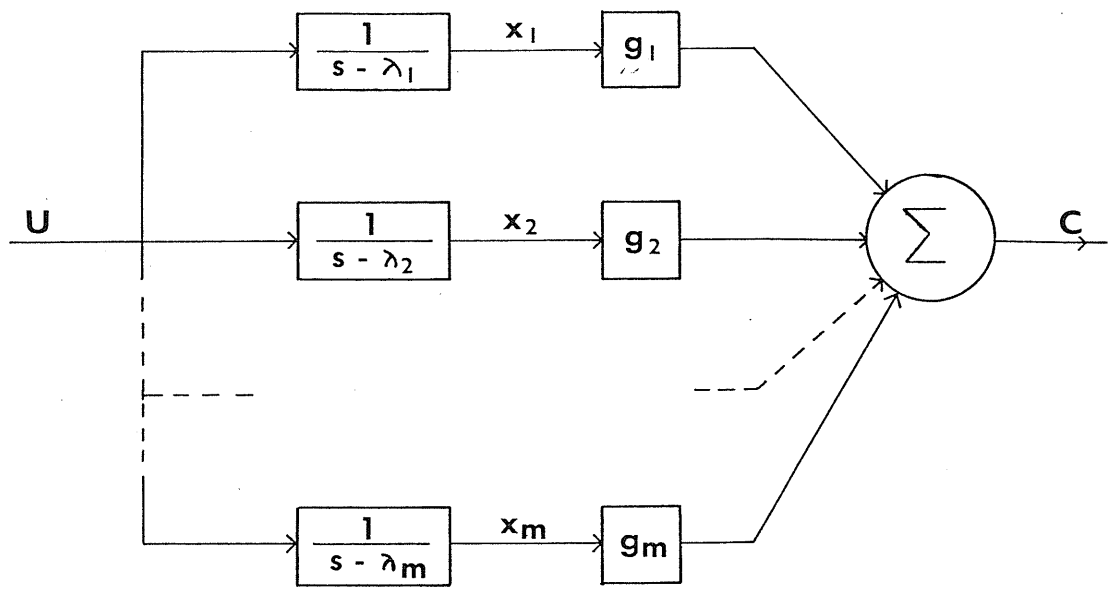

Figure 3.2.3 Distinct poles

Employing the impulse approximation, the plant response C to an excitation U of the form,

$$U(t) = (M\tau)\ \delta\ (t-nT) \qquad n = 0, 1, 2, \ldots \tag{3.2.16}$$

will be for $nT < t < (n + 1)T$,

$$c(t) = \sum_{i=1}^{m} g_i\ x_i\ (t) = \sum_{i=1}^{m} g_i\ (x_i(nT) + M\tau)e^{\lambda_i(t-nT)} \tag{3.2.17}$$

where the $x_i(nT)$ are the values of the state variables x_i at the times

nT^-. The distinction between nT^- and nT^+ is required as the x_i are discontinuous at the moments of impulse application.

Provided that the impulse period T is greater than the largest natural frequency of the plant, the response, $C(t)$ will appear as in Figure 3.2.4.

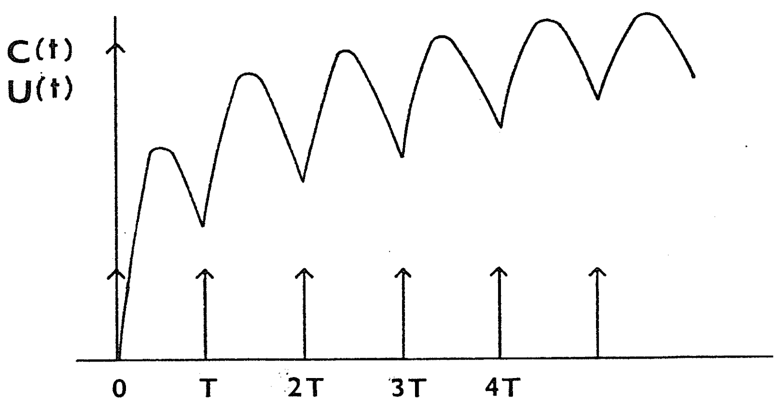

Figure 3.2.4 n^{th} order distinct, impulse response

In order to obtain an analytic expression for the impulse period T required to "minimax" the response, an expression for the steady-state maxima and minima is necessary.

Assuming the peak values of the response occurs in the interpulse period, the maxima are found by setting the first derivative of Equation (3.2.17) to zero.

$$\dot{C}(t) = 0 = \sum_{i=1}^{m} \lambda_i g_i \,(x_i(nT) + M\tau)e^{\lambda_i(t-nT)} \qquad (3.2.18)$$

Separation of the residues g_i into positive and negative components as follows,

$$g_k = g_i \qquad \text{for all } g_i > 0 \qquad r \text{ components}$$

$$g_i = (g_i) \qquad \text{for all } g_i < 0 \qquad s \text{ components}$$

results in an expression for the times of peak values of the response.

$$(t-nT) = \frac{1}{\sum\limits_j \lambda_j - \sum\limits_k \lambda_k} \ln\left[\frac{\prod\limits_k^r (|\lambda_k|\, g_k(X_k(nT)+M\tau))}{\prod\limits_j^s (|\lambda_j|\, g_i(X_j(nT)+M\tau))}\right] \qquad (3.2.20)$$

In this expression the λ_j, X_j are the eigenvalues and the state variables corresponding to the g_j. The λ_k and X_k have a similar relationship with the g_k.

The expression for the maximum of the steady-state response is then obtained by substituting (3.2.20) into Equation (3.2.17).

$$\underset{n \to \infty}{C_{max}(nT)} = \sum_{i=1}^{m} g_i \; (X_i(nT) + M\tau)\left[\frac{\prod\limits_k (|\lambda_k| g_k (X_k(nT) + M\tau))}{\prod\limits_j (|\lambda_j| g_j (X_j(nT) + M\tau))}\right]^{\frac{\lambda_i}{\sum\limits_j \lambda_j - \sum\limits_k \lambda_k}}$$

(3.2.21)

In this expression, the $X_i(nT)$ are the values of the state variables at the time nT^- as $n \to \infty$. As the minimums of the response occur at the times of impulse application, these values are also the minimum values.

In order to obtain a relationship between these minimum values of these state variables and the plant and pulse parameters, it is convenient to employ the Z transform as it gives an expression for the plant response at the impulse times. Employing the final value theorem in Z transform form results in an expression for the values of these state variables as follows,

$$\lim_{n \to \infty} X_i(nT) = \frac{M\tau}{1 - e^{\lambda_i T}} \tag{3.2.22}$$

Therefore the minimum value of the plant response C in the steady state will be

$$\lim_{n \to \infty} C_{min}(nT) = \sum_{i=1}^{m} g_i \frac{M\tau}{1 - e^{\lambda_i T}} \tag{3.2.23}$$

As with the first order plants, these values for C_{max}(3.2.21) and C_{min}(3.2.23) may be substituted into Equation 3.2.10 with $r(t)$ a unit step, and the impulse period T required to minimax the unit step regulator system response can be found. This requires the search for roots of a high order polynomial in e^{-T}.

For example, when a second order plant of the form

$$G(s) = \frac{1}{(S+1)(S+2)} \tag{3.2.24}$$

is excited by a train of unit impulses, the pulse period required for "minimax" is found to be $T = 0.549$ seconds.

3.2.2 Multiple Poles

The response of a plant containing one multiple pole of order r is examined in the section. The plant transfer function can be expressed in partial fraction form as follows,

$$G(S) = \frac{N(S)}{D(S)} = \frac{g_i}{(S-\lambda_1)^r} + \frac{g_2}{(S-\lambda_1)^{r-1}} \cdots \frac{g_r}{(S-\lambda_1)} + \frac{g_{r+1}}{(S-\lambda_{r+1})} \cdots \frac{g_m}{S-\lambda_m} \tag{3.2.25}$$

where

$$g_i = \frac{1}{(i-1)!}\frac{d^{i-1}}{ds^{i-1}}[(S-\lambda_1)^r G(S)]_{S=\lambda_1} \qquad (3.2.26)$$

$$i = 1, 2 \ldots r$$

and

$$g_i = \left.\frac{N(S)}{\frac{d}{ds}D(S)}\right|_{S=\lambda_i} \qquad i = r+1, \ldots m \qquad (3.2.27)$$

The general open loop system representation of Figure 3.2.1(a) can then be replaced by that of Figure 3.2.5.

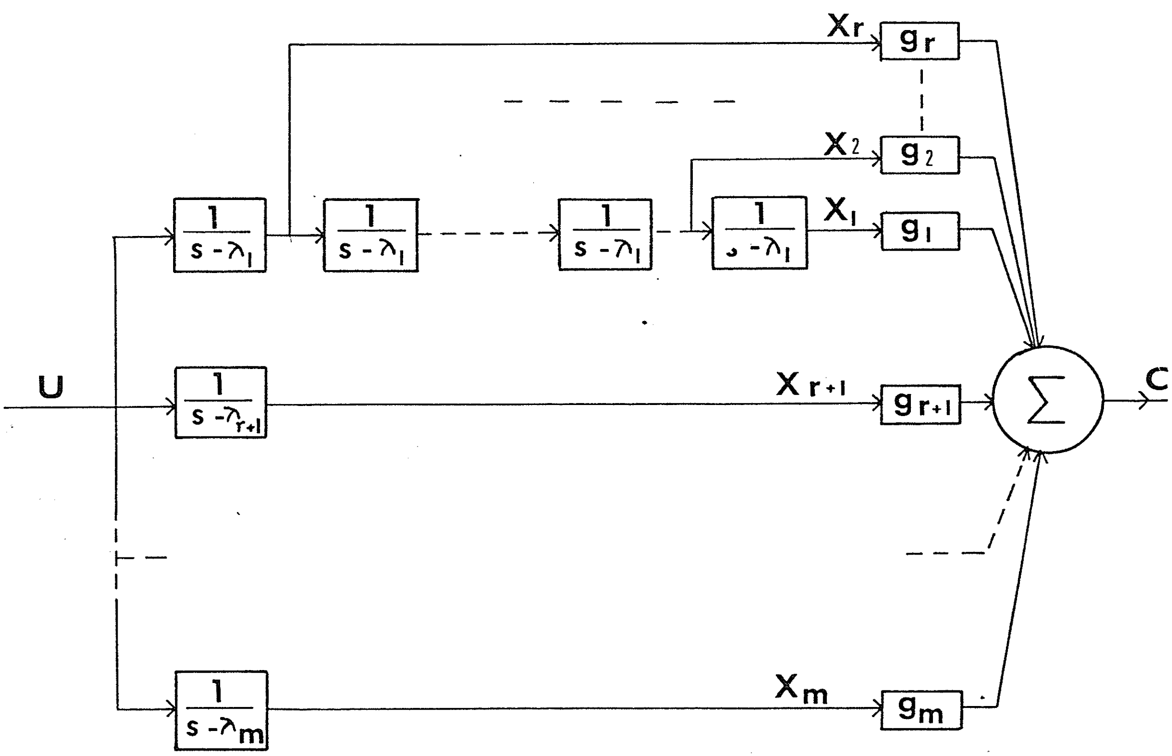

Figure 3.2.5 Multiple pole

If the plant is excited by a train of impulses T seconds apart and weight $M\tau$, then the plant response will be; for $nT < t < (n+1)T$

$$X_i(t) = (X_i(nT) + X_{i-1}(nT)(t-nT))e^{\lambda_i(t-nT)} + M\tau \frac{(t-nT)^{r-i}}{(r-i)!} e^{\lambda_i(t-nT)} \qquad i = 1, 2, \ldots r-1 \tag{3.2.27}$$

and

$$X_i(t) = (X_i(nT) + M\tau)\, e^{\lambda_i(t-nT)} \qquad i = r, r+1 \ldots m \tag{3.2.28}$$

The values $X_i(nT)$ are the magnitudes of the state variables at the times $t = nT$.

The plant response $C(t)$ will be equal to the sum of these state variables multiplied by their associated residues,

$$C(t) = \sum_{i=1}^{m} g_i \, X_i(t) \tag{3.2.29}$$

and will appear very similar to the response of the n^{th} order distinct plant, Figure 3.2.4.

Because the first $r-1$ state variables (Equation 3.2.27) contain time multiplied exponentials, it is much more difficult to develope general analytic expressions for the maxima, minima, and desired steady-state regulation pulse period T as was done in the first order and n^{th} order distinct cases. As this theoretical T is only used as a check against the modulator action, and is not a design factor, it was decided to be not sufficiently rewarding to develop an expression for this theoretical T.

3.3 Modulator Design, Control Strategy

The control strategy governing the signs of the emitted pulses has already been determined as being in direct correspondence with the sign of the reference input. The pulse frequency or emission time control law remains to be formulated. A reasonable approach to the determination of the pulse emission time is, as is done in PFM of the second kind, to use a threshold device which is activated when some function of the error signal reaches a threshold level (see Figure 3.3.1). Design of the modulator then consists of determining the functions to be applied to the system error to achieve the design objectives. Specifically, the design will require developing a threshold level filter (TLF), an error filter (EF), and a threshold device (TD) such that the regulator system error is "minimaxed".

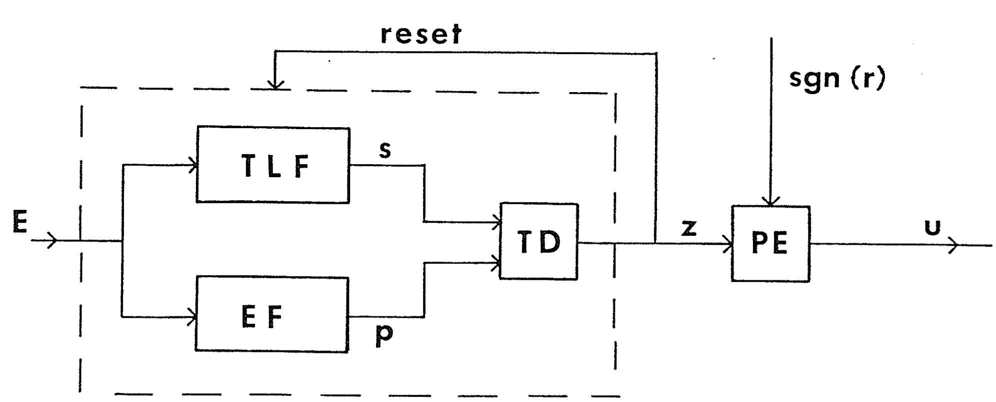

Figure 3.3.1 Block diagram of the modulator

By examining the open loop response curves presented in the previous section (Figures 3.2.2 and 3.2.4), it is seen that the plant pulse train response is characterized by its envelope. Minimization of the performance index equation 3.1.1 requires the modulator to emit pulses such that the steady-state response envelope is centred on the reference input and constant. For the pulse frequency modulator to accomplish this, it will require information not only on the system error, but also information about the steady-state error envelope. It was decided, therefore, to develop a threshold level filter set to some function of the error envelope, and to set the error filter for unity transmission. The modulator design problem therefore reduces to one of choosing the correct threshold level envelope and threshold device functions to achieve the "minimax" criteria.

When the PFM closed loop system is operating correctly, the error envelope in the steady-state should be symmetric about the zero axis. This would imply that, as the response minimums occur at the moments of pulse/impulse application, a threshold level equal to the mean value of the error envelope magnitude (2S in Figure 3.3.2) would maintain this envelope in its steady-state position. The emission of a pulse/impulse when the system error is equal to the threshold level (s) should force the error below its zero axis by a amount equal to what it was above. For the transient stage of the response it will be necessary to emit pulses if the error is greater than or equal to this threshold level and if a negative reference input is applied, this will read less than or equal to.

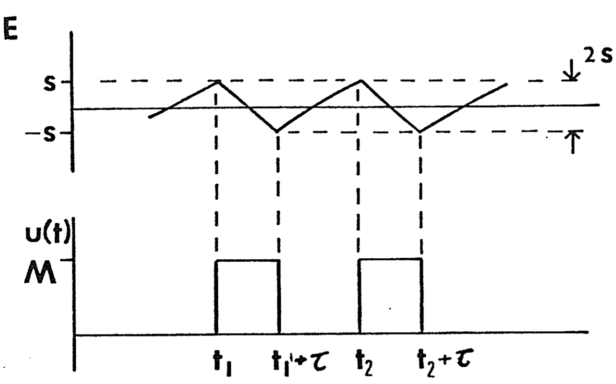

Figure 3.3.2 Steady-state threshold

Rather than design two modulators to cover the cases of positive and negative reference inputs, it was decided to modulate the error signal by the sign of the reference input.

i.e. $$E = \operatorname{sgn}(r)\,(r-c) \tag{3.3.1}$$

Then the threshold level filter (TLF in Figure 3.3.1) equation will be,

$$S = [E_{max}(\infty) - E_{min}(\infty)]/\ 2.0 \tag{3.3.2}$$

The error filter (EF) relation calls for unity transmission

$$p = E \tag{3.3.3}$$

and the threshold device (TD) equation is

$$\begin{aligned} Z &= 1 \quad \text{if} \quad E \geq S \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{3.3.4}$$

(a Z equal to 1 would initiate the emission of a pulse).

A modulator operating on this control strategy should minimax the steady - state error provided that the impulse approximation is valid, and $E_{max}(\infty)$ and $E_{min}(\infty)$ are available to the modulator.

Obtaining $E_{max}(\infty)$ and $E_{min}(\infty)$ is actually quite a simple matter if the system is in the steady-state. These values occur when the response velocity changes sign, an easily determined phenomenon. For the system to operate correctly from zero initial conditions, however, these steady-state values are required prior to being in the steady state. For the modulator to accomplish this, the following reasoning is employed. The magnitude of the error envelope during any particular interpulse interval is a function of the unit pulse response of the plant, plus the plant response to the initial conditions of state at the time of pulse application. When the system is in the steady - state, these initial conditions of state are identical for every pulse interval and the envelope will remain at a constant value. It would appear that the magnitude of the error envelope in the transient stage of the response could be used as an estimate of its own steady - state value. Inaccuracies of estimation result only from differing initial conditions of state at the time of pulse emission. As the system approaches the steady - state, these differences should vanish and the estimate become exact (see Figure 3.3.3). Consequently, the threshold level equation (3.3.2) need only be made recursive in nature to arrive at a solution to this problem. i.e.

$$S_{i+1} = [\max E_i(t) - \min E_i(t)]/2.0 \qquad (3.3.5)$$

In this Equation S_{i+1} is the threshold level for the $i+1^{th}$

pulse, and is a function of the system error during the time of the i^{th} pulse $E_i(t)$.

Because the threshold level filter requires the extremum values of the response, it will be necessary to block the threshold device from triggering a pulse until the system response has peaked. Therefore the threshold device Equation 3.3.4 will be modified to,

$$Z = 1 \quad \text{if} \quad E_i(t) \geq S_{i+1} \qquad \text{If} \quad \min E_i(t) \text{ has occured}$$
$$= 0 \quad \text{otherwise} \tag{3.3.6}$$

When a pulse of non negligible width is used to drive an n^{th} order distinct plant, the plant response will not be instantaneous (the minimum of the response will not occur at the moment of pulse initiation, Figure 3.3.4). Unless the threshold level filter takes this delay in action into account, the mean value of the steady-state error envelope will be offset from zero by an amount equal to the delay. The magnitude of the delay will be,

$$E(t_i) - \max E_i(t) \tag{3.3.7}$$

The delay compensated threshold level Equation 3.3.5 will then become,

$$S_{i+1} = E(t_i) - \max E_i(t) + [\max E_i(t) - \min E_i(t)]/2.0 \tag{3.3.8}$$

When there is no delay in the plant pulse response, $E(t_i)$ will be equal to $\max E_i(t)$ and the equation reduces to that of (3.3.5).

The use of a pulse frequency modulator operating by the control laws given above (Equations 3.3.1, 3.3.6, and 3.3.8) to drive any plant of the admissible class, should "minimax" the steady - state regulator system error (set the performance index to zero).

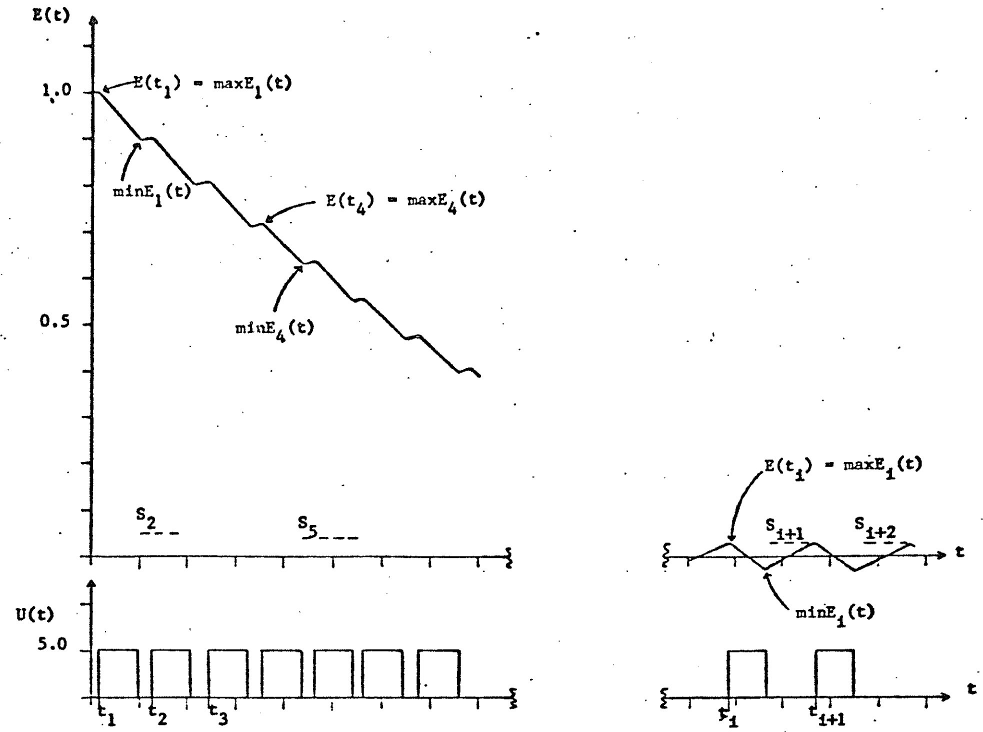

Figure 3.3.3 1^{st} order plant, system error E and control U.

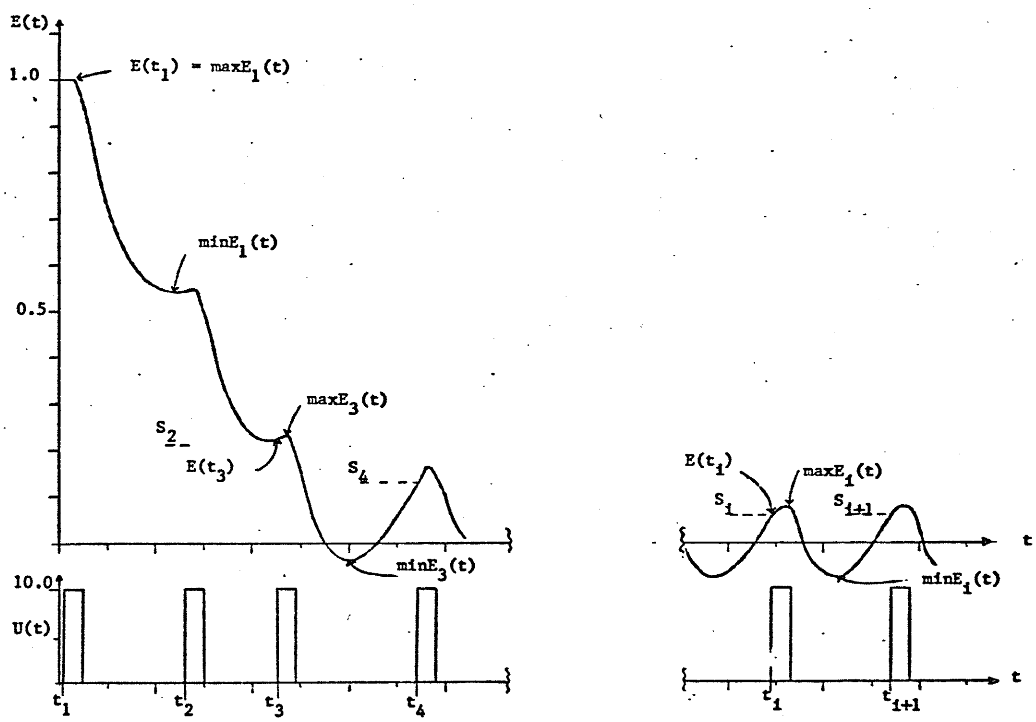

Figure 3.3.4 n^{th} order plant, system error E and control U.

Chapter IV

MODULATOR REALIZATION AND SYSTEM SIMULATION

4.1 Physical Realization of Control Strategy

The only component of any complexity in the modulator will be the threshold level filter, described by Equation (3.3.8). It will require the use of three memory elements plus a device which detects response extrema. The memory functions are easily realized by the use of sample and hold (S/H) devices which, due to recent advancements in hybrid linear integrated circuits, are inexpensive, fast, and accurate. To control the S/H devices (logic 1 places them in the track mode, 0 in the hold mode) to retain the maxima and minima, a device sensitive to the rate of change of the modulated system error (E) is required. The use of a differentiator for this purpose was rejected as these devices are quite noise sensitive. The comparator (CE), delay line (DELAY) circuit of Figure 4.1.1 was thought to be more reliable. The output of the comparator (y) assumes a logic value of 1 if the error is increasing, and is 0 otherwise. The signal y is fed into negative and positive edge triggered flipflops (FF_1 and FF_2 respectively) to obtain the required S/H control signals (b_1 and b_2). These memory elements are thereby programmed to track and hold the signal E. Upon initiation of a pulse (z=1) the flipflops are reset to logic 1 allowing the signal to be tracked again. The signal z also causes the third S/H device to track and hold the value of the error at the time of pulse initiation ($E(t_i)$). The threshold level s_{i+1} is obtained by a few simple operations on the outputs of these three sample and hold devices. The

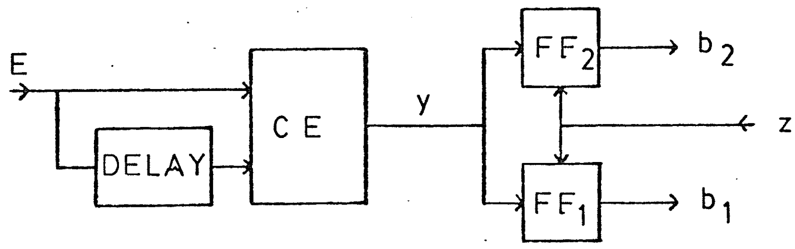

Figure 4.1.1 Device to indicate sgn($\frac{dE}{dt}$)

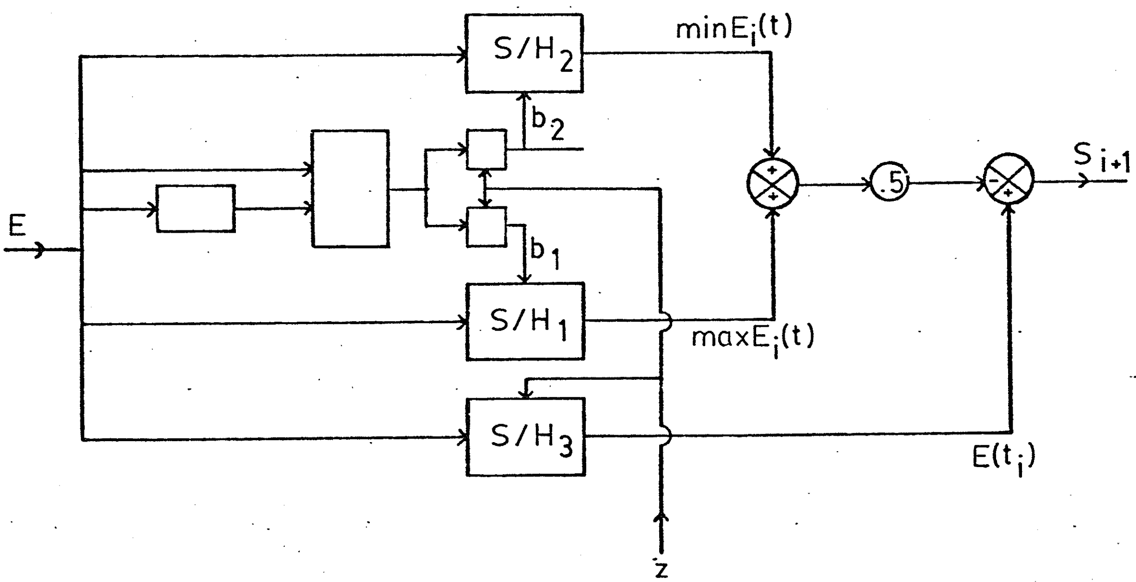

Figure 4.1.2 Threshold level filter

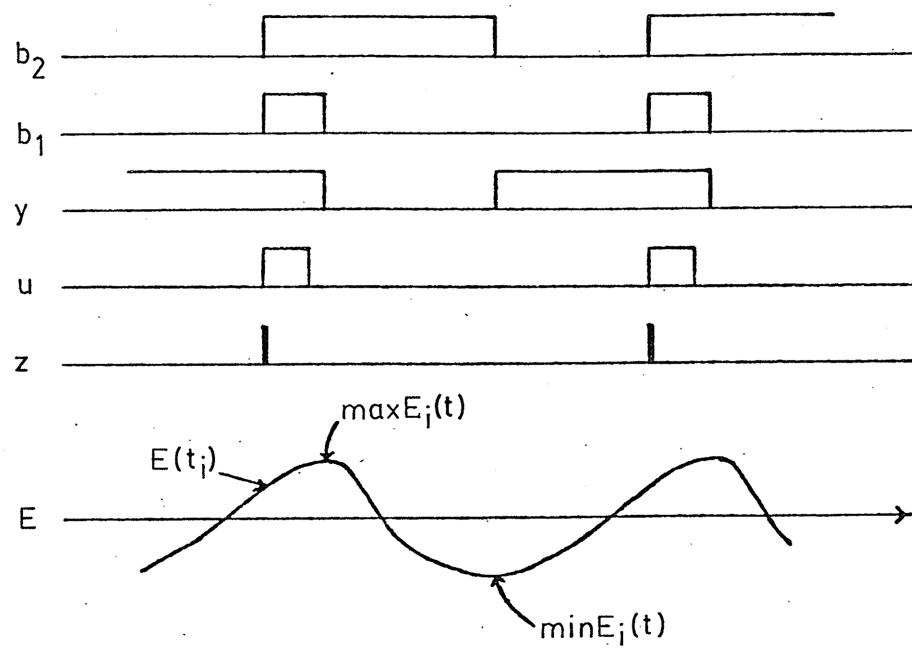

Figure 4.1.3 Threshold level filter timing diagram

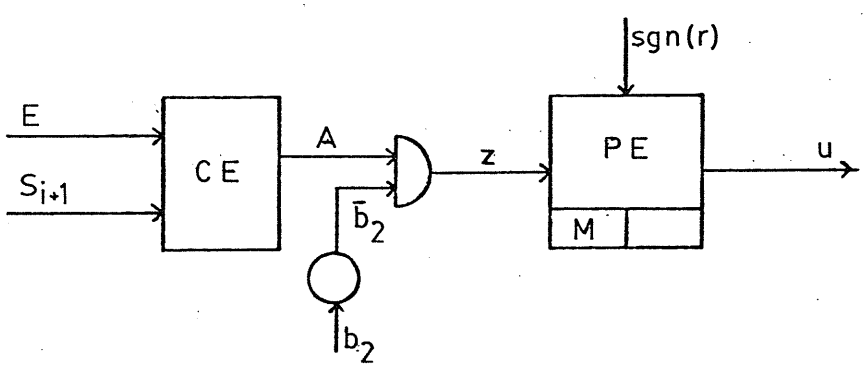

Figure 4.1.4 Threshold device and pulse element

block diagram of the threshold level filter and the timing diagram of the associated variables appears in Figures 4.1.2 and 4.1.3.

The threshold device relation (Equation 3.3.6) is implemented by the use of the comparator,AND gate configuration Figure 4.1.4. The comparator output A assumes a logic 1 if $E(t) \geq s_{i+1}$. This signal A is then ANDed with the complement of b_2 to ensure that no pulse is emitted before $\min E_i(t)$ has occurred. The output of the AND gate z is the signal which causes the actual pulse element (P.E.) to initiate a pulse of magnitude M, width τ and polarity sgn(r). At the same time the signal z resets the threshold level filter.

The complete PFM prototype and regulator system appears in Figure 4.1.5.

4.2 Modification of the Prototype

During the course of simulation trials of the prototype PFM regulator system it was found that under certain conditions of pulse size and plant parameters the system would not display the required zero of the performance index. For example, for the plant $G(s) = \dfrac{1}{s^2+2(.707)5s+5^2}$, when the pulse parameters were set at $M = 300.0$ units and $\tau = 0.02$ seconds, the regulator system response was as shown in Figure 4.2.1. As can be seen, although the modulator was not designed for plants with complex poles, the steady-state error is "minimaxed" as required. However, when the pulse magnitude was set at $M = 150.0$ units, the system response, illustrated in Figure 4.2.2, did not provide the required zero of the performance index, but rather, remained locked into the

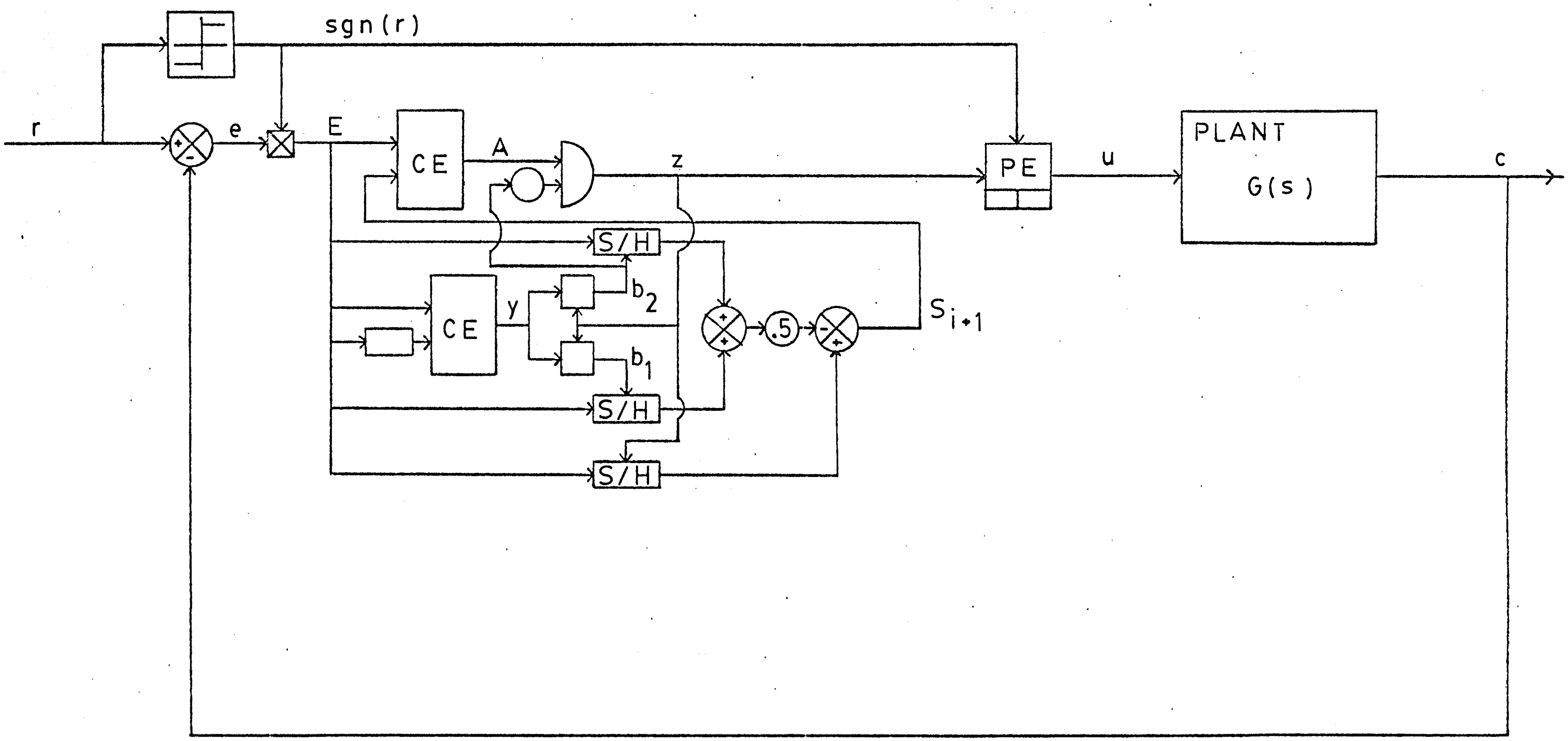

Figure 4.1.5 Prototype PFM control system

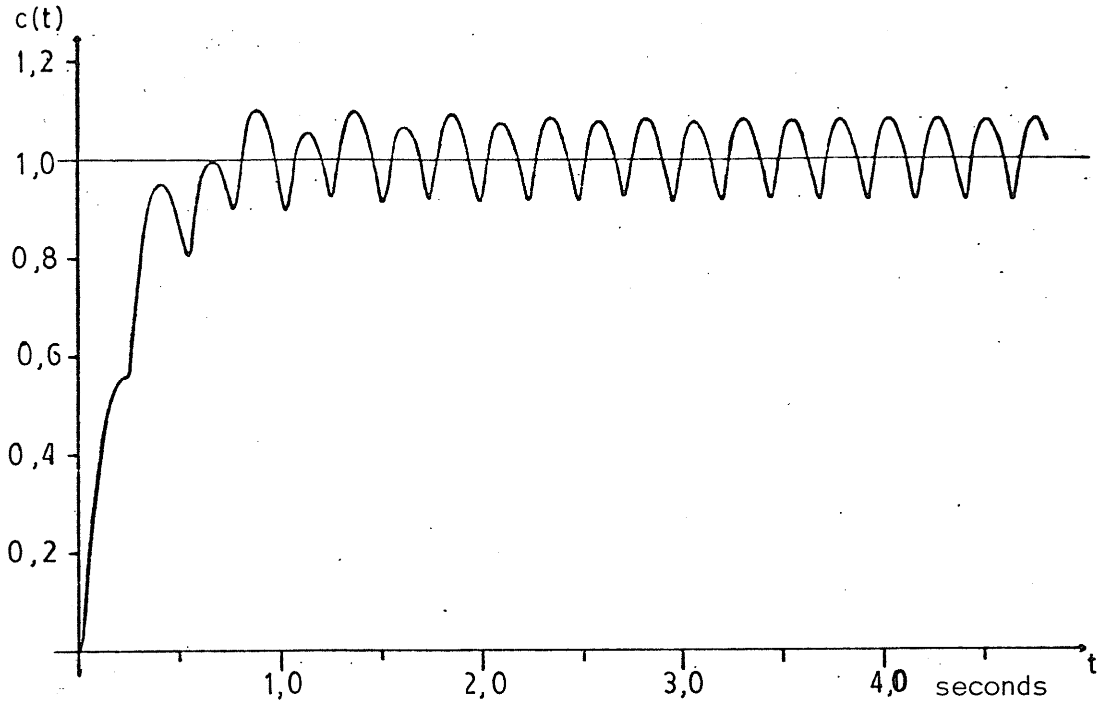

Figure 4.2.1 $G(s) = \dfrac{1}{s^2+2(0.707)5s+5^2}$, M = 300.0 units,

τ = 0.02 seconds

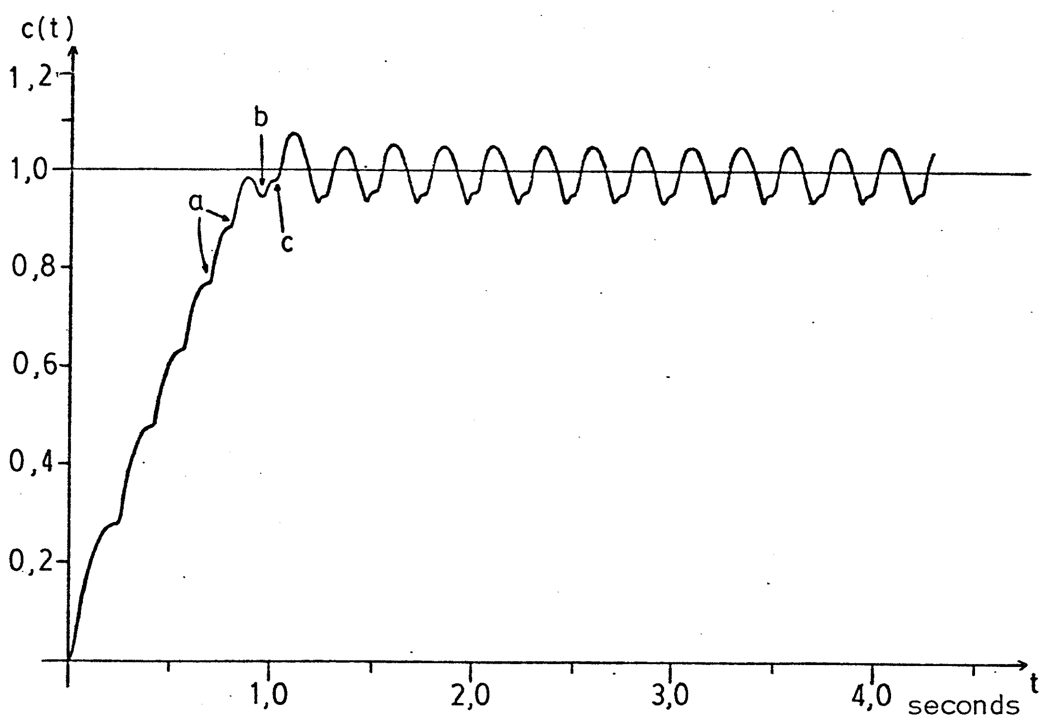

Figure 4.2.2 $G(s) = \dfrac{1}{s^2+2(0.707)5s+5^2}$, M = 150.0 units,

τ = 0.02 seconds

irregular oscillation shown. This phenomena was also seen to occur when the pulse magnitude was set at M = 200.0 units and M = 100.0 units, but failed to occur when M = 500.0 units.

It was reasoned that these irregular oscillations occurred because there was too sharp a transition between the envelope magnitude in the transient stage of the response, and its desired steady-state value. The emission of a pulse is controlled through the threshold level by the system output, but the system response to a single pulse can be quite sensitive to the rate of change of the system output.

Refering to Figure 4.2.2, in the transient stage of the response (point (a) in Figure), the plant is pulsed when its response velocity is near zero, consequently, the effect of these pulses is relatively large. In the transition from the transient to the steady - state stage of the response, point b, the threshold level is such that the plant is not pulsed until well after the response has peaked. The response velocity at this point is quite large in magnitude and when the small pulses are used (M = 100, 150, 200) their effect on the response envelope is significantly less then the previous pulse's. As a result, the estimated threshold level is quite small, and the plant is pulsed as soon as the plant response peaks, point c. As this latest pulse occurs when the response velocity is again near zero, it has a greater effect on the envelope magnitude. The estimate of the threshold level is then raised, causing the next pulse to be emitted when the response velocity is again significantly large. Consequently, the threshold level cycles between a large and a small value which never converge.

By inspection of the system response, it appeared that a threshold level value lying between these two extremes would be more appropriate. Therefore, a threshold level which was the average of the current estimate of the threshold level, $s_{i+1} = E(t_i) - (\max E_i(t) + \min E_i(t))$ and the previous threshold level s_i, was employed (i.e. $s = (\frac{s_i + s_{i+1}}{2})$. It should be emphasized that this average does not affect the original steady-state design conditions as in the steady-state s_i will be equal to s_{i+1}.

This modification to the control strategy only required a slight change in the existing physical realization of the modulator. The modified system appears in Figure 4.2.3 and the added elements appear in the dashed box.

This modified prototype was then used to drive the plant $G(s) = \frac{1}{s^2+2(.707)5s+5^2}$ with pulses of magnitude $M = 150.0$ units and $\tau = 0.02$ seconds. The system response is illustrated in Figure 4.2.4, and, as can be seen the system displays a "minimaxed" response. Comparing this response with that of the unmodified system Figure 4.2.2, it is also seen that the rise time of the response is largely unaffected. The modified system has a rise time of 1.055 seconds and the original system has a rise time of 1.025 seconds.

4.3 Simulation Results

The complete PFM control system of Figure 4.2.3 was simulated on a digital computer by means of the IBM S/360 Continuous System Modeling Program (S/360 CSMP). The simulation results fall into three categories. The first section presents system responses when excited by a unit step

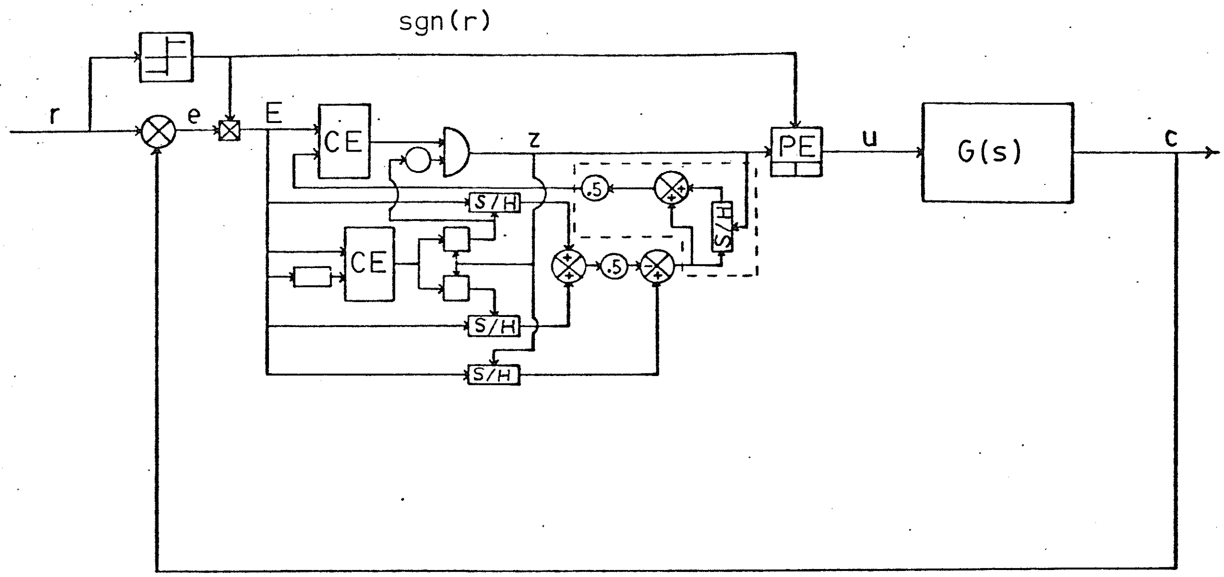

Figure 4.2.3 Modified PFM and regulator system

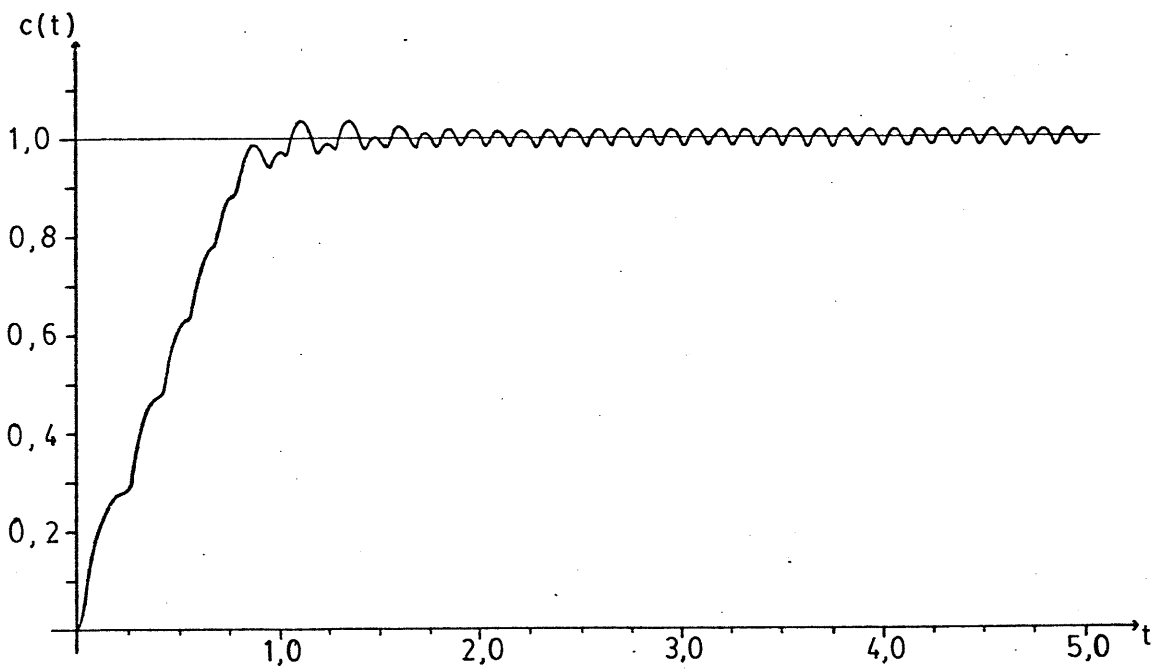

Figure 4.2.4 Modified system response, $G(s) = \dfrac{1}{s^2+2(.707)5s+5^2}$,

$M = 150.0$, $\tau = 0.02$

input. The systems involve type 0 plants from first to third order. A few examples of IPFM controlled systems are presented for comparison purposes.

In the second section the system input is allowed to vary with time, to be specific $r(t)$ is set equal to $\text{sine}(\frac{\pi}{6}t)$. The response of systems involving plants of first and second order is shown.

The final section presents the unit step regulator system response when a first order plant's gain and pole position are allowed to vary with time. Here again an example of an IPFM controlled system is presented for comparison purposes.

Regulator System With Unit Step Input

a) Trial 1 (Figure 4.3.1)

First order plant $G(s) = \frac{1}{s+2}$, pulse parameters, $M = 5.0$ units $\tau = 0.025$ seconds, dead time = 0.05 seconds. The plant and pulse parameters are the same as those used in Chapter 3 to calculate the theoretical pulse period T required for minimax. The experimental T was found to be 0.0625 seconds as opposed to the theoretical value of 0.062509 seconds. The difference between these two values was attributed to the effect of the fixed step size of the independent variable used in the computer program.

b) Trial 2 (Figure 4.3.2)

Second order plant, distinct poles $G(s) = \frac{1}{(s+1)(s+2)}$, pulse parameters; $M = 50.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds. The plant parameters are the same as those used in the second order example of Chapter 3, and the pulse area of 1 approximates the unit impulse.

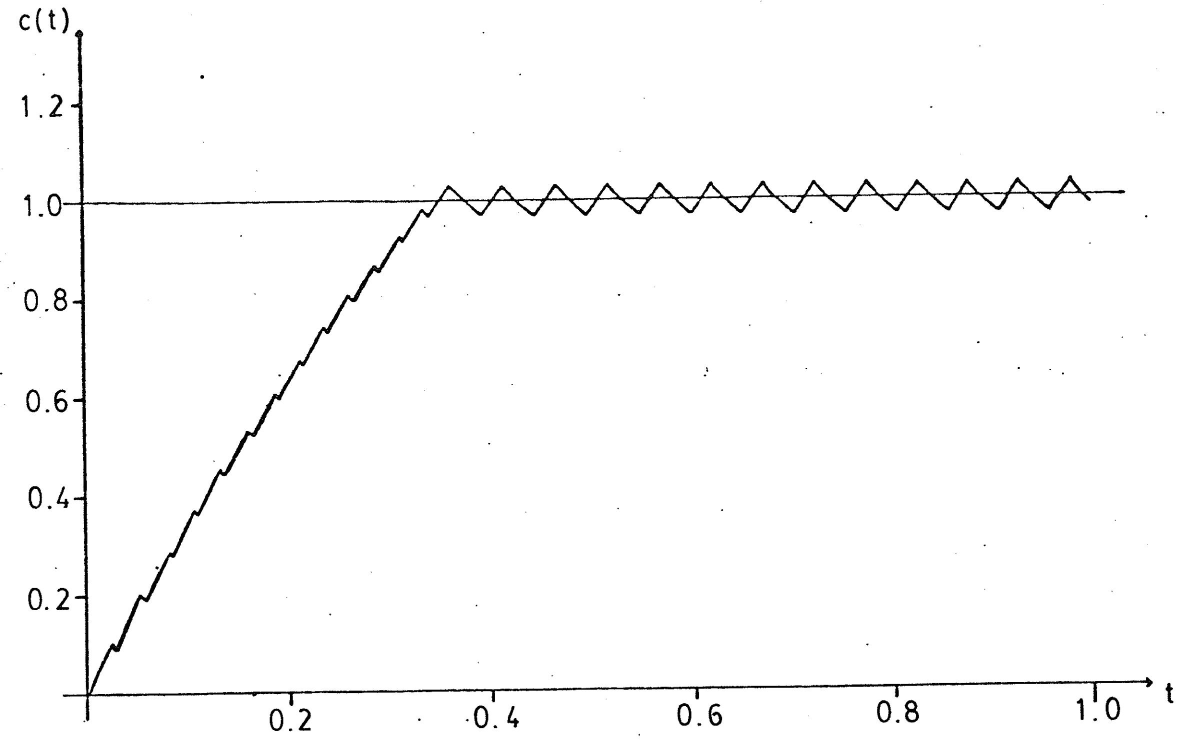

Figure 4.3.1 $G(s) = \frac{1}{s+2}$, $M = 5.0$, $\tau = 0.02$

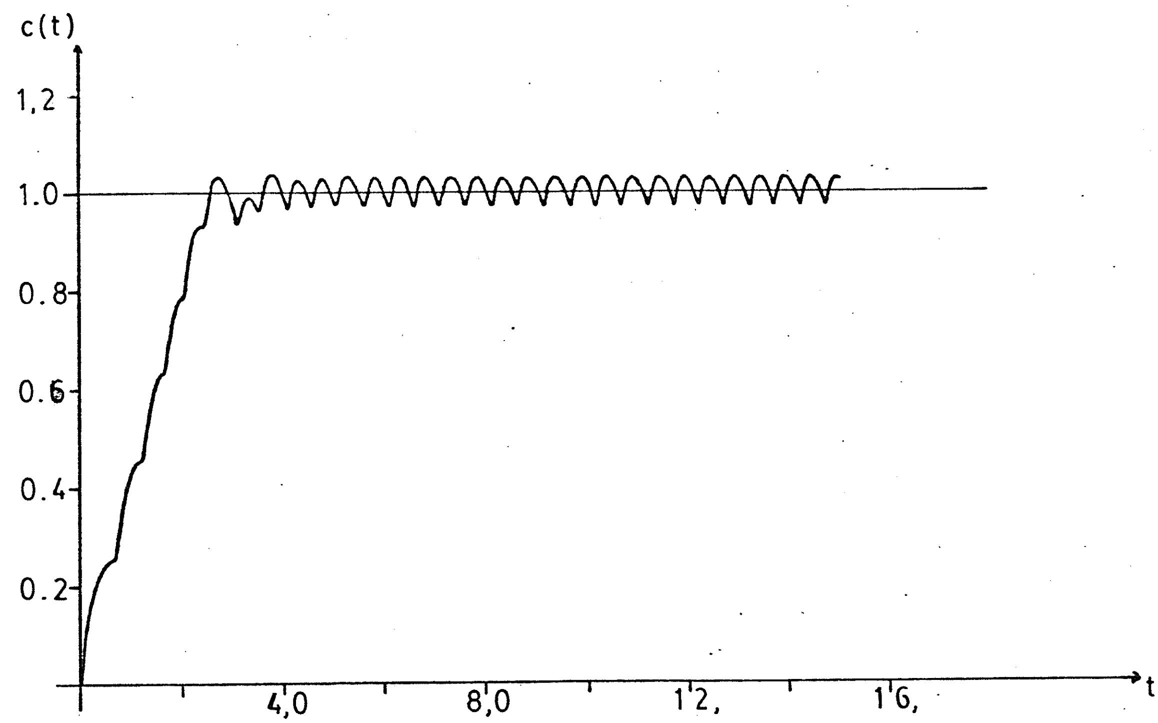

Figure 4.3.2 $G(s) = \frac{1}{(s+1)(s+2)}$, $M = 50.0$, $\tau = 0.02$

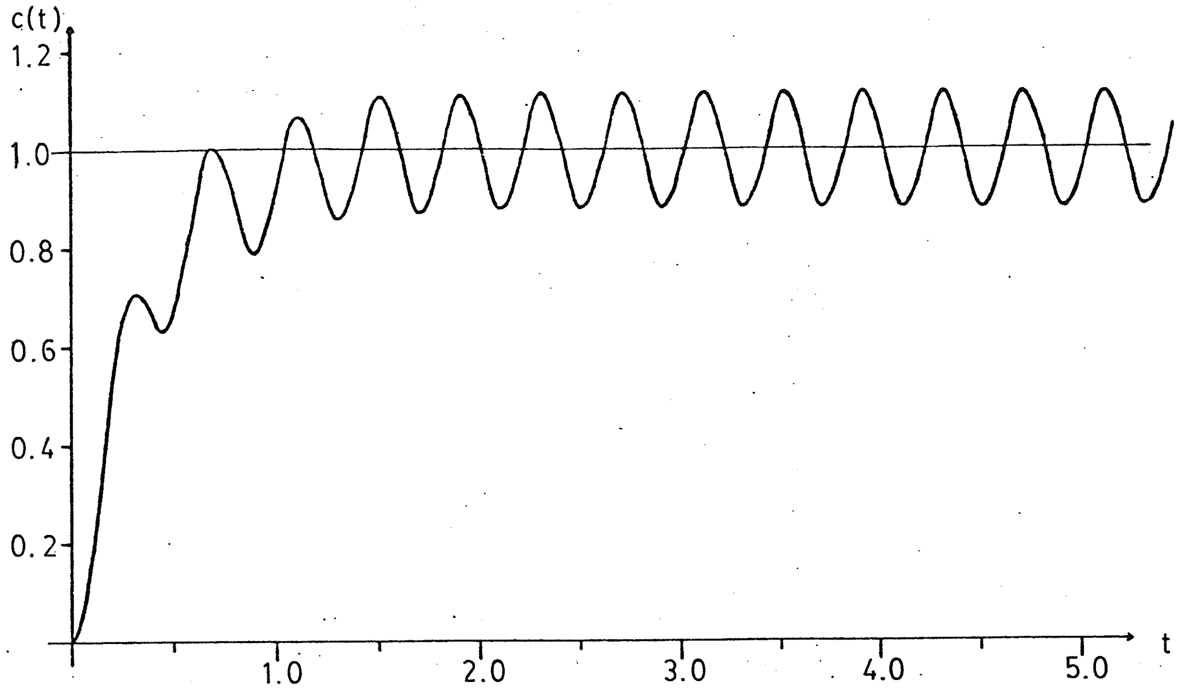

Figure 4.3.3 $G(s) = \frac{1}{(s+5)^2}$, M = 50.0, τ = 0.2

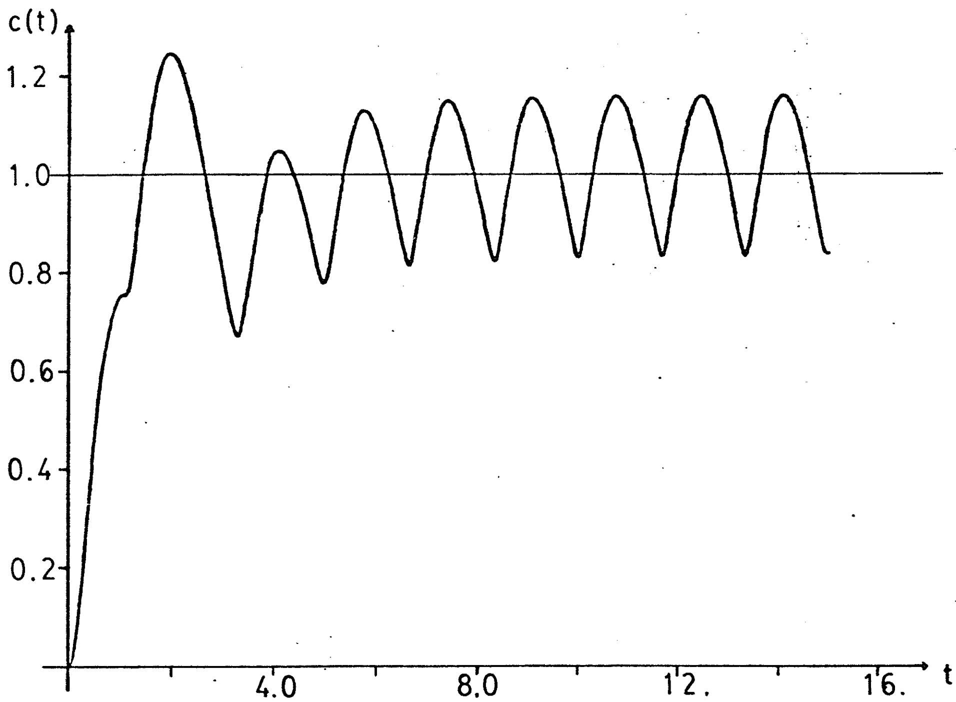

Figure 4.3.4 $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$, M = 500.0,

τ = 0.02

The T experimental was found to be 0.505 seconds as opposed to the theoretical value of 0.549 seconds. The discrepancy between these two values was attributed to the effect of the pulse, impulse approximation. Another simulation was run with the pulse parameters set at $M = 100.0$ units, $\tau = 0.01$ seconds, dead time = 0.05 seconds. In this case the T experimental was found to be 0.52 seconds.

c) Trial 3 (Figure 4.3.3)

Second order plant, multiple pole $G(s) = \dfrac{1}{(s+5)^2}$. Pulse parameters; $M = 50.0$ units, $\tau = 0.2$ seconds, dead time = 0.05 seconds.

d) Trial 4 (Figure 4.2.4)

Second order plant, complex poles $G(s) = \dfrac{1}{s^2+2(.707)5s+5^2}$.

Pulse parameters; $M = 150.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds. The result of this simulation has already been discussed in Section 2 of this chapter.

e) Trial 5 (Figure 4.3.4)

Third order plant, distinct poles $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$.

Pulse parameters; $M = 500.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds.

f) Trial 6 (Figure 4.3.5)

Third order plant, multiple poles $G(s) = \dfrac{1}{(s+10)^2(s+3)}$.

Pulse parameters; $M = 5000.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds.

g) Trial 7 (Figure 4.3.6)

Third order plant, complex poles $G(s) = \dfrac{1}{(s^2+2(0.5)5s+5^2)(s+8)}$.

Pulse parameters; $M = 500.0$ units, $\tau = 0.2$ seconds, dead time = 0.05 seconds.

h) Trials 8 and 9 (Figures 4.3.7 and 4.3.8 respectively)

IPFM controlled first order plant $G(s) = \frac{1}{s+2}$. Pulse parameters; $M = 5.0$ units, $\tau = 0.02$ seconds, deadtime = 0.002 seconds. Figure 4.3.7 shows the system response when the IPFM threshold level (A) is set at 0.001 units, and Figure 4.3.8 illustrates the response when the threshold level (A) is reduced to 0.0005 units. Note the irregular oscillations which occur when the threshold level is reduced to the lower value. These are the same plant and pulse parameters (except for dead time) as were used in trial 1, Figure 4.3.1.

i) Trials 10 and 11 (Figures 4.3.9 and 4.3.10 respectively)

IPFM controlled third order distinct plant $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$.

Pulse parameters ; $M = 500.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds. Figure 4.3.9 illustrates the response for an IPFM threshold level of 0.001 units, and Figure 4.3.10 illustrates the response for a level of 0.5 units. The plant and pulse parameters used in this trial are identical to those used in trial 5, Figure 4.3.4.

Sinusoidal Response, $r(t) = \text{sine}\,(\frac{\pi}{6}t)$

a) Trial 1 (Figure 4.3.11)

First order plant $G(s) = \frac{1}{(S+2)}$. Pulse parameters; $M = 5.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds.

b) Trial 2 (Figure 4.3.12)

Second order plant, distinct poles $G(s) = \frac{1}{(s+2)(s+8)}$.

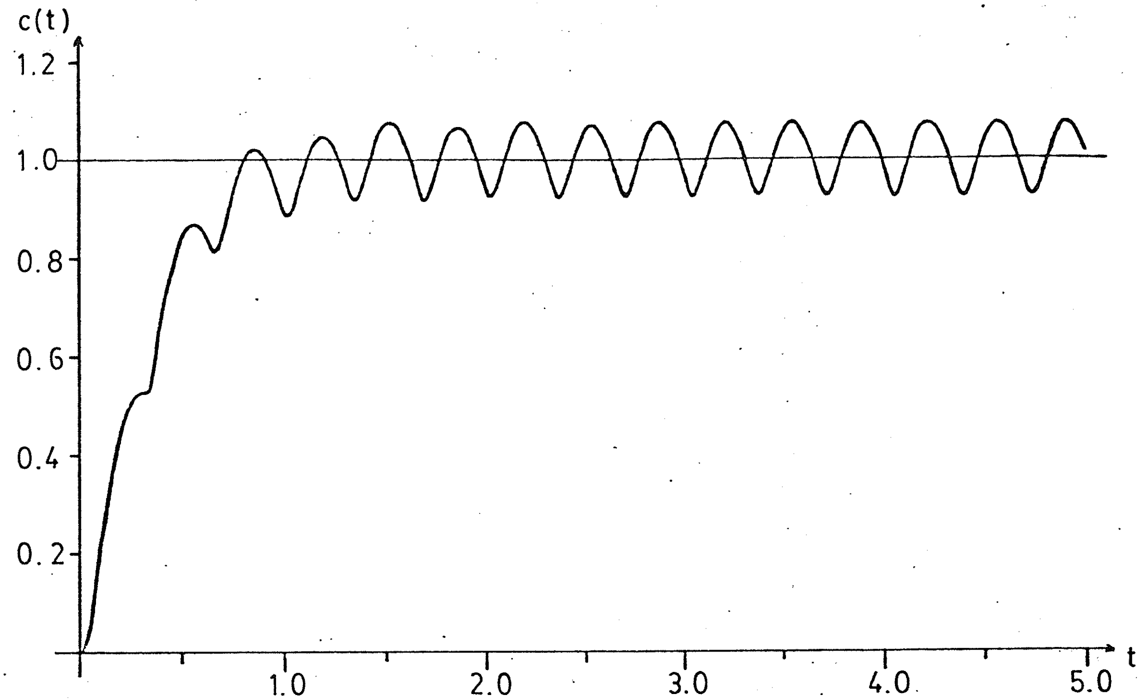

Figure 4.3.5 $G(s) = \dfrac{1}{(s+10)^2(s+3)}$, M = 5000.0

$\tau = 0.02$

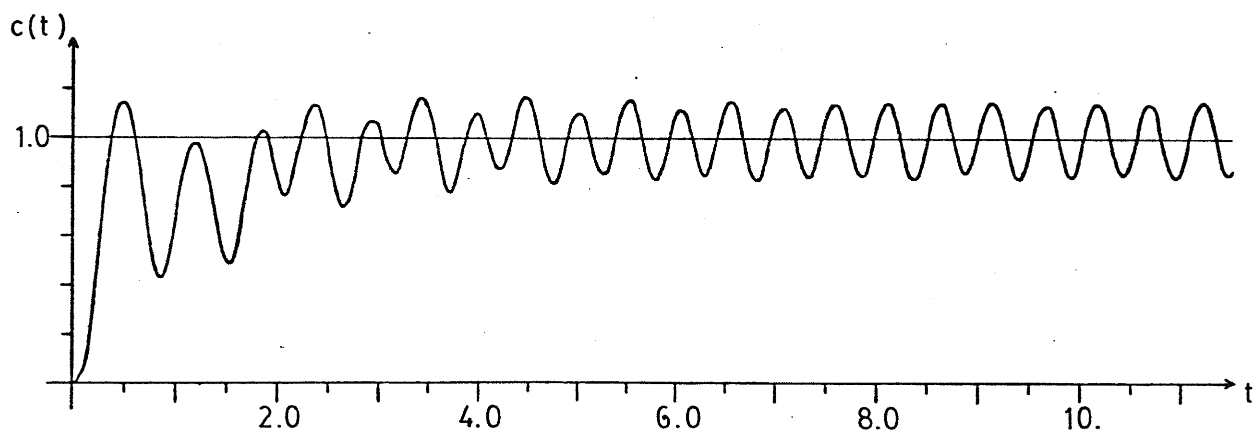

Figure 4.3.6 $G(s) = \dfrac{1}{(s^2+2(.5)5s+5^2)(s+8)}$,

$M = 500.0, \tau = 0.2$

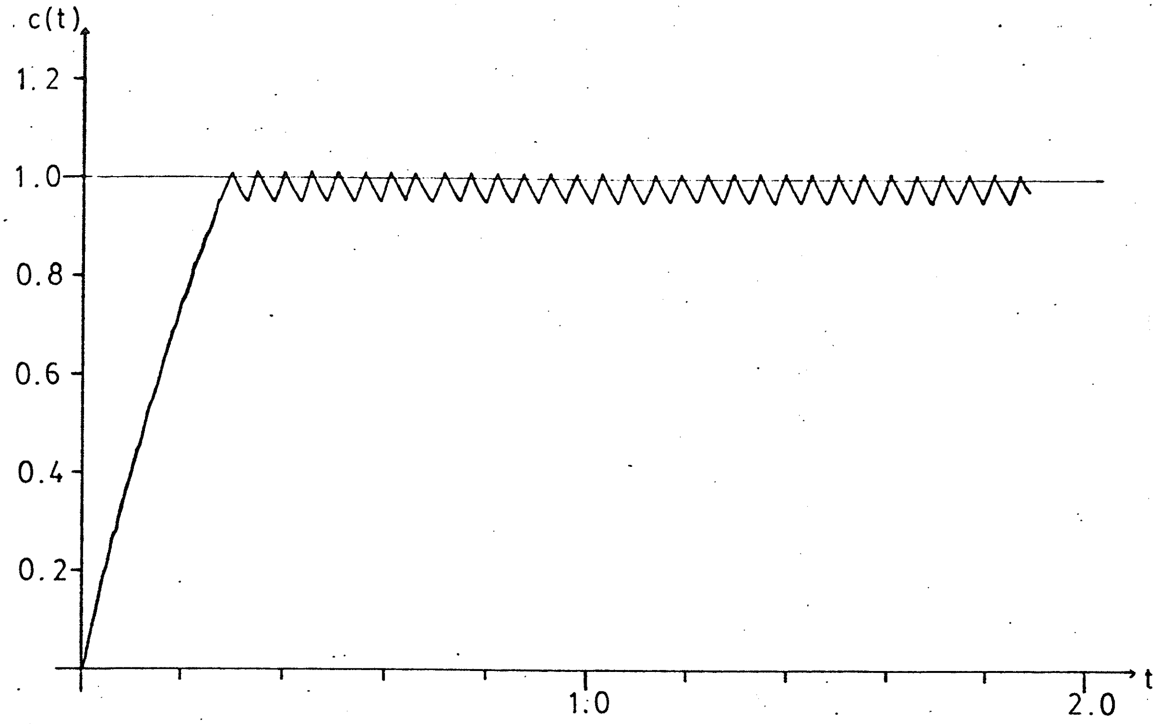

Figure 4.3.7 IPFM, A = 0.001, $G(s) = \frac{1}{(s+2)}$,

M = 5.0, τ = 0.02

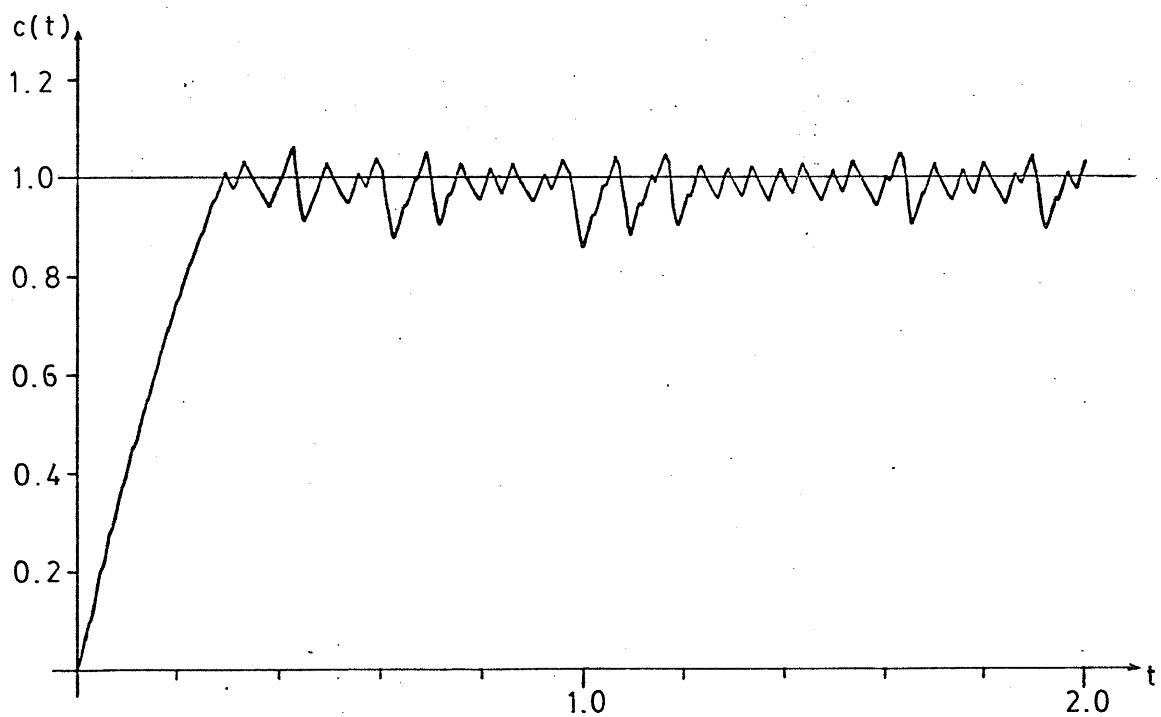

Figure 4.3.8 IPFM, A = 0.0005, $G(s) = \frac{1}{(s+2)}$,

M = 5.0, τ = 0.02

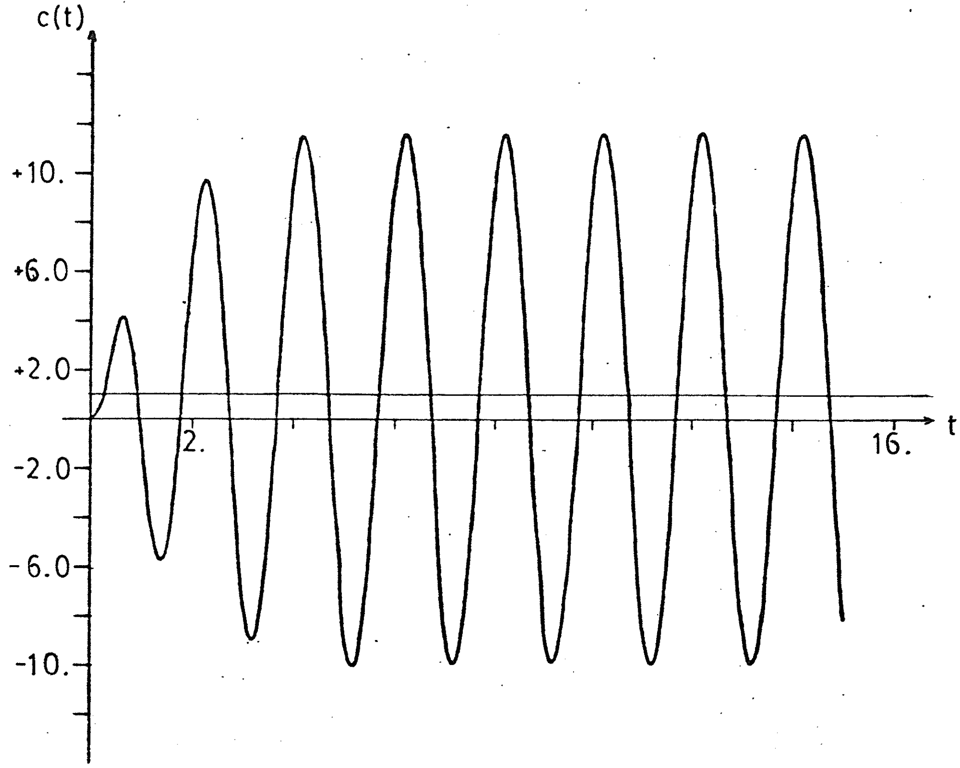

Figure 4.3.9 IPFM, A = 0.001 , $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$,

M = 500.0, τ = 0.02

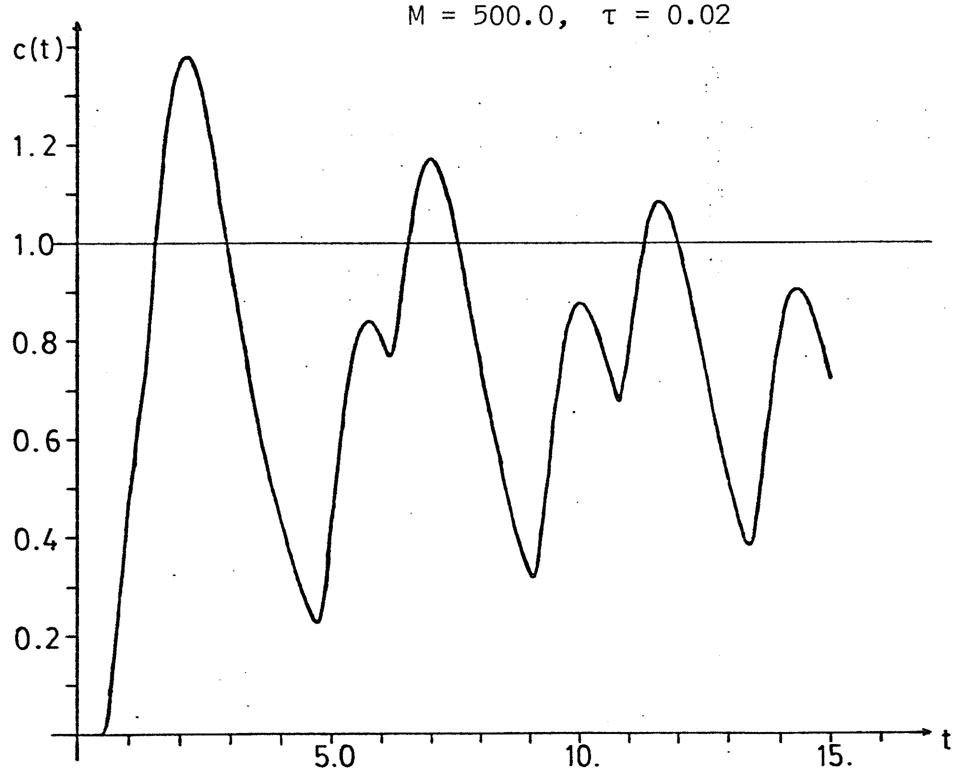

Figure 4.3.10 IPFM, A = 0.5, $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$,

M = 500.0, τ = 0.02

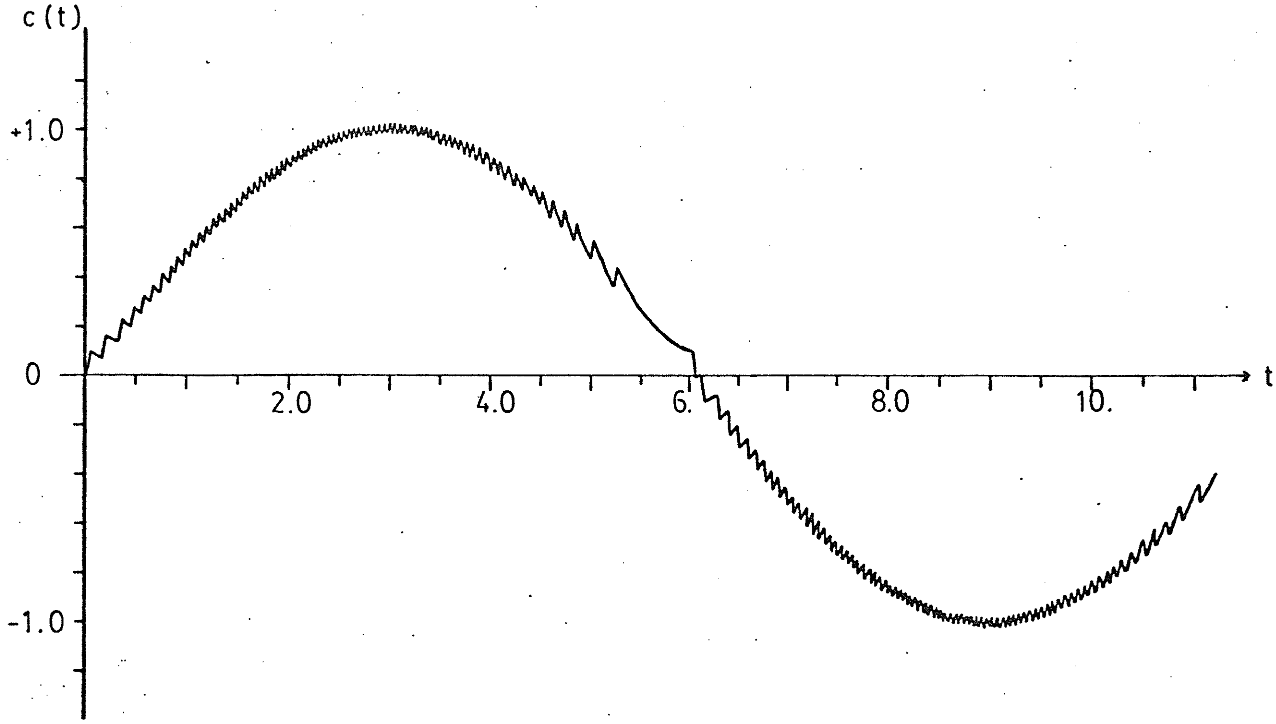

Figure 4.3.11 $G(s) = \frac{1}{(s+2)}$, $M = 5.0$, $\tau = 0.02$,

$r(t) = \text{sine}\ (\frac{\pi}{6}\ t)$

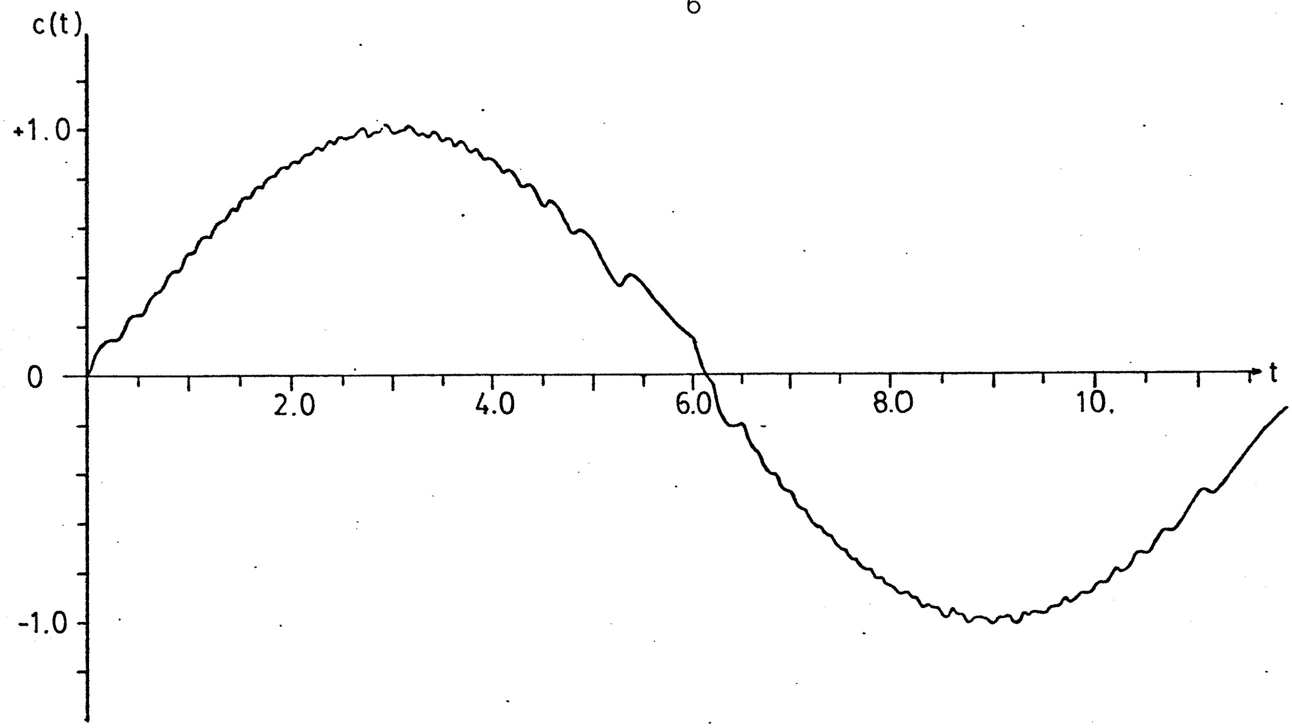

Figure 4.3.12 $G(s) = \frac{1}{(s+2)(s+8)}$, $M = 75.0$, $\tau = 0.02$,

$r(t) = \text{sine}\ (\frac{\pi}{6}\ t)$

Pulse parameters $M = 75.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds.

c) Trial 3 (Figure 4.3.13)

Second order plant, complex poles $G(s) = \frac{1}{s^2+2(.707)5s+5^2}$.

Pulse parameters; $M = 100.0$ units, $\tau = 0.02$ seconds, dead time = 0.05 seconds.

Parameter Variations

a) Trial 1 (Figure 4.3.14)

First order plant $G(s) = \frac{k}{(s+2)}$, $k = 1.0 + 0.4 \text{ sine } (\frac{2\pi}{3}t)$.

Pulse parameters; $M = 5.0$ units, $\tau = 0.02$ seconds, dead time = 0.002 seconds.

b) Trial 2 (Figure 4.3.15)

First order plant $G(s) = \frac{1}{(s+a)}$, $a = \frac{1}{0.5+0.2\text{sine}(2\pi\ t/3)}$

Pulse parameters; $M = 5.0$ units, $\tau = 0.02$ seconds, dead time = 0.002 seconds.

c) Trial 3 Figure 4.3.16)

IPFM controlled first order plant $G(s) = \frac{1}{(s+a)}$, $a = \frac{1}{0.5 + 0.2\text{sine}(2\pi t/3)}$ Pulse parameters; $M = 5.0$ units, $\tau = 0.02$ seconds, dead time = 0.002 seconds.

These are identical pulse and plant parameters as were used in the previous trial (2). The plant pole position varies about $S = -2.0$. The IPFM threshold level of 0.001 was used as for this value, and the same pulse parameters, the plant $G(s) = \frac{1}{s+2}$ exhibited a stable, well regulated

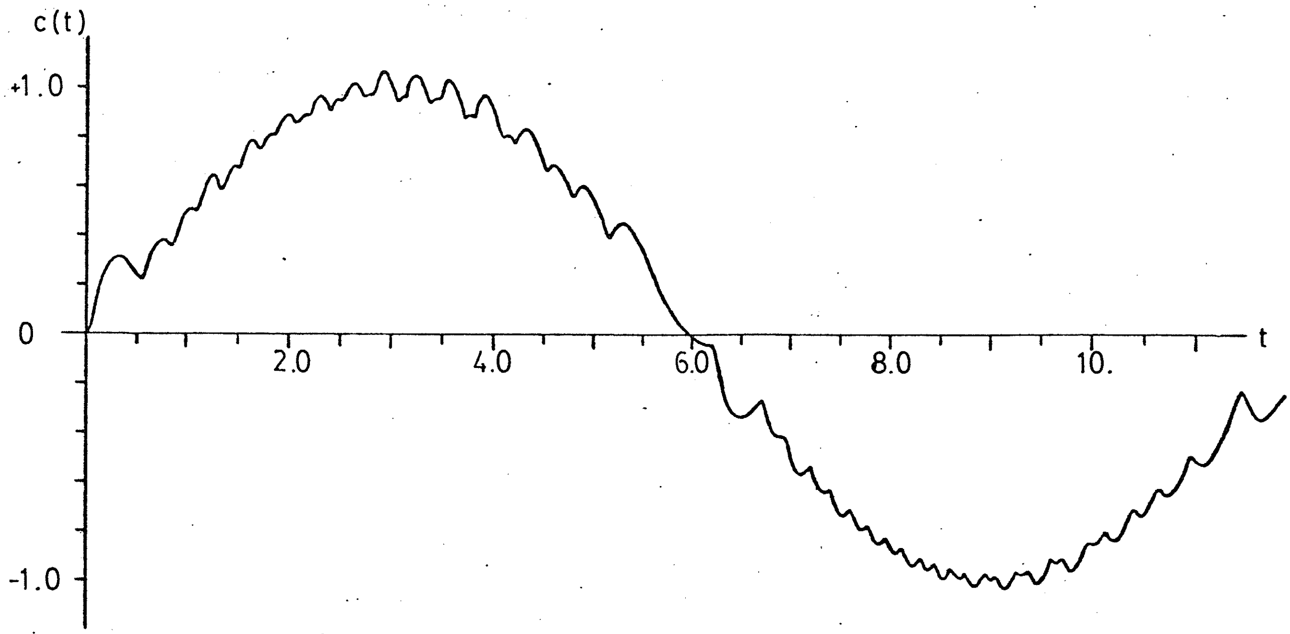

Figure 4.3.13 $G(s) = \frac{1}{s^2+2(0.6)4s+4^2}$, M = 100.0, τ = 0.02

$r(t) = \text{sine}(\frac{\pi}{6}t)$

9

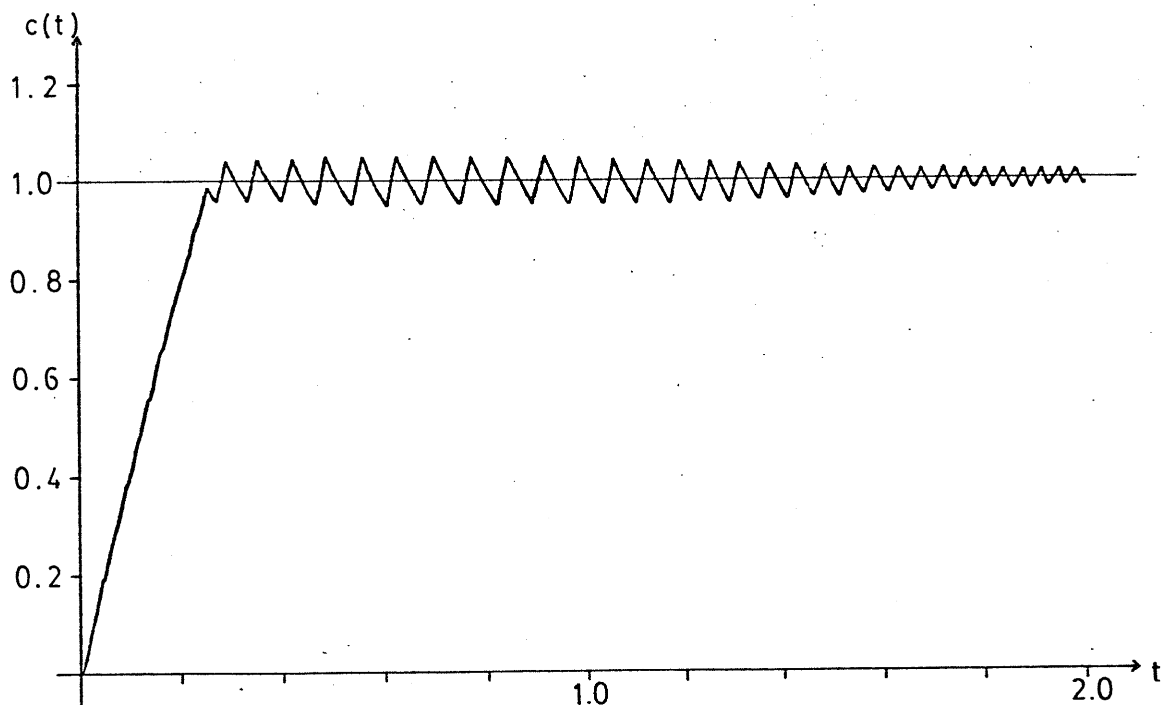

Figure 4.3.14 $G(s) = \frac{k}{(s+2)}$, K = 1.0 + 0.4 sine $(2\frac{\pi}{3}t)$,

M = 5.0, τ = 0.02

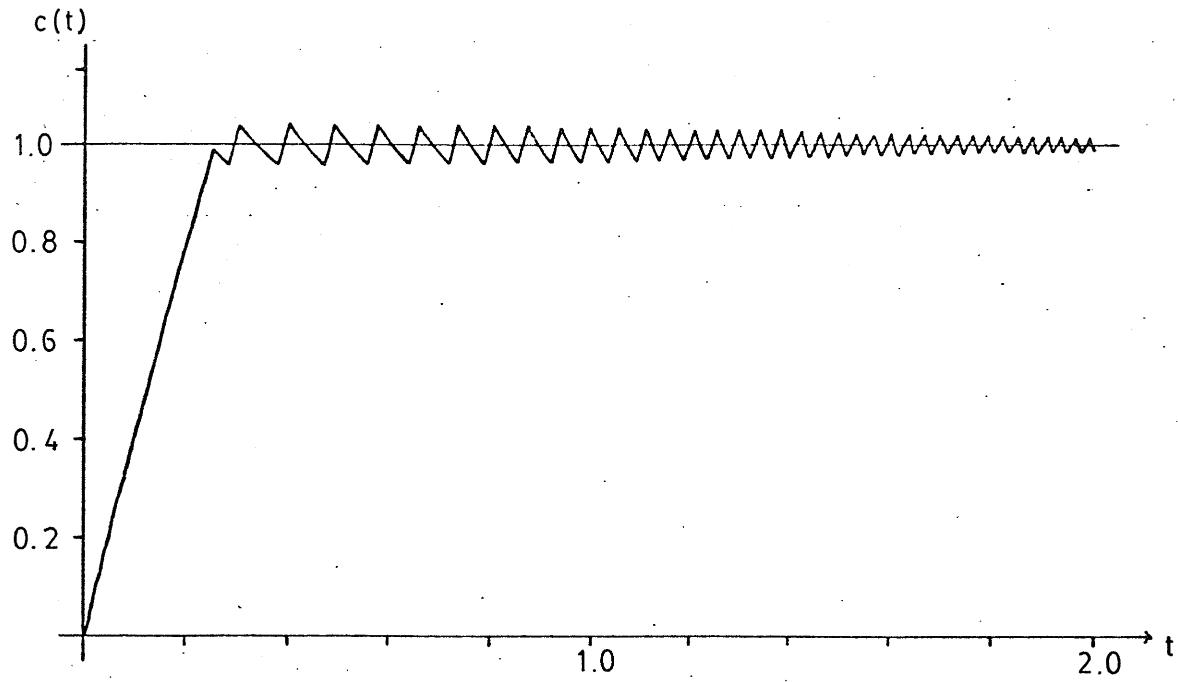

Figure 4.3.15 $G(s) = \frac{1}{s+a}$, $a = \frac{1}{0.5+0.2\text{sine}(\frac{2\pi}{3}t)}$,

$M = 5.0$, $\tau = 0.02$

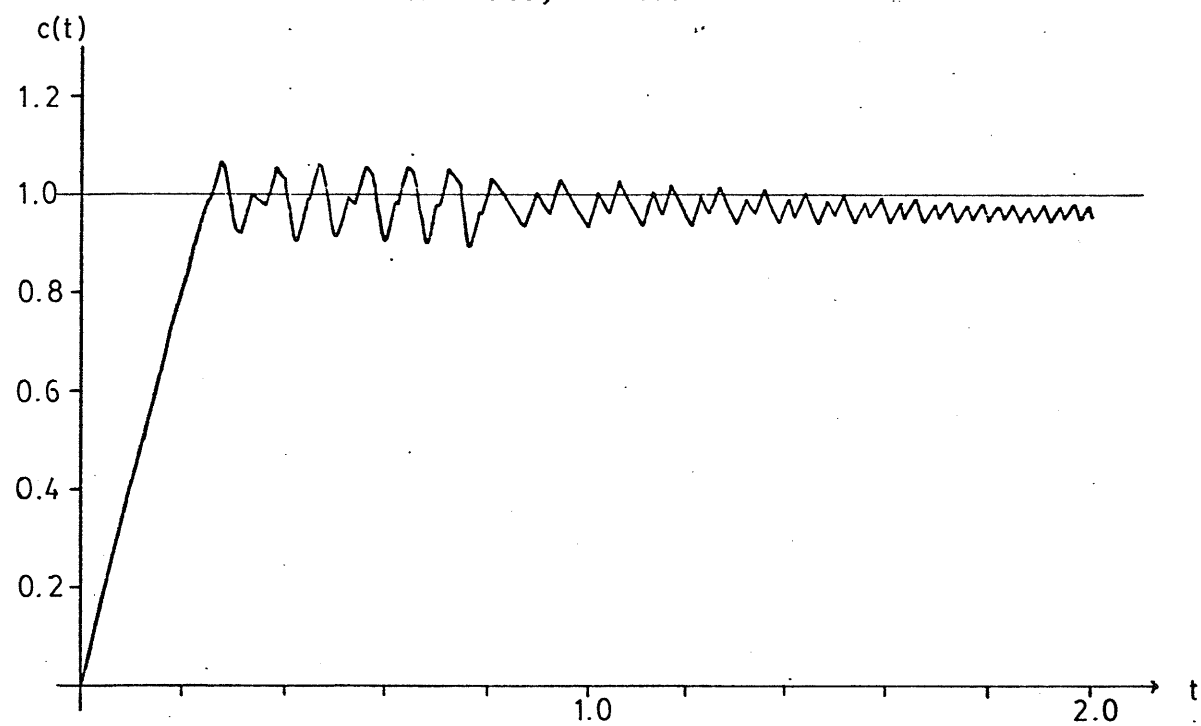

Figure 4.3.16 IPFM, $A = 0.001$, $G(s) = \frac{1}{(s+a)}$,

$a = \frac{1}{0.5+0.2\text{sine}(\frac{2\pi}{3}t)}$, $M = 5.0$, $\tau = 0.02$

response (trial 8 of the regulator response trials, Figure 4.3.7).

4.4 Discussion of Results

For regulator system, type 0 plant applications the PFM device developed in the work provides a "minimaxed" system response if the modulator parameters,pulse height and pulse width, are of sufficient magnitude to drive the plant to the desired reference level. This is demonstrated in the simulation results Figure 4.2.4 and Figures 4.3.1 to 4.3.6 which involve the regulation of type 0 plants of up to third order having distinct, multiple and lightly damped complex poles. If the modulator parameters, pulse height and pulse width are larger than the minimum required, the modulator will still maintain "minimax" and the modulator will have an "operating cushion" to be used in the event that the plant experiences any parameter variations or the reference input changes.

The effect of plant parameter variations on system response is illustrated in the simulation results of Figures 4.3.14 and 4.3.15. Here it is seen that the modulator has maintained "minimax" despite a 40% change in the magnitude of the plant gain (Fig. 4.3.14) and a 27% change in the plant pole position (Fig. 4.3.15).

System response to a time varying (sinusoidal) reference input is illustrated in Figures 4.3.11, 4.3.12 and 4.3.13, for plants of first, second and third order, respectively. Here it is demonstrated that the system is capable of tracking low frequency reference inputs but is subject to crossover distortion when the plant poles are real valued (Figures 4.3.11 and 4.3.12).

In order to place the performance of the designed modulator in perspective to that of an established PFM device a number of examples of comparable systems controlled by an IPFM device are presented (Figures 4.3.7, 4.3.8, 4.3.9, 4.3.10, 4.3.16). IPFM was chosen for the comparison as, of the standard types of PFM, it is the most straight forward to use. The "minimaxing" of the steady-state error is an optimum response and, the parameters of standard PFM modulators have to be adjusted or optimized to approach this response. IPFM, having only one parameter to optimize, (the pulse height and width are set to the same values as used in the designed modulator for comparable control situations) were consequently chosen over the other forms of PFM.

Figures 4.3.7 and 4.3.8 show the system response when the IPFM threshold is slightly above and slightly below the optimum value. It can be seen that the IPFM system response can be made to approach the "minimaxed" response of the designed modulator if the IPFM threshold level is chosen carefully. Depending on the degree to which this parameter diverges from the optimum value, irregular oscillations (Fig. 4.3.8) or even extreme oscillations (Fig. 4.3.9) can result.

The dependence of the optimum IPFM threshold level on a plant's parameters is illustrated in the example of Fig. 4.3.16. Here, the plant pole varies about the nominal value of -2. The IPFM threshold level and the pulse parameters were set to the same values used in the example of Fig. 4.3.7 (i.e. gave a stable response for the plant $G(s) = \frac{1}{s+2}$). It is seen that the dependence of optimum IPFM threshold level on plant parameters results in an irregular response which contrasts sharply with the "minimaxed" response of the same plant driven by the designed modulator,

Fig. 4.2.15.

Chapter V

CONCLUSIONS

The design of a PFM device to be used in the modulation of control system error is presented in this study. The device was required to complement the natural high power efficiency of pulse modulation by conserving the control effort, minimize some performance index of the steady-state error (independent of plant parameter values), and maintain acceptable transient performance. For physical and stability reasons the modulator design was directed towards regulator control systems involving type 0 plants. Upon examination of the simulation results, certain conclusions can be drawn:

a) For regulator systems the modulator will "minimax" the steady-state system error relatively independent of plant or pulse parameter values. It accomplishes this using only single signed pulses, thereby conserving the control effort. As to the transient performance, this is difficult to evaluate as both the magnitude of the steady-state error and the system rise time are directly dependent on the pulse size. The modulator does achieve as short a rise time as the satisfaction of the steady-state specification allows.

b) The performance of the modulator in non regulator systems is somewhat limited. The system will be able to track the reference input provided the rate of change of this signal is low. Recall that the modulator relies on the natural system (plant) decay to bring the response back in the direction of zero, and that the fixed pulse size places an upper limit on the response movement in the opposite direction. Also, when dealing with type 0 distinct and multiple pole plants, there will always be a certain amount of cross over distortion.

In summary, the designed modulator is an efficient, physically realizable device which will "minimax" the steady-state error of regulator systems with type 0 plants. It is easily employed in systems, requiring only the setting of the pulse parameters. Large pulses will decrease the transient system rise time, but will also increase the magnitude of the steady-state error. As the modulator threshold level is obtained from information gained from the system error, the modulator displays "adaptive" qualities. Consequently, the modulator recommends itself to systems in which plant parameter variations are expected, and systems where the reference input is slowly varying.

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APPENDIX A

CSMP System Simulation

CSMP [continuous system modelling program] is an application - oriented input language program that accepts problems expressed in the form of an analogue block diagram. The program includes a basic set of functional blocks (much like an analogue computer) with which the component of a continuous system may be represented, and accepts application-oriented statements for defining the connections between these functional blocks. A fixed format is provided for printing and print-plotting at selected increments of the independent variable. A typical CSMP simulation program of the PFM control system is shown in Fig. 1.

```
            ****CONTINUOUS SYSTEM MODELING PROGRAM****

                         *** VERSION 1.3 ***

INITIAL
PARAMETER K1=0.5,K2=0.5,XP=0.02,YP=0.002,ICI=0.0,IC2=0.0
PARAMETER C2=50.0
CONSTANT P1=1.0607,P2=1.4142
CONSTANT K3=1.02,K4-2.45,R=1.0
DYNAMIC
NOSORT
      Y=PULSE(XP,T1)
      ST=NOT(Y)
      C11=ZHOLD(ST,R)
      C1=INSW(C11,-C2,C2)
      A2=COMPAR(E,A1)
      ZT1=ZHOLD(T1,E)
      T1=X*A2*X4
      ED=DELAY(1,0.005,E)
      X4=COMPAR(E,ED)
      X41=NOT(X4)
      X31=FONSW(ZH1,0.0,0.0,X41)
      ZH1=RST(0.0,T1,X31)
      Z1=ZHOLD(ZH1,ED)
      X3=FCNSW(SH2,0.0,0.0,X4)
      ZH2=RST(0.0,Y,X3)
      Z2=ZHOLD(ZH2,E)
      A1=AT1-(S1+Z2)*0.5
      TRD=0.25-Y
      DT=MODINT(ICI,TRD,1.0,1.0)
      X=COMPAR(DT,YP)
      U=C1*Y
      C=CMPXPL(IC1,IC2,P1,P2,U)
      SGNR=INSW(R,-1.0,1.0)
      E=SGNR*(R-C)
METHOD ADAMS
TIMER DELT=0.0005,FINTIM=15.0,OUTDEL=0.005
PRTPLT C(U,ED,Z2)
END
STOP

OUTPUTS      INPUTS     PARAMS   INTEGS  +  MEM BLKS  FORTRAN  DATA CDS
33(500)    78(1400)   15(400)     3+    1=   4(300)   30(600)      8

END JOB
```

Figure 1   CSMP Simulation Program