APPLICATION OF SPECTRAL TECHNIQUES TO FAULT DETECTION AND ISOLATION

by

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ABSTRACT

Several spectral signatures for the isolation of faults at input lines have been developed. These signatures cover two of the most common types of faults, that is single stuck at and single bridging faults. Some bridging faults may produce the same effect on the network output as a stuck at fault and such faults cannot be isolated the same way as the others. The spectral conditions that lead to such situations are investigated and methods of isolating these faults are also considered. Finally several medium size industrial circuits are studied and their input fault isolation signatures are found.

LIST OF FREQUENTLY USED SYMBOLS

T^n	-- orthogonal transform (a matrix of size 2^n X 2^n)
F, G, H	-- boolean functions
$\hat{F}$, $\hat{G}$, $\hat{H}$	-- boolean functions realized by faulty circuits
f_i, g_i, h_i	-- entries of character sequences of F, G, H respectively
n	-- number of inputs of a circuit
X_i	-- an input to a circuit
R	-- the spectrum of a boolean function
r_a	-- a spectral coefficient, where 'a' is any combination of 0 - n
$\hat{R}$	-- the faulty spectrum of a boolean function
$\hat{r}_a$	-- a faulty spectral coefficient, where 'a' is any combination of 0 - n
S-A-0	-- stuck at 0
S-A-1	-- stuck at 1

CONTENTS

Chapter I

INTRODUCTION

A simple and straightforward way to test a combinational network is to apply all possible combinations to its inputs, measure the responses from the output and compare them to the expected results. For a circuit of n inputs there are 2^n combinations of inputs and 2^n responses to be checked at the output. The problem with this approach is that the 2^n expected results have to be stored in some media during testing and when n gets large these 2^n values are difficult to handle. It is almost impossible to test a large network network using such an approach.

Alternatively the circuit can be tested with a selected subset of the possible combination of inputs (test set) such that all faults of a given type are covered by this subset. This method eliminates the need of having to process the large volume of response data, (i.e. it achieves certain response data compression) but it has the drawback of having to compile the test set which in most cases is rather complicated and detailed knowledge of the circuit is required. This is especially true for large circuits.

Other techniques of testing combinational circuits have been developed by several researchers. Among them are

Hayes' "transitional count test" [6] and "one's count test" [7], the data compression technique deviced by Fujiwara and Kinoshita [3] etc. These techniques achieved the goal of response data compression but they all have to employ test sets which means that the combination of inputs cannot be implemented simply by means of a binary counter. Jacob Savir's syndrome test [17], [18], which is basically a count of minterms of the function that the circuit realizes eliminates the need for a test set and at the same time achieves response data compression. His approach is to identify all lines in the circuit which are syndrome untestable and make these problem lines unate [8]. To make a function F unate in a problem line g, extra gates and inputs are added to the circuit so that g appears in the new function F_u only in the complemented or non-complemented but not both forms. But this is a rather strict condition, and in some cases, the calculations involved makes this method prohibitive.

In the spectral approach of combinational circuit testing [13], [20] a subset of the spectrum of a circuit under test is measured and compared to those of the fault free circuit. The spectrum which may be visualized as attributes of a circuit is made up of 2^n entries for a circuit of n inputs. Each entry of the spectrum is called a spectral coefficient. Spectral coefficients can be obtained by a transformation on the characteristic sequence [see section 2.2 and 2.3] or they can be measured directly from a circuit [see section they can be measured directly from a circuit [see section 3.6]. The major advantage of the spectral approach is the elimination of test set while achieving good response data compression. In comparision to Savir's syndrome testing the spectral approach is more flexible. In [13] syndrome testing is generalized and it turned out to be a particular case of testing using spectral techniques.

Fault isolation is the technique of locating a fault when it is detected. In this thesis the spectral method is investigated to find its capability for fault isolation. Only isolation of faults at input lines were studied because faults at this level are repairable while faults at internal lines of a chip render the chip useless.

The traditional approach to fault isolation is to use simulation to construct a fault dictionary which lists each possible output response and the corresponding class of faults that lead to this response. In the spectral approach a signature for a given type of fault is found for the circuit. By measuring the signature of the circuit under test and comparing it to the fault free signature we can tell if the circuit is functioning normally or not. If it is not, then from the measured signature and a list of faulty ones calculated from the fault free spectrum we are able to pinpoint the location of the fault. Several signatures have been developed. Some of these signatures make use of the patterns of changes in the spectral coefficients of the signature when a fault occurs, thus by matching these patterns with the calculated faulty ones the fault can be isolated. There are other signatures that make use of the zero spectral coefficients in the signature and still others that make use of the actual values of the spectral coefficients of the signature. The last of these signatures was found to be the most useful when applied to 'real world' networks.

There are situations in which two or more faults may have the same effect on a circuit. It was found that these situations have a close relationship with symmetric properties of the function [9]. This is especially true for input stuck at faults. The spectral conditions for these situations to occur are investigated.

The background material that is used throughout this thesis is presented in chapters two and three. Chapter two discusses the properties of the orthogonal transform which is required for transforming a characteristic sequence of the function into its spectrum. The properties of the spectrum itself are discussed in the latter part of chapter two. Chapter three discusses the relationship between the spectrum of a fault free circuit and that of circuits with single input stuck at or bridging faults. The relationships are established using Tokmen's result discussed at the end of chapter two on the relationship between spectra of subfunctions and that of the function itself.

Two signatures for the isolation of single input bridging faults are discussed in chapter four. The first signature makes use of the patterns of changes in the spectral coefficients to identify a fault and the second one uses the actual values of the sectral coefficients. Chapter five is devoted to signatures of input stuck at fault isolation. Three isolation signatures are discussed, two of these signatures are similar to those for isolation of input bridging faults discussed in chapter four. Chapter six discusses the limitation of these signatures when applied to industrial circuits. A more practical approach for a combined signature for single input stuck at or bridging faults is presented. Signatures for multiple outputs which are common to most industrial circuits will also be discussed. This chapter is concluded with a study of several 'real world' circuits. Their combined spectral isolation signature for input stuck at or bridging faults are found. Chapter seven is the conclusion for this thesis.

Throughout this thesis discussions often involve column vectors e.g. the characteristic sequences of functions. For typographical reasons they are presented in the form of row vectors.

Chapter II

SOME PROPERTIES OF SPECTRAL COEFFICIENTS

In this chapter we look at the spectral approach to the study of boolean functions. We shall discuss the advantage of this approach over the traditional method. To use the spectral technique, it is necessary to transform the output of the circuit realizing the boolean function from the boolean domain to the spectral domain. We shall discuss briefly several of these transforms and examine some of the properties of one of them which will be used throughout this thesis. The transformation is carried out by matrix multiplication, but it can also be achieved in a more efficient way (fast transform). We shall examine how the number of computations are reduced by the fast transfrom.

The latter part of this chapter is devoted to the product of the transformation - the spectrum. We shall discuss the physical meaning of the spectral coefficients which constitute the spectrum and their properties. The spectral conditions for variable redundancy and symmetry between variables will also be examined. This chapter is concluded with a discussion of the applications of the spectral technique.

2.1 WHY SPECTRAL COEFFICIENTS

The traditional way of studying the behaviour of a digital logic network is by means of a truth table which is a binary tabulation of the outputs (0, 1) of the network for all possible combinations of input values. This binary tabulation is also called the characteristic sequence of the logic network or of the boolean function that is realized by the network. For a network of n inputs there would be 2^n entries in the characteristic sequence. One severe disadvantage of using the characteristic sequence is that each entry in it only carries a limited amount of information which is the state of the output of the network for a particular set of values for its inputs. It only gives local information about the function.

The spectrum of an n input digital network or of its boolean function is made up of 2^n spectral coefficients, and it is derived by applying an orthogonal transformation to the characteristic sequence. One nice feature of the spectrum is that each spectral coefficient carries different global information about the function. That is why in many cases we can study the functional aspects of a network by examining a small subset of the coefficients of the spectrum. It is in this sense that the use of spectral techniques in the study of digital networks is preferable to classical methods.

2.2 ORTHOGONAL TRANSFORM

The transformation of the characteristic sequence into the spectrum is carried out by multiplying it by an orthogonal matrix.

i.e.

$$| T^n | \, | F | = | R | \qquad \ldots\ldots\ldots\ldots 2.1$$

where

$| T^n |$ is the orthogonal matrix of dimension 2^n X 2^n

$| F |$ is the characteristic sequence

$| R |$ is the spectrum of the network or of its binary function

There is a lot of variation in the possible orthogonal transforms which T^n may assume, but only a few give a meaningful transformed product. Some of the simple ones among them are Walsh-Hadamard transform, Walsh-Kacmarz transform, Walsh-Paley transform and the Rademacher-Walsh transform [9], [12]. Throughout this thesis the Walsh-Hadamard transform will be used. It has a recursive definition as follows.

$$T^0 = | 1 | \qquad \ldots\ldots\ldots\ldots 2.2$$

$$T^n = \begin{vmatrix} T^{n-1} & T^{n-1} \\ T^{n-1} & -T^{n-1} \end{vmatrix} \qquad \ldots\ldots\ldots\ldots 2.3$$

So

$$T^1 = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$T^2 = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{vmatrix}$$

$$T^3 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{vmatrix}$$

The ordering of the spectral coefficients is as follows.

$$| T^n | \, | f_i | = \begin{vmatrix} r_0 \\ r_1 \\ r_2 \\ r_{12} \\ \circ \\ \circ \\ r_i \\ r_{1i} \\ r_{2i} \\ r_{12i} \\ r_{3i} \\ r_{13i} \\ r_{23i} \\ r_{123i} \\ \circ \\ \circ \\ r_n \\ r_{1n} \\ r_{2n} \\ \circ \\ \circ \\ r_{123\cdots n} \end{vmatrix} \qquad \text{..........} \; 2.4$$

<u>Example</u>

$$F = X_1X_2 + X_3$$

The characteristic sequence of F is :

{0, 0, 0, 1, 1, 1, 1, 1}

The spectrum of F is given by

$$
\begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{vmatrix}
\begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}
=
\begin{vmatrix} 5 \\ -1 \\ -1 \\ 1 \\ -3 \\ -1 \\ -1 \\ 1 \end{vmatrix}
\begin{matrix} -- & r_0 \\ -- & r_1 \\ -- & r_2 \\ -- & r_{12} \\ -- & r_3 \\ -- & r_{13} \\ -- & r_{23} \\ -- & r_{123} \end{matrix}
$$

The other transforms mentioned above actually contain the same set of rows as the Hadamard Transform but with a different ordering of the rows. Four of the common orderings are illustrated in Fig. 2.1. Therefore the spectra from these transforms are the same except for the ordering of the coefficients.

$$
\begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{vmatrix}
$$

(a)

$$
\begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1
\end{vmatrix}
$$

(b)

$$
\begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{vmatrix}
$$

(c)

$$
\begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{vmatrix}
$$

(d)

Transforms for $n = 3$
(a) Walsh-Hadamard Transform
(b) Walsh-Kacmarz Transform
(c) Walsh-Paley Transform
(d) Rademacher-Walsh Transform

Fig. 2.1

The ordering of the spectral coefficients is closely related to the ordering of the inputs to the network as we shall see later. We say that a spectral coefficient is associated with the input X_i if i appears in the subscript of that coefficient. We also define the order of a spectral coefficient as the number of variables it associates with. For example r_0 is of order 0, r_1, r_2, ... r_n are of order 1 and r_{12}, r_{13}, ... r_{ij} ... are of order 2 and so on.

The following are some useful properties of the Walsh-Hadamard transform T^n.

1. The inner product of row i and row j is equal to zero for i not equal to j and is equal to 2^n for i = j. This is a basic property of orthogonal matrices.

2. T^n is symmetric

   Proof by induction :

   $T^0 = | 1 |$ is symmetric

   Assume that T^k is symmetric i.e.

   $$T^k(i,j) = T^k(j,i)$$

   From the recursive definition of T^n

   $$T^{k+1} = \begin{vmatrix} T^k & T^k \\ T^k & -T^k \end{vmatrix}$$

   Divide T^{k+1} into 4 quadrants as shown naming the top right hand corner the first quadrant and counting counter clockwise. We want to prove that

   $$T^{k+1}(i, j) = T^{k+1}(j, i).$$

If $T^{k+1}(i,j)$ is in the 1^{st}, 2^{nd} or 3^{rd} quadrant then

$$T^{k+1}(i,j) = T^{k}(i^*,j^*) \quad \text{and}$$
$$T^{k+1}(j,i) = T^{k}(j^*,i^*)$$

where

$$i^* = i(\text{mod } 2^k) \quad \text{and}$$
$$j^* = j(\text{mod } 2^k)$$

But $T^k(i^*,j^*) = T^k(J^*,i^*)$ by hypothesis.

Therefore

$$T^{k+1}(i,j) = T^{k+1}(j,i)$$

If T^{k+1} is in the 4^{th} quadrant then

$$T^{k+1}(i,j) = - T^{k}(i^*,j^*) \quad \text{and}$$
$$T^{k+1}(j,i) = - T^{k}(j^*,i^*)$$

Again the left hand sides of the above two equations are equal. Therefore $T^{k+1}(i, j) = T^{k+1}(j, i)$ for all i and j <= 2^{k+1}. Therefore T^{k+1} is symmetric and so is T^n for n >= 0.

3.

$$(T^n)^{-1} = \frac{1}{2^n} T^n$$

Proof

Since T^n is symmetric

$$T^n = (T^n)^t \quad \text{the transpose of } T^n$$

$$T^n \circ T^n = T^n \circ (T^n)^t$$

$$= 2^n I$$

Therefore

$$\frac{1}{2^n} T^n = (T^n)^{-1} \quad \ldots\ldots\ldots\ldots \quad 2.5$$

2.3 FAST TRANSFORM

The method of finding the spectrum by matrix multiplication becomes quite inefficient as the number of inputs 'n' gets large because the number of operations invloved would be 2^n X 2^n. The fast transform [5] enables us to evaluate the same spectrum in n X 2^n operations.

If we take a look at the operations required for each spectral coefficient we will notice that a lot of the operations are common to several coefficients. For example let us look at a function of 3 variables.

$$F = \{a, b, c, d, e, f, g, h\}$$

where $\{a, b, c, d, e, f, g, h\} \in \{0, 1\}$

the spectrum of F is given by :

$$\begin{vmatrix} r_0 \\ r_1 \\ r_2 \\ r_{12} \\ r_3 \\ r_{13} \\ r_{23} \\ r_{123} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{vmatrix}$$

Expanding the above matrix equation we have

$$r_0 = (a+b) + (c+d) + (e+f) + (g+h) \quad \ldots\ldots \quad 2.6$$

$$r_1 = (a-b) + (c-d) + (e-f) + (g-h) \quad \ldots\ldots \quad 2.7$$

$$r_2 = (a+b) - (c+d) + (e+f) - (g+h) \quad \ldots \quad 2.8$$

$$r_{12} = (a-b) - (c-d) + (e-f) - (g-h) \quad \ldots \quad 2.9$$

$$r_3 = (a+b) + (c+d) - (e+f) - (g+h) \quad \ldots \quad 2.10$$

$$r_{13} = (a-b) + (c-d) - (e-f) - (g-h) \quad \ldots \quad 2.11$$

$$r_{23} = (a+b) - (c+d) - (e+f) + (g+h) \quad \ldots \quad 2.12$$

$$r_{123} = (a-b) - (c-d) - (e-f) + (g-h) \quad \ldots \quad 2.13$$

We see that the operations

$(a+b)$, $(c+d)$, $(e+f)$, $(g+h)$

are common to r_0, r_2, r3 and r_{23}.
Operations

$(a-b)$, $(c-d)$, $(e-f)$ and $(g-h)$

are common to r_1, r_{12}, r_{13} and r_{123}.
If we evaluate these partial sums/differences and rewrite eq. 2.6 - 2.13 using the following notations for these partial sums and partial differences

$i = a + b \qquad j = a - b$

$k = c + d \qquad l = c - d$

$m = e + f \qquad n = e - f$

$o = g + h \qquad p = g - h$

we have

$$r_0 = (i+k) + (m+o) \quad \ldots\ldots\ldots\ldots \quad 2.14$$

$$r_1 = (j+l) + (n+p) \quad \ldots\ldots\ldots\ldots \quad 2.15$$

$$r_2 = (i-k) + (m-o) \quad \ldots\ldots\ldots\ldots \quad 2.16$$

$$r_{12} = (j-l) + (n-p) \qquad \ldots\ldots\ldots\ldots \quad 2.17$$

$$r_{3} = (i+k) - (m+o) \qquad \ldots\ldots\ldots\ldots \quad 2.18$$

$$r_{13} = (j+l) - (n+p) \qquad \ldots\ldots\ldots\ldots \quad 2.19$$

$$r_{23} = (i-k) - (m-o) \qquad \ldots\ldots\ldots\ldots \quad 2.20$$

$$r_{123} = (j-l) - (n-p) \qquad \ldots\ldots\ldots\ldots \quad 2.21$$

Again we have some partial sums and partial differences common in eq. 2.14 to 2.21. If we evaluate these partial sums and partial differences and rewrite 2.14 - 2.21 using the following notations

$$q = i + k \qquad r = i - k$$

$$s = j + l \qquad t = j - l$$

$$u = m + o \qquad v = m - o$$

$$w = n + p \qquad x = n - p$$

we have

$$r_{0} = q + u \qquad \ldots\ldots\ldots\ldots \quad 2.22$$

$$r_{1} = s + w \qquad \ldots\ldots\ldots\ldots \quad 2.23$$

$$r_{2} = r + v \qquad \ldots\ldots\ldots\ldots \quad 2.24$$

$$r_{12} = t + x \qquad \ldots\ldots\ldots\ldots \quad 2.25$$

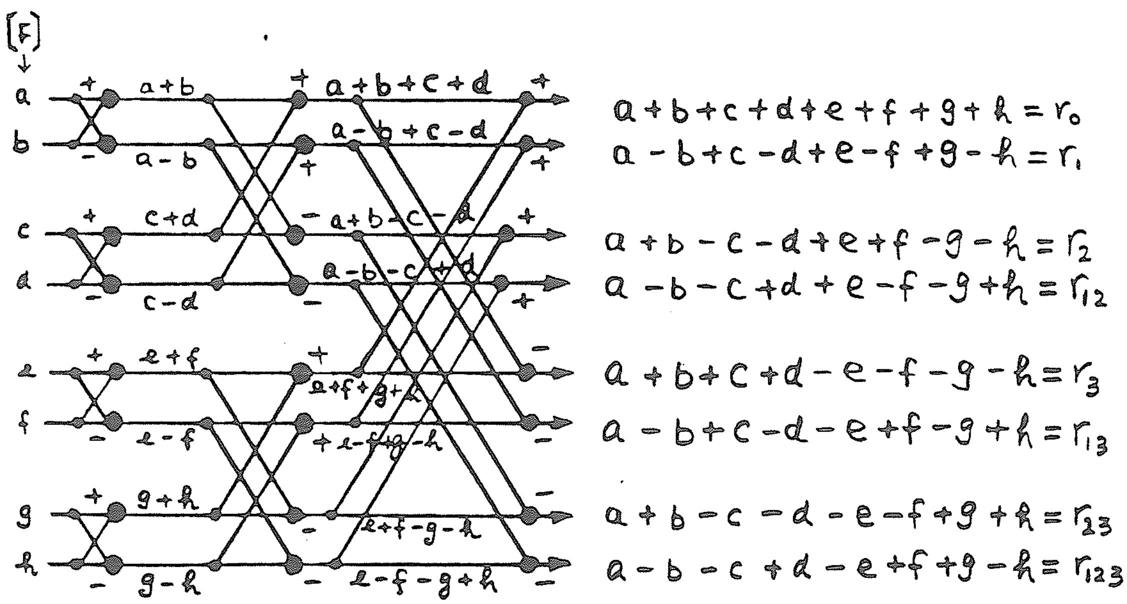
$$r_{3} = q - u \qquad \ldots\ldots\ldots\ldots \quad 2.26$$

$$r_{13} = s - w \qquad \ldots\ldots\ldots\ldots \quad 2.27$$

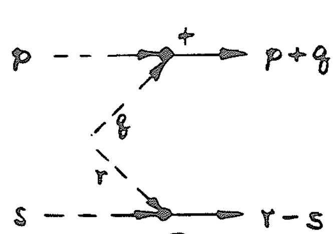
$$r_{23} = r - v \qquad \ldots\ldots\ldots\ldots \quad 2.28$$

$$r_{123} = t - x \qquad \ldots\ldots\ldots\ldots \quad 2.29$$

The operations that have to be done in this process are those involved in evaluating the partial sums and partial differences plus the operations for the final stage i.e. eq. 2.22 - 2.29. Therefore we have reduced the number of operations from $2^3 \times 2^3$ to 3×2^3. In general for functions of n variables there would be n-1 sets of partial sums and differences each of 2^n entries. Together with the operations at the final stage, the number of operations needed to derive the spectrum is $n \times 2^n$. The following is a graphical representation of such a fast transform.



LEGEND



2.4 PHYSICAL MEANING OF SPECTRAL COEFFICIENTS

If we examine the pattern of each row of the matrix T^n we notice that the first row which corresponds to r_0 is made up of all 1's. So r_0 is a count of the number of minterms of the function. The second row is made up of alternating +1's and -1's. So r_1 is the difference between the number of minterms in the function when $X_1 = 0$ and the number of minterms in the function when $X_1 = 1$. Similarly r_i is the difference between the number of minterms in the function when $X_i = 0$ and those when $X_i = 1$. Further observation of the row in T^n that corresponds to $r_{ijk\cdots m}$ reveals that a 1 in this row corresponds to $X_i \oplus X_j \oplus X_k \oplus\ldots\ldots\oplus X_m = 0$ and that a -1 in this row corresponds to $X_i \oplus X_j \oplus X_k \oplus\ldots\ldots\oplus X_m = 1$. Therefore $r_{ijk\cdots m}$ is the difference between the number of minterms in the function when the XOR of all variables associate with it is equal to 0 and those when the XOR of the variables is equal to 1.

Example

$$F(X_1, X_2, X_3) = X_1X_2 + X_3$$

From section 2.2 the spectrum of F is

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
5	-1	-1	1	-3	-1	-1	1

Let us look at the Karnaugh map of F (Fig. 2.2a) and compare it to Fig. 2.2b and Fig. 2.2c which are Karnaugh maps of

$G(X_1, X_2, X_3) = X_1$ and $H(X_1, X_2, X_3) = X_1 \oplus X_2$ respectively to see how we can derive spectral coefficients from them. In Fig. 2.2a there are 5 minterms and this is r_0 from above. Comparing Fig. 2.2a and 2.2b we will notice that

number of minterms in F corresponding to G = 1 is 3

number of minterms in F corresponding to G = 0 is 2

and

$$r_1 = 2 - 3 = -1$$

Comparing Fig. 2.2a and 2.2c

number of minterms in F corresponding to H = 1 is 2

number of minterms in F corresponding to H = 0 is 3

and

$$r_{12} = 3 - 2 = 1$$

In general

$$r_{ijk\cdots m} = N_0 - N_1$$

where

N_0 = number of minterms in the function corresponding to $X_i \oplus X_j \oplus X_k \oplus \ldots \oplus X_m = 0$

N_1 = number of minterms in the function corresponding to $X_i \oplus X_j \oplus X_k \oplus \ldots \oplus X_m = 1$

X_3 \ $X_2 X_1$	00	01	11	10
0			1	
1	1	1	1	1

$F = X_1 X_2 + X_3$

(a)

X_3 \ $X_2 X_1$	00	01	11	10
0		1	1	
1		1	1	

$G = X_1$

(b)

X_3 \ $X_2 X_1$	00	01	11	10
0		1		1
1		1		1

$H = X_1 \oplus X_2$

(c)

Fig 2.2

Therefore each coefficient carries information on different attributes of the function as a whole although it is not possible to tell from the spectrum the output of the function for a particular set of input values except through the use of the inverse transform to recalculate F explicitly as discussed later in this chapter.

2.5 UNIQUENESS OF THE SPECTRUM

The spectrum uniquely defines a given function i.e. distinct functions have distinct spectra. This is not difficult to prove.

Suppose two different functions F and G both have the same spectrum R.

i.e.

$$| T^n | \; | F | = | R | \qquad \text{..........} \quad 2.30$$

$$| T^n | \; | G | = | R | \qquad \text{..........} \quad 2.31$$

Combining 2.30 and 2.31 we have

$$| T^n | \; | F-G | = 0 \qquad \text{..........} \quad 2.32$$

Equation 2.32 has non-trivial solution if and only if T^n is singular. But T^n is never singular since its inverse always exists and is equal to $T^n/2^n$. Therefore $| F-G | = 0$ and so $F = G$. But this is contradictory to our initial assumption that F and G are different.

Therefore the spectrum uniquely represents a boolean function.

2.6 THE INVERSE TRANSFORM

Given the spectrum of a boolean function we can derive the characteristic sequence of F by applying an inverse transform to R.

$$| F | = | T^n |^{-1} \; | R |$$

$$= \frac{1}{2^n} \mid T^n \mid \mid R \mid \qquad \text{.......... 2.34}$$

This is a direct result of eq. 2.5.

Note that it is possible to choose an arbitrary vector of 2^n entries of 1's and 0's to form a characteristic sequence, but it is not possible to start from an arbitrarily chosen 'spectrum' and perform the reverse transform to get a valid characteristic sequence. To verify that a given set of numbers is a valid spectrum it is necessary to go through the inverse transform and ensure that the result is made up of 0's and 1's only.

2.7 PARITY OF SPECTRAL COEFFICIENTS

In a given spectrum, either all of the spectral coefficients are odd or all of them are even.

Proof :

If r_0 is odd then the number of minterms in the function must be odd.

From the previous section we have

$$r_{ij\cdots m} = N_0 - N_1$$

where N_0 = number of minterms in F when $X_i \oplus X_j \oplus \ldots \oplus X_m = 0$

N_1 = number of minterms in F when $X_i \oplus X_j \oplus \ldots \oplus X_m = 1$

But $N_0 + N_1 = r_0$ = total number of minterms in F.

Therefore either N_0 = odd and N_1 = even or N_0 = even and N_1 = odd

In either case $N_0 - N_1$ = odd.

Therefore other spectral coefficients are odd if r_0 is odd. If r_0 is even then it can similarly be proved that the rest of the spectral coefficients are also even.

2.8 REDUNDANCY OF VARIABLES

A function F is said to contain redundant variables if

$$F\ (X_1,\ X_2,\ \ldots\ X_n) = G\ (Y_1,\ Y_2,\ \ldots\ Y_m)$$

where

$m < n$ and

$Y_i \in \{X_1,\ X_2,\ \ldots\ X_n\}$

for each i $(1 <= i <= m)$

A variable X_i is said to be redundant in a function F if F is independent of X_i

i.e.

$$F(X_1,X_2\ldots X_{i-1},0,X_{i+1}\ldots X_n) = F(X_1,X_2\ldots X_{i-1},1,X_{i+1}\ldots X_n)$$

for all assignments to the other n-1 variables.

This can be detected by simplifying F using boolean algebraic techniques and checking whether F still involves X_i. If not then X_i is redundant and F will degenerate from a function of n variables to a function of n-1 or fewer variables.

For example the function

$$F = X_3 + \overline{X}_3X_1 + X_1\overline{X}_2$$

can be simplified to

$$F = X_3 + \overline{X}_3 X_1 + X_1 + X_1 \overline{X}_2$$
$$= X_3 + X_1$$

and therefore X_2 is redundant.

This procedure of simplication of function is a classical NP complete problem and consequently hard to implement by a computer program.

Detecting variable redundancy by spectral means turns out to be rather simple because if a variable X_i is redundant then all spectral coefficients that are associated with X_i are zero.

Proof :

$r_{ij\ldots m} = N_0 - N_1$

where N_0 and N_1 are defined as before.

But the number of minterms in F when $X_i = 0$ is equal to the number of minterms in F when $X_i = 1$ since F does not depend on X_i. Therefore $N_0 = N_1$. So $r_{ij\ldots m} = N_0 - N_1 = 0$.

Therefore all coefficients associated with X_i are zero.

Corollary :

If the last spectral coefficient is not zero then none of the variables in the boolean function are redundant.

The spectrum of the above function is :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}

6	-2	0	0	-2	-2	0	0

We see that all spectral coefficients associated with X_2 are zero.

The detection of redundancy of variables can thus be easily achieved with the spectrum and its implementation by computer program is straight forward.

2.9 SYMMETRIES

The traditional method of detecting symmetry between variables X_i and X_j in a binary function $F(X_1,..X_i,X_j,..X_n)$ is to interchange X_i and X_j and check to see if the function is invariant. If it is then the function is symmetric between X_i and X_j. This could be cumbersome even for a function with few variables.

Detection of symmetries by spectral techniques is more attractive. Actually the above method detects only one kind of symmetry, the non-equivalence symmetry as categorized by Hurst [9]. Using spectral methods several other kinds of symmetries can also be detected. They are

1. Equivalence symmetry between X_i and X_j $ES(X_i, X_j)$

   $F(X_1...0,0...X_n) = F(X_1...1,1...X_n)$

2. Non-equivalance symmetry between X_i and X_j $NES(X_i, X_j)$

   $F(X_1...0,1...X_n) = F(X_1...1,0...X_n)$

3. Multiform symmetries between X_i and X_j $MS(X_i, X_j)$

   $F(X_1...0,0...X_n) = F(X_1...1,1...X_n)$ and

   $F(X_1...0,1...X_n) = F(X_1...1,0...X_n)$

4. Single variable symmetry of X_i over $\bar{X}_j$ $\{SVS\ X_i\}\bar{X}_j$

$$F(X_1 \ldots 0, 0 \ldots X_n) = F(X_1 \ldots 1, 0 \ldots X_n)$$

5. Single variable symmetry of X_i over X_j $\{SVS\ X_i\}X_j$

$$F(X_1 \ldots 0, 1 \ldots X_n) = F(X_1 \ldots 1, 1 \ldots X_n)$$

It is not attempted here to derive the spectral conditions required for each type of symmetry which can be found in [9]. The results are quoted as follows.

Type of Symmetry	Necessary and Sufficient conditions in spectral terms
Equivalence symmetry	$r_{ia} + r_{ja} = 0$
Non-equivalence symmetry	$r_{ia} - r_{ja} = 0$
Multiform symmetry	$r_{ia} = r_{ja} = 0$
Single variable symmetry of X_i over $\bar{X}_j$	$r_{ia} + r_{ija} = 0$
Single variable symmetry of X_i over X_j	$r_{ia} - r_{ija} = 0$

where 'a' is any combination of spectral coefficient subscript not containing i or j.

To detect symmetry (say non equivalance symmetry) between X_i and X_j, it is necessary to compare r_{ia} and r_{ja} for all 'a' e.g. (X_{1i}, X_{1j}), (X_{12i}, X_{12j}), ... (X_{iklmn}, X_{jklmn}) etc.. Since 'a' does not contain i nor j, there are 2^{n-2} possible choices for each of r_{ia} and r_{ja}. Therefore $\frac{1}{2}$ of all the spectral coefficients have to be examined. Of course we can stop the comparision as soon as we find the first pair of r_{ia} and r_{ja} that do not satisfy the condition and conclude that the symmetry does not exist.

Again as in the case of redundancy of variables, the spectral approach of detecting symmetries is straightforward and can be incoporated in computer programs quite easily. One drawback of this approach is that it cannot be applied to partially specified functions (functions with don't cares). A more efficient method of detecting symmetries in fully and partially specified functions using the idea of compatiblity can be found in [14].

Symmetries have been applied to the synthesis of combinational circuits by several researchers, for example see [1], [10].

2.10 SPECTRUM OF SUBFUNCTIONS

A subfunction of a function $F(X_1.....X_n)$ is the function with a subset of the variables fixed. For example

$$F_0(X_1.....X_m) = F(X_1.....X_m,0,0, \ldots ,0,0,0)$$
$$F_1(X_1.....X_m) = F(X_1.....X_m,1,0, \ldots ,0,0,0)$$
$$F_2(X_1.....X_m) = F(X_1.....X_m,0,1, \ldots ,0,0,0)$$

etc.

are subfunctions of $F(X_1.....X_n)$

In general

$$F_u(X_1.....X_m) = F(X_1.....X_m,U_1....U_{n-m})$$

where

$$u = \sum_{i=1}^{n-m} U_i * 2^{i-1}$$

and $U_i \in \{0, 1\}$

An important relationship originated from Tokmen [21] connecting the spectrum of a function and its subfunctions is stated as follows.

$$| R_0 \ R_1 \ \ldots \ R_B | = \frac{1}{2^{n-m}} | R^0 \ R^1 \ \ldots \ R^B | \ T^{n-m} \quad .. \ 2.34$$

where $B = 2^{n-m} - 1$

R_u is the spectrum of F_u.

R^i is the partition of the spectrum of F as depicted in the following example.

For example if R is a subfunction with X_n and X_{n-1} fixed, then R would be partitioned into 4 parts with R_0 consisting of spectral coefficients that do not associate with either X_{n-1} or X_n, R^1 consisting of spectral coefficients that associate with X_{n-1} but not X_n etc. and we have the following relation from eq. 2.34

$$[R_0 \ R_1 \ R_2 \ R_3] = \tfrac{1}{4}[R^0 \ R^1 \ R^2 \ R^3]$$

where

$$R = \begin{vmatrix} R^0 \\ R^1 \\ R^2 \\ R^3 \end{vmatrix}$$

From the ordering of spectral coefficients in eq. 2.4 it is clear what spectral coefficients each partition R^i consists of. Rewriting equation 2.34 in another form

$$| R^0 \ R^1 \ \ldots \ R^B | = | R_0 \ R_1 \ \ldots \ R_B | \ T^{n-m} \quad .... \ 2.35$$

A proof of this relationship can be found in [21], here we shall demonstrate this using a simple example with $n-m = 2$.

Example

$$F = X_4(X_1 + X_2) + X_3\bar{X}_1X_2 + \bar{X}_3X_1X_2$$

The spectrum R of this function is :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
8	-2	-4	-2	0	-2	0	2

r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
-4	2	0	2	0	-2	0	2

Assume that the variables to be fixed are X_3 and X_4.

$$F_0 = F(X_1, X_2, 0, 0) = X_2X_1$$

$$F_1 = F(X_1, X_2, 1, 0) = X_2\bar{X}_1$$

$$F_2 = F(X_1, X_2, 0, 1) = X_2 + X_1$$

$$F_1 = F(X_1, X_2, 1, 1) = X_2 + X_1$$

$$R_0 = \text{spectrum of } F_0 = (1, -1, -1, 1)$$

$$R_1 = \text{spectrum of } F_1 = (1, 1, -1, -1)$$

$$R_2 = \text{spectrum of } F_2 = (3, -1, -1, -1)$$

$$R_3 = \text{spectrum of } F_3 = (3, -1, -1, -1)$$

So

$$| R_0, R_1, R_2, R_3 | = \begin{vmatrix} 1 & 1 & 3 & 3 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{vmatrix}$$

Now partition R into 2^{n-m} partitions as follows.

$$R^0 = r_0, r_1, r_2, r_{12} = 8, -2, -4, -2$$
$$R^1 = r_3, r_{13}, r_{23}, r_{123} = 0, -2, 0, 2$$
$$R^2 = r_4, r_{14}, r_{24}, r_{124} = -4, 2, 0, 2$$
$$R^3 = r_{34}, r_{134}, r_{234}, r_{1234} = 0, -2, 0, 2$$

Therefore $\frac{1}{4}|R^0\ R^1\ R^2\ R^3\ |\ T^2$

$$= \frac{1}{4} \begin{vmatrix} 8 & 0 & -4 & 0 \\ -2 & -2 & 2 & -2 \\ -4 & 0 & 0 & 0 \\ -2 & 2 & 2 & 2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{4} \begin{vmatrix} 4 & 4 & 12 & 12 \\ -4 & 4 & -4 & -4 \\ -4 & -4 & -4 & -4 \\ 4 & -4 & -4 & -4 \end{vmatrix}$$

$$= |\ R_0\ R_1\ R_2\ R_3\ |$$

Equation 2.34 is a very important relationship in the study of faults in logic circuits as will be appreciated later. It enables us to derive the spectra of faulty circuits from that of the fault free circuit without having to go through the process of finding the characteristic sequcnce of the faulty circuit and transforming these faulty sequences.

2.11 APPLICATIONS OF THE SPECTRUM

Spectral techniques have been used in classification of boolean functions and the synthesis of threshold logic networks [2]. In [9] Hurst devised several techniques to manipulate spectral coefficients so as to derive a simple 'core' function for a given boolean function in the synthesis of combinational circuits. Finally in the fault detection area several papers [13], [20] have appeared on testing combinational circuits using subsets of the spectra.

Chapter III

FAULTS IN NETWORKS AND THEIR RELATION TO SPECTRA

The most common types of faults in a digital logic circuit are stuck at faults and bridging faults. Stuck at faults occur when a line is held constant at either 1 or 0 irrespective of the changes at the input. This type of fault may happen when a wire is shorted to ground or the power line, when open circuit is present at the input of a TTL gate, when short circuits are present in diodes, transistors of other components. Bridging faults occur when two or more lines which are supposed to be insulated from each other are in electrical contact (shorted). This type of fault may happen at various hardware levels such as defective masking inside the chip, excessive solder between pads on the printed circuit board, breakdown of insulations etc.

In this chapter we shall examine how each type of fault can be modelled, then we shall discuss how the spectrum of a circuit can be affected by each type of fault. We limit ourselves to the study of both types of faults at input lines only since faults at input pins of a chip can be repaired but faults at internal lines render the chip useless and it has to be replaced.

3.1 FAULT MODELS

The modeling of an input stuck at fault is quite simple. If input X_i of a function $F(X_1, X_2, \ldots X_i, \ldots X_n)$ is stuck at 0 then X_i of F is assigned 0 i.e. the faulty function becomes a subfunction of F. i.e.

$$\hat{F}_{s-a-0} = F(X_1, X_2, \ldots ,0, \ldots X_n)$$

Similarly for X_i stuck at 1

$$\hat{F}_{s-a-1} = F(X_1, X_2, \ldots ,1, \ldots X_n)$$

For bridging faults there are several models that can be employed depending on the type of technology used for the integrated circuits. In [4] Garett showed that most logic families have a 'WIRED' logic property. For some logic families such as TTL the effect of a bridging fault between two or more lines is that the point where these lines are shorted behaves as an 'AND' gate. For other families it may behave as an 'OR', 'NAND' or 'NOR' gate.

Example

Let us consider the circuit realizing the function

$$F(X_1, X_2, X_3, X_4) = X_1\bar{X}_2 + X_3X_4$$

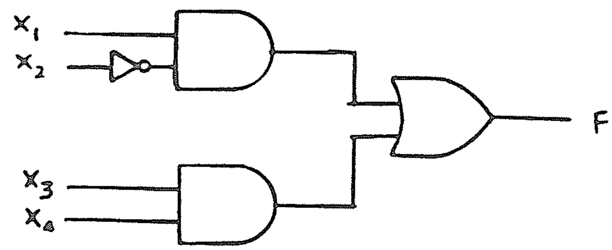


Fig. 3.1

The effect of a bridging between X_2 and X_4 is illustrated in Fig. 3.2 .

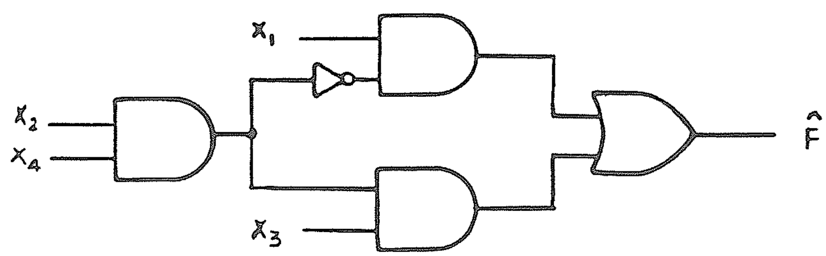


Fig. 3.2

For bridging faults between input lines the function

$$F(X_1, X_2, \ldots X_i, X_j, X_k, \ldots X_n)$$

which has a bridging fault between X_i and X_k will realize the function

$$\hat{F}_{ik} = F(X_1, X_2, \ldots ,X_iX_k, X_j, X_iX_k, \ldots X_n)$$

for the 'AND' model.

In some families the faulty lines may have an effect of an 'OR' gate and so the faulty function will realize the function

$$\hat{G}_{ik} = F(X_1, X_2, \ldots ,X_i+X_k, X_j, X_i+X_k, \ldots X_n)$$

for the 'OR' model.

Other models such as the 'NAND' and 'NOR' models can also be used depending on the internal construction of the gates. Formulae derived using one particular model can be transformed to another model using the duality property of boolean functions without much difficulty. Throughout this thesis the 'AND' model will be used.

3.2 FAULTY SPECTRUM (INPUT STUCK AT FAULT)

The spectrum of a circuit with the input X_i having a stuck at fault can be derived from that of the fault free circuit using Tokmen's result on the relationship between a function and its subfunctions [see section 2.10].

Let F be the fault free function

$\hat{F}$ be the faulty function with X_i stuck at fault

R be the spectrum of F

$\hat{R}$ be the spectrum of $\hat{F}$

r_i be a spectral coefficient of R

$\hat{r}_i$ be a spectral coefficient of $\hat{R}$

We want to find $\hat{r}_i$'s in terms of r_i's.

From equation 2.9 we have for the fault free circuit

$$\begin{vmatrix} R_0 & R_1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} R^0 & R^1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \qquad \ldots\ldots\ldots\ldots \quad 3.1$$

where R_0 is the spectrum of $F(X_1, X_2, \ldots X_{i-1}, 0, X_{i+1}, \ldots X_n)$

R_1 is the spectrum of $F(X_1, X_2, \ldots X_{i-1}, 1, X_{i+1}, \ldots X_n)$

R^0 is the partition of spectrum such that all spectral coefficients in this partition do not associate with X_i

R^1 is the partition of spectrum such that all spectral coefficients in this partition associate with X_i

Similarly for the faulty circuit

$$\begin{vmatrix} \hat{R}_0 & \hat{R}_1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} \hat{R}^0 & \hat{R}^1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \qquad \ldots\ldots\ldots\ldots \quad 3.2$$

or

$$\begin{vmatrix} \hat{R}^0 & \hat{R}^1 \end{vmatrix} = \begin{vmatrix} \hat{R}_0 & \hat{R}_1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \qquad \ldots\ldots\ldots\ldots \quad 3.3$$

where $\hat{R}_0$, $\hat{R}_1$, $\hat{R}^0$ and $\hat{R}^1$ are similarly defined for the faulty circuit as for the fault free circuit.

If X_i is stuck at 0 then

$$\hat{R}_0 = \hat{R}_1 = R_0$$

From eq 3.1

$$R_0 = \frac{1}{2} \begin{vmatrix} R^0 & R^1 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

Therefore 3.3 becomes

$$\left| \hat{R}^0 \quad \hat{R}^1 \right| = \frac{1}{2} \left| R^0 \quad R^1 \right| \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= \frac{1}{2} \left| R^0 \quad R^1 \right| \begin{vmatrix} 2 & 0 \\ 2 & 2 \end{vmatrix} \qquad \ldots\ldots\ldots \quad 3.4$$

Expanding 3.4 we have

$$\hat{R}^0 = R^0 + R^1 \qquad \ldots\ldots\ldots\ldots \quad 3.5 \quad \text{and}$$

$$\hat{R}^1 = 0$$

i.e.

$$\hat{r}_a = r_a + r_{ia} \qquad \ldots\ldots\ldots\ldots \quad 3.6 \quad \text{and}$$

$$\hat{r}_{ia} = 0$$

where 'a' is any subscript of a spectral coefficient that does not contain i. If X_i is stuck at 1 it can be similarly derived that

$$\hat{r}_a = r_a - r_{ia} \qquad \ldots\ldots\ldots\ldots \quad 3.7 \quad \text{and}$$

$$\hat{r}_{ia} = 0$$

3.3 FAULTY SPECTRUM (INPUT BRIDGING FAULT)

In this section we are going to derive the spectrum of a circuit with an input bridging fault in terms of the fault free spectrum. The 'AND' model will be used. For the 'OR' model the derivation is similar. Assume that the bridging is between X_i and X_j of the function

$$F(X_1, X_2, \ldots X_i, X_j, \ldots X_n).$$

We will make use of the relationship between a function and its subfunctions and also the fact that the subfunctions

$$\hat{F}(X_1, X_2, \ldots 0, 0, \ldots X_n)$$
$$\hat{F}(X_1, X_2, \ldots 0, 1, \ldots X_n)$$
$$\hat{F}(X_1, X_2, \ldots 1, 0, \ldots X_n)$$

of the faulty function are all equal to the subfunction

$$F(X_1, X_2, \ldots 0, 0, \ldots X_n)$$

of the fault free function. Also

$$\hat{F}(X_1, X_2, \ldots 1, 1, \ldots X_n) = F(X_1, X_2, \ldots 1, 1, \ldots X_n)$$

where F and $\hat{F}$ ary defined as in the previous section.

First partition the fault free spectrum R into 4 partitions R^0 (R^{00}), R^1 (R^{01}), R^2 (R^{10}) and R^3 (R^{11}).

Where R^0 consists of all coefficients that do not associate with X_i nor X_j

R^1 consists of all coefficients that only associate with X_i

R^2 consists of all coefficients that only associate with X_j

R^3 consists of all coefficients that associate with X_i and X_j

Applying eq. 2.9 to the fault free circuit we have

$$\left| R_0 \quad R_1 \quad R_2 \quad R_3 \right| = \frac{1}{4} \left| R^0 \quad R^1 \quad R^2 \quad R^3 \right| T^2 \quad \ldots\ldots\ldots \quad 3.8$$

where R_0 is the spectrum of the subfunction

$$F(X_1, X_2, \ldots 0, 0, \ldots X_n)$$

and R_1, R_2, and R_3 are similarly defined.

For the faulty function we have

$$\left| \hat{R}_0 \quad \hat{R}_1 \quad \hat{R}_2 \quad \hat{R}_3 \right| = \frac{1}{4} \left| \hat{R}^0 \quad \hat{R}^1 \quad \hat{R}^2 \quad \hat{R}^3 \right| T^2 \quad \ldots\ldots\ldots \quad 3.9$$

or

$$\left| \hat{R}^0 \quad \hat{R}^1 \quad \hat{R}^2 \quad \hat{R}^3 \right| = \left| \hat{R}_0 \quad \hat{R}_1 \quad \hat{R}_2 \quad \hat{R}_3 \right| T^2 \quad \ldots\ldots\ldots \quad 3.10$$

Where the $\hat{R}^i$'s and $\hat{R}_i$'s are similarly defined.

But $\hat{R}_0 = \hat{R}_1 = \hat{R}_2 = R_0$ and $\hat{R}_3 = R_3$ since their corresponding subfunctions are equal. Therefore eq. 3.10 becomes

$$\left| \hat{R}^0 \quad \hat{R}^1 \quad \hat{R}^2 \quad \hat{R}^3 \right| = \left| R_0 \quad R_0 \quad R_0 \quad R_3 \right| T^2 \quad \ldots\ldots\ldots \quad 3.11$$

Substituting R_0 and R_3 from 3.8 into 3.11

$$\left| \hat{R}^0 \quad \hat{R}^1 \quad \hat{R}^2 \quad \hat{R}^3 \right| = \frac{1}{4} \left| R^0 \quad R^1 \quad R^2 \quad R^3 \right| \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{4} \left| R^0 \quad R^1 \quad R^2 \quad R^3 \right| \begin{vmatrix} 4 & 0 & 0 & 0 \\ 2 & 2 & 2 & -2 \\ 2 & 2 & 2 & -2 \\ 4 & 0 & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 0 & 1 & 2 & 3 \\ R & R & R & R \end{vmatrix} \begin{vmatrix} 4 & 0 & 0 & 0 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 2 & 0 & 0 & 0 \end{vmatrix} \quad \ldots \quad 3.12$$

and we have the following result after expanding 3.12.

$$\hat{r}_a = r_a + \frac{1}{2}(r_{ia} + r_{ja}) + r_{ija} \quad \ldots\ldots\ldots\ldots \quad 3.13$$

$$\hat{r}_{ia} = \hat{r}ja = \frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots\ldots \quad 3.14$$

$$\hat{r}_{ija} = -\frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots\ldots \quad 3.15$$

Where 'a' is any spectral coefficient subscript not containing i or j.

3.4 SIGNATURE

A given fault is said to be covered by an attribute of a circuit if the attribute measured from a circuit with the fault differs from that measured from a fault free one. Tzidon et al [22] defined the signature of a combinational circuit as follows :

Let N be a combinational network, F be an arbitrary set of faults and S be an ordered set of attributes. S is called a signature of (N, F) iff any fault $\phi \in F$ is covered by at least one attribute in S.

That is to say any fault in F would cause at least one attribute in S to change. For example the parity of the number of minterms is a signature for multiple stuck at faults for fan-out-free networks (tree networks) composed of AND, OR, NAND, NOR, and NOT gates. This is obvious from the result of section 2.8 on redundant variables because the parity of the number of minterms on fan-out-free networks is always odd [22]. Any stuck at fault in such a network would result in one or more variables not having any influence on the circuit making them redundant. So one or more of the first order spectral coefficients will be zero. This forces the parity of the spectral coefficients to be even. Therefore the number of minterms (r_0) becomes even.

As another example, a NILE network is one in which the Number of Inverters along any Loop is Even and which is composed of AND, OR, NAND, NOR and NOT gates only. A loop here refers to two fan out paths which are convergent. Such networks can only realize functions in which any variable X_i can only appear either in its complemented or non-complemented but not both forms (unate functions) [8]. So any single stuck at fault in such a network would increase or decrease the number of minterms of the function. Therefore the number of minterms is a signature for single stuck at faults of NILE networks.

In the above signatures we can attach some physical meaning to the atrributes that form the signature. There are signatures which do not measure physical attributes as in the above examples. One such example is the Linear Feedback Shift Register [19] technique of deriving a signature from the characteristic sequence of a function. It is used by Hewlett Packard in integrated circuit testing.

Spectral coefficients can be used as signatures for certain types of faults such as stuck at faults and bridging faults in combinational circuits. One simple example would be a set S of non-zero spectral coefficients such that every variable X_i associates with at least one coefficient in S. Then S is a signature for multiple input stuck at faults for that circuit. This is obvious because such a fault would force at least one coefficient in S to go to zero. In [20] Susskind uses the same principle to form a signature for stuck at faults but he only uses the highest order (last) spectral coefficient.

3.5 SPECTRAL COEFFICIENT TESTABILITY OF A FAULT

A circuit is said to be r_a testable for a particular fault if the value of r_a in the presence of the fault differs from the value of r_a observed for the fault free circuit; otherwise it is r_a untestable. The notion of r_a testability originates from [13].

Assume 'a' does not contain i. For a single input stuck at fault, say at input X_i of a circuit, we know from section 3.2 that $\hat{r}_{ia} = 0$, so if $r_{ia} \neq 0$ then the circuit is r_a testable for a stuck at fault at X_i. Also for X_i stuck at 0

$$\hat{r}_a = r_a + r_{ia}$$

So if $r_{ia} \neq 0$ then $\hat{r}_a \neq r_a$ and the circuit is r_a testable for X_i stuck at 0. Similarly for X_i stuck at 1

$$\hat{r}_a = r_a + r_{ia}$$

Again if $r_{ia} \neq 0$ the circuit is r_a testable for X_i stuck at 1. Therefore the circuit is

r_{ia} testable if $r_{ia} \neq 0$ and

r_a testable if $r_{ia} \neq 0$

for both X_i stuck at 0 or X_i stuck at 1. In particular for a stuck at fault at input X_i, the circuit is r_0 testable or syndrome testable [17], [18] if $r_i \neq 0$.

For the case of an input bridging fault, say between inputs X_i and X_j of a circuit, using the result from secion 3.3 we have to consider 3 cases.

1.

$$\hat{r}_a = r_a + \tfrac{1}{2}(r_{ia} + r_{ja}) + r_{ija}$$

The circuit is r_a testable if

$$\tfrac{1}{2}(r_{ia} + r_{ja}) + r_{ija} \neq 0 \quad \text{or}$$

$$r_{ia} + r_{ja} + 2r_{ija} \neq 0$$

In particular it is r_0 testable if

$$r_i + r_j + 2r_{ij} \neq 0$$

2.

$$\hat{r}_{ia} = \hat{r}_{ja} = \frac{1}{2}(r_{ia} + r_{ja})$$

The circuit is r_{ia} and r_{ja} testable if

$$r_{ia} \neq \frac{1}{2}(r_{ia} + r_{ja}) \quad \text{or}$$

$$r_{ia} \neq r_{ja}$$

3.

$$\hat{r}_{ija} = -\frac{1}{2}(r_{ia} + r_{ja})$$

The circuit is r_{ija} testable if

$$\hat{r}_{ija} \neq -\frac{1}{2}(r_{ia} + r_{ja}) \quad \text{or}$$

$$r_{ia} + r_{ja} + 2r_{ija} \neq 0$$

Therefore for a single input bridging fault between inputs X_i and X_j, the circuit is r_a and r_{ija} testable if

$$r_{ia} + r_{ja} + 2r_{ija} \neq 0 \qquad \text{..........} \quad 3.16$$

and is r_{ia} and r_{ja} testable if

$$r_{ia} \neq r_{ja} \qquad \ldots\ldots\ldots\ldots \quad 3.17$$

Example

Consider the circuit realising the majority function

$$F(X_1, X_2, X_3) = X_3X_2 + X_3X_1 + X_2X_1$$

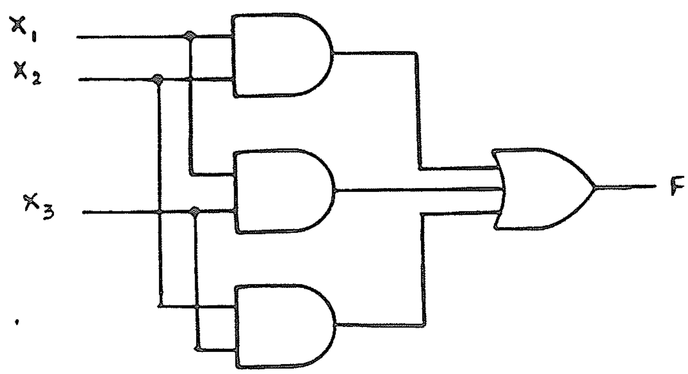


Fig. 3.3

The spectrum of F is

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
4	-2	-2	0	-2	0	0	2

We see that r_1, r_2 and r_3 are non-zero, therefore the circuit is r_0 testable for stuck at faults at X_1, X_2, or X_3. As a matter of fact the circuit is a NILE network, so the number of minterms changes for any stuck at fault [see section 3.4].

Let us look at a few spectral coefficients to see what effects a bridging fault between X_1 and X_2 has on them. We see that

$$r_1 + r_2 + 2r_{12} = -2 - 2 + 0 \neq 0$$

$$r_1 = r_2$$

$$r_{13} + r_{23} + 2r_{123} = 0 + 0 + 4 \neq 0$$

According to eq. 3.16 and 3.17, r_0, r_{12}, r_3 and r_{123} will change value but not r_1 nor r_2 for bridging between X_1 and X_2. The spectrum shown below is for the same circuit with X_1 and X_2 bridged.

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
2	-2	-2	2	0	0	0	0

By making use of these changes in the spectrum we are able to construct signatures for single input stuck at faults and single input bridging faults. For the above example we know that the circuit is r_0 testable for single stuck at faults (input and internal lines). For single input bridging faults we find a minimal subset of spctral coefficients such that at least one of the coefficients changes on any bridging fault. In this example we have

$$r_1 + r_2 + 2r_{12} \neq 0$$

$$r_1 + r_3 + 2r_{13} \neq 0$$

$$r_2 + r_3 + 2r_{23} \neq 0$$

Therefore the network is r_0 testable for all single input bridging faults. A signature for any single stuck at fault or single input bridging fault for this circuit is r_0.

3.6 MEASUREMENT OF SPECTRUM

The transformation of the characteristic sequence of a function into the spectrum is necessary only at the time the circuit is being studied. During the actual testing of the network the spectral coefficients of a circuit under test can be measured directly using the set up [15] shown in Fig. 3.4. The c_i's in fig. 3.4 are the control lines used to select the variables to be included in the function being generated. The spectral coefficients are measured by running the counter through all 2^n possible combinations of the input and the spectral coefficient can be read directly from the spectral coefficient accumulator (the up/down counter). With the clock running at 1 MHz a count of 2^{20} takes about 1 sec. therefore network testing using this approach is economical for medium size (about 20 inputs) circuits. For larger circuits the circuit has to be partitioned and each partition tested separately.

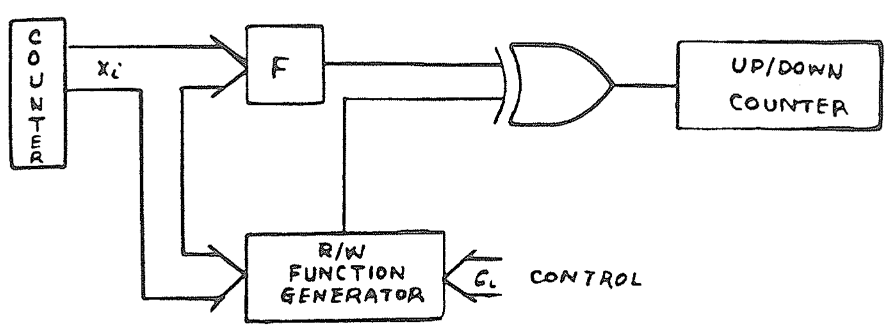
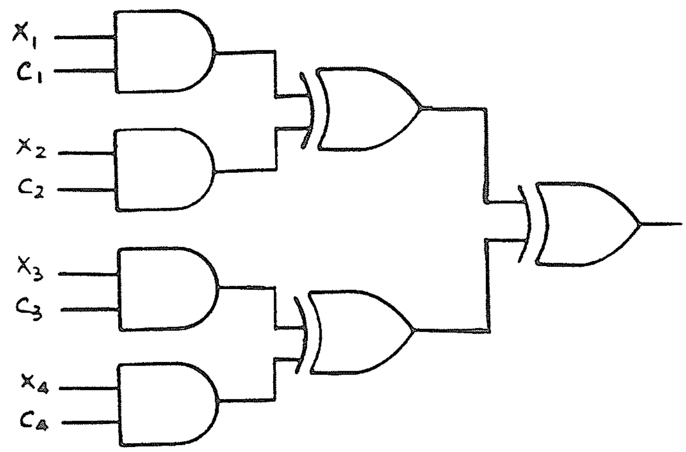


Fig. 3.4



A simple four variable R/W function generator
Fig. 3.5

3.7 CONCLUSION

We have seen how spectral coefficients can be used for constructing signatures. It should be noted that all signatures discussed in the previous sections are for fault detection. But they are not capable of isolating faults i.e. pinpointing their locations. In the following chapters we shall discuss techniques of finding signatures which are capable of locating faults at the input lines using the spectral approach.

Chapter IV

ISOLATION OF SINGLE INPUT BRIDGING FAULTS

In the previous chapter we have seen how spectral coefficients can be used to form fault detection signatures of combinational circuits by making use of their changes in the event of a fault. Such an approach has the advantage of being circuit independent (i.e. detailed knowledge of the circuit is not required to devise a test for the circuit) and usually results in a simpler solution than other approaches.

In this chapter we look at ways of finding signatures for combinational circuits which enable us to isolate single input bridging faults (i.e. faults that result from shorting of two input leads) using the spectral approach. The method is also based on the changes in spectral coefficients when a fault occurs. We shall use the 'AND' model for bridging faults. We also investigate the limitations of such signatures and the application of constrained testing to remove the limitation. Any fault mentioned in this chapter refers to a single input bridging fault if not explicitly stated.

4.1 SINGLE INPUT BRIDGING FAULT ISOLATION SIGNATURE

The two spectral properties that will be used here are :

1. a bridging fault between two input lines X_i and X_j is r_a testable and r_{ija} testable (for i, j not contained in a) if and only if [see sec. 3.5]

$$r_{ia} + r_{ja} + 2r_{ija} \neq 0 \qquad \text{..........} \quad 4.1$$

where 'a' can be any subscript of spectral coefficients including '' (null).

2. a bridging fault between two input lines X_i and X_j is r_{ia} and r_{ja} testable (for i, j not contained in a) if and only if [see sec. 3.5]

$$r_{ia} - r_{ja} \neq 0 \qquad \text{..........} \quad 4.2$$

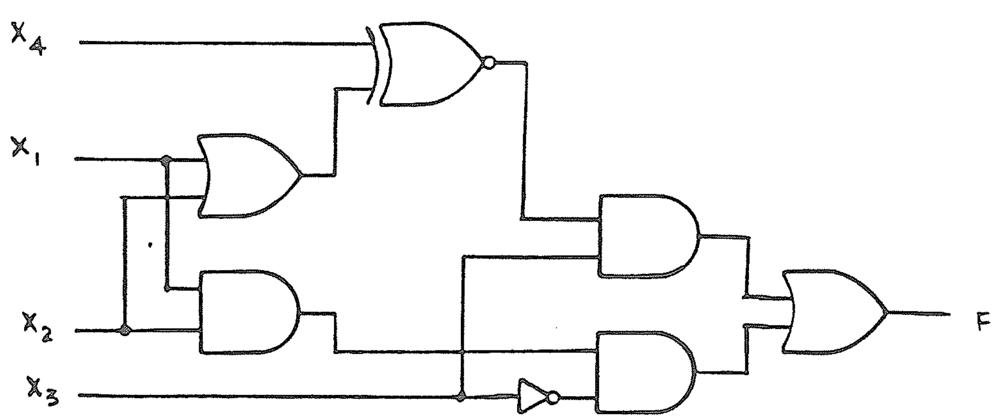
Based on these two properties we can find a signature of a combinational circuit for the detection of input bridging faults by finding for each possible bridging a set of spectral coefficients that will change if that particular bridging occurs. Then from these sets of coefficients find the minimum coverage for all bridgings to form a signature. Such a signature can determine a 'go' or 'no go' for a circuit i.e. whether the circuit is free from input bridging faults.

Not all bridging faults cause the same set of spectral coefficients to change value. By noting the pattern of changes in the spectral coefficients we are able to find a signature from which we can determine the location of the input bridging fault if it occurs. We also have to exhaustively go through each possible input bridging and find the set of spectral coefficients that will be changed by the bridging fault. Instead of finding a minimum coverage from these sets of coefficients, we choose from each set a minimum number of coefficients that can uniquely represent that bridging fault. Since we are making use of the pattern of changes in the spectral coefficients to distinguish the faults, this set of coefficients has to be chosen such that no two bridging faults would cause the same coefficients in the chosen set of spectral coefficients to change value. The resulting subset of spectral coefficients is a signature for isolation of single input bridging faults. Such a signature is not unique.

Example

Consider the circuit which realizes the function

$$F(X_1, X_2, X_3, X_4) = X_4X_3(X_2 + X_1) + \overline{X}_3X_2X_1 + \overline{X}_4X_3\overline{X}_2\overline{X}_1$$



The spectral coefficients of this circuit are :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
6	-2	-2	2	-2	-2	-2	2

r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
-2	2	2	2	2	-2	-2	-2

There are 6 possible single input bridging faults namely :

(X_1, X_2), (X_1, X_3), (X_1, X_4), (X_2, X_3) and (X_3, X_4)

Now let us find for each possible bridging fault which of the spectral coefficients will change if that fault occurs. Using eq. 4.1 and 4.2 we have the following for bridging between X_1 and X_2 :

check	coeff. affected	coeff. unaffected
$r_1 + r_2 + 2r_{12} = 0$		r_0, r_{12}
$r_{13} + r_{23} + 2r_{123} = 0$		r_3, r_{123}
$r_{14} + r_{24} + 2r_{124} \neq 0$	r_4, r_{124}	
$r_{134} + r_{234} + 2r_{1234} \neq 0$	r_{34}, r_{1234}	
$r_1 = r_2$		r_1, r_2
$r_{13} = r_{23}$		r_{13}, r_{23}
$r_{14} = r_{24}$		r_{14}, r_{24}
$r_{134} = r_{234}$		r_{134}, r_{234}

So the set of spectral coefficients that will change due to bridging between X_1 and X_2 are :

r_4, r_{34}, r_{124}, r_{1234}

Similarly the spectral coefficients for the remaining possible input bridgings are listed as follows :

BRIDGING	COEFFICIENTS CHANGED DUE TO BRIDGING
X_1, X_2	r_4, r_{34}, r_{124}, r_{1234}
X_1, X_3	r_0, r_2, r_{12}, r_{13}, r_{23}, r_{24}, r_{124}, r_{134}, r_{234}, r_{1234}
X_1, X_4	r_2, r_3, r_{13}, r_{23}, r_{34}, r_{123}, r_{124}, r_{134}, r_{234}, r_{1234}
X_2, X_3	r_0, r_1, r_{12}, r_{13}, r_{14}, r_{23}, r_{123}, r_{124}, r_{134}, r_{1234}
X_2, X_4	r_1, r_3, r_{13}, r_{23}, r_{34}, r_{123}, r_{124}, r_{134}, r_{234}, r_{1234}
X_3, X_4	r_1, r_2, r_{13}, r_{23}, r_{24}, r_{134}, r_{234}

Now that we have 6 sets of spectral coefficients each representing a particular bridging, we choose from each set a minimum number of coefficients which can identify the corresponding bridging. For example r_0 and r_2 are chosen for bridging between X_1 and X_3 because this is the only single input bridging fault that would cause these two coefficients to change value at the same time. The other bridging faults may cause either one of r_0 or r_2 to change but not both. For this example we may have the following choice :

BRIDGING	COEFFICIENTS CONTRIBUTED TO FORM SIGNATURE
X_1, X_2	r_4
X_1, X_3	r_0, r_2
X_1, X_4	r_2, r_3
X_2, X_3	r_0, r_1
X_2, X_4	r_1, r_3
X_3, X_4	r_1, r_2

A signature for the isolation of a single input bridging fault for the above circuit is :

$$(r_0, r_1, r_2, r_3, r_4)$$

To test the circuit for single input bridging, check to see that all spectral coefficients in the signature are the same as the one for the fault free circuit. If not, the circuit is faulty. To locate where the bridging fault is, find out which of the spectral coefficients in the signature change and then find out which bridging fault those spectral coefficients represent, thus locating the fault.

4.2 SIGNATURE WITH MINIMAL SPECTRAL COEFFICIENTS

The above method of choosing spectral coefficients from different sets (each representing a particular bridging) to form a signature is a straightforward approach and normally results in a set of low order coefficients which are easier to obtain than the higher order ones. But this approach does not guarantee a minimum number of spectral coefficients for the signature.

To find a signature with the minimum number of spectral coefficients, we build for each possible input bridging a bit array with 2^n entries where n is the number of inputs. Each entry in the array represents a spectral coefficient. A '1' indicates that the corresponding spectral coefficient changes due to the bridging and a '0' indicates that it does not. This forms an array of size 2^n by $\frac{1}{2}n(n-1)$, where each row associates with a spectral coefficient and each column associates with a possible bridging fault.

Choose from these 2^n rows a minimum number of rows such that they have a different bit pattern for each possible bridging and that none of the patterns contains all 0's (to distinguish from the fault free circuit). This is a minimum coverage problem. Algorithms can be found in quite a number of texts such as [11] to solve this problem. The spectral coefficients corresponding to the selected rows is the required signature.

Obviously the minimum number of spectral coefficients that have to be included in this signature must be greater than or equal to k where k is the smallest integer greater than or equal to

$$\log_2 (1 + n(n - 1)/2)$$

Using this approach for the above example we have the following array with each column associated with a possible bridging.

POSSIBLE BRIDGINGS

	X_1,X_2	X_1,X_3	X_1,X_4	X_2,X_3	X_2,X_4	X_3,X_4
r_0	0	1	0	1	0	0
r_1	0	0	0	1	1	1
r_2	0	1	1	0	0	1
r_{12}	0	1	0	1	0	0
r_3	0	0	1	0	1	0
r_{13}	0	1	1	1	1	1
r_{23}	0	1	1	1	1	1
r_{123}	0	0	1	1	1	0
r_4	1	0	0	0	0	0
r_{14}	0	0	0	1	0	0
r_{24}	0	1	0	0	0	1
r_{124}	1	1	1	1	1	0
r_{34}	1	0	1	0	1	0
r_{134}	0	0	1	1	1	1
r_{234}	0	1	1	0	1	1
r_{1234}	1	1	1	1	1	0

In this example n=4 and k=3, and we find that rows corresponding to r_1, r_2 and r_{34} give different bit patterns for each possible input bridging and there is at least one '1' in each of them. Thus (r_1, r_2, r_{34}) is a signature for single input bridging fault isolation with the minimum number of spectral coefficients.

Notice that the procedure for locating faults using this signature is different from that presented previously. In this case we cannot just use those spectral coefficients in the signature that change value, but all spectral coefficients in the signature must be used to identify a fault. For example a bridging fault between X_1 and X_2 cannot be represented by the change in value of r_{34} alone, but by a change in value of r_{34} and no change in value of r_1 or r_2 as well.

4.3 LIMITATION OF FAULT ISOLATION SIGNATURES

For some circuits a bridging between two inputs may cause all of the spectral coefficients to change value. This makes the selection of spectral coeffieicnts to represent such faults impossible if we are just using change of value rather than the actual value. If there is only one such fault for all possible single input bridging faults, we will still be able to find a fault isolation signature for the circuit. But if there is more than one such fault the signature for such a circuit can only differentiate between them as one class of faults although the rest of the faults can still be differentiated from one another. To isolate individual faults in this class, we have to use other techniques such as constrained testing which will be discussed later in this chapter.

To find a fault isolation signature for a circuit with one or more possible single input bridging faults that affects all spectral coefficients we carry out the same procedure as mentioned in section 4.1 and 4.2 except that we have one set of spectral coefficients for these problem bridgings which includes the whole spectrum. The value of k in section 4.2 would be the smallest integer greater than $\log_2 (n(n-1)/2 - p + 1)$ where p is the number of problem bridgings.

4.4 THE PROBLEM BRIDGINGS

We know from chapter 2 that in a given spectrum either all of the spectral coefficients are odd of all of them are even. There is a simple situation in which all spectral coefficients change value due to a single input bridging, that is when the number of minterms of the function changes from odd to even or vice versa when the bridging occurs. In this case the parity of r_0 changes and so do the rest of the spectral coefficients.

If the parity of the spectral coefficients is not affected by the bridging, is it possible for all the coefficients to change their values? This depends on whether there is any solution to the equation :

$$T^n * \left| f_i - g_i \right| = \left| k_i \right| \qquad \ldots\ldots\ldots\ldots \quad 4.3$$

where f and g are the characteristic sequences for the fault free and faulty circuit respectively. They can only assume values of 1 and 0. The k_i's must be nonzero even integers. Replacing $f_i - g_i$ by c_i equation 4.3 becomes :

$$T^n * \left| c_i \right| = \left| k_i \right| \qquad \ldots\ldots\ldots\ldots \quad 4.4$$

where the c_i's can only assume values of 0, -1 or +1.

Let us look at the case of circuits with 3 inputs. Without loss of generality assume that the bridging is between X_3 and X_2. The values of f_i's, g_i's and c_i's can be listed as follows :

X_3	X_2	X_1	f	g	c
0	0	0	f_0	f_0	$c_0 = 0$
0	0	1	f_1	f_1	$c_1 = 0$
0	1	0	f_2	f_0	c_2
0	1	1	f_3	f_1	c_3
1	0	0	f_4	f_0	c_4
1	0	1	f_5	f_1	c_5
1	1	0	f_6	f_6	$c_6 = 0$
1	1	1	f_7	f_7	$c_7 = 0$

We notice that c_0, c_1, c_6 and c_7 are all zero. This is because the bridging does not have any effect on f in these cases.

The values of c_2 to c_5 can be 0, 1 or -1 and the following restrictions apply to them.

1. If zeroes are present, there must be an even number of them, otherwise at least one of the k_i's would be odd, so there can be at most be 2 zeroes in the case with 3 variables.
2. If neither c_2 nor c_4 is zero, they must be of the same value. This is because if c_2 has a value say +1 and c_4 -1, then

$$f_2 - f_0 = 1 \Rightarrow f_0 = 0 \qquad \text{and}$$
$$f_4 - f_0 = -1 \Rightarrow f_0 = 1$$

which is contradictory.

The same restrictions also apply to c_3 and c_5. If there is no zero in c_2 to c_5 then either k_0 or k_1 will be zero. If there are two zeroes present then depending on the positions of the zeroes in c_2 to c_5, at least one of the k_i's will be zero. Therefore there is no solution to equation 4.3 for functions of 3 variables.

For functions of 4 variables, again without loss of generality assume that the bridging is between X_4 and X_3. The values of f_i, g_i, and c_i's are listed below.

X_4	X_3	X_2	X_1	f	g	c
0	0	0	0	f_0	f_0	$c_0 = 0$
0	0	0	1	f_1	f_1	$c_1 = 0$
0	0	1	0	f_2	f_2	$c_2 = 0$
0	0	1	1	f_3	f_3	$c_3 = 0$
0	1	0	0	f_4	f_0	c_4
0	1	0	1	f_5	f_1	c_5
0	1	1	0	f_6	f_2	c_6
0	1	1	1	f_7	f_3	c_7
1	0	0	0	f_8	f_0	c_8
1	0	0	1	f_9	f_1	c_9
1	0	1	0	f_{10}	f_2	c_{11}
1	0	1	1	f_{11}	f_3	c_{12}
1	1	0	0	f_{12}	f_{12}	$c_{12} = 0$
1	1	0	1	f_{13}	f_{13}	$c_{13} = 0$
1	1	1	0	f_{14}	f_{14}	$c_{14} = 0$
1	1	1	1	f_{15}	f_{15}	$c_{15} = 0$

With the same reasoning as presented previously, we see that if neither c_4 nor c_8 is zero then they must be equal. The same applies to c_5 and c_9, c_6 and c_{10}, c_7 and c_{11}. Now if we assign a zero to one of each pair of such c_i's, we are free to choose the other 4 c_i's. For example if we arbitrarily assign zeroes to c_4, c_5, c_{10} and c_{11} then we can assign any of {0, 1, or −1} to c_6, c_7, c_8 and c_9. We know that there are equal number of +1's and −1's in each column of column of T^n. Therefore if we choose the 4 c_i's in such a way that the number of +1's and -1's are odd then none of the k_i's will be zero or odd. One such example is presented below.

EXAMPLE

X_4	X_3	X_2	X_1	f_i	g_i	c_i
0	0	0	0	0	0	0
0	0	0	1	1	1	0
0	0	1	0	0	0	0
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	0	1	1	1	0
0	1	1	0	1	0	1
0	1	1	1	1	0	1
1	0	0	0	1	0	1
1	0	0	1	0	1	-1
1	0	1	0	0	0	0
1	0	1	1	0	0	0
1	1	0	0	D	D	0
1	1	0	1	D	D	0
1	1	1	0	D	D	0
1	1	1	1	D	D	0

The D's are don't cares but the don't cares of f_i and g_i's in the same row must be equal. One possible function which possesses this characteristic is :

$$F(X_1,X_2,X_3,X_4) = X_4\overline{X}_2\overline{X}_1 + \overline{X}_4\overline{X}_1X_2 + \overline{X}_4X_3X_2$$

The spectrum of the above function is :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
6	0	2	0	-2	0	2	0
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
4	-4	-2	-4	-2	0	2	0

The spectrum of the circuit with X_3 and X_4 bridged is :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
4	-2	4	-2	0	-2	0	-2
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
0	-2	0	-2	0	2	0	2

Notice that every one of the spectral coefficients changes value but their parity remains unchanged. Therefore solutions to equation 4.3 do exist and there are circuits that behave in such a nasty manner. The number of 4 variable functions that behave this way was estimated to be less than 1% of all possible functions. So hopefully the number of functions with more than one bridging behaving this way is comparatively small. This was not investigated, but ways to overcome this problem were examined and they are presented in the following sections.

4.5 CONSTRAINED TESTING

With the signatures introduced above, bridgings that cause all spectral coefficients to change value cannot be distinquished among themselves (the problem bridgings). In order to be able to do so we have to employ another technique of testing the circuits.

A constrained testing is a test performed on a subset of the inputs while the rest of the inputs are kept constant. Our aim is to fix a minimum number of inputs which are not involved in the problem bridgings and find a signature for these problem bridgings by which we are able to isolate one from the other.

One obvious objective would be to constrain the inputs in such a way that the parity of r_0 and that of the rest of the spectrum do not change when the problem bridging occurs in the constrained circuit. Let us look at the case of one bridging say between X_i and X_j. To preserve the parity of the spectral coefficients if it changes when the bridging occurs, it is sufficient to constrain any one of the inputs say X_k where X_k is not one of X_i nor X_j. The value that X_k assumes is important. By fixing X_k at one value, the parity of the r_i's will remain unchanged for bridging between X_i and X_j, but fixing X_k at the other value is bound to change the parity of all r_i's when the same bridging occurs.

The proof of this is rather simple. Suppose the parity of the r_i's changes from even to odd when the bridging occurs. Partition the minterms of the fault free circuit into 2 parts according to whether they associate with $X_k = 0$ or $X_k = 1$.

no. of minterms $= K_0$	$X_k = 0$

no. of minterms $= K_1$	$X_k = 1$

Since r_0 of the fault free circuit is even $K_0 + K_1$ must be even. There are two possibilities.

1. K_0 and K_1 are both even.
2. K_0 and K_1 are both odd.

Similarly we divide the minterms of the faulty circuit into two groups.

no. of minterms $= \hat{K}_0$	$X_k = 0$

no. of minterms $= \hat{K}_1$

$X_k = 1$

Since r_0 of the faulty circuit is odd, there are again two possibilities.

1. $\hat{K}_0$ is even and $\hat{K}_1$ is odd.
2. $\hat{K}_0$ is odd and $\hat{K}_1$ is even.

There are 4 different ways that K_0, K_1, $\hat{K}_0$ and $\hat{K}_1$ can assume their values.

1. K_0 and K_1 are both even, $\hat{K}_0$ is even and $\hat{K}_1$ is odd. In this case if we constrain X_k to 0 we are looking at a subfunction whose number of minterms for the fault free and faulty circuit are K_0 and $\hat{K}_0$ respectively and they are both even. Therefore the parity of the spectral coefficients is preserved.
2. K_0 and K_1 are both even, $\hat{K}_0$ is odd and $\hat{K}_1$ is even. Constraining X_k to 1 would preserve the even parity of the spectral coefficients.
3. K_0 and K_1 are both odd, $\hat{K}_0$ is odd and $\hat{K}_1$ is even. Constraining X_k at 0 would preserve the odd parity of the spectral coefficients.
4. K_0 and K_1 are both odd, $\hat{K}_0$ is even and $\hat{K}_1$ is odd.

Constraining X_k at 1 would preserve the even parity of the spectral coefficients.

On the other hand if the parity of the r_i's changes from odd to even the proof would be similar and so we can always constrain an input to preserve the parity of the spectral coefficients.

If the parity of the spectral coefficients does not change when the bridging fault occurs then for some of the inputs, constraining an input at any value ('0' or '1') will preserve the parity of the spectral coefficients. But for other inputs, constraining any one of them at either '1' or '0' will reverse the parity of the spectral coefficients. That is, which input to be constrained is important in this case.

For example, using the symbols already defined, if the parity of the spectral coefficients for the fault free and faulty circuit are both odd then we have the following possiblities for K_0, K_1, $\hat{K}_0$ and $\hat{K}_1$.

1. K_0 is odd and K_1 is even.
2. K_0 is even and K_1 is odd.
3. $\hat{K}_0$ is odd and $\hat{K}_1$ is even.
4. $\hat{K}_0$ is even and $\hat{K}_1$ is odd.

If 1 and 3 or 2 and 4 are satisfied then no matter what value X_k is constrained to, the even or odd parity will be preserved. If 1 and 4 or 2 and 3 are satisfied then constraining X_k at either '0' or '1' will reverse the parity.

To determine what value X_k should be fixed at, we need the spectrum of the fault free circuit and that of the faulty circuit. If we have both of them we calculate K_0, K_1 $\hat{K}_0$ and $\hat{K}_1$ as follows.

For the fault free circuit we have

$$r_0 = K_0 + K_1$$
$$r_1 = K_0 - K_1$$

and for the faulty circuit we have

$$\hat{r}_0 = \hat{K}_0 + \hat{K}_1$$
$$\hat{r}_1 = \hat{K}_0 - \hat{K}_1$$

So

$$K_0 = \frac{r_0 + r_1}{2} \qquad \ldots\ldots\ldots\ldots \quad 4.5$$

$$K_1 = \frac{r_0 - r_1}{2} \qquad \ldots\ldots\ldots\ldots \quad 4.6$$

and

$$\hat{K}_0 = \frac{\hat{r}_0 + \hat{r}_1}{2} \qquad \ldots\ldots\ldots\ldots \quad 4.7$$

$$\hat{K}_1 = \frac{\hat{r}_0 - \hat{r}_1}{2} \qquad \ldots\ldots\ldots\ldots \quad 4.8$$

If K_0, K_1, $\hat{K}_0$ and $\hat{K}_1$ are known we will be able to determine what value X_k should be constrained at.

To derive the faulty spectrum from the fault free one, one obvious method would be to do a reverse transform of the fault free spectrum back to its boolean domain, then do a transform again after changing the characteristic sequence using the relationship between the subfunction of the faulty and fault free functions discussed in beginning of section 3.3 . Notice that the fast transform can be used for both the reverse and forward transforms.

An alternative would be to derive $\hat{r}_0$ and $\hat{r}_k$ directly from the fault free spectrum using the results from section 3.3 . They are

$$\hat{r}_0 = r_0 + \frac{1}{2}(r_i + r_j) + r_{ij}$$

$$\hat{r}_k = r_k + \frac{1}{2}(r_{ik} + r_{jk}) + r_{ijk}$$

Notice that this way we do not have to derive the whole faulty spectrum but just the first order coefficients that associate with the variables that we want to constrain.

4.6 SIGNATURES FOR VERIFICATION OF SPECTRAL VALUES

The isolation signature discussed in sections 4.1 and 4.2 makes use of the pattern of changes in the spectral coeff-cients when a bridging occurs. Each spectral coefficient can therefore be used as a binary form of information i.e. whether it changes or not. As an alternative we may use the actual values to isolate the bridging faults. Since a spectral coefficient can assume a wide range of values, the number of coefficients required in the signature is usually smaller than that required by the techniques section 4.1 or 4.2 . In some cases one spectral coefficient may be enough to isolate all the input bridging faults.

To find such a signature we would have to evaluate the faulty spectrum of every possible bridging fault using the relationships between the faulty and fault free spectrum in section 3.3 . Then we build a matrix similar to the one in section 4.2 except that in this case each entry is an integer instead of a bit. Also included in this matrix is a column for the fault free spectrum. The matrix would be of size $2^n \times \frac{1}{2}n(n-1) + 1$. We find the minimum number of rows (i.e. spectral coefficients) that can cover all possibe faults. Again this is a minimum coverage problem.

Example

Using the circuit in section 4.1, we build the following matrix.

POSSIBLE FAULTS

	fault free	X_1, X_2	X_1, X_3	X_1, X_4	X_2, X_3	X_2, X_4	X_3, X_4
r_0	6	6	2	6	2	6	6
r_1	-2	-2	-2	-2	0	2	-4
r_2	-2	-2	0	2	-2	-2	-4
r_{12}	2	2	0	2	0	2	2
r_3	-2	-2	-2	-4	-2	-4	-2
r_{13}	-2	-2	2	0	0	-4	0
r_{23}	-2	-2	0	-4	2	0	0
r_{123}	2	2	0	0	0	0	2
r_4	-2	2	-2	-2	-2	-2	-2
r_{14}	2	2	2	2	0	2	0
r_{24}	2	2	0	2	2	2	0
r_{124}	2	-2	0	-2	0	-2	2
r_{34}	2	-2	2	0	2	0	2
r_{134}	-2	-2	-2	0	0	0	0
r_{234}	-2	-2	0	0	-2	0	0
r_{1234}	-2	2	0	0	0	0	-2

A signature for this circuit may be composed of r_1, r_2 and r_4. There might be other combinations of spectral coefficients that constitute such a signature. If this is the case then the one with lower order coefficients should be chosen, the reason being that lower order coefficients are easier to measure then higher order ones.

As in the case of the problem bridgings discussed earlier, there may be bridgings that this signature will not be able to isolate. In this case the spectra of the faulty circuits have to be identical, in other words the faulty circuits are functionally equivalent since a spectrum uniquely defines a circuit. First let us look at the case in which a bridging between X_i, X_j and a bridging between X_i, X_k result in the same spectrum, where i, j and k are all distinct.

We would tend to think that symmetry (non equivalence) between X_j and X_k would be the required condition. Actually it turns out to be a necessary but not a sufficient condition.

For example

$$F(X_1,X_2,X_3,X_4) = X_1X_2 + X_3X_4$$

F is symmetric between X_1 and X_2 but a bridging fault between X_1 and X_3 would result in the faulty function :

$$\begin{aligned}\hat{F}_{13} &= X_1X_2X_3 + X_1X_3X_4 \\ &= X_1X_3(X_2 + X_4)\end{aligned}$$

A bridging fault between X_2 and X_3 results in the faulty function :

$$\begin{aligned}\hat{F}_{23} &= X_1X_2X_3 + X_2X_3X_4 \\ &= X_2X_3(X_1 + X_4)\end{aligned}$$

Note that $\hat{F}_{13} \neq \hat{F}_{23}$, so that the symmetry between X_1 and X_2 is not a sufficient condition for two single bridging faults to result in functionally equivalent circuits.

To derive the necessary and sufficient condition we divide the faulty spectra into 8 groups as follows. Using the relationships from section 3.3 we have for the faulty circuit with X_i and X_j bridged :

$$\hat{r}_a = r_a + \frac{1}{2}(r_{ia} + r_{ja}) + r_{ija} \qquad \text{.......... } 4.9$$

$$\hat{r}_{ia} = \frac{1}{2}(r_{ia} + r_{ja}) \qquad \text{.......... } 4.10$$

$$\hat{r}_{ja} = \frac{1}{2}(r_{ia} + r_{ja}) \qquad \text{.......... } 4.11$$

$$\hat{r}_{ka} = r_{ka} + \frac{1}{2}(r_{ika} + r_{jka}) + r_{ijka} \qquad \text{....... } 4.12$$

$$\hat{r}_{ija} = -\frac{1}{2}(r_{ia} + r_{ja}) \qquad \text{.......... } 4.13$$

$$\hat{r}_{ika} = \frac{1}{2}(r_{ika} + r_{jka}) \qquad \text{.......... } 4.14$$

$$\hat{r}_{jka} = \frac{1}{2}(r_{ika} + r_{jka}) \qquad \text{.......... } 4.15$$

$$\hat{r}_{ijka} = -\frac{1}{2}(r_{ika} + r_{jka}) \qquad \text{.......... } 4.16$$

The faulty spectrum with X_i and X_k bridged is :

$$\hat{r}_a = r_a + \frac{1}{2}(r_{ia} + r_{ka}) + r_{ika} \qquad \text{.......... } 4.17$$

$$\hat{r}_{ia} = \frac{1}{2}(r_{ia} + r_{ka}) \qquad \text{.......... } 4.18$$

$$\hat{r}_{ja} = r_{ja} + \frac{1}{2}(r_{ija} + r_{jka}) + r_{ijka} \qquad \text{....... } 4.19$$

$$\hat{r}_{ka} = \frac{1}{2}(r_{ia} + r_{ka}) \qquad \text{.......... } 4.20$$

$$\hat{r}_{ija} = \frac{1}{2}(r_{ija} + r_{jka}) \qquad \text{.......... } 4.21$$

$$\hat{r}_{ika} = -\frac{1}{2}(r_{ia} + r_{ka}) \qquad \text{.......... } 4.22$$

$$\hat{r}_{jka} = \frac{1}{2}(r_{ija} + r_{jka}) \qquad \text{.......... } 4.23$$

$$\hat{r}_{ijka} = -\frac{1}{2}(r_{ija} + r_{jka}) \qquad \text{.......... } 4.24$$

for all 'a' where 'a' does not contain i, j or k.

If the faulty spectra are to be equal then each corresponding group of these spectral coefficients must also be the same.

Equating 4.9 and 4.17, 4.10 and 4.18 we have

$$r_{ja} = r_{ka} \qquad \text{.......... } 4.25$$

$$r_{ija} = r_{ika} \qquad \text{.......... } 4.26$$

Equation 4.25 and 4.26 imply that the necessary condition is that the network must be symmetric (non equivalence) between X_j and X_k.

Equating 4.11 and 4.19 we have

$$r_{ia} + r_{ja} = 2r_{ija} + r_{jka} + 2r_{ijka} \quad \text{or}$$

$$r_{ia} = r_{ja} + r_{ija} = r_{jka} + 2r_{ijka} \quad \ldots\ldots\ldots \quad 4.27$$

Equating 4.13 and 4.21 we have

$$- r_{ia} - r_{ja} = r_{ija} + r_{jka} \quad \ldots\ldots\ldots\ldots \quad 4.28$$

Substituting 4.28 into 4.27 we have

$$r_{ia} = r_{ja} - r_{ia} - r_{ja} + 2r_{ijka} \quad \text{or}$$

$$2r_{ia} = 2r_{ijka} \quad \text{or}$$

$$r_{ia} = r_{ijka} \quad \ldots\ldots\ldots\ldots \quad 4.29$$

The rest of the equations are redundant. The necessary and sufficient condition for bridging faults between X_i, X_j and X_i, X_k to produce functionally equivalent circuits are :

$$r_{ja} = r_{ka} \quad \ldots\ldots\ldots\ldots \quad 4.30$$

$$r_{ija} = r_{ika} \quad \ldots\ldots\ldots\ldots \quad 4.31$$

$$r_{ia} = r_{ijka} \quad \ldots\ldots\ldots\ldots \quad 4.32$$

$$r_{ia} + r_{ja} + r_{ija} + r_{jka} = 0 \quad \ldots\ldots\ldots\ldots \quad 4.33$$

An example of such a function is :

$$F(X_1,X_2,X_3) = X_1X_2 + \overline{X}_1\overline{X}_2X_3$$

The faulty function with X_1 and X_3 bridged is :

$$\hat{F}_{13} = X_1X_2X_3 + \overline{X_1X_3}X_2X_1X_3$$
$$= X_1X_2X_3$$

Similarly the faulty function with X_2 and X_3 bridged is :

$$\hat{F}_{23} = X_1X_2X_3$$

The relationship between the spectral coefficients of the fault free and faulty circuit shown in section 3.3 assumes the 'AND' model for a bridging fault. If other models were used in this derivation, the conditions 4.30 to 4.33 would be different. To generalize the above conditions for different models it is necessary to derive the relationships for each model. It is not attempted to derive them here, but the generalized conditions are listed as follows :

$$r_{ja} = r_{ka} \quad \text{..........} \quad 4.34$$

$$r_{ija} = r_{ika} \quad \text{..........} \quad 4.35$$

$$r_{ia} + (1 - V)r_{ija} + Vr_{ijka} = 0 \quad \text{..........} \quad 4.36$$

$$r_{ia} + r_{ja} + U(r_{ija} + r_{jka}) = 0 \quad \text{..........} \quad 4.37$$

where the values of U and V are different for different models used and their values are listed below.

Model	U	V
AND	1	1
OR	-1	1
NAND	1	-1
NOR	-1	-1

The conditions under which two different bridging faults X_i, X_j and X_k , X_l (i, j and k are distinct) produce functionally equivalent circuits can be similarly derived. The conditions are listed below.

$$r_{ia} + r_{ja} + 2r_{ija} = r_{ka} + r_{la} + 2r_{lka} \quad \ldots\ldots \quad 4.38$$

$$r_{ja} - r_{ia} = r_{ika} + r_{ila} + 2r_{ikla} \quad \ldots\ldots \quad 4.39$$

$$r_{la} - r_{la} = r_{jla} + r_{jka} + 2r_{ijka} \quad \ldots\ldots \quad 4.40$$

$$r_{ia} + r_{ja} + 2r_{ija} + r_{ijka} + r_{ijla} + 2r_{ijlka} = 0 \quad \ldots\ldots \quad 4.41$$

$$r_{jka} = r_{ila} \quad \ldots\ldots \quad 4.42$$

$$r_{jla} = r_{ika} \quad \ldots\ldots \quad 4.43$$

$$r_{ika} + r_{jka} + r_{ijka} + r_{ijla} = 0 \quad \ldots\ldots \quad 4.44$$

$$r_{ila} + r_{jla} + r_{ikla} + r_{jkla} = 0 \quad \ldots\ldots \quad 4.45$$

Notice that the conditions do not require symmetry between any variables and obviously the conditons are much stricter and thus harder to satisfy than the ones discussed previously in this section.
One example of such a circuit is :

$$F(X_1, X_2, X_3, X_4) = \overline{X}_1\overline{X}_2 + \overline{X}_1\overline{X}_3 + \overline{X}_2X_3X_4 + \overline{X}_1X_3X_4 + \overline{X}_2\overline{X}_3\overline{X}_4$$

A bridging fault between X_3 and X_4 or bridging fault between X_1 and X_2 produces the same faulty function

$$\hat{F} = \overline{X}_1 + \overline{X}_2$$

4.7 CONCLUSION

Basically we have discussed two isolation signatures, the first one makes use of the patterns of changes in the spectral coefficients and the second signature uses verification of the actual values of the spectral coefficients. The processes of finding these two signatures are very similar, Each has to explicitly go through the outcome of every possible fault. Also each requires the process of finding a minimal cover except that the first one requires less storage if implemented in a computer program and this signature usually requires more spectral coefficients than the other.

In the actual testing of circuits both signatures require stored information for faulty circuits. It might be in the form of bit patterns for the first signature or the actual values of the faulty spectral coefficients in the second signature.

Chapter V

ISOLATION OF INPUT STUCK AT FAULTS

Three methods of finding signatures for the isolation of single input stuck at faults will be discussed in this chapter. The first method is basically the same as the method presented in chapter 4 for single input bridging faults which involves the detection of changes in spectral coefficients when the fault occurs. The second method is a little different. It involves the detection of zeroes in the spectral coefficients in the signature. The third signature involves the values of the spectral coefficients and it is also similar to the signature discussed in section 4.6. The limitations of each signature are also discussed. A fault mentioned in this chapter refers to a single input stuck at fault if not explicitly stated.

5.1 SIGNATURES FOR DETECTION OF CHANGES IN SPECTRUM

The spectral property that will be used in this section is : An input line X_i is r_a and r_{ia} testable [13] for both stuck at 0 and stuck at 1 if and only if

$$r_{ia} \neq 0 \qquad \text{..........} \quad 5.1$$

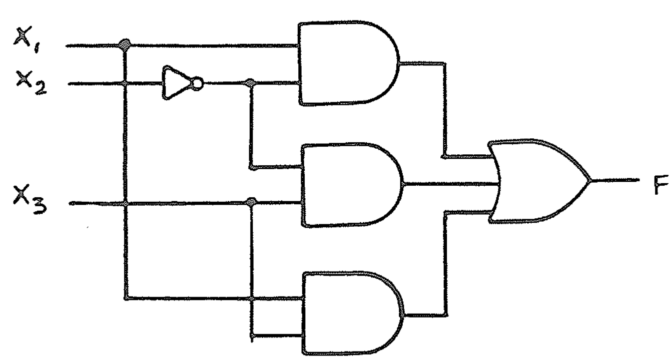
To find a signature for the isolation of single input stuck at faults we proceed as in the case of single input bridging faults. Using the relationships given in eq. 3.6, construct for each input (possible input stuck at fault) a subset which contains all spectral coefficients that change value on a stuck at fault at that input. When all inputs are processed, choose from each subset a minimum number of coefficients that represent the stuck at fault at the input. The resulting set of spectral coefficients is a signature for isolation of single stuck at fault for that circuit. As before this signature is not unique.

To find a signature with the minimum number of spectral coefficients, a process similar to that outlined in section 4.2 can be performed.

Example

Consider the circuit which realizes the function

$$F(X_1, X_2, X_3) = \overline{X}_2 X_1 + X_3 \overline{X}_2 + X_3 X_1$$



The spectrum of F is :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
4	-2	2	0	-2	0	0	-2

Using the condition given by eq. 5.1 we set up the following matrix similar to the one in section 4.2 with a '1' indicating a change in the spectral coefficient for both stuck at 1 and stuck at 0 and a '0' indicating that there is no change in the spectral coefficient.

Possible Faults

	X_1 s-a-f	X_2 s-a-f	X_3 s-a-f
r_0	1	1	1
r_1	1	0	0
r_2	0	1	0
r_{12}	0	0	1
r_3	0	0	1
r_{13}	0	1	0
r_{23}	1	0	0
r_{123}	1	1	1

legend : s-a-f = stuck at 1 or 0

We see that (r_1, r_2, r_3) is a signature for single input stuck at faults for the above circuit.

One obvious limitation of this signature is that when all the coefficients in the spectrum are non-zero then a stuck at fault at any input would cause every one of the spectral coefficients to change value. This is obvious from eq 5.1. Suppose for any input X_i, $r_{ia} \neq 0$ for all a. Then a stuck at fault at X_i is r_a and r_{ia} detectable, r_a covers half of the spectrum and r_{ia} covers the other half, forcing every coefficient to change its value. Therefore this signature is not suitable for circuits such as the tree networks whose spctral coefficients are all non-zero.

5.2 SIGNATURES FOR DETECTION OF ZEROES

In this section we look at a signature which enables us to locate single input stuck at faults by examining those spectral coefficients that go to zero when the fault occurs.

We know that if X_i is stuck at 0 or 1, all coefficients that associate with X_i will go to zero. In this signature, we are also interested in which of the spectral coefficients that do not associate with X_i will go to zero when X_i is stuck.

We know from section 3.1 that for X_i stuck at 0

$$\hat{r}_a = r_a + r_{ia} \qquad \text{..........} \quad 5.2$$

and for X_i stuck at 1

$$\hat{r}_a = r_a - r_{ia} \qquad \text{..........} \quad 5.3$$

where 'a' does not contain i.

Therefore $\hat{r}_a$ goes to zero if

$$r_a + r_{ia} = 0 \quad \text{and}$$

$$r_a - r_{ia} = 0$$

for X_i stuck at 0 and X_i stuck at 1 respectively.

With the above results, we build, for each possible fault, a list of spectral coefficients that do not go to zero if that particular fault occurs. There are two such lists for each input i.e. one for stuck at 0 and one for stuck at 1 since the conditions for stuck at 0 and stuck at 1 are not the same. To find a fault isolation signature for the circuit we use a technique similar to that employed in section 4.2. We represent each possible stuck at fault with a bit vector with each bit representing one spectral coefficient. A '0' indicates that the corresponding spectral coefficient goes to zero for that particular fault and a '1' indicates that it does not. We also must include a bit vector for the fault free circuit. In this vector a '0' indicates that the corresponding coefficient is 0 and a '1' indicates that it is 1. So there are 2n + 1 vectors for a function with n variables. Let K be the smallest integer greater then $\log_2(2n + 1)$ and start from the first K rows, finding the minimum number of rows M such that each of these 2n + 1 vectors have different bit patterns in these M rows. The spectral coefficients corresponding to these rows are a fault isolation signature for single input stuck at faults for the circuit.

EXAMPLE

Using the example in section 4.1, a fault isolation signature for this circuit can be found as follows. The spectrum for this circuit is :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
6	-2	-2	2	-2	-2	-2	2

r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
-2	2	2	2	2	-2	-2	-2

Notice that none of the spectral coefficients of this function is zero. The method presented in the last section fails to find a fault isolation signature for it. We now build for the fault free circuit and for each possible fault a list of coefficients that are not zero.

	<u>Non-zero coefficients</u>
Fault Free	all coefficients
X_1 s-a-0	r_0, r_3, r_{24}, r_{234}
X_1 s-a-1	r_0, r_2, r_{23}, r_{34}
X_2 s-a-0	r_0, r_3, r_{14}, r_{134}
X_2 s-a-1	r_0, r_1, r_{13}, r_{34}
X_3 s-a-0	r_0, r_1, r_2, r_{12}
X_3 s-a-1	r_0, r_4, r_{14}, r_{24}, r_{124}
X_4 s-a-0	r_0, r_{12}, r_{13}, r_{23}
X_4 s-a-1	r_0, r_1, r_2, r_3, r_{123}

The matrix indicating which coefficients go to zero for every possible input stuck at fault is listed below.

Possible faults

	Fault Free	X_1 s-a-0	X_1 s-a-1	X_2 s-a-0	X_2 s-a-1	X_3 s-a-0	X_3 s-a-1	X_4 s-a-0	X_4 s-a-1
r_0	1	1	1	1	1	1	1	1	1
r_1	1	0	0	0	1	1	0	0	1
r_2	1	0	1	0	0	1	0	0	1
r_{12}	1	0	0	0	0	1	0	1	0
r_3	1	1	0	1	0	0	0	0	0
r_{13}	1	0	0	0	1	0	0	1	0
r_{23}	1	0	1	0	0	0	0	1	0
r_{123}	1	0	0	0	0	0	0	0	1
r_4	1	0	1	0	1	0	1	0	0
r_{14}	1	0	0	1	0	0	1	0	0
r_{24}	1	1	0	0	0	0	1	0	0
r_{124}	1	0	0	0	0	0	1	0	0
r_{34}	1	0	1	0	1	0	0	0	0
r_{134}	1	0	0	1	0	0	0	0	0
r_{234}	1	1	0	0	0	0	0	0	0
r_{1234}	1	0	0	0	0	0	0	0	0

We need at least 4 coefficients to differentiate 9 cases so in this case $K = 4$. A minimal set of spectral coefficients to cover all these faults are

r_1, r_2, r_3 and r_{24}

Besides being able to isolate the faults, this signature also tells whether it is a s-a-0 or s-a-1 fault. If it is not necessary to differentiate whether the fault is s-a-0 or s-a-1 on the input line, it may be possible in some cases to eliminate some of the coefficients in the signature which are used only for this purpose.

5.3 SIGNATURES FOR VERIFICATION OF SPECTRAL VALUES

The procedure to find a signature which makes use of the actual values of the faulty spectral coefficients to isolate input stuck at faults is similar to the one presented in section 4.6. The relationship between the faulty and fault free circuit used in this case is discussed in section 3.2. The spectra for all possible single input stuck at faults are evaluated using these relationships and a minimum number of the spectral coefficients are found to cover these faults.

Using this method a signature for the circuit in section 4.1 is found to be

$$r_1, r_2, r_3, r_{24}$$

which is the same as that from the last section. The values of the fault free and faulty spectral coefficients for this signature are listed below.

Possible faults

	Fault Free	X_1 s-a-0	X_1 s-a-1	X_2 s-a-0	X_2 s-a-1	X_3 s-a-0	X_3 s-a-1	X_4 s-a-0	X_4 s-a-1
r_1	-2	0	0	0	-4	-4	0	0	-4
r_2	-2	0	-4	0	0	-4	0	0	-4
r_3	-2	-4	0	-4	0	0	0	0	-4
r_{24}	2	4	0	0	0	0	0	4	0

As before there are situations when this signature will not be able to isolate some of the faults i.e. when the faulty spectra are identical. The conditions such that a stuck at 0 at X_i produces a faulty spectrum identical to a stuck at 0 at X_j are derived as follows.

We divide the coefficients of the two faulty spectra into 4 groups. Making use of the results in section 3.2, for the faulty function with X_i stuck at 0

$$\hat{r}_a = r_a + r_{ia} \qquad \text{.......... } 5.4$$

$$\hat{r}_{ia} = 0 \qquad \text{.......... } 5.5$$

$$\hat{r}_{ja} = r_{ja} + r_{ija} \qquad \text{.......... } 5.6$$

$$\hat{r}_{ija} = 0 \qquad \text{.......... } 5.7$$

For the faulty function with X_j stuck at 0

$$\hat{r}_a = r_a + r_{ja} \qquad \text{.......... } 5.8$$

$$\hat{r}_{ia} = r_{ia} + r_{ija} \qquad \text{.......... } 5.9$$

$$\hat{r}_{ja} = 0 \qquad \text{.......... } 5.10$$

$$\hat{r}_{ija} = 0 \qquad \text{.......... } 5.11$$

for all 'a' where 'a' does not contain i or j.

The corresponding groups are equal if the faulty spectra are identical. Equating 5.4 amd 5.8 we have

$$r_a + r_{ia} = r_a + r_{ja} \quad \text{or}$$

$$r_{ia} = r_{ja} \qquad \ldots\ldots\ldots\ldots \quad 5.12$$

Equating 5.5 and 5.10

$$ria + rija = 0 \qquad \ldots\ldots\ldots\ldots \quad 5.13$$

Equation 5.12 and 5.13 are the conditions required for the faulty spectra to be identical. But these are the spectral conditions required for non equivalence symmetry between X_i, X_j and the conditions for single variable symmetry of X_i over $\bar{X}_j$ [section 2.9] respectively. These two conditions also imply single variable symmetry of X_j over $\bar{X}_i$.

Conditions for X_i stuck at 1 and X_j stuck at 1, X_i stuck at 0 and X_j stuck at 1 can be derived in a similar fashion. These conditions are listed below [see section 2.9].

Fault	Conditions required
X_i s-a-0, X_j s-a-0	NES(X_i, X_j) and {SVS X_i}$\bar{X}_j$
X_i s-a-1, X_j s-a-1	NES(X_i, X_j) and {SVS X_i}X_j
X_i s-a-0, X_j s-a-1	ES(X_i, X_j) and {SVS X_i}X_j

5.4 CONCLUSION

We have discussed three signatures for the isolation of input stuck at faults. Of the three signatures the one which detects zeroes has several characteristics that are worth mentioning. The use of this signature does not require any of the spectral coefficients of the fault free circuit during the actual testing of the circuits as opposed to the other two which require the spectral coefficients that form the signature to be present in order that the measured coefficients can be compared to the fault free ones. Also we know that when an input is stuck, one half (or more) of the spectral coefficients will go to zero. For stuck at faults at different inputs, different 'halves' will go to zero. It is unlikely that every coefficient will go to zero because if this were the case then r_a would have to be equal to $-r_{ia}$ for X_i stuck at 0 or equal to r_{ia} for X_i stuck at 1 for all 'a'. This makes such a signature more likely to be found than the one based on the detection of changes in spectral coefficients.

Chapter VI

FAULT ISOLATION ON INDUSTRIAL CIRCUITS

The theoretical approaches outlined in the previous chapters to finding isolation signatures for single input stuck at or bridging faults guarantee that the signatures are of minimum size if they exist. In general when these approaches are applied to practical circuits, the number of computations required renders them economically unfeasible. To make these approaches applicable to practical circuits we sacrifice the size of the signature for the efficiency in the process of finding it by working with a subset of the spectrum. That is, the search for the signature is not carried out on the entire spectrum but on a prescreened subset of it. The resulting signature will not be the minimum but will still be of reasonable size. In this chapter we shall examine the limitations of the theorectical approaches and introduce a more practical approach for finding isolation signatures for single input stuck at or bridging faults. We have discussed isolation signatures for stuck at faults and bridging faults separately. In practice a combined isolation signature for both types of faults is preferred. We shall examine how these signatures can be combined. We shall also examine signatures for circuits with multiple outputs which are a common feature in 'real world' circuits. Finally, using the approach discussed in this chapter we shall present case studies finding the combined signatures of several 7400-family circuits.

6.1 LIMITATIONS OF THE FAULT ISOLATION METHODS

The procedures for finding fault isolation signatures presented so far, whether by detecting changes of spectral coefficients or by detecting zeroes in the spectrum for stuck at faults, consist of 2 stages. First, the spectral coefficients for each possible fault are selected or evaluated, then a subset of the spectrum is searched to ensure that each possible fault can be uniquely identified. The major limitation of these methods lies in the searching process.

Let us estimate the number of operations required in the searching process to find that a signature does not exist. Assume for simplicity that we are comparing spectral coefficients and that it requires 1 operation for each comparision. For a circuit of n inputs there are $m = 2^n$ spectral coefficients. Assume that there are 'f' possible faults that we have to distinquish. We start off by using 1 spectral coefficient i.e. there would be ${}_mC_1$ choices or combinations. For each combination there are ${}_fC_2$ comparisions and so for the case of 1 spectral coefficient there are ${}_fC_2 * {}_mC_1$ comparisions. But for each combination we can stop as soon as we find two identical coefficients. Assume that on the average we stop at the half way point i.e. the total number of comparisions would be $\frac{1}{2}\ {}_fC_2 * {}_mC_1$. For the case of 2 spectral coefficients there would be ${}_mC_2$ combinations of 2 from a total of m (2^n) coefficients and the total number of comparisions is $\frac{2}{2}\ {}_fC_2 * {}_mC_2$. Similarly for k spectral coefficients the number of comparisions (operations) would be $\frac{k}{2}\ {}_fC_2 * {}_mC_k$. Therefore the number of operations N required to go through all the comparisions using 1, 2, 3,....,k,...2^n coefficients would be :

$$N = \frac{1}{2}\ {}_fC_2 * {}_mC_1 + \frac{2}{2}\ {}_fC_2 * {}_mC_2 + \ldots + \frac{k}{2}\ {}_fC_2 * {}_mC_k + \ldots + \frac{m}{2}\ {}_fC_2 * {}_mC_m$$

$$= \frac{1}{2}\ {}_fC_2\ [{}_mC_1 + 2\,{}_mC_2 + \ldots + k\,{}_mC_k + \ldots + m\,{}_mC_m] \quad .. \quad 6.1$$

$$N = \frac{1}{2}\ {}_fC_2\ [m\,{}_mC_m + (m-1)\,{}_mC_{m-1} + \ldots + 2\,{}_mC_2 + 1\,{}_mC_1] \quad .. \quad 6.2$$

Adding 6.1 and 6.2 and making use of the fact that

$${}_mC_k = {}_mC_{m-k}$$

we have

$$2N = {}_fC_2\ [m\,{}_mC_0 + m\,{}_mC_1 + \ldots + m\,{}_mC_m]$$

$$= {}_fC_2[m2^m]$$

$$= {}_fC_2[2^n 2^{2^n}]$$

So the number of operations is on the order of at least 2^{2^n} which is too large to handle even for small n. Also for large n say n > 20 the amount of memory required is also prohibitive for these methods.

For the signatures discussed in the last two chapters, the larger the number of distinct spectral coefficients (i.e. the more random the spectrum) the simpler is the signature. This is especially true for the signature which detects of changes in spectral coefficients. Industrial circuits in general realize functions which are rather regular and therefore do not often exhibit such randomness in their spectra.

6.2 A PRACTICAL APPROACH

The isolation signature that involves the verification of values of spectral coefficients is more appropriate for industrial circuits than others. The reason being that spectral coefficients can assume quite a wide range of values and this signature makes full use of each coefficient rather than using it as a binary form of information as in the other signatures. Threfore it does not depend so much on the randomness of spectral coefficients as the other signatures, although a wider spread (i.e. more non-zero coefficients) and a more random spectrum would result in a smaller signature.

But this method still has the problem of requiring too much computation. So instead of searching the entire spectrum for the signature, the spectral coefficients are prescreened and a portion of them is selected to be searched. The result is most unlikely to be the minimum signature but is of reasonable size and one important achievement is that the signature is derived at a reasonable cost. Further, for circuits that were formerly too large, we can now deduce a reasonable signature.

Obviously the choice of this subset of spectral coefficients is important. Let us first look at how to determine which coefficients are to be included for input bridging faults. Our purpose is to examine each spectral coefficient and select a subset such that the faulty coefficients corresponding to those in this subset are distinct for most of the possible input faults. It is these coefficients that will give good fault isolation. The relationship between the faulty and fault free function derived in section 3.3 is rewritten below.

$$\hat{r}_a = r_a + \frac{1}{2}(r_{ia} + r_{ja}) + r_{ija} \qquad \ldots\ldots\ldots\ldots \quad 6.3$$

$$\hat{r}_{ia} = \hat{r}_{ja} = \frac{1}{2}(r_{ia} + r_{ja}) \qquad \ldots\ldots\ldots\ldots \quad 6.4$$

$$\hat{r}_{ija} = -\frac{1}{2}(r_{ia} + r_{ja}) \qquad \ldots\ldots\ldots\ldots \quad 6.5$$

To determine whether a spectral coefficient r_B which is of order k is a suitable candidate we examine the following cases.

1. For bridging between X_i and X_j such that i and j are both in 'B', we know from eq. 6.5 that for these bridgings $\hat{r}_B$'s are determined by a group G_1 of k-1th order coefficients whose subscripts all appear in 'B'.

2. For bridgings that involve X_i and X_j such that only one of i or j is in 'B', we know from eq. 6.4 that for these bridgings $\hat{r}_B$'s are determined by a group G_2 of k^{th} order coefficients such that all the k subscripts k-1 of them appear in 'B'.

3. For bridgings involving X_i and X_j such that neither i nor j is in 'B', $\hat{r}_B$'s are determined from eq. 6.3 by r_B and a group G_3 of coefficients of order k+1 and k+2 such that 'B' appears in their subscripts.

If there is certain 'randomness' in these three groups (G_1, G_2 and G_3) of coefficients (i.e. they are made up of a large set of non-indentical integers) then r_B is a candidate. This prescreening process can be stopped as soon as we find a 'satisfactory' subset of coefficients, or it can be applied to every spcetral coefficient to choose the best subset. Obviously the latter would be more likely to give a shorter signature. Since lower order coefficients are easier to obtain than higher order ones, and also for reasons which will be appreciated later in section 6.4, lower order coefficients are prefered to higher order ones.

For example, to determine whether or not r_0 is a suitable candidate, we check all first and second order coefficients for randomness. For a first order coefficient r_k, we check all first order coefficients r_i, all second order coefficients r_{ik} and all third order coefficients r_{ijk} etc.

For input stuck at faults the prescreening is much simpler, since all coefficients associated with the input which is stuck go to zero. For this reason low order coefficients are prefered to higher order ones. From section 3.2 the faulty spectral coefficients are given by :

$$\hat{r}_a = r_a + r_{ia} \quad \text{for } X_i \text{ s-a-0} \quad \ldots\ldots\ldots\ldots \quad 6.6$$

$$\hat{r}_a = r_a - r_{ia} \quad \text{for } X_i \text{ s-a-1} \quad \ldots\ldots\ldots\ldots \quad 6.7$$

To determine whether a k^{th} order coefficient r_B is a suitable candidate, we check the absolute value of all $k+1^{th}$ order coefficients r_{iB} for randomness. The reason the absolute values are used instead of the actual value of the spectral coefficients themselves is obvious from eq. 6.6 and 6.7.

The number of spectral coefficients to be chosen for the subset depends on the randomness of the spectral coefficients. Obviously the larger the subset the more likely it is that a shorter signature will be found. For a circuit with n inputs a subset of approximately n spectral coefficients give signatures of reasonable size for the circuits we have examined.

If the subset of spectral coefficients chosen is not capable of isolating all faults, then the faults that are not isolated are checked to see if they actually can be isolated either by using the conditions given in sections 4.6 and 5.3 or by simply comparing the faulty spectra. If it is found that these faults can be isolated, then the spectral coefficients by which they can be isolated are added to the original subset of coefficients and the search for the signature is repeated. If it is found that they cannot be isolated at all (i.e. they produce functionally equivalent circuits), then other means have to be used to isolate them.

6.3 MULTIPLE OUTPUTS

It is not unusual for an industrial circuit to have more than one output and if this is the case then obviously the one that associates with all of the inputs will be chosen for testing. The other outputs may be used to discriminate faults that cannot be isolated by that output. Therefore a circuit with multiple outputs might give more fault isolation than the comparable single output circuits. A signature in this case may then be composed of spectral coefficients from more than one output.

An alternative approach to a multiple output signature is to use the lower order spectral coefficients of the spectrum of every output of the circuit in the search for the signature. The resulting signature would involve more outputs but this should not pose any problem in the actual testing of the circuits because the spectral coefficients at each output can be measured in parallel. One advantage of this signature is that it will be made up only of lower order coefficients and in some cases even of r_0's only. One such example using this approach can be found in the last section of this chapter (signature of 7449). The drawback of this approach is that in the prescreening process we may have to process more spectral coefficients than in the other approaches.

6.4 COMBINED SIGNATURE

So far signatures for input stuck at faults and input bridging faults have been discussed separately. In practice it is desirable to have a signature that isolates both types of faults. A combined signature for these two types of faults can be found without much extra processing. First we prescreen the spectral coefficients and select the subset for each type of fault. The search for a signature is carried out using the union of these two subsets of spectral coefficients and for both types of faults at the same time i.e. the columns of the matrix that facilitate the search would be composed of those that associate with stuck at faults and those that associate with bridging faults. The resulting signature is one that isolates both single input stuck at faults and single input bridging faults.

One problem that may arise from the combination of the two signatures is that there may be situations in which a bridging fault and a stuck at fault result in functionally equivalent circuits. Let us investigate the conditions required for this to happen. There are 4 cases we must examine.

1. $X_i\ X_j$ bridging fault = X_i stuck at 0

2. $X_i\ X_j$ bridging fault = X_i stuck at 1

3. $X_i\ X_j$ bridging fault = X_k stuck at 0

4. $X_i\ X_j$ bridging fault = X_k stuck at 1

where i, j and k are distinct.

We shall derive the conditions for case 1 and 3. The conditions for the other 2 cases are similar.

Case 1

We divide the spectral coefficients into 4 groups as follows.

For a bridging fault between X_i and X_j we have from eq. 3.13 - 3.15

$$\hat{r}_a = r_a + \frac{1}{2}(r_{ia} + r_{ja}) + r_{ija} \quad \ldots\ldots\ldots \quad 6.8$$

$$\hat{r}_{ia} = \frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots \quad 6.9$$

$$\hat{r}_{ja} = \frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots \quad 6.10$$

$$\hat{r}_{ija} = -\frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots \quad 6.11$$

For X_i stuck at 0 we have from eq. 3.6

$$\hat{r}_a = r_a + r_{ia} \quad \ldots\ldots\ldots \quad 6.12$$

$$\hat{r}_{ia} = 0 \quad \ldots\ldots\ldots \quad 6.13$$

$$\hat{r}_{ja} = r_{ja} + r_{ija} \quad \ldots\ldots\ldots \quad 6.14$$

$$\hat{r}_{ija} = 0 \quad \ldots\ldots\ldots \quad 6.15$$

for all 'a' where 'a' does not conatin i or j.

These 4 corresponding groups of spectral coefficients are identical for functionally equivalent circuits.

From 6.9 and 6.13 we have

$$\frac{1}{2}(r_{ia} + r_{ja}) = 0 \quad \ldots\ldots\ldots \quad 6.16$$

From 6.8, 6.12 and 6.16 we have

$$r_a + r_{ija} = r_a + r_{ia} \quad \text{or}$$

$$r_{ia} = r_{ija} \quad \ldots\ldots\ldots \quad 6.17$$

The conditions required for X_i stuck at 0 to produce a functionally equivalent circuit to that produced by a bridging between X_i and X_j are :

1. Equivalence symmetry between X_i and X_j (6.16)
2. Single variable symmetry of X_i over X_j (6.17)

Case 3

We divide the spectral coefficients into 8 groups as follows. For bridging fault between X_i and X_j we have

$$\hat{r}_a = r_a + \frac{1}{2}(r_{ia} + r_{ja}) + r_{ija} \quad \ldots\ldots\ldots \quad 6.18$$

$$\hat{r}_{ia} = \frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots \quad 6.19$$

$$\hat{r}_{ja} = \frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots \quad 6.20$$

$$\hat{r}_{ija} = -\frac{1}{2}(r_{ia} + r_{ja}) \quad \ldots\ldots\ldots \quad 6.21$$

$$\hat{r}_{ka} = r_{ka} + \frac{1}{2}(r_{ika} + r_{jka}) + r_{ijka} \quad \ldots\ldots \quad 6.22$$

$$\hat{r}_{ika} = \frac{1}{2}(r_{ika} + r_{jka}) \quad \ldots\ldots\ldots \quad 6.23$$

$$\hat{r}_{jka} = \frac{1}{2}(r_{ika} + r_{jka}) \quad \ldots\ldots\ldots \quad 6.24$$

$$\hat{r}_{ijka} = -\frac{1}{2}(r_{ika} + r_{jka}) \quad \ldots\ldots\ldots \quad 6.25$$

For X_k stuck at 0 we have

$$\hat{r}_a = r_a + r_{ka} \quad \ldots\ldots\ldots\ldots \quad 6.26$$

$$\hat{r}_{ia} = r_{ia} + r_{ika} \quad \ldots\ldots\ldots\ldots \quad 6.27$$

$$\hat{r}_{ja} = r_{ja} + r_{jka} \qquad \text{.......... } 6.28$$

$$\hat{r}_{ija} = r_{ija} + r_{ijka} \qquad \text{.......... } 6.29$$

$$\hat{r}_{ka} = 0 \qquad \text{.......... } 6.30$$

$$\hat{r}_{ika} = 0 \qquad \text{.......... } 6.31$$

$$\hat{r}_{jka} = 0 \qquad \text{.......... } 6.32$$

$$\hat{r}_{ijka} = 0 \qquad \text{.......... } 6.33$$

After simplifying and eliminating the redundant equations we have the following required conditions.

$$r_{ia} + r_{ja} + 2r_{ija} = 2r_{ka} \qquad \text{.......... } 6.34$$

$$r_{ia} - r_{ja} = 2r_{jka} \qquad \text{.......... } 6.35$$

$$r_{ika} + r_{jka} = 0 \qquad \text{.......... } 6.36$$

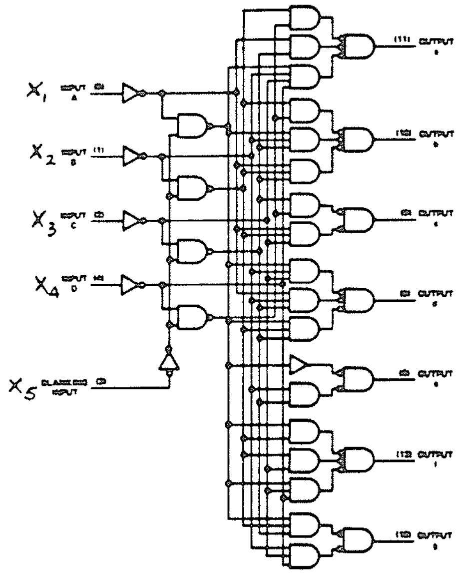
$$r_{ka} + r_{ijka} = 0 \qquad \text{.......... } 6.37$$

for all 'a' where 'a' does not contain i or j.

6.5 CASE STUDIES

In this section we shall apply the techniques discussed in the previous sections to several 7400-family medium scale integrated circuits to construct their combined signatures. The spectra of some of the circuits involved can be found in appendix A, while for the others only the part of spectrum which is involved in the derivation of the signature is listed because of the size of the complete spectrum.

1. BCD to seven segment decoder 7449.



The spectrum of this circuit at output 'a' (11) is made up of 0, ±2, ±4 and ±8 only. It is not too random but has a fairly wide spread. Let us examine first the lower order coefficients and see if they are good candidates for the chosen subset and try to construct a signature from them if they are. For bridging faults the corresponding groups of spectral coefficients for r_0, r_1, r_2, r_3, r_4 and r_5 are checked as outlined in section 6.2 and the values involved with each of them are listed below.

r_0	0, ±2, ±4, -8
r_1	0, ±2, ±4, -8
r_2	0, ±2, ±4, -8
r_3	0, ±2, ±4, -8
r_4	0, ±2, ±4, -8
r_5	0, ±2, ±4, -8

We see that $r_0 - r_5$ of the faulty spectra are involved in almost all values that make up the fault free spectrum. Therefore they are definitely candidates for the chosen subset. Similarly for stuck at faults we have the following tabulation.

r_0	2, 8
r_1	0, 2, 4
r_2	0, 2, 4
r_3	0, 2, 4
r_4	0, 2, 4
r_5	2

We see that r_5 will not give much fault isolation but the rest seem to be a reasonable subset to be searched for the signature. We now construct the matrix as shown below. For the sake of readability the matrix is listed with each column associated with a spectral coefficient and each row associated with a possible fault.

FAULT	r_0	r_1	r_2	r_3	r_4	r_5
normal	32	0	0	8	8	8
X_1X_2	8	0	0	6	2	-8
X_1X_3	12	0	4	0	4	-12
X_1X_4	8	0	2	6	0	-8
X_1X_5	5	-5	1	-1	1	-5
X_2X_3	10	2	2	2	-2	-10
X_2X_4	6	0	2	0	2	-6
X_2X_5	3	-1	-3	1	3	-3
X_3X_4	10	2	-2	2	2	-10
X_3X_5	3	-3	1	-3	1	-3
X_4X_5	3	-1	3	1	-3	-3
X_1 S-A-0	6	0	2	6	2	-6
X_1 S-A-1	10	0	2	-2	2	-10
X_2 S-A-0	10	-2	0	2	-2	-10
X_2 S-A-1	6	-2	0	2	6	-6
X_3 S-A-0	10	2	2	0	2	-10
X_3 S-A-1	6	-6	2	0	2	-6
X_4 S-A-0	10	-2	-2	2	0	-10
X_4 S-A-1	6	-2	6	2	0	-6
X_5 S-A-0	0	0	0	0	0	0
X_5 S-A-1	16	-4	4	4	4	0

Legend : X_iX_j = bridging between X_i and X_j

One signature for single input stuck at fault isolation and single input bridging fault isolation for the BCD to segment decoder is

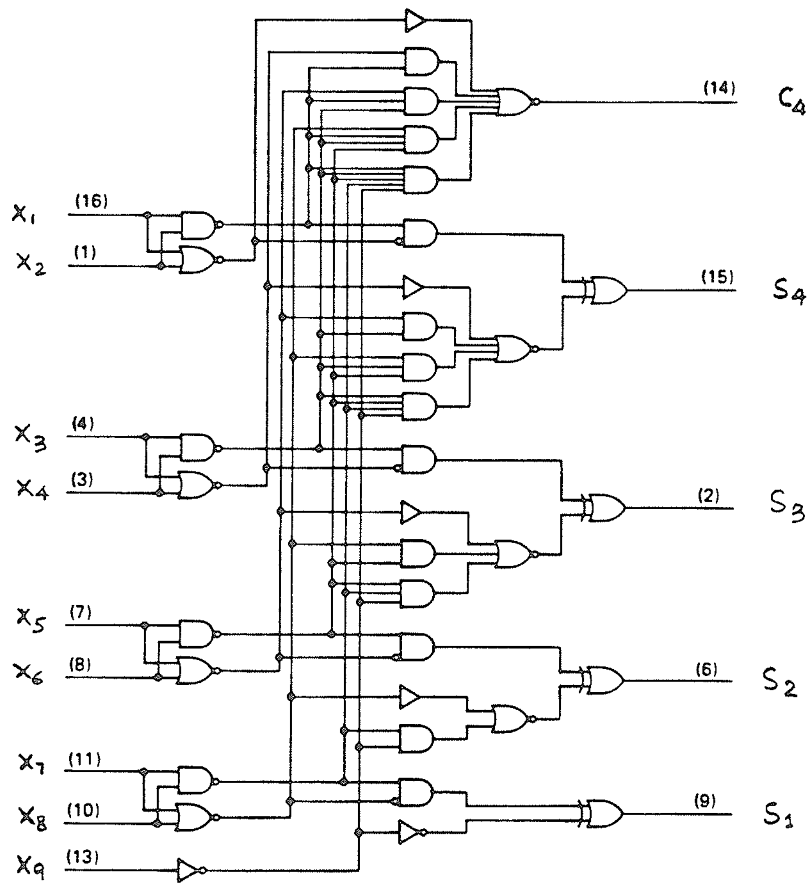
r_0, r_2, r_3 and r_4

Let us see if we can find an isolation signature for this circuit using the approach that utilises the multiple outputs of the circuit. The first and second order spectral coefficients of the spectrum at the 7 different outputs of this circuit involve quite a large range of numbers, so we consider whether we can construct a signature using r_0's only. From the following matrix which is constructed using the spectra at outputs 'a', 'b', 'c', 'd', 'e', 'f' and 'g' of the circuit, we see that a possible isolation signature for single input stuck at faults and single input bridging faults is

r_0 at output 'e' and r_0 at output 'f'

FAULT	r_0 (a)	r_0 (b)	r_0 (c)	r_0 (d)	r_0 (e)	r_0 (f)	r_0 (g)
X_1X_2	8	14	12	8	6	12	11
X_1X_3	12	10	8	14	12	8	11
X_1X_4	8	10	11	11	9	11	12
X_1X_5	5	4	6	4	0	3	5
X_2X_3	10	13	14	8	8	11	8
X_2X_4	6	9	13	9	5	10	9
X_2X_5	3	3	4	6	4	2	6
X_3X_4	10	13	9	11	7	6	9
X_3X_5	3	3	4	4	2	6	6
X_4X_5	3	3	3	5	3	5	7
X_1 S-A-0	6	10	8	12	12	12	14
X_1 S-A-1	10	8	12	8	0	6	10
X_2 S-A-0	10	12	12	8	4	14	12
X_2 S-A-1	6	6	8	12	8	4	12
X_3 S-A-0	10	12	12	12	8	6	12
X_3 S-A-1	6	6	8	8	4	12	12
X_4 S-A-0	10	12	14	10	6	8	10
X_4 S-A-1	6	6	6	10	6	10	14
X_5 S-A-0	0	0	0	0	0	0	0
X_5 S-A-1	16	18	20	20	12	18	24

2. Fast carry look ahead adder 7483.



If we examine the spectrum of this circuit at output S_4 we will notice the following points :

1. The spectrum does not possess a wide spread at all, a great portion of it being zeroes and the rest are made up of four numbers namely 64, -32 and ± 16. Therefore we would expect the signature to be composed of a large number of spectral coefficients.

2. The first, second and third order coefficients except r_{12}, r_{123}, r_{124}, r_{125}, r_{126}, r_{127}, r_{128} and r_{129} are

zero implying that r_0 and the first order coefficients except r_1 and r_2 do not give much fault isolation.

3. The rest of the higher order coefficients are zero except for some whose subscripts contain 1 and 2.

Consequently we would expect that r_1, r_2, r_{12}, r_{123}, r_{124}, r_{125}, r_{126}, r_{127}, r_{128} and r_{129} will give more fault isolation than the other spectral coefficients. We construct the matrix using this subset of coefficients.

FAULT	r_1	r_2	r_{12}	r_{123}	r_{124}	r_{125}	r_{126}	r_{127}	r_{128}	r_{129}
normal	0	0	-192	64	-32	-32	-16	-16	-16	-16
X_1X_2	0	0	0	0	0	0	0	0	0	0
X_1X_3	0	-32	-96	96	32	-16	-16	-8	-8	-8
X_1X_4	0	-32	-96	32	96	-16	-16	-8	-8	-8
X_1X_5	0	-128	-96	32	32	96	-16	-8	-8	-8
X_1X_6	0	-128	-96	32	32	-16	96	-8	-8	-8
X_1X_7	0	-112	-96	32	32	-16	-16	96	-8	-8
X_1X_8	0	-112	-96	32	32	-16	-16	-8	96	-8
X_1X_9	0	-112	-96	32	32	-16	-16	-8	-8	96
X_2X_3	-32	0	-96	96	32	-16	-16	-8	-8	-8
X_2X_4	-32	0	-96	32	96	-16	-16	-8	-8	-8
X_2X_5	-128	0	-96	32	32	96	-16	-8	-8	-8
X_2X_6	-128	0	-96	32	32	-16	96	-8	-8	-8
X_2X_7	-112	0	-96	32	32	-16	-16	96	-8	-8
X_2X_8	-112	0	-96	32	32	-16	-16	-8	96	-8
X_2X_9	-112	0	-96	32	32	-16	-16	-8	-8	96

X_3X_4	0	0	-64	64	64	-96	-96	-48	-48	-48
X_3X_5	0	0	-208	16	48	16	-48	-24	-24	-24
X_3X_6	0	0	-208	16	48	-48	16	-24	-24	-24
X_3X_7	0	0	-184	24	72	-48	-48	24	-24	-24
X_3X_8	0	0	-184	24	72	-48	-48	-24	24	-24
X_3X_9	0	0	-184	24	72	-48	-48	-24	-24	24
X_4X_5	0	0	-208	48	16	16	-48	-24	-24	-24
X_4X_6	0	0	-208	48	16	-48	16	-24	-24	-24
X_4X_7	0	0	-184	72	24	-48	-48	24	-24	-24
X_4X_8	0	0	-184	72	24	-48	-48	-24	24	-24
X_4X_9	0	0	-184	72	24	-48	-48	-24	-24	24
X_5X_6	0	0	-224	32	32	-32	-32	0	0	0
X_5X_7	0	0	-216	40	40	-24	-16	-24	-16	-16
X_5X_8	0	0	-216	40	40	-24	-16	-16	-24	-16
X_5X_9	0	0	-216	40	40	-24	-16	-16	-16	-24
X_6X_7	0	0	-216	40	40	-16	-24	-24	-16	-16
X_6X_8	0	0	-216	40	40	-16	-24	-16	-24	-16
X_6X_9	0	0	-216	40	40	-16	-24	-16	-16	-24
X_7X_8	0	0	-208	48	48	-32	-32	-16	-16	0
X_7X_9	0	0	-208	48	48	-32	-32	-16	0	-16
X_8X_9	0	0	-208	48	48	-32	-32	0	-16	-16
X_1-0	0	-192	0	0	0	0	0	0	0	0
X_1-1	0	192	0	0	0	0	0	0	0	0
X_2-0	-192	0	0	0	0	0	0	0	0	0
X_2-1	192	0	0	0	0	0	0	0	0	0
X_3-0	0	0	-128	0	128	-64	-64	-32	-32	-32
X_3-1	0	0	-256	0	0	0	0	0	0	0

X_4-0	0	0	-128	128	0	-64	-64	-32	-32	-32
X_4-1	0	0	-256	0	0	0	0	0	0	0
X_5-0	0	0	-224	32	32	0	-32	-16	-16	-16
X_5-1	0	0	-160	96	96	0	-32	-16	-16	-16
X_6-0	0	0	-224	32	32	-32	0	-16	-16	-16
X_6-1	0	0	-160	96	96	-32	0	-16	-16	-16
X_7-0	0	0	-208	48	48	-32	-32	0	-16	-16
X_7-1	0	0	-176	80	80	-32	-32	0	-16	-16
X_8-0	0	0	-208	48	48	-32	-32	-16	0	-16
X_8-1	0	0	-176	80	80	-32	-32	-16	0	-16
X_9-0	0	0	-208	48	48	-32	-32	-16	-16	0
X_9-1	0	0	-176	80	80	-32	-32	-16	-16	0

Legend : X_iX_j = bridging between X_i and X_j

X_i-0 = X_i stuck at 0

X_i-1 = X_i stuck at 1

We see that r_2, r_{12}, r_{123}, r_{124}, r_{125}, r_{126}, r_{127}, r_{128} and r_{129} are sufficient to isolate all the bridging and stuck at faults except for the following faults.

1. X_4 stuck at 1 and X_3 stuck at 1
2. X_7 X_8 bridging and X_9 stuck at 0
3. X_7 X_9 bridging and X_8 stuck at 0
4. X_8 X_9 bridging and X_7 stuck at 0

A check of the spectrum at S_4 of the circuit reveals that

$r_{3a} = r_{4a}$ and

$r_{3a} = r_{34a}$

This means that the faults X_4 stuck at 1 and X_3 stuck at 1 cannot be isolated at output S_4. Let us check the spectrum against eq. 6.34 - 6.37 with i = 7, j = 8 and k = 9.

1. Either $r_{7a} = r_{8a} = r_{9a} = 0$

   or $r_{7a} = r_{8a} = r_{79a} = 16$

   also $r_{78a} = 0$

   so eq. 6.34 and 6.35 are satisfied.

2. $r_{79a} = r_{89a} = 0$

   and so eq 6.36 is satisfied.

3. Either $r_{9a} = r_{89a} = 0$

   or $r_{9a} = 16$ and $r_{89a} = -16$

   or $r_{9a} = -16$ and $r_{89a} = 16$

   and so eq. 6.37 is satisfied.

Therefore a bridging fault between X_7 and X_8 cannot be distinquished from a stuck at zero at X_9. Similarly it can be shown that the other two pairs of faults cannot be isolated at this output. We turn to another output C_4 and see if we can isolate these faults from there.

From the spectrum of the circuit at output C_4 we see that $r_3 \neq r_{34}$ which means that r_3 can be used to isolate X_3 stuck at 1 and X_4 stuck at 1. Also r_0 is sufficient to isolate the other faults which cannot be isolated using output S_4 (see matrix below). Therefore an isolation signature for the 7483 is

r_2, r_{12}, r_{123}, r_{125}, r_{126}, r_{127}, r_{128} and r_{129}

at output S_4 plus

r_0 and r_3 at output C_4.

Actually, if we had worked with the spectrum at output C_4 in the first place, a signature could have been found more easily.

Applying the prescreening stretegy discussed in the previous section to the spectrum at C_4 we find that r_0 and the first order coefficients appear to give good fault isolation. So we shall see if we can get an isolation signature from these coefficients. Let us examine the following matrix which is constructed using the spectrum at output C_4.

FAULT	r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
normal	60	60	60	-32	-12	-12	-12	-12	-12	-8
X_1X_2	180	60	60	-96	-96	-36	-36	-36	-36	-24
X_1X_3	42	14	42	14	-42	-18	-18	-18	-18	-12
X_1X_4	42	14	42	-42	14	-18	-18	-18	-18	-12
X_1X_5	72	24	72	-48	-48	24	-24	-12	-12	-12
X_1X_6	72	24	72	-48	-48	-24	24	-12	-12	-12
X_1X_7	72	24	72	-48	-48	-12	-12	24	-24	-12
X_1X_8	72	24	72	-48	-48	-12	-12	-24	24	-12
X_1X_9	78	26	78	-48	-48	-18	-18	-18	-18	26
X_2X_3	42	42	14	14	-42	-18	-18	-18	-18	-12
X_2X_4	42	42	14	-42	14	-18	-18	-18	-18	-12
X_2X_5	72	72	24	-48	-48	24	-24	-12	-12	-12
X_2X_6	72	72	24	-48	-48	-24	24	-12	-12	-12
X_2X_7	72	72	24	-48	-48	-12	-12	24	-24	-12
X_2X_8	72	72	24	-48	-48	-12	-12	-24	24	-12
X_2X_9	78	78	26	-48	-48	-18	-18	-18	-18	26
X_3X_4	32	32	32	-32	-32	0	0	0	0	0

X_3X_5	38	38	38	-22	-18	-22	-14	-10	-10	-8
X_3X_6	38	38	38	-22	-18	-14	-22	-10	-10	-8
X_3X_7	38	38	38	-22	-18	-10	-10	-22	-14	-8
X_3X_8	38	38	38	-22	-18	-10	-10	-14	-22	-8
X_3X_9	40	40	40	-20	-22	-12	-12	-12	-12	-20
X_4X_5	38	38	38	-18	-22	-22	-14	-10	-10	-8
X_4X_6	38	38	38	-18	-22	-14	-22	-10	-10	-8
X_4X_7	38	38	38	-18	-22	-10	-10	-22	-14	-8
X_4X_8	38	38	38	-18	-22	-10	-10	-14	-22	-8
X_4X_9	40	40	40	-22	-20	-12	-12	-12	-12	-20
X_5X_6	44	44	44	-32	-32	-12	-12	-4	-4	0
X_5X_7	52	52	52	-32	-32	-12	-8	-12	-8	-8
X_5X_8	52	52	52	-32	-32	-12	-8	-8	-12	-8
X_5X_9	50	50	50	-32	-32	-10	-6	-10	-10	-10
X_6X_7	52	52	52	-32	-32	-8	-12	-12	-8	-8
X_6X_8	52	52	52	-32	-32	-8	-12	-8	-12	-8
X_6X_9	50	50	50	-32	-32	-6	-10	-10	-10	-10
X_7X_8	44	44	44	-32	-32	-4	-4	-12	-12	0
X_7X_9	50	50	50	-32	-32	-10	-10	-10	-6	-10
X_8X_9	50	50	50	-32	-32	-10	-10	-6	-10	-10
X_1-0	120	0	120	-64	-64	-24	-24	-24	-24	-16
X_1-1	0	0	0	0	0	0	0	0	0	0
X_2-0	120	120	0	-64	-64	-24	-24	-24	-24	-16
X_2-1	0	0	0	0	0	0	0	0	0	0
X_3-0	28	28	28	0	-28	-1 2	-12	-12	-12	-8
X_3-1	92	92	92	0	-36	-12	-12	-12	-12	-8
X_4-0	28	28	28	-28	0	-1 2	-12	-12	-12	-8

X_4-1	92	92	92	-36	0	-12	-12	-12	-12	-8
X_5-0	48	48	48	-32	-32	0	-16	-8	-8	-8
X_5-1	72	72	72	-32	-32	0	-8	-16	-16	-8
X_6-0	48	48	48	-32	-32	-16	0	-8	-8	-8
X_6-1	72	72	72	-32	-32	-8	0	-16	-16	-8
X_7-0	48	48	48	-32	-32	-8	-8	0	-16	-8
X_7-1	72	72	72	-32	-32	-16	-16	0	-8	-8
X_8-0	48	48	48	-32	-32	-8	-8	-16	0	-8
X_8-1	72	72	72	-32	-32	-16	-16	-8	0	-8
X_9-0	52	52	52	-32	-32	-12	-12	-12	-12	0
X_9-1	68	68	68	-32	-32	-12	-12	-12	-12	0

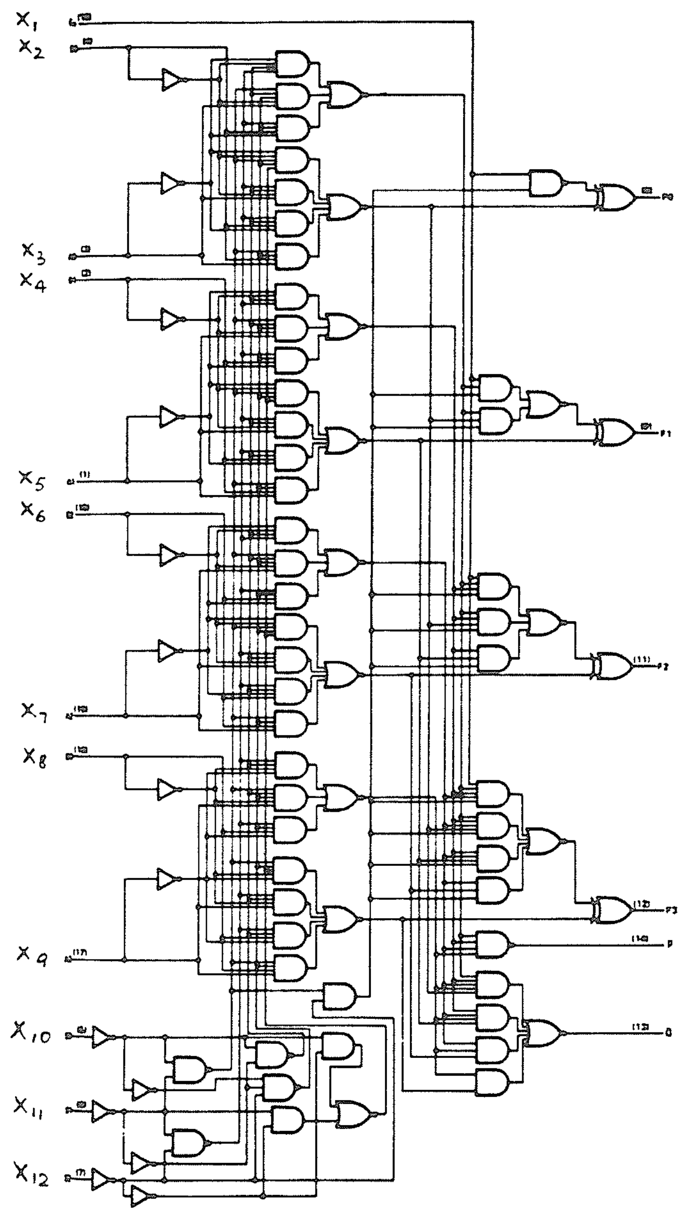
From the matrix above we see that r_0, r_1, r_3, r_6, r_7 and r_8 are sufficient to isolate all the faults except X_1 stuck at 1 and X_2 stuck at 1. The spectrum which is not listed due to its size shows that

$$r_{1a} = r_{2a} \quad \text{and}$$
$$r_{1a} = r_{12a}$$

Therefore these two faults cannot be isolated at this output. But we know from the previous discussion that they can be isolated by r_2 at output S_4. Therefore an isolation signature for the 7483 is

r_0, r_1, r_3, r_6, r_7 and r_8 at output C_4
and r_2 at output S_4

3. Arithmetic and Logic Unit 74381



The zero to third order coefficients of the spectrum at output G are listed in the appendix. Coefficients r_0 and the other first order coefficiens are a satisfactory subset to be searched for a signature. Due to typographical difficulties the matrix listed below is divided into two parts, the first part contains coefficients r_0 to $r_{(10)}$ and the second part contains $r_{(11)}$ to $r_{(12)}$. From this matrix we see that a combined signature for the arithmetic and logic unit is

r_0, r_3, r_5, r_6, r_8 and r_{10} at output G

FAULT	r0	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10
normal	2176	48	16	16	32	32	64	64	-384	-384	-640
X_1X_2	2208	32	32	32	32	32	64	64	-384	-416	-672
X_1X_3	2208	32	32	32	32	32	64	64	-384	-416	-640
X_1X_4	2216	40	16	16	40	48	64	64	-392	-424	-696
X_1X_5	2216	40	16	16	48	40	64	64	-392	-424	-632
X_1X_6	2232	56	16	16	32	32	56	80	-408	-440	-744
X_1X_7	2232	56	16	16	32	32	80	56	-408	-440	-616
X_1X_8	2024	-168	8	8	16	16	32	32	-168	-456	-696
X_1X_9	1960	-168	8	8	16	16	32	32	-424	-168	-696
X_1X_{10}	1864	-296	-8	24	-16	48	-32	96	-424	-456	-296
X_1X_{11}	2120	-40	24	-8	48	-16	96	-32	-424	-456	-600
X_1X_{12}	1928	-296	24	24	48	48	96	96	-168	-264	-600
X_2X_3	2192	64	16	16	32	32	64	64	-400	-400	-656
X_2X_4	2200	48	24	16	24	16	64	64	-408	-408	-712
X_2X_5	2200	48	24	16	16	24	64	64	-408	-408	-648
X_2X_6	2216	48	40	16	32	32	40	48	-424	-424	-760
X_2X_7	2216	48	40	16	32	32	48	40	-424	-424	-632
X_2X_8	1976	56	-184	8	16	16	32	32	-184	-472	-744
X_2X_9	1976	24	-184	8	16	16	32	32	-472	-184	-680
X_2X_{10}	1816	40	-312	24	-16	48	-32	96	-472	-408	-312
X_2X_{11}	2136	40	-56	-8	48	-16	96	-32	-408	-472	-584

X_2X_{12}	1880	72	-312	24	48	48	96	96	-216	-216	-648
X_3X_4	2200	48	16	24	24	16	64	64	-408	-408	-680
X_3X_5	2200	48	16	24	16	24	64	64	-408	-408	-616
X_3X_6	2216	48	16	40	32	32	40	48	-424	-424	-728
X_3X_7	2216	48	16	40	32	32	48	40	-424	-424	-600
X_3X_8	1976	56	8	-184	16	16	32	32	-184	-472	-648
X_3X_9	1976	24	8	-184	16	16	32	32	-472	-184	-712
X_3X_{10}	1880	40	-8	-312	-16	48	-32	96	-408	-472	-312
X_3X_{11}	2072	40	24	-56	48	-16	96	-32	-472	-408	-552
X_3X_{12}	1880	72	24	-312	48	48	96	96	-216	-216	-552
X_4X_5	2208	64	0	0	32	32	64	64	-416	-416	-672
X_4X_6	2224	48	16	16	48	32	48	32	-432	-432	-784
X_4X_7	2224	48	16	16	48	32	32	48	-432	-432	-656
X_4X_8	1968	56	8	8	-176	16	32	32	-176	-496	-784
X_4X_9	1968	24	8	8	-176	16	32	32	-496	-176	-656
X_4X_{10}	1776	40	-8	24	-304	48	-32	96	-496	-368	-304
X_4X_{11}	2160	40	24	-8	-48	-16	96	-32	-368	-496	-592
X_4X_{12}	1904	72	24	24	-304	48	96	96	-240	-240	-720
X_5X_6	2224	48	16	16	32	48	48	32	-432	-432	-720
X_5X_7	2224	48	16	16	32	48	32	48	-432	-432	-592
X_5X_8	1968	56	8	8	16	-176	32	32	-176	-496	-592
X_5X_9	1968	24	8	8	16	-176	32	32	-496	-176	-720
X_5X_{10}	1904	40	-8	24	-16	-304	-32	96	-368	-496	-304
X_5X_{11}	2032	40	24	-8	48	-48	96	-32	-496	-368	-528
X_5X_{12}	1904	72	24	24	48	-304	96	96	-240	-240	-528
X_6X_7	2240	64	0	0	0	0	64	64	-448	-448	-704
X_6X_8	1952	56	8	8	16	16	-160	32	-160	-544	-864

X_6X_9	1952	24	8	8	16	16	-160	32	-544	-160	-608
X_6X_{10}	1696	40	-8	24	-16	48	-288	96	-544	-288	-288
X_6X_{11}	2208	40	24	-8	48	-16	-32	-32	-288	-544	-608
X_6X_{12}	1952	72	24	24	48	48	-288	96	-288	-288	-864
X_7X_8	1952	56	8	8	16	16	32	-160	-160	-544	-480
X_7X_9	1952	24	8	8	16	16	32	-160	-544	-160	-736
X_7X_{10}	1952	40	-8	24	-16	48	-32	-288	-288	-544	-288
X_7X_{11}	1952	40	24	-8	48	-16	96	-32	-544	-288	-480
X_7X_{12}	1952	72	24	24	48	48	96	-288	-288	-288	-480
X_8X_9	1664	48	-16	-16	-32	-32	-64	-64	-384	-384	-640
X_8X_{10}	1536	64	-32	64	-64	128	-128	256	-512	-384	-512
X_8X_{11}	1792	64	64	-32	128	-64	256	-128	-256	-896	-512
X_8X_{12}	2048	96	0	0	0	0	0	0	-512	-128	-512
X_9X_{10}	1536	32	32	0	64	0	128	0	-384	-512	-512
X_9X_{11}	1792	32	0	32	0	64	0	128	-896	-256	-512
X_9X_{12}	2048	0	0	0	0	0	0	0	-128	-512	-512
$X_{10}X_{11}$	1920	16	16	16	32	32	64	64	-384	-384	-384
$X_{10}X_{12}$	1664	48	-48	48	-96	96	-192	192	-128	-128	-640
$X_{11}X_{12}$	1408	48	48	-48	96	-96	192	-192	-128	-128	-1152
X_1-0	2224	0	16	16	32	32	64	64	-368	-432	-656
X_1-1	2128	0	16	16	32	32	64	64	-400	-336	-624
X_2-0	2192	48	0	16	32	32	64	64	-400	-400	-688
X_2-1	2160	48	0	16	32	32	64	64	-368	-368	-592
X_3-0	2192	48	16	0	32	32	64	64	-400	-400	-624
X_3-1	2160	48	16	0	32	32	64	64	-368	-368	-656
X_4-0	2208	48	16	16	0	32	64	64	-416	-416	-736
X_4-1	2144	48	16	16	0	32	64	64	-352	-352	-544

X_5-0	2208	48	16	16	32	0	64	64	-416	-416	-608
X_5-1	2144	48	16	16	32	0	64	64	-352	-352	-672
X_6-0	2240	48	16	16	32	32	0	64	-448	-448	-832
X_6-1	2112	48	16	16	32	32	0	64	-320	-320	-448
X_7-0	2240	48	16	16	32	32	64	0	-448	-448	-576
X_7-1	2112	48	16	16	32	32	64	0	-320	-320	-704
X_8-0	1792	64	0	0	0	0	0	0	0	-512	-768
X_8-1	2560	32	32	32	64	64	128	128	0	-256	-512
X_9-0	1792	0	0	0	0	0	0	0	-512	0	-768
X_9-1	2560	96	32	32	64	64	128	128	-256	0	-512
X_{10}-0	1536	32	-32	32	-64	64	-128	128	-512	-512	0
X_{10}-1	2816	64	64	0	128	0	256	0	-256	-256	0
X_{11}-0	2048	32	32	-32	64	-64	128	-128	-512	-512	-512
X_{11}-1	2304	64	0	64	0	128	0	256	-256	-256	-768
X_{12}-0	1536	96	32	32	64	64	128	128	0	0	-512
X_{12}-1	2816	0	0	0	0	0	0	0	-768	-768	-768

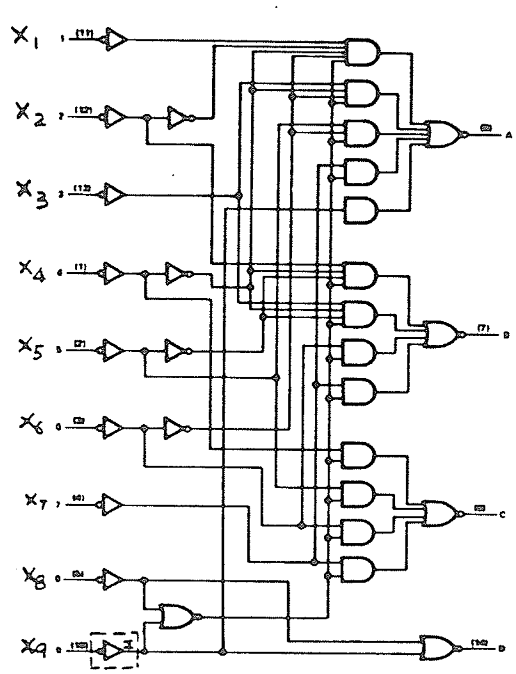
FAULT	r_{11}	r_{12}
normal	-128	-640
X_1X_2	-128	-608
X_1X_3	-160	-608
X_1X_4	-120	-600
X_1X_5	-184	-600
X_1X_6	-104	-584
X_1X_7	-232	-584
X_1X_8	-184	-408
X_1X_9	-184	-472
X_1X_{10}	-88	-568

X_1X_{11}	-40	-824
X_1X_{12}	-344	-296
X_2X_3	-144	-624
X_2X_4	-104	-616
X_2X_5	-168	-616
X_2X_6	-88	-600
X_2X_7	-216	-600
X_2X_8	-136	-456
X_2X_9	-200	-456
X_2X_{10}	-40	-616
X_2X_{11}	-56	-808
X_2X_{12}	-296	-312
X_3X_4	-136	-616
X_3X_5	-200	-616
X_3X_6	-120	-600
X_3X_7	-248	-600
X_3X_8	-232	-456
X_3X_9	-168	-456
X_3X_{10}	-72	-552
X_3X_{11}	-56	-872
X_3X_{12}	-392	-312
X_4X_5	-160	-608
X_4X_6	-80	-592
X_4X_7	-208	-592
X_4X_8	-80	-464
X_4X_9	-208	-464
X_4X_{10}	-16	-656

X_4X_{11}	-48	-784
X_4X_{12}	-272	-304
X_5X_6	-144	-592
X_5X_7	-272	-592
X_5X_8	-272	-464
X_5X_9	-144	-464
X_5X_{10}	-80	-528
X_5X_{11}	-48	-912
X_5X_{12}	-464	-304
X_6X_7	-192	-576
X_6X_8	32	-480
X_6X_9	-224	-480
X_6X_{10}	32	-736
X_6X_{11}	-32	-736
X_6X_{12}	-224	-288
X_7X_8	-352	-480
X_7X_9	-96	-480
X_7X_{10}	-96	-480
X_7X_{11}	-32	-992
X_7X_{12}	-608	-288
X_8X_9	-640	-128
X_8X_{10}	0	-256
X_8X_{11}	-256	-512
X_8X_{12}	-256	-512
X_9X_{10}	0	-256
X_9X_{11}	-256	-512
X_9X_{12}	-256	-512

$X_{10}X_{11}$	-384	-1408
$X_{10}X_{12}$	-896	-640
$X_{11}X_{12}$	-384	-384
X_1-0	-144	-592
X_1-1	-112	-688
X_2-0	-112	-624
X_2-1	-144	-656
X_3-0	-176	-624
X_3-1	-80	-656
X_4-0	-96	-608
X_4-1	-160	-672
X_5-0	-224	-608
X_5-1	-32	-672
X_6-0	-64	-576
X_6-1	-192	-704
X_7-0	-320	-576
X_7-1	64	-704
X_8-0	-256	-256
X_8-1	0	-1024
X_9-0	-256	-256
X_9-1	0	-1024
X_{10}-0	0	-512
X_{10}-1	-256	-768
X_{11}-0	0	-1024
X_{11}-1	0	-256
X_{12}-0	-512	0
X_{12}-1	256	0

4. Priority encoder 74147



In this example we shall demonstrate another advantage of having multiple outputs in a circuit for input fault isolation. We shall find the signature for stuck at faults at input and internal lines and then find the combined isolation signature for input stuck at and bridging faults and see what extra cost we have to pay in terms of additional spectral coefficients for fault isolation.

For a circuit with multiple outputs it is obvious that the signature for a stuck at fault (input and internal lines) would have to involve all outputs. Applying the technique of path analysis [13] to this circuit we find that only X_9 and the line marked I are candidates for syndrome untestability at output 'A' for a stuck at fault. That is, a stuck at fault at these lines may or may not cause r_0 measured at output 'A' to change value and this must be verified by other methods. Let us examine if the circuit is really syndrome untestable at 'A' with respect to a stuck at fault at the line marked 'I' and, if it is, then we shall have to use other coefficients to cover this fault. From [13] we know that a circuit with n inputs is syndrome testable with respect to an internal line G stuck at 0 if and only if

$$r_0 \neq \frac{1}{2}(\hat{r}_0 + \hat{r}_{n+1}) \qquad \ldots\ldots\ldots \quad 6.38$$

and with respect to G stuck at 1 if and only if

$$r_0 \neq \frac{1}{2}(\hat{r}_0 - \hat{r}_{n+1}) \qquad \ldots\ldots\ldots \quad 6.39$$

where $\hat{r}_0$ and $\hat{r}_{n+1}$ are the spectral coefficients of H which is depicted in the figure below.

$$F(X_1, X_2, \ldots X_n) = H(X_1, X_2, \ldots X_n, G(X_1, X_2, \ldots X_n))$$

When the circuit realizing F is functioning normally, H behaves as a function of n variables, but when G is stuck, H behaves as a function of n+1 variables with the $n+1^{th}$ input stuck. In the case of the 74147, G is the part of the circuit enclosed in the box and H is the rest of the circuit (refer to the circuit shown previously).

At output 'A' it is found that $r_0 = 171$, $\hat{r}_0 = 342$ and $\hat{r}_{10} = 342$ and we see that

$$r_0 \neq \frac{1}{2}(\hat{r}_0 + \hat{r}_{10})$$
$$\neq \frac{1}{2}(\hat{r}_0 - \hat{r}_{10})$$

Therefore the circuit is syndrome testable with respect to a stuck at fault on internal line I. Also it is syndrome testable with respect to a stuck at fault at X_9 because X_9 is connected to the internal line I by an inverter only. A stuck at fault (0, 1) at X_9 implies a stuck at fault (1, 0) at I.

Alternatively, we can check to see if the circuit is syndrome testable with respect to a stuck at fault at X_9 using the conditions given in section 3.5 and conclude whether it is syndrome testable with respect to a stuck at fault at I.

With the presence of multiple outputs the above calculation is actually not necessary. If we examine the circuit we notice that although X_9 and the internal line I are syndrome untestable candidates for output 'A', the circuit is syndrome testable at output 'D' with respect to a stuck at fault at these lines. This shows how multiple outputs can be beneficial to fault detection. Therefore a signature for stuck at faults at input and internal lines for a 74147 is

r_0 at outputs A, B, C and D

Since we have the 4 r_0's from each output for the signature for stuck at faults, it would be logical to construct the combined isolation signature based on these 4 coefficients. Let us see if they are sufficient to isolate all single input stuck at and bridging faults. We construct the matrix using these 4 coefficients as shown below.

FAULT	r_0 at A	r_0 at B	r_0 at C	r_0 at D
normal	171	410	392	128
X_1X_2	172	409	392	128
X_1X_3	170	409	392	128
X_1X_4	174	413	388	128
X_1X_5	166	413	388	128
X_1X_6	182	397	388	128
X_1X_7	150	397	388	128
X_1X_8	214	461	452	64
X_1X_9	86	461	452	64
X_2X_3	169	410	392	128
X_2X_4	173	414	388	128
X_2X_5	165	414	388	128
X_2X_6	181	398	388	128
X_2X_7	149	398	388	128
X_2X_8	213	462	452	64

X_2X_9	85	462	452	64
X_3X_4	175	414	388	128
X_3X_5	167	414	388	128
X_3X_6	183	398	388	128
X_3X_7	151	398	388	128
X_3X_8	215	462	452	64
X_3X_9	87	462	452	64
X_4X_5	163	410	392	128
X_4X_6	179	394	392	128
X_4X_7	147	394	392	128
X_4X_8	211	458	456	64
X_4X_9	83	458	456	64
X_5X_6	187	394	392	128
X_5X_7	155	394	392	128
X_5X_8	219	458	456	64
X_5X_9	91	458	456	64
X_6X_7	139	410	392	128
X_6X_8	203	474	456	64
X_6X_9	75	474	456	64
X_7X_8	235	474	456	64
X_7X_9	107	474	456	64
X_8X_9	43	410	392	128
X_1 S-A-0	170	410	392	128
X_1 S-A-1	172	410	392	128
X_2 S-A-0	172	408	392	128
X_2 S-A-1	170	412	392	128
X_3 S-A-0	168	416	392	128

X_3 S-A-1	174	404	392	128
X_4 S-A-0	176	416	384	128
X_4 S-A-1	166	404	400	128
X_5 S-A-0	160	416	384	128
X_5 S-A-1	182	404	400	128
X_6 S-A-0	192	384	384	128
X_6 S-A-1	150	436	400	128
X_7 S-A-0	128	384	384	128
X_7 S-A-1	214	436	400	128
X_8 S-A-0	256	512	512	0
X_8 S-A-1	86	308	272	256
X_9 S-A-0	0	512	512	0
X_9 S-A-1	342	308	272	256

A signature for isolation of single input stuck at faults and single input bridging faults for this circuit is :

r_0 at output A and

r_0 at output B

Therefore a signature for single stuck at faults (input and internal lines) and isolation of single input stuck at and single input bridging faults is :

r_0 at outputs A, B, C and D

The signature for input fault isolation is obtained at no extra cost.

Chapter VII

CONCLUSION

We have shown that spectral coefficients can be used for single fault isolation at inputs to a combinational circuit. Several approaches to the construction of an isolation signature for single stuck at or bridging faults have been presented. The isolation of faults with the signatures presented requires some form of fault dictionary. It may be in the form of bit patterns or in the form of vectors of integers. Each bit pattern or integer vector represents uniquely a particular fault or class of faults. It is in this respect that our approaches to fault isolation resemble the traditional method. But the derivation of the fault dictionary does not require any fault simulation in our case as opposed to the traditional approach. As a matter of fact it can be derived from the fault free spectrum without detailed knowledge of the circuit.

Of all the isolation signatures presented, the one which involves the verification of the spectral values is more practical than the others for the reasons mentioned at the beginning of chapter 6. In chapter 6 the approach of constructing such a signature was slightly modified to make it more practical. Signatures were found for several 'real world' circuits, and it was noted that the low order spectral coefficients of the circuits examined played an important part in their signatures. With the exception of the fast carry look ahead adder 7483, the signatures found for all of the circuits examined involved the zero and first order spectral coefficients. As a matter of fact a signature composed of zero and first order coefficients for the 7483 can be found using another output. This does not imply that the higher order spectral coefficients cannot be used in such signatures but it appears that the lower order ones are quite sufficient. In addition they have the advantage mentioned earlier that are not present in the high order coefficients.

Theoretically, the idea of fault isolation presented in the previous chapters can be extended to multiple faults. The faulty spectra of the circuits with multiple faults can be evaluated from the fault free one using a similar approach to that presented in chapter 3. In practice the number of possible cases of multiple faults to be considered is too large to make it economically feasible. For example let us examine the case of input stuck at faults for a circuit of n inputs. The number of possible stuck at faults that involve i ($i < n$) inputs regardless of stuck at 0 or stuck at 1 is ${}_iC_n$. But each one of the i inputs can be stuck at '0' or '1' and so the number of possible faults that involve i inputs is $2^i{}_iC_n$. The total number of possible multiple faults for a circuit of n inputs is

$$2^2\,{}_2C_n + 2^3\,{}_3C_n + \dots + 2^i\,{}_iC_n + \dots 2^n\,{}_nC_n$$

Even if we are not interested in which of the inputs is stuck at '1' or stuck at '0', the number of multiple faults would be

$${}_2C_n + {}_3C_n + \dots + {}_iC_n + \dots {}_nC_n$$
$$= 2^n$$

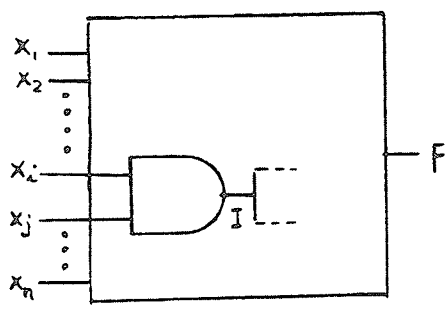
For large n (n > 10) this would be too large to allow for each fault to be studied individually.

In the case of bridging faults, the situation is even more complicated. For example if i of n inputs of a circuit are involved in a bridging fault, then this bridging fault may be composed of one, two, three, or up to k bridgings where k is the largest integer less than $\frac{i}{2}$. Therefore the number of possible faults again prohibits the isolation of each individual fault. More work is required in the area of multiple fault isolation.

The isolation of faults at internal lines can not be studied the same way as for input faults. An input line associates with half of the spectral coefficients in a spectrum. But the association of an internal line to spectral coefficients is not as straightforward as in the case of input lines [see section 2.2]. Fortunately, as mentioned earlier, the isolation of these faults is not often necessary, especially in the testing of integrated circuits. In practice, a signature for detection of internal faults and isolation of input faults is more desirable and economically feasible. The signature for the priority encoder 74147 presented in chapter 6 is one example of this.

With such a signature, the isolation of input faults is complicated by the fact that an internal fault on some occasions produces the same effect as a single or multiple input fault.

For example



In the above circuit X_i and X_j are inputs to an AND gate and I is the output from the AND gate which is an internal line. A stuck at 0 at I cannot be differentiated from a stuck at 0 at either or both of X_i and X_j. In such a case, if the fault is detected it can only be isolated by actually examining the circuit. Consequently there are several areas of fault isolation related to the work of this thesis which required further research for their solution.

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Appendix A

SPECTRAL COEFFICIENTS OF SOME INTEGRATED CIRCUITS

Spectrum of BCD to 7 segment decoder 7449 at output 'a' :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
8	-2	2	0	2	4	0	2
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
2	0	-4	2	0	2	-2	0
r_5	r_{15}	r_{25}	r_{125}	r_{35}	r_{135}	r_{235}	r_{1235}
-8	2	-2	0	-2	-4	0	-2
r_{45}	r_{145}	r_{245}	r_{1245}	r_{345}	r_{1345}	r_{2345}	r_{12345}
-2	0	4	-2	0	-2	2	0

Spectrum of BCD to 7 segment decoder 7449 at output 'b' :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
9	1	3	3	3	-1	1	-3
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
3	-1	-3	1	1	1	-1	-1
r_5	r_{15}	r_{25}	r_{125}	r_{35}	r_{135}	r_{235}	r_{1235}
-9	-1	-3	-3	-3	1	-1	3
r_{45}	r_{145}	r_{245}	r_{1245}	r_{345}	r_{1345}	r_{2345}	r_{12345}
-3	1	3	-1	-1	-1	1	1

Spectrum of BCD to 7 segment decoder 7449 at output 'c' :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
10	-2	2	2	2	-2	2	2
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
4	0	0	0	-4	0	0	0
r_5	r_{15}	r_{25}	r_{125}	r_{35}	r_{135}	r_{235}	r_{1235}
-10	2	-2	-2	-2	2	-2	-2
r_{45}	r_{145}	r_{245}	r_{1245}	r_{345}	r_{1345}	r_{2345}	r_{12345}
-4	0	0	0	4	0	0	0

Spectrum of BCD to 7 segment decoder 7449 at output 'd' :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
10	2	-2	-2	2	2	-2	6
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
0	0	0	0	0	0	0	0
r_5	r_{15}	r_{25}	r_{125}	r_{35}	r_{135}	r_{235}	r_{1235}
-10	-2	2	2	-2	-2	2	-6
r_{45}	r_{145}	r_{245}	r_{1245}	r_{345}	r_{1345}	r_{2345}	r_{12345}
0	0	0	0	0	0	0	0

Spectrum of BCD to 7 segment decoder 7449 at output 'e' :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
6	6	-2	-2	2	2	2	2
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
0	0	0	0	0	0	0	0

r_5	r_{15}	r_{25}	r_{125}	r_{35}	r_{135}	r_{235}	r_{1235}
-6	-6	2	2	-2	-2	-2	-2
r_{45}	r_{145}	r_{245}	r_{1245}	r_{345}	r_{1345}	r_{2345}	r_{12345}
0	0	0	0	0	0	0	0

Spectrum of BCD to 7 segment decoder 7449 at output 'f' :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
9	3	5	-1	1	3	-1	1
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
-1	1	-1	1	-1	1	-1	1
r_5	r_{15}	r_{25}	r_{125}	r_{35}	r_{135}	r_{235}	r_{1235}
-9	-3	-5	1	3	1	-1	-3
r_{45}	r_{145}	r_{245}	r_{1245}	r_{345}	r_{1345}	r_{2345}	r_{12345}
1	-1	1	-1	1	-1	1	-1

Spectrum of BCD to 7 segment decoder 7449 at output 'g' :

r_0	r_1	r_2	r_{12}	r_3	r_{13}	r_{23}	r_{123}
12	2	0	-2	0	-2	-4	2
r_4	r_{14}	r_{24}	r_{124}	r_{34}	r_{134}	r_{234}	r_{1234}
-2	0	-2	0	-2	0	-2	0
r_5	r_{15}	r_{25}	r_{125}	r_{35}	r_{135}	r_{235}	r_{1235}
-12	-2	0	2	0	2	4	-2
r_{45}	r_{145}	r_{245}	r_{1245}	r_{345}	r_{1345}	r_{2345}	r_{12345}
2	0	2	0	2	0	2	0

Spectrum of fast carry look ahead adder 7483 at output 'S_4':

r_0 = 256

r_{12} = -192

r_{123}, r_{124}, r_{1234} = 64

r_{125}, r_{1235}, r_{1245}, r_{12345}, r_{126},
r_{1236}, r_{1246}, r_{12346} = -32

r_{127}, r_{1237}, r_{1247}, r_{12347}, r_{128},
r_{1238}, r_{1248}, r_{12348}, r_{129}, r_{1239},
r_{1249}, r_{12349}, $r_{1256789}$, $r_{12356789}$, $r_{12456789}$,
$r_{123456789}$ = -16

r_{123567}, r_{124567}, $r_{1234567}$, r_{12568}, r_{123568},
r_{124568}, $r_{1234568}$, r_{12569}, r_{123569}, r_{124569},
$r_{1234569}$, r_{12789}, r_{123789}, r_{124789}, $r_{1234789}$ = 16

The rest of the spectral coefficients are zero.

Spectral coefficients of order 0, 1, 2 and 3 of fast carry look ahead adder 7483 at output 'C_4':

r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
60	60	60	-32	-32	-12	-12	-12	-12	-8

r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}
60	-32	-32	-12	-12	-12	-12	-8

r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}
-32	-32	-12	-12	-12	-12	-8

r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}
4	0	0	0	0	0

r_{45}	r_{46}	r_{47}	r_{48}	r_{49}
0	0	0	0	0

r_{56}	r_{57}	r_{58}	r_{59}
-4	4	4	0

r_{67}	r_{68}	r_{69}
4	4	0

r_{78}	r_{79}
-4	0

r_{89}
0

r_{123}	r_{124}	r_{125}	r_{126}	r_{127}	r_{128}	r_{129}
-32	-32	-12	-12	-12	-12	-8

r_{134}	r_{135}	r_{136}	r_{137}	r_{138}	r_{139}
4	0	0	0	0	0

r_{145}	r_{146}	r_{147}	r_{148}	r_{149}
0	0	0	0	0

r_{156}	r_{157}	r_{158}	r_{159}
-4	4	4	0

r_{167}	r_{168}	r_{169}
4	4	0

r_{178}	r_{179}
-4	0

r_{189}
0

r_{234}	r_{235}	r_{236}	r_{237}	r_{238}	r_{239}
4	0	0	0	0	0

r_{245}	r_{246}	r_{247}	r_{248}	r_{249}
0	0	0	0	0

r_{256}	r_{257}	r_{258}	r_{259}
-4	4	4	0

r_{267}	r_{268}	r_{269}
4	4	0

r_{278}	r_{279}
-4	0

r_{289}
0

r_{345}	r_{346}	r_{347}	r_{348}	r_{349}
12	12	12	12	8

r_{356}	r_{357}	r_{358}	r_{359}
0	0	0	0

r_{367}	r_{368}	r_{369}
0	0	0

r_{378}	r_{379}
0	0

r_{389}
0

r_{456}	r_{457}	r_{458}	r_{459}
0	0	0	0

r_{467}	r_{468}	r_{469}
0	0	0

r_{478}	r_{479}
0	0

r_{489}
0

r_{567}	r_{568}	r_{569}
4	4	8

r_{578}	r_{579}
4	0

r_{589}
0

r_{678}	r_{679}
4	0

r_{689}
0

r_{789}
8

Low order spectral coefficients of 74381 at output 'C':

r_0	r_1	r_2	r_3	r_4	r_5	r_6
2167	48	16	16	32	32	64
r_7	r_8	r_9	$r_{(10)}$	$r_{(11)}$	$r_{(12)}$	
64	-384	-384	-640	-128	-640	
r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}
0	0	0	0	0	0	16
r_{19}	$r_{1(10)}$	$r_{1(11)}$	$r_{1(12)}$			
-48	-16	-16	48			
r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}
0	0	0	0	0	-16	-16
$r_{2(10)}$	$r_{2(11)}$	$r_{2(12)}$				
-48	16	16				
r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}	$r_{3(10)}$
0	0	0	0	-16	-16	16
$r_{3(11)}$	$r_{3(12)}$					
-48	16					
r_{45}	r_{46}	r_{47}	r_{48}	r_{49}	$r_{4(10)}$	$r_{4(11)}$
0	0	0	-32	-32	-96	32
$r_{4(12)}$						
32						
r_{56}	r_{57}	r_{58}	r_{59}	$r_{5(10)}$	$r_{5(11)}$	$r_{5(12)}$
0	0	-32	-32	32	-96	32
r_{67}	r_{68}	r_{69}	$r_{6(10)}$	$r_{6(11)}$	$r_{6(12)}$	
0	-64	-64	-192	64	64	
r_{78}	r_{79}	$r_{7(10)}$	$r_{7(11)}$	$r_{7(12)}$		
-64	-64	64	-192	64		

r_{89}	$r_{8(10)}$	$r_{8(11)}$	$r_{8(12)}$
-128	-128	-128	384

$r_{9(10)}$	$r_{9(11)}$	$r_{9(12)}$
-128	-128	384

$r_{(10)(11)}$	$r_{(10)(12)}$
128	128

$r_{(11)(12)}$
-384

r_{123}	r_{124}	r_{125}	r_{126}	r_{127}	r_{128}	r_{129}
16	0	0	0	0	0	0

$r_{12(10)}$	$r_{12(11)}$	$r_{12(12)}$
0	0	0

r_{134}	r_{135}	r_{136}	r_{137}	r_{138}	r_{139}	$r_{13(10)}$
0	0	0	0	0	0	0

$r_{13(11)}$	$r_{13(12)}$
0	0

r_{145}	r_{146}	r_{147}	r_{148}	r_{149}	$r_{14(10)}$	$r_{14(11)}$
16	0	0	0	0	0	0

$r_{14(12)}$
0

r_{156}	r_{157}	r_{158}	r_{159}	$r_{15(10)}$	$r_{15(11)}$	$r_{15(12)}$
0	0	0	0	0	0	0

r_{167}	r_{168}	r_{169}	$r_{16(10)}$	$r_{16(11)}$	$r_{16(12)}$
0	0	0	0	0	0

r_{178}	r_{179}	$r_{17(10)}$	$r_{17(11)}$	$r_{17(12)}$
0	0	0	0	0

r_{189}	$r_{18(10)}$	$r_{18(11)}$	$r_{18(12)}$
16	16	16	16

$r_{19(10)}$	$r_{19(11)}$	$r_{19(12)}$
16	16	-48

$r_{1(10)(11)}$	$r_{1(10)(12)}$
-16	-16

$r_{1(11)(12)}$
-16

r_{234}	r_{235}	r_{236}	r_{237}	r_{238}	r_{239}	$r_{23(10)}$
0	0	0	0	0	0	0

$r_{23(11)}$	$r_{23(12)}$
0	0

r_{245}	r_{246}	r_{247}	r_{248}	r_{249}	$r_{24(10)}$	$r_{24(11)}$
-16	0	0	0	0	0	0

$r_{24(12)}$
0

r_{256}	r_{257}	r_{258}	r_{259}	$r_{25(10)}$	$r_{25(11)}$	$r_{25(12)}$
0	0	0	0	0	0	0

r_{267}	r_{268}	r_{269}	$r_{26(10)}$	$r_{26(11)}$	$r_{26(12)}$
0	0	0	0	0	0

r_{278}	r_{279}	$r_{27(10)}$	$r_{27(11)}$	$r_{27(12)}$
0	0	0	0	0

r_{289}	$r_{28(10)}$	$r_{28(11)}$	$r_{28(12)}$
-16	-16	48	-16

$r_{29(10)}$	$r_{29(11)}$	$r_{29(12)}$
48	-16	-16

$r_{2(10)(11)}$	$r_{2(10)(12)}$
16	-48

$r_{2(11)(12)}$
16

r_{345}	r_{346}	r_{347}	r_{348}	r_{349}	$r_{34(10)}$	$r_{34(11)}$
-16	0	0	0	0	0	0

$r_{34(12)}$
0

r_{356}	r_{357}	r_{358}	r_{359}	$r_{35(10)}$	$r_{35(11)}$	$r_{35(12)}$
0	0	0	0	0	0	0

r_{367}	r_{368}	r_{369}	$r_{36(10)}$	$r_{36(11)}$	$r_{36(12)}$
0	0	0	0	0	0

r_{378}	r_{379}	$r_{37(10)}$	$r_{37(11)}$	$r_{37(12)}$
0	0	0	0	0

r_{389}	$r_{38(10)}$	$r_{38(11)}$	$r_{38(12)}$
-16	48	-16	-16

$r_{39(10)}$	$r_{39(11)}$	$r_{39(12)}$
-16	48	-16

$r_{3(10)(11)}$	$r_{3(10)(12)}$
16	16

$r_{3(11)(12)}$
-48

r_{456}	r_{457}	r_{458}	r_{459}	$r_{45(10)}$	$r_{45(11)}$	$r_{45(12)}$
0	0	0	0	0	0	0

r_{467}	r_{468}	r_{469}	$r_{46(10)}$	$r_{46(11)}$	$r_{46(12)}$
0	0	0	0	0	0

r_{478}	r_{479}	$r_{47(10)}$	$r_{47(11)}$	$r_{47(12)}$
0	0	0	0	0

r_{489}	$r_{48(10)}$	$r_{48(11)}$	$r_{48(12)}$
-32	-32	96	-32

$r_{49(10)}$	$r_{49(11)}$	$r_{49(12)}$
96	-32	-32

$r_{4(10)(11)}$	$r_{4(10)(12)}$
32	-96

$r_{4(11)(12)}$
32

r_{567}	r_{568}	r_{569}	$r_{56(10)}$	$r_{56(11)}$	$r_{56(12)}$
0	0	0	0	0	0

r_{578}	r_{579}	$r_{57(10)}$	$r_{57(11)}$	$r_{57(12)}$
0	0	0	0	0

r_{589}	$r_{58(10)}$	$r_{58(11)}$	$r_{58(12)}$
-32	96	-32	-32

$r_{59(10)}$	$r_{59(11)}$	$r_{59(12)}$
-32	96	-32

$r_{5(10)(11)}$	$r_{5(10)(12)}$
32	32

$r_{5(11)(12)}$
-96

r_{678}	r_{679}	$r_{67(10)}$	$r_{67(11)}$	$r_{67(12)}$
0	0	0	0	0

r_{689}	$r_{68(10)}$	$r_{68(11)}$	$r_{68(12)}$
-64	-64	192	-64

$r_{69(10)}$	$r_{69(11)}$	$r_{69(12)}$
192	-64	-64

$r_{6(10)(11)}$	$r_{6(10)(12)}$
64	-192

$r_{6(11)(12)}$
64

r_{789}	$r_{78(10)}$	$r_{78(11)}$	$r_{78(12)}$
-64	192	-64	-64

$r_{79(10)}$	$r_{79(11)}$	$r_{79(12)}$
-64	192	-64

$r_{7(10)(11)}$	$r_{7(10)(12)}$
64	64

$r_{7(11)(12)}$
-192

$r_{89(10)}$	$r_{89(11)}$	$r_{89(12)}$
128	-384	128

$r_{8(10)(11)}$	$r_{8(10)(12)}$
128	128

$r_{8(11)(12)}$
128

$r_{9(10)(11)}$	$r_{9(10)(12)}$
128	128

$r_{9(11)(12)}$
128

$r_{(10)(11)(12)}$

-640

Low order spectral coefficients of 74147 at output 'A' :

r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
171	-1	1	-3	5	-11	21	-43	85	-171

r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}
1	1	1	1	1	1	1	1

r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}
-1	-1	-1	-1	-1	-1	-1

r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}
3	3	3	3	3	3

r_{45}	r_{46}	r_{47}	r_{48}	r_{49}
-5	-5	-5	-5	-5

r_{56}	r_{57}	r_{58}	r_{59}
11	11	11	11

r_{67}	r_{68}	r_{69}
-21	-21	-21

r_{78}	r_{79}
43	43

r_{89}
-85

Low order spectral coefficients of 74147 at output 'B' :

r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
410	0	-2	-2	6	6	-26	-26	102	102
r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}		
0	0	0	0	0	0	0	0		
r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}			
2	2	2	2	2	2	2			
r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}				
2	2	2	2	2	2				
r_{45}	r_{46}	r_{47}	r_{48}	r_{49}					
-6	-6	-6	-6	-6					
r_{56}	r_{57}	r_{58}	r_{59}						
-6	-6	-6	-6						
r_{67}	r_{68}	r_{69}							
26	26	26							
r_{78}	r_{79}								
26	26								
r_{89}									
-102									

Low order spectral coefficients of 74147 at output 'C' :

r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
392	0	0	0	-8	-8	-8	-8	120	120

r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}
0	0	0	0	0	0	0	0

r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}
0	0	0	0	0	0	0

r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}
0	0	0	0	0	0

r_{45}	r_{46}	r_{47}	r_{48}	r_{49}
8	8	8	8	8

r_{56}	r_{57}	r_{58}	r_{59}
8	8	8	8

r_{67}	r_{68}	r_{69}
8	8	8

r_{78}	r_{79}
8	8

r_{89}
-120

Low order spectral coefficients of 74147 at output 'D' :

r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
128	0	0	0	0	0	0	0	-128	-128
r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}		
0	0	0	0	0	0	0	0		
r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}			
0	0	0	0	0	0	0			
r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}				
0	0	0	0	0	0				
r_{45}	r_{46}	r_{47}	r_{48}	r_{49}					
0	0	0	0	0					
r_{56}	r_{57}	r_{58}	r_{59}						
0	0	0	0						
r_{67}	r_{68}	r_{69}							
0	0	0							
r_{78}	r_{79}								
0	0								
r_{89}									
128									