THE UNIVERSITY OF MANITOBA

A BRIEF INVESTIGATION INTO THE ANALYSIS OF STATIC AND DYNAMIC STRESSES IN RAILWAY TRACK WITH APPLICATION TO THE CANADIAN RAIL TRANSPORT PROBLEMS

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

WINNIPEG, MANITOBA

February 1976

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To

Hekmat and Hedda

ABSTRACT

At the present time, several problems confront the Canadian Railways development, which demand the full understanding of the stresses in the railway track. The analysis of these stresses is of a particular difficult nature and simple practical methods are always needed. The lack of publications in Canada about some of these procedures and their use in solving some Canadian planning and design difficulties, brought about the necessity for a self-contained guide on that concern.

Some of the problems confronting the Canadian railways today and in the future, and the use of the study of the track stresses in the proper decision-making for a solution to these problems, is the subject of Chapter 9, the last chapter. It is to be pointed out, however, that of necessity, this study has been limited in its scope, and only some obvious examples have been studied as fully as possible. Chapter 8 illustrates the application of the track stress analysis to a specific problem related to the grain movement economy in Canada.

The other parts of this thesis deal with the theories and the procedures for estimating the railway track stresses. The research in this field has been far more advanced and dynamic in Europe than in North America. In the last four decades, for example, series of publications have appeared in Germany which deal with these fundamentals and which conclude with useful formulas. Some of these have never been introduced in North America, although they are now in major use in the European Railway administrations. After a presentation of various theories and formulas that have been known since the beginning of the railways in the world, and briefly dealing with the theoretical and experimental analysis, in Chapters 2 to 6, Chapter 7 discusses the technical evaluation of the different procedures, and comparisons are made between the different methods. A useful table is established in the appendix for the convenience of the planner or designer, to give calculated values according to the recommended formulas. Chapter 2 deals briefly with a simplified elastic model of the track system as a basis for Chapter 3, which discusses the practical procedures of estimating the static bending moment exerted on railroad tracks. Chapter 3 also includes a historical review of the development of these techniques.

Chapter 4 complements Chapter 3, as it considers the dynamic effects resulting from train motion and vibration, which must be added to the bending moment equations presented in Chapter 3. Following the theoretical and imperical models in Chapter 3 and Chapter 4, Chapter 5 presents the analysis of stresses in each of the railway track elements. Chapter 5 also includes some relationships between different properties of the rail section, which may be useful to the railway engineer.

In Chapter 6 a brief discussion of the use of experimental techniques is presented with broad emphasis upon experiments that

lead to successful experimental stress analysis.

This study, in general, can be a useful guide for both the practicing engineer and the student or researcher in the subject of stresses in railway tracks and its application to the Canadian Railway problems.

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to his advisor, Professor A.M.Lansdown, for his many helpful suggestions, confidence, and patience. Professor Lansdown's superior knowledge on the subject is deeply appreciated.

Especial acknowledgement is due Professor K.R.McLachlan who read through the work and gave very valuable ideas and comments. His interest and help are sincerely acknowledged.

I wish to thank Professor O.Hawaleshka and Professor L. Domaschuk for their valuable advise at the final stage. Also, thanks to Mr. David Young who helped significantly in the computer programming.

Acknowledgement should also be made to the CN Rail and the CP Rail for their help during the collection of data. My sincere thanks to my father, Mr. Ali Abdel Razik, Mr. Nader El-Tawil of the University of Aachen, Germany, Mr. Hans Fricke of the German Book Club in Winnipeg, and the staff of the Interlibrary Loan at the University of Manitoba, for their help in the collection of the overseas publications.

Most of all, I thank my wife, Hedda, for her invaluable assistance at all points in the preparation of the thesis, and I acknowledge fully my debt to her loving patience, faith, and encouragement.

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CHAPTER 1: INTRODUCTION

Railways belong to the early technologies which founded the massive industrialization of western Europe and played a major role in the development of North America. From a status of near-monopoly until the beginning of this century, railways have been relegated to only one among several competing modes of transportation. This drastic change which came about as a result of technological innovation, has been recognized in Western Europe and Japan over the past 20 years, but - as we shall see - has yet to be faced up to in Canada.

- J. Lukasiewicz in The Institutionalization of Canadian Rail's Obsolescence.

1. General Considerations

The near future of the Canadian Railways is most likely to be full of activity. The expanding economy of Canada has called upon those in control of railroads and other forms of transport to exert every effort to meet the demand for carrying freight and passengers.

Railways are the most important means of surface transportation in agricultural-industrial countries such as Canada. Despite the growth of other modes of transportation, all indications point to railroads as continuing to be the backbone of the transportation industry in the future of this country. The dynamically developing railroad industry, in keeping pace with the current "state of the art", serves the growing economy and population of Canada. For this reason, the decisions of design and practice must be based on full economic costs, and modified only where safety governs.

The steel wheel on steel rail, which is the principle of guided surface transport, has four elements in it; the track, rolling stock, traction, and signalling. In all of these four elements of basic railway engineering, there are indications that a basis for modernization exists in Canada. The practices and procedures now in use are not the ultimate, for every phase of railroad engineering is subject to further study and improvement. However, the questions involved in the development in the first three of the elements mentioned above, are still basically:

(a) what rail speeds can be attained and what will they demand in track elements, rolling stock, and traction ?,

(b) what rail tonnages can be carried and what effect will they have on track elements, rolling stock, and traction ?

The answer to these two questions is deeply related to the understanding of the stresses exerted on the railway track by the weight of the rolling stock, and the effect of the speed on such a stress. Thus, the study of these stresses forms the major part of the work reported here.

This study may furthermore be of particular interest at the present time, as the discussion of the national railway policy has been gaining momentum in Canada. Daily reports speak of:

- The railway electrification of the Canadian main lines,
- The constant tendency in railway practice to increase the loading and the speed,
- The overloading or overstressing in some railway lines,

- The speed limitations or restrictions in certain railway lines,
- The introduction of passenger fast service trains,
- The abandonment of the light branch lines,
- The relocation of some railway lines,
- CP Rail's granting 1/2 million dollars for track structure research,
- The placement of the first reinforced concrete ties in Canada.
- The introduction of the LRC (Light,Rapid,Comfotable) train in Canada, using light alloys in the structure of locomotives and passenger cars.
- The introduction of the Grain Hopper Aluminum Cars.

Thus, a major part of this thesis is devoted to bringing together the information available in the world on track stresses and related problems, and show how this information may be applied to Canadian problems.

2. Track Elements Considerations

The entire subject matter is emmanently suitable for a comprehensive treatment, which is the aim of the present work. The track structure of today is the result of years of experience and of trial and error. The railroad track as a complete structural unit has not evolved through any process of straight forward design, unlike other branches of structural engineering, such as, bridges and other steel structures, for instance. The undetermined character of the track components largely account for this situation. Very little is known of the strength and forces in a roadbed. What is known, indicates some lack of uniformity in its strength, and other characteristics. The same holds true for the ballast. The ballast consists of randomly arranged stones with only limited grading, without any matrix or filler to keep each stone in constant relation to its neighbors. The ballast provide support to the tie by a very rudimentary form of mechanical interlocking of the stones themselves, which brings point-loading, friction, and indeed the shape of the stones into play. This interlocking slowly breaks down under traffic. Furthermore, the endless variations in the nature of the subgrade and the ballast materials, the methods used in their construction, the different topographical situations, the differences of rainfall, snowfall and drought from place to place and from year to year, leads to uncertainities which are not surprising.

Concerning the tie pressures distributed to the ballast and subgrade, it is known that ties possess the variability of all wood and are subject to further changes with weathering and use. The non-uniformity of ballast sections furthermore makes tie mechanics indeterminate.

The rail itself is difficult to analyze, especially in view of the non-uniformity of support and variable loading applied to it. Another great difficulty of railroad structure, is that one must deal with moving loads, especially with two load-kinds; the repeating load and the shock-type load. The effect of these loads can only be understood with great difficulty. Although the wheel loads of stationary vehicles are known quite accurately, they can only be estimated approximately for vehicles in motion.

Thus, uncertainty already prevails with regard to forces exerted upon and stresses induced in the rails. The determination of the stresses caused in the rails by the various forces, however, depends on values which fluctuate within wide limits and in certain cases are known only by their order of magnitude. This applies especially to the elastic deflection of the tie caused under the vertical loads transmitted throught the rails.

Bearing in mind all the difficulties reported above about the railroad track stress analysis, one can understand not only the difference between the railroad track and any other engineering structure, but also the great uncertainty prevailing in the railway track structure when the action of its several elements work as a complete structural unit.

A great deal of research has been done on the difficulties mentioned above, and while many theories have been put forth, the majority of them have not been recognized as wholly applicable. In addition, the computation methods should be as simple as possible for practical application. The lack of information in the Canadian libraries about some of the computation of the railway track stresses was a motive for the present work. The primary concern of almost all papers published so far on the subject in the United States or Europe has been restricted to the consideration of the technical details. In the present work , however, a wider horizon is attempted and the practical use of this information is illustrated.

This thesis is therefore designed, not to fill a gap in the experience of the line-engineer, but for all those concerned with practical work in the field of concern in Canada, specifically the planners and designers who are responsible for the improvement of our railway network. Moreover, in dealing with these factors, the thesis will point to the direction in which subsequent research may continue. Most of the problems of track stress analysis are dealt with from a practical viewpoint in the light of the most up-to-date knowledge available, in addition to the development of this knowledge. Furthermore, some problems confronting Canadian railways today, and in the future, and the use of the study of the track stress in the proper decision-making for a solution of these problems, is the secondary concern of this work.

This study concerns itself primarily with standard gauge railways, rails with expansion joints over wooden or .concrete ties. The loads considered are only the vertical loads on the track elements. The study deals only with the track superstructure (rails, ties and ballast) in addition to the vertical pressure distribution on the subgrade underneath. The form of the track discussed is the intercity rail transport track. The effect of the traffic volume on the stresses is left for other papers and research.

In the course of this work, major help was derived from the numerous publications on the subject in the United States and overseas, including several monographs in German. In attempting to form a comprehensive unit of all this material, it has been found convenient to state the units of the physical terms used in the study in the two international systems of units according to precise conversion factors. However, for the convenience of the reader, a conversion table is provided at the back of the Appendix, especially for the compound terms, such as bending moment, stress ...etc. It is important that the reader notice that the unit of weight "TON", is meant in Europe as 1000 Kilograms, while in North America it is 2000 Pounds.

In the Mathematical Notation of this text, two minor departures were made from the existing practice, in that Greek letters were avoided and also that subscripts were not used. The need for this arose from two facts;

(1) the entire manuscript was filed and printed by the Computer and these characters are not available in its editing facilities.

(2) the variability of the notations used by different engineering backgrounds.

The use of the English letters, whether individually or compound, however, was found to be a simple and effective way to achieve the purpose.

CHAPTER 2: PRELIMINARIES

THE ANALYSIS OF BEAMS ON ELASTIC FOUNDATION[1]

The analysis presented in this chapter is restricted to the analysis of beams of unlimited length (the infinite beam), and loading is restricted to that of concentrated loading. The above restrictions are imposed because they are the closest theoretical assumptions to the reality of the case of rolling stock loading on a railway track. In addition, a brief analysis is given at the end of the chapter for beams of finite length with free ends, which are loaded by two symmetrical forces, as is the case which exists in a tie under the action of rail pressure.

Another assumption which is important for the mathematical analysis, is that the elastic foundation is continuous[2] ,so that when the beam is deflected, the intensity of the continuously distributed reaction at any section is proportional to the deflection at that section. Mathematically,

$$p=k.y$$

---

[1]The material covered in this chapter is fairly standard "Strength of Materials" text book information. It was found convenient to report it here,because of the extensive dependence of the following chapters on this theory.
[2]The consistency of such an assumption with reality is discussed in Chapter 7.

where

p is the reaction per unit length of the beam,

y is the deflection,

and

k is a constant called the foundation constant.

This assumption is equivalent to the case of a beam resting on a continuously distributed set of springs, the stiffness of which is defined by the "foundation constant" [4]. The latter constant can be expressed as the reaction per unit length, provided that the deflection is equal to unity.

The assumption that the supporting medium is elastic, implies that its material follows Hooke's Law. Its elasticity, therefore, can be defined as the stress which will cause a deflection equal to unity. This constant of the supporting medium, k1(lbs/in^3 or kgs/cm^3) is called the "modulus of foundation"[1]. For a beam with a uniform cross-section, which is the case in railway tracks, the relation between the two constants mentioned above (k and k1)[1]can be stated as,

$$k = k1 \cdot b$$

where

k is the foundation constant (lb/in^2 or kg/cm^2)

k1 is the modulus of foundation (lb/in^3 or kg/cm^3)

and

---

[1] These two constants k, and k1, will be used where appropriate throughout this thesis.

b is the constant width of the beam (in or cm)

The well-known differential equation of deflection to be applied is [3]:

$$EI \; d^4y/dx^4 = q \qquad \ldots\ldots(2\text{-}1)$$

where,

E is the modulus of elasticity of the beam

I is the moment of inertia of the beam,

and

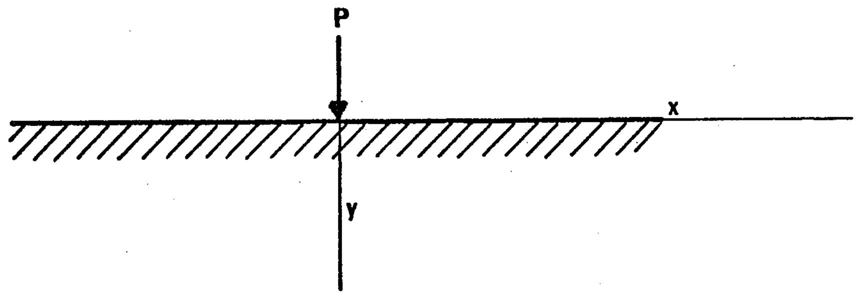
dx is the distance between two vertical cross sections on the beam under consideration.

For an unloaded portion of the beam, the only acting force will be a continuously distributed reaction from the foundation of intensity ky. then $q = -ky$ where q is the intensity of the load acting on the beam.

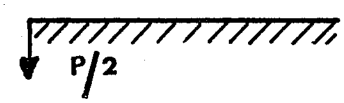
Hence the equation (2-1) becomes;

$$EI \; d^4y/dx^4 = -ky \qquad \ldots\ldots(2\text{-}2)$$

and represents the deflection curve of a beam supported on an elastic foundation. The general solution of Equation (2-2), can be derived using the boundary conditions which were introduced at the beginning of this chapter as restrictions, which are shown in Fig. (2-1) below.



A single concentrated load on an infinitely long bar, taking the origin of the coordinates at the point of application of the load



Only part of the bar to the right of the load need to be considered, due to symmetry.

Fig. (2-1)
A single concentrated load on an infinitely long bar

In this general solution, the deflection can be, finally, represented by [1],[2],[3]:

$$y = (P.v/2k).(e^{-vx}).(\cos vx + \sin vx) \qquad .....(2\text{-}3)$$

where,

$v = \sqrt[4]{k/4EI}$

e is the exponential.

Taking the successive derivatives of y, with respect to x in Equation (2-3), we obtain the expressions for the slope(s), the bending moment(M), and the shearing force(Q) on the right side of the beam as;

$$dy/dx = s = -(P.v^2/k).(e^{-vx}).(\sin vx) \qquad .....(2\text{-}4)$$

$$-EI.d^2y/dx^2 = M = (P/4v).e^{-vx}.(\cos vx - \sin vx) \quad \text{.....(2-5)}$$

and

$$-EI.d^3y/dx^3 = Q = -(P/2).e^{-vx}.\cos vx \quad \text{.....(2-6)}$$

In order to make the calculation of deflections, bending moments, and shearing forces as simple as possible, we introduce the symbols;

$$B1 = e^{-vx}(\cos vx + \sin vx) \quad \text{.....(2-7)}$$

$$B2 = e^{-vx}.\sin vx \quad \text{.....(2-8)}$$

$$B3 = e^{-vx}(\cos vx - \sin vx) \quad \text{.....(2-9)}$$

$$B4 = e^{-vx}.\cos vx \quad \text{.....(2-10)}$$

Hence, Equations (2-3), (2-4), (2-5), and (2-6) can be written as

$$y = (Pv/2k).B1 \quad \text{.....(2-11)}$$

$$s = (Pv^2/k).B2 \quad \text{.....(2-12)}$$

$$M = (P/4v).B3 \quad \text{.....(2-13)}$$

$$Q = -(P/2).B4 \quad \text{.....(2-14)}$$

In order to facilitate the application of the four functions of vx, (B1, B2, B3, and B4), numerical tables were first given by H.Zimmermann [1],[2],[3], in his principal book on this subject [26]. These tables (or a computer programme) can be used to calculate the values of the stresses in the track due to train static loading. However, in the third chapter of the thesis, much simpler techniques will be introduced for determining such stresses.

Equations (2-11), (2-12), (2-13), and (2-14) each have a wave form with gradually diminishing amplitude, as shown in Fig. (2-2). The length a, of these waves is given by the period of the functions cos vx and sin vx [2]. i.e.

$$a = 2\pi / v = 2\pi \sqrt[4]{4EI/k} \qquad .....(2-15)$$

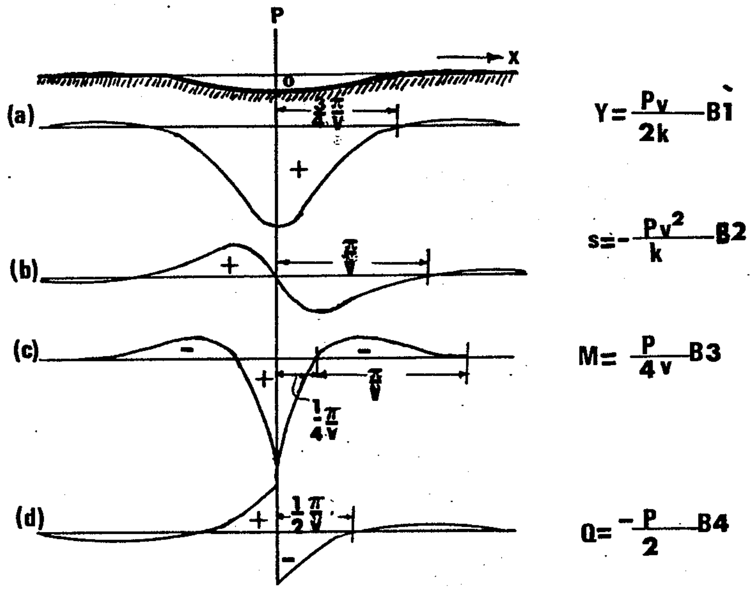


Fig. (2-2)

The deflection, angle of slope, bending moment, shearing force, diagrams of an infinite beam on elastic foundation, under a single concentrated load.

Infinitely close to the right of the point of application of the load (x = 0), we have the values:

$$y = Pv/2k \qquad .....(2-16)$$

$$s = 0 \qquad .....(2-17)$$

$$M = P/4v \qquad .....(2-18)$$

$$Q = -P/2 \qquad .....(2-19)$$

The deflection, bending moment, and shearing force which appeared above in (2-16), (2-18), and (2-19) are seen to be maximum values.

Further points to be observed, are the characteristics of the functions B1, B2, B3, and B4 in Equations (2-11), (2-12), (2-13), and (2-14) respectively. In Fig. (2-3) the functions B1 and B3 are shown graphically [2].

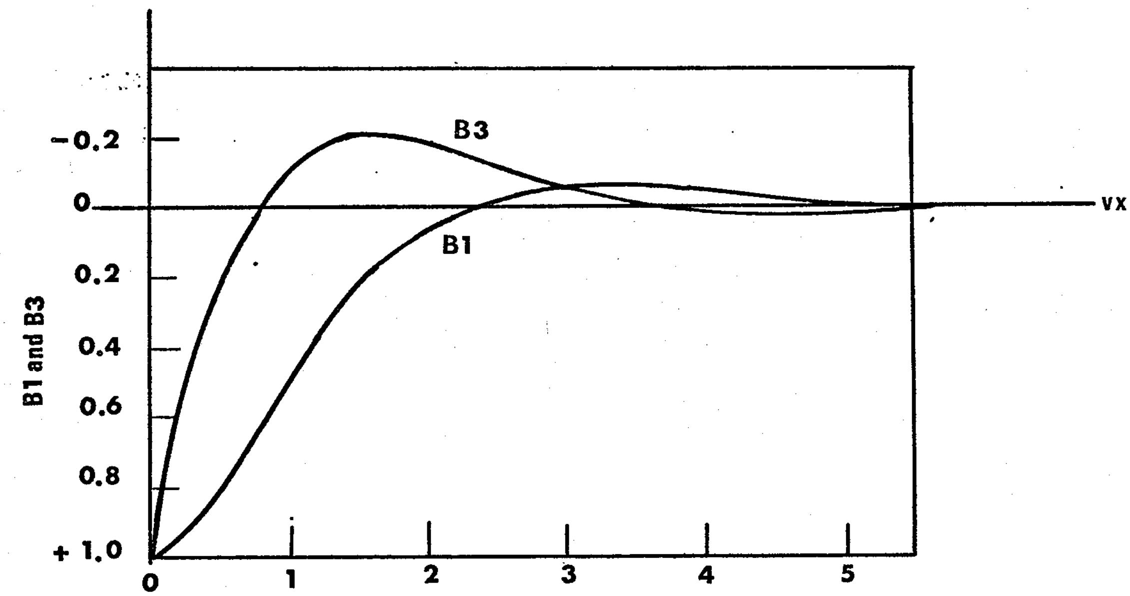


Fig. (2-3)

Graphical representation of the functions B1 and B3 for the determination of the deflection and the bending moment of the beam.

Given that n is any positive integer, the zero points for B1 are located at vx = (3/4 +n) π, for B2 at vx = nπ , for B3 at vx = (1/4 + n)π , and for B4 at vx = (1/2 + n)π. Similarly, the extreme values (maximum or minimum) of these functions are located, for B1 at vx = nπ, for B2 at vx = (1/4 + n)π , for B3 at $vx = (1/2 + n)\pi$, and for B4 at $vx = (3/4 + n)\pi$.

These functions are rapidly decreasing in amplitude. When $vx > 1.5\pi$, the value of any of the four functions is less than 0.01. This means if the beam is supported for a distance $x = \pm 1.5\pi/v$ from the point of application, the load will have only a small effect on the formation of the deflection line after this distance x, i.e. a beam of the length $L = 3\pi/v$ loaded with a concentrated force p at the middle, will have approximately the same deflection curve as the infinitely long beam shown in Fig. (2-2). This indicates, then, that it is possible to analyze railway track using infinite beam theory.

Principle of Superposition and the Reciprocity Theorem

From Equations (2-11), (2-12), (2-13), and (2-14) it can be seen that y, s, M, and Q are directly proportional to the load P. It follows therefore, that the "principle of superposition" and the "reciprocity theorem" are directly applicable to the system [1]. It is significant for studies of stresses in track that the reciprocity theorem applies to deflection, angular deflection, bending moment, and shearing force, particularly for the application to particular problems, when, in addition to the lateral forces due to wheel loading, axial forces or twisting moments may act on the beam. If we have forces P1, and P2 acting at points 1 and 2 respectively, it is apparent either from the curves or from the equations mentioned above that:

$$(y1,2) = (y2,1),$$

$$(s1,2) = \pm(s2,1),$$

$$(M1,2) = \pm(M2,1),$$

$$(Q1,2) = \pm(Q2,1).$$

where,

(y1,2), (s1,2), (M1,2), and (Q1,2) are the deflection, the slope, the bending moment, and the shearing force at point 2 due to a force P1 acting at point 1.

and

(y2,1), (s2,1), (M2,1), and (Q2,1) are the deflection, the slope, the bending moment, and the shearing force at point 1 due to a force P2 acting at point 2.

Thus the curves of y, s, M, and Q in Fig. (2-2) are at the same time "influence lines" for y, s, M, and Q.

The Approximation in the Theory

The foregoing analysis was entirely based on the assumption that the elastic foundation is continuous, so that when the beam is deflected, the intensity of the continuously distributed reaction at any section is proportional to the deflection at that section. Very seldom, however, does it happen that the foundation is actually constituted in this way.

A serious objection can be made to the simplifying assumption on which this elementary theory is based, because it is obvious that the reaction q of the foundation on the beam does not depend upon the local deflection y alone, but is also a function of all the other deflections of the foundation surface occuring at that moment. Biot [4] attempted to give a more comprehensive solution for deformations, taking into account this aspect. He carried out analyses, based on assumptions of a two-dimensional foundation and of a three-dimensional foundation, Fig. (2-4) a, and b.

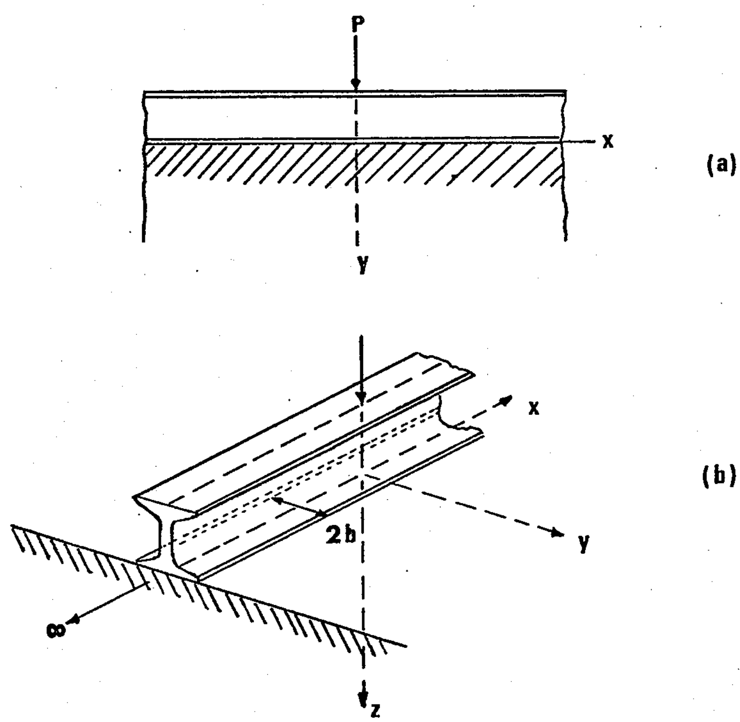


Fig. (2-4)

Graphical representations of a two-dimensional foundation and of a three-dimensional foundation

The two analyses of Biot differ quite fundamentally from one another. From the analysis of two-dimensional foundation , the maximum bending moment was found to be proportional to the one-third power of the beam stiffness EI, while from the analysis of three-dimensional foundation it was found proportional to the

0.277 power of the same quantity.

The maximum bending moment by the use of the approximate theory of Winkler and Zimmermann was found earlier in this chapter to be proportional to the 0.25 power of EI . This bending moment is, however, close enough to the results of Biot's exact theory for three-dimensional foundation, whereas it is not in agreement with the exact theory for two-dimensional foundation.

Beams of Finite Length

An aspect of importance in the study of stresses and deformations in track is the special case of a beam of finite length having free ends, which is loaded with two symmetrical forces P, Fig. (2-5-a). The described condition exists in the case of a tie under the action of rail pressures.

The solution for stresses in a finite beam can be obtained by solving the bending equation. Alternatively, and more easily, a solution can be obtained using the methods of superposition if the system may be assumed linear. Thus, as shown in Fig. (2-5-a and b), the infinite beam equation can be used if the appropriate equivalent force-couple systems are placed at points A and B, the end points of the finite beam.

This is a straight forward process and, hence, no further detailed description is justified here.

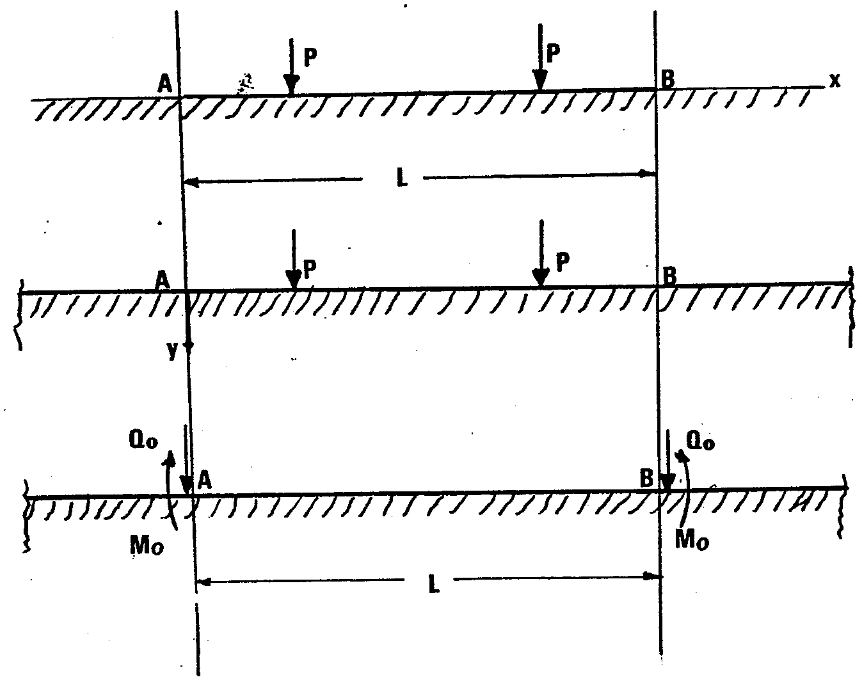


Fig. (2-5)

Analysis of a beam of finite length using the methods of superposition

The theory presented above in this chapter, with its two special cases of interest, was the key for all the research conducted in the area of railway-track stress-analysis. As we shall see in the following chapter, further simplifications for the use of the theory in railway engineering had been introduced over the years.

CHAPTER 3: ANALYSIS OF STATIC BENDING

MOMENTS AND DEFLECTIONS IN TRACK

The analysis of bending moments and deflections in a railway track has been developed over several decades. One early assumption, in which the rail is considered as a beam on a continuous elastic foundation, has been common to all the successful investigations. This brought about the applicability of the theory reported in the previous chapter to the analysis of railway track. In the present chapter a review of the relevant literature is presented. The two calculation procedures currently in use are presented thereafter.

1.Review of Literature

The analysis of stresses produced in rails has attracted the attention of engineers ever since the first railroads were built. In 1817, Barlow [5] wrote a book about his own "Strength of Material Experiments", in which he described his experiments with iron rails to measure the deflections under rapidly moving loads for the first time and compared these measurements with static results. He considered the rail as a beam on two supports and so (PL/4) was the value for the maximum bending moment [5]. In 1867, E. Winkler[27] published a book on the strength of materials , in which the bending of a beam on an elastic foundation is discussed for the first time and the applicability of the theory to the analysis of stresses in railroad tracks is indicated. The value of (0.189 PL) for the maximum bending moment was found. The formula of Winkler was scientifically rigorous within the assumptions made, and it survived for over one hundred years, inspite of a number of objections raised against it. The major objections were, [6]:

1.The support points were not fastened, as he assumed, but deflect under the load.

2.The distances of axles are in reality not equal and not as the calculation presumes them.

3.Often, in practice, the sizes of the axle loads are not equal. Therefore, when calculating the bending moment, a different influence of one axle load on another exists.

Winkler was the first to introduce the assumption that the reaction forces of the foundations are proportional at every point to the deflection of the beam at that point [1]. This assumption, a few years later, formed the basis of H.Zimmermann's classical work [26](published in 1888) on the analysis of the railroad track stresses. Zimmermann prepared tables for simplifying the Winkler analysis of a beam on an elastic foundation and applied the theory in calculating the deflection of the ties and the rail, which he considered to be a continuous beam on elastic supports. With conventional distances between the ties, the vertical rail load was distributed over several ties so that the isolated elastic supports could be replaced by an equivalent continuous elastic foundation. In Zimmermann and Winkler's theory, although soil was thought to be the chief supporting medium, and the theory applied by Zimmermann mainly to railroad tracks, it was later found that there were other fields where their assumptions were much more applicable. In particular, two other fields of application, were successful: the first was concerned with networks of beams for floor systems of ships, buildings and bridges, and the second dealt with thin shells of revolution for pressure vessels, boilers, containers, and reinforced concrete halls and domes of large spans [1]. The theory held rigidly for these other applications; but the railroad track application had to be considered as only a "practical approximation".

A. Foppl, in 1898, published a work on one of his classical experiments [28], from which it was proven, that for a large variety of soils one important assumption of Winkler's is true, that the foundation deforms only along the portion directly under loading.

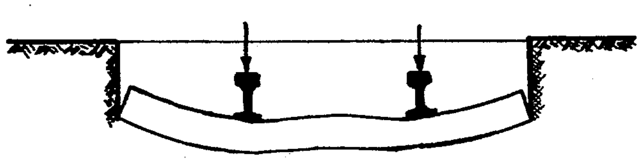


Fig. (3-1)

The foundation deforms only along the portion directly under loading.

In 1899, A. Wasiutynski[5] devised an optical method for stress analysis and succeeded in getting photographic records of bending strains and deflections in a rail under the wheels of a moving locomotive . He had a book published on the elastic deformation of track [29].

In 1913, the train administration of Oldenburg, Germany, put forth a proposal to introduce a unified and simplified standard calculation method of track superstructure work. These efforts were interrupted by the First World War. On the basis of experiments carried out by the Netherlands Railways in 1923, the railroad administration began to use the so-called axle formula, in the calculation of the track superstructure. The basic assumption for the setting up of this formula was that the cross-tie in the unloaded fields was not regarded as being present and the track in this region lifted itself without weight. Although the Netherlands formula yielded a maximum bending moment of 0.1875 PL, which is very close to the work of Winkler and Zimmermann, the results of the Netherland Railway axle formula remained unsatisfactory for the following reasons [6].

1. The influences of one axle load on the actions at a second point are not equal to those of a second load on the first point, since the two loads themselves are unequal.

2. Not considering the deflection of the cross-tie, led to assumptions which were significantly inconsistent with reality.

3. The effect of the neighboring loads on the moment appeared small as compared to the results of actual measurements.

After the war, in Germany again, in the meeting of the

Technical Committee in Heidelberg, 1922, it was decided to further pursue the problem. It was determined that one had to regard the unified track super-structure calculations through experiments; that is, one could not approach the problem of track super-structure calculation with theory alone. Stress measurements were carried out by the Netherlands Railroad and the German Railroad Administrations of Dresden, jointly. The results were reported in September 1930 [6]. The experimental results were not a success, but they indicated in which direction the research had to continue. The following points were recognized [6]:

1. The stresses of equal axle loads in different positions diverge essentially from one another and it is to be taken as certain that the basic reason for this divergence is to be found in the reciprocal influence of the neighboring axle loads.

2. The reciprocal influences of the neighboring loads are much greater than that which had been calculated by all the previous methods.

3. The influence of various cross-tie distances on the track stresses is significant. A variation of cross-tie spacing changes considerably the profile of the reciprocal influence of the neighboring axle loads.

According to the resolution of the Technical Committee of September 1930, it was decided to investigate the determination of the "modulus of foundation". The work was carried out by a joint-committee of the railroad of the Netherlands, the National Railroad of Switzerland , the German Central Train Office (Reichsbahn) of Munich, the German Train Administration of

Karlsruhe and the German Central Train Office of Berlin, known collectively as the United Middle European Train Administration. For the standardization of their experiments, they agreed on a vehicle, which had been designed by the Netherlands Railroad, and the track elements were standardized as well. Through experiments, in which the investigators set up extensive measurements for the investigation of modulus of foundation, "k1", it was hoped to attain a certain basis for the track-superstructure calculations. In these hopes they were disappointed, since the measured value of k1 oscillated between wide limits; namely, between 5 and 40 kgs/cm³ (181 - 1445 lb/in³) . Indeed, the k1 value in even apparantly similar ballast and subgrade combinations proved to be very different. Thus, it did not seem possible , according to these measurements, to assign a definite modulus of foundation for a given track section. The United Middle European Train Administration recommended, therefore, that the so-called axle- position formula, be used. This formula completely disregards the elastic deflection of the track and in regard to the stress distribution, rests upon fully arbitrary assumptions. With this decision, however, as Schramm states [11] "the baby was thrown out with the bath-water" . Normally, during experimentation , when one gets different results for similar cases, the experimenter should set a range of valid values rather than switching to another theory , which from the beginning neglects accurate assumptions. Measurements, in the meantime, have proved that the axle-position formula does not accurately attain the true stresses, especially in regard to the influences of the neighboring axles.

The German Train Administration (Reichsbahn) at this time did not introduce the axle-position formula, but instead introduced the calculation procedure of Jaehn, which can be regarded as a simplified calculation of Zimmermann [14], [11], [12], [15]. Jaehn's procedure satisfactorily agreed with the latest measurement results at that time [11], [14].

During this period, the well-known structural engineer, Stephen Timoshenko, had already published in Zurich his paper on the subject [7]. In his work, he followed Zimmermann's analysis, indicating a remarkable agreement between the calculations and the experimental results. Timoshenko's experiment was really more of an inverse problem, i.e. the calculating of the vertical forces produced by locomotive wheels on the rail provided that, either the deflections or the stresses in the rail have been determined by experiment. The difference between the sum of the calculated forces and the actual weight of the locomotive used, never exceeded 8 percent [7]. Timoshenko had also been the first to extend the simplified theory of Zimmermann for the Russian railroad system in 1915 [5]. Zimmermann's analysis was also used on the Polish railroad system by A. Wasiutynski [5].

In the United States, Timoshenko continued his research on the subject in a study made by the engineers of the Westinghouse Electric and Manufacturing Company. He published a major paper , co-authored by B.F. Langer in 1932 [8]. A few years earlier in 1913, the lack of knowledge concerning the track structure had been recognized in the United States. As a result, a joint committee under the auspices of the "American Railroad Engineering Association", "American Society of Civil Engineers", and the "Association of American Railroads", was formed to study the action of the track as one unit and the action of each of its component elements individually [9]. The Committee carried out their research under the leadership of Professor A. N. Talbot. The Committee reported its study in seven progress reports, the first of which was published in 1918 and the last in 1941 [30] . The study of the special committee of A.R.E.A., A.S.C.E., A.R.A., however, added almost nothing to the theory of Winkler and Zimmermann, but it was successful especially in obtaining the most stable values for the foundation constant, k, for different cases (1st and 6th Progress Reports[9],). Up to the present time, the report of the said joint Committee has been a valuable design manual for the American Railways.

Meanwhile, in Europe in 1935, Robert Hanker tried to solve the problem of the simulation of the rail to an infinite beam on continuous elastic support, since the distance between each two ties was believed to violate the continuity assumption. In order to solve this difficulty, Hanker imagined, for the sake of calculation, a turning around of the cross-ties to form continuous longitudinal ties. Hanker's analysis is worthwhile to be refered to as a "Design Method". He titled his publication, "Uniform Longitudinal Calculation of the Railway Track Superstructure "[31] . His work also appeared in two other publications [6], [10].

A consequence of the information in the review so far presented, is that there are only three methods for track super-structure stress calculation ; one by Zimmermann, one by Jaehn, and a third by Hanker. The method which was introduced by Jaehn, is currently being used in parts of Europe (especially in Germany) [14], [11]. This method was introduced later to the Middle East as well [12]. The other method, which could be referred to as a "more direct application" of Zimmermann's analysis, is currently being used in North America [9] and parts of Europe. In the next sections , a presentation of both Jaehn's and the Joint Committee's methods is made, and then later, Chapter 7 will deal in greater depth with technical comparisons between the two methods.

2. The Method of The Joint Committee (A.R.E.A.,A.S.C.E.,and A.A.R.)

As was mentioned earlier in this chapter, the method of the A.R.E.A., A.S.C.E., and A.A.R. jointly , which is being used in some parts of Europe as well, is no more than Zimmermann's method, with straight-forward simplification to make it practical for the railroad engineer.

The special committee on stresses in railroad track of the A.R.E.A., A.S.C.E., and A.R.A., had given, as results of experiments, values for the foundation constant, k, for various types and depths of ballast. The values of k and the corresponding description of ties, roadbed section, and rails are given in Table (3-1) below [9]:

Table (3-1) Values of k from
the First and the Sixth Progress Reports of the Special
Committee on Stresses in Railroad Track

Rail	Ties	Track and Ballast	k (lb./in²)
(1) 85 lb	7"x9"x8'-6" spaced 22"c. to c.	6" Fine cinder ballast, in poor condition on loam and clay subgrade.	530
(2) 85 lb	7"x9"x8'-6" spaced 22"c. to c.	6" Cinder ballast, in fair condition on loam and clay subgrade.	750
(3) 85 lb	6"x8"x8'-0" spaced 22"c. to c.	6" Limestone on loam and clay roadbed. Good before tamping.	970
(4) 85 lb	6"x8"x8'-0" spaced 22"c. to c.	6" Limestone on loam and clay roadbed. After tamping.	1080
(5) 85 lb	7"x9"x8'-0"	12" Limestone on loam and clay roadbed. Good before tamping.	1065
(6) 85 lb	7"x9"x8'-0"	12" Limestone on loam and clay roadbed. After tamping.	1090
(7) 85 lb	7"x9"x8'-6" spaced 22" c. to c.	24" Crushed limestone on loam and clay.	1200
(8) 130 lb	7"x9"x8'-6" spaced 22" c. to c.	24" Gravel Ballast plus 8" of heavy limestone on well-compacted roadbed.	2900-3000
(9) 110 lb RE	7"x9"x8'-0" spaced 22" c. to c. G.E.O. fastenings	Flint gravel ballast on wide, stable roadbed.	2500,2600 3600 Ave. 2900
(10) 110 lb RE	7"x9"x8'-0" spaced 22" c. G.E.O. fastenings	Limestone ballast on wide, stable roadbed.	3700,5500, 6200 Ave. 5100

Assuming that the rail is an infinitely long beam on continuous elastic support of foundation constant k, and using the same notations which were used in Chapter 2, p, y, and M are the upward pressure per unit of rail length, the deflection of the rail, and the bending moment at point of wheel load where x=0 respectively, and P is the applied wheel load. The distance from the wheel load to the first point of zero upward bending is taken as (x1), and the distance from the wheel load to the first point of zero upward pressure (zero deflection as well) is taken as (x2). Fig. (3-2) shows the above coordinates plotted on typical deflection and moment curves extracted from Fig. (2-2) in the second chapter of this work.

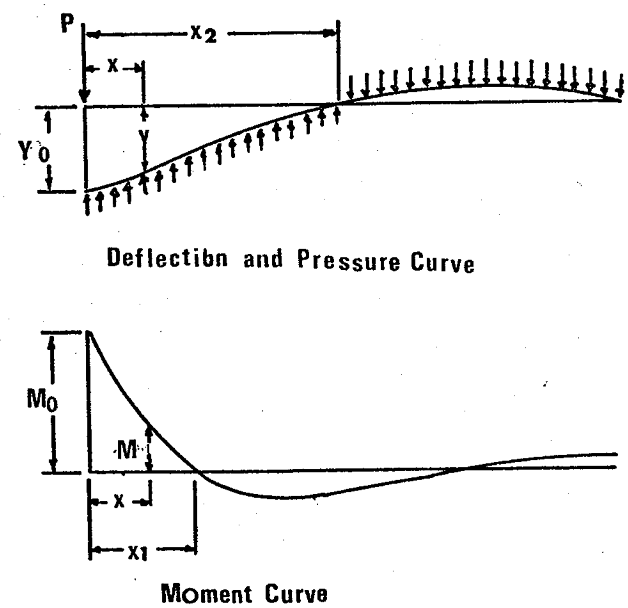


Fig. (3-2)

Single-wheel load deflections

and bending distribution

The derivation of the formulas used in the "Joint Committee's Method" are based on those shown in Chapter 2 (Equations 2-11 and 2-13), namely,

$$y = P \cdot v \cdot B1/2k$$

and $$M = P \cdot B3/4v \; .$$

These are the two equations representing the two curves in Fig. (3-2) above.

It was shown that the zero points for B1 are located at $vx = (3/4 + n)\pi$, and for B3 at $vx = (1/4 + n)\pi$.

Therefore, the distance from the wheel load to the first point of zero moment, (x1), can be expressed as

$$(x1) = \pi/4v$$

where

$$v = \sqrt[4]{k/4EI}$$

or

$$(x1) = (\pi/4) \cdot \sqrt[4]{4EI/k} \quad \ldots\ldots (3-1)$$

and the distance from the wheel load to the first point of zero upward pressure (zero deflection), (x2), can be expressed as

$$(x2) = (3/4 + 0)\pi /v$$

or

$$= (3\pi /4) \cdot \sqrt[4]{4EI/k}$$

From Equation (3-1) above, therefore

$$(x2) = 3(x1) \quad \ldots\ldots (3-2)$$

From Equation (2-18), the maximum bending moment at the point of wheel load (where x = 0), Mo, was expressed as

$$Mo = P/4v \qquad \text{or}$$

$$Mo = P \sqrt[4]{EI/64k} \qquad \text{.....(3-3)}$$

Substituting from Equation (3-1) into Equation (3-3) therefore,

$$Mo = 0.318P \;.\; (x1) \qquad \text{.....(3-4)}$$

From Equation (2-16), the maximum deflection at the point of wheel load (where $x = 0$), yo, was expressed as

$$yo = P \;.\; v/2k \qquad \text{or,}$$

$$yo = P \Big/ \sqrt[4]{64\ EI \;.\; k^3} \qquad \text{.....(3-5)}$$

Substituting from Equation (3-1) into Equation (3-5) therefore,

$$yo = 0.393P/(k \;.\; x1) \qquad \text{.....(3-6)}$$

Going back to the basic assumption, on which the theory of beam on elastic foundation was built, i.e.,

$$p = k \;.\; y$$

where p is the upward reaction per unit length of the beam. It is apparent that the upward reaction per unit of rail length at the point of wheel load (where $x = 0$), po, can be expressed as

$$po = k \;.\; yo \qquad \text{.....(3-7)}$$

where yo can be determined from Equation (3-6) above.

The said special committee thereafter plotted Equations (2-11) and (2-13) accurately as the so-called "Master Diagram", [9], as shown in Fig. (3-3) below. Recalling these two equations to plot the master diagram ;

Equation (2-11) $\quad y = P \;.\; v \;.\; B1/2k \quad$ and,

Equation (2-13) $\quad M = P \;.\; B3/4v$

considering the maximum values as unity, i.e.:

$$yo = P \cdot v/2k = 1 \qquad \text{and,}$$

$$Mo = P/4v = 1 .$$

In other words, the plotted equations were

$$y = 1\ B1 \quad \text{and} \quad M = 1\ B3.$$

The peaks of the curves (ordinate direction) were therefore, assigned a value of unity, and the ordinate consequently was as a fraction of unity. Meanwhile, the unit of scaling on the horizontal axis, was taken as the distance ($\pi/4$), called (x1), from the ordinate, as shown in Fig. (3-3) below.

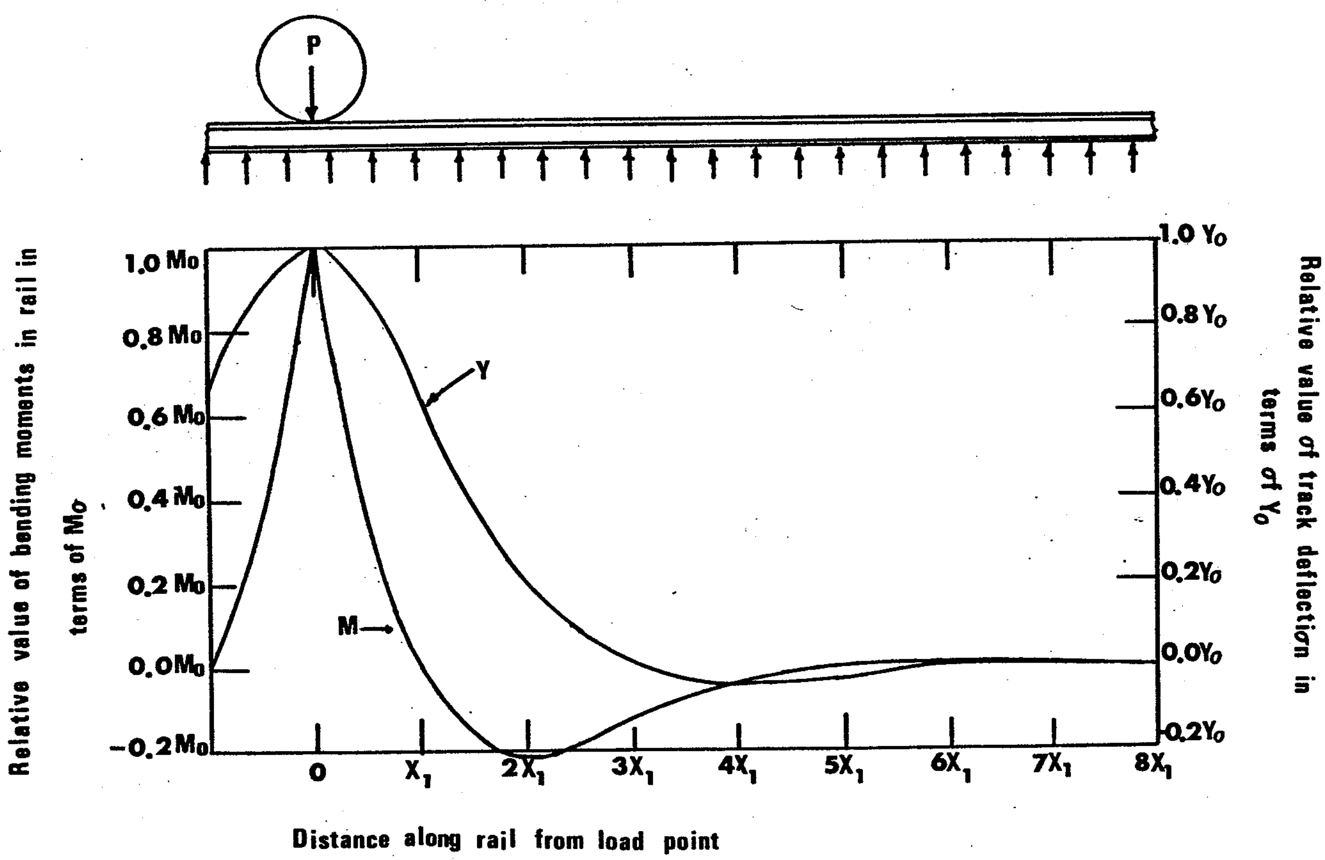


Fig. (3-3)
Master diagram for moments, and rail deflection under a single-wheel load

Using the Fig. (3-3) and Equations (3-1), (3-2), (3-4), (3-6), and (3-7), the calculation procedure for bending moment of two neighboring wheels can be summarized in the following steps:

(1) Determine k value from Table (3-1) above, using the characteristics of the rail, ties, ballast, and roadbed.

(2) Assuming the two wheel loads are P1 and P2 as shown in Fig. (3-4) and the bending moment is to be determined under the larger load P1, determine the value of (x1) from Equation (3-1).

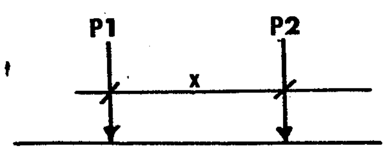


Fig. (3-4)

Two neighboring wheel-loads.

(3) Determine the maximum bending moment under each of the wheel loads, Mo1 and Mo2, using Equations (3-4), once for each load.

(4) The bending moment under the wheel load P1 will be = Mo1 + the bending effect of P2 under the load P1. Assuming that the distance from P2 to P1 is x, calculate the ratio of x/(x1).

(5) From Fig. (3-3) above, enter for the abscissa value of (x/(x1)) and determine the corresponding ordinate representing the moment ratio.

(6) The bending moment under P1 = Mo1 + the moment ratio x Mo2

3. Jaehn's Method of Analysis

Jaehn's method is an empirical relation [15], which is also based on Zimmermann's analysis [11], [12], [14]. The method is generally simpler than the "Joint Committee's Method" in the amount of calculations involved and in the procedure itself. The method proved to be sufficiently accurate for standard track gauges of 1435 mm (4.707 ft) and under the following conditions[11],[12] :

1. Rail weights exceeding 30 kg/m (20.16 lb/ft)
2. Tie spacing of between 60 and 80 cm (23.6 and 31.5 in)
3. Tie bearing areas of 4000 to 6000 cm^2 (620 to 930 in^2)
4. Modulus of foundation k1, of 10 to 20kg/cm^3 (361 to 722 lb/in^3)

Jaehn's method, when dealing with the effect of the adjacent loads under the considered wheel load, assumes that the two adjacent loads are equal to the considered one. This assumption was found not to cause any perceptible inaccuracy in actual results, as long as the differences between the adjacent loads and the considered one are within 20 per cent of the considered one [12]. The method also disregards the effect of any wheel loads except the two adjacent loads from both sides of the considered one, which is often the case in practice.

The formula for the bending moment is

$$M = H \cdot P \cdot a \qquad \ldots\ldots (3\text{-}8)$$

where

M is the bending moment under the middle load P

H is the coefficient of the wheel base

P is the middle wheel load considered

a is the tie spacing

The coefficient of the wheel base depends on the mean wheel base as follows, Fig. (3-5):

$$Cm = (Ca + Cb)/2 \qquad .....(3\text{-}9)$$

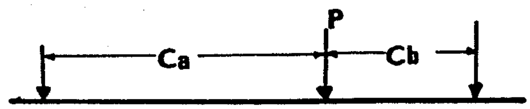


Fig. (3-5)

Wheel-loads and load distances.

The method further requires that, when applying Equation (3-9) above, if either Ca or Cb or both is greater than 280 centimetres (110.24 in), then the value of 280 is to be imposed in the equation (instead of the greater value). This implied condition holds always, even if one of the adjacent wheels is in an infinite distance. For example, in the case of an outer load (first or last in the train), two adjacent loads will be considered, one of which is in a distance of 280 centimetres, instead of infinity. From the above implied condition, the mean distance, Cm, will never be greater than 280 centimetres.

Having determined the mean distance, Cm, the coefficient of the wheel base, H, can be determined by one of the following three equations:

1. For Cm greater than or equal to 140 centimetres.

$$H = 0.057 + (Cm/1200) \qquad .....(3\text{-}10)$$

2. For Cm smaller than 140 centimetres and greater than or

equal to 112 centimetres.

$$H = 0.174 \qquad .....(3-11)$$

3. For Cm smaller than 112 centimetres and greater than or equal to 62 centimetres.

$$H = 0.434 - (Cm/430) \qquad .....(3-12)$$

Cm versus k represented by the three equations above is plotted in Fig. (3-6) below [12], [14].

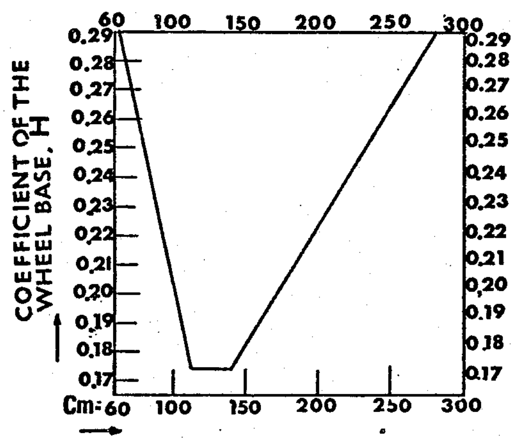


Fig. (3-6)
Coefficient of wheel base, H dependent on the mean wheel base, Cm.

An easier graphical representation, however, which is plotted in Fig. (3-7) below [11], relates Ca (abscissa) and Cb (ordinates) to H (diagonal) , directly without the use of Equation (3-9) to obtain the mean distance.

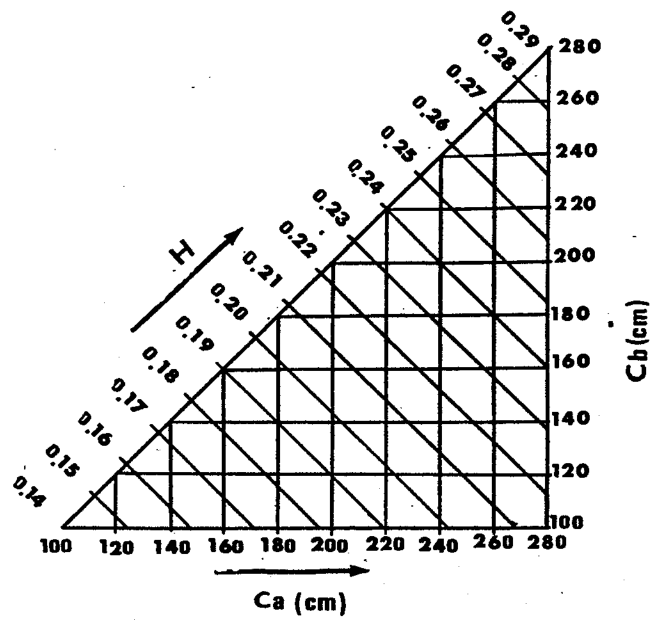


Fig. (3-7)

Coefficient of wheel base, H for given load distances, Ca & Cb.

In order to use this Figure, extend Ca and Cb by a vertical and a horizontal lines. From the point of intersection of these lines , take a diagonal to read H.

CHAPTER 4: SPEED FACTORS AND DYNAMIC

EFFECTS ON BENDING MOMENTS

The numerous experimental results taken since the investigations of railroad track stresses began, have shown that in general the stresses of the track grow larger with increasing speed. It was realized, therefore, that the dynamic deflection of the rail and the dynamic stresses under the action of the moving wheels of a vehicle differ from those calculated on the basis of the static formulas discussed in the previous chapter.

The most important of the various causes which may produce such an increase in deflection and stress are the following ones [7], [8].

(1) Variation in the forces acting on the rail caused by variable spring forces on the wheel.

(2) Vibration of the rails under moving loads.

(3) Different kinds of irregularities in the shape of the wheel or rail, such as flat spots on the rim, low spots on the rail, and discontinuities at the rail joints.

The problem was approached sometimes by establishing theoretical oriented analysis and applying experimental results to it thereafter, as in the work of S. Timoshenko and B. Langer [7], [8]; (Timoshenko established an equation for the dynamic deflection and concluded that the difference between its use and the use of the static one will always be less than 1/2 percent [7]). The other approach to the problem is the attempt to relate the dynamic influence in the stress to the speed linearly, (The Joint Committee of A.R.E.A., A.S.C.E., A.R.A. concluded experimentally that the stress increases 0.75 percent for each mile per hour increase of speed). A third approach which proved to be the most successful one, was a trial to relate empirically the increase in stresses to some speed functions of quadratic or cubic power.

1. Speed Factor, U

The speed factor introduced in the following section, is to be multiplied by the bending moment value obtained by either method in the previous chapter. The first formula was established by E. Winkler and H. Pihera [15] on the basis of theoretical considerations. The formula is:

$$U = 1/(1 - v^2/35500) \qquad \text{.....(4-1)}$$

where

U is the speed factor

and

v is the speed in km/hr

This formula is not valid, however, except for speeds less than 188 km/hr (117 mile/hr). Reaching 188 km/hr., the speed factor will increase to infinity, and then will be reversed thereafter. The German Central Train Office in Berlin, has from a great number of observations calculated the following mean values for U,[6]:

V in km/hr.	0	45	80
U, according to experiments of the German Train Administration, Dresden	1.00	1.13	1.34
U, according to the experiments of the Netherland Railway	1.00	1.05	1.20

According to the above speed factors, which were calculated from stress measurements, Formula (4-1) was adopted [10] later on as

$$U = 1/[1 - (0.00000007.P.a.v^2 /I)] \quad .....(4-2)$$

where

P is the wheel load in kg,

a is the distance c. to c. between ties in cm

V is the speed in km/hr

and

I is the moment of inertia of the rail

The Formula (4-2) still produced the same problem which was raised in the previous one. For medium tie distance, a, and common moment of inertia of rail, I, at speed V of about 200 km/hr (124 mile/hr), the speed factor, U is expected to reach infinity.

Such speed factors, stand in opposition with the experience, since speeds of 200 km/hr can occur without damage for the track, and without special increase in strength of the rail [6]. Experimental trips by the German Train Administration were carried out numerous times above this speed without excessive stresses, rail breaks, or similar things being noticed.

The Union of the Middle European Train Administrations introduced thereafter, attempts for the clarification of the range of U on an international scale [15]. A total of over 21,000 stress measurements were taken, in order to find out the speed factor, in the years 1930 to 1935 inclusively. It was only possible at that time, to carry out the measurements with low speeds. The stresses were measured at speeds of 5, 45, 90, and 100 km/hr (62 mile/hr), and the values for the higher speeds were extrapolated.

In 1936, the United Middle European Train Administrations, on the basis of the results for the above-mentioned experiments, made known a new formula for the calculation of the speed factor, which is

$$U = 1 + (v^2 /30000) \qquad .....(4-3)$$

The above equation was found very reasonable for speeds up to 100 km/hr., which is the range in which the speed grows quadratically. For higher speeds, however, this equation is no longer correct. For instance, if this equation were to be applied to a speed of 330 km/hr (205 mile/hr), which had been reached during test runs on ordinary tracks in France[1], U =

---

[1]Such a speed has been exceeded in parts of the "Tokaido Line" in Japan.

4.63 would result. This means that the increase of bending moment would be 4.63 times as much as static bending moment, which is incorrect as had been shown experimentally. Apparently, U, cannot be an ever-increasing quadratic function of speed [14].

In 1943, G. Schramm proposed a formula which eliminated the above defect, and which had real correspondence to actual facts [14]. Schramm's formula is :

$$U = 1 + (4.5V^2 /100,000) - (1.5V^3 /10,000,000) \quad .....(4-4)$$

The formula above approaches a maximum value of U = 1.6 at speed V = 200km/hr (124 mile/hr).

Since the establishment of Schramm's formula, it was used by the German Republic Railroad (DB rail) and still is in use at the present time [11], [14], [15]. This formula is also in use in England [17], and it was recently introduced to some railways in the Middle East as well [12].

A discussion of the practical value and significance of these formulas for speed factor is given later.

2. The Influence of the Wheel's Flat Spots

The shock effect of unrounded wheels on the track have been investigated under empirical considerations in the 1950's. The height of the flat spot and the speed were believed to be governing in this criterion [11]. In a maintenance statistical study which took place in West Germany over a period of seven months, out of 10,978 wheel rims investigated, 171 (or 1.56 percent) were discovered to have flat spots. Table (4-1) below shows the percentage of each height.

f = 0 to 1 mm	64 percent
f over 1 to 2 mm	23 percent
f over 2 to 3 mm	7 percent
f over 3 to 4 mm	2 percent
f over 4 mm	4 percent

Table (4-1)

In two cases, f was found = 8mm, and once even reached 31 mm.

In 1952 and 1953, Popp [32] and Rubin [33] of Germany carried out detailed theoretical investigations, which were approved as being correct through measurement by both the German and the American Railways [34] & [11]. The investigations mentioned above, proved that through a flat spot with height f, additional bending moment, Mf, which increases as a function of f, will be exerted. Furthermore, the said additional bending moment was found, to a certain extent, dependent on the springs of the vehicle, the wheel loads, and the speed V. The dependence on the speed V, was found to be very important, and the type of dependence was different for different springs and various heights of spots, f. On the average, Fig. (4-1) below, was found reliable for the additional bending moment's, Mf, dependence on speed [11].

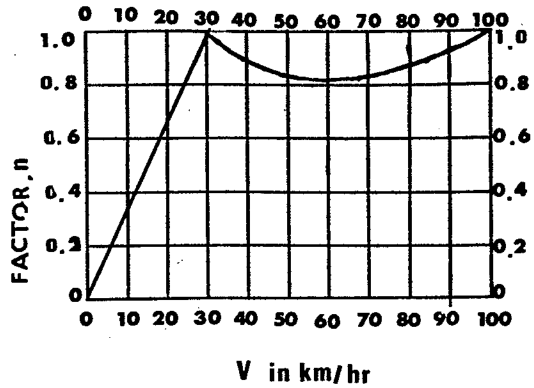


Fig. (4-1)

Influence of speed on the bending moments

as a result of wheels with flat spots.

Fig. (4-1) above gives a factor, n, (smaller than or equal to 1), versus speed. The additional bending moment, Mf, will increase with the increase of the factor, n, as will be explained.

The relation of the height of the flat spot, f, and the wheel load, P, to the additional bending moment, Mf, is shown in Fig. (4-2) below. After obtaining the additional bending moment from Fig. (4-2) below, the value is to be multiplied by the value of n, obtained from Fig. (4-1) above.

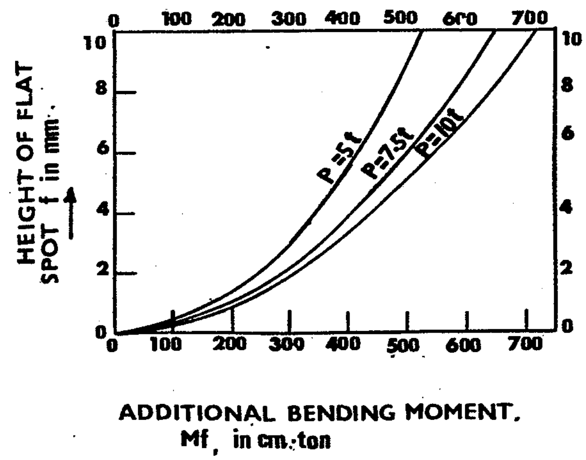


Fig. (4-2)

Additional bending moment as a result of wheels with flat spots.

In other words, in order to estimate the additional bending moment, one first enters Fig. (4-1) by the speed V, and determines the factor n, and then enters Fig. (4-2) with the flat spot height and the wheel load to obtain the additional bending moment, Mf , which must be multiplied by the factor n.

3. Inclusive Formula

According to the discussed dynamic effects in the two sections above, the total bending moment, Mt, can be determined from the following formula:

$$Mt = M \cdot U + n \cdot a \cdot Mf \cdot U \quad .....(4-5)$$

where

- Mt is the total bending moment exerted on the rail, including all the dynamic influences.

- M is the static bending moment as determined from Chapter 3.
- U is the speed factor as determined from equation (4-4).
- n is an additional moment factor as determined from Fig. (4-1).
- a is the distance c. to c. of the tie.
- M_f is the additional bending moment due to flat spots as determined by Fig. (4-2).

CHAPTER 5: ANALYSIS OF STRESSES

IN RAILWAY TRACK ELEMENTS

1. Stresses in Rail

Combining the methods demonstrated in Chapter 3 and Chapter 4 of this work, the maximum bending moment induced dynamically by vertical moving wheel loads can be estimated.

The maximum stress induced in the rail due to the above mentioned bending moments will occur at points of the rail cross-section farthest from the neutral axis. This stress is simply given by the formula:

$$S = M/Z \qquad .....(5-1)$$

where

S is the maximum bending stress in the rail (kg/cm^2 or lb/in^2)

M is the maximum bending moment induced by vertical moving loads (kg.cm or lb.in)

Z is the section modulus of the rail (cm^3 or in^3)

The rail section modulus is usually listed for the different rail sections used in tables of steel design manuals and the standard specifications. It is important sometimes, however, to calculate the section modulus, Z, in cases when a check of stress is to be done for a rail which had been worn, or a rail for which the value, Z, cannot be obtained from tables for any other reason. For simple and fairly accurate calculations, some empirical relations between the section modulus, the height of rail, and the weight of rail (in metric units) exists [14], [12].

The first relation assumes that the weight of rail and its height are both known,

$$Z = h.W/30.5 \qquad \ldots\ldots (5-2)$$

where

h is the rail height in mm.

W is the rail weight in kg/m.

Z is in cm^3

If only the height is known, we can use the formula

$$Z = 5.2h - 533 \qquad \ldots\ldots (5-3)$$

In the opposite case when the weight is known, the relation is

$$Z = (W^2 /35) + 3.6W \qquad \ldots\ldots (5-4)$$

A relation, therefore, exists between the weight and the height of rail

$$W = 156 - (16000/h) \qquad \ldots\ldots (5-5)$$

In case a check of stress is to be done for a rail which is worn, say the height of wear is dh mm, the formula to be used in order to determine the section modulus of the worn section, Z1 cm^3, is

$$Z1 = Z - (dh/30) \ . \ (W + 0.53(h- dh)) \qquad \ldots\ldots (5-6)$$

Z and W in the above relation represents the original section modulus and weight of the new rail, which can be obtained either from the tables or by using Equations (5-3) and (5-5) above.

Equation (5-1) for calculating the stress in the rail, if to be used to check whether a vehicle is inducing stresses within allowable limits in an existing track, the following permissible stresses are to be considered [11], [12], [14],

1500 kg/cm² or 21335 lb/in² for all through mainlines which sustain high speeds and carry heavy loads, and to a lesser extent, other very heavily loaded tracks, like those at humps of large marshalling yards.

1600 kg/cm² or 22757 lb/in² for tracks including through branch lines (through running lines and passing loops etc.) and with heavy or medium-heavy rails.

1800 kg/cm² or 25602 lb/in² for tracks from the above category but with lighter rails

2000 kg/cm² or 28447 lb/in² industrial tracks and all other tracks which are not included in the above category, for example, shunting, loadings storage sidings, and catch sidings.

While the above permissible stresses are used to check for an existing track, the criterion is a little bit different whenever the selection of a rail section is concerned. An initial guide always can be obtained by the empirical relation [14],

$$W = 4P \qquad \text{.....(5-7)}$$

where

W is the weight of the required rail in kg/m

P is the wheel load in ton (1000 kg)

Another more accurate calculation is possible by the use of Equation (5-1) in the form

$$Z = M/S$$

The value of M here is to be evaluated from either the method of the American Joint Committee (A.R.E.A.,A.S.C.E.,and A.A.R.) or the method of Jaehn mentioned in Chapter 3 of this work, multiplied by the speed coefficient U of the previous chapter. It is good practice, however, to use a design factor of safety, i.e. through consideration of the future wear of rail. Such a factor of safety was suggested [14] by using 0.8 times the permissible stresses only, and by neglecting the effect of the neighboring wheels which decrease the bending moment. In other words, when using the A.R.E.A.,A.S.C.E.,and A.A.R. method, simply use the Mo value, Equation (3-3), for the heaviest wheel. Meanwhile, when using Jaehn's method, consider both Ca and Cb are equal to 280 cm. in Equation (3-9).

Having determined the section modulus, Z, the weight and the height of the required section can be determined by the empirical relations given above in this chapter.

Another empirical relation was believed to be satisfactory [14] to give the weight of the rail directly:

$$W = 156 - (106000/(2P \cdot U + 67)) \quad \ldots\ldots(5\text{-}8)$$

where

U is the speed coefficient

P is the wheel load in ton (1000 kg)

W is the rail weight in kg/m

2. Stresses in the Tie

The function of the ties, as members in the railroad track structure is:

- To maintain the track gauge.
- To work as an elastic media between the rail and the ballast, which absorbs shocks and vibrations.
- To distribute the load on a larger area of the ballast.

When studying the stresses in ties[1], serious attention should be given especially to the tie length, which should be determined according to a suitable load distribution from the tie on the ballast, which allows the track gauge to remain always constant. Inadequate tie length can lead to serious "smooth-running" problems in addition to severe stresses in the tie. If a tie is too small, the track gauge will elongate and cause upward deflection on the tie [Fig. (5-1-a)]. Meanwhile, a tie which is too long will shorten the track gauge with a downward deflection tendency [Fig. (5-1-b)]. For these reasons, there is a zone at the middle of the tie under which the ballast has to be left without compaction so that no upward pressure would result in this zone. This uncompacted length is mainly a trial to make the resultant of the upward pressure from the ballast on the bottom of the tie coincide with the center line of the rail as shown in Fig. (5-1-c). This matter will protect against any

---

(1) This study will deal only with conventional cross-ties (wooden or concrete as used in Canada).

bending in the tie, which can cause a change in the gauge length.

This uncompacted length, g, as shown in Fig. (5-1-c) can be expressed as;

$$g = L - 2(L-t)$$

or

$$= 2t - L \qquad \text{.....(5-9)}$$

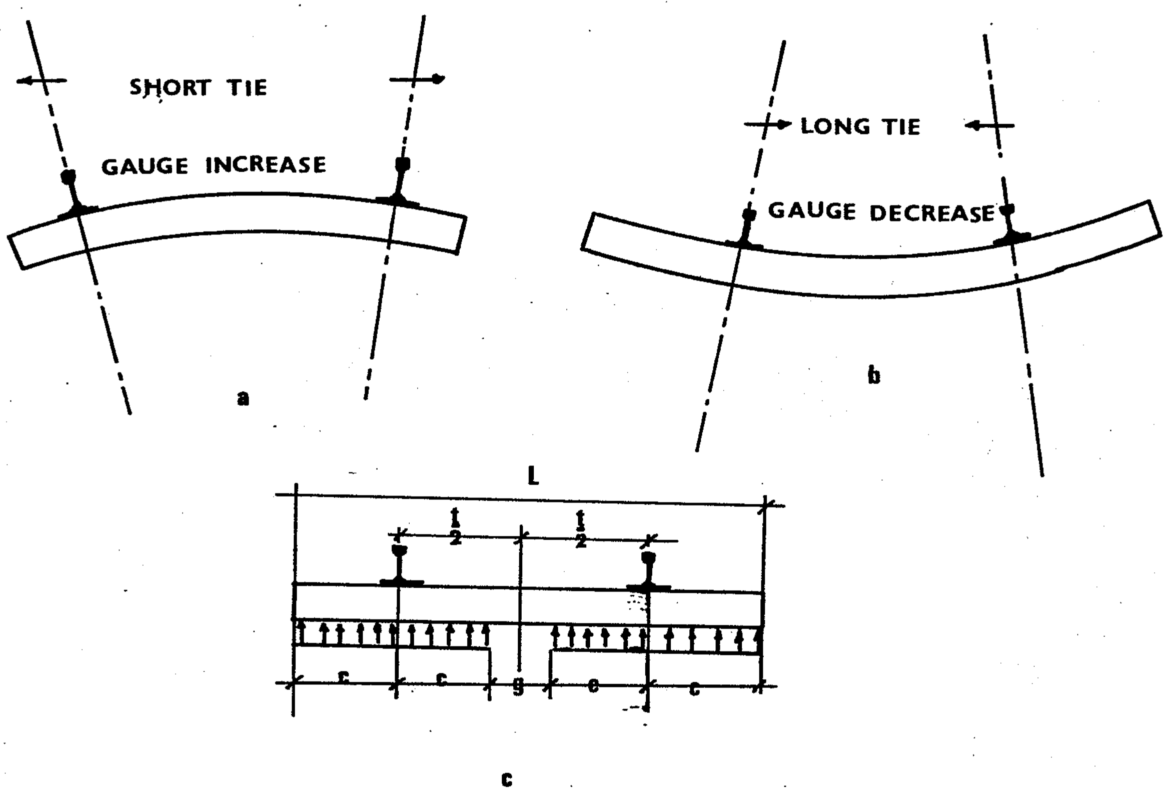


Fig. (5-1)

Effect of stresses on track gauge, in case of a tie which is either too short or too long.

where

t is the distance between the center lines of the two rails

L is the length of the tie

The ratio (q/t) was investigated experimentally and found, most suitable as "0.27", [12], [14].

using Equation (5-9), therefore,

$$(q/t) = 2 - (L/t)$$

from which we can reach

$$L = (2 - q/t) \cdot t$$

or

$$L = 1.73\ t \qquad \text{.....(5-10)}$$

The above is the equation for the most suitable tie length with respect to a given distance between center-lines of rails. In addition, the uncompacted length, q, can then be obtained as (0.27) times the distance between the center-lines of the rails.

If we assume now that the wheel load is = P, then the force which the rail will effect upon the tie, F, will be only a fraction of P, i.e.

$$F = T \cdot P \qquad \text{.....(5-11)}$$

where

T is a factor dependent upon

-Modulus of elasticity of materials of both the rail and the tie.

-Moment of inertia of both the rail and the tie

-Distance center to center of the ties

-Modulus of foundation, k1

-Speed of the passing trains

The factor, T, was possible to be calculated theoretically and found to be between .5 and .7 [12], [14], but according to experiments[1] which were done in England in 1942, it was found that after omitting some divergent results, the value of T lies between 0.4 and 0.8; therefore Equation (5-11) can be used as

$$F = .8P \qquad .....(5-12)$$

The maximum bending moment resulting in a tie , will be according to Fig. (5-2) as follows:

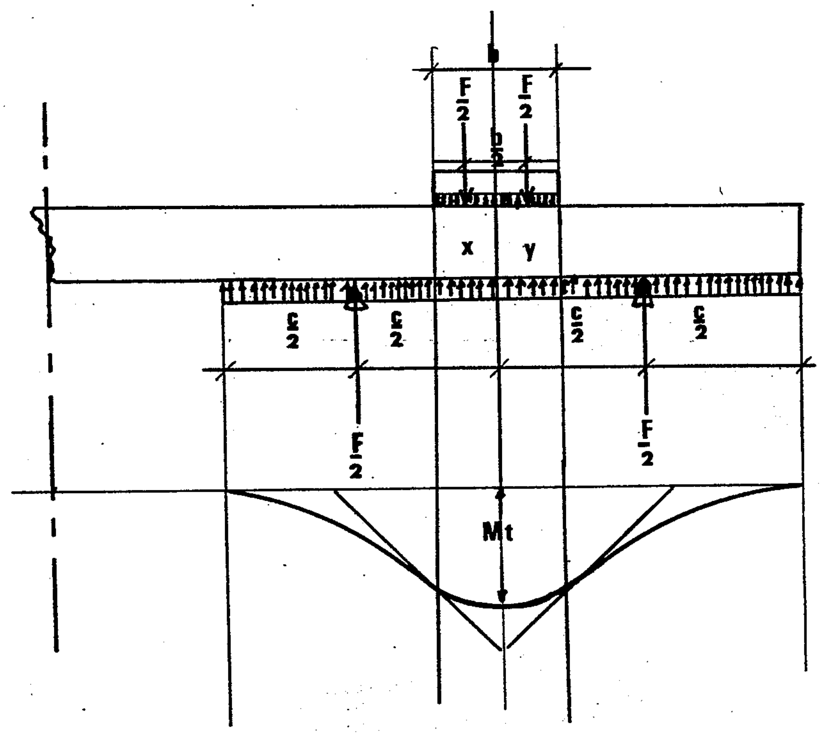


Fig. (5-2)

Load distribution and bending moment diagram of a loaded tie.

$$Mt = F/2 \;.\; c/2 - F/2 \;.\; b/4$$

or

---

(1) These experiments were done under moving loads, therefore the value of the factor T, resulting from the experiments, include the dynamic factor effect as well.

$$Mt = F/2\ (c/2 - b/4)$$

substituting for c from Fig. (5-1-c)

$$Mt = F/8(L-t-b) \qquad \text{.....}(5-13)$$

where

Mt is the maximum bending moment in the tie

b is the width of the rail base

Having determined the maximum bending moment in the tie, the maximum bending stress St, can therefore be determined by using the formula

$$St = Mt/Zt$$

where

Zt is the modulus of the tie section, which can be calculated as

$$Zt = L3 \cdot L2^2 / 6$$

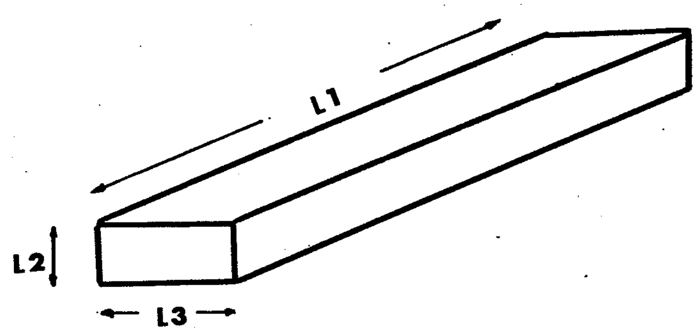


Fig. (5-3)

Tie dimensions.

The width b, in Equation (5-13) above, can be increased if safer stresses are needed by adding a wider base plate underneath the rail base.

The assumption, that the tie is only loaded on the ballast over length 4c, excluding a length q at the middle, see Fig. (5-1), on which the foregoing bending moment and stress analysis were based, is only a practical approximation. So too, is the assumption that the pressure acting at a length 2c at each side of the tie as being uniformly distributed, a practical approximation. In reality, the distribution of ballast pressures under a tie is as shown in Fig. (5-4) below.

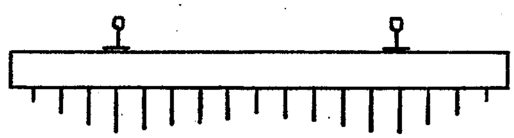


Fig. (5-4)

Distribution of ballast pressures under a tie.

Fig. (5-4) was indicated by Professor Talbot according to his experiments [9]. This distribution given by Talbot, is more realistic than the one assumed for the purpose of the mathematical analysis above and is more realistic than the classical assumptions of uniform pressure distribution over the entire length of the tie. However, the maximum bending moment obtained by Equation (5-13) above, would still be very close to the real case.

3. Pressure Distribution in the Ballast Section and on the Subgrade

The functions of the ballast are:

-Distributing tie loads with uniform pressure to subgrade

-Supporting and anchoring the track structure

-Providing drainage and reducing frost action

-Providing resilience for the track

-Facilitating maintenance operations

Of these functions, the first two, especially the first, are of the greatest importance. If ballast pressure is not fairly uniformly distributed on the subgrade, some ballast particles will penetrate into the subgrade causing the formation of pockets. In the field of railways, the uniformity of the support offered by the soil, is considered more important than the supporting value itself [13], [16].

As early as any other field of track stress research began, the ballast section was given a great deal of attention. The most important question to be answered was the depth of ballast needed to get a reasonably uniform pressure distribution on the subgrade at any particular tie spacing. Schubert in 1890, recommended, through a series of tests, that the depth of ballast equal the clear distance between the ties plus 8 inches. The Pennsylvania Railroad tests in 1912, using prototype size experiments, put forth a recommendation that the depth of ballast under the ties must not be less than the center to center tie spacing. Professor Talbot, with the American Joint Committee on Track, through their extensive studies and experiments, recommended a depth of ballast equal to tie spacing plus three to four additional inches. This is needed to get a reasonably uniform pressure on a subgrade. The joint committee also found that at depth of 6 inches below the center-line of a tie, the vertical pressure is 178 per cent of the average pressure over the tie.

In 1966, Salem [13], [16] analyzed the problem experimentally again, using much more developed pressure cells for his measurements and concluded that the depth of ballast section needed to get a fairly uniform pressure on the subgrade equals the tie spacing, minus three inches, and that the increasing of the depth more than this will not add very much more to the uniformity of pressure on the subgrade. Furthermore, he introduced two equations to determine the vertical pressure distribution below and to the right and left of the center-line of a tie. One of these two equations will be introduced in this section. Moreover, he recommended the use of a composite ballast section with the inferior and less expensive type being used as a sub-ballast, provided that the ballast used will not deteriorate in the long run. This recommendation was basically made after he realized that the three types of ballast which he used (chat, pit run gravel, and crushed slag), behaved in a similar manner as far as the magnitude of vertical pressures were concerned. One further important result of Salem's work was that the vertical pressure decreases rapidly with depth. It was found that the pressure at a depth of 6 inches is more than three times that at a depth of 18 inches and more than seven times that at a depth of 30 inches.

4. - A Mixed Theoretical, Empirical Relation for the Determination of the Transmission of the Pressure Through the Ballast using the Theory of Elasticity

Assumptions:

-The tie load is assumed to be uniformly distributed on the top of the ballast section.

-The ballast section is a perfectly homogeneous and elastic solid, which obeys Hook's Law.

It is known from the theory of elasticity that if a portion XY of the boundary of a semi-infinite mass is loaded with a strip of an intensity Pa per unit area, as shown in Fig. (5-5), the vertical pressure at point O is:

$$p = Pa/\pi \;\; (i + \sin i \; . \; \cos 2 w) \quad .....(5-14)$$

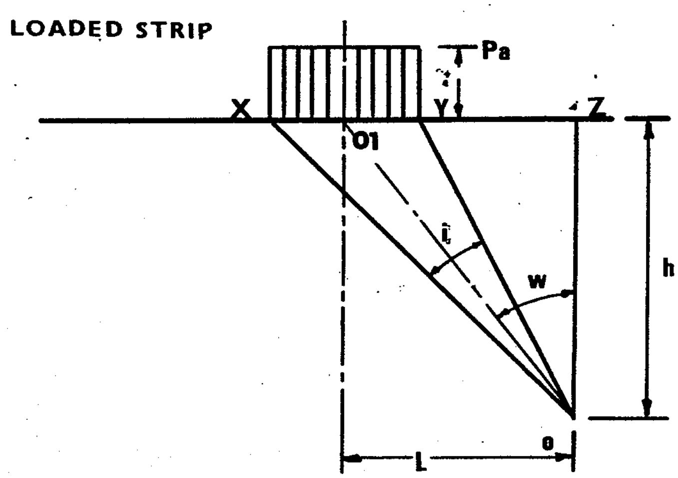


Fig. (5-5)

Transmission of the pressure through the ballast.

where

Pa = pressure per unit area on the top of the loaded mass

i = angle XOY

w = angle Z O 01

Because the assumptions on which the above analysis was based, are not perfectly true, a correction factor , CF, must be introduced to Equation (5-14) as follows:

$$p = CF \cdot Pa/\pi \; (i + \sin i \cdot \cos 2 w) \quad .....(5\text{-}15)$$

where

$$CF = p(\text{experimentally}) \;/\; p\;(\text{theoretically}),$$

in which p (experimentally) is the vertical pressure below the center-line of the tie determined by test, and p (theoretically) is the vertical pressure below the center-line of the tie determined by Equation number (5-14).

Salem [13], [16], after plotting the test results and the corresponding calculated values, concluded an average value of the correction factor, CF, as

$$CF = (48 - h)/22$$

Although it was noticed that the values of CF decrease with depth and at any particular depth the value of CF increases with tie load, the averaging effect of "CF" had a maximum error in the value of p, of only plus or minus ten per cent, if the general average value is to be used only when the depths ranging from 6 to 30 inches.

Equation (5-15), therefore becomes:

$$p = Pa.((48 - h)/22)(i + \sin i \cdot \cos 2 w)/\pi \quad .....(5\text{-}16)$$

The above equation, with the guidance of Fig. (5-5) is a very simplified procedure to estimate the pressure distribution anywhere in the ballast section or on the subgrade. The restriction of depth ranging 6 to 30 inches in that equation is not limiting any practical use of the equation, as will be discussed in a later chapter.

CHAPTER 6: COMMENTS ON EXPERIMENTAL APPROACHES

The field of railway track stresses involves a strong relationship between experimental stress analysis, and theoretical analysis. For every theoretical analysis which was done about this topic, some assumptions were really necessary to simplify the complicated track structure and the different properties of its materials. It was necessary, therefore, to prove that such assumptions are close to reality. Such a proof is only possible by experimentation. The next step was to compare the theoretical results with the experimental ones, and hence try to find reasons for any difference and introduce correction factors if possible.

In an area like the stress analysis of railway track, the research should be as complete as possible. One can never realize such an objective except by a proper method of the "making of experiments". Such an element is the theme of the following discussion.

The track stress experiments can always be classified as static or dynamic experiments. Each one of these can be either prototype size and circumstances or model size and circumstances.

Although the dynamic experiments are the most consistent with the reality of railroad track, the results are always impossible to interpret unless the stress criteria are to be divided into static stresses and dynamic influence. This often brought about the necessity of the static experiments. Other factors involved in dynamic versus static experiment choice, are the lower costs of static experiments, as compared to dynamic ones, and the lack of efficient instruments for dynamic experiments.

In prototype versus model experiments, the expenses and the size of a prototype experiment are tremendous. On the other hand, the simulation of the existing properties into a model could not always be complete and correct.

The arguments above may be understood from the following illustrations of some of the significant previous works of railroad track experimental techniques.

The action of a force on the rail can generally be expressed with an eccentric vertical load and a lateral load. By replacing the couples resulting from these loads around the centre of the rail base by two equal forces acting at the two points on the opposite edges of the base of the rail section, and taking the strain gauge measurements at the two points, the average of these two readings can be used for determining the magnitude of the vertical load. In order to get greater accuracy, without using any stress equation to obtain the load from the stress measured, a preliminary calibration of the track can be made by applying a known vertical load and taking readings at the two points. The effect of the adjacent vertical loads on the readings at the two mentioned points of a rail section can be determined by using a curve obtained by calibration instead of using any theoretical curve, such as the "Master Diagram" in Chapter 3 or "B3-curve" in Chapter 2.

In the field experiments, it is possible to attach magnetic strain gauges to the base of the rail in all the different positions [8]. Variable electric - resistance gauges were also used for the same purpose [18]. A rather interesting technique was the measuring of the thrust forces acting on rivets connecting the rail base to the ties, through the friction between the rivet and its hole [18]. These thrust forces can be used as indicators for the stresses in the two edges of the rail base.

In order to have an idea of the way in which these stresses, exerted on a rail by vertical loads, can be distributed through the different parts of the rail section, one can measure in the field, the stresses in the rail's web, for instance. Photoelastic methods can then be employed to find the relation between the web stress and the stresses in the other parts. Such photoelastic techniques are not considered as being very reliable. This is especially true when we try to analyze the stresses in the region of the base and the fillet at the bottom of the web, because the method of attachment of the base of the celluloid (or plastic) model could be in no way similar to that of an actual rail [8].

On the basis of the information obtained from such static tests, methods can always be devised whereby strain gauge measurements could be taken under dynamic conditions.

The apparatus to be used for the static calibration in order to determine the relationship between the vertical load and the average base stress, can be a loaded car to which hydraulic jacks would be rigidly attached. This is done in a way so that by moving the car, the jacks could quickly be spotted over the point on the track where the calibration is to be made. The car and the jacks can be first placed directly over the magnetic strain gauges and then later can be moved to different locations from the gauges.

In considering other elements of the track, when measuring static stresses on the ties or measuring the vertical pressure in the ballast section or on the subgrade, prototype size experiments can easily be prepared in the laboratory, i.e. not necessarily in the field as is the case when dealing with stresses in the rail or the track as a unit. Satisfactory results can always be obtained from such experiments, which are not dynamic, by simulating the actual field conditions in the lab. For instance, the vertical pressure distribution on the subgrade and in the ballast section can be experimentally determined in the laboratory by using the same types and depths of ballast used in the field, placing ties on top from the same size and material as used in the field, at same standard spacing as used in the field. The ballast section thereafter, can be placed on sufficient depth of subgrade material. The static load can always be applied by a hydraulic jack which can be used with a calibrated dynamometer to determine the load which is to be applied on a tie or set of ties in a group. Such a test can be done for different depths beneath the ties in the ballast section. Pressure cells can be used in order to determine the vertical pressure distribution [16]. Such pressure cells can always be calibrated under sand or air pressures and be placed thereafter on the subgrade or in the ballast section. In general, pressure cells are used to determine the pressure distribution within earth structures and foundations, it is considered an accurate pressure measuring device. It must be small enough to minimize its effect on the actions of the materials but large enough to measure average stress rather than localized stresses [16]. Salem, used in his study on the ballast and subgrade, thin cylindrical cells with parallel faces which are compressible along the cylindrical axis and have four strain gauges attached to the diaphram, in order to respond to the strains produced in it by the pressure [16].

When dealing with ballast and subgrade soils, all their physical properties and classifications should be experimentally studied with the known soil mechanics experiments. Soil properties of special importance to this study are water content, results of standard compaction tests, liquid limit, plastic limit, organic or inorganic, optimum moisture content, the maximum dry density, and soil classification according to any one of the three standard classifications. Meanwhile, the ballast sieve analysis must be done to check the uniformity of the gradation. All the above laboratory tests are of much significance to the topic of this study, since all the mentioned physical properties have definite contribution to the soil and ballast strength, which consequently define the amount of pressure they can take.

CHAPTER 7: TECHNICAL DISCUSSIONS AND COMPARISONS OF THE DIFFERENT METHODS OF TRACK STRESS ANALYSIS

The stresses, internal forces and displacements, which were introduced in Chapters 2, 3, and 4 of this work, are not the only forces exerted on a rail. Accompanying the vertical stresses on a rail of a track, are the lateral stresses exerted against the rail, and longitudinal stresses caused by temperature changes.

As far as the lateral stresses are concerned, they were found proportional to the vertical stresses. The special joint committee of A.R.E.A., A.S.C.E., and A.R.A. recommended use of 14 percent increase in vertical stress owing to lateral bending due to the horizontal component of wheel loading [9]. The longitudinal stresses, due to starting and braking, are only small and may therefore be disregarded [14]. The longitudinal forces caused by temperature changes are accurately known for long welded rails, but only to a limited degree of accuracy for tracks with expansion joints [14]. Such a temperature stress is significant, however, when compared to the normal stresses exerted by vertical loads.

From the above discussion we can see, that if we can set the permissible stresses in a way to cover a 14 percent increase in the normal stresses due to the vertical loads, which are calculated by any of the procedures previously presented, then only the stresses due to vertical loads are to be used. This approximation is very possible, however, especially if we realize that some uncertainties already exist in the calculation procedures due to the dependence on some imperfect assumptions and some values which fluctuate within wide limits and in some cases are known only by the order of magnitude, like the values of the modulus of foundation, k1. It would be a mistake, however, to expect more accuracy to be attained by any improvement in any method of bending moment calculations. The railway track is not an ordinary structure like a bridge or a building, in which stresses can simply be estimated by very general rules and formulas. The railway track consists of several elements of completely different materials, shapes and arrangements. These elements altogether interact and affect each other and form the track strength. Furthermore, the track is not perfectly "a beam on elastic foundation", and if it is so, the foundation is not continuous (due to the spaces between the ties). How close is a track to these very ideal conditions, which completely differ from one case to another due to various circumstances of the track components?

In rail and track stress calculations, it is always more important to obtain a reasonable approximation of general validity, than to determine an accuracy with a strictly limited validity. Calculations as an aid to intelligent comparison and logical deduction are the best we can get out of the previously mentioned techniques and formulas.

How the rail can be considered continuously elastically supported, how far each of the previously presented stress analysis methods is accurate if compared to each other, which ones to use, therefore, and how to go about them, will be the theme of the discussions to follow.

1. Assumptions of Ties Supplying Continuous Support to the Rail

In the cross-tie system, only the ties are supported continuously by the ballast, while the rail is only supported by the ties, i.e. closely spaced elastic support. A continuous elastic support was proven to be however, a good enough approximation, to replace the ties for calculation purposes [1]. In this way, the theory of beams on continuously elastic supports can be applicable to the rails themselves.

Assuming that the rail pressure (due to load P) on the tie is equal to F, and it causes a deflection Yt in the tie point of application, then the elasticity of support which results from one tie to the rail can be expressed as,

$$SC = F/Yt \qquad \text{.....(7-1)}$$

where

SC is the spring constant.

Replacing the sufficiently closely spaced ties by a continuously distributed foundation of constant, k, the foundation constant, k, will therefore be:

$$k = SC/a \qquad \text{.....(7-2)}$$

where

a is the spacing between the ties C. to C.

In general, separate elastic supports may be replaced by an imaginary continuous foundation if we have at least four of the supports in the characteristic wave length of the deflection line defined in Chapter 2 of the this work as (π/v). The formula, therefore, which must be satisfied, is

$$\sqrt[4]{64.a^3 \, . \, SC/EI} < \pi$$

The formula was obtained by substituting Equation (7-2) into the value of

$v = \sqrt[4]{k/4Ei}$, which was used in the analysis in Chapter 2 .

Corresponding to the above assumptions, the value of the "spring constant", SC, which consequently defines the value of the foundation constant, k, must be experimentally determined as follows:

Assuming that the spring constant, SC,is the same for every tie, and applying a load P to one of the ties, and also if we measure the deflection of the rail at every tie, it is obvious from the equilibrium of the system, that

$$P = SC\sum yi$$

or

$$SC = P/\sum yi \qquad .....(7\text{-}3)$$

Where

yi represents the deflection of the tie number i,

substituting the value of SC above into Equation (7-2), therefore

$$k = P/(a.\sum yi) \qquad .....(7\text{-}4)$$

This analysis, shows how one can go about the experimental determination of the foundation constant,k, in order to get the used approximation of the theory as close as possible to reality.

Although the previous allowance of considering the closely spaced ties as continuous support was first introduced by S. Timoshenko, in St. Petersburg in 1915, and proved experimentally by A. Wasiutynski in Warsaw in 1937, Robert Hanker, in 1935 [6],[10], went about dealing with such a problem in a completely different way.

Hanker assumed, theoretically, that the cross ties can be replaced by an imaginary pair of longitudinal ties supporting each rail continuously. The bearing area of each longitudinal "tie" was one half of the bearing area of the actual railway ties. Thus the actual bearing area per unit length was used to calculate the width of the imaginary continuous tie. This assumption led to the possibility of using the analysis of a continuous beam on elastic support for the rail and tie system. Having introduced such an imaginary assumption, it was necessary for Hanker to deal with the difference between his imaginary longitudinal-tie and the real cross-ties. Hanker therefore, in effect assumed that the longitudinal-tie would provide more stiffness for the track than that actually provided by the cross ties. To account for this, Hanker proposed an adjustment coefficient to be used in the classical beam on elastic foundation formula to reduce the calculated stiffness. Although the above is a rather interesting idea, the author believes that the formulas became so complicated that they were not really significant for practical use, especially since the assumptions were somewhat unrealistic.

2. Jaehn's Method Versus the Method of A.R.E.A., A.S.C.E., and A.A.R.

The two methods of analysing the stresses in rails, which were introduced in Chapter 3 were both established on the basis of Zimmermann's work. Jaehn's method, however, included some empirical reductions of the calculation procedures, while the method of the joint Committee of A.R.E.A.,A.S.C.E., and A.A.R. is no more than a direct simplification of Zimmermann's analysis.

It can be argued against Jaehn's method, that it has limited validity for certain conditions, such as ranges of ties and ballast and rail characteristics, as listed in Chapter 3. If one looks closely, however, one will find that the ranges of validity set by Jaehn are really covering the most common conditions existing in any track elements, with the exception of distance between the centres of the ties, which was limited by 60 - 80 cm (23.6 - 31.5 in) by Jaehn, while in North America, the most commonly used ties spacing are 22 and 24 inches. The second (24 inch) will exceed 60 cms. Therefore, it is in the right range anyway, while the spacing of 22 inches is, when converted, equivalent to 56 cm. The author believes that such a slight difference is insignificant, especially, as was mentioned at the beginning of this chapter, that any use for any of the two formulas is only a practical approximation, and that the best we can get out of them are intelligent comparisons. It was proved, however, through analysis of different vehicles [19], that using the two methods would result in different stresses for similar cases, but almost the same relative values of stresses of one vehicle to a different one. In the next chapter, such comparisons will be presented numerically.

Furthermore, the following argument can always be a helpful defense of Jaehn's method. The author believes that the simplicity of Jaehn's method does not cause any loss of accuracy. For instance, one of the disadvantages of Jaehn's method is that it simply includes two major terms: one is the wheel load and the other is the tie's spacing in addition to two empirical factors (for the adjacent wheels and the speed), with no mention at all for the foundation constant, k. Is ignoring the foundation constant really a severe loss of accuracy? Let us look at the maximum stress, which can be determined from the maximum bending moment as was done in the method of A.R.E.A., A.S.C.E., and A.A.R. jointly and in Zimmermann's analysis, Equation (2-18)

$$M_{max} = P/4v$$

or

$$P/4 \sqrt[4]{4EI/k}$$

If we put in such an equation the foundation constant = 2k instead of k, we find that the deviation of the maximum bending moment is only

$$(1-1/\sqrt[4]{2})$$

, i.e. 100 percent difference introduced in k causes only 16.5 percent deviation in the value of the maximum bending moment. From the above example, one can see that the withdrawal of k from a moment formula is not really significant. Moreover, not having introduced any value for the moment of inertia of the rail, is in fact one of the advantages of Jaehn's method. Using the method of A.R.E.A., A.S.C.E., and A.A.R. jointly may be possible, for the sake of time, if to be used for "check of stresses", in which case the rail moment of inertia is known, while if used for design, this value has to be assumed until the determination of the bending moment and the modulus of section which is required thereafter. Very seldom will the assumed value of the moment of inertia be the same one as the calculated, which would lead to several time consuming iterations.

Based on the previous discussions in this chapter, the writer recommends the introduction of Jaehn's method in the railways administrations, either as a single alternative or parallel with the method of the joint Committee of A.R.E.A., A.S.C.E., and A.A.R.. When using any stress formulas for design purposes,however, they can be in no way one hundred percent reliable.

Jaehn's method was, therefore, used together with Schramm's formula of the speed factor, to write a computer programme which estimates the bending moment (including the dynamic effect) on a railway track exerted by any particular train running at any particular speed. This programme was used thereafter for the determination of the said bending moment due to everyone of the 200 locomotives operating on the CNR lines. The locomotives' particulars and their dynamic bending moment were tabulated. The tables, and the source of the necessary data are listed in the Appendix.

3. The Speed Factor

Assuming that we have a perfectly elastic track with damping characteristics and a perfect vehicle, the rail bending stresses will decrease as the speed of the vehicle is increased. This is due to the fact that a certain amount of time is necessary for the complete development of the deflection of the ties and the bending of the rail, which may occur under a stationary vehicle. A decreasing fraction of that time is only present with increasing the speed of the vehicle. This same assumption may furthermore suggest that at a speed of infinity there is no stress at all on the rail.

The actual track is not perfectly elastic, neither is the actual vehicle perfect and variation from the predictions of a simple theoretical approach are to be expected. The formulas presented in Chapter 4 excluded the fact that the stress under a moving vehicle may not exceed the stress under the stationary vehicle. Schramm's formula, however, shows that he realized this fact for speeds exceeding 300 km/hr. As this decrease in stress is expected to happen at very high speeds, it may also occur at very low speeds. None of the investigators had seemed to realize this matter in any of the formulas, although Hanker[6] had stated,"The observation of the Empire Train Administration of Dresden is to be adhered to; that the stresses at resting load are about 15% greater than at a speed of 5 km/hr, and that only at a speed of about 40 km/hr do the stresses again become as great as those at the resting load". However, with the two impirical formulas present, it can be decided which is more suitable.

The formula introduced by the United Middle European Train

Administration, Equation (4-3), was based statistically on 21,000 stress measurements. When Schramm [11], [14], introduced his formula, which is still used in some parts of the world, one wonders, how a formula like Schramm's can be correct by having the speed factor as a quadratic function minus a cubic function of the speed. Meanwhile, the said Union's formula is only a quadratic function. It was necessary, therefore, to plot the values of the speed factor, corresponding to each speed using each of the two formulas between speeds zero and 350 km/hr, Fig. (7-1). From comparison of the two curves, we can see that there is some similarity between the values from speed zero and up to 100 km./hr. This range, however, was the only field for the experiments of the mentioned Union. The results of the Union's formula diverged from logic in a very quickly increasing way for speeds from 100 km./hr. on, which shows that such speeds were not taken into consideration. Meanwhile, Schramm's formula gives fairly reasonable values for the speed factor, even when the speed goes up as high as 200 km/hr. Schramm, however, did not mention anything about the speeds beyond 200 km/hr[11] & [14].
When considering the current and the future speeds, we can see from the figure that at very high speeds Schramm's Formula is no longer valid as well. It is recommended,however,that Schramm's formula be used, until further evidence is available from experimental results at higher speeds. Fig. (7-1) for Schramm's formula can be used to measure the speed factor for practical purposes. The figure is also provided with a conversion axis for the speed in mph.

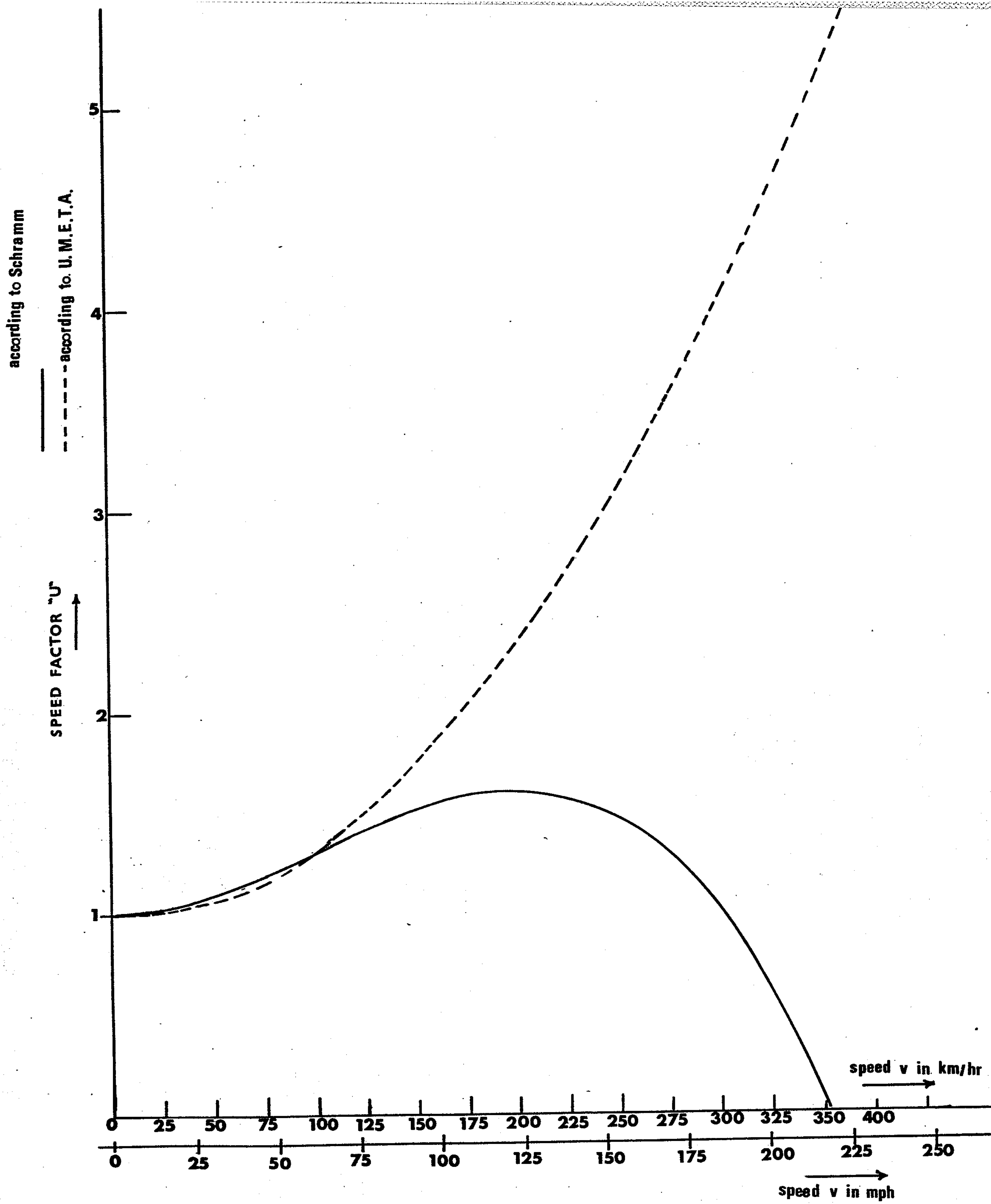


Fig. (7-1)

Comparison of speed factors (Schramm and U.M.E.T.A)

CHAPTER 8: APPLICATION OF TRACK STRESS ANALYSIS

TO A SPECIFIC PROBLEM

Traditionally it was thought that the type of study reported in the previous chapters, is only useful for designing track elements. However, the power and general usefulness of the track stress analysis will be demonstrated by a specific example [19]. This example is of particular interest because of its relevance to the grain movement economy in Canada.

GRAIN MOVEMENT ON BRANCH-LINE RAILWAYS--THE EFFECTS OF 12-WHEEL GRAIN HOPPER CARS :

In Canada, one of the major costs in grain handling is the cost of moving the grain from rural elevators to inland markets or terminals by the railways. This cost per bushel of grain is sensitive to the size (or the capacity) of the railroad cars used.

The wheat capacity of the standard 40-foot boxcars used in grain service is from 1330 to 2160 bushels. To improve its grain moving facilities, the Canadian Wheat Board purchased, in 1971, new grain hopper cars, with a capacity of 100 tons (200000 lbs), or 3365 bushels of wheat. The operating cost of the boxcars and hopper cars are similar, and thus the saving in using the latter is substantial. The increase in the grain capacity between the standard boxcars and the 100-ton grain hopper cars is 50 to 150 percent, while the increase in the car price is only 20 to 35 percent. Furthermore, the loading and unloading capabilities of the 100-ton grain hopper cars are much improved over those of the standard boxcars.

In Western Canada, however, many of the branch lines were built with light rails which are still in place and do not have the capacity to support the 100-ton hopper cars. This fact would cause an expensive rehandling procedure between the main lines and the branch lines.

One solution to this problem of light-capacity branch lines is to rebuild them to a heavier standard, replacing both the rail and the associated roadbed. The cost of so doing is significant. It was estimated to range from 13 Cents to 1.03 dollars per bushel of grain moved [19].

An alternative to this costly change is that of modifying the hopper cars such that their weight is more uniformly distributed over the track. In this way, their extra weight can be carried without exceeding safe stresses in the branch line tracks. The study proposed, therefore, a modification of 100-ton grain hopper cars by the provision of 6-wheeled trucks in place of the standard 4-wheel trucks used on regular freight equipment. This proposed modification, which has been estimated to cost only 0.5 cents per bushel of grain moved, is shown in Fig. (8-1) below.

In testing the feasibility of the modification proposed for grain hopper cars, the most important factors to be examined were: the live load stresses induced in bridges, the track, the ballast and ties, as well as the track deflections to be encountered. In addition, an estimate of the increased curve resistance, due to the longer, 6-wheeled trucks is required.

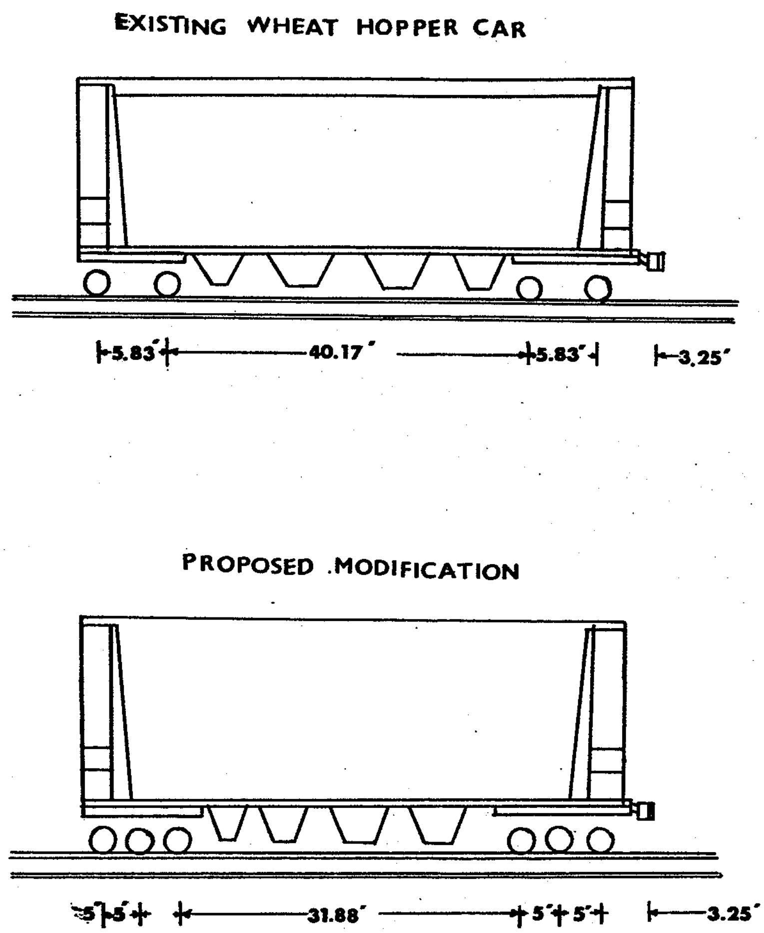


Fig. (8-1)

The existing wheat hopper car

and its proposed modification.

1. Bridge stresses

The live load shears and moments have been calculated for bridges of various span length for the following four cases:

(a) A string of standard boxcars; load on track = 177,000 lb/car;

(b) A string of Canadian Wheat Board Hopper Cars; load on track = 263,000lb/car;

(c) A string of modified hopper cars; load on track = 263,000 lb/car. and

(d) A pair of GMD-1 diesel electric locomotives (CN1000 to 1076) used on light branch lines; load on track = 240,000 lbs.

The live loads so obtained were then converted to equivalent Cooper's "E" values, which is the conventional live loading used on Canadian railroad bridges. It was found that the modified hopper car applies live loading to bridges which is 118 percent of that applied by 177,000 lb. boxcars pulled by GMD-1's. Such overstressing can normally be handled through a speed limitation which is sufficient to compensate for the train's "static overweight" on the bridge. The practice of speed restriction is followed on many light western Canadian branch lines.

In the worst case, however, when the practice of speed limitation cannot be used, we have to reduce the modified hopper cars weight by 18 percent through decreasing its capacity. However, each reduction in speed by 5 miles per hour results in an additional capacity of 135 bushels of wheat per car. Needless to say, that even with the reduced capacity, the saving due to using the modified grain hopper cars is still substantial. The modified hopper cars with reduced weight, will have a load on track = 224,000 lbs. instead of 263,000lbs.

2. Stresses in Track

The cases of loading to be compared, are the boxcar, the wheat hopper car, the modified wheat hopper cars full, and the modified wheat hopper cars light (reduced weight). Using values of physical properties representative of those likely to be encountered on typical light branch lines, the values of the track stresses for the four cases of loading listed above have been calculated twice. First, they were calculated by Jaehn's method, equations (3-8), (3-9), (3-10), (3-11) and (3-12). Then they were calculated by the method of the A.R.E.A., A.S.C.E., and A.A.R., Table (3-1) and equations (3-1), (3-4) and Fig. (3-3). These values are summarized in Table (8-1) below, and are also shown (in parenthesis) as a proportion of the values pertaining to loaded boxcars.

3. Deflections in Track

Static deflections in the track under the four cases of loading were calculated using equations (3-1), (3-2), (3-6), Table (3-1) and Fig. (3-3). Again, values of physical quantaties typically encountered on prairie branch lines were selected, and the results summarized in Table (8-1). Again, values have been compared to those for loaded boxcars.

4. Stresses in Ties

As was shown in Chapter 5, the stresses in the ties are considered to be proportional to the wheel load applied directly to them, and a function of rail, subgrade, ballast, and tie stiffness, in addition, the dimensions of both the rail and the tie. Therefore, for any particular track section, the tie stresses relate directly to the axle loads. This has been indicated in Table (8-1) again, with the loaded boxcars taken as unity.

5. Stresses in Ballast

It was shown, in Chapter 5, that the stresses in ballast are directly dependent on the wheel or axle load, as well as on some properties for the ballast profile and the ties. However, like tie stresses, for a particular section of branch line track, the relative ballast stresses will depend only on the wheel or axle loading. Thus, this is also indicated in Table (8-1), with the ballast stress under loaded boxcars considered as untiy.

6. Curve Resistance

The increased length of 6-wheeled trucks, as compared to standard 4-wheeled trucks leads to some increase in curve resistance of the modified cars. Several studies have been made of this problem. The mixed theoretical-empirical relationship which seems to be the most useful in this study is as follows [14], [12]:

$$Rc = (232.2 + 103.4a)/r$$

where

Rc = curve resistance (kg/ton or kg/1000kg).

a = distance between the farthest 2 axles in one truck, (metres); and

r = radius of curvature (metres).

FACTOR	BOXCAR	WHEAT HOPPER	MODIFIED WHEAT HOPPER (FULL)	MODIFIED WHEAT HOPPER (LIGHT)
Load limit (lb)	177,000	263,000	263,000	224,000
Wheat Capacity (bushels)	2,000	3,365	3,265	2,610
Load per Axle (kip)	44.2 (1.00)	65.8 (1.49)	43.8 (0.99)	37.3 (0.84)
Track Stress (Jaehn) in.kip	117 (1.00)	176 (1.50)	91 (0.77)	77 (0.66)
Track Stress (AAR-AREA-ASCE) in.kip	197 (1.00)	291 (1.48)	176 (0.89)	149 (0.76)
Track Deflections (in.)	0.29 (1.00)	0.41 (1.42)	0.36 (1.24)	0.31 (1.05)
Tie Stresses	(1.00)	(1.49)	(0.99)	(0.84)
Ballast Stresses	(1.00)	(1.49)	(0.99)	(0.84)
Curve Resistance (kg per ton or lb/kip)	405/r (1.00)	416/r (1.03)	547/r (1.35)	547/r (1.35)
Curve Resistance (per bushel)	35.9/r (1.00)	32.5/r (0.91)	44.0/r (1.23)	46.8/r (1.30)

Table (8-1)

Applying this expression to each of the four cases under consideration leads to the values of curve resistance in kg per ton (or pounds per kip) shown in Table (8-1). However, for comparative cost purposes, it is more meaningful to compare resistance in terms of curve resistance per bushel. This has also been shown in Table (8-1), and is derived from the expression:

Rcb = (Rc)(Load Limit)/Bushel Capacity

where, Rcb = the curve resistance per bushel.

Although the curve resistance per bushel is 30 percent greater for the lightly loaded modified hopper than for a conventional boxcar, the cost addition due to this factor can be shown [19] to be less than 1/10 cents per bushel in hauling grain from the rural elevator to ocean terminal, even with very severe assumptions of total curvature encountered on a round trip.

It was concluded that the modification is a very economical possibility for moving wheat on the prairies, when compared to branch-line upgrading, and that the benefit-cost ratio to the railways of making the modification is between 10 and 20, which is sufficiently high.

SUMMARY

Introducing higher capacity cars and ones which are more efficient in the loading and unloading procedures, is a feasible improvement for the freight rail operation. This improvement is of particular interest when it comes to the grain movement from rural elevators to inland markets or terminals. The light branch lines in Canada are a major problem facing such a type of operation. The fact that these heavier cars can be modified, as shown above, to run on the light branch lines, means that these lines do not necessarily need major upgrading or abandonment.

This is a typical cost-benefit analysis problem, which involves the railway track stress formulas. The example of wheat cars shows that an understanding of the stresses in the various elements of the track is an important component in railway planning as well as being of technical interest.

CHAPTER 9 : IMPLICATIONS OF TRACK STRESS ANALYSIS FOR RAIL-TRANSPORT PLANNING IN CANADA

Since the beginning of the Canadian railroads in 1835, an intensive development has been made, and some advanced operational techniques have been regularly introduced. However, the level of modern high-speed service which exists in Western Europe and Japan has not been reached in Canada.

The most important techniques, which must be urgently considered in Canada, are those which contribute to higher operating speed: the upgrading of the road beds and traffic controls, the elimination of level crossings, as well as the electrification of main lines [20], [21]. Such techniques are already in major use in some countries which have less demand for them than Canada. For instance, Canada is believed to have the second highest ton-miles of freight per capita in the world. In addition, for freight operations, the trains in Canada carry 3 to 5 times the loads common in Western Europe [20], [21]. Such a type of operation must maintain the stresses on the track within allowable limits, and consult the stress estimation procedures will be increasingly important in future developments.

It will now be shown that the passenger rail in Canada should play a much larger role in the future transportation spectrum than the state of its near obsolesence which now exists.

In the previous chapter it was shown that the application of stress estimation procedures in a specific problem, offers a satisfactory solution resulting in significant cost minimization. This should be the goal in planning the future of the rail-transport in Canada. A great deal of the Canadian rail-transport problems mentioned above may imply a variety of uses for the information collected in this thesis.

Some of the planning aspects of the Canadian railways and the part which track stress estimation might play in them are discussed in the following sections.

1. Intercity Passenger Rail Transportation in Canada

If we look at the Canadian intercity modes of travel, we find the automobile, the aircraft, and rail as major transportation modes. The fast passenger trains can be attractive relative to other modes for distances ranging between 150 and 400 miles [20]. The speed limitations of the highways (average 60 mph) and traffic congestion at both the origin and destination cities are unattractive features of the road mode, whereas congestion, especially in the airport access, is the main disadvantage of the air mode over the mentioned distances. From the point of view of the airlines, such short flights are no longer profitable with the increase in the fixed costs. It seems then, that there is demand for passenger train transportation if high speeds can be attained.

When speaking about high speed passenger trains, it is presently meant as 125 - 150mph. The near future will very likely provide techniques enabling running speeds of around 200 mph. The question then arises ; Can the existing Canadian tracks support such speeds inspite of all the restrictions of allowable stresses, curves, level crossings and stations?

The answer is no, and the only two alternative solutions are:

we build special trains[1] of higher speeds and lighter weights that can travel on the existing tracks with all their restrictions,

or we try to ease the speed restrictions of the existing track, either by remodelling of junctions, curves and relaying of the track, or even rebuilding it in some instances.

The investigations of the most economic and useful solution of such a problem produce answers to many fundamental questions, the most important of which are:

-How will the demand for rail transportation increase with the increase of train speed ?

-What are the costs of building new tracks capable of supporting the stresses induced by high speed trains ?

-What are the costs of eliminating the speed restrictions of the existing track by relocations, and hence, would the existing or relocated track be able to support the lightest available high speed trains at the desired speed, from the enduced stress point of view?

-What are the costs of building special trains which are capable of travelling on the existing tracks with high speeds?

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[1] Such trains should have high acceleration and deceleration rate in order to ease some of the speed restrictions, as for example, in stations, yards and level crossings. They have to take the form of multiple unit sets to ease the speed restriction of curves, and to be light enough to avoid over-stressing the track.

As we have seen above, in almost every single or multiple study of the solution alternatives, and in planning or designing the solution, the major part is to consult the stress estimation procedures, the subject of this work.

Now, we can never come to the study of high speed passenger trains in Canada without becoming involved in the rather interesting argument of electrification of the Canadian rail transport. In other words, electric railway versus existing diesel railways must be considered.

2. Electrification of the Canadian Railways

Between 1940 and 1960, the Canadian railways had gradually replaced the steam locomotives by diesel locomotives, lagging behind the United States by about six years [21]. This development in the railway traction technology which occured in Canada and the United States was very comparable with that which was taking place in the rest of the industrialized world, especially in Western Europe and Japan.

The development continued in Western Europe and in Japan by the electrification of the main lines. Canada and the United States seem to have stopped there in the development of the traction technology, although they developed some of the best systems in the world in some other railroad technologies, such as containerization, unit-train operation, computerized and automated handling of yard operations and automatic car location identification [21]. As important as the above technologies may be, there are some other railroad technologies in the industrialized world in which the Canadian railways are still lagging .

The most important of these techniques is the electrification of the main lines.

The major reasons for the Canadian and the American railways ignoring a major traction advanced technology such as electrification of mainlines, are no longer valid. Logically, these reasons were the availability of oil, and the costly "fixed plant" expenses of the railway electrification. However, as the oil of the world is vanishing, and as new techniques have been reduced the fixed-plant costs of electric locomotives, replacement of mainline diesel trains of Canada with electric trains becomes inceasingly attractive.

The following are the factors favouring the electrification of the mainlines in Canada.(1)

-The useful life of electric locomotives is estimated to be 30-35 years and exceeds that of diesels by a factor of 2 or 3. This is basically due to the rotating machinery of the electric locomotives, versus the reciprocating one in the diesel locomotives [21].

-Electric locomotives have lower maintenance costs than diesels, for the same reason above (rotating versus reciprocating machinery) [20],[21].

-The high level of maintenance required by diesel locomotives, reduces their availability for service by about 10 percent, if compared to the electrical locomotive's availability

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(1) This summary of electric traction versus diesel traction in Canada is based on recent literature - of particular use: J. Lukasiewicz in 1973 [20], J. Lukasiewicz 1974 [21], and D. Cass-Beggs in 1975 [24].

[20],[21]. Also, due to its reduced maintenance requirements, a typical electric locomotive can run 50 percent more mileage per month than the diesel [24].

-For the same weight, electric locomotives exert larger pull and develop more power than the diesel locomotives. The ratio of the tractive effort per unit weight and the horse power per unit weight between the electric and diesel locomotive is in the range of 2:1 to 5:1 [20],[21]. This fact is one of the major reasons which led to having the electric units always lighter than the diesel ones.

-Due to the superior work capacity of the electric locomotive as compared to the diesel one, substantial reduction in capital costs were also realized, [20], [21].

-The electric locomotives, due to their light, weight have larger pull on upgrades and better acceleration and deceleration performance than the diesel.

-The application of solid-state electronics which allows direct use of A-C power at commercial frequency and smooth control of traction current, through the use of high-power silicon rectifier on the locomotive, led to a drastic reduction in the costs of the fixed plant (catenary, substations and signalling equipment). Such costs were the major counter-argument point in the past against the railways electrification [20], [21].

-The problem of current collection was overcome and in France and Japan it has become practical up to speeds of 190 mph (about 300 km/h), [20], [21].

-Electrification allows flexibility on the use of primary energy source [21]. Of course, it would use non-petroleum primary energy resources [24].

-A sufficient level of traffic for electrification exists in Canada. Railways accounted for over 40 percent of freight ton-miles, which is the largest share among the various modes of transport and four times as much as that of trucks. Furthermore, if we take the railroad utilization measure known by the dynamic density, or gross ton-mile (travelled) per mile (track length), Canada's figures are very similar to those found in Western Europe [21]. Hence, the old counter argument of the great extent of Canada's railways and our small population is no longer valid [24].

-Out of total running track length of 23143 miles in Canada, about 22 percent carries the high density traffic. This percentage is about typical of the electrified portion in Western Europe [21].

-Electric locomotives have smaller environmental impact (noise and pollution) [20], than the diesel. In addition, they minimize the pollution exerted from the highways by more and more private automobile travel volume. This would be handled by rail.

-Although the order books of most companies manufacturing electric railway equipment are full for years ahead, it was suggested that, "Canada could expand its manufacturing facilities to meet the need. It may be that companies in Europe should be asked to develop factories here or provide under license the technology we need. A growing railway equipment industry in

Canada might well take over some of the labour and the plant of the automobile industry that the changing energy situation may leave redundant." [24].

-It was found feasible in some countries during railway electrification, to use the existing track, [24] and hence, construction of new lines may not always be needed.

-Static and dynamic loads in track and vehicles are reduced with electric traction because of the high adhesion, the large power/weight ratio, and the distribution of drive to a maximum number of wheels. Such lighter loads increase the eligibility for high speed operation [21].

In parallel with the electrification, other techniques must be introduced, such as "suspension" . It is particularly significant when the use of existing, standard, low quality lines is desired for fast passenger trains. Improvements in suspension can contribute to such desirable features as higher maximum speeds, improved passenger comfort, and reduced dynamic loads on tracks and vehicles. The features of the suspension that allow of these improvements are: low unsprung mass, and full articulation (the wheels to be steered by the car ahead in the direction of the curve), [21].

With the increasing of speeds after electrification, the construction of new tracks may be feasible and necessary. Therefore, some new methods and materials for track construction may be introduced, such as continuous reinforced concrete slab road- bed, concrete ties, and techniques of cleaning and reinforcing conventional ballast [21].

From the above summary about electrification and some of its expected consequences in Canada, we find that future technical challenges are really a capacity (weight) improvement, as well as a speed improvement. Such improvements will result if we can minimize the unloaded train weight, maximize the loaded train weight, maximize the speed, whilst keeping costs to a minimum. These challenges are complicated by the need for sophisticated optimization models. Therefore, the decision maker, the planner, and the designer, whether for track or equipment, may regularly need the knowledge of stresses in tracks.

3. The Use of Stress Estimation Procedures in Building Optimization and Simulation Models

When introducing any new transportation system (or service), the planner, or the decision maker knows, that economic optimization will not necessarily call for maximum speeds and maximum vehicle capacity. Techniques of linear and nonlinear programming and model building are often used for making such decisions, and the minimization of the total system costs is normally the goal of the decision maker in such cases. Of course, the speed and the capacity, in addition to affecting the total system cost, greatly influence the travel market share.

The total system costs for any railway service can be looked at from two different viewpoints.

First viewpoint : Cost to system operator and agency[1]:

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[1] In other transportation systems, these are under two different authorities, i.e. the operator is the "service carrier" and the agency is the "Government".

1. vehicle capital cost

2. vehicle operation and maintenance expenses:

   -fuel and oil expenses

   -vehicle crew and cabin attendants expenses

   -vehicle maintenance expenses

3. terminal construction cost

4. permanent-way construction cost

5. stations construction cost

6. terminals operation and maintenance expenses:

   -expenses of operation and maintenance of passenger (or freight) terminal area

   -vehicle servicing expenses

   -vehicle control expenses

7. permanent-way facilities maintenance expenses

Second viewpoint: Cost to system users

1. Fare and other out-of-pocket costs

2. Cost of time spent in travel via the system

In most of the cases, the decision maker will take the above two viewpoints of the cost together, forming a societal viewpoint [22]. The technique of optimization is to try to define each of the cost items above in terms of speed and capacity. All the expressions resulting thereafter will be combined together to form the "objective function" which is required to be minimized. The constraints for such an objective function are the upper and lower limits for the speed, the capacity, and that they be non-negative. The upper limits of the speed and the capacity are usually, either manufacturing restrictions or geometry of the line restrictions, while the lower limits are usually set in the range known for the other competing modes of transportation. The formulation of the model is therefore:

Minimize $\quad CPM = f(V,Q)$

subject to:

$$V_{min} \leq V \leq V_{max}$$

$$Q_{min} \leq Q \leq Q_{max}$$

and

V, Q are non-negatives

where,

CPM is the cost per passenger-mile (or ton-mile),

V is the speed in mph,

Q is the capacity (passenger or ton).

The objective function is normally non-linear, and its optimization must be achieved by an iterative procedure. One technique frequently employed for solving such problems, is the "Newton-Raphson Technique" [22]. The main idea of this technique is taking the partial derivative of the objective function two times, once with respect to the speed and then with respect to the capacity. The result of this operation will be two simultaneous equations, the solution of which, will determine the optimal values of the speed and the capacity. This is an iterative process which can be solved by the use of the computer.

Among the cost components previously listed, there are two components in which the cost is really proportional to the stress induced by the vehicle on the track, namely the permanent-way construction cost and the permanent-way facilities maintenance expenses. Therefore, for these two components, we can say that the cost will be a function of the stress, which is a function of the speed and the capacity. To emphasize the meaning of this, the part of the model related to the permanent-way construction cost will be discussed.

It is apparent that the construction cost of the supporting track is proportional to the stress induced in the rail, and consequently to the bending moment on the rail or to the wheel load, including the dynamic effect (Chapters 3 and 4). Using Equations (3-8) and (4-4), we can say that the construction costs of the supporting track α(P . U), where P is the axle load and U is the speed factor resulting from a speed V. The value of the axle load P is directly proportional to the capacity of the vehicle Q, and according to Equation (4-4),

$$U = 1 + (4.5v^2/10^5) - (1.5v^3/10^7)$$

Converting v into mph, Equation (4-4) becomes

$$U = 1 + (11.5\,v^2/10^5) - (6.15\,v^3/10^7)$$

Therefore

$$C = Q.(1+(11.5v^2/10^5) - (6.15v^3/10^7)).A+B$$

where

C: the construction cost of the supporting track/mile

A: cost of permanent-way construction/ton of train dynamic load/mile

B: the fixed cost of the permanent-way/mile

The discounted construction cost of the permanent-way (dollar/day)

$$= C . D . CRF/N$$

where

C: the construction cost of the supporting track/mile

D: the length of the track in miles

CRF: capital recovery factor for permanent-way construction costs.

N: number of utilization days per year

Dividing this last equation by the total daily expected passenger-miles, we can get the construction cost of the supporting track per passenger mile.

Similar expressions can be derived for each of the cost components listed above in terms of the speed and the capacity and putting them together thereafter, will result in the required objective function.

The values of the two constants A, and B, can be obtained by building a special probablistic model for this purpose, as follows:

-choose several other Canadian existing rail services where the climate, the land use, and the soil conditions are similar to those for the system under consideration.

-for each system collect the data about the construction cost of the supporting track/mile and the speed and the capacity of the vehicles.

-bring the cost for each system from the price of the year of construction up to the price of the planning year, using Canadian price indices.

-having everything known in the equation except the values of the parameters A and B, the observations can be fitted to a curve, using the multiple regression technique, and hence, the values for A and B can be determined.

The system design optimization model, as explained above, can be used successfully to explore the implications of a broad range of design and policy options. It can also provide a quantitative means of determining the likely impact of alternative designs on each of the variety of often conflicting viewpoints mentioned above.

There have been a number of important studies on similar problems. Of particular use, was the work of Snell [23], which was of a more general transportation system nature, and of Hamzawi [22], which was applicable to a STOL system (short take off and landing), between Montreal and Toronto.

4. SUMMARY

Through the track-elements stress estimation procedures, the Canadian railways can attain a very efficient guidance, not only for new track design purposes but also for planning, comparisons, and solving operational problems. In this chapter three examples of planning and operational techniques which can make use of the track-elements stress estimation procedures, have been discussed. They are :

Firstly, the passenger rail in Canada should play a much larger role in the future transportation spectrum. For distances ranging between 150 and 400 miles, fast passenger trains can be significantly superior to highway and aircraft transportation. The planning for such a fast service will include a cost-benefit analysis to compare between two major alternatives- laying new track or designing special trains that would be capable of running at high speeds on existing track. The study of these two alternatives will include a great deal of track stress analysis.

Secondly, several factors favour the replacement of main line diesel trains of Canada with electric trains at this time. In parallel with the electrification, other techniques must be introduced. If the existing tracks are to be used, we should introduce "suspension" . On the other hand, if the construction of new track is thought desirable, we may introduce the use of continuous reinforced concrete slab road-bed, concrete ties and new techniques of cleaning and reinforcing the ballast. The challenges accompanying electrification would need a cost benefit analysis involving speed and/or weight of trains. Again, track stress studies would be involved in such analysis.

Thirdly, economic maximization does not necessarily imply speed and weight maximization. There is always an optimum value for each of the latter factors, depending on the objective function which is usually the minimization of certain costs. The decision maker, in forming an optimization model to determine the optimal values of the speed and the weight of the train, which minimizes the costs, needs the formulas, relations, and the manners in which both the speed and the weight of the train affect the stresses resulting in the track.

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APPENDIX

TABLE A-1

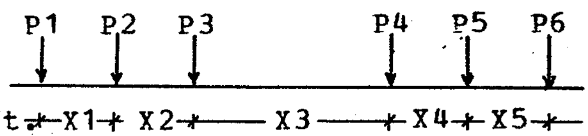
Canadian National Railway's diesel unit axle data (extracted from the diagrams in reference [25].

Each unit in the table consists of 3 lines;

Line-1 The title of the unit (Builder-Order Nr.-Model Nr.-Date Built)

Line-2 The axle loads in lbs.

Line-3 The axle distances in ft



N.B. P1----Front (Max.)

P6----Rear (Max.)

BUILDERS:

G.M.	-General Motors Diesel Ltd.
E.M.D.	-General Motors (Electro-Motive Division
C.L.C.	-Canadian Locomotive Company
M.L.W.	-Montreal Locomotive Works
Alco	-American Locomotive Company
G.E.	-General Electric Company

C.N.R. B-1 TO B-15 1964,5,6
61750 61750 61750 61750
8.00 14.00 8.00
G.E. AEM14857 1956
22000 22000 22000 22000
6.83 11.92 6.83
G.E. AEM19674 44TON 1957
23045 23045 23045 23045
6.83 11.92 6.83
G.E. 93297-TA 70TON 1950
34900 34900 34900 34900
6.83 12.58 6.83
C.N.R. 1964,5,6 300-314
61750 61750 61750 61750
8.00 14.00 8.00
G.M. C-313 GP-40 1966
66000 66000 66000 66000
9.00 31.50 9.00
INT. GE. 1-65777 B-B-94194 4GE-747 1948
24450 24450 24450 24450
7.50 9.50 7.50
G.M. C-208 G-, 1956
27688 27688 27688 27954 27954 27954
5.25 5.25 14.50 5.25 5.25
G.M. C-180 G-8 1954
28440 28440 28440 28440 28440 28440
5.25 5.25 14.50 5.25 5.25
G.M. C-131 NF-110 1952
37030 37030 37030 37030 37030 37030
6.58 6.58 15.84 6.58 6.58
G.M. C-153 NF-110 1953
37102 37102 37102 37102 37102 37102
6.58 6.58 15.84 6.58 6.58
G.M. C-207 NF-210 1956
38008 38008 38008 37888 37888 37888
6.58 6.58 15.84 6.58 6.58
G.M. C-246 NF-210 1958
37793 37793 37793 38227 38227 38227
6.58 6.58 15.84 6.58 6.58
G.M. C-271 NF-210 1960
37969 37969 37969 38044 38044 38044
6.58 6.58 15.84 6.58 6.58
G.M. C-236 G-12 1955,7
40500 40500 40500 40500
8.00 17.00 8.00
G.M. C-244 GMD-1 1958
40009 40009 40009 39994 39994 39994
5.25 5.25 20.50 5.25 5.25
G.M.D. C-255 GMD-1 1959
39658 39658 39658 39658 39658 39658
5.25 5.25 20.50 5.25 5.25
GMD C-258 GMD-1 1959
39948 39948 39948 39948 39948 39948

5.25 5.25 20.50 5.25 5.25
GMD G-262 GMD-1 1959
40030 40030 40030 40030 40030 40030
5.25 5.25 20.50 5.25 5.25
G.M. C-274 GMD-1 1960
39869 39869 39869 39757 39757 39757
5.25 5.25 20.50 5.25 5.25
G.M. C-190 SW-1200 (1204-1216) 1956
56250 56250 56250 56250
8.00 14.00 8.00
G.M. C-190 SW-1200 (1217-1221)
61500 61500 61500 61500
8.00 14.00 8.00
G.M. C-198 SW-1200 1956
56090 56090 57585 57585
8.00 14.00 8.00
G.M. C-215 SW-1200 1956,7
56603 56603 57248 57248
8.00 14.00 8.00
G.M. C-228 SW-1200 1957
56588 56588 56938 56938
8.00 14.00 8.00
G.M. C-240 SW-1200 1958
56978 56978 57028 57028
8.00 14.00 8.00
G.M. C-253 SW-1200 1958
55998 55998 56390 56390
8.00 14.00 8.00
GMD C-264 SW-1200 1959
55951 55951 55951 55951
8.00 14.00 8.00
GM C-273 SW-1200 1960
55818 55818 55893 55893
8.00 14.00 8.00
E.M.D 4270 SW-1200 1955
62095 62095 62095 62095
8.00 14.00 8.00
G.M. C-191 SW-1200 1955,6
61500 61500 61500 61500
8.00 14.00 8.00
E.M.D 4376 SW-1200 1957
61725 61725 61775 61775
8.00 14.00 8.00
E.MD. 4437 SW-1200 1960
61970 61970 61980 61980
8.00 14.00 8.00
C.L.C. C-622 H10-64 1951,2
38652 38652 38652 38652 38652 38652
5.42 5.42 19.17 5.42 5.42
C.L.C. C-622 H10-64 1951
38592 38592 38592 38592 38592 38592
5.42 5.42 19.17 5.42 5.42
C.L.C. C-625 H12-64 1952
39167 39167 39167 39167 39167 39167

5.42 5.42 19.17 5.42 5.42
C.L.C. C-623 H12-64 1953
38611 38611 38611 38611 38611 38611
5.42 5.42 19.17 5.42 5.42
C.L.C. C-633 H12-44 1955
55940 55940 55940 55940
8.83 16.67 8.83
C.L.C. C-637 H12-44 1956
55733 55733 55733 55733
8.83 16.67 8.83
M.L.W. SO.4202 RSC-13 1955
39736 39736 39736 39736 39736 39736
5.50 5.50 18.33 5.50 5.50
M.L.W. SO.4203 RSA-13 1955,6
39663 39663 39663 39663 39663 39663
5.50 5.50 18.33 5.50 5.50
M.L.W. SO.4204 RSA-13 1956,7
39958 39958 39958 39958 39958 39958
5.50 5.50 18.33 5.50 5.50
M.L.W. SO.4205 RSA-13 1957
40333 40333 40333 40333 40333 40333
5.50 5.50 18.33 5.50 5.50
M.L.W. 4211 RSC-24 1959
39742 39742 39742 39742 39742 39742
5.50 5.50 11.25 5.50 5.50
G.M. 1900-17 C-245,54 GMD-1 1958
61763 61763 61497 61497
8.00 23.00 8.00
ALCO SO.21072 RS-1 1957
61390 61390 62215 62215
9.33 21.67 9.33
M.L.W. SO 4911 CENTURY 630 1967
64167 64167 64167 64167 64167 64167
5.58 5.58 31.83 5.58 5.58
M.L.W SO 4912 CENTURY 630 1967,8
64667 64667 64667 64667 64667 64667
5.58 5.58 31.83 5.58 5.58
C.L.C. C-632-H H-16-44 1955
61750 61750 61750 61750
9.33 21.67 9.33
M.L.W.-W SO-4918 M 636 1970
64750 64750 64750 64750 64750 64750
5.58 5.58 32.21 5.58 5.58
MLW-W SO-4922 M636 1971
64750 64750 64750 64750 64750 64750
5.58 5.58 32.21 5.58 5.58
M.L.W. SO-4307 RS-3 1953
62250 62250 62250 62250
9.33 20.67 9.33
M.L.W SO.4310 RS-1600 1954
61750 61750 61750 61750
9.33 20.67 9.33
ALCO SO.20941 RS-3 1954
61623 61623 61623 61623

9.33 20.67 9.33
M.L.W. SO.4315 RS-10 1955
61250 61250 61250 61250
9.33 21.67 9.33
M.L.W. S/.4315 RD-10 1955
61250 61250 61250 61250
9.33 21.67 9.33
M.L.W. SO.4323 RS-10 1956,7
61000 61000 61000 61000
9.33 21.67 9.33
M.LW. 4310 RS-18 1959
57676 57676 57676 57676
8.83 22.17 8.83
M.L.W. 4310 RS18 1959
58750 58750 58750 58750
9.33 21.67 9.33
M.L.W. 4812 DL-718 1960,8
59750 59750 59750 59750
9.33 21.67 9.33
M.L.W. 4904 CENTURY 424 1964
65110 65110 65060 65060
9.33 23.17 9.33
M.L.W. 4907:4908 CENTURY 424 1966
65000 65000 65000 65000
9.33 23.17 9.33
M.L.W. 4909 CENTURY 424 1967
65000 65000 65000 65000
9.33 23.17 9.33
ALCO SO.21032 RS-11 1956
56500 56500 56500 56500
8.83 22.17 8.83
M.L.W. SO.4802 RS-11M 1957
62223 62223 61748 61748
9.33 21.67 9.33
M.L.W. SO.4805 RS-11M 1957
62223 62223 61748 61748
9.33 21.67 9.33
M.L.W. SO.4806 RS-11M 1957,8
61953 61953 61833 61833
8.83 22.17 8.83
M.L.W. SO.4808 RS-11M 1958
62200 62200 62393 62393
8.83 22.17 8.83
M.L.W. SO.4317 RS-10 1955
57000 57000 57000 57000
8.83 22.17 8.83
M.L.W. SO.4317 RS-10 1955
57250 57250 57250 57250
8.83 22.17 8.83
M.L.W. SO 4322 RS-10 1956
56955 56955 56955 56955
8.83 22.17 8.83
M.L.W 4810 RS-11M 1959
58500 58500 58500 58500

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          9.33     21.67       9.33
 M.L.W. 4810 RS-11M 1959
     57573     57573     57573     57573
          8.83     22.17       8.83
 M.L.W. 4812 DL-718 1960
     58650     58650     58650     58650
          9.33     21.67       9.33
 ALCO SO 20940 RS-3 1954
     62318     62318     62318     62318
          9.33     20.67       9.33
 G.M. C-301 CR-302 GP-35 1964
     64524     64524     64314     64314
          9.00     23.00       9.00
 G.M. C-315 GP-40 1966
     65178     65178     65178     65178
          9.00     25.00       9.00
 G.M. C-323 GP-40 1967
     65000     65000     65000     65000
          9.00     25.00       9.00
 G.M. C-235 GP-9 1957
     59250     59250     59250     59250
          9.00     22.00       9.00
 G.M. C-235 GP-9 1957
     59250     59250     59250     59250
          9.00     22.00       9.00
 G.M. C-235 GP-9 1957
     58680     58680     57910     57910
          8.00     23.00       8.00
 E.M.D. 5584 GP-9 1958
     61823     61823     61595     61595
          9.00     22.00       9.00
 G.M.D. C-263 GP-9 1959
     58900     58900     58900     58900
          9.00     22.00       9.00
 G.M. C-233 GP-1 1957
     58440     58440     57905     57905
          8.00     23.00       8.00
 G.M. C-239 GP-9 1958
     59000     59000     58210     58210
          8.00     23.00       8.00
 G.M. C-250 GP-9 1958
     58305     58305     57405     57405
          8.00     23.00       8.00
 GMD C-258 GP9R 1959
     57520     57520     57520     57520
          8.00     23.00       8.00
 GMD C-263 GP-9 1959
     57505     57505     57505     57505
          8.00     23.00       8.00
 G.M.D. A-184 GP-9 1955
     62035     62035     62035     62035
          9.00     22.00       9.00
 E.M.D. 5343 GP-9 1954
     62000     62000     62000     62000
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          9.00     22.00        9.00
E.M.D. 5445 GP-9R 1956
    61535    61535    62385    62385
          9.00     22.00        9.00
G.M.D. A-195 GP-9 1955,6
    62880    62880    62880    62880
          9.00     22.00        9.00
G.M. C-214 GP-9 1956
    60000    60000    60000    60000
          9.00     22.00        9.00
G.M.D. C-215 GP-9 1956,7
    62200    62200    61990    61990
          9.00     22.00        9.00
E.M.D. 5513 GP-9R 1957
    61398    61398    61398    61398
          9.00     22.00        9.00
E.M.D. 5512 GP-9R-D 1957
    61318    61318    61318    61318
          9.00     22.00        9.00
E.M.D. 5511 GP-9R 1957
    61095    61095    91095    61095
          9.00     22.00        9.00
G.M. C-234 GP-9 1957
    62030    62030    62380    62380
          9.00     22.00        9.00
G.M. C-238 GP-9 1957,8
    62108    62108    62258    62258
          9.00     22.00        9.00
E.M.D. 5612 GP-18 1960
    61870    61870    61540    61540
          9.00     22.00        9.00
G.M.D. A-168 A-169 GP-7 (4800,19) 1953
    60750    60750    60750    60750
          8.00     23.00        8.00
G.M.D. A-168 A-169 GP-7 (4820,3 ) 1953
    58913    58913    58913    58913
          8.00     23.00        8.00
G.M.D. E-958-A4 GP-7 1948
    59950    59950    59950    59950
          8.00     23.00        8.00
E.M.D. 5365 GP9 1954
    63500    63500    63500    63500
          9.00     22.00        9.00
E.M.D. 5444 GP-9R 1956
    62855    62855    63215    63215
          9.00     22.00        9.00
E.M.D. 5514 GP-9 1957
    63535    63535    62475    62475
          9.00     22.00        9.00
E.M.D. 5510 GP-9 1957
    63295    63295    62535    62535
          9.00     22.00        9.00
E.M.D. 5558 GP-9R 1957
    63500    63500    62260    62260
```

```
         9.00     22.00       9.00
 E.M.D. 5585 GP-9 1958
     62820    62820    63400    63400
         9.00     22.00       9.00
 E.M.D. 5613 GP-18 1960
     63535    63535    63100    63100
         9.00     22.00       9.00
 G.M. C-322 SD-40 1967
     64833    64833    64833    64833    64833    64833
         6.79     6.79     26.42    6.79     6.79
 G.M. C-324 C-325 SD-40 1967,8
     64583    64583    64583    64583    64583    64583
         6.79     6.79     26.42    6.79     6.79
 G.M. C-328 SD-40 1969
     64667    64667    64667    64667    64667    64667
         6.79     6.79     26.42    6.79     6.79
 G.M. C-330 SD-40 1969
     64667    64667    64667    64667    64667    64667
         6.79     6.79     26.42    6.79     6.79
 G.M. C-333 SD-40 1969,70,71
     64667    64667    64667    64667    64667    64667
         6.79     6.79     26.42    6.79     6.79
 G.M. C-338 SD-40 1971
     64833    64833    64833    64833    64833    64833
         6.79     6.79     26.42    6.79     6.79
 G.M. C-345 SD-40 1971
     64833    64833    64833    64833    64833    64833
         6.79     6.79     26.42    6.79     6.79
 E.M.D. 7186 SD-40 1969
     61167    61167    61167    61167    61167    61167
         6.79     6.79     26.42    6.79     6.79
 E.M.D. 7221 SD-40 1970
     61167    61167    61167    61167    61167    61167
         6.79     6.79     26.42    6.79     6.79
 E.M.D. 7289 SD-40 1970
     61167    61167    61167    61167    61167    61167
         6.79     6.79     26.42    6.79     6.79
 G.M. A-183 FP9A 1954,5
     64186    64186    64186    64186
         9.00     25.00       9.00
 G.M. C-183 FP9A 1955
     64186    64186    64186    64186
         9.00     25.00       9.00
 G.M. C-217 FP-9A 1957
     63991    63991    68674    68674
         9.00     25.00       9.00
 G.M. C-230 FP-9A 1957
     64275    64275    64515    64515
         9.00     25.00       9.00
 G.M. C-242 FP-9A 1958
     64955    64955    64776    64776
         9.00     25.00       9.00
 G.M. C-182 F-9B 1954,5
     64218    64218    64218    64218
```

9.00 21.00 9.00
G.M. C-182 F-9B 1965
64218 64218 64218 64218
9.00 21.00 9.00
G.M. C-218 F-9B 1967
64215 64215 64025 64025
9.00 21.00 9.00
G.M. G-231 F-9B 1957
64425 64425 64025 64025
9.00 21.00 9.00
G.M. C-243 F-9-B 1958
63935 63935 64315 64315
9.00 21.00 9.00
C.L.C. C-651-A GPA-16-5 1954,5
59729 59729 59729 59729 59729
7.75 7.75 18.58 9.33
M.L.W. SO.4407 FPA-2 1955
64850 64850 64850 64850
9.33 19.83 9.33
M.L.W. 4407 FPA-2 1955,8
65163 65163 64888 64888
9.33 19.83 9.33
M.L.W. SO.4408 FPA-4 1958
65060 65060 64600 64600
9.33 19.83 9.33
M.L.W. 4409 FPA-4 1959
66739 66739 66739 66739
9.33 19.83 9.33
C.L.C. C-651-B CPB-16-5 1954,5
60957 60957 60957 60957 60957
7.75 7.75 18.58 9.33
M.L.W. SO.4504 FPB-2 1956
64563 64563 64563 64563
9.33 19.83 9.33
M.L.W. 4504 FPB-2 1955,8
65580 65580 64720 64720
9.33 19.83 9.33
M.L.W. SO.4505 FPB-4 1958
64830 64830 64535 64535
9.33 19.83 9.33
M.L.W. 4506 FPB-4 1959
64438 64438 64438 64438
9.33 19.83 9.33
G.M.D. C-13B SW-9 1952
62015 62015 62015 62015
8.00 14.00 8.00
E.M.D. 4045 SW-9 1952
61725 61725 61725 61725
8.00 14.00 8.00
G.M.C. 4178 SW-9 1953
61653 61653 61653 61653
8.00 14.00 8.00
G.M.C. 4300 SW-9 1955
61635 61635 61635 61635

8.00 14.00 8.00
G.M. C-201 SW-1200 1956
62018 62018 61758 61758
8.00 14.00 8.00
G.M. C-216 SW-1200 1957
61768 61768 61688 61688
8.00 14.00 8.00
G.M.D. C-265 SW-1200 1959
61465 61465 61465 61465
8.00 14.00 8.00
G.M.D. C-112 SW-8 1951
58250 58250 58250 58250
8.00 14.00 8.00
G.M.D. C-125 SW-8 1951
58000 58000 58000 58000
8.00 14.00 8.00
G.M.D. C-173 SW-900 1953,4
57250 57250 57250 57250
8.00 14.00 8.00
E.M.D. 4577 SW-900 1956
58115 58115 57705 57705
8.00 14.00 8.00
G.M. C-229 SW-900 1957
57755 57755 58615 58615
8.00 14.00 8.00
G.M. C-241 SW-900 1958
62250 62250 62250 62250
8.00 14.00 8.00
E.M.D 4424 SW-900 1958
58045 58045 57960 57960
8.00 14.00 8.00
G.M. C-241 SW-900 1958
62250 62250 62250 62250
8.00 14.00 8.00
E.M.D. E-445 NW-2 1941,2
61675 61675 61675 61675
8.00 14.00 8.00
E.M.D. E-761 MW-2 1946
62250 62250 62250 62250
8.00 14.00 8.00
E.M.CORP. E-874 NW-2 1947,8
61675 61675 61675 61675
8.00 14.00 8.00
M.L.W. SO-4105 S-4 1951,2
57500 57500 57500 57500
8.00 14.50 8.00
ALCO 20854 S-4 1953
57425 57425 57425 57425
8.00 14.50 8.00
M.L.W. SO-4108 S-4 1954
57575 57575 57575 57575
8.00 14.50 8.00
ALCO SO-20959 S-4 1955
58475 58475 58475 58475

8.00 14.50 8.00
M.L.W. SO-4110 S-4 1955,6
57513 57513 57513 57513
8.00 14.50 8.00
M.L.W. SO-4110 S-4 1955
57760 57760 57760 57760
8.00 14.50 8.00
ALCO SO-20977 S-4 1955
58048 58048 58048 58048
8.00 14.50 8.00
ALCO SO-20978 S-4 1955
58098 58098 58098 58098
8.00 14.50 8.00
ALCO S-1848-3005-3035 S-2 1941,2,6,7
57500 57500 57500 57500
8.00 14.50 8.00
M.L.W. DM-556 S-2 1949
58575 58575 58575 58575
8.00 14.50 8.00
M.L.W. DM-562 S-2 1949,50
58575 58575 58575 58575
8.00 14.50 8.00
ALCO 20737 S-4 1951
58500 58500 58500 58500
8.00 14.50 8.00
M.L.W. SO.4112 S-4 1956,7
57175 57175 57175 57175
8.00 14.50 8.00
ALCO SO.21031 S-4 1956
57595 57595 57930 57930
8.00 14.50 8.00
M.L.W. SO.4113 S-7 1957
57085 57085 58025 58025
8.00 14.50 8.00
M.L.W. SO.4114 DL-410 1958
58380 58380 58110 58110
8.00 14.25 8.00
M.L.W. SO.4002 S-3 1951,2
49375 49375 49375 49375
8.00 14.00 8.00
M.L.W. SO.4007 S-3 1953
49675 49675 49675 49675
8.00 14.00 8.00
M.L.W. SO.4009 S-3 1954
49775 49775 49775 49775
8.00 14.00 8.00
M.L.W. 4122 DL-411 1959
58455 58455 58455 58455
8.00 14.25 8.00
M.L.W. 4123 DL-411A 1959
61763 61763 61763 61763
8.00 14.25 8.00
E.M.D. E 958 A F3A 1948
56450 56450 57900 57900

9.00 21.00 9.00
G.M. E 958 B F3B 1948
53605 53605 57555 57555
9.00 21.00 9.00
G.M. E-833 F3A 1948
56450 56450 57900 57900
9.00 21.00 9.00
G.M. C-114 F7A 1951
57705 57705 57705 57705
9.00 21.00 9.00
G.M. C-114 F7B 1951
58073 58073 58073 58073
9.00 21.00 9.00
G.M. C-137 F7A 1951,2
58235 58235 58235 58235
9.00 21.00 9.00
G.M. C-137 F7B 1951,2
57718 57718 57718 57718
9.00 21.00 9.00
G.M. C-145 F-7A 1952
62000 62000 62000 62000
9.00 21.00 9.00
C.L.C. C-625 1952
62525 62525 62525 62525
9.33 24.67 9.33
C.L.C. C-625 CFB-16-4 1952
61263 61263 61263 61263
9.33 24.67 9.33
C.L.C. 9312,14842 G.E.752 1952,3
62500 62500 62500 62500
9.33 24.67 9.33
M.L.W. D.M.568 F-1500 1950
61750 61750 61750 61750
9.33 17.83 9.33
M.L.W. SO 4400 F-1600 1951
60425 60425 60425 60425
9.33 19.83 9.33
M.L.W. SO 4500 F-1600 1951
60300 60300 60300 60300
9.33 19.83 9.33
M.L.W. SO 4402 F-1600 1952
61925 61925 61925 61925
9.33 19.83 9.33
M.L.W. SO 4501 F-1600 1952
61750 61750 61750 61750
9.33 19.83 9.33
M.L.W SO 4403 F-1600 1953
61925 61925 61925 61925
9.33 19.83 9.33
G.M. C-313 GP-40 TC 1966
66000 66000 66000 66000
9.00 31.50 9.00

TABLE A-2

Maximum bending moment exerted on the track by th CN units listed in Table (A-1)

UNIT	Max.Speed (mile/hr)	Speed Coef. U	Max. Bending Moment (lb.in)	Under Axle Nr.
C.N.R. B-1 TO B-15 1964,5,6	40	1.1465	230160	4
G.E. AEM14857 1956	35	1.1160	75511	4
G.E. AEM19674 44TON 1957	35	1.1160	79097	4
G.E. 93297-TA 70TON 1950	55	1.2485	134015	4
C.N.R. 1964,5,6 300-314	40	1.1465	230160	4
G.M. C-313 GP-40 1966	83	1.4454	324458	4
INT. GE. 1-65777 B-B-94194 4GE-747 1948	50	1.2132	94212	4
G.M. C-208 G-, 1956	60	1.2845	101923	6
G.M. C-180 G-8 1954	65	1.3207	106621	6
G.M. C-131 NF-110 1952	60	1.2845	144508	6
G.M. C-153 NF-110 1953	60	1.2845	144792	6
G.M. C-207 NF-210 1956	60	1.2845	148328	3
G.M. C-246 NF-210 1958	60	1.2845	149180	6
G.M. C-271 NF-210 1960	60	1.2845	148466	6
G.M. C-236 G-12 1955,7	60	1.2845	169131	4
G.M. C-244 GMD-1 1958	65	1.3207	149989	3
G.M.D. C-255 GMD-1 1959	65	1.3207	148675	6
GMD C-258 GMD-1 1959	65	1.3207	149766	6
GMD G-262 GMD-1 1959	65	1.3207	150072	6
G.M. C-274 GMD-1 1960	65	1.3207	149468	3
G.M. C-190 SW-1200 (1204-1216) 1956	65	1.3207	241523	4
G.M. C-190 SW-1200 (1217-1221)	65	1.3207	264062	4
G.M. C-198 SW-1200 1956	65	1.3207	247260	4
G.M. C-215 SW-1200 1956,7	65	1.3207	245811	4
G.M. C-228 SW-1200 1957	65	1.3207	244476	4
G.M. C-240 SW-1200 1958	65	1.3207	244864	4
G.M. C-253 SW-1200 1958	65	1.3207	242129	4
GMD C-264 SW-1200 1959	65	1.3207	240235	4
GM C-273 SW-1200 1960	65	1.3207	239989	4
E.M.D 4270 SW-1200 1955	65	1.3207	266618	4
G.M. C-191 SW-1200 1955,6	65	1.3207	264062	4
E.M.D 4376 SW-1200 1957	65	1.3207	265245	4
E.MD. 4437 SW-1200 1960	65	1.3207	266126	4
C.L.C. C-622 H10-64 1951,2	60	1.2845	142201	6
C.L.C. C-622 H10-64 1951	60	1.2845	141982	6
C.L.C. C-625 H12-64 1952	60	1.2845	144091	6
C.L.C. C-623 H12-64 1953	60	1.2845	142047	6
C.L.C. C-633 H12-44 1955	60	1.2845	242552	4
C.L.C. C-637 H12-44 1956	60	1.2845	241663	4
M.L.W. SO.4202 RSC-13 1955	60	1.2845	146796	6

Cont. TABLE A-2

UNIT	Max.Speed (mile/hr)	Speed Coef. U	Max. Bending Moment (lb.in)	Under Axle Nr.
M.L.W. SO.4203 RSC-13 1955,6	60	1.2845	146527	6
M.L.W. SO.4204 RSC-13 1956,7	60	1.2845	147619	6
M.L.W. SO.4205 RSC-13 1957	60	1.2845	149004	6
M.L.W. 4211 RSC-24 1959	65	1.3207	150957	6
G.M. 1900-17 C-245,54 GMD-1 1958	89	1.4824	297668	2
ALCO SO.21072 RS-1 1957	60	1.2845	274040	4
M.L.W. SO 4911 CENTURY 630 1967	75	1.3918	257931	6
M.L.W SO 4912 CENTURY 630 1967,8	75	1.3918	259943	6
C.L.C. C-632-H H-16-44 1955	70	1.3566	287260	4
M.L.W.-W SO-4918 M 636 1970	75	1.3918	260280	6
MLW-W SO-4922 M636 1971	75	1.3918	260280	6
M.L.W. SO-4307 RS-3 1953	75	1.3918	297098	4
M.L.W SO.4310 RS-1600 1954	75	1.3918	294710	4
ALCO SO.20941 RS-3 1954	75	1.3918	294099	4
M.L.W. SO.4315 RS-10 1955	74	1.3849	290860	4
M.L.W. S/.4315 RD-10 1955	75	1.3918	292321	4
M.L.W. SO.4323 RS-10 1956,7	75	1.3918	291132	4
M.LW. 4310 RS-18 1959	80	1.4258	277589	4
M.L.W. 4310 RS18 1959	80	1.4258	287234	4
M.L.W. 4812 DL-718 1960,8	92	1.4996	307252	4
M.L.W. 4904 CENTURY 424 1964	75	1.3918	310745	2
M.L.W. 4907:4908 CENTURY 424 1966	75	1.3918	310219	4
M.L.W. 4909 CENTURY 424 1967	75	1.3918	310219	4
ALCO SO.21032 RS-11 1956	65	1.3207	251882	4
M.L.W. SO.4802 RS-11M 1957	75	1.3918	296961	2
M.L.W. SO.4805 RS-11M 1957	75	1.3918	296961	2
M.L.W. SO.4806 RS-11M 1957,8	75	1.3918	291069	2
M.L.W. SO.4808 RS-11M 1958	75	1.3918	293130	4
M.L.W. SO.4317 RS-10 1955	75	1.3918	267794	4
M.L.W. SO.4317 RS-10 1955	75	1.3918	268975	4
M.L.W. SO 4322 RS-10 1956	75	1.3918	267587	4
M.L.W 4810 RS-11M 1959	75	1.3918	279200	4
M.L.W. 4810 RS-11M 1959	75	1.3918	270488	4
M.L.W. 4812 DL-718 1960	75	1.3918	279916	4
ALCO SO 20940 RS-3 1954	75	1.3918	297414	4
G.M. C-301 CR-302 GP-35 1964	65	1.3207	289832	2
G.M. C-315 GP-40 1966	65	1.3207	292774	4
G.M. C-323 GP-40 1967	65	1.3207	291971	4
G.M. C-235 GP-9 1957	89	1.4824	298730	4
G.M. C-235 GP-9 1957	89	1.4824	298730	4
G.M. C-235 GP-9 1957	89	1.4824	282803	2
E.M.D. 5584 GP-9 1958	83	1.4454	303920	2
G.M.D. C-263 GP-9 1959	65	1.3207	264570	4
G.M. C-233 GP-1 1957	65	1.3207	250923	2
G.M. C-239 GP-9 1958	65	1.3207	253327	2
G.M. C-250 GP-9 1958	65	1.3207	250345	2
GMD C-258 GP9R 1959	65	1.3207	246976	4

Cont. TABLE A-2

UNIT	Max.Speed (mile/hr)	Speed Coef. U	Max. Bending Moment (lb.in)	Under Axle Nr.
GMD C-263 GP-9 1959	65	1.3207	246909	4
G.M.D. C-184 GP-9 1955	65	1.3207	278652	4
E.M.D. 5343 GP-9 1954	65	1.3207	278493	4
E.M.D. 5445 GP-9R 1956	65	1.3207	280226	4
G.M.D. C-195 GP-9 1955,6	65	1.3207	282445	4
G.M. C-214 GP-9 1956	65	1.3207	269511	4
G.M.D. C-215 GP-9 1956,7	65	1.3207	279394	2
E.M.D. 5513 GP-9R 1957	83	1.4454	301828	4
E.M.D. 5512 GP-9R-D 1957	65	1.3207	275433	4
E.M.D. 5511 GP-9R 1957	65	1.3207	409184	3
G.M. C-234 GP-9 1957	65	1.3207	280207	4
G.M. C-238 GP-9 1957,8	65	1.3207	279652	4
E.M.D. 5612 GP-18 1960	65	1.3207	277909	2
G.M.D. C-168 C-169 GP-7 (4800,19) 1953	65	1.3207	260844	4
G.M.D. C-168 C-169 GP-7 (4820,3) 1953	65	1.3207	252958	4
G.M.D. E-958-A4 GP-7 1948	65	1.3207	257407	4
E.M.D. 5365 GP9 1954	65	1.3207	285237	4
E.M.D. 5444 GP-9R 1956	65	1.3207	283950	4
E.M.D. 5514 GP-9 1957	65	1.3207	285386	2
E.M.D. 5510 GP-9 1957	65	1.3207	284316	2
E.M.D. 5558 GP-9R 1957	65	1.3207	285237	2
E.M.D. 5585 GP-9 1958	83	1.4454	311669	4
E.M.D. 5613 GP-18 1960	83	1.4454	312330	2
G.M. C-322 SD-40 1967	65	1.3207	262835	6
G.M. C-324 C-325 SO-40 1967,8	65	1.3207	261825	6
G.M. C-328 SD-40 1969	65	1.3207	262164	6
G.M. C-330 SD-40 1969	65	1.3207	262164	6
G.M. C-333 SD-40 1969,70,71	65	1.3207	262164	6
G.M. C-338 SD-40 1971	65	1.3207	262835	6
G.M. C-345 SD-40 1971	65	1.3207	262835	6
E.M.D. 7186 SD-40 1969	65	1.3207	247971	6
E.M.D. 7221 SD-40 1970	65	1.3207	247971	6
E.M.D. 7289 SD-40 1970	65	1.3207	247971	6
G.M. C-183 FP9A 1954,5	89	1.4824	323618	4
G.M. C-183 FP9A 1955	89	1.4824	323618	4
G.M. C-217 FP-9A 1957	89	1.4824	346238	4
G.M. C-230 FP-9A 1957	89	1.4824	325274	4
G.M. C-242 FP-9A 1958	89	1.4824	327497	2
G.M. C-182 F-9B 1954,5	89	1.4824	323773	4
G.M. C-182 F-9B 1965	89	1.4824	323773	4
G.M. C-218 F-9B 1967	89	1.4824	323762	2
G.M. G-231 F-9B 1957	89	1.4824	324818	2
G.M. C-243 F-9-B 1958	89	1.4824	324262	4
C.L.C. C-651-A GPA-16-5 1954,5	92	1.4996	307139	5
M.L.W. SO.4407 FPA-2 1955	92	1.4996	333475	4
M.L.W. 4407 FPA-2 1955,8	92	1.4996	335084	2
M.L.W. SO.4408 FPA-4 1958	92	1.4996	334552	2

Cont. TABLE A-2

UNIT	Max.Speed (mile/hr)	Speed Coef. U	Max. Bending Moment (lb.in)	Under Axle Nr.
M.L.W. 4409 FPA-4 1959	92	1.4996	343190	4
C.L.C. C-651-B CPB-16-5 1954,5	92	1.4996	313454	5
M.L.W. SO.4504 FPB-2 1956	92	1.4996	332001	4
M.L.W. 4504 FPB-2 1955,8	92	1.4996	337227	2
M.L.W. SO.4505 FPB-4 1958	92	1.4996	333373	2
M.L.W. 4506 FPB-4 1959	92	1.4996	331355	4
G.M.D. C-13B SW-9 1952	40	1.1465	231146	4
E.M.D. 4045 SW-9 1952	40	1.1465	230061	4
G.M.C. 4178 SW-9 1953	40	1.1465	229798	4
G.M.C. 4300 SW-9 1955	40	1.1465	229732	4
G.M. C-201 SW-1200 1956	40	1.1465	231154	2
G.M. C-216 SW-1200 1957	40	1.1465	230226	2
G.M.D. C-265 SW-1200 1959	40	1.1465	229100	4
G.M.D. C-112 SW-8 1951	40	1.1465	217111	4
G.M.D. C-125 SW-8 1951	40	1.1465	216182	4
G.M.D. C-173 SW-900 1953,4	40	1.1465	213388	4
E.M.D. 4577 SW-900 1956	40	1.1465	216609	2
G.M. C-229 SW-900 1957	40	1.1465	218475	4
G.M. C-241 SW-900 1958	40	1.1465	232025	4
E.M.D 4424 SW-900 1958	40	1.1465	216346	2
G.M. C-241 SW-900 1958	40	1.1465	232025	4
E.M.D. E-445 NW-2 1941,2	40	1.1465	229880	4
E.M.D. E-761 MW-2 1946	40	1.1465	232025	4
E.M.CORP. E-874 NW-2 1947,8	40	1.1465	229880	4
M.L.W. SO-4105 S-4 1951,2	40	1.1465	214317	4
ALCO 20854 S-4 1953	40	1.1465	214037	4
M.L.W. SO-4108 S-4 1954	40	1.1465	214596	4
ALCO SO-20959 S-4 1955	40	1.1465	217949	4
M.L.W. SO-4110 S-4 1955,6	40	1.1465	214366	4
M.L.W. SO-4110 S-4 1955	40	1.1465	215286	4
ALCO SO-20977 S-4 1955	40	1.1465	216363	4
ALCO SO-20978 S-4 1955	40	1.1465	216544	4
ALCO S-1848-3005-3035 S-2 1941,2,6,7	40	1.1465	214317	4
M.L.W. DM-556 S-2 1949	40	1.1465	218327	4
M.L.W. DM-562 S-2 1949,50	40	1.1465	218327	4
ALCO 20737 S-4 1951	60	1.2845	244305	4
M.L.W. SO.4112 S-4 1956,7	40	1.1465	213109	4
ALCO SO.21031 S-4 1956	40	1.1465	215919	4
M.L.W. SO.4113 S-7 1957	40	1.1465	216272	4
M.L.W. SO.4114 DL-410 1958	40	1.1465	217595	2
M.L.W. SO.4002 S-3 1951,2	40	1.1465	184036	4
M.L.W. SO.4007 S-3 1953	40	1.1465	185153	4
M.L.W. SO.4009 S-3 1954	40	1.1465	185523	4
M.L.W. 4122 DL-411 1959	40	1.1465	217875	4
M.L.W. 4123 DL-411A 1959	40	1.1465	230209	4
E.M.D. E 958 A F3A 1948	65	1.3207	260074	4
G.M. E 958 B F3B 1948	65	1.3207	258529	4

Cont. TABLE A-2

UNIT	Max.Speed (mile/hr)	Speed Coef. U	Max. Bending Moment (lb.in)	Under Axle Nr.
G.M. E-833 F3A 1948	65	1.3207	260074	4
G.M. C-114 F7A 1951	65	1.3207	259202	4
G.M. C-114 F7B 1951	65	1.3207	260856	4
G.M. C-137 F7A 1951,2	65	1.3207	261579	4
G.M. C-137 F7B 1951,2	65	1.3207	259262	4
G.M. C-145 F-7A 1952	65	1.3207	278493	4
C.L.C. C-625 1952	70	1.3566	290860	4
C.L.C. C-625 CFB-16-4 1952	70	1.3566	284993	4
C.L.C. 9312,14842 G.E.752 1952,3	70	1.3566	290747	4
M.L.W. D.M.568 F-1500 1950	75	1.3918	294710	4
M.L.W. SO 4400 F-1600 1951	75	1.3918	288386	4
M.L.W. SO 4500 F-1600 1951	75	1.3918	287786	4
M.L.W. SO 4402 F-1600 1952	75	1.3918	295541	4
M.L.W. SO 4501 F-1600 1952	75	1.3918	294710	4
M.L.W SO 4403 F-1600 1953	75	1.3918	295541	4
G.M. C-313 GP-40 TC 1966	83	1.4454	324458	4

TABLE A-3

Conversion factors of units of the physical terms used in the study:

Unit of Physical term	Metric	Anglo-American	SI
1 metric ton (tonne)	1000 kg	2204.622 lb	
1 short ton	907.185 kg	2000 lb	
1 kg		2.2046 lb	9.8069 newton
1 lb	0.4536 kg		4.4484 newton
1 mile	1.6093 km		
1 km		0.6214 mile	
1 ft	30.4800 cm		
1 in	2.5400 cm		
1 cm		0.3937 in 0.0328 ft	
1 in^3 (modulus of section)	16.3872 cm^3		
1 cm^3 (")		0.0610 in^3	
1 in^4 (moment of inertia)	41.7522 cm^4		
1 cm^4 (")		0.0240 in^4	
1 mile/hr	1.6093 km/hr		
1 km/hr		0.6214 mile/hr	
1 lb.in (bending moment)	1.1521 kg.cm		0.1130 newton.meter
1 kg.cm		0.8680 lb.in	0.0981 newton.meter
1 lb/in^2 (stress)	0.0703 kg/cm^2		6895 newton/$meter^2$
1 kg/cm^2 (")		14.2233 lb/in^2	98069 newton/$meter^2$
1 bushel	35.2381 litre 35238.0 cm^3	8 dry gallons 1.24456 ft^3 2150.42 in^3	0.035238 $meter^3$